

years and experience in this errantry, they purchase their freedom by some tryall of skill in y<sup>r</sup> faculty w<sup>ch</sup> they perform in publick before y<sup>e</sup> Majistrates of y<sup>e</sup> place, w<sup>ch</sup> is testified by an instrument under y<sup>e</sup> seale of y<sup>e</sup> magistracy. I believe if we should deny freedom to all such as leave y<sup>r</sup> own country and come to plant among us, we should doe y<sup>m</sup> noe injury, for none of y<sup>m</sup> having undergone this tryall, they would be noe better y<sup>n</sup> journeymen at home, but by our naturall civility for strangers has our law run more in y<sup>r</sup> favor.’”

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Sir William Rowan Hamilton read a Paper on a new System of Roots of Unity, and of operations therewith connected: to which system of symbols and operations, in consequence of the geometrical character of some of their leading interpretations, he is disposed to give the name of the “ICO-SIAN CALCULUS.”

This Calculus *agrees* with that of the Quaternions, in three important respects: namely, 1st, that its three chief symbols,  $\iota$ ,  $\kappa$ ,  $\lambda$ , are (as above suggested) *roots of unity*, as  $i$ ,  $j$ ,  $k$  are certain *fourth roots* thereof: 2nd, that these new roots *obey* the *associative* law of multiplication; and 3rd, that they are *not* subject to the *commutative* law, or that their *places as factors* must *not* in general be *altered* in a *product*. And it *differs* from the Quaternion Calculus, 1st, by involving roots with *different exponents*; and 2ndly, by *not requiring* (so far as yet appears) the *distributive* property of multiplication. In fact, + and -, in these new calculations, enter *only as connecting exponents*, and *not as connecting terms*: indeed, *no terms*, or in other words, *no polynomes*, nor even binomes, have hitherto presented themselves, in these late researches of the author. As regards the *exponents* of the new roots, it may be mentioned that in the *principal system*,—for the new Calculus involves a *family of systems*,—there are adopted the equations,—

$$1 = \iota^2 = \kappa^3 = \lambda^5, \lambda = \iota\kappa; \quad (\text{A})$$

so that we deal, in it, with a *new square root, cube root, and fifth root, of positive unity*; the latter root being the *product* of the two former, when taken in an *order* assigned, but *not* in the opposite order. From these simple assumptions (A), a long train of consistent calculations opens itself out, for every result of which there is found a corresponding geometrical interpretation, in the theory of two of the celebrated solids of antiquity, alluded to with interest by Plato in the *Timæus*; namely, the Icosaedron, and the Dodecaedron: whereof the *angles may now be unequal*. By making  $\lambda^4 = 1$ , the author obtains other symbolical results, which are interpreted by the Octaedron and the Hexaedron. The Pyramid is, in *this* theory, almost too simple to be interesting: but it is dealt with by the assumption,  $\lambda^3 = 1$ , the other equations (A) being untouched. As one fundamental result of those equations (A), which may serve as a slight specimen of the rest, it is found that if we make  $\iota\kappa^2 = \mu$ , we shall have

$$\mu^5 = 1, \mu = \lambda\iota\lambda, \lambda = \mu\iota\mu;$$

so that this *new fifth root*  $\mu$  has relations of perfect *reciprocity* with the former fifth root  $\lambda$ . But there exist more *general* results, *including* this, and others, on which Sir W. R. H. hopes to be allowed to make a future communication to the Academy: as also on some applications of the principles already stated, or alluded to, which appear to be in some degree interesting.

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The following donations were presented:—

1. By Corry Connellan, Esq.:—A copy of Sir Martin A. Shee's portrait of the late Thomas Moore, Esq.

2. By Edward Bewley, M. D.:—An autograph letter of Dr. Charles Lucas, of which the following is a copy:—

“By this time, I may congratulate my worthy, honest friend, first, on his safe arrival with his fair convoy and then, on their kind reception and assured success, in Dublin. I am