ACCURACY OF A PROCEDURE FOR COMPUTING LOWER CONFIDENCE LITMITS ON SYSTEM RELIABILITY FOR VARIOUS FAILURE DISTRIBUTIONS

by

Richard Joseph Girouard

UNITED STATES
NAVAL POSTGRADUATE SCHOOL

THESIS

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RELIABILITY FOR VARIOUS FAILURE DISTRIBUTIONS

by

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ABSTRACT

A statistical model for obtaining a lower 100(1 - α)% confidence limit on system reliability was developed for the Department of Navy Special Projects Office in May 1965. This method is a part of NAVWEPS OD 29304. The accuracy of this model is examined by computer simulation for various failure time distributions. The simulation results are presented and discussed.

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CHAPTER I

INTRODUCTION

In the last few years reliability has become of primary concern in the development of most large weapon systems. This growing importance of reliability has produced a great need for methods of predicting the overall reliability of these systems. Because of the size and complexity of present day military weapons system, it is necessary for major contractors to have many sub-contractors located throughout the country. This fact complicates the problem of formulating a method for computing lower confidence limits on system reliability where the reliability of the system is computed as the product of the reliabilities for the components.

Recently a system reliability model for computing a lower confidence limit for the reliability of weapon systems has been developed for the Navy and is contained in "Guide Manual for Reliability Measurement Program," NAVWEPS OD 29304, 15 May 1965. The model was prepared for the Department of Navy Special Projects Office. The document states that the model can be utilized by all contractors for subsystem reliability measurement and by the Navy for weapon system reliability measurement. The model permits the combination of test data from all levels (from component to weapon system) and types of tests into meaningful component, equipment, and subsystem failure rates and reliability indices.

An underlying assumption to this statistical model is constant failure rate, i.e., the exponential failure law is assumed to hold:

$$R = e^{-\lambda} \qquad (1)$$

where

R = reliability

λ = failure rate per mission

It is the purpose of this thesis to examine the accuracy of the statistical model contained in OD 29304. The investigation that follows will examine the model when

(1) the failure distribution is other than exponential; in particular, when failure times have a Rayleigh distribution, i.e., increasing failure rate. Appendix 1 contains a graph of the Rayleigh distribution and its failure rate function.

(2) the failure distribution is a binomial distribution.

In later chapters, it will be explained that the $100(1-\alpha)$ percentile of the simulated distribution of the lower confidence limit, denoted by $A_{1-\alpha}$, and the true reliability of the system, R_s, should be equal. Therefore, the difference will be used as a measure of the accuracy of the model. As an example of some of the results, consider the system with 4 components, each having a reliability of .995. Samples of sizes of 50 and 500 components were tested with a planned test time of 2 mission units. The true reliability of the system, R_s, is .9801. The 80th percentile points of the distribution of the lower confidence limit for Rayleigh distributed failure times were computed to be:

α	n_i	$A_{1-\alpha}$	$\lvert A_{1-\alpha} - R_s \rvert$
.20	50	.957	.023
.20	500	.961	.019

When the failure times were binomially distributed and the test time was one mission unit, the 80th percentile points of the distribution of the lower confidence limit were computed to be:

α	n_i	$A_{1-\alpha}$	$\lvert A_{1-\alpha} - R_s \rvert$
.20	50	.9683	.0120
.20	500	.9800	.0001

Chapter II contains a complete explanation of the statistical model in OD 29304. Chapter III discusses the simulation procedures and presents the simulation results. It also discusses the two methods used to determine the accuracy of the model. Chapter IV discusses and summarizes the results and states the conclusions.

CHAPTER II

EXPLANATION OF STATISTICAL MODEL (1)

If a system consists of k components in logical series, the true system reliability, R_s, may be expressed as

$$R_s = \prod_{i=1}^{k} R_i \qquad (2)$$

where R_i is the true reliability of the ith component.

It is desired to find a lower $100(1-\alpha)\%$ confidence limit for R_s. That is, a statistic $\hat{R}_{s,L(\alpha)}$ such that whatever the actual values of R_i, $i = 1,2,\ldots,k$,

$$P[R_s \geq \hat{R}_{s,L(\alpha)}] = 1 - \alpha \qquad (3)$$

The proposed method to find this lower $100(1-\alpha)\%$ confidence limit for R_s as outlined in OD 29304 will be presented below to include:

(1) Operating assumptions.

(2) Estimation of failure rates.

(3) Statistical equations for confidence limits on failure rates and reliability.

<u>Operating Assumptions</u>

The reliability measurement system described in OD 29304 is predicated upon the general reasonableness of the following assumptions:

(1) Constant Failure Rate - the exponential failure law is assumed to hold.

(2) Additivity of Stress Effects - the failure rate induced by two simultaneously acting stresses is equal to the sum of the failure rates due to the two stresses acting sequentially. This assumption permits adding data from single environment tests together to simulate mission experience.

(3) Independence of Component Failures - independence of component failures is assumed because components are normally tested individually by type, and subsystem reliability is estimated using component and other applicable test results (e.g., equipment and subsystem).

(4) Failure Rate Constancy - the failure rate is considered a function of only the stress acting. In other words, the item under test has no memory.

Estimation of Failure Rates

Under the above assumptions, if component 1 has failure rate λ_1, and component 2 has failure rate λ_2, then the corresponding failure rate for the simple serial subsystem is:

$$\lambda_{subsystem} = \lambda_1 + \lambda_2 \qquad (4)$$

Since the mode of testing generally employed by subsystem contractors is to test until some planned test time, the failure rate estimates, $\hat{\lambda}_i$, contain bias. Therefore, an unbiased failure rate estimate has been developed in OD 29304 and is given as

$$\hat{\lambda}_i = \frac{f_i}{S_i}\left[\frac{2n_i}{2n_i + 1}\right] \qquad (5)$$

where

n_i = sample size for the ith component; i.e., the number of tests conducted on component i.

S_i = the sum of all test times accumulated on the n components of type i.

f_i = number of components of type i that did not complete their mission, i.e., failure time was less than the planned test time.

Under the OD 29304 method, failure rate estimates for subsystem reliability can be calculated from component, component-environment, or component-environment-test condition level data. That is

$$\hat{\lambda}_{component} = \Sigma\, \hat{\lambda}_{component\text{-}environment} \qquad (6)$$

$$\hat{\lambda}_{equipment} = \Sigma\, \hat{\lambda}_{component} \qquad (7)$$

$$\hat{\lambda}_{subsystem} = \Sigma\, \hat{\lambda}_{equipment} \qquad (8)$$

It permits the combination of test data from all levels (from component to weapon system) and types of tests into meaningful component, equipment, and subsystem failure rates.

Statistical Equations

All statistical estimates, based as they are upon fragmentary data, are subject to statistical uncertainty. In reliability work, it is usually of interest to compute upper limits on the failure rate and corresponding lower limits on the reliability. The following statistical equations for calculation of the lower confidence limit are based on normal theory and have been corrected to compensate for small values of λ. Detailed derivations of these formulas are contained in reference 1.

The method proposed is to put n_i items of component i on test under the environmental conditions defined in the mission and let each operate until failure or the planned test time, whichever occurs first. All time is measured in mission units. Then an unbiased estimate of failure rate, $\hat{\lambda}_i$, for each component i is obtained from equation (5). If failures have occured for any of the i components, an upper limit on failure rate can be calculated as follows:

$$\hat{\lambda}_u = \frac{2\hat{\lambda} + (\beta K)^2\hat{C} + \sqrt{4\hat{\lambda}(\beta K)^2\hat{C} + (\beta K)^4\hat{C}^2}}{2} \qquad (9)$$

where

$$\hat{\lambda} = \sum_{i=1}^{k} \hat{\lambda}_i \qquad (10)$$

$$\hat{C} = \frac{\Sigma(\hat{\lambda}_i/S_i)}{\hat{\lambda}} \qquad (11)$$

K = (1 - α)th percentile of the normal distribution

β = correction factor; Table 1 shows appropriate beta values for the 80th percent confidence limit

When no failures have occurred, the upper limit on failure rate is

$$\hat{\lambda}_u = \frac{(\beta K)^2}{k} \sum_{i=1}^{k} \frac{1}{S_i} \qquad (12)$$

where

k = number of components in the simulations

S_i = the sum of all test times accumulated on the n_i components of type i

Substitution of the upper limits on the failure rates previously obtained will generate corresponding lower confidence limits on reliability; thus:

$$\hat{R}_{s,L(\alpha)} = e^{-\hat{\lambda}_u} \qquad (13)$$

To determine the accuracy of the above model, one needs to look at the characteristics of $\hat{R}_{s,L(\alpha)}$. By definition, $\hat{R}_{s,L(\alpha)}$ is a lower $100(1-\alpha)\%$ confidence limit for R_s, the system reliability; i.e., from equation (3)

$$P[\hat{R}_{s,L(\alpha)} \leq R_s] = 1 - \alpha$$

This says that R_s is always the $(1-\alpha)$th percentile point of the probability distribution of $\hat{R}_{s,L(\alpha)}$. Thus if we construct the distribution of $\hat{R}_{s,L(\alpha)}$ by computer simulation, we should find that the $(1-\alpha)$th percentile point of our constructed distribution is R_s, _if_ equation (3) is correct; i.e., _if_ in fact $\hat{R}_{s,L(\alpha)}$ is a true $100(1-\alpha)\%$ lower confidence limit for R_s.

Thus a measure for the accuracy of the model is

$$\left| A_{1-\alpha} - R_s \right|$$

where

$A_{1-\alpha}$ = $(1-\alpha)$th percentile point of the distribution of $\hat{R}_{s,L(\alpha)}$

If equation (3) is correct, then $A_{1-\alpha}$ should equal R_s.

TABLE 1

BETA VALUES FOR 80% CONFIDENCE LIMIT

Number of Failures f_i	Beta Values β_i	f_i	β_i	f_i	β_i
0	1.507	10	1.173	20	1.129
1	1.369	11	1.167	21	1.126
2	1.309	12	1.161	22	1.124
3	1.272	13	1.156	23	1.122
4	1.246	14	1.153	24	1.119
5	1.227	15	1.146	25	1.117
6	1.212	16	1.144	26	1.115
7	1.200	17	1.139	27	1.113
8	1.190	18	1.135	28	1.111
9	1.180	19	1.132	29	1.110

CHAPTER III

SIMULATION PROCEDURE AND RESULTS

In order to evaluate the accuracy of the OD 29304 model, a computer program was written and used to simulate the distribution of $\hat{R}_{s,L(\alpha)}$. The distribution is constructed on a computer by generating 500 values of $\hat{R}_{s,L(\alpha)}$ for a given set of parameter values k, $n_i (i = 1,2,...,k)$, $R_i (i = 1,2,...,k)$, $T_{oi} (i = 1,2,...,k)$, and a given failure distribution; i.e., $\hat{R}_{s,L(\alpha)}$ is a function of

k: number of components in the system

n_i: number of items of component i tested

R_i: reliability of component i

T_{oi}: planned test time for component i

$f_i(t)$: failure distribution for component i

When a failure distribution for component i is specified, the computer program generates 500 random failure time variates, $T_{ij} (j = 1,2,...,500)$, for component i. If the generated time variate is less than the planned test time, T_{oi}, a failure is "counted" by the computer. With the total number of failures, f_i, counted for a set of 500 generated failure times, the computer program then computes the failure rate for component i, $\hat{\lambda}_i$, from equation (5). The above procedure is repeated for each component i, and the corresponding upper limit on the system failure rate, $\hat{\lambda}_u$, is then calculated from equations (9), (10), and (11). With this value of the system failure rate, the lower confidence limit on the system reliability, $\hat{R}_{s,L(\alpha)}$, is computed from equation (13). The above procedure is replicated, giving 500 values of $\hat{R}_{s,L(\alpha)}$. These 500 values are then ordered by a separate subroutine, and the computer "picks" the $(1-\alpha)$th percentile of these 500 values, denoted by $A_{1-\alpha}$. This value is then compared with the true system reliability R_s.

The above simulation procedure was carried out for the following two methods:

(1) Rayleigh distributed failure times

(2) Binomial distributed failure times

<u>Rayleigh Distributed Failure Times</u>

Since the Rayleigh distribution is a one parameter distribution (see appendix 1), it was decided to choose the value of this parameter (a) such that the reliability of a component for one mission unit is .995; i.e.,

$$R_i(1) = P[T_{ij} > 1] = .995 \tag{14}$$

The determination of the value of this parameter is shown in appendix 2. The resulting failure time distribution that will be assumed for each component i in the simulation is:

$$f_i(t,a) = \frac{1}{a^2} t e^{-\frac{t^2}{2a^2}} \quad , a = 10, t > 0 \tag{15}$$

For Rayleigh distributed failure times, the accuracy of the model was examined for five cases:

(1) Case 1: T_{oi} = 0.5 mission units

(2) Case 2: T_{oi} = 1.0 mission units

(3) Case 3: T_{oi} = 1.5 mission units, i = 1,2

1.35 mission units, i = 3,4

(4) Case 4: T_{oi} = 2.0 mission units

(5) Case 5: T_{oi} = 5.0 mission units

Determining the Case 4 planned test time, T_{oi}, is shown in appendix 2. For each case, the number of components k was equal to 4, the reliability R_i was equal to .995, and various values of n_i were used. The cases are numbered and the results shown in Table 2. The mean and variance of the distribution of $\hat{R}_{s,L(\alpha)}$ are also shown in Table 2. The quantity labeled TT in Table 2 is expressed by

$$TT = \sum_{i=1}^{k} n_i Q_i \tag{16}$$

where

$$Q_i = 1 - R_i = \text{the unreliability of component } i$$

This quantity, TT, is a function of the amount of testing relative to the unreliability or failure rate. The results of the cases in Table 2 imply that the model is more accurate as TT increases.

Binomial Distributed Failure Times

For this method, the failure rate for component i, $\hat{\lambda}_i$, is given as

$$\hat{\lambda}_i = \hat{q}_i = \frac{f_i}{n_i} \quad (17)$$

where

f_i = number of components of type i that failed

n_i = number of items of component i tested

Using a uniform random number generator, the computer generates a uniform random value between 0 and 1 for component i. If the value is greater than R_i, a failure is "counted" by the computer. This procedure is repeated n_i times for each R_i and for each component i. Thus the number of failures counted divided by n_i becomes an estimate of the failure rate for component i as given by equation (17) above. This process is repeated for all components. With these estimates, $\hat{\lambda}_i$, the computer simulation determines $A_{1-\alpha}$ and calculates the mean and variance of the 500 values of $\hat{R}_{s,L(\alpha)}$ as previously explained.

For this method, the accuracy of the model was examined for three cases:

(1) Case 1: $R_i = .995$

(2) Case 2: $R_i = .950$

(3) Case 3: $R_i = .900$

The sum of the failure times, S_i, was set equal to to n_i; i.e.,

$$S_i = \sum_{i=1}^{n_i} T_{ij} = n_i \quad , i = 1,2,\ldots,k \quad (18)$$

the number of components k was equal to 4, and various values of n_i were used. The cases are numbered and the results shown in Table 3. Again, the quantity, TT, is also listed in Table 3.

TABLE 2

RESULTS OF COMPUTER SIMULATION
(Rayleigh Distributed Failure Times)

CASE	k	T_{oi}	n_i	R_i	R_s	α	$A_{1-\alpha}$	Mean of $\hat{R}_{s,L(\alpha)}$	Standard Deviation of $\hat{R}_{s,L(\alpha)}$	TT
1	4	0.5	50	.995	.980	.20	.937	.926	.023	1.0
			100	.995	.980	.20	.968	.956	.016	2.0
			500	.995	.980	.20	.988	.981	.008	10.0
2	4	1.0	100	.995	.980	.20	.970	.959	.015	2.0
			300	.995	.980	.20	.977	.970	.008	6.0
			500	.995	.980	.20	.984	.973	.006	10.0
3	4	2.0	50	.995	.980	.20	.957	.936	.020	1.0
			100	.995	.980	.20	.961	.947	.015	2.0
			500	.995	.980	.20	.961	.956	.006	10.0
4	4	5.0	50	.995	.980	.20	.907	.892	.018	1.0
			100	.995	.980	.20	.909	.897	.013	2.0
			500	.995	.980	.20	.909	.897	.007	10.0
5	4	$T_{o1} = T_{o2} = 1.5$	50	.995	.980	.20	.956	.938	.023	1.0
		$T_{o3} = T_{o4} = 1.35$	100	.995	.980	.20	.957	.939	.017	2.0
			500	.995	.980	.20	.967	.961	.006	10.0

TABLE 3

RESULTS OF COMPUTER SIMULATION

CASE	k	$S_i = \Sigma T_{ij}$	R_i	n_i	R_s	α	$A_{1-\alpha}$	Mean of $\hat{R}_{s,L(\alpha)}$	Standard Deviation of $\hat{R}_{s,L(\alpha)}$	TT
1	4	n_i	.995	25	.980	.20	.937	.916	.033	0.5
				50	.980	.20	.968	.941	.025	1.0
				100	.980	.20	.970	.959	.016	2.0
				500	.980	.20	.980	.973	.004	10.0
2	4	n_i	.987	25	.950	.20	.937	.879	.048	1.3
				50	.950	.20	.942	.904	.036	2.6
				100	.950	.20	.946	.923	.024	5.2
				500	.950	.20	.948	.939	.004	26.0
3	4	n_i	.974	25	.900	.20	.887	.825	.063	2.6
				50	.900	.20	.896	.850	.044	5.2
				100	.900	.20	.894	.869	.031	10.4
				500	.900	.20	.899	.889	.004	52.0

CHAPTER IV

SUMMARY AND CONCLUSIONS

It was stated previously that a measure of the accuracy of the model was given by

$$\left| A_{1-\alpha} - R_s \right|$$

where $A_{1-\alpha}$ is the $(1-\alpha)$ percentile point of the distribution of $\hat{R}_{s,L(\alpha)}$, and R_s is the true reliability of the system. For the Rayleigh distributed failure times, this measure for the various cases is shown below for R_s equal to .995.

T_{oi}	α	n_i	R_s	$\left\| A_{1-\alpha} - R_s \right\|$
0.5	.20	50	.995	.043
		100	.995	.012
		500	.995	.008
1.0	.20	100	.995	.010
		300	.995	.003
		500	.995	.003
2.0	.20	50	.995	.023
		100	.995	.019
		500	.995	.019
5.0	.20	50	.995	.073
		100	.995	.071
		500	.995	.071

The above results bring out some rather interesting points. If the planned test time, T_{oi}, is large, the model gives pessimistic results; i.e., the accuracy is not very good, regardless of the sample size n_i. However, if the planned test time is small, the accuracy of the model is very good if a large sample of a particular component is tested. In other words, for small planned test times, the accuracy of the model is very good when the quantity, TT, is greater than 6.

For the Binomially distributed failure times, the accuracy of the model for the various cases is given below.

α	R_s	n_i	$\left\| A_{1-\alpha} - R_s \right\|$
.20	.980	25	.043
	.980	50	.012
	.980	100	.010
	.980	500	.000
.20	.950	25	.013
	.950	50	.008
	.950	100	.004
	.950	500	.001
.20	.900	25	.013
	.900	50	.004
	.900	100	.002
	.900	500	.001

The results clearly show that the accuracy of the model is extremely good, especially when the sample size, n_i, is large.

Obviously, the main reason for the results in the case of Rayleigh distributed failure times is the fact that the components have increasing failure rates. The underlying assumption in this model is that components under test have a constant failure rate. In practical applications, many components follow this assumption reasonably well, when infant mortality failures are deleted and wearout failures are not anticipated during the mission duration. However, if a new component is being tested, a constant failure rate may not be reasonable at all. Therefore, when this model is being used to obtain a lower confidence limit on the reliability of a weapon system, and it is known that a particular component has an increasing failure rate, then the points brought out above as a result of the simulation should be taken into consideration.

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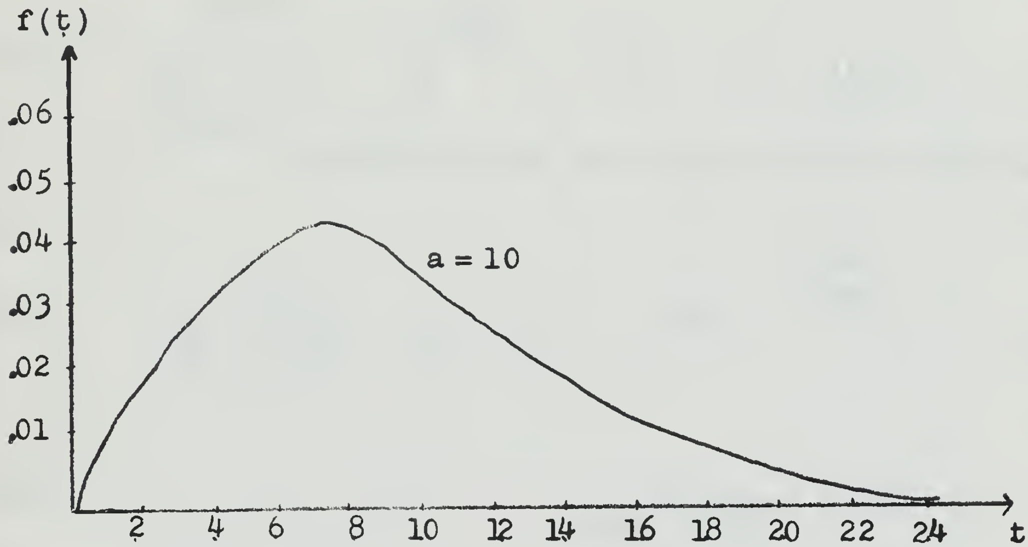
3. Lipow, M., and Lloyd, D. K., Reliability: Management, Methods, and Mathematics, Prentice-Hall, Inc., New Jersey, 1962.

APPENDIX 1

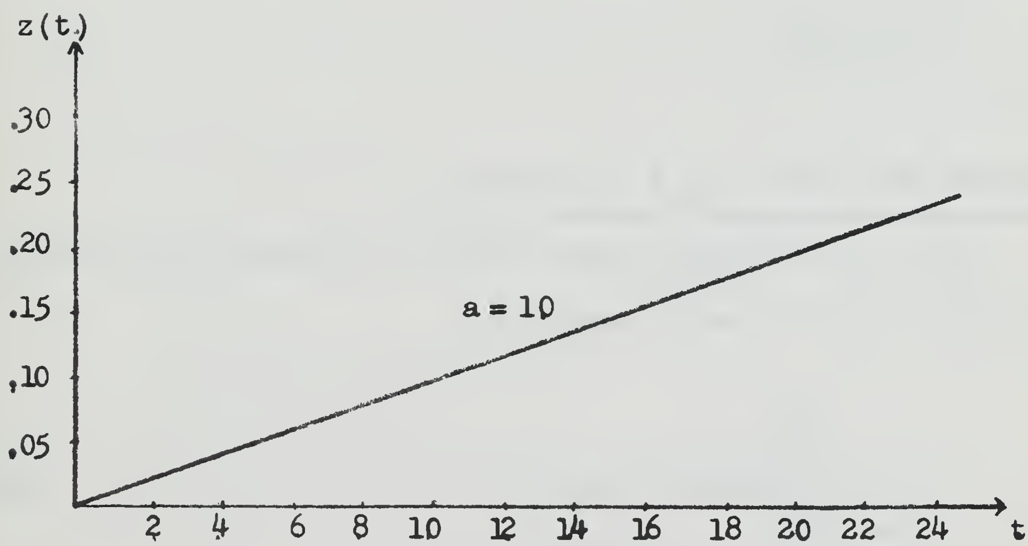
RAYLEIGH DISTRIBUTION

PDF: $F(t) = \frac{1}{a^2}\, t\, e^{-\frac{t^2}{2a^2}}$ CDF: $F(t) = 1 - e^{-\frac{1}{2}\frac{t^2}{a^2}}$

FR (failure rate) $z(t) = \frac{t}{a^2}$ (increasing failure rate)



Probability Density Function



Failure Rate Function

APPENDIX 2

Value of Parameter for Rayleigh Distribution

The value of the parameter, a, was chosen such that

$$R_i(1) = P[T > 1] = .995 \qquad (19)$$

where

$$R_i(t) = 1 - F(t) = e^{-\frac{t^2}{2a^2}} \qquad (20)$$

Equating equations (19) and (20), we obtain

$$e^{-\frac{1}{2a^2}} = .995 \quad , \; t = 1 \qquad (21)$$

or

$$a^2 = - \frac{1}{2(\ln .995)} \qquad (22)$$

hence

$$a = 10$$

Planned Test Time (T_{oi}) for Case 4

The Case 4 planned test time was chosen such that T_{oi} satisfied the following equation:

$$\frac{1}{T_{oi}} \int_0^{T_{oi}} Z(t)dt = Z(1) \qquad (23)$$

where

$$Z(t) = \frac{t}{a^2} \qquad (24)$$

This value of T_{oi} makes the "average" value of $Z(t)$ between o and T_{oi} equal $Z(1)$. Setting $t = 1$ in equation (24) and equating (23) and (24) gives

$$\frac{1}{T_{oi}} \int_o^{T_{oi}} \frac{t}{a^2} \, dt = \frac{1}{a^2} \qquad (25)$$

or

$$\frac{1}{T_{oi}} \left(\frac{T_{oi}^2}{2a^2} \right) = \frac{1}{a^2} \qquad (26)$$

hence

$$T_{oi} = 2.0$$

APPENDIX 3

COMPUTER PROGRAM

```
      DIMENSION PT(4,3),RSL(500),A(3),RSBAR(3),SRS(3),
      BETA(30),RS1(3),RSLBAR(3),RS(500),Q(4),NM(4)
      READ(5,10)(NM(J),J=1,4),(BETA(K),K=1,30),(Q(I),I=1,4)
   10 FORMAT(4I4/16F5.3/14F5.3/4F6.3)
      TPR = URN(0)
      DO 999 K = 1,4
      SS = NM(K)
      SRS(K) = 0.0
      A(K) = 0.0
      RSLBAR(K) = 0.0
      SRSL(K) = 0.0
      F2 = NM(K)
      B = (2.0 * F2)/(2.0 * F2) + 1.0)
```

This loop generates 500 time variates for each component

```
      DO 900 M = 1,500
      FS = 0.0
      VU = 0.0
      C = 0.0
      V1 = 0.0
      RSL(M) = 0.0
      RS(M) = 0.0
      DO 800 I = 1,4
      FF = 0.0
      VL = 0.0
      NL = NM(K)
      DO 700 J = 1,NL
      TPR = URN(1)
      IF(TPR.GT. Q(I))GO TO 605
      GO TO 700
  605 FF = FF + 1.0
  700 Continue
      VL = FF/F2
      V1 = V1 + VL
      FS = FS + FF
      C = C + (VL/SS)
      VU = VU + (1.0/SS)
  800 CONTINUE
```

This statement computes a value of $\hat{R}_s = e^{-\hat{\lambda}}$

```
      RS(M) = EXP(-V1)
      RSBAR(K) = RSBAR(K) + RS(M)
      IF(FS .GT. 0.0)GO TO 842
      BTA = BETA(1)
      VU = (VU*BTA)*(BTA*.177241)
      Go to 832
```

```
  842 C = C/V1
      IF(FS .GE. 30.0)GO TO 820
      L1 = FS + 1.0
      BTA = BETA(L1)
      GO TO 825
  820 BTA = 1.0
  825 B2 = (BTA * .708964) * BTA
      D = ((4.0 * V1)*(B2*C)) + ((B2*B2)*(C*C))
      D = SQRT(D)
      VU = (((2.0 * V1) + (B2*C)) + D)/(2.0)
```

This statement computes a value of $\hat{R}_{s,L(\alpha)} = e^{-\hat{\lambda}_u}$

```
  832 RSL(M) = EXP(-VU)
      RSLBAR(K) = RSLBAR(K) + RSL(M)
  900 CONTINUE
```

The following statements compute the mean and variance of $\hat{R}_s$ and $\hat{R}_{s,L}$.

```
      RSBAR(K) = RSBAR(K)/500.0
      RBAR = RSBAR(K)
      DO 905 M = 1,500
      SRS(K) = SRS(K) + (RS(M) - RBAR)**2
  905 CONTINUE
      SRS(K) = SQRT(SRS(K)/500.0)
      RSLBAR(K) = RSLBAR(K)/500.0
      RLBAR = RSLBAR(K)
      DO 2410 J = 1,500
      SRSL(K) = SRSL(K) + (RSL(J) - RSBAR)**2
 2410 CONTINUE
      SRSL(K) = SQRT(SRSL(K)/500.0)
```

The following statements order the 500 values of $\hat{R}_{s,L}$ and picks the 80th percentile of the 500 values, $A_{1-\alpha}$.

```
      II = 1
      DO 910 I = 1,101
      TEMP = RSL(1)
      DO 915 J = 1,500
      IF(TEMP - RSL(J))916,915,915
  916 TEMP = RSL(J)
      II = J
  915 CONTINUE
      RSL(II) = 0.0
      A(K) = TEMP
  9.0 CONTINUE
```

The following statements print out the results- $A_{1-\alpha}$, mean and standard deviation of $R_{s,L}$, and the mean and standard deviation of R_s.

```
      WRITE(6,1010) A(K),RSLBAR(K),SRSL(K),RSBAR(K),SRS(K)
 1010 FORMAT(//6F13.5)
  999 CONTINUE
      END
```

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13. ABSTRACT

A statistical model for obtaining a lower $100(1-\alpha)\%$ confidence limit on system reliability was developed for the Department of Navy Special Projects Office in May 1965. This method is a part of NAVWEPS OD 29304. The accuracy of this model is examined by computer simulation for various failure time distributions. The simulation results are presented and discussed.

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