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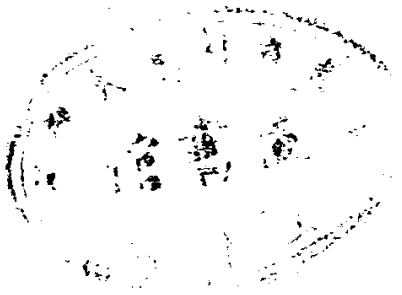
積分公式表

國立北平大學工學院

中華民國二十二年八月印

積分公式表



En Min

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中華民國二十二年八月印

萬有引力常數G　6.65×10^{-8}達因×厘米^2×克^2

在45°緯度之重力加速度g　$g=980.665$厘米×秒$^{-2}$

1大氣壓　1.01325×10^6達因×厘米$^{-2}$

在0°C時之水銀之比重　13.5955克×厘米$^{-3}$

積分公式一覽表

I. 含 $a+bx$ 者

1. $\int (a+bx)^n dx = \frac{(a+bx)^{n+1}}{b(n+1)}$ 但 $n \neq -1$

2. $\int \frac{dx}{a+bx} = \frac{1}{b}\log(a+bx)$

3. $\int \frac{xdx}{a+bx} = \frac{1}{b^2}\left\{(a+bx) - a\log(a+bx)\right\}$

4. $\int \frac{x^2dx}{a+bx} = \frac{1}{b^3}\left\{\frac{(a+bx)^2}{2} - 2a(a+bx) + a^2\log(a+bx)\right\}$

5. $\int \frac{dx}{x(a+bx)} = -\frac{1}{a}\log\frac{a+bx}{x}$

6. $\int \frac{dx}{x^2(a+bx)} = -\frac{1}{ax} + \frac{b}{a^2}\log\frac{a+bx}{x}$

7. $\int \frac{xdx}{(a+bx)^2} = \frac{1}{b^2}\left\{\log(a+bx) + \frac{a}{a+bx}\right\}$

8. $\int \frac{x^3dx}{(a+bx)^2} = \frac{1}{b^3}\left\{(a+bx) - 2a\log(a+bx) - \frac{a^2}{a+bx}\right\}$

9. $$\int \frac{dx}{x(a+bx)^2} = \frac{1}{a(a+bx)} - \frac{1}{a^2}\log\frac{a+bx}{x}$$

10. $$\int \frac{x\,dx}{(a+bx)^3} = \frac{1}{b^2}\left\{-\frac{1}{a+bx} + \frac{a}{2(a+bx)^2}\right\}$$

11. 含 a^2+x^2, a^2-x^2 及 $a+bx^n$ 者.

11. $$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

12. $$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x}$$

13. $$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\frac{x-a}{x+a}$$

14. $$\int \frac{dx}{a+bx^2} = \frac{1}{\sqrt{ab}}\tan^{-1}x\sqrt{\frac{b}{a}}$$

15. $$\int \frac{dx}{a^2-b^2x^2} = \frac{1}{2ab}\log\frac{a+bx}{a-bx}$$

16. $$\int x^m(a+bx^n)^p dx$$

$$= \frac{x^{m-n+1}(a+bx^n)^{p+1}}{b(np+m+1)} - \frac{a(m-n+1)}{b(np+m+1)}\int x^{m-n}(a+bx^n)^p dx$$

或

$$\int x^m(a+bx^n)^p dx$$

$$= \frac{x^{m+1}(a+bx^n)^p}{np+m+1} + \frac{anp}{np+m+1}\int x^m(a+bx^n)^{p-1}dx$$

17. $$\int \frac{dx}{x^m(a+bx^n)^p} = -\frac{1}{a(m-1)x^{m-1}(a+bx^n)^{p-1}}$$

$$-\frac{(m-n+np-1)b}{(m-1)a}\int \frac{dx}{x^{m-n}(a+bx^n)^p}$$

或

$$\int \frac{dx}{x^m(a+bx^n)^p} = \frac{1}{an(p-1)x^{m-1}(a+bx^n)^{p-1}}$$

$$+\frac{m-n+np-1}{an(p-1)}\int \frac{dx}{x^m(a+bx^n)^{p-1}}$$

18. $$\int \frac{(a+bx^n)^p}{x^m}dx = -\frac{(a+bx^n)^{p+1}}{a(m-1)x^{m-1}}$$

$$-\frac{b(m-n-np-1)}{a(m-1)}\int \frac{(a+bx^n)^p}{x^{m-n}}dx$$

或

$$\int \frac{(a+bx^n)^p}{x^m}dx = \frac{(a+bx^n)^p}{(np-m+1)x^{m-1}}$$

$$+\frac{anp}{np-m+1}\int \frac{(a+bx^n)^{p-1}dx}{x^m}$$

19. $$\int \frac{x^m dx}{(a+bx^n)^p} = \frac{x^{m-n+1}}{b(m-np+1)(a+bx^n)^{p-1}}$$

$$-\frac{a(m-n+1)}{b(m-np+1)}\int\frac{x^{m-n}dx}{(a+bx^n)^p}$$

或

$$\int\frac{x^m dx}{(a+bx^n)^p}=\frac{x^{m+1}}{an(p-1)(a+bx^n)^{p-1}}$$

$$-\frac{m+n-np+1}{an(p-1)}\int\frac{x^m dx}{(a+bx^n)^{p-1}}$$

20. $$\int\frac{dx}{(a^2+x^2)^n}=\frac{1}{2(n-1)a^2}\left\{\frac{x}{(a^2+x^2)^{n-1}}\right.$$

$$\left.+(2n-3)\int\frac{dx}{(a^2+x^2)^{n-1}}\right\}$$

21. $$\int\frac{dx}{(a+bx^2)^n}=\frac{1}{2(n-1)a}\left\{\frac{x}{(a+bx^2)^{n-1}}\right.$$

$$\left.+(2n-3)\int\frac{dx}{(a+bx^2)^{n-1}}\right\}$$

22. $$\int\frac{xdx}{(a+bx^2)^n}=\frac{1}{2}\int\frac{dz}{(a+bz)^n}$$ 但 $z=x^2$

23. $$\int\frac{x^2dx}{(a+bx^2)^n}=\frac{-x}{2b(n-1)(a+bx^2)^{n-1}}$$

$$+\frac{1}{2b(n-1)}\int\frac{dx}{(a+bx^2)^{n-1}}$$

24. $\int \frac{dx}{x(a+bx^n)} = \frac{1}{an}\log\left(\frac{x^n}{a+bx^n}\right)$

25. $\int \frac{dx}{x^2(a+bx^2)^n} = \frac{1}{a}\int \frac{dx}{x^2(a+bx^2)^{n-1}} - \frac{b}{a}\int \frac{dx}{(a+bx^2)^n}$

26. $\int \frac{xdx}{a+bx^2} = \frac{1}{2b}\log\left(x^2+\frac{a}{b}\right)$

27. $\int \frac{x^2dx}{a+bx^2} = \frac{x}{b} - \frac{a}{b}\int \frac{dx}{a+bx^2}$

28. $\int \frac{dx}{x(a+bx^2)} = \frac{1}{2a}\log\frac{x^2}{a+bx^2}$

29. $\int \frac{dx}{x^2(a+bx^2)} = -\frac{1}{ax} - \frac{b}{a}\int \frac{dx}{a+bx^2}$

30. $\int \frac{dx}{(a+bx^2)^3} = \frac{x}{2a(a+bx^2)} + \frac{1}{2a}\int \frac{dx}{a+bx^2}$

III. 含 $\sqrt{a+bx}$ 者.

31. $\int \sqrt{a+bx}\,dx = \frac{2}{3b}(a+bx)^{3/2}$

32. $\int x\sqrt{a+bx}\,dx = -\frac{2(2a-3bx)(a+bx)^{3/2}}{15b^2}$

$\int x^m\sqrt{a+bx}\,dx = \frac{2x^m(a+bx)^{\frac{3}{2}}}{b(2m+3)} - \frac{2am}{b(2m+3)}\int x^{m-1}\sqrt{a+bx}\,dx.$

$\int \frac{x^m dx}{\sqrt{a+bx}} = \frac{2x^m\sqrt{a+bx}}{b(2m+1)} - \frac{2am}{b(2m+1)}\int \frac{x^{m-1}dx}{\sqrt{a+bx}}$

33. $\int x^3\sqrt{a+bx}\,dx=\dfrac{2(8a^3-12abx+15b^2x^2)(a+bx)^{3/2}}{105b^3}$

34. $\int\dfrac{dx}{\sqrt{a+bx}}=\dfrac{2}{b}\sqrt{(a+bx)}$

35. $\int\dfrac{x}{\sqrt{a+bx}}dx=-\dfrac{2(2a-bx)}{3b^2}\sqrt{a+bx}$

36. $\int\dfrac{x^2dx}{\sqrt{a+bx}}=\dfrac{2(8a^2-4abx+3b^2x^2)}{15b^3}\sqrt{a+bx}$

37. $\int\dfrac{dx}{x\sqrt{a+bx}}=\dfrac{1}{\sqrt{a}}\log\dfrac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}}$ 但 $a>0$

38. $\int\dfrac{dx}{x\sqrt{a+bx}}=\dfrac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\dfrac{a+bx}{-a}}$ 但 $a<0$

39. $\int\dfrac{dx}{x^2\sqrt{a+bx}}=\dfrac{-\sqrt{a+bx}}{ax}-\dfrac{b}{2a}\int\dfrac{dx}{x\sqrt{a+bx}}$

40. $\int\dfrac{\sqrt{a+bx}}{x}dx=2\sqrt{a+bx}+a\int\dfrac{dx}{x\sqrt{a+bx}}$

IV. 含 $\sqrt{x^2+a^2}$ 者

41. $\int\sqrt{x^2+a^2}\,dx=\dfrac{x}{2}\sqrt{x^2+a^2}+\dfrac{a^2}{2}\log(x+\sqrt{x^2+a^2})$

42. $\int x\sqrt{x^2+a^2}\,dx=\dfrac{1}{3}(x^2+a^2)^{3/2}$

43. $\int x^3\sqrt{x^2+a^2}\,dx = \frac{x}{8}(2x^2+a^2)\sqrt{x^2+a^2} - \frac{a^4}{8}\log(x+\sqrt{x^2+a^2})$

44. $\int x^3\sqrt{x^2+a^2}\,dx = \frac{1}{15}(3x^2-2a^2)(x^2+a^2)^{3/2}$

45. $\int \frac{dx}{\sqrt{x^2+a^2}} = \log(x+\sqrt{x^2+a^2})$

46. $\int \frac{x\,dx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2}$

47. $\int \frac{x^2\,dx}{\sqrt{x^2+a^2}} = \frac{x}{2}\sqrt{x^2+a^2} - \frac{a^2}{2}\log(x+\sqrt{x^2+a^2})$

48. $\int \frac{dx}{x\sqrt{x^2+a^2}} = \frac{1}{a}\log\frac{x}{a+\sqrt{x^2+a^2}}$

49. $\int \frac{dx}{x^2\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{a^2x}$

50. $\int \frac{dx}{x^3\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{2a^2x^2} + \frac{1}{2a^3}\log\frac{a+\sqrt{x^2+a^2}}{x}$

51. $\int \frac{\sqrt{x^2+a^2}}{x}dx = \sqrt{x^2+a^2} - a\log\frac{a+\sqrt{x^2+a^2}}{x}$

52. $\int \frac{\sqrt{x^2+a^2}}{x^2}dx = -\frac{\sqrt{x^2+a^2}}{x} + \log(x+\sqrt{x^2+a^2})$

53. $\int (x^2+a^2)^{3/2}dx = \frac{x}{8}(2x^2+5a^2)\sqrt{x^2+a^2}$

$$+\frac{3a^4}{8}\log(x+\sqrt{x^2+a^2})$$

54. $\int x\,(x^2+a^2)^{3/2}dx = \frac{1}{5}(x^2+a^2)^{5/2}$

55. $\int (x^2+a^2)^{\frac{n}{2}}dx = \frac{x(x^2+a^2)^{\frac{n}{2}}}{n+1} + \frac{na^2}{n+1}\int (x^2+a^2)^{\frac{n}{2}-1}dx$

56. $\int x\,(x^2+a^2)^{\frac{n}{2}}dx = \frac{1}{n+2}(x^2+a^2)^{\frac{n+2}{2}}$

57. $\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{(x^2+a^2)}}$

58. $\int \frac{xdx}{(x^2+a^2)^{3/2}} = \frac{-1}{\sqrt{x^2+a^2}}$

59. $\int \frac{x^2dx}{(x^2+a^2)^{3/2}} = \frac{-x}{\sqrt{x^2+a^2}} + \log(x+\sqrt{x^2+a^2})$

60. $\int \frac{x^3dx}{(x^2+a^2)^{3/2}} = \frac{x^2+2a^2}{\sqrt{x^2+a^2}}$

V. 含 $\sqrt{x^2-a^2}$ 者.

61. $\int \sqrt{(x^2-a^2)}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log(x+\sqrt{x^2-a^2})$

62. $$\int (x^2-a^2)^{3/2}dx = \frac{x}{8}(2x^2-5a^2)\sqrt{x^2-a^2} + \frac{3a^4}{8}\log(x+\sqrt{x^2-a^2})$$

63. $$\int (x^2-a^2)^{\frac{n}{2}}dx = \frac{n(x^2-a^2)^{\frac{n}{2}}}{n+1} - \frac{na^2}{n+1}\int (x^2+a^2)^{\frac{n}{2}-1}dx$$

64. $$\int x\sqrt{(x^2-a^2)}\,dx = \frac{1}{3}(x^2+a^2)^{3/2}$$

65. $$\int x^2\sqrt{(x^2-a^2)}dx = \frac{x}{8}(2x^2-a^2)\sqrt{x^2-a^2} - \frac{a^4}{8}\log(x+\sqrt{x^2-a^2})$$

66. $$\int x(x^2-a^2)^{\frac{n}{2}}dx = \frac{(x^2-a^2)^{\frac{n}{2}+1}}{n+2}$$

67. $$\int \frac{dx}{\sqrt{(x^2-a^2)}} = \log(x+\sqrt{x^2-a^2})$$

68. $$\int \frac{dx}{(x^2-a^2)^{3/2}} = -\frac{x}{a^2\sqrt{(x^2-a^2)}}$$

69. $$\int \frac{xdx}{\sqrt{x^2-a^2}} = \sqrt{x^2-a^2}$$

70. $$\int \frac{x^2dx}{\sqrt{(x^2-a^2)}} = \frac{x}{2}\sqrt{x^2-a^2} + \frac{a^2}{2}\log(x+\sqrt{x^2-a^2})$$

71. $\int \frac{x^2 dx}{(x^2-a^2)^{3/2}} = -\frac{x}{\sqrt{x^2-a^2}} + \log(x+\sqrt{x^2-a^2})$

72. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\cos^{-1}\frac{a}{x} = \frac{1}{a}\sec^{-1}\frac{x}{a}$

73. $\int \frac{dx}{x^3\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2 x}$

74. $\int \frac{dx}{x^3\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{2a^2x^2} + \frac{1}{2a^3}\sec^{-1}\frac{x}{a}$

75. $\int \frac{\sqrt{x^2-a^2}}{x}dx = \sqrt{x^2-a^2} - a\cos^{-1}\frac{a}{x}$

76. $\int \frac{\sqrt{x^2-a^2}}{x^2}dx = -\frac{\sqrt{x^2-a^2}}{x} + \log(x+\sqrt{x^2-a^2})$

VI. 含 $\sqrt{a^2-x^2}$ 者.

77. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$

78. $\int (a^2-x^2)^{3/2}dx = \frac{x}{8}(5a^2-2x^2)\sqrt{a^2-x^2}$

$$+\frac{3a^4}{8}\sin^{-1}\frac{x}{a}$$

79. $\int x^2\sqrt{(a^2-x^2)}dx = \frac{x}{a}(2x^2-a^2)\sqrt{a^2-x^2}+$

$$\int u^m(a^2-u^2)^{\frac{n}{2}} \quad a^2(m-1)u^{m-1}(a^2-u^2)^{\frac{n}{2}} \quad a^2(m-1)\int u^{m-2}(a^2-u^2)^{\frac{n}{2}}$$

$$= \frac{1}{a^2(n-2)u^{m-1}(a^2-u^2)^{\frac{n}{2}-1}} + \frac{m+n-3}{a^2(n-2)}\int \frac{du}{u^m(a^2-u^2)^{\frac{n}{2}-1}}$$

$$+ \frac{a^4}{8}\sin^{-1}\frac{x}{a}$$

80. $$\int (a^2-x^2)^{\frac{n}{2}}dx = \frac{x(a^2-x^2)^{\frac{n}{2}}}{n+1} + \frac{a^2n}{n+1}\int (a^2-x^2)^{\frac{n}{2}-1}dx$$

81. $$\int x(a^2-x^2)^{\frac{n}{2}}dx = -\frac{(a^2-x^2)^{\frac{n}{2}+1}}{n+2}$$

82. $$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$$

83. $$\int \frac{dx}{(a^2-x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2-x^2}}$$

84. $$\int \frac{xdx}{\sqrt{a^2-x^2}} = -\sqrt{a^2-x^2}$$

85. $$\int \frac{x^2dx}{\sqrt{a^2-x^2}} = -\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$$

86. $$\int \frac{x^2dx}{(a^2-x^2)^{3/2}} = \frac{x}{\sqrt{a^2-x^2}} - \sin^{-1}\frac{x}{a}$$

87. $$\int \frac{x^mdx}{(a^2-x^2)^{1/2}} = -\frac{x^{m-1}}{m}\sqrt{a^2-x^2}$$

$$+ \frac{(m-1)a^2}{m}\int \frac{x^{m-2}}{\sqrt{a^2-x^2}}dx$$

$$\int \frac{(a^2-u^2)^{\frac{n}{2}}du}{u^m} = -\frac{(a^2-u^2)^{\frac{n}{2}+1}du}{a^2(m-1)u^{m-1}} + \frac{m-n-3}{a^2(m-1)}\int \frac{(a^2-u^2)^{\frac{n}{2}}du}{u^{m-2}}$$

$$= \frac{(a^2-u^2)^{\frac{n}{2}}}{(m-n-1)u^{m-1}} + \frac{a^2n}{m-n-1}\int \frac{(a^2-u^2)^{\frac{n}{2}-1}du}{u^m}$$

$\int (a^2-u^2)^{\frac{n}{2}} \quad a^2(n-2)(a^2-u^2)^{\frac{n}{2}-1} \quad a^2(n-2)\int (a^2-u^2)^{\frac{n}{2}-1}$

$$= \frac{-u^{m-1}}{(m-n+1)(a^2-u^2)^{\frac{n}{2}-1}} + \frac{a^2(m-1)}{m-n+1}\int \frac{u^{m-2}\,du}{(a^2-u^2)^{\frac{n}{2}}}$$

88. $$\int \frac{dx}{x\sqrt{a^2-x^2}} = \frac{1}{a}\log \frac{x}{a+\sqrt{a^2-x^2}}$$

89. $$\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2x}$$

90. $$\int \frac{dx}{x^3\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{2a^2x^2} + \frac{1}{2a^3}\log \frac{x}{a+\sqrt{a^2-x^2}}$$

91. $$\int \frac{\sqrt{a^2-x^2}}{x}dx = \sqrt{a^2-x^2} - a\log \frac{a+\sqrt{a^2-x^2}}{x}$$

92. $$\int \frac{\sqrt{a^2-x^2}}{x^2}dx = -\frac{\sqrt{a^2-x^2}}{x} - \sin^{-1}\frac{x}{a}$$

VII. 含 $\sqrt{2ax-x^2}$, $\sqrt{2ax+x^2}$ 者.

93. $$\int \sqrt{2ax-x^2}\,dx = \frac{x-a}{2}\sqrt{2ax-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x-a}{a}$$

94. $$\int \frac{dx}{\sqrt{2ax-x^2}} = \cos^{-1}\frac{a-x}{a}$$

95. $$\int \frac{dx}{\sqrt{2ax+x^2}} = \log(x+a+\sqrt{2ax+x^2})$$

96. $$\int x^m\sqrt{2ax-x^2}\,dx = -\frac{x^{m-1}(2ax-x^2)^{3/2}}{m+2} +$$

$$+\frac{(2m+1)a}{m+2}\int x^{m-1}\sqrt{2ax-x^2}\,dx$$

97. $$\int \frac{x^m dx}{\sqrt{2ax-x^2}} = -\frac{x^{m-1}\sqrt{2ax-x^2}}{m} + \frac{(2m-1)a}{m}\int \frac{x^{m-1}dx}{\sqrt{2ax-x^2}}$$

98. $$\int \frac{dx}{x^m\sqrt{2ax-x^2}} = -\frac{\sqrt{2ax-x^2}}{(2m-1)ax^m} + \frac{m-1}{(2m-1)a}\int \frac{dx}{x^{m-1}\sqrt{2ax-x^2}}$$

99. $$\int \frac{\sqrt{2ax-x^2}}{x^m}dx = -\frac{(2ax-x^2)^{3/2}}{(2m-3)ax^m} + \frac{m-2}{a(2m-3)}\int \frac{\sqrt{2ax-x^2}}{x^{m-1}}dx$$

100. $$\int x\sqrt{2ax-x^2}\,dx = -\frac{3a^2+ax-2x^2}{6}\sqrt{2ax-x^2} + \frac{a^3}{2}\operatorname{vers}^{-1}\frac{x}{a}$$

101. $$\int \frac{dx}{x\sqrt{2ax-x^2}} = -\frac{\sqrt{2ax-x^2}}{ax}$$

102. $$\int \frac{xdx}{\sqrt{2ax-x^2}} = -\sqrt{2ax-x^2} + a\operatorname{vers}^{-1}\frac{x}{a}$$

$\int \frac{\sqrt{2au-u^2}}{u^m}du = -\frac{(2au-u^2)^{\frac{3}{2}}}{a(2m-3)u^m} + \frac{m-3}{a(2m-3)}\int \frac{\sqrt{2au-u^2}}{u^{m-1}}du$

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103. $$\int \frac{x^3dx}{\sqrt{2ax-x^2}} = -\frac{x+3a}{2}\sqrt{2ax-x^2} + \frac{3}{2}a^2 \operatorname{vers}^{-1}\frac{x}{a}$$

104. $$\int \frac{\sqrt{2ax-x^3}}{x}dx = \sqrt{2ax-x^2} + a \operatorname{vers}^{-1}\frac{x}{a}$$

105. $$\int \frac{\sqrt{2ax-x^3}}{x^2}dx = -\frac{2\sqrt{2ax-x^2}}{x} - \operatorname{vers}^{-1}\frac{x}{a}$$

106. $$\int \frac{\sqrt{2ax-x^2}}{x^3}dx = -\frac{(2ax-x^2)^{3/2}}{3ax^3}$$

107. $$\int \frac{dx}{(2ax-x^2)^{3/2}} = \frac{x-a}{a^2\sqrt{2ax-x^2}}$$

108. $$\int \frac{xdx}{(2ax-x^2)^{3/2}} = \frac{x}{a\sqrt{2ax-x^2}}$$

109. $$\int f(x, \sqrt{2ax-x^2})dx = \int f(z+a, \sqrt{a^2-z^2})\,dz$$

但 $z = x-a$

110. $$\int f(x, \sqrt{2ax+x^2})dx = \int f(z-a, \sqrt{z^2-a^2})\,dz$$

但 $z = x+a$

VIII. 含 $a+bx+cx^2$ 者.

略記爲 $X=a+bx+cx^3$, $D=b^2-4ac$.

111. $$\int\frac{dx}{a+bx+cx^2}=\frac{1}{\sqrt{D}}\log\frac{2cx+b-\sqrt{D}}{2cx+b+\sqrt{D}}$$ 但 $D>0$

112. $$\int\frac{dx}{a+bx+cx^2}=\frac{2}{\sqrt{-D}}\tan^{-1}\frac{2cx+b}{\sqrt{-D}}$$ 但 $D<0$

113. $$\int\frac{dx}{a+bx-cx^2}=\frac{1}{\sqrt{b^2+4ac}}\log\frac{\sqrt{b^2+4ac}+2cx-b}{\sqrt{b^2+4ac}-2cx+b}$$

114. $$\int\frac{dx}{(a+bx+cx^2)^2}=-\frac{2cx+b}{DX}-\frac{2c}{D}\int\frac{dx}{X}$$

115. $$\int\frac{dx}{(a+bx+cx^2)^3}=-\frac{2cx+b}{D}\left(\frac{1}{2X^2}-\frac{3c}{DX}\right)+\frac{6c^2}{D^2}\int\frac{dx}{X}$$

116. $$\int\frac{dx}{(a+bx+cx^2)^n}=-\frac{2cx+b}{D(n-1)X^{n-1}}+\frac{2(2n-3)c}{(-D)^{n-1}}\int\frac{dx}{X^{n-1}}$$

117. $$\int\frac{xdx}{a+bx+cx^3}=\frac{1}{2c}\log X-\frac{b}{2c}\int\frac{dx}{X}$$

118. $$\int \frac{x^2dx}{(a+bx+cx^2)} = \frac{x}{c} - \frac{b}{2c^2}\log X + \frac{b^2-2ac}{2c^2}\int \frac{dx}{X}$$

119. $$\int \frac{xdx}{(a+bx+cx^2)^3} = \frac{bx+2a}{DX} + \frac{b}{D}\int \frac{dx}{X}$$

120. $$\int \frac{x^3dx}{(a+bx+cx^2)^2} = -\frac{(b^2-2ac)x+ab}{cDX} - \frac{2a}{D}\int \frac{dx}{X}$$

121. $$\int \frac{xdx}{(a+bx+cx^2)^n} = \frac{2a+bx}{D(n-1)X^{n-1}} + \frac{b(2n-3)}{D(n-1)}\int \frac{dx}{X^{n-1}}$$

122. $$\int \frac{x^m dx}{(a+bx+cx^2)^n} = -\frac{x^{m-1}}{(2n-m-1)cX^{n-1}} - \frac{b(n-m)}{c(2n-m-1)}\int \frac{x^{m-1}dx}{X^n} + \frac{a(m-1)}{c(2n-m-1)}\int \frac{x^{m-2}}{X^n}dx$$

123. $$\int \frac{dx}{x(a+bx+cx^2)} = \frac{1}{2a}\log \frac{x^2}{X} - \frac{b}{2a}\int \frac{dx}{X}$$

124. $$\int \frac{dx}{x^2(a+bx+cx^2)} = \frac{b}{2a^2}\log \frac{X}{x^2} - \frac{1}{ax} + \frac{b^2-2ac}{2a^2}\int \frac{dx}{X}$$

125. $$\int (a+bx+cx^2)^n dx = \frac{1}{2c(2n+1)}\left\{(b+2cx)X^n - nD\int X^{n-1}ax\right\}$$

126. $$\int \frac{dx}{x^m(a+bx+cx^2)^n} = -\frac{1}{a(m-1)x^{m-1}X^{n-1}}$$

$$-\frac{a(m+n-2)}{b(m-1)}\int \frac{dx}{x^{m-1}X^n} - \frac{c(m+2n-3)}{a(m-1)}\int \frac{dx}{x^{m-2}X^n}$$

127. $$\int \frac{(px+q)dx}{(x^2+bx+a)^n} = \frac{p(b^2-4a)+(2q-pb)(2x+b)}{2(n-1)(4a-b^2)(x^2+bx+a)^{n-1}}$$

$$+\frac{(2q-pb)(2n-3)}{(n-1)(4a-b^2)}\int \frac{dx}{(x^2+bx+a)^{n-1}}$$

IX. 含 $\sqrt{a+bx\pm cx^2}$ 者，但 $c>0$.

128. $$\int \sqrt{a+bx+cx^2}\,dx = \frac{2cx+b}{4c}\sqrt{a+bx+cx^2}$$

$$-\frac{b^2-4ac}{8c^{3/2}}\log(2cx+b+2\sqrt{c}\,\sqrt{a+bx+cx^2})$$

129. $$\int \sqrt{a+bx-cx^2}\,dx = \frac{2cx-b}{4c}\sqrt{a+bx-cx^2}$$

$$+\frac{b^2+4ac}{8c^{3/2}}\sin^{-1}\frac{2cx-b}{\sqrt{b^2+4ac}}$$

130. $$\int \frac{dx}{\sqrt{a+bx+cx^2}} = \frac{1}{\sqrt{c}}\log(2cx+b+2\sqrt{c}\,\sqrt{a+bx+cx^2})$$

131. $$\int \frac{dx}{\sqrt{a+bx-cx^2}} = \frac{1}{\sqrt{c}}\sin^{-1}\frac{2cx-b}{\sqrt{b^2+4ac}}$$

132. $$\int \frac{xdx}{\sqrt{a+bx+cx^2}} = \frac{\sqrt{a+bx+cx^2}}{c} - \frac{b}{2c^{3/2}}\log(2cx+b+2\sqrt{c}\sqrt{a+bx+cx^2})$$

133 $$\int \frac{xdx}{\sqrt{a+bx-cx^2}} = -\frac{\sqrt{a+bx-cx^2}}{c} + \frac{b}{2c^{3/2}}\sin^{-1}\frac{2cx-b}{\sqrt{b^2+4ac}}$$

134. $$\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}} = 2\log(\sqrt{x-\alpha}+\sqrt{x-\beta})$$

135. $$\int \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}} = 2\sin^{-1}\sqrt{\frac{x-\alpha}{\beta-\alpha}}$$

X. 含其他代數式之積分.

136 $$\int \frac{dx}{(a+bx)(a'+b'x)} = \frac{1}{ab'-a'b}\log\left(\frac{a'+b'x}{a+bx}\right)$$

137. $$\int \frac{xdx}{(a+bx)(a'+b'x)} = \frac{1}{ab'-a'b}\left\{\frac{a}{b}\log(a+bx) - \frac{a'}{b'}\log(a'+b'x)\right\}$$

138. $$\int \frac{dx}{(a+bx)^2(a'+b'x)} = \frac{1}{ab'-a'b}\left\{\frac{1}{a+bx} +\right.$$

$$+\frac{b'}{ab'-a'b}\log\frac{a'+b'x}{a+bx}\Big\}$$

139. $\int\sqrt{\frac{a+x}{b+x}}dx=\sqrt{(a+x)(b+x)}$

$$+(a-b)\log(\sqrt{a+x}+\sqrt{b+x})$$

140. $\int\sqrt{\frac{a-x}{b+x}}dx=\sqrt{(a-x)(b+x)}+(a+b)\sin^{-1}\sqrt{\frac{x+b}{a+b}}$

141. $\int\sqrt{\frac{a+x}{b-x}}dx=-\sqrt{(a+x)(b-x)}-(a+b)\sin^{-1}\sqrt{\frac{b-x}{a+b}}$

XI. 含三角函數者.

142. $\int\sin x dx=-\cos x$

143. $\int\sin^2 x dx=\frac{x}{2}-\frac{1}{4}\sin 2x$

144. $\int\sin^3 x dx=-\frac{1}{3}\cos x(\sin^2 x+2)$

145. $\int\sin^4 x dx=-\frac{1}{4}\sin^3 x\cos x-\frac{3}{8}\sin x\cos x+\frac{3}{8}x$

146. $\int\sin^n x dx=-\frac{\sin^{n-1}x\cos x}{n}+\frac{n-1}{n}\int\sin^{n-2}x dx$

147. $\int \cos x dx = \sin x$

148. $\int \cos^2 x dx = \frac{x}{2} + \frac{1}{4}\sin 2x$

149. $\int \cos^3 x dx = \frac{1}{3}\sin x(\cos^2 x + 2)$

150. $\int \cos^4 x dx = \frac{1}{4}\sin x \cos^3 x + \frac{3}{8}\sin x \cos x + \frac{3}{8}x$

151. $\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x dx$

152. $\int \tan x dx = -\log \cos x$

153. $\int \tan^2 x dx = \tan x - x$

154. $\int \tan^3 x dx = \frac{\tan^2 x}{2} + \log \cos x$

155. $\int \tan^4 x dx = \frac{\tan^3 x}{3} - \tan x + x$

156. $\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$

157. $\int \cot x dx = \log \sin x$

158. $\int \cot^2 x\,dx = -\cot x - x$

159. $\int \cot^3 x\,dx = -\frac{\cot^2 x}{2} - \log \sin x$

160. $\int \cot^4 x\,dx = -\frac{\cot^3 x}{3} + \cot x + x$

161. $\int \cot^n x\,dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x\,dx$

162 $\int \sec x\,dx = \log(\sec x + \tan x)$

163. $\int \sec^2 x\,dx = \tan x$

164. $\int \sec^3 x\,dx = \frac{\sin x}{2\cos^2 x} + \frac{1}{2}\log(\tan x + \sec x)$

165. $\int \sec^4 x\,dx = \tan x + \frac{\tan^3 x}{3}$

166. $\int \sec^n x\,dx = \frac{\sin x}{(n-1)\cos^{n-1} x} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$

167. $\int \operatorname{cosec} x\,dx = \log(\operatorname{cosec} x - \cot x)$

168. $\int \operatorname{cosec}^2 x\,dx = -\cot x$

169. $\int \operatorname{cosec}^3 x\,dx = \frac{1}{2}\log \tan\frac{x}{2} - \frac{\cos x}{2\sin^2 x}$

170. $\int \operatorname{cosec}^4 x\,dx = -\cot x - \frac{\cot^3 x}{3}$

171. $\int \operatorname{cosec}^n x\,dx = -\frac{\cos x}{(n-1)\sin^{n-1} x} + \frac{n-2}{n-1}\int \operatorname{cosec}^{n-2} x\,dx$

172. $\int \sin x \cos x\,dx = \frac{1}{2}\sin^2 x$

173. $\int \sin x \cos^n x\,dx = -\frac{\cos^{n+1} x}{n+1}$

174. $\int \sin^n x \cos x\,dx = \frac{\sin^{n+1} x}{n+1}$

175. $\int \sin^m x \cos^n x\,dx = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n}\int \sin^m x \cos^{n-2} x\,dx$

或

$$\int \sin^m x \cos^n x\,dx = -\frac{\sin^{m-1} x \cos^{n+1} x}{m+n} + \frac{m-1}{m+n}\int \sin^{m-2} x \cos^n x\,dx$$

176. $$\int \frac{\sin^m x\,dx}{\cos^n x} = -\frac{1}{m-n}\,\frac{\sin^{m-1}x}{\cos^{n-1}x} + \frac{m-1}{m-n}\int \frac{\sin^{m-2}x}{\cos^n x}\,dx$$

或

$$\int \frac{\sin^m x\,dx}{\cos^n x} = \frac{1}{n-1}\,\frac{\sin^{m+1}x}{\cos^{n-1}x} - \frac{m-n+2}{n-1}\int \frac{\sin^m x\,dx}{\cos^{n-2}x}$$

177 $$\int \frac{\cos^n x\,dx}{\sin^m x} = -\frac{\cos^{n+1}x}{(m-1)\sin^{m-1}x} + \frac{m-n-2}{m-1}\int \frac{\cos^n x}{\sin^{m-2}x}\,dx$$

或

$$\int \frac{\cos^n x\,dx}{\sin^m x} = -\frac{\cos^{n-1}x}{(m-n)\sin^{m-1}x} - \frac{n-1}{m-1}\int \frac{\cos^{n-2}x\,dx}{\sin^m x}$$

178. $$\int \frac{dx}{\sin^m x\cos^n x} = \frac{1}{n-1}\,\frac{1}{\sin^{m-1}x\cos^{n-1}x} + \frac{m+n-2}{n-1}\int \frac{dx}{\sin^m x\cos^{n-2}x}$$

或

$$\int \frac{dx}{\sin^m x\cos^n x} = -\frac{1}{m-1}\,\frac{1}{\sin^{m-1}x\cos^{n-1}x} + \frac{m+n-2}{m-1}\int \frac{dx}{\sin^{m-2}x\cos^n x}$$

179. $\int \frac{dx}{\sin^m x} = -\frac{1}{m-1}\frac{\cos x}{\sin^{m-1} x} + \frac{m-2}{m-1}\int \frac{dx}{\sin^{m-2} x}$

180. $\int \frac{dx}{\cos^m x} = \frac{1}{m-1}\frac{\sin x}{\cos^{m-1} x} + \frac{m-2}{m-1}\int \frac{dx}{\cos^{m-2} x}$

181. $\int \sin mx \sin nx dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)}$

182. $\int \sin mx \cos nx dx = -\frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)}$

183. $\int \cos mx \cos nx dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)}$

184. $\int x \sin dx = \sin x - x \cos x$

185. $\int x^2 \sin x dx = 2x \sin x - (x^2-2)\cos x$

186. $\int x^m \sin x dx = -x^m \cos x + m\int x^{m-1}\cos x dx$

187. $\int x \cos x dx = \cos x + x \sin x$

188. $\int x^2 \cos x dx = 2x \cos x + (x^2-2)\sin x$

189. $\int x^m \cos x dx = x^m \sin x - m\int x^{m-1}\sin x dx$

190. $$\int \frac{dx}{\sin x+\cos x}=\frac{1}{\sqrt{2}}\log\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)$$

191. $$\int \frac{dx}{a+b\sin x}=\frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\frac{a\tan\frac{x}{2}+b}{\sqrt{a^2-b^2}}$$

但 $a>b$

或

$$\int \frac{dx}{a+b\sin x}=\frac{1}{\sqrt{b^2-a^2}}\log\frac{a\tan\frac{x}{2}+b-\sqrt{b^2-a^2}}{a\tan\frac{x}{2}+b+\sqrt{b^2-a^2}}$$

但 $a<b$

192. $$\int \frac{dx}{a+b\cos x}=\frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)$$

但 $a>b$

或

$$\int \frac{dx}{a+b\cos x}=\frac{1}{\sqrt{b^2-a^2}}\log\frac{\sqrt{b-a}\tan\frac{x}{2}+\sqrt{b+a}}{\sqrt{b+a}\tan\frac{x}{2}-\sqrt{b+a}}$$

但 $a<b$

193. $\int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} = \frac{1}{ab}\tan^{-1}\left(\frac{b\tan x}{a}\right)$

XII. 含反三角函數者.

194. $\int \sin^{-1}x\,dx = x\sin^{-1}x + \sqrt{1-x^2}$

195. $\int (\sin^{-1}x)^2 dx = x(\sin^{-1}x)^2 - 2x + 2\sqrt{1-x^2}\sin^{-1}x$

196. $\int x\sin^{-1}x\,dx = \frac{1}{4}\left\{(2x^2-1)\sin^{-1}x + x\sqrt{1-x^2}\right\}$

197 $\int x^n\sin^{-1}x\,dx = \frac{1}{n+1}\left\{x^{n+1}\sin^{-1}x - \int \frac{x^{n+1}dx}{\sqrt{1-x^2}}\right.$

198. $\int \cos^{-1}x\,dx = x\cos^{-1}x - \sqrt{1-x^2}$

199. $\int (\cos^{-1}x)^2 dx = x(\cos^{-1}x)^2 - 2x + 2\sqrt{1-x^2}\cos^{-1}x$

200. $\int x^n\cos^{-1}x\,dx = \frac{1}{n+1}\left\{x^{n+1}\cos^{-1}x + \int \frac{x^{n+1}dx}{\sqrt{1-x^2}}\right.$

201. $\int \tan^{-1}x\,dx = x\tan^{-1}x - \frac{1}{2}\log(1+x^2)$

202. $\int x\tan^{-1}x\,dx = \frac{1}{2}\left\{(x^2+1)\tan^{-1}x - x\right\}$

203. $\int x^n \tan^{-1} x dx = \frac{1}{n+1}\left\{x^{n+1}\tan^{-1}x - \int \frac{x^{n+1}dx}{1+x^2}\right.$

204. $\int \cot^{-1} x dx = x\cot^{-1}x + \frac{1}{2}\log(1+x^2)$

205. $\int x\cot^{-1} x dx = \frac{1}{2}\left\{(x^2+1)\cot^{-1}x + x\right\}$

206. $\int x^n \cot^{-1} x dx = \frac{1}{n+1}\left\{x^{n+1}\cot^{-1}x + \int \frac{x^{n+1}dx}{1+x^2}\right\}$

207. $\int \sec^{-1} x dx = x\sec^{-1}x - \log(x+\sqrt{1+x^2})$

208. $\int x\sec^{-1} x dx = \frac{1}{2}\left\{x^2\sec^{-1}x - \sqrt{x^2-1}\right\}$

209. $\int \operatorname{cosec}^{-1} x dx = x\operatorname{cosec}^{-1}x + \log(x+\sqrt{1+x^2})$

210. $\int x\operatorname{cosec}^{-1} x dx = \frac{1}{2}\left\{x^2\operatorname{cosec}^{-1}x + \sqrt{x^2-1}\right\}$

XIII. 含指數函數者.

211. $\int e^{ax} dx = \frac{1}{a}e^{ax}$

212. $\int a^{bx} dx = \frac{a^{bx}}{b\log a}$

213. $\int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$

214. $\int x^n e^{ax}dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}dx$

215. $\int x^n a^{bx}dx = \frac{x^n a^{bx}}{b\log a} - \frac{n}{b\log a}\int x^{n-1}a^{bx}dx$

216. $\int \frac{e^{ax}}{x^n}dx = \frac{1}{n-1}\left\{\frac{-e^{ax}}{x^{n-1}} + a\int \frac{e^{ax}dx}{x^{n-1}}\right\}$

217. $\int \frac{a^x dx}{x^n} = \frac{-a^x}{(n-1)x^{n-1}} + \frac{\log a}{n-1}\int \frac{a^x dx}{x^{n-1}}$

218. $\int e^{ax}\sin mx dx = \frac{e^{ax}(a\sin mx - m\cos mx)}{a^2+m^2}$

219. $\int e^{ax}\cos mx dx = \frac{e^{ax}(a\cos mx + m\sin mx)}{a^2+m^2}$

220. $\int e^{ax}\sin^m bx dx = \frac{a\sin bx - mb\cos bx}{a^2+m^2b^2}e^{ax}\sin^{m-1}bx + \frac{m(m-1)b^2}{a^2+m^2b^2}\int e^{ax}\sin^{m-2}bx dx$

221. $\int e^{ax}\cos^m bx dx = \frac{a\cos bx + mb\sin b}{a^2+m^2b^2}e^{ax}\cos^{m-1}bx + \frac{m(m-1)b^2}{a^2+m^2b^2}\int e^{ax}\cos^{m-2}bx dx.$

XIV. 含對數函數者.

222. $\int \log x dx = x \log x - x$

223. $\int x^n \log x dx = x^{n+1} \left\{ \frac{\log x}{n+1} - \frac{1}{(n+1)^2} \right\}$

224. $\int (\log x)^n dx = x(\log x)^n - n \int (\log x)^{n-1} dx$

225. $\int e^{ax} \log x dx = \frac{1}{a} e^{ax} \log x - \frac{1}{a} \int \frac{e^{ax}}{x} dx$

226. $\int \frac{dx}{x \log x} = \log(\log x)$

227. $\int \frac{dx}{(\log x)^n} = -\frac{x}{(n-1)(\log x)^{n-1}} + \frac{1}{n-1} \int \frac{dx}{(\log x)^{n-1}}$

228. $\int \frac{x^m dx}{(\log x)^n} = -\frac{x^{m+1}}{(n-1)(\log x)^{n-1}} + \frac{m+1}{n-1} \int \frac{x^m dx}{(\log x)^{n-1}}$

$$\sin 2x = 2\sin x\cos x, \qquad \sin 3x = 3\sin x - 4\sin^3 x$$

$$\cos 2x = \cos^2 x - \sin^2 x, \quad \cdots \quad \cos 3x = 4\cos^3 x - 3\cos x$$

$$\sin^2 x = \frac{1-\cos^2 x}{2} \qquad \cos^2 x = \frac{1+\cos^2 x}{2}$$

$$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$$

$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$$

$$\sin x \pm \sin y = 2\sin\frac{x \pm y}{2}\cos\frac{x \mp y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\sin h\ x = \frac{1}{2}(e^x - e^{-x}), \qquad \cos h\ x = \frac{1}{2}(e^x + e^{-x})$$

設 $N(x) = (x-\alpha)^a(x-\beta)^b \cdots$

$$(x^2+px+q)^m(x^2+rx+s)^n \cdots$$

則

$$\frac{M(x)}{N(x)} = P(x) + \sum_{h=1}^{a}\frac{A_h}{(x-\alpha)^h} + \sum_{k=1}^{b}\frac{B_k}{(x-\beta)^k} + \cdots$$

$$+ \sum_{i=1}^{m}\frac{P_iX + Q_i}{(x^2+px+q)^i}$$

$$+ \sum_{j=1}^{n}\frac{R_jX + S_j}{(x^2+rx+s)^j} + \cdots$$

$$(1+x)^n = 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{r}x^r$$

$$+ \cdots \quad (|x| < 1)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \quad (-\infty < x < +\infty)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \quad (\quad ,, \quad)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \quad (\quad ,, \quad)$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \quad (-1 < x \leq 1)$$

51.5.11			
51.11.4			
51.12.28			

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