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Uses of Partial Fractions

Partial fraction

The process used to express a rational (polynomial) fraction into sum of reduced rational fractions.

Exmaple :

consider the fraction

$$\frac{X}{(X+1)(X+2)}$$

now we can express this fraction as a sum of reduced fractions as follows

$$\frac{X}{(X+1)(X+2)} = \frac{X}{(X+1)} - \frac{X}{(X+2)}$$

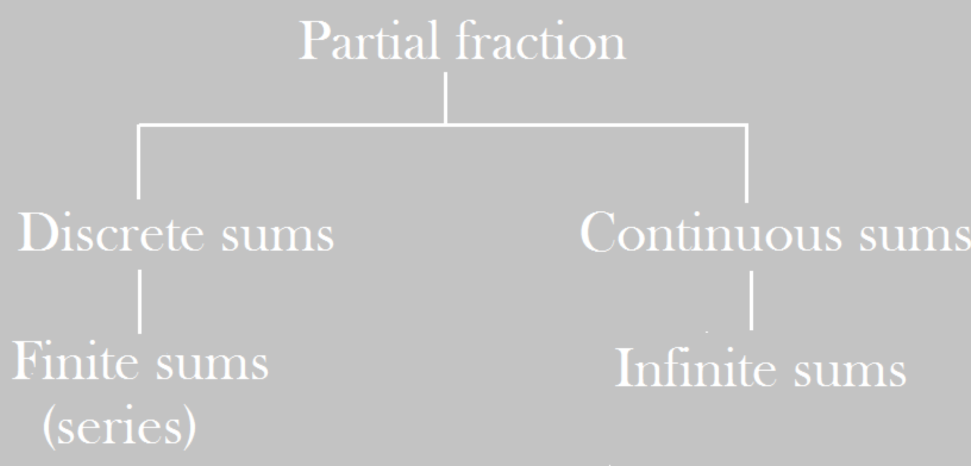
Similarly,

$$\frac{X}{(X+1)(X+2)(X+3)} = \frac{X}{(X+1)} - \frac{X}{(X+2)} - \frac{X}{2(X+1)} + \frac{X}{2(X+3)}$$

Uses of Partial Fraction

we describe uses of PF by a flow chart

PF are mainly used to find Sum of series



Continuous Sums

we will now discuss how partial fraction is used to find sums of different series

Type 1 :

Evaluate $$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

Solution :

now $$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{(n+1)}$$

Sum of n-terms $= 1 - \dfrac{1}{n+1}$

or $S_n = \dfrac{n}{n+1}$ ——— (A)

when $n \longrightarrow \infty$

$S_n \longrightarrow \sum_{n=1}^{\infty}$

taking $\lim_{n \to \infty}$ on bothsides of A

$$\lim_{n \to \infty} \left(S_n \right) = \lim_{n \to \infty} \left(\frac{n}{n+1} \right)$$

Hence

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$$

Type 2 :

Evaluate $\sum \frac{1}{(n+1)(n+2)(n+3)}$

note
now
when symbol
$\sum$
appears it
means infinite
sum

Solution:

we use partial fraction to find the sum

so $$\frac{1}{(n+1)(n+2)(n+3)} = \frac{1}{(n+1)(n+2)} - \frac{1}{(n+1)(n+3)}$$

using partial fractions

so apply summation on bothsides

$$\sum \frac{1}{(n+1)(n+2)(n+3)} = \sum\left(\frac{1}{(n+1)(n+2)}\right) - \sum\left(\frac{1}{(n+1)(n+3)}\right) \text{ ——— (B)}$$

Now

$$\sum \frac{1}{(n+1)(n+2)} = \frac{1}{2}$$ (see Type 1)

and

$$\sum \frac{1}{(n+1)(n+3)} = \frac{5}{12}$$

using these values in (B)

$$\sum \frac{1}{(n+1)(n+2)(n+3)} = \frac{1}{2} - \frac{5}{12}$$

$$S = \frac{1}{12}$$

similarly one can use partial fractions
to find sums of different series

s for
sum

Exercise for example

Evaluate

1) $$\sum \frac{n}{(n+1)(n+2)(n+3)}$$

2) $$\sum \frac{n(n+1)}{(n+2)(n+3)(n+4)(n+5)}$$

Thank you for reading