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INTERNAL REPORT

STEADY-STATE LAMINAR FLOW BOUNDARY CONDITIONS

FOR A STAINLESS STEEL COILED CAPILLARY VISCOSIMETER

BY

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BRANCH BRANCH OF FUNDAMENTAL RESEARCH

PROJECT NO. 5574

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STEADY-STATE LAMINAR FLOW BOUNDARY CONDITIONS FOR A STAINLESS STEEL COILED CAPILLARY VISCOSIMETER

by

R. A. Guereca,[1/] H. P. Richardson,[2/] and L. M. Walker[3/]

ABSTRACT

An analysis to determine criteria for steady-state, laminar flow boundary conditions for a coiled-tube gas viscosimeter is made by comparing experimental data from a thick-walled, stainless steel capillary in a horizontal, straight-tube condition with similar data taken after the same capillary was formed into a helix. Overlapping volumetric flowrates and pressure drops are covered for both configurations at pressure levels of 28, 225, and 1,000 psia at 300° K. Friction factor, Reynolds number, and Dean number correlation plots show conditions where the coiled-tube flow deviates from straight-tube flow. The coiled-tube data indicate departure from Poiseuille steady-

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state laminar flow at a Reynolds number close to 390 and a Dean number of about 15. Below these conditions and within the accuracy of the data reported, Dean's circulation is negligible, for all practical purposes. It is suggested that for coiled systems in general, an upper limit for steady-state laminar flow appears to be at a Dean number of about 10.

INTRODUCTION

In an earlier publication (7),[4/] the Bureau of Mines Helium Research Center presented accurate measurements of physical dimensions of a 19-foot section of stainless steel capillary tubing and developed a working equation for a gas viscosimeter to include the effects of temperature and pressure on the capillary dimensions. In a later publication (10), procedures were derived to correct for non-uniformity of various capillary-bore shapes; it was shown that the bore of the 19-foot capillary section was practically uniform.

This report presents criteria for establishing conservatively chosen Reynolds (Re) and Dean (Dn) numbers below which Poiseuille steady-state laminar flow apparently exists without perturbations due to secondary circulation known as Dean's circulation (4, 5, 12).

4/ Underlined numbers in parentheses refer to items in the list of references at the end of this report.

state laminar flow at a Reynolds number close to 390 and a Dean number of about 15. Below these conditions and within the accuracy of the data reported, Dean's circulation is negligible, for all practical purposes. It is suggested that for coiled systems in general, an upper limit for steady-state laminar flow appears to be at a Dean number of about 10.

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This report presents criteria for establishing conservatively chosen Reynolds (Re) and Dean (Dn) numbers below which Poiseuille steady-state laminar flow apparently exists without perturbations due to secondary circulation known as Dean's circulation (6, 9, 12).

Dean (4, 5) was the first to study the coiling problem from a theoretical viewpoint. His results indicated that the ratio of the mean velocities in a straight and a curved tube of the same dimensions is a function of

$$Re\ (d/D)^{1/2} \tag{1}$$

which is the Dean number; d is the capillary-bore internal diameter and D is twice the average radius of curvature of the helix. White (13) was the first to experimentally study Dean's relationship by plotting the friction factor ratio of the coiled-to-straight configuration against the Dean number for D/d ratios of 2,050, 50, and 15, the latter being the most tightly coiled system. In White's helices, the appearance of fully developed turbulence occurred later than in the corresponding straight tube and was at increasing Reynolds numbers as the D/d ratio decreased; however, the coil data deviated from strictly Poiseuille straight-tube laminar flow (64/Re) at decreasing Reynolds numbers. More recently, Creutz (3) determined the effect of coiling and apparent lack of fully developed turbulence for a wide range of D/d ratios. All results indicate that fully developed turbulence occurs at progressively smaller values of D/d. For porous media, the limiting practical case, the generally accepted Re for steady-state laminar flow is unity (11). Therefore, one might expect that coil fluid flow would start deviating from strictly straight-tube laminar flow at a Reynolds number between 1 and about 2,200, the Re representing the beginning of turbulence in a straight-tube system. It also would be expected that the departure from

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Berwald and Johnson (1) reported gas viscosities of 25 natural gases, using a coiled viscosimeter; according to Carr (2), their data are questionable because of a maximum variation of 25 percent in the results reported. Accuracies of viscosity values, per se, are not discussed in this report (our apparent viscosities are internally consistent to at least two percent). However, rapid changes in apparent viscosity are mentioned in discussing the experimental results where the coiled-tube flow deviates from strictly straight-tube flow. The secondary parameters of entrance, kinetic energy, gas slippage, and gas compressibility effects, although important for absolute gas viscosity determinations, were considered negligible in dealing with friction factors, Reynolds numbers, and Dean numbers for a capillary of the dimensions used and at the flowrates and pressure drops of these experiments. In no case were the experiments run in the diffuse flow region.

EXPERIMENTAL APPARATUS AND PROCEDURES

The apparatus, figure 1, consisted of a closed, constant-volume

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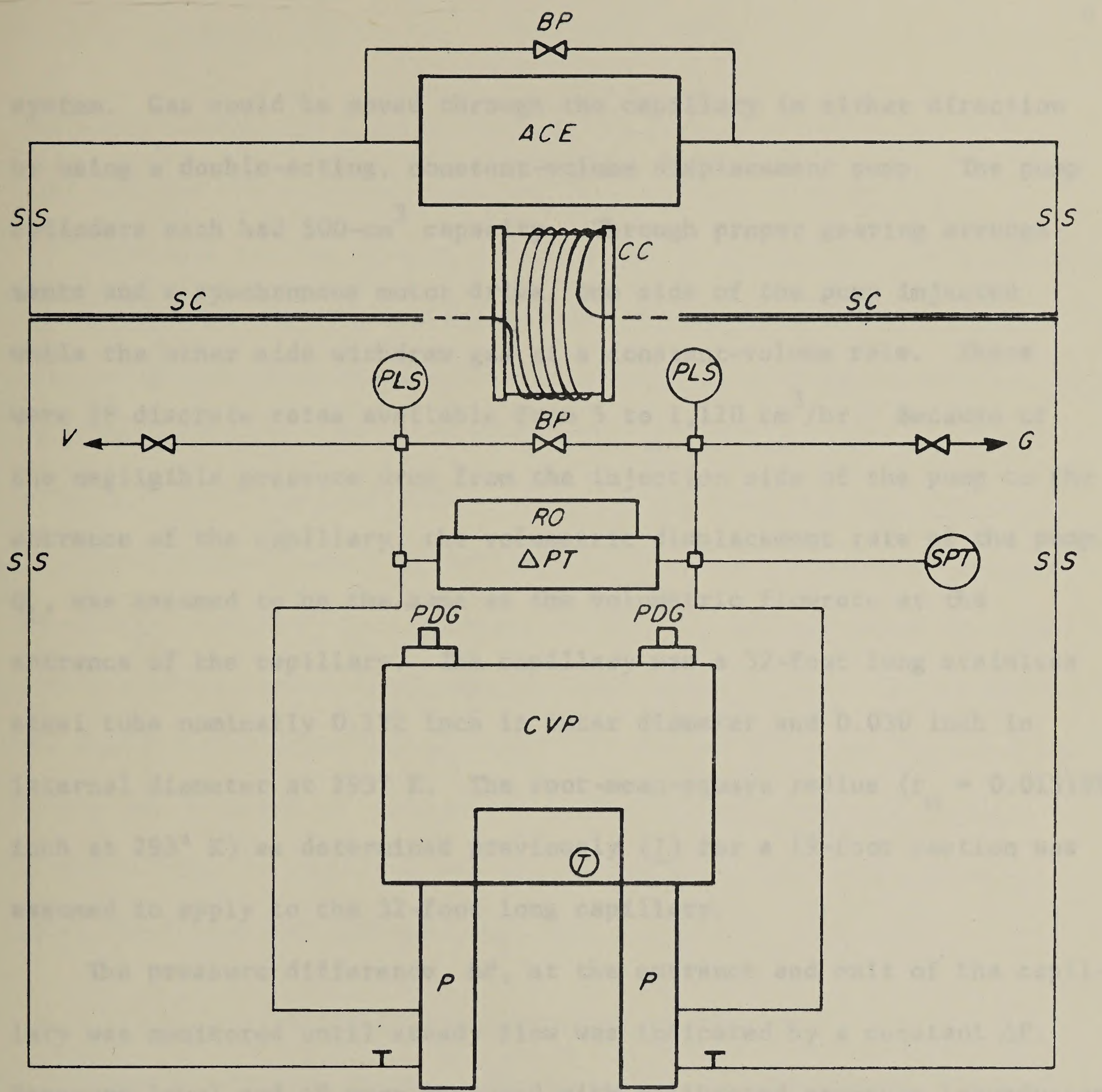
Figure 1.-Schematic diagram of experimental apparatus.



BP	Bypass valve
ACE	Auxiliary calibration equipment
SC	Straight capillary
CC	Coiled capillary
SS	Stainless steel tubing, 3/16 inch
PLS	Pressure limit switch
V	Vacuum
G	Filling gas
RO	Readout
ΔPT	Delta pressure transducer
SPT	System pressure transducer
PDG	Piston displacement gage
CVP	Constant volume pump
T	Timer
P	Piston, CVP

FIGURE 1.- Schematic Diagram of Experimental Apparatus.

system. Gas could be moved through the capillary in either direction by using a double-acting, constant-volume displacement pump. The pump cylinders each had 500-cm^3 capacity. Through proper gearing arrangements and a synchronous motor drive, one side of the pump injected while the other side withdrew gas at a constant-volume rate. There were 28 discrete rates available from 5 to 1,120 cm^3/hr. Because of the negligible pressure drop from the injection side of the pump to the entrance of the capillary, the volumetric displacement rate of the pump, Q_1, was assumed to be the same as the volumetric flowrate at the entrance of the capillary. The capillary was a 32-foot long stainless steel tube nominally 0.122 inch in outer diameter and 0.030 inch in internal diameter at 293° K. The root-mean-square radius (r_m = 0.015199 inch at 293° K) as determined previously (7) for a 19-foot section was assumed to apply to the 32-foot long capillary.

The pressure difference, ΔP, at the entrance and exit of the capillary was monitored until steady flow was indicated by a constant ΔP. Pressure level and ΔP were measured with calibrated pressure transducers. Bypass valves were used to facilitate gas handling and allow positive protection for the ΔP transducer.

To determine the effect of coiling the capillary, a series of runs was made with the capillary in a straight, horizontal configuration and duplicated with the capillary wound into a 20-inch diameter helix. Ambient laboratory temperatures were constant within 2 F°. The volumetric flowrates were known to within 0.965 x 10^{-6} cm^3/sec. The ΔP transducer was calibrated at 28 psia and 225 psia with a U-tube glass system. Gas could be moved through the capillary in either direction by using a double-acting, constant-volume displacement pump. The pump cylinders each had 500-cm^3 capacity. Through proper gearing arrangements and a synchronous motor drive, one side of the pump injected while the other side withdrew gas at a constant-volume rate. There were 28 discrete rates available from 3 to 1,120 cm^3/hr. Because of the negligible pressure drop from the injection side of the pump to the entrance of the capillary, the volumetric displacement rate of the pump, Q_1, was assumed to be the same as the volumetric flowrate at the entrance of the capillary. The capillary was a 32-foot long stainless steel tube nominally 0.122 inch in outer diameter and 0.030 inch in internal diameter at 293° K. The root-mean-square radius (r_m = 0.015199 inch at 293° K) as determined previously (7) for a 19-foot section was assumed to apply to the 32-foot long capillary.

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Nitrogen gas was used at pressure levels of 28, 225, and 1,000 psia, all at 300° K. For the 1,000-psia run, the 225-psia calibration was used, although there was evidence of a small zero-shift between pressure calibration levels. At given flowrates, Q_1, ΔP was reproducible to 0.001 psi; this was confirmed by plotting $\Delta P/L_T$ versus Q_m, where L_T was the corrected length at 300° K, and Q_m was the computed average volumetric flowrate (7). The mean radius, r_m, was corrected for temperature and pressure level changes, also shown in (7). Due to mechanical difficulties after the straight-tube run at 225 psia, the capillary tubing was reworked so that at the 1,000-psia run, the length was slightly less; in reworking the tubing for the coiled runs, another short length was cut off the capillary.

SUMMARY AND DISCUSSION OF RESULTS

Log-log plots of friction factor versus Reynolds number for both the straight and coiled configurations, and log-log plots of the friction factor ratio (coiled to straight) versus the Dean number are used to correlate experimental results. Tables 1 and 2, and figures 1a and 2

Figure 1a.-Steady-state boundary flow conditions, 1,000 psia and 300° K, friction factor versus Reynolds number.

Figure 2. -Steady-state boundary flow conditions, 1,000 psia and 300° K, friction factor ratio versus Dean number.

TABLE 1.-Experimental results, 1,000 psia and 300° K, coiled capillary

Q_m, cm^3/sec	$\Delta P/Q_m$	Apparent viscosity, η, micro-poises	Reynolds number, Re	Dean number, Dn	f_s, 64/Re	f_c	f_c/f_s
0.016666	3.072	191.7	113.028	4.42	0.56624	0.57203	1.0102
.019445	3.070	191.6	131.869	5.16	.48533	.48998	1.0096
.022223	3.051	190.4	150.709	5.90	.42466	.42608	1.0033
.027778	3.046	190.1	188.385	7.37	.33973	.34030	1.0017
.033335	3.060	191.0	226.069	8.85	.28310	.28487	1.0063
.044447	3.051	190.4	301.429	11.80	.21233	.21302	1.0032
.055560	3.060	191.0	376.793	14.75	.16986	.17091	1.0062
.066673	3.129	195.3	452.160	17.70	.14155	.14565	1.0290
.077787	3.173	198.0	527.530	20.65	.12133	.12659	1.0434
.088901	3.233	201.8	602.905	23.60	.10615	.11284	1.0630
.111131	3.364	209.9	753.670	29.51	.08491	.09393	1.1062
.133364	3.491	217.8	904.443	35.41	.07076	.08124	1.1481
.177836	3.718	232.0	1,206.040	47.22	.05306	.06488	1.2228
.222319	3.927	245.1	1,507.717	59.03	.04245	.05482	1.2914

Mean radius, r_m = 0.0152017 inch = 0.0386123 cm

Total length, L_T = 964.210 cm

Viscosity, $\eta = \frac{\pi r_m^4}{8L_T} \cdot \frac{\Delta P}{Q_m} = 62.41750 \frac{\Delta P}{Q_m}$, micropoises; $Q_m = Q_1(1+\frac{\Delta P}{2P_m})$

Density, $\rho = 0.07807 \frac{g}{cm^3}$, interpolated from Flynn (6)

Reynolds number, $Re = \frac{2\rho r_m}{\eta^* A} Q_m = 6781.767\ Q_m$; $A = \pi r_m^2$, cm^2; $\eta^* \cong 190$ micropoises, from Flynn (6)

Dean number, $Dn = Re\sqrt{\frac{2r_m}{D}} = 0.039149$ Re; D = helix diameter = 50.3865 cm

Friction factor, $f_s = \frac{64}{Re}$ = theoretical friction factor, straight tube

Friction factor, $f_c = \frac{4r_m A^2}{\rho L_T} \cdot \frac{\Delta P}{Q_m^2} = 3.103468 \times 10^{-3} \frac{\Delta P}{Q_m^2}$ = experimental friction factor, coil

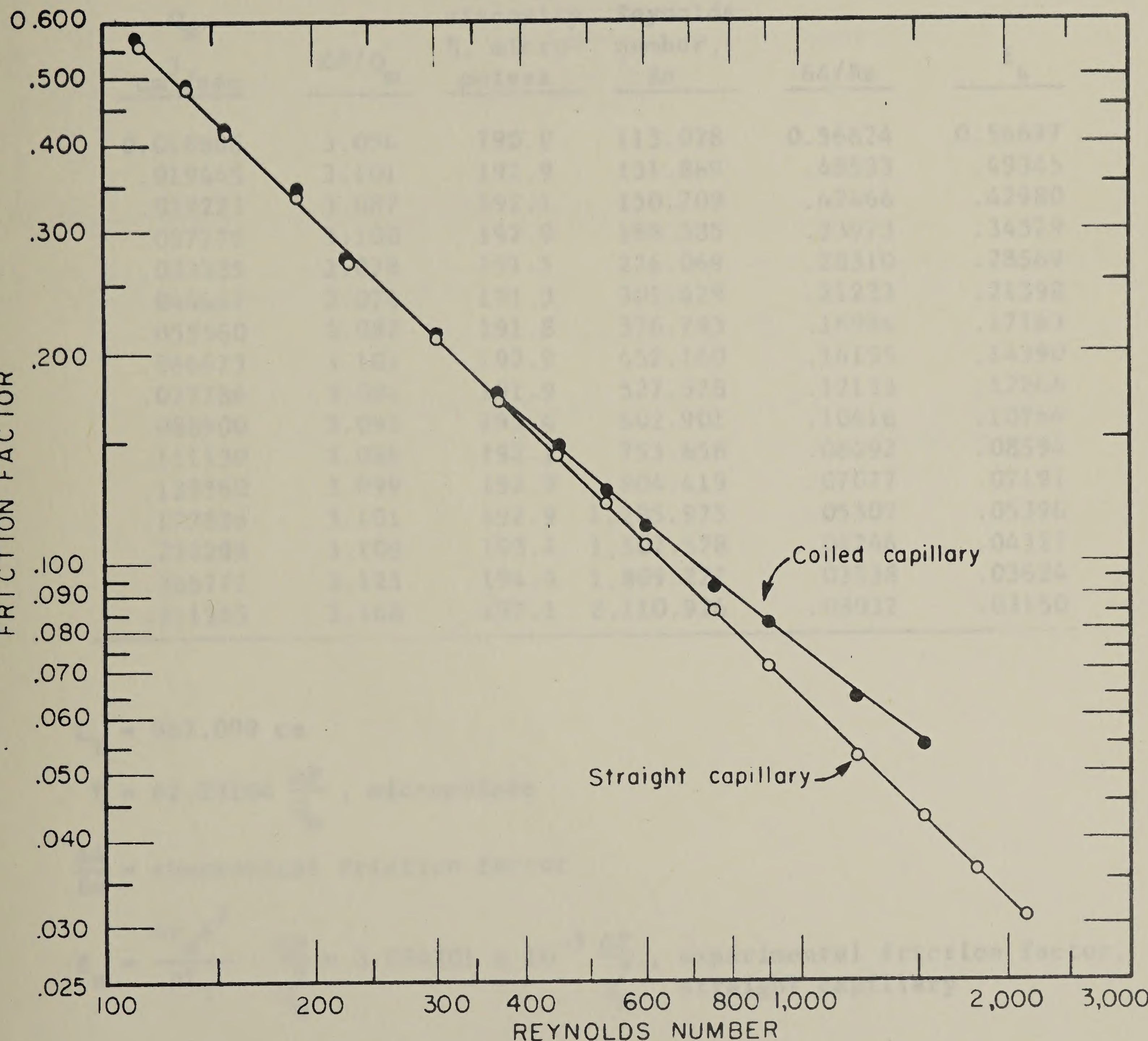

FIGURE 1a.—Steady-State Boundary Flow Conditions, 1,000 psia and 300 °K, Friction Factor Versus Reynolds Number.

TABLE 2.-Experimental results, 1,000 psia and 300° K, straight capillary

Q_m, cm^3/sec	$\Delta P/Q_m$	Apparent viscosity, η, micro-poises	Reynolds number, Re	64/Re	f_s
0.016666	3.054	190.0	113.028	0.56624	0.56697
.019445	3.101	192.9	131.869	.48533	.49345
.022223	3.087	192.1	150.709	.42466	.42980
.027778	3.100	192.9	188.385	.33973	.34529
.033335	3.078	191.5	226.069	.28310	.28569
.044447	3.074	191.3	301.429	.21233	.21398
.055560	3.082	191.8	376.793	.16986	.17163
.066673	3.101	192.9	452.160	.14155	.14390
.077786	3.084	191.9	527.528	.12133	.12266
.088900	3.093	192.4	602.901	.10616	.10764
.111130	3.086	192.1	753.658	.08492	.08594
.133360	3.099	192.9	904.419	.07077	.07191
.177826	3.101	192.9	1,205.975	.05307	.05396
.222299	3.109	193.4	1,507.578	.04246	.04327
.266777	3.125	194.4	1,809.221	.03538	.03624
.311265	3.168	197.1	2,110.924	.03032	.03150

L_T = 967.099 cm

$\eta = 62.23104 \frac{\Delta P}{Q_m}$, micropoises

$\frac{64}{Re}$ = theoretical friction factor

$f_s = \frac{4r_m A^2}{\rho L_T} \cdot \frac{\Delta P}{Q_m^2} = 3.094201 \times 10^{-3} \frac{\Delta P}{Q_m^2}$, experimental friction factor, straight capillary

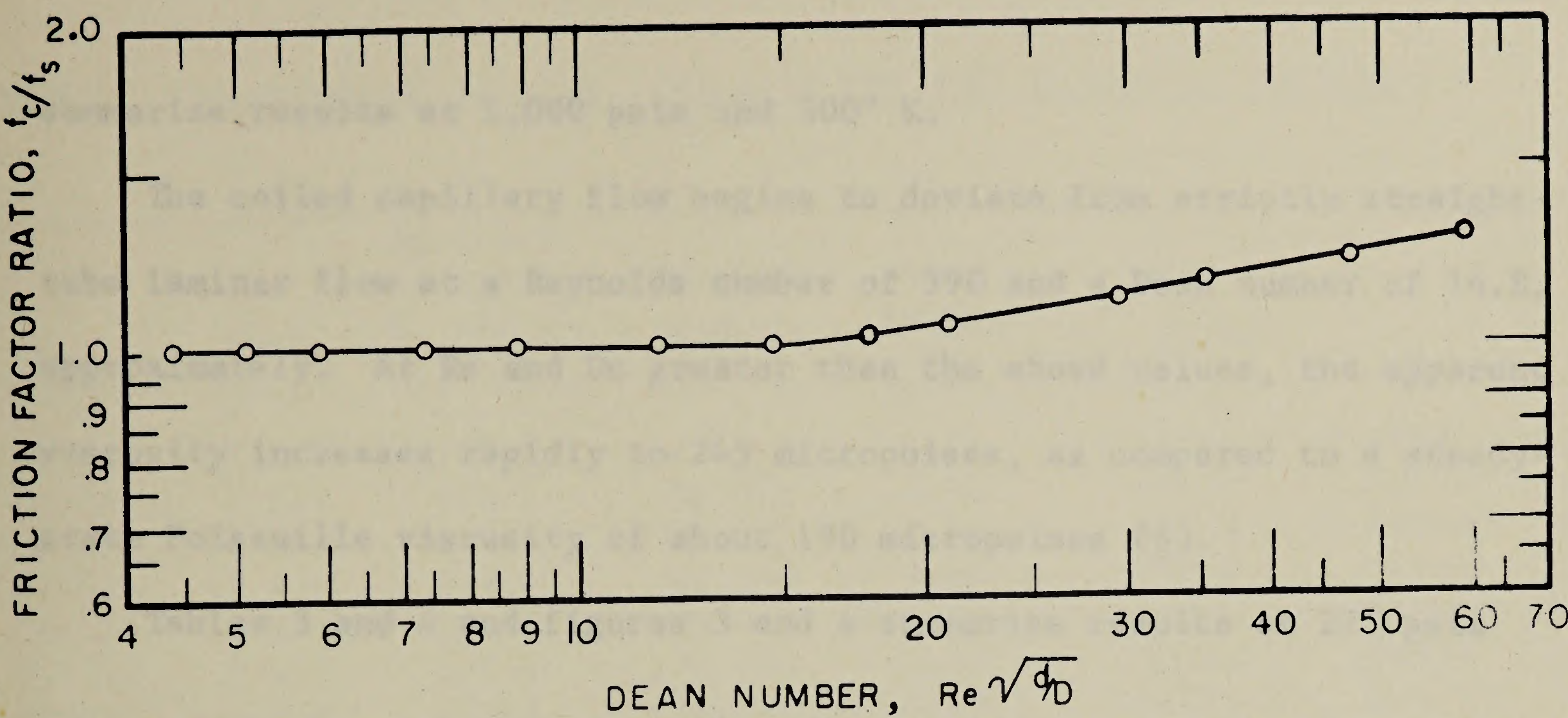

FIGURE 2.—Steady-State Boundary Flow Conditions, 1,000 psia and 300 °K, Friction Factor Ratio Versus Dean Number.

summarize results at 1,000 psia and 300° K.

The coiled capillary flow begins to deviate from strictly straight-tube laminar flow at a Reynolds number of 390 and a Dean number of 14.8, approximately. At Re and Dn greater than the above values, the apparent viscosity increases rapidly to 245 micropoises, as compared to a steady-state Poiseuille viscosity of about 190 micropoises (6).

Tables 3 and 4 and figures 3 and 4 summarize results at 225 psia

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Figure 3.-Steady-state boundary flow conditions, 225 psia and 300° K, friction factor versus Reynolds number.

Figure 4.-Steady-state boundary flow conditions, 225 psia and 300° K, friction factor ratio versus Dean number.

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and 300° K. The friction factor-Reynolds number plot does not show any obvious deviation, mainly because large changes in Re are reflected by small changes in the friction factor. However, the friction factor ratio-Dean number plot deviates at a Dean number of about 14.5 which corresponds to a Reynolds number of about 382. Also, the apparent viscosities in table 3 indicate a change in the type of fluid flow for the regions just mentioned. The coil data deviations at 1,000 and 225 psia should have occurred at the same Reynolds number because of the necessity of maintaining similitude in steady-state laminar flow (8). At the 1,000- and 225-psia pressure levels, it would have been interesting to have obtained higher Reynolds and Dean numbers in order

TABLE 3.-Experimental results, 225 psia and 300° K, coiled capillary

Q_m, cm^3/sec	$\Delta P/Q_m$	Apparent viscosity, η, micro-poises	Reynolds number, Re	Dean number, Dn	f_s, 64/Re	f_c	f_c/f_s
0.027782	2.879	179.7	45.015	1.76	1.42175	1.42494	1.0022
.033340	2.879	179.7	54.022	2.11	1.18472	1.18734	1.0022
.044457	2.879	179.6	72.033	2.81	.88848	0.89038	1.0021
.055575	2.879	179.6	90.048	3.52	.71073	.71221	1.0020
.066694	2.879	179.6	108.065	4.22	.59224	.59321	1.0020
.077816	2.879	179.6	126.085	4.93	.50760	.50857	1.0019
.088939	2.878	179.6	144.108	5.63	.44411	.44493	1.0018
.111190	2.878	179.6	180.162	7.05	.35524	.35584	1.0017
.133447	2.886	180.0	216.225	8.46	.29599	.29727	1.0043
.177981	2.894	180.5	288.383	11.28	.22193	.22353	1.0072
.222543	2.912	181.6	360.587	14.11	.17748	.17988	1.0135
.267135	2.950	184.1	432.839	16.94	.14786	.15181	1.0266
.311758	2.993	186.7	505.144	19.77	.12670	.13198	1.0417

r_m = 0.0152011 inch = 0.0386108 cm

L_T = 964.210 cm

$$\eta = \frac{\pi r_m^4}{8L_T} \cdot \frac{\Delta P}{Q_m} = 62.407802 \frac{\Delta P}{Q_m}, \text{micropoises}; \quad Q_m = Q_1\left(1+\frac{\Delta P}{2P_m}\right)$$

$\rho = 0.01762 \frac{g}{cm^3}$, interpolated from Flynn (6)

$$Re = \frac{2\rho r_m}{\eta^* A} Q_m = 1620.305\ Q_m; \quad A = \pi r_m^2,\ cm^2; \quad \eta^* \cong 179 \text{ micropoises, from Flynn (6)}$$

$$Dn = Re\sqrt{\frac{2r_m}{D}} = 0.039148\ Re; \quad D = 50.3865 \text{ cm}$$

f_s = theoretical friction factor, straight tube

$$f_c = \frac{4r_m A^2}{\rho L_T} \cdot \frac{\Delta P}{Q_m^2} = 0.013748 \frac{\Delta P}{Q_m^2}, \text{ experimental friction factor, coil}$$

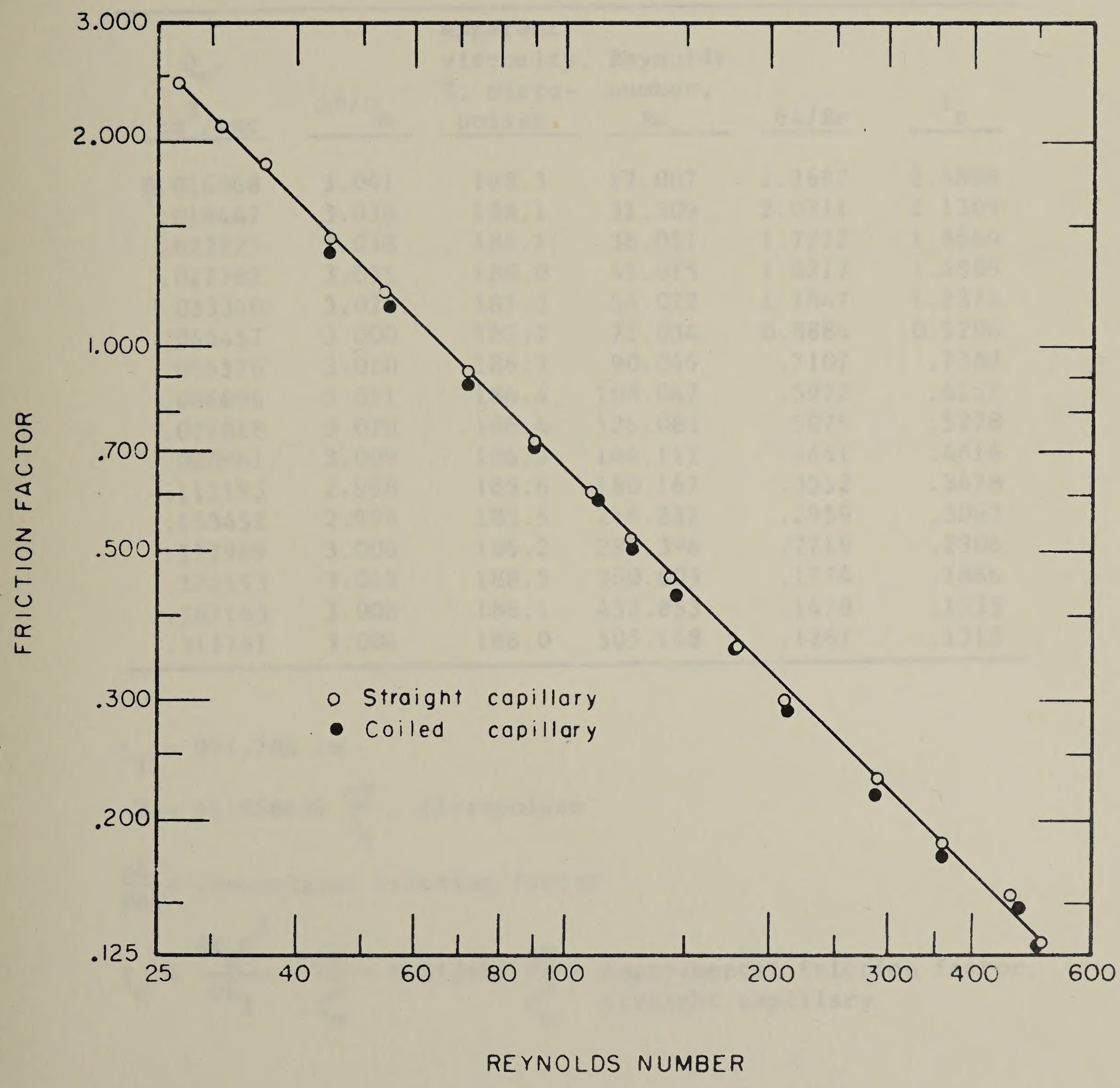

FIGURE 3.-Steady-State Boundary Flow Conditions, 225 psia and 300 °K, Friction Factor Versus Reynolds Number.

TABLE 4.-Experimental results, 225 psia and 300° K, straight capillary

Q_m, cm^3/sec	$\Delta P/Q_m$	Apparent viscosity, η, micropoises	Reynolds number, Re	64/Re	f_s
0.016668	3.041	188.3	27.007	2.3697	2.4886
.019447	3.038	188.1	31.509	2.0311	2.1309
.022225	3.038	188.1	36.011	1.7772	1.8644
.027782	3.035	188.0	45.015	1.4217	1.4905
.033340	3.024	187.3	54.022	1.1847	1.2374
.044457	3.000	185.7	72.034	0.8884	0.9206
.055576	3.010	186.3	90.049	.7107	.7389
.066696	3.011	186.4	108.067	.5922	.6157
.077818	3.010	186.4	126.085	.5075	.5278
.088941	3.009	186.3	144.112	.4441	.4616
.111193	2.998	185.6	180.167	.3552	.3678
.133452	2.996	185.5	216.232	.2959	.3063
.177989	3.006	186.2	288.396	.2219	.2306
.222553	3.045	188.5	360.603	.1774	.1866
.267143	3.006	186.1	432.853	.1478	.1535
.311761	3.004	186.0	505.148	.1267	.1315

L_T = 971.704 cm

$\eta = 61.926499 \frac{\Delta P}{Q_m}$, micropoises

$\frac{64}{Re}$ = theoretical friction factor

$f_s = \frac{4r_m A^2}{\rho L_T} \cdot \frac{\Delta P}{Q_m^2} = 0.013642 \frac{\Delta P}{Q_m^2}$ = experimental friction factor, straight capillary

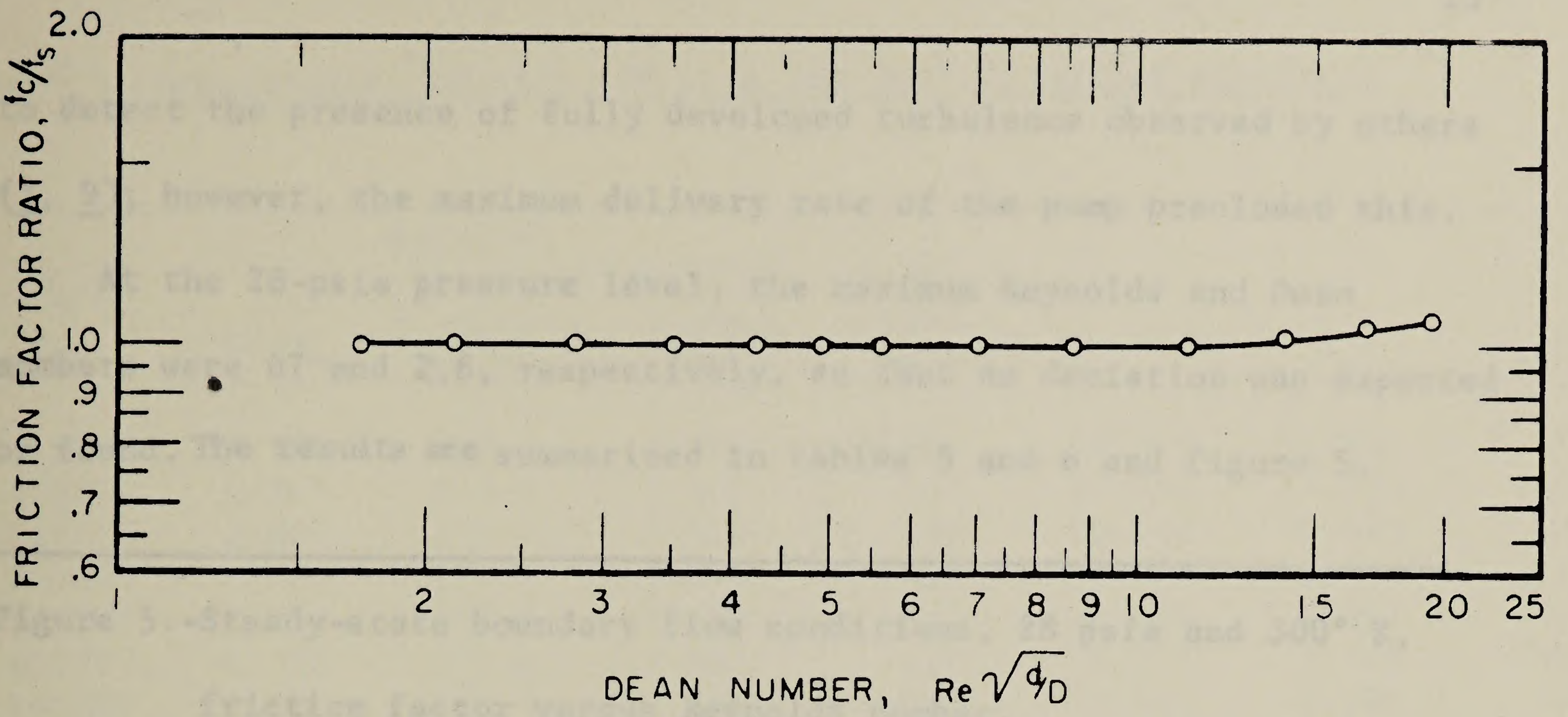

FIGURE 4.—Steady-State Boundary Flow Conditions, 225 psia and 300 °K, Friction Factor Ratio Versus Dean Number.

to detect the presence of fully developed turbulence observed by others (3, 9); however, the maximum delivery rate of the pump precluded this.

At the 28-psia pressure level, the maximum Reynolds and Dean numbers were 67 and 2.6, respectively, so that no deviation was expected or found. The results are summarized in tables 5 and 6 and figure 5.

Figure 5.-Steady-state boundary flow conditions, 28 psia and 300° K, friction factor versus Reynolds number.

At the start of some particular runs, unexplained variations in $\Delta P/Q_m$ were observed as well as variations of about two to three percent in the computed apparent viscosity, η. At the beginning of these, the slowest volumetric flowrates were being used and, apparently, the fluid flow system had not reached steady-state conditions. Also, the temperature control was not as precise as was desired; this was especially true during the straight-tube-configuration runs where the tubing was located at the laboratory ceiling.

CONCLUDING STATEMENTS

It is evident that at a Reynolds number greater than about 390 or at a Dean number larger than about 15, strictly steady-state Poiseuille laminar flow was measurably affected by Dean's circulation at 1,000 and 225 psia. The same behavior would have been observed at 28 psia if higher volumetric flowrates had been available and used. In any case, the data indicate that if one chooses a conservative upper limit of 10 to detect the presence of fully developed turbulence observed by others (3, 9); however, the maximum delivery rate of the pump precluded this.

At the 28-psia pressure level, the maximum Reynolds and Dean numbers were 67 and 2.6, respectively, so that no deviation was expected or found. The results are summarized in tables 5 and 6 and figure 5.

Figure 5.-Steady-state boundary flow conditions, 28 psia and 300° K, friction factor versus Reynolds number.

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TABLE 5.-Experimental results, 28 psia and 300° K, coiled capillary

Q_m, cm^3/sec	$\Delta P/Q_m$	Apparent viscosity, η, micro-poises	Reynolds number, Re	Dean number, Dn	f_s, 64/Re	f_c	f_c/f_s
0.019462	2.775	173.1	4.205	0.164	15.219	14.821	0.974
.022246	2.787	173.9	4.807	.187	13.313	13.024	.978
.027815	2.804	175.0	6.010	.233	10.648	10.481	.984
.033388	2.785	173.8	7.214	.280	8.872	8.673	.978
.044542	2.784	173.7	9.624	.374	6.650	6.497	.977
.055709	2.782	173.6	12.037	.468	5.317	5.192	.976
.066889	2.796	174.5	14.444	.562	4.431	4.345	.981
.078080	2.792	174.2	16.872	.657	3.793	3.717	.980
.089284	2.789	174.0	19.292	.750	3.317	3.247	.979
.111731	2.783	173.7	24.129	.939	2.652	2.590	.977
.134229	2.786	173.9	28.988	1.128	2.208	2.158	.977
.179378	2.787	173.9	38.762	1.508	1.651	1.615	.978
.224734	2.786	173.8	48.537	1.888	1.318	1.288	.977
.270315	2.797	174.5	58.418	2.272	1.096	1.076	.982
.316055	2.772	173.0	68.270	2.655	0.937	0.912	.973

r_m = 0.0152009 inch = 0.0386103 cm

L_T = 964.210 cm

$\eta = \frac{\pi r_m^4}{8L_T} \cdot \frac{\Delta P}{Q_m} = 62.404571 \frac{\Delta P}{Q_m}$, micropoises; $Q_m = Q_1(1+\frac{\Delta P}{2P_m})$

$\rho = 0.00233 \frac{g}{cm^3}$, extrapolated from Flynn (6)

$Re = \frac{2\rho r_m}{\eta^* A} Q_m = 216.073\ Q_m$; $A = \pi r_m^2$, cm^2; $\eta^* \cong 177$ micropoises, from Flynn (6)

$Dn = Re\sqrt{\frac{2r_m}{D}} = 0.038890\ Re$; D = 50.3865 cm

f_s = theoretical friction factor, straight tube

$f_c = \frac{4r_m A^2}{\rho L_T} \cdot \frac{\Delta P}{Q_m^2} = 0.103959 \frac{\Delta P}{Q_m^2}$, experimental friction factor, coil

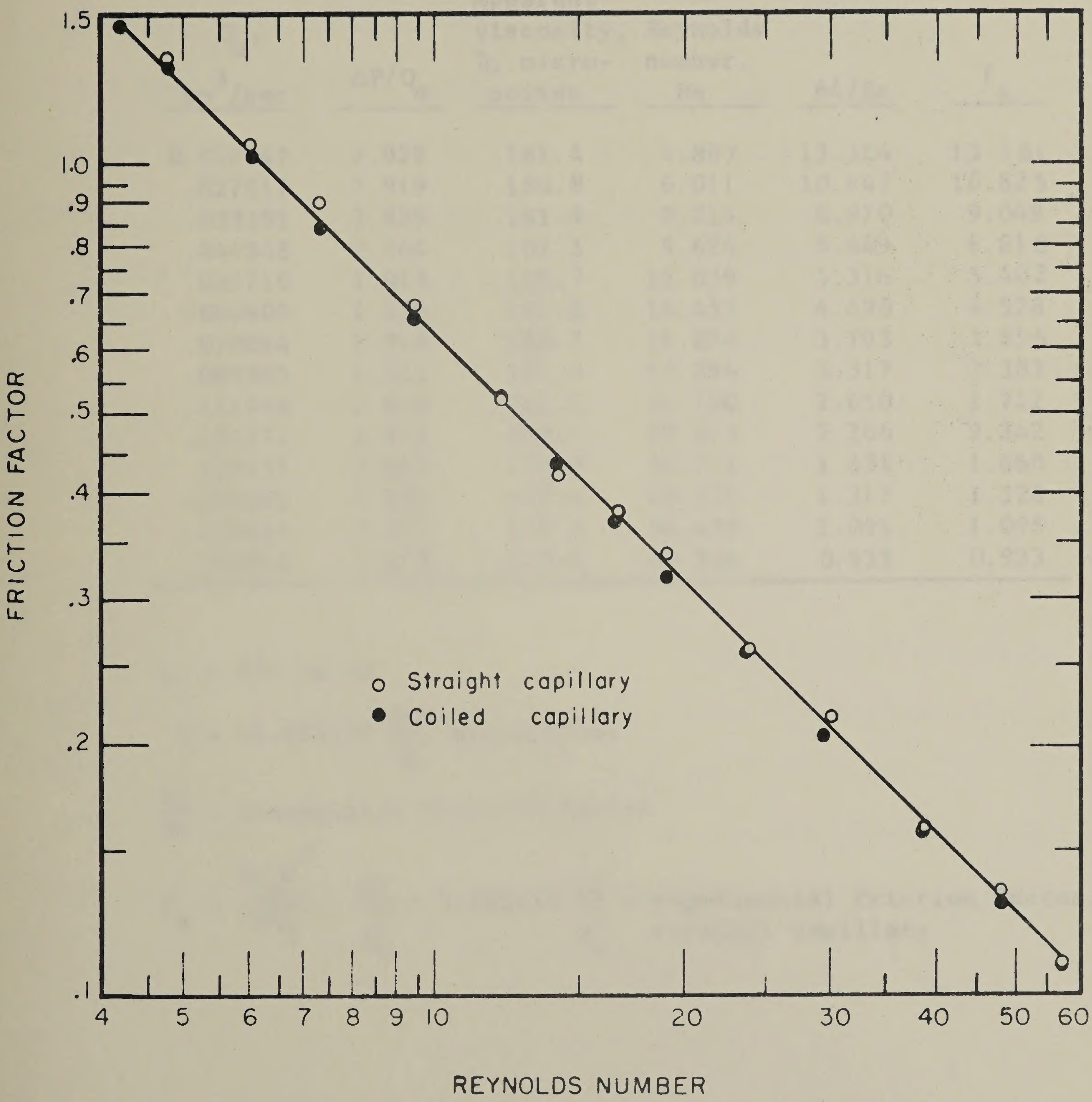

FIGURE 5.–Steady-State Boundary Flow Conditions, 28 psia and 300 °K, Friction Factor Versus Reynolds Number.

TABLE 6.-Experimental results, 28 psia and 300° K, straight capillary

Q_m, cm^3/sec	$\Delta P/Q_m$	Apparent viscosity, η, micro-poises	Reynolds number, Re	64/Re	f_s
0.022247	2.929	181.4	4.807	13.314	13.581
.027817	2.919	180.8	6.011	10.647	10.825
.033391	2.929	181.4	7.215	8.870	9.048
.044548	2.944	182.3	9.626	6.649	6.818
.055716	2.918	180.7	12.039	5.316	5.402
.066900	2.936	181.8	14.455	4.428	4.528
.078094	2.918	180.7	16.874	3.793	3.855
.089305	2.929	181.4	19.296	3.317	3.383
.111766	2.938	181.9	24.150	2.650	2.712
.134272	2.918	180.7	29.013	2.206	2.242
.179437	2.888	178.8	38.771	1.651	1.660
.224825	2.884	178.6	48.579	1.317	1.323
.270414	2.870	177.7	58.429	1.095	1.095
.316216	2.859	177.0	68.326	0.933	0.933

L_T = 971.04 cm

$\eta = 61.923293 \frac{\Delta P}{Q_m}$, micropoises

$\frac{64}{Re}$ = theoretical friction factor

$f_s = \frac{4r_m A^2}{\rho L_T} \cdot \frac{\Delta P}{Q_m^2} = 0.103158 \frac{\Delta P}{Q_m^2}$ = experimental friction factor, straight capillary

for the Dean number, Dean's circulation will be undetectable, within the experimental accuracies reported here.

In the works of White (13), Creutz (3), and Ito (9), the friction factor ratio versus Dean number plots all strongly suggest an upper limiting Dean number of about 10 for strictly steady-state Poiseuille flow.

This suggests that as long as

$$f_c/f_s = 1 \qquad (2)$$

and

$$Re\,(d/D)^{1/2} \leqq 10\,, \qquad (3)$$

one obtains steady-state laminar flow. It seems reasonable that for coiled systems in general, equation (3) holds. This presumes that the capillary tubing cross sections are not appreciably deformed by winding. Creutz (3) found a 94.5-percent reduction in the mean hydraulic radius in his tighter helices where D/d was in the order from 10 to 15. Our D/d was 660.

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