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# NAVAL POSTGRADUATE SCHOOL Monterey, California 



## THESIS

RECIPROCITY CALIBRATIO:
IV
A PLANE :TAVE RESONATOR
by

Charles Lyman Burmaster

December 1985

Dissertation Supervisor:
S. I. Garrett

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12 PERSONAL AUTHOR(S)
Charles Lyman Burmaster


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| cosati codes |  |  | 18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number) <br> Acoustic Calibration Reciprocity <br> Microphone Calibration Reciprocity Calibration <br> Condenser Microphone Resonant Recinrocity |  |  |
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3 ABSTRACT (Continue on reverse if necessary and identity by block number)
A non-standard method for the electroacoustic reciprocity calibration of a condenser microphone is theoretically developed and experimentally employed to calibrate a $\cdot \operatorname{FF} 640 \mathrm{AA}$ laboratory standard microphone. The average experimental calibration so abtained was found to be in absolute agreement with a pressure couoler comparison calibration of the same microphone made at the National Bureau of Standards to within an experimental uncertainty (sigma) of 0.03 dB over the frequency range of 245 to 1470 Hz . using a 70 cm . plane wave resonant cavity, and to within an experimental uncertainty (sigma) of 0.06 dB over the frequency range of 735 to 1470 Hz . using a 23 cm . Dlane wave resonant cavity. Above 1470 Hz. , the difference between the resonant plane wave reciorocity calibrations and the pressure coupler comparison calibration increased linearly with frequency to a maximum of 0.61 dB at 5145 Hz .

Beginning with theory previously published by Isadore Rudnick, reciprocity equations for the open circuit voltage reciving sensitivity are optimized for experimental

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Block 18 continued.
Plane wave resonator
Transfer Impedance
Computer aided test
Block 20 continued.
measurements in a plane wave resonant cavity to include the effects of finite microphone compliance and the non-adiabatic boundary conditions. Two right cylindrical plane wave resonant cavities of different dimensions were constructed to provide a self consistency cheak on the method.

A preliminary comparison of the theory for a free field reciprocity calibration, a pressure coupler reciprocity calibration, and a plane wave resonant reciprocity calibration is made to illustrate the cormon physics pertinent to the reciprocity principle that underlies the three methods.

Experimental calibrations based upon free field reciprocity were made alternately with plane wave resonant reciprocity calibrations to provide an ongoing experimental comparison when ombined with published diffraction effects for a standard mounting of a WE640AA laboratory standard microphone. The National Bureau of Standards comparison calibration was based upon an absolute pressure coupler reciprocity calibration and was obtained shortly after the resonant reciprocity calibration measurements were complete.

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Reciprocity Calibration in a Plane Wave Resonator
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by

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Charles Lyman Burmaster Lieutenant Commander, United States Navy B.S. Arizona State University, 1966 M.S. Naval Postgraduate School, 1978
Submitted in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY
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The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

## TABLE OF CONTENTS

I. THE PRINCIPLE OF ACOUSTICAL RECIPROCITY IN MICROPHONE CALIBRATION ..... 27
A. INTRODUCTION ..... 27
B. HISTORY ..... 30
C. A GENERAL THEORY OF ACOUSTICAL RECIPROCITY ..... 34
D. ACOUSTIC RECIPROCITY CALIBRATION OF MICROPHONE SENSITIVITY ..... 43

1. General Acoustic Reciprocity Calibration of Electroacoustic Transducers ..... 43
2. General Determination of The Reciprocity Factor "J" ..... 53
a. The Calculation of "J" for a Pressure Coupler Reciprocity Calibration ..... 57
b. The Calculation of "J" for a Free Field Reciprocity Calibration ..... 59
c. The Calculation of "J" for The Plane Wave Reciprocity Calibration ..... 62
3. A Summary of The Three Reciprocity Methods Compared in this Experiment ..... 78
a. Plane Wave Resonant Reciprocity Calibration ..... 78
b. Free Field Reciprocity Calibration ..... 79
c. Pressure Coupler Reciprocity Calibration ..... 80
II. EXPERIMENTAL CONSIDERATIONS FOR A PLANE WAVE RESONANT RECIPROCITY CALIBRATION ..... 81
A. INTRODUCTION ..... 81
B. DEVELOPMENT OF THE EXPERIMENTAL PLANE WAVE RECIPROCITY EQUATIONS USING CYLINDRICAL GEOMETRY ..... 83
4. Experimental Considerations ..... 83
5. Two Comparison Calibrations for Mierophone "C" ..... 100
6. The Comparison Calibration of Microphones "A" and "B" With Respect to Each .Other ..... 109
C. THE IMPRESSED PRESSURE CORRECTION ..... 111
III. EXPERIMENTAL PROCEDURES ..... 127
A. INTRODUCTION ..... 127.
B. EXPERIMENTAL APPARATUS ..... 129
7. Plane Wave Resonant Cavities ..... 129
8. The Microphone Translator for Free Field Calibrations ..... 137
C. SIGNAL FLOW AND COMPUTER CONTROL WITH PLANE WAVE RESONANT RECIPROCITY ..... 145
D. THE CALCULATION OF EXPERIMENTAL ERROR ..... 150
9. Introduction ..... 150
10. Measuring the Ratio el/il ..... 153
a. The Electrical Circuit ..... 153
b. Measuring The Input Capacitance of Preamplifiers ..... 159
c. The Calculation of el ..... 164
d. The Calculation of il ..... 169
e. Measuring The Signal Preamplifier Gain - ..... 175
f. Measuring Non-Linearity in The Lock-in Detector ..... 180
g. Computation of The Probable Error in el/il ..... 183
11. The Measurement of Vca/Vcb ..... 187
12. Calculating The Reciprocity Factor, "J" ---- ..... 189
a. Measuring Atmospheric Pressure ..... 190
b. Uncertainty in Resonant Frequency and Quality Factor ..... 193
C. Calculating The Effect of The Non- Adiabatic Boundary Conditions Upon The Stiffness of The Gas Within The Resonant Cavity ..... 199
d. Volume Measurements in the Plane Wave Resonant Cavitias ..... 207
e. Measuring Temperature and Relative Humidity ..... 212
13. A Summary of Experimental Uncertainty For The Plane Wave Resonant Reciprocity Experiment ..... 214
E. SIGNAL FLOW AND COMPUTER CONTROL FOR THE FREE FIELD COMPARISON CALIBRATION ..... 218
14. Introduction ..... 218
15. Experimental Considerations for a Free Field Comparison Calibration ..... 219
16. Measurement of the Sensitivity Ratio, M(640R)/M(688R) ..... 222
17. Free Field Reciprocity Measurements ..... 227
F. EXPERIMENTAL ERROR FOR THE FREE FIELD COMPARISON CALIBRATION ..... 235
18. Introduction ..... 235
19. Measuring The Open Circuit Receiving Voltage, e4 ..... 238
a. Analysis of The Electrical Circuit Used to Measure e4 ..... 238
b. Determination of Gain Uncertainty for The Ithaco 1201 Preamplifier ..... 240
c. Computation of The Uncertainty in The Measure of e4 ..... 242
20. Analytical Considerations for The Measure of "Vdrop" ..... 243
21. Analytical Considerations Made in The Measurement of R4 ..... 245
22. Measuring The "Acoustic" Separation Distance ..... 246
a. Introduction ..... 246
b. Determination of Spherical Spreading ..... 248
23. Calculating The Uncertairity in The Ratio Vb/Va ..... 251
24. A Summary of Experimental Error for The Free Field Comparison Calibration ..... 253
IV. SELF CONSISTENCY OF THE PLANE WAVE RESONANT RECIPROCITY CALIBRATION ..... 256
A. INTRODUCTION ..... 256
B. THE OBSERVATION OF EXPERIMENTAL PRECISION ..... 260
C. THE PRECISION FOR THE ROUND-ROBIN COMPARISON ..... 264
D. THE PRECISION ASSOCIATED WITH THE SWAP OF SIDE "A" AND SIDE "B" ELECTRONICS ..... 266
E. THE PRECISION OBTAINED BY REPLACING THE SIDE "B" RECIPROCAL MICROPHONE WITH ONE HAVING A SIGNIFICANTLY DIFFERENT SENSITIVITY (4 dB) ..... 267
F. THE PRECISION ASSOCIATED WITH REPLACING THE LONG TUBE WITH THE SHORT TUBE AS THE PLANE WAVE RESDNANT CAVITY ..... 268
G. A SUMMARY OF THE EXPERIMENTAL PRECISION FOUND FOR THE PLANE WAVE RESONANT RECIPROCITY CALIBRATION METHOD ..... 269
$v$. ABSOLUTE ACCURACY OF EXPERIMENTAL RESULTS ..... 271
A. INTRODUCTION ..... 271
B. CALCULATION OF THE IMPRESSED PRESSURE CORRECTION ..... 275
C. THE CORRECTION TO THE BULK MODULUS OF ELASTICITY WITHIN THE PLANE WAVE RESONANT CAVITY DUE TO THE NON-ADIABATIC BOUNDARY CONDITIONS ..... 284
D. THE ELECTRICAL DRIVING POINT IMPEDANCE CORRECTIUN ..... 286
E. THE CORRECTION DUE TO REVISED VALUES OF MICROPHONE AND TOTAL BIAS SUPPLY CAPACITANCE ..... 293
F. THE COUPLING CAPACITANCE CORRECTION. ..... 296
G. THE ABSOLUTE PLANE WAVE RESONANT RECIPROCITY CALIBRATION ..... 297
25. Introduction ..... 297
26. Experimental Results for The Plane Wave Resonant Reciprocity Calibration Compared With a NBS Pressure Coupler Comparison Calibration Obtained For The Same Microphone ..... 298
H. THE CORRECTED FREE FIELD COMPARISON CALIBRATION- ..... 304
I. A SUMMARY OF PLANE WAVE RESONANT RECIPROCITY CALIBRATIONS ..... 309
VI. CONCLUSION ..... 314
A. SUMMARY ..... 314
B. FURTHER EXPERIMENTS AND THEORETICAL INVESTIGATIONS ..... 318
APPENDIX A - HARMONICITY IN PLANE WAVE RESONANT RECIPROCITY CALIBRATION ..... 320
APPENDIX B - QNE EXAMPLE OF THE COMPUTER PROGRAM USED FOR PLANE WAVE RESONANT RECIPROCITY CALIBRATIONS ..... 326
A. INTRODUCTION ..... 326
B. FUNCTIONAL DESCRIPTION OF THE PROGRAM ..... 328
C. PROGRAM LISTING ..... 330
D. VARIABLE DEFINITIONS USED IN THE PROGRAM -- ..... 346
APPENDIX C - DESCRIPTION OF THE PROGRAM USED TD OBTAIN THE COMPARISON RATIO, Vb/Va IN THE FREE FIELD COMPARISON CALIBRATION ..... 352
A. INTRODUCTION ..... 352
B. FUNCTIONAL DESCRIPTION OF THE PROGRAM ..... 353
C. PROGRAM LISTING ..... 355
D. VARIABLE DEFINITIONS USED IN THE PROGRAM -- ..... 356
APPENDIX D - DESCRIPTION OF THE PROGRAM USED TO OBTAIN THE FREE FIELD COMPARISON CALIBRATION. ..... 357
A. INTRODUCTION ..... 357
B. FUNCTIONAL DESCRIPTION OF THE PROGRAM ..... 359
C. PROGRAM LISTING ..... 363
D. VARIABLE DEFINITIONS USED IN THE PROGRAM ..... 373
APPENDIX E - THE D. C. BLOCKING CAPACITOR CORRECTION ..... 377
APPENDIX F - THE DPEN CIRCUIT VOLTAGE SENSITIVITY CALIBRATION OBTAINED FROM OBSERVING A CHANGE IN MICROPHONE CAPACITANCE WITH A CHANGE IN BIAS VOLTAGE ..... 383
APPENDIX G - A PRINTOUT OF RAW DATA FOR THE PLANE WAVE RESONANT RECIPROCITY CALIBRATION IN BOTH A LONG AND A SHORT TUBE ..... 389
2 MAY DATA TAPE, LONG TUBE, A SIDE - 815 W/ET, B SIDE - 1248 ..... 391
2 MAY DATA TAPE, LONG TUBE, A SIDE - 1248, B SIDE - 815 W/ET ..... 397
2 MAY DATA TAPE, LONG TUBE, A SIDE - 1248, B SIDE - 1082 W/ET ..... 403
2 MAY DATA TAPE, LONG TUBE, A SIDE - 1082 W/ET, B SIDE - 815 ..... 409
22 NOV DATA TAPE, LONG TUBE, A SIDE - 1082, B SIDE - 815 W/ET ..... 415
22 NOV DATA TAPE, SHORT TUBE, A SIDE - 1243, B SIDE - 815 (ET ALONGSIDE) ..... 421
22 NOV DATA TAPE, SHORT TUBE, A SIDE - 1248, B SIDE - 1082 (ET ALONGSIDE) ..... 424
LIST OF REFERENCES ..... 427
INITIAL DISTRIBUTION LIST ..... 431

## LIST OF TABLES

3.1 Percent relative change from run to run under manual control as compared to computer control for a selected sample ..... 149
3.2 Measured capacitances of bias boxes and microphones ..... 158
3.3 Measured values of capacitances boxes used as "Cv" in the determination of preamplifier input capacitance ..... 161
3.4 Measured ratios for [a/b] ..... 162
3.5 Calculated input capacitances for signal preamplifiers used in experiment ..... 163
3.6 Measured change in capacitance due to the application of a bias voltage to a condenser microphone ..... 172
3.7 Linearity data obtained using ratio transformer serial \#304 ..... 181
3.8 Linearity data obtained using ratio transformer serial \#572 ..... 182
3.9 Temperature normalized resonance frequencies and the associated quality factors for three different modes ..... 196
3. 10 Volume measurements obtained for the plane wave cavity resonators ..... 208
3.11 Summary of probable error in experimental parameters ..... 216
3.12 A summary of probable error for the free field comparison calibration ..... 253
4.1 W.E. $640 A A$ calibration data ..... 263
4. 2 Round robin comparison between Ma and Mab to obtain the "modal" experimental precision ..... 265
4.3 A summary of precision in plane wave resonant reciprocity calibrations ..... 269
5. 1 Equivalent volumes for extreme W.E. 640 A laboratory standard condenser microphones ..... 278
5.2 Tabulated values of acoustic impedance for four W.E. 640 AA microphones ..... 279
5.3 Tabulated values of the impressed pressure correction obtained using equivalent volumes of extreme samples of the WE64OAA population and the difference between these corrections ..... 281
5.4 D. C. blocking capacitor corrections ..... 296
5.5 Absolute plane wave resonant reciprocity calibrations obtained using the 70 cm tube compared with the NBS comparison calibrations--- ..... 298
5.6 Absolute plane wave resonant reciprocity calibrations obtained using the 23 cm tube compared with the NBS comparison calibrations--- ..... 299
5.7 W.E.640AA Serial \#1248 calibrations ..... 302
5.8 NBS vs Free field data ..... 307
B. 1 Functional program listing for plane wave resonant reciprocity calibration ..... 329
C. 1 The functional program listing for Vratio which calculates the ratio of microphone sensitivities for free field reciprocity ..... 354
D. 1 Functional program listing for free field comparison calibration ..... 362
E. 1 Program listing for Ce correction ..... 380
E. 2 Comparison calulation between the $\mathrm{HP}-15 \mathrm{c}$ and
the IBM-XT with Cb and $\mathrm{Cc}=.01 \mathrm{Hf}---------\quad 381$
F. $1 \quad \mathrm{HP}-15 \mathrm{c}$ program for Open circuit voltage receiving sensitivity, Mo, obtained using the change in microphone capacitance with an increase in bias voltage387

## LIST OF FIGURES

1.1 Cavity with rigid walls into which sound is projected ..... 37
1.2 Electro-acoustic two port network ..... 43
1.3 Four port electro-acoustic network ..... 46
1.4 Reciprocal electroacoustic system ..... 48
1.5 A cylindrical plane wave resonator ..... 62
1.6 Modal resonances in a 23.3 cm , air filled brass pipe of 2.5 cm diameter at room temperature [22 deg C] ..... 66
1.7 Differential volume under consideration ..... 71
2.1 Modified four port network ..... 85
2.2 Geometry of the offset microphone used in the calculation of G1(n) ..... 87
2.3 Scheme used to obtain comparison voltages ..... 90
2.4 Reciprocal microphones mounted in the ends of a plane wave resonator ..... 111
2.5 Different terminations for the plane wave tubes ..... 118
2.6 A copy of the original engineering drawings showing the backplate design for the WE640AA condenser microphone ..... 125
3.1 Computer control of the microphone translator ..... 139
3.2 The comparison calibration between the Altec type 688 and the W.E. $640 A A$ microphones ..... 144
3.3 Schematic diagram of the signal flow for the plane wave resonant reciprocity calibration - ..... 145
3.4 Receiving signal flowgraph ..... 153
3.5 Receiving signal input circuit ..... 154
3.6 Simplified input circuit for acoustic signal ..... 157
3.7 Circuit used to measure the input capacitance of signal preamplifiers ..... 159
3.8 Simplified circuit for measuring the acoustic signal ..... 165
3.9 Analog to digital signal flow chart $-\ldots--$ - ..... 167
3.10 Circuit used to measure voltage across microphone when the magnitude of the source voltage is under computer control ..... 170
3. 11 Circuit used to measure amplifier gain ..... 176
3. 12 Measured gain for HP-465A serial \#95 ..... 177
3. 13 Measured gain for HP-465A serial \#93 ..... 178
3. 14 Circuit used to determine non-linearity in the lock-in detector ..... 180
3. 15 The pressure sensor signal path ..... 191
3.16 Pressure sensor vs. pressure reference ..... 192
3. 17 Data sample of a modal resonance from which a ravine Fn, Qn, and amplitude are obtained ..... 194
3.18 Fractional error in quality factor plotted on probability paper. The linear correlation coefficient is equal to .98 ..... 197
3. 19 Fractional error in resonant frequency plotted on probability paper. The linear correlation coefficient is equal to .97 ..... 198
3. 20 Correction to the ratio of specific heats due to the non-adiabatic boundary conditions within a right circular brass cavity 70.1 cm long and 1.73 cm radius, filled with air ---- 203
3.21 Correction to Mo due to the change in stiffness of the gas within the cavity caused by the non-adiabatic boundary conditions ..... 205
3.22 Signal flow used in measuring Vb/Va inside the anechoic chamber ..... 222
3. 23 The sensitivity ratio obtained for 640R/688R- ..... 224
3.24 Mo for the W.E. 640AA serial \#'s 1248 \& 609 (top) and the Altec type 688A (bottom) ..... 225
3. 25 Signal flow in the free field comparison calibration based upon free field reciprocity ..... 227
3.26 Raw data output from "N28" ..... 231
3.27 Quality control output from "N28" ..... 232
3. 28 A sample of the final distance versus Mo calculated by program "N28" ..... 233
3. 29 Measured gain for the Ithaco preamplifier ..... 241
3.30 Measuring the voltage drop [Vdrop] across the current resistor ..... 243
3.31 A plot showing the correction for the "acoustic" separation distance at 490 Hz ---- ..... 249
3. 32 The sensitivity ratio obtained for $640 \mathrm{R} / 688 \mathrm{R}-$ ..... 251
3.33 The variation of the 490 Hz microphone sensitivity with separation distance ..... 255
4.1 An illustration of the "electronics swap" ..... 257
4.2 The relative sizes of plane wave resonant cavities used in comparing calibrations ..... 258
4.3 Electromechanical configurations used in the plane wave resonant reciprocity calibrations- ..... 260
5.1 Raw data from both the long and short resonant tubes for plane wave resonant reciprocity calibrations ..... 272
5.2 Calculated values of the impressed pressure correction applicable to the "long" tube plane wave resonant reciprocity calibration - ..... 282
5.3 Calculated values of the impressed pressure correction applicable to the "short" tube plane wave resonant reciprocity calibration - ..... 283
5.4 The microphone plus bias box capacitance correction plotted for all six experimental combinations ..... 295
5.5 Plane wave resonant reciprocity calibration vs NBS comparison pressure coupler calibration of W.E. G40AA serial \#1248 type L laboratory standard microphone ..... 300
5.6 The corrected free field comparison calibration for W.E. $640 A A$ serial \#1248 condenser microphone ..... 305
5.7 Low frequency detail in the corrected free field comparison calibration ..... 306
5.8 Plane wave reciprocity calibrations for W.E. $640 A A$ serials \#815, and \#1082 listed top to bottom ..... 309
5.9 The effect of radial and azimuthal modes on the plane wave. resonant reciprocity calibration ..... 311
5. 10 Comparison calibration for the type BT-1751 Knowles subminiature transducer ..... 312
5.11 Manufacturer's calibration curves for the Type BT-1751 and Type BT-1757 Knowles subminiature transducers ..... 313
A. 1 Resonant frequency re. 20 degrees centigrade plotted vs. mode number ..... 322
A. 2 Harmonicity as measured with data obtained from condenser microphone W.E.640AA serial\# 1248 ..... 323
E. 1 The input electronics circuit for the acoustic signal ..... 378
E. 2 The bias voltage blocking capacitor correction to the open circuit voltage receiving sensitivity (plotted in dB) ..... 382

## LIST OF PHDTOGRAPHS

1.1 Some of the microphones used in this experiment ..... 33
1.2 Right cylindrical circular resonant cavity used to obtain the modal resonances shown in figure 1.6 ..... 65
2.1 In this photograph, the "ports" for the comparison microphone are mounted on the side wall of the cylinder ..... 88
2.2 The relative sizes of the Knowles subminiature transducer mounted alongside the one inch condenser microphone ..... 106
2.3 The relative size of the Knowles subminiature transducer alongside both a one inch W.E.g40AA and a one half inch General Radio microphone-- ..... 107
3.1 The 70.12 cm brass tube used as a plane wave resonant cavity ..... 130
3.2 Mounting ports, end view ..... 132
3.3 The "short" tube ..... 133
3.4 A closeup of the comparison microphone mounted in the wall of the short tube ..... 134
3.5 The microphone port in the wall of the short tube ..... 135
3.6 A comparison photograph showing the physical differences in the end microphone mounts between the short and the long tube ..... 136
3.7 The microphone translator ..... 138
3.8 The mounting of the drive motor and the optical shaft encoder ..... 140
3.9 The equipment setup at the entrance to the anechoic chamber ..... 142

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# 1. THE_PRINCIPLE_OF_ACOUSTICAL_RECIPROCITY 

## IN_MICROPHONE_CALIBRATION

## A. INTRODUCTION

In any experiment requiring absolute acoustic measurements there exists a need for an accurate calibrated standard microphone, or its equivalent, to be used to directly measure acoustic data or to be used as a sensitivity reference. The primary subject of this paper is an unconventional method for obtaining such a calibration on a standard icrophone. First described by Isadore Rudnick [Ref. 1], this unconventional method results in an acoustical reciprocity pressure calibration of a standard microphone. It is unconventional in that it uses a plane wave resonant cavity instead of a free field or a small pressure chamber for the calibration. The method of plane wave resonant reciprocity calibration [Ref. 1] has been satisfactorily employed by G.W. Swift, A. Migliori, S.L. Garrett, and J.C. Wheatley, to calibrate a dynamic pressure transducer from 0 to 400 hertz in a $1-\mathrm{MPa}$ helium gas. The dynamic pressure calibration so obtained was experimentally verified by another calibration method based upon a mercury
manometer to within an instrumental uncertainty of one percent [Ref. 2].

Conventional methods for obtaining acoustic pressure calibrations of microphones are published by The American National Standards, Inc. (ANSI)[Ref. 3], and include pressure coupler reciprocity calibration and free field reciprocity calibration. These two acoustic reciprocity microphone calibration techniques are well reported in the literature [Refs.4,5,6,7,8,9,10,11,12].

In this dissertation, the application of acoustical reciprocity to microphone sensitivity calibrations will include theory for all three types of acoustical reciprocity calibrations and experimental measurements of microphone sensitivity based upon plane wave resonant reciprocity, free field reciprocity, and pressure-reciprocity calibration performed in a closed coupler. The coupler calibrations provided here were performed by the National Bureau of Standards. Essential to the derivation of the general theory for a plane wave resonant reciprocity calibration is a "microphone that feels no impressed pressure" [Ref. 1]. This describes a perfectly rigid microphone for which the mechanical impedance is infinite. Here, the finite impedance of the microphone is included and a correction to the plane wave resonant reciprocity calibration is predicted and experimentally verified at low frequencies.

When calibrating microphones in a gas it is interesting to note, and will be shown, that all three methods of acoustic reciprocity calibration require sto within a multiplicative constant) the measurement of experimental variables that can be expressed in a similar way. These variables are; a volume, the frequency of sound, the barometric pressure, the ratio of specific heats, and basic electrical measurements. As a consequence of this common descriptive set, acoustical reciprocity calibrations are classified as priaary methods of acoustic microphone calibration after Bobber [Ref. 10], where a primary method is defined as requiring only basic measurements of voltage, current, electrical and acoustical impedance, length, mass(or density), and time(or frequency). Secondary methods are those in which a microphone, or some other reference, has been calibrated by a primary method and is used as a reference standard.

To appreciate the impact that acoustical reciprocity calibrations have had upon acoustical science, it is useful to review the history of acoustical reciprocity in the context of historical attempts to measure acoustic pressure, particle displacement, and particle velocity.
B. HISTORY

The acoustical reciprocity principle was introduced first in Lord Rayleigh's (John William Strutt) paper on "Some General Theorems relating to Vibration"[Ref. 13] in 1873. In that paper he gave the following example of reciprocity and credited Helmholtz with a proof in the case of a uniform fluid without friction:
"In a space occupied by air, let $A$ and $B$ be two sources of disturbance. The vibration excited at $A$ will produce at $B$ the same relative amplitude and phase as if the places were exchanged."[Ref. 13: p. 181]

In 1877, in his treatise on The Theory of Sound [Ref. 14], Lord Rayleigh gave as an example of the reciprocity theorem in acoustics the following:
"let $A$ and $B$ be two points of a space occupied by air, between which are situated obstacles of any kind. Then a sound originating at $A$ is perceived at $B$ with the same intensity as that with which an equal sound originating at $B$ would be perceived at A."[Ref. 14: Vol I, P. 134]

In various examples, Lord Rayleigh applied the reciprocity principle to a harmonic transverse force and a resulting displacement in a bar [Ref. 14: Vol I, p. 153], and to periodic electromotive forces and the resultant currents in electrical circuits [Ref. 14: Val I, P. 155].

Lord Rayleigh proposed to measure the acoustic particle velocity in a sound field with the "Rayleigh disk" in 1882 [Ref. 9: p. 148]. When the theory for the performance of the Rayleigh disk was set down by Koenig [Ref. 9: p. 148], it became widely used as tool in acoustical measurements. Other investigators measured the acoustic particle amplitude but in general these techniques yielded measurements which were accurate to within a few percent at best [Ref. 9: pp. 159-160].

A primary source of sound was invented by Gwozdz [Ref. 9: p. 169], a Russian engineer, in 1907 when he used a wire heated by electricity as a source of sound. This thermophone was improved on by Lange [Ref. 15] when he invented a thermophone with substantial acoustical output in 1914. H. D. Armold and I. B. Crandall [Ref. 16], developed a quantitative theory to explain the workings of the thermophone, and both S. Ballantine [Ref. 17] and E.C. Wente [Ref. 18] improved upon the theory. The thermophone remains today as a primary sound source with accuracies on the order of 1 dB re iV/ubar [Ref. 9: p. 171]. In 1917, developments in the field of vacuum tube amplifiers with their high input impedance made the design of a condenser microphone by Wente [Ref. 19] practical. The first calibration of Wente's condenser microphone was accomplished using a thermophone of his own construction [Ref. 19]. It remained for MacLean
[Ref. 4] and independently, Cook [Ref. 5] to first apply Rayleigh's reciprocity theorem to the electroacoustic system in a manner such that absolute acoustic reciprocity calibrations of the receiving (and transmitting) sensitivity of a microphone were obtained.
G.S.K. Wong and T.F.W. Embleton [Ref. 12], have predicted the best accuracies for a condenser microphone calibration to date using the pressure coupler reciprocity calibration method, with the uncertainty of the absolute open circuit sensitivity level expected to be less than . 005 dB re 1 V/ubar. This dissertation will theoretically and experimentally explain the calibration of the open circuit receiving sensitivity obtained with the method of plane wave resonant reciprocity for a standard Western Electric 640AA condenser microphone.


Photograph 1.1 Songe_of_the_mi $=r o p h o n e s=u s e d \_i n$ this_experiment.

In the photograph above, a W.E. G40AA one inch laboratory standard condenser microphone is shown on the left, in the center is a Knowles type BT 1751 subminiature transducer, and on the right is a General radio type 1434 condenser microphone. Both the W.E. G40AA and the GR 1434 condenser microphones have locally made electrical connectors for BNC connectors partially visable in this photo.
C. A GENERAL THEORY OF ACOUSTICAL RECIPROCITY

Prior to discussing acoustical reciprocity, the following symbols are defined:

$$
\mathscr{A}^{-} \text {velocity potential due source "A" }
$$

$$
\oiint_{B} \text { - velocity potential due source "B" }
$$

$$
P_{A} \text { - acoustic pressure field due source "A" }
$$

$$
\rho_{8}-\quad a c o u s t i c ~ p r e s s u r e ~ f i e l d ~ d u e ~ s o u r c e ~ " 8 " ~
$$

$$
\stackrel{u}{u}_{A} \text { - acoustic particle velocity due source "A" }
$$

$$
\vec{u}_{B} \text { - acoustic particle velocity due source "B" }
$$

$$
H_{A} \text { - volume velocity of source "A" }
$$

$$
\dot{U}_{8} \text { - volume velocity of source "S" }
$$

Po - equilibrium density of acoustic medium
$\omega$ - angular frequency of either monofrequency source
$e$ - open circuit receiving voltage
$i$ - short circuit transmitting current
$Z$ - general term for impedance [acoustic or electrical]
Mo - open circuit receiving sensitivity
$S_{i}-t r a n s m i t t i n g$ sensitivity (current)
$J$ - reciprocity factor (acoustic transfer admittance)
$\gamma$ - ratio of specific meats for the acoustic medium
$\gamma_{E}$ - ratio of specific heats for medium corrected for non-adiabatic boundary conditions and for the effects of relative humidity.
$P_{0}$ - equilibrium (ambient) pressure
Vo - cavity volume used in reciprocity scheme
$\lambda$ - acoustic wavelength

```
C - acoustic phase speed in unbounded medium
\(\langle\varepsilon\rangle_{t}\) - time averaged energy density
Qu - quality factor of the Nth resonance
\(\Delta \varepsilon\) - energy dissipated per cycle
    L - length of cylindrical plane wave resonator
    No - end cross sectional area of cylindrical plane wave
    \(N\) - the mode number of the longitudinal resonance
    \(f_{1}\) - fundamental frequency of longitudinal resonances
    \(f\) - frequency \(\{\mathrm{Hz}\}\)
    \(k\) - wavenumber, equal to ( \(2 *\) pi)/(wavelength)
    \(k_{N}\) - modal wavenumber \{for rigid boundary conditions in
    the ends of the cylindrical resonant cavity, equal
    to (mode number*pi)/(length of tube) \(\}\)
```

A theory of passive linear electroacoustics transducers was developed by L.L. Fold and H. Primakoff in a two part paper published in 1945 [Ref. 20] and in 1947 [Ref. 21]. In their theory, the pressure and normal velocity at each point on a transducer's surface and the voltage and current at a transducer's electrical terminals were shown to be related by a set of linear integral equations. Solving the wave equation using Green's functions for the medium in which the transducer was immersed and using the equations defining the electrical termination of the transducer, they showed that the behavior of a transducer could be completely characterized in terms of four parameters. These four
parameters were shown to be: the voltage across the electrical terminals of a transducer, the current flowing into these terminals, the acoustic pressure at any point on the surface of the transducer, and the normal velocity of the surface at every point on the face of the transducer [Ref. 20]. In part II of the paper, L.L. Foldy and $H$. Primakoff developed the conditions necessary for the validity of the electroacoustic reciprocity theorem in a wide variety of physical systems. They found that,


The four parameter transducer description of L.L. Foldy and H. Primakoff is used for a condenser microphone in this paper.

To demonstrate resonant reciprocity in a cavity, consider the volume shown in figure 1.1. The walls are rigid
and there are two reciprocal transducers flush mounted in the walls. In the ensuing discussion each transducer will be used alternately as a sound source and as a sound receiver.

 erojected.

When the sound fields within the cavity are irrotational, the acoustic particle velocity is given by the gradient of the velocity potential. Following kinsler, Frey, Coppens, and Sanders [Ref. 11: p.165], we apply a vector identity to the two velocity potentials associated with the two sources to obtain,

$$
\nabla \cdot\left(\phi_{A} \nabla \phi_{B}\right)=\nabla \phi_{A} \cdot \nabla \phi_{B}+\phi_{A} \nabla^{2} \phi_{B}
$$

When the vector identity is again applied, after reversing the order of the velocity potentials, we obtain,

$$
\nabla \cdot\left(\phi_{B} \nabla \phi_{A}\right)=\nabla \phi_{B} \cdot \nabla \phi_{A}+\phi_{B} \nabla^{2} \phi_{A}
$$

Equation 1.2

These two equations are now subtracted and the result $2 s$ integrated over the volume shown $1 n$ figure 1.1 . This yields the integral equation,

$$
\int_{V_{\text {ll }}} \nabla \cdot\left(\phi_{A} \nabla \phi_{B}-\phi_{B} \nabla \phi_{A}\right) d V=\int_{V o l .}\left(\phi_{D}{ }^{2} \phi_{B}-\phi_{B} \nabla^{2} \phi_{A}\right) d V
$$

Equation 1.

If the divergence theorem is applied to the left side we obtain Green's Theorem,

$$
\oint_{\text {Surface }}\left(\phi_{A} \nabla \phi_{B}-\phi_{B} \nabla \phi_{A}\right) \cdot \hat{N} d S=\int_{\text {Vol. }}\left(\phi_{A} \nabla^{2} \phi_{B}-\phi_{B} \nabla^{2} \phi_{A}\right) d V \quad \text { Equation i. } 4
$$

where $\hat{N}$ is the unit vector normal to the surface. Now, if we operate both sources at the same frequency,

$$
\nabla^{2} \phi_{A}=-k^{2} \phi_{A}
$$

Equation 1.5
and,

$$
\nabla^{2} \phi_{B}=-k^{2} \phi_{B}
$$

Equation 1.6
upon substituting equations 1.5 and 1.6 into equation 1.4, we obtain,

$$
\int_{\text {SURfACE }}\left(\phi_{A} \nabla \phi_{B}-\phi_{B} \nabla \phi_{A}\right) \cdot \hat{N} d S=\int_{\text {vol. }}\left(\phi_{A}\left[-k^{2} \phi_{B}\right]-\phi_{B}\left[-k^{2} \phi_{A}\right]\right) d V V_{\text {Equation } 1.7}
$$

and on the left side of equation 1.4 , all that remains is the integral.
$\bigoplus_{\text {surface }}\left(\phi_{A} \nabla \phi_{B}-\phi_{B} \nabla \phi_{A}\right) \cdot \hat{N} d S=0$
Equation 1.8

The acoustic particle velocity and the acoustic velocity potential are related by,
$\stackrel{\rightharpoonup}{u}=\nabla \phi$
Equation 1.9
and from the linearized Euler equation in a medium with constant density, we have,

$$
p=-\rho_{0} \frac{\partial \phi}{\partial t}=-j \omega \rho_{0} \phi
$$

Equation 1.10

Now, prior to substituting equations 1.9 and 1.10 into Equation 1.8, if we consider the integral in equation 1.8 to consist of one portion over the rigid walls and the other portion over the surfaces of each simple source exposed to the inside cavity volume, equation 1.8 can be simplified,

$$
\int_{\substack{\text { RIGID } \\ \text { WAllS }}}\left(\phi_{A} \nabla \phi_{B}-\phi_{B} \nabla \phi_{A}\right) \cdot \hat{N} d S+\int_{\substack{\text { SIMPLE } \\ \text { SOURCES }}}\left(\phi_{A} \nabla \phi_{B}-\phi_{B} \nabla \phi_{A}\right) \cdot \hat{N} d S=0 \text { Equation } 1.11^{\text {S qu }}
$$

Since the normal component of the particle velocity is zero at the rigid walls, the integral over the surface of the walls is zero and we are left with the integral over the exposed faces of the transducers. For harmonic velocity potentials, equations 1.9 and 1.10 are substituted into equation 1.8 where we obtain,

$$
\int\left(p_{A} \vec{u}_{B}-p_{B} \vec{u}_{A}\right) \cdot \hat{N} \cdot d S=0
$$

Equation 1.12
sources
faces

With the definition of the following parameters,
Fab $(\vec{r})=$ pressure at "A" when "B" is transmitting as a function of position, $\vec{r}$, on $A$ 's surface.
Mba( $\vec{r}$ ') $=$ pressure at " $B$ " when "A" $1 s$ transmitting as a function of position, $\vec{r}$, on $E \cdot s$ surface.
$\vec{u} a(\vec{r})=$ velocity of the "face" of source "A" as a function of position, $\vec{r}$.
$\vec{u} b(\vec{r}$ ") = velocity of the "face" of source "B" as a function of position, $\vec{r}$ '.

And provided the acoustic pressure over the face of the transducer acting as a receiver is uniform, we are able to write the following form of the statement of acoustic reciprocity:


When only simple sources are involved and each integral of the normal component of the velocity over the face of the source is replaced by the respective volume velocity this becomes,


Equation 1.14

This 15 analogous to Rayleigh's statement:

```
                        "....the vibration excited at A will produce
at B the same relative amplitude and phase as if
the places were exchanged."[Fief. 1J]
```

The principle of acoustic reciprocity will now be applied to electroacoustic transducers so that we can obtain what 15 termed a reciprocity calibration of the microphone open circuit receiving sensitivity.
D. ACOUSTIC RECIPROCITY CALIBRATIONS OF MICROPHONE SENSITIVITY

1. General__Acoustic__Recierocity__Calibration__of

## Electroacoustic_Transducers

When we describe the general properties of a simple source it is often correct to consider the simple source as a linear two port electroacoustic network [Ref. 20],


Figure 1.2 ElectrooAcoustic_Two_Port_Network.

From Fold and Primakoff [Refs. 20, 21], the equations describing the relationship between the open circuit receiving voltage, $e$, the pressure at the receiver surface, $p(\vec{r})$, the transmit current, $i$, and the transmitter normal velocity, $\vec{u}(\vec{r})$, at the face of the transducer are,

$$
\begin{aligned}
& e=\left[Z_{B}\right] i+\int_{\text {SuRfAce }} h^{\prime}\left(\vec{r}^{\prime}\right) \vec{u}_{N}\left(\vec{r}^{\prime}\right) d \vec{r}^{\prime} \\
& P=[h(\vec{r})] i+\int_{\text {SuRfAce }} z\left(\vec{r}^{\prime} \vec{r}^{\prime}\right) \vec{u}_{N}\left(\vec{r}^{\prime}\right) d \vec{r}^{\prime}
\end{aligned}
$$

The reciprocity relations that are required of any transducer used in a reciprocity calibration are [Ref. 21]:

$$
h(\vec{r}) / h\left(\vec{r}^{\prime}\right)=e^{i \alpha} ; \quad \begin{aligned}
& \alpha \text { a real function, } \\
& \text { Not dependent upon } \\
& \vec{r}
\end{aligned}
$$

$$
z\left(\vec{r}, \vec{r}^{\prime}\right)=Z\left(\vec{r}^{\prime}, \vec{r}\right)
$$

Equations 1.16

The variables used above are defined by L.L. Fold and $H$. Primakoff to be [Ref. 21]:

$$
\begin{aligned}
Z b= & \text { the blocked electrical impedance of } \\
& \text { the transducer } \\
h(\vec{r})= & \text { the speaker transfer impedance } \\
h \cdot\left(\vec{r}^{\prime}\right)= & \text { the microphone transfer impedance } \\
Z\left(\vec{r}, \vec{r}^{\prime}\right)= & \text { the generalized open circuit normal } \\
& \text { acoustic impedance of the transducer } \\
& \text { surface. }
\end{aligned}
$$

The first equation simply states that the speaker transfer impedance and the microphone transfer impedance are equal in magnitude and the phase angle between them is the same at all points of the transducer surface.

When equations 1.15 are recast in matrix form, $h(\vec{r})$ and $h \cdot\left(\vec{r}^{\prime}\right)$ are equal to $Z 21$ and $Z 12$ respectively, the generalized acoustic impedance is constant over the transducer's surface and is replaced with $Z 22$, the integrals of the normal velocity over the transducer surface are replaced by the volume velocity $\dot{U}$, and $Z b$ is replaced with 211.

$$
\left[\begin{array}{l}
e \\
p
\end{array}\right]=\left[\begin{array}{ll}
z_{11} & z_{12} \\
z_{21} & z_{22}
\end{array}\right]\left[\begin{array}{l}
i \\
\dot{u}
\end{array}\right]
$$

Equations 1.17

In particular, with the voltage-pressure and current-volume velocity formulation shown above, if 212 equals (+,-)Z21 then the transducer is said to be a reciprocal(+) or an antireciprocal(-) transducer [Ref. 22]. The impedances are now described by:

Z11 = open circuit voltage / short circuit current
Z12 = open circuit voltage / volume velocity
Z21 = received acoustic pressure / short circuit current Z22 = received acoustic pressure / volume velocity

When two such transducers are connected by an acoustic medium, a four port network will represent the system,


Figure 1.3 Four Port_Electrog-Acoustic_Network

If we choose all the receiving sensitivities to be the open circuit voltage receiving sensitivities and all the transmitting sensitivities to be the current transmitting sensitivities, we have the following definitions:

$$
\begin{aligned}
& M_{A}=e_{1} / p_{2} \\
& S_{A}=p_{3} / i 1 \\
& M_{B}=e_{4} / p_{3}
\end{aligned}
$$

$$
\text { Equation } 1.19
$$

$$
\text { Equation } 1.19
$$

Equation : . 20

$$
S_{B}=P 2 / i 4
$$

Dividing equation 1.18 by 1.19 we obtain,

$$
\frac{M_{A}}{S_{A}}=\frac{e_{1} i 1}{P_{2} P_{3}}
$$

Equation 1.22
and dividing equation 1.20 by equation 1.21 , we obtain,

$$
\frac{M_{B}}{S_{B}}=\frac{e 4 i 4}{P_{3} p_{2}}
$$

Equation 1.23

Where we define:
el = open circuit voltage at port (1) when transducer "E" is transmitting.
il = input current at port (1) when transducer "A" is transmitting.

P2 = acoustic pressure at port (2) when transducer "B" is transmitting.

P3 = acoustic pressure at port (ङ) when transducer "A" is transmitting.
e4 $=$ open circuit voltage at port (4) when transducer "A" is transmitting.

```
i4 = input current at port(4) when transducer "E"
    is transmitting.
```

Now, if both of the transducers and the medium are "reciprocal", meaning that $Z 12=221, \quad Z 23=232$, and $Z 34=243$, the equations representing the reciprocal system are,

$$
\left[\begin{array}{l}
e_{1} \\
e_{4}
\end{array}\right]=\left[\begin{array}{ll}
z_{11} & z_{14} \\
z_{41} & z_{44}
\end{array}\right]\left[\begin{array}{l}
i 1 \\
i 4
\end{array}\right]
$$




By definition, el and 11 do not coexist in time. Similarly, eq and 14 do not coexist in time. Thus if 1 receives when 4 transmits,

$$
e 1=\left[z_{11}\right](0)^{0}+\left[z_{14}\right][14]
$$

Equation 1.25
and when 1 transmits and 4 receives,

$$
e_{4}=\left[z_{41}\right][i 1]+\left[z_{44}\right](0) \quad \text { Equation } 1.26
$$

If the network described by equation 1.22 is reciprocal or antireciprocal, then Z14 $=(+-)$ Z41 so that,

$$
\frac{e 1}{i 4}=( \pm) \frac{e 4}{i 1}
$$

Equation 1.27

This is the electrical analog of Rayleigh's statement of acoustical reciprocity [Fief. 14]. Substituting this into equations 1.22 and 1.23 we obtain,

$$
\begin{gathered}
\frac{M_{A}}{S_{A}}=\frac{e_{1} i_{1}}{p_{2} p_{3}}=\frac{e_{4} i_{4}}{p_{3} p_{2}}=\frac{m_{B}}{S_{B}} \\
-49-
\end{gathered}
$$

Following L.L. Beranek [Ref. 9] and R.J. Bobber [Ref. 10] we define this ratio as the reciprocity factor, "J" and we have,

$$
\frac{M_{A}}{S_{A}}=\frac{M_{B}}{S_{B}} \equiv J
$$

Equation 1.29

Provided that the medium and both transducers are reciprocal, and if both transducers are ideal, we have shown that the reciprocity factor "J" depends only upon the medium (including its boundaries) and is independent of the transducers used.

To obtain a comparison between the two transducer's receiving sensitivities, a separate sound source 15 used to generate the same acoustic pressure that in turn 15 sampled at the same position by each of the receiving microphones. The ratio of the received voltages yields the desired comparison.

$$
p_{\text {(same })}=\frac{V_{A}}{M_{A}}=\frac{V_{B}}{m_{B}}
$$

Va and vb are the open circuit receiving voltages of transducers "A" and "B" respectively when the third separate sound source is transmitting. Equations 1.29 and 1. SO yield two expressions for Ma which we multiply together to obtain the square of the open circuit receiving sensitivity for transducer $A$.

$$
\left(m_{A}\right)^{2}=\left(m_{B} \frac{V_{A}}{V_{B}}\right)\left(S_{A} J\right)
$$

Equation 1.31

Applying the definitions of Sa and Mb found in equations 1.19 and 1.20 , we obtain the solution for the open circuit receiving sensitivity for microphone "A".

$$
J)^{1 / 2}
$$

Equation 1.52

Using a similar procedure for transducer "E" we obtain,

$$
M_{B}=\left(\frac{e l}{14} \frac{V_{B}}{V_{A}} J\right)^{1 / 2}
$$

Equation 1.J

If the voltages and currents in equations 1.32 and 1.33 can be measured, the only parameter remaining to be calculated in order to obtain a complete solution for the receiving sensitivities Ma and Mb is the reciprocity factor, "J". The physical meaning of the reciprocity factor "J" will be explained in the next section.
2. General Determination_of_The_Recigrocity_Eactor__"J." When the dimensions of the reciprocity factor, "J" are considered, we find that it can be interpreted as having dimensions of volume velocity over pressure. This is an acoustic admittance. After Rudnick [Ref. 1], we proceed to determine the solution for the reciprocity factor by considering as our reversible transducer one that is small compared to a wavelength, and is so noncompliant that its introduction at a point in the sound field riever alters the sound pressure at that point. By the same token, when used as a speaker its volume velocity is independent of the acoustic load. Let both transducers be identical and of this type. The restriction that they be identical can be lifted trivially by the introduction of a third receiver as was done previously in the development of equation 1.30. For "A" transmitting and "B" receiving we obtain, using equations 1.17 for transducer " $A$ ",

$$
\begin{aligned}
e 1 & =z_{11} i 1+z_{12} \dot{\Delta} \\
0 & =z_{21} i 1+z_{22} \dot{H}
\end{aligned}
$$

Equations 1.34

The second equation has a zero on the left because there is no impressed pressure and the ideal transducer does not feel the pressure that is self generated. For transducer "B" we
have,

$$
e_{4}=z_{44}(0)^{0}+z_{43} \dot{4} 3
$$

$$
P_{3}=z_{34}(0)^{0}+z_{33} \dot{U}_{3}
$$

since we have identical transducers in this example,

$$
\frac{e 4}{\rho 3}=M_{B}=M_{A}=\frac{Z_{12}}{Z_{22}}=\frac{Z_{43}}{Z_{33}} \quad \text { Equation } 1.36
$$

and,

$$
S_{A}=\frac{p 3}{i 1}
$$

Equation 1.37

We solve the second equation in 1.34 for the current, il,

$$
i 1=-\frac{z_{2} 2}{Z_{21}} \dot{U}_{2}=-\frac{\dot{U}}{M_{A}} \quad \text { Equation } 1.38
$$

Substituting the result for il into equation 1.37 and
manipulating the terms, we obtain equation 1.39 and thereby have shown that the reciprocity factor " 7 " $1=$ the acoustic transfer admittance.


Equation 1.39

Since the acoustic transfer admittance depends upon the medium through which the acoustic signal moves, and on the boundaries of this medium, the solutions obtained for the microphone voltage receiving sensitivities will only be valid if the medium and its boundaries are unchanged over the duration of the experimental measurements.

When setting out to calculate an analytical form for the reciprocity factor, different geometries of the medium between the two transducers will result in apparent differences in the forms found for "J". However, as mentioned earlier, it is important to note that stow within a multiplicative constant) all of these solutions for "J" will have the same form; The product of a volume and a frequency divided by the adiabatic bulk modulus of elasticity for the medium. To show this, three different "volumes" as shown in figure 1.1 will be utilized. For a gas medium, the adiabatic bulk modulus 15 expressed as the product of the
barometric pressure and the ratio of specific heats. Short descriptions of the dimensions involved in these different physical environments follow:

Volume 1: All dimensions are much smaller than the acoustic wavelength [Refs. 3, 9, 12].

Volume 2: Free field [Refs. 4, 5, 7,8, 9, 10, 111.

Volume 3: One dimension is greater than or equal to half an acoustic wavelength with the other two dimensions smaller than an acoustic wavelength (cylindrical cavity) [Refs. 1,2].

The solutions to be obtained for "J" in the next section will allow the equations for $M a$ and $M b$ to be defined in terms of basic electrical and physical measurements.
a. The Calculation of "J" for a Pressure Coupler Reciprocity Calibration

For all dimensions much smaller than an acoustical wavelength, "volume 1 " in the previous section, the magnitude and phase of the pressure will be essentially the same everywhere within the cavity. Using the adiabatic form of the ideal gas law,

$$
P V^{\gamma}=\text { CONSTANT }
$$

Equation 1.40
we take the natural $l o g$ of both sides and differentiate,

$$
\frac{d \theta}{\gamma}+\gamma \frac{d V}{V}=0
$$

Equation 1.41

If the acoustic pressure is harmonic then it can be written as,

$$
d P=-p=p_{0} e^{j \omega t}
$$

Equation

$$
1.42
$$

then considering the change in acoustic pressure with respect to time,

$$
\frac{d p}{d t}=j \omega p
$$

Recognizing that the volume velocity may be expressed as the time rate of change of the volume we can write the time derivative of equation 1.41 as ,

$$
\frac{j \omega p}{P_{0}}+\gamma \frac{\dot{\nu}}{V_{0}} \cong 0
$$

consequently,

$$
\frac{\dot{L} 3}{P_{2}}=\frac{-j \omega V_{0}}{\gamma P_{0}}
$$

This is the reciprocity factor for a "small" cavity. Without any manipulation $1 t 1$ seen to follow the general form given earlier for "J".
b. The Calculation of "J" for a Free Field Reciprocity Calibration

For the free field we make use of equation 1.14 which shows that the ratio of the acoustic pressure at the face of a transducer acting as a receiver to the volume velocity of that transducer acting as a source is the same for any simple source. We are therefore able to choose a simple sphere to represent the source. It can be shown [Ref. 11: p. 164], that for an oscillating sphere of radius "a", where "a" is much less than a wavelength, we have,

$$
j(\omega t-k r)
$$

$p(r, t) \doteq j \rho_{0} c u_{0}\left(\frac{a}{r}\right)(k a) e$
Equation 1.46

The volume velocity for the simple spherical source will be,

$$
\dot{U}=\left(4 \pi a^{2}\right) u_{0} e^{j \omega t}
$$

Equation 1.47

Combining equations 1.46 and 1.47 to obtain a form of the acoustic transfer admittance, we have,


Equation 1.48
substituting,

$$
k=\frac{2 \pi}{\lambda}
$$

Equation 1.49
we obtain for the magnitude of the free field reciprocity factor "J",

$$
\frac{\dot{\Delta} 3}{P 2}=\frac{2 \lambda r}{\rho_{0} C}
$$

Equation 1.50

When the numerator is multiplied by the wavelength, and the denominator is multiplied by the phase speed divided by the frequency, we see that,

$$
\frac{\dot{\cup} 3}{p 2}=\frac{2 f\left[\lambda^{2} r\right]}{p_{0} \gamma}
$$

Equation 1.51

This also is the general form given earlier for "J", where the effective volume is a rectangular solid with length and width equal to a wavelength and a height of $2 r$.
c. The Calculation of "J" for The Plane Wave Resonant Reciprocity Calibration

To illustrate the final case where the acoustic wavelength is much larger than two dimensions and equal to or smaller than twice the third dimension of the cavity, a plane wave cylindrical resonator is used. For this case, the reciprocal transducers are mounted in the ends.


Figure 1.5 A_Cylindrical_Plane_Wave_Regonator

Solutions of the wave equation within such a cavity predict three kinds of resonance modes; radial, azimuthal, and longitudinal. When the dimensions of the cylinder are chosen so that the lowest azimuthal and radial resonance modes occur at frequencies higher than the highest longitudinal resonance in the frequency range of interest,
then below this frequency, on ly longitudinal (or plane wave) resonances can occur. For a cavity with rigid walls and ends, the pressure field within the resonant cavity is given by,
$p(r, \theta, z) \propto \sin _{\sin }^{\cos }(m \theta) \cos \left(\frac{\omega_{z} z}{c}\right) J_{m}\left(\frac{\omega_{r} r}{c}\right) e^{j}$
Equation 1.52

Here, Jm() refers to a cylindrical Bessel function and not to the reciprocity factor.

From Morse [Ref. 22: p.398], the ratio of the fundamental azimuthal mode to the fundamental longitudinal mode is found to be .586 times the length to radius ratio of the cylindrical cavity. The ratio of the fundamental radial mode to the fundamental longitudinal mode is found to be 1.22 times the length to radius ratio of the cylindrical cavity. A photograph of the cylindrical plane wave resonant cavity used to experimentally observe the different resonant modes is shown next. All three different modes are then seen in figure 1.6 , as a relative plot of the microphone voltage output vs frequency. A Knowles Type BT-1751 subminiature transducer acted as the microphone and was mounted on the curved wall of the 23.3 cm long cylindrical
cavity shown in the photograph. The source was a W.E.b40AA condenser microphone, Serial \#1248, mounted on one end.


Photograph 1.2 Right_ㄷylindrical_resonant_cavity



Figure 1.p Modal_resonances_in_a_23.3_﹎ㅡ﹎_air_filled_brass


Following G.W. Swift,A. Migliori, S.L. Garrett, and J.C. Wheatley [Ref. 2], the calculation of acoustical transfer admittance is as follows:

The dot product of the acoustic particle velocity with both sides of the linearized Euler equation of motion yields,

$$
\vec{u} \cdot \rho_{0} \frac{\partial \vec{u}}{\partial t}=\vec{u} \cdot(-\nabla p)
$$

Next, the equation of continuity of mass, written to first order in the acoustic density, is multiplied on both sides by the acoustic pressure,

$$
\rho\left(\frac{1}{e_{0} c^{2}}\right) \frac{\partial p}{\partial t}=\rho(-\nabla \cdot \vec{u})
$$

adding equations 1.53 and 1.54 we have,

$$
\stackrel{\rightharpoonup}{u} \cdot \rho_{0} \frac{\partial \bar{u}}{\partial t}+\frac{p}{\rho_{0} t^{2}} \frac{\partial \rho}{\partial t}=\vec{u} \cdot(-\nabla \rho)+p(-\nabla \cdot \bar{u}) \quad \text { Equation } 1.55
$$

this equation can be rewritten as,

$$
\frac{\partial}{\partial t}\left(\frac{1}{2} \rho_{0} u^{2}+\frac{p^{2}}{2 \rho_{0} c^{2}}\right)=-\nabla \cdot\left(\varphi \psi^{-}\right) \quad \text { Equation } 1 . \text { So }
$$

This is an equation of continuity for energy density. The terms in the leftmost bracket are the kinetic energy censity and the potential energy density respectively. The product
of acoustic pressure and acoustic particle velocity is the acoustic intensity. . This equation is integrated over the volume within the resonant cavity to obtain,

$$
\int_{V O L} \frac{\partial}{\partial t}\left(\frac{1}{2} \rho_{0} u^{2}+\frac{p^{2}}{2 \rho_{0} c^{2}}\right) d v=-\int_{V O L} \nabla \cdot(p \vec{u}) d v
$$

Equation 1.57

Applying the divergence theorem to the right, hand side of this equation and reversing the order of integration and differentiation on the left hand side, the equation becomes,

$$
\frac{d}{d t}\left(E_{\text {total }}\right)=-\bigoplus_{\text {surface }} p \vec{u} \cdot \hat{N} d s
$$

For plane waves at a longitudinal resonance, the acoustic pressure amplitude is uniform over each end of the cylindrical cavity. Additionally, with the ideal receiving microphone (the mechanical impedance is infinite), the integral of the normal component of the particle velocity over the rigid wall of the cavity volume is zero everywhere except on the surface of the source microphone,

$$
\frac{d}{d t}\left[E_{\text {TOTAL }}\right]=-P_{\text {END }} \int_{\substack{\text { SOURCE } \\ \text { FACE }}} \sim \stackrel{\rightharpoonup}{u} \cdot \hat{N} d S
$$

Equation 1.59

Taking the average over one cycle yields,

$$
\frac{-1}{T} \int_{0}^{T} \frac{d E_{T}}{d t} d t=\frac{1}{T} \int_{0}^{T}\left(P_{E N D} L_{E N D}\right) d t
$$

Equation 1.60

Here we have obtained the incremental change per cycle of the plane wave energy within the cavity due to the work per cycle done by the source microphone.

$$
\Delta E=\frac{1}{f}\left[\frac{1}{T} \int_{0}^{T}\left(P_{E N D} \dot{U}_{E N D}\right) d t\right]
$$

Equation 1.61

Within the cavity, the energy dissipated per cycle equals the work done per cycle to drive the source when a steady state longitudinal resonance is maintained. Thus, the change in the total plane wave energy available above as a result of the average work done by the source over one cycle must be the energy input per cycle required to sustain the
plane wave resonance. Here the energy dissipated per cycle is expressed in terms of the source pressure, the source volume velocity and the sound frequency.

If the velocity of the driving face is considered to be harmonic and if the acoustical system being driven is at a longitudinal resonance, then the acoustic impedance at the source is purely resistive and the particle velocity and acoustic pressure are in phase at the face of the driving transducer. In this case we have,

$$
\begin{aligned}
& P_{\text {END }}=P_{0} \cos (\omega t) \\
& \dot{U}_{E N D}=\dot{\omega}_{0} \cos (\omega t)
\end{aligned}
$$

Equation 1.62

Now the energy per cycle required to sustain the resonance can be written as,

$$
\frac{\Delta E}{c y c l E}=\frac{P_{0} \dot{U}_{0}}{2 f}=\frac{P_{R m s} \dot{H}_{\text {Rms }}}{f}
$$

Equation 1.63

The terms for energy density found in equation 1.56 can now be used to calculate the total acoustical energy found within the cavity. Consider a differential volume of length
$d x$ and cross sectional area equal to that of the cylinder located at position "x" within the tube.


Figure 1.7 Differential_volume_under_congideration

For rigid ends, the pressure standing waves due to longitudinal resonances within the cavity are given by,

$$
P(x, t)=p_{0} \cos \left(k_{\omega} x\right) e^{j \omega t}
$$

Equation 1.64

From the linearized Euler equation in a medium with constant density we have,

$$
\rho_{0} \frac{\partial \stackrel{\rightharpoonup}{u}}{\partial t}=-\nabla-p
$$

Taking minus the gradient of equation 1.64 we obtain,

$$
-\nabla p=p_{0} \kappa_{N} \sin \left(\ell_{N} x\right) e^{j \omega_{N} t} \quad \text { Equation } 1.06
$$

Substituting into equation 1.65 and solving for the derivative of the particle speed with respect to time,

$$
\frac{\partial u}{\partial t}=\frac{p_{0} k_{N}}{\rho_{0}} \sin \left(k_{N} x\right) e^{j \omega_{N} t}
$$

Equation 1.67

Integrating, we obtain the acoustic particle speed,

$$
u(x, t)=\frac{p_{0} h_{N}}{j \omega_{N} \rho_{0}} \sin \left(h_{N} x\right) e^{j \omega_{N} t} \quad \text { Equation } 1.08
$$

Keeping the real part of the complex acoustic particle speed we have,

$$
u(x, t)=\frac{p_{0}}{e_{0} c_{0}} \sin \left(h_{N} x\right) \sin \left(\omega_{N} t\right)
$$

Equation 1.69

From equation 1.64 the real part of the complex acoustic pressure is,

$$
p(x, t)=p_{0} \cos \left(h_{N} x\right) \cos \left(\omega_{N} t\right) \quad \text { Equation } 1.70
$$

The total mechanical energy within the differential volume shown is the sum of the differential kinetic energy and the differential potential energy contained within.

$$
d E_{T}=d \varepsilon_{K E}+d \varepsilon_{P E}
$$

Equation 1.71

The differential kinetic energy from equation 1.56 is given by,

$$
d \varepsilon_{k E}=\frac{1}{2} \operatorname{lo}_{0} u^{2} A_{0} d x
$$

Equation 1.72
and the differential potential energy from equation 1.56 is given by,

$$
d \varepsilon_{\text {PE. }}=\frac{\rho^{2}}{2 \rho_{0} c^{2}} A_{0} d x
$$

Equation 1.73

Combining equations $1.69,1.70,1.72$ and 1.73, the differential of the total mechanical energy of the standing waves within the small slice of volume 15 obtained,

$$
\begin{aligned}
& d E_{T}=\frac{1}{2} \rho_{0}\left[\frac{\rho_{0}}{e_{0} C_{0}} \sin \left(h_{n} x\right) \sin \left(\omega_{N} t\right)\right]^{2} A_{0} d x+ \\
& \frac{1}{2} \frac{1}{\rho_{0} c_{0}^{2}}\left[\rho_{0} \cos \left(h_{N} x\right) \cos \left(\omega_{N} t\right)\right]^{2} A_{0} d x \quad \text { Equation } 1.74
\end{aligned}
$$

Integrating over the cavity volume we obtain tine integral form of the total mechanical energy within the plane wave resonant cavity,

$$
E_{T}=\frac{P_{0}^{2} A_{0}}{2 \rho_{0} C_{0}^{2}} \int_{x=0}^{x=L}\left[\operatorname{Sin}^{2}\left(K_{N} x\right) \sin ^{2}\left(\omega_{N} t\right)+\cos ^{2}\left(K_{n} x\right) \cos ^{2}\left(\omega_{N} t\right)\right] d x
$$

For rigid ends within the plane wave cavity, the modal wave number is,

$$
h_{N}=\frac{N \pi}{L}
$$

Equation 1.76

Upon substitution into equation 1.75 and integration we obtain,

$$
E_{T}=\frac{P_{0}^{2} A_{0} L}{4 l_{0} C_{0}^{2}}
$$

Equation 1.77

Using the root mean square value for the acoustic pressure,

$$
E_{T}=\frac{P_{\text {Rms }}^{2} A_{0} L}{\partial P_{0} C_{0}^{2}}
$$

Equation 1.78

Where "Q" is the quality factor of the plane wave longitudinal resonance we have [Fief. 11],

$$
\frac{E_{T}}{\Delta E / \text { cych }}=\frac{Q}{2 \pi}
$$

Equation 1.79
we can substitute the values previously determined to obtain,

$$
\frac{Q_{N}}{2 \pi}=\frac{\left(\frac{P_{R m S}^{2} A_{0} L}{2 l_{0} C_{0}^{2}}\right)}{\left(\frac{P_{R m s} \dot{L}_{R m S}}{f_{N}}\right)}
$$

Equation 1.80

From the ideal gas equation of state, we have,

$$
p_{0} \gamma=e_{0} c^{2}
$$

Equation 1.81

Since at a longitudinal resonance in a plane wave resonant cavity with rigid ends, the pressures at each end are equal in magnitude, we can write for the magnitude of the acoustical transfer admittance of the plane wave resonator,


Equation 1.82

Inasmuch as the quality factor for the Nth resonance is dimensionless and is computed from a set of basic electrical measurements at frequencies in the vicinity of resonance, we have once again the general form given earlier for "J".

The complete representation of the reciprocity equations which may be considered for experimental implementation in all three different geometries will next be summarized.
3. A_Summary_of_The_Three_Reciprocity_Methods_Compared_in

## ThisExxperiment

a. Plane Wave Resonant Reciprocity Calibration

For the case of the plane wave resonant cavity reciprocity calibration, we can combine equations 1.32 and 1.82 to obtain,
$M_{A}=\left(\begin{array}{ll}e 4 V_{A} & \pi V_{0} f_{N} \\ i 1 Y_{B} & P_{0} \gamma Q_{N}\end{array}\right)^{1 / 2}$
Equation 1.83

This is the initial form of the solution for the open circuit voltage receiving sensitivity found using the plane wave resonant reciprocity method of microphone calibration. In chapter II, the difficulties associated with experimentally obtaining the required measurements of the basic electrical and physical parameters used in the above equation will be addressed.
b. Free Field Reciprocity Calibration

For the case of the free field reciprocity calibration, equations 1.32 and 1.50 yield,
$M_{A}=\left(\frac{e_{4}}{i 1} V_{A} 2 \lambda r \quad V_{B} \rho_{0} C_{0} \quad\right)^{1 / 2}$
Equation 1.84

This is the solution for reciprocity calibration that was initially obtained by W.R. MacLean [Ref. 4] and independently by R.K. Cook [Ref. 5] in 1940.
c. Pressure Coupler Reciprocity Calibration

Finally, for the case of the pressure coupler reciprocity calibration, combining equations 1.32 and 1.45 we obtain,
$M_{A}=\left(\begin{array}{lll}e_{4} & V_{A} & 2 \pi f V_{0} \\ \hline i 1 & V_{B} & \rho_{0} \gamma\end{array}\right)^{1 / 2}$
Equation 1.85

With these pedagogical solutions for the open circuit receiving sensitivity complete, we are now able to consider the form of the solution shown in equation 1.83 with regard to the experimental methods used to obtain the basic electrical and physical measurements which, when substituted into equation 1.83, ultimately yield Ma.

## II. EXPERIMENTAL CONSIDERATIONS FOR A RESONANT RECIPROCITY CALIBRATION

## A. INTRODUCTION

The form of the equations derived in Chapter I for the microphone open circuit voltage receiving sensitivity found using the method of plane wave resonant reciprocity was not optimized for experimental implementation. Consider the following equations:

$$
\left.\begin{array}{l}
M_{A}=\left(\begin{array}{llll}
\frac{e 4}{} V_{A} & \pi V_{0} f_{N} \\
i 1 & V_{B} & p_{0} \gamma Q_{N}
\end{array}\right)^{1 / 2} \\
M_{B}=\left(\frac{e 1}{i 4} V_{B}\right. \\
\frac{i V_{0}}{} V_{N} \\
P_{0} \gamma Q_{N}
\end{array}\right)^{1 / 2}
$$

$$
\text { Equations } 2.1
$$

To reduce the amount of experimental error introduced by sine method of the experiment, it is useful to consider the impact of practical considerations upon the analytical form of equations 2.1. Therefore, the following questions will be addressed in this chapter:

1. How can the ratio Vaivb best be measured to reduce experimental error?
2. How can a self consistency check on the experimental results be incorporated into the experiment?
3. Is it possible to reduce the number of basic electrical measurements experimentally made?
4. What is the correction required in the experimentally obtained value for the open circuit voltage receiving sensitivity as a result of the non-ideal character of the compliant microphones used in the experiment?

Additionally, the experimental measurement of the quality factor of the nth resonance can be very difficult to obtain with any accuracy. Only with the experiment"under computer control using a technique such as that developed by D.V. Conte and S.L. Garrett [Ref. 23] or by J.B. Mehl [Ref. 24] is this source of error reduced.

With these considerations in mind, we will proceed $1 n$ the development of the equations required to experimentally measure the open circuit voltage receiving sensitivity of a microphone using the method of absolute plane wave reciprocity calibration.
B. DEVELOPMENT OF THE EXPERIMENTAL PLANE WAVE RECIPROCITY
EQUATIONS USING CYLINDRICAL GEOMETRY

## 1. Experimental_Considerations

When the method to be used in measuring the ratio Va/Vb is considered, two basic approaches are possible. In the first, the voltage ratio is measured as an experimental event unique unto itself. The final result is the ratio Va/Vb. The difficulties associated with this experimental approach are primarily related to the difficulty to be found in the precise repositioning of the microphones on one end of the resonant cavity to obtain the same reference pressure generated by a source at the other end. It is preferable to design a "hands-off" experiment that will eliminate the introduction of any such re-positioning error and will allow computer control of all the experimental measurements including those used to determine the quality factor, Q. When the reference source is mounted at a third port in the wall of the resonant cavity, the "hands-off" experiment is possible. However, there already exist two such sources of sound within the resonant cavity in the form of the reciprocal transducers mounted at each end. An experimentally productive alternative is to mount instead, a receiving microphone as a reference in the third port. It will be shown that by mounting a reference microphone in the wall of the resonant cavity, not only will we be able to
measure the ratio $V a / V b$, but we can also also obtain a comparison calibration of the reference microphone and a six way round-robin self consistency check of the experimental precision of the calibration for each mode. The calibrations of microphone receiving sensitivities so obtained will be as follows:
Ma - based upon absolute plane wave reciprocity.
Mb - based upon absolute plane wave reciprocity.
Mca - A comparison calibration of the reference

$\quad$ microphone based upon the absolute reciprocity

calibration of microphone A.

The modifications required for equations 2.1 which yield an analytical solution for resonant plane wave reciprocity sensitivity calibrations and which use a reference receiver mounted in the wall of the resonant cavity will now be derived.

When the development of the four port acoustical reciprocity network is expanded to include the comparison microphone mounted in the side of the cylinder, some
modifications to the analytical development for the solution of Ma are necessary. Consider figure 2.1 below.


Figure 2.1 Modified_four_port_network.

The relationships between the receiving and transmitting sensitivities for the two reciprocal microphones mounted on the ends can still be represented as before,

$$
\begin{aligned}
& M_{A}=S_{A} J \\
& M_{B}=S_{B} J
\end{aligned}
$$

Equations 2.2

To obtain a comparison between transducer "A" and transducer "B" with the geometry shown above, it is necessary to relate the pressure at the face of microphone "C" to the pressure
at each end. Thus, when " $A$ " is transmitting,

$$
M_{c}=\frac{V_{C A}}{P_{C A}}
$$

Equation 2.
and when " B " is transmitting,

$$
M_{C}=\frac{V_{C B}}{P_{C B}}
$$

Equation 2.4

At a longitudinal plane wave resonance, the fins acoustic pressure at the right end of the cavity can be related to the rms pressure "felt" by the comparison microphone, with a pressure standing wave correction factor, Glim), whiten depends upon the frequency of the plane wave resonance, and the position of the comparison microphone in the tube.

$$
\begin{equation*}
P_{C A}=\left[P_{3}\right][G 11(\omega)] \tag{Equation 2.3}
\end{equation*}
$$

The pressure standing wave correction factor, bin' is obtained of spatially averaging tine longitudinal variations
in acoustic pressure at a plane wave resonance over the face of the reference microphone assuming the microphone sensitivity is independent of position over the face.

It is worth noting that even though the comparison calibration of the microphone "C" sensitivity obtained with the previous assumption may be in error, this assumption has no effect upon the measurement of the absolute sensitivity of microphones "A" and "B". Refer to figure 2.2.
$[P 3] G 1(N)=\left[\frac{2 P_{1}}{m_{A R E A}}\right] \int_{x=A}^{x=b}\left[\cos \left(\frac{N \pi x}{L}\right)\right]\left[\left(\frac{b-a}{2}\right)^{2}-\left(x-\frac{b+a}{2}\right)^{2}\right]^{1 / 2} d x \quad$ Equation 2.6


Figure 2.2 Geogmetry_of_the_offset_microphone_used_in


In the figure above, the curvature of the sides is not involved since the spatial variation of the acoustic pressure occurs only in the longitudinal direction at a plane wave resonance.


Photograph 2.1 In_this the_cogmaarison_microphone_are_mounted_on_the_side_wall 드니므륻

The side wall mounts in the "long" tube were unused in the final experimental measurements since there was room to mount the Knowles subminiature transducers in the ends alongside the one inch condenser microphones. The side wall
mounts in the above photograph are entirely functional and were used in preliminary experiments using electret and dynamic microphones. In the "short" tube, the side wall mounts were necessary when a $1 / 2$ inch microphone was used as the comparison microphone.

To evaluate the relationship between the pressures at the two ends of the resonant cavity consider the general solution for standing plane waves in a cylinder driven at the left end at $x=0$ by transducer " $A$ ", with mechanical impedance $Z m a$, and terminated at the right end at $x=L$ by transducer "B", with mechanical impedance Zmb. The acoustic pressure in the cylindrical cavity can be represented as the superposition of a left going wave and a right going wave and will then be of the form,

$$
P(x, t)=A e^{j(\omega t+k[L-x])}+B e^{j(\omega t-k[L-x])}
$$

Equation 2.7



Fieferring to figure $2 . J$ above, when microphone " $A$ " is used as the source, the ratio of the pressure at $X=0$, ( P ? ') to the pressure at $X=L,(F J)$, will be,

$$
\frac{P_{2}^{\prime}}{P 3}=\frac{A e^{j k L}+B e^{-j k L}}{A+B}
$$

Since the force on the termination 15 pressure times the cross sectional area and the particle speed for a plane acoustic wave 15 given by,

$$
u=-\frac{1}{\rho_{0}} \int \frac{\partial P}{\partial x} d t
$$

We can use the definition for mechanical impedance to solve for the ratio of $A / B$ in equation 2.8. For the microphone mounted in a rigid end, the mechanical impedance will be in general,

$$
Z=\frac{\text { force APPLIED }}{\text { resulting speed }}
$$

Equation 2.10
which upon substitution of equations 2.7 and 2.9 into equation 2.10 at $X=L$, becomes,

$$
z_{m B}=\rho_{0} C A_{B}\left(\frac{A+B}{A-B}\right)
$$

Equation -. 11

Where Ab is the cross sectional area of inicroptiorie E. Solving for the ratio A/E we obtain,

$$
\frac{A}{B}=\frac{z_{m B}+l_{0} C A_{B}}{z_{M B}-l_{0} C A_{B}}
$$

Equation 2.12

Substituting into the equation for $F V^{\prime} / F=$ we obtain,

$$
\frac{P 2^{\prime}}{P 3}=\cos (k L)+j\left[\frac{\rho_{0} c A_{B}}{Z_{m B}}\right] \sin (k L)
$$

Equation 2.1.

Where $\mathrm{F}^{\prime}$ ' is the pressure at $x=0$, and $F=15$ the pressure at $x=L$ when transducer "A" is transmitting. With a similar but symmetrical development with transducer "E" transmitting we obtain,

$$
\frac{P 3^{\prime}}{P 2}=\cos (k L)+j\left[\frac{e_{0} C A_{A}}{z_{m A}}\right] \sin (k L)
$$

Equation 2.14

Where $F=$ is the pressure at the right end and $F=15$ the pressure at the left end when transducer "E" 15 transmitting. Considering the previous sketch of the two different situations, the open circuit receiving sensitivity for microphone $C$ can be found in each case,

$$
M_{C}=\frac{V_{C A}}{P_{C A}}=\frac{V_{C B}}{P_{C B}}
$$

When the standing wave scale factor is used to express the pressure at the offset comparison microphone as a function of the pressure at the end for both values of Mc, we have,

$$
\frac{V_{C A}}{P 3 G 1(N)}=\frac{V_{C B}}{P 3^{r} G 1(N)}
$$

Solving for the pressure ratio as a function of wavenumber,
$\frac{P 3}{P 2}=\frac{V_{C A}}{V_{C B}}\left[\cos (k L)+j \frac{l_{0} C_{A A}}{z_{M A}} \sin \left(h_{L}\right)\right]$ Equation 2.17

This pressure ratio will allow the ratio Ma/Mb to be expressed in terms of voltage ratios alone, provided the term in the brackets in equation 2.17 is equal to unity. If KL is equal to some multiple of pi, as it is with perfectly rigid ends, then this assertion is correct. Experimentally, the deviation of the value of KL from an exact multiple of pi is determined at a plane wave resonance by examining the harmonicity of the modal resonances in the plane wave resonant cavity. [See Appendix A.] If they are multiples of
the fundamental then $K L$ will be a multiple of pi with $\sin (K L)=0$ and $\cos (K L)=(+-)$. From the theory shown in Appendix A., the values of $K L$ at resonance are exact multiples of $p i$ only in the case of a perfectly rigid boundary or in the limit when the frequency goes to infinity. When the fundamental of the highest plane wave resonant frequency measured was calculated and compared to the lowest plane wave resonant frequency, the ratio so found was an estimate of how close the worst case KL came to being an exact multiple of pi. As shown in Appendix A., the worst case occurred as expected at the lowest plane wave resonance. In this worst case, the value of KL was measured as . 9927*pi, to four significant figures. Since the voltage ratio used in equation 2.18 comes in under the square root in the calculation for microphone sensitivity, the value of unity for cos(KL) will introduce an approximate . 001 db re 1 V/ubar worst case error. Since other experimental uncertainties will be shown to be larger than this value by at least one order of magnitude, equation 2.17 will be used in the form shown below.
$\frac{P 3}{P 2}=\frac{V_{C A}}{V_{C B}}$

Now the ratio of Ma/Mb becomes,

$$
\frac{M_{A}}{M_{B}}=\left(\frac{e 1}{p 2}\right)\left(\frac{p_{3}}{e 4}\right) \doteq\left(\frac{e 1}{e 4}\right)\left(\frac{V_{c A}}{V_{c B}}\right)
$$

Equation 2.19

Combining Equations 2.2 and 2.19 we obtain for the square of Ma,

$$
M_{*}^{2}=\left[M_{\theta}\left(\frac{e s}{e_{1}}\right)\left(\frac{v_{u}}{v_{v a}}\right)\right]\left[S_{n} J\right]
$$

Equation 2.20

When the definitions for $S a$ and Mb are substituted into equation 2.20 and we solve for Ma, we obtain,

$$
M_{A}=\left(\frac{e 1}{i 1} \frac{V_{C A}}{V_{C B}} \mathrm{~J}\right)^{1 / 2}
$$

Equation 2.21

Similarly,

$$
\begin{aligned}
M_{B}=\left(\frac{e^{4}}{i 4} \frac{V_{C B}}{V_{C A}}\right. & J)^{1 / 2} \\
& -95-
\end{aligned}
$$

By using the reciprocity relationship; e4i4 $=$ elil, we can eliminate the requirement to experimentally measure i4,

$$
M_{B}=\left(\left(\frac{e^{4}}{e^{1}}\right)\left(\frac{e_{4}^{4}}{i 1}\right)\left(\frac{V_{C B}}{V_{C A}}\right) J\right)^{1 / 2}
$$

Equation 2.2

Having considered the practical consequences and experimental advantages of the method chosen to measure VaハV, the consequences of a compliant microphone with regard to the calculation of the voltage ratio representing Ma/Mb, and having just reduced the number of basic electrical measurements required by one, consideration must now be given to the experimental determination of the reciprocity factor, "J". Consider the form of the acoustic transfer admittance derived in Chapter 1.

$$
J=\left[\frac{\pi V_{0} f_{N}}{\theta_{0} \gamma Q_{N}}\right]
$$

Equation 2.24

The temperature, pressure and density of the medium withim the resonant cavity were found to vary over the duration of
the experiment. Since the reciprocity factor "J" represents the acoustical transfer admittance of the medium within the resonant cavity during the measurement of parameters used to calculate a plane wave resonance reciprocity calibration, some modification in the value used for "J" must be made to properly account for medium changes. When "A" is transmitting this becomes,

$$
\frac{\dot{\dot{L}} 2}{\rho_{3}}=\frac{\pi V_{0} f_{N}}{P_{0} \gamma Q_{A T B R}}
$$

Equation 2.25

Where the subscripts "ATBR" stand for "A transmitting and B receiving". When "B" is transmitting we obtain,

$$
\frac{\dot{U}_{3}}{P_{2}}=\frac{\pi V_{0} f_{N}}{p_{0} \gamma Q_{B T A R}}
$$

Equation 2.26

Where the subscripts "BTAR" stand for "E transmitting and A receiving". Now Equations 2.21 and 2.25 are used to obtain the open circuit receiving sensitivity for transducer "A".

$$
M_{A}=\left(\begin{array}{llll}
e 1 & V_{C A} & \pi & V_{0} f_{N} \\
\hline i 1 & V_{C B} & \beta_{0} & \gamma Q_{A A B R}
\end{array}\right)
$$

Equation 2.27

Similarly, equations 2.23 and 2.26 are used to obtain the open circuit receiving sensitivity for transducer "E".

$$
M_{B}=\left(\begin{array}{llllll}
e_{4} & e 4 & V_{C B} & \pi & V_{0} & f_{N} \\
\hline e 1 & i 1 & V_{C A} & P_{0} & \gamma & Q_{B T A R}
\end{array}\right)^{1 / 2} \text { Equation } 2.28
$$

Equations 2.27 and 2.28 are the solutions we set out to obtain for a plane wave reciprocity calibration which uses a reference microphone mounted in a third port in the resonant cavity.

It is experimentally significant that the solutions for Ma and Mb using the method of plane wave reciprocity appear to be independent of the position used for the mounting of the reference microphone within the resonant cavity when, ir t fact they are rot. This 15 because it is possible to select a location for the reference microphone in the wall of the plane wave resonant cavity that will cause large experimental errors to occur in the solutions for Ma and Mb. Additionally, the error in the absolute calibration will cause any comparison Calibration made $1 \pi$ situs and based upon the absolute reciprocity calibration to be in error as well. These sources of potential error and the solutions

```
devised to avoid them will be discussed in the next
section.
```

2. Iwo_Comparison_Calibrations_for_Microphone_"C"

The need for a six way round-robin in situ experimental check on the precision of each modal calibration requires careful consideration of the experimental methods and calculations used to obtain any calibrations of the reference microphone. From an analytical point of view, two comparison calibrations of the reference microphone can be obtained that are based upon the absolute reciprocity solutions for Ma and Mb. Combining the reciprocity calibration obtained for Ma, the definition of Mc, and the pressure standing wave correction factor, we obtain Mc as a function of Ma and refer to this calibration as Ma.

$$
M_{c a}=\left(\frac{M_{a}}{e t}\right)\left(\frac{V_{c s}}{G_{1}\left(a_{n}\right)}\right) .
$$

Equation 2.29

Similarly, using the reciprocity calibration for Mb we obtain,

$$
M_{C B}=\left(\frac{m_{B}}{e_{4}}\right)\left(\frac{V_{C A}}{G 1(N)}\right)
$$

Equation 2.30

The diffarence in these two values for Mc is a measure of the experimental precision obtained.

With regard to the reference microphone, two different sizes and positions were used at different times. The first reference microphone was a B\&K type 4134 condenser microphone which had a $1 / 2$ inch diameter while the second was a Knowles subminiature transducer type BT-1751 with an outside diameter of. 055 inches. In separate configurations, both were used initially in the side of the cylindrical resonant cavity. The final configuration used the type BT-1751 mounted adjacent to the one inch WE640AA. This position was used in the larger cavity where the cavity inner diameter was sufficiently large to accomodate both microphones in the end cap. Experimental considerations yield two pragmatic reasons for selecting the end mounting in preference to the side mounting. When the cylindrical plane wave resonant cavity is driven at a longitudinal resonance, the standing waves within such a cavity, (when the ends are ~ rigid) will vary as cos (Kx), where $x$ gives the longitudinal position, $K$ equals $n * p i / L, n$ is the mode number, and $L$ is the length of the cylindrical cavity. At the fundamental mode, there will exist an acoustic standing wave pressure node at $x=L / 2$. When higher modal resonances occur, there will be "n" nodes found along the longitudinal axis of the tube. If the location of the reference

Bicrophone is at one of these pressure nodes, then no conparison voltages representing Ma/Mb can be obtained for that frequency. Additionally, if the microphone is mounted in the end, $G 1(n)$ becomes identically one, otherwise, there is a finite error associated with the calculation of $G 1(n)$ for any reference microphone position. While the end mounting removes the problem of finding a node at the reference microphone's position and the inaccuracies associated with computing G1(n), an additional but removable problem arises. The placement of the refererce microphone in the end of the cavity next to one of the reciprocal microphones will cause a decrease in the acoustical impedance at that end. For a given source strength, this will result in a decrease in the signal amplitude for the larger microphone. This situation will appear to yield a lower sensitivity calibration for the reciprocal microphone than that obtained in the absence of the immediate presence of the reference microphone. If the reciprocity calibration on which the comparison calibration is based is taken from the microphone mounted separately, this potential error is avoided.

An estimate of the magnitude of this effect can be made by comparing the volume velocity in this problem to the analogous current in an electrical circuit. Because the transverse dimensions in the end are much less than a
wavelength, the lumped parameter analogy to the electrical circuit is possible. When the reciprocal microphone is alone in one end, mounted in the rigid supporting wall, then the volume velocity through the plane of this end occurs only as a result of the motion of the diaphragm of this microphone. This is analogous to a current source providing all its output to a single impedance placed across its output terminals. When the additional reference microphone "C" is introduced adjacent to the reciprocal "microphone "日", the situation is analogous to the electrical current source having a second impedance placed in parallel across the first. When this is done, by current division, the magnitude of the current through the first impedance is reduced. Referring to equation 1.35 , it is seen that the open circuit signal voltage is directly proportional to the volume velocity of the microphone. Since the magnitude of the signal voltage is one of the measured electrical parameters used in the calculation of Mb, the value so obtained for $M b$ is lower than that obtained when only microphone "B" is mounted in the end. Since the change in the calculated microphone sensitivity is found to be proportional to the change in the acoustical impedance in the end of the cavity, the relative change in acoustic impedance is used to approximate the relative change in the calculated microphone sensitivity.

$$
\frac{\Delta z}{z}=\left(\frac{Z_{B}-\frac{Z_{B} Z_{C}}{Z_{B}+Z_{C}}}{Z_{B}}\right)=\frac{\Delta M}{M} \quad \text { Equation } 2.31
$$

If the assumption of rigid walls is correct and if there are no leaks in the mountings used for the microphones, and if the transverse dimensions of the microphones are much less than a wavelength, then the solution above is valid. To obtain an estimate of the magnitude of the relative change in sensitivity, consider the following: Since both transducers are operating well below their respective mechanical resonances, they are both operating in the stiffness control region. The stiffness of the volume of the Knowles subminiature transducer is modeled as that of a small Helmholtz resonator [Ref. 11: p.226], and the stiffness of a W.E. G40AA condenser microphone is well known in published literature [Ref. 3: p. 32]. When the mechanical impedance found using the stiffness is divided by the square of the cross sectional areas, the resulting acoustical impedances can be substituted into equation 2.31 to yield,

Since the volume within the Knowles subminiature microphone has roughly the same dimensions as the neck of the knowles subminiature type BT-1751 transducer which is cylindrical in shape with a neck diameter of 1.4 mm and a neck length of 1.57 mm , and the acoustic impedance of a W.E.640AA condenser microphone is $\sim 1.64 E+12$ NM^-5 (the stiffness of the 640 AA ) divided by the angular frequency,[Ref: 3: p.32] substitution into equation 2.32 using a density of 1.21 $\mathrm{kg} / \mathrm{M}^{\wedge} 3$ and a sound speed of $343 \mathrm{~m} / \mathrm{s}$ yields,


Equation 2.33

This corresponds to an expected decrease of 0.24 db re IV/ubar in the calculated microphone "B" sensitivity when microphone "C" is mounted alongside. Experimentally, a decrease of $0.22(+-) 0.10$ db re $1 v /$ bar was obtained by comparing the first ten plane wave reciprocity modal calibrations for W.E. 640AA serial \#815 with and without the Knowles type BT-1751 subminiature reference transducer mounted alongside. With this observed change, care must be made in the comparison calculations for Mc, to use only the
reciprocity calibrations obtained where the reciprocal microphone is mounted alone. The next two photographs show the relative sizes of the microphones involved in the calibration experiment.

W.E.640AA
microphone
diaphragm

Knowles BT-1751
microphone diaphragm

Photograph 2.2 The_relative_sizes_of_the_Knowles
 M1 드유ํㅡํㅡ․․

In the photograph above, the relative position of the small Knowles subminiature transducer is approximately at the "three o'clock" position relative to the one inch WE640AA condenser microphone. The end mount shown here was intended for use only in the larger (~70 cm) plane wave resonant cavity. The inner diameter of the smaller (~23 cm) plane wave cavity did not allow sufficient room for both to be mounted in the same end. As previously described, the
acoustic impedance in the end of the resonant cavity is slightly changed when the subminiature transducer is included alongside the larger one inch condenser microphone.


Photograph 2.3 The_relative_sizee_of_the_Knowles



In this photograph, the previous end mounting is removed and the microphone casings and diaphragms are visable. The subminiature transducer's FET preamplifier (inherent to the "type BT-1751"as obtained from the manufacturer) is in the extended case behind the neck and opening leading to the diaphragm. It is this portion that was previously modeled
as a small Helmholtz resonator to estimate the change in acoustic impedance at the end.

To complete the data required for a six way round robin check on the experimental precision for each modal calibration, we will next consider the comparison calibration of each reciprocal microphone based upon the absolute reciprocity calibrations already obtained for Ma and Mb.
3. The_Comparison_Calibration_of_(Microphones__"A"_-and_"B" With_Respectat으트드그_므늘

In order to complete the six way round robin comparison check on the experimental precision, comparison calibrations of the reciprocal microphones based upon each other are needed for each modal calibration. Refer to Equation 2.1日, which gives the ratio Ma/Mb. Solving in turn for each open circuit receiving sensitivity we obtain,

$$
M_{A B}=M_{B}\left(\frac{e_{1}}{e 4}\right)\left(\frac{V_{C A}}{V_{C B}}\right)
$$

and,

$$
M_{B A}=M_{A}\left(\frac{e 4}{e 1}\right)\left(\frac{V_{C B}}{V_{C A}^{\prime}}\right)
$$

Equation 2.35

The apparent change in sensitivity for microphone "B" resulting from the change in acoustical impedance at the end containing both microphones as described in the previous section, will not adversely effect the round robin precision of an individual modal calibration. The values of Mba and Mab shown above are still based upon eq, the measured signal
voltage of microphone "B". As such, the value for Mba will still be the "same" as that found for Mb. Similarly, Mab will be the "same" as Ma since while the apparent value of Mb goes down as a result of the change in the acoustical impedance, the ratio of e1/e4 will go up a proportionate amount. The relative precision of the experiment is determined by computing either (Ma-Mab)/(Ma+Mab), (Mb-Mba)/(Mb+Mba), or (Mca-Mcb)/(Mca+Mcb). All three computations result in the same value of the experimental precision.

While the equations developed for Ma, Mb, Mca, Mcb, Mab and Mba will allow a six way round robin consistency check of the experimental precision for each modal plane wave resonance, there remains one simplifying assumption upon which these equations are based, that remains to be discussed. This is the assumption, necessary for the solution for "J" given in Chapter $I$, that the microphones used "feel no impressed pressure". When the solution for "J" is modified to account for a microphone with a finite mechanical impedance, a correction to the solutions for Ma and Mb is required. This is the subject of the next section.
C. THE IMPRESSED PRESSURE CORRECTION

When the acoustic transfer admittance that is used in the reciprocity calibration was derived in chapter 1 , the derivation was unique to a microphone that felt no impressed pressure. The assumption of "zero pressure sensitivity" was then incorporated in equations 1.34. This was the same as saying that the mechanical impedance of the microphone was infinite. However, real microphones have a finite mechanical impedance and the effect of this finite impedance upon the reciprocity calibration must be considered. Refer to figure 2.4 for an illustration of two different but reciprocal condenser microphones mounted in the ends of a plane wave resonant cavity.


Figure 2.4 Reciprocal_microphones_mounted_in_the_ends


Referring to Equations 1.17, with interpretation of the coefficients as shown by Beranek [Ref.9:p.117] for the general case of both acoustical and electrical excitation, we can write for each reciprocal microphone,

$$
\left.\begin{array}{l}
V 1=\left(z_{11}\right)(i 1)+(b)\left(L_{2}\right) \\
P 2^{\prime}=(b)(i 1)+\left(z_{22}\right)\left(\dot{U}_{2}\right)
\end{array}\right\} \begin{aligned}
& \text { Acting As } \\
& \text { A SOURCE }
\end{aligned}
$$

```
Where: Z12 = Z21 = b, the transduction coefficient.
    V1 = voltage across the transducer's electrical
                        terminals when the transducer is used as a
                        speaker.
    I1 = current flowing in the transducer when it is
    used as a speaker.
    P2' = pressure at the speaker end of the resonant
    cavity when the driving frequency is at a
    longitudinal resonance.
    ei = open circuit receiving voltage.
    F2 = acoustic pressure at face of receiving
    microphone.
    U2 = volume velocity of speaker.
    U'2" = volume velocity of microphome.
    Z11 = blocked electrical impedance plus the electrical
        load (or generator) impedance.
    Z22 = the open circult acoustical impedance plus
        acoustic radiation impedance as seen at acoustic
        port #2.
```

Dividing the third and fourth equations, we obtain for Ma,

$$
M_{A}=\frac{b}{z 22}
$$

Solving the second equation in equations 2.36 for 11 ,


Equation 2.38

Substituting this value for il into the definition for sa we obtain,

$$
S_{A}=\frac{P_{3}}{i 1}=\frac{P_{3}}{\left(P 2^{p}-(z 22) \dot{U}\right) / b}
$$

Equation 2.39

We now define the previously derived transfer admittance for the longitudinally resonant plane wave cavity to be,

$$
J_{0}=\left|\frac{\omega 2}{p 3}\right|=\frac{\pi V_{0} f N}{\rho_{0} \gamma_{E} Q_{N}} \quad \text { Equation }=.40
$$

Where " $n$ " 15 the mode number of the longitudinal -esonance. Since, J equals Ma/Sa and at resonance within the plane wave resonant cavity, FZ and FJ are equal in magnitude, equation
2.39 can be substituted into equation 1.29 and reduced to,

$$
J=\left(\frac{-\dot{U} 2}{p 3}\right)\left\{1-\frac{\left(\frac{1}{z 22}\right)\left(\frac{p_{2}^{\prime}}{p 3}\right)}{\left(\frac{\dot{L} 2}{p 3}\right)}\right\}_{\text {Equation } 2.41}
$$

The first part of the above equation for the reciprocity factor "J", is the transfer impedance as derived in chapter one. Note however, that the assumption of a "rigid" ideal microphone is the same as stating that 222 is infinite. In this ideal case, the correction term goes to zero without the assumption that $P 2^{\prime}$ is necessarily zero. In fact, in a plane wave resonant cavity, as previousiy shown in equation 2.13, this 15 not the case. With a fimite impedance, it remains to be shown that the correction term multiplying (-Ú2/F3) is small. To do so requires the determination of the value of $Z 22$ as shown in the canonical equations wher the reciprocal macrophone la termiriated with an acouztic plarie wave resoriator.

The normal acoustic impedance (Z22) is shown oy Eeranek: [Ref.9:p.117] for the general case of simultaneous electric and acoustic excitation, as being equal to the sum of the open circult normal acoustic lmpedance and the acoustic radiation 1 mpedance. In the case of plane wave resonant reciprocity, the resonant acolistic cavity can be snown to be
"loading" the faces of the microphones mounted at each end. This impedance load for the transducer attached to a plane wave tube is given by [Ref. 11: p.201]:

$$
\frac{z_{m 0}}{\rho_{0} C A_{T}}=\frac{\frac{z_{M L}}{l_{0} C A_{T}}+j \operatorname{taN}\left(k^{\prime} L\right)}{1+j \frac{z_{M L}}{l_{0} C A_{T}}+A_{N}\left(k^{\prime} L\right)}
$$

where,

```
zmo = mechanical impedance of the tube seen at }x=
    in the plane wave resonant cavity.
zml = mechanical impedance at the X=L termination.
    P
    c = sound speed
    k = wavenumber [w/c]
    k' = complex wavenumber [k - נa]
        a = absorption coefficient
        L = length along the longrtudinal axis of the
        resonator.
    At = cross sectional area of tube & diaphragm.
```

If the cavity is a multiple of a half wavelength and is made to resonate longitudinally, the value of $k$ L has been shown (appendix A.) to be an integral multiple ot pi. Additionally, if there 15 a low loss (high $Q$ ) acoustic: system so that hyperbolic sine and cosine are well represented by the first term in a series expansion, then the mechanical impedance looking into the resonant tube 15 approximately equal to the mechanical impedance found at the opposite end.

When this load impedance is added to the open circuit mechanical impedance of the microphone and then expressed in terms of acoustical impedances, we have for 222:

$$
Z_{22} \cong\left[Z_{A}(x=0)+Z_{A}(x=L)\right]
$$

Equation 2.44

When this interpretation of 222 is combined with the results shown in equation 2.13, equation 2.41 is reduced to:

$$
J \cong J_{0}\left[1-\frac{1}{z 2 J_{0}}\right]
$$

Equation 2.45

The quantity "[1/(22 2*Jo)]" is the ratio of the normal acoustic admittance ot the microphone at acoustic port \#2 to the acoustic transfer admittance ot the medium When equation 2.4515 compared with equation $2.4 \%$, we see that the finite acoustic impedance of the transducer results in a
change in microphone sensitivity as compared to that expected for a perfectly rigid termination. If "Mao" is used to identify the magnitude of the open circuit receiving sensitivity calculated using equation 2.27, we have:


Equation 2.46

It is important to consider the acoustic system as a whole when the value of 222 is determined. The following considerations are in addition to those previously presented where the diaphragm area is equal to the tube area.

When equations 2.30 were employed to determine the impressed pressure correction, the four port network used as the system model, required 222 (or J22) to be associated with acoustic port $\# 2$. This means that the influence on 222 of the difference $i n$ the cross section area of the tube and the area of the microphone diaphragm must also be considered. This difference in areas will act as an acoustic transformer and will serve to increase the acoustic impedance of that port over that which would occur if the tube and diaphragm areas were the same [kef.9:p.125]. Tine multiplicative correction to Zmic that accounts for this increase $1 \pi$ acoustic 1 impedance at the end 15 the ratio of
the tube area to the diaphragm area. Consider the secuence of acoustic terminations shown in the next figure:


Figure 2. S Different terminations_for_the_giane wํㅡ르느트․

In the tirst tube, the area of the tube matches the area of the diaphragm. The acolistic impedance at the end of thls tube is simply the acoustic lmpedance of the microphone. In the bottom thbe, the majority of the end is a rigld wall. In this case, the limit of the acoustic inpedance tends to infinity as the area of the large tube becomes inumti freatgr than the area of the diaphragsi. While an exper.ment with
either the first or last situation is possible, it is the termination shown in tube \#2 that is of practical interest in the plane wave resonant cavities used in this experiment. To illustrate the influence of the different terminations, the definition of mechanical impedance is applied to the acoustic port at the end of the tube,


$$
\text { where: } \quad \begin{aligned}
\quad i m & =\text { mechanical impedance of the acoustic } \\
& \text { port at the end of the tube. } \\
\text { Fend } & =\text { acoustic pressure at the end of the tube. } \\
u & =\text { the resulting speed. } \\
\text { At } & =\text { cross section area at the end of the tube. }
\end{aligned}
$$

Since the only resulting speed at port \#2 is that of the microphone diaphragm, and rewriting equation 2.47 in terms of the acoustic $i m p e d a n i e$ of the microphone, we have:

where,

$$
\begin{aligned}
& \text { Ad }=\text { cross section area of the diaphragm. } \\
& \text { Ia }=\text { acoustic impedance. }
\end{aligned}
$$

When the effects of the acoustic transformer action due to the different areas (Beranek [Ref.9:p.125] and equation 2.48) and the loading due to longitudinal resonance (equation 2.43) are included in the value of 222 and provided that the "non-diaphragm" areas in the ends are approximately rigid, then:

$$
Z_{22} \cong\left[Z_{(\text {MICA }}\left(\frac{A_{T}}{A_{d A}}\right)+{\underset{(\text { Mics }}{ })}_{Z_{A}}\left(\frac{A_{T}}{A_{d \xi}}\right)\right] \cdot \text { Equation } 2.49
$$

It is this form of the value of 222 that must be used $1 \pi$ the computed correction to Mao. To illustrate the magnitude of this correction, assume for simplicity that both microphones are identical and that the cross section area of the tube is equal to the diaphragm area. With these simplifying assumptions,

$$
Z 22 \cong 2\left[R_{A}+j\left(w M_{A}-\frac{R_{A}}{w}\right)\right] \quad \text { Equation }=.50
$$

Where the subscript "a" refers to the lumped acoustical? parameters associated with the microphone being considered. If the acoustical impedance of a microphone 15 known for the
frequency range of interest，$J 22=1 / 222$ can be calculated and the value of $J 22 / J 0$ can be determined．

When the various reciprocity factors representing the different experimental geometries are examined，it is apparent that the magnitude of the impressed pressure correction will be different．Refer to equations 2.51 below．

$$
\begin{aligned}
& J_{0}\binom{\text { PRessure }}{\text { coupler }}=\left(2 \pi V_{0}\right)\left(\frac{f_{N}}{P_{0} \gamma_{E}}\right) \\
& J_{0}\left(\begin{array}{l}
\text { PlANE } \\
\text { WAVE V } \\
\text { RENAOR }
\end{array}\right)=\left(\frac{\pi V_{0}}{Q_{N}}\right)\left(\frac{f_{N}}{P_{0} \gamma_{E}}\right) \\
& J_{0}\binom{\text { FREE }}{\text { FIELD }}=\left(2 r \lambda^{2}\right)\left(\frac{f_{N}}{P_{0} \gamma_{E}}\right)
\end{aligned}
$$

The parameter values to be used for a rough numerical comparison are given below：

$$
\begin{aligned}
& \text { Ra }=\text { 3. } 27 E+7 \text { NSecMヘ-S (acoustic resistance, [Fief. ※: p. こ2] } \\
& \text { WoE. } 640 \mathrm{AA} \text { ) }
\end{aligned}
$$

$$
\begin{aligned}
& \text { Ka }=1.64 E+12 \text { NM }-5 \text { (average acoustic stiffness [same ref) } \\
& \text { Po = } 101330 \mathrm{pa} \text { (atmospheric pressure) } \\
& \gamma_{E}=1.39 \text { (ratio of specific heats[Fef 25]) } \\
& f_{n}=2450 \mathrm{~Hz} \text { (10th modal resonance in long tube) } \\
& c=343 \mathrm{M} / \mathrm{sec} \text { (phase speed in unbounded medium) } \\
& r=.20-.40 M \text { (separation distances for free field) } \\
& \text { on ~ } 165 \text { (quality factor for } 70 \mathrm{~cm} \text { plane wave } \\
& \text { resonant cavity) } \\
& \text { (volume of plane wave resonant cavity, ) } \\
& \text { (volume of pressure coupler [Fief. 刁: } \\
& \text { p.12]) }
\end{aligned}
$$

The case of the pressure coupler 15 similar to that of the plane wave resonant cavity. In the first case, due to the small dimensions of the cavity, $k L^{\sim} 0$, whereas in the plane wave resonant cavity $K L \sim N * p i$. In the case of the free field calibration, the correction must use a different value for 1/222. Substituting the appropriate values into equation 2.46 yields:
$\frac{M_{A}}{M_{A 0}}=\left\{\begin{array}{ll}\sim .95, & \text { Pressure Coupler } \\ \sim .99, & (K L<1)\end{array} \quad \begin{array}{ll}\sim & \text { Plane WAve resonator } \\ (K L=N \pi)\end{array} \quad\right.$ Equations 2.52


#### Abstract

As expected, these quantitatively different results simply reflect the fact that the effect of the compliance of the acoustic system is the greatest in the small pressure coupler cavity, least in the free field geometry, and somewhere in between in the larger plane wave resonant cavity geometry. These fractional compliance corrections, If ignored, correspond to microphone sensitivity changes of $\sim-.45, \sim-.10$, and $\sim-.0001$ db re iv/ubar respectively.


Since the parameter values used to obtain this comparison are only approximate, a different method of computing "J22/Jo" will now be used in the case of a plane wave resonant cavity to obtain a further comparison.

Consider the analytical form of the transduction coefficient found in the literature for an condenser microphone when the open circuit mechanical impedance of the microphone is so much greater than the radiation impedance that the radiation impedance is negligable [Ref. 11: p350]:

$$
b=\frac{C_{0} E_{0}}{2 \pi f_{v} \epsilon_{0} A_{e} A_{d}}
$$

Equation 2.53

Where: Co = microphone capacitance.
Ea = condenser microphone bias voltage.
$f_{n}=$ modal frequency of resonance.
$\epsilon_{0}=$ permittivity of free space.
Ae $=$ the effective backplate area of the condenser microphone.
and $\quad$ Ad $=$ the diaphragm area of the microphone.

If we can include the influence of the acoustic load due to the plane wave resonant cavity in the above solution for the electroacoustic transduction coefficient, we can use the previous equation determined for "Jo" to obtain an analytical form for the ratio "1/Z22*Jo".

With constant environmental conditions (atmospheric pressure, temperature, humidity, and density) the absolute pressure sensitivity under open circuit electrical conditions is a property of the microphone [Refs. 3 and 29]. Under the free field conditions that result in the value for "b" shown above, the value of 222 used in equation 2.37 is the open circuit acoustic impedance of the microphone. For identical microphones at each end of the plane wave resonant cavity, the invariance of Ma requires a [2*At/Ad] multiplicative correction to equation 2.53 to cancel a similar increase to the open circuit acoustic impedance as shown in equation 2.49. When this is done, the value of Ma calculated in equation 2.37 remains the same. The magnitude of the ratio "1/Z22*Jo" becomes:
$\frac{1}{Z_{22} J_{0}}=\frac{M_{A} P_{0} \gamma_{E} \epsilon_{0} A_{e} A_{d}^{2} Q_{N}}{A_{T} C_{0} E_{0} V_{0}}$
Equation 2.54

Since the quantities involved in equation 2.54 are known or can be experimentally measured, an estimate of the magnitude of this ratio and its effect upon the value computed for Ma can be made.


Figure 2.6 A_copy_of_the_original_engineering




When the ratio "J22/Jo" is now calculated for the long tube and substituted into Equation 2.46, we obtain essentially the same result as calculated previously (~ $1 \%$ relative change).

$$
\begin{aligned}
& \frac{M_{A}}{M_{1}} \cong 0.98 \text {, Plane Wive } \\
& M_{\text {nO }} \quad \text { RESONATOR( } k l=10 \pi \text { ) } \\
& \text { Equation } 2.55
\end{aligned}
$$

These different impressed pressure corrections may be significant if calibration accuracies to within (+-) ~0.5 db re $1 V / u b a r$ or better are desired. A more careful comparison between experimental results obtained using different cylindrical dimensions for the plane wave resonator will be discussed in the chapter on experimental corrections. In these discussions the ratio of areas given by At/Ad will be included in the calculation of 222.

## III. EXPERIMENTAL PROCEDURES

## A. INTRODUCTION

The experimental procedures and equipment required for two different methods of absolute reciprocity calibration of microphones are presented in this chapter. The method of primary interest was the plane wave resonant reciprocity calibration. The secondary free field comparison calibration, when corrected for diffraction [Ref. 3: p. 31], was used as a low frequency consistency check for the results of the primary method.

The separate apparatus built for these two methods are described first. The resonant reciprocity part of the experiment used two different right circular cylindrical cavities to obtain a self consistency chect on the associated plane wave resonant reciprocity calibrations. The free field calibration used a microphone translator that operated under computer control. Both methods used commercial electronics equipment which will be described in the appropriate section.

Following the description of the apparatus, the procedures and signalflow are presented for both calibration methods. Both a plane wave resonant reciprocity Galioration
of a laboratory standard type W.E. G40AA condenser microphone and a standard free field reciprocity calibration for an electrodynamic microphone are described. The free field reciprocity calibration of the electrodynamic microphone in turn yielded a comparison calibration of the previously calibrated W.E. $640 A A$ condenser microphone.

The basic experimental parameters obtained from measurements which occur in the experiment are; voltage, current, resistance, frequency, capacitance", temperature, length, barometric pressure, and relative humidity. The uncertainties in experimentally measured electrical and physical parameters are presented in an error analysis of these fundamental measurements. The propagation of error in the computed microphone open circuit valtage receiving sensitivity is then made resulting in an estimate of probable error. In addition, the experimental methods lised to Jetermine the system linearity, voltage transfer function(s), and the stability of different voliaye amplifiers will be explained. Eiperimentai procedures which result in a reduction of systematic error will be discussed whenever applicable. The final results for probable error will be summarized in the last section of this chapter.
B. EXPERIMENTAL APPARATUS

## 1. Plan트Wave_Resonant_Cavities

Two differently dimensioned plane wave resonant cavities were built to measure the self consistency in the method of plane wave resonant reciprocity calibration. Since the calibrations occurred at frequencies corresponding to longitudinal resonances, the first cavity was made one third the length of the second. Thus, the frequency of every third longitudinal resonance in the "long" tube matched the frequency of a longitudinal resonance in the "short" tube.

For each plane wave resonant calibration, three microphones were required. In the final calibration configuration, only two different size microphones were used, although each right circular cylindrical cavity was made of brass with ports constructed for the mounting of three different sizes of microphones. Two, one inch diameter reciprocal W.E. $640 A A$ condenser microphones were mounted in the opposite ends of the cavity while a one eighteenth inch ( 1.41 mm ) diameter comparison microphone was mounted in one end (for the long tube) or in the side of the cylinder wall (for the short tube). The comparison microphone was a Knowles type BT-1751 subminature transducer which had a small preamplifier built into the transducer
case. These brass tubes and the mounting ports for the various microphones are shown in the following photographs.


##  Rlanee_wa크﹎ressonant_cavity.

In this photograph, the left mount for the WE640AA microphone is dismantled and is shown in front of the tube. The tube itself is placed upon two wooden supports used for stability on the lab table. The rubber O-ring seal is easily seen in the dismantled mount and was provided for eventual calibrations in different gas media. The small pipe inlet in the center of the tube is intended for the
introduction of these different gases into the cavity. Since this capability was not employed in this e\%periment, the small hole intended as a gasport at the base of the center inlet pipe (radius ~ . 0004 meters) served only ta increase the acoustic losses during resonance measurements. The large ports for the $1 / 2$ inch comparison microphones are easily seen towards each end of the tube and are shown closed with the brass plugs shown in greater detail in photograph 3.2.


Photograph 3.2 Mounting_eorts_=_end_view

Here, one of the plexiglass mounts for the WE640AA 1 inch condenser microphone is shown dismantled. The port for the $1 / 2$ inch comparison microphone is also shown in a dismantled state. The nuts and bolts used to fasten the mount and WEb4OAA to the end are nonconductive nylon. The 0-ring seal and grove are easily seen. The markings shown inside the O-ring grove were used as reference marks for diameter measurements made using a calipers. In the final calibration configuration, the $1 / 2$ inch microphone ports were sealed with the plug snown in the lower right
foreground of photograph 3.2 and a Knowles type BT-1751 subminiature transducer was mounted in the end, alongside the WE640AA condenser microphone.


Photograph 3.3 The_"short"_tube.

Here is an end view of the short plane wave resonant cavity. Both the 1 inch plexiglass mount and the $1 / 2$ inch microphone mounts are removed. A gasport is placed in the center of the tube as it was with the long tube. During the final calibrations, the $1 / 2$ inch microphone port was again plugged and the Knowles type BT-1751 subminiature transducer was mounted in the wall of the cylinder. Slightly out of focus, the Knowles subminiature can be seen at the far end
of the above photograph. The markings on the near end were used as references when the major and minor axes of each end were measured with a calipers.


Photograph 3.4 A_closeup_of_the_comearisgn microghone_mounted_in_the_wall_offthe_short_tube.

Here the Knowles subminiature transducer is shown mounted in the wall of the short tube. Most of the casing shown houses the preamplifier that is an integral part of this type BT-175i subminiature transducer. The microphone is held in place only by the tape holding down the signal and power leads to the preamplifier.


Photograph 3.5 The_microphone_port_in_the_wall 1 _of the_short_tube.

This hole in the wall of the cylinder was machined so as to just allow the microphone opening to be flush with the inner wall of the cylinder with the casing of the preamplifier providing contact support. This is illustrated in the previous photograph. Notice that each microphone had its own plastic mount, the one shown in the photograph being designated "1082" which is for the WE640AA microphone of this serial number.


Photograph 3.6 A_comparison_Rhotggragh_showing_the
 short_and_the_l

Here the end views are shown for both the short and the long tubes. In the long tube, sufficient room at the end of the cavity remained for the Knowles type BT-1751 subminiature transducer to be mounted alongside the 1 inch WE64OAA whereas in the short tube, no such room existed and the Knowles was mounted in the wall of the cylinder.
2. The_Microphone_Translator_for Free_Field_Calibrations

Two experimental difficulties are hidden in the theory developed for the free field reciprocity calibration derived in chapter one. First, the free field theory requires that the source transducer appear as a point source in the far field of the receiving microphone. This does not appear to require more than one measurement of separation distance between source and receiver [Refer to equation 1.84]. Secondly, it is assumed that the reciprocal microphone will have sufficient strength to project sound at an adequate signal to noise level in the far field. These experimental facts are related as outlined below.

In the first case, free field reciprocity theory requires a point source so that the acoustic pressure amplitude falls off as $1 / R$ where $R$ is the "distance" between the source and the receiver. Experimentally, after anechoic conditions are obtained in the laboratory, the measured separation distance between the source face and the receiver diaphragm does not normally vary as 1/R! For a given operating frequency, a small difference is found between the physical separation distance and what is called the "acoustical separation distance",R'. The received pressure amplitude does fall off as $1 / R^{\prime}$. Experimentally, this requires a correction to the measured physical separation distance that is unique to a source/receiver pair and varies
as a function of frequency. The theoretical and experimental necessity of obtaining this 1/R' dependence requires the measurement of several received signals at different distances. A least squares error fit to the measured signal voltage versus the measured separation distance will yield the correction to the measured separation so that the acoustical separation distance may be calculated. The computer controlled microphone translator was constructed to increase the precision in the experimental measurement of the acoustical separation distance.


Photograph 3.7 Thé_microphonne_translator.

Notice the meter stick on the table next to the translator.
The total length of the translator was just over three




EF:GNR bE: Jw:

 trヨnミ1ヨヒロッ。

Here，the HF－gs computer controls the orive voltage to the screw metor which turns the threaded translator shaft．The total angie through which the smaft is turnea during an Jive rrapryel $1 \equiv$ ron？tred viatie ofticai shaft encousel．

This device provides an electronic pulse to the HP-5613 counter for each burst of light from an LED source that shines through a disk containing 1000 slits or "spokes". This provides a "count" measuring the total angular rotation between successive calibration measurements where each "count" represents $1 / 1000$ of a revolution. Measurement of the initial and final positions during any series of calibration stops allows each position to be precisely located after the fact. While the drive motor and the optical shaft encoder are shown in the schematic to be on opposite ends, they were actually both located on the same end as is shown in the next photograph.


Photograph 3.8 The_mounting_of_the_drive_motor_and the_optical_shaft_encoder.

The drive motor had a serew drive and is shown (on the bottom) mounted orthogonal to the threaded (13 threadsimeh, 60 degree pitch, $1 / 2$ inch diameter, 10 feet long) translator shaft. The gear box used to couple the screw motor to the shaft can just be seen through its housing. The Hewlett Packard HEDS-6000 incremental optical shaft encoder appears as a small black disc with a lead of computer ribbon cable looped over the top. The center shaft is the threaded translator shaft with precision ground stainless steel upper and lower shafts used to guide and support the moving microphone mounted on a precision ball carriage. The equipment rack with the equipment used in the previous schematic is shown in the next photograph.


Photograph 3.9 Ihe_eguipment_setup_at_the_entrance


The mobile rack was used to conveniently transport the equipment between the anechoic chamber and the resonant reciprocity laboratory. Missing in this photograph is the VHF switch used in resonant reciprocity to digitally switch equipments allowing the previous reciprocal source to become the receiver and vice versa. From top to bottom, the equipments shown are:

Frinceton Applied Research Model 5204 lock-in analyzer HP-3325-A synthesizer/function generator
HP- 3456 A digital voltmeter
HF-5316A universal counter
[blank space where $\mathrm{HP}-98 \mathrm{~B} 07 \mathrm{~A}$ VHF switch was normally located]
HP-85 computer
HP-7470A graphics plotter
Interface wiring for the $H P-3497 A$ [two panels]
HP-3497A data acquisition and control unit

On the floor to the right of the rack are two HP-467 power amplifiers used to amplify the HP-3325A signal for the Alter electrodynamic source.

The correction to the measured separation distance in free field reciprocity is illustrated in figure 3.31 where the measured separation distances and the acoustical separation distances are both plotted for the 490 Hz microphone calibration measurement. This necessity to vary the range between source and receiver to determine the acoustic separation distance is the indirect cause of the second experimental problem.

When the received microphone 51 gnal voltage is neasurec at increasing ranges, the low source leve, of tine withoam condenser microphone results in a very low signai to noise ratio in this far field. This low signal to noise level cari introduce significant measurement errors in the cal:bration procedure. The use of a different recifrocal scund source With an increased acoustic output solves the problem of a low signal to nolse ratic in the receavきd signal. In this experiment an Altec s88 electrodynamic micrephone was
selected as the reciprocal source since its acoustic output is significantly greater than that obtained for the condenser microphone. The schematic diagram of the setcip within the anechoic chamber used to obtain the comparison calibration between the Altec and wEG4OAA microphones is shown below.




The experimental results obtained with the above setup will Le d. scussed bater in the error analysis or the rree tield rompar: son Ealibration.
C. SIGNALFLOW AND COMPUTER CONTROL WITH PLANE WAVE RESONANT RECIPROCITY

The schematic diagram shown below illustrates the signal paths used for computer control of data acquisition. The basic data so acquired was then used to compute a plane wave resonant reciprocity calibration of the microphone open circuit valtage receiving sensitivity.


Figure 3.3. S드르르늳_diagram_of_the_signal_flow_for the_plane_wave_resonant_reciprocity_calibration

The controlling computer software was written to make the experiment entirely automated after the initial data is given to the computer. The following is a discussion of this procedure:

The computer program (see appendix B), written to control the plane wave reciprocity experiment, performs the fallowing steps beginning sequentially with the highest mode of interest and ending with the lowest mode of interest. Each of these steps (except for the initial input) are performed at each mode.

The initial input is obtained by operator responses to program questions. The program asks for the following data:

- the relative length of tube used (short or long tube)
- the starting relative humidity (assumed to remain constant)
- the highest mode of interest
- the lowest mode of interest
- the initial frequency band to search. This will be around the highest mode.
- the time constant for the PAR 5204 lock in analyzer.
- the valtage scale for the PAR 5204 lock in analyzer.

After this initial input, the computer begins the following sequence of equipment configuration and data acquisition subroutines:

Step 1. Set all equipment to the proper drive voltage and frequency. Then switch microphone " $A$ " to transmit and microphone "B" to receive.

Step 2. Perform a preliminary selection of the drive voltage amplitude so that the mid frequency range initial amplitude is approximately twenty percent of the lock in analyzer's full scale deflection.

Step 3. Sample 26 data points over the initial input frequency band. This band must bracket the center frequency of the mode of interest. Store the frequency and amplitude data and then analyze the data for the resonant frequency, $F 9$, the peak amplitude, $P 1$, and the quality factor of the resonance, Q1. These preliminary values are used as initial inputs to the least square error "Ravine" fit to a Rayleigh line shape. (Most initial values were found to be within approximately one or two percent of the final value.)

Step 4. Computing the "bandwidth" as the initial value for the resonant frequency divided by the initial value of the quality factor, adjust the frequency band of interest to include only plus or minus one "bandwidth" around the initial resonant frequency. Adjust the drive voltage so that the lock in analyzer is operating at ninety percent of ma\% scale at the modal resonance. Again sample 26 data points and store the data both in a memory array and on magnetic tape. Store both the average temperature and average atmospheric pressure found during the 26 point sample.

Step 5. Select microphone "B" to transmit and microphone "A" to receive. Ferform steps 2, 3 , and 4 for this
configuration.
Step 6. Perform a ravine search [Ref 2S: p.207], to find the least square error optimum values for F9, P1, and Q1 for both sets of data. Store these values on magnetic tape and in $\equiv$ memory array.

Step 7. Select microphone "A" to transmit using the previously obtained value for resonant frequency and the drive voltage selected when "A" transmitted earlier. Sele,t microphone "C" to receive. Measure the comparison voltage "ra ard storeit.

Gtep S. Select microphone "E" to transmit using the ravined value previously obtained for resonant frequency and the drive voltage selected when "E" transmitted earlier. Leave micropmone "C" in receive and measure the comparison voltage vob and store it.

Step 9. Using the equations developed for the $51 \%$ way rourd robin self consistency check, compute the $53 \%$ different open circuit receiving sensltivities. Store these values on magnetic tape and print these values for operator viewing.

Btep 10. Compute the frequencies of interest for the next lower mode and begin anew at step 1 until all the modes of interest have been sampled.

When these logical routines are completed, the computer automatically switches all experimental equipment into tre standby mode and awalts the next "initial" input of the operator. If the operator elects to stop data acquisition at this point, printouts of the experimental measurements and computations just completed are immediately available. The magnetic tape data base is avallable for future analysis and/or comparison when such a need develops.

The magnitude of the improvement in experimental precision using computer control as compared to manual measurements is illustrated in the table shown on the next page. Using measurements obtained for the 735 Hz . resonance (as a comparison sample), the percent reiative change in the source amplitude, resonant frequency, and computed quality factor were computed from one run to the next. The time required for the manual measurements (per resonance) averaged 25 minutes. The time required for the computer controlled measurements (per resonance) was slightiy less than 4 minutes.


Table 3.1 Percent_relative_change_from_run_tog_run
 s트르드tㅡd_sample.

It is apparent that a rough improvement in precision of from one to two orders of magnitude occurred due to the computer control of measurements. This was the typical result of using computer control of data acquisition.
D. THE CALCULATION OF EXPERIMENTAL ERROR

## 1. Introduction

When equation 2.27 for a plane wave resonant reciprocity calibration for the open circuit receiving sensitivity 15 examined, seven experimental variables exist and need to be measured. The method of their measurement and their place in the analytical formulae will determine their individual effect upon the total experimental error. Using the equation developed as the plane wave resonant reciprocity solution for Ma as an illustration,


Equation 3. 1
straightforward error analysis [Ref. 26], yields for the

> relative error,

The variables are,
eli - The ratio of the received signal voltage found across microphone "A" when $1 t: s$ used as a receiver

```
to the current driving microphone "A" when it is
used as a source.
Vca/Vcb - The ratio of the received comparison voltages seen by comparison microphone "C".
Vo - The product of the effective cross-sectional area of cylindrical cavity times its length.
Po - Atmospheric pressure within the cavity.
\(\gamma_{E}\) - gamma, the effective ratio of specific heats for the gas within the cavity, (accounting for the relative humidity and the non-adiabatic conditions at the boundary of the resonant cavity.)
Qn - The quality factor of the Nth resonance.
Fn - The frequency of the Nth resonance.
```

These values and their individual probable errors must be determined and included in the calculation of the probable error for Ma.

The necessity of obtaining absolute measurements of el and il prior to computing their ratio was avoided by using the lock-in detector to measure both $e 1$ and (indirectly), i1. Experimentally, the ratio e1/il was calculated as e1/(2*pi*f*C*v1). The variable $C$ is the capacitance of the condenser microphone, $v 1$ is the voltage drop measured across the condenser microphone when it is acting as a source, and el is the voltage across the condenser microphone when it is acting as a receiver. In this chapter, the term "condenser microphone" is used in the sense that includes the BNC electrical connection between the microphone cartridge and the external electronics. The frequency transfer
characteristics of the lock-in analyzer will cancel out in the ratio of the measured voltages. Any error invoived will result from nonlinearity in the lock-in analyzer and the inability to exactly measure capacitance and frequency.

The methods used in measuring these parameters and the determination of their individual contribution to the overall experimental error is presented next. A summary of the individual contributions to the probable error in the plane wave resonant reciprocity calibration is shown later in this chapter (in section $D$, part 5).
2. Meassuring_the_Ration_el/il
a. The Electrical Circuit

Figure 3.4 illustrates the signal flow involved in the determination of 01 , and figupe 3.5 is the schematic of the circuit used to measure el.


Figure 3.4 Receiviving_Signal_Flow_Chart.

Here we see the signal path from the received signal to the measured voltage. In electrical terms the input circuit is:



The circuit elements (with approilmate values) and the parameters of interest found in this circuit are:

```
el - Signal voltage desired.
Vout - Output voltage of the Signal Freamplifier.
Fib - Current limiting resistor in che sias Voltage
    Supply. (~10megohm)
Cc - D.C. Blocking capacıtor in the Eias Voltage
    Supply. (~.01uf)
Cb - Battery bypass for battery nolse. (~.0luf)
Cli - Connecting cable capacitance. (~40 pf:
Clo - Connectıng cable capacitance. (~gu pf)
Cl - Freamplifier lmput capacitance. (~20 pf)
E1 - Eias voltage (~11G volts)
Fi -- Freamplifier input resistance. (vig Megorm)
Vin - Irplit volEage to the preamplifier.
G - Gain uf the preamolifier. (~10)
```

Figure 3.5 is an abstract electrical model of the real physical system used to process the electrlcal signal generated by the condenser microphone. It represents a compromise between conflicting. factors of simplicity and accuracy that must be addressed from both the experimental and the theoretical standpoint. Numerically, using available computer software, it is possible to determine the "transfer function" of such a lumped parameter circuit with relative ease. The difficulty arises when accurate modeling is attempted and every stray capacitance, resistance, and inductance is measured and included in the model. Experimentally, such an approach is difficult to apply and is not necessary in every case.

The circuit analysis 15 slmplified winen the relatively small drop in signal voltage across the blocking capacitor, Cc, in figure $3.5, i s$ accounted for ty a one time correction to the firial microphone sensitivity. To see tiow ini $\quad$ is done, refer to figure $Ј .5$ and consider the sigmal vol iage that would be measured on each slde of the D.C. blocking capacitor, Cc. The effect of the blocking capacitor impedance on the magnitude of "Vin" depends upon the magnitude of the input impedance of the signal preamplifier formed by the parallel combination of "Clo", "Ci", arit "Fi:". By simple voltage division, wsing the iumped parametar

of Vs/vin remains constant to within ~.01 dB civer the frequency range used in this experiment. The dB signal loss across the blocking capacitor Cc is calculated in Appendix E and shows an approximate ~.03 dE loss across the entire frequency range used. This means that the blocking capacitor Ce may be ignored in the circuit analysis if a one time correction to the open circuit voltage receiving sensitivity of +.03 JE 15 made at the end. Thus, by measuring the bias box input capacitance (winen it $1=$ disconnected from the circuit), the cable capacitances, anc the input capecitance of the signal preamplifier, the circuit model can be simplified as shown in figure E.ó below.


Figure 3.6 Simplified_ingut_circuit_for_acoustic_signal

Here, the measured capacitances of the connecting cables, the bias $b \circ \%$, and the input of the signal preamplifier can be combined. The capacitances of the connecting cables and the bias boxes were measured directly with a General Fiadio Type $1615 A$ capacitance bridge in turn calibrated with reference to a General Fiadio type 1404 reference standard capacitor serial \#2507. This reference capacitor had a specification of 10 picofaradsito 20 ppm) for $1 k h z$ at $2 \bar{s}(+-1)$ deg $C$. In addition, the zero blas voltage capacitance values for the different w.e.buraf
laboratory standard microphones were measured with a Hewlect Fackard 4192A LF Impedance analyzer calibrated to the same reference. The results of these measurements are listed in table 3. 2 below.

|  | 〔sample | $\begin{aligned} & \text { precision } \\ & \text { sigma (pf) ] } \end{aligned}$ |
| :---: | :---: | :---: |
| Bias box: "A" plus the cable capacitance (BTAF Ct).......... | 151.900 pf | $.028 \quad(n=0)$ |
| Bias box "B" plus the cable capacitance (ATBR Ct).......... | 150.070 pf | .012 ( $0=5$ ) |
| W.E.g4OAA Serial \#1248........ (with the "\#1248" ENC connector) | 52.160 pf | $.002(n=13)$ |
| W.E.G4OAA Serial \#1082......... (with the "\#1082" ENC connector) | 51.974 pf | . 002 ( $n=13)$ |
| W.E.g40AA Serial \#0815.......... (with the "\#O815" ENC connector) | 49.954 pf | . 003 (n=13) |

##  

Since the input capacitance of the signal preanpliz:er will vary from amplifier to amplifler, the foliowing fiertac was used to calculate the imput azpacitan:e for eazh preamplafier used in the experiment.
b. Measuring The Input Capacitance of Preamplifiers

The specifications of the preamplifiers used in this experiment gave the input capacitance as a nominal 20 to 30 picofarads. Since the capacitance of the W.E.G40AA condenser microphone was approximately 50 picofarads, any error in the determination of the preamplifier input capacitance on the order of $1 / 10$ picofarad was significant and the rough value given in the equipment specifications was grossly inadequate. Refer to the circuit shown below.


Figure 3.7 Circuit_used_to_measure_the_input

## 

Straightforward voltage division will yield a solution for Vo. When this solution is combined with the gain, Vg=Zi*Vs*G/[Zi+Zv]. When this solution is inverted,
$\left[\frac{1}{V_{G}}\right]=\left[\frac{1}{j \omega Z_{i} V_{S} G}\right]\left[\frac{1}{C_{V}}\right]+\left[\frac{1}{V_{S} G}\right]$ Equation $2 . z$
$Z v$ is the impedance of the variable capacitor 10 the input \& $1 /[j w C \vee]$ ), and $Z i$ is the input 1 impedance of the cable and preamplifier combination ( $\mathrm{R}_{1} /\left[1+\mathrm{Ri}_{\mathrm{H}} \mathrm{j} \mathrm{w}_{\mathrm{*}}\{\mathrm{Ci}+\mathrm{Cl}\}\right]$ ). Typical circuit parameters used for this circuit were:
frequency $=1000 \mathrm{~Hz}$.
Cl ~ 4 pf
$\mathrm{Ci}_{\mathrm{i}}$ ~ 25 pf
G $\sim 10$
CV ~ 4-60 pf [see table 3. 3]
Vg ~ variable
Vs ~ variable, on the order of 10 millivolts.
$R_{1} \sim 10$ megohm, uncertainty estimated at $1 \%$.

This 15 the form of the equation for a straight line, $Y=a x+b$. If individual values of $1 / V g$ and $1 / C v$ are fit by the least squares error method to a straight line, the values of "a" and "b" can be calculated. The resultant absolute value of the ratio of $a / b$ equals the magnitude of $1 /[j w Z 1]$. Individual "boxes" with different values of $C V$ were constructed and calibrated so that as large a range of values as possible would be available for the least square error analysis. The following table lists the capacitance values measured for these individual boxes using the same General Radio Type $1615 A$ capacitance bridge previously wised to obtain the bias bo\% capacitances.
Box \# capacitance (pf)

(average) $\quad$| Precision |
| :---: |
| (Sigma using |
| (avo samples) |
| 1 |


 드르르드드느므드르․

The capacitors listed above were selected for their thermal stability and were observed to remain stable to within the precision obtained above for the laboratory temperature range of 19-22 degrees centigrade.

When the values for "a" and "b" are obtained via least square error analysis, the solution for the total input capacitance is given by:

$$
C_{T}=\left[\left(\frac{a}{b}\right)^{2}-\left(\frac{1}{\omega R_{i}}\right)^{2}\right]
$$

Equation 3.4

The uncertainty in the result obtained for Ct is given by [Ref. 28: Eqn. 3-2]:
$\delta C_{T}=\left\{\frac{\left(\frac{a}{b}\right)^{2}\left(\delta \frac{a}{b}\right)^{2}}{\left(\frac{a}{b}\right)^{2}-\frac{1}{\omega^{2} R_{i}^{2}}}+\frac{\left(\delta R_{i}\right)^{2}}{\left(\omega^{4} R_{i}^{6}\right)\left[\left(\frac{a}{b}\right)^{2}-\frac{1}{\omega^{2} R_{i}^{2}}\right]}\right\}^{1 / 2}$ Equation 3.5
The results obtained with the least square error linear fit using the method described above are given in the tables below. The ultimate uncertainties in Ct are due largely to the $1 \%$ estimated uncertainty in Ri.

The following data have $\approx 3$ significant figures and are in picofarads.

Ithaca 1201 Ser.\#63594 (X10 setting, Ri~100 Mohms)

$$
\begin{aligned}
& \text { chan "A" } \\
& \text { chan " } B \text { " }
\end{aligned}
$$

[a/b]= 22.9081
data run
\#2
data run \#1
[alb]= 24.6444
22.9463
24.6597

Ithaca 1201 Ser.\#61783 (X10 setting, Ri~100 Mohms)

$$
\begin{array}{lll}
\text { chan "A" } & {[a / b]=} & 23.6920 \\
\text { chan "B" } & {[a / b]=} & 26.0170
\end{array}
$$

Hewlett Packard
Type 465A Lab Serial \# 95
(lab $B$ side $\times 10$ setting)
[a/b]= 29.6300
29.6302

Hewlett Packard
Type 465A Lab Serial \# 93 (lab A side X10 setting)
$[a / b]=28.9561$
29.0806

Note: The value of the "cable" capacitance, $\mathrm{Cl}=1.926 \mathrm{Pf}$.

Table 3.4 Measured_ratios_for_[a/b]

With the experimentally determined values for [a/b], the value of the preamplifier input capacitance, $C i$ can be determined.

Signal Preamplifier/Ci(pf)/sigma(pf)/"standard sigma"/P.E. (ppm) values of Ci have ~ 3 significant figures

Ithaco 1201 Ser. \#63594
(X10 setting)

| chan A | 20.946 | .088 | 20 ppm | $\sim 4201$ |
| :--- | :--- | :--- | :--- | :--- |
| chan B | 22.675 | .088 | 20 ppm | $\sim 3880$ |

Ithaco 1201 Ser.\#61783
( X 10 setting)

| chan A | 21.793 | .088 | 20 ppm | $\sim 4038$ |
| :--- | :--- | :--- | :--- | :--- |
| chan B | 24.045 | .088 | 20 ppm | $\sim 3660$ |

Hewlett Packard
Type 465A Lab Serial \#95
(lab B side X10 setting) 23.067 . 148 20 ppm ~ 6416
Hewlett Packard
Type 465A Lab Serial \#93
(lab A side $\times 10$ setting) 22.338 . 145 20 ppm ~ 6491
Average probable error for the input capacitance ${ }^{-\bar{\eta}-\overline{4} \overline{7} \overline{8} \overline{0}} \mathrm{ppm}$

Table 3.5 Calal드lated_ingut_드르르드느므드́﹎for_signal ereamelifiers_used_in_experiment.

When the signal preamplifier input capacitances so determined are included in the circuit analysis, the circuit can be further simplified as shown in the following section.
c. The Calculation of $e 1$

Referring back to figure 3.6, the total input resistance and the total input capacitance can be combined in a complex impedance, $2 t$. The value of $Z t$ is given by,

$$
Z_{T}=\frac{R_{T}}{1+j \omega R_{T} C_{T}}
$$

The values for Rt and Ct are given by,

$$
R_{T}=\frac{R_{b} R_{i}}{R_{b}+R_{i}} ; \quad C_{T}=C_{e}+C_{i}
$$

$$
\text { Equation } 3.7
$$

Now the circuit becomes,


Figure 3.8 Simplified_circuit_for_measuring_the acoustic. signal

The solution for the ratio of vinlelis given by:

$$
\frac{V_{1 N}}{e 1}=\frac{1}{\{\text { Rear }\}+j\{\text { manalinary }\}}
$$

$$
\{R E A L\}=1+\frac{C T}{C_{m}}
$$

and the imaginary part is,

$$
\{\text { Imaginary }\}=\frac{-1}{\omega R_{T} C_{m}}
$$

The solution for the magnitude of el for this circuit is,

$$
e_{1}=\operatorname{Vin}\left(\left[1+\frac{c_{T}}{c_{m}}\right]^{2}+\left[\frac{1}{w R_{T} C_{m}}\right]^{2}\right)^{1 / 2}
$$

Nominal values for the parameters in this equation were:
Ct ~ 170 pf.
Cm ~ 50 pf.
Ki ~ 10 Megohms.

Fiefer ahead to figure 3.9 to relate the signal amplifier input to the final "voltage" sent via the "Hewlett packard Interface Bus" (HFIE) to the HF-85 computer. The output of the signal amplifier 15 equal to the input multiplied by tine amplifier gain. This same amplifier output voltage, Vout, 15 then the analog input voltage to the sack lock $1 \pi$
analyzer where it is scaled by the gain setting of the fafi 5204 lock-in analyzer, "B", and sent to the data acquisition system. Due to the design of the lock-in analyzer, the gain setting of "B" is selected so that the acquisition input has a range of values of from 0 to 1.15 volts. This data acquisition system is HPIB compatible and is in turn sampled to provide the digitized output, Vdata, to the HP-85 computer.


Figure 3.9 Analog_to_dıgital_slgnal_flow_chart.

The inverse of this path gain is computed to determine the value of Vin when Vdata is known. Thus, the analog voltage "G*Vin" 15 obtained by multiplying "Vdata" ty the gain setting, "E".

$$
[G]\left[V_{i}\right]=[B]\left[V_{d+N}\right]
$$

Solving for Vina and substituting into equation 3.11 we obtain the received signal voltage in terms of experimentally measured parameters:
$e_{1}=\left[\frac{B V_{\text {dATA }}}{G}\right]\left(\left[1+\frac{C_{T}}{C_{m}}\right]^{2}+\left[\frac{-1}{W R_{T} C_{m}}\right]^{2}\right)^{1 / 2}$ Equation 3.13

Prior to computing the ratio e1/il, we must next discuss the calculation of the transmitting current, 11.
d. The Calculation of il

The drive current for the source microphone is estimated from a knowledge of the microphone capacitance, the bias voltage, and the drive voltage seen across the source terminals.

$$
i 1=j \omega C_{m i c} V_{\text {dive }}
$$

Equation 3.14

Using a constant value of capacitance to model the microphone neglects the effect of the motional impedance on the ratio of [e1/il] for the condenser microphone. (After the fact, it was determined that the value of il should have been experimentally measured and not calculated. However, the following explains what was actually done.)

The drive voltage across the source microphone's terminals that is used for the computation of il is not the drive voltage that the computer program "asks" for but rather is a resultant of the "asked" for voltage and the signal path transfer function between the signal function generator and the microphone. This discrepancy is corrected by using an experimentally determined least squares fit to "ask" vs "get" data. The circuit shown below is the circuit used to obtain the "ask" vs "get" data needed.





Using the data experimentally measured in this fashion， we have a series of estimates of＂Vget＂that use acceotatle straight line appro\％imations to the transfer flinction for short segments of frequency．

Equation form
（）$ク 2<f<520 h z$

$1510 \rightarrow 2$ \＆ $2500 \rightarrow 2$

こらげいこくよく999の トこ

Vget $=[k(f)] * V a s k$

Vget $=[.9278+4.979 E-5 * f] *$（Jask （probable error $\sim 450$ ppm）
Vget $=$［．9516＋4．16さE－6＊f］＊Vask： iprobable error～玉10 ppmう
bget $=$［．954！＋1．7！4E－b＊f］＊＇vash． \｛probable error～ 410 ppm\}

$$
\text { \{probabie error } \sim 586 \text { ppin) }
$$

＇Iger $=[.9568+2$. コ24E－7＊f ］＊Vask \｛prooable error～ 269 ppm\}

The next correction we must include in order to accurately calculate the drive current 15 the observea increase in capacitance of the source microphone which occurs with an increase in bias voltage.

While the capacitance of the source microphone is primarily a function of its dimensions, the electrostatic force between the backplate and the diaphragm of the condenser microphone as a result of the applied b:as voltage, will cause a displacement of the microphone diaphragm away from the unbiased equilibrium position. This change in the separation between the diaphragm and the backplate will cause a slight increase in microphone capacitance relative to the "no bias" capacitance. This slight change ( $\sim .3 \%$ ) is easily measured and is tabulated below. The data were obtained using the internal bias supply voltage and the standard functions available on the Hewlett Fackard HF-4192A LF : mpedance analyzer.

Approwimate solutions for Mo are ottanney for the W.E. 40 AA microphones using the change in capacitance vs bias voltage data. The theory and results of such a calculation are shown in Appendi\% $F$.

For W.E. G40AA TYPE "L" laboratory standard microphones. Measured values are uncorrected $H F-4192 A$ readings.

Bias
(volts D.C.)

- Measured C/delta C

Blas"2
(all values 1 n Ff.)
ivalts 2 :
$\begin{aligned} & \text { W.E. G40AA Serial \#'s } \\ & \text { \#1248 } \text { \#0815 }\end{aligned}$
\#1082
$-3556.790 / .030$
$56.990 / .045 \quad 54.8421 .072$
$56.980 / .0554 .8201050 \quad 700$
-30 56.785/.025
-25 56.780/.020
$-20 \quad 56.770 \% .010$
-15 56.765/.005
$-1056.76 .3 / .003$
056.7601 .000
$+1056.763 / .003$
$+1556.765 / .005$
$+2056.7701 .010$
+25 56.775/.015
$+7.056 .780 / .020$
$+3556.790 \% .030$
$56.970 / .025$
$56.960 / .015$
56.955/.010
$56.950 / .005$
$56.945 / .000$
56.950/.005
56.955/.010
$56.958 / .013$
56.9701 .025 56.980/.055 56.9001 .045
-4.820..050 900
$54.805 / .035625$
$54.7901 .020 \quad 400$
54.785/.015 225
$54.775 / .005100$
54.7701 .000
$54.775 / .005100$
$54.788 / .018 \quad 225$
$54.793 / .02 \mathrm{E}$
$54.808 / .038$ 525
$54.923 / .053900$
$54.8421 .075 \quad 1225$
correction for mounting bracket $\quad=-4.704(p-1$
correat in " *at ショi:trat:on of 4f-4192A $=+0.008(p f$ )
Total correction required $=-4.786\left(p^{+}\right)$

Tatle 3.6 Measurec_change_in_들acitance_due_to
agel ication_offa_b1크_voltage_to_a_condenser_microphone

Due to the possibility of damaging the MF-4ig2A Le impedance analyzer, it was impractical to measure the sharigio in capacitance due to the applied baas when the W.E.bana micropnones were biased at their oxperimental bias of 1:1: volts. A linear least squares tit to the oata shown aoove was used to extrapolate an estiflate of the cinenge 1.2 inicrophone capacitance which resulted from the appiied $51=5$ oltage. The equations 50 ottained had the form,
$\Delta C_{M}=a\left[V_{B i A S}\right]^{2}+b$

Individual correlation coefficients of .985, .996, and .999 were obtained for the bias voltage capacitance correction equations given below for W.E. G40AA microphones with serial numbers \#1082, \#1248, and \#0815 respectively. The uncertainties in the slopes and intercepts so obtained are given as "delta a" and "delta b" respectively.

```
delta C1 = [2.4731 E-5]*V^2 + 3.1668 E-4 (pf)
(#1082) delta "a" ~ 1.2E-6, delta "b" ~ 8.03E-4
(#1248) delta "a" ~ 1.bE-6, delta "b" ~ 1.1E-4
delta CJ = [5.9254 E-5]*V^2 + 2.3190 E-4 (pf)
(#0815) delta "a" ~ 8.2E-7, delta "b" ~ 5.5E-4
```

At a nominal bias voltage of ~ 117 volts, the capacitance corrections expected due to the bias voltage are on the order of one percent and are significant to the overall calibration.

Thus, we can calculate the value of il as,

$$
i 1=\left(2 \pi f_{N}\right)\left[K(f) V_{A S K}\right]\left[C_{0 m}+\Delta C_{m}\right]
$$

Equation 3.18
$\frac{e_{1}}{i 1}=\frac{\left[\frac{B V_{\text {dATA }}}{G}\right]\left[\left(1+\frac{C_{T}}{C_{m}}\right)^{2}+\left(\frac{-1}{W_{N} R_{T} C_{m}}\right)^{2}\right]^{1 / 2}}{\left[2 \pi f_{N}\right]\left[K(f) V_{A S K}\right]\left[C_{o m}+\Delta C_{m}\right]}$ Equation 2.17

The probable error in the ratio elis may now be calculated provided the probable errors associated with the preamplifier gain, $G$, the measured resonant frequency, fin, and the non-linearity in el/vi due to the lock-in analyzer are available. An analysis of these probable' errors will be included in the next four sections.
e. Measuring The Signal Preamplifier Gain, G

Two different signal preamplifiers were available for use in this experiment. These were the Hewlett Packard type 465A amplifier and the Ithaco model 1201 low noise preamplifier. Both amplifiers were used at a nominal gain of "10" but were experimentally observed to have different gain stability characteristics. Since the duration of a typical experimental "run" averaged eight hours, it was of paramount importance that the drift in the signal gain be minimized over this time period. Both amplifiers had experimental advantages; the HP-465A had the lowest drift rate and was selected for use in the plane wave resonant reciprocity experiment. The Ithaco 1201 had an internal battery power supply which greatly reduced cross talk and 60 cycle hum during the later free field reciprocity calibrations. The circuit used to measure the gain experimentally was controlled by the HP-85 computer and is shown below.


Figure 3. 11 Circuit_used_tonmeasure_amelifier 브ํํ․

In the above circuit, after establishing a constant rins voltage output, the function generator was directed by computer to step through the frequency range of interest (200-6000Hz) at intervals of 100 Hz . At each frequency the output voltage was measured by the lock:in analyzerldata acquisition ane controi unit and stored $1 n$ a computer array. Ne\%t, the amplifier was included in the circuit and the procedure was repeated. The entire procedure was repeated several tines to obtain an estimate of precisiun for the array data. The galn of HF-46SA amplifiers was calculated by comparing the two arrays and the results are 3hown 1 n figures ふ. 12 and こ. 1 J below.





Since the drift of measured values over a single day is roughly $1 / 5$ the drift observed over an elght month fericd, the latter $i s$ used as the worst case estimate of uncertainty in $G$, while the former 15 used as the best case estimate. Thus, the range of precision in this data is from ~260ppm to ~1300 ppm. Since the computer controlled circuit measures the relative gain, the systematic error included in any one measurement is cancelled out and the precision of the data is used as the estimate for probable error in this parameter. The galn analysis for the Ithaco ampl:tiers will
be included later when the free field reciprocity experiment is discussed.

Next, the non-linearities associated with the lock-in detector signal path will be discussed.
f. Measuring Non-Linearity in The Lock-in Detector

The non-linearity in the lock-in detector was measured using the circuit shown in figure 3.14 below.


Figure 3.14 Circuity_used_to_determine


Experimental determination of the linearity of the lock-in detector found that the fractional error varied as a function of what portion of the scale was used to measure the data. Since the computer program dynamically sought to run the source until the receiver was at ~ $90 \%$ of full scale, the linearity at that scale position was measured and used in the error calculations. Two different ratio transformers were used to determine the linearity of the
system and the measured results are shown in the following tables.

All data was obtained at 1000 hz , so that the limits of the ratio transformers would not be exceeded.

Transformer serial\# 304
transformer ratio voltage ratio fractional error(ppm)

| .1 | .097117 | -29700 |
| :--- | :--- | :--- |
| .2 | .206257 | +30300 |
| .3 | .307339 | +23900 |
| .4 | .398773 | -3080 |
| .5 | .497208 | -5620 |
| .6 | .698417 | -3910 |
| .7 | .800050 | - |
| .8 | .900181 | + |
| .9 |  | + |

Table 3.7 Lingearity_data_obbtained_using_ratio transformer_serial_\#_304

As shown on the next page, a different ratio transformer was used to obtain a comparison.

All data was obtained at a frequency of 1000 he 50 that the limits of the ratlo transformer would not be exceeded.

Transformer serial\# 572
transformer ratio voltage ratio fractional error (ppm)

| .1 | .096869 | - | 32300 |
| :--- | :--- | :--- | ---: |
| .2 | .199362 | - | 3200 |
| .3 | .300154 | +300 |  |
| .4 | .399208 | - | 1980 |
| .5 | .599039 | - | 1920 |
| .6 | .700138 | - | 2315 |
| .7 | .798215 | + | 197 |
| .8 | .898320 | - | 2236 |
| .9 | - | 1970 |  |

Table ت̇. 8 Linearity data_obtained_using_ratig


Since the linearity of these ratio transformers nominally is on the order of 10 ppm, the fractional error due to monlinearity in the signal flow through the locki-in detector 15 from 220 to ~180) ppin jepending upon which ratio transformer is accurate. An estimate of 1400 ppm will be used for the probable error iri kine Inearaty of tile system near full scale and $\sim 700$ ppm near malf scale.
g. Computation of The Probable Error in eli

For the purpose of error analysis, the effect of the [w*Rt*Cm] term in equation 3.19 is relatively small since the neglected term is at worst one order of magnitude smaller and normally two orders of magnitude smaller than the $[1+C t / C m]$ term. As a result, its contribution to the overall error will also be smaller. Equation 3.19 is therefore rewritten for the purpose of error analysis as:

$$
\frac{e 1}{i 1} \cong \frac{\left(\frac{B}{G}\right)\left(\frac{V_{\text {daTA }}}{K(f) V_{A S K}}\right)\left(1+\frac{C_{T}}{C_{M}}\right)}{2 \pi f_{N} C_{m}}
$$

Equation 3.20

The relative error in eli can now be more easily computed as:

$$
\left.\frac{e_{1}}{i l}=\left(\frac{\delta G}{G}\right)^{2}+\left(\frac{\delta\left[C_{m}^{-1}+C_{T} C_{m}^{-2}\right]}{\left[C_{m}^{-1}+C_{T} C_{m}^{-2}\right]}\right)^{2}\right\}
$$

Equation 3.21

When it is noted that the contribution to the error due to the resonant frequency will not apply to the total error in the open circuit receiving voltage sensitivity, Mo, due to a cancellation from the resonant frequency in the numerator of Jo, we can exclude the relative error due to the resonant frequency, note that the scale factor $B$ is a constant, and
rewrite the relative uncertainty in $[e 1 / 11]$ as，

$$
\frac{\delta\left(\frac{e l}{i i}\right)}{\left(\frac{e l}{i 1}\right)}=\left\{\left(\frac{s\left(\frac{\text { dada ta }}{\text { VgeT }}\right)}{\left(\frac{V_{\text {daTa }}}{\text { Vg at }}\right)}\right)^{2}+\left(\frac{\delta G}{G}\right)^{2}+\left(\frac{\delta\left(C_{m}^{-1}+C_{T} C_{m}^{-2}\right)}{\left(C_{m}^{-1}+C_{T} C_{m}^{-2}\right)}\right)\right\}_{\text {Equation } 3.22}^{1 / 2}
$$

The error in the voltage ratio will come from two sources； the uncertainty in Vget，as a result of the inexactness of $K(f) * V a s k$, and the uncertainty due to the nonlinearity in the voltage ratio resulting from the non－linearity in the lock－in analyzer．Using the results from the previous section and the experimental determination of the inexactness found for $K(f) * V a s k$（shown in equations 3．15）， we Gave，

$$
\frac{\delta\left(\frac{V d A T A}{V g e t}\right)}{\left(\frac{V d A T A}{V g e t}\right)}=\left([1400 \mathrm{ppm}]^{2}+[365 \mathrm{ppm}]^{2}\right)^{1 / 2}=1447 \mathrm{ppm}_{\text {Equation }}=.2=
$$

The uncertainties due the capacitance terms are computed next．Let the result，$W$ ，be defined as：

$$
W=C_{m}^{-1}+C_{T} C_{m}^{-2}
$$

Straightforward error analysis yields,

$$
\delta w=\left\{\left(\frac{\partial w}{\partial C_{m}} \delta C_{m}\right)^{2}+\left(\frac{\partial W}{\partial C_{T}} \delta C_{T}\right)^{2}\right\}^{1 / 2} \quad \text { Equation }=.25
$$

with,

$$
\frac{\partial W}{\partial C_{m}}=-C_{m}^{-2}-2 C_{T} C_{m}^{-3}
$$

Equation Z. IS
and,

$$
\frac{\partial W}{\partial C_{T}}=C_{m}^{-2}
$$

Equation 3.ご
 and subsequent division by equation 3.24 , the fractional uncertainty due to capacitance terms 15 obtained.

$$
\frac{\delta W}{W}=\frac{\left\{\left[\left(-C_{m}^{-2}-2 C_{T} C_{m}^{-3}\right)\left(\delta C_{m}\right)\right]^{2}+\left[\frac{\delta C_{T}}{C_{m}^{2}}\right]^{2}\right\}^{1 / 2}}{\left(C_{m}^{-1}+C_{T} C_{m}^{-2}\right)}
$$

With nominal values of $\mathrm{Ci}, \mathrm{Cm}$, delta Ci and delta Cm taken from the combination of of data within tables 3.2 and 3.4 we obtain,

```
    CT ~178 pf (total of Clo, Cli, and Cm ref. figure ङ.ड)
    Cm ~ 52 pf
    delta CT ~ . 15 pf
    delta Cm ~ .002 pf
```

Upon substitution into the equation for the relative uncertainty due to the capacitance terms, these figures yield:

## $\frac{\delta W}{W} \cong 660 \mathrm{ppm}$

Calculating the uncertainty in eli due to uncertainty in capacitance, uncertainty in the voltage ratio and uncertainty in the preamplifier gain, we obtain,

$$
\frac{\delta\left(\frac{e l}{i 1}\right)}{\left(\frac{e l}{i}\right)}=\left[(660)^{2}+(1447)^{2}+(780)^{2}\right]^{1 / 2}=1770 \text { ppm }
$$

Next we will consider the measurement of the voltage ratio Vca/Vcb.

Refer to the comparison voltage signal flowpath shown in figure 3.3 and reproduced below.


Figure 3.3 Signalf $\mathfrak{l}$ ownepath_for_comparison

## ㄴoㄴtㅕges

Since the signal path for the measurement of both "Vca" and "Vab" is identical, only the ratio of these measured voltages is important. As such, the non-linearity of the lock in analyzer will determine the error contributed by this ratio. When the non-linearities measured using the Gertsch ratio transformers are examined in table 3.7 and
table 3.8, the average non-linearity is much worse than that obtained at a scale reading of 0.9. The previous use of the non-1inearity associated with a 0.9 scale reading for the ratio of Vdata/Vget in the calculation of el/i1 was possible only because the controlling program dynamically adjusted the transmitting sources until the receiving microphone gave an output in this scale region. No such dynamic adjustment was achieved for the comparison voltages Vca or Vcb and they typically were at half scale. Since the probable error due to non-linearity in the system at half scale is ~ 3770 ppm, this is the value of probable error used for the ratio Vca/Vcb.

## 

Refer to equation 2.40 which is reproduced immediately below.

$$
J_{0}=\frac{\pi V_{0} f_{N}}{P_{0} \gamma_{E} Q_{N}}
$$

Equation 2.40

Experimental calculation of the reciprocity factor, Jo, required the measurement of five experimental parameters. These are the atmospheric pressure, the resonant frequency, the quality factor of each modal resonance, the ratio of specific heats for the gas within the cavity, and the cavity volume. Two other experimental parameters varied over the course of the experiment and required measurement in real time. These parameters were the temperature and the relative humidity. The relative humidity and temperature will cause a change in the ratio of specific heats [Ref. 25], and any temperature change affects the speed of sound and hence the resonant frequency obtained. A discussion of the means used to measure all of these parameters will be presented next.
a. Measuring Atmospheric Pressure

Changes in the ambient pressure were monitored using a high precision MKS Baratron pressure head sensor, model 270 with a HS6-1500 purifying diffusion pump, type 162 all of which then was calibrated by comparison to the ambient pressure at the experimental altitude. The absolute reference was provided as data determined by the Meteorological Department at the Naval Postgraduate School. The data was available at 15 minute intervals and was compared over roughly one weeks time. The" correction so determined was applied to the output of the Baratron's associated electronics signal as measured by the Hewlett Packard 3456A digital voltmeter. Refer to figure 3.15 below for a sketch of the pressure sensor signal path.


Figure 3. 15 The_eressure_sensor_signal_path.

The data used to obtain the calibration for the output of the baratron pressure head is plotted in figure 3.16 and is a plot of the output of the Baratron system over approximately one weeks tine. The output of the MrS baratron pressure head is scaled to indicate $m \mathrm{mHg}$ (Torr) so that the correction calibration of +5.58 mbar becomes +4.25 $m m H g$ in the software implementation of this calibration.


Figure ふ. 16 Presssureㅡ_sensor_vs._presssure

## refererence

The precision of this pressure reading is observed as deviations from the least square fit to a straight line. The deviations range from ~. 1 mbar to ~ 1.2 mbar and the average value $15 \sim .25$ mbar. If the systematic error is taken as the claimed accuracy of the Naval Fostgraduate School reference, $\{\sim .2$ mbar\} the probable error $1 n$ pressure $15 \sim 319$ ppm.
b. Uncertainty in Resonant Frequency and Quality Factor Plane wave longitudinal resonances in the "short" cylindrical cavity were sampled and analyzed using the least square error ravine process referred to in step 6 of part $C$ in this chapter. The inaccuracies found in measuring the quality factor generally increase with the sharpness of the resonance. Since the short tube had higher Q's for its modal resonances, it was selected for this experimental determination of the precision of $F_{n}$ and $Q_{n}$. As such, the fractional error in $Q_{n}$ found using the short tube will be the upper bound on the determination of the fractional error in Qn for the experimental calibration. There is no experimental reason to expect any difference in the precision found for $F$ f determined in the long tube compared with that determined in the short tube. It must be noted, however, that the fractional error in the determination of resonant frequency does not enter into the calculation for the open circuit receiving voltage sensitivity shown in equation 3.1 since the resonant frequency in the denominator of the ratio found for el/il is cancelled by the resonant frequency found in the numerator of the reciprocity factor, J. The analysis that follows for the probable error in Fn is included only for completeness. The transducers used as source and receiver for these measurements were type W.E. $640 A A$ condenser microphones mounted in the ends of the plane wave resonant cavity.

Normally, the computer program that controlled the experiment caused first one condenser microphone to act as a source and the other as a receiver and then switched the roles of source and receiver so that reciprocity data could be obtained. To determine the precision of the numerical algorithm that obtained a least mean square error fit to a Rayleigh line shape, two such sets of data were still obtained but the program was not allowed to switch roles between source and receiver after the first data set was sampled. See figure 3.17 below for an illustration of how the Rayleigh line shape was sampled around resonance.


Figure 3.17 Data_sanmele_of_a_modal_resonance_from


With no change in the source or receiver, two such data sets were obtained one after the other. After both data
sets were ravined, an immediate comparison in resonant frequency and quality factor for the mode under inspection was made. When several modes were sampled many times, sufficient data was available to determine the precision of the ravine technique by direct comparison. Since the relative humidity varied within the laboratory on a time scale of hours while the temperature within the laboratory changed noticeably every few minutes, only the change in average temperature as measured by the thermistor shown in figure 3.3, was important for comparison pLirposes as the mean time between data set 1 and data set 2 was approximately 45 seconds. The fact that the speed of sound in an ideal gas is proportional to the square root of the absolute temperature was used to normalize all the ravined resonant frequencies to a common temperature of 20 degrees celsius for comparison purposes. These normalized resonant frequencies and the associated quality factors for three selected modes are given in table 3.9 below.

|  | $\begin{aligned} & \text { Data set } 1 \\ & \text { Fnl (Hz) } \end{aligned}$ | Qn 1 | $\begin{aligned} & \text { Data set ? } \\ & \text { Fn2 (hz) } \end{aligned}$ | On2 |
| :---: | :---: | :---: | :---: | :---: |
| Mode | 6 | ． |  |  |
|  | 4418.3168 | 136.0039 | 4418.375 | 1こ6． 2080 |
|  | 4418.3466 | 135.7561 | 4418.3059 | 135．9890 |
|  | 4418.2921 | 135.9326 | 4418.2692 | 135.9256 |
|  | 4418.2725 | 135.8651 | 4418.2368 | 135.8540 |
|  | 4418.2727 | 135.919. | 4418.2463 | 135.9010 |
|  | 4418.2624 | 135.9017 | 4418.2475 | 135.8697 |
| Mode |  |  |  |  |
|  | 7371.1629 | 127．7876 | 7371.3582 | 127.7575 |
|  | 7371.4691 | 127．7841 | 7371.4724 | 127．7953 |
|  | 7371.5582 | 127.8270 | 7371.5235 | 127.8268 |
|  | 7371.5865 | 127.8205 | 7371．5533 | 127.8211 |
|  | 7371．6375 | 127.8591 | 7371.6006 | 127.8483 |
|  | 7371.6222 | 127.8077 | 7371.5953 | 127.8442 |
|  | 7371.6297 | 127.8777 | 7371.5867 | 127.9813 |
|  | 7371.6051 | 127.8772 | 7ご1．5731 | 127.8668 |
|  | 7371.6250 | 127．9132 | フミフ1．5900 | 127.9086 |
|  | 7371.6005 | 127.8899 | 7371.5707 | 127.9129 |
|  | 7371.6039 | 127.8869 | 7371．5771 | 127.8824 |
|  | 7371.6055 | 127.9279 | ＂7371．5650 | 127.9199 |
|  | 7371.6589 | 127.8962 | 7ざ1．6こ69 | 127.7822 |
|  | 7371.6643 | 127．7987 | 7371．6503 | 127．3922 |
|  | 7．71．6897 | 127.9431 | 7371．66こ6 | 127.9454 |
| Mode |  |  |  |  |
|  | 16164.2499 | 184．85こ1 | 15：57．S2： | 197． $3: 47$ |
|  | 15155．1959 | 134.2745 | 16：55． 170 | 104．25tb |
|  | 16：55．47ー | ：34．ここここ | 15155．4553 | 184． 26 S |
|  | 16：65．5358 | 184．6545 | 15：55．5152 | 134.795 |
|  | 16165.755 | 185.2799 |  | ：ご，ここ |
|  | 15165.3517 | ：85．75心 | $\therefore 15$ ． 31.7 | 10.6000 |
|  |  | 126．34？ | $161 \leq 5.6702$ | 136.424 .6 |
|  | 15165.8540 | 186．9020 | 16165.7844 | 186.9 －7 |
|  | ：5165．9372 | 187．4才5 | 1615j．68こ： | 13？．E！¢ |
|  | 16165.9594 | 187．9696 | 16155．975 | 188．0こ： |

Table 3．9 Temperature＿normalized＿resoriance
freguencies＿and＿the＿associateg＿gualit ffector efor＿tiret
differentamodes．

In the following two graphs of the fractional error found between data set 1 and data ミet こ，the vertiad ni：iss is in parts per million（ppin）fractional errer and the
horizontal axis is taken from gaussian probability paper [Ref. 28: p.67] and is plotted as the percent of readings at or below the value found on the vertical scale. Such a plot illustrates the distribution of a data set with a gaussian distribution as a straight line. By calculating the correlation coefficient of the plotted data as a function of an arbitrary linear horizontal scale, the correlation to a straight line is determined.


Figure 3.18 Fractional_error_in_quality_factor
Plotted_on_probability_pager. The_linear_correlation_coefficient 15_egual_t므읩


Figure 3.19 Fractional_error_1ח_resonant_freguency
2lotted_on_probabilıty_pager. The_linear_correlation_coefficlerit is_egual_t으․97

All the data shown in table 3.9 was used. Since both of the probability plots seem to roughly fit a straight line, the conclusion 15 that the data sets have roughly/ a gaussian distribution and as such, the standard deviation is taken as a measure of the precision of the technique. For the quality factor, the probable error is then $\sim 180$ ppm. For the resonant frequency, the above results show a probable error of $\sim 7$ pin.
c. Calculating The Effect of The Non-adiabatic Boundary Conditions Upon The Stiffness of The Gas Within The Resonant Cavity

In chapter one, two methods of calculating the bulk modulus of elasticity of the gas within the plane wave resonant cavity are shown in equation 1.81. The starting point of this analysis is the adiabatic form of the bulk modulus of elasticity given as the product of the atmospheric pressure and the ratio of specific heats of the gas within the cavity. It will be shown that the effective ratio of specific heats of the gas within the plane wave resonant cavity will be determined by the gas content, the thermal properties of the physical boundaries, and the ratio of volume to surface area for the physical cavity.

The temperature and relative humidity change the ratio of specific heats in an empirical equation determined by Wong and Embleton [Ref.25]. Their result is given below. The relative humidity (the mole fraction of water vapor in humid air divided by the mole fraction of water vapor in saturated humid air) is given by "h" and the temperature in degrees centigrade is given by "t".

$$
\begin{aligned}
& J_{H}^{\prime}=1.39984-A(h+0.125) \\
& A=5.2 \times 10^{-4}+4 \times 10^{-5} t+7.5 \times 10^{-7} t^{2}+4.5 \times 10^{-8} t^{3} \quad \text { Equation } 3.31
\end{aligned}
$$

This equation will provide the value for gamma (the ratio of specific heats, cp/cv) for free field reciprocity calculations.

In order to consider the effect of the non-adiabatic conditions along the boundary of the plane wave resonant cavity, the influence of the thin layer of air in contact with the wall upon the stiffness of the gas within the cavity must be examined. Since the wall of the cavity is made of brass, with a thermal conductivity four orders of magnitude greater than air, and the heat capacity of the brass tube is four orders of magnitude greater than the heat capacity of the air contained within, the condition along the thin layer of air next to the wall is approximately isothermal. The stiffness of this small layer is therefore slightly less than that of the remainder of the air volume within the cavity. The overall bulk modulus must therefore be reduced slightly to account for this effect.

Since the ratio of specific heats given by Wong and Embleton is valid many thermal layer depths away from the wall and the ratio of specific heats for the isothermal case is equal to one, the effective gamma is first approximated by volume weighting the thermal layer with a gamma of one and the remaining air column with the value of gamma given in equation 3.31. To first order in thermal layer depth, this yields an effective gamma as shown below,

$$
\gamma_{E} \cong \gamma_{H}-2\left(\gamma_{H}-1\right) \frac{S_{T}}{r_{0}}
$$

Here, the thermal layer depth is given by [Ref. 27: pp. 225-6],

$$
\delta_{T}=\left\{\frac{2 K}{\rho \omega c_{p}}\right\}^{1 / 2}
$$

Equation 3.33

Where,

$$
\begin{aligned}
K & =\text { thermal conductivity [J/sec*M*degk] } \\
\ell & =\text { air density }\left[K g / M^{*} 3\right] \\
\mathrm{E} p & =\text { specific heat at constant pressure [J/Kg*degk] } \\
\omega & =2 * p i * \text { frequency [rad/sec] }
\end{aligned}
$$

thus,

$$
\begin{aligned}
& \delta_{T} \sim(2.5 E-\Sigma) / \text { sqr\{f\} [meters] } \\
& r_{0}=1.256 E-2 M\{s h o r t \text { tube\}, } 1.718 E-2 \text { Milong tube\} }
\end{aligned}
$$

If the end effects are not neglected, equation 3.32 must be modified as shown below where "L" is the length of the cavity.

$$
\begin{aligned}
& r_{0}<L \text {, } S_{T} \ll L
\end{aligned}
$$

Equation こ. こ. 4

When the ends are accounted for to first order in thermal. layer depth, the relative change in the corrected value of gamma over that which is calculated when the ends are neglected is . 003 percent and . O1 percent for the $10 n g$ and short tubes respectively. Since the simple model used to obtain equation 3.32 yields an approximate correction to gamma on the order of 0.5 percent, the ends are neglected in the following analysis.

To verify that the preceding analysis 15 correct to first order in thermal layer depth, a more precise analysis is required. When the change from adiabatic to isothermal is modeled as an exponential change following the known thermal characteristics within the right circular cylinder [Ref. 28], and when the boundary value for gamma is matched both at the wall of the cylinder and within the volume, then the volume weighting of the adiabatic bulk modulus 15 reflected in an effective value of gamma given by,


And to second order in thermal layer depth, the solution $2 \equiv$,

$$
\gamma_{E}=\gamma_{H}-2\left(\gamma_{H}-1\right) \frac{\delta_{T}}{r_{0}}+2\left(\gamma_{H}-1\right)\left(\frac{\delta_{T}}{r_{0}}\right)^{2}
$$

This solution agrees with that of equation 3.32 to first order in thermal layer depth. This correction is shown below as a function of frequency in figure 3.20.


Figure 3. 20 Correction_to_the_rationof_seecific
heats_due_to_the_non=adiabatic_ogundarızconditions_within_a_rigat
 with_air.

This correction, applied to the value of gamma and used with ambient pressure to calculate the adiabatic bulk: modulus, provides the correction to the adiabatic bulk modulus of elasticity due to the non-adiabatic boundary conditions within a specific (the "long" tube) right circular plane wave resonant cavity.

When the restrictions given for equation 3.34 apply, the effective ratio of specific heats, valid to first order in thermal layer depth, can be put in a more general form:

$$
\gamma_{E}=\gamma_{H}+\left(1-\gamma_{H}\right)\left\{\left[\frac{K}{\pi C_{P} l}\right]^{1 / 2}\left[\frac{\text { Surface AreA }}{\sqrt{f} \text { Volume }}\right]\right\} \text { Equation } 3.57
$$

In general, the independent parameter used to plot the magnitude of this correction to the open circuit voltage receiving sensitivity is given as "b", where:

$$
B=\left\{\left[\frac{k}{\pi \ell_{p} \rho}\right]^{1 / 2}\left[\frac{\text { Surface Areas }}{\sqrt{f} \text { Volume }}\right]\right\}=\text { Equation <compat>...s }
$$

The correction described by equation 3.37 in terms of " $B$ " 15 shown in the figure below.


Figure 3.21 The_correction_to Mo_due_to_the_change in_stiffness_of the_gas_within_the_cavity_caused_by_the non-adiabatic_boundary_conditions.

The correction shown above does not correct for heat conduction losses at the boundary of the tube. Heat conduction losses, as well as any acoustical loss, will be determined in the experimental measure of an which 15 then directly employed in the calculation of the acoustical transfer admittance, Jo.

Since the paper of Wong and Embleton was not avallabie when the computer program controlling the experiment was
written, a correction to the preliminary experimental results is needed. The experimental data that was obtalned using the program in appendi\% $B$, used a less accurate correction to the ratio of specific heats to account for the effects of changes in temperature, humidity, and the effect of the non-adiabatic boundary layer (See appendix B, lines 3770-3810). This approximate solution for the ratio of specific heats must be corrected to that obtained by wong and Embleton and the associated correction to the open circuit receiving sensitivity must also be made. Analytically, when the range of experimental temperatures 15 varied from 19 to 21 degrees centagrade and the range of experimental relative humidities is varied from $40 \%$ to $65 \%$, the experimental result for the absolute walue of the ooen cirsuit voltage receiving sensitivity, Mo, must be correctej by t. OOTaB throughout the trequency range used as a result of using the ratio of specific heats in equation उ. Bi. Wnen this correction is used, the e\%perimental uncertalnty in the ratio of specific heats 15 that obtaımed by wong anc Embleton and 15 given as 400pm [Ref. IS].
d. Volume Measurements in The Plane Wave Resonant Cavities

Two different right cylindrical cavity resonators were used in the plane wave resonant reciprocity calibrations. The lengths were selected so that the resonant fundamental in the long tube was one third the resonant fundamental in the short tube. This allowed direct comparison of experimental results obtained with two different resonant cavities at every third modal resonance of the long tube.

The actual experimental volume was measured in a straightforward way with a slight negative correction necessary due to the small protrusion of the W.E.640AA type microphones into the cavity. The microphones were adjusted in position so that the longitudinal equilibrium positions of the diaphragms were flush with the plane of the physical end of the cylinder. Since there were three different microphone pairs each with their own volumetric protrusion into the main volume, three different experimental corrections were measured and applied to the basic volume found for each resonant cavity. The results obtained using two different gauge calipers \{one Peacock caliper and one Kanon calipers are shown below in table 3.10.

> Long Cylinarical Cavity vs. Short Cylinarical Cavity

```
End area shape
average eccentricity
```

～Circular
e 〔．OSJ
～Elliptical e～ .094

The following data have four significant digits：

| End area | $9.2758 \mathrm{E}-4 \mathrm{M} 2$ | $4.9577 \mathrm{E}-4 \mathrm{M} 2$ |
| :--- | :--- | :--- |
| Length | $7.0120 \mathrm{E}-1 \mathrm{M}$ | $2.3372 \mathrm{E}-1 \mathrm{M}$ |
| Basic volumes | $6.5042 \mathrm{E}-4 \mathrm{M} 3$ | $1.1587 \mathrm{E}-4 \mathrm{M}$ M |

Individual microphone volumetric protrusions．
W．E．G40AA Serial－protrusion

| 1248 | 3．407E－7 M～2 |
| :---: | :---: |
| 1082 | 3．084E－7 M～2 |
| 0815 | 3．370E－7 |

Serial pair W．E． 640 AA ＇s
$1248-1082$
1248 －0815
$1082-0815$

Long tube volume
corrected value
6．4977E－4 M～3
$6.4974 \mathrm{E}-4 \mathrm{M}$ へ
$6.4977 \mathrm{E}-4 \mathrm{M}$ 3

Short tube volume corrected value

1．1522E－4 M゙こ
1．1519E－4 M J
1．152ミE－4 Mへ3

Table 3.10 Volume＿measurements＿obtained＿for＿the Qlane＿wave＿cavity＿resonators

When the ralative error in each of these volumes 15 calculated，the values obtained depend essentially upon which resonant cavity 15 being used．Since the thermal expansion coefficient for orass 15 ～1．9E－5／deg $C$ and the temperature range in the laboratory was from if to 29 Jegrees centigrade，the fractional error in length Jue to thermal effects was～jopm．Since the uncertarnties in
measuring the length of the long tube at least one order of magnitude greater, the thermal effects were neglected for calculations of volume.

For the long cavity, refer to equation 3. 39 .

$$
\frac{S V_{0}}{V_{0}}=\left\{\left(\frac{2 \delta D_{0}}{D_{i n} .}\right)^{2}+\left(\frac{\delta L}{L}\right)^{2}\right\}^{1 / 2}
$$

Equation 3.39

The uncertainties in the individual measurements were found to be,

The average length of long tube $=7.012 \mathrm{E}-1 \mathrm{M}$
The standard deviation in length $=2.08 E-4 \mathrm{M}$, based on three measurements made with a Kano vernier calipers ser. \#SkO14 from the USNPGS Mechanical Engineering Dept.

The average diameter of long tube $=3.437 E-2 \mathrm{M}$
The standard deviation in diameter $=2.85 E-5 \mathrm{M}$, based on eight measurements made with a Peacock vernier calipers from the USNPGS Physics Dept.

Next, use 15 made of the fact that with a random distribution of error in the measurements, the standard deviation divided by the square root of the number of samples is the standard deviation in the estimate of the sample mean. With these considerations and including the systematic error involved in neglecting the effects of thermal expansion, the uncertainty in the volume measured for the large tube was found to be $\sim 12 \mathrm{ppm}$.

When careful measurements of the dimensions of the small cavity were made, the model used to calculate the end area had to be that of an ellipse since definite major and minor axis were measured at each end of the tube. Using equation 3.40, the uncertainty in the volume was calculated. The semi-major axis is given by "a" and the semi-minor axis is given by "b".

$$
\frac{\delta V_{0}}{V_{0}}=\left\{\left(\frac{\delta a}{a}\right)^{2}+\left(\frac{\delta b}{b}\right)^{2}+\left(\frac{\delta L}{L}\right)^{2}\right\}^{1 / 2}
$$

Equation 3.40

The data used to calculate the experimental uncertainty in the volume of the small tube was measured exclusively with the Peacock gauge calipers and is shown below.

The average length of the small tube $=2.337 \mathrm{E}-1 \mathrm{M}$
The standard deviation in length $=5.63$ E-5 M, based upon twelve measurements made using the Peacock calipers.

The average semi -major axis of the end $=1.259 \mathrm{E}-2 \mathrm{M}$, based upon twelve measurements made using the Peacock calipers.

The standard deviation in the semi-major axis $=4.85 \mathrm{E}-5 \mathrm{M}$
The average semi-minor axis of the end $=1.254 \mathrm{E}-2 \mathrm{M}$, based upon fourteen measurements made using the Peacock calipers.

The standard deviation in the semi-minor axis = 5.18E-5M, based upon fourteen measurements made using the Peacock calipers.

Using the above values, the uncertainty found for the small tube is calculated using equation 3.40. As expected, the relative error was greater with the smaller volume and the uncertainty in the volume measured for the small tube (including the systematic error involved in ignoring thermal expansion) was calculated as ~ 1569 ppm.
e. Measuring Temperature and Relative Humidity

Since both the temperature and the relative humidity found within the lab directly affected the ratio of specific heats as shown by equation 3.31 and the temperature change affected the thermal expansion of the brass cylinder, these experimental parameters were also measured.

The measure of temperature occurred under program control before and after each basic 26 point data set was obtained. The two values were averaged and this value was stored with the acoustic data. Since the two temperature samples were obtained symmetric in time around the sample obtained for the center frequency of the modal resonance, the average value obtained is the estimate of temperature associated with that modal resonance. The equipment used to automate this measurement of temperature was a HP-3456A digital voltmeter sampling the output of HP0837-1064 thermistor as directed by the HP-85 computer. The equipment could easily track relative temperature changes on the order of . 001 degree centigrade. The useful temperature range for this setup was -80 to 130 degrees centigrade, well beyond the normal range of 19 to 22 degrees centigrade found in the laboratory. The thermistor was sealed and placed into a bath of icewater where its output was observed to be +. 20 degrees centigrade which was then calibrated to the triple point of water (. 01 degrees centigrade at 1 atmosphere) by subtracting . 19 degrees. Thus, the fractional error in
absolute temperature sampled by the छystem 15 estimated to be (+-). O1 degrees absolute or ~ 34 ppm for a standare laboratory absolute temperature of 293 degrees.

The percent relative humidity was sampled only at the beginning of each program run and was assumed to be constant for the duration of each run. A Durotherm relative humidity gauge built by Sussp Co. of West Germany was used to observe the value of relative humidity to ~ $1 \%$. This observed value was then manually input to the computer where it was used for all the calculations. During the course of severai months, the range of relative humidities within the laboratory was observed to vary from 40 to 65 percent relative humidity. An average daily variation of ${ }^{2}(+-$ ) $5 \%$ occurred during those portions of the day that e\%perimentai data was normally obtained. Dver the course of any one program run, the largest change in reiative numidity observed was $4 \%$ with an average change of $1 \%$ when the largest change 15 considered Lising equation jur the fractional error introduced into the open circuit receiving sensitivity by considering the relatıve numidity to be a known but constant value for the duration of the experiment was 28 ppm.
5. A_Summary_of_Experimental_Uncertainty_For_The_Plane Wave_Resonant_Reciprocity_Experiment

In addition to the sources of error previously discussed, two more sources were considered. First, the open circuit receiving sensitivity is directly proportional to the bias voltage used to bias the condenser microphones. The battery power supplies built to bias the condenser microphones provided an approximate D.C. voltage of ~ 120 volts. Since the battery voltage depended greatly upon the temperature, real time sampling of the bias voltage was done with each basic 26 point data set using the HP-3456A digital voltmeter. The specifications of the digital voltmeter claim a measurement accuracy of 40 ppm traceable to the National Bureau of Standards and this is the probable error used for the bias voltage.

Second, since the frequency of longitudinal resonance will vary linearly with the square root of absolute temperature, slight differences in the calibration frequency of the "Nth" mode were observed due to temperature variations within the laboratory from one day to the next. The range of temperatures in the lab was from $\sim 19$ to ${ }^{2} 22$ degrees centigrade which results in an potential 0.5\% maximum shift in the calibration frequency associated with a particular mode. The average slope of the open circuit voltage receiving sensitivity versus frequency for the highest 10 modes was ~ $-.00024 \mathrm{~dB} / \mathrm{Hz}$. For comparison
purposes, this corresponds to roughly a . 007 dE difference in sensitivity over a 28 Hz maximum frequency shift at the 23rd mode. This temperature dependent systematic error is neglected during measurements of experimental precision since this possible shift is roughly an order of magnitude smaller than other observed uncertainties.

A summary of the probable error found in experimental parameters is shown below in table 3.11 . Equation 3.2 has been modified by deleting fin and adding the uncertainty caused by the bias voltage to calculate the overall probable error. The result shows the relative uncertainty in the open circuit voltage receiving sensitivity and 15 given in equation 3.41.

$$
\begin{aligned}
& \left.\left[2 \frac{\delta E_{\text {Bit }_{0}}}{\varepsilon_{\text {bia }}}\right]^{2}+\left[\frac{\delta P_{0}}{P_{0}}\right]^{2}+\left[\frac{\delta \gamma_{E}}{\gamma_{E}}\right]^{2}+\left[\frac{\delta Q_{N}}{Q_{N}}\right]^{2}\right\}^{\frac{1}{2}}
\end{aligned}
$$

probable error（ppm） expected vs．range $3800(1900-5600)$
1．ratio Vca／Vcb \｛scalen． 5$\}$ \｛non－1 inearity\}

2．ratio e1／i1
a．ratio \｛Vdata／Vget\}; 1450 ppm
b．system capacitance； 660 ppm
c．preamplifier gain； 780 ppm
$1800(900-2500)$

玉．atmospheric pressure ．320
4．quality factor，Qn
5．resonant frequency，fn
6．ratio of specific meats
7．Savity volume，long tube
8．cavity volume，small tube
9．bias voltage

180
7
400
1200
（610－1700）
3600
$(1000-5700)$



With these probable errors，the expected unsertality in the＂long tube＂plane wave resonant reciprocity calibration is roughly 2200 ppm or $\sim .02 \mathrm{dE}$ re 1 V／ubar with a range of up to 3300 ppm or $\sim .0 こ$ dB re 1 V／ubar．In the short tuoe， the expected uncertainty $\quad$ in the plane wave resonant reciprocity calibration is roughly 2780 ppm or（rounding up） ～．03 dB re ！V／ubar with a range of up to 42こ0 pom or थ．． 4 do re 1 V／ubar．When the precision of tme prisliminary e\％perlmental results 15 presented and alscussed in chapter

IV, it will be seen that this eッpected e»perimental uncertainty is of the same order of magnitude as the precision observed in the experimental results.

The absolute accuracy of the final calitration will still require corrections to the raw program output to account for:

- the compliance of the microphones,
- the motional impedance of the microphones,
- the non-standard definition of the capacitance of the WEG4OAA microphone. which 1 ncludes the capacitance of the ENC electrical connector extending the microphone cartridge,
- the change of stiffness of the gas within the cavity due to non-adiabatic boundary conditions, and
- the corrections associated with accurate measurements of capacitance for the cables, the bias circuits, and the microphones (opposed to the approximate values used in the computer program).

In the next section, the experimental procedure and associated uncertainty in the comparıson calibration will ve examined.
E. SIGNAL FLOW AND COMPUTER CONTROL FOR THE FREE FIELD COMPARISON CALIBRATION

## 

The intent of this portion of the experiment was to obtain an accurate low frequency free field calibration for the W.E. G40AA condenser microphone as a check on the absolute accuracy of the resonant reciprocity calibration. At very low frequencies, the diffraction correction goes to zero [Ref. 3: p. 33, Fig. A2] and the free field results are useful for comparison with the plane wave resonant reciprocity calibration and the standard pressure coupler calibration. Since the W.E.G40AA condenser microphone had such a low acoustical output in the frequency range of interest, a free field reciprocity calibration for this microphone was not obtained. Instead, sufficient data was obtained to compute a free field reciprocity calibration for an Altec type 688 electrodynamic microphone which was then used to compute a free field comparison calibration for the W. E. 640 AA condenser microphone.
2. Experimental__Considerations_for__a__-_Free__Figld

## Conmarisonncalibration

The free field reciprocity equations developed in chapter one are difficult to use directly with a condenser microphone when used as a speaker at low frequency due to the low acoustic output of a condenser microphone. The Alter type 688 electrodynamic microphone performs well in this frequency range when used as a speaker. Using this feature to advantage, a free field comparison calibration for the W.E. $640 A A$ condenser microphone is more easily obtained.

Starting with equations 1.30 and 1.84 , we have,

$$
\frac{V_{A}}{M_{A}}=\frac{V_{B}}{M_{B}}
$$

and,

$$
M_{A}=\left\{\frac{e 4 V_{A} 2 \lambda r}{i \perp V_{B} \quad e_{0} c}\right\}^{1 / 2}
$$

Let "Ma" refer to the Alter type 688 electrodynamic microphone and "Mb" to the W.E.G40AA condenser microphone. Then, solving equation 1.30 for $M b$ and substituting the solution for Ma into the result, we obtain,

$$
M_{B}=\left\{\begin{array}{llll}
e^{4} & V_{B} & 2 r \\
i 1 & V_{A} & f e_{0}
\end{array}\right\}^{1 / 2}
$$

Equation 2.42

When the standard relationship between density, atmospheric pressure and temperature [Fief. 9: p.40] is substituted into equation 3.42 , we are able to obtain the form of the equation used in the experiment to obtain the free field comparison calibration for the W.E.g40 AA condenser microphone. The equation used in the computer program in appendix: $D$ differs only in the scale factor necessary for the units used.

$$
m_{B}=\left\{\left(\frac{e 4}{i 1}\right)\left(\frac{V_{B}}{V_{A}}\right)\left(\frac{T_{K} r}{P f}\right)\left(\frac{2 P_{0}}{\rho_{0} T_{0}}\right)_{\text {STR }}\right\}^{\frac{1}{2}} \quad \text { Equation } \therefore .48
$$

Here we have, in addition to the definitions :1sed groin chapter 1:

$$
\begin{aligned}
& \text { TK = temperature } 1 n \text { degrees kelvin. } \\
& \text { To }=\text { standard temperature. (27. deg k) } \\
& F \quad=\text { atmospheric pressure. } \\
& \text { Fo }=\text { standard atmospheric pressure. : } 1 \text { latinos: }{ }^{2} 11 \text { ali Fair } \\
& \rho_{0}=\text { density of all at standard pressure and tempersitu. }
\end{aligned}
$$

$$
\begin{aligned}
& r=\text { sepertitan in meter; between source ac micrnftione. }
\end{aligned}
$$

The above calibration was accomplished in two separate steps. First, a separate acoustic source was used to provide the same pressure field within an anechoic chamber for each of the frequencies of linterest for the later comparison calibration. The program that facilitated the measurement of these ratios was called "VRATIO" and 15 listed in appendix C. These comparısons were stored 1 n an array used in a subsequent computer program called "N28" that controlled the translation of the W.E.G4OAA condenser microphone while the 688 electrodynamic microphone performed as a stationary source. Program "N28" is listed in appendı\% D. The second step then consisted of running program "N28" at each frequency of interest. The next sections describe the operation of these two programs.
3. Measurement_of_The_Sensitivity_Ratios_M(64OR)/M(6日gR)

The ratio of the W.E.G40AA open circuit voltage receiving sensitivity to the Altec type 689A open circuit voltage receiving sensitivity \{M(640R)/M(688R)\} was measured using the circuit shown in figure 3.22 in the first part of a two step experiment to obtain the comparison calibration of the W.E.G4OAA condenser microphone. The figure shown below illustrates the the signal flowpath for the electrical equipment involved.


Figure 3.22 Signal_flow_used_in_measuring_브으﹎ㅡㄹ


The requirement that a free field calibration be carried out in a space that is free from surfaces which cause appreciable reflection of sound and free from background noise which may obscure the received signal [Fef J: F.19] was met by using the anechoic chamber located at the Naval Postgraduate School, Monterey, California. It was found to satisfy the dimensional requirements [Ref.3: p.19] for a measurement error of less than $\sim 0.1$ dB. As will later te shown, a variation of calibration sensitivity due to suspected reflections from apparatus $1 n$ the anechoic chamber were observed to be on the order of ~0.03 dB .

Each receiving microphone was mung at the same spat: al location "with a three wire support referer:ced to tme frort face of the microphones about five meters from the sourid source. This was rolighly three and one malf wavelengttis separat: on at the lowest frequency of interest. The nearest surface within the anechoic chamber was roighly two meters tway from the three wire mounting point.

Frogram "UFATIO" worked as follows:

Gtep 1. The first microphone was mounted in posicion in the anechoic chamber and all visable motion was allowed zo subside.

Etep 2. Frogram "VFATIO" was set 1 刀to operation. A separats speaker source $1 s$ thrned on by the computer program. The frogr am samples the received sigmal and averages twenty tivo aata pulnts per frequency of interest. These average salues are stored : zn array labeled $\hat{A}(1, M)$.

Etep 3. When the program has completed sampling the tirst microphone, it pauses and asks the operator to mount the second microphone. When the first microphone is removed and the second mounted, and all swaying stops, the operator indicates that the microphone exchange is complete by pressing CONT.

Step 4. The program samples and averages twenty five data points per frequency of interest and stores the average values 1 m the array $A(2, M)$.

Step 5. The program calculates and prints the ratio $A(1, M) / A(2, M)$ and stores the result in array $R(i)$. The standard deviation for each ratio is printed as "SQR(S)" and the program run is complete.

The results of the first step are given below.




To check the above results, consider the known sensitivity levels for type W.E.640AA and Altec type 688A microphones, shown below.




When the response of the 688 electrodynamic microphone is
divided point by point into the response of the 640 condenser microphone, quantitative agreement with the experimentally determined ratios shown in figure 3.23 is obtained. The values so obtained did result in the expected response for the condenser microphone. The actual free field calibrations obtained for the W.E.640AA serial\#1248 condenser microphone are shown as the larger circles in the W.E.g40AA sensitivity plot)
4. Frees_Figeld_Reciprocity_Measurements

A sketch of the experimental equipment used to make the final free field reciprocity measurements is given in the figure below.


Figure 3.25 Signal_flow_in_the_free_field comparison_calibration_based_upon_free_field_recigrocity.

A description of the operation of this final step in the free field comparison calibration is given below in the description of the operation of the controlling program.

The third computer program given in appendix $D$, was written to control the second half of the free field

```
comparison calibration based upon a reciprocity
calibration. After the initial operator inputs are made,
the data is sampled, analyzed and stored on magnetic tape.
Separate computer runs are required for each frequency of
interest. The operator must remain present during each data
run to respond to interactive computer inputs.
    The initial operator inputs are:
    - insert comparison ratios for each frequency of interest
    into the program. (See Appendix C for the program used
    to obtain these ratios.)
- input the plane wave tube modal number of the resonance frequency desired. This will allow later comparison with the plane wave pressure calibration results after the diffraction effects of the free field calibration are subtracted out.
- input the driving voltage to be used by the synthesizer/ function generator.
- input the 5204 lock in analyzer's sensitivity scale.
- input the 5204 lock in analyzer's time constant.
- measure and enter the starting separation distance (cm) between the source and the receiver.
- measure and enter the 4 - wire current limiting resistor used in the driving circuit. (ohms)
```

The computer program will then perform the following steps;
Step 1. The function generator/synthesizer is set to the proper drive voltage and frequency. The source is turned on and the receiver output is monitored.

Step 2. The program samples the peripheral equipment for atmospheric pressure, ambient temperature, and the receiver bias voltage. These are temporarily stored in computer memory.

Step 3. The counters for the optical revolution counter are initialized. (The resolution is 1000 "counts" per 1 complete turn. One complete turn of the threaded drive shaft moves the receiving microphone about $1 / 8$ of a centimeter.)

Step 4. The driving voltage for the main drive motor is turned on for exactly six seconds and then removed.

Step 5. The system waits fifteen seconds for all transverse motion to damp out. Then the program then takes thirty sequential samples (at intervals of three electronic time constants) of the received voltage. After averaging, the data is temporarily stored.

Step 6. When the eighth drive interval is complete, and the sequential sampling is complete for that interval, the program asks the operator to enter the anechoic chamber and measure and input to the program the separation between source and receiver. This will allow a spot check on the relative error of the measuring technique used by the operator.

Step 7. Return to step 4 and continue until twenty separation distances have been sampled. At the end of twenty intervals, the source is turned off, the bias voltage, ambient temperature, and atmospheric pressure are sampled and averages are obtained with the data obtained in step 2. These averages are then stored on magnetic tape.

Step 8. The program asks the operator to enter the anechoic chamber and measure the final separation and enter it into the computer.

Step 9. The initial and final operator entered distances allow the program to scale the counter registers. Arrays containing scaled values of received voltages and scaled values of the related separation distances are displayed to the operator and stored on magnetic tape.

Step 10. The program then performs a least squares fit of the received data to $\{V(r)=E O /[r+a]\} r$ is the computer measured distance corresponding to $V(r)$, and "a" is correction to the separation distance needed to obtain spherical spreading.

Step 11. The values for EO and "a" obtained by the least squares fit are printed for operator use and stored on magnetic tape.

Step 12. The program prints a plot of $V(r)$ vs $r$ for operator viewing.

Step 13. The program prints a plot of log[V(r)] vs log[r] for operator viewing. Here a straight line indicates the region in the data array where [1/r] spherical spreading occurred.

Step 14. The program calculates, prints and stores on magnetic tape the receiving sensitivity for this particular frequency as a function of range.

Step 15. The program shuts down awaiting a new set of initial operator inputs to go to the next frequency of interest.

The fallowing three figures illustrate the output available to the operator as the above program is run. The data shown below was obtained for the 735 hz comparison calibration for W.E. $640 A A$ serial \#1248. The 4 -wire resistance measurement obtained previously for the calculation of the source current, i1, is output just before the raw data is printed. The parameters, "vo", "a", and the correlation coefficient, "R" refer to the least squares fit to Vo/(r+a) for this data used by the program. "N" is the number associated with a particular distance; "RUN" refers to the distance in cm. travelled since the last measurement; "R(CM)" refers to the separation distance in cm. between source and receiver for a particular measurement; and "VOLTS" refers to output voltage measured across the microphone at a particular distance.

EここA TRAHE，EムGAK FOU

4．UIF：E－FI．UIH．M：$=$
TIME＝ ［IfiTE＝

Eた。

configuration ot microphanes Valle af チ－wire resictance dater＝2コ May 19E4

| H | F：HN | Firro | WL゙LT |
| :---: | :---: | :---: | :---: |
| 1 | 1．5゙こ | 1た．『ゼこ |  |
| － | 1．E\％ | 17，ご心 |  |
| 5 | 1．55\％ | 19．75 |  |
| 4 | 1．554 | 23．－こ |  |
| 5 | 1． $5+$ ¢ | こごごぼ曲 | 1．4ECもSE－E14 |
| $E$ | 1．Ef E | ご気 | 1． $3+31 E-1514$ |
| 7 | 1．E5以 | ご，ごフぼ |  |
| 8 | 1．551 | ご，こご |  |
| C | 1．55 | こも．4气可 | 1．150．30E－［194 |
| 15 | 1．55 | こ以． $5+5$ |  |
| 11 | 1．E5 | こ1． |  |
| 12 | 1．E54 | 33．150 |  |
| 13 | $1.55 \bar{i}$ | こ4．74 |  |
| 14 | 1．55 | 36．こテご | 8．3185E－以ら5 |
| 15 | 1．EEG | 37．¢ 3 ¢ | 8．4E141E－005 |
| 15 | 1．5E1 | ご |  |
| 17 | 1．554 | 4日 ¢5E | 7．64400E－805 |
| 18 | 1． 554 | 4 ¢．5bす |  |
| 19 | 1.547 | 44.055 | ？．0999TE－065 |
| 25 | 1．547 | 45.04 |  |

```
main data
N = \mp@code { n u m b e r ~ D f ~ r u m }
FUN = disst in cin for interval
    SImCE last measu.| Emment
Fi(CM)== total separatjon im
    Cm.
VOLTS = Open ct:t. rFEEjvEd
    voltage
```

Fesullte of 1 Encts squarf error
ancelysis for $V(r)=\forall o f r+a]$

Fesulltes of 1 terst squarf error ancely玉1s foir $v(r)=\forall o f l r+a]$

Figure 3． 26 EGaw＿data＿output＿from＿＂N28＂＂

| \＃DF MEABIREMENTE＝ <br> MEAE EHENFEDO＝ <br> EALE OIET（OH）＝ <br> 25． 92256645 | $\begin{aligned} & 20 \\ & 2 \in .75 \end{aligned}$ |
| :---: | :---: |
|  |  |
| GTAFT OTSTAM＝ | 14．5 |
| EHEIESTCH） | 4E， |
| 5204 SEHS | ． 6 可 1 |
| て1－MUSTE＝ | 12 Ec |
| TVTEL SOUHT＝ | 1ここ184 |
| FFE日 \＆ $\mathrm{H}=$ ） | 7 T |

Eighth interval＂spot＂checl： on total separation．
percent fractional error for measured separation vs． calculated separation at the eighth interval．．．

> 52G4 sens == scale selected on the FAF 5 oot locti-in analvaer
> T17 MvoLTS = driving voltage output from the function generator in millivolts

At this point a log／－10g plot is shown of the data．If the far field is a straight line with slope equal to +1 ， then the data is of． That $i=$ to say，the spreading loss goes as［1／r］（spherical spreading）．If the line varies about a＂straight＂line， then standing waves or some other difficulty is berng encountered and further experimental adjustment in the apparatus 15 required prior to taking data．

Figure 3.27 Quality＿control＿output＿from＿＂N28＂

DE FE 1 UEW2GGUEIHS＝ －4ヨ．
$34.7142 .0510 \mathrm{E}-6 \mathrm{y}$
［IE FE $1!\%$ IJEEROUEIHS＝ －49．E12586T15ふ4

DE RE 1\％．リEC2EजVEIAS＝ －47．1055．545こ24

|  | Here the last mirie calibration measurements are shown with： |
| :---: | :---: |
|  |  |
|  | distance（cin）－sensitivity |
| $-43.121353$ | （at～116 V bias） |
|  | and sensitivity level in |
| OB FE 1U， | ak re 1 V／ubar（at 200 V bias） |


IR RE 1U，リE：
－43 15：ここここ4ア8

ロG RE 1U，リBE2らGUEIHS＝
－42．138きこちごこ
45 万思
 $-49.1351531944$
I $1=$

UQ＝ごちも
$\mathrm{FQ}=$
天ロ
$11=$ the measured transmitting にurent
$\because \mathrm{Mr}_{1}=$
－ロこちここち5ー日46もー
F！UTEE 1U！UEQEGGVEIHS＝

ETHMA
1 ES125ご4ごEE－4 average ratio determmed mith
＜FATIO＝：
ごい1
$\because=$ least square error
reference voltage cistermined earliter．

Figure 3.28 A＿sample＿of＿the＿final＿distanceloversus


At each frequency there were twenty measured values of Mo corresponding to the twenty different measurement distances. The statistics shown at the end were computed using all twenty data points although the last figure only shows the last nine calibrations.

Before the final results can be plotted and interpreted, error analysis and potential corrections need to be considered. This will be done next.

## F. EXPERIMENTAL ERROR FOR THE FREE FIELD COMPARISON CALIBRATION

## 1. Introduction

The equation used for the receiving sensitivity based upon a free field reciprocity calibration is derived in the previous section and is given in equation 3.43 which is reproduced below.
$M_{B}=\left\{\begin{array}{lllll}e 4 & V_{B} & T_{K} & r & \partial \rho_{0} \\ \hline i 1 & V_{A} & P & f & \rho_{0}\end{array} T_{0} .1 / 2 . \quad\right.$ Equation 3.43

The source current, il, is not directly measured but is determined from the voltage drop across a resistor in the speaker circuit and a four wire measurement of the resistance of this resistor. When the ratio $V(d r o p) / R 4$ is substituted for il, we have,

$$
\Pi_{B}=\left\{\frac{(4.303) \text { et } R 4 V_{B} T_{K} \Gamma}{V_{\text {drop }} V_{A} \rho f}\right\}^{/ \alpha} \text { Equation } 3.44
$$

The expected probable error based upon this equation will be somewhat different than the probable error found for plane wave reciprocity.

$$
\begin{aligned}
\frac{\delta M_{B}}{m_{B}} & =\frac{1}{2}\left\{\left(\frac{\delta e \psi}{e \psi}\right)^{2}+\left(\frac{\delta V_{d r o p}}{V_{d r o p}}\right)^{2}+\left(\frac{\delta\left(\frac{V_{B}}{V_{A}}\right)}{\left(\frac{V_{B}}{V_{A}}\right)}\right)^{2}+\left(\frac{\delta E_{b_{i o}}}{E_{b i o}}\right)^{2}\right. \\
& \left.+\left(\frac{\delta T_{K}}{T_{K}}\right)^{2}+\left(\frac{\delta r}{r}\right)^{2}+\left(\frac{\delta \varnothing}{\rho}\right)^{2}+\left(\frac{\delta f}{f}\right)^{2}\right\}^{1 / 2} \text { Equation } 3.45
\end{aligned}
$$

The variables in equation 3.45 are defined as:
et - the received voltage measured with the FAR 5204 lock-in analyzer.

Vorop - the voltage drop across the current limiting resistor found in the driving circuit. Used to measure the driving current. This was measured with an HF-J4.38A digital multimeter.

RA

- The current limiting resistor used in the speaker circus This was measured with a 4-wire resistance measurement using the HF-3456A digital voltmeter.
volva - Comparison voltage ratio measured by the program "URATIO" described previously. Both of these voltages were measured on the FAR 5204 loct:-in analyzer.

T - Temperature in degrees centigrade. The MF-3456f digit. voltmeter was used in conjunction with a thermistor (accessory No. 44414 A ) to sample temperature.
r - The measured separation distance corrected for afoliatia centers.

FO - At.nospheric pressure (mmHG) monitored and averaged during the data run. This parameter was attaint isis y the same experimental setup as was used and described in the previous section for plane wave resonant reciprocity.
$f$ - The frequency (hz) of the source signal. These frequencies were selected as multiples of 245 Hz for ease of comparison with the plane wave resonant reciprocity calibrations.

Ebias - The bias voltage used tor the W.E.buiAA condenser microphone.

These parameters and their calculated probable errors must be measured and included in the calculation for "Mb". The error analysis for the value of the atmospheric pressure, the temperature, and the frequency are the same as previously done in the case of the plane wave resonant calibration. The experimental methods used to measure the remainder of these parameters is presented next.

a. Analysis of The Electrical Circuit Used to Measure e4

The received microphone voltage was measured in the manner shown in figure 3.25. The simplified circuit shown in figure 3.6 still applies to the analysis of the received signal. However, due to the different cables and cable capacitances involved, the values obtained for the transfer function will be slightly different. Equation 3.11 is still used to calculate magnitude of the received voltage. The value "el" has been replaced with "e4" to conform to the notation used in the free field experiment.
$e_{4}=\left(\frac{B}{G}\right)\left(V_{\text {data }}\right)\left\{\left(1+\frac{C T}{C_{m}}\right)^{2}+\left(\frac{1}{\omega R C_{m}}\right)^{2}\right\}^{1 / 2}$
Equation 3.11

Since the $[w * R * C m]$ term in the calculation of es is negligible for the purpose of error analysis, it will be neglected. The error analysis for et is given below.
$\frac{\delta e^{4}}{e_{4}}=\left\{\left[\frac{\delta\left[B V_{\text {clata }}\right]}{B V_{\text {data }}}\right]^{2}+\left[\frac{\delta G}{G}\right]^{2}+\left[\frac{\delta\left(1+\frac{C T}{C_{m}}\right)}{1+\frac{C_{T}}{C m}}\right]^{2}\right\}^{1 / 2}$ Equation 3.46

Since the fractional uncertainty in the capacitance term may be calculated from previous uncertainty analysis (with Ct~255 Pf.), and specifications for the PAR 5204 claim an uncertainty in the $B * V i n$ product to be $\sim_{1 \%}$ (although experimental measurement of consistency between attenuators and amplifiers showed a $0.2 \%$ accuracy), only the uncertainty in the amplifier gain for the Ithaco 1201 preamplifier remains to be be determined prior to calculating the total fractional uncertainty in e4.
b. Determination of Gain Uncertainty for The Ithaco 1201 Preamplifier

The Ithaco preamplifier was used in the anechoic chamber because of its self contained battery power supply. In the anechoic chamber, severe electrical noise was preventing experimental progress until the Ithaco preamplifier was employed. Acoustically, the anechoic chamber was quiet. Electrically, it had 60 cycle interference. The following is a plot of the gain characteristics obtained for the Ithaco preamplifier when operated on AC power as compared to operation with the internal battery.


Figure 3. 29 Measured_gain_for_the_Ithaco
preamelifier.

When the battery was fully charged, the galn appra\%imated that obtained using $A C$ power. After severial day use, a noticeable drop to the extent shown was ooserved. The fractional difference between battery power and $A C$ power is seen to be roughly $0.08 \%$. This 800 ppin fractional change is used as an estimate for the uncertalnty in the battery powered gain.
c. Computation of The Uncertainty in The Measure of es The following are a list of specifications/parameter averages for use in equation 3.46 to estimate the probable error in eq.

- typical value for es ~ 1.0 E-4 volts (obtained from figure 3.26)
- preamplifier gain, G ~ 10
- uncertainty in gain, delta $G$ ~ 800 ppm
- scale factor $x$ volts in product, Bin ~ 6. 1 E-S volts
- total capacitance, Ct ~ 255 pf
- microphone capacitance, Cm ~ 50 pf
- although the PAR 5204 specifications claim an uncertainty in BVin product to be delta Bin ~ $1 \%$ (if the magnitude option is factory installed), experimental measurement of the consistency found between attenuators and amplifiers shows an accuracy of $0.2 \%$.
- uncertainty in Ct obtained from previous section , delta Ct ~. 15 pf
- uncertainty in Cm obtained from the previous section, delta Cm ~. 002 pf

When these values are used to calculate the components of equation 3.46, an estimate for the probable error in et is obtained.

$$
\frac{\delta e 4}{e_{4}}=\left\{(2000)^{2}+(800)^{2}+(525)^{2}\right\}^{1 / 2} \text { Equation } 3.47
$$

Thus, the total probable error in eq is estimated to be ~ 2220 ppm.

Next, the analytical considerations for the experimental determination of the measure of [Vdrop] will be discussed.
3. Analytical_Considerations__for_The__Measurement_of Vgrop
a. The Measure of Vdrop

The voltage drop across the current limiting resistor was continuously monitored using a HP-3698 digital multimeter included across the circuit. See figure 3.30 below.


Figure 3.30 Meagsuring_the_valtage_drop_[vgrop] acㅡㅁㅡsㅡthe_current_resistor.

The determination of this voltage drop was straightforward and was done for each frequency of
interest. Unfortunately, unacceptable cross talk occurred when any attempt was made to use the same equipment in both sides of the system under computer control. As a result, the lock in analyzer could not be used in the transmitting circuit as it was already employed in the receiving circuit. To avoid the cross-talk problem, the voltage drop was calculated from a linear least square error fit to experimental measurements of [Vdrop $=a * V a s k+b]$ measured at each frequency of interest. The values of "a" and "b" so found are shown in appendix D in lines 4000 to 4070. The uncertainty in the voltage measured as "vdrop" was taken from the equipment specifications as ( 0.29 plus $163 / f r e q$ ) \% and is calculated as ~ 5120 ppm at 735 Hz .
4. Analytical_Considerations_Made_in The Measurement of

RY
The current limiting resistor was specially chosen for its low temperature coefficient over the temperature range expected within the anechoic chamber. It was mounted in a shielded box with permanent electrical connections wired in to facilitate a four-wire resistance measurement. Based upon the equipment specifications of measurement accuracy of (.0045 + 4/resistance)\% for the four wire resistance measurement obtained using the HP-3658 digital voltmeter, the probable error in R4 was calculated to be ~ 651 ppm. The thermal instability of the resistor used over the temperature range of $17<T<22$ degrees centigrade was experimentally measured and the data was fit using the method of least square error. The result is given below where $T$ is measured in degrees centigrade.
$R 4=\left\{\left(5.3282 \times 10^{-4}\right) T+66.658\right\}{ }^{\text {ohms }}{ }_{\text {Equation } 3.48}$

When the range of temperature from 17 to 21 degrees centigrade is used, the probable error due to the neglect of temperature variation in the value of $R 4 \sim 160$ ppm. Thus the total probable error in R4 is ~ 670 ppm.
5. Measuring_The_"Acoustic""_Separation_Distance
a. Introduction

The distance between the faces of the microphones was measured three times during each run of program "N28". The starting distance between the faces of the microphones was the first distance measured and manually input to the computer. A steel tape measure was attached to the source microphone support and could be rotated into position for distance measurements. When the acoustic data was being sampled, the steel tape was positioned behind the source microphone and did not significantly interfere with the acoustic data by introducing additional scattering in the system. The second distance was measured approximately halfway through the computer controlled spatial translation of the W.E. $640 A A$ microphone. The program stopped taking acoustical data and requested the operator to enter the anechoic chamber, rotate the steel tape into position, measure the distance, stow the tape, and seal the anechoic chamber. When this distance was manually entered into the computer, the program continued with the translation of the W.E.640AA and the acoustic sampling. The final distance was measured at the end of each translation run and manually entered into the computer. The spot check of measured distance as shown in figure 3.27 normally indicated a small discrepancy between the computer calculated distance which
was based upon the operator entered initial and final distances, and the measured distance at that point. The results usually were on the order of a few tenths of one percent relative error. An average value of this discrepancy was ~ $0.5 \%$. This value is used as the estimate of the precision in the measurement of "acoustic separation".
b. Determination of Spherical Spreading

A direct plot of $V(r)$ versus $r$ does not yield the desired [1/r] spreading loss. When a correction is added to the measured separation distance, $r$, to obtain the "acoustic" separation distance, $r^{\prime}, \quad a \quad p l o t$ of this "acoustic" separation versus $V(r)$ yields a perfect [1/r'] plot. The correction, "a", to the measured separation distance is obtained by a linear least square error fit applied to the inverse of $V(r)=V o /[r+a]$. Here "r" is the measured separation distance and [r+a] is the "acoustic" separation distance. $V(r)$ is the received open circuit signal voltage. The slope of the least square error fit to a straight line equals [1/Vo] and the intercept equals [a/Vo]. The correction "a" is obtained by dividing the "intercept" by the "slope". If this correction is not made, the fractional error in the free field calibration will be significantly larger. A plot of $V(r)$ versus both the measured separation distance and the acoustic separation distance is seen in the figure below for data measured at 490 Hz .

> | -7.8 |
| :--- |
| $\begin{array}{l}\text { The least square error } \\ \text { Correction to the measured } \\ \text { col } \\ \text { microphone separation is }\end{array}$ |
| minus 1.19 cm . |
| $\mathbf{0}-8.2$ |

Figure 3.31 A_El으_showing_the_correction for


The uncertainty of the correction applied to the measured distance was found to vary approximately ~5.0\% from one run to the next. With a correction magnitude of about ~ 2 cm., this yields an uncertainty in this correction of ~ 0.1 cm. The resolution of the tape measure was estimated to have a systematic error of ~0.05 cm. Finally, for a typical distance of 30 cm., the measured precision of
the separation distance of $\sim 0.5 \%$ yields a calculated uncertainty of $\sim 0.15 \mathrm{~cm}$.

The total probable error for the measure of the separation between microphones is calculated to be the square root of the sum of the squares of the uncertainty in $r$, the uncertainty in the correction, and the estimate of systematic error.

$$
\frac{\delta r^{\prime}}{r^{\prime}}=\frac{\left[(\delta r)^{2}+(\delta a)^{2}+(\delta \text { systematic })^{2}\right]^{1 / 2}}{r^{\prime}}
$$

Equation 3.49

Thus, the total probable error in the measure of the acoustical separation between microphones at a typical separation of 30 cm . was estimated to be ~ 6240 ppm.

The procedure used to measure this voltage ratio has already been discussed in section C.2. As was seen, the precision of the measure of this ratio varied from frequency to frequency. Figure 3.23 is reproduced below for convenience.


Figure 3.32 The_sensitivivty_ratiog_obtained_for


The precision of the different values measured for [Vb/Va] is seen to vary with frequency. When the average of the
different precisions is taken as an estimate of the probable error and the average fractional error is taken to represent the experimental uncertainty, delta [Vb/Va] was calculated as roughly ~ 6300 ppm.
7. A_Summary_of_Exegerimental_Error_for_-The__Free_-Figld

## Comparison_Calibration

The probable error for each measured parameter is summarized in the table shown below.

| parameter | probable error <br> (in ppm) |
| :--- | :---: |
| * frequency | 7 |
| * pressure | 320 |
| * temperature | 34 |
| * Ebias | 60 |
| e4 | 2220 |
| Vdrop | 5120 |
| R4 | 670 |
| r(acoustic) | 6240 |
| Vb/Va | 6300 |
| se probable errors are explained in an earlier chapter |  |
| or analysis for plane wave resonant reciprocity. |  |

* These probable errors are explained in an earlier chapter on error analysis for plane wave resonant reciprocity.

Table 3.12 A_summary_of_probable_error_for_the free_fielg_comearisgn_calibration

The total error in the free field sensitivity calibration is given by the equation shown below. Note that the uncertainty in the bias voltage is included as it was in the previous section for the plane wave resonant reciprocity calibration.

$$
\begin{aligned}
\frac{\delta M_{B}}{M_{B}}= & \frac{1}{2}\left\{\left(\frac{\delta e 4}{e^{4}}\right)^{2}+\left(\frac{\delta V_{d}}{V_{d}}\right)^{2}+\left(\frac{\delta R 4}{R 4}\right)^{2}+\left(\frac{\delta\left(\frac{V_{B}}{V_{A}}\right)}{\frac{V_{B}}{V_{A}}}\right)^{2}+\right. \\
& \left.\left(\frac{\delta T}{T}\right)^{2}+\left(\frac{\delta r^{\prime}}{r^{\prime}}\right)^{2}+\left(\frac{\delta P}{8}\right)^{2}+\left(\frac{\delta f}{f}\right)^{2}+\left(2 \frac{\delta E_{\text {bid }}}{E_{\text {biA }}}\right)\right\}^{1 / 2} \text { Equation } 3.50
\end{aligned}
$$

The total calculated probable error in the free field comparison calibration is taken as one half the square root of the sum of the squares of these individual probable errors and is calculated to be approximately ~ 5,300 ppm. This is on the order of $\sim 0.05 d B$ re 1 volt/ubar when applied to the sensitivity calibration.

However, one additional experimental uncertainty was observed when the free field sensitivities were plotted with respect to the distance, r.


Figure 3.33 The_variation_of_the_49으브르․ microghone_sensitivity_with_separation_distance.

The spatial variation observed in this plot is cyclic over roughly ~ 18 cm. This is not explained by standing waves between the source and receiver nor by reflection from any reflecting surface located along the acoustic axis. This suggests a more complex but unknown interference pattern as the source of this variation. When this observed variation of approximately ~. 035 dB (roughly 4040 ppm), is included with the 5,300 ppm probable error previously obtained, the overall uncertainty in the free field comparison calibration is calculated to be $\sim 6,700$ ppm or .06 dB re 1 volt/ubar.

## IV. _SELF_CONSISTENCY_OF_THE_PLANE_WAVE

## RESONANT_RECIPROCITY_CALIBRATION

## A. INTRODUCTION

Three different "experimental calibrations" of the same
microphone were examined to experimentally obtain the
precision associated with the plane wave resonant
reciprocity calibration provided by the output of the
computer program listed in appendix B.
First, the external electronics package normally
connected to the side "A" microphone was switched with the
external electronics package normally connected to the side
"B" microphone. the precision associated with this
electronics "swap" was then experimentally observed. this
exchange ls lliustrated ln the figure below.

further signal processing with the same signal path

Figure 4.1 An_illustration_of_the_"electronicssswag."

Seconc, two physically different right circular cylindrical plane wave resonant cavities were eacn used to Lailorate the same microphones. Here tne precision associated with the mechanical details of the construction of the Eavities and the physical remountirig of tie microphones was observed. A comparison between these two plane wave resonant cavities ls lllustrated in the figure below. Note that every third harmonic of the iong cavity is matched with she harmonics associated with the short cavity.

## LONG PLANE WAVE

 RESONANT CAVITY> length $=70.12 \mathrm{~cm}$
> inside diameter $=3.44 \mathrm{~cm}$ material $=$ brass
$\mathrm{Fo}=735 \mathrm{nz}$
SHORT
CAVITY

```
length = 23.37 cm
inside diameter = 2.51 cm
material = brass
```

Figure 4.2 The_relative_slㄹes_of_Rlane_wave resonant_cavities_used_1n_comearing_caliorations

Third, the reference microphone was calibrated opposite. reciprocal microphones of significantly aifferent sensitibltaes. In the case of the WEGUGAA microphomes wsea Mere, this difference in sensitivity level was rougnly + df over the frequency range considered.

Finally, whatever the configuration, each ti ne the plane wave resonant reciprocity calibration was calchlated, a 3 : way round robin self consıstency check was cotained ȧ described in chapter two. In this procedure, two fiterent.

involved in every plane wave rosonant reciprocit: calibration. One calibratıon was based bipon the absalub plane wave resonant reciprocity calibration of microprone A and the other calibration was based upon the absolute plane wave resonant reciprocity calibration of micropmone E. The entire "set" is listed below:

```
Ma - plane wave resonant reciprocity calibration of
    mic "A".
Mab - comparison calibration of mic "A" based upon the
                reciprocity calibration of mic "E".
Mb - plane wave resonant reciprocity calibration of
                mic "B".
Mba - comparison calibration of mic "E" based upon the
        reciprocity calibration of mic "A".
Mca - comparıson calibration of mic "C" based upon the
        reciprocity calitration of mic "A".
Mcb - comparison calibration of mic "C" based upen the
        reciprocity calitration of mic "E".
```

Five different electromechanical configurations were mecessary to observe the precision of these eaperimeni ì caibitations. A Jiscussicn Gf these contigurataons and the results obtaibed will be given mét.

## B. THE OBSERVATION OF EXPERIMENTAL PRECISION

The five electromechanical configurations used to determine the experimental precision are shown in the figure below:


Figure 4.3 Electromechanical_configurations_used in_the_elane_wave_resonant_reciprocity_caligbrations.

The "A" or "B" subscript to the WE640AA condenser microphone serial numbers refers to the external electronics configuration of bias box and preamplifier used for that microphone in a particular calibration run. Throughout the calibrations obtained using the above configurations,
microphone serial \#1248 was the "reference" micropticne minse plane wave resonant reciprocity calibrations were ultimately compared with a pressure coupler calıbration obtained for the same microphone at the National Bureau of Standards. In row I. of figure 4.3 above, the WE64OAA microphone serial \#1248 was calibrated in the long tube using side "B" electronics. In row II. of figure 4.3 , the referance microphone was calibrated using side "A" electronias. In row III. of figure 4.3 , the reference microphone 15 paired opposite WEb4OAA serial \#1082 which is approximately four aB less sensitive than the WEG4OAA serial \#815 previcusly used. In rows IV. \& $V$. of figure $4 . J$, the short tube 15 uṣed to pair the reference microphone with the serial \#SiS and serial \#1082 microphones, respectively. A compilation of the "raw calitration" program output for these different configurat:ons is tabulated below.
(All calibration values are for WEG4OAA serial \#:243 in cE re 1 volt/ubar.)
(The frequency of the calibration equals the mode \# tines $245 \mathrm{~F}=$

## Long tube

mode \# Configuration/average \& sigma

Short tube
Configuration/average long\& sigma average short
I. II. I.-II. III.
IV. $V$.

1. $-48.98-48.93-.05-48.89 /-48.93$ .05
2. $-48.93-48.93 \quad .00 \quad-48.93 i-48.93$
?. $-48.87-48.89+.02$
$-48.88 /-48.88-48.87-49.85 /-49.80-.15$
.01
3. $-48.90-48.88-.02$
$-48.88 /-48.89$
4. $-48.8 \geq-48.87+.04-48.86 /-48.85$
5. $-48.88-48.86-.02$
6. $-48.77-48.80+.03$
7. $-48.77-48.74-.05$
8. $-49.63-48.57+.06$
9. $-48.71-48.69-.02$
$11 .-48.60-48.66+.96$
10. -4 E. $6 \mathrm{Z}-48.61-.02$
11. $-48.53-48.58+.05$
12. $-48.58-48.58 \quad .90$
13. $-48.55-48.58+.05$
14. $-48.55-48.56+.05-48.53 /-48.54$
15. $-48.54-48.58+.94-43.551-48.56$
16. $-48.51-49.50+05-4 E .59-45.53-48.51-45.41,-49.45$
17. -48.5?-48.02 +.05 -42.60-45.61
```
20. \(-48.55-48.63+.08-48.59 /-48.59\)
\(21 . *-49.48 *-48.71-.77-48.771-48.74-48.70-48.58 /-48.59-.95\)
.04 .01
22. \(-48.75-48.78+.05-48.85 /-48.79\)
.05
23. \(-48.96-48.95-.01-49.001-48.97\)
.0ふ
note 1: The data for mode \#21 in column I 15 ignored in the statistics for that row as bad data.
note 2: The various configurations of microphones and preamplifier systems (I., II., etc,...) are found in figure 4. . .
```



When the statistics of the above data are determined, the sigma of the average modal calibration level is ${ }^{2} .02 \mathrm{~dB}$ for both the long and short tube data. Additionally, with the e:ception of the data obtained at mode \#9 (220S hz ), the short tube calibrations averaged ~.05 dE greater sensitivit, level when compared to the long tube calibrations. This $1 \equiv$ in the direction expected since the negative correction necessary to account for the finite compliance of the microphone is greater in magnitude ؛as Enuwn ir "rie ont chapter) when the microphone is mounted $1 \pi$ the sinile. piane wave resonant cavity.

Next, the round-rodin precision associated with each
modal calibration 15 discussed.

## C．THE PRECISION FOUND FDR THE FOUND－FROEIN COMPARISON

For each of the configurations shown in figure 4．ה，the six way round robin self consistency check was cotained by comparing the two different calibrations obtained for any one of the three microphones involved．The following table lists the results obtained for the relative error between these two calibrations．The relative error obtained using configurations II，III，and IV 15 presented below fer comparison．As shown in figure 4．3，configurations il and s II used the long tube while configuration IV used the short tube．In each case，the relative error was calculated as，
$\underset{\text { Error }}{\text { Relative }}(\%)=\left(\frac{M_{A}-M_{B}}{M_{A}+M_{B}}\right)(100)$
ここのr: surat: er.

```
Mode # frequency(Ma)
```

```
Mode # frequency(Ma)
```

$$
\because \text { relative er or }
$$

III.

| 245 | -.040 |
| :--- | :--- |
| 490 | +.085 |
| 725 | +.010 |
| 780 | +.007 |
| 1295 | +.011 |
| 1470 | +.004 |
| 1715 | +.083 |
| 1700 | +.008 |
| 2905 | -.000 |

$$
I I
$$

I . .
$+.00:$
＋．006
$+.0:$－－．104
＋． 101
$+\ldots 11$
－．． 9 4
－•・ーシ
－． 1.18
$+.1 \Xi$
$+.61$


| 10 | 2450 | -.009 | +.004 |  |
| :--- | :--- | :--- | :--- | :--- |
| 11 | 2695 | -.144 | -.000 |  |
| 12 | 2940 | -.017 | -.001 | -.070 |
| 13 | 3185 | -.006 | -.001 |  |
| 14 | 3430 | -.016 | -.009 |  |
| 15 | 3675 | -.006 | +.004 | -.023 |
| 16 | 3920 | -.017 | -.005 |  |
| 17 | 4165 | -.003 | +.055 |  |
| 18 | 4410 | -.011 | -.001 | -.022 |
| 19 | 4655 | -.011 | -.001 |  |
| 20 | 4900 | +.060 | +.058 |  |
| 21 | 5145 | -.021 | -.008 | -.010 |
| 22 | 5390 | -.012 | -.009 |  |
| 23 | 5635 | -.009 | -.065 |  |




The worst comparison between Ma and Mab was found for configuration II at mode 11. For this worst case, the fractional error was $0.144 \%$ This corresponds to a calibration difference of $\sim .013 \mathrm{~dB}$ in the sensitivity levels obtained for Ma and Mab. The average value of (Ma-Mab)/(Ma+Mab) was found to be $0.026 \%$ which corresponds to an average calibration difference in sensitivity levels of ~. 002 dB. Thus, the round-robin self consistency check shows an average difference betiveen ila arid Mab of ~.004 dB.

The next examination of experimental precision will deal with that precision found when the external combination of preamplifier and bias boxes on sides "A" and "B" are exchanged,
D. THE PRECISION ASSOCIATED WITH THE SWAP OF SIDE "A" AND SIDE "B" EXTERNAL ELECTRONICS

In table 4.1, a comparison of the results found for configurations I and II will yield an estimate of the experimental uncertainty due to inaccuracies in the electronics systems calibration. From the data so obtained, it is impossible to determine if there was a significant systematic error applied equally to the calibrations of both electronic systems. Any such error will appear in the absolute comparisons which will be given in chapter five. The average difference between configurations I and II was -. 018 dB . The standard deviation of this quantity was .035 dB. This indicates that the experimental limit in the temperature independent calibration of the electronics was reached. Any systematic error introduced as a result of calibration differences existing between the side "A" and side "B" electronics was masked in the standard deviation of Ma(1248) minus Mb(1248).

Next, the precision of the "A" side serial \#1248 microphone calibration is determined as the microphone on the "B" side is changed to one of significantly different sensitivity.
E. THE PRECISION OBTAINED gY FEFLACING THE SIDE "E" RECIFFOCAMICROPHONE WITH ONE HAVING A SIGNIFICANTLY DIFFEFENT SENSITIUITY (4 DB)

When the results obtained with configurations II and II I shown in table 4.1 are compared, the experimental uncertainty due to exchanging the side "B" reciprocal microphone is observed. The average difference here was -. O1 dB with a standard deviation in this difference of of dB. Again the observable systematic error shown by the average difference was less than the statistical uncertaimty in the procedure. This result shows that there was no statistical difference in the calibration of the reference microphone as the sensitivity of the side "E" microphone was changed.

[^0]F. THE PRECISION ASSOCIATED WITH REPLACING THE LONG TUBE WITH THE SHORT TUBE AS THE PLANE WAVE RESONANT CAVITY

When the results obtained with the long tube were compared with the results obtained with the short tube, the average absolute difference in calibration was . OS dB with a standard deviation of .03 dB . In six of the seven frequencies used for comparison of the calibration results (configuration II data vs configuration IV data), the sensitivity obtained using the long tube was less than that obtained using the smaller tube. This relative difference in raw sensitivities is expected as shown by the calculated magnitudes of the impressed pressure correction which are provided in chapter five.

In the following section, a summary of the experimental precision found for the plane wave resonant reciprocity method will be presented and the average uncertainty in the experimental precision will be computed.
G. A SUMMARY OF THE EXFEFIMENTAL PFECISION FOUND FDR THE FLANE WAVE RESONANT RECIFROCITY CALIERATION METHOD

The summary of experimental results is shown in the table below. All calibrations values are in $d E$ re ivolt/ubar and are for W.E. G40AA serial \#1248.

```
all dB re IV/ubar
```

Observation

Electronics swap
1248A/815B minus
$815 \mathrm{~A} / 1248 \mathrm{~B}$
Side "B" exchange of reciprocal microphone 1248A re 815 m minus
1248A re $1082 \mathrm{~B}-.010 \quad .029$
Long tube vs. Short tube 1243A/815B re long minus 1248A/815B re short -.032 .0.8

Round robin self consistency comparisons for configurations II, III, \& IU.
a0s(iMa-Mat)/(Ma+Mab)) .002 .00]


## resgonant_reciprocity_caligrations

From the data summarized in the above table, it is obvious that the comparison between Ma and Mab in the serf consistency chect for all the plane wave reミonant
calibrations is an order of magnitude smaller than the other


#### Abstract

observed uncertainties. This suggests that the repositioning of microphones and equipments in the different configurations may be a major source of the uncertainties. Since systematic differences are expected when comparing the long vs short tube results, the sigma for this difference is not a part of the overall uncertainty. The square root of the sum of the squares of the sigmas for the electronics swap and the exchange of side "B" microphones will be the experimental estimate of the overall precision. This estimate of the experimental precision for the method of plane wave resonant reciprocity is ~. 045 dB .

In the next chapter, corrections that are necessary for the absolute comparison of the plane wave resonant reciprocity calibration to the results of other calibration techniques will be discussed. The microphone used for all these absolute comparisons will be the WE640AA serial \#1248 condenser microphone.


## V. ABSㅇㄴㄴㅌㅌ_ACCURACY_OF_ExpERIMENTAL_RESULTS

## A. INTRODUCTION

With the self consistency of the calibrations using
plane wave resonant reciprocity established in chapter four,
several corrections to the raw computer output must be made
before comparisons can be made between plane wave resonant
reciprocity calibrations and NBS pressure coupler comparison
calibrations. The raw computer output shown below was
obtained using experimental configurations I through $V$
listed in chapter four, figure 4.3 . These are plane wave
resonant reciprocity calibrations for the w.E. $640 A A$ serial
\# 1248 condenser microphone.


Figure 5.1 Raw_data_from_both_the_long_[star]_and
short_[boge]_resonant_tubes_for_Rlane_wave_resonant_recierocity


To obtain the final values for the absolute plane wave resonant reciprocity calibration, four corrections must be made to the "raw" program calculation of Mo. The first two are general corrections to the "ideal" resonant reciprocity calculations incorporating equation 3.1 into the computer program. The third results from the experimental procedure employed. The forth has three parts and corrects for
deliberate errors introduced into the program for experimental convenience. All the corrections are summarized below.

1. The_iㅉำressed_pressure_correction. The correction due to the finite compliance of the microphone. (~.001 to ~. 14 dB )
2. The_correction_to-the_rationof_specifiçhegts.

The correction to the bulk modulus of elasticity for the air volume within the cavity due to the non-adiabatic boundary conditions at the walls of the cylindrical brass cavity expressed as a change in the ratio of specific heats. In this case, a correction of $\sim .007 \mathrm{~dB}$ is needed at all frequencies to correct for the inaccurate calculation of this correction that was used in the computer program.
3. The_driving_point_electrical_ignedance_correction. The driving point impedance correction for the Thevenin equivalent circuit used to represent the condenser microphone. (~ negligible)

a. A standard electrical definition for the microphone must be used prior to comparison with other calibration results. It is therefore necessary to subtract out the capacitance of the BNC electrical connection used as part of the microphone during this experiment. (~2.8pF) A correction to Mo due to this change in microphone capacitance will result. (~. 16 to ${ }^{\sim} .36 \mathrm{~dB}$ including the correction described next in part 4b.)
b. For experimental convenience, the values of the bias box and cable capacitances were fixed early in the writing of the operational computer program and are listed in lines 670-760 of the program in appendix B. Subsequent, more accurate measurements of these capacitances require a correction to Mo. (~. 16 to $\sim .36 \mathrm{~dB}$ including the correction described above in part 4a.)
c. The drop in signal voltage across the D.C. blocking capacitor in the bias box of the microphone preamplifier signal path was temporarily ignored in
chapter three. As shown in Appendix E, this loss is approximately independent of frequency over the range of frequencies employed in this experiment. The correction required for Mo to account for this "loss" in signal is roughly $\sim+0.03 \mathrm{~dB}$.

The effect of the compliance of the microphone on the plane wave resonant reciprocity open circuit voltage receiving sensitivity calibration will be discussed first.

## B. CALCULATION OF THE IMPRESSED PRESSURE CORRECTION

Using the standard method of pressure coupler reciprocity calibration [Ref. 3], the equivalent volume correction is required whenever the equivalent volume of the pair of microphones within the cavity exceeds 0.4 percent of the physical volume within the coupler [Ref. 3: p.15]. This accounts for the effect of the acoustic impedance of the microphone on the pressure coupler microphone calibration. This correction requires the accurate determination of the equivalent volume of the icrophone under calibration or the functional equivalent, its acoustic impedance [Ref. 3: p.81. The method of plane wave resonant reciprocity calibration requires the determination of the acoustic impedance for any microphone under calibration regardless of the volume of the cavity.

In part $C$ of chapter two, the correction required to account for the finite mechanical impedance of a microphone was derived. This impressed pressure correction accomplishes the same task for plane wave resonant reciprocity as the equivalent volume correction accomplishes in pressure coupler reciprocity. A systematic error in the plane wave resonant reciprocity calibration of condenser microphones on the order of $\sim 0.001$ to . 14 dB re $1 \mathrm{~d} / \mathrm{ubar}$ may result if the impressed pressure correction is ignored.

The correction is shown in equation 2.46 and the acoustic driving point impedance is given in equation 2.49. Both equations are reproduced below.
$M_{A}=M_{10}\left\{1-\frac{1}{Z 22 J}\right\}^{\frac{1}{2}}$
Equation 2.46


The acoustic impedances, Za(mic A) and Za(mic B), shown in equation 2.49 must be exactly measured if the correction is to be accurately determined. In the absence of an accurate determination of these acoustic impedances, an average value for the acoustic impedance of a WE640AA laboratory standard microphone was obtained from the ANSI standard method for the calibration of microphones [Ref. 3]. This value was used for both Za(mic A) and Za(mic B) in the determination of the magnitude of this correction at one frequency when it was illustrated in chapter two. While this will allow an approximation to be made in calculating the impressed pressure correction, no insight is obtained with regard to the possible range of corrections that result from extreme
samples (due to different values of $Z a)$ of the W.E.g40AA microphone population. The "equivalent volumes" of two W.E. 640 AA microphones that represent such extremes and which will provide such insight are provided below in table 5.1 [Ref. 29]. The term "equivalent volume" is often used in pressure coupler calibrations as a matter of convenience. It is simply another way to express the acoustic impedance of the microphone in a manner which simplifies pressure coupler reciprocity calculations. As such, it will generally have both a real and an imaginary part representing the dissipative and reactive portions of the microphone impedance. The exact relationship [Ref. 3: p.8] between the "equivalent volume" and "Za" must be used to derive the final impressed pressure correction as it is shown in equation 5.2.

Equivalent volumes of two condenser microphones [Ref. 29].

W.E. G40AA Serial\# 646

W.E.g40AA Serial\# 151

- all units MKS, M^3 -

| Frequency ( Hz ) | Re. volume | Im. volume | Re. volume | Im. volume |
| :---: | :---: | :---: | :---: | :---: |
| \{multiplied bys | \{E-7\} | \{E-9\} | \{E-7\} | \{E-9\} |
| 50.0 | 1.2207 | -. 93636 | . 43979 | -. 24779 |
| 100.0 | 1.2207 | -1.8728 | . 43976 | -. 49557 |
| 200.0 | 1.2203 | -3.7458 | . 43965 | -. 99101 |
| 300.0 | 1.2196 | -5.6195 | . 43946 | -1.4862 |
| 400.0 | 1.2187 | -7.4940 | . 43919 | -1.9809 |
| 500.0 | 1.2176 | -9.3697 | . 43885 | -2.4751 |
| 600.0 | 1.2162 | -11.247 | . 43843 | -2.9687 |
| 700.0 | 1.2145 | -13.125 | . 43794 | -3.4614 |
| 800.0 | 1.2125 | -15.006 | . 43737 | -3.9532 |
| 900.0 | 1.2103 | -16.889 | . 43673 | -4.4439 |
| 1000.0 | 1.2078 | -18.774 | . 43601 | -4.9333 |
| 1500.0 | 1.1909 | -28.234 | . 43131 | -7.3576 |
| 2000.0 | 1.1660 | -37.751 | . 42477 | -9.7314 |
| 2500.0 | 1.1321 | -47.284 | . 41646 | -12.038 |
| 3000.0 | 1.0878 | -56.746 | . 40645 | -14.262 |
| 4000.0 | . 96365 | -74.797 | . 38168 | -18.400 |
| 5000.0 | . 78900 | -89.957 | . 35134 | -22.034 |
| 6000.0 | . 57288 | -99.723 | . 31659 | -25.073 |
| 7000.0 | . 34154 | -102.28 | . 27874 | -27.454 |
| 8000.0 | . 12923 | -97.767 | . 23925 | -29.148 |
| 9000.0 | -. 038557 | -88.254 | . 19954 | -30.166 |
| 10000.0 | -. 15406 | -76.476 | . 16094 | -30.550 |
| 11000.0 | -. 22358 | -64.609 | . 12456 | -30.373 |
| 12000.0 | -. 25897 | -53.866 | . 091201 | -29.726 |
| 13000.0 | -. 27171 | -44.698 | . 061398 | -28.709 |
| 14000.0 | -. 27059 | -37.116 | . 035386 | -27.421 |
| 15000.0 | -. 26157 | -30.940 | . 013165 | -25.952 |
| 16000.0 | -. 24846 | -25.938 | -. 0054385 | -24.381 |
| 17000.0 | -. 23356 | -21.866 | -. 020718 | -22.770 |
| 18000.0 | -. 21825 | -18.529 | -. 033032 | -21.167 |
| 19000.0 | -. 20327 | -15.902 | -. 042759 | -19.609 |
| 20000.0 | -. 18903 | -13.691 | -. 050276 | -18.119 |

* The above values have only ~ three significant digits.[Ref. 29]

Table 5.1 Equivalent volumes_for_extreme_w.E.g40AA
laboratory_standard_condenser microphones

In addition to these values, the measured acoustical impedances for four particular W.E. 640AA laboratory standard type "L" microphones were obtained from the literature [Ref. 301.
W. E. G40AA

Serial\#

1087
1121
1134
0904
design
parameters
[Ref. 30]

Acoustic parameter
Stiffness (NM^-5)
Mass
(kgM^-4)
Resistance (NsecM^-5)
$1.95 \mathrm{E}+12 \quad 584$

1. $88 \mathrm{E}+12$

475
473
434
$1.2 \mathrm{E}+12$
420
3.80E+7
3. $63 \mathrm{E}+7$
3.12E+7
3.16E+7
$2.63 E+7$


## 

When equation 2.24 and equation 2.49 are combined, two analytical forms of the impressed pressure correction for plane wave resonant reciprocity may be computed. Choice between the two equations shown below is strictly a matter of convenience. The first expresses the correction using the acoustic impedance, and the second uses the equivalent volume of the microphone. In each case, it is assumed that identical microphones are mounted in the ends of the plane wave resonant cavity.

or,

where we define:
Ne $=$ equivalent volume; [ $\left.Y_{E P} P_{0}\right] /[j w Z a]$
$\gamma_{E}=$ ratio of specific heats
$f_{n}=$ frequency of Nth modal resonance
Ia $=$ acoustic driving-point impedance
of the microphone
Rn = quality factor of Nth modal resonance
Vo $=$ volume of plane wave resonant cavity
At $=$ area of tube cross section
Ad = area of microphone diaphragm

Provided the resonant frequencies and modal quality factors are available, the correction factor can be calculated for the case of plane wave resonant reciprocity calibrations.

Using equation 5.2, the range of the impressed pressure corrections can be calculated using experimental data for the quality factor of the modal resonance while the data provided by the National Bureau of Standards is used for the
equivalent volumes of the two extreme samples of the W.E.b40AA population. The atmospheric pressure is also provided to facilitate use of equation 5.1.

All values MKS units; Vo(short) ~ 1.159E-4, Vo(long) ~ 6.504E-4, Ad/At(short) ~.5435, Ad/At(long) ~. 2903

| freq | ( Hz ) : | * ${ }_{\text {* }} \mathrm{long}$ | tube Qn |  | $\begin{aligned} & \text { ** } \\ & \text { Po } \end{aligned}$ | Qn |  | $\begin{aligned} & \text { real } \\ & (E-7) \end{aligned}$ | \#646 <br> imag <br> (E-9 |  | $\begin{aligned} & \text { real } \\ & (E-7) \end{aligned}$ | $\begin{aligned} & \text { imag } \\ & (E-Q) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 735 | ; | 100007 | 97.5 | ; | 99843 | 42.4 | ; | 1.21 | $-13.8$ |  | . 438 | -3.6 |
| 1470 | , | 100027 | 116.8 | ; | 99838 | 76.0 | ! | 1.19 | -27.7 |  | . 432 | -7.2 |
| 2205 | ; | 100081 | 153.0 | ; | 99862 | 103.0 | ; | 1.15 | -41.7 |  | . 421 | $-10.6$ |
| 2940 | ' | 100106 | 193.7 | ; | 99872 | 115.1 | ; | 1.09 | -55.6 |  | . 408 | 14 |
| 3675 |  | 100076 | 218.7 | : | 99886 | 117.0 | ; | 1.00 | -68.9 |  | . 390 | -17 |
| 4410 |  | 100082 | 229.8 | ; | 99898 | 116.1 | ; | . 892 | -81.0 |  | . 369 | -1 |
| 5145 |  | 100039 | 239.5 | ; | 99910 | 114.0 | ; | . 758 | -91.4 |  | . 346 | -22 |

* 2 May data tape, records 26-46 (see appendix G)
** 22 Nov data tape, records 24-33 (see appendix G)
long tube $d B$ correction
short tube dB correction
freq ( Hz ): Ref \#646 Ref \#151 spread: Ref \#646 Ref \#151 spread

| 735 | -.0013 | -.0003 | .001 | -.0053 | -.0015 | .0039 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1470 | -.0031 | -.0008 | .0023 | -.0195 | -.0053 | .0142 |
| 2205 | -.0061 | -.0016 | .0045 | -.0406 | -.0108 | .0298 |
| 2940 | -.0104 | -.0026 | .0078 | -.0622 | -.0159 | .0463 |
| 3675 | -.0146 | -.0036 | .0110 | -.0802 | -.0199 | .0603 |
| 4410 | -.0180 | -.0044 | .0136 | -.0951 | -.0232 | .0719 |
| 5145 | -.0212 | -.0052 | .0160 | -.1068 | -.0259 | .0809 |

Table 5.3 Tabulated_values_of the_imeressed
eressure_correction_obtained_using_eguivalent_volumes_of_extreme Samples_offthe_w. corrections.


Figure 5.2 Calculated_values_of the_imeressed pressure_correction_apel icable_to the_"logn"_tube_plane_wave resonant_recigrocity_calibrations_[3_sigma_s_oog3_dB]

The above corrections are calculated using an average value of Za obtained from table 5.2, and experimental data for the other parameters in equation 5.1. When the same proceedure is used for corrections to Mo found using the "small" cavity, both the magnitude and the range of the corrections are slightly larger.

 pressure_correction_apglicable_to the_"short"__tube_plane_wave resonant_reciprocity_calibrationュ_[3]_sigma_shown]

The impressed pressure correction for the short tube (figure 5.3) is larger than that obtained for the long tube (figure 5.2). The impact of this difference on the absolute calibration results will be discussed in the next chapter.

The next section will discuss the correction to the bulk modulus of elasticity for the air within the resonant cavity due to the non-adiabatic boundary conditions.

# C. THE CORRECTION TO THE BULK MODULUS OF ELASTICITY WITHIN THE PLANE WAVE RESONANT CAVITY DUE TO THE NON-ADIABATIC ROUNDARY CONDITIONS 

In chapter three, the form of the adiabatic bulk modulus of elasticity for the air within the resonant cavity was given as the product of the atmospheric pressure and the ratio of specific heats. The general correction to account for the change in stiffness of the volume of air within the resonant cavity due to the non-adiabatic boundary conditions was given in equation 3.37 as a correction to the ratio of specific heats obtained under free field conditions of temperature and humidity. A correction to the sensitivity level that incorporated this correction was given in figure 3.21. In general, to account for effect of the change in stiffness of the volume of air within the resonant cavity due to non-adiabatic boundary conditions, the sensitivity level must be corrected by adding:

$$
\text { CORR }=10 \operatorname{LOG}\left(\frac{\gamma_{H}}{\gamma_{E}}\right)
$$

When the solution for the effective value of gamma is first obtained using the program solution, and next using equation 3.37, the program solution is found to be .007 dB too low
across a percent relative humidity and temperature range of 40\% $<H<65 \%$ and 19 deg C. < T < 21 deg C.

The raw sensitivity levels plotted in figure 5.1 must therefore be increased by $\sim .007 \mathrm{~dB}$ to correct the program results for the fundamentally wrong, but only slightly inaccurate solution for the effective gamma programmed in lines 3770-3810 of the program shown in appendix B.

## D. THE ELECTRICAL DRIVING POINT IMPEDANCE CORRECTION

When the electrical model for the W.E.640AA serial \#1248 condenser microphone was chosen, the effect of the medium upon the motion of the microphone diaphragm and consequently upon the electrical driving point impedance was assumed to be negligible. Thus, neglecting the electrical resistance of the dielectric in the back volume of the microphone, the electrical model chosen was that of a simple capacitance:

$$
Z_{M K} \cong Z_{E B} \cong \frac{1}{j \omega C_{M i C}} .
$$

The validity of such an assumption rests upon two conditions. First, that the length of the tube loading the diaphragm at a plane wave resonance is a multiple of a half wavelength. Second, that the termination at the opposite end of the tube is for all practical measurements, rigid. These conditions illustrate one method used to obtain the blocked electrical impedance (~capacitive) of a condenser microphone. If the termination at the opposite end is absolutely rigid and if the system has low dissipation (high Q), then the motion of the diaphragm is effectively blocked [see equation 2.43] and Zeb can be measured [Ref. 29]. Since
the plane wave resonator was anticipated to operate under these conditions, the assumption seemed reasonable. However, even though the system used had low dissipation (high Q), the termination at the opposite end was a finite impedance resulting from the combination of another W.E.g40AA and a semirigid plastic mount. The desired blocked mechanical conditions provided by an absolutely rigid end were not achieved.

A calculation of the notional impedance of the microphone and an analysis of the required correction (if any) is therefore necessary to examine the validity of the assumption that the experimental termination is essentially rigid.

$$
Z_{E}=Z_{E B}+Z_{\text {MOT }}
$$

where,

$$
\begin{aligned}
\text { Xe } & =\text { electrical driving point impedance. } \\
\text { Zed } & =\text { blocked electrical impedance. } \\
\text { Zmot } & =\text { motional impedance }
\end{aligned}
$$

When equation 2.21 was employed to calculate Ma, the value of el was calculated using equation 3.13 which in turn was based upon figure 3.8 and the simple electrical model given in equation 5.4. Using figure 3.8,

$$
\frac{V_{\text {out }}}{G}=\left[\frac{Z_{T}}{Z_{T}+Z_{E B}}\right] e 1 \text { Equation } 5.6
$$

where,
$G=$ gain of signal preamplifier
$Z t=$ total input impedance at the preamplifier.
Zeb $=$ blocked electrical impedance ( $1 / j w C$ ).

In terms of $Z t$ and $Z e b$, the program solution for $e 1$ was,


If the actual electrical driving point impedance had been measured and used in the calculation instead of Zeb, the solution for el would have been:

where Re is the electrical driving point impedance and is defined as the ratio of the voltage across the input
terminals of the microphone divided by the input current when the microphone is operating under the load of the plane wave resonant cavity.

The correction to the calculated value of Ma obtained using equation 3.13 which accounts for the effect of the motional impedance upon the computed value of $e 1$ is:


Equation 5.9

When similar considerations are made for the correct calculation of il using le instead of Zebu (using equation 3.14), the correction to Ma becomes:

$$
\underset{(\text { due I1) }}{\operatorname{corR~To~} M_{A}}=\left|\frac{Z_{e}}{Z_{E B}}\right|^{1 / 2}
$$

The product of these two corrections yields the total correction:

$$
\begin{aligned}
& \begin{array}{c}
\text { CORR TO } M_{A} \\
\text { duE } Z_{\text {Mot }}
\end{array}=\left|\left(\frac{Z_{e}}{Z_{E B}}\right)^{1 / 2}\left(\frac{1+\frac{Z_{e}}{Z_{T}}}{1+\frac{Z_{E S}}{Z_{T}}}\right)^{1 / 2}\right|_{\text {Equation } 5.11}^{1} \\
&-289-
\end{aligned}
$$

If $Z e, Z e b$, and $Z t$ were available, this correction could be calculated. However, only $Z e b$ and $Z t$ are available. While this does not allow an exact correction to be computed, a close approximation can be determined.

Equation 5.11 can be rewritten slightly to be of the form:
corr to $M_{A}=$ due Zmor

$$
\left|\left(1+\frac{z_{\text {MOT }}}{z_{E B}}\right)^{1 / 2}\left(1+\frac{\left(\frac{z_{\text {MOT }}}{z_{r}}\right)}{\left(1+\frac{z_{E B}}{z_{r}}\right)}\right)^{1 / 2}\right|=
$$

Equation 5.12

Since the real part of [Zeb/Zt] can be computed and can be shown to be very much greater than 1 , the correction becomes:

Equation 5.13

Beginning with equations 2.36 and dividing $V 1$ by $I 1$, the determination of Xe yields a solution for Zmot. This is the method shown by Hunt [Ref.34:p.96]. The traditional value of Zmot as obtained by Hunt [-b^2/Z22], is seen to be modified
by the square of the impressed pressure correction. When these results are combined with the transduction coefficient previously determined (see the text before equation 2.54), equation 5.13 is of the form:
$C_{\text {ORR }} t_{0} M_{A} \cong 11+j\left(\frac{2 A_{T} C_{0}}{\omega A_{d}}\right)\left(\frac{C_{0} E_{0}}{\epsilon_{0} A_{e}}\right)^{2} \frac{(I M P C)^{2}}{Z_{A}}| |$
Equation 5.14

Here the term IMPC refers to the impressed pressure correction previously derived. Since the largest magnitude of the impressed pressure correction is $\sim 0.98$, it is the magnitude of other terms found in the transduction coefficient and the acoustic impedance which will determine the magnitude of this correction. When the phase of the impressed pressure correction is ignored, the remaining terms yield the value of the maximum possible correction due to Zmot. This worst case magnitude is approximated by substitution of the following values into equation 5.14:

$$
\begin{aligned}
& \mathrm{Eo} \sim 117 \mathrm{volts} \\
& \mathrm{Co} \sim 49.14 \mathrm{pf} \\
& \mathrm{At} / \mathrm{Ad} \sim 3.44 \\
& \mathrm{Ae} \sim 1.29 \mathrm{E}-4 \mathrm{M} \mathrm{M}^{2} \\
& \mathrm{Ra} \sim 3.43 \mathrm{E}+7 \mathrm{NSM}-5 \\
& \mathrm{Ma} \sim 492 \mathrm{Kg} \mathrm{M}^{\wedge}-4 \\
& \mathrm{Ka} \sim 1.76 \mathrm{E}+12 \mathrm{NM} \wedge-5
\end{aligned}
$$

A straightforward computation shows this correction to be much less than . 001 dB and therefore negligible.

## E. THE CORRECTION DUE TO REVISED VALUES OF MICROPHONE AND TOTAL BIAS SUPPLY CAPACITANCE

At an early stage in the experiment, it was decided that the electrical definition of the "microphone" would for simplicity include the BNC connector that was fabricated for the electrical connection to the W.E.G40AA microphone cartridge. This resulted in the capacitance of the "extender" contributing to a slight increase in the capacitance measured for the "W.E.G40AA microphone" and at the same time contributing to an equal decrease in capacitance measured for the external system capacitance. When this "extender" capacitance is taken into account in the sensitivity calculations, a correction results for the open circuit voltage receiving sensitivity for the microphone.

Additionally, a considerable time passed from the first measurement of microphone capacitance and the final result. Consequently, the values of capacitance used for various capacitance terms in the analytical solutions for "Mo" that were programmed on the computer were "frozen" for computational purposes, knowing full well that a later correction would be necessary. These corrections and the "extender" correction were made using equation 5.15 below. Equation 5.15 is obtained from equation 3.11 and the fact that the open circuit voltage receiving sensitivity is

```
directly proportional to the received signal voltage, e1.
```

$\frac{M_{M} \text { (correct) }}{M_{R} \text { (PRog (mM) }}=\left[\frac{\left(1+\frac{C_{T}^{\prime \prime}}{C_{m^{\prime \prime}}}\right)^{2}+\left(\frac{1}{W R C_{m^{\prime \prime}}^{\prime \prime}}\right)^{2}}{\left(1+\frac{C_{T}}{C_{m}}\right)^{2}+\left(\frac{1}{W R C_{m}}\right)^{2}}\right]_{\text {Equation 5. } 15}^{1 / 2}$

```
Here we define,
    Ct \(=\) program value used for the cable and bias supply
    capacitance. (BTAR:177.28pf, ATBR:176.053pf)
    Ct. = corrected experimental measure of Ct.
    Ct'" = Ct' corrected to include the "extender" capacitance.
        (BTAR: 173.22pf, ATBR: 171.42pf)
    \(\mathrm{Cm}=\) program value used for the microphone capacitance.
        (Serial \#1248: 52.722pf)
    Cm' = corrected experimental measure of Cm.
    Cm" = Cm' corrected to exclude the "extender" capacitance.
        (Serial \#1248: 49.41pf) (\#1248 BNC Connector \(=2.75\) pf)
    \(R \quad=\) parallel combination of the bias blocking resistor and
        the input resistance of the signal preamplifier.
        (BTAR:~50470500hms, ATBR:~4987470ohms)
    \(\omega \quad=2 * \mathrm{pi} * \mathrm{Fn}\)
```

Since there were six experimental combinations of microphone-system pairs, there were six different sets of corrections that were calculated. As shown below, the magnitude of the correction required depended primarily upon the correction to the microphone capacitance.


 드으브므느느므므으

This correction would be unnecessary for experimental setups where the magnitude of the electrical parameters were well established and properly lncluded in the controlling computer program.

The final correction to be considered is a result of the approximation of the circuit shown in figure $3 . \Xi$ by the circuit shown in figure 3.4. This coupling capacitance correction 15 derived in appendix $E$ and will be summarized in the next section.

## F. THE COUPLING CAPACITANCE CORRECTION

When preliminary solutions for the received signal transfer function given by equation 3.11 were made, it was convenient to note that the signal loss due to the blocking capacitor $C$ e in the circuit shown in figure 3.3 was roughly constant across the frequency range of the resonant reciprocity calibration. The correction for the drop in the signal voltage across the blocking capacitor which was temporarily neglected and must now be included. In Appendix $E$, the calculation of the magnitude of the required correction was shown to be roughly constant at ~ +0.03dB for the calibration frequencies used in this experiment. The tabulated corrections are shown in the next table.

Frequency ( Hz ) correction ( dB )
245.039
490.032

735 . 031
980 - 2695 . 030
2940 - 5635 . 029
average $=.030$, sigma $=.002$

Table 5.4 D.․․․_Blocking_capacitor_corrections.

This concludes the descriptions of the corrections necessary to obtain an absolute calibration.
G. THE ABSOLUTE PLANE WAVE RESINANT RECIPROCITY CALIBRATION

## 1. Introduction

When the values obtained by the computer program for the open circuit voltage reciving sensitivity are corrected as indicated in three of the preceding four sections, the calibration is compared with a National Bureau of Standards pressure coupler calibration obtained for the same microphone. The lower six modal calibrations (below ~ 1500 Hz ) obtained using the long tube have an average discrepancy of $\sim .03 \mathrm{~dB}$ when compared with the NBS calibration data. The lower two modal calibrations (below ${ }^{2} 1500 \mathrm{~Hz}$ ) obtained using the short tube have an average discrepancy of ~0.06 dB when compared with the NBS calibration data. Above the frequencies indicated, the difference between NBS and plane wave resonant reciprocity calibrations are seen to increase beyond the experimental uncertainty of the plane wave resonant reciprocity calibrations.
2. Exexerimental__Results_for_-Ine__Plane__Wave_Resonant

Recigrocity_Calibration_Comeared_With_a_NBS_Pressure_Cougler
Cogmearison_Caliquration_obtained_For_The_Same_Mícrophone
The previous corrections are applied in the table shown belor

All dB re 1 V/ubar, all corrections in dB.

|  | 1********* corrections *********** |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| freq: \#1248 | iratio of | i* extender | 1 IMPC | Cc ifinal ${ }^{* *}$ NBS |
| (Hz): prog | ispecific | i \& Ct i | 1 | corri cal |
| output | lheats | ' corr \| | i | , |
| (sigma) | (.03 range) |  |  |  |
| 245 :-48.93(.05) | $1+.007$ | +.32 i | 1-. 0001 | .039:-48.56:-48.55 |
| 490 i-48.93(.00) | ; | +. 27 i | i-.001it | +.032i-48.62i-48.59 |
| 735 :-48.88(.01) | ; | +.26 i | i-.001 | 031:-48.58:-48.58 |
| 980 :-48.89(.01) | ; | i +.26 i | $i-.002 i+$ | 030:-48.60:-48.61 |
| 1225:-48.85(.02) | ; | +.25 i | $i-.0031$ | i-48.56:-48.61 |
| 1470:-48.86(.02) | i | ! | i-.003i | i-48.57:-48.63 |
| 1715:-48.79(.02) | 1 | ' | i-. 0041 | i-48.51:-48.63 |
| 1960:-48.75(.02) | 1 | ! | i-.005i | i-48.46:-48.63 |
| 2205:-48.66(.03) | 1 | ! | i-.007 | i-48. 38 i-48.64 |
| 2450:-48.69(.02) | ; | ; | i-.008i | " i-48.41:-48.65 |
| 2695:-48.63(.03) | 1 | , | i-.010 1 | i-48.35i-48.67 |
| 2940:-48.61(.02) | ; | , | $i-.011 i+$ | +.029:-48.33:-48.67 |
| 3185:-48.56(.03) | ; | ' | i-.013i | i-48. $28:-48.70$ |
| 34301-48.57(.01) | 1 | ' | i-.015i | i-48.30:-48.73 |
| 3675:-48.56(.02) | 1 | , | 1-.016 | i-48.29:-48.76 |
| 3920i-48.54(.02) | 1 | ! | 1-.018: | i-48.27:-48.79 |
| 4165:-48.56(.02) | \| " | ! | i-.020 | i-48.29i-48.84 |
| 4410:-48.53(.03) | 1 | ' | i-.021; | i-48.26i-48.90 |
| 4655:-48.60(.03) | ; | , | i-.023i | i-48.33i-48.96 |
| 4900:-48.59(.04) | - | , | i-.024i | i-48.33i-49.02 |
| 5145:-48.74(.04) | i | ' | i-.026i | i-48.48:-49.10 |
| 5390:-48.79(.05) | 1 | , | i-.028: | " i-48.53i-49.20 |
| 56351-48.97(.03) | ; " | , | i-. 029 i | i-48.71:-49.30 |

* The value of this correction depends upon the configuration used. ** interpolated from original data. See table 5.7.




A similar table showing the corrections applied to the raw program output for the short tube is next.

All dB re 1 V/ubar, all corrections in dB.
 * The value of this correction depends upon the configuration used. ** interpolated from original data. See table 5.7.

## Table 5.6 Absoㅡ느늘_plane_wave_reciprocity




When the results of the short tube and long tube are plotted with the NBS data, the agreement between both plane wave resonant reciprocity tubes is apparent.


Figure 5.5 Plane_wave_resonant recigrocity
calibration_vs_NBS_comparison_eressure_cougler_calibration_of


The triangles plot the NBS pressure coupler comparison calibration, the stars plot the long tube plane wave resonant reciprocity calibration and the diamonds show the short tube resonant reciprocity calibrations. The experimental uncertainties obtained for the data shown in the above figure is tabulated in the next table.

** The certified NBS calibration specified only 3 significant figures. The above data is before the roundoff provided in the formal report and shows the individual comparison calibrations actually obtained as referenced to two separate standard


The upper bound is one "experimental sigma" above the average sensitivity shown in the center column for both the long and short data sets. The lower bound is one "experimental sigma" below the same average. For each data set at each different calibration frequency, the sigma used is the square root of the sum of the squares of the experimental sigma found for Ma. The long tube results agree with the NBS calibration to within $\sim .03 \mathrm{~dB}$ up to 1470 Hz and the short tube results agree with the NBS calibration to within ~o. OG dB up to 1470 Hz . Above this frequency, the resonant reciprocity calibrations disagree with the NBS comparison calibration beyond the experimental uncertainty. However, both resonant reciprocity tubes provide consistant results throughout the range of frequencies used in the experiment.

Since the diaphragm area of the microphone occupies a significant portion of the cross sectional area of the short tube ( $\sim 54 \%$ as compared to $\sim 29 \%$ in the long tube), any deviation from the assumption that Za(mic A) equals Za(mic B) will adversely effect the accuracy of the impressed pressure correction in the short tube to a greater extent than the corresponding correction for the impressed pressure
in the long tube. Additionally, the fractional error in the volume of the short tube is greater than that obtained for the long tube (the tube volume is used in calculating "Jo" in the impressed pressure correction). Both of these contributions to increased absolute uncertainty apply for the short tube calibration. For these reasons, the microphone calibrations obtained using the long tube geometry are believed to be the more accurate of the two.

## H. THE CORRECTED FREE FIELD COMPARISON CALIBRATION

There were only two corrections to the free field comparison results given by program "N28". The first is calculated using equation 5.15 with a change of value for $C t$ and Ct." to 255.0 pF . When this is done the correction is essentially the same for all the frequencies sampled and is equal to +0.54 dB . The second correction is for the diffraction of the microphone and mount and is given in appendix $A$ to reference 3 . With both of the corrections applied, the diffraction corrected (pentagon plot) free field (star plot) comparison calibration is superimposed upon the NBS pressure coupler (triangle plot) calibration. In addition, the uncorrected free field results are shown with Professor Medwin's calibration results in figure 3.24. The corrected results are shown on the next page.


Figure 5.6 The_corrected_free_field_comparison


Since the diffraction correction obtained from data in reference $A$ is accurate only for a standard microphone mount and the mount used in this experiment was not standard, the diffraction correction so obtained was only approximate. This is seen in the $\sim 0.5 \mathrm{~dB}$ agreement between the NBS pressure coupler results and the diffraction "corrected" results.

In the next plot, the data is "blown-up" so that the low frequency detail is seen.


Figure 5.7 L으_freguency_detail_in_the_corrected free_field_comparison_calibration.

The data plotted in the previous two plots is given in the table below.

All data in dB re 1 Volt/ubar

NBS data
(pressure coupler) Freq run \#1 run\#2 Hz

Free field data for Serial \#1248
Free Ct Free diffraction diffraction field "raw"

Corr field correction* cal.
corrected calibration
$200-48.55-48.53$
245
$300-48.58-48.54$
$\begin{array}{llll}490 & -48.61 & -48.56 \\ 500 & -48.61\end{array}$
$700-48.61-48.54$
$\begin{array}{llll}735 & & \\ 980 & & \\ 1000 & -48.64 & -48.59 \\ 1470 & & \\ 1500 & -48.66 & -48.60 \\ 2000 & -48.65 & -48.61\end{array}$ 2205
2450
$2500-48.66-48.62$
$\begin{array}{llll}2695 & & & \\ 3000 & -48.69 & -48.66\end{array}$ 3185
3920
4000-48.79-48.80
4655
5000-49.03-49.06

| 5390 | -43.63 | 0.54 | -43.09 | -5.59 | -48.68 |
| :--- | :--- | :--- | :--- | :--- | :--- |

$6000-49.42-49.47$
$7000-49.97-50.01$
$7595 \quad-42.07 \quad 0.54 \quad-41.53 \quad-8.48 \quad-50.01$
$8000-50.55-50.55$
$9000-51.48-51.59$
9800
12250
14700

| -43.06 | 0.54 | -42.52 | -9.46 | -48.52 |
| :--- | :--- | :--- | :--- | :--- |
| -46.55 | 0.54 | -46.01 | -9.23 | -55.24 |
| -50.63 | 0.54 | -50.09 | -8.49 | -58.58 |

* L.R. interpolation to data listed in appendix A of Ref. 3

Table 5.8 NBS_vs_Free_field_data.

Since the anechoic chamber is designed to be reflection-free for plane waves, the normal specific acoustic impedance of the walls is intended to be the "rho-r" product for air. This is not a perfect impedance match to spherical acoustic waves which have a reactive component. The specific acoustic impedance of a spherical wave is given below.

$$
z_{A}=\frac{\rho_{0} C}{1-j / k r}
$$

When the speaker and microphone are "relatively" close to the wall, meaning $k r$ is small, there will be reflections due to the impedance mismatch even in a perfectly anechoic room. These reflections cause an error in the value of Mo. Additionally, the requirement [Ref. 3:p.19] that at low frequencies the acoustic impedances of the surfaces be within $2 r^{\prime} / r$ percent of "rhona" where $r$ " is the minimum separation distance from the surface to the microphone and $r$ is the microphone separation, is not met. The inability to meet these restrictions within the anechoic chamber is seen in the discrepancy in Mo observed at low frequency.
I. A SUMMARY OF PLANE WAVE RESONANT RECIPROCITY CALIBRATIONS

The plane wave resonant reciprocity calibrations shown so far have been those obtained for the W.E.g40AA serial \#1248 condenser mi crophone and were used for comparison with the NBS pressure coupler calibration. Additional calibrations for two more W.E.G40AA condenser microphomes and one small electret "hearing-aid" subminiature transducer were obtained.


Figure 5.8 Plane_wave_resonant_reciprocity
 므tํํ․․․

The absolute experimental uncertainty is essentially the same for all WE640AA microphones, ~ 0.03 dB below 1470 Hz , while above this frequency the difference between the resonant reciprocity calibration and the NBS comparison calibration climbs to a maximum of $\sim .5 \mathrm{~dB}$.

The theory for resonant reciprocity calibrations has been shown to absolutely agree with standard methods of calibration only for the case of plane wave resonance at low frequency. Experimental results that extend calibration frequencies into the region of radial and azimuthal resonances are expected to fail in accuracy. To illustrate, the calibration data plotted on the next page shows the agreement between short and long tube calibrations out to the 23rd modal resonance in the long tube (~5635 Hz ) which is the upper limit for plane wave resonances in that tube. The effect on the calibration due to the interference of radial and azimuthal modes with plane wave modes is apparent above the 23 rid mode. Here the discrepancy between short and long tube calibration results is seen to sharply increase.


Figure 5.9 The_effect_of_radial_and_azimuthal


These calibrations are listed as approximate since they are the "raw" program output.

The type BT-1751 Knowles subminiature transducer was used as the comparison microphone and the following comparison calibration was obtained:


Figure 5.10 Comparison_calibration_for_the_tyee


The sigma of the experimental precision is smaller than the size of the plotting symbols in this calibration. When this calibration is compared with the manufacturer specifications for this device, the sensitivity appears to be about $\sim 1.5 \mathrm{~dB}$ too low. However, the variations in the D.C. supply voltage for the Type BT-1751 preamplifier were not monitored. From Knowles technical bulletins, a change of just a few tenths of a volt (1.25 volts to 1.0 volts) can cause roughly a ~1.0
dB drop in sensitivity. Since the battery used to power the BT-1751 D.C. supply was not replaced during that portion of the experiment (~12 months), such a drop is probable.

 the_Type_1751_and_Type_1757_Knowles_subminiature_transducers.

This concludes the free field comparison and the plane wave resonant reciprocity calitration experiments.

## VI. CONCLUSION

## A. SUMMARY

In chapter $I$, the general subject of acoustic reciprocity calibrations of electroacoustic transducers is reviewed and the effect of the medium and boundary conditions surrounding the transducer are considered for three different experimental environments. Solutions for the acoustic transfer admittance for a pressure coupler calibration, a free field calibration, and a plane wave resonant reciprocity calibration are shown to differ only by a multiplicative constant. Derivations of the open circuit voltage receiving sensitivities are provided for all three techniques. In chapter II, considerations are made regarding experimental techniques used to implement plane wave resonant reciprocity calibrations. The resulting reciprocity equations provide a six way round robin check on experimental precision (plus or minus. . 002 dB ) and the number of experimental parameters required for the calibration is reduced by one. The derivation of a plane wave resonant reciprocity calibration is extended to include necessary conditions relating the acoustic impedance of the reciprocal microphone, the frequency, the speed of sound
within the gas, the gas density, and the dimensions of the plane wave resonant cavity. An impressed pressure correction is derived from the canonical equations for an electroacoustic transducer by making the reasonable assumption that the diaphragm of a condenser microphone does "move" under the influence of an acoustic pressure. This consideration of the finite impedance of the microphone results in a correction to the theoretical sensitivity of an ideal microphone and is shown to depend upon the ratio of the driving point acoustic admittance of the microphone to the transfer acoustic admittance of the medium. In chapter III, the improvement in experimental precision due to computer control of the data acquisition is measured and the probable errors associated with experimental measurements and calculations are determined. In chapter IV the self consistency of the plane wave resonant reciprocity calibrations are obtained and the overall experimental precision is found to be ~0.045 dB. In chapter $V$, calculations necessary to determine and correct for the impressed pressure, the effect of the non-adiabatic boundary conditions on the stiffness of the gas within the resonant cavity, circuit analysis approximations, and corrections to values of program constants in the computer program controlling the plane wave resonant reciprocity calibration are obtained. When the raw experimental calibration data is
corrected as outlined above, the absolute plane wave resonant reciprocity calibrations have close experimental agreement below ~ 1500 hertz (with an average absolute discrepancy in the long tube of $\sim .03 \mathrm{~dB}$ and in the short tube of ~. 06 dB$)$ with that portion of a pressure coupler comparison calibration performed in air (and hydrogen) an the same microphone by the National Bureau of Standards. Beyond this frequency, the difference (in dB) between the NBS comparison calibration and the resonant reciprocity calibrations increases linearly with frequency with a linear correlation coefficient of $r=0.991$.

Both the long and the short tube resonant reciprocity calibrations have an average absolute difference of ~. 029 $d B$ (.018 dB sigma) from 735 to 5145 Hz , the common frequency range of both calibrations. Although the average absolute difference in the resonant reciprocity calibrations and the diffraction corrected free field comparison calibrations (from 1470 to 5390 Hz ) is ~0.12 dB (sigma ~.09 dB) and the average absolute difference in the NBS comparison calibration and the diffraction corrected free field comparison calibration is ~ 0.32 dB (sigma ~. 22 dB ), the failure to use experimentally an exact replica of the standard mounting [Ref.3:P.21] for the WE640AA results in an unknown uncertainty for the diffraction correction and hence calls into question the validity of the absolute agreement
(sigma ~. 09 dB ) with the free field results observed from 1470 to 5390 Hz .

In conclusion it can be said that the results of this experiment show that the method of plane wave resonant reciprocity calibration is in absolute agreement with other laboratory standard techniques at low frequency, is under computer control, and has the potential of becoming a fully automated laboratory standard technique of reciprocity calibration for electrostatic transducers.
B. FURTHER EXPERIMENTS AND THEORETICAL INVESTIGATIONS

In the future, the practical convenience of combining under computer control all of the data acquisition necessary to calculate the icrophone's acoustical impedance, the driving point electrical impedance, and the plane wave resonant reciprocity calibration is the logical correction to and extension of this experimental method.

A more detailed experimental and theoretical study of the impressed pressure correction is needed with regard to more compliant electrodynamic microphones (and other transducer types), different gas atmospheres, a wider range of length to diameter ratios, and a wider range of diaphragm diameter to tube diameter ratios for the resonant cavities.

Any further experiment may also extend the upper frequency limit of plane wave resonant reciprocity calibrations by examining the selective employment of "clean" plane wave resonances in the region of azimuthal modal resonances and/or the use of highly symmetric mountings of the microphones on precisely machined cylindrical cavities. This suggestion is a result of two experimental observations. First, several higher frequency "short" tube plane wave resonant reciprocity calibrations appear to be asymptotically approaching the correct pressure calibration. The particular modes (twelve through sixteen)
used for these calibrations are seen in figure 1.6. The plane wave resonant reciprocity calibrations which result from the use of these modes are shown in figure 5.9 and tend towards agreement with the NBS pressure coupler comparison calibrations even though they exist in a frequency domain where azimuthal resonances are expected. Secondly, extended plane wave resonances were observed exclusive of any observable azimuthal or radial modes up to $\sim 50 \mathrm{Khz}$ when highly symmetric electret microphones were used in the early stages of the experiment. In light of the aforementioned experimental indications, these apparently "clean" plane wave resonances may possibly be employed with success.

The application of plane wave resonant reciprocity to a system of transducers coupled through a water filled cylindrical cavity with a "pressure-release" (styrofoam) boundary is another avenue of experimental and theoretical investigation which may be both interesting and significant. A preliminary investigation has already begun [Ref. 35] by applying the method to a "slow wave resonant calibration" where a compliant wall and fluid filled waveguide are used to obtain a low frequency reciprocity calibration in water.

## APPENDIX A

HARMONICITY IN FLANE WAVE RESONANT RECIPROCITY CALIERATION


#### Abstract

When the condenser microphones are mounted in the ends of the brass cylindrical cavity, the acoustical load at the ends is affected by the mechanical impedance of the microphones and the assumption that the microphones are non-compliant must be more carefully examined.

If the microphones were perfectly rigid, the boundary conditions at each end of the cavity require that the gradient of the acoustic pressure be equal to zero. If this is the case, then the resonant frequencies observed for modes greater than the first will simply be multiples of this fundamental resonance (KL $\left.=n^{*} p i\right)$. This appears to be the case when the resonant frequencies are plotted opposite the mode number of the resonance. Since the information plotted below was obtained over a ten hour period, the resonant frequencies were obtalned at different laboratory temperatures over a range of roughly 19 degrees centigrade to 22 degrees centigrade. Since the speed of sound in air at normal atmospheric pressure varies as the square root of the absolute temperature, and since the frequency of a plane progressive sound wave varies as the free space sound speed, the frequency of resonance varies also as the square root of


the absolute temperature within the plane wave resonant cavity. For comparison purposes, the temperature dependence of the different resonant frequencies was removed by referencing all the resonances to 20 degrees centigrade using equation A. 1 shown below.
$f\left(\right.$ Re. $\left.20^{\circ} \mathrm{C}\right)=f($ Re. TI $)\left(\frac{293.16}{273.16+T 1}\right)^{1 / 2}$ Equation A. 1

If this plot of the "corrected" resonant frequency $v s$. mode number were exactly a straight line, then the modal resonances would be perfect multiples of the fundamental resonance and the ends of the tube would be "rigid" (perfect harmonicity). This appears to be the case when figure A.: is "eye integrated".


Figure A. 1 Kegsonant_freguency_re._20_deg_C_vs_Mode
Number

As is seen above, any lack of "narmonicity" or deviation from a straight line is not readily apparent. The terminations in the plane wave resonant cavity appear essentially "rigid" with regard to their influence upon the resonant frequency obtained for the acoustic pressure. If the condenser microphones were absolutely noncompliant, then they would not function as microphones as there would be no change in capacitance with impressed acoustic pressure. Functionally, the microphones cannot be "rigia". To see the magnitude of the deviation from true harmonicity, the data
is normalized by dividing all the individual resonance frequencies by the "apparent" fundamental associated with the highest resonant frequency obtained (simulating the rigid limit), the non-harmonicity at the low resonant modes becomes apparent as a deviation from a value of one provided the scale is properly chosen. See figure A. 2.




With regard to the effect this measured non-harmonicity has upon the value of the receiving sensitivity in the plane wave reciprocity calibratıon, recall equation 2.13.

$$
\frac{p 2^{\prime}}{p 3}=\cos (k L)+j\left[\frac{\rho_{0} c A_{B}}{z_{m b}}\right] \sin (k L)
$$

Equation 2.13

Here we see that at modal resonances, for the compliance of the condenser microphones to be a negligible effect, we require that:

$$
\left|e_{0} c A_{B} \sin (k L)\right| \ll\left|z_{m B}\right|
$$

AND,

$$
\cos (K L) \cong 1
$$

In figure A. 2 , the ratio of the resonant frequency for the lowest mode to the normalized resonant frequency found for the highest mode was calculated to be .9927. This represents an estimate of how close the boundary conditions correspond to a value of $K L$ equals pi for the fundamental mode (worst case). When we express Equation 2.15 in terms of acoustic impedances, we see that,

$$
\frac{\rho_{2}^{\prime}}{\rho_{3}}=\cos (k L)+j\left[\frac{z_{A}(\text { Air })}{z_{A}(\text { END) })}\right] \sin (k L) \quad \text { Equation A. } 3
$$

Since the ratio of the acoustic impedance of the medium (air) to that of the semi-rigid composite ends (microphone plus rigid mount) is much less than one, we have upon substitution for $\operatorname{Cos(kl)}$ and $\operatorname{Sin}(k l)$ in Equation $A .3$,
$\frac{p_{2}}{p 3} \cong \cos [.9927 \pi] \cong-.9997$ Equation A. 4

This justifies the assumption that the microphones are essentially non-compliant with regard to their effect on the pressure distribution within the resonant cavity. Since the ratio of the pressures comes in under the square root in the calculation for the microphone open circuit voltage receiving sensitivity, this assumption will introduce, at the lowest mode, roughly a 20log(sqr[Cos\{.9927*pi\}]) or .001 do re lu/ubar error. Errors introduced due to this assumption are significantly lower at the higher modal resonances as seen in figure A. 2.

## APPENDIX B

## ONE EXAMPLE OF THE COMPUTER PROGRAM USED FOR

## PLANE WAVE RESONANT RECIPROCITY CALIBRATIDNS

## A. INTRODUCTION

The computer program shown on the following pages was written in Hewlett Packard series 80 Basic. It was used on the HP-85 and completed data acquisition and sensitivity calculations for three microphones at one modal resonance roughly every twenty minutes. The most time consuming portion of the program was that part which calculated the three parameter least mean square error fit to a Rayleigh line shape. Actual data acquisition was completed in roughly three minutes.

A careful examination of the particular program shown in part $C$ below shows that it is but one of six almost identical programs used to accommodate the various combinations of preamplifiers and microphones that were experimentally used. The decision to write six "different" programs was made for operational simplicity. Such variables as volumetric protrusion into the main cavity depended upon which two microphones were in place. The
voltage transfer for each "end" of the cylindrical cavity depended upon the amplifier and microphone combination used. Thus slightly different programs were used to accommodate(referencing W.E.640AA serial numbers); 1248-1082, 1248-815, 1082-815, and three more similar microphone setups with the "electronics" reversed. The particular program discussed in this appendix is the one with microphone \#1248 with the "A" side electronics and microphone \#1082 with the "B" side electronics.

The following pages are separated into three different parts for convenience. First, there is a functional description of the program shown in table B. 1. Next, there is the program listing and last there is a list of definitions for the variables used in the program. The functional outline for the program is shown next.

B．FUNCTIONAL DESCRIPTION OF THE FFOGRAM
The main program is divided into eight subsection outlined below．

1．Initial setup．
2．A transmit，$B$ receive data．（ATBR）
3．B transmit，A receive data．（ETAR）
4．A transmit，C receive comparison voltage．
5．B transmit，C receive comparison voltage．
6．Numerical analysis of all data．
7．Calculation of six different values of open circuit voltage receiving sensitivity．
8．Subroutines（17）

These eight functional descriptions are more fully described in table B． 1 below．

Beginnitig line number－Functional description
10 Program initiaiszation anc preliminary equipment setup．
240 Magnetic tape imitialization．
370 Cperator inputs for data run．
630 Dimension arrays and input measured capacitances，load resistances and numerical corrections for tine side meunted microphones．
2010 Input eavity volumes for $d i f f e r e n t$ configurations ans rearrange data．
2170 Begin program run．Switch system to ATBR．
2350 Search for initial drive voltaje for first bata sample．
ここ20 Iritial data sample．
工54 Analyze data for rough $0, F$, ，kA．
2S\％betain receiver bias voltage anc calculate Eine recuirec correction to microphone capaci＝ance．
2630 With the preliminary values of $0, F, \dot{d} \boldsymbol{d}$ ，ootain the first modal data set with sice E rereiving．Obtain rough Q，F，im for this data set．
2370 Switch system to ETAR．
2880 Obtain bias voltage for side A in receive and calculate the required correction to the microphone rapaci sance．
2940 Sample the modal resonance for side A recel．ing．Ot： roligh Q，F，\＆for this nata se：．
Z1SO Goto Favine sutrouti fe for Aibf コata．Store rauned vaiue＝．
Z270 Goto Kavine subroutime for Eififi data．Store ravinel ia．lles．
Z450 Based upon the requested frive voltage，aalrlilate the transint eurrent Lised for the＂A＂milgrophone $1 r$ fitiff．

3670 Switch and obtain the ATCR comparison voltage.
3720 Switch and obtain the BTCR comparison voltage.
3760 Calculate the effective value for specific heats used $=0$ correct the bulk modulus of elasticity for boundary conditions.
3820 Change atmospheric pressure from mmHg to MKS.
3860 Calculations for six way round robin determination of Ma(reciprocity), Mb(reciprocity), Mab(comparison), Mba(comparison), Mca(comparison), ${ }^{2}$ Mcb(comparison)
4020 Store results on magnetic tape and print out results for operator.
4390 Check for end of data runs desired.
4400 Based upon the quality factor for the ( $N+1$ ) th mode, calculate the expected bandwidth of Nth resonance to be used in the next computer controlled calibration.
4530 Subroutine used to ravine the modal data for accurate values of $Q, F, \& A$.
5470 Subroutine used to calculate the mean square error with regard to the Rayleigh line shape.
5580 Subroutine used for initial data collection.
5760 Subroutine used to adjustment the initial drive voltage and control the preliminary data sampling.
6270 Subroutine used to make a raw data computation of $Q, F, \dot{k} A$.
7020 Subroutine used to sample temperature.
7120 Subroutine used to sample atmospheric pressure.
7220 Subroutine used to obtain side A Dias voltage.
7320 Subroutine used to obtain side B blas voltage.
7410 Subroutine used to select system to ATMFR.
7540 Subroutine used to select system to ETAR.
7690 Subroutine used to select system to ATCR.
7810 Subroutine used to select system to ETCF.
7930 Subroutine used to sample voltage vca.
8080 Subroutine used to sample voltage Vco.
8220 Subroutine used to relate preamplifier gain, 5204 巨cale, and capacitive voltage division to the signai voitage va.
8310 Subroutine used to relate preamplifier gain, 3204 saale, and capacitive voltage division to the signai voitage it.
4440 End after all desired modes are sampled.

[^1]
## C. PROGRAM LISTING

```
i) CLEAR ! i248-"A":1082-"B"
20 ! INPUT TIME IF NOT ALREADY
3O OPTION BASE 1
40 L4=0
50 DIM G2(26),G4(26),B1(30),T1(90),T3(90),T4(90),V!(2),T5(2)
.P5(2),K1(2),G9(2),A7(2),A1(2,48)
6 0 \text { DIM G5(2,26),F5(2,26),F7(4),E1(5),A2(2),G1(2,50)}
70 DIM F(26),A(26),F1(26),B(3,3),X(3),C(3),V(2),Q(2),D(90),K
(3),F2(2),G(90),T(3),V2(2,3),A3(26)
80 DISP "SETTIME H*3600+M*60,MDD;...ONLY IF NECESSARY"
90 ! REVISED 28 AFRIL 84
100 ! RECIPROCITY PROG G91A
110 ! PRELIMINARY SETUP
111! 1248-1082 SETUP
120 OUTPUT 717 :"AM".1,"MR"
130 PRINT "PROGRAM G9:A OF 27 APRIL 84"
140 PRINTER IS 701,80
150 IMAGE 2A,2X,7A,X,5A,X,7A,X,7A,X,7A,4X,4A,7X,4A,9X,2A,1X.
6A
:60 PRINT USING i50 ; "N"."Frea HZ".."Tde`C"","OA", "QB","<MA>"
"<MR>"."<MC>",""%"."RECDRD"
i>0 IMAGE 2D,X.DDDDD.DD.X.DD.DDD,X,DDD.DDD.X,DDD.DDD,X.D.DDD
E,X,D.DDDE,X,D.DDDE.X,D.DDD,X,DD
180 DISP "THIS PROG PERFORMS A RECIPROCITY CAL AT DESIG MODA
FREGS AND PUTS DATA ON TAPE"
130 BEEP
2;0 Ci=1 ! CAUITY UOL-1248-1082
220 DISP "INPUT DDMMYY"
230 INPUT T(3)
240! INPUT TAPE STATUS
250 DISP "ENTER i=ERASE.REWIND &FORMAT: 2=TAPE HLREADY READY
&O BEEP
270 INPUT U7
2!30 IF U7=1 THEN 290 E!SE 400
290 DISP "ARE YIU CERTAIN YOU WANT TO ERASE THE TAPE?ว???"
300 DISP '" =YES.2=NO"
310 INOUT U7
320 IF U7=1 THEN 330 ELSE 400
3 3 0 ~ E R A S E T A P E ~ 9 ~ B E E P
340 ! PREPARE TAPE
350 CREATE "DAT1",100.96
360 CREATE "DAT2",i00.96
370 CREATE "DATV",100,.800
3 8 0 ~ C R E A T E ~ " D A T F " . 1 0 1 ] , 8 0 0 ~
390 ! ENTER RUN PARRAMETERS
400 DISP "SHORT DR LONG TUBE7"
410 DISP "SHORT=1.LONG =2"
420 BEEP
4 3 0 ~ I N P U T ~ C ~ ! ~
440 DISP "ENTER STARTING RECORDz'"
450 INPUT L4 ! INIT RECORD NUMBER
45月 DISP "ENTER HIGH MODE F "
```

```
4 7 0 ~ I N P U T ~
480 DISP "ENTER LOW MODE *"
4 9 0 ~ I N P U T ~ L , ~
5 0 0 ~ D I S P ~ " E N T E R ~ T H E ~ L O W ~ F R E Q ~ ( H Z ) ~ F O R ~ H I G H ~ M O D E " '
510 INPUT FI
5 2 0 ~ D I S P ~ " E N T E R ~ T H E ~ H I ~ F R E Q ~ ( H Z ) ~ F O R ~ H I G H ~ M O D E " '
530 INPUT F.2
540 DISP "ENTER 5204 TIME CONSTANT(SEC)"
550 INPUT T1
560 TH-TH*1000
5 7 0 \text { DISP ':ENTER THE } 5 2 0 4 \text { SENSITIVITY IN VOLTS"}
5 8 0 ~ I N P U T ~ B ~
590 M9=15: MAX # OF RAUINES
600 DISP -"ENTER REL HUMIDITY%"
6i0 BEEP
6 2 0 ~ I N P U T ~ R 1
630
640 ! DIMENSION ARRAYS
650 ! DATA ARRAYS
660 ! .........................
670 ! MEASURED CAPACITANCES
675 Ci(1)=52.722 ! PF $1248
690 C2=C1(1)
700 Ci(2)=177.2802 ! PF BTAR SYS
715 C1(3)=53.123 ! PF $1082
720 C3=C1(3)
730 C1(4)=176.0527 ! PF ATBR SYS
740 C1(5)=50.212 ! PF #815
750 C5=C1(5)
760
761 R8=5047050 ! A SIDE R EFF LOAD
7E2 R9=4987470 ! E SIDE R EFF LDAD
7 7 0 \text { ! GEOMETRY CORRECTION}
780 ! FOR SHORT TUBE
790 D(1)=.965115
8 0 0 D ( 2 ) = . 8 6 3 2 1 8 ~
810 D(3)=.702132
820 D(4)=.494203
830 D(5)=.255249
840 D(6)=.00325317
850 D(7)=.243115
860 D(8)=.466004
870 D(9)=.649799
880 D(10)=.782323
890 D(11)=.855722
900 D(12) =. 866966
910 D(13) =.817927
920 D(14)=.715056
930 D(15)=.568673
940 D(16)=.391959
950 D(17)=.179753
960 D(18)=.007256489
970 D(19)=.171237
```

```
980 D(20)=.323466
990 D(21)=.440099
1000 D(22)=.515326
1010 RAD ! CORR FOR SMALL OFFSET
1020 ! GOTD 1070
1030 FOR N=1.TO 22
1040 D (N) =ABS (COS (N*PI*1.46/23.37))
1050 NEXT N
1060 ! LONG TUBE GEO CORR...
1070 G(19)=.99602
1080 G(2)=.984117
1090G(3)=.964383
1100 G(4)-.936999
1110.G(5)=. }302206
1120G(6) =.860311
1130G(7)=.81168049
1:40 G(8)=, 75674208
1150 G(9)=.6959764
1160 G(10)=.6299141
1170 G(11)=.5591307
1180 G(12)=.4824135
1190G(13)=.4058943
1200G(14)=.3247655
1210 G(15)=.2415518
1220G(16)=.1569648
1230G(17)=.0717239
1240G(18)=.01345009
1250G(19)=.09784211
1260G(20)=.18074880877
1270G(21)=.2614853
1280 G(22) =. 3393913
1230 G(23) =.413837
1300 G(24) =.48422887
1310G(25)=.5500143
1320 G(26)=.6106872
1330 G(27)=.66579
1340 G(28)=..7149249
1350G(29)=.7577435
1360G(30)=.7939622
1370G(31)=.8233585
1080G(32)=.8457732
1390G(33)=.8611109
1400 G(34)=..8693409
$410 G(35)=.8704961
1420G(35)=.8646722
14.30G(37)=.852025
1440G(38)=.8327731
1450G(39)=.8071849
1460G(40)=..7755856
1470 G(41)=.7383485
1480G(42)=..6958912
1490 G(43)=.64867i5
i500 G(44)=.5971825
```

```
1510G(45)=.5419471
1520G(46)=.4835125
1530G(47)=.4224453
1540 G(48)=.3593248
1550G(49)=. 2947382
1560G(50)=.2292741
1980 G2=1
1990 A7(1)=.20134
2000 A7 (2) =. 147528
2010 V2(1,1)=.0001152232 ! METERS^3
2020 V2(1,2)=.0001151946 ! V2(C,C1)
2025V2(1,3)=.000 1152269! C C =1:1248-1082
2030 V2(2,1)=.0006498109 ! C1=2;1248-815
2040 V2 (2,2)=.0006497823 ! C1=3;1082-815
2045 V2(2,3)=.0006497779 ! P-III-60
2050 F9-1
2060 I9=1
2070 FOR M=1 TO 50
2080 G1 (2,M)=G(M)
2090 NEXT M
2100 FOR M=1 TO 22
2110 G1(1,M)=D(M)
2120 NEXT M
2130 FOR M=23 TO 50
2140 G1(1,M)=0
2150 NEXT M
2160
2170; BEGIN PROGRAM
2180 PRINTER IS 2
2190 PRINT "TIME DATE"
2200 IMAGE 2X.2D.2D.4X.4D
2210 FOR R=L TO L1 STEP -1
2220 PRINTER IS 2
2230 PRINT USING 2200 ; TIME/3600.DATE
2240 L=R
2250 PRINT "."
2260 IF C=1 THEN UT=3*L ELSE U7=L
2270 DISP "MODE NUMBER IS"".L
2280 DISP "LOW FREQ (HZ) IS",F1
2290 DISP "HI FREQ (HZ) IS".F2
2300 DISP "5204 SCALE SENS(V)IS",B
2310 CLEAR @ BEEP
2320 DISP "SELECT ATBR SWITCHING"
2330 GOSUB 7400! SELECT ATBR
2340 BEEP
2350 DISP "GET INITIAL DRIVE VOLTAGE"
2360 ! VOLTAGE
2370 Ji=0
2380 A1=5
2 3 9 0 ~ ! ~ I N I T I A L ~ S E A R C H ~ F O R ~ D R I V E ~ V O L T A G E ~
2400 OUTPUT 717 ;"FR".(F2+F1)/2,"HZ"
2410 OUTPUT 717 :"AM".A1,"MR"
2420 WAIT T1*3
```

```
2430 OUTPUT 709 ;"UT3"
2440 ENTER 709 : Q9
450 IF Q9>.25 THEN 2470 ELSE Al=Al+25
2460 GOTO 2410
2470 IF Q9<. }35\mathrm{ THEN 2490 ELSE AI=A1-20
2480 GOTO 2410
2490 BEEP
2500 DISP "PRELIM"DATA SAMPLE"
2510 DISP "USED TO EST BANDWIDTH"
2 5 2 0 \text { GOSUB 5570 : INITIAL DATA COLLECTION}
2530 BEEP
2540 DISP "ANALYIZE SAMPLE"
2550 GOSUB 6260 ! DATA ANALYSIS
2560 BEEP
2570 DISP "GET 'B' BIAS"
2580 GOSUB 7310 ! GET E2."B BIAS"
2590 DISP ".'B' BIAS =",E2
2600 !
2610 C1(3)=C3+.00016584+.000024894*E2^2 ! CORR FOR BIAS ON i
0 8 2
2520 BEEP
2630 DISP "GET ATBR DATA".
2640 GOSUB 5750 ! GET DATA ATBR
2650 GOSUB 4450 ! UHF TO NEUTRAL
2660 PRINT "ATBR DRIVE(MV)=",A1
2670 DISP "ATBR AVG TEMP=",TF(L)
2680 DISP "AUG ATMOS PRESS MMHG*".PS(1)
2690 DISP "SAVE & SCALE DATA".
2700 A9=A1
2710 A2=A1 ! SAVE ATBR DRIVE
2 7 2 0 ~ F O R ~ N = 1 ~ T O ~ 2 6
2730 GOSUB 8300 ! GET B2
2740 G5(1,N)=A(N)*B*B2
2750 A(N)=G5(1,N)
2760 NEXT N
2770 BEEP
2790 DISP "ANALYIZE ATBR DATA"
2790 GOSUB 6260! DATA ANALYSIS
2800 ! SAVE ROUGH VALUES
2810: OBTAINED WITH DATA
2820 ! ANALYSIS ROUTINE-1: POINT FIT
2830 V(1)=P1
2240 Q(i)=Q1
2850 F2(1)=F9
2960 BEEP
2870 DISP "SWITCH TO BTAR"
2880 DISP "GET 'A' BIAS"
2890 GOSUB 7530 ! SWITCH BTAR
2900 GOSUB 7210 ! GET E1."A BIAS"
2910 DISP ".A. BIAS=",E1
2920 C1(1)=C2+.0012485+.000036329*E1`? ! CORR FOR BIAS ON 12
```

```
2930 DISP "'GET BTAR DATA"
2 9 4 0 \text { GOSUB 5750 ! DATA COLLECTION}
2950 PRINT "BTAR DRIVE(MU)=",A1
2960 DISP "BTAR AVG TEMP=",T5(2)
2970 DISP "AVG ATMOS PRESS MMHG=",PS(2)
2980 : SAVE AND SCALE DATA
2990 FOR N=1 TO 26
3000 GOSUB 8210 ! GET 81
3010 G5(2,N)-A(N)*B*B1
3020 A(N)=G5(2,N)
3030 NEXT N
3040 GOSUB 6260 ! DATA ANALYSIS
3050 ! SAVE ROUGH VALUES
3050 V(2)=P1
3070 Q(2)=01
3080 F2(2)=F9
3090 ! RECALL ATBR DATA
3100 Q1=G(1)
3110 F9=F2(1)
3120 P1=V(1)
3130 FOR N=1 TO 26
3140 A(N)=G5(1,N)
3150 NEXT N
3160 DISP " RAVINE ATBR DATA"
3170 05=01
3180 GOSUB 4510 ! RAVINE DATA
3190 ! SAVE RAUINED ATBR
3200 ! VALUES
3210 Q(1)=Q1
3220 F2(1)=F9
3230 V(1)=P1
3240 GOSUB 5460 ! GET MSE
3250 ! NORMALIZE MSE
3260 K1(1)=K(1)/P1`2 ! END ATBR
RAUIN
E
3270 ! RECALL BTAR DATA
3280 Q1=Q(2)
3290 F9=F2(2)
3300 P1=V(2)
3310 FOR N=1 TO 26
3320 A(N)=G5(2,N)
3 3 3 0 ~ N E X T ~ N
3340 DISP " RAVINE BTAR DATA"
3350 Q5=Q1
3 3 6 0 ~ G O S U B ~ 4 5 1 0 ~ ! ~ R A V I N E ~ D A T A ~
3370 ! SAVE RAUINED BTAR
3380 ! VALUES
3390 Q(2)=01
3400 F2(2)=F9
3410 U(2)=P1
3420 GOSUB 5460 ! GET MSE
3430 ! NORMALIZE MSE
3440 K1(2)=K(1)/P:`2 ! END BTAR
```

3450
3460
3470
3480
3490
3500 ! ACTUAL DATA,ASK US GET.
3510 IF F2(2)<520 THEN 3515 ELSE 3520 : P-III-80
3515 B6=.92775+.00004979592*F2(2)
3517 GOTO 3550
3520 IF F2(2)<1020 THEN 3525 ELSE }353
3525 B6*.9516+.000004163265*F2(2)
3527 GOTO 3550
3530 IF F2(2)<1510 THEN 3535 ELSE }354
3535 B6=.95412+.000001714286*F2(2)
3537 GOTO 3550
3540 IF F2 (2)<2500 THEN-3545 ELSE }354
3545 B6=.95577+6.122449E-7*F2(2)
3547 GOTO 3550
3549 B6 =.9568022+.000000222449*F2(2)
3550 A2=A2=B6
3640 IT=2*PI*F2(i)*A2*.001*(C2+.00124885+.0000363329*E1^2)*.
000000000001
3650 DISP "A TRANS CURRENT(A) =", II
3660 BEEP
3670 DISP "GET ATCR COMPARISON*
3680 GOSUB 7680 ! SWITCH ATCR
3590 GOSUB }792
3700 DISP "ATCR VOLTAGE=",VI(1)
3710 BEEP
3 7 2 0 ~ D I S P ~ " ' G E T ~ B T C R ~ C O M P A R I S O N " '
3730 GOSUB 7800 ! SWITCH BTCR
3740 GOSUB 8070
3750 DISP "BTCR VOLTAGE=",V1(2)
3760 ! CALC RATIO OF SPECIFIC HEATS }J=1=BTAR,2=ATB

```
```

3770 FOR N=1 TO 2

```
3770 FOR N=1 TO 2
3780 D=R:/100*.625*10*(23.84-2948/(273.16+T5(N))-5.03*LGT(27
3780 D=R:/100*.625*10*(23.84-2948/(273.16+T5(N))-5.03*LGT(27
3.16+TS(N)))
3.16+TS(N)))
3790 G0=(0*1.324+(1000-0)*1.402432)/1000
3790 G0=(0*1.324+(1000-0)*1.402432)/1000
3800G9(N)=G0-2*(GO-1)*A7(C)/SQR(F2(N))+2*(GO-1)*A7(C)^2/F2(
3800G9(N)=G0-2*(GO-1)*A7(C)/SQR(F2(N))+2*(GO-1)*A7(C)^2/F2(
N)
N)
3810 NEXT N
3810 NEXT N
3820 ! CHG PRESS TO MKS
3820 ! CHG PRESS TO MKS
3830 FOR N=1 TO 2
3830 FOR N=1 TO 2
3840 P5 (N)=P5(N)*101330/760
3840 P5 (N)=P5(N)*101330/760
3 8 5 0 ~ N E X T ~ N
3 8 5 0 ~ N E X T ~ N
3 8 6 0 ~ ! ~ O P E N ~ C I R C U I T ~ S E N S I T I V I T Y ~ C A L C U L A T I O N S ~
3 8 6 0 ~ ! ~ O P E N ~ C I R C U I T ~ S E N S I T I V I T Y ~ C A L C U L A T I O N S ~
3870: M1=MA (ADJUSTMENT MADE SO REF BIAS IS 200 VOLT
3870: M1=MA (ADJUSTMENT MADE SO REF BIAS IS 200 VOLT
S)
S)
3880 ! M2=MB C=1=SHORT TUBE
3880 ! M2=MB C=1=SHORT TUBE
3890 ! M3=MCA C=2=LDNG TUBE
3890 ! M3=MCA C=2=LDNG TUBE
3900! M4=MCB
3900! M4=MCB
3910! M5=MAB Ci=1=1248T/1082R
```

3910! M5=MAB Ci=1=1248T/1082R

```
```

3920 ! M6-MBA
3930 M1=SQR(V(2)*V1(1)*PI*V2(C,C1)*F2(1)/(I|*V1(2)*G9(1)*PS(
1)*0(1)))
3940 M2=SQR(V(1)*V(1)*Vi(2)*PI*V2(C,Ci)*F2(2)/(I|*V(2)*Vi(i)
*G3(2)*P5(2)*Q(2)))
3948 Gi=Gi(C,L)
3950 IF C=2 THEN G1=1 ! LONG TUBE HAS THE REF MIC IN THE END
3960 M3=M1*V1(2)/(V(2)*G1)
3970 M4=M2*V1(1)/(V(1)*G1)
4000 M5-M2*V(2)*V1(1)/(V(1)*V1(2))
4010 MG-M1*V(%)*V1(2)/(V(2)*VT(1))
4 0 2 0 ~ ! ~ S T O R E ~ R E S U L T S ~ O N ~ T A P E ~
4 0 3 0 ~ B E E P
4 0 4 0 ~ B E E P
4050 PRINTER IS 2
4060 PRINT "MODE \#=",L
4070 PRINT "FREQ=",F2(1)
4080 PRINT "MA=".M1
4 0 9 0 ~ P R I N T ~ " M A B = " , M 5 ~
4100 PRINT "MMB=",M2
4110 PRINT "MBA=",M6
4120 PRINT "MCA=",M4
4130 PRINT "MCB=".M3
4131 PRINT "E1=",E!
4132 PRINT "E2=",E2
4140 PRINT "K1(1)=",K1(1)
4150 PRINT "'K1(2)="'K1(2)
4160 PRINTER IS 701.80
4190 ASSIGN" 1 TO "DAT1"
4200 PRINT\# 1,L4 : L,M1,M2.M3.M4.M5.M6.PS(1),P5(2),TS(1),T5(
2),C
4210 ASSIGN* 1 TO "DAT2"
4 2 2 0 ~ P R I N T \# ~ 1 . L 4 ~ ; ~ A 2 , V 1 ( 1 ) , V 1 ( 2 ) , G 3 ( 1 ) , G G ( 2 ) , V ( 1 ) , V ( 2 ) , Q ( 1 )
,Q(2).F2(1),F2(2),C1
4230 ASSIGN\# 1 TO "DATV"
4240 PRINT\# 1,L4;G5(.)
4250 ASSIGN\# i TO "DATF"
4260 PRINT* 1.L4 ; F5(.)
4270 U1=(M1+M5)/2
4280 U2=(M2+M6)/2
4290 U3 =(M4+M3)/2
4300 U4=(M1-M5)-100/(M1+M5)
4310 PRINT USING 170 ; L,F2(2).T5(2),Q(1).Q(2).U1,U2,U3.U4,L
4
4320 !
4 3 3 0 ~ : ~ I T E R A T E ~ R E C O R D ~ \# ~ A N D ~
4340 ! CHECK FOR END OF RUN
4350 L4=L4 + 1
4 3 6 0 ~ G O T O ~ 4 3 9 0 ~
4370 ! CHECK FOR LAST MODE
4 3 8 0 ! OTHERWISE END
4 3 9 0 ~ I F ~ L = 1 ~ T H E N ~ 4 4 4 0 ~ E L S E ~ 4 4 0 0 ~

```
```

4400 I9=(L-1)*F9/L
4410 F1=19-2*I9/(Qi*SQR(I9/F9))
4420 F2=I9+2*I9/(Q1*SQR(I3/Fg))
4 4 3 0 ~ N E X T ~ R ~
4440 END
4 4 5 0
4460 : PUT UHF SW IN NEUTRAL
4 4 7 0
4480 OUTPUT 716 USING "2A" ; "A3""
4490 OUTPUT 716 USING "2A" ; "B3"
4 5 0 0 ~ R E T U R N
4 5 1 0
4 5 2 0
4 5 3 0 ~ ! ~ R A Y I N E ~ S U B R O U T I N E ~
4540
4550 ! RAUINE Q
4 5 6 0
4570 FOR M=1 TO M9
4580 Q4=01/200
4590 Q3=01+04
4600 Q2=01
4610 Q1-Q1-Q4
4620 K(1)=0
4 6 3 0 ~ F D R ~ N = i ~ T O ~ 2 6 ~
4640 K1=(F(N)/Fg-F9/F(N))^2*Q1*Q1+1
4650 K2=SQR(K1)
4660 K(1)=K(1)+(A(N)-P1/K2)^2
4670 NEXT N
4680 K(2)=0
4 6 9 0 ~ F O R ~ N = 1 ~ T O ~ 2 6 ~
4700 K1=(F(N)/F9-F9/F(N))^2*Q2*Q2+1
4710 K2=SQR(K1)
4720K(2)=K(2)+(A(N)-P9/K2)^2
4 7 3 0 ~ N E X T ~ N
4740 K (3)=0
4750 FOR N=1 TO 26
4750 Kl=(F(N)/Fg-F9/F(N))^2*03*03+1
4770 K2=SQR(K1)
4780 K(3) =K(3)+(A(N)-P1/K2) `2
4 7 9 0 ~ N E X T ~ N
4800 K=Q4*(K(3)-K(1))/(2*(2*K(2)-K(1)-K(3)))
4310 Q1=Q2+K
4 8 2 0
4830 ! RAVINE PEAK AMPLITUDE
4 3 4 0
4850 P4=P1/500
4860 P3=P1+P4
4870 P2=P1
4880 P1=P1-P4
4 8 9 0 ~ K ( 1 ) = 0
4 9 0 0 ~ F O R ~ N = 1 ~ T O ~ 2 6
4910 K1=(F(N)/F9-F9/F(N))'2*O9*Q1+1
4920 K2=SQR(K1)

```
```

4930 K(1)=K(1)+(A(N)-P|/K2)^2
4 9 4 0 ~ N E X T ~ N ~
4950 K(2)=0
4960 FOR N=1 TO 26
4970 K1=(F(N)/F9-F9/F(N))^2*Q1*Q1+1
4980 K2-SQR(K1)
4990 K(2)-K(2)+(A(N)-P2/K2)^2
5000 NEXT N
5010 K(3)=0
5020 FOR N=1 TO 26
5030 Ki=(F(N)/Fg-F9/F(N))^2*Q1*Q1+1
5040 K2-SQR(K1)
5050 K(3)-K(3)+(A(N)-P3/K2)^2
5 0 6 0 ~ N E X T ~ N
5070 K=P4*(K(3)-K(1))/(2*(2*K(2)-K(1)-K(3)))
5080 P1-P2+K
5 0 9 0
5100 ! RAUINE FREQ
5110
5120 F8-F9/(Q1*2000)
5130 F7-F9+F8
5140 F6-F9
5150 F5-F9-F8
5160 K(1)=0
5170 FOR N-1 TO 26
5180 Ki=(F(N)/F5-F5/F(N))^2*Q1*Q1+1
5190 K2=SQR(K1)
5200 K(1)=K(1)+(A(N)-P1/K2)`2 5 2 1 0 ~ N E X T ~ N 5220 K(2)=0 5 2 3 0 ~ F O R ~ N = 1 ~ T O ~ 2 6 ~ 5240 Ki=(F(N)/FG-F6/F(N))^2*Q1*Q1+1 5250 K2=SQR(K1) 5260 K(2)=K(2)+(A(N)-P1/K2)`2
5 2 7 0 ~ N E X T ~ N
5280 K (3)=0
5290 FOR N=1 TO 26
5300 K1=(F(N)/F7-F7/F(N))`2*Q1*Q1+1
5310 K2=SQR(K1)
5320 K(3)=K(3)+(A(N)-P1/K2)^2
5 3 3 0 ~ N E X T ~ N
5340 K=F8*(K(3)-K(1))/(2*(2*K(2)-K(1)-K(3)))
5350 F9=F6+K
5360 IF ABS((Q1-Q5)/Q1)>.001 THEN 5370 ELSE 5410
5370 Q5=Q1
5380 DISP "Q=",Q1
5390 NEXT M
5400 GOTO 5450
5410 !
5420 DISP "SMOOTH Q=",Q1
5430 DISP "SMOOTH F=",FG
5440 DISP "SMOOTH A=",P1
5450 RETURN

```
```

5460 :
5480
5490
5500 K(i)=0 ! SUB TO GET MSE
5 5 1 0 ~ F O R ~ N = 1 ~ T O ~ 2 6
5520 Ki=(F(N)/Fg-F9/F(N))^2*Q1*Q1+1
5530 K2=SQR(K1)
5540 K(1)=K(1)+(A(N)-P1/K2)~2
5 5 5 0 ~ N E X T ~ N
5560 RETURN

```

```

5 5 8 0 ~ I N I T I A L ~ D A T A ~ C O L L E C T I O N
5590: SUBROUTINE
5600 !
5610 A5=0
5620 D1=(F2-F1)/25
5630 F=F1
5640 DUTPUT 717 ;"FR",F,"HZ"
5650 OUTPUT 717 :"AM",A1,"MR"
5660 FOR N=1 TO 26
5670 F(N)=F
5030 OUTPUT 717 ; "FR",F(N),"HZ"
5690 WAIT 4-T1
5700 OUTPUT 709 :"UT3"
5710 ENTER 709 ; A(N)
5720 F-F+D1
5 7 3 0 ~ N E X T ~ N
5 7 4 0 RETURN
5750 ! .............................
5760 : DRIVE ADJUSTMENT
5770: AND DATA COLLECTION
5780 ! ......................
5790 DUTPUT``17 :"FR",F9,"HZ"
5300 IF A1>3455 THEN 5810 ELSE 5830
5810 OUTPUT 717 :"AM",3455."MR"
5815 A1=3455
5220 GOTO 5910
5830 DUTPUT 717 ;"AM",A1,"MR"
5840 WATT 8*T1
5850 OUTPUT 709 :"VT3"'
5860 ENTER 709 ; Q9 ! FRACTIONAL VULTAGE OUTPUT FROM 5204-70
9
5870 IF Q9>.85 THEN 5890 ELSE A1=A 1+25
5880 GOTO 5800
5890 IF Q9<.95 THEN 5910 ELSE Al=A1-20
5900 GOTO 5800
53i0 DISP "DRIUE VOLTAGE(MV)IS",AI
5920 D1=(F2-F1)/25
5930 PRINT "REQUESTED DRIVE VOLTAGE =".AI
5940 F=F1
5950 QUTPUT 717 : "FR".F."HZ"
5960 OUTPUT 717 :"AM".A1."MR"

```
```

5970 GOSUB 7110 ! GET PRESS=P
5980 PS(j)=P
5990 T8-TIME/3600
6000 GOSUB 7010 ! GET TEMP=T
6010 T5(J)=T
6 0 2 0 ~ F O R ~ N = 1 ~ T O ~ 2 6 ~
6030 A(N)=0
6040 F(N)=F
6 0 5 0 ~ F 5 ( J , N ) = F
6060 OUTPUT 717 ;"FR",F(N),"HZ"
6070 WAIT 8-T1
6080 FOR Q6-1 TO 16
6090 OUTPUT 709 :"VT3"
6100 ENTER 709; B1(06)
6110 A(N)=A(N)+Bi(Q6)
6120 NEXT G6
6130 A(N)=A(N)/16
6140 F-F+D1
6150. NEXT N
6160 OUTPUT 717 :"AM"^.I."MR"
6 1 7 0 GOSUB 7010 ! GET TEMP=T
6i80 T5(J)=(T5(J)+T)/2 ! AVERAGE TEMP DURING DATA
RUN
6i90 T9-TIME/3600
6200 T(J)=(T8+T9)/2
6 2 1 0 GOSUB 7110 ! GET PRESS=P
6220 P5(J)={(P+P5(J))/2 ! AVERAGE
6230 P5(J)=100=PS(J) ! ADJ=MMHG
6240 P5(J)=P5(J)+4.2646 ! CAL
5 2 5 0 ~ R E T U R N
6260 ! \dot{RAW' DAOTM AMAOALYYSİS}
6280 ! SUBROUTINE
6 2 9 0 ~ !
6300 AS=AMAX(A)
6 3 1 0 ~ F O R ~ X = 1 ~ T O ~ 2 6 ~
6 3 2 0 ~ I F ~ A ( X ) = A 5 ~ T H E N ~ 6 3 4 0 ~ E L S E ~ 6 3 3 0
6 3 3 0 ~ N E X T ~ X ~
6340 A6 = X
6350 X =0
6360 <2=0
6370 X3=0
6380 \4=0
6390 Y Y =0
6400 Y2=0
6410 Y3-0
6420 FOR N-1 TO 7
6430 Fi(N)=F(A6-4+N)-F(A6-3)
6440 A3(N)=A(A6-4+N)-A(A6-3)
6 4 5 0 ~ N E X T ~ N
6460 FOR I=1 TO 7
6470 人4= 人4+F1(I)"4
6480 X 3 = X 3 +Fi(I) 3

```
```

0490 X2= X2+F1(I)^2
6500 X Y = X Y +Fi(I)
6510 Y1-Y1+A3(I)
6520 Y2-Y2+A3(I)=Fi(I)
6530 Y3-Y3+A3(I)=Fi(I)^2
6 5 4 0 ~ N E X T ~ I ~
6550 B(i, 1)=\4
6560 B(1,2)=\times3
6570 B(i,3)=X2
6580 B (2,1)=X3
6590 B (2,2)=x2
5600 B(2,3)=x1
6610 B(3,1)=X2
6620 B(3,2)=X1
6630 B(3,3)=7
6640 C(1)=Y3
6650 C(2)=Y2
6660 C(3)=Yi
6 6 7 0 ~ M A T ~ X = S Y S ( B , C ) ~
6680 F9=-(X(2)/X(9)*.5)+F(A6-3)
6690 Pi=X(3)+A(A6-3)-X(2)^2/(4*X(1))
6700 H7=P1/SQR(2)
6710 FOR I= 1 TO A6
6720 IF A(I)<H7 THEN 6730 ELSE"6790
6730 IF H7<A(I+1) THEN 6740 ELSE }679
6740 F3-F(I)
6750 H3=A(I)
6760 F4=F(I+1)
6770 H4=A(I+1)
6780 GOTO 6800
6 7 9 0 ~ N E X T ~ I ~
6 8 0 0 ~ F O R ~ N = A 6 ~ T O ~ 2 6 ~
6 8 1 0 ~ I F ~ A ( N ) > H 7 ~ T H E N ~ 6 8 2 0 ~ E L S E ~ 6 8 8 0 ~
6 8 2 0 IF H7>A(N+1) THEN 6830 ELSE 6880
6830 F5=F(N)
6340 H5=A(N)
6850 F6=F(N+1)
6 8 6 0 ~ H 6 = A ( N + 1 )
6870 GOTO 6890
6 8 8 0 ~ N E X T ~ N
6390 F7=F3+(H7-H3)-(F4-F3)/(H4-H3)
6900 F8=F5+(H5-H7)-(F6-F5)/(H5-H6)
6310 Q1-F9/(F8-F7)
6920 IF J1=0 THEN 6930 ELSE }595
6330 Fi=F9-F9/Q1
5940 F2-F9+F9/Q1
6950 Ji= Ji+1
6 9 6 0 ~ C L E A R ~ B E E P
6 9 7 0 ~ D I S P ~ " R O U G H ~ Q = " , G 1 ~
6 9 8 0 ~ D I S P ~ " R O U G H ~ F = " . F g ~
5990 DISP "ROUGH A=",P!
7 0 0 0 ~ R E T U R N ~ ! ~ E N D ~ O F ~ R A W ~ A N A L Y S I S ~
7010 !

```
```

7020 ! SUBROUTINE TO GET TEMP
7030 D DEGREES CENTIGRADE IN
7 0 4 0
7 0 5 0
7050 ! OUTPUT`709`;"DOM,,\mp@code{,}
7070 OUTPUT 722 ; "F4RIMETI"
7080 WAIT 5000
7080 WAIT 5000 ; T
7100 RETURN
7110
SUBROUTINE TO GET ATMOS
7130 ! PRESSURE IN MMHG = P
7140!
7 1 5 0 ~ O U T P U T ~ 7 0 9 ~ ; " D C 4 . 1 " '
7150 OUTPUT 709 :"DO4.2""
7170 OUTPUT 722 ; "FIRIMOTI"
7180 WAIT 1000
7190 ENTER 722 : P
7200 RETURN

```

```

7 2 2 0 ! ~ \ ~ S U B R O U U T I N E ~ O O O G E O T ~ S I D E E `7 2 3 0 ! ~ " A " ~ B I A S ~ V O L T A G E ~ = ~ E ! ~ 7 2 4 0 1250 OUTPUT 709 ;"DC4.1,2" 7260 OUTPUT 709:"DO4,3"' 7260 OUTPUT 709:"DO4, 3:" 7.280 WAIT 60000 7290 ENTER 722 ; EI 7300 RETURN 7310 !` SUUBROUUTINE YOO GE`T SIODE
7320! SUBROUTINE TO GET SIDE
7340
7350 OUTPUT 709 ;"DCA.1,2,3"
7 3 7 0 WAIT 60000
7380 ENTER 722 ; E2
7 3 9 0 ~ R E T U R N
7 4 0 0
7410 SEIECT
7420 ! ATBR
7430 OUTPUT 709 ;"AR"
7440 OUTPUT 709 ;"DO4,7.12.13"
7450 OUTPUT 709;"DC4,4,10.11"
7460 DISP "A-TRANS, B-RCU"
7470 DUTPUT 716 USING "2A"": "A4""
7470 DUTPUT 716 USING "2A"": "A4""
7490 WAIT 10000 ! ALLUWS TRANSCIENTS TO DIE OUT
7 5 0 0 ~ J = 1
7510 OUTPUT 709 : "AC1"
7 5 2 0 ~ R E T U R N
7530
R
7350 OUTPUT 709 ;"DC4.1,2,3"
7 4 1 0
SELECT
7430 OUTPUT 709
UARIABLE - T
\M......................
P
7 3 3 0
4 0 0 !
..................
! . . . . . . . . . . . . . . . . . . . . .

```
```

7540 ! BTAR
7550 ! SWITCHING
7 5 6 0
7570 OUTPUT 709 :"AR"
7 5 8 0 OUTPUT 709 :"DC4.4.10,11"*
7590 ! SW SOURCE TO B. PWR ON 8 IN-OUT RELAY
7600 OUTPUT 709 ;"DC4,7,12,13"
76:0 DISP "B-TRANS,A-RCUR'
7620 OUTPUT 716 USING "2A" ; "AI"
7630 OUTPUT 716 USING "2A" ; "B1"
7640 WAIT 10000
7650 J=2
7660 OUTPUT 709 : "AC10"
7670 RETURN
7680 ! .............................
7 6 9 0 ! ATCR SWITCHING
7 7 0 0
7710 OUTPUT 709 :"AR"
7720 OUTPUT 709:"DO4,0.7.12,13"
7730 OUTPUT 709 :"DC4,4.10,11"
7740 OUTPUT }716\mathrm{ USING" "2A"":."*A2""
7750 OUTPUT 716 USING "2A" ; ".02"
7 7 6 0 ! ATCR
7 7 7 0 OUTPUT 709 :"ACI"
7 7 8 0 WAIT 10000
7 7 9 0 ~ R E T U R N
7800 ! ..............................
7 8 1 0 ~ ! ~ B T C R ~ S W I T C H I N G ~
7820
7830 OUTPUT 709 :"AR"
7840 OUTPUT 709 :"DO4.4.10.19"
7850 QUTPUT 709 :"DC4.7,12.13"
7860 OUTPUT }716\mathrm{ USING" "2A"": "A2"'
7870 QUTPUT 716 USING "2A" : "B2"
7 8 8 0 ~ ! ~ B T C R ~
1890 OUTPUT 709 :"AC10"
7 9 0 0 ~ W A I T ~ 1 0 0 0 0 ~
7 9 1 0 ~ R E T U R N
7920 ! .............................
7930 ! SUBROUTINE TO GET
7940 ! ATCR=V1(1)
7950 OUTPUT 717 ;"FR".F2(1),"HZ"
7960 DUTPUT 717 ;"AM".,A9,"MR""
7970 S5=0
7980 FOR N=1 TO 36
790 WAIT 8.TI
3000 QUTPUT 709 ;'VUT3'*
8010 ENTER 709 : S4
8 0 2 0 ~ S 5 = S 5 + S 4 ~
8 0 3 0 ~ N E X T ~ N
8040 S5-S5*B/G2
8050 VI(1)=55/36
8060 RETURN

```
```

3070
8080
8090 OUTPUT 7177:"FR',F2(2),"HZ"
8100 OUTPUT }717\mathrm{ :"AM".A1,"MR"
8110 S7=0
3120 FOR N=1 TO 36
8130 WAIT 8-TI
8140 UUTPUT 709 :"UT3"
8150 ENTER 709 ; S6
8160 S7-S7+56
8 1 7 0 ~ N E X T ~ N
8180 S7-S7-B/G2
8190 V1(2)-S7/36
8200 RETURN

```

```

3 2 2 0 ~ ! ~ S U B R O U T I N E ~ T O ~ R E L A T E ~
8230 ! MIC "A"1248 RECEIVED
NPUT
8235 IF FS(2.N)<300 THEN 8240 ELSE }825
8240 B6-10.05431
8245 GOTO }827
8250 IF F5(2,N)<1200 THEN 8255 ELSE 8265
8255 B6-10.0683817-.00001030612*F5(2,N)
8260 GOTO 8270
8265 IF FS(2,N)<5500 THEN 8266 ELSE }826
8266 B6=10.062585-.00000262425-F5(2.N)
8267 GOTD }827
8268 B6=10.071968-.00000457469-F5(2.N)
8270 B3=(Cl(2)/C1(1)+1)-2 ! Ci(2)=BTAR PF;C1(1)=1248+BIAS PF
8271 B4=(1/(2-PI=F5(2,N)=R8*C1(1)*.000000000001)) ^2
8272 B5=SQR (B3+84)
8279 B1-B5/B6
3280 ! B1-A1(1,U7)/B6 ! XTRA
8 2 9 0 ~ R E T U R N ~
8300
83:0 ! SUBROUTINE TO RELATE
8320 ! MIC "B"1082 RECEIVED VOLTAGE TO 5204 INPUT
8325 IF FS(1,N)<300 THEN 8330 ELSE }834
3330 D5=9.625:5
8335 GOTO 8370
8340 IF F5(1.N)<1100 THEN 8345 ELSE 8355
8345 D6-9.642248-.00001940816-F5(1.N)
8350 GOTO 8370
8355 IF F5(1,N)<5500 THEN 8360 ELSE 8366
8360 D6=9.627574-.000003284514*FS(1,N)
3365 GOTO 9370
8366 D6=9.638486-.000005069388-F5(1.N)
8370 D3-(Ci(4)/C1(3)+i)*2 ! Ci(4)=ATBR PF;Ci(3)=1082+ BIAS P
F
3371 D4=(1/(2*PI*F5(1,N)*R9*C1(3)*.000000000001))`2
8372 D5-SOR(D3+D4)
8379 B2-05/D6
8380 ! B2*A1 (2.U7)/D6 ! XTRA
8390 RETURN

```
\(A[]\)－The microphone signal voltage corrected for the inflit transfer function to the lock－in analyzer and the analyzer＇s sensitivity．The argument is the number of one of the 26 data samples seen at each mode．
AS－Initialized variable always equal to zero．
A9－Temporary storage for drive voltage in milizvolts．
A1－The drive amplitude desired in millivolts．
\(A_{1}[]\)－not used．
A1［，］－not used．
\(A 7[]\)－An array used in the program．correction to the ratio of specific heats．The argument \(1 s\)＂\(C\)＂．
A6－Temporary variable used 1 n the raw data analysis routine to indicate the data location of the peak signal ootarnec
AJ［］－Storage of statistics in raw data analysis routine．
A2－Temporary storage for drive voltage in millivolis．
A2［］－not used．
B－The FAR 5204 sensitivity in volts．
E［］－mot used．
\(E[\),\(] －An array used to store varıaoles used in a least 引quare\) error fit to a second order polynomial．
E1［］－A temporary accumulator for the \(\equiv i g n a l\) averaging associated with each data point in the 20 polnt sampie．
B1－A temporary output used in relating the＂A side＂signa！ voltage to the PAR 5204 input voltage．
B6－Used in tre subroutine calculating orive current．Eo multiplied by the＂asked for＂voltage yields the＂ret oriving voltage for＂STAF＂．
 ＋hinction relating tie＂EtAジ＂microfione sigral to \(\because=\ldots\) ．． 5 EOH input．
E2 H temporary output used in relating＝ife a sice＇E－git vo．tage to the PAR 5 got 1 mput viciage．
C－A program indzcator of short tube \((\mathbb{C}=:\) ）or long tute （ \(\mathrm{C}=2\) ）．
C2－A dummy variable for microphone \＃124s capacitance．
［［］－Storage of statistics in raw data anaiysis routine．
CS－A dummy variable for microphone \＃Sis capacitance．
C1－An indicator of cavit，rolume \(\quad 1=: 248-1093\)
correction with regard to which \(2=1248-8: 5\)
microphones are mounted in the ends．\(=1082-3!\)

 capacitanres corrected tor bias voltagr．

O－－A dumin varıable for microphone fiveraparizance．
```

D[] - An array contalming the geometrical correction for the Knowles subminiature microphone mounted in tre wall of the short tube．The array argument is the mode number．
DS－Temporary variable used in calculating the＂ATEf＂sign＝l transfer to the input of the fAR 5204.
D4－Temporary variable used in calculating the＂ATEF＂Elgnal transfer to the input of the PAR 5204.
D1－The frequency increment in the initial data sample．
D6－＂D6＂multiplied by the＂asked for＂voltage yields the＂get driving voltage for＂ATER＂．
D3－Used as a temporary variable in the signal transfer function relating the＂ATBR＂microphone signal to the Fif 5204 input．
E1－＂A side＂bias voltage．
Ei［］－not used．
E2－＂B side＂bias voltage．
$F$－The frequency in the initial data collection routine．
F［］－Temporary storage used in initial data collection．
FG－Upper half power point calculated for the raw sata analysls．
FS－Temporary variable used in the raw data analysis for the quality factor Q1．
FS［，］－not used．
F4－Temporary variable used in the raw data analysis for che quality factor Q1．
F9－Current best estimate of resonant frequency．Usej aiso as the unperturbed value of frequency in the Ravine subroutine．
F1－The initial estimate of the low frequency boundary 0 the first modal resonance of interest．Later used as a program calculated estimate for the lower frequency of the initial search band for the the ne\％t lower modal resunance．
Fi［］－Storage of statistics $1 \pi$ raw data analysis routine．
F7－Lower half power polnt calculated ror che raw data analysts．
F7［］－not used．
F6－Unperturbed frequency in the fiaylne subroutine．
F3－Temporary variable used in the raw data analysis rutue．
F2－The initial estimate of the high frequency bourcary of the first modal resonance of interest．Later used as $二$ program calculated estimate for the higher frequency $=f$ the initial search band for the ne：t lower modal resonance．
F2［］－Storage array for the ravined estimate of frequenc；far 三 modal resonance．
G［1－The geometrical correction for the 12 arct mounted in the wall of tie tono the ：m3 ：


```
                                    argument is the point number of the aata sample (1 to 2b
G9[] - The effective gamma as calculated by the computer
program. The argument is "c".
G4[] - mot used.
G1 - Temporary storage for G1[,] in the calculation of the
various micropmone sensitivities.
G1[,]- A temporary storage for geometrical correction factors.
G1[2,m]=G[m] and G1[1,m]=D[m]. M = mode number.
G2 - Always equal to one.
G2[] - not used.
GO - Temporary variable used in the program correction to the
ratio of specific heats.
H5 - Used in initlal determination of quality factor, Q1,
    resonant frequenc,, F9, and peak amplitude for the
    resonance, P1.
H4 - Used in lnitial determination of quality factcr, こ1,
        resonant frequency, F9, and peak amplitude for tl:e
        resonance, F1.
H7 - Half power amplitude used in the raw cata anaiysis used
        for the initial determination of quality factur, Q:,
        resonant frequency, F9, and feak dimplitude fo: the
        resonance, Fi.
Hb - Used ln imitiaj jetermination of quality factor, D1,
        resoriant frequenc,, FG, arid pear ampilitude for tine
        rejonamにe, Fi.
HZ - Used in initial determination of quality factor, Q1,
        resonant frequency, F9, and peak amplitude for the
        resomance, F1.
I - A program counter.
I9 - The center frequency calculated for the ne%t lower modal
        resonance.
I1 - The tramsmitting current calculated using the aafa!:it..̈
        model for the microphone and the dilving viluage arros=
        the micropmone terminals.
J - A program flag used in the switching subroutines alic
        i ater used to lndicate BTAR or ArEF in the date.
J1 - A flag used to lndicate when the initial daca sample l:
        complete or not. If Jl% 1, then the initial sample: i=
        over.
k: Used in the Fiavine subroutzme as a temporary calcula*ic:
        of the "step" correctian to Elther Fi, Di, or FF% Jue E-
        previous perturbatlon.
K[] - Mean square error output by MSE subr sutire.
```





```
i. - The in:taal hrgt.mabe mbinomr.
```



```
    :nterest
!.i Trem imltial l jui mode nimber.
```

| MS |  | Comparison calioration of Mic＂A＂tased upen the reciprocity calibration of Mi＝＂E＂． |
| :---: | :---: | :---: |
| $M 4$ | － |  reciprocit，Ea之ituration of Mic＂こ＂． |
| $\therefore 7$ | － | The naximun number of＂ravines＂desiredir the i＝n三． square error analysis of $F, O$ ，aid A lising a $\%$ ojita， line shape． |
| M1 | － | Fieciprocit $\mathrm{fa}^{\text {alisration for＂A ミide＂microphone．}}$ |
| Me |  | Comparison calibration of $M_{1} c$＂El＂based upon the reciprocity calibration of Mic＂A＂． |
| MJ | － | Comparison calibration of Mic＂C＂based upon the reciprocity calibration of Mic＂A＂ |
| M2 | － | Reciprocity calibration for＂E side＂microphona． |
| N |  | A program counter． |
| 0 | － | The mass of water in the atmosphere per unit volume á calculated by reference to the temperature and rel at：e humidity．This 15 used in the computer correction to crie ratio of specific heats due to the non－adiabatic oouriaar： conditions within the resonant cavity． |
| P | － | The output variable in the subroutine used to measure atmospheric pressure． |
| F＇S［］ | － | The average atmospheric pressure obtained over one modal data sample．Used also as the 1 nitial pressure obtáned before the data sample is obtained． |
| F4 | － | The amount by which the peak volcage 15 perturbed 110 the Fiavine subroutine． |
| FI | － | $\sim 3.14159$ |
| F1 | － | The best estimate of the peak signal voltage．The lower perturbation in the signal voltage in the Favine subroutine． |
| $F=$ | － | The higher perturbation of the peak voltage in the fiavine subrout：ne． |
| P2 | － | The previous best estimate of the peal：voltage arid the current unperturbed value for the pear voltage in the Fiavine subroutine． |
| Q［．］ | － | Storage array for the ravined estimate of the qualit； factor for a modal resonance． |
| 05 | － | Temporary storage for quality factor used in Favine subroutine． |
| 0.4 | － | The size of the perturbation of＂Q＂used in the Favine subroutine． |
| Q9 | － | A sample of output voltage used in the seerch for tre optimum imitial drive voltage． |
| 06 | － | A counter in the signal averaging associated with cone point data sample． |
| Q1 | － | The current best estimate of the＂R＂．The lower perturbation of the value for＂Q＂in the Ravine sutroutine． |
| 103 | － | Ther righer perturbation of the salue tor＂D＂in the RA＂： subrautine． <br> The orevious best estimate and rurrent hiperturbed velue |

for the value of＂Q＂in the Ravine subroutine．
－A program counter for mode number under consicer乡tion．
R8－＂A side＂effective resistance including the infut preanp resistance and the bias blocking resistor．
R9－＂B slde＂effective resistance including the input preanp resistance and the bias blocking resistor．
F1－The relative numidity observed at the beginning of the experiment．
S5－An accumulator in the subroutine obtaining compari三on voltages．
S4－A sample variable in the subroutine obtaining comparison voltages．
57 －An accumulator in the subroutine obtaining comparison voltages．
So－A sample variabie in the subroutine obtaining compari三or voltages．
T－Temperature variable in the＂Get Temp＂subroutirie．Used as the temporary storage for the value of temperature obtained just after a data sample has been obtained．
Ti［］－The average temperature obtalned over one modal data sample．
TJ［］－not Lised．
T［］－not used．
「9．．－The time in decimal hcurs at the beginning of a sata sample．
TS［］－The value of modal temperature before the data samples are taken．Also used to store the average value of temperature associated with a modal calibration．
T4［］－not used．
T1－The FAR 5204 time constant（sec．）．
T9－The time in decimal hours at the end of a aata sample．
U4－The percent uncertainty between Ma and Mao．
U：－The average value of Ma and Mab．
17 －A program＋1ag； $1+U 7=1$ ，tmen erase the tapp； $1+\cup 7=2$ ， then do not erase the tape．Later in the program，this is used as a counter（see line 2260）logically reiated $: 2$ either the long or short tube as a count of mode number
U3－The average value of Mca and Mcb．
U2－The average value of Mb and Moa．
U［］－Storage array for ravined signal voltage at mada！ resonance．
Vi［］－Storage array for the comparison voltages；
1 ＝Vca，$Z=$ Vcb．
V2［］－not usec．
vi2［，］－An array storing the corrected volumes．The argunent are＂C＂and＂Cl＂．

$\times 4$－Storage of statistics $1 \Pi$ raw data anaiy三is rolue：ie．
$x 1$－Storage $0^{+}$statistics $1 \Pi$ raw data analisis rcutine．


```
x2 - Storage of statistics in raw aata analysis routine.
Y1 - Storage of statistiEs in raw data analysis routine.
Y3 - Storage of statistics ln raw data analysis rolitime.
Y2 - Storage of statisti=s im raw data analysis routine.
```

Note: A number of 1 terations of thas program were writteri both $1 n i t s$ development and to accommodate the tive different electroacoustic configurations described 1 n chapter four. As a result, not all the origimal variables were employed in the final program. Since computer memory was not a problem, the dimension statements for these varıaoles were left unchanged to facili $a \operatorname{lat}$ a returns to older configurations. This is why some of the variables are listed as "not used" in this particular version.

APPENDIX C

DESCRIPTION OF THE PROGRAM USED TO OBTAIN
THE COMPARISON RATID, Vb/Va IN THE FREE FIELD
COMPARISON CALIBRATION

## A. INTRODUCTION

The computer program written on the following page was written in Hewlett Packard series 80 basic. It was used on the HP-85 and obtained the comparison voltages for the free field reciprocity calibration.

The following pages are separated into three different parts for convenience. First, there is a functional description of the program shown in table C.1. Next there is the program listing and last there is a list of definitions for the variables used in the program. Figure 3.2 in the beginning of chapter III illustrates the equipment setup controlled by this program.

The functional outline is shown next.
B. FUNCTIONAL DESCRIPTION OF THE FFOGRAM

This program 15 divided 1 nto $53 \%$ subsections outlined below:

1. Initial setup using w.E. b40AA microphone.
2. Calculation of voltage division for condenser miciophone.
3. Speaker transmit, W.E. b4OAA recelve data.
4. Secondary setup using Altec 688A electrodynamic microphone.
5. Speaker transmit, Altec $688 A$ receive data.
6. Calculation of ratio 640R/688R vs. frequency.

These si\% functional descriptions are more fully described
in the table below.

10 Frogram initialization and equipment setup. The condenser microphone must be set up before proceeding with the run.
40 This operator modifiable line dictates the frequencies that will be examined for a signal voltage. In this e\%ample, we start with mode 19 and proceed through mode 31. These modes refer to the longitudinal resonances obtained in the long tube with plane wave resonant reciprocity.
52 Here the circult variables involved in the voltage division assoclated with the condenser microphone are used to calculate the signal transfer function.
60 Twenty five samples at each frequency of interest are averaged to reduce the error associated with a low signai to nolse ratio. The average and standard deviation are calculated and stored.
170 The operator is cued to enter the anechoic chamber to remove the condenser microphone and replace it with the Altec $088 A$ electrodynamic microphone. A three wire support was used to spatially relocate the Altec 10 the same position.
210 Data sampling for the Altec $688 A$ begins in the same fashion described in line bo above.
50 The HP-35 begins calculating and frimting the resuls to let the operator know that the precedure $1 \equiv$ finistiej. Both the average ratio and the standard deviation of tine ratio are computed.

Table C. 1 Théfurstional_prggraifoisting ror_Vratig.

```
10 DFT:OH SASE 1
20 DIM A(2,40),R(40),S(3,40)
30 OUTPUT 717 ;'AM".125."MR"
40 FOR M=19 TO 31 STEP }
50 S=0
51 Q=0
52 EI=1 16.12
53 C=52.722+.0012485+.000036329*E1"2
54 R8=3247563
55 B4=(1/(2*PI*245*M*R&*C*.000000000001))^2
G1=SQR((255/C+1)^2+84)
60 FOR N=1 TD 25
70 OUTPUT 717 ;"FR",M*245,"HZ"
80 WAIT 500
90 OUTPUT 709 :"UT3"
92 ENTER 709; SI
105 S1=S 1*.001*G1
i10 S = S +S 1
115Q=Q+S1^2
120 NEXT N
130 A(1.M)=S/25
135S(I,M)=(0-25*A(I,M)`-2)/24
140 PRINT '."'
145 PRINT "'FREQ =', M*245
150 PRINT "'A(1,M)=",A(1,M)
160 NEXT M
170 CLEAR BEEP
180 DISP "SET UP NEXT MIC NOW"
190 DISP "PRESS CONT WHEN DONE"
200 PAUSE
210 FOR M=19 TO 31 STEP 3
220 S=0
225 Q=0
230 FOR N=1 TO 25
240 OUTPUT 717 ;"FR",M*245."HZ"
250 WAIT }50
260 DUIPUT 709 ;"UT3"
270 ENTER 709; S:
275 S1=S1*.001
290 S = S +S S
205 O=0+S1`2
290 NEXT N
3DO A(2,M)=S/25
305S(2,M)=(Q-25*A(2,M)^2)/24
310 PRINT ..."
315 PRINT "FREQ =".M*245
320 PRINT "A(2,M)=",A(2,M)
3 3 0 ~ N E X T ~ M ~
340 PRINT ....
340 PRINT "..",
35U PRINT ....'SRAGE RATIOS"
3E,U PRINT ''
370 FOR M=:9 TO 31 STEP 3
330 PRINT "FOR",M*245,"HZ, R=",A(1,M)/A(2,M)
390 PRINT ...
3Э5 PRINT "'SQR(S) =", SQR(S(1,M)+S(2,M))
400 NEXT M
4 1 0 ~ C L E A R ~ O ~ E E E P ~
42O DISP "THE END"
430 END
```

D. VARIABLE DEFINITIONS USED IN THE FROGFAM "vratıo"

```
A[,] - Storage array used to store the individual values of the
        ratio. The first argument is 1 = condenser mic; 2 =
        electrodynamic mic and the second argument is the mode
        number, "M".
    B4 - Temporary storage for a portion of the transfer function
        of the input acoustic signal.
    C - The bias voltage corrected value of capacitance for the
        condenser microphone.
    E1 - This is an operator modifiable value of the measured
        bias voltage for the condenser microphone.
    G1 - The calculated value of the transfer function at the
        frequency of interest.
    M - Mode of interest.
    N - Sample counter.
    R8 - Parallel combination of the bias blocking resistor and
        the input resistance of the signal preamplifier.
    S - Accumulator for sample voltage.
    S1 - Sample voltage.
    S[,] - Array storage for individual ratio sigmas. The
        arguments are the same as used for A[,].
    Q - Accumulator for sample voltage छguared.
```


## APPENDIX D

## DESCRIPTION OF THE PROGRAM USED TO

## OBTAIN THE FREE FIELD COMPARISON CALIBRATION

## A. INTRODUCTION


#### Abstract

The computer program shown on the following pages is written in Hewlett Packard series 80 basic. It was used on the HP-85 and completed data acquisition and a comparison sensitivity calibration for the W.E.b40AA serial \#1248 condenser microphone. The most time consuming portion of the program is that associated with operator interaction. The operator is directed to enter the anechoic chamber and measure the separation distance between the diaphragm of the condenser microphone and the shielded front end of the Altec 688A electrodynamic speaker microphone at the beginning of the run, at a check point in the middle of the run, and at the end of the run. After these operator measured distances and a few other parameters are entered into the computer, the program does the rest.

At the end of each series of measurements at ever increasing separations, a "quality control" plot of the data is provided for operator viewing. Two plots are provided to the operator. The first is a simple plot of signal voltage


vs range and appears as a $1 / r$ type plot. The second plots the $\log (v)$ vs $-\log (r)$. If the slope of the log/-log plot is exactly one, the spreading is spherical. Of course, the slope is never exactly one, but variations due to standing waves and other difficulties are easily seen and prompt experimental repair prior to taking additional data.

The following pages are separated into three different parts for convenience. First, there is a functional description of the program which is listed in table D.1. Next, there is the program listing and last there is a list of definitions for the variables used in the program.

The functional listing for the program is next.

The main program $1 s$ divided 1 nto seven subsections aj outlined below:

1. Initial program setup with operator imputs. The W. E. 640AA is the microphone and the Altec 688 A is the speaker.
2. Motor drive activation and sequential sampling at intervals of separation beglns.
3. Program stops and cues operator to enter the anachoic chamber to obtain a "check" distance.
4. Motor drive is activated and sequential sampling at intervals of separations continues to the end of the data run. Total time $15 \sim 45$ minutes per freq.
5. The operator is again cued to enter the chamber and obtain the final distance.
6. After the operator enters the final distance, the program enters the calculation phase and outputs the "check" plot.
7. If the operator observes an absence of obvious standing waves and desires to luse the sampled data, then he answers a query regarding the type of electrodynamic microphone used (elther o88A or be己), inputs a check value for the current measuring resistor, and the program calculates the comparison calibration for the w.E.b40AA. A sample of tmis output is shown in figures J. 2b, J. 27, and J. 2B.

These sever functional descriptions are more flily ouri: ted :n tie table beiow.

Beginning line number－Functional description
10 Frogram initialization and equipment setup tegins．
40 The counter 15 reset to prepare for the new run．
50 The computer control is directed to send a＂motor off＂ command．
bu The operator 15 asked to set in the time if it is the first run of the day．
110 The arrays are dimensioned．
190 The subroutine to set up the recording magnetic tape 15 run．
190 The subroutine storing the preamplifier gain is run loading this info into the array $A[]$.
195 The least square error fit solutions relating ＂asked for＂driving voltage to＂measured＂drive current through the current measuring resistor are read into the proper arrays．
200 The operator is asked to enter the $1 m 1 t i a l$＂record＂ number where the data will be stored on the magnetic tape．
220 The first＂ruri＂prior to the first data sample（ $N=1$ ） is begun．
Note：regardless of what setup existed，the program printed＂b．3 Trans，o4OAA receive＂due to the fixed value of Fi finally adopted in line $\mathbf{i} 15$ ．A later query actually determines the value for＂Vratic＂ whach will be used．
540 E：as voltage $i s$ obtamed．
Eミュ Operator is asked to enter Jesired modj rurder anj gr：ung voltage to be usea．
こev Eperator ：ड asked to enter the Jesired drımny．．itage for the speaker．
S7s Operator is asked to enter the voltage drop acrose the current measuring resistor．
578 Subroutine entering the values of vo／Va geterminec in the program＂Vratio＂lnitializes the necessary arrays．
 volts．




 …．．．．．．．．．ジ．：！ensur：クy resissor．Sınce a
 made，the a．erage value 1 e everithally bised to＝omper sate for any slight temperature ceperdent changes tound i： the $r$ esistance．

800 The bias voltage is sampled. holdover from previous version)
810 The tabulated frequency dependent gain for the preamplifier $i s$ read 1 nto an array.
820 The "counter" output from the optical shaft encoder is initialized.
830 The temperature 15 sampled.
840 The atmospheric pressure 15 sampled.
870 The bias voltage 15 sampled.
910 Data sampling loop begins. The drive motor 15 activated, allowed to run $T 1$ milliseconds, and turned off. A delay of fifteen seconds 15 observed co allow swaying to cease.
970 Thirty data readings are sampled at this separation.
1120 At the eighth interval, the operator $i s$ asked to enter the anachoic cmamber and obtain a comparison separat. ch measurement. After this is entered 1 hitie frogram, tiu remalning separation measurements are made by the computer.
Lig: After all data has been measured, the bias voltage is again sampled.
1:70 Again the atmospheric pressure 15 sampled.
1200 Again the temperature 15 sampled.
1210 The averages are stored for the bias, pressure, and temperature.
1254 The operator 15 asked to measure anj enter the 4 -wire current measuring resistor and the final distinte.
1450 The data is printed for operator viowing and stored or. magnetic tape.
1760 An ordinary data plot is provided for oper ator evaluation.
1790 Details of operator measurements are primted.
2010 A log/log plot is provided for operatol avaihation.
2040 The operator 15 asted to decide if the rum was gopd or if $1 t$ must be repeated.
20gO Calibration sensitivities are printed jut fsarh range with statistics lncludec. 〈Sigra is in Vifa)
2420 The final form of the sensitivity caliurations are stored on tape.
2500. Subroutine to get least squares fit for $1 / \mathrm{r}$ gata.

2760 Subroutine to get temperature.
2830 Subroutine to get atmospheric pressure.
2920 Subroutine to get bi as voitage.
Boo Subroutine to get plot of tata.
24.0 Subroutinie to get array with preamp gein.

J690 Sutroutine to get woltage dazision for d.E.EA: Aa.


Eoce Subroutime taget arrats witn "Jratio' sutpit bäa.

figld_comparison_caliobrationo

Thas program was inaikasy sirtten to contral data adruisition for three separate free field reciprocity calculations. In this final version, two different comparison calitrations based upon the data necessary for one reciprocity calibration are possible. The final comparison calibration is based upon either the Altec beßA electrodynamic microphone or an older "bJこ type saltsháaelectrodynamic mierophone. The comparison calibratim results finally used are referenced to the filtec b8BA.

[^2]
## C. PROGRAM LISTING

```
10! REV 19 MAY 84
20! PROG N28-INVERSE R DATA COLLECTION. CPU CONTROLS MOTOR
RUN
30: AND DATA STORAGE/PLOT
40 DUTPUT 720 ;"RE" ! RESETS COUNTER
50 OUTPUT 709;"DO4,14,15"! MAKES SURE VOLTS TO MOTOR ARE D
FF
OO CLEAR
6i DISP "SETTIME?? H*3600+M*60.MDD"
62 DISP "PRESS CONT IF OK"
63 PAUSE
7 0 ~ D I S P ~ " H O O K ~ U P ~ M O T O R ~ V O L T S " '
80 BEEP
9 0 ~ B E E P ~
100 BEEP
110 DPTION BASE }
i20 DIM B(20),V(20),C(20),M0(20),A(40),E1(2),T1(3),P1(3)
130 DIM B1(20).V1(20),B8(20)
140 DIM B7(20),V7(20),V8(20)
150 DIM S4(9),E3(20),E4(20),A2(40),B2(40),R5(40),R6(40)
160 ! B WILL STORE RANGE
170 ! C WILL STORE DELTA RANGE
180 GOSUB 3770 ! TAPE SETUP
190 GOSUB 3430: PREAMP GAIN
195 GOSUB 4000 ! INITIALIZE II
200 DISP "ENTER BEGINNING RECORD NUMBER FOR 'DAT%' STUFF"
210 INPUT L4
220 FOR N=1 TO 20
230 MO(N)=0
240 B1 (N)=0
250 V1(N)=0
260 B7(N)=0
270 V7(N)=0
280 B8(N)=0
290 V8(N)=0
3 0 0 ~ N E X T ~ N
310 ! FOR R1=1 TO 3
315 R1=1
320 ! RI=1: 633T640R
330! R1=2: 688T640R
340 ! R1=3; 688T633R
350 OUTPUT 720 ;"RE" ! RESETS COUNTER
360 OUTPUT 709 :"DO4,14.15" ! MAKES SURE VDLTS TO MOTOR ARE
OFF
3 7 0 ~ I F ~ R 1 = 1 ~ T H E N ~ G O T D ~ 4 0 0 ~ ! ~ 6 3 3 T 6 4 0 R ~
380 IF R1=2 THEN GOTO 440 ! 638T640R
390 IF Rl=3 THEN GOTO 480 ! 688T633R
400 PRINT "633A TRANS,640AA RCV"
410 PRINT .."
420 DISP "633A TRANS,640AA RCV"
430 GOTD 510
440 PRINT "E88 TRANS.640AA RCV"
450 PRINT "'.'
```

```
4 6 0 ~ D I S P ~ " ' 6 8 8 ~ T R A N S , 6 4 0 A A ~ R C V " * )
470 GOTD 510
480 PRINT "688 TRANS, 633A RCV"
490 PRINT "."
500 DISP "688 TRANS, 633A RCV"
510 C9-52.722 ! PF $1248
520 R8-9247663 ! OHMS
540 GOSUB 2920 ! GET BIAS
550 C2=255 ! PF
5 5 2 \text { DISP "LONG TUBE PLANE WAVE MODE NUMBER FOR FREQ"}
5 5 4 ~ I N P U T ~ F
556 F9-F
558 F=F*245 ! FOR LATER COMPARISON WITH TUBE RECIPROCITY DAT
A"
5 6 0 ~ D I S P ~ " I N P U T ~ D R I V I N G ~ M V " ~
5 7 0 \text { INPUT AO ! ASKED FOR DRIVING VOLTAGE IN MV}
571 IF R1=1 THEN V2=AO
575! V0-A2(Fg)=U2+B2(F9)
576 DISP "INPUT U DROP"
577 INPUT vO
578 GOSUB 5000
5 8 0 ~ D I S P ~ " E N T E R ~ 5 2 0 4 ~ S E N S ~ I N ~ V O L T S " '
590 INPUT BO
600 DISP "ENTER 5204 TIME CONSTANT"
610 INPUT T2
620 T2-T2*1000
6 3 0 ~ D I S P ~ " M E A S U R E ~ A N D ~ E N T E R ~ T H E ~ S T A R T ~ D I S T A N C E ~ F O R ~ M I C ~ B . M I C ~
    A TO MIC B FACE(CM)"
6 4 0 ~ I N P U T ~ D 1 ~
630 T =10000 ! INTERVAL OF DRIVE(MOTOR) TIME IN MILLISEC
700 M1=20 ! zPOINTS,MAX=20'
710!
7 2 0 !
730 UUTPUT 717 : "FR".F."HZ"
740 OUTPUT 717 :"AM",AO."MR"
750 IF Ri=1 THEN 760 ELSE }80
760 DISP "MEASURE/ENTER 633 4-WIRE CURRENT LIMITING RESISTAN
CE"
7 7 0 ~ I N P U T ~ R Q ( 1 )
780 ! RG=74.705 ! DHMS
7 3 0 ~ P R I N T ~ " 4 W I R E - R ( O H M S ) = " , R Q ( 1 )
800 IF R1<3 THEN GOSUB 2920! GET BIAS
810 GOSUB 3690 ! GET G1
820 T=0 ! INITALIZE TOTAL COUNT
830 GOSUB 2760 ! GET
840 GOSUB 2830 ! GET P
860 IF R1 - 3 THEN 890 ELSE }87
870 GOSUB 2920 ! GET BIAS
380 E!(R1)-E1
890 P; (R1)=P
900 T1(R1)=T3
910 FOR N=1 TO M1 : GET DATA
```

```
920 OUTPUT 720 ;"FN12"
930 OUTPUT 709 ;"DC4,14,i5"
9 4 0 ~ W A I T ~ T 1 ~ T
950 OUTPUT 709 :"DO4,14,15"
960 HAIT 15000 ! STOP SHAYING
970 ENTER 720 ; Ai ! CDUNT THIS RUN
980 V(N)=0 ! INITALIZE VOLTS
9 9 0 ~ T - T + A i
1000 FOR M=1 TO 30 ! AVG 30 READINGS
1010 DUTPUT 709 ;"VT3"
1020 WAIT 3#T2 ! WAIT THREE 5204 TIME CONSTANTS
1030 ENTER 709; E
1040 V(N)=V(N)+E
1050 NEXT M
1060 V(N)=V(N)/30
1070 B(N)=T ! CUMULATIVE COUNT
1080 C(N)=A1 ! COUNT NTH RUN
1090 DISP A1,T,N,V(N)
1100 BEEP
1110 OUTPUT 720 ;"RE"
1120 IF N=INT(M1/2.3) THEN 1130 ELSE 1160
1130 BEEP 50,505
1140 DISP "MEAS&ENTER DIST(CM)"
1150 INPUT D7! CHECK DIST.
1160 NEXT N ! END OF GET DATA
1161 PRINT "TIME =", TIME/3600
1162 PRINT "DATE=",DATE
1170 IF R!=3 THEN i180 ELSE 1190
1180 GOSUB 2920 ! GET BIAS
1190 GOSUB 2830 ! GET P
1200 GOSUB 2760 ! GET T3
1210 IF Ri<3 THEN Ei(R1)=(E;(R1)+E|)/2
1220 P1(R1)=(P1(R1)+P)/2
1230 Ti (R1)=(T1 (R1)+T3)/2
1240 BEEP 50,505
1250 OUTPUT 717 ;"AM",1,"MR"
1253 IF R1=1 THEN 1254 ELSE 1260
1254 DISP "MEASURE/ENTER 633 4-WIRE CURRENT LIMITING RESISTA
NCE"
\255 INPUT R9(2)
1256 PRINT "WIRE RESISTANCE=",R9(2)
1257 R9=(RG(1)+R9(2))/2
1258 PRINT "AVG 4-WIRE OHMS=".R9
1260 DISP "MEASURE AND ENTER THE FINAL DIST(CM) "
1270 INPUT D2 ! FINAL DIST
1280 L=ABS(D2-D1) ! TOTAL DIST(CM)
1290 FOR N=1 TO M1 ! SCALE DATA
1300V(N)=U(N)*BO*Gi
1310 IF D1>D2 THEN 1320 ELSE 1340
1320 B(N)=D1-B(N)=L/T
1330 GOTO 1350
1340 B(N)=D1+B(N)*L/T
1350 C(N)=C(N)*1/T
```

```
1360 NEXT N : END OF SCALE DATA
1370 ITAGE 2D,1X,3D.DDD,1X,40.00D.1X,D.DDODDE
1380 PRINT " N RUN R(CM) volTS"
1390 FOR N=1 TO MI ! PRINY DATA
1400 PRINT USING 1370;N,C(N),B(N),V(N)
1410 NEXT N ! END OF PRINT DATA
1420 GOSUB 2500 ! GET L`2 FIT
1430 ASSIGN# I TO "DAT1"
1440 IF Ri<3 THEN 1450 ELSE 1480
1450 PRINT: I,L4 ; V(),B(),E\(Ri),P\(Ri),T\(Ri),VO,RG,F,A,B,
M1
1460 L4=L4+1
1470 GOTO 1500
1480 PRINT# 1,L4 : V(),B(),E1(2),P1(R1),T1(R1),VO,R9,F,A,B,M
1
1490 L4=L4+1
1500
i510 IF Ri=1 THEN 1540 ! STOREDATA
1520 IF R1=2 THEN 1610
1530 IF R l=3 THEN 1680
1540 FOR N=1 T0 M1 ! 633T-640R
1550 B1 (N)=B(N)
1560 V1(N)=V(N)
1570 NEXT N
1580 U6=1/A
1590 A6=B/A
1600 GOTD 1750
1610 FOR N=1 TO M1 ! 688T-640R
1820 B7(N)=B(N)
1E30 U7(N)=U(N)
1640 NEXT N
1050 V7=1/A
1660 A7=B/A
1670 GOTO i750
1680 FOR N=1 TO Mi ! 688T-633R
1690 B8(N)=B(N)
1700 V8(N)=V(N)
1710 NEXT N
1720 V8=1/A
1730 A8=B/A
1740 ! END STORE DATA
i750
1780 GOSUB 3000 : PLOT DATA
1770 COPY ! END PLOT I/R DATA
:780 PRINT ...
1790 PRINT ..." ! RECORD DETAILS
1800 PRINT "* OF MEASUREMENTS=".M1
1810 PRINT "MEAS CHECK (CM)=",D7
1820 PRINT "CALC DIST (CM)=",B(INT(M1/2.3))
1830 PRINT "....................."
1840 PRINT "DIST `%ERROR=",(B(INT(M1/2.3))-D7)*100/D7
1850 PRINT "......................
1860 PRINT "START DIST(CM)=".DI
```

```
1870 PRINT "END DIST(CM)=",D2
1880 PRINT "'5204 SENS ="",B0
1890 PRINT "717 MVOLTS=",AO
1900 PRINT "TOTAL COUNT=",T
1910 PRINT **FREQ (HZ) =",F
1920 PRINT "." ! END RECORD DETAILS
1930 FOR N=1 TO M1 ! SWAP LOG DATA
1940 E3(N)=1/B(N)
1950 E4(N)=V(N)
1960 NEXT N
1970 FOR N=1 TO M1
1980 B(N)=ABS(20-LGT(E3(N)))
1990V(N)=ABS(20-LGT(E4(N)))
2000 NEXT N
2010 GOSUB 3000 ! PLOT SUBROUTINE
2020 COPY ! END PLOT LOG DATA
2030 GCLEAR O BEEP
2040 DISP "RERUN LAST DATA ? ENTER 1= RERUN, 2=NO, CONTINUE"
050 INPUT }2
2060 IF Z9=1 THEN RI=R!-1
2070 ! NEXT RI
2075 Ri=1
2080 PRINT "MO(640) US RNG(CM)"
2090 PRINT "."
2100 IMAGE 3D.DDD,IX,D.DDDDE
2110 PRINT " RANGE(CM) MO(640)"
2120 M9=0 ! SUM MO
2130 R3=0 ! SUM R
2140 R4=0 ! SUM R^2
2150 K9=0 ! COUNTER
2160 M8=0 ! SUM MO^2
2170 ! FOR M=INT(M1/2) TO M1
2180! V4=1/(B1(M)/V7+A7/V7)
2190 ! V3=1/(B1(M)/V8+A8/V8)
2200: R3=R3+V4/V3
2210! R4=R4+(V4/V3)^2
2220 ! K9=K9+1
2225 ! PRINT "RATIO=",V4/V3
2230 ! NEXT , M
2240
2250
2254
2255 DISP "INPUT U DROP IN UOLTS"
2256 INPIJT VO
2257 DISP "INPUT 1=640R+633T"
2258 DISP "INPUT 2=640R+688T"
2259 INPUT N
2260 IF N=1 THEN R5=RS(F9) ELSE R5=R6(F9)
2265 FOR N=1 TO M1
2270 E4=\/I(N)
2280! I ! = (A2(FG)=V2+B2(F9))/R9
2284 II=VO/R9
```

```
2290 Z1-E4*R5*4.30356i*(273.16+TI(1))*(B1(N)+A6)
2300 22-IT*P1(1)-100*F
2310 MO(N)=SQR(Z1/Z2)
2320 PRINT USING 2T00; 81(N),MO(N)
2325 PRINT "DB RE IV/UBQ200VBIAS*",20*LGT(200*MO(N)/E!(1))-2
0
2330
2340
2350
2355 PRINT "II=", II
2356 PRINT "VO=",VO
2357 PRINT "R9=",R9
2360 MT=M9/M1 ! <MO(N)>
2370 PRINT "<MO>=",M7
2375 PRINT "AUGDB iU/UBQ200UBIAS=",20*LGT(200*M7/EI(1))-20
2380 M\delta=SQR(ABS (M8-M1*M7^2)/(M1-1))
2390 PRINT "SIGMA=",M6
2400 RRINT "<RATIO=>",R5
2405 : PRINT "PROG RATIO=".R5(F9)
2410 ! PRINT "SIGMA R=",R6
2411 G\XiSUB 2420
2417 DISP "END"
24:8 BEEP
2419 END
2420 ASSIGN* 1 TO "DAT2"
2430 PRINT* 1.L4; F.MO(),M7,M6
2440 ASSIGHE I TO *
2450 PRINT
2460 RETURN
2490 ! \cdotsM.......................
2500 : SUUBROUUTOINE OOO GOETMUEASSOT SQUUARES FIT (I/U)=(R/V)+(`D/V
)
2510 X1=0 ! SX
2520 X2=0 ! SX^2
2530 X3=0 ! SXY
2540 Y1=0! SY
2550 Y2=0 ! SY^2
2560 Y3=0 ! N
2570 FOR N=1 TO M1
2580 X1=X1+B(N)
2590 X 2 = X2+B(N)*B(N)
2600 X3=X3+B(N)/V(N)
26i0 Y1=Y1+i/N(N)
2620 Y2=Y2+1/(V(N)=V(N))
2630 Y = Y3+1
2640 NEXT N
2650 A=(Y3-X3-X1-Y1)/(Y3-X2-X1*X1)
2660 B=(Y9*X2-X1*X3)/(Y3*X2-X1-X1)
2670 Q5=Y3*X3-X1-Y1
2580 Q6=SOR(Y3*X2-X1~2)
2690 Q7=SOR(Y3-Y2-Y1^2)
2700 R=65/(06*07)
```

```
2710 PRINT "VO=(VOLTS)",1/A
2720 PRINT "'a=(CM)".B/A
2730 PRINT "R=",R
2740 RETURN
2750
2760 ! SUBROUTINE TO GET TEMP
2770 OUTPUT 709;"DO4,i"
2780 OUTPUT 722 : "F4R1MGT1"
2790 WAIT 5000
2800 ENTER 722 ; T3 : TEMP DEG C
2810 RETURN
2820 !
2830 : SUBROUUTINE YO GGEOT PRESSS
2840 OUTPUT 709 :"DCA.1"
2850 OUTPUT 709;"DO4.2"
2860 OUTPUT 722 :"FIRIMOT!"
2870 WAIT 5000
2880 ENTER 722 ; P ! PRESS
2890 P=100*P+4.2646! SCALE + CAL MMHG
2900 RETURN
2910
2920
2930
2940 OUTPUT 709 :"DO4,3"
2950 OUTPUT 722 :"FIRIMOT!"
2960 WAIT 5000
2970 ENTER 722 ; E! ! BIAS FOR 640
2980 RETURN
2990 ! ............................. PLOT SUBROUTINE
3000
3010 V=0! V WILL BE MAX VOLTAGE
3020 FBR N=1 TO M1
3030 IF U>U(N) THEN 3050 ELSE 3040
3 0 4 0 ~ V = V ( N )
3 0 5 0 ~ N E X T ~ N
3060 Z=100000000000
3070 FOR N=1 TO M1
3080 IF Z<V(N) THEN 3100 ELSE 3090
3090 Z=V(N) ! Z WILL BE MIN VOLT
3100 NEXT N
3i10 L1=ABS(B(M1)-B(1))/10
3i20 IF B(1)<B(M1) THEN 3140 ELSE 3130
3130 PRINT "PLOT IS BACKWARDS"
3140 SCALE B(1)-2*LI,B(M1)+2*L1,Z-2*(V-Z)/5,V+(V-Z)/5
3:50 Li=ABS (B(M1)-B(i))/10
3160 XAXIS Z,L1,B(1),B(M1)
3170 YAXIS B(i),(V-Z)/10.Z.V
3180 MOVE B(1)+L1*5,Z+3*(V-Z)/10
3190 LDIR 0
3200 LABEL "RANGE.CM"
3<10 MOVE B(M1)-5*L},V+(V-Z)/10
```

```
3220 LABEL "UMAX=",V
3230 PENUP
3240 MOVE B(1),V(1)
3250 VI=ABS((V-Z)/30)/2
3260 H1=ABS((B(M1)-B(1))/40)/2
3270 FOR I= 1 TO NII
3280 MOUE B(I),V(I)
3290 IDRAW H1,O
3300 IDRAW - (2*H1),0
3310 IDRAW HI,O
3320 IDRAW 0,V9
3330 IDRAW 0,-(2*V1)
3340 NEXT I
3350 LDIR 90
3360 FOR X=B(1) TO B(M1) STEP LI
3370 MOVE X,Z-2.5*(V-Z)/10
3380 LABEL UALS(INT(X))
3 3 9 0 ~ N E X T ~ X ~
3400 LDIR 0
3410 RETURN
3420
3430 ! PREAMP GAIN
3440 A(1)=9.94366
3450 A(2)=9.99608
3460 A(3)=9.99776
3470 A(4)=10.00223
3480 A(5)=10.0051
3490 A(6)=10.00272
3500 A(7)=10.00388
3510 A(8)=10.00242
3520 A(9)=10.00343 ! ?22??
3530 A(10)=10.00181
3540 A(11)=10.00178
3550 A(12)=9.99894
3560 A(13)=9.99823
3570 A(14)=9.99744
3580 A(15)=9.997
3590 A(16)=9.99744 ! 727
3600 A(17)=9.99681
3610 A(18)=9.99361
3620 A(19)=9.99248
3630 A(20)=9.99084
3640 A(21)=9.9896
3650 A(22)=9.98785
3651 A(23)=9.98611
3652 A(24)=9.98511
3653 A(25)=9.98411
3654 A(26)=9.98311
3655 A(27)=9.983
3656 A(28)=9.9825
3657 A(29)=9.982
```

```
3658 A(30)=9.9815
3659 A(31)=9.98125
3660 A(32)=9.981
3661 A(33)=9.98075
3662 A(34)=9.9805
3663 A(35)=9.98025
3664 A(36)=9.98005
3665 A(37)=9.97995
3670 RETURN
3680
3690 ! UÖLTAGGE DIUISION FOR S4O
3700 C=C C + . O012485+.000036329*E| ^2
3710 B3=(C2/C+1)^2
3720 84=(1/(2*PI*F*R8*C*.000000000001))^^2
3730 B5=SQR(B3+B4)
3740 IF R1=3 THEN G {=1/A(FG) ELSE Gi=B5/A(FG)
3750 RETURN
3760
3770 DISP "ENTER }1=|\mathrm{ ERASEMTAPE, 2=OLK TAPE"
! . . . . . . . . . . . . . . . . . . . . . . . . .
3 7 8 0 ~ I N P U T ~ U 7 ~
3790 IF U7=1 THEN 3800 ELSE 3870
3800 DISP "ARE U CERTAIN U WANT TO ERASE THE TAPE????`"
3810 DISP " }1=\mathrm{ YES,2=NO"
3 8 2 0 ~ I N P U T ~ U T ~
3830 IF UT=1 THEN 3840 ELSE 3870
3840 ERASETAPE © BEEP
3850 CREATE "DAT1".92,480
3860 CREATE "DAT2"'31,184
3870 RETURN
4 0 0 0 \text { ! SUBROUTINE TO SETUP COEFFICIENTS USED TO CALC I:}
4010 A2(2) =.0007352517
4 0 1 1 ~ A 2 ( 6 ) = . 0 0 0 7 3 7 4 8 9 5 ~
4012 A2(14)=.0007321923
4013 A2(18)=.0007241014
4015 A2 (9) =.0007159909
4 0 1 6 ~ B 2 ( 9 ) = . 0 0 0 5 5 0 9 1 ~
4017 A2(8)=.0007169636
40;8 B2(8)=.0003581818
4019 A2(11)=.0007124636
4020 A2(10)=.000713388;
4021 B2(11)=.0002854545
4 0 2 2 ~ A 2 ( 1 2 ) = . 0 0 0 7 1 2 8 5 4 5 ~
4 0 2 3 ~ B 2 ~ ( 1 2 ) ~ = - . 0 0 0 6 7 2 7 2 7 ~
4 0 3 0 ~ A 2 ( 2 3 ) = 0
4040 B2(2)=-.000713636
4041 B2(6)=.0001951515
4 0 4 2 ~ 8 2 ( 1 4 ) = - . 0 0 0 2 6 8 6 6 6 7 ~
4043 B2(18) =+.0003575758
4050 B2(10)=.0001727273
4 0 6 0 ~ B 2 ( 2 3 ) = 0
4070 RETURN
```

```
5000 : SUBROUTINE FOR RATIOS
5001
5002
5003
5010 R5(5)=47.04
5020 R5(6)=52.556
5030 R5(7)=50.03
5040 RS (8)=52.59
5050 RS(9)=49.6
5060 R5(10)=42.76
5070 R5(11)=39.24
5080 R5(12)=37.92
5090 R5(13)=38.18
5100 R5(14)=34.83
5110 R5(15)=32.17
5120 R5(16)=28.92
5130 RS(17)=27.25
5140 R5(18)=28.83
5150 R5(19)=21.33
5151 R5(20)=18.38
5152
5153! G40R/688R
5154 !
5160 R6(1)=14.57
5161 R6(2)=15.3
5162 R6(3)=20.71
5163 R6(4)=16.3
5164 R6(5)=18.9
5170 ! R6(5)=17.603
5180! R6(6)=16.889
5181 R6(6)=18.16
5190 R6(7)=20.158
5200 R6(8)=21.659
5210 R6(g)=22.463
5220 R6(10)=24.728
5230 R6(11)=21.587
5240 R6(12)=18.96
5250 R6(13)=22.276
5260 R6(14)=26.309
5270 R6(15)=23.404
5280 R6(16)=27.752
5290 R6(17)=25.069
5300 R6(18)=27.495
5310 ! R6(19)=26.237
5320 R6(20)=26.742
5321 R6(19)=27.03
5322 R6(22)=27.23
5323 R6(25)=33.42
5324 R6(28)=41.82
5325 RG(31)=45.51
5330 RETURN
```

 the program．（cm）
E－Temporary storage for one sample of signal vol：age． Thirty such readings are obtalned oefore an ：רdivialial ＂Jata＂point is considered complete．
E4－Temporary storage for the signal voltage used in the calculation of Mo．
E4［］－Temporary storage array used prior to plotting data．
E1－Sample storage for bias voltage．
E1［］－Array storage for values of bias voltage．
$E[$［ $]$－Temporary storage array used prior to plotting data．
$F$－The mode number（multiple of 245 Hz ）desired for a particular experimental run．Frogram modified to be the actual frequency of the former mode number．
F9－Temporary storage for＂F＂，the mode number（multiple of 245 Hz ）desired for a particular experimental run．
G1－Numerical value obtained by subroutine calculation for magnitude of the voltage division transter functior tar the condenser microphone input circuat．
H1－Variable used in the plotting routine．
I－A counter used in the plotting routine．
I1－Computer calculated value of driving current lsed in the calculation of Mo．
Kq－A counter used in the statistical analysis of Mo．
L4－The operator entered＂record number＂used to 1dentit； where on the magnetic tape storage 15 desired．
$L$－The total distance between the beginning position and i．： end for a particular data run．icm）
L1－One tenth tme total distance travelledin a particuiar calitration run．
Mg－Used in the statistical analysis of Mo：the 三um ot the squares of individual values of Mo．
$M$－ने counter．
Ma－！Jsed 1 －the statistical analysis of Mo：to 引in a－ lndividua！values of Mo．
M！－The rumber of＂different sefaration＂calitratian measurements desired．Normaliy set at 20.
M7－Temporary storage for the open circint voltige reaきi：i．ig sensitivity calculated $1 \pi$ volts／fascals．
Mo－Sigma for＂Mテ＂over the sequence ot ealiorations a：or＝ particular trequency．
Mo［］－Array storage for the 1 ndivicual values of＂MT＂．
N －A counter for Do． 100 s ．
Fi［］－Array storage for the final value of the atmospmer：c pressure lised $1 \pi$ a particular calloration calculatian．
F．－Sample value for atiospherif prescure．
G5－Teapcrary varıanle Li三ed in least 三quare error ミateraurt．

OS－Tenporary varıanle used 1 n least suliare error subral．t：in．
Fis－THe parallel combinatlore of tre bryit mesistaice－t tre



Y1 - Used in the statistics calculations for the least sudere fit of data to a $1 / \mathrm{r}$ plot. The sum of $Y$ valui
$y$ - Used in the statistics caiculations for the least squares fit of data to a $1 / \mathrm{r}$ plot. The counter.
Y2 - Used in the statistics calculations for the least squares fit of data to a $1 / r$ plot. The sum of $Y$ squared values.
29 - Operator entered decision variable; if $=1$, then data run must be redone.
Z1 - Temporary variable used in the calculation of Mo.
Z2 - Temporary variable used in the calculation of Mo.
2 -

- Variable used in the plotting routine. Equal to the minimum signal voltage measured in a particular cata rini.


## AF'F'ENDIX E

## THE <br> D.C. BLOCKING <br> CAFACITOR COFRECTION

Calculation of the signal loss due to the D.C. Elocking capacitor $C$ c shown below and in figure 3.3, was accomplished by comparing two different numerical solutions for the ratio [el/Vin]. These solutions were obtained for the tion different circuits using circuit analysis software. The first numerical solution used a homegrown circuits solution on the $H P-15 C$ hand held calculator using the complew mode. The significance of the comple\% mode on this particular hand held calculator is that complex numbers can be direc:iy employed in the impedance calculations. The second soluticn was obtalned on an IBM XT microprocessor using the "Pspice" circuit analysis software made availatie ty che Electrical Engineering Department at the United States Naval hcadem;. Both solutions were in substantial agreemert with ar average discrepancy of. oul dB. The input circuit for the acoust:e signal is shown telow:


Figure E. 1 The_ingut_circuit_for the_acoustic S19nal.

For comparison purposes, the first circuit analvsis neglects both Ce and Cb . In the second analysis both of these ${ }^{2} .01 \mathrm{~h}$ uFarad capacitors are included.

The first solution for the sigmal voltage el' is given as a function of the "observec" voltage, vin, using the signal transfer obtained for fre circilt shown ajever roglecting ce end Ct. This analysis parailels the sciut:on used in the compleer program. In the second computation.
both Cc and Cb are included and the signal voltage is given by el".

The correction to the program solution for Mo is given below in equation E. 1
$\underset{M_{0} \text { due } C_{c,}, C_{b}}{\operatorname{CorRECT}}=20 \log \left(\frac{e_{1}^{\prime \prime}}{e_{1}^{\prime \prime}}\right)$
Equation E. 1

If we define individual impedance terms as follows:

```
    Zm - microphone impedance, 1/jwCm
    Zc - coupling capacitor impedance, 1/jwCc
    Z1 - load impedance consisting of Ci, Clo, and Ri.
    Zb - bias impedance consisting of Rb, Cb, and Cli.
    Z1'- modified load impedance consisting of Ri, Rb, Ci, Clo,
        and Cli.
    Zt - (Zl + ZC) in parallel with Zb.
```

Then the correction may be rewritten as:
$\left.\begin{array}{c}\text { Correction To } \\ M_{0} \text { due } \mathrm{Ce}_{\mathrm{C}} \mathrm{Cb} \\ =20 \log \end{array} \frac{\left(1+\frac{z_{m}}{z_{t}}\right)\left(1+\frac{z_{c}}{z_{L}}\right)}{\left(1+\frac{z_{m}}{z_{L^{\prime}}}\right)}\right\}$ Equation E.2

This correction is solved numerically with the program listed in table E. 1.

Register usage
Variable - number

## Program Listing for HP-15c hand-held scientific calculator



* Wo $=245 *$ pi*2
~ 1539.3804
(read top to bottom, left to right)
F LBL B RCL . 2 X STO 0 RCL 0 STO X.O RCL 0 STO X. 5 RCL 0 STO X. 7 RCL . 3 GSB 1 STO . 3 RCL . 6 GSB 1 STO . 6 RCL . 9 GSB 1 STO . 9 RCL . 8 1/X RCL . 7 F[I] $1 / X$ 0 RCL . 6 F[I] $+$ 1/X RCL . 4
** program instructions are described in the HP-15C owners number of the longitudinal resonance desired. The output gives: the real component, the imaginary component, and finally the magnitude. From the magnitude, the correction in dB is obtained.

A comparison of the correction obtained using the above program was made with a numerical solution obtained using the IBM XT microprocessor and the "Pspice" circuit analysis software made available by the Electrical Engineering Department at the United States Naval Academy. The parameter variables were the same as listed in table E. 1 with the exception that $C c$ and $C b$ were given values of .01 uF for simplicity. The tabulated comparison of results is given in table E. 2.

| Mode \# - | HP-15c corr solution (dB) | $\begin{aligned} & \text { IBM-XT corr } \\ & \text { soluti on }(d B) \end{aligned}$ | discrepancy <br> (dB) |
| :---: | :---: | :---: | :---: |
| 1 | . 054 | . 056 | -. 002 |
| 2 | . 045 | . 045 | 0 |
| 3 | . 043 | . 044 | -. 001 |
| 4 | . 042 | . 044 | -. 002 |
| 5 | . 042 | . 044 | -. 002 |
| 6 | . 042 | . 043 | -. 001 |
| 7 | . 041 | . 040 | +. 001 |
| 8 | . 041 | . 040 | +. 001 |
| 9 | . 041 | . 040 | +. 001 |
| 10 | . 041 | . 040 | +. 001 |
| ******** | no further chan | through mode | $23 * * * * * * * * * *$ |




A plot of the proper correction is shown in figure E.2. The correction due to the temporary neglect of Cc in the analytical solution for [el/Vin] is constant to within a value less than . 01 dB .



## correstion

> This concludes the calcuiation or, ohe correction to account for the acoustic sighal dropped acooss the o. a. blucking capacitor. The correction is ath aimost conseant $+.05 \mathrm{JE}$.

THE OPEN CIRCUIT VOLTAGE SENSITIVITY
CALIBRATION OBTAINED FROM OBSERVING A
CHANGE IN MICROPHONE CAPACITANCE WITH A
CHANGE IN BIAS VOLTAGE

When the received signal voltage from a condenser microphone is simulated by a small change in the bias voltage and when the definition of the microphone open circuit voltage sensitivity is considered,

$$
M_{0}=\frac{\delta V}{\delta P}=\frac{\partial V}{\partial C} \frac{\partial C}{\partial P}=\frac{\partial V}{\partial C} \frac{\partial C}{\partial V^{2}} \frac{\partial V^{2}}{\partial P} \quad \text { Equation } F .1
$$

Next, the force on a parallel plate capacitor due to the charge on the plates is given by [Ref. 36]:

$$
F=\frac{Q^{2}}{2 \epsilon_{0} A}
$$

Equation F. 2

The electrostatic pressure is simply the force divided by the effective backplate area. This electrostatic pressure
is used to simulate the acoustic pressure.

$$
-P \sim \frac{F}{A \text { (bevelate) }}=\frac{Q^{2}}{2 \epsilon_{0} A^{2} \text { (brexplate) }}
$$

Equation F. 3

The electrostatic pressure shown here only acts on a small portion of the microphone diaphragm; where the charge is concentrated opposite the backplate. Since it is an acoustic pressure that is being simulated, an interpretation of the "area" in the above equation is necessary. If the electrostatic pressure were applied equally to the entire diaphragm, this would be a proper simulation of the acoustic pressure. Since the electrostatic pressure is only applied to a portion of the diaphragm, the use of the diaphragm area in the above equation results in a low estimate for the simulated acoustic pressure. Similarly, since the acoustic pressure is actually applied to an area greater than the backplate area, use of the backplate area in the above equation results in a high estimate for the simulated acoustic pressure. As an initial estimate, the average of the two areas is used with the appropriate uncertainty to simulate the acoustic pressure.

When the definition of capacitance as $C=Q / V$ is substituted into equation $F .3$, we obtain,

$$
p=\frac{C_{0}^{2} V_{0}^{2}}{2 \epsilon_{0} A_{e}^{2}}
$$

When $C=Q / V$ is differentiated, we obtain,

$$
\frac{\partial V}{\partial c}=-\frac{V_{0}}{C_{0}}
$$

Equation F. 5

Solving equation F. 4 for the square of the bias voltage,

$$
V_{0}^{2}=\frac{2 \epsilon_{0} A e^{2}}{C_{0}^{2}} p
$$

Equation F. 6

The partial derivative of $V^{\wedge} 2$ re $P$ is given by:

$$
\frac{\partial V^{2}}{\partial P}=\frac{2 \epsilon_{0} A e^{2}}{C_{0}^{2}}
$$

Equation F. 7

The slope of the experimental straight line fit obtained in equations 3.17 yields the magnitude of the partial
derivative of $C$ with respect to the square of the bias voltage. Thus, the magnitude of the open circuit voltage receiving sensitivity for a condenser microphone is given by combining equation F.1, equation F.S, the experimental slope, and equation F.7.
$M_{0} \sim \frac{V_{0}}{C_{0}^{3}}\left(\frac{\Delta C}{\Delta V^{2}}\right)\left(2 \epsilon_{0} A_{e}{ }^{2}\right)$
Equation F. 8

The fractional uncertainty in this sensitivity will be:
$\frac{\delta M_{0}}{M_{0}}=\left\{\left(\frac{\delta V_{0}}{V_{0}}\right)^{2}+\left(3 \frac{\delta C_{0}}{C_{0}}\right)^{2}+\left(\frac{\delta S L O P \varepsilon}{S L O P \varepsilon}\right)^{2}+\left(2 \frac{\delta A_{e}}{A_{e}}\right)^{2}\right\}^{\frac{1}{2}}$ Equation F. 9

Table F.1, shown below, gives the computer program used to evaluate the above equation.

Register usage


HP-15c scientific hand-held calculator program *

| $F$ LBL A | $X$ | $g$ LOG |
| :---: | :---: | :---: |
| RCL . 1 | RCL . 2 | 2 |
| 3 | $\times$ | 0 |
| $y^{\wedge} \mathrm{x}$ | RCL 3 | X |
| $1 / X$ | $g x^{\wedge} 2$ | 2 |
| RCL . 0 | X | 0 |
| X | RCL . 4 | - |
| 2 | X | $g$ RTN |

* Program instructions are described in the HP-15c owners handbook.
** The W.E. 640 AA microphones had the following basic capacitances (excluding the BNC connectors) and measured values of [dC/dV^2] (see equations 3.17):

Mic. "Co" w/o extender - [dC/dVへ2] sigma []
Serial \#1082 - 49.37 Pf 2.47E-17 1.2E-18
Serial \#1248 - 49.41 Pf 3.66E-17.1.6E-18
Serial \#815 - 46.48 Pf - 5.92E-17 8.2E-19
*** Ae is the effective area estimated by obtaining the average of the backplate area and the diaphragm area. The average fractional uncertainties in the above variables are: Ae ~.38, slope ~.04, Co ~ $\sim$. 01 , Vo assumed exact.

Table F. 1 HP=15드_program_for_Mo_colaculation

The values of Mo that result from the above equation
are:
W.E. $640 A A$

Serial \#1082 -------
Serial \#1248
Serial \#815
[dC/dV^2] Mo
$245 \mathrm{~Hz} \sim \mathrm{Mo}$
[figure 5.8]
$-55<$ ~ $-50<-45 d B \sim-49.5 d B$
$-52<\sim-47<-42 d B \sim-48.6 d B$
$-46<\sim-41<-36 \mathrm{~dB} \sim-45.7 \mathrm{~dB}$

These rough calculations are seen to be in agreement with more accurate calibrations shown above as obtained from figure 5.8.

## APPENDIX G

A PRINTOUT OF RAW DATA
FOR THE PLANE WAVE RESONANT
RECIPROCITY CALIERATION IN
BOTH A LONG AND A SHORT TUBE

After data acquisition, the computer program listed in appendix $B$ stored data on a magnetic tape. The data so stored is printed in this appendix without consideration of significant figures.

The format used in the different data sets is given below:

L7 - The mode number of the longitudinal resonance.

M1

M2 - Reciprocity calibration for the side B microphone \{V/pa\}.
M3 - Comparison calibration of "C" microphone based upon Mi \{v/pa\}.
M4 - Comparison calibration of "C" micropinone based upon M2 \{V/pa\}.
M5 - Comparison calibration of the side A microphone bised upon M2 \{V/pa\}.
M6 - Comparison calibration of the side $B$ microphone based upon M1 \{V/pa\}.
PS[1,N] - Atmospheric pressure \{pa\} for midtime of side A data.
PS[2,N] - Atmospheric pressure \{pa\} for midtime of side G data.
$T S[1, N]$ - Temperature $\{d e g C\}$ for midtime of side A data.
TS[2,N] - Temperature $\{d e g C\}$ for midtime of side B data.
C - \{long tube data\} System identifier used in program.
$C$ - \{short tube data\} $A$ side bias voltage \{Volts\}.
A2 - Drive voltage in RMS millivolts.
V1[1,N] - Comparison voltage, Vca \{Volts\}.
VI[2,N] - Comparison voltage, Vcb \{Volts;.
G9[1,N] - Calculated effective gamma at midtime of A side data. $G 9[2, N]$ - Calculated effective gamma at midtime of $B$ side data. $V[1, N]$ - Ravined signal voltage, A side receive \{RMS Volts\}. V[2,N] - Ravined signal voltage, B side receive \{FMS Volts?.

```
Q[1,N] - Ravined quality factor, A side receive.
Q[2,N] - Ravined quality factor, B side receive.
F2[1,N] - Ravined resonant frequency {hz}, A side receive.
F2[2,N] - Ravined resonant frequency {hz}, B side receive.
C1 - {long tube data} System identifier used in program.
C1 - {short tube data} B side bias voltage {Volts}.
```

The data that follow are grouped in seven sets. The
first five sets list "long tube data" and the last two list
"short tube data". At the beginning of each data set, the
following library data is provided:
[NAME OF STORAGE TAPE] [ARRAY LOCATION ON TAPE]
[LONG OR SHORT TUEE] [MODE NUMBERS OF DATA IN SET]
[SERIAL \# OF SIDE A MIC] [SERIAL \# OF SIDE B MIC]
*\{Side a bias voltage\} \{sigma side a bias voltage\}
*\{Side b bias voltage\} \{sigma side b bias voltage\}

* This format 15 included only for the long tube data.

2 MAY DATA TAPE，ARRAY STORAGE 1 to 23
LONG TUBE MODES 1 TO 23
A SIDE $=815 \mathrm{w} / E T$ ， B SIDE $=1248$
A SIDE BIAS $=116.334$ VOLTS，SIGMA $=.001 \mathrm{VOLTS}$ $B$ SIDE BIAS $=118.464$ VOLTS，SIGMA $=.007$ VOLTS

```
L`=
M1=2.235738733.37E-2
M2= こ.1324893007こE-2
M3= 7.94895356556E-3
M4= अ.947>58F262EE-3
MS=2.3.35.017%137E-2
M6= こ.13281794138E-2
P561. 21%=100083.387675
```



```
T5<1, 21;=21.377
T5:2, 21 \= 21.8625
C=2
LT=
M1=2.34日409日2187E-2
M2= 2.1177895:37EEE-2
MJ= 7.34137111396E-3
M4= 子.34こE.24@4945E-3
MS=2.940255@9773E-2
MG= こ.11745704879E-2
P5:1, こ2 % = 10以И58.188175
```



```
T5<1, こ2 = = 1.8285
TG&, 2こ == 21.8ここ5
C=2
LT= 1
M1 = 923518020569
M2= こ.192gこ156TSEE-こ
M3=5.745.0日V23414E-3
M4= 5 730848.70508E-3
M5= .928445こ37375
M6= 02108924589%
F5:1:23 %=19@128.189491
F5!2. 23 \=100日22.523611
T5<1, 23 y= 21.77.
T562, 23 = 21.7735
r:= 2
```



「1 = 2

U161．22 ）＝1．18949638889E－3
リ1く2，22 $=$＝29853611111E－4
万961， $22=1.39633713741$
GG（2．22）＝1．39630763786

U（2．22 ）＝3．59764483367E－3
日（1．22）＝78． 255767213
ゆ（2，22）＝ 78 9577945821
$F 2(1,22)=490.255804847$
$F 2<2,2 z y=496.299979984$
■1＝ 2
Rこ＝こ247． 23.2958
U1（1，23）＝，25259444444E－4

$\Gamma 961,23)=135418845558$
ज9r2． 23 ）＝1．3．7418796276
！！：23 ）＝2．E6132こ？日293E－3
いくこ，23 ）＝2．824788322こ．EE－3
Q（1，23）＝E2．1917484365
Q（2，2З）$=52.4975851541$
Fごく1，23 り＝243． 565388871
F2r2，23 $=243.81748575$
ri＝ 2

| $\begin{aligned} & L 7= \\ & M 11= \end{aligned}$ | こ 39139097899E－2 |
| :---: | :---: |
| M2＝ | $1571.5403913 E-2$ |
| M3 $=$ | $29815244316 E-3$ |
| M4 $=$ | 9．20627313686E－3 |
| MS $=$ | 2．99073944999E－2 |
| M6＝ | 15760141654E－2 |
| P5：1． | $17 \geqslant=100995.38728$ |
| P5くこ | $17 \times=10 \mathrm{MO89}$ ．920793 |
| T5i1 | 17 ）$=22.0965$ |
| T5＠ | 17 ）$=2.085$ |
| に＝2 |  |
| $\llcorner\vec{r}=\mathrm{E}$ | $E$ |
| M1＝ | －38917573899E－2 |
| $M 2=3$ | 2．130ETアE1447E－2 |
| $M 3=$ |  |
| $M 4=8$ | 8．7．3739972942E－3 |
| M5＝ | 2．38894159371E－2 |
| $M E=2$ | 2．130¢5027384E－2 |
| P5（1， | $13 y=150120.1198$ |
| P5i』． | 13 ）＝1061ころ．853011 |
| T5il． | $13 y=22.048$ |
| T5：2， | $18)=22.5355$ |
| $=2$ |  |

$L \bar{i}=5$

$M 2=$ ． 1214242646
$M J=$－56アヴ5И64812E－3
M4＝G．56617アS428GE－3
$\cdots 5=2.84948441254 E-2$
$M E=2.1429$ ज5555JE－2
PE © $1,193=1$ פ0694． 0.53371
PS\＆
T5（1，19）＝21．986

C．$=2$
$L \bar{i}=4$

$M 2=$ ご．125以29．38119E－2
$M 3=3.1933377736 E-3$
$M 4=8.19398292885 \mathrm{E}-3$
$M 5=$ 2．35598921786E－2
ME＝こ．124E581E13こE－2

F5（2，20 $\because=1$ 100G4．787E2G
T51，20＝21．927
T5ふ．ごツ
$\Gamma:=\Xi$

Аコ＝2ご5．31523138
$41(1,17)=.691637175$
U162． $17=1.2313083333 E-7$
G9（1，17）＝ 1.39884317379
$G 9(2.17)=1.39884363516$
$\because(1,17)=3.93612196973 E-3$

以（1．17）＝124． 0079030.55

F2（1，17）＝1717．92839515
F2！2．17：＝1717．86535535
$\Gamma 1=2$

$\because 1(1.13)=1.57981388389 E-3$

$G 9(1,18)=1.39851713733$
59（2，18 ）＝1．39861782378
U1． 18 ）＝3．8524939282E－3
$v(2,18)=4.05158635725 E-3$

$Q(2,18)=117.498178043$
$F 2\{1,13\}=1471.7617945$
F2く2， 18 ン $=1471.8013357$ ق
C1＝

リ1（1，1？）＝6015
U1（2． $19>=1.21258055550 \mathrm{E}-3$
曰9（1，19）＝ 1.3982689872
F9（2．19）＝ 1.39832645124
リ（1， 19 ） $341962322 E-\Xi$

曰（1．19）＝113．769日すら321
「こ． 19 ）＝113．74901541：
F261， $19 \div=1225.52253759$
F2《2．19 ）＝12ご．451ネ1399
$C 1=\Xi$
$\mathrm{AD}=272 \mathrm{Q} .4 \mathrm{ESOG7}$
$\because 161,2 日\}=1.48943722222 E-3$
U1（2．26）＝1．16455333．333E－．
ज9（1．2日 ）＝1．3979．2日9953
ज9\＆2，20 $=1.39793257259$
$\boldsymbol{J}(1.2 日)=3.86286495644 E-3$
$V(2,20)=4.05951667387 E-3$
Q（1．20）＝1月6．日GT13418

Fごく1，2ด ！＝980．1235日1844
F262， $20,=980.158060585$
$C 1=2$


|  |
| :---: |
| Ч1《2． 13 ン 961483875 |
| ज9：1． 3.3 ＝ 1.3994235428 |
| G9（2， 13 ）$=1.39942338225$ |
| $4(1.3)=$ 3．84229614104E－3 |
| $\psi(2,1 \Xi)=4.02959648857 E-3$ |
| Q（1， 1.3$)=125.335432342$ |
| Q（2，13）＝185．473935026 |
| $F 2$（1，13）＝2701．12073709 |
| F2（2．13）＝ごア1．15197878 |
| $C 1=2$ |
|  |
|  |
|  |
| G9＜1． 14 ＝$=39931823153$ |
| G9（2． 14 ）＝1．39331019348 |
| 4 （1，14）＝ $314.349652 \mathrm{E} 6 \mathrm{E}-3$ |
| U（2，14）＝4．0179192＠77ご－3 |
| b（1，14）＝156．14219529 |
| $Q(2,14)=160.1558398$ |
| F2 2,14 ）$=2454.59613366$ |
| F2゙2． 14 ？＝2454．6556日57亏 |
| $C 1=2$ |
| $\mathrm{HZ}=1$ アア5．4Eここ1955 |
|  |
| W1（2， 15 ＝ $36948611111 E-3$ |
| 59（1． $15 \%$＝ 3.3918144365 |
| 「962，15＝1．3951204244 |
| U1，15＝З 50377e5747E－3 |
| W－15 $=4.0553407014 \mathrm{E}$－3 |
| V（1． 15 ）＝153．420．53512 |
| $Q(2) \quad 15$ ） 4.3 .25576155 |
| F2：1． 15 ）2209． 22 219075 |
|  |
| に1＝こ |
|  |
|  |
| V1くこ． $16 \geqslant=1.2914 .561111 E-3$ |
| $G 9(1.16)=1.39992595884$ |
| G9¢2． 16 ¢ $=1.395026$ 9933 |
|  |
|  |
| Q（1．16）＝134．255157767 |
|  |
| $F 20.169$ ¢ 664.355954 |
| $F 262.16: 1964.40199164$ |
| C1＝2 |


$A 2=1201.81509816$
， 1 （1，ヨ）＝2．297955555．56E－3
い1（2，g）＝．001766125
曰9（1，g）＝1．39575669961
$59(2,9)=1.39975654545$
奴 1,9 ）$=3.87566246879 E-3$
$V(2, \quad-)=4$. 日6T152799日7E－3
E1． 9 ）$=213.955475391$
U（2， 9$)=219.147965522$
$F 2(1, \exists)=3683.5947849$
F2（2， 9 ）＝3683．50579893
に1＝2


V1（2，1以 ：＝1． $69646944444 E-$ E
ज9（1， 10 ）＝1．39969825988
ज！（2．1日 ）＝1．39568790．3．5
い（1．1（
$V(2,10)=4.97252565491 E-3$
Q（1，1日）＝211．日5日4450日8
区（2．16）＝211．133499315
F2：1，1日 $>=3437.8572222$
F2゙2．1日 ン＝ 3437.8827435
$E 1=2$
$\mathrm{HZ}=12 \overline{\mathrm{~F}} .491342 \mathrm{BE}$
$\because 1(1,11)=2.07049166667 E-3$


G？2， 11 ！＝ 2956839115
！（1，11）＝3 192456アフロ6E－3

亿1． 11 ）＝204． $565.31299 ?$

F2（1， 11 ＝$=192.0220345$
F2r2， $11 \therefore=3192.13719912$
$\because 1=2$
H2＝15E．16474G19

リ1《（ 12 ン $1.5424972222 こ E-3$
斤9（1， $12=1.39552119947$
G9（2． 12 ）$=1.39952150226$
4（1： 12 ）$=$ I． $5594966101 E-3$
い（こ． 12$)=4$ ．日7 $112332376 E-3$


F2ध1． 12 ）$=247.4534633 \because$
Fで（こ． 12 ）$=2947.491526$ 年
$[1=2$




112013n51934
$M 5=3.91194657396 E-2$
$M G=2.21063410568 E-2$
PE（1，5 ）＝10氏129．852313
P5（2． 5 ）$=100128.519524$
TS（1，5 ）＝21．127
TS（2． 5 ）$=21.1595$
$\Gamma:=2$

Lア＝1：

$M 2=-223392491 E-2$

M4＝1．34ごG8493532E－2
MS＝3． $12419757060 \mathrm{E}-2$

052839

T5（1，6 $=$ 21．28．
T5（2，E）＝21．ヨ115

M2＝2．215この上以以日ファEー2

M4＝1．こ．ご $115734 \mathrm{E}-2$
$M 5=2.127 E 5730829 E-2$

P5（1， 7 ）＝100095．20583

T51．$\quad 7=21.4115$
$5(2,7)=21.4375$
$L_{i}=1 E$

リ2＝ご．こ17478ロロ日8EEー？
M3＝1．35141459287E－2
M4＝1．351日जア413ロアE－き
MS＝？ $3385715651 E-2$
$M 6=2.218145386 \mathrm{E} 4 \mathrm{E}-2$
F5（1，3 ）＝195070．G83983

$T 5(2, g)=215495$
$I:=a$

Нこ＝11日に．2039691
$\because 1(1$ ，$\quad$ ， $2.212003333 .3 E-3$
！1（2． 5 ）＝1．70330555556E－3
GG（1，5 ）＝ 1.40 日ण゙241921
$G 9(2,5)=1.40650109154$
$\psi(1,5)=3.61$ 亿72 $529368 E-3$
$\psi(2,5)=4.9 日 988381666 E-\Xi$
17（1． 5 ）＝234．328428378
Q（2， 5 ）$=235.545019298$

$F 2(2,5)=4560.500 ア E 27:$
$\Gamma 1=2$
$A 2=1130.18596$
い1《1，曰 ）＝2． $32498333333 \mathrm{E}-3$
ज1（2， 6 ） 001300 B
G9（1，E $=1.3994872123$
G9（2，E）＝1．39944754351
$\psi(1,6)=3.34955778394 E-3$

®（1．6 ）＝229．695268876
$0(2,6)=223.937779421$
Fぎ，5 $)=4416.5969345$
$F 2(2,6)=4416.82232224$
ㄷ：$=2$
$\mathrm{H} 2=11 \mathrm{~B} .121626$
$\because 1(1, \quad 7)=2.32751388889 E-3$
V1i2，$\vec{i}=1.78875555556 \mathrm{~F}-\boldsymbol{7}$
万9（1，7）＝1．39989177449
G9（2， $7=1.3995906927$
$\because(1: 7)=3.31214664251 E-3$

R（1，$\quad 7)=297 \quad i 5553394$

F2ध1，$\vec{子}$ ） $4171.4 \xi 3225$
$F 2(2,7)=41 \vec{i} .6831117$
I：1＝2

リ1（1，$\because$ ）$=$ 日気 21775

ज9（1，3）＝1．29383160342

$W(1.3)=3.8 \underline{3} 424246814 \mathrm{E}-3$

可（1：3）＝222． 987426882

$F 2(1,3)=3$－

に1＝2


| $\llcorner\vec{F}=$ ご |
| :---: |
| $M E=25$ |
|  |
| M4＝9．95969ロロロ91iE－3 |
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$\mathrm{Hz}=117 \mathrm{~B} .41 \mathrm{n} 4 \mathrm{E} 27$

$\Downarrow 1(2,1)=1.21223333333 E-3$
โ9（1， 1 ）＝1．49016795388

リ（1． 1 ）＝3．33815482475E－3
い（こ，1）＝4． 185519552 24E－3
R（1： 1 ）＝243．552616581
Q（2． 1 ）$=243.839245121$
$F 2(1,1)=5641.989675$ Q3
$F こ(2,1)=5641.9592 .321$
$\Gamma 1=2$
$\vec{A}=113.442915$
リ1（1，2＝1．755897こ222こE－3
416，こ＝1．3T153611111E－3
T9（1，2 2 ＝ 1.40013307529
GS（2．2 ）＝1．40日13858673
$U(1,2)=5.81422475641 E-3$
$W(2, \quad 2)=4.91769680303 E-3$
Q（1，2）$=242.511154852$
曰（2，2 $)=242,639154593$
F2：1，2 $=5.95 .32587741$
$F 2(2,2)=5395.71$ 日60223
C． $1=2$

$\forall 1(1,3)=.61962 .375$

$G 961.3>=1.4015394676$
$G G(2, \quad)=1.40 \mathrm{G} 15 \mathrm{E} 4502$
$!1(1, ~ J)=$ ․ 3749859914E－3

Q（1．З ）＝24日．737451．393

$F=(1,3)=5149,3236136$
F2〔？，J）＝5149．6290547
$\therefore 1=\Xi$
Нご＝113日． 2414 国
$\because 1(1,4)=$ シ 1214 666657E－3
い1（2， 4 ）＝ 1 6665388388GE－J
上9（1．4）＝1．40日0541889
59（2． 4 ）＝ 1.4 日6区5234895
$4(1,4)=3.83317744898 E-3$
（2， 4 ）＝4．日9296815325E－こ
10（1．4）＝2．55． 35890517


F2！2， 4 ）$=49$ 可 15 亿54914
$\check{\square} 1=こ$

2 MAY DATA TAPE，ARRAY STORAGE 24 to 46 LONG TUBE MODES 1 TO 23
$A$ SIDE $=1248, B$ SIDE $=815 \mathrm{w} / E T$
A SIDE BIAS $=116.307$ VOLTS，SIGMA $=$ UNK．
B SIDE BIAS $=118.495$ VOLTS，SIGMA $=$ UNK．

| $\begin{aligned} & L \bar{r}= \\ & M 1= \end{aligned}$ | 2． $99642127291 E-2$ |
| :---: | :---: |
| $M 2=2$ | 2．89521237497E－2 |
| $m 3=?$ | 7． 3 33731 316 E －3 |
| M4 $=$ | －．930E8766135E－3 |
| $M 5=2$ | 2．W9FE1962688E－2 |
| $M \in=$ | 029003246071 |
| P5：1 | $443=109597.256845$ |
| FSく2． | $44 y=109066.856859$ |
| T5：1． | $44 y=22.5785$ |
| T5：2． | 44 \％22．679 |
| $C=2$ |  |
| LT＝ |  |
| $\cdots 1=2$ |  |
| $112=2$ | －9515658区Eご2 |
| $M 3=7$ | ？？6\％家14715E－3 |
| M4 $=$ i |  |
| $M 5=2$ | 2．9774？9849日．3E－2 |
| M6＝ 2 |  |
| F5 \％ 1 ． | 459 99964．7915763 |
| FS（ | $457=99967.2581618$ |
| T5ヶ1， | ， 45 ＝22．712 |
| T5く2． | $45:=22.718$ |
| $r=2$ |  |
| $L \vec{r}=1$ |  |
| $1 \cdot 11=3$ | 2．03015．514．59E－2 |
| $M 2=3$ | －888こ6893266E－2 |
| MS＝ | 5．73279017959E－3 |
| M4＝5 | 5． $3741576655 E-3$ |
| $M 5=2$ | 2． $1318354994 \mathrm{E}-2$ |
| $M E=3$ | 2．88594亿36829E－2 |
| P5＜1， | $45 \div=99987.3908229$ |
| Fらくご， | $46)=99482.5243263$ |
| TSC1， | ， 469 2 22.7235 |
| T562， | ， $46>=22.735$ |
| $\underline{=}$ ？ |  |



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Hご＝こごき このに343．34
$1141,403=1.22211944444 E-3$
U1〔2：4日 i＝1．65974166657E－3
G9〈1．4日〉＝1．33881117756 G9（2． 4 G$)=1.398819898 \mathrm{~B} 2$
$14(1.45)=3.94425302151 E-3$
U（2．48）＝3．3151859385EE－3
Q（1， 45$)=123.133202767$
时（2，40）$=123.614970089$
F261，4日3＝1719．05898422
$F 2(2,4 日)=1719.030291$ 亿
C1＝
$\mathrm{H}=$ ごヨG．この日16日34
！1＜1． 41 ソ
$41(2,41)=1.60018055556 E-3$
$\boxed{G}(1,41)=1.33853141163$
GG（2． 41 ）$=1.33858111401$
$4(1: 41)=7.45452338362 E-3$
䙺（ご，41）＝3．83134226245E－3
○（1．41）＝11E．78Й92136
Qथ， 41 ）＝116．316795597
$F 2(1,41)=1472.34557151$
F3！2， $41 \because=1473.51791455$
$E 1=2$



जヨ（1． 42 ）＝1．338286．3133
G9：2． 42 ＝ 1.39625986 亿2
いर $1: 42$ ）$=3.9545164633 E-3$

R（1． 42 ）＝112．272月558
曰， $4 \because$ ）＝ $113.3 E 35193$
$F \cong 1,42 \geqslant 1225.57383845$
$F こ 6,4 こ う=1226.64962842$
$G 1=2$



$L_{i}=11$
M1 = にじ14きヨロ1アき7
$M 2=3.64568169925-2$

- 1 M39以145こ195Eース
$M 4=1.69216504233 E-2$
$M 5=2.14918117845 E-2$
$M 6=3.04028642683 E-3$
$P 5(1,35)=160112.720043$
F(2. 36 ) = 100998.853833
TS(1, 36) $=22.431$
T5(2. 36)= 22.45
$B=2$
Li=1

Mぶ=1. ロコフ47261267E-2
$M 4=1.02765067332 E-2$
$M 5=2.13834346499 E-2$
M6= 3 (1)
(1) $=160$ 亿. 0.44484
F.5ic, $37=10101887.987524$
$T 5(1,37)=22.4095$
TS: $2.37=22.392$
$\Gamma=2$
$\llcorner\vec{i}=\overrightarrow{9}$
$M 1=$ 2. 13 397にテア52E-2

$M 3=9.92684844121 E-3$
$M 4=9.92799014118 E-3$
MS= 玉.1.ヨこここア8253E-2
ME= З 6以5 $162257 E-2$
F5 (1, 3 (

T5r1, 38 = 22.4405
T5(2. 38 ) = 2こ.4475
$L \vec{i}=8$
$M 1=0212510243$
$M 2=2.9859583279 E-2$
M3 = 3.4.57E5185951E-3
M4 = 9.456171726த5E-3
$M 5=2.1252 E 993495 E-2$
$M \in=$ こ. 986425509日GE-2
P5 (1, 39 ) = 100558.655155
F5:2. $\because 9 \%=10905.3216$
151, 39 = 22. 489
$r=2$



$\forall 1(2,32)=2.31394444444 E-3$
जG（1，32）＝ 1.3997206294
G9（2．32 ：＝1．39972031649
隹 $1,32=3.958999761$ 25E－3
W（2，32 ）＝3．81487681868E－3
Q（1，32）＝218．564849973
Q（2． 22$)=218.619873435$
$F 241, \quad 32)=3636.89584035$
F2（2，32 ）＝3686．935241日8
$C 1=2$
A2＝1225．GEE6451
$\because 1<1,33$ ？$\because$ ． $617333333 \Im E-3$
$\| 1$（2，3J＝2．22663888389E－3
［9（1．33）＝ 1.3995 .5111113
$G G(2,3.3=1.39965098525$
$\because(1: 33)=3.95545565692 E-3$
$\psi(2,3)=3.826318639 \in 9 E-3$
Q（1． 33$)=210.929$ 日 325
Q（2， 35$)=210.756278059$
F2（1，33）＝3441．1435127ヨ
F262，3．3）＝3441．13734394
C1＝2
$\mathrm{A} 2=1273.4922$ פ． 24
U1〔1． 34 ）＝ 1 ．5E2こ1388889E－3
V1（2． 34$)=$ ． 133516 E6EFE－3
G9（1． 34 ）＝1．39057416425

U（1，34）＝ヨ．984以フ725317E－3

1（1． 24 ）$=205.325852391$

$F 2(1,34)=3195$ 톤4539

$\therefore: 1=2$


$\because 1(2,35)=2.01594444444 E-3$
G981，35＝$=399436$ 33181
［9（2． 3.
$4(1,35)=3.94425902111 E-3$
W（2，35 ）＝3．81782319012E－3
Q（1： 3.5$)=193.741530115$
Q（ت，こ与 ）＝153．-212699 日G
$F 261,35:=2949.3368025$
F2《2， $35 \%=2949.9332486$
$11=2$



ひ1（2，28）＝2．27393838を89E－3
G3（1，28）$=1.39994365514$
E9（2．28）＝ 1.33934279627
W（1．28） $2.26625131902 E-3$
$U(2,28)=3.81529179986 E-3$
Q（1，23 ）＝235．50733316
円（2，28）＝235．389245916
$F 2$（1， $28, i=4659.3274373$
$F$ F（2， 28$)=4669.56046784$
C1＝こ
$\overline{H 2}=113$ 日 1886411
，1（1．29）＝1．735230555．56E－3
い1ヶ2，29 ）＝2．336930555．56E－き
G9（1，29 $\quad=1.33989329209$
［542，29 $=1.39989314313$
4（1．29）＝3． $93561680731 \mathrm{E}-3$
Vi2．29 ）＝3．85992996日6ロЕ－3
曰（1，29）＝229．748996147
区く2．29＝239． $2667049 \mathrm{G1}$
$F 2(1,2 \exists)=4424.53728158$
$F 2(2,2 \xi)=4424.69771542$
$C 1=2$
A2＝ 1136.123 ESBG
リ1ヶ1．ふ®＝

ज9 1.30 ）$=1.33934088567$
БGく2．



凸（2，च
$F$ ご 1,30 ソ＝4173． 2983292
$F 2$（2，
E1＝$=$

$41, ~=1,72411388939 E-3$

जG： 51 ）＝ 1.3957344854

$4(1 \cdot 31)=3.95842$ 亿88945E－3
Vi，31 ）＝3．82E65こ173ろ7E－3
17（1．31）＝222． 441338965

Fご，31 $\because=332.90774758$
$F 2 \Omega 231=3929.9494997$
L1＝？


FE＝ $1154.4 E 164223$
U1 1,24 ㄴ $1.15439638839 E-3$
U1（2， 24 ）$=1.595830 .55556 \mathrm{E}-3$
โ9（1， 24 ）$=1.40512160888$
59（2． 24 ）＝ 1.40512173892
b（1． 24$)=3.95544252194 E-3$
$V(2,24)=3.80548293712 E-3$
Q（1，24）$=243.559919769$
0（2， 24 ）＝24．5．5ア1713981
Fとて1，24 ）＝5649．5106390．3
F2イ2． 24 ）$=5 E 49.64119488$
C1＝
$\mathrm{H} 2=113$ ． 4452513
$41(1,25)=.00131405$
U1（2，25）＝ $1.78586944444 \mathrm{E}-\mathrm{T}$
G9（1． 25$)=1.45658551037$
G9（2，25）＝1．40008254191
U（1．25）$=3.95556579916 \mathrm{E}-3$
U（2，25）＝ $3.31753954396 \mathrm{E}-3$
Q（1，25）$=241.831716762$
$Q(2,25)=241.696819496$
$F 2(1,25)=5494.49815415$
$F こ(2,25)=5404.63294635$
C1＝2
Н2＝ 1130 3巨g9． 167
$\because 1$（1，26 $)=1.4597944444 \mathrm{E}-3$
V1：2．26 ）＝1．992416665E7E－3
Б9（1，26）＝1．40654039822
G962． $26=1.40$ 日03939679
U（1．26）$=4$ 日曰アS21606日2E－3

冋《1．26 ）＝2こ9．5こ27ア1819
Q（2． 2 E$)=233.335471196$
$F 2(1,26)=5159.32216106$
$F 2<$ 2． 26 ）$=5159.57467125$
$\Gamma: 1=2$
$A 己=1106 \cdot 3692308$
U161，27 $\quad=1.56541111111 \mathrm{E}-3$

Б961，27 ）＝1．39999191569
59（2，27）$=1.39999136144$
$U(1,27)=3.93293 .315566 \mathrm{E}-3$
U（2，2T ）＝3． 8 亿1616406994E－З
民（1．27）＝235．827172982
区（2，27 ）＝236．3947601022
F2（1，27）＝4915．513131044
F2！2，27 $1=4915.3$ И645211
R1＝2

2 MAY DATA TAPE，ARRAY STORAGE 47 to 69
LONG TUBE MODES 1 TO 23
A SIDE $=1248$ ，B SIDE $=1082 \mathrm{w} / E T$
A SIDE BIAS $=116.333$ VOLTS，SIGMA $=.002$ VOLTS
B SIDE BIAS $=118.513$ VOLTS，SIGMA $=.003 \mathrm{VOLTS}$


|  |
| :---: |
| V1（2，67 $=1.17574388839 \mathrm{E}-3$ |
| G9（1，67 ）＝1．39731598881 |
| $G 9(2,67)=1.39731652428$ |
| $U(1,67)=$ ？ $06.371454356 \mathrm{E}-3$ |
| W（2，67）＝3．10585768612E－3 |
| 历1． 5 ¢ $=93.4060 ⿴ 囗 6179$ |
| Qr2，F－7 $=93.4526957501$ |
| F2¢1，67 $=735.255284804$ |
| F2，2．57＞＝735．184日8゙心1 |
| $\mathrm{C} 1=1$ |
| H2＝こ23？BES44以E |
|  |
|  |
| 591． 63 ＝$=3.363542414$ |
| G9，6S $=1.396355968$ |
| 41． 63 ）＝2．4世2ア993938E－3 |
|  |
|  |
| 区（2， 68$)=79.881987842$ |
| $F 2$ ， 5 ，$=491.02763537$ |
|  |
| $\mathrm{C}=1$ |
| $\overline{\mathrm{H}}=3247 \mathrm{Se3} \mathrm{E} 45$ |
| $\because 1(1,69)=5.54873611111 E-4$ |
|  |
| 59 ¢189 |
|  |
| U（1，53 ）＝1．895555．5516E－3 |
| $V(2, E g)=1.942729995 E .5 E-3$ |
| $0 ¢ 1.69$ ¢ 62.5935503854 |
| Ø（2， 59 ）＝E2．57bu1S719 |
| $F 2$（1， $69 \%=244.231195194$ |
| $F 2$（2， 69$)=244.164963492$ |
| $\underline{C} 1=1$ |



 $U 1(2,63)=1.67539444444 E-3$


$(\|(1: 63)=3.32545544545 E-3$
$V(2, E J)=3.35851318101 E-3$
Q（1． 6.3$)=127.425135131$
Q（こ，GJ）＝126．8907249E9
F2 1,53$\}=1725.19316754$
$F 2$（2． 63$)=1$ ア2以． 15612 た $\Gamma: 1=1$

リ1（1，E4 ）＝ $1.5 E 40444444 E-3$

凸ヨ（1．54）＝1．33857741295
G9（2． 64$)=1.39857731419$
$\because(1, E 4)=3.73535469032 E-3$
$V(2,64)=3.84312149969 E-3$
O（1，64）＝ 119.582769732
 F2（1： 64$)=1474 . \overline{1532273}$ $F 262,64$ ）$=1474.12274979$ $C 1=1$

い1《1．
Uイ2，
59（1． $5.5=1.3982864159$
 $\because(1$－

日（1，65）＝115．549415323

$F 261.55)=1227.5253$
F2！$\quad E .5=1227.4489816$ $1=1$



ज9（1． $\left.66^{\circ}\right)=1.39739449398$
G9i2． 56 ＝$=1.39789493993$
$4(1.65)=3.3475444188 E-3$
U（2，6曰 ）＝こ．38745426334E－3
（1）इБ ）＝10？．日95215965
Q（2，Бた）＝107．110572782

 C1＝ 1

$L_{i}=11$
-22151549578
$M 3=.010375330287$

$M 5=2.151 E 610818 E-2$
ME= - ロ6107フ51313E-2
P(1, 59 ) $=9894.1285$.
$T 5(1,59)=22.545$
T5(2, 59 ) $=22.656 .5$
LT=1

$12=2 . \mathrm{E} 422 \mathrm{E} 24 \mathrm{GE}$ - 2
$M 3=1$. $2 E$ ED8S162E-2
$M 4=1.0267303967 E-2$

MБ = 2.04240848715E-2
P5<1, BQ $=99841.8622868$

T5:1, 50 ン $=2.760$

$L_{i}=9$


$\cdots 3=9$ 9204149494E-3

2. 4 4. $\because 72698 E-2$

FŚ . 51 : $=3945.5621299$

Tらく1.
TS:2. 61 i= 22.7515
$L_{i}=B$
$M 1=2.12942996459 E-2$
M2= 2.024209577日8E-2
$M 3=9.44553 .5698 E-3$
M4 = 9.4.31229243GE-3
$M 5=2.1286557166 E-2$
M6= 2. $92481258656 E-2$
P5:1, 52 9 $=9851.4616421$

$T 5(1.52)=22.8595$
$\Gamma=2$


$\mathrm{H} 2=1584.86591278$
1,1 （1，55 ）＝002354425 い1（2，5．5 ）＝2．3252こアフ7ア78E－3 G9（1， 55 ）＝1．39973179225 59（2， 55 ）$=1.3997313369$ $\psi(1,55)=3.21250 .3044 .59 E-3$
$U(2,55)=3.845 .5113812 \mathrm{E}-3$
Q（1． 5.5$)=228.854767296$ $0(2,55)=229.913414967$ $F 2(1,55)=3665.39442119$ $F 2(2,55)=3656.97913361$ C $1=1$

U141， 56 ＝ $2.29839444444 \mathrm{E}-3$
$41(25)=2.24510833333 \mathrm{E}-3$
GG（1， 56$)=1.39956054575$
G3（2， 56$)=1.39965984616$
$V(1,56)=3.83982679075 E-3$
V（2， $5 E)=3.86142364894 E-3$
日（1． 56 ）＝215．929541569
Q（2，56 ）＝218．823927991
$\mathrm{F} 2(1,56)=2441.48333037$
$F 2(2.56)=3441.58956932$
$C 1=1$
$\mathrm{H} 2=172 \mathrm{~B} 316 \mathrm{E} 1 \mathrm{~B}$


FE＝1847．89594613
い1（1．58＝ $3.09772222222 E-3$
V1（2．58）＝2．ロ4Eア1944444E－3
■9（1． 53 ＝ 1.39949411184
59（2．58）＝1．39949379323
U（1．55）＝3．83日3245．5167E－3
W（2，SG）＝3． $38144925427 E-3$
Q（1． 58 ）$=201.591162995$
$0(2,58)=201.659656053$
$F 2(1,5!)=2950.56475895$
$F 2(2,58)=2950.65391957$
C1＝ 1

$\overrightarrow{H 2}=1441.5483457$
$\because 161,51 \geqslant 2.282 .27222222 E-3$

G9（1，51）＝ 1.39996683598
$G 9(2.51)=1.39996589829$
$W(1,51)=3.555366488$ 日9E－3
$U(2,51)=3.88744228593 E-3$
$0(1.51)=248.975122902$
区（2，51）＝248．375889262
F （ 4,51 ）$=4667.64199183$
$F こ(2,51)=4667.267 G 1731$
$E 1=1$
A2＝1465．41ごらきゴ
$\because 1(1,52)=2.37598611111 \mathrm{E}-3$
W1《2．52 ）＝2．3799472こごきE－ヨ
万961． 52 ＝ 1.399914 亿8914
に9（2．52 ）＝1．39991293862
リ（1，52）＝さ．З2以ころ5526．53E－3
V（こ，5こ ）＝3．26958287542E－3
Q（1，52）＝ $442.536284950^{\circ}$
Q（2． 52$)=242.554852661$
$F 2(1,52)=4422.96669749$
$F 2(2.5 こ)=4423.09317435$
$C 1=1$
$\vec{H}=1489$ こア242753
$41(1,53)=.0024086$

GG（1，53）＝1．39965761951
G92，53 ？＝1．399856751区1
$\because<1.3$ ） $3.3048636912 E-3$

R（1，53）＝239．549873029

F261，S3 $=4177.27617474$
$F 2$（2． $53=4177.4278174$
C． $1=1$


U1：2． 54 ン 2．38818055556E－J
G9（1． 54 ）＝ $1.399796952 日 2$
G9（2．54 ）＝1．39979623743
$U(1.54)=3.31671636643 E-3$

D（1．54 ）＝233．173314243
Wとこ，54）＝2ここ． 167256147
F2（1，54 ）＝3．32．51653491
$F 2(2,54$ ）$=3932.64953807$
$C 1=1$

Lア= ご

M2= 日2こ1日452949
MJ= B.59593こ33767E-3
M4 = 8.70722964056E-3
$M 5=2.06568135681 E-2$
M6= 2.2日756911252E-2
F5 (1, 47) = 99923.5596158
P5 (2. 47) = 99924.7928921
T5i1, 47 $=22.336$
15:2, 47 ! $=22.3335$
Lア = ت

$M 2=2.198147943 E E-2$

M4 = 9.8984r.ç11tE-3
M5= 2.10日95E40055E-2
Mю = ご.19アフフこアビこ4E-2
F.5:1.48 3= 99920.7263592
F5 (2, 48$)=99918.5930961$
T5:2, $4 \mathrm{E}=2=2 \mathrm{~B} 4$
LT=


$m 4=1.1$ 194293793E-2


F.5: 49 = 49223.2596092

T5i1.49)=21.354
TSi. $49 \%=21.217$
に= こ
L $\vec{i}=$ ご
$M 1=$ - 1 -918この日ロ4E-?
Mコ= シ.17318475788E-2
$M 3=$ 日1214日242154
M4=1.21267617972E-2
M5 = 2.1614日939129E-2
ME= 2.175708日379EE-2


T5 (5日 ) = 21 日29
TE (2. 56) = 21.3555
C= こ

A2＝ 417 9292日4E
U1 1,47 ？$=1.5112$ W555．5EE－？

G9（1．47）＝1．40010888515
G9《2． 47 ？$=1.4$ 4010895631
$U(1,47)=3.83638526851 E-3$
U2， 47 ）$=3.87176243498 E-3$
曰（1．47）＝259．259765188 $0(2,47)=259.575489973$
$F 2 〔 1.47$ ）$=5656.10449798$ Fこ（2．47）＝5655．799938以9
$1: 1=1$




に9《2， 43 ）＝ 1.40 旬 532494
！（1．48）＝2． $25331578826 \mathrm{E}-3$
いこ， $4 \xi$ ）$=3.911$ F1252069E－3
Q（1，48）＝257．5E4117458

$F 241,48)=5407.45753503$
F2＜2． 48 ＝$=546.978$ i9158
C． $1=1$
A2＝1417．7E55255
$41(1.49)=1.3574166667 E-3$
い1くこ． 49 ）＝こ． 92299444444 Eーき
に9：4．49 $\because=1.40656558$

$U(1.49)=3.85787534215 E-3$

（1）4． 47 ） 255.471955729
（6）49＝255．391129914
$F 6$ F1， $4 \%$＝$=5158.75 E 7345$
Fこと（49＝5158．445こご4气5
$[:]=1$
$\mathrm{H}=141 \mathrm{~B}$ ．EAE1448

い1\＆2．5以＝2．1ア211656EETEー3
59（1．50 ン＝ 5 ．40日内1695811
GG（2，5月 ン＝1．40601582735
$\because(1,59)=3.82129758348 E-3$

ज（1．5以）＝249．9ワ262013

F？（1．50）＝4912．395．51683
$F こ(\Omega .5 \underline{\text { 月 }}$ ）$=4512.63068643$
$1: 1=1$

2 MAY DATA TAPE，ARRAY STORAGE 70 to 92 LONG TUBE MODES 1 TO 23
A SIDE $=1082 \mathrm{w} / E T, 6$ SIDE $=815$
A SIDE BIAS $=116.355$ VOLTS，SIGMA $=.004$ VOLTS
B SIDE BIAS $=118.530$ VOLTS，SIGMA $=.002$ VOLTS

|  | $L=3$ $M 1=15355544947 E-2$ |
| :---: | :---: |
|  | $M 2=29471$ E9N4．5EE－2 |
|  |  |
|  | M4＝7．9579EET2267E－3 |
|  | $M 5=1.93506922324 E-2$ |
|  | $M 6=2.97557109874 \mathrm{E}-2$ |
|  | PS（1， 90 ）$=1010122.319728$ |
|  | P5（2，90＝$=100117.719379$ |
|  | T5¢1．90\％＝22．924 |
|  | TS（2，90 $)=22.946$ |
|  | $C=2$ |
|  | $\llcorner\vec{i}=$ ご |
|  | $\cdots 11=1.35363544 E-2$ |
|  | M2＝2．951979756E－2 |
|  | $M J=$ ア．36635618694E－3 |
|  |  |
|  |  |
|  | ME＝2．959E－454275E－2 |
|  | PSく1，G1＝180113．986658 |
|  | FS¢． 31 y $=1$ ¢0106．586846 |
|  | T541． 31 ）＝23． 9475 |
|  | T5ヶ2．91－23． 965 |
|  | $\underline{C}=2$ |
|  | $L_{i}=1$ |
|  | $\cdots 1=1.9548502$ ¢8こ5E－2 |
|  | M2＝2．9481592つ54こE－2 |
|  | $M 3=5.7521750378 E-3$ |
|  | $M 4=5.75164168351 E-3$ |
|  | $M 5=013346798719$ |
|  | $M E=$ こ． $94843255814 \mathrm{E}-2$ |
|  | P5¢1，Эこ ）＝109らす1．987392 |
|  | FS¢．Gこ＝10日G\％2．4．4372 |
|  | T541． 92 ＝ 23.1545 |
|  | T5ヶ，G2＝23．1665 |
|  | $\underline{C}=2$ |

R2＝30S3．5575811

1.11 （1． 71 ）＝3．5פ516388839E－4

V1〔こ． $\mathrm{g}_{1}$ ）＝ $1.2516972 こ ゙ き こ ゙ き ー \Xi ~$
ज9（1． 31 ）$=1.39625185748$
$G G(2, G 1 y=1.39626130193$


Q（1． 31 ）$=78341813133$

$F 2$（1． 31 ）$=431.3255593$

$\mathrm{C}: 1=?$


リ1（2． 92 ）$=7.6854 E 3 E 6$ GGE－4
今9（1．92 ）＝1．39455957027
59ヶ2， $92, ~=1.3945583522 こ$
リर1． 92 ）＝2．E5737993425E－3
$V(2.32)=2.585475348 \mathrm{E}=3$

Q（2，Э2）＝巨こ．4S1341557
F2：1， 92 ＝244．457994698
Fこくこ，Эこ り＝244．418358こと
C． $1=3$


F2＝2ごア．2こゴミ1．3

U1？2，8E＝． 101772925
ज9（1． 36 ）$=1.39873064518$
G9（2，86）＝1．398730以5764
！$(1,8 E)=3.94639$ ค97599E－3
$\psi(2,8 E)=3.7896 こ 914126 E-3$
Q（1，35）$=125.261842168$
万（2， 36$)=125.171562957$
F2：1， 36 ）$=1729.59732656$
$F 2(2.35)=1720.58025431$
$C 1=3$
A2＝2444．25454日7E
U1： $87=0115995$
$\Downarrow 1<2.37)=1.63825555556 E-\Xi$
G9（1，37 $)=1.39849976534$
〔9〔．37＞＝1．39850045353
$\because(1.27)=3.9458234+15 E-3$
$U(2,87)=3.80579577883 E-3$
$5(1,87)=119.499711957$
D（2， 27$)=119.552265362$
F2（1，？ 2 ）＝ 1474.34889869
F2くこ：ぶ $)=1474.4175086$
$C 1=\underset{Z}{Z}$
R2＝こちEG．GE1アフETG


曰G（1，3：）＝1．37320981482



「（1，33）＝ 115.14276572


$F 2$（2． 28 ）$=1297.6590565$
$C 1=$ Z
$\vec{H}=2806.165059$


ज？
G9（2， $59 ?=1.39781752694$
世（1，$\because$ ）＝3．97836113087E－3

曰（1． 37 ）$=1$ 日6． 34882897
『（2， 29$)=106.92537415$


$\therefore 1=?$




Aこ＝1201．8176748G


リ1（1）79＝ 1 59978888889E－
V1（2， 79 ）$=2.3368583353 \mathrm{~J}-3$
$G 9(1.79)=1.39956842594$
$G 9(2.79)=1.3995624615 \approx$
$\psi(1,79)=$－ $96384798627 E-3$

W（1，79）＝210． 0.35962814
$0(2,79)=299.54663891$
$F 2\{1,79)=3444.55745679$
$F こ(2.79)=3444.5918$ ®294
$C 1=3$

$4161,30 \%=1.5275333 .33 E-3$
W1：E．

G9（2． 8 E ？$=1.3994959175$
U（1，3日）＝…711564日12こE－3
$1,3(3)=3.809565829] E-3$
R（1，З ）＝この


Fたく？81 ：＝3197 798．36135
C：1＝3
$\mathrm{H} 2=1$ GG．16E2以3GE
$41(1, B 1)=1.44507222222 E-3$
W1《こ． 81 ）$=2.1 こ 444$ 万2ごこごEーき
G90，51 ）＝1．39941391436
G9（2，$\quad 81 \geqslant 1.39941324551$
1.61 （ 61 ）＝ $94559445156 E-3$

い（2， 81 ）＝3．793こ6日ロ417こE－3

D（2，छ1 ）＝197．546758．48
$F 2(1, \quad 31)=2952.58816732$

$\Gamma 1=3$

LT=1:
MI = 2.11845911012E-2
06 34.514096Eース
$M 4=1.27959662356 E-2$
MS = 2.11879122852E-2
$M 5=$ ? $06655131536 E-2$
FE(1, 74 ) $=10$ D064.588293


T5く2. 74 ) = 22.57
$L_{i}=1 \because$
M1 = こ.193こ4こころち41E-こ
$M 2=3.11624613311 E-2$
M3=1.34日ツE577ア31E-2
M4 = 1.34953818095E-2
M5= こ.19408471192E-2
M6= 3.1日915ロ14419E-2


T5 (1. 75 ) $=22.7055$
T5 (2, 75 ン= ここ・•1
レア=17


$M=1.25397535 \mathrm{E}=2$
$114=$ 日135




TS (1, ア5 ン = 22.748

Lア=1白
$M 1=2.0832835255 E-2$
M2= 3.131? 1524 万EE-

M4 = .913506.22505
$M 5=2.58419831235 E-2$
ME= 3 13951519EらろE-こ
F5 (1, 77 = $=1$ 10093.187133

$1 \cdot 1, ?=-186$
に= こ
Aこ= 113 区. 25.5151


曰9 (1, 74 )=1.399862.34.515
「9(2. 74 ) $=1.39986172985$
$\forall(1,74)=4.09602455314 E-3$
$\forall(2,74)=3.834654375$ WिE-3
Q(1. 74) = 236.581503767
Q(ご, ア4 ) = 236.615こ882日6
F2:1, 74 ン $=4674.07117852$
F2イこ, 74 ン=4674.17日20793
K1=3
$\mathrm{F} 2=113 \mathrm{~B} .18925 \mathrm{G}$


G9(1, 75)=1.29981276325
GG(2, 75 ) = 1.39981255398
い《1: 75 > = 3.94764703575E-3

「人
Q(2, 75 ) $=231.529467959$


C1=


| サ161． |
| :---: |
| ，1ヶ2，76＝こ．45839444444E－̇ |
| G9¢，アE＝1．3F75912411 |
|  |
|  |
|  |
|  |
|  |
|  |
| $F 2(2, ~ F 6)=4182.371248$ g |
| E1＝3 |
| $\mathrm{H} 2=117 \mathrm{~F}$ 9488E5 |
| $1.141, \quad \geqslant \vec{i}$ |
|  |
| 回1，フ7 ン＝1．397611737 |
|  |
|  |
|  |
|  |
|  |
| $F 2 ¢ 1, ~>? ~ 勺=3.35 .7521671$ |
|  |
|  |


|  | Li $=2$ $M 1=315017131279 E-2$ |
| :---: | :---: |
|  | M2＝2．75934801927E－2 |
|  | M3＝8．54185673148E－3 |
|  | $M 4=8.65948502126 E-3$ |
|  | MS＝2．154515315．39E－2 |
|  | M6＝М27537339449 |
|  | P5（1，万0 ）＝99989．9249829 |
|  | P5くこ．70 ）＝99994．5905961 |
|  |  |
|  |  |
|  | $r=2$ |
|  | Lア＝ごご |
|  | $\cdots 1=21350267374 \mathrm{E}$ |
|  | M2＝ロごコ55198ここ |
|  |  |
|  | M4＝9．9＠8E7以？9672E－3 |
|  | M5＝2．13日38285761E－2 |
|  | M6＝こ．935日ど61194E－2 |
|  | P5（1， 71 ¢ $=1$ 日ดด23．8563 |
|  |  |
|  | T541， 71 ¢ $=2.504$ |
|  | T5：2．71：＝22．513 |
|  | に＝こ |
|  | LT＝ご |
|  | $11=.02133508$ |
|  | $M 2=$－ $9807449518 E-2$ |
|  |  |
|  |  |
|  | M5＝－13387994337E－2 |
|  | ME＝9ご86以16449 |
|  |  |
|  |  |
|  | T5（1，72 ）＝23．559 |
|  | T5：2． 72 ン＝22．569 |
|  | $\underline{r}=2$ |
|  | Lア $=$－回 |
|  |  |
|  | M2＝3．04251473911E－こ |
|  | $\cdots 3=1.21 .30335 .3 E-2$ |
|  | M4＝1．21363こ2073E－こ |
|  | MS＝2．12日き1日S日2EふEーで |
|  | ME＝3．0425こ705974E－2 |
|  | F5（1， 73 ）＝10065 ． 555149 |
|  |  |
|  | T5：1， 3 ，$=22.515$ |
|  | T542，アコ＝ここ．Eここら |
|  | $\square=?$ |

H2＝1178．4148以43E

U1： $2, ~ 7 \square=1.56026944444 E-3$
$G G(1,70)=1.40602718932$


い（こ，ア日）＝3．3E2日1423749E－3
々（1．7Q ）＝245．59625．3114
Q（2，ア日）$=244.585024644$
Fと〔1，7日 ン＝5658． 53117357
F262， 7 ？$=5653.00057526$
「1＝ 3
$\mathrm{H} 2=113 \mathrm{~B} .44721955$

$\because 1(2,71)=1.75586944444 \mathrm{E}-3$
［9（1， 71 ）＝1．39999207575
G9（2，71）$=1.39999163522$
$U(1,71)=2.9564554812 E-3$
V（2，ア1 ）＝3．79665251817E－3
㸚 $(1, ~ 子 1)=242.959542151$
Q（2， 31$)=242.874196919$
F2（1，ア1）＝5412． 92774954
$F 2(2,71)=5412.16912263$
C． $1=3$
$A 2=113$ ． 2825696
リ1（1，アこ ）＝1．4559166656アE－Z

曰9（1， 72 i＝ 1.399951 .56 .524
F9＜2， $72=1.39951679$ 亿4
U（1， 22 ）$=2$ 99698330312E－3

R（1，$T$ ）$=241.349811853$


F2Cこ．？こ $=5166.1 G E 385$
C1＝？
$\begin{aligned} & =1130.21824814\end{aligned}$
U1（1， 3 ）＝ $1.59436111111 \mathrm{E}-3$

曰9（1． 33 ）＝ 1.39990829951
T9（2， $73=1.39996793394$
U（1． 73 ）＝7 99697725745E－3
$W(2,73)=383553283691 E-3$
R（1，73）＝237． 317756215
（č， 7 ？$=237.8675112$
F2¢1，73 i＝492日．6901336
F2（2，73 $1=4926,7742839$
$\mathrm{C} 1=3$

22 NOV DATA TAPE, ARRAY STORAGE 1 to 23
LONG TUBE MODES 1 TO 23
A SIDE $=1082$, $B$ SIDE $=815 \mathrm{w} / E T$
A SIDE BIAS $=116.386$ VOLTS, SIGMA $=.001 \mathrm{VOLTS}$
B SIDE BIAS $=118.565$ VOLTS, SIGMA $=.001$ VOLTS




Нこ＝ご42．5285こ日き5

U1（2， 17 ン＝ 1.85594166 Б5フE－ 3
ज9（1，17）＝1．33871653539
GG（z． 17 ）＝ 1.3957161764 .3
$!(1,17)=3.94595523281 E-3$
$W(2,17)=$ ？． $81297676549 E-3$
に（1，17）＝124．5日内721238
D（2， $1 \vec{i})=124.548$ ह4596に
F2：1，17 $=1722.82113544$
F2？$\quad 17$ ？$=1722.79419113$
C． $1=3$
Aこ＝26S3．4017E41
$\because 1(1,13)=1.18424444444 E-3$

G9（1，13）＝1．39849286743
$G 9(3,18)=1.3954934915$
$U(1, \quad 13)=3.9543349948 E-3$
U（こ， 18$)=3.83179982045 E-3$
Q（1． 13$)=118.132917229$
Q（2， 15 ）$=113.145615917$
$F 2(1,18)=1476.264 .38439$

$\Gamma: 1=3$

$\because 1$（1． 19 ＝ $1.1769788333 E-3$
サ1：2． $1: \geqslant=1.76421944444 E-3$
G9（1． 19 ）＝ 1.35820594666


$\%(2,19)=$ 3499141255E－3
®（1，19）＝ 11155550133
区ー， 19 ＝ 111.73058565
F2《1，19 サ＝122马．
F2！2， $19:=1228.92692728$
$\mathrm{E}=\mathrm{Z}$

41：20 ．

G 9 （1，20）＝ 2037815798
凸3（2，2Q）＝1．3781654483
$\because(1.2 \square)=3.9586266884 E-3$
U（2，2日 $)=3.341947857$ ご
曰（1，2以 ）＝1日4 8514885日4 O（2，2曰）＝ 164.31196 .544
Fこ（1，2日 ）＝92．23119229
F2！2，zロ＝$=32.85565 .505$
C： $1=3$

$\mathrm{H} 2=554.544848$
$\because 1$ ！ $1,1 \Xi=1.42$ EES111111E－3

G9（1： 13$)=1.39939146678$
G9（2， 13 ）＝1．399301296125
$4(1,13)=3.9514163339 E-3$
ひ（2， 13 ）＝3．8196863E491E－3
$0(1,13)=135.153165891$

$F 2$ F1． $13=2308.77837697$
F2《2． 13 ）＝2ア98．7464275G
$\Gamma 1=$ ？
Hこ＝13ア1．47EE4ごこ
U161， $143=1.355258 .33 .33 .3 \mathrm{E}-3$

G941， 14 ）＝1．39919053918
G9＜2， $14 \% 1.39913961505$
い（1． 14 ）$=3.56$－ $49157924 E-3$
$U(2,14)=3.8255995658 E-3$
「（1， 14 ）＝ 159.9359649 日3
（1）（2， 14$)=159.811254933$
Fご1，14 り＝2461．33417326
F2イ3． 14 ？$=2451.4394321$ た
$C: 1=3$


|  |
| :---: |
|  |
|  |
| リ1．15 ） |
| VG，15＝3 Q2126838日4E－3 |
| 圌く1．15）＝152．92845069 |
| 込 15＝152． |
|  |
| $F 282,15=2216.65438451$ |
| C1＝3 |
|  |
| $\because 1$ ¢1，15 $=1.2579503333 \mathrm{E}$ |
| V1：2，1E＝区以1503 |
| 曰9（1，15\％＝1．398日312184 |
|  |
|  |
|  |
| ¢（1，15）＝15 596\％心132 |
| 门（2． 16 ）＝175．6174773才 |
| F2：1，15＝155，P－98151 |
|  |
| $\square 1=\square$ |


| $L^{7}=15$ $M 1=619.5157121$ |
| :---: |
| $M 2=3$ 日 $33201594.58 \mathrm{E}-2$ |
| $M J=1.354 .5259155 E-2$ |
| M4＝1．3＠75601372EE－2 |
| $M 5=$－ 19917874453 |
|  |
| P5（1， 3 ）＝100644．988933 |
|  |
| T5（1， 9 ）$=23.5125$ |
|  |
| $\Gamma=2$ |
| $L \vec{i}=14$ |
| $11=1.9823983054 \mathrm{E}-2$ |
|  |
|  |
| $M 4=1.2546792$ ご12E－2 |
| M5＝019823970616 |
| $M E=3.6598526448 E-2$ |
|  |
| F5（2，10 ）＝109063．554991 |
| T5：1，10）＝23．631 |
| T562．10＝$=3.6455$ |
| $\Gamma=2$ |
| $L_{i}=13$ |
|  |
| M2＝J．0472959045E－2 |
| $M \mathcal{O}=1.198 E 0.542395 E-2$ |
| M4＝1．1989J．J663GE－2 |
|  |
|  |
| P5i1． 11 \％$=100957.588524$ |
| F62． 11 ＝ 6 ¢059．3212 |
| T541，11 ）＝23．5， |
| T5ヶ2． 11 ＝23．5805 |
| $\underline{E}=2$ |
| LT＝12 |
| $\cdots 11$＝ 954985124 E |
|  |
| $\mathrm{MJ}=$ Q1134751175 |
| $M 4=1.13521423 .314 \mathrm{E}-2$ |
| $M 5=01955784308$ |
|  |
| F5（1，12 ）＝159以乌日．187451 |
| F．5．2． 12 ）$=1$ g【ng 254214 |
| T5：1．12＝2？．7らす5 |
| T5ヶ3．12 サ＝ご，アご |
| L＝ 2 |

$\mathrm{H}=1297.580501$
い1（1．$\because=$＝ 91695575
U（こ．Э ）＝こ．522E416ら6EアEーJ
G9（1，9）＝1．39963．344934
G9i2， 9 ）＝ 1.39963449481
$U(1 . G)=3.98533989236 E-3$
りくこ，Э＝＝342644789日1E－3
Q（1，-2$)=217.3$ 亿8486936
风（こ．？＝217．7479519め8
$F 2(1,5)=5694.4375697$
F2（2．$\ddagger$ ）＝
$11=3$
$A こ=1245847246$


「9（1，10）＝1．39955435351

$1,10(19)=35285828945 E-3$
$\psi(2.1 G)=3.83555416841 E-3$
日（1，19）＝21日．726ご36843
（6， 16$)=210.598183638$
Fこれ1，1日 り＝3447 92636748 $F 2(2,1$（1）$=344797513539$ $\therefore 1=?$
$\mathrm{H} 2=13 \Xi \mathrm{ESJES}$

| $\left.\begin{array}{ll}\because 141 . & 11 \\ \psi 1<2 . & 11\end{array}\right\}=2.51283888835 E-3$ |
| :---: |
|  |
| F9r2． 11 ¢ $=1.39455955$ |
|  |
|  |
|  |
|  |
| $F 24.11$ y＝ご心1．3582314 |
| F9！2， 11 ）＝321 375こ97．7 |
| $\Gamma 1=3$ |
| $\mathrm{A}=1512.7864$ |
|  |
|  |
| 5961． 12 ） |
| 上9（2．12）$=1.3995997319$ |
| $\because(1.12)=$ G3811117163E－3 |
| いこ，12＝こ SE157こ12日4EE－3 |
| D（1，12）＝192．72ア4ア24囚4 |
|  |
| Fご 1，12＝205． 3.2085 |
| F2¢2，12＝2955．8日0．52643 |
|  |

```
LT=13
```



```
M2= こ. -
\(M \Xi=.012786173898\)
M4 = 1.278009135E-2
M5 = 2 \(23723452486 E-2\)
\(M 6=3.06464 E 42\) ลロフEーこ
P5 (1, 5 ) = 9995. G238855
F5(2, 5) = 999.96.6571947
T501, 5 ) \(=2.3 .375\)
T5イこ. 5 )=23.725
\(\Gamma=\Omega\)
\(L \vec{i}=13\)
```




```
\(M 3=1.35156091 E-2\)
```






```
FSi2, \(5=10 \mathrm{y} 013.455642\)
T5! : 5 \(=23.594\)
```



```
\(\Gamma=2\)
\(1 \vec{i}=1 \vec{i}\)
```






```
\(M 5=2 . 以 13724958 E-2\)
```



```
F541. 7 = 10 2027.5こ2846
```



```
T5 (1, \(\vec{i})=23.565\)
```



```
I:=
\(\angle \vec{i}=1 E\)
```



```
Mミ= ? 5 - 51544535以1E-2
M3= 013430236559
M4 = 1.3440572255.5E-2
\(M 5=2.61072496394 E-2\)
ME= 3 日8104251EE4E-こ
```




```
T5 (1, ? \(=2354\)
```



```
\(I:=2\)
```


$1,11(1,5)=1.65431111111 E-3$
$V 1(2,5)=2.39576944444 E-3$
■9（1，5）＝1． 29984379349
โG（2， 5 ）$=1.3998443 .396$
1 （1， 5 ） 3 ． 5 E12443203E－3
$U(2, \quad 5)=3.815224 .2669 E-3$
是（1，5 ）＝234．967854442
$巾_{2}(2, \quad 5 \quad . \quad 233.99253691$
$F 2(1.5)=4581.51275575$
F2（2．5 ）＝4681．4449685：
ㄴ1＝3


$V 1(2,6)=2.50765555 E E-3$


$4(1.6)=3.96914518876 E-3$

民（1，6）＝ここS．S1こ688555
隹（2， 6 ）2ご． 19549566
$F 2(1, \quad$ E $)=4435.15829727$
$F 2(2, \quad)=443568656$
$\therefore 1=3$


|  |  |
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《1（1， 1 ）＝1．16174972222E－3

I9（1，1）＝1．29997726256
G9（2． 1 ）＝1．39997697904
W（1， 1 ）

过（1， 1 ）$=239.479942561$
0（2． 1 ）＝242．बアS18612こ
$F 2(1.1)=5 E 73.25498425$
Fごき， 1 ）＝55アふ． 68531491
C：1＝？
F2＝1178． 5104516
（1）（1，2） 2 2 $3035111111 E-3$

ज941， $2=1.39995978599$
59（2，2）＝1．39995220691


た（1．こ ）＝249．514195995

F2（1，2 $=$ 5425．2G679151
F2！2，z＝5424．ヨ729230う
$\because 1=3$


U1（2，$\quad=2.0518338889 E-3$




（1）$\quad 3=230.565453579$

F2！1： 3 ）＝ 5176.519660 .3

C．1＝ 3
H2＝117E 21543243

$11(2.4$ ）$=2.26036944444 \mathrm{E}-3$
G9（1． 4 ）＝1．39988490227
G9（2， 4 ）＝1．299866日3765
$\because(1,4)=59765137369 \mathrm{E}-3$
いこ． $4>=3.83815443516 E-\Xi$


$F 2(1,4)=4929.48453578$
$F 2(2,4)=4329.3345555$
$\mathrm{C} 1=3$

22 NOV DATA TAPE，ARRAY STORAGE 24 to 33 SHORT TUBE MODES 1 TO 10
A SIDE $=1248$ ，B SIDE $=815$（ET ALONGSIDE）

| LT $=2$ $M 1=2.1981162293 E-2$ |
| :---: |
| M2＝2．97542日ア94日7E－2 |
| $M J=8.5698755134 E-3$ |
| M4 $=6$ 674685344BE－3 |
| $115=2.110954 .34778 E-2$ |
| M6＝2．97377623449E－2 |
| F5¢1． 32 ＝ 99637.7290895 |
| $F 5(2.32)=99831.8626158$ |
| T54， 32 i＝22．454 |
| $T 5<2, ~ 22=22.442$ |
| $C=115.4119$ |
| $L T=1$ |
| $M 1=2.659579898 \mathrm{E}$－2 |
| M2＝2．9514143849EE－2 |
| $M J=8.621692113 日 7 E-3$ |
| M4 $=8 . \overline{6} 3841715247 \mathrm{E}-3$ |
| M5＝こ．1日122830717E－2 |
| M6＝て．945276263ロЗE－2 |
| P5（1，33 ）$=39842.9289184$ |
| F5¢2． 33$)=39846.3956634$ |
| T5＜1， 3 J ＇$=22.4235$ |
| TS¢E， 3 S i＝22．368 |
| $C=1164093$ |




LT＝

M2＝3． $9512258353 E-2$
MJ＝1．25S79629871E－き
$M 4=1.26937557462 E-2$
$M 5=2.1892357324 E-2$
$M E=$ S． $09375991414 E-2$
F5（1，29 ）＝ 39386.8608056
PS（2．29 ）＝ $29875.19439 こ 1$
T5！1，29＝23． 2705

$\Gamma=116.41 E 5$
LT＝4


$M J=1.104517231 E-2$
M4＝1．105Eめ594752E－？
M5＝2．15．玉こここったらEー2
M6＝З．
P5（1．3 ）＝99872．19462こ4

T5！． 30 ＝29．873

$\Gamma=116.4152$

| L $M 1$ | 2 |
| :---: | :---: |
| $=3$ | 0165469 ¢19E－ |
| $M 15=9$ | 6657973S539E－3 |
| M4 $=9$ | 6742日175755E－3 |
| $M S=2$ | 1392．324ア1日EE－2 |
| $M \epsilon=3$ | （1421以12058E－2 |
| PS 1 | $31 \%$ 日GЗ62．6616日26 |
| FS¢ | 31 ） 99864.3262145 |
| T5：1 | $\underline{\sim 1}$ ）$=22.5015$ |
| T562 |  |
| $\bar{\square}=11$ | 6.4127 |

$A 2=367$ 96559213
$11(1,23)=5.23991656567 E-4$
ね1（2．2S）＝6．69593888689E－4
$G 3(1.28)=1.39913663613$
G9（2，28）＝1．39916676211
$U(1,28)=3.94055472812 E-3$
$U(2,23)=3.89946145986 E-3$
$\Gamma_{1}(1,28)=115.077297331$
ロ（2．こも）＝115．968915996
$F 2 \div 1,28)=4447.59712346$ F2（2． 29$)=4447.33053342$ $\mathrm{C}=118$ 579
$\mathrm{HE}=387.838489862$
$11(1,29)=9.04597222222 E-4$ U1：2， $2 G)=1.23596111111 E-\Xi$
凸9（1，29 ）＝ 1 39897078．318
GG：2g＝＝1．39897358592
（1），29）＝268以ヒ789887E－3
$W(こ, 29)=$ ？23446895975E－3
凹（1．29）＝117． 5.55024283
『（2，2马 ）＝ 116.345425871
F2（1，29）＝3703．24109935
$F 2(2,26)=3702.87523621$
「： $1=118: 5$ 万デア
$\mathrm{HE}=411.76 \mathrm{~F} 58 \mathrm{E}$
U1（1． 3 日 $=1.03219194444 E-\Im$
$W 1(2, ~ 36)=1.41389611111 E-3$
GG1，30＝$=1.39365012959$

！（1．30）＝4． $2357404309 E-3$

并（1．已日 ）＝ 115.125197639
Q（2，$\because\binom{0}{$\hline}$=114.757918125$
Fご1，30＝2961 115以658？

C1＝113．5ア59
$\mathrm{FI}=483.34992151$
U1（1，J1）＝1．1日347138359E－3

上9：1， 31 ＝ 1.39822366634
G9（2． 31 ＝ 1.398223682 .34
$4(1, \quad こ 1)=4.13748587483 E-3$
$\because(2, J 1)=4$ ng24541278BE－3
曰（1．ت1）＝1日2． 352398438
（2， 21 ）$=102.31455516$
F261，こ1）＝ここ19．11165アウ1
Fこ！2：こ1＝ごこ1：9アアコここ17
「1＝ 1185 5

$\mathrm{HE}=579$. SES341231
リ1（1．24）＝2．72944160667E－4
リ1（2．24＝3．16833888889E－4
G9（1，24） 24.3997556644
ज962，24＝$=3997$ 亿899995
W（1，24）＝4．Й5日378194BE－3
U（2，24）＝3．9228692182こE－3
Qr1．24）＝118．3r9340747
Q（2． 24 ）＝113．427451995
$F 2 \div 1,24 \quad \because=7431.31692558$
F262．24＝ 3430.56979396
E1＝118．5933
$\mathrm{A} 2=5 \mathrm{~B} .893403$
$\because 1(1,25)=1.59542222222 E-4$

G34．25 ）＝1．39951444381
G962 25 ！＝1． 29961585.542
$W(1,25)=4.04890436865 E-3$
りぐ2，25＝3．91664105863E－3
日（1，25）$=114.45331137$
戸（2，ご
F2（1，25＝$=6582.97516396$
$F 2(2,25:=6582.69651431$
$\Gamma 1=112.5 \Omega 21$
$\mathrm{H} 2=459.838954 \mathrm{E} 1$
$41(1,26)=44311$ 66E6E57E－5 W1くご ごも $\because=9.42152777778 \mathrm{G}$巨9（1．こ6 ）＝$=39949680.373$ GG《2，$\approx \%=1.2994997957$


旦（1．2太）＝112．71908259
民く，こE $=112.8523879$
$F 201,25$＝593日． 95169154

$\Gamma 1=118.581$
H2＝411．9915．595

U1くこ．27 ン＝ミ．844838888B9E－4
$5961,27)=1.3993569064$
G9（2．27 ：＝ 3 393597SJ4J
U（1．27）＝3．2801日175695E－3

「（1，ご）＝ 114 ．ब以らら84511
बミ：27＝
$F 2$（1，27 $=5191.70958126$
$F 2<2, ~ 27)=5191.5663068$
に1＝ 115.58 S

22 NOV DATA TAPE，ARRAY STORAGE 34 to 43 SHORT TUBE MODES 1 TO 10 A SIDE $=1248$ ，$B$ SIDE $=1082$（ET ALONGSIDE）

|  |
| :---: |
| $M 2=1.9230391561 E-2$ |
| $M 3=8.5414152205 E-3$ |
| M4＝8．54152515999E－3 |
| M5＝2．116713G日0ロЗE－こ |
| ME＝1 923914．9279E－2 |
| P5i1．42＝90716．1997539 |
| F5¢2． $42 \%$ 93714．9997934 |
| TS¢1，42 $=23.13$ ¢5 |
| T56．4こ＝23．15 |
| $\underline{\square}=115.414$ |
| LV＝1 |
| $M 1=$－ $10199220.32 E-2$ |
| $M 2=1.89718549347 E-2$ |
| $M 3=7.9509398125 E-3$ |
| $14=7.96091996144 E-3$ |
| $M 5=2.1 日 18257224 E-2$ |
| M6＝1． $29716424818 E-2$ |
| F＇54，43＝99639．1339776 |
| F5（2，43 $=99681.1342488$ |
| T5 1． $43 \quad i=23.1535$ |
| TS¢ 2 E ＝23．196 |
| $E=11 \epsilon+154$ |


|  |
| :---: |
|  |
| 59 ¢ ，42＝1．39733043776 |
| F9， 42 ＝ |
|  |
|  |
|  |
|  |
| $F 2$（1，42，$=1431.16097950^{\circ}$ |
|  |
| に1＝113 5E41 |
|  |
| W1（1．4．3＝1．5935611111E－3 |
| U1： 43 ） $4.451630555 E-Z$ |
| ［9：1， 43 y $=1.39565757876$ |
|  |
| $U(1,4.3)=3.87217$ ¢ $\quad 3$ S56E－3 |
|  |
|  |
| Q（E． 43$)=40.4847613162$ |
| Fこと1． 43 ，$=740.1774519$ |
| Fご，4．$=$ 740．126653099 |
| $51=118.5654$ |



$\because 1(1,38)=$ ．$\because 6086775$
U1《2，3E）＝6．44009444444E－4
$G 9<1,38)=139916292925$
G9（2．З太 $=1.3991612662$ 2
， 1,33 ）$=3.9359663602 E-3$

D（1． 33$)=134.052315597$
［12，38 $)=134.661544924$
$F 2(1,33)=4439.38330796$ F2〔2．3By＝4445．1658日91．
に1＝113．5らった
$\overline{\mathrm{H}}=5 \mathrm{5} 7.541445834$





011 （1， $34=3.217257777 \mathrm{BE}-4$

G9（1，34）＝1．39971917232
け9く2，こ4 ：＝1．3997ご663214
U（1，34）＝3．90．359139425E－3
$U(2,34)=3.93326498791 E-3$
Q（1，34）＝ 12 ㅍ．541202897
区 $(2,34)=130.698108595$
$F 2$（1．34 ）＝ 7407.33566674
$F 2$（2． 34 ）＝ 7406.68396456
$C 1=118.5 E う$
$\mathrm{H} 2=5 \pm 1.847751575$

Q（2，35 ）＝12 2.291737492
 $F 2$（2， 3.5 ） 5662.36791147
C．1＝11G．5E17

$!1 \because 1, ~ J i \quad i=3.3294444444 E-5$
サ1く2．36 ン＝9．6656660655アE－5
5G（1，35 ）$=1.39958761204$
G9：2， $56=1.39950791874$
$\because(1.36)=3.92279239176 E-3$

（1）（1，ヨ气 ）＝128． 3 32814812

$F=1$（55＝ 519.3 .321732
 C1＝118．5®日8



ज9i1，37 ！＝ 399356942
GG8．
$4(1, ~ 37)=3.925053392 E-3$
w（2，37）$=3.95261$ 日2 SJE．3E－3
曰（1，Ј7）＝132． 143242952

F2（1，こ7 ）＝5179．ご5868816

F1＝113 55S

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[^0]:    Next, the precision associated with replacing the long tube with the smort tube is discussed.

[^1]:    Table B. 1 Functional_program_listing

[^2]:    The program listing is next.

