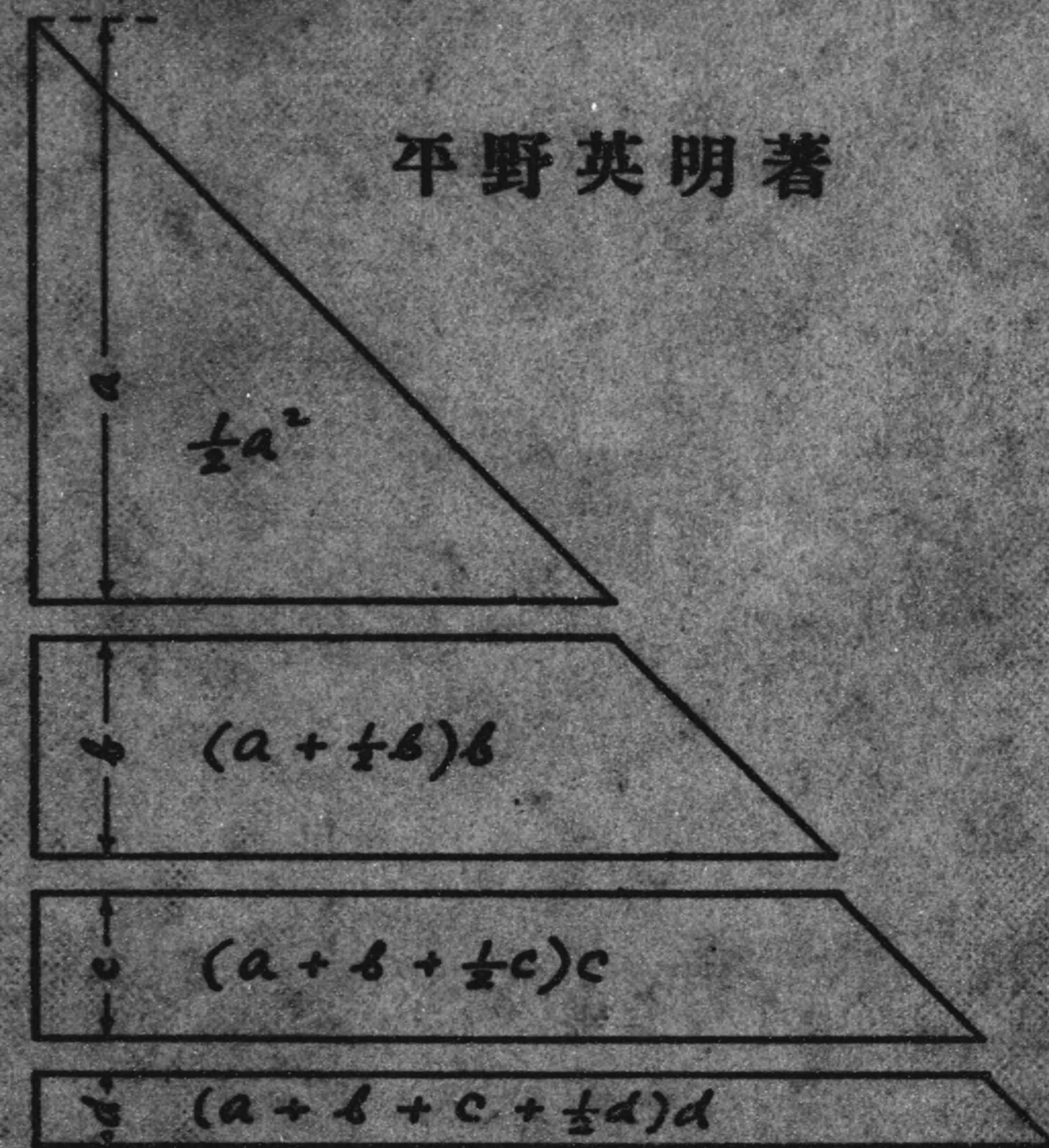
中等教育開平教授用書

平野英明・著

東京修文館

昭和８

中等教育
開平教授用書



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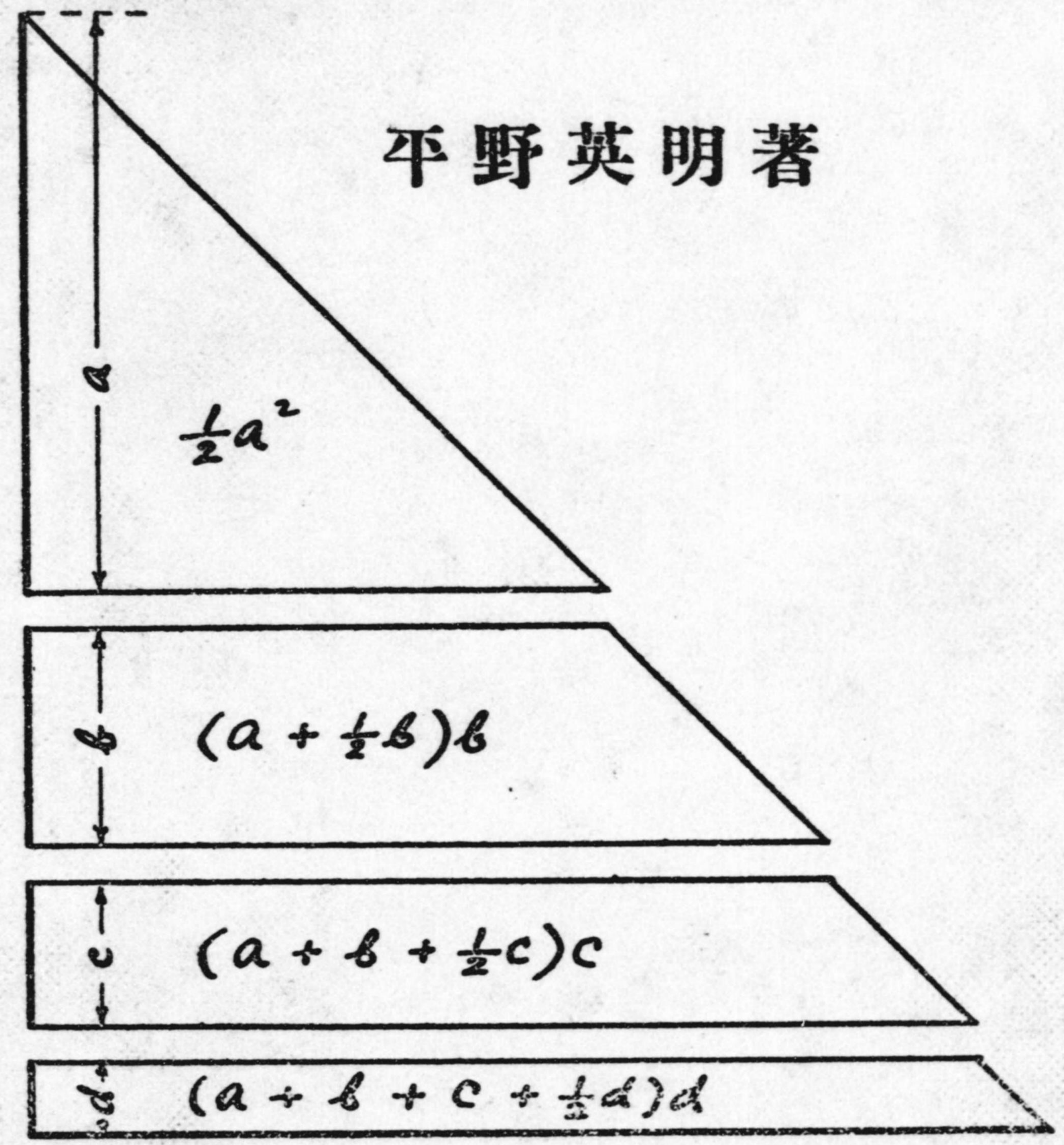

東京 大阪 修文館

中等教育

開平教授用書

平野英明著

東京 修文館 大阪

An Increment of $\sqrt{N}$

If a number N receives a positive increment of $100x$ %, its square root $\sqrt{N}$ also receives another but only between $(50x - 12.5x^2)$ and $50x$ %.

Eimei Hirano

Tôkyô, Oct. 19. '32.

An Increment of $\sqrt[m]{N}$

If a number N receives a positive increment of $100x$ %, its mth root $\sqrt[m]{N}$ also receives another but only between $100\left(\frac{x}{m}-\frac{x^2}{2m}+\frac{x^2}{2m^2}\right)$ and $100\frac{x}{m}$ %.

Eimei Hirano

Tōkyō, Oct. 19, '32.

Three Claims

Invention:—

The New Method for the Extraction of a Square Root and of a Cube Root is matchless.

Discovery:—

The New Theorem has shown that every Textbook of Mathematics in the world is wrong in point of Evolution.

By-Product:—

The New Method of Teaching Logarithms, which is a By-Product of the Invention and Discovery, is epoch-making in the line.

Eimei Hirano

Tōkyō, Sept. 7, '33.

序　　言

順天中學校出身平野英明君明治三十九年度五學年級A組ニ居タ時余ハ偶々一時間同級ニ教ヘ $a^0=1$ ヲ如何ニ說明スベキカト問ヒシニ答アリ $a^{n-n}=\frac{a^n}{a^n}=1$ ナリト，乃チ余ハ斯クテハ $a^{-n}=\frac{1}{a^n}$ トノ豫備知識ヲ要ス，依テ1ニ a ヲ0回乘ジタルモノ卽チ一回モ乘ゼザルモノナル故1ナリトノ說明法ヲ敎ヘタリ。平野君ハ此說明ヲ擴メテ

1ニ10ヲ半回乘ズレバ	$10^{\frac{1}{2}}=3.162,$	$\therefore$	$\log 3.162=0.500$
〃　$\frac{1}{4}$　〃	$10^{\frac{1}{4}}=1.778,$	$\therefore$	$\log 1.778=0.250$
〃　$\frac{3}{4}$　〃	$10^{\frac{3}{4}}=5.623,$	$\therefore$	$\log 5.623=0.750$
〃　$\frac{1}{8}$　〃	$10^{\frac{1}{8}}=1.334,$	$\therefore$	$\log 1.334=0.125$
	等		等

之ヨリ常用對數表ヲ生徒ニ作ラセタリト，如何ニモ捷徑法ナリ。

此對數表製作法ハ開平ノミヲ基トス，同君ハ更ニ新開平法ヲ發案セラレタリ，之ヲ見ルニ珠算ノ開平法ヲ筆算化シテ數術一度ニ行ヒ得ル如クシタルモノナリ，又同君赤字法ヲ創案シ，更ニ一般開方ニ關スル法則二ツヲ考ヘラレタリ，其第二ノ法則ニ至ツテハ賞讃ヲ吝ムベカラズ，入學試驗問題ノ平易化セラルルモノモ多々アルベシ，一般數學敎師ハ須ク硏究スベキモノニシテ頰冠リシテ通ルコトハ不可能ナリ，一言以テ序トナス。

順天中學校長　松　見　文　平

昭和八年三月十九日

HASIGAKI

Hatikô Kyôzyu Siio-Hitosi

Hirano-Eimei Kun wa Sûgaku no Tisudi wo ukete umare Nagoya no Kôtôkôgyô wo sotugyôsita Hito de, Ti no konomu Sûgaku wo yatte orareruga, sono Keireki ga kottôtekina Sûgaku wo yarasenai node, tuneni Ooyô no aru Hômen wo yattekita Hito de aru.

Watasi mo Sûgaku wo yatta mono da ga, Yûgi ni suginai yôna kottôtekina Sûgaku niwa Kyômi wo motanu node, Hirano Kun to aisiru yôni natte konokata Hanasi ga yoku au kara oioini sitasiku natte kita.

Hirano Kun wa hazime Keisanzyaku no Hatumei wo yari, tugini Taisû (Logarithms) no Sansyutu-hô tosite 10 no Renzoku-Kaihei Keisan wo yaru kotoni natte, Kaiheihô wo atukau koto ni nari, Soroban deno Kaiheihô wo Hissan ni utusu koto wo kangae, tugini itidoni Hutaketa mo Miketa mo sarani Sûketa mo tateru koto wo kangaedasi, sarani huno Sûzi (Negative Figures, Hirano Kun wa Akazi to yonde oru) wo tateru koto wo kangae dasareta.

Sorera no Keisan wo siteite, Gosa no koto wo atukau koto ni nari, koremade aru Sû N ga $2p$-keta tadasiku wakatte oraneba sono 2-zyôkon $\sqrt{N}$ (square root) wo p-keta tadasiku siru koto wa dekinai mono to ippanni ayamararete ita koto (Zissai wa p-keta areba yoi mono wo) ni kiduki, soreo aratameru Undô wo okosô to iu Kessin wo serareta.

Sono Hanasi wo kiita watasi wa sonna koto wa ippanni sirewatatte oru koto dyâ nai ka to kotaetaga, sono matigai ga odoroku hodo ôkuno Syomotu ni dete oru no wo misetuke rarete odoroita koto da! Kore wo mitukedasita no wa tasikani Hirano Kun no Otegara da.

Kono Koto ni tuite omoidasita koto wa Gosa no kuttuita Ryô no Sirusikata ga osiete nai koto de aru. Rikigaku no Mondai wo Seito ni yarasetemo Kinziti wo dasaseru Baai nado, 6-do 10-pun bakari (about 6° 10′) to iu kara, Gosa no Han'i wa ? to taduneruto, daitai desu, to ittari, 15-hun madewa nai to omoimasu, nado to itte oru. Dandan sirabete miruto tumari Gosa no kuttuita Ryô no Sirusikata ga wakkatte inai node aru. 2 no 2-zyôkon wa ? to kikeba, 1.4142 de mada Hasita ga aru, nado to iu konamaikina koto wa sitte oru ga, sate sore wo Kigô de arawase, to iuto Te ga denai. Gosa no aru Baai no ippanno Kakikata tosite $a<x<b$ wo osietemo mada $1.4142<2<1.4143$ wa nakanaka kakenai to iu Arisama da.

Kono koto wa Mekata demo Nagasa demo Zikan demo subete hakatte dasita Sûryô niwa sorezore Gosa ga tuite oru noda kara, honrai itumo dete kuru koto de aru ga, sono Sirusikata ga yokuwa osiete nai si, tukawarete oranai to iu Arisama da kara, nan toka seneba naranu to omotta Koto da. Kono Hirano Kun ga tumami ageta Koto datte sono hitotu no Eda (an instance) ni suginai no da.

Kore wa kekkyoku koremadeno Kigô dewa sono Ryôgawa ni saigono Hitoketa ka Hutaketa kawo nozokuno hokawa onazi Sûzi no

Tunagari de aru monowo kudokudosiku kakaneba naranai koto ya, sonomama Keisan no naka e orikonde yuku koto no dekinai Katati ni natte oru koto nado ga tukawarenu Riyû de arô. Sikasi Kigô ga warui dake no tameni hituyôna Koto ga kayôni osierarezu atukawarezuni sunde yuku to iu koto wa osii koto da kara, iroiro to sirabete An wo tate, Hirano Kun ya Nakano-Sizuka Sensei nado no Iken wo kiite kimeta no ga tugino Sirusikata de aru.

Gosa no Tuita Ryoo no Kakikata to Yomikata

Tatoeba $\sqrt{2} = 1.4142{\to}3$ to kaite $1.4142 \leq \sqrt{2} \leq 1.4143$ to iu koto wo arawasu, $a = 283{\to}95$ wa $283 \leq a \leq 295$ wo arawasi, $b = 5{\to}116$ wa $5 \leq b \leq 116$ wo arawasu. Sosite $236{\to}9 \times 3$ wa $(236{\to}9) \times 3$ no Imi de $708{\to}17$ ni hitosii to si, kore wo "Sitihyaku hati kara zyûsiti made" to yomu. $5.67{\to}8$ wa "Go ten roku siti kara hati made" to yomi; $5670{\to}80$ wa "Gosen roppyakû sitizyû kara hatizû made" to yomu. Tumari tugino yottuno Kimari ni sitagau:

(a) Kono Kigô wa Ya $\to$ de tunagareta tokorono hutatuno Bubun kara naritatte ite, tada hitotuno Sû wo arawasu. Sotogawa ni Kakko wa nai ga aru no to mattaku onazi de aru.

(b) Ya no maeno Bubun wa Kigô ga arawasu Sû no Saisyô-gendo wo arawasu.

(c) Ya no atono Bubun wa kono Kigô no daihyô suru Sû no Saidai-gendo no migino Hazi no Bubun wo simesu. Tumari Saidai-gendo wo arawasu Sûzi no uti Saisyô-gendo no to Hazime ni oite ketaketa sorotte mattaku hitosii Sûzi dake wa korewo habuita mono dearu.

(d) Yomikata wa madu hazime maeno Bubun wo yomi, tugini "kara" to irete, atono Bubun wo yomi, saigoni "made" to musubu.

Kokoni tyû-i subeki koto wa $\sqrt{2} = 1.4{\to}5$ de ari, mata $\sqrt{2} = 1.4142{\to}3$ de attemo $1.4{\to}5 = 1.4142{\to}3$ towa naranai koto de aru. Sikasi $1.4142{\to}3$ wa Mokuteki ni yottewa $1.4{\to}5$ to kaetemo yoimono de aru.

Kore nara tasyô no Mendô wa aru ga, Gosa no nai Baai to onaziyôni Hissan no nakae kumikonde yuku koto ga dekiru. Soredeite Gosa no Gendo ga arawarete oru kara tyôho da. Sosite kono Yomikata wa Denwa-bangô nado ni koremade kanari tukawarete ita mama da. 341→9 Marunouti, wa Monbusyô no Denwa de atte, 341, 342, 343, ..., 349 to kokonotu no Seisû wo arawasu. Tadasi tigau kotowa 341→9 wa tada hitotu no Sû de, sono Atae wa 341 to 349 tono Aida ni aru Seisû matawa Konsyôsû dearu.

Kono Syomotu ga dekiru koto ga Enisi to natte kayôna Mondai ga tokareta koto wa kono Syomotu no Seisitu ni husawasii, kono Syomotu no Kagayaki de aru. Hirano Kun no Tegara ni Sanzi wo osimanu.

(Siio-Hitosi)

Zimmukigen 2593n. 5gt. 19nt.

緒　　言

帝國內ノミカ外國ニテモ中等學校ニ於テ開平開立ヲ敎ヘル時ニハ次ノ樣ニ敎ヘタリ。

> $\sqrt[4]{3}$ ヲ小數第二位迄計算スルニハ先ヅ $\sqrt{3}$ ヲ 1.7320 迄用意セヨ, $\sqrt[6]{2}$ ヲ小數第二位迄トナラバ $\sqrt{2}$ ヲ 1.414213 迄用意セヨ, $\sqrt[8]{10}$ ヲ小數第五位迄トナラバ $\sqrt{10}$ ヲ 3.16227766016837933199 迄用意セヨ

此敎ヘ方ハ言語同斷ナルモノナリ、以上三例ハ夫々 $\sqrt{3}=1.73$, $\sqrt{2}=1.41$, $\sqrt{10}=3.162277$ 迄用意スレバ十分ナリ。

以上ノ誤見ハ甚ダ迷惑ナ弊害ヲ齎ラシタリ, 何トナレバ對數敎授法ニ於テ $10^{\frac{1}{8}}=1.33352$ 卽チ $\log 1.33352=0.125$ 等ヨリ對數ヲ敎フレバ極メテ簡單平凡明瞭ナルモノヲ從來ハ $10^{\frac{1}{8}}=1.33352$ ヲ中等學生ニハ困難ナ計算カノ如ク誤念セシメタルガ故ナリ、實ニ對數ハ原理モ數値算出モ易々タルモノナリ。

之レニ對シテ新定理一個ヲ發見シ, 之レガ證明ヲモ完成セリ, 新定理ハ一般開方ニ關スルモノナルヲ以テ下級生ノ爲ニハ適セズ, 之レヲ平易化シテ開平ニノミ適スベク作セリ, 左レド上級生ニハ之レヲ讀マシムルモ亦可ナランカ。

又一般開方, 殊ニ開平ノ際ノ根ノ位取リニ資センガ爲ニ法則第一ヲ出シタリ, 蓋シ簡單明瞭 (laconic) ナリト信ズ。

晉新法則第二ハ新定理ヲ應用ニ便利ナル如ク書キ直セシノミ。

別ニ新開平法ト新開立法トヲ考案セリ之レ亦從來ノ開平開立ノ勞ヲ半減ス。

以上ノ發見ト考案トヲ併用スレバ鬼ニ金棒ニシテ、高等學校ヤ專門學校ノ數學入學試驗問題中開平開立問題ニ悉ク應用スルコトヲ得、第四高等學校昭和七年度入學試驗數學問題〔4〕ノ如キ實ニ勞ヲ十分ノ一程ニ減ズ。

著者平素ヨリノ議論トシテ不盡根ハ實用上四桁マデヲ算シ以下ハ四捨五入トスルコト、開平剩除ハ一般ニハ出サザルベキコト、此點ヲ加味シテ筆ヲ採リタルコトヲ一言斷リ置クモノナリ。

而シテ之レヲ開平開立ニ關スル限リ教科書ニ代ヘラレタシ、世界中一般ノ數科書ハ開平開立ニ關シテ「間違ヒ」ナルヲ以テ當然本書ノ採用ヲ必要トスルモノナリ、敢ヘテ此强キ言ヲナス。

此發見ト考案トヲ外國雜誌ニ投書スベク英文ニテ先ヅ發表シタリ、依テ最初ノ數頁ハ其時ノ文ナリ、讀者諒セラレタシ。

著者　平野英明

2593 年 10 月 19 日

中等教育開平教授用書

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第　一　編

中等教育
開平教授用書

第一編　下級生用

第一章　總　論

1. 開平，平方根

或ル數ノ平方ガ與ヘラレタル數ニ等シキ時ハ，此或數ヲ與ヘラレタル數ノ平方根ナリト稱ス，例ヘバ 5 ノ平方ハ 25 ナルヲ以テ 5 ハ 25 ノ平方根ナリ。

與ヘラレタル數ノ平方根ヲ索ムルコトヲ其ノ數ヲ平方ニ開クト稱シ，平方ニ開クコトヲ開平ト稱ス。又換言スレバ開平トハ面積既知ノ正方形ノ一邊ノ長サヲ算出スル方法ナリ。

或ル數ノ平方ヲ書キ表ハスニハ其數ノ肩ニ小サキ 2 ノ字ヲ附ス，例ヘバ 5 ノ平方ハ 5^2 ト書ク。

或ル數ノ平方根ヲ書キ表ハスニハ符號 $\sqrt{\ }$ ヲ其數ニ冠ス，例ヘバ 25 ノ平方根ハ $\sqrt{25}$ ト書ク。

即チ　$5^2=25$　ナルガ故ニ　$\sqrt{25}=5$

1 ヨリ 5 マデノ平方ハ夫々 1, 4, 9, 16, 25 ナルヲ以テ

$\sqrt{1}=1,\ \sqrt{4}=2,\ \sqrt{9}=3,\ \sqrt{16}=4,\ \sqrt{25}=5$　ナリ。

2. 數ノ開平ノ概念

甲數ト乙數トノ和ノ平方ハ，甲數ノ平方ト，甲數ト乙數トノ積ノ二倍ト，乙數ノ平方トノ和ニ等シ。

之レ有名ナル定理ナリ，開平ハ之レヲ逆ニ利用シテ

與ヘラレタル數ノ平方根ノ最上位ノ數（之レヲＡトセヨ）ハ容易ニ知リ得ルモノナルニヨリ，Ａノ平方ヲ引法リ，剩餘ヲＡノ２倍ニテ除セバ其商ハ第二位ノ數（之レヲＢトセヨ）ヲ判斷シ得。

一般ニハ尙剩餘ヲ生ズベキニヨリ，以上ノ方法ニテ得タル二桁ノ數ヲ新シキＡトシ，第三位ノ數ヲ新シキＢトシテ演算ヲ進ム。

3. 本書ノ開平法

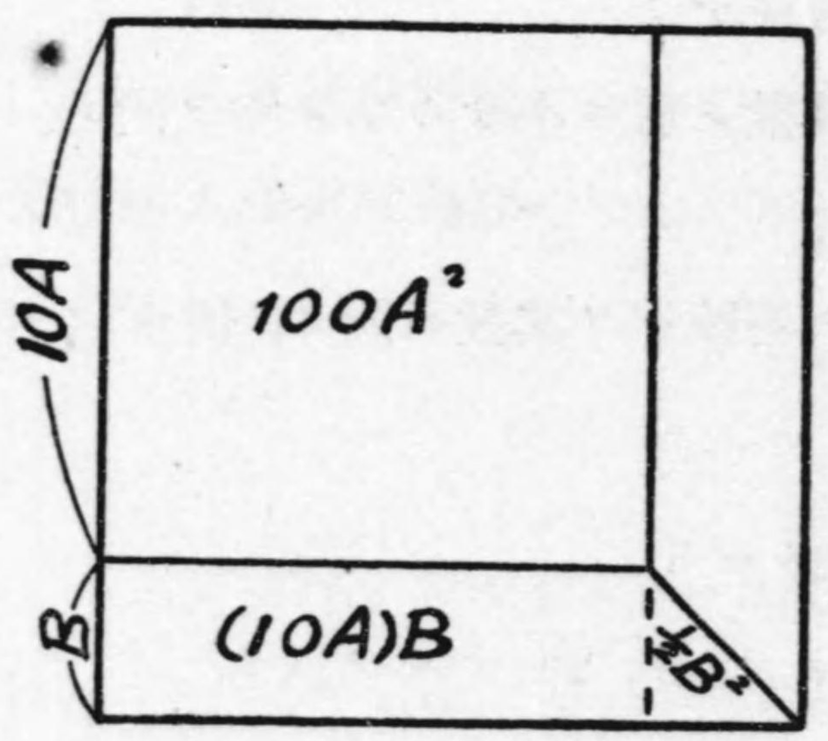
三桁乃至四桁ノ整數アリテ，之レガ平方根正商ヲ（10A+B）ナル形ヲナスモノト假定ス，A, B 共ニ一桁ノ整數ナリトス。

A ハ容易ニ知リ得, B ハ $B>2A$ ナル時ニハ少シク面倒ナリ。第四章乃至第七章ハ之レガ爲ニ生ジタルモノナリ。

a. 先ヅ原數ヨリ（$100A^2$）ヲ引キ，

b. 殘ヲ２ニテ除シ，商ヲ第一半剩餘ト稱ス。

c. 上記半剩餘ヲ（10A）ニテ除シテ其結果ヨリ B ヲ判斷ス。



$$
\begin{array}{l}
\quad 10A \quad + \quad B \\
\sqrt{0100A^2 + 2(10A)B + B^2} \\
\quad 100A^2 \\
\hline
\qquad (10A)B + \frac{1}{2}B^2
\end{array}
$$

4. 算術的平方根ト代數的平方根

前節ニ 25 ノ平方根ハ 5 ナリト云ヒシガ代數學ニヨレバ −5 ノ平方モ 25 ナリ，卽チ代數學ニテハ 5 モ −5 モ 25 ノ平方根ナリ，負根ヲ棄テ、正根ダケ採ルモノヲ算術的平方根ト稱ヘ，兩者ヲ採ルモノヲ代數的平方根ト稱シ $\pm\sqrt{25}$ 卽チ ±5 ト纒メテ書ク。

5. 百進法

本書ニ於テハ開平スベキ原數ヲ必ズ百進法ニテ讀ム，否考フルモノトス。但シ其求メタル平方根ハ十進法ニテ讀ム。

例 1.	7584	ハ之レヲ	75 84	ト考ヘ
2.	75840	ハ	07 58 40	ト考ヘ
3.	7.584	ハ	07. 58 40	ト考フ
4.	0.00007584	ハ	0. 00 00 75 84	ト考ヘ
5.	0.00000007584	ハ	0. 00 00 00 07 58 40	ト考フ

6. 第一有效數字

前節ニ於ケル五ツノ例ニ於テ百進法第一有效數字ハ夫々 75, 07, 07, 75, 07 ナリ，然ルニ十進法第一有效數字ハ皆 7 ニシテ皆一樣ナリ。開平ニハ先ヅ之レヲ見通スコト何ヨリモ大切ナリ。

7. 位取

本書ニ於テハ第一有效數字ガ小數點ヨリ左乃至右第幾桁目ニアルカヲ以テ定メ，「左何」,「右何」等ト稱ス。第 5 節ニアル五ツノ例ノ位取リハ夫々「左 2」,「左 3」,「左 1」,「右 3」,「右 4」ナリ。

8. 數例ヲ擧グレバ

<table>
<tr><th rowspan="2">數</th><th colspan="2">百進法</th><th colspan="2">十進法</th></tr>
<tr><th>位取</th><th>第一有效數字</th><th>位取</th><th>第一有效數字</th></tr>
<tr><td>3.1416</td><td>左 1</td><td>03</td><td>左 1</td><td>3</td></tr>
<tr><td>31.416</td><td>左 1</td><td>31</td><td>左 2</td><td>3</td></tr>
<tr><td>314160000</td><td>左 5</td><td>03</td><td>左 9</td><td>3</td></tr>
<tr><td>31416000</td><td>左 4</td><td>31</td><td>左 8</td><td>3</td></tr>
<tr><td>0.00031416</td><td>右 2</td><td>03</td><td>右 4</td><td>3</td></tr>
<tr><td>0.0031416</td><td>右 2</td><td>31</td><td>右 3</td><td>3</td></tr>
</table>

十進法ニテハ 3.1416 ト 31.416 トハ數字値 (digit value) ノ同ジキモノナリ，然ルニ百進法ニ於テハ此兩者ハ全然數字値ヲ異ニス。

$\sqrt{3.1416}$ ト $\sqrt{31.416}$ トハ半桁違ヒナリ，而シテ $\sqrt{3.1416}=1.77245$ 强　及ビ $\sqrt{31.416}=5.60500$ 弱　ナルヲ以テ 1.77245 ト 5.60500 トハ結局半桁違ヒナリ，10 ヲ乘ズレバ如何ナル數モ一桁進ムベキコトハ何人モ知ル所ナルガ，如何ナル數モ之レニ $\sqrt{10}$ ヲ乘ズレバ半桁進メラルベク，$\sqrt{10}$ ヲ二回乘ズレバ結局一桁進メラルベシ，故ニ $\sqrt{10}$ ノ代リニ $10^{\frac{1}{2}}$ トモ書クコトアリ，一般ニ $\sqrt{N}$ ノ代リニ $N^{\frac{1}{2}}$ トモ書ク。

$1.77245\sqrt{10}=5.60500$　　$17.7245\sqrt{10}=56.0500$

$5.60500\sqrt{10}=17.7245$　　$56.0500\sqrt{10}=177.245$

9. 本書中ノ規約

(a) 本書ニ於テハ別段ノ要求ナキ限リ開平剩餘ヲ出サズ。

(b) 本書ニ於テハ別段ノ要求ナキ限リ不盡根ハ上四桁計算シ，以下ハ四捨五入法ニヨリテ處理ス。時ニハ例外モアリ。

不盡根トハ開キ切レザル數ノ根ナリ。

(c) 別段ノ要求ナキ限リ正根ノミヲ求メタルモノトス。

(d) 本書中ノ諸例題ハ皆何レカノ教科書乃至雜誌，乃至上級學校入學試驗問題ヨリ採リタルモノナリ，特ニ(文)トアルハ文部省發行高等小學校第二學年用算術書ヨリ採リタルモノナリ。但シ本書獨特ノ主義ニヨリテ四桁マデ算出シタルモノ多シ。

10. 實際上ノ便宜

初學者ハ第二章ヨリ始メ第一章ヲ後廻シトスルヲ反ツテ解シヨキ方法ナリトセンカ。

第二章　最モ平凡ナル開平法

1. 平凡ナル方法

例　（1）$\sqrt{1764}$　ヲ計算セヨ。㊫

```
       1f
    4a 2e
 √ 1 7 6 4
   1 6b
     1 6 4c
       8 2d
       8 2g
         0h
```

a. 先ヅ　$40<\sqrt{1764}<50$　ナルヲ以テ 4 ヲ立テル

b. 4 ノ平方 16 ヲ左端ニ列ベテ記入

c. 殘 164 ヲ得.（實演ニハ記入不要）

d. 殘 164 ヲ 2 ニテ除シ 82 ヲ記入 此 82 ヲ半剰餘ト稱ス

e. 8,2÷4 ヲ行ヒ商ノ第一數字 2 ヲ採リ求ムル平方根ノ第二數字ト假想シ上ニ記入

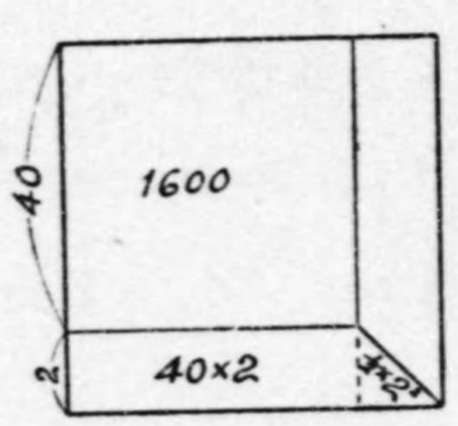
f. 其半バ 1 ヲ 2 ノ上邊ニ記入（之レモ省略シ得）

g. $4_1\times2=82$　下段ニ記入

h. 最後ノ半剰餘ガ此場合 0 ナリ 故ニ（a ト e）卽チ 42 ハ答ナリ。

浄書スレバ

```
     4 2
 √ 1 7 6 4
   1 4
       8 2
       8 2
         0
```



下圖ハ圖解ナリ。

以上ハ古來珠算ニテ行ヒシ所ニシテ，著者其レヲ筆算化シタリ。

（2）　$\sqrt{961}$　㊫

```
     65
    3 1
 √ 9 6 1
   9
     3 0 5
     3 0 5
```

答　3 1

（3）　$\sqrt{2025}$　㊫

```
     25
    4 5
 √ 2 0 2 5
   1 6
     2 1 2 5
     2 1 2 5
```

答　4 5

（4）　$\sqrt{588}$　㊫

```
   2 4. 2 5
 √ 5 8 8
   4
     9 4
     8 8     (∵2₂×4)
       6
       4 8 2 (∵24₁×2)
       1 1 8
```

答　2 4.2 5 弱

（5）　$\sqrt{250}$　㊫

```
   1 5. 8 1
 √ 2 5 0
   1
     7 5
     6 2 5   (∵1₂₅×5)
     1 2 5
     1 2 3 2 (∵15₄×8)
         1 8
```

答　1 5.8 1 强

（6）　$\sqrt{6499}$　㊫

```
     8 0. 6 2
 √ 6 4 9 9
   6 4
       4 9 5
       4 8 1 8
           1 3 2
```

答　8 0.6 2 弱

（7）　$\sqrt{77.7}$　㊫

```
   8. 8 1 5
 √ 7 7. 7
   6 4
     6 8 5
     6 7 2
       1 3
       8 8 0 5
       4 1 9 5
```

答　8.8 1 5 弱

(8) $\sqrt{\frac{13}{25}}$ (文)

$\sqrt{\frac{13}{25}}$ =

```
      .7211
  √ 0.52
     49
      15
      142
        8
        7205
         795
```

答 0.7211强

(9) $\sqrt{9999}$ (文)

```
     99.995弱
  √ 9999
    81
     9495
     8505
      99
      89505
       9495
       899505
        49995
        4999625
```

答 99.99强

小數第三位ノ 5 ハ弱ナリ，何ントトナレバ下ノ半剩除ガ僅カ不足シテ居レバナリ，之レガ强トナラバ切上ゲテ答ハ 100 トナリシモノナリ。(頁11見ヨ)

(10) $\sqrt[4]{6}$

```
    2.449强               1.565弱
  √ 6                   √ 2.449
    4                     1
    1                       7245
     88                     625
     12                      995
      968                    918
      232                     77
```

答 1.565弱

(3')，(4)，(5) 及ビ (10) ハ公式應用ニ適ス。

(4)，(9) 及ビ (10) ハ赤字法ニ適ス。

第三章 複 桁 法

1. 明瞭ナル時ハ平方根ノ第三第四數字等ハ二字一度ニ求ム

(1) 7 頁第 (4) ノ問題ハ次ノ如クナシ得。

```
    2 4.2 5
  √ 5 8 8
    4
      9 4
      8 8
        6 0
```

第 7 頁ニ費ヤセシ數字	19
此處ニ 〃 〃	14
差引節約數字數	5

第六章 D 公式ニヨレバ尙速シ

答 2 4.2 5 (∵ 60 ÷ 2,4 = 25)

(2) 頁第 (5) 問題ハ次ノ如クナシ得

```
    1 5.8 1
  √ 2 5 0
    1
      7 5
      6 2 5
      1 2 5
```

第 7 頁ニ費ヤセシ數字數	22
此處ニ 〃 〃	16
差引節約數字數	6

第五章 A 公式ニヨレバ尙速シ

答 1 5.8 1 (∵ 125 ÷ 1,5 = 81)

(3) 頁第 (6) ノ問題ハ

```
      8 0.6 2
  √ 6 4 9 9
    6 4
        4 9 5
```

第 7 頁ニ費ヤセシ數字	20
此處ニ 〃 〃	13
差引節約數字數	7

答 8 0.6 2 (∵ 495 ÷ 8 = 62)

（４）　頁第（7）ノ問題ハ　　（５）　同第（8）問題

```
   8.8 1 5            .7 2 1 1
√ 7 7.7            √ 0.5 2
  6 4                 4 9
  6 8 5                1 5
  6 7 2                1 4 2
    1 3                .   8
```

答　8.8 1 5　　答　7 2 1 1

萬一桁數ヲ四桁ヨリモ多ク精密ニ要スル場合ニハ

```
        07
   8.8 1 4 7 6 0 3        .7 2 1 1 1 0 2 5
√ 7 7.7                √ 0.5 2

     1 3                     8
     1 2 3 2 9 8             7 9 2 6 0 5
         6 7 0 2 0             7 3 9 5
```

蓋シ　$67020 \div 8{,}814 = 7603$　　$7395 \div 7{,}211 = 1025$

結局次ニハ四桁一度ニ立テルコトヲ得。

勿論更ニ次回ニハ八桁一度ニ立テ得。

著者ハ毎會二桁宛立テル方法ヲ百進法 (Centesimal Method) ト命名セリ。

毎會三桁宛立テル方法ヲ千進法 (Millesimal Method) ト命名セリ。

毎度好都合ナダケ宛立テル方法ヲ便宜法 (Expediential Method) ト命名セリ。

第四章　赤　字　法

或ル數ヲ立テタル時ニ原數ニ不足アルコトヲ知リタル場合ニモ別段立テ直シヲナス必要ハナキモノナリ，從ツテ不足ヲ最初ヨリ計劃的ニ作ルモ一策ナリ，之レヲ赤字法 (the Red Method) ト稱シ，本書中ノ重要ナル項目ノ一ツナリトス。

（１）　$\sqrt{888} = ?$　（２）　$\sqrt{48} = ?$　（３）　$\sqrt{35.81} = ?$

```
攴   3 0 2̄ 0̄     攴   7 0 7̄ 1̄     攴   6.0 1̄ 6̄
  √ 8 8 8          √ 4 8            √ 3 5.8 1
    9                4 9              3 6
     −6               −5                  −9 5
```

答　2 9.8 0　　答　6.9 2 9　　答　5.9 8 4 ガ答

```
攴    1 0 0 0 0 5̄ 0 0 0 1̄ 2̄ 5̄ 0 0
  √     9 9 9 9
      1
              −5
               5̄        1 2 5
                      −1 2 5
```

答　9 9.9 9 4 9 9 9 8 7 4 9 9…………

最後ノ例ハ初學者ニハ難題ナルベシ，後ト廻シトスベシ。

第五章 公式應用第一種

來ルベキ平方根ノ第二數字ガ 5 トナルガ如キ場合ニハ完全ニ一階梯速ク行ヒ得，卽チ次ノ公式ヲ應用ス。

$$(x+0.5)^2 \equiv x(x+1)+0.25 \qquad \text{(A)}$$

```
     4 5.              9. 5              1 5.
 √ 2 0 2 5         √ 9 0 2 5         √ 2 5 0
   2 0 2 5           9 0 2 5           2 2 5
   -------           -------           -------
         0                 0             1 2 5

     0. 6 5            7. 5                    1. 5
 √ 0. 4 3 2        √ 5 6. 5 9     √(7/3) = √ 2. 3 3 3 3
     4 2 2 5         5 6. 2 5                2 2 5
   ---------         --------              ---------
         4 7 5           1 7                   4 1 6
```

以上六問題皆(文)，但シ何レモ計算ノ出發ノミヲ示シ，之レヨリ幾桁カ計算ヲ進ムベキ用意ヲナシ半剩餘ヲ現ハシタリ。

開平剩餘ヲ求メラレタル場合ナラバ剩餘ヲ 2 ニテ除スコトナク其儘出スベシ，卽チ半剩餘ノ 2 倍ガ剩餘ナリ。

第六章ニアルモノハ此公式應用ノ擴張ニ過ギズ，而シテ第六章ハ左程ニモアラザレドモ本章ハ極メテ重要ナルモノナリトス。

第六章 公式應用第二種

來ルベキ平方根ノ第二數字ガ 5 ニアラズトモ 5 ニ近ケレバ次ノ如ク二回ニ引キテ尚若干速ク行ナフコトヲ得，左レド學生中學習ニ繁雜ナリトナスモノハ之レヲ學バズトモ大シタル支障ナシ。

$$(x+0.7)^2 \equiv (x+0.5)^2 + 0.2(2x+1.2) \qquad \text{(B)}$$

$$(x+0.6)^2 \equiv (x+0.5)^2 + 0.2(\ x+0.55) \qquad \text{(C)}$$

$$(x+0.4)^2 \equiv (x+0.5)^2 - 0.2(\ x+0.45) \qquad \text{(D)}$$

$$(x+0.3)^2 \equiv (x+0.5)^2 - 0.2(2x+0.8) \qquad \text{(E)}$$

```
B 式應用          C 式應用          D 式應用          E 式應用

   1. 7              2. 6              3. 4              4. 3
 √ 2. 9 1          √ 6. 9           √ 1 1. 6          √ 1 8 8 4
   2 2 5             6 2 5             1 2 2 5           2 0 2 5
   -------           -------           --------          ---------
     3 3               3 2 5            -3 2 5            -7 0 5
     3 2               2 5 5            -3 4 5            -8 8
   -------           -------           --------          ---------
       1                 7                 2                1 7 5
```

上記四公式ヲ書キ直シテ第七章ノ四式ト比較セヨ。

$$(x+0.7)^2 \equiv (x+0.5)^2 + 0.2\{2(x+0.5)+0.2\}$$

$$(x+0.6)^2 \equiv (x+0.5)^2 + 0.2\{(x+0.5)+0.05\}$$

$$(x+0.4)^2 \equiv (x+0.5)^2 - 0.2\{(x+0.5)-0.05\}$$

$$(x+0.3)^2 \equiv (x+0.5)^2 - 0.2\{2(x+0.5)-0.2\}$$

第七章　公式應用第三種

來ルベキ平方根ノ第二數字ガ 9, 8 ナル場合, 因ニ 2, 1 ナル場合モ此種ノ應用ナレドモ之レハ結局第二章ニ說キタルモノニ歸スルヲ以テ公式ヲ擧グルダケニ止ム, 從ツテ公式ニ番號ヲ付セズ。

$$(x+0.2)^2 \equiv x^2 + 0.2(2x+0.2)$$

$$(x+0.1)^2 \equiv x^2 + 0.2(x+0.05)$$

$$(x-0.1)^2 \equiv x^2 - 0.2(x-0.05) \qquad \text{(F)}$$

$$(x-0.2)^2 \equiv x^2 - 0.2(2x-0.2) \qquad \text{(G)}$$

F 公式應用　　　G 公式應用

```
  2            8            2            8
  1.9          7.9          1.8          7 8
√ 3.7        √ 6 3        √ 3.5        √ 6 1
  4            6 4          4            6 4
  -1 5         -5           -2 5         -1 5
    1 9 5        7 9 5        3 8          1 5 8
      4 5        2 9 5        1 3              8
```

次ノ如ク結局赤字法ニ過ギザルコトヲ知ル。

```
  2 1̄          8.1̄          2.2̄          8.2̄
√ 3.7        √ 6 3        √ 3.5        √ 6 1
  4            6 4          4            6 4
  -1̄ 5         -5̄           -2̄ 5         -1 5
    2̄   5        8   5        4 2          1̄ 6 2
      4 5        2 9 5        1 3              8
```


第八章　實　例

```
(1)             (2)             (3)
  1.4 1 4         1.7 3 2         2.2 3 6
√ 2             √ 3             √ 5
  2 2 5           2 2 5           4
    1̄ 2̄ 5̄           3 7 5           5
    1 4 5           3 2             4 2
      2 0             5 5             8

(4)             (5)             (6)
  2 4 5 0 5̄       2 6 4 6         3
√ 6             √ 7               2.8 2 8
  6 2 5           6 2 5         √ 8
    1̄ 2̄ 5̄           3 7 5         9
    2 4 5           2 5 5           5̄
    1 2             1 2             6̄ 2
    1 2 1 2 5       1 0 4 8           8
      -1 2            1 5 2

(6)             (7)             (8)
  2 8             3.1 6 2         3 4 6 4
√ 8             √ 1 0           √ 1 2
  4               9               1 2 2 5
  2               5                 1̄ 2̄ 5̄
  1 9 2           3 0 5             3 4 5
      8           1 9 5             2 2
```

(9)
```
    4 1 2 7
√ 1 5
  1 6
      5
      4 0 5
      1 0 5
```

(10)
```
     4.2 4 2
√ 1 8
  1 6
  1
     8 2
     1 8
```

(11)
```
    4 5 2 4-
√ 2 0
  2 0 2 5
      1 2 5

    4.4 7 2
```

(12)
```
    4 5 8 2
√ 2 1
  2 0 2 5
      3 7 5
      3 6 3 2
        1 1 8
```

(13)
```
    5.1 0 1
√ 2 4
  2 5
      5
      5 0 5
          5
```

(14)
```
    5 4 7 7
√ 3 0
  3 0 2 5
      1 2 5
      5 4 5
      4 2
```

(15)
```
  6.  8 4-
3 5
3 6
    5
```

(16)
```
      6 1 6 4
√ 3 8
  3 6
  1
      6 0 5
      3 9 5
```

(17)
```
    6.5 1 9+
√ 4 2
  4 2 2 5
      1 2 5
```

(18)
```
    6.7 0 8 2
√ 4 5
  4 2 2 5
    1 3 7 5
    1 3 2
        5 5
```

(19)
```
    7.0 7 2
√ 4 8
  4 9
      5
```

(20)
```
    7.0 7 1
√ 5 0
  4 9
      5
```


(21)
```
    7.7 4 6
√ 6 0
  5 6 2 5
    1 8 7 5
    1 5 2
      3 5 5
```

(22)
```
    8.0 6 2
√ 6 5
  6 4
      5
```

(23)
```
    8.3 6 7-
√ 7 0
  6 4
  3
  2 4 4 5
    5 5 5
```

(24)
```
    8.6 6 0
√ 7 5
  7 2 2 5
    1 3 7 5
      8 5 5
      5 2
```

(25)
```
    9.0 5 5
√ 8 0
  8 1
      5
```

(26)
```
    9.2 2 0
√ 8 5
  8 1
  2
    1 8 2
      1 8
```

(27)
```
    9.5 1 3+
√ 9 0
  9 0.2 5
      1 2 5
```

(28)
```
    9.7 4 7-
√ 9 5
  9 0 2 5
    2 3 7 5
    1 9 2
      4 5 5
```

(29)
```
    3.5 3 6
√ 1 2.5
  1 2 2 5
      1 2 5
```

(30)
```
    5.7 7 3
√ 3 3.3 3
  3 0 2 5
    1 5 4
    1 1 2
      4 2
```

(31)
```
    7.5 4 7-
√ 5 5.5 5
  5 6 2 5
      3 5
```

(32)
```
    8.1 6 5
√ 6 6.6 6
  6 4
    1 3 3
      8 0 5
      5 2 5
```

第九章　代數式ノ開平

文部省編纂ノ高等小學校第二學年用算術書第7頁乃至第11頁ヲ採レリ，サレド上級學校入學試驗準備ニモ是非共精讀スベシ。

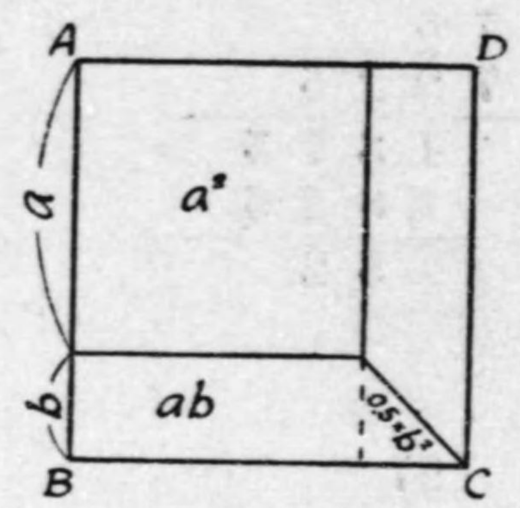
同算術書 7 頁問題(4)　$\sqrt{a^2+2ab+b^2}$

演　算

$$
\begin{array}{l}
\qquad\quad 0.5b \\
\; a \;+\;\; b \\
\sqrt{a^2 + 2ab + b^2} \\
\;\; '' \\
\hline
\qquad ab + 0.5b^2 \\
\qquad\; '' \;+\; '' \\
\hline
\qquad\qquad 0
\end{array}
$$

答　$a+b$

圖　解



說明　問題ハ面積 $a^2+2ab+b^2$ ナル正方形 ABCD ノ一邊 AB ノ長サヲ計算セヨトナリ。

（1）　先ヅ原式ノ第一項 a^2 ノ平方根卽チ a ヲ a^2 ノ直上ニ記入ス。

（2）　a ノ平方卽チ a^2 ヲ原式ヨリ引ク，同時殘リヲ 2 ニテ除シ演算ノ如ク半剩餘 $ab+0.5b^2$ ヲ記入ス。

（3）　ab ヲ(1)ニ於テ立テタル a ニテ除シ商卽チ b ヲ第二項トシテ立テル，此時 b, $2ab$, ab 皆一鉛直線上ニ來ル如ク行儀正シク書ク習慣ヲ作ルベシ。

（4）　以上ノ b ノ半分卽チ $0.5b$ ヲ其直上ニ記入スベシ。之レ亦 b ノ直上ニ行儀正シク書クベシ，（但シ最上層ノ $0.5b$ ハ省略シテ記臆ニ留メ置クモ可ナリ）

（5）（1）ニ於テ立テタル a ト（4）ニ於テ記入シタル $0.5b$ トノ和 $(a+0.5b)$ ＝（3）ニ於テ記入シタル b ヲ剩ジ　$(a+0.5b)b$ ヲ作リ，之レヲ(2)ニ於テ得タル半剩除ヨリ引クベシ，問題(4)ノ場合ニハ剩除ナシ，餘リハ 0 ナリ。

依テ　$a+b$　ヲ以テ答トナス。

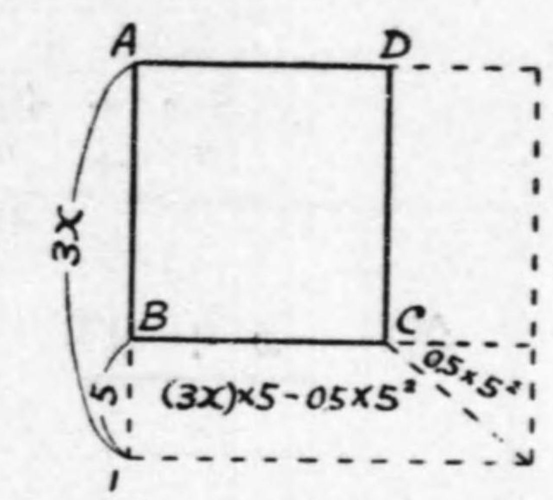
同算術書 8 頁問題(5)　$\sqrt{9x^2-30x+25}$

演　算

$$
\begin{array}{l}
\qquad\quad -\;2.5 \\
\; 3x \;-\; 5 \\
\sqrt{9x^2 - 30x + 25} \\
\;\; '' \\
\hline
\qquad -15x + 12.5 \\
\qquad\quad '' \;+\; '' \\
\hline
\qquad\qquad 0
\end{array}
$$

答　$3x-5$

圖　解



說　明

問題ハ面積 $9x^2-30x+25$ ナル正方形 ABCD ノ一邊 AB ノ長サヲ計算セヨトナリ。

（1）　先ヅ $9x^2$ ノ平方根 $3x$ ヲ $9x^2$ ノ直上ニ行儀正シク書ク。

（2）　$3x$ ノ平方 $9x^2$ ヲ原式ヨリ引キ，殘リヲ直ニ 2 ニテ除シ，半剩餘 $-15x+12.5$ ヲ上記演算ノ如ク行儀正シク書ク。

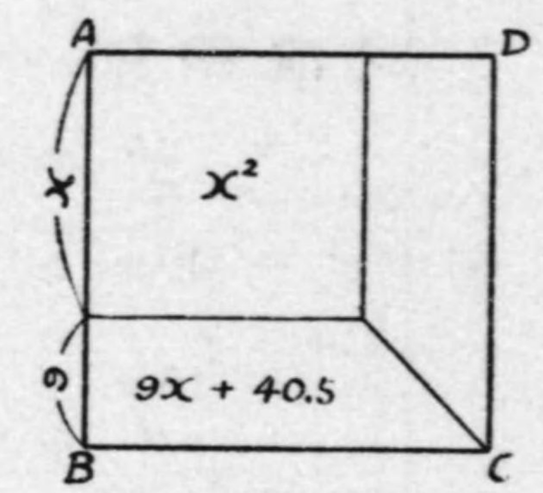
（3）　$-15x$ ヲ(1)ニ於テ立テタル $3x$ ニテ除シ，結果 -5 ヲ $-30x$ ノ直上ニ行儀正シク書ク。

（4）　(3)ニ於テ立テタル -5 ノ半分 -2.5 ヲ -5 ノ直上ニ行儀正シク書ク，（但シ -2.5 ハ略シテ記臆ニ留メ置クモ可ナリ）

（５） (1)ニ於テ立テタル $3x$ ト(4)ニ於テ作リタル -2.5 トノ代數和 $(3x-2.5)$ ニ(3)ニ於テ得タル -5 ヲ乘ジテ $(3x-2.5)(-5)$ ヲ(2)ニ於テ得タル半剩餘 $(-15x+12.5)$ ヨリ引クベシ，(5)ノ此問題ノ場合ニハ剩餘ナシ，卽チ $\underline{3x-5}$ ヲ以テ答トナス。

同算術書 8 頁問題(6)ノ〔イ〕 $\sqrt{x^2+18x+81}$

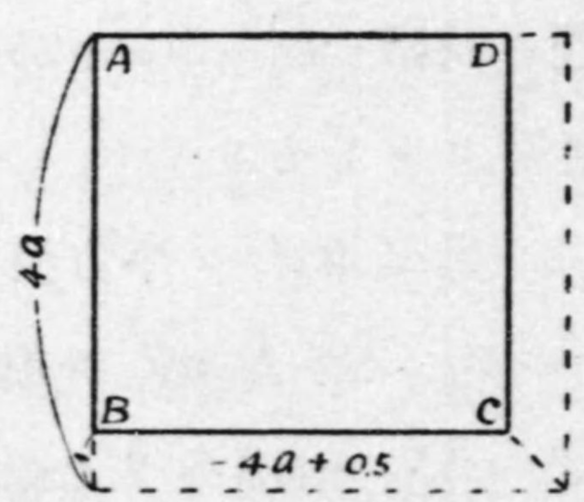
$$\begin{array}{l} \quad + \; 4.5 \\ x + 9 \\ \sqrt{x^2 + 18x + 81} \\ \;\; '' \\ \hline \qquad 9x + 40.5 \\ \qquad \;'' \; + \; '' \\ \hline \qquad\qquad 0 \end{array}$$



答　$\underline{x+9}$

同算術書 8 頁(6)ノ〔ロ〕 $\sqrt{16a^2-8a+1}$

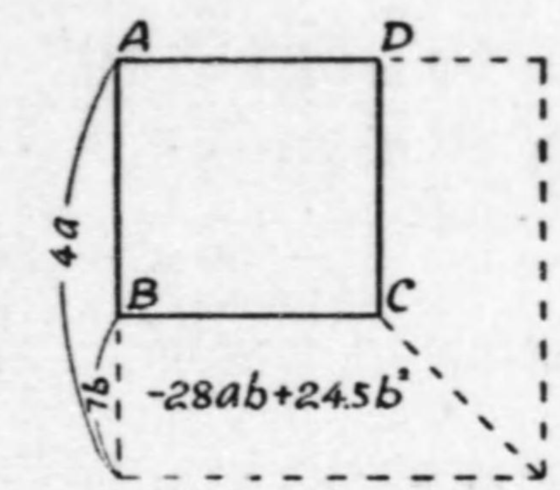
$$\begin{array}{l} \quad - \; 0.5 \\ 4a - 1 \\ \sqrt{16a^2 - 8a + 1} \\ \;\; '' \\ \hline \qquad -4a + 0.5 \\ \qquad \;\; '' \; + \; '' \\ \hline \qquad\qquad 0 \end{array}$$



答　$\underline{4a-1}$

同算術書 8 頁(6)ノ〔ハ〕 $\sqrt{16a^2-56ab+49b^2}$

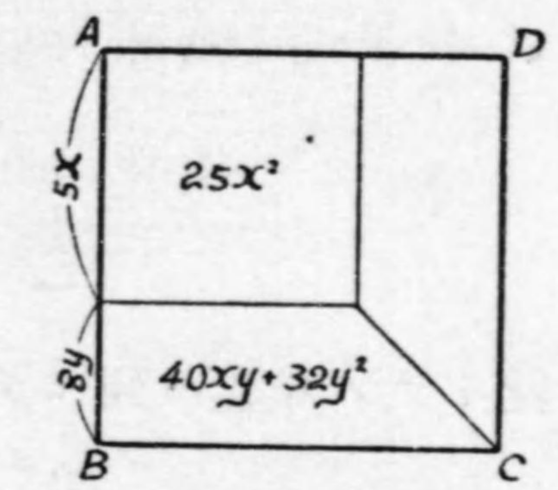
$$\begin{array}{l} \quad - \; 3.5b \\ 4a - 7b \\ \sqrt{16a^2 - 56ab + 49b^2} \\ \;\; '' \\ \hline \qquad -28ab + 24.5b^2 \\ \qquad\quad '' \; + \; '' \\ \hline \qquad\qquad 0 \end{array}$$



答　$\underline{4a-7b}$

(6)ノ〔ニ〕 $\sqrt{25x^2+80xy+64y^2}$

$$\begin{array}{l} \quad + \; 4y \\ 5x + 8y \\ \sqrt{25x^2 + 80xy + 64y^2} \\ \;\; '' \\ \hline \qquad 40xy + 32y^2 \\ \qquad\;\; '' \; + \; '' \\ \hline \qquad\qquad 0 \end{array}$$



答　$\underline{5x+8y}$

圖解ハ皆說明ノ爲ニ挿入シタルノミ，演算ニハ不必要ナリ。

同算術書 8 頁問題(7) $\sqrt{a^2+8a+25}$

$$\begin{array}{l} \quad + \; 2 \\ a + 4 \\ \sqrt{a^2 + 8a + 25} \\ \;\; '' \\ \hline \qquad 4a + 12.5 \\ \qquad \; '' \; + \; 8 \\ \hline \qquad\quad 4.5 \times 2 \end{array}$$

答　$\underline{a+4}$ 餘 9

餘リハ最後ニ出テ來タ半剩餘 4.5 ノ 2 倍ニシテ 4.5 其儘ニアラズ，之レ曩ニ第一ノ剩餘ヲ直ニ 2 ニテ除シ半剩餘 $(4a+12.5)$ ヲ下シタルガ故ナリ。

同算術書 8 頁問題(8)ノ〔イ〕 $\sqrt{81x^2 - 72x + 72}$

$$
\begin{array}{rrrrr}
 & & - & 2 & \\
 & 9x & - & 4 & \\
\sqrt{} & 81x^2 & - 72x & + 72 & \\
 & x & & & \\
\hline
 & & -36x & + 36 & \\
 & & {''} & + 8 & \\
\hline
 & & & 28 & \times 2
\end{array}
$$

答 $\underline{9x - 4}$ 餘 $\underline{56}$

同算術書 (8)ノ〔ロ〕 $\sqrt{49a^2 + 84ab + 10b^2}$

此問題ニハ三種異樣ノ答アリ。面白キ問題ナリ。

$$
\begin{array}{rrrrr}
 & & & 3b & \\
 & 7a & + & 6b & \\
\sqrt{} & 49a^2 & + 84ab & + 10b^2 & \\
 & {''} & & & \\
\hline
 & & 42ab & + 5b^2 & \\
 & & {''} & + 18b^2 & \\
\hline
 & & & -13b^2 & \times 2
\end{array}
$$

答甲 $\underline{7a + 6b}$ 餘 $\underline{-26b^2}$

b ニ就イテノ降冪ニ列ベテ行ヘバ

$$
\begin{array}{rrrrr}
 & & + & \frac{21}{\sqrt{10}}a & \\
 & \sqrt{10}\,b & + & \frac{42}{\sqrt{10}}a & \\
\sqrt{} & 10b^2 & + 84ab & + 49a^2 & \\
 & {''} & & & \\
\hline
 & & 42ab & + 24.5a^2 & \\
 & & {''} & + 88.2a^2 & \\
\hline
 & & & -63.7a^2 & \times 2
\end{array}
$$

答乙 $\underline{\sqrt{10}\,b + 4.2\sqrt{10}\,a}$ 餘 $\underline{-127.4a^2}$

ab ノ項ヲ餘セバ

$$(7a + \sqrt{10}\,b)^2 = 49a^2 + 14\sqrt{10}\,ab + 10b^2$$

$$84ab - 14\sqrt{10}\,ab = (84 - 14\sqrt{10})ab$$

$\therefore$ 答丙 $\underline{7a + \sqrt{10}\,b}$ 餘 $\underline{(84 - 14\sqrt{10})ab}$

第二編 上級生用

第十章 椎尾式誤差附ノ數ノ記法

第八高等學校敎授椎尾ひとし氏ノ創案ニ係ハル。

本章ハ本書ノ目的タル開平乃至開立ニハ直接ノ關係ナキモ之レガ本發明ノ言ハントスル所ヲ簡單明瞭ナラシムルノミナラズ遂ニハ著者ノ硏究其モノヲモ援助セシコトサヘ屢々アリタリ。

尙椎尾敎授ノ Hasigaki ヲ讀マレタシ。

$\sqrt{3} = 1.73{\to}4$ ト書キテ $1.73 < \sqrt{3} < 1.74$ ヲ表ハス。

$\sqrt{2} = 1.4142{\to}3$ 〃 $1.4142 < \sqrt{2} < 1.4143$ 〃

$a = 288{\to}9$ 〃 $288 < a < 289$ 〃

$\pi = 3.1415{\to}6$ 〃 $3.1415 < \pi < 3.1416$ 〃

$236{\to}9 \times 3$ トハ $(236{\to}9) \times 3$ ノ意味ニシテ $708{\to}17$ ニ等シ。

$236{\to}9$ ハ唯一無二ノ數ヲ表ハス，236 ヨリ大ニシテ 239 ヨリハ小ナリトダケ明瞭ニシテ其レ以上ハ不明ナリ，勿論コレダケ明瞭ナレバ十分ナル場合ニ用フ。

例一. $753{\to}4 + 294{\to}5 = ?$ 例二. $753{\to}4 - 294{\to}5 = ?$

$$
\begin{array}{rr}
 & 753{\to}4 \\
+ & 294{\to}5 \\
\hline
 & 1047{\to}9
\end{array}
\qquad\qquad
\begin{array}{rr}
 & 753{\to}4 \\
- & 294{\to}5 \\
\hline
 & 458{\to}60
\end{array}
$$

例三. $753{\to}4 \times 294{\to}5 = 221000{\to}3000$

例四. 753→4 ÷ 294→5 = 2.55→7

例五. $\dfrac{1}{4.318\to 9} = 0.23153\to 9$

演算中ノ原數ニ誤差付キノモノアル時ハ別段ノ指定ノナキ限リ出來ルダケ精密ニ計算シ，而カモ誤差ノ範圍ヲ最後ノ一桁以內ニ止ム。

例六. 7.53→4 × 2.94→5 = ？

答 22.1→3 トスベク

22.129→243 トセズ

22.12→25 モ不可ナリ

例七. $\dfrac{1}{4.318\to 9} = ?$

答 0.23153→9 トスベク

0.231535→89 トセズ

例八. $\sqrt{3.14\to 5} = ?$

答 1.772→5 トスベシ

1.772004→4824 ハ不可

1.772 モ近似値ニシテ 1.775 モ近似値ナリ，左レド 1.772→5 ハ近似値ニアラズ，1.772 ト 1,775 トノ間ニ値ヲ有スル一ツノ確實ナル數ニシテ只其値ヲ之レ以上精密ニ知ルニ由ナキカ，或ハ知ル必要ナシノ意ナリ。

誤差ノ範圍ガ1點ナル時ハ矢ノ後ニ來ル數字ヲ省略シテモ支障ナシ

例九. 1.4142→3 = 1.4142→ (= 1.4142………)

3.1415→6 = 3.1415→

3.14159→10 = 3.14159→

1047→9, 458→60, 2.55→7 等ハ省略スベカラズ

第十一章 一般開方ニ關スル新法則

法則 N ヲ 10^m 進法デ讀ミ第一有効數字ガ小數點ノ左乃至右第 n 位ニアレバ $\sqrt[m]{N}$ ノ十進法ニ於ケル第一有効數字ハ小數點ノ夫々左乃至右第 n 位ニアル。但シ N ハ正ノ數, m ハ正ノ整數ナリトスル。

N ヲ 10^m 進法デ表ハスニハ數字値 (digit value) 及ビ小數點 (10^m-esimal point) ニ對スル第一有効數字ノ所在ヲ別々ニ表ハス方ガ便利ナコトガ屢々アル。而シテ小數點ニ對スル第一有効數字ノ所在ハ之レヲ「位取」ト稱シテ「左 n」或ハ「右 n」ナル語ヲ以テ表ハス。故ニ 10^m 進法ニ於ケル N ガ「左 n」ナラバ十進法ニ於ケル $\sqrt[m]{N}$ モ亦「左 n」デアル。

從來位取ナル實語ハ數列ニ對スル小數點ノ位置即チ小數點ニ重キヲ置イタガ本書デハ小數點ニ對スル第一有効數字ノ關係的位置即チ第一有効數字ニ重キヲ置ク、但シ極簡單ナモノハ從來ノ考ヘ方デモヨイ

第十二章 平野式開平法

$$N = (a+b)^2 + R_2$$

$$\quad\quad \tfrac{1}{2}b$$

$$a \quad + \quad b$$

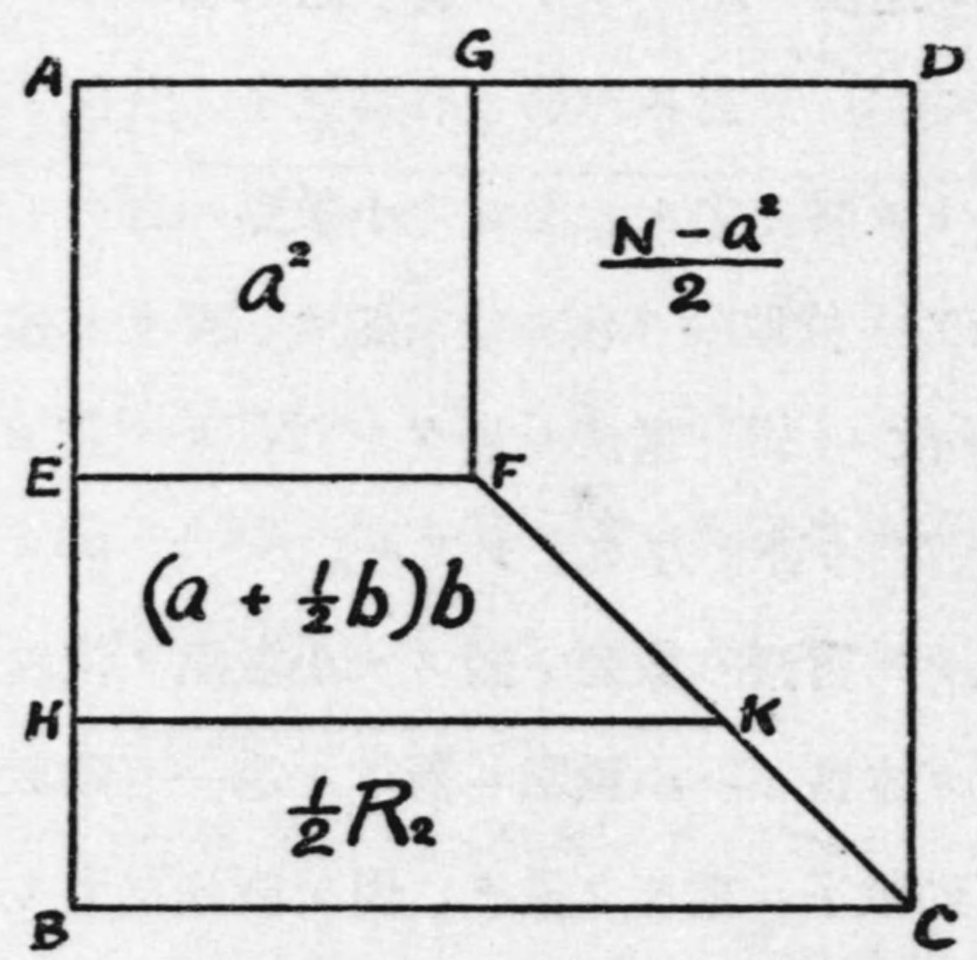
$$\sqrt{a^2 \; + \; 2ab \; + \; b^2 \; + \; R_2}$$

”

$$ab \; + \tfrac{1}{2}b^2$$

$$(a \quad + \tfrac{1}{2}b)b$$

$$\tfrac{1}{2}R_2$$



前頁ノ説明

ABCD ヲ面積 *N* ナル正方形トスル、*AB* ノ長サヲ計算スルコトガ終局ノ目的デアル、*AB* ヲ *a, b, c, d,* …… 等ト好都合ナ様ニ切ツテ一部分ヅツ計算シテ行ク方針ヲ採ル、先ヅ *AB* 上ニ $AE=a$ ト取ツテ *a* ハ之レヲ解ル限リ大キク採ル、次ニ正方形 *AEFG* ヲ完成スル、*FC* ヲ結ンデ梯形 *EBCF* ヲ完成スル、スルト其梯形ノ面積ハ $\frac{1}{2}(N-a^2)$ ニナル、次ニハ *EB* ノ長サヲ計算スルノガ新シイ目的トナル、偖テ *EB* 上ニ $EH=b$ ト取ルノデアルガ *b* ハ解ル限リ長ク採ルガヨイ、*HK* ヲ結ンデ梯形 *EHKF* ヲ完成スル、スルト其面積ハ $(a+\frac{1}{2}b)b$ ニナル、次ニハ *HB* ノ長サ即チ梯形 *HBCK* ノ高サヲ計算スルノガ第三ノ目的トナル、斯クノ如クシテ *c, d,* …… 等ヲモ計算スルノデアル。

多項式開平ノ例

$$\sqrt{4x^4 - 12x^3 + 29x^2 - 30x + 25} = ?$$

$$\quad\; - \; 1.5x \; + \; 2.5$$

$$2x^2 - 3x + 5 \longleftarrow \text{答}$$

$$\sqrt{4x^4 - 12x^3 + 29x^2 - 30x + 25}$$

”

$$- 6x^3 + 14.5x^2 - 15x + 12.5$$

$$” \; + 4.5x^2$$

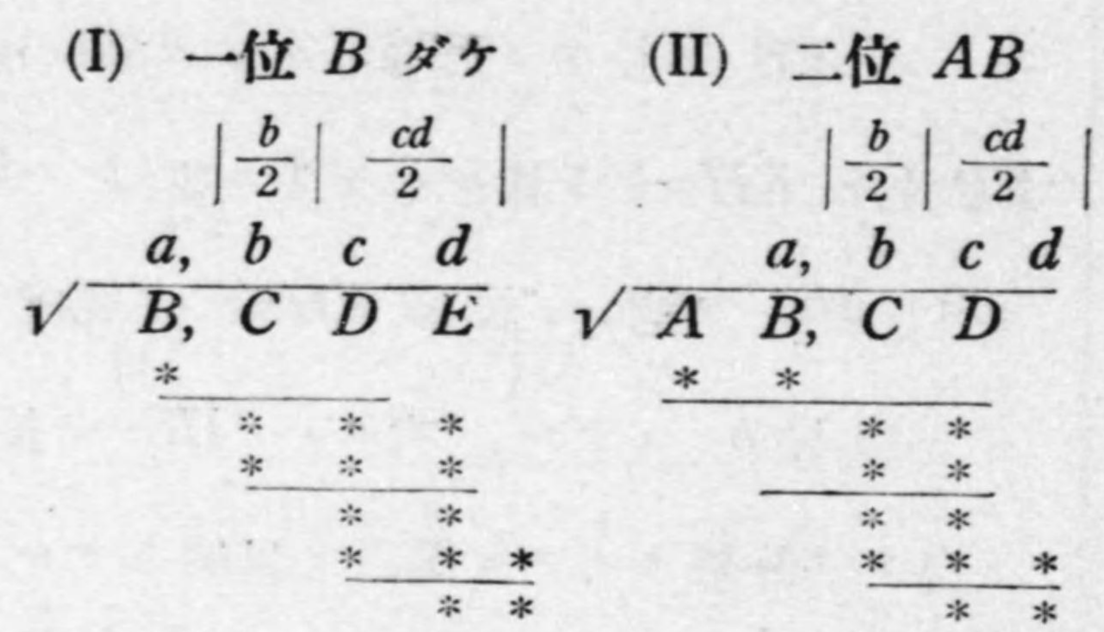
$$+ 10x^2$$

$$” \quad - 15x + 12.5$$

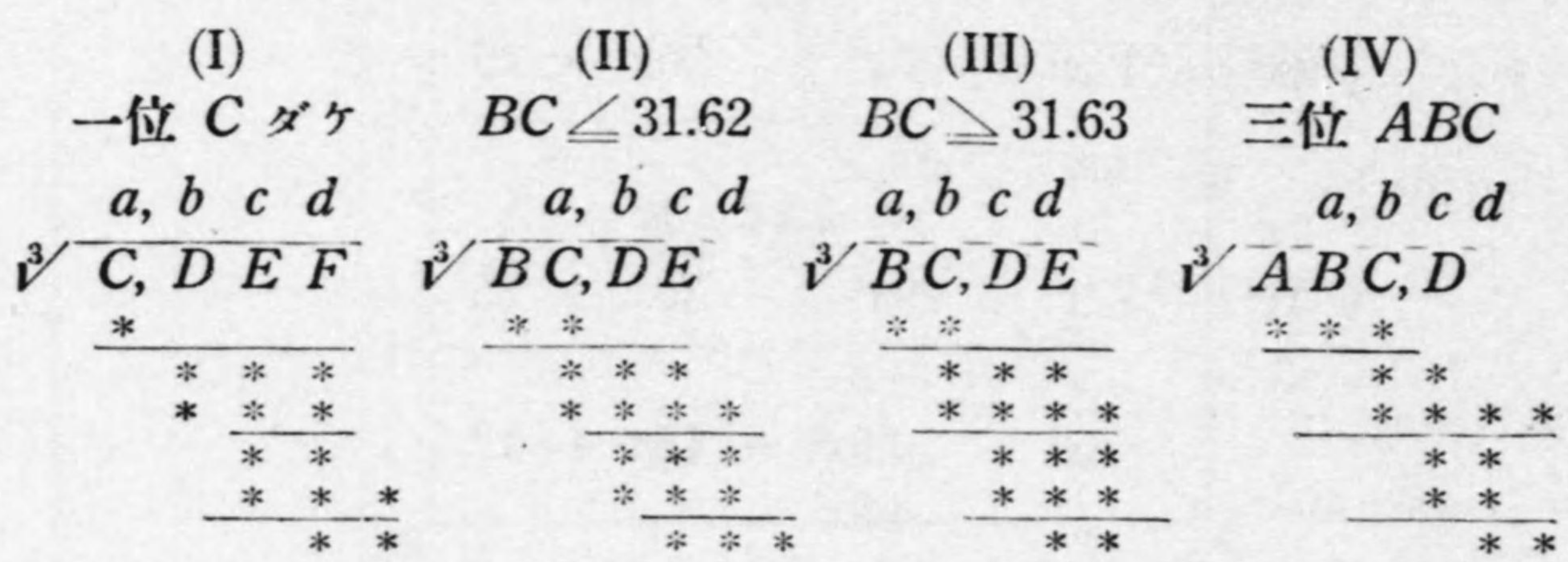
$$0$$

第十三章　平野式開平法ノ特徴（附開立法）

1. 平方根第一部分 a ノ平方ヲ原數 N カラ引キ殘ヲ 2 デ割ル。立方根ノ場合ハ 3 デ割ル。
2. a, b, c 等ハ一桁ト限ラズ幾桁デモ便利ナダケ採ル。
3. a, b, c 等ハ正ト限ラズ便宜上負ニデモ採ル。
4. $\sqrt{N}$ 乃至 $\sqrt[3]{N}$ ヲ p 桁ダケ求メルナラバ N ハ先ヅ p 桁ノミ用意シテ、萬一不足ヲ生ジタ時ハ臨機必要ナダケ補充スル。
5. 舊方法デハ $\sqrt{A\pm\sqrt{B}}$ ナル形ハ先ヅ之レヲ $(\sqrt{\alpha}\pm\sqrt{\beta})$ ナル形ニ變化シタモノデアルガ、本方法デハ其必要ナイ。
6. $\sqrt{A^2\pm B}$, $\sqrt{A^2\pm B^2}$ ナル形ハ A 或ハ $(A+B)$ ヲ直ニ探ル。
7. 試驗ニハ鉛筆一本デヤル。實用ニハ算盤，計算尺，計算器，平方表等ヲ應用スル。
8. 開平ニハ N ノ百進法第一數字ガ一位 B ダケデモ，二位 AB トアッテモ，$\sqrt{N}$ ノ第一數字 a ハ恒ニ B ノ直上ニ記入スルコト



9. 開立ニハ N ノ千進法第一數字ガ一位 B ダケナラバ $\sqrt{N}$ ノ第一數字 a ハ恒ニ C ノ直上ニ；二位 BC トアッテ 31.62 乃至尚小ナル時ハ C ノ直上ニ；31.63 乃至尚大ナル時ハ B ノ直上ニ；三位 ABC トアル時ハ恒ニ B ノ直上ニ記入スル。



10. a, b, c 等ハ凡テ詰メテ書ク。

第十四章　平野式開平法ノ分類

A　桁數ニヨル分類

1. 單桁法 (S)
   Simple Method

2. 複桁法 (M)
   Multiple Method
   - 百進法 Centesimal Method.
   - 千進法 Millesimal Method.
   - 臨機法 Expediential Method.

3. 公式應用法 (F)
   Formula Method

B.　負數字有無ニヨル分類

1. 黑字法 (B)
   Black Method

2. 赤字法 (R)
   Red Method

平野式開平法ノ例

$\sqrt{72.233001} = ?$

S. B,

```
  2 45 45
  8.4 9 9
√7 2.2 3 3 0 0 1
 6 4
   4 1 1 6 5
   3 2 8
     8 3 6 5
     7 6 0 0 5
       7 6 4 5 0 5
       7 6 4 5 0 5
                 0
```

M. B.

```
  8.4 9 9
√7 2.2 3 3 0 0 1
 6 4
   4 1 1 6 5
   3 2 8
     8 3 6 5   0 5
     8 3 6 5 0 0 5
                 0
```

M. R.

```
  8.5 0̄ 1
√7 2.2 3
 7 2 2 5
     -1
       8̄5̄     5
    -1 5 0 0 5
```

M. R.

```
  8.5 0 1̄
√7 2.2 3
 7 2 2 5
     -1
```

8.4 9 9 ガ答ナリ

但シ最後ノ演算ノ如ク此處マデ手ヲ省ク時ハ答ガ有理根ナリヤ或ハ不盡根ナリヤヲ知ルニ由ナシ，近似値ヲ求ムルニハ之レニ限ル，有理根ハ特別ナル場合ニシテ不盡根コソ普通ナリ。故ニ此方法最モ尊トシ。

下段左側ニ見ル方法ニ於テハ最後ノ半剩餘ノ2倍ガ省カレタル部分ニ等シ，故ニ答ハ有理根ニシテ

$$\sqrt{72.233001} = \underline{8.499}$$

第十五章　平方根ニ於ケル增分及ビ誤差

$$(x+1)^2 - x^2 \equiv 2x + 1$$

ナルヲ以テ正整數 x ガ 1 單位ノ增分ヲ受クル時 x^2 ノ受クル增分ハ x ノ 2 倍ニ 1 ヲ加ヘタルモノニ等シ。

x ガ小數乃至混小數ナル場合ニハ少シク上述ノ定理ヲ押擴メテ考フレバ可ナリ。

正整數 x ガ 1 ノ增分ヲ受ケタル時ニハ $\sqrt{x}$ ハ如何ナル增分ヲ受クルベキカ。

正數 N ガ $100x\%$ ノ正ノ增分ヲ受クレバ其平方根 $\sqrt{N}$ ハ $(50x - 12.5x^2)\%$ 以上 $50x\%$ 以下ノ增分ヲ受ク。

之レ重要ナル定理ナリ，特ニ $50x\%$ 以下ナルコトガ應用ノ爲ニ重要ナリ。

因ニ此定理ハ次ノ如ク書キ換ヘルコト勿論支障ナシ，只上記ノ儘ニ證明スベシ。

正數 N ガ $d\%$ ノ正ノ增分ヲ受クレバ其平方根 $\sqrt{N}$ ハ $(\frac{1}{2}d - \frac{1}{800}d^2)\%$ 以上 $\frac{1}{2}d\%$ 以下ノ增分ヲ受ク。

〔定理ノ證明〕

第十二章ニ於テ一般開方ニ就キテノ定理ヨリ又證明シ得ベキモ今ハ平方根ニ就キテノミ比較的平易ニ證明スベシ

$N > 0$ ニシテ $0 < x < 1$ トス,

二項定理ニヨリ

$$\sqrt{N(1+x)}$$

$$= \sqrt{N}\left[1+\frac{x}{2}-\frac{\frac{1}{2}(1-\frac{1}{2})}{1\times 2}x^2+\frac{\frac{1}{2}(1-\frac{1}{2})(2-\frac{1}{2})}{1\times 2\times 3}x^3-\cdots\cdots\cdots\right]$$

相隣レル二ツノ一般項第 $(r+1)$ 及第 $(r+2)$ 項ノ和ハ

$$-(-1)^r\frac{1}{2}\frac{(1-\frac{1}{2})}{2}\frac{(2-\frac{1}{2})}{3}\frac{(3-\frac{1}{2})}{4}\cdots\cdots\cdots\frac{(r-1-\frac{1}{2})}{r}x^r\left\{1-\frac{(r-\frac{1}{2})}{r+1}x\right\}$$

此組合項 (compound term) ノ各因數ハ皆正ニシテ 1 ヨリ小ナリ, 故ニ r ガ隅數ナル時ハ全項ガ負トナリ, 之レガ奇數ナル時ハ全項ガ正トナル, 而シテ組合セ項皆然ルヲ以テ當初ノ恒等式ノ第三項以下ノ總和ハ負ニシテ第四項以下ノ總和ハ正ナリ。故ニ

$$\sqrt{N}\left[1+\frac{x}{2}-\frac{\frac{1}{2}(1-\frac{1}{2})}{1\times 2}x^2\right] < \sqrt{N(1+x)} < \sqrt{N}\left[1+\frac{x}{2}\right]$$

之レヨリ

$$\left(\frac{x}{2}-\frac{x^2}{8}\right) < (\sqrt{1+x}-1) < \frac{x}{2}$$

故ニ以上ノ定理ヲ得。 Q. E. D.

例一. $\sqrt{10.00}$ = 3.1622→3 ヲ知ルトセヨ。

$\sqrt{10.01}$ = ?

定理ニヨリ右邊ハ最大限度ニ於テ 0.05 % ノ増分ヲ得ベク

$\sqrt{10.00}$ = 3.1622→3

増分 = 0.0015→6

∴ $\sqrt{10.01}$ = 3.1637→9

故ニ今別ニ $\sqrt{10.0075\cdots\cdots\cdots\cdots}$ ヲ小數第二位マデ求ム, 等ノ問題アリトスルモ原數ノ小數三位以下ハ全然無用ノ長物ナリ。

此事ハ帝國内ノミカ外國ノ教科書ニモ一般ニ誤リタル感念ヲ與ヘラレタルヲ以テ, 其蒙ヲ啓クノ必要アリ。

例二. $\sqrt{9.8692}$ = 3.14152……… 之レガ原數ニ

0.0001 即チ

0.00102% 弱 ヲ加フルモ

其平方根 3.14152………… ハ

0.00051% 弱 即チ

實増分 0.000002 以下 ヲ加ヘラレ

3.14155 以下ノ値トナル, 即チ

$\sqrt{9.8692\text{→}3}$ = 3.14152→5

結論トシテ

$\sqrt{9.8692\cdots\cdots\cdots\cdots}$ ヲ小數第四位マデ計算スル爲ニハ原數ノ小數五位以下ハ如何ナルモノタリトモ斷ジテ之レヲ明瞭ニスル必要ナシ即チ之レヲ切捨ツルトモ, 或ハ之レヲ切上グルトモ決シテ支障ヲ來ス程ノ誤差ヲ生ゼシメズ。

第十六章　開平原數ノ桁數ニ就キテノ法則

前章ノ新定理ハ次ノ如ク開平ニ應用ス。

> 法則　$\sqrt{N}$ ヲ最初ノ p 桁ダケ決定スル爲ニハ一般ニ N ハ最初ノ p 桁ダケ用意スレバ足ル．但シ N ノ第 p 桁目ニ 1 ヲ加ヘテ見タ時ニ $\sqrt{N}$ ノ第 p 桁目ニ 1 ノ増分ヲ來スガ如キ場合ニ限リ N ノ第 $(p+1)$ 桁目ヲ要ス．但第 $(p+1)$ 桁以下マデヲ要スルガ如キ例外ハ極メテ稀ナリ．此處ニ p ハ正整數．

$\sqrt{N}$ ガ $\sqrt[3]{N}$ ト代ル場合ニモ此法則ハ流用セラレ得，證明ハ第十二章ヨリ容易ニ爲シ得ベシ此處ニハ略ス。

此法則ハ舊來ノ開平法ヲ根本ヨリ改革スルモノナリ。

次ノ例ハ準教科書トモ云フベキ某參考書ヨリ採レリ。

例三．　$\dfrac{3}{7}$ ノ平方根ヲ小數第三位マデ計算セヨ。

前ニ述ベタ樣ニ先ヅ分母ノ根號ヲ取去ル手段ヲ考ヘテ

$$\sqrt{\frac{3}{7}} = \sqrt{\frac{3\times7}{7\times7}} = \frac{\sqrt{21}}{\sqrt{7^2}} = \frac{\sqrt{21}}{7}$$

$\sqrt{21} = 4.5825\cdots\cdots$　デアルカラ上式ハ

$$\sqrt{\frac{3}{7}} = \frac{4.582\cdots\cdots}{7} = 0.654$$

或ハ又次ノ方法ヲ取ツテモヨイ。

$$\frac{3}{7} = 0.42857142\cdots\cdots$$

デアルカラ $\dfrac{3}{7}$ ノ平方根ヲ小數第三位マデ求ムルニハ

0.42857142……ノ平方根ヲ小數第三位マデ求メレバヨイ。ソレニハコノ内ノ小數第六位マデヲ取ツタ數 0.428571 ノ平方根ヲ求メレバヨイカラ

```
0.42 | 85 | 71 | 0.654
  36 |    |    |   6
-----------    |------
   6  85       |  125
   6  25       |    5
-----------     ------
      60  71     1304
      52  16        4
      ------
       8  55
```

トシテ，答ハ 0.654 トスル。

中學生ノ中ニハ $\sqrt{\dfrac{3}{7}}$ ヲ小數第三位マデ計算スルト云フコトヲ

$\frac{3}{7}=0.428\cdots\cdots$ トシテ，$\frac{3}{7}$ ノ値ヲ小數第三位マデ出シテ其平方根ヲ求メタガルモノガ多イカラ注意シナケレバナラヌ。今コノ誤ツタ考ヘデ開平シタトスレバ

```
 0.4 2 | 8          | 0.6 5 4
   3 6 |            |   6
 ------             |-----------
     6   8 0        | 1 2 5
     6   2 5        |     5
     ---------      |-----------
         5 5 0 0    | 1 3 0 4
         5 2 1 6    |       4
         -------
           2 8 4
```

トナツテ矢張リ　答 0.654　ヲ得ルケレド，之レハ偶然ノ暗合デアツテ，一般ニハ眞ノ値ヨリモ小サクナリ勝ノモノデアル。

著者ヨリ先生ヘ謹ンデ呈言

$\sqrt{0.429}<0.655$ デアルカラ先生ノ御議論コソ誤リニテ中學生ノ方ハ正シ。偶然ノ暗合ニハアラズシテ，證明ノ出來ルコトナリ。

簡單ニ考フルモ　$\sqrt{0.428}$ ヲ以テスラ尚 0.000284 ヲ剩シ居ルニアラズヤ，原數中ノ 0.000571 ヲ放棄セリトテ何ンコトカアラン。

此認識不足ハ世界的ノモノニシテ先生御一人ニテハ無之。

因ニ著者工夫ノ新開平法ニテ行ヘバ

```
 ___       .6 5 4
/ 3  =  √ ----------
\/ 7     .4 2 8
          4 2 2 5
         ----------
              2 7 5→
              3 2 5
```


第十七章　新　定　理

> 正數 N ガ $100x\%$ ノ正ノ增分ヲ受クレバ其 m 乘根 $\sqrt[m]{N}$ ハ $100\left(\frac{x}{m}-\frac{x^2}{2m}+\frac{x^2}{2m^2}\right)$ % 以上 $100\frac{x}{m}$ % 以下ノ增分ヲ受ク

之レヲ開平ノミニ就キテ云ヘバ第十一章ノ如ク

> 正數 N ガ $100x\%$ ノ正ノ增分ヲ受クレバ其平方根 $\sqrt{N}$ ハ $(50x-12.5x^2)$ % 以上 $50x$ % 以下ノ增分ヲ受ク

又開立ノミニ就キテ云ヘバ

> 正數 N ガ $100x\%$ ノ正ノ增分ヲ受クレバ其立方根 $\sqrt[3]{N}$ ハ $\left(\frac{100}{3}x-\frac{100}{9}x^2\right)$ % 以上 $\frac{100}{3}x$ % 以下ノ增分ヲ受ク

簡單ナル定理ナレドモ應用價値ハ甚ダ大ナリ。

〔證明〕 正數 N ガ正增分 $100x$ % ヲ受ケ $N(1+x)$ トナル時 $m>1$ トスレバ,

$$\left(\frac{x}{m}-\frac{x^2}{2m}+\frac{x^2}{2m^2}\right)<\left(\sqrt[m]{1+x}-1\right)<\frac{x}{m}$$

ナルベシ,

斯クアレバ第一ノ定理ハ證明セラレタルナリ。

偖二項定理ニヨリテ

$$\sqrt[m]{N(1+x)}$$

$$=\sqrt[m]{N}(1+x)^{\frac{1}{m}}$$

$$=\sqrt[m]{N}\Bigg[1+\frac{x}{m}-\frac{\frac{1}{m}\left(1-\frac{1}{m}\right)}{1\times2}x^2+\frac{\frac{1}{m}\left(1-\frac{1}{m}\right)\left(2-\frac{1}{m}\right)}{1\times2\times3}x^3$$

$$-\cdots\cdots\cdots\cdots$$

$$-(-1)^r\frac{1}{m}\frac{\left(1-\frac{1}{m}\right)}{2}\frac{\left(2-\frac{1}{m}\right)}{3}\frac{\left(3-\frac{1}{m}\right)}{4}\cdots\cdots\frac{\left(r-1-\frac{1}{m}\right)}{r}x^r$$

$$+(-1)^r\frac{1}{m}\frac{\left(1-\frac{1}{m}\right)}{2}\frac{\left(2-\frac{1}{m}\right)}{3}\frac{\left(3-\frac{1}{m}\right)}{4}\cdots\cdots$$

$$\frac{\left(r-1-\frac{1}{m}\right)}{r}\frac{\left(r-\frac{1}{m}\right)}{(r+1)}x^{r+1}$$

$$-\cdots\cdots\cdots\cdots\Bigg]$$

今第四項以下ヲ二項宛括レバ

$$\sqrt[m]{N(1+x)}$$

$$=\sqrt[m]{N}\Bigg[1+\frac{x}{m}-\frac{1}{m}\frac{\left(1-\frac{1}{m}\right)}{2}x^2$$

$$+\frac{1}{m}\frac{\left(1-\frac{1}{m}\right)}{2}\frac{\left(2-\frac{1}{m}\right)}{3}x^3\left\{1-\frac{\left(3-\frac{1}{m}\right)}{4}x\right\}$$

$$+\frac{1}{m}\frac{\left(1-\frac{1}{m}\right)}{2}\frac{\left(2-\frac{1}{m}\right)}{3}\frac{\left(3-\frac{1}{m}\right)}{4}\frac{\left(4-\frac{1}{m}\right)}{5}x^5\left\{1-\frac{\left(5-\frac{1}{m}\right)}{6}x\right\}$$

$$+\cdots\cdots\cdots\cdots$$

$$+(-1)^{r+1}\frac{1}{m}\frac{\left(1-\frac{1}{m}\right)}{2}\frac{\left(2-\frac{1}{m}\right)}{3}\frac{\left(3-\frac{1}{m}\right)}{4}\cdots\cdots$$

$$\frac{\left(r-\frac{1}{m}\right)}{(r+1)}x^{r+1}\left\{1-\frac{\left(r+1-\frac{1}{m}\right)}{(r+2)}x\right\}$$

$$+\cdots\cdots\cdots\cdots\Bigg]$$

即チ r ヲ隅數トスレバ此式ノ右邊括弧內各項中ノ各因數ガ正ニシテ各項自體モ正ナリ，故ニ第四項以下ヲ省ケバ總體ノ値ハ減ジ

$$\sqrt[m]{N(1+x)}>\sqrt[m]{N}\left[1+\frac{x}{m}-\frac{1}{m}\frac{\left(1-\frac{1}{m}\right)}{2}x^2\right]$$

$$\sqrt[m]{N(1+x)}>\sqrt[m]{N}\left[1+\frac{x}{m}-\frac{x^2}{2m}+\frac{x^2}{2m^2}\right]$$

又第三項以下ヲ二項ツヽ括レバ

$$\sqrt[m]{N(1+x)}$$

$$=\sqrt[m]{N}\left[1+\frac{x}{m}-\frac{1}{m}\frac{\left(1-\frac{1}{m}\right)}{2}x^2\left\{1-\frac{\left(2-\frac{1}{m}\right)}{3}x\right\}\right.$$

$$-\frac{1}{m}\frac{\left(1-\frac{1}{m}\right)}{2}\frac{\left(2-\frac{1}{m}\right)}{3}\frac{\left(3-\frac{1}{m}\right)}{4}x^4\left\{1-\frac{\left(4-\frac{1}{m}\right)}{5}x\right\}$$

$$-\cdots\cdots\cdots\cdots$$

$$-(-1)^r\frac{1}{m}\frac{\left(1-\frac{1}{m}\right)}{2}\frac{\left(2-\frac{1}{m}\right)}{3}\frac{\left(3-\frac{1}{m}\right)}{4}\cdots\cdots$$

$$\cdots\cdots\frac{\left(r-1-\frac{1}{m}\right)}{r}x^r\left\{1-\frac{\left(r-\frac{1}{m}\right)}{(r+1)}x\right\}$$

$$\left.-\cdots\cdots\cdots\cdots\right]$$

即チ r ヲ隅數ナリトスレバ各項中ノ各因數皆正ニシテ各項自體ハ第三項以下皆負ナリ，故ニ之レヲ省ケバ總體ノ値ハ増加シ

$$\sqrt[m]{N(1+x)}<\sqrt[m]{N}\left[1+\frac{x}{m}\right]$$

故ニ $\left(1+\frac{x}{m}-\frac{x^2}{2m}+\frac{x^2}{2m^2}\right)<\sqrt[m]{1+x}<\left(1+\frac{x}{m}\right)$

或ハ $\left(\frac{x}{m}-\frac{x^2}{2m}+\frac{x^2}{2m^2}\right)<\left(\sqrt[m]{1+x}-1\right)<\frac{x}{m}$

Q. E. D.

$m=2$, $m=3$ ト置換スレバ容易ニ第二第三ノ定理ヲ證明シ得

第十八章　代數式ノ開平

例一． $\sqrt{a^2+2ab+b^2}=$ ？

說明ノ樣式

$$\begin{array}{lccccc}
 & & & \tfrac{1}{2}b & & \\
 & a & + & b & & \\
\sqrt{} & a^2 & + & 2ab & + & b^2 \\
 & a^2 & & & & \\
\hline
\div 2 & & & ab & + & \tfrac{1}{2}b^2 \\
(a+\tfrac{1}{2}b)b & & & (a & + & \tfrac{1}{2}b)b \\
\hline
 & & & & & 0
\end{array}$$

實際演算

$$\begin{array}{lccccc}
 & a & + & b & & \\
\sqrt{} & a^2 & + & 2ab & + & b^2 \\
 & a^2 & & & & \\
\hline
 & & & ab & + & 0.5b^2 \\
 & & & ab & + & 0.5b^2 \\
\hline
 & & & & & 0
\end{array}$$

答 $(a+b)$

例二． $\sqrt{a^2+2ab+2ac+b^2+2bc+c^2}=$ ？

$$\begin{array}{lccccccccccc}
 & a & + & b & + & c & & & & & & \\
\sqrt{} & a^2 & + & 2ab & + & 2ac & + & b^2 & + & 2bc & + & c^2 \\
 & a^2 & & & & & & & & & & \\
\hline
 & & & ab & + & ac & + & 0.5b^2 & + & bc & + & 0.5c^2 \\
 & & & ab & & & + & 0.5b^2 & & & & \\
\hline
 & & & & & ac & & & + & bc & + & 0.5c^2 \\
 & & & & & ac & & & + & bc & + & 0.5c^2 \\
\hline
 & & & & & & & & & & & 0
\end{array}$$

答 $(a+b+c)$

例三. $\sqrt{9x^2 - 30x + 25}$

$$\begin{array}{llllll}
 & 3x & - & 5 & & \\
\sqrt{} & 9x^2 & - & 30x & + & 25 \\
 & 9x^2 & & & & \\
\hline
 & & - & 15x & + & 12.5 \\
 & & - & 15x & + & 12.5 \\
\hline
 & & & & & 0
\end{array}$$

答 $(3x - 5)$

例四. $\sqrt{25x^2 + 80xy + 64y^2}$

$$\begin{array}{llllll}
 & 5x & + & 8y & & \\
\sqrt{} & 25x^2 & + & 80xy & + & 64y^2 \\
 & 25x^2 & & & & \\
\hline
 & & & 40xy & + & 32y^2 \\
 & & & 40xy & + & 32y^2 \\
\hline
 & & & & & 0
\end{array}$$

答 $(5x + 8y)$

例五. $\sqrt{9x^6 - 12x^3y + 4y^2}$

$$\begin{array}{llllll}
 & 3x^3 & - & 2y & & \\
\sqrt{} & 9x^6 & - & 12x^3y & + & 4y^2 \\
 & 9x^6 & & & & \\
\hline
 & & - & 6x^3y & + & 2y^2 \\
 & & - & 6x^3y & + & 2y^2 \\
\hline
 & & & & & 0
\end{array}$$

答 $(3x^3 - 2y)$

例六. $\sqrt{16a^2 - 56ab + 49b^2}$

$$\begin{array}{llllll}
 & 4a & - & 7b & & \\
\sqrt{} & 16a^2 & - & 56ab & + & 49b^2 \\
 & 16a^2 & & & & \\
\hline
 & & - & 28ab & + & 24.5b^2 \\
 & & - & 28ab & + & 24.5b^2 \\
\hline
 & & & & & 0
\end{array}$$

答 $(4a - 7b)$

例七. $\sqrt{x^4 - 4x^3 + 10x^2 - 12x + 9}$

$$\begin{array}{llllllllll}
 & x^2 & - & 2x & + & 3 & & & & \\
\sqrt{} & x^4 & - & 4x^3 & + & 10x^2 & - & 12x & + & 9 \\
 & x^4 & & & & & & & & \\
\hline
 & & - & 2x^3 & + & 5x^2 & - & 6x & + & 4.5 \\
 & & - & 2x^3 & + & 2x^2 & & & & \\
\hline
 & & & & & 3x^2 & - & 6x & + & 4.5 \\
 & & & & & 3x^2 & - & 6x & + & 4.5 \\
\hline
 & & & & & & & & & 0
\end{array}$$

答 $(x^2 - 2x + 3)$

例八. $\sqrt{x^4 - x^3 + \frac{11}{12}x^2 - \frac{1}{3}x + \frac{1}{9}}$

$$\begin{array}{llllllllll}
 & x^2 & - & \frac{1}{2}x & + & \frac{1}{3} & & & & \\
\sqrt{} & x^4 & - & x^3 & + & \frac{11}{12}x^2 & - & \frac{1}{3}x & + & \frac{1}{9} \\
 & x^4 & & & & & & & & \\
\hline
 & & - & \frac{1}{2}x^3 & + & \frac{11}{24}x^2 & - & \frac{1}{6}x & + & \frac{1}{18} \\
 & & - & \frac{1}{2}x^3 & + & \frac{1}{8}x^2 & & & & \\
\hline
 & & & & & \frac{1}{3}x^2 & & & & \\
 & & & & & \frac{1}{3}x^2 & - & \frac{1}{6}x & + & \frac{1}{18} \\
\hline
 & & & & & & & & & 0
\end{array}$$

答 $(x^2 - \frac{1}{2}x + \frac{1}{3})$

以上ノ六例ヲ「分離係數法」ニテ行ケバ

（三）

$$\begin{array}{rrr} & -2.5 & \\ 3 & -5 & \\ \hline \sqrt{9} & -30 & +25 \\ 9 & & \\ \hline & -15 & +12.5 \\ & -15 & +12.5 \\ \hline & & 0 \end{array}$$

∴　答　$(3x-5)$

（四）

$$\begin{array}{rrr} & 4 & \\ 5 & +8 & \\ \hline \sqrt{25} & +80 & +64 \\ 25 & & \\ \hline & 40 & +32 \\ & 40 & +32 \\ \hline & & 0 \end{array}$$

答　$(5x+8y)$

（五）

$$\begin{array}{rrr} 3 & -2 & \\ \hline \sqrt{9} & -12 & +4 \\ 9 & & \\ \hline & -6 & +2 \\ & -6 & +2 \\ \hline & & 0 \end{array}$$

答　$(3x^3-2y)$

（六）

$$\begin{array}{rrr} 4 & -7 & \\ \hline \sqrt{16} & -56 & +49 \\ 16 & & \\ \hline & -28 & +24.5 \\ & -28 & +24.5 \\ \hline & & 0 \end{array}$$

答　$(4a-7b)$

（七）

$$\begin{array}{rrrrr} 1 & -2 & +3 & & \\ \hline \sqrt{1} & -4 & +10 & -12 & +9 \\ 1 & & & & \\ \hline & -2 & +5 & -6 & +4.5 \\ & -2 & +2 & & \\ \hline & & 3 & & \\ & & 3 & -6 & +4.5 \\ \hline & & & & 0 \end{array}$$

答　(x^2-2x+3)

（八）

$$\begin{array}{rrrrr} 1 & -\frac{1}{2} & +\frac{1}{3} & & \\ \hline \sqrt{1} & -1 & +\frac{11}{12} & -\frac{1}{3} & +\frac{1}{9} \\ 1 & & & & \\ \hline & -\frac{1}{2} & +\frac{11}{24} & -\frac{1}{6} & +\frac{1}{18} \\ & -\frac{1}{2} & +\frac{1}{8} & & \\ \hline & & +\frac{1}{3} & & \\ & & +\frac{1}{3} & -\frac{1}{6} & +\frac{1}{18} \\ \hline & & & & 0 \end{array}$$

答　$(x^2-\frac{1}{2}x+\frac{1}{3})$

第十九章　例題一束（逆對數算出）

（A）　次ノ例題六十三個ハ最初ノヲ（1）トセズ（2）トセリ。

（B）　（2）$\sqrt{10}$，（3）$\sqrt[4]{10}$，（4）$\sqrt{10\sqrt{10}}$，（5）$\sqrt[4]{10\sqrt{10}}$，
（6）$\sqrt{10\sqrt{10\sqrt{10}}}$，tc.　tc.

（C）　何レモ唯一ノ原數 10 ヨリ來ル。

（D）　對數ト深キ關係アリ，例ヘバ　$\sqrt[4]{10\sqrt{10}}=10^{\frac{3}{8}}$ ナルヲ以テ

$$\log\sqrt[4]{10\sqrt{10}}=\frac{3}{8}$$

（E）　從ツテ此六十三個ノ結果ハ逆對數表ヲ作ル。

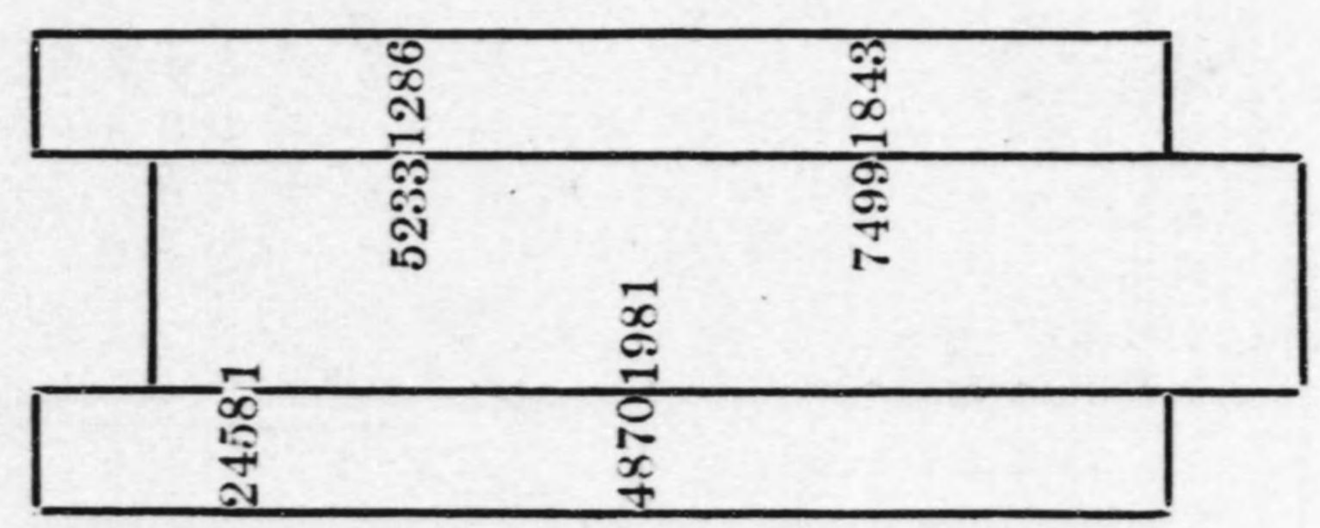
（F）　逆對數曲線ヲ作リ得，而シテ逆對數曲線ヲ裏返セバ對數曲線トナルヲ以テ對數曲線ヲ得，即チ對數表ヲモ作リ得ベシ。（附錄）

（G）　逆對數表ハ別紙附錄ノ如クスレバ「表計算尺」トナル。（附錄）

（H）　此六十三個ノ數ハ自然ニ程ヨキ數値ノ間隔ヲ保ツヲ以テ開平問題參考辭典トモ云フベシ。

（I）　中學生ニモ鉛筆ト紙トダケニテ平易ニ出來ルコト甚ダ面白シ

表計算尺使用例	$2.458\times1.981=$?	答　4.870
	〃　$\times5.233=$?	答　12.86
	〃　$\times7.499=$?	答　18.43



(1)

(2)

```
      3.1 6 2 3
  √ 1 0
    9
      5
      3 0 5
      1 9 5
      1 8 7 8
          7 2
```

(3)

```
    1.7 7 8 3
  √ 3.1 6 2 3
    1
    1 0 8 1
      9 4 5
      1 3 6 1 5
      1 2 1 4 5
        1 4 7
```

(4)

```
      5.6 2 3 4
  √ 3 1.6 2 3
    2 5
      3 3 1
      3 1 8
        1 3 1 5
        1 1 2 2
          1 9 3
```

(5)

```
    1.3 3 3 5
  √ 1.7 7 8 3
    1
      3 8 9
      3 4 5
        4 4 1 5
        3 9 4 5
          4 7
```

(6)

```
      4.2 1 7 0
  √ 1 7.7 8 3
    1 6
      8 9
      8 2
        7 1 5
        4 2 0 5
        2 9 4 5
```

(7)

```
    2.3 7 1 4
  √ 5.6 2 3 4
    4
      8 1 1 7
      6 4 5
      1 6 6 7
      1 6 3 4 5
          3 2 5
```

(8)

```
              ___
      7.5 0 1 1
  √ 5 6.2 3 4
    5 6 2 5
          -8
```

(9)

```
    1.1 5 4 8
  √ 1.3 3 3 5
    1 2 1
        6 1 7 5
        5 6 2 5
          5 5
```

(10)

```
      3.6 5 1 7
  √ 1 3.3 3 5
    1 2 2 5
        5 4 2 5
        3 5 5
        1 8 7 5
        1 8 1 2 5
```

(11)

```
    2.0 5 3 5
  √ 4.2 1 6 9
    4
      1 0 8 4 5
      1 0 1 2 5
          7 2
```

(12)

```
              ___
      6.5 0 6 2
  √ 4 2.1 6 9
    4 2 2 5
        -4 0 5
```

(13)

```
            _
    1.5 4 0 1
  √ 2.3 7 1 4
    2 2 5
        6 0 7
        6 0 8
          -1
```

(14)

```
      4.8 6 9 7
  √ 2 3.7 1 4
    1 6
      3 8 5 7
      3 5 2
        3 3 7
        2 8 9 8
          4 7 2
```

(15)

```
    2.7 3 8 4
  √ 7.4 9 8 9
    4
    1 7 4 9 4 5
    1 6 4 5
      1 0 4 4 5
        8 1 4 5
        2 3
```

(16)

```
              _
      8.6 6 0 4
  √ 7 4.9 8 9
    6 4
      5 4 9
      4 9 8
        5 1 4 5
        5 1 7 8
          -3 3
```

(17)

1.0746
√1.1548
1
774
7245
495

(18)

$3.40\overline{18}$
√11.548
9

(19)

1.2409
√1.5399
144
4995
488
115

(20)

$4.0\overline{7}\overline{58}$
√15.399
16
−3005
28245
−2295

(21)

1.4330
√2.0535
1
52675
48
4675
4245
430

(22)

4.5316
√20.535
2025
1425
13545
705

(23)

1.6548
√2.7384
225
244
155
892
8125
795

(24)

5.2330
√27.384
25
1192
102
172
15645
1555

(25)

1.9109
√3.6517
1
132585
1305
2085
1905
180

(26)

6.0429
√36.517
36
2585
2408
177

(27)

2.2067
√4.8697
4
43
42
1485

(28)

$7.0\overline{2}\overline{17}$
√48.697
49
−1515
14 2
−117

(29)

2.5483
√6.4938
625
1219
1008
211

(30)

8.0584
√64.938
64
469
40125
6775

(31)

$3.0\overline{5}\overline{73}$
√8.6596
9
−1702
$\overline{1}5125$
−2145

(32)

9.3057
√86.596
81
2798
2745
53

(33)

1.0366
```
√1.0746
 1
   373
   3045
     685
```

(34)

3.2781
```
√10.746
  9
   87
   62
   253
   22645
    2655
```

(35)

1.1140
```
√1.2409
 121
    1545
    1105
      440
```

(36)

3.5226
```
√12.409
 1225.
    795
    702
     93
```

(37)

$1.20\overline{29}$
```
√1.4330
 144
    -35
```

(38)

$3.7855\ +$
```
√14.330
  9
  2665
  2345
   320
   2992
    208
```

(39)

1.2864
```
√1.6540
 144
   1074
    992
     82
```

(40)

4.0679
```
√16.548
 16
    274
    2418
     322
```


(41)

1.3824
```
√1.9109
 1
  455
  345
  11045
  1072
    325
```

(42)

4.3714
```
√19.109
 16
  15545
  1245
   3095
   30345
     605
```

(43)

$1.5\bar{1}\overline{451}$
```
√2.2067
 225
   -2165
```
$\overline{15}05$
```
   -670
```

(44)

$4.70\overline{24}$
```
√22.067
 16
  30335
  3045
   -115
```

(45)

$1.60\overline{37}$
```
√3.5483
 1
  77415
  78
    -585
```

(46)

5.0481
```
√25.483
 25
   2415
   2008
    407
```

(47)

1.7154
```
√2.9427
 1
  971
  945
   2635
   1705
    930
```

(48)

5.4247
```
√29.427
 25
  221
  208
   1335
   1082
    253
```

(49)
```
    1.8434
  √3.3982
   1
   1199
   112
      791
      728
       63
```

(50)
```
    5.830 6̄
  √33.982
   25
    449
    432
     171
     17445
      -355
```

(51)
```
   2.0 2̄10
  √3.9242
   4
     -379
      4 2
       19
```

(52)
```
    6.2643
  √39.242
   36
    162
    122
     401
     3738
      272
```

(53)
```
   2.1288
  √4.5316
   4
    2658
    205
     608
     422
     186
```

(54)
```
    6.7317
  √45.316
   36
    4658
    4445
     213
     20145
      1155
```

(55)
```
   2.2876
  √5.2330
   4
    6165
    42
    1965
    1792
     173
```

(56)
```
    7.2339
  √52.330
   49
    1665
    142
     245
     21645
      2855
```

(57)
```
   2.4582
  √6.0429
   4
   102
   88
    14145
    12125
      202
```

(58)
```
    7.7736
  √60.429
   49
    57145
    5145
     5695
     54145
      2805
```

(59)
```
   2.6416
  √6.9782
   625
    364
    255
    1091
    1048
      43
```

(60)
```
    8.3536
  √69.782
   64
    2891
    2445
     446
     41625
      2975
```

(61)
```
   2.8387
  √8.0584
   4
   20292
   192
    109
     8445
     2455
```

(62)
```
    9.0 2̄ 3̄1̄
  √80.584
   81
   -208
    18 2
    -282
```

(63)
```
   3.0505 +
  √9.3057
   9
    15285
    15125
       16
```

(64)
```
    9.6466
  √93.057
   81
    60285
    558
     4485
     3848
      637
```

第二十章　$\sqrt{A\pm\sqrt{B}}$ ナル形ノ計算法改革論

新定理ハ此處ニモ重大ナル應用價値アリ，從來ハ $\sqrt{A\pm\sqrt{B}}$ ヲ $(\sqrt{\alpha}\pm\sqrt{\beta})$ ナル形ニ變形シテ，次ニ α 及ビ β ヲ別々ニ開平シ，最後ニ其和乃至差ヲ採ルコトトシタリ，之レ甚ダシク手間取ル愚策ナル上ニ，$\sqrt{A\pm\sqrt{B}}$ ハ必ズシモ $(\sqrt{\alpha}\pm\sqrt{\beta})$ ニ變形シ得ラルベキモノニアラズ，否變形シ得ラルルモノコソ例外的ノモノナリ，教科書ハ皆斯カル問題ノミヲ選定シ居ルハ滑稽ナリ，只斯カル變形ヲナシ得ル場合モアルモノナルコトヲ教フル點ニ於テ若干ノ意味アルモノニシテ，計算法トシテハ半文錢ノ價値モナキナリ。

新定理ハ次ノ如ク教フ，何ヲ苦ンデカ $(\sqrt{\alpha}\pm\sqrt{\beta})$ ニ變形センヤ

> $\sqrt{A\pm\sqrt{B}}$ ヲ p 桁計算スル爲ニハ，先ヅ B ヲ開平シテ $(A\pm\sqrt{B})$ ヲ大體 p 桁用意シ，必要ヲ生ジタル時ノミ第 $(p+1)$ 桁，乃至以上ヲ用意スベシ。

因ニ $\sqrt{A^2+B}$ ナル形ハ B ガ比較的小ナル場合ニハ甚ダ好都合ナリ。$\sqrt{A^2+B^2}$ モ亦 $\sqrt{(A+B)^2-2AB}$ ニ等シキヲ以テ $\sqrt{A^2+B}$ ニ準ス。

例　，$\sqrt{618^2+2.94}=?$

$$\begin{array}{r} 6\,1\,8.{\scriptstyle 2\,3\,8} \\ \sqrt{\quad 1.4\,7} \end{array}$$

答　<u>6 1 8.2 3 8</u>

例一.　$\sqrt{4+2\sqrt{3}}=?$

$$\begin{array}{r} 3.5\,\overline{3\,5} \\ \sqrt{1\,2\quad\quad} \\ 1\,2\,2\,5 \\ \hline -1\,2\,5 \end{array}$$

$$\begin{array}{r} 2.7\,3\,2 \\ \sqrt{7.4\,6\,4} \\ 4\quad\quad \\ \hline 1\,7\,3\,2 \\ 1\,6\,4\,5 \\ \hline 8\,7 \end{array}$$

答　<u>2.7 3 2</u>

例二.　$\sqrt{5+2\sqrt{6}}=?$

$$\begin{array}{r} 5.\bar{1}\,{\scriptstyle 0}\,\bar{1} \\ \sqrt{2\,4\quad\quad} \\ 2\,5\quad\quad \\ \hline -5\quad \\ \bar{5}\quad 5 \\ \hline -5 \end{array}$$

$$\begin{array}{r} 3.1\,4\,6 \\ \sqrt{9.8\,9\,8} \\ 9\quad\quad \\ \hline 4\,4\,9 \\ 3\,0\,5 \\ \hline 1\,4\,4 \end{array}$$

答　<u>3.1 4 6</u>

例三.　$\sqrt{15-2\sqrt{14}}$

$$\begin{array}{r} 7.5\,\overline{1\,6} \\ \sqrt{5\,6\quad\quad} \\ 5\,6\,2\,5 \\ \hline -1\,2\,5 \end{array}$$

$$\begin{array}{r} 2.7\,4\,2 \\ \sqrt{7.5\,1\,6} \\ 4\quad\quad \\ \hline 1\,7\,5\,8 \\ 1\,6\,4\,5 \\ \hline 1\,1\,3 \end{array}$$

答　<u>2.7 4 2</u>

例四.　$\sqrt{9+4\sqrt{5}}$

$$\begin{array}{r} 9.{\scriptstyle 0}\,\overline{5\,5} \\ \sqrt{8\,0\quad\quad} \\ 8\,1\quad\quad \\ \hline -5\quad \end{array}$$

$$\begin{array}{r} 4.2\,3\,6 \\ \sqrt{1\,7.9\,4\,4} \\ 1\,6\quad\quad \\ \hline 9\,7\,2 \\ 8\,2\quad \\ \hline 1\,5\,2 \end{array}$$

答　<u>4.2 3 6</u>

例五. $\sqrt{2-\sqrt{3}}$

```
  1.7 3 2 0
√ 3
  1
  1
    9 4 5
      5 5
      5 1 4 5
        3 5 5
```

```
     .5 1 7 6
√ 0.2 6 7 9
    2 5
        8 9 5
        5 0 5
        3 9 0
```

答 0.5 1 7 6

例六. $\sqrt{7-3\sqrt{5}}$

```
     6.7 0 8 2
√ 4 5
  3 6
    4 5
    4 4 4 5
        5 5
```

```
     0.5 4 0 2
√ 0.2 9 1 8
    2 5
      2 0 9
      2 0 8
          1
```

答 0.5 4 0 2

例七. $\sqrt{4\frac{1}{2}+\sqrt{8}}$

```
  2.8 2 8
√ 8
  4
  2
  1 9 2
      8
```

```
  2.7 0 7
√ 7.3 2 8
  4
  1 6 6 4
  1 6 4 5
      1 9
```

答 2.7 0 7

例八. $\sqrt{12+2\sqrt{35}}$

```
  1 1.8 3 2
√ 1 4 0
  1 2 1
      9 5
      9 1 2
        3 8
```

```
      4.8 8 2
√ 2 3.8 3 2
  1 6
      3 9 1 6
      3 5 2
        3 9 6
```

答 4.8 8 2

例九. $\sqrt{7+\sqrt{3}}$

```
  1.7 3 2
√ 3
  1
  1
    9 4 5
      5 5
```

```
  3.0 4̅ 5̅
√ 8.7 3 2
  9
  −1 3 4
```

答 2.9 5 5

例拾. $\sqrt{8+\sqrt{2}}$

```
  1.5 8̅ 5̅
√ 2
  2 2 5
  −1̲ 2 5
    1̅ 2 3 2
      −8 2
```

```
  3.0 7 2̅
√ 9.4 1 4
  9
    2 0 7
    2 1 2 4 5
      −5 4 5
```

答 3.0 6 8

例拾壹. $\sqrt{87.4938+\sqrt{7.5361{\to}2}}$

```
  2.7 4 5 1 6→9
√ 7.5 3 6 1
  4
  1 7 6 8
  1 6 4 5
    1 2 3 0 5
    1 0 8 8
      1 4 1 5→
          2 0
```

```
    9.5 0 0 5̅ 7̅→8
√ 9 0.2 3 8 9 6→9
  9 0 2 5
          −5 5 0 5→
                2 0
```

答 9.4 9 9 4 2→3

例一ハ何レノ方法ニヨルモ殆ンド同様ノ手間ヲ要ス。

例二乃至例八ニ至リテ新方法ノ優ルコトヲ知ルベシ。

例九乃至例十一ハ $\sqrt{\alpha}+\sqrt{\beta}$ ナル形ニハ成リ能ハザル問題ナリ。卽チ新方法ニ限ル。

例十一ノ如キ或ハ小數五位迄ノ計算ヲ不可能ナリト考ヘシ人モアラン。

次ノ二例ハ變形出來ル問題ナルモ新方法ノ優ルコトハ顯著ナリ

例拾貳.

$$\sqrt{6+2\sqrt{2}+2\sqrt{3}+2\sqrt{6}}$$

```
      6

   2.8 2 8 4
 √ 8
   4
   2
   1 9 2
       8
     5 6 2
     2 3 8

   3.4 6 4 1
 √ 1 2
     9
     1 5
     1 2 8
       2 2
       2 0 5 8
         1 4 2

   5.1̄ ₀ 1̄0̄2̄
 √ 2 4
   2 5
   -5̄  5
    5̄  5
       -5

   4.1 4 6 2 5+
 √ 1 7.1 9 1 5
   1 6
     5 9 5 7 5
     4 0 5
     1 9 0 7 5
     1 6 4 8
       2 5 8 5
```

答 4.1463

例拾參.

$$\sqrt{8+2\sqrt{2}+2\sqrt{5}+2\sqrt{10}}$$

```
   2.8 2 8 4
 √ 8.

   4.5 2̄ 7̄8̄
 √ 2 0
   2 0 2 5
    -1 2 5
       9 0 2
      -3 4 8

   6.3 2 4 5
 √ 4 0
   3 6
   2
   1 8 4 5
     1 5 5
     1 2 6 2
       2 8 8

   8.
   2.8 2 8 4
   4.4 7 2 1
   6.3 2 4 5
 2 1.6 2 5 0

   4.6 5 ₀ 3
 √ 2 1.6 2 5 0
   1 6
   2 8 1
   2 5 8
     2 3 2 5
     2 3 1 2 5
         1 2 5
```

答 4.6503

次ノ如キ問題モ結局新方法ガ優ル。

例拾四. $\sqrt{2}+\sqrt{3+2\sqrt{5+12\sqrt{3+2\sqrt{2}}}}$

新計算法

```
   2.8 2 8
 √ 8
   4
   2
   1 9 2
       8

   2.4 1 4
 √ 5.8 2 8
   4
     9 1
     8 8
       3 4

2.414 × 12 = 28.968

   5.8 2 8
 √ 3 3.9 6 8
   2 5
     4 4 8
     4 3 2
       1 6 4

   3.8 2 8
 √ 1 4.6 5 6
   9
   2 8 2 8
   2 7 2
     1 0 8

   2.4 1 4
 √ 5.8 2 8
   4
     9 1
     8 8
       3 4
```

答 2.414

舊計算法

$\sqrt{3+2\sqrt{2}}=\sqrt{2}+1$

$\sqrt{5+12\sqrt{3+\sqrt{2}}}$

$=\sqrt{5+12(\sqrt{2}+1)}$

$=\sqrt{17+12\sqrt{2}}$

$=\sqrt{17+2\sqrt{72}}$

$=3+\sqrt{8}$

$\sqrt{3+2(3+\sqrt{8})}$

$=\sqrt{9+2\sqrt{8}}$

$=\sqrt{8}+1$

$\therefore$ 原式 $=\sqrt{2+\sqrt{8}+1}$

$=\sqrt{3+\sqrt{8}}$

$=\sqrt{2}+1$

$=2.414$

答 2.414

此方法ハ一目スル所甚ダあつさりトシテ居ル如クナレドモ事實ハ中々ニ手間ヲ要ス，何レニシテモ新方法ニ限ルト云フベシ

前例ハ兎モ角 $(\sqrt{2}+\sqrt{\beta})$ ナル形ニ變化シ得ル場合ナレドモ，次ノ例ノ如キハ新方法ヨリ外ニ行リ方モナシ。

例拾五. $\sqrt{2+\sqrt{3+\sqrt{4+\sqrt{5+\sqrt{6+\sqrt{7+\sqrt{8}}}}}}}$

```
   2.8 2 8 4
√ 8
  4
  2
  1 9 2
      8
      5 6 2
      2 8 3

   3.1 3 5 0
√ 9.8 2 8 4
  9
    4 1 4
    3 0 5
    1 0 9 2
      9 3 4 5
      1 5 7 5

   3.0 2 2 4
√ 9.1 3 5 0
  9
      6 7 5
      6 0 2
        7 3 0

   2.8 3 2 3
√ 8.0 2 2 4
  4
  2 0 1
  1 9 2
      9 1 2
      8 4 4 5
        6 7 5
```

```
         3 8
   2.6 1 4
√ 6.8 3 2 3
  6 2 5
    2 9 1
    2 5 5
      3 6 1 5
      2 6 0 5
      1 0 1 0

   2.3 6 9 3
√ 5.6 1 4 8
  4       3
    8 0 7
    6 4 5
    1 6 2 9
          1
    1 3 9 8
      2 2 1 0

   2.0 0 0 3
√ 4.3 6 9 3
  4
    1 8 4 5
    1 8 0 5
        4 5
        6 0
```

答 2.0 6 0 3

大文字ノミニテ四桁ヲ與フ.

小文字マデ行ヘバ五桁迄知リ得ベシ。

例十六. $\sqrt{8+\sqrt{7+\sqrt{6+\sqrt{5+\sqrt{4+\sqrt{3+\sqrt{2}}}}}}}$

```
   1.4 1 4 2→
√ 2
  1
    5
    4 8
      2
      1 4 0 5
        5 9 5

   2.1 0 1 0−
√ 4.4 1 4 2→
  4 4 1
        2 1

   2.5 3̄ 0 0 0
√ 6.1 0 0 9→
  6 2 5
      −7 4 5 5
       7̄ 5̄ 4 5

   2.7 3 3 1
√ 7.4 7 0 0→
  4
  1 7 3 5
  1 6 4 5
      9 0
      8 1 4 5
        8 5 5
```

```
   3.0 4̄ 4̄ 7̄
√ 8.7 3 3 1
  9
   −1 3 3 4 5
     1̄ 2   8
    −1 4 2 5

   3.1 5 5 1→
√ 9.9 5 5 2→
  9
    4 7 7 6
    3 0 5
    1 7 2
    1 5 6 2 5
      1 6 3 5

   3.3 4 0 0 7̄
√ 1 1.1 5 5 1→
    9
    1 0 7 7 5 5
      9 4 5
      1 3 2 5 5
      1 3 2 8
          −2 5
```

答 3.3 3 9 9→

第二十一章　例　題

例十七．

$\sqrt{93-\sqrt{8640}}$ ヲ小數第二位マデ精確ニ求ム

先ヅ舊方法ニテハ

$\sqrt{93-\sqrt{8640}} = \sqrt{\alpha} - \sqrt{\beta}$

$\alpha + \beta = 93$

$\alpha\beta = \dfrac{8640}{4}$

$\alpha - \beta = \sqrt{93^2 - 8640}$

$= 3$

$\alpha = 48,\quad \beta = 45$

$\sqrt{93-\sqrt{8640}} = \sqrt{48} - \sqrt{45}$

右邊ノ計算ヲスベキナルガ．先ヅ兩項共小數第三位マデ計算シタリトセヨ

$\sqrt{48} = 6.928$

$\sqrt{45} = 6.708$

0.220

小數第二位ノ２ハ不確實ナリ，依ツテ尙一桁追加シテ

$\sqrt{48} = 6.9282$

$\sqrt{45} = 6.7082$

0.2200

尙モ該２ハ不確實，依テ

$\sqrt{48} = 6.92820$

$\sqrt{45} = 6.70820$

0.22000

尙モ該２ハ不確實，依テ

$\sqrt{48} = 6.928203$

$\sqrt{45} = 6.708203$

0.220000

尙モ該２ハ不確實，依テ

$\sqrt{48} = 6.9282032$

$\sqrt{45} = 6.7082039$

0.2199993

∴　該２ハ駄目，答 0.21


```
                      6.  9   2   8   2   0   3   2
                   √ 48.00000000000000
                      36
129                  1200
  9                  1161
1382                   3900
   2                   2764
13848                  113600
    8                  110784
138562                   281600
     2                   277124
13856403                   44760000
       3                   41569209
138564062                   319079100
        2                   277128124
                             41950976

                      6.  7   0   8   2   0   3   9
                   √ 45.00000000000000
                      36
127                    900
  7                    889
13408                   110000
    8                   107264
134162                    273600
     2                    268324
13416403                    52760000
       3                    40249209
134164069                   1251079100
        9                   1207476621
                              43602479
```

新方法ニテハ

```
      93.0483 ……                  .21 ……
  √ 8640                      √ 0.0483 ……
    81                             4
     270                            415
     2745                           205
      -45                           210
```

答 0.21

〔註〕 $\sqrt{0.0484} = 0.22$

∴　結局以上ダケ必要

次ノ例ハ "Studies on Higher Mathematics" ヨリ

例十八.　$\dfrac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$ ヲ小數第三位マデ計算セヨ。

〔解〕 分母ヲ有理化スレバ　$\dfrac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} = \sqrt{18} - \sqrt{12}$

之ヲ x トオキ平方スレバ

$$x^2 = 30-2\sqrt{18\times 12} = 30-\sqrt{864}$$

x ヲ小數第三位迄正シク求ムルニハ x^2 ヲ小數第六位マデ正シク求ムレバ可ナリ。ヨツテ次ノ如ク計算ス。

8,6 4	2 9.3 9 3 8 7 6
4	4 9
4 6 4	9
4 4 1	5 8 3
2 3 0 0	3
1 7 4 9	5 8 6 9
5 5 1 0 0	9
5 2 8 2 1	5 8 7 8 3
2 2 7 9 0 0	3
1 7 6 3 4 9	5 8 7 8 6 8
5 1 5 5 1 0 0	8
4 7 0 2 9 4 4	5 8 7 8 7 6 7
4 5 2 1 5 6 0 0	7
4 1 1 5 1 3 6 9	5 8 7 8 7 7 4 6
4 0 6 4 2 3 1 0 0	6
3 5 2 7 2 6 4 7 6	

$$x^2 = 30-29.393876\cdots\cdots = 0.606123\cdots\cdots$$

0.6 0,6 1,2 3	0.7 7 8
4 9	1 4 7
1 1 6 1	7
1 0 2 9	1 5 4 8
1 3 2 2 3	8
1 2 3 8 4	

Ans.　0.7 7 8

〔注意〕 $\sqrt{\alpha}$ ト $\sqrt{\beta}$ トヲ小數第三位マデ正シクトツテ差ヲ求ムレバ，誤差ハ 0.001 ヨリ小サクナリマスガ，ソレデモ小數第三位マデ果シテ正シイカ否カハ不明デス。ソレデ豫メ第何位マデ各ヲ求メテ計算スレバ充分カト云フ斷定ハ得ラレヌノデス。本問ニ於テハ $\sqrt{18} = 4.242\underline{6}\cdots\cdots$，$\sqrt{12} = 3.464\underline{1}\cdots\cdots$ デスカラ，各ヲ小數第三位マデトツテ以下切捨テヽ計算シテモ正シイ答ガ得ラレルノデスガ，コレハ一般的デハナイノデスカラ御注意下サイ。

───○───

著者ヨリ先生ヘ謹ンデ呈言

此解答ニ二ツノ缺點アリ，先ヅ x ヲ平方セラレシコト愚策ナリ第二ニ假リニ第一ハ之レヲ認メルトスルモ x^2 ヲ小數第六位マデ求メラレタルナド言語同斷ナリ，蓋シ小數第四位マデニテ十分ナリ否 $\sqrt{0.6068} = 0.778\cdots\cdots$ ナルヲ以テ 0.6061 ノ小數第四位ハ 8 ヲ超ヘザルコトヲ知ルノミニテ十分ナリ。換言スレバ $\sqrt{0.6061\to 8} = 0.778$ ナリ，$\sqrt{0.6069}$ ニ至リテ初メテ $\sqrt{0.6069} = 0.779$ ナリ。

但シ此認識不足ハ世界的ノモノニシテ先生御一人ノ御失策ニハ無之

（次頁ニ續ク）

新定理ニ從ヒ新方法ヲ以テ計算スレバ

第一方法　$\sqrt{18}$, $\sqrt{12}$　ヲ別々ニ計算スルトスレバ

```
      4.2 4 2 6→8
 √ 1 8
   1 6
     1
       8 2
       1 8
       1 7 7 2 8 2
             2 7 1

      3.4 6 4 1→2
 √ 1 2
     9
     1 5
     1 2 8
       2 2
       2 0 5 8
         1 4 2
```

```
  4.2 4 2 6→8
  3.4 6 4 1→2
```

答　0.7 7 8

第二方法　假リニ先ヅ x^2 ノ形ニスルトシテモ

```
     3 0.6̄ 0 6̄ 1̄→2̄
 √ 8 6 4
   9
   −1 8
     1 8 1 8
       −1 8
         1 8 3 6 1 8
             −3 6

         0.7 7 8→9
   √ 0.6 0 6 1
       4 9
         5 8 0 5
         5 1 4 5
           6 6→
             5
```

答　0.7 7 8

見ヲ在リノ儘主義ニ限ルコトヲ

原文ノ計算ニハ	178 數字	
第一方法ナレバ	47 〃	73.5%節約
第二方法ナレバ	51 〃	71.3%節約

第二十二章　入學試驗問題（一般ニ問題中ノ一部分ヲ採ル）

平凡ナル開平問題

海軍生徒採用試驗七年(1)

$x = \sqrt{\frac{2}{3}}$　小數三位迄

```
      .8 1 6
 √ 0.6 6 6 6
     6 4
       1 3 3
         8 0 5
         5 2 5
```

答　0.8 1 6

桐生高工七年(3)

$x = \frac{3}{4} \pm \sqrt{\frac{5}{16}}$　小數三位迄

$x = 0.75 \pm \sqrt{0.3125}$

```
      0.5 5 9
 √ 0.3 1 2 5
     3 0 2 5
         5
```

```
   0.7 5
 +   5 5 9
   1.3 0 9
   0.1 9 0
```

$x =$ 1.309　或ハ　0.190

福岡女子專門七年(1)

$x = \frac{3}{\sqrt{2}} = \sqrt{4.5}$　小數二位迄

```
    2.1 2
 √ 4.5
   4
     2 5
     2 0 5
       4 5
```

答　2.1 2

高知高等七年(1)

$x = -1 \pm \sqrt{13}$　四捨五入ニテ小數三位迄

```
     3.6 0 6
 √ 1 3
   1 2 2 5
       3 7 5
       3 5 5
         2
```

$x =$ 2.606　或ハ　−4.606

東京外語七年代〔1〕

$x = \sqrt{24}$ 四捨五入制ヨリ小數二位迄

```
      5. 1 0 1
  √ 2 4
    2 5
      -5
        5 0 5
          -5
```

答 4. 9 0

京城帝大豫科七年幾〔1〕

$x = 2\sqrt{1200} + 40\pi + \frac{20}{3}\pi$

```
      7 0. 7 1
  √ 4 8 0 0
    4 9
      -5
```

∴ $2\sqrt{1200} = 69.28\cdots\cdots$

廣島高師七年〔3〕

$x = 45 - \sqrt{675}$

```
    2 6. 0 2
  √ 6 7 5
    4
    1 3 7 5
    1 3 8
        -5
```

$x = 19.02$

陸軍士官七年代〔2〕

$2\sqrt{\frac{7}{3}}$ 米 トハ

```
    1. 5 2 7 5
  √ 2. 3 3 3
    2 2 5
        4 1 6
        3 0 2
        1 1 4
```

之レヨリハ次ノ方ガ早シ

```
    3. 0 5 5 弱
  √ 9. 3 3 3
    9
      1 6 6
```

答 3. 0 5 5 m. 弱

同 七年〔3〕(ii)

$x = -\sqrt{\frac{4}{3}}$ 小數第三位迄

```
    1. 1 5 4
  √ 1. 3 3 3
    1 2 1
        6 1
        5 6 2 5
          4 7
```

$x = -1.154$

山形高等八年〔6〕

$x = \sqrt{15}$ 小數二位迄

```
      4. 1 2 6
  √ 1 5
    1 6
      -5
        4   5
      -1 0 5
```

答 3. 8 7

廣島高等學校八年〔8〕

$m \leqq \sqrt[4]{5184} < (m+1)$

ヲ滿足サセル整數 m 如何

```
      7 2              8. 5
  √ 5 1 8 4        √ 7 2
    4 9                7 2. 2 5
      1 4 2
```

∴ $m = 8$

廣島高師八年角〔3〕

$x = \sqrt{6} + 3$

```
    2. 5 5 0 5
  √ 6
    6 2 5
     -1 2 5
      1 2 5
            1 2 5
          -1 2 5
```

答 5. 4 5 0

東京高等商船八年〔3〕

$x = 12.5 \pm \sqrt{43.75}$

```
      6. 6 1
  √ 4 3. 7 5
    4 2. 2 5
          7 5
        6 5 5
          9 5
```

$x = 19.11$ 或ハ 5.89

成城高等八年數物〔1〕

$x = \sqrt{6} + \sqrt{3}$ 小數二位迄

```
    2. 5 5 0          1. 7 3 2
  √ 6              √ 3
    6 2 5              1
   -1 2 5              1
                         9 4 5
                           5 5
```

$x = 4.18$

```
別法  x = √(9 + 2√18)

      8. 5 1            4. 1 8
  √ 7 2              √ 1 7. 4 8
    7 2                1 6
      -1 2 5                7 4
                            4 0 5
                            3 3→
                              4
```

第三高等八年〔1〕

$$x = \frac{5 + 2\sqrt{7}}{3} \quad \text{四捨五入}$$
$$x = \frac{20 + 8\sqrt{7}}{3} \quad \text{小數三位迄}$$

```
  2 1.1 6 6 0 強
√ 4 4 8
  4
  ─────
    2 4
    2 0 5
    ─────
      3 5
      3 3 7 2 8
      ─────────
        1 2 7 2
```

41.1660 ÷ 3 = 13.722

13.722 ÷ 4 = 3.4305

x = 3.431; x = 13.722

前者ハ後者ノ $\frac{1}{4}$ ナルヲ以テ後者ダケ計算シテ後4分シテ前者ヲ得。

海軍兵學校八年代〔2〕

$x = \sqrt{2}$ 小數二位迄

```
  1.5 8̄
√ 2
  2 2 5
  ───────
  −1 2 5
```

x = 1.41

明治專門八年數物〔1〕

$$\tan 2x = \pm\frac{4}{17}\sqrt{21}$$

```
  1 8.3 3              1.0 7 8
√ 3 3 6          17 │ 1 8.3 3
  1                 │ 1 7
  ─────               ─────
  1 1 8                 1 3 3
  1 1 2                 1 1 9
  ─────                 ─────
      6                   1 4
```

$\tan 2x = \pm 1.078$

和歌山高商八年代〔3〕

$$x = \frac{-1 + \sqrt{1001}}{2}$$
$$= -0.5 + \sqrt{250.25}$$

```
  1 5.8
√ 2 5 0.2 5
  2 2 5
  ─────
    1 2 6 2 5
    1 2 3 2
    ─────────
          3
```

x = 1 5.3

京城醫專八年〔4〕　參考

$$x = \frac{1 + \sqrt{37}}{2}$$

```
     6.0 8 2
√ 3 7
  3 6
  ───
    5
```

x = 3.5 4 1

松江高等　七年〔2〕

$$x = \sqrt{\frac{1}{10}} + \sqrt{\frac{1}{30}} \quad \text{小數三位迄}$$

```
    .3 1 6 2
√ 0.1 0
      9
    ───
      5
    3 0 5
    ─────
    1 9 5

    .1 8 2 5
√ 0.0 3 3 3 3
      1
    ─────
    1 1 6
    1 1 2
    ─────
        4 6
```

答　0.4 9 8

成蹊高等　七年〔3〕

$p = -2.5 \pm \sqrt{11.25}$

小數二位迄

```
    3.3 5 4
√ 1 1.2 5
    9
  ─────
  1 1 2 5
    9 4 5
  ───────
    1 8
```

p = 0.85 或ハ −5.85

同　同　〔4〕

$$\sqrt{\frac{175}{9}} = ?$$

```
   4.4 0 8
√ 1 9.4 3
  1 6
  ─────
    1 7 1 5
    1 6 8
    ───────
        3 5
```

答　4.4 0 8

京城醫專　八年〔4〕

$$x = \frac{1 + \sqrt{37}}{2}$$

```
     6.0 8 3
√ 3 7
  3 6
  ───
    5
```

x = 3.5 4 1

第二十三章　入學試驗問題（一般＝問題中ノ一部分ヲ採ル）

新定理ノ効果顯著ナル例

上田蠶絲專門　七年〔5〕

$x = \sqrt[4]{\frac{16}{3}}$

```
   2.3 0 9          1.5 1 9
 √ 5.3 3 3        √ 2.3 0 9
   4                2 2 5
     6 6 6              2 9 5
     6 4 5
       2 1
```

$x = \underline{1.519}$

神戸高工　七年〔1〕

$x = \sqrt{\frac{48 \times 8 \times 90000}{129}}$

小數切上

$x = 100\sqrt{27.6\cdots\cdots}$

```
      5.2 6
  √ 2 7.6
    2 5
      1 3
      1 0 2
        2 8
```

$x = \underline{526}$

鳥取高農　七年〔1〕　其儘

$\sqrt{472758049} = ?$

```
   2 1 7 4 3.
 √ 4 7 2 7 5 8 0 4 9.
   4
     3 6 3 7 9
     2 0 5
     1 5 8 7 9
     1 4 9 4 5
         9 3 4 0
         8 6 8 8
           6 5 2 2 4 5
           6 5 2 2 4 5
                     0
```

答　$\underline{21743}$

今原數ノ終四桁ヲ切捨テヨ

```
   2 1 7 4 3
 √ 4 7 2 7 5
   4
     3 6 3 7 5
     2 0 5
     1 5 8
     1 4 9 4 5
         9 3
         9 3 4 0 2 4 5
           - 4 0 2 4 5
```

不足ハ丁度切捨テタル部分ノ半ナリ

故ニ　　答　$\underline{21743}$

東京女高師　八年〔2〕

$\sqrt{88209} = ?$

```
   3 0 3̄
 √ 8 8 2 0 9
   9
     - 8 9 5 5
       9̄   4 5
             0
```

答　$\underline{297}$

前例ノ如ク終二桁ヲ切捨テナバ

```
   3 0 3̄
 √ 8 8 2
   9
     - 9
       9̄   4 5
         - 4 5
```

省カレタル部分ガ最後ノ不足ノ二倍ナリ故ニ答ハ正シ。

海軍機關學校　八年代〔1〕

$p = 1 - \sqrt{\frac{1}{3}}$　小數二位迄

```
    .5 7 7 2
 √ .3 3 3 3
    2 5
      4 1 6
      3 7 4 5
        4 1 5
```

$p = \underline{0.422}$

海軍兵學校　八年幾〔2〕

$V = \sqrt{\frac{625}{2}}$，$V_1 = \sqrt{\frac{1250}{9}}$

```
   1 7.7             1 1.8
 √ 3 1 2.5         √ 1 3 8.8
   1                 1 2 1
   1 0 6 2               8 9
     9 4 5
   1 1 7
```

$V = \underline{17.1}$　　$V_1 = \underline{11.8}$

第六臨數（女）　八年〔5〕

$x = \frac{\sqrt{50} - 5}{2}$　小數二位迄

```
     3.5 3
  √ 1 2.5
    1 2 2 5
        1 2 5
```

$x = \underline{1.03}$

上田蠶絲專門　八年〔1〕

$\sqrt[6]{46656} = ?$

```
   2 1 6                 6
 √ 4 6 6            ∛ 2 1 6
   4 4 1               2 1 6
     1 2 5
     1 2 7 8
       - 2 8
```

答　$\underline{6}$

第二高等學校理　八年〔6〕

$y = 5\sqrt{2} - \sqrt{3}$

```
   1.7 3 2          .5 1 7 7
√ 3             √.2 6 8
  1                 2 5
  1                     9
     9 4 5            5 0 5
       5 5            3 9 5
```

$y = \underline{2.588}$

東京府立高等　七年〔1〕

$x = \dfrac{1 \pm \sqrt{5 - \sqrt{27}}}{2}$　小數二位迄

```
      5.1̄ 0 1̄         .3 1
√ 2 4            √.1 0 1
  2 5                9
     -5                5 5→
      5̄  5             6 0
        -5
```

或ハ次ノ如ク計算スルモ可ナリ

$x = 0.5 \pm \sqrt{1.25 - \sqrt{1.5}}$

```
   1.2 2 4 7         .1 5 +
√ 1.5            √.0 2 5 2
  1 4 4               2 2 5
      3
      2 4 2
        5 8
```

$x = \underline{0.65 \text{ 或ハ } 0.34}$

臺南高工　八年〔1〕

$x = \sqrt{21 + 12\sqrt{3}}$

```
   2 0 7 8 5          6.5 3̄6̄
√ 4 3 2          √ 4 1.7 8 5
  4                 4 2 2 5
    1 6               -2 3 2
    1 4 2 4 5
      1 7 5 5
```

$x = \underline{6.464}$

假想問題

$\sqrt{2.947 + \sqrt{53618}} \cdots\cdots$

```
     7.3 2 2……
√ 5 3.6 1 8
  4 9
     2 3 0 9
     2 1 4 5
       1 6 4

     3.2 0 5
√ 1 0.2 6 9
  9
    6 3
    6 2
      1 4 5
      1 6 0 1 2 5
     -1 4 1 2 5
```

答　$\underline{3.205}$

勿論新方法ニ限ル問題ナリ。

東京商大專門部　八年〔2〕

> $8.02(\sqrt{582} - 24)$ ト 1 ト何レガ大ナルカ。

左ノヲ x トシ

$8.02(\sqrt{582} - 24) - 1$

$(\sqrt{588} - 24) - \dfrac{1}{8.02}$

$\sqrt{583} - (24 + \dfrac{1}{8.02})$

$\sqrt{582} - 24.124688$

之レガ正負ヲ檢スレバ足ル，2, 4, 1, 2, 4, 6, 8, 8 ヲ立テテ過剰乃至不足ヲ生ズルノヲ檢スレバ可ナルヲ以テ開平ニ當リ「次ニ何ガ立ツカ」ナドト考フル必要ナシ。

```
   2 4.1 2 4 6 8弱
√ 5 8 2
  4
    9 1
    8 8
      3
      2 8 8 7 2
        1 1 2 8
```

∴　$x < 1$　Q. E. I

別　法

$x - 1 = 8.02(\sqrt{582} - 24) - 1$

$= \sqrt{8.02^2 \times 582}$

$- 8.02 \times 24 - 1$

$= \sqrt{37434.4728} - 193.48$

之レガ正負ヲ檢スレバ足ル

```
   1 9 3.4 8      9̄
√ 3 7 4 3 4.4 7 2 8
  1
  1 3 7 1 7
  1 3 0 5
      6 6 7 2 3 6
      6 5 1 7 8
        1 5 4 5 6 4
        1 5 4 7 5 2
           -1 8 8
```

∴　$x - 1 < 0$

∴　$x < 1$　Q.E.I.

註

$\bar{9}$ ナル數字ハ最早必要ニアラズ，只下ニ不足ノ出タル事實ヨリ解決ハ明瞭ナリ。

何レニシテモ開平ノ良問題ナリ。新開平法ニモ御誂向キナリ。

大阪商大豫科　七年〔2〕

$$x = \frac{\sqrt{26 - 15\sqrt{3}}}{5\sqrt{2} - \sqrt{38 + 5\sqrt{3}}}$$

先ヅ舊方法ニテ行ヘバ

$$\sqrt{38 + 5\sqrt{3}} = \sqrt{\alpha} + \sqrt{\beta}$$

$$\alpha + \beta = 38$$

$$4\alpha\beta = 75$$

$$\alpha - \beta = \sqrt{38^3 - 75}$$

$$= \sqrt{1444 - 75}$$

$$= \sqrt{1369}$$

$$= 37$$

$$\alpha = \frac{75}{2}, \quad \beta = \frac{1}{2}$$

$$\therefore \text{分母} = 5\sqrt{2} - \sqrt{\frac{75}{2}} - \sqrt{\frac{1}{2}}$$

$$= 10\sqrt{\frac{1}{2}} - \sqrt{\frac{1}{2}} - \sqrt{\frac{75}{2}}$$

$$= 9\sqrt{\frac{1}{2}} - \sqrt{\frac{75}{2}}$$

次ニ

$$\sqrt{26 - 15\sqrt{3}} = \sqrt{a} - \sqrt{b}$$

$$a + b = 26,$$

$$4ab = 675$$

$$a + b = \sqrt{26^2 - 675}$$

$$= 1$$

$$a = \sqrt{\frac{27}{2}}, \quad b = \sqrt{\frac{25}{2}}$$

$$x = \frac{\sqrt{\frac{27}{2}} - \sqrt{\frac{25}{2}}}{9\sqrt{\frac{1}{2}} - \sqrt{\frac{75}{2}}}$$

$$= \frac{\sqrt{27} - \sqrt{25}}{9 - \sqrt{75}}$$

$$= \frac{3\sqrt{3} - 5}{9 - 5\sqrt{3}}$$

$$= \frac{(3\sqrt{3}-5)(5\sqrt{3}+9)}{81 - 75}$$

$$= \frac{\sqrt{3}}{3}$$

$$= \frac{1.732}{3}$$

$$= 0.577$$

答　0.577

〔註〕

分子分母ニ$(5\sqrt{2}+\sqrt{38+5\sqrt{3}})$ヲ乘ジ，次ニ$(12+5\sqrt{3})$ヲ乘ゼントスル行キ方ハ失敗スベシ。

上記ノ問題ヲ新計算法デ行フモ尚若干有利ナリ，而カモ舊方法ハ簡單化出來ザル問題ニハ困難ナルヲ新方法ハ何ンノ苦モナク之レヲ

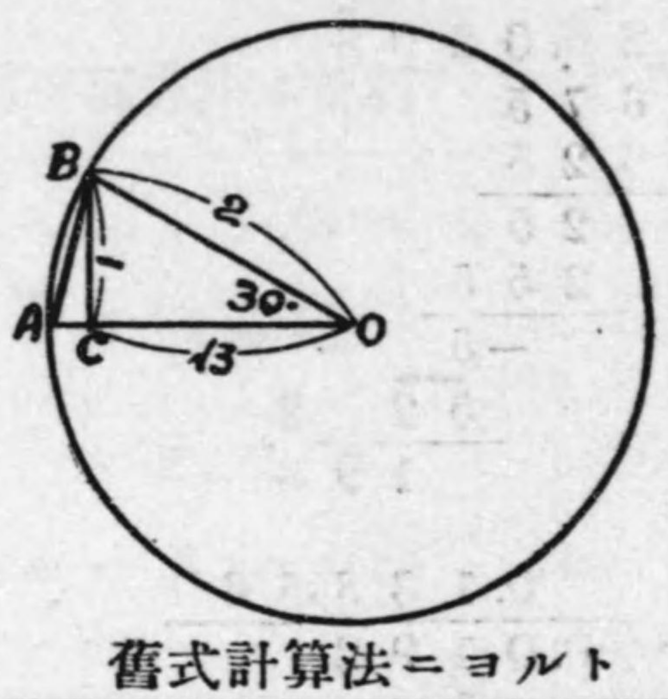
8.6603 √75 64 55 498 52 5178 22	26.0208̄ 675 625 25 255 −5 52　2 198
6.8308 √46.6603 36 533 512 2.1015 20445 570	0.13856 √0.0192 1 46 345 115 1072 78
7.0711 √50 49 5 49245 755	0.577 0.2403) 0.13856 12015 1841 16821 1589
7.0711 6.8308 0.2403	

答　0.577

左レド分母ヲ無理ノママニ進マントスルハ多少無理ト云フベシ，只從來ハ平方根ニ於ケル誤差ノ感念ニ錯覺アリテ必要以上ノ取越苦勞ヲナシタルヲ覺醒セシメントスルニアリ。

旅順工科大學昭和七年度入學試驗問題〔3〕

半徑 2 ナル圓ニ內切スル正十二邊形ノ一邊ノ長サヲ四捨五入法ニヨリテ小數第三位マデ計算セヨ。



圓Oノ半徑ヲ2トシ，ABヲ之レニ內切スル正十二邊形ノ一邊ナリトス，Bヨリ半徑OAニ垂線BCヲ下ス

$$AB=\sqrt{BC^2+AC^2}=\sqrt{1+(2-\sqrt{3})^2}$$
$$=\sqrt{3-\sqrt{48}}$$

舊式計算法ニヨルト

$$\sqrt{8-\sqrt{48}}=\sqrt{\alpha}-\sqrt{\beta}$$
$$8-\sqrt{48}=\alpha+\beta-2\sqrt{\alpha\beta}$$
$$\alpha+\beta=8$$
$$\alpha\beta=\frac{48}{4}$$
$$\alpha-\beta=\sqrt{8^2-48}=4$$
$$\alpha=\frac{8+4}{2}=6$$
$$\beta=\frac{8-4}{2}=2$$
$$\sqrt{8-\sqrt{48}}=\sqrt{6}-\sqrt{2}$$

```
            2. 4 4 9 4
          √6.0 0 0 0 0 0 0 0
           4
  4 4      2 0 0
   4       1 7 6
  4 8 4      2 4 0 0
     4       1 9 3 6
  4 8 8 9      4 6 4 0 0
        9      4 4 0 0 1
  4 8 9 8 4      2 3 9 9 0 0
          4      1 9 5 9 3 6
                   4 3 9 6 4

  2.4 4 9 4
  1.4 1 4 2
  1.0 3 5 2      AB = 1.035
```

```
              1.  4  1  4  2
            √2.0 0 0 0 0 0 0 0
             1
  2 4        1 0 0
    4          9 6
  2 8 1          4 0 0
      1          2 8 1
  2 8 2 4        1 1 9 0 0
        4        1 1 2 9 6
  2 8 2 8 2          6 0 4 0 0
          2          5 6 5 6 4
                       3 8 3 6
```

新式計算法デハ

```
     6.9 2 8 2……
  √ 4 8
    3 6
      6
    5 8 0 5
      1 9 5
      1 3 8 2
        5 6 8
        5 5 3 9 2
          1 4 0 8

    1.0 3 5
  √ 1.0 7 1 7 ……
    1
      3 5 8 5
      3 0 4 5
        5 4 0
```

$\therefore$ AB = 1.035

赤字法デ行ルト侚速イ、

四高　七年〔4〕

$\sqrt{1+x}-\sqrt{1-x}=y^3$ ニ於テ x ニ如何ナル實數値ヲ與フル時 y ガ極小トナルカ，而シテ其時ノ y ノ値ヲ小數第二位マデ計算セヨ。

舊式答案

y ガ極小テラバ y^3 モ極小，與ヘラレタ式ノ左邊モ極小。其左邊ハ x ガ小サキ程小サシ，而シテ x ハ -1 ヨリ小トハナリ得ズ，故ニ -1 トシ

$$y^3=-\sqrt{2},\quad \therefore\ y=-\sqrt[6]{2}$$

```
               1.  4  1  4  2  1  3
             √2.0 0 0 0 0 0 0 0 0 0 0 0
              1
  2 4         1 0 0
    4           9 6
  2 8 1           4 0 0
      1           2 8 1
  2 8 2 4         1 1 9 0 0
        4         1 1 2 9 6
  2 8 2 8 2           6 0 4 0 0
          2           5 6 5 6 4
  2 8 2 8 4 1           3 8 3 6 0 0
            1           2 8 2 8 4 1
  2 8 2 8 4 2 3         1 0 0 7 5 9 0 0
              3           8 4 8 5 2 6 9
                          1 5 9 0 6 3 1
```

〔滿點？〕

```
                   1.    1    2
              ∛ 1.4 1 4 2 1 3
                1
                  4 1 4
  3 0 0 × 1       3 0 0
    3 0 × 1         3 0
          1           1
                  8 3 2 1 3
  3 6 3 × 2       7 2 6
    1 2 × 1 1 0     1 3 2 0
        4 × 2             8
                    9 2 8 5
```

$\therefore\ y=-1.12$

◇證明ノ出來ルコトナリ

◇要求桁數ヲ增セバ難易ノ差ハ一層甚ダシクナル

新式答案

$$\left\{\begin{array}{l} y=-\sqrt[6]{2} \\ \text{迄ハ舊式ト} \\ \text{同樣ニ行フ} \end{array}\right\}$$

```
      1.4 1
   √ 2
      1
        5
        4 8
          2

      1.1 2
   ∛ 1.4 1
      1.3 3 1
          7 9
          2 6→
          3 0
```

$\therefore\ y=-1.12$

〔滿　點〕

a）要スルニ

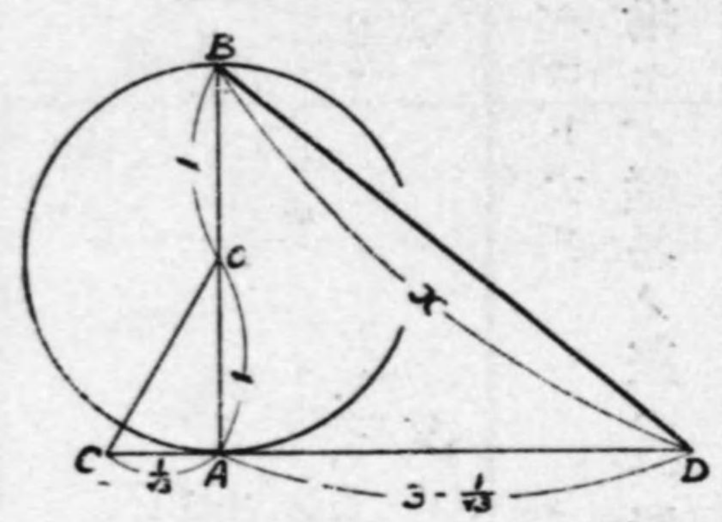
$$\sqrt[3]{1.42}<1.13$$

ヲ確カメテ置ク。

b）$1.1^3=1.331$ ヲ知ラザル時ニハ一手間多ク掛カル。

高知高等學校昭和八年度〔4〕

> 中心 O ナル圓ノ直徑 AB ノ一端 A ニ於ケル切線上ニ一點 C ヲトリ $\angle AOC = 30^\circ$ ナラシム。又 CA ノ延長上ニ一點 D ヲトリテ CD ヲ此圓ノ半徑ノ三倍ニ等シカラシムレバ，線分 DB ノ長サハ略々半圓周ノ長サニ等シキコトヲ證明セヨ。
>
> 但シ　$\sqrt{3} = 1.73205081$　トシテ計算スベシ。



半徑ヲ 1 トセヨ。

$$CA : AO = 1 : \sqrt{3}$$

$$\therefore \quad CA = \frac{1}{\sqrt{3}}$$

$$\therefore \quad AD = 3 - \frac{1}{\sqrt{3}}$$

而シテ　$AB = 2$

DB $= x$　トシ，$x = \sqrt{2^2 + \left(3 - \frac{1}{\sqrt{3}}\right)^2} = \sqrt{13.\dot{3} - 2\sqrt{3}}$

之レガ半圓周ニ近似スレバ可ナリ，

然ルニ $\sqrt{3} = 1.73205081$ ト與ヘラレタルヲ以テ

$x = \sqrt{9.86923169{\to}72}$

卽チ右側ノ如キ計算ニヨリ

$x = 3.141533334{\to}40$

```
            075
   3. 1  4  1  5  3  3  3  3  4→4 0
 √ 9. 8  6  9  2  3  1  6  9→7 2
   9
     4 3 4 6 1 5 8 4 5
     3 0 5
     1 2 9 6
     1 2 4 8
         4 8 1 5 8 4 5
         4 7 1 1 1 2 5
           1 0 4 7 2 0
           1 0 4 7 0 6
                   1 3→2 9
```


卽チ小數第九位ハ 4 以上トノミ明白ナルノミナレドモ第八位迄ハ明瞭ニ決定セラルベシ，之レヲ圓周率 3.14159265 ニ比スレバ四桁モ相違ス。之レ遠似ニシテ近似ニアラズ。

依ツテ結論トシテ言フ，$\sqrt{3}$ ヲ小數第四位マデ或ハソレ以下ヲ與ヘラレルカ或ハ別段ニ $\sqrt{3}$ ノ値ヲ與ヘラレザリセバ天下ノ良問題ナカルベカリシヲ殘念ナリキ。

本問題ニ就キテハ世上諸先生ノ解義ヲ與ヘラレタルモノヲ見ルニ一ツトシテ此點ニ注意セラレタルモノナシ，而シテ 3.1415 マデ計算シテ「故ニ半圓周ノ長サニ近似ス」ト結バレタリ。

今日迄ハ皆斯クノ如ク解クベク敎ヘラレタルナリ。

今假リニ $\sqrt{3}$ ノ値ヲ與ヘラレズトセンカ

```
      3.5 3̄ 5̄ 9̄
   √ 1 2
     1 2 2 5
        −1 2 5
          1 0 5 4 5
         −2 0 4 5

      1 3.3 3 3 3
 −        3.5 3 5 9
          9.8 6 9 2
```

```
     3.1 4 1 5 强
   √ 9.8 6 9 2
     9
       4 3 4 6
       3 0 5
       1 2 9 6
       1 2 4 8
           4 8
```

結果ハ半圓ノ長サ 3,1416 弱ニ近似ス。

FORWORD

Prof. Hitosi Siio,

The Eighth Higher School.

Mr. Eimei Hirano, who was born of a family of mathematicians, is a hard worker in the field of mathematics, though he was once graduated from Nagoya College of Technology. His career does not allow him to waste time in handling curio mathematics, or what we call mathematical philosophy ; and he has an interest in such branches only that have some value in practical application.

I myself have also made a special study of mathematics ; I am no more interested than he is in curio mathematics, which are nothing but a pastime. So naturally, since we came to know each other, we have come to make our friendship closer and closer.

Mr. Hirano first made inventions in slide rules, then, he intended to teach logarithms and the making or rather creation of the logarithmic tables by continual extraction of square roots of 10 (See page 17 of this book). This necessitated him to extract a square root so often. During the course of his hard work, he happened to hit upon a new idea of transforming the old Japanese method for the extraction of a square root by abacus to written arithmetic calculation. He also thought out to do the task, not only by digits, but also by groups of digits. Moreover he invented what he calls the "Red Method" in which either a negative digit or a group of negative difgits is ellected.

He also came to notice that almost all the teachers of mathematics in the world are wrong, or at least short of cognizance in the nature of a square root and a cube root. They fancied, "In order to determine p places of $\sqrt{N}$, $2p$ places of N must be prepared, when only p places thereof will generally do as well." At last, he came to discover his Theorem on an Increment of $\sqrt{N}$, and of $\sqrt[m]{N}$ at large, and the Hiranian Rule II for Evolution in General (See page 6 of this book).

When I heard this from him, I said: "Of course everybody does know that we require only p places of N instead of $2p$ places, to carry out $\sqrt{N}$ to p places! He brought me plenty of instances of this misconception, both in text books and in other papers. I was surprised! No one can deny him the honour of discovering the wide-spread misconception among teachers of mathematics.

This reminds me of the fact that there has been no laconic notation fit for a numeral quantity with the limits of error attached to itself. I once heard one of my students say in his answer to my question in Dynamics: "About 6 degrees and 10 minutes." I asked him what the limits of error were. He replied: "I think the error is less than 5 minutes." Closely testing the young man, I found that he had no means or convention to express a numeral quantity with the limits of error attached to itself, while he was very clever a young man. I tested him again by asking another quetion: "What is the value of $\sqrt{2}$?" His reply was "1.4142, and there is some more in lesser places." I bade him to write what he said in mathematical

convention only to find him utterly unable to do so. He understood $a < x < b$, yet he could not write down $1.4142 < \sqrt{2} < 1.4143$.

An error is absolutely unavoidable in any numeral quantity measured out by man, either in weight, in length or in time; it is understood that there *IS* an error. But the way of expressing a number with the limits of error clearly attached to itself, has not been properly introduced; and the students have no concrete idea about an error. I have very often thought that something must be done to answer the want. What Mr. Hirano has pointed out is nothing but an instance of the shortness of cognizance about an error.

In conclusion, I came to realise that the old way of expressing a number with the limits of its error explicit, i.e. an unequality, is of no avail. Because in the expressions such as $1.4142 < \sqrt{2} < 1.4143$ or $3.1415 < \pi < 3.1416$, etc., you have to repeat tediously the same groups of figures, 1.414's or 3.141's. Again because such unequalities can not be put, as they are, within written calculations. The inadequacy of an unequality as a means to express a number and its limits of error, must have been the reason why it has not been successfully taught in schools, and also why it has not been fully utilized. However, we must not leave the matter as it is, simply because the notation is inadequate or defective. The missing link must be discovered, in one way or another. I made a proposal, I consulted with Mr. Eimei Hirano and Prof. Sizuka Nakano who contributed much in the form of opinions. Finally I came to decide as follows:—

The Notation and its Reading of a Numder with the Limits of its Error Explicit

For instance, $\sqrt{2} = 1.4142{\to}3$ stands for $1.4142 < \sqrt{2} < 1.4143$; $a = 283{\to}95$ for $283 < a < 295$; $b = 5{\to}116$ for $5 < b < 116$. And $236{\to}9 \times 3$ is absolutely the same as $(236{\to}9) \times 3$, for $236{\to}9$ is undetatchable and represents one and one only number whose value lies between 236 and 239. $236{\to}9 \times 3$ equals $708{\to}17$ in value, which is read "Seven hundred and eight to seventeen." $5.67{\to}8$ is read "Five decimal six seven to eight;" and $5670{\to}80$ is read "Five thousand six hundred and seventy to eighty." So we have the following rules:—

(a) The notation cosisting of two parts connected by an arrow → is just the same as if there were a parenthesis enclosing the notaion, i.e. the parenthesis is always understood, and it represents one and one only number.

(b) The value of the numeral quantity represented by the notation is larger than the number embodified in the first part.

(c) The value of the numeral quantity represented by the notation is less than the number that is obtained by putting the part, a figure or a group of figures, after the arow →, in place of the last figure or the last group of so many figures.

(d) The first part is read first, and then with intercalation of "to" the second part is read continually. Note, though $\sqrt{2} = 1.4{\to}5$ and $\sqrt{2} = 1.4142{\to}3$, only $1.4142{\to}3$ can be replaced with $1.4{\to}5$ whenever the latter does as well. And this is the simpler.

This notation, though not without additional cost of time, can be put within a written calculation. And the limits of error are quite explicit and clear.

The way of reading has been in wide practice in daily life, as in a telephone directory, &c., though there is a little difference in convention. 341(9) Marunouti is a group of telephone numbers of the Dept. of Education, Tôkyô; and is to represent no less than nine integers, 341, 342, 343, ..., 349 But $341{\to}9$ represents one and one only numeral quantity whose value lies between 341 and 349. It might be an integer or it might have a fraction mixed in itself.

The publication of this book has been the inducement to the solution of the great great problem. The fact is very congruent to the nature of this work. It must be a tribute to the glory of this book. I have no hesitation in offering my tribute as to the success of Mr. Hirano.

(Hitosi Siio)

Nagoya May, 19, 1933.

PREFACE

Strange to say, almost all the mathematic teachers throughout the world have been rather careless and short of cognizance in the nature of evolution. They taught, "In order to determine p places of $\sqrt[m]{N}$, from $\{(m-1)\,p+1\}$ to mp places of N must be prepared." To calculate $\sqrt[4]{3}$ to two places in the decimal, they taught to prepare "$\sqrt{3} = 1.7320$"

Whence "$\sqrt[4]{3} = \sqrt{1.7320} = 1.31$"

But as $\sqrt{1.73} = 1.31$; $\sqrt{1.74} = 1.31$,

so $\sqrt[4]{3} = \sqrt{1.73{\to}4} = 1.31$

It is quite unnecessary or rather ridiculous to carry out $\sqrt{3}$ any further down than two places in the decimal.

To calculate $\sqrt[6]{2}$ to two places in the decimal, they taught

"$\sqrt[6]{2} = \sqrt[3]{1.414213} = 1.12$"

But as $\sqrt[3]{1.41} = 1.12$; $\sqrt[3]{1.42} = 1.12$

so $\sqrt[6]{2} = \sqrt[3]{1.41{\to}2} = 1.12$

Hence no digits after the second place in the decimal part of $\sqrt{2}$ are necessary to be clear. To calculate $\sqrt[8]{10}$ to six places in the decimal, they taught

"$\sqrt[8]{10} = \sqrt[4]{3.1622,7766,0168,3793,3199,8893}$

" $= \sqrt{1.7782,7941,0038}$

" $= 1.333521$"

But $\sqrt{10} = 3.162277$

and $\sqrt{3.162277{\to}8} = 1.778279$

and $\sqrt{1.778289{\to}90} = 1.333521$

It is crazy to prepare so many places of $\sqrt{10}$ as twenty-four! Hence the Hiranian Rule II.

The Hiranian Method for the Extraction of a Square Root and a Cube Root helped by the Hiranian Rule II is a super-dreadnaught against some questions in the entrance examinations to the higher schools.

EIMEI HIRANO,
Author.

Tokyo, Oct. 19th, 1932.

CONTENTS

THE HIRANIAN METHOD FOR THE EXTRACTION OF A SQUARE ROOT AND OF A CUBE ROOT

CHAPTER I. NEW RULES

Centesimal Scale. To calculate $\sqrt{N}$, it is newly proposed to read N centesimally or in the scale of notation of which 100 is the radix. So 3.67 and 36.7 are entirely different in digit value from each other. 67 in the 3.67 and 36 in the 36.7 are each *one* inseparable digit.

The First Useful Centesimal Digit. 3 is the first useful centesimal digit in 3.67; 36 in 36.7; 2 in 29475; 29 in 294753; 6 in 0.000618; 61 in 0.0000618.

Punctuation, Centesimal Point. What we have hitherto called a decimal point, is no longer to be so called; we shall call it a Centesimal Point, or simply a Point. The putting-in of the point is termed Punctuation.

Digit Value and Punctuation Separately. To express a number in the decimal scale or in any other scale, it is often more convenient to give the digit value and the punctuation separately.

Millesimal Scale and Other Scales. To calculate $\sqrt[3]{N}$, read N in the Millesimal Scale; to calculate $\sqrt[m]{N}$ generally, read N in the 10^m-esimal Scale, or in the scale of notation of which 10^m is the radix.

THE HIRANIAN RULE I FOR EVOLUTION

Rule I. If the first useful digit of N expressed in the scale of notation of which 10^m is the radix be at the nth place on the lefthand side or on the righthand side of the 10^m-esimal point, the first digit of $\sqrt[m]{N}$ in the decimal scale is at the nth place on the lefthand side or on the righthand side of the decimal point respectively.

$m=2$ for a square root. $m=3$ for a cube root.

To express N in the 10^m-esimal scale, it is often convenient to have the digit value and the location of the first useful digit with respect to the 10^m-esimal point separately. The latter shall be call:d the "punctuation," and is expressed as "n left" or "n right." Hence if N in the 10^m-esimal scale is of "n left," then $\sqrt[m]{N}$ in the decimal scale is also of "n left."

EXAMPLES EXPLAINING RULE I

$10^{10}\pi = 3\ 1\ 4\ 1\ 5\ 9\ 2\ 6\ 5\ 3\ 5 \bullet 8\ 9\ 7\ 9\ 3......$

Scale	Punc.	1st Useful Digit	Root
Centesimal	6 left	3	$\sqrt{10^{10}\pi}=177245.$ ...
Millesimal	4 „	3 1	$\sqrt[3]{10^{10}\pi}=3155.3......$
10^4-simal	3 „	3 1 4	$\sqrt[4]{10^{10}\pi}=421.00......$
10^5 „	3 „	3	$\sqrt[5]{10^{10}\pi}=125.72......$
10^6 „	2 „	3 1 4 1 5	$\sqrt[6]{10^{10}\pi}=56.172......$
10^7 „	2 „	3 1 4 1	$\sqrt[7]{10^{10}\pi}=31.593......$
10^8 „	2 „	3 1 4	$\sqrt[8]{10^{10}\pi}=20.518......$
10^9 „	2 „	3 1	$\sqrt[9]{10^{10}\pi}=14.667......$
10^{10} „	2 „	3	$\sqrt[10]{10^{10}\pi}=11.212......$
10^{11} „	1 „	31415926535	$\sqrt[11]{10^{10}\pi}=9.0009......$

$10^{-1}\pi = 0 \bullet 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 3\ 1\ 4\ 1\ 5\ 9......$

Scale	Punc.	1st Useful Digit	Root
Centes'l	5 rt.	3	$\sqrt{10^{-1}\pi}=0.000017..$
Melles'l	4 „	3 1 4	$\sqrt[3]{10^{1-}\pi}=0.000679..$
10^4-simal	3 „	3 1 4	$\sqrt[4]{10^{1-}\pi}=0.00421...$
10^5 „	2 „	3	$\sqrt[5]{10^{-1}\pi}=0.01257...$
10^6 „	2 „	3 1 4	$\sqrt[6]{10^{-1}\pi}=0.02607...$
10^7 „	2 „	3 1 4 1 5	$\sqrt[7]{10^{-1}\pi}=0.04389...$
10^8 „	2 „	3 1 4 1 5 9 2	$\sqrt[8]{10^{-1}\pi}=0.06488...$
10^9 „	2 „	3 1 4 1 5 9 2 6 5	$\sqrt[9]{10^{-1}\pi}=0.08792...$
10^{10} „	1 „	3	$\sqrt[10]{10^{-1}\pi}=0.11212...$
10^{11} „	1 „	3 1	$\sqrt[11]{10^{-1}\pi}=0.13680...$
10^{12} „	1 „	3 1 4	$\sqrt[12]{10^{-1}\pi}=0.16147...$
10^{13} „	1 „	3 1 4 1	$\sqrt[13]{10^{-1}\pi}=0.24122...$
10^{14} „	1 „	3 1 4 1 5	$\sqrt[14]{10^{-10}\pi}=0.20951...$

THE HIRANIAN RULE II FOR EVOLUTION

Rule II. In order to determine the first p places of $\sqrt[m]{N}$ it is sufficient as a rule to have only the first $(p-q)$ places of N, q being the characteristic of **log***m* common. In cases where the addition of one unit to the $(p-q)$th place of N, gives an increment of a unit to the pth place of $\sqrt[m]{N}$, more than $(p-q)$ places of digits of N must be had. The greater, however, the value of p, the rarer the exceptional cases. It is also a rare exception to require more than $(p-q+1)$ places of digits. p, m, $(p-q)$ are positive integers.

$m=2$ for a square root. $m=3$ for a cube root.

Examples showing the Verity of Rule II (例證)

(1) $\sqrt[4]{3}=?$ two places in the decimal.

$$\sqrt{3}=1.73\to 4$$

$$\sqrt{1.73\to 4}=1.31\to 2$$

$\therefore$ Ans. 1.31

35	15
1. 7 3→4	1. 3 1→2
√3	√1. 7 3→4
1	1
1	3 6 5
9 4 5	3 4 5
5 5	2 0→5

(2) $\sqrt[6]{2}=?$ two places in the decimal.

$$\sqrt{2}=1.41\to 2$$

$$\sqrt[3]{1.41\to 2}=1.12\to 3$$

$\therefore$ Ans. 1.12

(3) $\sqrt[6]{\frac{10000}{10252}}=?$ four places in the decimal.

$$\frac{10000}{10252}=0.9754\to 5$$

$$\sqrt{0.9754\to 5}=0.9876\to 7$$

$$\sqrt[3]{0.9876\to 7}=0.9958\to 9$$

$\therefore$ Ans. 0.9958

Reference: $0.9958=\sqrt[6]{0.9751\to 6}$

(4) $p=6$, $p-q=P$

$P=5$

$$\sqrt[10]{2.1219\to 20}=1.07813\to 4$$

$P=4$

$$\sqrt[100]{1.851\to 2\times 10^{3}}=1.07813\to 4$$

$P=3$

$$\sqrt[1000]{4.69\to 70\times 10^{32}}=1.07813\to 4$$

$P=2$

$$\sqrt[10000]{5.2\to 3\times 10^{326}}=1.07813\to 4$$

$P=1$

$$\sqrt[100000]{2\to 3\times 10^{3267}}=1.07813\to 4$$

$P=0$

$$\sqrt[1000000]{10^{22672\to 5}}=1.07813\to 4$$

In the last case no digit value is wanted to be clear; only the number of places in the integral part is to be known. That is

$$\sqrt[1000000]{N}=1.07813\to 4$$

so far as N is a number consisting of 32673 or 32674 or 32675 places of digits, irrespective the digit value of N.

CHAPTER II. A SQUARE ROOT

THE HIRANIAN METHOD FOR THE EXTRACTION OF A SQUARE ROOT 平野式開平法

$$N = (a+b)^2 + R_2$$

$$
\begin{array}{llllll}
 & & \tfrac{1}{2}b & & & \\
a & + & b & & & \\
\sqrt{a^2} & + & 2ab & + & b^2 & + R_2 \\
'' & & & & & \\
\hline
 & & ab & + \tfrac{1}{2}b^2 & & \\
 & & (a & + \tfrac{1}{2}b)b & & \\
\hline
 & & & & & \tfrac{1}{2}R_2
\end{array}
$$

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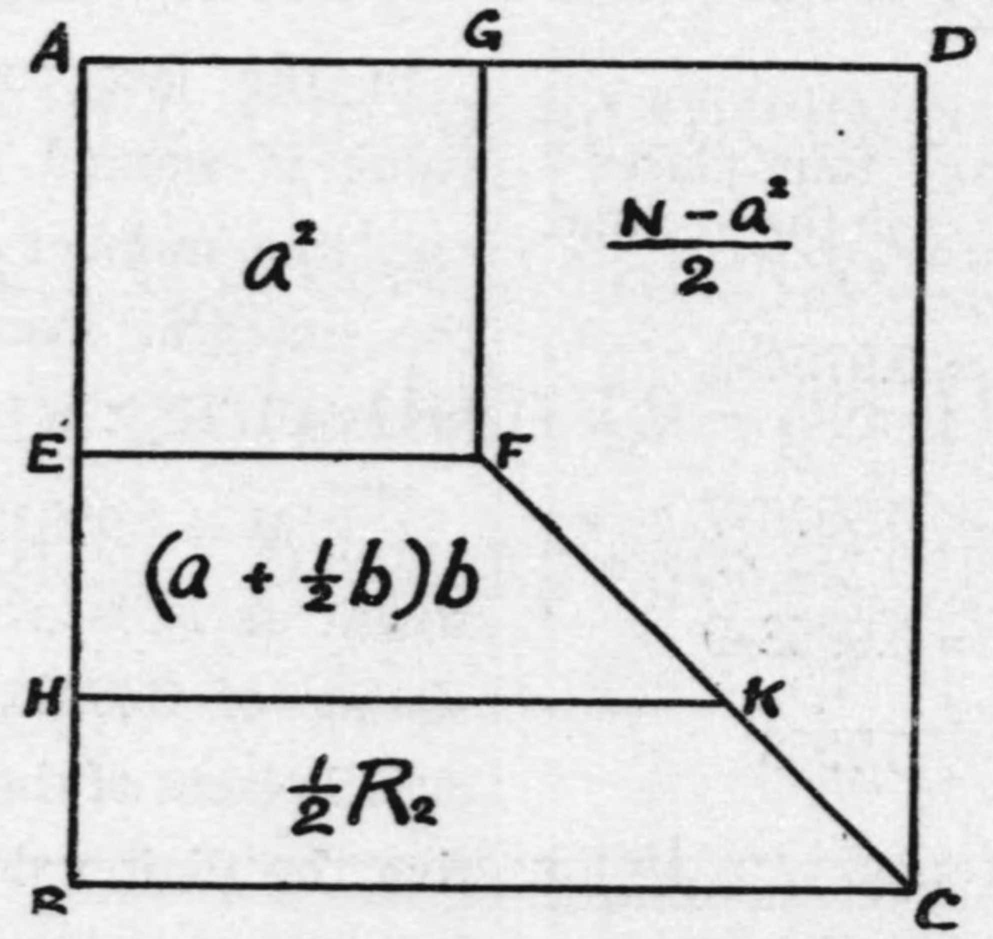

Explanation to Page 8

Let *ABCD* be a square whose area is *N*; to calculate *AB* is the final aim. *AB* is divided into parts, *a*, *b*, *c*, *d*, ... which are calculated one by one. Within *AB* take *AE*=*a*, the largest that you can find out; complete the square *AEFG*. Join *FC* to complete the trapezoid *EBCF* whose area must be $\frac{1}{2}(N-a^2)$. To calculate *EB* is the next aim. Within *EB* take *EH*=*b*, the largest that you can find out; complete the trapezoid *EHKF* whose area must be $(a+\frac{1}{2}b)b$. To calculate *HB*, the height of the new trapezoid *HBCK*, is a fresh aim. Find out *c*, *d*, ... similarly.

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An example of a polinomial expression :—

$$\sqrt{4x^4 - 12x^3 + 29x^2 - 30x + 25} = ?$$

$$
\begin{array}{llllll}
 & -\ 1.5 & +\ 2.5 & & & \\
2 & -\ 3 & +\ 5 & & & \\
\sqrt{4} & -\ 12 & +\ 29 & -\ 30 & +\ 25 & \\
'' & & & & & \\
\hline
 & -\ 6 & +\ 14.5 & -\ 15 & +\ 12.5 & \\
 & '' & +\ 4.5 & & & \\
\hline
 & & +\ 10 & & & \\
 & & '' & -\ 15 & +\ 12.5 & \\
\hline
 & & & & 0 &
\end{array}
$$

Ans. $2x^2 - 3x + 5$

THE NEW METHOD FOR $\sqrt{(A)B,\ CD\ EF......}$

$$\left|\ \frac{b}{2}\ \right|\ \frac{cd}{2}\ \left|\ \frac{efgh}{2}\ \right|$$

$$\begin{array}{l} \quad a,\ b\ c\ d\ e\ f\ g\ h \\ \sqrt{(A)\ B,\ C\ D\ E\ F\ G\ H} \\ \quad a^2 \\ \hline \quad \frac{1}{2}\times\{N-a^2\} \\ \quad \left(a\,\frac{b}{2}\right)\times b \\ \hline \quad \frac{1}{2}\times\{N-(ab)^2\} \\ \quad \left(ab\,\frac{cd}{2}\right)\times(cd) \\ \hline \quad \frac{1}{2}\times\{N-(abcd)^2\} \\ \quad \left(abcd\,\frac{efgh}{2}\right)\times(efgh) \\ \hline \quad \frac{1}{2}\times\{N-(abcdefgh)^2\} \end{array}$$

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DIAGRAMIC GUIDE TO PUNCTUATION

❸, "3 left;" ②, "2 right;" etc.

0 0 0 a b c d e

❸ ❷ ❶ ① ② ③ ④

$\sqrt{0\ 0\ 0\ 0\ 0\ 0\ (A)\ B\ \ CD\ \ EF\ \ GH\ \ I}$

❸ ❷ ❶ ① ② ③ ④

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THE OLD METHOD FOR COMPARISON

$$\begin{array}{l} \quad a\ b\ c\ d\ e\ f\ g\ h \\ \sqrt{(A)\ B\,C\,D\,E\,F\,G\,H\,I\,J\,K\,L\,M\,N\,O\,P} \\ \quad a^2 \\ \hline \quad N-a^2 \end{array}$$

$$\begin{array}{ll} 2\times a+b & 2\times a\times b+b^2 \\ \quad\quad b & \\ \hline & N-(ab)^2 \\ 2\times(ab)+c & 2\times(ab)\times c+c^2 \\ \quad\quad c & \\ \hline & N-(abc)^2 \\ 2\times(abc)+d & 2\times(abc)\times d+d^2 \\ \quad\quad d & \\ \hline & N-(abcd)^2 \\ 2\times(abcd)+e & 2\times(abcd)\times e+e^2 \\ \quad\quad e & \\ \hline & N-(abcde)^2 \\ 2\times(abcde)+f & 2\times(abcde)\times f+f^2 \\ \quad\quad f & \\ \hline & N-(abcdef)^2 \\ 2\times(abcdef)+g & 2\times(abcdef)\times g+g^2 \\ \quad\quad g & \\ \hline & N-(abcdefg)^2 \\ 2\times(abcdefg)+h & 2\times(abcdefg)+h+h^2 \\ \quad\quad h & \\ \hline & N-(abcdefgh^2 \end{array}$$

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THE METHOD IN FURTHER STATEMENT

$$N=(A)B,\ CD\ EF\ GH\ IJ\ KL\ MN......$$
$$\sqrt{N}=a,b\,c\,d\,e\,f\,g\,h\,i\,j\,k\,l\,m......$$
where $BCD=100\times B+10\times C+D$, etc.
$\neq B\times C\times D$ as is usually the case.

1. The original number, of which its square root is to be extracted, $(A)B,\ CD\ EF\ GH......$ is always read centesimally, or it is to be expressed in the scale of notation of which 100 is the radix. The first useful digit of the above is here represented either by B when it is less than 10, or by AB when it is not less than 10. The second digit (might be zero) is here represented by CD; the third by EF, and so forth. The square root, $a,b\,c\,d......$ is always read in the decimal scale. The first useful digit is here represented by a; the second (might happen to be zero) by b; the third by c, and so forth.

2. In calculation, either isolated digits, such as $a, b, c, d...$, or groups of digits, such as cd, $efgh$, $ijklmnop$, ..., are sought for for rapidity.

3. $[N-\{a(b)(c)\}^2]$ is to be divided by 2.

4. Any of the digits or groups of digits may be either positive or negative.

5. In order to carry out p places of $\sqrt{N}$, N is prepared first to p places which, when failing, is to be added afterwards with a $(p+1)$th digit. See the Hiranian Rule II.

6. In the old method to calculate the form of $\sqrt{A\pm\sqrt{B}}$, you used previously to transform it into the form of $(\sqrt{\alpha}\pm\sqrt{\beta})$. The Hiranian Rule II tells you that it is very silly, and that $\sqrt{B}$ is to be calculated first in order to prepare p places of $(A\pm\sqrt{B})$, and a $(p+1)$th place only when necessary.

7. To calculate $\sqrt{A^2\pm B}$, $\sqrt{A^2+B^2}$, either A or $(A+B)$ may be taken for $a(b)(c)$ at once.

8. If it not be for school examinations, an instrument such as an abacus, a slide rule, a calculating machine or a square table, may be cooperatively employed for convenience.

9. The Punctuation is to be done by following the Diagramic Guide to Punctuation, given in the middle of page 10.

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THE RED METHOD

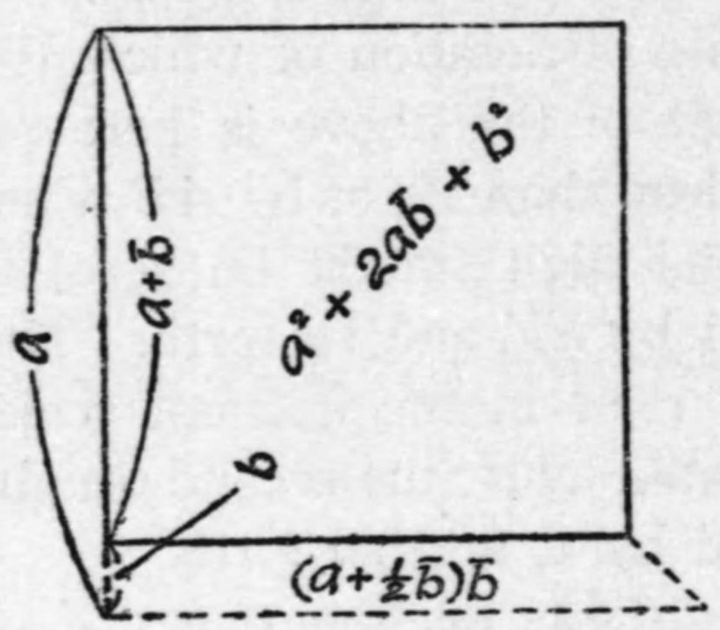
General Form

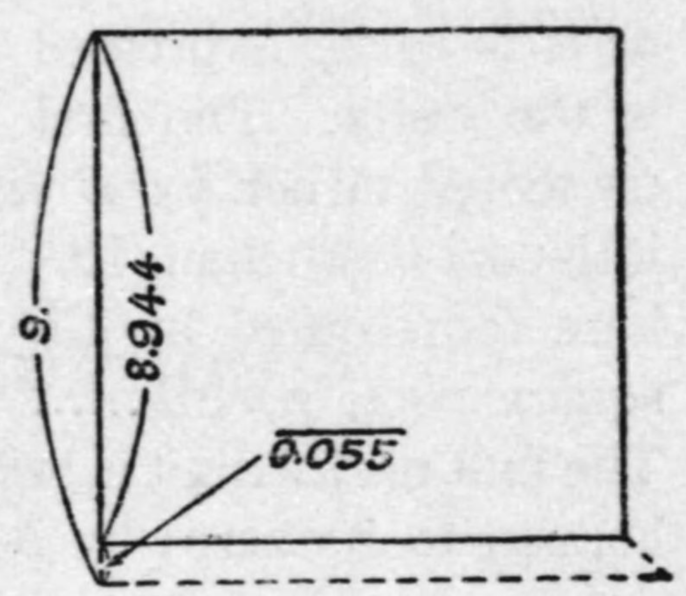
$$\begin{array}{l} \quad a \;+\; \bar{b} \\ \sqrt{a^2 + 2ab + b^2} \\ \text{,,} \\ \quad a\bar{b} + .5\bar{b}^2 \\ \quad \text{,,} \quad \text{,,} \\ \quad\quad 0 \end{array}$$

Example (1)

$$\begin{array}{l} 9.\;\; \bar{5}\,\bar{5} \leftarrow \text{Ans.} \\ \sqrt{8\,0} \quad = ? \\ 8\,1 \\ \quad \bar{5} \end{array}$$

Ans. 8. 9 4 4





$\bar{b} = -b$ $\qquad\qquad$ $\overline{55} = -55$

$$\begin{array}{l} \quad\;\; {}^{05} \qquad \overline{275} \\ \quad 9.1 \quad \overline{5\,5} \quad 5\,3\,2\,8\,6\,1\,8 \\ (2) \;\; \sqrt{8\,2.8} \qquad\qquad\qquad = ? \\ \quad 8\,1 \\ \qquad 9 \\ \qquad 9\,0\,5 \\ \qquad -5 \\ \qquad -5\,0\,0\,4\,8\,4\,8\,7\,5 \\ \qquad\qquad 4\,8\,4\,8\,7\,5 \end{array} \qquad \left\{ \begin{array}{l} 91 \times \overline{55} = -5005 \\ \overline{275} \times \overline{55} = + \quad 15125 \\ \qquad\qquad -500484875 \end{array} \right.$$

$$\therefore \sqrt{82.8} = 9.1\,0\,0\,\overline{5\,5}\,0\,5\,3\,2\,8\,6\,1\,8\ldots\ldots$$
$$= 9.0\,9\,9\,4\,5\,0\,5\,3\,2\,8\,6\,1\,8\ldots\ldots$$

$$\begin{array}{l} \quad 1.\,0\,1 \qquad\qquad\qquad\qquad \overline{4\,9\,5\,0\,4\,9\,5} \\ (3) \;\; \sqrt{1.\,0\,2\,0\,0\,9\,9\,9\,9\,9\,9\,9\,9\,9\,9} = ? \\ \quad 1.\,0\,2\,0\,1 \\ \qquad\qquad\qquad\qquad\qquad -5 \end{array}$$

$$\therefore \sqrt{1.\,0\,2\,0\,0\,9\,9\,9\,9\,9\,9\,9\,9\,9\,9}$$
$$= 1.\,0\,0\,9\,9\,9\,9\,9\,9\,9\,9\,9\,9\,9\,9\,5\,0\,4\,9\,5\,0\,5$$

(World Copyright)

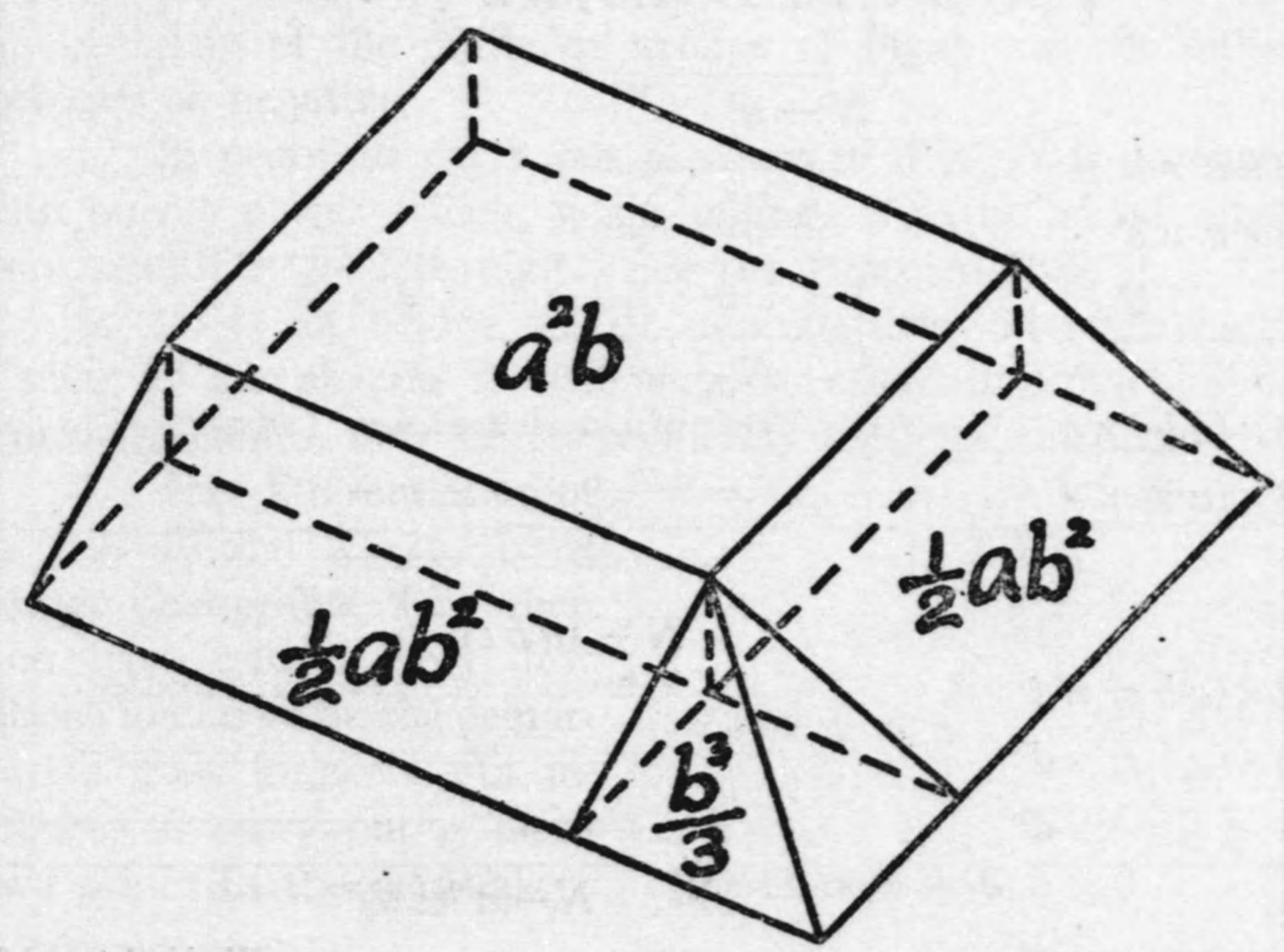
CHAPTER III. A CUBE ROOT

THE HIRANIAN METHOD FOR THE EXTRACTION OF A CUBE ROOT 平野式開立法

$$N = (a + b)^3 + R_2$$

$$\begin{array}{l} \quad a \;+\; b \\ \sqrt[3]{a^3 + 3a^2b + 3ab^2 + b^3 + R_2} \\ \text{,,} \\ \qquad a^2b + ab^2 + \tfrac{1}{3}b^3 \\ (a^2+\tfrac{1}{3}b^2) \quad (a^2 \qquad + \tfrac{1}{3}b^2)b \\ b^2 \qquad\qquad ab^2 \\ \qquad\qquad\qquad\qquad \tfrac{1}{3}R_2 \end{array}$$

(World Copyright)



THE NEW METHOD FOR $\sqrt[3]{(A)(B)C, DEF......}$

	When in Digit V. $N \leqq 10\sqrt{10}$	When in Digit V. $N > 10\sqrt{10}$
	$a,\ b\ c\ d\ ......$	$a,\ b\ c\ d\ e...$
	$\sqrt[3]{(B)\ C, D\,E\,F, ...}$	$\sqrt[3]{(A)\ B\,C, D\,E\,F...}$
	a^3	......
	$\frac{1}{3} \times \{N - a^3\}$	....
$\left(a^2 \frac{b^2}{3}\right) \times b$		
$a \times b^2$		
	$\frac{1}{3} \times \{N - (a\,b)^3\}$	....
$\left\{(a\,b)^2 \frac{(c\,d)^2}{3}\right\} \times (c\,d)$		
$(a\,b) \times (c\,d)^2$		
	$\frac{1}{3} \times \{N - (a\,b\,a)^3\}$	....

THE OLD METHOD FOR COMPARISON

	$a,\quad b\quad c\quad d$
	$\sqrt{(A)\,(B)\,C, D\,E\,F\,G\,H\,I\,J\,K\,L}$
	a^3
	$N - a^3$
$3 \times a^2 \times b$	
$3 \times a \times b^2$	
b^3	
	$N - (a\,b)^3$
$3 \times (a\,b)^2 \times c$	
$3 \times (a\,b) \times c^2$	
c^3	
	$N - (a\,b\,c)^3$
$3 \times (a\,b\,c)^2 \times d$	
$3 \times (a\,b\,c) \times d^2$	
d^3	
	$N - (a\,b\,c\,d)^3$

THE METHOD IN FURTHER STATEMENT

$$N = (A)(B)C,\ DEF\ GHI\ JKL\ MNO\ ......$$
$$\sqrt[3]{N} = a, b\,c\,d\,e\,f\,g\,h\,i\,j\,k\,l\,m\ ......$$
$$\text{where } CDEF = 1000 \times C + 100 \times D + 10 \times E + F, \text{ etc.}$$
$$\neq C \times D \times E \times F \text{ as is usually the case.}$$

1. The original number, of which the cube root is to be extracted, $(A)(B)C$, $DEF\ GHI$ ...... is always read millesimally or it is expressed in the scale of notation of which 1000 is the radix. The first useful digit is here represented either by C when it is less than 10, or by BC when it is not less than 10 but less than 100, or by ABC when it is not less than 100. The second digit (might be zero) is here represented by DEF; and the third GHI, and so forth. The cube root to be extracted, $a, b\,c\,d$ ...... is always read in the decimal scale. The first useful digit is represented by a; the second digit (might happen to be zero) is here represented by b; the third by c, and so forth.

2. In calculation, either isolated digits, such as a, b, c, d ..., or groups of digits, such as $d\,e$, $f\,g\,h\,i$, $j\,k\,l\,m\,n\,o\,p\,q$, ..., are sought for for rapidity.

3. $[N - \{a(b)(c)\}^3]$ is dividsd by 3.

4. Any of the digits or groups of digits may be either positive or negative.

5. In order to carry out p places of $\sqrt[3]{N}$, N is prepared first only p places which, when failing, is to be added afterwards with a $(p+1)$th digit. See the Hiranian Rule II.

6. If it not be for school examinations, an instrument, such as an abacus, a slide rule, a calculating machine or Barlow's Tables, may be cooperatively used for rapidity.

7. The differentiation of $N \leqq 10\sqrt{10}$ and $N > 10\sqrt{10}$ is rather changeable, for when you have only a very few places to carry out, the demarcation goes higher. For instance, to carry out $\sqrt[3]{32}$ to two places in the decimal:

	$3.\,1\ 7$
	$\sqrt[3]{3\ 2}$
	$2\ 7$
	$1\ 6\ 6\ 6\ \frac{2}{3}$
$9\ 0\ 0\ \frac{1}{3} \times 1$	$9\ 0\ 0\ \frac{1}{3}$
$3\ 0 \times 1$	$3\ 0$
	$7\ 3\ 6\ \frac{1}{3}$
$9\ 6\ 1\ 0\ 0 \times 7$	

CHAPTER IV. APPLICATIONS

PRACTICAL EXAMPLES 開平ノ實演

(1)

```
    | 35 |
   6 7.  ← Ans.
√ 4 4 8 9 = ?
  3 6
  4 4 4 5   (∴ 44÷6≒7)
  4 4 4 5   (∵ 63,5×7)
        0
```

(2)

```
    | 35 | 4
   6 7 8.  ← Ans.
√ 4 5 9 6 8 4 = ?
  3 6
  4 9 8 4 2   (∴ 49÷6,7≒7)
  4 4 4 5     (∵ 63,5×7)
    5 3 9 2   (∴ 539÷67≒8)
    5 3 9 2   (∵ 674×8)
          0
```

(3)

```
    | 25 | 335 |
   8 5 6 7.  ← Ans.
√ 7 3 3 9 3 4 8 9 = ?
  6 4
  4 6 9 6 7
  4 1 2 5
    5 7 1 7 4 4 5 (85335×67)
    5 7 1 7 4 4 5
                0
```

(3)

As $(N+0.5)^2=N(N+1)+0.25$

$8.5^2=8\times9+0.25$

$85^2=7225$

```
      | 335 |
   8 5 6 7.  ← Ans.
√ 7 3 3 9 3 4 8 9 = ?
  7 2 2 5
    5 7 1 7 4 4 5
    5 7 1 7 4 4 5
                0
```

(4) Tottori College of Aricul-ture [1], 1932.

```
    | 37 | 15
  2 1 7 4 3.  ← Ans.
√ 4 7 2 7 5 8 0 4 9 = ?
  4
    3 6 3 7 9
    2 0 5
    1 5 8 7 9
    1 5 8 1 3 8  (2137×74)
          6 5 2 2 4 5
          6 5 2 2 4 5
                    0
```

(5)

```
    | 45 | 49 |
   6 9 9 8.  ← Ans.
√ 4 8 9 7 2 0 0 4 = ?
  3 6
    6 4 8 6
    5 8 0 5
      6 8 1 0 0 2
      6 8 1 0 0 2
                0
```

(5)

By the Red Method 赤字法

```
          1̄
   7      2̄.  ← Ans.
√ 4 8 9 7 2 0 0 4 = ?
  4 9
        −2 7 9 9 6
        −1 3 9 9 8
     −( +        2
        −1 4
                 0
```

Ans. $700\overline{2}=6998$

Preparation for the Log. Table 對數表製作準備

(1) $10^{\frac{1}{2}} = \sqrt{10} = ?$

(2) $10^{\frac{1}{4}} = \sqrt{\sqrt{10}} = ?$

(3) $10^{\frac{3}{4}} = \sqrt{10\sqrt{10}} = ?$

(4) $10^{\frac{1}{8}} = \sqrt{\sqrt{\sqrt{10}}} = ?$

(5) $10^{\frac{3}{8}} = \sqrt{\sqrt{10\sqrt{10}}} = ?$

(6) $10^{\frac{5}{8}} = \sqrt{10\sqrt{\sqrt{10}}} = ?$

(7) $10^{\frac{7}{8}} = \sqrt{10\sqrt{10\sqrt{10}}} = ?$

(1)

```
    3. 1 6
 √1 0
    9
      5
      3 0 5
      1 9 5
```

log 3.1 6 = 0.5 0 0

(2)

```
   1. 7 8
 √3. 1 6
   1
   1 0 8
     9 4 5
     1 3 5
```

log 1.7 8 = 0.2 5 0

(3)

```
    5. 6 2
 √3 1. 6
  2 5
    3 3
    3 1 8
      1 2
```

log 5.6 2 = 0.7 5 0

(4)

```
   1. 3 3
 √1. 7 8
   1
     3 9
     3 4 5
       4 5
```

log 1.3 3 = 0.1 2 5

(5)

```
   2. 3 7
 √5. 6 2
   4
   1 6 2
     8 1
     6 4 5
     1 6 5
```

log 2.3 7 = 0.3 7 5

(6)

```
    4. 2 2
 √1 7. 8
  1 6
      9 0
      8 2
        8
```

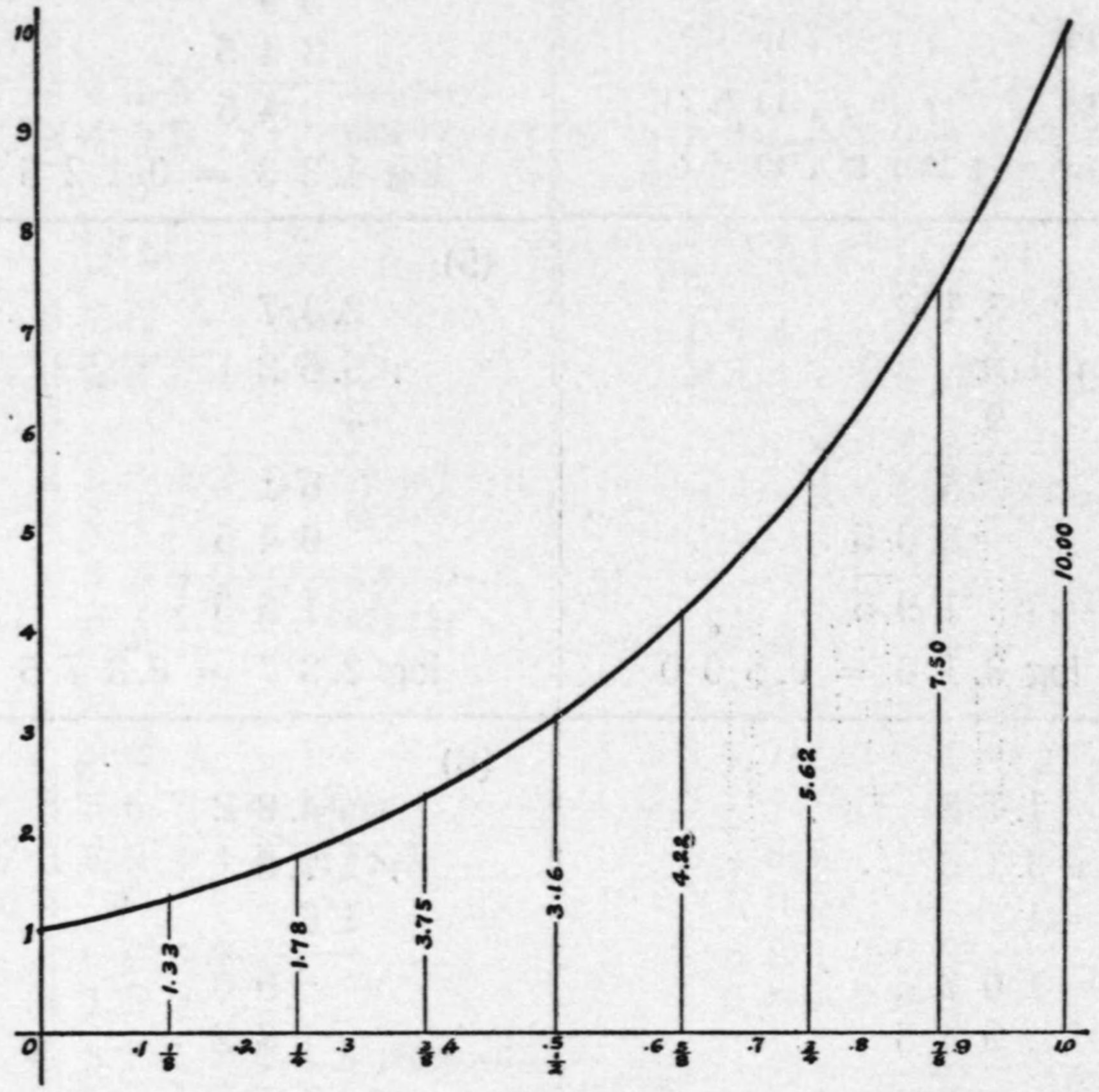
log 4.2 2 = 0.6 2 5

(7)

```
    7. 5 0
 √5 6. 2
  5 6 2 5
        2 5
```

log 7.5 0 = 0.8 7 5

The Anti-Logarithmic Curve



(The Vertical Scale is only $\frac{1}{10}$ of the Horizontal)

CHAPTER V NEW THEOREMS

AN INCREMENT OF $\sqrt[m]{N}$

If a positive number N receives a positive increment of 100 %, its mth root $\sqrt[m]{N}$ also receives another, but only between

$$100\left(\frac{x}{m}-\frac{x^2}{2m}+\frac{x^2}{2m^2}\right)\ \% \text{ and } 100\frac{x}{m}\ \%.$$

This theorem, which is the mother of the two following theorems, are proved on pages 21, 22 and 23.

AN INCREMENT OF $\sqrt{N}$

If a positive number N receives a positive increment of $100x$ %, its square root $\sqrt{N}$ also receives another, but only between $(50x-12.5x^2)$ % and $50x$ %.

AN INCREMENT OF $\sqrt[3]{N}$

If a positive number N receives a positive increment of $100x$%, its cube root $\sqrt[3]{N}$ also receives another, bnt only between $(33.\dot{3}x-11.\dot{1}x^2)$% and $33.\dot{3}x$%.

[Proof] It is sufficient to show

$$\left(\frac{x}{m} - \frac{x^2}{2m} + \frac{x^2}{2m^2}\right) < \left(\sqrt[m]{1+x} - 1\right) < \frac{x}{m}$$

in order to prove the first or the fundamental theorem, where N is a positive number, and

$m > 1$

Now by the Binomial Theorem:

$$\sqrt[m]{N(1+x)}$$

$$= \sqrt[m]{N}(1+x)^{\frac{1}{m}}$$

$$= \sqrt[m]{N}\left[1 + \frac{x}{m} - \frac{\frac{1}{m}\left(1-\frac{1}{m}\right)}{1 \times 2}x^2 + \frac{\frac{1}{m}\left(1-\frac{1}{m}\right)\left(2-\frac{1}{m}\right)}{1 \times 2 \times 3}x^3\right.$$

$$- \cdots\cdots\cdots\cdots\cdots\cdots$$

$$-(-1)^r\frac{1}{m}\frac{\left(1-\frac{1}{m}\right)}{2}\frac{\left(2-\frac{1}{m}\right)}{3}\frac{\left(3-\frac{1}{m}\right)}{4}\cdots\cdots\frac{\left(r-1-\frac{1}{m}\right)}{r}x^r$$

$$+(-1)^r\frac{1}{m}\frac{\left(1-\frac{1}{m}\right)}{2}\frac{\left(2-\frac{1}{m}\right)}{3}\frac{\left(3-\frac{1}{m}\right)}{4}\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\frac{\left(r-1-\frac{1}{m}\right)}{r}\frac{\left(r-\frac{1}{m}\right)}{(r+x)}x^{r+1}$$

$$\left. - \cdots\cdots\cdots\cdots\cdots\cdots\right]$$

Combine every two contiguous terms excepting the first three

$$\sqrt[m]{N(1+x)}$$

$$= \sqrt[m]{N}\Bigg[1 + \frac{x}{m} - \frac{1}{m}\frac{\left(1-\frac{1}{m}\right)}{2}x^2$$

$$+ \frac{1}{m}\frac{\left(1-\frac{1}{m}\right)}{2}\frac{\left(2-\frac{1}{m}\right)}{3}x^3\left\{1 - \frac{\left(3-\frac{1}{m}\right)}{4}x\right\}$$

$$+ \frac{1}{m}\frac{\left(1-\frac{1}{m}\right)}{2}\frac{\left(2-\frac{1}{m}\right)}{3}\frac{\left(3-\frac{1}{m}\right)}{4}\frac{\left(4-\frac{1}{m}\right)}{5}x^5\left\{1-\frac{\left(5-\frac{1}{m}\right)}{6}x\right\}$$

$$+ \cdots\cdots\cdots\cdots\cdots\cdots$$

$$+ (-1)^{r+1}\frac{1}{m}\frac{\left(1-\frac{1}{m}\right)}{2}\frac{\left(2-\frac{1}{m}\right)}{3}\frac{\left(3-\frac{1}{m}\right)}{4}\cdots\cdots$$

$$\cdots\cdots\frac{\left(r-\frac{1}{m}\right)}{(r+1)}x^{r+1}\left\{1 - \frac{\left(r+1-\frac{1}{m}\right)}{(r+2)}x\right\}$$

$$+ \cdots\cdots\cdots\cdots\cdots\cdots\Bigg]$$

If r is an even integer, every combined term is positive, because every factor thereof is positive (and less than unity). Hence if you cross off all the combined terms, the whole righthand side of the equation loses some in value, resulting

$$\sqrt[m]{N(1+x)} > \sqrt[m]{N}\left[1 + \frac{x}{m} - \frac{x^2}{2m} + \frac{x^2}{2m^2}\right]$$

Independently combine every two contiguous terms excepting the first two instead of three

$$\sqrt[m]{N(1+x)}$$

$$= \sqrt[m]{N}\Bigg[1 + \frac{x}{m} - \frac{1}{m}\frac{\left(1-\frac{1}{m}\right)}{2}x^2\left\{1 - \frac{\left(2-\frac{1}{m}\right)}{3}x\right\}$$

$$- \frac{1}{m}\frac{\left(1-\frac{1}{m}\right)}{2}\frac{\left(2-\frac{1}{m}\right)}{3}\frac{\left(3-\frac{1}{m}\right)}{4}x^4\left\{1 - \frac{\left(4-\frac{1}{m}\right)}{5}x\right\}$$

$$- \cdots\cdots\cdots\cdots\cdots\cdots$$

$$- (-1)^r\frac{1}{m}\frac{\left(1-\frac{1}{m}\right)}{2}\frac{\left(2-\frac{1}{m}\right)}{3}\frac{\left(3-\frac{1}{m}\right)}{4}\cdots\cdots$$

$$\cdots\cdots\frac{\left(r-1-\frac{1}{m}\right)}{r}x^r\left\{1 - \frac{\left(r-\frac{1}{m}\right)}{(r+1)}x\right\}$$

$$- \cdots\cdots\cdots\cdots\cdots\cdots\Bigg]$$

If r is an even integer, every combined term is negative because every factor thereof is positive, while every combined term has a negative sign. And if you cross off all the combined terms, the whole righthand side of the equation gains some in value, resulting

$$\sqrt[m]{N(1+x)} < \sqrt[m]{N}\left[1 + \frac{x}{m}\right]$$

In conclusion

$$\left(1 + \frac{x}{m} - \frac{x^2}{2m} + \frac{x^2}{2m^2}\right) < \sqrt[m]{1+x} < \left(1 + \frac{x}{m}\right)$$

$$\left(\frac{x}{m} - \frac{x^2}{2m} + \frac{x^2}{2m^2}\right) < \left(\sqrt[m]{1+x} - 1\right) < \frac{x}{m}$$

Q. E. D.

Put $m=2$, and $m=3$ to get the second and third theorems.

APPLICATION (OF THE SECOND THEOREM)

$$\sqrt{3.1415926\cdots\cdots} = ?$$

where $\sqrt{3.1410000} = 1.7722866\cdots\cdots$

New original 3.1410000

increment 0.0005926……

or 0.01886…………% | 0.00943…………% or

0.0001671…… increment

Hence $\sqrt{3.1415926\cdots\cdots} = 1.7724537{\to}9$

Cf. $\sqrt{\pi} = 1.7724538509055\cdots\cdots$

To carry out $\sqrt{\pi}$ to three places in the decimal, all teachers of mathematics in the world thought it necessary to prepare

$$\pi = 3.141592$$

But the new theorem says it is quite unnecessary. We require only

$$\pi = 3.141$$

Just note
$$\begin{cases} \sqrt{3.140} = 1.772\cdots\cdots \\ \sqrt{3.141} = 1.772\cdots\cdots \\ \sqrt{3.142} = 1.772\cdots\cdots \\ \sqrt{3.143} = 1.772\cdots\cdots \\ \sqrt{3.144} = 1.773\cdots\cdots \end{cases}$$

昭和八年九月十日印刷
昭和八年九月十五日發行

定價金壹圓

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東京市神田區表神保町二

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大阪市東區博勞町五丁目五六

印刷者 白井赫太郎
東京市神田區錦町三丁目一七

發行所

東京修文館
東京市神田區表神保町二
振替口座東京二六四四番

大阪修文館
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