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自修適用

# 漢譯溫氏高中三角法題解

附對數表



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## 序 言

溫德華氏係美國著名數學家，任中學教員有年。故其所編小代數，幾何，及三角三書適合中學校採用。其譯本前後經商務印書館出版，已風行國內。其小代數題解，前歲已由予演出。其幾何題解譯本已由商務印書館出版。惟其三角題解，雖經永定鄭輔維君譯出，但僅出一版。近來坊間索購仍不乏人。而予之學生及諸友好，多促予再譯。予有鑒于此，因于近一月中譯出，以供學者之參考。匆忙之中不無謬誤之處，祈閱者有以教我，幸甚。

譯者謹識于舊都，

九，一，二三。

## 凡 例

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書中所譯名詞，悉以商務印書館出版之顧裕魁君三角譯本爲宗。

溫氏原解內有缺圖處，今一一補之，以便參考

是書在一閱月譯成難免無謬誤之處尙祈海內名士指教是幸

譯者謹識于舊都

九,一,二三。

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# 溫氏高中平面三角法題解

## 例題 1

[設  $\pi = 3.1416$ ]

1. 化下列諸角爲真弧度。而以  $\pi$  之分數式表示其結果：

$60^\circ, 45^\circ, 150^\circ, 195^\circ, 11^\circ 15', 123^\circ 45', 37^\circ 30'$ .

$$60^\circ = \frac{60}{180}\pi = \frac{1}{3}\pi$$

$$45^\circ = \frac{45}{180}\pi = \frac{1}{4}\pi$$

$$150^\circ = \frac{150}{180}\pi = \frac{5}{6}\pi$$

$$195^\circ = \frac{195}{180}\pi = \frac{13}{12}\pi$$

$11^\circ 15' = \frac{11\frac{1}{4}}{180}\pi = \frac{1}{16}\pi$

$123^\circ 45' = \frac{123\frac{3}{4}}{180}\pi = \frac{11}{16}\pi$

$37^\circ 30' = \frac{37\frac{1}{2}}{180}\pi = \frac{5}{24}\pi$

於  $\frac{2}{3}\pi, \frac{3}{4}\pi, \frac{5}{8}\pi, \frac{15}{16}\pi, \frac{7}{15}\pi$

本位弧內，各有若干度。

$$\frac{2}{3}\pi \text{ 本位弧} = 180^\circ \times \frac{2}{3} = 120^\circ$$

$$\frac{3}{4}\pi \text{ 本位弧} = 180^\circ \times \frac{3}{4} = 135^\circ$$

$$\frac{5}{8}\pi \text{ 本位弧} = 180^\circ \times \frac{5}{8} = 112^\circ$$

$13'$

$$\frac{15}{16}\pi \text{ 本位弧} = 180^\circ \times \frac{15}{16} =$$

$168^\circ 45'$

$$\frac{7}{15}\pi \text{ 本位弧} = 180^\circ \times \frac{7}{15} = 84^\circ$$

3. 改  $1^\circ$  爲本位弧 其小數部分

若干。又  $1'$  若何。

$$1^\circ = \frac{\pi}{180} \text{ 本位弧} = \frac{3.1416}{180}$$

$$= 0.017453 \text{ 本位弧}$$

$$1' = \frac{0.017453}{60} \text{ 本位弧}$$

$$= 0.0002909 \text{ 本位弧}$$

4. 一個本位弧內有若干秒。

$$\begin{aligned} 1 \text{ 本位弧} &= 57^{\circ} 17' 45'' \\ &= 206,265'' \end{aligned}$$

5. 試以本位弧表示八角形及十二角形之一內角

因八角形諸內角之總和為

$$\begin{aligned} 8 \times 180^{\circ} - 360^{\circ} &= 8\pi - 2\pi \\ &= 6\pi \end{aligned}$$

故其每一內角 =  $\frac{6}{8}$  本位弧

$$= \frac{3}{4} \pi \text{ 本位弧}$$

因十二角形諸內角之和為  $12 \times 180^{\circ} - 360^{\circ} = 12\pi - 2\pi = 10\pi$

故其每一內角 =  $\frac{10}{12} \pi$  本位

$$\text{弧} = \frac{5}{6} \pi \text{ 本位弧}$$

6. 於 50 呎半徑之圓周上。截取 10 尺之弧。問此弧所對之圓心角若干度。

此弧乘  $\frac{10}{50}$  本位弧 =  $\frac{1}{5}$  本位弧

$$= \frac{57^{\circ} 17' 45''}{5} = 11^{\circ} 27' 33''$$

7. 地球赤道半徑約 3963 哩。設於赤道上有相距 1000 哩之兩

點。問經度之差。

$$\text{其經度之差} = \frac{1000}{3963} \text{ 本位弧}$$

$$\text{或} \frac{1000}{3963} \times 57^{\circ} 17' 45'' = 14^{\circ}$$

$$27' 28''$$

8. 赤道上兩點之徑度差為  $1^{\circ}$  問兩點間相距若干哩。

按第三題  $1^{\circ} = 0.017453$  本位弧

故在赤道上之一度即等於

$$0.017453 \times 3963 \text{ 哩} = 69.166$$

哩

9. 設  $1^{\circ}$  之圓心角所乘之弧為一呎。求圓半徑。

因弧之一度等於一呎

故 1 本位弧 (即  $57^{\circ} 17' 45''$ )

$$= 57 \frac{57^{\frac{3}{4}}}{60} \text{ 尺} = 57 \text{ 呎 } 3.55 \text{ 吋}$$

= 半徑

10 地球之旋轉而移赤道上。一點。由其周上經過與地球半徑相等之距離。問須幾小時。

因二十四小時內地球自轉  $360^{\circ}$ ,

即  $2\pi$  本位弧

故 自轉一本位弧需時  $\frac{34}{2\pi} =$

$$\frac{12}{3.1416} = 3 \text{ 小時 } 49 \text{ 分 } 11$$

秒。

II. 設時鐘分針之長為  $3\frac{1}{2}$  呎問其極端於 25 分鐘內所行之長 (令  $\pi = \frac{22}{7}$ 。下題仿此)。

因 六十分時所經之圓周 =

$$2\pi \times 3\frac{1}{2} \text{ 呎}$$

故 二十五分時所經之弧

$$= \frac{25}{60} \times 2\pi \times 3\frac{1}{2} \text{ 呎}$$

$$= \frac{45}{60} \times 2 \times \frac{22}{7} \times \frac{7}{2} \text{ 呎,}$$

$$= 9 \frac{1}{6} \text{ 呎} = 9 \text{ 呎 } 2 \text{ 吋.}$$

12. 一輪每秒鐘旋轉 15 次, 問旋過 4 本位弧須時幾何

因 此輪於  $\frac{1}{15}$  秒時內轉  $2\pi$  本位弧

故 轉 4 本位弧需時  $4 \left( \frac{1}{15} \text{ 秒} \right)$

$$\div 2\pi) \text{ 秒, 即 } \frac{7}{165} \text{ 秒.}$$

例 題 II

I. ABC 三角形 (2 圖) 內, 餘一銳角 B 之各函數為何.

$$\sin B = \frac{b}{c}, \quad \cos B = \frac{a}{c}.$$

$$\tan B = \frac{b}{a}, \quad \sec B = \frac{c}{a}.$$

$$\cot B = \frac{a}{b}, \quad \csc B = \frac{c}{b}.$$

試比較 A 及 B 之各函數而示其

$$\sin A = \cos B, \quad \sec A = \csc B.$$

$$\cos A = \sin B, \quad \csc A = \sec B,$$

$$\tan A = \cot B, \quad \text{vers } A =$$

overcs B,

$$\cot A = \tan B, \quad \text{covers } A =$$

vers B

$$\sin A = \frac{a}{c}, \quad \cos B = \frac{a}{c}.$$

$$\cos A = \frac{b}{c}, \quad \sin B = \frac{b}{c}.$$

$$\tan A = \frac{a}{b}, \quad \cot B = \frac{a}{b}.$$

$$\cot A = \frac{b}{a}, \quad \tan B = \frac{b}{a}.$$

$$\sec A = \frac{c}{b}, \quad \csc B = \frac{c}{b}.$$

$$\csc A = \frac{c}{a}, \quad \sec B = \frac{c}{a}.$$

$$\text{vers } A = \frac{c-b}{c}, \text{ covers } B.$$

$$= \frac{c-b}{c}.$$

$$\text{covers } A = \frac{c-a}{c}, \text{ vers } B$$

$$= \frac{c-a}{c}.$$

3. 設  $a, b, c$  有下之各值。試求  $A$  之各函數值。

(i) 3, 4, 5. (iv) 9, 40, 41.

(ii) 5, 12, 13. (v) 3, 9, 8, 8, 9.

(iii) 8, 15, 17. (vi) 1, 19, 1, 69,  
1, 69.

(i)

$$\sin A = \frac{3}{5}, \quad \cot A = \frac{4}{3}.$$

$$\cos A = \frac{4}{5}, \quad \sec A = \frac{5}{4}.$$

$$\tan A = \frac{3}{4}, \quad \csc A = \frac{5}{3}.$$

(ii)

$$\sin A = \frac{5}{13}, \quad \cot A = \frac{12}{5}.$$

$$\cos A = \frac{12}{13}, \quad \sec A = \frac{13}{12}.$$

$$\tan A = \frac{5}{12}, \quad \csc A = \frac{13}{5}.$$

(iii)

$$\sin A = \frac{8}{17}, \quad \cot A = \frac{15}{8}.$$

$$\cos A = \frac{15}{17}, \quad \sec A = \frac{17}{15}.$$

$$\tan A = \frac{8}{15}, \quad \csc A = \frac{17}{8}.$$

(iv)

$$\sin A = \frac{9}{41}, \quad \cot A = \frac{40}{9}.$$

$$\cos A = \frac{40}{41}, \quad \sec A = \frac{41}{40}.$$

$$\tan A = \frac{9}{40}, \quad \csc A = \frac{41}{9}.$$

(v)

$$\sin A = \frac{39}{89}, \quad \cot A = \frac{80}{39}.$$

$$\cos A = \frac{80}{89}, \quad \sec A = \frac{89}{80}.$$

$$\tan A = \frac{39}{80}, \quad \csc A = \frac{89}{39}.$$

(vi)

$$\sin A = \frac{119}{169}, \quad \cot A$$

$$= \frac{120}{119}.$$

$$\cos A = \frac{120}{169}, \quad \sec A = \frac{169}{120}.$$

$$\tan A = \frac{119}{120}, \quad \csc A = \frac{169}{119}$$

欲  $a, b, c$  (2圖) 三線。爲直角三角形之三邊。則各線必長若干而後可。又問 3 題內各數是否即所需之長。

三角形三邊相關爲  $a^2 + b^2 = c^2$

今依所與條件證明如下：

$$\begin{aligned} 3^2 + 4^2 &= 5^2 & 9^2 + 40^2 &= 41^2 \\ 5^2 + 12^2 &= 13^2 & 39^2 + 8^2 &= 41^2 \\ 8^2 + 15^2 &= 17^2 & 1.19^2 + 1.60^2 &= 2.00^2 \end{aligned}$$

5. 設  $a, b, c$  有以下之各值。

試求  $A$  之各函數值。

(i)  $2mn, m^2 - n^2, m^2 + n^2$

(ii)  $\frac{2xy}{x-y}, x+y, \frac{x^2+y^2}{x-y}$

(iii)  $pqr, qrs, rsp.$

(iv)  $\frac{mn}{pq}, \frac{mv}{sq}, \frac{nr}{ps}.$

(i)

$$\sin A = \frac{a}{c} = \frac{2mn}{m^2+n^2}$$

$$\cos A = \frac{b}{c} = \frac{m^2-n^2}{m^2+n^2}$$

$$\tan A = \frac{a}{b} = \frac{2mn}{m^2-n^2}$$

$$\cot A = \frac{b}{a} = \frac{m^2-n^2}{2mn}$$

$$\sec A = \frac{c}{b} = \frac{m^2+n^2}{m^2-n^2}$$

$$\csc A = \frac{c}{a} = \frac{m^2+n^2}{2mn}$$

(ii)

$$\sin A = \frac{2xy}{x-y} \times \frac{x-y}{x^2+y^2} = \frac{2xy}{x^2+y^2}$$

$$\begin{aligned} \cos A &= (x+y) \times \frac{x-y}{x^2+y^2} \\ &= \frac{x^2-y^2}{x^2+y^2} \end{aligned}$$

$$\tan A = \frac{2xy}{x-y} \times \frac{1}{x+y} = \frac{2xy}{x^2-y^2}$$

$$\begin{aligned} \cot A &= (x+y) \times \frac{x-y}{2xy} \\ &= \frac{x^2-y^2}{2xy} \end{aligned}$$

$$\begin{aligned} \sec A &= \frac{x^2+y^2}{x-y} \times \frac{1}{x+y} \\ &= \frac{x^2+y^2}{x^2-y^2} \end{aligned}$$

$$\begin{aligned} \csc A &= \frac{x^2+y^2}{x-y} \times \frac{x-y}{2xy} \\ &= \frac{x^2+y^2}{2xy} \end{aligned}$$

(iii)

$$\sin A = \frac{pqr}{rsp} = \frac{q}{s}$$

$$\cos A = \frac{qrs}{rsp} = \frac{q}{p},$$

$$\tan A = \frac{pqr}{qrs} = \frac{p}{s}.$$

$$\cot A = \frac{qrs}{pqr} = \frac{s}{p}.$$

$$\sec A = \frac{rsp}{qrs} = \frac{p}{q}.$$

$$\csc A = \frac{rsp}{pqr} = \frac{s}{q}.$$

(iv)

$$\sin A = \frac{mn}{pq} \times \frac{ps}{nr} = \frac{ms}{qr},$$

$$\cos A = \frac{mn}{sq} \times \frac{ps}{nr} = \frac{mpv}{nqr},$$

$$\tan A = \frac{mn}{pq} \times \frac{sq}{mv} = \frac{ns}{pv},$$

$$\cot A = \frac{mv}{sq} \times \frac{pq}{mn} = \frac{pv}{ns},$$

$$\sec A = \frac{nr}{ps} \times \frac{sq}{mv} = \frac{nqr}{mqv},$$

$$\csc A = \frac{nr}{ps} \times \frac{pq}{mn} = \frac{qr}{ms},$$

6, 求證前題 (i) 及 (ii) 內 a, b, c 之值。爲合於直角三角形之三邊。

(i)

$$a^2 + b^2 = c^2$$

$$(2mn)^2 + (m^2 - n^2)^2$$

$$= (m^2 + n^2)^2,$$

$$4m^4n^2 + m^4 - 2m^2n^2 + n^4$$

$$= m^4 + 2m^2n^2 + n^4$$

$$m^4 + 2m^2n^2 + n^4$$

$$= m^4 + 2m^2n^2 + n^4$$

(ii)

$$\left(\frac{2xy}{x-y}\right)^2 + (x+y)^2$$

$$= \left(\frac{x^2 + y^2}{x-y}\right)^2$$

$$\frac{4x^2y^2}{x^2 - 2xy + y^2} + x^2 + 2xy + y^2$$

$$= \frac{x^4 + 2x^2y^2 + y^4}{x^2 - 2xy + y^2}$$

$$4x^2y^2 + x^4 - 2x^2y^2 + y^4$$

$$= x^4 + 2x^2y^2 + y^4,$$

$$x^4 + 2x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4$$

7, 欲令 5 題 (iii) 及 (iv) 內 a, b, c 之各值。爲直角三角形之各邊。

問當立何種程式。

(iii)

$$p^2q^2r^2 + q^2r^2s^2 = n^2r^2,$$

$$\text{或 } p^2q^2 + q^2s^2 = p^2s^2,$$

(iv)

$$\frac{m^2n^2}{p^2q^2} + \frac{m^2v^2}{s^2q^2} = \frac{n^2r^2}{p^2s^2},$$

$$\text{或 } m^2n^2s^2 + m^2p^2v^2 = n^2q^2r^2.$$

已知  $a^2 + b^2 = c^2$ , 求下列六題中 A 及 B 之函數.

8.  $a = 14, b = 243.$

$$\begin{aligned} c &= \sqrt{24^2 + 143^2} \\ &= \sqrt{21025} \\ &= 145. \end{aligned}$$

$$\sin A = \frac{24}{145} = \cos B.$$

$$\cos A = \frac{143}{145} = \sin B.$$

$$\tan A = \frac{24}{143} = \cot B.$$

$$\cot A = \frac{143}{24} = \tan B.$$

$$\sec A = \frac{145}{143} = \csc B.$$

$$\csc A = \frac{145}{24} = \sec B.$$

9.  $a = 0.264, c = 0.2650$

$$\begin{aligned} b &= \sqrt{(c+a)(c-a)} \\ &= \sqrt{0.529 \times 0.001}. \\ &= 0.023. \end{aligned}$$

$$\sin A = \frac{a}{c} = \frac{264}{265} = \cos B.$$

$$\cos A = \frac{b}{c} = \frac{23}{265} = \sin B.$$

$$\tan A = \frac{a}{b} = \frac{264}{23} = \cot B.$$

$$\cot A = \frac{b}{a} = \frac{23}{264} = \tan B.$$

$$\sec A = \frac{c}{b} = \frac{265}{23} = \csc B.$$

$$\csc A = \frac{c}{a} = \frac{265}{264} = \sec B.$$

10.  $b = 9.5, c = 19.30$

$$\begin{aligned} a &= \sqrt{(c+a)(c-a)} \\ &= \sqrt{28.8 \times 9.8} \\ &= \sqrt{282.24} \\ &= 16.8. \end{aligned}$$

$$\sin A = \frac{a}{c} = \frac{168}{193} = \cos B.$$

$$\cos A = \frac{b}{c} = \frac{95}{193} = \sin B.$$

$$\tan A = \frac{a}{b} = \frac{168}{95} = \cot B.$$

$$\cot A = \frac{b}{a} = \frac{95}{168} = \tan B.$$

$$\sec A = \frac{c}{b} = \frac{193}{95} = \csc B.$$

$$\csc A = \frac{c}{a} = \frac{193}{168} = \sec B.$$

11.  $a = \sqrt{p^2 + q^2}, b = \sqrt{2pq}$

$$c = \sqrt{p^2 + q^2 + 2pq} = p + q$$

$$\sin A = \frac{a}{c} = \frac{\sqrt{p^2 + q^2}}{p + q} =$$

$$\cos B.$$

$$\cos A = \frac{b}{c} = \frac{\sqrt{2pq}}{p+q} =$$

sin B.

$$\tan A = \frac{a}{b} = \frac{\sqrt{p^2+q^2}}{\sqrt{2pq}} =$$

cot B.

$$\cot A = \frac{b}{a} = \frac{\sqrt{2pq}}{\sqrt{p^2+q^2}} =$$

tan B.

$$\sec A = \frac{c}{b} = \frac{p+q}{\sqrt{2pq}} =$$

csc B.

$$\csc A = \frac{c}{a} = \frac{p+q}{\sqrt{p^2+q^2}} =$$

sec B.

$$12 \quad a = \sqrt{p^2+pq}, \quad c = p+q.$$

$$b^2 = c^2 - a^2 = q^2 + pq$$

$$\therefore b = \sqrt{q^2 + pq}.$$

$$\sin A = \frac{a}{c} = \frac{\sqrt{q^2+pq}}{p+q} =$$

cos B.

$$\cos A = \frac{b}{c} = \frac{\sqrt{q^2+pq}}{p+q} =$$

sin B.

$$\tan A = \frac{a}{b} = \frac{\sqrt{p+pq}}{\sqrt{p+pq}} =$$

cot B.

$$\cot A = \frac{b}{a} = \frac{\sqrt{p^2+pq}}{\sqrt{p^2+pq}} =$$

tan B.

$$\sec A = \frac{c}{b} = \frac{p+q}{\sqrt{q^2+pq}} =$$

csc B.

$$\csc A = \frac{c}{a} = \frac{p+q}{\sqrt{p^2+pq}} =$$

sec B.

$$13 \quad b = 2\sqrt{pq}, \quad c = p+q;$$

$$a^2 = c^2 - b^2;$$

$$a^2 = p^2 + 2pq + q^2 - 4pq$$

$$= p^2 - 2pq + q^2.$$

$$\therefore a = p - q.$$

$$\sin A = \frac{a}{c} = \frac{p-q}{p+q} = \cos B.$$

$$\cos A = \frac{b}{c} = \frac{2\sqrt{pq}}{p+q} = \sin B.$$

$$\tan A = \frac{a}{b} = \frac{p-q}{2\sqrt{pq}} =$$

cot B.

$$\cot A = \frac{b}{a} = \frac{2\sqrt{pq}}{p-q} =$$

tan B.

$$\sec A = \frac{c}{b} = \frac{p+q}{2\sqrt{pq}} =$$

csc B.

$$\csc A = \frac{c}{a} = \frac{p+q}{p-q} =$$

sec B.



14.  $a=2b.$

因  $a=2b$ , 又  $a^2+b^2=c^2$

故  $4b^2+b^2=c^2$

$$5b^2=c^2.$$

$$c=b\sqrt{5}.$$

$$\sin A = \frac{a}{c} = \frac{2b}{b\sqrt{5}} = \frac{2}{\sqrt{5}}$$

$$\sqrt{5}.$$

$$\cos A = \frac{b}{c} = \frac{b}{b\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$\sqrt{5}.$$

$$\tan A = \frac{a}{b} = \frac{2b}{b} = 2$$

$$\cot A = \frac{b}{a} = \frac{b}{2b} = \frac{1}{2}$$

$$\sec A = \frac{c}{b} = \frac{b\sqrt{5}}{b} = \sqrt{5}$$

$$\csc A = \frac{c}{a} = \frac{b\sqrt{5}}{2b} = \frac{1}{2}\sqrt{5}$$

15.  $a = \frac{2}{3}c.$

因  $a = \frac{2}{3}c.$

$$b = \sqrt{c^2 - a^2}$$

$$= \sqrt{c^2 - \frac{4}{9}c^2}$$

$$= \frac{c}{3}\sqrt{5}.$$

$$\sin A = \frac{a}{c} = \frac{\frac{2}{3}c}{c} = \frac{2}{3}$$

$$\cos A = \frac{b}{c} = \frac{\frac{c}{3}\sqrt{5}}{c} = \frac{1}{3}\sqrt{5}$$

$$\tan A = \frac{a}{b} = \frac{\frac{2}{3}c}{\frac{c}{3}\sqrt{5}} = \frac{2}{5}\sqrt{5}$$

$$\cot A = \frac{b}{a} = \frac{\frac{c}{3}\sqrt{5}}{\frac{2}{3}c} = \frac{1}{2}\sqrt{5}$$

$$\sec A = \frac{c}{b} = \frac{c}{\frac{1}{3}c} = \frac{3}{2}$$

$$\csc A = \frac{c}{a} = \frac{c}{\frac{2}{3}c} = \frac{3}{2}$$

16  $a+b = \frac{5}{4}c.$

$$a+b = \frac{5}{4}c.$$

乘方  $a^2+b^2+2ab = \frac{25}{16}c^2$

$$a^2+b^2 = c^2$$

相減  $2ab = \frac{9}{16}c^2$

又自  $a^2+b^2 = c^2$

減  $2ab = \frac{9}{10}c^2$

---

$$a^2 + b^2 - 2ab = \frac{7}{16}c^2 = \frac{16 - 5\sqrt{7}}{9}.$$

開方  $a - b = \frac{c}{4}\sqrt{7}.$

$$\frac{a + b = \frac{5}{4}c.}{}$$

$$2b = \frac{5}{4}c - \frac{c}{4}\sqrt{7}.$$

$$b = \frac{c}{8}(5 - \sqrt{7}).$$

$$2a = \frac{5}{4}c + \frac{c}{4}\sqrt{7}.$$

$$a = \frac{c}{8}(5 + \sqrt{7}).$$

$$\sin A = \frac{a}{c} = \frac{\frac{c}{8}(5 + \sqrt{7})}{c} = \frac{5 + \sqrt{7}}{8}.$$

$$\cos A = \frac{b}{c} = \frac{\frac{c}{8}(5 - \sqrt{7})}{c} = \frac{5 - \sqrt{7}}{8}.$$

$$\tan A = \frac{a}{b} = \frac{5 + \sqrt{7}}{5 - \sqrt{7}} = \frac{16 + 5\sqrt{7}}{9}.$$

$$\cot A = \frac{b}{a} = \frac{5 - \sqrt{7}}{5 + \sqrt{7}}$$

$$\sec A = \frac{c}{b} = \frac{c}{\frac{c}{8}(5 - \sqrt{7})} = \frac{4}{9}(5 + \sqrt{7}).$$

$$\csc A = \frac{c}{a} = \frac{c}{\frac{c}{8}(5 + \sqrt{7})} = \frac{4}{9}(5 - \sqrt{7}).$$

17  $a - b = \frac{1}{4}c.$

$$a - b = \frac{1}{4}c.$$

方  $a^2 - 2ab + b^2 = \frac{1}{16}c^2.$

$$\frac{a^2 + b^2 = c^2.}{}$$

減  $2ab = \frac{15}{16}c^2.$

$$\frac{a^2 + b^2 = c^2.}{}$$

加  $a^2 + 2ab + b^2 = \frac{31}{16}c^2.$

開方  $a + b = \frac{c}{4}\sqrt{31}.$

$$a - b = \frac{c}{4}.$$

$$2a = \frac{c}{4}\sqrt{31} + \frac{c}{4}.$$

$$a = \frac{c}{8}(\sqrt{31} + 1).$$

$$2b = \frac{c}{4}\sqrt{31} - \frac{c}{4}.$$

$$b = \frac{c}{8}(\sqrt{31} - 1)$$

$$\begin{aligned} \sin A = \frac{a}{c} &= \frac{\frac{c}{8}(\sqrt{31} + 1)}{c} \\ &= \frac{\sqrt{31} + 1}{8} \end{aligned}$$

$$\begin{aligned} \cos A = \frac{b}{c} &= \frac{\frac{c}{8}(\sqrt{31} - 1)}{c} \\ &= \frac{(\sqrt{31} - 1)}{8}. \end{aligned}$$

$$\begin{aligned} \tan A = \frac{a}{b} &= \frac{\sqrt{21} + 1}{\sqrt{31} - 1} \\ &= \frac{16 + \sqrt{31}}{15}. \end{aligned}$$

$$\begin{aligned} \cot A = \frac{b}{a} &= \frac{\sqrt{31} - 1}{\sqrt{31} + 1} \\ &= \frac{16 - \sqrt{31}}{15}. \end{aligned}$$

$$\begin{aligned} \sec A = \frac{c}{b} &= \frac{8}{\sqrt{31} - 1} \\ &= \frac{4}{15}(\sqrt{31} + 1). \end{aligned}$$

$$\csc A = \frac{c}{a} = \frac{8}{\sqrt{31} + 1}$$

$$= \frac{4}{5}(\sqrt{31} - 1)$$

18, 設  $\sin A = \frac{3}{5}$  而  $c = 20.5$   
求  $a$ 。

$$\sin A = \frac{a}{c} = \frac{3}{5}$$

$$\frac{a}{20.5} = \frac{3}{5},$$

故  $a = 12.3$ ,

19, 設  $\cos A = 0.44$ ,  $c = 3.5$   
求  $b$ 。

$$\cos A = \frac{b}{c} = 0.44$$

$$\frac{b}{3.5} = 0.44,$$

故  $b = 1.54$ ,

20 設  $\tan A = \frac{11}{3}$ , 而  $b = 2\frac{5}{11}$ .

求  $a$ 。

$$\tan A = \frac{a}{b} = \frac{a}{2\frac{5}{11}} = \frac{11}{3}.$$

$$\text{故 } \frac{11a}{27} = \frac{11}{3},$$

$$a = 9,$$

21, 設  $\cot A = 4$ , 而  $a = 17$

求  $b$ 。

$$\cot A = \frac{b}{a} = \frac{b}{17} = 4$$

因得  $b = 78$ ,

22 設  $\sec A = 2$ , 而  $b = 20$

求  $c$ 。

$$\sec = \frac{c}{b} = \frac{c}{20} = 2.$$

$$\sec A = \frac{c}{b} = \frac{c}{20} = 2,$$

因得  $c=40$ ,

23. 設  $\csc A = 6.45$ , 而  $a = 35.6$   
求  $a_0$

$$\begin{aligned} \csc A &= \frac{c}{a} = \frac{c}{35.6} \\ &= 6.45 \end{aligned}$$

因得  $c = 229.62$ .

24. 有下列已知各值求作一直角三角形  $c=6, \tan A = \frac{3}{2}$

$$\text{因 } \tan A = \frac{a}{b} = \frac{3}{2}.$$

故  $a : b = 3 : 2$ ,

先作  $AB$  等於 2,

再作  $BC$  等於 3, 且令與  $AB$  直交。

連  $A, C$  兩點,

引長  $AC$  至  $D$ , 作  $AD=6$

作  $DE$  與  $AB$  之引長線直交

$ADE$  直角三角形與  $ABC$

正三角形相似,

$ADE$  即所求者之三角形,

25.  $a=35, \cos A = \frac{1}{2}$ .

先作  $A'B'C'$  直角三角形, 取  $b'=1, c'=2$ ,

$$\text{即得 } \cos A = \frac{1}{2},$$

再作  $ABC$  直角三角形與  $A'B'C'$  相似, 而作  $a=35$ .

$ABC$  即所求之三角形,

26.  $b=2, \sin A = 0.6$

先作  $A'B'C'$  直角三角形, 取  $a'=9, c'=10$ ,

$$\text{即得 } \sin A = 0.6$$

再作  $ABC$  直角三角形與  $A'B'C'$  相似, 而取  $b=2$ .

$ABC$  即所求之三角形

27.  $b=4, \csc A = 4$

先作  $A'B'C'$  直角三角形, 取  $c'=4, a'=1$ ,

再作  $ABC$  直角三角形與  $A'B'C'$  相似, 而取  $b=4$ ,

$ABC$  即所求之三角形.

28. 於直角三角形之  $c=2.5$  哩  $\sin A = 0.86$ . 求  $\cos A = 0.8$ , 求餘二邊。

$$\sin A = \frac{a}{c} \quad \left| \quad \cos A = \frac{b}{c}.$$

$$\text{即 } a = c \sin A \quad \left| \quad \text{按 } b = c \cos A \right. \\ = 1.5. \quad \left| \quad = 2.$$

29. 以分角器作 $20^\circ, 40^\circ$ 及 $70^\circ$ 之三角。由測其必需之綫，以決定其函數並以所得值與下表中所列較近各值比較之。

30. 由上表設  $A=20^\circ, C=1$ ，又設  $A=20, C=4$ ，試各求  $A=20^\circ, C=1$ 。直角三角形之餘二邊。

$$\sin A = \frac{a}{c},$$

即  $a = c \sin A,$

$$a = 0.342,$$

$$\cos A = \frac{b}{c},$$

即  $b = c \cos A,$

$$b = 0.940.$$

$$A = 20^\circ, c = 4.$$

$$a = 4 \times 0.342$$

$$= 1.368.$$

$$b = 4 \times 0.940.$$

$$= 3.760.$$

31. 由直立竿之水平影長度竿長。測時得太陽之仰角之正切為 0.82。同時有塔之水平影長 174.3 碼。求塔高。

$$\tan A = \frac{a}{b} = 0.82.$$

因得  $a = 0.82b,$

但  $b = 174.3$  碼。

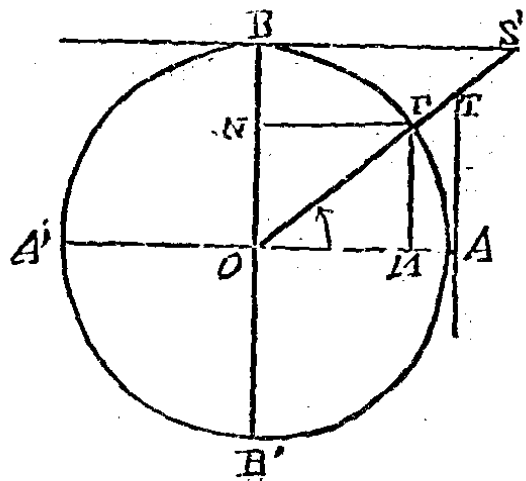
故  $= 0.82 \times 174.3$  碼，

$$= 142.926$$
 碼。

### 例題 III

1. 試以直綫表示與 3 圖內  $x$  較大之銳角之各函數。

設  $x$  為一銳角試明其



2.  $\sin x$  小於  $\tan x$ .

按第三圖,  $OM: MP = OA:$

$AT$

但  $OM < OA,$

故  $MP < AT.$

即  $\sin x < \tan x.$

3.  $\sec x$  大於  $\tan x,$

$$OT = \sec x, AB = \tan x.$$

OAT 直角三角形中

$$\text{有 } OT > AT.$$

$$\text{故 } \sec x > \tan x.$$

4.  $\csc x$  大於  $\cot x$ .

$$OS = \csc x, BS = \cot x.$$

BOS 直角三角形中

$$\text{有 } OS > RS.$$

$$\text{故 } \csc x > \cot x.$$

5. 求作  $x$  角。若  $\tan x = 3$ 。

以一為半徑作圓，其心為  $O$ ，於圓周  $A$  點作切線  $AT$  等於  $3AO$ 。連  $O, T$ 。

則  $AOT$  即所求之角。

6.  $\csc x = 2$ 。

以一為半徑作圓，其心為  $O$ ，於圓周  $B$  點作切線  $BS$  等於  $2AO$ 。連  $O, S$ 。

則  $AOS$  即所求之角。

$$7. \cos x = \frac{1}{2}.$$

取  $OM$  等於  $OA$  半徑之半。於  $M$  點作一垂線，與圓周相遇於  $P$ ，作  $OP$ 。

則  $POM$  即所求之角。

8.  $\sin x = \cos x$ 。

$$\text{設 } MP = \sin x, OM = \cos x.$$

但按題  $MP = OM$ 。

故（按幾何理） $x = 45^\circ$ 。

作一角等於  $45^\circ$  即所求之角

9.  $\sin x = 2\cos x$ 。

作  $PMO$  正角，取  $PM = 2OM$ ，連  $O, P$ 。

$POM$  即所求之角。

10.  $4\sin x = \tan x$ 。

在  $OA$  半徑上截取四分之一為  $M$ ，自  $M$  點作垂線遇圓周於  $P$ 。

作  $OP$ 。

$POM$  即所求之角。

11. 證明一角之正弦。必等於其倍角之弦之半。

已有  $POA$  角

再作  $POB$  角  $= 2POA$  角。

作  $PB$  弦，則必與  $OA$  正交。

而  $PM$ （即  $\frac{1}{2}PB$ ）即為  $POA$  之正弦。

故  $\sin x = 2x$  之弦二分

一.

12. 設  $\sin x$  等於內接正十  
角形一邊之半, 求  $x^\circ$

設  $AC$  為內切十邊形之  
一邊.

則  $\frac{360^\circ}{10} = 36^\circ$  即  $AOC$  角.

作  $OB$  平分  $AC$ .

則  $AOC$  角亦被平分.

且  $AOB$  角  $= 18^\circ$ .

但  $AOB$  之正弦.

$$= \frac{1}{2} AC,$$

或  $x$  (或  $AOB$ )  $= 18^\circ$ .

13. 已知  $x$  及  $y$ ,  $x+y$  小於  
 $90^\circ$ . 求作  $\sin(x+y) - \sin x$  之值。

設  $AB = \sin(x+y),$   
 $= \sin x,$

取  $B$  點為圓心, 以  $CD$  為  
半徑作弧, 與  $AB$  相遇於  $E$ .

則  $EA$  即所求值.

14.  $\tan(x+y) - \sin(x+y)$   
 $+ \tan x - \sin x$  之值。

設  $AB = \sin(x+y),$   
 $CD = \sin x;$

又設  $EF = \tan(x+y),$

$GF = \tan x,$

以  $AR$  為度, 自  $F$  度至  $H$ .

又以  $GF$  為度, 自  $H$  度至  $I$ .

又以  $CD$  為度, 自  $I$  反向度

至  $K$

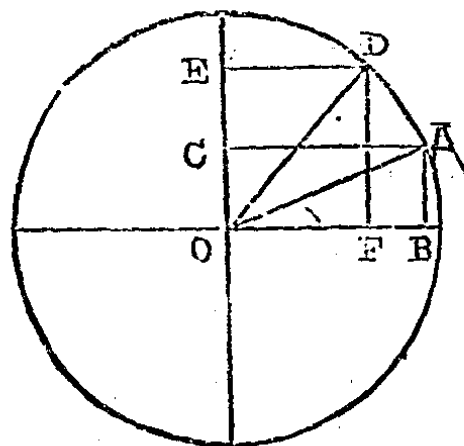
則  $EK$  即所求之值.

15.  $\sin y = 2 \sin x,$

設  $AB$  為  $x$  角之正弦 (在  
以  $O$  為心之圖中).

作  $AC$ , 直交垂徑.

則  $CO = AB.$



在垂徑上截取  $CF = CO$ .

作  $FD$  與垂徑直交, 而與  
圓周相遇於  $D$ .

作  $DE$  與  $OB$  正交; 作  
 $OD$ .

今  $OF = 2 BO$ , (依作法),  
 $ED = FO$ ,  
 $FO$  爲  $OD$  半徑之射影  
 故  $DE = 2 AB$   
 $DOB$  卽所求之角.

16.  $\cos y = \frac{1}{2} \cos x$

設  $OB = \cos AOB$ .  
 在  $OB$  之中點  $C$ , 作垂線  
 $CD$ , 與圓周相遇於  $C$ , 作  $CO$ .  
 則  $DOB$  卽所求之角.

17.  $\tan y = 3 \tan x$ .

設  $AB = \tan x$ ,  
 引長  $AB$  至  $C$ , 作  $AC$   
 $= 3 AB$ .  
 自圓心  $O$  點作  $OC$ .  
 則  $COA$  卽所求之角.

18.  $\sec y = \csc x$ .

因  $\sec y = \csc x$ ,

$$\frac{c}{b} = \frac{c}{a}$$

故  $a = b$   
 作一等腰正角三角形.  
 所求之角等於  $45^\circ$ .

19. 試以圖明  $2 \sin A > \sin 2A$ .

作  $BOC, COA$  兩角, 各與  
 已知  $A$  角相等.

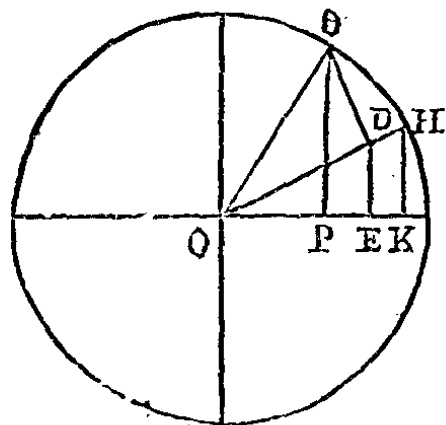
則  $AB = 2 \sin A$ ,  
 $AD$  (自  $A$  至  $OB$  之垂線)  
 $= \sin 2A$ ,

但  $AB > AD$ ,

故  $2 \sin A > \sin 2A$ .

20. 已知兩角  $A$  及  $B$ .  $A + B < 90^\circ$ . 試明  $\sin(A + B) < \sin A + \sin B$ .

求證  $\sin(A + B) < \sin A + \sin B$



作  $HOK = A$  角.

$COH = B$  角.

則  $\sin(A + B) = CP$ .

$\sin A = HK, \sin B$   
 $= CD$

但  $CP < CD + DE$ ,



$$HK > DE.$$

$$\text{故 } CP < CD + HK,$$

$$\sin(A+B) < \sin A + \sin B$$

21. 已知單位圓之  $\sin x$ 。求  $n$  半徑之圓內與  $\sin x$  相當位置之綫之長。

1:  $x = \sin x$ : 所求之綫。

故 所求之綫  $= r \sin x$ 。

22. 於直角三角形已知斜邊  $C$ 。又  $\sin A = m, \cos A = n$  求餘二邊。

$$\sin A = \frac{a}{c} = m,$$

$$\text{故 } a = cm.$$

$$\cos A = \frac{b}{c} = n,$$

$$\text{故 } b = cn.$$

## 例題 VI

1. 試求以下各函數之餘角之函數。

$$\sin 30^\circ, \quad \csc 18^\circ 10'.$$

$$\cos 45^\circ, \quad \cos 37^\circ 24'.$$

$$\tan 89^\circ, \quad \cot 82^\circ 19'.$$

$$\cot 15^\circ, \quad \csc 54^\circ 46'.$$

$$\sin 30^\circ = \cos(90^\circ - 30^\circ)$$

$$= \cos 60^\circ.$$

$$\cos 45^\circ = \sin(90^\circ - 45^\circ)$$

$$= \sin 45^\circ.$$

$$\tan 89^\circ = \tan(90^\circ - 1^\circ)$$

$$= \cot 1^\circ.$$

$$\cot 15^\circ = \tan(90^\circ - 15^\circ).$$

$$= \tan 75^\circ.$$

$$\csc 18^\circ 10' = \sec(90^\circ - 18^\circ 10')$$

$$= \sec 71^\circ 50'.$$

$$\cos 37^\circ 24' = \sin(90^\circ - 37^\circ 24')$$

$$= \sin 52^\circ 36'.$$

$$\cot 82^\circ 19' = \tan(90^\circ - 82^\circ 19')$$

$$= \tan 7^\circ 41'.$$

$$\csc 54^\circ 46' = \sec(90^\circ - 54^\circ 46')$$

$$= \sec 35^\circ 14'.$$

2. 試求小於  $45^\circ$  之一角之函數。

$$\sin 60^\circ, \quad \csc 69^\circ 2'.$$

$$\cos 75^\circ, \quad \cos 85^\circ 39'.$$

$$\tan 57^\circ, \quad \cot 89^\circ 59'.$$

$$\cot 84^\circ, \quad \csc 45^\circ 1'.$$

$$\sin 60^\circ = \cos(90^\circ - 60^\circ)$$

$$= \cos 30^\circ.$$

$$\cos 75^\circ = \sin(90^\circ - 75^\circ)$$

$$= \sin 15^\circ.$$

$$\begin{aligned}\tan 57^{\circ} &= \cot(90^{\circ} - 57^{\circ}) \\ &= \cot 33^{\circ}.\end{aligned}$$

$$\begin{aligned}\cot 84^{\circ} &= \tan(90^{\circ} - 84^{\circ}) \\ &= \tan 6^{\circ}.\end{aligned}$$

$$\begin{aligned}\csc 69^{\circ} 2' &= \sec(90^{\circ} - 69^{\circ} 2'.) \\ &= \sec 20^{\circ} 58'.\end{aligned}$$

$$\begin{aligned}\cos 85^{\circ} 39' &= \sin(90^{\circ} - 85^{\circ} 39'), \\ &= \sin 421'.\end{aligned}$$

$$\begin{aligned}\cot 89^{\circ} 59' &= \tan(90^{\circ} - 89^{\circ} 59') \\ &= \tan 0^{\circ} 1'.\end{aligned}$$

$$\begin{aligned}\csc 45^{\circ} 1' &= \sec(90^{\circ} - 45^{\circ} 1') \\ &= \sec 44^{\circ} 59'.\end{aligned}$$

3. 已知  $\tan 30^{\circ} = \frac{1}{3}\sqrt{3}$ ;  
求  $\cot 60^{\circ}$ .

$$\begin{aligned}\tan 30^{\circ} &= \cot(90^{\circ} \\ &- 30^{\circ}) = \cot 60^{\circ}.\end{aligned}$$

$$\text{故 } \cot 60^{\circ} = \frac{1}{3}\sqrt{3}.$$

4. 已知  $\tan A = \cot A$ ; 求  $A$ .

$$\tan A = \cot(90^{\circ} - A),$$

$$\text{又 } \tan A = \cot A.$$

$$\text{因知 } 90^{\circ} - A = A.$$

$$2A = 90^{\circ}.$$

$$\text{故 } A = 45^{\circ}.$$

5. 已知  $\cos A = \sin 2A$ ; 求  $A$ .

$$\cos A = \sin(90^{\circ} - A)$$

$$\text{又 } \cos A = \sin 2A,$$

$$\text{因知 } 2A = 90^{\circ} - A.$$

$$3A = 90^{\circ}.$$

$$\text{故 } A = 30^{\circ}.$$

6. 已知  $\sin A = \cos 2A$ ; 求  $A$ .

$$\sin A = \cos(90^{\circ} - A).$$

$$\text{又 } \sin A = \cos 2A.$$

$$\text{因知 } 2A = 90^{\circ} - A.$$

$$3A = 90^{\circ}$$

$$\text{故 } A = 30^{\circ}.$$

7. 已知  $\cos A = \sin(45^{\circ} -$

$$\frac{1}{2}A), \text{ 求 } A.$$

$$\cos A = \sin(90^{\circ} - A),$$

$$\text{又 } \cos A = \sin(45^{\circ} -$$

$$\frac{1}{2}A).$$

$$\text{故知 } 90^{\circ} - A = 45^{\circ} - \frac{1}{2}A.$$

$$180^{\circ} - 2A = 90^{\circ} - A.$$

$$A = 90^{\circ}.$$

8. 已知  $\cot \frac{1}{2}A = \tan A$  求  $A$ .

$$\tan A = \cot(90^{\circ} - A),$$

$$\frac{1}{2} = 90^{\circ} - A,$$

$$A = 180^{\circ} - 2A,$$

$$3A = 180^{\circ},$$

$$\text{故 } A = 60^{\circ}.$$

9. 已知  $\tan(45^\circ + A) = \cot A$ ,  
求  $A$ .

$$\begin{aligned}\cot A &= \tan(90^\circ - A) \\ 45^\circ + A &= 90^\circ - A. \\ 2A &= 75^\circ. \\ A &= 22^\circ 30' .\end{aligned}$$

10. 設  $\sin A = \cos 4A$ , 求  $A^\circ$   
 $\sin A = \cos(90^\circ - A)$ ,  
 $4A = 90^\circ - A$ ,  
 $5A = 90^\circ$ .  
 $A = 18^\circ$ .

11. 設  $\cot A = \tan 8A$ , 求  
 $A^\circ$   
 $\cot A = \tan(90^\circ -$   
 $A)$ ,

$$\begin{aligned}8A &= 90^\circ - A, \text{ 求} \\ 9A &= 90^\circ, \\ A &= 10^\circ .\end{aligned}$$

12. 設  $\cot A = \tan nA$ , 求  
 $A^\circ$

$$\begin{aligned}\cot &= \tan(90^\circ - A), \\ nA &= 90^\circ - A, \\ (n+1) &= 90^\circ, \\ A &= \frac{90^\circ}{n+1} .\end{aligned}$$

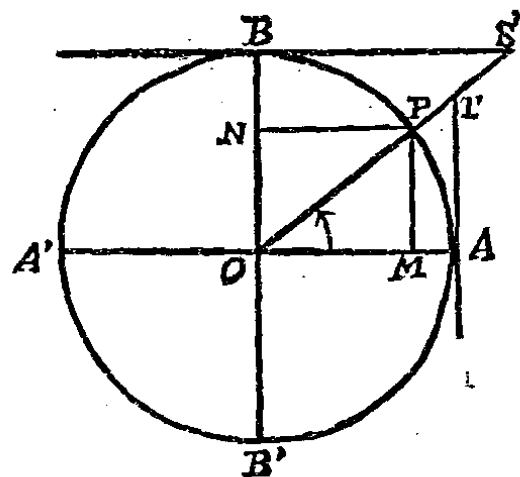
## 例題 V

I. 試以第三章所示單位圓內  
表函數之綫值。證明公式 [1],  
[2],[3],。

$$(1) \sin^2 A + \cos^2 A = 1,$$

$$(2) \tan A = \frac{\sin A}{\cos A} .$$

$$\begin{aligned}(3) \sin A \times \csc A &= 1, \\ \cos A \times \sec A &= 1, \\ \tan A \times \cot A &= 1.\end{aligned}$$



$$(1) \text{ 原有 } MP = \sin A, \\ OM = \cos A,$$

$$\text{且 } \overline{MP^2} + \overline{OM^2} = \overline{OP^2}; \\ \text{但 } \overline{OP^2} = 1.$$

$$\text{故 } \overline{MP^2} + \overline{BM^2} = 1.$$

$$\text{即 } \sin^2 A + \cos^2 A = 1.$$

$$(2) \text{ 原有 } MP = \sin A,$$

$$OM = \cos A,$$

$$AT = \tan A,$$

因  $OAT, OMP$  爲兩三角形相似,

$$\text{得 } TA: OA = PM: OM.$$

$$\text{或 } \frac{TA}{OA} = \frac{PM}{OM}.$$

$$\text{但 } OA = 1,$$

$$\text{故 } TA = \frac{PM}{OM}.$$

$$\text{即 } \tan A = \frac{\sin A}{\cos A}.$$

$$(3) \quad PM = \sin A,$$

$$OS = \csc A$$

因  $OSB, POM$  爲兩相似三角形,

$$\text{得 } OS: OB = OP: PM,$$

$$\text{或 } \frac{OS}{OB} = \frac{OP}{PM};$$

$$\text{但 } OB = 1,$$

$$OF = 1.$$

$$\text{故 } OS = \frac{1}{PM}.$$

$$OS \times PM = 1,$$

$$\text{即 } \csc A \times \sin A = 1.$$

$$\text{又 } \cos A = OM,$$

$$\sec A = OT.$$

因  $OTA, OPM$  爲兩相似

三角形.

$$\text{得 } OT: OA = OP: OM.$$

$$\text{或 } \frac{OT}{OA} = \frac{OP}{OM}.$$

$$\text{但 } OA = 1, OP = 1.$$

$$\text{故 } OT = \frac{1}{OM}.$$

$$OT \times OM = 1,$$

$$\text{即 } \sec A \times \cos A = 1,$$

$$\text{又 } \tan A = AT,$$

$$\cot A = BS.$$

自  $SOB, TAO$  兩相似三角形中

$$\text{得 } BS: BO = AO: AT.$$

$$\text{或 } \frac{BS}{BO} = \frac{AO}{AT}.$$

$$\text{但 } BO = 1, AO = 1.$$

$$\text{故 } BS = \frac{1}{AT}.$$

$$BS \times AT = 1.$$

$$\text{即 } \cot A \times \tan A = 1.$$

試證下列各式:

$$2. \quad 1 + \tan^2 A = \sec^2 A.$$

$$\tan A = \frac{a}{b}, \sec A = \frac{c}{b}.$$

$$a^2 + b^2 = c^2$$

以  $b^2$  除各項

$$\text{得 } \frac{a^2}{b^2} + \frac{b^2}{b^2} = \frac{c^2}{b^2}.$$

以  $\frac{a^2}{b^2}$  與  $\frac{c^2}{b^2}$  之相當  $\tan^2 A$

與  $\sec^2 A$  代入

$$\text{即得 } \tan^2 A + 1 = \sec^2 A$$

$$3. \quad 1 + \cot^2 A = \csc^2 A_0$$

$$\cot A = \frac{b}{a}, \quad \csc = \frac{c}{b}.$$

$$a^2 + b^2 = c^2$$

以  $a^2$  除各項

$$\text{得 } \frac{a^2}{b^2} + \frac{b^2}{a^2} = \frac{c^2}{a^2}.$$

以  $\frac{b^2}{a^2}$  與  $\frac{c^2}{a^2}$  之相當  $\cot^2 A$

與  $\csc^2 A$  代入

$$\text{即得 } 1 + \cot^2 A = \csc^2 A.$$

$$4. \quad \cot A = \frac{\cos A}{\sin A}.$$

$$\text{原有 } \cot A = \frac{b}{a},$$

$$\sin A = \frac{a}{c}, \quad \cos A = \frac{b}{c}$$

$$\text{代入得 } \frac{b}{a} = \frac{b}{c} \div \frac{a}{c}$$

$$\text{即 } \cot A = \frac{\cos A}{\sin A}.$$

$$5. \quad \sin A \sec A = \tan A.$$

$$\text{原有 } \sin A = \frac{a}{c} \sec A = \frac{c}{b}.$$

$$\tan A = \frac{a}{b}.$$

$$\text{代入得 } \frac{a}{c} \times \frac{c}{b} = \frac{a}{b}.$$

$$\text{即 } \sin A \sec A = \tan A.$$

$$6. \quad \sin A \cot A = \cos A.$$

$$\text{原有 } \sin A = \frac{a}{c}.$$

$$\cot A = \frac{a}{c},$$

$$\cos A = \frac{b}{c}.$$

$$\text{代入得 } \frac{a}{c} \times \frac{b}{a} = \frac{b}{c}.$$

$$\text{即 } \sin A \cot A = \cos A.$$

$$7. \quad \cos A \csc A = \cot A_0$$

$$\text{原有 } \cos A = \frac{b}{c},$$

$$\csc A = \frac{c}{a},$$

$$\cot A = \frac{b}{a},$$

$$\text{代入得 } \frac{b}{c} \times \frac{c}{a} = \frac{b}{a}.$$

$$\text{即 } \cos A \csc A = \cot A.$$

$$8. \quad \tan A \cos A = \sin A_0$$

原有  $\tan A = \frac{a}{b}$   
 $\cos A = \frac{b}{c}$ ,  
 $\sin A = \frac{a}{c}$ .

代入得  $\frac{a}{b} \times \frac{b}{c} = \frac{a}{c}$ ,

即  $\tan A \cos A = \sin A$ .

9.  $\sin A \sec A \cot A = 1$ .

原有  $\sin A = \frac{a}{c}$ ,  
 $\sec A = \frac{c}{b}$ ,  
 $\cot A = \frac{b}{a}$ ,

代入得  $\frac{a}{c} \times \frac{c}{b} \times \frac{b}{a} = 1$ .

即  $\sin A \sec A \cot A = 1$ .

10.  $\cos A \csc A \tan A = 1$ .

原有  $\cos A = \frac{b}{c}$ ,  
 $\csc A = \frac{c}{a}$ ,  
 $\tan A = \frac{a}{b}$ ,

代入得  $\frac{b}{c} \times \frac{c}{a} \times \frac{a}{b} = 1$ .

即  $\cos A \csc A \tan A = 1$ .

11.  $(1 - \sin^2 A) \tan^2 A$

$$= \cos^2 A \tan^2 A$$

按第一題,  $1 - \sin^2 A = \cos^2 A$ .

故  $(1 - \sin^2 A) \tan^2 A = \cos^2 A \tan^2 A$ ,

但按第八題,  $\cos A \tan A = \sin A$ .

$$\cos^2 A \tan^2 A = \sin^2 A.$$

故  $(1 - \sin^2 A) \tan^2 A = \sin^2 A$ .

12.  $\sqrt{1 - \cos^2 A} \cot A = \cos A$

$$= \cos A$$

按第一題  $\sqrt{1 - \cos^2 A} = \sin A$ .

$$\sqrt{1 - \cos^2 A} \cot A = \sin A \cot A$$

但按第六題  $\sin A \cot A = \cos A$ .

故得  $\sqrt{1 - \cos^2 A} \cot A = \cos A$ .

13.  $(1 + \tan^2 A) \sin^2 A = \sec^2 A \sin^2 A$

$$= \sec^2 A \sin^2 A$$

按第二題  $1 + \tan^2 A = \sec^2 A$ .

$$(1 + \tan^2 A) \sin^2 A = \sec^2 A \sin^2 A.$$

但按第五題  $\sec^2 A \sin^2 A$   
 $= \tan^2 A.$

故有  $(1 + \tan^2 A) \sin^2 A$   
 $= \tan^2 A.$

14.  $(1 - \sin^2 A) \csc^2 A$   
 $= \cot^2 A.$

按第一題  $1 - \sin^2 A$   
 $= \cos^2 A.$

$(1 - \sin^2 A) \cos^2 A = \csc^2 A$   
 $\csc^2 A$

但按第七題  $\cos A \csc A.$   
 $= \cot A.$

故得  $(1 - \sin^2 A) \csc^2 A$   
 $= \cot^2 A.$

15.  $\tan^2 A \cos^2 A + \cos^2 A.$   
 $= 1.$

按第八題  $\tan A \cos A$   
 $= \sin A.$

$\tan^2 A \cos^2 A = \sin^2 A.$

$\tan^2 A \cos^2 A + \cos^2 A$   
 $= \sin^2 A + \cos^2 A.$

按第一題  $\sin^2 A + \cos^2 A$   
 $= 1.$

故  $\tan^2 A \cos^2 A + \cos^2 A$   
 $= 1.$

16.  $(\sin^2 A - \cos^2 A)^2$

按第一題  $\sin^2 A + \cos^2 A$   
 $= 1.$

$(\sin^2 A + \cos^2 A)^2 = 1.$

但按代數理,

$(\sin^2 A - \cos^2 A)^2$   
 $= (\sin^2 A + \cos^2 A)^2 - 4$   
 $\sin^2 A \cos^2 A.$

故  $(\sin^2 A - \cos^2 A)^2$   
 $= 1 - 4 \sin^2 A \cos^2 A.$

17.  $(1 - \tan^2 A) = \sec^2 A - 4$   
 $\tan^2 A.$

按第二題  $1 - \tan^2 A$   
 $= \sec^2 A.$

故  $(1 - \tan^2 A)^2 = \sec^2 A.$

但按代數理,

$(- \tan^2 A)^2$   
 $= (1 - \tan^2 A)^2 - 4 \tan^2 A$

故  $(1 - \tan^2 A)^2 = \sec^2 A$   
 $- 4 \tan^2 A,$

18.  $\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$   
 $= \sec A \csc A.$

$\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} =$   
 $\frac{\sin^2 A + \cos^2 A}{\cos A \sin A}.$

但按第一題  $\sin^2 A + \cos^2 A$

=1.

又按第三題  $\frac{1}{\cos A} = \sec A,$

$$\frac{1}{\sin A} = \csc A.$$

故  $\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$   
 $= \sec A \csc A$

19.  $\sin^4 A - \cos^4 A = \sin^2 A$   
 $- \cos^2 A.$

$$\sin^4 A - \cos^4 A = (\sin^2 A + \cos^2 A)(\sin^2 A - \cos^2 A).$$

按第一題  $\sin^2 A + \cos^2 A$   
 $= 1.$

故  $\sin^4 A - \cos^4 A = \sin^2 A$   
 $- \cos^2 A,$

20.  $\sec A - \cos A = \frac{1}{\cos A}$   
 $- \cos A.$

按第三題  $\sec A = \frac{1}{\cos A}.$

$$\sec A - \cos A = \frac{1}{\cos A}$$

$$- \cos A = \frac{1 - \cos^2 A}{\cos A}.$$

按第一題  $1 - \cos^2 A$   
 $= \sin^2 A,$

故  $\sec A - \cos A = \frac{\sin^2 A}{\cos A}.$

又按第三題  $\frac{\sin A}{\cos A}$   
 $= \tan A.$

遂得  $\sec A - \cos A$   
 $= \sin A \tan A.$

21.  $\csc A - \sin A = \cos A$   
 $\cot A.$

按第三題  $\csc A = \frac{1}{\sin A}$

故  $\csc A - \sin A = \frac{1}{\sin A}$

$$- \sin A = \frac{1 - \sin^2 A}{\sin A}.$$

按第一題  $1 - \sin^2 A = \cos^2 A.$

故  $\csc A - \sin A = \frac{\cos^2 A}{\sin A}.$

又按第二題  $\frac{\cos A}{\sin A} = \cot A$

即得  $\csc A - \sin A = \cos A \cot A.$

22.  $\frac{\cos A}{1 - \sin A} = \frac{1 - \sin A}{\cos A}.$

化去分數,  $\cos^2 A = 1 - \sin^2 A.$

此式已於第一題內證實不謬.

故  $\frac{\cos A}{1 - \sin A} = \frac{1 - \sin A}{\cos A}.$



## 例題 VI

已知下列諸值。求其餘各函數之值。

$$1. \quad \sin A = \frac{12}{13}。$$

$$\sin^2 A + \cos^2 A = 1,$$

$$\cos^2 A = 1 - \left(\frac{12}{13}\right)^2.$$

$$\begin{aligned} \cos A &= \sqrt{1 - \left(\frac{12}{13}\right)^2} \\ &= \sqrt{\frac{25}{169}}. \end{aligned}$$

$$\text{故 } \cos A = \frac{5}{13},$$

$$\tan A = \frac{5}{12},$$

$$\cot A = \tan A \text{ 之倒數}$$

$$= \frac{12}{5},$$

$$\sec A = \cos A \text{ 之倒數}$$

$$= \frac{13}{5},$$

$$\csc A = \sin A \text{ 之倒數}$$

$$= \frac{13}{12}.$$

$$2. \quad \sin A = 0.8。$$

$$\sin^2 A + \cos^2 A = 1.$$

$$\cos^2 A = 1 - (0.8)^2.$$

$$\cos A = \sqrt{1 - (0.8)^2}.$$

$$\text{故 } \cos A = 0.6,$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{0.8}{0.6}$$

$$= 1.3333,$$

$$\cot A = \tan A \text{ 之倒數}$$

$$= \frac{0.6}{0.8} = 0.75,$$

$$\sec A = \frac{1}{0.6} = 1.6667,$$

$$\csc A = \frac{1}{0.8} = 1.25.$$

$$3. \quad \cos A = \frac{60}{61}。$$

$$\sin^2 A + \cos^2 A = 1.$$

$$\sin A = \sqrt{1 - \frac{3600}{3721}}$$

$$= \sqrt{\frac{121}{3721}} = \frac{11}{61}.$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{11}{60}.$$

$$\cot A = \frac{1}{\tan A} = \frac{60}{11}.$$

$$\sec A = \frac{1}{\cos A} = \frac{61}{60}.$$

$$\csc A = \frac{1}{\sin A} = \frac{61}{11}.$$

$$4. \quad \cos A = 0.28。$$

$$\sin^2 A + \cos^2 A = 1,$$

$$\begin{aligned}\sin A &= \sqrt{1 - (0.28)^2} \\ &= \sqrt{.9216} = 0.96.\end{aligned}$$

$$\begin{aligned}\tan A &= \frac{\sin A}{\cos A} = \frac{0.96}{0.28} \\ &= 3.4286.\end{aligned}$$

$$\begin{aligned}\cot A &= \frac{1}{\tan A} = \frac{1}{3.4286} \\ &= 0.2917.\end{aligned}$$

$$\begin{aligned}\sec A &= \frac{1}{\cos A} = \frac{1}{0.28} \\ &= 3.5714.\end{aligned}$$

$$\begin{aligned}\csc A &= \frac{1}{\sin A} = \frac{1}{0.96} \\ &= 1.0417\end{aligned}$$

$$5. \quad \tan A = \frac{4}{3}$$

有  $\tan A = \frac{4}{3},$

則  $\cot A = \frac{3}{4}.$

$$\tan A = \frac{\sin A}{\cos A},$$

代之  $\frac{4}{3} = \frac{\sin A}{\sqrt{1 - \sin^2 A}}.$

化分數  $3\sin A = 4\sqrt{1 - \sin^2 A},$

$$9\sin^2 A = 16 - 16\sin^2 A$$

$$5\sin A = 4.$$

即得  $\sin A = \frac{4}{5},$

$$\cos A = \frac{\sin A}{\tan A} = \frac{3}{5},$$

$$\sec A = \frac{1}{\cos A} = \frac{5}{3},$$

$$\csc A = \frac{1}{\sin A} = \frac{5}{4}.$$

6.  $\cot A = 1,$

$$\cot A = 1,$$

則  $\tan A = 1.$

$$\tan A = \frac{\sin A}{\cos A},$$

$$1 = \frac{\sin A}{\sqrt{1 - \sin^2 A}},$$

$$\sin A = \sqrt{1 - \sin^2 A},$$

$$\sin^2 A = 1 - \sin^2 A,$$

$$2\sin^2 A = 1,$$

$$\sin^2 A = \frac{1}{2}.$$

故  $\sin A = \sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{2}.$

$$\cos A = \frac{\sin A}{\tan A} = \frac{1}{2}\sqrt{2},$$

$$\sec A = \frac{1}{\cos A} = \frac{1}{\frac{1}{2}\sqrt{2}}$$

$$= \sqrt{2}.$$

$$7. \quad \cot A = 0.5.$$

$$\tan A = \frac{1}{\cot A} = \frac{1}{0.5}$$

$$= 2.$$

$$\tan A = \frac{\sin A}{\cos A} = 2.$$

$$2 \cos A = \sin A.$$

$$\text{方之 } 4 \cos^2 A - \sin^2 A = 0.$$

$$\cos^2 A + \sin^2 A = 1.$$

$$\text{加} \quad 5 \cos^2 A = 1.$$

$$\cos A = \sqrt{\frac{1}{5}} = 0.45$$

$$4 \cos^2 A + 4 \sin^2 A = 4,$$

$$\underline{4 \cos^2 A - \sin^2 A = 0.}$$

$$\text{減} \quad 5 \sin^2 A = 4.$$

$$\sin A = \sqrt{\frac{4}{5}} = 0.90.$$

$$\sec A = \frac{1}{\cos A} = 2.22.$$

$$\csc A = \frac{1}{\sin A} = 1.11.$$

$$8. \quad \sec A = 6.$$

$$\cos A = \frac{1}{\sec A} = \frac{1}{6},$$

$$\sin A = \sqrt{1 - \cos^2 A}$$

$$= \sqrt{1 - \frac{1}{36}} = \sqrt{\frac{35}{36}}.$$

$$\text{故 } \sin A = \frac{1}{2} \sqrt{3},$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2} \sqrt{3}}{\frac{1}{2}}$$

$$= \sqrt{3},$$

$$\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{3}}$$

$$= \frac{1}{3} \sqrt{3},$$

$$\csc A = \frac{1}{\sin A} = \frac{1}{\frac{1}{2} \sqrt{3}}$$

$$= \frac{2}{3} \sqrt{3},$$

$$9. \quad \csc A = \sqrt{2}.$$

$$\sin A = \frac{1}{\csc A} = \frac{1}{\sqrt{2}} = \frac{1}{2} \sqrt{2}.$$

$$\cos A = \sqrt{1 - \left(\frac{1}{2} \sqrt{2}\right)^2}$$

$$= \sqrt{1 - \frac{1}{2}}.$$

$$= \sqrt{\frac{1}{2}} = \frac{1}{2} \sqrt{2}.$$

$$\tan A = \frac{\frac{1}{2} \sqrt{2}}{\frac{1}{2} \sqrt{2}} = 1.$$

$$\cot A = \frac{1}{1} = 1.$$

$$\sec A = \frac{1}{\frac{1}{2}\sqrt{2}} = \sqrt{2}.$$

10.  $\sin A = m.$

$$\cos A = \sqrt{1 - \sin^2 A} \\ = \sqrt{1 - m^2}.$$

$$\tan A = \frac{\sin A}{\cos A} \\ = \frac{m}{\sqrt{1 - m^2}} \\ = \frac{m\sqrt{1 - m^2}}{1 - m^2}.$$

$$\cot A = \frac{1}{\tan A} \\ = \frac{1}{m} \sqrt{1 - m^2}.$$

$$\sec A = \frac{1}{\cos A} \\ = \frac{1}{\sqrt{1 - m^2}}.$$

$$\csc A = \frac{1}{\sin A} = \frac{1}{m}.$$

11.  $\sin A = \frac{2m}{1 + m^2}.$

$$\cos A = \sqrt{1 - \sin^2 A}.$$

故  $\cos A$

$$= \sqrt{1 - \frac{4m^2}{1 + 2m^2 + m^4}}.$$

$$= \sqrt{\frac{1 - 2m^2 + m^4}{1 + 2m^2 + m^4}}$$

$$= \frac{1 - m^2}{1 + m^2}.$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{2m}{1 - m^2}.$$

$$\cot A = \frac{1}{\tan A} = \frac{1 - m^2}{2m}.$$

$$\sec A = \frac{1}{\cos A} = \frac{1 + m^2}{1 - m^2}.$$

$$\csc A = \frac{1}{\sin A} = \frac{1 + m^2}{2m}.$$

12.  $\cos A = \frac{2mn}{m^2 + n^2}.$

$$\sin A = \sqrt{1 - \cos^2 A} \\ = \sqrt{1 - \frac{4m^2n^2}{m^2 + 2m^2n^2 + n^2}}$$

$$= \sqrt{\frac{m^4 - 2m^2n^2 + n^4}{m^4 + 2m^2n^2 + n^4}}$$

$$= \frac{m^2 - n^2}{m^2 + n^2}.$$

$$\tan A = \frac{\sin A}{\cos A} \\ = \frac{m^2 - n^2}{2mn}.$$

$$\cot A = \frac{1}{\tan A}$$

$$\begin{aligned}
 &= \frac{2mn}{m^2 - n^2} \\
 \sec A &= \frac{1}{\cos A} \\
 &= \frac{m^2 + n^2}{2mn} \\
 \csc A &= \frac{1}{\sin A} \\
 &= \frac{m^2 + n^2}{m^2 - n^2}
 \end{aligned}$$

13. 設  $\tan 45^\circ = 1$ , 求  $45^\circ$  之他函數。

$$\frac{\sin 45^\circ}{\cos 45^\circ} = \tan 45^\circ$$

$$\frac{\sin 45^\circ}{\cos 45^\circ} = 1 \quad (A)$$

$$\sin^2 45^\circ + \cos^2 45^\circ = 1 \quad (B)$$

按 (A)  $\sin 45^\circ = \cos 45^\circ$ .

按 (B)  $\cos^2 45^\circ + \cos^2 45^\circ = 1$ .

即  $2 \cos^2 45^\circ = 1$ .

$$\cos^2 45^\circ = \frac{1}{2}$$

$$\begin{aligned}
 \cos 45^\circ &= \sqrt{\frac{1}{2}} \\
 &= \frac{1}{2} \sqrt{2}
 \end{aligned}$$

$$\sin 45^\circ = \frac{1}{2} \sqrt{2}$$

$$\begin{aligned}
 \cot 45^\circ &= \frac{1}{\tan 45^\circ} \\
 &= \frac{1}{1} = 1
 \end{aligned}$$

$$\begin{aligned}
 \operatorname{csc} 45^\circ &= \frac{1}{\frac{1}{2} \sqrt{2}} \\
 &= \sqrt{2}
 \end{aligned}$$

$$\operatorname{csc} 45^\circ = \frac{1}{\frac{1}{2} \sqrt{2}} \sqrt{2}$$

14. 設  $\sin 30^\circ = \frac{1}{2}$ , 求  $30^\circ$  之

他函數。

$$\sin^2 30^\circ + \cos^2 30^\circ = 1$$

$$\cos 30^\circ = \sqrt{1 - \frac{1}{4}}$$

$$= \sqrt{\frac{3}{4}} = \frac{1}{2} \sqrt{3}$$

$$\tan 30^\circ = \frac{\frac{1}{2}}{\frac{1}{2} \sqrt{3}}$$

$$= \frac{1}{3} \sqrt{3}$$

$$\cot 30^\circ = \frac{1}{\frac{1}{3} \sqrt{3}} = \sqrt{3}$$

$$\sec 30^\circ = \frac{1}{\frac{1}{2} \sqrt{3}}$$

$$= \frac{2}{3}\sqrt{3}.$$

$$\csc 30^\circ = \frac{1}{\frac{1}{2}} = 2.$$

15. 設  $\csc 60^\circ = \frac{2}{3}\sqrt{3}$ ; 求

$60^\circ$  之他函數

$$\begin{aligned} \sin 60^\circ &= \frac{1}{\csc 60^\circ} \\ &= \frac{1}{\frac{2}{3}\sqrt{3}} = \frac{1}{2}\sqrt{3}. \end{aligned}$$

$$\begin{aligned} \cos 60^\circ &= \sqrt{1 - \sin^2 60^\circ} \\ &= \sqrt{1 - \left(\frac{1}{2}\sqrt{3}\right)^2} \\ &= \sqrt{1 - \frac{3}{4}} = \frac{1}{2}. \end{aligned}$$

$$\tan 60^\circ = \frac{\frac{1}{2}\sqrt{3}}{\frac{1}{2}} = \sqrt{3}$$

$$\cot 60^\circ = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}.$$

$$\sec 60^\circ = \frac{1}{\frac{1}{2}} = 2.$$

16. 設  $\tan 15^\circ = 2 - \sqrt{3}$ ; 求  
 $15^\circ$  之他函數.

$$\begin{aligned} \text{夫 } \tan 15^\circ &= \frac{\sin 15^\circ}{\cos 15^\circ} \\ &= 2 - \sqrt{3}. \end{aligned}$$

$$\sin^2 15^\circ + \cos^2 15^\circ = 1.$$

但  $\sin^2 15^\circ = (2 - \sqrt{3})\cos 15^\circ$ .

故  $[(2 - \sqrt{3})\cos 15^\circ]^2$

$$\begin{aligned} + \cos^2 15^\circ &= 1, (4 - 4\sqrt{3} \\ &+ 3)\cos^2 15^\circ + \cos^2 15^\circ \\ &= 1, (8 - 4\sqrt{3})\cos^2 15^\circ \\ &= 1 \end{aligned}$$

$$\begin{aligned} \cos^2 15^\circ &= \frac{1}{4(2 - \sqrt{3})} \\ &= \frac{2 + \sqrt{3}}{4}, \end{aligned}$$

$$\begin{aligned} \cos 15^\circ &= \sqrt{\frac{2 + \sqrt{3}}{4}} \\ &= \frac{1}{2}\sqrt{2 + \sqrt{3}} \end{aligned}$$

$$\begin{aligned} \sin^2 15^\circ &= 1 - \cos^2 15^\circ \\ &= 1 - \frac{2 + \sqrt{3}}{4} \\ &= \frac{2 - \sqrt{3}}{4}, \end{aligned}$$

$$\sin 15^\circ = \frac{1}{2}\sqrt{2 - \sqrt{3}}.$$

$$\cot 15^\circ = \frac{1}{\tan 15^\circ}$$

$$= \frac{1}{2 - \sqrt{3}}$$

$$= 2 + \sqrt{3}$$

$$\sec 15^\circ = \frac{1}{\frac{1}{2}\sqrt{2} + \sqrt{3}}$$

$$= 2(2 - \sqrt{3})$$

$$\sqrt{2 + \sqrt{3}}$$

$$\csc 15^\circ = \frac{1}{\frac{1}{2}\sqrt{2 - \sqrt{3}}}$$

$$= 2(2 + \sqrt{3})$$

$$\sqrt{2 - \sqrt{3}}$$

17. 設  $\cot 22^\circ 30' = \sqrt{2}$

+1; 求  $22^\circ 30'$  之他函數

$$\tan 22\frac{1}{2}^\circ = \frac{1}{\cot 22\frac{1}{2}^\circ}$$

$$= \frac{1}{\sqrt{2} + 1} = \sqrt{2} - 1$$

$$\frac{\sin 22\frac{1}{2}^\circ}{\cos 22\frac{1}{2}^\circ} = \tan 22\frac{1}{2}^\circ, \quad (1)$$

$$\cos 22\frac{1}{2}^\circ + \sin 22\frac{1}{2}^\circ = 1, \quad (2)$$

按 (1)  $\cos 22\frac{1}{2}^\circ \tan 22\frac{1}{2}^\circ$

$$= \sin 22\frac{1}{2}^\circ$$

方之  $\cos^2 22\frac{1}{2}^\circ \tan^2 22\frac{1}{2}^\circ$

$$= \sin^2 22\frac{1}{2}^\circ$$

按 (2)  $\cos^2 22\frac{1}{2}^\circ = \sin^2 22\frac{1}{2}^\circ$

$$+ 1,$$

相加  $\cos^2 22\frac{1}{2}^\circ \tan^2 22\frac{1}{2}^\circ$

$$+ \cos^2 22\frac{1}{2}^\circ = 1,$$

$$\cos^2 22\frac{1}{2}^\circ (\tan^2 22\frac{1}{2}^\circ + 1) = 1,$$

$$\cos^2 22\frac{1}{2}^\circ (4 - 2\sqrt{2}) = 1,$$

$$\cos 22\frac{1}{2}^\circ \sqrt{4 - 2\sqrt{2}} = 1$$

故  $\cos 22\frac{1}{2}^\circ = \frac{1}{\sqrt{4 - 2\sqrt{2}}}$

$$= \sqrt{\frac{4 + 2\sqrt{2}}{8}}$$

$$= \frac{1}{2} \sqrt{2 + \sqrt{2}}$$

$$\sin 22\frac{1}{2}^\circ = \sqrt{1 - \frac{2 + \sqrt{2}}{4}}$$

$$= \sqrt{\frac{4 - 2 - \sqrt{2}}{4}}$$

$$= \sqrt{\frac{2 - \sqrt{2}}{4}}$$

$$= \frac{1}{2} \sqrt{2 - \sqrt{2}}$$

$$\sec 22\frac{1}{2}^\circ = \frac{1}{\frac{1}{2} \sqrt{2 + \sqrt{2}}}$$

$$= (2 - \sqrt{2})$$

$$\sqrt{2 - \sqrt{2}}$$

18. 設  $\sin 0^\circ = 0$ ; 求  $0^\circ$  之他函數.

$$\begin{aligned}\cos 0^\circ &= \sqrt{1 - \sin^2 0^\circ} \\ &= \sqrt{1 - 0} \\ &= 1.\end{aligned}$$

$$\tan 0^\circ = \frac{\sin 0^\circ}{\cos 0^\circ} = \frac{0}{1} = 0.$$

$$\cot 0^\circ = \frac{1}{\tan 0^\circ} = \frac{1}{0} = \infty.$$

$$\sec 0^\circ = \frac{1}{\cos 0^\circ} = \frac{1}{1} = 1.$$

$$\csc 0^\circ = \frac{1}{\sin 0^\circ} = \frac{1}{0} = \infty.$$

19. 設  $\sin 90^\circ = 1$ ; 求  $90^\circ$  之他函數。

$$\sin 90^\circ = 1.$$

$$\cos 90^\circ = \sqrt{1 - \sin^2 90^\circ} = 0.$$

$$\tan 90^\circ = \frac{\sin 90^\circ}{\cos 90^\circ} = \frac{1}{0} = \infty.$$

$$\cot 90^\circ = \frac{1}{\tan 90^\circ} = \frac{1}{\infty} = 0.$$

$$\sec 90^\circ = \frac{1}{\cos 90^\circ} = \frac{1}{0} = \infty.$$

$$\csc 90^\circ = \frac{1}{\sin 90^\circ} = \frac{1}{1} = 1.$$

20. 設  $\tan 90^\circ = \infty$ . 求  $90^\circ$  之他函數.

$$(1) \tan 90^\circ = \infty.$$

$$\begin{aligned}(2) \cot 90^\circ &= \frac{1}{\tan 90^\circ} \\ &= \frac{1}{\infty} = 0.\end{aligned}$$

$$(3) \frac{\cos 90^\circ}{\sin 90^\circ} = \cot 90^\circ = 0.$$

$$\text{故 } \cos 90^\circ = 0.$$

$$(4) \sin^2 90^\circ + \cos^2 90^\circ = 1.$$

$$\text{故 } \sin^2 90^\circ = 1,$$

$$\sin 90^\circ = 1.$$

$$(5) \sec 90^\circ = \frac{1}{0} = \infty.$$

$$(6) \cos 90^\circ = 0.$$

試用下列各項表示其餘各函數之值。

$$21. \sin A_0$$

$$\cos A = \sqrt{1 - \sin^2 A}.$$

$$\tan A = \frac{\sin A}{\sqrt{1 - \sin^2 A}}.$$

$$\cot A = \frac{\sqrt{1 - \sin^2 A}}{\sin A}.$$

$$\sec A = \frac{1}{\sqrt{1 - \sin^2 A}}.$$

$$\csc A = \frac{1}{\sin A}.$$

$$22. \cos A_0$$



$$\sin A = \sqrt{1 - \cos^2 A}.$$

$$\tan A = \frac{\sqrt{1 - \cos^2 A}}{\cos A}.$$

$$\cot A = \frac{\cos A}{\sqrt{1 - \cos^2 A}}.$$

$$\sec A = \frac{1}{\cos A}.$$

$$\csc A = \frac{1}{\sqrt{1 - \cos^2 A}}$$

23.  $\tan A$ 。

(1)  $\cot A = \frac{1}{\tan A}.$

(2)  $\frac{a}{b} = \tan A.$

$$a = b \tan A.$$

$$a^2 = b^2 \tan^2 A.$$

$$a^2 - b^2 \tan^2 A = 0.$$

相減  $\frac{a^2 + b^2}{b^2(1 + \tan^2 A)} = 1.$

$$b^2 = \frac{1}{1 + \tan^2 A}$$

$$= \cos A.$$

故  $\cos A = \sqrt{\frac{1}{1 + \tan^2 A}}.$

(3)  $\sin A = \sqrt{1 - \cos^2 A}$

$$= \sqrt{1 - \frac{1}{1 + \tan^2 A}}$$

$$= \frac{\tan A}{\sqrt{1 + \tan^2 A}}$$

(4)  $\sec A = \frac{1}{\cos A}$

$$= \sqrt{1 + \tan^2 A}.$$

(5)  $\csc A = \frac{1}{\sin A}$

$$= \frac{\sqrt{1 + \tan^2 A}}{\tan A}$$

24.  $\cot A$ 。

(1)  $\tan A = \frac{1}{\cot A}.$

(2)  $\frac{\sin A}{\cos A} = \tan A.$

設  $x = \sin A, y = \cos A,$

則  $\frac{x}{y} = \frac{1}{\cot A}$

$$x \cot A = y.$$

$$x^2 \cot^2 A = y^2.$$

$$x^2 \cot^2 A - y^2 = 0.$$

加  $\frac{x^2}{x^2(1 + \cot^2 A)} + y^2 = 1.$

$$x^2 = \frac{1}{1 + \cot^2 A}.$$

即  $\sin A = \frac{1}{\sqrt{1 + \cot^2 A}}.$

(3)  $\cos A = \sqrt{1 - \sin^2 A}$

$$\begin{aligned}
 &= \sqrt{1 - \frac{1}{1 + \cot^2 A}} \quad \text{故} \\
 &= \sqrt{\frac{1 + \cot^2 A - 1}{1 + \cot^2 A}} \\
 &= \frac{\cot A}{\sqrt{1 + \cot^2 A}}.
 \end{aligned}$$

$$\begin{aligned}
 (4) \sec A &= \frac{1}{\cos A} \\
 &= \frac{\sqrt{1 + \cot^2 A}}{\cot A}.
 \end{aligned}$$

$$(5) \csc A = \frac{1}{\sin A} = \frac{1}{\sqrt{1 + \cot^2 A}}.$$

25. 設  $2\sin A = \cos A$ 。求  $\sin A$  及  $\cos A$ 。

$$\sin^2 + \cos^2 A = 1.$$

$$\sin^2 + 4\sin^2 A = 1.$$

$$5\sin^2 A = 1.$$

$$\sin^2 A = \frac{1}{5}.$$

$$\sin A = \frac{1}{5} = \frac{1}{5}\sqrt{5}.$$

$$\cos A = \frac{2}{5}\sqrt{5}.$$

26. 設  $4\sin A = \tan A$ 。求  $\sin A$  及  $\tan A$ 。

$$\tan A = \frac{\sin A}{\cos A}.$$

但題云  $\tan A = 4\sin A$ ,

$$4\sin A = \frac{\sin A}{\cos A},$$

$$4\sin A \times \cos A = \sin A,$$

$$\text{而} \quad \cos A = \frac{\sin A}{4\sin A} = \frac{1}{4}.$$

$$\text{又} \quad \sin^2 A + \cos^2 A = 1,$$

$$\text{故} \quad \sin A = \sqrt{1 - \frac{1}{16}}.$$

$$= \sqrt{\frac{15}{16}}$$

$$= \frac{1}{4}\sqrt{15}.$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{4}\sqrt{15}}{\frac{1}{4}}$$

$$= \sqrt{15}.$$

26. 若  $\sin A : \cos A = 9 : 40$ 。求  $\sin A$  及  $\cos A$ 。

$$40 \sin A = 9 \cos A,$$

$$\text{方之} \quad 1600 \sin^2 A = 81 \cos^2 A;$$

$$1600 \sin^2 A - 81 \cos^2 A = 0.$$

$$\text{但} \quad \sin^2 A + \cos^2 = 1.$$

以  $81$  乘之再與上式相加。

$$\text{得} \quad 1681 \sin^2 A = 81.$$

$$\text{故} \quad 41 \sin^2 A = 9.$$

$$\sin A = \frac{9}{41}.$$

又  $\sin^2 A + \cos^2 A = 1.$

$$\cos A = \sqrt{1 - \sin^2 A}.$$

故  $\cos = \sqrt{1 - \left(\frac{9}{41}\right)^2} = \frac{40}{41}.$

28. 試化  $\tan^2 A + \cot^2 A - \sin^2 A - \cos^2 A$  爲僅含  $\cos A$  之式。

$$\tan^2 A = \frac{\sin^2 A}{\cos^2 A} = \frac{1 - \cos^2 A}{\cos^2 A}.$$

$$\cot^2 A = \frac{\cos^2 A}{\sin^2 A} = \frac{\cos^2 A}{1 - \cos^2 A}.$$

$$\begin{aligned} & \frac{1 - \cos^2 A}{\cos^2 A} + \frac{\cos^2 A}{1 - \cos^2 A} \\ & - 1 + \cos^2 A = \cos^2 A \\ & = \frac{1 - 2\cos^2 A + 2\cos^4 A}{\cos^2 A} \\ & = \frac{-\cos^2 A + \cos^4 A}{-\cos^4 A} \\ & = \frac{1 - 3\cos^2 A + 3\cos^4 A}{\cos^2 A - \cos^4 A}. \end{aligned}$$

29. 試明  $\sin A + \cos A = (1 + \tan A)\cos A$ .

$$\frac{\sin A}{\cos A} = \tan A.$$

$$\sin A = \tan A \cos A.$$

$$\sin A + \cos A = \tan A \cos A + \cos A$$

$$= (1 + \tan A)\cos A.$$

40. 試明  $\tan A + \cot A = \sec A \times \csc A.$

$$\tan A = \frac{\sin A}{\cos A}.$$

$$\cot A = \frac{\cos A}{\sin A}.$$

$$\tan A + \cot A$$

$$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}.$$

$$= \frac{\sin^2 A + \cos^2 A}{\cos A \sin A}$$

但  $\sin A + \cos A = 1,$

故  $\tan A + \cot A$

$$= \frac{1}{\cos A \sin A}$$

$$= \sec A \csc A.$$

## 例題 VII

求解下之各方程式。

I.  $2 \cos x = \sec x$

$$2 \cos x = \frac{1}{\cos x}.$$

故  $2 \cos^2 x = 1.$

$$\cos x = \sqrt{\frac{1}{2}}.$$

$$x = 45^\circ$$

$$2. \quad 4 \sin x = \csc x,$$

$$4 \sin x = \frac{1}{\sin x}.$$

$$\text{故 } 4 \sin^2 x = 1,$$

$$\sin x = \frac{1}{2}$$

$$x = 30^\circ$$

$$3. \quad \tan x = 2 \sin x_0$$

$$\frac{\sin x}{\cos x} = 2 \sin x.$$

$$\sin x = 2 \sin x \cos x.$$

$$(1 - 2 \cos x) \sin x = 0.$$

$$\text{故 } \sin x = 0,$$

$$x = 0.$$

$$\text{又 } 1 - 2 \cos x = 0.$$

$$\cos x = \frac{1}{2}.$$

$$\text{而 } x = 60^\circ.$$

$$\text{故知 } x = 0^\circ \text{ 或 } 60^\circ.$$

$$4. \quad \sec x = \sqrt{2} \tan x_0$$

$$\frac{1}{\cos x} = \sqrt{2} \frac{\sin x}{\cos x}.$$

$$1 = \sqrt{2} \sin x.$$

$$\sin x = \frac{1}{\sqrt{2}}.$$

$$x = 45^\circ$$

$$5. \quad \sin^2 x = 3 \cos^2 x_0$$

$$\frac{\sin^2 x}{\cos^2 x} = 3.$$

$$\tan x = \sqrt{3}.$$

$$\text{故 } x = 60^\circ.$$

$$6. \quad 2 \sin^2 x + \cos^2 x = \frac{3}{2}_0$$

$$\sin^2 x + (\sin^2 x + \cos^2 x)$$

$$= \frac{3}{2}_0$$

$$\sin^2 x + 1 = \frac{3}{2}.$$

$$\sin^2 x = \frac{1}{2}.$$

$$\sin x = \frac{1}{\sqrt{2}}.$$

$$x = 45^\circ.$$

$$7. \quad 3 \tan^2 x - \sec^2 x = 1_0$$

$$3 \tan^2 x - (\tan^2 + 1) = 1.$$

$$2 \tan^2 x - 1 = 1.$$

$$2 \tan^2 x = 2.$$

$$\tan x = 1.$$

$$\text{故 } x = 45^\circ.$$

$$8. \quad \tan x + \cot x = 2_0$$

$$\tan x + \frac{1}{\tan x} = 2.$$

$$\tan^2 x - 2 \tan x + 1 = 0.$$

$$\tan x - 1 = 0.$$

$$\tan x = 1.$$

$$\text{故 } x = 45^\circ.$$

$$9. \quad \sin^2 x - \cos x = \frac{1}{4}.$$

$$(1 - \cos^2 x) - \cos x = \frac{1}{4}.$$

$$\cos^2 x - \cos x = \frac{3}{4}$$

$$\cos^2 x + \cos x + \frac{1}{4} = 1.$$

$$\text{開方} \quad \cos x + \frac{1}{2} = 1.$$

$$\cos x = \frac{1}{2}.$$

$$\text{故} \quad x = 60^\circ.$$

$$10. \quad \tan^2 x - \sec x = 1.$$

$$(\sec^2 x - 1) - \sec x = 1.$$

$$\sec^2 x - \sec x = 2.$$

$$\sec^2 x - \sec x + \frac{1}{4} = \frac{9}{4}.$$

$$\text{開方} \quad \sec x - \frac{1}{2} = \pm \frac{3}{2}$$

$$\sec x = 3.$$

$$\text{故} \quad x = 60^\circ.$$

$$11. \quad \sin x + \sqrt{3} \cos x = 2.$$

$$\sqrt{3} \cos x = 2 - \sin x.$$

$$3 \cos^2 x = (2 - \sin x)^2.$$

$$3(1 - \sin^2 x) = 4 - 4 \sin x + \sin^2 x.$$

$$4 \sin^2 x - 4 \sin x + 1 = 0.$$

$$\text{開方} \quad 2 \sin x - 1 = 0.$$

$$\sin x = \frac{1}{2}.$$

$$\text{故} \quad x = 30^\circ.$$

$$12. \quad \tan^2 x + \csc^2 x = 3.$$

$$\tan^2 x + (1 + \cot^2 x) = 3.$$

$$\tan^2 x - 2 + \cot^2 x = 0.$$

$$\text{開方} \quad \tan x - \cot x = 0.$$

$$\tan x - \left( \frac{1}{\tan x} \right) = 0.$$

$$\tan^2 x = 1.$$

$$\tan x = 1.$$

$$\text{故} \quad x = 45^\circ.$$

$$13. \quad 2 \cos x + \sec x = 3.$$

$$2 \cos x + \frac{1}{\cos x} = 3.$$

$$2 \cos^2 x - 3 \cos x + 1 = 0.$$

$$(2 \cos x - 1)(\cos x - 1) = 0.$$

$$\cos x = 1 \text{ 或 } \frac{1}{2}.$$

$$\text{故} \quad x = 0^\circ \text{ 或 } 60^\circ.$$

$$14. \quad \cos^2 x - \sin^2 x = \sin x.$$

$$(1 - \sin^2 x) - \sin^2 x = \sin x.$$

$$2 \sin^2 x + \sin x - 1 = 0.$$

$$(2 \sin x - 1)(\sin x + 1) = 0.$$

$$\sin x = \frac{1}{2} \text{ 或 } -1.$$

$$x = 30^\circ.$$

$$15. \quad 2 \sin x + \cot x = 1 + 2 \cos x.$$

$$2 \sin x + \frac{\cos x}{\sin x} = 1 + 2 \cos x.$$

$$\begin{aligned}
& 2\sin^2x + \cos x = \sin x \\
& + 2\cos x \sin x, \\
& 2\sin^2x - \sin x = 2\cos x \sin x \\
& - \cos x, \\
& \sin x(2\sin x - 1) = \cos x(2\sin x \\
& - 1), \\
& (\sin x - \cos x)(2\sin x - 1) = 0.
\end{aligned}$$

故  $\sin x = \cos x,$

而  $\tan x = 1,$

$$x = 45^\circ.$$

又因  $\sin x = \frac{1}{2}.$

而  $x = 30^\circ.$

故  $x = 30^\circ$  或  $45^\circ.$

16.  $\sin^2x + \tan^2x = 3\cos^2x.$

$$\sin^2x + \frac{\sin^2x}{\cos^2x} = 3\cos^2x.$$

$$\begin{aligned}
(1 - \cos^2x) + \frac{(1 - \cos^2x)}{\cos^2x} \\
= 3\cos^2x.
\end{aligned}$$

$$-4\cos^4x + \frac{1}{\cos^2x} = 0.$$

$$4\cos^4x = 1.$$

$$\cos x = \sqrt{\frac{1}{2}}.$$

故  $x = 45^\circ.$

17.  $\tan x + 2\cot x = \frac{5}{1}$

$$\cos x_0$$

$$\frac{\sin x}{\cos x} + 2 \frac{\cos x}{\sin x} = \frac{5}{2\sin x}$$

$$\sin^2x + 2\cos^2x = \frac{5}{2}\cos x.$$

$$(1 - \cos^2x) + 2\cos^2x = \frac{5}{2}\cos x.$$

$$\cos^2x - \frac{3}{2}\cos x + 1 = 0,$$

$$(\cos x - 2)(\cos x - \frac{1}{2}) = 0.$$

$$\cos x = 2 \text{ 或 } \frac{1}{2}.$$

故  $x = 60^\circ.$

## 例題 VII

1. 於例IV內。既得b後。求c之別種解法。

$$\cos A = \frac{b}{c}.$$

$$c \cos A = b.$$

故  $c = \frac{b}{\cos A}.$

2. 於例 III 內既得a後。求c之別種解法。

$$\sin A = \frac{a}{c}.$$

$$c \sin A = a.$$

故  $c = \frac{a}{\sin A}.$

3. 於例IV內。既得諸角。求  
b之別種解法。

$$\cos A = \frac{b}{c}.$$

故  $b = c \cos A.$

4. 於例V內。既得諸角。求  
c之別種解法。

$$\sin A = \frac{a}{c}.$$

$$c \sin A = a.$$

故  $c = \frac{a}{\sin A}.$

5. 已知 B及C。求A, a, b

$$A = 90^\circ - B.$$

$$\cos B = \frac{a}{c}.$$

故  $a = c \cos B.$

$$\sin B = \frac{b}{c},$$

故  $b = c \sin B.$

6. B及b。求A, a, c

$$A = 90^\circ - B.$$

$$\cot B = \frac{a}{b},$$

$$a = b \cot B.$$

$$\sin B = \frac{b}{c},$$

$$c = \frac{b}{\sin B}.$$

7. 已知 B及a。求A, b, c

$$A = 90^\circ - B.$$

$$\cot B = \frac{a}{b}.$$

$$a = b \cot B.$$

$$\cos B = \frac{a}{c},$$

$$c = \frac{a}{\cos B}.$$

8. 已知 b及c。求A, B, a

$$\cos A = \frac{b}{c}.$$

$$B = 90^\circ - A.$$

$$a = \sqrt{c^2 - b^2}$$

$$= \sqrt{(c+b)(c-b)}$$

9. 有直角三角形 已知下之  
各項求三餘項

$$\tan A = \frac{a}{b} = \frac{3}{4} = 0.7500.$$

故  $= 36^\circ 52',$  (見表)

$$B = 90^\circ - A = 53^\circ 8'.$$

$$c = \sqrt{a^2 + b^2} = 5.$$

10.  $a = 7, c = 13^\circ$

求  $A = 32^\circ 35', B = 57^\circ 25', b$

$$=10.954.$$

$$\sin A = \frac{a}{c} = \frac{7}{13} = 0.5385.$$

$$\text{故 } A = 32^{\circ}35'.$$

$$B = 90^{\circ} - A = 57^{\circ}25'.$$

$$b = \sqrt{(c-a)(c+a)}$$

$$= \sqrt{120} = 10.954.$$

$$\text{11. } a = 5.3, A = 12^{\circ}17'.$$

$$B = 90^{\circ} - A.$$

$$= 77^{\circ}43'.$$

$$\frac{b}{a} = \cot A$$

$$\text{故 } b = a \cot A$$

$$= 5.3 \times 4.5928.$$

$$= 24.342.$$

$$\frac{b}{c} = \sin A.$$

$$c = \frac{a}{\sin A} = \frac{5.3}{0.2127}$$

$$= 24.918.$$

$$\text{12. } a = 104, B = 43^{\circ}18'.$$

$$A = 90^{\circ} - B$$

$$= 46^{\circ}42'.$$

$$\frac{b}{a} = \tan B.$$

$$\text{故 } b = a \tan B.$$

$$= 104 \times 0.9424$$

$$= 9.800.$$

$$\frac{a}{c} = \cos B.$$

$$\text{故 } c = \frac{a}{\cos B}$$

$$= \frac{10.4}{0.7278}$$

$$= 14.290.$$

$$\text{13. } c = 26, A = 37^{\circ}42'.$$

$$B = 90^{\circ} - A$$

$$= 52^{\circ}18'.$$

$$\frac{a}{c} \sin A.$$

$$\text{故 } a = c \sin A$$

$$= 26 \times 0.6115$$

$$= 15.900.$$

$$\frac{b}{c} = \cos A.$$

$$\text{故 } b = c \cos A$$

$$= 26 \times 0.7912$$

$$= 20.572.$$

$$\text{14. } c = 140, B = 24^{\circ}12'.$$

$$A = 90^{\circ} - B = 65^{\circ}48'.$$

$$\frac{a}{c} = \cos B.$$

$$\text{故 } a = c \cos B = 140$$

$$\times 0.9121$$



$$=127.694.$$

$$\frac{b}{c} = \sin B.$$

$$\begin{aligned} \text{故 } b &= c \sin B = 140 \\ &\times 0.4099 \\ &= 57.386. \end{aligned}$$

$$15. \quad b=19, c=23;$$

$$\cos A = \frac{b}{c} = \frac{16}{23} = 0.8261.$$

$$\text{故 } A = 34^{\circ}18'.$$

$$B = 90^{\circ} - A = 55^{\circ}42'.$$

$$\begin{aligned} a &= \sqrt{(c-b)(c+b)} \\ &= \sqrt{168} = 12.961. \end{aligned}$$

$$16. \quad b=98, c=135.2;$$

$$\cos A = \frac{b}{c} = \frac{98}{135.2} = 0.7248.$$

$$\text{故 } A = 43^{\circ}33'.$$

$$B = 90^{\circ} - A = 46^{\circ}27'.$$

$$\begin{aligned} a &= \sqrt{(c-b)(c+b)} \\ &= \sqrt{8675.04} \\ &= 93.139. \end{aligned}$$

$$17. \quad b=42.4, A=32^{\circ}14'$$

$$B = 90^{\circ} - A = 57^{\circ}46'.$$

$$\frac{a}{b} = \tan A.$$

$$\begin{aligned} \text{故 } a &= b \tan A = 42.4 \\ &\times 0.6305 \end{aligned}$$

$$=26.733.$$

$$\frac{b}{c} = \cos A.$$

$$\begin{aligned} \text{故 } c &= \frac{b}{\cos A} = \frac{42.4}{0.8459} \\ &= 50.124. \end{aligned}$$

$$18. \quad b=200, B=46^{\circ}11'$$

$$A = 90^{\circ} - B = 43^{\circ}49'.$$

$$\frac{a}{b} = \cot B,$$

$$\begin{aligned} \text{故 } a &= b \cot B = 200 \\ &\times 0.9595 \\ &= 161.900. \end{aligned}$$

$$\frac{b}{c} = \sin B,$$

$$\begin{aligned} \text{故 } c &= \frac{b}{\sin B} = \frac{200}{0.7219} \\ &= 277.166. \end{aligned}$$

$$19. \quad a=95, b=37$$

$$\tan A = \frac{a}{b} = \frac{95}{37} = 2.5976.$$

$$\text{故 } A = 68^{\circ}43'.$$

$$B = 90^{\circ} - A = 21^{\circ}17'.$$

$$\begin{aligned} c &= \sqrt{a^2 + b^2} = \sqrt{10394} \\ &= 101.951. \end{aligned}$$

$$20. \quad a=6, c=103$$

$$\sin A = \frac{a}{c} = \frac{6}{103} = 0.0583.$$

$$\text{故 } A = 3^{\circ} 21'.$$

$$B = 90^{\circ} - A = 86^{\circ} 39'.$$

$$b = \sqrt{c^2 - a^2} = \sqrt{10573} \\ = 102.825.$$

$$21. \quad a = 3.12, B = 5^{\circ} 8'$$

$$A = 90^{\circ} - B = 84^{\circ} 52'.$$

$$\frac{b}{a} \tan A,$$

$$\text{故 } b = a \tan A = 3.12$$

$$\times 0.0898$$

$$= 0.280.$$

$$\frac{a}{c} = \cos B.$$

$$\text{故 } c = \frac{a}{\cos B} = \frac{3.12}{0.9960} \\ = 3.133.$$

$$22. \quad a = 17, c = 18$$

$$\tan \frac{1}{2} B = \sqrt{\frac{c-a}{c+a}} = \sqrt{\frac{1}{35}}$$

$$= 0.1690.$$

$$\text{故 } \frac{1}{2} B = 9^{\circ} 36'.$$

$$B = 19^{\circ} 12'.$$

$$A = 90^{\circ} - B = 70^{\circ} 48'.$$

$$b = \sqrt{(c-a)(c+a)}$$

$$= \sqrt{35} = 5.916.$$

$$23. \quad c = 57, A = 38^{\circ} 29'.$$

$$B = 90^{\circ} - A = 51^{\circ} 31'$$

$$\frac{a}{c} = \sin A.$$

$$\text{故 } a = c \sin A = 57$$

$$\times 0.6223 = 35.471.$$

$$\frac{b}{c} = \cos A$$

$$\text{故 } b = c \cos A = 57$$

$$\times 0.7828 = 44.620.$$

$$24. \quad a + c = 18, b = 12$$

$$c^2 - a^2 = b^2$$

$$(c-a)(c+a) = b^2$$

$$18(c-a) = 144.$$

$$c - a = 8.$$

$$\text{故 } c = 13.$$

$$a = 5.$$

$$\sin A = \frac{a}{c} = \frac{5}{13} = 0.3846.$$

$$\text{故 } A = 22^{\circ} 37'$$

$$B = 60^{\circ} - A.$$

$$= 67^{\circ} 23'.$$

$$25. \quad a + b = 9, c = 8$$

$$a^2 + b^2 = c^2 = 64.$$

$$a^2 + 2ab + b^2 = 9^2 = 81$$

$$\text{減 } 2ab = 17$$

$$a^2 - 2ab + b^2 = 64 - 17 = 47.$$

$$\text{開方, } a - b = \sqrt{47} = 6.856.$$

但  $a+b=9,$

故  $a=7,$

$$b=1.072.$$

$$\begin{aligned} \text{又 } \tan \frac{1}{2}B &= \sqrt{\frac{c-a}{c+a}} \\ &= \sqrt{\frac{0.072}{15.928}} \\ &= 0.0672. \end{aligned}$$

故  $\frac{1}{2}B=3^{\circ}51',$

$$B=7^{\circ}42',$$

$$A=90^{\circ}-B$$

$$=82^{\circ}18'.$$

### 例題 IX

1. 已知直角三角形之兩項如下(1-30)。試以對數求各角之值。及最近之分爲止。

$$\sin A = \frac{a}{c} = \frac{1}{2}.$$

故  $A=30^{\circ}.$

$$B=(90^{\circ}-A)$$

$$=60^{\circ}.$$

$$\frac{b}{c} = \cos A$$

故  $b=c \cos A,$

查表,  $\log \cos A = 9.93753$

$$\log 12 = 1.07918$$

$$\log b = 1.01671$$

反查,  $b=10.392.$

2.  $A=60^{\circ}, b=40$

$$B=90^{\circ}-A=30^{\circ}.$$

$$\frac{b}{c} = \cos A,$$

故  $c = \frac{b}{\cos A}.$

查表,  $\log b = 0.60206$

$$\text{colog } \cos = 0.30103$$

$$\log c = 0.90309.$$

反查  $c=8.$

$$c^2 = a^2 + b^2.$$

$$c^2 - a^2 = b^2 = 48.$$

$$\log 48 = \log a^2 = 1.68124.$$

故  $\log a = 0.84062.$

$$a = 6.9282.$$

3.  $A=30^{\circ}, a=3$

$$B=90^{\circ}-A=60^{\circ}.$$

$$\frac{a}{c} = \sin A,$$

故  $c = \frac{a}{\sin A}.$

查表  $\log a = .47712$

$$\text{colog } \sin a = 0.30103$$

$$\log c = 0.77815$$

反查  $c=6,$

$$c^2 = a^2 + b^2.$$

$$c^2 - a^2 = b^2 = 27.$$

$$\log 27 = \log b^2 = 1.43136.$$

$$\text{故 } \log b = 0.71568.$$

$$b = 5.1961$$

$$4. \quad a = 4, \quad b = 4$$

a與b既相等，則此三角形必等腰，故A與B亦必相等。

$$\text{故 } A = \frac{1}{2} \times 90^\circ = 45^\circ,$$

$$B = \frac{1}{2} \times 90^\circ = 45^\circ,$$

$$c^2 = a^2 + b^2 = 32.$$

$$\log 32 = \log c^2 = 1.50515$$

$$\text{故 } \log c = 0.75257$$

$$c = 5.6568.$$

$$5. \quad a = 2, \quad c = 2.82843$$

$$b = \sqrt{a^2 - c^2}$$

$$= \sqrt{(c+a)(c-a)}.$$

$$\log b^2 = \log(c+a)$$

$$+ \log(c-a).$$

$$\text{查表, } \log(c+a) = 0.68381.$$

$$\log(c-a) = 9.91826 - 10$$

$$\log b^2 = 0.60207.$$

$$\log b = 0.30103$$

$$\text{反查, } b = 2.$$

故 此三角形必為等腰直角三角形。

$$\text{因得 } A = B = 45.$$

$$6. \quad c = 627, \quad A = 23^\circ 30'$$

$$B = 90^\circ - A = 66^\circ 30'.$$

$$a = c \sin A$$

$$\log a = \log c + \log \sin A.$$

$$\log c = 2.79727$$

$$\log \sin A = 9.60070$$

$$\log a = 2.39797.$$

$$a = 250.02.$$

$$b = c \cos A$$

$$\log b = \log c + \log \cos A.$$

$$\log c = 2.79727$$

$$\log \cos A = 9.96240$$

$$\log b = 2.75967$$

$$b = 575.0$$

$$7. \quad c = 2280, \quad A = 28^\circ 5'$$

$$B = 90^\circ - A = 61^\circ 55'.$$

$$a = c \sin A$$

$$\log a = \log c + \log \sin A.$$

$$\log c = 3.35793$$

$$\log \sin A = 9.67280$$

$$\log a = 3.03073$$

$$a = 1073.3$$

$$b = c \cos A.$$

$$\log b = \log c + \log \cos A.$$

$$\log c = 3.35793.$$

$$\log \cos A = \underline{9.94560}$$

$$\log b = 3.30353'$$

$$b = 2011.5.$$

$$8. \quad c = 72.15, A = 39^\circ 34';$$

$$B = 90^\circ - A = 50^\circ 26'.$$

$$a = c \sin A.$$

$$\log a = \log c + \log \sin A.$$

$$\log c = 1.85824$$

$$\log \sin A = \underline{9.80412}$$

$$\log a = 1.66236$$

$$a = 45.958.$$

$$b = c \cos A,$$

$$\log b = \log c + \log \cos A.$$

$$\log c = 1.85824$$

$$\log \cos A = \underline{9.88699}$$

$$\log b = 1.74523$$

$$b = 55.620.$$

$$9. \quad c = 1, A = 36^\circ.$$

$$B = 90^\circ - A = 54^\circ.$$

$$\frac{a}{c} = \sin A$$

$$\text{故 } a = c \sin A.$$

$$\log a = \log c + \log \sin A.$$

$$\log c = 0.00000$$

$$\log \sin A = \underline{9.76922}$$

$$\log a = 9.76922 - 10$$

$$a = 0.58779.$$

$$\frac{b}{c} = \cos A,$$

$$\text{故 } b = c \cos A.$$

$$\log b = \log c + \log \cos A.$$

$$\log c = 0.00000$$

$$\log \cos A = \underline{9.90796}$$

$$\log b = 9.90796 - 10$$

$$b = 0.80902.$$

$$10. \quad c = 200, B = 21^\circ 47'.$$

$$A = 90^\circ - B = 68^\circ 23'.$$

$$\frac{a}{c} = \sin A.$$

$$\text{故 } a = c \sin A.$$

$$\log a = \log c + \log \sin A.$$

$$\log c = 2.30103$$

$$\log \sin A = \underline{9.96783}$$

$$\log a = 2.26886$$

$$a = 185.72$$

$$\frac{b}{c} = \cos A,$$

$$b = c \cos A.$$

$$\log b = \log c + \log \cos A.$$

$$\log c = 2.30103$$

$$\log \cos A = \underline{9.56949}$$

$$\log b = 1.87052$$

$$b=74.22.$$

$$11. \quad c=93.4, B=70^{\circ}25'$$

$$A=90^{\circ}-B=13^{\circ}35'.$$

$$a=c \sin A,$$

$$\log a = \log c + \log \sin A.$$

$$\log c = 1.97035$$

$$\log \sin A = \underline{9.37081}$$

$$\log a = 1.34116.$$

$$a = 21.936.$$

$$b = a \cot A.$$

$$\log b = \log a + \log \cot A.$$

$$\log a = 1.34116.$$

$$\log \cot A = \underline{0.61687}.$$

$$\log b = 1.95803.$$

$$b = 90.788.$$

$$12. \quad a=637, A=4^{\circ}35'$$

$$B=90^{\circ}-A=85^{\circ}25'.$$

$$b = a \cot A.$$

$$\log b = \log a + \log \cot A.$$

$$\log a = 2.80414.$$

$$\log \cot A = \underline{1.09601}.$$

$$\log b = 3.90015.$$

$$b = 7946.$$

$$c = \frac{a}{\sin A}.$$

$$\log c = \log a + \operatorname{colog} \sin A$$

$$\log a = 8.80414.$$

$$\operatorname{colog} \sin A = \underline{1.09740}.$$

$$\log c = 3.90154.$$

$$c = 7971.5.$$

$$13. \quad a=48.532, A=36^{\circ}44'$$

$$B=90^{\circ}-A=53^{\circ}16'.$$

$$\frac{a}{c} = \sin A,$$

$$\text{故 } c = \frac{a}{\sin A}.$$

$$\log c = \log a$$

$$+ \operatorname{colog} \sin A.$$

$$\log a = 1.68603.$$

$$\operatorname{colog} \sin A = \underline{0.22323}.$$

$$\log c = 1.90926.$$

$$c = 81.144.$$

$$\frac{b}{c} = \cos A,$$

$$\text{故 } b = c \cos A.$$

$$\log b = \log c + \log \cos A.$$

$$\log c = 1.90926.$$

$$\log \cos A = \underline{9.90386}.$$

$$\log b = 1.81312.$$

$$b = 65.031.$$

$$14. \quad a=0.0008, A=86^{\circ}.$$

$$B = 90^\circ - A = 4^\circ.$$

$$\frac{a}{c} = \sin A,$$

$$\text{故 } c = \frac{b}{\sin A}.$$

$$\log c = \log a \\ + \operatorname{colog} \sin A.$$

$$\log a = 6.90309 - 10.$$

$$\operatorname{colog} \sin A = 0.00106.$$

$$\log c = 6.90415 - 10.$$

$$c = 0.000802.$$

$$\frac{b}{c} = \cos A,$$

$$\text{故 } b = c \cos A.$$

$$\log b = \log c + \log \cos A,$$

$$\log c = 6.90415 - 10.$$

$$\log \cos A = 8.84358 - 10.$$

$$\log b = 5.7473 - 10.$$

$$b = 0.0000559.$$

$$15. \quad b = 50.937, B = 43^\circ 38'$$

$$A = 90^\circ - B = 46^\circ 12'.$$

$$\frac{a}{b} = \tan A,$$

$$\text{故 } a = b \tan A.$$

$$\log a = \log b + \log \tan A.$$

$$\log b = 1.70703.$$

$$\log \tan A = 0.01820.$$

$$\log a = 1.72523.$$

$$a = 53.116.$$

$$\frac{a}{c} = \sin A,$$

$$\text{故 } c = \frac{a}{\sin A}.$$

$$\log c = \log a \\ + \operatorname{colog} \sin A.$$

$$\log a = 1.72523.$$

$$\operatorname{colog} \sin A = 0.14161.$$

$$\log c = 1.86684.$$

$$c = 73.593.$$

$$16. \quad b = 2, B = 3^\circ 38'$$

$$A = 90^\circ - B = 86^\circ 32',$$

$$\frac{a}{b} = \tan A,$$

$$a = b \tan A.$$

$$\log a = \log b + \log \tan A.$$

$$\log b = 0.30103.$$

$$\log \tan A = 1.19723.$$

$$\log a = 1.49826.$$

$$a = 31.496.$$

$$\frac{a}{c} = \sin A,$$

$$\text{故 } c = \frac{a}{\sin A}.$$

$$\log c = \log a$$

$$+ \text{colog sin } A.$$

$$\log a = 1.49826.$$

$$\text{colog sin } A = \underline{0.00087}.$$

$$\log c = 1.49913.$$

$$c = 31.559.$$

$$17. \quad a = 992, \quad B = 76^\circ 19'$$

$$A = 90^\circ - B = 13^\circ 41'.$$

$$\frac{a}{c} = \sin A,$$

$$\text{故 } c = \frac{a}{\sin A}.$$

$$\log c = \log a + \text{colog sin } A.$$

$$\log a = 2.99651.$$

$$\text{colog sin } A = \underline{0.62607}.$$

$$\log c = 3.62258.$$

$$c = 4163.5.$$

$$\frac{b}{c} = \sin B,$$

$$b = c \sin B.$$

$$\log b = \log c + \log \sin B.$$

$$\log c = 3.62258.$$

$$\log \sin B = \underline{9.9750}.$$

$$\log b = 3.61008.$$

$$b = 4074.5.$$

$$18. \quad a = 73, \quad B = 68^\circ 52'$$

$$A = 90^\circ - B = 21^\circ 8'.$$

$$\frac{a}{c} = \sin A,$$

$$\text{故 } c = \frac{a}{\sin A}.$$

$$\log c = \log a + \text{colog sin } A.$$

$$\log a = 1.86332.$$

$$\text{colog sin } A = \underline{0.44305}.$$

$$\log c = 2.30637.$$

$$c = 202.47.$$

$$\frac{b}{c} = \sin B,$$

$$b = c \sin B.$$

$$\log c = 2.30637.$$

$$\log \sin B = \underline{9.96976}.$$

$$\log b = 2.27613.$$

$$b = 188.86.$$

$$19. \quad a = 2.189, \quad B = 45^\circ 25'$$

$$A = 90^\circ - B = 44^\circ 35'$$

$$\frac{a}{c} = \sin A,$$

$$\text{故 } c = \frac{a}{\sin A}.$$

$$\log c = \log a + \text{colog sin } A.$$



$$\begin{aligned} \log a &= 0.34025. \\ \text{colog sin } A &= \underline{0.15370}. \\ \log c &= 0.49395. \\ c &= 3.1185. \\ \frac{b}{c} &= \cos A \\ b &= c \cos A. \\ \log c &= 0.49395. \\ \log \cos A &= \underline{0.5262}. \\ \log b &= 0.34657. \\ b &= 2.2211. \\ 20. \quad b &= 4, A = 37^\circ 56'. \\ B &= 90^\circ - A = 52^\circ 4'. \\ \frac{b}{c} &= \cos A, \\ \text{故 } c &= \frac{b}{\cos A}. \\ \log c &= \log b + \text{colog} \\ &\quad \cos A. \\ \log b &= 0.00206. \\ \text{colog cos } A &= \underline{0.10307}. \\ \log c &= 0.70513. \\ c &= 5.0714. \\ \frac{a}{b} &= \tan A, \\ a &= b \tan A. \\ \log b &= 0.60206. \end{aligned}$$

$$\begin{aligned} \log \tan A &= \underline{0.89177}. \\ a &= 3.1176. \\ 21. \quad c &= 8590, a = 4475 \\ \sin A &= \frac{a}{c}. \\ \log \sin A &= \log a + \text{colog } c. \\ \log a &= 3.65089. \\ \text{colog } c &= \underline{6.06601}. \\ \log \sin A &= 9.71690 = 10. \\ A &= 31^\circ 24'. \\ B &= 90^\circ - A = 58^\circ 36'. \\ \frac{b}{a} &= \cot A, \\ b &= a \cot A. \\ \log a &= 3.65089. \\ \log \cot A &= \underline{0.21438}. \\ \log b &= 3.86527. \\ b &= 7332.8. \\ 22. \quad c &= 86.53, a = 71, \\ &\quad 78 \\ \sin A &= \frac{a}{c}. \\ \log \sin A &= \log a + \text{colog} \\ &\quad c. \\ \log a &= 1.85600. \\ \text{colog } c &= \underline{8.06283}. \\ \log \sin A &= 9.91883. \end{aligned}$$

$$A = 56^{\circ} 3'.$$

$$B = 90^{\circ} - A = 33^{\circ}$$

$$57'.$$

$$\frac{a}{b} = \cot A.$$

$$b = \frac{a}{\cot A}.$$

$$\log a = 1.85600.$$

$$\log \cot A = 9.82817.$$

$$\log b = 1.68417.$$

$$b = 48.324.$$

$$23. \quad c = 9.35, a = 8.49$$

$$\sin A = \frac{a}{c}.$$

$$\log \sin A = \log a + \text{colog}$$

$$c.$$

$$\log a = 0.92891.$$

$$\text{colog } c = 9.02919 - 10.$$

$$\log \sin A = 9.95810 - 10.$$

$$A = 65^{\circ} 14'.$$

$$B = 90^{\circ} - A = 24^{\circ}$$

$$46'.$$

$$\frac{b}{c} = \cos A,$$

$$b = c \cos A,$$

$$\log c = 0.97081,$$

$$\log \cos A = 9.62214.$$

$$\log b = 0.59295.$$

$$b = 3.917.$$

$$24. \quad c = 2194,$$

$$b = 1312.7$$

$$\cos A = \frac{b}{c}.$$

$$\log \cos A = \log b \text{ colog}$$

$$c.$$

$$\log b = 3.11816.$$

$$\text{colog } c = 6.66876 - 10.$$

$$\log \cos A = 9.77692 - 10$$

$$A = 53^{\circ} 15'.$$

$$B = 90^{\circ} - A = 36^{\circ}$$

$$45'.$$

$$\frac{a}{c} = \sin A,$$

$$a = c \sin A.$$

$$\log c = 3.34124.$$

$$\log \sin A = 9.90377.$$

$$\log a = 3.24501.$$

$$a = 1758.$$

$$25. \quad c = 30.69,$$

$$b = 18.256$$

$$\cos A = \frac{b}{c}.$$

$$\log \cos A = \log b$$

$$+ \text{colog } c.$$

$$\begin{aligned} \log b &= 1.26140. \\ \text{colog } c &= \underline{8.51300 - 10}. \\ \log \cos A &= 9.77440 - 10 \\ A &= 53^\circ 30'. \\ B &= 90^\circ - A \\ &= 36^\circ 30'. \\ \frac{a}{b} &= \tan A, \\ a &= b \tan A. \\ \log b &= 1.26140. \\ \log \tan A &= \underline{0.13079}. \\ \log a &= 1.39219. \\ a &= 24.671. \\ 26. \quad a &= 38.313, \\ b &= 19.522 \\ \tan A &= \frac{a}{b}. \\ \log \tan A &= \log a \\ &\quad + \text{colog } b. \\ \log a &= 1.58335. \\ \text{colog } b &= \underline{8.70948 - 10} \\ \log \tan A &= 10.29283 - \\ &\quad 10 \\ A &= 63^\circ. \\ B &= 90^\circ - A \\ &= 27^\circ. \end{aligned}$$

$$\begin{aligned} \frac{a}{c} &= \sin A, \\ \text{故} \quad c &= \frac{a}{\sin A}. \\ \log a &= 1.58335. \\ \text{colog } \sin A &= \underline{0.05012}. \\ \log a &= 1.63347. \\ c &= 43. \\ 27. \quad a &= 1.2291, \\ b &= 14.950 \\ \tan A &= \frac{a}{b}. \\ \log \tan A &= \log a \\ &\quad + \text{colog } b. \\ \log a &= 0.08959. \\ \text{colog } b &= \underline{8.82536 - 10} \\ \log \tan A &= 8.91495 - 10 \\ A &= 4^\circ 42'. \\ B &= 90^\circ \\ A &= 85^\circ 18'. \\ \frac{a}{c} &= \sin A, \\ \text{故} \quad c &= \frac{a}{\sin A}. \\ \log a &= 0.08959. \\ \text{colog } \sin A &= \underline{1.08651}. \\ \log c &= 1.17610. \end{aligned}$$

$$\begin{aligned}
 & c=15. \\
 28. \quad & a=415.38, \\
 & b=62.080 \\
 & \tan A = \frac{a}{b}. \\
 & \log \tan A = \log \\
 & \quad a + \text{colog } b. \\
 & \log a = 2.61845. \\
 & \text{colog } b = \underline{8.20705 - 10} \\
 & \log \tan A = 10.82550 - \\
 & \quad 10 \\
 & A = 81^\circ 30'. \\
 & B = 90^\circ - \\
 & A = 8^\circ 30'.
 \end{aligned}$$

$$\begin{aligned}
 & \sin A = \frac{a}{c}, \\
 \text{故} \quad & c = \frac{a}{\sin A}. \\
 & \log a = 2.61845. \\
 & \text{colog } \sin A = \underline{0.00480}. \\
 & \log c = 2.62325.
 \end{aligned}$$

$$\begin{aligned}
 & c = 42c. \\
 29. \quad & a = 13.690 \\
 & b = 16.926 \\
 & \tan A = \frac{a}{b}. \\
 & \log \tan A = \log
 \end{aligned}$$

$$\begin{aligned}
 & a + \text{colog } b. \\
 & \log a = 1.13640 \\
 & \text{colog } b = \underline{8.77144 - 10}. \\
 & \log \tan A = 9.90784 - 10. \\
 & A = 38^\circ 58'. \\
 & B = 90^\circ - A = 51^\circ 2'. \\
 & \sin A = \frac{a}{c}, \\
 \text{故} \quad & c = \frac{a}{\sin A}. \\
 & \log a = 1.13640. \\
 & \text{colog } \sin A = \underline{0.20144}. \\
 & \log c = 1.33784. \\
 & c = 21.769.
 \end{aligned}$$

$$\begin{aligned}
 30. \quad & c = 91.92, a = 2.19; \\
 & \sin A = \frac{a}{c}. \\
 & \log \sin A = \log a + \text{colog } c. \\
 & \log a = 0.34044. \\
 & \text{colog } c = \underline{8.03659 - 10}. \\
 & \log \sin A = 8.37763 - 10. \\
 & A = 1^\circ 22'. \\
 & B = 90^\circ - A = 88^\circ 38' \\
 & \frac{b}{c} = \cos A, \\
 & b = c \cos A. \\
 & \log c = 1.96341.
 \end{aligned}$$

$$\log \cos A = 9.99988.$$

$$\log b = 1.96329.$$

$$b = 91.894.$$

有下之各項。試計算其求知量。並求面積。

$$\tan A = \frac{a}{b}.$$

$$\log \tan A = \log a + \operatorname{colog} b.$$

$$\log a = 0.69897.$$

$$\operatorname{colog} b = 9.22185 - 10.$$

$$\log \tan A = 9.92082 - 10.$$

$$A = 39^\circ 48'.$$

$$B = 90^\circ - A = 50^\circ 12'.$$

$$\sin A = \frac{a}{c}.$$

故 
$$c = \frac{a}{\sin A}.$$

$$\log a = 0.29897.$$

$$\operatorname{colog} \sin A = 0.19375.$$

$$\log c = 0.89272.$$

$$c = 7.8112.$$

$$F (\text{即面積}) = \frac{ab}{2} = \frac{30}{2} = 15.$$

$$32. \quad a = 0.615, c = 70$$

$$\sin A = \frac{a}{c}.$$

$$\log \sin A = \log a + \operatorname{colog} c.$$

$$\log a = 9.78888 - 10.$$

$$\operatorname{colog} c = 8.15490 - 10.$$

$$\log \sin A = 7.94378 - 10.$$

$$A = 30^\circ 12'.$$

$$B = 90^\circ -$$

$$A = 89^\circ 29' 48''.$$

$$b = \sqrt{(c+a)(c-a)}.$$

$$\log b = \frac{\log(c+a) + \log(c-a)}{2}$$

$$\log(c+a) = 1.84890.$$

$$\log(c-a) = 1.84126.$$

$$2 \mid 3.69016.$$

$$\log b = 1.84508.$$

$$b = 69.997.$$

$$F (\text{即面積}) = \frac{1}{2} ab,$$

$$\log a = 9.78888 - 10.$$

$$\log b = 1.84508.$$

$$\operatorname{colog} 2 = 9.69097 - 10.$$

$$\log F = 1.33293.$$

$$F = 21.525.$$

$$33. \quad b = \sqrt[3]{2}, c = \sqrt{3}$$

$$\sqrt[3]{2} = 1.25992.$$

$$\sqrt{3} = 1.73205.$$

$$\cos A = \frac{b}{c}.$$

$$\log \cos A = \log b + \operatorname{colog} c.$$

$$\log b = 0.10034.$$

$$\begin{aligned}\operatorname{colog} c &= \underline{9.76144 - 10.} \\ \log \cos A &= 9.85178 - 10. \\ A &= 43^\circ 20'. \\ B &= 90^\circ - A = 46^\circ 40'.\end{aligned}$$

$$\frac{a}{c} = \sin A,$$

$$a = c \sin A,$$

$$\log c = 0.23856,$$

$$\log \sin A = \underline{9.83648.}$$

$$\log a = 0.07504.$$

$$a = 1.1886.$$

$$F = \frac{1}{2} ab.$$

$$\log a = 0.07504,$$

$$\log b = 0.10034.$$

$$\operatorname{colog} 2 = \underline{9.69897 - 10.}$$

$$\log F = 9.87435 - 10.$$

$$F = 0.74876.$$

$$34. \quad a = 7, \quad A = 18^\circ 14'.$$

$$B = 90^\circ - A = 71^\circ 46'.$$

$$\sin A = \frac{a}{c},$$

$$\text{故} \quad c = \frac{a}{\sin A}$$

$$\log c = \log a + \operatorname{colog}$$

$$\sin A,$$

$$\log a = 0.84510.$$

$$\operatorname{colog} \sin A = \underline{0.5046.}$$

$$\log c = 1.3407.$$

$$c = 22.372.$$

$$\tan A = \frac{a}{b},$$

$$\text{故} \quad b = \frac{a}{\tan A}$$

$$\log a = 0.84510.$$

$$\operatorname{colog} \tan A = \underline{0.48224 - 10.}$$

$$\log b = 1.32734.$$

$$b = 21,249.$$

$$F = \frac{1}{2} ab.$$

$$\log a = 0.84510.$$

$$\log b = 1.32734.$$

$$\operatorname{colog} 2 = \underline{9.69897 - 10.}$$

$$\log F = 1.87141,$$

$$F = 74.372.$$

$$35 \quad b = 12, \quad A = 29^\circ 8'.$$

$$B = 90^\circ - A = 60^\circ 52'.$$

$$\cos A = \frac{b}{c},$$

$$\text{故} \quad c = \frac{b}{\cos A}$$

$$\log c = \log b + \operatorname{colog}$$

$$\cos A$$

$$\begin{aligned} \log b &= 1.07918. \\ \text{colog } \cos A &= \underline{0.05874}. \\ \log c &= 1.13792. \\ c &= 13.738. \\ \frac{a}{c} &= \sin A, \\ a &= c \sin A. \\ \log c &= 1.13792. \\ \log \sin A &= \underline{0.68739}. \\ \log a &= 0.82531. \\ A &= 6.6882. \\ F &= \frac{1}{2}ab. \\ \log a &= 0.82531, \\ \log b &= 1.07918. \\ \text{colog } 2 &= \underline{9.69897. - 10.} \\ \log F &= 1.60346. \\ F &= 40.129. \\ 36. \quad c &= 68, A = 69^\circ 54' \\ B &= 90^\circ - A = 20^\circ 6'. \\ \frac{a}{c} &= \sin A, \\ a &= c \sin A, \\ \log a &= \log c + \log \sin A \\ \log c &= 1.83251, \\ \log \sin A &= \underline{9.97271}. \end{aligned}$$

$$\begin{aligned} \log a &= 1.80522, \\ a &= 63.859. \\ \frac{b}{c} &= \cos A, \\ b &= c \cos A. \\ \log c &= 1.83251. \\ \log \cos A &= \underline{9.53613}. \\ \log b &= 1.36854. \\ b &= 23.369. \\ F &= \frac{1}{2}ab. \\ \log a &= 1.80522. \\ \log b &= 1.36864. \\ \text{colog } 2 &= \underline{9.69897. - 10.} \\ \log F &= 2.87283. \\ F &= 746.15. \\ 37. \quad c &= 27, B = 44^\circ 4'; \\ A &= 90^\circ - B = 45^\circ 56' \\ a &= c \sin A, \\ \log a &= \log c + \log \sin A \\ \log c &= 1.4313. \\ \log \sin A &= \underline{9.85645}. \\ \log a &= 1.28781. \\ a &= 19.40. \\ b &= c \cos A. \\ \log b &= \log c + \log \cos A \\ \log c &= 1.43136, \end{aligned}$$

$$\begin{aligned} \log \cos A &= \underline{9.84429.} \\ \log b &= 1.27365. \\ b &= 18.778. \\ F &= \frac{1}{2} ab. \\ \log a &= 1.28781. \\ \log b &= 1.27365. \\ \text{colog } z &= \underline{9.69897 - 10.} \\ \log F &= 2.26043. \\ F &= 182.15. \\ 28. \quad a &= 47, B = 48^\circ 49' \\ A &= 90^\circ - B = 41^\circ 11' \\ b &= a \cot A, \\ \log b &= \log a + \log \cot \\ & \quad A, \\ \log a &= 1.67210, \\ \log \cot A &= \underline{10.05803.} \\ \log b &= 1.73013. \\ b &= 52.719. \\ c &= \frac{a}{\sin A}. \\ \log a &= 1.67210, \\ \text{colog } \sin A &= \underline{.18146.} \\ \log c &= 1.85356. \\ c &= 71.377. \\ F &= \frac{1}{2} ab, \end{aligned}$$

$$\begin{aligned} \log a &= 1.67210. \\ \log b &= 1.73013. \\ \text{colog } z &= \underline{9.69897 - 10.} \\ \log F &= 3.10120. \\ F &= 1262.4. \\ 39. \quad b &= 9, B = 34^\circ 44' \\ A &= 90^\circ - B = 55^\circ \\ & \quad 16'. \\ a &= b \tan A. \\ \log a &= \log b + \log \tan \\ & \quad A. \\ \log b &= 0.95424. \\ \log \tan A &= \underline{10.15908. - 10} \\ \log a &= 1.11332. \\ a &= 12.981. \\ c &= \frac{a}{\sin A}. \\ \log a &= 1.11332. \\ \text{colog } \sin A &= \underline{0.08523.} \\ \log c &= 1.19815. \\ c &= 15.795. \\ F &= \frac{1}{2} ab. \\ \log a &= 1.11332. \\ \log b &= 0.95424. \\ \text{colog } z &= \underline{9.69897 - 10.} \end{aligned}$$



$$\log F = 1.76653.$$

$$F = 58.416.$$

$$4. \quad c = 84.62, B = 86^\circ 4'$$

$$A = 90^\circ - B = 3^\circ 56'$$

$$a = c \sin A.$$

$$\log a = \log c + \log \sin A$$

$$\log c = 0.92747.$$

$$\log \sin A = \underline{8.83630 - 10.}$$

$$\log a = 9.76377 - 10.$$

$$a = 0.58046.$$

$$b = c \cos A.$$

$$\log c = 0.92747.$$

$$\log \cos A = \underline{0.90858.}$$

$$\log b = 0.92645.$$

$$b = 8.442.$$

$$F = \frac{1}{2} ab.$$

$$\log a = 9.76377 - 10.$$

$$\log b = 0.92645.$$

$$\text{colog } 2 = \underline{9.69897 - 10.}$$

$$\log F = 0.38919.$$

$$F = 2.4501.$$

41. 試以c及A表F之值

$$F = \frac{1}{2} ab.$$

$$\frac{a}{c} = \sin A.$$

故  $a = c \sin A.$

$$\frac{b}{c} = \cos A,$$

故  $b = c \cos A.$

$$F = \frac{1}{2} ab.$$

$$= \frac{1}{2} c^2 \sin A \cos A.$$

24. 試以a及A表F之值

$$F = \frac{1}{2} ab.$$

$$\frac{b}{c} = \cot A,$$

故  $b = a \cot A.$

$$F = \frac{1}{2} ab.$$

$$= \frac{1}{2} a^2 \cot A.$$

43. 試以b及A表F之值

$$F = \frac{1}{2} ab.$$

$$\frac{a}{b} = \tan A,$$

故  $a = b \tan A.$

$$F = \frac{1}{2} ab.$$

$$= \frac{1}{2} b^2 \tan A.$$

44. 試以a及c表F之值

$$F = \frac{1}{2} ab.$$

$$c^2 = a^2 + b^2.$$

$$b^2 = c^2 - a^2.$$

$$b = \sqrt{c^2 - a^2}.$$

故  $F = \frac{1}{2} a \sqrt{c^2 - a^2}.$

已知下之各項。試解各三角

形(45-48)。

45.  $F=58, a=10.$

$$F = \frac{1}{2}ab,$$

$$b = \frac{2F}{a}.$$

$$\log b = \log 2F + \operatorname{colog} a.$$

$$\log 2F = 2.06446.$$

$$\operatorname{colog} a = \underline{9.00000 - 10}.$$

$$\log b = 1.06446.$$

$$b = 11.6.$$

$$\tan A = \frac{a}{b}.$$

$$\log \tan A = \log a + \operatorname{colog} b.$$

$$\log a = 1.00000.$$

$$\operatorname{colog} b = 9.93554 - 10.$$

$$\log \tan A = \underline{9.93554 - 10}.$$

$$A = 40^\circ 45' 48''.$$

$$B = 90^\circ - A.$$

$$= 49^\circ 14' 12''.$$

$$c = \frac{a}{\sin A}.$$

$$\log a = 1.00000,$$

$$\operatorname{colog} \sin A = \underline{0.18513}.$$

$$\log c = 1.18513.$$

$$c = 15.315.$$

46.  $F=18, b=5$

$$F = \frac{1}{2}ab,$$

$$a = \frac{2F}{b}.$$

$$\log a = \log 2F + \operatorname{colog} b.$$

$$\log 2F = 1.55630.$$

$$\operatorname{colog} b = \underline{9.30103 - 10}.$$

$$\log a = 0.85733,$$

$$a = 7.2.$$

$$\tan A = \frac{a}{b}.$$

$$\log \tan A = \log a + \operatorname{colog} b.$$

$$\log a = 0.85733$$

$$\operatorname{colog} b = \underline{9.30103 - 10}$$

$$\log \tan A = 10.15836 - 10$$

$$A = 55^\circ 13' 20''.$$

$$B = 90^\circ - A$$

$$= 34^\circ 46' 40''.$$

$$c = \frac{a}{\sin A}.$$

$$\log a = 0.85733$$

$$\operatorname{colog} \sin A = \underline{0.08546}$$

$$\log c = 0.94279$$

$$c = 8.7658.$$

47.  $F=12, A=29^\circ$

$$B = 90^\circ - A = 61^\circ,$$

$$\frac{1}{2}ab = (F) = 12.$$

$$ab = 24.$$

$$a = \frac{24}{b}$$

$$\tan A = \frac{a}{b}$$

$$\tan 20^\circ = \frac{24}{b^2}$$

$$b^2 = \frac{24}{\tan 20^\circ}$$

$$\log b = \frac{1}{2}(\log 24 + \operatorname{colog} \tan 20^\circ)$$

$$\log 24 = 1.38021$$

$$\operatorname{colog} \tan 20^\circ = 0.25625$$

$$2 \mid 1.63646$$

$$\log b = 0.81823$$

$$b = 6.58.$$

$$\tan A = \frac{a}{b}$$

故  $a = b \tan 20^\circ.$

$$\log b = 0.81823.$$

$$\log \tan 20^\circ = 0.74375$$

$$\log a = 0.56193$$

$$a = 3.6474.$$

$$\tan A = \frac{a}{c}$$

故  $c = \frac{a}{\sin 20^\circ}$

$$\log a = 0.56198$$

$$\operatorname{colog} \sin 20^\circ = 0.31443$$

$$\log c = 0.87641$$

$$c = 7.5233.$$

48:  $F = 100, c = 220$

$$\frac{1}{2}ab = (F) = 100.$$

$$ab = 200.$$

$$a = \frac{200}{b}$$

$$a^2 = \frac{40000}{b^2}$$

$$a^2 + b^2 = (c^2) = 484.$$

代入,  $\frac{40000}{b^2} + b^2 = 484.$

$$40,000 + b^4 = 484b^2.$$

$$b^4 - 484b^2 = -40,000.$$

加以  $(242)^2$

$$b^4 - 484b^2 + (242)^2 = 18,564.$$

但  $\log \sqrt{18564} = \frac{1}{2}(4.26867)$   
 $= 2.13434.$

又  $21.3434 = \log 136.25.$

故 (將以上之方程開方) 得

$$b^2 - 242 = 136.25:$$

$$b^2 = 378.25$$

$$\log b = \frac{1}{2}(\log 378.25)$$

$$= 1.28889$$

$$b = 19.449$$

$$\cos A = \frac{b}{c}.$$

$$\log \cos A = \log b + \operatorname{colog} c.$$

$$\log b = 1.28889.$$

$$\operatorname{colog} c = \underline{8.65758} - 10$$

$$\log \cos A = 9.94647$$

$$A = 27^\circ 42'.$$

$$B = 90^\circ - A$$

$$= 62^\circ 8'.$$

$$\frac{a}{c} = \sin A,$$

$$a = c \sin A.$$

$$\log c = 1.34242$$

$$\log \sin A = \underline{9.66970}$$

$$\log a = 1.01212$$

$$a = 10.283$$

49. 設直角三角形之斜邊等於其一邊之三倍。求各角。

設  $c = \text{弦}$ ,  $a = \text{兩腰之一}$

則  $c = 3a$ .

$$\sin A = \frac{a}{c} = \frac{1}{3}.$$

$$\log \sin A = \log a$$

$$+ \operatorname{colog} c.$$

$$\log a = 0.00000$$

$$\operatorname{colog} c = \underline{8.52288} - 10$$

$$\log \sin A = 9.52288 - 10$$

$$A = 19^\circ 28' 17''.$$

$$B = 90^\circ - A$$

$$= 70^\circ 31' 43''$$

50. 設直角三角形之斜邊為 6。而其一角為餘一角之二倍。求餘二邊。

設  $c = \text{弦} = 6$ ,  $B = 2A$

則  $B = 60^\circ$ ,  $A = 30^\circ$ .

$$\frac{a}{c} = \sin A,$$

$$a = c \sin A$$

$$\log a = \log c + \log \sin A.$$

$$\log c = 0.77815$$

$$\log \sin A = \underline{9.69897}$$

$$\log a = 0.47712$$

$$a = 3.$$

$$\frac{b}{c} = \sin B,$$

$$b = c \sin B.$$

$$\log c = 0.77815$$

$$\log \sin B = \underline{9.93753}$$

$$\log b = 0.71568$$

$$b = 5.1951.$$

於  $\text{rt}\triangle$  內。已知  $C$  及  $A = nB$ 。  
求  $a$  及  $b$ 。

$$B = 90^\circ - A = 90^\circ - nB.$$

$$B(n+1) = 90^\circ,$$

$$\text{故 } B = \frac{90^\circ}{n+1}.$$

$$\frac{a}{c} = \cos B = \cos$$

$$\frac{90^\circ}{n+1},$$

$$\text{故 } a = c \cos \frac{90^\circ}{n+1}.$$

$$\frac{b}{c} = \sin B = \sin$$

$$\frac{90^\circ}{n+1},$$

$$\text{故 } b = c \sin \frac{90^\circ}{n+1}.$$

52. 於rt△內其斜邊與大邊之較。某於大小二邊之較求各角。

設  $a =$  大邊,  $b =$  小邊,  
 $c =$  斜。

$$\text{故 } c - a = a - b.$$

$$2a - b = c. \quad (1)$$

$$a^2 + b^2 = c^2 \quad (2)$$

以 (1) 乘方, 與 (2) 相減,

$$4a^2 - 4ab + b^2 = c^2.$$

$$\frac{a^2}{\quad} + b^2 = c^2.$$

$$3a^2 - 4ab = 0.$$

$$3a^2 = 4ab.$$

$$3a = 4b.$$

$$a = \frac{4}{3}b.$$

$$\tan A = \frac{a}{b} = \frac{4}{3}.$$

$$\log \tan A = \log 4 + \text{colog} 3$$

$$\log 4 = 0.60206$$

$$\text{colog} 3 = 0.52288 - 10$$

$$\log \tan A = 10.12494 - 10$$

$$A = 54^\circ 7' 48''.$$

$$B = 90^\circ - A$$

$$= 36^\circ 52' 12''.$$

53. 自尖塔底之水平距 120 呎之地測其頂之仰角為  $60^\circ 30'$  求塔高。

設  $a =$  塔高,  $b =$  離塔底之距,

$A =$  仰角.

$$\tan A = \frac{a}{b}.$$

$$a = b \tan A.$$

$$= 120 \times \tan 60^\circ 30'.$$

$$\log a = \log 120 + \log$$

$$\tan 60^\circ 30'.$$

$$\log 120 = 2.07918$$

$$\log \tan 60^\circ 30' = 10.24736 - 10$$

$$\log a = 2.32654$$

$$a = 212.1$$

即尖塔高 = 212.1 呎。

54. 一石垂直出於水面上 326 呎。自其頂測得一艇之仰角為  $24^\circ$ 。求此艇與石底之距。

設  $a$  = 石高,  $b$  = 船石之距,

$$A = \text{俯角.}$$

$$\frac{b}{a} = \cot A.$$

$$b = a \cot A.$$

$$\log b = \log a + \log \cot A.$$

$$\log a = 2.51322$$

$$\log \cot A = 10.35142 - 10$$

$$\log b = 2.86464$$

$$b = 732.22.$$

即 其距 = 732.22 呎。

55. 設一碑之高為 200 呎。而其頂之仰角測得為  $3^\circ 30'$ 。問自測點至碑之水平距。

設  $a$  = 碑高, = 碑距.

$$A = \text{仰角.}$$

$$\frac{b}{a} = \cot A,$$

$$b = a \cot A.$$

$$\log b = \log a + \log \cot A.$$

$$\log a = 2.30103$$

$$\log \cot A = 1.21351$$

$$\log b = 3.51454$$

$$b = 3270.$$

即碑之水平距 = 327 呎。

56. 沿江岸測得 AB 距為 9 呎。而 A 與隔岸一樹 C 相對。已知  $\angle C$  角為  $21^\circ 14'$  求江寬。

$$\tan A = AC \div AB.$$

$$\text{故 } AC = AB \times \tan B.$$

$$\log AC = \log AB + \log \tan B.$$

$$\log AB = 1.98227$$

$$\log \tan B = 9.58944$$

$$\log AC = 1.57171.$$

$$AC = 37.3.$$

即 其寬 = 37.3 尺。

57. 設一倚面。於水平距 40 呎之地升起 1 呎。則此面之仰角為何。

$$\tan A = \frac{a}{b} = \frac{1}{40}.$$

$$\log \tan A = \log a + \text{colog } b.$$

$$\log A = 0.00000$$

$$\text{colog } b = 8.36794 - 10$$

$$\log \tan A = 8.39794$$

$$A = 1^\circ 25' 56''.$$

= 仰角.

53. 高  $120^\circ$  之塔影(水平影)長  $70^\circ$ , 求太陽之高度。

$$\tan A = \frac{a}{b} = \frac{120}{70},$$

$$\log \tan A = \log a + \operatorname{colog} b,$$

$$\log a = 2.07613$$

$$\operatorname{colog} b = 8.15490 - 10$$

$$\log \tan A = 10.23108 - 10$$

$$A = 59^\circ 44' 35''.$$

= 仰角.

59. 樹影長 80 呎。太陽之高度為  $50^\circ$ 。求樹高。

設  $a$  = 樹高,  $b$  = 影長,  
 $A$  = 日之仰角.

$$\frac{a}{b} = \tan A,$$

$$a = b \tan A$$

$$= 80 \times \tan 50^\circ.$$

$$\log A = \log 80 + \log \tan 50^\circ$$

$$\log 80 = 1.90309$$

$$\log \tan 52 = 10.07619 - 10$$

$$\log a = 1.97928$$

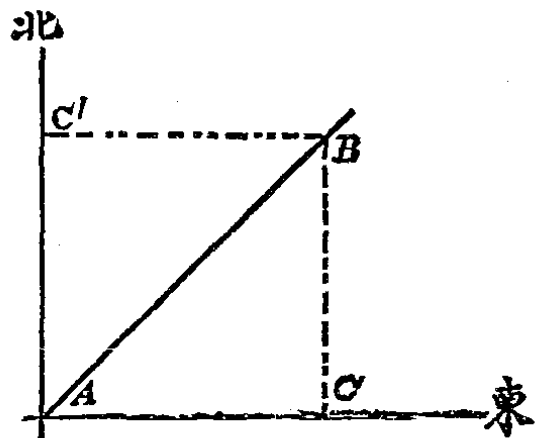
$$a = 95.34$$

即樹高 = 95.34 呎.

60. 一船向東北行。其速率為每小時 10 哩。求向東及向北之速率。

設  $AB$  為船之方向, 又等於一小時之速率即 10 哩。  
又  $AC$  = 每小時東行之速率

因此船正向東北  
故  $A = 45^\circ$ 。



$$\frac{AC}{AB} = \cos A,$$

$$AC = AB \times \cos A;$$

$$\log AC = \log AB + \log \cos A.$$

$$\log AB = 1.00000$$

$$\log \cos A = 9.84949$$

$$\log AC = 0.84949$$

$$AC = 7.0712$$

即 每小時東行7.0712哩。

船既正向東北即北行之哩

數必同。

61. 於20呎高之窗前。有6呎高之床。今以梯置於床邊。而適與窗齊。求梯長。

$$\tan A = \frac{a}{b} = \frac{20}{6}.$$

$$\log \tan A = \log 20 \\ + \text{colog } 6.$$

$$\log 20 = 1.30103$$

$$\text{colog } 6 = 9.22185 - 10$$

$$\log \tan A = 10.52288 - 10$$

$$A = 73^\circ 18'.$$

$$c = \frac{a}{\sin A}.$$

$$\log a = 1.30103$$

$$\text{colog } \sin A = 0.01871$$

$$\log c = 1.31974$$

$$c = 20.88.$$

即 梯之長 = 20.88呎。

62. 40呎長之梯。置於街之一邊。適與33呎高之窗齊。若置

於彼邊而不動其梯根。適與彼邊21呎高之窗齊。求街寬。

設  $B, B'$  = 梯倚兩窗所成之兩角。

$b, b'$  = 其相當之對邊。

$$\cos B = \frac{33}{40}.$$

$$\log \cos B = \log 33 + \text{colog } 40.$$

$$\log 33 = 1.51851$$

$$\text{colog } 40 = 8.39794 - 10$$

$$\log \cos B = 9.91645 - 10$$

$$B = 34^\circ 24' 45''.$$

$$\tan B = \frac{b}{33}.$$

故  $b = 33 \tan B.$

$$\log 33 = 1.51851$$

$$\log \tan B = 9.83571$$

$$\log b = 1.35422$$

$$b = 22.605$$

$$\sin B' = \frac{21}{40}.$$

$$\log 21 = 1.32222$$

$$\text{colog } 40 = 8.39794 - 10$$

$$\log \cos B' = 9.72016$$



$$B' = 58^{\circ} 19' 54''.$$

$$\tan B' = \frac{b'}{21}.$$

$$b' = 21 \tan B'.$$

$$\log 21 = 1.32222$$

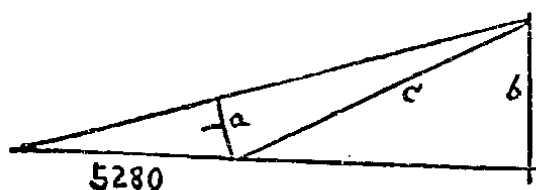
$$\log \tan B' = \underline{0.23982}$$

$$\log b' = 1.53204$$

$$b' = 34.044.$$

$$\text{街寬} = b + b' = 34.044.$$

63. 於山頂測平地上直路內表哩石之連續兩點。得仰角  $5^{\circ}, 15^{\circ}$  求山高



兩石之距 = 1 哩 = 5280 呎.

作垂線  $a$ .

$$\sin 5^{\circ} = \frac{a}{5280},$$

$$\text{故 } a = 5280 \sin 5^{\circ}.$$

$$\log 5280 = 3.72263.$$

$$\log \sin 5^{\circ} = \underline{8.94030 - 10}.$$

$$\log a = 2.66293.$$

$$\sin 10^{\circ} = \frac{a}{c},$$

$$\text{故 } c = \frac{a}{\sin 10^{\circ}},$$

$$\log a = 2.66293.$$

$$\text{colog } \sin 10 = \underline{0.76033}.$$

$$\log c = 3.42326.$$

$$\cos 75^{\circ} = \frac{b}{c},$$

$$\text{故 } b = c \cos 75^{\circ}.$$

$$\log c = 3.42326.$$

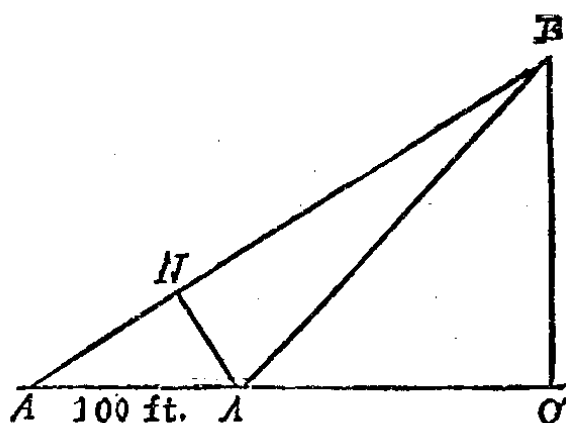
$$\log \cos 75 = \underline{9.41300 - 10}.$$

$$\log b = 2.83626.$$

$$b = 6.85.9.$$

即 山高 = 685.9 呎

94. 自平面內某點望礮台。測得仰角為  $30^{\circ}$  再自某點近礮台 100 呎 測之得  $45^{\circ}$  求礮台之高



設  $B$  = 礮臺,  $AC$  = 地平  
面,  $BC$  = 自臺至地之垂

則  $\widehat{BAC} =$  在定點測得之仰角  $= 30^\circ$ .  $\widehat{BA'C} =$  走近 100 呎測得之仰角  $= 45^\circ$ . 自  $A'$  作  $A'N$  與  $AB$  直交.

於  $\triangle A'N$  直角三角形中, 有  $\widehat{NAA'} = 30^\circ$ .  
 $\widehat{NA'A} = 60^\circ$ .

故  $NA' = \left( \frac{1}{2} \times 100 \text{ 呎} \right)$   
 $= 50 \text{ 呎},$

$$AN = \sqrt{(100)^2 - (50)^2}$$

$$= \sqrt{7500} = 50\sqrt{3}$$

$$= 86.602$$

於  $\triangle BNA'$  直角三角形中有

$$\frac{BN}{NA'} = \cot \widehat{NBA'} = \cot 15^\circ,$$

$$BN = NA' \cot 15^\circ.$$

$$\log NA' = 1.69897$$

$$\log \cot 15^\circ = 0.57195$$

$$\log BN = 2.27092.$$

$$BN = 186.60$$

$$\text{但 } AN = 86.60$$

$$\text{故 } AB = 273.28$$

於  $\triangle ABC$  直角三角形中有  $\widehat{BAC} = 30^\circ$ .

$$\widehat{ABC} = 60^\circ.$$

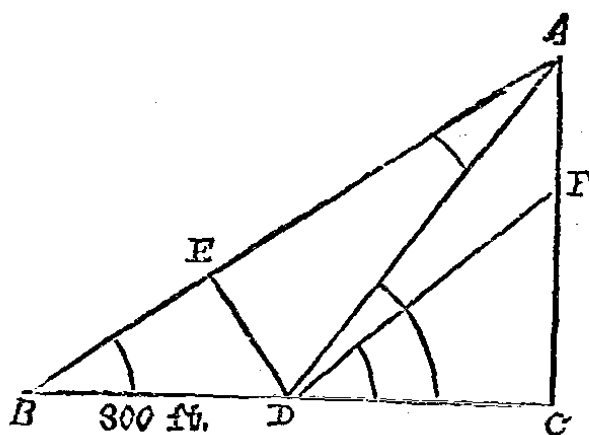
$$\text{故 } AB = \frac{1}{2} AB$$

$$= \frac{1}{2} = 273.20 \text{ 呎}$$

$$= 136.60 \text{ 呎}.$$

即駁臺之高  $= 136.60$  呎.

65. 自地上某點。測教堂鐘樓及塔頂。各得仰角  $40^\circ$  及  $51^\circ$ 。再自某點於水平線內遠行 300 呎測之得塔尖之仰角為  $33^\circ 45'$  求鐘樓及塔尖之距。



設  $F =$  鐘樓,  $A =$  塔尖,  
 $B$  與  $D$  為測量之點.

作  $DE =$  自  $D$  至  $AB$  之  
 垂線.

於  $\triangle BED$  直角三角形中,

$$\text{有 } \frac{ED}{BD} = \sin 33^\circ 45'.$$

$$\text{故 } ED = 300 \times \sin 33^\circ$$

$$45'.$$

$$\log 300 = 2.47712$$

$$\log \sin 33^\circ 45' = \underline{9.74474}$$

$$\log ED = 2.22186$$

$$\widehat{EAD} = 180^\circ - 33^\circ 45'$$

$$- (180^\circ - 51^\circ)$$

$$= 17^\circ 15'.$$

於 ADE 直角三角形中

$$\text{有 } \frac{ED}{AD} = \sin 17^\circ 15'.$$

$$\text{故 } AD = \frac{ED}{\sin 17^\circ 15'}.$$

$$\log ED = 2.22186$$

$$\text{cologsine } 17^\circ 15' = \underline{0.52791}$$

$$\log AD = 2.74977$$

於 ADC 直角三角形中

$$\text{有 } \frac{DC}{AD} = \cos 51^\circ.$$

$$\text{即 } DC = AD \cos 51^\circ.$$

$$\log AD = 2.74977$$

$$\log \cos 51^\circ = \underline{9.79897}$$

$$\log DC = 2.54864$$

$$\text{又有 } \frac{AC}{DC} = \tan 51^\circ.$$

$$\text{即 } AC = DC \tan 51^\circ.$$

$$\log DC = 2.54864$$

$$\log \tan 51^\circ = \underline{10.09163}$$

$$\log AC = 2.64027$$

$$AC = 436.79 \text{ 呎.}$$

於 FDC 直角三角形中

$$\text{有 } \frac{FC}{DC} = \tan 40^\circ.$$

$$\text{即 } FC = DC \tan 40^\circ.$$

$$\log DC = 2.54864$$

$$\log \tan 40^\circ = \underline{9.92381}$$

$$\log FC = 2.47245$$

$$FC = 296.79 \text{ 呎}$$

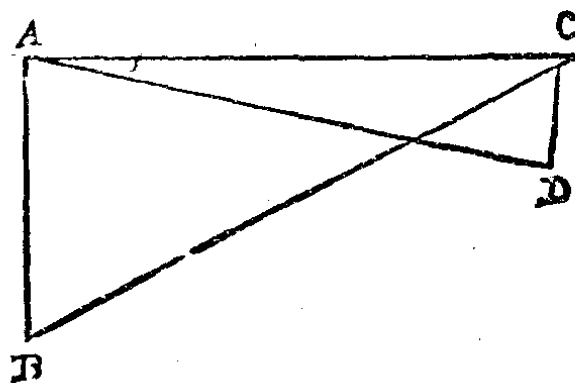
自鐘樓至塔尖之距離

$$= AC - FC$$

$$= 436.79 - 266.79$$

$$= 142 \text{ 呎.}$$

66. 自一點 A。測一不可到之礮台頂 C。得 C 之仰角為  $12^\circ$  再自正交 AC 之 AB 線內距 A 為 219 呎之一點 B 測之。得 ABC 角為  $61^\circ 45'$  求此礮台之高。



按圖 CD = 礮台之高。

於 CAB 直角三角形中，有

$$\frac{AB}{AC} = \cot ABC.$$

$$\text{即 } AC = \frac{AB}{\cot ABC}.$$

$$\log AC = \log AB \\ + \text{colog } \cot ABC.$$

$$\log AB = 2.34044$$

$$\text{colog } \cot ABC = \underline{0.26977}$$

$$\log AC = 2.61021$$

於 ADC 直角三角形中，

$$\text{有 } \frac{CD}{AC} = \sin CAD.$$

$$\text{即 } CD = AC \sin CAD.$$

$$\log AC = 2.61021$$

$$\log \sin CAD = \underline{9.31788 - 10}$$

$$\log CD = 1.9282$$

$$CD = 84.74.$$

即砲臺之高 = 84.74 呎。

### 例題 X

試解下列之等腰三角形。其角之值。核至最近之角

1. 已知 a 及 A 求 C, c, h。

$$C = 180^\circ - 2A = 2(90^\circ \\ - A).$$

$$\frac{\frac{1}{2}c}{a} = \cos A,$$

$$\text{即 } c = 2a \cos A.$$

$$\frac{h}{a} = \sin A,$$

$$\text{即 } h = a \sin A.$$

2. 已知 a 及 C, 求 A, c, h。

$$C + 2A = 180,$$

$$\text{即 } A = 90^\circ - \frac{1}{2}C.$$

$$\frac{\frac{1}{2}c}{a} = \cos A$$

$$\text{即 } c = 2a \cos A.$$

$$\frac{h}{a} = \sin A,$$

$$\text{即 } h = a \sin A.$$

3. 已知 c 及 A, 求 C, a, h。

$$C = 180^\circ - 2A = 2(90^\circ \\ - A).$$

$$\frac{\frac{1}{2}c}{a} = \cos A,$$

$$2a = \frac{c}{\cos A},$$

$$\text{即 } a = \frac{c}{2 \cos A}.$$

$$\frac{h}{a} = \sin A,$$

即  $h = a \sin A.$

4. 已知  $c$  及  $C$ , 求  $A, a, h.$

$$A = 90^\circ - \frac{1}{2}C.$$

$$\frac{\frac{1}{2}c}{a} = \cos A,$$

$$a = \frac{c}{2 \cos A}.$$

$$\frac{h}{a} = \sin A,$$

$$h = a \sin A,$$

5. 已知  $h$  與  $A$ , 試求

$$C, a, c.$$

$$C = 2(90^\circ - A).$$

$$\sin A = \frac{h}{a},$$

即  $a = \frac{h}{\sin A}.$

$$\cos A = \frac{\frac{1}{2}c}{a} = \frac{c}{2a},$$

即  $c = 2a \cos A.$

6. 已知  $h$  及  $C$ , 求  $A, a, c.$

$$A = 90^\circ - \frac{1}{2}C.$$

$$\sin A = \frac{h}{a},$$

即  $a = \frac{h}{\sin A}.$

$$\cos A = \frac{\frac{1}{2}c}{a} = \frac{c}{2a},$$

即  $c = 2a \cos A.$

7. 已知  $a$  及  $h.$  求  $A, C, c.$

$$\sin A = h \div a.$$

$$C = 2(90^\circ - A).$$

$$\cos A = \frac{\frac{1}{2}c}{a} = \frac{c}{2a},$$

即  $c = 2a \cos A.$

8. 已知  $c$  及  $h.$  求  $A, C, a.$

$$\tan A = \frac{h}{\frac{1}{2}c}.$$

$$C = 2(90^\circ - A).$$

$$\sin A = \frac{h}{a},$$

$$a = \frac{h}{\sin A}.$$

9. 已知  $a = 14.3, c = 11.0$  求  $A, C, h.$

$$\cos A = \frac{\frac{1}{2}c}{a}$$

$$\log \cos A = \log \frac{1}{2}c + \operatorname{colog} a,$$

$$\log \frac{1}{2}c = 0.74036$$

$$\operatorname{colog} a = \underline{8.84466 - 10}$$

$$\log \cos A = 9.58502 - 10$$

$$A = 67^\circ 22' 50''.$$

$$C = 2(90^\circ - A)$$

$$= 45^\circ 14' 20''.$$

$$\sin A = \frac{h}{a},$$

$$h = a \sin A.$$

$$\log a = 1.15534$$

$$\log \sin A = 9.96524$$

$$\log h = 1.12058.$$

$$h = 13.2.$$

10. 已知  $a = 0.2950$ ,  $A = 68^\circ 10'$ . 求  $c, h, F$ .

$$\sin A = \frac{h}{a},$$

$$h = a \sin A.$$

$$\log h = \log a + \log \sin A.$$

$$\log a = 9.46982 - 10$$

$$\log \sin A = \underline{.95767 - 10}$$

$$\log h = 9.43749 - 10$$

$$h = 0.27348.$$

$$\cos A = \frac{\frac{1}{2}c}{a}$$

$$\frac{1}{2}c = a \cos A.$$

$$\log a = 9.46982 - 10$$

$$\log \cos A = \underline{9.57044 - 10}$$

$$\log \frac{1}{2}c = 9.04026 - 10$$

$$\frac{1}{2}c = 0.109714,$$

$$c = 0.21943.$$

$$F = \frac{1}{2}ch,$$

$$2F = ch.$$

$$\log c = 9.34130 - 10$$

$$\log h = \underline{9.43749 - 10}$$

$$\log 2F = 8.77879 - 10$$

$$2F = 0.050689$$

$$F = 0.03004.$$

11. 已知  $c = 2.3520$ ,  $C = 69^\circ 49'$ . 求  $a, h, F$ .

$$\frac{1}{2}C = 34^\circ 54' 30''.$$

$$\sin \frac{1}{2}C = \frac{\frac{1}{2}c}{a},$$

$$a = \frac{\frac{1}{2}c}{\sin \frac{1}{2}C}$$

$$\log a = \log \frac{1}{2}c$$

$$+ \text{colog} \sin \frac{1}{2}C.$$

$$\log \frac{1}{2}c = 0.07041$$

$$\text{colog} \sin \frac{1}{2}C = 0.24240$$

$$\log a = 0.31281$$

$$a = 2.055.$$

$$\cos \frac{1}{2}C = \frac{h}{a},$$

$$h = a \cos \frac{1}{2}C.$$

$$\log a = 0.31281$$

$$\log \cos \frac{1}{2}C = 0.91385$$

$$\log h = 0.22666$$

$$h = 1.6852.$$

$$F = \frac{1}{2}ch,$$

$$2F = ch.$$

$$\log c = 0.37144$$

$$\log h = 0.22666$$

$$\log 2F = 0.59810.$$

$$2F = 3.9637$$

$$F = 1.9819.$$

12. 已知  $h = 7.4847$ ,  $A = 76^\circ 14'$ ; 求  $a, c, F$ .

$$\sin A = \frac{h}{a},$$

$$a = \frac{h}{\sin A}.$$

$$\log a = \log h + \text{colog} \sin A.$$

$$\log h = 0.87417$$

$$\text{colog} \sin A = 0.01266$$

$$\log a = 0.88683$$

$$a = 7.706.$$

$$\tan A = \frac{h}{\frac{1}{2}c},$$

$$\frac{1}{2}c = \frac{h}{\tan A}.$$

$$\log h = 0.87417$$

$$\text{colog} \tan A = 9.38918 - 10$$

$$\log \frac{1}{2}c = 0.26335$$

$$\frac{1}{2}c = 1.8338.$$

$$c = 3.6676.$$

$$F = \frac{1}{2}ch.$$

$$\log \frac{1}{2} c = 0.26335$$

$$\log h = 0.87417$$

$$\log F = 1.13752$$

$$F = 13.725.$$

13. 已知  $a=6.71$ ,  $h=6.6$ 。求  
A, C, c。

$$\sin A = \frac{h}{a}$$

$$\log \sin A = \log h + \text{colog } a.$$

$$\log h = 0.81954$$

$$\text{colog } a = 9.17328 - 10$$

$$\log \sin A = 9.99282 - 10,$$

$$A = 79^\circ 36' 30''.$$

$$\text{故 } C = 20^\circ 47'.$$

$$\cos A = \frac{\frac{1}{2}c}{a}.$$

$$\frac{1}{2}c = a \cos A.$$

$$\log a = 0.82572$$

$$\log \cos A = 9.25617 - 10$$

$$\log \frac{1}{2}c = 0.08289$$

$$\frac{1}{2}c = 1.2103$$

$$c = 2.4206.$$

14. 已知  $c=9$ ,  $h=20$ ; 試求  
A, C, a。

$$\tan \frac{1}{2}C = \frac{\frac{1}{2}c}{h}.$$

$$\log \tan \frac{1}{2}C = \log \frac{1}{2}c + \text{colog } h$$

$$\log \frac{1}{2}c = 0.65321$$

$$\text{colog } h = 8.69897 - 10.$$

$$\log \tan \frac{1}{2}C = 9.35218.$$

$$\frac{1}{2}C = 12^\circ 40' 49''.$$

$$C = 25^\circ 21' 38''.$$

$$2A = 180^\circ - C.$$

$$\text{故 } A = 77^\circ 19' 11''.$$

$$\sin A = \frac{h}{a},$$

$$\text{故 } a = \frac{h}{\sin A}.$$

$$\log h = 1.30103$$

$$\text{colog } \sin A = 0.01072$$

$$\log a = 1.31175$$

$$a = 20.5.$$

15. 已知  $c=147$ ,  $F=2572.5$

求 A, C, a, h。

$$\frac{1}{2}ch = F.$$

$$h = \frac{F}{\frac{1}{2}c} = \frac{2F}{c}.$$

$$\log h = \log^2 F + \text{colog } c.$$

$$\log 2F = 3.71139$$



$$\text{colog } c = 7.83268 - 10$$

$$\log h = 1.54407$$

$$h = 35.$$

$$\tan A = \frac{h}{\frac{r}{2}c}$$

$$\log h = 1.54407$$

$$\text{colog } \frac{1}{2}c = 8.13371 - 10$$

$$\log \tan A = 9.67778 - 10$$

$$A = 25^\circ 27' 47''.$$

$$\text{故 } C = (180^\circ - 2A)$$

$$= 129^\circ 4' 26''.$$

$$a = \frac{h}{\sin A}.$$

$$\log h = 1.54407$$

$$\log \sin A = 0.36661$$

$$\log a = 1.91068.$$

$$a = 81.41.$$

16. 已知  $h = 16.8$ ,  $F = 43.68$ ;

求  $A$ ,  $C$ ,  $a$ ,  $c$ .

$$\frac{1}{2}ch = F.$$

$$\frac{r}{2}c = \frac{F}{h}.$$

$$\log \frac{1}{2}c = \log F + \text{colog } h.$$

$$\log F = 1.64028$$

$$\text{colog } h = 8.77469 - 10$$

$$\log \frac{1}{2}c = 0.41497$$

$$\frac{1}{2}c = 2.60.$$

$$c = 5.2.$$

$$\tan A = \frac{h}{\frac{1}{2}c}.$$

$$\log h = 1.22531$$

$$\text{colog } \frac{1}{2}c = 9.58503 - 10$$

$$\log \tan A = 10.81034 - 10$$

$$A = 81^\circ 12' 9''.$$

$$\frac{1}{2}C = 8^\circ 47' 51''.$$

$$C = 17^\circ 35' 42''.$$

$$\cos A = \frac{\frac{1}{2}c}{a}.$$

$$a = \frac{\frac{1}{2}c}{\cos A}.$$

$$\log \frac{1}{2}c = 0.41497$$

$$\text{colog } A = 0.81547$$

$$\log a = 1.23044$$

$$a = 17$$

17. 試以  $a$  及  $c$ ; 表下值。

$$F = \frac{1}{2}ch.$$

$$h = \sqrt{a^2 - \frac{c^2}{4}}.$$

$$= \sqrt{\frac{4a^2 - c^2}{4}}$$

$$= \frac{1}{2}\sqrt{4a^2 - c^2}.$$

$$\text{故 } F = \frac{1}{2}c\left(\frac{1}{2}\sqrt{4a^2 - c^2}\right)$$

$$= \frac{1}{4}c\sqrt{4a^2 - c^2}$$

18. 試以  $a$  及  $c$  表下值。

$$F = \frac{1}{2}ch.$$

$$\frac{1}{2}c = a \sin \frac{1}{2}C.$$

$$h = a \cos \frac{1}{2}C.$$

$$\text{故 } F = a \sin \frac{1}{2}C \times a \cos \frac{1}{2}C.$$

$$= a^2 \sin \frac{1}{2}C \cos \frac{1}{2}C.$$

19. 試以  $a$  及  $A$  表下值。

$$F = \frac{1}{2}ch$$

$$\frac{1}{2}c = a \cos A.$$

$$h = a \sin A.$$

$$\text{故 } F = a \cos A \times a \sin A.$$

$$= a^2 \sin A \cos A.$$

20. 試以  $h$  及  $C$  表下值。

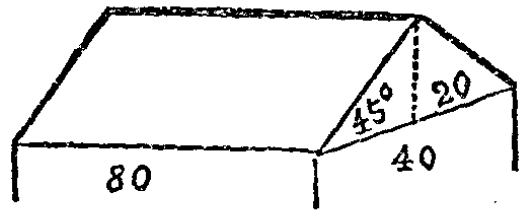
$$F = \frac{1}{2}ch.$$

$$\frac{1}{2}c = h \tan \frac{1}{2}C.$$

$$\text{故 } F = h(h \tan \frac{1}{2}C)$$

$$= h^2 \tan \frac{1}{2}C.$$

21. 有  $40 \times 80$  呎之穀倉。其屋頂之傾斜為  $45^\circ$ 。求桷長及屋頂之全面積。



依解等腰三角形之法解之。

$$40 \div 2 = 20 = \frac{1}{2}c.$$

$$\cos 45^\circ = \frac{\frac{1}{2}c}{a} = \frac{20}{a}.$$

$$20 = a \cos 45^\circ.$$

$$\text{故 } a = \frac{20}{\cos 45^\circ}.$$

$$\log a = \log 20 + \text{colog } \cos 45^\circ.$$

$$\log 20 = 1.30103$$

$$\text{colog } \cos 45^\circ = 0.15051$$

$$\log a = 1.45154$$

$$a = 28.284.$$

$$2 \times 28.284 \times 80 = 4525.44.$$

故其斜面長 28,284 呎。

倉頂面積等於 4525.44 方呎。

22. 問於單位圓內  $45^\circ$  圓心角所對之弦。長為若干。

$$\sin \frac{1}{2}C = \frac{\frac{1}{2}c}{a},$$

$$\frac{1}{2}c = a \sin \frac{1}{2}C.$$

$$\log \frac{1}{2}c = \log a + \log \sin \frac{1}{2}C.$$

$$\begin{aligned}\log a &= 0.00000 \\ \log \sin \frac{1}{2} C &= 9.58284 - 10 \\ \log \frac{1}{2} c &= 9.58284 - 10 \\ \frac{1}{2} c &= 0.38268 \\ c &= 0.76536. \\ &= \text{弦}.\end{aligned}$$

23. 設圓半徑爲30。弦長 $44^\circ$   
求此弦所對之圓心角。

$$\begin{aligned}\sin \frac{r}{2} C &= \frac{\frac{1}{2} c}{a} \\ \log \sin \frac{1}{2} C &= \log \frac{1}{2} c + \text{colog } a \\ \log \frac{1}{2} c &= 1.34242 \\ \text{colog } a &= 8.52288 - 10 \\ \log \sin \frac{1}{2} C &= 9.86530 - 10 \\ \frac{1}{2} C &= 47^\circ 10'. \\ C &= 94^\circ 20' .\end{aligned}$$

24. 弦長爲5。而其所對之圓  
心角爲 $133^\circ$ 。求圓半徑。

$$\begin{aligned}\sin \frac{r}{2} C &= \frac{\frac{1}{2} c}{a} \\ a &= \frac{\frac{1}{2} c}{\sin \frac{1}{2} C} \\ \log a &= \log \frac{1}{2} c + \text{colog } \sin \\ &\quad \frac{1}{2} C. \\ \log \frac{1}{2} c &= 0.39794 \\ \text{colog } \sin \frac{1}{2} C &= 0.03760\end{aligned}$$

$$\begin{aligned}\log a &= 0.43554 \\ a &= 2.7261.\end{aligned}$$

25. 設一弦等於其圓半徑之  
 $\frac{2}{3}$  求此弦所對之圓心角。

$$\begin{aligned}\text{設 } a &= 3, \\ \text{則 } c &= 2, \frac{1}{2} c = 1. \\ \sin \frac{1}{2} C &= \frac{1}{3} \\ \log \sin \frac{1}{2} C &= \log 1 + \text{colog } 3 \\ \log 1 &= 0.00000 \\ \text{colog } 3 &= 9.52288 - 10 \\ \log \sin \frac{1}{2} C &= 9.52288 - 10 \\ \frac{1}{2} C &= 19^\circ 28' 17''. \\ C &= 38^\circ 56' 33''. \\ &= \text{圓心角}.\end{aligned}$$

26. 設圓半徑爲12。扇形  
(Sector)角爲 $36^\circ$ 。求此扇形  
之面積。

$$\text{圓面積} = \pi \times R^2 (R = \text{半徑})$$

$$\text{圓分面積} = \frac{30\pi R^2}{360}.$$

$$\begin{aligned}\log(\text{圓分面積}) &= \log 30 + \text{colog} \\ &\quad 360 + \log \pi + 2 \\ &\quad \log R.\end{aligned}$$

$$\begin{aligned}\log 30 &= 1.47712 \\ \text{colog } 360 &= 7.44370 - 10 \\ \log \pi &= 0.49715\end{aligned}$$

$$2 \log R = \underline{2.15836}$$

$$(\text{圓分面積}) = 1.57633$$

$$\text{圓分面積} = 37.699$$

### 例題 XI

有正多角形。已知之兩項如下。求其餘各項。(1-6)

1.  $n = 10, c = 10.$

$$\frac{1}{2} C = \frac{180^\circ}{10} = 18^\circ.$$

$$\frac{1}{2} c = 0.5.$$

$$A = 90^\circ - \frac{1}{2} C = 72^\circ$$

$$h = \frac{1}{2} c \tan A.$$

$$\log h = \log \frac{1}{2} c + \log \tan A$$

$$\log \frac{1}{2} c = 9.69897 - 10$$

$$\log \tan A = \underline{10.48822 - 10}$$

$$\log h = 0.18719$$

$$h = 1.5388.$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\text{即 } r = \frac{\frac{1}{2} c}{\cos A} \quad (r = \text{半徑})$$

$$\log r = \log \frac{1}{2} c + \text{colog } \cos A.$$

$$\log \frac{1}{2} c = 9.69897 - 10$$

$$\text{colog } \cos A = \underline{0.51002}$$

$$\log r = 0.20899$$

$$r = 1.618$$

$$F = \frac{1}{2} hp, \quad (h = \text{高}, p = \text{諸邊之總})$$

$$2F = ph.$$

$$\log h = 0.18719$$

$$\log p = \underline{1.00000}$$

$$\log 2F = 1.18719$$

$$2F = 15.388.$$

$$F = 7.694.$$

2.  $n = 18, r = 10.$

$$\frac{1}{2} C = \frac{180^\circ}{18} = 10^\circ.$$

$$A = 90^\circ - \frac{1}{2} C = 80^\circ.$$

$$\frac{h}{r} = \sin A.$$

$$h = r \sin A$$

$$\log h = \log r + \log \sin A$$

$$\log r = 0.00000.$$

$$\log \sin A = \underline{9.99335 - 10}$$

$$\log h = 9.89335 - 10$$

$$h = 0.9848.$$

$$\frac{\frac{1}{2} c}{r} = \cos A.$$

$$\frac{1}{2} = r \cos A.$$

$$\log r = 0.00000$$

$$\log \cos A = \underline{9.23967 - 10}$$

$$\log \frac{1}{2} c = 9.23967 - 10$$

$$\frac{1}{2}c = 0.17365.$$

$$c = 0.34730.$$

$$p = 18 \times c = 6.2514.$$

$$F = \frac{1}{2}hp.$$

$$2F = hp.$$

$$\log h = 9.99335 - 10$$

$$\log p = \underline{0.79598}$$

$$\log 2F = 0.79933$$

$$2F = 6.1564.$$

$$F = 3.0782.$$

$$3. \quad n = 20, r = 20^\circ$$

$$\frac{1}{2}C = \frac{180^\circ}{20} = 9^\circ.$$

$$A = 90^\circ - 9^\circ = 81^\circ.$$

$$h = r \sin A.$$

$$\log r = 1.30103$$

$$\log \sin A = \underline{9.99462 - 10}$$

$$\log h = 1.29565$$

$$h = 19.754.$$

$$\frac{1}{2}c = r \cos A.$$

$$\log r = 1.30103$$

$$\log \cos A = \underline{9.19433 - 10}$$

$$\log \frac{1}{2}c = 0.49536$$

$$\frac{1}{2}c = 3.1286$$

$$c = 6.2572$$

$$p = 20 \times c = 125.14.$$

$$F = \frac{1}{2}hp.$$

$$2F = hp.$$

$$\log h = 1.29565$$

$$\log p = 2.09742$$

$$\log 2F = 3.39305.$$

$$2F = 2472.$$

$$F = 1236.$$

$$4. \quad n = 8, h = 10$$

$$\frac{1}{2}C = \frac{180^\circ}{8} = 22^\circ 30'.$$

$$\frac{\frac{1}{2}c}{h} = \tan \frac{1}{2}C.$$

$$\frac{1}{2}c = h \tan \frac{1}{2}C.$$

$$\log \frac{1}{2}c = \log h + \log \tan \frac{1}{2}C.$$

$$\log h = 0.00000$$

$$\log \tan \frac{1}{2}C = \underline{9.61722 - 10}$$

$$\log \frac{1}{2}c = 9.61722 - 10$$

$$\frac{1}{2}c = 0.41421.$$

$$c = 0.82842$$

$$\frac{h}{r} = \cos \frac{1}{2}C,$$

$$r = \frac{h}{\cos \frac{1}{2}C}.$$

$$\log h = 0.00000$$

$$\text{colog } \cos \frac{1}{2}C = \underline{0.03438}$$

$$\log r = 0.03438$$

$$r = 1.0824.$$

$$F = \frac{1}{2}hp = 3.3137.$$

$$5. \quad n = 11, F = 20.$$

$$\frac{1}{2}hp = (F) = 20.$$

$$hp = (2F) = 40.$$

$$h = \frac{40}{p}$$

$$c = \frac{p}{11}.$$

$$\frac{1}{2}C = \frac{180^\circ}{11} = 16^\circ 22',$$

$$\tan \frac{1}{2}C = \frac{1}{2}c \div h.$$

以  $h$  與  $c$  之值代入此式即

$$\text{得 } \frac{1}{2}C = \frac{p}{22} \div \frac{40}{p}$$

$$= \frac{p^2}{880}.$$

$$p^2 = 880 \times \tan \frac{1}{2}C.$$

$$p = \sqrt{880 \times \tan \frac{1}{2}C}.$$

$$\log p \doteq \frac{1}{2}(\log 880$$

$$+ \log \tan \frac{1}{2}C).$$

$$\log 880 = 2.94448.$$

$$\log \tan \frac{1}{2}C = 9.46788 - 10$$

$$2 | \underline{2.41236}$$

$$\log p = 1.20618$$

$$p = 1.6076.$$

$$c = \frac{p}{11} = 1.4615.$$

$$\frac{\frac{1}{2}c}{r} = \sin \frac{1}{2}C.$$

$$r = \frac{1}{2}c \div \sin \frac{1}{2}C.$$

$$\log r = \log \frac{1}{2}c$$

$$+ \log \sin \frac{1}{2}C.$$

$$\log \frac{1}{2}c = 9.86377 - 10$$

$$\text{cologs} \sin \frac{1}{2}C = 0.55008$$

$$\log r = 0.41385$$

$$r = 2.5933.$$

$$\frac{h}{r} = \cos \frac{1}{2}C.$$

$$h = r \cos \frac{1}{2}C.$$

$$\log r = 0.41385$$

$$\log \cos \frac{1}{2}C = 9.98204$$

$$\log h = 0.39587$$

$$h = 2.4882.$$

$$6. \quad n = 7, F = 7.$$

$$\frac{1}{2}hp = (F) = 7.$$

$$hp = (2F) = 14.$$

$$h = \frac{14}{p}.$$

$$c = \frac{p}{7}.$$

$$\frac{1}{2}C = \frac{180^\circ}{9} = 25^\circ 43'.$$

$$\tan \frac{1}{2}C = \frac{1}{2}c \div h.$$

以  $h$  與  $c$  之值代入此式,

$$\begin{aligned} \text{即得 } \tan \frac{1}{2}C &= \frac{p}{14} \div \frac{p}{14} \\ &= \frac{p^2}{196}. \end{aligned}$$

$$p^2 = 196 \times \tan \frac{1}{2}C.$$

$$p = \sqrt{196 \times \tan \frac{1}{2}C}.$$

$$\begin{aligned} \log p &= \frac{1}{2}(\log 196 \\ &\quad + \log \tan \frac{1}{2}C) \end{aligned}$$

$$\log 196 = 2.29226.$$

$$\log \tan \frac{1}{2}C = \underline{0.68271 - 10}$$

$$2 \underline{1.97497}$$

$$\log p = 0.98749.$$

$$p = 9.716.$$

$$c = \frac{p}{7} = 0.674.$$

$$\frac{\frac{1}{2}c}{h} = \tan \frac{1}{2}C.$$

$$h = \frac{\frac{1}{2}c}{\tan \frac{1}{2}C}.$$

$$\log h = \log \frac{1}{2}c$$

$$+ \text{co } \log \tan \frac{1}{2}C.$$

$$\log \frac{1}{2}c = 9.84136 - 10$$

$$\text{colog } \tan \frac{1}{2}C = \underline{0.31729}$$

$$\log h = 0.15865$$

$$h = 1.441.$$

$$\frac{\frac{1}{2}c}{r} = \sin \frac{1}{2}C.$$

$$r = \frac{\frac{1}{2}c}{\sin \frac{1}{2}C}.$$

$$\log \frac{1}{2}c = 9.84136 - 10$$

$$\text{colog } \sin \frac{1}{2}C = \underline{0.36259}$$

$$\log r = 0.20395$$

$$r = 1.5994.$$

7. 求單位圓之內接正十角形之一邊。

$$\frac{1}{2}C = \frac{180^\circ}{10} = 18^\circ.$$

$$\frac{\frac{1}{2}c}{r} = \sin \frac{1}{2}C,$$

$$c = 2 \sin \frac{1}{2}C.$$

$$\log c = \log 2 + \log \sin \frac{1}{2}C.$$

$$\log 2 = 0.30103$$

$$\log \sin \frac{1}{2}C = \underline{9.48998}$$

$$\log c = 9.79101 - 10$$

$$c = 0.61803.$$

8. 求單位圓之外切正十角形之一邊。

$$\frac{1}{2}C' = 18^\circ.$$

$$\frac{\frac{1}{2}c}{h} = \tan \frac{1}{2}C.$$

$$\frac{1}{2}c = h \tan \frac{1}{2}C.$$

$$\log \frac{1}{2}c = \log h + \log \tan \frac{1}{2}C.$$

$$\log h = 0.00000$$

$$\log \tan \frac{1}{8}C = 9.51178$$

$$\log \frac{1}{2}c = 9.51178$$

$$\frac{1}{2}c = 0.32492$$

$$c = 0.64984$$

9. 設內接正六角形之一邊爲 10 求內接正十二角形之一邊。

設  $O =$  圓心.

$BC =$  正六邊形之一邊.

$BA =$  正十二邊形之一邊.

又  $OD$  爲  $BA$  之垂線.

則  $OB = BC = r.$

$$\angle BOD = \frac{180^\circ}{12} = 15^\circ.$$

在  $ODB$  直角三角形中,

有  $\sin \angle BOD = \frac{1}{2}AB \div OB,$

故  $AB = 2OB \times \sin \angle BOD$

$$\log AB = \log 2 OB + \log \sin \angle BOD.$$

$$\log 2 OB = 0.30203$$

$$\log \sin \angle BOD = 9.41300$$

$$\log AB = 9.71403$$

$$AB = 0.51764.$$

10. 已知  $n$  及  $c$  又設  $b$  爲內接邊數之正多形之邊。試以  $n$  及  $c$  表  $b$  之值。

設  $O =$  圓心.

$BC = n$  邊多邊形之邊.

$BA = 2n$  邊多邊形之邊.

則  $OA$  必與  $BC$  直交於其中點  $D.$

$$\widehat{BOA} = \frac{360^\circ}{2n} = \frac{180^\circ}{n}.$$

$$\widehat{OBC} = 90^\circ - \frac{180^\circ}{n}.$$

但  $BOA$  爲等腰三角形.

故  $\widehat{OBA} = \frac{1}{2} \left( 180^\circ - \frac{180^\circ}{n} \right) = 90^\circ - \frac{90^\circ}{n}.$

而  $\widehat{ABC} = \widehat{OBA} - \widehat{OBC}$   
 $= \left( 90^\circ - \frac{90^\circ}{n} \right)$



$$-\left(90^\circ - \frac{180^\circ}{n}\right)$$

$$= \frac{90^\circ}{n},$$

$$\frac{\frac{1}{2}c}{b} = \cos \frac{90^\circ}{n}.$$

$$\frac{1}{2}c = b \cos \frac{90^\circ}{n}.$$

即 
$$b = \frac{\frac{1}{2}c}{\cos \frac{90^\circ}{n}}$$

$$= \frac{c}{2 \cos \frac{90^\circ}{n}}.$$

II. 設正八角形及正九角形之周各爲16。求面積之較。

$$(1) \quad \frac{1}{2}c = \frac{h}{2n} = \frac{16}{16}$$

$$A = \frac{180^\circ}{n} = 22^\circ 30'.$$

$$\log h = \log \frac{1}{2}c + \log \cot A.$$

$$\log \frac{1}{2}c = 0.00000$$

$$\log \cot A = 10.38278 - 10$$

$$\log h = 0.38278$$

$$\log F = \log h + \log \frac{1}{2}h.$$

$$\log h = 0.38278$$

$$\log \frac{1}{2}p = 0.90309$$

$$\log F = 1.28587$$

$$F = 19.3139.$$

$$(2) \quad \frac{1}{2}c' = \frac{p}{2n'} = \frac{16}{18}$$

$$= 0.8889.$$

$$A' = \frac{180^\circ}{n'} = 20^\circ$$

$$\log h' = \log \frac{1}{2}c' + \log \cot A'.$$

$$\log \frac{1}{2}c' = 9.94885 - 10$$

$$\log \cot A' = 10.43893 - 10$$

$$\log h' = 0.38778$$

$$\log F' = \log h' + \log \frac{1}{2}p.$$

$$\log h' = 0.38778$$

$$\log \frac{1}{2}p = 0.90309$$

$$\log F' = 1.29087$$

$$F' = 1.5377.$$

$$F' - F = 19.5377$$

$$- 1.3139$$

$$= 0.2238$$

12. 設正五角及正六角形

之面積各爲12。求兩周之較。

$$(1) \quad F = 12, \quad n = 5.$$

$$\frac{1}{2}c = \frac{180^\circ}{5} = 36^\circ.$$

$$F = \frac{1}{2}hp,$$

$$h = \frac{24}{p}$$

$$\frac{1}{2}c = \frac{p}{2n} = \frac{p}{10}$$

$$\tan \frac{1}{2}C = \frac{\frac{1}{2}c}{h} = \frac{\frac{p}{10}}{\frac{24}{p}}$$

$$= \frac{p^2}{240}$$

$$p^2 = 240 \tan \frac{1}{2}C,$$

$$\log 240 = 2.38021.$$

$$\log \tan \frac{1}{2}C = 9.86126 - 10$$

$$2 \mid \underline{2.24147}$$

$$\log p = 1.12074$$

$$p = 13.205.$$

$$(2) \quad \frac{1}{2}C' = 30, n' = 6.$$

$$\tan \frac{1}{2}C' = \frac{\frac{p'}{2}}{\frac{p'}{2n'}} = \frac{p'/2}{p'/12}$$

$$p'^2 = 288 \tan \frac{1}{2}C',$$

$$\log 288 = 2.45939.$$

$$\log \tan \frac{1}{2}C' = 9.76144 - 10$$

$$2 \mid \underline{2.22083}$$

$$\log p' = 1.11042$$

$$p' = 12.895.$$

$$p - p' = 13.205 - 12.895.$$

$$= 0.310.$$

13. 截去以  $1$  為邊之正方形之各角。而成一正八角形。求此形之面積。

$$h = \frac{1}{2}.$$

$$\frac{1}{2}C = \frac{1}{2} \left( \frac{390^\circ}{8} \right)$$

$$= 22^\circ 30'.$$

$$A = 90^\circ - \frac{1}{2}C = 67^\circ 30'.$$

$$\tan A = \frac{h}{\frac{1}{2}c}.$$

$$\text{有 } \frac{1}{2}c = \frac{h}{\tan A}.$$

$$\log \frac{1}{2}c = \log h + \text{colog } \tan A.$$

$$\log h = 9.69897 - 10$$

$$\text{colog } \tan A = 9.61722 - 10$$

$$\log \frac{1}{2}c = 9.31619 - 10$$

$$p = \frac{1}{2}c \times 2n = nc.$$

$$F = \frac{1}{2}ph = \frac{1}{2}c \times \frac{1}{2}n.$$

$$\log F = \log \frac{1}{2}c + \log \frac{1}{2}n.$$

$$\log \frac{1}{2}c = 9.31617 - 10.$$

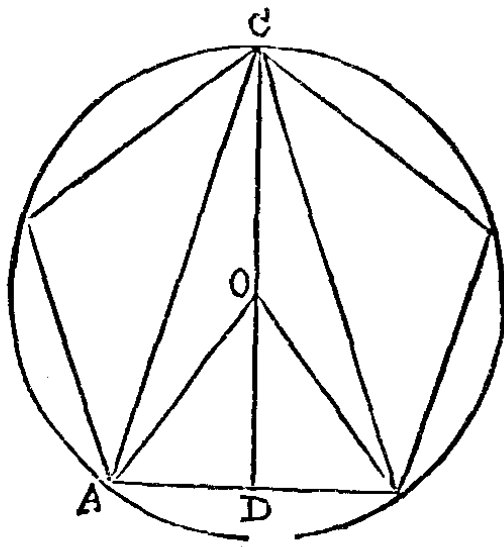
$$\log \frac{1}{2}n = 0.60206$$

$$\log F = 9.91825 - 10$$

$$F = 0.82842.$$

14. 求對角綫各等於  $12$  之

正五角形之面積。



$$\widehat{AOD} = \frac{180^\circ}{n} = 36^\circ.$$

$$\widehat{AOC} = 180^\circ - 36^\circ = 144^\circ$$

$$\widehat{ACD} = \frac{1}{2} (180^\circ - 144^\circ) \\ = 18^\circ = \widehat{CAO}$$

$$\widehat{OAD} = 90^\circ - \widehat{AOD} = 54^\circ$$

$$\widehat{DAC} = 54^\circ + 18^\circ = 72^\circ.$$

$$\cos DAC = \frac{AD}{AC} = \frac{\frac{1}{2}c}{12}.$$

$$\frac{1}{2}c = 12 \cos DAC.$$

$$\log \frac{1}{2}c = \log 12 + \log \cos \\ DAC.$$

$$\log 12 = 1.07918$$

$$\log \cos 72^\circ = \underline{9.48998}$$

$$\log \frac{1}{2}c = 0.56916$$

$$\tan DAO = \frac{h}{\frac{1}{2}c}.$$

$$h = \frac{1}{2}c \tan DAO.$$

$$\log \frac{1}{2}c = 0.56916$$

$$\log \tan 54^\circ = \underline{10.13874 - 10}$$

$$\log h = 0.70790$$

$$p = \frac{1}{2}c \times 2n.$$

$$F = \frac{1}{2}ph = \frac{1}{2}c \times nh.$$

$$\log F = \log \frac{1}{2}c + \log n + \log h.$$

$$\log \frac{1}{2}c = 0.56916$$

$$\log n = 0.69897$$

$$\log h = 0.70790$$

$$\log F = 1.97603$$

$$F = 94.03.$$

15. 設圓內接正五角形之面積爲331.8。問圓內接之正十一角形。面積若干。

設 AB = 內切正五角形之一邊, AD = 內切正十一角形之一邊。

又 r = 圓之半徑, O = 圓心, h 與 h' 爲兩多角形之垂距。

又 p 與 p' 爲兩形之周界 c 與 c' 爲兩形之一邊。

$$F = \frac{1}{2}ph.$$

$$331.8 = \frac{1}{2}ph.$$

$$ph = 663.6.$$

$$h = \frac{663.6}{p}.$$

$$c = \frac{p}{5}.$$

$$\frac{1}{2}c = \frac{p}{10}.$$

$$\text{AOE 角} = 36^\circ.$$

$$\begin{aligned} \tan 36^\circ &= \frac{\frac{1}{2}c}{h} = \frac{p}{10} \times \frac{p}{663.6} \\ &= \frac{p^2}{663.6}. \end{aligned}$$

$$p^2 = 663.6 \tan 36^\circ.$$

$$\log p^2 = \log 663.6 + \log \tan 36^\circ$$

$$\log 663.6 = 3.82191$$

$$\log \tan 36^\circ = 9.86126 - 10$$

$$\log p^2 = 3.68317$$

$$\log p = 1.84159$$

$$\text{因 } \frac{1}{2}c = \frac{p}{10},$$

$$\text{故 } \log \frac{1}{2}c = 0.84159$$

$$\sin \text{AOE} = \frac{\frac{1}{2}c}{r}.$$

$$r = \frac{\frac{1}{2}c}{\sin \text{AOE}}$$

$$\log r = \log \frac{1}{2}c + \text{colog } 36^\circ.$$

$$\log \frac{1}{2}c = 0.84159$$

$$\text{colog } \sin 36^\circ = 0.23078$$

$$\log r = 1.07237$$

$$\widehat{\text{AOC}} = \frac{360^\circ}{22} = 16^\circ 21'$$

$$49''.$$

$$\sin \text{AOC} = \frac{1}{2}c' \div r.$$

$$\frac{1}{2}c' = r \times \sin \text{AOC}.$$

$$\log r = 1.07237$$

$$\log \sin \text{AOC} = 9.44984 - 10$$

$$\log \frac{1}{2}c' = 0.52221$$

$$\tan \text{AOC} = \frac{\frac{1}{2}c'}{h'}.$$

$$h' = \frac{1}{2}c' \div \tan \text{AOC}.$$

$$\log \frac{1}{2}c' = 0.52221$$

$$\text{colog } \tan \text{AOC} = 0.53221$$

$$\log h' = 1.05442$$

$$F = \frac{1}{2}p'h'.$$

$$= \frac{1}{2}c' \times 11 \times h'.$$

$$\log \frac{1}{2}c' = 0.52221$$

$$\log 11 = 1.04139$$

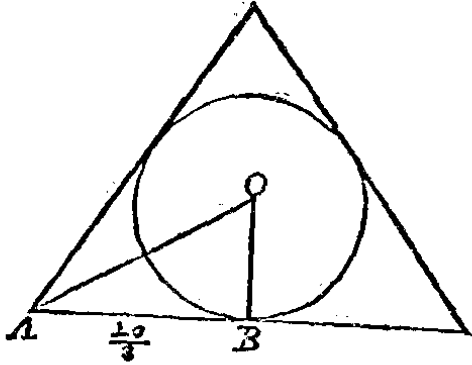
$$\log h' = 1.05142$$

$$\log F = 2.61802$$

$$F = 414.97.$$

16. 今有內切於正三角形之

圓。若三角形之周界為20。求圓之面積。



周界 = 20.

$$AB = \frac{1}{3} \times 20 = \frac{10}{3}.$$

$$\widehat{OAB} = \frac{1}{3} \times 180^\circ = 30^\circ.$$

$$\tan 30^\circ = \frac{r}{AB}.$$

$$r = AB \tan 30^\circ.$$

$$\log AB = 0.52288$$

$$\log \tan 30^\circ = 9.76144 - 10$$

$$\log r = 0.28432.$$

$$\text{面積} = \pi r^2$$

$$\log \pi = 0.49715$$

$$\log r^2 = 0.56864$$

$$\log \text{面積} = 1.06579.$$

$$\text{面積} = 1.0636.$$

17. 設圓內切正十六角形之面積為100。求圓內切十五角形之面積。

$$\frac{1}{2}C = \frac{360^\circ}{32} = 11^\circ 15',$$

$$\frac{1}{2}C' = \frac{360^\circ}{30} = 12^\circ.$$

設  $AC = h, AB = r, BC = \frac{1}{2}c.$

則  $F = \frac{1}{2}ph.$

$$100 = \frac{1}{2}ph.$$

$$h = \frac{200}{p}.$$

$$\tan \frac{1}{2}C = \frac{p}{32} \div \frac{200}{p}.$$

$$p^2 = 6400 \tan \frac{1}{2}C.$$

$$\log 6400 = 3.80618$$

$$\log \tan \frac{1}{2}C = 9.29866 - 10$$

$$2 \mid 3.10484$$

$$\log p = 1.55242$$

$$p = 35.68.$$

$$\frac{1}{2}c = 35.68 \div 32.$$

$$= 1.115.$$

$$\sin \frac{1}{2}C = \frac{\frac{1}{2}c}{r} = \frac{1.115}{r}.$$

$$r = \frac{1.115}{\sin \frac{1}{2}C}.$$

$$\log 1.115 = 0.04727$$

$$\text{colog} \sin \frac{1}{2}C = 0.70976.$$

$$\log r = 0.75703$$

$$\frac{h'}{r} = \cos \frac{1}{2} C' (I_2).$$

$$h' = r \cos \frac{1}{2} C'.$$

$$\log r = 0.75703$$

$$\log \cos \frac{1}{2} C' = \underline{9.99040 - 10}$$

$$\log h' = 0.74743$$

$$\frac{\frac{1}{2}c'}{r} = \sin \frac{1}{2} C'$$

$$\frac{1}{2}c' = r \times \sin \frac{1}{2} C'.$$

$$\log r = 0.75703$$

$$\log \sin \frac{1}{2} C' = \underline{9.31788}$$

$$\log \frac{1}{2}c' = 0.07491.$$

$$F = \frac{1}{2} \left( \frac{c'}{2} \times 2nh' \right).$$

$$\log F = \log \frac{1}{2} c' + \log n \\ + \log h'.$$

$$\log \frac{1}{2}c' = 0.07491$$

$$\log 15 = 1.17609$$

$$\log h' = \underline{0.74743}$$

$$\log F = 1.99843$$

$$F = 99.640.$$

18. 某圓之周界爲  $r$  求外切正十二角形之周界。

即已知內切圓周界等於一,  $n = 12$ , 求  $p$ .

$$2\pi r = \text{圓周界.}$$

$$r = \frac{\text{圓周界}}{2\pi}.$$

$$\frac{1}{2}C = \frac{360^\circ}{24} = 15^\circ.$$

$$\tan 15^\circ = \frac{\frac{1}{2}c}{r} = \pi C.$$

$$C = \frac{\tan 15^\circ}{3.1416}$$

$$\log \tan 15^\circ = 9.42805.$$

$$\text{colog } 3.1416 = \underline{9.50285 - 10}$$

$$\log c = 8.13099 - 10$$

$$\log 12 = 1.07918$$

$$\log p = 0.01008$$

$$p = 1.0235.$$

19. 正二十五角形之面積爲 40 求其內切及外接兩圓間所含環形之面積。

$$\frac{1}{2}ch = \frac{40}{25} = 1.6.$$

$$\frac{1}{2}C = 7^\circ 12'.$$

$$\frac{\frac{1}{2}c}{h} = \tan \frac{1}{2}C,$$

$$\text{或 } \frac{\frac{1}{2}ch}{h^2} = \tan \frac{1}{2}C.$$

$$\text{故 } h^2 = \frac{1.6}{\tan \frac{1}{2}C}.$$

$$\log 1.6 = 0.20412$$

$$\text{colog } \tan \frac{1}{2}C = \underline{0.89850}$$

$$\log h^2 = 1.10262$$

$$\log h = 0.55131.$$

$$\frac{h}{r} = \cos \frac{1}{2}C.$$

$$r = \frac{h}{\cos \frac{1}{2}C}.$$

$$\log h = 0.55131$$

$$\text{colog } \cos \frac{1}{2}C = \underline{0.00344}$$

$$\log r = 0.55475$$

$$\log r^2 = 1.10950$$

$$\text{外接圓之面積} = \pi r^2.$$

$$\log \pi = 0.49715$$

$$\log r^2 = \underline{1.10950}$$

$$\log F = 1.60665$$

$$F = 40.425.$$

$$\text{內切圓之面積} = \pi h^2.$$

$$\log \pi = 0.49715$$

$$\log h^2 = 1.10262$$

$$\log F' = 1.59977$$

$$F' = 39.790.$$

$$\text{環面積} = F - F' = 0.635.$$

## 例題 XII

1. 試作象限 II 內一角之各

函數。且定其符號。

凡正弦與正割在橫坐標上者為正，在橫坐標下者為負。

凡餘弦與餘切向直徑之右者為正，向直徑之左者為負。

正割之號與餘弦同。

餘割之號與正弦同。如下。

sin 與 csc 為 +

cos 與 sec 為 -。

tan 與 cot 為 -。

2. 試作象限 III 內一角之各函數。且定其符號。

sin 與 csc 之號為 -。

cos 與 sec 之號為 -。

tan 與 cot 之號為 +。

3. 試作象限 IV 內一角之各函數。且定其符號。

sin 與 csc 之號為 -。

cos 與 sec 之號為 +。

tan 與 cot 之號為 -。

4. 下列各角之函數之符號為何。

$340^\circ$ ,  $239^\circ$ ,  $145^\circ$ ,  $400^\circ$ 。

$700^\circ, 1200^\circ, 3800^\circ$ .

$340^\circ$  之角在第一象限.

$$\sin = -. \tan = -. \sec = +.$$

$$\cos = +. \cot = -. \csc = -.$$

$239^\circ$  之角在第三象限.

$$\sin = -. \tan = +. \sec = -.$$

$$\cos = -. \cot = +. \csc = -.$$

$145^\circ$  在第二象限.

$$\sin = +. \tan = -. \sec = -.$$

$$\cos = -. \cot = -. \csc = +.$$

$400^\circ = 360^\circ + 40^\circ$ ,

故  $400^\circ$  乃在第一象限.

$$\sin = +. \tan = +. \sec = +.$$

$$\cos = +. \cot = +. \csc = +.$$

$700^\circ = 360^\circ + 340^\circ$ ,

故  $700^\circ$  乃在第四象限.

$$\sin = -. \tan = -. \sec = +.$$

$$\cos = +. \cot = -. \csc = -.$$

$1200^\circ = 3 \times 360^\circ + 120^\circ$ ,

故  $1200^\circ$  乃在第二象限.

$$\sin = +. \tan = -. \sec = -.$$

$$\cos = -. \cot = -. \csc = +.$$

$3800^\circ = 10 \times 360^\circ + 200^\circ$ .

故  $3800^\circ$  乃在第三象限.

$$\sin = -. \tan = +. \sec = -.$$

$$\cos = -. \cot = +. \csc = -.$$

5. 小於  $360^\circ$  而其正弦等於  $\frac{5}{7}$  者問有若干角 $^\circ$ 並問在何象限內。

其正弦之號既爲 (+) 故按第二十二節此諸角必皆在第一與第二象限內。

按第二十二節，第一象限正弦自 0 漸增至 1；第二象限之正弦自 1 漸減至 0。

故小於  $360^\circ$  之諸角中惟有兩角之正弦等於  $+\frac{5}{7}$ ：一在第一象限，一在第二象限。

6.  $\cos x = +\frac{2}{30}$  問此  $x$  角小於  $720^\circ$ 。能有若干值。並問在何象限內。

$720^\circ$  之  $360^\circ$  之二倍，故半徑移動成角時適經兩週。

但正號之餘弦必在第一與第四象限，故  $x$  可有四值。二在第一象限，二在第四象限。

7. 有下之各件。問小於  $180^\circ$  之  $x$  角。各有若干值。

$$\begin{aligned} \sin x &= \frac{8}{7}, \text{ as } x = \frac{1}{5}, \cos x \\ &= -\frac{1}{5}, \tan x = \frac{2}{3}, \cot x \\ &= -\frac{3}{2}. \end{aligned}$$



(1) 其號既爲+, 則此角可在第一或第二象限.

故  $x$  可有二值小於  $180^\circ$ .

(2) 其號既爲+, 則此角可在第一或第四象限.

故  $x$  僅有一值小於  $180^\circ$ .

(3) 其號既爲-, 則此角可在第二或第三象限.

故  $x$  僅有一值小於  $180^\circ$ .

(4) 其號既爲-, 則此角可在第一或第三象限.

故  $x$  僅有一值小於  $180^\circ$ .

(5) 其號既爲-, 則此角可在第二或第四象限.

故  $x$  僅有一值小於  $180^\circ$ .

8. 有下之各件。問各角  $x$  在何界內°

$$\cos x = -\frac{2}{9}, \cot x = 4,$$

$$\sec x = 80, \csc x = -3.$$

(設  $x < 360^\circ$ )

若  $\cos x = -\frac{2}{9}$ , 則  $x$  角必在第二或第三象限內, 即在  $90^\circ$  與  $270^\circ$  之間.

若  $\cot x = 4$ , 則  $x$  角必在  $0^\circ$  與  $90^\circ$  之間, 或  $180^\circ$  與

$270^\circ$  之間.

若  $\sec x = 80$ , 則  $x$  角必在  $0^\circ$  與  $90^\circ$  之間, 或  $270^\circ$  與  $360^\circ$  之間.

若  $\csc x = -3$ , 則  $x$  角必在  $180^\circ$  與  $360^\circ$  之間

9. 設一角之正弦及餘弦均爲負或餘弦及正切均爲負。或餘切爲正及正弦爲負。問各角在何象限內。

(1) 負號之正弦必在第二或第三象限, 負號之餘弦必在第三第四象限. 故正弦餘弦皆爲負號之角必在第三象限.

(2) 負號之餘弦必在第二或第三象限, 負號之正切必在第二或第四象限. 故餘弦正切皆爲負號之角必在第二象限

(3) 正號之餘切必在第一或第三象限, 負號之正弦必在第三或第四象限. 故餘切爲正, 正弦爲負之角必在第三象限.

10. 於 $0^\circ$ 及 $360^\circ$ 之間其正弦有絕對值 $\frac{3}{5}$ 。問有若干角。又此諸正弦中，若干為正若干為負。

$0^\circ$ 與 $360^\circ$ 之間有十週，每週中正弦等於 $\frac{3}{5}$ 之角有四。

故角之總數為四十。

第一與第二象限之正弦為正，

第三與第四象限之正弦為負，

故正弦為正之角有二十，正弦為負之角亦有二十。

11. 設由  $\cos x = \pm$

$$\sqrt{1 - \sin^2 x}$$

求  $\cos x$ ，如何而取正號。如何而取負號。

因第一與第四象限諸角之餘弦為正，故凡  $x$  角在此限內時必用正號。

因第二與第三象限諸角之餘弦為負，故凡  $x$  角在此限內之時必用負號。

12. 設  $\cos x = -\sqrt{\frac{1}{2}}$ 。而  $x$  在象限 II 內求餘各函數。

$$\sin^2 x + \cos^2 x = 1$$

$$\sin x = \sqrt{1 - \cos^2 x}$$

$$= \sqrt{1 - \left(-\sqrt{\frac{1}{2}}\right)^2}$$

$$= \sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{2}$$

$$\cos x = \frac{1}{\sin x} = \frac{1}{\frac{1}{2}\sqrt{2}}$$

$$= \sqrt{2}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{-\frac{1}{2}\sqrt{2}}$$

$$= -\sqrt{2}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\sqrt{\frac{1}{2}}}{-\frac{1}{2}\sqrt{2}}$$

$$= -1$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{-1}$$

$$= -1$$

13. 設  $\tan x = \sqrt{3}$ 。而  $x$  在象限 III 內。求餘之函數。

$$\tan x = \sqrt{3}$$

$$\cot x = \frac{1}{\sqrt{3}} = \frac{1}{3}\sqrt{3}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\tan x \times \cos x = \sin x$$

$$\sqrt{3} \times \cos x = \sin x.$$

$$3 \cos^2 x - \sin^2 x = 0.$$

$$\frac{\cos^2 x + \sin^2 x = 1}{4 \cos^2 x} = 1.$$

$$\cos^2 x = \frac{1}{4}.$$

$$\cos x = \pm \frac{1}{2}.$$

即是第三象限之角，故此  
餘弦必為負。

$$\text{即 } \cos x = -\frac{1}{2}.$$

$$\begin{aligned} \sin x &= \sqrt{1 - \left(-\frac{1}{2}\right)^2} \\ &= \sqrt{\frac{3}{4}} = \pm \frac{1}{2} \sqrt{3}. \end{aligned}$$

此正弦亦必為負。

$$\text{即 } \sin x = -\frac{1}{2} \sqrt{3}.$$

$$\sec x = \frac{1}{-\frac{1}{2}} = -2.$$

$$\begin{aligned} \csc x &= \frac{1}{-\frac{1}{2} \sqrt{3}} \\ &= -\frac{2}{\sqrt{3}}. \end{aligned}$$

14. 已知  $\sec = +7$  及  $\tan x$   
為負；求  $x$  之餘各函數。

$x$  角必在第四象限內，

故正弦餘割正切餘切皆為  
負，而餘弦為正。

$$\cos x = \frac{1}{\sec} = \frac{1}{7}.$$

$$\sin x = \pm \sqrt{1 - \frac{1}{49}}$$

$$= \pm \sqrt{\frac{48}{49}}$$

$$= -\frac{4}{7} \sqrt{3}.$$

$$\csc x = \frac{1}{\sin x} = \frac{1}{-\frac{4}{7} \sqrt{3}}$$

$$= -\frac{7}{12} \sqrt{3}.$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{4}{7} \sqrt{3}}{\frac{1}{7}}$$

$$= -4 \sqrt{3}.$$

$$\cot x = \frac{1}{\tan x} = -\frac{1}{4 \sqrt{3}}$$

$$= -\frac{1}{12} \sqrt{3}.$$

15. 已知  $\cot x = -3$ ，求各函  
數可有之值。

按公式[3]， $\tan x = -\frac{1}{3}$ ，此  
可在第二或第四象限。

按公式[1]，

$$\sin^2 x = 1 - \cos^2 x.$$

$$\sin x = \sqrt{1 - \cos^2 x}.$$

按公式[2]，

$$-\frac{1}{3} = \frac{\sqrt{1 - \cos^2 x}}{\cos x}.$$

$$\frac{1}{9} = \frac{1 - \cos^2 x}{\cos^2 x}$$

$$\cos^2 x = 9 - 9 \cos^2 x$$

$$\cos^2 x = \frac{9}{10}$$

$$\cos x = \frac{3}{\sqrt{10}} = \frac{3}{10} \sqrt{10}$$

在第二象限爲負，在第四爲正。

按公式[1]，

$$\sin x = \sqrt{1 - \frac{9}{10}} = \sqrt{\frac{1}{10}}$$

$$= \frac{1}{10} \sqrt{10}$$

在第二象限爲正，在第四爲負。

$$\sec x = \frac{\sqrt{10}}{3} = \frac{1}{3} \sqrt{10}$$

$$\csc x = \sqrt{10}$$

在第二象限爲正，在第四爲負。

16. 三角形一角之何函數可爲負。又此函數爲負時。其角如何。

若三角形之角爲銳角，

則其諸函數皆爲正。

若三角形之角爲鈍角，

則其諸函數皆與第二象限內相同。

故正弦與餘割必常爲正，餘弦正切餘切正割可以爲負。

17.  $\csc 360^\circ$  或可等於  $+\infty$  或等於  $-\infty$ 。何故。

凡銳角愈近  $0^\circ$  則其餘切之正值必愈大。凡角愈近  $360^\circ$  則其餘切之負值必愈大。角度等於  $0^\circ$  或  $360^\circ$  之時其餘切適與橫徑平行必不能與之相遇。但  $360^\circ$  之餘切可謂正向又可謂反向，故其值可謂  $+\infty$  又可謂  $-\infty$ 。

18. 試以公式[1]-[3]求餘各函數已知。

$$(1) \tan 90^\circ = \infty.$$

$$(2) \cos 180^\circ = -1.$$

$$(3) \cot 270^\circ = 0.$$

$$(4) \csc 360^\circ = \pm \infty.$$

$$(1) \tan 90^\circ = \infty = \frac{1}{0}.$$

$$\cot 80^\circ = \frac{1}{\infty} = 0.$$

$$\frac{\sin 90^\circ}{\cos 90^\circ} = \frac{1}{0},$$

$$\cos^2 90^\circ = 0 \quad \sin 90^\circ = 0.$$

$$\cos^2 90^\circ + \sin^2 90^\circ = 1,$$

$$\sin^2 90^\circ = 1,$$

$$\sin 90^\circ = 1.$$

$$\sec 90^\circ = \frac{1}{\cos 90^\circ} = \frac{1}{0}$$

$$= \infty.$$

$$\csc 90^\circ = \frac{1}{\sin 90^\circ} = \frac{1}{1}$$

$$= 1.$$

$$(2) \quad \cos 180^\circ = -1.$$

$$\sin^2 180^\circ + \cos^2 180^\circ = 1,$$

$$\sin^2 180^\circ + 1 = 1,$$

$$\sin 180^\circ = 0.$$

$$\tan 180^\circ = \frac{\sin 180^\circ}{\cos 180^\circ} = \frac{0}{-1}$$

$$= 0.$$

$$\cot 180^\circ = \frac{\cos 180^\circ}{\sin 180^\circ} = \frac{-1}{0}$$

$$= -\infty.$$

$$\sec 180^\circ = \frac{1}{\cos 180^\circ} = \frac{1}{-1}$$

$$= -1.$$

$$\csc 180^\circ = \frac{1}{\sin 180^\circ} = \frac{1}{0}$$

$$= \infty.$$

$$(3) \quad \cot 270^\circ = 0.$$

$$\tan 270^\circ = \frac{1}{0} = \infty.$$

$$\frac{\cos 270^\circ}{\sin 270^\circ} = 0,$$

$$\cos 270^\circ = 0 \quad \sin 270^\circ = 0.$$

$$\sin^2 270^\circ + \cos^2 270^\circ = 1.$$

$$\sin^2 270^\circ + 0 = 1,$$

$$\sin 270^\circ = 1.$$

$$\sec 270^\circ = \frac{1}{\cos 270^\circ} = \frac{1}{0}$$

$$= \infty.$$

$$\csc 270^\circ = \frac{1}{\sin 270^\circ} = \frac{1}{1}$$

$$= 1.$$

$$(4) \quad \csc 360^\circ = -\infty.$$

$$\sin 360^\circ = \frac{1}{-\infty} = 0.$$

$$\sin^2 360^\circ + \cos^2 360^\circ = 1,$$

$$\cos^2 360^\circ = 1,$$

$$\cos 360^\circ = 1,$$

$$\tan 360^\circ = \frac{0}{1} = 0.$$

$$\cot 360^\circ = \frac{1}{0} = \infty.$$

$$\sec 360^\circ = \frac{1}{\cos 360^\circ} = \frac{1}{1}$$

$$= 1.$$

19. 求  $\sin 45^\circ$ ,  $\tan 540^\circ$ ,  
 $\cos 630^\circ$ ,  $\cot 720^\circ$ ,  $\sin 810^\circ$ ,

$\cos 900^\circ$  之各值

$$\begin{aligned}\sin 450^\circ &= \sin(360^\circ + 90^\circ) \\ &= \sin 90^\circ = 1.\end{aligned}$$

$$\begin{aligned}\tan 540^\circ &= \tan(360^\circ + 180^\circ) \\ &= \tan 180^\circ = 0.\end{aligned}$$

$$\begin{aligned}\cos 630^\circ &= \cos(360^\circ + 270^\circ) \\ &= \cos 270^\circ = 0.\end{aligned}$$

$$\begin{aligned}\cot 720^\circ &= \cot(360^\circ + 360^\circ) \\ &= \cot 360^\circ = \infty.\end{aligned}$$

$$\begin{aligned}\sin 810^\circ &= \sin(2 \times 360^\circ \\ &+ 90^\circ) = \sin 90^\circ = 1.\end{aligned}$$

$$\begin{aligned}\csc 900^\circ &= \csc(2 \times 360^\circ \\ &+ 180^\circ) = \csc 180^\circ = \infty.\end{aligned}$$

20. 計算下式之各值。 $a \sin 0^\circ + b \cos 90^\circ - c \tan 180^\circ$

$$\sin 0^\circ = 0.$$

$$\cos 90^\circ = 0.$$

$$\tan 180^\circ = 0.$$

以此諸值代入，得

$$a \times 0 + b \times 0 - c \times 0 = 0.$$

21.  $a \cos 90^\circ - b \tan 180^\circ + c \cot 90^\circ$

$$\cos 90^\circ = 0.$$

$$\tan 180^\circ = 0.$$

$$\cot 90^\circ = 0.$$

故  $a \times 0 - b \times 0 + c \times 0 = 0$ .

22.  $a \sin 90^\circ - b \cos 360^\circ + (a - b) \cos 180^\circ$

$$\sin 90^\circ = 1.$$

$$\cos 360^\circ = 1.$$

$$\cos 180^\circ = -1.$$

代入即得。

$$a \times 1 - b \times 1 + (a - b) \times (-1) = 0.$$

23.  $(a^2 - b^2) \cos 360^\circ - 4cb \sin 270^\circ$

$$\cos 360^\circ = 1.$$

$$\sin 270^\circ = -1.$$

$$\begin{aligned}\text{故 } (a^2 - b^2) \times 1 - 4cb \\ \times (-1) &= a^2 - b^2 + 4cb.\end{aligned}$$

## 例題 X

1. 試以小於  $45^\circ$  之一正角之函數表示  $\sin 250^\circ$ .

$$\sin 250^\circ = \sin(270^\circ - 20^\circ)$$

2. 試以小於  $45^\circ$  之角函數表示下函數。

$$\sin 172^\circ.$$

$$\begin{aligned}\sin 172^\circ &= \sin(180^\circ - 8^\circ) \\ &= \sin 8^\circ.\end{aligned}$$

3.  $\cos 100^\circ$ .

$$\begin{aligned}\cos 100^\circ &= \cos(90^\circ + 10^\circ) \\ &= -\sin 10^\circ.\end{aligned}$$

4.  $\tan 125^\circ = \tan 6^\circ$ .  
 $\tan 125^\circ = \tan(90^\circ + 35^\circ)$   
 $= -\cot 35^\circ$ .
5.  $\cot 91^\circ$ .  
 $\cot 91^\circ = \cot(90^\circ + 1^\circ)$   
 $= -\tan 1^\circ$ .
6.  $\sec 110^\circ$ .  
 $\sec 110^\circ = \sec(90^\circ + 20^\circ)$   
 $= -\csc 20^\circ$ .
7.  $\csc 157^\circ$ .  
 $\csc 157^\circ = \csc(180^\circ - 23^\circ)$   
 $= \csc 23^\circ$ .
8.  $\sin 204^\circ$ .  
 $\sin 204^\circ = \sin(180^\circ + 24^\circ)$   
 $= -\sin 24^\circ$ .
9.  $\cos 359^\circ$ .  
 $\cos 359^\circ = \cos(360^\circ - 1^\circ)$   
 $= \cos 1^\circ$ .
10.  $\tan 300^\circ$ .  
 $\tan 300^\circ = \tan(270^\circ + 30^\circ)$   
 $= -\cot 30^\circ$ .
11.  $\cot 264^\circ$ .  
 $\cot 264^\circ = \cot(270^\circ - 6^\circ)$   
 $\csc 92^\circ 25' = \csc(90^\circ + 2^\circ 25') = \sec 2^\circ 25'$ .
12.  $\sec 244^\circ$ .  
 $\sec 244^\circ = \sec(270^\circ - 26^\circ)$   
 $= -\csc 26^\circ$ .
13.  $\csc 271^\circ$ .  
 $\csc 271^\circ = \csc(270^\circ + 1^\circ)$   
 $= -\sec 1^\circ$ .
14.  $\sin 163^\circ 49'$ .  
 $\sin 163^\circ 49' = \sin(180^\circ - 16^\circ 11')$   
 $= \sin 16^\circ 11'$ .
15.  $\cos 195^\circ 33'$ .  
 $\cos 195^\circ 33' = \cos(180^\circ + 15^\circ 33')$   
 $= -\cos 15^\circ 33'$ .
16.  $\tan 269^\circ 15'$ .  
 $\tan 269^\circ 15' = \tan(270^\circ - 45')$   
 $= \cot 45'$ .
17.  $\cot 139^\circ 17'$ .  
 $\cot 139^\circ 17' = \cot(180^\circ - 40^\circ 43')$   
 $= -\cot 40^\circ 43'$ .
18.  $\sec 299^\circ 45'$ .  
 $\sec 299^\circ 45' = \sec(270^\circ + 29^\circ 45')$   
 $= -\csc 29^\circ 45'$ .
19.  $\csc 92^\circ 25'$ .
20. 試以小於  $45^\circ$  之正角之函數表下列負角之各函數。  
 $\sin(-75^\circ) = \sin(270^\circ + 15^\circ) = -\cos 15^\circ$ .

$$\begin{aligned}\cos(-75^\circ) &= \cos(270^\circ + 15^\circ) = \sin 15^\circ. \\ \tan(-75^\circ) &= \tan(270^\circ + 15^\circ) = -\cot 15^\circ. \\ \cot(-75^\circ) &= \cot(270^\circ + 15^\circ) = -\tan 15^\circ.\end{aligned}$$

31.  $-127^\circ$

$$\begin{aligned}\sin(-127^\circ) &= \sin(270^\circ - 37^\circ) = -\cos 37^\circ. \\ \cos(-127^\circ) &= \cos(270^\circ - 37^\circ) = -\sin 37^\circ. \\ \tan(-127^\circ) &= \tan(270^\circ - 37^\circ) = \cot 37^\circ. \\ \cot(-127^\circ) &= \cot(270^\circ - 37^\circ) = \tan 37^\circ.\end{aligned}$$

22.  $-200^\circ$

$$\begin{aligned}\sin(-200^\circ) &= \sin(180^\circ - 20^\circ) = \sin 20^\circ. \\ \cos(-200^\circ) &= \cos(180^\circ - 20^\circ) = -\cos 20^\circ. \\ \tan(-200^\circ) &= \tan(180^\circ - 20^\circ) = -\tan 20^\circ. \\ \cot(-200^\circ) &= \cot(180^\circ - 20^\circ) = -\cot 20^\circ.\end{aligned}$$

23.  $-345^\circ$

$$\begin{aligned}\sin(-345^\circ) &= \sin(360^\circ - 345^\circ) = \sin 15^\circ. \\ \cos(-345^\circ) &= \cos(360^\circ - 345^\circ) = \cos 15^\circ. \\ \tan(-345^\circ) &= \tan(360^\circ - 345^\circ) = \tan 15^\circ. \\ \cot(-345^\circ) &= \cot(360^\circ - 345^\circ) = \cot 15^\circ.\end{aligned}$$

24.  $-52^\circ 37'$

$$\begin{aligned}\sin(-52^\circ 37') &= \sin(270^\circ + 37^\circ 23') = -\cos 37^\circ 23'. \\ \cos(-52^\circ 37') &= \cos(270^\circ + 37^\circ 23') = \sin 37^\circ 23'. \\ \tan(-52^\circ 37') &= \tan(270^\circ + 37^\circ 23') = -\cot 37^\circ 23'. \\ \cot(-52^\circ 37') &= \cot(270^\circ + 37^\circ 23') = -\tan 37^\circ 23'.\end{aligned}$$

25.  $-196^\circ 54'$

$$\begin{aligned}\sin(-196^\circ 54') &= \sin(180^\circ - 16^\circ 54') = \sin 16^\circ 54'. \\ \cos(-196^\circ 54') &= \cos(180^\circ - 16^\circ 54') = -\cos 16^\circ 54'.\end{aligned}$$



$$\tan(-196^{\circ}54') = \tan(180^{\circ} - 16^{\circ}54') = -\tan 16^{\circ}54'.$$

$$\cot(-196^{\circ}54') = \cot(182^{\circ} - 16^{\circ}54') = -\cot 16^{\circ}54'.$$

26. 求  $120^{\circ}$  之各函數.

$$\sin 120^{\circ} = \sin(90^{\circ} + 30^{\circ}) = \cos 30^{\circ} = \frac{1}{2}\sqrt{3}.$$

$$\cos 120^{\circ} = \cos(90^{\circ} + 30^{\circ}) = -\sin 30^{\circ} = -\frac{1}{2}.$$

$$\tan 120^{\circ} = \tan(90^{\circ} + 30^{\circ}) = -\cot 30^{\circ} = -\sqrt{3}.$$

$$\cot 120^{\circ} = \cot(90^{\circ} + 30^{\circ}) = -\tan 30^{\circ} = -\frac{1}{\sqrt{3}}.$$

27. 求  $135^{\circ}$ 。下列諸角之各函數.

$$\sin 135^{\circ} = \sin(90^{\circ} + 45^{\circ}) = \cos 45^{\circ} = \frac{1}{\sqrt{2}}.$$

$$\cos 135^{\circ} = \cos(90^{\circ} + 45^{\circ}) = -\sin 45^{\circ} = -\frac{1}{\sqrt{2}}.$$

$$\tan 135^{\circ} = \tan(90^{\circ} + 45^{\circ}) = -\cot 45^{\circ} = -1.$$

$$\cot 135^{\circ} = \cot(90^{\circ} + 45^{\circ}) = -\tan 45^{\circ} = -1.$$

28.  $150^{\circ}$ 。

$$\sin 150^{\circ} = \sin(180^{\circ} - 30^{\circ}) = \sin 30^{\circ} = \frac{1}{2}.$$

$$\cos 150^{\circ} = \cos(180^{\circ} - 30^{\circ}) = -\cos 30^{\circ} = -\frac{1}{2}\sqrt{3}.$$

$$\tan 150^{\circ} = \tan(180^{\circ} - 30^{\circ}) = -\tan 30^{\circ} = -\frac{1}{\sqrt{3}}.$$

$$\cot 150^{\circ} = \cot(180^{\circ} - 30^{\circ}) = -\cot 30^{\circ} = -\sqrt{3}.$$

29.  $210^{\circ}$ 。

$$\sin 210^{\circ} = \sin(180^{\circ} + 30^{\circ}) = -\sin 30^{\circ} = -\frac{1}{2}.$$

$$\cos 210^{\circ} = \cos(180^{\circ} + 30^{\circ}) = -\cos 30^{\circ} = -\frac{1}{2}\sqrt{3}.$$

$$\tan 210^{\circ} = \tan(180^{\circ} + 30^{\circ}) = \tan 30^{\circ} = \frac{1}{\sqrt{3}}.$$

$$\cot 210^{\circ} = \cot(180^{\circ} + 30^{\circ}) = \cot 30^{\circ} = \sqrt{3}.$$

30.  $225^{\circ}$ 。

$$\sin 225^{\circ} = \sin(180^{\circ} + 45^{\circ}) = -\sin 45^{\circ} = -\frac{1}{\sqrt{2}}.$$

$$\cos 225^\circ = \cos (180^\circ + 45^\circ) = -\cos 45^\circ = -\frac{1}{2}\sqrt{2}.$$

$$\tan 225^\circ = \tan (180^\circ + 45^\circ) = \tan 45^\circ = 1.$$

$$\cot 225^\circ = \cot (180^\circ + 45^\circ) = \cot 45^\circ = 1.$$

31.  $240^\circ$ .

$$\sin 240^\circ = \sin (270^\circ - 30^\circ) = -\cos 30^\circ = -\frac{1}{2}\sqrt{3}.$$

$$\cos 240^\circ = \cos (270^\circ - 30^\circ) = -\sin 30^\circ = -\frac{1}{2}.$$

$$\tan 240^\circ = \tan (270^\circ - 30^\circ) = \cot 30^\circ = \sqrt{3}.$$

$$\cot 240^\circ = \cot (270^\circ - 30^\circ) = \tan 30^\circ = \frac{1}{\sqrt{3}}.$$

32.  $300^\circ$

$$\sin 300^\circ = \sin (270^\circ + 30^\circ) = -\cos 30^\circ = -\frac{1}{2}\sqrt{3}.$$

$$\cos 300^\circ = \cos (270^\circ + 30^\circ) = \sin 30^\circ = \frac{1}{2}.$$

$$\tan 300^\circ = \tan (270^\circ + 30^\circ) = -\cot 30^\circ = -\sqrt{3}.$$

$$\cot 300^\circ = \cot (270^\circ + 30^\circ) = -\tan 30^\circ = -\frac{1}{\sqrt{3}}.$$

33.  $-30^\circ$

$$\sin (-30^\circ) = -\sin 30^\circ = -\frac{1}{2}.$$

$$\cos (-30^\circ) = \cos 30^\circ = \frac{1}{2}\sqrt{3}.$$

$$\tan (-30^\circ) = -\tan 30^\circ = -\frac{1}{\sqrt{3}}.$$

$$\cot (-30^\circ) = -\cot 30^\circ = -\sqrt{3}.$$

34.  $-225^\circ$

$$-225^\circ = 90^\circ - 45^\circ$$

$$\sin (-225^\circ) = \sin (90^\circ - 45^\circ) = \cos 45^\circ = \frac{1}{2}\sqrt{2}.$$

$$\cos (-225^\circ) = \cos (90^\circ - 45^\circ) = -\sin 45^\circ = -\frac{1}{2}\sqrt{2}.$$

$$\tan (-225^\circ) = \tan (90^\circ - 45^\circ) = \cot 45^\circ = 1.$$

$$\cot (-225^\circ) = \cot (90^\circ - 45^\circ) = -\tan 45^\circ = -1.$$

35. 已知  $\sin x = -\frac{1}{2}$  及  $\cos x$  爲負。求  $x$  之其餘各函數及

x之值.

因  $\sin 45^\circ = -\frac{1}{\sqrt{2}}$ , 又  $\cos 45^\circ$  爲負, 故 x 角必在第三象限內, 且必等於  $(180^\circ + 45^\circ)$  即  $225^\circ$ .

$$\cos (180^\circ + 45^\circ) = -\cos 45^\circ = -\frac{1}{\sqrt{2}}$$

$$\tan (180^\circ + 45^\circ) = \tan 45^\circ = 1.$$

$$\cot (180^\circ + 45^\circ) = \cot 45^\circ = 1.$$

36. 已知  $\cot x = -\sqrt{3}$  及 x 在象限 II 內°求 x 之其餘各函數及 x 之值。

因  $\cot 30^\circ = \sqrt{3}$ , 且其號爲負, 故 x 角必爲  $(180^\circ - 30^\circ)$  即  $150^\circ$ .

$$\sin 150^\circ = \sin (180^\circ - 30^\circ) = \sin 30^\circ = \frac{1}{2}.$$

$$\cos 150^\circ = \cos (180^\circ - 30^\circ) = -\cos 30^\circ = -\frac{1}{2}\sqrt{3}.$$

$$\tan 150^\circ = \tan (180^\circ - 30^\circ) = -\tan 30^\circ = -\frac{1}{\sqrt{3}}.$$

37. 求  $3540^\circ$  之各函數.

$$3540^\circ = 9 \times 360^\circ + 300^\circ.$$

$$\sin 300^\circ = \sin (270^\circ + 30^\circ) = -\cos 30^\circ = -\frac{1}{2}\sqrt{3}.$$

$$\cos 300^\circ = \cos (270^\circ + 30^\circ) = \sin 30^\circ = \frac{1}{2}.$$

$$\tan 300^\circ = \tan (270^\circ + 30^\circ) = -\cot 30^\circ = -\frac{1}{\sqrt{3}}.$$

$$\cot 300^\circ = \cot (270^\circ + 30^\circ) = -\tan 30^\circ = -\frac{1}{\sqrt{3}}.$$

38. 何角之正弦等於  $-\frac{1}{2}$  又問何角之正切等於  $-\sqrt{3}$ . (二角均須小於  $360^\circ$ ).

(1) 因  $\sin 30^\circ = \frac{1}{2}$ , 而其號爲負, 故所求之角必在第三或第四象限內, 且必等於  $(180^\circ + 30^\circ)$  即  $210^\circ$  或  $(360^\circ - 30^\circ)$  即  $330^\circ$ .

(2) 因  $\tan 60^\circ = \sqrt{3}$ , 又其號數爲負, 故所求之角必在第

二或第四象限內，且必等於  $(180^\circ - 60^\circ)$  即  $120^\circ$  或  $(360^\circ - 60^\circ)$  即  $300^\circ$ 。

39. 問於 27—34 題中。之何角之餘弦爲  $\frac{1}{2}\sqrt{2}$ 。又何角之餘切爲  $-\sqrt{3}$ 。

(1) 因  $\cos 45^\circ = \frac{1}{\sqrt{2}}$ ，而所言之號又爲負，故所求之角必在第二或第三象限內，且必等於  $(180^\circ - 45^\circ)$  即  $135^\circ$  或  $(180^\circ + 45^\circ)$  即  $225^\circ$ 。又  $-225^\circ$  之函數與  $(360^\circ - 225^\circ)$  即  $135^\circ$  之函數相同。故餘弦爲  $-\frac{1}{\sqrt{2}}$  之諸角即  $135^\circ, -225^\circ$ 。與  $-225^\circ$ 。

(2) 因  $\cot 30^\circ = \sqrt{3}$ ，而所言之號又爲負，故其角必在第二或第四象限內，且必等於  $(180^\circ - 30^\circ)$  即  $150^\circ$  或  $(360^\circ - 30^\circ)$  即  $330^\circ$ 。故餘切爲  $-\sqrt{3}$  之諸角即  $150^\circ$  與  $330^\circ$ 。

40. 問  $x$  於  $0^\circ$  及  $720^\circ$  間爲何值。方能合於  $\sin x = +\frac{1}{2}$ 。

因  $\sin 30^\circ = \frac{1}{2}$ ，而題言之號又爲正，故於第一週內  $x$  角必在第一或第二象限內，且必等於  $30^\circ$  或  $(180^\circ - 30^\circ)$  即  $150^\circ$ 。於第二週內各角同加  $360^\circ$ 。故  $x$  角之值爲  $30^\circ, 150^\circ, 或 510^\circ$ 。

41. 已知  $\sin 12^\circ, \cos 26^\circ, \tan 45^\circ, \cot 72^\circ; \sin 191^\circ; \cos 120^\circ, \tan 244^\circ, \cot 357^\circ$  外。求  $0^\circ$  與  $360^\circ$  間與此相當函數(符號含內)。同值之他角。

其號既須相同，則

$\sin 12^\circ$  必在第二象限，即等於  $\sin (180^\circ - 12^\circ) = \sin 168^\circ$ 。

$\cos 26^\circ$  必在第四象限，即等於  $\cos (360^\circ - 26^\circ) = \cos 334^\circ$ 。

$\tan 45^\circ$  必在第三象限，即等於  $\tan (180^\circ + 45^\circ) = \tan 225^\circ$ 。

$\cot 72^\circ$  必在第三象限，即等於  $\cot (180^\circ + 72^\circ) = \cot 252^\circ$ 。

$\sin 191^\circ$  必在第四象限, 即等於  $\sin (360^\circ - 1^\circ) = \sin 349^\circ$ .

$\cos 120^\circ$  必在第三象限, 即等於  $\cos (180^\circ + 60^\circ) = \cos 240^\circ$ .

$\tan 244^\circ$  必在第一象限, 即等於  $\tan (244^\circ - 180^\circ) = \tan 64^\circ$ .

$\cot 357^\circ$  必在第二象限, 即等於  $\cot (357^\circ - 180^\circ) = \cot 177^\circ$ .

42. 已知  $\tan 238^\circ = 1.6$ ; 求  $\sin 122^\circ$ .

$$\tan 238^\circ = \tan (180^\circ + 58^\circ) = \tan 58^\circ.$$

$$\sin 122^\circ = \sin (180^\circ - 58^\circ) = \sin 58^\circ.$$

但  $\tan 238^\circ = 1.6$ .

故  $\tan 58^\circ = 1.6$ .

$$\tan 58^\circ = \frac{\sin 58^\circ}{\cos 58^\circ}.$$

$$1.6 = \frac{\sin 58^\circ}{\sqrt{1 - \sin^2 58^\circ}}.$$

$$2.56 - 2.56 \sin^2 58^\circ = \sin^2 58^\circ.$$

$$3.56 \sin^2 58^\circ = 2.56.$$

$$\sin 58^\circ = \sqrt{\frac{2.56}{3.56}} = 0.8480.$$

43. 已知  $\cos 333^\circ = 0.89$  求  $\tan 117^\circ$ .

$$\cos 333^\circ = 0.89 = \cos (270^\circ + 63^\circ) = \sin 63^\circ.$$

$$\tan 117^\circ = \tan (180^\circ - 63^\circ) = -\tan 63^\circ.$$

$$\sin^2 63^\circ + \cos^2 63^\circ = 1.$$

$$(0.89)^2 + \cos^2 63^\circ = 1.$$

$$\cos^2 63^\circ = 0.2079.$$

$$\cos 63^\circ = 0.4559.$$

$$-\tan 23^\circ = -\frac{\sin 63^\circ}{\cos 63^\circ} = -\frac{0.89}{0.4559} = -1.9522.$$

試將下列各式簡單之 (44-50).

$$44. a \cos (90^\circ - x) + b \cos (90^\circ + x)。$$

$$\begin{aligned} a \cos (90^\circ - x) + b \cos (90^\circ + x) &= a \sin x + b(-\sin x) \\ &= (a - b) \sin x. \end{aligned}$$

$$45. m \cos (90^\circ - x) \sin (90^\circ - x)。$$

$$m \cos (90^\circ - x) \sin (90^\circ - x) = m \sin x \cos x.$$

$$46. (a - b) \tan (90^\circ - x) + (a + b) \cot (90^\circ + x)。$$

$$\tan (90^\circ - x) = \cot x,$$

$$\cot (90^\circ + x) = -\tan x.$$

故上式等於  $(a - b) \cot x - (a + b) \tan x$ .

$$47. a^2 + b^2 - 2ab \cos^\circ (180 - x)。$$

$$\begin{aligned} a^2 + b^2 - 2ab \cos (180^\circ - x) &= a^2 + b^2 - 2ab(-\cos x) \\ &= a^2 + b^2 + 2ab \cos x. \end{aligned}$$

$$48. \sin (90^\circ + x) \sin (180^\circ + x) + \cos (90^\circ + x) \cos (180^\circ - x)$$

$$\sin (90^\circ + x) \sin (180^\circ + x) + \cos (90^\circ + x) \cos (180^\circ - x)$$

$$= (\cos x)(-\sin x) + (-\sin x)(-\cos x)$$

$$= -\sin x \cos x + \sin x \cos x = 0.$$

$$49. \cos (180^\circ + x) \cos (270^\circ - y) - \sin (180^\circ + x) \sin$$

$$(270^\circ - y)$$

$$\text{因 } \cos (180^\circ + x) = -\cos x.$$

$$\cos (270^\circ - y) = -\sin y.$$

$$\sin (180^\circ + x) = -\sin x.$$

$$\sin (270^\circ - y) = -\cos y.$$

故上式等於  $\cos x \sin y - \sin x \cos y$ .

$$50. \tan x + \tan(-y) - \tan (180^\circ - y)。$$

因  $\tan(-y) = -\tan y$ .  $-\tan(180-y) = \tan y$ .

故 上式等於  $\tan x$ .

51. 問  $x$  爲何值。則  $\sin x + \cos x$  式爲正。又何值則爲負。

若  $x$  在第一象限則  $\sin x + \cos x$  必爲正，因  $\sin x$  與  $\cos x$  皆爲正故也。

在第二象限內  $\sin x$  爲正而  $\cos x$  爲負，故若  $\sin x$  大於  $\cos x$  或等於  $\cos x$  則  $\sin x + \cos x$  仍爲正，但已過此象限之中線 ( $135^\circ$ ) 之後  $\cos x$  恒大於  $\sin x$  故  $\sin x + \cos x$  卽爲負。

在第三象限內  $\sin x$  與  $\cos x$  皆爲負，故其相加之和亦必爲負。

在第四象限內  $\sin x$  爲負  $\cos x$  爲正，當  $315^\circ$  時二者皆等，以後則  $\cos x$  大於  $\sin x$ ，故  $\sin x + \cos x$  在  $135^\circ$  與  $315^\circ$  之間與爲負，在  $0^\circ$  與  $135^\circ$  或  $315^\circ$  與  $360^\circ$  之間則爲正。

52. 問 51 題之式爲  $\sin x - \cos x$  時。答數爲何。

若  $x$  角自  $0^\circ$  漸增至  $45^\circ$ ，則  $\sin x$  亦漸增，而  $\cos x$  則漸減。當  $45^\circ$  時  $\sin x = \cos x$ ，故自  $0^\circ$  至此  $\sin x - \cos x$  必爲負。過  $45^\circ$  以後  $\sin x$  恒大於  $\cos x$ ，故  $\sin x - \cos x$  亦卽爲正。

在第二象限  $\sin x$  爲正  $\cos x$  爲負，故  $\sin x - \cos x$  必常爲正。

在第三象限  $\sin x$  與  $\cos x$  皆爲負。故  $\sin x$  小於  $\cos x$  時(卽自  $180^\circ$  至  $225^\circ$ )  $\sin x - \cos x$  必爲正；自  $225^\circ$  以後  $\sin x$  恒大於  $\cos x$  故  $\sin x - \cos x$  爲負。

在第四象限  $\sin x$  爲負  $\cos x$  爲正，故  $\sin x - \cos x$  必常爲正。總之在  $0^\circ$  與  $45^\circ$  或  $225^\circ$  與  $360^\circ$  之間則常爲負，在  $45^\circ$  與  $225^\circ$  之間則常爲正。

53. 求  $x - 90^\circ$  之各函數。以  $x$  之函數表之。

$$x - 90^\circ = 360^\circ - (90^\circ - x) = 270^\circ + x.$$

$$\sin(x - 90^\circ) = \sin(270^\circ + x) = -\cos x,$$

$$\cos(x - 90^\circ) = \cos(270^\circ + x) = \sin x,$$

$$\tan(x - 90^\circ) = \tan(270^\circ + x) = -\cot x,$$

$$\cot(x - 90^\circ) = \cot(270^\circ + x) = -\tan x.$$

54. 求  $x - 180^\circ$  之各函數。以  $x$  之函數表之。

$$x - 180^\circ = 360^\circ - (180^\circ - x) = 180^\circ + x.$$

$$\sin(x - 180^\circ) = \sin(180^\circ + x) = -\sin x$$

$$\cos(180^\circ) = \cos(180^\circ + x) = -\cos x,$$

$$\tan(x - 180^\circ) = \tan(180^\circ + x) = \tan x,$$

$$\cot(x - 180^\circ) = \cot(180^\circ + x) = \cot x,$$

## 例題 XIV

1. 設  $\sin x = \frac{3}{5}$ ,  $\cos x = \frac{4}{5}$ ,  $\sin y = \frac{5}{13}$ ,  $\cos y = \frac{12}{13}$ , (求  $x + y$ )

之正弦及餘弦之值。

$$\sin(x + y) = \sin x \cos y + \cos x \sin y,$$

$$= \left(\frac{3}{5} \times \frac{12}{13}\right) + \left(\frac{4}{5} \times \frac{5}{13}\right) = \frac{36}{65} + \frac{20}{65} = \frac{56}{65}.$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y,$$

$$= \left(\frac{4}{5} \times \frac{12}{13}\right) - \left(\frac{3}{5} \times \frac{5}{13}\right) = \frac{48}{65} - \frac{15}{65} = \frac{33}{65}.$$



2. 設由公式[8]及[9]。令  $x=90^\circ$ 。求  $\sin(90^\circ-y)$  及  $\cos(90^\circ-y)$  之值。

$$\begin{aligned}\sin(90^\circ-y) &= \sin 90^\circ \cos y - \cos 90^\circ \sin y \\ &= (1 \times \cos y) - (0 \times \sin y) = \cos y.\end{aligned}$$

$$\begin{aligned}\cos(90^\circ-y) &= \cos 90^\circ \cos y + \sin 90^\circ \sin y \\ &= (0 \times \cos y) + (1 \times \sin y) = \sin y.\end{aligned}$$

設由公式[4] - [11]。求下各題之首四個函數°

3.  $90^\circ+y$

$$\begin{aligned}\sin(90^\circ+y) &= \sin 90^\circ \cos y + \cos 90^\circ \sin y \\ &= (1 \times \cos y) + (0 \times \sin y) = \cos y.\end{aligned}$$

$$\begin{aligned}\cos(90^\circ+y) &= \cos 90^\circ \cos y - \sin 90^\circ \sin y \\ &= (0 \times \cos y) - (1 \times \sin y) = -\sin y.\end{aligned}$$

$$\tan(90^\circ+y) = -\frac{\cos y}{\sin y} = -\cot y.$$

$$\cot(90^\circ+y) = -\frac{\sin y}{\cos y} = -\tan y.$$

4.  $180^\circ-y$

$$\begin{aligned}\sin(180^\circ-y) &= \sin 180^\circ \cos y - \cos 180^\circ \sin y \\ &= (0 \times \cos y) - (-1 \times \sin y) = \sin y.\end{aligned}$$

$$\begin{aligned}\cos(180^\circ-y) &= \cos 180^\circ \cos y + \sin 180^\circ \sin y \\ &= (-1 \times \cos y) + (0 \times \sin y) = -\cos y.\end{aligned}$$

$$\tan(180^\circ-y) = -\frac{\sin y}{\cos y} = -\tan y.$$

$$\cot(180^\circ-y) = -\frac{\cos y}{\sin y} = -\cot y.$$

5.  $180^\circ+y$

$$\begin{aligned}\sin(180^\circ+y) &= \sin 180^\circ \cos y + \cos 180^\circ \sin y \\ &= (0 \times \cos y) + (-1 \times \sin y) = -\sin y,\end{aligned}$$

$$\begin{aligned}\cos (180^{\circ} + y) &= \cos 180^{\circ} \cos y - \sin 180^{\circ} \sin y \\ &= (-1 \times \cos y) - (0 \times \sin y) = -\cos y.\end{aligned}$$

$$\tan (180^{\circ} + y) = \frac{-\sin y}{-\cos y} = \tan y.$$

$$\cot (180^{\circ} + y) = \frac{-\cos y}{-\sin y} = \cot y.$$

6.  $270^{\circ} - y$ .

$$\begin{aligned}\sin (270^{\circ} - y) &= \sin 270^{\circ} \cos y - \cos 270^{\circ} \sin y \\ &= (-1 \times \cos y) - (0 \times \sin y) = -\cos y.\end{aligned}$$

$$\begin{aligned}\cos (270^{\circ} - y) &= \cos 270^{\circ} \cos y + \sin 270^{\circ} \sin y \\ &= (0 \times \cos y) + (-1 \times \sin y) = -\sin y.\end{aligned}$$

$$\tan (270^{\circ} - y) = \frac{-\cos y}{-\sin y} = \cot y.$$

$$\cot (270^{\circ} - y) = \frac{-\sin y}{-\cos y} = \tan y.$$

7.  $270^{\circ} + y$ .

$$\begin{aligned}\sin (270^{\circ} + y) &= \sin 270^{\circ} \cos y + \cos 270^{\circ} \sin y \\ &= (-1 \times \cos y) + (0 \times \sin y) = -\cos y.\end{aligned}$$

$$\begin{aligned}\cos (270^{\circ} + y) &= \cos 270^{\circ} \cos y - \sin 270^{\circ} \sin y \\ &= (0 \times \cos y) - (-1 \times \sin y) = \sin y.\end{aligned}$$

$$\tan (270^{\circ} + y) = \frac{-\cos y}{-\sin y} = \cot y.$$

$$\cot (270^{\circ} + y) = \frac{-\sin y}{-\cos y} = \tan y.$$

8.  $360^{\circ} - y$

$$\begin{aligned}\sin (360^{\circ} - y) &= \sin 360^{\circ} \cos y - \cos 360^{\circ} \sin y \\ &= (0 \times \cos y) - (1 \times \sin y) = -\sin y.\end{aligned}$$

$$\begin{aligned}\cos(360^\circ - y) &= \cos 360^\circ \cos y + \sin 360^\circ \sin y \\ &= (1 \times \cos y) + (0 \times \sin y) = \cos y.\end{aligned}$$

$$\tan(360^\circ - y) = \frac{-\sin y}{\cos y} = -\tan y.$$

$$\cot(360^\circ - y) = \frac{\cos y}{-\sin y} = -\cot y.$$

9.  $360^\circ + y^\circ$

$$\begin{aligned}\sin(360^\circ + y) &= \sin 360^\circ \cos y + \cos 360^\circ \sin y \\ &= (0 \times \cos y) + (1 \times \sin y) = \sin y.\end{aligned}$$

$$\begin{aligned}\cos(360^\circ + y) &= \cos 360^\circ \cos y - \sin 360^\circ \sin y \\ &= (1 \times \cos y) - (0 \times \sin y) = \cos y.\end{aligned}$$

$$\tan(360^\circ + y) = \frac{\sin y}{\cos y} = \tan y.$$

$$\cot(360^\circ + y) = \frac{\cos y}{\sin y} = \cot y.$$

10.  $x - 90^\circ$

$$\begin{aligned}\sin(x - 90^\circ) &= \sin x \cos 90^\circ - \cos x \sin 90^\circ \\ &= (0 \times \sin x) - (1 \times \cos x) = -\cos x.\end{aligned}$$

$$\begin{aligned}\cos(x - 90^\circ) &= \cos x \cos 90^\circ + \sin x \sin 90^\circ \\ &= (0 \times \cos x) + (1 \times \sin x) = \sin x.\end{aligned}$$

$$\tan(x - 90^\circ) = \frac{-\cos x}{\sin x} = -\cot x.$$

$$\cot(x - 90^\circ) = \frac{\sin x}{-\cos x} = -\tan x.$$

11.  $x - 180^\circ$

$$\begin{aligned}\sin(x - 180^\circ) &= \sin x \cos 180^\circ - \cos x \sin 180^\circ \\ &= \sin x \times (-1) - \cos x \times 0 = -\sin x.\end{aligned}$$

$$\begin{aligned}\cos(x - 180^\circ) &= \cos x \cos 180^\circ + \sin x \sin 180^\circ \\ &= \cos x (-1) + \sin x \times 0 = -\cos x.\end{aligned}$$

$$\tan(x - 180^\circ) = \frac{-\sin x}{-\cos x} = \tan x.$$

$$\cot(x - 180^\circ) = \frac{-\cos x}{-\sin x} = \cot x.$$

12.  $x - 270^\circ$

$$\begin{aligned}\sin(x - 270^\circ) &= \sin x \cos 270^\circ - \cos x \sin 270^\circ \\ &= \sin x \times 0 - \cos x + (-1) = \cos x.\end{aligned}$$

$$\begin{aligned}\cos(x - 270^\circ) &= \cos x \cos 270^\circ + \sin x \sin 270^\circ \\ &= \cos x \times 0 + \sin x \times (-1) = -\sin x.\end{aligned}$$

$$\tan(x - 270^\circ) = \frac{\cos x}{-\sin x} = -\cot x,$$

$$\cot(x - 270^\circ) = \frac{-\sin x}{\cos x} = -\tan x.$$

13.  $-y$

$$\begin{aligned}\sin(0^\circ - y) &= \sin 0^\circ \cos y - \cos 0^\circ \sin y \\ &= (0 \times \cos y) - (1 \times \sin y) = -\sin y.\end{aligned}$$

$$\begin{aligned}\cos(0^\circ - y) &= \cos 0^\circ \cos y + \sin 0^\circ \sin y \\ &= (1 \times \cos y) + (0 \times \sin y) = \cos y.\end{aligned}$$

$$\tan(0^\circ - y) = \frac{-\sin y}{\cos y} = -\tan y.$$

$$\cot(0^\circ - y) = \frac{\cos y}{-\sin y} = -\cot y.$$

14.  $45^\circ - y$

$$\begin{aligned}\sin(45^\circ - y) &= \sin 45^\circ \cos y - \cos 45^\circ \sin y \\ &= \frac{1}{\sqrt{2}} \cos y - \frac{1}{\sqrt{2}} \sin y = \frac{1}{\sqrt{2}}\end{aligned}$$

$$(\cos y - \sin y).$$

$$\begin{aligned}\cos (45^\circ - y) &= \cos 45^\circ \cos y + \sin 45^\circ \sin y \\ &= \frac{1}{2}\sqrt{2} \cos y + \frac{1}{2}\sqrt{2} \sin y \\ &= \frac{1}{2}\sqrt{2} (\cos y + \sin y).\end{aligned}$$

$$\operatorname{tsn} (45^\circ - y) = \frac{\cos y - \sin y}{\cos y + \sin y} = \frac{1 - \tan y}{1 + \tan y}.$$

$$\operatorname{cot} (45^\circ - y) = \frac{\cos y + \sin y}{\cos y - \sin y} = \frac{\cot y + 1}{\cot y - 1}.$$

15.  $45^\circ + y$ .

$$\begin{aligned}\sin (45^\circ + y) &= \sin 45^\circ \cos y + \cos 45^\circ \sin y \\ &= \frac{1}{2}\sqrt{2} \cos y + \frac{1}{2}\sqrt{2} \sin y \\ &= \frac{1}{2}\sqrt{2} (\cos y + \sin y).\end{aligned}$$

$$\begin{aligned}\cos (45^\circ + y) &= \cos 45^\circ \cos y - \sin 45^\circ \sin y \\ &= \frac{1}{2}\sqrt{2} \cos y - \frac{1}{2}\sqrt{2} \sin y \\ &= \frac{1}{2}\sqrt{2} (\cos y - \sin y).\end{aligned}$$

$$\tan (45^\circ + y) = \frac{\cos y + \sin y}{\cos y - \sin y} = \frac{1 + \tan y}{1 - \tan y}.$$

$$\operatorname{cot} (45^\circ + y) = \frac{\cos y - \sin y}{\cos y + \sin y} = \frac{\cot y - 1}{\cot y + 1}.$$

16.  $30^\circ + y$ .

$$\begin{aligned}\sin (30^\circ + y) &= \sin 30^\circ \cos y + \cos 30^\circ \sin y \\ &= \frac{1}{2}(\cos y + \sqrt{3} \sin y).\end{aligned}$$

$$\begin{aligned}\cos (30^\circ + y) &= \cos 30^\circ \cos y - \sin 30^\circ \sin y \\ &= \frac{1}{2}(\cos y - \sqrt{3} \sin y).\end{aligned}$$

$$\tan (30^\circ + y) = \frac{\cos y + \sqrt{3} \sin y}{\sqrt{3} \cos y - \sin y}; \text{以 } \sqrt{3} \cos y \text{ 同除分子}$$

$$\text{分母} = \frac{\frac{1}{3}\sqrt{3} + \tan y}{1 - \frac{1}{3}\sqrt{3} \tan y}$$

$$\begin{aligned} \cot(39^\circ + y) &= \frac{\sqrt{3} \cos y - \sin y}{\cos y + \sqrt{3} \sin y}; \text{以 } \sin y \text{ 同除分母分子,} \\ &= \frac{\sqrt{3} \cot y - 1}{\cot y + \sqrt{3}} \end{aligned}$$

17.  $60^\circ - y^\circ$

$$\begin{aligned} \sin(60^\circ - y) &= \sin 60^\circ \cos y - \cos 60^\circ \sin y \\ &= \frac{1}{2}(\sqrt{3} \cos y - \sin y). \end{aligned}$$

$$\begin{aligned} \cos(60^\circ - y) &= \cos 60^\circ \cos y + \sin 60^\circ \sin y \\ &= \frac{1}{2}(\cos y + \sqrt{3} \sin y). \end{aligned}$$

$$\tan(60^\circ - y) = \frac{\sqrt{3} \cos y - \sin y}{\cos y + \sqrt{3} \sin y} = \frac{\sqrt{3} - \tan y}{1 + \sqrt{3} \tan y}$$

$$\cot(60^\circ - y) = \frac{\cos y + \sqrt{3} \sin y}{\sqrt{3} \cos y - \sin y} = \frac{\frac{1}{3}\sqrt{3} \cot y + 1}{\cot y - \frac{1}{3}\sqrt{3}}$$

18. 求以  $\sin x$  表  $\sin 3x$

$$\begin{aligned} \sin 3x &= \sin(2x + x) \\ &= \sin 2x \cos x + \\ &\quad \cos 2x \sin x. \end{aligned}$$

$$\begin{aligned} \sin 2x &= 2 \sin x \cos x, \\ \cos 2x &= \cos^2 x - \sin^2 x. \end{aligned}$$

$$\begin{aligned} \text{故 } \sin 3x &= 2 \sin x \cos^2 x \\ &\quad + \sin x \cos^2 x \\ &\quad - \sin^3 x. \\ &= 3 \sin x \cos^2 x \end{aligned}$$

$$- \sin^3 x.$$

但  $\cos^2 x = 1 - \sin^2 x.$

$$\begin{aligned} \text{故 } \sin 3x &= 3 \sin x - 3 \sin^3 x \\ &\quad - \sin^3 x = 3 \sin x \\ &\quad - 4 \sin^3 x. \end{aligned}$$

19. 求以  $\cos x$  表  $\cos 3x.$

$$\begin{aligned} \cos 3x &= \cos(2x + x) \\ &= \cos 2x \cos x \\ &\quad - \sin 2x \sin x. \end{aligned}$$

$$\sin 2x = 2 \sin x \cos x.$$

$$\cos 2x = \cos^2 x - \sin^2 x.$$

$$\text{故 } \cos 3x = \cos^2 x - \sin^2 x$$

$$\begin{aligned} \cos x - 2 \sin^2 x \cos x. \\ = \cos^2 x - 3 \sin^2 x \\ \cos x. \end{aligned}$$

$$\text{但 } \sin^2 x = 1 - \cos^2 x.$$

$$\begin{aligned} \text{故 } \cos 3x &= \cos^3 x - 3 \cos x \\ &+ 3 \cos^3 x = 4 \cos^3 x \\ &- 3 \cos x. \end{aligned}$$

20. 已知  $\tan \frac{1}{2}x = 1$ ; 求  $\cos x_0$

$$\tan \frac{1}{2}x = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

$$1 = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

$$1 = \frac{1 - \cos x}{1 + \cos x}$$

$$1 + \cos x = 1 - \cos x.$$

$$4 \cos x = 0.$$

$$\cos x = 0.$$

21. 已知  $\cot \frac{1}{2}x = \sqrt{3}$ . 求  $\sin x$ .

$$\cos \frac{1}{2}x = \sqrt{\frac{1 + \cos x}{1 - \cos x}}$$

$$\sqrt{3} = \sqrt{\frac{1 + \cos x}{1 - \cos x}}$$

$$3 = \frac{1 + \cos x}{1 - \cos x}.$$

$$3 - 3 \cos x = 1 + \cos x.$$

$$-4 \cos x = -2.$$

$$\cos x = \frac{1}{2}.$$

$$\sin^2 x = 1 - \cos^2 x.$$

$$= 1 - \frac{1}{4} = \frac{3}{4}.$$

$$\sin x = \sqrt{\frac{3}{4}} = \frac{1}{2}\sqrt{3}.$$

22. 已知  $\sin x = 0.2$ . 求  $\sin \frac{1}{2}x$  及  $\cos \frac{1}{2}x_0$

$$\sin x = 0.2.$$

$$\cos^2 x = 1 - \sin^2 x.$$

$$= 1 - 0.04.$$

$$\cos x = \sqrt{0.96}.$$

$$\sin \frac{1}{2}x = \sqrt{\frac{1 - \cos x}{2}}$$

$$= \sqrt{\frac{1 - \sqrt{0.96}}{2}}$$

$$= \sqrt{\frac{1 - 0.4\sqrt{6}}{2}}$$

$$= 0.10051.$$

$$\cos \frac{1}{2}x = \sqrt{\frac{1 + \cos x}{2}}$$

$$= \sqrt{\frac{1 + 0.4\sqrt{6}}{2}}$$

$$= 0.99493.$$

23. 已知  $\cos x = 0.5$ 。求  $\cos 2x$  及  $\tan 2x$ 。

$$\cos 2x = \cos^2 x - \sin^2 x.$$

$$\sin x = \sqrt{1 - 0.5^2} = \frac{1}{2}\sqrt{3}.$$

$$\text{故 } \cos 2x = 0.25 - 0.75.$$

$$= -0.5x = -\frac{1}{2}.$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{1}{2}\sqrt{3}}{\frac{1}{2}}$$

$$= \sqrt{3}.$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x} = \frac{2\sqrt{3}}{1 - 3}$$

$$= -\sqrt{3}.$$

24. 已知  $\tan 45^\circ = 1$ ；求  $22^\circ 30'$  之各函數。

$$\text{設 } x = 45^\circ.$$

$$\text{則 } \tan x = \frac{\sin x}{\cos x} = 1.$$

$$\text{故 } \sin x = \cos x.$$

$$\sin^2 x + \cos^2 x = 1.$$

$$2 \sin^2 x = 1.$$

$$\sin^2 x = \frac{1}{2}.$$

$$\sin x = \frac{1}{2}\sqrt{2} = \cos x.$$

$$\sin 22^\circ 40' = \sin \frac{1}{2}x$$

$$= \sqrt{\frac{1 - \frac{1}{2}\sqrt{2}}{2}}$$

$$= \frac{1}{2}\sqrt{2 - \sqrt{2}}$$

$$= 0.3827.$$

$$\cos 22^\circ 30' = \cos \frac{1}{2}x$$

$$= \sqrt{\frac{1 + \frac{1}{2}\sqrt{2}}{2}}$$

$$= \frac{1}{2}\sqrt{2 + \sqrt{2}}$$

$$= 0.9239.$$

$$\tan 22^\circ 30' = \tan \frac{1}{2}x$$

$$= \sqrt{\frac{1 - \frac{1}{2}\sqrt{2}}{1 + \frac{1}{2}\sqrt{2}}}$$

$$= \sqrt{\frac{(1 - \frac{1}{2}\sqrt{2})^2}{1 - \frac{1}{2}}}$$

$$= \sqrt{2(1 - \frac{1}{2}\sqrt{2})^2}$$

$$= (1 - \frac{1}{2}\sqrt{2})\sqrt{2}$$

$$= \sqrt{2} - 1 = 0.4142.$$

$$\cot 22^\circ 36' = \cot \frac{1}{2}x$$

$$= \sqrt{\frac{1 + \frac{1}{2}\sqrt{2}}{1 - \frac{1}{2}\sqrt{2}}}$$

$$= \sqrt{\frac{(1 + \frac{1}{2}\sqrt{2})^2}{1 + \frac{1}{2}}}$$

$$= \sqrt{2(1 + \frac{1}{2}\sqrt{2})^2}$$

$$= (1 + \frac{1}{2}\sqrt{2})\sqrt{2}$$



$$= \sqrt{2} + 1 = 2.4142.$$

25. 已知  $\sin 30^\circ = 0.5$ ; 求  $15^\circ$  之各函數。

$$\sin 30^\circ = 0.5 = \frac{1}{2}.$$

$$\begin{aligned} \text{故 } \cos 30^\circ &= \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} \\ &= \frac{1}{2}\sqrt{3}. \end{aligned}$$

$$\text{因 } \sin \frac{1}{2}x = \sqrt{\frac{1 - \cos x}{2}}.$$

$$\begin{aligned} \text{故 } \sin 15^\circ &= \sqrt{\frac{1 - \frac{1}{2}\sqrt{3}}{2}} \\ &= \frac{1}{2}\sqrt{2 - \sqrt{3}} \\ &= 0.2588. \end{aligned}$$

$$\begin{aligned} \cos 15^\circ &= \sqrt{\frac{1 + \frac{1}{2}\sqrt{3}}{2}} \\ &= \frac{1}{2}\sqrt{2 + \sqrt{3}} \\ &= 0.9659. \end{aligned}$$

$$\begin{aligned} \tan 15^\circ &= \sqrt{\frac{1 - \frac{1}{2}\sqrt{3}}{1 + \frac{1}{2}\sqrt{3}}} \\ &= \sqrt{\frac{2 - \sqrt{3}}{2 + \sqrt{3}}} \\ &= \sqrt{\frac{(2 - \sqrt{3})^2}{4 - 3}} \\ &= 2 - \sqrt{3} = 0.2679 \end{aligned}$$

$$\cot 15^\circ = \sqrt{\frac{1 + \frac{1}{2}\sqrt{3}}{1 - \frac{1}{2}\sqrt{3}}}$$

$$= 2 + \sqrt{3} = 3.7321$$

26. 求證  $\tan 18^\circ$

$$= \frac{\sin 33^\circ + \sin 3^\circ}{\cos 33^\circ + \cos 3^\circ}.$$

$$\text{設 } x = 18^\circ.$$

$$\text{則 } y = 15^\circ.$$

$$(1) \quad 2 \sin x \cos y = \sin(x+y) + \sin(x-y).$$

$$(2) \quad 2 \cos x \cos y = \cos(x+y) + \cos(x-y).$$

以 (2) 除 (1), 得

$$\begin{aligned} \tan x &= \frac{\sin(x+y)}{\cos(x+y)} \\ &\quad + \frac{\sin(x-y)}{\cos(x-y)}. \end{aligned}$$

將  $x$  與  $y$  之值代入, 得

$$\tan 18^\circ = \frac{\sin 33^\circ + \sin 3^\circ}{\cos 33^\circ + \cos 3^\circ}.$$

求證下之諸公式:

$$27. \quad \sin 2x$$

$$= \frac{2 \tan x}{1 + \tan^2 x}.$$

$$\sin 2x = 2 \sin x \cos x$$

$$= \frac{2 \sin x}{\cos x} \cos^2 x$$

$$= \frac{2 \sin x}{\cos x} \times \frac{1}{\sec^2 x}.$$

但按 5/2,  $\sec^2 = 1 + \tan^2 x$ .

$$\text{故 } \sin 2x = \frac{2 \tan x}{1 + \tan^2 x}.$$

$$28. \cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$\begin{aligned} \cos 2x &= \frac{\sin 2x}{\tan 2x} \\ &= \frac{2 \sin x \cos x}{\frac{2 \tan x}{1 - \tan^2 x}} \\ &= \frac{(1 - \tan^2 x)(2 \sin x \cos x)}{2 \tan x} \end{aligned}$$

$$= (1 - \tan^2 x) \cos^2 x.$$

但按  $\sec^2 = 1 + \tan^2 x$

$$= \frac{1}{\cos^2 x}$$

$$\text{故 } \cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}.$$

$$29. \tan \frac{1}{2}x = \frac{\sin x}{1 + \cos x}.$$

$$31. \quad \sin \frac{1}{2}x \pm \cos \frac{1}{2}x = \sqrt{1 \pm \sin x}.$$

$$\sin \frac{1}{2}x \pm \cos \frac{1}{2}x = \sqrt{(\sin \frac{1}{2}x \pm \cos \frac{1}{2}x)^2}$$

$$= \sqrt{\sin^2 \frac{1}{2}x + \cos^2 \frac{1}{2}x \pm 2 \sin \frac{1}{2}x \cos \frac{1}{2}x}$$

$$= \sqrt{1 \pm 2 \sin \frac{1}{2}x \cos \frac{1}{2}x}$$

$$= \sqrt{1 \pm \sin x}.$$

$$\tan \frac{1}{2}x = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

$$= \sqrt{\frac{(1 - \cos x)(1 + \cos x)}{(1 + \cos x)^2}}$$

$$= \sqrt{\frac{1 - \cos^2 x}{(1 + \cos^2 x)}}$$

$$= \frac{\sin x}{1 + \cos x}.$$

$$30. \cot \frac{1}{2}x = \frac{\sin x}{1 - \cos x}.$$

$$\cot \frac{1}{2}x = \sqrt{\frac{1 + \cos x}{1 - \cos x}}$$

$$= \sqrt{\frac{(1 + \cos x)(1 - \cos x)}{(1 - \cos x)^2}}$$

$$= \frac{\sqrt{1 - \cos^2 x}}{(1 - \cos x)^2}$$

$$= \frac{\sin x}{1 - \cos x}.$$

$$32. \frac{\tan x \pm \tan y}{\cot x \pm \cot y} = \pm \tan x \tan y.$$

$$\begin{aligned} \frac{\tan x \pm \tan y}{\cot x \pm \cot y} &= \frac{\frac{\sin x}{\cos x} \pm \frac{\sin y}{\cos y}}{\frac{\cos x}{\sin x} \pm \frac{\cos y}{\sin y}} \\ &= \frac{\sin x \cos y \pm \cos x \sin y}{\cos x \cos y} \\ &\quad \times \frac{\sin x \sin y}{\cos x \sin y \pm \sin x \cos y} \\ &= \pm \frac{\sin x \sin y}{\cos x \cos y} = \pm \tan x \tan y \end{aligned}$$

$$33. \tan(45^\circ - x) = \frac{1 - \tan x}{1 + \tan x}.$$

$$\tan(45^\circ - x) = \frac{\tan 45^\circ - \tan x}{1 + \tan 45^\circ \tan x} = \frac{1 - \tan x}{1 + \tan x}.$$

設 A, B, C, 爲三角形之三角。求証。

$$34. \sin A + \sin B + \sin C = 4 \cos \frac{1}{2}A \cos \frac{1}{2}B \cos \frac{1}{2}C.$$

$$\begin{aligned} &\sin A + \sin B + \sin C \\ &= \sin A + \sin B + \sin(180^\circ - A + B) \\ &= \sin A + \sin B + \sin(A + B). \end{aligned}$$

$$\begin{aligned} \text{按公式[20]與[21],} &= 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B) + 2 \sin \frac{1}{2} \\ &(A + B) \cos \frac{1}{2}(A + B) = 2 \sin \frac{1}{2}(A + B) \\ &[\cos \frac{1}{2}(A - B) + \cos \frac{1}{2}(A + B)]. \end{aligned}$$

$$\begin{aligned} \text{按公式[22].} &= 2 \sin \frac{1}{2}(A + B) 2 \cos \frac{1}{2}A \cos \frac{1}{2}B \\ &= 4 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}A \cos \frac{1}{2}B. \end{aligned}$$

$$\text{但} \quad \cos \frac{1}{2}C = \cos[90^\circ - \frac{1}{2}(A + B)] = \sin \frac{1}{2}(A + B).$$

故  $\sin A + \sin B + \sin C = 4 \cos \frac{1}{2}A \cos \frac{1}{2}B \cos \frac{1}{2}C.$

35.  $\cos A + \cos B + \cos C = 1 + 4 \sin \frac{1}{2}A \sin \frac{1}{2}B \sin \frac{1}{2}C$   
 $\cos C = \cos [180^\circ - (A + B)] = -\cos (A + B).$

故  $\cos A + \cos B + \cos C$   
 $= \cos A + \cos B - \cos (A + B).$

按公式[22],  $= 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B) - \cos \frac{1}{2}(A + B).$

按公式[13],  $= 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B) - 2 \cos^2 \frac{1}{2}$   
 $(A + B) + 1$

$= [2 \cos \frac{1}{2}(A + B)] \times [\cos \frac{1}{2}(A - B) - \cos \frac{1}{2}$   
 $(A + B)] + 1$

按公式[23],  $= [2 \cos \frac{1}{2}(A + B)] \times [2 \sin \frac{1}{2}A \sin \frac{1}{2}B] + 1$

$= (2 \sin \frac{1}{2}C) (2 \sin \frac{1}{2}A \sin \frac{1}{2}B) + 1$

$= 1 + 4 \sin \frac{1}{2}A \sin \frac{1}{2}B \sin \frac{1}{2}C.$

36.  $\tan A + \tan B + \tan C = \tan A \times \tan B \times \tan C.$

因  $A + B + C = 180^\circ$ , 故  $C = 180^\circ - (A + B).$

即  $\tan C = \tan [180^\circ - (A + B)] = -\tan (A + B).$

按公式[6],  $\tan A + \tan B = \tan (A + B) (1 - \tan A \tan B)$

$= \tan (A + B) - \tan (A + B) \tan A \tan B.$

故  $\tan A + \tan B + \tan C = \tan (A + B) - \tan (A + B)$

$- \tan (A + B) \tan A \tan B$

$= -\tan (A + B) \tan A \tan B$

$= \tan A \times \tan B \times \tan C.$

37.  $\cot \frac{1}{2}A + \cot \frac{1}{2}B + \cot \frac{1}{2}C = \cot \frac{1}{2}A \times \cot \frac{1}{2}B \times \cot \frac{1}{2}C.$

$\frac{1}{2}A + \frac{1}{2}B + \frac{1}{2}C = 90^\circ$ , 故  $\frac{1}{2}C = 90^\circ - \frac{1}{2}(A + B).$

$\cot \frac{1}{2}C = \tan \frac{1}{2}(A + B);$

$\cot \frac{1}{2}B = \tan \frac{1}{2}(A + C),$

$$\cot \frac{1}{2}A = \tan \frac{1}{2}(B + C)$$

故  $\cot \frac{1}{2}A + \cot \frac{1}{2}B + \cot \frac{1}{2}C$

$$= \tan \frac{1}{2}(A + B) + \tan \frac{1}{2}(A + C) + \tan \frac{1}{2}(B + C)$$

按 36 題  $= \tan \frac{1}{2}(A + B) \times \tan \frac{1}{2}(A + C) \times \tan \frac{1}{2}(B + C)$

$$= \cot \frac{1}{2}A \times \cot \frac{1}{2}B \times \cot \frac{1}{2}C.$$

求化下各題爲便於對數計算式.

38.  $\cot x + \tan x$

$$= 2 \cot 2x.$$

$$\cot x + \tan x$$

$$= \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$$

$$= \frac{2(\cos^2 x + \sin^2 x)}{2 \sin x \cos x}$$

按公式[12],  $= \frac{2}{\sin 2x}.$

39.  $\cot x - \tan x$

$$\cot x - \tan x$$

$$= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$$

$$= \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$$

按公式[13],  $= \frac{\cos 2x}{\sin x \cos x}$

$$= \frac{2 \cos 2x}{2 \sin x \cos x}.$$

按公式[12],  $= \frac{2 \cos 2x}{\sin 2x}$

40.  $\cot x + \tan y$

$$\cot x + \tan y$$

$$= \frac{\cos x}{\sin x} + \frac{\sin y}{\cos y}$$

$$= \frac{\cos x - \cos y}{\sin x}$$

$$\frac{\sin y}{\cos y}$$

按公式[9],  $= \frac{\cos(x - y)}{\sin x \cos y}.$

41.  $\cot x - \tan y$

$$\cot x - \tan y$$

$$= \frac{\cos x}{\sin x} - \frac{\sin y}{\cos y}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{\sin x \cos y}$$

$$\frac{\cos(x - y)}{\sin x \cos y}$$

按公式[5],  $= \frac{\cos(x + y)}{\sin x \cos y}.$

42.  $\frac{1 - \cos 2x}{1 + \cos 2x}.$

$$\frac{1 - \cos 2x}{1 + \cos 2x} = \frac{\frac{1 - \cos x}{2}}{\frac{1 + \cos 2x}{2}}$$

按公式 [16], [17],

$$= \frac{\sin^2 x}{\cos^2 x}$$

$$= \tan^2 x.$$

43.  $1 + \tan x \tan y$

$$1 + \tan x \tan y$$

$$= 1 + \frac{\sin x}{\cos x} \times \frac{\sin y}{\cos y}$$

$$= \frac{\cos x \cos y + \sin x \sin y}{\cos x \cos y}$$

$$\frac{\sin y}{\sin y}$$

按公式 [9],  $= \frac{\cos(x-y)}{\cos x \cos y}$

44.  $1 - \tan x \tan y$

$$1 - \tan x \tan y$$

$$= 1 - \frac{\sin x \sin y}{\cos x \cos y}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y}$$

$$\frac{\sin y}{\sin y}$$

按 [4],  $= \frac{\cos(x+y)}{\cos x \cos y}$

45.  $\cot x \cot y + 1$

$$\cot x \cot y + 1$$

$$= \frac{\cos x}{\sin x} \times \frac{\cos y}{\sin y} + 1$$

$$= \frac{\cos x \cos y + \sin x \sin y}{\sin x \sin y}$$

按 [9],  $= \frac{\cos(x-y)}{\sin x \sin y}$

46.  $\cot x \cot y - 1$

$$\cot x \cot y - 1$$

$$= \frac{\cos x \cos y}{\sin x \sin y} - 1$$

$$= \frac{\cos x \cos y - \sin x \sin y}{\sin x \sin y}$$

按 [5],  $= \frac{\cos(x+y)}{\sin x \sin y}$

47.  $\frac{\tan x + \tan y}{\cot x + \cot y}$

$$\frac{\tan x + \tan y}{\cot x + \cot y}$$

$$\frac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{\frac{\cos x}{\sin x} + \frac{\cos y}{\sin y}}$$

$$\frac{\frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y}}{\frac{\cos x \sin y + \sin x \cos y}{\sin x \sin y}}$$

$$\frac{\frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y}}{\frac{\sin x \cos y + \cos x \sin y}{\sin x \sin y}}$$

$$= \frac{\sin x \sin y}{\cos x \cos y}$$

$$= \tan x \tan y.$$

### 例題 XV

1. 求下各函數之一切值。

$$\sin^{-1}\frac{1}{2}\sqrt{3}, \tan^{-1}\frac{1}{3}\sqrt{3}, \text{vers}^{-1}\frac{1}{2},$$

$$\cos^{-1}(-\frac{1}{2}\sqrt{2}), \csc^{-1}\sqrt{2}, \tan^{-1}\infty,$$

$$\sec^{-1}2, \cos^{-1}(-\frac{1}{2}\sqrt{3})^{\circ}$$

$$\sin^{-1}\frac{1}{2}\sqrt{3}=60^{\circ}+2n\pi$$

或  $120^{\circ}+2n\pi$ .

$$\tan^{-1}\frac{1}{3}\sqrt{3}=30^{\circ}+2n\pi$$

或  $210^{\circ}+2n\pi$

$$\text{vers}^{-1}\frac{1}{2}=60^{\circ}+2n\pi$$

或  $300^{\circ}+2n\pi$

$$\cos^{-1}(-\frac{1}{2}\sqrt{2})=135^{\circ}+2n\pi$$

或  $225^{\circ}+2n\pi$ .

$$\csc^{-1}\sqrt{2}=45^{\circ}+2n\pi$$

或  $135^{\circ}+2n\pi$ .

$$\tan^{-1}\infty=90^{\circ}+2n\pi$$

或  $270^{\circ}+2n\pi$ .

$$\sec^{-1}2=60^{\circ}+2n\pi$$

或  $300^{\circ}+2n\pi$ .

$$\cos^{-1}(-\frac{1}{2}\sqrt{3})=150^{\circ}+2n\pi$$

或  $210^{\circ}+2n\pi$ .

2. 求證  $\sin^{-1}(-x) = -\sin^{-1}x$

$$\cos^{-1}(-x) = \pi - \cos^{-1}x$$

$$\sin^{-1}(-x) = (\text{sine 爲 } -x \text{ 之角})$$

$$= -(\text{sine 爲 } x \text{ 之角})$$

$$= \sin^{-1}x.$$

$$\cos^{-1}(-x) = (\text{cosine 爲 } -x \text{ 之角})$$

$$= \pi - (\text{cosine 爲 } x \text{ 之角}) = \pi - \cos^{-1}x.$$

3. 設  $\sin^{-1}x + \sin^{-1}y = \pi$ ,  
求證  $x = y$ .

若兩角之和等於  $180^{\circ}$ , 則其  
一角之正弦必等餘一角之  
正弦.

$$\text{故 } \sin(\sin^{-1}x) = \sin(\sin^{-1}y)$$

即  $x = y$ .

4. 設  $y = \sin^{-1}\frac{1}{3}$ , 求  $\tan y$ .

$$y = \sin^{-1}\frac{1}{3},$$

$$\text{故 } \sin y = \frac{1}{3}.$$

$$\cos y = \sqrt{1 - \sin^2 y}$$

$$= \sqrt{\frac{8}{9}} = \frac{2}{3}\sqrt{2}.$$

$$\text{即 } \tan y = \frac{\frac{1}{3}}{\frac{2}{3}\sqrt{2}} = \frac{2}{2\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}.$$

5. 求證  $\cos(\sin^{-1}x) = \sqrt{1-x^2}$ .

設  $\sin^{-1}x = y$ ;

則  $x = \sin y$ ,

$$\sqrt{1-x^2} = \cos y$$

$$= \cos(\sin^{-1}x).$$

6. 求證  $\cos(2\sin^{-1}x) = 1-2x^2$ .

設  $\sin^{-1} x = y$ ;

證  $x = \sin y$ ,

$$\begin{aligned}\cos(2 \sin^{-1} x) &= \cos 2y \\ &= \cos^2 y - \sin^2 y \\ &= 1 - \sin^2 y - \sin^2 y \\ &= 1 - 2 \sin^2 y \\ &= 1 - 2x^2.\end{aligned}$$

7. 求證  $\tan(\tan^{-1} x + \tan^{-1} y)$

$$= \frac{x+y}{1-xy}.$$

設  $\tan^{-1} x = u$ ,

$\tan^{-1} y = v$ .

則  $x = \tan u$ ,

$y = \tan v$ .

$$\begin{aligned}\tan(\tan^{-1} x + \tan^{-1} y) &= \tan(v+u) \\ &= \frac{\tan u + \tan v}{1 - \tan u \tan v} \\ &= \frac{x+y}{1-xy}.\end{aligned}$$

8. 設  $x = \frac{1}{\sqrt{2}}$ , 試求  $\sin^{-1} x + \cos^{-1} x$  之一切值。

$$\sin^{-1} \frac{1}{\sqrt{2}} = 45^\circ + 2n\pi$$

$$\text{或 } 135^\circ + 2n\pi$$

$$\cos^{-1} \frac{1}{\sqrt{2}} = 45^\circ + 2n\pi$$

$$\text{或 } -45^\circ + 2n\pi.$$

故  $\sin^{-1} \frac{1}{\sqrt{2}} + \cos^{-1} \frac{1}{\sqrt{2}}$

$$= 90^\circ + 2n\pi$$

或  $2n\pi$ ,

$$\text{或 } 180^\circ + 2n\pi,$$

$$\text{或 } 90^\circ + 2n\pi,$$

$$= 0^\circ \text{ 或 } 90^\circ \text{ 或 } 180^\circ.$$

9. 求證  $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

$$= \sin^{-1} x.$$

設  $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = y$ .

則  $\frac{x}{\sqrt{1-x^2}} = \tan y$ .

按 5/2,

$$\sec^2 y = 1 + \tan^2 y$$

$$= 1 + \left(\frac{x}{\sqrt{1-x^2}}\right)^2$$

$$= \frac{1}{1-x^2}.$$

故  $\cos^2 y = \frac{1}{\sec^2 y} = 1 - x^2$ ,

$$\sin^2 y = 1 - \cos^2 y = x^2,$$

$$\sin y = x.$$

故  $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

$$= \sin^{-1} x.$$

10. 求  $\sin(\tan^{-1} \frac{5}{12})$  之值。



設  $\tan^{-1}\frac{5}{12}=x$ .

則  $\frac{5}{12}=\tan x$ .

按  $5/12$ ,

$$\sec^2 x = 1 + \tan^2 x$$

$$= 1 + \left(\frac{5}{12}\right)^2$$

$$= \frac{144}{169}$$

故  $\cos^2 x = \frac{144}{169}$ ,

$$\sin^2 x = \frac{25}{169}$$

$$\sin x = \pm \frac{5}{13}$$

II. 求  $\cot(2\sin^{-1}\frac{3}{5})$  之值。

設  $\sin^{-1}\frac{3}{5}=x$ ;

則  $\sin x = \frac{3}{5}$ ,

$$\cos x = \pm \frac{4}{5}$$

$$\cot x = \pm \frac{4}{3}$$

$$\cot(2\sin^{-1}\frac{3}{5})$$

$$= \cot 2x.$$

按 [5],  $\frac{\cot^2 x - 1}{2 \cot x}$

$$= \frac{\frac{16}{9} - 1}{\pm \frac{8}{3}}$$

$$= \pm \frac{7}{24}$$

12. 求  $\sin(\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3})$

之值。

設  $\tan^{-1}\frac{1}{2}=x$ ,

$$\tan^{-1}\frac{1}{3}=y,$$

則  $\tan x = \frac{1}{2}$ ,

$$\tan y = \frac{1}{3}.$$

但  $\sec^2 x = 1 + \tan^2 x$

$$= \frac{5}{4},$$

$$\cos^2 x = \frac{4}{5},$$

$$\cos x = \frac{\pm 2}{\sqrt{5}},$$

$$\sin x = \frac{\pm 1}{\sqrt{5}};$$

又  $\cos y = \frac{\pm 3}{\sqrt{10}}$ ,

$$\sin y = \frac{\pm 1}{\sqrt{10}}.$$

故  $\sin(\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3})$

$$= \sin(x+y)$$

$$= \sin x \cos y + \cos x \sin y$$

$$\sin y = \frac{1}{\sqrt{5}} \times \frac{3}{\sqrt{10}}$$

$$+ \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{10}}$$

$$= \frac{\pm 5}{\sqrt{50}}$$

$$= \pm \frac{1}{2\sqrt{2}}.$$

13. 設  $\sin^{-1}x = 2 \cos^{-1}x$ ,  
求  $x_0$

$$\sin^{-1}x = 90^\circ - \cos^{-1}x.$$

$$\begin{aligned} \text{故 } 90^\circ - \cos^{-1}x \\ = 2 \cos^{-1}x. \end{aligned}$$

$$3 \cos^{-1}x = 90^\circ.$$

$$\begin{aligned} \cos^{-1}x = 30^\circ \text{ 或 } 150^\circ \text{ 或 } \\ 270^\circ. \end{aligned}$$

$$x = \pm \frac{1}{2} \sqrt{3} \text{ 或 } 0.$$

14. 求証  $\tan(2 \tan^{-1}x)$

$$= \frac{2x}{1-x^2}.$$

設  $\tan^{-1}x = y$ ,

則  $x = \tan y$ .

$$\begin{aligned} \tan(2 \tan^{-1}x) \\ = \tan 2y \\ = \frac{2 \tan y}{1 - \tan^2 y} \\ = \frac{2x}{1 - x^2}. \end{aligned}$$

15.  $x$  求証  $\sin(2 \tan^{-1}x)$

$$= \frac{2x}{1+x^2}.$$

設  $\tan^{-1}x = y$ .

則  $x = \tan y$ .

$$\sin(2 \tan^{-1}x) = \sin 2y$$

$$= 2 \sin y \cos y$$

$$= 2 \frac{\sin y}{\cos y} \cos^2 y$$

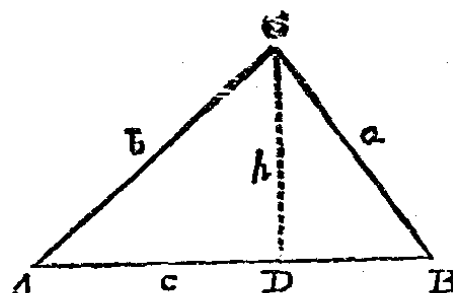
$$= \frac{2 \tan y}{\sec^2 y}$$

$$= \frac{2 \tan y}{1 + \tan^2 y}$$

$$= \frac{2x}{1+x^2}.$$

### 例題 XVI

問於第三十四章內。若有一角爲直角。則其公式變爲若何。



若  $C$  角爲直角,

$$\text{則 } \frac{a}{c} = \frac{\sin A}{\sin C} = \sin A;$$

$$\frac{b}{c} = \frac{\sin B}{\sin C} = \sin B;$$

$$\frac{a}{b} = \frac{\sin A}{\sin B} = \tan A$$

$$\frac{a}{\sin A} = \frac{c}{\sin C} = c;$$

$$\frac{b}{\sin B} = \frac{c}{\sin C} = c.$$

2. 試以正弦之定律證三角形一角之一等分綫。分對邊爲兩分。與其兩鄰邊成正比例。

設  $CD$  平分  $C$  角。

$$\text{則 } \frac{AD}{CD} = \frac{\sin \frac{1}{2} C}{\sin A},$$

$$\frac{DB}{CD} = \frac{\sin \frac{1}{2} C}{\sin B}.$$

將兩式相除, 得

$$\frac{AD}{DB} = \frac{\sin B}{\sin A}.$$

但  $\frac{\sin B}{\sin A} = \frac{b}{a},$

故  $\frac{AD}{DB} = \frac{b}{a}.$

3. 設  $A=90^\circ$ , 或  $A=0^\circ$  或  $A=180^\circ$  問公式 [26] 及其三角形各當變爲若何。

公式 [26] 卽

$$a^2 = b^2 + c^2 - 2bc \cos A,$$

(1)  $A=90^\circ, \cos A=0,$

故  $a^2 = b^2 + c^2.$

(2)  $A=0^\circ, \cos A=1.$

故  $a^2 = b^2 + c^2 - 2bc.$

(3)  $A=180^\circ, \cos A=-1.$

故  $a^2 = b^2 + c^2 + 2bc.$

(1')  $ABC$  爲正三角形。

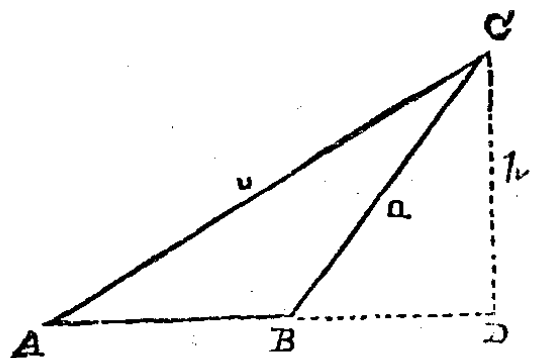
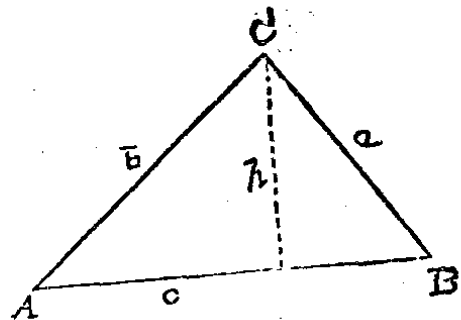
(2') 成一直綫如下。

$$\begin{array}{ccc} A & B & C \\ \hline a = BC & & c = AB, \\ b = AC & & a = b - c. \end{array}$$

(3') 成一直綫如下。

$$\begin{array}{ccc} A & B & C \\ \hline a = BC & & c = BA, \\ & = AC & a = b + c. \end{array}$$

4. 不論  $B$  角之 或銳或鈍 求証  $C$  等於  $a \cos B + b \cos A$  (55 及 56 圖) 若變其字母將得如何之兩個對稱式。又設  $B=90^\circ$ , 則其公式變爲若何。



(1) 若  $B$  爲銳角

則  $\cos B = \frac{DB}{a}.$  (見一圖)

$$\cos A = \frac{AD}{b}.$$

$$\text{故 } DB = a \cos B,$$

$$AD = b \cos A.$$

$$DB + AD = a \cos B + b \cos A.$$

但  $DB + AD = c$ .

$$\text{故 } c = a \cos B + b \cos A,$$

(2) 若  $B$  爲鈍角

$$\text{則 } \frac{AD}{b} = \cos A. (\text{見二圖})$$

$$\frac{BD}{a} = \cos(180^\circ - B)$$

$$= -\cos B.$$

$$\text{故 } AD = b \cos A,$$

$$BD = -a \cos B.$$

$$AD - BD = b \cos A + a \cos B.$$

但  $AD - BD = c$

$$\text{故 } c = a \cos A + b \cos A.$$

(3) 相稱之二公式即

$$b = a \cos C + c \cos A,$$

$$\text{與 } a = b \cos C + c \cos B.$$

(4) 若  $B = 90^\circ$ ,

$$\text{則 } \cos A = \frac{c}{b}.$$

$$\text{故 } c = b \cos A.$$

5. 試由下列三方程式(前題所求得者) 證第三十五章之定

理。

$$c = a \cos B + b \cos A,$$

$$b = a \cos C + c \cos A,$$

$$a = b \cos C + c \cos B.$$

---


$$c^2 = ac \cos B + bc \cos A.$$

(1)

$$b^2 = ab \cos C + bc \cos A,$$

(2)

$$a^2 = ab \cos C + ac \cos B.$$

(3)

以 (2) (3) 相加,

$$a^2 + b^2 = 2 ab \cos C + bc \cos$$

$$A + ac \cos B. \quad (4)$$

自 (1) 減 (4),

$$c^2 - a^2 - b^2 = 2 ab \cos C.$$

$$\text{故 } c^2 = a^2 + b^2 - 2 ab \cos C.$$

仿此可證

$$a^2 = b^2 + c^2 - 2 bc \cos A,$$

$$b^2 = a^2 + c^2 - 2 ac \cos B.$$

6. 於公式[27]內,  $\frac{1}{2}(A - B)$

之最大值爲何。

$$\frac{a - b}{a + b} = \frac{\tan \frac{1}{2}(A - B)}{\tan \frac{1}{2}(A + B)}.$$

$A - B$  之限爲  $180^\circ$

故  $\frac{1}{2}(A - B)$  之最大值

$$= \frac{180^\circ}{2} = 90^\circ.$$

依下(i)(ii)之所設。求用公式(27)導出之式。並記其三角形之狀態。

及  $B=C$  向 [27] 化爲何式且三角形有何變更

$$(1) \quad C=90^\circ.$$

$$(2) \quad A-B=90^\circ, \text{ 又 } B=C.$$

公式 [27] 即

$$\frac{a-b}{a+b} = \frac{\tan \frac{1}{2}(A-B)}{\tan \frac{1}{2}(A+B)}$$

$$(1) \quad C=90^\circ,$$

$$A+B=90^\circ.$$

$$B=90^\circ - A$$

$$\frac{a-b}{a+b} = \frac{\tan \frac{1}{2}[A - (90^\circ - A)]}{\tan 45^\circ}$$

$$= \frac{\tan (A - 45^\circ)}{1}$$

$$= \tan (A - 45^\circ).$$

C 既是正角則 ABC 必爲正三角形。

$$(2) \quad A-B=90^\circ, B=C.$$

$$\frac{a-b}{a+b} = \frac{\tan \frac{1}{2}(A-B)}{\tan \frac{1}{2}(A+B)}$$

$$A+B+C=180^\circ$$

$$A+2B=180^\circ$$

$$A - B = 90^\circ$$

$$\text{故 } 3B = 90^\circ.$$

$$B = 30^\circ.$$

$$C = 30^\circ.$$

$$A = 120^\circ.$$

$$\frac{a-b}{a+b} = \frac{\tan 45^\circ}{\tan 75^\circ}$$

$$= \frac{\tan 45^\circ}{\cot 15^\circ}$$

$$= \frac{1}{2 + \sqrt{3}}$$

$$\text{故 } a+b = (a-b)(2 + \sqrt{3}).$$

B 既等於 C 則 ABC 必爲等腰三角形。

## 例題 XVII

I. 設 求

$$a = 500, \quad c = 123^\circ 12',$$

$$A = 10^\circ 12', \quad b = 2051.5,$$

$$B = 46^\circ 36', \quad c = 2362.6.$$

$$a = 500.$$

$$A = 10^\circ 12'$$

$$B = 46^\circ 36'$$

$$A + B = 56^\circ 48'$$

$$\text{故 } C = 123^\circ 12'.$$

$$b = \frac{a \sin B}{\sin A}.$$

$$\log a = 2.69897$$

$$\text{colog } \sin A = 0.75182$$

$$\log \sin B = \underline{9.86128}$$

$$\log b = 3.31207$$

$$b = 2051.5.$$

$$c = \frac{a \sin C}{\sin A},$$

$$\log a = 2.69897$$

$$\text{colog } \sin A = 0.75182$$

$$\log \sin C = \underline{9.92260}$$

$$\log c = 3.37339$$

$$c = 2362.6.$$

2. 設 求

$$a = 795, \quad C = 55^\circ 20',$$

$$A = 79^\circ 59', \quad b = 567.69,$$

$$B = 44^\circ 41'; \quad c = 663.99.$$

$$a = 795.$$

$$A = 79^\circ 59'$$

$$B = \underline{44^\circ 41'}$$

$$A + B = 124^\circ 40'$$

$$\text{故 } C = 55^\circ 20'.$$

$$b = \frac{a \sin B}{\sin A},$$

$$\log a = 2.90037$$

$$\text{colog } \sin A = 0.00667$$

$$\log \sin B = 9.84707,$$

$$\log b = 2.75411$$

$$b = 567.69.$$

$$c = \frac{a \sin C}{\sin A},$$

$$\log a = 2.90037$$

$$\text{colog } \sin A = 0.00667$$

$$\log \sin C = \underline{9.91512}$$

$$\log c = 2.82216$$

$$c = 663.99.$$

3. 設 求

$$a = 804, \quad C = 35^\circ 4',$$

$$A = 99^\circ 55', \quad b = 577.31,$$

$$B = 45^\circ 1'; \quad c = 468.93.$$

$$a = 804.$$

$$A = 99^\circ 55'$$

$$B = \underline{45^\circ 1'}$$

$$A + B = 144^\circ 56'$$

$$\text{故 } C = 35^\circ 4'.$$

$$b = \frac{a \sin B}{\sin A},$$

$$\log a = 2.90526$$

$$\log \sin B = 9.84961$$

$$\text{colog } \sin A = \underline{0.00654}$$

$$\log b = 2.76141$$

$$b = 577.31.$$

$$c = \frac{a \sin C}{\sin A}$$

$$\begin{aligned}\log a &= 2.90526 \\ \log \sin C &= 9.75931 \\ \text{colog} \sin A &= \underline{0.00654}\end{aligned}$$

$$\begin{aligned}\log c &= 2.67111 \\ c &= 468.93\end{aligned}$$

4. 設 求

$$\begin{aligned}a &= 820, & C &= 25^\circ 12', \\ A &= 12^\circ 49', & b &= 2276.6, \\ B &= 141^\circ 59'; & c &= 1573.90\end{aligned}$$

$$a = 820$$

$$A = 12^\circ 4'$$

$$B = \underline{141^\circ 59'}$$

$$A + B = 154^\circ 48'$$

故  $C = 25^\circ 12'.$

$$b = \frac{a \sin B}{\sin A},$$

$$\log a = 2.91381$$

$$\log \sin B = .78950$$

$$\text{colog} \sin A = \underline{0.65398}$$

$$\log b = 3.35729$$

$$b = 2276.6.$$

$$c = \frac{a \sin C}{\sin A},$$

$$\log a = 2.91381$$

$$\log \sin C = 9.62918$$

$$\text{colog} \sin A = \underline{0.65398}$$

$$\log c = 3.19697$$

$$c = 1573.9.$$

5. 設 求

$$c = 1005, \quad C = 47^\circ 14',$$

$$A = 78^\circ 19', \quad a = 1340.6,$$

$$B = 54^\circ 27'; \quad b = 1113.80$$

$$c = 1005.$$

$$A = 78^\circ 19'$$

$$B = \underline{54^\circ 27'}$$

$$A + B = 132^\circ 46'$$

故  $C = 47^\circ 14'.$

$$a = \frac{c \sin A}{\sin C},$$

$$\log c = 3.00217$$

$$\log \sin A = 9.99091$$

$$\text{colog} \sin C = \underline{0.13423}$$

$$\log a = 3.12731$$

$$a = 1340.6.$$

$$b = \frac{c \sin B}{\sin C},$$

$$\log c = 3.00217$$

$$\log \sin B = \underline{9.91042}$$

$$\text{colog} \sin C = \underline{0.13423}$$

$$\log b = 3.04682$$

$$b = 1113.8.$$

6. 設 求

$$\begin{aligned} b &= 13.57, & A &= 18^\circ 50', \\ B &= 13^\circ 57', & a &= 53.276, \\ C &= 57^\circ 13'; & c &= 47.324. \end{aligned}$$

$$b = 13.57.$$

$$B = 13^\circ 57'$$

$$C = \underline{57^\circ 13'}$$

$$B + C = 71^\circ 10'$$

$$\text{故 } A = 108^\circ 50'.$$

$$a = \frac{b \sin A}{\sin B},$$

$$\log b = 1.13258$$

$$\log \sin A = 9.97610$$

$$\text{colog } \sin B = \underline{0.61785}$$

$$\log a = 1.72653$$

$$a = 53.276$$

$$c = \frac{b \sin C}{\sin B},$$

$$\log b = 1.13258$$

$$\log \sin C = 9.92465$$

$$\text{colog } \sin B = \underline{0.61785}$$

$$\log c = 1.67508$$

$$c = 47.324.$$

7. 設 求

$$a = 6412, \quad B = 56^\circ 56';$$

$$A = 70^\circ 55', \quad b = 5685.9,$$

$$C = 52^\circ 9'; \quad c = 5357.50$$

$$a = 6412.$$

$$A = 70^\circ 55'$$

$$C = \underline{52^\circ 9'}$$

$$A + C = 123^\circ 4'$$

$$\text{故 } B = 56^\circ 56'.$$

$$b = \frac{a \sin B}{\sin A},$$

$$\log a = 3.80699$$

$$\log \sin B = 9.92326$$

$$\text{colog } \sin A = \underline{0.02455}$$

$$\log b = 3.75480$$

$$b = 5685.9.$$

$$C = \frac{a \sin C}{\sin A},$$

$$\log a = 3.80699$$

$$\log \sin C = 9.89742$$

$$\text{colog } \sin A = \underline{0.02455}$$

$$\log c = 3.72896$$

$$c = 5357.5.$$

8. 設 求

$$b = 999, \quad B = 77^\circ,$$

$$A = 37^\circ 58', \quad a = 630.7,$$

$$C = 65^\circ 2'; \quad c = 929.480$$

$$b = 999.$$

$$A = 37^\circ 58'$$

$$c = \underline{65^\circ 2'}$$



$$A + C = 103^\circ$$

$$\text{故 } B = 77^\circ.$$

$$a = \frac{b \sin A}{\sin B},$$

$$\log b = 2.99957$$

$$\log \sin A = 9.78902$$

$$\text{colog } \sin B = \underline{0.01128}$$

$$\log a = 2.79987$$

$$a = 630.77.$$

$$c = \frac{b \sin C}{\sin B},$$

$$\log b = 2.99957$$

$$\log \sin C = 9.95739$$

$$\text{colog } \sin B = \underline{0.01128}$$

$$\log c = 2.96824$$

$$C = 929.48$$

9. 設於某點 B. 測敵壘 A° 得 BC 線及 A BC A CB 兩角爲 322.55 碼及  $60^\circ 34'$ ,  $56^\circ 10'$ . 求 AB 距。

$$a = 322.55.$$

$$B = 60^\circ 34',$$

$$C = \underline{56^\circ 10'}$$

$$B + C = 116^\circ 44'$$

$$\text{故 } A = 63^\circ 16',$$

$$c = \frac{a \sin C}{\sin A},$$

$$\log a = 2.50850$$

$$\log \sin C = 9.91942$$

$$\text{colog } \sin A = \underline{0.01910}$$

$$\log c = 2.47712$$

$$c = 300.$$

$$AB = 300 \text{ 碼},$$

10. ABC 三角形之 B 及 C 爲  $50^\circ 30'$  及  $122^\circ 9'$  而 BC 爲 90 哩。求 AB 及 AC。

$$C = 122^\circ 9'$$

$$B = \underline{50^\circ 30'}$$

$$B + C = 172^\circ 39'$$

$$\text{故 } A = 7^\circ 21'.$$

$$b = \frac{BC \sin B}{\sin A},$$

$$\log BC = 0.5424$$

$$\log \sin B = 9.88741$$

$$\text{colog } \sin A = \underline{0.89303}$$

$$\log b = 1.73468$$

$$b = 54.285.$$

$$\text{即 } AC = 54.285 \text{ 哩.}$$

$$c = \frac{Bc \sin C}{\sin A},$$

$$\log Bc = 0.95424$$

$$\log \sin C = 9.92771$$

$$\text{colog } \sin A = \underline{0.89303}$$

$$\log c = 1.77498$$

$$c = 59.564.$$

即  $AB = 59.564$  哩。

1. 二測者於一平面上距5哩之處相對而立。測一同直立面內之輕氣球。各得仰角 $55^\circ$ 及 $85^\circ$ 求球與測者之各距。並距平面之高。

同測其仰視角為 $55^\circ$ 與 $58^\circ$ ；試求氣球之高并求二人與球之距。

設  $C$  為氣球， $AB$  為測地者

$$B = 58^\circ.$$

$$A = 55^\circ.$$

$$A + B = 113^\circ$$

故  $C = 67^\circ.$

$$a = \frac{c \sin A}{\sin C}.$$

$$\log c = 0.69897$$

$$\log \sin A = 9.91336$$

$$\text{colog } \sin C = 0.03597$$

$$\log a = 0.64830$$

$$a = 4.4494.$$

即  $BC = 4.4494$  哩。

$$b = \frac{c \sin B}{\sin C}.$$

$$\log c = 0.69897$$

$$\log \sin B = 9.92842$$

$$\text{colog } \sin C = 0.03597$$

$$\log b = 0.66336$$

$$b = 4.6064.$$

即  $AC = 4.6064$  哩。

$$\frac{h}{a} = \sin B.$$

$$h = a \sin B.$$

$$\log a = 0.64830$$

$$\log \sin B = 9.92842$$

$$\log h = 0.57672$$

$$h = 3.7733.$$

即 高 = 3.7733 哩。

12. 已知平行四邊形之一對角線  $d$  及此對角與兩邊所成之角  $x$  及  $y$ 。求兩邊。設  $a = 11.237$ ， $x = 19^\circ 1'$  而  $y = 42^\circ 54'$ 。求兩邊兩值。

$$\frac{a}{d} = \frac{\sin x}{\sin z}.$$

故  $a = \frac{a \sin x}{\sin z}.$

$$\frac{c}{d} = \frac{\sin y}{\sin z}.$$

故  $c = \frac{d \sin y}{\sin z}.$

$$d = 11.237,$$

$$x = 19^{\circ} 1'$$

$$y = 42^{\circ} 54'$$

$$x + y = 61^{\circ} 55'$$

$$\text{故 } z = 118^{\circ} 5'.$$

$$\log d = 1.05065$$

$$\log \sin x = 9.51301$$

$$\text{colog } \sin z = 0.05440$$

$$\log a = 0.61806$$

$$a = 4.1501.$$

$$\log d = 1.05065$$

$$\log \sin y = 9.83297$$

$$\text{colog } \sin z = 0.05440$$

$$\log c = 0.93802$$

13. 自船測燈塔。爲北  $34^{\circ}$  東。船向南駛行三哩。再測之。爲北  $23^{\circ}$  東。求塔與船兩次各距。

[注意] 北  $34^{\circ}$  東。言自船至塔之視線。在地平之東北。而與直北綫作  $34^{\circ}$  之角也。

$$A = 21^{\circ}$$

$$B = (180 - 34^{\circ}) = 146^{\circ}$$

$$A + C = 169^{\circ}$$

$$\text{故 } C = 11^{\circ}.$$

$$a = \frac{c \sin A}{\sin C},$$

$$\log c = 0.47712$$

$$\log \sin A = 9.59188$$

$$\text{colog } \sin c = 0.71940$$

$$\log a = 0.78840$$

$$a = 6.1433,$$

$$b = \frac{c \sin B}{\sin C},$$

$$\log c = 0.47712$$

$$\log \sin B = 9.74756$$

$$\text{colog } \sin C = 0.71949$$

$$\log b = 0.94408$$

$$b = 8.7918.$$

故二次離塔之距爲 6.1433

哩與 8.7918 哩。

14. 已知梯形之平行兩邊  $a$ ,  $b$ 。及平行邊一端之兩角  $x, y$ 。求不平行之兩邊。設  $a = 15$ ,  $b = 7$ ,  $x = 70^{\circ}$ ,  $y = 40^{\circ}$ 。計算其結果。

已知平行邊

$$AB = 7 \quad DC = 15.$$

$$\text{且 } \angle ABC = 40^{\circ}. \quad \angle BCD = 70^{\circ}.$$

求  $AD$  與  $BC$ 。

作  $AE$  與  $BC$  平行。

則  $AR = EC$ 。

$$\begin{aligned} DE &= DC - AB \\ &= 15 - 7 = 8. \end{aligned}$$

又  $\angle AED = (\angle BCD) = 70^\circ$ .

故  $\angle DAE = 180^\circ - (240^\circ + 70^\circ) = 70^\circ$

但  $\angle AED = \angle DAE = 70^\circ$ ,

故  $\triangle AED$  必為等腰三角形,

即  $DA = DE = 8$ .

今有  $AE = BC$ , 再欲求  $BC$

如下,

$$\frac{AE}{DE} = \frac{\sin \angle ADE}{\sin \angle DAE}.$$

$$\log DE = 0.90309$$

$$\log \sin \angle ADE = 9.80807$$

$$\text{colog} \sin \angle DAE = \underline{0.02701}$$

$$\log AE = 0.73817$$

$$\text{即 } BC = (AE) = 5.4723.$$

解下列各題。而不用對數。

15. 設求  $b = 7.07107$ ,  $A = 30^\circ$ ,  $C = 105^\circ$ 。及  $a$  及  $c$ 。

自  $C$  作垂線至  $c$ 。

設  $p$  與  $q$  為其所分之兩線段。

$$A = 30^\circ,$$

$$C = 105^\circ.$$

$$\text{故 } B = 45^\circ.$$

$$\frac{a}{b} = \frac{\sin A}{\sin B} = \frac{\frac{1}{2}}{\frac{1}{2}\sqrt{2}}$$

$$a = \frac{b}{\sqrt{2}}$$

$$= \frac{7.07107}{1.41421} = 5.$$

$$\frac{p}{b} = \cos A = \frac{1}{2}\sqrt{3}$$

$$= 0.86603.$$

$$p = b \times 0.86603$$

$$= 7.07107$$

$$\times 0.86603$$

$$= 6.12376.$$

$$\frac{q}{a} = \cos b = \frac{1}{2}\sqrt{2}$$

$$= 0.70711$$

$$q = a \times 0.70711$$

$$= 5 \times 0.70711$$

$$= 3.53555.$$

$$c = p + q$$

$$6.12376 + 3.53555$$

$$= 9.6593.$$

16. 設  $c = 9.562$ ,  $A = 45^\circ$ ,  $B = 60^\circ$ ; 求  $a$  及  $b$ 。

$$C = 75^\circ.$$

$$a = \frac{c \sin A}{\sin C}.$$

$$\begin{aligned}\sin C &= \sin(45^\circ + 30^\circ) \\ &= \sin 45^\circ \cos 30^\circ \\ &\quad + \cos 45^\circ \sin 30^\circ. \\ &= \frac{1}{2}\sqrt{2} \times \frac{\sqrt{3}}{2} \\ &\quad + \frac{1}{2}\sqrt{2} \times \frac{1}{2} \\ &= \frac{1}{4}(\sqrt{6} + \sqrt{2}).\end{aligned}$$

故

$$\begin{aligned}a &= \frac{9.562 \times \frac{1}{2}\sqrt{2}}{\frac{1}{4}(\sqrt{6} + \sqrt{2})} \\ &= \frac{19.124 \times \sqrt{2}}{\sqrt{6} + \sqrt{2}} \\ &= \frac{(19.124 \times \sqrt{2})\sqrt{6} - \sqrt{2}}{6 - 2} \\ &= 9.562 \times (\sqrt{3} \\ &\quad - 1) = 6.99986 = 7.\end{aligned}$$

$$\begin{aligned}b &= \frac{a \sin B}{\sin A} \\ &= \frac{7 \times \frac{1}{2}\sqrt{3}}{\frac{1}{2}\sqrt{2}} \\ &= \frac{7\sqrt{3}}{\sqrt{2}} = \frac{7\sqrt{6}}{2} \\ &= 3.5\sqrt{6} = 8.573.\end{aligned}$$

17. 三角形之底爲 600 呎兩  
底角爲  $30^\circ$  及  $120^\circ$ . 求餘兩邊及  
高。

$$AB = 600.$$

$$A = 30^\circ.$$

$$B = 120^\circ.$$

故  $C = 30^\circ.$

$$a = c = 600 \text{ 呎.}$$

$$b = \frac{a \sin B}{\sin A}$$

$$= \frac{600 \times \sin(180^\circ - 60^\circ)}{\sin 30^\circ}$$

$$= \frac{600 \times \frac{1}{2}\sqrt{3}}{\frac{1}{2}}$$

$$= 600 \times 1.732051$$

$$= 1039.2 \text{ 呎}$$

$$h = a \sin B = 600$$

$$\times \frac{1}{2}\sqrt{3}$$

$$= 519.6 \text{ 呎}$$

18. 三角形之二角爲  $20^\circ$  及  
 $40^\circ$ . 求其二對邊之比。

設  $x = 20^\circ,$

$$y = 40^\circ,$$

又  $a$  與  $b$  爲其對邊。

則  $\frac{\sin x}{\sin y} = \frac{a}{b}.$

$$\sin x = 0.3420. \text{ (真數)}$$

$$\sin y = 0.6428. \text{ (真數)}$$

故  $a:b = 3420:6428$

$$= 855:1607.$$

19. 三角形之三角爲  $5:10:$

21. 而其最小角之對邊爲 3。求餘二邊。

因 A, B, C 之比例爲 5:10:21,

$$\begin{aligned} \text{故 } A &= \frac{5}{36} \times 180^\circ \\ &= 25^\circ. \end{aligned}$$

$$\begin{aligned} B &= \frac{10}{36} \times 180^\circ \\ &= 50^\circ. \end{aligned}$$

$$\begin{aligned} C &= \frac{21}{36} \times 180^\circ \\ &= 105^\circ. \end{aligned}$$

$$\begin{aligned} b &= \frac{a \sin B}{\sin A} \\ &= \frac{3 \times 0.7660}{0.4226} \\ &= 5.438. \end{aligned}$$

$$\begin{aligned} c &= \frac{a \sin C}{\sin A} \\ &= \frac{3 \times 0.9656}{0.4226} \\ &= 6.857. \end{aligned}$$

23. 已知三角形之一邊爲 27。而其兩鄰角各爲  $30^\circ$ 。求外接圓之半徑 (參看第三十四章)

$$2R = \frac{a}{\sin A},$$

$$\begin{aligned} \sin A &= \sin 120^\circ \\ &= \sin(180^\circ - 120^\circ) \\ &= \sin 60^\circ. \end{aligned}$$

$$\sin 60^\circ = \frac{1}{2}\sqrt{3}.$$

$$\begin{aligned} \text{故 } 2R &= \frac{27}{\frac{1}{2}\sqrt{3}} = \frac{54}{\sqrt{3}} \\ &= 18\sqrt{3}. \\ R &= 9\sqrt{3} = 15.588. \end{aligned}$$

### 例題 XVIII

I. 下列各項。問有解法若干。

(i)  $a=80, b=100, A=30^\circ$ .

按題,  $a < b$ ,

但  $a > b \sin A = 100 \times \frac{1}{2}$ ,

又  $A < 90^\circ$ .

故有二解法。

(ii)  $a=50, b=100, A=30^\circ$ .

按題,  $a = b \sin A = 100 \times \frac{1}{2}$ .

故僅有一解法。

(iii)  $a=40, b=100, A=30^\circ$ .

按題,  $a < b \sin A$   
 $= 100 \times \frac{1}{2},$

又  $A > 90^\circ.$

故 無解法.

(iv)  $a = 13.9, b = 11.46, A$   
 $= 77^\circ 20'.$

按題,  $a < b.$

故 祇有一解法.

(v)  $a = 70, b = 75, A = 60^\circ.$

按題,  $a < b,$

但  $a > b \sin A = 75$   
 $\times \frac{1}{2} \sqrt{3},$

又  $A < 90.$

故 有二解法.

(vi)  $a = 134.16, b = 84.54,$   
 $B = 52^\circ 9' 11''.$

$b < a,$

$E < 90^\circ.$

$\sin B = 0.7896. (\text{真數})$

$84.54 < 134.16$

$\times 0.7896.$

故  $b < a \sin B.$

即 無解法.

(vii)  $a = 200, b = 100,$

$A = 30^\circ.$

因  $a > b,$

故 祇有一解法.

2. 設 求

$a = 840, B = 12^\circ 13' 34''$

$b = 485, C = 146^\circ 15' 26'',$

$A = 21^\circ 31'; c = 1272.1.$

$$\sin B = \frac{b \sin A}{a},$$

$$\log b = 2.68574$$

$$\log \sin A = 9.56440$$

$$\text{colog } a = \underline{7.07572 - 10}$$

$$\log \sin B = 9.32586$$

$$B = 12^\circ 13' 34''.$$

故  $C = 146^\circ 15' 26''.$

$$c = \frac{a \sin C}{\sin A},$$

$$\log a = 3.92428$$

$$\log \sin C = 9.74466$$

$$\text{colog } \sin A = \underline{0.43560}$$

$$\log c = 3.10454$$

$$c = 1272.1.$$

3. 設 求

$a = 9.399 \quad B = 57^\circ 23' 40''.$

$b = 9.197 \quad C = 2^\circ 1' 20'',$

$A = 100^\circ 35'; c = 0.38525.$

$$\sin B = \frac{b \sin A}{a},$$

$$\log b = 0.96365$$

$$\log \sin A = 9.93495$$

$$\text{colog } a = \underline{9.02692 - 10}$$

$$\log \sin B = 9.92552$$

$$B = 57^\circ 23' 40''.$$

$$\text{故 } C = 2^\circ 1' 20''.$$

$$c = \frac{a \sin C}{\sin A},$$

$$\log a = 0.97308$$

$$\log \sin C = 8.54761$$

$$\text{colog } \sin A = \underline{0.06505}$$

$$\log c = 9.58574 - 10$$

$$c = 0.38525.$$

4. 設 求

$$a = 91.05, \quad B = 41^\circ 13',$$

$$b = 77.04, \quad C = 57^\circ 37' 04'',$$

$$A = 51^\circ 9' 16''; c = 116.720$$

$$\sin B = \frac{b \sin A}{a},$$

$$\log b = 1.78672$$

$$\log \sin A = 9.89193$$

$$\text{colog } a = \underline{8.04067 - 10}$$

$$\log \sin B = 9.81882$$

$$B = 41^\circ 13'.$$

$$\text{而 } C = 87^\circ 37' 54''.$$

$$c = \frac{a \sin C}{\sin A},$$

$$\log a = 1.95933$$

$$\log \sin C = 9.99963$$

$$\text{colog } \sin A = \underline{0.10857}$$

$$\log c = 2.06753$$

$$c = 116.82.$$

5. 設 求

$$a = 55.55, \quad A = 54^\circ 31' 13''$$

$$b = 66.66, \quad C = 47^\circ 44' 7''$$

$$B = 77^\circ 44' 20''; c = 50.4810$$

$$\sin A = \frac{a \sin B}{b},$$

$$\log a = 1.74468$$

$$\log \sin B = 9.98999$$

$$\text{colog } b = \underline{8.17613 - 10}$$

$$\log \sin A = 9.91080$$

$$A = 54^\circ 31' 13''.$$

$$\text{而 } C = 48^\circ 44' 7''.$$

$$c = \frac{a \sin C}{\sin A},$$

$$\log a = 1.74468$$

$$\log \sin C = 9.86925$$

$$\text{colog } \sin A = \underline{0.0820}$$

$$\log c = 1.70313$$



6. 設  $c=50.481$ ,  
 $a=309, b=360$ ,  
 $A=21^{\circ}14'25''$ ;  
 求  $B=24^{\circ}57'54''$ ,  
 $C=133^{\circ}47'41''$ ,  
 $c=615.67$ ,  
 $B'=155^{\circ}2'6''$ ,  
 $C'=3^{\circ}43'29''$ ,  
 $c'=55.41$ .

按題,  $a < b$ ,

但  $a > b \sin A$ ,

又  $A < 90$ .

故此題有二解法.

(1)  $\sin B = \frac{b \sin A}{a}$ ,  
 $\log b = 2.55630$   
 $\log \sin A = 9.55904$   
 $\operatorname{colog} a = \underline{7.51004 - 10}$   
 $\log \sin B = 9.62538$   
 $B = 24^{\circ}57'54''$ .

而  $C = 133^{\circ}27'41''$ .

$$c = \frac{a \sin C}{\sin A},$$

$$\log a = 2.48996$$

$$\log \sin C = 9.85843$$

$$\operatorname{colog} \sin A = \underline{0.44096}$$

$$\log c = 2.78935$$

$$c = 615.67.$$

(2)  $B' = 180^{\circ} - B = 155^{\circ}2'6''$ .

$$C' = B - A = 3^{\circ}43'29''$$

$$c' = \frac{a \sin C'}{\sin A},$$

$$\log a = 2.48996$$

$$\log \sin C' = 8.81267$$

$$\operatorname{colog} \sin A = \underline{0.44096}$$

$$\log c' = 1.74359$$

$$c' = 55.41.$$

7. 設  $a=8.716, b=9.787$ ,

$$A=38^{\circ}14'12'';$$

求  $B=44^{\circ}1'28''$ ,

$$C=97^{\circ}44'20''$$

$$c=13.954,$$

$$B'=135^{\circ}58'32''$$

$$C'=5^{\circ}47'16''$$

$$c'=1.4272.$$

因  $A < 90^{\circ}, a < b, a > b \sin A$ ,

故有二解法如下,

(1)  $\sin B = \frac{b \sin A}{a}$ ,

$$\begin{aligned}\log b &= 0.99065 \\ \log \sin A &= 9.79163 \\ \text{colog } a &= \underline{9.05968 - 10}\end{aligned}$$

$$\begin{aligned}\log \sin B &= 9.84196 \\ B &= 44^\circ 1' 28''.\end{aligned}$$

$$\begin{aligned}\text{而 } C &= 97^\circ 41' 20'' \\ c &= \frac{a \sin C}{\sin A},\end{aligned}$$

$$\begin{aligned}\log a &= 0.94032 \\ \log \sin C &= 9.99602 \\ \text{colog } \sin A &= \underline{0.20837}\end{aligned}$$

$$\begin{aligned}\log c &= 1.14471 \\ c &= 13.954\end{aligned}$$

$$\begin{aligned}(2) \quad B' &= 180^\circ - B \\ &= 135^\circ 58' 32''. \\ C' &= B - A = 5^\circ 47' \\ &16''.\end{aligned}$$

$$c' = \frac{a \sin C'}{\sin A},$$

$$\begin{aligned}\log a &= 0.94032 \\ \log \sin C' &= 9.00365 \\ \text{colog } \sin A &= \underline{0.20837}\end{aligned}$$

$$\begin{aligned}\log c' &= 0.15234 \\ c' &= 1.4202.\end{aligned}$$

$$\begin{aligned}8. \quad \text{設} \quad & \text{求} \\ a &= 4.4, \quad B = 90^\circ,\end{aligned}$$

$$\begin{aligned}b &= 5.21, \quad C = 32^\circ 22' 43'', \\ A &= 57^\circ 37' 17''; c = 2.79010\end{aligned}$$

$$\sin B = \frac{b \sin A}{a},$$

$$\begin{aligned}\log b &= 0.71684 \\ \log \sin A &= 9.92661 \\ \text{colog } a &= \underline{9.35655 - 10}\end{aligned}$$

$$\begin{aligned}\log \sin B &= 10.03000 \\ B &= 90^\circ.\end{aligned}$$

$$\begin{aligned}\text{而 } C &= 32^\circ 22' 43''. \\ c &= b \cos A,\end{aligned}$$

$$\begin{aligned}\log b &= 0.71684 \\ \log \cos A &= \underline{9.72877} \\ \log c &= 0.44561\end{aligned}$$

$$c = 2.7901.$$

$$9. \quad \text{設} \quad \begin{aligned}a &= 34, \quad b = 22, \\ B &= 30^\circ 20';\end{aligned}$$

$$\begin{aligned}\text{求} \quad A &= 51^\circ 18' 27'', \\ C &= 98^\circ 21' 33'', \\ c &= 43.098,\end{aligned}$$

$$A' = 128^\circ 41' 33''$$

$$C' = 20^\circ 58' 27'',$$

$$c' = 15.5930$$

$$\text{因 } b < a, b > a \sin B,$$

$$B < 90^\circ,$$

故有二解法如下,

$$(1) \sin A = \frac{a \sin B}{b},$$

$$\log a = 1.53148$$

$$\log \sin B = 9.70332$$

$$\operatorname{colog} b = \underline{8.65758 - 10}$$

$$\log \sin A = 9.89238$$

$$A = 51^\circ 18' 27''.$$

$$C = 98^\circ 21' 33''.$$

$$c = \frac{a \sin C}{\sin A},$$

$$\log a = 1.53148$$

$$\log \sin C = 9.99536$$

$$\operatorname{colog} \sin A = \underline{0.10762}$$

$$\log c = 1.63446$$

$$c = 43.098.$$

$$(2) A' = 180^\circ - A = 128^\circ$$

$$41' 33''.$$

$$C' = A - B = 20^\circ 58'$$

$$27''.$$

$$c' = \frac{a \sin C'}{\sin A},$$

$$\log a = 1.53148$$

$$\log \sin C' = 9.55382$$

$$\operatorname{colog} \sin A = \underline{0.10762}$$

$$\log c' = 1.19292$$

$$c' = 15.593.$$

10. 設  $b = 19, c = 18,$

$$C = 15^\circ 49';$$

求  $B = 16^\circ 43' 13'',$

$$A = 147^\circ 27' 47'',$$

$$a = 35.519,$$

$$B' = 163^\circ 16' 47'',$$

$$A' = 0^\circ 54' 13'',$$

$$a' = 1.04150$$

因  $c < b, c > b \sin C,$   
 $c < 90^\circ,$

故有二解法如下,

$$(1) \sin B = \frac{b \sin C}{c},$$

$$b = 1.27875$$

$$\log \sin C = 9.43546$$

$$\operatorname{colog} c = \underline{8.74473 - 10}$$

$$\log \sin B = 9.45894$$

$$B = 16^\circ 43' 13''.$$

而  $A = 147^\circ 27' 47''.$

$$a = \frac{b \sin A}{\sin B}.$$

$$\log b = 1.27875$$

$$\log \sin A = 9.73065$$

$$\operatorname{colog} \sin B = \underline{0.54106}$$

$$\log a = 1.55046$$

$$a = 35.519.$$

$$(2) B' = 180^\circ - B$$

$$= 163^\circ 16' 47''.$$

$$A' = B - C = 0^\circ 54'$$

$$13''.$$

$$a' = \frac{b \sin A'}{\sin B},$$

$$\log b = 1.27875$$

$$\log \sin A' = 8.19784$$

$$\text{colog } \sin B = \underline{0.54106}$$

$$\log a' = 0.01765$$

$$a' = 1.0415.$$

II. 設  $a = 75, b = 29,$

$$B = 16^\circ$$

$15'36''$ 。求相當兩三角形面積之較。

其兩三角形之較所成之三角形高等於  $a \sin B$ , 其二腰各等於 29.

$$\log \sin B = 9.44715$$

$$\log a = \underline{1.87506}$$

$$\log (a \sin B) = 1.32221$$

$$a \sin B = 21.$$

$$\begin{aligned} 29^2 - 21^2 &= (29 - 21)(29 + 21) \\ &= 8 \times 50 = 400. \end{aligned}$$

$$\text{故 } \sqrt{29^2 - 21^2} = 20.$$

$$\begin{aligned} \text{即此三角形之底} &= 2 \times 20 \\ &= 40, \text{ 其高} = 21; \end{aligned}$$

$$\text{而其面積} = \frac{1}{2} \times 40 \times 21 = 420.$$

12. 已知平行四邊形之一邊為  $a$ , 對角綫  $d$ , 及兩對角綫之交角  $A$ 。求第二對角綫。

特例  $a = 35, d = 63, A = 21^\circ 36'30''$ 。

$$a = 35.$$

$$\frac{1}{2}d = 31.5.$$

$$A = 21^\circ 36'30''.$$

$$\sin B = \frac{\frac{1}{2}d \sin A}{a},$$

$$\log \frac{1}{2}d = 1.49831$$

$$\log \sin A = 9.56615$$

$$\text{colog } a = \underline{8.45593}$$

$$\log \sin B = 9.52039$$

$$B = 19^\circ 21'20''.$$

$$C = 139^\circ 2'10''.$$

$$\frac{1}{2}d = \frac{a \sin C}{\sin A},$$

$$\log a = 1.54407$$

$$\log \sin C = 9.81663$$

$$\text{colog } \sin A = \underline{0.43385}$$

$$\log \frac{1}{2}d' = 1.79455$$

$$\frac{1}{2}d' = 62.3086.$$

$$d' = 124.617.$$

例題 XIX

1. 設 求

$$a=77.99, \quad A=51^{\circ}15',$$

$$b=83.39, \quad B=56^{\circ}30',$$

$$C=72^{\circ}15'; \quad c=95.24$$

$$b+a=161.38.$$

$$b-a=5.2.$$

$$B+A=107^{\circ}45'.$$

$$\frac{1}{2}(B+A)=53^{\circ}52'30''.$$

$$\tan \frac{1}{2}(B-A) = \frac{(b-a) \times \tan \frac{1}{2}(B+A)}{b+a}$$

$$\log(b-a)=0.73239$$

$$\log \tan \frac{1}{2}(B+A)$$

$$=0.13675$$

$$\text{colog}(b+a)=\underline{7.79215-10}$$

$$\log \tan \frac{1}{2}(A-B)$$

$$=8.66129$$

$$\frac{1}{2}(B-A)=2^{\circ}37'30''.$$

$$\frac{1}{2}(B-A)=2^{\circ}37'30''.$$

$$A=51^{\circ}15'.$$

$$B=56^{\circ}30'.$$

$$c = \frac{b \sin C}{\sin B},$$

$$\log b=1.92111$$

$$\log \sin C=9.97882$$

$$\text{colog} \sin B=\underline{0.07889}$$

$$\log c=1.97882$$

$$c=95.24.$$

2. 設 求

$$b=872.5, \quad B=60^{\circ}45'2''$$

$$c=632.7, \quad C=39^{\circ}14'58''$$

$$A=80^{\circ}; \quad a=984.83$$

$$b-c=239.8.$$

$$b+c=1505.2.$$

$$B+C=100^{\circ}.$$

$$\frac{1}{2}(B+C)=50^{\circ}.$$

$$\log(b-c)=2.37985,$$

$$\text{colog}(b+c)=6.8224, \quad -10$$

$$\log \tan \frac{1}{2}(B+C)$$

$$=\underline{0.07619}$$

$$\log \tan \frac{1}{2}(B-C)$$

$$=9.27844$$

$$\frac{1}{2}(B-C)=10^{\circ}45'2''.$$

$$B=60^{\circ}45'2''.$$

$$C=39^{\circ}14'58''.$$

$$\log b=2.94077$$

$$\log \sin A=9.99335$$

$$\text{colog} \sin B=\underline{0.05924}$$

$$\log a=2.99336$$

$$a=984.83.$$

3. 設 求

$$a=17, \quad A=77^{\circ}12'53'',$$

$$b=12, \quad B=43^{\circ}30'7'',$$

$$C=59^{\circ}17'; c=14.987$$

$$a+b=29.$$

$$a-b=5.$$

$$A+B=120^{\circ}43'.$$

$$\frac{1}{2}(A+B)=60^{\circ}21'30''.$$

$$\tan \frac{1}{2}(A-B)$$

$$= \frac{(a-b)\tan \frac{1}{2}(A+B)}{a+b}$$

$$\log(a-b)=0.69897$$

$$\log \tan \frac{1}{2}(A+B)$$

$$=10.24486$$

$$\text{colog}(a+b)=\underline{8.53760-10}$$

$$\log \tan \frac{1}{2}(A-B)$$

$$=9.48143$$

$$\frac{1}{2}(A-B)=16^{\circ}51'23''$$

$$A=77^{\circ}12'53''.$$

$$B=43^{\circ}30'7''.$$

$$c = \frac{b \sin C}{\sin B},$$

$$\log b=1.07618$$

$$\log \sin C=9.93435$$

$$\text{colog} \sin B=\underline{0.16217}$$

$$\log c=1.17570$$

$$c=14.987.$$

4. 設 求

$$a=\sqrt{5}, \quad B=93^{\circ}28'36''$$

$$c=\sqrt{3}, \quad C=50^{\circ}30'24''$$

$$A=35^{\circ}53'; a=1.3131$$

$$\sqrt{5}=2.2361.$$

$$\sqrt{3}=1.7321.$$

$$b+c=3.9681.$$

$$b-c=0.5040.$$

$$B+C=144^{\circ}7'.$$

$$\frac{1}{2}(B+C)=72^{\circ}3'30''.$$

$$\tan \frac{1}{2}(B-C)$$

$$= \frac{(b-c)\tan \frac{1}{2}(B+C)}{b+c}$$

$$\log(b-c)=9.70243-10$$

$$\log \tan \frac{1}{2}(B+C)$$

$$=10.48073-10$$

$$\text{colog}(b+c)=\underline{9.40145-10}$$

$$\log \tan \frac{1}{2}(B-C)$$

$$=9.59358.$$

$$\frac{1}{2}(B-C)=21^{\circ}25'6''.$$

$$B=93^{\circ}28'36''.$$

$$C=50^{\circ}38'24''.$$

$$a = \frac{c \sin A}{\sin C},$$

$$\log c=0.23856$$

$$\log \sin A = 9.76800$$

$$\operatorname{colog} \sin C = 0.11172$$

$$\log a = 0.11828$$

$$a = 1.3131.$$

5. 設 求

$$a = 0.917, \quad A = 132^\circ 18' 27'',$$

$$b = 0.312, \quad B = 14^\circ 34' 24''$$

$$C = 33^\circ 7' 9'', \quad c = 6.67750.$$

$$a + b = 1.229.$$

$$a - b = 0.605.$$

$$A + B = 146^\circ 52' 51''.$$

$$\frac{1}{2}(A + B) = 73^\circ 26' 25''.$$

$$\tan \frac{1}{2}(A - B)$$

$$= \frac{(a - b) \tan \frac{1}{2}(A + B)}{a + b},$$

$$\log (a - b) = 9.78176 - 10$$

$$\log \tan \frac{1}{2}(A + B)$$

$$= 10.52674$$

$$\operatorname{colog} (a + b) = 9.91045 - 10$$

$$\log \tan \frac{1}{2}(A - B)$$

$$= 10.21895 - 10.$$

$$\frac{1}{2}(A - B) = 58^\circ 52' 2''.$$

$$A = 132^\circ 18' 27''.$$

$$B = 14^\circ 34' 24''.$$

$$c = \frac{b \sin C}{\sin B},$$

$$\log b = 9.49415 - 10$$

$$\log \sin C = 9.73750$$

$$\operatorname{colog} \sin B = 0.59926$$

$$\log c = 9.83091 - 10$$

$$c = 0.6775.$$

6. 設 求

$$a = 13.715, \quad A = 118^\circ 55' 49'',$$

$$s = 11.214, \quad C = 45^\circ 41' 35'',$$

$$B = 15^\circ 22' 36''; \quad b = 4.1554$$

$$a - c = 2.501.$$

$$a + c = 24.926.$$

$$A + C = 164^\circ 37' 24''.$$

$$\frac{1}{2}(A + C) = 82^\circ 18' 42''.$$

$$\tan \frac{1}{2}(A - C)$$

$$= \frac{(a - c) \tan \frac{1}{2}(A + C)}{a + c},$$

$$\log (a - c) = 0.39811$$

$$\log \tan \frac{1}{2}(A + C)$$

$$= 10.86968 - 10$$

$$\operatorname{colog} (a + c) = 8.60330 - 10$$

$$\log \tan \frac{1}{2}(A - C)$$

$$= 9.87109$$

$$\frac{1}{2}(A - C) = 36^\circ 37' 7''.$$

$$A = 118^\circ 55' 49''.$$

$$C = 45^\circ 41' 35''.$$

$$b = \frac{a \sin B}{\sin A},$$

$$\log a = 1.13720$$

$$\log \sin B = 9.42352$$

$$\text{colog} \sin A = \underline{0.05789}$$

$$\log b = 0.61861$$

$$b = 4.1554.$$

7. 設 求

$$b = 5030.9, \quad B = 65^\circ 13' 51'',$$

$$c = 1578.2, \quad C = 28^\circ 42' 5'',$$

$$A = 85^\circ 4' 4''; \quad a = 3297.20$$

$$b + c = 4588.1.$$

$$b - c = 1413.7.$$

$$B + C = 93^\circ 55' 56''.$$

$$\frac{1}{2}(B + C) = 46^\circ 57' 58''.$$

$$\tan \frac{1}{2}(B - C)$$

$$= \frac{(b - c) \tan \frac{1}{2}(B + C)}{b + c},$$

$$\log (b - c) = 3.15036$$

$$\log \tan \frac{1}{2}(B + C)$$

$$= 10.02983 - 10$$

$$\text{colog} (b + c) = \underline{6.33837 - 10}$$

$$\log \tan \frac{1}{2}(B - C)$$

$$= 9.51856 - 10$$

$$\frac{1}{2}(B - C) = 18^\circ 15' 53''.$$

$$C = 28^\circ 42' 5''.$$

$$B = 65^\circ 13' 51''.$$

$$a = \frac{b \sin A}{\sin B},$$

$$\log b = 3.47726$$

$$\log \sin A = 9.99898$$

$$\text{colog} \sin B = \underline{0.04191}$$

$$\log a = 3.51815$$

$$a = 3297.2.$$

8. 設 求

$$a = 4527, \quad A = 68^\circ 29' 15''.$$

$$b = 3465, \quad B = 45^\circ 24' 18'',$$

$$C = 66^\circ 6' 27''; \quad c = 44490$$

$$a + b = 7992.$$

$$a - b = 1062,$$

$$A + B = 113^\circ 53' 33''.$$

$$\frac{1}{2}(A + B) = 56^\circ 56' 47''.$$

$$\tan \frac{1}{2}(A - B)$$

$$= \frac{(a - b) \tan \frac{1}{2}(A + B)}{a + b}$$

$$\log (a - b) = 3.02612$$

$$\log \tan \frac{1}{2}(A + B)$$

$$= 10.08659 - 10$$

$$\text{colog} (a + b) = \underline{6.09734 - 10}$$

$$\log \tan \frac{1}{2}(A - B)$$

$$= 9.31005 - 10$$

$$\frac{1}{2}(A - B) = 11^\circ 32' 28''.$$

$$A = 68^\circ 29' 15''.$$



$$B = 45^{\circ} 24' 28''.$$

$$c = \frac{a \sin C}{\sin A},$$

$$\log a = 3.65581$$

$$\log \sin C = 9.96109$$

$$\text{colog } \sin A = \underline{0.03136}$$

$$\log c = 3.64826$$

$$c = 4449.$$

9. 設 求

$$a = 55.14, \quad A = 117^{\circ} 24' 32'',$$

$$b = 33.09, \quad B = 32^{\circ} 11' 28'',$$

$$C = 30^{\circ} 24'; \quad c = 31.4310$$

$$a + b = 88.23$$

$$a - b = 22.05.$$

$$A + B = 149^{\circ} 36'.$$

$$\frac{1}{2}(A + B) = 74^{\circ} 48''.$$

$$\tan \frac{1}{2}(A - B)$$

$$= \frac{(a-b)\tan \frac{1}{2}(A+B)}{a+b},$$

$$\log(a-b) = 1.34341$$

$$\log \tan \frac{1}{2}(A+B)$$

$$= 10.56592 - 10$$

$$\text{colog}(a+b) = \underline{8.05438 - 10}$$

$$\log \tan \frac{1}{2}(A-B)$$

$$= 9.96371$$

$$\frac{1}{2}(A-B) = 42^{\circ} 36' 32''.$$

$$A = 117^{\circ} 24' 32''.$$

$$B = 32^{\circ} 11' 28''.$$

$$c = \frac{b \sin C}{\sin B},$$

$$\log b = 1.51970$$

$$\log \sin C = 9.70418$$

$$\text{colog } \sin B = \underline{0.27348}$$

$$\log c = 1.49736$$

$$c = 31.431.$$

10. 設 求

$$a = 47.99, \quad A = 2^{\circ} 46' 8'',$$

$$b = 53.14, \quad B = 1^{\circ} 54' 42'',$$

$$C = 175^{\circ} 19' 10''; \quad c = 81.0640$$

$$a + b = 81.13.$$

$$a - b = 14.85.$$

$$A + B = 4^{\circ} 40' 50''.$$

$$\frac{1}{2}(A + B) = 2^{\circ} 20' 25''.$$

$$\tan \frac{1}{2}(A - B)$$

$$= \frac{(a-b)\tan \frac{1}{2}(A+B)}{a+b},$$

$$\log(a-b) = 1.17173$$

$$\log \tan \frac{1}{2}(A+B)$$

$$= 8.61138$$

$$\text{colog}(a+b) = \underline{8.09082 - 10}$$

$$\log \tan \frac{1}{2}(A-B)$$

$$= 7.87393$$

$$\frac{1}{2}(A-B) = 0^{\circ} 25' 43''.$$

$$A = 2^{\circ}46'8''.$$

$$B = 1^{\circ}54'42''.$$

$$c = \frac{b \sin C}{\sin B},$$

$$\log b = 1.52035$$

$$\log \sin C = 8.91169$$

$$\text{colog } \sin B = 1.47680$$

$$\log c = 1.90884$$

$$c = 81.056.$$

11. 設三角形之二邊各等於 6。而其夾角為  $60^{\circ}$ 。求第三邊

$$\text{因 } a = b,$$

$$\text{故 } A = B,$$

$$A + B = 120^{\circ}.$$

$$\text{即 } A = B = C = 60^{\circ},$$

$$\text{即 } a = b = c = 6.$$

12. 設三角形之二邊各等於 6。而其夾角為  $120^{\circ}$ 。求第三邊。

$$A + B = 60^{\circ},$$

$$\text{故 } A = B = 30^{\circ},$$

$$a = b = 6.$$

$$c = \frac{a \sin C}{\sin A},$$

$$\log a = 0.77815$$

$$\log \sin C = 9.93753$$

$$\text{colog } \sin A = 0.30103$$

$$\log c = 1.01671$$

$$c = 10.392.$$

13. 試以第一解法。用於二等邊三角形。

$$\text{因 } a = b,$$

$$\text{故 } \tan \frac{1}{2}(A - B)$$

$$= \frac{a - b}{a + b} \tan \frac{1}{2}(A + B)$$

變為  $\tan \frac{1}{2}(A - B) = 0.$

$$A - B = 0.$$

$$A = B.$$

$$= \frac{1}{2}(180^{\circ} - C)$$

$$= 90^{\circ} - \frac{1}{2}C.$$

$$C = \frac{a \sin C}{\sin A}$$

14. 設三角形之二邊為 10,

11 夾角為  $50^{\circ}$  求第三邊

$$a + b = 21.$$

$$a - b = 1.$$

$$A + B = 130^{\circ}$$

$$\frac{1}{2}(A + B) = 65^{\circ}.$$

$$\tan \frac{1}{2}(A - B)$$

$$= \frac{(a - b) \tan \frac{1}{2}(A + B)}{a + b},$$

$$\log(1 - b) = 0.00000$$

$$\log \tan \frac{1}{2}(A + B) = 10.33133$$

$$\text{colog}(a+b) = \underline{8.67778 - 10}$$

$$\log \tan \frac{1}{2}(A-B) = 9.00911$$

$$\frac{1}{2}(A-B) = 5^\circ 49' 55''.$$

$$A = 70^\circ 49' 55''.$$

$$B = 59^\circ 10' 9''.$$

$$c = \frac{b \sin C}{\sin B},$$

$$\log b = 1.00000$$

$$\log \sin C = 6.88425$$

$$\text{colog} \sin B = \underline{0.6617}$$

$$\log c = 0.95042.$$

$$c = 8.9212.$$

15. 設 三角形之二邊爲

$$43.301,$$

25. 夾角爲  $30^\circ$ 。求第三邊。

$$a+b = 68.301$$

$$a-b = 18.301,$$

$$A+B = 150^\circ.$$

$$\frac{1}{2}(A+B) = 75^\circ.$$

$$\tan \frac{1}{2}(A-B)$$

$$= \frac{(a-b) \tan \frac{1}{2}(A+B)}{a+b}.$$

$$\log(a-b) = 1.26427$$

$$\log \tan \frac{1}{2}(A+B)$$

$$= 10.57195$$

$$\text{colog}(a+b) = 8.16557 - 10$$

$$\log \tan \frac{1}{2}(A-B) = 9.99999$$

$$\frac{1}{2}(A-B) = 45^\circ.$$

$$A = 120^\circ.$$

$$B = 30^\circ.$$

B, C 既等, 則 ABC 爲等腰三角形。

$$\text{即得 } c = (b) = 25.$$

16. 自某處 C, 測 A, B 兩點。

得 CA = 3825 碼, CB =

3475.6 碼。而  $\angle ACB =$

$62^\circ 31'$  求(B距。

$$b+a = 7300.6.$$

$$b-a = 349.4.$$

$$B+A = 117^\circ 26'.$$

$$\frac{1}{2}(A+B) = 58^\circ 44' 30''.$$

$$\tan \frac{1}{2}(A-B)$$

$$= \frac{(b-a) \tan \frac{1}{2}(B+A)}{b+a},$$

$$\log(b-a) = 2.54332$$

$$\log \tan \frac{1}{2}(B+A)$$

$$= 10.21680$$

$$\text{colog}(b+a) = \underline{6.13664 - 10}$$

$$\log \tan \frac{1}{2}(B-A) = 8.89676$$

$$\frac{1}{2}(B-A) = 4^\circ 30' 29''.$$

$$B = 63^\circ 14' 59''.$$

$$A = 54^{\circ}14'1''.$$

$$c = \frac{b \sin C}{\sin B}$$

$$\log b = 3.58263$$

$$\log \sin C = 9.94799$$

$$\text{colog} \sin B = 0.04916$$

$$\log c = 3.57008$$

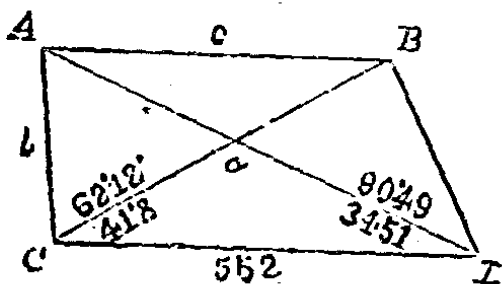
$$c = 3000.$$

即  $AB = 3000$  碼

有不可近之兩物A與B。自A  
B綫之同綫相距562碼之兩站  
CD望之。得 $\angle ACB = 62^{\circ}12'$

$\angle BCD = 41^{\circ}8'$ ,  $\angle ABD =$   
 $60^{\circ}49'$ ,  $\angle CBC = 34^{\circ}51'$ 。求AB  
距。

角 $= 60^{\circ}49'$ ,  $\angle ADC$ 角 $= 34^{\circ}$   
 $51''$ 。試求AB之距。



於  $\triangle ACD$  三角形中, 有  
 $A = 180^{\circ} - (C + D)$   
 $= 42^{\circ}40'$ .

$$\frac{b}{562} = \frac{\sin 34^{\circ}51'}{\sin 41^{\circ}49'}$$

故  $b = \frac{562 \sin 34^{\circ}51'}{\sin 41^{\circ}49'}$ .

$$\log 562 = 2.74974$$

$$\log \sin 34^{\circ}51' = 9.75696$$

$$\text{colog} \sin 41^{\circ}49'$$

$$= 0.17609$$

$$\log b = 2.68274$$

$$b = 481.66.$$

於  $\triangle ACB$  三角形中, 有

$$\tan \frac{1}{2}(A - B)$$

$$= \frac{a - b}{a + b} \tan \frac{1}{2}(A + B).$$

$$\frac{1}{2}(A + B) = \frac{1}{2}(180^{\circ} - C)$$

$$= 58^{\circ}54'.$$

$$a - b = 816.98 - 481.66$$

$$= 335.32.$$

$$a + b = 816.98 + 481.66$$

$$= 1298.64,$$

$$\tan \frac{1}{2}(A - B)$$

$$= \frac{(a - b) \tan \frac{1}{2}(A + B)}{a + b},$$

$$\log (a - b) = 2.52548$$

$$\log \tan \frac{1}{2}(A + B)$$

$$= 10.21951$$

$$\text{colog} (a + b) = 6.88651 - 10$$

$$\begin{aligned}\log \tan \frac{1}{2}(A-B) &= 9.63150 \\ \frac{1}{2}(A-B) &= 23^{\circ} 10' 26'' \\ A &= 82^{\circ} 4' 26'' \\ c &= \frac{a \sin C}{\sin A}\end{aligned}$$

$$\begin{aligned}\log a &= 2.91221 \\ \log \sin C &= 9.94674 \\ \text{colog } \sin A &= 0.09417 \\ \log c &= 2.86312 \\ c &= 729.67.\end{aligned}$$

即  $AB = 729.67$  碼。

18. 於同時同站開車各沿成  $30^{\circ}$  角之兩直軌進行。其一車速率為每點鐘 30 哩。又一車每點鐘 40 哩。問半點鐘後。二車距遠若干。

$$\begin{aligned}a + b &= 35. \\ a - b &= 5. \\ A + B &= 150^{\circ} \\ \frac{1}{2}(A + B) &= 75^{\circ}. \\ \tan \frac{1}{2}(A - B) &= \frac{(a-b)\tan \frac{1}{2}(A+B)}{a+b}, \\ \log (a-b) &= 0.69897 \\ \log \tan \frac{1}{2}(A + B) &= 10.57195 \\ \text{colog}(a + b) &= 8.4559 - 10\end{aligned}$$

$$\begin{aligned}\log \tan \frac{1}{2}(A-B) &= 9.72685 \\ \frac{1}{2}(A-B) &= 28^{\circ} 3' 52'' \\ B &= 46^{\circ} 56' 8'' \\ A &= 103^{\circ} 3' 52'' \\ c &= \frac{b \sin C}{\sin B}\end{aligned}$$

$$\begin{aligned}\log b &= 1.17609 \\ \log \sin C &= 9.69897 \\ \text{colog } \sin B &= 0.13633 \\ \log c &= 1.01139 \\ c &= 10.266.\end{aligned}$$

即兩車相距 10.266 哩。

19. 已知平行四邊形之兩對角線為 5 與 6。而其所成之角為  $49^{\circ} 18'$ 。求兩邊。

設  $ABDE$  為平行方形  
 $EB = 6, AD = 5,$   
 又  $\angle BCA = 49^{\circ} 18'.$   
 於  $\triangle ACB$  三角形中有  
 $BC = a = 3.$   
 $AC = b = 2.5.$

求  $AB$  即  $c.$   
 $a - b = 0.5.$   
 $a + b = 5.5.$   
 $A + B = 130^{\circ} 42'.$

$$\begin{aligned}\frac{1}{2}(A+B) &= 65^{\circ}21'. \\ \tan \frac{1}{2}(A-B) &= \frac{(a-b)\tan \frac{1}{2}(A+B)}{a+b},\end{aligned}$$

$$\log(a-b) = 9.69897 - 10$$

$$\log \tan \frac{1}{2}(A+B)$$

$$= 10.33829$$

$$\operatorname{colog}(a+b) = \underline{9.25964 - 10}$$

$$\log \tan \frac{1}{2}(A-B)$$

$$= 9.29590$$

$$\frac{1}{2}(A-B) = 11^{\circ}12'20''.$$

$$A = 76^{\circ}33'20''.$$

$$B = 54^{\circ}8'40''.$$

$$c = \frac{a \sin C}{\sin A},$$

$$\log a = 0.47712$$

$$\log \sin C = 9.87975$$

$$\operatorname{colog} \sin A = \underline{0.01207}$$

$$\log c = 0.36894.$$

$$AB = c = 2.3385.$$

於 ABC 三角形中有

$$EC = a = 3,$$

$$AC = b = 2.5,$$

$$\text{ACE 角} = 130^{\circ}42'.$$

$$A + E = 49^{\circ}18'$$

$$\frac{1}{2}(A+E) = 24^{\circ}39'.$$

$$\begin{aligned}\tan \frac{1}{2}(A-E) &= \frac{(a-b)\tan \frac{1}{2}(A+E)}{a+b}\end{aligned}$$

$$\log(a-b) = 9.69897 - 10$$

$$\log \tan \frac{1}{2}(A+E)$$

$$= 9.66171$$

$$\operatorname{colog}(a+b) = \underline{9.25964 - 10}$$

$$\log \tan \frac{1}{2}(A-E)$$

$$= 8.62032$$

$$\frac{1}{2}(A-E) = 2^{\circ}23'20''.$$

$$A = 27^{\circ}2'20''.$$

$$c = \frac{a \sin C}{\sin A},$$

$$\log a = 0.47712$$

$$\log \sin C = 9.87975$$

$$\operatorname{colog} \sin A = \underline{0.34238}$$

$$\log c = 0.69925$$

$$EA = c = 5.0032.$$

20. 三角形兩邊之比爲 5:9°  
而其夾角爲 139°54'。求餘兩角°

$$a = 9.$$

$$b = 5.$$

$$a + b = 14.$$

$$a - b = 4.$$

$$A + B = 40^{\circ}6'.$$

$$\frac{1}{2}(A+B) = 20^{\circ}3',$$

$$\tan \frac{1}{2}(A - B)$$

$$= \frac{(a - b) \tan \frac{1}{2}(A + B)}{a + b}$$

$$\log(a - b) = 0.60206$$

$$\log \tan \frac{1}{2}(A + B)$$

$$= 9.56224$$

$$\text{colog}(a + b) = 8.85387 - 10$$

$$\log \tan \frac{1}{2}(A - B)$$

$$= 9.01817$$

$$\frac{1}{2}(A - B) = 5^\circ 57' 10''.$$

$$A = 26^\circ 0' 10''.$$

$$B = 14^\circ 5' 50''.$$

21. 今欲求隔池之兩物A與B之距。故擇一C點。量得CA=426碼, CB=322.4碼。∠ACB=68。42''求AB距。

$$b + a = 748.4.$$

$$b - a = 193.6.$$

$$B + A = 111^\circ 18''.$$

$$\frac{1}{2}(B + A) = 55^\circ 39''.$$

$$\tan \frac{1}{2}(B - A)$$

$$= \frac{(b - a) \tan \frac{1}{2}(B + A)}{b + a}$$

$$\log(b - a) = 2.01536$$

$$\log \tan \frac{1}{2}(B - A)$$

$$= 10.16530$$

$$\text{colog}(b + a) = 7.12587 - 10$$

$$\log \tan \frac{1}{2}(B - A)$$

$$= 9.30653$$

$$\frac{1}{2}(B - A) = 11^\circ 27' 1''.$$

$$B = 67^\circ 6' 1''.$$

$$c = \frac{b \sin C}{\sin B}$$

$$\log b = 2.62941.$$

$$\log \sin C = 9.96927$$

$$\text{colog} \sin B = 0.03565$$

$$\log c = 2.63433$$

$$c = 430.85.$$

$$\text{即 } AB = 430.84 \text{ 碼.}$$

## 例題 XX

已知三邊。試解下列各三角形。

$$a = 51$$

$$b = 65$$

$$c = 20$$

---


$$2s = 136.$$

$$s = 68.$$

$$s - a = 17.$$

$$s - b = 3.$$

$$s - c = 48.$$

$$\tan \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\log(s-b) = 0.47712$$

$$\log(s-c) = 1.68124$$

$$\operatorname{colog} s = 8.16749 - 10$$

$$\operatorname{colog}(s-a) = 8.76955 - 10$$

$$\underline{2) 19.09540 - 20}$$

$$\log \tan \frac{1}{2}A = 9.54770$$

$$\frac{1}{2}A = 19^\circ 26' 24''.$$

$$\text{故 } A = 38^\circ 52' 48''$$

$$\tan \frac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$\log(s-a) = 1.28045$$

$$\log(s-c) = 1.68124$$

$$\operatorname{colog} s = 8.16749 - 10$$

$$\operatorname{colog}(s-b) = 9.52288 - 10$$

$$\underline{2) 20.60206 - 20}$$

$$\log \tan \frac{1}{2}B = 10.30103$$

$$\frac{1}{2}B = 63^\circ 26' 6''.$$

$$\text{故 } B = 126^\circ 52' 12''.$$

$$\text{而 } C = 14^\circ 15'.$$

$$2. \quad 78, 101, 290$$

$$a = 78$$

$$b = 101$$

$$\underline{c = 29}$$

$$2s = 208$$

$$s = 104.$$

$$s - a = 26.$$

$$s - b = 3.$$

$$s - c = 75.$$

$$\tan \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\log(s-b) = 0.47712$$

$$\log(s-c) = 1.87506$$

$$\operatorname{colog} s = 7.98297 - 10$$

$$\operatorname{colog}(s-a) = 8.58503 - 10$$

$$\underline{2) 18.92018 - 20}$$

$$\log \tan \frac{1}{2}A = 9.46009$$

$$\frac{1}{2}A = 16^\circ 5' 27''.$$

$$\text{故 } A = 32^\circ 10' 55''.$$

$$\tan \frac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$\log(s-a) = 1.41497$$

$$\log(s-c) = 1.87506$$

$$\operatorname{colog} s = 7.98297 - 10$$

$$\operatorname{colog}(s-b) = 9.52288 - 10$$

$$\underline{2) 20.79588 - 20}$$

$$\log \tan \frac{1}{2}B = 10.39794$$

$$\frac{1}{2}B = 68^\circ 11' 55''.$$

$$\text{故 } B = 136^\circ 23' 50''.$$



$$A + B = 168^{\circ} 34'45''.$$

$$\text{而 } C = 11^{\circ}25'15''.$$

$$3. \quad 111, 145, 40$$

$$a = 111$$

$$b = 145$$

$$c = 40$$

$$2s = 295$$

$$s = 148.$$

$$s - a = 37.$$

$$s - b = 3.$$

$$s - c = 108.$$

$$\tan \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\log(s-b) = 0.47712$$

$$\log(s-c) = 2.03342$$

$$\text{colog } s = 7.82974 - 10$$

$$\text{colog}(s-a) = \frac{8.43180 - 10}{2}$$

$$= 18.77208 - 20$$

$$\log \tan \frac{1}{2}A = 9.38604$$

$$\frac{1}{2}A = 13^{\circ}40'16''.$$

$$A = 27^{\circ}20'32''.$$

$$\tan \frac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$\log(s-a) = 1.56820$$

$$\log(s-c) = 2.03342$$

$$\text{colog } s = 7.82974 - 10$$

$$\text{colog}(s-b) = \frac{9.52288 - 10}{2}$$

$$= 20.95124 - 20$$

$$\log \tan \frac{1}{2}B = 10.47712$$

$$\frac{1}{2}B = 71^{\circ}33'54''.$$

$$B = 143^{\circ}7'48''.$$

$$B + A = 170^{\circ}28'20''.$$

$$C = 9^{\circ}31'40''.$$

$$4. \quad 21, 26, 31$$

$$a = 21$$

$$b = 26$$

$$c = 31$$

$$2s = 78$$

$$s = 39$$

$$s - a = 18.$$

$$s - b = 13.$$

$$s - c = 8.$$

$$\tan \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\log(s-b) = 1.11394.$$

$$\log(s-c) = 0.90309$$

$$\text{colog } s = 8.40894 - 10$$

$$\text{colog}(s-a) = \frac{8.74473 - 10}{2}$$

$$= 19.17070 - 20$$

$$\log \tan \frac{1}{2}A = 9.58535$$

$$\frac{1}{2}A = 21^{\circ}3'6.3''.$$

$$A = 42^{\circ}6'13''.$$

$$\tan \frac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$\log(s-a) = 1.25527$$

$$\log(s-c) = 0.90309$$

$$\operatorname{colog} s = 8.40894 - 10$$

$$\operatorname{colog}(s-b) = 8.88606 - 10$$

$$2) 19.45336 - 20$$

$$\log \tan \frac{1}{2}B = 9.72668$$

$$\frac{1}{2}B = 28^\circ 3' 18''.$$

$$B = 56^\circ 6' 36''.$$

$$A + B = 98^\circ 12' 49''.$$

$$C = 81^\circ 47' 11''.$$

5. 19, 34, 49.

$$a = 19$$

$$b = 34$$

$$c = \underline{49}$$

$$2s = 102$$

$$s = 51.$$

$$s - a = 32.$$

$$s - b = 17.$$

$$s - c = 2.$$

$$\tan \frac{1}{2}A = \sqrt{\frac{(s-d)(s-c)}{s(s-a)}}$$

$$\log(s-b) = 1.33045$$

$$\log(s-c) = 0.30103$$

$$\operatorname{colog} s = 8.29243 - 10$$

$$\operatorname{colog}(s-a) = 8.49485 - 10$$

$$2) 18.31876 - 20$$

$$\log \tan \frac{1}{2}A = 9.15938$$

$$\frac{1}{2}A = 8^\circ 12' 48''.$$

$$A = 16^\circ 25' 36''.$$

$$\tan \frac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$\log(s-a) = 1.50515$$

$$\log(s-c) = 0.30103$$

$$\operatorname{colog} s = 8.29243 - 10$$

$$\operatorname{colog}(s-b) = 8.76955 - 10$$

$$2) 18.86816 - 20$$

$$\log \tan \frac{1}{2}B = 9.43408$$

$$\frac{1}{2}B = 15^\circ 12'.$$

$$B = 30^\circ 24'.$$

$$C = 133^\circ 10' 24''.$$

6. 43, 50, 57.

$$a = 43$$

$$b = 50$$

$$c = \underline{57}$$

$$2s = 150$$

$$s = 75.$$

$$s - a = 32.$$

$$s - b = 25.$$

$$s - c = 18.$$

$$\tan \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}},$$

$$\log(s-b) = 1.39794$$

$$\log(s-c) = 1.25527$$

$$\text{colog } s = 8.12494 - 10$$

$$\text{colog } (s-a) = \underline{8.49485 - 10}$$

$$2) \underline{19.27300 - 20}$$

$$\log \tan \frac{1}{2}A = 9.63650$$

$$\frac{1}{2}A = 23^\circ 24' 47.6''.$$

$$A = 46^\circ 49' 35''.$$

$$\tan \frac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}},$$

$$\log(s-a) = 1.50515$$

$$\log(s-c) = 1.25527$$

$$\text{colog } s = 8.12494 - 10$$

$$\text{colog } (s-b) = \underline{8.60206 - 10}$$

$$2) \underline{19.48742 - 20}$$

$$\log \tan \frac{1}{2}B = 9.74371$$

$$\frac{1}{2}B = 28^\circ 59' 52''.$$

$$B = 57^\circ 59' 44''.$$

$$C = 75^\circ 10' 41''.$$

7. 37, 58, 79.

$$a = 37$$

$$b = 58$$

$$c = 79$$

$$2s = 174$$

$$s = 87.$$

$$s - a = 50.$$

$$s - b = 29.$$

$$s - c = 8.$$

$$\tan \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}},$$

$$\log(s-b) = 1.46240$$

$$\log(s-c) = 0.90309$$

$$\text{colog } s = 8.06048 - 10$$

$$\text{colog } (s-a) = \underline{8.30103 - 10}$$

$$2) \underline{18.72700 - 20}$$

$$\log \tan \frac{1}{2}A = 9.36350$$

$$\frac{1}{2}A = 13^\circ 0' 14.5''.$$

$$A = 26^\circ 0' 29''.$$

$$\tan \frac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}},$$

$$\log(s-a) = 1.69897$$

$$\log(s-c) = 0.90309$$

$$\text{colog } s = 8.06048 - 10$$

$$\text{colog } (s-b) = \underline{8.53760 - 10}$$

$$2) \underline{19.20014 - 20}$$

$$\log \tan \frac{1}{2}B = 9.60007$$

$$\frac{1}{2}B = 21^\circ 42' 40''.$$

$$B = 43^\circ 25' 20''.$$

$$C = 110^{\circ} 34' 11''.$$

$$8. \quad 73, 82, 91_0$$

$$a = 73$$

$$b = 82$$

$$c = 91$$

$$2s = 246$$

$$s = 123.$$

$$s - a = 50.$$

$$s - b = 41.$$

$$s - c = 32.$$

$$\tan \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\log(s-b) = 1.61278$$

$$\log(s-c) = 1.50515$$

$$\operatorname{colog} s = 7.91009 - 10$$

$$\operatorname{colog}(s-a) = 8.30103 - 10$$

$$2) 19.32905 - 20$$

$$\log \tan \frac{1}{2}A = 9.66453$$

$$\frac{1}{2}A = 24^{\circ} 47' 29''.$$

$$A = 49^{\circ} 34' 58''.$$

$$\tan \frac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$\log(s-a) = 1.69897$$

$$\log(s-c) = 1.50515$$

$$\operatorname{colog} s = 7.91009 - 10$$

$$\operatorname{colog}(s-b) = 8.38722 - 10$$

$$2) 19.50143 - 20$$

$$\log \tan \frac{1}{2}B = 9.75072$$

$$\frac{1}{2}B = 29^{\circ} 23' 29''.$$

$$B = 58^{\circ} 46' 58''.$$

$$C = 71^{\circ} 38' 4''.$$

$$9. \quad 14.493, 55.4363, 66.9129.$$

$$a = 14.493$$

$$b = 55.4363$$

$$c = 66.9129$$

$$2s = 136.8422$$

$$s = 68.4211.$$

$$s - a = 53.9281$$

$$s - b = 12.9848$$

$$s - c = 15.082$$

$$\tan \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\log(s-b) = 1.11343$$

$$\log(s-c) = 0.17846$$

$$\operatorname{colog}(s-a) = 8.26819 - 10$$

$$2) 17.72489 - 20$$

$$\log \tan \frac{1}{2}A = 8.86245$$

$$\frac{1}{2}A = 4^{\circ} 10' 07''.$$

$$A = 8^{\circ} 20' 1''.$$

$$\frac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$\log(s-a) = 1.73181$$

$$\log(s-c) = 0.17846$$

$$\operatorname{cologs} = 8.16481 - 10$$

$$\operatorname{colog}(s-b) = \underline{8.88657 - 10}$$

$$2) \underline{18.96165 - 10}$$

$$\operatorname{logtan} \frac{1}{2}B = 9.48082$$

$$\frac{1}{2}B = 16^\circ 50' 26''.$$

$$B = 33^\circ 40' 5''.$$

$$C = 137^\circ 50' 54''.$$

$$10. \quad \sqrt{5}, \sqrt{9},$$

$$\sqrt{7}$$

$$a = \sqrt{5} = 2.1361$$

$$b = \sqrt{6} = 2.4495$$

$$c = \sqrt{7} = 2.6458$$

$$2s = 7.3314$$

$$s = 3.6657$$

$$s-a = 1.4296$$

$$s-b = 1.2162$$

$$s-c = 1.0199$$

$$\operatorname{tan} \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}},$$

$$\log(s-b) = 0.08500$$

$$\log(s-c) = 0.00856$$

$$\operatorname{cologs} = 9.43585 - 10$$

$$\operatorname{colog}(s-a) = \underline{9.84478 - 10}$$

$$2) \underline{19.37419 - 20}$$

$$\operatorname{logtan} \frac{1}{2}A = 9.68709$$

$$\frac{1}{2}A = 25^\circ 56' 36''.$$

$$A = 51^\circ 53' 12''.$$

$$\operatorname{tan} \frac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}.$$

$$\log(s-a) = 0.15522$$

$$\log(s-c) = 0.00856$$

$$\operatorname{cologs} = 9.43585 - 10$$

$$\operatorname{colog}(s-b) = \underline{9.91500 - 10}$$

$$2) \underline{19.51463 - 20}$$

$$\operatorname{logtan} \frac{1}{2}B = 9.75732$$

$$\frac{1}{2}B = 29^\circ 45' 45''.$$

$$B = 56^\circ 31' 48''.$$

$$C = 98^\circ 5'.$$

已色下之各件求各角(11-16)。

$$11. \quad a=6, b=8, c=10.$$

$$a=6$$

$$b=8$$

$$c=10$$

$$1) \quad s=24$$

$$s=12.$$

$$s-a=6.$$

$$s-b=4.$$

$$s-c=1.$$

$$\operatorname{tan} \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}},$$

$$\log(s-b) = 0.60206$$

$$\log(s-c) = 0.30103$$

$$\text{cologs} = 8.92082 - 10$$

$$\text{colog}(s-a) = 9.22180 - 10$$

$$2) 19.04526 - 20$$

$$\log \tan \frac{1}{2}A = 9.52288$$

$$\frac{1}{2}A = 18^\circ 26' 6''.$$

$$A = 36^\circ 52' 12''.$$

因  $10^2 = 6^2 + 8^2$

故 ABC 爲正三角形,

$$C = 90^\circ,$$

$$B = 53^\circ 7' 48''.$$

12.  $a=6, b=6, c=10^\circ$

$$a=6$$

$$b=6$$

$$c=10$$

$$2c=22$$

$$s=11.$$

$$s-a=5.$$

$$s-b=5.$$

$$s-c=1.$$

$$\tan \frac{1}{2}C = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}},$$

$$\log(s-a) = 0.69897$$

$$\log(s-b) = 0.69897$$

$$\text{colgs} = 8.95861 - 10$$

$$\text{colog}(s-c) = 10.00000 - 10$$

$$2) 20.39655 - 20$$

$$\log \tan \frac{1}{2}C = 10.17827$$

$$\frac{1}{2}C = 56^\circ 26' 33''.$$

$$C = 112^\circ 35' 6''$$

A B C 乃等腰三角形

故  $A=B=\frac{1}{2}(180^\circ - C)$

$$= 33^\circ 83' 27''$$

13.  $a=6, b=6, c=6.$

A B C 既爲平邊三角形, 則亦必爲等角三角形.

故  $A=B=C\frac{1}{3}\times 180^\circ = 60^\circ$

14.  $a=6, b=9, c=12.$

$$a=6.$$

$$b=9$$

$$c=12$$

$$2s=27$$

$$s=13.5.$$

$$s-a=7.5$$

$$s-b=4.5$$

$$s-c=1.5.$$

$$\tan A = \sqrt{\frac{(s-b)(s-c)}{(s-a)}},$$

$$\log(s-b) = 0.65321$$

$$\log(s-c) = 0.17609$$

$$\text{colog}(s-a) = 9.12494 - 10$$

$$2) \underline{18.82391 - 20}$$

$$\log \tan \frac{1}{2}A = 941196 - 10$$

$$A = 14^\circ 28' 39''.$$

$$= 28^\circ 57' 18''$$

$$\tan \frac{1}{2}B = \sqrt{\frac{(-a)(s-c)}{s(s-b)}},$$

$$\log(s-a) = 0.87506$$

$$\log(s-c) = 0.17609$$

$$\text{cologs} = 8.86967 - 10$$

$$\text{colog}(s-b) = \underline{9.34679 - 10}$$

$$2) \underline{19.26761 - 20}$$

$$\log \tan \frac{1}{2}B = 9.63380 - 10$$

$$\frac{1}{2}A = 23^\circ 17' 3''.$$

$$B = 46^\circ 34' 6''.$$

$$C = 104^\circ 28' 36''.$$

$$15. \quad a=2, b = \sqrt{6}, c =$$

$$\sqrt{3} - 1$$

$$a=2.$$

$$b = \sqrt{6} = 2.44947$$

$$c = \sqrt{3} - 1 = \underline{0.73205}$$

$$2s = 5.18152$$

$$s = 2.59076$$

$$s-a = 0.59076$$

$$s-b = 0.13129$$

$$s-c = 1.85871$$

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

$$r^2 = \frac{(s-a)(s-b)(s-c)}{s}$$

$$\log(s-a) = 9.77141 - 10$$

$$\log(s-b) = 9.15011 - 10$$

$$\log(s-c) = 0.26921.$$

$$\text{cologs} = \underline{9.58657 - 10}$$

$$\log r^2 = \underline{18.77730 - 20}$$

$$\log r = 9.38865 - 10$$

按公式(22)得

$$\log \tan \frac{1}{2}A = 9.61724$$

$$\log \frac{1}{2}B = 10.23851$$

$$\log \tan \frac{1}{2}C = 9.11944$$

$$\frac{1}{2}A = 22^\circ 30'.$$

$$\frac{1}{2}B = 60^\circ.$$

$$\frac{1}{2}C = 7^\circ 30''.$$

$$A = 45^\circ.$$

$$B = 120^\circ.$$

$$C = 15^\circ.$$

$$16. \quad a=2, b = \sqrt{6}, c =$$

$$\sqrt{3} + 10$$

$$a=2.$$

$$b = \sqrt{6} = 2.44947$$

$$c = \sqrt{3} + 1 = \underline{2.73205}$$

$$2s = 7.18152$$

$$s = 3.59076.$$

$$s - a = 1.59076.$$

$$s - b = 1.14129.$$

$$s - c = 0.85871$$

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}},$$

$$\log(s-a) = 0.20160$$

$$\log(s-b) = 0.05740$$

$$\log(s-c) = 9.93385 - 10$$

$$\text{colog } s = \underline{9.44481 - 10}$$

$$2) \underline{19.63766 - 20}$$

$$\log r = 9.81883 - 10$$

按公式 [32] 得

$$\log \tan \frac{1}{2}A = 9.61723$$

$$\log \tan \frac{1}{2}B = 9.76134$$

$$\log \tan \frac{1}{2}C = 9.88498$$

$$\frac{1}{2}A = 22^\circ 30'.$$

$$\frac{1}{2}B = 30^\circ.$$

$$\frac{1}{2}C = 37^\circ 30'.$$

$$A = 45^\circ.$$

$$B = 60^\circ.$$

$$C = 75^\circ.$$

17. ABC 三城之相距為 AB = 165哩。AC = 72哩。BC = 185哩其B在A之直東。問C在A之

何方。並有若何二答。

$$a = 185$$

$$b = 72$$

$$c = 165$$

$$2s = 422$$

$$s = 211.$$

$$s - a = 26$$

$$s - b = 139.$$

$$s - c = 46.$$

$$\tan \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}},$$

$$\log(s-b) = 2.14301$$

$$\log(s-c) = 1.66276$$

$$\text{colog } s = 7.67572 - 10$$

$$\text{colog}(s-a) = \underline{8.58503 - 10}$$

$$2) \underline{20.06652 - 20}$$

$$\log \tan \frac{1}{2}A = 10.03326$$

$$\frac{1}{2}A = 47^\circ 11' 31''.$$

$$A = 94^\circ 23' 2''.$$

BAC角 =  $91^\circ 23' 2''$ . 減去  $90^\circ$  (即自東至北之象限) 得  $4^\circ 23' 2''$ , 即C在A之北偏西  $4^\circ 23' 2''$ .

但C亦可在A之南, 故有兩答, 即北偏西與南偏西是也.



18. 測者視 7 呎長之一物°  
自目距此物兩端之視線爲 5 呎,  
8 呎求此兩視線之交角。

$$a=5$$

$$b=8$$

$$c=7$$

$$2s=20$$

$$s=10.$$

$$s-a=5.$$

$$s-b=2.$$

$$s-c=3.$$

$$\tan \frac{1}{2}C = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

$$\log (s-a) = 0.69897$$

$$\log (s-b) = 0.30103$$

$$\operatorname{colog} s = 9.00000 - 10$$

$$\operatorname{colog} (s-c) = 9.52288 - 10$$

$$2) \quad 19.52288 - 20$$

$$\log \tan \frac{1}{2}C = 9.76144$$

$$\frac{1}{2}C = 30^\circ$$

$$C = 60^\circ$$

19. 設用公式 (28) 以夾一  
角。何以於例 II 內遇之。並無  
別義。

因 諸邊皆已知，又此角

祇能有一值，故以公式  
[28] 求角之同數時不致  
有含糊之數。於第二類  
中角之對邊爲未知，又  
可有二值，故其角亦可  
有二值。

20. 三角形之三邊爲 3, 4, 6。

求最大角之正弦。

$$a=3$$

$$b=4$$

$$c=6$$

$$2s=13$$

$$s=6.5.$$

$$s-a=3.5.$$

$$s-b=2.5.$$

$$s(s-c)=3.25.$$

$$\tan \frac{1}{2}C = \sqrt{\frac{s-a)(s-b)}{s(s-c)}}$$

$$\lg (s-a) = 0.54407$$

$$\log (s-b) = 0.39794$$

$$\operatorname{colog} s(s-c) = 9.48812 - 10$$

$$2) \quad 20.43013 - 20$$

$$\log \tan \frac{1}{2}C = 10.21507$$

$$\frac{1}{2}C = 58^\circ 88' 25''.$$

$$C = 117^\circ 16' 50''.$$

$$\log \sin C = 9.94879$$

$$\sin C = 0.88877$$

21. 有ABC三市, A距B 200哩, 距C 184哩。而B在C之直北。相距150哩。問A在C之北若干哩。

$$a = 150.$$

$$b = 184$$

$$c = \underline{200}$$

$$2s = 534$$

$$s = 267.$$

$$s - a = 117.$$

$$s - b = 83.$$

$$s - c = 67.$$

$$\tan \frac{1}{2}C = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

$$\log(s-a) = 2.06819$$

$$\log(s-b) = 1.91908$$

$$\operatorname{colog} s = 7.57349 - 10$$

$$\operatorname{colog}(s-c) = \underline{8.17393 - 10}$$

$$2) \underline{19.73469 - 20}$$

$$\log \tan \frac{1}{2}C = 9.86735$$

$$\frac{1}{2}C = 36^\circ 22' 58''.$$

$$C = 72^\circ 45' 56''.$$

自A至BC作垂線。求a'即垂線所分成之一段。

$$a' = b \cos C$$

$$\log b = 2.26482$$

$$\log \cos C = \underline{9.47171}$$

$$\log a' = 1.73653$$

$$a' = 54.516$$

即A在C之北相距54.516哩。

22. 三角形之三邊爲78.9,

65.4, 97.3。求最大角。

$$a = 78.9$$

$$b = 65.4$$

$$c = \underline{97.3}$$

$$2s = 241.6$$

$$s = 120.8$$

$$s - a = 41.9.$$

$$s - b = 55.4.$$

$$s - c = 23.5.$$

$$\tan \frac{1}{2}C = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

$$\log(s-a) = 1.62221$$

$$\log(s-b) = 1.74351$$

$$\operatorname{colog} s = 7.91793 - 10$$

$$\operatorname{colog}(s-c) = \underline{8.62893 - 10}$$

$$2) \underline{19.91258 - 20}$$

$$\log \tan \frac{1}{2}C = 9.95629 - 10$$

$$\frac{1}{2}C = 42^\circ 7' 17''.$$

$$C = 84^\circ 14' 34''.$$

23. 三角形之三邊爲487.25, 512.33, 544.37。求最小角。

$$a = 487.25$$

$$b = 512.33$$

$$c = 544.37$$

$$2s = 1543.95$$

$$s = 771.975$$

$$s - a = 284.725.$$

$$s - b = 259.645.$$

$$s - c = 227.605.$$

$$\tan \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}},$$

$$\log (s-b) = 2.41438$$

$$\log (s-c) = 2.35718$$

$$\text{colog } (s-a) = 7.54557 - 10$$

$$\text{colog } s = 7.11239 - 10$$

$$2) \quad 19.42952 - 20$$

$$\log \tan \frac{1}{2}A = 9.71476 - 10$$

$$\frac{1}{2}A = 27^\circ 24' 27''.$$

$$A = 54^\circ 48' 54''.$$

24. 設 三角形之三邊爲

$$\frac{\sqrt{3+1}}{2\sqrt{2}}, \frac{\sqrt{3-1}}{2\sqrt{2}}, \frac{\sqrt{3}}{2} \quad \text{求各}$$

角。

$$\frac{\sqrt{3+1}}{2\sqrt{2}} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$= \frac{2.44947 + 1.41421}{4}$$

$$= \frac{3.86368}{4}$$

$$= 0.96592.$$

$$\frac{\sqrt{3-1}}{2\sqrt{2}} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$= \frac{2.44947 - 1.41421}{4}$$

$$= 0.25882.$$

$$\frac{\sqrt{3}}{2} = \frac{1.73205}{2}$$

$$= 0.86603.$$

$$a = 0.96592$$

$$b = 0.25882$$

$$c = 0.86603$$

$$2s = 2.09077$$

$$s = 1.04538.$$

$$s - a = 0.07946.$$

$$s - b = 0.78656.$$

$$s - c = 0.17935.$$

$$r = \frac{(s-a)(s-b)(s-c)}{s},$$

$$\log (s-a) = 8.90015 - 10$$

$$\log (s-b) = 9.89573 - 10$$

$$\log (s-c)=9.25370-10$$

$$\operatorname{colog} s=9.98072-10$$

$$\log r^2=18.03030-20$$

$$\log r=9.01515-10$$

按公式(32),

$$\log \tan \frac{1}{2}A=10.11500-10$$

$$\log \tan \frac{1}{2}B=9.11940-10$$

$$\log \tan \frac{1}{2}C=9.76145-10$$

$$\frac{1}{2}A=52^{\circ}30'.$$

$$\frac{1}{2}B=7^{\circ}30'.$$

$$\frac{1}{2}C=30^{\circ}.$$

$$A=105^{\circ}.$$

$$B=15^{\circ}.$$

$$C=60^{\circ}.$$

25. 三角形之三邊爲14.6吋  
16.7吋, 18.8吋。求自最大角之  
頂點其對邊所作垂線之長。

$$a=18.8$$

$$b=16.7$$

$$c=14.6$$

$$2s=50.1$$

$$s=25.05.$$

$$s-a=6.25.$$

$$s-b=8.35.$$

$$s-c=10.45.$$

$$\tan \frac{1}{2}B=\sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$\log (s-a)=0.79588$$

$$\log (s-c)=1.01912$$

$$\operatorname{colog} s=8.60119-10$$

$$\operatorname{colog} (s-b)=9.07831-10$$

$$2) 19.49450-20$$

$$\log \tan \frac{1}{2}B=9.74725-10$$

$$\frac{1}{2}B=29^{\circ}11'46''.$$

$$B=58^{\circ}23'32''.$$

設 AB 爲自 A 至 BC 之垂

線, 則  $\frac{AD}{14.6}=\sin B,$

$$AD=14.6 \sin B$$

$$\log 14.6=1.16435$$

$$\log \sin B=9.93026$$

$$\log AD=1.09461$$

$$AD=12.434$$

### 例題 XXI

已知下各事。求面積(1-10)。

$$1. a=4474.5, b=2164.5,$$

$$C=116^{\circ}30'20''.$$

$$F=\frac{1}{2}ab \sin C.$$

$$\log a=3.65075$$

$$\log b=3.33536$$

$$\text{colog } 2 = 9.69897 - 10$$

$$\log \sin C = \underline{9.95177}$$

$$\log F = 6.63685$$

$$F = 4.333,600$$

$$2. \quad b = 21.66, \quad c = 36.94,$$

$$A = 66^\circ 4' 19''.$$

$$F = \frac{1}{2}bc \sin A.$$

$$\log b = 1.33566$$

$$\log c = 1.56750$$

$$\text{colog } 2 = 9.69897 - 10$$

$$\log \sin A = \underline{9.96097}$$

$$\log F = 2.56310$$

$$F = 365.68.$$

$$3. \quad a = 510, \quad c = 173, \quad B = 162^\circ$$

$$30' 28''.$$

$$F = \frac{1}{2}a c \sin B.$$

$$\log a = 2.70757$$

$$\log c = 2.23805$$

$$\text{colog } 2 = 9.69897 - 10$$

$$\log \sin B = \underline{9.47795}$$

$$\log F = 4.12254$$

$$F = 13.260$$

$$4. \quad a = 408, \quad b = 41, \quad c = 401.$$

$$a = 408$$

$$b = 41$$

$$c = \underline{401}$$

$$2s = 850$$

$$s = 425.$$

$$s - a = 17.$$

$$s - b = 384.$$

$$s - c = 24.$$

$$F = \sqrt{s(s-a)(s-b)(s-c)},$$

$$\log s = 2.62839$$

$$\log(s-a) = 1.23045$$

$$\log(s-b) = 2.58433$$

$$\log(s-c) = \underline{1.38021}$$

$$2) \underline{7.82338}$$

$$\log F = 3.91169$$

$$F = 8160.$$

$$5. \quad a = 40, \quad b = 13, \quad c = 37.$$

$$a = 40$$

$$b = 13$$

$$c = 37$$

$$2s = 90.$$

$$s = 45.$$

$$s - a = 5.$$

$$s - b = 32$$

$$s - c = 8.$$

$$F = \sqrt{s(s-a)(s-b)(s-c)},$$

$$\log s = 1.65321$$

$$\log(s-a) = 0.69897$$

$$\log(s-b) = 1.50515$$

$$\log(s-c) = \underline{0.90309}$$

$$2) \underline{4.76042}$$

$$\log F = 2.38021$$

$$F = 240.$$

6.  $a=624, b=205, c=445$

$$a=624$$

$$b=205$$

$$c=445$$

$$2s = 1274$$

$$s = 637$$

$$s-a = 13.$$

$$s-b = 432.$$

$$s-c = 192.$$

$$F = \sqrt{s(s-a)(s-b)(s-c)},$$

$$\log s = 2.80414$$

$$\log(s-a) = 1.11394$$

$$\log(s-b) = 2.63548$$

$$\log(s-c) = \underline{2.28330}$$

$$2 \log F = 8.83686$$

$$\log F = 4.41843$$

$$F = 26.208.$$

7.  $b=149, A=70^\circ 42' 36''$ ,

$$B=39^\circ 18' 28''.$$

$$A=70^\circ 42' 30''.$$

$$B=39^\circ 18' 28''.$$

故  $C=69^\circ 59' 2''.$

$$a = \frac{s \sin A}{\sin B},$$

$$\log b = 2.17319$$

$$\log \sin A = 9.97490$$

$$\operatorname{colog} \sin B = \underline{0.19827}$$

$$\log a = 2.34636$$

$$F = \frac{1}{2} ab \sin C,$$

$$\log a = 2.34635$$

$$\log b = 2.17319$$

$$\operatorname{colog} c = \underline{9.69897 - 10}$$

$$\log F = 4.16146$$

$$F = 15.540.$$

8.  $a=215.9, c=307.7,$

$$A=25^\circ 9' 31''$$

因  $a < c, a > c \sin A,$

$$A < 90^\circ.$$

故有二解法.

$$\sin C = \frac{c \sin A}{A},$$

$$\log c = 2.48814$$

$$\log \sin A = 9.62852$$

$$\operatorname{colog} a = \underline{7.66575 - 10}$$

$$\log \sin C = 9.78240$$

$$C = 37^\circ 17' 38''.$$

$$B = 117^\circ 32' 51''.$$

又  $C' = 142^\circ 42' 22''.$

$$B' = 12^\circ 8' 17''.$$

$$F = \frac{1}{2}ac \sin B.$$

$$\operatorname{colog} 2 = 9.69897 - 10$$

$$\log a = 2.33425$$

$$\log c = 2.48813$$

$$\log \sin B = \underline{9.94774}$$

$$\log F = 4.46909$$

$$F = 29.450.$$

$$F = \frac{1}{2}ac \sin B'$$

$$\operatorname{colog} 2 = 9.69897 - 10$$

$$\log a = 2.23425$$

$$\log c = 2.48813$$

$$\log \sin B' = \underline{9.32268}$$

$$\log F' = 3.84403$$

$$F' = 6982.8.$$

9.  $b=8, c=5, A=60^\circ$

$$F = \frac{1}{2}bc \sin A$$

$$= \frac{1}{2}(8 \times 5)$$

$$(0.86603)$$

$$= 20 \times 0.86603$$

$$= 17.3206.$$

10.  $a=7, c=3, A=60^\circ$

$$\sin C = \frac{c \sin A}{a},$$

$$\log c = 0.47712$$

$$\log \sin A = 9.93753$$

$$\operatorname{colog} a = \underline{9.15490 - 10}$$

$$\log \sin C = 9.56955$$

$$C = 21^\circ 47' 12''.$$

而  $B = 98^\circ 12' 48''.$

$$F = \frac{1}{2}ac \sin B$$

$$\operatorname{colog} 2 = 9.69897 - 10$$

$$\log a = 0.84510$$

$$\log c = 0.47712$$

$$\log \sin B = \underline{9.99552}$$

$$\log F = 1.01671$$

$$\log F = 10.392.$$

11. 已知  $a=60, B=40^\circ$

$35' 12''$ . 面積 = 12. 內切圓半徑。

$$\frac{1}{2}ac \sin B = 12$$

$$C = \frac{24}{a \sin B}$$

$$\log 24 = 1.38021$$

$$\operatorname{colog} a = 8.22185 - 10$$

$$\operatorname{colog} \sin B = \underline{0.18669}$$

$$\log c = 9.98875 - 10$$

$$c = 0.61483.$$

$$a - c = 59.39517.$$

$$a + c = 60.61483.$$

$$A + C = 139^\circ 24' 48''.$$

$$\frac{1}{2}(A + C) = 69^\circ 42' 24''.$$

$$\tan \frac{1}{2}(A - C) = \frac{(a - c) \tan \frac{1}{2}(A + C)}{a + c}$$

$$\log(a - c) = 1.77368.$$

$$\text{colog}(a + c) = 8.21742 - 10$$

$$\log \tan \frac{1}{2}(A + C)$$

$$= 10.43206$$

$$\log \tan \frac{1}{2}(A - C)$$

$$= 10.42316$$

$$\frac{1}{2}(A - C) = 69^\circ 19' 19''.$$

$$A = 139^\circ 1' 43''.$$

$$b = \frac{a \sin B}{\sin A}.$$

$$\log a = 1.77815$$

$$\log \sin B = 9.81331$$

$$\text{colog} \sin A = 0.18331$$

$$\log b = 1.77477$$

$$b = 59.534$$

$$a = 60.$$

$$b = 59.534$$

$$c = 0.61483$$

$$2s = 120.14883.$$

$$\xi = 60.07442$$

$$F = rs.$$

$$\text{故 } r = \frac{F}{s} = \frac{12}{60.07442}$$

$$\log F = 1.07918$$

$$\text{colog } s = 8.22131 - 10$$

$$\log r = 9.30049 - 10$$

$$r = 0.19975.$$

12. 求以平行四邊形之兩鄰邊及夾角表面積之公式。

按幾何學，平行方形之面積 = 底 × 高。本題之面積為  $bh$ ,

$$\text{但 } h = a \sin A,$$

故 平行方形之面積 =  $ab \sin A$ .

13. 求二等邊梯形之兩平行邊及一銳角表面積之公式。

設  $a$  = 下底,

$b$  = 上底,

$h$  = 高,

$$p = \frac{1}{2}(a - b),$$

$A$  = 下底線之角.

$$\text{則 } F = \frac{1}{2}(a + b)h.$$

$$\text{但 } \frac{h}{\frac{1}{2}(a - b)}$$



$$= \tan A$$

故  $h = \frac{1}{2}(a-b)\tan A$

而  $F = \frac{1}{2}(a+b) \times \frac{1}{2}(a-b)$

$$\tan A = \frac{1}{2}(a^2 - b^2)\tan A.$$

14. 三角形之兩邊及夾角爲 2416, 1712,  $30^\circ$ . 又一三角形之兩邊及夾角爲 1948, 2848,  $150^\circ$ . 求此兩形面積之和。

設  $a = 2416, c = 1712,$

$$B = 30^\circ.$$

$$F = \frac{1}{2}ac \sin B.$$

$$\text{colog } 2 = 9.69897 - 10$$

$$\log a = 3.38310$$

$$\log c = 3.23350$$

$$\log \sin B = 9.69897$$

$$\log F = 6.01454$$

$$F = 1,034,000.$$

設  $c' = 1948,$

$$c' = 2848, B'$$

$$= 150^\circ.$$

$$F' = \frac{1}{2}a' c' \sin B'.$$

$$\text{colog } 2 = 0.69897 - 10$$

$$\log a' = 3.28959$$

$$\log c' = 3.45454$$

$$\log \sin B' = 9.69897$$

$$\log F' = 6.14207$$

$$= 1,387,000.$$

故  $F + F' = 2,421,000$

15. 二等邊三角形之底爲 20, 其面積爲  $100 \div \sqrt{3}$ . 求各角.

$$a = b.$$

$$b = 20.$$

$$F = \frac{1}{2}ch.$$

$$F = 100 \div \sqrt{3}.$$

$$\frac{1}{2}ch = 100 \div \sqrt{3}.$$

$$10h = \frac{100}{\sqrt{3}}.$$

$$h = \frac{10}{\sqrt{3}}.$$

$$\frac{h}{\frac{1}{2}c} = \tan A.$$

$$\log h = 0.76144$$

$$\text{colog } \frac{1}{2}c = 9.00000 - 10$$

$$\log \tan A = 9.76144$$

$$A = 30^\circ.$$

$$B = 30^\circ.$$

而  $C = 120^\circ.$

16. 四邊形之面積。等於其兩對角線乘夾角正弦之半。求證。

設  $a$  與  $b$  爲其二對角線之長,  $C$  爲其夾角. 又設  $a_1, a_2$  與

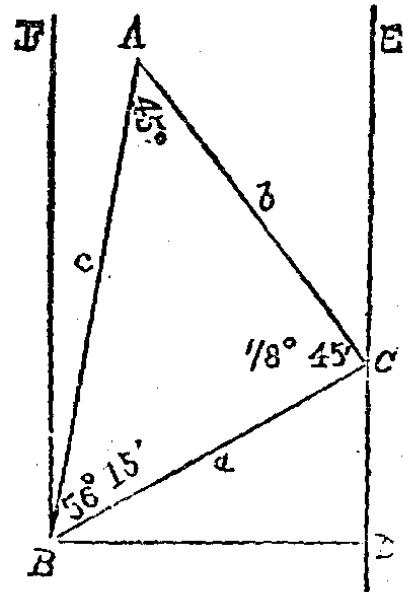
$b_1, b_2$  爲二邊角線交點所分之四段。

此四邊形之面積必等於其四三角形諸面積之總，如下，

$$\begin{aligned} F &= \frac{1}{2}a_1 b_2 \sin C \\ &+ \frac{1}{2}a_2 b_2 \sin C \\ &+ \frac{1}{2}a_2 b_1 \sin C \\ &+ \frac{1}{2}a_1 b_1 \sin C \\ &= \frac{1}{2}(a_1 b_2 + a_2 b_2 \\ &+ a_2 b_1 + a_1 b_1) \\ &\times \sin C \\ &= \frac{1}{2}(a_1 + a_2)(b_1 \\ &+ b_2) \sin C \\ &= \frac{1}{2}ab \sin C. \end{aligned}$$

### 例題 XXII

1. 一船駛於英國海峽。自船測一石。爲北  $33^\circ 45'$  西船向南  $67^\circ 30'$  西行 18 哩再測之。爲北  $11^\circ 15'$  東。求石與船兩次之距。



設 A 爲裏的石。

C 爲第一次測處。

B 爲第二次測處。

$a = 18$  哩。

$\angle ACE = 33^\circ 45'$ 。

$\angle DCB = 67^\circ 30'$ 。

$\angle ABF = 11^\circ 15'$ 。

$\angle ACB = 180^\circ - (\angle ACE$

$+ \angle DCB) = 78^\circ 45'$ 。

$\angle CBD = 90^\circ - \angle DCB$

$= 22^\circ 30'$ 。

$\angle ABC = 90^\circ - (\angle CBD$

$+ \angle ABF) = 56^\circ 15'$ 。

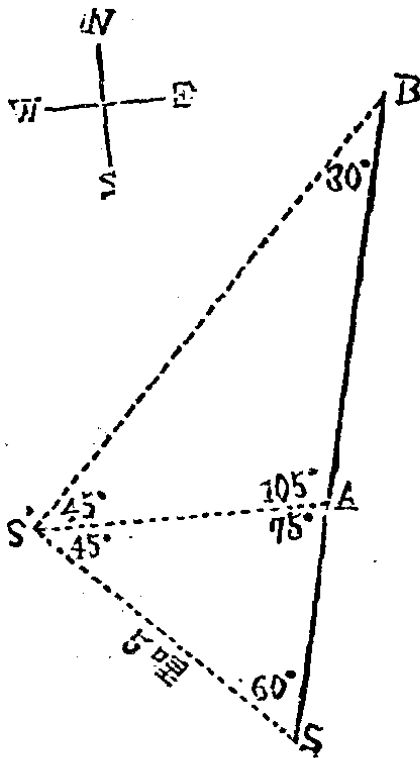
故  $\angle ABC = 45^\circ$ 。

$$b = \frac{a \sin B}{\sin C}.$$

$$\begin{aligned} \log a &= 1.25527 \\ \log \sin B &= 9.91985 \\ \text{colog } \sin A &= 0.15051 \\ \log b &= 1.32563 \\ b &= 21.166. \\ b &= \frac{a \sin C}{\sin A}. \\ \log a &= 1.25527 \\ \log \sin C &= 9.99157 \\ \text{colog } \sin A &= 0.15051 \\ \log c &= 1.39735 \\ c &= 24.966. \end{aligned}$$

即得二處相石之距為 21.166 哩與 24.966 哩。

2. 自一船同時測 A, B 兩物。在北 15° 東之一線內。船向西北行 5 哩。測得 A 在正東。B 在東北。求 AB 距。

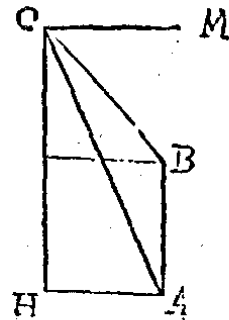


設 AB 為二物. s, s' 為二次測量之處。

$$\begin{aligned} \frac{S'A}{S S'} &= \frac{\sin A S S'}{\sin S' A S}. \\ \log S S' &= 0.69897 \\ \log \sin A S S' &= 9.93753 \\ \text{colog } \sin S' A S &= 0.01506 \\ \log S' A &= 0.65156 \\ \frac{A B}{S' A} &= \frac{\sin B S' A}{\sin S' B A}. \\ \log S' A &= 0.65156 \\ \log \sin B S' A &= 9.84949 \\ \text{colog } \sin S' B A &= 0.30103 \\ \log A B &= 0.80208 \\ A B &= 6.3399. \end{aligned}$$

即自 A 至 B 之距為 6.3399 哩。

3. 一堡與一碑。在同一水平面內。自堡頂望之。得碑尖與碑底之俯角為 40° 與 80° 但堡高為 140 呎。求碑高。



HC = 礮臺之高.

AB = 功牌之高.

$$\angle MCB = 40^\circ.$$

$$\angle HCA = 10^\circ.$$

$$\angle HAC = 80^\circ.$$

$$HC = 140 \text{ 呎.}$$

$$AC = \frac{140}{\sin A}.$$

$$\log 140 = 2.14613$$

$$\text{colog } \sin A = \underline{0.00665}$$

$$\log AC = 2.15278$$

$$\angle HCA = 10^\circ.$$

$$\angle MCB = 40^\circ.$$

故  $\angle AGB = 40^\circ.$

$$\angle CAB = 10^\circ.$$

而  $\angle ABC = 130^\circ.$

$$AB = \frac{AC \sin C}{\sin B}.$$

$$\log AC = 2.15278$$

$$\log \sin C = 9.80807$$

$$\text{colog } \sin B = \underline{0.11575}$$

$$\log AB = 2.07660$$

$$AB = 119.29.$$

即 之高 = 119.29 呎

4. 設太陽之高度為  $60^\circ$  問一杖與水平綫當作何角。能令

其影在水平面上為最長。

必使竿與日之光線正交始能得之影。

設  $BC = \text{竿.}$

$AC = \text{地平面.}$

$$B = 90^\circ.$$

$$A = 60^\circ.$$

故  $C = 30^\circ.$

5. 設太陽之高度為  $30^\circ$  問 10 呎長有竿長之一杖在水平面上所照最長影之長。

設  $a = \text{與光線正交之竿.}$

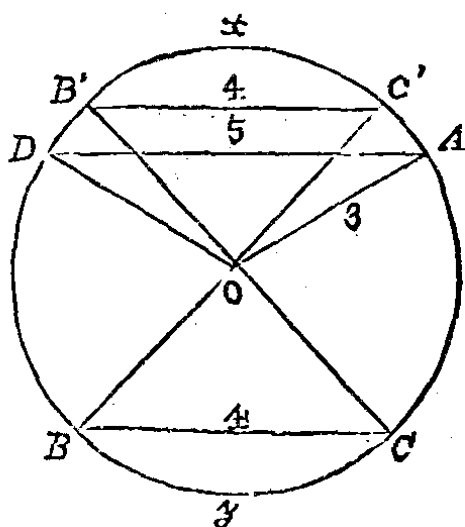
$c = \text{其最長之影.}$

$$\frac{a}{c} = \sin A = \frac{1}{2}.$$

$$c = 2 \quad a = 20.$$

即最長之影 = 20 呎.

6. 於半徑為 3 之一圓內。有長為 4 與 5 之平行兩弦。求此兩弦間所含之面積。(有兩解法)(有二解法)。



於 BOC 三角形中,

$$h = \sqrt{3^2 - 2^2}$$

$$= \sqrt{5}$$

$$F = \frac{1}{2} \times \sqrt{5} \times 4$$

$$= 2\sqrt{5}$$

$$= 4.4722$$

$$\sin \frac{1}{2} BOC = \frac{2}{3}$$

$$\log 2 = 0.30103$$

$$\text{colog } 3 = \underline{9.52288 - 10}$$

$$\log \sin \frac{1}{2} BOC$$

$$= 9.82391$$

$$\frac{1}{2} BOC = 41^\circ 48' 38''$$

$$BOC = 83^\circ 37' 16''$$

$$\text{圓面積} = \pi \times 3^2$$

$$\log \pi = 0.49715$$

$$\log 3^2 = \underline{0.95424}$$

$$\log (\text{圓面積}) = 1.45319$$

$$\text{圓面積} = 28.275$$

BOC 圓分之面積

$$= \frac{83^\circ 37' 16''}{360^\circ}$$

$$\times 28.274$$

$$= \frac{301036}{1296000}$$

$$\times 28.274$$

$$= \frac{15259}{324000}$$

$$\times 28.274$$

$$\log 75259 = 4.87656$$

$$\log 28.274 = 1.45139$$

$$\text{colog } 324000 = \underline{4.48945 - 10}$$

$$\log (\text{圓分面積}) = 0.81740$$

BOC 圓分之面積

$$= 6.5675$$

ByC 弧分之面積

$$= 6.5675 - 4.4722$$

$$= 2.0953$$

於 DOA 三角形中

$$h = \sqrt{3^2 - 2.5^2}$$

$$= 1.6583$$

$$F = \frac{1}{2} \times 1.6583 \times 5$$

$$= 4.1458$$

$$\sin \frac{1}{2} DOA = \frac{5}{6}$$

$$\log 5 = 0.69897$$

$$\text{colog } 6 = \underline{9.22185 - 10}$$

$$\log \sin \frac{1}{2}DOA$$

$$= 9.9282$$

$$\frac{1}{2}DOA = 56^\circ 26' 33''.$$

$$DOA = 112^\circ 53' 7''.$$

DOA圓分之面積

$$= \frac{112^\circ 53' 7''}{360^\circ}$$

$$\times 28.274$$

$$= \frac{406387}{1296000}$$

$$\times 28.274$$

$$\log 406387 = 5.60894$$

$$\log 28.274 = 1.45139$$

$$\text{colog } 129600 = \underline{3.88739 - 10}$$

$\log$  (圓分面積)

$$= 0.94772$$

$$\text{圓分面積} = 8.8658.$$

D x A弧分之面積

$$= 8.8658 - 4.1458$$

$$= 4.72.$$

得 DACB之面積

$$= \text{圓面積} - (By C + DxA)$$

$$= 28.274 - (2.0953 + 4.72)$$

$$= 21.4587.$$

得 DAÇ' B'之面積

$$= Dx A - B' x C'$$

$$= 4.72 - 2.0953$$

$$= 2.6247.$$

### 例題 XXIII

1. 塔之仰角爲  $48^\circ 19' 14''$ 。而其底距測點 95 呎。求塔高及頂與測點之距。

$$\text{已知 } A = 48^\circ 19' 14''.$$

$$b = 95 \text{ 呎.}$$

求 a 與 c.

$$a = b \tan A.$$

$$c = b \sec A.$$

$$\log b = 1.9772$$

$$\log \tan A = \underline{10.05045}$$

$$\log a = 2.02817$$

$$a = 106.70.$$

$$\log b = 1.97772$$

$$\log \sec A = \underline{0.17720}$$

$$\log c = 2.15492$$

$$c = 142.86.$$

即 塔之高 = 106.70 呎。自測處至塔頂之距 = 142.86 呎。

2. 自 1000 呎高之山上。測得一船之俯角爲  $77^\circ 35' 11''$ 。求船與山頂之距。

已知  $B=12^{\circ}24'49''$ ,  
 $a=100$  英尺, 求  $c$ .

$$c=a \sec B.$$

$$\log a=3.00000$$

$$\log \sec B=\underline{0.01027}$$

$$\log c=3.01027$$

$$c=1023.9.$$

即所求之距 = 1023.9 呎。

3. 90 呎高之旗竿。於水平面上照影 117 呎。求太陽之高度。

已知  $a=90$  呎,  $b=117$  呎, 求  $A$ .

$$\tan A=\frac{a}{b}.$$

$$\log a=1.95424$$

$$\operatorname{colog} b=\underline{7.93181-10}$$

$$\log \tan A=9.88605$$

$$A=37^{\circ}34'5''.$$

即 太陽之高 =  $37^{\circ}34'5''$ .

4. 設太陰之光落於地上某點。太陰對於通過某點之地球半徑。其所對之角, 為  $57,3''$ 。其地球半徑為 3956.2 哩。求太陰與地心之距。

設  $C$  = 任一處,  $A$  = 月,  
 $B$  = 地心。

於  $ABC$  正三角形中, 已知  $A=57'3''$ ,  $a=39562$  哩, 求  $c$ .

$$c=a \csc A.$$

$$\log a=3.59728$$

$$\log \csc A=\underline{1.78004}$$

$$\log c=5.37732$$

$$c=238,410.$$

即 月之遠 = 238,410 哩。

5. 太陽半徑所對之地心角為  $16.2''$ 。而太陽距為 9240000 哩。求太陽之直徑。

設  $A$  = 地心,  $B$  = 日心,

$C$  = 日輪邊上之一点。

於  $ABC$  正三角形中, 已知  $A=16'2''$ ,  $c=92,400,000$  哩, 求  $2a$ .

$$a=c \sin A.$$

$$\log c=7.96567$$

$$\log \sin A=\underline{7.66874}$$

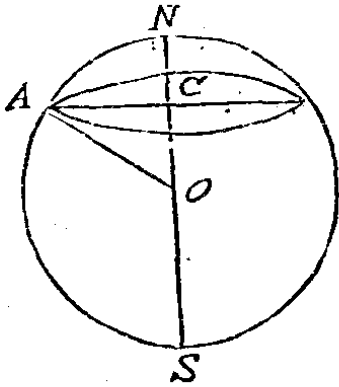
$$\log a=5.63441$$

$$a=430,930.$$

$$2a=861,860.$$

即 日徑 = 861,860 哩。

6. 開姆白利其之緯度為 $42^{\circ} 22' 49''$ 。求此緯度平行綫之半徑長。



設  $O$  = 地心,  $NS$  = 地軸,  
 $NAS$  = 堪見力智之子午綫,  
 $A$  = 堪見力智之地位,  
 $C$  = 緯度圈之中心.

於  $OAC$  正三角形中, 已知  $O = 90^{\circ} - 42^{\circ} 22' 49'' = 47^{\circ} 37' 11''$ ,  $OA = 3956.2$  哩.

求  $AC$ .

$$AC = AO \sin O,$$

$$\log AO = 3.59728$$

$$\log \sin O = 9.86846$$

$$\log AC = 3.45674$$

$$AC = 2922.4.$$

即緯度圈之半徑 = 2922.4 哩.

17. 問何處緯度平行綫之周。等於赤道周之半。

此緯度圈之半徑必等於地半徑之半.

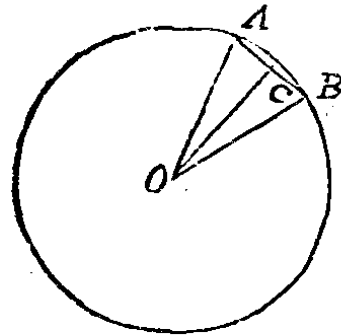
依第六題之圖, 已知  $AB = \frac{1}{2}AO$ , 求  $90^{\circ} - O$  角即  $A$  角.

$$\cos A = \frac{AC}{AO} = \frac{1}{2}.$$

故  $A = 60^{\circ}$ .

所求之緯度即  $60^{\circ}$ .

8. 於半徑 6.7 之圓內。接一正十三邊形。求每邊長。



設  $O$  = 外切切心,  
 $AB$  = 多邊形之一邊,  
 $C$  = 其中點.

於  $OCB$  正三角形中, 已知

$$O = \frac{360^{\circ}}{26} = 13^{\circ} 50' 46'',$$

$OB = 6.7$ . 求  $AB$  即  $2CB$ .

$$CB = OB \sin BOC.$$

$$\log OB = 0.82607$$

$$\log \sin BOC = 9.37897$$



$$\log CB = 0.20504$$

$$CB = 1.6034.$$

$$AB = 3.2068.$$

即 多邊形一邊之長  
= 3.2068.

9. 圓內接正七邊形之一邊  
為 5.73, 求圓半徑。

依第八題之圖, 已知  $BC$   
=  $\frac{1}{2} \times 5.73 = 2.856$ , 又  $BOC$

$$\text{角} = \frac{360^\circ}{14} = 25^\circ 42' 51''. \text{ 求}$$

$OB$ .

$$OB = BC \csc BOC.$$

$$\log BC = 0.45712$$

$$\log \csc BOC = \underline{0.36263}$$

$$\log OB = 0.81975$$

$$OB = 6.6031.$$

即圓半徑 = 6.6031.

10. 有 93.97 呎高之一塔在  
江邊。其對邊一物之俯角為  $25^\circ$   
 $12' 54''$ 。求江寬。

已知  $A = 90^\circ - 15^\circ 12' 54''$   
=  $64^\circ 47' 6''$ ,  $b = 93.97$ , 即  $a$ .

$$a = b \tan A.$$

$$\log b = 1.97299$$

$$\log \tan A = \underline{10.32708}$$

$$\log a = 2.30007.$$

$$a = 199.56.$$

即河之寬 = 199.56 呎。

11. 自 58 呎高之一塔上。測  
測得與塔底同平面內之二物之  
仰角為  $30^\circ 13' 18''$  及  $45^\circ 46'$   
 $14''$  求二物之距。

(1) 已知  $A = 90^\circ - 30^\circ$

$$13' 18'' = 59^\circ 46' 42'',$$

$$b = 58. \text{ 求 } a.$$

$$a = b \tan A.$$

$$\log b = 1.76343$$

$$\log \tan A = \underline{10.23469}$$

$$\log a = 1.99812$$

$$a = 99.568.$$

(2) 已知  $A' = 90^\circ - 45^\circ$

$$46' 14'' = 44' 13' 46'',$$

$$b = 58. \text{ 求 } a'.$$

$$a' = b \tan A'.$$

$$\log b = 1.76343$$

$$\log \tan A' = \underline{9.98832}$$

$$\log a' = 1.75175$$

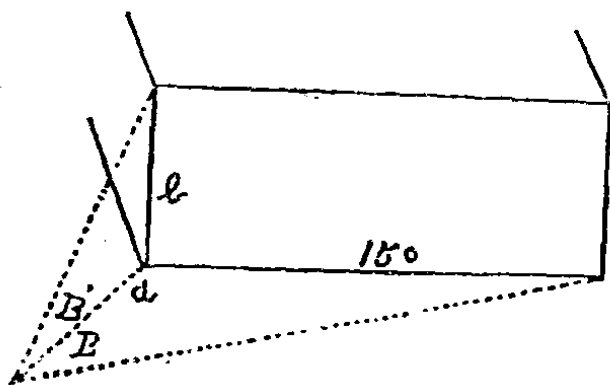
$$a' = 56.461.$$

$$a - a' = 43.107.$$

即兩物間之距 = 43.107 呎。

12. 一人直立於 105 呎長之

平頂屋一角之前。測得長所對之水平角之餘弦為 $\sqrt{\frac{1}{5}}$  及高所對之直立角之正弦為 $\frac{3}{\sqrt{34}}$ 。求屋高。



設  $a$  = 測量人與屋角之距。  
 $b$  = 屋之高。  
 $B$  = 其長所對之角。  
 $B'$  = 其高所對之角。

則  $a = 150 \cot B,$   
 $b = a \tan B'$   
 $= 150 \cot B \tan B$

但  $\cos B = \sqrt{\frac{1}{5}},$   
 故  $\sin B = \sqrt{1 - \frac{1}{5}}$   
 $= \frac{2}{\sqrt{5}}.$

而  $\cot B = \frac{\cos B}{\sin B} = \frac{1}{2}.$

又  $\sin B' = \frac{3}{\sqrt{34}}$

故  $\cos B' = \frac{5}{\sqrt{34}}.$

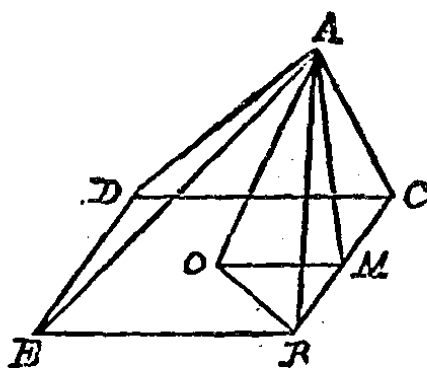
而  $\tan B' = \frac{\sin B'}{\cos B'} = \frac{3}{5}$

代回即得

$$b = 150 \times \frac{1}{2} \times \frac{3}{5} = 45.$$

即 屋之高 = 45呎,

13. 有底為正方形之正稜錐體。其旁稜長, 150呎。而底之一邊為 200 呎。求此面與底之倚度。



設  $A$  為稜錐之頂,  $BCDE$  為其底,  $O$  為其底之心,  $M$  為  $BC$  邊之中點, 求  $AMO$

角。於AOB正三角形中，

$$AB = 150,$$

$$OB = \frac{1}{2}BD = 100\sqrt{2}$$

$$\begin{aligned} \text{故 } AO &= \sqrt{AB^2 - OB^2} \\ &= 50. \end{aligned}$$

於 AOM正三角形中，

$$\begin{aligned} \tan OMA &= \frac{AO}{OM} = \frac{50}{100} \\ &= 0.5. \end{aligned}$$

$$OMA = 26^\circ 34'.$$

即 斜度 =  $26^\circ 34'$

14. 自36呎寬之溝之一邊。測對邊一牆之仰角得  $62^\circ 29' 10''$ 。設以梯自測點達至牆頂。需長幾何。

已知  $b = 36$ ,  $A = 62^\circ 29' 10''$ , 求  $c$ .

$$c = b \sec A.$$

$$\log b = 1.55630$$

$$\log \sec A = 0.33783$$

$$\log c = 1.89413$$

$$c = 78.367.$$

即 梯之長 = 78.367呎。

15. 旗杆折斷。一部分倒地。距杆根15呎。設其折斷部分為

39呎。求全杆之長。

已知  $c = 39$ ,  $b = 15$ , 求  $c + a$

$$a = \sqrt{(c+b)(c-b)}$$

$$= \sqrt{(39+15)(39-15)}$$

$$= \sqrt{54 \times 24}$$

$$= \sqrt{6^2 \times 9 \times 4}$$

$$= 36.$$

$$c + a = 39 + 36 = 75.$$

即 全竿之長 = 75呎

19. 自一城直上之輕氣球。

測得又一城之俯角為  $10^\circ 149''$ 。其兩城相離8哩。求氣球之高。

已知  $A = 90^\circ - 10^\circ 14' 9''$

$= 79^\circ 45' 51''$ ,  $a = 8$ . 求  $b$ .

$$b = a \cot A.$$

$$\log a = 0.90309$$

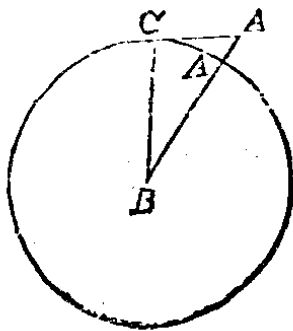
$$\log \cot A = 9.25666$$

$$\log b = 0.15975$$

即氣球之高 = 1.4446呎。

17. 自3哩高之山頂。測地

球上面可見之最遠物。得其仰角為  $2^\circ 13' 60''$  求地球之直徑。



設 A' 為山頂, C 為所測之物, B 為地心.

已知 B 角 =  $90^\circ - A = 2^\circ 13' 50''$ ,  $AD = 3$ . 求 a.

$$BC = AB \cos B.$$

$$a = (a + 3) \cos B.$$

$$\text{故 } a(1 - \cos B) = 3 \cos B.$$

$$a = \frac{3 \cos B}{1 - \cos B}.$$

$$\text{按 [16], } = \frac{3 \cos B}{2 \sin^2 \frac{B}{2}}.$$

$$\log \frac{3}{2} = 0.17609$$

$$\log \cos B = 9.99967$$

$$\text{colog } \sin^2 \frac{B}{2} = 3.42154$$

$$\log a = 3.59730$$

$$a = 3959.4.$$

$$2a = 7912.8.$$

即 地徑 = 7912.8 哩.

18. 40 呎長之梯。倚於街一

邊 33 呎高之窗。設移梯頂於街之對邊而不動梯根。則見其倚於 21 呎高之窗。求街之寬。

$$\text{街寬之一分} = \sqrt{40^2 - 33^2}$$

$$= \sqrt{511}$$

$$= 22.605.$$

$$\text{街寬之餘一分} = \sqrt{40^2 - 21^2}$$

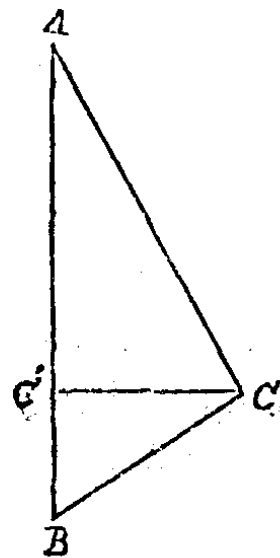
$$= \sqrt{1159}$$

$$= 34.044.$$

$$22.605 + 34.044 = 56.649.$$

即 全街之寬 = 56.649 呎.

19. 屋之高度。對一直角於其街之他邊之窗上。自同點測得屋頂之仰角為  $60^\circ$  而街寬為 30 呎。求屋之高。



已知  $CC' = 30$ ,  $\angle CC' = 60^\circ$ ,

$\angle BCC' = 30^\circ$ . 求  $AB$ .

$\angle CAB = 30^\circ$ .

故  $AC = 2 \times CC' = 60$ .

$$\cos A = \frac{AC}{AB}$$

$$AB = \frac{AC}{\cos A} = \frac{60}{\frac{1}{2}\sqrt{3}} = 40\sqrt{3} = 69.282.$$

即 屋之高 = 69.282 呎

20. 54 呎高之一燈臺。在磐石上。自一船測得燈臺及磐石之仰角。為  $4^\circ 52'$  與  $4^\circ 2'$ 。求石之高及船與石之距。

設  $h$  = 石之高。

$a$  = 石與舟之距。

$$\text{則 } \frac{h+54}{h} = \frac{\tan 4^\circ 52'}{\tan 4^\circ 2'}$$

$$1 + \frac{54}{h} = \frac{\tan 4^\circ 52'}{\tan 4^\circ 2'}$$

$$\frac{54}{h} = \frac{\tan 4^\circ 52' - \tan 4^\circ 2'}{\tan 4^\circ 2'}$$

$$h = 54 \frac{\tan 4^\circ 2'}{\tan 4^\circ 52' - \tan 4^\circ 2'}$$

$$= 54 \frac{\cos 4^\circ 52' \sin 4^\circ 2'}{\sin (4^\circ 52' - 4^\circ 2')}$$

$$= 54 \frac{\cos 4^\circ 52' \sin 4^\circ 2'}{\sin 50'}$$

$$\log 50 = 1.73239.$$

$$\log \cos 4^\circ 52' = 9.99843$$

$$\log \cos 4^\circ 2' = 8.84718$$

$$\text{colog } \sin 50' = 1.83732$$

$$\log h = 2.41532$$

$$h = 260.21.$$

$$\text{又 } a = h \cdot \cot 4^\circ 2'$$

$$\log h = 2.41532$$

$$\log \cot 4^\circ 2' = 11.15174$$

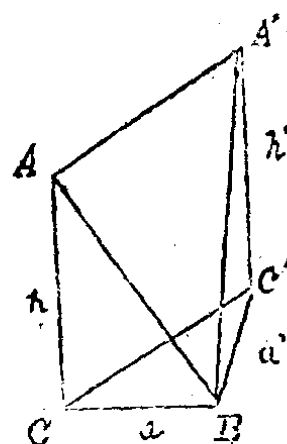
$$\log a = 3.56706$$

$$a = 3690.3.$$

即 石之高 = 260.21 呎

舟石之距 = 3690.3 呎

21. 一人在輕氣球測得地上在南一物之俯角  $35^\circ 30'$ 。其氣球向東平行  $2\frac{1}{2}$  哩。得同物之俯角  $23^\circ 14'$ 。求氣球之高。



設. A 與 A' 爲兩次測量之處  
C 與 C' 爲自 A A' 至地面之  
垂線底, B 爲所測之物.

$$\text{則 } A = 54^{\circ} 30',$$

$$A' = 66^{\circ} 46',$$

$$CC' = AA' = 2\frac{1}{2}.$$

$$a = h \tan A,$$

$$a' = h \tan A'.$$

$$a' - a = (2\frac{1}{2})^2.$$

$$h^2 \tan^2 A' - h^2 \tan^2 A = (2\frac{1}{2})^2,$$

$$h^2 = \frac{(2\frac{1}{2})^2}{\tan^2 A' - \tan^2 A}.$$

$$h = \frac{2\frac{1}{2}}{\sqrt{\tan^2 A' - \tan^2 A}}.$$

但  $\tan^2 A' - \tan^2 A$

$$= (\tan A' + \tan A)(\tan A' - \tan A)$$

$$= \frac{\sin(A' + A)}{\cos(A' + A)} \times \frac{\sin(A' - A)}{\cos(A' - A)}$$

$$= \frac{\sin(A' + A)\sin(A' - A)}{\cos^2 A' \cos^2 A}$$

$$\text{故 } h = \frac{2\frac{1}{2} \cos A' \cos A}{\sqrt{\sin(A' + A)\sin(A' - A)}}.$$

$$\log 2\frac{1}{2} = 0.39793$$

$$\log \cos A' = 9.59602$$

$$\log \cos A = 9.76395$$

$$\text{colog } \sqrt{\sin(A' + A)}$$

$$= 0.03408$$

$$\text{colog } \sqrt{\sin(A' - A)}$$

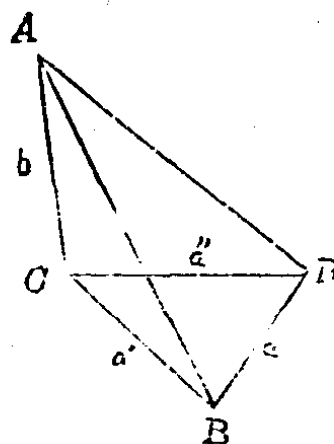
$$= 0.33636$$

$$\log h = 0.12835$$

$$h = 1.3438.$$

即 氣球之高 = 13438 哩.

22. 一人立於同一水平面上之一塔之南。測其仰角得  $54^{\circ} 16'$ 。後乃東行 100 碼。得仰角  $50^{\circ} 8'$ 。求塔高。



設 AC 爲樓, B 與 B' 爲二次  
測量之處

$$\text{則 } BB' = 100.$$

$$a' = b \cot A B C,$$

$$a'' = b \cot A B' C,$$

$$a'^2 - a'^2 = a^2.$$

$$b^2(\cot^2 AB'C - \cot^2 ABC) = 100^2.$$

$$b = \frac{100}{\sqrt{\cot^2 50^\circ 8' - \cot^2 54^\circ 16'}}$$

$$= \frac{100 \sin 54^\circ 16' \sin 50^\circ 8'}{\sqrt{\sin 104^\circ 24' \sin 4^\circ 8'}}$$

$$\log 100 = 2.00000$$

$$\log \sin 54^\circ 16' = 9.90942$$

$$\log \sin 50^\circ 8' = 9.88510$$

$$\text{colog} \sqrt{\sin 104^\circ 24'}$$

$$= 0.00693$$

$$\text{colog} \sqrt{\sin 4^\circ 8'}$$

$$= 0.57110 \quad 0.57110$$

$$\log b = 2.37255$$

$$b = 235.81.$$

即樓之高 = 235.81 碼。

23. 自塔南之某點 A。測得塔之仰角為  $30^\circ$ 。又自塔西相距為 a 之一點 B 測之。得仰角  $18^\circ$ 。

試證塔高為  $\frac{a}{\sqrt{2+2\sqrt{5}}}$ 。其  $18^\circ$

之正切為  $\frac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}$ 。

依前題之圖及所代之字，

$$b = \frac{a}{\sqrt{\cot^2 18^\circ - \cot^2 30^\circ}}$$

$$\text{但 } \cot^2 18^\circ = \frac{10+2\sqrt{5}}{6-2\sqrt{5}}$$

$$= \frac{(10+2\sqrt{5})(6+2\sqrt{5})}{6^2 - (2\sqrt{5})^2}$$

$$= 5+2\sqrt{5}.$$

$$\text{又 } \cot^2 30^\circ = 3.$$

$$\text{故 } b = \frac{a}{\sqrt{2+2\sqrt{5}}}.$$

24. 一竿定於堤頂。竿之仰角及俯角。為  $60^\circ$  及  $30^\circ$ 。試證竿長為堤高之二倍。

設  $l$  - 竿之長。

$h$  = 土堆之高。

$a$  = 測者與堆之距。

$$\text{則 } h = a \tan 30^\circ.$$

$$h + l = a \tan 60^\circ.$$

$$\frac{h+l}{h} = \frac{\tan 60^\circ}{\tan 30^\circ}$$

$$= \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = 3.$$

$$h+l = 3h.$$

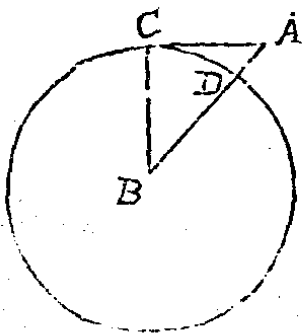
$$\text{故 } l = 2h.$$

25. 自與塔底相距為 a 之某點。測得塔頂之仰角  $A$ 。為其塔

頂上旗竿之仰角之餘角。試證竿長為  $2a \cot 2A$ 。

設  $h =$  樓之高。  
 $l =$  旗竿之長。  
 則  $h = a \tan A$ 。  
 $h + l = a \cot A$   
 $l = a (\cot A - \tan A)$   
 $= a \frac{\cot^2 A - 1}{\cot A}$   
 $= 2a \cot 2A$ 。

26. 真實的水平線者。線上各點與地心等距離者也。真實的水平線上任何點之切線。為可見的水平線。設於任何點作此二線。而延長至 1 哩。求他端之距離。



已知  $CA = 1$  哩,  $BC =$  地半徑  $= 3956.2$  哩。求  $AA' = AB - C$  B. 所求之距太小, 依常法甚難求得, 其最易之法如下,

$$\overline{AC}^2 = \overline{AB}^2 - \overline{BC}^2$$

$$= (AB - BC)(AB + BC).$$

故  $AB - BC = \frac{\overline{AC}^2}{AB + BC}$

今  $AB$  臺  $BC$  相差極微。若與  $BC^2$  相比則二者皆甚大, 故可取其略近之數, 設  $AB = BC$ 。

則  $AA' = AB - BC$   
 $= \frac{\overline{AC}^2}{2 BC}$   
 $= \frac{1}{7912.4}$  哩  
 $= \frac{5280 \times 12}{6912.4}$  吋。

$\log 5280 = 3.72263$

$\log 12 = 1.07918$

$\text{colog } 7912.4 = 6.10169 - 10$

$\log AA' = 0.90350$

$AA' = 8.0076$  吋。

即 所求之距  $= 8.0076$  吋

27. 於本例題之問題 1。問計算仰角或距離之錯誤。對於塔高有何影響。

依第一題所代之字, 設角



度之錯爲  $c_1$ , 又其距之錯爲  $c_2$ ,

則  $a = b \tan A$  變爲  
 $a = (b + c_2) \tan(A + c_1).$

而  $c = b \sec A$  變爲  
 $c = (b + c_2) \sec(A + c_1).$

今將  $a$  之答數所差之數演述於下,

$$\begin{aligned} (b + c_2) \tan(A + e_1) - b \tan A &= b \{ \tan(A + c_1) \\ &= \tan A \} \\ &+ e_2 \tan(A + e_1) \\ &= \frac{b \sin c_1}{\cos(A + c_1) \cos A} \\ &+ e_2 \tan(A + e_1). \end{aligned}$$

其略近數爲  $\frac{bc_1}{\cos^2 A} + c_2 \tan A,$

$e_1$  卽徑率之數.

今將  $c$  之答數所差之數演述於下,

$$\begin{aligned} (b + e_2) \sec(A + c_1) - b \sec A &= b \{ \sec(A + e_1) \\ &- \sec A \} \\ &+ e_2 \sec(A + e_1). \end{aligned}$$

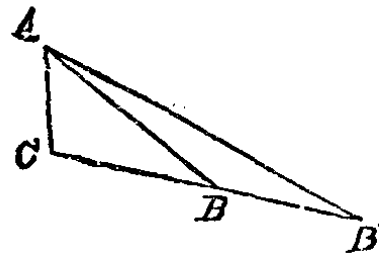
$$\begin{aligned} &= \frac{b \{ \cos A - \cos(A + c_1) \}}{\cos(A + e_1) \cos A} \\ &+ e_2 \sec(A + e_1) \end{aligned}$$

按[23],  $= \frac{2b \sin(A + \frac{1}{2}e_1)}{\cos(A + c_1) \cos A} \sin(\frac{1}{2}c_1) + c_2 \sec(A + c_1).$

其略近數爲

$$\begin{aligned} &\frac{be \sin A}{\cos^2 A} + e_2 \sec A \\ &= (be_1 \tan A + e_2) \sec A. \end{aligned}$$

28 由不可即近之某物之底。在同直綫內之兩點。測得此物之仰角及兩點距。試決定此物之高。



設  $AC$  爲遠物,  $B$  與  $B'$  爲所擇之兩處.

已知  $B'$  角,  $ABB$  角, 與  $B B'$  邊. 求  $AC$ .

$$AB = B B' \frac{\sin B'}{\sin BAB'}$$

$$= BB' \frac{\sin B'}{\sin(ABC-B')}.$$

$$AC = AB \sin ABC$$

$$= BB' \frac{\sin B' \sin ABC}{\sin(ABC-B')}.$$

29. 水平線上一不可近之塔之仰角爲  $63^{\circ}26'$ 。又遠行距塔底500呎測之。得某頂之仰角爲  $32^{\circ}14'$ 。求塔高。

按前題所得之公式。

$$AC = 500 \frac{\sin 32^{\circ}14' \sin 63^{\circ}26'}{\sin(63^{\circ}26' - 32^{\circ}14')}$$

$$= 500 \frac{\sin 32^{\circ}14' \sin 63^{\circ}26'}{\sin 31^{\circ}12'}$$

$$\log 500 = 2.69897$$

$$\log \sin 32^{\circ}14'$$

$$= 9.72703$$

$$\text{colog} \sin 31^{\circ}12'$$

$$= 0.28565$$

$$\log AC = 2.66319$$

$$AC = 460.46.$$

即樓之高 = 460.46 呎。

30. 有一塔築在河邊。自對邊測之。得其仰角  $60^{\circ}13'$ 。再遠46呎測之。得仰角  $50^{\circ}19'$ 。求江之寬。

依第二十八題之圖，

$$CB = AB \cos ABC$$

$$= BB' \frac{\sin B' \cos ABC}{\sin(ABC-B')}$$

$$\text{故 } CB = 40 \frac{\sin 50^{\circ}19' \cos 60^{\circ}13'}{\sin 9^{\circ}54'}$$

$$\log 40 = 1.60206$$

$$\log \sin 50^{\circ}19'$$

$$= 9.88626$$

$$\log \cos 60^{\circ}13'$$

$$= 9.69611$$

$$\text{colog} \sin 9^{\circ}54'$$

$$= 0.76465$$

$$\log CB = 1.94908$$

$$CB = 88.936.$$

即河之寬 = 88.936 呎。

31. 向北進行之船。見相距8哩之兩燈室於正西線內。歷一小時。其一臺之方位在南西。而又一臺在南南西。求此船每小時之速率。

依第二十八題之圖，設B與B'爲二燈塔，C爲第一次舟之地位，A爲第二次舟之地位。

$$\text{已知 } \angle CAB = 22^{\circ}30', \angle CAB'$$

$=45^\circ$ . 故  $\angle ABC + 67^\circ 30'$ ,

$B' = 45^\circ$ . 求  $AC$ .

按第二十八題,

$$AC = \frac{\sin 45^\circ \sin 67^\circ 30'}{\sin 22^\circ 30'}$$

$$= 8 \sin 45^\circ \cot 22^\circ 30'.$$

$$\log 8 = 0.90309$$

$$\log \sin 45^\circ$$

$$= 9.84949$$

$$\log \cot 22^\circ 30'$$

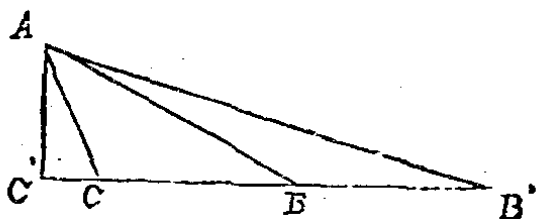
$$= 10.38278$$

$$\log AC = 1.13536$$

$$AC = 13.657$$

即 舟之速率每小時 13.657 哩.

32. 試決定斜面一個可近物之高。



設  $CBB'$  爲斜面,  $AC$  爲物,  $B$  與  $B'$  爲測量之點,  $AC'$  爲自  $A$  至  $CBB'$  之垂線.

已知  $CB$ ,  $BB'$ ,  $\angle ABC$  角,  $B'$  角. 求  $AC$ .

按第二十八題之公式.

$$AC' = BB' \frac{\sin B' \sin \angle ABC}{\sin (\angle ABC - B')}.$$

$$\text{又 } C'B = BB' \frac{\sin B' \cos \angle ABC}{\sin (\angle ABC - B')}.$$

但  $C'C = C'B - CB$ ,

$$\text{故 } AC = \sqrt{AC'^2 + C'C^2}.$$

33. 距斜面塔底 40 呎之處。塔身所對之角爲  $41^\circ 19'$  再遠 60 呎。塔身所對之角爲  $23^\circ 45'$ 。求塔之高。

按第三十二題之解法,

$$AC' = 60 \frac{\sin 23^\circ 45' \sin 41^\circ 19'}{\sin 17^\circ 34'}$$

$$C'B = 60 \frac{\sin 22^\circ 45' \cos 41^\circ 49'}{\sin 17^\circ 34'}.$$

$$\log 60 = 1.77815$$

$$\log \sin 23^\circ 45' = 9.60503$$

$$\log \sin 41^\circ 19' = 9.81969$$

$$\text{colog } \sin 17^\circ 34'$$

$$= 0.52026$$

$$\log AC' = 1.72313$$

$$AC' = 52.860.$$

$$\log \sin 23^\circ 45' = 9.60503$$

$$\log \cos 41^\circ 19' = 9.87568$$

$$\text{colog } \sin 17^\circ 34'$$

$$= 0.52026$$

$$\log C'B = 1.77912$$

$$C'B = 60.134.$$

$$C'C = 60.134 - 40$$

$$= 20.134.$$

$$\tan ACC' = \frac{AC'}{C'C}.$$

$$\log AC' = 1.72313$$

$$\text{colog } C'C = 8.69607 - 10$$

$$\log \tan ACC' = 0.41920$$

$$ACC' = 69^\circ 8' 55''.$$

$$AC = AC' \csc ACC',$$

$$\log AC' = 1.72313$$

$$\log \csc ACC' = 0.02941$$

$$\log AC = 1.75254$$

$$AC = 56.564.$$

即 樓之高 = 56.564 呎。

34. 一塔與其所在之斜面成  $113^\circ 12'$  之角又向下距塔成 89 呎之地。得塔身所對之角為  $23^\circ 27'$ 。求塔高。

於  $ACB$  三角形中，已知  $CB = 89$  呎，  $C = 113^\circ 12'$ ，  $B = 23^\circ 27'$ ，求  $AC$ 。（見三十二題之圖）。

$$A = 180^\circ - (B + C)$$

$$= 43^\circ 21'.$$

$$AC = CB \frac{\sin B}{\sin A}.$$

$$\log 89 = 1.94939$$

$$\log \sin 23^\circ 27' = 9.59983$$

$$\text{colog } \sin 43^\circ 21'$$

$$= 0.16339$$

$$\log AC = 1.71261$$

$$AC = 51.595.$$

即 樓之高 = 51.595 呎。

35. 自 42 尺高之屋之頂及底。測得竿尖之仰角為  $14^\circ 13'$  及  $23^\circ 19'$ 。求竿高。

設  $A$  為竿頂，  $B$  為屋頂，  $B'$  為屋底，  $C$  為自  $A$  至  $B$  之垂線底。求  $B'C$ 。

依 (28) 與 (30) 二題之式，

$$CB = BB' \frac{\sin AB'C \cos ABC}{\sin (ABC - AB'C)}$$

$$= 42 \frac{\sin 66^\circ 41' \cos 75^\circ 47'}{\sin 9^\circ 6'}.$$

$$\log 42 = 1.62325$$

$$\log \sin 66^\circ 41' = 9.96300$$

$$\text{colog } \sin 9^\circ 6'$$

$$= 0.80091$$

$$\log CB = 1.77737$$

$$CB = 59.892.$$

$$\begin{aligned} B'C &= CB + BB' \\ &= 59.892 + 42 \\ &= 101.892. \end{aligned}$$

即 竿之高 = 101.892 呎。

36. 三角形之三邊爲 17, 21 及 28。試證平分最大邊至其對角所作之綫之長爲 13。

設  $a=28$ ,  $b=21$ ,  $c=17$ ,

則按公式 [26],

$$17^2 = 28^2 + 21^2 - 2 \times 28 \times 21$$

$\cos C$ . 求證

$$13^2 = 14^2 + 21^2 - 2 \times 14 \times 21$$

$\cos C$ .

將第二式倍之, 減去第一式, 得  $2 \times 13^2 - 17^2 = 2 \times 14^2 - 28^2 + 21^2 = 21^2 - 2 \times 14^2 = 7^2(3^2 - 2^3) = 7^2$ .

$$2 \times 169 - 289 = 49,$$

$$49 = 49.$$

此式既不錯, 則中綫之爲 13 亦不錯。

37. 一探船在港之南西。見一敵船由港駛行。其方向爲南  $80^\circ$  東。而速率爲每小時 9 哩。問此探船須行何方向及何速率始能於  $1\frac{1}{2}$  小時內捕獲敵船。

設 A 爲港, B 爲捕敵之艦停泊處, C 爲追及敵船之處。

已知 A 角 =  $125^\circ$ ,  $b = 13\frac{1}{2}$ .

$C = 10$ , 求 B 與  $\frac{a}{1\frac{1}{2}}$ .

$$\tan \frac{1}{2}(B - C) = \frac{b - c}{b + c} \tan \frac{1}{2}(B + C)$$

$$= \frac{3.5}{23.5} \tan 27^\circ 30'$$

$$= \frac{7}{47} \tan 27^\circ 30'.$$

$$\log 7 = 0.84510$$

$$\text{colog } 47 = 8.32790 - 10$$

$$\log \tan 27^\circ 30' = 9.71648$$

$$\log \tan \frac{1}{2}(BC) = 8.88948$$

$$\frac{1}{2}(B - C) = 4^\circ 26'.$$

$$B - C = 8^\circ 52'.$$

$$B + C = 55^\circ.$$

故  $B = 31^\circ 56'$ .

$$a = \frac{b \sin A}{\sin B}$$

$$= \frac{13.5 \sin 125}{\sin 31^\circ 56'}.$$

$$\log 13.5 = 1.13033$$

$$\log \sin 125^\circ = 9.91336$$

$$\text{colog } \sin 31^\circ 56'$$

$$=0.27660$$

$$\log a = 1.32029$$

$$a = 20.907.$$

$$\frac{a}{1\frac{1}{2}} = 13.938$$

即得此艦之方向當為東北偏東  $31^{\circ}56'$ ，即北偏東  $76^{\circ}56'$ 。其速率當為每小時  $13.938$  哩。

38. 一人自江邊上行斜坡  $70$  碼。測得對岸一物之俯角為  $2\frac{1}{4}^{\circ}$ 。此斜坡之底。距江邊與高若  $3\frac{1}{2}$  與  $1$ 。求江寬。

設  $A$  與  $B$  為該人兩次所立之地位， $C$  為河之對岸。

已知  $c = 70$ ， $c$  角 =  $2\frac{1}{4}^{\circ}$ 。

$$A = 180^{\circ} - \tan^{-1} \frac{1}{3\frac{1}{2}} \text{ 求 } b.$$

$$A = 180^{\circ} - \tan^{-1} \frac{2}{7} \\ = 180^{\circ} - \tan^{-1} 0.$$

$$2857 = 180^{\circ} - 15^{\circ} 56'$$

$$40'' = 164^{\circ} 3' 20''.$$

$$B = 180^{\circ} - (A + C)$$

$$= 13^{\circ} 41' 40''.$$

$$b = \frac{c \sin B}{\sin C}$$

$$\log 70 = 1.84510$$

$$\log \sin 13^{\circ} 41' 40''$$

$$= 9.37428$$

$$\operatorname{colog} \sin 2^{\circ} 15' = 1.40605$$

$$\log b = 2.62543$$

$$b = 422.11.$$

即 河之寬 =  $42.211$  碼。

39. 在某點由湖長所對之角為  $46^{\circ}24'$ 。而此點至湖之兩端。為  $346$  呎與  $290$  呎。求湖長。

已知  $A = 46^{\circ}24'$ ， $b = 346$ ， $c = 290$ 。求  $a$ 。

$$\tan \frac{1}{2}(B - C) \frac{b - c}{b + c} \tan \frac{1}{2}(B + C) \\ = \frac{56}{636} \tan 66^{\circ} 48'$$

$$\log 56 = 1.74819$$

$$\operatorname{colog} 635 = 7.19654 - 10$$

$$\log \tan 66^{\circ} 48' = 10.36795$$

$$\log \tan \frac{1}{2}(B - C) \\ = 9.31268.$$

$$\frac{1}{2}(B - C) = 11^{\circ} 36' 33''.$$

$$B - C = 23^{\circ} 13' 6''$$

$$B + C = 133^{\circ} 36'.$$

$$\text{即 } B = 78^{\circ} 24' 33''.$$

$$a = \frac{b \sin A}{\sin B}$$

$$= \frac{346 \sin 46^{\circ} 13'}{\sin 78^{\circ} 24' 33''}$$

$$\begin{aligned} \log 346 &= 2.53908 \\ \log \sin 46^{\circ} 24' &= 9.85984 \\ \text{colog } \sin 78^{\circ} 24' 33'' & \\ &= \underline{0.00895} \\ \log a &= 2.40787 \\ a &= 255.78. \\ \text{即 湖之長} &= 255.78 \text{ 呎.} \end{aligned}$$

40. 二船相距一哩。自此船測得彼船與岸上礮臺之角距為  $35^{\circ} 14' 10''$ 。自彼船測得此船與礮臺之角距為  $42^{\circ} 11' 53''$ 。求兩船與礮臺之各距。以呎數表之

$$\begin{aligned} \text{已知 } B &= 35^{\circ} 14' 20'', \\ C &= 42^{\circ} \end{aligned}$$

$$\begin{aligned} 11' 53'', a &= 5280. \text{ 求 } b \text{ 與 } c. \\ A &= 180^{\circ} - (B + C) \\ &= 102^{\circ} 33' 57''. \end{aligned}$$

$$b = \frac{a \sin B}{\sin A}$$

$$\begin{aligned} \log 5280 &= 3.72263 \\ \log \sin 35^{\circ} 14' 10'' & \\ &= 9.76114 \\ \text{colog } \sin 102^{\circ} 33' 57'' & \\ &= \underline{0.01053} \\ \log b &= 3.49430 \end{aligned}$$

$$\begin{aligned} b &= 3121.1. \\ c &= \frac{a \sin C}{\sin A} \\ \log 5280 &= 3.72263 \\ \log \sin 42^{\circ} 11' 53'' & \\ &= 9.82717 \\ \text{colog } \sin 102^{\circ} 33' 57'' & \\ &= \underline{0.01052} \\ \log c &= 3.56033 \\ c &= 3633.5 \\ \text{即 此船與砲臺之距} & \\ &= 3121.1 \text{ 呎.} \\ \text{彼船與砲臺之距} & \\ &= 3633.5 \text{ 呎.} \end{aligned}$$

41. 沿江邊作500呎之基綫。自綫之此端。測得彼端與對邊一物之角距為  $53^{\circ}$ 。又自彼端測得此端與同物之角距為  $79^{\circ} 12'$ 。求江寬。

$$\begin{aligned} \text{已知 } B &= 53^{\circ}, C = 79^{\circ} 12', \\ a &= 500. \text{ 求 } p, \text{ 即自 } A \text{ 至 } a \text{ 之} \\ &\text{垂線.} \end{aligned}$$

$$\begin{aligned} A &= 180^{\circ} - (B + C) \\ &= 47^{\circ} 48'. \\ b &= \frac{a \sin B}{\sin A}. \end{aligned}$$

$$p = b \sin C$$

$$= a \frac{\sin E \sin C}{\sin A}$$

$$= 500 \frac{\sin 53^\circ \sin 79^\circ 12'}{\sin 47^\circ 48'}$$

$$\log 500 = 2.69897$$

$$\log \sin 53^\circ = 9.90235$$

$$\log \sin 79^\circ 12' = 9.99224$$

$$\text{colog} \sin 47^\circ 48'$$

$$= \underline{0.13030}$$

$$\log p = 2.72386$$

$$y = 529.49.$$

即 江之寬 = 529.49 呎。

42. 與地平傾斜  $15^\circ$  之斜面上。直立一塔。一人自塔底上行斜面 80 呎。得塔身所對之角  $30^\circ$ 。求塔高。

設 A 與 B 爲塔之頂與底，C 爲測量之點。

已知  $a = 80$ ,  $B = 75^\circ$ ,

$C = 20^\circ$  求 c.

$$A = 180^\circ - (B + C)$$

$$= 75^\circ$$

$$C = \frac{a \sin A}{\sin B}$$

$$= \frac{80 \sin 30^\circ}{\sin 65^\circ}$$

$$\log 80 = 1.90309$$

$$\log \sin 30^\circ = 9.69897$$

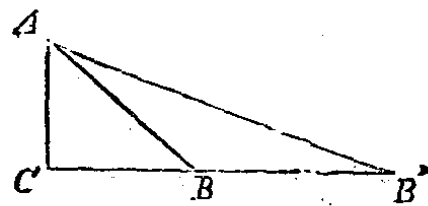
$$\text{colog} \sin 75^\circ = \underline{0.01506}$$

$$\log c = 1.61712$$

$$c = 41.411.$$

即 塔之高 = 41.411 呎。

43. 在某點由斜面上。一塔所對之角爲  $42^\circ 17'$ 。向下遠行 325 呎。得  $21^\circ 47'$ 。而面之傾斜爲  $8^\circ 53'$  求塔高。



$$AB = B B' \frac{\sin B'}{\sin (B - B')}$$

$$= B B' \frac{\sin B'}{\sin (B - B')}$$

$$AC = \frac{A B \sin B}{\sin C}$$

$$= B B' \frac{\sin B \sin B'}{\sin C \sin (B - B')}$$

$$= 325 \frac{\sin 42^\circ 17' \sin 21^\circ 47'}{\sin 98^\circ 53' \sin 20^\circ 30'}$$

$$\log 325 = 2.51188$$



$$\log \sin 42^{\circ} 17' = 9.82788$$

$$\log \sin 21^{\circ} 47' = 9.56949$$

$$\text{colog } \sin 98^{\circ} 53'$$

$$= 0.00524$$

$$\text{colog } \sin 20^{\circ} 30'$$

$$= 0.45567$$

$$\log AC = 2.37016$$

$$AC = 234.51.$$

即 塔之高 = 234.51 英尺。

44. 自船望見一土角在北微東。船向西北行30哩。則土角在正東。求此角與第二測點相距。

設A爲土角, B與C爲兩次測量之處。

已知  $B = 56^{\circ} 15'$ ,  $C = 45^{\circ}$ ,

$a = 30$ . 求  $b$ .

$$A = 180^{\circ} - (B + C)$$

$$= 78^{\circ} 45'.$$

$$b = \frac{a \sin B}{\sin A}$$

$$= \frac{30 \sin 56^{\circ} 15'}{\sin 78^{\circ} 45'}.$$

$$\log 30 = 1.47712$$

$$\log \sin 56^{\circ} 15' = 9.91985$$

$$\text{colog } \sin 78^{\circ} 45'$$

$$= 0.00843$$

$$\log b = 1.40540$$

$$b = 25.433.$$

第二次測量處距角 25.433 哩。

45. 兩測者相對立於雲之兩旁。相距700呎。測得仰角爲  $44^{\circ} 56'$  及  $36^{\circ} 4'$ 。求雲高。

已知  $A = 44^{\circ} 56'$ ,  $B = 36^{\circ} 4'$ ,  $C = 700$ . 求自 C 至 c 之垂線 p.

$$C = 180^{\circ} - (B + A) = 99^{\circ}.$$

$$b = \frac{c \sin B}{\sin C}.$$

$$p = b \sin A = \frac{c \sin B \sin A}{\sin C}$$

$$= \frac{700 \sin 36^{\circ} 4' \sin 44^{\circ} 56'}{\sin 99^{\circ}}.$$

$$\log 700 = 2.84510$$

$$\log \sin 36^{\circ} 4' = 9.76991$$

$$\log \sin 44^{\circ} 56' = 9.84898$$

$$\text{colog } \sin 99^{\circ} = 0.00538$$

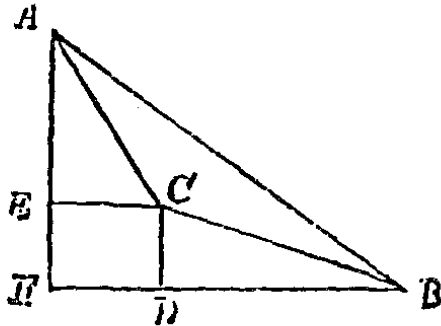
$$\log b = 2.46937$$

$$p = 295.09.$$

即 雲之高 = 294.69 呎。

46. 自山足之一點 B。得其 A 頂之仰角爲  $60^{\circ}$ 。再自與地平  $30^{\circ}$  傾斜之山路上行一哩。達於

C點。得  $\angle ACB$  角為  $135^\circ$ 。求以呎數表此山之高。



$$CD = CB \sin CBD$$

$$= 5280 \times \frac{1}{2} = 2640$$

$$AC = \frac{CB \sin CBA}{\sin CAB}$$

$$AE = AC \sin ECA$$

$$= \frac{CB \sin CBA \sin ECA}{\sin CAB}$$

$$= \frac{5280 \sin 30^\circ \sin 75^\circ}{\sin 15^\circ}$$

$$= \frac{5280 \times \frac{1}{2} \cos 15^\circ}{\sin 15^\circ}$$

$$= 2640 \cot 15^\circ.$$

$$\log 2640 = 3.42160$$

$$\log \cot 15^\circ = 10.57195$$

$$\log AE = 3.99355$$

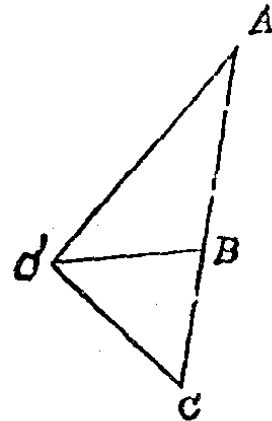
$$AE = 9852.6$$

$$AF = AE + CD$$

$$= 12,492.6.$$

即 山之高 = 12,492.6 呎。

47. 自船見二石與船在同一直線內。其方位為北  $15^\circ$  東。船向西北行 5 哩。則其第一石在直東。第二石在東北。求二石距。



設 A 與 B 為兩石，C 與 C' 為舟兩次之地位。

已知  $C = 60^\circ$ ,  $CC'B = 45^\circ$ ,  $CC'A = 90^\circ$ ,  $CC' = 5$ . 求 AB.

$$AC = CC' \sec C = 5 \times 2$$

$$= 10.$$

$$BC = CC' \frac{\sin BC'C}{\sin CBC'}$$

$$= 5 \frac{\sin 45^\circ}{\sin 70^\circ}.$$

$$\log 5 = 0.69897$$

$$\log \sin 45^\circ = 9.84949$$

$$\text{colog } \sin 70^\circ = 0.01506$$

$$\log BC = 0.56352$$

$$BC = 3.6603.$$

$$AB = AC - BC$$

$$= 6.3297.$$

即 二石相距 6.3397 哩。

48. 自與尖塔底同平面上之一窗。測得塔之仰角為  $40^\circ$  又自 18 呎高之窗測之。得仰角  $37^\circ 30'$ 。求此尖塔之高。

設 A 與 B 為兩窗, C 為塔頂。

已知  $c = 18$ ,  $A = 50^\circ$ ,  
 $B = 127^\circ 30'$ , 求塔高。

$$h = b \sin 40^\circ$$

$$C = 180^\circ - (A + B)$$

$$= 2^\circ 30'.$$

$$b - c = \frac{\sin B}{\sin C}$$

$$= 18 \frac{\sin 127^\circ 30'}{\sin 2^\circ 30'}.$$

$$\text{故 } h = 18 \frac{\sin 127^\circ 30' \sin 40^\circ}{\sin 2^\circ 30'}.$$

$$\log 18 = 1.25527$$

$$\log \sin 127^\circ 30'$$

$$= 9.89947$$

$$\log \sin 40^\circ = 9.80807$$

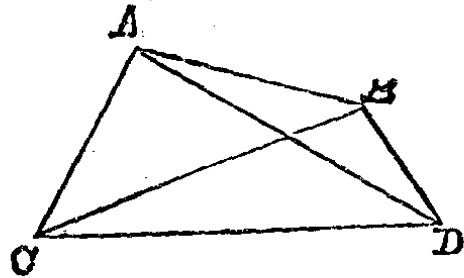
$$\text{cologsine } 2^\circ 30' = 1.36032$$

$$\log h = 2.32313$$

$$h = 210.44.$$

即 塔之高 = 210.44 呎。

49. 設測角於已知長之一線之兩端而決定不可近之兩物距。



設 A 與 B 為兩物, C 與 D 為已知線之兩端。

已知 CD,  $\angle ACD$  角,  $\angle BCD$  角,  $\angle ADC$  角,  $\angle BDC$  角. 求 AB.

$$AC = CD \frac{\sin \angle ADC}{\sin \angle CAD}.$$

$$BC = CD \frac{\sin \angle BDC}{\sin \angle CBD}.$$

今於  $\triangle CAB$  三角形內已知兩邊與其夾角, 則其第三邊可按常法求之。

50. 將以決定隔江禮拜堂 A 及一塔 B 之距離。沿江先測一線  $CD = 600$  呎 (C 大約與 A 相對)。又得  $\angle ACB = 58^\circ 20'$ ,  $\angle ACD = 95^\circ 20'$ ,  $\angle ADB = 50^\circ 33'$ ,  $\angle BDC = 98^\circ 45'$ 。問所求之距離。

按前題之解法.

$$AC = CD \frac{\sin ADC}{\sin CAD}$$

$$= 600 \frac{\sin 45^\circ 15'}{\sin 39^\circ 24'}$$

$$BC = CD \frac{\sin BDC}{\sin CBD}$$

$$= 900 \frac{\sin 88^\circ 45'}{\sin 44^\circ 15'}$$

$$\log 600 = 2.77815$$

$$\log \sin 45^\circ 15' = 9.85137$$

$$\text{colog} \sin 39^\circ 23'$$

$$= \underline{0.19726}$$

$$\log AC = 2.82678$$

$$AC = 671.09.$$

$$\log 900 = 2.95424$$

$$\log \sin 88^\circ 45' = 9.99492$$

$$\text{colog} \sin 44^\circ 15'$$

$$= \underline{0.15627}$$

$$\log BC = 2.92934$$

$$BC = 849.84.$$

$$\tan \frac{1}{2}(CAB - CBA)$$

$$= \frac{BC - AC}{BC + AC}$$

$$\tan \frac{1}{2}(CAB + CBA)$$

$$= \frac{178.75}{1520.93}$$

$$\tan 60^\circ 50'.$$

$$\log 178.751 = 2.25124$$

$$\text{colog} 1520.93 = 6.81789 - 10$$

$$\log \tan 60^\circ 50' = \underline{10.25327}$$

$$\log \tan \frac{1}{2}(CAB - CBA)$$

$$= 9.32340$$

$$\frac{1}{2}(CAB - CBA)$$

$$= 11^\circ 53' 28''.$$

$$\frac{1}{2}(CAB + CBA)$$

$$= \underline{60^\circ 55'}.$$

$$CBA = 72^\circ 43' 28''.$$

$$AB = BC \frac{\sin ACB}{\sin CAB}$$

$$= 849.84 \frac{\sin 58^\circ 20'}{\sin 72^\circ 43' 28''}$$

$$\log 849.84 = 2.92934$$

$$\log \sin 58^\circ 20' = 9.92999$$

$$\text{colog} \sin 72^\circ 43' 28''$$

$$= 0.02005$$

$$\log AB = 2.87938$$

$$AB = 757.50.$$

即 所求之距 = 757.50 呎.

51. 將求空中一點 A 之高度。先測一水平的基線 CD 440 碼。自 C 得 A 之仰角  $37^\circ 18'$ 。及 AD 間之水平角  $76^\circ 18'$ 。自 D

得AC間之水平角 $67^{\circ}14'$ 。求高。

設  $A'$  為自  $A$  至地面之垂線底，與  $CD$  同在一平面於  $A'CC$  三角形內

$$A'C = CD \frac{\sin C}{\sin A'}$$

$$= 440 \frac{\sin 67^{\circ}14'}{\sin 36^{\circ}28'}$$

$$A'A = A'C \tan \angle CA'$$

$$= 440 \frac{\sin 67^{\circ}14'}{\sin 36^{\circ}28'}$$

$$\tan 37^{\circ}18'$$

$$\log 440 = 2.64345$$

$$\log \sin 67^{\circ}14' = 9.96477$$

$$\log \tan 37^{\circ}18' = 9.88184$$

$$\text{colog} \sin 36^{\circ}28'$$

$$= 0.22595$$

$$\log A'A = 2.71601$$

$$A'A = 520.01.$$

即 所求高 = 520.01 碼。

52. 自相距 3000 呎之兩站測一氣球。自此站得球與彼站間之水平角  $75^{\circ}25'$ 。及球之仰角  $18^{\circ}$ 。自彼站得球與此站間之水平角  $64^{\circ}30'$ 。求氣球之高。

設  $B$  為此站， $C$  為彼站， $A$

為氣球所在之位， $A'$  為自球至地之垂線底與  $BC$  同在地平面上。則  $AA' = A'B \tan \angle A'BA$

$$= BC \frac{\sin \angle A'CB}{\sin \angle BA'C}$$

$$\tan \angle A'BA$$

$$= 3000 \frac{\sin 64^{\circ}30'}{\sin 40^{\circ}5'}$$

$$\tan 18^{\circ}.$$

$$\log 3000 = 3.47712$$

$$\log \sin 64^{\circ}30' = 9.95549$$

$$\log \tan 18^{\circ} = 9.51178$$

$$\text{colog} \sin 40^{\circ}5'$$

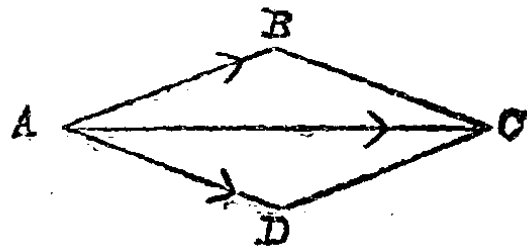
$$= 0.19118$$

$$\log AA' = 3.13557$$

$$AA' = 1366.4.$$

即 氣球之高 = 1366.4.

53. 有一為 410 磅一為 320 磅之二力。成  $51^{\circ}37'$  之角。求其結果之集力及方向。



設  $AB$  與  $AD$  為二力， $AC$  為

其集力。

則於 ABC 三角形中已知  
 $c=410$ ,  $a=320$ ,  $B=180^\circ$   
 $-51^\circ 37' = 128^\circ 23'$ . 求 B 與  
 A.

$$\begin{aligned} \tan \frac{1}{2}(C-A) &= \frac{c-a}{c+a} \tan \frac{1}{2} \\ &= \frac{90}{730} \tan 25^\circ 48' \\ & \quad 30''. \end{aligned}$$

$$\log 90 = 1.95424$$

$$\log \tan 25^\circ 48' 30''$$

$$= 9.68448$$

$$\operatorname{colog} 730 = \underline{7.13668 - 10}$$

$$\log \frac{1}{2}(C-A) = 8.77540$$

$$\frac{1}{2}(C-A) = 3^\circ 24' 43''.$$

$$\frac{1}{2}(C+A) = \underline{25^\circ 48' 30''}$$

$$A = 22^\circ 23' 47''.$$

$$b = a \frac{\sin B}{\sin A}$$

$$= 320 \frac{\sin 128^\circ 23'}{\sin 22^\circ 23' 47''}.$$

$$\log 320 = 2.50515$$

$$\log \sin 128^\circ 23'$$

$$= 9.89425$$

$$\operatorname{colog} \sin 22^\circ 23' 47''$$

$$= \underline{0.41906}$$

$$\log b = 2.81846$$

$$b = 658.39.$$

即 集力 = 658.36 磅, 集力  
 與第一力之偏度為  $22^\circ 23'$   
 $47''$ .

54. 一未知力。與 128 磅之一  
 力相併。得 200 磅之結果。而此  
 結果與已知力成  $18^\circ 24'$  之角。求  
 未知力之集力及方向。

依前題之圖, 於 ABC 三角  
 形中已知  $c=128$ ,  $A=18^\circ$   
 $24'$ ,  $b=200$ . 求 a 與 B.

$$\tan \frac{1}{2}(B-C)$$

$$= \frac{b-c}{b+c} \tan \frac{1}{2}$$

$$(B+C)$$

$$= \frac{72}{328} \tan 80^\circ 48'.$$

$$\log 72 = 1.84733$$

$$\log \tan 80^\circ 48' = 10.79058$$

$$\operatorname{colog} 328 = \underline{7.48413 - 10}$$

$$\log \tan \frac{1}{2}(B-C) = 10.13204$$

$$\frac{1}{2}(B - C) = 53^\circ 34' 44''$$

$$\frac{1}{2}(B + C) = 80^\circ 48'$$

$$B = 134^\circ 22' 44''.$$

$$180^\circ - B = 45^\circ 37' 16''.$$

$$a = \frac{b \sin A}{\sin B}$$

$$= 200 \frac{\sin 18^\circ 24'}{\sin 134^\circ 22' 44''}$$

$$\log 200 = 2.30103$$

$$\log \sin 18^\circ 24' = 9.49920$$

$$\text{coloo} \sin 134^\circ 22' 44''$$

$$= 0.14586$$

$$\log a = 1.94609$$

$$a = 88.326$$

即得 所求之力 = 88.326

磅, 兩原力所成之角為 45。

37' 16''.

55. 自兩站由風箏之高所乘之角同為 A。而自此站由風箏與彼站之聯線所乘之角為 B。其兩站距為 a。試顯風箏之高為  $\frac{1}{2} a \sin A \sec B$ 。

設 C 為風箏所在之點, D 與 E 為兩站 C' 為自 C 至地面之垂線底。

兩站測得之仰視角既等故

CDE 為等腰三角形, 即

$$CD = CE = \frac{1}{2} a \sec B.$$

$$\text{又 } CC' = CD \sin A$$

$$= \frac{1}{2} a \sin A \sec B.$$

56. 水平面上有二塔。相距 120 呎。一人連續立於其底。測得一塔之仰角。為又一塔仰角之二倍。但於兩塔間之中點測之。得二仰角互為餘角。求證二塔之高為 90 呎及 40 呎。

設 A 與 B 為兩塔之頂, A' 與 B' 為其底, 為兩塔中間之點。則 AA'C 與 BB'C 為相似三角形, 而

$$\frac{AA'}{B'C} = \frac{A'C}{BB'}$$

$$AA' \times BB' = B'C \times A'C$$

$$= 3600.$$

$$\text{又 } AB'A' = 2BA'B'.$$

$$\text{故 } \tan A'BA'$$

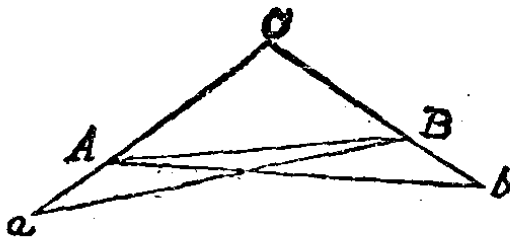
$$= \frac{2 \tan BA'B'}{1 - \tan^2 BA'B'}$$

$$\text{或 } \frac{AA'}{120} = \frac{2 \frac{BB'}{120}}{1 - \frac{BB'^2}{120^2}}$$

$$\begin{aligned}
 &= \frac{240 \overline{BB'}}{120^2 - \overline{BB'^2}} \\
 AA'(120^2 - \overline{BB'^2}) & \\
 &= 120 \times 240 \overline{BB'} \\
 \frac{3600}{\overline{BB'}} (120^2 - \overline{BB'^2}) & \\
 &= 120 \times 240 \overline{BB'} \\
 120^2 - \overline{BB'^2} &= 8\overline{BB'^2} \\
 \overline{BB'^2} &= 40^2 \\
 \overline{BB'} &= 40 \\
 AA' &= 90.
 \end{aligned}$$

即塔之高為 90 與 40 呎。

57. 設無測角器。而欲求 A、B 兩點與一不可近之 C 點間之距。延長 CA 至 a, CB 至 D 又聯 AB, Ab, Ba。測得 AB 500, aA100, aB56°, bB100, AB550。計算 AC 及 BC 兩距。



於 aAB 三角形中,

$$\begin{aligned}
 s &= \frac{1}{2}(500 + 100 + 560) \\
 &= 580,
 \end{aligned}$$

$$\tan \frac{1}{2}aAB = \sqrt{\frac{80 \times 480}{580 \times 20}} = \sqrt{\frac{26}{29}}$$

$$\log 96 = 1.98227$$

$$\text{colog } 29 = \frac{8.53760 - 10}{2(0.51987)}$$

$$2(0.51987)$$

$$\log \tan \frac{1}{2}aAB$$

$$= 10.25993$$

$$\frac{1}{2}aAB = 61^\circ 12' 20''.$$

$$aAB = 122^\circ 24' 40''.$$

$$CAB = 57^\circ 35' 20''.$$

於 bAB 三角形中,

$$S = \frac{1}{2}(500 + 550 + 100) = 575.$$

$$\tan \frac{1}{2}bBA = \sqrt{\frac{75 \times 475}{575 \times 25}}$$

$$= \sqrt{\frac{57}{23}}$$

$$\log 57 = 1.75587$$

$$\text{colog } 23 = \frac{8.63827 - 10}{2(0.39413)}$$

$$2(0.39413)$$

$$\log \tan \frac{1}{2}bBA$$

$$= 10.19707$$

$$\frac{1}{2}bBA = 57^\circ 34' 30''.$$

$$bBA = 115^\circ 9'.$$

$$CBA = 64^\circ 51'.$$

於 ABC 三角形中,



$$A = 57^{\circ}35'20''$$

$$B = 64^{\circ}51'$$

$$C = 57^{\circ}33'40''$$

$$BC = AB \frac{\sin A}{\sin C}$$

$$= 500 \frac{\sin 57^{\circ}35'20''}{\sin 57^{\circ}33'40''}$$

$$AC = AB \frac{\sin B}{\sin C}$$

$$= 500 \frac{\sin 64^{\circ}51'}{\sin 57^{\circ}33'40''}$$

$$\log 500 = 2.6097$$

$$\log \sin 57^{\circ}35'20''$$

$$= 9.92646$$

$$\text{colog} \sin 57^{\circ}33'40''$$

$$= 0.07368$$

$$\log BC = 2.69911$$

$$BC = 500.16$$

$$\log 500 = 2.69897$$

$$\log \sin 64^{\circ}51' = 9.95674$$

$$\text{colog} \sin 57^{\circ}33'40''$$

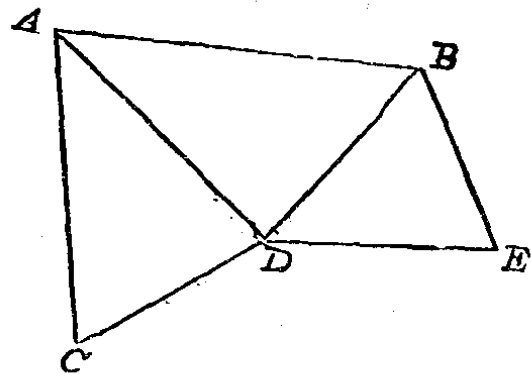
$$= 0.07368$$

$$\log AC = 2.72939$$

$$AC = 536.28$$

58. 有不可近之二物 A 及 B。於 D 點外。無他點可俱見

之。任取一點 C。可望見 A 及 D。測得 CD 290 呎，ADC 89°，ADC 5030。再取一點 E。可望見 D 及 B。得 DE 200 呎。BDE 54° 30'，BFD 88° 30'。自 D 得 ADB 72° 30'。計算 AB 距。



$$AD = CD \frac{\sin ACD}{\sin CAD}$$

$$= 290 \frac{\sin 50^{\circ}30'}{\sin 40^{\circ}30'}$$

$$\log 290 = 2.30103$$

$$\log \sin 50^{\circ}30' = 9.88741$$

$$\text{colog} \sin 40^{\circ}30'$$

$$= 0.18746$$

$$\log AD = 2.37590$$

$$AD = 237.63$$

$$BD = DE \frac{\sin BED}{\sin DBE}$$

$$= 200 \frac{\sin 88^{\circ}30'}{\sin 37'}$$

$$\log 200 = 2.30103$$

$$\log \sin 88^{\circ} 30' = 9.99985$$

$$\text{colog} \sin 37^{\circ} = 0.22054$$

$$\log B D = 2.52142$$

$$B D = 332.22.$$

$$\text{tah} \frac{1}{2}(D A B - D B A)$$

$$= \frac{B D - A D}{B D + A D}$$

$$\text{tan} \frac{1}{2}(D A B + D B A)$$

$$= \frac{94.59}{569.5} \text{tan} 53^{\circ} 45'$$

$$\log 94.59 = 1.97585$$

$$\text{colog} 56.985 = 7.24424$$

$$\log \text{tan} 53^{\circ} 45' = 10.13476$$

$$\log \text{tan} \frac{1}{2}(D A B - D B A)$$

$$= 9.35485$$

$$\frac{1}{2}(D A B - D B A)$$

$$= 12^{\circ} 45' 22''.$$

$$\frac{1}{2}(D A B + D B A)$$

$$= 53^{\circ} 35'$$

$$D A B = 66^{\circ} 30' 21''.$$

$$A B = B D \frac{\sin A D B}{\sin D A B}$$

$$= 332.22 \frac{\sin 72^{\circ} 30'}{\sin 66^{\circ} 33' 21''}.$$

$$\log 332.22 = 2.52142$$

$$\log \sin 72^{\circ} 30' = 9.97942$$

$$\text{colog} \sin 66^{\circ} 30' 21''$$

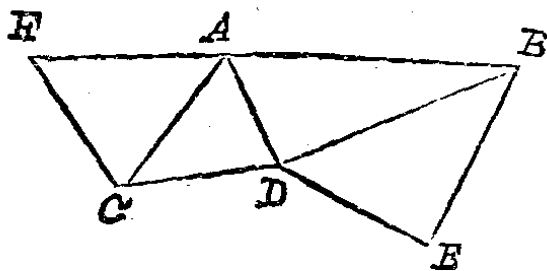
$$= 0.03758$$

$$\log A B = 2.53842$$

$$A B = 345.48'$$

即 A B 之距 = 345.48 呎

59. 有不可近之二物 A 及 B 無一點可俱見之。取兩點 C 及 D 相距 200 碼。今其由 C 可見 A。由 D 可見 B。自 C 至可見 A 之 F 點。及自 D 至可見 B 之 E 點。測得 CF 及 DF 各為 200 碼。又得 AFC 83°, ACD 53° 3'。 ACF 54° 31' BDE 54° 30' BDC 156° 25', D EB 88° 30'。計算 AB 之水平距。



$$A C = C F \frac{\sin A F C}{\sin C A F}$$

$$= 200 \frac{\sin 33^{\circ}}{\sin 42^{\circ} 29'}$$

$$\log 200 = 2.30103$$

$$\log \sin 83^{\circ} = 9.99675$$

$$\begin{aligned}\text{colog sin } 42^{\circ}29' \\ = 0.17045\end{aligned}$$

$$\begin{aligned}\log AC = 2.46823 \\ AC = 239.92.\end{aligned}$$

$$\begin{aligned}BD = DE \frac{\sin B E D}{\sin D B E} \\ = 200 \frac{\sin 88^{\circ}30'}{\sin 37^{\circ}}\end{aligned}$$

$$\begin{aligned}\log 200 = 2.30103 \\ \log \sin 88^{\circ}30' = 9.99985 \\ \text{colog sin } 37^{\circ} = 0.22054\end{aligned}$$

$$\log BD = 2.52142$$

$$BD = 332.22.$$

$$\begin{aligned}\tan \frac{1}{2}(ADC - CAD) \\ = \frac{AC - CD}{AC + CD}\end{aligned}$$

$$\begin{aligned}\tan \frac{1}{2}(ADC + CAD) \\ = \frac{93.92}{493.92} \tan 63^{\circ}15'.\end{aligned}$$

$$\begin{aligned}\log 93.92 = 1.97276 \\ \text{colog } 493.92 = 7.30634 - 10 \\ \log \tan 63^{\circ}15' = 10.29753 \\ \log \tan \frac{1}{2}(ADC - CAD) \\ = 9.57663\end{aligned}$$

$$\frac{1}{2}(ADC - CAD) = 20^{\circ}40'8''$$

$$\frac{1}{2}(ADC + CAD) = 63^{\circ}15'$$

$$ADC = 83^{\circ}55'8''.$$

$$\begin{aligned}AD = AC \frac{\sin ACD}{\sin ADC} \\ = 293.92 \frac{\sin 53^{\circ}30'}{\sin 83^{\circ}55'8''}.\end{aligned}$$

$$\log 292.92 = 2.46823$$

$$\log \sin 53^{\circ}20' = 9.90518$$

$$\begin{aligned}\text{colog sin } 83^{\circ}55'8'' \\ = 0.00245\end{aligned}$$

$$\log AD = 2.37585$$

$$AD = 237.61.$$

$$\begin{aligned}BDA = BDC - ADC \\ = 156^{\circ}25' - 83^{\circ}55'8'' \\ = 72^{\circ}29'52''\end{aligned}$$

$$\begin{aligned}\tan \frac{1}{2}(DAB - DBA) \\ = \frac{BD - AD}{BD + AD}\end{aligned}$$

$$\begin{aligned}\tan \frac{1}{2}(DAB + DBA) \\ = \frac{94.61}{569.83} \tan 53^{\circ}45'4''.\end{aligned}$$

$$\begin{aligned}\log 94.61 = 1.97594 \\ \text{colog } 569.93 = 7.24426 - 10 \\ \log \tan 52^{\circ}45.4'' \\ = 20.13478\end{aligned}$$

$$\log \frac{1}{2}(DAB - DBA)$$

$$= 9.35498$$

$$\frac{1}{2}(DAB - DBA)$$

$$= 12^{\circ}45'35''$$

$$\frac{1}{2}(DAB + DBA)$$

$$= 53^{\circ}45'4''$$

$$DAB = 66^{\circ}30'39''.$$

$$AB = BD \frac{\sin ADB}{\sin BAD}$$

$$= 332.22 \frac{\sin 72^{\circ}29'52''}{\sin 66^{\circ}30'39''}$$

$$\log 332.22 = 2.52142$$

$$\log \sin 72^{\circ}29'52''$$

$$= 9.97941$$

$$\operatorname{colog} \sin 66^{\circ}30'39''$$

$$= 0.03757$$

$$\log AB = 2.53840$$

$$AB \approx 345.46.$$

即 AB 之距 = 345.46 碼。

60 北溫帶內之一柱在測者之東南東而正午時。其影之一端在其東北。其影長 80 呎。而自測者之位置得柱之仰角為  $45^{\circ}$  求柱高。

設 A 為測此所之地位，B 為影之尖點，C 為柱底。已知 A

$= 67^{\circ}30'$ 。C =  $67^{\circ}30'$ ，a = 80。求 b。

$$b = a \frac{\sin B}{\sin A}$$

$$= 80 \frac{\sin 45^{\circ}}{\sin 67^{\circ}30'}$$

$$\log 80 = 1.9309$$

$$\log \sin 45^{\circ} = 9.81949$$

$$\operatorname{colog} \sin 67^{\circ} = 30'$$

$$= 0.03438$$

$$\log b = 1.78696$$

$$b = 61.23.$$

設 B' 為柱頂。

則  $A = B = 45^{\circ}$ ，

而 AB' C 為等腰三角形

故 柱高 = 61.23 呎

61，自山頂測得山底同水平面內兩物之俯角為  $45^{\circ}$  及  $30^{\circ}$ 。而其兩物間之水平角為  $30^{\circ}$ 。試証山高等於兩物之距。

設 A 為山頂，A' 為自 A 至山底平面之垂線底，B 與 C 為二物。

則  $A'B = A'A$ 。

$$A'C = A'A \tan 60^{\circ}$$

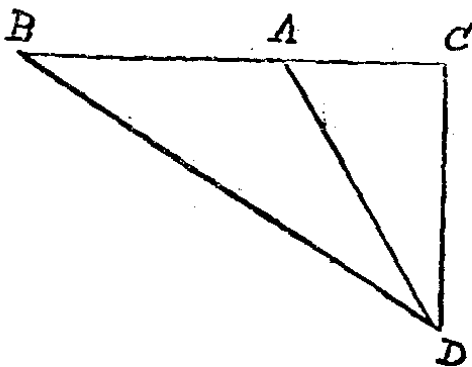
$$= \sqrt{3} A'A,$$

$$\begin{aligned} \overline{BC^2} &= \overline{A'B^2} + \overline{A'C^2} \\ &\quad - 2A'B \times A'C \\ &\quad \cos BA'C \\ &= \overline{A'A^2} + \overline{3A'A} \\ &\quad + 2A'A \times \sqrt{3}A'A \\ &\quad \times \frac{1}{2}\sqrt{3} \\ &= \overline{A'A^2} + \overline{3A'A^2} \\ &\quad - \overline{3A'A^2} = \overline{A'A^2} \end{aligned}$$

$$BC = A'A.$$

即 山之高 = 二物之距。

62. 將以知江面寬  $AB^\circ$  於  $B$  之延線內  $^\circ$  取  $AC$  爲 100 碼。又於  $AC$  上過  $C$  點之垂綫內。取  $CD$  爲 200 碼。求  $BDA$  角爲  $37^\circ, 18' 30''$  求  $AB$ 。



$$\tan ADC = \frac{AC}{CD} = \frac{100}{200} = \frac{1}{2}.$$

$$\log \tan ADC = 9.69897$$

$$ADC = 26^\circ 33' 54''.$$

$$\begin{aligned} BDC &= ADB + ADC \\ &= 63^\circ 52' 24''. \\ BC &= CD \tan BDC \\ &= 200 \tan 63^\circ 52' 24'' \end{aligned}$$

$$\log 200 = 2.30103$$

$$\log \tan 63^\circ 52' 24''$$

$$= 10.30939$$

$$\log BC = 2.61042$$

$$BC = 407.77$$

$$AB = BC - AC$$

$$= 307.77$$

$$AB = 307.77$$

63. 三角形之三邊和爲 180。但知  $\angle A = 2\angle B, \angle C = 2\angle C^\circ$ 。求三邊。

$$B = 2C.$$

$$A = 2B = 4C.$$

$$A + B + C = 7C = 180^\circ.$$

$$\text{故 } C = 25^\circ 42' 51\frac{2}{7}''.$$

$$B = 51^\circ 25' 42\frac{6}{7}''.$$

$$A = 102^\circ 51' 25\frac{5}{7}''.$$

$$\frac{a}{c} = \frac{\sin A}{\sin C}$$

$$\log \sin A = 9.98897$$

$$\text{colog } \sin C = 0.36263$$

$$\log \frac{a}{c} = 0.35160$$

$$\frac{a}{c} = 2.247$$

$$a = 2.247 c.$$

$$\frac{b}{c} = \frac{\sin B}{\sin C}$$

$$\log \sin B = 9.89311$$

$$\text{colog } \sin C = 0.36263$$

$$\log \frac{b}{c} = 0.25574$$

$$\frac{b}{c} = 1.802.$$

$$b = 1.802 c$$

$$a + b + c = (2.247 + 1.802 + 1)c \\ = 5.049 c.$$

$$\text{故 } c = \frac{100}{5.049} = 19.806.$$

$$a = 2.247 c = 44.$$

$$504$$

$$b = 1.802 c = 35.$$

$$\underline{690}$$

$$a + b + c = 100.000$$

其三邊爲 19.8, 35.7, 44.5.

64. 設  $\sin^2 A + 5 \cos^2 A = 3$ .  
求 A。

$$\sin^2 A + 5 \cos^2 A = 3.$$

$$\sin^2 A + 5 - 5 \sin^2 A = 3.$$

$$4 \sin^2 A = 2.$$

$$\sin^2 A = \frac{1}{2}.$$

$$\sin A = \pm \sqrt{\frac{1}{2}}.$$

$$\text{故 } A = \pm 45^\circ, \pm 135^\circ.$$

65. 設  $\sin^2 A = m \cos A = n$ .

求  $\cos A$

$$\sin^2 A = m \cos A = n.$$

$$1 - \cos^2 A = m \cos A = n.$$

$$\cos^2 A + m \cos A = n + 1.$$

$$A \cos^2 A + ( ) + m^2 = m^2 + 4 \\ (n + 1).$$

$$2 \cos A + m = \pm \sqrt{m^2 + 4}$$

$$\text{故 } \cos A = \frac{1}{2} \left[ -m \pm \sqrt{m^2 + 4(n + 1)} \right].$$

66. 已知  $\sin A = m \sin B$ .  
及  $\tan A = n \tan B$ . 求  $\sin A$   
及  $\cos B$ .

$$\tan A = n \tan B.$$

$$\frac{\sin A}{\cos A} = n \frac{\sin B}{\cos B}.$$

$$\frac{m \sin B}{\cos A} = \frac{n \sin B}{\cos B}.$$

$$\cos A = \frac{m}{n} \cos B.$$

$$\cos^2 A = \frac{m^2}{n^2} \cos^2 B.$$

$$\sin^2 A = m^2 \sin^2 B.$$

$$1 = \frac{m^2}{n^2} \cos^2 B$$

$$+ m^2 \sin^2 B.$$

$$\frac{m^2}{n^2} \cos^2 B + m^2(1 - \cos^2 B) = 1.$$

$$\begin{aligned} \cos^2 B &= \frac{1 - m^2}{\frac{m^2}{n^2} - m^2} \\ &= \frac{(1 - m^2)n^2}{(1 - n^2)m^2}. \end{aligned}$$

$$\cos B = \frac{n}{m} \sqrt{\frac{1 - m^2}{1 - n^2}}.$$

$$\cos^2 A = \frac{m^2}{n^2} \cos^2 B = \frac{1 - m^2}{1 - n^2}.$$

$$\sin^2 A = 1 - \frac{1 - m^2}{1 - n^2} = \frac{m^2 - n^2}{1 - n^2}.$$

$$\sin A = \sqrt{\frac{m^2 - n^2}{1 - n^2}}.$$

67. 設  $\tan^2 A + 4 \sin^2 A = 6$ .  
求  $A$ .

$$\tan^2 A + 4 \sin^2 A = 6.$$

$$\frac{\sin^2 A}{1 - \sin^2 A} + 4 \sin^2 A = 6.$$

$$\begin{aligned} \sin^2 A + 4 \sin^2 A - 4 \sin^4 A \\ = 6 - 6 \sin^2 A \end{aligned}$$

$$4 \sin^4 A - 11 \sin^2 A + 6 = 0.$$

$$(4 \sin^2 A - 3)(\sin^2 A - 2) = 0.$$

$$\sin^2 A = \frac{3}{4} \text{ 或 } 2.$$

$$\sin A = \pm \frac{1}{2} \sqrt{3}.$$

故  $A = \pm 60^\circ, \pm 120^\circ$ .

68. 設  $\sin A = \sin 2A$ . 求  $A$ .

$$\begin{aligned} \sin A &= \sin 2A \\ &= 2 \sin A \cos A. \end{aligned}$$

$$\text{故 } \sin A (1 - 2 \cos A) = 0.$$

故  $\sin A = 0$ ;

或  $1 - 2 \cos A = 0$ .

$$\cos A = \frac{1}{2}.$$

$$A = 0^\circ, 180^\circ, \pm 60^\circ.$$

69. 設  $\tan 2A = 3 \tan A$ .  
求  $A$ .

$$\tan 2A = 3 \tan A.$$

$$\frac{2 \tan A}{1 - \tan^2 A} = 3 \tan A.$$

$$2 \tan A = 3 \tan A - 3 \tan^3 A$$

$A$ .

$$3 \tan^3 A - \tan A = 0.$$

$$\tan A (3 \tan^2 A - 1) = 0.$$

$$\tan A = 0.$$

$$\text{或 } 3 \tan^2 A - 1 = 0.$$

$$\text{故 } \tan A = \pm \frac{1}{\sqrt{3}}.$$

$$A = 0^\circ, 180^\circ, 30^\circ, 150^\circ, 210^\circ, 330^\circ.$$

$$70. \text{ 求證 } \tan 50^\circ + \cot 50^\circ \\ = 2 \sec 10^\circ.$$

$$\tan 50^\circ + \cot 50^\circ = \tan 50^\circ$$

$$+ \frac{1}{\tan 50^\circ}$$

$$= \frac{\tan^2 50^\circ + 1}{\tan 50^\circ}$$

$$= \frac{\sec^2 50^\circ}{\tan 50^\circ}$$

$$= \frac{1}{\sin 50^\circ \cos 50^\circ}$$

$$= \frac{2}{2 \sin 50^\circ \cos 50^\circ}$$

$$= \frac{2}{\sin 100^\circ}$$

$$= \frac{2}{\cos 10^\circ}$$

$$= 2 \sec 10^\circ.$$

71. 已知正 $n$ 邊形之一邊爲 $a$ 。求以 $n$ 及 $a$ 示其內切及外接圓之半徑之式。

設  $P, H, D$  爲內接正五邊形六邊形, 其十邊形之邊。

(1) 每邊 $a$ 所對之圓心角爲

$\frac{360^\circ}{n}$ 。若 $R$ 爲外接圓之半徑,

$r$ 爲內切圓之半徑, 則

$$\frac{\frac{1}{2}a}{R} = \sin \frac{180^\circ}{n},$$

$$\frac{\frac{1}{2}a}{r} = \tan \frac{180^\circ}{n}.$$

$$\text{故 } R = \frac{a}{2} \csc \frac{180^\circ}{n},$$

$$r = \frac{a}{2} \cot \frac{180^\circ}{n}.$$

(2) 設  $R = r$

$$\text{則 } P = 2 \sin 36^\circ,$$

$$H = 2 \sin 30^\circ = r.$$

$$D = 2 \sin 18^\circ.$$

$$\text{求證 } P^2 = H^2 + D^2,$$

$$\text{或 } 4 \sin^2 36^\circ = r + 4 \sin^2 18^\circ$$

$$\sin 36^\circ = \cos 54^\circ,$$

$$\text{或 } \sin (2 \times 18^\circ)$$

$$= \cos (3 \times 18^\circ).$$

按公式 [12], 又演題十四

$$\text{題問 } 19. 2 \sin 18^\circ \cos 18^\circ$$

$$= 4 \cos^3 18^\circ - 3$$



$$\begin{aligned} & \cos 18^\circ. \\ 2 \sin 18^\circ &= 4 \cos^2 18^\circ - 3 \\ &= 4 - 4 \sin^2 18^\circ - 3 \\ &= 1 - 4 \sin^2 18^\circ. \end{aligned}$$

$$\begin{aligned} \text{故 } 4 \sin^2 18^\circ &= 1 - 2 \sin 18^\circ \\ &= 1 - 2 \cos 72^\circ. \\ 1 + 4 \sin^2 18^\circ &= 2 - 2 \cos 72^\circ \\ &= 2 (1 - \cos 72^\circ) \end{aligned}$$

$$\text{按 [16], } = 4 \sin^2 36^\circ.$$

72. 試以三角形之兩邊為  $b$ ,  $c$ , 及夾角  $A$  表面積公式。

設  $P$  為自  $B$  至  $b$  之垂線,

$$\text{則 } F = \frac{1}{2}pb.$$

$$\text{但 } P = c \sin A,$$

$$\begin{aligned} \text{故 } F &= \frac{1}{2}c \sin A \times b \\ &= \frac{1}{2}bc \sin A. \end{aligned}$$

73. 試以三角形之兩角  $A, B$  及夾邊  $c$  表面積公式。

$$a = \frac{c \sin A}{\sin C}.$$

$$b = \frac{c \sin B}{\sin C}.$$

$$\begin{aligned} F &= \frac{1}{2}ab \sin C \\ &= \frac{1}{2}c^2 \frac{\sin A \sin B}{\sin C} \end{aligned}$$

$$= \frac{1}{2}c^2 \frac{\sin A \sin B}{\sin(A+B)}$$

74. 試以三角形之三邊表面積公式。

$$F = \frac{1}{2}ac \sin B$$

按公式 [12],

$$\sin B = 2 \sin \frac{1}{2}B \cos \frac{1}{2}B$$

按公式 (28),

$$\sin \frac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{ac}}$$

按公式 [29],

$$\cos \frac{1}{2}B = \sqrt{\frac{s(s-b)}{ac}},$$

$$\text{故 } \sin B = \frac{2}{ac} \sqrt{s(-a)}$$

$$\frac{(s-b)(s-c),}{}$$

$$\text{故 } F = \sqrt{s(s-a)(s-b)(s-c)}$$

75. 設  $a$  為等邊三角形之邊。求證其面積為  $\frac{a^2\sqrt{3}}{4}$ 。

$$\begin{aligned} F &= \frac{1}{2}bc \sin A \\ &= \frac{1}{2}a^2 \sin 60^\circ \\ &= \frac{1}{2}a^2 \times \frac{1}{2}\sqrt{3} \\ &= \frac{a^2\sqrt{3}}{4} \end{aligned}$$

76. 矩形之兩鄰邊為 52.25

鎖及38.24鎖。求面積。

$$\text{面積} = 52.25 \times 38.24.$$

$$\log 52.25 = 1.71809$$

$$\log 38.24 = \underline{1.58252}$$

$$\log (\text{面積}) = 330061$$

$$\text{面積} = 1998.$$

$$1998 \text{ 方鎖} = 199 \text{ 英畝} 8 \text{ 方鎖}$$

77. 平行四邊形之兩邊爲59.8鎖及37.05鎖。而其夾角爲 $72^\circ 10'$ 。求面積。

$$\text{面積} = 59.8 \times 37.05 \sin 72^\circ 10'.$$

$$\log 59.8 = 1.77670$$

$$\log 37.05 = 1.56879$$

$$\log \sin 72^\circ 10' = \underline{9.97861}$$

$$\log (\text{面積}) = 3.32410$$

$$\text{面積} = 2109.1.$$

$$2109.1 \text{ 方鎖} = 210 \text{ 英畝.}$$

$$9.1 \text{ 方鎖.}$$

78. 平行四邊形之兩邊爲15.36鎖及11.46鎖。而其夾角爲 $47^\circ 30'$ 。求面積。

$$\text{面積} = 15.36 \times 11.46 \sin 47^\circ 30'$$

$$\log 15.36 = 1.18639$$

$$\log 11.46 = 1.05918$$

$$\log \sin 47^\circ 30' = \underline{9.86763}$$

$$\log (\text{面積}) = 2.11320$$

$$\text{面積} = 129.78 \text{ 方鎖}$$

$$= 12 \text{ 英畝} 9.78 \text{ 方鎖.}$$

79. 三角形之兩邊爲12.38鎖及6.78鎖。而其夾角爲 $46^\circ 24'$ 。求面積。

$$\text{面積} = \frac{1}{2} \times 12.38 \times 6.$$

$$78 \sin 46^\circ 24'.$$

$$\log 6.19 = 0.79169$$

$$\log 6.78 = 0.83123$$

$$\log \sin 46^\circ 24' = \underline{9.85984}$$

$$\log (\text{面積}) = 1.48276$$

$$\text{面積} = 30.392.$$

$$30.392 \text{ 方鎖} = 3 \text{ 英畝} 392 \text{ 方鎖.}$$

80. 直角三角形成直角之兩邊18.37鎖及13.44鎖。求面積。

$$\text{面積} = \frac{1}{2} \times 18.37 \times 13.$$

$$44$$

$$\text{colog } 2 = 9.69867 - 10$$

$$\log 18^\circ 37' = 1.26411$$

$$\log 13^\circ 44' = \underline{1.12840}$$

$$\log (\text{面積}) = 2.39251$$

$$\text{面積} = 123.45.$$

$$123.45 \text{ 方鎖} = 12 \text{ 英畝} 3.45 \text{ 方}$$

鎖。

81. 三角形之兩角為 $76^{\circ}54'$ 及 $57^{\circ}33'12''$ 。而其夾邊為9鎖。求面積。

按公式 (34),

$$\text{面積} = \frac{9^2 \sin 76^{\circ}54' \sin 57^{\circ}33'12''}{2 \sin 134^{\circ}27'12''}$$

$$\log 81 = 1.90849$$

$$\log \sin 76^{\circ}54' = 9.98855$$

$$\log \sin 57^{\circ}33'12''$$

$$= 9.92629$$

$$\text{colog } 2 = 9.69897 - 10$$

$$\text{colog } \sin 134^{\circ}27'12''$$

$$= 0.14641$$

$$\log \text{面積} = 1.66871$$

$$\text{面積} = 46.634.$$

46.634 方鎖 = 4 英畝 6.634 方鎖。

82. 設 三角形之兩邊為 19.74 鎖及 17.34 鎖。其一在北  $82^{\circ}30'$  西。而又一在南  $24^{\circ}15'$  東。求面積。

$$\text{其夾角} = 121^{\circ}45'.$$

$$\text{面積} = \frac{1}{2} \times 19.74 \times 17$$

$$34 \sin 121^{\circ}45'.$$

$$\text{colog } 2 = 9.69897 - 10$$

$$\log 19.74 = 1.29535$$

$$\log 17.34 = 1.23905$$

$$\log \sin 121^{\circ}45'$$

$$= 9.92960$$

$$\log (\text{面積}) = 2.16297$$

$$\text{面積} = 145.54$$

$$45.54 \text{ 方鎖} = 14 \text{ 英畝 } 5.54 \text{ 方}$$

鎖。

83. 三角形之三邊為 49 鎖, 50.25 鎖及 25.69 鎖。求面積。

$$\text{面積} = \sqrt{s(s-a)(s-b)(s-c)}.$$

$$s = \frac{1}{2}(49 + 50.25 + 25.69)$$

$$= 62.47.$$

$$s - a = 13.47.$$

$$s - b = 12.22.$$

$$s - c = 36.78.$$

$$\log 62.47 = 1.79567$$

$$\log 13.47 = 1.12937$$

$$\log 12.22 = 1.08707$$

$$\log 36.78 = 1.56561$$

$$2) 5.57772$$

$$\log (\text{面積}) = 2.78886$$

$$\text{面積} = 614.97.$$

614.97 方鎖 = 61 英畝 4.97 方鎖.

84. 三角形之三邊爲 10.64 鎖 12.28 鎖 9 鎖。求面積。

$$\text{面積} = \sqrt{s(s-a)(s-b)(s-c)}.$$

$$s = \frac{1}{2} (10.64 + 12.28 + 9) \\ = 15.96.$$

$$s - a = 5.32.$$

$$s - b = 3.68$$

$$s - c = 6.96$$

$$\log 15.96 = 1.20303$$

$$\log 5.32 = 0.72591$$

$$\log 3.68 = 0.56585$$

$$\log 6.96 = 0.84261$$

$$2) \underline{3.33740} \quad .$$

$$\log (\text{面積}) = 1.66870$$

$$\text{面積} = 46.633.$$

46.633 方鎖 = 4 英畝 6.633 方鎖.

85. 有三角之田。面積共 14 英畝。其三邊之比。若 3:5:7。求三邊。

設 其三邊爲  $3x$  鎖  $5x$  鎖  $7x$  鎖。

14 英畝 = 140 方鎖。

$$s = \frac{1}{2} (3x + 5x + 7x$$

$$= 7.5x.$$

$$s - a = 4.5x$$

$$s - b = 2.5x$$

$$s - c = 0.5x$$

$$140 = \sqrt{7.5x \times 4.5x \times 2.5x}$$

$$\times 0.5x = \frac{x^2}{4} \sqrt{15 \times 9 \times 5}$$

$$= \frac{15x^2}{4} \sqrt{3}.$$

$$\text{故 } x = \frac{4 \times 140}{15\sqrt{3}} = \frac{112}{3\sqrt{3}}$$

$$\log 112 = 2.04922$$

$$\text{colog } 3\sqrt{3} = 9.28432 - 10$$

$$2) \underline{1.33354}$$

$$\log x = 0.66677$$

$$x = 4.6427$$

$$3x = 13.9281$$

$$5x = 23.2135$$

$$7x = 32.4989$$

即得三邊爲 13.93 鎖 23.21 鎖 32.50 鎖。

86. 於 ABCD 四邊形。已知 AB = 17.22 鎖。AD = 7.45 鎖。CD = 14.10 鎖。EC = 5.25 鎖。及對角線 AC = 15.40 鎖。求面積。

於 ABC 三角形中，

$$s = \frac{1}{2}(17.22 + 5.25 + 15.04) \\ = 18.755.$$

$$s - a = 1.535$$

$$s - b = 13.505.$$

$$s - c = 3.715.$$

$$\log 18.755 = 1.27312$$

$$\log 1.535 = 0.18611$$

$$\log 13.505 = 1.13049$$

$$\log 3.715 = 0.56936$$

$$2) \underline{3.15968}$$

$$\log(\text{面積}) = 1.57984$$

$$\text{面積} = 39.005.$$

於 ACD 三角形中,

$$s = \frac{1}{2}(15.04 + 14.10 + 7.45) = 18.295.$$

$$s - a = 3.255$$

$$s - b = 4.195$$

$$s - c = 10.845$$

$$\log 18.295 = 1.26233$$

$$\log 3.255 = 0.51255$$

$$\log 4.195 = 0.62273$$

$$\log 10.845 = 1.03523$$

$$2) \underline{3.43284}$$

$$\log(\text{面積}) = 1.71642$$

$$\text{面積} = 52.050.$$

$$\text{ABC之面積} = 38.005$$

ABCD之面積

$$= 52.050$$

ABCD之面積

$$= 90.055$$

$$90.055 \text{ 方鎊} = 9 \text{ 英畝 } 0.055$$

方鎊

87. 四邊形之兩對角線爲  $a$  及  $b$ 。其交角爲  $D$ 。試證其面積爲  $\frac{1}{2} ab \sin D$ 。

設  $a_1, b_1$  與  $a_2, b_2$  爲對角線被交點所分之四段；即

$$a = a_1 + a_2, \quad b = b_1 + b_2.$$

則對角線所成四三角形之面積爲

$$\frac{1}{2} a_1 b_1 \sin D, \quad \frac{1}{2} a_2 b_1 \sin D,$$

$$\frac{1}{2} a_1 b_2 \sin D, \quad \frac{1}{2} a_2 b_2 \sin D.$$

即得 四邊形之面積

$$= \frac{1}{2} a_1 (a_1 + b_2) \sin D$$

$$+ \frac{1}{2} a_2 (a_1 + a_2) (a_1 + b_2)$$

$$\sin D$$

$$\frac{1}{2} (a_1 + a_2) (a_1 + b_2)$$

$$\sin D$$

$$\frac{1}{2} ab \sin D.$$

88. 四邊形之兩對角線爲 34 與 56, 其交角爲  $67^\circ$ 。求面

積。

$$\text{面積} = \frac{1}{2} \times 34 \times 56 \times \sin 67^\circ.$$

$$\log 17 = 1.23045$$

$$\log 56 = 1.74819$$

$$\log \sin 67^\circ = 9.96403$$

$$\log(\text{面積}) = 2.94267$$

$$\text{面積} = 876.34.$$

89. 四邊形之兩對角綫爲 75 與 49。其交角爲 42°。求面積。

$$\text{colog } 2 = 9.69897 - 10$$

$$\log 75 = 1.87506$$

$$\log 49 = 1.69020$$

$$\log \sin 42^\circ = 9.82551$$

$$\log(\text{面積}) = 3.08974$$

$$\text{面積} = 1229.5.$$

90. 正  $n$  邊形之一邊爲  $a$ 。試證其面積爲  $\frac{na^2}{4} \cot \frac{180^\circ}{n}$ 。

自各角作線至心，分原形爲  $n$  等腰三角形各以  $a$  爲

底又各以  $\frac{360^\circ}{n}$  爲頂角。各

三角形之高即

$$i = \frac{a}{2} \cot \frac{180^\circ}{n},$$

而其面積即

$$\frac{1}{2} ab = \frac{a^2}{4} \cot \frac{180^\circ}{n}.$$

故 多邊形之面積等於

$$\frac{na^2}{4} \cot \frac{180^\circ}{n}.$$

91. 正五邊形之一邊爲 25。求面積。

$$\text{面積} = \frac{5 \times 25^2}{4} \cot \frac{180^\circ}{5}$$

$$= 781.25 \cot 36^\circ.$$

$$\log 781.25 = 2.89279$$

$$\log \cot 36^\circ = 10.13874$$

$$\log(\text{面積}) = 3.03153$$

$$\text{面積} = 1075.3.$$

92. 正六邊形之一邊爲 32°。求面積。

$$\text{面積} = \frac{6 \times 32^2}{4} \cot \frac{180^\circ}{6}$$

$$= 1536 \cot 30^\circ.$$

$$\log 1536 = 2.18639$$

$$\log \cot 30^\circ = 10.23856$$

$$\log(\text{面積}) = 3.42495$$

$$\text{面積} = 2660.4.$$

93. 正十邊形之一邊爲 46°。求面積。

$$\text{面積} = \frac{10 \times 46^2}{4} \cot \frac{180^\circ}{10}$$

$$= 5290 \cot 18^\circ.$$

$$\log 5290 = 3.72346$$

$$\log \cos 18^\circ = 10.48822$$

$$(\log \text{面積}) = 4.21168$$

$$\text{面積} = 16,281.$$

94. 求周為 74 呎之圓面積。

$$2\pi r = 74.$$

$$\text{故 } r = \frac{37}{\pi}.$$

$$\text{面積} = \pi r^2 = \frac{37^2}{\pi}.$$

$$\log 37^2 = 3.13640$$

$$\text{colog } \pi = 9.50285 - 10$$

$$\log(\text{面積}) = 2.63925$$

$$\text{面積} = 435.76$$

即 面積 = 435.76 方呎

95. 半徑為 125 呎之圓面積。

$$\text{面積} = \pi \times 125^2$$

$$\log 125^2 = 4.19382$$

$$\log \pi = 0.49715$$

$$\log(\text{面積}) = 4.69097$$

$$\text{面積} = 49.088.$$

即 面積 = 49.088 方呎。

96. 於徑為 125 呎之圓內。  
求  $22^\circ$  弧之扇形面積。

扇形面積: 圓面積 = 22:

360.

$$\text{故 扇形面積} = \frac{25}{360} \pi \left( \frac{125}{2} \right)^2$$

$$= \frac{11 \times 125^2}{720} \pi,$$

$$\log 11 = 1.04193$$

$$\log 125^2 = 4.19382$$

$$\text{colog } 720 = 7.04267 - 10$$

$$\log \pi = 0.49715$$

$$\log(\text{面積}) = 2.87503$$

$$\text{扇形面積} = 749.95.$$

即 扇形面積 = 749.95 方呎。

97. 於半徑為 44 呎之圓內。  
求  $25^\circ$  弧之扇形面積。

$$\text{扇形面積} = \frac{25}{360} \pi 44^2$$

$$= \frac{1210}{9} \pi.$$

$$\log 1210 = 3.08279$$

$$\log \pi = 0.49715$$

$$\text{colog } 9 = 9.04576 - 10$$

$$\log(\text{面積}) = 2.62570$$

$$\text{扇形面積} = 422.38.$$

即 扇形面積 = 422.38 方呎。

98. 於徑爲 50 呎之圓內。求  
280° 弧之弧矢形面積。

此弧矢形面積 = 同弧之扇形面積 + 兩邊爲半徑夾角爲 80° 之三角形面積。

$$\begin{aligned} \text{扇形面積} &= \frac{280}{360} \pi 25^2 \\ &= \frac{4375\pi}{9} \end{aligned}$$

$$\log 4375 = 3.64098$$

$$\log \pi = 0.49715$$

$$\text{colog } 9 = 9.04576 - 10$$

$$\log(\text{面積}) = 3.18389$$

$$\text{扇形面積} = 1527.2.$$

$$\begin{aligned} \text{三角形面積} &= \frac{1}{2} \times 25^2 \sin 80^\circ \\ &= 312.5 \sin 80^\circ. \end{aligned}$$

$$\log 312.5 = 2.49485$$

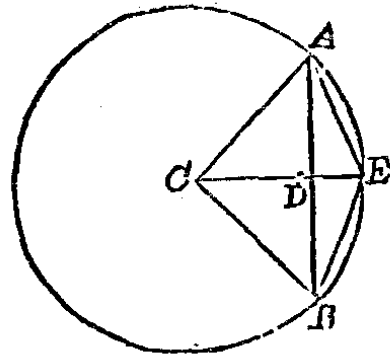
$$\log \sin 80^\circ = 9.99335$$

$$\log(\text{面積}) = 2.48820$$

$$\text{三角形面積} = 307.75.$$

故弧矢形面積 = 1834.95 方呎。

99. 有弧矢形 (小於半圓)  
其弦爲 20。求此弦與小弧之中  
點距爲 9。求此弧矢形之面積。



AEB 弧矢形面積 = CAB 扇形面積 - CAB 三角形面積。

$$\tan AED = \frac{10}{2} = 4.$$

$$\log \tan AED = 10.69897.$$

$$AED = 78^\circ 41' 24''.$$

$$\begin{aligned} ACD &= 180^\circ - 2AED \\ &= 22^\circ 37' 12''. \end{aligned}$$

$$\begin{aligned} AC &= AC \csc ACD \\ &= 10 \csc 22^\circ 37' 12''. \end{aligned}$$

$$\log 10 = 1.00000$$

$$\begin{aligned} \log \csc 22^\circ 37' 12'' \\ &= 0.41497 \end{aligned}$$

$$\log AC = 1.41497$$

$$AC = 26.$$

$$\begin{aligned} ACB &= 45^\circ 14' 24'' \\ &= 45.24^\circ. \end{aligned}$$

CAB 扇形面積



$$= \frac{ACB}{360^\circ} \pi AC^2$$

$$= \frac{45.24}{360} \pi 26^2$$

$$= \frac{370}{3000} \pi 26^2$$

$$\log 377 = 2.57634$$

$$\log \pi = 0.49715$$

$$\log 26^2 = 2.82994$$

$$\text{colog } 3000 = 6.52288 - 10$$

$$\log(\text{面積}) = 2.42631$$

$$\text{扇形面積} = 266.88.$$

CAB三角形面積

$$= AD \times CD$$

$$= 0(26 - 2)$$

$$= 240.$$

故弧矢形面積 = 26.88.

100. 設  $r$  爲圓半徑。則外切正  $n$  邊形之面積爲  $nr^2 \tan$

$\frac{180^\circ}{n}$ 。內接正  $n$  邊形之面積  $\frac{n}{2} r^2$

$\sin \frac{360^\circ}{n}$ 。

正內接多邊形之面積爲  $\frac{n}{2}$

$$r^2 \sin \frac{360^\circ}{n}.$$

自多邊形之各角作線至共心

分原形爲  $n$  相等等腰三角形，各以多邊形之邊爲底而頂角爲  $\frac{360^\circ}{n}$ 。

於正外切多邊形內各邊等

$2r \tan \frac{180^\circ}{n}$ ，其各三角形之

高則爲  $r$ 。故各三角形之面積

等於  $r^2 \tan \frac{180^\circ}{n}$ ，而多邊形之

面積爲  $nr^2 \tan \frac{180^\circ}{n}$ 。

於有法內切多邊形內各邊

等  $2r \sin \frac{180^\circ}{n}$ ，其各三角形

之高則爲  $r \cos \frac{180^\circ}{n}$ 。故各三

角形之面積等於  $r^2 \sin \frac{180^\circ}{n}$

$\sin \frac{180^\circ}{n} = \frac{r^2}{2} \sin \frac{360^\circ}{n}$ ，而

多邊形之面積爲  $\frac{nr^2}{n} \sin \frac{360^\circ}{n}$ 。

101. 設  $a$  爲正  $n$  邊形之一

邊。則內切圓之面積爲  $\frac{\pi a^2}{4} \cot^2$

$\frac{180^\circ}{n}$ 。外接圓之面積爲  $\frac{\pi a^2}{4} \csc$

$\frac{2180^\circ}{n}$ 。

外接圓之面積等於  $\frac{\pi a^2}{4} \csc^2 \frac{180^\circ}{n}$ .

設  $r$  為內切圓之半徑，則

$$a = 2r \tan \frac{180^\circ}{n},$$

故  $r = \frac{a}{2} \cot \frac{180^\circ}{n}$ .

$$\pi r^2 = \frac{\pi a^2}{4} \cot^2 \frac{180^\circ}{n}.$$

設  $R$  為外接圓之半徑，則

$$a = 2R \sin \frac{180^\circ}{n}.$$

故  $R = \frac{a}{2} \csc \frac{180^\circ}{n}$ .

$$\pi R^2 = \frac{\pi a^2}{4} \csc^2 \frac{180^\circ}{n}.$$

102. 圓內接正多邊形之面積。與外切等邊數之正多邊形之面積相比。如3:4。求邊數。

$$\frac{\frac{n}{2} r^2 \sin \frac{360^\circ}{n}}{\pi r^2 \tan \frac{180^\circ}{n}} = 3:4.$$

$$2nr^2 \sin \frac{360^\circ}{n} = 3nr^2 \tan \frac{180^\circ}{n}.$$

$$2 \sin \frac{360^\circ}{n} = 3 \tan \frac{180^\circ}{n}.$$

$$4 \sin \frac{180^\circ}{n} \cos \frac{180^\circ}{n}$$

$$= 3 \frac{\sin \frac{180^\circ}{n}}{\cos \frac{180^\circ}{n}}$$

$$4 \cos^2 \frac{180^\circ}{n} = 3.$$

$$\cos \frac{180^\circ}{n} = \frac{\sqrt{3}}{2}.$$

$$\frac{180^\circ}{n} = 30^\circ.$$

$$n = 6,$$

103. 圓內接正多邊形之面積。為內接及外切半邊數之正多邊形兩面積之等比中項。

$$\begin{aligned} 2n \text{邊內接正多邊形之面積} \\ = nr^2 \sin \frac{180^\circ}{n}. \end{aligned}$$

$$\begin{aligned} n \text{邊內接正多邊形之面積} \\ = \frac{n}{2} r^2 \sin \frac{360^\circ}{n}. \end{aligned}$$

$$\begin{aligned} n \text{邊外切正多邊形三面積} \\ = nr^2 \tan \frac{180^\circ}{n}. \end{aligned}$$

$$\begin{aligned} \frac{\frac{n}{2} r^2 \sin \frac{360^\circ}{n}}{nr^2 \tan \frac{180^\circ}{n}} \\ = \frac{\frac{n^2 r^4}{2} \sin \frac{360^\circ}{n}}{nr^2 \tan \frac{180^\circ}{n}} \end{aligned}$$

$$\begin{aligned}
 &= n^2 r^4 \sin \frac{180^\circ}{n} \\
 &\frac{\cos \frac{180^\circ}{n} \sin \frac{180^\circ}{n}}{\cos \frac{180^\circ}{n}} \\
 &= n^2 r^4 \sin \frac{180^\circ}{n} \\
 &= \left( nr^2 \sin \frac{180^\circ}{n} \right)^2
 \end{aligned}$$

104. 圓外切之正多邊形之面積。爲內接等邊數之正多邊形與外切半邊數之正多邊形兩面積之調和中項。

2n邊外切正多邊形之面積

$$= a = 2nr^2 \tan \frac{90^\circ}{n}$$

2n邊內接正多邊形之面積

$$= b = nr^2 \sin \frac{180^\circ}{n}$$

n邊外切正多邊形之面積

$$= c = nr^2 \tan \frac{180^\circ}{n}$$

求講

$$\frac{2}{a} = \frac{1}{b} + \frac{1}{c}$$

$$\frac{1}{b} + \frac{1}{c} = \frac{1}{nr^2 \sin \frac{180^\circ}{n}}$$

$$\begin{aligned}
 &+ \frac{1}{nr^2 \tan \frac{180^\circ}{n}} \\
 &= \frac{1 + \cos \frac{180^\circ}{n}}{nr^2 \sin \frac{180^\circ}{n}} \\
 &= \frac{2 \cos^2 \frac{90^\circ}{n}}{2nr^2 \sin \frac{90^\circ}{n} \cos \frac{90^\circ}{n}} \\
 &= \frac{2 \cos^2 \frac{50^\circ}{n}}{2nr^2 \sin \frac{90^\circ}{n} \cos \frac{90^\circ}{n}} \\
 &= \frac{2 \cos \frac{90^\circ}{n}}{2nr^2 \sin \frac{90^\circ}{n}} \\
 &= \frac{2}{a}
 \end{aligned}$$

105. 外切等邊三角形之周。爲內接等邊三角形之周之二倍。

外切有等邊三角形之每邊

$$= 2r \tan 60^\circ = 2\sqrt{3}r$$

內接等邊三角形之每邊

$$= 2r \sin 60^\circ = \sqrt{3}r$$

106. 外切正方形。爲內接正十二邊形之三分之四。

外切正方形之面積  $4r^2 =$   
內接十二邊形之面積

$$= \frac{12}{2} r^2 \sin \frac{360^\circ}{12}$$

$$= 6r^2 \sin 30^\circ = 3r^2.$$

107. 三角形之兩邊爲 3 與 12。夾角爲  $30^\circ$ 。求與此形等面積之二等邊直角三角形之斜邊。

已知之三角形面積

$$= \frac{1}{2} \times 3 \times 12 \times \sin 30^\circ$$

$$= 9.$$

所求三角形之一腰

$$= \sqrt{2 \times 9} = 3\sqrt{2}.$$

所求三角形之斜邊。

$$= \sqrt{2(3\sqrt{2})^2} = \sqrt{36}$$

$$= 6.$$

即 所求之斜邊 = 6.

108. 今地球赤道直徑爲 7925.6 哩。試以呎數表大圓之一分弧之長。

大圓周 =  $\pi 7925.6$ .

弧 1' 之呎數

$$= \frac{\pi \times 7925.6 \times 5280}{360 \times 60}$$

$$= \frac{7926.5 \times 5280\pi}{21600}$$

$$\log 7925.6 = 3.89903$$

$$\log 5280 = 3.72263$$

$$\log \pi = 0.49715$$

$$\text{co } \log 21600 = 5.66555 - 10$$

$$3.78436$$

弧 1' 之長 = 60864 呎

109. 一船自南緯  $43^\circ 45'$  駛行。航路爲北微東  $2345$  哩。求所達之緯度。及所成之經距。

行向 = 北偏東  $11^\circ 15'$ .

緯度之差 =  $2345 \times \cos$   
 $11^\circ 15'$ .

所經之距 =  $2345 \times \sin$   
 $11^\circ 15'$ .

$$\log 2345 = 3.37014$$

$$\log \cos 11^\circ 15'$$

$$= 9.99157$$

$$\log(\text{緯度之差}) = 3.36171$$

緯度之差 =  $2299.9'$   
 $= 38^\circ 19' 54''.$

$$\log 2345 = 3.37014$$

$$\log \sin 11^\circ 15' = 9.20024$$

$$\log(\text{所經之距}) = 2.66038$$

所經之距 = 457.49.

即所在之緯度南緯  $5^{\circ}25'15''$ ，其子午之距等於 457.49 哩。

110. 一船自北緯  $1^{\circ}45'$  駛行。航路為南東微東。而達於南緯  $2^{\circ}31'$ 。求距及經距。

行向 = 南偏東  $56^{\circ}15'$ 。

緯度之差 =  $4^{\circ}16' = 256$  哩。

所經之距 =  $256 \sec 5^{\circ}15'$ 。

子午距 =  $256 \tan 56^{\circ}15'$ 。

$\log 256 = 2.40824$

$\log \sec 56^{\circ}15' = 0.25526$

$\log(\text{所經之距}) = 2.66350$

所經之距 = 460.79。

$\log 256 = 2.40824$

$\log \tan 56^{\circ}15' = 10.17511$

$\log(\text{子午距}) = 2.58335$

子午距 = 383.13。

所經之距 = 460.79 哩。

子午距 = 383.13 哩。

111. 一船自南緯  $13^{\circ}17'$  駛行。航路為北東微東。直至經距為 207 哩。求距及所達之緯度。

行向 =  $64^{\circ}41'15''$ 。

子午距 = 207 哩。

所經之距 =  $207 \csc 64^{\circ}41'15''$ 。

緯度之差 =  $207 \cos 64^{\circ}41'15''$ 。

$\log \csc 64^{\circ}41'15'' = 0.04384$

$\log(\text{所經之距}) = 2.35981$

所經之距 = 228.98。

$\log 207 = 2.31597$

$\log \cot 64^{\circ}41'15'' = 9.674.83$

$\log(\text{緯度之差}) = 1.99080$

緯度之差 =  $96.904' = 1^{\circ}37'54''$ 。

$13^{\circ}17' - 1^{\circ}37'54''$

$= 11^{\circ}39'6''$ 。

所經之距 = 228.98 哩。

所到緯度 = 南緯  $11^{\circ}39'6''$ 。

112. 一船之航路。在南東之間。駛行 244 哩。但知始於南緯  $2^{\circ}52'$  而達於南緯  $5^{\circ}8'$ 。求航路及經距。

緯度之差 =  $2^{\circ}16'136 =$  哩

所經之距 = 244 哩。

$$\cos(\text{行向}) = \frac{136}{244}$$

$$\begin{aligned} \text{子午距} &= \sqrt{244^2 - 136^2} \\ &= \sqrt{(244 + 136)(244 - 136)} \\ &= \sqrt{380 \times 108}. \end{aligned}$$

$$\log 136 = 2.13354$$

$$\text{colog } 244 = 7.61261 - 10$$

$$\log(\text{行向}) = 9.74615$$

$$\text{行向} = 56^\circ 7' 32''.$$

$$\log 380 = 2.57978$$

$$\log 180 = 2.03342$$

$$2) 4.61320$$

$$\log(\text{子午距}) = 2.30660$$

$$\text{子午距} = 202.58.$$

行向爲南偏東  $56^\circ 7' 32''$ ,

子午距等於 202.58 哩。

113. 一船自北緯  $32^\circ 18'$  在北西之間駛行 344 哩。其經距爲 103 哩。求航路及所達之緯度。

午距適爲 103 海里 求其行向及所到之緯度。

$$\text{路距} = 344 \text{ 哩。}$$

$$\text{子午距} = 103 \text{ 哩。}$$

$$\sin(\text{行向}) = \frac{103}{344}.$$

$$\text{緯度之差} = \sqrt{344^2 - 103^2}$$

$$= \sqrt{(344 + 103)(344 - 103)}$$

$$= \sqrt{447 \times 241}.$$

$$\log 103 = 2.01284$$

$$\text{colog } 344 = 7.46344 - 10$$

$$\log \sin(\text{行向}) = 9.47628$$

$$\text{行向} = 17^\circ 25' 12''.$$

$$\log 447 = 2.65031$$

$$\log 241 = 2.38202$$

$$2) 5.03233$$

$$\log(\text{緯度之差}) = 2.51616$$

$$\text{緯度之差} = 328.22'$$

$$= 5^\circ 28' 13''.$$

$$32^\circ 18' + 5^\circ 28' 13''$$

$$= 37^\circ 46' 13''.$$

行向爲北偏西  $17^\circ 25' 22''$

所至之緯度爲北緯  $37^\circ 46'$

$13''$

114. 一船在南東之間駛行。得緯度差 136 哩及經距 203 哩。求距及航路。

$$\text{緯度之差} = 136 \text{ 哩。}$$

$$\text{子午距} = 203 \text{ 哩。}$$

$$\tan(\text{行向}) = \frac{203}{136}.$$

$$\begin{aligned} \log 203 &= 2.30750 \\ \operatorname{colog} 136 &= \underline{7.86646 - 10} \\ \log \tan(\text{行向}) &= 10.17396 \\ \text{行向} &= 56^\circ 10' 49''. \\ \text{所經之距} &= 203 \operatorname{csc} 56^\circ 10' \\ & \qquad \qquad \qquad 49'' \\ \log 203 &= 2.30750 \\ \log \operatorname{csc} 56^\circ 10' 49'' & \\ &= \underline{0.08051} \\ \log(\text{所經之距}) &= 2.38801 \\ \text{所經之距} &= 244.35. \\ \text{行向爲南偏東 } &56^\circ 10' 49'' \\ \text{所經之距} &= 244.35 \text{ 哩.} \end{aligned}$$

115. 一船向北駛行終日。其速率每小時15哩。問起點與所達之地點。在一直線內相距若干。(今地球半徑爲3962.8哩。)

一日所行之哩數

$$= 24 \times 15 \text{ 哩} = 360 \text{ 哩}$$

$$= \frac{360}{2\pi \times 3962.8} \times 360^\circ$$

$$= \frac{64800^\circ}{3962.8\pi}$$

$$\log 64800 = 4.81158$$

$$\operatorname{colog} 3962.8 = \underline{6.40200 - 10}$$

$$\operatorname{colog} \pi = \underline{9.50285 - 00}$$

$$\log(\text{所行之路}) = 0.71643$$

$$\text{所行之路} = 5.2051^\circ$$

$$= 5^\circ 12' 18''.$$

所行之弧之通弦

$$= 2 \times 3962.8 \sin 2^\circ 36' 9''.$$

$$= 7925.6 \sin 2^\circ 36' 9''.$$

$$\log 79256 = 3.89903$$

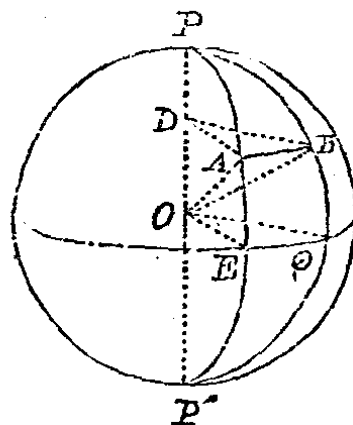
$$\log \sin 2^\circ 36' 9'' = \underline{8.65712}$$

$$\log(\text{弦通}) = 2.55615$$

$$\text{弦通} = 359.87$$

所求之距 = 359.87 哩。

116. 已知任何兩經線間於任何緯線上之經距。求於兩經線上之任何兩點之經度差。



於  $ODA$  正三角形中， $AO D$  角  $= 90^\circ - (\text{緯度})$ 。

$$\text{故 } \frac{DA}{OA} = \sin(90^\circ - \text{緯度})$$

$$= \cos \text{緯度}.$$

但  $\triangle DAB$  與  $\triangle OEQ$  爲相以三角形

$$\text{故 } \frac{DA}{OE} = \frac{AB}{EQ}, \text{ 或 } \frac{DA}{OA} = \frac{AB}{EQ}.$$

$$\text{相代, 得 } \cos(\text{緯度}) = \frac{AB}{EQ}$$

$$\text{故 } EQ = \frac{AB}{\cos \text{緯度}} = AB \times \sec \text{緯度}.$$

即 經度差 = 子午距  $\times$   $\sec$  緯度.

117. 一船自北緯  $42^{\circ}16'$  西經  $72^{\circ}16'$  向東駛行 149 哩。問所至處之位置。

$$\text{經度之差} = 149 \sec 42^{\circ}16'.$$

$$\log 149 = 2.17319$$

$$\log \sec 42^{\circ}16' = 0.13076$$

$$\log(\text{經度差}) = 2.30395$$

$$\text{經度差} = 201.35 = 3^{\circ}21'21''.$$

所至之經度 = 西經  $68^{\circ}54'39''$ .

118. 一船自南緯  $44^{\circ}49'$  東經  $119^{\circ}42'$  向西駛行直至東經

$117^{\circ}16'$  求駛行距。

$$\text{緯度差} = 2^{\circ}26' = 146'.$$

$$\text{子午距} = 146 \cos 44^{\circ}49'.$$

$$\log 146 = 2.11435$$

$$\log \cos 44^{\circ}49' = 9.85087$$

$$\log(\text{子午距}) = 2.01522$$

$$\text{子午距} = 103.57$$

即 所行之路 = 103.57 哩。

119. 一船自北緯  $31^{\circ}14'$  西經  $42^{\circ}19'$  向北東駛行 325 哩。求所到之位置。

$$\text{行向} = \text{北偏東 } 67^{\circ}30',$$

$$\text{緯度之差} = 325 \cos 67^{\circ}30'.$$

$$\log 325 = 2.51188$$

$$\log \cos 67^{\circ}30' = 9.58284$$

$$\log(\text{緯度差}) = 2.09472$$

$$\text{緯度差} = 124.37' = 2^{\circ}4'$$

$$22'', \text{ 中緯度}$$

$$= 31^{\circ}14' + \frac{1}{2}(2^{\circ}4'22'')$$

$$4'22''$$

$$= 32^{\circ}35'30''.$$

$$\text{子午距} = 325 \sin 67^{\circ}30'.$$

經度差

$$= 325 \sin 67^{\circ}30'$$

$$\sec 32^{\circ}16'11''.$$



$$\begin{aligned}\log 325 &= 2.51188 \\ \log \sin 67^\circ 30' &= 9.96562 \\ \log \sec 32^\circ 16' 11'' & \\ &= \underline{0.07282} \\ \log (\text{經度差}) &= 2.55036 \\ \text{經度差} &= 355.11' = 5^\circ 55' 7'' \\ &7''.\end{aligned}$$

所至之處即北緯  $33^\circ 18' 22''$   
西經  $86^\circ 23' 53''$ .

120. 求自哈佛安那至夸角之方位及距離 (哈佛安那在北緯  $23^\circ 9'$  西經  $82^\circ 22'$ 。夸角在北緯  $42^\circ 2'$  西經  $70^\circ 3'$ )。

$$\begin{aligned}\text{經度差} &= 12^\circ 19' = 739'. \\ \text{緯度差} &= 18^\circ 53' \\ &= 1133'.\end{aligned}$$

$$\text{中緯度} = 23^\circ 9' + \frac{1}{2} (18^\circ 53') = 32^\circ 35' 30''.$$

$$\begin{aligned}\text{子午距} &= \text{經度差} \times \cos \text{中緯度} \\ &= 739 \times \cos 32^\circ 35' 30''.\end{aligned}$$

$$\begin{aligned}\tan (\text{行向}) &= \frac{\text{子午距}}{\text{緯度差}} \\ &= \frac{739 \cos 32^\circ 35' 30''}{1133}\end{aligned}$$

$$\log 739 = 2.86864$$

$$\begin{aligned}\log \cos 32^\circ 35' 30'' & \\ &= 9.92559 \\ \text{colog } 1133 &= \underline{6.94577 - 10} \\ \log \tan (\text{行向}) &= 9.74000 \\ \text{行向} &= 28^\circ 47' 26''. \\ \text{兩處之距} &= \text{緯度差} \times \sec \\ \text{行向} &= 1133 \times \sec 28^\circ 47' 26''. \\ \log 113 &= 3.05423 \\ \log \sec 28^\circ 47' 26'' & \\ &= \underline{0.05730}\end{aligned}$$

$$\log (\text{兩處之距}) = 3.11153$$

$$\text{兩處之距} = 1292.8.$$

其方向為北偏東  $28^\circ 47' 26''$ ，其距等於 1292.8 哩。

121. 一船離北緯  $49^\circ 57'$  西經  $15^\circ 16'$ 。向南西間駛至某處。其經距為 194 哩。緯度為  $47^\circ 18'$  北。求江寬，距離，及某處之經度。

$$\text{緯度差} = 2^\circ 39' = 159 \text{ 哩.}$$

$$\begin{aligned}\text{中緯度} &= 49^\circ 57' - \frac{1}{2} (2^\circ 39') \\ &= 48^\circ 37' 30'',\end{aligned}$$

$$\text{子午距} = 194 \text{ 哩.}$$

$$\text{經度差} = 194 \text{ sec } 48^{\circ} 37' 30''.$$

$$\log 194 = 2.28780$$

$$\log \sec 28^{\circ} 37' 30'' \\ = 0.17981$$

$$\log(\text{經度差}) = 2.46761$$

$$\text{緯度差} = 293.50' \\ = 4^{\circ} 53' 30''.$$

$$\tan(\text{行向}) = \frac{194}{159}$$

$$\log 194 = 2.28780$$

$$\text{colog } 159 = 7.79860 - 10$$

$$\log \tan(\text{行向}) = 10.08640$$

$$\text{行向} = 50^{\circ} 39' 44''.$$

$$\text{所行之路} = 159 \text{ sec } 50^{\circ} 39' 44''.$$

$$\log 159 = 2.20140$$

$$\log \sec 50^{\circ} 39' 44'' \\ = 0.19799$$

$$\log(\text{所行之路}) = 2.39939$$

$$\text{所行之路} = 250.84'$$

其方向為南偏西  $50^{\circ} 39' 44''$

其所行之路 = 250.84 哩。

其所至之經度為西經  $20^{\circ} 9' 30'''$

122. 一船離北緯  $42^{\circ} 30'$  西

經  $58^{\circ} 51'$ 。向南東微南駛行 300 哩。求所到處之位置。

$$\text{行向} = 33^{\circ} 45',$$

$$\text{緯度之差} = 300 \cos 33^{\circ} 45'.$$

$$\log 300 = 2.47712$$

$$\log \cos 33^{\circ} 45' = 9.91985$$

$$\log(\text{緯度差}) = 2.39697$$

$$\text{緯度差} = 249.44' \\ = 4^{\circ} 9' 26''.$$

$$\text{中緯度} = 40^{\circ} 25' 17''.$$

$$\text{子午距} = 300 \sin 33^{\circ} 45'.$$

經度之差

$$= 300 \sin 33^{\circ} 45' \sec 40^{\circ} 25' 17''.$$

$$\log 300 = 2.47712$$

$$\log \sin 33^{\circ} 45' = 9.74474$$

$$\log \sec 40^{\circ} 25' 17'' \\ = 0.11845$$

$$\log(\text{經度差}) = 2.34031$$

$$\text{經度差} = 218.93' \\ = 3^{\circ} 38' 56''.$$

所至之處即北緯  $38^{\circ} 20' 34''$ 。

西經  $55^{\circ} 12' 4''$ 。

123. 一船離北緯  $49^{\circ} 57'$  西

經  $30^{\circ}$ 。向南 39 西駛至北緯  $47^{\circ}$

44'。求距及所到地之經度。

$$\text{行向} = 39^\circ.$$

$$\text{緯度差} = 2^\circ 13' = 133 \text{ 哩}.$$

$$\text{中緯度} = 48^\circ 50' 30''.$$

$$\text{所行之路} = 133 \sec 39^\circ.$$

$$\log 133 = 2.12385$$

$$\log \sec 39^\circ = 0.10950$$

$$\log (\text{路}) = 2.23335$$

$$\text{所行之路} = 171.14.$$

$$\text{子午距} = 133 \tan 39^\circ.$$

經度差

$$= 133 \tan 39^\circ \sec 48^\circ 50' 30''$$

$$\log 133 = 2.12385$$

$$\log \tan 39^\circ = 9.90837$$

$$\log \sec 48^\circ 50' 30''$$

$$= 0.18198$$

$$\log(\text{經度差}) = 2.21390$$

$$\text{經度差} = 163.64'$$

$$= 2^\circ 43' 38''.$$

所行之路 = 171.14 哩。所至之經度爲西經  $32^\circ 43' 38''$

124. 一船離北緯  $37^\circ$  西經  $32^\circ 19'$ 。向北西間駛行 300 哩。至北緯  $41^\circ$ 。求航路及所到地之經度。

$$\text{緯度差} = 41 = 240 \text{ 哩}$$

$$\text{中緯度} = 39^\circ.$$

$$\text{所行之路} = 300.$$

$$\cos(\text{行向}) = \frac{240}{300}.$$

$$\log 240 = 2.38021$$

$$\text{colog } 300 = 7.52288 - 10$$

$$\log \cos \text{行向} = 9.90209$$

$$\text{行向} = 36^\circ 52' 12''.$$

$$\text{子午距} = \sqrt{300^2 - 240^2}$$

$$= \sqrt{60 \times 540}$$

$$= 180.$$

$$\text{經度差} = 180 \sec 39^\circ.$$

$$\log 180 = 2.5527$$

$$\log \sec 39^\circ = 0.10950$$

$$\log(\text{經度差}) = 2.36477$$

$$\text{經度差} = 231.62'$$

$$= 3^\circ 51' 27''.$$

其行向爲北偏西  $36^\circ 52' 12''$ ，所至之經度爲西經  $36^\circ 7' 37''$

125. 一船離南緯  $50^\circ 10'$  東經  $30^\circ$ 。向東南東駛行。成 160 哩之經距。求距及所到之位置。

$$\text{行向} = 67^\circ 30'.$$

$$\text{子午路} = 160 \text{ 哩}$$

$$\text{所行之路} = 160 \csc 67^\circ 30'.$$

$$\text{緯度差} = 160 \cot 67^\circ 30'$$

$$\log 60 = 2.20412$$

$$\log \csc 67^\circ 30' = \underline{0.03438}$$

$$\log(\text{路}) = 2.23850$$

$$\text{所行之路} = 173.18.$$

$$\log 160 = 2.20412$$

$$\log \cot 67^\circ 30' = \underline{9.61722}$$

$$\log(\text{緯度差}) = 1.82134$$

$$\text{緯度差} = 66.273'$$

$$= 1^\circ 6' 16''$$

$$\text{所至之緯度} = 51^\circ 16' 16''.$$

$$\text{中緯度} = 50^\circ 43' 8''.$$

$$\text{經度差} = 160 \sec 50^\circ 43' 8''$$

$$\log 160 = 2.20412$$

$$\log \sec 50^\circ 43' 8''$$

$$= \underline{0.19851}$$

$$\log(\text{經度差}) = 2.40294$$

$$\text{經度差} = 252.71$$

$$= 4^\circ 12' 43''$$

所行之路為 73.18 哩所至之處  
為南緯  $51^\circ 16' 16''$  東經  $34^\circ 12' 43''$

126 一船離北緯  $49^\circ 30'$  西  
經  $25^\circ$  向南東間駛行 215 哩成  
167 哩之經距求航路及所到之  
位置。

$$\sin(\text{行向}) = \frac{167}{215}$$

$$\log 167 = 2.22272$$

$$\text{colog } 215 = \underline{7.66756 - 10}$$

$$\log \sin(\text{行向}) = 9.89028$$

$$\text{行向} = 50^\circ 57' 48''.$$

$$\text{緯度差} = \sqrt{215^2 - 167^2}$$

$$= \sqrt{48 \times 382}$$

$$\log 48 = 1.68124$$

$$\log 382 = \underline{2.58206}$$

$$2) \underline{42.6330}$$

$$\log(\text{緯度差}) = 2.13165$$

$$\text{緯度差} = 135.41'$$

$$= 2^\circ 15' 25''.$$

$$\text{中緯度} = 48^\circ 22' 18''.$$

$$\text{經度差} = 167 \sec 48^\circ 22' 18''.$$

$$\log 167 = 2.22272$$

$$\log \sec 48^\circ 22' 18''$$

$$= \underline{0.17764}$$

$$\log(\text{經度差}) = 2.40036$$

$$\text{經度差} = 251.39'$$

$$= 4^\circ 11' 23''.$$

其行向為南偏東  $50^\circ 57' 48''$   
所至之處即北緯  $47^\circ 14' 35''$  西

經 $20^{\circ}48'37''$

127. 一船離南緯 $43^{\circ}$ 西經 $21^{\circ}$ 駛行273哩至南緯 $40^{\circ}17'$ 。問何二航路經度 $^{\circ}$

所求之兩行向在一子午線之兩邊成相等之兩角。

$$\text{緯度差} = 2^{\circ}43' = 163 \text{ 哩}$$

$$\text{所行之路} = 273 \text{ 哩}$$

$$\cos(\text{行向}) = \frac{163}{273}$$

$$\log 163 = 2.21219$$

$$\text{colog } 273 = 7.56384 - 10$$

$$\log \cos(\text{行向}) = 9.77603$$

$$\text{行向} = 53^{\circ}20'21''$$

$$\text{子午距} = \sqrt{273^2 - 163^2}$$

$$= \sqrt{110 \times 436}$$

$$\text{中緯度} = 41^{\circ}38'30''$$

度差經

$$= \sqrt{110 \times 436} \sec 41^{\circ}38'30''$$

$$\log \sqrt{110} = 1.02069$$

$$\log \sqrt{436} = 1.31975$$

$$\log \sec 41^{\circ}38'30''$$

$$= 0.12950$$

$$\log(\text{經度差}) = 2.46694$$

$$\text{經度差} = 293.05'$$

$$= 4^{\circ}53'3''$$

(1) 行向爲北偏東 $53^{\circ}20'21''$ ，所至之經度爲西經 $16^{\circ}6'57''$ ，

(2) 行向爲北偏西 $53^{\circ}20'21''$

所到經度爲西經 $25^{\circ}53'3''$ ，

128. 一船自北緯 $17^{\circ}$ 東經 $119^{\circ}$ 駛行219哩。而成162哩。問此答有何者四種。

四行向與子午線所成之角皆等。

$$\sin(\text{行向}) = \frac{162}{219}$$

$$\log 162 = 2.20952$$

$$\text{colog } 219 = 7.65956 - 10$$

$$\log \sin(\text{行向}) = 9.86908$$

$$\text{行向} = 47^{\circ}42'33''$$

$$\text{緯度差} = \sqrt{219^2 - 162^2}$$

$$= \sqrt{57 \times 381}$$

$$\log 57 = 1.75587$$

$$\log 381 = 2.58092$$

$$2) 4.33679$$

$$\log(\text{緯度差}) = 2.16840$$

$$\text{緯度差} = 147.37'$$

$$= 2^{\circ}27'22''$$

$$(1) \text{ 中度差} = 18^{\circ}13'41''.$$

$$\text{經度差} = 162 \text{ sec}$$

$$18^{\circ}13'41''.$$

$$\log 162 = 2.20952$$

$$\log \sec 18^{\circ}13'41''$$

$$= 0.02236$$

$$\log(\text{經度差}) = 3.23188$$

$$\text{經經度} = 170.56'$$

$$= 2^{\circ}50'34''.$$

$$(2) \text{ 中度差} = 15^{\circ}46'19''$$

$$\text{經度差} = 162 \text{ sec}$$

$$15^{\circ}46'19''.$$

$$\log 162 = 2.20952$$

$$\log \sec 15^{\circ}46'19''$$

$$= 0.01667$$

$$\log(\text{經度差}) = 2.22619$$

$$\text{經度差} = 168.34'$$

$$= 2^{\circ}48'20''.$$

(1) 行向爲北偏東  $47^{\circ}42'33''$ , 所至之處即北緯  $19^{\circ}27'22''$  東經  $121^{\circ}59'34''$ .

行向爲北偏西  $47^{\circ}42'33''$ , 其所至之處即北  $19^{\circ}27'22''$  東經  $116^{\circ}9'26''$ .

(2) 行向爲南偏東  $41^{\circ}42'$

$33''$ , 所至之處即北緯  $14^{\circ}32'38''$  東經  $121^{\circ}48'29''$ .

行向爲南偏西  $47^{\circ}42'33''$ , 所至之處即北緯  $14^{\circ}32'38''$  東經  $116^{\circ}11'40''$ .

129. 一船自緯度  $30^{\circ}$  向東駛行 360 哩。求起點與所到處之最短距。又設緯線爲  $45^{\circ}$  及  $60^{\circ}$ 。試解之。

按 116 題,

緯度圈之半徑 =  $3962.8 \cos(\text{緯度})$ . 所經弧之度數

$$= \frac{360 \times 360}{2\pi \times 3962.8 \cos(\text{緯度})}$$

$$\log 360^2 = 5.11260$$

$$\text{colog}^2 \pi = 9.20182 - 10$$

$$\text{colog } 3962.8 = 6.40200 - 14$$

$$0.71642$$

所經弧之度數 =  $5.205 \text{ sec}(\text{緯度})$ .

所行弧之分數 =  $312.3(\text{緯度})$

弧之通弦

$$= 2(\text{緯度圈半徑}) \sin(\frac{1}{2}\text{弧})$$

$$= 2 \times 3962.8 \cos(\text{緯度})$$

$$\times \sin(156.15' \text{ sec 緯度})$$

$$= 7925.6 \cos \text{緯度}$$

$\sin(156.15 \text{ sec 緯度}).$

(1) 緯度 =  $30^\circ$ .

$$\log 156.15 = 2.19356$$

$$\log \sec 30^\circ = 0.06247$$

$$\log \frac{1}{2} \text{弧} = 2.25604$$

$$\frac{1}{2} \text{弧} = 180.30'$$

$$= 3^\circ 0' 18''.$$

$$\log 7925.6 = 3.89903$$

$$\log \cos 30^\circ = 9.93753$$

$$\log \sin 3^\circ = 0' 18''$$

$$= 8.71652$$

$$\log \text{通弦} = 2.55603$$

$$\text{通弦} = 359.82.$$

(2) 緯度 =  $45^\circ$ .

$$\log 156.15 = 2.19354$$

$$\log \sec 45^\circ = 0.15051$$

$$\log \frac{1}{2} \text{弧} = 2.34405$$

$$\frac{1}{2} \text{弧} = 220.82'$$

$$= 3^\circ 60.9''$$

$$\log 7925.6 = 3.89903$$

$$\log \cos 45^\circ = 9.84949$$

$$\log \sin 3^\circ 40' 49''$$

$$= 8.80746$$

$$\log \text{通弦} = 2.55598$$

$$\text{通弦} = 359.73.$$

(3) 緯度 =  $60^\circ$

$$\sec \text{ 緯度} = 2.$$

$$\frac{1}{2} \text{弧} = 212.30'$$

$$= 5^\circ 12' 18''.$$

$$\log 7925.6 = 3.89903$$

$$\log \sin 5^\circ 12' 18''$$

$$\log \cos 60^\circ = 9.69897$$

$$\log \text{通弦} = 2.55570$$

$$\text{通弦} = 359.50.$$

緯度  $30^\circ$ , 最短之距為 359.82 哩.

緯度  $45^\circ$ , 最短之距為 359.73 哩.

緯度  $60^\circ$ , 最短之距為 359.50 哩.

其常式為  $7925.6 \cos(\text{緯度}) \sin(156.15 \text{ sec 緯度})$

130. 一船離南緯  $37^\circ 16'$  西徑  $18^\circ 42'$  駛行向東北行 104 哩. 再向北微西行 60 哩. 再向西南行 216 哩. 求所到之位置. 及自起點之方位及距離.

第一行方向 =  $45^\circ$ .

$$\dots \text{度差} = 104 \cos 45^\circ.$$

$$\text{子午距} = 104 \sin 45^\circ.$$

$$\log 104 = 2.01763$$

$$\log \cos 45^\circ = 9.84949$$

$$\begin{aligned} \log \text{緯度差} &= 1.86652 \\ \text{緯度差} &= \text{N.} 75.54. \\ \text{子午距} &= \text{E.} 73.54. \\ \text{第二行方向} &= 22^\circ 30'. \\ \text{緯度差} &= 60 \cos 22^\circ 30'. \\ \text{子午距} &= 60 \sin 22^\circ 30'. \\ \log 50 &= 1.778815. \\ \log \cos 22^\circ 30' &= \underline{9.95562} \\ \log \text{緯度差} &= 1.74373. \\ \text{緯度差} &= \text{北 } 55.434 \\ \log 60 &= 1.77815 \\ \log \sin 22^\circ 30' &= \underline{9.58284} \\ \log \text{子午距} &= 1.36099 \\ \text{子午距} &= \text{W.} 22.951. \\ \text{第三行方向} &= 78^\circ 45'. \\ \text{緯度差} &= 216 \cos 78^\circ 45'. \\ \text{子午距} &= 219 \sin 78^\circ 45'. \\ \log 216 &= 2.33445 \\ \log \cos 78^\circ 45' &= \underline{9.29024} \\ \log \text{緯度差} &= 1.6246 \\ \text{緯度差} &= \text{S.} 42.14. \\ \log 216 &= 2.33445 \\ \log \sin 78^\circ 46' &= \underline{9.99157} \\ \log \text{子午距} &= 2.32602 \\ \text{子午距} &= \text{W.} 211.85. \end{aligned}$$

$$\begin{aligned} \text{各緯差之和} &= \text{N.} 86.834' \\ &= \text{N.} 1^\circ 26' 50''. \\ \text{所到之緯度} &= \text{S.} 35^\circ 49' 10''. \\ \text{中緯度} &= 36^\circ 32' 35''. \\ \text{各子午距之和} &= \text{W.} 161.271. \\ \text{經度差} &= 161.271 \sec 36^\circ 32' 35'' \\ \log 161.271 &= 2.20755 \\ \log \sec 36^\circ 32' 35'' &= \underline{0.09506} \\ \log \text{經差} &= 2.30261 \\ \text{經度差} &= 200.73' \\ &= 3^\circ 20' 44''. \\ \text{所到之經度} &= \text{W.} 22^\circ 2' 44''. \\ \tan(\text{方向}) &= \frac{161.271}{86.834} \\ \log 161.271 &= 2.20755 \\ \text{colog } 86.834 &= \underline{8.06131} - 10 \\ \log \tan(\text{方向}) &= 10.26886 \\ \text{方向} &= 61^\circ 42'. \\ \text{離原處之遠} &= 86.834 \sec 61^\circ 42'. \\ \log 86.834 &= 1.93869 \\ \log \sec 61^\circ 42' &= \underline{0.32414} \\ \log(\text{距}) &= 2.26283 \\ \text{離原處之遠} &= 183.16. \\ \text{所到之處在原處之北偏西 } &61^\circ \end{aligned}$$



42', 其遠爲 183.16 海里, 其緯度爲南  $35^{\circ}49'10''$ , 其經度爲西  $22^{\circ}2'44''$

131. 一船離神角(120題) 駛行 $\circ$ 向南東微南  $114$  哩 $\circ$  再向北微東  $94$  哩 $\circ$  再向西北  $42$  西哩 $\circ$  如 130 題解之 $\circ$

第一行方向 =  $33^{\circ}45'$ .

緯度差 =  $114 \cos 33^{\circ}45'$ .

子午距 =  $114 \sin 33^{\circ}45'$ .

$\log 114 = 2.05690$

$\log \cos 33^{\circ}45' = 9.91985$

$\log(\text{緯度差}) = 1.97675$

緯度差 = S.  $94.787$ .

$\log 114 = 2.05690$

$\log \sin 33^{\circ}45' = 9.74474$

$\log(\text{子午距}) = 1.80164$

子午距 = E.  $63.334$ .

第二行方向 =  $11^{\circ}15'$ .

緯度差 =  $94 \cos 11^{\circ}15'$ .

子午距 =  $94 \sin 11^{\circ}15'$ .

$\log 94 = 1.97313$

$\log \cos 11^{\circ}15' = 9.99157$

$\log(\text{緯度差}) = 1.99470$ .

緯度差 = N.  $92.164$ .

$\log 94 = 1.97413$

$\log \sin 11^{\circ}15' = 9.29024$

$\log \text{子午距} = 1.26337$

子午距 = E.  $18.339$ .

第三行方向 =  $67^{\circ}30'$

緯度差 =  $42 \cos 67^{\circ}30'$ .

子午距 =  $42 \sin 67^{\circ}30'$ .

$\log 42 = 1.62325$

$\log \cos 67^{\circ}30' = 9.58284$

$\log \text{緯度差} = 1.20609$

緯度差 = N.  $16.073$ .

$\log 42 = 1.62325$

$\log \sin 67^{\circ}30' = 9.96562$

$\log \text{子午距} = 1.58887$

子午距 = W.  $38.804$ .

各緯度差之和 = N.  $13.48'$

= N.  $13'29''$ .

確德角之緯度 =  $42^{\circ}2'$ .

所到之緯度 = N.  $42^{\circ}15'29''$ .

中緯度 =  $42^{\circ}8'44''$ .

子午距之和 = E.  $42.869$ .

經度差 =  $42.869 \sec 42^{\circ}8'44''$ .

$\log 42.869 = 1.63214$

$\log \sec 42^{\circ}8'44''$

$$=0.12992$$

$$\log \text{經度差} = 1.76206$$

$$\text{經度差} = E.47.817'$$

$$= E.57'49''.$$

$$\text{確德角之經度} = W.73^{\circ}3'.$$

$$\text{所到之經度} = W.69^{\circ}5'11''.$$

$$\tan(\text{方向}) = \frac{42.869}{13.48}$$

$$\log 42.869 = 1.63214$$

$$\text{colog } 13.48 = 8.87031 - 10$$

$$\log \tan(\text{方向}) = 10.50245$$

$$\text{方向} = 72^{\circ}32'40''.$$

離原處之遠

$$= 13.48 \sec 72^{\circ}32'40''.$$

$$\log 13.38 = 1.12969$$

$$\log \sec 72^{\circ}32'40''$$

$$= 0.52293$$

$$= 1.65262$$

$$\text{離原處之遠} = 44.939.$$

所到之處在原處之北偏東

$72^{\circ}32'40''$ ，其遠為 44.939

海里，其緯度為北  $42^{\circ}15'$

$29''$ ，其經為西  $59^{\circ}5'11''$ 。

132. 一船離好望角 (南緯

$34^{\circ}22'$ ，東經  $18^{\circ}30'$ ) 駛行。向北西 126 浬。再向北微東  $184'$  再向西南西 217 浬。如 130 題解之。

第一行方向 =  $45^{\circ}$ 。

$$\text{緯度差} = 126 \cos 45^{\circ}.$$

$$\text{子午距} = 126 \sin 45^{\circ}.$$

$$\log 126 = 2.10037$$

$$\log \cos 45^{\circ} = 9.84949$$

$$\log \text{緯度差} = 1.94986$$

$$\text{緯度差} = N.89.096.$$

$$\text{子午距} = W.89.096.$$

第二行方向 =  $11^{\circ}15'$ 。

$$\text{緯度差} = 84 \cos 11^{\circ}15'.$$

$$\text{子午距} = 84 \sin 11^{\circ}15'.$$

$$\log 84 = 1.92428$$

$$\log \cos 11^{\circ}15' = 9.99157$$

$$\log \text{緯度差} = 1.91585$$

$$\text{緯度差} = N.82.386.$$

$$\log 84 = 1.92428$$

$$\log \sin 11^{\circ}15' = 9.29024$$

$$\log \text{子午距} = 1.21452$$

$$\text{子午距} = E16.388.$$

第三行方向 =  $67^{\circ}30'$ 。

$$\text{緯度差} = 217 \cos 67^{\circ}30'$$

$$\text{子午距} = 217 \sin 67^{\circ}30'$$

$$\begin{aligned} \log 217 &= 2.33646 \\ \log \cos 67^\circ 30' &= \underline{9.58284} \\ \log \text{緯度差} &= 1.91930 \\ \text{緯度差} &= S.83.042. \\ \log 217 &= 2.33646 \\ \log \sin 67^\circ 30' &= \underline{9.96562} \\ \log \text{子午距} &= 2.30208 \\ \text{子午距} &= W.200.49. \\ \text{各緯差之和} &= N.88440' \\ &= N.1^\circ 28' 26'' \\ \text{所到之緯度} &= S.32^\circ 53' 34''. \\ \text{中緯度} &= 33^\circ 30' 47''. \\ \text{子午距之和} &= W.273.198. \\ \text{經度差} &= 273.198 \text{ sec} \\ &= 33^\circ 37' 47''. \\ \log 273.198 &= 2.43648 \\ \log \sec 33^\circ 37' 47'' &= \underline{0.07954} \\ \log \text{經度差} &= 2.51602 \\ \text{經度差} &= 328.11' \end{aligned}$$

$$\begin{aligned} &= 5^\circ 28' 7''. \\ \text{所至之經度} &= E.13^\circ 1' 53'. \\ \tan (\text{方向}) &= \frac{273.198}{88.44}. \\ \log 273.198 &= 2.43648 \\ \text{colog } 88.44 &= \underline{8.05335 - 10} \\ \log \tan (\text{方向}) &= 10.48983 \\ \text{方向} &= 72^\circ 3' 4''. \\ \text{離原處之遠} &= 88.44 \sec 72^\circ 3' 4'' \\ \log 88.44 &= 1.94665 \\ \log \sec 72^\circ 3' 43'' &= \underline{0.51147} \\ &= 2.45812 \\ \text{離原處之遠} &= 287.16 \\ \text{所至之處在原處之北偏西} &72^\circ 3' 43'', \text{其遠爲 } 287.16 \\ \text{海里, 其緯度爲南 } &32^\circ 53' 34, \text{其經度爲東 } 13^\circ 1' 53''. \end{aligned}$$

## 例題 XXIV

### 測角法內之問題

1. 試證  $\sin x + \cos x = \sqrt{2} \cos (x - \frac{1}{4}\pi)$ .

按 [9],  $\cos (x - \frac{1}{4}\pi) = \cos x \cos \frac{1}{4}\pi + \sin x \sin \frac{1}{4}\pi$

$$\begin{aligned}
 &= \frac{1}{2}\sqrt{2} \cos x + \frac{1}{2}\sqrt{2} \sin x \\
 &= \frac{1}{2}\sqrt{2} (\cos x + \sin x).
 \end{aligned}$$

故得  $\sin x + \cos x = \sqrt{2} \cos(x + \frac{1}{4}\pi)$ .

2.  $\sin x - \cos x = -\sqrt{2} \cos(x + \frac{1}{4}\pi)$ 。

按 [5],  $\cos(x + \frac{1}{4}\pi) = \cos x \cos \frac{1}{4}\pi - \sin x \sin \frac{1}{4}\pi$

$$\begin{aligned}
 &= \frac{1}{2}\sqrt{2} \cos x - \frac{1}{2}\sqrt{2} \sin x \\
 &= \frac{1}{2}\sqrt{2} (\cos x - \sin x).
 \end{aligned}$$

故得  $\sin x - \cos x = -\sqrt{2} \cos(x + \frac{1}{4}\pi)$ .

3.  $\sin x + \sqrt{3} \cos x = 2 \sin(x + \frac{1}{3}\pi)$ 。

按 [4],  $\sin(x + \frac{1}{3}\pi) = \sin x \cos \frac{1}{3}\pi + \cos x \sin \frac{1}{3}\pi$

$$\begin{aligned}
 &= \frac{1}{2} \sin x + \frac{1}{2}\sqrt{3} \cos x \\
 &= \frac{1}{2}(\sin x + \sqrt{3} \cos x).
 \end{aligned}$$

即得  $\sin x + \sqrt{3} \cos x = 2 \sin(x + \frac{1}{3}\pi)$

4.  $\sin(x + \frac{1}{3}\pi) + \sin(x - \frac{1}{3}\pi) = \sin x$ 。

按 [4],  $\sin(x + \frac{1}{3}\pi) = \sin x \cos \frac{1}{2}\pi + \cos x \sin \frac{1}{3}\pi$

$$= \frac{1}{2} \sin x + \frac{1}{2}\sqrt{3} \cos x.$$

按 [8],  $\sin(x - \frac{1}{3}\pi) = \frac{1}{2} \sin x - \frac{1}{2}\sqrt{3} \cos x$ 。

即得  $\sin(x + \frac{1}{3}\pi) + \sin(x - \frac{1}{3}\pi)$

$$= \sin x.$$

5.  $\cos(x + \frac{1}{6}\pi) + \cos(x - \frac{1}{6}\pi) = \sqrt{3} \cos x$ 。

按 [5],  $\cos(x + \frac{1}{6}\pi) = \frac{1}{2}\sqrt{3} \cos x - \frac{1}{2} \sin x$ 。

按 [9],  $\cos(x - \frac{1}{6}\pi) = \frac{1}{2}\sqrt{3} \cos x + \frac{1}{2} \sin x$ 。

即得  $\cos(x + \frac{1}{6}\pi) + \cos(x - \frac{1}{6}\pi)$

$$= \sqrt{3} \cos x,$$

$$6. \quad \tan x + \sec x = \tan\left(\frac{1}{2}x + \frac{1}{4}\pi\right).$$

$$\begin{aligned} \tan x + \sec x &= \frac{\sin x}{\cos x} + \frac{1}{\cos x} = \frac{\sin x + 1}{\cos x} \\ &= \frac{1 - \cos\left(x + \frac{1}{2}\pi\right)}{\sin\left(x + \frac{1}{2}\pi\right)} \end{aligned}$$

$$\begin{aligned} \text{按 [16] 及 [12],} \quad &= \frac{2 \sin^2 \frac{1}{4}\left(x + \frac{1}{2}\pi\right)}{2 \sin \frac{1}{2}\left(x + \frac{1}{2}\pi\right) \cos \frac{1}{2}\left(x + \frac{1}{2}\pi\right)} \\ &= \frac{\sin\left(\frac{1}{2}x + \frac{1}{4}\pi\right)}{\cos\left(\frac{1}{2}x + \frac{1}{4}\pi\right)} \\ &= \tan\left(\frac{1}{2}x + \frac{1}{4}\pi\right). \end{aligned}$$

$$7. \quad \tan x + \sec x = \frac{1}{\sec x - \tan x}.$$

$$\begin{aligned} \text{按} \quad & \sec^2 x = 1 + \tan^2 x, \\ & \sec^2 x - \tan^2 x = 1. \end{aligned}$$

$$(\sec x + \tan x)(\sec x - \tan x) = 1.$$

$$\text{即得} \quad \tan x + \sec x = \frac{1}{\sec x - \tan x}.$$

$$8. \quad \frac{1 - \tan x}{1 + \tan x} = \frac{\cot x - 1}{\cot x + 1}.$$

$$\frac{\cot x - 1}{\cot x + 1} = \frac{\frac{1}{\tan x} - 1}{\frac{1}{\tan x} + 1} = \frac{1 - \tan x}{1 + \tan x}.$$

$$9. \quad \frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x} = 2 \cos x.$$

$$\begin{aligned} \frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x} &= \frac{\sin^2 x + (1 + \cos x)^2}{\sin x(1 + \cos x)} \\ &= \frac{\sin^2 x + \cos^2 x + 2 \cos x + 1}{\sin x(1 + \cos x)} \end{aligned}$$

$$\begin{aligned}
&= \frac{1 + 2 \cos x + 1}{\sin x (1 + \cos x)} \\
&= \frac{2 (1 + \cos x)}{\sin x (1 + \cos x)} \\
&= \frac{2}{\sin x} \\
&= 2 \csc x.
\end{aligned}$$

10.  $\tan x + \cot x = 2 \csc 2x$

$$\begin{aligned}
\tan x + \cot x &= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \\
&= \frac{1}{\sin x \cos x} = \frac{2}{2 \sin x \cos x} = \frac{2}{2 \sin 2x}
\end{aligned}$$

按 [12],  $= 2 \csc 2x$ .

11.  $\cot x - \tan x = 2 \cot 2x$

$$\begin{aligned}
\cot x - \tan x &= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \frac{\cos^2 x - \sin^2 x}{\sin x \cos x} \\
&= \frac{2 \cos 2x}{\sin 2x} = 2 \cot 2x.
\end{aligned}$$

12.  $1 + \tan x \tan 2x = \sec 2x$

$$\begin{aligned}
1 + \tan x \tan 2x &= 1 + \frac{\sin x \sin 2x}{\cos x \cos 2x} \\
&= 1 + \frac{2 \sin^2 x \cos x}{\cos x (1 - 2 \sin^2 x)} \\
&= 1 + \frac{2 \sin^2 x}{1 - 2 \sin^2 x} \\
&= \frac{1}{1 - 2 \sin^2 x} = \frac{1}{\cos 2x} = \sec 2x.
\end{aligned}$$

$$13. \quad \sec 2x = \frac{\sec^2 x}{2 - \sec^2 x}$$

$$\begin{aligned} \sec 2x &= \frac{1}{\cos^2 x} = \frac{1}{2 \cos^2 x - 1} + \frac{\frac{1}{\cos^2 x}}{2 - \frac{1}{\cos^2 x}} \\ &= \frac{\sec^2 x}{2 - \sec^2 x} \end{aligned}$$

$$14. \quad 2 \sec 2x = \sec(x + 45^\circ) \sec(x - 45^\circ)$$

$$\begin{aligned} 2 \sec 2x &= \frac{2}{\cos^2 x} = \frac{2}{\cos^2 x - \sin^2 x} \\ &= \frac{2}{(\cos x - \sin x)(\cos x + \sin x)} \\ &= \frac{2}{2 \cos(x + 45^\circ) \cos(x - 45^\circ)} \\ &= \sec(x + 45^\circ) \sec(x - 45^\circ). \end{aligned}$$

按 24/1, 2,

$$15. \quad \tan 2x + \sec 2x = \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$\begin{aligned} \tan 2x + \sec 2x &= \frac{\sin 2x}{\cos 2x} + \frac{1}{\cos 2x} \\ &= \frac{\sin 2x + 1}{\cos 2x} \\ &= \frac{1 - \cos(2x + 90^\circ)}{\sin(2x + 90^\circ)} \end{aligned}$$

$$\begin{aligned} \text{按 [16] 及 [12],} &= \frac{2 \sin^2(x + 45^\circ)}{2 \sin(x + 45^\circ) \cos(x + 45^\circ)} \\ &= \frac{\sin(x + 45^\circ)}{\cos(x + 45^\circ)} \end{aligned}$$

$$\begin{aligned}
 \text{按 [4] 及 [5],} &= \frac{\sin x \cos 45^\circ + \cos x \sin 45^\circ}{\cos x \cos 45^\circ - \sin x \sin 45^\circ} \\
 &= \frac{\frac{1}{2} \sqrt{2} \sin x + \frac{1}{2} \sqrt{2} \cos x}{\frac{1}{2} \sqrt{2} \cos x - \frac{1}{2} \sqrt{2} \sin x} \\
 &= \frac{\cos x + \sin x}{\sin x - \cos x}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad \sin 2x &= \frac{2 \tan x}{1 + \tan^2 x} \\
 &= \frac{2 \tan x}{1 + \tan^2 x} = \frac{2 \tan x}{\sec^2 x} = 2 \tan x \cos^2 x = 2 \sin x \cos x \\
 &= \sin 2x.
 \end{aligned}$$

$$17. \quad 2 \sin x + \sin 2x = \frac{2 \sin^2 x}{1 - \cos x}.$$

$$2 \sin x + \sin 2x = 2 \sin x + 2 \sin x \cos x = 2 \sin x (1 + \cos x).$$

$$\text{但} \quad 1 - \cos^2 x = \sin^2 x,$$

$$\text{故} \quad 1 + \cos x = \frac{\sin^2 x}{1 - \cos x}.$$

$$\text{而} \quad 2 \sin x + \sin 2x = 2 \sin x \frac{\sin^2 x}{1 - \cos x} = \frac{2 \sin^3 x}{1 - \cos x}.$$

$$18. \quad \sin 3x = \frac{\sin^2 2x - \sin^2 x}{\sin x}.$$

$$\text{按 [20], } \sin 2x + \sin x = 2 \sin \frac{3}{2}x \cos \frac{1}{2}x.$$

$$\text{按 [21], } \sin 2x - \sin x = 2 \cos \frac{3}{2}x \sin \frac{1}{2}x.$$

$$\text{故} \quad \sin^2 2x - \sin^2 x = 2 \sin \frac{3}{2}x \cos \frac{3}{2}x.$$

$$\text{按 [12],} \quad 2 \sin \frac{1}{2}x \cos \frac{1}{2}x$$

$$= \sin 3x \sin x.$$

$$\text{故} \quad \sin 3x = \frac{\sin^2 2x - \sin^2 x}{\sin x}.$$



$$19. \quad \tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} \circ$$

$$\tan 3x = \tan(2x + x)$$

$$\text{按 [6],} \quad = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$

$$\begin{aligned} \text{按 [14],} \quad &= \frac{\frac{2 \tan x}{1 - \tan^2 x} + \tan x}{1 - \frac{2 \tan x}{1 - \tan^2 x} \tan x} \\ &= \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} \circ \end{aligned}$$

$$20. \quad \frac{\tan 2x + \tan x}{\tan 2x - \tan x} = \frac{\sin 3x}{x \sin x} \circ$$

$$\text{按 [24],} \quad \frac{\sin A + \sin B}{\sin A - \sin B} = \frac{\sin \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}$$

以合理與分理改之,得

$$\frac{\sin A}{\sin B} = \frac{\tan \frac{1}{2}(A+B) + \tan \frac{1}{2}(A-B)}{\tan \frac{1}{2}(A+B) - \tan \frac{1}{2}(A-B)}$$

以  $3x$  代  $A$ ,  $x$  代  $B$ , 則

$$\frac{\sin 3x}{\sin x} = \frac{\tan 2x + \tan x}{\tan 2x - \tan x}$$

$$21. \quad \sin(x+y) + \cos(x-y) = 2 \sin(x + \frac{1}{4}\pi) \sin(y + \frac{1}{4}\pi) \circ$$

$$\text{按 [4],} \quad \sin(x+y) = \sin x \cos y + \cos x \sin y.$$

$$\text{按 [9],} \quad \cos(x+y) = \cos x \cos y + \sin x \sin y.$$

$$\sin(x+y) + \cos(x-y) = (\sin x + \cos x) \cos y + (\cos x + \sin x) \sin y$$

$$\sin y = (\sin x + \cos x)(\cos y + \sin y).$$

$$\begin{aligned} \text{但} \quad \sin x + \cos x &= \sqrt{2} \left( \frac{1}{2} \sqrt{2} \sin x + \frac{1}{2} \sqrt{2} \cos x \right) \\ &= \sqrt{2} \sin(x + \frac{1}{4}\pi) \end{aligned}$$

仿此,  $\sin y + \cos y = \sqrt{2} \left( \sin y + \frac{1}{2}\pi \right)$ .

故  $\sin(x+y) + \cos(x-y) = 2 \sin \left( x + \frac{1}{2}\pi \right) \sin \left( y + \frac{1}{2}\pi \right)$ .

$$22. \quad \sin(x+y) - \cos(x-y) = -2 \sin \left( x - \frac{1}{2}\pi \right) \sin \left( y - \frac{1}{2}\pi \right).$$

按 [4],  $\sin(x+y) = \sin x \cos y + \cos x \sin y$ .

按 [9],  $\cos(x-y) = \cos x \cos y + \sin x \sin y$ .

$$\begin{aligned} \sin(x+y) - \cos(x-y) &= (\sin x - \cos x) \cos y + (\sin x - \cos x) \\ &\quad \sin y = (\sin x - \cos x)(\cos y - \sin y) \\ &= -2 \sin \left( x - \frac{1}{2}\pi \right) \sin \left( y - \frac{1}{2}\pi \right). \end{aligned}$$

$$23. \quad \tan x + \tan y = \frac{\sin(x+y)}{\cos x \cos y}.$$

$$\begin{aligned} \tan x + \tan y &= \frac{\sin x}{\cos x} + \frac{\sin y}{\cos y} \\ &= \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y} \end{aligned}$$

$$\text{按 [4],} \quad = \frac{\sin(x+y)}{\cos x \cos y}.$$

$$24. \quad \tan(x+y) = \frac{\sin 2x + \sin 2y}{\cos 2x + \cos 2y}.$$

按 [20],  $\sin 2x + \sin 2y = 2 \sin(x+y) \cos(x-y)$ .

按 [22],  $\cos 2x + \cos 2y = 2 \cos(x+y) \cos(x-y)$ .

$$\begin{aligned} \text{故} \quad \frac{\sin 2x + \sin 2y}{\cos 2x + \cos 2y} &= \frac{2 \sin(x+y) \cos(x-y)}{2 \cos(x+y) \cos(x-y)} \\ &= \tan(x+y). \end{aligned}$$

$$25. \quad \frac{\sin x + \cos y}{\sin x - \cos y} = \frac{\tan \left[ \frac{1}{2}(x+y) + 45^\circ \right]}{\tan \left[ \frac{1}{2}(x-y) - 45^\circ \right]}.$$

$$\sin x + \cos y = \sin x + \sin(y + 90^\circ)$$

$$\text{按 [20],} \quad = 2 \sin \frac{1}{2}(x+y+90^\circ) \cos \frac{1}{2}(x-y-90^\circ).$$

按 [21],  $\sin x - \cos y = 2 \cos \frac{1}{2}(x+y+90^\circ) \sin \frac{1}{2}(x-y-90^\circ)$ .

$$\begin{aligned} \text{故 } \frac{\sin x + \cos y}{\sin x - \cos y} &= \frac{\tan \frac{1}{2}(x+y+90^\circ)}{\tan \frac{1}{2}(x-y-90^\circ)} \\ &= \frac{\tan[\frac{1}{2}(x+y)+45^\circ]}{\tan[\frac{1}{2}(x-y)-45^\circ]}. \end{aligned}$$

26.  $\sin 2x + \sin 4x = 2 \sin 3x \cos x$ .

按 [20],  $\sin 2x + \sin 4x = 2 \sin 3x \csc x$ .

27.  $\sin 4x = 4 \sin x \cos^3 x - 8 \sin^3 x \cos x = 8 \cos^3 x \sin x - 4 \cos x \sin^3 x$

按 [12],  $\sin 4x = 2 \sin 2x \cos 2x$ .

$$\begin{aligned} \text{按 [12], [13], } &= 4 \sin x \cos x (2 - 2 \sin^2 x) \\ &= 4 \sin x \cos x - 8 \sin^3 x \cos x. \end{aligned}$$

$$\begin{aligned} \text{又 } \sin 4x &= 4 \sin x \cos x (2 \cos^2 x - 1) \\ &= 8 \cos^3 x \sin x - 4 \sin x \cos x. \end{aligned}$$

28.  $\cos 4x - 1 = 8 \cos^2 x + 8 \cos^4 x = 1 - 8 \sin^2 x + 8 \sin^4 x$ .

$$\begin{aligned} \text{按 [13], } \cos 4x &= 2 \cos^2 2x - 1 \\ &= 2 (2 \cos^2 x - 1)^2 - 1 \\ &= 8 \cos^4 x - 8 \cos^2 x + 2 - 1 \\ &= 1 - 8 \cos^2 x + 8 \cos^4 x. \end{aligned}$$

$$\begin{aligned} \text{又 } \cos 4x &= 1 - 2 \sin^2 2x \\ \text{按 [12], } &= 1 - 2 (4 \sin^2 x \cos^2 x) \\ &= 1 - 8 \sin^2 x \cos^2 x \\ &= 1 - 8 \sin^2 x + 8 \cos^4 x. \end{aligned}$$

29.  $\cos 2x + \cos 4x = 2 \cos 3x \cos x$ .

按 [22],  $\cos 2x + \cos 4x = 2 \cos 3x \cos x$ .

30.  $\sin 3x - \sin x = 2 \cos 2x \sin x$ .

按 [21],  $\sin 3x - \sin x = 2 \cos 2x \sin x$ .

31.  $\sin^3 x \sin 3x - \cos^3 x \cos 3x = \cos^2 2x$   
 $\sin^2 x \sin 3x = \sin x \sin^2 x \sin 3x$   
 $= \sin x (1 - \cos^2 x) \sin 3x$   
 $= \sin x \sin 3x = \sin x \cos^2 x \sin 3x$ .

$\cos^3 x \cos 3x = \cos x \cos^2 x \cos 3x$   
 $= \cos x (1 - \sin^2 x) \cos 3x$   
 $= \cos x \cos 3x - \cos \sin^2 x \cos 3x$

故  $\sin^3 x \sin 3x + \cos^3 x \cos 3x$   
 $= \sin x \sin 3x + \cos x \cos 3x$   
 $- \sin x \cos^2 x \sin 3x - \cos x \sin^2 x \cos 3x$ .

按 [9].  $= \cos(3x - x)$   
 $= \sin x \cos x (\cos x \sin 3x + \sin x \cos 3x)$   
 $= \cos(3x - x)$

按 [4],  $= \cos 2x - \sin x \cos \sin(3x + x)$

按 [12],  $= \cos 2x - \frac{1}{2} \sin 2x \sin 4x$

按 [12],  $= \cos 2x - \sin^2 2x \cos 2x$

$= \cos 2x (1 - \sin^2 2x)$

按 [13],  $= \cos^3 2x$ .

32.  $\cos^4 x - \sin^4 x = \cos 2x$

$\cos^4 x - \sin^4 x = (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$   
 $= 1 \times \cos 2x = \cos 2x$ .

33.  $\cos^4 x + \sin^4 x = 1 - \frac{1}{2} \sin^2 2x$

$\cos^4 x + \sin^4 x = (\cos^2 x + \sin^2 x) - 2 \sin^2 x \cos^2 x$   
 $= 1 - 2 \sin^2 x \cos^2 x = 1 - \frac{1}{2} \sin^2 2x$

$$34. \quad \cos^6 x - \sin^6 x = (1 - \sin^2 x \cos^2 x) \cos 2x^0$$

$$\cos^6 x - \sin^6 x = (\cos^2 x - \sin^6 x)(\cos^4 x + \cos^2 x \sin^2 x \sin^4 x)$$

按 [13],

$$= \cos 2x \{ (\cos^2 x + \sin^2 x)^2 - \cos^2 x \sin^2 x \}$$

$$= \cos 2x (1 - \cos^2 x \sin^2 x).$$

$$35. \quad \cos^6 x + \sin^6 x = 1 - 3 \sin^2 x \cos^2 x_0$$

$$\cos^6 x + \sin^6 x = (\cos^2 x + \sin^2 x)(\cos^4 x - \cos^2 x \sin^2 x + \sin^4 x)$$

$$= \cos^4 x - \cos^2 x \sin^2 x + \sin^4 x,$$

$$= (\cos^2 x + \sin^2 x)^2 - 3 \cos^2 x \sin^2 x$$

$$= 1 - 3 \cos^2 x \sin^2 x.$$

$$36. \quad \frac{\sin 3x + \sin 5x}{\cos 3x - \cos 5x} = \cot x$$

按 [20],  $\sin 3x + \sin 5x = 2 \sin 4x \cos x.$

按 [23],  $\cos 3x - \cos 5x = 2 \sin 4x \sin x.$

故  $\frac{\sin 3x + \sin 5x}{\cos 3x - \cos 5x} = \frac{\cos x}{\sin x} = \cot x'$

$$37. \quad \frac{\sin 3x + \sin 5x}{\sin x + \sin 3x} = 2 \cos 2x_0$$

按 [20],  $\sin 3x + \sin 5x = 2 \sin 4x \cos x.$

按 [20],  $\sin x + \sin 3x = 2 \sin 2x \cos x.$

故  $\frac{\sin 3x + \sin 5x}{\sin x + \sin 3x} = \frac{\sin 4x}{\sin 2x}$

按 [12],  $= \frac{2 \sin 2x \cos 2x}{\sin 2x}$

$$= 2 \cos 2x.$$

$$38. \quad \csc x - 2 \cot 2x \cos x = 2 \sin x^{\circ}$$

$$\csc x - 2 \cot 2x \cos x = \csc x - 2 \frac{\cos 2x}{\sin 2x} \cos x$$

$$\text{按 [12],} \quad = \csc x - \frac{2 \cos 2x \cos x}{2 \sin x \cos x}$$

$$= \frac{1}{\sin x} - \frac{\cos 2x}{\sin x} = \frac{1 - \cos 2x}{\sin x}$$

$$\text{按 [13],} \quad = \frac{2 \sin^2 x}{\sin x} = 2 \sin x.$$

$$39. \quad (\sin 2x - \sin 2y) \tan(x+y) = 2(\sin^2 x - \sin^2 y)^{\circ}$$

$$\text{按 [21],} \quad \sin^2 x - \sin^2 y = 2 \cos(x+y) \sin(x-y).$$

$$\text{故} \quad (\sin^2 x - \sin^2 y) \tan(x+y) = 2 \sin(x+y) \sin(x-y).$$

$$\text{按 [20],} \quad \sin x + \sin y = 2 \sin \frac{1}{2}(x+y) \cos \frac{1}{2}(x-y).$$

$$\text{按 [20],} \quad \sin x - \sin y = 2 \cos \frac{1}{2}(x+y) \sin \frac{1}{2}(x-y).$$

$$\text{故} \quad \sin^2 x - \sin^2 y = 4 \sin \frac{1}{2}(x+y) \cos \frac{1}{2}(x+y) \sin \frac{1}{2}(x-y) \cos \frac{1}{2}(x-y)$$

$$\text{按 [*2],} \quad = \sin(x+y) \sin(x-y).$$

$$\text{故} \quad 2(\sin^2 x - \sin^2 y) = 2 \sin(x+y) \sin(x-y)$$

$$= (\sin 2x - \sin 2y) \tan(x+y).$$

$$40. \quad (1 + \cot x + \tan x)(\sin x - \cos x) = \frac{\sec x}{\csc^2 x} - \frac{\csc x}{\sec^2 x}$$

$$(1 + \cot x + \tan x)(\sin x - \cos x) = \sin x - \cos x + \cos x:$$

$$= \frac{\cos^2 x}{\sin x} + \frac{\sin^2 x}{\cos x} = \sin x -$$

$$= \frac{\sin^2 x}{\cos x} - \frac{\cos^2 x}{\sin x} = \frac{\sec x}{\csc^2 x} - \frac{\csc x}{\sec^2 x}$$

$$41. \quad \sin x + \sin 3x + \sin 5x = \frac{\sin^2 3x}{\sin x}.$$

按 [20],  $\sin x + \sin 5x = 2 \sin 3x \cos 2x$ ,

故  $\sin x + \sin 3x + \sin 5x = \sin 3x + 2 \sin 3x \cos 2x$   
 $= \sin 3x (1 + 2 \cos 2x).$

按 [21],  $\sin 3x - \sin x = 2 \cos 2x \sin x$ .

$$\frac{\sin 3x}{\sin x} - 1 = 2 \cos 2x.$$

$$1 + 2 \cos 2x = \frac{\sin 3x}{\sin x}.$$

故  $\sin x + \sin 3x + \sin 5x = \sin 3x \frac{\sin 3x}{\sin x} = \frac{\sin^2 3x}{\sin x}.$

$$42. \quad \frac{3 \cos x + \cos 3x}{3 \sin x - \sin 3x} = \cot^2 x.$$

$$3 \cos x + \cos 3x = 2 \cos x + (\cos x + \cos 3x)$$

按 [22],  $= 2 \cos x + 2 \cos x \cos 2x$   
 $= 2 \cos x (1 + \cos 2x)$

按 [17],  $= 4 \cos^3 x.$

$$3 \sin x - \sin 3x = 2 \sin x - (\sin 3x - \sin x)$$

按 [21],  $= 2 \sin x - 2 \sin x \cos 2x$   
 $= 2 \sin x (1 - \cos 2x)$

按 [16],  $= 4 \sin^3 x.$

故  $\frac{3 \cos x + \cos 3x}{3 \sin x - \sin 3x} = \frac{4 \cos^3 x}{4 \sin^3 x} = \cot^3 x.$

$$43. \quad \sin 3x = 4 \sin x \sin (60^\circ + x) \sin (90^\circ - x).$$

按 [4],  $\sin (60^\circ + x) = \frac{1}{2} \sqrt{3} \cos x + \frac{1}{2} \sin x.$

按 [8],  $\sin (60^\circ - x) = \frac{1}{2} \sqrt{3} \cos x - \frac{1}{2} \sin x.$

故  $\sin(60^\circ + x) \sin(60^\circ - x) = \frac{3}{4} \cos^2 x - \frac{1}{4} \sin^2 x$

$$= \frac{3(1 - \sin^2 x) - \sin^2 x}{4}$$

$$= \frac{3 - 4 \sin^2 x}{4}.$$

$$4 \sin x \sin(60^\circ + x) \sin(60^\circ - x) = \sin x (3 - 4 \sin^2 x)$$

$$= 3 \sin x - 4 \sin^3 x$$

按 14/18,  $= \sin 3x.$

44.  $\sin 4x = 2 \sin x \cos 3x + \sin 2x.$

按 [21],  $\sin 4x - \sin 2x = 2 \cos 3x \sin x.$

故  $\sin 4x = 2 \cos 3x \sin x + \sin 2x.$

45.  $\sin x + \sin(x - \frac{2}{3}\pi) + \sin(\frac{1}{3}\pi - x) = 0.$

按 [20],  $\sin(x - \frac{2}{3}\pi) + \sin(\frac{1}{3}\pi - x)$

$$= 2 \sin(-\frac{1}{6}\pi) \cos(x - \frac{1}{2}\pi)$$

$$= -2 \sin \frac{1}{6}\pi \sin x$$

$$= -\sin x.$$

故  $\sin x + \sin(\frac{2}{3}\pi) + \sin(\frac{1}{3}\pi - x) = 0.$

46.  $\cos x \sin(y - z) + \cos y \sin(z - x) + \cos z \sin(x - y) = 0.$

按 [8],  $\cos x \sin(y - x) = \cos x \sin y \cos z - \cos x \cos y \sin z$

$$\sin y \sin(z - x) = \cos y \sin z \cos x - \cos y \cos z \sin x.$$

$$\cos z \sin(x - y) = \cos z \sin x \cos y - \cos z \cos x \sin y.$$

故  $\cos x \sin(y - z) + \cos y \sin(z - x) + \cos z \sin(x - y) = 0.$

47.  $\cos(x + y) \sin y - \cos(x + z) \sin z = \sin(x + y)$

$$\cos y - \sin(x + z) \cos z.$$

按 [4] 及 [5]

$$\sin(x + y) \cos y - \cos(x + y) \sin y = \sin x (\cos^2 y + \sin^2 y)$$

$$= \sin x.$$



$$\begin{aligned}\sin(x+z)\cos z - \cos(x+z)\sin z &= \sin x(\cos^2 z + \sin^2 z) \\ &= \sin x.\end{aligned}$$

故  $\sin(x+y)\cos y - \cos(x+y)\sin y = \sin(x+z)\cos z - \cos(x+z)\sin z,$

$$\cos(x+y)\sin y - \cos(x+z)\sin x = \sin(x+y)\cos y - \sin(x+z)\cos z.$$

48.  $\cos(x+y+z) + \cos(x+y-z) + \cos(x-y+z) + \cos(y+z-x) = 4\cos x\cos y\cos z_0$

按 [22],  $\cos[(x+y)+z] + \cos[(x+y)-z] = 2\cos(x+y)\cos z.$

$$\cos[x+(x-y)] + \cos[z-(x-y)] = 2\cos z\cos(x-y).$$

故  $\begin{aligned}\cos(x+y+z) + \cos(x+y-z) + \cos(x-y+z) + \cos(y+z-x) &= 2\cos(x+y)\cos z + 2\cos(x-y)\cos z \\ &= 2\cos z[\cos(x+y) + \cos(x-y)] \\ &= 2\cos z(2\cos x\cos y) \\ &= 4\cos x\cos y\cos z.\end{aligned}$

49.  $\sin(x+y)\cos(x-y) + \sin(x+z)\cos(x-z) + \sin(z+x)\cos(z-x) = \sin 2x + \sin 2y + \sin 2z_0$

按 [20],  $\sin(x+y)\cos(x-y) = \frac{1}{2}(\sin 2x + \sin 2y),$

$$\sin(y+z)\cos(y-z) = \frac{1}{2}(\sin 2y + \sin 2z).$$

$$\sin(z+x)\cos(z-x) = \frac{1}{2}(\sin 2z + \sin 2x).$$

故  $\begin{aligned}\sin(x+y)\cos(x-y) + \sin(y+z)\cos(y-z) + \sin(z+x)\cos(x-z) &= \sin 2x + \sin 2y + \sin 2z.\end{aligned}$

50.  $\frac{\sin 75^\circ + \sin 15^\circ}{\sin 75^\circ - \sin 15^\circ} = \tan 60^\circ_0$

按 [20],  $\sin 75^\circ + \sin 15^\circ = 2 \sin 45^\circ \cos 30^\circ$ .

$$\sin 75^\circ - \sin 15^\circ = 2 \cos 45^\circ \sin 30^\circ.$$

故 
$$\frac{\sin 75^\circ + \sin 15^\circ}{\sin 75^\circ - \sin 15^\circ} = \frac{2 \sin 45^\circ \cos 30^\circ}{2 \cos 45^\circ \sin 30^\circ}$$

$$= \tan 45^\circ \cot 30^\circ = \tan 60^\circ.$$

51.  $\cos 20^\circ + \cos 100^\circ + \cos 140^\circ = 0.$

按 [22],  $\cos 20^\circ + \cos 100^\circ = 2 \cos 60^\circ \cos 40^\circ = \cos 40^\circ$ .

又  $\cos 140^\circ = \cos (180^\circ - 40^\circ) = -\cos 40^\circ$ .

故  $\cos 20^\circ + \cos 100^\circ + \cos 140^\circ = 0.$

52.  $\cos 36^\circ + \sin 36^\circ = \sqrt{2} \cos 9^\circ.$

證  $24/1,$

$$\cos 36^\circ + \sin 36^\circ = \sqrt{2} \cos (36^\circ - \frac{1}{4}\pi) = \sqrt{2} \cos (-9^\circ)$$

$$= \sqrt{2} \cos 9^\circ.$$

53.  $\tan 11^\circ 15' + 2 \tan 22^\circ 30' + 4 \tan 45^\circ$   
 $= \cot 11^\circ 15'.$

按  $24/11,$

$$\cot 11^\circ 15' - \tan 11^\circ 15' = 2 \cot 22^\circ 30'.$$

$$2 \cot 22^\circ 30' - 2 \tan 22^\circ 30' = 4 \cot 45^\circ.$$

故  $\cot 11^\circ 15' - \tan 11^\circ 15' - 2 \tan 22^\circ 30'$   
 $= 4 \cot 45^\circ = 4 \tan 45^\circ.$

故  $\tan 11^\circ 15' + 2 \tan 22^\circ 30' + 4 \tan 45^\circ$   
 $= \cot 11^\circ 15'.$

設  $A, B, C$  爲平面三角形之各角求證

54  $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C.$

$$A + B + C = 180^\circ.$$

按 [20],  $\sin 2A + \sin 2B = 2 \sin(A+B) \cos(A-B)$

$$= 2 \sin C \cos(A - B),$$

按 [12],  $\sin 2C = 2 \sin C \cos C.$

故  $\sin 2A + \sin 2B + \sin 2C$   
 $= 2 \sin C \cos(A - B) + 2 \sin C \cos C$

$$= 2 \sin C [\cos(A - B) + \cos(A + B)]$$

按 [23],  $= 2 \sin C [-2 \sin A \sin(-B)]$

$$= 4 \sin A \sin B \sin C,$$

55.  $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C.$

按 [22],  $\cos 2A + \cos 2B = 2 \cos(A + B) \cos(A - B)$

$$= 2 \cos C \cos(A - B).$$

故  $\cos 2A + \cos 2B + \cos 2C = -2 \cos C \cos(A - B) + \cos 2C.$

按 [13],  $= -2 \cos C \cos(A - B) + 2 \cos^2 C - 1$

$$= 2 \cos C [\cos C - \cos(A - B)] - 1$$

$$= 2 \cos C [-\cos(A + B) - \cos(A - B)]$$

$$- 1$$

按 [22],  $= 2 \cos C (-2 \cos A \cos B) - 1$

$$= -1 - 4 \cos A \cos B \cos C.$$

56.  $\sin 3A + \sin 3B + \sin 3C = -4 \cos \frac{3A}{2} \cos \frac{3B}{2} \cos \frac{3C}{2}$

按 [20],  $\sin 3A + \sin 3B = 2 \sin \frac{3}{2}(A + B) \cos \frac{3}{2}(A - B)$

$$= 2 \sin \frac{3}{2}(180^\circ - C) \cos \frac{3}{2}(A - B)$$

$$= -2 \cos \frac{3}{2} C \cos \frac{3}{2}(A - B).$$

按 [12],  $\sin 3C = 2 \sin \frac{3}{2} C \cos \frac{3}{2} C$

$$= 2 \sin \frac{3}{2}(180^\circ - A - B) \cos \frac{3}{2} C$$

$$= -2 \cos \frac{3}{2}(A + B) \cos \frac{3}{2} C.$$

故  $\sin 3A + \sin 3B + \sin 3C = -2 \cos \frac{3}{2} C [\cos \frac{3}{2}(A - B) + \cos \frac{3}{2}(A + B)]$

B)]

$$= -2\cos\frac{3}{2}C(2\cos\frac{3}{2}A\cos\frac{3}{2}B)$$

$$= -4\cos\frac{3}{2}A\cos\frac{3}{2}B\cos\frac{3}{2}C.$$

$$57. \cos^2 A - \cos^2 B + \cos^2 C = 1 - 2\cos A \cos B \cos C.$$

$$\text{按 [16],} \quad \cos^2 A = \frac{1 + \cos 2A}{2}.$$

$$\cos^2 B = \frac{1 + \cos 2B}{2}.$$

$$\cos^2 C = \frac{1 + \cos 2C}{2}.$$

$$\text{故 } \cos^2 A + \cos^2 B + \cos^2 C = \frac{3 + \cos 2A + \cos 2B + \cos 2C}{2}$$

$$\text{按 } \quad 24/55,$$

$$\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A \cos B \cos C.$$

$$\text{故 } \cos^2 A + \cos^2 B + \cos^2 C = \frac{3 - 1 - 4\cos A \cos B \cos C}{2}$$

$$= 1 - 2\cos A \cos B \cos C.$$

$$58. \tan A \tan B + \tan B \tan C + \tan C \tan A = 1$$

$$\tan A \tan B + \tan B \tan C + \tan C \tan A$$

$$= \tan A \tan B + (\tan A + \tan B) \tan C$$

$$= \tan A \tan B + \frac{\tan A + \tan B}{\tan(A+B)}$$

$$\text{按 [6],} \quad = \tan A \tan B + \frac{\tan A + \tan B}{\frac{\tan A + \tan B}{1 - \tan A \tan B}}$$

$$= \tan A \tan B + 1 - \tan A \tan B = 1.$$

$$59. \sin^2 A + \sin^2 B + \sin^2 C = 1 - 2\sin A \sin B \sin C$$

$$\sin C = \cos(A+B)$$

故 (5),  $\sin C + \sin A \sin B = \cos A \cos B - \sin A \sin B.$

$$\sin C + \sin A \sin B = \cos A \cos B,$$

$$\sin^2 C + 2 \sin A \sin B \sin C + \sin^2 A \sin^2 B$$

$$= \cos^2 A \cos^2 B.$$

$$\sin^2 C + 2 \sin A \sin B \sin C = \cos^2 A \cos^2 B - \sin^2 A \sin^2 B$$

$$= (1 - \sin^2 A)(1 - \sin^2 B) - \sin^2 A \sin^2 B$$

$$= 1 - \sin^2 A - \sin^2 B.$$

故  $\sin^2 A + \sin^2 B + \sin^2 C = 1 - 2 \sin A \sin B \sin C.$

60.  $\sin 2A + \sin 2B + \sin 2C = 4 \cos A \cos B \cos C.$

故 [20],  $\sin 2A + \sin 2B = 2 \sin(A+B) \cos(A-B)$

$$= 2 \cos C \cos(A-B).$$

按 (12),  $\sin 2C = 2 \sin C \cos C$

故  $\sin 2A + \sin 2B + \sin 2C = 2 \cos C \cos(A-B) + 2 \sin C \cos C$

$$= 2 \cos C [\cos(A-B) + \sin C]$$

$$= 2 \cos C [\cos(A-C) + \cos(A+B)]$$

按 [22],  $= 4 \cos A \cos B \cos C.$

試証下列諸式.

61.  $\sin(\sin^{-1}x + \sin^{-1}y)$

$$= x\sqrt{1-y^2} + y\sqrt{1-x^2}$$

按 [4],  $\sin(\sin^{-1}x + \sin^{-1}y) = \sin(\sin^{-1}x) \cos(\sin^{-1}y)$

$$+ \cos(\sin^{-1}x) \sin(\sin^{-1}y)$$

$$= x\sqrt{1-y^2} + y\sqrt{1-x^2}.$$

62.

$$\tan(\tan^{-1}x + \tan^{-1}y)$$

$$= \frac{x+y}{1-xy}$$

$$\text{按 [6], } \tan(\tan^{-1}x + \tan^{-1}y) = \frac{\tan(\tan^{-1}x) + \tan(\tan^{-1}y)}{1 - \tan(\tan^{-1}x)\tan(\tan^{-1}y)} = \frac{x+y}{1-xy}.$$

$$63. \quad 2 \sin^{-1}x = \sin^{-1} \frac{2x}{1-x^2}.$$

$$\text{按 [14], } \tan(2 \tan^{-1}x) = \frac{2 \tan(\tan^{-1}x)}{1 - \tan^2(\tan^{-1}x)} = \frac{2x}{1-x^2}.$$

$$\text{故} \quad 2 \tan^{-1}x = \tan^{-1} \frac{2x}{1-x^2}.$$

$$64. \quad 2 \sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2}).$$

$$\text{按 [12], } \sin(2 \sin^{-1}x) = 2 \sin(\sin^{-1}x)\cos(\sin^{-1}x) = 2x\sqrt{1-x^2}.$$

$$\text{故} \quad 2 \sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2}).$$

$$65. \quad \text{求證} \quad 2 \cos^{-1}x = \cos^{-1}(2x^2 - 1).$$

$$\text{按 [13], } \cos(2 \cos^{-1}x) = 2 \cos^2(\cos^{-1}x) - 1 = 2x^2 - 1.$$

$$\text{故} \quad 2 \cos^{-1}x = \cos^{-1}(2x^2 - 1).$$

$$66. \quad 3 \tan^{-1}x = \tan^{-1} \frac{3x - x^3}{1 - 3x^2}.$$

$$\tan(3 \tan^{-1}x) = \tan(\tan^{-1}x + 2 \tan^{-1}x)$$

$$\text{按 [6],} \quad = \frac{\tan(\tan^{-1}x) + \tan(2 \tan^{-1}x)}{1 - \tan(\tan^{-1}x)\tan(2 \tan^{-1}x)}$$

$$= \frac{x + \tan(2 \tan^{-1}x)}{1 - x \tan(2 \tan^{-1}x)}$$

$$\text{按 63 題,} \quad = \frac{x + \frac{2x}{1-x^2}}{1 - x \frac{2x}{1-x^2}} = \frac{3x - x^3}{1 - 3x^2}.$$

$$\text{故} \quad 3 \tan^{-1}x = \tan^{-1} \frac{3x - x^3}{1 - 3x^2}.$$

$$67. \sin^{-1} \sqrt{\frac{x}{y}} = \tan^{-1}$$

$$\sqrt{\frac{x}{y-x}}$$

$$\text{設 } \sin^{-1} \sqrt{\frac{x}{y}} = n.$$

$$\text{則 } \sqrt{\frac{x}{y}} = \sin n.$$

$$\sqrt{\frac{y-x}{y}} = \cos n.$$

$$\sqrt{\frac{x}{y-x}} = \tan n.$$

$$\text{故 } n = \tan^{-1} \sqrt{\frac{x}{y-x}}.$$

$$\text{而 } \sin^{-1} \sqrt{\frac{x}{y}} = \tan^{-1} \sqrt{\frac{x}{y-x}}.$$

$$68. \sin^{-1} \sqrt{\frac{x-y}{x-z}} = \tan^{-1}$$

$$\sqrt{\frac{x-y}{y-z}}$$

$$\text{設 } \sin^{-1} \sqrt{\frac{x-y}{x-z}}$$

$$= n.$$

$$\text{則 } \sqrt{\frac{x-y}{x-z}} = \sin n.$$

$$\sqrt{\frac{y-z}{x-z}} = \cos n.$$

$$\sqrt{\frac{x-y}{y-z}} = \tan n.$$

$$n = \tan^{-1} \sqrt{\frac{x-y}{y-z}}.$$

$$\text{故 } \sin^{-1} \sqrt{\frac{x-y}{x-z}}$$

$$= \tan^{-1} \sqrt{\frac{x-y}{y-z}}.$$

$$69. \sin^{-1} x = \sec^{-1} \frac{1}{\sqrt{1-x^2}}$$

$$\text{設 } \sin^{-1} x = n.$$

$$\text{則 } x = \sin n.$$

$$\sqrt{1-x^2} = \cos n.$$

$$\frac{1}{\sqrt{1-x^2}} = \sec n.$$

$$n = \sec^{-1} \frac{1}{\sqrt{1-x^2}}.$$

$$\text{故 } \sin^{-1} x = \sec^{-1} \frac{1}{\sqrt{1-x^2}}.$$

$$70. 2 \sin^{-1} x = \tan^{-1}$$

$$\frac{2\sqrt{x^2-1}}{2-x^2}.$$

$$\text{證 } 2 \sec^{-1} x = n.$$

$$\text{則 } x = \sec \frac{1}{2}n.$$

$$\frac{1}{x} = \cos \frac{1}{2}n.$$

按 [13]

$$2\left(\frac{1}{x}\right)^2 - 1 = \cos n.$$

$$\frac{4x^2 - 4}{(2 - x^2)^2} = \tan^2 n.$$

$$\frac{2 - x^2}{x^2} = \cos n.$$

$$\tan n = \frac{2\sqrt{x^2 - 1}}{2 - x^2}.$$

$$\frac{x^2}{2 - x^2} = \sec n.$$

$$n = \tan^{-1} \frac{2\sqrt{x^2 - 1}}{2 - x^2}$$

按 2/5:

$$\left(\frac{x^2}{2 - x^2}\right)^2 - 1 = \tan^2 n.$$

$$\text{故 } 2 \sec^{-1} x = \tan^{-1} \frac{2\sqrt{x^2 - 1}}{2 - x^2}.$$

71.  $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = 45^\circ$

按 [6],  $\tan(\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3})$

$$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}} = 1.$$

故  $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = \tan^{-1} 1 = 45^\circ.$

72. 求證

$$\tan^{-1} \frac{3}{5} + \tan^{-1} \frac{1}{5}$$

$$= \tan^{-1} \frac{4}{7}.$$

按 [6],  $\tan(\tan^{-1} \frac{3}{5} + \tan^{-1} \frac{1}{5})$

$$= \frac{\frac{3}{5} + \frac{1}{5}}{1 - \frac{3}{5} \times \frac{1}{5}} = \frac{4}{7}.$$

73.

$$\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{12}{13}$$

$$= \sin^{-1} \frac{16}{65}$$

按 [4],

$$\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{12}{13}$$

$$= \sin(\sin^{-1} \frac{3}{5}) \cos(\sin^{-1} \frac{12}{13}) + \cos(\sin^{-1} \frac{3}{5})$$

$$\sin(\sin^{-1} \frac{12}{13}) = \frac{3}{5} \times \frac{5}{13} + \frac{4}{5} \times \frac{12}{13} = \frac{63}{65}.$$

證  $\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{12}{13} = \sin^{-1} \frac{16}{65}.$



$$74. \quad \sin^{-1} \frac{1}{\sqrt{82}} + \sin^{-1} \frac{4}{\sqrt{41}} = 45^\circ.$$

$$\begin{aligned} \text{按 [4], } \sin \left( \sin^{-1} \frac{1}{\sqrt{82}} + \sin^{-1} \frac{4}{\sqrt{41}} \right) \\ &= \frac{1}{\sqrt{82}} \times \frac{5}{\sqrt{41}} + \frac{9}{\sqrt{82}} \times \frac{4}{\sqrt{41}} \\ &= \frac{5}{41\sqrt{2}} = \frac{36}{41\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1}{2} \sqrt{2}. \end{aligned}$$

$$\begin{aligned} \text{故 } \sin^{-1} \frac{1}{\sqrt{82}} + \sin^{-1} \frac{4}{\sqrt{41}} \\ &= \sin^{-1} \frac{1}{2} \sqrt{2} = 45^\circ. \end{aligned}$$

$$75. \quad \sec^{-1} \frac{5}{3} + \sec^{-1} \frac{13}{12} = 75^\circ 45'.$$

$$\sec^{-1} \frac{5}{3} + \sec^{-1} \frac{13}{12} = \cos^{-1} \frac{3}{5} + \cos^{-1} \frac{12}{13}.$$

$$\begin{aligned} \text{按 [5], } \cos \left( \cos^{-1} \frac{3}{5} + \cos^{-1} \frac{12}{13} \right) \\ &= \frac{3}{5} \times \frac{12}{13} - \frac{4}{5} \times \frac{5}{13} = \frac{16}{65}. \end{aligned}$$

$$\text{故 } \cos^{-1} \frac{3}{5} + \cos^{-1} \frac{12}{13} = \cos^{-1} \frac{16}{65}.$$

$$\sec^{-1} \frac{5}{3} + \sec^{-1} \frac{13}{12} = \sec^{-1} \frac{65}{16} = 75^\circ 45'.$$

$$76. \quad \tan^{-1}(2 + \sqrt{3}) - \tan^{-1}(2 - \sqrt{3}) = \sec^{-1} 2.$$

$$\text{設 } \tan^{-1}(2 + \sqrt{3}) - \tan^{-1}(2 - \sqrt{3}) = n.$$

$$\begin{aligned} \text{則刻, 按 [10], } \tan n &= \frac{(2 + \sqrt{3}) - (2 - \sqrt{3})}{1 + (2 + \sqrt{3})(2 - \sqrt{3})} \\ &= \frac{2\sqrt{3}}{2} = \sqrt{3}. \end{aligned}$$

$$\text{故 } n = 60^\circ.$$

$$\sec n = 2.$$

$$\text{而 } \tan^{-1}(2 + \sqrt{3}) - \tan^{-1}(2 - \sqrt{3}) = \sec^{-1} 2.$$

$$77. \quad \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} = 45^\circ.$$

$$\text{設 } \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} = n,$$

$$\text{又 } \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} = v.$$

$$\text{則, 按 [6], } \tan n = \frac{\frac{1}{3} + \frac{1}{5}}{1 - \frac{1}{3} \times \frac{1}{5}} = \frac{4}{7}.$$

$$\tan v = \frac{\frac{1}{7} + \frac{1}{8}}{1 + \frac{1}{7} \times \frac{1}{8}} = \frac{3}{11}.$$

$$\tan(n + v) = \frac{\frac{4}{7} + \frac{3}{11}}{1 - \frac{4}{7} \times \frac{3}{11}} = 1.$$

$$n + v = \tan^{-1} 1 = 45^\circ.$$

$$\text{即 } \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} = 45^\circ.$$

$$78. \quad \tan^{-1} \frac{1}{1 - 2x + 4x^2} + \tan^{-1} \frac{1}{1 + 2x + 4x^2} = \tan^{-1} \frac{1}{2x^2}.$$

$$\text{按 [6], } \tan \left( \tan^{-1} \frac{1}{1 - 2x + 4x^2} + \tan^{-1} \frac{1}{1 + 2x + 4x^2} \right)$$

$$= \frac{\frac{1}{1 + 2x + 4x^2} + \frac{1}{1 + 2x + 4x^2}}{1 - \frac{1}{(1 - 2x + 4x^2)(1 + 2x + 4x^2)}}$$

$$= \frac{1 + 2x + 4x^2 + 1 - 2x - 4x^2}{(1 - 2x + 4x^2)(1 + 2x + 4x^2) - 1}$$

$$= \frac{2 + 8x^2}{4x^2 + 16x^4} = \frac{1}{2x^2}.$$

$$\text{故 } \tan^{-1} \frac{1}{1 - 2x + 4x^2} + \tan^{-1} \frac{1}{1 + 2x + 4x^2} = \tan^{-1} \frac{1}{2x^2}.$$

$$79. \quad \text{設 } \cos x = \frac{3}{5}, \text{ 求 } \sin \frac{1}{2}x \text{ 及 } \cos \frac{1}{2}x.$$

按 [16],  $\sin \frac{1}{2}x = \pm \sqrt{\frac{1 - \frac{3}{5}}{3}} = \pm \sqrt{\frac{1}{5}} = \pm \frac{1}{5} \sqrt{5}.$

按 [17],  $\cos \frac{1}{2}x = \pm \sqrt{\frac{1 + \frac{3}{5}}{2}} = \pm \sqrt{\frac{4}{5}} = \pm \frac{2}{5} \sqrt{2}.$

80. 設  $\tan x = \frac{1}{2}$ , 求  $\tan \frac{1}{2}x$ .

按 [14],  $\tan x = \frac{2 \tan \frac{1}{2}x}{1 - \tan^2 \frac{1}{2}x}.$

$$\frac{1}{2} = \frac{2 \tan \frac{1}{2}x}{1 - \tan^2 \frac{1}{2}x}.$$

$$1 - \tan^2 \frac{1}{2}x = 4 \tan \frac{1}{2}x.$$

$$\tan^2 \frac{1}{2}x + 4 \tan \frac{1}{2}x = 1.$$

$$\tan^2 \frac{1}{2}x + 4 \tan \frac{1}{2}x + 4 = 5$$

$$\tan \frac{1}{2}x + 2 = \pm \sqrt{5}$$

故  $\tan \frac{1}{2}x = \pm \sqrt{5} - 2.$

81. 設  $\sin x + \cos x = \sqrt{\frac{1}{2}}$ , 求  $\cos 2x$ .

$$\sin x + \cos x = \sqrt{\frac{1}{2}}.$$

$$\sin^2 x + 2 \sin x \cos x + \cos^2 x = \frac{1}{2}.$$

$$1 + 2 \sin x \cos x = \frac{1}{2}.$$

$$2 \sin x \cos x = -\frac{1}{2}.$$

按 [12],  $\sin 2x = -\frac{1}{2}.$

$$\begin{aligned} \cos 2x &= \pm \sqrt{1 - \left(-\frac{1}{2}\right)^2} \\ &= \pm \sqrt{\frac{3}{4}} = \pm \frac{1}{2} \sqrt{3}. \end{aligned}$$

82. 設  $\tan 2x = \frac{24}{7}$ , 求  $\sin x$ .

$$\tan 2x = \frac{24}{7}.$$

按 14/19,  $\sec^2 2x = 1 + \tan^2 2x = \frac{625}{59}$ .

$$\cos^2 2x = \frac{49}{625}.$$

$$\cos 2x = \pm \frac{7}{25}.$$

$$1 - 2 \sin^2 x = \pm \frac{7}{25}.$$

$$\sin^2 x = \frac{9}{25} \text{ 或 } \frac{16}{25}.$$

故  $\sin x = \pm \frac{3}{5} \text{ 或 } \pm \frac{4}{5}.$

83. 設  $\cos 3x = \frac{23}{27}$ . 設求  $\tan x$ .

按 14/193,  $\cos 3x = 4 \cos^3 x - 3 \cos x$ .

$$4 \cos^3 x - 3 \cos x = \frac{23}{27}.$$

$$108 \cos^3 x - 81 \cos x = 23.$$

$$108 \cos^3 x - 81 \cos x - 23 = 0.$$

$$(3 \cos x + 1)(36 \cos^2 x - 12 \cos x - 23) = 0.$$

(1)  $3 \cos x + 1 = 0,$

$$\cos x = -\frac{1}{3}.$$

(2)  $36 \cos^2 x - 12 \cos x - 23 = 0.$

$$36 \cos^2 x - 12 \cos x + 1 = 24.$$

$$6 \cos x - 1 = \pm 2\sqrt{6}.$$

$$\cos x = \frac{1}{6}(1 \pm 2\sqrt{6}).$$

$$\sin x = \pm \sqrt{1 - \cos^2 x}$$

$$= \pm \sqrt{1 - \left(-\frac{1}{3}\right)^2}, \text{ 或 } \pm \sqrt{1 - \frac{1}{6}(1 \pm 2\sqrt{6})}$$

$$= \pm \sqrt{\frac{8}{9}}, \text{ 或 } \pm \frac{1}{36} \sqrt{(11 \mp 4\sqrt{6})}$$

$$= \pm \frac{2}{3} \sqrt{2}, \text{ 或 } \frac{1}{6} (\sqrt{3} \mp 2\sqrt{2})$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$= \frac{\pm \frac{2}{3} \sqrt{2}}{-\frac{1}{3}}, \text{ or } \frac{\pm \frac{1}{6} (\sqrt{3} \mp 2\sqrt{2})}{\frac{1}{6} (1 \pm 2\sqrt{6})}$$

$$= \mp 2\sqrt{2}, \text{ 或 } \frac{\pm (\sqrt{3} \mp 2\sqrt{2})}{1 \pm 2\sqrt{6}}$$

$$= \mp 2\sqrt{2}, \text{ 或 } \frac{\pm (3 - 2\sqrt{2})(1 - 2\sqrt{6})}{(1 + 2\sqrt{6})(1 \pm 2\sqrt{6})}$$

$$\text{或 } \frac{\pm (\sqrt{3} + 2\sqrt{2})(1 + 2\sqrt{6})}{(1 - 2\sqrt{6})(1 + 2\sqrt{6})}$$

$$= \mp 2\sqrt{2}, \text{ 或 } \frac{\pm (9\sqrt{3} - 8\sqrt{2})}{-23}$$

$$\text{或 } \frac{\pm (9\sqrt{3} + 8\sqrt{2})}{-23}$$

故  $\tan x = \pm 2\sqrt{2}, \text{ 或 } \pm \frac{1}{23} (9\sqrt{3} + 8\sqrt{2}),$

或  $\frac{1}{23} (9\sqrt{3} - 8\sqrt{2}).$

84. 設  $5 \csc x - \cot x$

$\sqrt{3}$ , 求  $\sin \frac{1}{2}x$ .

$$2 \csc x - \cot x = \sqrt{3}.$$

$$\frac{2}{\sin x} - \frac{\cos x}{\sin x} = \sqrt{3}.$$

$$2 = \cos x = \sqrt{3} \sin x.$$

$$4 - 4 \cos x + \cos^2 x = 3 \sin^2 x$$

$$= 3 - 3 \cos^2 x.$$

$$4 \cos^2 x - 4 \cos x + 1$$

$$= 0.$$

$$2 \cos x - 1 = 0.$$

$$\cos x = \frac{1}{2}.$$

$$\text{故 [16], } \sin \frac{1}{2}x = \pm \sqrt{\frac{1 - \frac{1}{2}}{2}}.$$

$$\text{故 } \sin \frac{1}{2}x = \pm \frac{1}{2}.$$

85. 求  $\sin 18^\circ$  及  $\cos 36^\circ$ .

$$(1) \quad 54^\circ = 90^\circ - 36^\circ.$$

$$3 \times 18^\circ = 90^\circ - 2 \times 18^\circ.$$

$$\cos(3 \times 18^\circ) = \sin(2 \times 18^\circ).$$

按 14/18,

$$\begin{aligned} 4 \cos^3 18^\circ - 3 \cos 18^\circ \\ = 8 \sin 18^\circ \cos \end{aligned}$$

$$18^\circ.$$

$$4 \cos^2 18^\circ - 3 = 2 \sin 18^\circ.$$

$$\begin{aligned} 4 - 4 \sin^2 18^\circ - 3 \\ = 2 \sin 18^\circ. \end{aligned}$$

$$\begin{aligned} 4 \sin^2 18^\circ + 2 \sin 18^\circ \\ = 1. \end{aligned}$$

$$\begin{aligned} 16 \sin^2 18^\circ + ( ) - 1 \\ = 5. \end{aligned}$$

$$4 \sin 18^\circ + 1 = \pm \sqrt{5}.$$

$$4 \sin 18^\circ = \pm \sqrt{5} - 1.$$

$$\text{故 } \sin 18^\circ = \frac{\sqrt{5} - 1}{4}.$$

$$\begin{aligned} (2) \quad \cos 36^\circ &= 1 - 2 \sin^2 18^\circ \\ &= 1 - 2 \end{aligned}$$

$$\begin{aligned} &\left(\frac{\sqrt{5} - 1}{4}\right)^3 \\ &= \frac{\sqrt{5} - 1}{4}. \end{aligned}$$

求次之各值。

86. 試求  $a \sec x + b \csc x$ ,

$$\text{已知 } \tan x = \sqrt[3]{\frac{b}{a}}.$$

$$\tan x = \frac{b^{\frac{1}{3}}}{a^{\frac{1}{3}}}$$

按 5/2,

$$\begin{aligned} \sec^2 x &= 1 + \frac{b^{\frac{2}{3}}}{a^{\frac{2}{3}}} \\ &= \frac{a^{\frac{2}{3}} + b^{\frac{2}{3}}}{a^{\frac{2}{3}}}. \end{aligned}$$

$$\cot x = \frac{a^{\frac{1}{3}}}{b^{\frac{1}{3}}}.$$

按 5/3,

$$\cos^2 x = \frac{a^{\frac{2}{3}} + b^{\frac{2}{3}}}{b^{\frac{2}{3}}}.$$

$$\text{故 } \sec x = \frac{(a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{1}{3}}}{a^{\frac{1}{3}}}.$$

$$\csc x = \frac{(a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{1}{3}}}{b^{\frac{2}{3}}}$$

$$a \sec x + b \csc x.$$

$$= a^{\frac{2}{3}} (a^{\frac{2}{3}} + b^{\frac{2}{3}})$$

$$\frac{1}{2} + b^{\frac{2}{3}} (a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{1}{3}}$$

$$= (a^{\frac{2}{3}} + b^{\frac{2}{3}}) (a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{1}{3}}$$

$$= (a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{2}{3}}.$$

87. 已知  $\sin 2x = \sqrt{1-m^2}$

$$\sin 3x.$$

$$\sin 2x = \sqrt{1-m^2}.$$

$$\cos^2 2x = m^2.$$

$$\cos 2x = \pm m.$$

$$1 - 2 \sin^2 x = \pm m.$$

$$2 \sin^2 x = 1 \pm m.$$

$$\sin x = \sqrt{\frac{1 \pm m}{2}}.$$

按 14/8,

$$\sin 3x = 3 \sin x - 4$$

$$\sin^3 x$$

$$= 3 \left( \frac{1 \pm m}{2} \right)^{\frac{1}{3}} - 4$$

$$\left( \frac{1-m}{2} \right)^{\frac{3}{2}}$$

$$= \left( \frac{1-m}{2} \right)^{\frac{1}{2}}$$

$$\left( 3 - 4 \frac{[1 \pm m]}{2} \right)$$

$$= \left( \frac{1 \pm m}{2} \right)^{\frac{1}{2}}$$

$$(1 \pm 2m).$$

88.  $\sin x$ , 已知  $\tan^2 x + 3$

$$\cot^2 x = 4.$$

$$\tan^2 x + 3 \cot^2 x = 4.$$

$$\tan^2 x + \frac{3}{\tan^2 x} = 4.$$

$$\tan^4 x - 4 \tan^2 x + 3 = 0.$$

$$(\tan^2 x - 1)(\tan^2 x - 3) = 0.$$

故

$$\tan^2 x = 1 \text{ 或 } 3.$$

$$\cot^2 x = 1 \text{ 或 } \frac{1}{3}.$$

$$\csc^2 x = 2 \text{ 或 } \frac{4}{3}.$$

$$\sin^2 x = \frac{1}{2} \text{ 或 } \frac{3}{4}.$$

故  $\sin x = \pm \frac{1}{2} \sqrt{2}$  或  $\pm \frac{1}{2} \sqrt{3}$ .

89.  $\frac{\csc^2 x - \sec^2 x}{\csc^2 x - \sec^2 x}$ , 已知  $\tan$

$$x = \sqrt{\frac{1}{7}},$$

$$\tan x = \sqrt{\frac{1}{7}},$$

$$\sec^2 x = 1 + \frac{1}{7} = \frac{8}{7}.$$

$$\cot x = \sqrt{7}.$$

$$\csc^2 x = 1 + 7 = 8.$$

$$\begin{aligned} \text{故 } \frac{\csc^2 x - \sec^2 x}{\csc^2 x + \sec^2 x} \\ = \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}} = \frac{3}{4}. \end{aligned}$$

$$90. \cos x, \text{ 已知 } 5 \tan x +$$

$$\sec x = 5.$$

$$5 \tan x + \sec x = 5.$$

$$\frac{5 \sin x}{\cos x} + \frac{1}{\cos x} = 5.$$

$$5 \sin x + 1 = 5 \cos x.$$

$$5 \sin x = 5 \cos x - 1.$$

$$25(1 - \cos^2 x)$$

$$\begin{aligned} = 25 \cos^2 x - 10 \cos x \\ + 1. \end{aligned}$$

試將下列各式簡單之。

$$50 \cos^2 x - 10 \cos x - 24 = 0.$$

$$25 \cos^2 x - 5 \cos x - 12 = 0.$$

$$(5 \cos x - 4)(5 \cos x + 3) = 0.$$

$$5 \cos x = 4 \quad \text{或} \quad -3$$

$$\text{故 } \cos x = \frac{4}{5} \quad \text{或} \quad -\frac{3}{5}.$$

$$91. \sec x, \text{ 已知 } \tan x$$

$$= \frac{a}{\sqrt{2a+1}}$$

$$\tan x = \frac{a}{\sqrt{2a+1}}$$

$$\sec^2 x = 1 + \frac{a^2}{2a+2}$$

$$= \frac{a^2 + 2a + 1}{2a + 2}$$

$$\text{故 } \sec x = \pm \frac{a+1}{\sqrt{2a+1}}$$

$$92. \frac{(\cos x + \cos y)^2 + (\sin x + \sin y)^2}{\cos^2(x-y)}.$$

按 [22] 及 [20],

$$\frac{(\cos x + \cos y)^2 + (\sin x + \sin y)^2}{\cos^2 \frac{1}{2}(x-y)}$$

$$= \frac{[2 \cos \frac{1}{2}(x+y) \cos \frac{1}{2}(x-y)]^2 + [2 \sin \frac{1}{2}(x+y) \cos \frac{1}{2}(x-y)]^2}{\cos^2 \frac{1}{2}(x-y)}$$

$$= 4 \cos^2 \frac{1}{2}(x+y) + 4 \sin^2 \frac{1}{2}(x+y) = 4.$$

$$93. \frac{\sin(x+2y) - 2 \sin(x+y) + \sin x}{\cos(x+2y) - 2 \cos(x+y) + \cos x}.$$



$$\begin{aligned} & \frac{\sin(x+2y) - 2\sin(x+y) + \sin x}{\cos(x+2y) - 3\cos(x+y) + \cos x} \\ &= \frac{[\sin(x+2y) + \sin x] - 2\sin(x+y)}{[\cos(x+2y) + \cos x] - 2\cos(x+y)} \\ \text{按 [20], [22]} &= \frac{2\sin(x+y)\cos y - 2\sin(x+y)}{2\cos(x+y)\cos y - 2\cos(x+y)} \\ &= \frac{\sin(x+y)(\cos y - 1)}{\cos(x+y)(\cos y - 1)} = \frac{\sin(x+y)}{\cos(x+y)} \\ &= \tan(x+y). \end{aligned}$$

$$\begin{aligned} 94. & \frac{\sin(x-z) + 2\sin x + \sin(x+z)}{\sin(y-z) + 2\sin y + \sin(y+z)} \\ &= \frac{\sin(x-z) + 2\sin x + \sin(x+z)}{\sin(y-z) + 2\sin y + \sin(y+z)} \\ &= \frac{[\sin(x+z) + \sin(x-z)] + 2\sin x}{[\sin(y+z) + \sin(y-z)] + 2\sin y} \\ \text{按 [20],} &= \frac{2\sin x \cos z + 2\sin x}{2\sin y \cos z + 2\sin y} \\ &= \frac{\sin x(\cos z + 1)}{\sin y(\cos z + 1)} = \frac{\sin x}{\sin y}. \end{aligned}$$

$$\begin{aligned} 95. & \frac{\cos 6x - \cos 4x}{\sin 6x + \sin 4x} \\ &= \frac{\cos 6x - \cos 4x}{\sin 6x + \sin 4x} = \frac{-2\sin 5x \sin x}{2\sin 5x \cos x} = \frac{-\sin x}{\cos x} = -\tan x. \end{aligned}$$

$$\begin{aligned} 95. & \tan^{-1}(2x+1) + \tan^{-1}[(2x-1)] \\ & \tan[\tan^{-1}(2x+1) + \tan^{-1}(2x-1)] \end{aligned}$$

$$\begin{aligned} \text{按 [6],} &= \frac{2x+1+2x-1}{1-(2x+1)(2x-1)} = \frac{4x}{2-4x^2} \\ &= \frac{2x}{1-2x^2} \end{aligned}$$

$$\text{故 } \tan^{-1}(2x+1) + \tan^{-1}(2x-1) = \tan^{-1} \frac{2x}{1-2x^2}$$

$$\begin{aligned} 97. & \frac{1}{1+\sin^2x} + \frac{1}{1+\cos^2x} + \frac{1}{1+\sec^2x} + \frac{1}{1+\cos^2x} \\ & \frac{1}{1+\sin^2x} + \frac{1}{1+\cos^2x} + \frac{1}{1+\sec^2x} + \frac{1}{1+\csc^2x} \\ & = \left( \frac{1}{1+\sin^2x} + \frac{1}{1+\csc^2x} \right) + \left( \frac{1}{1+\cos^2x} + \frac{1}{1+\sec^2x} \right) \\ & = \left( \frac{1}{1+\sin^2x} + \frac{\sin^2x}{1+\sin^2x} \right) + \left( \frac{1}{1+\cos^2x} + \frac{\cos^2x}{1+\cos^2x} \right) \\ & = \frac{1+\sin^2x}{1+\sin^2x} + \frac{1+\cos^2x}{1+\cos^2x} = 1+1=2. \end{aligned}$$

$$\begin{aligned} 98. & 2 \sec^2x - \sec^4x - 2 \csc^2x + \csc^4x \\ & 2 \sec^2x + \sec^4x - 2 \csc^2x + \csc^4x \\ & = 1 - 2 \csc^2x + \csc^4x - 1 + 2 \sec^2x - \sec^4x \\ & = (\csc^2x - 1)^2 - (\sec^2x - 1)^2 \\ & = \cot^2x - \tan^2x. \end{aligned}$$

試解下列之各方程式。

$$92. \quad \sin x = 2 \sin\left(\frac{1}{3}\pi + x\right).$$

$$\begin{aligned} \text{按 [4]} \quad \sin x &= 2 \sin\frac{1}{3}\pi \cos x + 2 \cos\frac{1}{3}\pi \sin x \\ &= \sqrt{3} \cos x + \sin x \end{aligned}$$

$$\sqrt{3} \cos x = 0.$$

$$\cos x = 0.$$

$$\text{故 } x = \frac{1}{2}\pi \text{ 或 } \frac{3}{2}\pi.$$

$$100. \quad \sin 2x = 2 \cos x.$$

$$\sin 2x = 2 \cos x,$$

$$2 \sin x \cos x = 2 \cos x.$$

$$2 \cos x (\sin x - 1) = 0.$$

$$(1) \quad 2 \cos x = 0.$$

$$\cos x = 0.$$

故  $x = 90^\circ$  或  $270^\circ$ .

$$(2) \quad \sin x - 1 = 0.$$

$$\sin x = 1.$$

$$x = 90^\circ.$$

故  $x = 90^\circ$  或  $270^\circ$

$$101. \quad \cos 2x = 2 \sin x.$$

$$\cos 2x = 2 \sin x.$$

$$1 - 2 \sin^2 x = 2 \sin x.$$

$$2 \sin^2 x + 2 \sin x = 1.$$

$$4 \sin^2 x + 4 \sin x = 3.$$

$$2 \sin x - 1 = \pm \sqrt{3},$$

$$2 \sin x = -1 \pm \sqrt{3}.$$

$$2 \sin x = \frac{-1 \pm \sqrt{3}}{2}.$$

$$= \frac{-1 \pm 1.7320}{2}$$

$$= 0.3660 \text{ 或 } -1.3660$$

故  $x = 21^\circ 28'$  或  $158^\circ 32'$ .

$$102. \quad \sin x + \cos x = 1.$$

$$\sin x + \cos x = 1.$$

$$\sin^2 x + 2 \sin x \cos x + \cos^2 x = 1.$$

$$2 \sin x \cos x = 0.$$

$$(1) \quad \sin x = 0,$$

$$x = 0^\circ \text{ 或 } 180^\circ$$

$$(2) \quad \cos x = 0,$$

$$x = 90^\circ \text{ 或 } 270^\circ$$

故  $x = 0^\circ, 90^\circ, 180^\circ, \text{ 或 } 270^\circ.$

但  $(x = 180^\circ)$  與  $(x = 270^\circ)$  皆與本題不合,

故  $x = 0^\circ, \text{ 或 } 90^\circ$

$$103. \quad \sin x + \cos 2x = 4 \sin^2 x.$$

$$\sin x + \cos^2 x = 4 \sin^2 x.$$

$$\sin x + 1 - 2 \sin^2 x = 4 \sin^2 x.$$

$$6 \sin^2 x - \sin x - 1 = 0.$$

$$(2 \sin x - 1)(3 \sin x + 1) = 0.$$

$$2 \sin x - 1 = 0.$$

$$2 \sin x = 1.$$

$$x = 30^\circ \text{ 或 } 150^\circ.$$

$$3 \sin x + 1 = 0,$$

$$3 \sin x = -1$$

$$\sin x = -\frac{1}{3}.$$

$$x = 199^\circ 28' \text{ 或 } 340^\circ 32'.$$

故  $x = 30^\circ, 150^\circ, 199^\circ 28', \text{ 或 } 340^\circ 32'.$

$$104 \quad 4 \cos 2x + 3 \cos x = 1$$

$$4 \cos 2x + 3 \cos x = 1.$$

$$8 \cos^2 x - 4 + 3 \cos x = 1.$$

$$8 \cos^2 x + 3 \cos x - 5 = 0.$$

$$(\cos x + 1)(8 \cos x - 5) = 0.$$

$$(1) \quad \cos x + 1 = 0.$$

$$\cos x = -1.$$

$$x = 180^\circ.$$

$$(2) \quad 8 \cos x - 5 = 0.$$

$$8 \cos x = 5.$$

$$\cos x = \frac{5}{8} = 0.6250.$$

$$x = 51^\circ 19' \text{ 或 } 308^\circ 41'.$$

故

$$x = 51^\circ 19' 180^\circ \text{ 或 } 308, 41'.$$

$$105. \quad \sin x + \sin 2x = \sin 3x.$$

按 [2], 及 14/18,

$$\sin x + 2\sin x \cos x = 3\sin x - 4\sin^3 x$$

$$4\sin^3 x - 2\sin x + 2\sin x \cos x = 0,$$

$$\sin x(2\sin^2 x - 1 + \cos x) = 0.$$

$$\sin x(1 - 2\cos^2 x + \cos x) = 0.$$

$$\sin x(1 - \cos x)(1 + 2\cos x) = 0.$$

$$(1) \quad \sin x = 0.$$

$$x = 0^\circ \text{ 或 } 180^\circ.$$

$$(2) \quad 1 - \cos x = 0.$$

$$\cos x = 1.$$

$$x = 0^\circ.$$

$$(3) \quad 1 + 2\cos x = 0.$$

$$2 \cos x = -1$$

$$\cos x = -\frac{1}{2}.$$

$$x = 120^\circ \text{ 或 } 240^\circ.$$

故

$$x = 0^\circ, 120^\circ, 180^\circ, \text{ 或 } 240^\circ.$$

$$108. \quad \sin x = 2 = 3 \sin^2 x - \cos^2 x.$$

按 [12],  $2\sin x \cos x = 3\sin^2 x - \cos^2 x.$

$$3\sin^2x - 2\sin x \cos x - \cos^2x = 0.$$

$$(3\sin x + \cos x)(\sin x - \cos x) = 0.$$

$$3\sin x + \cos x = 0.$$

$$3\tan x + 1 = 0.$$

$$\tan x = -\frac{1}{3}.$$

故  $x = \tan^{-1}\left(-\frac{1}{3}\right) = 161^\circ 34', \text{或} 341^\circ 34'.$

(2)  $\sin x - \cos x = 0.$

$$\sin x = \cos x.$$

$$x = 45^\circ \text{或} 225^\circ.$$

故  $x = 45^\circ, 161^\circ 34', 225^\circ, \text{或} 341^\circ 34'.$

107.

$$\cos \theta = \frac{1}{3}\tan \theta^\circ$$

$$\cot \theta = \frac{1}{3}\tan \theta.$$

$$\frac{1}{\tan \theta} = \frac{\tan \theta}{3}.$$

$$\tan^2 \theta = 3.$$

$$\tan \theta = \pm\sqrt{3}.$$

故  $\theta = 60^\circ, 120^\circ, 240^\circ, \text{或} 300^\circ.$

[增註]  $\theta$  爲希臘第八字母°讀書音爲“賽塔”

108

$$2\sin \theta = \cos \theta^\circ$$

$$2\sin \theta = \cos \theta.$$

$$4\sin^2 \theta = \cos^2 \theta.$$

$$4\sin^2 \theta - 1 = \sin^2 \theta.$$

$$5\sin^2 \theta = 1.$$

$$\sin^2 \theta = \frac{1}{5}.$$

$$\sin \theta = \pm\frac{1}{\sqrt{5}} = \pm 0.4472.$$

故  $\theta = 26^\circ 34', 153^\circ 26', 206^\circ 34', \text{或} 333^\circ$

但  $26'$ .  
 $(\theta = 153^\circ 26')$  及  $(\theta = 333^\circ 26')$  皆與本題  
 不合.

故  $\theta = 26^\circ 34'$ , 或  $206^\circ 24'$ .  
 109.  $2 \sin^2 x + 5 \sin x = 3$ .

$$2 \sin^2 x + 5 \sin x = 3.$$

$$2 \sin^2 x + 5 \sin x - 3 = 0.$$

$$(\sin x + 3)(2 \sin x - 1) = 0.$$

$$(1) \quad \sin x + 3 = 3.$$

$$\sin x = -3.$$

故  $x$  不成能角.

$$(2) \quad 2 \sin x - 1 = 0.$$

$$2 \sin x = 1.$$

$$\sin x = \frac{1}{2}.$$

故  $x = 30^\circ$  或  $150^\circ$ .

$$110. \quad \tan x \sec x = \sqrt{2}$$

$$\tan x \sec x = \sqrt{2}.$$

$$\tan^2 x \sec^2 x = 2.$$

故  $5/2$

$$\tan^2 x (1 + \tan^2 x) = 2.$$

$$\tan^2 x \tan^4 x = 2.$$

$$\tan^4 x + \tan^2 x - 2 = 0.$$

$$(\tan^2 x - 1)(\tan^2 x + 2) = 0.$$

$$(1) \quad \tan^2 x - 1 = 0.$$

$$\tan^2 x = 1.$$

$$\tan x = \pm 1.$$

$$x = 45^\circ, 135^\circ, 225^\circ, \text{ 或 } 315^\circ.$$

$$(2) \quad \tan^2 x + 2 = 0.$$

$$\tan^2 x = -2.$$

$$\tan x = \pm \sqrt{-2}.$$

故  $x$  不能成角

故  $x$  祇可等於  $45^\circ, 135^\circ, 225^\circ, 315^\circ$ .

但  $(x = 225^\circ)$  及  $(x = 315^\circ)$  皆與本題不合

故  $x = 45^\circ$ , 或  $135^\circ$ .

$$\text{III} \quad \sin x = \cos 2x.$$

$$\sin x = \cos 2x.$$

$$\text{按 [13],} \quad \sin x = \cos^2 x - \sin^2 x.$$

$$= 1 - \sin^2 x - \sin^2 x$$

$$2 \sin^2 x + \sin x - 1 = 0.$$

$$(\sin x + 1)(2 \sin x - 1) = 0.$$

$$\sin x + 1 = 0.$$

$$\sin x = -1.$$

$$x = 270^\circ.$$

$$2 \sin x - 1 = 0.$$

$$2 \sin x = 1.$$

$$\sin x = \frac{1}{2}.$$

$$x = 30^\circ, \text{ 或 } 150^\circ.$$

故  $x = 30^\circ, 150^\circ, \text{ 或 } 270^\circ$ .

$$\text{II2.} \quad \tan x \tan 2x = 2.$$

$$\tan x \tan 2x = 2.$$



按[14],  $\tan x \frac{2 \tan x}{1 - \tan^2 x} = 2.$

$$2 \tan^2 x = 2 - 2 \tan^2 x.$$

$$4 \tan^2 x = 2.$$

$$\tan^2 x = \frac{1}{2}.$$

$$\tan x = \pm \sqrt{\frac{1}{2}} = \pm \frac{1}{2} \sqrt{2} = \pm 0.7071.$$

故

$$x = 35^\circ 16', 144^\circ 44', 215^\circ 16', \text{或 } 334^\circ 54'.$$

113

$$\sec x = 4 \csc x.$$

$$\sec x = 4 \csc x.$$

$$\frac{1}{\cos x} = \frac{4}{\sin x}.$$

$$\sin x = 4 \cos x.$$

$$\sin^2 x = 16 \cos^2 x.$$

$$1 = \cos^2 x = 16 \cos^2 x.$$

$$17 \cos^2 x = 1.$$

$$\cos x = \pm \frac{1}{\sqrt{17}} = \pm 0.2425.$$

故

$$x = 75^\circ 58', 104^\circ 2', 255^\circ 58', \text{或 } 284^\circ 2',$$

但

( $x = 104^\circ 2'$ )及( $x = 284^\circ 2'$ )皆與本題不合

故

$$x = 77^\circ 58' \text{ 或 } 255^\circ 58'.$$

114.  $\cos \theta + \cos 2\theta = 0.$

按 [13].  $\cos \theta + \cos^2 \theta - \sin^2 \theta = 0.$

$$\cos \theta + \cos^2 \theta - 1 + \cos^2 \theta = 0.$$

$$2 \cos^2 \theta + \cos \theta - 1 = 0.$$

$$(\cos \theta + 1)(2 \cos \theta - 1) = 0.$$

(1)  $\cos \theta + 1 = 0.$

$$\cos \theta = -1.$$

$$\theta = -180^\circ.$$

$$(2) \quad 2 \cos \theta - 1 = 0.$$

$$2 \cos \theta = 1.$$

$$\cos \theta = \frac{1}{2}.$$

$$\theta = 60^\circ \text{ 或 } 300^\circ.$$

故

$$\theta = 60^\circ, 180^\circ, \text{ 或 } 300^\circ.$$

$$115. \quad \cot \frac{1}{2}\theta + \csc \theta = 2.$$

$$\text{按 [19]'} \quad \pm \sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} + \frac{1}{\sin \theta} = 2.$$

$$\pm \sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} = 2 - \frac{1}{\sin \theta}.$$

$$\frac{1 + \cos \theta}{1 - \cos \theta} = 4 - \frac{4}{\sin \theta} + \frac{1}{\sin^2 \theta}.$$

$$\frac{1 + 2 \cos \theta + \cos^2 \theta}{1 - \cos^2 \theta} = 4 - \frac{4}{\sin \theta} - \frac{1}{\sin^2 \theta}$$

$$\frac{1 + 2 \cos \theta + \cos^2 \theta}{\sin^2 \theta} = 4 - \frac{4}{\sin \theta} - \frac{1}{\sin^2 \theta}.$$

$$\text{開方, } \frac{1 + \cos \theta}{\sin \theta} = 2 - \frac{1}{\sin \theta}.$$

$$1 + \cos \theta = 2 \sin \theta - 1.$$

$$2 + \cos \theta = 2 \sin \theta.$$

$$= 2\sqrt{1 - \cos^2 \theta}$$

$$4 + 4 \cos \theta + \cos^2 \theta = 4 - 4 \cos^2 \theta.$$

$$5 \cos^2 \theta + 4 \cos \theta = 0.$$

$$\cos \theta (5 \cos \theta + 4) = 0.$$

故

$$\cos \theta = 0, \text{ 或 } -0.8$$

又

$$2 + \sqrt{1 - \sin^2 \theta} = 2 \sin \theta,$$

$$1 - \sin^2 \theta = 4 \sin^2 \theta - 8 \sin \theta + 4.$$

$$5 \sin^2 \theta - 8 \sin \theta + 3 = 0.$$

$$(\sin \theta - 1)(5 \sin \theta - 3) = 0.$$

故  $\sin \theta = 1$  或  $0.6$ .

$$\cos \theta = 0$$
 或  $-0.8$ .

故  $\theta = 90^\circ$  或  $143^\circ 8'$ .

116.  $\cot x \tan 2x = 3.$

$$\cot x \tan 2x = 3.$$

按 [14],  $\frac{1}{\tan x} \times \frac{2 \tan x}{1 - \tan^2 x} = 3$

$$\frac{2}{1 - \tan^2 x} = 3.$$

$$2 = 3 - 3 \tan^2 x.$$

$$3 \tan^2 x = 1.$$

$$\tan^2 x = \frac{1}{3}.$$

$$\tan x = \pm \sqrt{\frac{1}{3}} = \pm \frac{1}{\sqrt{3}}.$$

故  $x = 30^\circ, 150^\circ, 210^\circ$ , 或  $330^\circ$ .

117.  $\sin x \sec 2x = 1.$

$$\sin x \sec 2x = 1.$$

$$\frac{\sin x}{\cos 2x} = 1.$$

$$\sin x = \cos 2x.$$

按 [13],  $\sin x = \cos^2 x - \sin^2 x.$

$$\sin x = 1 - 2 \sin^2 x.$$

$$2 \sin^2 x + \sin x - 1 = 0.$$

$$(\sin x + 1)(2 \sin x - 1) = 0.$$

$$(1) \quad \sin x + 1 = 0.$$

$$\sin x = -1.$$

$$x = 270^\circ.$$

$$(2) \quad 2\sin x - 1 = 0,$$

$$2 \sin x = 1.$$

$$\sin x = \frac{1}{2}.$$

$$x = 30^\circ \text{ 或 } 150^\circ.$$

故

$$x = 30^\circ, 150^\circ, \text{ 或 } 270^\circ.$$

$$118 \quad \sin^2 x + \sin 2x = 1.$$

$$\sin^2 x + \sin 2x = 1.$$

$$\text{按 [12], } \sin^2 x + 2 \sin x \cos x = 1.$$

$$\sin^2 x + 2 \sin x \sqrt{1 - \sin^2 x} = 1.$$

$$\sin^2 x + 2 \sin x \sqrt{1 - \sin^2 x} = 1.$$

$$2 \sin x \sqrt{1 - \sin^2 x} = 1 - \sin^2 x.$$

$$4 \sin^2 x - 4 \sin^4 x = 1 - 2 \sin^2 x + \sin^4 x.$$

$$5 \sin^4 x - 6 \sin^2 x + 1 = 0.$$

$$(\sin^2 x - 1)(\sin^2 x - \frac{1}{5}) = 0.$$

$$\sin^2 x - 1 = 0.$$

$$\sin^2 x = 1.$$

$$\sin x = \pm 1.$$

$$x = 90^\circ \text{ 或 } 270^\circ.$$

$$(2) \quad 5 \sin^2 x - 1 = 0.$$

$$5 \sin^2 x = 1.$$

$$\sin^2 x = \frac{1}{5}$$

$$\sin x = \pm \sqrt{\frac{1}{5}} = \pm \frac{1}{\sqrt{5}} \approx \pm 0.447.$$

$$x = 26^{\circ}34', 153^{\circ}26', 206^{\circ}34', \text{或} 333^{\circ}26'.$$

故  $x = 26^{\circ}34', 90^{\circ}, 153^{\circ}26', 206^{\circ}34', 270^{\circ}, \text{或} 333^{\circ}26'.$

但  $(x = 153^{\circ}26')$  及  $(x = 333^{\circ}26')$  皆與本題不合。

故  $x = 26^{\circ}34', 90^{\circ}, 206^{\circ}34' \text{或} 270^{\circ}.$

119. 求解  $\cos x \sin 2x \csc x = 1.$

$$\cos x \sin 2x \csc x = 1.$$

按 [12],  $\cos x (2 \sin x \cos x) \times \frac{1}{\sin x} = 1.$

$$2 \cos^2 x = 1.$$

$$\cos^2 x = \frac{1}{2}.$$

$$\cos x = \pm \sqrt{\frac{1}{2}} = \pm \frac{1}{2} \sqrt{2}$$

故

$$x = 45^{\circ}, 135^{\circ}, 225^{\circ}, \text{或} 315^{\circ}.$$

120.  $\cot x \tan 2x = \sec 2x.$

按 [14], [13],  $\frac{1}{\tan x} \times \frac{2 \tan x}{1 - \tan^2 x} = \frac{1}{\cos^2 x - \sin^2 x}.$

$$\frac{2}{1 - \tan^2 x} = \frac{1}{\cos^2 x - \sin^2 x}.$$

$$2 \cos^2 x - 2 \sin^2 x = 1 - \frac{\sin^2 x}{1 - \sin^2 x}.$$

$$1 - 4 \sin^2 x = - \frac{\sin^2 x}{1 - \sin^2 x}.$$

$$1 - 5 \sin^2 x + 4 \sin^4 x = - \sin^2 x.$$

$$1 - 4 \sin^2 x + 4 \sin^4 x = 0.$$

開方,

$$1 - 2 \sin^2 x = 0.$$

$$2 \sin^2 x = 0.$$

$$\sin^2 = \frac{1}{2}$$

$$\sin^2 x = \pm \sqrt{\frac{1}{2}} = \pm \frac{1}{2} \sqrt{2}.$$

故

$$x = 45^\circ, 135^\circ, 225^\circ, \text{ 或 } 315^\circ.$$

I21.

$$\sin 2x = \cos 4x.$$

$$\sin 2x = \cos 4x.$$

$$\begin{aligned} \text{按 [13], } \sin 2x &= \cos^2 2x - \sin^2 2x \\ &= 1 - \sin^2 2x - \sin^2 2x. \end{aligned}$$

$$2 \sin^2 2x + \sin 2x - 1 = 0.$$

$$(\sin 2x + 1)(2 \sin 2x - 1) = 0.$$

$$(1) \quad \sin 2x + 1 = 0.$$

$$\sin 2x = -1.$$

$$2x = 270^\circ \text{ 或 } 630^\circ.$$

$$x = 135^\circ \text{ 或 } 315^\circ.$$

$$(2) \quad 2 \sin 2x - 1 = 0.$$

$$2 \sin 2x = 1.$$

$$\sin 2x = \frac{1}{2}.$$

$$2x = 30^\circ, 150^\circ, 390^\circ, \text{ 或 } 510^\circ.$$

$$x = 15^\circ, 75^\circ, 195^\circ, \text{ 或 } 255^\circ.$$

故

$$x = 15^\circ, 75^\circ, 135^\circ, 195^\circ, \text{ 或 } 255^\circ.$$

$$I22. \sin 2z \cot z - \sin^2 z = \frac{1}{2}.$$

$$\sin 2z \cot z - \sin^2 z = \frac{1}{2}.$$

$$\text{按 [13], } 2 \sin z \cos z \times \frac{\cos z}{\sin z} - \sin^2 z = \frac{1}{2}.$$

$$2 \cos^2 z - \sin^2 z = \frac{1}{2}.$$

$$2 - \sin^2 z - \sin^2 z = \frac{1}{2}.$$

$$3 \sin^2 z = \frac{3}{2}.$$

$$\sin^2 z = \frac{1}{2}.$$

$$\sin z = \pm \sqrt{\frac{1}{2}} = \pm \frac{1}{\sqrt{2}}.$$

故  $z = 45^\circ, 135^\circ, 225^\circ, \text{ 或 } 315^\circ.$

123.  $\tan x + \tan 2x = \tan 3x.$

按 [14], 及 24/19,

$$\tan x + \frac{2 \tan x}{1 - \tan^2 x} = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}.$$

$$\tan x \left( 1 + \frac{1}{1 - \tan^2 x} - \frac{3 - \tan^2 x}{1 - 3 \tan^2 x} \right) = 0.$$

(1)  $\tan x = 0.$

$$x = 0^\circ \text{ 或 } 180^\circ.$$

(2)  $1 + \frac{1}{1 - \tan^2 x} - \frac{3 - \tan^2 x}{1 - 3 \tan^2 x} = 0.$

$$1 - 4 \tan^2 x + 3 \tan^2 x + 2 - 6 \tan^2 x - 3 + 4 \tan^2 x - \tan^4 x = 0.$$

$$2 \tan^4 x - 6 \tan^2 x = 0.$$

$$\tan^2 x (\tan^2 x - 3) = 0.$$

$$\tan^2 x = 0 \text{ 或 } 3.$$

$$\tan x = 0.$$

$$x = 0^\circ, \text{ 或 } 180^\circ.$$

$$\tan x = \pm \sqrt{3}.$$

$$x = 60^\circ, 120^\circ, 240^\circ, \text{ 或 } 300^\circ.$$

故  $x = 0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ,$   
或  $300^\circ.$

124.  $\cot x - \tan x = \sin x + \cos x.$

$$\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \sin x + \cos x.$$

$$\frac{\cos^2 x - \sin^2 x}{\sin x \cos x} = \sin x + \cos x.$$

$$\cos^2 x - \sin^2 x = (\sin x + \cos x) \sin x \cos x.$$

$$(\sin x + \cos x)(\cos x - \sin x - \sin x \cos x) = 0.$$

$$(1) \quad \sin x + \cos x = 0.$$

$$\sin x = -\cos x.$$

$$x = 135^\circ \text{ 或 } 315^\circ.$$

$$(2) \quad \cos x - \sin x - \sin x \cos x = 0.$$

$$\cos x - \sin x = \sin x \cos x.$$

方之,  $x \cos^2 x + \sin^2 x - 2 \sin x \cos x = \sin^2 x \cos^2 x$

$$\sin^2 x \cos^2 x + 2 \sin x \cos x = 1.$$

$$\sin^2 x \cos^2 x + 2 \sin x \cos x + 1 = 2.$$

開方,  $\sin x \cos x + 1 = \pm \sqrt{2}.$

$$\sin x \cos x = -1 \pm \sqrt{2}.$$

$$2 \sin x \cos x = -2 \pm 2\sqrt{2}.$$

按 [12],  $\sin 2x = -2 \pm 2\sqrt{2}.$

$$= 0.8284 \text{ 或 } 4.8284.$$

$$2x = 55^\circ 56', 124^\circ 4', 415^\circ 56', \text{ 或 } 484^\circ 4'.$$

$$x = 27^\circ 58', 62^\circ 2', 207^\circ 58', \text{ 或 } 242^\circ 2'.$$

故  $x = 27^\circ 58', 62^\circ 2', 135^\circ, 207^\circ, 58', 242^\circ 2', \text{ 或 } 315^\circ$

但  $(x = 62^\circ 2')$  及  $(x = 207^\circ 58')$  皆與本題不合

故  $x = 27^\circ 58', 135^\circ, 242^\circ 2', \text{ 或 } 315^\circ.$

125.  $\tan^2 x = \sin 2x.$

$$\tan^2 x = 2x.$$

按 [12],  $\tan^2 x = 2 \sin x \cos x.$

$$\tan^2 x = 2 \tan x \cos^2 x.$$



$$\tan^2 x = \frac{2 \tan x}{\sec^2 x}.$$

按 5/2,

$$\tan^2 x = \frac{2 \tan x}{1 + \tan^2 x}.$$

$$\tan^2 x + \tan^4 x = 2 \tan x.$$

$$\tan x (\tan^3 x + \tan x + 2) = 0.$$

$$\tan x (\tan x - 1)(\tan^2 x + \tan x + 2) = 0.$$

$$(1) \quad \tan x = 0.$$

$$x = 0 \text{ 或 } 180^\circ.$$

$$(2) \quad \tan x - 1 = 0.$$

$$\tan x = 1.$$

$$x = 45^\circ \text{ 或 } 225^\circ.$$

$$(3) \quad \tan^2 x + \tan x - 2 = 0.$$

$$4 \tan^2 x + ( ) + 1 = -7.$$

開方,

$$2 \tan x + 1 = \pm \sqrt{-7}.$$

$$\tan x = \frac{1}{2}(-1 \pm \sqrt{-7}).$$

故

$x$  不能成角.

自 (1) (2),

$$x = 0^\circ, 45^\circ, 180^\circ, \text{ 或 } 225^\circ.$$

126.

$$\tan x + \cot x = \tan 2x.$$

按 [14],

$$\tan x + \frac{1}{\tan x} = \frac{2 \tan x}{1 - \tan^2 x}.$$

$$\frac{\tan^2 x + 1}{\tan x} = \frac{2 \tan x}{1 - \tan^2 x}.$$

$$1 - \tan^4 x = 2 \tan^2 x.$$

$$\tan^4 x + 2 \tan^2 x = 1.$$

$$\tan^4 x + 2 \tan^2 x + 1 = 2.$$

$$\tan^2 x + 1 = \pm\sqrt{2}.$$

$$\tan^2 x = -1 \pm\sqrt{2}.$$

$$= 0.4142 \text{ 或 } -2.4142.$$

$$\tan x = \pm 0.6436.$$

故  $x = 32^\circ 46', 147^\circ 14', 212^\circ 46', \text{ 或 } 327^\circ 14'.$

127. 
$$\frac{1 - \tan x}{1 + \tan x} = \cos 2x_0$$

$$\frac{1 - \tan x}{1 + \tan x} = \cos 2x.$$

按 [13], 
$$\frac{\cos x - \sin x}{\cos x + \sin x} = \cos^2 x - \sin^2 x = (\cos x - \sin x)(\cos x + \sin x).$$

$$\cos x - \sin x = (\cos x - \sin x)(\cos x + \sin x)^2.$$

$$(\cos x - \sin x) [1 - (\cos x + \sin x)^2] = 0.$$

(1) 
$$\cos x - \sin x = 0.$$

$$\cos x = \sin x.$$

$$x = 45^\circ \text{ 或 } 225^\circ$$

(2) 
$$1 - (\cos x + \sin x)^2 = 0.$$

$$1 - (\cos^2 x + \sin^2 x + 2 \sin x \cos x) = 0.$$

$$1 - (1 + 2 \sin x \cos x) = 0.$$

$$2 \sin x \cos x = 0.$$

$$\sin x \cos x = 0.$$

$$x = 0^\circ, 90^\circ, 180^\circ, \text{ 或 } 270^\circ.$$

故  $x = 0^\circ, 45^\circ, 90^\circ, 180^\circ, 225^\circ \text{ 或 } 270^\circ.$

128. 
$$\sin x + \sin 2x = 1 - \cos 2x_0$$

按 [12], 
$$\sin x + 2 \sin x \cos x = 1 - \cos 2x.$$

按 [16], 
$$\sin x + 2 \sin x \cos x = 2 \sin^2 x.$$

$$\sin x (1 + 2 \cos x - 2 \sin x) = 0;$$

$$(1) \quad \sin x = 0.$$

$$x = 0^\circ \text{ 或 } 180^\circ,$$

$$(2) \quad 1 + 2 \cos x - 2 \sin x = 0.$$

$$\sin x - \cos x = \frac{1}{2}.$$

$$\sin^2 x + \cos^2 x - 2 \sin x \cos x = \frac{1}{4}.$$

$$1 - 2 \sin x \cos x = \frac{1}{4}.$$

$$2 \sin x \cos x = \frac{3}{4}.$$

按 [12],  $\sin 2x = \frac{3}{4} = 0.75.$

$$2x = 48^\circ 36', 131^\circ 24', 408^\circ 36', \text{ 或 } 491^\circ 24'.$$

$$x = 24^\circ 18', 65^\circ 42', 204^\circ 18', \text{ 或 } 245^\circ 42'.$$

故  $x = 0^\circ, 24^\circ 18', 65^\circ 42', 180^\circ, 204^\circ 18' \text{ 或 } 245^\circ 42'.$

但  $(x = 24^\circ 18')$  及  $(x = 245^\circ 42')$  皆與本題不合

故  $x = 0^\circ, 65^\circ 42', 180^\circ, \text{ 或 } 204^\circ 18'.$

129.  $\sec 2x + 1 = 2 \cos x.$

$$\sec 2x + 1 = 2 \cos x.$$

$$\frac{1}{\cos x} + 1 = 2 \cos x$$

$$1 + \cos 2x = 2 \cos x \cos 2x.$$

按 [13],  $1 + \cos^2 x - \sin^2 x = 2 \cos x (\cos^2 x - \sin^2 x).$

$$1 + \cos^2 x - 1 + \cos^2 x = 2 \cos x (\cos^2 x - 1 + \cos^2 x).$$

$$2 \cos^2 x = 2 \cos x (2 \cos^2 x - 1).$$

$$\cos x (2 \cos^2 x - \cos x - 1) = 0.$$

$$\cos x (\cos x - 1) (2 \cos x + 1) = 0.$$

$$(1) \quad \cos x = 0.$$

$$x = 90^\circ, \text{ 或 } 270^\circ,$$

$$(2) \quad \cos x - 1 = 0.$$

$$\cos x = 1.$$

$$x = 0^\circ.$$

$$(3) \quad 2 \cos x + 1 = 0,$$

$$2 \cos x = -1.$$

$$\cos x = -\frac{1}{2}.$$

$$x = 120^\circ \text{ 或 } 240^\circ.$$

故  $x = 0^\circ, 90^\circ, 120^\circ, 240^\circ, \text{ 或 } 270^\circ.$

$$130. \quad \tan 2x + \tan 3x = 0.$$

$$\tan 2x + \tan 3x = 0.$$

$$\tan 2x = -\tan 3x = \tan(-3x).$$

故  $2x = -3x, \text{ 或 } 180^\circ - 3x.$

$$(1) \quad 5x = 0^\circ + n 360^\circ.$$

$$x = 0^\circ, 72^\circ, 144^\circ, 216^\circ, \text{ 或 } 288^\circ.$$

$$2) \quad 5x = 180^\circ + n 360^\circ.$$

$$x = 36^\circ, 180^\circ, 180^\circ + 252^\circ, \text{ 或 } 324^\circ.$$

故  $x = 0^\circ, 36^\circ, 72^\circ, 180^\circ, 144^\circ, 180^\circ, 216^\circ, 252^\circ, 288^\circ, \text{ 或 } 324^\circ$

$$131. \quad \tan\left(\frac{1}{4}\pi + x\right) + \tan\left(\frac{1}{4}\pi - x\right) = 4.$$

$$\tan\left(\frac{1}{4}\pi + x\right) + \tan\left(\frac{1}{4}\pi - x\right) = 4.$$

按 [6], [10],  $\frac{1 + \tan x}{1 - \tan x} + \frac{1 - \tan x}{1 + \tan x} = 4.$

$$1 + 2 \tan x + \tan^2 x + 1 - 2 \tan x + \tan^2 x = 4 - 4 \tan^2 x.$$

$$6 \tan^2 x = 2.$$

$$\tan^2 x = \frac{1}{3}.$$

$$\tan x = \pm \sqrt{\frac{1}{3}} = \pm \frac{1}{\sqrt{3}}.$$

故  $x = 30^\circ, 150^\circ, 210^\circ, \text{ 或 } 330^\circ.$

$$132. \quad \sqrt{1 + \sin x} - \sqrt{1 - \sin x} = 2 \cos x.$$

方之.  $1 + \sin x - 2\sqrt{1 - \sin^2 x} + 1 - \sin x = 4 \cos^2 x.$

$$2 - 2\sqrt{1 - \sin^2 x} = 4 \cos^2 x.$$

$$1 - \sqrt{\cos^2 x} = 2 \cos^2 x.$$

$$1 - \cos x = 2 \cos^2 x.$$

$$2 \cos x + \cos x - 1 = 0.$$

$$(\cos x + 1)(2 \cos x - 1) = 0.$$

$$\cos x + 1 = 0.$$

$$\cos x = -1.$$

$$x = 180^\circ.$$

$$2 \cos x - 1 = 0.$$

$$\cos x = \frac{1}{2}.$$

$$x = 60^\circ, \text{ 或 } 300^\circ.$$

故  $x = 60^\circ, 180^\circ, \text{ 或 } 300^\circ$

但  $(x = 180^\circ)$  及  $(x = 300^\circ)$  皆與本題不合.

故  $x = 60^\circ.$

$$133. \quad \tan x \tan 3x = -\frac{2}{5}.$$

$$\tan x \tan 3x = -\frac{2}{5}.$$

按  $24/19,$

$$\tan x \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} = -\frac{2}{5}.$$

$$\frac{3 \tan^2 x - \tan^4 x}{1 - 3 \tan^2 x} = -\frac{2}{5}.$$

$$15 \tan^2 x - 5 \tan^4 x = -2 + 6 \tan^2 x.$$

$$5 \tan^4 x - 9 \tan^2 x - 2 = 0.$$

$$(\tan^2 x - 2)(5 \tan^2 x + 1) = 0.$$

$$(1) \quad \tan^2 x - 2 = 0.$$

$$\tan^2 x = 2.$$

$$\tan x = \pm\sqrt{2} = \pm 1.4142.$$

$$x = 54^\circ 44', 125^\circ 16', 234^\circ 44', \text{ 或 } 305^\circ 16'.$$

$$(2) \quad 5 \tan^2 x + 1 = 0,$$

$$5 \tan^2 x = -1.$$

$$\tan^2 x = -\frac{1}{5}.$$

$$\tan x = \pm\sqrt{-\frac{1}{5}}$$

成

x 不能成角

故自 (1),  $x = 54^\circ 44', 125^\circ 16', 234^\circ 44', \text{ 或 } 305^\circ 16'.$

$$134. \quad \sin(45^\circ + x) + \cos(45^\circ + x) = 1.$$

$$\sin(45^\circ + x) + \cos(45^\circ - x) = 1.$$

但  $45^\circ + x = 90^\circ - (45^\circ - x).$

故  $\cos(45^\circ - x) + \cos(45^\circ - x) = 1.$

$$2 \cos(45^\circ - x) = 1,$$

$$\cos(45^\circ - x) = \frac{1}{2}.$$

$$45^\circ - x = 60^\circ \text{ 故 } 300^\circ.$$

故

$$x = 15^\circ \text{ 或 } -255^\circ.$$

$$x = 105^\circ \text{ 或 } 345^\circ.$$

135.

$$\tan x + \sec x = a.$$

$$\tan x + \sec x = a.$$

$$\sec x = a - \tan x.$$

$$\sec^2 x = a^2 - 2a \tan x + \tan^2 x.$$

按

$$5/2 \quad 1 + \tan^2 x = a^2 - 2a \tan x + \tan^2 x,$$

$$2a \tan^2 x = a^2 - 1.$$

$$\tan x = \frac{a^2 - 1}{2a}$$

故

$$x = \tan^{-1} \frac{a^2 - 1}{2a}.$$

136.

$$\cos 2x = a(1 - \cos x).$$

$$\cos 2x = a(1 - \cos x).$$

按 [13],

$$\cos^2 x - \sin^2 x = a(1 - \cos x).$$

$$\cos^2 x - 1 + \cos^2 x = a - a \cos x.$$

$$2 \cos^2 x + a \cos x = a + 1.$$

$$15 \cos^2 x + ( ) + a^2 = a^2 + 8a + 8.$$

$$4 \cos x + a = \pm \sqrt{a^2 + 8a + 8}.$$

$$\cos x = \frac{-a \pm \sqrt{a^2 + 8a + 8}}{4}.$$

$$x = \cos^{-1} \left( \frac{-a \pm \sqrt{a^2 + 8a + 8}}{4} \right).$$

137.

$$(1 - \tan x) \cos 2x = a(1 + \tan x).$$

$$(1 - \tan x) \cos 2x = a(1 + \tan x).$$

$$\cos 2x = \frac{1 + \tan x}{1 - \tan x}.$$

$$\cos 2x = \frac{a(\cos x + \sin x)}{\cos x - \sin x}.$$

按 [13],

$$\cos^2 x - \sin^2 x = \frac{a(\cos x + \sin x)}{\cos x - \sin x}.$$

$$(\cos x - \sin x)^2 (\cos x + \sin x) - a(\cos x + \sin x) = 0.$$

$$(\cos x + \sin x) [(\cos x - \sin x)^2 - a] = 0.$$

(1)

$$\cos x + \sin x = 0.$$

$$\sin x = -\cos x$$

$$x = 135^\circ \text{ 或 } 315^\circ.$$

$$(2) \quad (\cos x = \sin x)^2 - a = 0.$$

$$\cos^2 x + \sin^2 x - 2 \sin x \cos x = a.$$

$$1 - 1 \sin x \cos x = a.$$

$$2 \sin x \cos x = 1 - a.$$

$$\sin 2x = 1 - a$$

$$2x = \sin^{-1}(1 - a).$$

$$x = \frac{1}{2} \sin^{-1}(1 - a).$$

故

$$x = 135^\circ, 315^\circ, \text{ 或 } \frac{1}{2} \sin^{-1}(1 - a)$$

$$138. \quad \sin^6 x + \cos^6 x = \frac{1}{12} \sin^2 2x.$$

$$\sin^6 x + \cos^6 x = \frac{7}{12} \sin^2 2x.$$

$$(\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x) = \frac{7}{12} \sin^2 2x.$$

$$\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x = \frac{7}{12} \sin^2 2x.$$

$$(\sin^4 x + 2 \sin^2 x \cos^2 x + \cos^4 x) - 3 \sin^2 x \cos^2 x = \frac{1}{3} \sin^2 x \cos^2 x.$$

$$(\sin^2 x + \cos^2 x)^2 - \frac{16}{3} \sin^2 x \cos^2 x.$$

$$1 = \frac{16}{3} \sin^2 x \cos^2 x.$$

$$16 \sin^2 x \cos^2 x = 3$$

$$4 \sin^2 x \cos^2 x = \frac{3}{4}$$

$$\sin^2 x = \pm \frac{1}{2} \sqrt{\frac{3}{4}}$$

$$2x = 60^\circ, 120^\circ, 240^\circ, 300^\circ, 420^\circ, 480^\circ, 600^\circ, \text{ 或 } 660^\circ.$$

$$x = 30^\circ, 60^\circ, 120^\circ, 250^\circ, 210^\circ, 240^\circ, 300^\circ, \text{ 或 } 330^\circ.$$

$$139. \quad \cos 3x + 8 \cos^2 x = 0.$$

$$\cos 3x + 8 \cos^3 x = 0.$$

按 14119.

$$4 \cos^3 x - 3 \cos x + 8 \cos^3 x = 0.$$

$$12 \cos^3 x - 3 \cos x = 0.$$



$$\cos x (4 \cos^2 x - 1) = 0:$$

$$\cos x = 0.$$

$$(1) \quad x = 90^\circ \text{ 或 } 270^\circ.$$

$$(2) \quad 4 \cos^2 x - 1 = 0.$$

$$4 \cos^2 x = 1.$$

$$\cos^2 x = \frac{1}{4}$$

$$\cos = \pm \frac{1}{2}.$$

$$x = 60^\circ, 120^\circ, 240^\circ, \text{ 或 } 300^\circ.$$

$$\text{故 } x = 60^\circ, 90^\circ, 120^\circ, 240^\circ, 270^\circ, \text{ 或 } 300^\circ.$$

$$140. \quad \sec(x + 120^\circ) + \sec(x + 120^\circ) = 2 \cos x_0$$

$$\sec(x + 120^\circ) + \sec(x - 120^\circ) = 2 \cos x.$$

$$\frac{1}{\cos(x + 120^\circ)} + \frac{1}{\cos(x - 120^\circ)} = 2 \cos x,$$

$$\frac{\cos(x - 120^\circ) + \cos(x + 120^\circ)}{\cos(x - 120^\circ) \cos(x + 120^\circ)} = 2 \cos x.$$

$$\text{按 [22], } \frac{2 \cos x \cos 120^\circ}{\cos(x + 120^\circ) \cos(x - 120^\circ)} = 2 \cos x,$$

按 [5] 及 [9],

$$\frac{2 \cos x \cos 120^\circ}{(\cos x \cos 120^\circ - \sin x \sin 120^\circ)(\cos x \cos 120^\circ + \sin x \sin 120^\circ)} = 2 \cos x.$$

$$\frac{2 \cos x \cos 120^\circ}{\cos^2 x \cos^2 120^\circ - \sin^2 x \sin^2 120^\circ} = 2 \cos x.$$

$$\text{但 } \sin 120^\circ = \frac{1}{2}\sqrt{3}, \quad \cos 120^\circ = -\frac{1}{2}.$$

$$\text{故 } \frac{-\cos x}{\frac{1}{4}\cos^2 x - \frac{3}{4}\sin^2 x} = 2 \cos x,$$

$$-\cos x = \frac{1}{2}\cos x (\cos^2 x - 3 \sin^2 x).$$

$$-2 \cos x = \cos x (4 \cos^2 x - 3).$$

$$\cos x (4 \cos^2 x - 1) = 0.$$

$$(1) \quad \cos x = 0.$$

$$x = 90^\circ, \text{ 或 } 270^\circ.$$

$$(2) \quad 4 \cos^2 x - 1 = 0.$$

$$4 \cos^2 x = 1.$$

$$\cos = \pm \frac{1}{2}.$$

$$x = 60^\circ, 120^\circ, 240^\circ, \text{ 或 } 300^\circ.$$

$$\text{故 } x = 60, 90^\circ, 120^\circ, 240^\circ, 270^\circ, \text{ 或 } 300^\circ.$$

$$141. \quad \csc x = \cot x + \sqrt{3}$$

$$\csc x = \cot x + \sqrt{3}$$

$$\csc^2 x = \cot^2 x + 2\sqrt{3} \cot x + 3.$$

$$\text{按 513, } 1 + \cot^2 x = \cot^2 x + 2\sqrt{3} \cot x + 3.$$

$$2\sqrt{3} \cot x = -2.$$

$$\cot x = -\frac{1}{\sqrt{3}} = -\frac{1}{3}\sqrt{3}$$

$$x = 120^\circ, \text{ 或 } 300^\circ.$$

但  $x = 300^\circ$  與本題不合.

$$\text{故 } x = 120^\circ.$$

$$142. \quad 4 \cos 2x + 6 \sin x = 5.$$

$$4 \cos 2x + 6 \sin x = 5.$$

$$\text{按 [13], } 4(1 - 2\sin^2 x) + 6\sin x = 5.$$

$$4 - 8\sin^2 x + 6\sin x = 5.$$

$$8 \sin^2 x - 6\sin x + 1 = 0.$$

$$(2 \sin x - 1)(4 \sin x - 1) = 0.$$

$$(1) \quad 2 \sin x - 1 = 0.$$

$$2 \sin x = 1.$$

$$\sin x = \frac{1}{2}.$$

$$x = 30^\circ \text{ 或 } 150^\circ.$$

$$(2) \quad 4 \sin x - 1 = 0.$$

$$4 \sin x = 1$$

$$\sin x = \frac{1}{4} = 0.25$$

$$x = 14^\circ 29', 165^\circ, 31'$$

$$\text{故} \quad x = 14^\circ 29', 30^\circ, 150^\circ \text{ 或 } 165^\circ 31'.$$

$$143. \quad \cos x - \cos 2x = 1.$$

$$\cos x - \cos 2x = 1$$

$$\text{按 [13], } \cos x - \cos^2 x + \sin^2 x = 1$$

$$\cos x - \cos^2 x + 1 - \cos^2 x = 1.$$

$$2 \cos^2 x - \cos x = 0.$$

$$(1) \quad \cos x = 0.$$

$$x = 90^\circ \text{ 或 } 270^\circ$$

$$(2) \quad 2 \cos x - 1 = 0.$$

$$2 \cos x = 1.$$

$$\cos x = \frac{1}{2}.$$

$$x = 60^\circ \text{ 或 } 300^\circ.$$

$$\text{故} \quad x = 60^\circ, 90^\circ, 270^\circ, \text{ 或 } 300^\circ$$

$$144. \quad \sin 4x - \sin 2x = \sin x$$

$$\sin 4x - \sin 2x = \sin x.$$

$$\text{按 [21], } 2 \cos 3x \sin x = \sin x.$$

$$\sin x (2 \cos 3x - 1) = 0.$$

$$(1) \quad \sin x = 0.$$

$$x = 0^\circ \text{ 或 } 180^\circ.$$

$$(2) \quad 2 \cos 3x = 1 = 0.$$

$$2 \cos 3x = 1,$$

$$\cos 3x = \frac{1}{2}.$$

$$3x = 60^\circ + n \cdot 360^\circ, \text{ 或 } 300^\circ + n \cdot 360^\circ.$$

$$x = 20^\circ, 142^\circ, \text{ 或 } 260^\circ. \text{ 又 } 100^\circ, 220^\circ, \text{ 或 } 340^\circ.$$

$$\text{故 } x = 0^\circ, 20^\circ, 100^\circ, 140^\circ, 180^\circ, 220^\circ, 260^\circ \text{ 或 } 340^\circ$$

$$145. \quad 2 \sin^2 x + \sin^2 2x = 2.$$

$$2 \sin^2 x + \sin^2 2x = 2$$

$$\text{按 [12], } 2 \sin^2 x + 4 \sin^2 x \cos^2 x = 2$$

$$\sin^2 x + 2 \sin^2 x (1 - \sin^2 x) = 1.$$

$$\sin^2 x + 2 \sin^2 x - 2 \sin^4 x = 1$$

$$2 \sin^4 x - 3 \sin^2 x + 1 = 0$$

$$(\sin^2 x - 1)(2 \sin^2 x - 1) = 0.$$

$$(1) \quad \sin^2 x - 1 = 0$$

$$\sin^2 x = 1$$

$$\sin x = \pm 1$$

$$x = 0^\circ, 90^\circ, 180^\circ \text{ 或 } 270^\circ.$$

$$(2) \quad 2 \sin^2 x - 1 =$$

$$2 \sin^2 x = 1$$

$$2 \sin^2 x = \frac{1}{2}$$

$$\sin x = \pm \sqrt{\frac{1}{4}} = \pm \frac{1}{2} \sqrt{2}$$

$$x = 45^\circ, 135^\circ, 225^\circ, \text{ 或 } 315^\circ.$$

$$\text{故 } x = 0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, \text{ 或 } 315^\circ.$$

$$\text{但 } (x = 0^\circ) \text{ 及 } (x = 180^\circ) \text{ 皆與本題不合}$$

$$\text{故 } x = 45^\circ, 90^\circ, 135^\circ, 225^\circ, 270^\circ, \text{ 或 } 315^\circ.$$

$$146. \quad \cos 5x = \cos 3x + \cos x = 0^\circ$$

$$\cos 5x + \cos 3x + \cos x = 0$$

按 [22],  $2 \cos 4x \cos x + \cos x = 0$

$$\cos x(2 \cos 4x + 1) = 0$$

$$(1) \quad \cos x = 0$$

$$x = 90^\circ, \text{ 或 } 270^\circ$$

$$(2) \quad 2 \cos 4x + 1 = 0$$

$$2 \cos 4x = -1$$

$$\cos 4x = -\frac{1}{2}$$

$$4x = 120^\circ + n 360^\circ, \text{ 或 } 240^\circ + n 360^\circ.$$

$$x = 30^\circ + n 120^\circ, 210^\circ, \text{ 或 } 300^\circ. \text{ 又 } 60^\circ, 150^\circ, 240^\circ, \text{ 或 } 330^\circ.$$

故  $x = 3^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ, 210^\circ, 240^\circ, 270^\circ,$   
 $300^\circ, \text{ 或 } 330^\circ.$

$$147. \quad \sec x - \cot x = \cos x - \tan x^\circ$$

$$\sec x - \cot x = \csc x - \tan x.$$

$$\frac{1}{\csc x} - \frac{\cos x}{\sin x} = \frac{1}{\sin x} - \frac{\sin x}{\cos x}.$$

$$\sin x - \cos^2 x = \cos x - \sin^2 x.$$

$$(\sin x - \cos x) + (\sin^2 x - \cos^2 x) = 0.$$

$$(\sin x - \cos x)(1 + \sin x + \cos x) = 0.$$

$$(1) \quad \sin x - \cos x = 0$$

$$\sin x = \cos x.$$

$$x = 45^\circ \text{ 或 } 225^\circ.$$

$$(2) \quad 1 + \sin x + \cos x = 0$$

$$\sin x + \cos x = -1.$$

$$\sin^2 x + 2 \sin x \cos x + \cos^2 x = 1.$$

$$1 + 2 \sin x \cos x = 1$$

$$2 \sin x \cos x = 0$$

$$\sin 2x = 0$$

$$2x = 0^\circ, 180^\circ, 360^\circ, \text{或} 540^\circ$$

$$x = 0^\circ, 90^\circ, 180^\circ, \text{或} 270^\circ$$

$$x = 0^\circ, 45^\circ, 90^\circ, 180^\circ, 270^\circ.$$

故

$$148. \quad \tan^2 x + \cot^2 x = \frac{10}{3}$$

$$\tan^2 x + \cot^2 x = \frac{10}{3}$$

$$\tan^2 x + \frac{1}{\tan^2 x} = \frac{10}{3}$$

$$3 \tan^4 x + 3 = 10 \tan^2 x$$

$$3 \tan^4 x - 10 \tan^2 x + 3 = 0.$$

$$(\tan^2 x - 3)(3 \tan^2 x - 1) = 0.$$

$$(1) \quad \tan^2 x - 3 = 0.$$

$$\tan^2 x = 3.$$

$$\tan x = \pm \sqrt{3}$$

$$x = 60^\circ, 120^\circ, 240^\circ, \text{或} 300^\circ$$

$$(2) \quad 3 \tan^2 x - 1 = 0.$$

$$3 \tan^2 x = 1$$

$$\tan^2 x = \frac{1}{3}$$

$$\tan x = \pm \sqrt{\frac{1}{3}} = \pm \frac{1}{\sqrt{3}}$$

$$x = 30^\circ, 150^\circ, 210^\circ, \text{或} 330^\circ$$

故

$$x = 30^\circ, 60^\circ, 120^\circ, 150^\circ, 210^\circ, 240^\circ,$$

$$300^\circ, \text{或} 330^\circ.$$

$$149 \quad \sin 4x - \cos 3x = \sin 2x_0$$

$$\sin 4x - \cos 3x = \sin 2x.$$

$$\sin 4x - \sin 2x - \cos 3x = 0$$

$$\text{按 } 2\cos 3x \sin x - \cos 3x = 0$$

$$\cos 3x(2 \sin x - 1) = 0$$

$$\cos 3x = 0$$

$$(1) \quad 3x = 90^\circ + n \cdot 360^\circ, \text{ 或 } 270^\circ + n \cdot 360^\circ.$$

$$x = 30^\circ, 150^\circ, \text{ 或 } 270^\circ, \text{ 又 } 90^\circ, 210^\circ, \text{ 或 } 330^\circ.$$

$$(2) \quad 2 \sin x + 1 = 0$$

$$2 \sin x = -1$$

$$\sin x = -\frac{1}{2}.$$

$$x = 30^\circ, \text{ 故 } 250^\circ.$$

$$\text{故 } x = 30^\circ, 90^\circ, 150^\circ, 210^\circ, 270^\circ, \text{ 或 } 330^\circ.$$

$$150. \quad \sin x + \cos x = \sec x$$

$$\sin x + \cos x = \sec x.$$

$$\sin x + \cos x = \frac{1}{\cos x}$$

$$\sin x \cos x + \cos^2 x = 1.$$

$$\sin x \cos x + (\cos^2 x - 1) = 0.$$

$$\sin x \cos x - \sin^2 x = 0.$$

$$\sin x(\cos x - \sin x) = 0.$$

$$(1) \quad \sin x = 0$$

$$x = 0^\circ \text{ 或 } 180^\circ$$

$$(2) \quad \cos x - \sin x = 0,$$

$$\cos x = \sin x$$

$$x = 45^\circ, \text{ 或 } 225^\circ.$$

$$\text{故 } x = 0^\circ, 45^\circ, 180^\circ, \text{ 或 } 225^\circ.$$

$$151. \quad 2 \cos x \cos 3x + 1 = 0$$

$$2 \cos x \cos 3 = 0.$$

按  $14/19,$

$$2 \cos x (4 \cos^3 x - 3 \cos x) + 1 = 0$$

$$8 \cos^4 x - 6 \cos^2 x + 1 = 0$$

$$(4 \cos^2 x - 1)(2 \cos^2 x - 1) = 0.$$

$$(1) \quad 4 \cos^2 x - 1 = 0$$

$$4 \cos^2 x = 1.$$

$$\cos^2 x = \frac{1}{4}$$

$$\cos x = \pm \sqrt{\frac{1}{4}} = \pm \frac{1}{2}$$

$$(2) \quad x = 60^\circ, 120^\circ, 240^\circ, \text{或 } 300^\circ.$$

$$2 \cos^2 x - 1 = 0$$

$$2 \cos^2 x = 1$$

$$\cos^2 x = \frac{1}{2}$$

$$\cos x = \pm \sqrt{\frac{1}{2}} = \pm \frac{1}{\sqrt{2}}$$

$$x = 45^\circ, 135^\circ, 225^\circ, \text{或 } 315^\circ.$$

故  $x = 45^\circ, 60^\circ, 120^\circ, 135^\circ, 225^\circ, 240^\circ, 300^\circ, \text{或 } 315^\circ.$

$$152 \quad \cos 3x - 2 \cos 2x + \cos x = 0.$$

$$(\cos 3x + \cos x) - 2 \cos 2x = 0.$$

$$\cos 2x(\cos x - 1) = 0$$

$$(1) \quad \cos 2x = 0$$

$$2x = 90^\circ, 270^\circ, 450^\circ, \text{或 } 630^\circ,$$

$$x = 45^\circ, 135^\circ, 225^\circ; \text{或 } 315^\circ,$$

$$(2) \quad \cos x - 1 = 0.$$

$$\cos x = 1.$$

$$x = 0^\circ$$



故  $x=0^\circ, 45^\circ, 135^\circ, 225^\circ$ , 故  $315^\circ$ .

153.  $\tan 2x \tan x = 1$

$$\tan 2x \tan x = 1.$$

$$\tan 2x = \frac{1}{\tan x} \cot x$$

$$2x = 90^\circ - x \text{ 故 } 270^\circ - x$$

$$3x = 90^\circ + n360^\circ \text{ 或 } 270^\circ + n360^\circ$$

$$x = 30^\circ, 150^\circ, \text{ 或 } 270^\circ, \text{ 又 } 90^\circ, 210^\circ, \text{ 或 } 330^\circ.$$

故  $x = 30^\circ, 90^\circ, 150^\circ, 210^\circ, 270^\circ, \text{ 或 } 330^\circ.$

154  $\sin(x + 12^\circ) + \sin(x - 8^\circ) = \sin 20^\circ$

$$\sin(x + 12^\circ) + \sin(x - 8^\circ) = \sin 20^\circ.$$

按 [20],  $2 \sin(x + 2^\circ) \cos 10^\circ = \sin 20^\circ.$

$$2 \sin(x + 2^\circ) \cos 10^\circ = 2 \sin 10^\circ \cos 10^\circ.$$

$$2 \sin(x + 2^\circ) = 2 \sin 10^\circ.$$

$$\sin(x + 2^\circ) = \sin 10^\circ.$$

$$x + 2^\circ = 10^\circ \text{ 或 } 170^\circ.$$

故  $x = 8^\circ \text{ 或 } 168^\circ.$

255.  $\tan(60^\circ + x) \tan(60^\circ - x) = -2.$

$$\tan(60^\circ + x) \tan(60^\circ - x) = -2.$$

按 [6] 及 [10].

$$\frac{\tan 60^\circ + \tan x}{1 - \tan 60^\circ \tan x} \times \frac{\tan 60^\circ - \tan x}{1 + \tan 60^\circ \tan x} = -2.$$

$$\frac{\tan^2 60^\circ - \tan^2 x}{1 - \tan^2 60^\circ \tan^2 x} = -2.$$

$$\frac{3 - \tan^2 x}{1 - 3 \tan^2 x} = -2.$$

$$3 - \tan^2 x = -2 + 6 \tan^2 x.$$

$$7 \tan^2 x = 5.$$

$$\tan^2 x = \frac{5}{7} = 0.71428571$$

$$\tan x = \pm 0.8451.$$

故  $x = 40^\circ 12' 139^\circ 18', 220^\circ 12',$  或  $319^\circ 48'.$

$$156. \quad \sin(x + 120^\circ) + \sin(x + 60^\circ) = \frac{3}{2}$$

$$\sin(x + 120^\circ) + \sin(x + 60^\circ) = \frac{3}{2}$$

$$\text{按 [20], } 2 \sin(x + 90^\circ) \cos 30^\circ = \frac{3}{2}$$

$$2 \cos x \cos 30^\circ = \frac{3}{2}.$$

$$2 \cos x \left(\frac{1}{2}\sqrt{3}\right) = \frac{3}{2}.$$

$$\cos x = \frac{3}{2\sqrt{3}} = \frac{1}{2}\sqrt{3}$$

故  $x = 30^\circ$  或  $330^\circ.$

$$157 \quad \sin(x + 30^\circ) \sin(x - 30^\circ) = \frac{1}{4}.$$

$$\sin(x + 30^\circ) \sin(x - 30^\circ) = \frac{1}{4}.$$

$$\text{按 [23], } -\frac{1}{2}(\cos 2x - \cos 60^\circ) = \frac{1}{4}.$$

$$\cos 2x - \cos 60^\circ = 1.$$

$$\cos 2x - \frac{1}{2} = 1.$$

$$2x = 120^\circ, 240^\circ, 480^\circ, \text{ 或 } 600^\circ.$$

故  $x = 60^\circ, 120^\circ, 240^\circ, \text{ 或 } 300^\circ.$

$$158. \quad \sin^4 x + \cos^4 x = \frac{5}{8}$$

$$\sin^4 x + \cos^4 x = \frac{5}{8}.$$

$$\sin^4 x + 2 \sin^2 x \cos^2 x + \cos^4 x = 2 \sin^2 x \cos^2 x + \frac{5}{8}.$$

$$(\sin^2 x + \cos^2 x)^2 = 2 \sin^2 x \cos^2 x + \frac{5}{8}$$

$$1 = 2 \sin^2 x \cos^2 x + \frac{5}{8}$$

$$2 \sin^2 x \cos^2 x = \frac{3}{8}$$

$$4 \sin^2 x \cos^2 x = \frac{3}{4}$$

按 [12],  $\sin^2 2x = \frac{3}{4}$ .

$$\sin^2 x = \pm \frac{1}{2} \sqrt{3}$$

$$2x = 60^\circ, 120^\circ, 240^\circ, 300^\circ, 420^\circ, 480^\circ, 600^\circ, \text{ 或 } 660^\circ,$$

$$\text{故 } x = 30^\circ, 60^\circ, 120^\circ, 150^\circ, 210^\circ, 240^\circ, 300^\circ, \text{ 或 } 330^\circ,$$

159.  $\sin^4 x - \cos^4 x = \frac{7}{25}$

$$\sin^2 x - \cos^2 x = \frac{7}{25}$$

$$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = \frac{7}{25}$$

$$1 \times (\sin^2 x - \cos^2 x) = \frac{7}{25}$$

$$\cos^2 x - \sin^2 x = -\frac{7}{25}$$

按 [13],  $\cos 2x = -\frac{7}{25}$

$$2x = 106^\circ 16', 253^\circ 44', 466^\circ 16', \text{ 或 } 613^\circ 44',$$

$$\text{故 } x = 53^\circ 8', 126^\circ 52', 233^\circ 8', \text{ 或 } 306^\circ 52'$$

160.  $\tan(x + 30^\circ) = 2 \cos x$

設  $x + 30^\circ = y$

則  $x = y - 30^\circ$ .

相代  $\tan y = 2 \cos(y - 30^\circ)$

按 [9],  $\tan y = 2 \cos y \cos 30^\circ + 2 \sin y \sin 30^\circ$

$$\frac{\sin y}{\cos y} = \sqrt{3} \cos y + \sin y.$$

$$\sin y = \sqrt{3} \cos^2 y + \sin y \cos y.$$

$$\sin y(1 - \cos y) = \sqrt{3} \cos^2 y.$$

$$\sin^2 y(1 - \cos y)^2 = 3 \cos^4 y.$$

$$(1 - \cos^2 y)(1 - 2 \cos y + \cos^2 y) = 3 \cos^4 y.$$

$$1 - 2 \cos y + 2 \cos^3 y - \cos^4 y = 3 \cos^4 y.$$

$$4 \cos^4 y - 2 \cos^3 y + 2 \cos y - 1 = 0.$$

$$(2 \cos y - 1)(2 \cos^3 y + 1) = 0.$$

$$(1) \quad 2 \cos y - 1 = 0.$$

$$2 \cos y = 1.$$

$$\cos y = \frac{1}{2}$$

$$y = 60^\circ \text{ 或 } 300^\circ.$$

$$(2) \quad x = 30^\circ \text{ 或 } 270^\circ.$$

$$2 \cos^3 y + 1 = 0.$$

$$2 \cos^3 y = -1.$$

$$\cos^3 y = -\frac{1}{2}.$$

$$\cos y = -\sqrt[3]{\frac{1}{2}}$$

(2)式內之餘根甚為複雜茲姑不論

$$\cos y = -0.7937$$

$$y = 127^\circ 28' \text{ 或 } 232^\circ 32'$$

$$x = 97^\circ 28' \text{ 或 } 202^\circ 32'$$

$$x = 30^\circ, 97^\circ 28', 202^\circ 32', \text{ 或 } 270^\circ.$$

故

但  $(x = 97^\circ 28')$ ,  $(x = 202^\circ 32')$ , 及  $(x = 270)$  皆與本題不合

故

$$x = 30^\circ.$$

161.

$$\sec x = 2 \tan x + \frac{1}{4}.$$

$$\sec x - 2 \tan x + \frac{1}{4}$$

$$\sec^2 x = 4 \tan^2 x + \tan x + \frac{1}{16}$$

按  $5/2$ ,  $1 + \tan^2 x = 4 \tan^2 x + \tan x + \frac{1}{16}$

$$3 \tan^2 x + \tan x - \frac{15}{16} = 0$$

$$48 \tan^2 x + 16 \tan x - 15 = 0$$

$$(2 \tan x + 3)(12 \tan x - 5) = 0$$

$$4 \tan^2 x + 3 = 0$$

(1)  $4 \tan x = -3$

$$\tan x = -\frac{3}{4} = -0.75.$$

$$x = 143^\circ 8', \text{ 或 } 323^\circ 8'.$$

(2)  $12 \tan x - 5 = 0$

$$12 \tan x = 5$$

$$\tan x = \frac{5}{12} = 0.4167$$

$$x = 22^\circ 37', \text{ 或 } 202^\circ 37'.$$

故  $x = 22^\circ 37', 143^\circ 8', 202^\circ 37', 323^\circ 8'.$

但  $(x = 202^\circ 37')$  及  $(x = 323^\circ 8')$  皆與本題不合.

故  $x = 22^\circ 37',$  或  $143^\circ 8'.$

162.  $\sin 11x \sin 4x + \sin 5x \sin 2x = 0$

$$\sin 11x \sin 4x + \sin 5x \sin 2x = 0.$$

按 [23],  $\sin 11x \sin 4x = -\frac{1}{2}(\cos 15x - \cos 7x),$

又  $\sin 5x \sin 2x = -\frac{1}{2}(\cos 7x - \cos 3x).$

故  $-\frac{1}{2}(\cos 15x - \cos 7x) - \frac{1}{2}(\cos 7x - \cos 3x) = 0$

$$\cos 15x - \cos 7x + \cos 7x - \cos 3x = 0$$

$$\cos 15x - \cos 3x = 0.$$

按 [23],  $-2 \sin \frac{1}{2}(15x+3x) \sin \frac{1}{2}(15x-3x) = 0$   
 $-\sin 9x \sin 6x = 0.$

$$\sin 9x \sin 6x = 0.$$

按  $14/18$ ,  $\sin 9x = 3 \sin 3x - 4 \sin^3 3x.$

按 [16],  $\sin 6x = \pm \sqrt{\frac{1 - \cos 3x}{2}}.$

相代,  $(3 \sin 3x - 4 \sin^3 3x) \left( \pm \sqrt{\frac{1 - \cos 3x}{2}} \right) = 0.$

$$(\sin 3x)(3 - 4 \sin^2 3x) \left( \pm \sqrt{\frac{1 - \cos 3x}{2}} \right) = 0.$$

(1)  $\sin 3x = 0$

$$3x = 0^\circ, 180^\circ, 360^\circ, 540^\circ, 720^\circ, \text{或 } 900^\circ.$$

$$x = 0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ, \text{或 } 300^\circ.$$

(2)  $3 - 4 \sin^2 3x = 0.$

$$4 \sin^2 3x = 0.$$

$$4 \sin 3x = \pm \sqrt{3}$$

$$\sin 3x = \pm \frac{1}{2} \sqrt{3}$$

故  $3x = 60^\circ, 120^\circ, 240^\circ, 300^\circ, 420^\circ, 480^\circ, 600^\circ, 660^\circ, 780^\circ,$   
 $840^\circ \text{或 } 960^\circ.$

$$x = 20^\circ, 40^\circ, 80^\circ, 100^\circ, 140^\circ, 160^\circ, 200^\circ, 220^\circ, 260^\circ, 280^\circ,$$
  
 $320^\circ \text{或 } 340^\circ.$

(3)  $\pm \sqrt{\frac{1 - \cos 3x}{2}} = 0.$

$$\frac{1 - \cos 3x}{2} = 0.$$

$$1 - \cos 3x = 0.$$

$$\cos 3x = 1.$$

$$3x = 0^\circ, 360^\circ, \text{或 } 720^\circ.$$

$$x = 0^\circ, 120^\circ, \text{或 } 240^\circ.$$

故  $x = 0^\circ, 20^\circ, 40^\circ, 60^\circ, 80^\circ, 100^\circ, 120^\circ, 140^\circ, 160^\circ, 180^\circ,$   
 $200^\circ, 220^\circ, 240^\circ, 260^\circ, 280^\circ, 300^\circ, 320^\circ, \text{或 } 340^\circ.$

$$163. \cos x + \cos 3x + \cos 5x + \cos 7x = 0.$$

$$\cos x + \cos 3x + \cos 5x + \cos 7x = 0.$$

$$(\cos 7x + \cos 5x) + (\cos 3x + \cos x) = 0.$$

按 [22],  $2 \cos 6x \cos x + 2 \cos 2x \cos x = 0.$

$$\cos x (\cos 6x + \cos 2x) = 0.$$

按 [22],  $\cos x \times 2 \cos 4x \cos 2x = 0.$

$$\cos x \cos 2x \cos 4x = 0.$$

(1)  $\cos x = 0.$

$$x = 90^\circ, \text{或 } 270^\circ.$$

(2)  $\cos 2x = 0.$

$$2x = 90^\circ, 270^\circ, 450^\circ, \text{或 } 630^\circ.$$

$$x = 45^\circ, 135^\circ, 225^\circ, \text{或 } 315^\circ.$$

(3)  $\cos x = 0.$

$$4x = 90^\circ, 270^\circ, 450^\circ, 630^\circ, 810^\circ, 990^\circ, 1170^\circ, \text{或 } 1350^\circ.$$

$$x = 22\frac{1^\circ}{2}, 67\frac{1^\circ}{2}, 112\frac{1^\circ}{2}, 157\frac{1^\circ}{2}, 202\frac{1^\circ}{2}, 247\frac{1^\circ}{2},$$

$$292\frac{1^\circ}{2}, \text{或 } 337\frac{1^\circ}{2}.$$

故  $x = 22\frac{1^\circ}{2}, 45^\circ, 67\frac{1^\circ}{2}, 90^\circ, 112\frac{1^\circ}{2}, 135^\circ, 157\frac{1^\circ}{2}, 202\frac{1^\circ}{2},$

$$225^\circ, 247\frac{1^\circ}{2}, 270^\circ, 292\frac{1^\circ}{2}, 315^\circ, \text{或} 337\frac{1^\circ}{2}$$

$$164. \quad \sin(x+12^\circ)\cos(x-12^\circ)=\cos 33^\circ \sin 57^\circ$$

$$\sin(x+12^\circ)\cos(x+12^\circ)=\cos 33^\circ \sin 57^\circ,$$

$$\text{按 [20], } \frac{1}{2}(\sin 2x + \sin 24^\circ) = \frac{1}{2}(\sin 90^\circ + \sin 24^\circ).$$

$$\sin 2x + \sin 24^\circ = \sin 90^\circ + \sin 24^\circ.$$

$$\sin 2x = \sin 90^\circ.$$

$$2x = 90^\circ + n 360^\circ.$$

故

$$x = 45^\circ \text{或} 225^\circ.$$

$$165. \quad \sin^{-1}x + \sin^{-1}\frac{1}{2}x = 120^\circ.$$

$$\sin^{-1}x + \sin^{-1}\frac{1}{2}x = 120.$$

$$\sin(\sin^{-1}x + \sin^{-1}\frac{1}{2}x) = \sin 120^\circ = \cos 30^\circ = \frac{1}{2}\sqrt{3}.$$

$$\text{按 [4], } \sin(\sin^{-1}x)\cos(\sin^{-1}\frac{1}{2}x) + \cos(\sin^{-1}x)\sin(\sin^{-1}\frac{1}{2}x) = \frac{1}{2}\sqrt{3}.$$

$$x\sqrt{1-\frac{1}{4}x^2} + \frac{1}{2}x\sqrt{1-x^2} = \frac{1}{2}\sqrt{3}$$

$$x\sqrt{4-x^2} + x\sqrt{2-x^2} = \sqrt{3}$$

$$x\sqrt{4-x^2} = \sqrt{3} - x\sqrt{1-x^2}.$$

$$x^2(4-x^2) = 3 - 2\sqrt{3}x\sqrt{1-x^2} + x^2$$

$$(1-x^2)$$

$$4x^2 - 4x^2 = 3 - 2\sqrt{3}x\sqrt{1-x^2} + x^2 - x^2$$

$$3x^2 - 3 = -2\sqrt{3}x\sqrt{1-x^2}.$$

方之,

$$9x^4 - 18x^2 + 9 = 12x^2 - 12x^4.$$

$$21x^4 - 30x^2 + 9 = 0.$$

$$7x^4 - 10x^2 - 3 = 0.$$

$$(x^2-1)(7x^2-3)=0.$$



$$(1) \quad x^2 - 1 = 0.$$

$$x^2 = 1.$$

$$x = \pm 1.$$

$$(2) \quad 7x^2 - 3 = 0$$

$$7x^2 = 3$$

$$x^2 = \frac{3}{7}$$

$$x = \pm \sqrt{\frac{3}{7}} = \pm \frac{1}{7} \sqrt{21}$$

$$\text{故} \quad x = \pm 1 \text{ 或 } \pm \frac{1}{7} \sqrt{21}.$$

$$166. \quad \tan^{-1}x + \tan^{-1}2x = \tan^{-1}3\sqrt{3}.$$

$$\tan^{-1}x + \tan^{-1}2x = \tan^{-1}3\sqrt{3}$$

$$\tan(\tan^{-1}x + \tan^{-1}2x) = 3\sqrt{3}$$

$$\text{按 (6),} \quad \frac{x+2x}{1-2x^2} = 3\sqrt{3}$$

$$\frac{3x}{1-2x^2} = 3\sqrt{3}$$

$$\frac{x}{1-2x^2} = \sqrt{3}$$

$$x = \sqrt{3} - 2\sqrt{3x^2}.$$

$$2\sqrt{3x^2} + x - \sqrt{3} = 0.$$

$$x^2 + \frac{1}{6}\sqrt{3}x - \frac{1}{2} = 0.$$

$$(x - \frac{1}{2}\sqrt{3})(x + \frac{1}{2}\sqrt{3}) = 0.$$

$$\text{故} \quad x = \frac{1}{2}\sqrt{3} \text{ 或 } 3 - \frac{1}{2}\sqrt{3}.$$

$$167. \quad \sin^{-1}x + 2 \cos^{-1}x = \frac{2}{3}\pi.$$

$$\sin^{-1}x + 2 \cos^{-1}x = \frac{2}{3}\pi$$

$$\text{但} \quad \sin^{-1}x + \cos^{-1}x = \frac{1}{2}\pi$$

$$\text{相減} \quad \cos^{-1}x = \frac{1}{6}\pi$$

故  $x = \frac{1}{2}\sqrt{3}$ .

168.  $\sin^{-1}x + 3 \cos^{-1}x = 210^\circ$

$$\sin^{-1}x + 3 \cos^{-1}x = 210^\circ.$$

但  $\sin^{-1}x + \cos^{-1}x = 90^\circ$ ,

相減  $2 \cos^{-1}x = 120^\circ$ .

$$\cos^{-1}x = 60^\circ.$$

故  $x = \frac{1}{2}$ .

169.  $\tan^{-1}x + 2 \cot^{-1}x = 135^\circ$

$$\tan^{-1}x + 2 \cot^{-1}x = 135^\circ.$$

但  $\tan^{-1}x + \cot^{-1}x = 90^\circ$ .

相減  $\cot^{-1}x = 45^\circ$ .

故  $x = 1$ .

170.  $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}2x$

$$\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}2x.$$

$$\tan[\tan^{-1}(x+1) + \tan^{-1}(x-1)] = 2x.$$

按 [6],  $\frac{x+1+x-1}{1-(x+1)(x-1)} = 2x$ .

$$\frac{2x}{1-x^2} = 2x$$

$$x = 2x - x^3$$

$$x^3 - x = 0$$

$$x(x^2 - 1) = 0$$

171.  $\tan^{-1}\frac{x+2}{x+1} + \tan^{-1}\frac{x-2}{x-1} = \frac{3}{4}\pi$

$$\tan^{-1}\frac{x+2}{x+1} + \tan^{-1}\frac{x-2}{x-1} = \frac{3}{4}\pi$$

$$\tan \left( \tan^{-1} \frac{x+2}{x+1} + \tan^{-1} \frac{x-2}{x-1} \right) = \tan \frac{3}{4} \pi = -1.$$

按 [6],

$$\frac{\frac{x+2}{x+1} + \frac{x-2}{x-1}}{1 - \frac{x+2}{x+1} \times \frac{x-2}{x-1}} = -1.$$

$$\frac{2x^2 - 4}{3} = -1.$$

$$2x^2 - 4 = -3$$

$$2x^2 = 1.$$

$$x^2 = \frac{1}{2}$$

故

$$x = -\frac{1}{2}\sqrt{2}$$

172.

$$\tan^{-1} \frac{2x}{1-x^2} = 60^\circ.$$

$$\tan^{-1} \frac{2x}{1-x^2} = 60^\circ$$

按 [14],

$$2 \tan^{-1} x = 60^\circ$$

$$\tan^{-1} x = 30^\circ \text{ 或 } 210^\circ$$

$$x = \tan 30^\circ, \text{ 或 } \tan 210^\circ,$$

$$x = \frac{1}{2}\sqrt{3}$$

$$173: \cos 2\theta \sec \theta + \sec \theta + 1 = 0.$$

$$\cos 2\theta \sec \theta + \sec \theta + 1 = 0$$

按 [13]  $\frac{\cos^2 \theta - \sin^2 \theta}{\cos \theta} + \sec \theta + 1 = 0$

$$\frac{\cos^2 \theta - \sin^2 \theta}{\cos \theta} + \frac{1}{\cos \theta} + 1 = 0$$

$$\frac{2 \cos^2 \theta - 1}{\cos \theta} + \frac{1}{\cos \theta} + 1 = 0.$$

$$2 \cos^2 \theta - 1 + 1 + \cos \theta = 0.$$

$$2 \cos^2 \theta + \cos \theta = 0.$$

$$\cos \theta (2 \cos \theta + 2) = 0.$$

$$(1) \quad \cos \theta = 0$$

$$\theta = 90^\circ \text{ 或 } 270^\circ.$$

$$(2) \quad 2 \cos \theta + 2 = 0.$$

$$2 \cos \theta = -2.$$

$$\cos \theta = -1.$$

$$\theta = 180^\circ, \text{ 或 } 360^\circ$$

故  $\theta = 90^\circ, 120^\circ, 240^\circ, \text{ 或 } 270^\circ,$

但  $(\theta = 90^\circ)$  及  $(\theta = 270^\circ)$  皆與本題不合

故  $\theta = 120^\circ \text{ 或 } 240^\circ$

$$174. \quad \sin x \cos 2x \tan x \cot 2x \sec x \csc 2x = 1$$

$$\sin x \cos 2x \tan x \cot 2x \sec x \csc 2x = 1$$

按 [13],  $\cos 2x = \cos^2 x - \sin^2 x$

按 [15],  $\cot 2x = \frac{\cot^2 x - 1}{2 \cot x}$

$$\begin{aligned} & \frac{\cos^2 x}{\sin^2 x} - 1 = \frac{\cos^2 x - \sin^2 x}{\sin^2 x} \\ & = \frac{\cos^2 x - \sin^2 x}{2 \sin x \cos x} \end{aligned}$$

按 [3], 及 [13],  $\csc 2x = \frac{1}{\sin 2x} = \frac{1}{2 \sin x \cos x}$

以各同數代入原式

$$\sin x (\cos^2 x - \sin^2 x) \left( \frac{\sin x}{\cos x} \right) \left( \frac{\cos^2 x - \sin^2 x}{2 \sin x \cos x} \right) \left( \frac{1}{\cos x} \right)$$

$$\left(\frac{1}{2 \sin x \cos x}\right) = 1.$$

$$\frac{(\cos^2 x - \sin^2 x)^2}{4 \cos^4 x} = 1$$

$$\frac{(2 \cos^2 x - 1)^2}{4 \cos^4 x} = 1$$

$$\frac{2 \cos^2 x - 1}{2 \cos^2 x} = \pm 1$$

故  $2 \cos^2 x - 1 = \pm 2 \cos^2 x$

$$4 \cos^2 x = 1.$$

$$\cos^2 x = \frac{1}{4}$$

$$\cos x = \pm \frac{1}{2}$$

$$x = 60^\circ, 120^\circ, 240^\circ, \text{ 或 } 300^\circ.$$

175.  $\sin \frac{1}{2}x (\cos 2x - 2)(1 - \tan^2 x) = 0.$

$$\sin \frac{1}{2}x (\cos^2 x - 2)(1 - \tan^2 x) = 0.$$

(1)  $\sin \frac{1}{2}x = 0.$

$$\frac{1}{2}x = 0^\circ \text{ 或 } 180^\circ.$$

(2)  $x = 0^\circ.$

$$\cos 2x - 2 = 0.$$

$$\cos 2x = 2.$$

故  $x$  不能成角.

(3)  $1 - \tan^2 x = 0$

$$\tan^2 x = 1$$

$$\tan x = \pm 1$$

$$x = 45^\circ, 135^\circ, 225^\circ, \text{ 或 } 315^\circ.$$

$$x = 0^\circ, 45^\circ, 135^\circ, 225^\circ, 315^\circ.$$

故

176

$$\sin 3x = \cos 2x = 1.$$

$$\sin 3x = \cos 2x - 1.$$

按  $4/18$ , 與 [13].  $3\sin x - 4\sin^3 x = 1 - 2\sin^2 x - 1.$

$$4\sin^3 x - 2\sin^2 x - 3\sin x = 0.$$

$$\sin x(4\sin^2 x - 2\sin x - 3) = 0.$$

$$(1) \quad \sin x = 0.$$

$$x = 0^\circ, \text{ 或 } 180^\circ.$$

$$(2) \quad 4\sin^2 x - 2\sin x - 3 = 0.$$

$$16\sin^2 x - 8\sin x + 1 = 14.$$

$$4\sin x - 1 = \pm\sqrt{13}.$$

$$4\sin x = 1 \pm \sqrt{13}.$$

$$\sin x = \frac{1}{4}(1 \pm \sqrt{13}).$$

$$\sin x = 1.1514, \text{ 或 } 0.6514.$$

$$x = 220^\circ 39', \text{ 或 } 319^\circ 21'.$$

故  $x = 0^\circ, 180^\circ, 220^\circ 39', \text{ 或 } 319^\circ 21'.$

$$177. \quad \tan x + \tan 2x = 0.$$

$$\tan x + \tan 2x = 0.$$

$$\text{按 [14], } \tan x + \frac{2\tan x}{1 - \tan^2 x} = 0$$

$$\tan x - \tan^3 x + 2\tan x = 0$$

$$\tan x(\tan^2 x - 3) = 0$$

$$(1) \quad \tan x = 0$$

$$x = 0^\circ \text{ 或 } 180^\circ.$$

$$(2) \quad \tan^2 x - 3 = 0$$

$$\tan^2 x = 3$$

$$\tan x = \pm\sqrt{3}$$

$$x = 60^\circ, 120^\circ, 240^\circ, \text{ 或 } 300^\circ.$$

故  $x=0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ, \text{或 } 300^\circ.$   
 178.  $\sin 2\theta = \cos 3\theta.$

$$\sin 2\theta = \cos 3\theta.$$

按 [12], 及 14/19,  $2 \sin \theta \cos \theta = 4 \cos^3 \theta - 3 \cos \theta.$

$$4 \cos^3 \theta - 3 \cos \theta - 2 \sin \theta \cos \theta = 0.$$

$$\cos \theta (4 \cos^2 \theta - 3 - 2 \sin \theta) = 0.$$

$$(1) \quad \cos \theta = 0.$$

$$\theta = 90^\circ \text{ 或 } 270^\circ.$$

$$(2) \quad 4 \cos^2 \theta - 3 - 2 \sin \theta = 0.$$

$$4 - 4 \sin^2 \theta - 3 - 2 \sin \theta = 0.$$

$$4 \sin^2 \theta + 2 \sin \theta = 1.$$

$$16 \sin^2 \theta + 8 \sin \theta + 1 = 5.$$

$$4 \sin \theta + 1 = \pm \sqrt{5}$$

$$4 \sin \theta = -1 \pm \sqrt{5}$$

$$\sin \theta = \frac{1}{4}(-1 \pm \sqrt{5}).$$

$$= \frac{1}{4}(-1 \pm 2.23606.)$$

$$= 0.3090 \text{ 或 } -0.8090.$$

$$\theta = 18^\circ, \text{ 或 } 162^\circ. \text{ 又 } 234^\circ \text{ 或 } 306^\circ.$$

故  $\theta = 18^\circ, 90^\circ, 162^\circ, 234^\circ, 270^\circ, \text{ 或 } 306^\circ.$

$$179. (3 - 4 \cos^2 x) \sin 2x = 0.$$

$$(3 - 4 \cos^2 x) \sin 2x = 0.$$

$$(1) \quad 3 - 4 \cos^2 x = 0.$$

$$4 \cos^2 x = 3.$$

$$2 \cos x = \pm \sqrt{3}$$

$$\cos x = \pm \frac{1}{2} \sqrt{3}$$

$$x=30^\circ, 150^\circ, 210^\circ, \text{ 或 } 330^\circ.$$

$$(2) \quad \sin 2x=0.$$

$$2x=0^\circ, 180^\circ, 360^\circ, \text{ 或 } 540^\circ.$$

$$x=0^\circ, 90^\circ, 180^\circ, \text{ 或 } 270^\circ.$$

$$\text{故 } x=0^\circ, 30^\circ, 90^\circ, 150^\circ, 180^\circ, 210^\circ, 270^\circ, \text{ 或 } 330^\circ.$$

$$180. \sin x + \sin 2x + \sin 3x = 0.$$

$$\sin x + \sin 2x + \sin 3x = 0.$$

$$(\sin 3x + \sin x) + \sin 2x = 0.$$

$$\text{按 [20]}, 2\sin 2x \cos x + \sin 2x = 0.$$

$$\sin 2x (2 \cos x + 1) = 0.$$

$$(1) \quad \sin 2x = 0.$$

$$2x=0^\circ, 180^\circ, 360^\circ, \text{ 或 } 540^\circ.$$

$$x=0^\circ, 90^\circ, 180^\circ, \text{ 或 } 270^\circ.$$

$$\text{故 } x=0^\circ, 60^\circ, 90^\circ, 120^\circ, 180^\circ, 240^\circ, 270^\circ, \text{ 或 } 300^\circ.$$

$$\text{但 } (x=60^\circ, ) \text{ 與 } (x=300^\circ) \text{ 均與本題不合.}$$

$$\text{故 } x=0^\circ, 90^\circ, 120^\circ, 180^\circ, 240^\circ, \text{ 或 } 270^\circ.$$

$$181. \sin \theta + 2 \sin 2\theta + 3 \sin 3\theta = 0.$$

$$\sin \theta + 2 \sin 2\theta + 3 \sin 3\theta = 0.$$

$$\text{按 [13] 及 14/18,} \quad \sin \theta + 4 \sin \theta \cos \theta + 9 \sin \theta - 12 \sin^3 \theta = 0.$$

$$10 \sin \theta + 4 \sin \theta \cos \theta - 12 \sin^3 \theta = 0.$$

$$\sin \theta (5 + 2 \cos \theta - 6 \sin^2 \theta) = 0.$$

$$(1) \quad \sin \theta = 0.$$

$$\theta = 0^\circ \text{ 或 } 180^\circ.$$

$$(2) \quad 5 + 2 \cos \theta - 6 \sin^2 \theta = 0.$$



$$5 + 2 \cos \theta - 6 + 6 \cos^2 \theta = 0.$$

$$6 \cos^2 \theta + 2 \cos \theta = 1.$$

$$36 \cos^2 \theta + 12 \cos \theta + 1 = 7.$$

$$6 \cos \theta + 1 = \pm \sqrt{7}$$

$$6 \cos \theta = -1 \pm \sqrt{7}$$

$$\cos \theta = \frac{1}{6}(-1 \pm \sqrt{7})$$

$$= \frac{1}{6}(-1 \pm 2.64575)$$

$$= 0.2743 \text{ 或 } -0.6076.$$

$$x = 74^\circ 5' \text{ 或 } 285^\circ 55'.$$

$$= 127^\circ 25', \text{ 或 } 232^\circ 35'.$$

又

故  $\theta = 0^\circ, 74^\circ 5', 127^\circ 25', 180^\circ, 232^\circ 35', \text{ 或 } 285^\circ 55'.$

182.  $\sin^2 x \cos^2 x - \cos^2 x - \sin^2 x + 1 = 1.$

$$\sin^2 x \cos^2 x - \cos^2 x - \sin^2 x + 1 = 1.$$

$$\sin^2 x \cos^2 x - \cos^2 x + \sin^2 x + 1 = 0.$$

$$\sin^2 x \cos^2 x - 1 + 1 = 0.$$

$$\sin^2 x \cos^2 x = 0.$$

$$\sin x \cos x = 0.$$

$$2 \sin x \cos x = 0.$$

$$\sin 2x = 0.$$

$$2x = 0^\circ, 180^\circ, 360^\circ, \text{ 或 } 540^\circ.$$

$$x = 0^\circ, 90^\circ, 180^\circ, \text{ 或 } 270^\circ.$$

183.

$$\sin x + \sin 3x = \cos x - \cos 3x.$$

$$\sin x + \sin 3x = \cos x - \cos 3x.$$

按 14/18, 19,

$$\sin x + 3 \sin x - 4 \sin^3 x = \cos x - 4 \cos^3 x + 3$$

cos x.

$$4 \sin^3 x - 4 \cos^3 x - 4 \sin x + \cos x = 0.$$

$$(\sin^3 x - \cos^3 x) - (\sin x - \cos x) = 0.$$

$$(\sin x - \cos x) (\sin^2 x + \sin x \cos x + \cos^2 x) - (\sin x - \cos x) = 0.$$

$$(\sin x - \cos x) (1 + \sin x \cos x) - (\sin x - \cos x) = 0.$$

$$(\sin x - \cos x) (1 + \sin x \cos x - 1) = 0.$$

$$(\sin x - \cos x) \sin x \cos x = 0.$$

$$\sin x - \cos x = 0.$$

(1)  $\sin x = \cos x;$

$$x = 45^\circ \text{ 或 } 225^\circ.$$

(2)  $\sin x = 0.$

$$x = 0^\circ \text{ 或 } 180^\circ.$$

(3)  $\cos x = 0.$

$$x = 90^\circ \text{ 或 } 270^\circ.$$

故  $x = 0^\circ, 45^\circ, 90^\circ, 180^\circ, 225^\circ, \text{ 或 } 270^\circ.$

184.  $(1 - \sqrt{1 - \tan^2 x}) \cos 2x \operatorname{vers} 3x = 0.$

$$(1 - \sqrt{1 - \tan^2 x}) \cos 2x \operatorname{vers} 3x = 0.$$

(1)  $1 - \sqrt{1 - \tan^2 x} = 0.$

$$1 = \sqrt{1 - \tan^2 x}.$$

$$1 = 1 - \tan^2 x.$$

$$\tan^2 x = 0.$$

$$\tan x = 0.$$

$$x = 0^\circ \text{ 或 } 180^\circ.$$

(2)  $\cos 2x = 0.$

$$2x = 90^\circ, 270^\circ, 450^\circ, \text{ 或 } 630^\circ.$$

$$x=45^\circ, 135^\circ, 225^\circ, \text{或} 315^\circ.$$

(3)

$$\cos 3x=0.$$

$$1-\cos 3x=0.$$

$$\cos 3x=1.$$

$$3x=0^\circ, 360^\circ, \text{或} 720^\circ.$$

故  $x=0^\circ, 45^\circ, 120^\circ, 135^\circ, 180^\circ, 225^\circ, 240^\circ, \text{或} 315^\circ.$

185.

$$\tan(\theta+45^\circ)=8 \tan \theta^\circ$$

$$\tan(\theta+45^\circ)=8 \tan \theta$$

按 [6],

$$\frac{\tan \theta + \tan 45^\circ}{1 - \tan \theta \tan 45^\circ} = 8 \tan \theta$$

$$\frac{\tan \theta + 1}{1 - \tan \theta} = 8 \tan \theta.$$

$$\tan \theta + 1 = 8 \tan \theta - 8 \tan^2 \theta$$

$$8 \tan^2 \theta - 7 \tan \theta = -1$$

$$256 \tan^2 \theta - ( \quad ) + 49 = 17$$

$$16 \tan \theta - 7 = \pm \sqrt{7}.$$

$$16 \tan \theta = 7 \pm \sqrt{17}.$$

$$\tan \theta = \frac{1}{16} (7 \pm \sqrt{17})$$

$$= \frac{1}{16} (7 \pm 4.1231)$$

$$= 0.6952 \text{或} 0.1798.$$

$$\theta = 34^\circ 48' \text{或} 214^\circ 48'.$$

$$= 10^\circ 12', \text{或} 190^\circ 12'.$$

又

故

$$\theta = 10^\circ 12' 34^\circ 48', 190^\circ, 12', \text{或} 214^\circ 48',$$

186

$$\sin(x-30^\circ) = \frac{1}{2} \sqrt{3} \sin x^\circ$$

$$\sin(x-30^\circ) = \frac{1}{2}\sqrt{3}\sin x.$$

按[8],  $\sin x \cos 30^\circ - \cos x \sin 30^\circ = \frac{1}{2}\sqrt{3}\sin x.$

$$\frac{1}{2}\sqrt{3}\sin x - \frac{1}{2}\cos x = \frac{1}{2}\sqrt{3}\sin x.$$

$$-\frac{1}{2}\cos x = 0.$$

$$\cos x = 0.$$

$$x = 90^\circ \text{ 或 } 270^\circ.$$

187.  $\tan(\theta+45^\circ) \tan \theta = 2.$

$$\tan(\theta+45^\circ) \tan \theta = 2.$$

按 [6]  $\frac{\tan \theta + \tan 45^\circ}{1 - \tan \theta \tan 45^\circ} \times \tan \theta = 2.$

$$\frac{(\tan \theta + 1) \tan \theta}{1 - \tan \theta} = 2.$$

$$\frac{\tan^2 \theta + \tan \theta}{1 - \tan \theta} = 2.$$

$$\tan^2 \theta + \tan \theta = 2 - 2 \tan \theta$$

$$\tan^2 \theta + 3 \tan \theta = 2.$$

$$4 \tan^2 \theta + ( ) + 9 = 17.$$

$$2 \tan \theta + 3 = \pm \sqrt{17}.$$

$$2 \tan \theta = -3 \pm \sqrt{17}$$

$$\tan \theta = \frac{1}{2}(-3 \pm \sqrt{17})$$

$$= \frac{1}{2}(-3 \pm 4.1231)$$

$$= 0.5616 \text{ 或 } -3.5616.$$

$$\theta = 29^\circ 16' \text{ 或 } 206^\circ 19'.$$

$$= 105^\circ 41' \text{ 或 } 285^\circ 42'.$$

又

$$\sin^{-1} \frac{1}{2} x = 30^\circ.$$

$$\sin^{-1} \frac{1}{2} x = 30^\circ$$

$$\sin(\sin^{-1}\frac{1}{2}x) = 30^\circ$$

$$\frac{1}{2}x = \frac{1}{2}$$

故

$$x = 1.$$

189. 解下之兩方程式。以求x及y.

$$x \sin x + y \sin B = a,$$

$$x \sin x + y \cos B = b$$

以  $\cos x$  乘(1),  $x \sin x \cos x + y \sin B \cos x$   
 $= a \cos x$

以  $\sin x$  乘(2),  $x \sin x \cos x + y \cos B \sin x$   
 $= b \sin x$  4

自 (3) 減 (4),  $y (\sin B \cos x - \cos B \sin x)$   
 $= a \cos x - b \sin x.$

按 [8],  $y \sin(B-x) = a \cos x - b \sin x.$

故  $y = \frac{a \cos x - b \sin x}{\sin(B-x)}$

以  $\cos B$  乘(1),  $x \sin x \cos B + y \sin B \cos B$   
 $= a \cos B.$  5

以  $\sin B$  乘(2),  $x \cos x \sin B + y \sin B \cos B$   
 $= b \sin B.$  (6)

自 (6) 減 (5),  $x(\cos x \sin B - \sin x \cos B)$   
 $= b \sin B - a \cos B.$

按 [8],  $x \sin(B-x) = b \sin B - a \cos B$

$$x \sin(B-x) = b \sin B - a \cos B.$$

故  $x = \frac{b \sin B - a \cos B}{\sin(B-x)}$

190. 試解下式。以求x及y. (1)

$$\sin x + \sin y = a, \quad (2)$$

$$\cos x + \cos y = b.$$

按 [20], 化 (1) 式,  $2 \sin \frac{1}{2}(x+y) \cos \frac{1}{2}(x-y) = a$  (3)

按 [22], 化 (2) 式,  $2 \cos \frac{1}{2}(x+y) \cos \frac{1}{2}(x-y) = b$  (4)

以 (4) 除 (3),  $\tan \frac{1}{2}(x+y) = \frac{a}{b}$  (5)

按 [2],  $\frac{\sin \frac{1}{2}(x+y)}{\cos \frac{1}{2}(x+y)} = \frac{a}{b}.$

$$\frac{\sin \frac{1}{2}(x+y)}{\sqrt{1 - \sin^2 \frac{1}{2}(x+y)}} = \frac{a}{b}.$$

方之  $\frac{x \sin^2 \frac{1}{2}(x+y)}{1 + \sin^2 \frac{1}{2}(x+y)} = \frac{a^2}{b^2}$

$$b^2 \sin^2 \frac{1}{2}(x+y) = a^2 - a^2 \sin^2 \frac{1}{2}(x+y).$$

$$(a + b^2) \sin^2 \frac{1}{2}(x+y) = a^2$$

$$\sin^2 \frac{1}{2}(x+y) = \frac{a^2}{a^2 + b^2}$$

$$\sin \frac{1}{2}(x+y) = \frac{a}{\sqrt{a^2 + b^2}}. \quad (6)$$

以  $\sin \frac{1}{2}(x+y)$  之同數代入 (3) 式.

$$\frac{2a}{\sqrt{a^2 + b^2}} \cos \frac{1}{2}(x-y) = a$$

$$\cos \frac{1}{2}(x-y) = \frac{1}{2} \sqrt{a^2 + b^2} \quad (8)$$

自 (5),  $x+y = 2 \tan^{-1} \frac{a}{b}$  或 (9)

自 (8),  $x-y = 2 \cos^{-1} \frac{1}{2} \sqrt{a^2 + b^2}$ . (10)

以 (9) (10) 相如, 又以 2 除之,

$$x = \tan^{-1} \frac{a}{b} + \cos^{-1} \frac{1}{2} \sqrt{a^2 + b^2}.$$

以 (9) (10) 相減, 又以 2 除之,

$$y = \tan^{-1} \frac{a}{b} - \cos^{-1} \frac{1}{2} \sqrt{a^2 + b^2}.$$

191. 試解下式以求  $r$  及  $\theta$ 。

$$r \sin \theta = a, \quad (1)$$

$$r \cos \theta = b. \quad (2)$$

以 (2) 除 (1),  $\tan \theta = \frac{a}{b}.$  (3)

$$\theta = \tan^{-1} \frac{a}{b}. \quad (4)$$

以 (1) 乘方,  $r^2 \sin^2 \theta = a^2.$  (5)

以 (2) 乘方,  $r^2 \cos^2 \theta = b^2.$  (6)

以 (5)(6) 相加,  $r^2 (\sin^2 \theta + \cos^2 \theta) = a^2 + b^2,$

故  $r^2 = a^2 + b^2,$   
 $r = \sqrt{a^2 + b^2}.$

192. 試解下式以求  $r$  及  $\theta$ 。

$$r \sin(\theta_0 + \alpha) = a, \quad (1)$$

$$r \sin(\theta_0 + \beta) = b. \quad (2)$$

按 [5], 化 (1) 式,

$$r \sin \theta \cos \alpha + r \cos \theta \sin \alpha = a \quad (3)$$

按 [5], 化 (2) 式,

$$r \cos \theta \cos \beta - r \sin \theta \sin \beta = b$$

以  $\cos \beta$  乘 (3),

$$r \sin \theta \cos \alpha \cos \beta + r \cos \theta \sin \alpha \cos \beta = a \cos \beta. \quad (5)$$

以  $\sin \beta$  乘 (4),

$$-r \sin \theta \sin \alpha \sin \beta + r \cos \theta \sin \alpha \cos \beta = b \sin \beta \quad (6)$$

自 (5) 減 (6),

$$r \sin \theta \cos x \cos \beta + \sin x \sin \beta = b \sin x$$

$$r \sin \theta \cos(x - \beta) = a \cos \beta - b \sin x.$$

按 [9],

$$\text{故} \quad r \sin \theta = \frac{a \cos \beta - b \sin x}{\cos(x - \beta)} \quad (7)$$

以  $\sin \beta$  乘 (3),

$$r \sin \theta \cos x \sin \beta + r \cos \theta \sin x \sin \beta = x \sin \beta \quad (8)$$

以  $\cos x$  乘 (4)

$$-r \sin \theta \cos x \sin \beta + r \cos \theta \cos x \cos \beta = b \cos x \quad (9)$$

以 (8)(9) 相加

$$r \cos \theta (\sin x \sin \beta + \cos x \cos \beta) = a \sin \beta + b \cos x.$$

按 (9),  $r \cos \theta \cos(x - \beta) = a \sin \beta + b \cos x.$

$$\text{故} \quad r \cos \theta = \frac{a \sin \beta + b \cos x}{\cos(x - \beta)}. \quad (10)$$

$$\text{以 (10) 除 (7),} \quad \tan \theta = \frac{a \cos \beta - b \sin x}{a \sin \beta + b \cos x} \quad (11)$$

$$\text{故} \quad \theta = \tan^{-1} \frac{a \cos \beta - b \sin x}{a \sin \beta + b \cos x} \quad (12)$$

$$\text{由 (1) 與 (2)} \quad r = \frac{a}{\sin(\theta + x)} = \frac{b}{\cos(\theta + \beta)} \quad (13)$$

以 (12) 式內  $\theta$  之同數代入 (13),

$$\begin{aligned} r &= \frac{a}{\sin\left(\tan^{-1} \frac{a \cos \beta - b \sin x}{a \sin \beta + b \cos x} + x\right)} \\ &= \frac{b}{\cos\left(\tan^{-1} \frac{a \cos \beta - b \sin x}{a \sin \beta + b \cos x} + \beta\right)}. \end{aligned}$$

193. 試解下式以求  $r, \theta$  及  $\phi$ 。



$$r \cos \phi \sin \theta = a, \quad (1)$$

$$r \cos \phi \cos \theta = b, \quad (2)$$

$$r \sin \phi = c. \quad (2)$$

以 (2) 除 (1),  $\tan \theta = \frac{a}{b} \quad (4)$

$$\theta = \tan^{-1} \frac{a}{b}.$$

以 (1) 乘方,  $r^2 \cos^2 \phi \sin^2 \theta = a^2. \quad (6)$

以 (2) 乘方  $r^2 \cos^2 \phi \cos^2 \theta = b^2. \quad (7)$

(6) 加 (7),  $r^2 \cos^2 \phi (\sin^2 \theta + \cos^2 \theta) = a^2 + b^2.$

故  $r^2 \cos^2 \phi = a^2 + b^2. \quad (8)$

$$r \cos \phi = \sqrt{a^2 + b^2} \quad (9)$$

以 (9) 除 (3),  $\tan \phi = \frac{c}{\sqrt{a^2 + b^2}}$

故  $\theta \phi = \tan^{-1} \frac{c}{\sqrt{a^2 + b^2}} \quad (10)$

以 (3) 乘方,  $r^2 \sin^2 \phi = c^2 \quad (11)$

(11) 如 (8),  $r^2 (\sin^2 \phi + \cos^2 \phi) = a^2 + b^2 + c^2.$

故  $r^2 = a^2 + b^2 + c^2.$

而  $r = \sqrt{a^2 + b^2 + c^2}$

194. 試解下列方程式之各羣以求  $r, \theta, \phi, x$  及  $y$ 。

$$x \sin 21^\circ + y \cos 44^\circ = 179.70, \quad (1)$$

$$x \cos 21^\circ + y \sin 44^\circ = 232.30. \quad (2)$$

$$0.3584 x + 0.7193 y = 179.70 \quad (3)$$

$$0.9336 x + 0.6947 y = 232.30 \quad (4)$$

$$3584 x + 7193 y = 1797000 \quad (5)$$

$$9336 x + 6947 y = 2323000 \quad (6)$$

$$\text{以 } 1167 \text{ 乘 (5), } 4182528x + 8394231y = 2097099000 \quad (7)$$

$$\text{以 } 448 \text{ 乘 (6), } 4182528x + 312256y = 1040704000 \quad (8)$$

$$\text{自 (7) 減 (8), } 5281975y = 1056395000$$

$$\text{故 } y = 200$$

以  $y$  之同數代入 (5) 式.

$$3584x + 1438600 = 1797000$$

$$8584x = 3584000$$

$$\text{故 } x = 100.$$

$$195. \quad \sin x - \sin y = 0.7038. \quad (1)$$

$$\cos x - \cos y = -0.7245. \quad (2)$$

接 [21], 化 (1) 式,

$$2 \cos \frac{1}{2}(x+y) \sin \frac{1}{2}(x-y) = 0.7038. \quad (3)$$

接 [23], 化 (2) 式,

$$-2 \sin \frac{1}{2}(x+y) \sin \frac{1}{2}(x-y) = -0.7245. \quad (4)$$

以 (3) 除 (4)

$$\tan \frac{1}{2}(x+y) = \frac{0.7245}{0.7038}. \quad (5)$$

$$\frac{\sin \frac{1}{2}(x+y)}{\cos \frac{1}{2}(x+y)} = \frac{0.7245^2}{0.7038^2}.$$

$$\frac{\sin \frac{1}{2}(x+y)}{\sqrt{1 - \sin^2 \frac{1}{2}(x+y)}} = \frac{0.7245^2}{0.7038^2}.$$

$$0.7038^2 \sin^2 \frac{1}{2}(x+y) = 0.7245^2 - 0.7245^2 \sin^2 \frac{1}{2}(x+y).$$

$$\sqrt{0.7038^2 + 0.7245^2} \sin \frac{1}{2}(x+y) = \pm 0.7245^2$$

$$\sin \frac{1}{2}(x+y) = \frac{\pm 0.7245}{\sqrt{0.7038^2 + 47245^2}}$$

$$\sin \frac{1}{2}(x+y) = \frac{\pm 0.7245}{1.0100667}$$

$$\sin \frac{1}{2}(x+y) = \pm 0.7172$$

以  $\sin \frac{1}{2}(x+y)$  之同數代入 (4) 式,

$$2 (\pm 0.7172) \sin \frac{1}{2}(x-y) = 0.7245.$$

$$\sin \frac{1}{2}(x-y) = \frac{0.7245}{\pm 1.4344} \quad (7)$$

$$\sin \frac{1}{2}(x-y) = \pm 0.5051. \quad (7)$$

由 (6),  $\frac{1}{2}(x+y) = 45^\circ 50', 134^\circ 10', 225^\circ 50',$   
或  $314^\circ 10'. (8)$

由 (7),  $\frac{1}{2}(x-y) = 30^\circ 20', 149^\circ 40', 210^\circ 20',$   
或  $329^\circ 40'. (9)$

(8) 加 (9),  $x = 76^\circ 10', 283^\circ 50', 436^\circ 10,$   
或  $643^\circ 50'.$

自 (8) 減 (9),  $y = 15^\circ 30', 344^\circ 30', 15^\circ 30',$   
或  $344^\circ 30'.$

故  $x = 76^\circ 10'$  或  $283^\circ 50'.$   
 $y = 15^\circ 30'$  或  $344^\circ 30'.$

但  $(x = 283^\circ 50')$  及  $(y = 344^\circ 30')$  均與本題不合.

故  $x = 76^\circ 10'.$   
 $y = 15^\circ 30'.$

196.  $r \sin \theta = 92.344 \quad (1)$

$$r \cos \theta = 205.309. \quad (2)$$

以 (2) 除 (1)  $\tan \theta = \frac{92.344}{205.309} = 0.4498$

故  $\theta = 24^\circ 13'$  或  $204^\circ 13' \quad (6)$

由 (1) 
$$r = \frac{92.344}{\sin \theta} = \frac{92.344}{\pm 0.4102} = \pm 225.12.$$

故 
$$\theta = 24^{\circ} 13', r = 225.12.$$

按 
$$\theta = 204^{\circ} 13', r = 225.12. \quad (1)$$

197. 
$$r \sin(\theta - 19^{\circ} 18') = 59.4034. \quad (1)$$

$$r \cos(\theta - 30^{\circ} 54') = 147.9347, \quad (2)$$

按 [8], 化 (1) 式.

$$r \sin \theta \cos 19^{\circ} 18' - r \cos \theta \sin 19^{\circ} 18' = 59.4034. (3)$$

按 [9], 化 (2) 式.

$$r \cos \theta \cos 30^{\circ} 54' + r \sin \theta \sin 30^{\circ} 54' = 147.9347. (4)$$

以  $\sin 30^{\circ} 54'$  乘 (3), 得

$$\begin{aligned} r \sin \theta \sin 30^{\circ} 54' \cos 19^{\circ} 18' - r \cos \theta \cos 30^{\circ} 54' \sin 19^{\circ} 18' \\ = 59.4034 \sin 30^{\circ} 54'. \end{aligned} \quad (5)$$

以  $\cos 19^{\circ} 18'$  乘 (4), 得

$$\begin{aligned} r \sin \theta \sin 30^{\circ} 54' \cos 19^{\circ} 18' - \cos \theta \cos 30^{\circ} 54' \cos 19^{\circ} 18' \\ = 147.9347 \cos 19^{\circ} 18'. \end{aligned} \quad (6)$$

自 (6) 減 (5), 得

$$\begin{aligned} r \cos \theta (\cos 19^{\circ} 18' \cos 30^{\circ} 54' + \sin 19^{\circ} 18' \sin 30^{\circ} 54') \\ = 147.9347 \cos 19^{\circ} 18' \\ - 59.4034 \sin 30^{\circ} 54' \end{aligned}$$

按 [9],  $r \cos \theta \cos(30^{\circ} 54' - 19^{\circ} 18')$

$$\begin{aligned} = 147.9347 \cos 19^{\circ} 18' \\ - 59.4034 \sin 30^{\circ} 54'. \end{aligned}$$

故  $r \cos \theta = \frac{147.9347 \cos 19^{\circ} 18' - 59.4034 \sin 30^{\circ} 54'}{\cos 10^{\circ} 36'}. (7)$

以  $\cos 30^{\circ} 54'$  乘 (3), 得

$$r \sin \theta \cos 30^{\circ} 54' \cos 19^{\circ} 18' - r \cos \theta \cos 30^{\circ} 54' \sin 19^{\circ} 18' = 59.4034 \cos 30^{\circ} 54'. \quad (8)$$

以  $\sin 19^{\circ} 18'$  乘 (4), 得

$$r \sin \theta \sin 30^{\circ} 54' \sin 19^{\circ} 18' + r \cos \theta \cos 30^{\circ} 54' \sin 19^{\circ} 18' = 147.9347 \sin 19^{\circ} 18'.$$

按 (9),  $r \sin \theta \cos(30^{\circ} 54' - 19^{\circ} 18')$

$$= 147.9347 \sin 19^{\circ} 18' + 59.4034 \cos 30^{\circ} 54'.$$

$$\text{故 } r \sin \theta = \frac{147.9347 \sin 19^{\circ} 18' + 59.4034 \cos 30^{\circ} 54'}{\cos 10^{\circ} 36'} \quad (10)$$

以 (7) 除 (10)

$$\begin{aligned} \tan \theta &= \frac{147.9347 \sin 19^{\circ} 18' - 59.4034 \cos 30^{\circ} 54'}{147.9347 \cos 19^{\circ} 18' - 59.4034 \sin 30^{\circ} 54'} \\ &= \frac{147.9347 \times 0.3305 + 59.4034 \times 0.8581}{147.9347 \times 0.9438 - 59.4034 \times 0.5135} \\ &= \frac{48.89241835 + 50.97405754}{139.62076986 - 30.50364590} \\ &= \frac{99.86647589}{109.11712396} \\ &= 0.9152. \end{aligned}$$

$$\text{故 } \theta = 42^{\circ} 28' \text{ 或 } 222^{\circ} 28' \quad (11)$$

以 (11) 式內  $\theta$  之同數代入 (1) 式; 得

$$r \sin(42^{\circ} 28' - 19^{\circ} 18') = 59.4034. \text{ 或 } r \sin(222^{\circ} 28' + 9^{\circ} 18') = 59.4034$$

$$r \sin 23^{\circ} 10' = 59.4034. \text{ 或 } r \sin 203^{\circ} 10' = 59.4034.$$

$$r = \frac{59.4034}{\sin 23^{\circ} 10'}, \text{ 或 } r = \frac{59.4034}{\sin 203^{\circ} 10'}$$

$$r = \frac{59.4034}{0.3934} \text{ 或 } r = \frac{59.4034}{-0.3934}$$

$$r = 151, \text{ 或 } r = -151.$$

故 差  $\theta = 42^\circ 28'$ , 則  $r = 151$ ,

差  $\theta = 222^\circ 28'$  則  $r = -151$ .

$$198, \quad r \cos \phi \cos \theta = -146.7654.$$

$$r \sin \phi \cos \theta = 81. \quad (2)$$

$$r \sin \theta = -54. \quad (3)$$

以 (1) 除 (2), 得  $\tan \phi = \frac{81}{-46.7654} = -1.7320$ .

故  $\phi = 120^\circ$  或  $300^\circ$

而  $\cos \phi = \pm \frac{1}{2}$ ,

以 (4) 式內  $\phi$  之同數代入(1)式, 得

$$r \left(\mp \frac{1}{2}\right) \cos \theta = -46.7654$$

$$r \cos \theta = \pm 93.5308 \quad (5)$$

以 (5) 除 (3), 得  $\tan \theta = \frac{-54}{\pm 93.5308} = \pm 0.5773$ .

故  $\theta = 150^\circ$ , 或  $330^\circ$ , 又  $30^\circ$  或  $210^\circ$ .

$$\sin \theta = \frac{1}{2} \text{ 或 } -\frac{1}{2} \text{ 又 } \frac{1}{2} \text{ 或 } -\frac{1}{2} \quad (6)$$

以(6) 式內  $\sin \theta$  之同數代入(3)式, 得

$$r \left(\pm \frac{1}{2}\right) = -54$$

故  $r = \mp 108$ .

即得  $r = 108, \phi = 120^\circ, \theta = 330^\circ$ . 或  $r = 108, \phi = 300^\circ, \theta = 210^\circ$ .

或  $r = 108, \phi = 120^\circ, \theta = 150^\circ$ . 或  $r = 108, \phi = 300^\circ, \theta = 30^\circ$ .

199. 於下式消去  $\theta$

$$x = r(\theta - \sin \theta) \quad (1)$$

$$y = r(1 - \cos \theta). \quad (2)$$

$$1 - \cos \theta = \text{vers } \theta.$$

以(3)式內  $1 - \cos \theta$  之同數代入(2)式得  $(3)$

$$y = r \operatorname{vers} \theta$$

$$\operatorname{vers} \theta = \frac{y}{r}$$

$$\text{故} \quad \theta = \operatorname{vers}^{-1} \frac{y}{r} \quad (4)$$

$$\text{目 (2)} \quad 1 - \cos \theta = \frac{y}{r}$$

$$1 - \sqrt{1 - \sin^2 \theta} = \frac{y}{r}$$

$$1 - \frac{y}{r} = \sqrt{1 - \sin^2 \theta}$$

$$1 - \frac{2y}{r} + \frac{y^2}{r^2} = \sin^2 \theta$$

$$\sin^2 \theta = \frac{2y}{r} - \frac{y^2}{r^2} = \frac{2ry - y^2}{r^2}$$

$$\text{故} \quad \sin \theta = \pm \frac{r}{r} \sqrt{2ry - y^2} \quad (5)$$

以 (4) 內式  $\theta$  之同數及 (5) 式內  $\sin \theta$  之同數代入 (1) 式內, 得

$$x = \theta \operatorname{vers}^{-1} \frac{y}{r} \mp \sqrt{2ry - y^2}$$

$$x = \pm \sqrt{2ry - y^2} + r \operatorname{vers}^{-1} \frac{y}{r}$$

## 例 題 XXV

1. 已知  $\log_{10} 2 = 0.30103$ ,  $\log_{10} 3 = 0.47712$ ,  $\log_{10} 7 = 0.84510$  求  
 $\log_{10} 6$ ,  $\log_{10} 14$ ,  $\log_{10} 21$ ,  $\log_{10} 4$ ,  $\log_{10} 12$ ,  $\log_{10} 5$ ,  $\log_{10} \frac{1}{2}$ ,  $\log_{10} \frac{1}{4}$ ,  
 $\log_{10} \frac{7}{9}$ ,  $\log_{10} 0$

$$\log_{10} 6 = \log_{10} 2 + \log_{10} 3$$

$$\log_{10} 2 = 0.30103$$

$$\log_{10} 3 = \underline{0.47712}$$

$$\text{故 } \log_{10} 6 = 0.7715$$

$$\log_{10} 21 = \log_{10} 3 + \log_{10} 7$$

$$\log_{10} 3 = 0.47712$$

$$\log_{10} 7 = \underline{0.84510}$$

$$\text{故 } \log_{10} 21 = 1.32222$$

$$\log_{10} 12 = \log_{10} 3 + \log_{10} 4$$

$$\log_{10} 3 = 0.47712$$

$$\log_{10} 4 = \underline{0.60206}$$

$$\text{故 } \log_{10} 12 = 1.07918$$

$$\log_{10} \frac{1}{2} = \log_{10} 1 - \log_{10} 2$$

$$\log_{10} 1 = 0.00000$$

$$\log_{10} 2 = \underline{0.30103}$$

$$\text{故 } \log_{10} \frac{1}{2} = 1.69897$$

$$\log_{10} 14 = \log_{10} 2 + \log_{10} 7$$

$$\log_{10} 2 = 0.30103$$

$$\log_{10} 7 = 0.84510$$

$$\text{故 } \log_{10} 14 = 1.14613$$

$$\log_{10} 4 = 2 \log_{10} 2$$

$$\log_{10} 2 = 0.30103$$

$$\text{故 } \log_{10} 4 = 0.60206$$

$$\log_{10} 5 = \log_{10} 10 - \log_{10} 2$$

$$\log_{10} 10 = 1.00000$$

$$\log_{10} 2 = \underline{0.30103}$$

$$\text{故 } \log_{10} 5 = 0.69897$$

$$\log_{10} \frac{1}{4} = 2 \log_{10} \frac{1}{2}$$

$$\log_{10} \frac{1}{2} = 1.69897$$

$$\text{故 } \log_{10} \frac{1}{4} = 1.39794$$

$$\log_{10} \frac{7}{9} = \log_{10} 7 - \log_{10} 9$$

$$\log_{10} 7 = 0.84510$$

$$\log_{10} 3^2 = \underline{0.95424}$$

$$\text{故 } \log_{10} \frac{7}{9} = 1.89086$$

$$\log_{10} \frac{21}{20} = \log_{10} 21 - \log_{10} 20$$

$$\log_{10} 21 = 1.32222$$

$$\log_{10}(10 \times 2) = \underline{1.30103}$$

$$\text{故 } \log_{10} \frac{21}{20} = 0.02119$$

2 以1題之已知各件求  $\log_2 10$ ,  $\log_3 5$ ,  $\log_7 \frac{1}{2}$ ,  $\log_5 \frac{9}{343}$ 。

$$\log_2 10 = \frac{\log_{10} 10}{\log_{10} 2} = \frac{1}{0.30103} = 3.3219.$$



$$\log_2 5 = \frac{\log_{10} 5}{\log_{10} 2} = \frac{0.69897}{0.30103} = 2.3219.$$

$$\log_3 5 = \frac{\log_{10} 5}{\log_{10} 3} = \frac{0.69897}{0.47712} = 1.4650.$$

$$\log_7 \frac{1}{2} = \frac{\log_{10} \frac{1}{2}}{\log_{10} 7} = \frac{-0.30103}{0.84510} = -0.3562.$$

$$\begin{aligned} \log_5 \frac{9}{343} &= 2 \log_5 3 - \log_5 7 \\ &= \frac{2 \log_{10} 3 - 3 \log_{10} 7}{\log_{10} 5} \\ &= \frac{0.95424 - 2.53530}{0.69897} \\ &= -2.2620 \end{aligned}$$

3. 已知  $\log_{10} e = 0.43429$ , 求  $\log_e 2, \log_e 3, \log_e 5, \log_e 7, \log_e 8$   
 $\log_e 9, \log_e \frac{2}{3}, \log_e \frac{4}{5}, \log_e \frac{35}{7}, \log_e \frac{7}{600}$

$$\log_e 2 = \frac{\log_{10} 2}{\log_{10} e} = \frac{0.30103}{0.43429} = 0.69315.$$

$$\log_e 3 = \frac{\log_{10} 3}{\log_{10} e} = \frac{0.47712}{0.43429} = 1.09862.$$

$$\log_e 5 = \frac{\log_{10} 5}{\log_{10} e} = \frac{0.69896}{0.43429} = 1.60945.$$

$$\log_e 7 = \frac{\log_{10} 7}{\log_{10} e} = \frac{0.84510}{0.43429} = 1.94596.$$

$$\log_e 8 = \frac{3 \times \log_{10} 2}{\log_{10} e} = \frac{0.90309}{0.43429} = 2.07943.$$

$$\log_e 9 = \frac{2 \times \log_{10} 3}{\log_{10} e} = \frac{0.95424}{0.43429} = 2.19727.$$

$$\log_e \frac{2}{3} = \log_e 2 - \log_e 3 = -0.40547.$$

$$\log_e \frac{4}{5} = 2 \log_e 2 - \log_e 5 = -0.22315.$$

$$\log_e \frac{35}{29} = \log_e 5 + \log_e 7 - 3 \log_e 3 = 0.25952.$$

$$\begin{aligned} \log_e \frac{7}{60} &= \log_e 7 - (\log_e 5 + \log_e 3 + 2 \log_e 2) \\ &= -2.14845. \end{aligned}$$

4. 於下之各方程式求  $x$   $5^x = 12$ ,  $16^x = 10$ ,  $27^x = 4$ .

$$5^x = 12.$$

故  $x \log_{10} 5 = \log_{10} 12.$

$$x = \frac{\log_{10} 12}{\log_{10} 5} = \frac{1.07918}{0.69897} = 1.54396.$$

$$16^x = 10.$$

故  $\log_{10} 16 = \log_{10} 10.$

$$x = \frac{\log_{10} 10}{\log_{10} 16} = \frac{1.00000}{1.20412} = 0.83048.$$

$$27^x = 4$$

故  $x \log_{10} 27 = \log_{10} 4$

$$x = \frac{\log_{10} 4}{\log_{10} 27} = \frac{0.60206}{1.43136} = 0.42062.$$

## 例 題 XXVI

1. 計算  $\log_e 3$  至小數五位止。

(茲特將本題演至十位小數以備第四題之用)

原有公式

$$\log_e \frac{Z+1}{Z} = 2 \left( \frac{1}{2Z+1} + \frac{1}{3(2Z+1)^3} + \frac{1}{5(2Z+1)^5} + \dots \right).$$

今設  $Z = \frac{1}{2}$ ,

別  $\frac{Z+1}{Z} = 3$ , 又  $2Z+1 = 2$ .

則  $\log_e 3 = 2 \left( \frac{1}{2} + \frac{1}{3 \times 2^3} + \frac{1}{5 \times 2^5} + \dots \right).$

2	<u>2.0000000000</u>	
4	<u>1.0000000000</u>	$\div 1 = 1.0000000000$
4	<u>0.2500000000</u>	$\div 3 = 0.0833343333$
4	<u>0.0625000000</u>	$\div 5 = 0.0125000000$
4	<u>0.0156250000</u>	$\div 7 = 0.00223214286$
4	<u>0.0039062500</u>	$\div 9 = 0.04343402778$
4	<u>0.0009766250</u>	$\div 11 = 0.00008877811$
4	<u>0.00024414062</u>	$\div 13 = 0.00001878005$
4	<u>0.00006103515</u>	$\div 15 = 0.00000406901$
4	<u>0.00001525879</u>	$\div 17 = 0.00000089758$
4	<u>0.00000381470</u>	$\div 19 = 0.0000002077$
4	<u>0.00000095367</u>	$\div 21 = 0.00000004541$
4	<u>0.00000023842</u>	$\div 23 = 0.00000001037$
4	<u>0.00000005960</u>	$\div 25 = 0.00000000238$
4	<u>0.00000001490</u>	$\div 27 = 0.00000000055$
4	<u>0.00000000372</u>	$\div 29 = 0.00000000013$
4	<u>0.00000000093</u>	$\div 31 = 0.00000000003$
	<u>0.00000000023</u>	$\div 33 = 0.00000000001$
		<u>1.09861228867</u>

即  $\log_e 3 = 1.09861228867$ .

2. 計算  $\log_e 5$  至小數五位止。

茲特先演  $\log_e 2$  之同數至八位小數以爲本題與後數題之用。

設  $Z=1$ .

則  $Z+1=2$ . 又  $2Z+1=3$ .

故  $\log_e 2 = 2 \left( \frac{1}{3} + \frac{1}{3 \times 3^3} + \frac{1}{5 \times 3^5} + \dots \right)$ .

$$\begin{array}{r}
3 \overline{) 2.00000000} \\
9 \overline{) 0.66666667} \div 1 = 0.66666667 \\
9 \overline{) 0.07407407} \div 3 = 0.024691358 \\
9 \overline{) 0.008230453} \div 5 = 0.001646091 \\
9 \overline{) 0.000914495} \div 7 = 0.000130642 \\
9 \overline{) 0.000101611} \div 9 = 0.000011290 \\
9 \overline{) 0.000011290} \div 11 = 0.000001026 \\
9 \overline{) 0.000011254} \div 13 = 0.00000096 \\
\quad \underline{0.000000139} \div 15 = 0.00000009 \\
\quad \underline{0.000000015} \div 17 = 0.00000001 \\
\qquad \qquad \qquad 0.693147180
\end{array}$$

即  $\log_e 2 = 0.693147180$ .

今設  $Z = 4$ .

則  $Z + 1 = 5$ . 又  $2Z + 1 = 9$ .

故  $\log_e \frac{5}{4} = 2 \left( \frac{1}{9} + \frac{1}{3 \times 9^3} + \frac{1}{5 \times 9^5} + \dots \right)$ .

$$\begin{array}{r}
9 \overline{) 2.000000} \\
9 \overline{) 0.222222} \div 1 = 0.222222 \\
9 \overline{) 0.024691} \\
9 \overline{) 0.000743} \div 3 = 0.000913 \\
9 \overline{) 0.000305} \\
\quad \underline{0.000034} \div 5 = 0.000001 \\
\qquad \qquad \qquad 0.223143
\end{array}$$

試  $\log_e \frac{5}{4} = 0.22314$ .

$$\begin{aligned}
\log_e 5 &= 0.22314 + \log_e 4 \\
&= 0.22314 + 2 \times \log_e 2 \\
&= 0.22314 + 2 \times 0.69315 \\
&= 1.60944.
\end{aligned}$$

3. 計算  $\log_e 7$  至小數五位止。

設  $Z = 6$ .

則  $Z+1=7$ . 又  $2Z+1=13$ .

$$\text{故 } \log_e \frac{7}{6} = 2 \left( \frac{1}{13} + \frac{1}{3 \times 13^3} + \frac{1}{5 \times 13^5} + \dots \right).$$

$$\begin{array}{r} 13 \overline{) 2.000000} \\ 13 \underline{0.153846} \div 1 = 0.153846 \\ 13 \underline{0.011834} \\ 13 \underline{0.000916} \div 3 = 0.000303 \\ 13 \underline{0.000070} \\ \quad 0.000005 \div 5 = 0.000001 \\ \qquad \qquad \qquad 0.154150 \end{array}$$

即

$$\log_e \frac{1}{6} = 0.15415.$$

$$\log_e 7 = 0.15415 + \log_e 6$$

$$= 0.15415 + \log_e 2 + \log_e 3$$

$$= 0.15415 + 0.693147 + 1.098612$$

$$= 1.94591.$$

4. 計算  $\log_e 10$  至小數十位止。

$$\text{設 } Z=9$$

則  $Z+1=10$ . 又  $2Z+1=19$ .

$$\text{故 } \log_e \frac{10}{9} = 2 \left( \frac{1}{19} + \frac{1}{3 \times 19^3} + \frac{1}{5 \times 19^5} + \dots \right).$$

$$\begin{array}{r} 19 \overline{) 2.0000000000} \\ 19 \underline{0.10526315789} \div 1 = 0.10526315789 \\ 19 \underline{0.00554016620} \\ 19 \underline{0.00029158769} \div 3 = 0.00009719590 \\ 19 \underline{0.00001534672} \\ 19 \underline{0.00000080772} \div 5 = 0.0000004251 \\ 19 \underline{0.00000004251} \\ \quad 0.0000000224 \div 7 = 0.0000000032 \\ \qquad \qquad \qquad 0.10536051565 \end{array}$$

即

$$\log_e \frac{10}{9} = 0.1053605156.$$

$$\log_e 10 = 0.1053605156 + 2 \log_e 3.$$

5. 計算  $\log_{10} 2$ ,  $\log_{10} e$ ,  $\log_{10} 11$  各至小數五位止。

$$\log_{10} 2 = \frac{\log_e 2}{\log_e 10} = \frac{0.693147}{2.302585} = 0.30103$$

$$\log_e e = \frac{\log_e e}{\log_e 10} = \frac{1}{2.302585} = 0.43429.$$

今欲演  $\log_e 11$ ,設  $Z = 10$ .則  $Z + 1 = 11$ . 又  $2Z + 1 = 21$ .

$$\text{故 } \log_{10} \frac{11}{10} = 2 \log_{10} e \left( \frac{1}{21} + \frac{1}{3 \times 21^3} + \frac{1}{5 \times 21^5} + \dots \right).$$

$$\begin{array}{r} 21 \overline{) 2.000000} \\ 21 \overline{) 0.095238} + 1 = 0.065239 \\ 21 \overline{) 0.004535} \\ \hline 0.000216 \div 3 = 0.000072 \\ \hline 0.095310 \end{array}$$

即

$$\begin{aligned} \log_{10} \frac{11}{10} &= 0.09531 \times \log_{10} e \\ &= 0.09531 \times 0.43429 \\ &= 0.04139. \end{aligned}$$

$$\begin{aligned} \log_{10} 11 &= 0.04139 + \log_{10} 10 \\ &= 1.04139. \end{aligned}$$

## 例題 XXVII

設  $\pi = 3.141592653589$ .1. 計算  $\sin 1'$ ,  $\cos 2'$ , 及  $\tan 1'$  至小數十一位止。

$$\begin{aligned}
 I' &= \frac{\pi}{180 \times 60} \text{徑率} = \frac{\pi}{10800} = \frac{3.141592653489}{10800} \\
 &= 0.0002908882 +
 \end{aligned}$$

又原書中既求得

$$\sin I' = 0.00029088 +,$$

故知

$$\begin{aligned}
 \sin I' &> \sqrt{1 - (0.00029089)^2} \\
 &> \sqrt{1 - 0.000000084617} \\
 &> \sqrt{0.999999915383} \\
 &> 0.999999957691. \quad (1)
 \end{aligned}$$

又

$$\begin{aligned}
 \cos I' &< \sqrt{1 - (0.00029088)^2} \\
 &< \sqrt{1 - 0.000000084611} \\
 &< \sqrt{0.999999915389} \\
 &< 0.999999957694. \quad (2)
 \end{aligned}$$

詳察 (1)(2) 二數有十一位小數相同,

故

$$\cos I' = 0.99999995769.$$

按第四十五節,

$$\sin x > x \cos x$$

故

$$\begin{aligned}
 \sin I' &> 0.0002908882 \times 0.99999995769 \\
 &> 0.0002908882 \\
 &\quad (1 - 0.00000004231) \\
 &> 0.000290882 - 0.000000000012 \\
 &> 0.00029088818
 \end{aligned}$$

故  $\sin I'$  必在 0.000290388818 與 0.000290882 之間.

其首九位小數必無可疑, 即

$$\sin I' = 0.000290888.$$

以後之數爲 18, 19, 20, 或 21.

得  $\sin I'$  未後之同數再依前法推演演可得十一位小數.

$$\sin 1' = 0.00029088820.$$

按  $\sin 1'$  與  $\cos 1'$  之同數, 特

$$\begin{aligned}\tan 1' &= \frac{\sin 1'}{\cos 1'} \\ &= \frac{0.00029088820}{0.99999995769} \\ &= 0.000290888212.\end{aligned}$$

試以同法及公式  $\sin 2x = 2 \sin x \cos x$  計草  $\sin 2'$  至小數九位止。又此兩結果。能一致否。

$$2' = \frac{\pi}{5400} = \frac{3.141592653589}{5400} = 0.0005817764+.$$

(1)

故  $\sin 2'$  必在 0 與 0.0005817765 之間。

$$\begin{aligned}\text{又} \quad \cos 2' &> \sqrt{1 - (0.0005817765)^2} \\ &> \sqrt{1 - 0.0000003384638959} \\ &> \sqrt{0.9999996615361040} \\ &> 0.999999830768\end{aligned}$$

但  $\sin x > x \cos x,$

$$\begin{aligned}\text{故} \quad \sin x &> 0.0005817764 \times 0.99999983076 \\ &> 0.0005817764(1 - 0.00000016924) \\ &> 0.0005817764 - 0.000000000698 \\ &> 0.0005817763.\end{aligned} \quad (2)$$

由 (1) (2) 觀之可知其首九位小數必無可疑, 即

$$\sin 2' = 0.000581776.$$

又法

$$\begin{aligned}\sin 2' &= 2 \sin 1' \cos 1' \\ &= 2 \times 0.00029088820 \times 0.99999995769\end{aligned}$$



$$=0.00058177640(1-0.00000004231)$$

$$=0.00058177640-0.00000000025$$

$$=0.000581776+.$$

故 兩法首九位之小數皆同。

3. 計算試  $\sin 1^\circ$  至小數四位止。

$$1^\circ = \frac{\pi}{180} = \frac{3.141592653589}{180}$$

$$=0.01745329 \text{ 徑率}$$

$$\text{故 } \cos 1^\circ > \sqrt{1-(0.01746)^2}$$

$$> \sqrt{0.99969515}$$

$$> 0.999847.$$

$$\text{但 } \sin x > x \cos x,$$

$$\text{故 } \sin 1^\circ > 0.017453$$

$$\times 0.999847$$

$$> 0.017453$$

$$(1-0.000153)$$

$$> 0.017453-0.0000027$$

$$> 0.0174503.$$

即得其首四位小數為

$$\sin 1' = 0.0175.$$

4. 試由公式  $\cos x =$

$$1-2\sin^2 \frac{x}{2}$$

$$\text{顯 } \cos x > 1 - \frac{x^2}{2}$$

按四十五節  $\sin x < x$ .

$$\text{故 } \sin \frac{x}{2} < \frac{x}{2}$$

$$\sin^2 \frac{x}{2} < \frac{x^2}{4}$$

$$1-2\sin^2 \frac{x}{2} < 1 - \frac{x^2}{2}$$

$$\text{故 } \cos x < 1 - \frac{x^2}{2}$$

5 試以自然正弦表。顯小

於  $4^\circ 40'$  之一切角。其  $\sin x$  及  $x$  常有四位小數一致。

$$4^\circ 40' = \frac{280}{10800} \pi = \frac{7\pi}{270}$$

$$= \frac{7 \times 3.141592653589}{270}$$

$$= 0.0814487 \text{ 徑率.}$$

$$4^\circ 41' = 0.081448 + 0.0002909$$

$$= 0.0817396 \text{ 徑率}$$

查表,

$$\sin 4^{\circ}40' = 0.0814.$$

$$\sin 4^{\circ}41' = 0.0816.$$

故知凡角等於  $4^{\circ}40'$  或小於  $4^{\circ}40'$  則  $\sin x$  與  $x$  之首四位小數相同. 若角大於  $4^{\circ}40'$  則否.

6. 設  $\log x$  及  $\log \sin x$  小數五位一致. 試由表內求  $x$  可有之最大值.

設  $x$  爲角之秒數,

$$\text{則 } x'' = \frac{\pi x}{648000}.$$

$\frac{\pi x}{648000}$  之對數爲

$$\begin{aligned} & \log x'' + (\log \pi - \log 648000) \\ &= \log x'' + (0.49715 - 5.81158) \\ &= \log x'' - 5.31443 \end{aligned}$$

$$= \log x'' + 4.68557 - 10.$$

但依原書第四表表前之解釋凡表中之  $\log \sin$  皆已加 10, 故知

$$\begin{aligned} \log \sin x + 10 &= \log x'' + s. \\ \text{即 } \log \sin x &= \log x'' + s - 10. \end{aligned}$$

故, 若  $\log \sin x$  與  $x$  首五位之小數相等, 則

$$\begin{aligned} \log x'' + 4.68557 - 10 \\ &= \log x'' + s - 10. \end{aligned}$$

$$\text{即 } s = 4.68557.$$

但按表  $s$  之同數所可有之最大角爲  $2409''$ .

故  $\log x$  與  $\log \sin x$  所可有之最大角亦爲

$$2409'' = 40'9''.$$

### 例 題 XXVIII

1 算計  $\sin 6'$  及  $\cos 6'$  至小數七位<sup>o</sup>

$$\text{由 } 27/2, \quad \sin 2' = 0.000581776$$

$$\text{又由 } 27/1, \quad \cos 1' = 0.999999958,$$

$$\sin 1' = 0.000290888.$$

$$\text{故 } \sin 3' = 2 \sin 2' \cos 1' - \sin 1'$$

$$= 2 \times 0.000581776(1 - 0.000001)$$

$$- 0.000290888$$

$$= 2 \times 0.000581776 - 0.000290888$$

$$=0.000872664.$$

$$\begin{aligned}\cos 2' &= 2 \cos^2 1' - 1 \\ &= 2(0.999999958) - 1 \\ &= 0.999999916 \times 2 - 1 \\ &= 0.999999832.\end{aligned}$$

$$\begin{aligned}\cos 3' &= 2 \cos 2' \cos 1' - \cos 1' \\ &= \cos 1' (2 \cos 2' - 1) \\ &= 0.999999958(2 \times 0.999999832 - 1) \\ &= (1 - 0.000000042)(0.999999664) \\ &= 0.999999622.\end{aligned}$$

按 [12],  $\sin 6' = 2 \sin 3' \cos 3'$

$$\begin{aligned}&= 2 \times 0.000872664 \times 0.999999622 \\ &= 0.001744327.\end{aligned}$$

按 [13],  $\cos 6' = 2 \cos^2 3' - 1$

$$\begin{aligned}&= 2(0.999999622)^2 - 1 \\ &= 2 \times 0.999999244 - 1 \\ &= 0.999998488.\end{aligned}$$

於公式(1).令  $y = 1^\circ$ . 假定

$$\sin 1^\circ = 0.017454 +, \quad \cos 1^\circ = 0.9998484 + ^\circ$$

2. 計算二度之正弦及餘弦。

$$\begin{aligned}\sin 2^\circ &= 2 \sin 1^\circ \cos 1^\circ \\ &= 2 \times 0.017454 \times 0.999848 \\ &= 0.034902.\end{aligned}$$

$$\begin{aligned}\cos 2^\circ &= 2 \cos^2 1^\circ - 1 \\ &= 2 \times (0.999848)^2 - 1\end{aligned}$$

$$= 2 \times (0.999696) - 1$$

$$= 0.999392.$$

### 3 計算三度之正弦及餘弦<sup>o</sup>

$$\sin 3^{\circ} = 2 \sin 2^{\circ} \cos 1^{\circ} - \sin 1^{\circ}$$

$$= 2 \times 0.034902 \times 0.999848 - 0.017454$$

$$= 0.052339.$$

$$\cos 3^{\circ} = (2 \cos 2^{\circ} - 1) \cos 1^{\circ}$$

$$= 0.99784 \times 0.999848$$

$$= 0.998632$$

### 4. 計算四度之正弦及餘弦

$$\sin 4^{\circ} = 2 \sin 3^{\circ} \cos 1^{\circ} - \sin 2^{\circ}$$

$$= 2 \times 0.052339 \times 0.999848 - 0.034902$$

$$= 0.069760$$

$$\cos 4^{\circ} = 2 \cos 3^{\circ} \cos 1^{\circ} - \cos 2^{\circ}$$

$$= 2 \times 0.998630 \times 0.999848 - 0.999392$$

$$= 1.996956 - 0.999392$$

$$= 0.997564.$$

### 5. 計算五度之正弦及餘弦

$$\sin 5^{\circ} = 2 \sin 4^{\circ} \cos 1^{\circ} - \sin 3^{\circ}$$

$$= 2 \times 0.069760 \times 0.999848 - 0.052339$$

$$= 0.087160.$$

$$\cos 5^{\circ} = 2 \cos 4^{\circ} \cos 1^{\circ} - \cos 3^{\circ}$$

$$= 2 \times 0.997564 \times 0.999848 - 0.998632$$

$$= 1.994825 - 0.998632$$

$$= 0.996193.$$

## 例題 XXIX

1. 求  $-1$  及  $+1$  之六方根

$$-1 = \cos 180^\circ + i \sin 180^\circ$$

$$+1 = \cos 0^\circ + i \sin 0^\circ$$

故  $-1$  之六方諸根如下,

$$\cos 30^\circ + i \sin 30^\circ = \frac{\sqrt{3} + i}{2},$$

$$\cos 90^\circ + i \sin 90^\circ = i,$$

$$\cos 150^\circ + i \sin 150^\circ = \frac{-\sqrt{3} + i}{2},$$

$$\cos 210^\circ + i \sin 210^\circ = \frac{-\sqrt{3} - i}{2},$$

$$\cos 270^\circ + i \sin 270^\circ = -i,$$

$$\cos 330^\circ + i \sin 330^\circ = \frac{\sqrt{3} - i}{2}.$$

又  $+1$  之六方諸根如下,

$$\cos 0^\circ + i \sin 0^\circ = +1,$$

$$\cos 50^\circ + i \sin 60^\circ = \frac{1 + \sqrt{-3}}{2},$$

$$\cos 120^\circ + i \sin 120^\circ$$

$$= \frac{-1 + \sqrt{-3}}{2},$$

4. 試以  $\sin \theta$  及  $\cos \theta$  表  $\sin 4\theta$  及  $\cos 4\theta$ 。

按第四十七節

$$\sin 4\theta = 4 \cos^3 \theta \sin \theta - \frac{4 \times 3 \times 2}{3} \cos \theta \sin^3 \theta$$

$$= 4 \cos^3 \theta \sin \theta - 4 \cos \theta \sin^3 \theta,$$

$$\cos 180^\circ + i \sin 180^\circ = -1,$$

$$\cos 240^\circ + i \sin 240^\circ$$

$$= \frac{-1 - \sqrt{-3}}{2},$$

$$\cos 300^\circ + i \sin 300^\circ = \frac{-1 + \sqrt{-3}}{2},$$

2. 求  $i$  之三個立方根。

$$i = \cos 90^\circ + i \sin 90^\circ.$$

故  $i$  之三立方根如下,

$$\cos 30^\circ + i \sin 30^\circ = \frac{\sqrt{3} + i}{2},$$

$$\cos 150^\circ + i \sin 150^\circ = \frac{-\sqrt{3} + i}{2}$$

$$\cos 270^\circ + i \sin 270^\circ = -i.$$

3. 求  $-i$  之四個方諸根。

$$-i = \cos 270^\circ + i \sin 270^\circ.$$

故  $-i$  之四方諸根如下,

$$\cos 67\frac{1}{2}^\circ + i \sin 67\frac{1}{2}^\circ,$$

$$\cos 157\frac{1}{2}^\circ + i \sin 157\frac{1}{2}^\circ,$$

$$\cos 247\frac{1}{2}^\circ + i \sin 247\frac{1}{2}^\circ,$$

$$\cos 337\frac{1}{2}^\circ + i \sin 337\frac{1}{2}^\circ.$$

$$\begin{aligned} \text{又 } \cos 4\theta &= \cos^4 \theta - \frac{4 \times 3}{2} \cos^2 \theta \sin^2 \theta + \frac{4 \times 3 \times 2 \times 1}{4} \sin^4 \theta \\ &= \cos^4 \theta - 6 \cos^2 \theta \sin^2 \theta + \sin^4 \theta. \end{aligned}$$

### 例題 XXX

1. 試以適纜所得之級數。證明  $\sin^2 x + \cos^2 x = 1$ ,

$$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \dots$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320} - \dots$$

$$\text{故 } \sin^2 x = x^2 - \frac{x^4}{3} + \frac{2x^6}{45} - \frac{x^8}{315} + \dots$$

$$\text{又 } \cos^2 x = 1 - x^2 + \frac{x^4}{3} - \frac{2x^6}{45} + \frac{x^8}{315} - \dots$$

$$\text{即得 } \sin^2 x + \cos^2 x = 1.$$

2.  $\sin(-x) = -\sin x$ , 及  $\cos(-x) = \cos x$ .

正弦之級數其  $x$  之指數皆為奇數, 故其號與  $x$  同變.

餘弦之級數其  $x$  指數皆為偶數, 故其號不與  $x$  之號同變.

3.  $\sin 2x = 2 \sin x \cos x$ .

$$\sin 2x = 2x - \frac{(2x)^3}{6} + \frac{(2x)^5}{120} - \frac{(2x)^7}{5040} + \dots$$

$$= 2x - \frac{4x^3}{3} + \frac{4x^5}{15} - \frac{8x^7}{315} + \dots$$

$$= 2 \left( x - \frac{2x^3}{3} + \frac{4x^5}{15} - \frac{8x^7}{315} + \dots \right)$$

$$\text{又 } \sin x \cos x = x - \frac{2x^3}{3} + \frac{2x^5}{15} - \frac{4x^7}{315} + \dots$$

故  $\sin 2x = 2 \sin x \cos x$

4.  $\cos 2x = 1 - 2 \sin^2 x$

$$\sin^2 x = x^2 - \frac{x^4}{3} + \frac{2x^6}{45} - \frac{x^8}{315} + \dots$$

$$\begin{aligned} \text{故 } 1 - 2 \sin^2 x &= 1 - 2x^2 + \frac{2x^4}{3} - \frac{4x^6}{45} + \frac{2x^8}{315} - \dots \\ &= 1 - \frac{(2x)^2}{2} + \frac{(2x)^4}{24} - \frac{(2x)^6}{720} + \frac{(2x)^8}{40320} - \dots \\ &= \cos 2x. \end{aligned}$$

5. 求  $\sec x$  之級數直至含有  $x$  之方之項為止。

$$\begin{aligned} \sec x &= \frac{1}{\cos x} = 1 \div \left( 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots \right) \\ &= 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \dots \end{aligned}$$

6. 求  $x \cos x$  之級數(令其  $x \cot x = \frac{x}{\sin x} - \cos x$ )

$$x \cos x = x - \frac{x^3}{2} + \frac{x^5}{24} - \frac{x^7}{720} + \dots$$

$$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \dots$$

$$x \cot x = \frac{x \cos x}{\sin x} = 1 - \frac{x^2}{3} + \frac{x^4}{45} - \frac{2x^6}{945} + \dots$$

7. 計算  $\sin 10^\circ$  及  $\cos 10^\circ$  至小數五位。

$$10^\circ = 10 \times \frac{\pi}{180} \text{ 徑率}$$

$$\text{故 } \sin 10^\circ = \frac{\pi}{28} - \frac{1}{6} \left( \frac{\pi}{18} \right)^3 + \frac{1}{120} \left( \frac{\pi}{18} \right)^5 + \dots$$

$$\cos 10^\circ = 1 - \frac{1}{2} \left( \frac{\pi}{18} \right)^2 + \frac{1}{24} \left( \frac{\pi}{18} \right)^4 - \frac{1}{720} \left( \frac{\pi}{18} \right)^6 + \dots$$

故  $\pi = 3.141592653589$ . 即

$$\pi = 3.141593, \frac{\pi}{18} = 0.174533.$$

$$\pi^2 = 9.869604, \frac{\pi^2}{2 \times 18^2} = 0.015231,$$

$$\pi^3 = 31.006277, \frac{\pi^3}{6 \times 18^3} = 0.000886,$$

$$\pi^4 = 97.409010, \frac{\pi^4}{24 \times 18^4} = 0.000039,$$

$$\pi^5 = 306.019685, \frac{\pi^5}{120 \times 18^5} = 0.000001.$$

$$\begin{aligned} \text{故 } \sin 10^\circ &= 0.174633 - 0.000886 + 0.000001. \\ &= 0.173648. \end{aligned}$$

$$\cos 10^\circ = 1 - 0.015231 + 0.000039$$

$$= 0.984808.$$

計算  $\tan 15^\circ$  至小數五位.

$$15^\circ = \frac{\pi}{12} \text{ 徑率}$$

$$\tan 15^\circ = \frac{\pi}{12} + \frac{1}{3} \left( \frac{\pi}{12} \right)^3 + \frac{2}{15} \left( \frac{\pi}{12} \right)^5 + \frac{17}{315} \left( \frac{\pi}{12} \right)^7 + \dots$$

$$\pi = 3.141593, \frac{\pi}{12} = 0.261799,$$

$$\pi^3 = 31.006277, \frac{\pi^3}{3 \times 12^3} = 0.005981.$$

$$\pi^5 = 306.019685, \frac{\pi^5}{15 \times 12^5} = 0.000164.$$

$$\pi^7 = 3020.293227, \frac{\pi^7}{315 \times 12^7} = 0.000005.$$

$$\text{故 } \tan 15^\circ = 0.267949.$$



9. 由  $\sin x$  之指數值。顯

$$\cos 3x = 4 \cos^3 x - 3 \cos x$$

按 §48,

$$\cos x = \frac{1}{2}(e^{xi} + e^{-xi})$$

$$\text{故 } \cos 3x = \frac{1}{2}(e^{3xi} + e^{-3xi})$$

$$= \frac{1}{2}(e^{xi} + e^{-xi})(e^{2xi} - 1 + e^{-2xi})$$

$$= \cos x \left[ \left( 4 \times \frac{1}{2}(e^{xi} + e^{-xi}) \right) - 3 \right]$$

$$= \cos x (4 \cos^2 x - 3)$$

$$= 4 \cos^3 x - 3 \cos x$$

10. 由  $\sin x$  之之指數值。顯,

$$\sin 3x = 3 \sin x - 4 \sin^3 x$$

按 §48,

$$\sin 3x = \frac{1}{2i}(e^{3xi} - e^{-3xi})$$

$$= \frac{1}{2i}(e^{xi} - e^{-xi})(e^{2xi} + 1 + e^{-2xi})$$

$$= \sin x \left[ (-4) \left\{ \frac{1}{2i}(e^{xi} - e^{-xi}) \right\}^2 + 3 \right]$$

$$= \sin x (-4 \sin^2 x + 3)$$

$$= 3 \sin x - 4 \sin^3 x.$$

# TABLE I

THE

## COMMON OR BRIGGS LOGARITHMS

OF THE

### NATURAL NUMBERS

From 1 to 10000.

#### 1-100

N	log	N	log	N	log	N	log	N	log
1	0.00000	21	1.32222	41	1.61278	61	1.78533	81	1.90849
2	0.30103	22	1.34242	42	1.62325	62	1.79239	82	1.91381
3	0.47712	23	1.36173	43	1.63347	63	1.79934	83	1.91908
4	0.60206	24	1.38021	44	1.64345	64	1.80618	84	1.92428
5	0.69897	25	1.39794	45	1.65321	65	1.81291	85	1.92942
6	0.77815	26	1.41497	46	1.66276	66	1.81954	86	1.93450
7	0.84310	27	1.43136	47	1.67210	67	1.82607	87	1.93952
8	0.90309	28	1.44716	48	1.68124	68	1.83251	88	1.94448
9	0.95424	29	1.46240	49	1.69020	69	1.83885	89	1.94939
10	1.00000	30	1.47712	50	1.69897	70	1.84510	90	1.95424
11	1.04139	31	1.49136	51	1.70757	71	1.85126	91	1.95904
12	1.07918	32	1.50515	52	1.71600	72	1.85733	92	1.96379
13	1.11394	33	1.51851	53	1.72428	73	1.86332	93	1.96848
14	1.14613	34	1.53148	54	1.73239	74	1.86923	94	1.97313
15	1.17609	35	1.54407	55	1.74036	75	1.87506	95	1.97772
16	1.20412	36	1.55630	56	1.74819	76	1.88081	96	1.98227
17	1.23045	37	1.56820	57	1.75587	77	1.88649	97	1.98677
18	1.25527	38	1.57978	58	1.76343	78	1.89209	98	1.99123
19	1.27875	39	1.59106	59	1.77085	79	1.89763	99	1.99564
20	1.30103	40	1.60206	60	1.77815	80	1.90309	100	2.00000
N	log	N	log	N	log	N	log	N	log

#### 1-100

## 100-150

N	0	1	2	3	4	5	6	7	8	9
100	00 000	00 043	00 087	00 130	00 173	00 217	00 260	00 303	00 346	00 389
101	00 432	00 475	00 518	00 561	00 604	00 647	00 689	00 732	00 775	00 817
102	00 860	00 903	00 945	00 988	01 030	01 072	01 115	01 157	01 199	01 242
103	01 284	01 326	01 368	01 410	01 452	01 494	01 536	01 578	01 620	01 662
104	01 703	01 745	01 787	01 828	01 870	01 912	01 953	01 995	02 036	02 078
105	02 119	02 160	02 202	02 243	02 284	02 325	02 366	02 407	02 449	02 490
106	02 531	02 572	02 612	02 653	02 694	02 735	02 776	02 816	02 857	02 898
107	02 938	02 979	03 019	03 060	03 100	03 141	03 181	03 222	03 262	03 302
108	03 342	03 383	03 423	03 463	03 503	03 543	03 583	03 623	03 663	03 703
109	03 743	03 782	03 822	03 862	03 902	03 941	03 981	04 021	04 060	04 100
110	04 139	04 179	04 218	04 258	04 297	04 336	04 376	04 415	04 454	04 493
111	04 532	04 571	04 610	04 650	04 689	04 727	04 766	04 805	04 844	04 883
112	04 922	04 961	04 999	05 038	05 077	05 115	05 154	05 192	05 231	05 269
113	05 308	05 346	05 385	05 423	05 461	05 500	05 538	05 576	05 614	05 652
114	05 690	05 729	05 767	05 805	05 843	05 881	05 918	05 956	05 994	06 032
115	06 070	06 108	06 145	06 183	06 221	06 258	06 296	06 333	06 371	06 408
116	06 446	06 483	06 521	06 558	06 595	06 633	06 670	06 707	06 744	06 781
117	06 819	06 856	06 893	06 930	06 967	07 004	07 041	07 078	07 115	07 151
118	07 188	07 225	07 262	07 298	07 335	07 372	07 408	07 445	07 482	07 518
119	07 555	07 591	07 628	07 664	07 700	07 737	07 773	07 809	07 846	07 882
120	07 918	07 954	07 990	08 027	08 063	08 099	08 135	08 171	08 207	08 243
121	08 279	08 314	08 350	08 386	08 422	08 458	08 493	08 529	08 565	08 600
122	08 636	08 672	08 707	08 743	08 778	08 814	08 849	08 884	08 920	08 955
123	08 991	09 026	09 061	09 096	09 132	09 167	09 202	09 237	09 272	09 307
124	09 342	09 377	09 412	09 447	09 482	09 517	09 552	09 587	09 621	09 656
125	09 691	09 726	09 760	09 795	09 830	09 864	09 899	09 934	09 968	10 003
126	10 037	10 072	10 106	10 140	10 175	10 209	10 243	10 278	10 312	10 346
127	10 380	10 415	10 449	10 483	10 517	10 551	10 585	10 619	10 653	10 687
128	10 721	10 755	10 789	10 823	10 857	10 890	10 924	10 958	10 992	11 025
129	11 059	11 093	11 126	11 160	11 193	11 227	11 261	11 294	11 327	11 361
130	11 394	11 428	11 461	11 494	11 528	11 561	11 594	11 628	11 661	11 694
131	11 727	11 760	11 793	11 826	11 860	11 893	11 926	11 959	11 992	12 024
132	12 057	12 090	12 123	12 156	12 189	12 222	12 254	12 287	12 320	12 352
133	12 385	12 418	12 450	12 483	12 516	12 548	12 581	12 613	12 646	12 678
134	12 710	12 743	12 775	12 808	12 840	12 872	12 905	12 937	12 969	13 001
135	13 033	13 066	13 098	13 130	13 162	13 194	13 226	13 258	13 290	13 322
136	13 354	13 386	13 418	13 450	13 481	13 513	13 545	13 577	13 609	13 640
137	13 672	13 704	13 735	13 767	13 799	13 830	13 862	13 893	13 925	13 956
138	13 988	14 019	14 051	14 082	14 114	14 145	14 176	14 208	14 239	14 270
139	14 301	14 333	14 364	14 395	14 426	14 457	14 489	14 520	14 551	14 582
140	14 613	14 644	14 675	14 706	14 737	14 768	14 799	14 829	14 860	14 891
141	14 922	14 953	14 983	15 014	15 045	15 076	15 106	15 137	15 168	15 198
142	15 229	15 259	15 290	15 320	15 351	15 381	15 412	15 442	15 473	15 503
143	15 534	15 564	15 594	15 625	15 655	15 685	15 715	15 746	15 776	15 806
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145	16 137	16 167	16 197	16 227	16 256	16 286	16 316	16 346	16 376	16 406
146	16 435	16 465	16 495	16 524	16 554	16 584	16 613	16 643	16 673	16 702
147	16 732	16 761	16 791	16 820	16 850	16 879	16 909	16 938	16 967	16 997
148	17 026	17 056	17 085	17 114	17 143	17 173	17 202	17 231	17 260	17 289
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150	17 609	17 638	17 667	17 696	17 725	17 754	17 782	17 811	17 840	17 869
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152	18 184	18 213	18 241	18 270	18 298	18 327	18 355	18 384	18 412	18 441
153	18 469	18 498	18 526	18 554	18 583	18 611	18 639	18 667	18 696	18 724
154	18 752	18 780	18 808	18 837	18 865	18 893	18 921	18 949	18 977	19 005
155	19 033	19 061	19 089	19 117	19 145	19 173	19 201	19 229	19 257	19 285
156	19 312	19 340	19 368	19 396	19 424	19 451	19 479	19 507	19 535	19 562
157	19 590	19 618	19 645	19 673	19 700	19 728	19 756	19 783	19 811	19 838
158	19 866	19 893	19 921	19 948	19 976	20 003	20 030	20 058	20 085	20 112
159	20 140	20 167	20 194	20 222	20 249	20 276	20 303	20 330	20 358	20 385
160	20 412	20 439	20 466	20 493	20 520	20 548	20 575	20 602	20 629	20 656
161	20 683	20 710	20 737	20 763	20 790	20 817	20 844	20 871	20 898	20 925
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163	21 219	21 245	21 272	21 299	21 325	21 352	21 378	21 405	21 431	21 458
164	21 484	21 511	21 537	21 564	21 590	21 617	21 643	21 669	21 696	21 722
165	21 748	21 775	21 801	21 827	21 854	21 880	21 906	21 932	21 958	21 985
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167	22 272	22 298	22 324	22 350	22 376	22 401	22 427	22 453	22 479	22 505
168	22 531	22 557	22 583	22 608	22 634	22 660	22 686	22 712	22 737	22 763
169	22 789	22 814	22 840	22 866	22 891	22 917	22 943	22 968	22 994	23 019
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171	23 300	23 325	23 350	23 376	23 401	23 426	23 452	23 477	23 502	23 528
172	23 553	23 578	23 603	23 629	23 654	23 679	23 704	23 729	23 754	23 779
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175	24 304	24 329	24 353	24 378	24 403	24 428	24 452	24 477	24 502	24 527
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177	24 797	24 822	24 846	24 871	24 895	24 920	24 944	24 969	24 993	25 018
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179	25 285	25 310	25 334	25 358	25 382	25 406	25 431	25 455	25 479	25 503
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187	27 184	27 207	27 231	27 254	27 277	27 300	27 323	27 346	27 370	27 393
188	27 416	27 439	27 462	27 485	27 508	27 531	27 554	27 577	27 600	27 623
189	27 646	27 669	27 692	27 715	27 738	27 761	27 784	27 807	27 830	27 852
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192	28 330	28 353	28 375	28 398	28 421	28 443	28 466	28 488	28 511	28 533
193	28 556	28 578	28 601	28 623	28 646	28 668	28 691	28 713	28 735	28 758
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196	29 226	29 248	29 270	29 292	29 314	29 336	29 358	29 380	29 403	29 425
197	29 447	29 469	29 491	29 513	29 535	29 557	29 579	29 601	29 623	29 645
198	29 667	29 688	29 710	29 732	29 754	29 776	29 798	29 820	29 842	29 863
199	29 885	29 907	29 929	29 951	29 973	29 994	30 016	30 038	30 060	30 081
200	30 103	30 125	30 146	30 168	30 190	30 211	30 233	30 255	30 276	30 298
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202	30 535	30 557	30 578	30 600	30 621	30 643	30 664	30 685	30 707	30 728
203	30 750	30 771	30 792	30 814	30 835	30 856	30 878	30 899	30 920	30 942
204	30 963	30 984	31 005	31 027	31 048	31 069	31 091	31 112	31 133	31 154
205	31 175	31 197	31 218	31 239	31 260	31 281	31 302	31 323	31 345	31 366
206	31 387	31 408	31 429	31 450	31 471	31 492	31 513	31 534	31 555	31 576
207	31 597	31 618	31 639	31 660	31 681	31 702	31 723	31 744	31 765	31 785
208	31 806	31 827	31 848	31 869	31 890	31 911	31 931	31 952	31 973	31 994
209	32 015	32 035	32 056	32 077	32 098	32 118	32 139	32 160	32 181	32 201
210	32 222	32 243	32 263	32 284	32 305	32 325	32 346	32 366	32 387	32 408
211	32 428	32 449	32 469	32 490	32 510	32 531	32 552	32 572	32 593	32 613
212	32 634	32 654	32 675	32 695	32 715	32 736	32 756	32 777	32 797	32 818
213	32 833	32 853	32 873	32 893	32 913	32 934	32 954	32 974	33 001	33 021
214	33 041	33 062	33 082	33 102	33 122	33 143	33 163	33 183	33 203	33 224
215	33 244	33 264	33 284	33 304	33 325	33 345	33 365	33 385	33 405	33 425
216	33 445	33 465	33 486	33 506	33 526	33 546	33 566	33 586	33 606	33 626
217	33 646	33 666	33 686	33 706	33 726	33 746	33 766	33 786	33 806	33 826
218	33 846	33 866	33 885	33 905	33 925	33 945	33 965	33 985	34 005	34 025
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220	34 242	34 262	34 282	34 301	34 321	34 341	34 361	34 380	34 400	34 420
221	34 439	34 459	34 479	34 498	34 518	34 537	34 557	34 577	34 596	34 616
222	34 635	34 655	34 674	34 694	34 713	34 733	34 753	34 772	34 792	34 811
223	34 830	34 850	34 869	34 889	34 908	34 928	34 947	34 967	34 986	35 005
224	35 025	35 044	35 064	35 083	35 102	35 122	35 141	35 160	35 180	35 199
225	35 218	35 238	35 257	35 276	35 295	35 315	35 334	35 353	35 372	35 392
226	35 411	35 430	35 449	35 468	35 488	35 507	35 526	35 545	35 564	35 583
227	35 603	35 622	35 641	35 660	35 679	35 698	35 717	35 736	35 755	35 774
228	35 793	35 813	35 832	35 851	35 870	35 889	35 908	35 927	35 946	35 965
229	35 984	36 003	36 021	36 040	36 059	36 078	36 097	36 116	36 135	36 154
230	36 173	36 192	36 211	36 229	36 248	36 267	36 286	36 305	36 324	36 342
231	36 361	36 380	36 399	36 418	36 436	36 455	36 474	36 493	36 511	36 530
232	36 549	36 568	36 586	36 605	36 624	36 642	36 661	36 680	36 698	36 717
233	36 736	36 754	36 773	36 791	36 810	36 829	36 847	36 866	36 884	36 903
234	36 922	36 940	36 959	36 977	36 996	37 014	37 033	37 051	37 070	37 088
235	37 107	37 125	37 144	37 162	37 181	37 199	37 218	37 236	37 254	37 273
236	37 291	37 310	37 328	37 346	37 365	37 383	37 401	37 420	37 438	37 457
237	37 475	37 493	37 511	37 530	37 548	37 566	37 585	37 603	37 621	37 639
238	37 658	37 676	37 694	37 712	37 731	37 749	37 767	37 785	37 803	37 822
239	37 840	37 858	37 876	37 894	37 912	37 931	37 949	37 967	37 985	38 003
240	38 021	38 039	38 057	38 075	38 093	38 112	38 130	38 148	38 166	38 184
241	38 202	38 220	38 238	38 256	38 274	38 292	38 310	38 328	38 346	38 364
242	38 382	38 399	38 417	38 435	38 453	38 471	38 489	38 507	38 525	38 543
243	38 561	38 578	38 596	38 614	38 632	38 650	38 668	38 686	38 703	38 721
244	38 739	38 757	38 775	38 792	38 810	38 828	38 846	38 863	38 881	38 899
245	38 917	38 934	38 952	38 970	38 987	39 005	39 023	39 041	39 058	39 076
246	39 094	39 111	39 129	39 146	39 164	39 182	39 199	39 217	39 235	39 252
247	39 270	39 287	39 305	39 322	39 340	39 358	39 375	39 393	39 410	39 428
248	39 445	39 463	39 480	39 498	39 515	39 533	39 550	39 568	39 585	39 602
249	39 620	39 637	39 655	39 672	39 690	39 707	39 724	39 742	39 759	39 777
250	39 794	39 811	39 829	39 846	39 863	39 881	39 898	39 915	39 933	39 950
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251	39 967	39 985	40 002	40 019	40 037	40 054	40 071	40 088	40 106	40 123
252	40 140	40 157	40 175	40 192	40 209	40 226	40 243	40 261	40 278	40 295
253	40 312	40 329	40 346	40 364	40 381	40 398	40 415	40 432	40 449	40 466
254	40 483	40 500	40 518	40 535	40 552	40 569	40 586	40 603	40 620	40 637
255	40 654	40 671	40 688	40 705	40 722	40 739	40 756	40 773	40 790	40 807
256	40 824	40 841	40 858	40 875	40 892	40 909	40 926	40 943	40 960	40 976
257	40 993	41 010	41 027	41 044	41 061	41 078	41 095	41 111	41 128	41 145
258	41 162	41 179	41 196	41 212	41 229	41 246	41 263	41 280	41 296	41 313
259	41 330	41 347	41 363	41 380	41 397	41 414	41 430	41 447	41 464	41 481
260	41 497	41 514	41 531	41 547	41 564	41 581	41 597	41 614	41 631	41 647
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264	42 160	42 177	42 193	42 210	42 226	42 243	42 259	42 275	42 292	42 308
265	42 325	42 341	42 357	42 374	42 390	42 406	42 423	42 439	42 455	42 472
266	42 488	42 504	42 521	42 537	42 553	42 570	42 586	42 602	42 619	42 635
267	42 651	42 667	42 684	42 700	42 716	42 732	42 749	42 765	42 781	42 797
268	42 813	42 830	42 846	42 862	42 878	42 894	42 911	42 927	42 943	42 959
269	42 975	42 991	43 008	43 024	43 040	43 056	43 072	43 088	43 104	43 120
270	43 136	43 152	43 169	43 185	43 201	43 217	43 233	43 249	43 265	43 281
271	43 297	43 313	43 329	43 345	43 361	43 377	43 393	43 409	43 425	43 441
272	43 457	43 473	43 489	43 505	43 521	43 537	43 553	43 569	43 584	43 600
273	43 616	43 632	43 648	43 664	43 680	43 696	43 712	43 727	43 743	43 759
274	43 775	43 791	43 807	43 823	43 838	43 854	43 870	43 886	43 902	43 917
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276	44 091	44 107	44 122	44 138	44 154	44 170	44 185	44 201	44 217	44 232
277	44 248	44 264	44 279	44 295	44 311	44 326	44 342	44 358	44 373	44 389
278	44 404	44 420	44 436	44 451	44 467	44 483	44 498	44 514	44 529	44 545
279	44 560	44 576	44 592	44 607	44 623	44 638	44 654	44 669	44 685	44 700
280	44 716	44 731	44 747	44 762	44 778	44 793	44 809	44 824	44 840	44 855
281	44 871	44 886	44 902	44 917	44 932	44 948	44 963	44 979	44 994	45 010
282	45 025	45 040	45 056	45 071	45 086	45 102	45 117	45 133	45 148	45 163
283	45 179	45 194	45 209	45 225	45 240	45 255	45 271	45 286	45 301	45 317
284	45 332	45 347	45 362	45 378	45 393	45 408	45 423	45 439	45 454	45 469
285	45 484	45 500	45 515	45 530	45 545	45 561	45 576	45 591	45 606	45 621
286	45 637	45 652	45 667	45 682	45 697	45 712	45 728	45 743	45 758	45 773
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291	46 389	46 404	46 419	46 434	46 449	46 464	46 479	46 494	46 509	46 523
292	46 538	46 553	46 568	46 583	46 598	46 613	46 627	46 642	46 657	46 672
293	46 687	46 702	46 716	46 731	46 746	46 761	46 776	46 790	46 805	46 820
294	46 835	46 850	46 864	46 879	46 894	46 909	46 923	46 938	46 953	46 967
295	46 982	46 997	47 012	47 026	47 041	47 056	47 070	47 085	47 100	47 114
296	47 129	47 144	47 159	47 173	47 188	47 202	47 217	47 232	47 246	47 261
297	47 276	47 290	47 305	47 319	47 334	47 349	47 363	47 378	47 392	47 407
298	47 422	47 436	47 451	47 465	47 480	47 494	47 509	47 524	47 538	47 553
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300	47 712	47 727	47 741	47 756	47 770	47 784	47 799	47 813	47 828	47 842
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300-350

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300	47 712	47 727	47 741	47 756	47 770	47 784	47 799	47 813	47 828	47 842
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303	48 144	48 159	48 173	48 187	48 202	48 216	48 230	48 244	48 259	48 273
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311	49 276	49 290	49 304	49 318	49 332	49 346	49 360	49 374	49 388	49 402
312	49 415	49 429	49 443	49 457	49 471	49 485	49 499	49 513	49 527	49 541
313	49 554	49 568	49 582	49 596	49 610	49 624	49 638	49 651	49 665	49 679
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326	51 322	51 335	51 348	51 362	51 375	51 388	51 402	51 415	51 428	51 441
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335	52 504	52 517	52 530	52 543	52 556	52 569	52 582	52 595	52 608	52 621
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337	52 763	52 776	52 789	52 802	52 815	52 827	52 840	52 853	52 866	52 879
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340	53 148	53 161	53 173	53 186	53 199	53 212	53 224	53 237	53 250	53 263
341	53 275	53 288	53 301	53 314	53 326	53 339	53 352	53 364	53 377	53 390
342	53 403	53 415	53 428	53 441	53 453	53 466	53 479	53 491	53 504	53 517
343	53 529	53 542	53 555	53 567	53 580	53 593	53 605	53 618	53 631	53 643
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<b>550</b>	74 036	74 044	74 052	74 060	74 068	74 076	74 084	74 092	74 099	74 107
551	74 115	74 123	74 131	74 139	74 147	74 155	74 162	74 170	74 178	74 186
552	74 194	74 202	74 210	74 218	74 225	74 233	74 241	74 249	74 257	74 265
553	74 273	74 280	74 288	74 296	74 304	74 312	74 320	74 327	74 335	74 343
554	74 351	74 359	74 367	74 374	74 382	74 390	74 398	74 406	74 414	74 421
<b>555</b>	74 429	74 437	74 445	74 453	74 461	74 469	74 476	74 484	74 492	74 500
556	74 507	74 515	74 523	74 531	74 539	74 547	74 554	74 562	74 570	74 578
557	74 586	74 593	74 601	74 609	74 617	74 624	74 632	74 640	74 648	74 656
558	74 663	74 671	74 679	74 687	74 695	74 702	74 710	74 718	74 726	74 733
559	74 741	74 749	74 757	74 764	74 772	74 780	74 788	74 796	74 803	74 811
<b>560</b>	74 819	74 827	74 834	74 842	74 850	74 858	74 865	74 873	74 881	74 889
561	74 896	74 904	74 912	74 920	74 927	74 935	74 943	74 950	74 958	74 966
562	74 974	74 981	74 989	74 997	75 005	75 012	75 020	75 028	75 035	75 043
563	75 051	75 059	75 066	75 074	75 082	75 089	75 097	75 105	75 113	75 120
564	75 128	75 136	75 143	75 151	75 159	75 166	75 174	75 182	75 189	75 197
<b>565</b>	75 205	75 213	75 220	75 228	75 236	75 243	75 251	75 259	75 266	75 274
566	75 282	75 289	75 297	75 305	75 312	75 320	75 328	75 335	75 343	75 351
567	75 358	75 366	75 374	75 381	75 389	75 397	75 404	75 412	75 420	75 427
568	75 435	75 442	75 450	75 458	75 465	75 473	75 481	75 488	75 496	75 504
569	75 511	75 519	75 526	75 534	75 542	75 549	75 557	75 565	75 572	75 580
<b>570</b>	75 587	75 595	75 603	75 610	75 618	75 626	75 633	75 641	75 648	75 656
571	75 664	75 671	75 679	75 686	75 694	75 702	75 709	75 717	75 724	75 732
572	75 740	75 747	75 755	75 762	75 770	75 778	75 785	75 793	75 800	75 808
573	75 815	75 823	75 831	75 838	75 846	75 853	75 861	75 868	75 876	75 884
574	75 891	75 899	75 906	75 914	75 921	75 929	75 937	75 944	75 952	75 959
<b>575</b>	75 967	75 974	75 982	75 989	75 997	76 005	76 012	76 020	76 027	76 035
576	76 042	76 050	76 057	76 065	76 072	76 080	76 087	76 095	76 103	76 110
577	76 118	76 125	76 133	76 140	76 148	76 155	76 163	76 170	76 178	76 185
578	76 193	76 200	76 208	76 215	76 223	76 230	76 238	76 245	76 253	76 260
579	76 268	76 275	76 283	76 290	76 298	76 305	76 313	76 320	76 328	76 335
<b>580</b>	76 343	76 350	76 358	76 365	76 373	76 380	76 388	76 395	76 403	76 410
581	76 418	76 425	76 433	76 440	76 448	76 455	76 462	76 470	76 477	76 485
582	76 492	76 500	76 507	76 515	76 522	76 530	76 537	76 545	76 552	76 559
583	76 567	76 574	76 582	76 589	76 597	76 604	76 612	76 619	76 626	76 634
584	76 641	76 649	76 656	76 664	76 671	76 678	76 686	76 693	76 701	76 708
<b>585</b>	76 716	76 723	76 730	76 738	76 745	76 753	76 760	76 768	76 775	76 782
586	76 790	76 797	76 805	76 812	76 819	76 827	76 834	76 842	76 849	76 856
587	76 864	76 871	76 879	76 886	76 893	76 901	76 908	76 916	76 923	76 930
588	76 938	76 945	76 953	76 960	76 967	76 975	76 982	76 989	76 997	77 004
589	77 012	77 019	77 026	77 034	77 041	77 048	77 056	77 063	77 070	77 078
<b>590</b>	77 085	77 093	77 100	77 107	77 115	77 122	77 129	77 137	77 144	77 151
591	77 159	77 166	77 173	77 181	77 188	77 195	77 203	77 210	77 217	77 225
592	77 232	77 240	77 247	77 254	77 262	77 269	77 276	77 283	77 291	77 298
593	77 305	77 313	77 320	77 327	77 335	77 342	77 349	77 357	77 364	77 371
594	77 379	77 386	77 393	77 401	77 408	77 415	77 422	77 430	77 437	77 444
<b>595</b>	77 452	77 459	77 466	77 474	77 481	77 488	77 495	77 503	77 510	77 517
596	77 525	77 532	77 539	77 546	77 554	77 561	77 568	77 576	77 583	77 590
597	77 597	77 605	77 612	77 619	77 627	77 634	77 641	77 648	77 656	77 663
598	77 670	77 677	77 685	77 692	77 699	77 706	77 714	77 721	77 728	77 735
599	77 743	77 750	77 757	77 764	77 772	77 779	77 786	77 793	77 801	77 808
<b>600</b>	77 815	77 822	77 830	77 837	77 844	77 851	77 859	77 866	77 873	77 880
N	0	1	2	3	4	5	6	7	8	9

600-650

N	0	1	2	3	4	5	6	7	8	9
600	77 815	77 822	77 830	77 837	77 844	77 851	77 859	77 866	77 873	77 880
601	77 887	77 893	77 902	77 909	77 916	77 924	77 931	77 938	77 945	77 952
602	77 960	77 967	77 974	77 981	77 988	77 996	78 003	78 010	78 017	78 025
603	78 032	78 039	78 046	78 053	78 061	78 068	78 075	78 082	78 089	78 097
604	78 104	78 111	78 118	78 125	78 132	78 140	78 147	78 154	78 161	78 168
605	78 176	78 183	78 190	78 197	78 204	78 211	78 219	78 226	78 233	78 240
606	78 247	78 254	78 262	78 269	78 276	78 283	78 290	78 297	78 305	78 312
607	78 319	78 326	78 333	78 340	78 347	78 355	78 362	78 369	78 376	78 383
608	78 390	78 398	78 405	78 412	78 419	78 426	78 433	78 440	78 447	78 455
609	78 462	78 469	78 476	78 483	78 490	78 497	78 504	78 512	78 519	78 526
610	78 533	78 540	78 547	78 554	78 561	78 569	78 576	78 583	78 590	78 597
611	78 604	78 611	78 618	78 625	78 633	78 640	78 647	78 654	78 661	78 668
612	78 675	78 682	78 689	78 696	78 704	78 711	78 718	78 725	78 732	78 739
613	78 746	78 753	78 760	78 767	78 774	78 781	78 789	78 796	78 803	78 810
614	78 817	78 824	78 831	78 838	78 845	78 852	78 859	78 866	78 873	78 880
615	78 888	78 895	78 902	78 909	78 916	78 923	78 930	78 937	78 944	78 951
616	78 958	78 965	78 972	78 979	78 986	78 993	79 000	79 007	79 014	79 021
617	79 029	79 036	79 043	79 050	79 057	79 064	79 071	79 078	79 085	79 092
618	79 099	79 106	79 113	79 120	79 127	79 134	79 141	79 148	79 155	79 162
619	79 169	79 176	79 183	79 190	79 197	79 204	79 211	79 218	79 225	79 232
620	79 239	79 246	79 253	79 260	79 267	79 274	79 281	79 288	79 295	79 302
621	79 309	79 316	79 323	79 330	79 337	79 344	79 351	79 358	79 365	79 372
622	79 379	79 386	79 393	79 400	79 407	79 414	79 421	79 428	79 435	79 442
623	79 449	79 456	79 463	79 470	79 477	79 484	79 491	79 498	79 505	79 511
624	79 518	79 525	79 532	79 539	79 546	79 553	79 560	79 567	79 574	79 581
625	79 588	79 595	79 602	79 609	79 616	79 623	79 630	79 637	79 644	79 650
626	79 657	79 664	79 671	79 678	79 685	79 692	79 699	79 706	79 713	79 720
627	79 727	79 734	79 741	79 748	79 754	79 761	79 768	79 775	79 782	79 789
628	79 796	79 803	79 810	79 817	79 824	79 831	79 837	79 844	79 851	79 858
629	79 865	79 872	79 879	79 886	79 893	79 900	79 906	79 913	79 920	79 927
630	79 934	79 941	79 948	79 955	79 962	79 969	79 975	79 982	79 989	79 996
631	80 003	80 010	80 017	80 024	80 030	80 037	80 044	80 051	80 058	80 065
632	80 072	80 079	80 085	80 092	80 099	80 106	80 113	80 120	80 127	80 134
633	80 140	80 147	80 154	80 161	80 168	80 175	80 182	80 188	80 195	80 202
634	80 209	80 216	80 223	80 229	80 236	80 243	80 250	80 257	80 264	80 271
635	80 277	80 284	80 291	80 298	80 305	80 312	80 318	80 325	80 332	80 339
636	80 346	80 353	80 359	80 366	80 373	80 380	80 387	80 393	80 400	80 407
637	80 414	80 421	80 428	80 434	80 441	80 448	80 455	80 462	80 468	80 475
638	80 482	80 489	80 496	80 502	80 509	80 516	80 523	80 530	80 536	80 543
639	80 550	80 557	80 564	80 570	80 577	80 584	80 591	80 598	80 604	80 611
640	80 618	80 625	80 632	80 638	80 645	80 652	80 659	80 665	80 672	80 679
641	80 686	80 693	80 699	80 706	80 713	80 720	80 726	80 733	80 740	80 747
642	80 754	80 760	80 767	80 774	80 781	80 787	80 794	80 801	80 808	80 814
643	80 821	80 828	80 835	80 841	80 848	80 855	80 862	80 868	80 875	80 882
644	80 889	80 895	80 902	80 909	80 916	80 922	80 929	80 936	80 943	80 949
645	80 956	80 963	80 969	80 976	80 983	80 990	80 996	81 003	81 010	81 017
646	81 023	81 030	81 037	81 043	81 050	81 057	81 064	81 070	81 077	81 084
647	81 090	81 097	81 104	81 111	81 117	81 124	81 131	81 137	81 144	81 151
648	81 158	81 164	81 171	81 178	81 184	81 191	81 198	81 204	81 211	81 218
649	81 224	81 231	81 238	81 245	81 251	81 258	81 265	81 271	81 278	81 285
650	81 291	81 298	81 305	81 311	81 318	81 325	81 331	81 338	81 345	81 351
N	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
650	81 291	81 298	81 305	81 311	81 318	81 325	81 331	81 338	81 345	81 351
651	81 358	81 365	81 371	81 378	81 385	81 391	81 398	81 405	81 411	81 418
652	81 425	81 431	81 438	81 445	81 451	81 458	81 465	81 471	81 478	81 485
653	81 491	81 498	81 505	81 511	81 518	81 525	81 531	81 538	81 544	81 551
654	81 558	81 564	81 571	81 578	81 584	81 591	81 598	81 604	81 611	81 617
655	81 624	81 631	81 637	81 644	81 651	81 657	81 664	81 671	81 677	81 684
656	81 690	81 697	81 704	81 710	81 717	81 723	81 730	81 737	81 743	81 750
657	81 757	81 763	81 770	81 776	81 783	81 790	81 796	81 803	81 809	81 816
658	81 823	81 829	81 836	81 842	81 849	81 856	81 862	81 869	81 875	81 882
659	81 889	81 895	81 902	81 908	81 915	81 921	81 928	81 935	81 941	81 948
660	81 954	81 961	81 968	81 974	81 981	81 987	81 994	82 000	82 007	82 014
661	82 020	82 027	82 033	82 040	82 046	82 053	82 060	82 066	82 073	82 079
662	82 086	82 092	82 099	82 105	82 112	82 119	82 125	82 132	82 138	82 145
663	82 151	82 158	82 164	82 171	82 178	82 184	82 191	82 197	82 204	82 210
664	82 217	82 223	82 230	82 236	82 243	82 249	82 256	82 263	82 269	82 276
665	82 282	82 289	82 295	82 302	82 308	82 315	82 321	82 328	82 334	82 341
666	82 347	82 354	82 360	82 367	82 373	82 380	82 387	82 393	82 400	82 406
667	82 413	82 419	82 426	82 432	82 439	82 445	82 452	82 458	82 465	82 471
668	82 478	82 484	82 491	82 497	82 504	82 510	82 517	82 523	82 530	82 536
669	82 543	82 549	82 555	82 562	82 569	82 575	82 582	82 588	82 595	82 601
670	82 607	82 614	82 620	82 627	82 633	82 640	82 646	82 653	82 659	82 666
671	82 672	82 679	82 685	82 692	82 698	82 705	82 711	82 718	82 724	82 730
672	82 737	82 743	82 750	82 756	82 763	82 769	82 776	82 782	82 789	82 795
673	82 802	82 808	82 814	82 821	82 827	82 834	82 840	82 847	82 853	82 860
674	82 866	82 872	82 879	82 885	82 892	82 898	82 905	82 911	82 918	82 924
675	82 930	82 937	82 943	82 950	82 956	82 963	82 969	82 975	82 982	82 988
676	82 995	83 001	83 008	83 014	83 020	83 027	83 033	83 040	83 046	83 052
677	83 059	83 065	83 072	83 078	83 085	83 091	83 097	83 104	83 110	83 117
678	83 123	83 129	83 136	83 142	83 149	83 155	83 161	83 168	83 174	83 181
679	83 187	83 193	83 200	83 206	83 213	83 219	83 225	83 232	83 238	83 245
680	83 251	83 257	83 264	83 270	83 276	83 283	83 289	83 296	83 302	83 308
681	83 315	83 321	83 327	83 334	83 340	83 347	83 353	83 359	83 366	83 372
682	83 378	83 385	83 391	83 398	83 404	83 410	83 417	83 423	83 429	83 436
683	83 442	83 448	83 455	83 461	83 467	83 474	83 480	83 487	83 493	83 499
684	83 506	83 512	83 518	83 525	83 531	83 537	83 544	83 550	83 556	83 563
685	83 569	83 575	83 582	83 588	83 594	83 601	83 607	83 613	83 620	83 626
686	83 632	83 639	83 645	83 651	83 658	83 664	83 670	83 677	83 683	83 689
687	83 696	83 702	83 708	83 715	83 721	83 727	83 734	83 740	83 746	83 753
688	83 759	83 765	83 771	83 778	83 784	83 790	83 797	83 803	83 809	83 816
689	83 822	83 828	83 835	83 841	83 847	83 853	83 860	83 866	83 872	83 879
690	83 885	83 891	83 897	83 904	83 910	83 916	83 923	83 929	83 935	83 942
691	83 948	83 954	83 960	83 967	83 973	83 979	83 985	83 992	83 998	84 004
692	84 011	84 017	84 023	84 029	84 036	84 042	84 048	84 055	84 061	84 067
693	84 073	84 080	84 086	84 092	84 098	84 105	84 111	84 117	84 123	84 130
694	84 136	84 142	84 148	84 155	84 161	84 167	84 173	84 180	84 186	84 192
695	84 198	84 205	84 211	84 217	84 223	84 230	84 236	84 242	84 248	84 255
696	84 261	84 267	84 273	84 280	84 286	84 292	84 298	84 305	84 311	84 317
697	84 323	84 330	84 336	84 342	84 348	84 354	84 361	84 367	84 373	84 379
698	84 386	84 392	84 398	84 404	84 410	84 417	84 423	84 429	84 435	84 442
699	84 448	84 454	84 460	84 466	84 473	84 479	84 485	84 491	84 497	84 504
700	84 510	84 516	84 522	84 528	84 535	84 541	84 547	84 553	84 559	84 566
N	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
700	84 510	84 516	84 522	84 528	84 535	84 541	84 547	84 553	84 559	84 566
701	84 572	84 578	84 584	84 590	84 597	84 603	84 609	84 615	84 621	84 628
702	84 634	84 640	84 646	84 652	84 658	84 665	84 671	84 677	84 683	84 689
703	84 696	84 702	84 708	84 714	84 720	84 726	84 733	84 739	84 745	84 751
704	84 757	84 763	84 770	84 776	84 782	84 788	84 794	84 800	84 807	84 813
705	84 819	84 825	84 831	84 837	84 844	84 850	84 856	84 862	84 868	84 874
706	84 880	84 887	84 893	84 899	84 905	84 911	84 917	84 924	84 930	84 936
707	84 942	84 948	84 954	84 960	84 967	84 973	84 979	84 985	84 991	84 997
708	85 003	85 009	85 016	85 022	85 028	85 034	85 040	85 046	85 052	85 058
709	85 065	85 071	85 077	85 083	85 089	85 095	85 101	85 107	85 114	85 120
710	85 126	85 132	85 138	85 144	85 150	85 156	85 163	85 169	85 175	85 181
711	85 187	85 193	85 199	85 205	85 211	85 217	85 224	85 230	85 236	85 242
712	85 248	85 254	85 260	85 266	85 272	85 278	85 285	85 291	85 297	85 303
713	85 309	85 315	85 321	85 327	85 333	85 339	85 345	85 352	85 358	85 364
714	85 370	85 376	85 382	85 388	85 394	85 400	85 406	85 412	85 418	85 425
715	85 431	85 437	85 443	85 449	85 455	85 461	85 467	85 473	85 479	85 485
716	85 491	85 497	85 503	85 509	85 516	85 522	85 528	85 534	85 540	85 546
717	85 552	85 558	85 564	85 570	85 576	85 582	85 588	85 594	85 600	85 606
718	85 612	85 618	85 625	85 631	85 637	85 643	85 649	85 655	85 661	85 667
719	85 673	85 679	85 685	85 691	85 697	85 703	85 709	85 715	85 721	85 727
720	85 733	85 739	85 745	85 751	85 757	85 763	85 769	85 775	85 781	85 788
721	85 794	85 800	85 806	85 812	85 818	85 824	85 830	85 836	85 842	85 848
722	85 854	85 860	85 866	85 872	85 878	85 884	85 890	85 896	85 902	85 908
723	85 914	85 920	85 926	85 932	85 938	85 944	85 950	85 956	85 962	85 968
724	85 974	85 980	85 986	85 992	85 998	86 004	86 010	86 016	86 022	86 028
725	86 034	86 040	86 046	86 052	86 058	86 064	86 070	86 076	86 082	86 088
726	86 094	86 100	86 106	86 112	86 118	86 124	86 130	86 136	86 141	86 147
727	86 153	86 159	86 165	86 171	86 177	86 183	86 189	86 195	86 201	86 207
728	86 213	86 219	86 225	86 231	86 237	86 243	86 249	86 255	86 261	86 267
729	86 273	86 279	86 285	86 291	86 297	86 303	86 308	86 314	86 320	86 326
730	86 332	86 338	86 344	86 350	86 356	86 362	86 368	86 374	86 380	86 386
731	86 392	86 398	86 404	86 410	86 415	86 421	86 427	86 433	86 439	86 445
732	86 451	86 457	86 463	86 469	86 475	86 481	86 487	86 493	86 499	86 504
733	86 510	86 516	86 522	86 528	86 534	86 540	86 546	86 552	86 558	86 564
734	86 570	86 576	86 581	86 587	86 593	86 599	86 605	86 611	86 617	86 623
735	86 629	86 635	86 641	86 646	86 652	86 658	86 664	86 670	86 676	86 682
736	86 688	86 694	86 700	86 705	86 711	86 717	86 723	86 729	86 735	86 741
737	86 747	86 753	86 759	86 764	86 770	86 776	86 782	86 788	86 794	86 800
738	86 806	86 812	86 817	86 823	86 829	86 835	86 841	86 847	86 853	86 859
739	86 864	86 870	86 876	86 882	86 888	86 894	86 900	86 906	86 911	86 917
740	86 923	86 929	86 935	86 941	86 947	86 953	86 958	86 964	86 970	86 976
741	86 982	86 988	86 994	86 999	87 005	87 011	87 017	87 023	87 029	87 035
742	87 040	87 046	87 052	87 058	87 064	87 070	87 075	87 081	87 087	87 093
743	87 099	87 105	87 111	87 116	87 122	87 128	87 134	87 140	87 146	87 151
744	87 157	87 163	87 169	87 175	87 181	87 186	87 192	87 198	87 204	87 210
745	87 216	87 221	87 227	87 233	87 239	87 245	87 251	87 256	87 262	87 268
746	87 274	87 280	87 286	87 291	87 297	87 303	87 309	87 315	87 320	87 326
747	87 332	87 338	87 344	87 349	87 355	87 361	87 367	87 373	87 379	87 384
748	87 390	87 396	87 402	87 408	87 413	87 419	87 425	87 431	87 437	87 442
749	87 448	87 454	87 460	87 466	87 471	87 477	87 483	87 489	87 495	87 500
750	87 506	87 512	87 518	87 523	87 529	87 535	87 541	87 547	87 552	87 558
N	0	1	2	3	4	5	6	7	8	9



N	0	1	2	3	4	5	6	7	8	9
750	87 506	87 512	87 518	87 523	87 529	87 535	87 541	87 547	87 552	87 558
751	87 564	87 570	87 576	87 581	87 587	87 593	87 599	87 604	87 610	87 616
752	87 622	87 628	87 633	87 639	87 645	87 651	87 656	87 662	87 668	87 674
753	87 679	87 685	87 691	87 697	87 703	87 708	87 714	87 720	87 726	87 731
754	87 737	87 743	87 749	87 754	87 760	87 766	87 772	87 777	87 783	87 789
755	87 795	87 800	87 806	87 812	87 818	87 823	87 829	87 835	87 841	87 846
756	87 852	87 858	87 864	87 869	87 875	87 881	87 887	87 892	87 898	87 904
757	87 910	87 915	87 921	87 927	87 933	87 938	87 944	87 950	87 955	87 961
758	87 967	87 973	87 978	87 984	87 990	87 996	88 001	88 007	88 013	88 018
759	88 024	88 030	88 036	88 041	88 047	88 053	88 058	88 064	88 070	88 076
760	88 081	88 087	88 093	88 098	88 104	88 110	88 116	88 121	88 127	88 133
761	88 138	88 144	88 150	88 156	88 161	88 167	88 173	88 178	88 184	88 190
762	88 195	88 201	88 207	88 213	88 218	88 224	88 230	88 235	88 241	88 247
763	88 252	88 258	88 264	88 270	88 275	88 281	88 287	88 292	88 298	88 304
764	88 309	88 315	88 321	88 326	88 332	88 338	88 343	88 349	88 355	88 360
765	88 366	88 372	88 377	88 383	88 389	88 395	88 400	88 406	88 412	88 417
766	88 423	88 429	88 434	88 440	88 446	88 451	88 457	88 463	88 468	88 474
767	88 480	88 485	88 491	88 497	88 502	88 508	88 513	88 519	88 525	88 530
768	88 536	88 542	88 547	88 553	88 559	88 564	88 570	88 576	88 581	88 587
769	88 593	88 598	88 604	88 610	88 615	88 621	88 627	88 632	88 638	88 643
770	88 649	88 655	88 660	88 666	88 672	88 677	88 683	88 689	88 694	88 700
771	88 705	88 711	88 717	88 722	88 728	88 734	88 739	88 745	88 750	88 756
772	88 762	88 767	88 773	88 779	88 784	88 790	88 795	88 801	88 807	88 812
773	88 818	88 824	88 829	88 835	88 840	88 846	88 852	88 857	88 863	88 868
774	88 874	88 880	88 885	88 891	88 897	88 902	88 908	88 913	88 919	88 925
775	88 930	88 936	88 941	88 947	88 953	88 958	88 964	88 969	88 975	88 981
776	88 986	88 992	88 997	89 003	89 009	89 014	89 020	89 025	89 031	89 037
777	89 042	89 048	89 053	89 059	89 064	89 070	89 076	89 081	89 087	89 092
778	89 098	89 104	89 109	89 115	89 120	89 126	89 131	89 137	89 143	89 148
779	89 154	89 159	89 165	89 170	89 176	89 182	89 187	89 193	89 198	89 204
780	89 209	89 215	89 221	89 226	89 232	89 237	89 243	89 248	89 254	89 260
781	89 265	89 271	89 276	89 282	89 287	89 293	89 298	89 304	89 310	89 315
782	89 321	89 326	89 332	89 337	89 343	89 348	89 354	89 360	89 365	89 371
783	89 376	89 382	89 387	89 393	89 398	89 404	89 409	89 415	89 421	89 426
784	89 432	89 437	89 443	89 448	89 454	89 459	89 465	89 470	89 476	89 481
785	89 487	89 492	89 498	89 504	89 509	89 515	89 520	89 526	89 531	89 537
786	89 542	89 548	89 553	89 559	89 564	89 570	89 575	89 581	89 586	89 592
787	89 597	89 603	89 609	89 614	89 620	89 625	89 631	89 636	89 642	89 647
788	89 653	89 658	89 664	89 669	89 675	89 680	89 686	89 691	89 697	89 702
789	89 708	89 713	89 719	89 724	89 730	89 735	89 741	89 746	89 752	89 757
790	89 763	89 768	89 774	89 779	89 785	89 790	89 796	89 801	89 807	89 812
791	89 818	89 823	89 829	89 834	89 840	89 845	89 851	89 856	89 862	89 867
792	89 873	89 878	89 883	89 889	89 894	89 900	89 905	89 911	89 916	89 922
793	89 927	89 933	89 938	89 944	89 949	89 955	89 960	89 966	89 971	89 977
794	89 982	89 988	89 993	89 998	90 004	90 009	90 015	90 020	90 026	90 031
795	90 037	90 042	90 048	90 053	90 059	90 064	90 069	90 075	90 080	90 086
796	90 091	90 097	90 102	90 108	90 113	90 119	90 124	90 129	90 135	90 140
797	90 146	90 151	90 157	90 162	90 168	90 173	90 179	90 184	90 189	90 195
798	90 200	90 206	90 211	90 217	90 222	90 227	90 233	90 238	90 244	90 249
799	90 255	90 260	90 266	90 271	90 276	90 282	90 287	90 293	90 298	90 304
800	90 309	90 314	90 320	90 325	90 331	90 336	90 342	90 347	90 352	90 358
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<b>800</b>	90 309	90 314	90 320	90 325	90 331	90 336	90 342	90 347	90 352	90 358
801	90 363	90 369	90 374	90 380	90 385	90 390	90 396	90 401	90 407	90 412
802	90 417	90 423	90 428	90 434	90 439	90 445	90 450	90 455	90 461	90 466
803	90 472	90 477	90 482	90 488	90 493	90 499	90 504	90 509	90 515	90 520
804	90 526	90 531	90 536	90 542	90 547	90 553	90 558	90 563	90 569	90 574
<b>805</b>	90 580	90 585	90 590	90 596	90 601	90 607	90 612	90 617	90 623	90 628
806	90 634	90 639	90 644	90 650	90 655	90 660	90 666	90 671	90 677	90 682
807	90 687	90 693	90 698	90 703	90 709	90 714	90 720	90 725	90 730	90 736
808	90 741	90 747	90 752	90 757	90 763	90 768	90 773	90 779	90 784	90 789
809	90 795	90 800	90 806	90 811	90 816	90 822	90 827	90 832	90 838	90 843
<b>810</b>	90 849	90 854	90 859	90 865	90 870	90 875	90 881	90 886	90 891	90 897
811	90 902	90 907	90 913	90 918	90 924	90 929	90 934	90 940	90 945	90 950
812	90 956	90 961	90 966	90 972	90 977	90 982	90 988	90 993	90 998	91 004
813	91 009	91 014	91 020	91 025	91 030	91 036	91 041	91 046	91 052	91 057
814	91 062	91 068	91 073	91 078	91 034	91 089	91 094	91 100	91 105	91 110
<b>815</b>	91 116	91 121	91 126	91 132	91 137	91 142	91 148	91 153	91 158	91 164
816	91 169	91 174	91 180	91 185	91 190	91 196	91 201	91 206	91 212	91 217
817	91 222	91 228	91 233	91 238	91 243	91 249	91 254	91 259	91 265	91 270
818	91 275	91 281	91 286	91 291	91 297	91 302	91 307	91 312	91 318	91 323
819	91 328	91 334	91 339	91 344	91 350	91 355	91 360	91 365	91 371	91 376
<b>820</b>	91 381	91 387	91 392	91 397	91 403	91 408	91 413	91 418	91 424	91 429
821	91 434	91 440	91 445	91 450	91 455	91 461	91 466	91 471	91 477	91 482
822	91 487	91 492	91 498	91 503	91 508	91 514	91 519	91 524	91 529	91 535
823	91 540	91 545	91 551	91 556	91 561	91 566	91 572	91 577	91 582	91 587
824	91 593	91 598	91 605	91 609	91 614	91 619	91 624	91 630	91 635	91 640
<b>825</b>	91 645	91 651	91 656	91 661	91 666	91 672	91 677	91 682	91 687	91 693
826	91 698	91 703	91 709	91 714	91 719	91 724	91 730	91 735	91 740	91 745
827	91 751	91 756	91 761	91 766	91 772	91 777	91 782	91 787	91 793	91 798
828	91 803	91 808	91 814	91 819	91 824	91 829	91 834	91 840	91 845	91 850
829	91 855	91 861	91 866	91 871	91 876	91 882	91 887	91 892	91 897	91 903
<b>830</b>	91 908	91 913	91 918	91 924	91 929	91 934	91 939	91 944	91 950	91 955
831	91 960	91 965	91 971	91 976	91 981	91 986	91 991	91 997	92 002	92 007
832	92 012	92 018	92 023	92 028	92 033	92 038	92 044	92 049	92 054	92 059
833	92 065	92 070	92 075	92 080	92 085	92 091	92 096	92 101	92 106	92 111
834	92 117	92 122	92 127	92 132	92 137	92 143	92 148	92 153	92 158	92 163
<b>835</b>	92 169	92 174	92 179	92 184	92 189	92 195	92 200	92 205	92 210	92 215
836	92 221	92 226	92 231	92 236	92 241	92 247	92 252	92 257	92 262	92 267
837	92 273	92 278	92 283	92 288	92 293	92 298	92 304	92 309	92 314	92 319
838	92 324	92 330	92 335	92 340	92 345	92 350	92 355	92 361	92 366	92 371
839	92 376	92 381	92 387	92 392	92 397	92 402	92 407	92 412	92 418	92 423
<b>840</b>	92 428	92 433	92 438	92 443	92 449	92 454	92 459	92 464	92 469	92 474
841	92 480	92 485	92 490	92 495	92 500	92 505	92 511	92 516	92 521	92 526
842	92 531	92 536	92 542	92 547	92 552	92 557	92 562	92 567	92 572	92 578
843	92 583	92 588	92 593	92 598	92 603	92 609	92 614	92 619	92 624	92 629
844	92 634	92 639	92 645	92 650	92 655	92 660	92 665	92 670	92 675	92 681
<b>845</b>	92 686	92 691	92 696	92 701	92 706	92 711	92 716	92 722	92 727	92 732
846	92 737	92 742	92 747	92 752	92 758	92 763	92 768	92 773	92 778	92 783
847	92 788	92 793	92 799	92 804	92 809	92 814	92 819	92 824	92 829	92 834
848	92 840	92 845	92 850	92 855	92 860	92 865	92 870	92 875	92 881	92 886
849	92 891	92 896	92 901	92 906	92 911	92 916	92 921	92 927	92 932	92 937
<b>850</b>	92 942	92 947	92 952	92 957	92 962	92 967	92 973	92 978	92 983	92 988
<b>N</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>

N	0	1	2	3	4	5	6	7	8	9
850	92 942	92 947	92 952	92 957	92 962	92 967	92 973	92 978	92 983	92 988
851	92 993	92 998	93 003	93 008	93 013	93 018	93 024	93 029	93 034	93 039
852	93 044	93 049	93 054	93 059	93 064	93 069	93 075	93 080	93 085	93 090
853	93 095	93 100	93 105	93 110	93 115	93 120	93 125	93 131	93 136	93 141
854	93 146	93 151	93 156	93 161	93 166	93 171	93 176	93 181	93 186	93 192
855	93 197	93 202	93 207	93 212	93 217	93 222	93 227	93 232	93 237	93 242
856	93 247	93 252	93 258	93 263	93 268	93 273	93 278	93 283	93 288	93 293
857	93 298	93 303	93 308	93 313	93 318	93 323	93 328	93 334	93 339	93 344
858	93 349	93 354	93 359	93 364	93 369	93 374	93 379	93 384	93 389	93 394
859	93 399	93 404	93 409	93 414	93 420	93 425	93 430	93 435	93 440	93 445
860	93 450	93 455	93 460	93 465	93 470	93 475	93 480	93 485	93 490	93 495
861	93 500	93 505	93 510	93 515	93 520	93 526	93 531	93 536	93 541	93 546
862	93 551	93 556	93 561	93 566	93 571	93 576	93 581	93 586	93 591	93 596
863	93 601	93 606	93 611	93 616	93 621	93 626	93 631	93 636	93 641	93 646
864	93 651	93 656	93 661	93 666	93 671	93 676	93 682	93 687	93 692	93 697
865	93 702	93 707	93 712	93 717	93 722	93 727	93 732	93 737	93 742	93 747
866	93 752	93 757	93 762	93 767	93 772	93 777	93 782	93 787	93 792	93 797
867	93 802	93 807	93 812	93 817	93 822	93 827	93 832	93 837	93 842	93 847
868	93 852	93 857	93 862	93 867	93 872	93 877	93 882	93 887	93 892	93 897
869	93 902	93 907	93 912	93 917	93 922	93 927	93 932	93 937	93 942	93 947
870	93 952	93 957	93 962	93 967	93 972	93 977	93 982	93 987	93 992	93 997
871	94 002	94 007	94 012	94 017	94 022	94 027	94 032	94 037	94 042	94 047
872	94 052	94 057	94 062	94 067	94 072	94 077	94 082	94 086	94 091	94 096
873	94 101	94 106	94 111	94 116	94 121	94 126	94 131	94 136	94 141	94 146
874	94 151	94 156	94 161	94 166	94 171	94 176	94 181	94 186	94 191	94 196
875	94 201	94 206	94 211	94 216	94 221	94 226	94 231	94 236	94 240	94 245
876	94 250	94 255	94 260	94 265	94 270	94 275	94 280	94 285	94 290	94 295
877	94 300	94 305	94 310	94 315	94 320	94 325	94 330	94 335	94 340	94 345
878	94 349	94 354	94 359	94 364	94 369	94 374	94 379	94 384	94 389	94 394
879	94 399	94 404	94 409	94 414	94 419	94 424	94 429	94 433	94 438	94 443
880	94 448	94 453	94 458	94 463	94 468	94 473	94 478	94 483	94 488	94 493
881	94 498	94 503	94 507	94 512	94 517	94 522	94 527	94 532	94 537	94 542
882	94 547	94 552	94 557	94 562	94 567	94 571	94 576	94 581	94 586	94 591
883	94 596	94 601	94 606	94 611	94 616	94 621	94 626	94 630	94 635	94 640
884	94 645	94 650	94 655	94 660	94 665	94 670	94 675	94 680	94 685	94 689
885	94 694	94 699	94 704	94 709	94 714	94 719	94 724	94 729	94 734	94 738
886	94 743	94 748	94 753	94 758	94 763	94 768	94 773	94 778	94 783	94 787
887	94 792	94 797	94 802	94 807	94 812	94 817	94 822	94 827	94 832	94 836
888	94 841	94 846	94 851	94 856	94 861	94 866	94 871	94 876	94 880	94 885
889	94 890	94 895	94 900	94 905	94 910	94 915	94 919	94 924	94 929	94 934
890	94 939	94 944	94 949	94 954	94 959	94 963	94 968	94 973	94 978	94 983
891	94 988	94 993	94 998	95 002	95 007	95 012	95 017	95 022	95 027	95 032
892	95 036	95 041	95 046	95 051	95 056	95 061	95 066	95 071	95 075	95 080
893	95 085	95 090	95 095	95 100	95 105	95 109	95 114	95 119	95 124	95 129
894	95 134	95 139	95 143	95 148	95 153	95 158	95 163	95 168	95 173	95 177
895	95 182	95 187	95 192	95 197	95 202	95 207	95 211	95 216	95 221	95 226
896	95 231	95 236	95 240	95 245	95 250	95 255	95 260	95 265	95 270	95 274
897	95 279	95 284	95 289	95 294	95 299	95 303	95 308	95 313	95 318	95 323
898	95 328	95 332	95 337	95 342	95 347	95 352	95 357	95 361	95 366	95 371
899	95 376	95 381	95 386	95 390	95 395	95 400	95 405	95 410	95 415	95 419
900	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468
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N	0	1	2	3	4	5	6	7	8	9
000	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468
901	95 472	95 477	95 482	95 487	95 492	95 497	95 501	95 506	95 511	95 516
902	95 521	95 525	95 530	95 535	95 540	95 545	95 550	95 554	95 559	95 564
903	95 569	95 574	95 578	95 583	95 588	95 593	95 598	95 602	95 607	95 612
904	95 617	95 622	95 626	95 631	95 636	95 641	95 646	95 650	95 655	95 660
005	95 665	95 670	95 674	95 679	95 684	95 689	95 694	95 698	95 703	95 708
906	95 713	95 718	95 722	95 727	95 732	95 737	95 742	95 746	95 751	95 756
907	95 761	95 766	95 770	95 775	95 780	95 785	95 789	95 794	95 799	95 804
908	95 809	95 813	95 818	95 823	95 828	95 832	95 837	95 842	95 847	95 852
909	95 856	95 861	95 866	95 871	95 875	95 880	95 885	95 890	95 895	95 899
010	95 904	95 909	95 914	95 918	95 923	95 928	95 933	95 938	95 942	95 947
911	95 952	95 957	95 961	95 966	95 971	95 976	95 980	95 985	95 990	95 995
912	95 999	96 004	96 009	96 014	96 019	96 023	96 028	96 033	96 038	96 042
913	96 047	96 052	96 057	96 061	96 066	96 071	96 076	96 080	96 085	96 090
914	96 095	96 099	96 104	96 109	96 114	96 118	96 123	96 128	96 133	96 137
015	96 142	96 147	96 152	96 156	96 161	96 166	96 171	96 175	96 180	96 185
916	96 190	96 194	96 199	96 204	96 209	96 213	96 218	96 223	96 227	96 232
917	96 237	96 242	96 246	96 251	96 256	96 261	96 265	96 270	96 275	96 280
918	96 284	96 289	96 294	96 298	96 303	96 308	96 313	96 317	96 322	96 327
919	96 332	96 336	96 341	96 346	96 350	96 355	96 360	96 365	96 369	96 374
020	96 379	96 384	96 388	96 393	96 398	96 402	96 407	96 412	96 417	96 421
921	96 426	96 431	96 435	96 440	96 445	96 450	96 454	96 459	96 464	96 468
922	96 473	96 478	96 483	96 487	96 492	96 497	96 501	96 506	96 511	96 515
923	96 520	96 525	96 530	96 534	96 539	96 544	96 548	96 553	96 558	96 562
924	96 567	96 572	96 577	96 581	96 586	96 591	96 595	96 600	96 605	96 609
025	96 614	96 619	96 624	96 628	96 633	96 638	96 642	96 647	96 652	96 656
926	96 661	96 666	96 670	96 675	96 680	96 685	96 689	96 694	96 699	96 703
927	96 708	96 713	96 717	96 722	96 727	96 731	96 736	96 741	96 745	96 750
928	96 755	96 759	96 764	96 769	96 774	96 778	96 783	96 788	96 792	96 797
929	96 802	96 806	96 811	96 816	96 820	96 825	96 830	96 834	96 839	96 844
030	96 848	96 853	96 858	96 862	96 867	96 872	96 876	96 881	96 886	96 890
931	96 895	96 900	96 904	96 909	96 914	96 918	96 923	96 928	96 932	96 937
932	96 942	96 946	96 951	96 956	96 960	96 965	96 970	96 974	96 979	96 984
933	96 988	96 993	96 997	97 002	97 007	97 011	97 016	97 021	97 025	97 030
934	97 035	97 039	97 044	97 049	97 053	97 058	97 063	97 067	97 072	97 077
035	97 081	97 086	97 090	97 095	97 100	97 104	97 109	97 114	97 118	97 123
936	97 128	97 132	97 137	97 142	97 146	97 151	97 155	97 160	97 165	97 169
937	97 174	97 179	97 183	97 188	97 192	97 197	97 202	97 206	97 211	97 216
938	97 220	97 225	97 230	97 234	97 239	97 243	97 248	97 253	97 257	97 262
939	97 267	97 271	97 276	97 280	97 285	97 290	97 294	97 299	97 304	97 308
040	97 313	97 317	97 322	97 327	97 331	97 336	97 340	97 345	97 350	97 354
941	97 359	97 364	97 368	97 373	97 377	97 382	97 387	97 391	97 396	97 400
942	97 405	97 410	97 414	97 419	97 424	97 428	97 433	97 437	97 442	97 447
943	97 451	97 456	97 460	97 465	97 470	97 474	97 479	97 483	97 488	97 493
944	97 497	97 502	97 506	97 511	97 516	97 520	97 525	97 529	97 534	97 539
045	97 543	97 548	97 552	97 557	97 562	97 566	97 571	97 575	97 580	97 585
946	97 589	97 594	97 598	97 603	97 607	97 612	97 617	97 621	97 626	97 630
947	97 635	97 640	97 644	97 649	97 653	97 658	97 663	97 667	97 672	97 676
948	97 681	97 685	97 690	97 695	97 699	97 704	97 708	97 713	97 717	97 722
949	97 727	97 731	97 736	97 740	97 745	97 749	97 754	97 759	97 763	97 768
050	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813
N	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
<b>950</b>	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813
951	97 818	97 823	97 827	97 832	97 836	97 841	97 845	97 850	97 855	97 859
952	97 864	97 868	97 873	97 877	97 882	97 886	97 891	97 896	97 900	97 905
953	97 909	97 914	97 918	97 923	97 923	97 932	97 937	97 941	97 946	97 950
954	97 955	97 959	97 964	97 968	97 973	97 978	97 982	97 987	97 991	97 996
<b>955</b>	98 000	98 005	98 009	98 014	98 019	98 023	98 028	98 032	98 037	98 041
956	98 046	98 050	98 055	98 059	98 064	98 068	98 073	98 078	98 082	98 087
957	98 091	98 096	98 100	98 105	98 109	98 114	98 118	98 123	98 127	98 132
958	98 137	98 141	98 146	98 150	98 155	98 159	98 164	98 168	98 173	98 177
959	98 182	98 186	98 191	98 195	98 200	98 204	98 209	98 214	98 218	98 223
<b>960</b>	98 227	98 232	98 236	98 241	98 245	98 250	98 254	98 259	98 263	98 268
961	98 272	98 277	98 281	98 286	98 290	98 295	98 299	98 304	98 308	98 313
962	98 318	98 322	98 327	98 331	98 336	98 340	98 345	98 349	98 354	98 358
963	98 363	98 367	98 372	98 376	98 381	98 385	98 390	98 394	98 399	98 403
964	98 408	98 412	98 417	98 421	98 426	98 430	98 435	98 439	98 444	98 448
<b>965</b>	98 453	98 457	98 462	98 466	98 471	98 475	98 480	98 484	98 489	98 493
966	98 498	98 502	98 507	98 511	98 516	98 520	98 525	98 529	98 534	98 538
967	98 543	98 547	98 552	98 556	98 561	98 565	98 570	98 574	98 579	98 583
968	98 588	98 592	98 597	98 601	98 605	98 610	98 614	98 619	98 623	98 628
969	98 632	98 637	98 641	98 646	98 650	98 655	98 659	98 664	98 668	98 673
<b>970</b>	98 677	98 682	98 686	98 691	98 695	98 700	98 704	98 709	98 713	98 717
971	98 722	98 726	98 731	98 735	98 740	98 744	98 749	98 753	98 758	98 762
972	98 767	98 771	98 776	98 780	98 784	98 789	98 793	98 798	98 802	98 807
973	98 811	98 816	98 820	98 825	98 829	98 834	98 838	98 843	98 847	98 851
974	98 856	98 860	98 865	98 869	98 874	98 878	98 883	98 887	98 892	98 896
<b>975</b>	98 900	98 905	98 909	98 914	98 918	98 923	98 927	98 932	98 936	98 941
976	98 945	98 949	98 954	98 958	98 963	98 967	98 972	98 976	98 981	98 985
977	98 989	98 994	98 998	99 003	99 007	99 012	99 016	99 021	99 025	99 029
978	99 034	99 038	99 043	99 047	99 052	99 056	99 061	99 065	99 069	99 074
979	99 078	99 083	99 087	99 092	99 096	99 100	99 105	99 109	99 114	99 118
<b>980</b>	99 123	99 127	99 131	99 136	99 140	99 145	99 149	99 154	99 158	99 162
981	99 167	99 171	99 176	99 180	99 185	99 189	99 193	99 198	99 202	99 207
982	99 211	99 216	99 220	99 224	99 229	99 233	99 238	99 242	99 247	99 251
983	99 255	99 260	99 264	99 269	99 273	99 277	99 282	99 286	99 291	99 295
984	99 300	99 304	99 308	99 313	99 317	99 322	99 326	99 330	99 335	99 339
<b>985</b>	99 344	99 348	99 352	99 357	99 361	99 366	99 370	99 374	99 379	99 383
986	99 388	99 392	99 396	99 401	99 405	99 410	99 414	99 419	99 423	99 427
987	99 432	99 436	99 441	99 445	99 449	99 454	99 458	99 463	99 467	99 471
988	99 476	99 480	99 484	99 489	99 493	99 498	99 502	99 506	99 511	99 515
989	99 520	99 524	99 528	99 533	99 537	99 542	99 546	99 550	99 555	99 559
<b>990</b>	99 564	99 568	99 572	99 577	99 581	99 585	99 590	99 594	99 599	99 603
991	99 607	99 612	99 616	99 621	99 625	99 629	99 634	99 638	99 642	99 647
992	99 651	99 656	99 660	99 664	99 669	99 673	99 677	99 682	99 686	99 691
993	99 695	99 699	99 704	99 708	99 712	99 717	99 721	99 726	99 730	99 734
994	99 739	99 743	99 747	99 752	99 756	99 760	99 765	99 769	99 774	99 778
<b>995</b>	99 782	99 787	99 791	99 795	99 800	99 804	99 808	99 813	99 817	99 822
996	99 826	99 830	99 835	99 839	99 843	99 848	99 852	99 856	99 861	99 865
997	99 870	99 874	99 878	99 883	99 887	99 891	99 896	99 900	99 904	99 909
998	99 913	99 917	99 922	99 926	99 930	99 935	99 939	99 944	99 948	99 952
999	99 957	99 961	99 965	99 970	99 974	99 978	99 983	99 987	99 991	99 996
<b>1000</b>	00 000	00 004	00 009	00 013	00 017	00 022	00 026	00 030	00 035	00 039
N	0	1	2	3	4	5	6	7	8	9

TABLE II. — LOGARITHMS OF CONSTANTS.

Circumference of the Circle in degrees..... = 360 Circumference of the Circle in minutes..... = 21 600 Circumference of the Circle in seconds..... = 1 296 000 If the radius $r = 1$ , half the Circumference of the Circle is $\pi = 3.14 159 265 358 979 323 846 264 338 328$ .....			log 2.55 630 250 4.33 445 375 6.11 260 500 0.49 714 987
Also : $2\pi = 6.28 318 531$ $4\pi = 12.56 637 061$ $\frac{\pi}{2} = 1.57 079 633$ $\frac{\pi}{3} = 1.04 719 755$ $\frac{4\pi}{3} = 4.18 879 020$ $\frac{\pi}{4} = 0.78 539 816$ $\frac{\pi}{6} = 0.52 359 878$ $\frac{1}{2}\pi = 0.31 830 989$ $\frac{1}{2\pi} = 0.15 915 494$ $\frac{3}{2\pi} = 0.95 492 966$ $\frac{4}{\pi} = 1.27 323 954$ $\frac{8}{4\pi} = 0.23 873 241$	log 0.79 817 987 1.09 920 986 0.19 611 938 0.02 002 862 0.62 263 861 9.89 508 988 - 10 9.71 899 862 - 10 9.50 285 013 - 10 9.20 182 013 - 10 9.97 997 138 - 10 0.10 491 012 9.37 791 139 - 10	$\pi^2 = 9.86 960 440$ $\frac{1}{\pi^2} = 0.10 132 118$ $\sqrt{\pi} = 1.77 245 385$ $\frac{1}{\sqrt{\pi}} = 0.56 418 958$ $\sqrt{\frac{\pi}{2}} = 0.97 720 502$ $\sqrt{\frac{2}{\pi}} = 1.12 837 917$ $\sqrt[3]{\pi} = 1.46 459 189$ $\frac{1}{\sqrt[3]{\pi}} = 0.68 278 406$ $\sqrt[3]{\pi^2} = 2.14 502 940$ $\frac{2}{\sqrt[3]{\pi}} = 0.62 035 049$ $\frac{8}{\sqrt[3]{\pi}} = 0.80 599 598$	log 0.99 429 975 9.00 570 025 - 10 0.24 857 494 9.75 142 506 - 10 9.98 998 569 - 10 0.05 245 506 0.16 571 662 9.83 428 338 - 10 0.33 143 325 9.79 263 713 - 10 9.90 633 287 - 10
Arc $a$ , whose length is equal to the radius $r$ , is : in degrees ..... $a^\circ = \frac{180}{\pi} = 57.29 577 951^\circ$ in minutes ..... $a' = \frac{10 800}{\pi} = 3 437.74 677'$ in seconds ..... $a'' = \frac{648 000}{\pi} = 206 264.806''$			log 1.75 812 263 3.53 627 388 5.31 442 513
Arc $2a$ , whose length is equal to twice the radius, $2r$ , is : in degrees ..... $2a^\circ = \frac{360}{\pi} = 114.59 155 903^\circ$ in minutes ..... $2a' = \frac{21 600}{\pi} = 6 875.49 354'$ in seconds ..... $2a'' = \frac{1 296 000}{\pi} = 412 529.612''$			log 2.05 915 263 3.83 730 388 5.61 545 513
If the radius $r = 1$ , the length of the arc is : for 1 degree ..... $\frac{1}{a^\circ} = \frac{\pi}{180} = 0.01 745 329$ for 1 minute ..... $\frac{1}{a'} = \frac{\pi}{10 800} = 0.00 029 089$ for 1 second ..... $\frac{1}{a''} = \frac{\pi}{648 000} = 0.00 000 485$ for $\frac{1}{2}$ degree ..... $\frac{1}{2a^\circ} = \frac{\pi}{360} = 0.00 872 665$ for $\frac{1}{2}$ minute ..... $\frac{1}{2a'} = \frac{\pi}{21 600} = 0.00 014 544$ for $\frac{1}{2}$ second ..... $\frac{1}{2a''} = \frac{\pi}{1 296 000} = 0.00 000 242$			log 8.24 187 737 - 10 6.46 372 612 - 10 4.68 557 487 - 10 7.94 084 737 - 10 6.16 269 612 - 10 4.38 454 487 - 10
Sin $1''$ in the unit circle ..... = 0.00 000 485...			4.68 557 487 - 10

TABLE III.

THE LOGARITHMS

OF TAN

TRIGONOMETRIC FUNCTIONS:

From 0° to 0° 3', or 89° 57' to 90°, for every second;  
 From 0° to 2°, or 88° to 90°, for every ten seconds;  
 From 1° to 89°, for every minute.

NOTE. To all the logarithms -10 is to be appended.

log sin				0°		log tan = log sin log cos = 10.00000			
"	0'	1'	2'	"	"	0'	1'	2'	"
0	—	6.46373	6.76476	80	30	6.16270	6.63982	6.86167	80
1	4.68557	6.47090	6.76836	59	31	6.17694	6.61462	6.86455	29
2	4.98660	6.47797	6.77193	58	32	6.19072	6.61936	6.86742	28
3	5.16270	6.48492	6.77548	57	33	6.20409	6.65406	6.87027	27
4	5.28763	6.49175	6.77900	56	34	6.21705	6.65870	6.87310	26
5	5.38454	6.49849	6.78248	55	35	6.22964	6.66330	6.87591	25
6	5.46373	6.50512	6.78595	54	36	6.24153	6.66785	6.87870	24
7	5.53067	6.51165	6.78938	53	37	6.25378	6.67235	6.88147	23
8	5.58866	6.51808	6.79278	52	38	6.26536	6.67680	6.88423	22
9	5.63982	6.52442	6.79616	51	39	6.27664	6.68121	6.88697	21
10	5.68557	6.53067	6.79952	50	40	6.28763	6.68557	6.88969	20
11	5.72697	6.53683	6.80285	49	41	6.29836	6.68990	6.89240	19
12	5.76476	6.54291	6.80615	48	42	6.30882	6.69418	6.89509	18
13	5.79952	6.54890	6.80943	47	43	6.31904	6.69841	6.89776	17
14	5.83170	6.55481	6.81268	46	44	6.32903	6.70261	6.90042	16
15	5.86167	6.56064	6.81591	45	45	6.33879	6.70676	6.90306	15
16	5.88969	6.56639	6.81911	44	46	6.34833	6.71088	6.90568	14
17	5.91602	6.57207	6.82230	43	47	6.35767	6.71496	6.90829	13
18	5.94085	6.57767	6.82545	42	48	6.36682	6.71900	6.91088	12
19	5.96433	6.58320	6.82859	41	49	6.37577	6.72300	6.91346	11
20	5.98660	6.58866	6.83170	40	50	6.38454	6.72697	6.91602	10
21	6.00779	6.59406	6.83479	39	51	6.39315	6.73090	6.91857	9
22	6.02800	6.59939	6.83786	38	52	6.40158	6.73479	6.92110	8
23	6.04730	6.60465	6.84091	37	53	6.40985	6.73865	6.92362	7
24	6.06579	6.60983	6.84394	36	54	6.41797	6.74248	6.92612	6
25	6.08351	6.61499	6.84694	35	55	6.42594	6.74627	6.92861	5
26	6.10055	6.62007	6.84993	34	56	6.43376	6.75003	6.93109	4
27	6.11694	6.62509	6.85289	33	57	6.44145	6.75376	6.93355	3
28	6.13273	6.63006	6.85584	32	58	6.44900	6.75746	6.93599	2
29	6.14797	6.63496	6.85876	31	59	6.45643	6.76112	6.93843	1
30	6.16270	6.63982	6.86167	30	60	6.46373	6.76476	6.94085	0
"	89'	88'	87'	"	"	89'	88'	87'	"

log cot = log cos  
log sin = 10.00000

89°

log cos

°	log sin	log tan	log cos	°	°	log sin	log tan	log cos	°
0	—	—	10.00000	0.60	100	7.46373	7.46373	10.00000	0.50
10	5.68557	5.68557	10.00000	50	10	7.47090	7.47091	10.00000	50
20	5.98660	5.98660	10.00000	40	20	7.47797	7.47797	10.00000	40
30	6.16270	6.16270	10.00000	30	30	7.48491	7.48492	10.00000	30
40	6.28763	6.28763	10.00000	20	40	7.49175	7.49176	10.00000	20
50	6.38454	6.38454	10.00000	10	50	7.49849	7.49849	10.00000	10
1	6.46373	6.46373	10.00000	0.50	110	7.50512	7.50512	10.00000	0.49
10	6.53067	6.53067	10.00000	50	10	7.51165	7.51165	10.00000	50
20	6.58866	6.58866	10.00000	40	20	7.51808	7.51809	10.00000	40
30	6.63982	6.63982	10.00000	30	30	7.52442	7.52443	10.00000	30
40	6.68557	6.68557	10.00000	20	40	7.53067	7.53067	10.00000	20
50	6.72697	6.72697	10.00000	10	50	7.53683	7.53683	10.00000	10
2	6.76476	6.76476	10.00000	0.59	120	7.54291	7.54291	10.00000	0.48
10	6.79952	6.79952	10.00000	50	10	7.54890	7.54890	10.00000	50
20	6.83170	6.83170	10.00000	40	20	7.55481	7.55481	10.00000	40
30	6.86167	6.86167	10.00000	30	30	7.56064	7.56064	10.00000	30
40	6.88969	6.88969	10.00000	20	40	7.56639	7.56639	10.00000	20
50	6.91602	6.91602	10.00000	10	50	7.57206	7.57207	10.00000	10
3	6.94085	6.94085	10.00000	0.57	130	7.57767	7.57767	10.00000	0.47
10	6.96433	6.96433	10.00000	50	10	7.58320	7.58320	10.00000	50
20	6.98660	6.98661	10.00000	40	20	7.58866	7.58867	10.00000	40
30	7.00779	7.00779	10.00000	30	30	7.59406	7.59406	10.00000	30
40	7.02800	7.02800	10.00000	20	40	7.59939	7.59939	10.00000	20
50	7.04730	7.04730	10.00000	10	50	7.60465	7.60466	10.00000	10
4	7.06579	7.06579	10.00000	0.56	140	7.60985	7.60986	10.00000	0.46
10	7.08351	7.08352	10.00000	50	10	7.61499	7.61500	10.00000	50
20	7.10055	7.10055	10.00000	40	20	7.62007	7.62008	10.00000	40
30	7.11694	7.11694	10.00000	30	30	7.62509	7.62510	10.00000	30
40	7.13273	7.13273	10.00000	20	40	7.63006	7.63006	10.00000	20
50	7.14797	7.14797	10.00000	10	50	7.63496	7.63497	10.00000	10
5	7.16270	7.16270	10.00000	0.55	150	7.63982	7.63982	10.00000	0.45
10	7.17694	7.17694	10.00000	50	10	7.64461	7.64462	10.00000	50
20	7.19072	7.19073	10.00000	40	20	7.64936	7.64937	10.00000	40
30	7.20409	7.20409	10.00000	30	30	7.65406	7.65406	10.00000	30
40	7.21705	7.21705	10.00000	20	40	7.65870	7.65871	10.00000	20
50	7.22964	7.22964	10.00000	10	50	7.66330	7.66330	10.00000	10
6	7.24188	7.24188	10.00000	0.54	160	7.66784	7.66785	10.00000	0.44
10	7.25378	7.25378	10.00000	50	10	7.67235	7.67235	10.00000	50
20	7.26536	7.26536	10.00000	40	20	7.67680	7.67680	10.00000	40
30	7.27664	7.27664	10.00000	30	30	7.68121	7.68121	10.00000	30
40	7.28763	7.28764	10.00000	20	40	7.68557	7.68558	9.99999	20
50	7.29836	7.29836	10.00000	10	50	7.68989	7.68990	9.99999	10
7	7.30882	7.30882	10.00000	0.53	170	7.69417	7.69418	9.99999	0.43
10	7.31904	7.31904	10.00000	50	10	7.69841	7.69842	9.99999	50
20	7.32903	7.32903	10.00000	40	20	7.70261	7.70261	9.99999	40
30	7.33879	7.33879	10.00000	30	30	7.70676	7.70677	9.99999	30
40	7.34833	7.34833	10.00000	20	40	7.71088	7.71088	9.99999	20
50	7.35767	7.35767	10.00000	10	50	7.71496	7.71496	9.99999	10
8	7.36682	7.36682	10.00000	0.52	180	7.71900	7.71900	9.99999	0.42
10	7.37577	7.37577	10.00000	50	10	7.72300	7.72301	9.99999	50
20	7.38454	7.38455	10.00000	40	20	7.72697	7.72697	9.99999	40
30	7.39314	7.39315	10.00000	30	30	7.73090	7.73090	9.99999	30
40	7.40158	7.40158	10.00000	20	40	7.73479	7.73480	9.99999	20
50	7.40985	7.40985	10.00000	10	50	7.73865	7.73866	9.99999	10
9	7.41797	7.41797	10.00000	0.51	190	7.74248	7.74248	9.99999	0.41
10	7.42594	7.42594	10.00000	50	10	7.74627	7.74628	9.99999	50
20	7.43376	7.43376	10.00000	40	20	7.75003	7.75004	9.99999	40
30	7.44145	7.44145	10.00000	30	30	7.75376	7.75377	9.99999	30
40	7.44900	7.44900	10.00000	20	40	7.75745	7.75746	9.99999	20
50	7.45643	7.45643	10.00000	10	50	7.76112	7.76113	9.99999	10
100	7.46373	7.46373	10.00000	0.50	200	7.76475	7.76476	9.99999	0.40
°	log cos	log cot	log sin	°	°	log cos	log cot	log sin	°

°	log sin	log tan	log cos	°	°	log sin	log tan	log cos	°
200	7.76475	7.76476	9.99999	040	300	7.94034	7.94056	9.99998	030
10	7.76836	7.76837	9.99999	50	10	7.94325	7.94326	9.99998	50
20	7.77193	7.77194	9.99999	40	20	7.94564	7.94566	9.99998	40
30	7.77549	7.77549	9.99999	30	30	7.94802	7.94804	9.99998	30
40	7.77899	7.77900	9.99999	20	40	7.95039	7.95040	9.99998	20
50	7.78248	7.78249	9.99999	10	50	7.95274	7.95276	9.99998	10
210	7.78594	7.78595	9.99999	030	310	7.95508	7.95510	9.99998	020
10	7.78938	7.78938	9.99999	50	10	7.95741	7.95743	9.99998	50
20	7.79278	7.79279	9.99999	40	20	7.95973	7.95974	9.99998	40
30	7.79616	7.79617	9.99999	30	30	7.96203	7.96205	9.99998	30
40	7.79952	7.79952	9.99999	20	40	7.96432	7.96434	9.99998	20
50	7.80284	7.80285	9.99999	10	50	7.96660	7.96662	9.99998	10
220	7.80615	7.80615	9.99999	038	320	7.96887	7.96889	9.99998	028
10	7.80942	7.80943	9.99999	50	10	7.97113	7.97114	9.99998	50
20	7.81268	7.81269	9.99999	40	20	7.97337	7.97339	9.99998	40
30	7.81591	7.81591	9.99999	30	30	7.97560	7.97562	9.99998	30
40	7.81911	7.81912	9.99999	20	40	7.97782	7.97784	9.99998	20
50	7.82229	7.82230	9.99999	10	50	7.98003	7.98005	9.99998	10
230	7.82545	7.82546	9.99999	037	330	7.98223	7.98225	9.99998	027
10	7.82859	7.82860	9.99999	50	10	7.98442	7.98444	9.99998	50
20	7.83170	7.83171	9.99999	40	20	7.98660	7.98662	9.99998	40
30	7.83479	7.83480	9.99999	30	30	7.98876	7.98878	9.99998	30
40	7.83786	7.83787	9.99999	20	40	7.99092	7.99094	9.99998	20
50	7.84091	7.84092	9.99999	10	50	7.99305	7.99308	9.99998	10
240	7.84393	7.84394	9.99999	036	340	7.99520	7.99522	9.99998	026
10	7.84694	7.84695	9.99999	50	10	7.99732	7.99734	9.99998	50
20	7.84992	7.84994	9.99999	40	20	7.99943	7.99946	9.99998	40
30	7.85289	7.85290	9.99999	30	30	8.00154	8.00156	9.99998	30
40	7.85583	7.85584	9.99999	20	40	8.00363	8.00365	9.99998	20
50	7.85876	7.85877	9.99999	10	50	8.00571	8.00574	9.99998	10
250	7.86166	7.86167	9.99999	035	350	8.00779	8.00781	9.99998	025
10	7.86455	7.86456	9.99999	50	10	8.00985	8.00987	9.99998	50
20	7.86741	7.86743	9.99999	40	20	8.01190	8.01193	9.99998	40
30	7.87025	7.87027	9.99999	30	30	8.01395	8.01397	9.99998	30
40	7.87309	7.87310	9.99999	20	40	8.01598	8.01600	9.99998	20
50	7.87590	7.87591	9.99999	10	50	8.01801	8.01803	9.99998	10
260	7.87870	7.87871	9.99999	034	360	8.02002	8.02004	9.99998	024
10	7.88147	7.88148	9.99999	50	10	8.02203	8.02205	9.99998	50
20	7.88423	7.88424	9.99999	40	20	8.02402	8.02405	9.99998	40
30	7.88697	7.88698	9.99999	30	30	8.02601	8.02604	9.99998	30
40	7.88969	7.88970	9.99999	20	40	8.02799	8.02801	9.99998	20
50	7.89240	7.89241	9.99999	10	50	8.02996	8.02998	9.99998	10
270	7.89509	7.89510	9.99999	033	370	8.03192	8.03194	9.99997	023
10	7.89776	7.89777	9.99999	50	10	8.03387	8.03390	9.99997	50
20	7.90041	7.90043	9.99999	40	20	8.03581	8.03584	9.99997	40
30	7.90305	7.90307	9.99999	30	30	8.03775	8.03777	9.99997	30
40	7.90568	7.90569	9.99999	20	40	8.03967	8.03970	9.99997	20
50	7.90829	7.90830	9.99999	10	50	8.04159	8.04162	9.99997	10
280	7.91088	7.91089	9.99999	032	380	8.04350	8.04353	9.99997	022
10	7.91346	7.91347	9.99999	50	10	8.04540	8.04543	9.99997	50
20	7.91602	7.91603	9.99999	40	20	8.04729	8.04732	9.99997	40
30	7.91857	7.91858	9.99999	30	30	8.04918	8.04921	9.99997	30
40	7.92110	7.92111	9.99998	20	40	8.05105	8.05108	9.99997	20
50	7.92362	7.92363	9.99998	10	50	8.05292	8.05295	9.99997	10
290	7.92612	7.92613	9.99998	031	390	8.05478	8.05481	9.99997	021
10	7.92861	7.92862	9.99998	50	10	8.05663	8.05666	9.99997	50
20	7.93108	7.93110	9.99998	40	20	8.05848	8.05851	9.99997	40
30	7.93354	7.93356	9.99998	30	30	8.06031	8.06034	9.99997	30
40	7.93599	7.93601	9.99998	20	40	8.06214	8.06217	9.99997	20
50	7.93842	7.93844	9.99998	10	50	8.06396	8.06399	9.99997	10
300	7.94084	7.94085	9.99998	030	400	8.06578	8.06581	9.99997	020
°	log cos	log cot	log sin	°	°	log cos	log cot	log sin	°



°	log sin	log tan	log cos	°	°	log sin	log tan	log cos	°
400	8.06578	8.06551	9.99997	020	500	8.16268	8.16273	9.99995	010
10	8.06738	8.06761	9.99997	50	10	8.16413	8.16417	9.99995	50
20	8.06938	8.06941	9.99997	40	20	8.16557	8.16561	9.99995	40
30	8.07117	8.07120	9.99997	30	30	8.16700	8.16705	9.99995	30
40	8.07295	8.07299	9.99997	20	40	8.16843	8.16848	9.99995	20
50	8.07473	8.07476	9.99997	10	50	8.16986	8.16991	9.99995	10
410	8.07650	8.07653	9.99997	010	510	8.17128	8.17133	9.99995	00
10	8.07826	8.07829	9.99997	50	10	8.17270	8.17275	9.99995	50
20	8.08002	8.08005	9.99997	40	20	8.17411	8.17416	9.99995	40
30	8.08176	8.08180	9.99997	30	30	8.17552	8.17557	9.99995	30
40	8.08350	8.08354	9.99997	20	40	8.17692	8.17697	9.99995	20
50	8.08524	8.08527	9.99997	10	50	8.17832	8.17837	9.99995	10
420	8.08696	8.08700	9.99997	018	520	8.17971	8.17976	9.99995	08
10	8.08868	8.08872	9.99997	50	10	8.18110	8.18115	9.99995	50
20	8.09040	8.09043	9.99997	40	20	8.18249	8.18254	9.99995	40
30	8.09210	8.09214	9.99997	30	30	8.18387	8.18392	9.99995	30
40	8.09380	8.09384	9.99997	20	40	8.18524	8.18530	9.99995	20
50	8.09550	8.09553	9.99997	10	50	8.18662	8.18667	9.99995	10
430	8.09718	8.09722	9.99997	017	530	8.18798	8.18804	9.99995	07
10	8.09886	8.09890	9.99997	50	10	8.18935	8.18940	9.99995	50
20	8.10054	8.10057	9.99997	40	20	8.19071	8.19076	9.99995	40
30	8.10220	8.10224	9.99997	30	30	8.19206	8.19212	9.99995	30
40	8.10386	8.10390	9.99997	20	40	8.19341	8.19347	9.99995	20
50	8.10552	8.10555	9.99996	10	50	8.19476	8.19481	9.99995	10
440	8.10717	8.10720	9.99996	016	540	8.19610	8.19616	9.99995	06
10	8.10881	8.10884	9.99996	50	10	8.19744	8.19749	9.99995	50
20	8.11044	8.11048	9.99996	40	20	8.19877	8.19883	9.99995	40
30	8.11207	8.11211	9.99996	30	30	8.20010	8.20016	9.99995	30
40	8.11370	8.11373	9.99996	20	40	8.20143	8.20149	9.99995	20
50	8.11531	8.11535	9.99996	10	50	8.20275	8.20281	9.99994	10
450	8.11693	8.11696	9.99996	015	550	8.20407	8.20413	9.99994	05
10	8.11853	8.11857	9.99996	50	10	8.20538	8.20544	9.99994	50
20	8.12013	8.12017	9.99996	40	20	8.20669	8.20675	9.99994	40
30	8.12172	8.12176	9.99996	30	30	8.20800	8.20806	9.99994	30
40	8.12331	8.12335	9.99996	20	40	8.20930	8.20936	9.99994	20
50	8.12489	8.12493	9.99996	10	50	8.21060	8.21066	9.99994	10
460	8.12647	8.12651	9.99996	014	560	8.21189	8.21195	9.99994	04
10	8.12804	8.12808	9.99996	50	10	8.21319	8.21324	9.99994	50
20	8.12961	8.12965	9.99996	40	20	8.21447	8.21453	9.99994	40
30	8.13117	8.13121	9.99996	30	30	8.21576	8.21581	9.99994	30
40	8.13272	8.13276	9.99996	20	40	8.21703	8.21709	9.99994	20
50	8.13427	8.13431	9.99996	10	50	8.21831	8.21837	9.99994	10
470	8.13581	8.13585	9.99996	013	570	8.21958	8.21964	9.99994	03
10	8.13735	8.13739	9.99996	50	10	8.22085	8.22091	9.99994	50
20	8.13888	8.13892	9.99996	40	20	8.22211	8.22217	9.99994	40
30	8.14041	8.14045	9.99996	30	30	8.22337	8.22343	9.99994	30
40	8.14193	8.14197	9.99996	20	40	8.22463	8.22469	9.99994	20
50	8.14344	8.14348	9.99996	10	50	8.22588	8.22595	9.99994	10
480	8.14495	8.14500	9.99996	012	580	8.22713	8.22720	9.99994	02
10	8.14646	8.14650	9.99996	50	10	8.22838	8.22844	9.99994	50
20	8.14796	8.14800	9.99996	40	20	8.22962	8.22968	9.99994	40
30	8.14945	8.14950	9.99996	30	30	8.23086	8.23092	9.99994	30
40	8.15094	8.15099	9.99996	20	40	8.23210	8.23216	9.99994	20
50	8.15243	8.15247	9.99996	10	50	8.23333	8.23339	9.99994	10
490	8.15391	8.15395	9.99996	011	590	8.23456	8.23462	9.99994	01
10	8.15538	8.15543	9.99996	50	10	8.23578	8.23585	9.99994	50
20	8.15685	8.15690	9.99996	40	20	8.23700	8.23707	9.99994	40
30	8.15832	8.15836	9.99996	30	30	8.23822	8.23829	9.99993	30
40	8.15978	8.15982	9.99995	20	40	8.23944	8.23950	9.99993	20
50	8.16123	8.16128	9.99995	10	50	8.24065	8.24071	9.99993	10
500	8.16268	8.16273	9.99995	010	600	8.24186	8.24192	9.99993	00
°	log cos	log cot	log sin	°	°	log cos	log cot	log sin	°

i	log sin	log tan	log cos	i	i	log sin	log tan	log cos	i
0 0	8.24186	8.24192	9.99993	0 60	100	8.30879	8.50883	9.99991	0 50
10	8.24306	8.24313	9.99993	50	10	8.30983	8.30992	9.99991	50
20	8.24426	8.24433	9.99993	40	20	8.31086	8.31095	9.99991	40
30	8.24546	8.24553	9.99993	30	30	8.31189	8.31198	9.99991	30
40	8.24665	8.24672	9.99993	20	40	8.31291	8.31300	9.99991	20
50	8.24785	8.24791	9.99993	10	50	8.31393	8.31403	9.99991	10
1 0	8.24903	8.24910	9.99993	0 59	110	8.31495	8.31505	9.99991	0 49
10	8.25023	8.25029	9.99993	50	10	8.31597	8.31606	9.99991	50
20	8.25140	8.25147	9.99993	40	20	8.31699	8.31708	9.99991	40
30	8.25258	8.25265	9.99993	30	30	8.31800	8.31809	9.99991	30
40	8.25375	8.25382	9.99993	20	40	8.31901	8.31911	9.99991	20
50	8.25493	8.25500	9.99993	10	50	8.32002	8.32012	9.99991	10
2 0	8.25609	8.25616	9.99993	0 58	120	8.32103	8.32112	9.99990	0 48
10	8.25726	8.25733	9.99993	50	10	8.32203	8.32213	9.99990	50
20	8.25842	8.25849	9.99993	40	20	8.32303	8.32313	9.99990	40
30	8.25958	8.25965	9.99993	30	30	8.32403	8.32413	9.99990	30
40	8.26074	8.26081	9.99993	20	40	8.32503	8.32513	9.99990	20
50	8.26189	8.26196	9.99993	10	50	8.32602	8.32612	9.99990	10
3 0	8.26304	8.26312	9.99993	0 57	130	8.32702	8.32711	9.99990	0 47
10	8.26419	8.26426	9.99993	50	10	8.32801	8.32811	9.99990	50
20	8.26533	8.26541	9.99993	40	20	8.32899	8.32909	9.99990	40
30	8.26648	8.26655	9.99993	30	30	8.32998	8.33008	9.99990	30
40	8.26761	8.26769	9.99993	20	40	8.33096	8.33106	9.99990	20
50	8.26875	8.26882	9.99993	10	50	8.33195	8.33205	9.99990	10
4 0	8.26988	8.26996	9.99992	0 56	140	8.33292	8.33302	9.99990	0 46
10	8.27101	8.27109	9.99992	50	10	8.33390	8.33400	9.99990	50
20	8.27214	8.27221	9.99992	40	20	8.33488	8.33498	9.99990	40
30	8.27326	8.27334	9.99992	30	30	8.33585	8.33595	9.99990	30
40	8.27438	8.27446	9.99992	20	40	8.33682	8.33692	9.99990	20
50	8.27550	8.27558	9.99992	10	50	8.33779	8.33789	9.99990	10
5 0	8.27661	8.27669	9.99992	0 55	150	8.33875	8.33886	9.99990	0 45
10	8.27773	8.27780	9.99992	50	10	8.33972	8.33982	9.99990	50
20	8.27883	8.27891	9.99992	40	20	8.34068	8.34078	9.99990	40
30	8.27994	8.28002	9.99992	30	30	8.34164	8.34174	9.99990	30
40	8.28104	8.28112	9.99992	20	40	8.34260	8.34270	9.99989	20
50	8.28215	8.28223	9.99992	10	50	8.34355	8.34366	9.99989	10
6 0	8.28324	8.28332	9.99992	0 54	160	8.34450	8.34461	9.99989	0 44
10	8.28434	8.28442	9.99992	50	10	8.34546	8.34556	9.99989	50
20	8.28543	8.28551	9.99992	40	20	8.34640	8.34651	9.99989	40
30	8.28652	8.28660	9.99992	30	30	8.34735	8.34746	9.99989	30
40	8.28761	8.28769	9.99992	20	40	8.34830	8.34840	9.99989	20
50	8.28869	8.28877	9.99992	10	50	8.34924	8.34935	9.99989	10
7 0	8.28977	8.28986	9.99992	0 53	170	8.35018	8.35029	9.99989	0 43
10	8.29085	8.29094	9.99992	50	10	8.35112	8.35123	9.99989	50
20	8.29193	8.29201	9.99992	40	20	8.35206	8.35217	9.99989	40
30	8.29300	8.29309	9.99992	30	30	8.35299	8.35310	9.99989	30
40	8.29407	8.29416	9.99992	20	40	8.35392	8.35403	9.99989	20
50	8.29514	8.29523	9.99992	10	50	8.35485	8.35497	9.99989	10
8 0	8.29621	8.29629	9.99992	0 52	180	8.35578	8.35590	9.99989	0 42
10	8.29727	8.29736	9.99991	50	10	8.35671	8.35682	9.99989	50
20	8.29833	8.29842	9.99991	40	20	8.35764	8.35775	9.99989	40
30	8.29939	8.29947	9.99991	30	30	8.35856	8.35867	9.99989	30
40	8.30044	8.30053	9.99991	20	40	8.35949	8.35959	9.99989	20
50	8.30150	8.30158	9.99991	10	50	8.36040	8.36051	9.99989	10
9 0	8.30255	8.30263	9.99991	0 51	190	8.36131	8.36143	9.99989	0 41
10	8.30359	8.30368	9.99991	50	10	8.36223	8.36235	9.99988	50
20	8.30464	8.30473	9.99991	40	20	8.36314	8.36326	9.99988	40
30	8.30568	8.30577	9.99991	30	30	8.36405	8.36417	9.99988	30
40	8.30672	8.30681	9.99991	20	40	8.36496	8.36508	9.99988	20
50	8.30776	8.30785	9.99991	10	50	8.36587	8.36599	9.99988	10
100	8.30879	8.30883	9.99991	0 50	200	8.36678	8.36689	9.99988	0 40
1 1 1	log cos	log cot	log sin	1 1 1	1 1 1	log cos	log cot	log sin	1 1 1

°	log sin	log tan	log cos	°	°	log sin	log tan	log cos	°
200	8.36678	8.36689	9.99988	040	300	8.41792	8.41807	9.99985	080
10	8.36768	8.36780	9.99988	50	10	8.41872	8.41887	9.99985	50
20	8.36858	8.36870	9.99988	40	20	8.41952	8.41967	9.99985	40
30	8.36948	8.36960	9.99988	30	30	8.42032	8.42048	9.99985	30
40	8.37038	8.37050	9.99988	20	40	8.42112	8.42127	9.99985	20
50	8.37128	8.37140	9.99988	10	50	8.42192	8.42207	9.99985	10
210	8.37217	8.37229	9.99988	089	810	8.42272	8.42287	9.99985	029
10	8.37306	8.37318	9.99988	50	10	8.42351	8.42366	9.99985	50
20	8.37395	8.37408	9.99988	40	20	8.42430	8.42446	9.99985	40
30	8.37484	8.37497	9.99988	30	30	8.42510	8.42525	9.99985	30
40	8.37573	8.37585	9.99988	20	40	8.42589	8.42604	9.99985	20
50	8.37662	8.37674	9.99988	10	50	8.42667	8.42683	9.99985	10
220	8.37750	8.37762	9.99988	038	320	8.42746	8.42762	9.99984	028
10	8.37839	8.37850	9.99988	50	10	8.42825	8.42840	9.99984	50
20	8.37926	8.37939	9.99988	40	20	8.42903	8.42919	9.99984	40
30	8.38014	8.38026	9.99987	30	30	8.42982	8.42997	9.99984	30
40	8.38101	8.38114	9.99987	20	40	8.43060	8.43075	9.99984	20
50	8.38189	8.38202	9.99987	10	50	8.43138	8.43154	9.99984	10
230	8.38276	8.38289	9.99987	037	330	8.43216	8.43232	9.99984	027
10	8.38363	8.38376	9.99987	50	10	8.43293	8.43309	9.99984	50
20	8.38450	8.38463	9.99987	40	20	8.43371	8.43387	9.99984	40
30	8.38537	8.38550	9.99987	30	30	8.43448	8.43464	9.99984	30
40	8.38624	8.38636	9.99987	20	40	8.43526	8.43542	9.99984	20
50	8.38710	8.38723	9.99987	10	50	8.43603	8.43619	9.99984	10
240	8.38796	8.38809	9.99987	036	340	8.43680	8.43696	9.99984	026
10	8.38882	8.38895	9.99987	50	10	8.43757	8.43773	9.99984	50
20	8.38968	8.38981	9.99987	40	20	8.43834	8.43850	9.99984	40
30	8.39054	8.39067	9.99987	30	30	8.43910	8.43927	9.99984	30
40	8.39139	8.39153	9.99987	20	40	8.43987	8.44003	9.99984	20
50	8.39225	8.39238	9.99987	10	50	8.44063	8.44080	9.99983	10
250	8.39310	8.39323	9.99987	035	350	8.44139	8.44156	9.99983	025
10	8.39395	8.39408	9.99987	50	10	8.44216	8.44232	9.99983	50
20	8.39480	8.39493	9.99987	40	20	8.44292	8.44308	9.99983	40
30	8.39565	8.39578	9.99987	30	30	8.44367	8.44384	9.99983	30
40	8.39649	8.39663	9.99987	20	40	8.44443	8.44460	9.99983	20
50	8.39734	8.39747	9.99986	10	50	8.44519	8.44536	9.99983	10
260	8.39818	8.39832	9.99986	034	360	8.44594	8.44611	9.99983	024
10	8.39902	8.39916	9.99986	50	10	8.44669	8.44686	9.99983	50
20	8.39986	8.40000	9.99986	40	20	8.44745	8.44762	9.99983	40
30	8.40070	8.40083	9.99986	30	30	8.44820	8.44837	9.99983	30
40	8.40153	8.40167	9.99986	20	40	8.44895	8.44912	9.99983	20
50	8.40237	8.40251	9.99986	10	50	8.44969	8.44987	9.99983	10
270	8.40320	8.40334	9.99986	033	370	8.45044	8.45061	9.99983	023
10	8.40403	8.40417	9.99986	50	10	8.45119	8.45136	9.99983	50
20	8.40486	8.40500	9.99986	40	20	8.45193	8.45210	9.99983	40
30	8.40569	8.40583	9.99986	30	30	8.45267	8.45285	9.99983	30
40	8.40651	8.40665	9.99986	20	40	8.45341	8.45359	9.99982	20
50	8.40734	8.40748	9.99986	10	50	8.45415	8.45433	9.99982	10
280	8.40816	8.40830	9.99986	032	380	8.45489	8.45507	9.99982	022
10	8.40898	8.40913	9.99986	50	10	8.45563	8.45581	9.99982	50
20	8.40980	8.40995	9.99986	40	20	8.45637	8.45655	9.99982	40
30	8.41062	8.41077	9.99986	30	30	8.45710	8.45728	9.99982	30
40	8.41144	8.41158	9.99986	20	40	8.45784	8.45802	9.99982	20
50	8.41225	8.41240	9.99986	10	50	8.45857	8.45875	9.99982	10
290	8.41307	8.41321	9.99985	031	390	8.45930	8.45948	9.99982	021
10	8.41388	8.41403	9.99985	50	10	8.46003	8.46021	9.99982	50
20	8.41469	8.41484	9.99985	40	20	8.46076	8.46094	9.99982	40
30	8.41550	8.41565	9.99985	30	30	8.46149	8.46167	9.99982	30
40	8.41631	8.41646	9.99985	20	40	8.46222	8.46240	9.99982	20
50	8.41711	8.41726	9.99985	10	50	8.46294	8.46312	9.99982	10
300	8.41792	8.41807	9.99985	030	400	8.46366	8.46385	9.99982	020
°	log cos	log cot	log sin	°	°	log cos	log cot	log sin	°

°	log sin	log tan	log cos	°	°	log sin	log tan	log cos	°
400	8.46366	8.46385	9.99982	020	500	8.50504	8.50527	9.99978	010
10	8.46439	8.46457	9.99982	50	10	8.50570	8.50593	9.99978	50
20	8.46511	8.46529	9.99982	40	20	8.50636	8.50658	9.99978	40
30	8.46583	8.46602	9.99981	30	30	8.50701	8.50724	9.99978	30
40	8.46655	8.46674	9.99981	20	40	8.50767	8.50789	9.99977	20
50	8.46727	8.46745	9.99981	10	50	8.50832	8.50855	9.99977	10
410	8.46799	8.46817	9.99981	019	510	8.50897	8.50920	9.99977	09
10	8.46870	8.46889	9.99981	50	10	8.50963	8.50985	9.99977	50
20	8.46942	8.46960	9.99981	40	20	8.51028	8.51050	9.99977	40
30	8.47013	8.47032	9.99981	30	30	8.51092	8.51115	9.99977	30
40	8.47084	8.47103	9.99981	20	40	8.51157	8.51180	9.99977	20
50	8.47155	8.47174	9.99981	10	50	8.51222	8.51245	9.99977	10
420	8.47226	8.47245	9.99981	018	520	8.51287	8.51310	9.99977	08
10	8.47297	8.47316	9.99981	50	10	8.51351	8.51374	9.99977	50
20	8.47368	8.47387	9.99981	40	20	8.51416	8.51439	9.99977	40
30	8.47439	8.47458	9.99981	30	30	8.51480	8.51503	9.99977	30
40	8.47509	8.47528	9.99981	20	40	8.51544	8.51568	9.99977	20
50	8.47580	8.47599	9.99981	10	50	8.51609	8.51632	9.99977	10
430	8.47650	8.47669	9.99981	017	530	8.51673	8.51696	9.99977	07
10	8.47720	8.47740	9.99980	50	10	8.51737	8.51760	9.99976	50
20	8.47790	8.47810	9.99980	40	20	8.51801	8.51824	9.99976	40
30	8.47860	8.47880	9.99980	30	30	8.51864	8.51888	9.99976	30
40	8.47930	8.47950	9.99980	20	40	8.51928	8.51952	9.99976	20
50	8.48000	8.48020	9.99980	10	50	8.51992	8.52015	9.99976	10
440	8.48069	8.48090	9.99980	016	540	8.52055	8.52079	9.99976	06
10	8.48139	8.48159	9.99980	50	10	8.52119	8.52143	9.99976	50
20	8.48208	8.48228	9.99980	40	20	8.52182	8.52206	9.99976	40
30	8.48278	8.48298	9.99980	30	30	8.52245	8.52269	9.99976	30
40	8.48347	8.48367	9.99980	20	40	8.52308	8.52332	9.99976	20
50	8.48416	8.48436	9.99980	10	50	8.52371	8.52396	9.99976	10
450	8.48485	8.48505	9.99980	015	550	8.52434	8.52459	9.99976	05
10	8.48554	8.48574	9.99980	50	10	8.52497	8.52522	9.99976	50
20	8.48622	8.48643	9.99980	40	20	8.52560	8.52584	9.99976	40
30	8.48691	8.48711	9.99980	30	30	8.52623	8.52647	9.99975	30
40	8.48760	8.48780	9.99979	20	40	8.52685	8.52710	9.99975	20
50	8.48828	8.48849	9.99979	10	50	8.52748	8.52772	9.99975	10
460	8.48896	8.48917	9.99979	014	560	8.52810	8.52835	9.99975	04
10	8.48965	8.48985	9.99979	50	10	8.52872	8.52897	9.99975	50
20	8.49033	8.49053	9.99979	40	20	8.52935	8.52960	9.99975	40
30	8.49101	8.49121	9.99979	30	30	8.52997	8.53022	9.99975	30
40	8.49169	8.49189	9.99979	20	40	8.53059	8.53084	9.99975	20
50	8.49236	8.49257	9.99979	10	50	8.53121	8.53146	9.99975	10
470	8.49304	8.49325	9.99979	013	570	8.53183	8.53208	9.99975	03
10	8.49372	8.49393	9.99979	50	10	8.53245	8.53270	9.99975	50
20	8.49439	8.49460	9.99979	40	20	8.53306	8.53332	9.99975	40
30	8.49506	8.49528	9.99979	30	30	8.53368	8.53393	9.99975	30
40	8.49574	8.49595	9.99979	20	40	8.53429	8.53455	9.99975	20
50	8.49641	8.49662	9.99979	10	50	8.53491	8.53516	9.99974	10
480	8.49708	8.49729	9.99979	012	580	8.53552	8.53578	9.99974	02
10	8.49775	8.49796	9.99979	50	10	8.53614	8.53639	9.99974	50
20	8.49842	8.49863	9.99978	40	20	8.53675	8.53700	9.99974	40
30	8.49908	8.49930	9.99978	30	30	8.53736	8.53762	9.99974	30
40	8.49975	8.49997	9.99978	20	40	8.53797	8.53823	9.99974	20
50	8.50042	8.50063	9.99978	10	50	8.53858	8.53884	9.99974	10
490	8.50108	8.50130	9.99978	011	590	8.53919	8.53945	9.99974	01
10	8.50174	8.50196	9.99978	50	10	8.53979	8.54005	9.99974	50
20	8.50241	8.50263	9.99978	40	20	8.54040	8.54066	9.99974	40
30	8.50307	8.50329	9.99978	30	30	8.54101	8.54127	9.99974	30
40	8.50373	8.50395	9.99978	20	40	8.54161	8.54187	9.99974	20
50	8.50439	8.50461	9.99978	10	50	8.54222	8.54248	9.99974	10
500	8.50504	8.50527	9.99978	010	600	8.54282	8.54308	9.99974	00
°	log cos	log cot	log sin	°	°	log cos	log cot	log sin	°

	log sin	log tan	log cot	log sec	
	8	8	11	9	
0	24 186	24 192	75 808	99 993	60
1	24 903	24 910	75 090	99 993	59
2	25 609	25 616	74 384	99 993	58
3	26 304	26 312	73 688	99 993	57
4	26 988	26 996	73 004	99 992	56
5	27 661	27 669	72 331	99 992	55
6	28 324	28 332	71 668	99 992	54
7	28 977	28 986	71 014	99 992	53
8	29 621	29 629	70 371	99 992	52
9	30 255	30 263	69 737	99 991	51
10	30 879	30 888	69 112	99 991	50
11	31 495	31 505	68 495	99 991	49
12	32 103	32 112	67 883	99 990	48
13	32 702	32 711	67 289	99 990	47
14	33 292	33 302	66 698	99 990	46
15	33 875	33 886	66 114	99 990	45
16	34 450	34 461	65 539	99 989	44
17	35 018	35 029	64 971	99 989	43
18	35 578	35 590	64 410	99 989	42
19	36 131	36 143	63 857	99 989	41
20	36 678	36 689	63 311	99 988	40
21	37 217	37 229	62 771	99 988	39
22	37 750	37 762	62 238	99 988	38
23	38 276	38 289	61 711	99 987	37
24	38 796	38 809	61 191	99 987	36
25	39 310	39 323	60 677	99 987	35
26	39 818	39 832	60 168	99 986	34
27	40 320	40 334	59 666	99 985	33
28	40 816	40 830	59 170	99 985	32
29	41 307	41 321	58 679	99 985	31
30	41 792	41 807	58 198	99 985	30
31	42 272	42 287	57 713	99 985	29
32	42 746	42 762	57 238	99 984	28
33	43 216	43 232	56 768	99 984	27
34	43 680	43 696	56 304	99 984	26
35	44 139	44 156	55 844	99 983	25
36	44 594	44 611	55 389	99 983	24
37	45 044	45 061	54 939	99 983	23
38	45 489	45 507	54 493	99 982	22
39	45 930	45 948	54 052	99 982	21
40	46 366	46 385	53 615	99 982	20
41	46 799	46 817	53 183	99 981	19
42	47 226	47 245	52 755	99 981	18
43	47 650	47 669	52 331	99 981	17
44	48 069	48 089	51 911	99 980	16
45	48 485	48 505	51 495	99 980	15
46	48 896	48 917	51 083	99 979	14
47	49 304	49 325	50 675	99 979	13
48	49 708	49 729	50 271	99 979	12
49	50 108	50 130	49 870	99 978	11
50	50 504	50 527	49 473	99 978	10
51	50 897	50 920	49 080	99 977	9
52	51 287	51 310	48 690	99 977	8
53	51 673	51 696	48 304	99 977	7
54	52 055	52 079	47 921	99 976	6
55	52 434	52 459	47 541	99 976	5
56	52 810	52 835	47 165	99 975	4
57	53 183	53 208	46 792	99 975	3
58	53 552	53 578	46 422	99 974	2
59	53 919	53 945	46 055	99 974	1
60	54 282	54 308	45 692	99 974	0
	log cos	log cot	log tan	log sin	

	log sin	log tan	log cot	log sec	
	8	8	11	9	
0	54 282	54 303	45 692	99 974	60
1	54 642	54 669	45 331	99 973	59
2	54 999	55 027	44 973	99 973	58
3	55 354	55 382	44 618	99 972	57
4	55 705	55 734	44 266	99 972	56
5	56 054	56 083	43 917	99 971	55
6	56 400	56 429	43 571	99 971	54
7	56 743	56 773	43 227	99 970	53
8	57 084	57 114	42 886	99 970	52
9	57 421	57 452	42 548	99 969	51
10	57 757	57 788	42 212	99 969	50
11	58 069	58 121	41 879	99 968	49
12	58 419	58 451	41 549	99 968	48
13	58 747	58 779	41 221	99 967	47
14	59 072	59 105	40 895	99 967	46
15	59 395	59 428	40 572	99 967	45
16	59 715	59 749	40 251	99 966	44
17	60 033	60 068	39 932	99 966	43
18	60 349	60 384	39 616	99 965	42
19	60 662	60 698	39 302	99 964	41
20	60 973	61 009	38 991	99 964	40
21	61 282	61 319	38 681	99 963	39
22	61 589	61 626	38 374	99 963	38
23	61 894	61 931	38 069	99 962	37
24	62 196	62 234	37 766	99 962	36
25	62 497	62 535	37 465	99 961	35
26	62 795	62 834	37 166	99 961	34
27	63 091	63 131	36 869	99 960	33
28	63 385	63 426	36 574	99 960	32
29	63 678	63 718	36 282	99 959	31
30	63 968	64 009	35 991	99 959	30
31	64 256	64 298	35 702	99 958	29
32	64 543	64 585	35 415	99 958	28
33	64 827	64 870	35 130	99 957	27
34	65 110	65 154	34 846	99 956	26
35	65 391	65 435	34 565	99 956	25
36	65 670	65 715	34 285	99 955	24
37	65 947	65 993	34 007	99 955	23
38	66 223	66 269	33 731	99 954	22
39	66 497	66 543	33 457	99 954	21
40	66 769	66 816	33 184	99 953	20
41	67 039	67 087	32 913	99 952	19
42	67 308	67 356	32 644	99 952	18
43	67 575	67 624	32 376	99 951	17
44	67 841	67 890	32 110	99 951	16
45	68 104	68 154	31 846	99 950	15
46	68 367	68 417	31 583	99 949	14
47	68 627	68 678	31 322	99 949	13
48	68 886	68 938	31 062	99 948	12
49	69 144	69 196	30 804	99 948	11
50	69 400	69 453	30 547	99 947	10
51	69 654	69 708	30 292	99 946	9
52	69 907	69 962	30 038	99 946	8
53	70 159	70 214	29 786	99 945	7
54	70 409	70 465	29 535	99 944	6
55	70 658	70 714	29 286	99 944	5
56	70 905	70 962	29 038	99 943	4
57	71 151	71 208	28 792	99 942	3
58	71 395	71 453	28 547	99 942	2
59	71 638	71 697	28 303	99 941	1
60	71 880	71 940	28 060	99 940	0
	log cos	log cot	log tan	log sin	

°	log sin	log tan	log cot	log cos	'
	8	8	11	9	
0	71 880	71 910	23 060	99 940	00
1	72 120	72 181	27 819	99 940	59
2	72 359	72 420	27 580	99 939	58
3	72 597	72 659	27 341	99 938	57
4	72 834	72 896	27 104	99 938	56
5	73 069	73 132	26 868	99 937	55
6	73 303	73 366	26 634	99 936	54
7	73 535	73 600	26 400	99 936	53
8	73 767	73 832	26 168	99 935	52
9	73 997	74 063	25 937	99 934	51
10	74 226	74 292	25 708	99 934	50
11	74 454	74 521	25 479	99 933	49
12	74 680	74 748	25 252	99 932	48
13	74 906	74 974	25 026	99 932	47
14	75 130	75 199	24 801	99 931	46
15	75 353	75 423	24 577	99 930	45
16	75 575	75 645	24 355	99 929	44
17	75 795	75 867	24 133	99 929	43
18	76 015	76 087	23 913	99 928	42
19	76 234	76 306	23 694	99 927	41
20	76 451	76 525	23 475	99 926	40
21	76 667	76 742	23 258	99 926	39
22	76 883	76 958	23 042	99 925	38
23	77 097	77 173	22 827	99 924	37
24	77 310	77 387	22 613	99 923	36
25	77 522	77 600	22 400	99 923	35
26	77 733	77 811	22 189	99 922	34
27	77 943	78 022	21 978	99 921	33
28	78 152	78 232	21 768	99 920	32
29	78 360	78 441	21 559	99 920	31
30	78 568	78 649	21 351	99 919	30
31	78 774	78 855	21 145	99 918	29
32	78 979	79 061	20 939	99 917	28
33	79 183	79 266	20 734	99 917	27
34	79 386	79 470	20 530	99 916	26
35	79 588	79 673	20 327	99 915	25
36	79 789	79 875	20 125	99 914	24
37	79 990	80 076	19 924	99 913	23
38	80 189	80 277	19 723	99 913	22
39	80 388	80 476	19 524	99 912	21
40	80 585	80 674	19 326	99 911	20
41	80 782	80 872	19 128	99 910	19
42	80 978	81 068	18 932	99 909	18
43	81 173	81 264	18 736	99 909	17
44	81 367	81 459	18 541	99 908	16
45	81 560	81 653	18 347	99 907	15
46	81 752	81 846	18 154	99 906	14
47	81 944	82 038	17 962	99 905	13
48	82 134	82 230	17 770	99 904	12
49	82 324	82 420	17 580	99 904	11
50	82 513	82 610	17 390	99 903	10
51	82 701	82 799	17 201	99 902	9
52	82 888	82 987	17 013	99 901	8
53	83 075	83 175	16 825	99 900	7
54	83 261	83 361	16 639	99 899	6
55	83 446	83 547	16 453	99 898	5
56	83 630	83 732	16 268	99 898	4
57	83 813	83 916	16 084	99 897	3
58	83 996	84 100	15 900	99 896	2
59	84 177	84 282	15 718	99 895	1
60	84 358	84 464	15 536	99 894	0
	log cos	log cot	log tan	log sin	

°	log sin	log tan	log cot	log cos	'
	8	8	11	9	
0	84 358	84 464	15 536	99 894	60
1	84 539	84 646	15 354	99 893	59
2	84 718	84 826	15 174	99 892	58
3	84 897	85 006	14 994	99 891	57
4	85 075	85 185	14 815	99 891	56
5	85 252	85 363	14 637	99 890	55
6	85 429	85 540	14 460	99 889	54
7	85 605	85 717	14 283	99 888	53
8	85 780	85 893	14 107	99 887	52
9	85 955	86 069	13 931	99 886	51
10	86 128	86 243	13 757	99 885	50
11	86 301	86 417	13 583	99 884	49
12	86 474	86 591	13 409	99 883	48
13	86 645	86 763	13 237	99 882	47
14	86 816	86 935	13 065	99 881	46
15	86 987	87 106	12 894	99 880	45
16	87 156	87 277	12 723	99 879	44
17	87 325	87 447	12 553	99 879	43
18	87 494	87 616	12 384	99 878	42
19	87 661	87 785	12 215	99 877	41
20	87 829	87 953	12 047	99 876	40
21	87 995	88 120	11 880	99 875	39
22	88 161	88 287	11 713	99 874	38
23	88 326	88 453	11 547	99 873	37
24	88 490	88 618	11 382	99 872	36
25	88 654	88 783	11 217	99 871	35
26	88 817	88 948	11 052	99 870	34
27	88 980	89 111	10 889	99 869	33
28	89 142	89 274	10 726	99 868	32
29	89 304	89 437	10 563	99 867	31
30	89 464	89 598	10 402	99 866	30
31	89 625	89 760	10 240	99 865	29
32	89 784	89 920	10 080	99 864	28
33	89 943	90 080	9 920	99 863	27
34	90 102	90 240	9 760	99 862	26
35	90 260	90 399	9 601	99 861	25
36	90 417	90 557	9 443	99 860	24
37	90 574	90 715	9 285	99 859	23
38	90 730	90 872	9 128	99 858	22
39	90 885	91 029	8 971	99 857	21
40	91 040	91 185	8 815	99 856	20
41	91 195	91 340	8 660	99 855	19
42	91 349	91 495	8 505	99 854	18
43	91 502	91 650	8 350	99 853	17
44	91 655	91 803	8 197	99 852	16
45	91 807	91 957	8 043	99 851	15
46	91 959	92 110	7 890	99 850	14
47	92 110	92 262	7 738	99 849	13
48	92 261	92 414	7 586	99 847	12
49	92 411	92 565	7 435	99 846	11
50	92 561	92 716	7 284	99 845	10
51	92 710	92 866	7 134	99 844	9
52	92 859	93 016	6 984	99 843	8
53	93 007	93 165	6 835	99 842	7
54	93 154	93 313	6 687	99 841	6
55	93 301	93 462	6 538	99 840	5
56	93 448	93 609	6 391	99 839	4
57	93 594	93 756	6 244	99 838	3
58	93 740	93 903	6 097	99 837	2
59	93 885	94 049	5 951	99 836	1
60	94 030	94 195	5 805	99 834	0
	log cot	log cot	log tan	log sin	

<i>r</i>	log sin 8	log tan 8	log cot 11	log cos 9	<i>r</i>
0	94 030	94 195	05 805	99 834	00
1	94 174	94 340	05 660	99 833	59
2	94 317	94 483	05 515	99 832	58
3	94 461	94 630	05 370	99 831	57
4	94 603	94 773	05 227	99 830	56
5	94 746	94 917	05 083	99 829	55
6	94 887	95 060	04 940	99 828	54
7	95 029	95 202	04 798	99 827	53
8	95 170	95 344	04 656	99 825	52
9	95 310	95 486	04 514	99 824	51
10	95 450	95 627	04 373	99 823	50
11	95 589	95 767	04 233	99 822	49
12	95 728	95 908	04 092	99 821	48
13	95 867	96 047	03 953	99 820	47
14	96 005	96 187	03 813	99 819	46
15	96 143	96 325	03 675	99 817	45
16	96 280	96 464	03 536	99 816	44
17	96 417	96 602	03 398	99 815	43
18	96 553	96 739	03 261	99 814	42
19	96 689	96 877	03 123	99 813	41
20	96 825	97 013	02 987	99 812	40
21	96 960	97 150	02 850	99 810	39
22	97 095	97 285	02 715	99 809	38
23	97 229	97 421	02 579	99 808	37
24	97 363	97 556	02 444	99 807	36
25	97 496	97 691	02 309	99 806	35
26	97 629	97 825	02 175	99 804	34
27	97 762	97 959	02 041	99 803	33
28	97 894	98 092	01 908	99 802	32
29	98 026	98 225	01 775	99 801	31
30	98 157	98 358	01 642	99 800	30
31	98 288	98 490	01 510	99 798	29
32	98 419	98 622	01 378	99 797	28
33	98 549	98 753	01 247	99 796	27
34	98 679	98 884	01 116	99 795	26
35	98 808	99 015	00 985	99 793	25
36	98 937	99 145	00 855	99 792	24
37	99 066	99 275	00 725	99 791	23
38	99 194	99 405	00 595	99 790	22
39	99 322	99 534	00 466	99 788	21
40	99 450	99 662	00 338	99 787	20
41	99 577	99 791	00 209	99 786	19
42	99 704	99 919	00 081	99 785	18
43	99 830	00 046	99 954	99 783	17
44	99 956	00 174	99 826	99 782	16
45	00 082	00 301	99 699	99 781	15
46	00 207	00 427	99 573	99 780	14
47	00 332	00 553	99 447	99 778	13
48	00 456	00 679	99 321	99 777	12
49	00 581	00 805	99 195	99 776	11
50	00 704	00 930	99 070	99 775	10
51	00 828	01 055	98 945	99 773	9
52	00 951	01 179	98 821	99 772	8
53	01 074	01 303	98 697	99 771	7
54	01 196	01 427	98 573	99 769	6
55	01 318	01 550	98 450	99 768	5
56	01 440	01 673	98 327	99 767	4
57	01 561	01 796	98 204	99 765	3
58	01 682	01 918	98 082	99 764	2
59	01 803	02 040	97 960	99 763	1
60	01 923	02 162	97 838	99 761	0
<i>r</i>	log cos 9	log cot 10	log tan 9	log sin 9	<i>r</i>

<i>r</i>	log sin 9	log tan 9	log cot 10	log cos 9	<i>r</i>
0	01 923	02 162	97 838	99 761	60
1	02 043	02 283	97 717	99 760	59
2	02 163	02 404	97 596	99 759	58
3	02 283	02 525	97 475	99 757	57
4	02 402	02 645	97 355	99 756	56
5	02 520	02 766	97 234	99 755	55
6	02 639	02 885	97 115	99 753	54
7	02 757	03 005	96 995	99 752	53
8	02 874	03 124	96 876	99 751	52
9	02 992	03 242	96 758	99 749	51
10	03 109	03 361	96 639	99 748	50
11	03 226	03 479	96 521	99 747	49
12	03 342	03 597	96 403	99 745	48
13	03 458	03 714	96 286	99 744	47
14	03 574	03 832	96 168	99 742	46
15	03 690	03 948	96 052	99 741	45
16	03 805	04 065	95 935	99 740	44
17	03 920	04 181	95 819	99 738	43
18	04 034	04 297	95 703	99 737	42
19	04 149	04 413	95 587	99 736	41
20	04 262	04 528	95 472	99 734	40
21	04 376	04 643	95 357	99 733	39
22	04 490	04 758	95 242	99 731	38
23	04 603	04 873	95 127	99 730	37
24	04 715	04 987	95 013	99 728	36
25	04 828	05 101	94 899	99 727	35
26	04 940	05 214	94 786	99 726	34
27	05 052	05 328	94 672	99 724	33
28	05 164	05 441	94 559	99 723	32
29	05 275	05 553	94 447	99 721	31
30	05 386	05 666	94 334	99 720	30
31	05 497	05 778	94 222	99 718	29
32	05 607	05 890	94 110	99 717	28
33	05 717	06 002	93 998	99 716	27
34	05 827	06 113	93 887	99 714	26
35	05 937	06 224	93 776	99 713	25
36	06 046	06 335	93 665	99 711	24
37	06 155	06 445	93 555	99 710	23
38	06 264	06 556	93 444	99 708	22
39	06 372	06 666	93 334	99 707	21
40	06 481	06 775	93 225	99 705	20
41	06 589	06 885	93 115	99 704	19
42	06 696	06 994	93 006	99 702	18
43	06 804	07 103	92 897	99 701	17
44	06 911	07 211	92 789	99 699	16
45	07 018	07 320	92 680	99 698	15
46	07 124	07 428	92 572	99 696	14
47	07 231	07 536	92 464	99 695	13
48	07 337	07 643	92 357	99 693	12
49	07 442	07 751	92 249	99 692	11
50	07 548	07 858	92 142	99 690	10
51	07 653	07 964	92 036	99 689	9
52	07 758	08 071	91 929	99 687	8
53	07 863	08 177	91 823	99 686	7
54	07 968	08 283	91 717	99 684	6
55	08 072	08 389	91 611	99 683	5
56	08 176	08 495	91 505	99 681	4
57	08 280	08 600	91 400	99 680	3
58	08 383	08 705	91 295	99 678	2
59	08 486	08 810	91 190	99 677	1
60	08 589	08 914	91 086	99 675	0
<i>r</i>	log cos 9	log cot 10	log tan 9	log sin 9	<i>r</i>



7°					8°					
	log sin	log tan	log cot	log cos		log sin	log tan	log cot	log cos	
	0	0	10	0		0	0	10	0	
0	08 589	08 914	91 086	99 675	60	14 556	14 769	85 220	99 575	60
1	08 692	09 019	90 981	99 674	59	14 445	14 872	85 128	99 574	59
2	08 795	09 123	90 877	99 672	58	14 535	14 963	85 037	99 572	58
3	08 897	09 227	90 773	99 670	57	14 624	15 054	84 946	99 570	57
4	08 999	09 330	90 670	99 669	56	14 714	15 145	84 855	99 568	56
5	09 101	09 434	90 566	99 667	55	14 803	15 236	84 764	99 566	55
6	09 202	09 537	90 463	99 666	54	14 891	15 327	84 673	99 565	54
7	09 304	09 640	90 360	99 664	53	14 980	15 417	84 583	99 563	53
8	09 405	09 742	90 258	99 663	52	15 069	15 508	84 492	99 561	52
9	09 506	09 845	90 155	99 661	51	15 157	15 598	84 402	99 559	51
10	09 606	09 947	90 053	99 659	50	15 245	15 688	84 312	99 557	50
11	09 707	10 049	89 951	99 658	49	15 333	15 777	84 223	99 556	49
12	09 807	10 150	89 850	99 656	48	15 421	15 867	84 133	99 554	48
13	09 907	10 252	89 748	99 655	47	15 508	15 956	84 044	99 552	47
14	10 006	10 353	89 647	99 653	46	15 596	16 046	83 954	99 550	46
15	10 106	10 454	89 546	99 651	45	15 683	16 135	83 865	99 548	45
16	10 205	10 555	89 445	99 650	44	15 770	16 224	83 776	99 546	44
17	10 304	10 656	89 344	99 648	43	15 857	16 312	83 688	99 545	43
18	10 402	10 756	89 244	99 647	42	15 944	16 401	83 599	99 543	42
19	10 501	10 856	89 144	99 645	41	16 030	16 489	83 511	99 541	41
20	10 599	10 956	89 044	99 643	40	16 116	16 577	83 423	99 539	40
21	10 697	11 056	88 944	99 642	39	16 203	16 665	83 335	99 537	39
22	10 795	11 155	88 845	99 640	38	16 289	16 753	83 247	99 535	38
23	10 893	11 254	88 746	99 638	37	16 374	16 841	83 159	99 533	37
24	10 990	11 353	88 647	99 637	36	16 460	16 928	83 072	99 532	36
25	11 087	11 452	88 548	99 635	35	16 545	17 016	82 984	99 530	35
26	11 184	11 551	88 449	99 633	34	16 631	17 103	82 897	99 528	34
27	11 281	11 649	88 351	99 632	33	16 716	17 190	82 810	99 526	33
28	11 377	11 747	88 253	99 630	32	16 801	17 277	82 723	99 524	32
29	11 474	11 845	88 155	99 629	31	16 886	17 363	82 637	99 522	31
30	11 570	11 943	88 057	99 627	30	16 970	17 450	82 550	99 520	30
31	11 666	12 040	87 960	99 625	29	17 055	17 536	82 464	99 518	29
32	11 761	12 138	87 862	99 624	28	17 139	17 622	82 378	99 517	28
33	11 857	12 235	87 765	99 622	27	17 223	17 708	82 292	99 515	27
34	11 952	12 332	87 668	99 620	26	17 307	17 794	82 206	99 513	26
35	12 047	12 428	87 572	99 618	25	17 391	17 880	82 120	99 511	25
36	12 142	12 525	87 475	99 617	24	17 474	17 965	82 035	99 509	24
37	12 236	12 621	87 379	99 615	23	17 558	18 051	81 949	99 507	23
38	12 331	12 717	87 283	99 613	22	17 641	18 136	81 864	99 505	22
39	12 425	12 813	87 187	99 612	21	17 724	18 221	81 779	99 503	21
40	12 519	12 909	87 091	99 610	20	17 807	18 306	81 694	99 501	20
41	12 612	13 004	86 996	99 608	19	17 890	18 391	81 609	99 499	19
42	12 706	13 099	86 901	99 607	18	17 973	18 475	81 525	99 497	18
43	12 799	13 194	86 806	99 605	17	18 055	18 560	81 440	99 495	17
44	12 892	13 289	86 711	99 603	16	18 137	18 644	81 356	99 494	16
45	12 985	13 384	86 616	99 601	15	18 220	18 728	81 272	99 492	15
46	13 078	13 478	86 522	99 600	14	18 302	18 812	81 188	99 490	14
47	13 171	13 573	86 427	99 598	13	18 383	18 896	81 104	99 488	13
48	13 263	13 667	86 333	99 596	12	18 465	18 979	81 021	99 486	12
49	13 355	13 761	86 239	99 595	11	18 547	19 063	80 937	99 484	11
50	13 447	13 854	86 146	99 593	10	18 628	19 146	80 854	99 482	10
51	13 539	13 948	86 052	99 591	9	18 709	19 229	80 771	99 480	9
52	13 630	14 041	85 959	99 589	8	18 790	19 312	80 688	99 478	8
53	13 722	14 134	85 866	99 588	7	18 871	19 395	80 605	99 476	7
54	13 813	14 227	85 773	99 586	6	18 952	19 478	80 522	99 474	6
55	13 904	14 320	85 680	99 584	5	19 033	19 561	80 439	99 472	5
56	13 994	14 412	85 588	99 582	4	19 113	19 643	80 357	99 470	4
57	14 085	14 504	85 496	99 581	3	19 193	19 725	80 275	99 468	3
58	14 175	14 597	85 403	99 579	2	19 273	19 807	80 193	99 466	2
59	14 266	14 688	85 312	99 577	1	19 353	19 889	80 111	99 464	1
60	14 356	14 780	85 220	99 575	0	19 433	19 971	80 029	99 462	0
	0	0	10	0		0	0	10	0	
	log cos	log cot	log tan	log sin		log cos	log cot	log tan	log sin	



9°					10°					
	log sin	log tan	log cot	log cos		log sin	log tan	log cot	log cos	
'	0	0	10	0	'	0	0	10	0	'
0	19 433	19 971	00 029	99 462	00	23 967	24 632	75 368	99 335	00
1	19 513	20 053	79 947	99 460	59	24 039	24 706	75 294	99 333	59
2	19 592	20 134	79 866	99 458	58	24 110	24 779	75 221	99 331	58
3	19 672	20 216	79 784	99 456	57	24 181	24 853	75 147	99 328	57
4	19 751	20 297	79 703	99 454	56	24 253	24 926	75 074	99 326	56
5	19 830	20 378	79 622	99 452	55	24 324	25 000	75 000	99 324	55
6	19 909	20 459	79 541	99 450	54	24 395	25 073	74 927	99 322	54
7	19 988	20 540	79 460	99 448	53	24 466	25 146	74 854	99 319	53
8	20 067	20 621	79 379	99 446	52	24 536	25 219	74 781	99 317	52
9	20 145	20 701	79 299	99 444	51	24 607	25 292	74 708	99 315	51
10	20 223	20 782	79 218	99 442	50	24 677	25 365	74 635	99 313	50
11	20 302	20 862	79 138	99 440	49	24 748	25 437	74 563	99 310	49
12	20 380	20 942	79 058	99 438	48	24 818	25 510	74 490	99 308	48
13	20 458	21 022	78 978	99 436	47	24 888	25 582	74 418	99 306	47
14	20 535	21 102	78 898	99 434	46	24 958	25 655	74 345	99 304	46
15	20 613	21 182	78 818	99 432	45	25 028	25 727	74 273	99 301	45
16	20 691	21 261	78 739	99 429	44	25 098	25 799	74 201	99 299	44
17	20 768	21 341	78 659	99 427	43	25 168	25 871	74 129	99 297	43
18	20 845	21 420	78 580	99 425	42	25 237	25 943	74 057	99 294	42
19	20 922	21 499	78 501	99 423	41	25 307	26 015	73 985	99 292	41
20	20 999	21 578	78 422	99 421	40	25 376	26 086	73 914	99 290	40
21	21 076	21 657	78 343	99 419	39	25 445	26 158	73 842	99 288	39
22	21 153	21 736	78 264	99 417	38	25 514	26 229	73 771	99 285	38
23	21 229	21 814	78 186	99 415	37	25 583	26 301	73 699	99 283	37
24	21 306	21 893	78 107	99 413	36	25 652	26 372	73 628	99 281	36
25	21 382	21 971	78 029	99 411	35	25 721	26 443	73 557	99 278	35
26	21 458	22 049	77 951	99 409	34	25 790	26 514	73 486	99 276	34
27	21 534	22 127	77 873	99 407	33	25 858	26 585	73 415	99 274	33
28	21 610	22 205	77 795	99 404	32	25 927	26 655	73 345	99 271	32
29	21 685	22 283	77 717	99 402	31	25 995	26 726	73 274	99 269	31
30	21 761	22 361	77 639	99 400	30	26 063	26 797	73 203	99 267	30
31	21 836	22 438	77 562	99 398	29	26 131	26 867	73 133	99 264	29
32	21 912	22 516	77 484	99 396	28	26 199	26 937	73 063	99 262	28
33	21 987	22 593	77 407	99 394	27	26 267	27 008	72 992	99 260	27
34	22 062	22 670	77 330	99 392	26	26 335	27 078	72 922	99 257	26
35	22 137	22 747	77 253	99 390	25	26 403	27 148	72 852	99 255	25
36	22 211	22 824	77 176	99 388	24	26 470	27 218	72 782	99 252	24
37	22 286	22 901	77 099	99 385	23	26 538	27 288	72 712	99 250	23
38	22 361	22 977	77 023	99 383	22	26 605	27 357	72 643	99 248	22
39	22 435	23 054	76 946	99 381	21	26 672	27 427	72 573	99 245	21
40	22 509	23 130	76 870	99 379	20	26 739	27 496	72 504	99 243	20
41	22 583	23 206	76 794	99 377	19	26 806	27 566	72 434	99 241	19
42	22 657	23 283	76 717	99 375	18	26 873	27 635	72 365	99 238	18
43	22 731	23 359	76 641	99 372	17	26 940	27 704	72 296	99 236	17
44	22 805	23 435	76 565	99 370	16	27 007	27 773	72 227	99 233	16
45	22 878	23 510	76 490	99 368	15	27 073	27 842	72 158	99 231	15
46	22 952	23 586	76 414	99 366	14	27 140	27 911	72 089	99 229	14
47	23 025	23 661	76 339	99 364	13	27 206	27 980	72 020	99 226	13
48	23 098	23 737	76 263	99 362	12	27 273	28 049	71 951	99 224	12
49	23 171	23 812	76 188	99 359	11	27 339	28 117	71 883	99 221	11
50	23 244	23 887	76 113	99 357	10	27 405	28 186	71 814	99 219	10
51	23 317	23 962	76 038	99 355	9	27 471	28 254	71 746	99 217	9
52	23 390	24 037	75 963	99 353	8	27 537	28 323	71 677	99 214	8
53	23 462	24 112	75 888	99 351	7	27 602	28 391	71 609	99 212	7
54	23 535	24 186	75 814	99 348	6	27 668	28 459	71 541	99 209	6
55	23 607	24 261	75 739	99 346	5	27 734	28 527	71 473	99 207	5
56	23 679	24 335	75 665	99 344	4	27 799	28 595	71 405	99 204	4
57	23 752	24 410	75 590	99 342	3	27 864	28 662	71 338	99 202	3
58	23 823	24 484	75 516	99 340	2	27 930	28 730	71 270	99 200	2
59	23 895	24 558	75 442	99 337	1	27 995	28 798	71 202	99 197	1
60	23 967	24 632	75 368	99 335	0	28 060	28 865	71 135	99 195	0
	log cos	log cot	log tan	log sin		log cos	log cot	log tan	log sin	

11°					12°					
	log sin	log tan	log cot	log cos		log sin	log tan	log cot	log cos	
	9	9	10	9		9	9	10	9	
0	23 060	28 865	71 135	99 195	60	31 788	32 747	67 253	99 040	60
1	28 125	28 933	71 067	99 192	59	31 847	32 810	67 190	99 038	59
2	28 190	29 000	71 000	99 190	58	31 907	32 872	67 128	99 035	58
3	28 254	29 067	70 933	99 187	57	31 966	32 933	67 067	99 032	57
4	28 319	29 134	70 866	99 185	56	32 025	32 995	67 005	99 030	56
5	28 384	29 201	70 799	99 182	55	32 084	33 057	66 943	99 027	55
6	28 448	29 268	70 732	99 180	54	32 143	33 119	66 881	99 024	54
7	28 512	29 335	70 665	99 177	53	32 202	33 180	66 820	99 022	53
8	28 577	29 402	70 598	99 175	52	32 261	33 242	66 758	99 019	52
9	28 641	29 468	70 532	99 172	51	32 319	33 303	66 697	99 016	51
10	28 705	29 535	70 465	99 170	50	32 378	33 365	66 635	99 013	50
11	28 769	29 601	70 399	99 167	49	32 437	33 426	66 574	99 011	49
12	28 833	29 668	70 332	99 165	48	32 495	33 487	66 513	99 008	48
13	28 896	29 734	70 266	99 162	47	32 553	33 548	66 452	99 005	47
14	28 960	29 800	70 200	99 160	46	32 612	33 609	66 391	99 002	46
15	29 024	29 866	70 134	99 157	45	32 670	33 670	66 330	99 000	45
16	29 087	29 932	70 068	99 155	44	32 728	33 731	66 269	98 997	44
17	29 150	29 998	70 002	99 152	43	32 786	33 792	66 208	98 994	43
18	29 214	30 064	69 936	99 150	42	32 844	33 853	66 147	98 991	42
19	29 277	30 130	69 870	99 147	41	32 902	33 913	66 087	98 989	41
20	29 340	30 195	69 805	99 145	40	32 960	33 974	66 026	98 986	40
21	29 403	30 261	69 739	99 142	39	33 018	34 034	65 966	98 983	39
22	29 466	30 326	69 674	99 140	38	33 075	34 095	65 905	98 980	38
23	29 529	30 391	69 609	99 137	37	33 133	34 155	65 845	98 978	37
24	29 591	30 457	69 543	99 135	36	33 190	34 215	65 785	98 975	36
25	29 654	30 522	69 478	99 132	35	33 248	34 276	65 724	98 972	35
26	29 716	30 587	69 413	99 130	34	33 305	34 336	65 664	98 969	34
27	29 779	30 652	69 348	99 127	33	33 362	34 396	65 604	98 967	33
28	29 841	30 717	69 283	99 124	32	33 420	34 456	65 544	98 964	32
29	29 903	30 782	69 218	99 122	31	33 477	34 516	65 484	98 961	31
30	29 966	30 846	69 154	99 119	30	33 534	34 576	65 424	98 958	30
31	30 028	30 911	69 089	99 117	29	33 591	34 635	65 365	98 955	29
32	30 090	30 975	69 025	99 114	28	33 647	34 695	65 305	98 953	28
33	30 151	31 040	68 960	99 112	27	33 704	34 755	65 245	98 950	27
34	30 213	31 104	68 896	99 109	26	33 761	34 814	65 186	98 947	26
35	30 275	31 168	68 832	99 106	25	33 818	34 874	65 126	98 944	25
36	30 336	31 233	68 767	99 104	24	33 874	34 933	65 067	98 941	24
37	30 398	31 297	68 703	99 101	23	33 931	34 992	65 008	98 938	23
38	30 459	31 361	68 639	99 099	22	33 987	35 051	64 949	98 936	22
39	30 521	31 425	68 575	99 096	21	34 043	35 111	64 889	98 933	21
40	30 582	31 489	68 511	99 093	20	34 100	35 170	64 830	98 930	20
41	30 643	31 552	68 448	99 091	19	34 156	35 229	64 771	98 927	19
42	30 704	31 616	68 384	99 088	18	34 212	35 288	64 712	98 924	18
43	30 765	31 679	68 321	99 086	17	34 268	35 347	64 653	98 921	17
44	30 826	31 743	68 257	99 083	16	34 324	35 405	64 595	98 919	16
45	30 887	31 806	68 194	99 080	15	34 380	35 464	64 536	98 916	15
46	30 947	31 870	68 130	99 078	14	34 436	35 523	64 477	98 913	14
47	31 008	31 933	68 067	99 075	13	34 491	35 581	64 419	98 910	13
48	31 068	31 996	68 004	99 072	12	34 547	35 640	64 360	98 907	12
49	31 129	32 059	67 941	99 070	11	34 602	35 698	64 302	98 904	11
50	31 189	32 122	67 878	99 067	10	34 658	35 757	64 243	98 901	10
51	31 250	32 185	67 815	99 064	9	34 713	35 815	64 185	98 898	9
52	31 310	32 248	67 752	99 062	8	34 769	35 873	64 127	98 896	8
53	31 370	32 311	67 689	99 059	7	34 824	35 931	64 069	98 893	7
54	31 430	32 373	67 627	99 056	6	34 879	35 989	64 011	98 890	6
55	31 490	32 436	67 564	99 054	5	34 934	36 047	63 953	98 887	5
56	31 549	32 498	67 502	99 051	4	34 989	36 105	63 895	98 884	4
57	31 609	32 561	67 439	99 048	3	35 044	36 163	63 837	98 881	3
58	31 669	32 623	67 377	99 046	2	35 099	36 221	63 779	98 878	2
59	31 728	32 685	67 315	99 043	1	35 154	36 279	63 721	98 875	1
60	31 788	32 747	67 253	99 040	0	35 209	36 336	63 664	98 872	0
	log sin	log tan	log cot	log cos		log sin	log tan	log cot	log cos	

13°					14°					
<i>r</i>	log sin D	log tan D	log cot 10	log cos D	<i>r</i>	log sin D	log tan D	log cot 10	log cos D	<i>r</i>
0	35 209	36 336	63 664	98 872	0	38 368	39 677	60 323	98 690	0
1	35 263	36 391	63 606	98 869	1	38 418	39 731	60 269	98 687	1
2	35 318	36 452	63 548	98 867	2	38 469	39 785	60 215	98 684	2
3	35 373	36 509	63 491	98 864	3	38 519	39 838	60 162	98 681	3
4	35 427	36 566	63 434	98 861	4	38 570	39 892	60 108	98 678	4
5	35 481	36 624	63 376	98 858	5	38 620	39 945	60 055	98 675	5
6	35 535	36 681	63 319	98 855	6	38 670	39 999	60 001	98 671	6
7	35 590	36 738	63 262	98 852	7	38 721	40 052	59 948	98 668	7
8	35 644	36 795	63 205	98 849	8	38 771	40 106	59 894	98 665	8
9	35 698	36 852	63 148	98 846	9	38 821	40 159	59 841	98 662	9
10	35 752	36 909	63 091	98 843	10	38 871	40 212	59 788	98 659	10
11	35 806	36 966	63 034	98 840	11	38 921	40 266	59 734	98 656	11
12	35 860	37 023	62 977	98 837	12	38 971	40 319	59 681	98 652	12
13	35 914	37 080	62 920	98 834	13	39 021	40 372	59 628	98 649	13
14	35 968	37 137	62 863	98 831	14	39 071	40 425	59 575	98 646	14
15	36 022	37 193	62 807	98 828	15	39 121	40 478	59 522	98 643	15
16	36 075	37 250	62 750	98 825	16	39 170	40 531	59 469	98 640	16
17	36 129	37 306	62 694	98 822	17	39 220	40 584	59 416	98 636	17
18	36 182	37 363	62 637	98 819	18	39 270	40 636	59 364	98 633	18
19	36 236	37 419	62 581	98 816	19	39 319	40 689	59 311	98 630	19
20	36 289	37 476	62 524	98 813	20	39 369	40 742	59 258	98 627	20
21	36 342	37 532	62 468	98 810	21	39 418	40 795	59 205	98 623	21
22	36 395	37 588	62 412	98 807	22	39 467	40 847	59 153	98 620	22
23	36 449	37 644	62 356	98 804	23	39 517	40 900	59 100	98 617	23
24	36 502	37 700	62 300	98 801	24	39 566	40 952	59 048	98 614	24
25	36 555	37 756	62 244	98 798	25	39 615	41 005	58 995	98 610	25
26	36 608	37 812	62 188	98 795	26	39 664	41 057	58 943	98 607	26
27	36 660	37 868	62 132	98 792	27	39 713	41 109	58 891	98 604	27
28	36 713	37 924	62 076	98 789	28	39 762	41 161	58 839	98 601	28
29	36 766	37 980	62 020	98 786	29	39 811	41 214	58 786	98 597	29
30	36 819	38 035	61 965	98 783	30	39 860	41 266	58 734	98 594	30
31	36 871	38 091	61 909	98 780	31	39 909	41 318	58 682	98 591	31
32	36 924	38 147	61 853	98 777	32	39 958	41 370	58 630	98 588	32
33	36 976	38 202	61 798	98 774	33	40 006	41 422	58 578	98 584	33
34	37 028	38 257	61 743	98 771	34	40 055	41 474	58 526	98 581	34
35	37 081	38 313	61 687	98 768	35	40 103	41 526	58 474	98 578	35
36	37 133	38 368	61 632	98 765	36	40 152	41 578	58 422	98 574	36
37	37 185	38 423	61 577	98 762	37	40 200	41 629	58 371	98 571	37
38	37 237	38 479	61 521	98 759	38	40 249	41 681	58 319	98 568	38
39	37 289	38 534	61 466	98 756	39	40 297	41 733	58 267	98 565	39
40	37 341	38 589	61 411	98 753	40	40 346	41 784	58 216	98 561	40
41	37 393	38 644	61 356	98 750	41	40 394	41 836	58 164	98 558	41
42	37 445	38 699	61 301	98 746	42	40 442	41 887	58 113	98 555	42
43	37 497	38 754	61 246	98 743	43	40 490	41 939	58 061	98 551	43
44	37 549	38 808	61 192	98 740	44	40 538	41 990	58 010	98 548	44
45	37 600	38 863	61 137	98 737	45	40 586	42 041	57 959	98 545	45
46	37 652	38 918	61 082	98 734	46	40 634	42 093	57 907	98 541	46
47	37 703	38 972	61 028	98 731	47	40 682	42 144	57 856	98 538	47
48	37 755	39 027	60 973	98 728	48	40 730	42 195	57 805	98 535	48
49	37 806	39 082	60 918	98 725	49	40 778	42 246	57 754	98 531	49
50	37 858	39 136	60 864	98 722	50	40 825	42 297	57 703	98 528	50
51	37 909	39 190	60 810	98 719	51	40 873	42 348	57 652	98 525	51
52	37 960	39 245	60 755	98 715	52	40 921	42 399	57 601	98 521	52
53	38 011	39 299	60 701	98 712	53	40 968	42 450	57 550	98 518	53
54	38 062	39 353	60 647	98 709	54	41 016	42 501	57 499	98 515	54
55	38 113	39 407	60 593	98 706	55	41 063	42 552	57 448	98 511	55
56	38 164	39 461	60 539	98 703	56	41 111	42 603	57 397	98 508	56
57	38 215	39 515	60 485	98 700	57	41 158	42 653	57 347	98 505	57
58	38 266	39 569	60 431	98 697	58	41 205	42 704	57 296	98 501	58
59	38 317	39 623	60 377	98 694	59	41 252	42 755	57 245	98 498	59
60	38 368	39 677	60 323	98 690	60	41 300	42 805	57 195	98 494	60
<i>r</i>	log sin D	log tan D	log cot 10	log cos D	<i>r</i>	log sin D	log tan D	log cot 10	log cos D	<i>r</i>
<i>r</i>	log cos D	log cot 10	log tan D	log sin D	<i>r</i>	log cos D	log cot 10	log tan D	log sin D	<i>r</i>

<i>i</i>	log sin 0	log tan 0	log cot 10	log cos 0	<i>i</i>
0	41 300	42 805	57 195	98 494	60
1	41 347	42 856	57 144	98 491	59
2	41 394	42 906	57 094	98 488	58
3	41 441	42 957	57 043	98 484	57
4	41 488	43 007	56 993	98 481	56
5	41 535	43 057	56 943	98 477	55
6	41 582	43 108	56 892	98 474	54
7	41 628	43 158	56 842	98 471	53
8	41 675	43 208	56 792	98 467	52
9	41 722	43 258	56 742	98 464	51
10	41 768	43 308	56 692	98 460	50
11	41 815	43 358	56 642	98 457	49
12	41 861	43 408	56 592	98 453	48
13	41 908	43 458	56 542	98 450	47
14	41 954	43 508	56 492	98 447	46
15	42 001	43 558	56 442	98 443	45
16	42 047	43 607	56 393	98 440	44
17	42 093	43 657	56 343	98 436	43
18	42 140	43 707	56 293	98 433	42
19	42 186	43 756	56 244	98 429	41
20	42 232	43 806	56 194	98 426	40
21	42 278	43 855	56 145	98 422	39
22	42 324	43 905	56 095	98 419	38
23	42 370	43 954	56 046	98 415	37
24	42 416	44 004	55 996	98 412	36
25	42 461	44 053	55 947	98 409	35
26	42 507	44 102	55 898	98 405	34
27	42 553	44 151	55 849	98 402	33
28	42 599	44 201	55 799	98 398	32
29	42 644	44 250	55 750	98 395	31
30	42 690	44 299	55 701	98 391	30
31	42 735	44 348	55 652	98 388	29
32	42 781	44 397	55 603	98 384	28
33	42 826	44 446	55 554	98 381	27
34	42 872	44 495	55 505	98 377	26
35	42 917	44 544	55 456	98 373	25
36	42 962	44 592	55 408	98 370	24
37	43 008	44 641	55 359	98 366	23
38	43 053	44 690	55 310	98 363	22
39	43 098	44 738	55 262	98 359	21
40	43 143	44 787	55 213	98 356	20
41	43 188	44 836	55 164	98 352	19
42	43 233	44 884	55 116	98 349	18
43	43 278	44 933	55 067	98 345	17
44	43 323	44 981	55 019	98 342	16
45	43 367	45 029	54 971	98 338	15
46	43 412	45 078	54 922	98 334	14
47	43 457	45 126	54 874	98 331	13
48	43 502	45 174	54 826	98 327	12
49	43 546	45 222	54 778	98 324	11
50	43 591	45 271	54 729	98 320	10
51	43 635	45 319	54 681	98 317	9
52	43 680	45 367	54 633	98 313	8
53	43 724	45 415	54 585	98 309	7
54	43 769	45 463	54 537	98 306	6
55	43 813	45 511	54 489	98 302	5
56	43 857	45 559	54 441	98 299	4
57	43 901	45 606	54 394	98 295	3
58	43 946	45 654	54 346	98 291	2
59	43 990	45 702	54 298	98 288	1
60	44 034	45 750	54 250	98 284	0
<i>i</i>	log cos	log cot	log tan	log sin	<i>i</i>

<i>i</i>	log sin 0	log tan 0	log cot 10	log cos 0	<i>i</i>
0	44 034	45 750	54 250	98 284	60
1	44 078	45 797	54 203	98 281	59
2	44 122	45 845	54 155	98 277	58
3	44 166	45 892	54 108	98 273	57
4	44 210	45 940	54 060	98 270	56
5	44 253	45 987	54 013	98 266	55
6	44 297	46 035	53 965	98 262	54
7	44 341	46 082	53 918	98 259	53
8	44 385	46 130	53 870	98 255	52
9	44 428	46 177	53 823	98 251	51
10	44 472	46 224	53 776	98 248	50
11	44 516	46 271	53 729	98 244	49
12	44 559	46 319	53 681	98 240	48
13	44 602	46 366	53 634	98 237	47
14	44 646	46 413	53 587	98 233	46
15	44 689	46 460	53 540	98 229	45
16	44 733	46 507	53 493	98 226	44
17	44 776	46 554	53 446	98 222	43
18	44 819	46 601	53 399	98 218	42
19	44 862	46 648	53 352	98 215	41
20	44 905	46 694	53 306	98 211	40
21	44 948	46 741	53 259	98 207	39
22	44 992	46 788	53 212	98 204	38
23	45 035	46 835	53 165	98 200	37
24	45 077	46 881	53 119	98 196	36
25	45 120	46 928	53 072	98 192	35
26	45 163	46 975	53 025	98 189	34
27	45 206	47 021	52 979	98 185	33
28	45 249	47 068	52 932	98 181	32
29	45 292	47 114	52 886	98 177	31
30	45 334	47 160	52 840	98 174	30
31	45 377	47 207	52 793	98 170	29
32	45 419	47 253	52 747	98 166	28
33	45 462	47 299	52 701	98 162	27
34	45 504	47 346	52 654	98 159	26
35	45 547	47 392	52 608	98 155	25
36	45 589	47 438	52 562	98 151	24
37	45 632	47 484	52 516	98 147	23
38	45 674	47 530	52 470	98 144	22
39	45 716	47 576	52 424	98 140	21
40	45 758	47 622	52 378	98 136	20
41	45 801	47 668	52 332	98 132	19
42	45 843	47 714	52 285	98 129	18
43	45 885	47 760	52 240	98 125	17
44	45 927	47 806	52 194	98 121	16
45	45 969	47 852	52 148	98 117	15
46	46 011	47 897	52 103	98 113	14
47	46 053	47 943	52 057	98 110	13
48	46 095	47 989	52 011	98 106	12
49	46 136	48 035	51 965	98 102	11
50	46 178	48 080	51 920	98 098	10
51	46 220	48 126	51 874	98 094	9
52	46 262	48 171	51 829	98 090	8
53	46 303	48 217	51 783	98 087	7
54	46 345	48 262	51 738	98 083	6
55	46 386	48 307	51 693	98 079	5
56	46 428	48 353	51 647	98 075	4
57	46 469	48 398	51 602	98 071	3
58	46 511	48 443	51 557	98 067	2
59	46 552	48 489	51 511	98 063	1
60	46 594	48 534	51 466	98 060	0
<i>i</i>	log cos	log cot	log tan	log sin	<i>i</i>

17°					18°					
<i>r</i>	log sin 0	log tan 0	log cot 10	log cos 0	<i>r</i>	log sin 0	log tan 0	log cot 10	log cos 0	<i>r</i>
0	46 594	48 534	51 466	93 060	30	48 998	51 178	48 822	97 821	60
1	46 635	48 579	51 421	93 056	59	49 037	51 221	48 779	97 817	59
2	46 676	48 624	51 376	93 052	58	49 076	51 264	48 736	97 812	58
3	46 717	48 669	51 331	93 048	57	49 115	51 306	48 694	97 808	57
4	46 758	48 714	51 286	93 044	56	49 153	51 349	48 651	97 804	56
5	46 800	48 759	51 241	93 040	55	49 192	51 392	48 608	97 800	55
6	46 841	48 804	51 196	93 036	54	49 231	51 435	48 565	97 796	54
7	46 882	48 849	51 151	93 032	53	49 269	51 478	48 522	97 792	53
8	46 923	48 894	51 106	93 029	52	49 308	51 520	48 480	97 788	52
9	46 964	48 939	51 061	93 025	51	49 347	51 563	48 437	97 784	51
10	47 005	48 984	51 016	93 021	50	49 385	51 605	48 394	97 779	50
11	47 045	49 029	50 971	93 017	49	49 424	51 648	48 352	97 775	49
12	47 086	49 073	50 927	93 013	48	49 462	51 691	48 309	97 771	48
13	47 127	49 118	50 882	93 009	47	49 500	51 734	48 266	97 767	47
14	47 168	49 163	50 837	93 005	46	49 539	51 776	48 224	97 763	46
15	47 209	49 207	50 793	93 001	45	49 577	51 819	48 181	97 759	45
16	47 249	49 252	50 748	92 997	44	49 615	51 861	48 139	97 754	44
17	47 290	49 296	50 704	92 993	43	49 654	51 903	48 097	97 750	43
18	47 330	49 341	50 659	92 989	42	49 692	51 946	48 054	97 746	42
19	47 371	49 385	50 615	92 986	41	49 730	51 988	48 012	97 742	41
20	47 411	49 430	50 570	92 982	40	49 768	52 031	47 969	97 738	40
21	47 452	49 474	50 526	92 978	39	49 806	52 073	47 927	97 734	39
22	47 492	49 519	50 481	92 974	38	49 844	52 115	47 885	97 729	38
23	47 533	49 563	50 437	92 970	37	49 882	52 157	47 843	97 725	37
24	47 573	49 607	50 393	92 966	36	49 920	52 200	47 800	97 721	36
25	47 613	49 652	50 348	92 962	35	49 958	52 242	47 758	97 717	35
26	47 654	49 696	50 304	92 958	34	49 996	52 284	47 716	97 713	34
27	47 694	49 740	50 260	92 954	33	50 034	52 326	47 674	97 708	33
28	47 734	49 784	50 216	92 950	32	50 072	52 368	47 632	97 704	32
29	47 774	49 828	50 172	92 946	31	50 110	52 410	47 590	97 700	31
30	47 814	49 872	50 128	92 942	30	50 148	52 452	47 548	97 696	30
31	47 854	49 916	50 084	92 938	29	50 185	52 494	47 506	97 691	29
32	47 894	49 960	50 040	92 934	28	50 223	52 536	47 464	97 687	28
33	47 934	50 004	49 996	92 930	27	50 261	52 578	47 422	97 683	27
34	47 974	50 048	49 952	92 926	26	50 298	52 620	47 380	97 679	26
35	48 014	50 092	49 908	92 922	25	50 336	52 661	47 339	97 674	25
36	48 054	50 136	49 864	92 918	24	50 374	52 703	47 297	97 670	24
37	48 094	50 180	49 820	92 914	23	50 411	52 745	47 255	97 666	23
38	48 133	50 223	49 777	92 910	22	50 449	52 787	47 213	97 662	22
39	48 173	50 267	49 733	92 906	21	50 486	52 829	47 171	97 657	21
40	48 213	50 311	49 689	92 902	20	50 523	52 870	47 130	97 653	20
41	48 252	50 355	49 645	92 898	19	50 561	52 912	47 088	97 649	19
42	48 292	50 398	49 602	92 894	18	50 598	52 953	47 047	97 645	18
43	48 332	50 442	49 558	92 890	17	50 635	52 995	47 005	97 640	17
44	48 371	50 485	49 515	92 886	16	50 673	53 037	46 963	97 636	16
45	48 411	50 529	49 471	92 882	15	50 710	53 078	46 922	97 632	15
46	48 450	50 572	49 428	92 878	14	50 747	53 120	46 880	97 628	14
47	48 490	50 616	49 384	92 874	13	50 784	53 161	46 839	97 623	13
48	48 529	50 659	49 341	92 870	12	50 821	53 202	46 798	97 619	12
49	48 568	50 703	49 297	92 866	11	50 858	53 244	46 756	97 615	11
50	48 607	50 746	49 254	92 861	10	50 896	53 285	46 715	97 610	10
51	48 647	50 789	49 211	92 857	9	50 933	53 327	46 673	97 606	9
52	48 686	50 833	49 167	92 853	8	50 970	53 368	46 632	97 602	8
53	48 725	50 876	49 124	92 849	7	51 007	53 409	46 591	97 597	7
54	48 764	50 919	49 081	92 845	6	51 043	53 450	46 550	97 593	6
55	48 803	50 962	49 038	92 841	5	51 080	53 492	46 508	97 589	5
56	48 842	51 005	48 995	92 837	4	51 117	53 533	46 467	97 584	4
57	48 881	51 048	48 952	92 833	3	51 154	53 574	46 426	97 580	3
58	48 920	51 092	48 908	92 829	2	51 191	53 615	46 385	97 575	2
59	48 959	51 135	48 865	92 825	1	51 227	53 656	46 344	97 571	1
60	48 998	51 178	48 822	92 821	0	51 264	53 697	46 303	97 567	0

19°					20°					
	log sin	log tan	log cot	log cos		log sin	log tan	log cot	log cos	
	0	0	10	0		0	0	10	0	
0	51 264	53 697	46 303	97 567	60	53 405	56 107	43 893	97 299	60
1	51 301	53 738	46 262	97 563	59	53 440	56 146	43 854	97 294	59
2	51 338	53 779	46 221	97 558	58	53 475	56 185	43 815	97 289	58
3	51 374	53 820	46 180	97 554	57	53 509	56 224	43 776	97 285	57
4	51 411	53 861	46 139	97 550	56	53 544	56 264	43 736	97 280	56
5	51 447	53 902	46 098	97 545	55	53 578	56 303	43 697	97 276	55
6	51 484	53 943	46 057	97 541	54	53 613	56 342	43 658	97 271	54
7	51 520	53 984	46 016	97 536	53	53 647	56 381	43 619	97 266	53
8	51 557	54 025	45 975	97 532	52	53 682	56 420	43 580	97 262	52
9	51 593	54 065	45 935	97 528	51	53 716	56 459	43 541	97 257	51
10	51 629	54 106	45 894	97 523	50	53 751	56 498	43 502	97 252	50
11	51 666	54 147	45 853	97 519	49	53 785	56 537	43 463	97 248	49
12	51 702	54 187	45 813	97 515	48	53 819	56 576	43 424	97 243	48
13	51 738	54 228	45 772	97 510	47	53 854	56 615	43 385	97 238	47
14	51 774	54 269	45 731	97 506	46	53 888	56 654	43 346	97 234	46
15	51 811	54 309	45 691	97 501	45	53 922	56 693	43 307	97 229	45
16	51 847	54 350	45 650	97 497	44	53 957	56 732	43 268	97 224	44
17	51 883	54 390	45 610	97 492	43	53 991	56 771	43 229	97 220	43
18	51 919	54 431	45 569	97 488	42	54 025	56 810	43 190	97 215	42
19	51 955	54 471	45 529	97 484	41	54 059	56 849	43 151	97 210	41
20	51 991	54 512	45 488	97 479	40	54 093	56 887	43 113	97 206	40
21	52 027	54 552	45 448	97 475	39	54 127	56 926	43 074	97 201	39
22	52 063	54 593	45 407	97 470	38	54 161	56 965	43 035	97 196	38
23	52 099	54 633	45 367	97 466	37	54 195	57 004	42 996	97 192	37
24	52 135	54 673	45 327	97 461	36	54 229	57 042	42 958	97 187	36
25	52 171	54 714	45 286	97 457	35	54 263	57 081	42 919	97 182	35
26	52 207	54 754	45 246	97 453	34	54 297	57 120	42 880	97 178	34
27	52 242	54 794	45 206	97 448	33	54 331	57 158	42 842	97 173	33
28	52 278	54 835	45 165	97 444	32	54 365	57 197	42 803	97 168	32
29	52 314	54 875	45 125	97 439	31	54 399	57 235	42 765	97 163	31
30	52 350	54 915	45 085	97 435	30	54 433	57 274	42 726	97 159	30
31	52 385	54 955	45 045	97 430	29	54 466	57 312	42 688	97 154	29
32	52 421	54 995	45 005	97 426	28	54 500	57 351	42 649	97 149	28
33	52 456	55 035	44 965	97 421	27	54 534	57 389	42 611	97 145	27
34	52 492	55 075	44 925	97 417	26	54 567	57 428	42 572	97 140	26
35	52 527	55 115	44 885	97 412	25	54 601	57 466	42 534	97 135	25
36	52 563	55 155	44 845	97 408	24	54 635	57 504	42 496	97 130	24
37	52 598	55 195	44 805	97 403	23	54 668	57 543	42 457	97 126	23
38	52 634	55 235	44 765	97 399	22	54 702	57 581	42 419	97 121	22
39	52 669	55 275	44 725	97 394	21	54 735	57 619	42 381	97 116	21
40	52 705	55 315	44 685	97 390	20	54 769	57 658	42 342	97 111	20
41	52 740	55 355	44 645	97 385	19	54 802	57 696	42 304	97 107	19
42	52 775	55 395	44 605	97 381	18	54 835	57 734	42 266	97 102	18
43	52 811	55 434	44 566	97 376	17	54 869	57 772	42 228	97 097	17
44	52 846	55 474	44 526	97 372	16	54 903	57 810	42 190	97 092	16
45	52 881	55 514	44 486	97 367	15	54 936	57 849	42 151	97 087	15
46	52 916	55 554	44 446	97 363	14	54 969	57 887	42 113	97 083	14
47	52 951	55 593	44 407	97 358	13	55 003	57 925	42 075	97 078	13
48	52 986	55 633	44 367	97 353	12	55 036	57 963	42 037	97 073	12
49	53 021	55 673	44 327	97 349	11	55 069	58 001	41 999	97 068	11
50	53 056	55 712	44 288	97 344	10	55 102	58 039	41 961	97 063	10
51	53 092	55 752	44 248	97 340	9	55 136	58 077	41 923	97 059	9
52	53 126	55 791	44 209	97 335	8	55 169	58 115	41 885	97 054	8
53	53 161	55 831	44 169	97 331	7	55 202	58 153	41 847	97 049	7
54	53 196	55 870	44 130	97 326	6	55 235	58 191	41 809	97 044	6
55	53 231	55 910	44 090	97 322	5	55 268	58 229	41 771	97 039	5
56	53 266	55 949	44 051	97 317	4	55 301	58 267	41 733	97 035	4
57	53 301	55 989	44 011	97 312	3	55 334	58 304	41 696	97 030	3
58	53 336	56 028	43 972	97 308	2	55 367	58 342	41 658	97 025	2
59	53 370	56 067	43 933	97 303	1	55 400	58 380	41 620	97 020	1
60	53 405	56 107	43 893	97 299	0	55 433	58 418	41 582	97 015	0
	0	0	10	0		0	0	10	0	
	log cos	log cot	log tan	log sin		log cos	log cot	log tan	log sin	

21°					22°				
	log sin	log tan	log cot	log cos		log sin	log tan	log cot	log cos
0	0	0	10	0	0	0	0	10	0
1	55 433	58 418	41 582	97 015	1	57 358	60 641	39 359	96 717
2	55 466	58 455	41 545	97 010	2	57 389	60 677	39 323	96 711
3	55 499	58 493	41 507	97 005	3	57 420	60 714	39 286	96 706
4	55 532	58 531	41 469	97 001	4	57 451	60 750	39 250	96 701
5	55 564	58 569	41 431	96 996	5	57 482	60 786	39 214	96 696
6	55 597	58 606	41 394	96 991	6	57 514	60 823	39 177	96 691
7	55 630	58 644	41 356	96 986	7	57 545	60 859	39 141	96 686
8	55 663	58 681	41 319	96 981	8	57 576	60 895	39 105	96 681
9	55 695	58 719	41 281	96 976	9	57 607	60 931	39 069	96 676
10	55 728	58 757	41 243	96 971	10	57 638	60 967	39 033	96 670
11	55 761	58 794	41 206	96 966	11	57 669	61 004	38 996	96 665
12	55 793	58 832	41 168	96 962	12	57 700	61 040	38 960	96 660
13	55 826	58 869	41 131	96 957	13	57 731	61 076	38 924	96 655
14	55 858	58 907	41 093	96 952	14	57 762	61 112	38 888	96 650
15	55 891	58 944	41 056	96 947	15	57 793	61 148	38 852	96 645
16	55 923	58 981	41 019	96 942	16	57 824	61 184	38 816	96 640
17	55 956	59 019	40 981	96 937	17	57 855	61 220	38 780	96 634
18	55 988	59 056	40 944	96 932	18	57 885	61 256	38 744	96 629
19	56 021	59 094	40 906	96 927	19	57 916	61 292	38 708	96 624
20	56 053	59 131	40 869	96 922	20	57 947	61 328	38 672	96 619
21	56 085	59 168	40 832	96 917	21	57 978	61 364	38 636	96 614
22	56 118	59 205	40 795	96 912	22	58 008	61 400	38 600	96 608
23	56 150	59 243	40 757	96 907	23	58 039	61 436	38 564	96 603
24	56 182	59 280	40 720	96 903	24	58 070	61 472	38 528	96 598
25	56 215	59 317	40 683	96 898	25	58 101	61 508	38 492	96 593
26	56 247	59 354	40 646	96 893	26	58 131	61 544	38 456	96 588
27	56 279	59 391	40 609	96 888	27	58 162	61 579	38 421	96 582
28	56 311	59 429	40 571	96 883	28	58 192	61 615	38 385	96 577
29	56 343	59 466	40 534	96 878	29	58 223	61 651	38 349	96 572
30	56 375	59 503	40 497	96 873	30	58 253	61 687	38 313	96 567
31	56 408	59 540	40 460	96 868	31	58 284	61 722	38 278	96 562
32	56 440	59 577	40 423	96 863	32	58 314	61 758	38 242	96 556
33	56 472	59 614	40 386	96 858	33	58 345	61 794	38 206	96 551
34	56 504	59 651	40 349	96 853	34	58 375	61 830	38 170	96 546
35	56 536	59 688	40 312	96 848	35	58 406	61 865	38 135	96 541
36	56 568	59 725	40 275	96 843	36	58 436	61 901	38 099	96 535
37	56 599	59 762	40 238	96 838	37	58 467	61 936	38 064	96 530
38	56 631	59 799	40 201	96 833	38	58 497	61 972	38 028	96 525
39	56 663	59 835	40 165	96 828	39	58 527	62 008	37 992	96 520
40	56 695	59 872	40 128	96 823	40	58 557	62 043	37 957	96 514
41	56 727	59 909	40 091	96 818	41	58 588	62 079	37 921	96 509
42	56 759	59 946	40 054	96 813	42	58 618	62 114	37 886	96 504
43	56 790	59 983	40 017	96 808	43	58 648	62 150	37 850	96 498
44	56 822	60 019	39 981	96 803	44	58 678	62 185	37 815	96 493
45	56 854	60 056	39 944	96 798	45	58 709	62 221	37 779	96 488
46	56 886	60 093	39 907	96 793	46	58 739	62 256	37 744	96 483
47	56 917	60 130	39 870	96 788	47	58 769	62 292	37 708	96 477
48	56 949	60 166	39 834	96 783	48	58 799	62 327	37 673	96 472
49	56 980	60 203	39 797	96 778	49	58 829	62 362	37 638	96 467
50	57 012	60 240	39 760	96 772	50	58 859	62 398	37 602	96 461
51	57 044	60 276	39 724	96 767	51	58 889	62 433	37 567	96 456
52	57 075	60 313	39 687	96 762	52	58 919	62 468	37 532	96 451
53	57 107	60 349	39 651	96 757	53	58 949	62 504	37 496	96 445
54	57 138	60 386	39 614	96 752	54	58 979	62 539	37 461	96 440
55	57 169	60 422	39 578	96 747	55	59 009	62 574	37 426	96 435
56	57 201	60 459	39 541	96 742	56	59 039	62 609	37 391	96 429
57	57 232	60 495	39 505	96 737	57	59 069	62 645	37 355	96 424
58	57 264	60 532	39 468	96 732	58	59 098	62 680	37 320	96 419
59	57 295	60 568	39 432	96 727	59	59 128	62 715	37 285	96 413
60	57 326	60 605	39 395	96 722	60	59 158	62 750	37 250	96 408
	0	0	10	0		0	0	10	0
	log cos	log cot	log tan	log sin		log cos	log cot	log tan	log sin



23°					24°					
	log sin	log tan	log cot	log csc		log sin	log tan	log cot	log csc	
	9	9	10	9		9	9	10	9	
0	59188	62785	37215	96403	60	60931	64858	35142	96073	60
1	59218	62820	37180	96397	59	60960	64892	35108	96067	59
2	59247	62855	37145	96392	58	60988	64926	35074	96062	58
3	59277	62890	37110	96387	57	61016	64960	35040	96056	57
4	59307	62926	37074	96381	56	61045	64994	35006	96050	56
5	59336	62961	37039	96376	55	61073	65028	34972	96045	55
6	59366	62996	37004	96370	54	61101	65062	34938	96039	54
7	59396	63031	36969	96365	53	61129	65096	34904	96034	53
8	59425	63066	36934	96360	52	61158	65130	34870	96028	52
9	59455	63101	36899	96354	51	61186	65164	34836	96022	51
10	59484	63135	36865	96349	50	61214	65197	34803	96017	50
11	59514	63170	36830	96343	49	61242	65231	34769	96011	49
12	59543	63205	36795	96338	48	61270	65265	34735	96005	48
13	59573	63240	36760	96333	47	61298	65299	34701	96000	47
14	59602	63275	36725	96327	46	61326	65333	34667	95994	46
15	59632	63310	36690	96322	45	61354	65366	34634	95988	45
16	59661	63345	36655	96316	44	61382	65400	34600	95982	44
17	59690	63379	36621	96311	43	61411	65434	34566	95977	43
18	59720	63414	36586	96305	42	61438	65467	34533	95971	42
19	59749	63449	36551	96300	41	61466	65501	34500	95965	41
20	59778	63484	36516	96294	40	61494	65535	34465	95960	40
21	59808	63519	36481	96289	39	61522	65568	34432	95954	39
22	59837	63553	36447	96284	38	61550	65602	34398	95948	38
23	59866	63588	36412	96278	37	61578	65636	34364	95942	37
24	59895	63623	36377	96273	36	61606	65669	34331	95937	36
25	59924	63657	36343	96267	35	61634	65703	34297	95931	35
26	59954	63692	36308	96262	34	61662	65736	34264	95925	34
27	59983	63726	36274	96256	33	61689	65770	34230	95920	33
28	60012	63761	36239	96251	32	61717	65803	34197	95914	32
29	60041	63796	36204	96245	31	61745	65837	34163	95908	31
30	60070	63830	36170	96240	30	61773	65870	34130	95902	30
31	60099	63865	36135	96234	29	61800	65904	34096	95897	29
32	60128	63899	36101	96229	28	61828	65937	34063	95891	28
33	60157	63934	36066	96223	27	61856	65971	34029	95885	27
34	60186	63968	36032	96218	26	61883	66004	33996	95879	26
35	60215	64003	35997	96212	25	61911	66038	33962	95873	25
36	60244	64037	35963	96207	24	61939	66071	33929	95868	24
37	60273	64072	35928	96201	23	61966	66104	33896	95862	23
38	60302	64106	35894	96196	22	61994	66138	33862	95856	22
39	60331	64140	35860	96190	21	62021	66171	33829	95850	21
40	60359	64175	35825	96185	20	62049	66204	33796	95844	20
41	60388	64209	35791	96179	19	62076	66238	33762	95839	19
42	60417	64243	35757	96174	18	62104	66271	33729	95833	18
43	60446	64278	35722	96168	17	62131	66304	33696	95827	17
44	60474	64312	35688	96162	16	62159	66337	33663	95821	16
45	60503	64346	35654	96157	15	62186	66371	33629	95815	15
46	60532	64381	35619	96151	14	62214	66404	33596	95810	14
47	60561	64415	35585	96146	13	62241	66437	33563	95804	13
48	60589	64449	35551	96140	12	62268	66470	33530	95798	12
49	60618	64483	35517	96135	11	62296	66503	33497	95792	11
50	60646	64517	35483	96129	10	62323	66537	33463	95786	10
51	60675	64552	35448	96123	9	62350	66570	33430	95780	9
52	60704	64586	35414	96118	8	62377	66603	33397	95775	8
53	60732	64620	35380	96112	7	62405	66636	33364	95769	7
54	60761	64654	35346	96107	6	62432	66669	33331	95763	6
55	60789	64688	35312	96101	5	62459	66702	33298	95757	5
56	60818	64722	35278	96095	4	62486	66735	33265	95751	4
57	60846	64756	35244	96090	3	62513	66768	33232	95745	3
58	60875	64790	35210	96084	2	62541	66801	33199	95739	2
59	60903	64824	35176	96079	1	62568	66834	33166	95733	1
60	60931	64858	35142	96073	0	62595	66867	33133	95728	0
	log csc	log cot	log tan	log sin		log csc	log cot	log tan	log sin	



25°					26°					
<i>i</i>	log sin D	log tan D	log cot 10	log cos D	<i>i</i>	log sin D	log tan D	log cot 10	log cos D	<i>i</i>
0	62 595	66 867	33 133	95 728	60	64 184	68 613	31 182	95 366	60
1	62 622	66 900	33 100	95 722	59	64 210	68 850	31 150	95 360	59
2	62 649	66 933	33 067	95 716	58	64 236	68 882	31 118	95 354	58
3	62 676	66 966	33 034	95 710	57	64 262	68 914	31 086	95 348	57
4	62 703	66 999	33 001	95 704	56	64 288	68 946	31 054	95 341	56
5	62 730	67 032	32 968	95 698	55	64 313	68 978	31 022	95 335	55
6	62 757	67 065	32 935	95 692	54	64 339	69 010	30 990	95 329	54
7	62 784	67 098	32 902	95 686	53	64 365	69 042	30 958	95 323	53
8	62 811	67 131	32 869	95 680	52	64 391	69 074	30 926	95 317	52
9	62 838	67 163	32 837	95 674	51	64 417	69 106	30 894	95 310	51
10	62 865	67 196	32 804	95 668	50	64 442	69 138	30 862	95 304	50
11	62 892	67 229	32 771	95 663	49	64 468	69 170	30 830	95 298	49
12	62 918	67 262	32 738	95 657	48	64 494	69 202	30 798	95 292	48
13	62 945	67 295	32 705	95 651	47	64 519	69 234	30 766	95 286	47
14	62 972	67 327	32 673	95 645	46	64 545	69 266	30 734	95 279	46
15	62 999	67 360	32 640	95 639	45	64 571	69 298	30 702	95 273	45
16	63 026	67 393	32 607	95 633	44	64 596	69 329	30 671	95 267	44
17	63 052	67 426	32 574	95 627	43	64 622	69 361	30 639	95 261	43
18	63 079	67 458	32 542	95 621	42	64 647	69 393	30 607	95 254	42
19	63 106	67 491	32 509	95 615	41	64 673	69 425	30 575	95 248	41
20	63 133	67 524	32 476	95 609	40	64 698	69 457	30 543	95 242	40
21	63 159	67 556	32 444	95 603	39	64 724	69 488	30 512	95 236	39
22	63 186	67 589	32 411	95 597	38	64 749	69 520	30 480	95 229	38
23	63 213	67 622	32 378	95 591	37	64 775	69 552	30 448	95 223	37
24	63 239	67 654	32 346	95 585	36	64 800	69 584	30 416	95 217	36
25	63 266	67 687	32 313	95 579	35	64 826	69 615	30 385	95 211	35
26	63 292	67 719	32 281	95 573	34	64 851	69 647	30 353	95 204	34
27	63 319	67 752	32 248	95 567	33	64 877	69 679	30 321	95 198	33
28	63 345	67 785	32 215	95 561	32	64 902	69 710	30 290	95 192	32
29	63 372	67 817	32 183	95 555	31	64 927	69 742	30 258	95 185	31
30	63 398	67 850	32 150	95 549	30	64 953	69 774	30 226	95 179	30
31	63 425	67 882	32 118	95 543	29	64 978	69 805	30 195	95 173	29
32	63 451	67 915	32 085	95 537	28	65 003	69 837	30 163	95 167	28
33	63 478	67 947	32 053	95 531	27	65 029	69 868	30 132	95 160	27
34	63 504	67 980	32 020	95 525	26	65 054	69 900	30 100	95 154	26
35	63 531	68 012	31 988	95 519	25	65 079	69 932	30 068	95 148	25
36	63 557	68 044	31 956	95 513	24	65 104	69 963	30 037	95 141	24
37	63 583	68 077	31 923	95 507	23	65 130	69 995	30 005	95 135	23
38	63 610	68 109	31 891	95 500	22	65 155	70 026	29 974	95 129	22
39	63 636	68 142	31 858	95 494	21	65 180	70 058	29 942	95 122	21
40	63 662	68 174	31 826	95 488	20	65 205	70 089	29 911	95 116	20
41	63 689	68 206	31 794	95 482	19	65 230	70 121	29 879	95 110	19
42	63 715	68 239	31 761	95 476	18	65 255	70 152	29 848	95 103	18
43	63 741	68 271	31 729	95 470	17	65 281	70 184	29 816	95 097	17
44	63 767	68 303	31 697	95 464	16	65 306	70 215	29 785	95 090	16
45	63 794	68 336	31 664	95 458	15	65 331	70 247	29 753	95 084	15
46	63 820	68 368	31 632	95 452	14	65 356	70 278	29 722	95 078	14
47	63 846	68 400	31 600	95 446	13	65 381	70 309	29 691	95 071	13
48	63 872	68 432	31 568	95 440	12	65 406	70 341	29 659	95 065	12
49	63 898	68 465	31 535	95 434	11	65 431	70 372	29 628	95 059	11
50	63 924	68 497	31 503	95 427	10	65 456	70 404	29 596	95 052	10
51	63 950	68 529	31 471	95 421	9	65 481	70 435	29 565	95 046	9
52	63 976	68 561	31 439	95 415	8	65 506	70 466	29 534	95 039	8
53	64 002	68 593	31 407	95 409	7	65 531	70 498	29 502	95 033	7
54	64 028	68 626	31 374	95 403	6	65 556	70 529	29 471	95 027	6
55	64 054	68 658	31 342	95 397	5	65 580	70 560	29 440	95 020	5
56	64 080	68 690	31 310	95 391	4	65 605	70 592	29 408	95 014	4
57	64 106	68 722	31 278	95 384	3	65 630	70 623	29 377	95 007	3
58	64 132	68 754	31 246	95 378	2	65 655	70 654	29 346	95 001	2
59	64 158	68 786	31 214	95 372	1	65 680	70 685	29 315	94 995	1
60	64 184	68 818	31 182	95 366	0	65 705	70 717	29 283	94 988	0
<i>i</i>	log cos D	log cot 10	log tan D	log sin D	<i>i</i>	log cos D	log cot 10	log tan D	log sin D	<i>i</i>

27°					28°				
	log sin	log tan	log cot	log cos		log sin	log tan	log cot	log cos
°	'	0	10	0	°	'	0	10	0
0	65 705	70 717	29 283	94 988	60	67 161	72 567	27 433	94 593
1	65 729	70 748	29 252	94 982	59	67 185	72 598	27 402	94 597
2	65 754	70 779	29 221	94 975	58	67 208	72 628	27 372	94 580
3	65 779	70 810	29 190	94 969	57	67 232	72 659	27 341	94 573
4	65 804	70 841	29 159	94 962	56	67 256	72 689	27 311	94 567
5	65 828	70 873	29 127	94 956	55	67 280	72 720	27 280	94 560
6	65 853	70 904	29 096	94 949	54	67 303	72 750	27 250	94 553
7	65 878	70 935	29 065	94 942	53	67 327	72 780	27 220	94 546
8	65 902	70 966	29 034	94 936	52	67 350	72 811	27 189	94 540
9	65 927	70 997	29 003	94 930	51	67 374	72 841	27 159	94 533
10	65 952	71 028	28 972	94 923	50	67 398	72 872	27 128	94 526
11	65 976	71 059	28 941	94 917	49	67 421	72 902	27 098	94 519
12	66 001	71 090	28 910	94 911	48	67 445	72 932	27 068	94 513
13	66 025	71 121	28 879	94 904	47	67 468	72 963	27 037	94 506
14	66 050	71 153	28 847	94 898	46	67 492	72 993	27 007	94 499
15	66 075	71 184	28 816	94 891	45	67 515	73 023	26 977	94 492
16	66 099	71 215	28 785	94 885	44	67 539	73 054	26 946	94 485
17	66 124	71 246	28 754	94 878	43	67 562	73 084	26 916	94 479
18	66 148	71 277	28 723	94 871	42	67 586	73 114	26 886	94 472
19	66 173	71 308	28 692	94 865	41	67 609	73 144	26 856	94 465
20	66 197	71 339	28 661	94 858	40	67 633	73 175	26 825	94 458
21	66 221	71 370	28 630	94 852	39	67 656	73 205	26 795	94 451
22	66 246	71 401	28 599	94 845	38	67 680	73 235	26 765	94 445
23	66 270	71 431	28 569	94 839	37	67 703	73 265	26 735	94 438
24	66 295	71 462	28 538	94 832	36	67 726	73 295	26 705	94 431
25	66 319	71 493	28 507	94 826	35	67 750	73 326	26 674	94 424
26	66 343	71 524	28 476	94 819	34	67 773	73 356	26 644	94 417
27	66 368	71 555	28 445	94 813	33	67 796	73 386	26 614	94 410
28	66 392	71 586	28 414	94 806	32	67 820	73 416	26 584	94 404
29	66 416	71 617	28 383	94 799	31	67 843	73 446	26 554	94 397
30	66 441	71 648	28 352	94 793	30	67 866	73 476	26 524	94 390
31	66 465	71 679	28 321	94 786	29	67 890	73 507	26 493	94 383
32	66 489	71 709	28 291	94 780	28	67 913	73 537	26 463	94 376
33	66 513	71 740	28 260	94 773	27	67 936	73 567	26 433	94 369
34	66 537	71 771	28 229	94 767	26	67 959	73 597	26 403	94 362
35	66 562	71 802	28 198	94 760	25	67 982	73 627	26 373	94 355
36	66 586	71 833	28 167	94 753	24	68 006	73 657	26 343	94 349
37	66 610	71 863	28 137	94 747	23	68 029	73 687	26 313	94 342
38	66 634	71 894	28 106	94 740	22	68 052	73 717	26 283	94 335
39	66 658	71 925	28 075	94 734	21	68 075	73 747	26 253	94 328
40	66 682	71 955	28 045	94 727	20	68 098	73 777	26 223	94 321
41	66 706	71 986	28 014	94 720	19	68 121	73 807	26 193	94 314
42	66 731	72 017	27 983	94 714	18	68 144	73 837	26 163	94 307
43	66 755	72 048	27 952	94 707	17	68 167	73 867	26 133	94 300
44	66 779	72 078	27 922	94 700	16	68 190	73 897	26 103	94 293
45	66 803	72 109	27 891	94 694	15	68 213	73 927	26 073	94 286
46	66 827	72 140	27 860	94 687	14	68 237	73 957	26 043	94 279
47	66 851	72 170	27 830	94 680	13	68 260	73 987	26 013	94 273
48	66 875	72 201	27 799	94 674	12	68 283	74 017	25 983	94 266
49	66 899	72 231	27 769	94 667	11	68 305	74 047	25 953	94 259
50	66 922	72 262	27 738	94 660	10	68 328	74 077	25 923	94 252
51	66 946	72 293	27 707	94 654	9	68 351	74 107	25 893	94 245
52	66 970	72 323	27 677	94 647	8	68 374	74 137	25 863	94 238
53	66 994	72 354	27 646	94 640	7	68 397	74 166	25 834	94 231
54	67 018	72 384	27 616	94 634	6	68 420	74 196	25 804	94 224
55	67 042	72 415	27 585	94 627	5	68 443	74 225	25 774	94 217
56	67 066	72 445	27 555	94 620	4	68 466	74 255	25 744	94 210
57	67 090	72 476	27 524	94 614	3	68 489	74 286	25 714	94 203
58	67 113	72 506	27 494	94 607	2	68 512	74 316	25 684	94 196
59	67 137	72 537	27 463	94 600	1	68 534	74 345	25 655	94 189
60	67 161	72 567	27 433	94 593	0	68 557	74 375	25 625	94 182
	log cos	log cot	log tan	log sin		log cos	log cot	log tan	log sin

29°					30°					
	log sin	log tan	log cot	log sec		log sin	log tan	log cot	log sec	
	0	0	10	0		0	0	10	0	
0	68 557	74 375	25 625	94 182	60	69 897	76 144	23 856	93 753	60
1	68 580	74 405	25 595	94 175	59	69 919	76 173	23 827	93 746	59
2	68 603	74 435	25 565	94 168	58	69 941	76 202	23 798	93 738	58
3	68 625	74 465	25 535	94 161	57	69 963	76 231	23 769	93 731	57
4	68 648	74 494	25 506	94 154	56	69 984	76 261	23 739	93 724	56
5	68 671	74 524	25 476	94 147	55	70 006	76 290	23 710	93 717	55
6	68 694	74 554	25 446	94 140	54	70 028	76 319	23 681	93 709	54
7	68 716	74 583	25 417	94 133	53	70 050	76 348	23 652	93 702	53
8	68 739	74 613	25 387	94 126	52	70 072	76 377	23 623	93 695	52
9	68 762	74 643	25 357	94 119	51	70 093	76 406	23 594	93 687	51
10	68 784	74 673	25 327	94 112	50	70 115	76 435	23 565	93 680	50
11	68 807	74 702	25 298	94 105	49	70 137	76 464	23 536	93 673	49
12	68 829	74 732	25 268	94 098	48	70 159	76 493	23 507	93 665	48
13	68 852	74 762	25 238	94 090	47	70 180	76 522	23 478	93 658	47
14	68 875	74 791	25 209	94 083	46	70 202	76 551	23 449	93 650	46
15	68 897	74 821	25 179	94 076	45	70 224	76 580	23 420	93 643	45
16	68 920	74 851	25 149	94 069	44	70 245	76 609	23 391	93 636	44
17	68 942	74 880	25 120	94 062	43	70 267	76 639	23 361	93 628	43
18	68 965	74 910	25 090	94 055	42	70 288	76 668	23 332	93 621	42
19	68 987	74 939	25 061	94 048	41	70 310	76 697	23 303	93 614	41
20	69 010	74 969	25 031	94 041	40	70 332	76 725	23 275	93 606	40
21	69 032	74 998	25 002	94 034	39	70 353	76 754	23 246	93 599	39
22	69 055	75 028	24 972	94 027	38	70 375	76 783	23 217	93 591	38
23	69 077	75 058	24 942	94 020	37	70 396	76 812	23 188	93 584	37
24	69 100	75 087	24 913	94 012	36	70 418	76 841	23 159	93 577	36
25	69 122	75 117	24 883	94 005	35	70 439	76 870	23 130	93 569	35
26	69 144	75 146	24 854	93 998	34	70 461	76 899	23 101	93 562	34
27	69 167	75 176	24 824	93 991	33	70 482	76 928	23 072	93 554	33
28	69 189	75 205	24 795	93 984	32	70 504	76 957	23 043	93 547	32
29	69 212	75 235	24 765	93 977	31	70 525	76 986	23 014	93 539	31
30	69 234	75 264	24 736	93 970	30	70 547	77 015	22 985	93 532	30
31	69 256	75 294	24 706	93 963	29	70 568	77 044	22 956	93 525	29
32	69 279	75 323	24 677	93 955	28	70 590	77 073	22 927	93 517	28
33	69 301	75 353	24 647	93 948	27	70 611	77 101	22 899	93 510	27
34	69 323	75 382	24 618	93 941	26	70 633	77 130	22 870	93 502	26
35	69 345	75 411	24 589	93 934	25	70 654	77 159	22 841	93 495	25
36	69 368	75 441	24 559	93 927	24	70 675	77 188	22 812	93 487	24
37	69 390	75 470	24 530	93 920	23	70 697	77 217	22 783	93 480	23
38	69 412	75 500	24 500	93 912	22	70 718	77 246	22 754	93 472	22
39	69 434	75 529	24 471	93 905	21	70 739	77 274	22 726	93 465	21
40	69 456	75 558	24 442	93 898	20	70 761	77 303	22 697	93 457	20
41	69 479	75 588	24 412	93 891	19	70 782	77 332	22 668	93 450	19
42	69 501	75 617	24 383	93 884	18	70 803	77 361	22 639	93 442	18
43	69 523	75 647	24 353	93 876	17	70 824	77 390	22 610	93 435	17
44	69 545	75 676	24 324	93 869	16	70 846	77 418	22 582	93 427	16
45	69 567	75 705	24 295	93 862	15	70 867	77 447	22 553	93 420	15
46	69 589	75 735	24 265	93 855	14	70 888	77 476	22 524	93 412	14
47	69 611	75 764	24 236	93 847	13	70 909	77 505	22 495	93 405	13
48	69 633	75 793	24 207	93 840	12	70 931	77 533	22 467	93 397	12
49	69 655	75 822	24 178	93 833	11	70 952	77 562	22 438	93 390	11
50	69 677	75 852	24 148	93 826	10	70 973	77 591	22 409	93 383	10
51	69 699	75 881	24 119	93 819	9	70 994	77 619	22 381	93 375	9
52	69 721	75 910	24 090	93 811	8	71 015	77 648	22 352	93 367	8
53	69 743	75 939	24 061	93 804	7	71 036	77 677	22 323	93 360	7
54	69 765	75 969	24 031	93 797	6	71 058	77 706	22 294	93 352	6
55	69 787	75 998	24 002	93 789	5	71 079	77 734	22 266	93 344	5
56	69 809	76 027	23 973	93 782	4	71 100	77 763	22 237	93 337	4
57	69 831	76 056	23 944	93 775	3	71 121	77 791	22 209	93 329	3
58	69 853	76 086	23 914	93 768	2	71 142	77 820	22 180	93 322	2
59	69 875	76 115	23 885	93 760	1	71 163	77 849	22 151	93 314	1
60	69 897	76 144	23 856	93 753	0	71 184	77 877	22 123	93 307	0
	log cos	log cot	log tan	log sin		log cos	log cot	log tan	log sin	

31°					32°					
<i>r</i>	log sin S	log tan T	log cot C	log cos O	<i>r</i>	log sin S	log tan T	log cot C	log cos O	<i>r</i>
0	71 184	77 877	22 123	93 307	60	72 421	79 579	20 421	92 512	60
1	71 205	77 906	22 074	93 299	59	72 441	79 607	20 393	92 834	59
2	71 226	77 935	22 065	93 291	58	72 461	79 635	20 365	92 826	58
3	71 247	77 963	22 037	93 284	57	72 482	79 663	20 337	92 818	57
4	71 268	77 992	22 008	93 276	56	72 502	79 691	20 309	92 810	56
5	71 289	78 020	21 980	93 269	55	72 522	79 719	20 281	92 803	55
6	71 310	78 049	21 951	93 261	54	72 542	79 747	20 253	92 795	54
7	71 331	78 077	21 923	93 253	53	72 562	79 776	20 224	92 787	53
8	71 352	78 106	21 894	93 246	52	72 582	79 804	20 196	92 779	52
9	71 373	78 135	21 865	93 238	51	72 602	79 832	20 168	92 771	51
10	71 393	78 163	21 837	93 230	50	72 622	79 860	20 140	92 763	50
11	71 414	78 192	21 808	93 223	49	72 643	79 888	20 112	92 755	49
12	71 435	78 220	21 780	93 215	48	72 663	79 916	20 084	92 747	48
13	71 456	78 249	21 751	93 207	47	72 683	79 944	20 056	92 739	47
14	71 477	78 277	21 723	93 200	46	72 703	79 972	20 028	92 731	46
15	71 498	78 306	21 694	93 192	45	72 723	80 000	20 000	92 723	45
16	71 519	78 334	21 666	93 184	44	72 743	80 028	19 972	92 715	44
17	71 539	78 363	21 637	93 177	43	72 763	80 056	19 944	92 707	43
18	71 560	78 391	21 609	93 169	42	72 783	80 084	19 916	92 699	42
19	71 581	78 419	21 581	93 161	41	72 803	80 112	19 888	92 691	41
20	71 602	78 448	21 552	93 154	40	72 823	80 140	19 860	92 683	40
21	71 622	78 476	21 524	93 146	39	72 843	80 168	19 832	92 675	39
22	71 643	78 505	21 495	93 138	38	72 863	80 195	19 805	92 667	38
23	71 664	78 533	21 467	93 131	37	72 883	80 223	19 777	92 659	37
24	71 685	78 562	21 438	93 123	36	72 902	80 251	19 749	92 651	36
25	71 705	78 590	21 410	93 115	35	72 922	80 279	19 721	92 643	35
26	71 726	78 618	21 382	93 108	34	72 942	80 307	19 693	92 635	34
27	71 747	78 647	21 353	93 100	33	72 962	80 335	19 665	92 627	33
28	71 767	78 675	21 325	93 092	32	72 982	80 363	19 637	92 619	32
29	71 788	78 704	21 296	93 084	31	73 002	80 391	19 609	92 611	31
30	71 809	78 732	21 268	93 077	30	73 022	80 419	19 581	92 603	30
31	71 829	78 760	21 240	93 069	29	73 041	80 447	19 553	92 595	29
32	71 850	78 789	21 211	93 061	28	73 061	80 474	19 526	92 587	28
33	71 870	78 817	21 183	93 053	27	73 081	80 502	19 498	92 579	27
34	71 891	78 845	21 155	93 046	26	73 101	80 530	19 470	92 571	26
35	71 911	78 874	21 126	93 038	25	73 121	80 558	19 442	92 563	25
36	71 932	78 902	21 098	93 030	24	73 140	80 586	19 414	92 555	24
37	71 952	78 930	21 070	93 022	23	73 160	80 614	19 386	92 546	23
38	71 973	78 959	21 041	93 014	22	73 180	80 642	19 358	92 538	22
39	71 994	78 987	21 013	93 007	21	73 200	80 669	19 331	92 530	21
40	72 014	79 015	20 985	92 999	20	73 219	80 697	19 303	92 522	20
41	72 034	79 043	20 957	92 991	19	73 239	80 725	19 275	92 514	19
42	72 055	79 072	20 928	92 983	18	73 259	80 753	19 247	92 506	18
43	72 075	79 100	20 900	92 976	17	73 278	80 781	19 219	92 498	17
44	72 096	79 128	20 872	92 968	16	73 298	80 808	19 192	92 490	16
45	72 116	79 156	20 844	92 960	15	73 318	80 836	19 164	92 482	15
46	72 137	79 185	20 815	92 952	14	73 337	80 864	19 136	92 473	14
47	72 157	79 213	20 787	92 944	13	73 357	80 892	19 108	92 465	13
48	72 177	79 241	20 759	92 936	12	73 377	80 919	19 081	92 457	12
49	72 198	79 269	20 731	92 929	11	73 396	80 947	19 053	92 449	11
50	72 218	79 297	20 703	92 921	10	73 416	80 975	19 025	92 441	10
51	72 238	79 326	20 674	92 913	9	73 435	81 003	18 997	92 433	9
52	72 259	79 354	20 646	92 905	8	73 455	81 030	18 970	92 425	8
53	72 279	79 382	20 618	92 897	7	73 474	81 058	18 942	92 416	7
54	72 299	79 410	20 590	92 889	6	73 494	81 086	18 914	92 408	6
55	72 320	79 438	20 562	92 881	5	73 513	81 113	18 887	92 400	5
56	72 340	79 466	20 534	92 874	4	73 533	81 141	18 859	92 392	4
57	72 360	79 495	20 505	92 866	3	73 552	81 169	18 831	92 384	3
58	72 381	79 523	20 477	92 858	2	73 572	81 196	18 804	92 376	2
59	72 401	79 551	20 449	92 850	1	73 591	81 224	18 776	92 367	1
60	72 421	79 579	20 421	92 842	0	73 611	81 252	18 748	92 359	0
	0	0	10	0		0	0	10	0	
<i>r</i>	log sec	log cot	log tan	log sin	<i>r</i>	log sec	log cot	log tan	log sin	<i>r</i>

33°					34°				
	log sin	log tan	log cot	log cos		log sin	log tan	log cot	log cos
0	73 611	81 252	18 748	92 359	60	74 756	82 899	17 101	91 857
1	73 630	81 279	18 721	92 351	59	74 775	82 926	17 074	91 849
2	73 650	81 307	18 693	92 343	58	74 794	82 953	17 047	91 840
3	73 669	81 335	18 665	92 335	57	74 812	82 980	17 020	91 832
4	73 689	81 362	18 638	92 326	56	74 831	83 008	16 992	91 823
5	73 708	81 390	18 610	92 318	55	74 850	83 035	16 965	91 815
6	73 727	81 418	18 582	92 310	54	74 868	83 062	16 938	91 806
7	73 747	81 445	18 555	92 302	53	74 887	83 089	16 911	91 798
8	73 766	81 473	18 527	92 293	52	74 906	83 117	16 883	91 789
9	73 785	81 500	18 500	92 285	51	74 924	83 144	16 856	91 781
10	73 805	81 528	18 472	92 277	50	74 943	83 171	16 829	91 772
11	73 824	81 556	18 444	92 269	49	74 961	83 198	16 802	91 763
12	73 843	81 583	18 417	92 260	48	74 980	83 225	16 775	91 755
13	73 863	81 611	18 389	92 252	47	74 999	83 252	16 748	91 746
14	73 882	81 638	18 362	92 244	46	75 017	83 280	16 720	91 738
15	73 901	81 666	18 334	92 235	45	75 036	83 307	16 693	91 729
16	73 921	81 693	18 307	92 227	44	75 054	83 334	16 666	91 720
17	73 940	81 721	18 279	92 219	43	75 073	83 361	16 639	91 712
18	73 959	81 748	18 252	92 211	42	75 091	83 388	16 612	91 703
19	73 978	81 776	18 224	92 202	41	75 110	83 415	16 585	91 695
20	73 997	81 803	18 197	92 194	40	75 128	83 442	16 558	91 686
21	74 017	81 831	18 169	92 186	39	75 147	83 470	16 530	91 677
22	74 036	81 858	18 142	92 177	38	75 165	83 497	16 503	91 669
23	74 055	81 886	18 114	92 169	37	75 184	83 524	16 476	91 660
24	74 074	81 913	18 087	92 161	36	75 202	83 551	16 449	91 651
25	74 093	81 941	18 059	92 152	35	75 221	83 578	16 422	91 643
26	74 113	81 968	18 032	92 144	34	75 239	83 605	16 395	91 634
27	74 132	81 996	18 004	92 136	33	75 258	83 632	16 368	91 625
28	74 151	82 023	17 977	92 127	32	75 276	83 659	16 341	91 617
29	74 170	82 051	17 949	92 119	31	75 294	83 686	16 314	91 608
30	74 189	82 078	17 922	92 111	30	75 313	83 713	16 287	91 599
31	74 208	82 106	17 894	92 102	29	75 331	83 740	16 260	91 591
32	74 227	82 133	17 867	92 094	28	75 350	83 768	16 232	91 582
33	74 246	82 161	17 839	92 086	27	75 368	83 795	16 205	91 573
34	74 265	82 188	17 812	92 077	26	75 386	83 822	16 178	91 565
35	74 284	82 215	17 785	92 069	25	75 405	83 849	16 151	91 556
36	74 303	82 243	17 757	92 060	24	75 423	83 876	16 124	91 547
37	74 322	82 270	17 730	92 052	23	75 441	83 903	16 097	91 538
38	74 341	82 298	17 702	92 044	22	75 459	83 930	16 070	91 530
39	74 360	82 325	17 675	92 035	21	75 478	83 957	16 043	91 521
40	74 379	82 352	17 648	92 027	20	75 496	83 984	16 016	91 512
41	74 398	82 380	17 620	92 018	19	75 514	84 011	15 989	91 504
42	74 417	82 407	17 593	92 010	18	75 533	84 038	15 962	91 495
43	74 436	82 435	17 565	92 002	17	75 551	84 065	15 935	91 486
44	74 455	82 462	17 538	91 993	16	75 569	84 092	15 908	91 477
45	74 474	82 489	17 511	91 985	15	75 587	84 119	15 881	91 469
46	74 493	82 517	17 483	91 976	14	75 605	84 146	15 854	91 460
47	74 512	82 544	17 456	91 968	13	75 624	84 173	15 827	91 451
48	74 531	82 571	17 429	91 959	12	75 642	84 200	15 800	91 442
49	74 549	82 599	17 401	91 951	11	75 660	84 227	15 773	91 433
50	74 568	82 626	17 374	91 942	10	75 678	84 254	15 746	91 425
51	74 587	82 653	17 347	91 934	9	75 696	84 280	15 720	91 416
52	74 606	82 681	17 319	91 925	8	75 714	84 307	15 693	91 407
53	74 625	82 708	17 292	91 917	7	75 733	84 334	15 666	91 398
54	74 644	82 735	17 265	91 908	6	75 751	84 361	15 639	91 389
55	74 662	82 762	17 238	91 900	5	75 769	84 388	15 612	91 381
56	74 681	82 790	17 210	91 891	4	75 787	84 415	15 585	91 372
57	74 700	82 817	17 183	91 883	3	75 805	84 442	15 558	91 363
58	74 719	82 844	17 156	91 874	2	75 823	84 469	15 531	91 354
59	74 737	82 871	17 129	91 866	1	75 841	84 496	15 504	91 345
60	74 756	82 899	17 101	91 857	0	75 859	84 523	15 477	91 336
	0	9	10	0		0	9	10	0
	log cos	log cot	log tan	log sin		log cos	log cot	log tan	log sin

35°				36°						
<i>i</i>	log sin 0	log tan 0	log cot 10	log cos 0	<i>i</i>	log sin 0	log tan 0	log cot 10	log cos 0	<i>i</i>
0	75 859	84 523	15 477	91 336	00	76 922	86 126	13 874	90 705	00
1	75 877	84 550	15 450	91 328	59	76 939	86 153	13 847	90 787	59
2	75 895	84 576	15 424	91 319	58	76 957	86 179	13 821	90 777	58
3	75 913	84 603	15 397	91 310	57	76 974	86 206	13 794	90 768	57
4	75 931	84 630	15 370	91 301	56	76 991	86 232	13 768	90 759	56
5	75 949	84 657	15 343	91 292	55	77 009	86 259	13 741	90 750	55
6	75 967	84 684	15 316	91 283	54	77 026	86 285	13 715	90 741	54
7	75 985	84 711	15 289	91 274	53	77 043	86 312	13 688	90 731	53
8	76 003	84 738	15 262	91 266	52	77 061	86 338	13 662	90 722	52
9	76 021	84 764	15 236	91 257	51	77 078	86 365	13 635	90 713	51
10	76 039	84 791	15 209	91 248	50	77 095	86 392	13 608	90 704	50
11	76 057	84 818	15 182	91 239	49	77 112	86 418	13 582	90 694	49
12	76 075	84 845	15 155	91 230	48	77 130	86 445	13 555	90 685	48
13	76 093	84 872	15 128	91 221	47	77 147	86 471	13 529	90 676	47
14	76 111	84 899	15 101	91 212	46	77 164	86 498	13 502	90 667	46
15	76 129	84 925	15 075	91 203	45	77 181	86 524	13 476	90 657	45
16	76 146	84 952	15 048	91 194	44	77 199	86 551	13 449	90 648	44
17	76 164	84 979	15 021	91 185	43	77 216	86 577	13 423	90 639	43
18	76 182	85 006	14 994	91 176	42	77 233	86 603	13 397	90 630	42
19	76 200	85 033	14 967	91 167	41	77 250	86 630	13 370	90 620	41
20	76 218	85 059	14 941	91 158	40	77 268	86 656	13 344	90 611	40
21	76 236	85 086	14 914	91 149	39	77 285	86 683	13 317	90 602	39
22	76 253	85 113	14 887	91 141	38	77 302	86 709	13 291	90 592	38
23	76 271	85 140	14 860	91 132	37	77 319	86 736	13 264	90 583	37
24	76 289	85 166	14 834	91 123	36	77 336	86 762	13 238	90 574	36
25	76 307	85 193	14 807	91 114	35	77 353	86 789	13 211	90 565	35
26	76 324	85 220	14 780	91 105	34	77 370	86 815	13 185	90 555	34
27	76 342	85 247	14 753	91 096	33	77 387	86 842	13 158	90 546	33
28	76 360	85 273	14 727	91 087	32	77 405	86 868	13 132	90 537	32
29	76 378	85 300	14 700	91 078	31	77 422	86 894	13 106	90 527	31
30	76 395	85 327	14 673	91 069	30	77 439	86 921	13 079	90 518	30
31	76 413	85 354	14 646	91 060	29	77 456	86 947	13 053	90 509	29
32	76 431	85 380	14 620	91 051	28	77 473	86 974	13 026	90 499	28
33	76 448	85 407	14 593	91 042	27	77 490	87 000	13 000	90 490	27
34	76 466	85 434	14 566	91 033	26	77 507	87 027	12 973	90 480	26
35	76 484	85 460	14 540	91 023	25	77 524	87 053	12 947	90 471	25
36	76 501	85 487	14 513	91 014	24	77 541	87 079	12 921	90 462	24
37	76 519	85 514	14 486	91 005	23	77 558	87 106	12 894	90 452	23
38	76 537	85 540	14 460	90 996	22	77 575	87 132	12 868	90 443	22
39	76 554	85 567	14 433	90 987	21	77 592	87 158	12 842	90 434	21
40	76 572	85 594	14 406	90 978	20	77 609	87 185	12 815	90 424	20
41	76 590	85 620	14 380	90 969	19	77 626	87 211	12 789	90 415	19
42	76 607	85 647	14 353	90 960	18	77 643	87 238	12 762	90 405	18
43	76 625	85 674	14 326	90 951	17	77 660	87 264	12 736	90 396	17
44	76 642	85 700	14 300	90 942	16	77 677	87 290	12 710	90 386	16
45	76 660	85 727	14 273	90 933	15	77 694	87 317	12 683	90 377	15
46	76 677	85 754	14 246	90 924	14	77 711	87 343	12 657	90 368	14
47	76 695	85 780	14 220	90 915	13	77 728	87 369	12 631	90 358	13
48	76 712	85 807	14 193	90 906	12	77 744	87 396	12 604	90 349	12
49	76 730	85 834	14 166	90 896	11	77 761	87 422	12 578	90 339	11
50	76 747	85 860	14 140	90 887	10	77 778	87 448	12 552	90 330	10
51	76 765	85 887	14 113	90 878	9	77 795	87 475	12 525	90 320	9
52	76 782	85 913	14 087	90 869	8	77 812	87 501	12 499	90 311	8
53	76 800	85 940	14 060	90 860	7	77 829	87 527	12 473	90 301	7
54	76 817	85 967	14 033	90 851	6	77 846	87 554	12 446	90 292	6
55	76 835	85 993	14 007	90 842	5	77 862	87 580	12 420	90 282	5
56	76 852	86 020	13 980	90 832	4	77 879	87 606	12 394	90 273	4
57	76 870	86 046	13 954	90 823	3	77 896	87 633	12 367	90 263	3
58	76 887	86 073	13 927	90 814	2	77 913	87 659	12 341	90 254	2
59	76 904	86 100	13 900	90 805	1	77 930	87 685	12 315	90 244	1
60	76 922	86 126	13 874	90 796	0	77 946	87 711	12 289	90 235	0
	0	0	10	0		0	0	10	0	
<i>i</i>	log cos	log cot	log tan	log sin	<i>i</i>	log cos	log cot	log tan	log sin	<i>i</i>



	log sin	log tan	log cot	log csc			log sin	log tan	log cot	log csc	
	0	0	10	0			0	0	10	0	
0	77946	87711	12289	90235	60	0	78934	89231	10719	89653	60
1	77963	87738	12262	90225	59	1	78950	89307	10693	89643	59
2	77980	87764	12236	90216	58	2	78967	89333	10667	89633	58
3	77997	87790	12210	90206	57	3	78983	89359	10641	89624	57
4	78013	87817	12183	90197	56	4	78999	89385	10615	89614	56
5	78030	87843	12157	90187	55	5	79015	89411	10589	89604	55
6	78047	87869	12131	90178	54	6	79031	89437	10563	89594	54
7	78063	87895	12105	90168	53	7	79047	89463	10537	89584	53
8	78080	87922	12078	90159	52	8	79063	89489	10511	89574	52
9	78097	87948	12052	90149	51	9	79079	89515	10485	89564	51
10	78113	87974	12026	90139	50	10	79095	89541	10459	89554	50
11	78130	88000	12000	90130	49	11	79111	89567	10433	89544	49
12	78147	88027	11973	90120	48	12	79128	89593	10407	89534	48
13	78163	88053	11947	90111	47	13	79144	89619	10381	89524	47
14	78180	88079	11921	90101	46	14	79160	89645	10355	89514	46
15	78197	88105	11895	90091	45	15	79176	89671	10329	89504	45
16	78213	88131	11869	90082	44	16	79192	89697	10303	89495	44
17	78230	88158	11842	90072	43	17	79208	89723	10277	89485	43
18	78246	88184	11816	90063	42	18	79224	89749	10251	89475	42
19	78263	88210	11790	90053	41	19	79240	89775	10225	89465	41
20	78280	88236	11764	90043	40	20	79256	89801	10199	89455	40
21	78296	88262	11738	90034	39	21	79272	89827	10173	89445	39
22	78313	88289	11711	90024	38	22	79288	89853	10147	89435	38
23	78329	88315	11685	90014	37	23	79304	89879	10121	89425	37
24	78346	88341	11659	90005	36	24	79319	89905	10095	89415	36
25	78362	88367	11633	89995	35	25	79335	89931	10069	89405	35
26	78379	88393	11607	89985	34	26	79351	89957	10043	89395	34
27	78395	88420	11580	89976	33	27	79367	89983	10017	89385	33
28	78412	88446	11554	89966	32	28	79383	90009	99991	89375	32
29	78428	88472	11528	89956	31	29	79399	90035	99965	89364	31
30	78445	88498	11502	89947	30	30	79415	90061	99939	89354	30
31	78461	88524	11476	89937	29	31	79431	90086	99914	89344	29
32	78478	88550	11450	89927	28	32	79447	90112	99888	89334	28
33	78494	88577	11423	89918	27	33	79463	90138	99862	89324	27
34	78510	88603	11397	89908	26	34	79478	90164	99836	89314	26
35	78527	88629	11371	89898	25	35	79494	90190	99810	89304	25
36	78543	88655	11345	89888	24	36	79510	90216	99784	89294	24
37	78560	88681	11319	89879	23	37	79526	90242	99758	89284	23
38	78576	88707	11293	89869	22	38	79542	90268	99732	89274	22
39	78592	88733	11267	89859	21	39	79558	90294	99706	89264	21
40	78609	88759	11241	89849	20	40	79573	90320	99680	89254	20
41	78625	88786	11214	89840	19	41	79589	90346	99654	89244	19
42	78642	88812	11188	89830	18	42	79605	90371	99628	89233	18
43	78658	88838	11162	89820	17	43	79621	90397	99603	89223	17
44	78674	88864	11136	89810	16	44	79636	90423	99577	89213	16
45	78691	88890	11110	89801	15	45	79652	90449	99551	89203	15
46	78707	88916	11084	89791	14	46	79668	90475	99525	89193	14
47	78723	88942	11058	89781	13	47	79684	90501	99499	89183	13
48	78739	88968	11032	89771	12	48	79699	90527	99473	89173	12
49	78756	88994	11006	89761	11	49	79715	90553	99447	89162	11
50	78772	89020	10980	89752	10	50	79731	90578	99422	89152	10
51	78788	89046	10954	89742	9	51	79746	90604	99396	89142	9
52	78805	89073	10927	89732	8	52	79762	90630	99370	89132	8
53	78821	89099	10901	89722	7	53	79778	90656	99344	89122	7
54	78837	89125	10875	89712	6	54	79793	90682	99318	89112	6
55	78853	89151	10849	89702	5	55	79809	90708	99292	89101	5
56	78869	89177	10823	89693	4	56	79825	90734	99266	89091	4
57	78886	89203	10797	89683	3	57	79840	90759	99241	89081	3
58	78902	89229	10771	89673	2	58	79856	90785	99215	89071	2
59	78918	89255	10745	89663	1	59	79872	90811	99189	89060	1
60	78934	89281	10719	89653	0	60	79887	90837	99163	89050	0
	log sin	log tan	log cot	log csc			log sin	log tan	log cot	log csc	

39°					40°						
	log sin	log tan	log cot	log cos		log sin	log tan	log cot	log cos		
°	'	0	10	0	°	'	0	10	0		
0		79 887	90 837	07 163	89 050	0		80 207	92 381	07 619	88 425
1		79 903	90 863	09 137	89 040	1		80 222	92 407	07 593	88 415
2		79 918	90 889	09 111	89 030	2		80 237	92 433	07 567	88 404
3		79 934	90 914	09 085	89 020	3		80 252	92 458	07 542	88 394
4		79 950	90 940	09 060	89 009	4		80 267	92 484	07 516	88 383
5		79 965	90 966	09 034	88 999	5		80 282	92 510	07 490	88 372
6		79 981	90 992	09 008	88 989	6		80 297	92 535	07 465	88 362
7		79 996	91 018	08 982	88 978	7		80 312	92 561	07 439	88 351
8		80 012	91 043	08 957	88 968	8		80 327	92 587	07 413	88 340
9		80 027	91 069	08 931	88 958	9		80 342	92 612	07 388	88 330
10		80 043	91 095	08 905	88 948	10		80 357	92 638	07 362	88 319
11		80 058	91 121	08 879	88 937	11		80 372	92 663	07 337	88 308
12		80 074	91 147	08 853	88 927	12		80 387	92 689	07 311	88 298
13		80 089	91 172	08 828	88 917	13		81 002	92 715	07 285	88 287
14		80 105	91 198	08 802	88 906	14		81 017	92 740	07 260	88 276
15		80 120	91 224	08 776	88 896	15		81 032	92 766	07 234	88 266
16		80 136	91 250	08 750	88 886	16		81 047	92 792	07 208	88 255
17		80 151	91 276	08 724	88 875	17		81 061	92 817	07 183	88 244
18		80 166	91 301	08 699	88 865	18		81 076	92 843	07 157	88 234
19		80 182	91 327	08 673	88 855	19		81 091	92 868	07 132	88 223
20		80 197	91 353	08 647	88 844	20		81 106	92 894	07 106	88 212
21		80 213	91 379	08 621	88 834	21		81 121	92 920	07 080	88 201
22		80 228	91 404	08 596	88 824	22		81 136	92 945	07 055	88 191
23		80 244	91 430	08 570	88 813	23		81 151	92 971	07 029	88 180
24		80 259	91 456	08 544	88 803	24		81 166	92 996	07 004	88 169
25		80 274	91 482	08 518	88 793	25		81 180	93 022	06 978	88 158
26		80 290	91 507	08 493	88 782	26		81 195	93 048	06 952	88 148
27		80 305	91 533	08 467	88 772	27		81 210	93 073	06 927	88 137
28		80 320	91 559	08 441	88 761	28		81 225	93 099	06 901	88 126
29		80 336	91 585	08 415	88 751	29		81 240	93 124	06 876	88 115
30		80 351	91 610	08 390	88 741	30		81 254	93 150	06 850	88 105
31		80 366	91 636	08 364	88 730	31		81 269	93 175	06 825	88 094
32		80 382	91 662	08 338	88 720	32		81 284	93 201	06 799	88 083
33		80 397	91 688	08 312	88 709	33		81 299	93 227	06 773	88 072
34		80 412	91 713	08 287	88 699	34		81 314	93 252	06 748	88 061
35		80 428	91 739	08 261	88 688	35		81 328	93 278	06 722	88 051
36		80 443	91 765	08 235	88 678	36		81 343	93 303	06 697	88 040
37		80 458	91 791	08 209	88 668	37		81 358	93 329	06 671	88 029
38		80 473	91 816	08 184	88 657	38		81 372	93 354	06 646	88 018
39		80 489	91 842	08 158	88 647	39		81 387	93 380	06 620	88 007
40		80 504	91 868	08 132	88 636	40		81 402	93 406	06 594	87 996
41		80 519	91 893	08 107	88 626	41		81 417	93 431	06 569	87 985
42		80 534	91 919	08 081	88 615	42		81 431	93 457	06 543	87 975
43		80 550	91 945	08 055	88 605	43		81 446	93 482	06 518	87 964
44		80 565	91 971	08 029	88 594	44		81 461	93 508	06 492	87 953
45		80 580	91 996	08 004	88 584	45		81 475	93 533	06 467	87 942
46		80 595	92 022	07 978	88 573	46		81 490	93 559	06 441	87 931
47		80 610	92 048	07 952	88 563	47		81 505	93 584	06 416	87 920
48		80 625	92 073	07 927	88 552	48		81 519	93 610	06 390	87 909
49		80 641	92 099	07 901	88 542	49		81 534	93 636	06 364	87 898
50		80 656	92 125	07 875	88 531	50		81 549	93 661	06 339	87 887
51		80 671	92 150	07 850	88 521	51		81 563	93 687	06 313	87 877
52		80 686	92 176	07 824	88 510	52		81 578	93 712	06 288	87 866
53		80 701	92 202	07 798	88 499	53		81 592	93 738	06 262	87 855
54		80 716	92 227	07 773	88 489	54		81 607	93 763	06 237	87 844
55		80 731	92 253	07 747	88 478	55		81 622	93 789	06 211	87 833
56		80 746	92 279	07 721	88 468	56		81 636	93 814	06 186	87 822
57		80 762	92 304	07 696	88 457	57		81 651	93 840	06 160	87 811
58		80 777	92 330	07 670	88 447	58		81 665	93 865	06 135	87 800
59		80 792	92 356	07 644	88 436	59		81 680	93 891	06 109	87 789
60		80 807	92 381	07 619	88 425	60		81 694	93 916	06 084	87 778
		0	0	10	0			0	0	10	0
		log cos	log cot	log tan	log sin			log cos	log cot	log tan	log sin



41°					42°					
	log sin	log tan	log cot	log cos		log sin	log tan	log cot	log cos	
	9	9	10	9		9	9	10	9	
0	81 694	93 916	06 084	87 778	80	82 551	95 444	04 556	87 107	60
1	81 709	93 942	06 058	87 767	59	82 565	95 469	04 531	87 096	59
2	81 723	93 967	06 033	87 756	58	82 579	95 495	04 505	87 085	58
3	81 738	93 993	06 007	87 745	57	82 593	95 520	04 480	87 073	57
4	81 752	94 018	05 982	87 734	56	82 607	95 545	04 455	87 062	56
5	81 767	94 044	05 956	87 723	55	82 621	95 571	04 429	87 050	55
6	81 781	94 069	05 931	87 712	54	82 635	95 596	04 404	87 039	54
7	81 796	94 095	05 905	87 701	53	82 649	95 622	04 378	87 028	53
8	81 810	94 120	05 880	87 690	52	82 663	95 647	04 353	87 016	52
9	81 825	94 146	05 854	87 679	51	82 677	95 672	04 328	87 005	51
10	81 839	94 171	05 829	87 668	50	82 691	95 698	04 302	86 993	50
11	81 854	94 197	05 803	87 657	49	82 705	95 723	04 277	86 982	49
12	81 868	94 222	05 778	87 646	48	82 719	95 748	04 252	86 970	48
13	81 882	94 248	05 752	87 635	47	82 733	95 774	04 226	86 959	47
14	81 897	94 273	05 727	87 624	46	82 747	95 799	04 201	86 947	46
15	81 911	94 299	05 701	87 613	45	82 761	95 825	04 175	86 936	45
16	81 926	94 324	05 676	87 601	44	82 775	95 850	04 150	86 924	44
17	81 940	94 350	05 650	87 590	43	82 789	95 875	04 125	86 913	43
18	81 955	94 375	05 625	87 579	42	82 802	95 901	04 099	86 902	42
19	81 969	94 401	05 599	87 568	41	82 816	95 926	04 074	86 890	41
20	81 983	94 426	05 574	87 557	40	82 830	95 952	04 048	86 879	40
21	81 998	94 452	05 548	87 546	39	82 844	95 977	04 023	86 867	39
22	82 012	94 477	05 523	87 535	38	82 858	96 002	03 998	86 855	38
23	82 026	94 503	05 497	87 524	37	82 872	96 028	03 972	86 844	37
24	82 041	94 528	05 472	87 513	36	82 885	96 053	03 947	86 832	36
25	82 055	94 554	05 446	87 501	35	82 899	96 078	03 922	86 821	35
26	82 069	94 579	05 421	87 490	34	82 913	96 104	03 896	86 809	34
27	82 084	94 604	05 396	87 479	33	82 927	96 129	03 871	86 798	33
28	82 098	94 630	05 370	87 468	32	82 941	96 155	03 845	86 786	32
29	82 112	94 655	05 345	87 457	31	82 955	96 180	03 820	86 775	31
30	82 126	94 681	05 319	87 446	30	82 968	96 205	03 795	86 763	30
31	82 141	94 706	05 294	87 434	29	82 982	96 231	03 769	86 752	29
32	82 155	94 732	05 268	87 423	28	82 996	96 256	03 744	86 740	28
33	82 169	94 757	05 243	87 412	27	83 010	96 281	03 719	86 728	27
34	82 184	94 783	05 217	87 401	26	83 023	96 307	03 693	86 717	26
35	82 198	94 808	05 192	87 390	25	83 037	96 332	03 668	86 705	25
36	82 212	94 834	05 166	87 378	24	83 051	96 357	03 643	86 694	24
37	82 226	94 859	05 141	87 367	23	83 065	96 383	03 617	86 682	23
38	82 240	94 884	05 116	87 356	22	83 078	96 408	03 592	86 670	22
39	82 255	94 910	05 090	87 345	21	83 092	96 433	03 567	86 659	21
40	82 269	94 935	05 065	87 334	20	83 106	96 459	03 541	86 647	20
41	82 283	94 961	05 039	87 322	19	83 120	96 484	03 516	86 635	19
42	82 297	94 986	05 014	87 311	18	83 133	96 510	03 490	86 624	18
43	82 311	95 012	04 988	87 300	17	83 147	96 535	03 465	86 612	17
44	82 326	95 037	04 963	87 288	16	83 161	96 560	03 440	86 600	16
45	82 340	95 062	04 938	87 277	15	83 174	96 586	03 414	86 589	15
46	82 354	95 088	04 912	87 266	14	83 188	96 611	03 389	86 577	14
47	82 368	95 113	04 887	87 255	13	83 202	96 636	03 364	86 565	13
48	82 382	95 139	04 861	87 243	12	83 215	96 662	03 338	86 554	12
49	82 396	95 164	04 836	87 232	11	83 229	96 687	03 313	86 542	11
50	82 410	95 190	04 810	87 221	10	83 242	96 712	03 288	86 530	10
51	82 424	95 215	04 785	87 209	9	83 256	96 738	03 262	86 518	9
52	82 439	95 240	04 760	87 198	8	83 270	96 763	03 237	86 507	8
53	82 453	95 266	04 734	87 187	7	83 283	96 788	03 212	86 495	7
54	82 467	95 291	04 709	87 175	6	83 297	96 814	03 186	86 483	6
55	82 481	95 317	04 683	87 164	5	83 310	96 839	03 161	86 472	5
56	82 495	95 342	04 658	87 153	4	83 324	96 864	03 136	86 460	4
57	82 509	95 368	04 632	87 141	3	83 338	96 890	03 110	86 448	3
58	82 523	95 393	04 607	87 130	2	83 351	96 915	03 085	86 436	2
59	82 537	95 418	04 582	87 119	1	83 365	96 940	03 060	86 425	1
60	82 551	95 444	04 556	87 107	0	83 378	96 966	03 034	86 413	0
	log cos	log cot	log tan	log sin		log cos	log cot	log tan	log sin	

43°				44°						
<i>i</i>	log sin D	log tan D	log cot 10	log cos D	<i>i</i>	log sin D	log tan D	log cot 10	log cos D	<i>i</i>
0	83 378	96 966	03 034	86 413	60	84 177	98 484	01 516	85 693	60
1	83 392	96 991	03 009	86 401	59	84 190	98 509	01 491	85 681	59
2	83 405	97 016	02 984	86 389	58	84 203	98 534	01 466	85 669	58
3	83 419	97 042	02 958	86 377	57	84 216	98 560	01 440	85 657	57
4	83 432	97 067	02 933	86 366	56	84 229	98 585	01 415	85 645	56
5	83 446	97 092	02 908	86 354	55	84 242	98 610	01 390	85 632	55
6	83 459	97 118	02 882	86 342	54	84 255	98 635	01 365	85 620	54
7	83 473	97 143	02 857	86 330	53	84 269	98 661	01 339	85 608	53
8	83 486	97 168	02 832	86 318	52	84 282	98 686	01 314	85 596	52
9	83 500	97 193	02 807	86 306	51	84 295	98 711	01 289	85 583	51
10	83 513	97 219	02 781	86 295	50	84 308	98 737	01 263	85 571	50
11	83 527	97 244	02 756	86 283	49	84 321	98 762	01 238	85 559	49
12	83 540	97 269	02 731	86 271	48	84 334	98 787	01 213	85 547	48
13	83 554	97 295	02 705	86 259	47	84 347	98 812	01 188	85 534	47
14	83 567	97 320	02 680	86 247	46	84 360	98 838	01 162	85 522	46
15	83 581	97 345	02 655	86 235	45	84 373	98 863	01 137	85 510	45
16	83 594	97 371	02 629	86 223	44	84 385	98 888	01 112	85 497	44
17	83 608	97 396	02 604	86 211	43	84 398	98 913	01 087	85 485	43
18	83 621	97 421	02 579	86 200	42	84 411	98 939	01 061	85 473	42
19	83 634	97 447	02 553	86 188	41	84 424	98 964	01 036	85 460	41
20	83 648	97 472	02 528	86 176	40	84 437	98 989	01 011	85 448	40
21	83 661	97 497	02 503	86 164	39	84 450	99 015	00 985	85 436	39
22	83 674	97 523	02 477	86 152	38	84 463	99 040	00 960	85 423	38
23	83 688	97 548	02 452	86 140	37	84 476	99 065	00 935	85 411	37
24	83 701	97 573	02 427	86 128	36	84 489	99 090	00 910	85 399	36
25	83 715	97 598	02 402	86 116	35	84 502	99 116	00 884	85 386	35
26	83 728	97 624	02 376	86 104	34	84 515	99 141	00 858	85 374	34
27	83 741	97 649	02 351	86 092	33	84 528	99 166	00 834	85 361	33
28	83 755	97 674	02 326	86 080	32	84 540	99 191	00 809	85 349	32
29	83 768	97 700	02 300	86 068	31	84 553	99 217	00 783	85 337	31
30	83 781	97 725	02 275	86 056	30	84 566	99 242	00 758	85 324	30
31	83 795	97 750	02 250	86 044	29	84 579	99 267	00 733	85 312	29
32	83 808	97 776	02 224	86 032	28	84 592	99 293	00 707	85 299	28
33	83 821	97 801	02 199	86 020	27	84 605	99 318	00 682	85 287	27
34	83 834	97 826	02 174	86 008	26	84 618	99 343	00 657	85 274	26
35	83 848	97 851	02 149	85 996	25	84 630	99 368	00 632	85 262	25
36	83 861	97 877	02 123	85 984	24	84 643	99 394	00 606	85 250	24
37	83 874	97 902	02 098	85 972	23	84 656	99 419	00 581	85 237	23
38	83 887	97 927	02 073	85 960	22	84 669	99 444	00 556	85 225	22
39	83 901	97 953	02 047	85 948	21	84 682	99 469	00 531	85 212	21
40	83 914	97 978	02 022	85 936	20	84 694	99 495	00 505	85 200	20
41	83 927	98 003	01 997	85 924	19	84 707	99 520	00 480	85 187	19
42	83 940	98 029	01 971	85 912	18	84 720	99 545	00 455	85 175	18
43	83 954	98 054	01 946	85 900	17	84 733	99 570	00 430	85 162	17
44	83 967	98 079	01 921	85 888	16	84 745	99 596	00 404	85 150	16
45	83 980	98 104	01 896	85 876	15	84 758	99 621	00 379	85 137	15
46	83 993	98 130	01 870	85 864	14	84 771	99 646	00 354	85 125	14
47	84 006	98 155	01 845	85 851	13	84 784	99 672	00 328	85 112	13
48	84 020	98 180	01 820	85 839	12	84 796	99 697	00 303	85 100	12
49	84 033	98 206	01 794	85 827	11	84 809	99 722	00 278	85 087	11
50	84 046	98 231	01 769	85 815	10	84 822	99 747	00 253	85 074	10
51	84 059	98 256	01 744	85 803	9	84 835	99 773	00 227	85 062	9
52	84 072	98 281	01 719	85 791	8	84 847	99 798	00 202	85 049	8
53	84 085	98 307	01 693	85 779	7	84 860	99 823	00 177	85 037	7
54	84 098	98 332	01 668	85 766	6	84 873	99 848	00 152	85 024	6
55	84 112	98 357	01 643	85 754	5	84 885	99 874	00 126	85 012	5
56	84 125	98 383	01 617	85 742	4	84 898	99 899	00 101	84 999	4
57	84 138	98 408	01 592	85 730	3	84 911	99 924	00 076	84 986	3
58	84 151	98 433	01 567	85 718	2	84 923	99 949	00 051	84 974	2
59	84 164	98 458	01 542	85 706	1	84 936	99 975	00 025	84 961	1
60	84 177	98 484	01 516	85 693	0	84 949	00 000	00 000	84 949	0
<i>i</i>	log cos D	log cot D	log tan 10	log sin D	<i>i</i>	log cos D	log cot D	log tan 10	log sin D	<i>i</i>

## TABLE IV.

FOR DETERMINING WITH GREATER ACCURACY THAN CAN BE DONE BY  
MEANS OF TABLE III.:

1.  $\log \sin$ ,  $\log \tan$ , and  $\log \cot$ , when the angle is between  $0^\circ$  and  $2^\circ$ ;
2.  $\log \cos$ ,  $\log \tan$ , and  $\log \cot$ , when the angle is between  $88^\circ$  and  $90^\circ$ ;
3. The value of the angle when the logarithm of the function does not lie between the limits 8. 54 684 and 11. 45 316.

FORMULAS FOR THE USE OF THE NUMBERS S AND T.

I. When the angle  $\alpha$  is between  $0^\circ$  and  $2^\circ$ :

$$\begin{array}{l|l} \log \sin \alpha = \log a'' + S. & \log a'' = \log \sin \alpha - S, \\ \log \tan \alpha = \log a'' + T. & = \log \tan \alpha - T, \\ \log \cot \alpha = \text{colog } \tan \alpha. & = \text{colog } \cot \alpha - T. \end{array}$$

II. When the angle  $\alpha$  is between  $88^\circ$  and  $90^\circ$ :

$$\begin{array}{l|l} \log \cos \alpha = \log (90^\circ - \alpha)'' + S. & \log (90^\circ - \alpha)'' = \log \cos \alpha - S, \\ \log \cot \alpha = \log (90^\circ - \alpha)'' + T. & = \log \cot \alpha - T, \\ \log \tan \alpha = \text{colog } \cot \alpha. & = \text{colog } \tan \alpha - T, \\ & \text{and } \alpha = 90^\circ - (90^\circ - \alpha). \end{array}$$

## VALUES OF S AND T.

$a''$	S	$\log \sin \alpha$	$a''$	T	$\log \tan \alpha$	$\alpha$	T	$\log \tan \alpha$
0		—	0		—	5 146		8. 39 713
2 409	4. 68 557	8. 06 740	200	4. 68 557	6. 98 660	5 424	4. 68 567	8. 41 999
3 417	4. 68 556	8. 21 920	1 725	4. 68 558	7. 92 263	5 689	4. 68 568	8. 44 072
3 823	4. 68 555	8. 26 795	2 432	4. 68 559	8. 07 156	5 941	4. 68 569	8. 45 955
4 190	4. 68 555	8. 30 776	2 976	4. 68 560	8. 15 924	6 184	4. 68 570	8. 47 697
4 810	4. 68 554	8. 37 038	3 434	4. 68 561	8. 23 142	6 417	4. 68 571	8. 49 305
5 414	4. 68 553	8. 41 904	3 838	4. 68 562	8. 26 973	6 642	4. 68 572	8. 50 802
5 932	4. 68 552	8. 45 872	4 204	4. 68 563	8. 30 930	6 859	4. 68 573	8. 52 200
6 408	4. 68 551	8. 49 223	4 540	4. 68 564	8. 34 270	7 070	4. 68 574	8. 53 516
6 633	4. 68 550	8. 50 721	4 629	4. 68 565	8. 35 766	7 173	4. 68 575	8. 54 145
6 851	4. 68 550	8. 52 125	4 853	4. 68 565	8. 37 167	7 274	4. 68 575	8. 54 753
7 267	4. 68 549	8. 54 684	5 146	4. 68 566	8. 39 713			
$a''$	S	$\log \sin \alpha$	$a''$	T	$\log \tan \alpha$	$\alpha$	T	$\log \tan \alpha$

**TABLE V.--CIRCUMFERENCES AND AREAS OF CIRCLES. 51**

If  $N$  = the radius of the circle, the circumference =  $2\pi N$ .

If  $N$  = the radius of the circle, the area =  $\pi N^2$ .

If  $N$  = the circumference of the circle, the radius =  $\frac{1}{2\pi} N$ .

If  $N$  = the circumference of the circle, the area =  $\frac{1}{4\pi} N^2$ .

N	$2\pi N$	$\pi N^2$	$\frac{1}{2\pi} N$	$\frac{1}{4\pi} N^2$	N	$2\pi N$	$\pi N^2$	$\frac{1}{2\pi} N$	$\frac{1}{4\pi} N^2$
0	0.00	0.0	0.000	0.00	50	314.16	7854	7.96	198.94
1	6.28	3.1	0.159	0.08	51	320.44	8171	8.12	206.98
2	12.57	12.6	0.318	0.32	52	326.73	8495	8.28	215.18
3	18.85	28.3	0.477	0.72	53	333.01	8825	8.44	223.53
4	25.13	50.3	0.637	1.27	54	339.29	9161	8.59	232.05
5	31.42	78.5	0.796	1.99	55	345.58	9503	8.75	240.72
6	37.70	113.1	0.955	2.86	56	351.86	9852	8.91	249.55
7	43.98	153.9	1.114	3.90	57	358.14	10207	9.07	258.55
8	50.27	201.1	1.273	5.09	58	364.42	10568	9.23	267.70
9	56.55	254.5	1.432	6.45	59	370.71	10936	9.39	277.01
10	62.83	314.2	1.592	7.96	60	376.99	11310	9.55	286.48
11	69.12	380.1	1.751	9.63	61	383.27	11690	9.71	296.11
12	75.40	452.4	1.910	11.46	62	389.56	12076	9.87	305.90
13	81.68	530.9	2.069	13.45	63	395.84	12469	10.03	315.84
14	87.96	615.8	2.228	15.60	64	402.12	12868	10.19	325.95
15	94.25	706.9	2.387	17.90	65	408.41	13273	10.35	336.21
16	100.53	804.2	2.546	20.37	66	414.69	13685	10.50	346.64
17	106.81	907.9	2.706	23.00	67	420.97	14103	10.66	357.22
18	113.10	1017.9	2.865	25.78	68	427.26	14527	10.82	367.97
19	119.38	1134.1	3.024	28.73	69	433.54	14957	10.98	378.87
20	125.66	1256.6	3.183	31.83	70	439.82	15394	11.14	389.93
21	131.95	1385.4	3.342	35.07	71	446.11	15837	11.30	401.15
22	138.23	1520.5	3.501	38.52	72	452.39	16286	11.46	412.53
23	144.51	1661.9	3.661	42.10	73	458.67	16742	11.62	424.07
24	150.80	1809.6	3.820	45.84	74	464.96	17203	11.78	435.77
25	157.08	1963.5	3.979	49.74	75	471.24	17671	11.94	447.62
26	163.36	2123.7	4.138	53.79	76	477.52	18146	12.10	459.64
27	169.65	2290.2	4.297	58.01	77	483.81	18627	12.25	471.81
28	175.93	2463.0	4.456	62.39	78	490.09	19113	12.41	484.15
29	182.21	2642.1	4.615	66.92	79	496.37	19607	12.57	496.64
30	188.50	2827.4	4.775	71.62	80	502.65	20105	12.73	509.30
31	194.78	3019.1	4.934	76.47	81	508.94	20612	12.89	522.11
32	201.06	3217.0	5.093	81.47	82	515.22	21124	13.05	535.08
33	207.35	3421.2	5.252	86.66	83	521.50	21642	13.21	548.21
34	213.63	3631.7	5.411	91.99	84	527.79	22167	13.37	561.50
35	219.91	3848.5	5.570	97.48	85	534.07	22698	13.53	574.95
36	226.19	4071.5	5.730	103.13	86	540.35	23235	13.69	588.55
37	232.48	4300.8	5.889	108.94	87	546.64	23779	13.85	602.32
38	238.76	4536.5	6.048	114.91	88	552.92	24328	14.01	616.25
39	245.04	4778.4	6.207	121.04	89	559.20	24885	14.16	630.33
40	251.33	5026.5	6.366	127.32	90	565.49	25447	14.32	644.58
41	257.61	5281.0	6.525	133.77	91	571.77	26016	14.48	658.98
42	263.89	5541.8	6.685	140.37	92	578.05	26590	14.64	673.51
43	270.18	5808.8	6.844	147.14	93	584.34	27172	14.80	688.27
44	276.46	6082.1	7.003	154.06	94	590.62	27759	14.96	703.15
45	282.74	6361.7	7.162	161.14	95	596.90	28353	15.12	718.19
46	289.03	6647.6	7.321	168.39	96	603.19	28953	15.28	733.39
47	295.31	6939.8	7.480	175.79	97	609.47	29559	15.44	748.74
48	301.59	7238.2	7.639	183.35	98	615.75	30172	15.60	764.26
49	307.88	7543.0	7.799	191.07	99	622.04	30791	15.76	779.94
50	314.16	7854.0	7.958	198.94	100	628.32	31416	15.92	795.77
N	$2\pi N$	$\pi N^2$	$\frac{1}{2\pi} N$	$\frac{1}{4\pi} N^2$	N	$2\pi N$	$\pi N^2$	$\frac{1}{2\pi} N$	$\frac{1}{4\pi} N^2$

TABLE VI — NATURAL FUNCTIONS.

°	0°		1°		2°		3°		4°		°
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	0000	1.000	0175	9998	0349	9994	0523	9986	0698	9976	60
1	0003	1.000	0177	9998	0352	9994	0526	9986	0700	9975	59
2	0006	1.000	0180	9998	0355	9994	0529	9986	0703	9975	58
3	0009	1.000	0183	9998	0358	9994	0532	9986	0706	9975	57
4	0012	1.000	0186	9998	0361	9993	0535	9986	0709	9975	56
5	0015	1.000	0189	9998	0364	9993	0538	9986	0712	9975	55
6	0017	1.000	0192	9998	0366	9993	0541	9985	0715	9974	54
7	0020	1.000	0195	9998	0369	9993	0544	9985	0718	9974	53
8	0023	1.000	0198	9998	0372	9993	0547	9985	0721	9974	52
9	0025	1.000	0201	9998	0375	9993	0550	9985	0724	9974	51
10	0029	1.000	0204	9998	0378	9993	0554	9985	0727	9974	50
11	0032	1.000	0207	9998	0381	9993	0558	9985	0729	9973	49
12	0035	1.000	0209	9998	0384	9993	0558	9984	0732	9973	48
13	0038	1.000	0212	9998	0387	9993	0561	9984	0735	9973	47
14	0041	1.000	0215	9998	0390	9992	0564	9984	0738	9973	46
15	0044	1.000	0218	9998	0393	9992	0567	9984	0741	9973	45
16	0047	1.000	0221	9998	0396	9992	0570	9984	0744	9972	44
17	0049	1.000	0224	9997	0398	9992	0573	9984	0747	9972	43
18	0052	1.000	0227	9997	0401	9992	0576	9983	0750	9972	42
19	0055	1.000	0230	9997	0404	9992	0579	9983	0753	9972	41
20	0058	1.000	0233	9997	0407	9992	0581	9983	0756	9971	40
21	0061	1.000	0236	9997	0410	9992	0584	9983	0758	9971	39
22	0064	1.000	0239	9997	0413	9991	0587	9983	0761	9971	38
23	0067	1.000	0241	9997	0416	9991	0590	9983	0764	9971	37
24	0070	1.000	0244	9997	0419	9991	0593	9982	0767	9971	36
25	0073	1.000	0247	9997	0422	9991	0596	9982	0770	9970	35
26	0076	1.000	0250	9997	0425	9991	0599	9982	0773	9970	34
27	0079	1.000	0253	9997	0427	9991	0602	9982	0776	9970	33
28	0081	1.000	0256	9997	0430	9991	0605	9982	0779	9970	32
29	0084	1.000	0259	9997	0433	9991	0608	9982	0782	9969	31
30	008	1.000	0262	9997	0436	9990	0610	9981	0785	9969	30
31	0090	1.000	0265	9996	0439	9990	0613	9981	0787	9969	29
32	0093	1.000	0268	9996	0442	9990	0616	9981	0790	9969	28
33	0096	1.000	0270	9996	0445	9990	0619	9981	0793	9968	27
34	0099	1.000	0273	9996	0448	9990	0622	9981	0796	9968	26
35	0102	9999	0276	9996	0451	9990	0625	9980	0799	9968	25
36	0105	9999	0279	9996	0454	9990	0628	9980	0802	9968	24
37	0108	9999	0282	9996	0457	9990	0631	9980	0805	9968	23
38	0111	9999	0285	9996	0459	9989	0634	9980	0808	9967	22
39	0113	9999	0288	9996	0462	9989	0637	9980	0811	9967	21
40	0116	9999	0291	9996	0465	9989	0640	9980	0814	9967	20
41	0119	9999	0294	9996	0468	9989	0642	9979	0816	9967	19
42	0122	9999	0297	9996	0471	9989	0645	9979	0819	9966	18
43	0125	9999	0300	9996	0474	9989	0648	9979	0822	9966	17
44	0128	9999	0302	9995	0477	9989	0651	9979	0825	9966	16
45	0131	9999	0305	9995	0480	9988	0654	9979	0828	9966	15
46	0134	9999	0308	9995	0483	9988	0657	9978	0831	9965	14
47	0137	9999	0311	9995	0486	9988	0660	9978	0834	9965	13
48	0140	9999	0314	9995	0488	9988	0663	9978	0837	9965	12
49	0143	9999	0317	9995	0491	9988	0666	9978	0840	9965	11
50	0145	9999	0320	9995	0494	9988	0669	9978	0843	9964	10
51	0148	9999	0323	9995	0497	9988	0671	9977	0845	9964	9
52	0151	9999	0326	9995	0500	9987	0674	9977	0848	9964	8
53	0154	9999	0329	9995	0503	9987	0677	9977	0851	9964	7
54	0157	9999	0332	9995	0506	9987	0680	9977	0854	9963	6
55	0160	9999	0334	9994	0509	9987	0683	9977	0857	9963	5
56	0163	9999	0337	9994	0512	9987	0686	9976	0860	9963	4
57	0166	9999	0340	9994	0515	9987	0689	9976	0863	9963	3
58	0169	9999	0343	9994	0518	9987	0692	9976	0866	9962	2
59	0172	9999	0346	9994	0520	9986	0695	9976	0869	9962	1
60	0175	9998	0349	9994	0523	9986	0698	9976	0872	9962	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	89°		88°		87°		86°		85°		

/	5°		6°		7°		8°		9°		/
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	0872	9962	1045	9945	1219	9925	1392	9903	1564	9877	60
1	0874	9962	1048	9945	1222	9925	1395	9902	1567	9876	59
2	0877	9961	1051	9945	1224	9925	1397	9902	1570	9876	58
3	0880	9961	1054	9944	1227	9924	1400	9901	1573	9876	57
4	0883	9961	1057	9944	1230	9924	1403	9901	1576	9875	56
5	0886	9961	1060	9944	1233	9924	1406	9901	1579	9875	55
6	0889	9960	1063	9943	1236	9923	1409	9900	1582	9874	54
7	0892	9960	1066	9943	1239	9923	1412	9900	1584	9874	53
8	0895	9960	1068	9943	1241	9923	1415	9899	1587	9873	52
9	0898	9960	1071	9942	1245	9922	1418	9899	1590	9873	51
10	0901	9959	1074	9942	1248	9922	1421	9899	1593	9872	50
11	0903	9959	1077	9942	1250	9922	1423	9898	1596	9872	49
12	0906	9959	1080	9942	1253	9921	1426	9898	1599	9871	48
13	0909	9959	1083	9941	1256	9921	1429	9897	1602	9871	47
14	0912	9958	1086	9941	1259	9920	1432	9897	1605	9870	46
15	0915	9958	1089	9941	1262	9920	1435	9897	1607	9870	45
16	0918	9958	1092	9940	1265	9920	1438	9896	1610	9869	44
17	0921	9958	1094	9940	1268	9919	1441	9896	1613	9869	43
18	0924	9957	1097	9940	1271	9919	1444	9895	1616	9869	42
19	0927	9957	1100	9939	1274	9919	1446	9895	1619	9868	41
20	0929	9957	1103	9939	1276	9918	1449	9894	1622	9868	40
21	0932	9956	1106	9939	1279	9918	1452	9894	1625	9867	39
22	0935	9956	1109	9938	1282	9917	1455	9894	1628	9867	38
23	0938	9956	1112	9938	1285	9917	1458	9893	1630	9866	37
24	0941	9956	1115	9938	1288	9917	1461	9893	1633	9866	36
25	0944	9955	1118	9937	1291	9916	1464	9892	1636	9865	35
26	0947	9955	1120	9937	1294	9916	1467	9892	1639	9865	34
27	0950	9955	1123	9937	1297	9916	1469	9891	1642	9864	33
28	0953	9955	1126	9936	1299	9915	1472	9891	1645	9864	32
29	0956	9954	1129	9936	1302	9915	1475	9891	1648	9863	31
30	0958	9954	1132	9936	1305	9914	1478	9890	1650	9863	30
31	0961	9954	1135	9935	1308	9914	1481	9890	1653	9862	29
32	0964	9953	1138	9935	1311	9914	1484	9889	1656	9862	28
33	0967	9953	1141	9935	1314	9913	1487	9889	1659	9861	27
34	0970	9953	1144	9934	1317	9913	1490	9888	1662	9861	26
35	0973	9953	1146	9934	1320	9913	1492	9888	1665	9860	25
36	0976	9952	1149	9934	1323	9912	1495	9888	1668	9860	24
37	0979	9952	1152	9933	1325	9912	1498	9887	1671	9859	23
38	0982	9952	1155	9933	1328	9911	1501	9887	1673	9859	22
39	0985	9951	1158	9933	1331	9911	1504	9886	1676	9859	21
40	0987	9951	1161	9932	1334	9911	1507	9886	1679	9858	20
41	0990	9951	1164	9932	1337	9910	1510	9885	1682	9858	19
42	0993	9951	1167	9932	1340	9910	1513	9885	1685	9857	18
43	0996	9950	1170	9931	1343	9909	1515	9884	1688	9857	17
44	0999	9950	1172	9931	1346	9909	1518	9884	1691	9856	16
45	1002	9950	1175	9931	1349	9909	1521	9884	1693	9856	15
46	1005	9949	1178	9930	1351	9908	1524	9883	1696	9855	14
47	1008	9949	1181	9930	1354	9908	1527	9883	1699	9855	13
48	1011	9949	1184	9930	1357	9907	1530	9882	1702	9854	12
49	1013	9949	1187	9929	1360	9907	1533	9882	1705	9854	11
50	1016	9948	1190	9929	1363	9907	1536	9881	1708	9853	10
51	1019	9948	1193	9929	1366	9906	1538	9881	1711	9853	9
52	1022	9948	1196	9928	1369	9906	1541	9880	1714	9852	8
53	1025	9947	1198	9928	1372	9905	1544	9880	1716	9852	7
54	1028	9947	1201	9928	1374	9905	1547	9880	1719	9851	6
55	1031	9947	1204	9927	1377	9905	1550	9879	1722	9851	5
56	1034	9946	1207	9927	1380	9904	1553	9879	1725	9850	4
57	1037	9946	1210	9927	1383	9904	1556	9878	1728	9850	3
58	1039	9946	1213	9926	1386	9903	1559	9878	1731	9849	2
59	1042	9946	1216	9926	1389	9903	1561	9877	1734	9849	1
60	1045	9945	1219	9925	1392	9903	1564	9877	1736	9848	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
/	84°		83°		82°		81°		80°		/

°	10°		11°		12°		13°		14°		°
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0	1736	9848	1908	9816	2079	9781	2250	9744	2419	9703	60
1	1739	9848	1911	9816	2082	9781	2252	9743	2422	9702	59
2	1742	9847	1914	9815	2085	9780	2255	9742	2425	9702	58
3	1745	9847	1917	9815	2088	9780	2258	9742	2428	9701	57
4	1748	9846	1920	9814	2090	9779	2261	9741	2431	9700	56
5	1751	9846	1922	9813	2093	9778	2264	9740	2433	9699	55
6	1754	9845	1925	9813	2096	9778	2267	9740	2436	9699	54
7	1757	9845	1928	9812	2099	9777	2269	9739	2439	9698	53
8	1759	9844	1931	9812	2102	9777	2272	9738	2442	9697	52
9	1762	9843	1934	9811	2105	9776	2275	9738	2445	9697	51
10	1765	9843	1937	9811	2108	9775	2278	9737	2447	9696	50
11	1768	9842	1939	9810	2110	9775	2281	9736	2450	9695	49
12	1771	9842	1942	9810	2113	9774	2284	9736	2453	9694	48
13	1774	9841	1945	9809	2116	9774	2286	9735	2456	9694	47
14	1777	9841	1948	9808	2119	9773	2289	9734	2459	9693	46
15	1779	9840	1951	9808	2122	9772	2292	9734	2462	9692	45
16	1782	9840	1954	9807	2125	9772	2295	9733	2464	9692	44
17	1785	9839	1957	9807	2127	9771	2298	9732	2467	9691	43
18	1788	9839	1959	9806	2130	9770	2300	9732	2470	9690	42
19	1791	9838	1962	9806	2133	9770	2303	9731	2473	9689	41
20	1794	9838	1965	9805	2136	9769	2306	9730	2476	9689	40
21	1797	9837	1968	9804	2139	9769	2309	9730	2478	9688	39
22	1799	9837	1971	9804	2142	9768	2312	9729	2481	9687	38
23	1802	9836	1974	9803	2145	9767	2315	9728	2484	9687	37
24	1805	9836	1977	9803	2147	9767	2317	9728	2487	9686	36
25	1808	9835	1979	9802	2150	9766	2320	9727	2490	9685	35
26	1811	9835	1982	9802	2153	9765	2323	9726	2493	9684	34
27	1814	9834	1985	9801	2156	9765	2326	9726	2495	9684	33
28	1817	9834	1988	9800	2159	9764	2329	9725	2498	9683	32
29	1819	9833	1991	9800	2162	9764	2332	9724	2501	9682	31
30	1822	9833	1994	9799	2164	9763	2334	9724	2504	9681	30
31	1825	9832	1997	9799	2167	9762	2337	9723	2507	9681	29
32	1828	9831	1999	9798	2170	9762	2340	9722	2509	9680	28
33	1831	9831	2002	9798	2173	9761	2343	9722	2512	9679	27
34	1834	9830	2005	9797	2176	9760	2346	9721	2515	9679	26
35	1837	9830	2008	9796	2179	9760	2349	9720	2518	9678	25
36	1840	9829	2011	9796	2181	9759	2351	9720	2521	9677	24
37	1842	9829	2014	9795	2184	9759	2354	9719	2524	9676	23
38	1845	9828	2016	9795	2187	9758	2357	9718	2526	9676	22
39	1848	9828	2019	9794	2190	9757	2360	9718	2529	9675	21
40	1851	9827	2022	9793	2193	9757	2363	9717	2532	9674	20
41	1854	9827	2025	9793	2196	9756	2366	9716	2535	9673	19
42	1857	9826	2028	9792	2198	9755	2368	9715	2538	9673	18
43	1860	9826	2031	9792	2201	9755	2371	9715	2540	9672	17
44	1862	9825	2034	9791	2204	9754	2374	9714	2543	9671	16
45	1865	9825	2036	9790	2207	9753	2377	9713	2546	9670	15
46	1868	9824	2039	9790	2210	9753	2380	9713	2549	9670	14
47	1871	9823	2042	9789	2213	9752	2383	9712	2552	9669	13
48	1874	9823	2045	9789	2215	9751	2385	9711	2554	9668	12
49	1877	9822	2048	9788	2218	9751	2388	9711	2557	9667	11
50	1880	9822	2051	9787	2221	9750	2391	9710	2560	9667	10
51	1882	9821	2054	9787	2224	9750	2394	9709	2563	9666	9
52	1885	9821	2056	9786	2227	9749	2397	9709	2566	9665	8
53	1888	9820	2059	9786	2230	9748	2399	9708	2569	9665	7
54	1891	9820	2062	9785	2233	9748	2402	9707	2571	9664	6
55	1894	9819	2065	9784	2235	9747	2405	9706	2574	9663	5
56	1897	9818	2068	9784	2238	9746	2408	9706	2577	9662	4
57	1900	9818	2071	9783	2241	9746	2411	9705	2580	9662	3
58	1902	9817	2073	9783	2244	9745	2414	9704	2583	9661	2
59	1905	9817	2076	9782	2247	9744	2416	9704	2585	9660	1
60	1908	9816	2079	9781	2250	9744	2419	9703	2588	9659	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	79°		78°		77°		76°		75°		

°	15°		16°		17°		18°		19°		°
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1	2591	9659	2759	9612	2926	9562	3093	9510	3258	9454	59
2	2594	9658	2762	9611	2929	9561	3096	9509	3261	9453	58
3	2597	9657	2765	9610	2932	9560	3098	9508	3264	9452	57
4	2599	9656	2768	9609	2935	9560	3101	9507	3267	9451	56
5	2602	9655	2770	9609	2938	9559	3104	9506	3269	9450	55
6	2605	9655	2773	9608	2940	9558	3107	9505	3272	9449	54
7	2608	9654	2776	9607	2943	9557	3110	9504	3275	9449	53
8	2611	9653	2779	9606	2946	9556	3112	9503	3278	9448	52
9	2613	9652	2782	9605	2949	9555	3115	9502	3280	9447	51
10	2616	9652	2784	9605	2952	9555	3118	9502	3283	9446	50
11	2619	9651	2787	9604	2954	9554	3121	9501	3286	9445	49
12	2622	9650	2790	9603	2957	9553	3123	9500	3289	9444	48
13	2625	9649	2793	9602	2960	9552	3126	9499	3291	9443	47
14	2628	9649	2795	9601	2963	9551	3129	9498	3294	9442	46
15	2630	9648	2798	9600	2965	9550	3132	9497	3297	9441	45
16	2633	9647	2801	9600	2968	9549	3134	9496	3300	9440	44
17	2636	9646	2804	9599	2971	9548	3137	9495	3302	9439	43
18	2639	9646	2807	9598	2974	9548	3140	9494	3305	9438	42
19	2642	9645	2809	9597	2977	9547	3143	9493	3308	9437	41
20	2644	9644	2812	9596	2979	9546	3145	9492	3311	9436	40
21	2647	9643	2815	9596	2982	9545	3148	9492	3313	9435	39
22	2650	9642	2818	9595	2985	9544	3151	9491	3316	9434	38
23	2653	9642	2821	9594	2988	9543	3154	9490	3319	9433	37
24	2656	9641	2823	9593	2990	9542	3156	9489	3322	9432	36
25	2658	9640	2826	9592	2993	9542	3159	9488	3324	9431	35
26	2661	9639	2829	9591	2996	9541	3162	9487	3327	9430	34
27	2664	9639	2832	9591	2999	9540	3165	9486	3330	9429	33
28	2667	9638	2835	9590	3002	9539	3168	9485	3333	9428	32
29	2670	9637	2837	9589	3004	9538	3170	9484	3335	9427	31
30	2672	9636	2840	9588	3007	9537	3173	9483	3338	9426	30
31	2675	9636	2843	9587	3010	9536	3176	9482	3341	9425	29
32	2678	9635	2846	9587	3013	9535	3179	9481	3344	9424	28
33	2681	9634	2849	9586	3015	9535	3181	9480	3346	9423	27
34	2684	9633	2851	9585	3018	9534	3184	9480	3349	9423	26
35	2686	9632	2854	9584	3021	9533	3187	9479	3352	9422	25
36	2689	9632	2857	9583	3024	9532	3190	9478	3355	9421	24
37	2692	9631	2860	9582	3026	9531	3192	9477	3357	9420	23
38	2695	9630	2862	9582	3029	9530	3195	9476	3360	9419	22
39	2698	9629	2865	9581	3032	9529	3198	9475	3363	9418	21
40	2700	9628	2868	9580	3035	9528	3201	9474	3365	9417	20
41	2703	9628	2871	9579	3038	9527	3203	9473	3368	9416	19
42	2706	9627	2874	9578	3040	9527	3206	9472	3371	9415	18
43	2709	9626	2876	9577	3043	9526	3209	9471	3374	9414	17
44	2712	9625	2879	9577	3046	9525	3212	9470	3376	9413	16
45	2714	9625	2882	9576	3049	9524	3214	9469	3379	9412	15
46	2717	9624	2885	9575	3051	9523	3217	9468	3382	9411	14
47	2720	9623	2888	9574	3054	9522	3220	9467	3385	9410	13
48	2723	9622	2890	9573	3057	9521	3223	9466	3387	9409	12
49	2726	9621	2893	9572	3060	9520	3225	9466	3390	9408	11
50	2728	9621	2896	9572	3062	9520	3228	9465	3393	9407	10
51	2731	9620	2899	9571	3065	9519	3231	9464	3396	9406	9
52	2734	9619	2901	9570	3068	9518	3234	9463	3398	9405	8
53	2737	9618	2903	9569	3071	9517	3236	9462	3401	9404	7
54	2740	9617	2907	9568	3074	9516	3239	9461	3404	9403	6
55	2742	9617	2910	9567	3076	9515	3242	9460	3407	9402	5
56	2745	9616	2913	9566	3079	9514	3245	9459	3409	9401	4
57	2748	9615	2915	9566	3082	9513	3247	9458	3412	9400	3
58	2751	9614	2918	9565	3085	9512	3250	9457	3415	9399	2
59	2754	9613	2921	9564	3087	9511	3253	9456	3417	9398	1
60	2756	9613	2924	9563	3090	9511	3256	9455	3420	9397	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	74°		73°		72°		71°		70°		





	26°		27°		28°		29°				
	sin	cos	sin	cos	sin	cos	sin	cos			
0	4226	9063	4384	8938	4540	8910	4693	8829	4848	8746	60
1	4229	9062	4386	8987	4542	8909	4697	8828	4851	8745	59
2	4231	9061	4389	8985	4545	8907	4700	8827	4853	8743	58
3	4234	9059	4392	8984	4548	8906	4702	8825	4856	8742	57
4	4237	9058	4394	8983	4550	8905	4705	8824	4858	8741	56
5	4239	9057	4397	8982	4553	8903	4708	8823	4861	8739	55
6	4242	9056	4399	8980	4555	8902	4710	8821	4863	8738	54
7	4245	9054	4402	8979	4558	8901	4713	8820	4866	8736	53
8	4247	9053	4405	8978	4561	8899	4715	8819	4868	8735	52
9	4250	9052	4407	8976	4563	8898	4718	8817	4871	8733	51
10	4253	9051	4410	8975	4566	8897	4720	8816	4874	8732	50
11	4255	9050	4412	8974	4568	8895	4723	8814	4876	8731	49
12	4258	9048	4415	8973	4571	8894	4726	8813	4879	8729	48
13	4260	9047	4418	8971	4574	8893	4728	8812	4881	8728	47
14	4263	9046	4420	8970	4576	8892	4731	8810	4884	8726	46
15	4266	9045	4423	8969	4579	8890	4733	8809	4886	8725	45
16	4268	9043	4425	8967	4581	8889	4736	8808	4889	8724	44
17	4271	9042	4428	8966	4584	8888	4738	8806	4891	8722	43
18	4274	9041	4431	8965	4586	8886	4741	8805	4894	8721	42
19	4276	9040	4433	8964	4589	8885	4743	8803	4896	8719	41
20	4279	9038	4436	8962	4592	8884	4746	8802	4899	8718	40
21	4281	9037	4439	8961	4594	8882	4749	8801	4901	8716	39
22	4284	9035	4441	8960	4597	8881	4751	8799	4904	8715	38
23	4287	9035	4444	8958	4599	8879	4754	8798	4907	8714	37
24	4289	9033	4446	8957	4602	8878	4756	8796	4909	8712	36
25	4292	9032	4449	8956	4605	8877	4759	8795	4912	8711	35
26	4295	9031	4452	8955	4607	8875	4761	8794	4914	8709	34
27	4297	9030	4454	8953	4610	8874	4764	8792	4917	8708	33
28	4300	9028	4457	8952	4612	8873	4766	8791	4919	8706	32
29	4302	9027	4459	8951	4615	8871	4769	8790	4922	8705	31
30	4305	9026	4462	8949	4617	8870	4772	8788	4924	8704	30
31	4308	9025	4465	8948	4620	8869	4774	8787	4927	8703	29
32	4310	9023	4467	8947	4623	8867	4777	8785	4929	8701	28
33	4313	9022	4470	8945	4625	8866	4779	8784	4932	8699	27
34	4316	9021	4472	8944	4628	8865	4782	8783	4934	8698	26
35	4318	9020	4475	8943	4630	8863	4784	8781	4937	8696	25
36	4321	9018	4478	8942	4633	8862	4787	8780	4939	8695	24
37	4323	9017	4480	8940	4636	8861	4789	8778	4942	8694	23
38	4326	9016	4483	8939	4638	8859	4792	8777	4944	8692	22
39	4329	9015	4485	8938	4641	8858	4795	8776	4947	8691	21
40	4331	9013	4488	8936	4643	8857	4797	8774	4950	8689	20
41	4334	9012	4491	8935	4646	8855	4800	8773	4952	8688	19
42	4337	9011	4493	8934	4648	8854	4802	8771	4955	8686	18
43	4339	9010	4496	8932	4651	8853	4805	8770	4957	8685	17
44	4342	9008	4498	8931	4654	8851	4807	8769	4960	8683	16
45	4344	9007	4501	8930	4656	8850	4810	8767	4962	8682	15
46	4347	9006	4504	8928	4659	8849	4812	8766	4965	8681	14
47	4350	9004	4506	8927	4661	8847	4815	8764	4967	8679	13
48	4352	9003	4509	8926	4664	8846	4818	8763	4970	8678	12
49	4355	9002	4511	8925	4666	8844	4820	8762	4972	8676	11
50	4358	9001	4514	8923	4669	8843	4823	8760	4975	8675	10
51	4360	8999	4517	8922	4672	8842	4825	8759	4977	8673	9
52	4363	8998	4519	8921	4674	8840	4828	8757	4980	8672	8
53	4365	8997	4522	8919	4677	8839	4830	8756	4982	8670	7
54	4368	8996	4524	8918	4679	8838	4833	8755	4985	8669	6
55	4371	8994	4527	8917	4682	8836	4835	8753	4987	8668	5
56	4373	8993	4530	8915	4684	8835	4838	8752	4990	8666	4
57	4376	8992	4532	8914	4687	8834	4840	8750	4992	8665	3
58	4378	8990	4535	8913	4690	8832	4843	8749	4995	8663	2
59	4381	8989	4537	8911	4692	8831	4846	8748	4997	8662	1
60	4384	8988	4540	8910	4695	8829	4848	8746	5000	8660	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	64°		63°		62°		61°		60°		

NATURAL SINES AND COSINES.

/	30°		31°		32°		33°		34°		/
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	5000	8660	5150	8572	5299	8480	5446	8387	5592	8290	60
1	5003	8659	5153	8570	5302	8479	5449	8385	5594	8289	59
2	5005	8657	5155	8569	5304	8477	5451	8384	5597	8287	58
3	5008	8656	5158	8567	5307	8476	5454	8382	5599	8285	57
4	5010	8654	5160	8566	5309	8474	5456	8380	5602	8284	56
5	5013	8653	5163	8564	5312	8473	5459	8379	5604	8283	55
6	5015	8652	5165	8563	5314	8471	5461	8377	5606	8281	54
7	5018	8650	5168	8561	5316	8470	5463	8376	5609	8279	53
8	5020	8649	5170	8560	5319	8468	5466	8374	5611	8277	52
9	5023	8647	5173	8558	5321	8467	5468	8372	5614	8276	51
10	5025	8646	5175	8557	5324	8465	5471	8371	5616	8274	50
11	5028	8644	5178	8555	5326	8463	5473	8369	5618	8272	49
12	5030	8643	5180	8554	5329	8462	5476	8368	5621	8271	48
13	5033	8641	5183	8552	5331	8460	5478	8366	5623	8269	47
14	5035	8640	5185	8551	5334	8459	5480	8364	5626	8268	46
15	5038	8638	5188	8549	5336	8457	5483	8363	5628	8266	45
16	5040	8637	5190	8548	5339	8456	5485	8361	5630	8264	44
17	5043	8635	5193	8546	5341	8454	5488	8360	5633	8263	43
18	5045	8634	5195	8545	5344	8453	5490	8358	5635	8261	42
19	5048	8632	5198	8543	5346	8451	5493	8356	5638	8259	41
20	5050	8631	5200	8542	5348	8450	5495	8355	5640	8258	40
21	5053	8630	5203	8540	5351	8448	5498	8353	5642	8256	39
22	5055	8628	5205	8539	5353	8446	5500	8352	5645	8254	38
23	5058	8627	5208	8537	5356	8445	5502	8350	5647	8253	37
24	5060	8625	5210	8536	5358	8443	5505	8348	5650	8251	36
25	5063	8624	5213	8534	5361	8442	5507	8347	5652	8249	35
26	5065	8622	5215	8532	5363	8440	5510	8345	5654	8248	34
27	5068	8621	5218	8531	5366	8439	5512	8344	5657	8246	33
28	5070	8619	5220	8529	5368	8437	5515	8342	5659	8245	32
29	5073	8618	5223	8528	5371	8435	5517	8340	5662	8243	31
30	5075	8616	5225	8526	5373	8434	5519	8339	5664	8241	30
31	5078	8615	5227	8525	5375	8432	5522	8337	5666	8240	29
32	5080	8613	5230	8523	5378	8431	5524	8336	5669	8238	28
33	5083	8612	5232	8522	5380	8429	5527	8334	5671	8236	27
34	5085	8610	5235	8520	5383	8428	5529	8332	5674	8235	26
35	5088	8609	5237	8519	5385	8426	5531	8331	5676	8233	25
36	5090	8607	5240	8517	5388	8425	5534	8329	5678	8231	24
37	5093	8606	5242	8516	5390	8423	5536	8328	5681	8230	23
38	5095	8604	5245	8514	5393	8421	5539	8326	5683	8228	22
39	5098	8603	5247	8513	5395	8420	5541	8324	5686	8226	21
40	5100	8601	5250	8511	5398	8418	5544	8323	5688	8225	20
41	5103	8600	5252	8510	5400	8417	5546	8321	5690	8223	19
42	5105	8599	5255	8508	5402	8415	5548	8320	5693	8221	18
43	5108	8597	5257	8507	5405	8414	5551	8318	5695	8220	17
44	5110	8596	5260	8505	5407	8412	5553	8316	5698	8218	16
45	5113	8594	5262	8504	5410	8410	5556	8315	5700	8216	15
46	5115	8593	5265	8502	5412	8409	5558	8313	5702	8215	14
47	5118	8591	5267	8500	5415	8407	5561	8311	5705	8213	13
48	5120	8590	5270	8499	5417	8406	5563	8310	5707	8211	12
49	5123	8588	5272	8497	5420	8404	5565	8308	5710	8210	11
50	5125	8587	5275	8496	5422	8403	5568	8307	5712	8208	10
51	5128	8585	5277	8494	5424	8401	5570	8305	5714	8207	9
52	5130	8584	5279	8493	5427	8399	5573	8303	5717	8205	8
53	5133	8582	5282	8491	5429	8398	5575	8302	5719	8203	7
54	5135	8581	5284	8490	5432	8396	5577	8300	5721	8202	6
55	5138	8579	5287	8488	5434	8395	5580	8299	5724	8200	5
56	5140	8578	5289	8487	5437	8393	5583	8297	5726	8198	4
57	5143	8576	5292	8485	5439	8391	5585	8295	5729	8197	3
58	5145	8575	5294	8484	5442	8390	5587	8294	5731	8195	2
59	5148	8573	5297	8482	5444	8388	5590	8292	5733	8193	1
60	5150	8572	5299	8480	5446	8387	5592	8290	5736	8192	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	30°		31°		32°		33°		34°		

°	35°		36°		37°		38°		39°		°
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	5736	8192	5878	8090	6018	7986	6157	7880	6293	7771	60
1	5738	8190	5880	8088	6020	7985	6159	7878	6295	7770	59
2	5741	8188	5883	8087	6023	7983	6161	7877	6298	7768	58
3	5743	8187	5885	8085	6025	7981	6163	7875	6300	7766	57
4	5745	8185	5887	8083	6027	7979	6166	7873	6302	7764	56
5	5748	8183	5890	8082	6030	7978	6168	7871	6305	7762	55
6	5750	8181	5892	8080	6032	7976	6170	7869	6307	7760	54
7	5752	8180	5894	8078	6034	7974	6173	7868	6309	7759	53
8	5755	8178	5897	8076	6037	7972	6175	7866	6311	7757	52
9	5757	8176	5899	8075	6039	7971	6177	7864	6314	7755	51
10	5760	8175	5901	8073	6041	7969	6180	7862	6316	7753	50
11	5762	8173	5901	8071	6044	7967	6182	7860	6318	7751	49
12	5764	8171	5906	8070	6046	7965	6184	7859	6320	7749	48
13	5767	8170	5908	8068	6048	7964	6186	7857	6323	7748	47
14	5769	8168	5911	8066	6051	7962	6189	7855	6325	7746	46
15	5771	8166	5913	8064	6053	7960	6191	7853	6327	7744	45
16	5774	8165	5915	8063	6055	7958	6193	7851	6329	7742	44
17	5776	8163	5918	8061	6058	7956	6196	7850	6332	7740	43
18	5779	8161	5920	8059	6060	7955	6198	7848	6334	7738	42
19	5781	8160	5922	8058	6062	7953	6200	7846	6336	7737	41
20	5783	8158	5925	8056	6065	7951	6202	7844	6338	7735	40
21	5786	8156	5927	8054	6067	7950	6205	7842	6341	7733	39
22	5788	8155	5930	8052	6069	7948	6207	7841	6343	7731	38
23	5790	8153	5932	8051	6071	7946	6209	7839	6345	7729	37
24	5793	8151	5934	8049	6074	7944	6211	7837	6347	7727	36
25	5795	8150	5937	8047	6076	7942	6214	7835	6350	7725	35
26	5798	8148	5939	8045	6078	7941	6216	7833	6352	7724	34
27	5800	8146	5941	8044	6081	7939	6218	7832	6354	7722	33
28	5802	8145	5944	8042	6083	7937	6221	7830	6356	7720	32
29	5805	8143	5946	8040	6085	7935	6223	7828	6359	7718	31
30	5807	8141	5948	8039	6088	7934	6225	7826	6361	7716	30
31	5809	8139	5951	8037	6090	7932	6227	7824	6363	7714	29
32	5812	8138	5953	8035	6092	7930	6230	7822	6365	7713	28
33	5814	8136	5955	8033	6095	7928	6232	7821	6368	7711	27
34	5816	8134	5958	8032	6097	7926	6234	7819	6370	7709	26
35	5819	8133	5960	8030	6099	7925	6237	7817	6372	7707	25
36	5821	8131	5962	8028	6101	7923	6239	7815	6374	7705	24
37	5824	8129	5965	8026	6104	7921	6241	7813	6376	7703	23
38	5826	8128	5967	8025	6106	7919	6243	7812	6379	7701	22
39	5828	8126	5969	8023	6108	7918	6246	7810	6381	7700	21
40	5831	8124	5972	8021	6111	7916	6248	7808	6383	7698	20
41	5833	8123	5974	8020	6113	7914	6250	7806	6385	7696	19
42	5835	8121	5976	8018	6115	7912	6252	7804	6388	7694	18
43	5838	8119	5979	8016	6118	7910	6255	7802	6390	7692	17
44	5840	8117	5981	8014	6120	7909	6257	7801	6392	7690	16
45	5842	8116	5983	8013	6122	7907	6259	7799	6394	7688	15
46	5845	8114	5986	8011	6124	7905	6262	7797	6397	7687	14
47	5847	8112	5988	8009	6127	7903	6264	7795	6399	7685	13
48	5850	8111	5990	8007	6129	7902	6266	7793	6401	7683	12
49	5852	8109	5993	8006	6131	7900	6268	7792	6403	7681	11
50	5854	8107	5995	8004	6134	7898	6271	7790	6406	7679	10
51	5857	8106	5997	8002	6136	7896	6273	7788	6408	7677	9
52	5859	8104	6000	8000	6138	7894	6275	7786	6410	7675	8
53	5861	8102	6002	7999	6141	7893	6277	7784	6412	7674	7
54	5864	8100	6004	7997	6143	7891	6280	7782	6414	7672	6
55	5866	8099	6007	7995	6145	7889	6282	7781	6417	7670	5
56	5868	8097	6009	7993	6147	7887	6284	7779	6419	7668	4
57	5871	8095	6011	7992	6150	7885	6286	7777	6421	7666	3
58	5873	8094	6014	7990	6152	7884	6289	7775	6423	7664	2
59	5875	8092	6016	7988	6154	7882	6291	7773	6426	7662	1
60	5878	8090	6018	7986	6157	7880	6293	7771	6428	7660	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	54°		53°		52°		51°		50°		

NATURAL SINES AND COSINES.

°	40°		41°		42°		43°		44°		°
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	6428	7660	6561	7547	6691	7431	6820	7314	6947	7193	60
1	6430	7659	6563	7545	6693	7430	6822	7312	6949	7191	59
2	6432	7657	6565	7543	6696	7428	6824	7310	6951	7189	58
3	6435	7655	6567	7541	6698	7426	6826	7308	6953	7187	57
4	6437	7653	6569	7539	6700	7424	6828	7306	6955	7185	56
5	6439	7651	6572	7538	6702	7422	6831	7304	6957	7183	55
6	6441	7649	6574	7536	6704	7420	6833	7302	6959	7181	54
7	6443	7647	6576	7534	6706	7418	6835	7300	6961	7179	53
8	6446	7645	6578	7532	6709	7416	6837	7298	6963	7177	52
9	6448	7644	6580	7530	6711	7414	6839	7296	6965	7175	51
10	6450	7642	6583	7528	6713	7412	6841	7294	6967	7173	50
11	6452	7640	6585	7526	6715	7410	6843	7292	6970	7171	49
12	6455	7638	6587	7524	6717	7408	6845	7290	6972	7169	48
13	6457	7636	6589	7522	6719	7406	6848	7288	6974	7167	47
14	6459	7634	6591	7520	6722	7404	6850	7286	6976	7165	46
15	6461	7632	6593	7518	6724	7402	6852	7284	6978	7163	45
16	6463	7630	6596	7516	6726	7400	6854	7282	6980	7161	44
17	6466	7629	6598	7515	6728	7398	6856	7280	6982	7159	43
18	6468	7627	6600	7513	6730	7396	6858	7278	6984	7157	42
19	6470	7625	6602	7511	6732	7394	6860	7276	6986	7155	41
20	6472	7623	6604	7509	6734	7392	6862	7274	6988	7153	40
21	6475	7621	6607	7507	6737	7390	6865	7272	6990	7151	39
22	6477	7619	6609	7505	6739	7388	6867	7270	6992	7149	38
23	6479	7617	6611	7503	6741	7387	6869	7268	6995	7147	37
24	6481	7615	6613	7501	6743	7385	6871	7266	6997	7145	36
25	6483	7613	6615	7499	6745	7383	6873	7264	6999	7143	35
26	6486	7612	6617	7497	6747	7381	6875	7262	7001	7141	34
27	6488	7610	6620	7495	6749	7379	6877	7260	7003	7139	33
28	6490	7608	6622	7493	6752	7377	6879	7258	7005	7137	32
29	6492	7606	6624	7491	6754	7375	6881	7256	7007	7135	31
30	6494	7604	6626	7490	6756	7373	6884	7254	7009	7133	30
31	6497	7602	6628	7488	6758	7371	6886	7252	7011	7130	29
32	6499	7600	6631	7486	6760	7369	6888	7250	7013	7128	28
33	6501	7598	6633	7484	6762	7367	6890	7248	7015	7126	27
34	6503	7596	6635	7482	6764	7365	6892	7246	7017	7124	26
35	6506	7595	6637	7480	6767	7363	6894	7244	7019	7122	25
36	6508	7593	6639	7478	6769	7361	6896	7242	7022	7120	24
37	6510	7591	6641	7476	6771	7359	6898	7240	7024	7118	23
38	6512	7589	6644	7474	6773	7357	6900	7238	7026	7116	22
39	6514	7587	6646	7472	6775	7355	6903	7236	7028	7114	21
40	6517	7585	6648	7470	6777	7353	6905	7234	7030	7112	20
41	6519	7583	6650	7468	6779	7351	6907	7232	7032	7110	19
42	6521	7581	6652	7466	6782	7349	6909	7230	7034	7108	18
43	6523	7579	6654	7464	6784	7347	6911	7228	7036	7106	17
44	6525	7578	6657	7463	6786	7345	6913	7226	7038	7104	16
45	6528	7576	6659	7461	6788	7343	6915	7224	7040	7102	15
46	6530	7574	6661	7459	6790	7341	6917	7222	7042	7100	14
47	6532	7572	6663	7457	6792	7339	6919	7220	7044	7098	13
48	6534	7570	6665	7455	6794	7337	6921	7218	7046	7096	12
49	6536	7568	6667	7453	6797	7335	6924	7216	7048	7094	11
50	6539	7566	6670	7451	6799	7333	6926	7214	7050	7092	10
51	6541	7564	6672	7449	6801	7331	6928	7212	7053	7090	9
52	6543	7562	6674	7447	6803	7329	6930	7210	7055	7088	8
53	6545	7560	6676	7445	6805	7327	6932	7208	7057	7085	7
54	6547	7559	6678	7443	6807	7325	6934	7206	7059	7083	6
55	6550	7557	6680	7441	6809	7323	6936	7203	7061	7081	5
56	6552	7555	6683	7439	6811	7321	6938	7201	7063	7079	4
57	6554	7553	6685	7437	6814	7319	6940	7199	7065	7077	3
58	6556	7551	6687	7435	6816	7318	6942	7197	7067	7075	2
59	6558	7549	6689	7433	6818	7316	6944	7195	7069	7073	1
60	6561	7547	6691	7431	6820	7314	6947	7193	7071	7071	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	40°		41°		42°		43°		44°		

	0°		1°		2°		3°		4°		
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	0000	Infinite	0175	57.2900	0349	28.6363	0524	19.0811	0699	14.3607	60
1	0003	3437.75	0177	56.3506	0352	28.3994	0527	18.9755	0702	14.2411	59
2	0006	1718.87	0180	55.4415	0355	28.1664	0530	18.8711	0705	14.1821	58
3	0009	1145.92	0183	54.5613	0358	27.9372	0533	18.7678	0708	14.1235	57
4	0012	859.436	0186	53.7066	0361	27.7117	0536	18.6656	0711	14.0655	56
5	0015	687.549	0189	52.8821	0364	27.4899	0539	18.5645	0714	14.0079	55
6	0017	572.957	0192	52.0807	0367	27.2715	0542	18.4645	0717	13.9507	54
7	0020	491.106	0195	51.3032	0370	27.0566	0544	18.3655	0720	13.8940	53
8	0023	429.718	0198	50.5485	0373	26.8450	0547	18.2677	0723	13.8378	52
9	0026	381.971	0201	49.8157	0375	26.6367	0550	18.1703	0726	13.7821	51
10	0029	343.774	0204	49.1039	0378	26.4316	0553	18.0750	0729	13.7267	50
11	0032	312.521	0207	48.4121	0381	26.2296	0555	17.9802	0731	13.6719	49
12	0035	286.478	0209	47.7395	0384	26.0307	0559	17.8863	0734	13.6174	48
13	0038	264.441	0212	47.0853	0387	25.8348	0562	17.7934	0737	13.5634	47
14	0041	245.552	0215	46.4489	0390	25.6418	0565	17.7015	0740	13.5098	46
15	0044	229.182	0218	45.8294	0393	25.4517	0568	17.6106	0743	13.4566	45
16	0047	214.858	0221	45.2261	0396	25.2644	0571	17.5205	0746	13.4039	44
17	0049	202.219	0224	44.6386	0399	25.0798	0574	17.4314	0749	13.3515	43
18	0052	190.984	0227	44.0661	0402	24.8978	0577	17.3432	0752	13.2996	42
19	0055	180.932	0230	43.5091	0405	24.7185	0580	17.2558	0755	13.2480	41
20	0058	171.885	0233	42.9641	0407	24.5418	0582	17.1693	0758	13.1969	40
21	0061	163.700	0236	42.4335	0410	24.3675	0585	17.0837	0761	13.1461	39
22	0064	156.259	0239	41.9158	0413	24.1957	0588	16.9990	0764	13.0958	38
23	0067	149.465	0241	41.4106	0416	24.0263	0591	16.9150	0767	13.0458	37
24	0070	143.237	0244	40.9174	0419	23.8593	0594	16.8319	0769	12.9962	36
25	0073	137.507	0247	40.4358	0422	23.6945	0597	16.7496	0772	12.9469	35
26	0076	132.219	0250	39.9655	0425	23.5321	0600	16.6681	0775	12.8981	34
27	0079	127.321	0253	39.5059	0428	23.3718	0603	16.5874	0778	12.8496	33
28	0081	122.774	0256	39.0568	0431	23.2137	0606	16.5075	0781	12.8014	32
29	0084	118.540	0259	38.6177	0434	23.0577	0609	16.4283	0784	12.7536	31
30	0087	114.589	0262	38.1885	0437	22.9038	0612	16.3499	0787	12.7062	30
31	0090	110.892	0265	37.7686	0440	22.7519	0615	16.2722	0790	12.6591	29
32	0093	107.426	0268	37.3579	0442	22.6020	0617	16.1952	0793	12.6124	28
33	0096	104.171	0271	36.9560	0445	22.4541	0620	16.1190	0796	12.5660	27
34	0099	101.107	0274	36.5627	0448	22.3081	0623	16.0435	0799	12.5199	26
35	0102	98.2179	0276	36.1776	0451	22.1640	0626	15.9687	0802	12.4742	25
36	0105	95.4895	0279	35.8006	0454	22.0217	0629	15.8945	0805	12.4288	24
37	0108	92.9085	0282	35.4313	0457	21.8813	0632	15.8211	0808	12.3838	23
38	0111	90.4633	0285	35.0695	0460	21.7426	0635	15.7483	0810	12.3390	22
39	0113	88.1436	0288	34.7151	0463	21.6056	0638	15.6762	0813	12.2946	21
40	0116	85.9398	0291	34.3678	0466	21.4704	0641	15.6048	0816	12.2505	20
41	0119	83.8435	0294	34.0273	0469	21.3369	0644	15.5340	0819	12.2067	19
42	0122	81.8476	0297	33.6935	0472	21.2049	0647	15.4638	0822	12.1632	18
43	0125	79.9434	0300	33.3662	0475	21.0747	0650	15.3943	0825	12.1201	17
44	0128	78.1263	0303	33.0453	0477	20.9460	0653	15.3254	0828	12.0772	16
45	0131	76.3900	0306	32.7303	0480	20.8188	0655	15.2571	0831	12.0346	15
46	0134	74.7292	0308	32.4213	0483	20.6932	0658	15.1893	0834	11.9923	14
47	0137	73.1390	0311	32.1181	0486	20.5691	0661	15.1222	0837	11.9504	13
48	0140	71.6151	0314	31.8205	0489	20.4465	0664	15.0557	0840	11.9087	12
49	0143	70.1533	0317	31.5284	0492	20.3253	0667	14.9898	0843	11.8673	11
50	0146	68.7501	0320	31.2416	0495	20.2056	0670	14.9244	0846	11.8262	10
51	0148	67.4019	0323	30.9599	0498	20.0872	0673	14.8596	0849	11.7853	9
52	0151	66.1055	0326	30.6833	0501	19.9702	0676	14.7954	0851	11.7448	8
53	0154	64.8580	0329	30.4116	0504	19.8546	0679	14.7317	0854	11.7045	7
54	0157	63.6567	0332	30.1446	0507	19.7403	0682	14.6685	0857	11.6645	6
55	0160	62.4992	0335	29.8823	0509	19.6273	0685	14.6059	0860	11.6243	5
56	0163	61.3829	0338	29.6245	0512	19.5156	0688	14.5438	0863	11.5853	4
57	0166	60.3058	0340	29.3711	0515	19.4051	0690	14.4823	0866	11.5461	3
58	0169	59.2659	0343	29.1220	0518	19.2959	0693	14.4212	0869	11.5072	2
59	0172	58.2612	0346	28.8771	0521	19.1879	0696	14.3607	0872	11.4685	1
60	0175	57.2900	0349	28.6363	0524	19.0811	0699	14.3007	0875	11.4301	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
	89°		88°		87°		86°		85°		

	5°		6°		7°		8°		9°		
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	0875	11.4391	1051	9.5144	1225	8.1443	1405	7.1154	1584	6.3133	00
1	0878	11.3919	1054	9.4978	1231	8.1248	1408	7.1004	1587	6.3019	59
2	0881	11.3440	1057	9.4614	1234	8.1054	1411	7.0855	1590	6.2901	58
3	0884	11.3163	1060	9.4352	1237	8.0860	1414	7.0706	1593	6.2783	57
4	0887	11.2789	1063	9.4090	1240	8.0667	1417	7.0558	1596	6.2666	56
5	0890	11.2417	1066	9.3831	1243	8.0476	1420	7.0410	1599	6.2549	55
6	0892	11.2048	1069	9.3572	1246	8.0285	1423	7.0264	1602	6.2432	54
7	0895	11.1681	1072	9.3315	1249	8.0095	1426	7.0117	1605	6.2316	53
8	0893	11.1316	1075	9.3060	1251	7.9906	1429	6.9972	1608	6.2200	52
9	0901	11.0954	1078	9.2805	1254	7.9718	1432	6.9827	1611	6.2085	51
10	0904	11.0594	1080	9.2553	1257	7.9530	1435	6.9682	1614	6.1970	50
11	0907	11.0237	1083	9.2302	1260	7.9344	1438	6.9538	1617	6.1856	49
12	0910	10.9882	1086	9.2052	1263	7.9158	1441	6.9395	1620	6.1742	48
13	0913	10.9529	1089	9.1803	1266	7.8973	1444	6.9252	1623	6.1628	47
14	0916	10.9178	1092	9.1555	1269	7.8789	1447	6.9110	1626	6.1515	46
15	0919	10.8829	1095	9.1309	1272	7.8606	1450	6.8969	1629	6.1402	45
16	0922	10.8481	1098	9.1065	1275	7.8424	1453	6.8828	1632	6.1290	44
17	0925	10.8139	1101	9.0821	1278	7.8242	1456	6.8687	1635	6.1178	43
18	0923	10.7797	1104	9.0579	1281	7.8062	1459	6.8548	1638	6.1066	42
19	0931	10.7457	1107	9.0338	1284	7.7883	1462	6.8406	1641	6.0955	41
20	0934	10.7119	1110	9.0098	1287	7.7704	1465	6.8269	1644	6.0844	40
21	0936	10.6783	1113	8.9860	1290	7.7525	1468	6.8131	1647	6.0734	39
22	0939	10.6450	1116	8.9623	1293	7.7348	1471	6.7994	1650	6.0624	38
23	0942	10.6118	1119	8.9387	1296	7.7171	1474	6.7856	1653	6.0514	37
24	0945	10.5789	1122	8.9152	1299	7.6996	1477	6.7720	1656	6.0405	36
25	0948	10.5462	1125	8.8919	1302	7.6821	1480	6.7584	1658	6.0296	35
26	0951	10.5136	1128	8.8686	1305	7.6647	1483	6.7448	1661	6.0188	34
27	0954	10.4813	1131	8.8455	1308	7.6473	1486	6.7313	1664	6.0080	33
28	0957	10.4491	1134	8.8225	1311	7.6301	1489	6.7179	1667	5.9972	32
29	0950	10.4172	1136	8.7996	1314	7.6129	1492	6.7045	1670	5.9865	31
30	0963	10.3854	1139	8.7769	1317	7.5958	1495	6.6912	1673	5.9758	30
31	0966	10.3538	1142	8.7542	1319	7.5787	1497	6.6779	1676	5.9651	29
32	0959	10.3224	1145	8.7317	1322	7.5618	1500	6.6646	1679	5.9545	28
33	0972	10.2913	1148	8.7093	1325	7.5449	1503	6.6514	1682	5.9439	27
34	0975	10.2602	1151	8.6870	1328	7.5281	1506	6.6383	1685	5.9333	26
35	0978	10.2294	1154	8.6648	1331	7.5113	1509	6.6252	1688	5.9228	25
36	0981	10.1988	1157	8.6427	1334	7.4947	1512	6.6122	1691	5.9124	24
37	0983	10.1683	1160	8.6203	1337	7.4781	1515	6.5992	1694	5.9019	23
38	0986	10.1381	1163	8.5989	1340	7.4615	1518	6.5863	1697	5.8915	22
39	0989	10.1080	1166	8.5772	1343	7.4451	1521	6.5734	1700	5.8811	21
40	0992	10.0780	1169	8.5555	1346	7.4287	1524	6.5606	1703	5.8708	20
41	0995	10.0483	1172	8.5340	1349	7.4124	1527	6.5478	1706	5.8605	19
42	0998	10.0187	1175	8.5126	1352	7.3962	1530	6.5350	1709	5.8502	18
43	1001	9.9893	1178	8.4913	1355	7.3800	1533	6.5223	1712	5.8400	17
44	1004	9.9601	1181	8.4701	1358	7.3639	1536	6.5097	1715	5.8298	16
45	1007	9.9310	1184	8.4490	1361	7.3479	1539	6.4971	1718	5.8197	15
46	1010	9.9021	1187	8.4280	1364	7.3319	1542	6.4846	1721	5.8095	14
47	1013	9.8734	1189	8.4071	1367	7.3160	1545	6.4721	1724	5.7994	13
48	1016	9.8448	1192	8.3863	1370	7.3002	1548	6.4596	1727	5.7894	12
49	1019	9.8164	1195	8.3656	1373	7.2844	1551	6.4472	1730	5.7794	11
50	1022	9.7882	1198	8.3450	1376	7.2687	1554	6.4348	1733	5.7694	10
51	1025	9.7601	1201	8.3245	1379	7.2531	1557	6.4225	1736	5.7594	9
52	1028	9.7322	1204	8.3041	1382	7.2375	1560	6.4103	1739	5.7495	8
53	1030	9.7044	1207	8.2838	1385	7.2220	1563	6.3980	1742	5.7396	7
54	1033	9.6768	1210	8.2636	1388	7.2066	1566	6.3859	1745	5.7297	6
55	1036	9.6499	1213	8.2434	1391	7.1912	1569	6.3737	1748	5.7199	5
56	1039	9.6226	1216	8.2234	1394	7.1759	1572	6.3617	1751	5.7101	4
57	1042	9.5949	1219	8.2035	1397	7.1607	1575	6.3496	1754	5.7004	3
58	1045	9.5679	1222	8.1837	1399	7.1455	1578	6.3376	1757	5.6906	2
59	1048	9.5411	1225	8.1640	1402	7.1304	1581	6.3257	1760	5.6809	1
60	1051	9.5144	1228	8.1443	1405	7.1154	1584	6.3138	1763	5.6713	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
	84°		83°		82°		81°		80°		



°	10°		11°		12°		13°		14°		°
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	1763	5.6713	1941	5.1146	2126	4.7045	2309	4.3315	2493	4.0103	00
1	1766	5.6617	1947	5.1366	2129	4.6979	2312	4.3257	2496	4.0038	59
2	1769	5.6521	1950	5.1286	2132	4.6912	2315	4.3200	2499	4.0009	58
3	1772	5.6425	1953	5.1207	2135	4.6845	2318	4.3143	2503	3.9979	57
4	1775	5.6330	1956	5.1128	2138	4.6779	2321	4.3086	2506	3.9910	56
5	1778	5.6234	1959	5.1049	2141	4.6712	2324	4.3029	2509	3.9861	55
6	1781	5.6140	1962	5.0970	2144	4.6646	2327	4.2972	2512	3.9812	54
7	1784	5.6045	1965	5.0892	2147	4.6580	2330	4.2916	2515	3.9763	53
8	1787	5.5951	1968	5.0814	2150	4.6514	2333	4.2859	2518	3.9714	52
9	1790	5.5857	1971	5.0736	2153	4.6448	2336	4.2803	2521	3.9665	51
10	1793	5.5764	1974	5.0658	2156	4.6382	2339	4.2747	2524	3.9617	50
11	1796	5.5671	1977	5.0581	2159	4.6317	2342	4.2691	2527	3.9568	49
12	1799	5.5578	1980	5.0504	2162	4.6252	2345	4.2635	2530	3.9520	48
13	1802	5.5485	1983	5.0427	2165	4.6187	2349	4.2580	2533	3.9471	47
14	1805	5.5393	1986	5.0350	2168	4.6122	2352	4.2524	2537	3.9423	46
15	1808	5.5301	1989	5.0273	2171	4.6057	2355	4.2468	2540	3.9375	45
16	1811	5.5209	1992	5.0197	2174	4.5993	2358	4.2413	2543	3.9327	44
17	1814	5.5118	1995	5.0121	2177	4.5928	2361	4.2358	2546	3.9279	43
18	1817	5.5026	1998	5.0045	2180	4.5864	2364	4.2303	2549	3.9232	42
19	1820	5.4936	2001	4.9969	2183	4.5800	2367	4.2248	2552	3.9184	41
20	1823	5.4845	2004	4.9894	2186	4.5736	2370	4.2193	2555	3.9136	40
21	1826	5.4755	2007	4.9819	2189	4.5673	2373	4.2139	2558	3.9089	39
22	1829	5.4665	2010	4.9744	2193	4.5609	2376	4.2084	2561	3.9042	38
23	1832	5.4575	2013	4.9669	2196	4.5546	2379	4.2030	2564	3.8995	37
24	1835	5.4486	2016	4.9594	2199	4.5483	2382	4.1976	2568	3.8947	36
25	1838	5.4397	2019	4.9520	2202	4.5420	2385	4.1922	2571	3.8900	35
26	1841	5.4308	2022	4.9446	2205	4.5357	2388	4.1868	2574	3.8854	34
27	1844	5.4219	2025	4.9372	2208	4.5294	2392	4.1814	2577	3.8807	33
28	1847	5.4131	2028	4.9298	2211	4.5232	2395	4.1760	2580	3.8760	32
29	1850	5.4043	2031	4.9225	2214	4.5169	2398	4.1706	2583	3.8714	31
30	1853	5.3955	2035	4.9152	2217	4.5107	2401	4.1653	2586	3.8667	30
31	1856	5.3868	2038	4.9078	2220	4.5045	2404	4.1600	2589	3.8621	29
32	1859	5.3781	2041	4.9006	2223	4.4983	2407	4.1547	2592	3.8575	28
33	1862	5.3694	2044	4.8933	2226	4.4922	2410	4.1493	2595	3.8528	27
34	1865	5.3607	2047	4.8860	2229	4.4860	2413	4.1441	2599	3.8482	26
35	1868	5.3521	2050	4.8788	2232	4.4799	2416	4.1388	2602	3.8436	25
36	1871	5.3435	2053	4.8716	2235	4.4737	2419	4.1335	2605	3.8391	24
37	1874	5.3349	2056	4.8644	2238	4.4676	2422	4.1282	2608	3.8345	23
38	1877	5.3263	2059	4.8573	2241	4.4615	2425	4.1230	2611	3.8299	22
39	1880	5.3178	2062	4.8501	2244	4.4555	2428	4.1178	2614	3.8254	21
40	1883	5.3093	2065	4.8430	2247	4.4494	2432	4.1126	2617	3.8208	20
41	1887	5.3008	2068	4.8359	2251	4.4434	2435	4.1074	2620	3.8163	19
42	1890	5.2924	2071	4.8288	2254	4.4374	2438	4.1022	2623	3.8118	18
43	1893	5.2839	2074	4.8218	2257	4.4313	2441	4.0970	2627	3.8073	17
44	1896	5.2755	2077	4.8147	2260	4.4253	2444	4.0918	2630	3.8028	16
45	1899	5.2672	2080	4.8077	2263	4.4194	2447	4.0867	2633	3.7983	15
46	1902	5.2588	2083	4.8007	2266	4.4134	2450	4.0815	2636	3.7938	14
47	1905	5.2505	2086	4.7937	2269	4.4075	2453	4.0764	2639	3.7893	13
48	1908	5.2422	2089	4.7867	2272	4.4015	2456	4.0713	2642	3.7848	12
49	1911	5.2339	2092	4.7798	2275	4.3956	2459	4.0662	2645	3.7804	11
50	1914	5.2257	2095	4.7729	2278	4.3897	2462	4.0611	2648	3.7760	10
51	1917	5.2174	2098	4.7659	2281	4.3838	2465	4.0560	2651	3.7715	9
52	1920	5.2092	2101	4.7591	2284	4.3779	2469	4.0509	2655	3.7671	8
53	1923	5.2011	2104	4.7522	2287	4.3721	2472	4.0459	2658	3.7627	7
54	1926	5.1929	2107	4.7453	2290	4.3662	2475	4.0408	2661	3.7583	6
55	1929	5.1848	2110	4.7385	2293	4.3604	2478	4.0358	2664	3.7539	5
56	1932	5.1767	2113	4.7317	2296	4.3546	2481	4.0308	2667	3.7495	4
57	1935	5.1686	2116	4.7249	2299	4.3488	2484	4.0257	2670	3.7451	3
58	1938	5.1605	2119	4.7181	2303	4.3430	2487	4.0207	2673	3.7408	2
59	1941	5.1526	2123	4.7114	2306	4.3372	2490	4.0158	2676	3.7364	1
60	1944	5.1446	2126	4.7046	2309	4.3315	2493	4.0108	2679	3.7321	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
	70°		78°		77°		76°		75°		



°	15°		16°		17°		18°		19°		°
	an	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	2679	3.7321	2867	3.4874	3057	3.2709	3249	3.0777	3443	2.9042	60
1	2683	3.7277	2871	3.4836	3060	3.2675	3252	3.0746	3447	2.9015	59
2	2686	3.7234	2874	3.4793	3064	3.2641	3256	3.0716	3450	2.8987	58
3	2689	3.7191	2877	3.4760	3067	3.2607	3259	3.0686	3453	2.8960	57
4	2692	3.7148	2880	3.4722	3070	3.2573	3262	3.0655	3456	2.8933	56
5	2695	3.7105	2883	3.4684	3073	3.2539	3265	3.0625	3460	2.8905	55
6	2698	3.7062	2886	3.4646	3076	3.2506	3269	3.0595	3463	2.8878	54
7	2701	3.7019	2890	3.4608	3080	3.2472	3272	3.0565	3466	2.8851	53
8	2704	3.6976	2893	3.4570	3083	3.2438	3275	3.0535	3469	2.8824	52
9	2708	3.6933	2896	3.4533	3086	3.2405	3278	3.0505	3473	2.8797	51
10	2711	3.6891	2899	3.4495	3089	3.2371	3281	3.0475	3476	2.8770	50
11	2714	3.6848	2902	3.4458	3092	3.2338	3285	3.0445	3479	2.8743	49
12	2717	3.6806	2905	3.4420	3096	3.2305	3288	3.0415	3482	2.8716	48
13	2720	3.6764	2908	3.4383	3099	3.2272	3291	3.0385	3486	2.8689	47
14	2723	3.6722	2912	3.4346	3102	3.2238	3294	3.0356	3489	2.8662	46
15	2726	3.6680	2915	3.4308	3105	3.2205	3298	3.0326	3492	2.8636	45
16	2729	3.6638	2918	3.4271	3108	3.2172	3301	3.0296	3495	2.8609	44
17	2733	3.6596	2921	3.4234	3111	3.2139	3304	3.0267	3499	2.8582	43
18	2736	3.6554	2924	3.4197	3115	3.2106	3307	3.0237	3502	2.8556	42
19	2739	3.6512	2927	3.4160	3118	3.2073	3310	3.0208	3505	2.8529	41
20	2742	3.6470	2931	3.4124	3121	3.2041	3314	3.0178	3508	2.8502	40
21	2745	3.6429	2934	3.4087	3124	3.2008	3317	3.0149	3512	2.8476	39
22	2748	3.6387	2937	3.4050	3127	3.1975	3320	3.0120	3515	2.8449	38
23	2751	3.6346	2940	3.4014	3131	3.1943	3323	3.0090	3518	2.8423	37
24	2754	3.6305	2943	3.3977	3134	3.1910	3327	3.0061	3522	2.8397	36
25	2758	3.6264	2946	3.3941	3137	3.1878	3330	3.0032	3525	2.8370	35
26	2761	3.6222	2949	3.3904	3140	3.1845	3333	3.0003	3528	2.8344	34
27	2764	3.6181	2953	3.3868	3143	3.1813	3336	2.9974	3531	2.8318	33
28	2767	3.6140	2956	3.3832	3147	3.1780	3339	2.9945	3535	2.8291	32
29	2770	3.6100	2959	3.3796	3150	3.1748	3343	2.9916	3538	2.8265	31
30	2773	3.6059	2962	3.3759	3153	3.1716	3346	2.9887	3541	2.8239	30
31	2776	3.6018	2965	3.3723	3156	3.1684	3349	2.9858	3544	2.8213	29
32	2780	3.5978	2968	3.3687	3159	3.1652	3352	2.9829	3548	2.8187	28
33	2783	3.5937	2972	3.3652	3163	3.1620	3356	2.9800	3551	2.8161	27
34	2786	3.5897	2975	3.3616	3166	3.1588	3359	2.9772	3554	2.8135	26
35	2789	3.5856	2978	3.3580	3169	3.1556	3362	2.9743	3558	2.8109	25
36	2792	3.5816	2981	3.3544	3172	3.1524	3365	2.9714	3561	2.8083	24
37	2795	3.5776	2984	3.3509	3175	3.1492	3369	2.9686	3564	2.8057	23
38	2798	3.5736	2987	3.3473	3179	3.1460	3372	2.9657	3567	2.8032	22
39	2801	3.5696	2991	3.3438	3182	3.1429	3375	2.9629	3571	2.8006	21
40	2805	3.5656	2994	3.3402	3185	3.1397	3378	2.9600	3574	2.7980	20
41	2808	3.5616	2997	3.3367	3188	3.1366	3382	2.9572	3577	2.7955	19
42	2811	3.5576	3000	3.3332	3191	3.1334	3385	2.9544	3581	2.7929	18
43	2814	3.5536	3003	3.3297	3195	3.1303	3388	2.9515	3584	2.7903	17
44	2817	3.5497	3006	3.3261	3198	3.1271	3391	2.9487	3587	2.7878	16
45	2820	3.5457	3010	3.3226	3201	3.1240	3395	2.9459	3590	2.7852	15
46	2823	3.5418	3013	3.3191	3204	3.1209	3398	2.9431	3594	2.7827	14
47	2827	3.5379	3016	3.3156	3207	3.1178	3401	2.9403	3597	2.7801	13
48	2830	3.5339	3019	3.3122	3211	3.1146	3404	2.9375	3600	2.7776	12
49	2833	3.5300	3022	3.3087	3214	3.1115	3408	2.9347	3604	2.7751	11
50	2836	3.5261	3026	3.3052	3217	3.1084	3411	2.9319	3607	2.7725	10
51	2839	3.5222	3029	3.3017	3220	3.1053	3414	2.9291	3610	2.7700	9
52	2842	3.5183	3032	3.2983	3223	3.1022	3417	2.9263	3613	2.7675	8
53	2845	3.5144	3035	3.2948	3227	3.0991	3421	2.9235	3617	2.7650	7
54	2849	3.5105	3038	3.2914	3230	3.0961	3424	2.9208	3620	2.7625	6
55	2852	3.5067	3041	3.2880	3233	3.0930	3427	2.9180	3623	2.7600	5
56	2855	3.5028	3045	3.2845	3236	3.0899	3430	2.9152	3627	2.7575	4
57	2858	3.4989	3048	3.2811	3240	3.0868	3434	2.9125	3630	2.7550	3
58	2861	3.4951	3051	3.2777	3243	3.0838	3437	2.9097	3633	2.7525	2
59	2864	3.4912	3054	3.2743	3246	3.0807	3440	2.9070	3636	2.7500	1
60	2867	3.4874	3057	3.2709	3249	3.0777	3443	2.9042	3640	2.7475	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
	74°		78°		72°		71°		70°		

1	20°		21°		22°		23°		24°		1
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	3640	2.7475	3839	2.6051	4040	2.4751	4245	2.3559	4452	2.2460	60
1	3643	2.7450	3842	2.6028	4044	2.4730	4248	2.3539	4456	2.2443	59
2	3646	2.7425	3845	2.6006	4047	2.4709	4252	2.3520	4459	2.2425	58
3	3650	2.7400	3849	2.5983	4050	2.4689	4255	2.3501	4463	2.2408	57
4	3653	2.7375	3852	2.5961	4054	2.4668	4258	2.3483	4466	2.2390	56
5	3656	2.7351	3855	2.5938	4057	2.4648	4262	2.3464	4470	2.2373	55
6	3659	2.7326	3859	2.5916	4061	2.4627	4265	2.3445	4473	2.2355	54
7	3663	2.7302	3862	2.5893	4064	2.4606	4269	2.3426	4477	2.2338	53
8	3666	2.7277	3865	2.5871	4067	2.4586	4272	2.3407	4480	2.2320	52
9	3669	2.7253	3869	2.5848	4071	2.4566	4276	2.3388	4484	2.2303	51
10	3673	2.7228	3872	2.5826	4074	2.4545	4279	2.3369	4487	2.2286	50
11	3676	2.7204	3875	2.5804	4078	2.4525	4283	2.3351	4491	2.2268	49
12	3679	2.7179	3879	2.5782	4081	2.4504	4286	2.3332	4494	2.2251	48
13	3683	2.7155	3882	2.5759	4084	2.4484	4289	2.3313	4498	2.2234	47
14	3686	2.7130	3885	2.5737	4088	2.4464	4293	2.3294	4501	2.2216	46
15	3689	2.7106	3889	2.5715	4091	2.4443	4296	2.3276	4505	2.2199	45
16	3693	2.7082	3892	2.5693	4095	2.4423	4300	2.3257	4508	2.2182	44
17	3696	2.7058	3895	2.5671	4098	2.4403	4303	2.3238	4512	2.2165	43
18	3699	2.7034	3899	2.5649	4101	2.4383	4307	2.3220	4515	2.2148	42
19	3702	2.7009	3902	2.5627	4105	2.4362	4310	2.3201	4519	2.2130	41
20	3706	2.6985	3906	2.5605	4108	2.4342	4314	2.3183	4522	2.2113	40
21	3709	2.6961	3909	2.5583	4111	2.4322	4317	2.3164	4526	2.2096	39
22	3712	2.6937	3912	2.5561	4115	2.4302	4320	2.3146	4529	2.2079	38
23	3716	2.6913	3916	2.5539	4118	2.4282	4324	2.3127	4533	2.2062	37
24	3719	2.6889	3919	2.5517	4122	2.4262	4327	2.3109	4536	2.2045	36
25	3722	2.6865	3922	2.5495	4125	2.4242	4331	2.3090	4540	2.2028	35
26	3726	2.6841	3926	2.5473	4129	2.4222	4334	2.3072	4543	2.2011	34
27	3729	2.6818	3929	2.5452	4132	2.4202	4338	2.3053	4547	2.1994	33
28	3732	2.6794	3932	2.5430	4135	2.4182	4341	2.3035	4550	2.1977	32
29	3736	2.6770	3936	2.5408	4139	2.4162	4345	2.3017	4554	2.1960	31
30	3739	2.6746	3939	2.5386	4142	2.4142	4348	2.2998	4557	2.1943	30
31	3742	2.6723	3942	2.5365	4146	2.4122	4352	2.2980	4561	2.1926	29
32	3745	2.6699	3946	2.5343	4149	2.4102	4355	2.2962	4564	2.1909	28
33	3749	2.6675	3949	2.5322	4152	2.4083	4359	2.2944	4568	2.1892	27
34	3752	2.6652	3953	2.5300	4156	2.4063	4362	2.2925	4571	2.1876	26
35	3755	2.6628	3956	2.5279	4159	2.4043	4365	2.2907	4575	2.1859	25
36	3759	2.6605	3959	2.5257	4163	2.4023	4369	2.2889	4578	2.1842	24
37	3762	2.6581	3963	2.5236	4166	2.4004	4372	2.2871	4582	2.1825	23
38	3765	2.6558	3966	2.5214	4169	2.3984	4376	2.2853	4585	2.1808	22
39	3769	2.6534	3969	2.5193	4173	2.3964	4379	2.2835	4589	2.1792	21
40	3772	2.6511	3973	2.5172	4176	2.3945	4383	2.2817	4592	2.1775	20
41	3775	2.6488	3976	2.5150	4180	2.3925	4386	2.2799	4596	2.1758	19
42	3779	2.6464	3979	2.5129	4183	2.3906	4390	2.2781	4599	2.1742	18
43	3782	2.6441	3983	2.5108	4187	2.3886	4393	2.2763	4603	2.1725	17
44	3785	2.6418	3986	2.5086	4190	2.3867	4397	2.2745	4607	2.1708	16
45	3789	2.6395	3990	2.5065	4193	2.3847	4400	2.2727	4610	2.1692	15
46	3792	2.6371	3993	2.5044	4197	2.3828	4404	2.2709	4614	2.1675	14
47	3795	2.6348	3996	2.5023	4200	2.3808	4407	2.2691	4617	2.1659	13
48	3799	2.6325	4000	2.5002	4204	2.3789	4411	2.2673	4621	2.1642	12
49	3802	2.6302	4003	2.4981	4207	2.3770	4414	2.2655	4624	2.1625	11
50	3805	2.6279	4006	2.4960	4210	2.3750	4417	2.2637	4628	2.1609	10
51	3809	2.6256	4010	2.4939	4214	2.3731	4421	2.2620	4631	2.1592	9
52	3812	2.6233	4013	2.4918	4217	2.3712	4424	2.2602	4635	2.1576	8
53	3815	2.6210	4017	2.4897	4221	2.3693	4428	2.2584	4638	2.1560	7
54	3819	2.6187	4020	2.4876	4224	2.3673	4431	2.2566	4642	2.1543	6
55	3822	2.6165	4023	2.4855	4228	2.3654	4435	2.2549	4645	2.1527	5
56	3825	2.6142	4027	2.4834	4231	2.3635	4438	2.2531	4649	2.1510	4
57	3829	2.6119	4030	2.4813	4234	2.3616	4442	2.2513	4652	2.1494	3
58	3832	2.6096	4033	2.4792	4238	2.3597	4445	2.2496	4656	2.1478	2
59	3835	2.6074	4037	2.4772	4241	2.3578	4449	2.2478	4660	2.1461	1
60	3839	2.6051	4040	2.4751	4245	2.3559	4452	2.2460	4663	2.1445	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
1	60°		68°		67°		66°		65°		1

1	25°		26°		27°		28°		29°		1
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	4663	2.1445	4877	2.0503	5095	1.9626	5317	1.8907	5543	1.8040	60
1	4667	2.1429	4881	2.0488	5099	1.9612	5321	1.8794	5547	1.8028	59
2	4670	2.1413	4885	2.0473	5103	1.9598	5325	1.8781	5551	1.8016	58
3	4674	2.1396	4888	2.0458	5106	1.9584	5328	1.8768	5555	1.8003	57
4	4677	2.1380	4892	2.0443	5110	1.9570	5332	1.8755	5558	1.7991	56
5	4681	2.1364	4895	2.0428	5114	1.9556	5336	1.8741	5562	1.7979	55
6	4684	2.1348	4899	2.0413	5117	1.9542	5340	1.8728	5566	1.7966	54
7	4688	2.1332	4903	2.0398	5121	1.9528	5343	1.8715	5570	1.7954	53
8	4691	2.1315	4906	2.0383	5125	1.9514	5347	1.8702	5574	1.7942	52
9	4695	2.1299	4910	2.0368	5128	1.9500	5351	1.8689	5577	1.7930	51
10	4699	2.1283	4913	2.0353	5132	1.9486	5354	1.8676	5581	1.7917	50
11	4702	2.1267	4917	2.0338	5136	1.9472	5358	1.8663	5585	1.7905	49
12	4706	2.1251	4921	2.0323	5139	1.9458	5362	1.8650	5589	1.7893	48
13	4709	2.1235	4924	2.0308	5143	1.9444	5366	1.8637	5593	1.7881	47
14	4713	2.1219	4928	2.0293	5147	1.9430	5369	1.8624	5596	1.7868	46
15	4716	2.1203	4931	2.0278	5150	1.9416	5373	1.8611	5600	1.7856	45
16	4720	2.1187	4935	2.0263	5154	1.9402	5377	1.8598	5604	1.7844	44
17	4723	2.1171	4939	2.0248	5158	1.9388	5381	1.8585	5608	1.7832	43
18	4727	2.1155	4942	2.0233	5161	1.9375	5384	1.8572	5612	1.7820	42
19	4731	2.1139	4946	2.0219	5165	1.9361	5388	1.8559	5616	1.7808	41
20	4734	2.1123	4950	2.0204	5169	1.9347	5392	1.8546	5619	1.7796	40
21	4738	2.1107	4953	2.0189	5172	1.9333	5396	1.8533	5623	1.7783	39
22	4741	2.1092	4957	2.0174	5176	1.9319	5399	1.8520	5627	1.7771	38
23	4745	2.1076	4960	2.0160	5180	1.9306	5403	1.8507	5631	1.7759	37
24	4748	2.1060	4964	2.0145	5184	1.9292	5407	1.8495	5635	1.7747	36
25	4752	2.1044	4968	2.0130	5187	1.9278	5411	1.8482	5639	1.7735	35
26	4755	2.1028	4971	2.0115	5191	1.9265	5415	1.8469	5642	1.7723	34
27	4759	2.1013	4975	2.0101	5195	1.9251	5418	1.8456	5646	1.7711	33
28	4763	2.0997	4979	2.0086	5198	1.9237	5422	1.8443	5650	1.7699	32
29	4766	2.0981	4982	2.0072	5202	1.9223	5426	1.8430	5654	1.7687	31
30	4770	2.0965	4986	2.0057	5206	1.9210	5430	1.8418	5658	1.7675	30
31	4773	2.0950	4989	2.0042	5209	1.9196	5433	1.8405	5662	1.7663	29
32	4777	2.0934	4993	2.0028	5213	1.9183	5437	1.8392	5665	1.7651	28
33	4780	2.0918	4997	2.0013	5217	1.9169	5441	1.8379	5669	1.7639	27
34	4784	2.0903	5000	1.9999	5220	1.9155	5445	1.8367	5673	1.7627	26
35	4788	2.0887	5004	1.9984	5224	1.9142	5448	1.8354	5677	1.7615	25
36	4791	2.0872	5008	1.9970	5228	1.9128	5452	1.8341	5681	1.7603	24
37	4795	2.0856	5011	1.9955	5232	1.9115	5456	1.8329	5685	1.7591	23
38	4798	2.0840	5015	1.9941	5235	1.9101	5460	1.8316	5688	1.7579	22
39	4802	2.0825	5019	1.9926	5239	1.9088	5464	1.8303	5692	1.7567	21
40	4806	2.0809	5022	1.9912	5243	1.9074	5467	1.8291	5696	1.7556	20
41	4809	2.0794	5026	1.9897	5246	1.9061	5471	1.8278	5700	1.7544	19
42	4813	2.0778	5029	1.9883	5250	1.9047	5475	1.8265	5704	1.7532	18
43	4816	2.0763	5033	1.9868	5254	1.9034	5479	1.8253	5708	1.7520	17
44	4820	2.0748	5037	1.9854	5258	1.9020	5482	1.8240	5712	1.7508	16
45	4823	2.0732	5040	1.9840	5261	1.9007	5486	1.8228	5715	1.7496	15
46	4827	2.0717	5044	1.9825	5265	1.8993	5490	1.8215	5719	1.7485	14
47	4831	2.0701	5048	1.9811	5269	1.8980	5494	1.8202	5723	1.7473	13
48	4834	2.0686	5051	1.9797	5272	1.8967	5498	1.8190	5727	1.7461	12
49	4838	2.0671	5055	1.9782	5276	1.8953	5501	1.8177	5731	1.7449	11
50	4841	2.0655	5059	1.9768	5280	1.8940	5505	1.8165	5735	1.7437	10
51	4845	2.0640	5062	1.9754	5284	1.8927	5509	1.8152	5739	1.7426	9
52	4849	2.0625	5066	1.9740	5287	1.8913	5513	1.8140	5743	1.7414	8
53	4852	2.0609	5070	1.9725	5291	1.8900	5517	1.8127	5746	1.7402	7
54	4856	2.0594	5073	1.9711	5295	1.8887	5520	1.8115	5750	1.7391	6
55	4859	2.0579	5077	1.9697	5298	1.8873	5524	1.8103	5754	1.7379	5
56	4863	2.0564	5081	1.9683	5302	1.8860	5528	1.8090	5758	1.7367	4
57	4867	2.0549	5084	1.9669	5306	1.8847	5532	1.8078	5762	1.7355	3
58	4870	2.0533	5088	1.9654	5310	1.8834	5535	1.8065	5766	1.7344	2
59	4874	2.0518	5092	1.9640	5313	1.8820	5539	1.8053	5770	1.7332	1
60	4877	2.0503	5095	1.9626	5317	1.8807	5543	1.8040	5774	1.7321	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
	64°		63°		62°		61°		60°		

	30°		31°		32°		33°		34°		
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	5774	1.7321	6009	1.6643	6249	1.6003	6494	1.5399	6745	1.4826	60
1	5777	1.7309	6013	1.6632	6253	1.5993	6498	1.5389	6749	1.4816	59
2	5781	1.7297	6017	1.6621	6257	1.5983	6502	1.5379	6754	1.4807	58
3	5785	1.7286	6020	1.6610	6261	1.5972	6506	1.5369	6758	1.4798	57
4	5789	1.7274	6024	1.6599	6265	1.5962	6511	1.5359	6762	1.4788	56
5	5793	1.7262	6028	1.6588	6269	1.5952	6515	1.5350	6766	1.4779	55
6	5797	1.7251	6032	1.6577	6273	1.5941	6519	1.5340	6771	1.4770	54
7	5801	1.7239	6036	1.6566	6277	1.5931	6523	1.5330	6775	1.4761	53
8	5805	1.7228	6040	1.6555	6281	1.5921	6527	1.5320	6779	1.4751	52
9	5808	1.7216	6044	1.6545	6285	1.5911	6531	1.5311	6783	1.4742	51
10	5812	1.7205	6048	1.6534	6289	1.5900	6536	1.5301	6787	1.4733	50
11	5816	1.7193	6052	1.6523	6293	1.5890	6540	1.5291	6792	1.4724	49
12	5820	1.7182	6056	1.6512	6297	1.5880	6544	1.5282	6796	1.4715	48
13	5824	1.7170	6060	1.6501	6301	1.5869	6548	1.5272	6800	1.4705	47
14	5828	1.7159	6064	1.6490	6305	1.5859	6552	1.5262	6805	1.4696	46
15	5832	1.7147	6068	1.6479	6310	1.5849	6556	1.5253	6809	1.4687	45
16	5836	1.7136	6072	1.6469	6314	1.5839	6560	1.5243	6813	1.4678	44
17	5840	1.7124	6076	1.6458	6318	1.5829	6565	1.5233	6817	1.4669	43
18	5844	1.7113	6080	1.6447	6322	1.5818	6569	1.5224	6822	1.4659	42
19	5847	1.7102	6084	1.6436	6326	1.5808	6573	1.5214	6826	1.4650	41
20	5851	1.7090	6088	1.6426	6330	1.5798	6577	1.5204	6830	1.4641	40
21	5855	1.7079	6092	1.6415	6334	1.5788	6581	1.5195	6834	1.4632	39
22	5859	1.7067	6096	1.6404	6338	1.5778	6585	1.5185	6839	1.4623	38
23	5863	1.7056	6100	1.6393	6342	1.5768	6590	1.5175	6843	1.4614	37
24	5867	1.7045	6104	1.6383	6346	1.5757	6594	1.5166	6847	1.4605	36
25	5871	1.7033	6108	1.6372	6350	1.5747	6598	1.5156	6851	1.4596	35
26	5875	1.7022	6112	1.6361	6354	1.5737	6602	1.5147	6856	1.4586	34
27	5879	1.7011	6116	1.6351	6358	1.5727	6606	1.5137	6860	1.4577	33
28	5883	1.6999	6120	1.6340	6363	1.5717	6610	1.5127	6864	1.4568	32
29	5887	1.6988	6124	1.6329	6367	1.5707	6615	1.5118	6869	1.4559	31
30	5890	1.6977	6128	1.6319	6371	1.5697	6619	1.5108	6873	1.4550	30
31	5894	1.6965	6132	1.6308	6375	1.5687	6623	1.5099	6877	1.4541	29
32	5898	1.6954	6136	1.6297	6379	1.5677	6627	1.5089	6881	1.4532	28
33	5902	1.6943	6140	1.6287	6383	1.5667	6631	1.5080	6886	1.4523	27
34	5906	1.6932	6144	1.6276	6387	1.5657	6636	1.5070	6890	1.4514	26
35	5910	1.6920	6148	1.6265	6391	1.5647	6640	1.5061	6894	1.4505	25
36	5914	1.6909	6152	1.6255	6395	1.5637	6644	1.5051	6899	1.4496	24
37	5918	1.6898	6156	1.6244	6399	1.5627	6648	1.5042	6903	1.4487	23
38	5922	1.6887	6160	1.6234	6403	1.5617	6652	1.5032	6907	1.4478	22
39	5926	1.6875	6164	1.6223	6408	1.5607	6657	1.5023	6911	1.4469	21
40	5930	1.6864	6168	1.6212	6412	1.5597	6661	1.5013	6916	1.4460	20
41	5934	1.6853	6172	1.6202	6416	1.5587	6665	1.5004	6920	1.4451	19
42	5938	1.6842	6176	1.6191	6420	1.5577	6669	1.4994	6924	1.4442	18
43	5942	1.6831	6180	1.6181	6424	1.5567	6673	1.4985	6929	1.4433	17
44	5945	1.6820	6184	1.6170	6428	1.5557	6678	1.4975	6933	1.4424	16
45	5949	1.6808	6188	1.6160	6432	1.5547	6682	1.4966	6937	1.4415	15
46	5953	1.6797	6192	1.6149	6436	1.5537	6686	1.4957	6942	1.4406	14
47	5957	1.6786	6196	1.6139	6440	1.5527	6690	1.4947	6946	1.4397	13
48	5961	1.6775	6200	1.6128	6445	1.5517	6694	1.4938	6950	1.4388	12
49	5965	1.6764	6204	1.6118	6449	1.5507	6699	1.4928	6954	1.4379	11
50	5969	1.6753	6208	1.6107	6453	1.5497	6703	1.4919	6959	1.4370	10
51	5973	1.6742	6212	1.6097	6457	1.5487	6707	1.4910	6963	1.4361	9
52	5977	1.6731	6216	1.6087	6461	1.5477	6711	1.4900	6967	1.4352	8
53	5981	1.6720	6220	1.6076	6465	1.5468	6716	1.4891	6972	1.4344	7
54	5985	1.6709	6224	1.6066	6469	1.5458	6720	1.4882	6976	1.4335	6
55	5989	1.6698	6228	1.6055	6473	1.5448	6724	1.4872	6980	1.4326	5
56	5993	1.6687	6233	1.6045	6478	1.5438	6728	1.4863	6985	1.4317	4
57	5997	1.6676	6237	1.6034	6482	1.5428	6732	1.4854	6989	1.4308	3
58	6001	1.6665	6241	1.6024	6486	1.5418	6737	1.4844	6993	1.4299	2
59	6005	1.6654	6245	1.6014	6490	1.5408	6741	1.4835	6998	1.4290	1
60	6009	1.6643	6249	1.6003	6494	1.5399	6745	1.4826	7002	1.4281	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
	59°		58°		57°		56°		55°		

°	35°		36°		37°		38°		39°		°
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	7002	1.4281	7265	1.3764	7536	1.3270	7813	1.2799	8098	1.2349	60
1	7006	1.4273	7270	1.3755	7540	1.3262	7818	1.2792	8103	1.2342	59
2	7011	1.4264	7274	1.3747	7545	1.3254	7822	1.2784	8107	1.2334	58
3	7015	1.4255	7279	1.3739	7549	1.3246	7827	1.2776	8112	1.2327	57
4	7019	1.4246	7283	1.3730	7554	1.3238	7832	1.2769	8117	1.2320	56
5	7024	1.4237	7288	1.3722	7558	1.3230	7836	1.2761	8122	1.2312	55
6	7028	1.4229	7292	1.3713	7563	1.3222	7841	1.2753	8127	1.2305	54
7	7032	1.4220	7297	1.3705	7568	1.3214	7846	1.2746	8132	1.2298	53
8	7037	1.4211	7301	1.3697	7572	1.3206	7850	1.2738	8136	1.2290	52
9	7041	1.4202	7306	1.3688	7577	1.3198	7855	1.2731	8141	1.2283	51
10	7046	1.4193	7310	1.3680	7581	1.3190	7860	1.2723	8146	1.2276	50
11	7050	1.4185	7314	1.3672	7586	1.3182	7865	1.2715	8151	1.2268	49
12	7054	1.4176	7319	1.3663	7590	1.3175	7869	1.2708	8156	1.2261	48
13	7059	1.4167	7323	1.3655	7595	1.3167	7874	1.2700	8161	1.2254	47
14	7063	1.4158	7328	1.3647	7600	1.3159	7879	1.2693	8165	1.2247	46
15	7067	1.4150	7332	1.3638	7604	1.3151	7883	1.2685	8170	1.2239	45
16	7072	1.4141	7337	1.3630	7609	1.3143	7888	1.2677	8175	1.2232	44
17	7076	1.4132	7341	1.3622	7613	1.3135	7893	1.2670	8180	1.2225	43
18	7080	1.4124	7346	1.3613	7618	1.3127	7898	1.2662	8185	1.2218	42
19	7085	1.4115	7350	1.3605	7623	1.3119	7902	1.2655	8190	1.2210	41
20	7089	1.4106	7355	1.3597	7627	1.3111	7907	1.2647	8195	1.2203	40
21	7094	1.4097	7359	1.3588	7632	1.3103	7912	1.2640	8199	1.2196	39
22	7098	1.4089	7364	1.3580	7636	1.3095	7916	1.2632	8204	1.2189	38
23	7102	1.4080	7368	1.3572	7641	1.3087	7921	1.2624	8209	1.2181	37
24	7107	1.4071	7373	1.3564	7646	1.3079	7926	1.2617	8214	1.2174	36
25	7111	1.4063	7377	1.3555	7650	1.3072	7931	1.2609	8219	1.2167	35
26	7115	1.4054	7382	1.3547	7655	1.3064	7935	1.2602	8224	1.2160	34
27	7120	1.4045	7386	1.3539	7659	1.3056	7940	1.2594	8229	1.2153	33
28	7124	1.4037	7391	1.3531	7664	1.3048	7945	1.2587	8234	1.2145	32
29	7129	1.4028	7395	1.3522	7669	1.3040	7950	1.2579	8238	1.2138	31
30	7133	1.4019	7400	1.3514	7673	1.3032	7954	1.2572	8243	1.2131	30
31	7137	1.4011	7404	1.3506	7678	1.3024	7959	1.2564	8248	1.2124	29
32	7142	1.4002	7409	1.3498	7683	1.3017	7964	1.2557	8253	1.2117	28
33	7146	1.3994	7413	1.3490	7687	1.3009	7969	1.2549	8258	1.2109	27
34	7151	1.3985	7418	1.3481	7692	1.3001	7973	1.2542	8263	1.2102	26
35	7155	1.3976	7422	1.3473	7696	1.2993	7978	1.2534	8268	1.2095	25
36	7159	1.3968	7427	1.3465	7701	1.2985	7983	1.2527	8273	1.2088	24
37	7164	1.3959	7431	1.3457	7706	1.2977	7988	1.2519	8278	1.2081	23
38	7168	1.3951	7436	1.3449	7710	1.2970	7992	1.2512	8283	1.2074	22
39	7173	1.3942	7440	1.3440	7715	1.2962	7997	1.2504	8287	1.2066	21
40	7177	1.3934	7445	1.3432	7720	1.2954	8002	1.2497	8292	1.2059	20
41	7181	1.3925	7449	1.3424	7724	1.2946	8007	1.2489	8297	1.2052	19
42	7186	1.3916	7454	1.3416	7729	1.2938	8012	1.2482	8302	1.2045	18
43	7190	1.3908	7458	1.3408	7734	1.2931	8016	1.2475	8307	1.2038	17
44	7195	1.3899	7463	1.3400	7738	1.2923	8021	1.2467	8312	1.2031	16
45	7199	1.3891	7467	1.3392	7743	1.2915	8026	1.2460	8317	1.2024	15
46	7203	1.3882	7472	1.3384	7747	1.2907	8031	1.2452	8322	1.2017	14
47	7208	1.3874	7476	1.3375	7752	1.2900	8035	1.2445	8327	1.2009	13
48	7212	1.3865	7481	1.3367	7757	1.2892	8040	1.2437	8332	1.2002	12
49	7217	1.3857	7485	1.3359	7761	1.2884	8045	1.2430	8337	1.1995	11
50	7221	1.3848	7490	1.3351	7766	1.2876	8050	1.2423	8342	1.1988	10
51	7226	1.3840	7495	1.3343	7771	1.2869	8055	1.2415	8346	1.1981	9
52	7230	1.3831	7499	1.3335	7775	1.2861	8059	1.2408	8351	1.1974	8
53	7234	1.3823	7504	1.3327	7780	1.2853	8064	1.2401	8356	1.1967	7
54	7239	1.3814	7508	1.3319	7785	1.2846	8069	1.2393	8361	1.1960	6
55	7243	1.3806	7513	1.3311	7789	1.2838	8074	1.2386	8366	1.1953	5
56	7248	1.3798	7517	1.3303	7794	1.2830	8079	1.2378	8371	1.1946	4
57	7252	1.3789	7522	1.3295	7799	1.2822	8083	1.2371	8376	1.1939	3
58	7257	1.3781	7526	1.3287	7803	1.2815	8088	1.2364	8381	1.1932	2
59	7261	1.3772	7531	1.3278	7808	1.2807	8093	1.2356	8386	1.1925	1
60	7265	1.3764	7536	1.3270	7813	1.2799	8098	1.2349	8391	1.1918	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
	54°		53°		52°		51°		50°		

°	40°		41°		42°		43°		44°		°
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	8391	1.1918	8693	1.1504	9004	1.1106	9323	1.0724	9657	1.0355	50
1	8376	1.1910	8698	1.1497	9009	1.1100	9331	1.0717	9663	1.0349	59
2	8401	1.1903	8703	1.1490	9015	1.1093	9336	1.0711	9668	1.0343	58
3	8406	1.1896	8708	1.1483	9020	1.1087	9341	1.0705	9674	1.0337	57
4	8411	1.1889	8713	1.1477	9025	1.1080	9347	1.0699	9679	1.0331	56
5	8416	1.1882	8718	1.1470	9030	1.1074	9352	1.0692	9685	1.0325	55
6	8421	1.1875	8724	1.1463	9036	1.1067	9358	1.0686	9691	1.0319	54
7	8426	1.1868	8729	1.1456	9041	1.1061	9363	1.0680	9696	1.0313	53
8	8431	1.1861	8734	1.1450	9046	1.1054	9369	1.0674	9702	1.0307	52
9	8436	1.1854	8739	1.1443	9052	1.1048	9374	1.0668	9708	1.0301	51
10	8441	1.1847	8744	1.1436	9057	1.1041	9380	1.0661	9713	1.0295	50
11	8446	1.1840	8749	1.1430	9062	1.1035	9385	1.0655	9719	1.0289	49
12	8451	1.1833	8754	1.1423	9067	1.1028	9391	1.0649	9725	1.0283	48
13	8456	1.1826	8759	1.1416	9073	1.1022	9396	1.0643	9730	1.0277	47
14	8461	1.1819	8765	1.1410	9078	1.1016	9402	1.0637	9736	1.0271	46
15	8466	1.1812	8770	1.1403	9083	1.1009	9407	1.0630	9742	1.0265	45
16	8471	1.1806	8775	1.1396	9089	1.1003	9413	1.0624	9747	1.0259	44
17	8476	1.1799	8780	1.1389	9094	1.0996	9418	1.0618	9753	1.0253	43
18	8481	1.1792	8785	1.1383	9099	1.0990	9424	1.0612	9759	1.0247	42
19	8486	1.1785	8790	1.1376	9105	1.0983	9429	1.0606	9764	1.0241	41
20	8491	1.1778	8796	1.1369	9110	1.0977	9435	1.0599	9770	1.0235	40
21	8496	1.1771	8801	1.1363	9115	1.0971	9440	1.0593	9776	1.0229	39
22	8501	1.1764	8806	1.1356	9121	1.0964	9446	1.0587	9781	1.0224	38
23	8506	1.1757	8811	1.1349	9126	1.0958	9451	1.0581	9787	1.0218	37
24	8511	1.1750	8816	1.1343	9131	1.0951	9457	1.0575	9793	1.0212	36
25	8516	1.1743	8821	1.1336	9137	1.0945	9462	1.0569	9798	1.0206	35
26	8521	1.1736	8827	1.1329	9142	1.0939	9468	1.0562	9804	1.0200	34
27	8526	1.1729	8832	1.1323	9147	1.0932	9473	1.0556	9810	1.0194	33
28	8531	1.1722	8837	1.1316	9153	1.0926	9479	1.0550	9816	1.0188	32
29	8536	1.1715	8842	1.1310	9158	1.0919	9484	1.0544	9821	1.0182	31
30	8541	1.1708	8847	1.1303	9163	1.0913	9490	1.0538	9827	1.0176	30
31	8546	1.1702	8852	1.1296	9169	1.0907	9495	1.0532	9833	1.0170	29
32	8551	1.1695	8858	1.1290	9174	1.0900	9501	1.0526	9838	1.0164	28
33	8556	1.1688	8863	1.1283	9179	1.0894	9506	1.0519	9844	1.0158	27
34	8561	1.1681	8868	1.1276	9185	1.0888	9512	1.0513	9850	1.0152	26
35	8566	1.1674	8873	1.1270	9190	1.0881	9517	1.0507	9856	1.0147	25
36	8571	1.1667	8878	1.1263	9195	1.0875	9523	1.0501	9861	1.0141	24
37	8576	1.1660	8884	1.1257	9201	1.0869	9528	1.0495	9867	1.0135	23
38	8581	1.1653	8889	1.1250	9206	1.0862	9534	1.0489	9873	1.0129	22
39	8586	1.1647	8894	1.1243	9212	1.0856	9540	1.0483	9879	1.0123	21
40	8591	1.1640	8899	1.1237	9217	1.0850	9545	1.0477	9884	1.0117	20
41	8596	1.1633	8904	1.1230	9222	1.0843	9551	1.0470	9890	1.0111	19
42	8601	1.1626	8910	1.1224	9228	1.0837	9556	1.0464	9896	1.0105	18
43	8606	1.1619	8915	1.1217	9233	1.0831	9562	1.0458	9902	1.0099	17
44	8611	1.1612	8920	1.1211	9239	1.0824	9567	1.0452	9907	1.0094	16
45	8617	1.1606	8925	1.1204	9244	1.0818	9573	1.0446	9913	1.0088	15
46	8622	1.1599	8931	1.1197	9249	1.0812	9578	1.0440	9919	1.0082	14
47	8627	1.1592	8936	1.1191	9255	1.0805	9584	1.0434	9925	1.0076	13
48	8632	1.1585	8941	1.1184	9260	1.0799	9590	1.0428	9930	1.0070	12
49	8637	1.1578	8946	1.1178	9266	1.0793	9595	1.0422	9936	1.0064	11
50	8643	1.1571	8952	1.1171	9271	1.0786	9601	1.0416	9942	1.0058	10
51	8647	1.1565	8957	1.1165	9276	1.0780	9606	1.0410	9948	1.0052	9
52	8652	1.1558	8962	1.1158	9282	1.0774	9612	1.0404	9954	1.0047	8
53	8657	1.1551	8967	1.1152	9287	1.0768	9618	1.0398	9959	1.0041	7
54	8662	1.1544	8972	1.1145	9293	1.0761	9623	1.0392	9965	1.0035	6
55	8667	1.1538	8978	1.1139	9298	1.0755	9629	1.0385	9971	1.0029	5
56	8672	1.1531	8983	1.1132	9303	1.0749	9634	1.0379	9977	1.0023	4
57	8678	1.1524	8988	1.1126	9309	1.0742	9640	1.0373	9983	1.0017	3
58	8683	1.1517	8994	1.1119	9314	1.0736	9646	1.0367	9988	1.0012	2
59	8688	1.1510	8999	1.1113	9320	1.0730	9651	1.0361	9994	1.0006	1
60	8693	1.1504	9004	1.1106	9325	1.0724	9657	1.0355	1.000	1.0000	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
	40°		45°		47°		48°		45°		

TABLE VII.—TRAVERSE TABLE.

Bearing.	Distance 1.		Distance 2.		Distance 3.		Distance 4.		Distance 5.		Bearing.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
0 15	1.000	0.004	2.000	0.009	3.000	0.013	4.000	0.017	5.000	0.022	89 45
30	1.000	0.009	2.000	0.017	3.000	0.026	4.000	0.035	5.000	0.044	30
45	1.000	0.013	2.000	0.026	3.000	0.039	4.000	0.052	5.000	0.065	15
1 0	1.000	0.017	2.000	0.035	3.000	0.052	3.999	0.070	4.999	0.087	89 0
15	1.000	0.022	2.000	0.044	2.999	0.065	3.999	0.087	4.999	0.109	45
30	1.000	0.026	1.999	0.052	2.999	0.079	3.999	0.105	4.998	0.131	30
45	1.000	0.031	1.999	0.061	2.999	0.092	3.998	0.122	4.998	0.153	15
2 0	0.999	0.035	1.999	0.070	2.998	0.105	3.998	0.140	4.997	0.174	88 0
15	0.999	0.039	1.998	0.079	2.998	0.118	3.997	0.157	4.996	0.196	45
30	0.999	0.044	1.998	0.087	2.997	0.131	3.996	0.174	4.995	0.218	30
45	0.999	0.048	1.998	0.096	2.997	0.144	3.995	0.192	4.994	0.240	15
3 0	0.999	0.052	1.997	0.103	2.996	0.157	3.995	0.209	4.993	0.262	87 0
15	0.998	0.057	1.997	0.113	2.995	0.170	3.994	0.227	4.992	0.283	45
30	0.998	0.061	1.996	0.122	2.994	0.183	3.993	0.244	4.991	0.305	30
45	0.998	0.065	1.996	0.131	2.994	0.196	3.991	0.262	4.989	0.327	15
4 0	0.998	0.070	1.995	0.140	2.993	0.209	3.990	0.279	4.988	0.349	86 0
15	0.997	0.074	1.995	0.148	2.992	0.222	3.989	0.296	4.986	0.371	45
30	0.997	0.078	1.994	0.157	2.991	0.235	3.988	0.314	4.985	0.392	30
45	0.997	0.083	1.993	0.166	2.990	0.248	3.986	0.331	4.983	0.414	15
5 0	0.996	0.087	1.992	0.174	2.989	0.261	3.985	0.349	4.981	0.436	85 0
15	0.996	0.092	1.992	0.183	2.987	0.275	3.983	0.366	4.979	0.458	45
30	0.995	0.096	1.991	0.192	2.986	0.288	3.982	0.383	4.977	0.479	30
45	0.995	0.100	1.990	0.200	2.985	0.301	3.980	0.401	4.975	0.501	15
6 0	0.995	0.105	1.989	0.209	2.984	0.314	3.978	0.418	4.973	0.523	84 0
15	0.994	0.109	1.988	0.218	2.982	0.327	3.976	0.435	4.970	0.544	45
30	0.994	0.113	1.987	0.226	2.981	0.340	3.974	0.453	4.968	0.566	30
45	0.993	0.118	1.986	0.235	2.979	0.353	3.972	0.470	4.965	0.588	15
7 0	0.993	0.122	1.985	0.244	2.978	0.366	3.970	0.487	4.963	0.609	83 0
15	0.992	0.126	1.984	0.252	2.976	0.379	3.968	0.505	4.960	0.631	45
30	0.991	0.131	1.983	0.261	2.974	0.392	3.966	0.522	4.957	0.653	30
45	0.991	0.135	1.982	0.270	2.973	0.405	3.963	0.539	4.954	0.674	15
8 0	0.990	0.139	1.981	0.278	2.971	0.418	3.961	0.557	4.951	0.696	82 0
15	0.990	0.143	1.979	0.287	2.969	0.430	3.959	0.574	4.948	0.717	45
30	0.989	0.148	1.978	0.296	2.967	0.443	3.956	0.591	4.945	0.739	30
45	0.988	0.152	1.977	0.304	2.965	0.456	3.953	0.608	4.942	0.761	15
9 0	0.988	0.156	1.975	0.313	2.963	0.469	3.951	0.626	4.938	0.782	81 0
15	0.987	0.161	1.974	0.321	2.961	0.482	3.948	0.643	4.935	0.804	45
30	0.986	0.165	1.973	0.330	2.959	0.495	3.945	0.660	4.931	0.825	30
45	0.986	0.169	1.971	0.339	2.957	0.508	3.942	0.677	4.928	0.847	15
10 0	0.985	0.174	1.970	0.347	2.954	0.521	3.939	0.695	4.924	0.868	80 0
15	0.984	0.178	1.968	0.356	2.952	0.534	3.936	0.712	4.920	0.890	45
30	0.983	0.182	1.967	0.364	2.950	0.547	3.933	0.729	4.916	0.911	30
45	0.982	0.187	1.965	0.373	2.947	0.560	3.930	0.746	4.912	0.933	15
11 0	0.982	0.191	1.963	0.382	2.945	0.572	3.927	0.763	4.908	0.954	79 0
15	0.981	0.195	1.962	0.390	2.942	0.585	3.923	0.780	4.904	0.975	45
30	0.980	0.199	1.960	0.399	2.940	0.598	3.920	0.797	4.900	0.997	30
45	0.979	0.204	1.958	0.407	2.937	0.611	3.916	0.815	4.895	1.018	15
12 0	0.978	0.208	1.956	0.416	2.934	0.624	3.913	0.832	4.891	1.040	78 0
15	0.977	0.212	1.954	0.424	2.932	0.637	3.909	0.849	4.886	1.061	45
30	0.976	0.216	1.953	0.433	2.929	0.649	3.905	0.866	4.881	1.082	30
45	0.975	0.221	1.951	0.441	2.926	0.662	3.901	0.883	4.877	1.103	15
13 0	0.974	0.225	1.949	0.450	2.923	0.675	3.897	0.900	4.872	1.125	77 0
15	0.973	0.229	1.947	0.458	2.920	0.688	3.894	0.917	4.867	1.146	45
30	0.972	0.233	1.945	0.467	2.917	0.700	3.889	0.934	4.862	1.167	30
45	0.971	0.238	1.943	0.475	2.914	0.713	3.885	0.951	4.857	1.188	15
14 0	0.970	0.242	1.941	0.484	2.911	0.726	3.881	0.968	4.851	1.210	76 0
15	0.969	0.246	1.938	0.492	2.908	0.738	3.877	0.985	4.846	1.231	45
30	0.968	0.250	1.936	0.501	2.904	0.751	3.873	1.002	4.841	1.252	30
45	0.967	0.255	1.934	0.509	2.901	0.764	3.868	1.018	4.835	1.273	15
15 0	0.966	0.259	1.932	0.518	2.898	0.776	3.864	1.035	4.830	1.294	75 0
0 15	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	0 15

75°—90°



Bearing	Distance 6.		Distance 7.		Distance 8.		Distance 9.		Distance 10.		Bearing
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
0 15	6.000	0.026	7.000	0.031	8.000	0.035	9.000	0.039	10.000	0.044	80 45
30	6.000	0.052	7.000	0.061	8.000	0.070	9.000	0.079	10.000	0.087	30
45	5.999	0.079	6.999	0.092	7.999	0.105	8.999	0.118	9.999	0.131	15
1 0	5.999	0.105	6.999	0.122	7.999	0.140	8.999	0.157	9.999	0.175	80 0
15	5.999	0.131	6.998	0.153	7.998	0.175	8.998	0.196	9.998	0.218	45
30	5.998	0.157	6.998	0.183	7.997	0.209	8.997	0.236	9.997	0.262	30
45	5.997	0.183	6.997	0.214	7.996	0.244	8.996	0.275	9.995	0.305	15
2 0	5.996	0.209	6.996	0.244	7.995	0.279	8.995	0.314	9.994	0.349	88 0
15	5.995	0.236	6.995	0.275	7.994	0.314	8.993	0.353	9.992	0.393	45
30	5.994	0.262	6.993	0.305	7.992	0.349	8.991	0.393	9.991	0.436	30
45	5.993	0.288	6.992	0.336	7.991	0.384	8.990	0.432	9.989	0.480	15
3 0	5.992	0.314	6.990	0.366	7.989	0.419	8.988	0.471	9.986	0.523	87 0
15	5.990	0.340	6.989	0.397	7.987	0.454	8.986	0.510	9.984	0.567	45
30	5.989	0.366	6.987	0.427	7.985	0.488	8.983	0.549	9.981	0.611	30
45	5.987	0.392	6.985	0.458	7.983	0.523	8.981	0.589	9.979	0.654	15
4 0	5.985	0.419	6.983	0.488	7.981	0.558	8.978	0.628	9.976	0.698	86 0
15	5.984	0.445	6.981	0.519	7.978	0.593	8.975	0.667	9.973	0.741	45
30	5.982	0.471	6.978	0.549	7.975	0.628	8.972	0.706	9.969	0.785	30
45	5.979	0.497	6.976	0.580	7.973	0.662	8.969	0.745	9.966	0.828	15
5 0	5.977	0.523	6.973	0.610	7.970	0.697	8.966	0.784	9.962	0.872	85 0
15	5.975	0.549	6.971	0.641	7.966	0.732	8.962	0.824	9.958	0.915	45
30	5.972	0.575	6.968	0.671	7.963	0.767	8.959	0.863	9.954	0.959	30
45	5.970	0.601	6.965	0.701	7.960	0.802	8.955	0.902	9.950	1.002	15
6 0	5.967	0.627	6.962	0.732	7.956	0.836	8.951	0.941	9.945	1.045	84 0
15	5.964	0.653	6.958	0.762	7.952	0.871	8.947	0.980	9.941	1.089	45
30	5.961	0.679	6.955	0.792	7.949	0.906	8.942	1.019	9.936	1.132	30
45	5.958	0.705	6.951	0.823	7.945	0.940	8.938	1.058	9.931	1.175	15
7 0	5.955	0.731	6.948	0.853	7.940	0.975	8.933	1.097	9.926	1.219	83 0
15	5.952	0.757	6.944	0.883	7.936	1.010	8.928	1.136	9.920	1.262	45
30	5.949	0.783	6.940	0.914	7.932	1.044	8.923	1.175	9.914	1.305	30
45	5.945	0.809	6.936	0.944	7.927	1.079	8.918	1.214	9.909	1.349	15
8 0	5.942	0.835	6.932	0.974	7.922	1.113	8.912	1.253	9.903	1.392	82 0
15	5.938	0.861	6.928	1.004	7.917	1.148	8.907	1.291	9.897	1.435	45
30	5.934	0.887	6.923	1.035	7.912	1.182	8.901	1.330	9.890	1.478	30
45	5.930	0.913	6.919	1.065	7.907	1.217	8.895	1.369	9.884	1.521	15
9 0	5.926	0.939	6.914	1.095	7.902	1.251	8.889	1.408	9.877	1.564	81 0
15	5.922	0.964	6.909	1.125	7.896	1.286	8.883	1.447	9.870	1.607	45
30	5.918	0.990	6.904	1.155	7.890	1.320	8.877	1.485	9.863	1.651	30
45	5.913	1.016	6.899	1.185	7.884	1.355	8.870	1.524	9.856	1.694	15
10 0	5.909	1.042	6.894	1.216	7.878	1.389	8.863	1.563	9.848	1.737	80 0
15	5.904	1.068	6.888	1.246	7.872	1.424	8.856	1.601	9.840	1.779	45
30	5.900	1.093	6.883	1.276	7.866	1.458	8.849	1.640	9.833	1.822	30
45	5.895	1.119	6.877	1.306	7.860	1.492	8.842	1.679	9.825	1.865	15
11 0	5.890	1.145	6.871	1.336	7.853	1.526	8.835	1.717	9.816	1.908	79 0
15	5.885	1.171	6.866	1.366	7.846	1.561	8.827	1.756	9.808	1.951	45
30	5.880	1.196	6.859	1.396	7.839	1.595	8.819	1.794	9.799	1.994	30
45	5.874	1.222	6.853	1.425	7.832	1.629	8.811	1.833	9.791	2.036	15
12 0	5.869	1.247	6.847	1.455	7.825	1.663	8.803	1.871	9.782	2.079	78 0
15	5.863	1.273	6.841	1.485	7.818	1.697	8.795	1.910	9.772	2.122	45
30	5.858	1.299	6.834	1.515	7.810	1.732	8.787	1.948	9.763	2.164	30
45	5.852	1.324	6.827	1.545	7.803	1.766	8.778	1.986	9.753	2.207	15
13 0	5.846	1.350	6.821	1.575	7.795	1.800	8.769	2.025	9.744	2.250	77 0
15	5.840	1.375	6.814	1.604	7.787	1.834	8.760	2.063	9.734	2.292	45
30	5.834	1.401	6.807	1.634	7.779	1.868	8.751	2.101	9.724	2.335	30
45	5.828	1.426	6.799	1.664	7.771	1.902	8.742	2.139	9.713	2.377	15
14 0	5.822	1.452	6.792	1.693	7.762	1.935	8.733	2.177	9.703	2.419	76 0
15	5.815	1.477	6.785	1.723	7.754	1.969	8.723	2.215	9.692	2.462	45
30	5.809	1.502	6.777	1.753	7.745	2.003	8.713	2.253	9.682	2.504	30
45	5.802	1.528	6.769	1.782	7.736	2.037	8.703	2.291	9.671	2.546	15
15 0	5.796	1.553	6.761	1.812	7.727	2.071	8.693	2.329	9.659	2.588	75 0
0 1	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	0 1
Bearing	Distance 6.		Distance 7.		Distance 8.		Distance 9.		Distance 10.		Bearing



Bearing.	Distance 1.		Distance 2.		Distance 3.		Distance 4.		Distance 5.		Bearing.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
15 15	0.965	0.263	1.930	0.526	2.894	0.789	3.859	1.052	4.824	1.315	74 45
30	0.964	0.267	1.927	0.534	2.891	0.802	3.855	1.069	4.818	1.336	30
45	0.962	0.271	1.925	0.543	2.887	0.814	3.850	1.086	4.812	1.357	15
16 0	0.961	0.276	1.923	0.551	2.884	0.827	3.845	1.103	4.806	1.378	74 0
15	0.960	0.280	1.920	0.560	2.880	0.839	3.840	1.119	4.800	1.399	45
30	0.959	0.284	1.918	0.568	2.876	0.852	3.835	1.136	4.794	1.420	30
45	0.958	0.288	1.915	0.576	2.873	0.865	3.830	1.153	4.788	1.441	15
17 0	0.956	0.292	1.913	0.585	2.869	0.877	3.825	1.169	4.782	1.462	73 0
15	0.955	0.297	1.910	0.593	2.865	0.890	3.820	1.186	4.775	1.483	45
30	0.954	0.301	1.907	0.601	2.861	0.902	3.815	1.203	4.769	1.504	30
45	0.952	0.305	1.905	0.610	2.857	0.915	3.810	1.220	4.762	1.524	15
18 0	0.951	0.309	1.902	0.618	2.853	0.927	3.804	1.236	4.755	1.545	72 0
15	0.950	0.313	1.899	0.626	2.849	0.939	3.799	1.253	4.748	1.566	45
30	0.948	0.317	1.897	0.635	2.845	0.952	3.793	1.269	4.742	1.587	30
45	0.947	0.321	1.894	0.643	2.841	0.964	3.788	1.286	4.735	1.607	15
19 0	0.946	0.326	1.891	0.651	2.837	0.977	3.782	1.302	4.728	1.628	71 0
15	0.944	0.330	1.888	0.659	2.832	0.989	3.776	1.319	4.720	1.648	45
30	0.943	0.334	1.885	0.668	2.828	1.001	3.771	1.335	4.713	1.669	30
45	0.941	0.338	1.882	0.676	2.824	1.014	3.765	1.352	4.706	1.690	15
20 0	0.940	0.342	1.879	0.684	2.819	1.026	3.759	1.368	4.698	1.710	70 0
15	0.938	0.346	1.876	0.692	2.815	1.038	3.753	1.384	4.691	1.731	45
30	0.937	0.350	1.873	0.700	2.810	1.051	3.747	1.401	4.683	1.751	30
45	0.935	0.354	1.870	0.709	2.805	1.063	3.741	1.417	4.676	1.771	15
21 0	0.934	0.358	1.867	0.717	2.801	1.075	3.734	1.433	4.668	1.792	69 0
15	0.932	0.362	1.864	0.725	2.796	1.087	3.728	1.450	4.660	1.812	45
30	0.930	0.367	1.861	0.733	2.791	1.100	3.722	1.466	4.652	1.833	30
45	0.929	0.371	1.858	0.741	2.786	1.112	3.715	1.482	4.644	1.853	15
22 0	0.927	0.375	1.854	0.749	2.782	1.124	3.709	1.498	4.636	1.873	68 0
15	0.926	0.379	1.851	0.757	2.777	1.136	3.702	1.515	4.628	1.893	45
30	0.924	0.383	1.848	0.765	2.772	1.148	3.696	1.531	4.619	1.913	30
45	0.922	0.387	1.844	0.773	2.767	1.160	3.689	1.547	4.611	1.934	15
23 0	0.921	0.391	1.841	0.781	2.762	1.172	3.682	1.563	4.603	1.954	67 0
15	0.919	0.395	1.838	0.789	2.756	1.184	3.675	1.579	4.594	1.974	45
30	0.917	0.399	1.834	0.797	2.751	1.196	3.668	1.595	4.585	1.994	30
45	0.915	0.403	1.831	0.805	2.746	1.208	3.661	1.611	4.577	2.014	15
24 0	0.914	0.407	1.827	0.813	2.741	1.220	3.654	1.627	4.568	2.034	66 0
15	0.912	0.411	1.824	0.821	2.735	1.232	3.647	1.643	4.559	2.054	45
30	0.910	0.415	1.820	0.829	2.730	1.244	3.640	1.659	4.550	2.073	30
45	0.908	0.419	1.816	0.837	2.724	1.256	3.633	1.675	4.541	2.093	15
25 0	0.906	0.423	1.813	0.845	2.719	1.268	3.625	1.690	4.532	2.113	65 0
15	0.904	0.427	1.809	0.853	2.713	1.280	3.618	1.706	4.522	2.133	45
30	0.903	0.431	1.805	0.861	2.708	1.292	3.610	1.722	4.513	2.153	30
45	0.901	0.434	1.801	0.869	2.702	1.303	3.603	1.738	4.503	2.172	15
26 0	0.899	0.438	1.798	0.877	2.696	1.315	3.595	1.753	4.494	2.192	64 0
15	0.897	0.442	1.794	0.885	2.691	1.327	3.587	1.769	4.484	2.211	45
30	0.895	0.446	1.790	0.892	2.685	1.339	3.580	1.785	4.475	2.231	30
45	0.893	0.450	1.786	0.900	2.679	1.350	3.572	1.800	4.465	2.250	15
27 0	0.891	0.454	1.782	0.908	2.673	1.362	3.564	1.816	4.455	2.270	63 0
15	0.889	0.458	1.778	0.916	2.667	1.374	3.556	1.831	4.445	2.289	45
30	0.887	0.462	1.774	0.923	2.661	1.385	3.548	1.847	4.435	2.309	30
45	0.885	0.466	1.770	0.931	2.655	1.397	3.540	1.862	4.425	2.328	15
28 0	0.883	0.469	1.766	0.939	2.649	1.408	3.532	1.878	4.415	2.347	62 0
15	0.881	0.473	1.762	0.947	2.643	1.420	3.524	1.893	4.404	2.367	45
30	0.879	0.477	1.758	0.954	2.636	1.431	3.515	1.909	4.394	2.386	30
45	0.877	0.481	1.753	0.962	2.630	1.443	3.507	1.924	4.384	2.405	15
29 0	0.875	0.485	1.749	0.970	2.624	1.454	3.498	1.939	4.373	2.424	61 0
15	0.872	0.489	1.745	0.977	2.617	1.466	3.490	1.954	4.362	2.443	45
30	0.870	0.492	1.741	0.985	2.611	1.477	3.481	1.970	4.352	2.462	30
45	0.868	0.496	1.736	0.992	2.605	1.489	3.473	1.985	4.341	2.481	15
30 0	0.866	0.500	1.732	1.000	2.598	1.500	3.464	2.000	4.330	2.500	60 0
o /	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	o /
Bearing.	Distance 1.		Distance 2.		Distance 3.		Distance 4.		Distance 5.		Bearing.

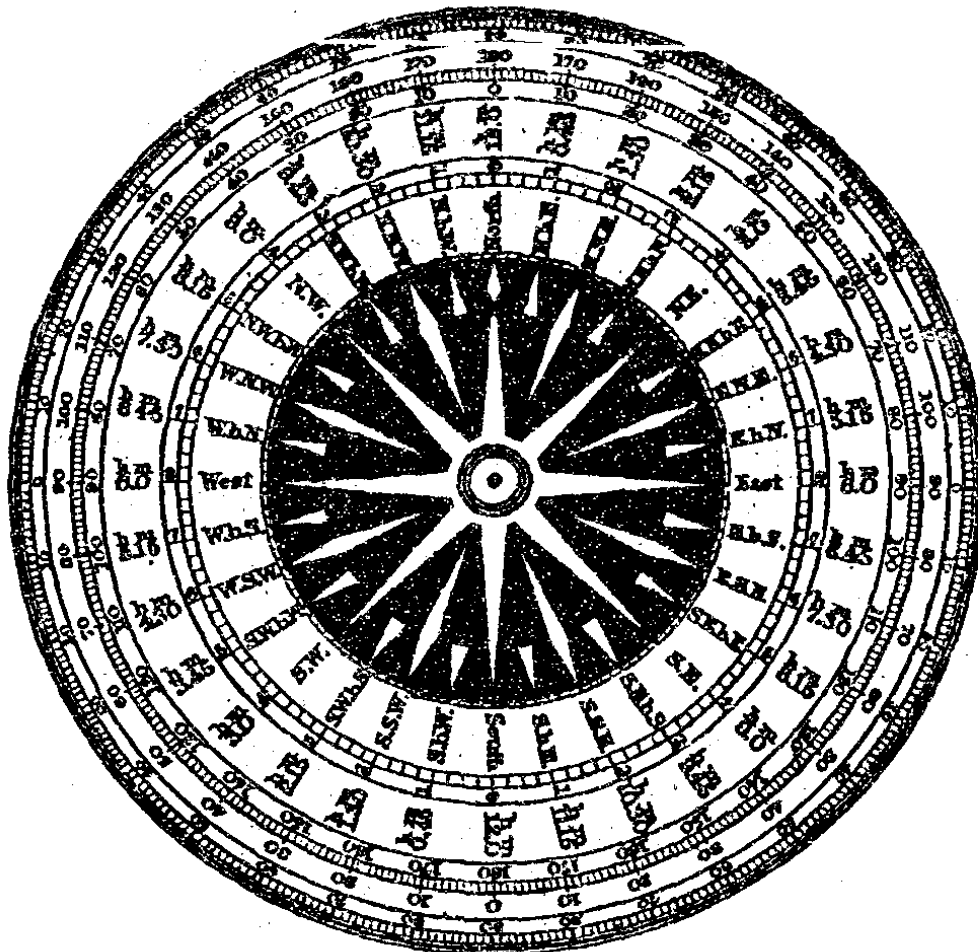
Bearing.	Distance 6.		Distance 7.		Distance 8.		Distance 9.		Distance 10.		Bearing.
° /	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	° /
15 15	5.789	1.578	6.754	1.811	7.718	2.104	8.683	2.367	9.648	2.630	74 45
30	5.782	1.603	6.745	1.871	7.709	2.138	8.673	2.405	9.636	2.672	30
45	5.775	1.629	6.737	1.900	7.700	2.172	8.662	2.443	9.625	2.714	15
16 0	5.768	1.654	6.729	1.929	7.690	2.205	8.651	2.481	9.613	2.756	74 0
15	5.760	1.679	6.720	1.959	7.680	2.239	8.640	2.518	9.601	2.798	45
30	5.753	1.704	6.712	1.988	7.671	2.272	8.629	2.556	9.588	2.840	30
45	5.745	1.729	6.703	2.017	7.661	2.306	8.618	2.594	9.576	2.882	15
17 0	5.738	1.754	6.694	2.047	7.650	2.339	8.607	2.631	9.563	2.924	73 0
15	5.730	1.779	6.685	2.076	7.640	2.372	8.595	2.669	9.550	2.965	45
30	5.722	1.804	6.676	2.105	7.630	2.406	8.583	2.706	9.537	3.007	30
45	5.714	1.829	6.667	2.134	7.619	2.439	8.572	2.744	9.524	3.049	15
18 0	5.706	1.854	6.657	2.163	7.608	2.472	8.560	2.781	9.511	3.090	72 0
15	5.698	1.879	6.648	2.192	7.598	2.505	8.547	2.818	9.497	3.132	45
30	5.690	1.904	6.638	2.221	7.587	2.538	8.535	2.856	9.483	3.173	30
45	5.682	1.929	6.629	2.250	7.575	2.572	8.522	2.893	9.469	3.214	15
19 0	5.673	1.953	6.619	2.279	7.564	2.605	8.510	2.930	9.455	3.256	71 0
15	5.665	1.978	6.609	2.308	7.553	2.638	8.497	2.967	9.441	3.297	45
30	5.656	2.003	6.598	2.337	7.541	2.670	8.484	3.004	9.426	3.338	30
45	5.647	2.028	6.588	2.365	7.529	2.703	8.471	3.041	9.412	3.379	15
20 0	5.638	2.052	6.578	2.394	7.518	2.736	8.457	3.078	9.397	3.420	70 0
15	5.629	2.077	6.567	2.423	7.506	2.769	8.444	3.115	9.382	3.461	45
30	5.620	2.101	6.557	2.451	7.493	2.802	8.430	3.152	9.367	3.502	30
45	5.611	2.126	6.546	2.480	7.481	2.834	8.416	3.189	9.351	3.543	15
21 0	5.601	2.150	6.535	2.509	7.469	2.867	8.402	3.225	9.336	3.584	69 0
15	5.592	2.175	6.524	2.537	7.456	2.900	8.388	3.262	9.320	3.624	45
30	5.582	2.199	6.513	2.566	7.443	2.932	8.374	3.299	9.304	3.665	30
45	5.573	2.223	6.502	2.594	7.430	2.964	8.359	3.335	9.288	3.706	15
22 0	5.563	2.248	6.490	2.622	7.417	2.997	8.345	3.371	9.272	3.746	68 0
15	5.553	2.272	6.479	2.651	7.404	3.029	8.330	3.408	9.255	3.787	45
30	5.543	2.296	6.467	2.679	7.391	3.061	8.315	3.444	9.239	3.827	30
45	5.533	2.320	6.455	2.707	7.378	3.094	8.300	3.480	9.222	3.867	15
23 0	5.523	2.344	6.444	2.735	7.364	3.126	8.285	3.517	9.205	3.907	67 0
15	5.513	2.368	6.432	2.763	7.350	3.158	8.269	3.553	9.188	3.947	45
30	5.502	2.392	6.419	2.791	7.336	3.190	8.254	3.589	9.171	3.988	30
45	5.492	2.416	6.407	2.819	7.322	3.222	8.238	3.625	9.153	4.028	15
24 0	5.481	2.440	6.395	2.847	7.308	3.254	8.222	3.661	9.136	4.067	66 0
15	5.471	2.464	6.382	2.875	7.294	3.286	8.206	3.696	9.118	4.107	45
30	5.460	2.488	6.370	2.903	7.280	3.318	8.190	3.732	9.100	4.147	30
45	5.449	2.512	6.357	2.931	7.265	3.349	8.173	3.768	9.081	4.187	15
25 0	5.438	2.536	6.344	2.958	7.250	3.381	8.157	3.804	9.063	4.226	65 0
15	5.427	2.559	6.331	2.986	7.236	3.413	8.140	3.839	9.045	4.266	45
30	5.416	2.583	6.318	3.014	7.221	3.444	8.123	3.875	9.026	4.305	30
45	5.404	2.607	6.305	3.041	7.206	3.476	8.106	3.910	9.007	4.345	15
26 0	5.393	2.630	6.292	3.069	7.190	3.507	8.089	3.945	8.988	4.384	64 0
15	5.381	2.654	6.278	3.096	7.175	3.538	8.072	3.981	8.969	4.423	45
30	5.370	2.677	6.265	3.123	7.160	3.570	8.054	4.016	8.949	4.462	30
45	5.358	2.701	6.251	3.151	7.144	3.601	8.037	4.051	8.930	4.501	15
27 0	5.346	2.724	6.237	3.178	7.128	3.632	8.019	4.086	8.910	4.540	63 0
15	5.334	2.747	6.223	3.205	7.112	3.663	8.001	4.121	8.890	4.579	45
30	5.322	2.770	6.209	3.232	7.096	3.694	7.983	4.156	8.870	4.618	30
45	5.310	2.794	6.195	3.259	7.080	3.725	7.965	4.190	8.850	4.656	15
28 0	5.298	2.817	6.181	3.286	7.064	3.756	7.947	4.225	8.829	4.695	62 0
15	5.285	2.840	6.166	3.313	7.047	3.787	7.928	4.260	8.809	4.733	45
30	5.273	2.863	6.152	3.340	7.031	3.817	7.909	4.294	8.788	4.772	30
45	5.260	2.886	6.137	3.367	7.014	3.848	7.891	4.329	8.767	4.810	15
29 0	5.248	2.909	6.122	3.394	6.997	3.878	7.872	4.363	8.746	4.848	61 0
15	5.235	2.932	6.107	3.420	6.980	3.909	7.852	4.398	8.725	4.886	45
30	5.222	2.955	6.093	3.447	6.963	3.939	7.833	4.432	8.704	4.924	30
45	5.209	2.977	6.077	3.474	6.946	3.970	7.814	4.466	8.682	4.962	15
30 0	5.196	3.000	6.062	3.500	6.928	4.000	7.794	4.500	8.660	5.000	60 0
° /	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	° /
Bearing.	Distance 6.		Distance 7.		Distance 8.		Distance 9.		Distance 10.		Bearing.

30° - 45°

Bearing.	Distance 1.		Distance 2.		Distance 3.		Distance 4.		Distance 5.		Bearing.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
30 15	0.864	0.504	1.728	1.008	2.592	1.511	3.455	2.015	4.319	2.519	59 45
30 30	0.862	0.508	1.723	1.015	2.585	1.523	3.447	2.030	4.308	2.538	30
30 45	0.859	0.511	1.719	1.023	2.578	1.534	3.438	2.045	4.297	2.556	15
31 0	0.857	0.515	1.714	1.030	2.572	1.545	3.429	2.060	4.286	2.575	59 0
31 15	0.855	0.519	1.710	1.038	2.565	1.556	3.420	2.075	4.275	2.594	45
31 30	0.853	0.522	1.705	1.045	2.558	1.567	3.411	2.090	4.263	2.612	30
31 45	0.850	0.526	1.701	1.052	2.551	1.579	3.401	2.105	4.252	2.631	15
32 0	0.848	0.530	1.696	1.060	2.544	1.590	3.392	2.120	4.240	2.650	58 0
32 15	0.846	0.534	1.691	1.067	2.537	1.601	3.383	2.134	4.229	2.668	45
32 30	0.843	0.537	1.687	1.075	2.530	1.612	3.374	2.149	4.217	2.686	30
32 45	0.841	0.541	1.682	1.082	2.523	1.623	3.364	2.164	4.205	2.705	15
33 0	0.839	0.545	1.677	1.089	2.516	1.634	3.355	2.179	4.193	2.723	57 0
33 15	0.836	0.548	1.673	1.097	2.509	1.645	3.345	2.193	4.181	2.741	45
33 30	0.834	0.552	1.668	1.104	2.502	1.656	3.336	2.208	4.169	2.760	30
33 45	0.831	0.556	1.663	1.111	2.494	1.667	3.326	2.222	4.157	2.778	15
34 0	0.829	0.559	1.658	1.118	2.487	1.678	3.316	2.237	4.145	2.796	56 0
34 15	0.827	0.563	1.653	1.126	2.480	1.688	3.306	2.251	4.133	2.814	45
34 30	0.824	0.566	1.648	1.133	2.472	1.699	3.297	2.266	4.121	2.832	30
34 45	0.822	0.570	1.643	1.140	2.465	1.710	3.287	2.280	4.108	2.850	15
35 0	0.819	0.574	1.638	1.147	2.457	1.721	3.277	2.294	4.096	2.868	55 0
35 15	0.817	0.577	1.633	1.154	2.450	1.731	3.267	2.309	4.083	2.886	45
35 30	0.814	0.581	1.628	1.161	2.442	1.742	3.257	2.323	4.071	2.904	30
35 45	0.812	0.584	1.623	1.168	2.435	1.753	3.246	2.337	4.058	2.921	15
36 0	0.809	0.588	1.618	1.176	2.427	1.763	3.236	2.351	4.045	2.939	54 0
36 15	0.806	0.591	1.613	1.183	2.419	1.774	3.226	2.365	4.032	2.957	45
36 30	0.804	0.595	1.608	1.190	2.412	1.784	3.215	2.379	4.019	2.974	30
36 45	0.801	0.598	1.603	1.197	2.404	1.795	3.205	2.393	4.006	2.992	15
37 0	0.799	0.602	1.597	1.204	2.396	1.805	3.195	2.407	3.993	3.009	53 0
37 15	0.796	0.605	1.592	1.211	2.388	1.816	3.184	2.421	3.980	3.026	45
37 30	0.793	0.609	1.587	1.218	2.380	1.826	3.173	2.435	3.967	3.044	30
37 45	0.791	0.612	1.581	1.224	2.372	1.837	3.163	2.449	3.953	3.061	15
38 0	0.788	0.616	1.576	1.231	2.364	1.847	3.152	2.463	3.940	3.078	52 0
38 15	0.785	0.619	1.571	1.238	2.356	1.857	3.141	2.476	3.927	3.095	45
38 30	0.783	0.623	1.565	1.245	2.348	1.868	3.130	2.490	3.913	3.113	30
38 45	0.780	0.626	1.560	1.252	2.340	1.878	3.120	2.504	3.899	3.130	15
39 0	0.777	0.629	1.554	1.259	2.331	1.888	3.109	2.517	3.886	3.147	51 0
39 15	0.774	0.633	1.549	1.265	2.323	1.898	3.098	2.531	3.872	3.164	45
39 30	0.772	0.636	1.543	1.272	2.315	1.908	3.086	2.544	3.858	3.180	30
39 45	0.769	0.639	1.538	1.279	2.307	1.918	3.075	2.558	3.844	3.197	15
40 0	0.766	0.643	1.532	1.286	2.298	1.928	3.064	2.571	3.830	3.214	50 0
40 15	0.763	0.646	1.526	1.292	2.290	1.938	3.053	2.584	3.816	3.231	45
40 30	0.760	0.649	1.521	1.299	2.281	1.948	3.042	2.598	3.802	3.247	30
40 45	0.758	0.653	1.515	1.306	2.273	1.958	3.030	2.611	3.788	3.264	15
41 0	0.755	0.656	1.509	1.312	2.264	1.968	3.019	2.624	3.774	3.280	49 0
41 15	0.752	0.659	1.504	1.319	2.256	1.978	3.007	2.637	3.759	3.297	45
41 30	0.749	0.663	1.498	1.325	2.247	1.988	2.996	2.650	3.745	3.313	30
41 45	0.746	0.666	1.492	1.332	2.238	1.998	2.984	2.664	3.730	3.329	15
42 0	0.743	0.669	1.486	1.338	2.229	2.007	2.973	2.677	3.716	3.346	48 0
42 15	0.740	0.672	1.480	1.345	2.221	2.017	2.961	2.689	3.701	3.362	45
42 30	0.737	0.676	1.475	1.351	2.212	2.027	2.949	2.702	3.686	3.378	30
42 45	0.734	0.679	1.469	1.358	2.203	2.036	2.937	2.715	3.672	3.394	15
43 0	0.731	0.682	1.463	1.364	2.194	2.046	2.925	2.728	3.657	3.410	47 0
43 15	0.728	0.685	1.457	1.370	2.185	2.056	2.913	2.741	3.642	3.426	45
43 30	0.725	0.688	1.451	1.377	2.176	2.065	2.901	2.753	3.627	3.442	30
43 45	0.722	0.692	1.445	1.383	2.167	2.075	2.889	2.766	3.612	3.458	15
44 0	0.719	0.695	1.439	1.389	2.158	2.084	2.877	2.779	3.597	3.473	46 0
44 15	0.716	0.698	1.433	1.396	2.149	2.093	2.865	2.791	3.582	3.489	45
44 30	0.713	0.701	1.427	1.402	2.140	2.103	2.853	2.804	3.566	3.505	30
44 45	0.710	0.704	1.420	1.408	2.131	2.112	2.841	2.816	3.551	3.520	15
45 0	0.707	0.707	1.414	1.414	2.121	2.121	2.828	2.828	3.536	3.536	45 0
o /	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	o /
Bearing.	Distance 1.	Distance 2.	Distance 3.	Distance 4.	Distance 5.	Bearing.					Bearing.

45° - 60°

Bearing.	Distance 6.		Distance 7.		Distance 8.		Distance 9.		Distance 10.		Bearing.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
30 15	5.183	3.023	6.047	3.526	6.911	4.010	7.775	4.334	8.638	5.038	59 45
30 30	5.170	3.045	6.031	3.553	6.893	4.060	7.755	4.563	8.616	5.075	30
45	5.156	3.068	6.016	3.579	6.875	4.090	7.735	4.602	8.594	5.113	15
31 0	5.143	3.070	6.000	3.605	6.857	4.120	7.715	4.635	8.572	5.150	59 0
15	5.129	3.113	5.984	3.631	6.839	4.150	7.694	4.669	8.549	5.188	45
30	5.116	3.135	5.968	3.657	6.821	4.180	7.674	4.702	8.526	5.225	30
45	5.102	3.157	5.952	3.683	6.803	4.210	7.653	4.736	8.504	5.262	15
32 0	5.088	3.180	5.936	3.709	6.784	4.239	7.632	4.769	8.481	5.299	58 0
15	5.074	3.202	5.920	3.735	6.766	4.269	7.612	4.802	8.457	5.336	45
30	5.060	3.224	5.904	3.761	6.747	4.298	7.591	4.836	8.434	5.373	30
45	5.046	3.246	5.887	3.787	6.728	4.328	7.569	4.869	8.410	5.410	15
33 0	5.032	3.268	5.871	3.812	6.709	4.357	7.548	4.902	8.387	5.446	57 0
15	5.018	3.290	5.854	3.838	6.690	4.386	7.527	4.935	8.363	5.483	45
30	5.003	3.312	5.837	3.864	6.671	4.416	7.505	4.967	8.339	5.519	30
45	4.989	3.333	5.820	3.889	6.652	4.445	7.483	5.000	8.315	5.556	15
34 0	4.974	3.355	5.803	3.914	6.632	4.474	7.461	5.033	8.290	5.592	56 0
15	4.960	3.377	5.786	3.940	6.613	4.502	7.439	5.065	8.266	5.628	45
30	4.945	3.399	5.769	3.965	6.593	4.531	7.417	5.098	8.241	5.664	30
45	4.930	3.420	5.752	3.990	6.573	4.560	7.395	5.130	8.217	5.700	15
35 0	4.915	3.441	5.734	4.015	6.553	4.589	7.372	5.162	8.192	5.736	55 0
15	4.900	3.463	5.716	4.040	6.533	4.617	7.350	5.194	8.166	5.772	45
30	4.885	3.484	5.699	4.065	6.513	4.646	7.327	5.226	8.141	5.807	30
45	4.869	3.505	5.681	4.090	6.493	4.674	7.304	5.258	8.116	5.843	15
36 0	4.854	3.527	5.663	4.115	6.472	4.702	7.281	5.290	8.090	5.878	54 0
15	4.839	3.548	5.645	4.139	6.452	4.730	7.258	5.322	8.064	5.913	45
30	4.823	3.569	5.627	4.164	6.431	4.759	7.235	5.353	8.039	5.948	30
45	4.808	3.590	5.609	4.188	6.410	4.787	7.211	5.385	8.013	5.983	15
37 0	4.792	3.611	5.590	4.213	6.389	4.815	7.188	5.416	7.986	6.018	53 0
15	4.776	3.632	5.572	4.237	6.368	4.842	7.164	5.448	7.960	6.053	45
30	4.760	3.653	5.554	4.261	6.347	4.870	7.140	5.479	7.934	6.088	30
45	4.744	3.673	5.535	4.286	6.326	4.898	7.116	5.510	7.907	6.122	15
38 0	4.728	3.694	5.516	4.310	6.304	4.925	7.092	5.541	7.880	6.157	52 0
15	4.712	3.715	5.497	4.334	6.283	4.953	7.068	5.572	7.853	6.191	45
30	4.696	3.735	5.478	4.358	6.261	4.980	7.043	5.603	7.826	6.225	30
45	4.679	3.756	5.459	4.381	6.239	5.007	7.019	5.633	7.799	6.259	15
39 0	4.663	3.776	5.440	4.405	6.217	5.035	6.994	5.664	7.772	6.293	51 0
15	4.646	3.796	5.421	4.429	6.195	5.062	6.970	5.694	7.744	6.327	45
30	4.630	3.816	5.401	4.453	6.173	5.089	6.945	5.725	7.716	6.361	30
45	4.613	3.837	5.382	4.476	6.151	5.116	6.920	5.755	7.688	6.394	15
40 0	4.596	3.857	5.362	4.500	6.128	5.142	6.894	5.785	7.660	6.428	50 0
15	4.579	3.877	5.343	4.523	6.106	5.169	6.869	5.815	7.632	6.461	45
30	4.562	3.897	5.323	4.546	6.083	5.196	6.844	5.845	7.604	6.495	30
45	4.545	3.917	5.303	4.569	6.061	5.222	6.818	5.875	7.576	6.528	15
41 0	4.528	3.936	5.283	4.592	6.038	5.248	6.792	5.905	7.547	6.561	40 0
15	4.511	3.956	5.263	4.615	6.015	5.275	6.767	5.934	7.518	6.594	45
30	4.494	3.976	5.243	4.638	5.992	5.301	6.741	5.964	7.490	6.626	30
45	4.476	3.995	5.222	4.661	5.968	5.327	6.715	5.993	7.461	6.659	15
42 0	4.459	4.015	5.202	4.684	5.945	5.353	6.688	6.022	7.431	6.691	48 0
15	4.441	4.034	5.182	4.707	5.922	5.379	6.662	6.051	7.402	6.724	45
30	4.424	4.054	5.161	4.729	5.898	5.405	6.635	6.080	7.373	6.756	30
45	4.406	4.073	5.140	4.752	5.875	5.430	6.609	6.109	7.343	6.788	15
43 0	4.388	4.092	5.119	4.774	5.851	5.456	6.582	6.138	7.314	6.820	47 0
15	4.370	4.111	5.099	4.796	5.827	5.481	6.555	6.167	7.284	6.852	45
30	4.352	4.130	5.078	4.818	5.803	5.507	6.528	6.195	7.254	6.884	30
45	4.334	4.149	5.057	4.841	5.779	5.532	6.501	6.224	7.224	6.915	15
44 0	4.316	4.168	5.035	4.863	5.755	5.557	6.474	6.252	7.193	6.947	46 0
15	4.298	4.187	5.014	4.885	5.730	5.582	6.447	6.280	7.163	6.978	45
30	4.280	4.206	4.993	4.906	5.706	5.607	6.419	6.303	7.133	7.009	30
45	4.261	4.224	4.971	4.928	5.681	5.632	6.392	6.336	7.102	7.040	15
45 0	4.243	4.243	4.950	4.950	5.657	5.657	6.364	6.364	7.071	7.071	45 0



**A TABLE OF THE ANGLES**

*Which every Point and Quarter Point of the Compass makes with the Meridian.*

North.		Points.	° ' "	Points.	South.	
N. by E.	N. by W.	0-1/4	2 45 45	0-1/4	S. by E.	S. by W.
		0-1/2	5 37 30	0-1/2		
		0-3/4	8 29 15	0-3/4		
		1	11 15 0	1		
N.N.E.	N.N.W.	1-1/4	14 8 45	1-1/4	S.S.E.	S.S.W.
		1-1/2	18 32 30	1-1/2		
		1-3/4	19 41 15	1-3/4		
		2	22 50 0	2		
N.E. by N.	N.W. by N.	2-1/4	25 18 45	2-1/4	S.E. by S.	S.W. by S.
		2-1/2	28 7 30	2-1/2		
		2-3/4	30 56 15	2-3/4		
		3	33 45 0	3		
N.E.	N.W.	3-1/4	36 33 45	3-1/4	S.E.	S.W.,
		3-1/2	39 22 30	3-1/2		
		3-3/4	42 11 15	3-3/4		
		4	45 0 0	4		
N.E. by E.	N.W. by W.	4-1/4	47 48 45	4-1/4	S.E. by E.	S.W. by W.
		4-1/2	50 37 30	4-1/2		
		4-3/4	53 26 15	4-3/4		
		5	56 15 0	5		
E.N.E.	W.N.W.	5-1/4	59 3 45	5-1/4	E.S.E.	W.S.W.
		5-1/2	61 52 30	5-1/2		
		5-3/4	64 41 15	5-3/4		
		6	67 30 0	6		
E. by N.	W. by N.	6-1/4	70 18 45	6-1/4	E. by S.	W. by S.
		6-1/2	73 7 30	6-1/2		
		6-3/4	75 56 15	6-3/4		
		7	78 45 0	7		
East.	West.	7-1/4	81 33 45	7-1/4	East.	West.
		7-1/2	84 22 30	7-1/2		
		7-3/4	87 11 15	7-3/4		
		8	90 0 0	8		

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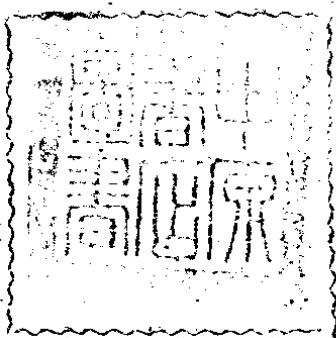
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