

UNITED STATES DEPARTMENT OF THE INTERIOR BUREAU OF MINES HELIUM ACTIVITY HELIUM RESEARCH CENTER

INTERNAL REPORT

STEADY-STATE LAMINAR FLOW BOUNDARY CONDITIONS

FOR A STAINLESS STEEL COILED CAPILLARY VISCOSIMETER

BY

R. A. Guereca

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L. M. Walker

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STEADY-STATE LAMINAR FLOW BOUNDARY CONDITIONS FOR A STAINLESS STEEL COILED CAPILLARY VISCOSIMETER

by

R. A. Guereca, $\frac{1}{}$ H. P. Richardson, $\frac{2}{}$ and L. M. Walker $\frac{3}{}$

ABSTRACT

An analysis to determine criteria for steady-state, laminar flow boundary conditions for a coiled-tube gas viscosimeter is made by comparing experimental data from a thick-walled, stainless steel capillary in a horizontal, straight-tube condition with similar data taken after the same capillary was formed into a helix. Overlapping volumetric flowrates and pressure drops are covered for both configurations at pressure levels of 28, 225, and 1,000 psia at 300° K. Friction factor, Reynolds number, and Dean number correlation plots show conditions where the coiled-tube flow deviates from straight-tube flow. The coiled-tube data indicate departure from Poiseuille steady-

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STEADY-STATE LAMIRAR FLOW BOUNDARY CONDITIONS FOR A STATULESS STEEL COLLED CAPILLARY VISCOSIMELER

and L. M. Walker²/

ASSTRACT

Ad analyses to depermine criteria for accessive, imminaries boundary conditions for a colled-tube gas viscontrater is made by comparing experimental data from a thick-walled, stainless steel capillary in a horizontal, arraight-tube condition with similar data taken after the same capillary was formed into a helix. Overlapping volumetric flowrates and pressure drops are covered for both configurations at pressure levels of 28, 225, and 1,000 pais at J00° K. Sticking factor, Raycolds number, and Dean number correlation plats show conditions where the colled-tube flow destates from straight-tube flow. The colled-tube data indicate departure from Poisseuffle stady-

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Work on wanterigt completed October 1966.

state laminar flow at a Reynolds number close to 390 and a Dean number of about 15. Below these conditions and within the accuracy of the data reported, Dean's circulation is negligible, for all practical purposes. It is suggested that for coiled systems in general, an upper limit for steady-state laminar flow appears to be at a Dean number of about 10.

INTRODUCTION

In an earlier publication (7), $\frac{4}{}$ the Bureau of Mines Helium

4/ Underlined numbers in parentheses refer to items in the list of references at the end of this report.

Research Center presented accurate measurements of physical dimensions of a 19-foot section of stainless steel capillary tubing and developed a working equation for a gas viscosimeter to include the effects of temperature and pressure on the capillary dimensions. In a later publication (<u>10</u>), procedures were derived to correct for non-uniformity of various capillary-bore shapes; it was shown that the bore of the 19foot capillary section was practically uniform.

This report presents criteria for establishing conservatively chosen Reynolds (Re) and Dean (Dn) numbers below which Poiseuille steady-state laminar flow apparently exists without perturbations due to secondary circulation known as Dean's circulation (4, 5, 12). state laminar flow at a Reynolds number close to 390 and a Dean number of about 15. Beice these conditions and within the acturacy of the data reported. Dean's circulation is negligible. for all practical purposes. It is suggested that for colled systems is general, an upper limit for steady-state leminar flow appears to be us a been outber of about 10

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Dean (4, 5) was the first to study the coiling problem from a theoretical viewpoint. His results indicated that the ratio of the mean velocities in a straight and a curved tube of the same dimensions is a function of

Re
$$(d/D)^{1/2}$$
 (1)

which is the Dean number; d is the capillary-bore internal diameter and D is twice the average radius of curvature of the helix. White (13) was the first to experimentally study Dean's relationship by plotting the friction factor ratio of the coiled-to-straight configuration against the Dean number for D/d ratios of 2,050, 50, and 15, the latter being the most tightly coiled system. In White's helices, the appearance of fully developed turbulence occurred later than in the corresponding straight tube and was at increasing Reynolds numbers as the D/d ratio decreased; however, the coil data deviated from strictly Poiseuille straight-tube laminar flow (64/Re) at decreasing Reynolds numbers. More recently, Creutz (3) determined the effect of coiling and apparent lack of fully developed turbulence for a wide range of D/d ratios. A11 results indicate that fully developed turbulence occurs at progressively smaller values of D/d. For porous media, the limiting practical case, the generally accepted Re for steady-state laminar flow is unity (11). Therefore, one might expect that coil fluid flow would start deviating from strictly straight-tube laminar flow at a Reynolds number between 1 and about 2,200, the Re representing the beginning of turbulence in a straight-tube system. It also would be expected that the departure from

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Re (6/2)

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strictly straight-tube flow would occur sooner with helices that were more tightly wound (smaller D/d ratios). Theoretically, a curved tube should always be more resistant to fluid flow that the corresponding straight tube; the slight difference, however, would have required much finer measurements than those employed here and very probably would have been absorbed in the friction factor.

Berwald and Johnson (1) reported gas viscosities of 25 natural gases, using a coiled viscosimeter; according to Carr (2), their data are questionable because of a maximum variation of 25 percent in the results reported. Accuracies of viscosity values, per se, are not discussed in this report (our apparent viscosities are internally consistent to at least two percent). However, rapid changes in apparent viscosity are mentioned in discussing the experimental results where the coiled-tube flow deviates from strictly straight-tube flow. The secondary parameters of entrance, kinetic energy, gas slippage, and gas compressibility effects, although important for absolute gas viscosity determinations, were considered negligible in dealing with friction factors, Reynolds numbers, and Dean numbers for a capillary of the dimensions used and at the flowrates and pressure drops of these experi-In no case were the experiments run in the diffuse flow region. ments.

EXPERIMENTAL APPARATUS AND PROCEDURES

The apparatus, figure 1, consisted of a closed, constant-volume

Figure 1.-Schematic diagram of experimental apparatus.

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Bypass valve
Auxiliary calibration equipment
Straight capillary
Coiled capillary
Stainless steel tubing, 3/16 inch
Pressure limit switch
Vacuum
Filling gas
Readout
Delta pressure transducer
System pressure transducer
Piston displacement gage
Constant volume pump
Timer
Piston, CVP

FIGURE I.- Schematic Diagram of Experimental Apparatus.



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TOURE In Salemathe Gradien of Experimental Apadidius.

Gas could be moved through the capillary in either direction system. by using a double-acting, constant-volume displacement pump. The pump cylinders each had 500-cm³ capacity. Through proper gearing arrangements and a synchronous motor drive, one side of the pump injected while the other side withdrew gas at a constant-volume rate. There were 28 discrete rates available from 5 to 1,120 cm³/hr. Because of the negligible pressure drop from the injection side of the pump to the entrance of the capillary, the volumetric displacement rate of the pump, Q_1 , was assumed to be the same as the volumetric flowrate at the entrance of the capillary. The capillary was a 32-foot long stainless steel tube nominally 0.122 inch in outer diameter and 0.030 inch in internal diameter at 293° K. The root-mean-square radius ($r_m = 0.015199$ inch at 293° K) as determined previously (7) for a 19-foot section was assumed to apply to the 32-foot long capillary.

The pressure difference, $\triangle P$, at the entrance and exit of the capillary was monitored until steady flow was indicated by a constant $\triangle P$. Pressure level and $\triangle P$ were measured with calibrated pressure transducers. Bypass valves were used to facilitate gas handling and allow positive protection for the $\triangle P$ transducer.

To determine the effect of coiling the capillary, a series of runs was made with the capillary in a straight, horizontal configuration and duplicated with the capillary wound into a 20-inch diameter helix. Ambient laboratory temperatures were constant within 2 F°. The volumetric flowrates were known to within 0.965 x 10^{-6} cm³/sec. The ΔP transducer was calibrated at 28 psia and 225 psia with a U-tube glass ayatem. Gas could is never through the capilitary in either circuiton by using a double-socius, constants-volume displacement pump. The pump cylinders each had 500-cm² capacity "Entropy prepar geneticg arrangements and a synchronom votor form, mas side of the pump injerted while the other side withdraw gas at a constant-volume rate. There were 28 discrete rates available from 1 to 1.120 cm²/hr means of the rang the segligible transmue drop from the significant ands at the poop to the emitance of the capiliers, the volumetric Singlitement value of the rang outrance of the capiliers, the volumetric Singlitement value of the rang destrance of the capiliers, the volumetric Singlitement value of the rang outrance of the capiliers, the volumetric Singlitement value of the rang destrance of the capiliers. The solutent of a 12-foot long stated at antrance of the respiliers in the solution and a state of destrand familier [12] for the solution of the volumetric flowmane at the destrance of the respiliers in the toor essen-square radius (r. = 3.03319 intege at 293° K) as familier for the foot essen-square radius (r. = 3.03319 include at 293° K) as familier form for form foot of the solution was assumed to apply to the 13-foot long cathing of

The pressure difference, AP, at the patrance and exit of the capitlary was menitored until steady flow was indicated by a constant 2P. Pressure level and AP were measured with calibrated pressure transdocers Bypass valves were used to facilitate gas handitag and allow positive protection for the AP transducer.

To determine the effect of colling the capillary, a saries of come was made with the capillary in a-straight, horizontal configuration bod duplicated with the capillary wound tate a 20-unch diameter halin. Ambient laboratory temperatures were constant within 2 P⁴. The volumatric flowrotes were known to 0.955 x 10⁻⁶ co³/sec. The AP transducer was calibrated at 28 pais and 225 pais with a 0-tube giass manometer containing n-butyl alcohol; transducer accuracy was at least 0.01 psi for all pressure drops and reproducible to 0.001 psi.

Nitrogen gas was used at pressure levels of 28, 225, and 1,000 psia, all at 300° K. For the 1,000-psia run, the 225-psia calibration was used, although there was evidence of a small zero-shift between pressure calibration levels. At given flowrates, Q_1 , ΔP was reproducible to 0.001 psi; this was confirmed by plotting $\Delta P/L_T$ versus Q_m , where L_T was the corrected length at 300° K, and Q_m was the computed average volumetric flowrate (7). The mean radius, r_m , was corrected for temperature and pressure level changes, also shown in (7). Due to mechanical difficulties after the straight-tube run at 225 psia, the capillary tubing was reworked so that at the 1,000-psia run, the length was slightly less; in reworking the tubing for the coiled runs, another short length was cut off the capillary.

SUMMARY AND DISCUSSION OF RESULTS

Log-log plots of friction factor versus Reynolds number for both the straight and coiled configurations, and log-log plots of the friction factor ratio (coiled to straight) versus the Dean number are used to correlate experimental results. Tables 1 and 2, and figures 1a and 2

- Figure la.-Steady-state boundary flow conditions, 1,000 psia and 300° K, friction factor versus Reynolds number.
- Figure 2. -Steady-state boundary flow conditions, 1,000 psia and 300° K, friction factor ratio versus Dean number.

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Mitrogen gas vas uses at pressure tevels of 18, 22, and 1, 200 pate all st 100° N. For the 1,000 pate run, the 225-pair calibration was used, although there was evidence of a small zero-shift buluesh pressure calibration levels. At given fluwrates, 0_1 , ΔF was reproducible to be 001 out, this was confirmed by plotting CPU, wersus 0_2 , where 1_T was the corrected length at 100° X, and 0_2 was the coopered every volumetric flowrate (D. The mean radius, r_{11} , was corrected to the personal distribution is after the straight-two out is 0_2 . Due to mean is difficulties after the straight-two out is 0_2 best, the length are stiphtly lass; is remerking the tubing for the rolling and alightly lass; is remerking the tubing for the rolling and short length was cut off the expective intervence off the expective

SURVEY AND DISCOSSION OF RESCURE

Log-log plots of frietion factor versus Remaids (maker for both the straight and colied configurations, and the los class of the frietion factor ratio (colled to straight) versus the face base public for used to correlate experimental results. Tables 1 and 2 and figures 18 and 2

Figure la Steady store boundire fiou conditions; 1,000 pata and 300° K.

Figure 2 -Steady-state burndary fiow conditions, 1.000 min and 300° E.

TABLE 1Experimental results, 1,000 psia and 500 K, colled capili	ABLE 1	E 1Experimental	results,	1,000 psia	and 300°	K, coiled	capilla	iry
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Q _m , cm ³ /sec	۵₽/Q _m	Apparent viscosity, Ŋ, micro- poises	Reynolds number, Re	Dean number, Dn	f _s , 64/Re	f	f _c /f _s
0.016666	3.072	191.7	113.028	4.42	0.56624	0.57203	1.0102
.019445	3.070	191.6	131.869	5.16	.48533	.48998	1.0096
.022223	3.051	190.4	150.709	5.90	.42466	.42608	1.0033
.027778	3.046	190.1	188.385	7.37	.33973	.34030	1.0017
.033335	3.060	191.0	226.069	8.85	.28310	.28487	1.0063
.044447	3.051	190.4	301.429	11.80	.21233	.21302	1.0032
.055560	3.060	191.0	376.793	14.75	.16986	.17091	1.0062
.066673	3.129	195.3	452.160	17.70	.14155	.14565	1.0290
.077787	3.173	198.0	527.530	20.65	.12133	.12659	1.0434
.088901	3.233	201.8	602.905	23.60	.10615	.11284	1.0630
.111131	3.364	209.9	753.670	29.51	.08491	.09393	1.1062
.133364	3.491	217.8	904.443	35.41	.07076	.08124	1.1481
.177836	3.718	232.0	1,206.040	47.22	.05306	.06488	1.2228
.222319	3.927	245.1	1,507.717	59.03	.04245	.05482	1.2914

Mean radius, $r_m = 0.0152017$ inch = 0.0386123 cm

Total length, $L_T = 964.210 \text{ cm}$ Viscosity, $\eta = \frac{\pi r_m^4}{8L_T} \cdot \frac{\Delta P}{Q_m} = 62.41750 \frac{\Delta P}{Q_m}$, micropoises; $Q_m = Q_1(1+\frac{\Delta P}{2P_m})$ Density, $\rho = 0.07807 \frac{B}{cm^3}$, interpolated from Flynn (6) Reynolds number, $Re = \frac{2\rho r_m}{\eta^* A} Q_m = 6781.767 Q_m$; $A = \pi r_m^2$, cm^2 ; $\eta^* \cong 190$ micropoises, from Flynn (6) Dean number, $Dn = Re \sqrt{\frac{2r_m}{D}} = 0.039149 \text{ Re}$; D = helix diameter = 50.3865 cmFriction factor, $f_s = \frac{64}{Re} = \text{theoretical friction factor, straight tube}$ Friction factor, $f_c = \frac{4r_m A^2}{\rho L_T} \cdot \frac{\Delta P}{Q_m^2} = 3.103468 \times 10^{-3} \frac{\Delta P}{Q_m^2} = \text{experimental friction factor, coil}$

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TABLE 1. - Experimental passits, 1,000 pate and 300" K. colled cepillary

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	52120				3.927	
Mar Street	and the first of	the second second second				and the second s

Mean Endine. r .. 0.01:2017 Ench = 0.0388123 cm

forst langth, La 964.210 cm

Macouley,
$$0 = \frac{m_{m}}{3L_{m}} \cdot \frac{\Delta P}{Q} = 62.41750 \frac{\Delta P}{Q}$$
, micropolses: $0_{m} = A_{1}(r, \frac{\Delta P}{2})$

Dansity, p = 0.07807 - Interpolated from Flyen (b)

Reyable number, He = $\frac{1}{2}$ A q_{μ} = (781.707 q_{μ} t Å = rr_{μ}^{2} , rn^{2} ; $T \approx 190$ micropolases.

Pean mater, Dn = Re $\sqrt{\frac{1}{0}}$ = 0.039149, Re; D = hulls dismeter = 30.3865 em

Ertecton factor,
$$f_{c} = \frac{h_{m}^{2} h}{\rho_{T}} \cdot \frac{h_{m}^{2}}{\rho_{T}} = 3.103468 \times 10^{-3} \frac{h_{m}}{\rho_{T}^{2}} = experimental friction$$



FIGURE Ia.—Steady-State Boundary Flow Conditions, 1,000 psia and 300 °K, Friction Factor Versus Reynolds Number.



FIGURE In-Steady-State Boundary Flaw Conditions, 1000 parts

TABLE 2.	-Experimental	results,	1,000 psia and	300° K, 8	straight c	apillary

Q _m , cm ³ /sec	∆P/Q _m	Apparent viscosity η, micro- poises	, Reyn olds number, <u>Re</u>	_64/Re	fs
0.016666	3.054	190.0	113.028	0.56624	0.56697
.019445	3.101	192.9	131.869	.48533	.49345
.022223	3.087	192.1	150.709	. 42466	.42980
027778	3.100	192.9	188.385	. 33973	.34529
.033335	3.078	191.5	226.069	.28310	.28569
.044447	3.074	191.3	301.429	.21233	.21398
.055560	3.082	191.8	376.793	.16986	.17163
.066673	3.101	192.9	452.160	.14155	.14390
.077786	3.084	191.9	527.528	.12133	.12266
.088900	3.093	192.4	602.901	.10616	.10764
.111130	3.086	192.1	753.658	.08492	.08594
.133360	3.099	192.9	904.419	.07077	.07191
.177826	3.101	192.9	1,205.975	. 05307	.05396
.222299	3.109	193.4	1,507.578	.04246	.04327
.266777	3.125	194.4	1,809.221	. 03538	.03624
.311265	3.168	197.1	2,110.924	. 03032	.03150

$$\begin{split} & L_{\rm T} = 967.099 \ {\rm cm} \\ & \eta = 62.23104 \ \frac{\Delta P}{Q_{\rm m}} \ , \ {\rm micropoises} \\ & \frac{64}{{\rm Re}} = {\rm theoretical friction factor} \\ & f_{\rm s} = \frac{4r_{\rm m}A^2}{\rho L_{\rm T}} \cdot \frac{\Delta P}{Q_{\rm m}^2} = 3.094201 \ {\rm x} \ 10^{-3} \ \frac{\Delta P}{Q_{\rm m}^2} \ , \ {\rm experimental friction factor}, \\ & {\rm straight capillary} \end{split}$$

T = 967.009 cm T = 62.23164 (AP 9) - 62.23164 (AP

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, experimental friction facto







summarize results at 1,000 psia and 300° K.

The coiled capillary flow begins to deviate from strictly straighttube laminar flow at a Reynolds number of 390 and a Dean number of 14.8, approximately. At Re and Dn greater than the above values, the apparent viscosity increases rapidly to 245 micropoises, as compared to a steadystate Poiseuille viscosity of about 190 micropoises (6).

Tables 3 and 4 and figures 3 and 4 summarize results at 225 psia

Figure 3.-Steady-state boundary flow conditions, 225 psia and 300° K, friction factor versus Reynolds number.

Figure 4.-Steady-state boundary flow conditions, 225 psia and 300° K, friction factor ratio versus Dean number.

and 300° K. The friction factor-Reynolds number plot does not show any obvious deviation, mainly because large changes in Re are reflected by small changes in the friction factor. However, the friction factor ratio-Dean number plot deviates at a Dean number of about 14.5 which corresponds to a Reynolds number of about 382. Also, the apparent viscosities in table 3 indicate a change in the type of fluid flow for the regions just mentioned. The coil data deviations at 1,000 and 225 psia should have occurred at the same Reynolds number because of the necessity of maintaining similitude in steady-state laminar flow ($\underline{8}$). At the 1,000- and 225-psia pressure levels, it would have been interesting to have obtained higher Reynolds and Dean numbers in order summarize results at 1,060 pais and 300° K.

The cotiad capillary flow begins to deviate from stricely straighttube idminar flow at a Maynoids mumber of 390 and a Dean number of 14.8, approximately. At He and Dn greater than the above values, the apparent viscosity increases rapidly to 245 micropoises, as compared to a steadystate Poiseuille viscosity of about 190 micropoises (6).

Figure 3.-Steady-erate boundary flow conditions, 225 psis and 300° K, friction factor versus Reynolds number. Figure 4.-Steady-state boundary flow conditions, 225 psis and 300° K.

and 300° K. The Frierian factor-Seynolds unsher plot does not show any obviaus deviation, mainly because large changes in Re are reflected by small changes in the friction factor. However, the friction factor ratio-Deak number plut deviates at a beam number of about 14.5 which corresponds to a Reynolds number of about 182. Also, the apparent viscosities in table 1 indicate a change in the type of fluid flow for the regions just mentiomed. The outly data deviations at 1,000 and 225 pain should have occurred at the same Reynolds number because of the mecasity of maintaining statificade in steady-state insider flow (3). At the 5,000- and 225-pain pressure levels, it would have been

TABLE	3Exper	imental	results,	225	psia	and	300°	Κ,	coiled	capil	lary	1
				the second se	the second se	the second se						

Q _m , <u>cm³/sec</u>	∆₽/Q _m	Apparent viscosity, Ŋ, micro- poises	Reynolds number, Re	Dean number, 	f _s , 64/Re	f	f _c /f _s
0.027782	2.879	179.7	45.015	1.76	1.42175	1.42494	1.0022
.033340	2.879	179.7	54.022	2.11	1.18472	1.18734	1.0022
.044457	2.879	179.6	72.033	2.81	.88848	0.89038	1.0021
.055575	2.879	179.6	90.048	3.52	.71073	.71221	1.0020
.066694	2.879	179.6	108.065	4.22	.59224	.59321	1.0020
.077816	2.879	179.6	126.085	4.93	.50760	.50857	1.0019
.088939	2.878	179.6	144.108	5.63	.44411	.44493	1.0018
.111190	2.878	179.6	180.162	7.05	.35524	.35584	1.0017
.133447	2.886	180.0	216.225	8.46	.29599	.29727	1.0043
.177981	2.894	180.5	288.383	11.28	.22193	.22353	1.0072
.222543	2.912	181.6	360.587	14.11	.17748	.17988	1.0135
.267135	2,950	184.1	432.839	16.94	.14786	.15181	1.0266
.311758	2.993	186.7	505.144	19.77	.12670	.13198	1.0417

 $\begin{aligned} \mathbf{r}_{m} &= 0.0152011 \text{ inch } = 0.0386108 \text{ cm} \\ \mathbf{L}_{T} &= 964.210 \text{ cm} \\ \eta &= \frac{\pi r_{m}^{4}}{8 \mathbf{L}_{T}} \cdot \frac{\Delta P}{Q_{m}} = 62.407802 \frac{\Delta P}{Q_{m}}, \text{micropoises; } Q_{m} = Q_{1}(1+\frac{\Delta P}{2P_{m}}) \\ \rho &= 0.01762 \frac{g}{cm3}, \text{ interpolated from Flynn (6)} \\ \text{Re} &= \frac{2\rho r_{m}}{\eta^{*} A} Q_{m} = 1620.305 Q_{m}; A = \pi r_{m}^{2}, cm^{2}; \eta^{*} \cong 179 \text{ micropoises, from Flynn (6)} \\ \text{Dn} &= \text{Re} \sqrt{\frac{2r_{m}}{D}} = 0.039148 \text{ Re; } D = 50.3865 \text{ cm} \\ \mathbf{f}_{s} &= \text{theoretical friction factor, straight tube} \\ \mathbf{f}_{c} &= \frac{4r_{m}A^{2}}{\rho \mathbf{L}_{T}} \cdot \frac{\Delta P}{Q_{m}^{2}} = 0.013748 \frac{\Delta P}{Q_{m}^{2}}, \text{ experimental friction factor, coil} \end{aligned}$

TABLE 3. -Exparts were a sulla wais and 100 K. colled capitiery

		900 Ge		
1.6017				
			5.005	

- - 0.0152011 Inch - 0.0186108 cm

L. = 504,210 cm

$$\eta = \frac{\pi r_m}{8 t_T} \cdot \frac{\Delta P}{q_m} = 62.407603 \frac{\Delta P}{q_m} \text{ mix reputses; } q_m = 0_1 (1 \frac{\Delta P}{20})$$

a = 0.01763 -2 interpoleted from Flynn (5

$$B_{2} = \frac{2\pi^{2}}{2} \frac{0}{4} = 1620.205 \frac{0}{2}: A = \pi^{2}_{m}, m^{2}: T = 179 \text{ microprizes, from Flyan (6)}$$

f = theoretical friction factor, wrought the

expurimental frietion factor, coll



REYNOLDS NUMBER

FIGURE 3.-Steady-State Boundary Flow Conditions, 225 psia and 300 °K, Friction Factor Versus Reynolds Number.

FRICTION FACTOR



Freight 3.-Stady-State Boundary From Conditions, 223 psia and

	• • • • • • • • • • • • • • • • • • •	Apparent	r		
Ο,		viscosity	, Reynolds		
m	ARIO	η, micro-	number,		f
cm ³ /sec	ΔF/Q m	poises	Re	<u>64/Re</u>	S
0.016668	3.041	188.3	27.007	2.3697	2.4886
.019447	3.038	188.1	31.509	2.0311	2.1309
.022225	3.038	188.1	36.011	1.7772	1.8644
.027782	3.035	188.0	45.015	1.4217	1.4905
.033340	3.024	187.3	54.022	1.1847	1.2374
.044457	3.000	185.7	72.034	0.8884	0.9206
.055576	3.010	186.3	90.049	.7107	.7389
.066696	3.011	186.4	108.067	.5922	.6157
.077818	3.010	186.4	126.085	. 5075	.5278
.088941	3.009	186.3	144.112	.4441	.4616
.111193	2.998	185.6	180.167	.3552	.3678
.133452	2.996	185.5	216.232	.2959	.3063
.177989	3.006	186.2	288.396	.2219	.2306
.222553	3.045	188.5	360.603	.1774	.1866
.267143	3.006	186.1	432.853	.1478	.1535
.311761	3.004	186.0	505.148	.1267	.1315

TABLE 4. - Experimental results, 225 psia and 300° K, straight capillary

$$\begin{split} \mathbf{L}_{\mathbf{T}} &= 971.704 \text{ cm} \\ \eta &= 61.926499 \, \frac{\Delta P}{Q_{\mathbf{m}}} \text{, micropoises} \\ \frac{64}{Re} &= \text{theoretical friction factor} \\ \mathbf{f}_{\mathbf{s}} &= \frac{4\mathbf{r}_{\mathbf{m}}^{\mathbf{A}^{2}}}{\rho \mathbf{L}_{\mathbf{T}}} \cdot \frac{\Delta P}{Q_{\mathbf{m}}^{2}} = 0.013642 \, \frac{\Delta P}{Q_{\mathbf{m}}^{2}} = \text{experimental friction factor, straight capillary} \end{split}$$

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		0.881	
	: C12-p-1		

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 $\frac{\Delta r_{c}^{A}}{\mu r_{c}} = \frac{\Delta P}{2} = 0.011 \text{ for all the second seco$



FIGURE 4.—Steady—State Boundary Flow Conditions, 225 psia and 300 °K, Friction Factor Ratio Versus Dean Number.

element volumetric finarates were build used and, apparently, the field flow system had not reached atendy state conditions. Also, the temperature fure control was not as provise as was cantred; thus was especially true during the straight-tube monfiguration runs where the tubing was located at the laboratory calling.

a Deep minist larger than about 15, earthering a ready-state Palanetille



DEAN NUMBER, REV PO

FRUME 4.-Steally-State Boundary Flow Conditions, 225 psia and 300 °K, Friction Fastor Ratio Versus Deor Number

to detect the presence of fully developed turbulence observed by others (3, 9); however, the maximum delivery rate of the pump precluded this.

At the 28-psia pressure level, the maximum Reynolds and Dean numbers were 67 and 2.6, respectively, so that no deviation was expected or found. The results are summarized in tables 5 and 6 and figure 5.

Figure 5.-Steady-state boundary flow conditions, 28 psia and 300° K, friction factor versus Reynolds number.

At the start of some particular runs, unexplained variations in $\Delta P/Q_m$ were observed as well as variations of about two to three percent in the computed apparent viscosity, η . At the beginning of these, the slowest volumetric flowrates were being used and, apparently, the fluid flow system had not reached steady-state conditions. Also, the temperature control was not as precise as was desired; this was especially true during the straight-tube-configuration runs where the tubing was located at the laboratory ceiling.

CONCLUDING STATEMENTS

It is evident that at a Reynolds number greater than about 390 or at a Dean number larger than about 15, strictly steady-state Poiseuille laminar flow was measurably affected by Dean's circulation at 1,000 and 225 psia. The same behavior would have been observed at 28 psia if higher volumetric flowrates had been available and used. In any case, the data indicate that if one chooses a conservative upper limit of 10 to detect the presence of fully dreatoped turbulence observed by others (3, 9); however, the maximum delivery rate of the nump precluded this.

At the 28-paid pressure level, the maximum Reynolds and Dean numbers ware 67 and 2.0, respectively, so that no deviation was expected or found. The results are summarized in tables 5 and 6 and figure 5.

Figure 5.-Steady-scars boundary flow conditions, 25 pais and 300° K. Friction factor versus Revnolds number.

At the start of some particular rune, unexplained variations in $\Delta P/R_{m}$ were observed as well as variations of about two to three percent in the computed apparent viscosity, \mathbb{T} . At the beginning of these, the slowest volumetric flowrotes were being used and, apparently, the fluid flow system had not reached standy-state conditions. Also, the temperature control was not as gracise as was dualed; this was especially true during the straight-cube-contiguration runs where the tubing was located at the laboratory celline.

COMOLEDIMG STATEMENTS

It is evident that at a Seynalds number greater than about 390 or at a Dean number larger than about 15, strictly steady-state Poissuille laminar flow was measurably affected by Dean's circulation at 1,000 and 225 pais. The same behavior would have been observed at 78 pais if higher volumetric flowrates had been available and used. In any case, the date indicate that if one chooses a conservative upper limit of 10

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TABLE	5	-Exp	periment	al rea	sults	, 28	psia	and	300°	Κ,	coiled	cap	illa	cv
		and the second sec	and the second sec	Contract of the local day of the local day in the local d	and the second s	a fill water and a state of the second	and the second sec	and the second se	and the second sec					-

	den demonitoren in soli den se anten se a						
		Apparent	1 - Carlos and				
Q _m ,		viscosity,	Reynolds	Dean			
2	AP/O	Ŋ, micro-	number,	number,	E CLIDA	e	c 1c
cm /sec	m m	poises	Re	Dn	s, 04/ke	r c	I / I C S
		Caranter and the second s	dimention dan sector and an and a sector and a				
0.019462	2.775	173.1	4.205	0.164	15.219	14.821	0.974
.022246	2.787	173.9	4.807	.187	13.313	13.024	.978
.027815	2.804	175.0	6.010	.233	10.648	10.481	.984
.033388	2.785	173.8	7.214	.280	8.872	8.673	.978
:044542	2.784	173.7	9.624	.374	6.650	6.497	.977
.055709	2.782	173.6	12.037	.468	5.317	5.192	.976
.066889	2.796	174.5	14.444	.562	4.431	4.345	.981
.078080	2.792	174.2	16.872	.657	3.793	3.717	.980
.089284	2.789	174.0	19.292	.750	3.317	3.247	.979
.111731	2.783	173.7	24.129	.939	2.652	2.590	.977
.134229	2.786	173.9	28.988	1.128	2.208	2.158	.977
.179378	2.787	173.9	38.762	1.508	1.651	1.615	.978
. 2247 34	2.786	173.8	48.537	1.888	1.318	1.288 .	. 97 7
.270315	2.797	174.5	58.418	2.272	1.096	1.076	.982
.316055	2.772	173.0	68.270	2.655	0.937	0.912	.973

$$\begin{split} \mathbf{r}_{\mathbf{m}} &= 0.0152009 \text{ inch } = 0.0386103 \text{ cm} \\ \mathbf{L}_{\mathbf{T}} &= 964.210 \text{ cm} \\ \eta &= \frac{\pi r_{\mathbf{m}}^{4}}{8L_{\mathbf{T}}} \cdot \frac{\Delta P}{Q_{\mathbf{m}}} = 62.404571 \frac{\Delta P}{Q_{\mathbf{m}}}, \text{ micropoises; } Q_{\mathbf{m}} = Q_{1}(1 + \frac{\Delta P}{2P_{\mathbf{m}}}) \\ \rho &= 0.00233 - \frac{g}{cm}_{3}, \text{ extrapolated from Flynn (6)} \\ \mathbf{Re} &= \frac{2\rho r_{\mathbf{m}}}{\eta^{*} A} Q_{\mathbf{m}} = 216.073 Q_{\mathbf{m}}; A = \pi r_{\mathbf{m}}^{2}, \text{ cm}^{2}; \eta^{*} \cong 177 \text{ micropoises, from Flynn (6)} \\ \mathbf{Dn} &= \mathbf{Re} \sqrt{\frac{2r_{\mathbf{m}}}{D}} = 0.038890 \text{ Re; } D = 50.3865 \text{ cm} \\ \mathbf{f}_{\mathbf{s}} &= \text{ theoretical friction factor, straight tube} \\ \mathbf{f}_{\mathbf{c}} &= \frac{4r_{\mathbf{m}}A^{2}}{\rho L_{\mathbf{T}}} \cdot \frac{\Delta P}{Q_{\mathbf{m}}^{2}} = 0.103959 \frac{\Delta P}{Q_{\mathbf{m}}^{2}}, \text{ experimental friction factor, coil} \end{split}$$

		15-5		8#6710. 10-140.

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(a) antrapolated fight 5 antrapolate (b)

$$\alpha = \frac{2\pi}{\sqrt{4}} \frac{1}{\sqrt{4}} = 210.073 \frac{1}{\sqrt{2}} + 210.073 \frac{1}{\sqrt{2}} + 270 \frac{2}{\sqrt{2}} \frac{1}{\sqrt{2}} = 177 \text{ micropoleon, from Plane$$

. thearerical priction factor, accelling toba



FIGURE 5.-Steady-State Boundary Flow Conditions, 28 psia and 300 °K, Friction Factor Versus Reynolds Number.



NBEWON, SOTONALM

FIGURE 5--Bready-State Boundary Flaw Conditions, 28 nord and 300 %. Frighton Factor Versus Reynolds Number.

TABLE 6.-Experimental results, 28 psia and 300° K, straight capillary

		Apparent			
Q_,		viscosity,	Reynolds		
3	AP/O	η, micro-	number,		f
_cm /sec	<u> </u>	poises	Re	<u>64/Re</u>	s
0.022247	2.929	181.4	4.807	13.314	13.581
.027817	2.919	180.8	6.011	10.647	10.825
.033391	2.929	181.4	7.215	8.870	9.048
.044548	2.944	182.3	9.626	6.649	6.818
.055716	2.918	180.7	12.039	5.316	5.402
.066900	2.936	181.8	14.455	4.428	4.528
.078094	2.918	180.7	16.874	3.793	3.855
.089305	2.929	181.4	19.296	3.317	3.383
.111766	2.938	181.9	24.150	2.650	2.712
.134272	2.918	180.7	29.013	2.206	2.242
.179437	2.888	178.8	38.771	1.651	1.660
.224825	2.884	178.6	48.579	1.317	1.323
.270414	2.870	177.7	58.429	1.095	1.095
.316216	2.859	177.0	68.326	0.933	0.933

$$\begin{split} & L_{T} = 971.04 \text{ cm} \\ & \eta = 61.923293 \frac{\Delta P}{Q_{m}}, \text{ micropoises} \\ & \frac{64}{Re} = \text{theoretical friction factor} \\ & f_{s} = \frac{4r_{m}A^{2}}{\rho L_{T}} \cdot \frac{\Delta P}{Q_{m}^{2}} = 0.103158 \frac{\Delta P}{Q_{m}^{2}} = \text{experimental friction factor,} \\ & straight capillary \end{split}$$

.....

	'	

- experimental friction factor

for the Dean number, Dean's circulation will be undetectable, within the experimental accuracies reported here.

In the works of White $(\underline{13})$, Creutz $(\underline{3})$, and Ito $(\underline{9})$, the friction factor ratio versus Dean number plots all strongly suggest an upper limiting Dean number of about 10 for strictly steady-state Poiseuille flow.

This suggests that as long as

$$f_c/f_s = 1$$
 (2)

and

Re
$$(d/D)^{1/2} \leq 10$$
, (3)

one obtains steady-state laminar flow. It seems reasonable that for coiled systems in general, equation (3) holds. This presumes that the capillary tubing cross sections are not appreciably deformed by winding. Creutz (3) found a 94.5-percent reduction in the mean hydraulic radius in his tighter helices where D/d was in the order from 10 to 15. Our D/d was 660. for the Dean muchar, Dean's circulation will be undersciable, within the experimental accoracies reported here.

In the works of White (13), Grouce (1), and Its (2), the friction factor ratio versus Dean number plots all strangly anglest as upper limiting Ream number of about 10 for scrictly strate Poiscuille

This suggests this as long as

one obtains standy-state isminar flow. It seems reasonable that for colled swatems is general, sayution (3) bolds. This presumes that the capillary tobing cross sections are not septectably deformed by winding Grants (3) found a 96.5-percent reduction in the mean hydraulic radius in his tighter bolices where 0/2 was to the order from 10 to 15. Our

D/d was 650.

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