

Geometry Teacher's Edition - Differentiated Instruction

[CK-12 Foundation](#)

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Chapter 1

Geometry TE - Differentiated Instruction

Geometry- Teacher's Edition- Differentiated Instruction

Introduction

Differentiated Instruction is a term that teachers often hear regarding instructing and successfully educating students. In the context of this teacher's edition, differentiated instruction refers to instruction that does not simply present material, but works to engage every student through multiple methods. As educators, we know that students all learn in different ways whether it is visually or kinesthetically or interpersonally. Today, there are also many students who come into our classrooms with learning challenges that are not addressed by traditional teaching methods. Differentiated instruction is about reaching each one of these different learners. In our classrooms, students can have multiple options for learning new ideas and concepts, while teachers can utilize many methods of presenting these ideas to the students. Differentiated instruction can assist all students in using their gifts and talents to learn, and as educators, we can assist students in developing and discovering these gifts and talents. By providing students with things like flexible grouping, projects and interactive activities, our students will become the active explorers of the information presented. This flexbook is designed to be a guide for you, the educator to assist you in differentiating the material in this geometry text. There are many suggestions and ideas in this flexbook, not all of them are meant to be incorporated into every lesson. Select those things that work best for you and your students so that the world of mathematics can really come to life.

1.1 Basics of Geometry

Points, Lines and Planes

I. Section Objectives

- Understand the undefined terms *point*, *line* and *plane*.
- Understand defined terms, including *space*, *segment* and *ray*.
- Identify and apply basic postulates of points, lines and planes.
- Draw and label terms in a diagram

II. Multiple Intelligences

This section is designed to assist educators in differentiating instruction with the multiple intelligences in mind.

- Visual Learners- one way to assist visual learners with this lesson is to use the actual objects mentioned in the lesson. Where there is a map or a globe mentioned, use an actual map and a globe. This will also assist students with special needs in making a connection with the material.
- Kinesthetic Learners- allow move time so that students can walk around the classroom identifying points, lines and planes in their surroundings. Request that students make a list of the things that they find.
- Interpersonal Learners- have students work in pairs or small groups to discuss their findings from the “walk around” activity. This engages students who need to talk about their work to gain a better understanding of a concept.

III. Special Needs/Modifications

This section is designed to assist with any modifications or to assist students who face learning challenges.

- Be sure that all of the vocabulary words are written on a board or overhead as they are presented and discussed. Request that students copy this information into a notebook. Reading the terms, hearing them discussed, seeing them written again and writing the words themselves assists students in retaining information.
- Write each postulate on the board as it is discussed.
- Example 7- Expand this for all learners.

The goal here is to assist students in grasping and learning each term/postulate and its definition. The more students interact with each term and concept, the more they will remember what has been taught.

- Draw an example of each vocabulary word. Example, draw three collinear points.
- Draw an example that illustrates each postulate.
- Allow students to have an interpersonal connection by discussing their drawings with a peer.

IV. Alternative Assessment

There are many ways to assess student understanding during a lesson. This section provides a few ideas for that.

- Walk around and observe students as they work. Are students on task? Are they working diligently? Is the conversation appropriate to what is being taught?
- Use peers to assess each other. With the activity in Example 7, have the students assess the accuracy of each other’s work and correct any inconsistencies. If time allows, you could even have a presentation part where students share their findings.

Segments and Distance

I. Section Objectives

- Measure distances using different tools
- Understand and apply the ruler postulate to measurement
- Understand and apply the segment addition postulate to measurement
- Use endpoints to identify distances on a coordinate grid

II. Multiple Intelligences

- Activity with Example 3- This activity will address the following intelligences- visual, interpersonal, kinesthetic, logical- mathematical.

The students work in pairs. One member in the pair draws a line segment that he or she has measured to find the distance. The other member also draws a line segment that he/she has measured. Then they switch drawings. Student A must figure out the length of student B's line segment, and student B must figure out the length of student A's line segment.

- Extension with postulates. In this lesson, there are two key postulates. One is the Ruler Postulate and the other is the Segment Addition Postulate. The students then take their work and determine which line segments are examples of the postulates. They can even exchange with another group to accomplish this.

This extension includes visual, interpersonal, kinesthetic, logical- mathematical intelligences.

- Discussion of activity- by having the students share their examples, answers and reasoning with the entire class or in small groups, the intrapersonal intelligence is included as students share their personal insights into their work.
- Activity with Example 5 and 6- hand out grid paper. Ask students to draw a coordinate plane and provide given distances on the board/overhead. Then allow the student time to draw a line segment with this distance. Then provide a time for sharing/feedback from the exercise.

III. Special Needs/Modifications

- Begin class with a brief review of previously learned material. This can be done with words on the board/overhead or as a class discussion.
- Review what distance means and what it means to estimate.
- Write all vocabulary on the board as it is brought up in the lesson. Request that students take the time to copy this information in their notebooks.
- Ruler Postulate defined on board.
- Segment Addition Postulate defined on board.

- Be sure that students are given plenty of time to think through their work and be sure that all students have finished examples before going over the answers. Sometimes, special needs students require more time to complete tasks and will stop working if the answers to a particular question are given before they have finished.

IV. Alternative Assessment

- Observe students as they work in groups. Notice which students need assistance or seem lost. Make a note of who each student is and set aside a time to check in with each of these students.
- Create an observation checklist of things to watch for when students are completing exercises in a group.
- Pay close attention to student thinking during discussions before and after an activity.

Rays and Angles

I. Section Objectives

- Understand and identify rays.
- Understand and classify angles.
- Understand and apply the protractor postulate.
- Understand and apply the angle addition postulate.

II. Multiple Intelligences

- Activity with identifying rays and angles.

Have students work in small groups. Assign one group rays and the other group angles. Using rulers, the students need to design a series of either rays or angles. You can use index cards for this activity. Then have the groups switch cards. The angle group needs to name all of the rays that the other group has drawn. The ray group needs to name all of the angles that the angle group has drawn. Then the groups exchange answers and check each other's work. This involves discussion and peer tutoring as well.

Addresses the following intelligences

- – Linguistic- students discuss their answers and thinking
- Logical- mathematical- students draw their angles and rays
- Spatial- visual- students draw angles and rays
- Interpersonal- students share their thinking in a group
- Intrapersonal- students explain their answers in a group
- Activity with Protractors

Provide students with drawings of several different angles. You can use the angles that were drawn in the previous activity. Have students measure their angles using protractors. Then have the students all share in a class discussion.

III. Special Needs/Modifications

- Begin each lesson with a review of previously learned vocabulary words and information. This helps students to recall what they have learned in a previous lesson. It also decreases the number of confused students once an assignment has been given.
- Write all vocabulary on the board or overhead. Request that students write these terms in their notebooks.
- Vocabulary for this lesson
 - Ray- include symbol notation and an example
 - Angle- include symbol notation and a diagram with sides and vertex labeled.
 - Right angle- include drawing
 - Perpendicular- include symbol
 - Acute angle- include drawing
 - Obtuse angle- include drawing
 - Straight angle- include drawing
 - Protractor Postulate
 - Angle Addition Postulate

IV. Alternative Assessment

- Use an observation checklist to observe students as they work.
- Pay attention to the questions asked during the lesson.
- Make a note of students who are having difficulty. Consider flexible grouping to assist these students in their work.

Segments and Angles

I. Section Objectives

- Understand and identify congruent line segments
- Identify the midpoint of line segments
- Identify the bisector of a line segment
- Understand and identify congruent angles
- Understand and apply the Angle Bisector Postulate

II. Multiple Intelligences

- We can differentiate this lesson by organizing the content into a table. This is done as part of a class discussion. It is not done ahead of time and then presented. Creating the chart is meant to be interactive. Since this lesson works with line segments and angles, we can use these as the two columns of our table. Here is a sample of a table and how to organize it for the students.

Table 1.1

Line segment	Angles
Congruent (show example)	Congruent (show example)
Segment midpoint	Show vertex and sides
Show symbols	Show symbols
Segment midpoint postulate	Angle bisector postulate

- Be sure to explain each concept and how they are different and similar depending on whether you are working with line segments or angles.
- This helps the students to see the connections between the concepts.

II. Multiple Intelligences: Linguistic, logical- mathematical, spatial- visual, interpersonal, intrapersonal

III. Special Needs/Modifications

- Review previously learned information. One way to do this is with students working in pairs to quiz each other.
- Write all vocabulary on the board/overhead. Request that students copy this information in their notebooks.
- Vocabulary
 - Congruent with symbol
 - Segment
 - Midpoint
 - Segment Midpoint Postulate
 - Segment bisector
 - Angle bisector postulate

IV. Alternative Assessment

- When creating the table, be sure to include all students in the discussion.
- Refer students back to the information in the lesson to assist with adding in the information.
- Make a note of which students have a strong grasp of the material. Be sure to pair those students up with students that seem to be having difficulty when working on in class assignments.

Angle Pairs

I. Section Objectives

- Understand and identify complementary angles
- Understand and identify supplementary angles
- Understand and utilize the Linear Pair Postulate

- Understand and identify vertical angles

II. Multiple Intelligences

- In this lesson, students are going to work on understanding the relationship between pairs of angles. One way to assist students in doing this is to create a chart that compares and contrasts the different relationships.
 - Go through all of the material in the lesson first. Be sure that the students have a basic understanding of the terms and concepts in this lesson. You want to use the activity to expand student knowledge and understanding.
 - To do this, students are going to work in small groups. Review what that compare means to look at the similarities between things, and that contrast means to look at the differences between things.
 - Hand out chart paper and markers to each group.
 - Request that students compare and contrast supplementary angles, complementary angles, linear pairs and vertical angles. Ask them to include drawings to justify what they are comparing and contrasting.
- Then allow time for the students to share their chart work with the rest of the class.
- II. Multiple Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, spatial- visual, interpersonal, intrapersonal

III. Special Needs/Modifications

- Sometimes, special needs students will have difficulty remembering how to do previously learned skills. Here are some prerequisite skills to review prior to beginning this lesson.
 - Solving one- step equations
 - Solving multi- step equations
 - Combining like terms to solve an equation
- Write all vocabulary words on the board/overhead. Request that students copy this information in their notebooks. Students will need this information to complete the activity.
- Vocabulary
 - Adjacent
 - Congruent
 - Complementary angles
 - Supplementary angles
 - Linear pairs
 - Line pair postulate
 - Vertical angles
 - Vertical angles theorem

IV. Alternative Assessment

- This is a great lesson to use an observation checklist. Make the checklist prior to teaching the lesson. Then use it while groups complete and present their charts. It will provide you with clear things to look and listen for when teaching this lesson.

Classifying Triangles

I. Section Objectives

- Define triangles
- Classify triangles as acute, right, obtuse, equiangular
- Classify triangles as scalene, isosceles, or equilateral

II. Multiple Intelligences

- This activity is a drawing activity that involves students creating a design and then classifying the triangles within the design.
 - The students are given plain paper and a ruler. They are told to create a page of triangles created by intersecting lines.
 - Once they have finished, ask them to create a key and to color (use crayons or colored pencils) to color in the different triangles found in the design.
 - Students can be asked to finish this design for homework.
- Multiple Intelligences- logical- mathematical and spatial- visual

III. Special Needs/Modifications

- Provide students with a diagram of a triangle with the vertices labeled, the sides labeled and the angles labeled. Be sure that students understand where to find the interior angle. They will need this to classify the triangles.
- Write all vocabulary words on the board/overhead. Request that students copy these notes into their notebooks.
- Vocabulary for classifying by angles
 - Right
 - Obtuse
 - Acute
 - Equiangular
- Vocabulary for classifying by side lengths
 - Scalene
 - Isosceles
 - Equilateral

IV. Alternative Assessment

- Use observation to assess students as they work. Most students will need assistance creating a key to show how their design has been colored. You may want to provide an example of this and then see how the students do following directions.
- You can pair students up to work together too. This may help students who are having a difficult time with the activity.

Classifying Polygons

I. Section Objectives

- Define polygons
- Understand the difference between convex and concave polygons
- Classify polygons by the number of sides
- Use the distance formula to find side lengths on a coordinate grid

II. Multiple Intelligences

- This activity has students begin working alone and then they work in a small group. The purpose is to assist students with developing a deeper understanding of concave and convex figures.
 - Students begin alone. They can choose to draw either four concave polygons or four convex polygons. They don't tell anyone else what they have chosen.
 - When finished, the students join a small group. Then they exchange papers and they must show, by drawing lines, whether the figures they have been given or concave or convex.
 - Students need to justify their answers.
 - Peers correct each other's work.
- Multiple Intelligences- linguistic, logical- mathematical, spatial- visual, interpersonal, intrapersonal

III. Special Needs/Modifications

- Polygon drawing with sides and vertices labeled on the board.
- Write all vocabulary words on the board. Request that students copy these words in their notebooks.
- Complete the distance formula examples slowly on the board/overhead. Be sure that the students are following along.
- Add another example using the distance formula. Use exercise 5 and find the length of \overline{BA} , \overline{AD} and \overline{DC} .

IV. Alternative Assessment

- Collect all student work when the groups have finished. Review their work and see how the students have justified whether their figure was concave or convex. This will show you a lot about how students were thinking as they worked on the assignment.

Problem Solving in Geometry

I. Section Objectives

- Read and understand given problem situations

- Use multiple representations to restate problem situations
- Identify problem- solving plans
- Solve real- world problems using planning strategies

II. Multiple Intelligences

- The great thing about this lesson is that each way of solving a problem can be identified with a specific intelligence. This lesson can assist each student in understanding how he/she works best.
- Begin by presenting all of the information in the lesson. Request that students take notes too.
- Then go through a brief discussion on multiple intelligences. Ask the students to try to identify how they learn best. Have them write this down on a piece of paper. You can refer back to this discussion throughout your teaching and help students to further define the ways that each of them learns best.
- Once students have identified how they learn best, reorganize the class according to learning styles.
- Then ask each group (you may need to subdivide if groups are large) to solve the exercises at the end of the section according to how the group learns.
- Students will easily leap into this, but if not help them with an example or two.

III. Special Needs/Modifications

- Review concept- the Pythagorean Theorem- it is mentioned in the lesson, but not reviewed.
- Write the steps to simplifying a problem on the board. Review what it means to “simplify” something.
 - What is this problem asking for?
 - What do I need to know to find the answer?

IV. Alternative Assessment

- Make notes about the groups of students when organized according to how they learn best.
- This can be very valuable when assisting students in learning.
- For example, a visual learner could better understand a concept presented verbally by drawing a picture. When you are aware of which category each student falls in, you can better address his/her needs when teaching.

1.2 Reasoning and Proof

Inductive Reasoning

I. Section Objectives

- Recognize visual patterns and number patterns

- Extend and generalize patterns
- Write a counterexample to a pattern rule

II. Multiple Intelligences

- Include group work in this lesson. Rather than explaining all of the information in the lesson and then assigning group work, intersperse the group work with the lesson.
- Begin by going over visual patterns.
- Then have students work in pairs. Each student draws a visual pattern. Then they exchange papers with their partner. Our next step is to write a rule for the pattern they have been given, and to extend the pattern two steps.
- Next, go over number patterns.
- Then have the students work in pairs. Each student writes a number pattern. Then they exchange papers with their partner. Our next step is to write a rule (an equation) for the number pattern and to extend the pattern two steps.
- Finally teach about conjectures and counterexamples.
- Have students work with the patterns that they have previously worked with and write a conjecture and a counterexample for each pattern.
- Multiple Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, spatial- visual, interpersonal, intrapersonal

III. Special Needs/Modifications

- Write all vocabulary on the board. Request that students copy this information down in their notebooks.
- Vocabulary
 - Conjecture
 - Counterexample

IV. Alternative Assessment

- Collect student papers.
- Review each student's work to assess understanding.
- Use this to review at the beginning of the next class. You can use different student patterns in the beginning of the next class to review the previously learned material.
- This will be especially helpful to special needs students who require a lot of review to recall previously learned concepts.

Conditional Statements

I. Section Objectives

- Recognize if- then statements
- Identify the hypothesis and conclusion of an if-then statement
- Write the converse, inverse and contrapositive of an if-then statement
- Understand a biconditional statement

II. Multiple Intelligences

- Students need to develop a good understanding of conditional statements and related statements in this section. Because of this, teach all of the material in the lesson and then you can use the following activity to expand student understanding.
- Divide students into four groups. Assign each group one of the following: converse, inverse, contrapositive and biconditional.
- Then write a conditional statement on the board/overhead.
- Each group needs to work together to write a related statement for the given conditional statement.
- When finished, go over the student answer as a class. Use other peers in the class to do any correcting that is needed.
- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, visual- spatial, interpersonal, intrapersonal

III. Special Needs/Modifications

- This is a difficult lesson for special needs students to understand because the language and symbols are so verbal. Many students with language based learning disabilities will find this challenging. Here is one option on how to scaffold the information in this text.
- Alter these definitions and provide an example (in words not symbols) for each.
- Converse- switch the hypothesis and the conclusion of the conditional statement.
- Inverse- add not to the conditional statement to negate it.
- Contrapositive- add not to the converse to negate it.
- Biconditional statement- combine the conditional statement and its converse together.
- Allow time for the students to work with these definitions and to copy them into their notebooks.

IV. Alternative Assessment

- Use the answers from each group to assess student learning.
- If the students are having difficulty with the activity, then after the first example add another example.
- Repeat as necessary until the students have a good grasp of the information.

Deductive Reasoning

I. Section Objectives

- Recognize and apply some basic rules of logic
- Understand the different parts that inductive reasoning and deductive reasoning play in logical reasoning
- Use truth tables to analyze patterns of reasoning

II. Multiple Intelligences

- Teach the concepts in this lesson prior to completing the activity. This activity involves music and will assist students in a concrete example of how to write a conclusion using inductive reasoning and then use deductive reasoning to prove their conclusion.
- Prepare a five or six music samples for students to listen to. Be sure that all of the samples are from the same genre of music, for example, all jazz. You can even make a few of them by the same artist. The students will have a couple of different options to draw a conclusion from.
- Then allow students time to work in a pair and to write a conclusion about the samples that they have heard.
- Have each pair exchange their conclusion with another pair. Then the students must use deductive reasoning to prove that the statement is correct.
- Intelligences: linguistic, logical- mathematical, musical, interpersonal, intrapersonal

III. Special Needs/Modifications

- Begin the class with a review of the following terms. Knowledge and understanding of these terms is implied in this lesson.
 - Linear pair
 - Adjacent angles
 - Vertical angles
 - Supplementary angles
 - Complementary angles
- Write the following new definitions on the board/overhead. Request that students copy this information down in their notebooks.
 - Law of Detachment
 - Law of Syllogism
 - Inductive Reasoning
 - Deductive Reasoning
- Truth Tables- go over this information very slowly. Be sure that the students understand all of the symbols prior to going over the section in the textbook. You could even create a chart of symbols and explanations on the board for easier understanding.

IV. Alternative Assessment

- Notice how students react to the music cues. Then notice the conclusions that they write about the music. Be sure that they prove their statements clearly. If they have difficulty, then provide examples or use another peer group to coach these students.

Algebraic Properties

I. Section Objectives

- Identify and apply properties of equality
- Recognize properties of congruence “inherited” from the properties of equality
- Solve equations and cite properties that justify the steps in the solution
- Solve problems using properties of equality and congruence

II. Multiple Intelligences

- There is a lot of information given in this lesson. Rather than using an activity, here are some suggestions on how to differentiate all of this information so that all learners are engaged.
- Begin lesson by writing the intention of the lesson on the board/overhead. Intention- “to combine geometric building blocks with reasoning.”
- Throughout the lesson, refer back to this statement. Identify the geometric building blocks for the students. For example, when there is an example on angles, refer back to student notes on angles. Ask students to brainstorm a few things that they have learned about angles. Then continue with the lesson.
- Review equality definition.
- Write all Properties on one half of the board.
- Write the statements of congruence on the other half of the board.
- Show students how to combine these two together visually. Use different colored chalk or pens (on a whiteboard) to illustrate how combining these statements together can help to prove the given statement.
- Practice this with a few examples. Encourage class participation.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- If you have many special needs students in the class, you may want to break up this lesson over two days.
- Day one- review properties and statements of congruence. Review basics of geometry.
- Day two- show students how to combine the two together.

- Here are some steps for combining statements and properties.
1. Look at whether you are working with line segments or angles. This will help you choose a statement of congruence.
 2. Choose a property that explains the given statement.
 3. Combine the statement of congruence and the property together for a final answer.

IV. Alternative Assessment

- Verbally and visually check- in with students to be sure that they are following the lesson. If necessary, go back over previously learned information so that you are sure to have everyone following along.

Diagrams

I. Section Objectives

- Provide the diagram that goes with a problem or proof.
- Interpret a given diagram.
- Recognize what can be assumed from a diagram and what can not be
- Use standard marks for segments and angles in diagrams.

II. Multiple Intelligences

- Complete this activity half-way through the lesson. Once you have gone over the definitions of the eleven postulates, divide the students into eleven groups.
- Assign each group a postulate.
- Request that each group design a diagram that best proves their given postulate.
- When finished, have each group share and justify their diagram.
- Request that they explain how the diagram illustrates the proof.
- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, visual- spatial, interpersonal, intrapersonal.
- When finished, go back to the text and work on the section where we combine the postulates, properties and statements of congruence together.

III. Special Needs/Modifications

- Review all properties from the previous lesson.
- Review the statements of congruence.
- Review the steps for combining the two together.
- Write all of the postulates on the board/overhead. Request that the students make a three column chart for each.

Table 1.2

Postulate	Definition	example
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IV. Alternative Assessment

- Work with the student groups to understand proving each postulate.
- Assessment is done through observation and verbal questions.
- Provide students with plenty of “think time” so that you receive the most accurate response.

Two- Column Proof

I. Section Objectives

- Draw a diagram to help set up a two- column proof.
- Identify the given information and statement to be proved in a two- column proof.
- Write a two- column proof.

II. Multiple Intelligences

- To work through this lesson, allow students to work in pairs and discuss their work. There are logical-mathematical, visual, linguistic, interpersonal and intrapersonal aspects to working with a peer.
- Since many students have a difficult time with proofs, one suggestion is to provide students with a fill-in the blank proof before working on the exercises at the end of the section.
- You can use the proof on page 101- 102 to do this or write one of your own.
- Provide the students with a series of statements on the board. The students fill- in the reasons.
- Provide students with a two- column proof where some of the statements are blank and some of the reasons are blank. Where there are blank statements, the students will need to use the reason to write a statement. Where the reason is blank, the students will need to use the statement to write the reason.
- This will assist the students in interacting with the information in the proof and discussing it will help with retention.
- Now students should be able to complete number fifteen in the exercises which requires that they write a two- column proof without any assistance.

III. Special Needs/Modifications

- Be sure that the students have a current list of postulates, properties and vocabulary where they can access it easily.
 - Here are some helpful hints for students in working on two- column proofs.
1. Draw a diagram to better understand the vocabulary in the given. For example, if you are working on proving that points are collinear, then draw a diagram of the collinear points. Then look at what statements and reasons you can write about it.
 2. Look at the vocabulary in the given. Example 2 has the word “bisects” in it. Therefore, you will need a statement and reason that explains bisects.

Example 2 also has a congruent symbol in it. Therefore, you will need a statement and a reason that addresses congruency.

IV. Alternative Assessment

- Give a homework assignment where students write their own two- column proof based on a common given.
- The next day review the assignment and answers with the students. In small groups, have them write one “best” proof for the given.

Segment and Angle Congruence Theorems

I. Section Objectives

- Understand basic congruence properties.
- Prove theorems about congruence.

II. Multiple Intelligences

- The best way to address different learning styles in this lesson is to use diagrams and to teach this lesson as a class discussion.
 - The students will need to break down the concepts provided to gain an excellent understanding of the material.
 - Prior to teaching the lesson, write the intention on the board or overhead. This will assist all visual learners and help special needs students too.
 - “To prove congruence properties, we turn congruence statements into number statements, and use properties of equality.
 - Here are some steps to write on the board:
1. Take the given and notice whether you are working with segments or angles.

2. Think of converting to measurement. For example $\overline{AB} \cong \overline{AB}$, as a statement, we can say that $AB = AB$. We are working with the measurement or length of the segment here. We have changed this to numbers. With angles, change to show the measurement of the angles is equal.

III. Special Needs/Modifications

- Be sure that students have a page of notes out that explain the properties of equality.
- Review each of the properties and what each means.
- Show students how to draw a diagram to illustrate a given statement. A picture often helps special needs students.
- Explain that postulates don't need to be proven.
- Explain that theorems need to be proven.

IV. Alternative Assessment

- Use an observation checklist to observe students as they work.
- Notice who is having difficulty and who isn't. Be sure to make a note of those students so that they can be offered assistance or a peer tutor.
- When possible, use peers to help explain concepts. The student teaching and the student learning both benefit greatly.

Proofs about Angle Pairs

I. Section Objectives

- State theorems about special pairs of angles.
- Understand proofs of the theorems about special pairs of angles.
- Apply the theorems in problem solving.

II. Multiple Intelligences

- Use the following activity to assist students in understanding theorems about special pairs of angles.
- Teach the content in the lesson. Be sure that the students have a good understanding of the concepts in the lesson. Then divide students into groups.
- Each group must make a diagram that illustrates each of the following theorems. They need to write a two column proof to show how the diagram illustrates the theorem.
- When all have finished, allow students time to share their work with the class.
- If time does not allow for students to work on all four of the theorems, assign each group a different theorem to work with.
- Students can complete their work using colored pencils, rulers and large chart paper.

- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Review the basic definitions of right angles, supplementary angles, complementary angles, and vertical angles.
- Draw an example of each on the board/overhead.
- Explain to students how to move from the basic definition of each angle relationship to the theorem.
- Write each of the theorems on the board/overhead and request that students copy this information in their notebooks.

IV. Alternative Assessment

- Examine the work that the students completed on their chart paper. Notice which groups were more accurate than others.
- Pay attention to which groups expressed their reasoning well verbally.
- Notice which groups expresses their reasoning well in the diagram.
- If a letter grade is needed, assign the same assignment for homework and then grade student work the following day. This will give you an excellent understanding of which students have a good grasp of the concepts and who still needs more practice.

1.3 Parallel and Perpendicular Lines

Lines and Angles

I. Section Objectives

- Identify parallel lines, skew lines, and perpendicular lines
- Know the statement of and use the Parallel Line Postulate.
- Know the statement of and use the Perpendicular Line Postulate.
- Identify angles made by transversals.

II. Multiple Intelligences

- Teach half of the lesson, complete the walk around activity, and then finish the rest of the lesson.
- Write the vocabulary on the board for the visual- spatial learners. Include the following.
 - parallel lines with symbol
 - perpendicular lines with symbol
 - parallel planes

- skew lines
- Have students walk around the room and make a list of all of the places where they can locate each type of lines.
- When finished, allow time for students to share their findings.
- Go on to the postulates. Break each postulate down into simple steps. This will assist visual- spatial learners and special needs students. Here are some suggestions.
- Parallel Line Postulate
 - Given a line and a point not on that line
 - One line parallel to the given line goes through that point
- Perpendicular Line Postulate
 - Given a line and a point not on that line
 - One line perpendicular to the line passes through that point
- When working with transversals, use color to indicate the different angles in a diagram. Use color for definitions too. This will help students to keep things clear.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Before working on the section of the lesson on transversals, review the meaning of the following words: adjacent, vertical, interior, exterior, corresponding and consecutive.
- Break down the transversal section. Use a diagram too.
- Teach adjacent and vertical angles first.
- Then teach interior, exterior and corresponding
- Finally, teach alternate interior, alternate exterior and consecutive angles.

Parallel Lines and Transversals

I. Section Objectives

- Identify angles formed by two parallel lines and a non- perpendicular transversal.
- Identify and use the Corresponding Angles Postulate.
- Identify and use the Alternate Interior Angles Theorem.
- Identify and use the Alternate Exterior Angles Theorem.
- Identify and use the Consecutive Interior Angles Theorem.

II. Multiple Intelligences

- This activity engages several of the intelligences, but also will demonstrate student understanding of the postulates and theorems in this lesson.
- Divide the students into a group of four.
- Assign each of the four students a different topic- Corresponding Angles Postulate, Alternate Interior Angles Theorem, Alternate Exterior Angles Theorem, Consecutive Interior Angles Theorem.
- Let the students know that their assignment is to prepare a lesson and teach the other students in the group about their topic. They can use their notes, a diagram, real life examples, a poem, a song, whatever they would like to make the topic clear. The other students in the groups will let the “teacher” know what he/she did well and also offer suggestions to improve the presentation.
- Intelligences- linguistic, logical- mathematical, musical, bodily- kinesthetic, visual- spatial, interpersonal, intrapersonal

III. Special Needs/Modifications

- Review the angles formed by a transversal
- Write all new postulates and theorems on the board/overhead. Request that the students copy this information down in their notebooks.
- Be sure to place students in a group where they will be well supported by their peers. Some groups are more encouraging than others.

IV. Alternative Assessment

- Create a rubric that covers the points that you want each student presentation to have.
- Share this rubric with the students.
- Observe students as they present their material.
- Include group feedback as part of the rubric.
- This could be used to calculate a quiz/classwork grade if necessary.

Proving Lines Parallel

I. Section Objectives

- Identify and use the Converse of the Corresponding Angles Postulate.
- Identify and use the Converse of Alternate Interior Angles Theorem.
- Identify and use the Converse of Alternate Exterior Angles Theorem.
- Identify and use the Converse of Consecutive Interior Angles Theorem.
- Identify and use the Parallel Lines Property.

II. Multiple Intelligences

- The best way to differentiate this lesson is to do so as part of a discussion. You want the students to make connections between the parallel lines, the transversal, and proving that the lines are parallel.
- Demonstrate that it is possible to draw two lines and a transversal and have the lines not be parallel. This is where things being true or not true comes into the lesson.
- The students are going to work with you as you work on the board/overhead. Begin by drawing two parallel lines and a transversal on the board.
- Request that the students mirror this work at their seats. They will need paper, rules, pencils and protractors.
- Then go through measuring each pair of angles.
- Once this is finished, go through each postulate and theorem and demonstrate proving that the lines are parallel using the postulates and theorems.
- Remind students that they are “proving” the accuracy of the statement.
- Completing this lesson this way engages the following intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal

III. Special Needs/Modifications

- Review that what a conditional statement is and how to write the converse of a conditional statement.
- Practice writing converse statements from conditional statements by using real life examples.
- Be sure that the students understand this concept before moving on to the material in the lesson.
- Review the Transitive Property.
- Write all new terms on the board. Request that the students copy these notes in their notebooks.

IV. Alternative Assessment

- The best way to assess student understanding is through observation.
- Ask probing questions, allow plenty of think time, and listen carefully to student responses during the work of the lesson.

Slopes of Lines

I. Section Objectives

- Identify and compute slope in the coordinate plane.
- Use the relationship between slopes of parallel lines.
- Use the relationship between slopes of perpendicular lines.
- Plot a line on a coordinate plane using different methods.

II. Multiple Intelligences

- For this lesson, be sure that students have grid paper, rulers and colored pencils.
- Complete each of the exercises in the text as a whole class.
- This will assist the students in practicing the constructions of lines and finding the slope of a line as you work with them.
- It might even make sense to have a list of the ordered pairs in each example prepared ahead of time and not use the text at first. This way, you can go through each example with the students constructing lines and figuring slopes on their own without the answers presented in the text.
- When you get to the parallel line section, you can introduce the theorem, and then have them construct the line and a line parallel to that given line.
- When you get to the perpendicular line section, you can introduce the theorem and work with the students to draw in a line perpendicular to the line drawn.
- Finally, work with students on using ordered pairs to graph different lines and to find the slopes of the lines. This is from the section on graphing strategies.
- Intelligences: linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal, bodily-kinesthetic

III. Special Needs/Modifications

- Review the following prior to beginning the lesson.
- Drawing a coordinate grid
- Labeling the coordinate grid
- How to locate the x and y axis'
- Review the origin as $(0, 0)$
- Review ordered pairs (x, y)
- Review finding the reciprocal of a number/fraction
- Write the two new theorems on the board and request that the students copy this information in their notebooks.

IV. Alternative Assessment

- Alternative Assessment in this lesson can be done through observation. As the students work on the exercises, walk around the room and observe them as they work.
- This is also a good time to notice students who need assistance.
- If several students are needing assistance, consider allowing students to work in pairs.

Equations of Lines

I. Section Objectives

- Identify and write equations in slope- intercept form.
- Identify equations of parallel lines.
- Identify equations of perpendicular lines.
- Identify and write equations in standard form.

II. Multiple Intelligences

- In this lesson, one way to differentiate this lesson is to break the material down into sections.
- There is only one example per section, so you may want to include more than one so that the students have a chance to practice the concept before moving on to something new.
- Include constructions whenever possible.
- When working with slope- intercept form, show how in the equation $y = mx + b$, that m is the slope of the line and that the b is the y intercept.
- Explain that the y intercept is where the line intersects with the y axis.
- When figuring out which equation represents a line parallel to a line already graphed, provide students with these steps.

1. Find the slope of the graphed line.
2. Look at the equation choices
3. Any equation with the same slope will be parallel to the graphed line.

- To figure out the equation for a line perpendicular to a graphed line, follow these steps.

1. Find the slope of the graphed line.
2. Find the reciprocal of the slope.
3. Any equation with the reciprocal as the slope is perpendicular to the line.

- Intelligences- linguistic, logical- mathematical, visual- spatial

III. Special Needs/Modifications

- Use the information above to scaffold this lesson for the students.
- Ex 4- Standard Form- begin with a simpler example first.
- Possible example $2y = 6x + 12$
- Write all steps on the board and request that the students copy those notes into their notebooks.

IV. Alternative Assessment

- Observe students as they work. If there is a lot of confusion, review concepts more than once to be certain that students are following the material.

Perpendicular Lines

I. Section Objectives

- Identify congruent linear pairs of angles.
- Identify the angles formed by perpendicular intersecting lines.
- Identify complementary adjacent angles.

II. Multiple Intelligences

- Begin this lesson by reviewing the definition of perpendicular lines and the symbol for perpendicular lines.
- List the description for each type of linear pair on the board.
- Then ask the students to use the description to draw an example of the pair being described.
- Example- Draw a linear pair of angles.
- Next, show students how a triangle can be drawn into their diagram.
- Request that they mark the right angle in the triangle.
- Then assign one of the other angles in the triangle a measurement.
- Request that students work to figure out the measurement of the missing angle.
- Explain to students that because they know that the interior angles of a triangle add up to be 180° that they can use this to figure out the missing angle of a triangle.
- To expand upon this, ask the students to draw a diagram for a peer to solve. They need to include angle measurements too.
- Then have them switch papers and solve each other's problems.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal

III. Special Needs/Modifications

- Review the following vocabulary prior to beginning the lesson.
- Congruent
- Perpendicular
- Complementary
- Supplementary
- Vertical angles
- Adjacent angles
- Linear pair

IV. Alternative Assessment

- Walk around the room as students work.
- Collect the problems/diagrams that they created for a peer to solve.
- Use these as a classwork grade or to assess student understanding.

Perpendicular Transversals

I. Section Objectives

- Identify the implications of perpendicular transversals on parallel lines.
- Identify the converse theorems involving perpendicular transversals and parallel lines.
- Understand and use the distance between parallel lines.

II. Multiple Intelligences

- Prior to teaching this lesson, provide students with another example. Request that they draw it out and then discover the 90° angles for themselves. Then move on to the text.
- Having the students engage right away and draw conclusions about the material will help to reaffirm the new information in their minds.
- Assist students in completing as many constructions as they can throughout this lesson. Drawing out the examples will expand student understanding.
- Have students practice one or two more examples of finding the distance between straight parallel lines before moving on to slanted parallel lines.
- When you begin working on slanted parallel lines, list these steps on the board. Request that the students copy these steps into their notebooks.

1. Choose two points on one of the lines.
2. Use those points to find the slope of the line.
3. Take the slope and find the slope of the segment perpendicular to the line- the opposite of the reciprocal.
4. Select a point on the line and use the slope to draw a segment perpendicular to the line.
5. Take the coordinates of where this segment intersects both lines.
6. Use the distance formula and these coordinates to find the distance between the parallel lines.

Writing out the steps in this fashion assists students with verbally seeing something, writing it down, and talking about it. Students will tend to remember the information better.

III. Special Needs/Modifications

- Review the following vocabulary.

- Perpendicular lines
- Parallel lines
- Transversals
- Corresponding angles
- Alternate interior angles
- Alternate exterior angles
- Consecutive interior angles
- Review converse and remind students that this is working backwards in a way.

Non- Euclidean Geometry

I. Section Objectives

- Understand non- Euclidean geometry concepts.
- Find taxicab distances.
- Identify and understand taxicab circles.
- Identify and understand taxicab midpoints.

II. Multiple Intelligences

- This is a great lesson to differentiate because taxicab geometry lends itself to creative problems.
- Go through the material in this lesson, and then divide students into groups. Assign each group either a taxicab distance or a taxicab circle.
- The students then work to write a problem to determine a taxicab distance or a taxicab circle.
- After writing the problem, the students need to draw out a diagram that shows their problem and the solution.
- Finally, the students act out the problem and the solution to the problem. They can use desks or different markers in the room to represent the area or “blocks” that they are working with.
- Be sure that students understand the directions and whether or not they will be graded on their work.
- If you are using a rubric for grading, share it with the students prior to their work.
- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, visual- spatial, interpersonal, in- trapersonal.

III. Special Needs/Modifications

- Review circles including radius
- Write a definition for taxicab geometry, taxicab circles, taxicab distance and taxicab midpoint on the board.

- Request that students copy the information down in their notebooks.

IV. Alternative Assessment

- This is a great opportunity to design a rubric for the activity.
- The rubric can be divided into three sections: the problem itself, the diagram and the skit.
- Students have many opportunities to excel in this assignment because of all of the different ways to participate.
- This could be used as a quiz grade or a classwork grade.

1.4 Congruent Triangles

Triangle Sums

I. Section Objectives

- Identify interior and exterior angles in a triangle.
- Understand and apply the Triangle Sum Theorem.
- Utilize the complementary relationship of acute angles in a right triangle.
- Identify the relationship of the exterior angles in a triangle.

II. Multiple Intelligences

- Begin this lesson by talking about interior and exterior angles of a triangle. Request that students draw an example in their notebooks of a triangle with interior and exterior angles.
- Request that they label the interior angles red and the exterior angles in blue.
- Move on to the Triangle Sum Theorem.
- Remind students that this is information that they already know, but that it has now been written as a theorem.
- One way to make this part of the lesson interactive is to draw a diagram of a triangle on the board and to play “fill in the blank” with the students. You can even have them work on teams and see which team can complete the addition the quickest.
- Another fun way to do this is to use only mental math. This is great for students to practice their thinking skills. The big thing to watch for is students calling out- remind them to raise a hand when done or to stand up.
- If you wish to keep this more traditional, then have students draw the triangles at their seats but use a protractor to measure angles and solve for missing angles.

III. Special Needs/Modifications

- Review the following terms.
- Interior angles
- Triangle
- Exterior angles
- Write new theorems on the board.
- Request that students copy this information down in their notebooks.
- Allow time for questions to check on student understanding.

IV. Alternative Assessment

- The best way to assess this lesson is through observation.
- If you are playing the game, observe which students are actively participating and which ones aren't.
- If completing seat work, walk around and check in with students as they work. Answer questions and offer assistance as needed.

Congruent Figures

I. Section Objectives

- Define congruence in triangles.
- Create accurate congruence statements.
- Understand that if two angles of a triangle are congruent to two angles of another triangle, the remaining angles will also be congruent.
- Explore properties of triangle congruence.

II. Multiple Intelligences

- A way to differentiate this lesson is to work with the determining the congruence of triangles in a hands- on way.
- Begin by teaching the material in the lesson, then move on to the activity.
- Students begin by drawing two congruent triangles. They should use letters to label the vertices of each triangle.
- Then they need to use a protractor to be certain that the two triangles are congruent.
- Once they have determined congruency, the student adds in the tic marks to show congruent sides.
- Finally, they exchange papers with a peer. Once they have exchanged papers, the students need to write three statements that demonstrate and explain that the two triangles are congruent.
- Check student work as part of a class discussion.

- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Review what the word congruent means.
- Review the properties of a triangle- meaning corresponding sides and corresponding angles being congruent.
- Explain the use of tic marks to show congruency. Be sure that students understand that they may see tic marks on other figures as well.
- Write the congruence properties on the board. Draw the similarities between these properties and the properties of equality.
- Allow time for any questions.

IV. Alternative Assessment

- Complete an assessment of student understanding by reviewing student diagrams and congruence statements during the class discussion.

Triangle Congruence Using SSS

I. Section Objectives

- Use the distance formula to analyze triangles on a coordinate grid.
- Understand and apply the SSS postulate of triangle congruence.

II. Multiple Intelligences

- After reviewing the first example, ask students to participate in this activity.
- Student one draws a triangle on the coordinate grid and then passes their paper to the right. All students are doing this simultaneously.
- The next student takes the triangle passed to them and uses the distance formula to figure out the lengths of each side of the triangle. Then he/she passes the paper to the right.
- The next student takes the measurements and draws a triangle congruent to the first triangle somewhere on the coordinate grid. Then he/she passes the paper to the right.
- The final student checks the work of all of the others.
- Discuss work when all have finished.
- This is great practice for the students and keeps them engaged because of the paper passing.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal

III. Special Needs/Modifications

- Review the definition for a postulate.
- Review distance formula and how to use it. Provide students with two examples.
- Practice drawing triangles on the coordinate grid.
- Review ordered pairs and how to use the ordered pairs with the distance formula.
- Write the SSS Triangle Congruence Postulate on the board/overhead. Request that students copy these notes down in their notebooks.

IV. Alternative Assessment

- Students can be assessed during the class discussion.
- Also, walk around during the paper passing exercise. You will be able to observe students as they work.
- Allow time for questions.
- Make a note of any students who are having difficulty during the lesson.

Triangle Congruence Using ASA and AAS

I. Section Objectives

- Understand and apply the ASA Congruence Postulate.
- Understand and apply the AAS Congruence Postulate.
- Understand and practice two- column proofs.
- Understand and practice flow proofs.

II. Multiple Intelligences

- When you introduce the ASA Congruence Postulate, review that a postulate is assumed true.
- Then go through the directions in the text, but have the students follow along with you and do the steps themselves in their seats.
- Once students have a good grasp on the ASA Congruence Postulate, then move on to the AAS Congruence Theorem. Be sure that the students understand that a theorem can be proved.
- Demonstrate the example.
- Ask the students what they can notice about the ASA Congruence Postulate and the AAS Congruence Theorem. Write their ideas on the board.
- You want the students to realize that they can be used equally. If the students aren't making this connection on their own, use an example from the text to guide them in discovering it. Having them discover it on their own is much more valuable than telling them the information.

- Move on to the two- column proofs. Go through the material. Request that the students participate in completing the proof.
- Flow proofs- go through the material.
- If time allows, have students write their own flow proofs and share them in small groups. You could also assign this as a homework assignment.

III. Special Needs/Modifications

- Write the new terms and vocabulary on the board/overhead.
- ASA Congruence Postulate
- AAS Congruence Theorem
- Review the difference between a postulate and a theorem.
- Review the Third Angle Theorem
- Review two- column proofs
- Allow time for questions.

IV. Alternative Assessment

- The best way to assess student learning in this lesson is through question and answer sessions.
- Be sure that you allow time for the students to participate in the lesson. Do not assume that they understand the material. Verify that they do through their responses.

Proof Using SAS and HL

I. Section Objectives

- Understand and apply the SAS Congruence Postulate.
- Identify the distinct characteristics and properties of right triangles.
- Understand and apply the HL Congruence Theorem.
- Understand that SSA does not necessarily prove triangles are congruent.

II. Multiple Intelligences

- Write these two points on the board/overhead. Write that students are going to know all of the theorems and postulates that can prove congruence, and that they are going to understand all of the combinations of sides and angles that do not prove congruence.
- Use the uncooked spaghetti throughout this lesson with the protractors.
- As each exercise is described in the text, walk the students through using the uncooked spaghetti and the protractors to test out each theorem.

- Then use the uncooked spaghetti to show how the AAA does not prove congruence but similarity.
- Demonstrate this by creating two different size triangles that have the same angle measurements. Then the students will see that although the angle measurements are the same, the triangles are not congruent.
- Teach the Pythagorean Theorem. Connect this theorem with the HL Congruence Theorem.
- Make this lesson as interactive as possible by using the protractors and the uncooked spaghetti to model each part of the lesson.
- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, visual- spatial, interpersonal.

III. Special Needs/Modifications

- Write all of the theorems on the board/overhead as they are taught. Request that the students copy these notes down in their notebooks.
- Review the different types of triangles: acute, obtuse, equilateral, right.
- Review the parts of a triangle- sides and hypotenuse
- Review Pythagorean Theorem
- Show students how to connect the Pythagorean Theorem to the HL Congruence Theorem.

IV. Alternate Assessment

- Observe students as they work through the lesson with the uncooked spaghetti.
- Allow time for student thinking and feedback.

ing Congruent Triangles

I. Section Objectives

- Apply various triangles congruence postulates and theorems.
- Know the ways in which you can prove parts of a triangle congruent.
- Find distances using congruent triangles.
- Use construction techniques to create congruent triangles.

II. Multiple Intelligences

- Divide the students into seven groups. Each group is assigned one of the theorems from the review.
- Then each group must create a diagram to show how each illustrates or does not illustrate congruence.
- Allow time for the students to work.
- When they have finished, allow time for each group to present their work to the class.

- When completing constructions, be sure that students have both a compass and a straightedge.
- Review the steps in the text on drawing a perpendicular bisector of the segment.
- Expand Example 4- Have the students practice drawing segments and practice drawing perpendicular bisectors of each segment.
- Students could work in pairs on this lesson.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- Complete the Congruence Theorem Review. Create a chart with all of this information and be sure that students copy it into their notebooks.
- Write out the notes for proving parts congruent.
- Be sure students understand that they need to use the distance formula, and the reflexive property of congruence, but that using a protractor does not necessarily mean that the triangles are congruent but similar.

IV. Alternative Assessment

- Create a rubric to grade students on their group presentations.
- Share the rubric with them prior to assigning the group work.
- Then use the rubric to give students a quiz or class work grade.

Isosceles and Equilateral Triangles

I. Section Objectives

- Prove and use the Base Angles Theorem.
- Prove that an equilateral triangle must also be equiangular.
- Use the converse of the Base Angles Theorem.
- Prove that an equiangular triangle must also be equilateral.

II. Multiple Intelligences

- In this lesson, the students are going to work with the Base Angles Theorem.
- They are going to need to prove the Base Angles Theorem with both isosceles and equilateral triangles.
- Prior to teaching the lesson, ask students to recall information about isosceles and equilateral triangles. Ask them to make a list of the characteristics of each in their notebooks.
- When finished, use a class discussion to generate a list of characteristics for both isosceles and equilateral triangles on the board.

- Then present the information in the text.
- As you teach about the Base Angles Theorem, point out which characteristics apply when working with this theorem.
- Do this for both the isosceles triangle and the equilateral triangle.
- Be sure that the students take notes on both triangle examples.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal.

III. Special Needs/Modifications

- Review isosceles triangles and their parts on the board.
- Present the material in words and in a diagram.
- Review converse statements. Be sure that students understand converse statements.
- Review equilateral triangles and their parts on the board.
- Present the material in words and in a diagram.
- Write out any points or conclusions that you make while discussing this lesson with the students.
- Write this information out on the board and request that the students copy these notes down in their notebooks.

IV. Alternative Assessment

- Alternative Assessment can be completed through observation and listening during the brainstorming session and during the discussion.

Congruence Transformations

I. Section Objectives

- Identify and verify congruence transformations.
- Identify coordinate notation for translations.
- Identify coordinate notation for reflections over the axes.
- Identify coordinate notation for rotations about the origin.

II. Multiple Intelligences

- To differentiate this lesson, begin by teaching all of the content in the lesson. The content will be necessary to do this activity.
- Begin the activity by giving students the coordinates of one triangle on the coordinate grid.
- Have students use colored pencils and a ruler during this lesson.

- Request that the students graph this triangle on the coordinate grid in one color.
- Then have students take this triangle and draw each of the following by using this triangle as a starting point.
- Students are going to have multiple pages of diagrams when they are finished.
- One page consists of a translation or slide.
- One page consists of a reflection or flip.
- One page consists of a rotation or turn.
- One page consists of a dilation.
- Stress the point that a dilation is the only image where the two images are not congruent. The two images are similar.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Write all new vocabulary words on the board.
- Request that students copy these words in their notebooks.
- Review the meaning of clockwise and counterclockwise.
- Review the difference between things that are congruent and things that are similar.

IV. Alternative Assessment

- Collect the packet of diagrams/drawings that the students have created during this lesson.
- This packet will allow you to assess student understanding of the information presented in this lesson.
- If letter grades are used, than a classwork or quiz grade can be given for this packet.

1.5 Relationships Within Triangles

Midsegments of a Triangle

I. Section Objectives

- Identify the midsegment of a triangle.
- Apply the Midsegment Theorem to solve problems involving side lengths and midsegments of triangles.
- Use the Midsegment Theorem to solve problems involving variable side lengths and midsegments of triangles.

II. Multiple Intelligences

- The best way to differentiate this lesson is to be sure to draw out each of the diagrams in this text on the board/overhead.
- Begin by reviewing the Parallel Postulate.
- Review with a few examples.
- Introduce students to the Midsegment Theorem through a diagram first.
- Draw the diagram on the board. Ask students to brainstorm some of the conclusions that they can draw about the diagram.
- List these conclusions on the board.
- Then take the conclusions that have been generated and use them to write the Midsegment Theorem.
- Include the two statements that are proven in the lesson.
- 1. Parallel to the third side
- 2. Half as long as the third side
- Show students how to prove these two statements.
- Use the examples in the text to do this, but be sure that the students are following through on the examples.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- Review symbols for parallel and congruent.
- The notes on how numbers 1 and 2 are proven are written out in paragraph form. Rewrite these notes in step form so that the students can follow it easier. This will assist students who have any problem with visually tracking information.
- Review solving multi- step equations.
- Show students how to take the diagram and write an equation from the information.
- Then review solving the equation.
- Show how to apply solving to equation to Example 2 on page 268.

IV. Alternative Assessment

- In this lesson, alternative assessment is done through observation.

Perpendicular Bisectors in Triangles

I. Section Objectives

- Construct the perpendicular bisector of a line segment.
- Apply the Perpendicular Bisector Theorem to identify the point of concurrency of the perpendicular bisectors of the sides (the circumcenter).
- Use the Perpendicular Bisector Theorem to solve problems involving the circumcenter of triangles.

II. Multiple Intelligences

- Begin by reviewing notes on the perpendicular bisector of a line segment.
- Show how the bisector divides the line segment into two congruent segments.
- Show how it intersects the line at a right angle.
- Ask students to draw two different line segments, measure them and draw in the perpendicular bisector.
- Either walk around and check student work or do a peer check. It is important to establish understanding before moving on to the next section.
- Go over the Perpendicular Bisector Theorem and its converse.
- Have students use a compass and colored pencils in the next activity.
 1. Students draw a triangle of their own design.
 2. Students draw in the perpendicular bisectors of each line segment.
 3. Students use a compass to draw in a circle that encompasses the triangle.
 4. Students label the circumcenter of the diagram.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic

III. Special Needs/Modifications

- Review the definitions of perpendicular and bisector.
- Write the Theorems on the board.
- Be sure that students copy those notes into their notebooks.
- Define circumcenter.
- Demonstrate how to draw in the segment bisectors and label the circumcenter.
- Review using a compass.

IV. Alternative Assessment

- Collect and examine student drawings/diagrams to assess student understanding.
- Be sure to allow time for student questions.
- If students are having a difficult time with the in class assignments, allow them the option of working with a peer.
- Be sure that peer work is on task through observation and walking around.

Angle Bisectors in Triangles

I. Section Objectives

- Construct the bisector of an angle.
- Apply the Angle Bisector Theorem to identify the point of concurrency of the perpendicular bisectors of the sides (the incenter).
- Use the Angle Bisector Theorem to solve problems involving the incenter of triangles.

II. Multiple Intelligences

- Write the intention of this lesson on the board/overhead.
- The intention is to inscribe circles in triangles.
- Go through the material in the lesson.
- After teaching the material in the lesson, give the students an opportunity to work with large triangles and inscribe circles in these triangles.
- This can be a lot of fun.
- Encourage students to use colored pencils and to make their diagram as colorful as they wish.
- Also use large chart paper.
- Allow students the option of working in a small group or by themselves.
- Then ask them to draw a diagram that shows the Concurrency of Angle Bisector Theorem.
- Be sure that they label each part of the diagram.

III. Special Needs/Modifications

- Review how to bisect an angle.
- Review that the bisector of an angle is the ray that divides the angle into two congruent angles.
- Remind students that with the Concurrency of Angle Bisectors Theorem, that we are going to show the point of intersection.
- Write these steps on the board.
 1. Draw in angle bisectors.
 2. Draw in perpendicular bisectors of each line segment.
 3. Show the point of intersection
 4. Use a compass to inscribe the circle inside the triangle.

IV. Alternative Assessment

- Allow time for the students to present their work to the class or in small groups.
- Walk around and listen to the students discuss and explain their work.
- Use this as a way to assess student understanding.

Medians in Triangles

I. Section Objectives

- Construct the medians of a triangle.
- Apply the Concurrency of Medians Theorem to identify the point of concurrency of the medians of the triangle (the centroid).
- Use the Concurrency of Medians Theorem to solve problems involving the centroid of triangles.

II. Multiple Intelligences

- Go through the initial material in this lesson first.
- Review the median of a triangle and how to find it.
- Show a diagram on the board that introduces the students to the Concurrency of Medians Theorem.
- Use the diagram to show each median being drawn in then show the point of intersection, the centroid.
- Introduce the vocabulary word as the material is covered.
- You may want to allow time for the students to try this with a triangle of their own creation.
- This will give them a good understanding of the concepts.
- Then complete the second part of the lesson.
- Pg. 300 -Complete the activity in Example 2 together as a class.
- Ask the students to write down the answers to the two questions.
- Then open up the discussion to a brainstorming session.
- Write student responses on the board.

III. Special Needs/Modifications

- Write all definitions on the board/overhead.
- Define median of a triangle.
- Define concurrent
- Define centroid and show the connection between concurrent and centroid.
- Review the midpoint formula.
- Review the distance formula.
- Review using the Geometer's Sketchpad.
- Define Napoleon's Theorem.

IV. Alternative Assessment

- Assessment is completed through observation and discussion.

Altitudes in Triangles

I. Section Objectives

- Construct the altitude of a triangle.
- Apply the Concurrency of Altitudes Theorem to identify the point of concurrency of the altitudes of the triangle (the orthocenter).
- Use the Concurrency of Altitudes Theorem to solve problems involving the orthocenter of triangles.

II. Multiple Intelligences

- Begin this lesson with a real life example about altitude. You could use a plane and review the meaning of the word altitude with the students. This will help them to draw associations as they work with the concept.
- Define altitude.
- Use these steps to find the altitude of a triangle.
 - 1. Identify the vertex you are using.
 - 2. Find the side of the triangle opposite the vertex or where this side should be located.
 - 3. Draw a straight line from the vertex to that opposite side, draw the side in if it does not exist in the original triangle.
- Show the two examples with the acute triangle and the obtuse triangle. Have students draw these two examples in their notebooks.
- Define the Concurrency of Triangles Altitude Theorem
- Define orthocenter
- Pg. 307 Example 1- do each step of the example with the students.
- “What do you observe?” Request that the students write their observations in their notebooks. Then allow some time for sharing and write these notes on the board.
- 8- What do you observe about the four points? Repeat the brainstorming activity to expand student thinking.
- Have students draw conjectures about whether or not the points would be collinear for all other kinds of triangles.
- Expand this as an extra credit assignment or homework assignment for students to draw different triangles and demonstrate whether or not the four points are collinear. Request that they include a writing piece in their assignment and describe their findings in words.
- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, visual- spatial, intrapersonal, interpersonal

III. Special Needs/Modifications

- Write all vocabulary words on the board and request that students copy these notes into their notebooks.

- Allow a lot of time for questions.
- Be sure that the students have copied down the steps for locating the altitude of a triangle in their notes.
- Students are most likely to be concerned when the altitude is drawn outside of the triangle. Use some extra time to show students why this is the case.

IV. Alternative Assessment

- Assess student learning through observation and through feedback during brainstorming sessions.
- Be alert to students who are not writing during the independent writing times.
- Be sure that all students are following the lesson. Review concepts or use peer tutoring if students are having challenges.

Inequalities in Triangles

I. Section Objectives

- Determine relationships among the angles and sides of a triangle.
- Apply the Triangle Inequality Theorem to solve problems.

II. Multiple Intelligences

- Teach the information in this lesson, and then expand it using a coordinate grid.
- The activity has the students draw triangles with different side lengths to prove the Triangle Inequality Theorem.
- Students use graph paper, colored pencils and rulers for this activity.
- Give students the dimensions for five different triangles on the board/overhead. For example, can we have a triangle with the lengths 6, 7, 12.
- Students need to draw out the triangle on the coordinate grid to demonstrate whether it can be a triangle or not.
- Then they also need to write an inequality demonstrating whether or not these side lengths work for a triangle.
- When finished, allow time for the students to prove their findings.
- Include the different theorems into their explanations when they occur. Point these out to the students.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- (Sides) Theorem- walk through the proof step by step. Provide students with a brief explanation of each step.

- Review briefly each previously learned term as it is introduced. Example- ruler postulate, angle addition, substitution
- (Angle) Theorem- on board
- Review what is meant by indirect reasoning- by assumption or conjecture
- Define corollary

IV. Alternative Assessment

- Collect student work on with the triangles on the coordinate grid.
- Use these diagrams and the inequality statements to assess student understanding.
- You can use this as a classwork grade.
- It is also useful to assess the students that are in need of assistance.

Inequalities in Two Triangles

I. Section Objectives

- Determine relationships among the angles and sides of two triangles.
- Apply the SAS and SSS Triangle Inequality Theorems to solve problems.
- Multiple Intelligences
- Expand this lesson by creating a model for students to use throughout the lesson.
- Before beginning, have students use three long strips of paper. Put fasteners to connect the strips of paper into a long strip.
- The fasteners will be moveable so that the students can manipulate the pieces into different shaped triangles. This is how they can demonstrate proving each of the theorems in the lesson.
- Have each student work with a partner since we will be working with two triangles.
- After going through the first example on SAS Inequality Theorem, have students work in pairs to test it out themselves.
- Offer time for feedback.
- Then go through the other examples in the lesson, after each example, have students work to test out the theorems with their model.
- Offer time for feedback.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Review inequalities in one triangle first.

- Review congruent triangles.
- Write all of the theorems on the board/overhead.
- Request that students copy these notes into their notebooks.
- Allow time for questions.

IV. Alternative Assessment

- Assess student understanding through observation of the partner work.
- Interact with students as they work through proving each theorem.
- Listen to student feedback and correct any unclear information.
- Expand this lesson into a writing assignment by having students write their observations and conclusions about the theorems in narrative form.
- If time allows, have students share their conclusions with the whole class or in small groups.

Indirect Proofs

I. Section Objectives

- Reason indirectly to develop proofs of statement.

II. Multiple Intelligences

- This is a short lesson but scaffold it into three sections. This will work for both multiple intelligences and for special needs students.
- Begin by defining an indirect proof.
- Define conjecture and what is meant by a conjecture.
- 1. Begin by writing if- then statements using real life examples.
- For example- “If Mary plays soccer then she is an athlete.”
- Request that the students write three if- then statements in their notebooks.
- Allow time for the students to share their work.
- 2. Algebraic Examples- use the one in the text to begin with.
- Then have students write three more algebraic examples.
- Exchange papers with a partner.
- Each partner must prove the if- then statement as true or false.
- Allow time for students to share their work.
- This helps students to make the connection between if- then statements and whether the statement is true or false.

- 3. Geometric Examples- use the example in the text.
- Then divide students into small groups.
- Request that they prove the following using the same diagram from the text.
- $\angle 2 = \angle 3$
- After students are finished writing the proof, allow time for sharing.
- Take the best parts of each written proof to compose a proof on the board.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Write all theorems on the board.
- Use the above activity to scaffold this lesson for the students.

IV. Alternative Assessment

- Prior to teaching the lesson, compose a list of essential elements for the proof that the students are going to write.
- When composing the group proof on the board, be sure that the final example has each of these elements in it.
- Request that students copy this proof into their notebooks.

1.6 Quadrilateral

Interior Angles

I. Section Objectives

- Identify the interior angles of convex polygons.
- Find the sums of interior angles in convex polygons.
- Identify the special properties of interior angles in convex quadrilaterals.

II. Multiple Intelligences

- Divide this lesson into two parts. The first part is going to focus on finding the sum of interior angles of polygons. The second part is going to focus on finding the sum of the interior angles of a quadrilateral.
- Define convex.
- Define polygon.
- Define interior angles.

- Note that the number of interior angles matches the number of sides of the polygon.
- Show how to use the Triangle Sum Theorem to divide the polygon into triangles.
- Demonstrate using an equation to solve for the sum of the measure of the interior angles.
- Have students do this with two new polygons in their notebooks- a pentagon and a decagon.
- Allow time for students to share their work when finished.
- Be sure student answers were found using the equation.
- Part 2 – move onto quadrilaterals
- Test out the equation with a rectangle and a trapezoid.
- Intelligences- logical- mathematical, visual- spatial, interpersonal, intrapersonal

III. Special Needs/Modifications

- Define all terms on the board. Request that students copy these notes into their notebooks.
- Here are the steps to solving these problems.
- 1. Divide the figure into triangles.
- 2. Use an equation to find the sum of the angles.
- 3. With a quadrilateral- check does the sum equal 360° ?

IV. Alternative Assessment

- Check student work. Be sure that the students are writing an equation when solving for the sum of the interior angles of both polygons and quadrilaterals.
- Provide correction when necessary.

Exterior Angles

I. Section Objectives

- Identify the exterior angles of convex polygons.
- Find the sums of exterior angles in convex polygons.

II. Multiple Intelligences

- Each of the examples in this lesson can be used as an extension to include multiple intelligences as you teach this lesson.
- I recommend not using the text with the students but teaching this lesson as an exploration.
- Have students work with rulers, protractors, paper and pencils at their desks in small groups.

- Have students work in pairs and/or groups of three.
- Then, present each section of the lesson.
- Have the students explore each example in their seats.
- For example, you draw the diagram on the board or overhead. Present a leading question, and then ask the students to work with the figure to solve the dilemma.
- When finished, allow time for class discussion.
- After going through each example in the text. Provide students with the notes for the lesson. They will have an experiential understanding of the information through working with each example.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Write all vocabulary and terms on the board/overhead.
- Define exterior angles.
 1. Formed by extending the side of a polygon.
 2. Two possible exterior angles for any given vertex.
- Define supplementary angles.
- Define vertical angles.
- Exterior Angle Sum

IV. Alternative Assessment

- Alternative Assessment is done through observation of student group work.
- Use an observation checklist to be sure that the groups are working to discover the big ideas of this lesson.

Classifying Quadrilaterals

I. Section Objectives

- Identify and classify a parallelogram.
- Identify and classify a rhombus.
- Identify and classify a rectangle.
- Identify and classify a square.
- Identify and classify a kite.
- Identify and classify a trapezoid.
- Identify and classify an isosceles trapezoid.

- Collect the classifications in a Venn diagram.
- Identify how to classify shapes on a coordinate grid.

II. Multiple Intelligences

- To differentiate this lesson, I recommend beginning by having students design a Venn diagram to classify the quadrilaterals. Do this BEFORE teaching the lesson.
- Begin by going through a brief explanation of a Venn diagram. Most students will be familiar with them.
- Then tell students that they are going to work with the text in small groups and design a Venn diagram to classify the following quadrilaterals.
- List the quadrilaterals on the board and provide students with chart paper and colored pencils.
- Students have been working with these figures for a long time. This exercise allows you an opportunity to walk around and assess student understanding about these figures.
- Allow students to devise their own classifications system to engage higher level thinking.
- When finished, ask students to explain how and why they chose to classify the figures the way that they did.
- Then move to the text.
- Go through the material in the text with the students having a deeper understanding of different quadrilaterals.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Define each type of quadrilateral on the board.
- Help students to create a chart of each with a drawing and a brief description.
- Define Opposite Sides of Parallelogram Theorem
- Define Opposite Angles of Parallelogram Theorem
- Review distance formula
- Review slope

IV. Alternative Assessment

- Use the Venn diagram exercise to assess student learning and understanding.
- Look at how the students have chosen to classify the different figures- does this make sense? What adjustments are needed?
- Did the students classify according to sides and angles?
- Provide feedback to expand student understanding.

Using Parallelograms

I. Section Objectives

- Describe the relationships between opposite sides in a parallelogram.
- Describe the relationship between opposite angles in a parallelogram.
- Describe the relationship between consecutive angles in a parallelogram.
- Describe the relationship between the two diagonals in a parallelogram.

II. Multiple Intelligences

- Begin by handing out pieces of string to each student. Some of the strings can be the same length and some can be different lengths.
- Have the students use these strings to work through the examples at their seats.
- This is a hands- on way to demonstrate the power of congruent side lengths.
- Then have the students design a quadrilateral on the coordinate grid. They can each decide whether it is a parallelogram or not.
- Exchange papers with a peer. Each peer needs to use the distance formula to test out whether the figure that they have been given is a parallelogram or not.
- Allow time for sharing.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, in- trapersonal.

III. Special Needs/Modifications

- Review the meaning of congruent.
- List out the following description of a parallelogram. Request that the students copy this information down in their notebooks.
- Parallelogram
 1. Quadrilateral with 2 pairs of parallel sides.
 2. Opposite sides are congruent.
 3. Opposite angles are congruent.
 4. Consecutive angles are supplementary.
 5. Diagonals bisect each other.
- Walk through each step of filling in the proofs.
- Provide a review of each “Reason” as it is presented.

IV. Alternative Assessment

- Listen to the student sharing and assess whether students understand what makes a parallelogram a parallelogram.
- Allow time for questions.

Proving Quadrilaterals are Parallelograms

I. Section Objectives

- Prove a quadrilateral is a parallelogram given congruent opposite sides.
- Prove a quadrilateral is a parallelogram given congruent opposite angles.
- Prove a quadrilateral is a parallelogram given that the diagonals bisect each other.
- Prove a quadrilateral is a parallelogram if one pair of sides is both congruent and parallel.

II. Multiple Intelligences

- To differentiate this lesson, begin by going through the material in the lesson and stop when you get to the diagram in Example 4.
- Divide students into five groups.
- Each group is going to use this diagram to PROVE that the figure is a parallelogram.
- Each group is assigned a characteristic of a parallelogram to prove. Explain that they will also need to prove the converse of each statement.
- Group 1- quadrilateral with two pairs of parallel sides.
- Group 2- opposite sides are congruent
- Group 3- opposite angles are congruent
- Group 4- consecutive angles are supplementary
- Group 5- Diagonals bisect each other.
- When finished, have students present their work.
- As a class decide if the group was successful in proving their statement.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- List characteristics of a parallelogram on the board.
- 1. Quadrilateral with two pairs of parallel sides.
- 2. Opposite sides are congruent.
- 3. Opposite angles are congruent.
- 4. Consecutive angles are supplementary.
- 5. Diagonals bisect each other.
- Be sure that students understand how to find each characteristic in a diagram.
- Walk through each proof.

- Explain each “Reason” as it is covered. This will require students to review previously learned information.

IV. Alternative Assessment

- Students will provide the assessment in this lesson when they decide whether each group has successfully proven their statement.

Rhombi, Rectangles, and Squares

I. Section Objectives

- Identify the relationship between the diagonals in a rectangle.
- Identify the relationship between the diagonals in a rhombus.
- Identify the relationship between the diagonals and opposite angles in a rhombus.
- Identify and explain biconditional statements.

II. Multiple Intelligences

- Break down the information in this lesson to provide students with the following notes on rectangles and rhombi.
- Rectangle
 1. Demonstrate diagonals are congruent using the distance formula
- Provide students with an example on the overhead that they can they figure out on grid paper using the distance formula. You can even divide the class in half. Ask one half of the class to work on one diagram and the other half of the class to work on another diagram.
- Rhombi
 1. Diagonals are perpendicular bisectors of each other.
 2. Diagonals bisect the interior angles.
- Define a biconditional statement as a conditional statement that also has a true converse. “if and only if”
- In pairs, have students write a biconditional statement for a rectangle and a biconditional statement for a rhombus.
- Allow time for students to share their statements.
- The class decides whether it is true biconditional statement or not.
- If not, provide counterexamples.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- Walk through each proof in the lesson.
- Provide a brief explanation of each “Reason” as it is presented. Do not assume students remember the definitions of each.
- Review the following:
 - Distance formula
 - Definition of Perpendicular bisector
 - Review finding the slope of a line.
 - Review conditional statements.

IV. Alternative Assessment

- Create a checklist of what would be acceptable correct biconditional statements. Use this checklist to assist students in evaluating the biconditional statements written by each pair.

Trapezoids

I. Section Objectives

- Understand and prove that the base angles of isosceles trapezoids are congruent.
- Understand and prove that if base angles in a trapezoid are congruent, it is an isosceles trapezoid.
- Understand and prove that the diagonals in an isosceles trapezoid are congruent.
- Understand and prove that if the diagonals in a trapezoid are congruent, the trapezoid is isosceles.
- Identify the median of a trapezoid and use its properties.

II. Multiple Intelligences

- Break down the information in this lesson into sections to assist student understanding.
- Define a trapezoid.
 1. One pair of parallel sides
 2. NOT parallelograms
- Define an Isosceles trapezoid.
 1. One pair of non- parallel sides that are the same length.
 2. Base angles are congruent.
 3. Diagonals are congruent.
- Activity- have students draw two trapezoids and two isosceles trapezoids.
- With the isosceles trapezoids request that they do the following.

- 1. Label angles, sides and diagonals to show that it is an isosceles trapezoid.
- 2. Write a statement and its converse for each label to explain it.
- Notes of Trapezoid Medians
 1. Connects the medians of the non- parallel sides in a trapezoid.
 2. Located half-way between the bases in a trapezoid.
- Theorem— $\frac{\text{sum of base lengths}}{2}$
- Use this equation with the example in the text. Request that students practice this as well.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Write all notes on the board/overhead. Request that students copy this information in their notebooks.
- Define symmetry.
- Use an example so that students understand symmetry in connection with trapezoids.

IV. Alternative Assessment

- Collect student diagrams.
- Assess understanding based on labels and statements.

Kites

I. Section Objectives

- Identify the relationship between diagonals in kites.
- Identify the relationship between opposite angles in kites.

II. Multiple Intelligences

- Write out the following notes on kites.
 1. No parallel sides.
 2. Two pairs of congruent sides adjacent to each other.
 3. Two vertex angles
 4. Two non- vertex angles that are congruent
 5. Diagonals are perpendicular
- Have students design a kite on a coordinate grid.

- They must prove it is a kite by providing proof that the diagonals are perpendicular, and that the non-vertex angles are congruent.
- They can do this through statements or by writing a proof with reasons.
- Finally, once their diagram has been approved, they can use it to design and decorate their own kite. Students can use chart paper and colored pencils to do this.
- Hang all work up in the class.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Define adjacent.
- Define vertex angles.
- Define non- vertex angles.
- Walk through each proof.
- Explain each “Reason” in the proof.

IV. Alternative Assessment

- Approve all students kite diagrams before students design their final kite.
- Be sure that the points have been successfully proven.

1.7 Similarity

Ratios and Proportions

I. Section Objectives

- Write and simplify ratios.
- Formulate proportions.
- Use ratios and proportions in problem solving.

II. Multiple Intelligences

- Differentiate this lesson by providing opportunities to expand each example for the students. This will provide students with a way to practice the concepts as the information is presented.
- Ratio is a fraction that compares two things.
- Three ways to write a ratio. As a fraction, with a colon or using the word to.

- Expand Example 1: What is the ratio of everything bagels to sesame bagels?
- Answer: $\frac{50}{25} = \frac{2}{1}$
- Expand the equation example with the dancers and the singers.
- “What if the ratio of dancers to singers was 5 to 4 and there were forty- five dancers? How many singers are there? How many dancers are there?”
- Answer: $5n = \text{dancers}$
- $4n = \text{singers}$
- $5n + 4n = 45$
- $9n = 45$
- $n = 5$
- $5(5) = 25$ dancers
- $4(5) = 20$ singers
- Proportion- an equation that compares two equal ratios.
- Expand Barn Dimensions example.
- “What if the water line was actually 20 ft instead of ten? What would the length be on a scale drawing?”
- Answer $x = 5$ inches
- Intelligences- linguistic, logical- mathematical, visual- spatial

III. Special Needs/Modifications

- Write each definition and its examples on the board. Request students write down the information in their notebook.
- Explain the Cross Multiplication Theorem as something that the students already know from previous classes about how to solve a proportion. This is a formal way of writing it.

IV. Alternative Assessment

- Throughout the expansion of each exercise, allow students to contribute their answers to class discussion.

Properties of Proportions

I. Section Objectives

- Prove theorems about proportions.
- Recognize true proportions.
- Use proportions theorem in problem solving.

II. Multiple Intelligences

- This is a brief lesson used to explain the corollary theorems associated with the Cross Multiplication Theorem.
- To make this lesson more interesting and interactive, use group work.
- Divide the students into five groups.
- After teaching the lesson and going through the material, assign each group one of the corollary theorems.
- Then put an example of two similar triangles (with measurements) on the board/overhead.
- Ask each group to use the example on the board to create an example that illustrates their corollary theorem.
- Allow time for the students to work and then have each group present their example to the class.
- When finished, encourage the other students to ask questions to see how well the students in the group can answer them.
- Answering questions is a great way to assess student understanding.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Write these notes on the board/overhead. Request that students copy them down.
- Cross Multiplication Theorem- defining property of proportions.
- Subtheorems are called corollary theorems.
- Review which terms in a proportion are the means and the extremes.
- Corollary 1- swap means.
- Corollary 2- swap extremes.
- Corollary 3- flip it upside down.
- Corollary 4 - $\frac{a+b}{b} = \frac{c+d}{d}$
- Corollary 5 - $\frac{a-b}{b} = \frac{c-d}{d}$

IV. Alternative Assessment

- Assessment can be done through student questions and answers following the exercise.

Similar Polygons

I. Section Objectives

- Recognize similar polygons.
- Identify corresponding angles and sides of similar polygons from a statement of similarity.
- Calculate and apply scale factors.

II. Multiple Intelligences

- Activity to differentiate this lesson.
- Ask students to draw a polygon and label the lengths of the sides of their polygon.
- Then ask the students to exchange polygons with someone else. Students may exchange more than once just be sure that everyone has a different polygon than the one that they started with.
- Students need to complete the following with this new polygon.
 1. Draw a similar polygon to the one that you have been given.
 2. Write proportions to demonstrate that the side lengths are similar.
 3. Determine the scale factor.
 4. Determine the ratio of the perimeters.
- When finished, divide into small groups to share their findings.
- Use peers to correct any errors in the work of each individual.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal

III. Special Needs/Modifications

- Define similar.
- Similar in the context of polygons.
 1. Same number of sides
 2. For each angle there is a corresponding angle in the other polygon that is congruent.
 3. Lengths of all corresponding sides are proportional.
- Write all assignment directions on the board so that students can refer back to what is needed for each step.
- Use flexible grouping to assist students in understanding the activity.
- Ratios of similar perimeters- same as scale factor- be sure that students understand these two concepts.

IV. Alternative Assessment

- Walk around and listen in on group discussions.
- Interject important information, offer feedback or constructive criticism when needed.

Similarity by AA

I. Section Objectives

- Determine whether triangles are similar.
- Understand AAA and AA rules for similar triangles.
- Solve problems about similar triangles.

II. Multiple Intelligences

- The technology integration is a nice way to differentiate this lesson by adding the interactive element of technology.
- In addition, after covering the material in the lesson, the shadow problems with the similar triangles are a fun way to help the students to gain a deeper understanding of the concepts in the lesson.
- You could have the students write their own problems and solve each other's using diagrams and drawings.
- You could also take the students outside, use a tree or a flagpole, the height of a student, and their shadow to measure and actually create a real life problem.
- This can be a fun way to bring the outdoors, nature and real life into the math classroom.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal

III. Special Needs/Modifications

- Walk students through the directions on how to use the technology.
- AAA Rule- if angles of a triangle are congruent to the corresponding angles of another triangle, then the triangles are similar.
- SAME AS: The AA Triangle Similarity Postulate except that it uses two angles and not three.
- Both are true and both work.
- Indirect measurement- provide students with a visual example of the two similar triangles and the proportions.

IV. Alternative Assessment

- Alternative Assessment in this lesson is done through observation and through interacting with the students during the activity.

Similarity by SSS and SAS

I. Section Objectives

- Use SSS and SAS to determine whether triangles are similar.
- Apply SSS and SAS to solve problems about similar triangles.

II. Multiple Intelligences

- There are two great ways to differentiate this lesson included in the text. One is the technology integration and one is the hands- on activity.
- Here is how we can take the hands- on activity and expand it a bit to include an interactive part.
- Complete the hands- on activity as it has been written in the text.
- When finished, ask students to write a few observations that they have made into their notebooks.
- Then ask students to contribute their observations to a class discussion.
- Write all of the student observations on the board. Point out the SSS for Similar Triangles.
- SSS for Similar Triangles- if the lengths of the sides of the two triangles are proportional, the triangles are similar.
- SAS for Similar Triangles- use Cheryl's examples to create your own example to illustrate the SAS for Similar Triangles.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- Write the intention of the lesson on the board.
- Intention is to explore relationships between proportional side lengths and congruent angles of similar triangles.
- Review the directions for using the technology.
- Write the directions to the hands- on activity on the board.

IV. Alternative Assessment

- Observe student work through the technology integration activity.
- Observe student work through the hands- on activity.

Proportionality Relationships

I. Section Objectives

- Identify proportional segments when two sides of a triangle are cut by a segment parallel to the third side.
- Divide a segment into any given number of congruent parts.

II. Multiple Intelligences

- Complete the technology integration as one way to differentiate this lesson.
- After working through the technology integration ask students to write their observations down in their notebooks.
- Conduct a sharing session and write student observations on the board.
- Midsegment of a Triangle- midsegment that divides the sides of a triangle proportionately.
- Activity- have the student use construction paper to design a triangle.
- Then using a ruler and a pencil, draw in the midsegment of the triangle.
- Ask students to make notes about the measurements of the triangle and how they have been altered with the midsegment.
- Then, use scissors to actually divide the triangle into two proportionate sections.
- Request that students write their observations down in their notebooks.
- The Lined Notebook Paper Corollary- demonstrate this as a whole class. Ask students to use notebook paper and a ruler.
- Write student observations on the board.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal.

III. Special Needs/Modifications

- Review ways to figure out if two triangles are similar using side lengths and angles.
- Define midsegment of a triangle on the board.
- Write out the Triangle Proportionality Theorem.
- Go through each step of each proof and explain each “Reason” as it is presented.
- Do not assume that the students remember previously learned information.

IV. Alternative Assessment

- Create an observation checklist to use during both activities.
- For the technology piece, make a list of things that you would like students to gain from the activity.
- For the hands- on piece, make a list of things that you would like students to gain.
- Notice during the discussion sessions whether or not these goals have been met.
- If not, make these points to the students and explain how they were discovered.

Similarity Transformations

I. Section Objectives

- Draw a dilation of a given figure.
- Plot the image of a point when given the center of dilation and scale factor.
- Recognize the significance of the scale factor of a dilation.

II. Multiple Intelligences

- Teach the information in this lesson.
- Then use the information on dilations to complete the following activity.
- Divide the students into groups and have them measure the classroom.
- Each group is working on the same assignment. Having each group work on the same assignment will produce different diagrams to explain the same information.
- Students are going to complete a drawing that shows the measurement of the classroom. They can create a scale to represent the measurement of the room.
- Next, students create a dilation of the classroom where the scale factor is $\frac{1}{2}$.
- Students need to draw a diagram of this and label it with the correct measurements.
- Then, students create a dilation of the classroom with a scale factor of $\frac{1}{4}$.
- Next, students create a dilation of the classroom with a scale factor of 3.
- Then have students complete the area and the perimeter of the room.
- Find a perimeter with a scale factor of $\frac{1}{3}$.
- Find an area with a scale factor of $\frac{1}{3}$.
- Allow time for the students to share their work and any observations.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Review congruence transformations that preserve length- translations, rotations, and reflections.
- Define a dilation.
- Define scale factor.
- Review finding the perimeter of a figure.
- Review finding the area of a figure.

IV. Alternative Assessment

- Walk around and observe students as they work.
- Collect diagrams and use for a classwork or a quiz grade.

Self- Similarity (Fractals)

I. Section Objectives

- Appreciate the concept of self- similarity.
- Extend the pattern in a self- similar figure.

II. Multiple Intelligences

- Differentiate this lesson by completing a construction of each of the fractals. Use rulers and measure to ensure accuracy.
- Here are the steps.
 - 1. Begin with a segment.
 - 2. Divide the segment into three congruent parts.
 - 3. Remove the middle part leaving two parts.
 - 4. Divide each segment into three congruent parts.
 - 5. Remove the middle part of each segment.
- Allow students to work in groups.
- You can extend this lesson by having students work with both horizontal lines and vertical lines.
- With the pattern for the Sierpinski Triangle, have students see which other polygons can be used to create a similar pattern.
- Allow students to work in pairs or groups on this assignment.
- Display student work.

III. Special Needs/Modifications

- Write the steps for each fractal on the board/overhead.
- Pattern for Sierpinski Triangle.
 - 1. Draw a triangle.
 - 2. Connect the midpoints of the sides of the triangle. Shade in the center triangle.
 - 3. Repeat this process with each triangle.

1.8 Right Triangle Trigonometry

The Pythagorean Theorem

I. Section Objectives

- Identify and employ the Pythagorean Theorem when working with right triangles.
- Identify common Pythagorean triples.
- Use the Pythagorean Theorem to find the area of isosceles triangles.
- Use the Pythagorean Theorem to derive the distance formula on a coordinate grid.

II. Multiple Intelligences

- This lesson uses the Pythagorean Theorem in several different ways.
- You can differentiate this lesson by expanding each of the examples in the lesson.
- Begin by drawing and labeling the parts of right triangle. Be sure that students understand which are the legs and the hypotenuse.
- Prove the Pythagorean Theorem
 1. Start with a right triangle.
 2. Construct the altitude
 3. Use it in an example.
- Start with an example where the hypotenuse is missing.
- Ask students to use the Pythagorean Theorem to find the length of the hypotenuse.
- Example: leg 1 = 4, leg 2 = 6
- Answer— $c = 7.2$
- Be sure that students understand that they will probably need to round to the nearest tenth.
- Move to finding a missing side length.
- Example, leg 1 = a , leg 2 = 4, hypotenuse = 5
- Have students solve this for leg a .
- Answer is 3.
- Introduce the concept of a Pythagorean Theorem. Show the difference between the first example where we did not have a perfect square and needed to round, and the second example where our answer was a perfect square.
- Return to the text and demonstrate the other Pythagorean Triples.
- Move on to finding the area of an isosceles triangle.
- Walk through this example in the text.

- Complete the exercise on the board step by step.
- Then allow time for student questions.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- Review constructing an altitude.
- Review symbol for similar.
- Review finding square roots/radicals.
- Review the concept of a perfect square.

IV. Alternative Assessment

- Observe students as they work.
- Check in periodically throughout the lesson to be sure that students understand the material.
- Review any information that is not clear.

Converse of the Pythagorean Theorem

I. Section Objectives

- Understand the converse of the Pythagorean Theorem.
- Identify acute triangles from side measures.
- Identify obtuse triangles from side measures.
- Classify triangles in a number of different ways.

II. Multiple Intelligences

- Begin by teaching all of the information in this lesson.
- You will need to prepare this activity by creating triangles of different sizes to put around the room. Be sure that there are some acute, obtuse and right triangles.
- Then, let students know that they are going to go on a triangle hunt. They are going to search around the room and locate different triangles.
- Each student needs to find a triangle and test it out to figure out if the triangle is an acute, obtuse or right triangle.
- They need to be prepared to justify their answer.
- The students should repeat this process with three different triangles.
- When finished, have the students gather in small groups to share their findings.

- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Define the Converse of the Pythagorean Theorem.
- Review that a converse statement switches the if and the then part of a conditional statement.
- Write out the formula for finding out whether a triangle is right triangle, an acute triangle or an obtuse triangle.

IV. Alternative Assessment

- Listen in on the group discussions.
- Be sure to ask questions and probe into student thinking.
- Also check each student's work on the triangles.
- This will help you to assess the accuracy of the student work.
- You may want to collect the work for a classwork grade.

Using Similar Right Triangles

I. Section Objectives

- Identify similar triangles inscribed in a larger triangle.
- Evaluate the geometric mean of various objects.
- Identify the length of an altitude using the geometric mean of a separated hypotenuse.
- Identify the length of a leg using the geometric mean of a separated hypotenuse.

II. Multiple Intelligences

- One way to differentiate this lesson is to have the students teach the concepts in the lesson.
- You can do this by dividing up the content as follows.
 1. Group 1- teaches a review of arithmetic mean.
 2. Group 2- teaches geometric mean
 3. Group 3- teaches finding the length of an altitude
 4. Group 4- teaches finding the length of a leg
- If you have a large class, you can assign one group the same topic for a different perspective.
- If you choose to do this activity, DO NOT teach the content first.

- Assign students the text and let them decipher it.
- This will also give you an opportunity to observe student understanding.
- Allow time for group work and request that students use diagrams in their presentations.
- When finished, each group “teaches” their concept to the others.
- Allow time for feedback, questions and clarification.
- Intelligences- logical- mathematical, linguistic, visual- spatial, interpersonal, intrapersonal, bodily-kinesthetic

III. Special Needs/Modifications

- Inscribed- remind the students of the circles
- Define altitude.
- Review definition for similar objects.
- Review finding the arithmetic mean.
- Provide students with these notes to help clarify the material.
- 1. To find the length of the altitude- take the length of the segments of the divided hypotenuse and find the geometric mean.
- 2. To find the length of the leg- multiple line segment of divided hypotenuse times the length of the hypotenuse and take the square root of the product.

IV. Alternative Assessment

- Provide feedback during presentations.
- Assess student learning during group work and presentations.

Special Right Triangles

I. Section Objectives

- Identify and use the ratios involved with right isosceles triangles.
- Identify and use the ratios involved with 30-60-90 triangles.
- Identify and use ratios involved with equilateral triangles.
- Employ right triangle ratios when solving real- world problems.

II. Multiple Intelligences

- Begin this lesson with an exploration about what happens when you divide up different shapes. Do this before teaching the content of the lesson.

- Start by having the students draw an equilateral triangle.
- Pose the question “What happens when you divide an equilateral triangle in half?”
- Have students actually cut their triangles in half using scissors.
- Then brainstorm answers to the questions.
- Then begin a new exploration.
- Have students draw a square.
- Pose the question, “What happens when you cut a square in half along the diagonal?”
- Have students cut their squares along the diagonal using scissors.
- Then brainstorm answers to the question.
- You should be able to create two columns of this information on the board.
- Label one side $45 - 45 - 90$ and the other side $30 - 60 - 90$
- Tell students that these are the concepts that you are going to be working with in the lesson.
- As you teach the lesson, keep referring back to the information that the students have already discovered during the exploration at the beginning of the class.
- Then move on to the content in the text.
- Intelligences- linguistic, logical-mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- List this description of the right isosceles triangle on the board/overhead.
- Two sides the same length
- Congruent base angles of 45° .
- One right angle
- Review the Pythagorean Theorem.

IV. Alternative Assessment

- Have students work through the problems in their notebooks as they are covered in the text.
- Then allow time for questions and answers.

Tangent Ratios

I. Section Objectives

- Identify the different parts of right triangles.
- Identify and use the tangent ratio in a right triangle.
- Identify complementary angles in right triangles.
- Understand tangent ratios in special right triangles.

II. Multiple Intelligences

- To differentiate this lesson, keep it active by including students in designing triangles to determine the tangent ratio.
- Begin by covering the material in the lesson.
- When finished, ask students to work with a partner and design three different right triangles.
- Ask students to measure and label the side lengths of each triangle, and label each angle.
- Then have the students exchange papers.
- The students each find the tangent ratios for each angle in each of the three triangles.
- Once this is completed, ask them to compare their answers with the chart which gives the angle measure for different special triangles.
- Have the students note if any of the triangles drawn fall into the category of these special triangles.
- When finished, ask the students to check each other's work.
- Allow time for whole class feedback.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- Write on board "Trigonometric ratios show the relationship between the sides of a triangle and the angles inside it."
- Define Tangent Ratio- of an angle is

$$\text{Tan } x = \frac{\text{Length of opposite side}}{\text{Length of adjacent side}}$$

- The x refers to the angle we are focused on.

IV. Alternative Assessment

- Collect student work and use the triangles to assess student understanding.
- Listen to student comments following the activity.
- Allow time for student questions.

Sine and Cosine Ratios

I. Section Objectives

- Review the different parts of right triangles.
- Identify and use the sine ratio in a right triangle.
- Identify and use the cosine ratio in a right triangle.
- Understand sine and cosine ratios in special right triangles.

II. Multiple Intelligences

- To differentiate this lesson, teach the material in the lesson and then complete a “working backwards” activity with the students.
- In the last lesson, we worked with a right triangle and wrote out the tangent ratios for the angles in the triangle.
- In this lesson, we are going to start with the cosine and sine for the different angles of a right triangle. Then the students need to take these ratios and design a triangle that matches the ratios.
- In this way, the activity is called “working backwards.”
- The angles of the triangle are A , B and C .
- $\sin A = \frac{6}{7}$
- $\sin B = \frac{5}{7}$
- $\cos A = \frac{5}{7}$
- $\cos B = \frac{6}{7}$
- Allow time for the students to work with these ratios and draw a right triangle that matches the ratios.
- When finished, allow time for student sharing.
- Intelligences- logical- mathematical, linguistic, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Review the parts of a triangle.
- Remind students that the x in the cosine and sine ratios refers to the angle that we are focusing on.
- Break down the two formulas and write them on the board. Request that the students copy them down in their notebooks.
- Allow extra time for questions.

IV. Alternative Assessment

- Collect student work after the activity.
- Use this to assess student understanding and provide extra support for students who are having difficulty.

Inverse Trigonometric Ratios

I. Section Objectives

- Identify and use the arctangent ratio in a right triangle.
- Identify and use the arcsine ratio in a right triangle.
- Identify and use the arccosine ratio in a right triangle.
- Understand the general trends of trigonometric ratios.

II. Multiple Intelligences

- The best way to differentiate this lesson is to break down the information in the lesson. This will help all students.
- Here are some notes to give the students as you teach the information in the lesson.
- Inverse of a trigonometric function has the word arc in front of it.
- Inverse Tangents
- Convert measurement to degrees in two ways.
 1. Use a table of trigonometric ratios.
 2. Use a calculator with “arctan”, “atan” or “tan⁻¹”
- This will give you the measure of the angle in degrees.
- Notice that we use the approximately symbol for measurements that are not exact.
- Point this out for students in the lesson examples.
- Inverse Sine
- You can find the arcsine by the same two methods as the arctangent.
- This converts the measurement to degrees.
- Inverse Cosine
- You can find the arccosine the same two ways.
- This will convert the measurement to degrees.

III. Special Needs/Modifications

- Begin with some work on inverses.
- Be sure that students understand an inverse of an operation undoes the operation.
- Use a one- step equation to show students this.
- Then use the notes in the Multiple Intelligences section to break down the content for the students.

IV. Alternative Assessment

- Observe students through this lesson.
- Allow plenty of time for the students to ask questions.
- Repeat examples or information that seems unclear.

Acute and Obtuse Triangles

I. Section Objectives

- Identify and use the Law of Sines.
- Identify and use the Law of Cosines.

II. Multiple Intelligences

- There is a lot of information in this chapter. I recommend breaking it down and going through the examples slowly so that students are given a visual aid, an auditory aid and a chance to verbally ask questions.
- Intention of lesson- to apply the sine and cosine ratios to angles in acute and obtuse triangles.
- Law of Sines- is constant. It can be used to find the missing lengths in triangles.
- Review using a calculator to find the value of sines.
- Law of Cosines- works on acute, obtuse and right triangles.
- If the students seem lost during this lesson, break them up into small groups. Assign each group either the Law of Sines or the Law of Cosines and have them create a poster explaining the steps to using these laws in an example.
- The students can even use one of the examples in the text.
- By having the students create a poster to explain the information, the students will learn to assimilate the information themselves.
- Allow time for each group to explain their poster when finished.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal

III. Special Needs/Modifications

- Begin with a review of sines and cosines.
- Allow time for student questions.
- Review acute triangles have all angles that are less than 90.
- Review that obtuse triangles have one angle that is greater than 90.
- Review using a calculator to find the value of sines.

IV. Alternative Assessment

- Walk around while the students are working on their posters.
- Listen to the conversation in the groups.
- Are the students on task? Are they having difficulty?
- Often if the conversation has strayed from the content of the assignment, the students are lost and not sure what to do next.
- Assess student understanding through posters and presentations. Clarify points that have been missed or are incorrect.

1.9 Circles

About Circles

I. Section Objectives

- Distinguish between radius, diameter, chord, tangent, and secant of a circle.
- Find relationships between congruent and similar circles.
- Examine inscribed and circumscribed polygons.
- Write the equation of a circle.

II. Multiple Intelligences

- To differentiate this lesson, break the lesson down into two sections. In the first section, cover all of the basic information about circles.
- Have the students work on creating a diagram of a circle. Their diagram must have the following things labeled: radius, chord, diameter, secant, tangent line.
- Encourage students to make their diagrams colorful.
- Allow time for sharing when students are finished.
- The second part of the lesson involves more of the operations associated with circles.
- For this lesson, be sure that students have graph paper to work with.
- Complete the examples on the board and walk through each of the examples and all of the steps needed to complete each one.
- Point out where to find the radius and the ordered pair in the equation.
- Make this section interactive so that you work through the example on the board/overhead while the students work through it in their notebooks.
- Working through this as a whole class will help the students to follow the steps of each problem.
- Intelligences- linguistic, visual- spatial, logical- mathematical, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Include the following notes for students.
- Two circles are congruent if they have the same radius. Two circles are similar if they have different radii. Their similarity is shown through a ratio.
- When writing similarity ratios be sure to simplify.
- Remember that you can write the ratios in three ways. The text uses a colon, but you can use a fraction or the word “to”.
- Define chord.
- Define diameter.
- Define secant.
- Tangent line- touches a circle at one point. This is called the point of tangency.
- Inscribed polygon- convex polygon inside circle.
- Circumscribed polygon- convex polygon around circle.
- Review convex polygons.
- Equations with graphing- work through slowly.
- Concentric circles- practice drawing them.

IV. Alternative Assessment

- Assess this lesson through student drawings.
- If the students can draw and label the parts of the circle, then they have an understanding of it.
- If the students can graph the circles, then they have an understanding of the process.

Tangent Lines

I. Section Objectives

- Find the relationship between a radius and a tangent to a circle.
- Find the relationship between two tangents draw from the same point.
- Circumscribe a circle.
- Find equations of concentric circles.

II. Multiple Intelligences

- Review tangents of a circle and the point of tangency.
- Tangent to a Circle Theorem- work with this theorem by first having the students complete an exploration.

- Have them begin by drawing a circle, the radius and a tangent line.
- Then ask what they can observe about the relationship between the radius and the tangent line. You want them to discover the theorem on their own and then you can put a name to it for them.
- When working with the Pythagorean Theorem and finding the hypotenuse of the triangle, ask students “How can we find the length of the hypotenuse?”
- Once again, you want the students to come up with the Pythagorean Theorem on their own.
- Show Pythagorean Theorem in Example 1.
- When working with the Tangent Segments from a Common External Point Theorem- begin with an exploration.
- Have the students draw it out and then record student observations on the board.
- You want them to discover the theorem on their own.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal

III. Special Needs/Modifications

- There is a lot of skills/vocabulary to review in this lesson. Do not assume that the students remember.
- Review radius.
- Define circumscribe.
- Define concentric circles.
- Review what is meant by a contradiction.
- Example 1- rather than beginning by proving this theorem through a contradiction, use the exploration first. Then go through and show how to use a contradiction. Beginning with the contradiction can confuse many students.
- Review the Pythagorean Theorem.
- Review converse statements.
- Example 5- show students where the hypotenuse is located.
- Review how to rationalize a denominator.
- Review HL congruence.

IV. Alternative Assessment

- Review student thinking by observing their work during the observations.
- Are the students able to come to conclusions about each theorem before it is actually taught?
- Is there higher level thinking involved?
- Can the students verbalize the steps to working through an example?

Common Tangents and Tangent Circles

I. Section Objectives

- Solve problems involving common internal tangents of circles.
- Solve problems involving common external tangents of circles.
- Solve problems involving externally tangent circles.
- Solve problems involving internally tangent circles.
- Common tangent

II. Multiple Intelligences

III. Special Needs/Modifications

- This lesson focuses on problem solving.
- These notes will work for multiple intelligences and special needs students.
- The work that we are going to do to differentiate this lesson is to be sure that the steps to working through the problems are clear and understood.
- List out these steps as you teach the lesson. Request that the students copy these notes into their notebooks.
- Define common internal tangent.
- Define common external tangent.
- Steps to common External Tangents
 - 1. Label the diagram.
 - 2. Draw a line segment that joins the centers of the two circles.
 - 3. Draw in the perpendicular segments.
 - 4. Look for any polygons – Example = rectangle.
- Steps to Common Internal Tangents
 - 1. Look for similarity. Here in Example 2, there are similar triangles.
 - “Who can we find the length of x ? Think back to our work on similar triangles.”
- Lead students to discover working with ratios.
 - 2. Then we can find the length of the hypotenuse of the two triangles to identify the distance between the two circles.
- Use the Pythagorean Theorem to do this.
- Intersecting circles- define internally and externally tangent
- Walk through each example with the students. Rely on previously learned material from this lesson.

IV. Alternative Assessment

- Be sure that the students are making the connections between the circles, perpendicular segments and where polygons and right triangles can help them in solving each problem.
- The students need to combine previously learned material to be successful with this lesson.
- Review information and skills when necessary.

Arc Measures

I. Section Objectives

- Measure central angles and arcs of circles.
- Find relationships between adjacent arcs.
- Find relationships between arcs and chords.

II. Multiple Intelligences

- Define Central angle
- Define arc
- In each part of this lesson, work with each diagram and then have the students brainstorm different observations about each diagram. Then give them the information from each diagram.
- For example, begin with the diagram that demonstrates the Arc Addition Postulate. It is common sense that the two arcs would add up to be the total. Give students the name of the postulate and ask them to come up with the meaning of the postulate given the diagram. Then walk them through it.
- Do the same with the diagram on the congruent chords.
- In Example 1- show that all angles must add to be 360° .
- Use the straight line to show the 180° , then talk to the students about working through the puzzle of figuring out the measure of each angle inside the circle.
- Keep this lesson interactive and engage students in participating in the discussion.
- Refrain from simply presenting the material to them.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- Define semicircle with a diagram.
- Define major arc with a diagram- named with three letters.
- Define minor arc with a diagram- named with two letters (endpoints).
- Review solving multi- step equations.

- Present diagrams first and then engage the students in a discussion about the diagram.

IV. Alternative Assessment

- Create a checklist of important points that you want students to discover in the lesson.
- Then listen for these points while you discuss the information and present the material to the students.
- During the practice exercises, pair students up to work together. This will help students to clarify the given information.

Chords

I. Section Objectives

- Find the lengths of chords in a circle.
- Find the measure of arcs in a circle.

II. Multiple Intelligences

- To differentiate this lesson, have the students work on an activity in small groups.
- The students are going to create a diagram to teach a theorem to the other students in the class.
- Divide the students into groups of four. Each group is assigned a different theorem.
- Group 1- The perpendicular bisector of a chord is the diameter.
- Group 2- the perpendicular bisector of a chord bisects the arc intercepted by the chord.
- Group 3- Congruent chords in the same circle are equidistant from the center of a circle.
- Group 4- Two chords equidistant from the center of a circle are congruent.
- Allow time for the students to work and then have each group teach the class about their theorem.
- Allow time for students to ask questions.
- From this activity, move to the longer examples in the text. The students should have an easier time working through these examples now that the theorems are very clear.
- Intelligences- logical- mathematical, linguistic, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal

III. Special Needs/Modifications

- Begin by defining a chord as a line segment whose endpoints are both on a circle.
- Show students a diagram to define a chord.
- Write out each theorem on the board. Request that students write the notes in their notebooks.
- Review the terms diameter, bisector, perpendicular, congruent.

- Point of the significance of the word “equidistant” in two of the theorems.

IV. Alternative Assessment

- Use flexible grouping to engage all learners.
- Walk around and observe students as they work on preparing their lesson.
- Be sure that each presentation accurately teaches the content of the lesson.
- Provide correction and feedback when necessary.

Inscribed Angles

I. Section Objectives

- Find the measure of inscribed angles and the arcs they intercept.

II. Multiple Intelligences

- Begin by teaching the material in the first part of this lesson. Stop before you get to the practical examples where students are actually figuring out angle measures.
- To expand student understanding, make the corollary section interactive.
- Have students work in pairs to draw out an example of each corollary.
- Tell students that you will be collecting the examples at the end of the class.
- Then move on to the actual examples in the lesson.

III. Special Needs/Modifications

- Provide students with the following notes.
- Inscribed angles- vertex on circles, sides are chords, intercepts an arc of the circle.
- Review parts of an angle.
- Review definition of a chord.
- Measure of inscribed angle is $\frac{1}{2}$ of the measure of the arc it intercepts.
- Measure of center angle is 2 (measure of inscribed angle)
- List out the inscribed angle corollaries.
- When working on the multi-step examples, use color to help students to differentiate between which angles are being worked with and which ones aren't being worked with. The color will help students to focus on the appropriate part of the diagram.

IV. Alternative Assessment

- Walk around and help students as they work.
- Collect student work from the corollaries activity.
- Examine each example and see if it clearly demonstrates or shows the corollary.
- Provide students with feedback/correction in the next class.

Angles of Chords, Secants and Tangents

I. Section Objectives

- Find the measures of angles formed by chords, secants and tangents.

II. Multiple Intelligences

- In this lesson, you are going to be working with three main theorems. The students need to learn these theorems and then prove each of the theorems.
- Here are some notes to help students to break down each theorem.
- Theorem- the measure of an angle formed by a chord and a tangent that intersects on the circle equals half of the measure of the intercepted arc.
- $\text{mangle} = \frac{1}{2} \text{marc}$
- Look at the first diagram and label the chord, tangent, intercepted arc and possible angles to measure.
- Theorem- angles inside a circle
- $\text{mangle} = \frac{1}{2}(\text{marc1} + \text{marc2})$
- Theorem- angles outside circle
- $\text{mangle} = \frac{1}{2}(\text{arc}^\circ 1 + \text{arc}^\circ 2)$
- Once the students understand the three theorems, go through the example and proofs in the lesson. Ask the students to point out where the theorems are illustrated and explained in each.
- Discuss each example and proof.
- Allow time for questions.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- Review chords.
- Review secants.
- Review tangents.
- Review interior angles.
- Review exterior angles.

- Review what an intercepted arc is and how to locate it.

IV. Alternative Assessment

- As you work through each example, have the students work through the example in their seats.
- When all have finished, ask the students to explain how they solved the problem.
- Then provide feedback.
- You can verbally check in with the students by having them raise their hands if they had the same answer. This will give you a visual cue of how many students were successful and how many were not.

Segments of Chords, Secants and Tangents

I. Section Objectives

- Find the lengths of segments associated with circles.

II. Multiple Intelligences

- Begin this lesson by having the students draw a circle. Then they need to use previously learned information and their text to draw in the following.
- Tangent
- Chord
- Secant
- Tangent segment
- Chord segment
- Secant segment
- Have them use color to draw in each item.
- When students are finished, explain that we are going to be using these diagrams to illustrate three different theorems.
- When working through each theorem, give students the measurements for each section of the circle, and then have them work to figure things out.
- Theorem- If two chords intersect, the product of segments of chord1 = product of segments of chord2.
- Then we create two similar triangles.
- Similar triangles- ratios
- Add in measures and solve for the missing segment length.
- Theorem- If two secants are drawn to a common point, $a(a + b) = c(c + d)$
- The a 's = 1st secant

- The c 's = 2nd secant
- Draw in two triangles inside the circle.
- Similar triangles- ratios
- Use formula to find the length of the segment of the secant.
- Theorem- tangent and secant $-a(a + b) = c^2$
- The a 's = secant
- The c = the tangent
- Use it to find the value of the missing tangent length.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, bodily- kinesthetic, intrapersonal.

III. Special Needs/Modifications

- Review chords.
- Review tangents.
- Review secants.
- Review squaring a number.
- Review the distributive property.

IV. Alternative Assessment

- Assess student learning through observation and class discussion.

1.10 Perimeter and Area

Triangles and Parallelograms

I. Section Objectives

- Understand the basic concepts of the meaning of area.
- Use formulas to find the area of specific types of polygons.

II. Multiple Intelligences

- Teach the material in this lesson, and then move on to the activity.
- To complete this activity, you will need to prepare enough drawings for one- half of the students in the class. One- half of the class receives a drawing of a complex figure.
- You will need to prepare the area measurements for this complex figure for the other half of the class.

- Then hand out one to each student. Some students will receive drawings and some will receive measurements.
- The students will need to figure out the measure of their figure and find the person in the room who has the correct area measurement for their figure.
- The activity is complete when both persons are sure that they have been matched up correctly.
- This is a noisy activity, but the students will have a lot of fun doing it. It also has a lot of movement in it which is excellent for kinesthetic learners.
- Then repeat this activity. Be sure that those who received drawings get area measurements and those who had measurements receive drawings.
- Intelligence- linguistic, logical- mathematical, bodily- kinesthetic, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Provide students with notes to refer to throughout the lesson and activity.
- Area of rectangle = $l \times w$
- Area of parallelogram = bh
- Area of triangle = $\frac{1}{2}bh$
- Write out the Congruent Area Postulate
- Write out the Area of Whole is Sum of Parts Postulate.
- Be sure that students copy these notes down in their notebooks.

IV. Alternative Assessment

- Student assessment is done through the activity. Were the students able to find the correct “match-up?”
- Assist students who have difficulty with the assignment.

Trapezoids, Rhombi and Kites

I. Section Objectives

- Understand the relationships between the areas of two categories of quadrilaterals: basic quadrilaterals and special quadrilaterals.
- Derive area formulas for trapezoids, rhombi and kites.
- Apply the area formula for these special quadrilaterals.

II. Multiple Intelligences

- Be sure to teach the material in this lesson in an interactive way. I would recommend going through the material without the text first. That way each student can explore the different characteristics of the three figures without using the information in the text as a guide.
- Then go back to the text and go over the information in it. The students will see this in a new way because they will have already “discovered” it.
- Here are some notes on each figure in the section. This will help the students to “break down” the content.
- Trapezoid- quadrilateral with one pair of parallel sides.
- Formula- $\frac{1}{2}(b_1 + b_2)h$
- Finding the Area of a Rhombus
 1. Frame a rhombus in a rectangle.
 2. Notice all of the triangles.
 3. 4 triangles to fill in the rhombus
 4. 8 triangles fill in the rectangle.
 5. 4 is half of 8
 6. Area of rhombus = $\frac{1}{2}$ area of a rectangle.
- Formula = $A = \frac{1}{2}d_1d_2$
- Finding the Area of a Kite
 1. Frame in a rectangle.
 2. Notice the similarities with the rhombus.
 3. Use the same formula as a rhombus.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal.

III. Special Needs/Modifications

- Review finding the area of a rectangle.
- Review finding the area of a parallelogram.
- Review finding the area of a triangle.
- Show how more than one figure can be combined together to create a new figure.
- Example- a rectangle and a triangle together.
- Review finding the area of such a figure.

IV. Alternative Assessment

- Observe students as they work through the explorations.
- Allow a lot of time for students to speculate and share their findings.

Area of Similar Polygons

I. Section Objectives

- Understand the relationship between the scale factor of similar polygons and their areas.
- Apply scale factors to solve problems about areas of similar polygons.
- Use scale models or scale drawings.

II. Multiple Intelligences

- There are several different components to this lesson.
- First, we can start with the basic information to write the formula for finding the area of similar polygons. In working through this section, use an example on the board and take the students through each step in the text as you do the work out on the board. This will help them to “see” where the formula really comes from.
- The next section is on scale drawings and scale models.
- One of the best ways for the students to understand scale drawings is to complete one.
- You could break the students off into pairs and have them create a scale drawing of the classroom. Allow students to use chart paper, rulers, tape measures, colored pencils and to create their own scale for the diagram.
- Have students work in pairs to complete the table. You could also expand this activity and add Mt. Everest to the table.
- The section on the giant can be fun. Ask the students to create a drawing to show how there aren't any twelve foot giants. They can use the information in the text as a guide.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, in-trapersonal.

III. Special Needs/Modifications

- Review perimeter.
- Review area of different figures.
- Review Pythagorean Theorem.
- Review finding the area of a rhombus.
- Define squaring a number.

IV. Alternative Assessment

- Use observation and student work product to assess student understanding.
- You can collect work for a class work grade when students are finished.

Circumference and Arc Length

I. Section Objectives

- Understand the basic idea of a limit.
- Calculate the circumference of a circle.
- Calculate the length of an arc of a circle.

II. Multiple Intelligences

- When working through this lesson, it is a good idea to begin by reviewing previously learned information about a circle.
- Have students brainstorm a list and then write them on the board.
- These include the labels for radius, diameter, center angle, arc, interior, etc.
- Also review that there are 360° in a circle.
- Then move on to the measurement for pi and having students understand the measurement for pi.
- Use the exploration in the text for this.
- The activity to differentiate this lesson comes in the example where the circle is inscribed inside the square on the graph paper.
- The students can count the units to figure out that the length of the side of the square is also the diameter of the square.
- Here is the activity.
- 1. Have students draw their own circle inscribed in a square.
- 2. Exchange papers with a partner.
- 3. Each student must label the length of the diameter.
- 4. Find the circumference of the circle.
- Allow time for sharing when students have finished.
- Walk through the section on how to find the arc measures. Be sure that the students understand the $\frac{60}{360}$ ratio and how it makes sense to multiply the diameter with the measure of the arc to find the arc measure.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Write formulas for finding the diameter and the radius of a circle on the board.
- You can even do a few examples to have students practice finding these measures.
- Write the formula for circumference on the board.

- Allow time for student questions.

IV. Alternative Assessment

- Observe students as they work on the circle dilemma.
- Offer assistance when needed.
- Listen for how the students solved the dilemmas when the students are sharing their work after the activity.

Circles and Sectors

I. Section Objectives

- Calculate the area of a circle.
- Calculate the area of a sector.
- Expand understanding of the limit concept.

II. Multiple Intelligences

- Teach the material in this lesson and then differentiate it with the following activity.
- Activity- have students work in pairs.
- Students begin by drawing a square on graph paper and then inscribing a circle within the square. The students can decide how much of the square is taken up by the circle.
- Have students shade in the area of the square around the circle.
- Exchange papers.
- Students work with each other's papers.
- They need to find the area of each circle.
- Then they need to find the area of the shaded region of each circle.
- Request that students write out the steps that they did to complete this assignment.
- Allow time for students to share their work at the end of the activity.
- When working with the sectors, be sure that the students understand what is meant by a sector.
- Use a diagram to show students a sector in a circle.
- Then show how it has an arc measure and how it also has a measure of the area of the circle.
- This will help students to make sense of the formula to find the area of the sector.
- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Review finding the circumference of the circle.
- Review pi.
- Review the concept of the limit and how it leads to pi.
- Write out the formulas on the board. Request students write these notes in their notebooks.

IV. Alternative Assessment

- Collect the student worksheet from the activity.
- Read all of the steps that the students wrote and check their process.
- Is there anything missing? Do the students understand where each part of the formula comes from?
- Is there higher level thinking here or are students just “using” the formula?

Regular Polygons

I. Section Objectives

- Recognize and use the terms involved in developing formulas for regular polygons.
- Calculate the area and perimeter of a regular polygon.
- Relate area and perimeter formulas for regular polygons to the limit process in prior lessons.

II. Multiple Intelligences

- When working through this lesson, be sure to explain each formula and how it was arrived at slowly and with detail.
- I recommend beginning the lesson without the text.
- Use the text as a teacher guide and break down the information in it for the students.
- Use the board/overhead to show each step.
- Begin by labeling the regular polygon and its parts in different colors.
- You can use these colors to track through to the formulas.
- For example, if you used red for the n in the diagram, then whenever the n is presented in a formula, you can put it in red.
- Color will help the students to track the information from the diagram to the examples and back again.
- Allow plenty of time for student questions and repeat material as necessary.
- Intelligences- linguistic, logical- mathematical, visual- spatial

III. Special Needs/Modifications

- Go to the simplest version of each of these formulas for the students to make sense of this unit.
- Because there is so much processing in this lesson, special needs students will have difficulty following all of the different possible options.
- Simplify it as much as possible.
- You want the students to understand the core concepts involved.
- Using color, as suggested above, will help special needs students.

IV. Alternative Assessment

- Ask questions and answer a lot of questions in this lesson.
- Since most of this lesson is about student process, be sure that the students are following the lesson.

Geometric Probability

I. Section Objectives

- Identify favorable outcomes and total outcomes.
- Express geometric situations in probability terms.
- Interpret probabilities in terms of lengths and areas.

II. Multiple Intelligences

- Review the basics of probability.
- Review the ratio for probability.
- Activity 1- Basic Probability- have the students use two number cubes and figure out what the probability would be to roll an even number.
- Students can work in pairs during this activity.
- Allow time for them to share their work when finished.
- Are there any surprising results?
- How did the students arrive at their answers?
- Activity 2- Geometric Probability
- Students work in pairs again.
- The students work together to design their own problem for determining geometric probability.
- Have the students write their problems out, use diagrams and not solve the problems.
- You can use these problems at a later date.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Provide the students with some notes on probability.
- Review how to convert fractions, decimals and percentages.

IV. Alternative Assessment

- Collect student problems.
- Check them for accuracy.
- Reassign them to students for a homework or classwork assignment.
- They could also be used for an extra credit problem.
- Have students explain their answers and write them out in words not just show a solution.

1.11 Surface Area and Volume

The Polyhedron

I. Section Objectives

- Identify polyhedral.
- Understand the properties of polyhedral.
- Use Euler's formula to solve problems.
- Identify regular (Platonic) polyhedral.

II. Multiple Intelligences

- Teach the material in this lesson, and then differentiate it by having the students work in groups to test out Euler's formula.
- Provide the students with actual solids or with diagrams of different polyhedral.
- The students need to use the solids or diagrams to label each polyhedra and to come up with a way to demonstrate how Euler's formula works.
- Allow students time to choose one solid that they are going to use for their "teaching" session.
- When students are finished preparing, allow time for them to teach the other students in the class how Euler's formula works.
- They need to be clear on the number of faces, edges and vertices of their solid.
- Allow time for the students to answer questions.
- Provide feedback to the students.

- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal

III. Special Needs/Modifications

- Provide students with notes on polyhedrons.
- Polyhedrons
- 3D
- Made of polygons and only polygons- faces
- Polygons join at the edges.
- Edges meet in points called vertices.
- No gaps between them.
- Review the definition of a polygon.
- Write out Euler's Formula.

IV. Alternative Assessment

- Pay close attention to the teaching session.
- Do the students prove Euler's formula or simply state reasons.
- Ask questions and stretch the students to really demonstrate how Euler's formula works and makes sense.

Representing Solids

I. Section Objectives

- Identify isometric, orthographic, cross- sectional views of solids.
- Draw isometric, orthographic, cross- sectional views of solids.
- Identify, draw and construct nets for solids.

II. Multiple Intelligences

- Make this lesson very interactive by giving students the following hands- on tasks.
- Students may work in small groups for this activity.
- In the activity, be sure that students have graph paper, dot paper, plain paper, rulers, tape and colored pencils.
- Students are going to choose a solid to work with. You can provide students with a model of a solid if you have them.

- Then students are going to create four different things.
- 1. Create an orthographic projection of their solid.
- 2. Create a cross- section of the solid.
- 3. Create a net for the solid.
- 4. Use the net to create an actual model of the solid.
- Intelligences- linguistic, visual- spatial, bodily- kinesthetic, logical- mathematical, interpersonal, in- trapersonal.

III. Special Needs/Modifications

- Review polyhedral.
- Review faces, edges and bases.
- Define isometric.
- Define perspective.
- Define orthographic projection.
- Define cross- section.
- When students draw an orthographic projection, they will need the following views:
 - 1. Top
 - 2. Left side
 - 3. Back
 - 4. Right side
 - 5. Front
 - 6. Bottom

IV. Alternative Assessment

- An assessment of student understanding can easily be completed by looking at the student work.
- Were the students able to create a net that built a solid?
- Did they use the same solid for all of the pieces of the activity?
- Where did students have challenges?

Prisms

I. Section Objectives

- Use nets to represent prisms.
- Find the surface area of a prism.
- Find the volume of a prism.

II. Multiple Intelligences

- When differentiating this lesson, you want to divide it into two sections. The first section is going to be on surface area and the second section is going to be on volume.
- The first thing to do is to present the information on surface area.
- When you teach surface area, have the students create a net of a prism (on graph paper) to work with.
- Once the students have created their nets, then use the formulas for surface area to show students how to calculate the surface area of the prism that they created.
- When finished, allow time for the students to complete their work.
- Then move on to volume.
- Use the same net to calculate the volume of the solid.
- Have the students see where the formula for finding the volume of a solid comes from.
- Building the lesson in this way connects the last few lessons together. We have connected surface area and volume with polyhedra.
- Allow time for students to share their work in the large class or in small groups.
- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, interpersonal, intrapersonal

III. Special Needs/Modifications

- Write out all new terms and information on the board. Be sure that students copy this information in their notebooks.
- Define prism. What makes a prism a prism?
- Show students the difference between a right prism and an oblique prism.
- Define Area Congruence Postulate
- Define Area Addition Postulate
- Define Surface Area.
- Review formulas for area of a triangle, parallelogram, rectangle and square.
- Show the difference between Lateral area and surface area.
- Define Volume.

- Define Volume Congruence Postulate.
- Define Volume Addition Postulate.
- Show how these two postulates are similar to the ones on surface area.
- Be sure that students know that the capital B in the volume formula means the area of the base not the length of the base.

IV. Alternative Assessment

- Listen to student responses during class discussions.
- More students will have a chance to share their work in small groups.
- If you use small groups, walk around and listen in on each group.
- Make notes of any students who are having difficulties.

Cylinders

I. Section Objectives

- Find the surface area of cylinders.
- Find the volume of cylinders.
- Find the volume of composite three- dimensional figures.

II. Multiple Intelligences

- To differentiate this lesson, you can make it very hands- on by using some different cylinders. Example- Quaker Oats containers, soda cans, etc.
- Teach the material in the lesson.
- Then have the students work to find the surface area and volume of each cylinder.
- The students will need string to determine circumference, rulers, colored pencils and paper.
- Have students draw a net for their cylinder and label all of the measurements involved.
- Then they complete their work.
- Provide time for students to share their work in small groups.

III. Special Needs/Modifications

- Review finding the area and circumference of a circle.
- Define cylinders.
- Show students the difference between right cylinders and oblique cylinders.

- Write out the formulas for surface area and volume of cylinders.
- Steps to Working with Composite Solids
 1. Break each composite solid into its smaller solid parts.
 2. Select the correct formula for either surface area or volume based on the problem.
 3. Find the surface area or volume of each smaller solid.
 4. Add/subtract the results of the surface area or volume based on the question.

IV. Alternative Assessment

- Observe students as they work in small groups.
- Have the students share their work from the cylinder activity when finished.
- Check student work for accuracy.
- Provide assistance and feedback when necessary.

Pyramids

I. Section Objectives

- Identify pyramids.
- Find the surface area of a pyramid using a net or formula.
- Find the volume of a pyramid.

II. Multiple Intelligences

- Differentiate this lesson by having students complete the following activities.
- Students are going to be working with a pyramid of their choosing.
- Begin by dividing students into groups. While the students will be working with different pyramids, they will have the support of the other students in the group as they work.
- Each student is to choose a type of pyramid: triangular, square, pentagonal, hexagonal, etc.
- It is fine if two students in the group choose the same pyramid.
- Next, the students need to draw a net for their pyramid. Have them include measurements of each part of the pyramid.
- Then, students need to find the surface area of the pyramid.
- The lateral area of the pyramid
- The volume of the pyramid
- Have students show all of their work.

- When finished, they need to check the work of one other student in their group.
- Finally, collect student work to assess levels of understanding.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modification

- Define regular pyramid.
- Identify types of pyramids according to each base.
- Define slant height.
- Write the formulas for surface area, lateral area and volume on the board.
- Be sure students understand how to find each measurement.

IV. Alternative Assessment

- Collect and review student work.
- Is the net correctly drawn and labeled?
- Is the surface area of each net correct?
- Is the lateral area of each correct?
- Is the volume correct?
- Have there been any corrections to the work of the students by a peer?

Cones

I. Section Objectives

- Find the surface area of a cone using a net or formula.
- Find the volume of a cone.

II. Multiple Intelligences

- Begin by having the students work to build cones.
- Students begin by drawing a half circle and measure the top of it.
- This top measurement becomes the circumference of the cone.
- Then they cut out the half circle.
- Have the students turn it into a cone.
- Then use the descriptions of the parts of the cone in the text to help the students to understand the parts of the cone.

- After this teach the material in the lesson and then move on to the next activity.
- Next, divide the students into groups.
- In each group, ask the students to choose either the surface area formula or the volume formula.
- With each formula, the students need to find a way to teach how the formula is put together for the other students in the class.
- The students will demonstrate the meaning of each part of the formula and present an example of how to use the formula.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Define cone as a single curved base that tapers to a single point. This point is called the **apex**.
- Base can be a circle or an oval.
- Right cone- apex in the center
- Oblique cone- apex not in the center
- Write the formulas for surface area and volume on the board.

IV. Alternative Assessment

- Create a checklist of the things that the students should be teaching in their lessons.
- Use these checklists during the presentations.
- Make notes of the things that the students cover.
- Be sure to provide feedback for the things that the students miss in their presentations.

Spheres

I. Section Objectives

- Find the surface area of a sphere.
- Find the volume of a sphere.

II. Multiple Intelligences

- Begin by showing students some spheres of different sizes. You can use a baseball, a globe and a basketball for example.
- Often the measurement of a ball is given according to the diameter. Have the diameter of any object that you show the students close by. Use one- for example, a fourteen inch basketball to demonstrate the following.

- Parts of a circle
- 1. O = center point
- 2. r = radius ($\frac{1}{2}$ of diameter)
- 3. d = diameter
- 4. Chord- intersects the center of the circle or sphere.
- 5. Secant- line, ray or line segment that intersects in two places and extends OUTSIDE the sphere
- 6. Tangent- intersects the sphere at only one point.
- SA of a Sphere- use the formula and then use the examples to have students work to find the SA of one or more of the given objects.
- For example, find the SA of the 14" basketball.
- Volume of a sphere- use the formula and then use the examples to have students work to find the volume of each given object.
- Allow time for students to share their work.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Review circles.
- Compare circles to spheres.
- Show how the parts of a circle relate to the parts of a sphere.
- Review the meaning of surface area.
- Review the definition of volume.

IV. Alternative Assessment

- Assess student understanding of the material through the discussion and through student answers when working with the given objects.

Similar Solids

I. Section Objectives

- Find the volumes of solids with bases of equal areas.

II. Multiple Intelligences

- To differentiate this lesson, begin by teaching the content in the lesson.

- Ask the student's to draw an example of Cavalieri's Principle (Volume of a solid postulate)
- Have students share their example with a peer and then allow time for student sharing.
- The class participation will give you time to see if the students understand the principle.
- Then move on to working with similar solids. The students are going to draw a pair of similar solids and then work to problem solve with the similar solids.
- Tell students to draw a solid that is similar to a rectangular prism with a depth of 4, a width of 6, and a height of 9.
- Students should begin by drawing this given rectangular prism and then draw one similar to it.
- Once this is similar, ask them to write ratios to demonstrate that the prisms are similar.
- Next, have the students find the surface area of each prism and demonstrate that they are similar through the Similar Solids Postulate.
- Finally, ask students to find the volume of each prism and demonstrate that they are similar through the Similar Solids Postulate.
- Allow time for the students to share their work.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Review surface area.
- Review volume.
- Review scale factor.
- Write Cavalieri's Principle on the board. Rename it as the Volume of a Solid Postulate.
- Review similar solids and writing equal ratios.

IV. Alternative Assessment

- Walk around as students work and assess their understanding through observation.
- You can collect student work to use as a classwork grade.
- Offer assistance to students who are in need of help.
- Use flexible grouping to assist these students.

1.12 Transformations

Translations

I. Section Objectives

- Graph a translation in a coordinate plane.
- Recognize that a translation is an isometry.
- Use vectors to represent a translation.

II. Multiple Intelligences

- To differentiate this lesson, make it very interactive.
- Be sure that students have graph paper, colored pencils and rulers at their seats. Work through this lesson on the overhead projector with graph paper yourself so that the students can model the examples and work them out themselves at their seats.
- Begin by reviewing some information about translation.
- Have students draw a translation. Use the example from the text or create one of your own.
- Use the distance formula as was done in the text to review finding the coordinates to graph. Go the extra step and graph an example with the students.
- Define isometry- explain how the distance between the two points of an image is the same as the distance between the two images.
- When you look at the example that the students have just drawn, illustrate this.
- Then name it with the Translation Isometry Theorem.
- Move on to vectors. Begin by having students graph two line segments on a coordinate grid. You can use the same line segments as in Example 2, or you can create your own.
- The key is that you want the students to understand that the vector is the horizontal and vertical direction connected with each graphed line segment.
- Drawing in vectors is a great opportunity for students to use color to differentiate the vectors.
- Then ask students to name the horizontal component and the vertical component of the graphed line segments.
- Expand Example 3. Read through it with the students and then give them time to graph the two triangles and to explore what happens to the triangles.
- Allow time for student sharing.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- Review translations and what makes a translation.
- Review using the distance formula with a translation.
- Define isometry.
- Define the Translation Isometry Theorem.
- Define Vector.

IV. Alternative Assessment

- Assess student understanding through discussion.
- Allow time for student feedback.
- Assess understanding again with the work done to expand Example 3. How did the students do with this? Do they understand vectors?
- What conclusions did they draw from the example?

Matrices

I. Section Objectives

- Use the language of matrices.
- Add matrices.
- Apply matrices to translations.

II. Multiple Intelligences

- Begin by teaching the material in the lesson. Then move on to the activity.
- Students will need graph paper, rulers and colored pencils.
- Students can work with a partner for this activity.
- Ask students to draw a triangle, a square or a rectangle on the coordinate grid.
- Then have them create a matrix of the coordinates of their polygon.
- Next, students are going to create a translation of the polygon that is two units down and three units to the right.
- *Note if this doesn't work with the student's image change it to two units up and three units to the left.*
- Then have the students design a matrix to represent this translation.
- Finally students will add the two matrices together.
- Ask them to exchange papers with a peer for a check of their work.
- After their peer review, make any necessary changes.
- Allow time for student's to share their work.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal.

III. Special Needs/Modifications

- Notes on Matrices
- 1. A multidimensional way to show data.

- 2. They have their own arithmetic.
- 3. In brackets, a matrix is an array of numbers.
- 4. Numbers are arranged in rows and columns.
- You add the elements of a matrix by adding the value in each place in one matrix with the matching value in the same place in the other matrix.
- Matrices can represent real- life data.
- Matrices can represent the vertices of a polygon.
- Operation with matrices and translations = ADDITION

IV. Alternative Assessment

- Collect student work and use it as a way to check student understanding.

Reflections

I. Section Objectives

- Find the reflection of a point in a line on a coordinate plane.
- Multiple matrices.
- Apply matrix multiplication to reflections.
- Verify that a reflection is an isometry.

II. Multiple Intelligences

- To differentiate this lesson, teach the material in the lesson first, and then use this activity to give the students a hands- on way of practicing multiplying matrix reflections.
- Have the students work in pairs.
- They will need graph paper, rulers and colored pencils.
- Students may choose a polygon and draw it on the coordinate grid.
- Then have them show that it is reflected by the line $y = x$.
- Students take the vertices of their polygon to create a matrix for it.
- Then multiply the matrix of the polygon by the matrix represented by $y = x$.
- Finally, students show the product in a new matrix.
- Allow time for student sharing in a whole class discussion or in small groups.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal.

III. Special Needs/Modifications

- Operation with reflections = MULTIPLICATION
- Matrix multiplication-
- Multiply firsts by firsts and seconds by seconds- then add the products
- You can't multiply a smaller matrix by a larger one.
- You can multiply a larger matrix by a smaller one.

IV. Alternative Assessment

- Observe students as they work through the assignment.
- Offer assistance when necessary.
- Check for student understanding when discussing the activity in the whole class discussion.
- If students are discussing in small groups, walk around and check in with them.

Rotations

I. Section Objectives

- Find the image of a point in a rotation in a coordinate plane.
- Recognize that a rotation is an isometry.
- Apply matrix multiplication to rotations.

II. Multiple Intelligences

- To differentiate this lesson, teach the material in the lesson first, and then let students practice working with rotations.
- The students are going to play a game called "Pass the Image"
- To prepare this activity, you will need to prepare some small square images. You can design a square where it is divided on the diagonal and one- half of it is blue and the other half is red, etc.
- Students are going to be given an image. They need to draw the image according to the rotation specified in the instruction.
- For example, a student is given an image card, they start by drawing the image as it is.
- Then they are told to draw it at 180° rotation.
- Then they draw it at a 90 degree rotation.
- A 45 degree rotation.
- A 270 degree rotation.
- Then when finished, they pass the image and are given a new one.

- You can do this several times and the students can then compare their work with other students who had the same images.
- Break up students in pairs to have them compare and discuss their work.
- Next, you can move to working with an image on the coordinate plane.
- You can have students draw their own or work with the exercises in the text.
- If you do this activity first, the students will have an excellent understanding of a rotation before moving to the coordinate grid.
- Intelligences- linguistic, logical- mathematical, interpersonal, intrapersonal, visual- spatial, bodily-kinesthetic.

III. Special Needs/Modifications

- Review the basics of matrices.
- Review how to multiply a matrix.
- Review how to draw a matrix using the vertices of a polygon.
- Define rotation
 1. Center at the origin with an angle of rotation of n° .
 2. Point moves counterclockwise along an arc of a circle.

IV. Alternative Assessment

- Create a “key” of what each image looks like after it is rotated.
- Then use this key to check student work.

Composition

I. Section Objectives

- Understand the meaning of composition.
- Plot the image of a point in a composite transformation.
- Describe the effect of a composition on a point or polygon.
- Supply a single transformation that is equivalent to a composite of two transformations.

II. Multiple Intelligences

- Begin by introducing the concept of a composition.
- A composition is when transformations are “put together”. In this lesson, we will be putting together translations, reflections and rotations.

- Glide Reflection- a composition of a reflection and a translation. The translation is in a direction parallel to the line of reflection.
- Expand Example 1- Before moving to the matrix, have the students draw out this glide reflection. This will give them a hands- on way to see the two images without first moving to the matrices. This will keep it in a visual way, before moving to an arithmetic way.
- Once students have practices drawing in the glide reflection, move to using the matrix to figure out the same information. At this point, you can refer back to the text.
- The technology integration in this chapter is also a great way to provide students with a visual and hands- on way of working with the material.
- Provide time for feedback, discussion and questions after completing the work with technology.
- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, visual spatial, interpersonal.

III. Special Needs/Modifications

- Review translations, reflections and rotations.
- Review matrices.
- Review the matrix for a 180° rotation $-\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
- Review the matrix for a 90° rotation $-\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

IV. Alternative Assessment

- Allow plenty of time for the students to ask questions during this lesson.
- Spend time on reviewing previously learned skills (ie. How to multiply a matrix) if necessary.

Tessellations

I. Section Objectives

- Understand the meaning of tessellation.
- Determine whether or not a given shape will tessellate.
- Identify the regular polygons that will tessellate.
- Draw your own tessellation.

II. Multiple Intelligences

- This is a fun lesson and students usually love working with tessellations.
- Provide students with some information on tessellations and then have them work on two activities.
- The first activity, you will need to prepare.

- Students will work in groups, so you will need a few polygons/shapes for each group. If you can provide different polygons/shapes for each group- great.
- Give each group their polygons and tell them that they will need to prove whether each one tessellates or not.
- Students need to demonstrate that it has no gaps, no overlapping shapes, that the entire plane is covered in all directions.
- Also have students demonstrate that it surrounds a point.
- Allow students time for this exploration and then have students share their work.
- The second activity is to have students create their own tessellation.
- Encourage students to be creative and design a colorful tessellation of their own creation.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Write the notes of how to tell if a shape tessellates on the board.
- Provide students with a few visual examples of some tessellations.
- Have them use actual pattern blocks to explore.

IV. Alternative Assessment

- Grade student tessellations.
- You will be able to tell if the students executed the concept well by looking at their tessellations.
- Be sure to include design and color in your evaluation.
- Also check the edges of the plane- did the students successfully fill- in partial images?

Symmetry

I. Section Objectives

- Understand the meaning of symmetry.
- Determine all the symmetries for a given plane figure.
- Draw or complete a figure with a given symmetry.
- Identify planes of symmetry for three- dimensional figures.

II. Multiple Intelligences

- To differentiate this lesson, divide the students into groups.

- Ask each group to come up with an example to explain line symmetry, rotational symmetry, point symmetry and planes of symmetry.
- When finished, have each group share their images.
- Then move on to the next part of the activity.
- Ask each group to draw half of an image that has line symmetry, rotational symmetry, point symmetry. Students can use objects in the room to help them brainstorm which image to draw for each. Students may use examples from biology as well.
- Then they are going to pass their papers to a group near them.
- The next group must finish the drawings according to each description.
- When finished, allow time for sharing.
- Intelligences- linguistic, logical-mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Two- dimensional
 1. Line symmetry- left- right symmetry. Divides the figure into two congruent halves. When flipped over the line of symmetry, it is exactly the same.
 2. Rotational symmetry- rotated image looks exactly like it did before the rotation.
 3. Point symmetry- looks the same right side up and upside down. It looks the same from the left and from the right.
- Three dimensional
 - Planes of symmetry- divide a *3D* figure into two parts that are reflections of each other. Think of a cylinder or cube.

IV. Alternative Assessment

- There are several ways to assess student understanding in this lesson.
- The first way is with the images to represent each type of symmetry.
- The second is with the partial images and completions.
- Collect student work and check to see that student work is complete and accurate.
- Provide students with feedback or corrections.

Dilations

I. Section Objectives

- Use the language of dilations.
- Calculate and apply scalar products.
- Use scalar products to represent dilations.

II. Multiple Intelligences

- To differentiate this lesson, begin by teaching the concepts in the lesson to the students.
- Then, students are going to create their own dilations using scalar multiplication.
- Students will need graph paper, rulers and colored pencils.
- Ask the students to show all of their work.
- Here are the steps to the activity.
- 1. Draw a polygon of choice on the coordinate grid.
- 2. Use the vertices of the polygon to create a matrix.
- 3. Select or use a given scale factor.
- 4. Multiply the scale factor with the matrix.
- 5. The product is a new matrix- the new matrix is the vertices of the dilated matrix.
- 6. Draw in the figure on the coordinate grid.
- Allow time for the students to share their work when finished.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal

III. Special Needs/Modifications

- Review that a dilation is an image “blown up” or decreased in size.
- Transformations are also dilations.
- Dilations can be written as a matrix.
- Review scale factor.
- Scalar Multiplication- Take the real number and multiply it with each element in a matrix. The product is a new matrix.
- To create a dilation on the coordinate grid
- 1. Design a matrix based on the vertices of a polygon drawn on the coordinate grid.
- 2. Decide on a scale factor for the dilation.
- 3. Multiply the scale factor with the matrix.

- 4. The product is a new matrix that is the vertices of the dilated figure.
- 5. Draw in the new figure on the coordinate grid.

IV. Alternative Assessment

- Collect student work.
- Check to be sure that the scalar multiplication is accurate.
- Be sure that the images match the elements of each matrix.
- Assign students a classwork grade based on their work.