

NPS67-80-003

# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



AERO-OPTICS AT SHORTER WAVELENGTHS

by

Allen E. Fuhs  
Distinguished Professor

March 1980

Approved for public release; distribution unlimited

Prepared for:  
Air Force Weapons Laboratory/ARLB  
Kirtland Air Force Base, NM, 87117

FEDDOCS  
D 208.14/2:NPS-67-80-003

DUDLEY KNOX LIBRARY  
NAVAL POSTGRADUATE SCHOOL  
MONTEREY, CA 93943-5101

NAVAL POSTGRADUATE SCHOOL

Monterey, California

Rear Admiral J. J. Ekelund  
Superintendent

Jack R. Borsting  
Provost

The work reported herein was supported by the Air Force Weapons Laboratory,  
Code ARLB, Kirtland AFB, NM, 87117.

Reproduction of all or part of this report is authorized.

This report was prepared by:

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS67-80-003	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Aero-optics at Shorter Wavelengths		5. TYPE OF REPORT & PERIOD COVERED Progress Report Dec. 1979-March 1980
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Allen E. Fuhs		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Aeronautics Naval Postgraduate School Monterey, CA, 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS MIPR # 80-MP-084
11. CONTROLLING OFFICE NAME AND ADDRESS AFWL/ARLB Kirtland AFB, NM, 87117		12. REPORT DATE 31 March 1980
		13. NUMBER OF PAGES 29
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES Prepared for the Flow Control Workshop held at AFWL, 10-12 March 1980.		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Lasers, Laser Turrets, Laser Propagation, Turbulence		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Using the optical system equation, the impact of a change in wavelength is determined. The rms phase variation at output aperture is critical and must be reduced by an amount equivalent to reduction in wavelength. If turret size is scaled with wavelength, the distortion due to the inviscid flow remains constant. Decreased wavelength offers the flexibility of greatly reduced turret weight, volume, and aerodynamic drag. Multiple turrets become feasible on an aircraft. The distortion due to the turbulent boundary layer becomes magnified by		

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

reduction of wavelength; however, if the turbulent boundary layer originates on the turret and if the turret is scaled, the boundary layer thickness decreases. Distortion becomes less significant in this case.

DD Form 1473  
1 Jan 73  
S/N 0102-014-6601

UNCLASSIFIED

ii SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

ABSTRACT

Using the optical system equation, the impact of a change in wavelength is determined. The rms phase variation at output aperture is critical and must be reduced by an amount equivalent to reduction in wavelength. If turret size is scaled with wavelength, the distortion due to the inviscid flow remains constant. Decreased wavelength offers the flexibility of greatly reduced turret weight, volume, and aerodynamic drag. Multiple turrets become feasible on an aircraft. The distortion due to the turbulent boundary layer becomes magnified by reduction of wavelength; however, if the turbulent boundary layer originates on the turret and if the turret is scaled, the boundary layer thickness decreases. Distortion becomes less significant in this case.

## TABLE OF CONTENTS

I. INTRODUCTION. . . . .	1
II. FAR-FIELD INTENSITY . . . . .	1
III. SCALING TURRET WEIGHT, VOLUME, AND DRAG . . . . .	5
IV. SCALING OPTICAL DISTORTION DUE TO INVISCID FLOW . . . . .	6
V. VISCOUS FLOW PHENOMENA LEADING TO OPTICAL DISTORTION. . . . .	9
A. Types of Viscous Flow Phenomena . . . . .	9
B. Turbulent Boundary Layers . . . . .	10
C. Scaling Propagation Through a Turbulent Boundary Layer of Fixed Thickness . . . . .	14
D. Scaling Propagation Through a Scaled Turbulent Boundary Layer .	15
VI. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS . . . . .	17
APPENDIX A. . . . .	19
REFERENCES. . . . .	21

## ACKNOWLEDGMENT

The author wishes to thank his wife, Emily, for typing this report.

LIST OF FIGURE AND TABLES

FIGURE

1. Diffraction angle for which the intensity is one half the maximum value. . . . . 15

TABLES

- I. Parameters for Turret Weight . . . . . 5
- II. Change of Half Angle with Change in Wavelength . . . . . 16





# AERO-OPTICS AT SHORTER WAVELENGTHS

by

Distinguished Professor Allen E. Fuhs  
Department of Aeronautics  
Department of Physics and Chemistry  
Naval Postgraduate School  
Monterey, California, 93940

## I. INTRODUCTION

An extensive technological base for aero-optics has been developed by the laser community. The research and development work was oriented to the CO<sub>2</sub> laser with wavelength of 10.6 microns. In recent times, lasers operating at much shorter wavelengths, e.g., iodine laser at 1.315 microns [1] or the excimer lasers in the blue end of the visible spectrum [2], have shown potential for superior performance. The information concerning laser turrets on aircraft needs to be reexamined for shorter wavelengths.

Section II discusses the far-field intensity using an equation which was developed by Holmes and Avizonis [3]. The equation highlights the impact of a change in wavelength. Section III discusses scaling of turret weight, volume, and drag. Section IV discusses the scaling of phase distortion due to inviscid flow. Section V considers phase distortion to aircraft generated turbulence. Finally Section VI summarizes the paper and states conclusions relative to unknowns or areas of uncertainty.

## II. FAR-FIELD INTENSITY

An equation for far-field intensity has been developed by Holmes and Avizonis [3]. The equation, which is useful for systems analysis, is

$$I_{ff} = \left( \frac{\pi D^2 P}{4 \lambda^2 Z^2} \right) \frac{R^m T_w T_d T_a K \exp[-(2\pi \delta_{rms} / \lambda)^2]}{\left( \frac{D}{FD_0} \right)^2 \left[ 1 + \left( \frac{5\sigma D}{3\lambda} \right)^2 \right]} \quad (1)$$

One additional term,  $D/FD_0$ , has been added by Fuhs [4] to account for turbulence. The meaning of each term will be discussed now.

The first term is the far-field intensity on axis for an Airy diffraction pattern. The mirror diameter is  $D$ ; the laser power at the output aperture, i.e., laser telescope primary mirror, is  $P$ ; the wavelength of the laser is  $\lambda$ ; and the range to the focal plane is  $Z$ . The shorter wavelength increases  $I_{ff}$ ; however, one may want to decrease mirror size, thereby decreasing turret weight and volume. Further, a decrease in mirror diameter,  $D$ , will decrease aerodynamic drag. Consequently, the ratio of  $D$  to  $\lambda$  will be assumed to be constant, i.e.,

$$S_1 = \lambda/D = \text{constant} \quad (2)$$

$S_1$  is a scaling factor. The turret will have a characteristic dimension  $L$ . An additional assumption is

$$S_2 = L/D = \text{constant} \quad (3)$$

where  $S_2$  is another scaling parameter.

The term for jitter is

$$1 + \left( \frac{5\sigma D}{3\lambda} \right)^2 = 1 + \left( \frac{5\sigma}{3S_1} \right)^2 \quad (4)$$

When  $S_1$  is constant, the jitter term depends on the line-of-sight rms angular beam jitter,  $\sigma$ . Many different phenomena influence the magnitude of  $\sigma$  including the following:

- aerodynamic inputs to airframe
- aerodynamic input directly to the laser turret
- aerodynamic input due to control surface deflections
- unsteady aerodynamics in vicinity of laser turret

unsteady aerodynamics due to open port

vibration due to aircraft components

electrical noise within servo system

laser turret mechanical jitter

interaction of turret aerodynamics with remainder of aircraft

Each item will not be discussed. When the laser turret shrinks in size to maintain  $S_1$  constant, the value of  $\sigma$  must be maintained constant to avoid increased detrimental effects.

From the causes of jitter listed above, the problem due to open port may be eliminated or decreased. With another wavelength, i.e., 1.3 microns instead of 10.6 microns, and a much smaller size, a window may become feasible.

The quantity  $R^m$  is the loss due to the reflectivity of  $m$ -mirrors in the optical train.  $T_w$  is the transmission of the window mentioned in the preceding paragraph. The quantity  $T_d$  is diffractive spillage around mirrors in the optical train.  $T_d$  is a function of Fresnel number

$$N_F = \frac{r_m^2}{\lambda \ell} = \frac{D_m^2}{4\lambda \ell} = \frac{D_m}{4S_1 \ell} \quad (5)$$

where  $r_m$  is the radius of mirrors in the optical train, and  $\ell$  is the spacing between two mirrors. If one scales the primary mirror by ratio  $S_1$ , then the diameter of a mirror in the optical train,  $D_m$ , undoubtedly will be scaled to a smaller size. The consequence is increased spillage, i.e., a smaller value for  $T_d$ . One assumes that  $\ell$  remains fixed.

The transmission of the atmosphere is  $T_a$  and is given by

$$T_a = e^{-\gamma Z} \quad (6)$$

where  $\gamma$  is an extinction coefficient. The extinction coefficient is the sum of four terms

$$\gamma = \alpha_a + \alpha_s + k_a + k_s \quad (7)$$

where  $\alpha_a$  is molecular absorption,  $\alpha_s$  is molecular scattering,  $k_a$  is aerosol absorption, and  $k_s$  is aerosol scattering. Each of the terms in equation (7) is dependent on wavelength.

The factor  $K$  is a consequence of nonuniform intensity,  $I$ , watts/cm<sup>2</sup>, at the primary mirror of the laser turret. If  $S_1$  is fixed, the intensity in the far-field does not change; see Appendix A.

Aero-optics plays an important role in the factor  $\delta_{\text{rms}}/\lambda$ . Of course,  $\delta_{\text{rms}}$  is the result of all optical aberrations starting within the cavity of the laser device. The aerodynamic window, the mirrors in the optical train, and the primary telescope mirror affect  $\delta_{\text{rms}}$ . The beam propagates through a turbulent boundary layer and the variable density field generated by the inviscid flow over the turret. Both external flows contribute to  $\delta_{\text{rms}}$ .

Obviously, when  $\lambda$  is decreased,  $\delta_{\text{rms}}$  must be decreased by a like amount. Otherwise,  $I_{\text{ff}}$  is reduced. When  $\lambda$  is reduced from 10.6 microns to 1.3 microns, i.e., by a factor of 8,  $\delta_{\text{rms}}$  must be reduced also by a factor of 8. The most difficult aspect of scaling to shorter wavelengths will be to reduce  $\delta_{\text{rms}}$ .

A connection exists between  $\psi(\xi, \eta)$  in equation (A-1) and  $\delta_{\text{rms}}/\lambda$  in equation (1). For more details concerning the relationship between  $\psi$  and  $\delta_{\text{rms}}$ , see Born and Wolfe [5].

A term,  $D/FD_0$ , appears in equation (1) to account for turbulence. The term is adequate for systems analysis but is not appropriate for a detailed examination of aero-optics. The question of turbulent flow is deferred to Section IV.

### III. SCALING TURRET WEIGHT, VOLUME, AND DRAG

Turret weight,  $W$ , can be scaled using the equation

$$\frac{W_2}{W_1} = \left[ \frac{D_2}{D_1} \right]^n \quad (8)$$

where  $n$  is a number depending on the turret optics and  $D$  is output aperture.

Table I gives the value of  $n$ ,  $W_1$ , and  $D_1$  for three turrets.

Table I. Parameters for Turret Weight

Type of Turret	$n$	$W_1$ , pounds	$D_1$ , cm
on-gimbal Cassegrain	2.50	1637	60
heliostat	1.93	1788	60
coelostat	2.40	2161	60

Changing from a  $CO_2$  laser to an iodine laser gives a wavelength ratio of  $1/8$ . Assuming  $S_1$  is fixed, the value of  $D$  can be reduced by a factor of  $1/8$ ; mirror diameter can be reduced from 60 cm to 7.5 cm. Use the numbers in equation (8) for an on-gimbal Cassegrain telescope. The result is

$$W_2 = 1637 \left[ \frac{1}{8} \right]^{2.5} = 9.04$$

The range of validity of the scaling law of equation (8) may have been exceeded; even so, the qualitative conclusion is that turret weight is very significantly reduced.

Turret volume,  $V$ , follows a scaling law of

$$\frac{V_2}{V_1} = \left( \frac{D_2}{D_1} \right)^3 \quad (9)$$

Once again, significant reductions occur.

Aerodynamic forces are given by an equation of the form

$$F = \frac{1}{2}\rho u^2 AC_F \quad (10)$$

where  $C_F$  is a force coefficient. The drag is

$$D = \frac{1}{2}\rho u^2 AC_D \quad (11)$$

where  $C_D$  is the drag coefficient. In equation (11),  $A$  is the cross sectional area of the turret exposed to the flow. Hence the drag varies as mirror diameter squared.

When laser turrets weigh in tons, only one turret can be used on an aircraft. However, when the turret is reduced in weight and volume by a factor of 100 or so, many turrets can be used. Instead of attempting to cover  $2\pi$ -steradians with one turret, each of the small turrets can cover a much smaller solid angle. Furthermore, the turrets can be located at many different places on the aircraft. In the enthusiasm to add turrets, consideration must be given to fire control, pointing, and tracking.

#### IV. SCALING OPTICAL DISTORTION DUE TO INVISCID FLOW

Three lengths are of relevance to the scaling of optical distortion due to inviscid flow; these are beam diameter,  $D$ ; wavelength,  $\lambda$ ; and characteristic turret dimension,  $L$ . One length is selected as a reference length to form ratios. Equations (2) and (3) indicate  $D$  is used as the reference.

Compressible flow over a laser turret generates a spatially varying density field. A relation exists between refractive index and density

$$n = 1 + (\kappa\rho/\rho_0) \quad (12)$$

The fractional changes are

$$\frac{\Delta n}{n} = \kappa \frac{\Delta\rho}{\rho} \quad (13)$$

By solving the equations of motion for flow over a turret, the density field can be obtained. Knowing the density field, the refractive index can be calculated.

The optical path length, OPL, is defined as

$$\text{OPL} = \int n(x,y,z) ds \quad (14)$$

where  $s$  is distance along a ray. The optical path length is a geometrical length weighted by the local refractive index. The change in OPL, known as optical path difference, OPD, is

$$\delta = \text{OPD} = \int n_i(x,y,z) ds_i - \int n_r(x,y,z) ds_r \quad (15)$$

where subscripts  $i$  and  $r$  indicate the  $i$ -th ray and a reference ray. The quantity  $\delta$  is equal to the optical path difference.

Define nondimensional variables

$$x = x'D$$

$$y = y'D$$

$$z = z'D$$

$$s = s'D$$

Substitution of the nondimensional variables into equation (14) gives

$$\text{OPL} = D \int n(x',y',z') ds' \quad (16)$$

Likewise the optical path difference becomes

$$\delta = D \int n_i ds'_i - D \int n_r ds'_r \quad (17)$$

Hence OPL and  $\delta$  scale as mirror diameter  $D$ . One must show that the integrals in equations (16) and (17) are constant when turret size varies so that  $S_1$  and  $S_2$  are constant.

For blunt turrets, the equation for the velocity potential in compressible flow is

$$\frac{1}{a^2} \frac{\partial \phi}{\partial x_i} \frac{\partial \phi}{\partial x_k} \frac{\partial^2 \phi}{\partial x_i \partial x_k} = \frac{\partial^2 \phi}{\partial x_j \partial x_j} \quad (18)$$

Introduce nondimensional variables

$$\phi = \phi' u_\infty D$$

$$u = u' u_\infty$$

$$a = a' a_\infty$$

into equation (14). The result is

$$\frac{M_\infty^2}{a'^2} \frac{\partial \phi'}{\partial x'_i} \frac{\partial \phi'}{\partial x'_k} \frac{\partial^2 \phi'}{\partial x'_i \partial x'_k} = \frac{\partial^2 \phi'}{\partial x'_j \partial x'_j} \quad (19)$$

A length scale parameter does not appear in equation (19). Although not demonstrated here, the boundary conditions can be nondimensionalized without a residual length scale. Consequently, the solution  $\phi'$  is independent of turret scale.

The local Mach number is related to freestream Mach number by

$$M = \frac{u}{a} = \frac{u' u_\infty}{a' a_\infty} = M_\infty M' \quad (20)$$

The local Mach number determines the local refractive index. Since  $M'$  is independent of turret scale, the identical value of refractive index exists at corresponding points in two flows with different scale. Consequently, the integrals in equations (16) and (17) are identical for the two flows at different scale.

The phase distortion, also known as optical path difference, relative to wavelength is

$$\frac{\delta}{\lambda} = \frac{D}{\lambda} \int n_i ds'_i - \frac{D}{\lambda} \int n_r ds'_r \quad (21)$$



Since the integrals are independent of scale, and since  $S_1$  is fixed, the phase distortion does not vary. Likewise  $\delta_{\text{rms}}/\lambda$  does not vary with turret scale provided  $M_\infty$ ,  $S_1$ , and  $S_2$  are fixed.

For small perturbation turrets, a similar argument can be made.

## V. VISCOUS FLOW PHENOMENA LEADING TO OPTICAL DISTORTION

### A. Types of Viscous Flow Phenomena

Viscosity dominates and determines, at least in part, the gross characteristics of several different types of flow. These include boundary layers, shear layers, open port cavities, wakes, and shock waves. Although not a flow of the same nature as those listed above, jet engine exhausts should be included; exhausts can have a detrimental effect on beam propagation. At the speeds and geometric scales involved with aircraft, the viscous flows lead to turbulence. Extensive literature [6-21] exists on the propagation of electromagnetic waves through a turbulent medium.

Boundary layers surround the aircraft surfaces. The boundary layer on leading edges and forward surfaces is laminar with transition occurring at a critical Reynolds number. Flow separation leads to shear layers. Shear layers separate two flow regions with different flow velocities.

Due to the extremely high irradiance in the laser beam, the turret may have an open port. Windows are not available which can survive the high irradiance. The interior of the turret is a cavity in which unsteady, turbulent flow exists. The flow in a cavity may have regularity in the sense that discrete vortices can be identified.

Wakes are turbulent and adversely affect propagation. When selecting a location for a laser turret, any location within or near a wake should be avoided.

Shock waves create large changes in density and refractive index. Shock waves which originate or reflect from boundary layers create intense turbulence. Depending on conditions, a triple point may occur in reflection of an oblique shock. Downstream of the triple point, a shear layer exists. Also downstream of a reflected shock, a separated flow region may occur.

Two cases can be identified. In one case, the laser beam propagates through a region disturbed by a shock wave. In the other case, the shock wave originates or reflects near the aperture of the turret. The latter case causes the greater difficulty with optical distortion.

#### B. Turbulent Boundary Layers

The boundary layer may originate on aircraft surfaces far upstream of the aperture. If this is the case, then the boundary layer will be thick. When the laser turret protrudes into the flow, the boundary layer may begin on the turret itself. In that case, the boundary layer is thin. One can state unequivocally that the thinner the boundary layer, the lesser the adverse influence on beam quality.

In addition to the distance which the boundary layer has traversed, other factors influence the boundary layer. Mach number characterizes the compressibility effects. At higher Mach numbers, the boundary layer tends to grow in thickness. Wall curvature, which usually is accompanied by pressure gradients, has an influence on boundary layer thickness and velocity profile. When the wall is concave to the external flow, Gortler vortices may occur. Cooling or heating can alter boundary layer properties. In fact, cooling may be one approach to flow control. Surface roughness may amplify turbulent intensities. Surface roughness may be due to rivet heads, structural joints, insects which have smeared the surface, etc.

Pressure gradients have been mentioned earlier. A gradient in which the pressure increases in the flow direction is termed an adverse gradient. Conversely, a gradient with decreasing pressure in the flow direction is a favorable gradient. Adverse pressure gradients lead to flow separation as well as thicker boundary layers. Favorable pressure gradients delay transition from laminar to turbulent flow. Favorable pressure gradients tend to cause thinner boundary layers. A boundary layer in a favorable pressure gradient rarely separates.

Shock wave, boundary layer interaction has been mentioned. If the interaction occurs at the aperture or immediately upstream of the aperture, severe beam degradation is likely.

For turbulent incompressible boundary layers, the thickness,  $\delta$ , is given by [22]

$$\frac{\delta}{x} = 0.37 \text{Re}_x^{-0.2} \quad (22)$$

According to equation (22),  $\delta$  increases as  $x$  to + 0.8 power. For a laminar, compressible boundary layer, the thickness is given by [23]

$$\delta = C_1 \text{Re}_x^{0.5} \left[ 1 + C_2 \frac{\gamma - 1}{2} M^2 \right] \quad (23)$$

where  $C_1$  and  $C_2$  are constants. Turbulent compressible boundary layers show similar trends with increasing Mach number.

A boundary layer in compressible flow has a temperature gradient across the boundary layer. The gradient in temperature is due to viscous dissipation. A quantity known as the recovery factor  $r$  is defined for an adiabatic wall;  $r$  is

$$r = \frac{T_w - T_\infty}{T_0 - T_\infty} = \text{Pr} \quad (24)$$

The temperature of the wall is  $T_w$ , the temperature of the air at the outer edge of the boundary layer is  $T_\infty$ , and the stagnation temperature is  $T_0$ . Equation (24) shows  $r$  equal to the Prandtl number,  $Pr$ . An effective Prandtl number can be defined for a turbulent boundary layer and will have a value near unity. Hence the wall temperature will be equal to the stagnation temperature when  $Pr = 1.0$ .

The rms density fluctuations in the turbulent boundary layer are related to the difference in density at the wall,  $\rho_w$ , and at the edge of the boundary layer,  $\rho_\infty$ . According to Sutton [24]

$$\rho_{rms} \cong 0.1(\rho_\infty - \rho_w) \quad (25)$$

for boundary layers with supersonic external flow. For subsonic boundary layers, the constant is larger than 0.1 shown in equation (25). The density at the wall is given by

$$\rho_w = \frac{P_\infty}{RT_w} \quad (26)$$

The rms fluctuation in refractive index is related to density by

$$\frac{n_{rms}}{n} = \kappa \frac{\rho_{rms}}{\rho} \quad (27)$$

where  $\kappa$  is the constant from equation (12).

An attenuation coefficient,  $\alpha$ , can be defined for propagation through a layer of a turbulent medium. The intensity ratio is

$$\frac{I}{I_0} = e^{-\alpha\delta} \quad (28)$$

where the layer thickness is  $\delta$ , and  $I_0$  is the intensity of the beam entering the layer. The attenuation is due to scattering by the turbulent boundary layer. Sutton [25] gives a formula for  $\alpha$  as follows:

$$\alpha = 2k^2 n_{\text{rms}}^2 \Lambda \quad (29)$$

where  $k$  is the wavenumber equal to  $2\pi/\lambda$  and  $\Lambda$  is the macroscale of the turbulence. The macroscale is obtained from

$$\Lambda = \int_0^{\infty} C(r) dr \quad (30)$$

where  $C(r)$  is the correlation function for refractive index in an isotropic turbulent medium. The correlation function is given by

$$C(r) = \frac{\int_0^{\infty} \Delta n(r+R) \Delta n(r) 4\pi R^2 dR}{\int_0^{\infty} [\Delta n(R)]^2 4\pi R^2 dR} \quad (31)$$

where  $\Delta n$  is the deviation from the average,  $\bar{n}$ ,

$$\Delta n = n(r) - \bar{n} \quad (32)$$

Sutton [25] reports values of the scale of turbulence equal to about  $0.1 \delta$ , i.e., the scale of turbulence is approximately 1/10 of the boundary layer thickness.

For propagation through turbulent boundary layers, several characteristic lengths occur as follows:

- $\Lambda$  macroscale of turbulence
- $\delta$  boundary layer thickness
- $D$  aperture for laser beam
- $\lambda$  wavelength of laser radiation

Many regions for turbulent propagation can be characterized by the length scales along with the extinction number  $\alpha\delta$ .

### C. Scaling Propagation Through a Turbulent Boundary Layer of Fixed Thickness

Two parameters can be used to characterize the degradation in beam quality due to turbulence. One is the decrease of intensity,  $\text{watts/cm}^2$ , on the axis of the beam in the far-field. For large values of  $D/\Lambda$ , equation (28) provides a means to calculate  $I/I_0$ . The second parameter is the half-intensity angle for the beam. The half-intensity angle,  $\theta_{1/2}$ , defines a cone with a vertex angle equal to  $\theta_{1/2}$ . On the surface of the cone, the intensity is one-half of the peak intensity.

Figure 1 shows the half-intensity angle as a function of the ratio  $D/\Lambda$ . The ordinate is the variable  $\theta_{1/2}/(1.129\lambda/D)$ . This variable compares the half-intensity angle for the scattered beam with the half-intensity angle for a diffraction limited beam.

Consider a  $\text{CO}_2$  laser beam at 10.6 microns from a  $D = 60$  cm mirror. Assume the boundary layer thickness,  $\delta$ , is 10 cm, and the macroscale,  $\Lambda$ , is 1 cm. For  $M = 1.5$  flow with turbulent Prandtl number of unity and  $p_\infty = 1$  atmosphere,  $\rho_\infty = 1.23 \text{ kg/m}^3$ , and  $T_\infty = 290^\circ\text{K}$ , the values of  $\rho_w$  and  $\rho_{\text{rms}}$  are  $0.848 \text{ kg/m}^3$  and  $0.038 \text{ kg/m}^3$ . Using  $\kappa = 2.4 \times 10^{-4}$ , the value of  $n_{\text{rms}}/n$  is  $7.44 \times 10^{-6}$ . The extinction coefficient, which is calculated using equation (29), is 0.39 per meter. The extinction number,  $\alpha\delta$ , is 0.039, and  $D/\Lambda$  is 60. The corresponding point can be plotted in Figure 1, giving  $\theta_{1/2}/(1.129\lambda/D)$  almost on the dotted line.

Consider now an iodine laser at 1.315 microns with a turret for which  $D/\lambda = S_1$  has been maintained constant. Hence  $D = 7.44$  cm; also,  $D/\Lambda$  is 7.44. The extinction coefficient grows to

$$\alpha = (0.39)(8.06)^2 = 25.34$$

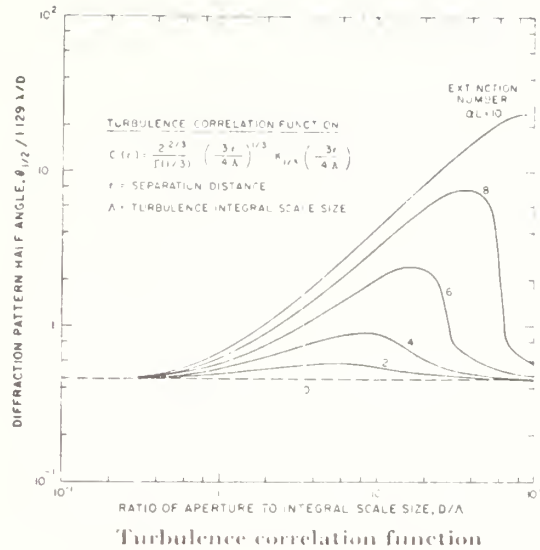


Figure 1. Diffraction angle for which the intensity is one half the maximum value. Reproduced from Sutton [25].

and the extinction number is 2.53. The point for the iodine laser can be plotted in Figure 1, giving a half angle parameter of about 0.6. The increase in half angle is about 20 per cent.

For a thicker turbulent boundary layer with  $\delta = 20$  cm, the results are more dramatic as shown in Table II. The half angle parameter increases from 0.48 to 1.2, which is a factor of 2.5.

It should be emphasized that equation (28) does not apply when  $D/\Lambda$  is less than approximately 10. The attenuation is less than that predicted by equation (28) when  $D/\Lambda < 10$ .

#### D. Scaling Propagation Through a Scaled Turbulent Boundary Layer

This subsection discusses the case for which the turbulent boundary layer originates on the laser turret instead of an aircraft surface. From equation (22), the ratio of boundary layer thicknesses is

Table II. Change of Half Angle with Change in Wavelength

wavelength, $\lambda$ , micron	10.6	1.31
boundary layer thickness, $\delta$ , cm	20	20
macroscale, $\Lambda$ , cm	2	2
extinction coefficient, $\alpha$ , per meter	0.78	50.67
extinction number, $\alpha\delta$	0.156	10.13
ratio $D/\Lambda$	30	3.72
ratio $D/\lambda$	56,603	56,603
Mach number $M$	1.5	1.5
pressure, $p_\infty$ , atmosphere	1	1
density, $\rho_\infty$ , $\text{kg/m}^3$	1.23	1.23
temperature, $T_\infty$ , $^\circ\text{K}$	290	290

$$\frac{\delta_2}{\delta_1} = \left(\frac{x_2}{x_1}\right)^{0.8} = \left(\frac{L_2}{L_1}\right)^{0.8} \quad (33)$$

Further, the ratio of macroscales of turbulence is

$$\frac{\Lambda_2}{\Lambda_1} = \frac{\delta_2}{\delta_1} \quad (34)$$

The extinction number scales as

$$\frac{\alpha\delta)_2}{\alpha\delta)_1} = \frac{\lambda_1^2}{\lambda_2^2} \frac{\Lambda_2}{\Lambda_1} = \frac{L_1^2}{L_2^2} \left(\frac{L_2}{L_1}\right)^{0.8} = \left(\frac{L_1}{L_2}\right)^{1.2} \quad (35)$$

Equation (35) assumes  $n_{\text{rms}}$  is not changed. The ratio  $D/\Lambda$  scales as



$$\frac{D/\Lambda}_2 = \frac{L_2}{L_1} \frac{\Lambda_1}{\Lambda_2} = \frac{L_2}{L_1} \left(\frac{L_1}{L_2}\right)^{0.8} = \left(\frac{L_2}{L_1}\right)^{0.2} \quad (36)$$

Using equations (35) and (36) along with data of Table II, the following values are found for  $\lambda = 1.315$  microns:

$$D/\Lambda = 19.8$$

$$\alpha\delta = 1.89$$

Plotting the preceding points in Figure 1 yields a  $\theta_{1/2}$  about 8 per cent larger. The impact of reduced  $\lambda$  is much less when the boundary layer originates on the turret.

## VI. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The analysis has assumed that  $S_1 = \lambda/D$  and  $S_2 = L/D$  were maintained constant as  $\lambda$  was decreased. The conclusions are valid for this case.

Using the optical system equation of Holmes and Avizonis [3], various consequences of reducing  $\lambda$  were examined. In summary, the following should be noted:

- $\delta_{\text{rms}}$  Phase distortion must be reduced by a factor of 8 for a change from  $\text{CO}_2$  to iodine laser.
- $K$  A factor which depends on the spatial distribution of intensity at the output aperture will not vary.
- $T_d$  Diffractive spillage will tend to increase reducing the value of  $T_d$ .
- $\sigma$  rms jitter can be maintained constant without change in  $I_{\text{ff}}$ .

Detailed comments were not made about atmospheric extinction coefficient; however, scattering tends to increase as  $\lambda$  decreases.

Turret weight and volume as well as turret aerodynamic drag will be reduced dramatically. Much greater flexibility in the number and location of turrets is a consequence. Further, the possibility of obtaining a suitable window should improve; open port problems are thereby circumvented.

The analysis shows that the phase distortion  $\delta/\lambda$  for inviscid flow is given by

$$\frac{\delta}{\lambda} = (\text{constant}) \frac{D}{\lambda} \quad (37)$$

Since  $D/\lambda$  is constant, the phase distortion due to inviscid flow is independent of wavelength. The statement is true provided  $S_1$  and  $S_2$  are fixed. Nonetheless severe degradation due to the inviscid flow field can occur; hence, the problem cannot be ignored.

For turbulent boundary layers, the value of the extinction number,  $\alpha\delta$ , increases as  $\lambda$  decreases. Also, the ratio  $D/\lambda$  decreases as  $\lambda$  decreases. Since extinction and scattering depend on  $\alpha\delta$  and  $D/\lambda$ , the trends are not favorable. The half-intensity angle increases, and extinction increases as  $\lambda$  decreases. Turbulent boundary layers require more attention.

Any change which causes a thinner turbulent boundary layer tends to improve beam quality. Mounting the laser turret at points farther forward decreases  $\delta$ . Cooling the wall in the supersonic case also should be an improvement. Finally, attention to surface roughness should help reduce scattering and attenuation.

When the turbulent boundary layer originates on the laser turret, the boundary layer thickness decreases as the turret decreases in size. Hence, the impact of the turbulent boundary layer on  $\theta_{1/2}$  is almost unchanged by a change in  $\lambda$ ; however, serious degradation in intensity due to  $\alpha\delta$  remains.

APPENDIX A

Consider the change in intensity at some point P in the far-field of an aperture illuminated in phase and amplitude by

$$F(\xi, \eta) = A(\xi, \eta) e^{j\psi(\xi, \eta)} \quad (\text{A-1})$$

Silver [26] on page 173 shows the amplitude and phase of the wave at point P is given by

$$U_P = \frac{j}{\lambda R} e^{-jkR} \int_A F(\xi, \eta) \exp[jk \sin \theta (\xi \sin \phi + \eta \cos \phi)] d\xi d\eta \quad (\text{A-2})$$

Intensity at point P is

$$I_P = U_P U_P^* \quad (\text{A-3})$$

Introduce nondimensional variables

$$\xi = \xi' D \quad \eta = \eta' D \quad (\text{A-4})$$

where D is the diameter of the aperture. The integration in equation (A-2) extends over  $A = \pi D^2/4$ . Examine first the exponent in the integrand of equation (A-2); it becomes

$$jk \sin \theta [\xi \sin \phi + \eta \cos \phi] = jkD \sin \theta [\xi' \sin \phi + \eta' \cos \phi] \quad (\text{A-5})$$

Note that

$$kD = 2\pi/S_1 \quad (\text{A-6})$$

The complex function  $F(\xi, \eta)$  is exactly the same as equation (A-1) except primes are added to  $\xi$  and  $\eta$ . The integral in equation (A-2), which is represented by  $I_n$ , becomes

$$D^2 I_n = D^2 \int_{A'} F(\xi', \eta') \exp[j(2\pi/S_1) \sin \theta (\xi' \sin \phi + \eta' \cos \phi)] d\xi' d\eta' \quad (\text{A-7})$$

The intensity at point P is obtained by combining equations (A-2), (A-3), and (A-7)

$$I_P = \frac{D^2 I_n^2}{\lambda^2 R^2} = \frac{I_n^2}{S_1^2 R^2} \quad (\text{A-8})$$

Consider two cases 1 and 2 which differ in the wavelength of radiation but for which  $S_1$  is constant. The ratio of intensities becomes

$$\frac{I_{P2}}{I_{P1}} = \frac{I_{n2}^2}{I_{n1}^2} \frac{R_1^2}{R_2^2} \quad (\text{A-9})$$

In equation (A-9), the integrals,  $I_n$ , cancel when P is located at the same point specified by  $R, \theta, \phi$ . Since  $R_1 = R_2$ , the ratio of intensities is identical. Consequently, the factor K in equation (1) is identical for cases 1 and 2.

## REFERENCES

1. W. E. McDermott, N. R. Pchelkin, D. J. Bernard, and R. R. Bousek, "An Electronic Transition Chemical Laser," Applied Physics Letters, Vol. 32, pp. 469-470, 1978.
2. C. K. Rhodes, Editor, Excimer Lasers, Springer-Verlag, Berlin, 1979.
3. D. A. Holmes and P. V. Avizonis, "Approximate Optical System Model," Applied Optics, Vol. 15, pp. 1075-1082, 1976.
4. A. E. Fuhs, High Energy Laser System Design, Naval Postgraduate School, Lecture Notes, 1979.
5. M. Born and E. Wolf, Principles of Optics, Pergamon Press, New York, Second Revised Edition, 1964.
6. H. B. Booker, "Radio Scattering in the Lower Ionosphere," Journal of Geophysical Research, Vol. 64, pp. 2164-2177, 1959.
7. R. E. Hufnagel and N. R. Stanley, "Modulation Transfer Function Associated with Image Transmission Through Turbulent Media," Journal of the Optical Society of America, Vol. 54, pp. 52-61, 1964.
8. D. A. deWolf, "Saturation of Irradiance Fluctuations due to Turbulent Atmosphere," Journal of the Optical Society of America, Vol. 58, pp. 461-466, 1968.
9. D. L. Fried, "Optical Resolution Through a Randomly Inhomogeneous Medium for Very Long and Very Short Exposures," Journal of the Optical Society of America, Vol. 56, pp. 1372-1379, 1966.
10. W. P. Brown, Jr., "Propagation in Random Media: Cumulative Effect of Weak Inhomogeneities," IEEE Transactions on Antennas and Propagation, Vol. AP-15, No. 1, pp. 81-89, 1967.
11. D. A. deWolf, "Wave Propagation Through Quasi-Optical Irregularities," Journal of the Optical Society of America, Vol. 55, pp. 812-817, 1965.
12. L. S. Taylor, "Decay of Mutual Coherence in Turbulent Media," Journal of the Optical Society of America, Vol. 57, pp. 304-308, 1967.
13. Yu. N. Barabanenkov, Yu. A. Krautsov, S. M. Rytov, and V. I. Tatarsky, "Status of the Theory of Propagation of Waves in a Randomly Inhomogeneous Medium," Usp. fiz. Nauk, Vol. 102, 1, 1970.
14. A. M. Prokhorov, F. V. Bunkin, K. S. Gochelashvily, and V. I. Shishov, "Laser Irradiance Propagation in Turbulent Media," Proceedings of the IEEE, Vol. 63, pp. 790-811, 1975.
15. R. M. Scott, "The Practical Applications of Modulation Transfer Functions," Journal of the Soc. Photo. Instr. Engrs., Vol. 2, p. 132, 1964.

16. F. Dow Smith, "Optical Image Evaluation and the Transfer Function," Applied Optics, Vol. 2, p. 335, 1963.
17. F. H. Perrin, "Manipulation and Significance of Sine Wave Response Functions," Journal of the Society of Motion Picture Television Engineers, Vol. 69, p. 239, 1960.
18. R. L. Lamberts, G. C. Higgins, and R. N. Wolfe, "Measurements and Analysis of the Distribution of Energy in Optical Images," Journal of the Optical Society of America, Vol. 48, pp. 487-490, 1958.
19. L. A. Chernov, Wave Propagation in a Random Medium, Translated from the Russian by R. A. Silverman, McGraw-Hill Book Co., New York, 1960.
20. B. J. Uscinski, The Elements of Wave Propagation in Random Media, McGraw-Hill Book Co., New York, 1977.
21. H. Tennekes and J. L. Lumley, A First Course in Turbulence, The MIT Press, Cambridge, 1972.
22. H. Schlichting, Boundary-Layer Theory, Sixth Edition, McGraw-Hill Book Co., New York, 1968.
23. H. W. Liepmann and A. Roshko, Elements of Gasdynamics, Wiley, New York, 1967.
24. G. W. Sutton, "On Optical Imaging Through Aircraft Turbulent Boundary Layers," AVCO Everett Research Laboratory, AMP 334, June, 1971.
25. G. W. Sutton, "Effect of Turbulent Fluctuations in an Optically Active Fluid Medium," AIAA Journal, Vol. 7, pp. 1737-1743, 1968.
26. S. Silver, Microwave Antenna Theory and Design, McGraw-Hill Book Co., 1949.

INITIAL DISTRIBUTION LIST

	Number of Copies
1. Defense Technical Information Center Cameron Station Alexandria, VA, 22314	2
2. Library, Code 0142 Naval Postgraduate School Monterey, CA, 93940	2
3. Distinguished Professor Allen E. Fuhs Naval Postgraduate School Monterey, CA, 93940	15
4. Captain Richard deJonckheere AFWL/ARLB Kirtland AFB, NM, 87117	50
5. Department Chairman, Code 67 Department of Aeronautics Naval Postgraduate School Monterey, CA, 93940	1
6. Department Chairman, Code 69 Department of Mechanical Engineering Naval Postgraduate School Monterey, CA, 93940	1
7. Department Chairman, Code 61 Department of Physics and Chemistry Naval Postgraduate School Monterey, CA, 93940	1
8. Dr. Roman J. Kramarsic R & D Associates International Airport ATO 9377 Albuquerque, NM, 87119	1
9. Major Eric J. Jumper, Ph.D. Department of Aeronautics US Air Force Academy, CO, 80840	2
10. Chairman Department of Aeronautics US Naval Academy Annapolis, MD, 21402	1
11. Professor Garret N. Vanderplaats Naval Postgraduate School Monterey, CA, 93940	1





U191242

DUDLEY KNOX LIBRARY - RESEARCH REPORTS



5 6853 01068121 6

~~U19124~~