Case SSV part I

EE4 – Building a SSV – Team PM9

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Introduction

The objective of this project is to design and build an SSV. In order to design an SSV we have to make intelligent design choices of designing elements (frame, wheels, shaft, drive structure, guiding system, positioning of the solar panel, total weight), and combine them to get an optimal result. To reach a motivated result, it is necessary to make a simulation before we start building.
In this document three subjects are treated: first we determined the solar panel characteristics, second we made a simulation of the race in Simulink and Matlab and as third we made a determination of the first second of the race using the bisection method.
On all measured values an appropriate error analysis is applied.

1. Characteristics and diode factor of the solar panel

The Characteristics of the solar panel can be determined by experiment. For the experiment we putted the solar panel beneath a strong light source, more specific a halogen lamp. In this way we could determine the diode factor and the characteristics of the panel which give the ideal working point of the DC motor. By combining the U-I graph of the solar panel and the DC-motor, we can define the rotates per minute of this ideal working point.

1.1 Process

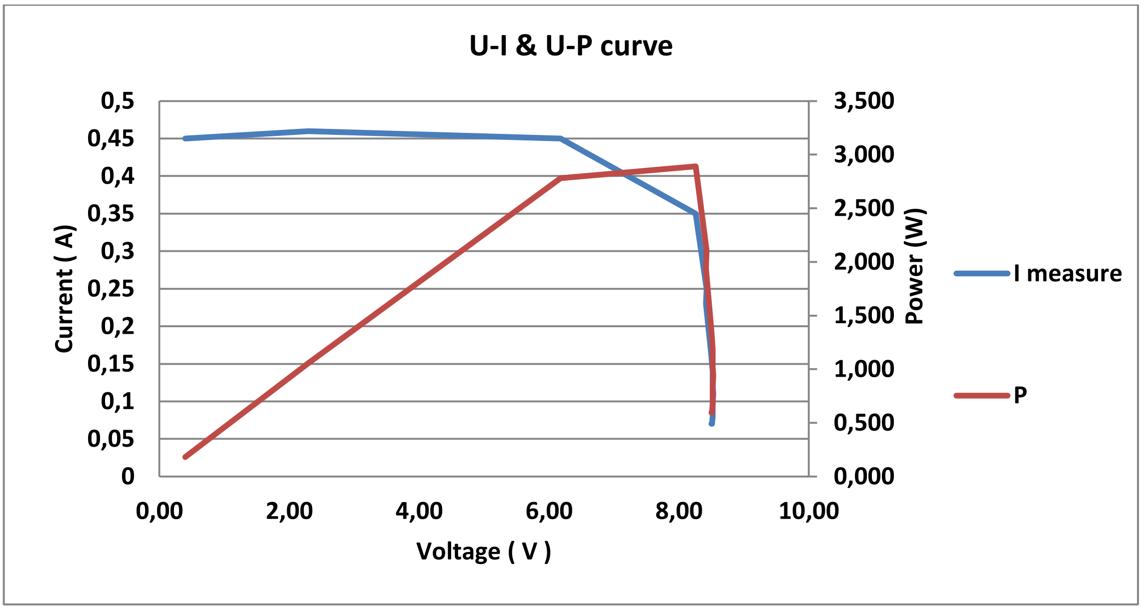
2. Define the short circuit current and open circuit voltage

First we defined the short circuit current by placing a current meter in serial with the solar panel. Because of the very low internal resistance of the meter, the dissipating energy in the meter is negligible. This means that the meter should measure the short circuit current.

The open circuit voltage can be obtained by connecting the voltmeter in parallel to the solar panel.

3. Measure the working points

When we want to define voltage-current and voltage-power characteristics (to define the working point where the motor has its maximal power), we need to know the current by each voltage. This can be measured by a voltmeter in parallel and current meter in series with a variable resistance , which is connected on the solar panel. When we set the resistance on an particular value, the meters will give a certain value. The table below (table 1) shows the measured values U and I by a certain resistance.



Graph 1 : Measured current in function of measured voltage and power in function of measured voltage.

4. Calculate m-values (diode factor) and the average m

Following formula describes current (I)in function of voltage (U):

$$I = I_{sc} - I_s(e^{\frac{U}{mNU_R}} - 1)$$

Explanation elements:

I_{sc} → short circuit current (A)

$I_s = 10^{-7}\ A$ → saturation current

U → output voltage (V)

$U_r = 25{,}7\ mV$ → thermal voltage at 25°C

m → diode factor

$e = 2{,}71828$ → methematical constant

N → number of solar cells in series

The diode factor m can be excluded in the above formula to get de following:

$$m = \frac{U}{\ln\left(\frac{I - Isc}{I_s} + 1\right) U_R N}$$

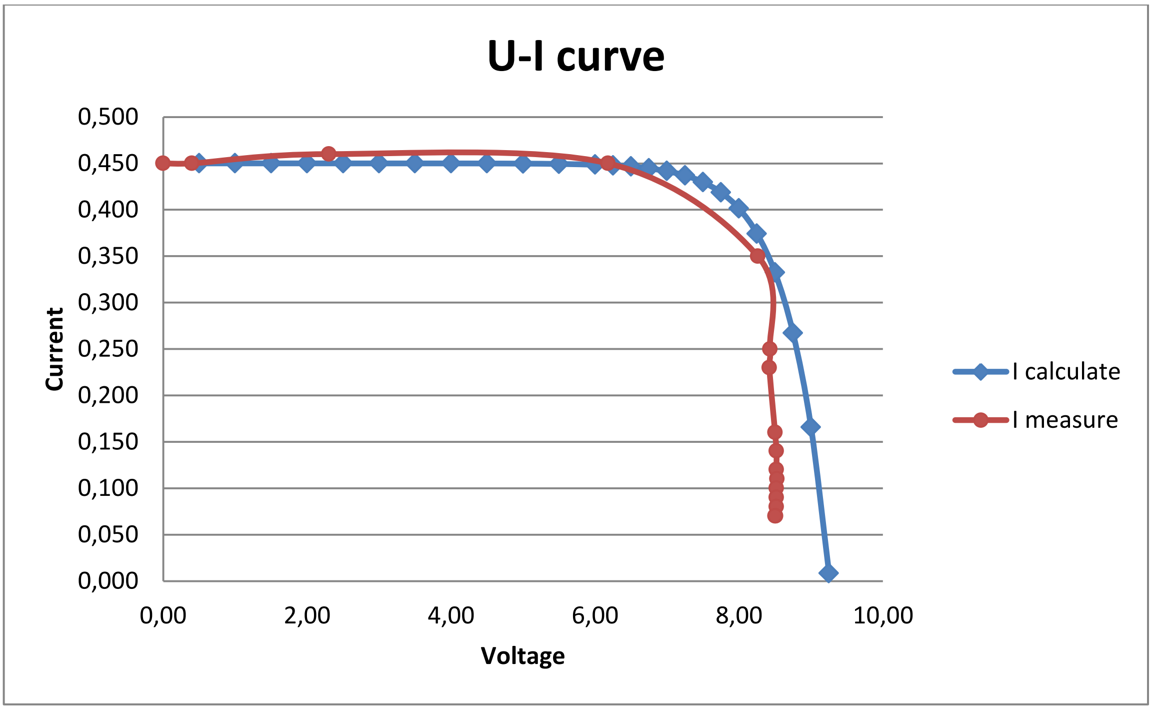
When we insert the measured data, we can calculate for each voltage and current the diode factor.
The final value can be computed by the average of all measurements.
The error on this value is calculated in the next chapter.
The measured values can be found in table 1, the next values are taken as a constant:
I_{sc} = 0.45 A [1] ; I_s = 1e-7A ; N = 16 ; Ur = 0,026 V

[1] For the simulations in Simulink, I_{sc} = 0,88 A had to be used instead of I_{sc} = 0.45 A because this approaches more the realistic short current we will get with the real sun.

U(V) (measured)	I(A) (measured)	P (W)	m
0,40	0,45	0,18	/
2,30	0,46	1,06	/
6,18	0,45	2,78	/
8,26	0,35	2,89	1,454
8,43	0,25	2,11	1,413
8,42	0,23	1,94	1,402
8,50	0,16	1,36	1,389
8,52	0,14	1,19	1.386
8,52	0,12	1,02	1,380
8,53	0,11	0,94	1,379
8,52	0,10	0,85	1,375
8,52	0,09	0,77	1,372
8,52	0,08	0,68	1,370
8,51	0,07	0,60	1,366
8,50	0,07	0,60	1,364
Average m			**1,39**

Table 1: Measured values

Control: The m value must situate between 1 and 5. This is the case, so the m value can be realistic.



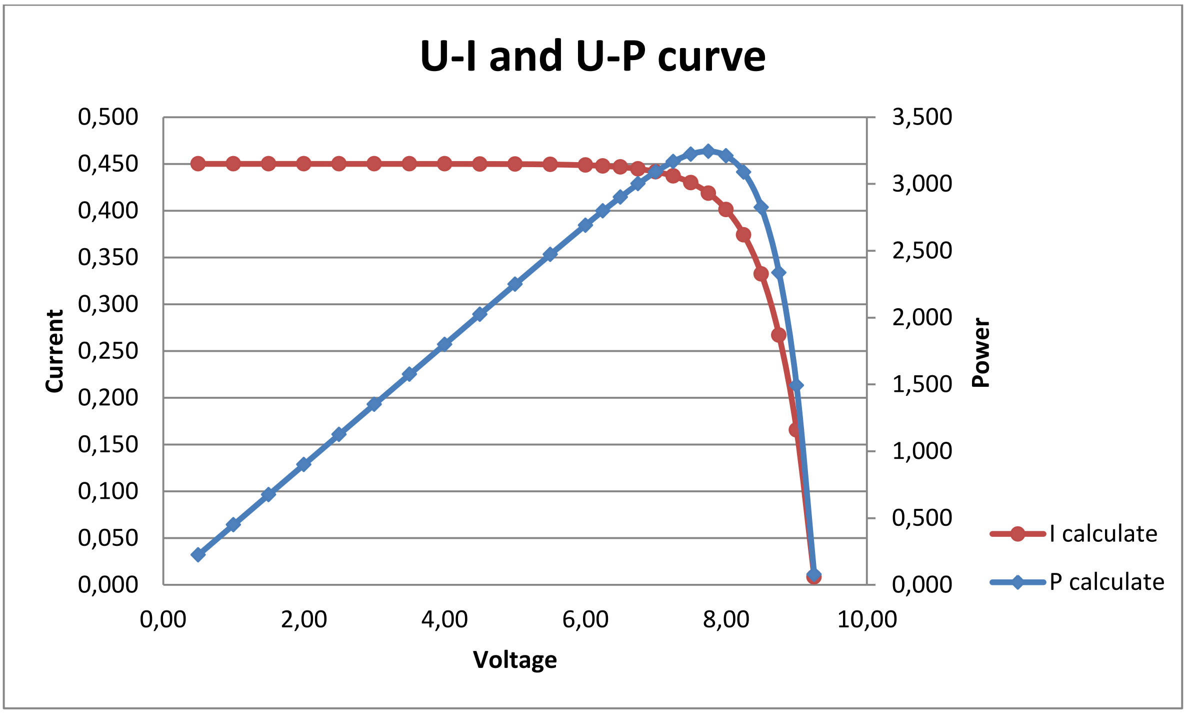
Graph 2 : Comparison measured curve and calculated curve

To obtain graph 2 we have plotted our curve with our average m value, then this value is always adjusted so that we got the graph that best matches to our measured points. we can see that the measured curve (red) correspond now approximately with the calculated curve (blue).

1.2 Discussion of the U-I and U-P characteristic

Using the calculated values U and I we can plot the U-I and U-P characteristic of the solar panel.

As we can see, the solar cell works like a current source when the connected resistance is low. A higher resistance leads to a decrease in current. This occurs due to the internal diode of the solar panel (figure 1). When the voltage over the resistor and thus over the diode, is too high and rises the breakdown voltage, the diode starts conducting. In this case a part of the current stays in the solar panel and can't be used. This value of voltage is approximately 8,5V.



Graph 3: U-I and U-P with calculated m value

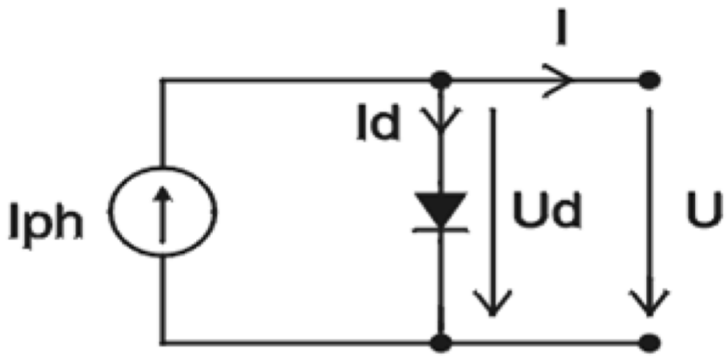


Figure 1: Ideal solar cel

The U-P characteristic can be created by multiplying the voltage and current. The characteristic gives that the power increases by increasing voltage. When the voltage becomes too high, the power drops sharply because off the internal diode which starts conducting by the breakdown voltage. The power output that the solar panel gives with our measurements is approximately 3,22 W by 7,50 V but practically it is dependent on the height of the sun.

1.3 Error analysis

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We don't work in ideal conditions and thus we have to define the error on our measurements.
In this part we will define the error on the diode factor using the min-max method:

$$\partial X = \frac{Xmax - Xmin}{2}$$

As we mentioned before, we can calculate the diode factor out of this formula:

$$m = \frac{U}{\ln\left(\frac{Isc - I}{I_s} + 1\right) * U_R * N}$$

The error on this value becomes:

$$\partial m = \frac{m, max - m, min}{2}$$

We used a multimeter "Tenma 72 – 7925" to define the voltage and current. We can find the information needed to define the error in the manual of the appliance[2].

Error = percentage of the measurement + number of digits

The voltmeter was set in the range of 20 V DC. The best accuracy is thereby given by ± (0.5% + 1).
The current meter was set in the range of 20 mA DC. The best accuracy is thereby given by ± (2% + 5).

Sample preparation : error on the voltage meter with U = 8,50 V :

$$\partial U = (\tfrac{0,5}{100} * 8,5 + 1 * 0,01) \text{ V}$$

$$\partial U = 0,05 \text{ V}$$

U = (8,50 ± 0,05) V

Sample preparation : error on the current meter with I = 0,16 A :

$$\partial I = (\tfrac{2}{100} * 0,16 + 5 * 0,01) \text{ A}$$

$$\partial I = 0,05 \text{ A}$$

I = (0,16 ± 0,05) A

=[2] http://www.farnell.com/datasheets/1661993.pdf consulted on (20/02/2014)

$$m,max = \frac{8{,}26 - 0{,}05}{\ln\left(\frac{0{,}45 - (0{,}35 + 0{,}06)}{10\text{^}(-7)} + 1\right) 0{,}027\,.\,16} = 1{,}4733$$

$$m,min = \frac{8{,}50 + 0{,}05}{\ln\left(\frac{0{,}45 - (0{,}07 - 0{,}05)}{10\text{^} - 7} + 1\right) 0{,}027\,.\,16} = 1{,}2958$$

$$m = \ 1{,}39 \ \pm 0{,}0888$$

We chose the highlighted errors in that way that they would indeed give us the highest or lowest values for m. Looking at the table below helps understanding this choice.

Table 2 lists up the measured values with their appropriate error:

U		Error	I		Error	Diode factor (m)
0,00			0,45	±	0,06	/
0,40	±	0,01	0,45	±	0,06	/
2,30	±	0,02	0,46	±	0,06	/
6,18	±	0,04	0,45	±	0,06	/
8,26	±	0,05	0,35	±	0,06	1,45
8,43	±	0,05	0,25	±	0,06	1,41
8,42	±	0,05	0,23	±	0,05	1,40
8,50	±	0,05	0,16	±	0,05	1,39
8,52	±	0,05	0,14	±	0,05	1,39
8,52	±	0,05	0,12	±	0,05	1,38
8,53	±	0,05	0,11	±	0,05	1,38
8,52	±	0,05	0,10	±	0,05	1,38
8,52	±	0,05	0,09	±	0,05	1,37
8,52	±	0,05	0,08	±	0,05	1,37
8,51	±	0,05	0,07	±	0,05	1,37
8,50	±	0,05	0,07	±	0,05	1,37
						1,39

Table 2 : measured values with their appropriate error

The error on the diode factor amounts ± 0,0888 . We can see that the measurements are between this marge, we conclude that the experiment was performed accurate.
The percentage error is 4,6 %.

1.3.1 Possible causes of errors

We had only ten minutes to do all the measurements. Therefore we had to work quickly and that's why the readings weren't always accurate.
During the experiments the solar panel heated up and influenced the measurements, because of the changing U_R value. This is also a cause of errors.

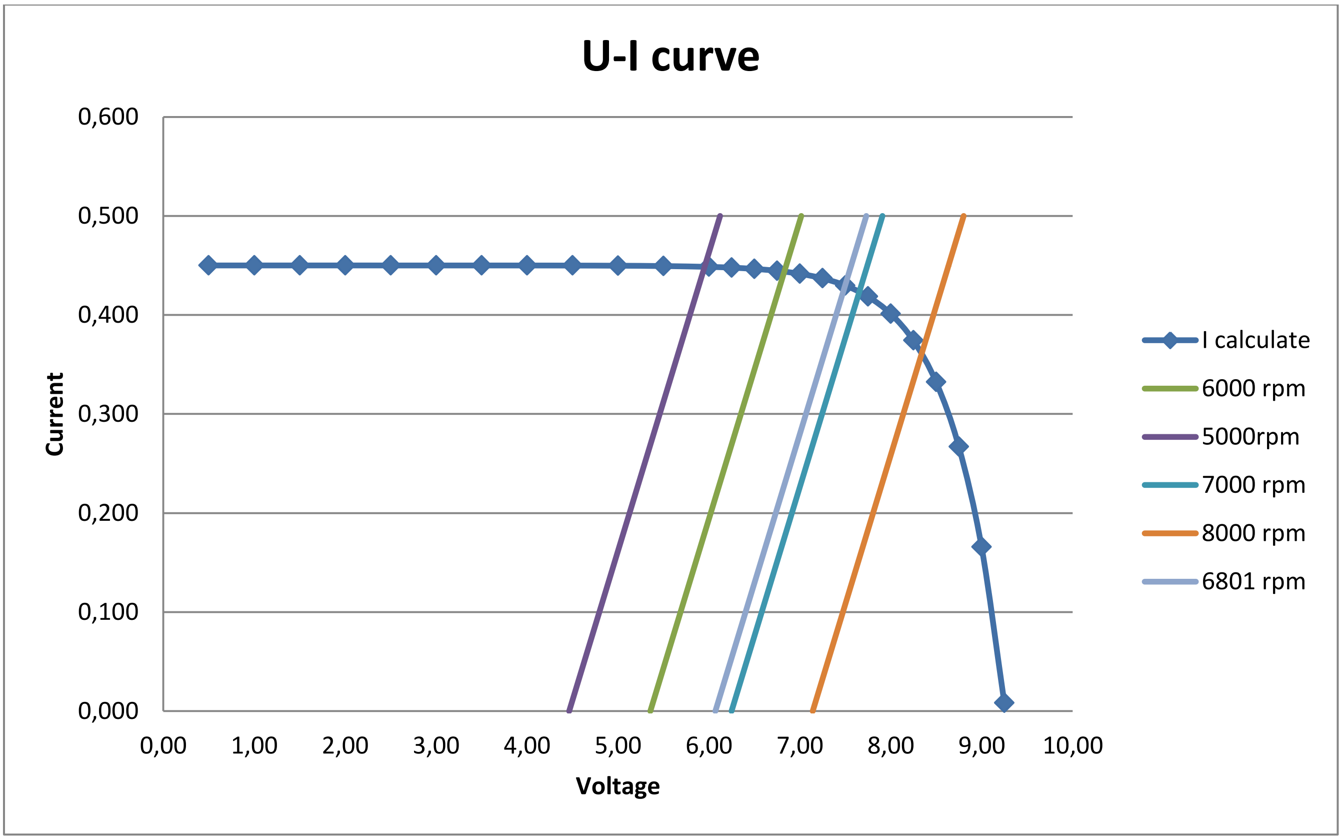
1.3 Working points of the DC motor

The values of U and I by maximum power are respectively 7,50 V and 0.43 A. Together with the data of the DC motor (terminal resistance=3.32Ω) we can compute the optimal revolution speed with the formula below.

$$U = I * R + \frac{n}{1120} \quad => n = (U - IR) * 1120 = (7{,}50 - 0.43 * 3.32) * 1120 = 6801\, rpm$$

The optimal revolution of the DC-motor is 6801 rpm when de DC-motor is connected on the solar panel. Graph 4 plots different revolutions, the working points are the intersections of the motor characteristic and the diode characteristic.

The curve of 6801 rpm intersects the solar characteristic where maximal power is achieved. This is the value that we want to continue working.



Graph 4 : U-I curve of the solar panel and the motor characteristic

2. Optimal gear ratio and optimal mass

2.1 Analytical calculations: optimal mass

- Starting from the force on the output axes, we get:

$$F_{as} = \frac{T_{out}}{R_w} = \frac{I.T_{in}}{R_w} \qquad \text{with} \qquad [\ i = \frac{T_{out}}{T_{in}}\ ]$$

- Assuming a constant acceleration we can use the following dynamic equation. This assuming is only valid within a very short time interval, but it gives us an idea to define the optimal mass of the SSV.

$$v^2 = v_0^2 + 2a(x - x_0) \quad with\ [\ v_0 = 0\ and\ x_0 = 0\ ]$$

$$v_{SSV}^2 = 2ax => v_{SSV} = \sqrt{2ax} \qquad (1)$$

- Out of Newton's second law we get:

$$F_{as} = ma => a = \frac{F_{as}}{m} = \frac{kT_{in}}{m} \qquad (2)$$

- Combining (1)=(2), gives:

$$\frac{v_{SSV}^2}{2x} = \frac{kT_{in}}{m} => k = \frac{v_{SSV}^2 m}{2xT_{in}} \qquad (3)$$

- $v_{SSV} = r_w.w_{out} \quad with \ \frac{w_{in}}{w_{out}} = i$

$$\left.\begin{array}{l} \\ k = \frac{i}{r_w} => i = k.r_w \end{array}\right\} => w_{out} = \frac{w_{in}}{k.r_w}$$

- $v_{SSV} = r_w.\frac{w_{in}}{r_w.k} = \frac{w_{in}}{k} => k = \frac{w_{in}}{v_{SSV}} \qquad (4)$
- Out of (3)=(4), we get:

$$v_{SSV}^3 = \frac{w_{in}.2x.T_{in}}{m} => v_{SSV} = \sqrt[3]{\frac{w_{in}.2x.T_{in}}{m}}$$

- When we fill in this value of v_{SSV} in the following equation of the velocity of the ball:

$$v_{ball} = \frac{[(m_{ball} - m_{SSV}).v_{ball(0)} + 2.m_{SSV}.v_{SSV}]}{m_{SSV} + m_{ball}} \qquad with\ [\, v_{ball(0)} = 0 \,]$$

$$=> \ v_{ball} = \frac{2.m_{SSV}.\sqrt[3]{\frac{w_{in}.2x.T_{in}}{m_{SSV}}}}{m_{SSV} + m_{ball}}$$

- To get the value of the maximum velocity of the ball we set the derivative equal to zero:

$$\frac{dv}{dm_{SSV}} = 0$$

- When we fill in this command in maple[3] we get a value for m_{SSV} whereby the ball will achieve is maximal velocity:

$$m_{SSV} = 2m_{ball} = 1.470 \text{ kg}$$

[3] http://www.maplesoft.com/products/Maple/students/ consulted on (14/03/2014)

2.2 Analytical calculations: optimal height of the ball

- By assuming an elastic collision between the SSV and the ball so that no energy is lost, there is conservation of energy and conservation of momentum:

$$\frac{m_{ball}}{v_{ball}^2} = m_{ball} g h$$

- $$h = \frac{v_{ball}^2}{2g} = \frac{\sqrt[3]{(\frac{w_{in} \cdot 2x.T_{in}}{m})^2}}{2g}$$

- When we fill this equation in in maple we get the optimal height of de ball, by assumption that the optimal mass of the ball is 1,470 kg (see 2.1):

$$h = 1{,}12 \text{ m}$$

2.3 Analytical Calculations optimal gear ratio

- $i = \frac{w_{in}}{w_{out}}$ with $\begin{cases} v_{SSV} = r_w . w_{out} \\ v_{SSV} = \sqrt[3]{\frac{w_{in} . 2x.T_{in}}{m}} \end{cases}$

$$i = \frac{0{,}025 . w_{in}}{v_{Svv}}$$

- When we fill in this equation in maple, we get:

$$i = 4$$

The advantage of doing analytical calculations is to give you an indication of the desired values. If we should take into consideration all factors in reality (friction, losses,…)it would be too complex to calculate analytically. That is why we make assumptions to make these calculations simpler and soluble.

As a result, our analytical values (ideal values) will differ from the actual values.
Analytical values give a good picture, but we must keep in mind that these values are in perfect condition.

3 Simulations Simulink

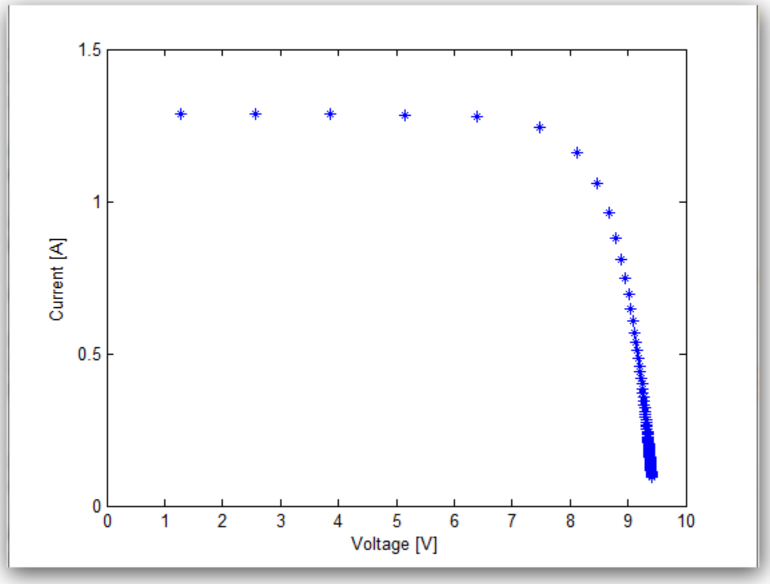
To obtain the ideal mass, we performed a simulation in Matlab-Simulink.
The project manager made available a Matlab template where we had to fill in our parameters and which we had to link to our SSV model in Simulink. In the program, these parameters are entered into a differential equation which is then solved. The following table lists the parameters that we have filled into the program.

	Value	Unit
Solar Power		
Ir	1000	
Isc	0,88	A
Is	1 e-7	A
Voc	9.25	V
Ir0	700 (Irradiance used for measurements)	
m	1,39*16 = 22,24	
DC-motor		
Ra	3,32	Ω
Km	0.00855	
L	0.22	mH
Cm	8.93 e-4	
Gearbox		
efficiency	-0.3	
SSV		
Cw	0,5	
A	0,029	m2
ρ	1,293	kg/m3
g	9,81	N/kg
Crr	0,012	
M_ball	0,735	kg
r_w	0,03	m

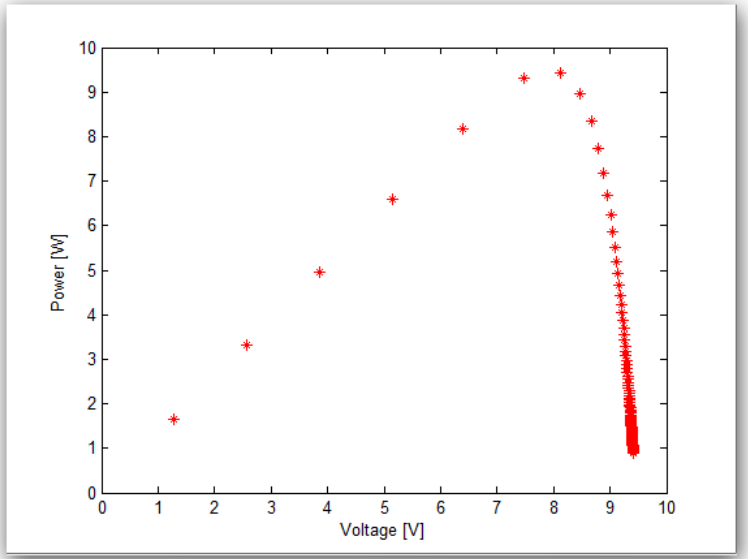
Table 3: List of filled in parameters in Simulink

3.1.1 Optimal resistor

We simulated the behavior of our solar panel linked to a resistor between 1 and 100 Ohm.



Graph 5: I-U relation for different resistor values



Graph 6 : Delivered power for different resistor values

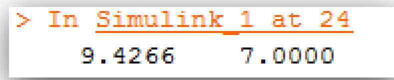
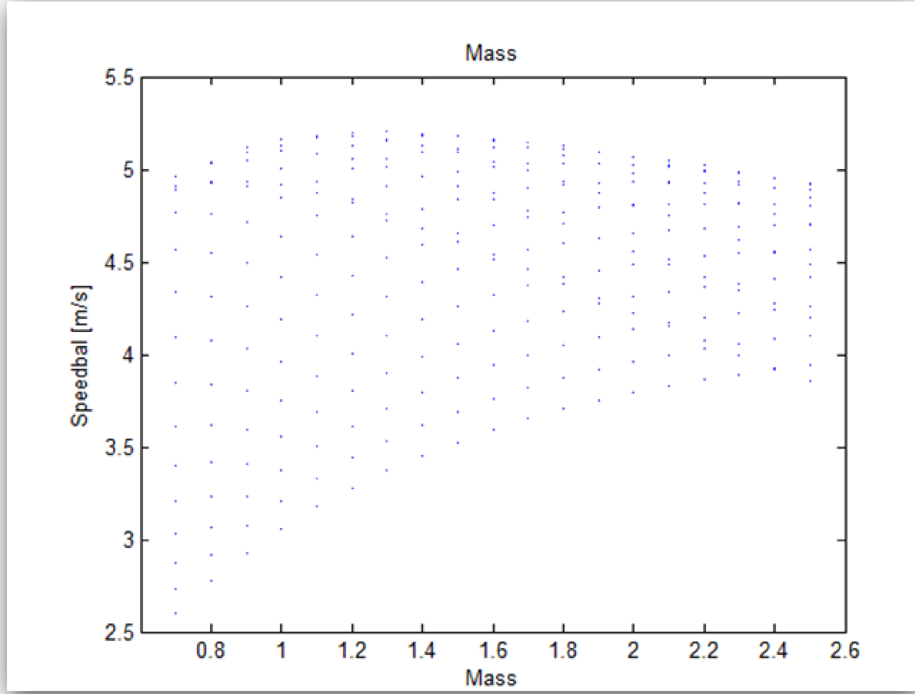


Figure 2 : Maximal power by optimal resistor

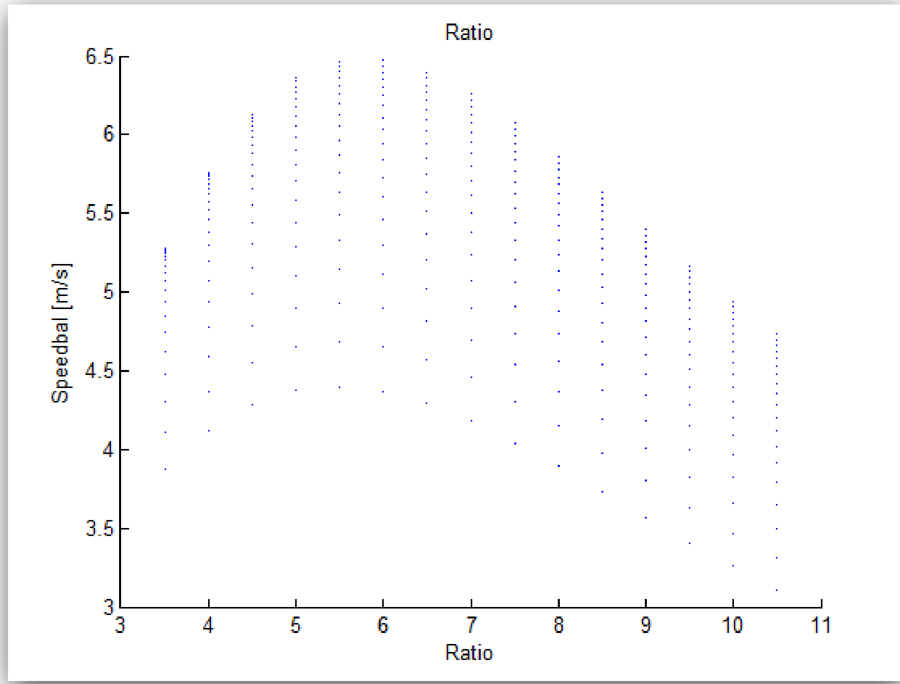
As we can see in figure 2, we get a maximal power of 9,4266 W with a resistor of 7 Ohm.
This value differs from the analytical value because there we worked with an Isc of 0,45A but for the the simulations we had to work with an Isc of 0,88 A.

3.1.2 Optimal mass



Graph 7 : Speed of the ball for different masses of the SSV

3.1.3 Optimal gear ratio



Graph 8: Optimal gear ratio when the ball has its maximal speed

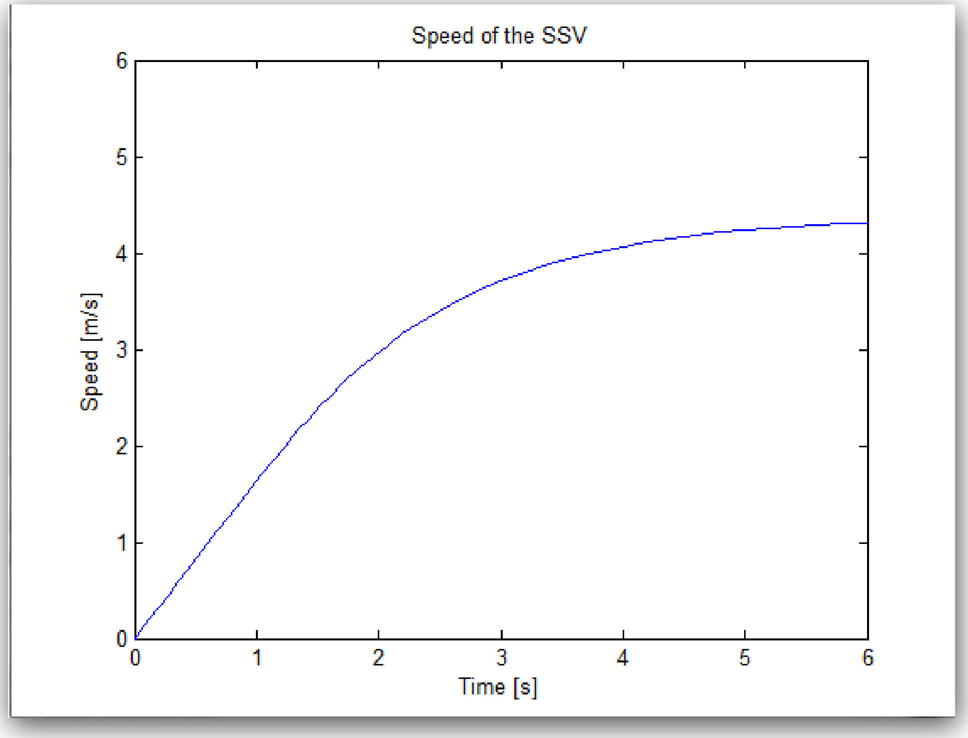
We simulated the speed of the ball in function of different gear ratios from 3.5 to 10.5 in steps of 0,5 (Graph 8) and for different masses from 0.7 Kg to 2,5 Kg in steps of 0,1 (Grapgh 7);
We can see on graph 8 that the ball achieves maximal speed with a gear ratio of 6 and in Graph 7 we can see that the SSv achieves maximal speed with a mass of 1.2 kg. This matches approximately our analytical approach. This gear ratio we will use in our SSV car.

We can find the maximal height of the ball and the thereby linked mass and gear ratio in the simulated table 4 in Simulink.
This gives 1,3802 m with a ratio of 6 and a mass of 1.2 kg.
This value is again higher than the analytical one, this is again related to the use of a different value of Isc.

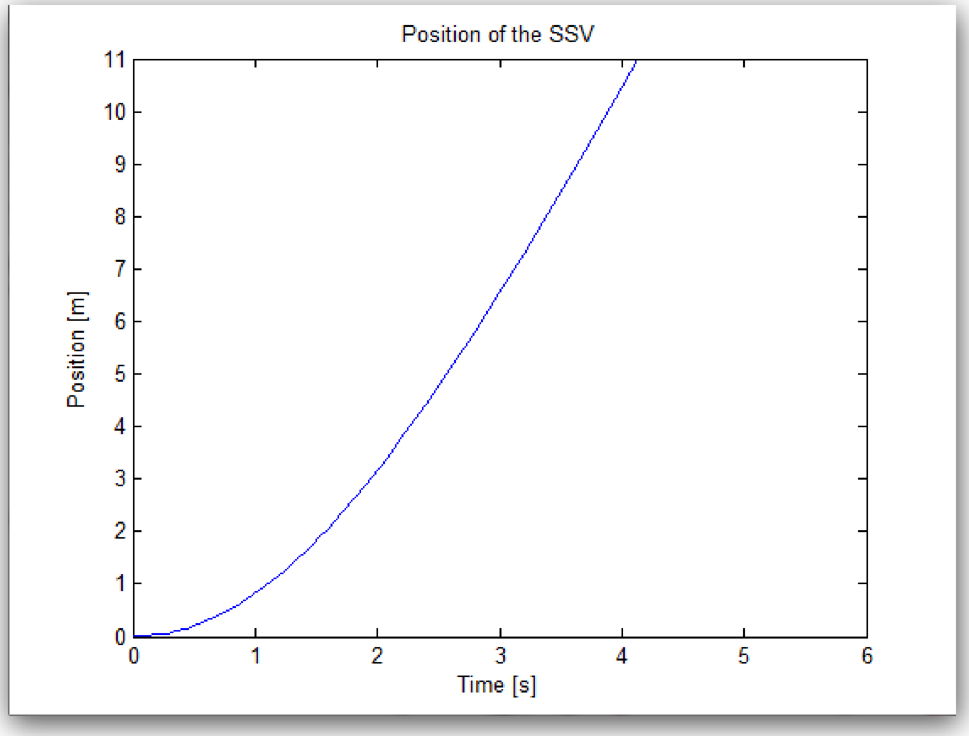
	1	2	3	4	5	6	7	8	9	10
1	1.2198	1.2534	1.2264	1.1577	1.0625	0.9576	0.8523	0.7534	0.6655	0.5893
2	1.2400	1.2965	1.2918	1.2377	1.1550	1.0538	0.9483	0.8456	0.7511	0.6672
3	1.2419	1.3193	1.3338	1.2978	1.2268	1.1331	1.0293	0.9251	0.8268	0.7380
4	1.2308	1.3243	1.3577	1.3383	1.2780	1.1951	1.0954	0.9931	0.8933	0.8006
5	1.2100	1.3183	1.3673	1.3623	1.3166	1.2416	1.1494	1.0500	0.9501	0.8558
6	1.1831	1.3043	1.3668	1.3755	1.3418	1.2784	1.1928	1.0968	0.9994	0.9038
7	1.1523	1.2828	1.3532	$\boxed{1.3802}$	1.3578	1.3038	1.2264	1.1361	1.0411	0.9461
8	1.1179	1.2550	1.3415	1.3738	1.3657	1.3210	1.2522	1.1673	1.0746	0.9823
9	1.0843	1.2260	1.3221	1.3662	1.3671	1.3308	1.2692	1.1917	1.1040	1.0129
10	1.0485	1.1939	1.2964	1.3522	1.3596	1.3364	1.2821	1.2094	1.1264	1.0389
11	1.0124	1.1612	1.2704	1.3347	1.3510	1.3356	1.2885	1.2227	1.1454	1.0609
12	0.9768	1.1274	1.2420	1.3133	1.3389	1.3306	1.2921	1.2330	1.1586	1.0786
13	0.9431	1.0916	1.2107	1.2895	1.3223	1.3223	1.2910	1.2382	1.1699	1.0932
14	0.9101	1.0583	1.1803	1.2643	1.3060	1.3097	1.2876	1.2406	1.1760	1.1030
15	0.8776	1.0256	1.1489	1.2388	1.2872	1.2981	1.2809	1.2386	1.1802	1.1119
16	0.8448	0.9935	1.1183	1.2110	1.2664	1.2852	1.2726	1.2368	1.1813	1.1180
17	0.8145	0.9621	1.0881	1.1849	1.2422	1.2670	1.2611	1.2302	1.1816	1.1207
18	0.7856	0.9306	1.0560	1.1552	1.2213	1.2492	1.2491	1.2252	1.1797	1.1234
19	0.7565	0.8990	1.0273	1.1275	1.1980	1.2317	1.2366	1.2173	1.1752	1.1230

Table 4 : Maximal height of the ball

3.1.4 Example plots



Graph 9: Speed of the SSS in function of time



Graph 10 : Position of the SSV in function of time

4 Bisection method

In this chapter we will calculate the first second of the replacement- and speed characteristic by using the bisection method. First there will be worked with steps of 0.1 seconds. When making these calculations, Matlab can't be used. To become more familiar with the bisection method, we have to apply this method first on an example function. The purpose of the bisection method is that the approximate zero-point of a function can be found. The method is supported by the fact that a function which is continue within a certain interval and of which the y-value of the start- and endpoint of the interval has a different sign, at least has one zero-point within this interval. The method works as following, the interval will be divided in two parts and there will be looked at which interval the y-values of the endings of signs differ. With this interval will we continue to work and the process starts again from the beginning.

4.1 Example exercise

To become more familiar with the bisection method, we will search the zero-point of the next function (search within the interval x ϵ [0, 10]).

$$y = \frac{1}{2} + \sin\left(\frac{x}{2}\right) e^{\sin\frac{x}{3}}$$

Step 1

$X \in [0; 10]$

$StartingPoint: [0.5; -0.292] \rightarrow contains\ Zero\ point$

Step 2

$X \in [0; 5]$

$StartingPoint: [0.5; 2.19] \rightarrow\ Zero\ point\ at\ [5,10]$

Step 3

$X \in [5; 7.5]$

$[2.119; -0.53] \rightarrow contains\ Zero\ point$

Step 4

$X \in [5; 6.25]$

$[2.119; 0.53] \rightarrow No\ Zero\ point \rightarrow [6.25; 7.5]$

Step 5

$X \in [6.25; 6.87]$

$[0.539; -0.118] \rightarrow contains\ Zero\ point$

Step 6

$X \in [6.56; 6.875]$

$[0.1879; -0.118] \rightarrow contains\ Zero\ point$

Step 7

$X \in [6.718; 6.875]$

$[0.027; -0.118] \rightarrow contains\ Zero\ point$

Step 8

$X \in [6.797; 6.875]$

$[0.027; -0.118] \rightarrow No\ Zeropoint \rightarrow Zero\ point\ between: [6.718; 6.797]$

Step 9

$X \in [6.718; 6.7575]$

$[0.027; -0.01073] \rightarrow contains\ Zero\ point$

Step 10

$X \in [6.738; 6.7478]$

$[0.00819; -0.01073] \rightarrow contains\ Zero\ point$

Step 11

$X \in [6.738; 6.7478]$

$[0.0081; -0.00149] \rightarrow contains\ Zero\ point$

Step12

Zero point =6.7429

4.2 Bisection method on the motor and solar panel

When we will approach the first second of the displacement-velocity characteristic, we use the bisection method by using the formula of the solar panel and the DC-motor.

$$\mathrm{I} = \mathrm{I_{sc}} - \mathrm{I_s}\left(\mathrm{e}^{\frac{\mathrm{U}}{\mathrm{mNU_R}}} - 1\right)$$

$$I_a = \frac{\mathrm{U} - \mathrm{Kt.}\,\omega}{\mathrm{Ra}}$$

We become the formula of the curve by subtracting these formulas. Now it is possible to calculate the zero point for each interval of the curve.

$$\mathrm{I} - I_a = \mathrm{I_{sc}} - \mathrm{I_s}\left(\mathrm{e}^{\frac{\mathrm{U}}{\mathrm{mNU_R}}} - 1\right) - \frac{\mathrm{U} - \mathrm{Kt.}\,\omega}{\mathrm{Ra}}$$

On the depart of the SSV (t=0seconds) the motor is not working yet and this means ω is zero. We know the equation is equal to zero when we will calculate the zero points.

$$\mathrm{I} - \mathrm{Ia} = 0{,}45 - 10^{-7}.\left(e^{\frac{U}{1{,}39.16.0.0257}} - 1\right) - \frac{\mathrm{U} - 8{,}55\,.\,10^{-3}.\,0}{3{,}32} = 0$$

By using the bisection method we find the value of U.

$$U = 1{,}49\ V$$

We calculate the current using the DC motor formula or the solar panel formula.

$$\mathrm{I} = 0{,}45 - 10^{-7}.\left(e^{\frac{1{,}49}{1{,}39.16.0.0257}} - 1\right) = 0{,}450\ A$$

With these value of I the torque of the motor can be find by using the following formula.

$$T_m = K_t\,.\,I\ = 8{,}55\,.\,10^{-3}.\ 0.45 = 3{,}85\,.\ 10^{-3} Nm$$

By multiplying the torque of the motor with the gear ratio, we get the torque of the wheel shaft. The optimal gear ratio for our SSV calculated with maple is 6,5 .

$$T_w = T_m\,.\ i = \ 3{,}85\,.\ 10^{-3}.\ 6{,}5 = \ 25{,}0\,.\ 10^{-3}\ Nm$$

The torque T_w on the wheel shaft will give a force $F_w$on the wheels. It can become by dividing the torque with the radius of the wheels.

$$F_w = \frac{T_w}{R_w} = \frac{25{,}0\,.\ 10^{-3}}{0.03} = 0{,}834\ N$$

The law of Newton tell us that the sum of the forces is equal to the product of the mass and the acceleration. Because the motor is not working yet on 0seconds and the SSV isn't moving, there are no frictions. So the sum of forces exists only F_w. By reforming we can calculate the acceleration as follow:

$$a = \frac{F_w}{m} = \frac{0{,}834}{1{,}20} = 0{,}694\, m/s^2$$

We want to now the velocity and distance driven after 0,1 second of the SSV. These values can be compute by following formulas.

Velocity of the SSV after 0,1s:

$$v = a.t = 0{,}695\,.\,0{,}1 = 0{,}0694\frac{m}{s}$$

The velocity of the SSV after 0,1s give us the possibility to calculate the rotation speed of the wheels. If we do over the previous calculation backwards, we can compute the currents again for this interval.

$$\omega_w = \frac{v}{t} = \frac{0{,}0694}{0{,}1} = 2{,}313\, rad/s$$

Now we can calculate the angular velocity of the wheel ω_w by a time of 0,1second . After this we can become the angular velocity of the motor ω_m by multiplying with the gear ratio .

$$\omega_m = \omega_w\,.\,i = 2{,}313\,.\,6.5 = 15{,}04\, rad/s$$

The angular velocity of the motor is needed in the next step when we will calculate the velocity and distance of the SSV on point of time 0,2s.
Now , we will start with the formula we used before. In this part we will fill in the known values, equalize to zero and search the zero point by using the bisection method.

$$\mathrm{I} - \mathrm{Ia} = \mathrm{I_{sc}} - \mathrm{I_s}\left(\mathrm{e}^{\frac{\mathrm{U}}{\mathrm{mNU_R}}} - 1\right) - \frac{\mathrm{U} - \mathrm{Kt}.\,\omega}{\mathrm{Ra}} = 0$$

$$<=> \ 0{,}45 - 10^{-7}.\left(e^{\frac{U}{1{,}39.16.0{,}0257}} - 1\right) - \frac{\mathrm{U} - 8{,}55\,.\,10^{-3}.\,15{,}04}{3{,}32} = 0$$

$$U = 1{,}62\, V$$

The following four calculations are similar with the calculation before for the interval of t = [0;0.1] but now for interval [0,1;0,2].

$$\mathrm{I} = 0{,}45 - 10^{-7}.\left(e^{\frac{1{,}62}{1{,}39.16.0.0257}} - 1\right) = 0{,}45\ \mathrm{A}$$

$$T_m = K_t\,.\,I = 8{,}55\,.\,10^{-3}.\ 0.45 = 3{,}85\,.\ 10^{-3} Nm$$

$$T_w = T_m\,.\ i = 3{,}85\,.\ 10^{-3}.\ 6{,}5 = 25{,}0\,.\ 10^{-3}\ Nm$$

$$F_w = \frac{T_w}{R_w} = \frac{25{,}0\,.\ 10^{-3}}{0.03} = 0{,}834\ N$$

At the moment the SSV moves which means there are friction forces, more specifically air resistance and rolling resistance. These two forces can be written in the formula of Newton. For the air resistance, we use the velocity which was calculated on 0,1 seconds and a drag coefficient (c_w) of 0,5. In the rolling resistance formula we use a rolling resistance coefficient (c_{rr}) equal on 0,012

$$F_{rolling} = c_{rr} \,.\, g \,.\, m = 0{,}012 \,.\, 9{,}81 \,.\, 1{,}2 = 0{,}141\ N$$

$$F_{air} = \frac{c_w \,.\, A \,.\, \rho \,.\, v^2}{2} = \frac{0{,}5 \,.\, 0{,}02 \,.\, 1{,}293 \,.\, 0{,}0694^2}{2} = 3{,}11 \,.\, 10^{-5}\ N$$

According Newton's law we can write:

$$a = \frac{F_w - F_{rolling} - F_{air}}{m} = \frac{0{,}834 - 0{,}141 - 3{,}11 \,.\, 10^{-5}}{1.20} = 0{,}577\ m/s^2$$

We can calculate the speed 0,1 second further than previous calculation. With other words, we take a step of 0,1 seconds.

$$v = v_0 + a.t = 0.0694 + 0{,}577 \,.\, 0{,}1 = 0{,}127\ m/s$$

Now we can calculate the angular velocity of the wheel ω_w by a time of 0,2second . After this we can become the angular velocity of the motor ω_m by multiplying with the gear ratio .

$$\omega_w = \frac{v}{r} = \frac{0{,}127}{0{,}03} = 4{,}23\ rad/s$$

$$\omega_m = \omega_w \,.\, i = 4{,}23 \,.\, 6{,}5 = 27{,}5\ rad/s$$

Now we have the values of two intervals of 0,1 second. We calculated the following intervals in excel. The results are given in the table below.

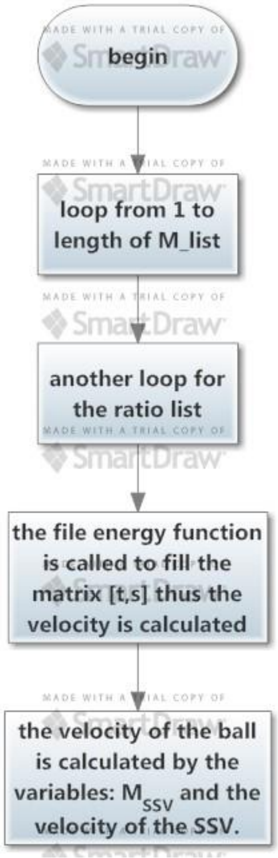
t	U	I	Fwiel	Fair	F Rolling	a	v	ω wiel	ω motor
0,3	1,73	0,45	0,834	0,000	0,141	0,577	0,185	6,2	40,0
0,4	1,84	0,45	0,834	0,000	0,141	0,577	0,242	8,1	52,5
0,5	1,94	0,45	0,834	0,000	0,141	0,577	0,300	10,0	65,0
0,6	2,05	0,45	0,834	0,001	0,141	0,576	0,358	11,9	77,5
0,7	2,16	0,45	0,834	0,001	0,141	0,576	0,415	13,8	90,0
0,8	2,26	0,45	0,834	0,001	0,141	0,576	0,473	15,8	102,5
0,9	2,37	0,45	0,834	0,001	0,141	0,576	0,530	17,7	114,9
1	2,48	0,45	0,834	0,002	0,141	0,575	0,588	19,6	127,4

Tabel 5 : Further elaboration of the bisection method

5. Matlab

5.1 MatLab questions

a. Draw a flow chart of the relation between these files.



b. Explain the following line in your own words. What are t and s?

```
[t,s] = ode15s(@Energy_func,(t0:tf/100:tf),x0,options);
```

Ode15s is used to replace old values between the brackets with new values. This line is used to change the time interval every time the loop begins. It starts with t=0 and ends with t=10 with an interval of 0.1 s. These values are put in the matrix [t,s].

c. What is done here and why?

```
index=find(s(:,1)>=10,1);
speed(i,j) = s(index-1,2)+(s(index,2)-s(index-1,2))*(10-s(index-1,1))/(s(index,1)-s(index-1,1));
```

A value is given to the parameter index. It searches the values in the first collom of the matrix s, who are larger or equal to 10. Then the speed of the SSV is calculated and put in a matrix speed (i,j). This is calculated by using differences between positions. These differences are divided by a time interval and thus a speed is found.

d. What is the function of this file? (energy_func)

This file is used to define your parameters you have to calculated. Then it uses the parameters to calculate the energy function.

e. What are dx, t and x? Why are they in this line? Does there exact name matter for the program?

Dx is equal to the integral of the energy function, this energy function variables are t and x. t is the time and x is the distance.

f. Describe these parameters and give their units

	Value	Unit	Describe
Solar Panel			
Isc	0,88	A	short-circuit current
Is	1 e-7	A	Saturation current
Ur	0,0257	V	Thermical voltage (25 °C)
m	1,39		Diode factor solar panel
N	16		number of sun cells
DC-motor			
R	3,32	Ω	Internal resistance motor
Ce	8,9285e-4	V/rpm	speed constant DC motor
Air resistance			
Cw	0,5		coefficient of air resistance
A	0,02	m²	frontal surface area
ρ	1,293	kg/m3	density of the air
Rolling resistance			
Crr	0,012		the coefficient of rolling resistance
g	9.81	N/kg	gravity constant
SSV			
r	0,03	m	wheel radius

Table 6 : Parameters in Matlab

g. What is x(2)?

This is the second order derivative of the displacement, this is equal to the acceleration of the car (a(t)= x’’(t))

Question I) What is TolFun? What is fzero and why do we call it here? What are sol and f?

Fzero is a function to determine the root of a nonlinear function thus fun(x)=0.

Question J) Explain the energy equations. What is the difference?

*dx(2) = -Crr*g + 60*Ce*ratio/(2*pi*r)*I/M*

*dx(2) = -Crr*g + (E*I)/(M*x(2)) - Cw*A*rho*x(2)*x(2)/(2*M)*

these equitations are an expression for the acceleration a(t). it’s Newton’s second law of motion and it’s a second order differential equitation. The difference is that the second equitation also describes the input work, so there has to be a load connected to the solar panel to make this approach.

h. What is the function of this file. How is it used? (func)

This file contains the global variables that are used in the file energy_solver. It also contains the formula for the voltage. So the voltage will be a function of other variables such as the gear ratio, diode factor ,…

i. What is f?

F is equal to the voltage of the motor.

5.2 MatLab Simulation

After we filled in the parameters of our SSV and the energy function we used MatLab to find the optimal mass and gear ratio. We have to use at least 10 different gear ratios and masses. If we used 2D-plots this would mean that we had to look at you have to look at 200 different graphs. This is why we chose to use a 3D-plot because there we can see the velocity of the ball in function of the gear ratio and the mass of the SSV.

After that we can see plots of the displacement as a function of the time. We want the highest possible velocity of the car after the 10 meters. We can check the ratio and mass we found in the first simulation and see if the value is the same for the velocity of the car function. More information will be given at the graphs.

5.2.1 Parameters

Isc= 0.88;	short-circuit current
Is=1e-8;	Saturation current
Ur=0.0257;	Thermical voltage (25 °C)
m= 1.1;	Diode factor solar panel
N= 16 ;	number of sun cells
% DC-motor	
R= 3.32 ;	Internal resistance motor
Ce= 8.9285e-4;	speed constant DC motor
% air resistance	
Cw= 0.5 ;	coefficient of air resistance
A= 0.02 ;	frontal surface area
rho= 1.293;	density of the air
%	rolling resistance
g=9.81;	gravity constant
Crr=0.012;	the coefficient of rolling resistance
% SSV	
r = 0.03 ;	wheel radius

5.2.2 Goal function

```
v(i,j) = (2*M*speed(i,j))/(M + M_ball)
```

this is because:

$$v_{ball} = [(m_{ball}-m_{SSV})*v_{ball} + 2*m_{SSV}*v_{SSV}]/(m_{SSV} + m_{ball})$$

But v_{ball} is equal to zero, so the equation can be simplified.

The goal function we want to optimize is obviously the velocity of the ball. That is the value we want to maximize. We try to find an equitation were the velocity of the ball is in function of parameters we can choose of the car. If we think about the design process we can find that the mass and the gear ratio of the car are the variables we can choose.

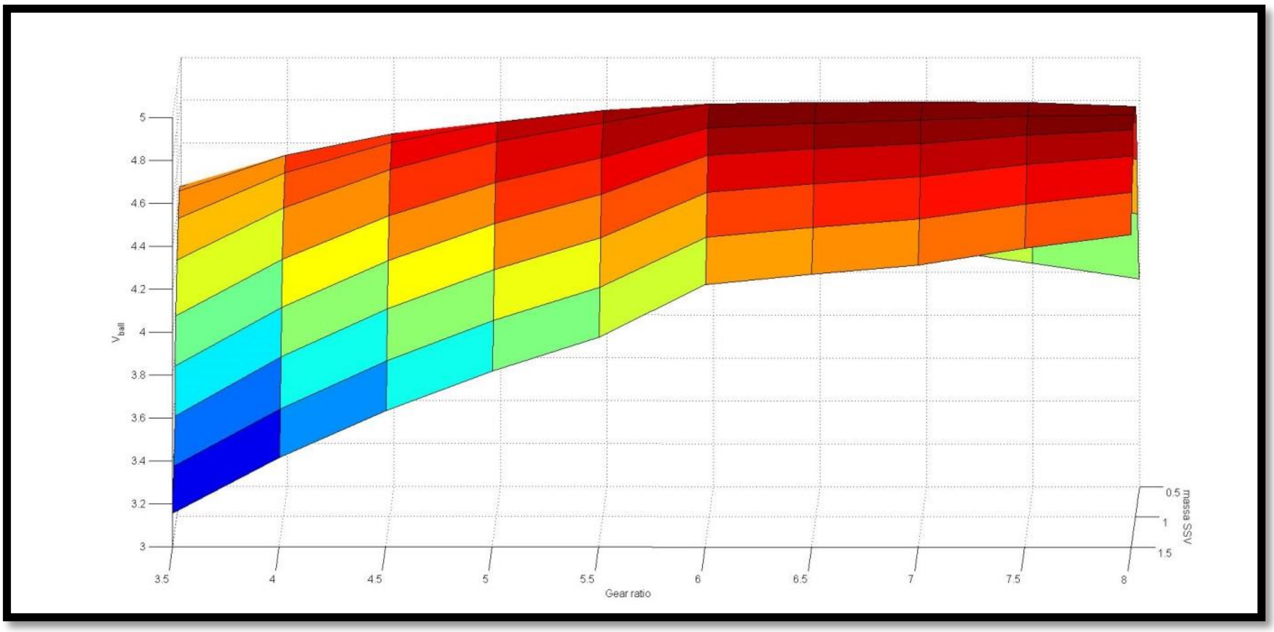


Figure 3: 3D-plot v_ball optimasation

It is hard to look at a 3D graph without the possibility to rotate the picture. So it is hard to see the top but as you can see the optimal mass of the SSV lays around 1.2 kilograms. If we then look at the gear ratio side, we can see that the optimal gear ratio lays around a value of 6.5. Both these graphs are obtained by using the function “surf” in MatLab. These function needs three inputs. These inputs has to be vectors to create a 3D surface.

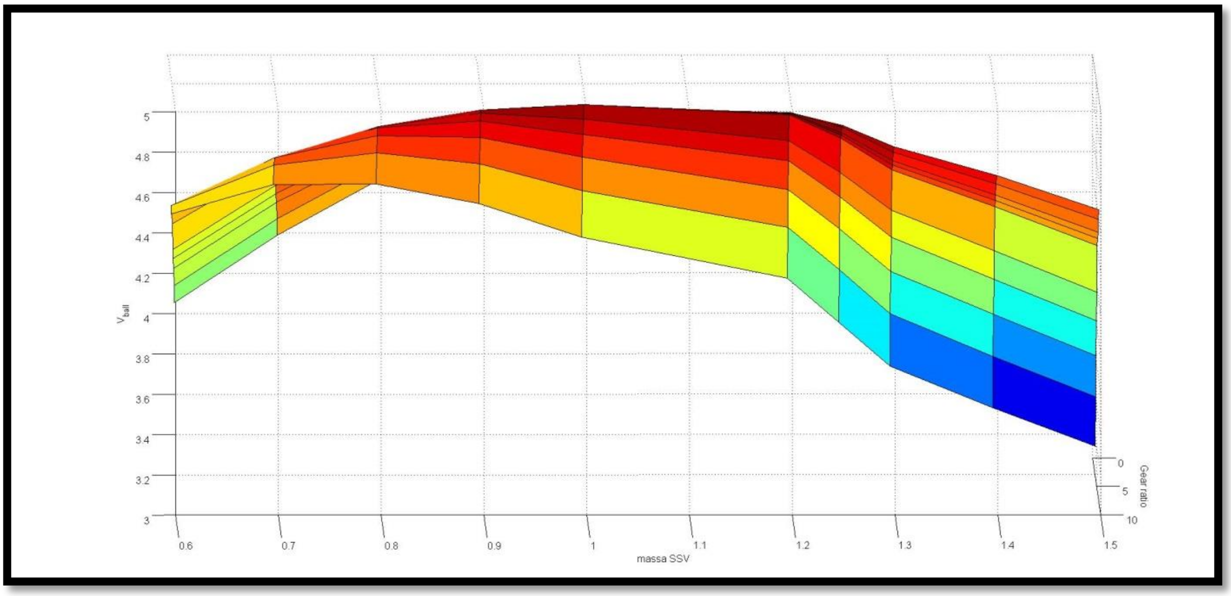


Figure 4: 3D-PLOT V_BALL OPTIMASATION

The last 2 graphs were showed sideways, let us now look at the next graph. Here you can see that isometric lines create a landscape. High levels off velocity are showed by the red area. So the highest v_{ball} will be situated in the dark red area of the graph. This is a confirmation that the optimal mass has a value around 1.1-1.2 kilograms. And that the gear ratio is around 6 - 6.5.

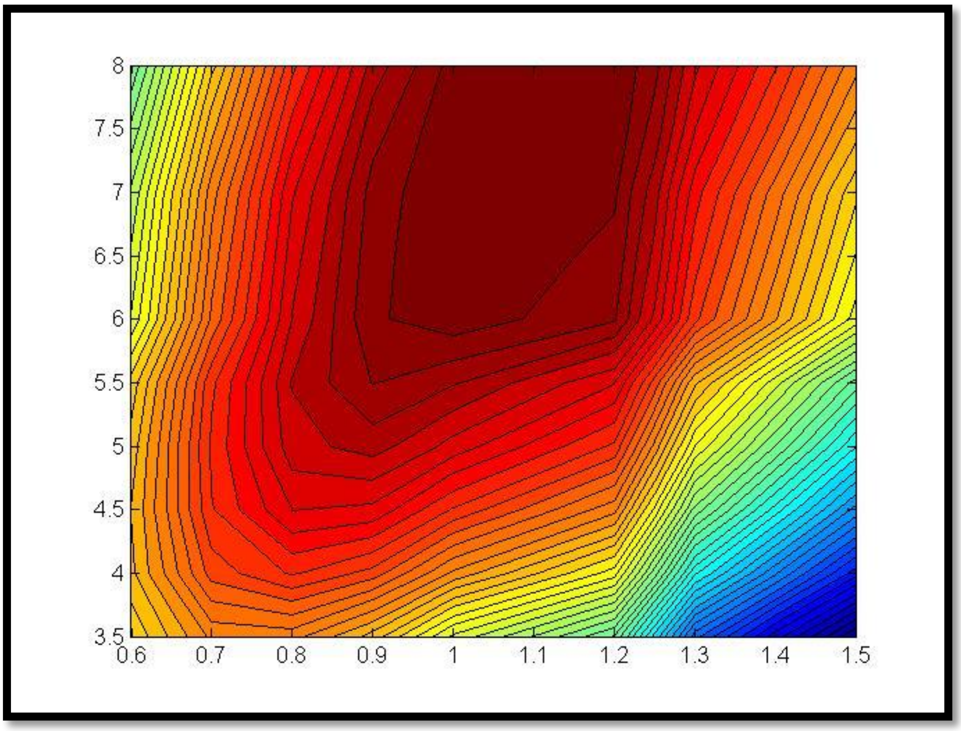
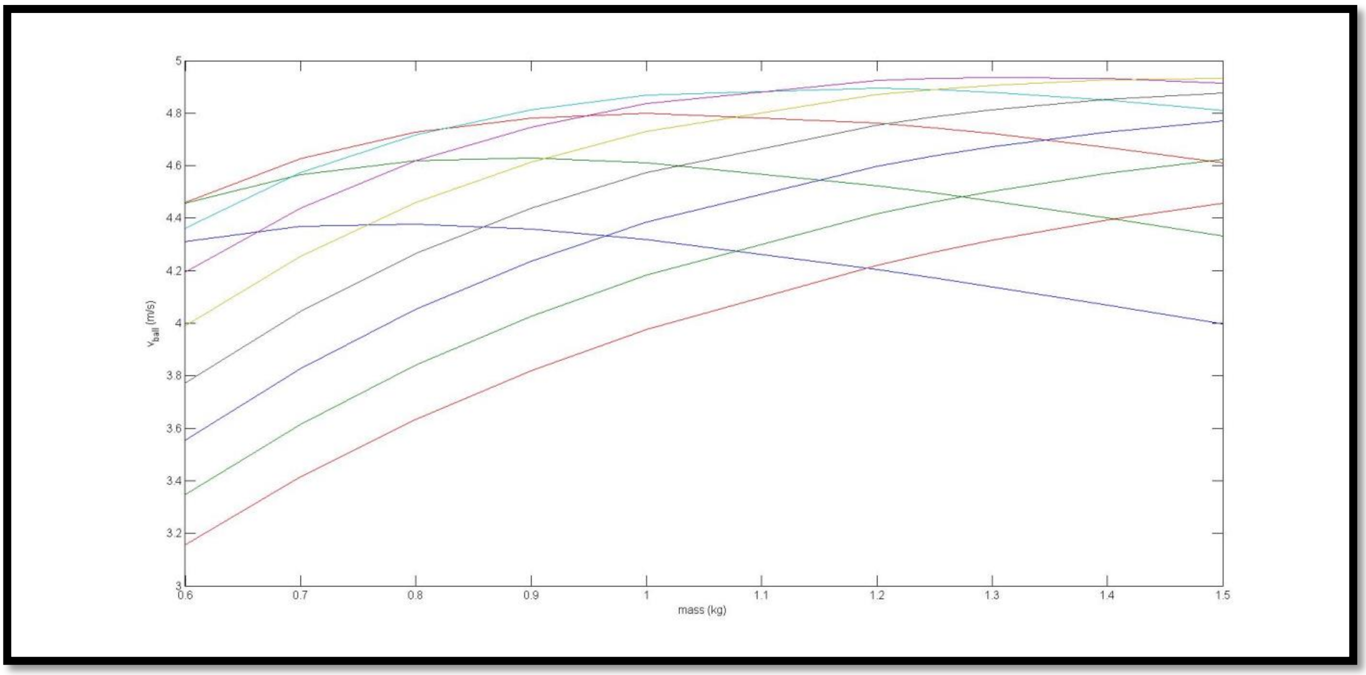
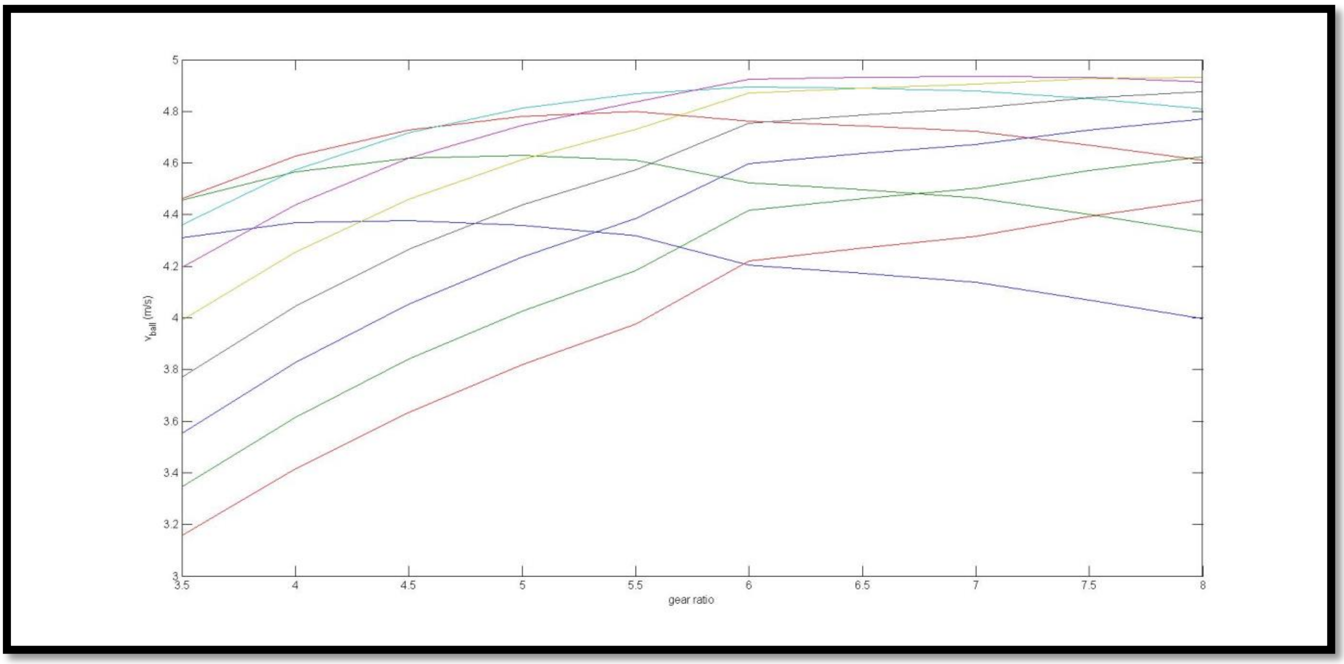


Figure 5: 3D-PLOT V_BALL OPTIMASATION highest level

To be really sure we can search for the optimal gear ratio for different masses and the optimal mass for the different gear ratios. This is shown in the 2D graphs.

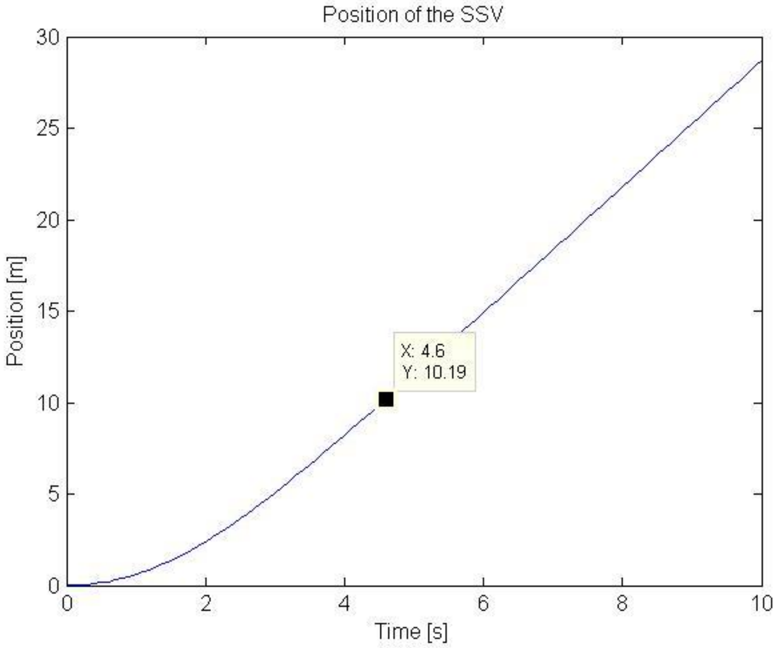


The lines with different colors are each from another gear ratio or mass. As we compare the two 2D-graphs we can see that the two pink lines (5th from above) has a peak and they peak at 1.2 kilogram and at 6.5 for the gear ratio. The gear ratio of 6.5 is the peak value for the simulation but is it a useful ratio?

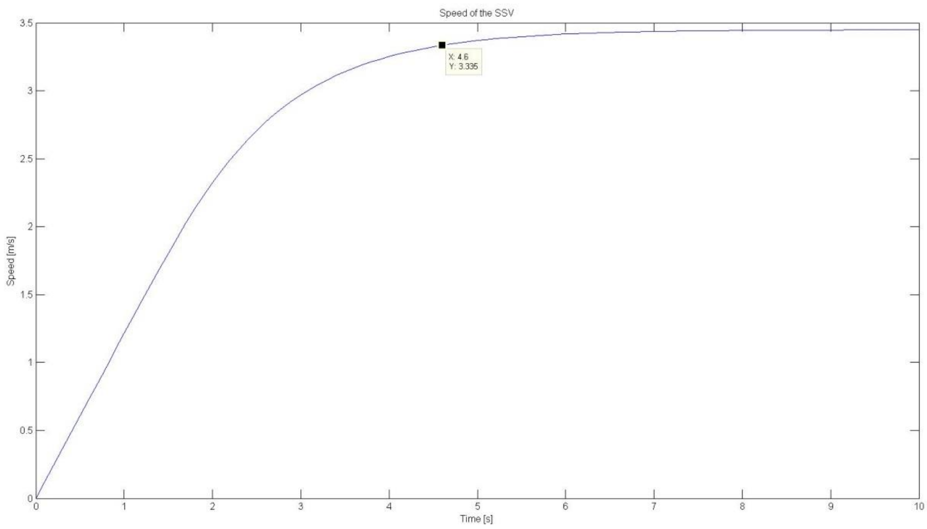


So we already found the optimal gear ratio and mass for our Smal Solar Vehicle. The next step is to calculate how long our vehicle accelerates towards the ball of 0.74 kilograms. This is shown in the next graph. The graph shows a 2D-picture, where the distance is in function of the time. We obtained this graph with the function “plot” in MatLab. There is also a tool available in MatLab: the Datacursor, wich we can use to look at data points of the graph . We can see that the SSV needs around 4.6 seconds to reach the 10 meters of the racing track.

5.2.3 Velocity and displacement simulation



The next graph shows the velocity of the SSV as a function of the time. What we like to obtain is a peak velocity after 4.6 seconds, because then the kinetic energy is at his highest point. As you can see this is absolutely true because after 4.6 seconds the graph (almost) reaches its top.



Graph 11 : velocity of the car in function of time

Now the height of the ball can be calculated:

$$h = v^2/2g \qquad [v= 3.3\ m/s]$$

$$h = 0.6\ m$$

5.2.4 Assumptions

In this model there have been a few assumptions otherwise the calculations would be too complicated. The first major assumption is the fact that we assume that at the point of impact, thus at 10 meters, there will also be maximum power. This simplified the calculation between the motor characteristic's and the acceleration and the velocity of the wheels.

Furthermore we also assume for our analytical calculations that the acceleration of the car is a constant in very small time intervals. This also simplifies our calculations. The collision is also assumed to be perfect. This means that there are no energy losses during the collision, in real life this would obviously be impossible. So we assume that all the kinetic energy from the Small Solar Vehicle is transmitted by the collision and will give an equal potential energy to the ball. There is thus conservation of energy when the collision happens.

The next assumption is the fact that we assume that the slope is not very steep. We do not use the vectors v_x or v_y , there is only one vector hence equal to v. This means that we do not take the angle of the slope into account.