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ESTIMATION OF EXPECTED CASUALTIES
USING ALIVENESS ADJUSTMENTS

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Abstract

In military operations research, it is often desired to estimate the expected casualties that would accrue to each side in a battle between opposing forces. One way to obtain credible estimates is to use field tests in which battles with engagements between battle units (such as tanks, armored personnel carriers and ground-to-ground missile systems) are simulated. One common feature of such simulated battles is the use of "real time casualty assessment" to determine the outcome of each engagement. Real time casualty assessment uses pre-set probabilities of kill, or "Pk" values; a Bernoulli trial with a Pk appropriate for the conditions of the engagement determines whether the battle unit fired upon is killed and thus removed from further play in the battle. For various reasons, it may be desired to estimate the expected numbers of battle units of given types that would be killed for Pk values different from those used in the experiment. This can be accomplished, using adjustments to the estimates obtained for the original experiment. Such estimators can be based on the computed "aliveness" of surviving battle units. We discuss two formulations of the aliveness concept, and compare the resulting estimators.

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1. Introduction

The performance characteristics of battle units or "platforms," such as tanks and armored personnel carriers, are of great interest in military operations research. The Department of Defense requires that major new platform designs must be evaluated in a series of test and evaluation experiments. The early experiments, involving "development tests and evaluations," are essentially engineering feasibility tests. As the design of a weapon system becomes mature, through early testing and consequent improvements, it is prudent to test the system or platform in more realistic "operational test and evaluation" experiments. These experiments are designed to provide estimates of the performance characteristics of the platform in a variety of situations, including various combat scenarios.

Data relating to the performance of a platform may come from a variety of sources, including war games, battle models, training exercises and field experiments. We wish to discuss the latter, and in particular how field experiments generally are run to simulate battles. An operational field experiment is often carried out in a series of "force-on-force" trials. In each trial, opposing forces engage in a simulated battle, usually using free play within a general scenario. Conditions of the trials (such as force sizes and scenarios) are varied in accordance with the experimental design.

A central feature of the typical operational evaluation field experiment is the use of real time casualty assessment (RTCA). Using laser, microwave, radio and radar instrumentation linked to a central computer, the locations of individual units and platforms are monitored, as well as data concerning platform parameters such as movement, ammunition remaining and intervisibility with other platforms. When a platform "engages" a target (i.e., detects, identifies, aims and fires one imaginary round or a burst of imaginary rounds at the target), a laser on the firing platform sends a coded message along the aim path of the weapon. Laser sensors on potential target platforms continually "listen" for such messages; when one is received, the identities of the firer (contained in the laser code of the firer) and target are transmitted to the central computer. The central computer contains a function, loaded before the battle begins, giving the probability a target is "killed" in an engagement, as a function of engagement conditions such as firer weapon and ammunition characteristics, relative motion of the firer and target, aspect and exposure of the target relative to the firer and firer-target range. Typically this function is implemented through a table of "Pk" (for "probability of kill") values.

The RTCA system is designed to simulate casualties during the battle. When an engagement occurs in a trial, the central computer carries out a Bernoulli trial to determine whether to judge the target killed by the firer. This is done by calling a random number generator and comparing the number returned with the Pk value appropriate for the engagement. If the Uniform(0,1) outcome is less than Pk, the target is declared killed, and the target is informed to cease activities. In some cases, a killed platform simulates its status by issuing cues such as smoke. Since the outcomes of engagements are computed and relayed in near real time (usually within a second), the process is called "real time casualty assessment." Realistic simulation of casualties is important, as casualties shape the battle, forcing the individual combatants ("players") and tactical commanders to react in realistic ways during the battle [ref. 1].

The force-on-force portion of the evaluation of the Sergeant York air defense system provides a good example of this type of experimentation, and we shall draw on the example in what follows. The SGT York experiment (called a "follow on evaluation" because other operational tests had preceded it) was conducted at Fort Hunter Liggett, CA during the period 2 April to 22 May, 1985. The experiment was designed to facilitate comparisons of the relative abilities of three air defense systems to provide air protection to an armored task force in several scenarios. The main mission performance criterion for the comparison was the proportion of losses of the Blue force, which possessed the air defense systems, to engagements by Red air platforms. As is well known, the results of the experiment led Secretary of Defense Weinberger to cancel the SGT York program. It may not be so well known, however, that the analysis of the SGT York test data was complicated by the fact that some of the Pk's used for RTCA were found to be incorrect, after the experiment had been completed. One of the most common forms of discrepancy in the Pk's was one in which engagements occurring in a trial were not transmitted to, or evaluated by, the central computer. Engagements that did not go to real time assessment due to instrumentation or computer errors were discovered through post test analyses of video and audio recordings made during the trials. One can model such nonassessment errors in terms of incorrect Pk's; the RTCA used Pk=0 for engagements it did not consider, when the actual Pk's were positive. It was estimated that forty to fifty percent of certain types of engagements in the SGT York experiment did not go to real time assessment [ref. 4].

We are concerned with the problem of how to "adjust" estimates of expected casualties, such as Blue losses in the SGT York experiment, to account for post-test changes in Pk values. It is clear that the use of incorrect Pk's in the RTCA process can have a profound effect on the battles in the experiment and thus on the estimates of expected casualties based on data from the experiment. For example, if the Pk used (PKU) in RTCA for a given engagement is less than the actual Pk (PKA), the target platform will tend to survive too long, possibly giving it a chance of inflicting subsequent casualties that should not have occurred. Similarly, if PKU > PKA for a given type of engagement, the target tends to be killed too soon and consequently is denied chances to fire on its opponents as often as it should have. Thus, errors in the Pk's used in RTCA have cascading, interactive effects on the battle. Estimators of expected casualties which do not account for such effects are likely to provide poor estimates.

2. The Problem

A force-on-force field experiment generates opportunities for engagements in accordance with a stochastic process. This process is sampled by the players through the opportunities that are taken, giving a point process of engagements. The RTCA system further samples this engagement process to determine the occurrences of kills that result from engagements. Because players and tactical commanders react to engagements and kills, characteristics of the process of engagement opportunities change through time in response to outcomes on the engagement and kill processes.

During a trial, the sample record of the engagement opportunity process is observed, up to a truncation point determined by the end of the battle. This point may depend on the outcome on the kill process, as well as battle time. For example, a trial in an experiment might terminate after two hours or when 60 percent of either force has been killed. The engagement process also depends on the kill process; a would-be firer does not engage targets after it has been declared killed, for example.

The problem we wish to consider is: given the portions of the sample records of the engagement and killed processes observed in the trials of the experiment using PKU values, estimate the expected casualties that would accrue on each side with PKA values.

3. A Proposed Solution

The concept of "aliveness" has been under development for several years, principally with the work of Marion Bryson at the U.S. Army's Combat Development and Experimentation Center (CDEC) and Carl Russell at the Army's Operational Test and Evaluation Agency (OTEA) [ref. 3]. Generally, the type of problems addressed by aliveness are those that can be stated in terms of post-test changes in the values of kill probabilities used in the RTCA process. Russell describes aliveness as "an arithmetic adjustment for cumulative differences between PKA's and PKU's which is applied [in estimating expected casualties]."

Bryson and Russell have suggested several improvements in their initial aliveness adjustment algorithms, and have apparently settled on a version which appears to work well in practice. To date, the justification for this algorithm appears to be essentially intuitive in nature, although there is increasing empirical evidence that it provides useful results. Generally, there appears to be widespread skepticism about the method, within the Army test and evaluation community. This may be due in part to the lack of published developments of a firm theoretical foundation for the method.

The Bryson-Russell (B-R) model can be described in terms of updates to a matrix K and a vector A, after each engagement. Suppose the potential firers in a force-on-force experiment are associated with rows of the matrix K and potential targets are associated with its columns. Entries of the matrix K represent the accumulated amounts of kill credited the firers against the targets. Thus, at a given point in the battle, the i,j<u>th</u> entry of K, say k_{ij}, is the cumulative amount of kill of target j credited to firer i up to that point in the battle. The k_{ij}'s can be any non-negative real numbers, and represent extensions of the "sum of Pk's" method of estimating expected casualties [ref. 3]. This method estimates the mean number of casualties inflicted by a firer by the sum of Pk's over all engagements by this firer. (We return to a discussion of this and related estimators below.)

The vector A has entries which are "aliveness" values; the j<u>th</u> component of A at a given point in the battle, say a_j, is the aliveness of the j<u>th</u> platform at that point in the battle. The aliveness values are used in computing K. The initial values, at the beginning of the battle, of K and A are 0 and 1, respectively. Suppose that at a certain point in the battle, platform i engages target j and the outcome is adjudicated by RTCA using PKU. Suppose, however, that RTCA should have used PKA for this engagement. Then the j<u>th</u> component of A and the ij<u>th</u> element K are updated as follows:

$$k_{ij}(\text{revised}) = k_{ij} + a_j(1 - (1 - \text{PKA})^{a_i})$$

$$a_j(\text{revised}) = \begin{cases} a_j * (1 - \text{PKA})^{a_i} / (1-\text{PKU}), & \text{if the target survived in RTCA} \\ 0, & \text{otherwise} \end{cases} \qquad (1)$$

At the end of the battle, the expected casualties inflicted by the i<u>th</u> platform against the j<u>th</u> target is estimated by k_{ij}. The expected number of casualties inflicted by the i<u>th</u> platform against all opposing platforms is estimated by $\sum_{j \epsilon J} k_{ij}$, where J is the index set of platforms opposing platform i. Similarly, the expected number of platforms of a given type that are killed by opposing platforms of a certain type would be estimated by

$$\sum_{i \epsilon I} \sum_{j \epsilon J} k_{ij}$$

where I and J are index sets corresponding to the types of firer and target platforms under consideration.

Russell [ref. 3] gives the following justification for this algorithm:

* It reduces to sums of Pk when RTCA uses the correct Pk's (i.e., when PKU = PKA throughout the battle),
* It adjusts in the correct direction when PKA $\neq$ PKU and $a_i = 1$,
* It seems to perform well in practice.

He admits, however, that the aliveness methodology proved very hard to sell for use in the SGT York analyses. Reasons cited for this resistance include:

* The method seems "too complicated",
* Aliveness values greater than 1 are possible,
* Credited numbers of kills greater than the total number of starting platforms are possible,
* The sum of credited kills and expected number of survivors rarely equals the total starting force.

The statistical characteristics of the B-R aliveness adjustment algorithm have recently been investigated by an NPS student, U.S. Army Major Ted Janosko, as part of his Master's thesis research [ref. 2]. Janosko's work provides an expository account of the use of the B-R model in a variety of realistic situations, and gives an intuitive motivation for the use of this approach. Janosko investigated the bias and variance of B-R aliveness adjusted estimates of expected battle casualties. This investigation was accomplished through simulation of two-sided battles between relatively small, homogeneous forces. Janosko concluded that the method provides remarkably good casualty estimates, with variance characteristics somewhat better than those given by body count estimators. However, the B-R based estimator does exhibit some bias, especially under certain conditions of degree of adjustment required in the Pk's, and the method of simulating the selection of targets by each firer.

The B-R model appears to generate incorrect values of expected casualties in some very simple test cases. Consider, for example*, a "battle" with two platforms, B1 and B2 on the Blue side and one platform, R1, on the Red side. In what follows, let us index K and A with the platforms in the order B1, B2, R1. Suppose B1 fires on R1 with probability PKA of killing R1, but suppose RTCA uses PKU to adjudicate the engagement. If R1 is killed the battle terminates; if R1 survives this engagement, suppose he fires on B2 with probability of kill PK, whereupon the battle terminates. In the field experiment, the expected number of Red casualties to be observed is PKU and the expected number of Blue to be observed is (1 - PKU) PK. However, for an actual battle, these means are PKA and (1 - PKA) PK, respectively.

Now let us consider how the B-R model adjusts the observed data to obtain estimates of the latter (true) expected casualties. R1 starts the battle with aliveness 1, and after

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* A similar example was suggested independently by Tukey [ref. 5].

being engaged by B1 has aliveness 0, if he is killed, or (1 - PKA) / (1 - PKU) if he survives. B1 is credited with PKA kills of R1, so after the first engagement the matrix K has all zero elements except for the value PKA in the (1,3) position. Now, if R1 survives this engagement, he engages B2 and is credited with kills equal to B2's aliveness (1) times

$$\left(1 - (1 - PK)^{(1-PKA)/(1-PKU)}\right),$$

the exponent representing the aliveness of the firer. Thus, at the end of the battle, we have

$$A = \begin{cases} (1,\ 1,\ (1-PKA)/(1-PKU)), & \text{with probability } (1-PKU)(1-PK) \\ (1,\ 1,\ 0), & \text{with probability } PKU \\ (1,\ 0,\ (1-PKA)/(1-PKU)), & \text{with probability } (1-PKU)PK, \end{cases}$$

and

$$K = \begin{cases} \begin{pmatrix} 0 & 0 & PKA \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \text{with probability } PKU \\ \begin{pmatrix} 0 & 0 & PKA \\ 0 & 0 & 0 \\ 0 & 1-(1-PK)^{(1-PKA)/(1-PKU)} & 0 \end{pmatrix}, & \text{with probability } (1-PKU). \end{cases}$$

The estimate k_{13} of expected Red casualties, PKA, is correct in either case. However, the expected outcome on the estimator k_{32} of the mean number of Blue casualties is

$$(1\text{-PKU})(1-(1\text{-PK})^{(1\text{-PKA})/(1\text{-PKU})}) \ .$$

Obviously, this can be quite different from the true value, (1 - PKA) PK. (There are also conditions where these are close. For example, the values are approximately the same when PKA $\approx$ PKU, or when PK is small and PKA is much smaller than PKU.)

In the next section, we shall discuss conditions under which the B-R algorithm can be derived, and motivate a second model which is a revision of the B-R algorithm. These two models are compared in the section following.

4. A Motivating Example; Sums of PK's Types of Estimators

Consider a one-sided "duel", as follows: A single firer shoots at a single target, until the target is killed or k shots have been fired, whichever is first. Suppose the probability the target is killed on shot n is PKA(n); n=1, 2, ..., k. The expected number of casualties is

$$\text{PKA}(1) + (1\text{-PKA}(1))\text{PKA}(2) + \ldots + \prod_{n=1}^{k-1} (1\text{-PKA}(n))\text{PKA}(k) \tag{2}$$

$$= 1 - \prod_{n=1}^{k} (1-PKA(n)) = c, \text{ say} .$$

Suppose one did not know this value and decided to estimate it with the sums of PK estimator, $\hat{C} = \sum_{n=1}^{N} PKA(n)$, where N is the (random) number of shots fired in the duel. This estimator has mass function as follows:

$\hat{c}$	$p(\hat{c})$
PKA(1)	PKA(1)
PKA(1)+PKA(2)	(1-PKA(1))PKA(2)
.	.
.	.
.	.
$\sum_{n=1}^{k-1} PKA(n)$	$\prod_{n=1}^{k-2} (1-PKA(n))PKA(k-1)$
$\sum_{n=1}^{k} PKA(n)$	$\prod_{n=1}^{k-1} (1-PKA(n)\ PKA(k) + \prod_{n=1}^{k} (1-PKA(n))$

Thus, the expected value of the estimator $\hat{C}$ is

$$E(\hat{C}) = PKA(1).PKA(1) + [PKA(1)+PKA(2)]\ (1-PKA(1))PKA(2)+\ldots$$

$$+ \left[\sum_{n=1}^{k} PKA(n)\right] \left[\prod_{n=1}^{k-1} (1-PKA(n))PKA(k) + \prod_{n=1}^{k} (1-PKA(n))\right] .$$

Factoring out first PKA(1), then PKA(2), etc., this sum can be written precisely in the form of eq. (2), so it follows that the sums of PKA estimator $\hat{C}$ is unbiased for estimating the mean number c of casualties in the duel.

Now suppose the duel is terminated by a sequence of Bernoulli "side experiments" with probabilities PKU(n);n = 1, 2, ... k, of stopping after the n<u>th</u> shot. The sums of PKA estimator is generally biased for this situation:

$$E(\hat{C}) = E\left(\sum_{n=1}^{N} PKA(n)\right) = PKA(1)+PKA(2)(1-PKU(1)) +\ldots+ PKA(k) \prod_{n=1}^{k-1} (1-PKU(n))$$
$$\neq 1- \prod_{n=1}^{k} (1-PKA(n)) = c \ . \qquad (3)$$

However, one could modify the sums of PKA estimator by weighing each term of the sum in eq. (3) by a ratio of the form

$$\Pi(1-PKA)/\Pi(1-PKU)$$

which would in effect "convert" the $\Pi(1 - PKU)$ terms to $\Pi(1 - PKA)$ terms. More precisely, consider the weighted sums of PK estimator

$$\tilde{C} = \sum_{n=1}^{k} PKA(n) \ . \prod_{m=1}^{n-1} \left[\frac{1-PKA(m)}{1-PKU(m)} \right] .$$

The expected value of $\tilde{C}$, written in a form similar to (3), is seen to be c. (Of course, the weights were contrived to make this true.)

This formally suggests adjusting PKA at each observed engagement by a factor of the form $\Pi(1 - PKA)/\Pi(1 - PKU)$ when RTCA uses PKU's in determining outcomes of duels and a sums of PK type of estimator is used. Indeed, the analogy with force-on-force battles simulated with RTCA may be close, since a battle can be considered to consist of a sequence of such small "duels". We will motivate the use of these weight factors, using a different approach, in the next section.

5. Development

For convenience of notation, let us temporarily index the values in K and A by a serial count of the engagement number. Consider an engagement of firer i against target j at the beginning of the battle, when i's aliveness has its initial value, $a_i(1) = 1$, and suppose the actual kill probability in this first engagement is PKA(1), so j would actually survive the engagement with probability 1 - PKA(1). Suppose RTCA adjudicates the engagement using kill probability PKU(1). From the point of view of the simulated battle, the target will have no further value if RTCA judges it to be killed in the engagement; let us imagine it to have "value" $a_j(1)$ (as yet undetermined) if it is judged to survive the engagement. We wish to interpret these "values" in terms of platform equivalents, so the expected number of survivors of the engagement simulated by RTCA is

$$0.PKU(1) + a_j(1)(1-PKU(1)) .$$

Now it is desired to reconstruct an expected number of survivors in the simulated engagement equal to the number that would have been expected in an actual engagement, by assigning an appropriate survival value, $a_j(1)$. This simply amounts to setting

$$0 \bullet PKU(1) + a_j(1) \bullet (1-PKU(1)) = 1-PKA(1) ,$$

so

$$a_j(1) = (1-PKA(1))/(1-PKU(1)) .$$

Given the platform j survives the first engagement, suppose it is engaged again by a firer with aliveness $a_{i'}(2) = 1$, with actual kill probability PKA(2), and with RTCA using PKU(2). Again, we wish to define a "value", measured in units equivalent to a hypothetical number of survivors, $a_j(2)$, such that the expected number of survivors (or survivor equivalents) in the RTCA adjudicated engagement equals the expected number of survivors (or survivor equivalents) that would be observed in an actual battle. This gives

$$0 \bullet PKU(2) + a_j(2)(1-PKU(2)) = 0 \bullet PKA(2) + a_j(1)(1-PKA(2)) ,$$

so

$$a_j(2) = a_j(1) \bullet (1-PKA(2))/(1-PKU(2))$$

$$= \frac{1-PKA(1)}{1-PKU(1)} \bullet \frac{1-PKA(2)}{1-PKU(2)} ,$$

given the target is judged (by RTCA) as having survived the engagement. In a similar way, if a target with "value" equivalent to $a_j(n-1)$ platforms (we henceforth call such a value the "aliveness" of platform j) is engaged by platform i'' having $a_i''(n-1) = 1$, then the aliveness of j after the $n \geq 1$st engagement is , by equating expected survivors with RTCA and actual battle conditions,

$$a_j(n) = \begin{cases} a_j(n-1) \bullet \frac{1-PKA(n)}{1-PKU(n)} , \text{ if the target survives RTCA} \\ 0, \text{ otherwise} \end{cases} \quad (4)$$

where we have defined $a_j(0) = 1$, as an initial condition. Note the upper expression is well defined, since $PKU(n) = 1$ implies the lower expression almost surely holds.

So far, we have considered only attacks by platforms having aliveness 1. The aliveness of the firer can be incorporated in various ways. Let us first consider a point of view that leads to the B-R model. We have asserted that the aliveness of a platform is a measure of its expected surviving strength, in units of platforms. Thus, a platform with aliveness α at a given point in the battle is viewed as if it were "worth" α platforms as a target. Suppose a firing platform with aliveness α is viewed as if it were "worth" α platforms all firing independently at the same target, with identical PK's. Equivalently, one could imagine the attacker firing a salvo of α rounds at its target. Then the probability the target survives the engagement is $(1 - PKA)^{\alpha}$. If such values accounting for the aliveness $a_i(n-1)$ of the firer are used in place of (1 - PKA) in the arguments leading up to eqs. (4), the B-R expression for aliveness in eqs. (1) result.

Now consider the kill to be credited to a firer with the B-R formulation. The amount of kill credited to a firer in an engagement is the reduction in the expected number of survivors (i.e., expected aliveness) as a result of the engagement. For the n $\geq$ 1st engagement, with some platform i firing on j, the expected aliveness surviving is, as described above,

$$a_j(n-1)(1-PKA(n))^{a_i(n-1)} ,$$

so the reduction in expected aliveness with this engagement is

$$a_j(n-1) - a_j(n-1)(1-PKA(n))^{a_i(n-1)}$$
$$= a_j(n-1)(1-(1-PKA(n))^{a_i(n-1)}) ,$$

as shown in eqs. (1). The amount of kill of platform j accumulated by the opposing side through k engagements of j is thus

$$\sum_{n=1}^{k} a_j(n-1)\left(1-(1-PKA(n))^{a_i(n-1)}\right)$$

$$= \sum_{n=1}^{k} [a_j(n-1)-a_j(n)\bullet(1-PKU(n))]$$

by (1), so

$$\sum_{i\epsilon I} k_{ij} = \sum_{n=1}^{k} a_j(n).PKU(n) + (1-a_j(n)) \tag{5}$$

where I is the index set of platforms firing on j through the k<u>th</u> engagement of j. We see, then, that the estimated expected number of kills of j, accumulated through n engagements, is essentially a weighted sums of Pk estimate, where the weights are aliveness values of the <u>target</u>. (Note the firer aliveness values do not appear explicitly in eq. (5). If the a_j's are all 1, eq. (5) reduces to a sums of Pk estimator.)

A second point of view of the effects of the aliveness α of a firer can be derived in terms of engagement rates that would be expected in a battle, had RTCA used PKA's instead of PKU's. Let E denote the event that i engages j in the n<u>th</u> engagement of the battle. Now

$$P(E|i\&j \text{ both not killed and PKA's are used}) =$$

$$P(E|i\&j \text{ both not killed and PKU's used}) \tag{6}$$

because occurrence of E does not depend on which PK's are used, so long as neither i nor j has been killed in the preceding n-1 engagements. Thus, rewriting the conditional probabilities,

$$P(E \mid i\&j \text{ both not killed and PKA's used}) = \frac{P(E \mid \text{PKA's used})}{P(i\&j \text{ both not killed} \mid \text{PKA's used})}$$

since $E \subset$ [i and j both not killed]. A similar relation holds for the other expression in (6), so

$$P(E \mid \text{PKA's used}) = P(E \mid \text{PKU's used}) \cdot \frac{P[i\&j \text{ not both killed} \mid \text{PKA's used}]}{P[i\&j \text{ not both killed} \mid \text{PKU's used}]} \quad (7)$$

The second factor on the right hand side of eq. (7) is the quotient

$$\frac{P[i\&j \text{ survive attacks against them in the first n-1 engagements of the battle} \mid \text{PKA's used}]}{P[i\&j \text{ survive attacks against them in the first n-1 engagements of the battle} \mid \text{PKU's used}]}$$

$$= \frac{1-PKA(\tau_1)}{1-PKU(\tau_1)} \bullet \frac{1-PKA(\tau_2)}{1-PKU(\tau_2)} \bullet\bullet\bullet \frac{1-PKA(\tau_m)}{1-PKU(\tau_m)},$$

where $\tau_1, \tau_2, \ldots, \tau_m$ are the engagements at which i or j were attacked. If we define aliveness by eq. (4), the latter expression is just $a_i(n-1)a_j(n-1)$. With this definition, eq. (7) is just

$$P[E|PKA\text{'s used}] = a_i a_j \; P[E|PKU\text{'s used}] , \qquad (8)$$

which implies that the engagement process with PKA's should average $a_i a_j$ as many engagements as when it uses PKU's. Thus, to "reconstruct" the experiment for PKA's, we should treat each engagement in the RTCA governed battle as $a_i a_j$ engagements. This can be interpreted as a_i "copies" of an engagement against a target with "value" equivalent to a_j platforms, using the interpretation of a_j as before. With this interpretation, the aliveness a_i of the firer is the number of copies of the engagement the firer is credited with, each time it fires.

Let us now construct updating algorithms for K and A, similar to eqs. (1), for this approach. Eq. (4) is used to compute the remaining aliveness of each target after it is engaged. (Note this equation does not involve the aliveness of the firer.) The kills credited the firer in one copy of the n<u>th</u> engagement is the reduction in the expected number of survivors,

$$a_j(n-1) - a_j(n-1)(1-PKA(n)) = a_j(n-1) \cdot PKA(n) .$$

Thus the expected loss in $a_i(n-1)$ copies of the engagement is $a_i(n-1)a_j(n-1)PKA(n)$. As a result of the n<u>th</u> engagement, K is updated by

$$k_{ij}(n) = k_{ij}(n-1) + a_i(n-1) \; a_j(n-1)PKA(n) . \qquad (9)$$

As for the B-R model, the amount of kill of platform j accumulated by the opposing side through n engagements is found by summing (9):

$$\sum_{i \in I} k_{ij} = \Sigma \, \Sigma \, a_i a_j \text{ PKA}$$

so the estimate is a weighted sums of P's, this time in terms of the PKA's.

In summary, the main difference between the B-R model and the alternate model (which Bryson and Russell also considered at one time and which we shall call the "Survival Ratio (S-R) Model" because the aliveness given in eq. (4) is of the form of a ratio of survival probabilities) can be stated in terms of how the aliveness of the firer is interpreted. For the B-R model, it is assumed that

* the firer gets a burst of a_i rounds at a group of a_j targets.

By "a group of targets", we mean that a round "kills" all a_j targets or it kills none of them -- consistent with a_j's interpretation as a "value" of a single target. Bryson and Russell interpret a_i as the "potency" of firer i. The S-R model, defined by eqs. (4) and (9), results with any one of the following assumptions:

* the firer gets a_i copies of an engagement with one round fired at a group of a_j targets;
* each of a_i firers shoots independently at a group of a_j

targets;

* the firer gets a_i copies of an engagement with one round fired at each of a_j independent targets; and
* each of a_i firers shoots independently at each of a_j independent targets.

The equivalence of the expected amount of kills in engagements under the four conditions is seen as follows. We have already argued that the first interpretation gives $a_i a_j$ PKA expected kills. The second would provide a_i Bernoulli (PKA) trials, each with prize a_j, that is, Na_j kills, where $N \sim b(a_i, PKA)$. The expected kills would be $E(N)a_j = a_i PKA a_j$, as in the first case. The third interpretation gives a_i times the expected kills in a $b(a_j, PKA)$ experiment, so again the expected kills in the engagement is $a_i[a_j PKA]$. Finally, the fourth interpretation gives total kills which is a sum of a_i binomial(a_j,PKA) outcomes; the expected value is $a_i[a_j PKA]$.

6. A Comparison of the B-R and S-R Estimators

A comparison of expected casualty estimates with the B-R and S-R models was undertaken, with an approach similar to that used by Janosko [ref. 2]. A simple computer program for simulating force-on-force battles was adapted by Janosco from Russell to accommodate variations in the method of simulating choice of targets by firers. Four target selection methods were considered:

1. choose a target at random from among the surviving opponents (this was Russell's method);

2. fire at the most "potent" surviving target (defined in terms of B-R aliveness);

3. fire at the most killable surviving target (highest firer PKU); and

4. fire at the most dangerous surviving target (target with the highest PKU against the firer).

A total of 360 battles, between a Blue defending force with 4 platforms and a Red attacking force with 12 platforms, were

Table 1. PK's used in the simulations.

	Blue				Red			
PK pair	PKA	PKU	E(C)	V(C)	PKA	PKU	E(C)	V(C)
1	.5	.5	3.37	1.03	.2	.2	8.43	11.89
2	.25	.5	3.37	1.03	.2	.2	8.43	11.89
3	.5	.25	3.93	0.13	.2	.2	4.91	9.58
4	.5	.5	3.37	1.03	.1	.2	8.43	11.89
5	.5	.5	2.20	1.75	.2	.1	11.02	4.78
6	.5	.5	3.37	1.03	.3	.2	8.43	11.89
7	.5	.5	3.79	0.38	.2	.3	6.32	11.85
8	.75	.5	3.37	1.03	.2	.2	8.43	11.89
9	.5	.75	2.72	1.63	.2	.2	10.19	7.98

simulated. The battles terminated when all platforms on one side or the other were killed. The sides took turns firing at their adversaries. Nine sets of PK values were used, as shown in Table 1.

The means and variances shown in Table 1 were computed with a random walk on the plane model, with absorbing regions corresponding to cases where all platforms on either side were killed. The means and variances calculated from simulated data vary from these values slightly more than would be expected from their theoretical standard errors, because the simulation routine used a "jitter" factor on the input PK's (see ref. 2). This caused the PK's used in the simulations to vary, from platform to platform on a given side, around the nominal PK values shown in Table 1. The theoretical means and variances, and those estimated from the simulation results from the set of 360 trials, are shown in Table 2. The level of agreement between the theoretical and simulated values tends to verify that the simulation software contains no serious bugs.

In what follows, we compare the B-R and S-R and simulation estimates of expected casualties for the Red and Blue sides, under 36 conditions. These conditions correspond to the cells in a (9 PK pairs) X (4 target selection methods) design matrix; 10 replications of battles were simulated under each condition. The B-R and S-R estimates incorporate adjustments related to changes from the PKU's to the PKA's, shown in Table 1. The simulation estimates use the "correct" PKA's, and thus constitute "ground truth"; the simulation estimates are average frequencies of casualties, over the replications with each PKA set. Thus, the simulation estimates are correct, to within sampling error.

Analyses of variance (AOV's) were conducted on the errors of the S-R estimates relative to the simulation estimates, and on the difference between B-R and S-R estimates, for the Blue and Red forces, using target selection method and PK pair as factors. Janosko [2] performed an extensive analysis for the values obtained with the B-R method, and errors of the B-R estimates relative to the simulation estimates. As was found by Janosko for his comparisons, generally both of the main effects were highly significant in the AOV's for our comparisons. This

Table 2. Means and standard errors of the mean from random walk calculations and simulation.

	Overall		By PK Pairs		By TGT SEL METH	
	Blue	Red	Blue	Red	Blue	Red
E(C)(Random Walk Theory)	3.28	8.28				
Sample Mean (Simulation)	3.37	8.12				
Std error of $\bar{x}$ (Random Walk Theory)	.053	.167				
Sample std error (Simulation)	.050	.173				
Standard error for BR-SR			.082	.39	.055	.26
Standard error for BR-SIM			.15	.39	.23	.58
Standard error for SR-SIM			.15	.38	.23	.57
Sample size for means	360		40		90	

suggests that accuracy of the aliveness adjustments generally depends on the amount of adjustment (differences between PKU's and PKA's), as well as other battle factors such as the selection of targets by gunners. Simultaneous multiple range tests for the differences between B-R and S-R casualty estimates, for each of the factors in the AOV, are shown in Tables 3 and 4. Similar results for the difference between the B-R and simulation estimates are given in Tables 5 and 6, and the differences between S-R and simulation estimates are summarized similarly in Tables 7 and 8.

While there are significant statistical differences in the estimates generated by the B-R and S-R methods, the values exhibit general agreement, as demonstrated by the plots of B-R vs. S-R estimates shown in Figures 1 and 2. One very large value for estimated Red casualties (48.9) was obtained with the B-R method; this generates a discrepant point in Figure 1 and shows up in other figures and summary table values. The slope of the scatter of points in Figures 1 and 2 is nearly 1, although there is some indication the B-R method may tend to give larger estimates than does the S-R method. This is borne out by the plots shown in Figures 3 and 4, which show the differences in B-R and S-R estimates, plotted for each of the 36 sets of conditions, labeled "cells" in these figures. (The cell code is PK pair + 10 * (target selection method).)

Finally, comparison of B-R and S-R estimates with simulation "ground truth" are summarized for each PK pair and target

Table 3. Subsets of conditions for which B-R and S-R Blue casualty estimates are similar.

Multiple range analysis for (B-R) - (S-R) by PKPAIR

Method: 90 Percent Scheffe

Level	Count	Average	Homogeneous Groups
2	40	-.3330000	+
[illegible]	[illegible]	[illegible]	+
4	40	-.1202500	+
[illegible]	40	.0370000	++
1	40	.2547500	++
[illegible]	40	.3175000	+
5	40	.3615000	+
3	40	.3645000	+
3	40	.7717500	+

Multiple range analysis for (BR) - (S-R) by TGTSELMETH

Method: 90 Percent Scheffe

Level	Count	Average	Homogeneous Groups
3	90	-.1143333	+
1	90	.1522222	+
4	90	.1774444	+
2	90	.4156669	+

Table 4. Subsets of conditions for which B-R and S-R Red casualty estimates are similar.

Multiple range analysis for (B-R) - (S-R) by PKPAIR

Method: 95 Percent Scheffe

Level	Count	Average	Homogeneous Groups
[illegible]	[illegible]	[illegible]	*
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	**
[illegible]	[illegible]	[illegible]	***
[illegible]	40	[illegible]	***
[illegible]	[illegible]	[illegible]	***
[illegible]	[illegible]	[illegible]	**
[illegible]	[illegible]	[illegible]	*
[illegible]	40	[illegible]	*

Multiple range analysis for (B-R) - (S-R) by TGTSELMETH

Method: 95 Percent Scheffe

Level	Count	Average	Homogeneous Groups
[illegible]	[illegible]	[illegible]	*
[illegible]	[illegible]	[illegible]	*
[illegible]	[illegible]	[illegible]	*
[illegible]	[illegible]	[illegible]	*

Table 5. Subsets of conditions for which B-R and simulation Blue casualty estimates are similar.

Multiple range analysis for (B-R) - SIM by TGTSELMETH

Method: [illegible] Percent Scheffe

Level	Count	Average	Homogeneous Groups
2	80	-.2140000	*
[illegible]	[illegible]	[illegible]	**
[illegible]	[illegible]	[illegible]	*
[illegible]	[illegible]	[illegible]	[illegible]

Multiple range analysis for (B-R) - SIM by PKPAIR

Method: [illegible] Percent Scheffe

Level	Count	Average	Homogeneous Groups
[illegible]	[illegible]	[illegible]	*
[illegible]	[illegible]	[illegible]	**
6	40	[illegible]	**
[illegible]	[illegible]	[illegible]	**
[illegible]	40	[illegible]	**
[illegible]	[illegible]	[illegible]	**
[illegible]	40	[illegible]	**
[illegible]	[illegible]	[illegible]	*
[illegible]	[illegible]	[illegible]	*

Table 6. Subsets of conditions for which B-R and simulation Red casualty estimates are similar.

Multiple range analysis for (B-R) - SIM by TGTSELMETH

Method: 90 Percent Scheffe

Level	Count	Average	Homogeneous Groups
4	90	-.3090000	*
[illegible]	[illegible]	[illegible]	*
[illegible]	[illegible]	-.2880000	*
[illegible]	[illegible]	[illegible]	[illegible]

Multiple range analysis for (B-R) - SIM by PKPAIR

Method: 90 Percent Scheffe

Level	Count	Average	Homogeneous Groups
[illegible]	[illegible]	[illegible]	*
[illegible]	[illegible]	[illegible]	**
2	40	-.3470000	**
[illegible]	[illegible]	[illegible]	*
3	40	-.5625000	*
[illegible]	40	[illegible]	*
[illegible]	40	[illegible]	**
[illegible]	[illegible]	[illegible]	**
4	[illegible]	[illegible]	*

Table 7. Subsets of conditions for which S-R and simulation Blue casualty estimates are similar.

Multiple range analysis for (S-R) - SIM by TGTSELMETH

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

Multiple range analysis for (S-R) - SIM by PKPAIR

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

Table 8. Subsets of conditions for which S-R and simulation Red casualty estimates are similar.

Multiple range analysis for (S-R) - SIM by TGTSELMETH

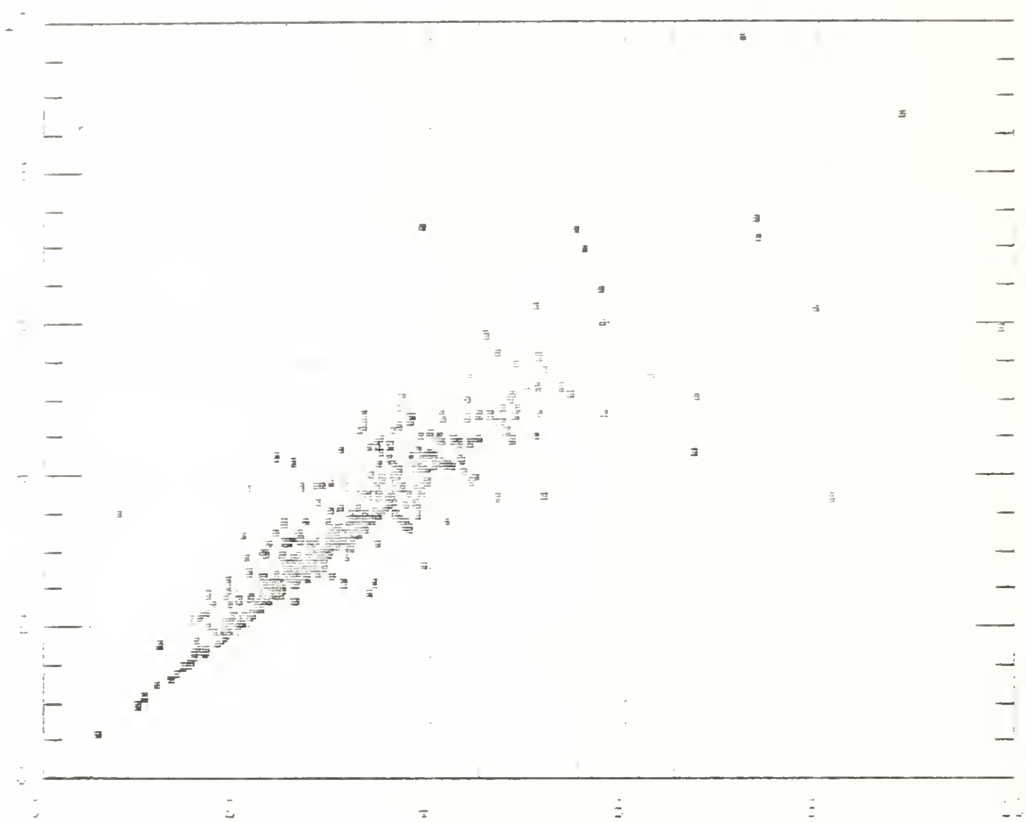
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

Multiple range analysis for (S-R) - SIM by PKPAIR

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

Figure 1. Plot of B-R estimates versus S-R estimates of Blue casualties.

B-R



S-R

Figure 2. Plot of B-R versus S-R Red casualty estimates.

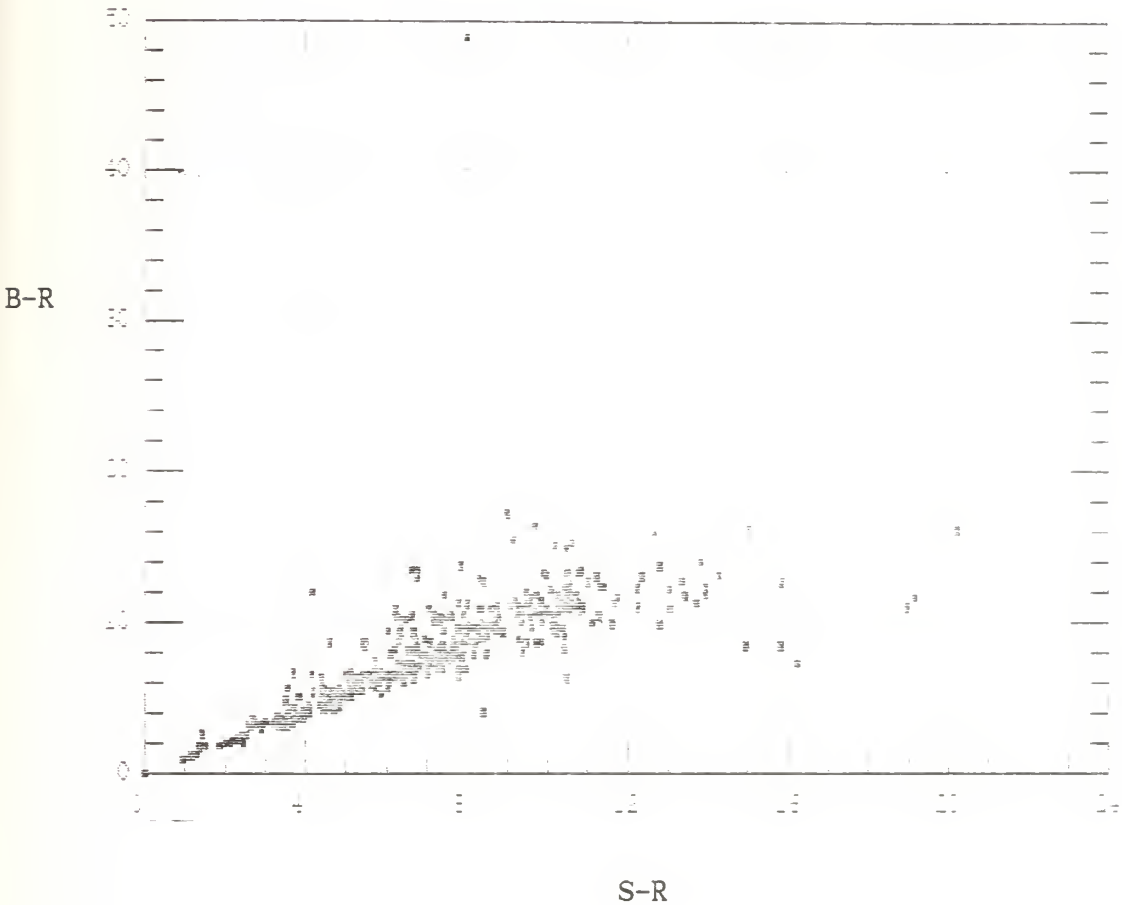


Figure 3. Summary of B-R and S-R differences, for estimated Blue casualties.

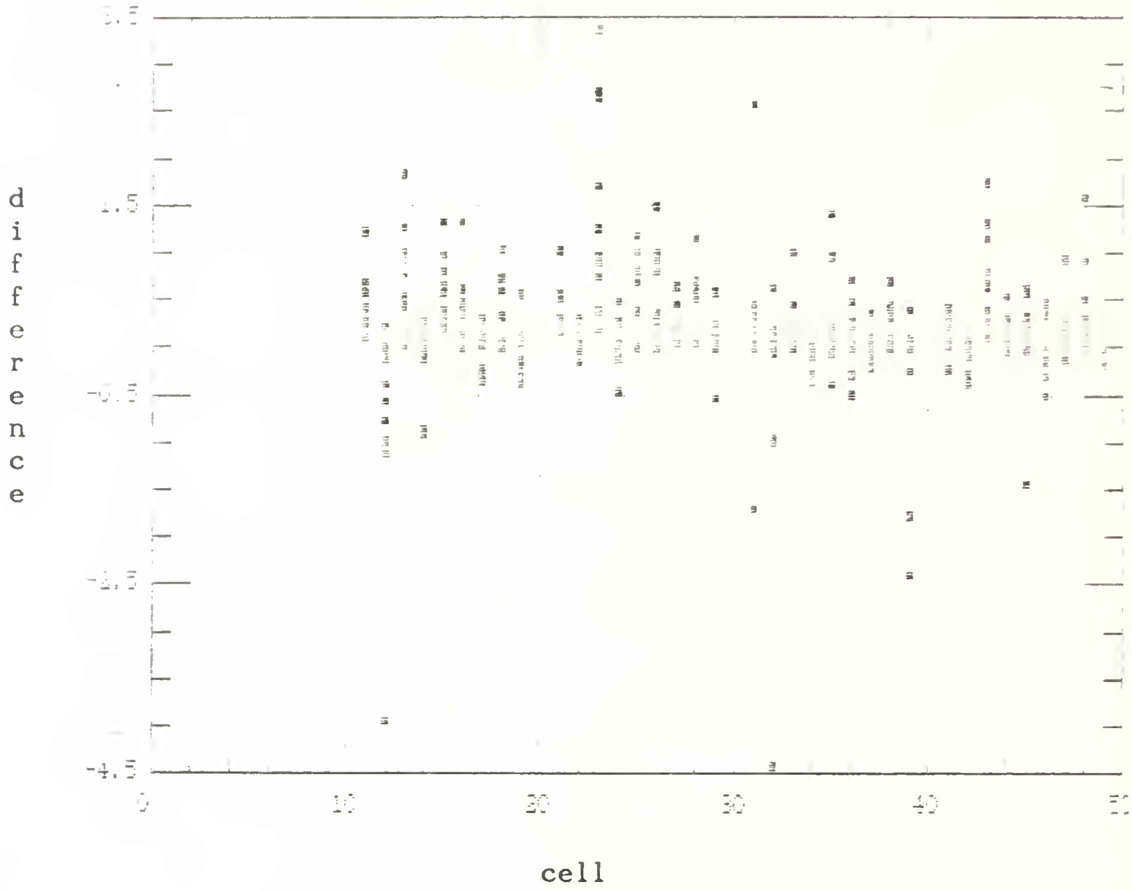


Figure 4. Summary of B-R and S-R differences, for estimated Red casualties.

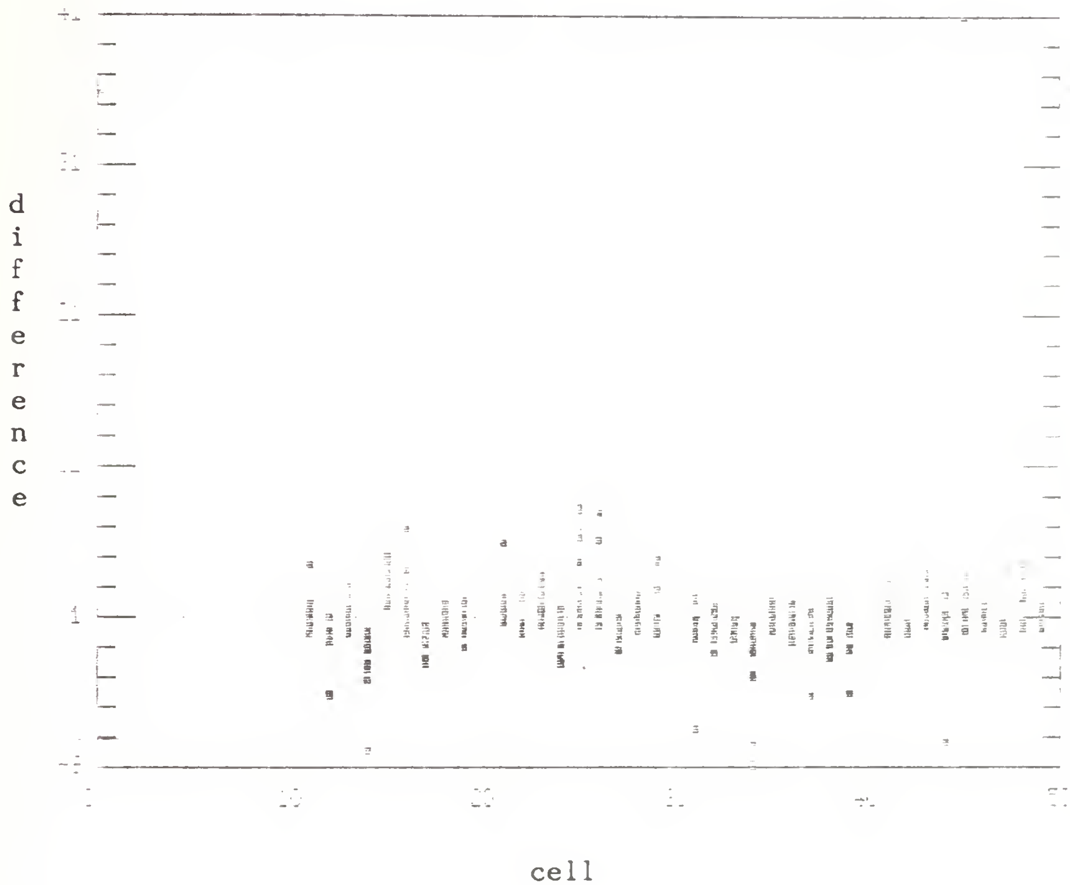


Table 9. Comparisons of three estimates of expected Blue casualties, by target selection method and PK Pair.

Comparisons by target selection methods

T1:	S-R<Sim<B-R	.041 .122
T2:	Sim<B-R<S-R	.214 .436
T3:	B-R<S-R<Sim	.015 .021
T4:	Sim<S-R<B-R	.259 .177

Comparisons by PK Pair

P1:	Sim<S-R<B-R	.050 .265
P2:	S-R<Sim<B-R	.36 .41
P3:	B-R<Sim<S-R	.122 .241
P4:	S-R<B-R<Sim	.362 .005
P5:	Sim<B-R<S-R	.294 .02
P6:	S-R<B-R<Sim	.09 .07
P7:	S-R<B-R<Sim	.317 .328
P8:	Sim<B-R<S-R	.016 .070
P9:	S-R<Sim<B-R	.159 .205

Table 10. Comparisons of three estimates of expected Red casualties, by target selection method and PK pair.

Comparisons by target selection method

T1:	S-R<Sim<B-R<Sim	.74 .29
T2:	S-R<Sim<B-R	.36 1.12
T3:	B-R<S-R<Sim	.15 .45
T4:	S-R<B-R<Sim	1.01 .91

Comparisons by PK Pairs

P1:	S-R<B-R<Sim	.99 .33
P2:	S-R<B-R<Sim	1.23 .85
P3:	S-R<B-R<Sim	.12 .56
P4:	S-R<Sim<B-R	.004 2.57
P5:	B-R<S-R<Sim	1.20 1.13
P6:	B-R<S-R<Sim	.12 1.04
P7:	S-R<Sim<B-R	.19 1.48
P8:	S-R<Sim<B-R	.15 .36
P9:	S-R<B-R<Sim	1.17 .68

selection method in Tables 9 and 10. Orderings and values of differences between averages in each cell are shown in these tables. The numerical value under each inequality symbol in Tables 9 and 10 is the difference between the respective mean estimates. For example, in Table 9, for target selection method 1, the entry is

$$\begin{array}{ccccc} \text{S-R} & < & \text{Sim} & < & \text{B-R} \\ & .041 & & .122 & \end{array}$$

This indicates that the average (over 90 battles) of the S-R adjusted Blue casualty estimate was .041 casualties below the average simulation ("true") estimate, which was in turn .122 casualties below the average B-R estimate.

7. Conclusions and Recommendations

The summaries shown above, in particular Tables 9 and 10, show there is not a consistent relationship in size between the B-R and S-R estimators, for the cases studied. Nor is either estimator consistently "better" in terms of bias and accuracy (closeness to the simulation estimates). Indeed, it appears that both estimators are performing surprisingly well, in general.

Perhaps choice between these competitors should be made on the basis of the sets of assumptions, discussed in Section 5, that lead to the two respective adjustment techniques. The assumptions supporting the S-R model seem somewhat more plausible than those leading to the B-R model, from the author's point of view. Since this is an issue of considerable importance, it might be worthwhile to examine the relative accuracy of these estimators using simulation runs with a high resolution combat model. This could be accomplished by making some runs with PKU's and other runs with PKA's. The aliveness adjustments could be applied to the runs using PKU's, to adjust to the PKA case. The results could then be compared with the PKA results, which represent "ground truth". (The same set of runs could be used for a second comparison, by reversing the roles of the PK's. That is, apply aliveness adjustments to adjust the casualties in PKA runs to the PKU conditions, and compare with observed PKU casualties (or sums of PK estimates).)

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