A COMPARISON OF CONFIDENCE INTERVAL PROCEDURES IN CENSORED LIFE TESTING PROBLEMS

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THESIS

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A Comparison of Confidence Interval Procedures in Censored Life Testing Problems

by

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ABSTRACT

Obtaining a confidence interval for a parameter λ of an exponential distribution is a frequent occurrence in life testing problems. Oftentimes the test plan used is one in which all the observations are censored at the same time point t_o. Several approximate confidence interval procedures are available in the statistical literature; however, to the knowledge of the author, the performance characteristics of the various approximations used in these procedures have not been established analytically. The purpose of this paper is to report the results of an empirical study of the performance of four of these procedures with respect to the expected length of the interval, the variance of the interval length, and the coverage probability.

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I. INTRODUCTION

In life testing applications, it is frequently desired to obtain a confidence interval for the parameter λ of an exponential distribution. In case a test plan is used for which all the observations are censored at the same time point t_o, several approximate confidence interval procedures are available in the statistical literature. To the knowledge of the author, the goodness of the various approximations used in these procedures, and hence the goodness of the procedures themselves, have not been established analytically. The purpose of this paper is to report the results of an empirical study of the performances of four of these procedures with respect to the expected length of the interval, the variance of the interval length and the coverage probability.

The general setting of the problem is as follows: suppose the random variables $T_1, T_2, \ldots, T_n$ are independent and identically distributed with common exponential distributions

$$f_i(t_i) = e^{-\lambda t_i}, \quad t_i > 0; \; i = 1, 2, \ldots, n.$$

The random variables that will actually be observed are $X_1, X_2, \ldots, X_n$, where

$$X_i = \begin{cases} T_i & \text{if } T_i < t_o \\ t_o & \text{if } T_i \geq t_o; \; i = 1, 2, \ldots, n. \end{cases}$$

These random variables are sometimes referred to as "censored" exponential variables, and t_o is called the "censoring point". Define the random variables $Y_1, Y_2, \ldots Y_n$ by

$$Y_i = \begin{cases} 1 & \text{if } X_i < t_o \\ o & \text{if } X_i \geq t_o; \; i = 1, 2, \ldots, n. \end{cases}$$

It is easily seen that the random variables $Y_1, Y_2, \ldots, Y_n$ are independent and have Bernoulli distributions with parameter $p = 1-e^{-\lambda t_o}$. In Section II, four confidence interval procedures for λ based on the X_i and Y_i are discussed.

II. CONFIDENCE INTERVAL PROCEDURES

A. PROCEDURE 1

This procedure is obtained as a special case of a solution to a more general problem that has been derived by Halperin [1]. The random variable $Y = \Sigma Y_i$ has a binomial distribution with unknown parameter p. Suppose y is the observed value of the random variable Y, then the 100 (1-α)% confidence bounds p_U and p_L may be obtained by solving the equations

$$P\ [Y < y] = \sum_{i=o}^{y} \binom{n}{i} p^i\ (1-p)^{n-i} = \alpha/2$$

and

$$P\ [Y > y] = \sum_{i=y}^{n} \binom{n}{i} p^i\ (1-p)^{n]i} = \alpha/2$$

for p. Due to the discreteness of the random variable Y, these equations do not yield an exact 100(1-α)% confidence interval. The confidence coefficient is at least 100(1-α)%. The confidence interval for λ is easily computed by inverting the relation $p = 1-e^{-\lambda t_o}$ to get $\lambda_L = -\ln(1-p_L)/t_o$ and $\lambda_U = -\ln(1-p_U)/t_o$.

A useful tool in solving for p_L and p_U is the Incomplete Beta function,

$$F(x;\alpha,\beta) = \frac{(\alpha+\beta+1)!}{\alpha!\beta!} \int_o^x t^\alpha\ (1-t)^\beta dt = 1 - \sum_{i=o}^{\alpha} \binom{\alpha+\beta+1}{i} x^i\ (1-x)^{\alpha+\beta+1-i}.$$

The upper confidence bound, p_U, is obtained by solving

$$F(p_U;\ y,n-y-1) = 1-\alpha/2;$$

and the lower bound, p_L, is obtained from

$$F(p_L;\ y,n-y-1) = \alpha/2.$$

The simplicity of the computation involved in this method is very apparent; however, it should be noted that the observed values of the random variables $X_1, X_2, \ldots, X_n$ have been disregarded. Intuitively it would appear that this procedure will not produce as good results as one which incorporates all the information of the experiment. However, due to its simplicity, this procedure merits consideration.

B. PROCEDURE 2

For $\lambda t_o << 1$, $Y = \Sigma Y_i$ is nearly a Poisson random variable with parameter $\lambda\Sigma X_i$. Rubenstein [2] shows that $\hat{\lambda} = \frac{Y}{\Sigma X_i}\left[1 + \frac{1}{2n}\right]^{-1}$ is a nearly unbiased estimator for λ, where the second factor is used to correct for the bias of the otherwise minimum variance estimator $Y/\Sigma X_i$. Wilks [3] states that for a Poisson random variable P with parameter ρ, the following relationship is approximately true: $\rho - P = Z\sqrt{\rho}$ where Z is the standard normal random variable. Rubenstein applies this reasoning to the random variable Y along with his modified estimator $\hat{\lambda}$ to obtain the relationship $\lambda - \hat{\lambda} = Z\sqrt{\lambda}$. This equation is solved for λ_U and λ_L by using the appropriate standard normal percentage point corresponding to a $100(1-\alpha)\%$ confidence level. The resulting solutions for λ_L and λ_U are

$$\lambda_L = \left[2\hat{\lambda} + z^2 C - (4\hat{\lambda} z^2 C + z^4 C^2)^{1/2}\right]/2$$

and

$$\lambda_U = \left[2\hat{\lambda} + z^2 C + (4\hat{\lambda} z^2 C + z^4 C^2)^{1/2}\right]/2 \text{ where } C = (\Sigma X_i)^{1/2}.$$

C. PROCEDURE 3

This procedure is a modification of one which has been suggested by Birnbaum [4] for application to uncensored life testing problems. Using the terminology commonly used in the literature of life testing to

facilitate the explanation, imagine that the random variables X_1, X_2, ..., X_n are observed sequentially. That is, a randomly selected item is put on test and is replaced with a similar item at failure or after a period of time t_o has elapsed, whichever occurs first. Then the arrival process of failures is a Poisson process. Suppose the experiment is continued until a total of k failures have been obtained. It is well known that the individual inter-arrival times of failures are exponentially distributed and the time to the k^{th} failure has a gamma distribution. Consider a test plan in which the experiment is stopped after a random amount of time (as would be the case for example, if n items were to be tested). The number K of failures is then a random variable. However, it would appear that, given K=k, the distribution of the time, W_k, until k failures have arrived can be approximated by a gamma distribution. More precisely, suppose the observed value of the random variable $K = \sum_{i=1}^{k} Y_i$ is k, and let $W_k = \sum_{i=1}^{k} X_i$. Then the conditional distribution of W_k, given that K=k, is approximately the gamma distribution

$$f(w) = \frac{\lambda^k}{\Gamma(k)} w^{k-1} e^{-\lambda w}; \ w > o.$$

It follows that $V = 2\lambda W_k$ can be approximated by a Chi-square variable with 2k degrees of freedom. Thus, if $\chi^2_{1-\alpha/2}$ and $\chi^2_{\alpha/2}$ are the upper and lower $\alpha/2$ percentage points of the Chi-square distribution with 2k degrees of freedom, then $\left(\frac{\chi^2_{\alpha/2}}{2W_k}, \frac{\chi^2_{1-\alpha/2}}{2W_k}\right)$ constitutes an approximate $100(1-\alpha)\%$ confidence interval for λ.

Birnbaum also suggests an estimator for λ which is merely the average of the upper and lower confidence bounds. This estimator has also been computed and tabulated.

D. PROCEDURE 4

In Procedure 3 the distribution of the random variable W_k, the waiting time until the k^{th} failure, is approximated by a gamma distribution. Since the test is terminated after n items are tested, the maximum value that W_k can attain is nt_o. Consider the conditional probability

$$P\,[W_k \leq w | W_k \leq nt_o] = \frac{P\,[W_k \leq w \text{ and } W_k \leq nt_o]}{P\,[W_k \leq nt_o]} = \frac{P\,[W_k \leq w]}{P\,[W_k \leq nt_o]} \,.$$

This yields a truncated gamma distribution having density function

$$f_T(w) = \begin{cases} \dfrac{f(w)}{F(nt_o)} & o \leq w \leq nt_o \\ o & w < o,\ w > nt_o \end{cases}$$

Where f(w) is the density function of the gamma distribution and $F(nt_o) = \int_o^{nt_o} f(w)\,dw$. Intuitively, this new function would seem to approximate the probability distribution of W_k in a censored test. An obvious drawback of this method is the difficulty of computation since both λ and $F(nt_o)$ which depends on λ are unknown. Thus, given tables of the Chi-square distribution function one is forced to use an iterative method for obtaining the desired λ such that

$$\frac{P\,[\chi^2 < 2\lambda w_k]}{P\,[\chi^2 < 2\lambda nt_o]} = \alpha/2 \qquad \frac{P\,[\chi^2 < 2\lambda w_k]}{P\,[\chi^2 < 2\lambda nt_o]} = 1-\alpha/2$$

to obtain λ_L and λ_U respectively.

The estimator for λ described in Procedure 3 is applicable to this procedure and is also tabulated.

III. COMPARISON OF PROCEDURES

A Monte Carlo study has been made to compare Procedures 1, 2, and 3 described above. One thousand samples of size n (n = 30, 40, 50) from an exponential distribution with parameter λ (λ = .1, .2, .8, 3, 5, 10) have been generated. For each sample, confidence intervals for λ have been obtained by using each procedure for various censoring points t_o and confidence coefficients $(1-\alpha)$ (α = .05, .01). The tabulated quantities are the average length of the confidence interval, the sample variance of these lengths, and the empirical coverage probability of the intervals (i.e., the proportion of intervals which actually covered λ).

An abbreviated Monte Carlo study has been made to compare Procedures 3 and 4. The lengthy computation of Procedure 4 required that the number of repetitions be reduced to 100 and that fewer combinations of λ, t_o, n, and α be used.

IV. CONCLUSIONS

The results of this study differentiate each procedure as to its merits and shortcomings with respect to certain life testing situations. Each method is discussed below in order to define the situations in which it could be used. The following discussion includes only comparisons for Procedures 1, 2, and 3. Procedures 3 and 4 are compared separately and to a lesser degree due to the differences in the Monte Carlo studies made.

Procedure 1 performed as expected; it generally gave less accurate results than the other procedures with respect to all three quantities--expected interval length, sample variance of this length, and empirical coverage probability. However, since the empirical coverage probability of this procedure tends to be conservative and since the computation required for this procedure is minimal, Procedure 1 would be favored in cases when a quick but dependable confidence interval is needed or when a rough estimate is needed for use in more sophisticated procedures. Disregarding the observed value of the random variable ΣX_i is the main reason for the conservative results.

The overall performance of Procedure 2 seems to rank it first among those studied. However, when t_o is near $1/\lambda$ Procedure 3 appears to give comparable results with less computation.

Procedure 3 performs very well in cases where t_o is approximately equal to $1/\lambda$; however, when $t_o << 1/\lambda$ few failures tend to occur and the random interspersing of censored times causes the sample variance of the procedure to be high. In general, the empirical coverage probability for this procedure is close to the chosen confidence coefficient. The actual computations needed are comparable to those for procedure 1.

The estimators for λ defined under Procedures 2 and 3 have been tabulated; the one given under Procedure 2 seems to be a nearly unbiased estimator for λ as stated. The estimator arising from Procedure 3 is consistently greater than the true value of λ.

The comparison of Procedures 3 and 4 are based on an abbreviated Monte Carlo study. This is due to the accuracy limitations of the computer when computing small values of λ and also the added dependence upon the time till k^{th} failure in the iterative step of the computation. A more extensive study will be necessary to obtain more meaningful results. It appears that the interval length obtained from using Procedure 4 is not as good as that of Procedure 3; however, the empirical coverage probability for Procedure 4 seems to be close to the desired confidence coefficient and the estimator for λ appears to provide a nearly unbiased estimator.

V. COMPUTATIONAL PROCEDURES

The IBM/System 360 Model 67 computer with the Fortran IV programming language has been used for all computations.

The exponentially distributed random sample is obtained by first generating random numbers from a uniform distribution on the interval (0, 1) and then by inverting these numbers to get a random sample from an exponential distribution with parameter λ. This is done by using the fact that if a random variable Z has an exponential distribution with distribution function

$$F(Z) = P[Z \leq z] = \begin{cases} o & z < o \\ 1-e^{-\lambda z} & o \leq z \leq 1 \\ 1 & z > 1 \end{cases}$$

the rando variable $X = 1-e^{-\lambda z}$ has a uniform distribution on the interval (0, 1). Thus, if x is a random number from the interval (0, 1), the number $z = -\ln (1-x)/\lambda$ is a random sample number from an exponential distribution with parameter λ. The subroutine RANDU of the IBM Scientific Library is used to generate these random numbers.

The second step in the computing procedure requires a simulation of censored testing. Therefore, each random number z is compared with the pre-determined censoring time t_o and a counter is used to obtain the number of failures $k = \Sigma Y_i$. For Procedure 1, only the value of k is needed; however, Procedures 2, 3, and 4 require additional information. If the value of z exceeds the value of t_o, it is disregarded and replaced by t_o; these values of t_o and z which are less than t_o are summed to obtain the observed value of the random variable ΣX_i for use in Procedure 2. The necessity of order in Procedures 3 and 4 require the random numbers to be dimensioned in an array to maintain the order in which

they are generated. This array contains a random ordering of z's and t_o's which are summed to the point of the k^{th} failure after the entire sample of size n has been generated. This sum contains some of the truncated times t_o but generally not all and it contains all $z < t_o$. The value of this sum is the observed value of the random variable W_k of Procedures 3 and 4.

The computation of Procedure 2 is trivial after the required normal deviate is read from a data card. However, due to the high number of possibilities for the value of k ($0 \leq k \leq n$) subroutines of the Beta and Chi-square distribution functions are used. These subroutines are written as cumulative distribution functions and give only the resulting probability, given the required input parameter. Since the input parameter λ is unknown, it has been required to write an additional function subprogram to iterate toward the desired parameter given the confidence coefficient. The restrictions on the input parameters for these subroutines force the cases of $k = 0$ and $k = n$ to be ignored. This is done by disregarding the sample which produces k and by accounting for it in the value of the number of repetitions. The IBM Scientific Library subroutines BDTR and CDTR along with the function subprograms written by the author are used for these computations.

For each of the thousand samples the confidence interval length is computed and is tested to see if it actually covers λ. The final number of those covering λ is divided by the number of repetitions yielding the empirical coverage probability. The sample variance is computed by using the relation $\sigma^2 = \left(\Sigma\ (\bar{x}-x_i)^2\right)/(m-1)$ where $\bar{x}$ is the average length of the intervals, x_i is the length of the i^{th} interval and m is the number of repetitions.

Table 1 CONFIDENCE COEFFICIENT $1-\alpha = .95$

$\_{o}$	n= / PROC.	30 AVG.	30 VAR.	30 C.P.	40 AVG.	40 VAR.	40 C.P.	50 AVG.	50 VAR.	50 C.P.
3	1.	.159	.001	.988	.136	.000	.971	.120	.000	.978
	2.	.148	.001	.963	.127	.000	.959	.112	.000	.970
	3.	.157	.001	.951	.132	.001	.953	.116	.000	.960
10	1.	.107	.000	.961	.090	.000	.977	.080	.000	.963
	2.	.092	.000	.951	.080	.000	.951	.071	.000	.955
	3.	.093	.000	.954	.081	.000	.945	.071	.000	.958
3.2	1.	.234	.002	.978	.200	.001	.963	.176	.001	.964
	2.	.213	.001	.961	.183	.001	.958	.163	.000	.952
	3.	.217	.001	.954	.186	.001	.961	.165	.000	.945
82	1.	.943	.036	.966	.799	.017	.959	.703	.011	.961
	2.	.855	.023	.942	.732	.012	.948	.647	.007	.945
	3.	.871	.026	.936	.743	.012	.954	.657	.008	.936
11	1.	4.54	.822	.954	3.84	.423	.966	3.43	.269	.957
	2.	4.23	.664	.946	3.60	.349	.955	3.22	.227	.954
	3.	4.56	7.10	.940	3.73	.473	.949	3.32	.292	.948
33	1.	3.18	.428	.970	2.73	.227	.972	2.38	.122	.962
	2.	2.77	.196	.963	2.41	.113	.956	2.12	.066	.957
	3.	2.80	.222	.959	2.42	.118	.957	2.14	.071	.961
12	1.	6.04	1.29	.974	5.16	.704	.962	4.56	.420	.957
	2.	5.50	.874	.951	4.75	.499	.955	4.22	.311	.955
	3.	5.62	1.00	.951	4.84	.562	.959	4.28	.338	.955
06	1.	12.1	5.39	.970	10.2	2.60	.966	9.07	1.56	.965
	2.	11.1	3.70	.950	9.39	1.85	.961	8.40	1.15	.964
	3.	11.4	4.14	.949	9.55	2.02	.956	8.54	1.29	.962
1	1.	10.7	5.47	.961	9.08	2.62	.971	7.98	1.63	.959
	2.	9.32	2.46	.945	7.99	1.30	.957	7.08	.798	.950
	3.	9.43	2.74	.939	8.05	1.32	.958	7.12	.827	.950

AVG. = AVERAGE LENGTH

VAR. = SAMPLE VARIANCE

C.P. = COVERAGE PROBABILITY

Table 2 CONFIDENCE COEFFICIENT $1-\alpha = .99$

t_o	n= / PROC.	30 AVG.	30 VAR.	30 C.P.	40 AVG.	40 VAR.	40 C.P.	50 AVG.	50 VAR.	50 C.P.
3	1.	.206	.002	.998	.177	.001	.998	.156	.000	.996
	2.	.202	.001	.996	.172	.001	.990	.152	.000	.989
	3.	.209	.002	.993	.176	.001	.993	.153	.000	.994
10	1.	.139	.001	.991	.117	.000	.998	.104	.000	.991
	2.	.123	.000	.988	.106	.000	.992	.094	.000	.990
	3.	.123	.000	.988	.106	.000	.995	.094	.000	.992
3.2	1.	.304	.003	.998	.260	.002	.996	.232	.001	.994
	2.	.286	.002	.995	.245	.001	.993	.220	.001	.986
	3.	.287	.002	.994	.246	.001	.995	.220	.001	.989
.82	1.	1.22	.060	.988	1.04	.030	.989	.913	.018	.991
	2.	1.15	.040	.987	.979	.020	.988	.862	.013	.993
	3.	1.15	.046	.988	.980	.022	.988	.867	.015	.988
.11	1.	5.90	1.38	.996	4.99	.713	.995	4.46	.454	.994
	2.	5.78	1.11	.989	4.87	.587	.989	4.34	.385	.990
	3.	6.07	14.6	.992	4.94	.813	.985	4.39	.503	.988
.33	1.	4.13	.729	.993	3.55	.384	.997	3.12	.238	.994
	2.	3.70	.342	.992	3.21	.197	.994	2.84	.132	.987
	3.	3.70	.385	.989	3.20	.204	.994	2.83	.135	.986
.12	1.	7.84	2.18	.995	6.70	1.19	.995	5.92	.710	.997
	2.	7.41	1.50	.988	6.36	.859	.991	5.64	.536	.994
	3.	7.43	1.73	.987	6.39	.970	.992	5.64	.583	.994
.06	1.	15.7	9.16	.993	13.4	4.68	.993	11.8	3.05	.991
	2.	14.8	6.20	.985	12.7	3.38	.991	11.2	2.14	.992
	3.	14.9	7.31	.986	12.7	3.79	.991	11.2	2.31	.993
.1	1.	14.0	9.33	.989	11.8	4.44	.997	10.4	2.49	.998
	2.	12.5	4.28	.981	10.6	2.26	.992	9.44	1.34	.996
	3.	12.4	4.74	.983	10.6	2.28	.994	9.43	1.44	.994

TABLE III

		N =	30				40				50			
			α = .05		.01		.05		.01		.05		.01	
λ	t_o	Method	Avg.	Var.	Avg.	Var.	Avg.	Var.	Avg.	Var.	Avg.	Var.	Avg.	Var.
.1	3	1	.100	.001	.100	.001	.101	.001	.101	.001	.100	.001	.100	.001
		3	.128	.002	.142	.002	.121	.001	.131	.001	.116	.001	.124	.001
.1	10	1	.100	.000	.100	.000	.100	.000	.100	.000	.100	.000	.100	.000
		3	.110	.000	.115	.001	.108	.000	.112	.000	.106	.000	.109	.000
.2	3.2	1	.197	.003	.197	.003	.198	.002	.198	.002	.197	.002	.201	.002
		3	.224	.003	.238	.003	.218	.002	.229	.002	.214	.002	.226	.002
.8	.82	1	.806	.051	.806	.051	.800	.034	.800	.034	.792	.027	.792	.027
		3	.914	.059	.968	.062	.882	.038	.922	.039	.861	.031	.892	.032
3	.11	1	2.99	1.10	2.99	1.10	2.94	.773	2.94	.773	3.00	.632	3.00	.632
		3	3.81	2.83	4.19	4.55	3.50	.954	3.76	.986	3.46	.731	3.65	.751
3	.33	1	2.98	.458	2.98	.458	3.03	.355	3.03	.355	2.98	.262	3.01	.298
		3	3.29	.534	3.44	.558	3.25	.384	3.37	.396	3.16	.285	3.28	.322
5	.12	1	5.00	1.87	5.00	1.87	5.04	1.43	5.04	1.43	5.03	1.12	5.03	1.12
		3	5.73	2.14	6.09	2.23	5.59	1.60	5.86	1.64	5.46	1.21	5.68	1.24
10	.06	1	10.1	7.95	9.98	7.70	9.89	5.29	10.0	5.62	9.96	4.11	9.99	4.53
		3	11.6	8.92	12.2	9.26	11.0	5.79	11.6	6.45	10.9	4.52	11.3	4.99
10	.1	1	10.1	5.69	10.1	5.69	10.1	4.09	10.1	4.09	10.0	3.19	10.0	3.03
		3	11.1	6.61	11.7	6.90	10.8	4.34	11.2	4.47	10.6	3.40	10.9	3.42

Avg. = Average Value of Estimate   Var. = Sample Variance

TABLE IV

α = .05 n = 30

λ	t_o	Proc.	Avg.	Var.	c.p.	$\hat{\lambda}$	Var. $\hat{\lambda}$
.8	.82	3	.884	.026	.96	.931	.055
		4	1.13	.005	.96	.805	.075
3	.11	3	4.59	1.06	.91	3.92	1.55
		4	5.92	4.42	.95	3.14	1.30
3	.33	3	2.72	.192	.95	3.15	.475
		4	3.06	.030	.94	2.98	.626
5	.06	3	7.67	3.44	.96	6.16	3.98
		4	9.63	6.41	.98	4.96	3.15
5	.12	3	5.69	1.05	.95	5.79	2.07
		4	7.46	.123	.95	4.89	2.76
10	.06	3	10.9	3.45	.96	10.8	7.44
		4	14.4	.642	.98	9.02	9.16

Avg. = Average Interval Length

Var. = Sample Variance

c.p. = Coverage Probability

$\hat{\lambda}$ = Estimate of λ

Var. $\hat{\lambda}$ = Variance of $\hat{\lambda}$

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MAIN PROGRAM FOR COMPUTING CONFIDENCE INTERVALS, SAMPLE VARI
ANCE, AND EMPIRICAL PROBABILITY FOR PROCEDURES 1,2, AND 3.
NN  THE NUMBER OF DIFFERENT COMBINATIONS OF CONFIDENCE
COEFFICIENTS AND SAMPLE SIZE
NCOM  THE NUMBER OF DIFFERENT COMBINATIONS OF LAMDA AND
CENSORING TIMES
N  SAMPLE SIZE
LREP  NUMBER OF REPETITIONS
IRAND  INPUT PARAMETER FOR SUBROUTINE RANDU
PROB  1-ALPHA/2  ALPHA IS THE CONFIDENCE COEF.
XNORM  THE NORMAL DEVIATE
AMDA  THE PARAMETER LAMDA
T  CENSORING TIME
      DIMENSION BETA(101),AMD(1000),AMDD(1000),AHAT(1000),XX
(100),
     1CHI(100),AM(1000),CHIL(100),AML(1000),AMDL(1000),BETAL
(101),
     1AMDDL(1000),CAMDA(1000),CC(1000),BB(1000),O(1000)
      READ(5,13)NN
   13 FORMAT(I4)
      DO25 II=1,NN
      READ(5,3)NCOM,N,LREP,IRAND,PROB,XNORM
    3 FORMAT(4I7,2X,2F10.6)
      NREP=LREP
      DO110 I=1,N
      CHIL(2*I)=0.
  110 CHI(2*I)=0.
      DO111 I=1,N
      BETAL(I)=0.
  111 BETA(I)=0.
      PROBL=1.-PROB
      DO25KK=1,NCOM
      LREP=NREP
      SCAM=0.
      MCUN=0
      LOUN=0
      KCUN=0
      MOUNT=0.
      AH=0.
      LCUNT=0
      KOUNT=0
      CH=0.
      NC=0
      OD=0.
      NO=0
      BIN=0.
      NB=0
      READ(5,2)AMDA,T
    2 FORMAT(2F20.8)
      DO15I=1,LREP
      K=0
      SMALT=0.
      SQNOR=XNORM*XNORM
GENERATE THE SAMPLE
      DO7L=1,N
      CALL RANDU(IRAND,IYRAN,X)
      IRAND=IYRAN
      X=-ALOG(1.-X)/AMDA
      IF(X-T)4,5,5
    4 K=K+1
      GO TO 6
    5 X=T
    6 SMALT=SMALT+X
      XX(L)=X
    7 CONTINUE
      IF(K-N)50,45,50
   45 LREP=LREP-1
      I=I-1
      GO TO 15
   50 CONTINUE
      IF(K)49,45,49
   49 TIME=0.
```

```
      DO55IT=1,N
      IF(XX(N-IT+1)-T)51,55,55
   51 JT=N-IT+1
      GO TO 56
   55 CONTINUE
   56 DO 60 IX=1,JT
      TIME=TIME+XX(IX)
   60 CONTINUE
C     COMPUTATION FOR PROC.3
      IF(CHI(2*K))65,61,65
   61 G=2*K
      CHI(2*K)=CVAL(PROB,G)
   65 CONTINUE
      AM(I)=CHI(2*K)/(2.*TIME)
      IF(AM(I)-AMDA)67,66,66
   66 MOUNT=MOUNT+1
   67 CONTINUE
      IF(CHIL(2*K))165,161,165
  161 G=2*K
      CHIL(2*K)=CVAL(PROBL,G)
  165 CONTINUE
      AML(I)=CHIL(2*K)/(2.*TIME)
      IF(AML(I)-AMDA)166,166,167
  166 MOUN=MOUN+1
  167 CONTINUE
      CC(I)=AM(I)-AML(I)
      CH=CH+CC(I)
      IF(MOUN+MOUNT-2)171,170,171
  170 NC=NC+1
  171 MOUN=0
      MOUNT=0
      CAMDA(I)=(CHI(2*K)+CHIL(2*K))/(4.*TIME)
      SCAM=SCAM+CAMDA(I)
C     COMPUTATION FOR PROC.2
      FN=N
      AMDA1=(K/SMALT)/(1.+1./(2.*FN))
      AH=AH+AMDA1
      AHAT(I)=AMDA1
      C=1./SMALT
      B=C*C
      AMD(I)=(SQNOR*C+2.*AMDA1+SQRT(SQNOR*SQNOR*B+4.*AMDA1*S
     QNOR*C))/2
      IF(AMD(I)-AMDA)9,8,8
    8 KOUNT=KOUNT+1
    9 CONTINUE
      AMDL(I)=(SQNOR*C+2.*AMDA1-
     1                             SQRT(SQNOR*SQNOR*B+4.*AMDA1*S
     QNOR*C))/2
      IF(AMDL(I)-AMDA)108,108,109
  108 KOUN=KOUN+1
  109 CONTINUE
      O(I)=AMD(I)-AMDL(I)
      OD=OD+O(I)
      IF(KOUN+KOUNT-2)173,172,173
  172 NO=NO+1
  173 KOUN=0
      KOUNT=0
C     COMPUTATION FOR PROC.1
      K=K+1
      IF(BETA(K))205,200,205
  200 B1=N-K+1
      A1=K
      BET=XVAL(PROB,A1,B1)
      IF(BET)201,202,202
  201 GO TO 25
  202 CONTINUE
      BETA(K)=BET
  205 PUP=BETA(K)
      AMDD(I)=-ALOG(1.-PUP)/T
      IF(AMDD(I)-AMDA)11,10,10
   10 LOUNT=LOUNT+1
   11 CONTINUE
```

```
      IF(BETAL(K))305,300,305
  300 B1=N-K+2
      A1=K-1
      BET=XVAL(PRCBL,A1,B1)
      IF(BET)201,302,302
  302 CONTINUE
      BETAL(K)=BET
  305 PLP=BETAL(K)
      AMDDL(I)=-ALOG(1.-PLP)/T
      IF(AMDDL(I)-AMDA)310,310,311
  310 LOUN=LOUN+1
  311 CONTINUE
      BB(I)=AMDD(I)-AMDDL(I)
      BIN=BIN+BB(I)
      IF(LOUN+LOUNT-2)175,174,175
  174 NB=NB+1
  175 LOUN=0
      LOUNT=0
   15 CONTINUE
      REP=LREP
      VAR3=0.
      VAR1=0.
      VAR2=0.
      AVH=0.
      SH=0.
      OD=OD/REP
      BIN=BIN/REP
      CH=CH/REP
      SCAM=SCAM/REP
      AH=AH/LREP
      DO20J=1,LREP
      SH=SH+(CAMDA(J)-SCAM)**2
      AVH=AVH+(AHAT(J)-AH)**2
      VAR1=VAR1+(O(J)-OD)**2
      VAR2=VAR2+(PB(J)-BIN)**2
      VAR3=VAR3+(CC(J)-CH)**2
   20 CONTINUE
      PB=NB/REP
      PC=NC/REP
      PO=NO/REP
      SH=SH/(REP-1.)
      VAR1=VAR1/(REP-1.)
      VAR2=VAR2/(REP-1)
      VAR3=VAR3/(REP-1.)
      AVH=AVH/(REP-1.)
      WRITE(6,21)
   21 FORMAT(1X,'REPS',3X,'SAMPLE SIZE LAMDA CUTOFF TIME PRO
BABILITY')
      WRITE(6,22) LREP,N,AMDA,T,PROB
   22 FORMAT(//,2I8,1X,3F10.5,//)
      WRITE(6,23)
   23 FORMAT(14X,'O.D. PROC',7X,'BIN. METH',6X,'CHI-SQ. METH
')
      WRITE(6,26)OD,BIN,CH,VAR1,VAR2,VAR3,PO,PB,PC
   26 FORMAT(1X,'AVERAGE',3(5X,F10.5),//,1X,'VARIANCE',3(5X,
F10.5),//,
     11X,'COVERAGE PROB',3(5X,F10.5))
      WRITE(6,24)AH,AVH
      WRITE(6,24) SCAM,SH
   24 FORMAT(1X,'AVG. LAMDA HAT=',F12.6,'VARIANCE LAMDA HAT=
',F12.6)
   25 CONTINUE
      END
XVAL ITERATES TOWARD LAMDA FOR PROCEDURE 1


      FUNCTION XVAL(ALPHA,A1,B1)
      DATA EPS/.5E-07/,N/1000/
      A=A1
      B=B1
      AL=ALPHA
    4 XLAST=-.1
```

```
      M=0
      X=.1
      DX=.1
   11 CALL BDTR(X,A,B,P,D,IER)
      TEST=P-AL
      IF(TEST)13,5,13
   13 IF(ABS(TEST)-.5E-04) 12,12,6
   12 IF(ABS((TEST-XLAST)/XLAST)-EPS)5,5,6
    5 XVAL=X
      RETURN
    6 IF(TEST*XLAST)7,8,8
    7 DX=-.5*DX
    8 X=X+DX
      XLAST=TEST
      M=M+1
      IF(M-N)11,9,9
    9 WRITE(6,15)
   15 FORMAT(1X,'NO CONVERGENCE')
      XVAL=-1.
      RETURN
      END
 CVAL ITERATES TOWARD LAMDA FOR PROCEDURE 3


      FUNCTION CVAL(ALPHA,G)
      DATA EPS/.5E-07/,N/1000/
      AL=ALPHA
    4 XLAST=-.1
      M=0
      X=.1
      DX=.1
   11 CALL CDTR(X,G,P,D,IER)
      TEST=P-AL
      IF(TEST)13,5,13
   13 IF(ABS(TEST)-.5E-04) 12,12,6
   12 IF(ABS((TEST-XLAST)/XLAST)-EPS)5,5,6
    5 CVAL=X
      RETURN
    6 IF(TEST*XLAST)7,8,8
    7 DX=-.5*DX
    8 X=X+DX
      XLAST=TEST
      M=M+1
      IF(M-N)11,9,9
    9 WRITE(6,15)
   15 FORMAT(1X,'NO CONVERGENCE')
      CVAL=-1.
      RETURN
      END
```

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13. ABSTRACT

Obtaining a confidence interval for a parameter λ of an exponential distribution is a frequent occurrence in life testing problems. Oftentimes the test plan used is one in which all the observations are censored at the same time point t_o. Several approximate confidence interval procedures are available in the statistical literature; however, to the knowledge of the author, the performance characteristics of the various approximations used in these procedures have not been established analytically. The purpose of this paper is to report the results of an empirical study of the performance of four of these procedures with respect to the expected length of the interval, the variance of the interval length, and the coverage probability.

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