PHILOSOPHICAL
TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCXLI.

PART I.

LONDON:
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MDCCCXLI.

ADVERTISEMENT.

The Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions,* take this opportunity to acquaint the Public, that it fully appears, as well from the Council-books and Journals of the Society, as from repeated declarations which have been made in several former *Transactions,* that the printing of them was always, from time to time, the single act of the respective Secretaries till the Forty-seventh Volume; the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions*; which was accordingly done upon the 26th of March 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgement of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon any subject, either of Nature or Art, that comes before them. And therefore the thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

A List of Public Institutions and Individuals, entitled to receive a copy of the Philosophical Transactions of each year, on making application for the same directly or through their respective agents, within five years of the date of publication.

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The Observatory at Dublin.
The Observatory at Armagh.
The Observatory at the Cape of Good Hope.
The Observatory at Paramatta.
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ROYAL MEDALS.

HER MAJESTY QUEEN VICTORIA, in restoring the Foundation of the Royal Medals, has been graciously pleased to approve of the following regulations for the award of them:

That the Royal Medals be given for such papers only as have been presented to the Royal Society, and inserted in their Transactions.

That the triennial Cycle of subjects be the same as that hitherto in operation: viz.

1. Astronomy; Physiology, including the Natural History of Organized Beings.
2. Physics; Geology or Mineralogy.
3. Mathematics; Chemistry.

That, in case no paper, coming within these stipulations, should be considered deserving of the Royal Medal, in any given year, the Council have the power of awarding such Medal to the author of any other paper on either of the several subjects forming the Cycle, that may have been presented to the Society and inserted in their Transactions; preference being given to the subjects of the year immediately preceding: the award being, in such case, subject to the approbation of Her Majesty.

The Council propose to give one of the Royal Medals in the year 1841 for the most important unpublished paper in Mathematics, communicated to the Royal Society for insertion in their Transactions after the termination of the Session in June 1838, and prior to the termination of the Session in June 1841.

The Council propose also to give one of the Royal Medals in the year 1841 for the most important unpublished paper in Chemistry, communicated to the Royal Society for insertion in their Transactions after the termination of the Session in June 1838, and prior to the termination of the Session in June 1841.

The Council propose to give one of the Royal Medals in the year 1842 for the most important unpublished paper in Astronomy, communicated to the Royal Society for insertion in their Transactions after the termination of the Session in June 1839, and prior to the termination of the Session in June 1842.

The Council propose also to give one of the Royal Medals in the year 1842 for the most important unpublished paper in Physiology, including the Natural History of Organized Beings, communicated to the Royal Society for insertion in their Transactions after the termination of the Session in June 1839, and prior to the termination of the Session in June 1842.

The Council propose to give one of the Royal Medals in the year 1843 for the most important unpublished paper in Physics, communicated to the Royal Society for insertion in their Transactions after the termination of the Session in June 1840, and prior to the termination of the Session in June 1843.

The Council propose also to give one of the Royal Medals in the year 1843 for the most important unpublished paper in Geology or Mineralogy, communicated to the Royal Society for insertion in their Transactions after the termination of the Session in June 1840, and prior to the termination of the Session in June 1843.

The Council propose to give one of the Royal Medals in the year 1844 for the most important unpublished paper in Mathematics, communicated to the Royal Society for insertion in their Transactions after the termination of the Session in June 1841, and prior to the termination of the Session in June 1844.

The Council propose also to give one of the Royal Medals in the year 1844 for the most important unpublished paper in Chemistry, communicated to the Royal Society for insertion in their Transactions after the termination of the Session in June 1841, and prior to the termination of the Session in June 1844.

CONTENTS.

PHILOSOPHICAL TRANSACTIONS.

I. *Supplement to a Paper "On the Theoretical Explanation of an apparent new Polarity in Light." By* G. B. Airy, *Esq. M.A. F.R.S., Astronomer Royal.*

Received October 24,—Read November 19, 1840.

In the Second Part of the Transactions of the year 1840, the Royal Society has published a memoir by me, explaining, on the undulatory theory of light, the apparent new polarity observed by Sir David Brewster; which explanation is based upon the assumption that the spectrum is viewed out of focus; an assumption which corresponded to the circumstances of my own observations, and to those of some other persons. Since the publication of that memoir, I have been assured by Sir David Brewster that the phenomenon was most certainly observed with great distinctness when the spectrum was viewed so accurately in focus that many of Fraunhofer's finer lines could be seen. This observation appeared to be contradictory to those of Mr. Talbot, cited by me in page 226 of the memoir, as well as to my own. With the view of removing the obscurity that still appeared to embarrass this subject, I have continued the theoretical investigation for that case which was omitted in the former memoir, namely, when the spectrum is viewed in focus, or when $a = 0$ (page 229); and I have arrived at a result which appears completely to reconcile the seemingly conflicting statements.

In the following investigation I shall use the symbols and the formulæ of the former memoir (as far as they apply) without further reference.

The value of ϱ in page 228 becomes, on making $a = 0$,

$$\varrho = e - \frac{b}{e} y,$$

and the disturbance of ether, on the point of the retina whose distance from the geometrical image is b, produced by a small portion δy of the front of the wave, is

$$\delta y \times \sin \frac{2\pi}{\lambda} (v t - \varrho)$$

or

$$\delta y \times \sin \frac{2\pi}{\lambda} \left(v t - e + \frac{b}{e} y\right),$$

and therefore the whole disturbance of ether on the point of the retina, produced by that part of the pupil which is not covered by any retarding plate, is

$$\int_y \sin\frac{2\pi}{\lambda}\left(vt - e + \frac{b}{e}y\right),$$

the limits of the integral being the values of y corresponding to the boundaries of the part of the pupil not covered by a retarding plate.

But if a portion of the pupil be covered by a plate producing the retardation R (expressed as an angle) in the phase of the wave, the expression to be integrated through the limits proper for the covered part will be

$$\int_y \sin\left(\frac{2\pi}{\lambda}(vt - e) - \mathrm{R}\right)$$

or

$$\int_y \sin\left(\frac{2\pi}{\lambda}\left(vt - e + \frac{b}{e}y\right) - \mathrm{R}\right).$$

Let the limits of the pupil be from $-h$ to $+h$, without regard to the other ordinate upon its surface (which amounts to supposing the form of the pupil to be a parallelogram), and let the part which depends on R be taken between the limits 0 and $+h$ (which amounts to supposing that half of the pupil to be covered which is on the side on which b is considered positive). Then the whole disturbance of the ether is

$$\int_y \sin\frac{2\pi}{\lambda}\left(vt - e + \frac{b}{e}y\right) \text{ from } y = -h \text{ to } y = 0$$

$$+\int_y \sin\left(\frac{2\pi}{\lambda}\left(vt - e + \frac{b}{e}y\right) - \mathrm{R}\right) \text{ from } y = 0 \text{ to } y = +h$$

$$= \frac{\lambda e}{2\pi b}\left\{\cos\frac{2\pi}{\lambda}\left(vt - e - \frac{bh}{e}\right) - \cos\frac{2\pi}{\lambda}(vt - e) + \cos\left(\frac{2\pi}{\lambda}(vt - e) - \mathrm{R}\right)\right.$$

$$\left. - \cos\left(\frac{2\pi}{\lambda}\left(vt - e + \frac{bh}{e}\right) - \mathrm{R}\right)\right\}.$$

The coefficient of $\cos\frac{2\pi}{\lambda}(vt - e)$ is

$$\frac{\lambda e}{2\pi b}\left\{\cos\frac{2\pi}{\lambda}\cdot\frac{bh}{e} - 1 + \cos\mathrm{R} - \cos\left(\frac{2\pi}{\lambda}\cdot\frac{bh}{e} - \mathrm{R}\right)\right\}$$

$$= -\frac{\lambda e}{2\pi b}\left\{1 - \cos\frac{2\pi}{\lambda}\cdot\frac{bh}{e} - \cos\mathrm{R} + \cos\frac{2\pi}{\lambda}\cdot\frac{bh}{e} \times \cos\mathrm{R} + \sin\frac{2\pi}{\lambda}\cdot\frac{bh}{e} \times \sin\mathrm{R}\right\}$$

$$= -\frac{\lambda e}{2\pi b}\left\{\left(1 - \cos\frac{2\pi}{\lambda}\cdot\frac{bh}{e}\right) \times (1 - \cos\mathrm{R}) + \sin\frac{2\pi}{\lambda}\cdot\frac{bh}{e} \times \sin\mathrm{R}\right\}$$

$$= -\frac{\lambda e}{2\pi b} 4\sin\frac{\pi}{\lambda}\cdot\frac{bh}{e} \times \sin\frac{\mathrm{R}}{2} \times \left\{\sin\frac{\pi}{\lambda}\cdot\frac{bh}{e} \times \sin\frac{\mathrm{R}}{2} + \cos\frac{\pi}{\lambda}\cdot\frac{bh}{e} \times \cos\frac{\mathrm{R}}{2}\right\}$$

$$= -\frac{2\lambda e}{\pi b}\cdot\sin\frac{\pi bh}{\lambda e}\cdot\sin\frac{\mathrm{R}}{2}\cdot\cos\left(\frac{\pi bh}{\lambda e} - \frac{\mathrm{R}}{2}\right);$$

and the coefficient of $\sin \frac{2\pi}{\lambda}(vt - e)$ is

$$\frac{\lambda e}{2\pi b}\left\{\sin \frac{2\pi}{\lambda} \cdot \frac{bh}{e} + \sin \mathrm{R} + \sin\left(\frac{2\pi}{\lambda} \cdot \frac{bh}{e} - \mathrm{R}\right)\right\}$$

$$= \frac{\lambda e}{2\pi b}\left\{\sin \frac{2\pi}{\lambda} \cdot \frac{bh}{e} \times (1 + \cos \mathrm{R}) + \sin \mathrm{R} \times \left(1 - \cos \frac{2\pi}{\lambda} \cdot \frac{bh}{e}\right)\right\}$$

$$= \frac{2\lambda e}{\pi b} \cdot \sin \frac{\pi b h}{\lambda e} \cdot \cos \frac{\mathrm{R}}{2} \cdot \left\{\cos \frac{\pi b h}{\lambda e} \cdot \cos \frac{\mathrm{R}}{2} + \sin \frac{\mathrm{R}}{2} \cdot \sin \frac{\pi b h}{\lambda e}\right\}$$

$$= \frac{2\lambda e}{\pi b} \cdot \sin \frac{\pi b h}{\lambda e} \cdot \cos \frac{\mathrm{R}}{2} \cdot \cos\left(\frac{\pi b h}{\lambda e} - \frac{\mathrm{R}}{2}\right).$$

And the intensity of light on the point of the retina, which is represented by the sum of the squares of these coefficients, is

$$\frac{4\lambda^2 e^2}{\pi^2 b^2} \cdot \sin^2 \frac{\pi b h}{\lambda e} \cdot \cos^2\left(\frac{\pi b h}{\lambda e} - \frac{\mathrm{R}}{2}\right).$$

For convenience, put $\frac{\pi b h}{\lambda e} = w$, and omit the constant factor $4h^2$; the expression becomes then

$$\left(\frac{\sin w}{w}\right)^2 . \cos^2\left(w - \frac{\mathrm{R}}{2}\right),$$

where it must be borne in mind that w is a multiple of the distance, of the point of the retina at which the intensity is sought, from the geometrical image of the point of light. It must also be borne in mind that this expression gives the intensity on that point of the retina produced by a single point of light, or a single line of light parallel to the bounding edge of the retarding plate.

The following Table contains the values of $\left(\frac{\sin w}{w}\right)^2 \cos^2\left(w - \frac{\mathrm{R}}{2}\right)$ for every 10° of w, and for every 60° of R. In computing them, w has been expressed in degrees: and the last figure of the numbers contained in the Table is the eighth decimal place.

Table of $\left(\frac{\sin w}{w}\right)^2 \cdot \cos^2\left(w - \frac{R}{2}\right)$.

Values of w.	Values of R.					
	0°	60°	120°	180°	240°	300°
− 175°	25	20	8	0	4	17
− 165	229	230	123	17	17	123
− 155	610	738	499	133	6	245
− 145	1050	1553	1286	515	12	280
− 135	1372	2560	2560	1372	184	184
− 125	1413	3527	4261	2882	767	33
− 115	1109	4168	6164	5101	2043	47
− 105	567	4231	7896	7896	4231	567
− 95	84	3618	9032	10912	7378	1964
− 85	104	2453	9216	13631	11282	4519
− 75	1111	1111	8294	15475	15475	8294
− 65	3473	148	6396	15970	19294	13046
− 55	7298	169	3962	14884	22013	18221
− 45	12346	1654	1654	12346	23038	23038
− 35	18020	4796	204	8835	22060	26652
− 25	23474	9402	217	5104	19175	28360
− 15	27778	14887	1994	1994	14886	27778
− 5	30154	20389	5427	231	9996	24959
+ 5	30154	24959	9996	231	5427	20388
+ 15	27778	27778	14887	1994	1994	14887
+ 25	23474	28360	19175	5104	217	9402
+ 35	18020	26651	22060	8835	204	4794
+ 45	12346	23038	23038	12346	1654	1654
+ 55	7298	18220	22013	14884	3962	169
+ 65	3473	13045	19294	15970	6396	148
+ 75	1111	8294	15475	15475	8294	1111
+ 85	104	4519	11282	13631	9216	2453
+ 95	84	1964	7378	10912	9032	3618
+ 105	567	567	4231	7896	7896	4231
+ 115	1109	47	2043	5101	6164	4168
+ 125	1413	33	767	2882	4261	3527
+ 135	1372	184	184	1372	2560	2560
+ 145	1050	279	12	515	1286	1553
+ 155	610	245	6	133	499	738
+ 165	229	123	16	17	123	230
+ 175	25	17	4	0	8	20

It has been deemed unnecessary to continue the Table beyond the values of w − 175° and + 175°, because the values of $\left(\frac{\sin w}{w}\right)^2$ become very small. The greatest maximum of this quantity occurs when $w = 0$; its value (expressing w in terms of the radius) is then $= 1$; the second maximum occurs when $w = \frac{3\pi}{2}$ nearly; its value is then $\left(\frac{2}{3\pi}\right)^2$ nearly $= \frac{1}{22}$ nearly; the amount of which will probably produce an inconsiderable influence on the expressions which we are now about to consider.

The curves in the annexed figure represent, by their ordinates, the values of $\left(\frac{\sin w}{w}\right)^2 \cdot \cos^2\left(w - \frac{R}{2}\right)$; the values of R being continued as far as 720°, in order to exhibit more distinctly to the eye the successive displacements of the principal bows of the curves. The ordinates, therefore, represent the intensity of light on different points of that small diffused image on the retina which is formed by the light coming from a single point, even when it is seen accurately in focus; the extreme breadth of the image represented in the figure corresponds to 360° of w, or is $= \frac{2\lambda e}{h}$.

If we express the area of each of the curves by summing the ordinates and dividing the sum by thirty-six, we find the following values:

R =	
$\overset{\circ}{0}$,	area is represented by 7234
60,	area is represented by 7055
120,	area is represented by 6696
180,	area is represented by 6517
240,	area is represented by 6696
300,	area is represented by 7055.

I shall proceed now to apply these numbers to the explanation of the phenomena in question.

Light is supposed to be incident on the eye from different points of a spectrum, formed in any way: the characteristic of the spectrum as concerned in the present investigation being, that the order of position of the different colours is the same as the order of the successive values of R.

First. Suppose the value of $\frac{2\lambda e}{h}$ to be small, at least in comparison with the distance between those points of the image of the spectrum in which R has changed by 360°.

1. Let $\frac{2\lambda e}{h}$ be exceedingly small. Since the same form of curve recurs for every change of 360° in R and not oftener, it is evident that the succession of bands (if there are any) in the visible image will depend on the changes of 360° in R. Our supposition, therefore, amounts to this; that the extent of the small diffused image is exceedingly less than the interval between the bands (if there are any). Here it is plain that the formation of the broad bands cannot depend on the inequalities of light in the narrow diffused image, but must depend on the quantity of light in the whole of each narrow diffused image considered as a total light from one point of the spectrum. Now the total light is equal for all points. For, as the intensity of light coming from one luminous point and falling on a point of the retina is represented by $\left(\frac{\sin w}{w}\right)^2 . \cos^2\left(w - \frac{R}{2}\right)$, the whole light coming from that luminous point is $\int_w \left(\frac{\sin w}{w}\right)^2 . \cos^2\left(w - \frac{R}{2}\right)$, the limits of the integral being $\pm \infty$. Now this definite integral is independent of R. For

$$\cos^2\left(w - \frac{R}{2}\right) = \frac{1}{2} - \frac{1}{2}\cos R + \cos R . \cos^2 w + \sin R . \cos w . \sin w,$$

and therefore

$$\left(\frac{\sin w}{w}\right)^2 \cos^2\left(w-\frac{R}{2}\right) = \left(\frac{1}{2}-\frac{1}{2}\cos R\right)\left(\frac{\sin w}{w}\right)^2 + \cos R\left(\frac{\cos w \,.\, \sin w}{w}\right)^2$$

$$+ \sin R \,.\, \frac{\cos w \,.\, \sin^3 w}{w^2};$$

and the whole intensity of light is represented by

$$\left(\frac{1}{2}-\frac{1}{2}\cos R\right)\int_w \left(\frac{\sin w}{w}\right)^2 + \cos R \int_w \left(\frac{\cos w \,.\, \sin w}{w}\right)^2 + \sin R \int_w \frac{\cos w \,.\, \sin^3 w}{w^2},$$

the limits of integration being $\pm\,\infty$. The last term, changing sign when w changes sign, evidently makes its definite integral $= 0$: the two former may be put in the shape

$$\left(\frac{1}{2}-\frac{1}{2}\cos R\right)\int_w \left(\frac{\sin w}{w}\right)^2 + \frac{1}{2}\cos R \int_{2w} \left(\frac{\sin 2w}{2w}\right)^2.$$

If $\int_w \left(\frac{\sin w}{w}\right)^2$ from $-\infty$ to $+\infty$ be $=$ S, then $\int_{2w} \left(\frac{\sin 2w}{2w}\right)^2$ from $-\infty$ to $+\infty$ is also $=$ S, and the expression becomes

$$\left(\frac{1}{2}-\frac{1}{2}\cos R\right)\cdot S + \frac{1}{2}\cos R \,.\, S \quad \text{or} \, \frac{1}{2}\,S,$$

which is independent of R. The total light, therefore, is independent of R, or is equal at all points; and therefore no bands are produced.

2. But if $\frac{2\lambda e}{h}$, though small, is not exceedingly small, the principal impression may be made upon the eye by the central patch of light from each source, included between the values $w = -180°$, $w = +180°$; while those parts of the light which extend beyond the central patch may be in fact aggregated with the central patches of light from the sources at a small distance on each side. And if the amounts in the central patches from different sources are unequal, while the whole amounts from the different sources are equal, it is evident that a bright central patch from one source may be combined with bright detached parts from another source, while a fainter central patch from that second source may be combined with an insignificant detached part from the first source, and thus the whole inequality of light may be double the inequality of the central patches. Now the amount of the light in the central patch, as we have found, is greatest, and represented by 7234, when $R = 0$ or $= 2n\pi$, and is least, and is represented by 6517, when $R = \pi$ or $= \overline{2n+1} \,.\, \pi$. The difference of these is $\frac{1}{10}$th of the whole; and therefore the difference of the whole light on each part of the retina, formed by combining the central patch formed by one source with the detached light formed by another source, will be nearly $\frac{1}{5}$th of the whole. This inequality of light is amply sufficient to form conspicuous bands.

The bars thus formed depend upon nothing but the changes in the value of R: it is wholly indifferent whether R increases or diminishes towards the side on which b

is considered positive; that is, it is indifferent whether the retarding plate is applied on the same side as the red end or the violet end of the spectrum. These appear to be the bars seen by Mr. TALBOT and myself when the spectrum was viewed in focus. They require that $\frac{2\lambda e}{h}$ be not large, that is, that the aperture of the pupil ($2h$), or the aperture of the telescope used be not very small; and that the changes of R be not very rapid; that is, that the plate of mica, &c. be thin. These circumstances held in my own experiment. I may add that the dark bands were not black, but merely dusky; as indicated by the numbers above.

Secondly. Suppose the value of $\frac{\lambda e}{h}$ to be comparable with the distance between those points of the image of the spectrum in which R has changed by 360°; for instance, suppose $\frac{\lambda e}{h}$ to be equal to that distance.

1. Let the red end of the external spectrum be on the same side as the retarding plate, that is, on the side on which b is considered positive. Then on the retina the violet end is on that side; or R increases towards the positive side. Let k be the ordinate measured from a fixed point on the retina to the centre of the diffused image of any colour (k being therefore a function of λ), and l the ordinate measured from the same fixed point to the point at which the intensity is to be ascertained; then $k + b = l$, or $b = l - k$, and the intensity produced by any one kind of light is represented by

$$\frac{\sin^2 \frac{\pi h}{\lambda e}(l-k)}{\left\{\frac{\pi h}{\lambda e}(l-k)\right\}^2} \cos^2\left(\frac{\pi h l}{\lambda e} - \frac{\pi h k}{\lambda e} - \frac{\mathrm{R}}{2}\right).$$

The sum of the intensities on one point of the retina produced by all the different kinds of light from the adjacent portions of the spectrum will be found by varying k in this expression, and adding together all the values so produced. Now if R increases when k increases (as occurs when the red end of the external spectrum is on the same side as the retarding plate), the last factor $\cos^2\left(\frac{\pi h l}{\lambda e} - \frac{\pi h k}{\lambda e} - \frac{\mathrm{R}}{2}\right)$ will undergo very great changes from the combined changes of $\frac{\pi h k}{\lambda e}$ and $\frac{\mathrm{R}}{2}$, whatever be the value of l, and the succession of values which it receives will not differ materially for different values of l; the first factor $\frac{\sin^2 \frac{\pi h}{\lambda e}(l-k)}{\left\{\frac{\pi h}{\lambda e}(l-k)\right\}^2}$ will also undergo great changes, but nearly the same for different values of l: and in consequence the aggregate of all the values for different values of k, exhibiting the total intensity of light upon the point l, will be nearly the same.

This aggregation will be represented graphically by supposing the second curve in the diagram to be moved towards the right hand, the third to be moved further to the right, &c., and taking the sum of the ordinates of the various curves which are then placed vertically one below the other; it is clear that the large ordinates of one curve will be added to the small ones of another, so as to produce in every part an approximate mean value. If we perform the same operation numerically, combining the last number of the first column in the Table with the last but three in the second column, the last but six in the third column, and so on, to the twelfth column (observing that the numbers in the columns recur after the sixth, or that they may be supposed to recur before the first), and if we remark that by adding the numbers from twelve columns we do in fact combine the intensities from all the diffused images that are in any degree superposed; and if we then divide by twelve, we find the following numbers to represent the intensities:

6884, 6882, 6881, 6879, 6875, 6872, 6870, 6868, 6867, 6868, &c.,

the greatest number being 6884 and the least 6868. It is plain that no bands will be visible here.

2. Let the violet end of the external spectrum be on the same side as the retarding plate. The same algebraic expression holds as in the other case, but there is this important difference in the interpretation, that R (which increases towards the violet end of the spectrum) is greatest in the spectrum on the retina on that side on which k is negative, or when k increases R diminishes. And if $\frac{\lambda e}{h}$ be equal to the change of k corresponding to a change of 2π in R, or if $\frac{\pi h}{\lambda e} \cdot \frac{\lambda e}{h}$ (or π) be equal to the change of $\frac{\pi h k}{\lambda e}$ corresponding to a change of 2π in R, or of π in $\frac{R}{2}$; then the changes of $\frac{\pi h k}{\lambda e}$ and of $\frac{R}{2}$ exactly destroy each other; $\frac{\pi h k}{\lambda e} + \frac{R}{2} =$ a constant C, and the whole intensity of light on a given point will be found by aggregating all the quantities

$$\frac{\sin^2 \frac{\pi h}{\lambda e}(l-k)}{\left\{\frac{\pi h}{\lambda e}(l-k)\right\}^2} \cdot \cos^2\left(\frac{\pi h l}{\lambda e} - C\right),$$

giving different values to k. As the second factor is independent of k, and as the changes of the first caused by changing the values of k will be similar (to the extent to which the light is sensible), whatever be the value of l, it follows that the aggregate will be expressed by the form $B \cos^2\left(\frac{\pi h l}{\lambda e} - C\right)$. This expression denotes that there will be light of all degrees of intensity from the brightest B to zero or total darkness; and that the whole of the changes will recur (or the dark bands will recur) when $\frac{\pi h l}{\lambda e}$ has changed by 2π, or when l has changed by $\frac{2\lambda e}{h}$.

This combination will be represented graphically by drawing back the second curve of the diagram by 30°, the third by 60°, and so on; and taking the sum of the ordinates which are then vertically one below the other. It is evident that the ordinates *zero* correspond throughout. If we perform the same operation numerically, combining the first number of the first column in the table with the fourth number of the second column, the seventh number of the third column, &c., and if we then divide the sum by twelve, we find the following numbers:

13646, 12829, 11295, 9227, 6875, 4524, 2456, 921, 105, 105, 921, &c.;

the greatest number being 13646, or a little greater, and the least being 0.

It is evident that these numbers denote the formation of most vivid black and bright bands.

The case which we have taken $\left(\text{when } \frac{\lambda e}{h} \text{ is exactly equal to the change of } k \text{ corresponding to a change of } 2\pi \text{ in R}\right)$ is the most favourable for the production of bands; but it will easily be understood that, in consequence of the small extent of the diffused image, conspicuous bands may be formed when the change of k corresponding to a change of 2π in R is sensibly greater or less than $\frac{\lambda e}{h}$.

The interval between the bands is $\frac{2\lambda e}{h}$, and is, therefore, usually small. They will, however, be made broader by making h small, that is, by contracting the aperture of the pupil, or by using a telescope with a limited object-glass. The value of R changes through 2π with no greater change in the quality of light than that produced by passing from one part of the spectrum to another part distant (on the retina) by $\frac{2\lambda e}{h}$, and therefore the retarding plate must be comparatively thick.

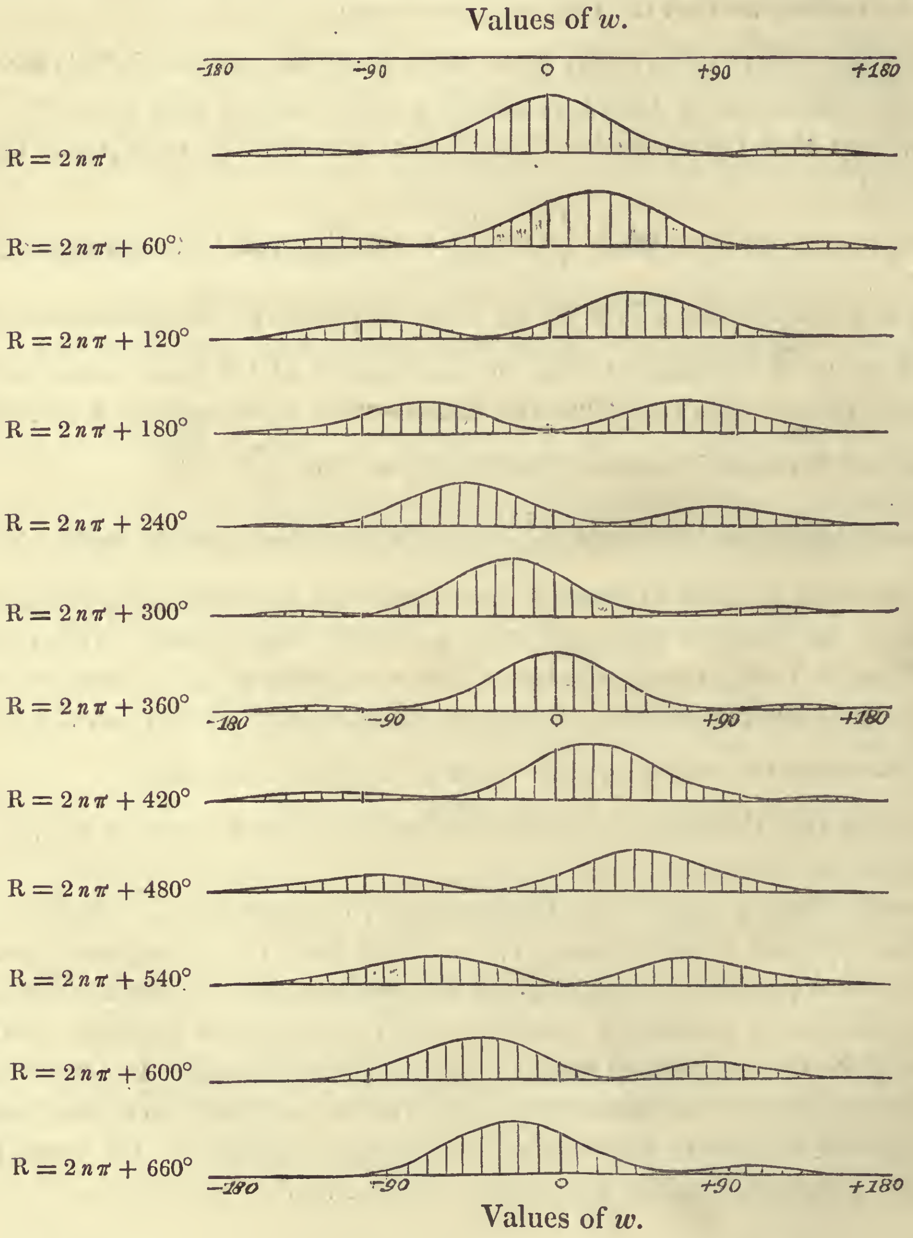
It is evident that these are the bands seen by Sir David Brewster when the spectrum was viewed in focus.

The investigation, as regards the explanation of the formation or non-formation of bands under different circumstances, when a thin plate of a transparent medium is placed to cover a portion of the pupil, and the eye is turned to view a spectrum, may now be considered as sufficiently complete, and (I conceive) as perfectly satisfactory. Some change in the expressions would undoubtedly be produced by introducing the consideration of the circular form of the pupil, the inclined position of the transparent plate adopted by Sir David Brewster in some experiments, &c., but none, I apprehend, which would at all affect the general explanation.

G. B. Airy.

Royal Observatory, Greenwich,
Oct. 23, 1840.

Curves representing by their ordinates the values of $\left(\frac{\sin w}{w}\right)^2 \cos^2\left(w - \frac{R}{2}\right)$.



II. *Contributions to Terrestrial Magnetism.—No.* II.

By Lieut.-Colonel Edward Sabine, *R.A. V.P.R.S.*

Received February 4,—Read February 11, 1841.

§ 3. *Captain* Belcher's *Observations on the west Coast of America, and the adjacent Islands.* § 4. *New Determination of the Magnetic Elements at Otaheite.*

§ 3. *Captain* Belcher's *Observations on the west Coast of America, and the adjacent Islands.*

THE observations, an account of which is now presented to the Society, were made by Captain Edward Belcher, R.N. and the officers of H.M.S. Sulphur, employed in the years 1837 to 1840 in surveying portions of the west coast of North America. The account has been drawn up from the official reports transmitted to the Admiralty, and placed in my hands by the Hydrographer, Captain Beaufort. The services which Captain Belcher and his officers may be expected to render to magnetical science are not terminated, as the Sulphur has not yet returned to England: but the portion now communicated forms a complete series, comprising the results of their labours up to the period of their final departure from the coast of America. The zeal, perseverance, and care with which these have been conducted will be best appreciated by an examination of the details.

Horizontal Intensity.—Captain Belcher joined the Sulphur at Panama in the spring of 1837, receiving from his predecessor, Captain Beechey, a six-inch inclination instrument by Robinson, and several needles for experiments on the horizontal intensity by the method of vibration. He had taken with him from England a nine-inch altitude and azimuth instrument with attached needles, and a five-inch theodolite, both by Cary, which he had employed in former surveys in determining declinations, and had had reason to confide in. Before his departure from Panama on a surveying cruize, which might furnish opportunities of magnetic observation at several stations on the west coast of America between Behring Strait and Peru, the times of vibration of the horizontal needles, eleven in number, were carefully observed, in March 1837, at a convenient spot near the ruins of the Convent of St. Francisco; and these observations were repeated at the same spot on the return of the Sulphur to Panama in October 1838, after an absence of eighteen months. By comparing the times of vibration in March 1837 and October 1838, as given in the subjoined Table, it will be seen that the magnetism of several of the needles had greatly altered in the interim:

Table I.—Comparison of Intensity Needles at Panama in March 1837, and October 1838. Commencing arcs 40°.

Periods.	Designation of the Needles.										
	1.	3.	4.	5.	6.	7.	8.	9.	11.	12.	13.
	s	s	s	s	s	s	s	s	s	s	s
March 1837	625·1	600·9	775·7	472·7	512·6	532·7	470·4	434·3	453·9	373·8	375·2
October 1838....	673·1	608·6	864·0	475·2	514·6	536·8	471·4	439·5	475·8	403·7	395·5

Nos. 5. 6. 7. and 8. had each undergone a small and comparatively insignificant loss of force; but the changes sustained by the other needles, especially by Nos. 1. 4. 11. 12. and 13, were too great to justify the deduction of results, either from a mean of the times of vibration at the two periods, or on the principle of an uniform loss corresponding to equal intervals of time. Unfortunately, Nos. 1. 3. and 4. were amongst those which had been most frequently employed at the stations visited in the cruize; and as an attentive examination of the observations made with them has not furnished, as it sometimes does, the means of discovering when and in what manner the alterations of magnetism took place, I have not attempted to draw from these observations conclusions which could not be otherwise than unsatisfactory. Happily several of the stations were revisited in 1839, when the apparatus was in more perfect order, and the observers having improved by practice, the results are such as leave no other regret for the failure on the first occasion, than what is due to the loss of time and pains. At those stations of the first cruize which were not subsequently visited, we may still derive results from the observations with Nos. 5. 6. 7. and 8, which, though not entitled to equal confidence in respect to precision with the determinations made in the subsequent voyage, are nevertheless well deserving of regard and record. It may be convenient, however, in the relation, to invert the order of succession, and to commence with an account of the second, or principal *magnetic*, voyage.

Having occasion to remain at Panama and its neighbourhood for some months after the needles had been vibrated as above noticed in October 1838, Captain Belcher repeated the observations with the needles specified in the next Table a third time, at the same place as before, on the 16th of March 1839. The times of vibration inserted in this Table were on both occasions in arcs commencing with 40°, which had been the uniform practice with all the needles at the stations visited in the first voyage. Having heard from Captain Beaufort of the attention which Captain Belcher and his officers were giving to magnetic observations, and having been permitted to examine the reports of the observations of the first voyage which had reached the Admiralty on the 1st of January 1839, I wrote to Captain Belcher to recommend that in future he should commence the vibrations at an arc of 20°. This letter was received in Panama early in March, and a double series of observations were made in consequence on the 16th of March, one series commencing with 40° to compare with those of October 1838, and a second commencing with 20°, to correspond with all the observations which should be subsequently made. The times of vibration commencing with 40° are those inserted in this Table; the series commencing with 20° will be found in its due succession.

Table II.—Comparison of the Intensity Needles at Panama in October 1838, and March 1839*.

Periods.	Designation of the Needles.						
	5.	7.	8.	9.	11.	12.	13.
	s	s	s	s	s	s	s
October 1838....	475·2	536·8	471·4	439·5	475·8	403·7	395·5
March 1839	476·3	536·1	471·4	438·2	474·8	404·6	394·5

This comparison having shown that the seven needles specified in the Table were in a steady magnetic state, Captain **Belcher** despatched Nos. 7. 8. and 9. to England, to have their times of vibration observed there, and to be returned to him on the coast of California; purposing by this means to attach his series of relative determinations to the great body of results obtained by other observers. The needles were received by me in August 1839, and were vibrated on the 12th and 13th of August at a suitable place near Woolwich, where I also observed the dip at the same time. They reached Captain **Belcher** again in the following November at Mazatlan. In the meantime the Sulphur had quitted Panama, having on board Nos. 5. 11. 12. and 13, and had visited successively Cocos Island, Oahu one of the Sandwich Islands, Kodiack and Sitka on the north-west coast of America, Fort Vancouver and Baker's Bay in Columbia River, Port Bodega, San Francisco, Monterey, S^{ta} Barbara, San Pedro, San Diego, San Quentin, San Bartholomew, Magdalena Bay, and St. Lucas Bay, arriving at Mazatlan in November. At each of the above-named stations the times of vibration of one or more of the needles were observed, and occasionally of all the four. On the arrival of the three needles which had been sent to England, their times of vibration were observed, in comparison with the others, first at Mazatlan, and a few days afterwards at San Blas, where, more time being available, the comparison was repeated on two different days, viz. on the 6th and 19th of December. From San Blas

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* Nos. 1. 3. and 4. are not included in this Table, because the observations on the 16th of March 1839 showed that they were still losing magnetism, and they were not therefore subsequently employed. No. 6. is also omitted, and the cause is explained by a memorandum of Captain Belcher's to the following effect: "No. 6. kept well during the first twenty-one months, and changed suddenly during an excursion to Conchagua in November and December 1838. It was vibrated on the 20th and 22nd of November, and gave consistent results: on the 27th it was carried on horseback up the Amapola hill, 3000 feet above the sea, and on its return on the 22nd of December was found to have lost magnetism equivalent to upwards of 12 seconds in 510 seconds. The surface rock on the Amapola hill was so highly magnetic that no satisfactory observations could be obtained there with the needle." An examination of the subsequent observations of No. 6, compared with those of the other needles, shows that its magnetism was unsteady for many months after this accident, becoming gradually weaker. I have not, therefore, taken into account the observations with this needle, as they do not yield independent results of equal value with the other needles, and there are enough consistent determinations without them.

the Sulphur proceeded to the islands of Socorro and Clarion, to Martins Island one of the Marquesas, and lastly to Bow Island, with which the stations on the west coast of America and its adjacent islands may be considered to have terminated.

To have made this series of magnetic determinations thoroughly complete, the needles should have been taken back to Panama, and their times of vibration should have been re-examined there at the close of the operations; but this proceeding did not consist with other duties. We are, therefore, without that direct evidence of the steady magnetism of the needles, subsequently to the observations at Panama in March 1839, which might have been furnished thereby; but where so many needles are employed, evidence of scarcely inferior weight may be obtained by their intercomparison; especially at stations where the opportunities of observation are favourable, and the probable error of the result with each needle is further diminished by its being derived from repetitions on different days. The observations at Mazatlan and San Blas, on the return of Nos. 7. 8. and 9. from England, furnish one good occasion of this nature; and we may take as a second the observations at Martins Island, being the last station at which they were repeated on different days.

If we divide the squares of the times of vibration of the several needles at Panama by the squares of their times of vibration at Mazatlan, we obtain quotients, which, if the needles were unchanged relatively to each other in the interim, should be identical; or as nearly so as the ordinary errors of observation permit, including therein the diurnal and irregular variations of the magnetic force itself. The times of vibration at San Blas and Panama, similarly treated, supply a similar comparison, in which, however, the quotients will differ in absolute value from the preceding ones, inasmuch as the horizontal intensity of the earth's magnetism is not precisely the same at Mazatlan and San Blas[*]; but the degree of accordance *with each other* of the second series of quotients will furnish, as in the former case, the required evidence; which is of greater weight in the instance of San Blas than in that of Mazatlan, because the times of vibration at San Blas were derived from observations on two different days, and at Mazatlan from those of a single day only.

TABLE III.—Intercomparison of the Intensity Needles at Mazatlan and San Blas.

	Quotients.								
	5.	7.	8.	9.	11.	12.	13.	Mean, omitting No. 8.	Weights[†].
Panama and Mazatlan	0·930	0·930	0·923	0·924	0·931	0·928	0·925	0·928	2
Panama and San Blas	0·951	......	0·950	0·956	0·959	0·962	0·956	0·958	3
Difference of each needle from the mean	+ ·002	+ ·002	– ·005	– ·004	+ ·003	·000	– ·003	Panama and Mazatlan.	
	+ ·001	....	– ·008	– ·002	+ ·001	+ ·004	– ·002	Panama and San Blas.	
Mean difference..............	+ ·001	+ ·002	– ·007	– ·003	+ ·002	+ ·002	– ·002	Allowing the respective weights.	

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[*] The quotients are, in fact, in the two cases, the respective values of the horizontal intensity at Mazatlan and San Blas relatively to the force at Panama taken as unity.

[†] These are arbitrary weights, assigned according to the number of *days* employed in each comparison.

No. 8. is the only needle which presents a difference from the other needles exceeding in value a five-hundredth part of the time of vibration. If therefore the magnetism of Nos. 5. 7. 9. 11. 12. and 13. suffered any change in the interval comprehended by the comparison, the alteration must have taken place to an equivalent amount in each of the needles: a coincidence the less probable, because they had all been previously exposed to greater extremes of natural temperature than were experienced at the stations visited between Panama and San Blas.

The difference which No. 8. presents from the mean of the other needles is equivalent to $1^{s}\cdot7$ in 480 seconds, its time of vibration, or to a proportional loss of magnetism. It does happen occasionally that results with the same needle corrected for temperature, will differ from each other to this amount, in cases when the subsequent return to the original time of vibration manifests that the magnetism of the needle has undergone no change, or at least no permanent change: but as the difference exceeds the probable error of observation, as will be presently shown, and as moreover nearly the same difference appears at all the subsequent stations, when the results with No. 8. are compared with those of the other needles, I have regarded it as an indication of an actual loss of magnetism sustained by No. 8., at some time between the observations at Panama in March 1839, and those at Mazatlan in November of the same year, rendering that needle less fit than the others for intermediate deductions: and I have allowed for this loss at all the stations subsequently to San Blas, by deducting ·00312 from the logarithm of the square of its time of vibration.

The observations at Mazatlan with No. 7, for the comparison of its time of vibration with that of the other needles, were made on the 29th of November. On the 30th of November and 2nd of December, this needle was employed in experiments to ascertain the effect on its time of vibration of differences of temperature, by vibrating it in air of the natural temperature, and in air heated by means of boiling water. No memorandum has accompanied the observations of any accident having occurred, either in putting the needle away after the conclusion of the observations of the 29th, or before the commencement of those of the 30th, but a comparison of the results on the three days manifests that the magnetism of the needle sustained an alteration in that interval:

November 29. Corrected time of vibration 550·9 seconds
November 30. Corrected time of vibration 556·7 seconds
December 2. Corrected time of vibration 555·9 seconds.

The observations at the next station, San Blas, confirm this direct evidence of a change, as is seen in the following statement, which shows the quotients of No. 7. at Mazatlan and San Blas compared with the mean quotients of the other needles.

	1839.	Needle 7.	Mean of the Needles.	Difference.
Mazatlan	Nov. 29	0·934	0·933	+ 0·001
	Nov. 30 and Dec. 2	0·916	0·933	− 0·017
San Blas	Dec. 6 and Dec. 19	0·940	0·959	− 0·019

I have allowed, therefore, a loss of magnetism in this needle equivalent to 5^{s}·4 in 556 seconds at all the stations subsequent to Mazatlan, and have accordingly deducted ·00848 from the logarithms of the squares of the times of vibration of No. 7. at those stations.

We now proceed to a similar general intercomparison of the needles at Martins Island.

Table IV.—Intercomparison of the Intensity Needles at Martins Island.

	Quotients.							
	5.	7.	8.	9.	11.	12.	13.	Mean.
Panama and Martins Island	0·975	0·983	0·981	0·981	0·981	0·983	0·978	0·980
Differences from the Mean	− ·005	+ ·003	+ ·001	+ ·001	+ ·001	+ ·003	− ·002	

Here, with the exception of No. 5, which appears to have sustained a slight loss of magnetism, the agreement of the quotients shows the general steadiness of the magnetic condition of the needles. In the case of No. 5. the difference does not exceed the limits of *occasional* error of observation; the results with this needle subsequently to San Blas may not deserve to be regarded as *fully* equal in value to those of the other needles; but the amount of error in the final determinations hazarded by retaining its results independent of correction is insignificant.

As this Table includes the whole interval between the observations at Panama in 1839, and those at Martins Island in 1840, we may regard it as substantiating the general steady magnetic condition of the needles in the whole of that interval, with the exception of the changes already noticed in Nos. 7. and 8, which have been traced to the period of their occurrence, and their amount examined and allowed for.

The times of vibration at all the stations visited subsequently to March 1839 were taken in arcs commencing with 20°: the time of the chronometer was noted at every 10th vibration during 300, and the mean time of 200 vibrations derived from ten partial results, i. e. from the 0th and 200th, the 10th and 210th, the 20th and 220th.

No satisfactory experiments having been made to determine the individual coefficients in the correction for temperature of these needles, I have taken an arbitrary coefficient for that purpose, being the arithmetical mean of the coefficients experimentally ascertained for the twenty-nine needles specified in the following list:—

·000165	HANSTEEN			Phil. Trans., 1828. Art. I.
·00019	LENZ	Needle .	1	Mém. de l'Acad. Imp. de St. Pétersbourg, 1824.
·00025	LENZ	Needle .	5	
·00026	LENZ	Needle .	2	
·00029	LENZ	Needle .	4	
·00016	LLOYD	Needle	L (4)	Trans. R. I. A., vol. xvii.
·000254	LLOYD	Needle	L (*a*)	Brit. Assoc. Report, 1835.
·000248	LLOYD	Needle	L (*b*)	

·00022	Sabine	Needle	L (3)	Account of the Euphrates Expedition. App.
·00027	Sabine	Needle	L (4)	
·00038	Sabine	Needle	E (1)	
·00041	Sabine	Needle	E (4)	
·000068	Sabine	Needle	FitzRoy's.	Voyage of the Beagle. App.
·00030	Sabine	Needle	(3)	Phil. Trans., 1840. Art. IV.
·000055	Ross and Sabine.	Needle	R L (4)	Brit. Assoc. Report, 1838.
·000436	Bache	Needle	(1)	Trans. Am. Phil. Soc., 1836.
·000423	Bache	Needle	(2)	
·000277	Bache	Needle	(3)	
·000117	Bache	Needle	(A)	
·000052	Bache	Needle	(C)	
·000357	Bache	Needle	(3 B)	
·000359	Christie	Needle	(1)	Phil. Trans., 1836. Art. XIX.
·000302	Christie	Needle	(3)	
·000177	Christie	Needle	Lozenge	
·000227	Christie	Needle	II	
·00036	Duperrey	Needle	I	Mém de L' Acad. Roy. de Bruxelles, tom. xiii.
·000625	Quetelet	Needle	II	
·00020	Forbes	Needle	I	Trans. R. S. Edin., vol. xiv.
·00013	Forbes	Needle	Flat	
·00026				

Whence,

$$T' = T\,[1 + {\cdot}00026\,(60^\circ - t)],$$

in which T is the time of vibration at any station, t the actual temperature in degrees of Fahr., and T′ is the equivalent time at the temperature of 60°.

The application of this correction gives the "corrected time" in Table V. In the few cases where the rate of the chronometer exceeded an insignificant amount, a correction for the rate is also included in the "corrected time," and a memorandum of the rate itself is inserted in the column of remarks.

Table V. contains an abstract of the observations at the different stations with the needles which have been specified: it includes every observation recorded to have been made with these needles between the 16th of March 1839 at Panama, and the 22nd of March 1840 at Bow Island, except, 1st, two incomplete observations, one with No. 11. at Fort Vancouver, and one with No. 13. at San Francisco, in which either the vibration was interrupted, or the needles came to rest, before the usual and requisite number of vibrations had been made; and 2nd, some observations at Tepic in the neighbourhood of San Blas and at Mazatlan, in which the needles, for the sake of experiment, were vibrated in air artificially heated, or alternately in the sun and in the shade.

Table V.—Abstract of observations with the Intensity Needles between the 16th of March 1839, and the 22nd of March 1840.

Station.	1839.	Needle.	Time.	Therm.	Corrected Time.	Remarks.
			s	°	s	
Panama	March 16	5	472·6	84	469·4	
	March 16	5	472·3	85		
	March 16	7	535·2	80	531·3	
	March 16	7	534·1	88		
	March 16	8	470·2	88	466·2	
	March 16	8	468·9	86		
	March 16	9	437·8	80	433·8	
	March 16	9	435·5	90		
	March 16	11	473·3	87	470·0	
	March 16	12	403·2	87	400·4	
	March 16	13	393·2	88	390·4	
Cocos Island	April 8	5	466·4	83	463·9	
	April 8	5	466·9	82		
	April 8	11	465·6	78	463·5	
	April 8	11	465·8	78		
	April 8	12	397·4	77	395·6	
	April 8	12	397·5	78		
	April 8	13	390·0	78	388·2	
	April 8	13	390·0	78		
Oahu	June 4	5	512·1	80	510·3	
	June 4	5	513·8	90		
	June 9	5	514·6	89		
	June 9	5	515·2	91		
	June 8	11	515·5	79	512·6	
	June 8	11	515·8	88		
	June 8	12	439·8	87	436·5	
	June 8	12	439·4	88		
	June 8	13	430·0	87	427·5	
	June 8	13	430·9	87		
Kodiack	July 7	5	688·5	79	685·1	
Sitka	July 18	5	730·3	61	730·1	
	July 18	11	730·4	62	730·1	
	July 18	12	624·0	64	623·4	
	July 18	13	611·2	67	610·1	
Woolwich	Aug. 12 and 13	7	766·4	66	766·2	Observed by Lieut.-Colonel Sabine.
	Aug. 12 and 13	7	766·8	66		
	Aug. 12 and 13	7	767·6	61		
	Aug. 12 and 13	8	670·7	68	669·7	
	Aug. 12 and 13	8	669·9	68		
	Aug. 12 and 13	8	671·7	63		
	Aug. 12 and 13	9	627·3	65	626·9	
	Aug. 12 and 13	9	627·8	61		
	Aug. 12 and 13	9	627·0	62		
Fort Vancouver	Aug. 13	5	616·8	59	616·9	Chron. G. $8^{s}\cdot5$
	Aug. 13	5	619·8	72		Chron. G. $8^{s}\cdot5$
	Aug. 14	5	616·9	65		Chron. G. $8^{s}\cdot5$
	Aug. 14	5	617·1	69		Chron. G. $8^{s}\cdot5$
	Aug. 14	5	619·8	69		Chron. G. $8^{s}\cdot5$
	Aug. 15	5	617·5	63		Chron. G. $8^{s}\cdot5$
	Aug. 15	5	617·2	63		Chron. G. $8^{s}\cdot5$
	Aug. 15	11	620·7	66	619·8	Chron. G. $8^{s}\cdot5$
	Aug. 13	12	527·5	82	526·9	Chron. G. $8^{s}\cdot5$
	Aug. 15	12	530·1	66		Chron. G. $8^{s}\cdot5$
	Aug. 13	13	515·6	81	515·2	Chron. G. $8^{s}\cdot5$
	Aug. 15	13	518·5	66		Chron. G. $8^{s}\cdot5$
Baker's Bay	September 13	5	622·4	60	622·6	
	September 13	5	624·4	70		

Table V. (Continued.)

Station.	1839.	Needle.	Time.	Therm.	Corrected Time.	Remarks.
			s	°	s	
Bodega	September 25	5	560·4	63	559·9	
San Francisco	September 30	5	556·8	76	555·0	
	September 30	5	557·2	71		
	September 30	11	558·7	64	558·1	
	September 30	12	476·4	62	476·2	
Monterey	October 5	5	549·7	65	549·0	
S^{ta} Barbara	October 10	5	539·0	77	536·6	
San Pedro	October 12	5	538·4	74	536·5	
San Diego	October 17	5	528·2	70	526·8	
San Quentin	October 24	5	515·3	77	513·1	
San Bartholomew	October 29	5	503·3	74	501·5	
Magdalena Bay	November 1	5	490·1	73	488·5	
Bay of St. Lucas	November 21	5	487·9	83	485·0	
Mazatlan	November 28	5	488·1	71	486·7	
	November 28	11	488·5	72	487·0	
	November 28	12	416·9	72	415·6	
	November 28	13	407·3	72	406·0	
	November 29	7	552·8	73	550·9	
	November 30	7	557·7	72	556·4	
	November 30	7	559·4	73		
	December 2	7	558·2	76		
	November 29	8	485·1	72	485·3	
	November 30	8	487·8	73		
	December 2	8	488·1	76		
	November 29	9	452·8	73	451·3	
	November 30	9	453·3	71		
	December 2	9	452·6	76		
San Blas	December 6	5	482·3	88	479·5	
	December 19	5	482·5	78		
	December 6	7	550·3	85	548·4	
	December 19	7	551·8	78		
	December 19	7	551·8	78		
	December 6	8	480·4	87	478·4	
	December 19	8	481·9	77		
	December 6	9	445·9	86	443·6	
	December 19	9	446·2	76		
	December 6	11	482·7	86	480·0	
	December 19	11	483·1	81		
	December 6	12	410·5	81	408·3	
	December 19	12	411·0	84		
	December 6	13	401·3	81	399·2	
	December 19	13	402·0	84		
Socorro Island	December 26	5	478·1	83	474·9	
	December 26	5	477·7	85		
Clarion Island	December 29	5	481·4	84	478·7	
	December 29	5	482·0	85		
	1840.					
Martins Island (Marquesas)	January 23	5	477·5	88	475·4	
	January 23	5	477·9	89		
	January 28	5	479·2	85		
	January 28	5	480·1	86		
	January 29	5	479·0	86		
	January 25	7	545·0	86	541·0	
	January 27	7	544·6	88		
	January 25	8	476·0	87	472·5	
	January 27	8	475·7	83		
	January 25	9	441·5	90	438·1	
	January 27	9	440·8	86		

TABLE V. (Continued.)

Station.	1840.	Needle.	Time.	Therm.	Corrected Time.		Remarks.
			s	°	s		
Martins Island (Marquesas	January 25	11	477·8	90	474·6		
	January 27	11	478·1	88			
	January 25	12	406·9	87	403·6		
	January 27	12	406·2	88			
	January 25	13	397·7	88	394·8		
	January 27	13	397·4	88			
Bow Island..........	February 6 ..	5	484·1	87	480·5	480·9	
	February 6 ..	5	483·6	87			Village.
	February 22 ..	5	483·4	82	480·1		
	February 22 ..	5	483·5	89			Observatory.
	February 25 ..	5	485·6	90	482·1		
	February 25 ..	5	485·8	90			
	February 26 ..	5	485·0	81			S.E. Cocoanut Grove.
	February 26 ..	5	485·3	88	481·7		Chron. L. 5·3.
	February 26 ..	5	484·9	88			West Cocoanut Grove.
	February 27 ..	5	484·1	85	480·8		
	February 27 ..	5	484·0	83			West Rock.
	February 28 ..	5	484·4	86			
	February 29 ..	5	483·5	87	479·8		
	February 29 ..	5	482·8	81			Village.
	March 20 and 21	5	484·3	83	481·4		
	March 22......	7	550·7	75	548·9		Mean of 22 sets. Observatory. Chron. G. 5·8.
	March 22......	7	551·1	76			
	March 22......	7	551·9	76			
	March 22......	8	480·8	76	479·4		
	March 22......	8	482·6	77			
	March 22......	8	481·5	76			
	March 22......	8	481·2	78			
	March 22......	9	446·4	79	443·9		
	March 22......	9	446·6	81			
	March 22......	9	446·4	86			
	March 22......	11	482·8	87	479·5		
	March 22......	11	483·1	82			
	March 22......	11	482·6	87			
	March 22......	12	411·0	86	408·2		
	March 22......	12	411·1	87			
	March 22......	12	411·2	88			
	March 22......	13	402·2	87	399·1		
	March 22......	13	401·9	87			
	March 22......	13	401·8	85			

By means of the observations in the preceding Table, we obtain the ratio of the horizontal intensity at each station to that at each of the others specified in the Table. The *absolute* horizontal intensity was nowhere observed, because Captain BELCHER was not furnished with an instrument for the purpose, and no such instrument has yet been carried to any of the stations which he visited. For the purpose of expressing the ratio determined by the observations, we may select any one of the stations as a base-station, and assign an arbitrary value for the horizontal intensity at that station. I have chosen Panama, and have made the horizontal intensity there = 1000, because that is the value which it bears at Panama in M. GAUSS's theoretical map of this element*, and those who may desire it will thus be enabled to

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* Atlas des Erdmagnetismus nach den elementen der theorie entworfen, Plate XI.; and Taylor's Scientific Memoirs, vol. ii. Plate XXII.

compare *directly* the horizontal intensities observed by Captain Belcher, with the computed intensities of M. Gauss's theory. Table VI. exhibits the observed values.

Table VI.—Observed Values of the Horizontal Intensity.

Station.	Needle.	Corrected Time.	Horizontal Intensity.		Number of		
					Needles.	Days.	Observations.
		s.					
Panama	5	469·4	1000	1000	7	1	11
	7	531·3	1000				
	8	466·2	1000				
	9	433·8	1000				
	11	470·0	1000				
	12	400·4	1000				
	13	390·4	1000				
Cocos Island	5	463·9	1024	1022	4	1	8
	11	463·5	1028				
	12	395·6	1025				
	13	388·2	1012				
Oahu	5	510·3	846	841	4	3	10
	11	512·6	841				
	12	436·5	841				
	13	427·5	834				
Kodiack	5	685·1	470	470	1	1	1
Sitka	5	730·1	413	412	4	1	4
	11	730·1	414				
	12	623·4	413				
	13	610·1	410				
Woolwich	7	766·2	480·9	480	2	2	6
	9	626·9	478·8				
Fort Vancouver	5	616·9	579	576	4	3	12
	11	619·8	575				
	12	526·9	577				
	13	515·2	574				
Baker's Bay	5	622·6	569	569	1	1	2
Bodega	5	559·9	703	703	1	1	1
San Francisco	5	555·0	716	711	3	1	4
	11	558·1	709				
	12	475·4	707				
Monterey	5	549·0	731	731	1	1	1
S^{ta} Barbara	5	536·6	765	765	1	1	1
San Pedro	5	536·5	766	766	1	1	1
San Diego	5	526·8	794	794	1	1	1
San Quentin	5	513·1	837	837	1	1	1
San Bartholomew	5	501·5	876	876	1	1	1
Magdalena Bay	5	488·5	924	924	1	1	1
Bay of St. Lucas	5	485·0	937	937	1	1	1
Mazatlan	5	486·7	930	928	6	3	11
	7	550·9	930				
	9	451·3	924				
	11	487·0	931				
	12	415·6	928				
	13	407·3	925				
San Blas	5	479·5	959	958	5	2	10
	7	548·4	957				
	9	443·6	956				
	11	480·0	959				
	12	408·3	962				
	13	400·2	956				
Socorro Island	5	474·9	977	977	1	1	2

Table VI. (Continued.)

Station.	Needle.	Corrected Time.	Horizontal Intensity.		Number of		
					Needles.	Days.	Observations.
		s					
Clarion Island....	5	478·7	962	962	1	1	2
Martins Island	5	475·4	975	980	7	5	17
	7	541·0	983				
	8	472·5	981				
	9	438·1	981				
	11	474·6	981				
	12	403·6	984				
	13	394·8	978				
Bow Island....	5	480·9	953	957	7	8	55
	7	548·9	955				
	8	479·4	953				
	9	443·9	955				
	11	479·5	961				
	12	408·2	962				
	13	399·1	957				

In this Table each needle has been given an equal influence on the mean result, without reference to the number of observations made with it. Where the observations do not afford certain and independent evidence of the unchanged state of each of the needles in respect to magnetism, weights assigned from other considerations must necessarily be arbitrary and uncertain. For example, at the last station in the Table, Bow Island, thirty-six observations were made with No. 5, and not more than three or four with each of the other six needles. But we have already seen, on intercomparison, reason to suspect that No. 5. may have sustained a slight loss of magnetism at the station preceding Bow Island, and it is the only needle in which any change of the kind is indicated subsequently to the general comparison at San Blas. Whilst, therefore, on the one hand, we might not be justified, without more clear and decided evidence, in altogether setting aside the result with No. 5, so on the other hand we should not obtain the most probable final deduction, by giving to that result a weight, in comparison with that of each of the other needles, proportioned to the number of observations, and resting on the probable error of a single observation,—apart from changes of magnetism in the needle itself.

We will now revert to the stations visited in the first voyage which were not subsequently revisited, and at which the values of the horizontal intensity may be derived by means of Nos. 5, 6, 7, or 8. Table VII. contains an abstract of the observations with these needles, in all of which the times of vibration were obtained in arcs commencing with 40°. The column entitled "Corrected Times," shows the mean time of vibration reduced to a standard temperature of 60°. The arithmetical mean of the times at Panama in March 1837, October 1839, and March 1839, has been taken as the approximate time of vibration at Panama throughout the interval; and the ratio of the horizontal intensity at the other stations has been computed accordingly, as shown in Table VIII. In this Table, as in Table VI., each needle has been given an equal influence on the general mean, without reference to the number of observations which were made with it.

Table VII.—Abstract of observations with the Intensity Needles, Nos. 5, 6, 7, and 8, at the undermentioned Stations.

Station.	1837.	Needle.	Time.	Therm.	Corrected Time.	Remarks.
			s	°	s	
Panama	March 10	5	471·7	79	470·4	
	March 10	5	473·7	79		
	March 10	6	512·5	80	510·4	
	March 10	6	512·6	72		
	March 12	7	532·7	71	531·2	
	March 12	8	470·4	76	468·4	
Port Etches	August 28	5	736·7	50	738·7	Chron. L. 8s·6.
	August 28	6	792·4	50	794·5	Chron. L. 8s·6.
	1838.					
Acapulco	January 17	6	508·1	88	504·5	
	January 17	7	530·7	91	526·5	
Callao	June 20 and 21	5	487·1	68	486·1	Mean of 80 observations.
	June 25	6	525·0	79	522·4	
	June 27	7	546·1	72	544·6	
	June 27	8	483·8	70	482·6	
Puna Island (Guayaquil)	September 23	5	476·1	78	473·7	Mean of 90 observations.
	September 23	6	514·3	79	510·5	Pt Spanola.
	September 23	6	515·3	78		
	September 17	6	517·5	99		Pa Arena.
	September 17	6	517·4	99		
	September 19	6	517·1	93		Pt Barranca.
	September 20	6	509·7	86		Town of Puna.
	September 20	6	510·8	88		
Panama	October 28	5	475·3	82?	472·6	Therm. not recorded.
	October 28	6	514·6	82?	511·7	
	October 28	7	536·8	82?	533·7	
	October 28	8	471·4	82?	468·7	
	1839.					
Panama	March 16	5	476·4	84	473·2	
	March 16	5	476·1	85		
	March 16	7	536·8	80	532·9	
	March 16	7	535·3	88		
	March 16	8	471·8	88	468·1	
	March 16	8	471·0	86		

Table VIII.—Observed values of the Horizontal Intensity.

Station.	Needle.	Corrected Time.	Horizontal Intensity. Panama = 1000.		Remarks.
		s			
Port Etches	5	738·7	408	411	
	6	794·5	414		
Acapulco	6	504·5	1026	1024	
	7	526·5	1023		
Callao	5	486·1	943	950	
	6	522·4	957		
	7	544·6	957		
	8	482·6	942		
Puna Island	5	473·7	993	998	
	6	510·5	1002		

At four of Captain Belcher's stations in North America, he was preceded in observations of the horizontal intensity by Mr. David Douglas, who visited California and the Columbia River in the years 1830 to 1833. It may not be out of place to examine here the degree of accordance in the results obtained by the two experimenters at the four stations, Fort Vancouver, San Francisco, Monterey, and S^{ta} Barbara; and the comparison will be found instructive. Mr. Douglas's observations were made with two pairs of needles, which, before his departure for America, were vibrated in the environs of London, at intervals of several months, with consistent results. One pair of needles, numbered 3. and 4, were returned to England from San Francisco in 1831 to have their magnetic state re-examined: they arrived safely, and were vibrated in 1836, when, on a comparison with their rates in 1828 and 1829, No. 3. was found to have slightly gained, and No. 4. to have slightly lost magnetism; the consequence, probably, of their having been kept in constant contact with each other (No. 4. being a more powerful magnet than No. 3.), except when used in observation, when both needles were always vibrated, and their combined results considered as one determination. The mean of the times of vibration of these needles in 1828–1829, and in 1836, consequently furnishes a satisfactory London rate for the intervening years. The second pair of needles, numbered 5. and 6, were in Mr. Douglas's possession at the period of his untimely death at Owhyhee in 1834, as his letters contain the notice of observations made with them at the summit of Mowna Kaah, and in the crater of Kiraueah, but they have not been found amongst his effects sent to England. The steadiness of this pair of needles can only be judged of, therefore, by their accordance everywhere with the results of Nos. 3. and 4.

Mr. Douglas's papers are in the Colonial Office; an account of his magnetic observations, which I drew up at the request of Lord Glenelg, then Secretary of State for the Colonies, was presented by His Lordship to the Royal Society, and was read in May 1837, but was not printed. The results of the horizontal intensity which will be now referred to, are taken from that account; they are also immediately deducible from the Table of the *total* intensities and dips observed by Mr. Douglas in North America, published in 1838 in my memoir on the magnetic Intensity of the Earth, in the Seventh Report of the British Association for the Advancement of Science: they are as follows:

Horizontal intensity: London = 1000.

	Nos. 5. and 6.		Nos. 3. and 4.	
Fort Vancouver	1830	1238;	1830	1220
San Francisco	1831 and 1833	1517;	1831	1511
Monterey	1831 and 1832	1566;	1831	1542
S^{ta} Barbara	1831	1636;	Not observed.	

or, if we regard London and Woolwich as identical in respect to the value of the horizontal intensity, and express this value by 480, which Captain Belcher's observations give as its ratio in August 1839 to 1000 at Panama, we have Mr. Douglas's determinations in immediate comparison with those of Captain Belcher as follows:

	Douglas, 1830, 1833.		Belcher, 1839.
Fort Vancouver	Nos. 5. and 6. 594 ;	Nos. 3. and 4. 586 . . .	576
San Francisco	Nos. 5. and 6. 728 ;	Nos. 3. and 4. 726 . . .	711
Monterey .	Nos. 5. and 6. 752 ;	Nos. 3. and 4. 740 . . .	731
S^{ta} Barbara .	Nos. 5. and 6. 785 ;	Nos. 3. and 4. not obs^{d}.	765

It has been assumed in this comparison that the horizontal intensity in London had the same representative value in the years to which Mr. Douglas's observations correspond as in the year to which Captain Belcher's correspond. But we know that the secular decrease of the dip in London causes a corresponding increase in the horizontal magnetic force at that station, and we are sufficiently acquainted with the average amount of the yearly diminution of the dip to introduce it as an element of calculation. Mr. Douglas's observations with Nos. 5. and 6. correspond to November 1828, when those needles were vibrated in London; and with Nos. 3. and 4. to January 1832, being the middle time between the observations before his departure, and those made with the same needles in June 1836, when returned to England. Captain Belcher's determination corresponds to August 1839, when his needles were vibrated at Woolwich. Taking the annual decrease of the dip in London in the interval at 2′·6[*], and the value of the horizontal intensity at 480·0 in August 1839, we have its value 472·7 in January 1832, and 469·7 in November 1828; omitting the consideration of the secular change of the intensity itself, of which we know extremely little at present. Adopting these values of the horizontal intensity at the respective epochs, the American determinations become as follows, being all relative to 480 in August 1839.

	Douglas.		Belcher.
Fort Vancouver	Nos. 5. and 6. 581 ;	Nos. 3. and 4. 577 . .	576
San Francisco .	Nos. 5. and 6. 712 ;	Nos. 3. and 4. 714 . .	711
Monterey . .	Nos. 5. and 6. 736 ;	Nos. 3. and 4. 729 . .	731
S^{ta} Barbara . .	Nos. 5. and 6. 768 ;	Nos. 3. and 4. not obs^{d}.	765

There are still involved in the comparison the secular change of dip at the American stations, and the secular changes of the total intensity both there and in London: none of these are known sufficiently to make them proper elements of calculation, though we have reason to believe that the effect of each of these causes on the comparative numbers would be considerably less than that of the decrease of dip in London[†]. But enough has been said to show the large proportion which in such

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[*] Eighth Report of the British Association, pp. 62. 66.

[†] By comparing Captain Belcher's observed inclinations with M. Hansteen's map of that element in 1780, we perceive that the inclination is annually increasing on the west coast of North America, but the amount of the annual change is apparently considerably less than that of the annual decrease in Europe:—with an annual increase of inclination we should have a decrease in the horizontal intensity; this corresponds with the remaining differences between the determinations of Captain Belcher and Mr. Douglas.

determinations, the "corrections for epoch" and the uncertainties consequent thereupon, bear to the probable amount of the combined instrumental differences and observation errors of independent careful observers; and, consequently, the importance of the *synchronism* so much insisted upon, of the observations which are to concur in determining the magnetic state of the globe at the present period.

Inclination.—The inclinations contained in Table IX. were observed with a six-inch instrument by Robinson, with a needle by the same artist. The poles of the needle were reversed on every occasion, and the observations were repeated in the eight different positions of the circle and needle. Usually five repetitions were made in each position, and the arcs at both ends of the needle being read, the inclinations in the Table are generally a mean of eighty readings.

Table IX.—Observations of the Inclination.

Station.	Date.	Poles.		Inclination.		Remarks.
		Direct.	Reversed.			
	1837.					
Panama	March 8......	30° 58′·8	32° 53′·2	31° 51′·0	31° 51′·9	Near the Ruins of the Convent of San Francisco.
	March 8......	30 54·6	32 51·0	31 52·8		
Magnetic Island	March 20......	30 27·5	32 01·5	31 14·5	31 11·9	
	March 22......	30 08·5	32 10	31 09·3		
Oahu	July 20 and 24.	40 43·4	42 27·6		41 35·1	
Port Etches......	August 26 and 28	76 02·2	76 03·6		76 02·9	
Sitka	September 20 ..	75 47·5	75 53·8	75 50·6	75 51·5	Observation Island.
	September 22 ..	75 49·1	75 55·5	75 52·3		
San Blas......	December 21 ..	44 20·2	46 30·2	45 25·2	45 24·3	Palm Island.
	December 22 ..	44 12·8	46 34·0	45 23·4		
	December 22 ..	43 59·2	45 26·3		44 42·7	Beach near the Arsenal.
San Francisco....	December 29 .	61 18·9	62 28·7		61 53·8	Yerba Buena.
	1838.					
Acapulco	January 18	36 51·2	39 03·7		37 57·4	Near the Castle.
Realejo	March 19 and 21	33 13·6	36 00·2		34 36·9	Cardon Island.
Cocos Island	April 6	22 29·2	24 36·9		23 33·2	
Callao..........	June 19 and 23	−4 20·9	−8 07·7		− 6 14·3	Plaza de los Muertes.
Puna Island .. (Guayaquil) ..	September 17 ..	8 09·4	9 11·1		8 40·2	Punta Arena.
	September 20 ..	8 58·1	10 15·8		9 36·9	Town of Puna.
	September 21 ..	8 22·3	10 27·9	9 25·1	9 08·0	P^{t} Spanola.
	September 24 ..	8 10·5	9 31·2	8 50·9		
	1839.					
Cocos Island	March 7......	21 26·6	24 24·9		22 55·7	
Oahu	June 1	40 24·9	42 15·0	41 19·9	41 16·8	
	June 6	40 12·5	42 09·7	41 11·1		
	June 7	40 36·2	42 02·6	41 19·4		
Kodiack	July 7	72 28·1	72 57·6		72 42·9	Near P^{t} Greville.
Sitka	July 19	75 43·4	75 54·9		75 49·1	
Baker's Bay	September 13 ..	69 27·5	69 26·2		69 26·9	
Fort Vancouver	August 12	69 15·9	69 23·6	69 19·8	69 22·2	
	August 12	69 20·2	69 28·9	69 24·6		
Port Bodega	September 25 ..	62 50·6	62 56·2		62 53·4	On the Sandy Neck.
San Francisco .	September 30 ..	61 50·2	62 16·0	62 03·1	62 05·0	
	September 30 ..	61 37·1	62 16·0	61 56·6		
	October 1	61 52·3	62 25·7	62 09·0		
	October 1	61 52·3	62 22·9	62 07·6		
	October 2	61 50·2	62 25·7	62 08·0		

Table IX. (Continued.)

Station.	Date.	Poles.		Inclination.		Remarks.
		Direct.	Reversed.			
	1839.					
Monterey	October 5	60° 59′·6	61° 07′·5		61° 03′·6	
Sta Barbara......	October 10	58 53·2	58 55		58 54·1	
San Pedro	October 12	58 14·7	58 28·1		58 21·4	Fossil Island.
San Diego	October 17	56 49·5	57 22·7		57 06·1	
San Quentin	October 24	54 01·9	54 58		54 29·9	
San Bartholomew	October 29	51 12·7	52 09·4		51 41·0	
Magdalena Bay	November 1 ..	45 22·1	47 40·5	46 31·3	46 34·0	
	November 1 ..	45 37·5	47 36·0	46 36·7		
San Lucas Bay ..	November 21 ..	44 37·5	46 41·0		45 39·3	
Mazatlan	November 29 ..	45 42·7	47 30·0	46 36·3	46 38·5	
	December 2 ..	45 43·6	47 37·7	46 40·7		
San Blas......	December 6 ..	43 34·5	45 22·8	44 28·7	44 32·5	Beach.
	December 19 ..	43 40·8	45 31·7	44 36·3		
	December 8 ..	43 39·9	45 39·4	44 39·6	44 38·3	Tepic.
	December 8 ..	43 38·4	45 39·4	44 38·9		
	December 10 ..	43 42·7	45 29·9	44 36·3		
	December 10 ..	43 39·9	45 29·9	44 34·9		
	December 11 ..	43 42·7	45 40·6	44 41·6		
Socorro Island ..	December 26 ..	39 31·8	41 55·5		40 43·7	
Clarion Island ..	December 29 ..	35 21·4	38 44·6		37 03·0	
	1840.					
Martins Island	January 23	−13 20·2	−14 53·6	−14 06·9	−14 06·0	Anna Maria Bay.
	January 25	−13 12·3	−14 50·6	−14 01·5		
	January 27	−13 22·5	−14 56·5	−14 09·5		
Bow Island....	February 6....	−30 13·9	−30 19·7	−30 16·8	−30 19·0	Entrance Village.
	February 29....	−30 18·0	−30 24·4	−30 21·2		
	February 25....	−30 07·1	−31 08·6		−30 37·9	S.E. point.
	February 26....	−30 00	−30 38·2		−30 19·1	S.W. point.
	February 27....	−29 43·4	−30 08·2		−29 55·8	Western extreme.
	February 22....	−30 04·8	−30 13·4	−30 09·1	−30 15·3	Observatory.
	March 14......	−29 44·6	−30 37·5	−30 11·1		
	March 20......	−29 27·7	−31 11·1	−30 19·4		
	March 21......	−29 28·7	−31 14·8	−30 21·7		

Total Intensity.—The values of the total intensity of Captain Belcher's American stations are deducible from the values of the horizontal intensity in Table VI., and of the inclination in Table IX., by the formula

$$I' = I \cdot \frac{h' \sec i'}{h \sec i},$$

where i, h, and I are the inclination, horizontal and total intensities at Woolwich, and i', h', and I′ the values of the same at any other station. Regarding Woolwich and London as identical, we have I = 1·372, the conventional number by which the total intensity in London is usually expressed. The values of the total intensity in Table X. have been thus computed.

Declination.—The declinations were observed with a nine-inch altitude and azimuth instrument by Cary, having a four-inch magnetic needle attached, which was read at both extremities. Each determination is stated to be the mean result of several observations, both of the true meridian, and of the magnetic direction.

Table X.—General Table of the Results of Captain Belcher's Magnetic Observations on the West Coast of America, and the adjacent Islands. The longitudes in this Table are east of Greenwich; the declinations east; the values of the horizontal intensity are expressed relatively to 1000 at Panama; and the total intensities relatively to 1·372 at London.

Station.	Date.	Latitude.	Longitude.	Declination.	Inclination.	Intensity.		Remarks.
						Horizontal.	Total.	
Port Etches	1837	+ 60° 21′	213° 19′	31° 38′·5	+ 76° 02′·9	411	1·728	
Kodiack	1839	+ 57 20	207 09	26 43·5	+ 72 42·9	470	1·603	
Sitka	1837	+ 57 03	224 34	27 42·0	+ 75 51·5			
Sitka	1839	+ 57 03	224 38	29 32·5	+ 75 49·1	412	1·704	
Baker's Bay	1839	+ 46 17	235 58	19 11·0	+ 69 26·9	569	1·643	
Fort Vancouver	1839	+ 45 37	237 24	19 22·0	+ 69 22·2	576	1·657	
Port Bodega	1839	+ 38 18	236 58	15 20·0	+ 62 53·4	703	1·563	
San Francisco	1837	+ 37 48	237 37	15 20·0	+ 61 53·8			
San Francisco	1839	+ 37 48	237 37	15 20·0	+ 62 05·8	711	1·540	
Monterey	1839	+ 36 36	238 07	14 13·0	+ 61 03·6	731	1·531	
S^{ta} Barbara	1839	+ 34 24	240 19	13 28·0	+ 58 54·1	765	1·501	
San Pedro	1839	+ 33 43	241 45	13 08·5	+ 58 21·4	766	1·480	
San Diego	1839	+ 32 41	242 47	12 20·6	+ 57 06·1	794	1·482	
San Quentin	1839	+ 30 22	244 02	12 06	+ 54 29·9	837	1·461	
San Bartholomew	1839	+ 27 40	245 07	10 46	+ 51 41·0	876	1·432	
Magdalena Bay	1839	+ 24 38	247 53	9 15	+ 46 34·0	924	1·362	
Mazatlan	1839	+ 23 11	253 36	9 24	+ 46 38·5	928	1·370	
San Lucas Bay	1839	+ 22 52	250 07	8 37·5	+ 45 39·3	937	1·359	
San Blas	1837	+ 21 32	254 44	8 34	+ 45 24·3	....	....	Palm Island.
San Blas	1839	+ 21 32	254 44	9 00	+ 44 32·5	958	1·362	Beach.
Oahu Island	1837	+ 21 17	202 00	10 39·5	+ 41 35·1			
Oahu Island	1839	+ 21 17	202 00	....	+ 41 16·8	841	1·134	
Socorro Island	1839	+ 18 43	249 06	....	+ 40 43·7	977	1·307	
Clarion Island	1839	+ 18 21	245 19	....	+ 37 03·0	962	1·222	
Acapulco	1838	+ 16 50	260 05	8 23	+ 37 57·4	1024	1·316	
Realejo	1838	+ 12 28	272 48	7 53·5	+ 34 36·9			
Panama	1837	+ 8 37	280 31	....	+ 31 51·9	1000	1·193	
Magnetic Island	1837	+ 8 04	278 15	7 37·5	+ 31 11·9			
Cocos Island	1838	+ 5 53	272 58	8 24	+ 23 33·2			
Cocos Island	1839	+ 5 53	272 58	....	+ 22 55·7	1022	1·125	
Puna Island	1838	− 2 47	280 05	8 56	+ 9 08	998	1·024	
Martins Island	1840	− 8 56	220 20	....	− 14 06·0	980	1·024	
Callao	1838	− 12 04	282 52	....	− 6 14·3	950	0·968	
Bow Island	1840	− 18 05	219 07	....	− 30 16·0	957	1·123	

The declination was observed by Captain Belcher at Socorro, Clarion, Martin, and Bow Islands, but the record of the observations has not yet been sent home: an early opportunity will be taken of supplying this deficiency in the Table when the observations shall have been received.

§ 4. *New Determination of the Magnetic Elements at Otaheite.*

In M. Gauss's "General Theory of Terrestrial Magnetism*," there is a note to the following effect:—"Otaheite is a station of the highest importance for the future improvement of the magnetic elements: the difference between the two determinations of intensity made there by different observers, viz. Erman 1·172 in 1830, and FitzRoy 1·017 in 1835, is much greater than can with any degree of probability be attributed to yearly changes, and considerably exceeds the greatest difference between the computed and observed intensities at the eighty-six stations at which the theory has been compared with observations."

The importance which M. Gauss attached to a more exact determination of the magnetic elements at Otaheite was communicated to Captain Belcher in a letter from Captain Beaufort, which also conveyed to him permission from the Admiralty to touch at that Island on his homeward voyage. Quitting Bow Island in March 1840, Captain Belcher arrived at Otaheite in April, and made there the observations contained in Tables XI. and XIII., of which the results are given in Tables XII. and XIV.

Table XI.—Observations with the Intensity Needles at Otaheite.

1840.	Needle.	Time.	Therm.	Corrected Time.	1840.	Needle.	Time.	Therm.	Corrected Time.
		s	°	s			s	°	s
April 4	5	483·9	87	479·5	May 6	8	478·9	88	476·3
April 4	5	483·7	87		May 6	8	479·1	86	
April 4	5	484·2	88		May 7	8	479·6	77	
May 3	5	482·8	90		May 7	8	478·6	76	
May 3	5	482·1	90		May 6	9	443·3	85	441·6
May 4	5	480·8	74		May 6	9	443·4	84	
May 4	5	482·3	86		May 7	9	444·2	75	
May 4	5	482·2	88		May 6	11	480·6	72	478·0
May 4	5	482·2	89		May 6	11	480·4	80	
May 5	5	481·6	76		May 6	11	480·7	89	
May 6	5	481·3	73		May 6	12	408·8	72	406·8
April 17	5	480·1	77	478·2†	May 6	12	409·1	85	
April 17	5	480·6	77		May 6	12	409·2	88	
May 6	7	547·9	94	544·2	May 6	13	399·8	73	398·2
May 6	7	547·9	94		May 6	13	400·7	86	
May 6	7	548·4	77		May 6	13	400·6	86	

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* Resultate für 1838, I.

† Observed at Papeite: the other observations with No. 5, as well as all the observations with other needles, were made at Point Venus.

Table XII.—Observed values of the Horizontal Intensity at Otaheite.

	Needle.	Corrected Time.	Horizontal Intensity.	
		s		
Point Venus	5	479·5	958·6	965·4
	7	544·2	971·8	
	8	476·3	965·1	
	9	441·6	965·1	
	11	478·0	966·8	
	12	406·8	968·9	
	13	398·2	961·4	
Papeite	5	478·2	963·8	

Table XIII.—Observations of the Inclination at Otaheite.

1840.	Poles		Inclination.		Place of Observation.
	Direct.	Reversed.			
May 2	29° 48'·4	30° 54'·0	30° 21'·2	30° 17'·7	Point Venus.
May 2	29 46·7	30 52·7	30 19·7		
May 4	29 45·0	30 46·2	30 15·6		
May 5	29 48·0	30 43·9	30 15·9		
May 6	29 50·4	30 41·3	30 51·9		
April 11	26 17·0	28 01·0	........27 09		Papeite.

Table XIV.—General Results of Captain Belcher's magnetic observations at Otaheite.

Place of Observation.	Declination.	Inclination.	Intensity.	
			Horizontal.	Total.
Point Venus	6° 30' E.	+ 30° 17'·7	965	1·133
Papeite	not observed.	+ 27 09	964	1·098

Horizontal Intensity.—Captain BELCHER's determination of the horizontal intensity falls between those of MM. ERMAN and FITZROY.

Horizontal Intensity according to	ERMAN	1005
	FITZROY . . .	874
	BELCHER . . .	965

But the differences of the three determinations far exceed in amount the errors, either instrumental or of observation, to which such experienced observers, provided with needles of steady magnetism, are liable; neither should we be justified in ascribing them to fluctuations in the magnetic force itself, especially at a station where the dip is small, and its variations have comparatively little influence on the horizontal intensity. The only known cause to which we can, with any degree of probability, attribute them, is to *station error*, in an island of which the basis is a volcanic rock: and to the same cause we must also refer the difference of three degrees in the inclination observed by Captain BELCHER at Papeite and Point Venus, stations within seven geographical miles of each other, with an instrument of which the probable error, as derived from the observations at Point Venus, does not exceed the same number of minutes. In such localities it is well known to observers, that disturbing influences, producing differences as great as and even greater than those above stated, frequently occur at places not many yards apart. We know the intensity observations at Point Venus, with Captain BELCHER's seven needles, to have been made at one spot, and it is worth while, therefore, to examine the degree of accordance with each other which their results present. Assuming the horizontal intensities determined by each of these needles to have an equal and independent value, and taking, therefore, the arithmetical mean as their most probable result, we have the errors of the needles as follows:—

No. 5.	Horizontal Intensity 958·6;	Error − 6·8
No. 7.	Horizontal Intensity 971·8;	Error + 6·4
No. 8.	Horizontal Intensity 965·1;	Error − 0·3
No. 9.	Horizontal Intensity 965·1;	Error − 0·3
No. 11.	Horizontal Intensity 966·8;	Error + 1·4
No. 12.	Horizontal Intensity 968·9;	Error + 3·5
No. 13.	Horizontal Intensity 961·4;	Error − 4·0
	Mean . . . . . 965·4	

Consequently the mean error ε_2* is

$$\varepsilon_2 = \sqrt{\frac{117{\cdot}59}{6}} = 4{\cdot}425,$$

whence the *probable error*, *r*, of a determination with one needle is

$$r = \varepsilon_2 \, . \, \varrho \, \sqrt{2} = 0{\cdot}674489 \, \varepsilon_2 = 3{\cdot}0,$$

and the probable error of a determination with seven needles $\frac{r}{\sqrt{7}} = 1{\cdot}1$.

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* ENCKE, Astron. Jahr. 1834, and Scientific Memoirs, vol. ii. Art. X.

M. Erman's determination differs 40·0, and Captain FitzRoy's 91·0 from Captain Belcher's; the former thirteen times, and the latter thirty times, the probable error of a determination with a single needle of steady magnetism, *where the spot of observation is the same.*

We have considered Captain Belcher's seven needles as giving *equal* and *independent* results for the ratio of the horizontal intensity at Otaheite and Panama; but the result with No. 8. is not strictly an *independent* one, inasmuch as at San Blas that needle received a small correction assigned from its comparison with the others; and the claim of the result of No. 5. to be considered as of *equal* value with that of each of the remaining needles is impaired by the probability of that needle having sustained a slight loss of magnetism at or before Martins Island (page 16.). If, therefore, we were to exclude the results of Nos. 5. and 8, and to derive the horizontal intensity at Otaheite from the five needles, which we may consider as of strictly independent and equal authority, we should have as their mean 966·8, with a probable error of 1·2, and the probable error of a determination with a single needle 2·6. It is true that the number of partial results from which this amount of the probable error is derived is small; but the probability of its being an approximately just representation of the errors of instrument and observation in this method, with needles of steady magnetism, is strengthened, if we examine in the same manner the results with the same five needles at the four stations preceding Otaheite; by so doing we obtain the probable error of a single needle from each as follows:—

Bow Island . . .	2·2
Martins Island . .	1·6
San Blas	1·5
Mazatlan	2·2

The integers in these quantities represent hundredth parts of the space comprised between two adjacent lines of horizontal intensity in M. Gauss's theoretical map of that element.

Uncertainties in respect to the magnetism of needles need no longer prove a source of vexatious anxiety and embarrassment even to travelling observers; with the simple apparatus described by M. Weber* the magnetic state of a needle may be examined at pleasure, and its magnetism may be altogether eliminated in the result.

With this advantage, however,—and it will be scarcely less valued by the confidence it creates whilst the observations are in progress, than by the independency it confers on their results,—and with a probable error of observation of extremely small amount, the magnetic traveller has still two serious sources of error to contend with: 1st, the values of the magnetic elements which he determines may not be *mean values*, by reason of the periodical or irregular fluctuations of the magnetic direction or

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* Resultate für 1837, and Scientific Memoirs, vol. ii. Art. IV.

force possibly prevailing at the time of observation; and, 2nd, they may not be true measures of the magnetic elements corresponding to the geographical position in which the observations are made, by reason of those local disturbing magnetic influences which are included under the name of station error.

For the first of these sources of error a remedy is presented, whenever the observations can be made in connexion with those of a fixed magnetic observatory, situated within such distance that the magnetic elements are subject to the same periodical and irregular variations. The particular advantage possessed by the absolute determinations of the fixed observatories,—that of being *mean* values of the quantities sought,—may thus be indefinitely extended.

Against the more formidable evil of *station error* the connexion with a fixed observatory affords the magnetic traveller no security; nor can it furnish him with a correction,—for to error from this source the absolute determinations of fixed observatories are themselves no less liable; and no continuance, or frequency of repetition at the spot itself, will lead to its discovery or assign its correction. The magnetic survey of the British Islands, and more especially of its Scottish and Irish portions, has shown that such disturbances are not confined to localities, which, like Otaheite, consist chiefly of volcanic rocks, but may exist unsuspected and productive of error of serious amount, wherever the igneous rocks rise through, or approach the superficial soil. It is this source of error which presents a practical difficulty to the determination of the elements of the theory of terrestrial magnetism from exact observations at a few chosen positions on the globe. The remedy is to be found in the combination of fixed magnetic observatories and magnetic surveys; the observations of the survey being based on and executed in concert with the regular observations of a fixed observatory; the country surveyed being also sufficiently extensive to neutralise district anomalies, as well as those of a more local nature. The observations of the survey, corrected to mean determinations by their connexion with those of an observatory, and combined in the manner described by Mr. Lloyd in the third section of the Survey of the British Islands, will furnish in their turn the correction for the station error, if any, of the fixed observatory.

Total Intensity.—From the value of the total intensity at Otaheite as now determined by Captain Belcher, we learn that the *southerly inflection* of the isodynamic lines, in and about the meridian of the Society Islands,—which was pointed out as one of the characteristic features of the general configuration of those lines in the southern hemisphere*,—is even more strongly marked in the latitude of those islands than I had ventured to draw it, under the circumstances of the unusual discordance in the only observations which we then possessed.

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* Seventh Report of the British Association, p. 73.

Declination.—I have collected in the subjoined Table all the recorded observations of the declination at Matavai Bay with which I am acquainted, from the earliest discovery of the island to the present time.

TABLE XV.—Declination observed at Matavai Bay, Otaheite.

Year.	Month.	Observer.	Declination. (East.)	Year.	Month.	Observer.	Declination. (East.)
1765	June........	Byron	5° 00′	1792	January	Vancouver	6° 12′
1767	July........	Wallis	5 36	1823	May........	Duperrey........	6 40
1769	June........	Cook	4 46	1824	March	Kotzebue........	6 50
1773	August......	Wales and Bayley	5 40	1826	April	Beechey	7 33
1774	April and May	Wales and Bayley	5 46	1835	November ..	FitzRoy	7 34
1777	December ..	Cook	5 34	1840	May........	Belcher	6 30

A first glance at these observations shows that the easterly declination has been increasing at Otaheite from the time of the first discovery of that island. It is scarcely probable that the progression has been strictly uniform throughout the whole period, but the deficiency of determinations in the years that form the middle portion of the interval, renders the data that we possess unsuitable for deducing the variation in the rate of the secular change; and we must be content with that approximate representation which may be given by an uniform rate. Assuming, therefore, the change of declination to be proportional to the time, I have computed by the method of least squares from the data contained in the Table, the following formula for the declination δ at Otaheite:

$$\delta = 6^\circ\ 11'{\cdot}85 + 1'{\cdot}656\, t,$$

t being the interval of time elapsed since January 1, 1800, expressed in terms of a year.

The declinations computed according to this formula, and the differences from the observed declinations, are as follows:—

	Computed.	Differences.		Computed.	Differences.
1765	5° 15′	+ 15′	1792	5° 59′	− 13′
1767	5 18	− 18	1823	6 51	+ 11
1769	5 21	+ 35	1824	6 52	+ 2
1773	5 28	+ 12	1826	6 55	− 38
1774	5 29	− 17	1835	7 12	− 22
1777	5 35	+ 1	1840	7 19	+ 49

It will be seen that the discordances with each other of the observations of recent date are as great, and even greater, than those of the earlier observers; which ought to be an indication that the larger discrepances are occasioned rather by local disturbing influences than by errors of observation. The probable error of a single determination, as resulting from the tabulated differences, would be about fifteen minutes.

Inclination.—The observations of the inclination made in the voyages of Captain Cook are entitled to much consideration, in respect both to the experience and skill of the observers, and to the goodness of their instruments. The English dipping needles of that period were made with much more care, and were much superior, especially in their axles, to those subsequently supplied to the government expeditions up to a very recent date. I have therefore placed in the subjoined Table the observations of Mr. Bayley in 1773, 1774, and 1777; and have combined them with the determinations of recent observers, for the purpose of exhibiting the secular change of the inclination at Otaheite, as deduced from the most unexceptionable data that we possess.

Table XVI.—Observations of the Inclination at Point Venus, Otaheite.

Year.	Month.	Observer.	Inclination.
			° ′
1773	August....	Bayley ..	−29 43
1774	May......	Bayley..	−29 59
1777	December..	Bayley ..	−29 47
1823	May......	Duperrey	−30 03
1830	September	Erman ..	−30 29·5
1835	November	FitzRoy	−30 14·5
1840	April	Belcher	−30 17·7

Whence, by the method of least squares, we obtain for the inclination the formula

$$I = -30^\circ\ 01'\!\cdot\!1 - 0'\!\cdot\!447\,t,$$

t being, as before, the interval of time elapsed since January 1, 1800, expressed in parts of a year. The inclination in January 1840, computed by the formula, is $-30^\circ\ 19'\!\cdot\!0$.

No observation recorded to have been made at Otaheite by Captain Belcher or his officers, has been omitted in the foregoing account: the manuscript records of the observations on the west coast of America and the adjacent islands, as well as those at Otaheite, are deposited in the Hydrographic Office of the Admiralty.

III. *On Ground Gru, or Ice formed, under peculiar circumstances, at the bottom of Running Water.* *By* JAMES FARQUHARSON, *LL.D. F.R.S., Minister of the Parish of Alford.*

Received March 11,—Read March 25, 1841.

IN a paper of mine on Ground Gru, or ice formed at the bottom of running water, which was honoured with a place in the Philosophical Transactions*, I had inferred, from a great many conditions attending a remarkable occurrence of the phenomenon in the rivers Don and Leochal, in the beginning of January 1835, as well as from its occurring only when the air is at the time quite clear, that it is caused, when the water has gone down in temperature to the freezing point, by the bottom of the water being cooled to a still lower temperature, in the same manner as the surface of the dry land, under a clear sky, is cooled down below the temperature of the air, as first demonstrated by the experiments of Dr. WELLS.

As the accuracy of the conclusion at which I arrived respecting the question has been controverted, I respectfully request the Royal Society to permit me to present to them brief notices of some recent occurrences of ground gru, in the same rivers to which I formerly referred, the conditions of which seem to me strongly to confirm the accuracy of the views I presented regarding the cause of the phenomenon; and also to answer some of the objections which have been brought against it.

Cold weather commenced on the 20th December 1840 (on which night the thermometer went down to 31°), and continued with frost every night, yet never below 26°, and with frost also through most of the day, till the 31st of the same month. By the 26th December, surface ice in considerable quantity was formed on the edges of the small river Leochal, and the temperature of the water was down to the freezing point. Down to the evening of the 28th the weather was cloudy, and there was no appearance in the river of anything resembling ground gru; but on that night the sky suddenly became clear; and before the morning of the 29th, the bottoms of all the rapids of the little river were thickly coated by the ground gru. The gru disappeared as speedily as it had formed, when, on the 29th, a close cloud, depositing slight showers of snow, again covered the whole sky, and continued till the temperature of the day and night rose above freezing.

In comparison with this, I would refer to a series of frosty days from the 1st to the 11th of February 1841, with a temperature the same as from the 22nd to 31st December, 1840, never descending below 26°. The water of the river descended to the freezing

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* Part II. for 1835, p. 329.

temperature, and surface ice was formed in large quantity on the edges of both the Leochal and the Don. A dense cloud covered the sky during the eleven days and nights, and no ground gru appeared in the rivers.

A remarkable occurrence of ground gru took place in both the rivers from the evening of the 7th to the morning of the 9th January 1841, with a completely clear sky during the time. The thermometer was at 2° below zero on the night of the 7th, at 9° at mid-day on the 8th, and at 7° below zero on the night of the 8th. I examined particularly the state of the Don, during this extreme and clear frost, before it abated on the morning of the 9th. The bottom of the river was everywhere coated by an immense quantity of ground gru, excepting where it was partially shaded by bridges, or lofty banks close to the stream. In the partially shaded places the bottom was clear of gru. Thus this remarkable formation of ground gru took place under exactly such circumstances as those in which hoar-frost or dew takes place on the dry land, when the surface of the earth becomes colder than the air, (which we explain by a radiation of heat from the surface of the earth into the clear sky, or by impulses of cold from the sky to the earth,) with only this difference, that there was an additional transparent fluid over the bottom of the river, namely, the water; and thus also a shade prevented the formation of ground gru in the river, as it does that of hoar-frost or dew on the land.

In noticing the objections to the explanation I have given of the cause of ground gru, I shall confine myself to those brought forward by a writer in the Penny Cyclopædia, under the name of Ground Gru, which I have seen only very lately, although I believe they have been published for some years. He says the explanations of the formation of ground gru, given by Dr. Farquharson and Mr. Eisdale, are least of all satisfactory, and adds, "The former gentleman says it is the result of radiation, and endeavours to substantiate his reasoning upon the principles of the formation of dew, seeming to forget entirely, that Dr. Wells maintains expressly, that wind and shade are alike obstacles to radiation; and that consequently a body of moving water so deep as to be impervious to light, and particularly when covered, as in the case of the Neva, with a sheet of ice three feet thick, and as much more snow, must present an insurmountable obstacle to the radiation of heat from the bottom of the river."

Now, in the first place, with respect to shade; I was so far from forgetting that it is an obstacle to radiation, that, on the contrary, in my observations in 1835, I had shown by very many instances, that shade had prevented the formation of ground gru, just as it prevents dew. Wherever shade intervened to prevent radiation from the bottoms of the rivers Don and Leochal, there no ground ice was formed; while the unshaded parts of the bottoms were coated with it. My explanation thus mainly rested upon the fact that shade prevents radiation. In the next place, with respect to wind; the writer in the Cyclopædia himself forgets the difference of the statical conditions of air and water in connexion with temperature. Air becomes heavier by diminution of temperature. Water under 39° Fahr. becomes lighter by diminution of temperature. During wind, on the land, the cold air at the surface of the earth is continually mixed with, or displaced by, the warmer air above; and by this process both the earth and air in contact with it are prevented from being reduced to a very low temperature by radiation. But in a body of moving water, whose temperature is under 39°, the eddies of the current throw down the coldest parts, which in still water would remain at the surface, to come into contact with the bottom. This last circumstance is the explanation of M. ARAGO; and it well accounts for the formation of ground gru taking place first in the most rapid parts of the streams; although neither by itself, nor when taken in conjunction with the other two circumstances to which he refers, namely, aptitude to formation of crystals on asperities at the bottom, and less impediment to the formation of crystals in a slower motion, will it account for the formation of ground gru, as all these circumstances are present when the water forms only surface ice. The formation of ground gru requires for its explanation an additional element, namely, the radiation, into the clear sky, of heat from the bottom of the river; and the formation never occurs but under a clear sky.

As to the ground gru, observed by Colonel JACKSON in the Neva under three feet of ice and three feet of snow, that can form no valid objection to the explanation I have given, unless it were ascertained that the gru was formed after the surface ice and the fall of the snow, and not before them. All rivers issuing from lakes, like the Neva, have very clear waters to admit of radiation through them, although as deep as it is; and all rivers are very clear during frost, owing to the freezing up of the little land rills that would convey earthy particles into them. Ground gru formed in the Neva would be much more permanent than in our rivers. The mean temperature of Alford is 45° FAHR., and that of the earth of course the same; and on the remission of its cause, the ground gru is here speedily detached from the bottom, by the transmission of heat from below. Not so in the Neva. There, according to KUPFFER, the mean temperature is only 38°·75; and under the action of a frost so severe as to form three feet depth of ice, although the condition of the clear sky might not continue, previously formed gru would, at that mean temperature, be of great permanence. This applies also to the Siberian rivers.

IV. *On a Remarkable Property of the Diamond.*

By Sir David Brewster, *K.H. D.C.L. F.R.S. and V.P.R.S.Ed.*

Received February 15,—Read March 4, 1841.

HAVING had occasion, some years ago, to examine the structure of a diamond plano-convex lens which gave triple images of minute microscopic objects, I discovered, by a particular method of observation, that the whole of its plane surface was covered with hundreds of minute bands, some reflecting more and some less light; and I naturally drew the inference that this diamond consisted of a great number of layers of different reflective, and consequently refractive, powers, from which arose all its imperfections as a single microscope. In this case the veins or layers lay parallel, or nearly so, to the axis of the lens, so as to produce the worst effect upon the refracted pencil; for if the axis of the lens had been perpendicular to the surfaces of these veins, its performance as a microscope would scarcely have been injured by them.

In repeating Mr. Airy's experiments on the action of the diamond in modifying Newton's rings near the polarising angle, I was led to re-examine the flat surface of the diamond above mentioned; but though I found my former observations perfectly correct, yet I was induced to suspect the accuracy of the inference which I drew from them, and which I could not but draw in the circumstances under which the phenomenon was presented to me.

In order that the Society may be able to judge of the new results at which I have arrived, I have given in Plate I. fig. 1. as accurate a drawing as I am able to make of the appearance of the flat surface of the diamond under consideration, as seen by light incident upon it nearly perpendicularly. The flat surface of the diamond is 0·058, or $\frac{1}{17}$th of an inch in diameter, and owing to the great convexity of its other surface, the light reflected by it does not interfere with the examination of the structure above mentioned.

The appearance shown in the figure is that which I observed some years ago; but upon shifting the line of illumination, I was surprised to perceive that *all the dark bands became light ones, and all the light bands became dark ones,* a phenomenon which placed it beyond a doubt that all *the bands were the edges of veins or laminæ whose visible terminations were inclined at different angles, not exceeding two or three seconds to the general surface.* Had this surface been an original face of the crystal there would have been nothing surprising in its structure, excepting the exceeding minuteness of the strata and the slight inclination of their terminal planes to each other; but being a surface ground and polished by art, the phenomenon which it presents is one extremely interesting.

The mineralogist will have no hesitation in admitting that this diamond is part of a composite crystal consisting of a great number of individual crystals, like certain specimens of *feldspar, carbonate of lime,* and other minerals; but it is more difficult to conceive that the terminal planes of these individual crystals should retain their relative inclination after undergoing the operations of grinding and polishing upon a lapidary's wheel.

To many persons such a result may appear inadmissible; but there are several physical facts, which, when well considered, cannot fail to diminish its improbability. If we grind and polish a surface of *mother-of-pearl* obliquely to the strata of which it is composed, we shall find it impossible to produce a perfectly flat surface: even if we grind it on the finest and softest hone, and polish it with the smoothest powder, the termination of each stratum will remain; and while the general surface reflects a white image, the grooves or striæ will give rise to the beautiful prismatic images produced by interference*.

Another analogous fact presented itself to me many years ago in examining *calcareous spar.* Having had occasion to form an artificial face upon one of the edges of the rhomb containing the obtuse angle, I used a coarse file without water, and found that it exposed faces of cleavage which had never been previously seen, and which were inclined to the general surface produced by the file†.

In examining the optical figures produced by the disintegration of crystallized surfaces, I have found that by coarse sandstone, or the action of a rasp, or large-toothed file, we can expose surfaces of crystallization with their natural polish differently inclined to the general surface‡.

In all these cases the faces, exposed by the mechanical action of grinding or filing, preserve their natural surfaces and polish, and will preserve them more perfectly and readily if they are faces of easy cleavage. The facility of exposing such faces by the action of grinding must increase as the veins or strata become thinner, and it is probable that their exceeding minuteness in the diamond may have aided in the production of the structure which has been described.

I have found it quite impossible to measure the inclination of any of the faces by the goniometer; but I have succeeded, though with some difficulty, in taking an impression of the grooved surface upon wax.

This structure sufficiently explains the existence of three images when the lens was used as a microscope, without supposing that the veins had different refractive powers. Faces of different inclinations would, of course, converge the rays to different foci on the retina, as effectually as if there had been only a variation in their refractive indices.

St. Leonard's, St. Andrew's,
February 11*th,* 1841.

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* See Philosophical Transactions, 1814.

† Edinburgh Journal of Science, Oct. 1828, vol. ix. p. 312.

‡ Trans. Royal Soc. Edin. vol. xiv.

V. *On the Phenomena of Thin Plates of Solid and Fluid Substances exposed to Polarized Light. By Sir* David Brewster, *K.H. D.C.L. F.R.S. and V.P.R.S.Ed.*

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Having received from Dr. Joseph Reade one of his beautiful instruments called the *Iriscope,* and made several experiments with it, I soon perceived that it might be advantageously employed in various investigations in physical optics. This instrument consists mainly of a plate of highly polished black glass, having its surface smeared with a solution of fine soap, and subsequently dried by rubbing it clean with a piece of chamois leather. If we breathe upon the glass surface, thus prepared, through a glass tube, the vapour is deposited in brilliant coloured rings, the outermost of which is black, while the innermost has various colours, or no colour at all, in proportion to the quantity of vapour deposited. The colours in these rings, when seen by common light, correspond with Newton's *reflected rings,* or those which have *black centres,* the only difference being, that in the plate of vapour, which is thickest in the middle, the rings in the iriscope have black circumferences*. By using a large system of rings, or depositing the vapour in straight lines in the plane of incidence, we can at once observe the phenomena of the coloured rings or bands at various angles of incidence.

The first person who investigated the modification of Newton's rings in reference to polarized light was M. Arago, who has given an account of his observations in a beautiful and highly interesting memoir, in the third volume of the *Mémoires d'Arcueil,* published in 1817. Without knowing what had been done by M. Arago, Professor Airy entered upon the same inquiry in 1831 and 1832; but the phenomena which he observed were the same as those which had been previously discovered by M. Arago, with the exception of the modification of the rings when formed by a lens pressed against the surface of a diamond.

When Newton's rings are formed by a lens pressed against a surface of glass, M. Arago observed that they were black centred, as usual; and whether viewed with the eye or with a doubly refracting rhomb of Iceland spar, that the single or the

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* These rings may be formed upon almost all transparent bodies with more or less brilliancy, though I have found several substances, and occasionally pieces of glass, that will not absorb the soap. The rings are produced upon natural as well as artificial surfaces, that is, upon transparent surfaces produced by fusion or crystallization, as well as upon those polished by art. The soap being gradually dissolved by the vapour, requires to be frequently renewed. I find that other substances, particularly some of the oils, produce the same effect as soap. The rings disappear quickly by evaporation, and their brilliancy and purity of colour depend on the relative temperature of the vapour and the glass.

double system of rings had the same colours and the same diameters, the rings being completely polarized at the polarizing angle of the glass.

When the lens, however, was pressed against a metallic mirror, and examined with a doubly refracting rhomboid, two images perfectly similar appeared between a perpendicular incidence, and that of 55° or the polarizing angle of glass. One of the images disappeared entirely at this angle of 55°, when the principal section of the rhomboid was perpendicular or parallel to the plane of reflexion; but reappeared at greater incidences, with this remarkable peculiarity, that the colour of each of the rings which composed it was complementary to that of the corresponding rings in the image which had disappeared.

M. Arago likewise remarks that we may easily perceive with the eye, naked and without the assistance of any crystal, that at a certain angle near 55° the rings are composed of two distinct sets having unequal diameters, the rhomboid separating in a great measure the two sets of rings, because they are very unequally polarized. He likewise found that these phenomena were not produced when the rings were formed upon *native sulphur* and *diamond.*

"If the presence of a metallic mirror," says M. Arago, "is necessary for the production of the phenomenon in question when the rings are formed upon a plate of air, the case is otherwise when the thin body has much more density, and is in contact by one of its faces with another medium of sufficient refractive power. Thus *coal* presents often in its cleavages very bright colours, produced by an extremely thin substance, and which are decomposed into two complementary images when they are examined with a rhomboid under sufficiently oblique incidences. The colours which are formed artificially by the progress of evaporation, on thin films of alcohol or oil of sassafras, deposited upon coal or any other analogous substance, give rise also to two images, dissimilar, and of opposite tints*."

In order to investigate the phenomena of the rings of vapour in the iriscope, I illuminated them with light polarized in an azimuth of 90°, or perpendicularly to the plane of incidence, and examined them by a magnifying glass, when the centre of the rings was seen by light reflected at about 53° 11′, the polarizing angle of water. The effect, which was very striking, is shown in Plate I. fig. 2. The central part, A B, of the system of rings, C D E F, was without rings and colours of any kind: the upper half, C D, was part of a system of rings with *white* circumferences, and was formed by polarized light incident on the film at an angle *greater* than the polarizing angle of water; while the under half, E F, was part of a system of rings with *black* circumferences like those seen by common light, and was formed by polarized light incident on the film at an angle *less* than the polarizing angle of water.

The absence of rings in the middle portion, A B, was of course owing to there being no light reflected from the *first* surface of the film with which that reflected from the *second* surface could interfere; and the reason of there being light reflected

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* Mémoires de Physique et de Chimie de la Société d'Arcueil, tom. iii. p. 363. Paris, 1817.

from the second surface was, that the light reflected from it was not incident at its polarizing angle.

I have elsewhere shown*, that when a film of water is laid upon glass whose refractive index is above 1·508, there is no angle of incidence upon the first surface of the film which will allow the refracted ray to fall upon the glass at the polarizing angle; and hence at every angle of incidence on the film, the refracted light is reflected from the glass *at angles less than the polarizing angle of the united media*, or less than an angle whose tangent is equal to $\frac{m}{m'}$, m being the refractive index of the glass, and m' that of the water. When the refractive index of the glass is 1·508, the angle of incidence on the film must be 90° exactly, in order that the refracted ray may fall upon the glass at the polarizing angle whose tangent is equal to $\frac{m}{m'}$.

Now as the portion of the coloured rings at C D, fig. 2., is formed by the interference of two pencils, C A, D E B, fig. 3., one of which, C A, is reflected at an angle, P C A, *above* the polarizing angle of water, and the other, E B, at an angle *below* or *less* than that angle; while the portion E F, fig. 2., is formed by the interference of two pencils, which are both reflected at angles *below* or *less* than that angle, we may suppose that in the formation of the rings with a white circumference, analogous to those with a white centre, there is a loss of half an undulation, while that loss takes place in the interference of common light, or of two pencils reflected on the same side of the polarizing angle.

When the rings are seen at angles between 0° of incidence and 53° 11′, the polarizing angle of water, they are *black* in the circumference, like the portion shown at E F, fig. 2.; and when they are seen at incidences between 53° 11′ and 90°, they are *white* in the circumference, like the portion shown at C D, fig. 2.

If the rings of vapour are formed upon a polished surface of *fluor spar*, additional phenomena will be exhibited. At all incidences, from 0° to about 78°, rings of the same character will be seen as already described; but the ratio of the refractive powers of water and fluor spar is such, that at an incidence of 78° 4′ upon the surface of the vapour, the light incident on the spar will be reflected at the polarizing angle of the united media. Thus if $m = 1·437$, the refractive index of fluor spar, and $m' = 1·336$, the refractive index of water, then $\frac{m}{m'} = 1·0716$, the refractive index of the united media, or of their separating surface. The polarizing angle for this surface will therefore be an angle whose tangent is 1·0756 or 47° 5′, and the angle of incidence on the first surface of the watery film corresponding to the angle of refraction 47° 5′, which is the angle of incidence on the second surface, is 78°·4.

At an incidence of 78° 4′, therefore, the rings will disappear altogether, as at 53° 11′, because the pencil incident on the spar will not be reflected. At incidences greater

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* Philosophical Transactions, 1815, p. 138.

than 78° 4′ the system of rings with the black circumference will again appear as at incidences below 53° 11′, and will be visible up to 90° of incidence, the interfering pencils being now both reflected at angles above the polarizing angle of the surfaces which reflect them.

This experiment with vapour and fluor spar I have not made; and it may be difficult to see the rings at such an oblique incidence. If the rings are formed by soap upon *plate glass*, or by *alcohol* upon *fluor spar*, the second disappearance of the rings may be seen:

$$\frac{\text{Plate glass}}{\text{soap}}\ \frac{m}{m'} = \frac{1\cdot510}{1\cdot487} = 1\cdot0154.$$

Polarizing angle at second surface of the soap . . . . 45° 26′
Angle of incidence on the first surface . . . . 71° 45′

$$\frac{\text{Fluor spar}}{\text{alcohol}}\ \frac{m}{m'} = \frac{1\cdot437}{1\cdot370} = 1\cdot049.$$

Polarizing angle at second surface of alcohol . . . . . 46° 22′
Angle of incidence on the first surface . . . . 82° 32′

If we call m, m' the indices of refraction of the two substances, viz. the *film* and the *surface* upon which it rests, m being the larger index, then a ray incident at 90° will fall upon the common surface of the two media at the polarizing angle of that surface, when the angle of refraction at the first surface is equal to the tangent, or cotangent of the polarizing angle, according as the refractive power of the film is less or greater than that of the body upon which it rests.

Hence we have $\sin i' = \frac{1}{m}$ or $\frac{1}{m'}$,

and

$$\tan i' = \frac{m}{m'}, \text{ or } \cot i' = \frac{m}{m'},$$

and

$$m = \frac{m'}{\sqrt{m'^2 - 1}}, \text{ and } m' = \frac{m}{\sqrt{m^2 - 1}},$$

when a ray incident at 90° is polarized at the second surface, or falls upon it at the polarizing angle.

These formulæ enable us to discover between what limits of refractive power the second disappearance of the rings can take place, and consequently what substances we should employ in order to observe it. In this manner we obtain the following results for the mean rays of the spectrum:—

Values of m'.	Values of $\frac{m'}{\sqrt{m'^2 - 1}}$, or m.
3·000	1·061
2·500	1·090
2·000	1·154
1·900	1·176

Values of m'.	Values of $\frac{m'}{\sqrt{m'^2 - 1}}$, or m.
1·800	1·202
1·700	1·236
1·600	1·281
1·554	1·307
1·508	1·336
1·500	1·341
1·400	1·428
1·336	1·508
1·307	1·554

The limits, therefore, between which the *second* disappearance of the rings can take place are 1·554, the index for *quartz* and *flint glass*, and 1·307, the index for *ice*. But though the range is very limited, it nevertheless includes a considerable variety of solid and fluid bodies. I have omitted the indices of Tabasheer, and the fluids produced by the compression of gaseous bodies, because, though their refractive powers are beneath 1·307, they cannot be used in the present inquiry.

When m and m' are thus related, the *white centred rings* will just disappear when $i = 90°$, the light being then incident on the second surface at its polarizing angle. But if we use a film of still less refractive power in relation to the second body, the refracted rays will fall on the second surface at an angle *greater* than the polarizing angle (i being still 90°), and consequently *the black centred rings will reappear*, and there will be some angle of incidence I on the film, less than 90°, at which the angle of refraction i' will be equal to the polarizing angle of the second surface. This angle will be found from the expression

$$\sin \mathrm{I} = \frac{m\,m'}{\sqrt{m^2 + m'^2}}.$$

When $m = m'$ no rings whatever will be formed, as no light is reflected at the common surface; but if $m = m'$ only for a particular colour in the spectrum of each substance, and if these indices differ considerably for another colour, rings will be formed in which that colour predominates, in which $m > m'$, or $m < m'$. This takes place in a remarkable manner with *oil of cassia* and *flint glass*, in which $m = m'$ for the *red* rays, but $m > m$ for the *blue* rays. The consequence of this is, that a quantity of *blue* light is reflected from the separating surface of the oil and the glass; and hence if a sufficiently thin film of oil of cassia is laid upon the glass, *blue* would greatly predominate in the system of rings.

Hitherto the azimuth of the polarized light has been 90°, or perpendicular to the plane of reflexion. Let us now suppose that its azimuth is gradually changed from 90° to 0° by the rotation of the polarizing surface or crystal.

At all azimuths, from 90° to 0°, the rings with the black circumference are seen, between the angles of 0° and 53° 11′, and at the incidence of 53° 11′. But at incidences between 53° 11′ and 90°, in the case of the iriscope, very interesting phenomena appear. We shall first describe what takes place at 56° 45′, the polarizing angle of the black glass. At this angle none of the polarized light is reflected when the azimuth is 90°, and the rings with the *white* circumference are beautifully seen on the dark ground of the glass, which now reflects no light. As the azimuth is changed to 87°, 88°, &c., the black glass reflects a little light, and the two surfaces of the film a little more light, the rings gradually become fainter and fainter, till at an azimuth of about 79° 0′ they disappear exactly as they did at 53° 11′, and in the azimuth 90°. When this disappearance takes place, the light reflected from the glass seems to be exactly equal to the light reflected from both surfaces of the film. At other angles of incidence the rings disappeared at different azimuths, varying from 90° to about 45°, as the angle of incidence varied from 53° 11′ to 90°. I found it difficult, however, to measure these azimuths with any accuracy, as the rings were not permanent; and I was therefore obliged to form the colours of thin plates upon highly refracting substances, such as *diamond*, *chromate of lead*, *artificial realgar*, and *greenockite* (the most refractive of all bodies), which had high polarizing angles. A solution of fine soap gave brilliant colours when dried, and in this way I obtained the following results with the surface of a very fine diamond. The index of refraction of the soap was 1·475, and that of the diamond 2·44, and their respective polarizing angles 55° 52′, and 67° 43′.

Angle of Incidence of the Polarized Light.	Azimuth of the Plane of Polarization at which the rings disappear.	
	Observed.	Calculated.
55° 52′	90° 0′	90° 0′
60	73 0	74 27
65	68 30	67 49
67 43	66 20	65 10
70	63 30	63 14
75	59 15	58 23
90		46 30

As the disappearance of the rings was not owing to the extinction of one of the interfering pencils, as at 55° 52′, for a sufficient quantity of polarized light was reflected from both surfaces of the film, there was reason to believe that it might arise from the two pencils being polarized at right angles to each other, in conformity with the law relating to the action of the second surfaces of plates which I have given in a former paper*.

Calling x the azimuth of primitive polarization, i the angle of incidence on the *first* surface of the film, i' the corresponding angle of refraction, and consequently the angle of incidence on the *second* surface, i'' the angle of refraction at the *second* surface,

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* Philosophical Transactions, 1830, pp. 148, 149.

and φ = the inclination of the plane of polarization of the reflected pencil C A, fig. 3.
φ' = that of the refracted pencil C D,
φ'' = that of the reflected pencil D E, and
φ''' = that of the refracted pencil E B, with which C A interferes; then by FRESNEL's formula we have for the ray C A,

$$\tan\varphi = \tan x \,.\, \frac{\cos(i+i')}{\cos(i-i')};$$

and by my formulæ* we have

$$\cot\varphi' = \cot x\cos(i-i')$$

$$\tan\varphi' = \tan x \,.\, \frac{1}{\cos(i-i')}$$

$$\tan\varphi'' = \tan x' \,.\, \frac{\cos(i'+i'')}{\cos(i'-i'')}.$$

But, after one refraction,

$$\tan x' = \tan\varphi = \tan x \,.\, \frac{1}{\cos(i-i')};$$

hence

$$\tan\varphi'' = \tan x \,.\, \frac{1}{\cos(i-i')} \cdot \frac{\cos(i'+i'')}{\cos(i'-i'')}$$

and

$$\cot\varphi'' = \frac{1}{\tan x} \,.\, \cos(i-i') \,.\, \frac{\cos(i'-i'')}{\cos(i'+i'')}.$$

And multiplying this by $\cos(i-i')$ for the change of plane produced by the second refraction at E, we have for the ray E B,

$$\cot\varphi''' = \cot x\cos^2(i-i') \,.\, \frac{\cos(i'-i'')}{\cos(i'+i'')}.$$

Now the two pencils which interfere, viz. C A and E B, have their planes of polarization inclined at angles φ and φ''' to the plane of reflexion; but in order that these angles may be complementary to each other, or may together make 90°, we must have $\tan\varphi = \cot\varphi'''$, or

$$\tan x\,\frac{\cos(i+i')}{\cos(i-i')} = \cot x\cos^2(i-i') \cdot \frac{\cos(i'-i'')}{\cos(i'+i'')};$$

and consequently

$$\tan^2 x = \cos^2(i-i') \cdot \frac{\cos(i-i')}{\cos(i+i')} \cdot \frac{\cos(i'+i'')}{\cos(i'-i'')};$$

and

$$\tan x = \cos(i-i') \,.\, \sqrt{\left(\frac{\cos(i-i')}{\cos(i+i')} \cdot \frac{\cos(i'-i'')}{\cos(i'+i'')}\right)}.$$

When the angle of incidence is 90°, $\cos(i+i') = \sin i'$, and $\cos(i-i') = \sin i'$, and hence

$$\tan x = \frac{1}{m}\sqrt{\frac{\cos(i'-i'')}{\cos(i'+i'')}}.$$

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* Philosophical Transactions, 1830.

If we now calculate by these formulæ the values of x for the different angles of incidence in the preceding Table, and subtract them from 90°, we shall have the numbers in the third column of the Table, which agree with those observed within the limits of the errors of observation. In the case of *water* and *glass*, too, where the azimuth of disappearance was observed to be about 79° or 11°, the formula gives 79° 28′, or 10° 32′, at an incidence of 56° 45′.

In order to ascertain the relation between the mutual inclination of the planes of polarization of the interfering pencils when they produced *black-centred* or *white-centred* rings, I have computed the following Table for an incidence of 56° 45′.

Azimuth of Polarized Light.	$+\phi$	$-\phi'''$	Film of *water* and *glass*. Inclination of Planes ϕ and ϕ'''.	
° ′	° ′	° ′	° ′	
90	90 0	90 0	180 0	*White*-centred rings.
87 30	74 43	82 45	157 28	
85 0	49 30	75 4	124 34	
79 28	28 26	61 34	90 0	*No rings.*
70 0	15 28	43 19	58 47	*Black*-centred rings.
45 0	5 45	18 57	24 42	
35 0	4 3	13 3	17 6	
20 0	2 6	7 7	9 13	
0 0	0 0	0 0	0 0	

By taking ϕ *positive*, or on the *right*-hand side of the plane of reflexion, then ϕ''' must be *negative*, or on the left-hand side of that plane[*]; hence $+\phi$, $-\phi'''$ will be the mutual inclinations of the planes of polarization of the interfering pencils, and we obtain the important law,

That when two polarized pencils reflected from the surfaces of a thin plate lying on a reflecting surface of a different refractive power interfere, half an undulation is not lost, and WHITE-*centred rings are produced, provided the mutual inclination of their planes of polarization is greater than* 90°*; and that when this inclination is less than* 90°, *half an undulation is lost, and* BLACK-*centred rings are produced; when the inclination is exactly* 90°, *the pencils do not interfere, and no rings are produced.*

At an incidence of 45° upon water and glass, where the signs of ϕ and ϕ''' are the same, the maximum difference in the planes of polarization is 23° 12′, which takes place in azimuth 70° 30′; and at an incidence of 10° the greatest difference is 2° 16′, which takes place at an azimuth of about 45°.

In the case of *soap* and *plate glass*, where the black-centred rings appear beyond the incidence of 71° 45′, the difference of inclination in the planes of the two pencils is also less than 90°.

I was now desirous of examining the phenomena of a perfect system of rings when the film had a greater refractive power than the substance upon which it was laid;

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[*] See Philosophical Transactions, 1830, p. 70, fig. 1.

after many ineffectual attempts to obtain such a system, I succeeded by laying a very small portion of *oil of laurel* upon *water* placed in a black vessel, or on the surface of diluted or real ink. The rings thus produced are splendid beyond description, and exhibit the various phenomena with singular beauty. As the polarizing angle of the oil *exceeds* that of the water, the *black*-centred rings are seen at the polarizing angle of the water, when the reflected light disappears. They continue to be seen till we reach the polarizing angle of the oil, when the rings disappear, and the white-centred ones commence, and continue till we reach the incidence of 90°*.

In forming thin films upon metallic surfaces, I employed many of the metals, and found the phenomena nearly the same upon them all, and differing very little from those produced upon transparent bodies. On a fine specimen of *specular iron ore*, I found a system of rings ready formed, with three orders of colours. The azimuth of the polarized light being inclined 90° to the plane of reflexion, the system of rings disappeared wholly at an angle of incidence of 58° 36′, which is therefore the polarizing angle of the unknown substance of which it was formed: consequently its index of refraction is about 1·638. Between this angle and 90° of incidence, the *white*-centred rings appeared; but at 72° 39′, the polarizing angle of the iron (which gives its refractive power for the *red* rays 3·200), the rings were singularly fine, being seen on a beautiful blue ground, produced by the disappearance of the *red* light, which is polarized at that angle. I now measured the azimuth of the plane of polarization when the rings disappeared, which was 59° 25′, whereas by the formula it is 57° 59′; a discrepancy not to be wondered at, when we consider that the index of refraction for the red rays, viz. 3·200, was used, in place of that for the mean ray, which is not known. The inclination of the planes of polarization of the two interfering pencils, when calculated by the previous formulæ, is + 32° 7′, and − 57° 53′; so that these planes being inclined 90° to each other, as in the case of soap and diamond, no interference takes place, and the rings disappear.

In the fine specimens of *oligist iron ore* from Elba, I have found crystals covered with the most beautiful coloured films, both of uniform and variable thickness. These films are not acted upon by the ordinary acids, like the coloured films upon steel, and appear, from their optical properties, to be of a metallic nature. When they are exposed to a polarized ray, they exhibit generally the same phenomena as the films already described; but there is no angle of incidence at which the colours disappear, either in the azimuth of 90°, at the polarizing angle of the first surface of the film,

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* These thin plates of oil of laurel exhibit some curious phenomena, which I believe have not been noticed. If we wet with water, alcohol, or the oil of laurel itself, the extremity of a short piece of wire, such as a large pin, and hold the pin in the hand, so that its head may be above, and almost touching the film, the film will recede in little waves of a circular shape, which form a new system of coloured rings; and they become covered with the vapour from the fluid on the head of the pin in such small particles that they reflect no light, and the rings appear to be blackened. By withdrawing the pin, the film is restored to its former state. The same effect is produced by heating the pin, or the fluid upon it, to promote evaporation.

or in those azimuths where the pencils, from the first and second surface, have their planes of polarization inclined 90° to each other. This, no doubt, arises from the high dispersive power of the film, in consequence of which the different homogeneous rays are polarized at angles considerably different from each other.

The phenomena of transparent films of low refractive power, when laid upon the polished surfaces of metals, and exposed to polarized light, are not very different from those which are exhibited when the film rests upon a transparent surface. I at first used a solution of soap, which produced pretty good tints on speculum metal; but at last I fell upon a method of laying down the most beautiful systems of coloured rings upon all surfaces of all forms, whether metallic, transparent, or opake. For this purpose I used the *oil of laurel*, which, when placed upon the surface of water, expands into a film, which gives the finest system of coloured rings. Having laid the plate of polished metal in a small porous wooden tray, such as is used for holding minerals, I poured water into it, so as to cover the metallic surface to the depth of the 50th part of an inch. I then formed a film of the oil upon the water, immediately above the metallic surface. In a short time the absorption of the water by the porous tray allowed the film of oil to descend and rest upon the metallic surface*. When the adhering moisture was removed by evaporation, the film was extremely beautiful; and if protected from dust may be preserved for any length of time.

Having laid a film of this kind upon *speculum metal*, I obtained the following results. The coloured rings disappeared almost completely at 56°, the polarizing angle of the oil. The *black-centred* rings appeared at all angles less than 56°, and the *white-centred* rings at all angles above it. Both the systems of rings were exceedingly distinct at the greatest angles of incidence, whereas on transparent surfaces of low refractive power, they can scarcely be seen at such angles. When the azimuth of the polarized ray varies from 90° to 0°, the rings disappear at different angles of incidence; or when the angles of incidence vary, the rings disappear in different azimuths. I measured these azimuths when the polarized ray was incident upon speculum metal, and obtained the following numbers:—

	Azimuth in which the rings disappear.		
Angles of Incidence.	Observed.	Calculated.	Difference.
° ′	° ′	° ′	
90		40 23	
71 50	56 25	57 22	− 0 57
60 0	65 45	65 4	+ 0 41
56 5	90 0	90 0	

In computing column third from the formula in p. 49, I used 1·49 as the index of refraction of *oil of laurel*, and 4·011 as the index of refraction for *speculum metal*, as deduced from my experiments on its elliptic polarization†.

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* The same effect is produced more slowly by evaporation; or the water may be sucked out of the tray by a tube, or run off by an aperture.

† Philosophical Transactions, 1830, p. 324.

I have made similar experiments when the rings were transferred to *silver*, whose elliptical polarization approaches nearest to circular polarization; and to *grain tin*, which appears to have the highest refractive power of any of the metals; but I found it very difficult to ascertain with any accuracy the azimuths in which the rings disappear.

If we use common in place of polarized light in the preceding experiments, and analyse the reflected light by a rhomb of calcareous spar, the very same phenomena will be exhibited.

When the films or thin plates are not laid upon the surfaces of fluid or solid bodies, the phenomena are of an entirely different kind. At all angles of incidence, and in all azimuths, the colours and character of the rings are the same, whether we use common or polarized light. In obtaining this result I stretched thin films of various oils, such as *oil of laurel, oil of cassia, oil of turpentine,* and many others, across circular apertures, and examined them in light polarized in different azimuths. The rings of course vanished at the polarizing angle of the oil, and the brilliancy of the colours varied with the angles of azimuth and incidence, but the complementary rings never appeared, the rings being always those with the black centre*.

In order to understand the cause of this, we must inquire into the state of polarization of the interfering pencils. The ratio of refraction being the same at both surfaces of the film, we have

$$\tan\varphi = \tan x \,.\, \frac{\cos(i+i')}{\cos(i-i')}, \text{ and } \cot\varphi''' = \cot x \,.\, \frac{\cos^3(i-i')}{\cos\ (i+i')};$$

and when $\tan\varphi = \cot\varphi'''$, which is the case when $\varphi + \varphi''' = 90°$, we have

$$\tan x = \frac{\cos^2(i-i')}{\cos(i+i')}.$$

When $i = 90°$, $\tan\varphi = A$, or the azimuth of the polarized ray, and $\cot\varphi''' = \frac{\cos^3 i'}{\sin\ i'}$.

If we now compute the values of φ and φ''' at different angles of incidence and in different azimuths of the polarized light, we shall obtain the results in the following Table. In azimuths 0° and 90°, φ and $\varphi''' = 0$.

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* The physical phenomena exhibited in these attenuated films are very remarkable. A current of fluid is projected from the margin and centre of the ring of fluid across the fluid surface, resembling the top of a pine apple. This movement makes the film thinner at some places than others, and hence arises an irregular system of coloured bands, with an incessant play of varying tints, as if the fluid were animated. The bands of colour are serrated with salient points, from which the fluid seems to shoot across the film. In the oils of cinnamon, naphtha, spearmint, wormwood, rapeseed, nutmegs, bergamot, savine, rosemary, &c., the phenomena are peculiarly beautiful. With poppy oil, the *red* and *green* tints of the 4th, 5th, and 6th orders were also seen.

Inclination of the planes of polarization of the two pencils, ϕ and ϕ'''.

Angles of Incidence.	Azimuth 22° 30′.		Azimuth 45°.		Azimuth 67° 30′.		Azimuth 80°.	
	Pencil from *first* surface.	Pencil from *second* surface.	Pencil from *first* surface.	Pencil from *second* surface.	Pencil from *first* surface.	Pencil from *second* surface.	Pencil from *first* surface.	Pencil from *second* surface.
0	22° 30′	22° 30′	45° 0′	45° 0′	67° 30′	67° 30′	80° 0′	80° 0′
10	21 42	22 5	43 51	44 24	66 40	67 4	79 36	79 48
20	19 11	19 34	40 13	40 38	64 13	64 14	78 13	78 23
30	15 25	15 55	33 40	34 33	58 7	58 58	75 10	75 38
40	10 18	11 1	23 41	25 11	43 21	48 37	68 6	68 6
50	4 18	4 52	10 18	11 37	23 41	26 24	45 52	49 23
56 in 45	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
60	2 6	2 35	5 4	6 13	12 5	14 44	26 42	31 42
70	7 54	11 52	18 32	26 53	39 0	50 45	62 16	70 49
80	15 11	24 41	33 13	47 58	57 41	69 32	74 56	80 58
85	18 40	33 34	39 12	58 2	63 5	75 30	77 48	83 43
90	22 30	43 57	45 0	66 44	67 30	79 54	80 0	85 40

The results in this Table, which may be considered as those of observation*, exhibit at one glance the general phenomena at all angles of incidence and azimuth.

The two interfering pencils are in every case reflected at angles either *both above* or *both below* the polarizing angle, and hence their planes of polarization are always on the same side of the plane of reflexion and in the same quadrant, and consequently they never can be at right angles to each other so as to prevent interference. For the same reason the inclination of the planes never can exceed 90°, so as to produce the complementary white-centred rings, in conformity with the law previously given.

If, for example, we compute the value of x in the preceding formula at an incidence of 70°, we shall find it 66° 25′, at which azimuth the inclinations ϕ and ϕ''' of the planes of polarization are 40° 47′, and 49° 53′; but though the sum of these angles is 90°, yet the real inclination of the planes is $\phi''' - \phi = 9° 6'$.

This property of parallel transparent films, of giving by reflexion pencils polarized in planes at various inclinations, when the incident light is polarized in different azimuths, enables us to obtain two pencils of polarized light, inclined at any angle, varying from 0° to 21° 44′ in glass, and to study the phenomena which such pencils exhibit, either in their mutual action, or in their relations to other properties of light.

But the phenomena become more varied and interesting when the second surface of the plate is *inclined* to the first. In this way we may produce effects analogous to those produced by a change in the refractive power of the second surface by contact with another refracting surface, and obtain pencils inclined 90° to each other, and therefore exhibiting the white-centred rings. The phenomena will in this case resemble those of a film of oil upon water.

When the refractive index of a parallel film exceeds 1·508, the ray is incident on the second surface at an angle less than the polarizing angle; but by inclining the

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* See Philosophical Transactions, 1830, pp. 74, 138.

second surface we can make it fall upon it at a greater angle than the polarizing angle. The phenomena may be still more varied by inclining the surface of emergence to the surface of incidence*; but as it is not easy to obtain films with faces suitably inclined to each other, it is unnecessary to pursue this branch of the subject any further.

Such are the phenomena of *thin* and *thick* plates when viewed by polarized light, or by common light subsequently analysed by a doubly refracting rhomb. But if we use polarized light, and subsequently analyse the light transmitted through the thin plates, we shall obtain a series of very interesting and instructive phenomena, analogous to those produced by plates of doubly refracting crystals which exhibit the polarized tints. In both these cases, the film is interposed between a polarizing plate and an analyzing rhomb. If the film is too thick to produce colours, it will depolarize the polarized ray, in a manner analogous to that of a crystallized plate, which is not thin enough to give the polarized tints; and if the film is sufficiently thin to produce uniform tints, a coloured band or system of rings, with black or white centres. Their action is analogous to that of thin crystallized plates, which either produce uniform tints like the laminæ of sulphate of lime, or uniaxal or biaxal systems of rings.

It would be unprofitable to describe minutely the great variety of phenomena which thin plates thus exhibit, as they vary with the refractive power of the fluid or solid upon which they are laid, so that I shall confine myself to the case in which a thin plate of oil of laurel rests on the surface of a specimen of *artificial realgar*. In common light, the colours of this film are very beautiful, but when examined in polarized light by an analyzing rhomb, they are brilliant beyond description.

1. *When the azimuth of the polarized light* is 90°, and the incidence of the polarized ray 56° 5′, the polarizing angle of oil of laurel.

When the film is viewed without the polarizing rhomb, no rings are seen, as there is no light reflected from the first surface of the film, and consequently no interference.

When the film is viewed with the polarizing rhomb, having its principal section in the plane of incidence, no rings appear, either in its ordinary or extraordinary image. But if the plane of polarization is less or more than 90°, by even a small quantity, then after the rhomb has been turned round nearly 90° towards the right, a system of *black-centred* rings is seen for an instant, and these, after disappearing, are followed by a system of *white-centred* ones, the white-centred rings appearing first if the rhomb is turned to the left. The same phenomena are repeated in every quadrant of the circular motion of the rhomb.

2. *When the azimuth of the polarized light varies* from 90° to 0°, the incidence, being 56° 5′, as before.

At 90° azimuth the phenomena are as above described.

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* Philosophical Transactions, 1830, p. 147, fig. 3.

At 67½°. Rhomb 0°, no rings.

Rhomb turning to the right, the white-centred rings appear, then vanish, when the azimuth of the rhomb is less than 67½°; then black-centred rings appear, which vanish at 180°; then succeed the white-centred ones, which vanish at about 210°; then the black-centred, which continue to 360°.

At 45°, 22½°. The very same phenomena appear at these and other azimuths, the azimuths of the rhomb at which the rings disappear out of the plane of incidence being a little less than the azimuths of the polarized light.

At 0°. The evanescence of the rings takes place when the azimuths of the rhomb are 0°, 90°, 180°, and 270°, the *white-centred* rings appearing in the *first* and *third*, and the *black-centred* ones in the *second* and *fourth* quadrant.

3. *Azimuth of polarized light* 90°.

Incidence of polarized light 68° 3′, the polarizing angle of realgar.

At this angle all the light reflected from the realgar has disappeared, excepting a dark bluish purple, in the middle of which is seen, without using the rhomb, a splendid system of richly-coloured rings, with a *white centre*. When the rhomb is applied as before, and performs a complete revolution, the white-centred rings are seen all round, disappearing at 90° and 270°.

4. *When the azimuth of the polarized light varies from* 90° to 0°, the incidence being 68° 3′, as before.

At 90° azimuth, the phenomena are as above described.

At 80°, and all other azimuths, the *white-centred rings* are seen when the rhomb is at 0°; but they disappear at azimuths of the rhomb a little less than the azimuths of polarization, and are then succeeded by the *black-centred rings*.

At 0° azimuth, the rings disappear when the rhomb is at 0° and 180°, and are *black-centred* all round.

Without using the rhomb, the rings always disappear at the azimuth x, at which the planes of polarization of the interfering pencils are rectangular.

At incidences above 68° 3′, the phenomena are of the same character. The rings are *white-centred* in 90° of azimuth, and when the rhomb is at 0°. They become very brilliant about 45°. Near 90° of rotation the rings vanish, and immediately the black-centred system appears, which quickly vanishes, and is succeeded by the white-centred system.

5. *Angles of incidence less than* 56° 5′.

In 90° of azimuth of the polarized ray, and the rhomb being at 0°, the black-centred rings are seen, and continue to be seen during a complete revolution of the rhomb. In all azimuths, from 90° to 0°, the rings disappear by turning the rhomb to the left, the arch diminishing from 90° to 0°; but in azimuths of an intermediate magnitude, the disappearance of the rings is followed by the appearance of the *white-centred system*, which quickly disappears, and is succeeded by the *black-centred system*. This phenomenon is seen best near 45° of azimuth.

When the plates or films are too thick to give the coloured rings, the phenomena of the differently polarized pencils may be finely seen by using *coloured glasses*, in which the pencils reflected from both surfaces may be observed. If the glass is *green*, for example, the pencil or image of a small aperture or luminous body will be *green*, while that reflected from the first surface, though in reality colourless, will appear *red*, from the physiological action of the green light upon the retina. Hence the two differently polarized pencils will have different colours, as if they were the tints of polarized light. If these coloured glasses are laid upon, or cemented on one side to, metals or highly refracting substances, the polarization of the coloured pencils which they reflect will be modified according to the principles already explained, and they will exhibit many interesting phenomena, varying with the colours of the glasses, as if the colours were produced by the absorption of polarized light.

In order to convey a general idea of the different classes of phenomena described in the preceding paper, I have represented two of the most important in figs. 4 and 5.

1. *Glass and Water.*—When a film of aqueous vapour is laid upon glass whose index of refraction is 1·508, the rings disappear at 53° 11′, the polarizing angle of the water, and also in the various azimuths where the two interfering pencils are polarized in planes at right angles to each other. At all azimuths greater than these, and at angles of incidence above the polarizing angle, the *white-centred* rings appear; and at all azimuths less than these, and at all incidences (except those at which the white-centred rings are seen), the *black*-centred rings appear.

The following Table shows the values of x, or the azimuths of disappearance of the rings, as computed from the formula in p. 49:—

Angles of Incidence.	Azimuths.	Complements.
° ′	° ′	° ′
53 11	90 0	0 0
55 0	82 8	7 52
60 0	76 52	13 8
65	75 15	14 45
67	75 10	14 50
70	75 30	14 30
73	76 18	13 42
74	76 42	13 18
75	77 9	12 51
76	77 36	12 24
80	80 0	10 0
85	84 15	5 45
90	90 0	0 0

If we now conceive A B, fig. 4, to be the section of the plane of incidence, having the different incidences marked upon it from 90° to 53° 11′, and if round a centre in A B prolonged, where 0° of incidence falls, we describe the azimuthal circle Z A Z, then the complements of the azimuths of the polarized light being set off from the corresponding angles of incidence on each side of A B, the curves A C B, A C B passing through these points will show at what angles of incidence and azimuth the rings disappear, in consequence of the planes of polarization of the two pencils being at these places rectangular.

At all incidences, and in all azimuths within the shaded space A C B C, the *white-centred rings* are seen, and at all other azimuths and incidences the *black-centred* rings are seen.

2. *Fluor Spar and Water.*—I have taken this combination as a specimen of the phenomena which take place at some incidences less than 90°, when the refracted ray falls on the second surface of the film, at angles greater than its polarizing angle. The following Table shows the values of x and their complements:—

Angles of Incidence.	Azimuths.	Complements.
° ′	° ′	° ′
53 11	0 0	0 0
55	82 35	7 25
60	77 47	12 13
63	76 54	13 6
65	76 41	13 19
67	77 6	12 54
70	78 9	11 51
75	82 0	8 0
78	88 41	11 9
78 4	90 0	0 0
80	83 28	6 32
85	77 31	12 29
90	74 14	15 46

By projecting these values, as is done in fig. 5, we obtain a double set of curves which unite at D, where the angle of incidence is 78° 4′, at which the refracted ray falls upon the second surface at its polarizing angle.

At all incidences, and in all azimuths within the shaded portions of the figure Z A Z D, D C B C, the *white-centred rings* are seen. At all azimuths and incidences corresponding with the outlines of the curves Z D Z, D C B C, the *rings disappear*; and at all azimuths and incidences without the shaded portions of the figure, the *black-centred rings* are seen*.

St. Leonard's, St. Andrew's,
April 8, 1841.

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* No reference is made in these figures to the phenomena which are seen by using both polarized light and the analyzing rhomb.

VI. *Memoir of the Case of a Gentleman born blind, and successfully operated upon in the* 18*th year of his age, with Physiological Observations and Experiments. By* J. C. August Franz, *of Leipzic, M.D., M.R.C.S., &c. Communicated by Sir* Benjamin C. Brodie, *Bart., F.R.S., &c.*

Received April 21,—Read May 6, 1841.

Mr. F. J., the subject of the present memoir, is the son of a physician; of scrofulous diathesis, but otherwise of robust constitution; of irritable temperament, but of contented and happy disposition; and endowed with an excellent understanding, quick power of conception, and retentive memory. In both the eyes of his father, cataract (with the addition, I suspect, of glaucoma) has manifested itself within the last four years, after a severe attack of influenza. The relatives on the paternal side are predisposed to diseases of the eye, but in the mother, and in the relatives on her side, no such predisposition can be traced. With regard to the cause of the ophthalmic affections which form the subject of this paper, the mother seemed to lay much stress on the following circumstance, which, although it may possibly have had some share in the cause of one of them, can have had no influence, in my opinion, in producing the other. She stated to me that in the eighth month of her pregnancy, which up to this period had proceeded favourably, she received from her youngest child, which she was carrying in her arms, a severe blow on the eye. This accident caused inflammation of the eye, accompanied with a curious visual illusion, viz. that all objects which she saw, but especially those situated on the ground, appeared of a deep concave form; an illusion which lasted for several months. The fright experienced from the accident also brought on convulsions, which, recurring several times, extended even to the fœtus. The recurrence of these convulsions produced in the mind of the mother a continual anxiety and fear for the health of the child, while the pain arising from the ophthalmia, together with the visual illusion just mentioned, gave her fears a direction more especially towards its eyes. Delivery took place at the proper period, when the eyes of the infant, which was otherwise healthy and well-formed, were found to present a twofold defect of organization. The father, to whose statement, on account of his professional knowledge, more weight is to be attached, informed me that both eyes were turned inwards to such an extent that a portion of the cornea was hidden by the inner canthus, and that in both pupils a yellowish-white discoloration was to be observed, which, being situated behind the iris, could not be the pupillary membrane. That the strabismus and cataract of both eyes in this case were congenital, is evident from the testimony both of the parents and of the nurse, whom I have closely questioned on this subject. The latter, who can distinctly remember all the circumstances of the case, told me that when the child was a few months old, she held a light before its eyes, of which it took no notice. I ascertained also from her that the eye-balls had not that restless motion which is generally observed in those who are born blind, but that both eyes were always turned inwards, and that but rarely either the one or the other was moved from the internal canthus.

It was also stated to me, that towards the end of the second year the operation of keratonyxis was performed on the right eye, upon which a severe iritis ensued, terminating in atrophy of the eye-ball. Within the next four years two similar operations were performed on the left eye, which did not indeed destroy the organ, but at the same time did not remove the opacity in the pupil. The colour of the opacity became in time, however, of a clearer white; and the patient acquired a certain sensation of light, which he did not seem to have had before the operation. Both eyes for a long time retained a disposition to inflammation, and suffered repeatedly from conjunctivitis, whence the vessels of the conjunctiva were increased in number and size to such an extent, that it was necessary they should be several times excised.

At the end of June 1840, the patient, being then seventeen years of age, was brought to me by my friend Dr. Swaine, for the purpose of consulting me with regard to the congenital double strabismus, and at the same time to hear my opinion on the more severe ophthalmic affection, which up to this period had been considered incurable; the patient himself regarded his case as hopeless. The following are the particulars elicited on an attentive and careful examination:—On the right side, the eyelids and parts adjacent appeared contracted; they were less in size, and the eye itself was situated deeper in the orbit than the left. At each act of winking spasms of the eyelids were induced, and, when the left eye was turned outwards, the spasmodic twitchings extended over that half of the face. Both eyes were so much inverted, that nearly one half of the cornea was hidden by the inner canthus. The left eye he could move voluntarily outwards or in any direction with certainty, but not without exertion; it returned immediately inwards, when the influence of the will had ceased. The motion of the right eye upwards and downwards the patient had under his control, but not so the movement towards the external canthus, in effecting which he only succeeded after many attempts. The left eye-ball was of the natural size and elasticity; the right, on the other hand, was at least a third smaller, and felt soft, and like dough; it was also, in the neighbourhood of the rectus internus, flat, or rather pressed inwards. The cornea was less convex, somewhat smaller, but not in proportion to the diminished size of the globe itself; it was clear, and free from opacity, except in the centre, where the keratonyxis had left an opake spot. The fibrous structure of the iris was irregular; its colour, which was brown, rather lighter than that of the left eye. Different degrees of light produced no effect on the motion of the iris; but when the eye was moved in a horizontal direction outwards, the pupil, in passing the centre of the orbit, contracted a little, and, when approaching the outer canthus, expanded again to the size it held when the eye was in its usual inverted position. On looking from the temporal side into the pupil, a large portion of the opake capsule was observed in the posterior chamber. The interior of the eye-ball presented a brownish-black appearance. The patient had not the slightest perception of light with this eye; it was perfectly amaurotic. The left eye presented in the conjunctiva, especially at the inner canthus, a number of varicose vessels, and in the sclerotica a fine vascularity around the outer half of the cornea; this latter membrane was regularly convex, clear, and perfectly pellucid The fibres of the iris were rather irregular; its brown colour not equally diffused. The pupil, which was uncommonly large, was not round, but drawn angularly downwards and inwards, neither altering in dimension with the movements of the eye, nor from the stimulus of light. On examining the eye by looking straight into it through the pupil, the anterior wall of the capsule was observed undestroyed, rendered opake in its whole extent, hypertrophied in several places, and of a colour and lustre like mother-of-pearl. On looking from the temporal side in an oblique direction into the pupil, there was visible in the anterior wall of the capsule a very small perpendicular cleft of about one line and a quarter in length. This cleft was situated so far from the centre of the pupil that it was entirely covered by the iris, and the inferior border being united to the uvea, it was kept a little open, so that the aqueous humour had free entrance into the cavity of the capsule. Except at the spot where the union of the capsule with the uvea took place (the cause of the angular form of the pupil), these two membranes were not in contact with each other. The patient only complained of an occasional sensation of pressure in the interior of the eye; otherwise the organ was free from pain. With this eye he had a perception of light, and was even capable of perceiving colours of an intense and decided tone. He believed himself moreover able to perceive about one third of a square inch of any bright object, if held at the distance of half an inch or an inch from the eye, and obliquely in such a direction as to reflect the light strongly towards the pupil. But this I am convinced was a mere delusion; for, from the state of the interior of the eye, as just described, it is evident that all rays of light falling in the direction of the optic axis in the pupil must be intercepted, and reflected by the opake capsule. By these rays, therefore, a perception of light indeed might be conveyed, but certainly no perception of objects. On the other hand, it seems probable that the lateral cleft in the capsule permitted rays of light to pass into the interior of the eye. But as this small aperture was situated entirely behind the iris, those rays only would have permeated which came in a very oblique direction from the temporal side. Admitting then these rays of light to pass through the cleft, still on account of their obliquity they must fall at a place situated about midway between the ciliar ligament and the centre of the posterior hemisphere, where, from the laws of optics, they could produce but a very imperfect image; and, owing to this imperfection of the image, it was impossible that the portion of the retina upon which these rays impinged could have obtained that acuteness of sensation which is essential for the mental perception of the image caused by an object. Nevertheless, we will assume that the cleft in the capsule held the same relation to the eye in this instance, as a small hole in a card placed immediately before a healthy eye; in this case the patient would not only have seen an object at the distance of half an inch or an inch, but even at a much greater distance. That he was incapable of this I have satisfied myself by repeated experiments, which have led me to the conclusion that his belief that he really saw objects resulted solely from his imagination, combined with his power of reasoning. In feeling an object and bringing it in contact with the eyelids and the cheek, while holding it close before his eye, by his refined sense of touch an idea of the object was produced, which was judged of and corrected according to the experience he had gained by constant practice. This opinion is confirmed by the observations of those who have known and watched him for years, and also by a fact which I have myself frequently observed, viz. that all well-educated blind persons, who are not absolutely amaurotic, endeavour to persuade others that they see more than they really can, in order to conceal as much as possible their deficiency in the noblest of the senses, and from a reluctance to be regarded as objects of compassion.

On terminating this inquiry into the condition of the visual organ and the actual state of vision, I may here be allowed to mention that the patient's sense of touch had attained an extraordinary degree of perfection, and that in order to examine an object minutely he conveyed it to his lips. The sensation produced by silk stuffs was most pleasing to him. He was said to possess the power of distinguishing colours by the touch, but this assertion was not confirmed by his own testimony.

After the examination above detailed, I gave my opinion that the defect of the right eye was irremediable; that the patient might obtain sight with the left eye by an operation; and that the disfigurement caused by the inversion of the eyes might also be removed by operation. Though the left eye had been considered incurable like the right, there appeared to me reasonable grounds to hope for a cure, provided I could succeed in keeping down inflammation, which is not easy to be done in an eye already several times operated upon, and especially in a young plethoric subject. The operation was then resolved upon.

On the 10th of July 1840, in the presence of Dr. Swaine, and with the kind assistance of Messrs. F. Fowke and F. Steinhaeuser, I made an incision in the cornea upwards, and introducing a pair of fine curved forceps, armed with teeth, into the posterior chamber, I seized the anterior wall of the capsule by passing one of the blades of the forceps into its small aperture, and attempted by pulling it slowly to separate it from its adhesion with the uvea and its peripheral connexion, in which I succeeded without producing a prolapsus of the vitreous body, or tearing the capsule, which I now removed. After this proceeding, a large piece of the lens of an opake colour, probably the nucleus, presented itself in the pupil, which was easily removed from the eye by means of DAVIEL's spoon; the pupillary aperture then appeared perfectly clear and black. The patient was now turned with his back to the light, for the purpose of trying a few experiments as to his sight, but from these I was obliged to desist on account of the pain which the light produced in the organ. Both eyes were then closed with narrow strips of court-plaster, and the patient carried to bed. Venesection, local bleeding, fomentations with iced water, continued without intermission for about forty-eight hours, together with the scrupulous observance of the most severe regimen, barely succeeded in keeping down the inflammation, the effects of which in this case, where but one eye offered hope, were much to be dreaded, if it should surpass that degree which was necessary for the healing of the wound in the cornea. This process went on and terminated so favourably, that the cicatrix, situated close to the sclerotica, is now scarcely visible. The patient suffered from muscæ volitantes and from a considerable intolerance of light, pain being produced by even a mild degree of light falling on the closed lids. The muscæ volitantes were greatly mitigated, and the intolerance of light ceased, after the lapse of a few weeks, by the use of proper pharmaceutical remedies, by local bleeding, change of air, &c., and the employment of the ophthalmic fountain of Professor JUNGKEN, which I have fully described in the Medical Gazette, vol. xxvii. p. 444. To promote the development of the power of vision, the use of the fountain was continued twice daily, with Pyrmont-water and latterly with simple spring-water, for the space of three months, when it was discontinued, as it began to irritate the eye.

Before I proceed further, I must again refer to the condition of sight previous to the operation. The right eye was completely amaurotic; in the left the power of vision existed, but, on account of the mechanical defect in the visual apparatus, was very little developed for the perception of light, and not at all for the perception of objects. It appeared to me, therefore, of the greatest interest to observe attentively the progressive development of the sensibility of the retina as regarded direct, refracted, reflected, and coloured rays of light; and also the progress of the visual perception in respect of the form, dimensions, and distance of objects. I was the more induced to undertake these physiological observations from having the opportunity of conducting them with an individual, who from his age, mental endowments, and education, offered peculiar advantages for such experiments.

On opening the eye for the first time on the third day after the operation, I asked the patient what he could see; he answered that he saw an extensive field of light, in which everything appeared dull, confused, and in motion. He could not distinguish objects. The pain produced by the light forced him to close the eye immediately. Two days afterwards, the eye, which had been kept closed by means of court-plaster, was again opened. He now described what he saw as a number of opake watery spheres, which moved with the movements of the eye, but, when the eye was at rest, remained stationary, and then partially covered each other. Two days after this the eye was again opened; the same phenomena were again observed, but the spheres were less opake and somewhat transparent; their movements more steady;

they appeared to cover each other more than before. He was now for the first time capable, as he said, to look through the spheres, and to perceive a difference, but merely a difference, in the surrounding objects. When he directed his eye steadily towards an object, the visual impression produced by the object was painful and very imperfect, and no clear visual perception of it took place, because the eye, on account of the intolerance of light, could not be kept open long enough for the formation of the idea as derived from visual sensation. The appearance of spheres diminished daily; they became smaller, clearer, and more pellucid, allowed objects to be seen more distinctly, and disappeared entirely after two weeks. The muscæ volitantes, which had the form of black, immoveable, and horizontal stripes, appeared, every time the eye was opened, in a direction upwards and inwards. When the eye was closed, he observed, especially in the evening, in an outward and upward direction, an appearance of dark blue, violet, and red colours; these colours became gradually less intense, were shaded into bright orange, yellow, and green, which latter colours alone eventually remained, and in the course of five weeks disappeared entirely.

As soon as the intolerance of light had so far abated that the patient could regard an object without pain and for a sufficient time to gain an idea of it, the following experiments were made in the presence of Dr. Swaine. The first experiments were of that class in which the idea of a visible object is derived merely from pure visual sensation; the succeeding, of that kind in which the idea, in ordinary cases, depends upon the sense of sight combined with the sense of touch, and is gained by reflecting on the impressions made on the organs of both senses. It was necessary to perform these experiments on different days, as otherwise they would have distressed the eye too much.

1st Experiment. Silk ribands of different colours, fastened on a black ground, were employed to show, first the primitive, and then the complementary colours. The patient recognized the different colours, with the exception of yellow and green, which he frequently confounded, but could distinguish when both were exhibited at the same time. He could point out each colour correctly when a variety was shown him at the same time. Grey pleased him best, because this colour he said produced an agreeable and grateful sensation; the effect of red, orange, and yellow was painful, but not disagreeable; that of violet and brown not painful, but very disagreeable; the latter he called ugly. Black produced subjective colours, and white occasioned the recurrence of muscæ volitantes in a most vehement degree.

2nd Experiment. The patient sat with his back to the light, and kept his eye closed. A sheet of paper, on which two strong black lines had been drawn, the one horizontal, the other vertical, was placed before him, at the distance of about three feet. He was now allowed to open the eye, and, after attentive examination, he called the lines by their right denominations. When I asked him to point out with his finger the horizontal line, he moved his hand slowly, as if feeling, and pointed to the vertical, but after a short time, observing his error, he corrected himself. The outline in black of a square, six inches in diameter, within which a circle had been drawn, and within the latter a triangle, was, after careful examination, recognized and correctly described by him. When he was asked to point out either of the figures, he never moved his hand directly and decidedly, but always as if feeling, and with the greatest caution; he pointed them out, however, correctly. A line consisting of angles, or in other words, a zigzag, and a spiral line, both drawn on a sheet of paper, he observed to be different, but could not describe them otherwise than by imitating their forms with his finger in the air. He said he had no idea of these figures.

3rd Experiment. The windows of the room were darkened, with the exception of one, towards which the patient, closing his eye, turned his back. At the distance of three feet and on a level with the eye, a solid *cube* and a *sphere*, each of four inches diameter, were placed before him. Allowing him to move the head in a lateral direction no further than was necessary to compensate the point of view of the right amaurotic eye, I now let him open his eye, and requested him to state decidedly what he observed. After attentively examining these bodies, he said he saw a *quadrangular* and a *circular* figure, and after some consideration he pronounced the one a *square* and the other a *disc*. His eye being then closed, the cube was taken away, and a disc of equal size substituted and placed next to the sphere. On again opening his eye, he observed no difference in these objects, but regarded them both as discs. The solid cube was now placed in a somewhat oblique position before the eye, and close beside it a figure cut out of pasteboard, representing a plane outline prospect of the cube when in this position. Both objects he took to be something like flat quadrates. A pyramid, placed before him with one of its sides towards his eye, he saw as a plain triangle. This object was now turned a little, so as to present two of its sides to view, but rather more of one side than of the other; after considering and examining it for a long time, he said that this was a very extraordinary figure; it was neither a triangle, nor a quadrangle, nor a circle; he had no idea of it, and could not describe it; "in fact," said he, "I must give it up." On the conclusion of these experiments, I asked him to describe the sensations the objects had produced, whereupon he said that immediately on opening his eye, he had discovered a difference in the two objects, the cube and the sphere, placed before him, and perceived that they were not drawings; but that he had not been able to form from them the idea of a square and a disc, until he perceived a sensation of what he saw in the points of his fingers, as if he really touched the objects. When I gave the three bodies (the sphere, cube, and pyramid) into his hand, he was much surprised that he had not recognized them as such by sight, as he was well acquainted with these solid mathematical figures by his touch. These experiments prove the correctness of the hypothesis I have advanced elsewhere on the well-known question put by Mr. MOLYNEUX to LOCKE, which was answered by both these gentlemen in the negative, and has been much discussed since their time.

4th Experiment. In a vessel, containing water to about the depth of one foot, was placed a musket-ball, and on the surface of the water a piece of pasteboard, of the same form, size, and colour as the ball. The patient could perceive no difference in the position of these bodies; he believed both to be upon the surface of the water.

Pointing to the ball, I desired him to take up this object; he made an attempt to take it from the plane of the water, but when he found he could not grasp it there, he said he had deceived himself, the objects were lying in the water; upon which I informed him of their real position. I now desired him to touch the ball, which lay in the water, with a small rod; he attempted this several times, but always missed his aim; he could never touch the object at the first movement of his hand towards it, but only by feeling about with the rod. On being questioned with respect to reflected light, he said that he was always obliged to bear in mind, that the looking-glass was fastened to the wall, in order to correct his idea of the apparent situation of objects behind the glass.

When the patient first acquired the faculty of sight, all objects appeared to him so near that he was sometimes afraid of coming in contact with them, though they were in reality at a great distance from him. He saw everything much larger than he had supposed from the idea obtained by his sense of touch. Moving, and especially living objects, such as men, horses, &c., appeared to him very large. If he wished to form an estimate of the distance of objects from his own person, or of two objects from each other, without moving from his place, he examined the objects from different points of view by turning his head to the right and to the left. Of perspective in pictures he had of course no idea; he could distinguish the individual objects in a painting, but could not understand the meaning of the whole picture; it appeared to him unnatural, for instance, that the figure of a man represented in the front of the picture should be larger than a house or a mountain in the background. All objects appeared to him perfectly flat; thus, although he very well knew by his touch that the nose was prominent, and the eyes sunk deeper in the head, he saw the human face only as a plane. Though he possessed an excellent memory, this faculty was at first quite deficient as regarded visible objects; he was not able, for example, to recognize visitors, unless he heard them speak, till he had seen them very frequently. Even when he had seen an object repeatedly, he could form no idea of its visible qualities in his imagination, without having the real object before him. Heretofore, when he dreamed of any persons, of his parents, for instance, he felt them and heard their voices, but never saw them; but now, after having seen them frequently, he saw them also in his dreams. The human face pleased him more than any other object presented to his view; the eyes he thought most beautiful, especially when in motion; the nose disagreeable, on account of its form and great prominence; the movement of the lower jaw in eating he considered very ugly. Although the newly-acquired sense afforded him many pleasures, the great number of strange and extraordinary sights was often disagreeable and wearisome to him; he said that he saw too much novelty which he could not comprehend. And even though he could see both near and remote objects very well, he would nevertheless continually have recourse to the use of the sense of touch.

On the 21st of September I operated, in the presence of several medical gentlemen, in one sitting, on both eyes for the congenital strabismus. The lids were fixed by the fingers of an assistant, the ball of the eye by a pair of forceps, and the tendon of the muscle divided by a pair of curved scissors. The rectus internus of the right eye was, like the organ itself, atrophied. The conjunctiva of the left eye was thickened at the inner angle; the muscle was uncommonly broad and thick; its tendon had a very broad attachment to the ball, and behind it was a separate bundle of muscular fibres attached to the sclerotica. The pupils of both eyes assumed immediately after the operation their proper position in the orbits. No inflammation ensued; not even in the left eye, which, from the prior operation, was still rather sensitive. The muscæ volitantes became less irksome, and the violent spasms which previously had affected not only the eyelids, but also the whole left side of the face, disappeared entirely. The right eye, which had been amaurotic, gained by this operation the power of perceiving light, so that when the left eye is closed, the patient can now distinguish light and shade, on the hand being moved before this eye. The sight of the left eye likewise was considerably improved in acuteness and clearness, both as regarded near and distant objects, but especially the latter. Objects now, however, appeared in a different situation to that which they really held; when, for instance, he directed his eye to an object situated immediately before him, he saw it more to his right, and, if he attempted to grasp it, he moved his hand in this wrong direction*. For this reason in walking across a room he always took a direction to the right, and consequently often came unawares in contact with articles of furniture, &c. This appearance of objects in false positions lasted for two months, after which time he was also capable of walking forwards in a straight direction. The right atrophied eye, which before the operation was deeply sunk in the orbit, is now more prominent, and appears therefore fuller and larger, so that the difference of the two eyes is less perceptible; he has consequently gained considerably in personal appearance. On one occasion when I was honoured with a visit from Mr. LAWRENCE, Dr. WATSON, Dr. KERRISON, and several other medical gentlemen at my residence, I introduced him to them for examination.

In the middle of October I let him try several pair of spectacles at Mr. A. Ross's, in Regent Street. With a double convex lens of $5\frac{1}{4}$ inch focus, he saw both near and remote objects of large size most clearly and distinctly, but for small objects he could find no glasses that improved his sight. He recognized the capitals of a large print with his naked eye, and on looking through a pin-hole made in a card held close before the eye, he could distinguish even the small letters of a very minute print. He had not yet learned to read. The reason for the condition of his sight with respect to small objects, and that his vision is better on cloudy days, is no doubt to be sought in the enlarged pupil and the immobility of the iris.

In the middle of November he was able without spectacles to read the names over the windows of the shops in the streets, and to tell the time to the minute by St.

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* This phenomenon I have observed in all eyes operated upon for strabismus of a great degree and long standing, when the other eye was closed. I have mentioned it in the Medical Gazette for June 1840, vol. xxvi. p. 540, where I have also given an explanation of the physiological cause.

Paul's clock. Walking alone in the crowded streets, especially in the City, he found very tedious. He said, seeing so many different things, and the quick movements of the multitude of people, carriages, &c., confused his sight to such a degree, that at last he could see nothing; that the sensation produced by the object last seen had not yet disappeared from the retina, when the next object made its impression thereon, by which means confusion of ideas, great anxiety, and even vertigo were occasioned, from which he could only free himself by closing his eyes for a few moments.

In the middle of December an experiment was again made with spectacles. A lens of seven inches focus was now of the same service as one of 5¼ inches had been two months before. After the operation for the strabismus he was accustomed, in speaking with any person, to turn his eye away from the face, as otherwise he said he felt disturbed by the looks of the person; he had now at length learned to look at the eyes of those with whom he conversed. The old habit of using the sense of touch to examine objects he had not yet entirely lost.

In the middle of February 1841, a third experiment was made with spectacles. A lens of ten inches focus was of the same service as one of seven inches had been on the last occasion, and one of 5¼ inches four months ago. This proves a slow, but positive amelioration of sight, and permits us to expect a still greater improvement, the more so as the patient has not passed the period of puberty. If the employment of spectacles were begun at the present period, although it is now more than seven months since the operation was performed, there would be no further amelioration of sight; the development of the visual apparatus would be arrested. I am therefore of opinion that the use of spectacles is not to be permitted, until it is, as it were, mathematically demonstrated by similar experiments with lenses, that the sight is no longer improved; by which means the faculty may in time, perhaps, reach such a degree of perfection as not to require any lens at all for remote objects.

This is the only case on record within my knowledge wherein, with a person born blind and afterwards successfully operated upon at a period of life as far advanced as in this instance, such experiments have ever been made. In the well-known case of Cheselden, published in the Philosophical Transactions for the year 1728 (page 447), the patient was only in the fourteenth year of his age, and although the case contains many highly interesting physiological observations, no series of systematic experiments was instituted. Beer has also made some interesting observations, which, however, like those made in rather a superficial manner by Janin and Daviel, tend principally to describe the impressions which the newly-acquired sense had made on the mind of the person operated upon. In Ware's case the patient was not born blind, but had become so at an early period of life. In the present paper I have merely given the simple history of the case, without making any remark on several points interesting to the pathologist and physiologist, to which I shall advert on a future occasion; the explanation and philosophy of the foregoing experiments as to the sense of sight I shall attempt in another paper, which I purpose to lay before this Society.

VII. *Additional Note on the Contraction of Voluntary Muscle in the Living Body.* *By* William Bowman, *Esq., F.R.S., Societ. Philomath. Paris. Corresp., Demonstrator of Anatomy in King's College, London, and Assistant Surgeon to the King's College Hospital.*

Received April 15,—Read April 29, 1841.

In my paper of June last, published in the Philosophical Transactions*, I showed, by observations on the *Rigor Mortis*, that contraction, in voluntary muscle, essentially consists of an approximation and change of form of the minute particles composing its structure; the phenomena of contraction in living Monoculi and Arguli were also briefly adverted to, but it remained undecided in what manner these minute movements are employed in the higher animals, in the production of motion *during life*. The almost insurmountable difficulty of submitting the living muscle of the Vertebrata to high powers of the microscope, so much enhances the value of any facts bearing on this obscure point, that I am induced to lay before the Society a short account of some recent examinations of human tetanic muscle, which, with the considerations accompanying them, appear to me to afford conclusive evidence on the subject.

Two opportunities have lately occurred to me of carefully observing the conditions of the muscular system, in cases of fatal tetanus, and the following has been the result:

1. Many muscles appear healthy in all respects.
2. Parts of certain muscles present a remarkably pale gray aspect, arising, doubtless, from their blood having been pressed out by the contraction, a state of which the appearance has been aptly compared by my friend Professor Budd, to that of the flesh of fishes.
3. In other situations, the muscles have lost in a great measure their fine fibrous character, and present a soft mottled surface, which readily tears, or receives an impression from the contact of the finger, a condition with which may be associated,
4. Extensive ecchymoses, often contrasting strangely with the pallor of contiguous portions.

On microscopic examination, while the other affected muscles appear natural, the primitive fasciculi of those which have lost their texture or are ecchymosed (3. 4.), are by no means so, but present at certain points characteristic marks of a high degree of contraction; they are swollen into a fusiform shape, and have their transverse striæ very much closer together than usual (Plate II. (*a*)). Elsewhere these primitive fasciculi are, on the contrary, diminished in diameter, and their

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* Part II., 1840.

transverse striæ either greatly widened and deranged (*b*), or altogether obliterated (*c*, *c*, *c*), in consequence of the whole texture of the organ being broken up into those primitive elements, of which the discs are constructed; and here the primitive fasciculi are frequently broken across, with or without a corresponding rupture of the sarcolemma (*d*, *e*). The extent of the swollen or contracted parts seems liable to great variety; the one selected for delineation (Pl. II.) contains upwards of sixty striæ, but others on contiguous primitive fasciculi were more extensive. Some primitive fasciculi in the neighbourhood, which at the point examined presented no rupture, had a very unusual diversity in the proximity of their striæ at different points, but everywhere preserved, like the rest, that proportion which I have shown to obtain between the diameter of the primitive fasciculi and the closeness of their transverse striæ. These I conclude to have been ruptured at a point further on.

Although the bare detail of these appearances may seem to warrant the conclusion that contractions have taken place in the situation of the fusiform or belly-like swellings, the effect of which has been to stretch and even to disorganize the remaining parts of the primitive fasciculi, yet I shall endeavour to confirm and illustrate it by the following considerations.

1. *The Contraction of a Muscle is the essential cause of its own rupture.*

This is best exemplified in a fragment of a primitive fasciculus of a reptile or fish removed from the body, and contracting between plates of glass. The contraction commences at its extremities, which, becoming swollen, receive the pressure of the upper plate, and may be fixed by it. If so, the intermediate part is stretched and torn as contraction proceeds; and if an isolated contraction occurs in the centre, the parts between it and the two extremities are similarly affected*, the conditions of the rupture being, 1. a partial contraction of the ruptured muscle, and, 2. a force superior to the tenacity of the uncontracted part, holding the ends of the fragment asunder.

The same conditions apply in the healthy living subject, where it is impossible, in consequence of the admirable adaptation of mechanical arrangements to the extensibility of muscles, that any rupture can take place solely from the action of antagonists. For example, no force of the flexors of the knee can by itself rupture the extensors, because the structure of the joint prevents flexion being carried beyond a point which the extensors, if relaxed, readily allow. And yet antagonist muscles may and do play a conspicuous part in most muscular ruptures; but it is only by affording a resistance to the approximation of the ends of the ruptured muscle, greater than the tenacity of its uncontracted parts,—such a resistance, in fact, as might be offered by any power mechanically adapted to produce the same effect. I say *uncontracted* parts, and the propriety of supposing them in the living subject appears from the examination of the tetanic muscle; for putting aside the impossibility of any rupture happening in a muscle of which no part was physically weaker than another,

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* *Loc. cit.*, p. 490.

by a contraction consisting of an *absolutely simultaneous and uniform approximation* of all its elementary parts to one another, and supposing for a moment that though the contraction was everywhere present, yet some feebleness of structure, or trifling diminution of the contractile force at one part, determined a rupture to take place there, the resulting appearances would necessarily be altogether different from those that have been detailed. The rupture would be definite and abrupt, without extensive stretching and consequent disorganization, and the whole retracting fibre would bear traces of an uniform and universal contraction, instead of an unequal and partial one, very limited in extent, and similar in every essential character to that which I have delineated in dying muscle*.

2. *There is no Repellent Force between the Contractile Elements of Muscular Fibre.*

When muscle is taken from the body after all irritability has subsided, and when no stimulant has been previously applied capable of disordering its action during the rigor mortis, the distance between the transverse striæ will usually be found nearly uniform at every part; but when partial contractions occur in a fragment that has been removed prior to the cessation of its irritability, the contracted parts remain permanently distinguished from the rest of the primitive fasciculus, by the closeness of their transverse striæ. In the former case, every primitive fasciculus having its extremities held apart by its proper antagonists, the contractile efforts constituting the rigor mortis are uniformly and gradually expended, and no inordinate amount of contraction can leave its vestiges in any part; but, in the latter, no such antagonizing power being exerted, the contractions remain wherever and to whatever degree they may have been present.

This explanation involves and rests upon the above principle, and it follows, that whatever prevents a muscular fibre from being elongated, when its contractile energy subsides, must cause it to retain that arrangement of its parts which was assumed during contraction. Now, in the examples under consideration, the *rupture* prevented such an elongation, and the result is, that *the organ has been, as it were, surprised in the very act of contraction, and retains in its structure the permanent impression of that act,*—a view strongly corroborated by the uniformity of the distance between the transverse striæ in those muscles, which had been likewise convulsed, but had escaped rupture.

It may be further remarked, that the occurrence of ecchymoses entirely accords with the idea of partial contractions, while it is inconsistent with that of an universal one, for how could the vessels be torn in tetanic spasm if this were merely a strong and enduring, but uniform approximation of all the elements of the primitive fasciculi to one another? They would be compressed indeed, but not dragged asunder; whereas, such would be the natural effect of excessive partial contractions, oscillating from place to place, and continually drawing in opposite directions, and in an irregular manner, the uncontracted portions; for the capillaries take a longitudinal course be-

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* *Loc. cit.*, Plate XIX. figs. 83. 88., No. 3. &c.

tween the primitive fasciculi, and inosculate with one another by very frequent transverse branches, which complete the vascular web, and serve to attach its several parts to the primitive fasciculi which occupy its interstices.

Lastly, all the specimens of bloodless tetanic muscle which I have seen, have presented this striking peculiarity:—that the pallor has not occupied the whole muscle, but patches of it, comprehending a portion of many primitive fasciculi, but not the entire length of any,—a fact tending to the same conclusion.

What has now been advanced, seems to render it—as it appears to me—certain, that the tetanic spasm has consisted in contractions engaging only parts of each primitive fasciculus at a time, and if so, of course changing their place, in order to bring every portion into use in its turn. Whether the primitive fasciculi alternate with one another in their contractions, is an obscure question, on which these observations shed no light.

It may be urged, however, that, even granting this conclusion true, it is unsafe to argue concerning the healthy and moderate actions of an organ from the phenomena it presents when in a morbid state. But the weight of this objection is more apparent than real, for in a physiological point of view, the contractions of tetanus differ from those properly termed voluntary, only in being uncontrollable by the will and excessive in amount and duration. In violent tetanic spasm, I have myself ascertained that the peculiar sound of voluntary contraction is audible in the part, and identical with the normal sound; and the appearance and feel of a muscle thus rigidly convulsed, can be perfectly simulated for a short period by an act of volition*.

I therefore conclude, from the whole of the preceding remarks, combined with the facts and arguments advanced in my former paper, that *the contraction of voluntary muscle is not a sustained act of the whole congeries of contractile elements composing it; but a rapid series of partial acts, in which all duly share, becoming by turns contracted and relaxed.*

King's College, London,
April 6, 1841.

The figure represents a portion of a primitive fasciculus, taken from among many others, from the complexus muscle, where it was ecchymosed and had lost in a great measure its fibrous appearance. At (*a*) a fusiform contracted portion, with the striæ remarkably close. On either hand the sarcous elements are much stretched (*b*), or even entirely disarranged (*c*, *c*, *c*), while at (*d*) there is a transverse rupture within the sarcolemma, and at (*e*) this sheath itself has given way.

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* I yesterday, in company with my friend, Professor Todd, discovered the same appearances of *partial* contractions in the recti muscles of the abdomen, ruptured by violent straining in diarrhœa. The voluntary muscles of the whole body were infested with the *Trichina Spiralis*, and were enfeebled.—*August* 2, 1841.

VIII. *Note on an inequality in the Height of the Barometer, of which the Argument is the Declination of the Moon. By Sir* J. W. Lubbock, *Bart., Treas. and V.P.R.S.*

Received March 16,—Read March 18, 1841.

In the Companion to the British Almanac for 1839, I inserted some results which were obtained with a view of ascertaining the influence of the moon on the barometer and on the dew-point. Mr. Luke Howard's researches on this subject having recalled my attention to that paper, I find some results which I then gave seem to indicate that the moon's position in declination influences the barometer. In order to render this more manifest, I shall now combine all the observations given in p. 3*, (and here recapitulated) in three categories. These observations correspond in part to different angular distances of the moon from the sun (or times of transit); but as the inequality of the Ocean, of which the argument is the moon's declination, is independent (or very nearly so) of the time of the moon's transit, it is probable that so also is that in the height of the barometer. In this case we may with propriety combine in the same category observations which correspond to similar *declinations*, although to different times of transit.

The following are the results:

No. of Observations.	Moon's Declination.	Moon's Parallax.	Height of Barometer.	Thermometer Attached.
	°	′	inch.	
78	21·1	57·4	30·063	69·0
167	15·1	56·9	30·000	67·7
93	4·4	56·5	29·060	67·7

This seems to indicate an elevation of nearly one-tenth of an inch for seventeen degrees of declination. The inequality has a contrary sign to the inequality of the same argument in the Tides of the Ocean.

First Category.

No. of Observations.	Moon's Declination.	Moon's Parallax.	Height of Barometer.	Thermometer Attached.
	°	′	inch.	
12	21·7	58·3	30·022	69·7
13	21·6	58·5	30·091	69·9
13	20·2	56·8	30·057	67·2
16	21·8	56·4	30·106	67·6
12	21·2	56·7	30·037	69·4
12	20·3	58·0	30·068	70·5
78	126·8	344·7	180·381	414·3
Average..	21·1	57·4	30·063	69·0

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* Companion to the British Almanac, 1839.

Second Category.

No. of Observations.	Moon's Declination.	Moon's Parallax.	Height of Barometer.	Thermometer Attached.
	°	′	inch.	
15	18·9	57·6	30·078	67·3
13	16·9	57·5	30·078	69·1
12	18·5	56·5	29·926	67·3
14	19·8	57·1	30·036	67·3
14	15·1	56·4	29·901	68·5
15	17·5	57·3	30·048	68·4
14	13·0	57·1	30·057	69·2
14	10·1	56·4	29·932	68·3
16	14·5	56·3	29·921	65·9
13	12·8	56·3	29·932	66·6
15	10·3	57·5	29·976	67·2
12	14·8	57·9	30·119	68·2
167	182·2	683·9	360·004	813·3
Average. .	15·1	56·9	30·000	67·7

Third Category.

No. of Observations.	Moon's Declination.	Moon's Parallax.	Height of Barometer.	Thermometer Attached.
	°	′	inch.	
15	7·0	56·7	30·007	69·7
15	·2	56·4	30·009	69·3
16	4·3	56·4	29·986	68·4
17	9·0	56·4	29·880	66·3
14	1·7	56·2	29·941	65·9
16	4·2	56·9	29·939	66·6
93	26·4	339·0	179·762	406·2
Average. .	4·4	56·5	29·960	67·7

IX. *On the Calculation of Attractions, and the Figure of the Earth.*

By C. J. HARGREAVE, *B.A., of University College, London. Communicated by* JOHN T. GRAVES, *Esq., A.M., F.R.S., of the Inner Temple.*

Received January 14,—Read February 11, 1841.

THE principal object of the calculations contained in the following paper, is to investigate the figure which a fluid, consisting of portions varying in density according to any given law, would assume, when every particle is acted upon by the attraction of every other and by a centrifugal force arising from rotatory motion. To what extent this may have been the original condition of the earth, is a doubtful question; and although observation does not fully warrant this supposition of the regular arrangement of parts, it has necessarily been made the foundation of most of the mathematical calculations connected with the investigation. Before proceeding to this problem, it is necessary to calculate the attraction of a body of any given figure, and consisting of strata, varying in their densities according to any given law; and it is in this problem that the principal difficulty lies. The elegant method of solution discovered by LAPLACE is well known; and I have followed his steps as far as the point where the equation, known by his name, first appears. In order to illustrate the nature of the deviation which I have there made, it will be necessary to mention some of the principal steps of the two methods.

By means of a theorem, which LAPLACE laid down as true of all spheroids that differ but little from spheres, and the properties of the integral of the equation referred to, he was enabled to substitute the easy rules of differentiation for the more complicated inverse processes, and thus to compute the attraction of that class of figures. It has, however, been since discovered by Mr. IVORY, that this theorem is true only of spheroids of a particular kind; and, consequently, to this kind the solution of the problem is restricted. This defect, and the indirectness of his analysis, led other mathematicians to consider the question; and, in 1811, Mr. IVORY published his method, which has the great advantage of being more direct, though equally limited.

The method given in the following paper does not appear to be confined in its operation to any particular class of spheroids; since the coefficients of the series, into which the required function is developed, are determined absolutely, without any reference to the form of the spheroid to which they are about to be applied. The principal change consists in the different manner of treating this partial differential equation. LAPLACE and the subsequent writers on this equation, both as applied to the calculation of attractions and the mathematical theory of electricity, suppose the coefficients of every term of the series to be expanded into another series of the sines and cosines of multiple arcs; and they avail themselves of the property which these terms possess of vanishing, in certain cases, when integrated between certain limits. The success of this plan, however, depends upon the restricting hypothesis above referred to, that the radius vector of the surface of the body is capable of expansion in a series of terms, each of which satisfies LAPLACE's equation. The following method shows that the coefficient of the general term of the first series is independent of one of the variables, and thus dispenses with the second series of expansions. This result I have arrived at, by first obtaining the integral of LAPLACE's equation in its most general form, and deducing the arbitrary functions introduced therein, from considerations which enter previous to the equation of the surface of the attracting body. These coefficients being known, it is evident that the attraction of any homogeneous body on a point within or without it may be immediately found when the equation of its surface is given, since it then depends only on a series of explicit and definite integrations of known functions, which can always be effected, at least approximately. From this, the attraction of a heterogeneous body, similarly circumstanced, may be found by the usual method of dividing it into concentric layers, and summing the several attractions of these, deduced as above.

By substituting the attraction so obtained, in the equation of equilibrium of a fluid body, CLAIRAUT's theorem is immediately deduced; and, from a peculiarity in the functions representing the attraction, it will be seen, that the same principles with longer processes may be carried on indefinitely, without the necessity of actually determining the precise form of those functions.

The restricted species of spheroid above referred to, comprises all surfaces of revolution; so that it is sufficiently extensive for most practical purposes; but the integration of LAPLACE's equation renders the analysis more direct, and the theory more complete.

On the General Problem of Attractions.

1. Let ϱ represent the density of a body at the point (x, y, z); and let f, g, h be the coordinates of a particle attracted by the body, parallel respectively to the axes x, y, z; then, if the power of attraction be inversely as the square of the distance, the resolved part of the attraction of the body, parallel to

$$x \text{ is} \iiint \frac{\varrho(f-x)\,dx\,dy\,dz}{\{(f-x)^2+(g-y)^2+(h-z)^2\}^{\frac{3}{2}}},$$

$$y \text{ is} \iiint \frac{\varrho(g-y)\,dx\,dy\,dz}{\{(f-x)^2+(g-y)^2+(h-z)^2\}^{\frac{3}{2}}},$$

$$z \text{ is} \iiint \frac{\varrho(h-z)\,dx\,dy\,dz}{\{(f-x)^2+(g-y)^2+(h-z)^2\}^{\frac{3}{2}}},$$

the limits of integration being determined by the equation to the surface of the body.

2. Let V represent the sum of the products of each particle by the reciprocal of its distance from the attracted point.

Then $V = \iiint \frac{\varrho\, dx\, dy\, dz}{\{(f-x)^2 + (g-y)^2 + (h-z)^2\}^{\frac{1}{2}}}$; and, by differentiating V, we obtain the well-known property $\frac{d^2 V}{d f^2} + \frac{d^2 V}{d g^2} + \frac{d^2 V}{d h^2} = 0$, or $-4\pi\varrho'$, according as the attracted particle is not or is within the attracting mass; ϱ' being in the latter case the density of the attracted particle*. By transforming these equations to polar coordinates, we obtain

$$\frac{d^2 V}{d r^2} + \frac{2}{r}\frac{d V}{d r} + \frac{1}{r^2}\frac{d^2 V}{d \theta^2} + r^2 \cot\theta \frac{d V}{d\theta} + \frac{1}{r^2 \sin^2\theta}\frac{d^2 V}{d\varphi^2} = 0, \text{ or } -4\pi\varrho',$$

and

$$V = \int_0^r \int_0^\pi \int_0^{2\pi} \frac{\varrho\, r'^2\, dr' \sin\theta'\, d\theta'\, d\varphi'}{\{r^2 + r'^3 - 2 r r' (\cos\theta \cos\theta' + \sin\theta \sin\theta' \cos(\varphi - \varphi'))\}^{\frac{1}{2}}};$$

where $r^2 = f^2 + g^2 + h^2$, $\cos\theta = \frac{h}{\sqrt{(f^2 + g^2 + h^2)}}$, $\tan\varphi = \frac{g}{f}$; and similar expressions are true of r' θ' and φ' in terms of x, y, z.

Put $\cos\theta = \mu$, and $\cos\theta' = \mu'$, and they become

$$r\frac{d^2 (r V)}{d r^2} + \frac{d}{d\mu}\left((1-\mu^2)\frac{d V}{d\mu}\right) + \frac{1}{1-\mu^2}\frac{d^2 V}{d\varphi^2} = 0, \text{ or } -4\pi\varrho' r^2 \dagger. \quad . \quad . \quad . \quad . \quad (1.)$$

$$V = \int_0^r \int_{-1}^1 \int_0^{2\pi} \frac{\varrho\, r'^2\, dr'\, d\mu'\, d\varphi'}{\{r^3 + r'^2 - 2 r r' (\mu\mu' + \sqrt{(1-\mu^2)}\sqrt{(1-\mu'^2)} \cos(\varphi - \varphi'))\}^{\frac{1}{2}}} \quad . \quad (2.)$$

3. Expansion by the binomial theorem shows that

$$\{r^2 + r'^2 - 2 r r' (\mu\mu' + \sqrt{(1-\mu^2)}\sqrt{(1-\mu'^2)} \cos(\varphi - \varphi'))\}^{\frac{1}{2}}$$

may be expressed either in powers of r or of r'; thus

$$P_0 \frac{1}{r'} + P_1 \frac{r}{r'^2} + \ldots + P_n \frac{r^n}{r'^{n+1}} + \ldots, \text{ or } P_0 \frac{1}{r} + P_1 \frac{r'}{r^2} + P_2 \frac{r'^2}{r^3} + \ldots + P_n \frac{r'^n}{r^{n+1}} + \ldots,$$

where P_n is a symmetrical function of μ, $\sqrt{(1-\mu^2)} \cos\varphi$, $\sqrt{(1-\mu^2)} \sin\varphi$ on the one part, and μ', $\sqrt{(1-\mu'^2)} \cos\varphi'$, $\sqrt{(1-\mu'^2)} \sin\varphi'$ on the other.

By substituting the first expansion in (2.), and the value of V so obtained in (1.), we have a series of equations

$$\int_0^r \int_{-1}^1 \int_0^{2\pi} \varrho \left\{\frac{d}{d\mu}\left((1-\mu^2)\frac{d P_n}{d\mu}\right) + \frac{1}{1-\mu^2}\frac{d^2 P_n}{d\varphi^2} + n(n+1) P_n\right\} \frac{dr'\, d\mu'\, d\varphi'}{r'^{n-1}}$$

$$= 0, \text{ or } -4\pi\varrho',$$

except when $n = 2$; and in all cases

$$\frac{d}{d\mu}\left((1-\mu^2)\frac{d P_n}{d\mu}\right) + \frac{1}{1-\mu^2}\frac{d^2 P_n}{d\varphi^2} + n(n+1) P_n = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.)$$

which is the equation of LAPLACE's coefficients.

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* Vide Pratt. Mec. Phil., § 168. LAPLACE, Méc. Cél. liv. iii.

† Vide Pratt. Mec. Phil., § 169.

4. This equation was not integrated; but by a skilful use of its properties, the problem of attractions was greatly simplified by LAPLACE. He laid down a theorem, respecting the surfaces of all spheroids of small deviation, that their radii vectores might be developed into series, every term of which would satisfy the above equation; and he also gave a method of expansion. By means of this theorem, the problem could be solved for spheroidal bodies which differ but little from spheres; but its generality has been greatly restricted by the researches of subsequent writers*, by whom it has been shown that it is true only for bodies whose radii are expressible in *rational and integral functions of* μ', $\surd(1-\mu'^2)\cos\varphi'$, $\surd(1-\mu'^2)\sin\varphi'$. Among these are the ellipsoid and elliptical spheroid, and a large class of other spheroids. In these papers I have adopted a different proceeding; I integrate the equation itself generally, and determine the arbitrary functions contained in the integrals by the circumstances of the problem itself. In consequence of the peculiar form which P_n then takes, V may be found by effecting the operations indicated, which are only explicit integrations.

5. I shall now proceed to integrate this equation.

Consider μ and φ as functions of two new variables X and Y, to be determined from the equations,

$$dX = \frac{dX}{d\varphi}(d\varphi + k\,d\mu)$$

$$dY = \frac{dY}{d\varphi}(d\varphi + k'\,d\mu),$$

where k and k' are the roots of the equation $(1-\mu^2)\,k^2 + \frac{1}{1-\mu^2} = 0$. These roots are $\frac{\pm\sqrt{-1}}{1-\mu^2}$, whence we obtain

$$X = \varphi + \tfrac{1}{2}\sqrt{-1}\log\frac{1+\mu}{1-\mu}, \text{ and } Y = \varphi - \tfrac{1}{2}\sqrt{-1}\log\frac{1+\mu}{1-\mu}. \quad . \quad . \quad (4.)$$

$$\frac{d^2P_n}{d\mu^2} = \left(\frac{dX}{d\mu}\right)^2\frac{d^2P_n}{dX^2} + 2\frac{dX}{d\mu}\frac{dY}{d\mu}\frac{d^2P_n}{dX\,dY} + \left(\frac{dY}{d\mu}\right)^2\frac{d^2P_n}{dY^2} + \frac{d^2X}{d\mu^2}\frac{dP_n}{dX} + \frac{d^2Y}{d\mu^2}\frac{dP_n}{dY},$$

$$= -\frac{1}{(1-\mu^2)^2}\frac{d^2P_n}{dX^2} + \frac{2}{(1-\mu^2)^2}\frac{d^2P_n}{dX\,dY} - \frac{1}{(1-\mu^2)^2}\frac{d^2P_n}{dY^2} + \frac{2\mu\sqrt{-1}}{(1-\mu^2)^2}\left(\frac{dP_n}{dX} - \frac{dP_n}{dY}\right);$$

$$\frac{dP_n}{d\mu} = \frac{dX}{d\mu}\frac{dP_n}{dX} + \frac{dY}{d\mu}\frac{dP_n}{dY} = \frac{\sqrt{-1}}{1-\mu^2}\left(\frac{dP_n}{dX} - \frac{dP_n}{dY}\right);$$

$$\frac{d^2P_n}{d\varphi^2} = \left(\frac{dX}{d\varphi}\right)^2\frac{d^2P_n}{dX^2} + 2\frac{dX}{d\varphi}\frac{dY}{d\varphi}\frac{d^2P_n}{dX\,dY} + \left(\frac{dY}{d\varphi}\right)^2\frac{d^2P_n}{dY^2} + \frac{d^2X}{d\varphi^2}\frac{dP_n}{dX} + \frac{d^2Y}{d\varphi^2}\frac{dP_n}{dY},$$

$$= \frac{d^2P_n}{dX^2} + 2\frac{d^2P_n}{dX\,dY} + \frac{d^2P_n}{dY^2}.$$

Substituting these in (3.), we obtain

$$\frac{4}{1-\mu^2}\frac{d^2P_n}{dX\,dY} + n(n+1)P_n = 0.$$

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* See two articles by Mr. IVORY in the Philosophical Transactions, 1812.

From (4.), by subtraction,

$$X - Y = \sqrt{-1}\log\frac{1+\mu}{1-\mu};$$

whence

$$\mu = \frac{\varepsilon^{-(X-Y)\sqrt{-1}} - 1}{\varepsilon^{-(X-Y)\sqrt{-1}} + 1},$$

and

$$1 - \mu^2 = \frac{4\,\varepsilon^{-(X-Y)\sqrt{-1}}}{\left(\varepsilon^{-(X-Y)\sqrt{-1}} + 1\right)^2} = \frac{4}{\left(\varepsilon^{-\frac{1}{2}(X-Y)\sqrt{-1}} + \varepsilon^{\frac{1}{2}(X-Y)\sqrt{-1}}\right)^2} = \frac{1}{\cos^2\frac{X-Y}{2}};$$

consequently

$$\frac{d^2 P_n}{dX\,dY} + \frac{n(n+1)P_n}{4\cos^2\frac{X-Y}{2}} = 0. \quad \text{Let } n(n+1) = a.$$

Let

$$\frac{dP_n}{dY} = v, \text{ then } \frac{dv}{dX} + \frac{a P_n}{4\cos^2\frac{X-Y}{2}} = 0, \text{ and } P_n = -\frac{dv}{dX}\frac{4}{a}\cos^2\frac{X-Y}{2};$$

$$v = \frac{dP_n}{dY} = -\frac{d^2 v}{dX\,dY}\frac{4}{a}\cos^2\frac{X-Y}{2} - \frac{dv}{dX}\frac{4}{a}\cos\frac{X-Y}{2}\sin\frac{X-Y}{2},$$

or

$$\frac{d^2 v}{dX\,dY} + \frac{dv}{dX}\tan\frac{X-Y}{2} + \frac{av}{4\cos^2\frac{X-Y}{2}} = 0.$$

Let

$$\frac{dv}{dY} + v\tan\frac{X-Y}{2} = t.$$

Then

$$\frac{d^2 v}{dX\,dY} + \frac{dv}{dX}\tan\frac{X-Y}{2} + \frac{v}{2\cos^2\frac{X-Y}{2}} = \frac{dt}{dX},$$

and

$$\frac{dt}{dX} + \frac{a-2}{4}\frac{v}{\cos^2\frac{X-Y}{2}} = 0; \text{ or } v = -\frac{4}{a-2}\cos^2\frac{X-Y}{2}\frac{dt}{dX};$$

whence

$$\frac{dv}{dY} = -\frac{4}{a-2}\cos^2\frac{X-Y}{2}\frac{d^2 t}{dX\,dY} - \frac{4}{a-2}\cos\frac{X-Y}{2}\sin\frac{X-Y}{2}\frac{dt}{dX};$$

$$t = -\frac{4}{a-2}\cos^2\frac{X-Y}{2}\frac{d^2 t}{dX\,dY} - \frac{8}{a-2}\cos\frac{X-Y}{2}\sin\frac{X-Y}{2}\frac{dt}{dX},$$

or

$$\frac{d^2 t}{dX\,dY} + \frac{dt}{dX}2\tan\frac{X-Y}{2} + t\frac{a-2}{4\cos^2\frac{X-Y}{2}} = 0.$$

Let

$$\frac{dt}{dY} + 2t\tan\frac{X-Y}{2} = q,$$

and by repeating a similar process, we obtain

$$\frac{d^2 q}{dX\,dY} + \frac{dq}{dX}\,3\tan\frac{X-Y}{2} + q\,\frac{a-6}{4\cos^2\frac{X-Y}{2}} = 0.$$

By observing the assumptions here made, and the results obtained, we find that in the first assumption $\left(\frac{dP_n}{dY} = v\right)$, the coefficient of $P_n \tan\frac{X-Y}{2}$ is 0; in the second, that of $v\tan\frac{X-Y}{2} = 1$; and so on, in the order of the natural numbers; and in the results, the numerical coefficients are $1, \frac{a}{4}$; $2, \frac{a-2}{4}$; $3, \frac{a-6}{4}$ ... generally $n, \frac{a-n(n-1)}{4}$. I shall prove this in the general case, by showing that if it is true of one value of n (as we see it is), it is true of the next value, and so on. Let the $(n-1)$th substitution give

$$\frac{d^2\varrho}{dX\,dY} + \frac{d\varrho}{dX}(n-1)\tan\frac{X-Y}{2} + \varrho\,\frac{a-(n-1)(n-2)}{4\cos^2\frac{X-Y}{2}} = 0,$$

and let

$$\frac{d\varrho}{dY} + (n-1)\,\varrho\tan\frac{X-Y}{2} = s:$$

then, as before,

$$\frac{d^2\varrho}{dX\,dY} + (n-1)\tan\frac{X-Y}{2}\cdot\frac{d\varrho}{dX} + \frac{n-1}{2}\,\frac{1}{\cos^2\frac{X-Y}{2}}\cdot\varrho = \frac{ds}{dX};$$

and, therefore,

$$\frac{ds}{dX} + \varrho\,\frac{a-(n-1)n}{4\cos^2\frac{X-Y}{2}} = 0, \text{ and } \varrho = -\frac{ds}{dX}\,\frac{4}{a-(n-1)n}\cos^2\frac{X-Y}{2};$$

$$\frac{d\varrho}{dY} = \frac{d^2 s}{dX\,dY}\,\frac{4}{a-(n-1)n}\cos^2\frac{X-Y}{2} - \frac{ds}{dX}\,\frac{4\cos\frac{X-Y}{2}\cdot\sin\frac{X-Y}{2}}{a-(n-1)n}.$$

Consequently

$$-\frac{d^2 s}{dX\,dY}\,\frac{4}{a-(n-1)n}\cos^2\frac{X-Y}{2} - \frac{ds}{dX}\,\frac{4}{a-(n-1)n}\cos\frac{X-Y}{2}\cdot\sin\frac{X-Y}{2}$$
$$-\frac{ds}{dX}\,\frac{4(n-1)}{a-(n-1)n}\cos\frac{X-Y}{2}\cdot\sin\frac{X-Y}{2} = s,$$

or

$$\frac{d^2 s}{dX\,dY}\,\frac{4}{a-(n-1)n}\cos^2\frac{X-Y}{2} + \frac{ds}{dX}\,\frac{4n}{a-(n-1)n}\cos\frac{X-Y}{2}\cdot\sin\frac{X-Y}{2} + s = 0;$$

that is,

$$\frac{d^2 s}{dX\,dY} + \frac{ds}{dX}\,n\tan\frac{X-Y}{2} + s\,\frac{a-n(n-1)}{4}\cos^2\frac{X-Y}{2} = 0,$$

and, therefore, the law of coefficients, as above stated, is correct.

Restoring the value of a, we get

$$\frac{d^2 s}{d\mathrm{X}\,d\mathrm{Y}} + \frac{ds}{d\mathrm{X}}\,n \tan\frac{\mathrm{X}-\mathrm{Y}}{2} + \frac{s\,.\,n}{2}\,\frac{1}{\cos^2\frac{\mathrm{X}-\mathrm{Y}}{2}} = 0,$$

which is integrable; and its integral is $\frac{ds}{d\mathrm{Y}} + n\,.\,s \tan\frac{\mathrm{X}-\mathrm{Y}}{2}$ = some arbitrary function of Y, as $\chi\,\mathrm{Y}$. Integrate again, then

$$s = \varepsilon^{-\int n \tan\frac{\mathrm{X}-\mathrm{Y}}{2}\,d\mathrm{Y}}\left(\int \varepsilon^{\int n \tan\frac{\mathrm{X}-\mathrm{Y}}{2}\,d\mathrm{Y}}\,\chi\mathrm{Y}\,d\mathrm{Y} + \psi\mathrm{X}\right),$$

where ψ is arbitrary. Effecting these integrations, and reducing,

$$s = \cos^{-2n}\frac{\mathrm{X}-\mathrm{Y}}{2}\left(\int \cos^{2n}\frac{\mathrm{X}-\mathrm{Y}}{2}\,.\,\chi\mathrm{Y}\,d\mathrm{Y} + \psi\mathrm{X}\right).$$

To return to P_n, we have the following systems of equations:

$\mathrm{P}_n = \int v\,d\mathrm{Y}$,

$$\begin{aligned}
v &= \varepsilon^{-\int \tan\frac{\mathrm{X}-\mathrm{Y}}{2}\,d\mathrm{Y}}\left(\int t\,\varepsilon^{\int \tan\frac{\mathrm{X}-\mathrm{Y}}{2}\,d\mathrm{Y}}\,d\mathrm{Y}\right) = \cos^{-2}\frac{\mathrm{Y}-\mathrm{X}}{2}\int t\cos^2\frac{\mathrm{Y}-\mathrm{X}}{2}\,d\mathrm{Y},\\
t &= \qquad\qquad\qquad\qquad\qquad\qquad\quad \cos^{-4}\frac{\mathrm{Y}-\mathrm{X}}{2}\int q\cos^4\frac{\mathrm{Y}-\mathrm{X}}{2}\,.\,d\mathrm{Y},\\
&\ \vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots \qquad\qquad \vdots\\
\varrho &= \qquad\qquad\qquad\qquad\qquad\qquad\quad \cos^{-2(n-1)}\frac{\mathrm{Y}-\mathrm{X}}{2}\int s\cos^{2(n-1)}\frac{\mathrm{Y}-\mathrm{X}}{2}\,.\,d\mathrm{Y},\\
s &= \qquad\qquad\qquad\qquad\qquad\qquad\quad \cos^{-2n}\frac{\mathrm{Y}-\mathrm{X}}{2}\left(\int \cos^{2n}\frac{\mathrm{Y}-\mathrm{X}}{2}\,.\chi\mathrm{Y}\,d\mathrm{Y} + \psi\mathrm{X}\right);
\end{aligned}$$

whence

$$\mathrm{P}_n = \ldots\ldots\int \cos^{-2}\frac{\mathrm{Y}-\mathrm{X}}{2}\int \cos^{-2}\frac{\mathrm{Y}-\mathrm{X}}{2}\left(\int \cos^{2n}\frac{\mathrm{X}-\mathrm{Y}}{2}\,\chi\mathrm{Y}\,d\mathrm{Y} + \psi\mathrm{X}\right)d\mathrm{Y}\,d\mathrm{Y}\ldots(n \text{ times.})$$

Now $\cos\frac{\mathrm{Y}-\mathrm{X}}{2} = \cos\left(\sqrt{-1}\,\log\sqrt{\frac{1+\mu}{1-\mu}}\right) = \frac{1}{2}\left(\sqrt{\frac{1+\mu}{1-\mu}} + \sqrt{\frac{1-\mu}{1+\mu}}\right)$,

and $\cos^2\frac{\mathrm{Y}-\mathrm{X}}{2} = \frac{1}{1-\mu^2}$; and the complete integral will be expressed, by substituting for X and Y in terms of μ and ϕ.

6. But an important point yet remains to be determined. The original equation, being a partial differential equation of the second order, can only involve in its integral two arbitrary functions. But here, after $\chi\,\mathrm{Y}$ and $\psi\,\mathrm{X}$ have come in by two integrations, we have n integrations to perform with respect to Y. It would seem, therefore, that no constant or arbitrary function of X must be added in these integrations.

Such is not the case. At each integration a function of X must be added, and these functions determined by reference to the original differential equation*.

7. Returning to the value of P_n, we have

$$P_0 = \chi_1\left(\varphi - \frac{1}{2}\sqrt{-1}\log\frac{1+\mu}{1-\mu}\right) + \psi\left(\varphi + \frac{1}{2}\sqrt{-1}\log\frac{1+\mu}{1-\mu}\right).$$

Now in the calculation of attractions, where P_0 is the coefficient of r^0 in the expansion of

$$\left\{r^2 + r'^2 - rr'\left(\mu\mu' + \sqrt{(1-\mu^2)}\sqrt{(1-\mu'^2)}\cos(\varphi - \varphi')\right)\right\}^{-\frac{1}{2}},$$

we know that it is 1; consequently

$$\chi_1\left(\varphi - \frac{1}{2}\sqrt{-1}\log\frac{1+\mu}{1-\mu}\right) + \psi\left(\varphi + \frac{1}{2}\sqrt{-1}\log\frac{1+\mu}{1-\mu}\right) = 1,$$

and expanding by TAYLOR's theorem, we get

$$\left.\begin{aligned}
&\chi_1\varphi - \chi'_1\varphi\,\frac{1}{2}\sqrt{-1}\log\frac{1+\mu}{1-\mu} - \frac{\chi''_1\phi}{2}\left(\frac{1}{2}\log\frac{1+\mu}{1-\mu}\right)^2\\
&\qquad + \frac{\chi'''_1\phi}{2\,.\,3}\sqrt{-1}\left(\frac{1}{2}\log\frac{1+\mu}{1-\mu}\right)^3 + \&c.\\
&+ \psi\varphi + \psi'\varphi\,\frac{1}{2}\sqrt{-1}\log\frac{1+\mu}{1-\mu} - \frac{\psi''\phi}{2}\left(\frac{1}{2}\log\frac{1+\mu}{1-\mu}\right)^2\\
&\qquad - \frac{\psi'''\phi}{2\,.\,3}\sqrt{-1}\left(\frac{1}{2}\log\frac{1+\mu}{1-\mu}\right)^3 + \&c.
\end{aligned}\right\} = 1.$$

By equating the coefficients of the same powers of $\frac{1}{2}\log\frac{1+\mu}{1-\mu}$, we have $\chi_1\varphi + \psi\varphi = 1$, and $\psi'\varphi - \chi'_1\varphi = 0$, or $\psi\varphi - \chi_1\varphi = \text{constant}$.

Therefore $\psi\varphi$ and $\chi_1\varphi$ are absolute constants, and their sum is 1; whence it follows that $\chi\varphi = 0$. Let $\psi\phi = \frac{c}{2}$, then

$$P_n = c\left(\ldots\int\cos^{-2}\frac{Y-X}{2}\int\cos^{-2}\frac{Y-X}{2}\int\frac{1}{2}\cos^{-2}\frac{Y-X}{2}\,dY\,dY\ldots(n\text{ times})\right).$$

Effecting these integrations, we find that P_n consists of a series of powers of

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* The common differential equation $(1-\mu^2)\frac{d^2P_n}{d\mu^2} - 2\mu\frac{dP_n}{d\mu} + n(n+1)P_n = 0$ will illustrate this point. Let $P_n = \frac{d^{-n}z}{d\mu^{-n}}$, and after substitution, differentiate n times; then $(1-\mu^2)\frac{d^2z}{d\mu^2} - 2(n+1)\mu\frac{dz}{d\mu} = 0$, whence $z = \int\frac{k\,d\mu}{(1-\mu^2)^{n+1}} + m$. It is clear that no more *arbitrary* constants than k and m can be introduced; and yet if the integrals were left indefinite, we might obtain an integral of an expression which should differ from the integral of the same expression obtained by a slightly different process, by a constant. By another integration this would cease to be a constant, and we should obtain thus different values for P_n. The fact is, that constants must be added at each integration, and recourse had to the original equation, to determine them in terms of m, k, and μ.

$\tan \frac{\mathrm{Y}-\mathrm{X}}{2}$, whose coefficients may, for anything we yet know, be functions of X. The following process shows that constants as coefficients will satisfy the original equation, and determines them. The integration itself gives the coefficient of the highest power.

Let then*

$$\mathrm{P}_n = c \left\{ \begin{array}{l} \frac{2^{n-1}}{[n]} \tan^n \frac{\mathrm{Y}-\mathrm{X}}{2} + c' \tan^{n-1} \frac{\mathrm{Y}-\mathrm{X}}{2} + c'' \tan^{n-2} \frac{\mathrm{Y}-\mathrm{X}}{2} \\ + \ldots + c^{(n-2)} \tan^2 \frac{\mathrm{Y}-\mathrm{X}}{2} + c^{(n-1)} \tan \frac{\mathrm{Y}-\mathrm{X}}{2} + c^{(n)} \end{array} \right\}.$$

Then

$$\frac{d\,\mathrm{P}_n}{d\,\mathrm{Y}} = c \left[\begin{array}{l} \frac{2^{n-2}}{[n-1]} \tan^{n-1} \frac{\mathrm{Y}-\mathrm{X}}{2} + \frac{n-1}{2} c' \tan^{n-2} \frac{\mathrm{Y}-\mathrm{X}}{2} \\ + \frac{n-2}{2} c'' \tan^{n-3} \frac{\mathrm{Y}-\mathrm{X}}{2} + \ldots . + \frac{2}{2} c^{(n-2)} \tan \frac{\mathrm{Y}-\mathrm{X}}{2} \\ + \frac{1}{2} c^{(n-1)} \end{array} \right] \left(1 + \tan^2 \frac{\mathrm{Y}-\mathrm{X}}{2}\right),$$

$$= c \left\{ \begin{array}{l} \frac{2^{n-2}}{[n-1]} \tan^{n+1} \frac{\mathrm{Y}-\mathrm{X}}{2} + \frac{n-1}{2} c' \tan^n \frac{\mathrm{Y}-\mathrm{X}}{2} + \left(\frac{n-2}{2} c'' + \frac{2^{n-2}}{[n-1]}\right) \tan^{n-1} \frac{\mathrm{Y}-\mathrm{X}}{2} \\ \quad + \left(\frac{n-3}{2} c''' + \frac{n-1}{2} c'\right) \tan^{n-2} \frac{\mathrm{Y}-\mathrm{X}}{2} \\ + \left(\frac{n-4}{2} c^{\mathrm{iv}} + \frac{n-2}{2} c''\right) \tan^{n-3} \frac{\mathrm{Y}-\mathrm{X}}{2} + \ldots + \left(\frac{3}{2} c^{(n-3)} + \frac{5}{2} c^{(n-5)}\right) \tan^4 \frac{\mathrm{Y}-\mathrm{X}}{2} \\ \quad + \left(\frac{2}{2} c^{(n-2)} + \frac{4}{2} c^{(n-4)}\right) \tan^3 \frac{\mathrm{Y}-\mathrm{X}}{2} \\ + \left(\frac{1}{2} c^{(n-1)} + \frac{3}{2} c^{(n-3)}\right) \tan^2 \frac{\mathrm{Y}-\mathrm{X}}{2} + \frac{2}{2} c^{(n-2)} \tan \frac{\mathrm{Y}-\mathrm{X}}{2} + \frac{1}{2} c^{(n-1)} \end{array} \right\}.$$

$$\frac{d^2\,\mathrm{P}_n}{d\,\mathrm{X}\,d\,\mathrm{Y}} = - \frac{c}{\cos^2 \frac{\mathrm{Y}-\mathrm{X}}{2}} \left\{ \begin{array}{l} \frac{(n+1)\,2^{n-3}}{[n-1]} \tan^n \frac{\mathrm{Y}-\mathrm{X}}{2} + \frac{n\,(n+1)}{4} c' \tan^{n-1} \frac{\mathrm{Y}-\mathrm{X}}{2} \\ + \frac{n-1}{2} \left(\frac{n-2}{2} c'' + \frac{2^{n-2}}{[n-1]}\right) \tan^{n-2} \frac{\mathrm{Y}-\mathrm{X}}{2} \\ + \frac{n-2}{2} \left(\frac{n-3}{2} c''' + \frac{n-1}{2} c'\right) \tan^{n-3} \frac{\mathrm{Y}-\mathrm{X}}{2} \\ + \ldots + \frac{4}{2} \left(\frac{3}{2} c^{(n-3)} + \frac{5}{2} c^{(n-5)}\right) \tan^3 \frac{\mathrm{Y}-\mathrm{X}}{2} \\ + \frac{3}{2} \left(\frac{2}{2} c^{(n-2)} + \frac{4}{2} c^{(n-4)}\right) \tan^2 \frac{\mathrm{Y}-\mathrm{X}}{2} \\ + \frac{2}{2} \left(\frac{1}{2} c^{(n-1)} + \frac{3}{2} c^{(n-3)}\right) \tan \frac{\mathrm{Y}-\mathrm{X}}{2} - \frac{1}{2} \frac{2}{2} c^{(n-2)} \end{array} \right\},$$

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* $[n] = 1 . 2 . 3 . 4 \ldots . n.$

$$n\frac{n+1}{4}\mathrm{P}_n\frac{1}{\cos^2\frac{\mathrm{Y}-\mathrm{X}}{2}}=\frac{c}{\cos^3\frac{\mathrm{Y}-\mathrm{X}}{2}}\left\{\begin{array}{l}\frac{(n+1)2^{n-3}}{[n-1]}\tan^n\frac{\mathrm{Y}-\mathrm{X}}{2}+\frac{n(n+1)}{4}c'\tan^{n-1}\frac{\mathrm{Y}-\mathrm{X}}{2}\\ +\frac{n(n+1)}{4}c''\tan^{n-2}\frac{\mathrm{Y}-\mathrm{X}}{2}+\ldots\\ \ldots+\frac{n(n+1)}{4}c^{(n-1)}\tan\frac{\mathrm{Y}-\mathrm{X}}{2}+\frac{n(n+1)}{4}c^{(n)}\end{array}\right\}.$$

Whence we obtain $c'=0$, $c'''=0$, $c^{\mathrm{v}}=0$, &c.

Also

$$\left(\frac{n(n+1)}{4}-\frac{(n-1)(n-2)}{4}\right)c''=\frac{2^{n-3}}{[n-2]};\quad\left(\frac{n(n+1)}{4}-\frac{(n-3)(n-4)}{4}\right)c^{\mathrm{iv}}=\frac{(n-3)(n-2)}{4}c'';$$

$$\left(\frac{n(n+1)}{4}-\frac{(n-5)(n-6)}{4}\right)c^{\mathrm{vi}}=\frac{(n-5)(n-4)}{4}c^{\mathrm{iv}};\ \text{\&c. \&c.}$$

Consequently

$$\mathrm{P}_n=c\left\{\begin{array}{l}\frac{2^{n-1}}{[n]}\tan^n\frac{\mathrm{Y}-\mathrm{X}}{2}+\frac{2^{n-1}}{2[n-2](2n-1)}\tan^{n-2}\frac{\mathrm{Y}-\mathrm{X}}{2}\\ +\frac{2^{n-1}}{2.4.[n-4](2n-1)(2n-3)}\tan^{n-4}\frac{\mathrm{Y}-\mathrm{X}}{2}\\ +\frac{2^{n-1}\tan^{n-6}\frac{\mathrm{Y}-\mathrm{X}}{2}}{2.4.6.[n-6].(2n-1)(2n-3).(2n-5)}+\ldots\end{array}\right\}.$$

Now

$$\frac{\mathrm{Y}-\mathrm{X}}{2}=-\tfrac{1}{2}\sqrt{-1}\log\frac{1+\mu}{1-\mu}=\tan^{-1}\left(-\mu\sqrt{-1}\right),\ldots\tan\frac{\mathrm{Y}-\mathrm{X}}{2}$$
$$=\left(-\mu\sqrt{-1}\right);$$

whence, finally,

$$\mathrm{P}_n=2^{n-1}c\left(\frac{(-\mu\sqrt{-1})^n}{[n]}+\frac{(-\mu\sqrt{-1})^{n-2}}{2.[n-2].(2n-1)}+\frac{(-\mu\sqrt{-1})^{n-4}}{2.4.[n-4](2n-1)(2n-3)}+\ldots\right),$$

a remarkable result, showing that in this instance P_n is independent of ϕ.

P_n being free from ϕ, is a perfectly symmetrical function of μ and μ'; and μ' is a constant with respect to μ; therefore

$$\mathrm{P}_n=\mathrm{K}_n\left(\frac{(-\mu\sqrt{-1})^n}{[n]}+\frac{(-\mu\sqrt{-1})^{n-2}}{2[n-2](2n-1)}+\frac{(-\mu\sqrt{-1})^{n-4}}{2.4.[n-4](2n-1)(2n-3)}+\ldots\right)\times$$
$$\left(\frac{-\mu'\sqrt{-1})^n}{[n]}+\frac{(-\mu'\sqrt{-1})^{n-2}}{2.[n-2](2n-1)}+\ldots\right).$$

To determine K_n for any particular value of n, we refer to the expression from which the two series were deduced; namely,

$$\left\{r^2+r'^2-2rr'\left(\mu\mu'+\sqrt{(1-\mu^2)}\sqrt{(1-\mu'^2)}\cos(\phi-\phi')\right)\right\}^{-\frac{1}{2}}.$$

When μ and μ' are each 1, then $\mathrm{P}_n=1$, which gives an equation to find K_n.

8. Returning to the theory of attractions, we have, when the particle is internal,

$$V = \int_r^R \int_{-1}^1 \int_0^{2\pi} \varrho\, r' \left(P_0 + P_1 \frac{r}{r'} + \ldots + P_n \frac{r^n}{r'^n} + \ldots\right) dr'\, d\mu'\, d\varphi'$$

$$= 2\pi \int_r^R \int_{-1}^1 \varrho \left(P_0 r' + P_1 r + P_2 \frac{r^2}{r'} + \ldots + P_n \frac{r^n}{r'^{n-1}} + \ldots\right) dr'\, d\mu',$$

for that portion of the body which is comprised between a sphere of radius r, and the surface of the body, supposed to be a surface of revolution round the axis of z; for, in that case R, the value of r' at the surface, is independent of φ'.

9. Suppose, for example, we wish to find the attraction of a homogeneous spheroid, on a point within it. In this case ϱ is constant, and

$$R^2 = \left(\frac{1-\mu'^2}{a^2} + \frac{\mu'^2}{c^2}\right)^{-1};$$

a being the semi-major, and c the semi-minor axis.

First, all the even terms vanish; for the general even term is

$$2\pi\varrho \int_r^R \int_{-1}^1 P_{2n+1} \frac{r^{2n+1}}{r'^{2n}}\, dr'\, d\mu' = -\frac{2\pi\varrho\, r^{2n+1}}{2n-1} \int_{-1}^1 P_{2n+1} \left(\frac{1-\mu'^2}{a^2} + \frac{\mu'^2}{c^2}\right)^{\frac{2n-1}{2}} d\mu'$$

$$+ \frac{2\pi\varrho\, r^2}{2n-1} \int_{-1}^1 P_{2n+1}\, d\mu'.$$

Now P_{2n+1} consists of odd powers of μ'; and $\left(\frac{1-\mu'^2}{a^2} + \frac{\mu'^2}{c^2}\right)^{\frac{2n-1}{2}}$ can be expanded in even powers of μ'; therefore the integral of the product (which is an odd function), taken from $\mu' = -1$ to $\mu' = 1$, is 0. Also $\int_{-1}^1 P_{2n+1}\, d\mu' = 0$*. All the odd terms above the third vanish; for the $(2n + 1)$th term is

$$2\pi\varrho \int_r^R \int_{-1}^1 P_{2n} \frac{r^{2n}}{r'^{2n-1}}\, dr'\, d\mu' = -\frac{\pi\varrho}{n-1} \int_{-1}^1 P_{2n}\, r^{2n} \left(\frac{1-\mu'^2}{a^2} + \frac{\mu'^2}{c^2}\right)^{n-1} d\mu'$$

$+ \frac{\pi\rho}{n-1} \int_{-1}^1 P_{2n}\, r^2\, d\mu'$, for it may be shown that $\int_{-1}^1 \int_0^{2\pi} P_\iota P_{\iota'}\, d\mu'\, d\varphi' = 0$, if ι and ι' be different integers. Now when n is greater than 1, $\left(\frac{1-\mu'^2}{a^2} + \frac{\mu'^2}{c^2}\right)^{n-1}$ is a rational and entire function of μ', and, therefore, capable of being expressed in a series of LAPLACE's coefficients†, the highest of which will be of the $(2n - 2)$th order; and therefore no term of this expansion can be of the same order as P_{2n}; and the integral of the product of any two of different orders, between these limits, vanishes. So the second member of this vanishes.

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* See Pratt. Mec. Phil., § 180.

† See Pratt. Mec. Phil., § 176. POISSON, Théorie Math. de la Chal., chap. viii. LAPLACE, Méc. Cél. liv. iii. chap. ii.

The first term is

$$2\pi\varrho\int_r^{\mathrm{R}}\int_{-1}^{1} r'\,dr'\,d\mu' = \pi\varrho\int_{-1}^{1}\left(\frac{1-\mu'^2}{a^2}+\frac{\mu'^2}{c^3}\right)^{-1}d\mu' - \pi\varrho r^2\int_{-1}^{1}d\mu'$$

$$= 2\pi\varrho\left(a^2\frac{\sqrt{1-e^2}}{e} - r^2\right),\ (e\text{ being the eccentricity}).$$

The third term is

$$\frac{\pi}{2}\mathrm{K}_2\varrho\int_r^{\mathrm{R}}\int_{-1}^{1}\left(\frac{1}{3}-\mu^2\right)\left(\frac{1}{3}-\mu'^2\right)\frac{r^2}{r'}\,dr'\,d\mu'$$

$$= -\frac{\pi\varrho}{4}\mathrm{K}_2\int_{-1}^{1}\left(\frac{1}{3}-\mu^2\right)\left(\frac{1}{3}-\mu'^2\right)r^2\log\left(\frac{1-\mu'^2}{a^2}+\frac{\mu'^2}{c^2}\right)d\mu',$$

$$= -\pi\varrho\frac{\mathrm{K}_2}{2}\left(\frac{1}{3}-\mu^2\right)r^2\left(\frac{2}{3}\frac{c\,a^2}{(a^2-c^2)^{\frac{3}{2}}}\tan^{-1}\frac{\sqrt{(a^2-c^2)}}{c} - \frac{2}{3}\frac{c^2}{a^2-c^2} - \frac{4}{9}\right),$$

$$= -3\pi\varrho\left(\frac{1}{3}-\mu^2\right)r^2\left(\frac{1}{3}-\frac{1}{e^2}+\frac{\sqrt{(1-e^2)}}{e^3}\sin^{-1}e\right),\ \text{for } \mathrm{K}_2 = 9.$$

The value of V for the sphere of radius r, calculated by the usual method, is $\frac{4\pi\rho r^2}{3}$[*]; consequently, for the whole ellipsoid, the value of V is

$$\frac{4\pi\rho r^2}{3} + 2\pi\varrho\left(a^2\frac{\sqrt{(1-e^2)}}{e}\sin^{-1}e - r^2\right) - 3\pi\varrho\left(\frac{1}{3}-\mu^2\right)r^2\left(\frac{1}{3}-\frac{1}{e^2}+\frac{\sqrt{(1-e^2)}}{e^3}\sin^{-1}e\right).$$

Differentiate to f, by means of the equations $r^2 = f^2 + g^2 + h^2$ and $\mu = \frac{h}{r}$, and we have

$$\left.\begin{aligned} -\frac{d\mathrm{V}}{df} &= \text{attraction in } x = +2\pi\varrho f\left(1-\frac{1}{e^2}+\frac{\sqrt{(1-e^2)}}{e^3}\sin^{-1}e\right); \\ \text{so } -\frac{d\mathrm{V}}{dg} &= \text{attraction in } y = +2\pi\varrho g\left(1-\frac{1}{e^2}+\frac{\sqrt{(1-e^2)}}{e^3}\sin^{-1}e\right); \\ \text{so } -\frac{d\mathrm{V}}{dh} &= \text{attraction in } y = +4\pi\varrho h\left(+\frac{1}{e^2}-\frac{\sqrt{(1-e^2)}}{e^3}\sin^{-1}e\right); \end{aligned}\right\}$$

which are the common expressions otherwise found[†].

Also

$$-\frac{d\mathrm{V}}{dr} = \text{attraction to centre} = +4\pi\varrho\left\{\frac{r}{3}+\frac{3}{2}r\left(\frac{1}{3}-\mu^2\right)\left(\frac{1}{3}-\frac{1}{e^2}+\frac{\sqrt{(1-e^2)}}{e^3}\sin^{-1}e\right)\right\}.$$

10. By a similar process, I have deduced the attraction of an oblate spheroid, on a point within it; the density varying inversely as the distance from the centre. The corresponding expressions are

$$-\frac{d\mathrm{V}}{df} = +2\pi\varrho\frac{f}{r} - \frac{3}{4}\frac{\pi\varrho f}{a(1-e^2)}\left\{\frac{1}{e^2}-\frac{1}{3}-\left(\frac{(1-e^2)^2}{2e^3}+\frac{2}{3}\frac{1-e^2}{e}\right)\log\frac{1+e}{1-e}\right\};$$

$$-\frac{d\mathrm{V}}{dg} = +2\pi\varrho\frac{g}{r} - \frac{3}{4}\frac{\pi\varrho g}{a(1-e^2)}\left\{\frac{1}{e^2}-\frac{1}{3}-\left(\frac{(1-e^2)^2}{2e^3}+\frac{2}{3}\frac{1-e^2}{e}\right)\log\frac{1+e}{1-e}\right\};$$

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[*] See Pratt. Mec. Phil., § 172.

[†] Ibid. § 158.

$$-\frac{d\mathrm{V}}{dh} = +2\pi\varrho\frac{h}{r} + \frac{3}{2}\frac{\pi\varrho h}{a(1-e^2)}\left\{\frac{1}{e^2} - \frac{1}{3} - \left(\frac{(1-e^2)^2}{2e^3} + \frac{2}{3}\frac{1-e^2}{e}\right)\log\frac{1+e}{1-e}\right\};$$

$$-\frac{d\mathrm{V}}{dr} = +2\pi\varrho + \frac{3}{4}\frac{\pi\varrho(3\mu^2-1)}{a(1-e^2)}\left\{\frac{1}{e^2} - \frac{1}{3} - \left(\frac{(1-e^2)^2}{2e^3} + \frac{2}{3}\frac{1-e^2}{e}\right)\log\frac{1+e}{1-e}\right\}.$$

11. When the particle attracted is external, the series in (3.) does not give a finite expression. Instead of taking it separately, I make it a case of a general theorem which follows.

12. To find the attraction of a spheroid on a point within it, the density being any function of the distance from the centre, and the eccentricity being small.

Let $\varrho\,\varphi\,r'$ represent the law of density; then the value of V, for the portion comprised between the surface and a sphere of radius r, is

$$2\pi\varrho\int_r^{\mathrm{R}}\int_{-1}^{1}\varphi r'.r'\left(\mathrm{P}_0 + \mathrm{P}_1\frac{r}{r'} + \ldots + \mathrm{P}_n\frac{r^n}{r'^n} + ..\right)dr'\,d\mu'.$$

Integrating by parts we have

$$\int\varphi r'.r'^{1-n}.dr' = \frac{\phi_{\prime}r'}{r'^{n-1}} + (n-1)\frac{\phi_{\prime\prime}r'}{r'^n} + (n-1)n\frac{\phi_{\prime\prime\prime}r'}{r'^{n+1}} + (n-1)n(n+1)\frac{\phi_{\text{iv}}r'}{r'^{n+1}} + \&\text{c}.$$

Therefore the $(n+1)$th term of V is

$$2\pi\varrho r^n\int_{-1}^{1}\mathrm{P}_n\left(\frac{\phi_{\prime}\mathrm{R}}{\mathrm{R}^{n-1}} + (n-1)\frac{\phi_{\prime\prime}\mathrm{R}}{\mathrm{R}^n} + (n-1)n\frac{\phi_{\prime\prime\prime}\mathrm{R}}{\mathrm{R}^{n+1}} + (n-1)n(n+1)\frac{\phi_{\text{iv}}\mathrm{R}}{\mathrm{R}^{n+2}} + ..\right)d\mu'$$

$$-2\pi\varrho r^n\int_{-1}^{1}\mathrm{P}_n\left(\frac{\phi_{\prime}r}{r^{n-1}} + (n-1)\frac{\phi_{\prime\prime}r}{r^n} + (n-1)n\frac{\phi_{\prime\prime\prime}r}{r^{n+1}} + (n-1)n(n+1)\frac{\phi_{\text{iv}}r}{r^{n+2}} + \ldots\right)d\mu'.$$

Now $\mathrm{R} = \left\{\frac{1}{a^2} + \left(\frac{1}{c^2} - \frac{1}{a^2}\right)\mu'^2\right\}^{-\frac{1}{2}} = a(1+e^2\mu'^2)^{-\frac{1}{2}} = a\left(1 - \frac{e^2}{2}\mu'^2\right)$, rejecting e^4 and higher powers of e; and $\varphi_{(m)}\mathrm{R} = \varphi_{(m)}a - \varphi_{(m-1)}a.\frac{ae^2\mu'^2}{2}$ to the same degree of accuracy. The last member of the expression for V need only be calculated when $n = 0$; for all the rest of the terms $\left(\text{involving}\int_{-1}^{1}\mathrm{P}_n\,d\mu' \text{ where } n > 0\right)$ vanish.

The first member need only be calculated when $n = 0$, and when $n = 2$; for when n is odd, it vanishes as before; and also when n is even and greater than 2: for the functions of R involve no higher powers of μ' than the square; and consequently they vanish, when multiplied by P_4, P_6, .. &c., and integrated with respect to μ', from -1 to $+1$*.

When $n = 0$, the term is

$$2\pi\varrho\int_{-1}^{1}(\mathrm{R}\varphi_{\prime}\mathrm{R} - \varphi_{\prime\prime}\mathrm{R})\,d\mu' - 2\pi\varrho\int_{-1}^{1}(r\varphi_{\prime}r - \varphi_{\prime\prime}r)\,d\mu',$$

$$= 2\pi\varrho\int_{-1}^{1}\left(a\varphi_{\prime}a - \varphi a.\frac{a^2e^2\mu'^2}{2} - \varphi_{\prime\prime}a\right)d\mu' - 4\pi\varrho(r\varphi_{\prime}r - \varphi_{\prime\prime}r),$$

$$= 4\pi\varrho\left(a\varphi_{\prime}a - \varphi_{\prime\prime}a - \frac{e^2}{6}a^2\varphi a - (r\phi_{\prime}r - \phi_{\prime\prime}r)\right).$$

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* See Art. 9.

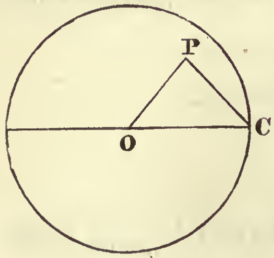
When $n = 2$, the term is

$$\frac{9}{2}\pi\varrho r^2\left(\mu^2-\frac{1}{3}\right)\int_{-1}^{1}\left(\mu'^2-\frac{1}{3}\right)\left(\frac{\phi_{\prime}\mathrm{R}}{\mathrm{R}}+\frac{\phi_{\prime\prime}\mathrm{R}}{\mathrm{R}^2}+2\,\frac{\phi_{\prime\prime\prime}\mathrm{R}}{\mathrm{R}^3}+2\,.\,3\,\frac{\phi_{\mathrm{iv}}\mathrm{R}}{\mathrm{R}^4}+\ldots\right)d\mu'$$

$$=\frac{9}{2}\pi\varrho r^2\left(\mu^2-\frac{1}{3}\right)\Sigma_1^{\infty}[m-1]\int_{-1}^{1}\left(\mu'^2-\frac{1}{3}\right)\frac{\phi_m\mathrm{R}}{\mathrm{R}^m}\,d\mu',$$

$$=\frac{9}{2}\pi\varrho r^2\left(\mu^2-\frac{1}{3}\right)\Sigma_1^{\infty}[m-1]\int_{-1}^{1}\left(\mu'^2-\frac{1}{3}\right)\left(\frac{\phi_m a}{a^m}-\frac{\phi_{m-1}a}{a^{m-1}}\cdot\frac{e^2\mu'^2}{2}+\frac{\phi_m a}{a^m}\cdot\frac{m\,e^2\mu'^2}{2}\right)d\mu',$$

$$=\frac{2}{5}\pi\varrho r^2\left(\mu^2-\frac{1}{3}\right)e^2\,\Sigma_1^{\infty}[m-1]\left(\frac{\phi_m a}{a^m}\,.\,m-\frac{\phi_{m-1}a}{a^{m-1}}\right)=-\frac{2}{5}\pi\varrho r^2\left(\mu^2-\frac{1}{3}\right)e^2\varphi a.$$



To calculate V for the sphere whose radius is r.

Let the sphere be referred to polar coordinates, the centre being the pole.

Let $\mathrm{OC} = r$, $\mathrm{OP} = r_1$, and $\mathrm{POC} = \theta$; then $\mathrm{PC} = \surd(r^2+r_1^2-2rr_1\cos\theta)$.

Mass of the element at $\mathrm{P} = \varrho\, r_1^2\,\varphi r_1\,dr_1\sin\theta\,d\theta\,d\omega$, and

$$\mathrm{V}=\int_0^r\int_0^{\pi}\int_0^{2\pi}\frac{\rho r_1^2\phi r_1\,dr_1\sin\theta\,d\theta\,d\omega}{\surd(r^2+r_1^2-2rr_1\cos\theta)}=2\pi\varrho\int_0^r\int_0^{\pi}\frac{r_1^2\phi r_1\sin\theta\,dr_1\,d\theta}{\surd(r^2+r_1^2-2rr_1\cos\theta)}$$

$$=2\pi\varrho\int_0^r\frac{r_1\phi_1 r_1}{r}\Big((r+r_1)-(r-r_1)\Big)dr_1=\frac{4\pi\rho}{r}\int_0^r r_1^2\varphi r_1\,dr_1$$

$$=\frac{4\pi\rho}{r}(r^2\varphi_{\prime}r-2r\varphi_{\prime\prime}r+2\varphi_{\prime\prime\prime}r-\mathrm{K}),$$

K being the value of $r_1^2\varphi_1 r_1-2r_1\varphi_{\prime\prime}r_1+2\varphi_{\prime\prime\prime}r_1$, when $r_1=0$.

The whole value of V then is

$$4\pi\varrho\left\{a\phi_{\prime}a-\phi_{\prime\prime}a-\frac{1}{6}e^2a^2\phi a-\left(\phi_{\prime\prime}r-\frac{2}{r}\phi_{\prime\prime\prime}r+\frac{\mathrm{K}}{r}\right)-\frac{1}{10}r^2\left(\mu^2-\frac{1}{3}\right)e^2\phi a\right\}.$$

And the attraction toward the centre

$$=-\frac{d\mathrm{V}}{dr}=4\pi\varrho\left\{\frac{\psi r}{r^2}+\frac{1}{5}\left(\mu^2-\frac{1}{3}\right)r\,e^2\phi a\right\},$$

where

$$\psi r=2\phi_{\prime\prime\prime}r-2r\phi_{\prime\prime}r+r^2\phi_{\prime}r-\mathrm{K}=\int_0^r r^2\phi r\,dr.$$

13. To find the attraction of the same spheroid on a particle without it.

The series (Art. 3.) is

$$2\pi\varrho\int_0^{\mathrm{R}}\int_{-1}^{1}\phi r'\left(\mathrm{P}_0\frac{r'^2}{r}+\mathrm{P}_1\frac{r'^3}{r^2}\,.+\,.\,.\right)dr'\,d\mu'.$$

Now

$$\int\phi r'\,.\,r'^{n+1}\,.\,dr'=r'^{n+1}\phi_{\prime}r'-(n+1)r'^n\phi_{\prime\prime}r'+n(n+1)r'^{n-1}\varphi_{\prime\prime\prime}r'-\&c.\ (\mathrm{A});$$

and the general (nth) term of V is

$$\frac{2\pi\rho}{r^n}\int_{-1}^{1}\mathrm{P}_{n-1}\left(r'^{n+1}\varphi_{\prime}r'-(n+1)r'^n\phi_{\prime\prime}r'+n(n+1)r'^{n-1}\phi_{\prime\prime\prime}r'-\&c.\right)d\mu'$$

$$-\frac{2\pi\rho}{r^n}\int_{-1}^{1}\mathrm{P}_{n-1}\,\mathrm{C}\,d\mu',$$

where C is the value of (A) when $r' = 0$. As before, the last term need be calculated only when $n = 1$; and the first when $n = 1$, and $n = 3$.

When $n = 1$, it is

$$\frac{2\pi\rho}{r}\int_{-1}^{1}(\mathrm{R}^2\,\phi_{\prime}\,\mathrm{R} - 2\,\mathrm{R}\,\phi_{\prime\prime}\,\mathrm{R} + 2\,\phi_{\prime\prime\prime}\,r - \mathrm{K})\,d\,\mu' = \frac{2\pi\rho}{r}\int_{-1}^{1}\psi\,r\,d\,\mu'$$

$$= \frac{4\pi\rho}{r}\left(\psi\,a - \frac{e^2}{6}\,a^3\,\phi\,a\right).$$

When $n = 3$, it is

$$\frac{9}{2}\left(\mu^2 - \frac{1}{3}\right)\frac{\pi\rho}{r^3}\int_{-1}^{1}d\,\mu'\left(\mu'^2 - \frac{1}{3}\right)\Big(\mathrm{R}^4\,\phi_{\prime}\,\mathrm{R} - 4\,\mathrm{R}^3\,\phi_{\prime\prime}\,\mathrm{R} + 12\,\mathrm{R}^2\,\phi_{\prime\prime\prime}\,\mathrm{R} - 24\,\mathrm{R}\,\phi_{\text{iv}}\,\mathrm{R}$$

$$+ 24\,\phi_{\text{v}}\,\mathrm{R}\Big) = -\frac{2}{5}\left(\mu^2 - \frac{1}{3}\right)\frac{\pi\rho\,a^5}{r^3}\,\phi\,a\,.\,e^2.$$

The whole value of V is

$$\frac{4\pi\rho}{r}\left\{\psi\,a - \frac{e^2}{6}\,a^3\,\phi\,a - \frac{1}{10}\left(\mu^2 - \frac{1}{3}\right)\frac{a^5}{r^2}\,\phi\,a\,.\,e^2\right\}.$$

And

$$-\frac{d\,\mathrm{V}}{d\,r} = \frac{4\pi\rho}{r^2}\left\{\psi\,a - \frac{e^2}{6}\,a^3\,\phi\,a - \frac{3}{10}\left(\mu^2 - \frac{1}{3}\right)\frac{a^5}{r^2}\,\phi\,a\,.\,e^2\right\}.$$

14. Instead of the eccentricity $e = \sqrt{\left(1 - \frac{c^2}{a^2}\right)}$ it will be more convenient to employ the ellipticity $\varepsilon = 1 - \frac{c}{a}$*. These give $\varepsilon = \frac{e^2}{2}$. And the values of V become for an internal point,

$$4\pi\varrho\left\{a\,\phi_{\prime}\,a - \phi_{\prime\prime}\,a - \left(\phi_{\prime\prime}\,r - \frac{2}{r}\,\phi_{\prime\prime\prime}\,r + \frac{\mathrm{K}}{r}\right) - \frac{\varepsilon}{3}\,a^2\,\phi\,a - \frac{r^2}{5}\left(\mu^2 - \frac{1}{3}\right)\varepsilon\,\phi\,a\right\};$$

for an external point,

$$4\pi\varrho\frac{1}{r}\left\{\psi\,a - \frac{\varepsilon}{3}\,a^3\,\phi\,a - \frac{3}{5}\,\frac{1}{r^2}\left(\mu^2 - \frac{1}{3}\right)a^5\,\phi\,a\right\}.$$

15. To find the attraction on the supposition that the body is composed of spheroidal layers, homogeneous in themselves, but differing from one another in density and ellipticity.

First, on an internal point.

Let r', as before, be the radius vector of any layer; a' its equatorial radius; $\varrho\,\phi\,a'$ its density, and ε' its ellipticity, being some function of a' as $\chi\,a'$. Then

$$a' = r'\,(1 + \varepsilon'\,\mu'^2) \text{ and } \phi\,a' = \phi\,r' + r'\,\phi'\,r'\,\chi\,r'\,\mu'^2 = \phi\,r' + \mathrm{F}\,r'\,.\,\mu'^2, \text{ suppose.}$$

Consequently to the term before produced in (12.) by $\phi\,r'$ we must add a term similarly produced by $\mathrm{F}\,r'\,.\,\mu'^2$. Also, instead of taking, in the first instance, the portion comprised between the surface and a sphere of radius r, we must take the por-

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* See PUISSANT, vol. i. p. 259, where the word ellipticity is used in this sense.

tion between the surface and that spheroid on which the point lies, whose ellipticity is ε_1. The $(n+1)$th term of V now becomes

$$2\pi\varrho r^n \int_{-1}^{1} \mathrm{P}_n \, d\mu' \left\{ \int_r^{\mathrm{R}} \frac{\phi r'}{r'^{n-1}} \, dr' + \mu'^2 \int_r^{\mathrm{R}} \frac{\mathrm{F} r'}{r'^{n-1}} \, dr' \right\}.$$

The first part of this gives $4\pi\varrho \left\{ \begin{array}{l} a\varphi_{,} a - \varphi_{,,} a - \frac{\varepsilon}{3} a^2 \varphi a - \frac{\varepsilon}{5}\left(\mu^2 - \frac{1}{3}\right) r^2 \varphi a \\ -\left(\mathrm{a}\varphi_{,}\mathrm{a} - \varphi_{,,}\mathrm{a} - \frac{\varepsilon_1}{3}\mathrm{a}^2 \varphi \mathrm{a} - \frac{\varepsilon_1}{5}\left(\mu^2 - \frac{1}{3}\right) r^2 \varphi \mathrm{a}\right) \end{array} \right\}$,

a being semiaxis major of the stratum on which r lies. To determine the other part, it is necessary to compute it when $n = 0$ and $n = 2$, which gives

$$4\pi\varrho \left\{ \frac{1}{3} \int_{\mathrm{a}}^{a} a' \mathrm{F} a' \, da' + \frac{1}{5} r^2 \left(\mu^2 - \frac{1}{3}\right) \int_{\mathrm{a}}^{a} \frac{\mathrm{F} a'}{a'} \, da' \right\}.$$

To the sum of these we must add the value of V for the inner spheroid; and for this purpose we have to obtain V for an external point.

To the expression in (13.) we must add

$$\frac{2\pi\varrho}{r^n} \int_{-1}^{1} d\mu' \left(\mu'^2 \mathrm{P}_{n-1} \int_0^{\mathrm{R}} \mathrm{F} r' \, r'^{n+1} \, dr' \right),$$

to be calculated when $n = 1$ and $n = 3$. This is

$$4\pi\varrho \left\{ \frac{1}{3r} \int_0^{a} \mathrm{F} a' . a'^2 \, da' + \frac{\mu^2 - \frac{1}{3}}{5r^3} \int_0^{a} \mathrm{F} a' . a'^4 \, da' \right\}.$$

The whole value of V is

$$\frac{4\pi\varrho}{r} \left\{ \begin{array}{l} \psi a - \frac{\varepsilon}{3} a^3 \varphi a - \frac{1}{5}\left(\mu^2 - \frac{1}{3}\right) \varepsilon \frac{a^5}{r^2} \varphi a + \frac{1}{3} \int_0^{a} \mathrm{F} a' . a'^2 \, da' \\ + \frac{1}{5} \frac{\mu^2 - \frac{1}{3}}{r^2} \int_0^{a} \mathrm{F} a' . a'^4 \, da' \end{array} \right\}.$$

After writing a for a, and ε_1 for ε, add this to the other value of V, and apply the equations

$$\int_{\mathrm{a}}^{a} \mathrm{F} a' . a' \, da' = \int_{\mathrm{a}}^{a} a'^2 \varepsilon' \, d\varphi a' = a^2 \varepsilon \varphi a - \mathrm{a}^2 \varepsilon_1 \varphi \mathrm{a} - \int_{\mathrm{a}}^{a} \varphi a' \, d(a'^2 \varepsilon'),$$

$$\frac{1}{r} \int_0^{\mathrm{a}} \mathrm{F} a' . a'^2 \, da' = \frac{1}{r} \int_0^{\mathrm{a}} a'^3 \varepsilon' \, d\varphi a' = \mathrm{a}^2 \varepsilon_1 \varphi \mathrm{a} - \frac{1}{r} \int_0^{\mathrm{a}} \varphi a' \, d(a'^3 \varepsilon'),$$

and similar equations for the other integrals; and we shall obtain

$$\mathrm{V} = 4\pi\varrho \left\{ \begin{array}{l} a\varphi_{,} a - \varphi_{,,} a - \mathrm{a}\varphi_{,}\mathrm{a} + \varphi_{,,}\mathrm{a} + \frac{\psi \mathrm{a}}{r} - \frac{1}{3}\left\{ \int_{\mathrm{a}}^{a} \varphi a' \, d(a'^2 \varepsilon') + \frac{1}{r} \int_0^{\mathrm{a}} \varphi a' \, d(a'^3 \varepsilon') \right\} \\ - \frac{r^2}{5}\left(\mu^2 - \frac{1}{3}\right) \left\{ \int_{\mathrm{a}}^{a} \varphi a' \, d\varepsilon' + \frac{1}{r^5} \int_0^{\mathrm{a}} \varphi a' \, d(a'^5 \varepsilon') \right\} \end{array} \right\}.$$

16. To find the equation of equilibrium of a heterogeneous spheroidal mass of fluid, revolving about its axis, with an angular velocity ω.

By the principles of hydrostatics the general equation is $\int \frac{dp}{\varrho'} = \int (\mathrm{X}\,dx + \mathrm{Y}\,dy + \mathrm{Z}\,dz)$, ϱ' being the density at the point (x, y, z), p the pressure, and X, Y, Z the sums of the resolved parts of the forces; which are $-\frac{d\mathrm{V}}{dx}$, $-\frac{d\mathrm{V}}{dy}$, $-\frac{d\mathrm{V}}{dz}$, and the centrifugal force. Let the axis of z be that of rotation; then the centrifugal force is $\omega^2 x$ along x, and $\omega^2 y$ along y. Let us express ω in terms of the ratio of the centrifugal force at the equator to the equatorial gravity. Call this ratio m, which is small in the case of the earth, being of the same order as ε. Then

$$m = \frac{\omega^2 a^3}{\text{mass}} = \frac{\omega^2 a^3}{4\pi\varrho\psi a}, \text{ or } \omega^2 = \frac{4\pi\varrho m\psi a}{a^3}.$$

Therefore

$$\mathrm{X} = \frac{d\mathrm{V}}{dx} + \frac{4\pi\varrho m\psi a . x}{a^3},\ \mathrm{Y} = \frac{d\mathrm{V}}{dy} + \frac{4\pi\varrho m\psi a . y}{a^3},\ \mathrm{Z} = \frac{d\mathrm{V}}{dz},$$

and

$$\int \frac{dp}{\varrho'} = \mathrm{V} + \frac{2\pi\varrho m\psi a}{a^3}(1 - \mu^2)\, r^2.$$

Now $\int \frac{dp}{\rho'}$ is a constant for a level surface. Hence for any stratum we have

$$\mathrm{C} = \mathrm{V} + \frac{2\pi\rho m\psi a}{a^3}(1 - \mu^2)\, r^2.$$

At the surface this is

$$\mathrm{C} = \frac{\psi a}{r} - \frac{1}{3r}\int_0^a \phi a'\, d\,(a'^3 \varepsilon') - \frac{1}{5}\left(\mu^2 - \frac{1}{3}\right)\frac{1}{r^3}\int_0^a \phi a'\, d\,(a'^5 \varepsilon') + \frac{m}{2}\frac{\psi a}{a^3}(1 - \mu^2) r^2,$$

$$= \frac{1}{3r}\,\mathrm{M}a - \frac{1}{5r^3}\left(\mu^2 - \frac{1}{3}\right)\mathrm{N}a + \frac{1}{3}\frac{mr^2}{2a^3}\,\mathrm{M}a\,(1 - \mu^2),$$

where

$$\mathrm{M}a = \int_0^a \phi a' \frac{d\,(a'^3(1 - \varepsilon'))}{da'}\, da', \text{ and } \mathrm{N}a = \int_0^a \phi a' \frac{d\,(a'^5 \varepsilon')}{da'}\, da'.$$

For r write $a\,(1 - \varepsilon\mu^2)$, then

$$\mathrm{C} = \frac{\mathrm{M}a}{3a}(1 + \varepsilon\mu^2) - \frac{1}{5}\left(\mu^2 - \frac{1}{3}\right)\frac{\mathrm{N}a}{a^3} + \frac{1}{6}m\,(1 - \mu^2)\frac{\mathrm{M}a}{a}.$$

Equate the coefficients of μ^2, then

$$\frac{\mathrm{M}a}{a}\left(\varepsilon - \frac{m}{2}\right) = \frac{3\,\mathrm{N}a}{5a^3} \ldots . \text{ (B)}.$$

17. By differentiating and changing the sign of $\int \frac{dp}{\rho'}$, we obtain the amount of gravity which acts towards the centre; which, to the order we are now considering, is the same as the whole force of gravity; since the cosine of the angle of the vertical differs from unity only by terms of a higher order.

Consequently

$$g = 4\pi\varrho\left\{\frac{\mathrm{M}a}{3r^2} - \frac{3}{5}\left(\mu^2 - \frac{1}{3}\right)\frac{\mathrm{N}a}{r^4} - \frac{m}{3}(1-\mu^2)\frac{r\mathrm{M}a}{a^3}\right\},$$

$$= 4\pi\varrho\left\{\frac{\mathrm{M}a}{3a^2}(1 + 2\varepsilon\mu^2) - \frac{3}{5}\left(\mu^2 - \frac{1}{3}\right)\frac{\mathrm{N}a}{a^4} - \frac{m}{3}(1-\mu^2)\frac{\mathrm{M}a}{a^2}\right\},$$

$$= 4\pi\varrho\frac{\mathrm{M}a}{3a^2}\left\{1 + 2\varepsilon\mu^2 + (1 - 3\mu^2)\left(\varepsilon - \frac{m}{2}\right) - m(1-\mu^2)\right\}\text{(by B.)},$$

$$= 4\pi\varrho\frac{\mathrm{M}a}{3a^2}\left\{1 + \varepsilon - \frac{3m}{2} + \mu^2\left(\frac{5}{2}m - \varepsilon\right)\right\} = \mathrm{G}\left\{1 + \sin^2 l\left(\frac{5}{2}m - \varepsilon\right)\right\},$$

where G is the equatorial gravity and l the latitude.

18. From this it appears that up to terms of the 1st order, $\mathrm{R} = a(1 - \varepsilon\sin^2 l)$ is the equation of the curve which generates the surface of equilibrium, where the value of ε depends on m, or on the velocity of rotation: but as the coefficients of higher powers of $\sin l$ may be considerable, it will be useful to find the surface of equilibrium to a greater degree of exactness. For this purpose we must introduce the fourth power of $\sin l$, whose coefficient will be of the second order. Let the equation of the strata be $a' = r'(1 + \varepsilon'\mu^2 + \mathrm{A}'\mu^4)$, or $r' = a'\left(1 - \varepsilon'\mu^2 + (\varepsilon'^2 - \mathrm{A}')\mu^4\right)$, where ε' and A' are functions of a' as $\chi a'$ and $\theta a'$. Then

$$\phi a' = \phi r' + \mathrm{F}r'\mu^2 + r^2\frac{d(\overline{\chi r}^3 . \phi' r)}{dr}\frac{\mu^4}{2} + r\phi' r\,\theta r\,\mu^4 = \phi r + \mathrm{F}r.\mu^2 + \Pi r.\mu^4, \text{ suppose.}$$

The $(n+1)$th term of V for an internal point now becomes

$$2\pi\varrho r^n\int_{-1}^{1} d\mu'\,\mathrm{P}_n\left\{\int_r^{\mathrm{R}}\frac{\phi r'}{r'^{n-1}}dr' + \mu'^2\int_r^{\mathrm{R}}\frac{\mathrm{F}r'}{r'^{n-1}}dr' + \mu'^4\int_r^{\mathrm{R}}\frac{\Pi r'}{r'^{n-1}}dr'\right\}.$$

By writing for r and R their values in terms of a and a, it is easily found that

$$\int_r^{\mathrm{R}}\frac{\phi r'}{r'^{n-1}}dr' \text{ equals } \int_0^a\frac{\phi a'}{a'^{n-1}}da' + \left(\frac{\phi a}{a^{n-2}}\right)\left(-\varepsilon\mu'^2 + \mu'^4(\varepsilon^2 - \mathrm{A}') - \mu'^4\frac{n-1}{2}\varepsilon^2\right)$$

$$+ \left(\frac{\phi' a}{a^{n-3}}\right)\varepsilon^2\frac{\mu'^4}{2} - \text{the same functions of } \mathrm{a} \text{ and } \varepsilon_1.$$

And that similar equations are true for the two remaining integrals.

1st. Let $n = 0$, and we have

$$2\pi\varrho\int_{-1}^{1}d\mu'\left\{\begin{array}{l}\displaystyle\int_0^a a'\phi a'\,da' + a^2\phi a\left(-\varepsilon\mu'^2 + \mu'^4\left(\frac{3}{2}\varepsilon^2 - \mathrm{A}\right)\right) + a^3\phi' a\frac{\varepsilon^2\mu'^4}{2} \\ \displaystyle + \mu'^2\int_0^a a'.\mathrm{F}a'\,da' + \mu'^2 a^2\mathrm{F}a(-\varepsilon\mu'^2) + \mu'^4\int_0^a a'\Pi a'\,da' \\ - \text{same function of } \mathrm{a} \text{ and } \varepsilon_1,\end{array}\right\},$$

which gives

$$4\pi\varrho\left\{\int_0^a a'\varphi a'\,da' + a^2\varphi a\left(-\frac{\varepsilon}{3} + \frac{1}{5}\left(\frac{3}{2}\varepsilon^2 - \mathrm{A}\right)\right) + a^3\varphi' a\frac{\varepsilon^2}{10} + \frac{1}{3}\int_0^a \mathrm{F}a'.a'.da'\right.$$

$$\left. - a^2\mathrm{F}a\frac{\varepsilon}{5} + \frac{1}{5}\int_0^a a'.\Pi a'\,da'\right\} - 4\pi\varrho\,(\text{same function of } \mathrm{a}_1\,\varepsilon_1 \text{ and } \mathrm{A}_1).$$

2nd. Let $n = 2$, and we get

$$\frac{9}{4} 2\pi\varrho\left(\mu^2 - \frac{1}{3}\right) r^2 \int_{-1}^{1} d\mu' \left(\mu'^2 - \frac{1}{3}\right)\left\{\int_{\mathrm{a}}^{a} \frac{\varphi a'}{a'} d a' + (\varphi a - \varphi \mathrm{a})\left(-\varepsilon' \mu'^2 + \mu'^4\left(\frac{\varepsilon'^2}{2} - \mathrm{A}'\right)\right)\right.$$

$$\left. + (a\varphi' a - \mathrm{a}\varphi' \mathrm{a})\frac{\varepsilon'^2\mu'^4}{2} + \int_{\mathrm{a}}^{a} \frac{\mathrm{F} a'}{a'} d a' + (\mathrm{F} a - \mathrm{F}\,\mathrm{a})(-\varepsilon' \mu'^2) + \int_{\mathrm{a}}^{a} \Pi a' . a' . d a'\right\},$$

which gives

$$4\pi\varrho\left(\mu^2 - \frac{1}{3}\right) r^2\left\{\varphi a\left(-\frac{\varepsilon}{5} + \frac{6}{35}\left(\frac{\varepsilon^2}{2} - \mathrm{A}\right)\right) + a\varphi' a \frac{3}{35}\varepsilon^2 + \frac{1}{5}\int_{0}^{a} \frac{\mathrm{F} a'}{a'} d a' - \frac{6}{35}\mathrm{F} a . \varepsilon\right.$$

$$\left. + \frac{6}{35}\int_{0}^{a} \frac{\Pi a'}{a'} d a'\right\} - 4\pi\varrho\left(\mu^2 - \frac{1}{3}\right) r^2 \text{ (same function of a, } \varepsilon_1 \text{ and } \mathrm{A}_1).$$

3rd. Let $n = 3$, and we have

$$\frac{1225}{64}\left(\mu^4 - \frac{6}{7}\mu^2 + \frac{3}{35}\right) 2\pi\varrho r^4 \int_{-1}^{1} d\mu' \left(\mu'^4 - \frac{6}{7}\mu'^2 + \frac{3}{35}\right)\left\{\int_{\mathrm{a}}^{a} \frac{\phi a'}{a'^3} d a' + \left(\frac{\phi a}{a^2} - \frac{\phi \mathrm{a}}{\mathrm{a}^2}\right)\right.$$

$$\left(-\varepsilon' \mu'^2 - \mu'^4\left(\frac{\varepsilon'^2}{2} + \mathrm{A}'\right)\right) + \left(\frac{\phi' a}{a} - \frac{\varphi' \mathrm{a}}{\mathrm{a}}\right)\varepsilon'^2\frac{\mu'^4}{2} + \int_{\mathrm{a}}^{a} \frac{\mathrm{F} a'}{a'^3} d a' + \left(\frac{\mathrm{F} a}{a^2} - \frac{\mathrm{F}\mathrm{a}}{\mathrm{a}^2}\right)(-\varepsilon' \mu'^2)$$

$$\left. + \int_{\mathrm{a}}^{a} \frac{\Pi a'}{a'^3} d a'\right\},$$

or

$$\left(\mu^4 - \frac{6}{7}\mu^2 + \frac{3}{35}\right) 4\pi\varrho r^4\left\{-\frac{\phi a}{a^2}\frac{1}{9}\left(\frac{\varepsilon^2}{2} + \mathrm{A}\right) + \frac{\phi' a}{a}\frac{\varepsilon^2}{18} - \frac{\varepsilon}{9}\frac{\mathrm{F} a}{a^2} + \frac{1}{9}\int_{0}^{a} \frac{\Pi a'}{a'^3} d a'\right\}$$

$$- \left(\mu^4 - \frac{6}{7}\mu^2 + \frac{3}{35}\right) 4\pi\varrho r^4 \text{ (same function of a, } \varepsilon_1 \text{ and } \mathrm{A}_1).$$

19. To the sum of these must be added V for the inner spheroid, for which we shall have to find the value of V generally, when the attracted point is without the body.

The general term of the series for V is

$$\frac{2\pi\varrho}{r^{n-1}}\int_{-1}^{1} d\mu' \mathrm{P}_{n-2}\left\{\int_{0}^{\mathrm{R}} r'^n . \varphi r' . d r' + \mu'^2 \int_{0}^{\mathrm{R}} r'^n \mathrm{F} r' . d r' + \mu'^4 \int_{0}^{\mathrm{R}} r'^n . \Pi r' . d r'\right\},$$

and

$$\int_{0}^{\mathrm{R}} r'^n \varphi r' d r' = \int_{0}^{a} a'^n . \varphi a' . d a' + a^{n+1}\varphi a\left(-\varepsilon\mu^2 + \mu^4(\varepsilon^2 - \mathrm{A}) + \mu^4\frac{n}{2}\varepsilon^2\right)$$

$$+ a^{n+2}\varphi' a \frac{\varepsilon^2\mu^4}{2};$$

and similar equations hold for the other functions.

1st. Let $n = 2$, then we have

$$\frac{2\pi\varrho}{r}\int_{-1}^{1} d\mu' \left\{\int_{0}^{a} a'^2 \varphi a' d a' + a^3\varphi a\left(-\varepsilon\mu'^2 + \mu'^4(2\varepsilon^2 - \mathrm{A})\right) + a^4\varphi' a \frac{\varepsilon^2}{10}\right.$$

$$\left. + \int_{0}^{a} a'^2 \mathrm{F} a' d a' + a^3 \mathrm{F} a(-\varepsilon\mu'^2) + \int_{0}^{a} a'^2 \Pi a' d a'\right\},$$

which is equal to

$$\frac{4\pi\varrho}{r}\left\{\int_{0}^{a} a'^2 \varphi a' d a' + a^3\varphi a\left(-\frac{\varepsilon}{3} + \frac{1}{5}(2\varepsilon^2 - \mathrm{A})\right) + a^4\varphi' a\frac{\varepsilon^2}{10} + \frac{1}{3}\int_{0}^{a} a'^2 \mathrm{F} a' . d a'\right.$$

$$\left. - \frac{1}{5}a^3 \mathrm{F} a . \varepsilon + \frac{1}{5}\int_{0}^{a} a'^2 \Pi a' d a'\right\}.$$

2nd. Let $n = 4$, and the term is

$$\frac{9}{4}\left(\mu^2 - \frac{1}{3}\right)\frac{2\pi\varrho}{r^3}\int_{-1}^{1} d\,\mu'\left(\mu'^2 - \frac{1}{3}\right)\left\{\int_0^a a'^4\,\varphi a'\,d\,a' + a^5\,\varphi\,a\left(-\varepsilon\,\mu'^2 + \mu'^4\,(3\,\varepsilon^2 - \mathrm{A})\right)\right.$$

$$\left. + a^6\,\varphi'\,a\,\frac{\varepsilon^2\,\mu'^4}{2} + \int_0^a a'^4\,\mathrm{F}\,a'\,d\,a' - a^5\,\mathrm{F}\,a\,\varepsilon\,\mu'^2 + \int_0^a a'^4\,\Pi\,a'\,d\,a'\right\},$$

or

$$4\,\pi\,\varrho\,\frac{\mu^2 - \frac{1}{3}}{r^3}\left\{a^5\,\varphi\,a\left(-\frac{\varepsilon}{5} + \frac{6}{35}\,(3\,\varepsilon^2 - \mathrm{A})\right) + a^6\,\varphi'\,a\,\frac{3}{35}\,\varepsilon^2 + \frac{1}{5}\int_0^a a'^4\,\mathrm{F}\,a'\,d\,a'\right.$$

$$\left. - \frac{6}{35}\,a^5\,\mathrm{F}\,a\,.\,\varepsilon + \frac{6}{35}\int_0^a a'^4\,\Pi\,a'\,d\,a'\right\}.$$

3rd. Let $n = 6$, and we get

$$\frac{1225}{64}\left(\mu^4 - \frac{6}{7}\,\mu^2 + \frac{3}{35}\right)\frac{4\,\pi\,\rho}{r^5}\int_{-1}^{1} d\,\mu'\left(\mu'^4 - \frac{6}{7}\,\mu'^2 + \frac{3}{35}\right)\left\{\int_0^a a'^6\,\varphi\,a'\,d\,a'\right.$$

$$+ a^7\,\varphi\,a\left(-\varepsilon\,\mu'^2 + \mu'^4\,(4\,\varepsilon^2 - \mathrm{A})\right) + a^8\,\varphi'\,a\,\frac{\varepsilon^2\,\mu'^4}{2} + \int_0^a a'^6\,\mathrm{F}\,a'\,d\,a' - a^7\,\mathrm{F}\,a\,\varepsilon\,\mu'^2$$

$$\left. + \int_0^a a'^6\,\Pi\,a'\,d\,a'\right\},$$

which is

$$\frac{4\,\pi\,\varrho}{r^5}\left(\mu^4 - \frac{6}{7}\,\mu^2 + \frac{3}{35}\right)\left\{a^7\,\varphi\,a\,\frac{1}{9}\,(4\,\varepsilon^2 - \mathrm{A}) + a^8\,\varphi'\,a\,\frac{1}{18}\,\varepsilon^2 - \frac{\varepsilon}{9}\,a^7\,\mathrm{F}\,a + \frac{1}{9}\int_0^a a'^6\,\Pi\,a'\,d\,a'\right\}.$$

20. In these three terms write a for a, ε_1 for ε, and A_1 for A, and add them to the other value of V, and apply the equations

$$\int_0^a a'^2\,\Pi\,a'\,d\,a' = \frac{1}{2}\int_0^a a'^4\,d\,(\chi\,a'^2\,\varphi'\,a') + \int_0^a a'^3\,\theta\,a'\,d\,(\varphi\,a') = \frac{1}{2}\,a^4\,\varepsilon^2\,\varphi'\,a + a^3\,\mathrm{A}\,\varphi\,a$$

$$- 2\int_0^a a'^3\,\chi\,a'^2\,.\,d\,\varphi\,a' - \int_0^a \varphi\,a'\,d\,(a'^3\,\mathrm{A}') = \frac{1}{2}\,a^4\,\varepsilon^2\,\varphi'\,a + a^3\,\mathrm{A}\,\varphi\,a - 2\,a^3\,\varepsilon^2\,\varphi\,a$$

$$+ 2\int_0^a \varphi\,a'\,d\,(a'^3\,\varepsilon'^2) - \int_0^a \varphi\,a'\,d\,(a'^3\,\mathrm{A}'),$$

and similar equations for the other integrals, and the value of V for an internal point becomes

$$4\pi\varrho\left\{\begin{bmatrix} \displaystyle\int_{\mathrm{a}}^{a}\varphi a'\,\frac{d\left(a'^2\left\{\frac{1}{2} - \frac{1}{3}\varepsilon' + \frac{1}{5}\left(\frac{3}{2}\,\varepsilon'^2 - \mathrm{A}'\right)\right\}\right)}{d\,a'}\,d\,a' + \frac{1}{3\,r}\int_0^{\mathrm{a}}\varphi\,a'\,\frac{d\left(a'^3\left\{1 - \varepsilon' + \frac{3}{5}(2\,\varepsilon'^2 - \mathrm{A}')\right\}\right)}{d\,a'}\,d\,a' \\ \displaystyle -\frac{1}{5}\left(\mu^2 - \frac{1}{3}\right)\left(r^2\int_{\mathrm{a}}^{a}\varphi\,a'\,\frac{d\left(\varepsilon' - \frac{3}{7}(\varepsilon'^2 - 2\,\mathrm{A}')\right)}{d\,a'}\,d\,a' + \frac{1}{r^3}\int_0^{\mathrm{a}}\varphi a'\,\frac{d\left(a'^5\left\{\varepsilon' - \frac{6}{7}(3\,\varepsilon'^2 - \mathrm{A}')\right\}\right)}{d\,a'}\,d\,a'\right) \\ \displaystyle +\frac{1}{9}\left(\mu^4 - \frac{6}{7}\mu^2 + \frac{3}{35}\right)\left(r^4\int_{a}^{\mathrm{a}}\varphi a'\,\frac{d\,\frac{1}{a'^2}\left(-\frac{\varepsilon'^2}{2} - \mathrm{A}'\right)}{d\,a'}\,d\,a' + \frac{1}{r^5}\int_0^{\mathrm{a}}\varphi a'\,\frac{d\left(a'^7\,(4\,\varepsilon'^2 - \mathrm{A}')\right)}{d\,a'}\,d\,a'\right) \end{bmatrix}\right\}$$

$$= 4\,\pi\,\varrho\left\{\mathrm{M}'\,a - \mathrm{M}'\,\mathrm{a} + \frac{1}{3\,r}\,\mathrm{M}\,\mathrm{a} + \frac{1}{5}\left(\mu^2 - \frac{1}{3}\right)\left(r^2\,\mathrm{N}'\,a - r^2\,\mathrm{N}'\,\mathrm{a} + \frac{1}{r^3}\,\mathrm{N}\,\mathrm{a}\right)\right.$$

$$\left. + \frac{1}{9}\left(\mu^4 - \frac{6}{7}\,\mu^2 + \frac{3}{35}\right)\left(r^4\,\mathrm{P}'\,a - r^4\,\mathrm{P}'\,\mathrm{a} + \frac{1}{r^5}\,\mathrm{P}\,\mathrm{a}\right)\right\},$$

where the functions M, N, P, &c. are substituted for the corresponding integrals.

21. The equation of equilibrium will be formed exactly as in (16.), except that the expression for the velocity must be found more accurately. By (17.), the force of gravity at the equator is $4\,\pi\,\varrho\,\frac{\mathrm{M}\,a}{3\,a^2}\left(1+\varepsilon-\frac{3\,m}{2}\right)$; and the centrifugal force is $\omega^2\,a$; therefore $m = \dfrac{\omega^2\,a}{4\,\pi\,\rho\,\frac{\mathrm{M}\,a}{3\,a^2}\left(1+\varepsilon-\frac{3\,m}{2}\right)}$, and $\omega^2 = 4\,\pi\,\varrho\,\frac{\mathrm{M}\,a}{3\,a^3}\left(m+m\,\varepsilon-\frac{3\,m^2}{2}\right)$.

The equation of equilibrium becomes

$$\int\frac{dp}{\rho'} = \mathrm{C} = \mathrm{V} + 2\,\pi\,\varrho\,r^2\,(1-\mu^2)\,\frac{\mathrm{M}\,a}{3\,a^3}\,m\left(1+\varepsilon-\frac{3\,m}{2}\right),$$

which at the surface becomes

$$\mathrm{C}' = \frac{\mathrm{M}\,a}{3\,r} - \frac{1}{5}\left(\mu^2-\frac{1}{3}\right)\frac{\mathrm{N}\,a}{r^3} + \frac{1}{9}\left(\mu^4-\frac{6}{7}\mu^2+\frac{3}{35}\right)\frac{\mathrm{P}\,a}{r^5} + \frac{1}{2}(1-\mu^2)\,r^2\,\frac{\mathrm{M}\,a}{3\,a^3}\left(m+m\,\varepsilon-\frac{3\,m^2}{2}\right),$$

$$= \frac{\mathrm{M}\,a}{3\,a}\,(1+\varepsilon\,\mu^2+\mathrm{A}\,\mu^4) - \frac{1}{5}\left(\mu^2-\frac{1}{3}\right)(1+3\,\varepsilon\,\mu^2)\,\frac{\mathrm{N}\,a}{a^3} + \frac{1}{9}\left(\mu^4-\frac{6}{7}\mu^2+\frac{3}{35}\right)\frac{\mathrm{P}\,a}{a^5}$$

$$+\frac{1-\mu^2}{2}\,(1-2\,\varepsilon\,\mu^2)\,\frac{\mathrm{M}\,a}{3\,a^3}\left(m+m\,\varepsilon-\frac{3\,m^2}{2}\right).$$

Equate the coefficients of μ^2 and those of μ^4, and we have two equations for determining ε and A, thus showing that equilibrium is possible. These are,

$$\frac{\mathrm{M}\,a}{3\,a}\left(\varepsilon-\frac{m}{2}-\frac{3}{2}\,m\,\varepsilon+\frac{3}{4}\,\mu^2\right) - \frac{\mathrm{N}\,a}{a^3}\left(\frac{1}{5}-\frac{\varepsilon}{5}\right) - \frac{2}{21}\,\frac{\mathrm{P}\,a}{a^5} = 0,$$

and
$$\frac{\mathrm{M}\,a}{3\,a}\,(\mathrm{A}+m\,\varepsilon) - \frac{\mathrm{N}\,a}{a^3}\,\frac{3\,\varepsilon}{5} + \frac{1}{9}\,\frac{\mathrm{P}\,a}{a^5} = 0;$$

whence
$$\frac{\mathrm{M}\,a}{3\,a}\left(\varepsilon-\frac{m}{2}+\frac{6\,\mathrm{A}}{7}+\frac{3}{4}\,m^2+\frac{m\,\varepsilon}{7}-\frac{11}{7}\,\varepsilon^2\right) = \frac{\mathrm{N}\,a}{5\,a^3},$$

and
$$\frac{\mathrm{M}\,a}{3\,a}\left(\mathrm{A}-3\,\varepsilon^2+\frac{5\,m\,\varepsilon}{2}\right) = -\frac{1}{9}\,\frac{\mathrm{P}\,a}{a^5}.$$

22. The resultant attraction in the direction of r, is obtained by differentiating $\int\frac{dp}{\rho'}$, as found in (21.), and changing its sign. This produces

$$4\pi\varrho\left\{\frac{\mathrm{M}\,a}{3\,r^2} - \frac{3}{5}\left(\mu^2-\frac{1}{3}\right)\frac{\mathrm{N}\,a}{r^4} + \frac{5}{9}\left(\mu^4-\frac{6}{7}\mu^2+\frac{3}{35}\right)\frac{\mathrm{P}\,a}{r^6} - r\,(1-\mu^2)\left(m+m\,\varepsilon-\frac{3\,m^2}{2}\right)\frac{\mathrm{M}\,a}{3\,a^3}\right\},$$

since the terms arising from differentiating the expression for V in (20.) with respect to a, cancel each other.

For r write its value, and arrange the result according to powers of μ, and this formula becomes

$$4\,\pi\,\varrho\left\{\begin{array}{l}\frac{\mathrm{M}\,a}{3\,a^2}\left(1-m+m\,\varepsilon-\frac{3\,m^2}{2}+\mu^2\left(2\,\varepsilon+m+2\,m\,\varepsilon-\frac{3\,m^2}{2}\right)+\mu^4\,(2\,\mathrm{A}+\varepsilon^2-m\,\varepsilon)\right)\\ \quad -\frac{\mathrm{N}\,a}{a^4}\left(-\frac{1}{5}+\mu^2\left(\frac{3}{5}-\frac{4}{5}\,\varepsilon\right)+\mu^4\,\frac{12}{5}\,\varepsilon\right)+\frac{\mathrm{P}\,a}{a^6}\left(\frac{1}{21}-\mu^2\,\frac{10}{21}+\mu^4\,\frac{5}{9}\right)\end{array}\right\}$$

$$= 4\,\pi\,\varrho\,\frac{\mathrm{M}\,a}{3\,a^2}\left\{1+\varepsilon-\frac{3\,m}{2}-\frac{27}{14}\,m\,\varepsilon+\frac{9}{4}\,m^2-\frac{2}{7}\,\varepsilon^2+\frac{3}{7}\,\mathrm{A}\right.$$

$$\left.+\mu^2\left(\frac{5}{2}\,m-\varepsilon+\frac{72}{7}\,m\,\varepsilon-\frac{15}{4}\,m^2-\frac{29}{7}\,\varepsilon^2+\frac{12}{7}\,\mathrm{A}\right)+\mu^4\left(4\,\varepsilon^2-3\,\mathrm{A}-\frac{15}{2}\,m\,\varepsilon\right)\right\},$$

by applying the two equations deduced in last article. The total attraction normal to the surface, or the force of gravity, is found from this by dividing by the cosine of the angle of the vertical, or by $1 - 2\varepsilon^2(\mu^2 - \mu^4)$; whence

$$g = 4\pi\varrho\frac{\mathrm{M}a}{3a^2}\left\{1 + \varepsilon - \frac{3m}{2} - \frac{27}{14}m\varepsilon + \frac{9}{4}m^2 - \frac{2}{7}\varepsilon^2 + \frac{3}{7}\mathrm{A}\right.$$

$$\left. + \mu^2\left(\frac{5}{2}m - \varepsilon + \frac{72}{7}m\varepsilon - \frac{15}{4}m^2 - \frac{15}{7}\varepsilon^2 + \frac{12}{7}\mathrm{A}\right) + \mu^4\left(2\varepsilon^2 - 3\mathrm{A} - \frac{15}{2}m\varepsilon\right)\right\}.$$

Let λ be the sine of the real latitude; then as $\mu^2 = \lambda^2 - 4\varepsilon\overline{\lambda^2 - \lambda^4}$, we get

$$g = 4\pi\varrho\frac{\mathrm{M}a}{3a^2}\left\{1 + \varepsilon - \frac{3m}{2} - \frac{27}{14}m\varepsilon + \frac{9}{4}m^2 - \frac{2}{7}\varepsilon^2 + \frac{3}{7}\mathrm{A}\right.$$

$$\left. + \lambda^2\left(\frac{5}{2}m - \varepsilon + \frac{2}{7}m\varepsilon - \frac{15}{4}m^2 + \frac{13}{7}\varepsilon^2 + \frac{12}{7}\mathrm{A}\right) - \lambda^4\left(2\varepsilon^2 + 3\mathrm{A} - \frac{5}{2}m\varepsilon\right)\right\}.$$

Let G represent the equatorial gravity; then

$$g = \mathrm{G}\left\{1 + \lambda^2\left(\frac{5}{2}m - \varepsilon - \frac{26}{7}m\varepsilon + \frac{20}{7}\varepsilon^2 + \frac{12}{7}\mathrm{A}\right) - \lambda^4\left(2\varepsilon^2 + 3\mathrm{A} - \frac{5}{2}m\varepsilon\right)\right\}$$

$$= \mathrm{G}\left\{1 + \sin^2 l\left(\frac{5}{2}m - \varepsilon - \frac{17}{14}m\varepsilon + \frac{6}{7}\varepsilon^2 - \frac{9}{7}\mathrm{A}\right) + \sin^2 l\cos^2 l\left(2\varepsilon^2 + 3\mathrm{A} - \frac{5}{2}m\varepsilon\right)\right\},$$

which is an extension of CLAIRAUT's theorem.

23. In this process A indicates the amount of deviation of the required surface from the surface represented by $r = a(1 - \varepsilon\mu^2)$. If the equation had been taken $a = r\left\{1 + \mathrm{e}\mu^2 - \left(\frac{3}{2}\mathrm{e}^2 + \mathrm{B}\right)\left(\mu^4 - \mu^2\right) + \mathrm{e}^2\mu^4\right\}$, B would have been the index of deviation from an elliptic spheroid.

To apply CLAIRAUT's theorem to this surface, we have

$$\varepsilon = \mathrm{e} + \frac{3}{2}\mathrm{e}^2 + \mathrm{B}, \text{ and } \mathrm{A} = -\frac{\mathrm{e}^2}{2} - \mathrm{B};$$

whence

$$g = \mathrm{G}\left\{1 + \sin^2 l\left(\frac{5}{2}m - \mathrm{e} + \frac{2}{7}\mathrm{B} - \frac{17}{14}m\mathrm{e}\right) - \sin^2 l\cos^2 l\left(\frac{5}{2}m\mathrm{e} - \frac{\mathrm{e}^2}{2} + 3\mathrm{A}\right)\right\},$$

which is the same expression as that obtained by Mr. AIRY in the Philosophical Transactions, 1826, except that instead of e or $\frac{a-c}{a}$, the symbol is used to represent $\frac{a-c}{c}$.

24. The circumstance of the terms arising from the differentiation of V with respect to a vanishing, affords an easy method of extending CLAIRAUT's theorem indefinitely, without calculating the value of V.

It may be shown independently, that these terms cancel each other in all cases.

The $(n+1)$th term of V for the portion including the point is

$$2\pi\varrho r^n\int_{-1}^{1}d\mu'\,\mathrm{P}_n\left\{\int_r^{\mathrm{R}}\frac{\phi r'}{r'^{n-1}}dr' + \mu'^2\int_r^{\mathrm{R}}\frac{\mathrm{F}r'}{r'^{n-1}}dr' + \mu'^4\int_r^{\mathrm{R}}\frac{\Pi r'}{r'^{n-1}}dr' + \ldots\right\}.$$

The corresponding term for the other portion is

$$\frac{2\pi\rho}{r^{n+1}}\int_{-1}^{1}d\mu'\,\mathrm{P}_n\left\{\int_0^r r'^{n+2}\,\varphi r'\,dr' + \mu'^2\int_0^r r'^{n+2}\,\mathrm{F}r'\,dr' + \mu'^4\int_0^r \Pi r'.r'^{n+2}\,dr' + \ldots\right\};$$

and it is evident, by inspection of these functions, that that portion of the sum of their partial differential coefficients, which arises from differentiating with respect to r under the signs of integration, is equal to zero. This being the case, it is not necessary to know the forms of the functions $\mathrm{M}\,a$, $\mathrm{N}\,a$, $\mathrm{P}\,a$, &c., nor their numerical coefficients; but only the functions of μ, by which they are respectively multiplied; and these are LAPLACE's coefficients.

Thus the equation of equilibrium at the surface would be

$$\mathrm{C} = \frac{\mathrm{M}\,a}{3\,r} - \frac{1}{5}\left(\mu^2 - \frac{1}{3}\right)\frac{\mathrm{N}\,a}{r^3} + \frac{1}{9}\left(\mu^4 - \frac{6}{7}\mu^2 + \frac{3}{35}\right)\frac{\mathrm{P}\,a}{r^5} - \left(\mu^6 - \frac{15}{11}\mu^4 + \frac{5}{11}\mu^2 - \frac{5}{231}\right)\frac{\mathrm{Q}\,a}{r^7}$$

$$+ \frac{1}{2}(1 - \mu^2)\,r^2\,m\,\mathrm{G}, \text{ where } \frac{1}{r} = \frac{1}{a}\left(1 + \varepsilon\,\mu^2 + \mathrm{A}\,\mu^4 + \mathrm{D}\,\mu^6\right), \text{ suppose.}$$

Expand r, recollecting that $\mathrm{N}\,a$, $\mathrm{P}\,a$, and $\mathrm{Q}\,a$ are of the 1st, 2nd, and 3rd orders respectively, and we have three equations to determine ε, A and D.

By differentiating C with respect to r, and eliminating $\mathrm{N}\,a$, $\mathrm{P}\,a$, and $\mathrm{Q}\,a$ by these three equations, we have the resolved force in r, which divided by the cosine of the angle of the vertical gives g exactly as in (23.).

It is evident that this may be carried on indefinitely; and to any order, without finding g for the next lower order.

METEOROLOGICAL JOURNAL,

KEPT BY THE ASSISTANT SECRETARY,

AT THE APARTMENTS OF THE

ROYAL SOCIETY,

BY ORDER OF

THE PRESIDENT AND COUNCIL.

OBSERVANDA.

Height of the Cistern of the Barometer above the plinth at Waterloo Bridge....83 feet 2 inches.
———————— above the mean level of the sea ....97 feet.
Height of the receiver of the Rain Gauge above the court of Somerset House ..79 feet.
The External Thermometer is 2 feet higher than the Barometer Cistern.
The Thermometers are graduated to Fahrenheit's scale.
The Barometer is divided into inches and tenths.
The Hours of Observation are of Mean Time, the day beginning at Midnight.
The *daily* observations of the Barometer are *not* corrected.
The *monthly means* are corrected for capillarity and temperature by the Table contained in Mr. Baily's paper in *Phil. Trans.* for 1837.

METEOROLOGICAL JOURNAL FOR JULY AND AUGUST, 1840.

1840.	9 o'clock, A.M.			3 o'clock, P.M.			Dew Point at 9 A.M., deg. Fahr.	Diff. of Wet and Dry Bulb Thermometer.	External Thermometers.				Rain in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	REMARKS.
	Barometer uncorrected.		Att. Ther.	Barometer uncorrected.		Att. Ther.			Fahrenheit.		Self-registering				
	Flint Glass.	Crown Glass.		Flint Glass.	Crown Glass.				9 A.M.	3 P.M.	Lowest	Highest			
W 1	29.922	29.916	65.5	29.846	29.840	66.0	58	07.1	62.0	65.2	52.4	68.2	.133	S	Cloudy—light wind throughout the day. Evening, Overcast.
T 2	29.774	29.768	63.7	29.728	29.722	65.9	58	05.8	61.2	63.7	59.6	66.6		SE	A.M. Overcast—light rain—brisk wind, as also high wind throughout the night. P.M. Overcast—light wind. Ev. Slight rain.
F 3	29.540	29.532	64.4	29.534	29.530	65.9	58	05.2	58.8	62.9	56.5	66.5	.033	W	A.M. Overcast—light rain and wind. P.M. Fine—light clouds and wind. Evening, Overcast—light wind.
S 4	29.878	29.874	81.5	29.902	29.896	67.5	54	09.0	61.3	67.4	53.7	69.2	.080	W	A.M. Fine—lt. clds. & wind. P.M. Cldy.—brisk wind. Ev. The like.
⊙ 5	29.732	29.728	72.0	29.706	29.700	66.4	56	08.7	63.3	63.4	58.7	68.6		W	A.M. Cloudy—brisk wind throughout the night. P.M. Overcast—brisk wind, with showers. Evening, Fine and starlight.
M 6	29.810	29.804	68.0	29.686	29.678	66.9	55	07.5	62.6	63.8	53.3	75.3	.016	S var.	Cloudy—brisk wind throughout the day. Ev. Light rain—high wind.
T 7	29.634	29.630	77.2	29.706	29.698	65.8	55	07.5	59.7	64.6	50.6	71.7	.138	W	A.M. Dark heavy clouds—brisk wind, with light showers. P.M. Fine—light clouds—brisk wind. Evening, Overcast.
W 8	29.822	29.816	64.8	29.780	29.772	64.7	55	07.1	61.2	62.7	53.8	66.3	.025	SW	A.M. Cloudy—light wind. P.M. Light rain and wind. Ev. Overcast—heavy rain.
T 9	29.878	29.872	69.0	29.956	29.948	64.9	53	07.1	59.4	65.7	51.3	67.3	.500	NW	A.M. Overcast—light wind and showers. P.M. Cloudy—light wind. Evening, Fine and starlight.
F 10	29.996	29.988	64.2	29.960	29.952	63.9	53	05.9	60.3	62.0	51.3	67.7		W	A.M. Cloudy—light wind. P.M. Fine—light clouds. Ev. Cloudy.
S 11	30.002	29.998	75.0	29.976	29.970	65.2	54	08.3	60.2	62.3	52.6	66.8		NW	Fine—light clouds and wind throughout the day. Ev. The like.
⊙ 12	30.012	30.004	72.4	29.982	29.974	64.8	53	08.0	58.3	61.7	52.2	69.2		NW	A.M. Fine—lt. clouds & wind. P.M. Cloudy—lt. wind. Ev. The like.
M 13	30.012	30.004	68.3	30.088	30.080	62.9	53	05.9	55.7	59.5	49.4	62.7		NW	A.M. Cloudy—light wind. P.M. Light rain and wind. Evening, Fine and moonlight.
)T 14	30.316	30.310	71.2	30.294	30.286	63.7	50	07.1	59.4	68.7	50.2	70.5		SW	Fine—light clouds and wind throughout the day. Evening, Fine and moonlight.
W 15	30.304	30.300	79.9	30.220	30.212	65.8	54	09.0	63.7	72.3	57.0	77.2		S	Fine—nearly cloudless, with light breeze throughout the day. Evening, Fine and moonlight.
T 16	30.152	30.148	70.7	30.026	30.018	67.2	56	09.3	66.5	72.4	56.0	74.3		S	Cloudy—light breeze throughout the day. Evening, The like.
F 17	29.998	29.992	75.7	29.926	29.918	67.3	55	08.3	62.7	67.7	52.8	74.6		SW var.	Fine—light clouds and breeze throughout the day. Ev. Overcast.
S 18	29.778	29.772	64.4	29.784	29.776	66.2	60	05.1	63.7	65.7	58.0	69.7		W	A.M. Lightly overcast—light breeze, with very fine rain. P.M. Overcast. Evening, The like.
⊙ 19	29.700	29.694	66.3	29.622	29.618	68.2	60	06.1	63.7	69.0	58.7	68.6		S	Cloudy—light wind throughout the day. Ev. Overcast—lt. rain—high [wind.
M 20	29.566	29.560	68.3	29.548	29.544	67.3	58	07.1	63.3	65.8	53.2	70.0	.091	S var.	A.M. Cloudy—light wind, with occasional showers. P.M. Cloudy—brisk wind. Evening, Fine and moonlight.
T 21	29.630	29.626	76.2	29.660	29.652	67.8	59	07.6	63.7	61.7	55.3	76.0	.066	W var.	A.M. Fine—light clouds—brisk wind. P.M. Overcast—light rain. Evening, Fine and starlight.
W 22	29.820	29.816	66.2	29.858	29.850	66.6	56	05.4	61.4	65.8	52.7	70.2	.305	W	Cloudy—light wind throughout the day. Evening, Fine & starlight.
T 23	30.032	30.024	62.8	30.056	30.048	64.0	52	06.2	59.7	60.2	53.5	72.4		W	A.M. Cloudy—very slight rain early. P.M. Cloudy—light wind. Evening, Overcast.
F 24	30.066	30.058	65.0	30.014	30.006	64.5	54	07.6	61.7	64.7	55.0	64.5		S	Cloudy—light wind throughout the day. Ev. Overcast—brisk wind.
S 25	29.814	29.806	62.6	29.772	29.764	66.0	58	05.0	60.3	66.0	58.5	67.4	.016	S	A.M. Overcast—high wind—rain early—high wind throughout the night. P.M. Slight rain. Evening, The like.
⊙ 26	29.694	29.688	63.0	29.696	29.690	64.0	56	04.4	58.8	60.0	56.3	70.2	.291	SE	A.M. Lightly overcast—heavy rain during the night. P.M. Overcast—light rain. Evening, Overcast—light showers.
M 27	29.956	29.950	64.0	29.958	29.950	65.0	58	06.9	61.0	70.2	55.3	62.4	.105	NW	A.M. Overcast—light wind. P.M. Fine—light clouds and wind. Evening, Fine and starlight.
T 28	30.084	30.076	63.2	30.106	30.098	65.7	57	05.0	61.7	69.7	55.6	71.4		S	A.M. Overcast—lt. wind. P.M. Fine—lt. clouds & wind. Ev. Cloudy.
W 29	30.260	30.256	78.3	30.234	30.228	67.7	57	08.4	64.7	69.6	60.2	71.8		NW	A.M. Fine—nearly cloudless—lt. breeze. P.M. Cloudy. Ev. Overcast.
T 30	30.148	30.140	67.2	30.098	30.090	68.6	61	05.6	65.4	70.9	60.2	72.0		SSW	Cloudy—light wind throughout the day. Ev. Overcast—slight rain.
F 31	34.210	30.204	72.5	30.212	30.204	69.4	60	07.3	62.3	66.7	58.7	73.2	.016	NW	Fine—light clouds & wind throughout the day. Ev. Fine & starlight.
EAN.	29.921	20.915	69.1	29.901	29.894	66.0	56	06.9	61.5	65.5	54.9	69.8	Sum. 1.815		Mean Barometer corrected .... 9 A.M. 3 P.M. F. 29.817 .. 29.805 C 29.810 .. 29.797
S 1	30.278	30.272	73.9	30.216	30.210	68.0	56	07.2	63.7	71.5	53.2	72.6		NW	Fine—light clouds & wind throughout the day. Ev. Fine & starlight.
⊙ 2	30.254	30.250	76.6	30.200	30.192	69.3	60	08.0	66.7	78.0	55.3	79.0		S	Fine—lt. clouds & breeze throughout the day. Ev. Fine & starlight.
M 3	30.244	30.240	82.9	30.180	30.172	72.3	62	07.6	70.0	79.3	61.0	79.3		S	Fine—nearly cloudless throughout the day. Ev. Fine and starlight.
T 4	30.188	30.184	76.5	30.132	30.126	72.0	58	07.0	67.5	77.2	61.0	80.0		NW	A.M. Light fog. P.M. Fine—light clouds. Evening, The like.
W 5	30.150	30.196	71.0	30.098	30.094	71.0	61	05.5	67.2	71.7	59.0	68.5		SE	Fine—light clouds throughout the day. Evening, The like.
T 6	30.034	30.030	74.6	29.992	29.986	72.0	60	06.8	69.6	76.6	60.0	73.0		ENE	Fine—light clouds throughout the day. Evening, The like.
F 7	30.020	30.016	74.0	30.024	30.020	73.0	61	09.7	71.3	72.0	64.0	78.5		NE	A.M. Fine—light clouds and breeze. P.M. Lightly overcast—brisk wind. Evening, The like.
S 8	30.150	30.144	69.0	30.148	30.142	71.0	59	05.0	65.4	73.0	59.0	76.0		NNE	Fine—lt. clouds—brisk wind throughout the day. Ev. The like.
⊙ 9	30.292	30.286	70.0	30.322	30.318	69.8	54	04.4	60.2	75.5	52.4	75.5		N	A.M. Lightly overcast—light wind. P.M. Fine—light clouds and wind. Evening, The like.
M 10	30.098	30.094	74.0	29.984	29.978	72.0	58	07.3	69.5	76.5	60.0	77.0		SW	A.M. Overcast—lt. fog. P.M. Fine—lt. clouds. Ev. Lightly overcast.
T 11	29.614	29.610	68.0	29.574	29.570	69.5	60	03.0	63.5	69.0	62.0	79.7	.058	S	A.M. Ovct.—lt. wind. P.M. Fine—lt. clouds. Ev. Lightly overcast.
W 12	29.654	29.650	71.0	29.636	29.632	70.0	57	08.1	65.5	67.3	57.0	70.5	.069	WSW	A.M. Fine—light clouds and wind. P.M. Lightly overcast. Evening, Overcast—heavy rain.
)T 13	29.656	29.652	70.0	29.621	30.620	68.5	54	07.0	63.0	67.6	55.0	70.0	.027	SW	A.M. Overcast—light wind. P.M. Fine—light clouds—brisk wind. Evening, The like.
F 14	29.530	29.526	66.0	29.604	29.600	67.2	54	05.0	61.0	61.0	56.0	70.0	.058	W	A.M. Overcast. P.M. Overcast—heavy showers. Evening, The like.
S 15	29.818	29.814	65.0	29.832	29.828	66.0	54	05.5	61.2	65.8	53.5	67.0	.213	SW var.	A.M. Heavy clouds—brisk wind. P.M. Fine—light clouds, with occasional showers. Evening, Overcast.
⊙ 16	29.968	29.962	66.0	29.904	29.900	66.0	55	06.5	63.0	68.2	54.5	67.5	.072	W	Fine—light clouds nearly the whole of the day—heavy shower at 3½, P.M. Evening, Fine—light clouds.
M 17	29.206	29.202	63.0	29.264	29.260	64.5	59	02.0	59.8	59.5	58.0	72.0	.159	SE var.	A.M. Overcast—light rain—high wind. P.M. Fine—light clouds—brisk wind. Rainbow ½ p. 5. Evening, Overcast.
T 18	29.374	29.370	61.5	29.552	29.548	62.0	54	05.0	59.5	61.8	52.5	63.2	.175	W	Overcast—brisk wind throughout the day, as also the evening.
W 19	29.712	29.708	60.0	29.766	29.762	63.0	57	00.0	57.0	67.5	57.0	63.8	.369	SE	Overcast—light rain & wind nearly throughout the day. Evening, Overcast.
T 20	30.016	30.012	63.5	30.002	29.998	66.0	56	05.5	66.0	72.8	60.0	69.0	.061	S	Lightly overcast throughout the day. Evening, The like.
F 21	29.954	29.950	69.2	29.894	29.888	69.0	58	06.4	69.0	75.5	60.0	70.5		SE	A.M. Lightly overcast—light fog. P.M. Fine—light clouds and breeze. Evening, Fine—light clouds.
S 22	29.836	29.832	67.2	29.824	29.818	69.0	62	02.2	63.5	71.6	62.5	77.8		SW	A.M. Overcast—light wind. P.M. Fine—light clouds and wind. Evening, Lightly overcast.
⊙ 23	29.998	29.994	72.0	29.982	29.978	69.0	59	05.5	63.0	70.5	56.0	72.5		WSW	A.M. Heavy clouds—light wind. P.M. Fine—light clouds and wind. Evening, Dark heavy clouds—brisk wind.
M 24	30.050	30.044	70.0	29.996	29.990	68.6	57	04.0	62.5	70.2	56.0	72.5		SW	A.M. Lightly overcast—lt. wind. P.M. Fine—lt. clouds. Ev. The like.
T 25	30.038	30.034	72.0	30.000	29.994	68.2	55	06.5	63.5	70.0	54.4	73.5		WSW	A.M. Fine—light clouds. P.M. Lightly overcast. Ev. Overcast.
W 26	30.060	30.054	66.0	30.070	30.066	67.3	61	03.5	65.0	68.5	61.0	73.0		NW	Overcast—light wind throughout the day. Evening, The like.
T 27	30.110	30.104	65.0	30.052	30.048	67.2	59	03.0	61.5	69.5	56.7	70.8		W	Overcast throughout the day, as also the evening.
F 28	30.060	30.054	65.8	30.080	30.076	67.0	61	03.0	63.5	69.6	60.0	72.0		W	A.M. Lightly overcast. P.M. Fine—lt. clouds. Evening, Lightly overcast.
S 29	30.204	30.198	66.0	30.200	30.194	67.0	60	01.1	60.9	67.2	60.0	72.0	.011	E	A.M. Overcast—light fog and haze. P.M. Lightly overcast. Evening, Light fog.
⊙ 30	30.128	30.124	68.5	30.086	30.082	69.0	62	04.5	66.8	77.5	58.0	69.0		E	A.M. Overcast—light fog. P.M. Fine—light clouds. Ev. Overcast.
M 31	30.184	30.180	66.8	30.124	30.120	68.7	59	04.0	63.0	72.0	60.0	79.5		NE	A.M. Overcast—light wind. P.M. Fine—light clouds—brisk wind. Evening, Cloudy.
EAN.	29.964	29.961	69.2	29.947	29.942	68.5	58	05.2	64.3	70.8	57.9	72.7	Sum. 1.272		Mean Barometer corrected.............. 9 A.M. 3 P.M. F. 29.859 .. 29.844 C. 29.855 .. 29.838

METEOROLOGICAL JOURNAL FOR SEPTEMBER AND OCTOBER, 1840.

	1840.	9 o'clock, A.M.			3 o'clock, P.M.			Dew Point at 9 A.M., deg. Fahr.	Diff. of Wet and Dry Bulb Thermometer.	External Thermometers.				Rain in Inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	REMARKS.
		Barometer uncorrected.		Att. Ther.	Barometer uncorrected.		Att. Ther.			Fahrenheit.		Self-registering				
		Flint Glass.	Crown Glass.		Flint Glass.	Crown Glass.				9 A.M.	3 P.M.	Lowest	Highest			
	T 1	30.034	30.026	66.7	29.904	29.896	69.7	63	04.0	65.9	74.0	54.3	72.9		N	A.M. Cloudy—light wind. P.M. Fine—nearly cloudless—light wind. Evening, Fine and starlight.
	W 2	29.756	29.748	77.9	29.674	29.668	71.6	66	06.8	69.8	70.7	63.9	76.2		S	A.M. Fine—nearly cloudless. P.M. Fine—light clouds and wind. Evening, Overcast—heavy rain.
	T 3	29.664	29.656	66.8	29.748	29.740	68.2	61	06.0	60.3	62.7	56.3	75.7	.236	W	A.M. Overcast—lt. wind—rain early. P.M. Fine—lt. clouds and wind. Ev. Fine & starlight.
	F 4	29.826	29.818	69.7	29.826	29.818	65.3	58	07.2	61.8	60.4	52.6	64.2		S	A.M. Cloudy—light wind. P.M. Cloudy—light rain and wind. Ev. Fine and starlight.
	S 5	30.012	30.004	66.3	30.042	30.034	65.3	57	05.7	60.0	65.2	51.2	64.8	.122	W	A.M. Light haze and wind. P.M. Cloudy—light wind. Evening, Fine and starlight.
	⊙ 6	30.276	30.268	69.0	30.228	30.220	64.7	55	05.6	59.8	68.3	50.8	67.2		S	A.M. Fine—nearly cloudless. P.M. Fine—light clouds. Ev. Cloudy.
	M 7	30.132	30.124	68.5	30.088	30.080	65.7	58	06.3	62.6	65.8	57.3	69.8		SW	A.M. Fine—light clouds and wind. P.M. Cloudy—light rain and wind. Evening, Cloudy.
	T 8	30.180	30.172	68.0	30.158	30.150	65.0	56	04.5	58.4	64.7	52.3	68.0		NW	A.M. Fine—nearly cloudless—light fog. P.M. Thick haze. Evening, Cloudy—very slight rain.
	W 9	30.044	30.036	62.8	29.996	29.988	63.8	57	05.2	62.3	65.8	56.7	66.7		S	Overcast—brisk wind throughout the day, as also the evening.
	T 10	30.014	30.006	64.0	30.020	30.012	65.2	59	05.2	61.3	65.7	60.7	68.0	.041	WNW	A.M. Overcast—rain early—high wind throughout the night. P.M. Fine—light clouds. Evening, Fine and starlight.
	○ F 11	29.994	29.986	61.9	29.952	29.944	63.7	55	03.8	60.0	62.8	51.9	66.8		SW	Fine—light clouds and breeze throughout the day. Evening, Fine and starlight.
	S 12	30.024	30.016	66.4	29.966	29.958	61.5	52	05.9	56.8	60.8	47.2	65.3		W	Fine—lt. clouds and wind throughout the day. Ev. Fine and clear.
	⊙ 13	29.900	29.894	64.9	29.764	29.758	60.0	48	05.9	53.8	58.6	45.2	62.2		SW	A.M. Light Fog. P.M. Fine—light clouds. Evening, The like.
	M 14	29.474	29.466	55.9	29.308	29.300	57.9	49	06.1	54.3	53.2	46.0	60.7		S	A.M. Overcast—light wind. P.M. Overcast—light rain and wind—great depression in barometer. Evening, Heavy rain.
SEPTEMBER	T 15	29.314	29.306	67.3	29.306	29.300	57.4	50	04.0	51.2	54.3	45.9	59.4	.283	W	Fine—light clouds and wind throughout the day. Evening, Overcast—light rain—high wind.
	W 16	28.892	28.888	62.4	28.858	28.852	57.7	49	06.8	55.7	53.8	46.9	60.8	.130	S	A.M. Fine—lt. clouds & wind. P.M. Overcast—lt. showers—brisk wind. Ev. Fine and starlight.
	T 17	29.550	29.542	60.7	29.660	29.652	56.6	48	06.2	55.3	58.2	45.7	60.0	.111	SW var.	Fine—light clouds—brisk wind throughout the day. Evening, Fine and starlight.
	F 18	29.896	29.890	55.3	29.888	29.880	55.0	45	05.3	51.3	57.7	41.7	59.3		W	A.M. Fine—light clouds and fog. P.M. Fine—light clouds & wind—½ past 5, rainbow, with slight rain. Evening, Fine and clear.
	S 19	29.880	29.872	56.3	29.908	29.900	56.2	49	04.6	53.3	54.6	49.3	59.7		NNW	A.M. Fine—light clouds—high wind. P.M. Cloudy—brisk wind. Ev. Fine and starlight.
	⊙ 20	30.036	30.028	52.3	30.036	30.030	54.0	44	05.9	51.3	55.0	42.4	56.6		WNW	A.M. Fine—nearly cloudless—lt. fog. P.M. Overcast—lt. wind. Ev. Fine and starlight.
	M 21	30.026	30.018	55.0	29.946	29.938	56.0	45	05.3	54.5	58.8	45.0	57.8		S	A.M. Fine—light clouds and wind. P.M. Cloudy—light brisk wind. Evening, Cloudy—light rain.
	T 22	29.500	29.494	55.4	29.500	29.496	55.3	50	04.8	55.2	49.7	52.4	62.5	.083	S	Overcast—lt. rain—high wind nearly the whole day. Ev. Cloudy.
	W 23	29.402	29.396	52.3	29.498	29.492	54.7	45	04.8	51.4	57.0	44.6	56.7	.333	WSW	A.M. Fine—light clouds—brisk wind. P.M. Fine—dark clouds—brisk wind. Evening, Fine and starlight.
	T 24	29.530	29.522	53.0	29.574	29.566	54.9	48	04.8	53.2	56.3	48.0	58.8		E	A.M. Fine—light clouds and fog. P.M. Cloudy, with occasional light showers. Evening, Overcast—light rain.
	● F 25	29.796	29.788	53.3	29.944	29.936	51.8	49	02.3	50.2	54.3	50.0	59.3	.097	NW	A.M. Overcast—lt. rain & wind. P.M. Cloudy—brisk wind. Ev. Lt. fog.
	S 26	30.062	30.054	53.4	29.966	29.958	55.0	46	03.0	50.7	57.7	45.4	56.5	.019	SW	A.M. Fine—light clouds and wind. P.M. Cloudy—light rain and wind. Evening, Overcast—light rain and wind.
	⊙ 27	30.014	30.006	55.7	29.980	29.976	57.0	48	04.5	55.4	61.8	49.0	60.6	.130	S	A.M. Fine—lt. clouds. P.M. Lightly overcast. Ev. Fine & starlight.
	M 28	29.790	29.782	55.4	29.654	29.648	57.2	52	04.4	56.3	58.0	51.3	63.8		SE	Overcast—high wind, with light rain nearly the whole of the day, as also the evening.
	T 29	29.624	20.618	58.2	29.630	29.624	58.5	53	04.4	55.5	56.3	50.2	60.3	.269	S	A.M. Fine—nearly cloudless—brisk wind. P.M. Fine—light clouds. Evening, Fine and starlight.
	W 30	29.998	29.990	55.5	30.004	29.996	57.2	49	04.7	52.8	56.4	46.3	59.0		W	A.M. Fine—light clouds and wind. P.M. Cloudy—light wind. Evening, Dark heavy clouds.
	MEAN.	29.821	29.814	61.0	29.801	29.794	60.2	52	05.1	57.3	60.0	50.0	63.3	Sum. 1.854		Mean Barometer corrected .... 9 A.M. 3 P.M. F. 29.738 .. 29.721 C. 29.730 .. 29.713
	T 1	30.022	30.016	55.2	30.044	30.036	56.2	50	04.6	55.3	57.3	51.7	57.7		S	Overcast—light fog and wind throughout the day, as also the evening.
	F 2	30.172	30.164	55.3	30.172	30.164	57.6	52	04.1	53.7	57.7	51.3	58.7		W	A.M. Cloudy—light wind. P.M. Fine—light clouds. Evening, Fine and starlight.
	S 3	30.194	30.186	53.8	30.148	30.140	54.9	46	03.3	47.7	52.5	44.3	59.0		NW	A.M. Fine—lt. clouds & wind. P.M. Cloudy—lt. wind. Ev. The like.
	⊙ 4	30.084	30.076	53.3	30.080	30.074	55.5	49	02.8	51.2	55.0	46.3	54.6		NNW	A.M. Lightly overcast—slight rain—brisk wind. P.M. Heavy clouds—light wind. Evening, Fine and starlight.
	M 5	30.100	30.092	52.2	30.078	30.070	53.4	48	03.0	49.5	49.4	46.4	57.2		NW	A.M. Ovct.—lt. wind. P.M. Cloudy—lt. showers. Ev. Fine & starlight.
	T 6	30.126	30.118	51.0	30.118	30.110	53.2	45	02.6	45.4	52.4	42.6	55.3	.116	NW	Cloudy—light fog and wind throughout the day. Evening, Fine and starlight—frosty.
	W 7	30.114	30.106	49.5	30.116	30.108	51.2	42	02.6	43.7	52.8	39.0	53.6		SSW	A.M. Fine—light clouds, with light fog and wind. P.M. Cloudy—light wind. Evening, Fine and starlight—light fog.
	T 8	30.266	30.258	48.0	30.252	30.244	48.7	41	02.4	40.7	50.3	36.8	53.9		S	A.M. Fine & cloudless—lt. fog. P M. Foggy. Ev. Fine & moonlight.
	F 9	30.326	30.318	46.7	30.282	30.276	48.0	39	02.4	40.3	53.8	40.0	51.6		WSW	A.M. Thick fog—light wind. P.M. Fine—nearly cloudless. Ev. Fine and moonlight—light fog.
	S 10	30.318	30.310	46.7	30.292	30.284	48.8	41	03.4	45.7	55.2	39.8	55.4		N	A.M. Thick fog—lt. wind. P.M. Fine—nearly cloudless. Ev. Fine and starlight—light fog.
	○ ⊙ 11	30.400	30.392	49.3	30.408	30.402	51.8	46	03.4	52.5	57.5	44.4	56.7		NE	A.M. Lightly overcast—light fog and wind. P.M. Fine—lt. wind. Ev. Fine & moonlight.
	M 12	30.604	30.596	49.6	30.592	30.584	51.6	46	02.9	50.3	55.7	44.7	59.3		N	Fine—nearly cloudless—light wind throughout the day. Evening, Fine and moonlight—light fog.
	T 13	30.590	30.582	48.8	30.530	30.522	50.7	43	02.1	43.2	55.2	42.2	57.3		NNW	A.M. Thick fog—light wind. P M. Fine—light clouds and wind. Evening, Fine and moonlight—light fog.
	W 14	30.404	30.398	48.6	30.308	30.300	50.4	42	01.2	43.2	58.8	39.7	56.6		SW	Fine—nearly cloudless—light fog throughout the day. Evening, Fine and moonlight—light fog.
OCTOBER	T 15	30.258	30.250	50.4	30.152	30.144	51.3	42	02.3	43.7	53.8	41.9	60.7		S	A.M. Fine—light fog. P.M. Hazy—light fog. Ev. Cloudy—light fog.
	F 16	29.936	29.928	50.3	29.830	29.822	51.8	46	03.5	48.3	55.3	42.0	54.8		S	A.M. Light clouds and wind. P.M. Overcast—light wind. Evening, Overcast—light rain.
	S 17	29.812	29.804	53.0	29.900	29.892	53.8	49	02.5	54.8	53.3	48.0	56.7	.066	NW	Lightly overcast—light wind throughout the day. Evening, Overcast—deposition.
	⊙ 18	30.026	30.018	52.2	29.936	29.932	53.0	47	02.5	50.8	53.5	49.0	56.8		SSW	Overcast throughout the day. Ev. Overcast—light rain—brisk wind.
	M 19	29.704	29.698	53.3	29.726	29.718	54.3	48	05.2	53.5	54.8	49.0	55.7	.133	NW	A.M. Cloudy—brisk wind. P.M. Fine—light clouds—brisk wind. Evening, Fine and starlight—high wind.
	T 20	30.100	30.094	51.3	30.104	30.096	52.2	45	04.8	47.7	52.8	45.8	56.6		NW	A.M. Fine and cloudless—brisk wind. P.M. Fine—light clouds and wind. Evening, Fine and starlight.
	W 21	30.156	30.150	49.7	30.120	30.112	51.0	41	04.0	45.7	51.8	42.2	54.6		NW	Fine—light clouds and wind throughout the day. Ev. Overcast.
	T 22	29.904	29.898	50.0	29.856	29.848	50.8	45	02.1	48.8	51.7	44.8	52.8		S	Overcast—slight rain—light wind throughout the day. Evening, Fine and starlight.
	F 23	29.960	29.952	49.6	29.856	29.848	52.0	42	02.4	45.2	52.2	43.7	52.7	.036	SW	A.M. Fine—lt. clouds & wind. P.M. Overcast—light rain and wind. Ev. The like.
	S 24	29.556	29.548	49.6	29.556	29.548	50.2	43	03.2	45.2	47.2	42.5	53.2	.286	W	A.M. Fine—light clouds—brisk wind. P.M. Fine—light clouds—at 2 P.M. heavy shower of hail. Evening, Fine and starlight
	● ⊙ 25	29.784	29.776	47.0	29.832	29.826	47.7	38	03.2	42.2	48.2	39.3	50.6	.102	NW	Fine—nearly cloudless—light wind throughout the day. Evening. Fine and starlight.
	M 26	29.886	29.878	44.3	29.826	29.818	46.2	37	02.0	39.2	47.8	35.4	49.3		W	A.M. Overcast—lt. fog and wind. P.M. Lightly overcast. Ev. Overcast—lt. rain.
	T 27	29.178	29.172	48.7	29.152	29.146	49.5	45	01.9	51.3	49.3	38.6	52.8	.302	S	A.M. Overcast—light rain and wind, as also throughout the night. P.M. Cloudy—light wind. Evening, Fine and starlight.
	W 28	29.136	29.130	46.8	29.210	29.202	49.0	42	02.0	42.7	47.9	41.3	52.3	.061	S	A.M. Fine—light clouds & wind—rain early. P.M. Lightly cloudy. Evening, Overcast—light rain.
	T 29	29.340	29.336	46.7	29.310	29.304	47.7	43	01.2	45.6	47.2	41.4	49.4	.033	NE	Overcast—very fine rain—light wind nearly the whole of the day. Ev. Overcast—lt. fog.
	F 30	29.522	29.514	47.4	29.528	29.522	49.7	44	02.3	47.3	51.7	40.3	49.6	.338	E	A.M. Fine—light clouds and wind. P.M. Fine, with occasional showers. Evening, Fine and starlight.
	S 31	29.564	29.558	48.5	29.561	29.558	50.7	45	02.0	47.3	51.5	43.2	52.8		ENE	A.M. Lightly overcast—light wind. P.M. Fine—light clouds. Evening, Fine and starlight.
	MEAN.	29.985	29.978	50.0	29.932	29.924	51.4	44	02.8	47.2	52.7	43.2	54.8	Sum. 1.473		Mean Barometer corrected .... 9 A.M. 3 P.M. F. 29.931 .. 29.875 C. 29.923 .. 29.866

METEOROLOGICAL JOURNAL FOR NOVEMBER AND DECEMBER, 1840.

1840.	9 o'clock, A.M.			3 o'clock, P.M.			Dew Point at 9 A.M., deg. Fahr.	Diff. of Wet and Dry Bulb Ther.	External Thermometers.				Rain in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	REMARKS.
	Barometer uncorrected.		Att. Ther.	Barometer uncorrected.		Att. Ther.			Fahrenheit.		Self-registering				
	Flint Glass.	Crown Glass.		Flint Glass.	Crown Glass.				9 A.M.	3 P.M.	Lowest	Highest			
NOVEMBER															
⊙ 1	29.644	29.638	48.8	29.488	29.484	50.0	47	01.7	47.2	50.5	44.9	53.7		E	Overcast—deposition—lt. wind throughout the day. Evening, Light rain—high wind.
M 2	29.538	29.532	50.4	29.544	29.538	51.8	46	03.5	51.2	54.7	46.5	53.3	.050	SE	A.M. Fine—lt. clouds—high wind—very high wind throughout the night. P.M. Fine—lt. clouds and wind. Ev. Fine & starlight.
T 3	29.432	29.428	51.3	29.398	29.394	52.4	48	02.0	51.5	54.0	47.6	56.4		NE	A.M. Lightly overcast. P.M. Fine—light clouds. Ev. Overcast.
W 4	29.288	29.282	51.8	29.292	29.286	53.2	49	01.3	50.3	53.3	49.0	55.6	.052	S	A.M. Overcast—deposition—lt. fog and wind—rain during the night. P.M. Fine—light clouds. Evening, Fine and starlight.
T 5	29.202	29.196	51.9	29.296	29.292	53.4	49	02.1	50.3	52.5	48.8	55.8	.302	S	A.M. Dark heavy clouds—high wind—heavy rain early. P.M. Fine—light clouds and wind. Ev. Overcast—light rain—brisk wind.
F 6	29.056	29.050	50.6	28.980	28.976	51.4	45	02.0	47.3	48.7	45.8	54.4	.133	SE	A.M. Overcast—light rain—high wind, with occasional rain. P.M. Fine—light clouds and wind. Ev. Overcast—rain—high wind.
S 7	29.138	29.132	51.4	29.168	29.162	52.3	47	02.3	48.8	50.4	48.0	51.7	.286	SSW	A.M. Fine—light clouds and wind—very high wind throughout the night. P.M. Fine—lt. clouds—occasional rain. Ev. Fine & starlight.
⊙ 8	29.360	29.352	49.7	29.358	29.352	51.0	45	02.9	47.0	51.5	44.7	55.3	.083	S	Fine & cloudless throughout the day. Ev. Fine & moonlight—lt. wind.
☽ M 9	29.106	29.100	49.9	29.142	29.136	51.2	46	03.5	48.8	51.7	46.0	53.8	.091	S	A.M. Fine—nearly cloudless—brisk wind—very high wind early, with rain. P.M. Cloudy—light rain. Ev. Overcast—light rain.
T 10	29.074	29.070	49.3	29.166	29.158	50.2	45	01.9	44.5	48.8	44.2	53.5	.105	SSW	Fine—light clouds & wind throughout the day. Ev. Fine & starlight.
W 11	29.332	29.326	47.3	29.166	29.160	48.8	43	01.9	44.2	45.3	42.2	50.3		S	A.M. Lt. fog & wind—P.M. Overcast—lt. rain & wind. Ev. Overcast.
T 12	29.528	29.520	46.6	29.534	29.528	47.4	40	01.7	41.3	46.8	40.6	49.8	.114	S	Fine—light clouds and wind throughout the day. Ev. Lightly overcast.
F 13	28.828	28.822	47.8	28.606	28.600	50.2	45	02.0	48.7	51.7	41.7	51.0	.272	S	A.M. Fine—lt. clds.—hi. wind—rain, with very hi. wind throughout the night. P.M. Dk. clds.—hail & rain—hi. wi. Ev. Overct.—lt. rain.
S 14	29.044	29.038	48.7	29.160	29.154	49.4	44	03.1	46.2	48.4	45.0	53.4	.230	S	Fine—nearly cloudless throughout the day. Ev. Fine and moonlight.
⊙ 15	29.512	29.504	46.2	29.550	29.546	47.0	39	01.6	39.3	45.2	38.6	50.6		S	A.M. Fine—nearly cloudless—light wind. P.M. Overcast. Ev. Overcast—light rain.
M 16	29.084	29.076	48.7	29.200	29.194	52.0	49	01.9	55.5	57.3	38.5	57.9	.083	SW var.	A.M. Overcast—light rain—very high wind. P.M. Fine—light clouds. Ev. Lightly overcast.
T 17	29.600	29.594	51.3	29.454	29.448	51.3	47	02.6	48.2	45.3	46.7	60.6		S var.	Overcast—light wind, with occasional rain throughout the day. Ev. Light rain—very high wind.
W 18	29.814	29.806	49.0	29.752	29.748	47.3	43	03.1	41.2	36.3	41.4	54.6	.447	NW	A.M. Overcast—light rain—brisk wind. P.M. Snow and rain—brisk wind. Ev. Rain and wind.
T 19	29.800	29.796	44.4	29.852	29.846	45.6	41	01.4	39.8	43.3	37.2	42.4	.450	N	A.M. Overcast—light rain and wind. P.M. Cloudy—light wind. Ev. Fine and starlight.
F 20	30.112	30.104	42.0	30.090	30.082	43.6	33	02.9	35.7	42.8	35.0	44.7		NW	Fine—light clouds and wind throughout the day. Ev. fine & starlight.
S 21	29.526	29.518	42.8	29.218	29.210	44.6	41	01.9	43.3	46.0	35.0	44.3	.033	S	A.M. Overcast—light rain—high wind. P.M. Cloudy—high wind. Ev. Overcast—light rain—very high wind.
⊙ 22	29.794	29.786	43.2	29.920	29.916	44.0	38	03.3	40.3	42.3	40.0	47.3	.211	NNW	Fine—light clouds—high wind throughout the day—very high wind during the night. Ev. fine and starlight.
M 23	30.070	30.062	41.2	30.000	29.994	43.0	33	02.6	38.4	45.3	35.6	43.5		S	Overcast—light wind and rain nearly the whole of the day. Ev. lt. fog.
● T 24	30.140	30.134	46.3	30.220	30.212	47.9	43	01.6	50.0	52.0	38.2	51.2		W	A.M. Cloudy—light fog and wind. P.M. Fine—light clouds and wind. Ev. Fine and starlight.
W 25	30.424	30.416	45.0	30.426	30.418	46.0	40	01.6	41.0	46.0	38.3	54.3		N	A.M. Thick fog—light wind. P.M. Cloudy—light wind. Ev. Fine and starlight.
T 26	30.460	30.452	42.0	30.416	30.408	42.3	35	02.3	37.4	43.3	36.0	47.0		E	A.M. Light fog & wind. P.M. Fine—light clouds. Ev. Fine & starlight.
F 27	30.430	30.424	41.0	30.412	30.404	41.4	35	01.5	34.5	43.8	34.3	44.0		N	A.M. Thick fog—light wind—white frost. P.M. Fine—light clouds. Ev. Light fog.
S 28	30.406	30.398	39.7	30.374	30.366	39.2	35	01.0	32.8	37.7	32.8	44.5		NE	Thick fog nearly the whole of the day. Ev. Dense fog—sharp frost.
⊙ 29	30.336	30.328	37.3	30.248	30.242	38.6	32	01.1	30.8	42.2	30.3	39.3		NE	A.M. Thick fog—white frost. P.M. Overcast—light fog. Ev. Cloudy.
M 30	30.084	30.076	40.3	30.054	30.048	42.8	35	02.2	46.7	51.8	30.3	47.7		S	Overcast—deposition—light wind throughout the day, as also the evening.
MEAN.	29.639	29.632	46.5	29.616	29.610	47.6	42	02.2	44.1	47.6	40.8	50.7	Sum. 2.942		Mean Barometer corrected .... 9 A.M. 3 P.M. F 29.595 .. 29.569 C. 29.587 .. 29.562
DECEMBER															
T 1	29.960	29.952	47.5	30.000	29.992	48.9	45	01.4	51.7	53.5	45.8	52.8		SE var.	Overcast——light wind throughout the day, as also the evening.
W 2	30.188	30.180	47.9	30.248	30.240	48.0	42	02.8	44.6	46.2	44.7	55.5		WNW	Cloudy—light wind throughout the day. Evening, Fine & starlight.
T 3	30.538	30.530	43.0	30.036	30.028	43.3	36	01.9	36.3	40.5	35.7	47.4		NW	A.M. Fine—nearly cloudless—lt. wind. P.M. Light fog. Evening, Fine and moonlight.
F 4	30.480	30.472	40.3	30.436	30.428	40.6	33	01.3	32.8	38.3	32.5	42.3		SW	A.M. Light fog—white frost. P.M. Light fog. Ev. Overcast—lt. fog.
S 5	30.466	30.458	40.9	30.306	30.298	41.6	34	02.4	39.4	42.8	32.7	41.0		SW	Light fog and wind throughout the day—air frosty. Evening, Overcast—light fog.
⊙ 6	30.186	30.178	41.8	30.100	30.094	42.2	35	01.4	40.8	42.2	38.6	43.5		E	A.M. Thick fog—light wind. P.M. Overcast—lt. wind. Evening, Fine and starlight.
M 7	29.808	29.800	40.9	29.646	29.640	41.5	35	02.0	39.2	38.3	35.2	43.7		E	Lightly overcast—lt. brisk wind throughout the day, also the evening.
T 8	29.230	29.224	39.9	29.254	29.248	41.3	34	02.6	39.7	43.2	34.2	40.3		E	Overcast—light brisk wind, with occasional light rain during the day. Evening, Fine and moonlight.
☽ W 9	29.738	29.730	40.4	29.854	29.848	41.0	35	01.3	36.3	41.3	36.2	44.3	.066	S	Fine—lt. clouds and wind throughout the day. Ev. Fine & moonlight.
T 10	29.970	29.962	40.0	29.936	29.928	41.3	38	01.3	40.8	43.2	36.2	42.0		E	Light fog, with lt. deposition throughout the day. Ev. Lightly cloudy.
F 11	29.958	29.950	40.2	29.940	29.934	39.9	36	01.9	37.6	36.0	35.2	45.0		ENE	Overcast—brisk wind throughout the day, as also the evening.
S 12	30.124	30.116	39.5	30.176	30.168	39.8	34	02.6	38.5	38.3	36.0	39.2		N	Light fog and wind throughout the day. Evening, The like.
⊙ 13	30.282	30.276	38.2	30.280	30.272	37.9	33	02.5	33.8	34.3	33.7	39.7		N	Overcast—brisk wind throughout the day. Ev. The like, with lt. fog.
M 14	30.236	30.228	34.0	30.220	30.212	33.2	25	*frozen*	27.3	29.8	27.3	35.4		N	A.M. Light clouds and wind—sharp frost. P.M. Fine—light clouds and wind. Evening, Overcast—sharp frost.
T 15	30.276	30.268	32.3	30.242	30.234	32.3	25	*ditto*	27.0	27.3	27.3	31.3		N	A.M. Overcast—light snow. P.M. Fine—light clouds. Evening, Fine and starlight.
W 16	30.132	30.124	29.8	29.966	29.958	30.8	25	*ditto*	28.3	30.3	24.0	28.5		NW	Overcast—light snow—brisk wind throughout the day. Ev. The like.
T 17	29.772	29.766	32.5	29.766	29.758	32.8	28	*ditto*	31.4	24.3	27.7	34.6		NE	A.M. Overcast—light snow—brisk wind. P.M. Fine—light clouds and wind. Evening, Overcast—sharp frost.
F 18	29.614	29.608	29.6	29.494	29.488	31.8	25	*ditto*	28.3	35.2	21.2	31.7		N	A.M. Overcast—brisk wind—slight thaw. P.M. Overcast—continued thaw. Evening, Overcast—snow and wind.
S 19	29.588	29.580	31.8	29.568	29.560	33.0	27	*ditto*	31.3	35.0	29.2	35.5		N	Overcast—light wind, with frost throughout the day. Evening, Overcast—thaw.
⊙ 20	29.840	29.832	34.0	29.950	29.944	34.9	29	02.4	35.8	36.3	29.3	36.3		N	Overcast—light brisk wind throughout the day. Evening, The like.
M 21	30.262	30.254	35.9	30.278	30.270	36.7	30	02.2	35.6	36.2	35.0	37.2		NE	A.M. Overcast—high wind. P.M. Fine—light clouds—high wind. Evening, Overcast—high wind.
T 22	30.344	30.336	35.4	30.298	30.290	34.8	28	*frozen*	32.2	32.3	32.3	37.3		ENE	A.M. Cloudy—high wind—sharp frost. P.M. Fine—light clouds and wind. Evening, Overcast—light fog.
● W 23	30.176	30.170	31.7	30.094	30.086	33.2	25	*ditto*	27.5	33.8	26.9	32.8		NNW	A.M. Light fog & wind. P.M. Fine—lt. clouds. Ev. Fine & starlight—lt. clouds.
T 24	30.100	30.092	32.0	30.120	30.112	33.5	28	*ditto*	32.3	35.8	27.0	34.4		NNW	A.M. Overcast—brisk wind—sharp frost. P.M. Fine—light clouds. Evening, Fine and starlight.
F 25	30.328	30.320	34.0	30.342	30.338	33.0	28	*ditto*	30.0	33.0	28.9	36.7		N	Light fog and wind, with sharp frost throughout the day. Evening, Thick fog—sharp frost.
S 26	30.562	30.554	30.7	30.574	30.568	31.0	25	*ditto*	29.0	30.2	25.0	33.4		NW	Overcast—light wind throughout the day. Ev. Lt. fog—sharp frost.
⊙ 27	30.656	30.650	31.5	30.590	30.584	32.0	26	*ditto*	32.3	34.4	29.2	32.6		NW	A.M. Light fog and wind—sharp frost. P.M. Overcast—thaw. Evening, Overcast.
M 28	30.408	30.400	32.3	30.342	30.334	32.9	28	*ditto*	30.3	32.8	29.4	35.3		E	A.M. Overcast—lt. wind. P.M. Light fog & wind. Ev. The like.
T 29	30.256	30.248	31.2	30.246	33.240	31.8	26	*ditto*	26.4	32.3	24.2	33.0		S	Light fog and wind, with sharp frost throughout the day. Evening, Overcast—rapid thaw—rain.
W 30	30.406	30.398	31.8	30.310	30.304	32.3	27	*ditto*	27.8	34.2	26.2	37.6		S	A.M. Light fog & wind. P.M. Fine—lt. clouds. Ev. Overcast—lt. rain.
T 31	29.886	29.878	34.0	29.930	29.924	36.5	30	01.3	41.3	43.0	28.5	42.0	.347	W	A.M. Overcast—light fog and rain. P.M. Fine—light clouds and wind. Evening, Overcast—light rain.
MEAN.	30.121	30.114	36.3	30.082	30.075	36.9	31	01.9	34.4	36.6	31.5	38.8	Sum. .413		Mean Barometer corrected .... 9 A.M. 3 P.M. F. 30.104 .. 30.064 C. 30.096 .. 30.056

Fig. 2.

Fig. 3

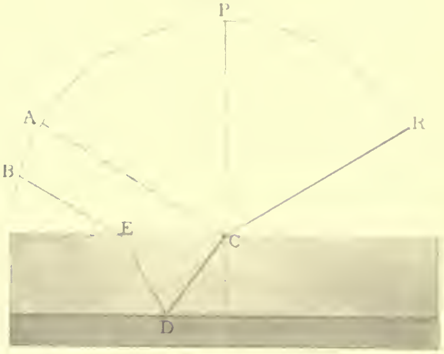


Fig. 1.

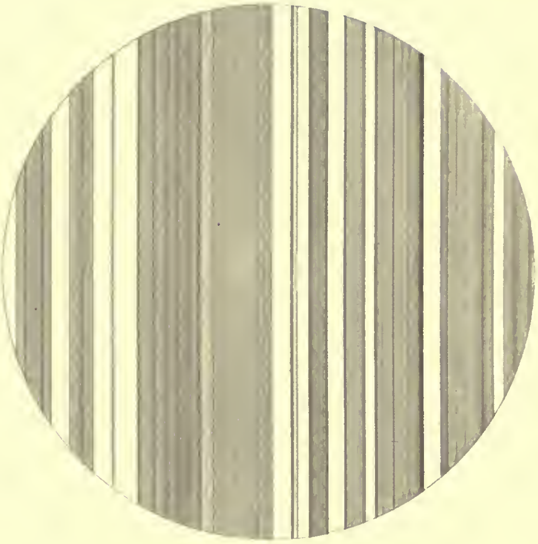
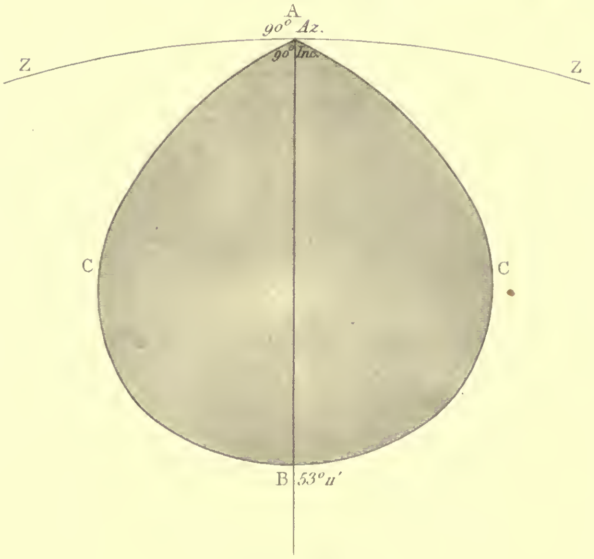
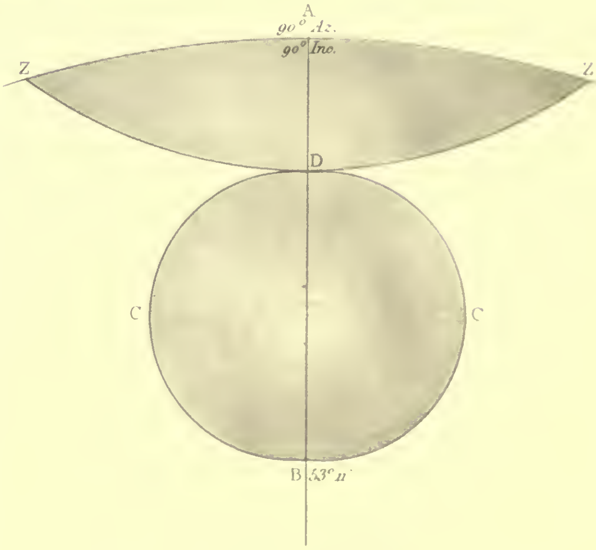


Fig. 4.



Glass & Water.

Fig. 5.



Fluor Spar & Water.

Tetanus - part of P. Fasc. of Complexus (ecchymosed) Feb. 19. 1841.

Magnified about 300 diameters.

W. Bowman ad. nat. del.

CONTENTS.

APPENDIX.

ERRATUM.

Page 229, line 2, *for* to prolong themselves into lymph, *read* to prolong themselves into the lymph.

ADJUDICATION of the MEDALS of the ROYAL SOCIETY for the year 1841 by the PRESIDENT and COUNCIL.

The COPLEY MEDAL to Dr. G. S. OHM, of Nuremberg, for his researches into the Laws of Electric Currents, contained in various memoirs, published in SCHWEIGGER'S Journal, POGGENDORFF'S Annalen, and in a separate work, entitled "Die galvanische Kette mathematisch bearbeitet."

The ROYAL MEDAL in the department of Chemistry, to ROBERT KANE, M.D., M.R.I.A., for his memoir entitled, "The Chemical History of Archil and Litmus," published in the Philosophical Transactions for 1840.

The other ROYAL MEDAL, not having been awarded in the department of Mathematics, was awarded in that of Physics, to EATON HODGKINSON, Esq., F.R.S., for his Paper entitled, "Experimental Researches on the Strength of Pillars of Cast Iron and other Materials," published in the Philosophical Transactions for 1840.

PHILOSOPHICAL TRANSACTIONS.

X. The Bakerian Lecture.—*On the Organs of Reproduction, and the Development of the Myriapoda.—First Series. By* George Newport, *Esq., Member of the Royal College of Surgeons, and of the Entomological Society of London. Communicated by* Peter Mark Roget, *M.D., Sec. R.S. &c. &c.*

Received June 17,—Read June 17, 1841.

THE development of the Myriapoda has hitherto been only partially investigated. It is, nevertheless, a subject of great importance to the comparative anatomist, from the remarkable fact that it takes place in a manner entirely different from that of most of the higher Articulata, to some of which the Myriapoda are closely allied both in habits and structure. The true Insecta arrive at their perfect state by an aggregation or apparent diminution in the number of their segments, but the Myriapoda, on the contrary, by a repeated increase of these parts, which in many instances are multiplied to several times their original number. This addition of segments, during the growth of the animal, occurs throughout the whole class, and is one of its chief characteristics. This fact was first noticed long ago by **Degeer**, but since the period of his observations nothing further was added to our knowledge until it was fully confirmed by the careful investigations of **Savi**, and also by the more recent labours of **Brandt**, **Gervais**, and **Waga**. But excellent as are the observations of these naturalists, some of the most important circumstances connected with them have been entirely overlooked, both as regards the condition of the embryo on leaving the ovum, and also as regards the manner in which the new segments are developed. M. **Gervais**[*] has pointed out a circumstance in which the Scolopendradæ differ from the Iulidæ in the development of the legs, but no precise account, so far as I have been able to ascertain, has been given of the production of the segments. In the observations which I now have the honour of submitting to the Royal Society, I propose, first, to examine the organs of reproduction, and then to show the various changes as they occur in the development of *Iulus terrestris*, one of the commonest species of the Iulidæ of this country.

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[*] Annales des Sciences Natur., tom. vii. Janvier 1837, p. 35, &c.

1. *Organs of Reproduction.*

The reproductive parts in Iulus are exceedingly interesting on account of their simplicity of structure. Treviranus* has described them in the male as two elongated tubes which terminate in separate orifices behind the seventh pair of legs, without any external organ of intromission. In the female, he says, they are composed of a long ovary, formed of two knots of eggs which extend from its outlet in the fourth segment, to its termination beneath the alimentary canal, near the anus; but in this account he has entirely overlooked the essential parts of these organs in both sexes.

In the male of *Iulus terrestris,* the reproductive organs (Plate III. fig. 1.) are two elongated, and sometimes partially convoluted tubes, placed side by side beneath the alimentary canal, immediately above the nervous system, and between the two large salivary vessels. Anteriorly (*a*), they terminate in two organs of intromission, which pass out at the under surface of the seventh segment by distinct orifices, behind the seventh pair of legs. Posteriorly, they extend backwards as far as the middle of the colon. In the anterior third of their course they lie close together (*b*), but afterwards separate, become smaller, and have developed from their sides, at short distances, a number of minute glandular cæca (*c c*). Soon after separating they are again connected transversely by three short ducts (*d d d*), by means of which the two organs communicate freely with each other. Two of these ducts are situated anterior, and the third posterior to the first pair of cæca. In the posterior part of the body, each tube is divided into two portions, covered with cæca, and between them there also appear to be some transverse communications, as in the anterior parts of the tubes, but of this I am not fully satisfied. The cæca are of an hour-glass form, and may perhaps be regarded as the proper testes of the animal. They are each connected to the larger, or efferential tubes, by a short narrow duct (*e*), the proper efferential vessel of each cæcum or testis. The delicacy and transparency of these parts in an immature individual, allow their structure to be examined with great facility (fig. 2.). They are simple constricted sacs, lined with a thickened mucous membrane (*f*), and are folds or intussusceptions of the whole muscular, as well as mucous lining of the greater tubes, and from the structure of their interior seem to perform a secretory or glandular function. I have been unable to trace any tubes continued from their larger or cæcal terminations, but have seen many minute vessels distributed over their exterior surface (*g*), that appear to convey a fluid, probably for the purposes of secretion. At first I regarded all these as tracheal vessels, but this cannot indeed be the case, as many of them anastomose with other similar vessels connected with the adipose tissue. On examining the sacs by transmitted light, their interior is distinctly seen to be filled with a fine granulous fluid (*h*). In those sacs which are nearest the posterior extremity of the organs, the fluid is thin and transparent; but that which is contained in the large tubes, near their anterior termi-

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* Vermischte Schriften Anatomischen und Physiologischen Inhalt. Bremen, 1817.

nation, is thicker and more opake the nearer it lies to their outlet. In those individuals which have not arrived at their full growth, many of the sacs are only partially developed. Some of them are narrow, elongated, and have not yet begun to secrete (*i*); and others are only beginning to make their appearance at the sides of the tubes (*k*). In all of them, the interior or mucous lining is distinctly seen to be continuous with that of the large efferential tubes, and in those which appear to be secreting, the base of each vesicle is constricted by the mucous lining within, which seems to form a valve (*e*).

The general structure of the generative organs is very simple throughout the whole of the Iulidæ, but many curious peculiarities occur in the larger species. Thus, in one of the African types, the double organ of intromission (fig. 3. *a*) is prehensile, each part having the form of a distinct claw, between the moveable joints of which passes out the elongated half-corneous penis. These parts are covered in anteriorly by a horny valve, somewhat of a triangular form, and the whole occupies an oval space on the under surface of the seventh segment, corresponding to that usually occupied by the legs. Anteriorly, the two large efferential ducts (*b*) terminate in the penis in very fine tubes, beyond which they become more and more dilated, but instead of being close together, they are placed at some distance from each other throughout their whole course, and lie, one on each side, above the nervous cord, but are connected by transverse ducts from near their very outlet in the penis to their lateral development into cæca (*c*). In this part of their course they would seem to perform the office, not only of efferential ducts, but also of receptacles for the seminal fluid, with which both the transverse ducts (*d*), and certain short dilatations from their inner side, are greatly distended. In one species, *Iulus* ——? I found fourteen of these transverse ducts, and in another, more than twenty. In other respects, these organs of reproduction, in the larger species, resemble those of *Iulus terrestris*, being covered with an abundance of short cæca. In this part of their structure they approach closely to some of the *Annelida*, in which, as in the common Leech, the cæca of the male organs are regarded as accessory vesiculæ seminales. But the great number of these cæca in Iulus, and the presumed absence of other structures which may be regarded as the proper testes of the animal, together with the greatly distended condition of the transverse ducts and efferential tubes towards their anterior part, have inclined me to the opinion now advanced respecting their true nature; although from their analogy with the Annelida, and also from the circumstance of my not having yet been able to follow out the main ducts to their commencement, the subject must still be regarded as open to further inquiry. With regard to the product of secretion in these organs, I have never yet found anything but a granulous fluid in the cæca, apparently similar to the granules in the higher animals, from which spermatozoa are produced; but this might have arisen from the immature recent specimens I was alone able to obtain. It would be interesting to ascertain whether these germs of spermatozoa are produced in the cæca, as there seems reason to believe, as we shall presently find that the ova in the female are secreted in sacs which appear to be analogous to these cæca in the male organs. I am inclined to think that the spermatozoa are not developed until the granulous fluid has passed into the efferential ducts at the season of impregnation.

In the female the organs of reproduction (fig. 4.) are as simple as those of the male. They consist of a single elongated bag, or oviduct, covered on its exterior surface with a very great number of ovisacs or cæca, of various sizes, each of which secretes but a single ovum. This oviduct extends backwards beneath the alimentary canal from its double vaginal outlet (*a a*) in the fourth segment behind the second pair of legs, as far as the posterior part of the rectum, close to the anus, where it ends in a cul de sac (*d*). It is most nearly in contact with the alimentary canal on its upper surface, but is separated from it by adipose tissue; in the pregnant female it is smooth and distended with ova, that have passed into it from the ovisacs, and are ready to be deposited immediately after intercourse with the male. The ova at the anal extremity of the duct are as perfect as those near the vaginal outlets. The oviduct contains in its cavity at least from seventy to eighty of these perfect eggs, awaiting impregnation, arranged in two or more irregular rows, and greatly distending its sides. In some of the larger species of the genus there are four, and in others five rows of eggs, the number of which is much greater than in our native species. Treviranus* merely described the ovary as formed of two rows of eggs, but the proper ovaries or ovisacs entirely escaped his observation. The ovisacs are distributed thickly, to the number of many hundreds, over the whole exterior of the oviduct (fig. 4 and 5. *c c*), from its posterior or cæcal extremity to within a short distance of its vaginal outlets. Each ovisac, whatever be its state of development, contains but a single ovum, every part of which is produced in it, from the germinal vesicle, in the most rudimentary form, to the yelk, albuminous fluid and shell. This fact deserves particular consideration. A large proportion of the ova in their ovisacs never arrive at maturity, but are retarded in their growth by the more rapid development of others that are near them; so that on examining an oviduct partially distended with ova, the greater number of ovisacs, in different states of development, are at the sides, and on the under surface of the duct, in parts which correspond to the interstices between the fully developed eggs that have passed into the oviduct, or are still forming on its exterior. One row of ovisacs usually exists on each side of the duct, near its upper part, but most of the ovisacs in the course of development are at its sides. The structure of the duct, and of the numerous ovisacs, is best seen in those specimens which have not yet arrived at maturity, or in those which have just deposited one laying of eggs. In these individuals the oviduct (*b, c*), to within a short distance of its division into two outlets, is studded with minute ovisacs, each filled with the rudiments of its minute ovum. Its general appearance in a female that has recently deposited its eggs, is completely botruoidal (fig. 5.), very like the ovary of

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* *Loc. cit.*

Birds, some ova being always fully developed, and ready to pass into the oviduct, while others are in various stages of development, many of which are imperceptible, excepting with the aid of a powerful lens.

But the most remarkable condition of the female organs is their double vaginal outlet (*a a*), as in Crustacea, although the oviduct itself is a single tube until near its termination (*b*), where it is divided into two short canals, which, from a slight opacity at their base, where they join the single duct, appear, when seen by transmitted light, to be separated from it by a valve, or duplicature of the lining mucous membrane. The vaginal orifices (*a a*) are simply two nipple-shaped portions of the tegument, with somewhat oval apertures, surrounded by a corneous ring, from which is developed a circle of minute hairs. Internally these apertures are closed by a soft thickened membrane: they are situated on the under surface of the fourth segment of the body, and correspond in that to the insertions of the legs in the third segment.

In their general structure the organs of reproduction in the female Iulus present some analogies to those of *Melöe* and *Forficula* among insects, in which the single ovisacs are arranged on the outside of a large oviduct, into which all the matured ova are passed, to be deposited at one act, as in *Iulus*; but they differ from these genera in the structure of their vaginal outlets being double, as in *Crustacea* and *Arachnida*; and differ again from the last in the remarkable fact, that throughout nearly its whole extent the oviduct is a single sac, and divides only into two canals, one of which passes on each side of the nervous cord, immediately before it arrives at its termination. It is difficult to give a reason for this peculiar conformation. May it be connected with a necessity for great rapidity in the act of depositing the ova required by this singular tribe of Invertebrata, the only apparent explanation of this peculiarity?

The situation of these organs beneath the alimentary canal, and their separation from it by the interposition of the adipose tissue, extended in the manner of a peritoneal coat, has already been stated. This adipose tissue, formed of vesicles filled with fat, as in insects, is most extended in the course of the muscular bands that pass down from the dorsal vessel, on each side of the segments, until they arrive at the upper surface of the oviduct. In the course of these bands there seem reasons for suspecting that vessels also exist, as they are proved to do in the *Annelida*; and that the ovaries are supplied, by these means, with the circulatory fluid directly from the dorsal vessel. The division of the dorsal vessel at its anterior part into distinct vessels, and the connexion of these with a large vessel extending along the upper surface of the nervous cord in *Scolopendra*, as seen both by Mr. LORD* and myself†, strongly support this opinion.

The simplicity of these organs in Iulus beautifully exemplifies the remarkable similarity of structure which the labours of anatomists have shown to exist between these parts in the two sexes. In both male and female the outlet to the organs is

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* Medical Gazette, March 3, 1838, vol. xxi. p. 892.

† Ibid. March 17, 1838, p. 970.

double, and also the excretory duct, for a short distance. In the female the ducts are soon united, and continue so throughout their whole course, forming a single large sac, covered externally with a large number of cæca, or ovisacs, in each of which the rudiments of a germ are secreted. In the male the ducts continue single, but communicate with each other transversely, thus forming a common cavity with a double outlet, as in the female. Externally they are also covered with cæca, apparently the analogues of the ovisacs; for like them they contain their peculiar fluid, and perhaps future spermatozoon that is believed to be essential to excite the development of the germ in the female. This uniformity of the organs in the two sexes, is further illustrated by the circumstance, that when the secretory process is complete, the fluid passes into the large efferential ducts in the male, as the ova are passed into the great oviduct of the female. These parts agree still more curiously in the entire absence of a *spermatheca* in the female, for the reception of the male fluid, and of distinct *vesiculæ seminales* in the male, for retaining it after it is secreted, the want of which in the former sex seems to indicate that the ova are deposited immediately after, or at the moment of impregnation; since, in true insects, in most of those instances in which the ova are deposited quickly after the coitus, there is either no spermatheca, or one that is but imperfectly developed.

2. *Structure of the Ovum.*

The existence of the ovisacs in Iulus as single, isolated capsules, on the exterior of the oviduct, in each of which a single egg is produced, is particularly favourable to a minute examination of the ovum in all its states, especially as ova are found at the same time in every stage of development (fig. 5. *c c*). The most rudimentary condition of the ovisacs I have yet seen was observed on examining, by transmitted light, part of an oviduct that had been placed for twenty-four hours in spirits of wine, and afterwards dissected in water. In this (fig. 6.) the smallest ovisacs appeared like very minute glandiform bodies, developed, as it were, directly from the structure of the duct itself (*c c*), in which the rudiments of the future egg had begun to be produced. The smallest of these bodies were of an elongated shape, and not more than three, or at most four, blood-globules in diameter. They appeared to have distinct parietes, and to be filled with very minute graniform cells of a uniform size, slightly opake, and of a yellow colour. The diameter of these cells, as nearly as I could ascertain by direct comparison, was equal to about one third of that of a blood-globule. In the midst of these cells there was a larger, but much more delicate structure, of a circular form (*a*), and equal in size to about two of the cells, but whether this was the proper germinal vesicle, or its macula, [?] it was difficult to determine. Other sacs in the duct, which were twice the size of these, were filled with similar contents. From the opacity and yellow colour of these graniform cells, it was evident that they constituted the yelk in one of its earliest stages. I have never yet seen these peculiar cells absent in any of the ovisacs, even in their most rudimentary condition. They have always exhibited the same yellow colour, with only a slight difference of opacity. At a later stage of development the whole appear to be inclosed in a distinct membrane, the *membrana vitelli* (fig. 11. *d*), and even at this early period (fig. 8.) the membrane may be regarded as in the course of formation, as seems to be indicated by the fact, that the cells always cohere together when the specimen has been placed in spirits of wine, and afterwards allowed to remain for some time in water. In these cases the ovisac becomes distended by the imbibition of fluid, as is proved by the existence of a clear transparent space between the interior of the ovisac (figs. 7 to 11. *d*) and the yelk (*e*), the cells of which do not separate, but together retain the form of a single mass unaltered, even when pressed in different directions between plates of talc beneath the microscope. When the ova are a little further advanced (fig. 9.), the same yellow-coloured graniform cells compose the yelks; but they are a little darker in colour, larger, and more distinctly exhibit a granular structure in their interior: they closely resemble in form and structure the vesicles of the yelk of the higher animals, as described in the excellent researches of Dr. MARTIN BARRY*. The membrane of the yelk (*d*) at this period is more strongly marked, and exhibits a distinct outline, when treated as above described; and the germinal vesicle is much more apparent (*a*, *b*). At this stage of the ovum, the outline of the vesicle is more distinct than in the previous stages, and this body is slightly larger. The *macula* (*a*) is of a perfectly globular form, and is apparently covered by a separate membrane†. It is very distinct, and is formed by an aggregation of minute cells, or vesicles, surrounded by fluid. In a more advanced stage of development the vesicle is surrounded at a little distance by an outer ring, which exists in all the ova in the succeeding stages, but is not seen in the first stages of the ovum.

When a perfect egg that has passed into the oviduct, and is ready to be deposited, has been placed in spirits of wine, and is afterwards examined beneath the microscope, there are seen on the outside of its opake shell one or more large spots (Plate IV. fig. 5. *b*. *c c*), which appear to be occasioned by some circumstance connected with the formation of the shell in the oviduct or ovary. These spots are large and oval, are formed by concentric rings, and appear to be the result of incomplete depositions of the different layers of material of which the shell is composed. In the vicinity of the spots within the egg I have usually found some aggregated oil-globules. The yelk is of a light yellow colour, and occupies nearly the whole of the interior, there being only a very small space around it, and a very slight quantity of albumen. The form of the yelk within the shell, when the egg has remained for some time in spirits of wine, is irregular. At one end it becomes obtuse, and rounded; at the other incurved, and a little pointed. On one surface it is large and convex, and on the opposite concave or excavated. In the middle of the concavity is the transparent globular vesicle, the proper germ vesicle, considerably enlarged, and presenting the

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* Philosophical Transactions, 1838, 1839, 1840.

† See WAGNER's Physiology, translated by Dr. WILLIS, Part I. 1841, p. 41, note 48.

appearance of an air-bubble; it projects from the surface of the yelk, and the cavity in which it is inserted. When viewed laterally, in those ova which have been first placed in spirit, and afterwards in water, it is very much elevated above the yelk into part of the clear space. In one instance I observed on the upper surface of this vesicle an aggregation of globules, apparently oil-globules, about six in number. Around the yelk, after the egg has been placed in water, there is a free space, as in the immature ovum. When the perfect ovum, soon after its impregnation and deposition, is placed for a few minutes in rectified spirits of wine, it begins to collapse on one side in the long axis of the egg: that surface is thus rendered concave. Sometimes the egg contracts also at one extremity, but this is only to a small extent when compared with the other. On watching this contraction of the egg and its yelk beneath the microscope, while it remains in rectified spirits of wine, the yelk is seen to retire from the shell towards the centre of the egg; and its external surface acquires a translucent appearance, resembling the albuminous space in the egg of the Bird. During this contraction, it may be seen, through the sides of the yelk, that the cells are of an uniform size, and retire slowly towards the centre. When the egg has remained about half an hour in rectified spirit, the whole interior has become translucent, although it is perfectly opake in the natural state, excepting a small part near the centre, the locality of the germinal vesicle. That those parts of the shell which collapse are the thinnest is proved by several observations. If the egg, when entirely contracted, is put into cold water, it again becomes opake, and the shell expands to its original dimensions and form; but the yelk does not change its appearance, it retains the shape it has taken in the spirit. From this expansion of the shell we may reasonably conclude that fluid is imbibed through it, and occupies the space between it and the yelk, and that this space is lined by distinct membranes; the *membrana vitelli* before noticed in the early stages of the ovum, and the *membrana externa* or *chorion* (*b*) that lines the interior of the shell. These membranes exist in the egg at the period when the embryo is complete; the membrana externa, or proper chorion, still lining the interior of the shell, and the membrana vitelli constituting the amnion which incloses the young animal. We are thus able to identify in these low forms of Articulata the parts that are known to exist in the most perfect animals, and in addition to them, the external envelope or shell. It is worthy of remark, that the whole of these structures are formed in the ovisac, in which, as before shown, only one egg is secreted at the same time, and is then passed, complete in all its parts, into the oviduct, to be impregnated preparatory to the production of the new being.

3. *Deposition of the Ova, and Habits of the Species.*

I have never yet seen the Iulidæ *in coitu*, but from the fact that the oviduct in the female, at the season of depositing her eggs, is always completely filled with them, all equally developed, as well as from the entire absence of a spermatheca, we may reasonably conclude that the act is of short duration, and that oviposition takes place very soon after impregnation. From the circumstance that many eggs are far advanced when the animals are collected in the autumn, I at first supposed that season to be the proper period for depositing the eggs ; but the facts now about to be stated induce me to believe that this process takes place very early in the spring.

In order to observe the habits of these animals, I collected from thirty to forty of them in the beginning of October, when they were preparing to hybernate, and placed them in a tin box, covered at the bottom with clay and sand mixed with vegetable mould, the soil in which they were captured. I fed them, according to the directions of Professor Waga in his interesting experiments*, with decaying leaves and slices of apple. In this way they were preserved perfectly healthy, and as winter approached, gradually passed into a state of hybernation. At the end of December, when the weather was severe, and the temperature of the room in which my specimens were kept was between 30° Fahr. and 40° Fahr., they were collected together in a heap, each coiled up in a circle between the folds of dried leaves, as if in a perfect state of hybernation. They were not aroused by the opening of the box. The temperature in which they had been kept, up to this time, had seldom been lower than 38° Fahr. On the 13th of January, when the temperature of the room was 41° Fahr., some of them were in a much more complete state of hybernation. A few were still coiled up between the dried leaves, and on opening the box became slightly disturbed, and crawled about very slowly; but others had penetrated the moistened clay at the bottom of the box, each one having formed for itself a circular cavity, in which it lay coiled up in a spiral form, in a state of rest, from which it was aroused with much difficulty. In this situation they remained for many days of exceedingly cold weather, during which the temperature of the room in which they were placed was for some time so low as 28° Fahr. On the 24th of February they were still in the same state, some of them being coiled up between the leaves, while others remained in their circular holes in the clay. A few days after this, the weather having suddenly become warmer, they began to arouse from their hybernation, moved actively about, and again took food. On the sixth of March they were still active and healthy, and, much to my satisfaction, I now discovered, in a little circular cavity at the bottom of one of the holes in the clay, a large packet of eggs, from sixty to seventy in number, that adhered loosely together, and seemed to have been very recently deposited. They were of a yellowish white colour, of an obtuse oval shape, and about the size of the seeds of the wild poppy, which they much resemble. A second packet of eggs was deposited in a similar manner on the 25th of March, and subsequently to this many other packets through the months of March, April and May, as was also observed by Professor Waga in his investigations; so that the spring months seem to be the season of oviposition.

The habits of Iulidæ in regard to the deposition of their ova are somewhat peculiar.

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* Revue Zoologique, par la Société Cuvierienne, No. 3, Mars 1839, p. 76.

Waga has already remarked, that although a moist locality is necessary to the health of these animals, it is prejudicial to them at the season of undergoing their changes, and is then even more fatal to them than a very dry one. I am fully satisfied of the correctness of this observation. Deterred by this instinct, the Iulus will not deposit her eggs, when in confinement, if the soil be too much moistened, nor if it be allowed to become too dry, while she is particularly careful to exclude them from light, by depositing them in a burrow in the middle of the soil, as far as possible removed from the sides of her prison, if this happen to be a glass vessel. She is also exceedingly anxious to prevent the accession of atmospheric air, and carefully closes the entrance to her burrow. The manner in which she proceeds with her labours is curious. Having excavated a little cylindrical hole to the depth of about an inch, and only just large enough to admit her body, she forms at the bottom of it a little circular cell or chamber, by digging out the soil grain by grain with her mandibles and anterior pair of feet. I have seen her busily employed in this part of her labours. When she has excavated the burrow to its proper depth, she remains for a few minutes with her head and the anterior half or two thirds of her body in the hole, as if resting from her toils, with the posterior part exposed on the surface, to enable her to cling by her feet to its margin, and thus afford her support in bringing up the soil she is removing from the bottom. Having continued in this situation for a few minutes, she again resumes her labours. In about a minute she gradually withdraws herself backwards from the hole, bringing up with her a little pellet of moistened clay, which she holds between her first pair of legs and under surface of the head. As soon as it is brought to the margin of the hole, it is passed backwards by these to the second pair, and so onwards to the next in succession as it reaches them, until it is removed entirely out of the way. She then immediately re-enters the hole, and this operation is repeated many times, until she has excavated a chamber at the bottom sufficient for her purpose. In each instance the pellet of clay is saturated with fluid, which appears to be supplied for the purpose by the large salivary glands of the animal, the chief function of which thus appears to be to furnish a great quantity of solvent fluid, to enable the parent the more easily to excavate the chamber intended for the residence of her future progeny in their most helpless, and, as we shall presently see, almost vegetative condition. Having accomplished this part of her labour she remains for some time at rest, with the greater portion of her body concealed in the burrow, and soon afterwards begins to deposit her eggs. When this is finished she immediately sets about the completion of her labours, by carefully closing up the entrance of the burrow. This she does with clay thoroughly moistened to form a thick paste, which she gently presses into the entrance, and fills up nearly to a level with the surface of the soil, thus not only preventing the intrusion of enemies, but also protecting the eggs from the prejudicial influence of the atmosphere, by exposure to which they quickly perish.

When the egg is examined immediately after it has been deposited, it is highly translucent, but soon becomes more opake. Its shell is soft and elastic, and, when exposed to the air, quickly becomes dry and shrivelled. It is on this account very difficult to preserve it to watch the changes in the embryo. Of at least a dozen packets of eggs which I obtained this spring, I have succeeded in rearing only one, which happened to be the packet first deposited, as just described. It is owing, probably, to this difficulty in rearing the eggs and the young of the Iulidæ, that the changes of these animals have been so imperfectly known, and that so much want of precision has been found in the observations of those who have most attended to this subject. Thus Degeer, who first watched the development of these animals, describes them as possessing six legs when newly hatched*, while M. Savi states that they are completely apodal†; and Waga‡, in his excellent observations on this class, confirms the statements of Savi, but is exceedingly vague in parts of his account of the changes, and has entirely overlooked many most important facts in regard to the production of the new segments.

In my own investigations these difficulties were in part guarded against by filling a thin glass tube with moistened clay, in which a little cavity was formed for the eggs close to the glass, and when the eggs had been carefully removed into it the hole was closed up with clay, and also the tube with a cork. By this means the eggs could be watched from day to day, with a common lens, through the glass, without being exposed to the atmosphere. In this way the changes were easily observed; and one or more eggs were removed from time to time for closer inspection beneath the microscope, care being taken always to close up immediately the hole in the clay. The specimens were examined both in the recent state, and also after they had remained a few hours in spirit of wine, so that the changes were accurately noticed.

The time occupied by the development of these animals extends through many weeks. It is divisible into separate *periods*, which are most distinctly marked, but the extent of which is influenced by external circumstances; and although in general of a certain duration, it is more or less hastened, as in many true insects, by an abundance or deficiency of food, and above all by the temperature of the surrounding medium. I have been unable, from accidental loss of specimens, as well as from the time required for their completion necessarily extending far beyond the present period, to observe the whole of the changes as they occur throughout the entire life of the species, but nevertheless have been able to extend my investigations sufficiently far to enable me to record some remarkable facts, the occurrence of which, I have distinctly ascertained, continues to the full development of the species, and indicates the existence of one general law or principle in the growth of this class of animals.

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* Mémoires, tom. vii. p. 582.

† Observazioni per servire alla storia di una specie di Iulus communissima, *Bologna*, 1817. Bulletin des Sciences Naturelles, tom. xii. Dec. 1823. Memorie Scientificke de Paolo Savi Professore, &c. Decade 1ma Pisa, 1828.

‡ *Loc. cit.*

4. *Evolution of the Embryo.*

The *first period* in the evolution of the embryo extends from the deposition of the egg to the gradual bursting of the shell, and exposure of the embryo within it.

In my observations this period extended from the morning of the 6th of March, a few hours after the eggs had been laid, to the 31st of the same month, a period of twenty-five entire days. The atmosphere of the room in which the eggs remained, varied throughout the whole time only from about 50° to 60° Fahr. It has been already seen that the egg of Iulus contains all the primary parts of the ovum of Vertebrata while still in the ovisac, and that the chief alteration the egg undergoes, after it has acquired a size large enough to be recognizable by the naked eye, is simply an increase of its whole bulk.

I can offer but few observations on the earlier changes in the egg after impregnation, a subject already deeply investigated in other classes. I noticed, however, that there was considerable alteration in the appearance of the graniform cells of the yelk within the *first day* after the egg was deposited. They varied much in size towards the end of the day, some of them being much larger than others. The smaller ones were exceedingly numerous, less globular in form, more opake and granular, and adhered together, as if by a tenacious mucus, more firmly than in the unimpregnated egg, but I could not yet observe any trace of the embryo. The shell was still soft and elastic, and the egg became shrivelled and dried up if exposed to the air only for a few minutes. On the *second day* there was still no marked difference in the external appearance, but in the interior the whole yelk had become firmer, and the cells cohered more together, and I thought I could perceive some faint indications of the future embryo, composed entirely of large and small cells. On the *third day* the egg exhibited no further perceptible change; but on the *fourth* its contents were distinctly less fluid than at any preceding period, and the cells were more globular and larger. This was very distinct both in regard to the larger as well as the smaller cells. The whole of them were more closely aggregated together, and much more distinct beneath the microscope. Up to this period I had not satisfactorily observed the outline of the embryo, but there was now a little granular mass on one side of the shell, which I was inclined to regard as the future being. The exterior of the shell had also become more firm and less elastic. Unfortunately, at this period I was accidentally prevented from continuing my observations, which I was unable to resume until the 25th of March, the *nineteenth day* after the egg was deposited. There was now a complete alteration in its form. It was more obtuse at both ends, and had become much larger. This enlargement of the impregnated egg, I have constantly observed in the eggs of *Melolontha*, *Melöe* and *Athalia*, in all of which the shell is elastic. At this period the outline of the embryo coiled up within the shell, and nearly filling the whole interior, was very distinct (Plate IV. fig. 2.). On one side there was a clear transparent space extending about half way across the shell, indicating the distance between the inflected head and tail of the embryo, the body of which was composed entirely of cells, a little larger, and more closely aggregated together than in the preceding examination; but there were no rudiments of limbs, or even of a division of the body into distinct segments. The shell had acquired more firmness, but the egg still contracted, and dried up quickly when exposed to the air for a few minutes. On the following day, the *twentieth* from the deposition of the egg, the outline of the embryo was more apparent (fig. 3.), and on its concave, or ventral surface, there were faint traces of a division of the body into six segments. Up to this period I was unable to detect any funis or umbilical cord to the embryo, a structure of which I was particularly in search, and which I was aware had been seen by RATHKE in Crustacea. When the egg was examined in rectified spirit, the outline of the embryo was rendered more distinct, especially at its extremities, and its whole body was contracted so as to leave a space between it and the shell. It was still formed of cells of different sizes. From this time, to the bursting of the shell, the egg became every day larger, until the morning of the 31st of March, the *twenty-fifth day* from the deposition of the egg, when it was greatly distended, and began to assume a kidney-shaped appearance (fig. 4.); that side of it which corresponded to the ventral surface of the embryo then became a little concave, and the opposite, which corresponded to the dorsal surface, much more convex. The shell was now bursting longitudinally in the median line of the dorsal surface, and the back of the soft and perfectly white embryo was gradually pressing through the opening. The young being was now entering a new state. Its body had exhausted the nourishment supplied to it by the yelk, and it had thus arrived at the termination of the first period of its development, a period of twenty-five days.

In the *second period of development* the embryo is exposed to a new medium, and perhaps derives the means of its further growth from external sources, although it is still enveloped in the fœtal membranes, and retains its connexion with the shell.

The apodal condition of the young Iulus at the bursting of its shell, has already been noticed by SAVI and WAGA, the latter of whom refers also to its extremely delicate and motionless state; but both of these naturalists have overlooked the remarkable fact of its remaining for many days connected to the shell by means of a distinct funis (*d*), and also of its being still inclosed in the *amnion*, the proper fœtal covering.

The liberation of the embryo by the bursting of its shell is a remarkably slow process, as compared with the escape of other animals from the egg. In my observations from ten to twelve hours elapsed before the body of the embryo was so far liberated as to remain only partially inclosed between the two halves of the shell (fig. 5.), to the interior of which it was still attached by its pedicle or funis (*d*). So remarkable is its condition at this period, that it strongly resembles the expansion of the germ in the seed of a plant, rather than the evolution of a living animal. The embryo is perfectly motionless, and the bursting of its shell appears to be effected, not by any direct effort of its own, since, up to this period, it has acquired only the form and external semblance of a living animal; but by the force of expansion of the growing body, the development of which being greatest along the dorsal, or larger curvature, exerts, in consequence, a greater degree of force against the middle of the dorsal than the corresponding part of the ventral surface; the head and tail of the embryo acting as a fulcrum against the ventral surface only at the ends of the shell, and thus bending it to the kidney-shaped form it assumes, while the dorsal surface of the embryo is gradually pressed through the opening. From the comparative rapidity of its enlargement immediately after the shell is fissured, it seems as if the stimulus given to it by exposure to a new medium, atmospheric air, is the great means of exciting its evolution. The embryo is now formed of eight distinct segments (fig. 5. *a*), including the head, the ninth or anal segment being still indistinct. The head is more defined in its outline, and firmer in texture than other parts of the body, and is inflected against the under surface of the prothorax (2.), or second segment, from which it is divided on the upper part by a deep transverse line. At its sides it exhibits a faint trace of the future antennæ. The four thoracic segments also exhibit on their ventral surface little nipple-shaped extensions, three of which, on each side, are the rudiments of the future legs. When viewed from above, the body of the embryo appears compressed and wedge-shaped, its greatest diameter being in the second and third segments, while each succeeding segment is more and more contracted. I was unable at this period to detect any separate internal organs, the whole embryo being still a congeries of vesicles or cells, in the midst of which there seemed to be some faint traces of the commencement of an alimentary canal.

It has already been stated, that certain markings exist on the egg at the time it is deposited. These are all on the external surface of the shell. They are seen, in the empty shell (fig. 5. *b*), to arise from some deficiency in the several layers of which it is formed. Some of these markings (*c c*) show that the shell is composed of at least four concentric layers or coverings, the deficiency in which, when viewed on the outside of the egg, presents an appearance which at first may readily be mistaken for the germinal vesicle and spot, seen through the shell. Each layer of the shell appears to be thicker the nearer it is to the surface. On the inner surface of the shell there is also often a slight marking (*d*), at that part which has not been separated, but forms a valve or connexion between the two halves. From the circumstance of its being near the part where the funis joins the external membrane (5. *a. e*) that lines the shell, and is ruptured with it, there is some reason to believe that this may have been the situation of the germinal vesicle at the period of impregnation. The fissure in the shell, although always in its longitudinal axis, is often rough and uneven, and never entirely separates the two halves.

At the *end of the first day* (fig. 5. *a*) I carefully removed the embryo and shell into diluted spirits of wine, and on examination beneath the microscope, found the body of the embryo covered with an exceedingly delicate cuticle, through which the cells it is formed of were distinctly visible. It was also completely inclosed in a smooth and perfectly transparent membrane (*c*), which seemed to contain a clear fluid, interposed between it and the embryo. This membrane I regard as the analogue of the *amnion*, the *vitelline*, or investing membrane of the embryo in the higher animals, and identical with the *membrana vitelli*, before described as the proper membrane of the yelk in the egg of Iulus. It is a shut sac, that completely invests the embryo, except at its funnel-shaped prolongation at the extremity of the body (*d*), where it is constricted, and, together with another membrane (*e*), which in the unburst egg is external to this, and lines the interior of the shell, assists to form the cord or proper funis (*d*). The *funis* enters the body of the embryo at the posterior part of the dorsal surface of the future penultimate segment, where the *mucro* or spine exists, in the adult animal, and not at the dorsal surface of the thoracic region, as seen by Rathke in the Crustacea. The proper anal or terminal segment (9) is as yet but imperfectly developed. In the funis (*d*) I also observed some exceedingly delicate structures that exhibited all the appearance of vessels. They seemed to enter the body by two sets, that were spread over, and entirely lost in the membrane (*e*). Whether these were, indeed, vessels, or merely folds of the membrane, I am not certain. The membrane (*e*) in which they appeared adheres closely to the shell, and retains the embryo in connexion with it by means of the funis. In the unburst egg this is also a shut sac, like the amnion, and forms the *membrana externa* or *chorion* [?], the second or outer investing membrane of the ovum lining the interior of the shell.

The detection of these two investing membranes of the embryo in Myriapoda may be regarded with some interest in reference to the analogies which they bear to similar structures in Vertebrata, since they show the persistence of one universal law in the mode of development of the germ.

On the *second day* after the bursting of the egg, the segments of the body were more distinctly marked, and the form of the second, the prothoracic segment, was slightly altered, and the rudiments of the legs were enlarged.

On the *third day* the embryo had considerably increased in size (fig. 6.), but was still perfectly motionless, and attached to the shell by the funis. This attachment continues for many days, during which the embryo remains partially protected by the two halves of the shell (fig. 5 and 8.), and in so far as it has ceased to derive nourishment from an internal source, although still an embryo, it may perhaps, physiologically considered, be regarded as placed in the same relative state of extra-uterine existence, as the fœtus of the Kangaroo in the marsupium of its parent. When examined at this period in the recent state, all the parts of the body are still indistinct; but in specimens that have been for some time in spirits of wine, the divisions between the segments are well marked, and the altered form of the prothoracic segment, with its smooth rounded inferior surface, is well defined. The rudiments of the legs are more developed, but those of the second and third segments less than the fifth; so that not only at the bursting of its shell, as first noticed by Savi, but also for several days afterwards, the embryo is completely apodal, the future limbs existing only in a rudimentary state. Posteriorly to the fifth segment, the body is more soft and delicate, and the segments less clearly defined. This results from the circumstance that it is at this part of the body that the future new segments are to be produced.

On the *fourth day* (fig. 7.) I first observed some faint traces of a single eye, or ocellus, on each side of the head. The embryo had now further increased in size, and the rudiments of its future legs had become larger and more obtuse (6. *a*); an appearance which the newly-formed limbs of the Articulata often exhibit previously to their further elongation. Traces of the formation of internal organs were now evident through the tegument at the posterior part of the body, and the funis (*d*) was contracted, as if about to separate. Internally, the body was still formed of cells aggregated together, but differing more in size than at any previous period, as if they were becoming fused into separate tissues; and in the midst of them, and closely surrounded on all sides, was the newly-formed alimentary canal. The canal was now more opake, and when pressed out of the body, more firmly adhered together, than any other internal structure, and was distinctly composed of an aggregation of very minute cells. Around the sides of the body, muscular structure was also in the course of development, but as yet was exceedingly indistinct. I could observe no perfect fibre. This fact will sufficiently account for the entire absence of spontaneous motion in the embryo up to this period. A new process was now about to commence, —the development of new segments. We have seen that on the third day the posterior part of the body is less distinctly divided into segments than the anterior, the first five segments being most distinctly marked. The sixth and seventh are now more defined. It is in the membrane (*f*), that connects the seventh with the eighth segment, at the posterior margin of which the funis (*d*) enters, and which segment is permanent as the *penultimate* throughout the life of the animal, that the formation of new segments is taking place. At this period, it is only a little ill-defined space that unites the seventh and eighth segments into one mass; but in proportion as the anterior parts of the body become developed, this part is also enlarged, not as a single structure, but as a multiplication or repetition of separate similar structures.

On the *ninth day* (fig. 8.) the changes have advanced much further. Not only have the future new segments become more distinct (*f*), but transverse depressions are also seen on the dorsal surface of the original segments, showing their division into double ones, as in the perfect animal. The rudiments of the legs are now further developed (6. *a*), and their transparent distal extremities are seen through the investing membrane applied closely together, and extended along the ventral surface of the body, as in the nymphs or pupæ of true insects. The antennæ and ocelli are more apparent, and the embryo itself has increased at least one-third of its original dimensions. It is still attached by the funis to the shell; but this attachment is daily becoming more fragile, and is now separated by very slight causes. The embryo has thus continued to grow through nine succeeding days, since the bursting of its shell, without any visible means of nourishment, the nutriment supplied by the yelk having been exhausted before that occurrence. Hence it becomes matter of inquiry, from whence it now derives its means of growth? Whether it has already sufficient materials derived from the egg, and stored up within itself for its further development; or whether the external inclosing membrane may not still contribute to the function of nutrition by absorbing fluid condensed from the air of the humid locality in which it resides? The probability of this last supposition is somewhat countenanced by the fact, that I have constantly observed the membranes of the embryo at this period covered with microscopic drops of fluid; but whether this is fluid condensed on the membranes from the atmosphere of the dwelling, or whether it results from the transudation of that which was contained in the amnion, remains for future inquiry.

Up to this period the embryo gives not the slightest evidence of spontaneous or voluntary motion. Internally, it is still composed of cells of different sizes, that are now in the course of producing muscular and other structures. In some parts of its body no arrangement of them seems as yet to have taken place, the cells being merely aggregated together. Cells of three very distinct sizes now exist. The diameter of the largest of these is nearly three times that of the second size; and the second again are nearly twice and a half the size of the smallest. The smallest-sized cells fill up the interspaces of the others, and appear as if breaking down to form interstitial or cellular substance, while the second-sized cells are arranged in rows to form particular structures. In the midst of these cells the *alimentary canal* is now nearly complete, but I have been unable to observe its connexion with the funis; and at its anal extremity it is a little dilated, and extends forward as a short straight intestine, the *rectum,* until it arrives at a part where a valve seems about to be formed. The diameter of the canal is there enlarged, and on its surface are three distinct longitudinal muscular bands. The so-called *hepatic vessels* also exist as distinct tubes, inserted, one on each side, into the alimentary canal at the constricted or valve-like part above noticed. The canal is then continued forwards until it is again dilated into the proper stomach, and terminates, or rather commences, in a narrow œsophagus. It is much longer than the body of the embryo, being convoluted or folded upon itself, in its lower portion, to adapt it to the changes the body undergoes in the enlargement and elongation of its segments. It is not yet separated from the now forming structures by any distinct investment, either adipose or peritoneal, except only what belongs to itself; but it is closely surrounded by cells of the second and third size.

On the *tenth day,* the great circulatory, or dorsal vessel, was most distinctly seen through the amnion and skin. This doubtless had existed much earlier, although not observed. It was exceedingly well marked, but I was unable to observe any motion in it. The head of the embryo had now begun to assume the peculiar corneous appearance common to the larvæ of true insects; its body had much increased in size, and the amnion was still covered with microscopic drops of fluid.

On the *eleventh day* the head was more distinct, and the antennæ appeared at its sides like short crescent-shaped clubs, with their terminations directed forwards. Above them the single ocelli were distinctly seen. All the segments, posteriorly to the third, exhibited the transverse line that indicated the division into double segments, and the posterior segments were much increased in size.

On the *sixteenth day* none of the embryos had left the egg-shell, nor exhibited any signs of motion. I now thought I could perceive some irregular movements in the dorsal vessel through the skin and amnion. The whole body of the animal was greatly enlarged, particularly in the posterior region, and more especially in that part in which the new segments were making their appearance. The head had acquired a slightly brownish colour, and the future eyes were more visible, being also slightly tinged. The first period after the bursting of the shell was now soon to close, as the amnion was greatly distended, and in a few of the specimens the funis was ruptured. In the whole of them it had become so fragile as to be separated by the slightest motion.

On the morning of the *seventeenth day* (fig. 9.) all the embryos were ready to leave the amnion. Some of them were already detached from the shell, others were still connected to it. Their increase of bulk within the last few hours had been very great. The body was now more straightened, the head less inflected under the thorax, and the eye was a dark-coloured spot above and behind the antenna. The segments of the body were divided by distinct reduplicatures of the proper tegument, and the legs, folded side by side against the ventral surface, were much further extended beneath the amnion (6. *a*). The transverse divisions of the first six segments strongly marked the original segments; and the amnion (*c*), now ready to burst, was tightly extended over the dorsal surface, and by the elongation of the body was rendered more distinct on the ventral. The great increase in the length of the animal was mainly occasioned by the growth of the posterior segments, more especially those in the ante-penultimate space,—the proper *germinal space* or *membrane* (*f*),—the faint divisions of which into new segments were now distinctly seen through the amnion. The seven anterior segments, including the head, were greatly enlarged, and the hitherto minute anal and penultimate segments (8. 9.),—in the first of which the remains of the funis (*d*) forms a rudimentary anal spine,—had also been enlarged, and were now fast acquiring the form they afterwards retain throughout the life of the animal. Some of the specimens soon threw off their covering, and entered the *third period of development.*

The animal was now greatly enlarged, and possessed three pairs of legs, but it still lay with these newly developed limbs coiled up without voluntary motion. The amnion had been fissured at its anterior dorsal surface, and slipped off backwards from the posterior segments, and lay at the anal extremity, while the animal itself with its limbs coiled up appeared as if exhausted with these, its first, spontaneous efforts. No other signs of animal existence were given than occasional slight movements of the antennæ. A second, a third, and then a fourth specimen gradually escaped from their coverings, and these were followed by others in quick succession; but so feeble at first were their efforts, after throwing off the envelope, that scarcely any motion could be perceived in them. The embryos thus passed from their apparently inanimate to an animated state of existence; from a condition in which they seemed merely to vegetate, endowed with no voluntary or instinctive powers, but, like the vegetable, formed entirely of an aggregation of cells, totally incapable of spontaneous motion,—to one in which they became active beings, gradually acquiring voluntary and instinctive faculties, both as regards the means of procuring nourishment and of preserving themselves from injury. Short indeed is the transition from a mass of uniform and inanimate cells, to the development into an active being, endowed with its peculiar instincts.

In less than ten hours from the commencement of this last change, the whole of the embryos had burst and thrown off the amnion. Those in which the funis had not been separated, now left their covering still connected by it to the shell.

In *about an hour* after leaving the amnion (fig. 10.), the young Iulus exhibited a marked change. Its head was elongated on the prothorax (2.), the parts of the mouth were distinctly moveable, and the eye, a single ocellus on each side of the head, acquired a darker colour. Each antenna was composed of six distinct joints, of which the third was much the longest, and the two apical ones were short, and sunk one within the other. The whole body had been increased at least one-fourth in bulk since leaving the amnion. It now measured about a line in length, and exhibited very distinctly the nine original segments. The seven anterior of these were strongly marked. In the *germinal space* (7. *f*), between the original seventh and eighth segments, *six new segments* were now developed. These were still very small, the length of the whole being equal only to that of one of the original segments. At the present time they did not form independent divisions of the body, but were covered by the common tegument, and thus appeared like supplementary parts of the seventh segment, produced from the germinal membrane, and interposed between the seventh and the penultimate segment (8.), which, as before stated, is a permanent segment throughout the life of the animal. This latter fact shows that it is not merely by an elongation and division of the terminal segment that the body of the Iulus is developed, but that it arrives at its perfect state by an actual production of entirely new segments; that these are new growths or formations which are in progress long before they are apparent to the eye; and that the original segments of the ovum, into which the animal is first moulded, are permanent segments throughout its whole life. But still more curious is it, that not only have new segments been formed as described, but that the common tegument by which they are now covered, and which also invests the whole body as the true skin, has already begun to be detached, preparatory to its being thrown off, as is shown in the fact that the new segments are now seen beneath it; and it is further remarkable, that this deciduation of the first skin of the animal had actually commenced before the bursting of the amnion. These circumstances explain the cause of the very quiescent state of the young Iulus, and its almost, and perhaps entire abstinence from food while this skin remains on its body. It is not until this skin is thrown off that the new segments become elongated, and the Iulus then appears suddenly to have acquired six new divisions to its body. The production of new legs is equally curious. Up to the present period the animal has but six legs (6. *a*). But four additional pairs are nevertheless in the course of formation. These at present exist only as eight minute nipple-shaped prominences on the under surface of the sixth and seventh segments (*b*, *c*), four on each, covered by the common tegument, which we have seen is becoming deciduous. The three single pairs of legs that now exist as the only locomotive organs, are attached, one pair to the prothoracic or second segment, one to the third, and one to the fifth segment. The fourth, or segment intermediate between these, never possesses any legs, but in the female contains the outlets of the organs of generation. In pursuing the analogies between these segments and the thorax of insects, the first two seem to represent the *pro-* and *mesothorax*, while the fourth and fifth becoming united, answer to the *metathorax*: this is analogous to the fusion of the fourth and fifth segments in the changes of true insects. The general appearance of the animal has now become less delicate, the head has acquired a darker colour, and a faint broad brown patch (*p*) is now making its appearance at the anterior part of the seventh segment. This patch, which is permanent through all the earlier changes of the animal, is of the greatest utility in determining the production of the new segments. It is in the segment immediately posterior to this that the male organs find their outlet, a circumstance the more remarkable, from the fact that this outlet is in the anterior part of the original germinal space, and that at the bursting of the egg this is very near the termination of the body. This was the condition of the young Iulus *one hour* (fig. 10.) after leaving the amnion. It soon began to exhibit its animal powers, to show the instinct peculiar to its species, and to be sensibly affected by external causes. In less than six hours from the bursting of the amnion, those specimens which had first undergone the changes were in motion. At first the antennæ were the organs employed. They were moved slowly to and fro, and seemed to gain power by use; in a short time the limbs began to be extended, and the animal slowly raised itself upon them for the first time. Its first efforts at locomotion were exceedingly feeble, but it gradually gained strength. At the end of twelve hours the whole of the embryos were in motion, crawling about slowly, but moving the antennæ briskly. On exposing them to a strong light, a marked effect was produced in their movements. They evidently were greatly affected by it, and seemed instinctively to shun it, retiring out of the way. This was the first marked exhibition of instinct. Locomotion was at first performed very slowly, but with instinctive care. The anal segment previously to each step was expanded like the anal leg of the larva of an insect, and being first attached like a true proleg, and its step, as it were, measured, its body was carried forwards by an effort that extended, as in insects, from segment to segment.

At *twenty-four hours* (fig. 11.) after escaping from the amnion, the young animals were lying together in a heap, but when disturbed, seemed to have acquired more power of moving; they remained quiet except when aroused, and had not yet taken food. The only marked difference in their appearance, excepting that they had still further increased in size, was in the nipple-shaped protuberances on the sixth and seventh segments, the rudiments of future legs (*b*, *c*). These were now more distinct and mammiform. Ten hours later in the day, they assumed still more the appearance of nipples projecting from the under surface of the segments. When examined in specimens that had been placed in spirit of wine, it became evident that these projections were occasioned by the development under the deciduous tegument of four new but exceedingly minute legs, complete in all their parts, each covered by its proper skin. The claws to the legs of the other segments were also more strongly marked. The new segments (7.*f*) were now more developed, although still covered by the common tegument, and, as in the preceding state, forming only one division of the body, while a small space behind them (*g*) indicated the point from which other new segments were to be produced.

On the *nineteenth day* (fig. 12.) these animals had acquired a little darker colour, but were still remaining quiet in their cells, and did not appear to have taken food. The enlargement of the body had not extended to the prothorax, which did not increase in size in proportion to the rest. The eye was more distinct, and the margins of the segments were bordered with short red points. The double pairs of new legs to the sixth and seventh segments, were now distinctly visible through the external tegument, which had begun to be separated from the under surface of the old segments, to which, up to this period, it had closely adhered. The patch on the side of the seventh segment had become darker, and the new segments were further advanced.

On the *twentieth day* (fig. 13.), although the animal continued almost motionless, it had acquired much strength; its limbs were much larger, and the claws at their apex more distinct. In a specimen that had been hardened in spirit of wine, the rudiments of the two double pairs of legs of the sixth and seventh segments (*b*, *c*), each encased in its own proper skin, were seen coiled up, with all the articulations perfect, while still further enlargement had taken place in the new segments. The common tegument was extended over the new segments, as in the previous observations, and the animals were still collected together among the burst envelopes of the eggs, and the cast-off amnions, remaining, as it were, in a kind of pupa state.

On the *twenty-first day* (fig. 14.) they were in the same condition, coiled up, perfectly quiescent, with their legs disposed side by side along the under surface of the body, like the pupæ of Lepidoptera. The new legs had now considerably increased in size, as well as the whole animal, although it had not taken food. The animal was still partially coiled up, but the skin that covered its body was greatly distended, more especially along the ventral surface. It was less able to move than before, the period of throwing off this skin being fast approaching; the double legs of the sixth and seventh segments, inclosed in their proper skin, were now more elongated, and very much enlarged, and the new segments were further developed, as well as the future germinal membrane (*g*). The external tegument was more extensively separated from the whole body, especially at the posterior part, and the head was retracted within it, and bent on the under part of the thorax. It was thus evident that this tegument was not of recent formation, that it simply inclosed the animal, as the whole had previously been inclosed in the amnion, as is proved by the circumstance that it extended smoothly over the whole body, antennæ and legs, and did not follow the inflexion or reduplication of the proper surface of the animal like the folds of the true skin beneath it, as in the original segments (14. 7.), but passed directly over them, and was simply protruded or extended by the growth of parts beneath, as in the instance of the new legs (*b*, *c*). Up to this period the young Iulus must still be regarded as in the embryo condition, although for a day or two after bursting the amnion, it possessed the power of locomotion, and evinced some development of its peculiar instinct. At its next change of skin, when it enters what I regard as the fourth period of its development, and when it has acquired fourteen pairs of legs, it assumes for the first time a condition analogous to the larva state of true insects on bursting from the ovum; the difference between the two being, that the analogue of this tegument of the embryo in insects is slipped off at the bursting of the amnion on leaving the shell, while that of the Myriapod is not thrown off until some days after it has entirely left the ovum. The correctness of this analogy is confirmed by the fact, that the double legs, which may be regarded as the analogues of the abdominal legs of the Caterpillar, are not acquired until a late period in the development of the embryo in insects, as I have seen in the embryo of the Caterpillar of one of the Saw-flies, *Athalia centifoliæ*, in which the first legs developed are the thoracic, and at a little later period, while the larva is yet in the egg, the abdominal ones. The permanent state of Iulus is thus strictly analogous to the transitory condition of the insect in the larva state, the relative development of the two being very similar. The Iulidæ and other Myriapoda are thus connected, on the one hand, with insects in the larva state; and, on the other, still more closely with the Annelides by the reproduction of segments of the body; the repetition of the segments in Iulus and other Myriapoda being one of the last occurrences in the higher forms of Articulata, in which distinct segments, or principal, and vital portions of the body are formed after leaving the ovum. This phenomenon almost entirely ceases in the higher families of Myriapoda, the Scolopendradæ, in which the number of segments produced is gradually diminished. I may here also remark, in proof of the persistence of the same principles in the development of the egg in true insects, that in *Athalia centifoliæ* I have found the animal in its shell inclosed in a distinct amnion, and that the funis enters the body of the embryo at a part precisely similar to that of the entrance of the funis in Iulus, the posterior margin of the penultimate segment, the prolongation of which segment in some larvæ of insects, the *Sphingidæ*[*], &c., may be regarded, as in the Iulus, as the remnant and representative of the funis.

This embryo condition of the animal will thus explain the circumstance of its first acquiring a slight power of locomotion, and then remaining perfectly quiescent, without taking food, to prepare for this change,—the third period of its embryo life. It is then to prepare for this last change that it now lies perfectly quiet, absolute rest being necessary to its proper evolution, which may probably be retarded by the slightest disturbance.

The lower portion of the alimentary canal is now distinctly visible through the new segments. When examined by transmitted light, it exhibits a corrugated or folded appearance, being folded to allow of its sudden extension at the period of throwing off the skin, and elongation of the segments. The colon is of a very dark colour, and exhibits its thickened peculiar structure, with its longitudinal muscular bands. Around its posterior part I observed an aggregation of what appeared to be globular cells. They seemed to be part of the organs of generation in the course of development. At first I regarded them as hepatic vessels, but this could hardly be the case, from the fact that each of these organs directly enters the canal as a straight vessel; but they might be vessels folded up to be unfolded suddenly, as in the case of the alimentary canal.

On the *twenty-second day* (fig. 15.) but little further advance is made in the development of the animal, save only that the original segments more distinctly exhibit the appearance of the segments of the perfect Myriapod. This appearance was more distinct than on the previous day, and consisted of transverse markings that divided each segment into two parts, the posterior of which was now impressed with longitudinal striæ. From the length of the posterior portion of each segment being slightly shorter than the anterior, it is evident that the segments had not yet acquired their fullest development, although they were more advanced than on the previous day. The dark patch of colour on the sides of the seventh segment, which first began to show itself on the seventeenth day, was now much more distinct, and became each day more and more apparent. The head of the animal was a little more bent on the thorax; the limbs more straightened; the new segments much further developed; and the whole indicated that the deciduation of the covering was now rapidly approaching.

The *fourth period of development* is as distinctly marked as the third. The young Iulus now has seven pairs of legs, and fifteen segments to its body.

On the *twenty-sixth day* (fig. 16.) nearly the whole of my specimens had changed their condition. In the morning some of them had already undergone the change, and were now briskly moving their antennæ, but still remained collected together. The antennæ were elongated at least one-third of their original length, and exhibited six distinct joints. The eye still consisted of a single ocellus, but this was now surrounded by a darker coloured portion of the tegument. The new legs (*b, c*) were

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[*] Philosophical Transactions, 1834, Part II. Plate XIV. figs. 11 and 12. (12.)

equal in size and length to the original ones, but were evidently more feeble. The transverse markings on the seven anterior segments (2. 7.) were very distinct, and the large brown patch (*p*) on the seventh segment was much darker in colour. The whole body of the animal was considerably elongated. This was produced chiefly by the extension of the new segments (7. *f*) formed from the germinal membrane at the posterior part of the seventh, and which, in the early part of the last period, seemed to form a single distinct segment covered by the common tegument. The most anterior of these new segments (8.), now the eighth of the whole body, had acquired an extent equal on its upper surface to the preceding segment, but was shorter on its ventral surface. Like the preceding original segments, it was divided into two regions by a transverse depressed line. The next segment in succession to this, the ninth, had also become enlarged to about one-third of the eighth, and was like it marked transversely. The next four segments were each more developed than in the preceding state, but not to so great an extent as the others. The two remaining segments (14. 15.), the penultimate and anal, had undergone no change. They had simply acquired a little extension at the apex of the segment, and were now covered with a few scattered hairs. It is thus proved that the body is elongated, not by the division of already formed segments into others, but always by the formation of new ones in the germinal membrane that extends from the posterior margin of the antepenultimate segment, to the penultimate, which last segment, with the anal, undergoes no change. That segment is always furthest advanced which is immediately posterior to the last segment that possesses legs; and then the next in succession, until we arrive at the terminal ones, the penultimate and anal, which never bear legs. The body of the Iulus is thus formed of fifteen segments. In this respect it affords a further analogy to those already pointed out with the larva of insects at their first coming from the egg, only that the Iulus is one grade lower. The usual number of segments in insects is thirteen, but this is not constant. It varies in accordance with the higher or lower state of development of the species. Thus, in some, the thirteenth is only a very rudimental one, even less developed than the fifteenth at this period in Iulus. But in some of the apodal larvæ, which approach closely in their rudimental condition of development to Iulus, the number is fourteen, besides a minute anal tubercle, analogous to the anal or fifteenth of Iulus. This has already been elsewhere shown both by Westwood* and myself†, and seems to confirm the view here advanced with regard to the comparative development of these animals; though in Iulus all the segments are double.

On the *twenty-eighth day* I found all the specimens, now in the fourth period of development, still lying collected together. On moistening the soil in which they were placed, they soon moved briskly their antennæ, as if seeking nourishment; their motions were still exceedingly feeble. This could not have arisen from too reduced

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* Transactions of the Entomological Society, vol. ii. p. 124.

† Cyclopædia of Anatomy and Physiology. Insecta, vol. ii. p. 871.

a temperature of the surrounding medium, as the last few days had been exceedingly warm, and at the time of making this observation the temperature of the atmosphere was 73° Fahr., and was not lower than 72° Fahr. during the whole day. I am greatly inclined to think that the elevated temperature of this and the following day, very much accelerated the subsequent changes in some of the specimens.

On the *twenty-ninth day* (fig. 17.) the temperature ranged between 67° and 70° Fahr. The young Iulidæ were now moving about very briskly, with their antennæ in constant motion. They now partook very freely of food, which consisted of the nutritious matter found by them in the moistened clay in which they were placed. Many of them had completely gorged themselves with it, as was evident from the darkened and distended state of the alimentary canal and colon, which were distinctly seen through the teguments of the body. The warmth and moisture at this period seems to have been extremely beneficial to them. Some of them appeared already to have acquired their peculiar instinct of burrowing in the clay, as several were very busily at work in a little round hole at the bottom of the cell: their bodies seemed also to have acquired a degree of strength sufficient for this purpose. The external tegument had become of a much darker colour, and assumed the appearance of horn. The divisions of all the segments were very distinct. The limbs had acquired much strength, and the anal segment was expanded and employed, as before stated, like the anal proleg of the larva of Coleoptera. The six new segments had grown very much and were fast acquiring their full size, and the germinal membrane at the margin of the antepenultimate (13. *g*) filling the *germinal space*, was beginning again to be developed. The large patch on the seventh segment was now deeper in colour, and in a line with this there was a minute spot on each of the five succeeding segments, indicating the existence of the *foramina repugnatoria* (8 to 12.) of Waga, or entrance to the little sacs in the body that secrete an offensive fluid. The first of these sacs is situated in the seventh segment, its outlet being in the large patch just noticed; and one of them is also developed at the anterior part of each of the succeeding segments, to the twelfth inclusive; but none is as yet seen in the thirteenth or last of the new segments, which may be called the *germinal segment*, and not only at the present time, but in each of the succeeding periods, always is more delicate than the others, is shorter, and has no repugnatory foramen. The terminal segments are still covered with short scattered hairs. The eye now exhibits a peculiar appearance. The dark circle around it has changed its form, and become somewhat triangular, and the single ocellus in the centre seems as if formed of several eyes grouped together.

On the *thirtieth day* I removed the specimens into a new habitation, a small phial, the bottom of which was filled with mould, and the top secured by a cork. On putting some macerated boiled meat, or animal fibre, into the bottle, they fed most voraciously upon it. They had now acquired much strength, and were of a much darker brownish colour. The alimentary canal was still seen through the teguments, filled with food. The new segments were still soft and delicate, and the anal segments were still employed in locomotion, as above stated. The germinal space now showed indications of the formation of other new segments. Each of the new segments had been much increased in size, from the eighth to the thirteenth inclusive; those most anterior being the most perfect. The fourteenth, or penultimate segment, as before shown, still maintained a great superiority over the rest in point of size and colour, although the whole had become darker, and were thus more clearly distinguished from the thirteenth or ante-penultimate, and germinal space, which are always whitish and delicate. The annulus around the ocellus was now of a more triangular form. There were a few scattered hairs on the anal segment and the under part of the body, as well as on the antennæ and parts of the mouth. The temperature of the atmosphere was about 70° Fahr.

On the *thirty-first day* (fig. 18.) the new segments had acquired a darker colour, and become further developed, and were almost equal in size to the original ones; but the thirteenth, and the germinal space, were still whitish and delicate. No additional legs had yet been developed, but there seemed to be a little distention of the under surface of the eighth segment in some of the largest and most active specimens, and in the whole of them a slight alteration of position had taken place in the six true legs. The prothoracic legs (2.), approximated at their base, were situated more closely to the anterior of the mesothoracic segment (3.), while those of the latter, in like manner, approached the generative or fourth segment; and those of the fifth segment, which I regard as the proper metathorax, had advanced a little forwards to the fourth. In other respects the animal remained the same as on the previous day.

On the *thirty-third day* (fig. 19.) it was evident that the high temperature of the atmosphere had accelerated the changes which in some of them seemed about to take place. The temperature of the atmosphere was now 67° Fahr. One specimen, that had undergone its change a few hours before, now exhibited two additional pairs of legs to the eighth segment (8. 18.), which on the previous days was enlarged on the under surface; several specimens at this period underwent a change, but I am not certain whether they actually shed their skins, or whether the legs were simply developed from the eighth segment by the extension of the old skin, as I did not actually witness the supposed exuviation of this covering. In other respects these individuals seemed to remain in the same condition as those specimens that had not cast their tegument, saving that the body was a little more extended, and the germinal space (13. *g*) was more developed. I cannot help regarding this change, therefore, as a *pseudo-change*, which takes place only under certain circumstances, such as repletion with food and moisture, and high temperature of the surrounding medium, operating, perhaps, on those specimens which were furthest advanced and first developed from the egg; since there are strong reasons for believing that those which are developed earliest from the egg undergo their changes most rapidly, while those last developed are later than the others at each succeeding change, so that some specimens may be entering their fifth period when others have not yet passed their fourth. This I have no doubt was the case with these Iulidæ, some of which acquired their two additional pairs of legs on the thirty-first day, others on the thirty-second, and some not until the thirty-fifth, and thirty-sixth and seventh; while the remainder did not undergo this change at all, but continued to feed and remain active, and instead of now acquiring two pairs of legs, acquired ten pairs at their next change, or fifth period of development.

On the *forty-fifth day* (fig. 20.) the whole of the remaining specimens were preparing to undergo their transformation. This appears to have been their proper period of change. The variation in the shedding of the skin just noticed, includes, from the time when the first specimen changed, to the completion of that process by the last, a period of six days. The specimens had now acquired a much darker colour, and the marking on the seventh segment was becoming paler. This was one day before the change. The temperature of the room was now 65° Fahr. What renders it more probable that the preceding was a pseudo-change, is, that I was unable to rear any of the specimens which underwent it, while others that attempted to change a little subsequently to those at the period noticed, perished in the attempt. The proper period was now approaching. On the forty-fourth day the specimens had ceased to take food, seemed torpid, and lay coiled up in a spiral form; the tegument of the body now began to assume a whitish crustaceous appearance, and the animals secreted themselves beneath any dry covering, but avoided parts too wet. The principal changes in their general appearance were in the eyes, each ocellus being much more distinct; and in the germinal space (*g*), which was now developed to its greatest extent, and distinctly exhibited the six new segments.

The casting off the skin, as in insects, is a tedious and eventful occurrence to the young Iulus. Waga states that the skin of the Iulus bursts on the under surface of the body in the thoracic region, *between the single pairs of legs*; that the head is first withdrawn, and afterwards the anterior segments, and then the rest of the body. I have been unable to confirm his account as to the part at which the skin is fissured. According to my own observations, when the young Iulus is about to change its skin, it bends its body in a semicircular form, with its head inflected against the under surface of the second segment. In this condition it remains for several hours, with its legs widely separated, and the dorsal surface of the segments extended. The head is then more forcibly bent on the sternum, and a longitudinal fissure takes place in the middle of the epicranium, and is immediately extended outwards on each side posteriorly to the antennæ, in the course of other sutures, the analogues of which I have elsewhere described in insects as the *triangular* and *epicranial sutures.* Through the opening thus formed in its covering the head is first carefully withdrawn, and with it the antennæ and part of the mouth, and afterwards the anterior segments and single pairs of legs. The first, and apparently the most difficult part of the shedding of the skin by *Iulus*, is its detachment from the posterior segments of the body, and from the interior of the colon. To effect this, the animal, which has previously been lying coiled up in a circular form, first straightens its whole body; it then forcibly contracts and shortens its body, especially at the posterior part, and by this means becomes greatly enlarged in bulk at its middle portion, but smaller at its extremities. During these efforts, which are some of the most powerful it is able to make, the skin becomes loosened from its posterior parts, and while still contracting its segments, the anal extremity, and with it the lining of the colon, become entirely detached, and from these it gently withdraws itself within the old skin in which the body is incased, as from the finger of a glove. This is precisely what takes place in the shifting of the skin in insects. Having effected this part of its labour, all the posterior segments are again shortened, the animal again disposes itself in a circular form, and after repeated exertions succeeds in bursting the tegument of the head in the part just described. As in the case of true insects, the young Iulus entirely empties the alimentary canal by voiding its fæces, and ceasing to eat for one or two days preparatory to undergoing each transformation. When examined immediately before the change, there are no other symptoms of new legs than slight elevations of the skin, and this perhaps accounts for the length of time occupied in the change, the new legs requiring time for further development before the old skin is thrown off.

When these changes have been effected, the animal again arranges its legs along the ventral surface of the body, and coils itself up in a circular form, in which state it remains for several hours, often with the skin partially covering the posterior segments. In these transformations, as in those of insects, the whole of the structures undergo alteration; the lining membrane of the colon and lower intestines comes away attached to the posterior, as that of the mouth and œsophagus does to the anterior part. It is not, therefore, by the bursting of the skin on the under surface of the anterior segments that the change is effected, as stated by WAGA, but by a separation of the natural sutures of the covering of the head. Indeed it is almost impossible to conceive how the legs of the thorax and covering of the mandibles could be thrown off if the change took place as stated by WAGA.

It has been supposed that the Iulus devours its cast skin, as is done by some larvæ of insects. I certainly have seen it nibbling at the skin some hours after the change, but although there were several cast skins in the vessel, and no food, there seemed no disposition on the part of the animal to devour it.

The *fifth period of development* being now attained, the young Iulus has three ocelli on each side of the head, seven joints to the antennæ, thirty-four legs, and twenty-one segments to its body.

On the *forty-eighth day* (fig. 21.) the young Iulus has entered this period, and exhibits a marked alteration in its appearance. The antennæ are considerably longer than the head, with seven distinct joints, and, as in the adult, the apical one is short and inserted into the sixth. The length of these organs has been increased chiefly by the elongation of the second basilar joint, which is now narrower and longer than the others. The single ocellus has disappeared, and in its stead three distinct ocelli, arranged in a triangle, have been developed. The apex of the triangle is directed upwards. The ocelli are of two sizes, the largest, a single one, being at the posterior angle. The development of an increased number of ocelli in Iulus at successive periods was discovered by M. Gervais, but the precise time of their appearance has not before been indicated. The new segments of the body produced at the former change of the animal, from the eighth to the twelfth inclusive, are now of the same size as the original ones, and each has developed from it two additional pairs of legs, so that the whole number of legs is now thirty-four. The thirteenth, or if we may so term it, *germinal* segment of the last period, is less developed than the preceding ones, and is distinguished from them by the circumstance that it is smaller, possesses no legs, and has no lateral spot, which exists, as above stated, on each of the preceding segments, to the seventh, marking the existence of the *foramina repugnatoria.* The large patch on the seventh segment is now larger and darker than heretofore, and the spots on the succeeding segments have been increased in size. The *germinal space* (13. *g*), which existed in the preceding period, and was then seen to be forming segments, is now developed into six new apodal segments, from the fourteenth to the nineteenth inclusive, very much smaller and shorter than the rest; and a germinal space (*h*) is again forming between the last of these and the penultimate segment of the body, which, as above stated, undergoes no marked change. The whole body is thus composed of twenty-one segments, including the head. The first twelve of these are now perfectly developed, as well as the last two, the intermediate ones being only in their preparatory states. The antennæ, parts of the mouth, legs, and anal segments, are still covered with minute hairs. At this period I gave my specimens for food some decayed leaves and rotten bark of the elm, as also some uncooked potatoe, on which they seemed to feed voraciously. They seemed to thrive most rapidly on the decaying bark, and grew daily, especially at the posterior extremity of the body.

On the *sixty-second day* (fig. 22.) all the specimens had been lying quietly coiled up in a circle for nearly twenty-four hours, preparatory to again changing their skin. The segments, from the thirteenth to the eighteenth inclusive, which had been partially developed at the last change of skin, were now nearly completed, and exhibited the transverse impression of the perfect animal; and the future spots on the sides of these segments were now shining faintly through the old skin, which had become whitened and dried, exhibiting the peculiar appearance of the approaching change. Besides this, instead of the three ocelli developed at the last change, there were now *five*, which appeared through the tegument, still arranged in a triangle; one of these was larger than the last, and distinctly formed of two parts, so that *six ocelli* were now about to appear. The number of legs still continued the same as at the last change. The nineteenth segment, which, at the period of transformation, was but a very soft fold of the tegument, with a slight division in the middle of it attached to the eighteenth, was now almost as much developed as the preceding ones, and the white germinal membrane, extended from it, showed the formation below it of six new segments. In this state the young animal lay coiled up awaiting its change.

On the *sixty-third day* the animal again changed its skin, and entered its *sixth period of development*. It then had acquired twenty-seven segments to its body, which had greatly increased in size, and was of a brown colour. It had six distinct ocelli on each side of the head, and all the segments, to the eighteenth inclusive, were furnished with legs, of which it had now fifty-eight. Six additional new segments had also been developed to its body, as in the preceding changes, anterior to the penultimate segment; and the germinal membrane behind them (*i*) was still in further course of development, the penultimate segment still remaining unchanged. The six segments from which legs had now been developed had also the *foramina repugnatoria* marked with small spots, while the spots on the preceding six had become larger and darker in colour. The chief difference now consisted in the appearance of the thorax, which is of a lighter colour than the rest of the body. The animal may now be regarded as having acquired all the essential parts of its body. Time and circumstances prevented me from following its transformations still further; but sufficient, I trust, has already been observed to claim from naturalists a little more attention to the remarkable series of phenomena connected with its growth, and to add to the importance of watching the development of this greatly neglected, but most singular group of animals.

Recapitulation and Conclusions.

The conclusions to which the facts detailed in this paper seem to lead, are, I think, as interesting to the zoologist, in reference to the situation which this remarkable class, the Myriapoda, ought to occupy in the arrangement of animals, as to the comparative anatomist, and physiological inquirer. The evident conformity to one type of the organs of reproduction in the two sexes, is in accordance with the views now advocated by the best anatomists. It has been seen that the Iulidæ, in some parts of their organization, as in the organs of reproduction, approach in their internal structure to the true insect, in maintaining, although in a simple state, a perfect form of development; while, in the external parts of the same organs, as in the double outlets of the female, and double organs of intromission of the male, they again recede to the type of those in which these organs exist in one of the lowest forms of development.

The structure of the ovum in Iulus approximates to that of the higher classes, and is in accordance with the observations of WAGNER, BISCHOFF, and of Dr. MARTIN BARRY, whose invaluable researches on this subject have so recently enriched the Transactions of the Royal Society. The same reasons that induced this last inquirer to advocate the existence of the *membrana vitelli* and *chorion* in the earlier stages of the ovum of higher animals, have also led me to believe in its existence in these lower forms, the Myriapoda; an inference entirely in accordance with the facts subsequently ascertained respecting the membranes that invest the embryo at the bursting of the ovum.

Moreover, the few facts detailed in this paper show that the habits of these creatures, although hitherto comparatively neglected, are as interesting to the naturalist as those of the more extensively investigated divisions of other Articulata.

But it is in the evolution of the embryo that the facts ascertained appear to be of the greatest interest. In conformity with the views of Schwann and Schleiden, and of our own accurate observer, Dr. Martin Barry, the embryo of Iulus in its earlier stages is found to be composed entirely of a congeries of cells, thus assimilating in origin the animal to the vegetable creation. But in the higher animals, as is well known, in which, chiefly, this subject has been studied, the changes which the future being undergoes in this stage of its existence are so exceedingly rapid, that it is with great difficulty that the facts connected with them are ascertained. In the embryo of the Myriapoda these changes are more gradual, and the transitions more slowly marked. Besides confirming the statements of Savi and Waga regarding the apodal condition of the embryo at the bursting of its shell, and its hexapodal state at a later period, I have noticed the important additional facts of the detection of the *amnion* and *chorion* [?] which inclosed the embryo, and also the insertion of the funis at the posterior margin of the penultimate segment of the body, instead of at the dorsal part of the thoracic region, as seen by Rathke in Crustacea; thus more closely identifying the structures of the embryo, as well as those of the ovum in Myriapoda, with similar structures in the ovum, and its development in the higher classes, and further illustrating the persistence of one general law or principle in the development of animated beings.

But not less interesting is the fact, that the growth of the animal takes place by the addition of entirely new segments, developed in the germinal membrane that connects the penultimate with the *then* ante-penultimate segment. This mode of increase by the generation of new segments, and not by the extension or division of those already formed, closely connects the Myriapoda with the Annelides, and somewhat resembles the growth of segments in the fissiparous *Naïades* of the latter class, as remarked to me by my friend Dr. Baly, when examining the specimen beneath the microscope. But it differs from the reproduction in these animals, in the circumstance that the segments produced are not the terminal segments of the body, but are new formations in the germinal membrane interposed between the newly formed ante- and the permanent penultimate segment.

The development of segments is one of the first changes in the embryo, and commences even before it bursts from the amnion. It is repeated with corresponding numbers at each change of tegument. In the *Iulus terrestris*, during the earlier transformations, the addition is *sextuple* at each change, a ratio that agrees most curiously with the number of segments found in the adult state. But it is not to be inferred, from this fact, that this is the number of segments produced at each change in all the *Chilognatha*, or that the new segments are produced in a corresponding part of the body in the other divisions of this class, the *Chilopoda*. In these two orders the parts in which the new segments appear differ greatly, and most distinctly mark these two divisions of the Myriapoda. In the *Chilognatha* the segments are always produced as above stated, but in the Chilopoda, the proper Scolopendradæ, according to GERVAIS*, the new parts are developed between *each* of the original segments. These are remarkable differences in the modes of growth of the two orders of this aberrant and most singular class. Although the new segments make their appearance at the same part of the body in all the Chilognatha, they differ in number in different animals of this group, and from the few observations I have yet made, I am inclined to believe that this difference is characteristic of different genera. Thus in *Iulus terrestris* it is sextuple through the earlier periods of life, but in a well-defined genus, *Blaniulus*, nearly related to Iulus, the number of segments at each reproduction appears to be *quadruple*, while in the young of another genus, which I believe to be *Polydesmus*, closely connected with the above, the number of segments appears to be only twofold at each change. But in each of these instances the number of new segments is similar throughout the earlier changes of the animal.

The development of legs takes place subsequently to that of new segments, which, when first produced, are always apodal, the legs being developed to the new segments at the next change of tegument. But as regards the number of legs produced, this is less regular than that of the segments. Thus in the hexapodous condition of the animal, legs are being produced to only two of the original apodal segments beneath the common tegument, while six new segments are in progress beyond them. When the skin is thrown off the legs are elongated, and the segments become more developed. In the mean time new legs are being formed beneath the tegument for those new segments, while other new segments are being produced beyond the last of these organs of locomotion. In like manner the eyes make their appearance as one of the last commenced changes, while the antennæ are the parts that earliest attain their full development. Such are the conclusions at which I have arrived in these investigations, which I propose to continue at a future period.

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* *Loc. cit.*

XIII. *Researches, tending to prove the Non-vascularity and the peculiar uniform Mode of Organization and Nutrition of certain Animal Tissues, viz. Articular Cartilage, and the Cartilage of the different Classes of Fibro-Cartilage; the Cornea, the Crystalline Lens, and the Vitreous Humour; and the Epidermoid Appendages.*

By JOSEPH TOYNBEE, *Esq., Member of the Royal College of Surgeons, London, and late Assistant to the Conservators of the Museum of that Institution. Communicated by Sir* BENJAMIN C. BRODIE, *Bart. F.R.S. &c. &c.*

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Introduction.

IT is now generally acknowledged that the process of nutrition in most animal tissues consists in changes undergone by the nutrient liquor sanguinis, which has exuded into them through the coats of the capillaries ramifying throughout them. The vessels themselves vary in number in different structures: in muscle, the capillaries are very numerous, and the spaces between them very small; whilst in tendon and ligament, on the other hand, the latter are comparatively large; but in all structures, whatever may be the degree of their vascularity, the tissue the furthest removed from the vessel is nourished equally well with that which is in immediate contact with it.

In all vascular structures, therefore, there is of necessity a considerable extent of tissue which is nourished without being in contact with blood-vessels, and the knowledge of this fact forms a necessary introduction to the study of the process of nutrition in those organs, into which, whilst in a healthy state, anatomists have never succeeded in tracing blood-vessels. The organized tissues, constituting such non-vascular organs, may be divided into three classes:

The first, comprehending articular cartilage, and the cartilage of the different classes of fibro-cartilage;

The second, the cornea, the crystalline lens, and the vitreous humour;

The third, the epidermoid appendages, viz. the epithelium, the epidermis, nails and claws, hoofs, hair and bristles, feathers, horn, and teeth.

It is to these tissues that the investigations I have now to communicate relate: I shall endeavour to prove that no vessel ever enters them when they are perfectly developed, and in a healthy state, and to demonstrate the manner in which they are nourished.

In the first place, no anatomist has ever been able to trace vessels into these tissues in the adult healthy state; and it appears to me, that the due action of the organs into the composition of which these tissues enter is incompatible with vascularity; in other words, that the presence of blood-vessels within them is a sign of disease.

The first class, for instance, of the non-vascular tissues, comprising cartilage and fibro-cartilage, is subject, from its situation in the joints, &c., to repeated concussions, and to constant compression and attrition, which in these unyielding tissues would necessarily be destructive of the integrity of blood-vessels.

Those of the *second class* are required to be perfectly transparent for the due transmission of the rays of light, which would be impossible if the circulation of a coloured fluid were carried on throughout their substance*.

The tissues of the *third class* are unceasingly exposed to friction, laceration and incision; and hence, of course, it is necessary that they should not be traversed by vessels.

In the numerous attempts which I have made to inject these tissues, I have never been able to trace a blood-vessel into any one of them: on the contrary, my injections prove that the vessels which previous anatomists had traced no further than their circumference (supposing them to be continued into these tissues, either as serous vessels, or as red blood-vessels too minute for injection), actually terminate in veins, without the limits of these tissues. The terminations of these vessels in the immediate vicinity of the non-vascular tissues, present certain convolutions, dilatations, plexuses and other peculiarities, which differ in various parts, but in all instances enable a large quantity of blood to circulate slowly in the neighbourhood of these tissues, from which it may be inferred that they are subservient to the nutrition of the latter; and their existence certainly constitutes another argument against the presence of vessels within them.

All these non-vascular tissues are in structure very analogous: they all contain corpuscles or cells, of which some of them are almost entirely made up; while in others, as the cornea, a few only are present.

I am induced to ascribe to these corpuscles very important functions, and I shall therefore make a few observations upon them. SCHLEIDEN has lately concluded, from the researches which he has conducted, that all vegetable tissues are developed from cells. SCHWANN has arrived at similar conclusions on the mode of development of animal tissues, and he has proved, I think satisfactorily, that tissues which, when perfectly formed, have a structure so different from each other as articular cartilage and muscular fibre, are developed in a similar manner, viz. from cells. M. SCHWANN ascribes to these cells, during development, a vital function, and believes that they must have the power, not only of attracting, but also of chemically changing, the substances brought into contact with them†.

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* I believe that the blood discs circulate through all vessels, and MÜLLER allows that the existence of serous vessels (viz. those carrying the liquor sanguinis only) has never been demonstrated.

† The British and Foreign Medical Review, vol. ix. p. 523. It is to the very valuable article in this

To the elaborate researches of these physiologists, and to those of MÜLLER, VALENTIN, PURKINJE, HENLE, &c., I must refer for information upon the nature and intimate structure of these corpuscles.

These cells are modifications of two principal forms, the one being either round or oval, the other compressed in the form of a scale; and their existence in one of these two forms has lately been detected in most organs of the body.

In a circular form they appear to be the principal components of bone, cartilage, muscular and nervous fibre, and of the parenchyma of glands*; in the form of a flattened cell or scale, they constitute the epidermis and its appendages, the epithelium of mucous and serous membranes, and of the inner tunic of the vascular system.

When the almost universal presence of these cells is considered, I think it cannot be doubted but that they perform important parts in the functions of nutrition and secretion.

I am induced to agree with M. SCHWANN, that these cells have vital actions, and I believe that they not only possess them in the tissues during development, but also in subsequent periods of life. I ascribe to them the function of circulating, and of perhaps changing the nature of, the nutrient fluid which is brought to the circumference of the solid non-vascular tissues, and I believe them in some measure to compensate for the absence of the internal vascularity possessed by other structures.

In proof that they possess vital properties, I may allude to the changes undergone in the structure of the cornea and the crystalline lens, without the penetration of them by any vessels.

I must here observe, that in some of the non-vascular tissues, as in the cornea and the vitreous humour, where only a small quantity of corpuscles exists, the laxity of their consistence admits of their ready penetration by the nutrient fluid brought to their circumference.

The only difference which appears to me to exist between the mode of nutrition in the vascular and the non-vascular animal tissues is, that *the former* derive their nutrient fluid from the blood which circulates through the capillaries contained in their substance; and that *the latter* are penetrated by the nutrient fluid which exudes from the large vessels by which they are surrounded, and that its distribution through them is assisted by the vital properties of the corpuscles which they contain; in *both classes*, the particles of which the tissues are composed attract from this fluid the elements which nourish them†.

Journal, and to some additional pages by Dr. BALY, in the second edition of his Translation of MÜLLER'S Elements of Physiology, that I am indebted for my knowledge of SCHWANN'S and SCHLEIDEN'S labours.

* VALENTIN and PURKINJE.

† In reference to this subject, I beg to direct particular attention to the following quotation from Professor

THE FIRST CLASS OF NON-VASCULAR ANIMAL TISSUES.

Of Articular Cartilage and Fibro-Cartilage.

The tissues which I have placed in the first class are articular cartilage, and fibro-cartilage; but in reference to the latter, the cartilaginous portion only can be considered as non-vascular. These tissues are analogous to each other in their situation, structure, mode of nutrition and functions. Each of them forms a part of joints, and is subject, in the performance of its functions, to concussion and compression, and is composed of corpuscles or cells possessing similar characters. Although they are properly considered as non-vascular tissues, they appear to be pervaded by blood-vessels at an early period of their development, or perhaps it would be more correct to say, that as growth proceeds, the cartilage increases, so as to occupy the space which had previously been permeated by vessels.

I have been able to demonstrate that vessels are never found within these cartilages when fully developed, but at that period vessels form convolutions in their immediate vicinity. These vessels are separated from articular cartilage at adult age by a layer of bone, and in fibro-cartilage, at the same period, they uniformly terminate within the boundary of its fibrous tissue. Over a certain portion of the free surface of both of these tissues blood-vessels extend, but they do not penetrate into their substance.

The investigations which are about to be detailed, lead, I think, to the certain conclusion, that articular cartilage in the adult state is principally nourished by fluid derived from the vessels of the cancelli of the bone to which it is attached, which exudes through the coats of those vessels, and makes its way into the substance of the cartilage through the intermediate lamella of bone. The cartilage of fibro-cartilage is nourished in like manner by liquor sanguinis, derived from vessels situated in the contiguous fibrous portion. The vessels ramifying in a certain extent of the free synovial surface of both these species of cartilage contribute doubtless to their nutrition, but not to near the same extent as do the vessels of the opposite side. With respect to the actual process of nutrition in these cartilages, I shall only observe here, that the cells of these structures must be regarded as having the function which has been ascribed to those of all non-vascular tissues, viz. that of promoting the circulation of, and modifying, the nutrient liquor. In connection with this process, however, it will be seen, that articular cartilage presents in its adult state very minute canals, which may be regarded as existing for the reception of the nutrient fluid, and

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OWEN'S Odontography:—"But since every secretive process, and the development of the primordial cells of every tissue are due to changes produced in the liquor sanguinis, transuded from and beyond the sphere of the ultimate capillaries, the absence of these vessels in the dense dental substance is as little conclusive against its vital and organized nature, as it would be to prove the inert condition of the germinal membrane of the ovum before the thirtieth hour of incubation."—p. 13.

for its circulation throughout the mass of the cartilage; their presence is especially required in this particular form of cartilaginous structure, from the great degree of density which it possesses.

1. *Articular Cartilage.*

Articular cartilage is situated either on the rounded extremities of long bones, or on the surfaces of flat and irregular bones. The portion of bone upon which it rests, is in some instances formed by the ossification of a distinct cartilaginous epiphysis. In non-epiphysal bones, the extremity of the shaft of the bone performs the same functions with regard to the articular cartilage situated upon it, as do the epiphyses in those bones which are provided with them. There is this difference in articular cartilage, with regard to its nutrition during and after its development; that in the former state there is no positive separation of it from the cartilage which is subsequently converted into bone, and in which its nourishing vessels are contained; whilst, in the latter state, these vessels are separated from it by an osseous lamella. The free surface of articular cartilage during, as well as after, its development, is covered by synovial membrane, to which it is attached by cellular tissue.

Around every joint, from a very early period of development, there are found numerous arteries and veins, "the articular vessels," by which their nutrition is effected.

With respect to vascularity, the nutrition of articular cartilage during its development may be divided into two stages; viz. the early one, during which no vessels enter any of the structures of the joints; and the subsequent one, in which the cartilage of the bone on the one surface, and the synovial membrane on another, are supplied with blood-vessels. In adult age, after development is completed, the same vessels continue the process of nutrition; those of the bone being situated in the cancelli, as before described, whilst those of the synovial membrane are considerably diminished in size.

In order to illustrate the nutrition of cartilage during its earliest stage, I have made the following dissections.

A. *The First Stage of Development of Articular Cartilage.*

Dissections of Articular Cartilage during the more early periods of fœtal development, before Blood-vessels enter into any of the Structures of the Joints.—a. In a fœtal Calf, which measured twelve lines from the vertex of the head to the commencement of the caudal vertebræ, the rudiments of the shaft of the os femoris were wholly cartilaginous, and measured one line in length.

The corpuscles or cells of which this cartilage was composed, were large, round, and loosely connected together.

Its extremities were smooth, and appeared to be covered by a synovial membrane, but the cells here did not present any difference from those of the shaft.

In no portion of either shaft or extremities could any vessels or canals be observed.

b. In a fœtal Calf, which measured fifteen lines between the points above indicated, the rudimentary os femoris was a line and a quarter in length, and one fourth of a line in breadth in its middle part; the extremities were somewhat expanded.

Excepting a small osseous ring at its median part, this rudimentary os femoris was entirely cartilaginous, and its cartilage was composed of large rounded cells, loosely connected together by a gelatinous substance. The cells of the articular margin differed from those in the Calf of twelve lines in length, described in the preceding dissection, in being elongated, and in having their long axis parallel with the free surface of the cartilage. The synovial surface of these cells presented a defined border, beyond which were flattened scales, each having an elongated process. There was no appearance of canals or vessels in any part of this cartilage.

c. In a fœtal Calf, measuring seventeen lines from the vertex of the head to the commencement of the caudal vertebræ, the cartilage of the inferior femoral epiphysis presented no appearance of canals. Its cells were large and round, and their connecting medium lax and easily compressible. On compressing a thick section of the extremity of the cartilaginous femur, the flattened cells of the epithelium of its synovial membrane were distinctly seen; they were as large as those of the epithelium of the mouth in the adult human subject; one of their surfaces faced the articular cartilage, the other the cavity of the joint.

d. In a fœtal Calf, measuring 2½ inches between the points above alluded to, the os humeri, which was two lines and a half in length, was ossified to the extent of three fourths of a line at its central position. Its articular surfaces were smooth and defined, and it presented no appearance of canals. The cells towards the articular surface, as in the preceding dissections, were elongated and flat.

e. In a fœtal Calf, measuring three inches in length, the epiphysal extremities of the cartilaginous femur presented no canals, and the arrangement of the cells of the cartilage was the same as in the above dissections.

The fœtal Calves in all the above dissections had been injected, and in each case the extremities of the cartilaginous rudimentary bone were found to be surrounded by large ramifications of sanguiferous vessels.

The above observations on the nutrition of articular cartilage during the earliest periods of its development have been principally confined to preparations of fœtal Calves, from my not having been able to procure those of the human fœtus sufficiently numerous and varied for my purpose. My more limited examinations, however, of the human fœtus, have led me also to the conclusions, that during the most early periods, the cartilage of the epiphysal extremities of bones does not contain any blood-vessels, and that notwithstanding their absence, the cells of this cartilage are developed, and its growth carried on; and that at the same time the cells of the epiphysal and the articular cartilage are formed and developed without the presence of vessels.

I think it may be naturally deduced from the facts demonstrated in the foregoing dissections,—

First. That during the most early periods of fœtal life, the growth of cartilage takes place, and that its component cells or corpuscles undergo certain progressive changes in their form and size, without the presence in its substance of any blood-vessels.

Secondly. That the vessels encircling the cartilage contribute to effect such changes in its corpuscles, and that the changes are facilitated by the softness of the substance of the cartilage.

Thirdly. That at the more early period of fœtal development the synovial surface of cartilage does not contain blood-vessels.

B. *The Second Stage of Development of Articular Cartilage.*

In the stage of development described in the preceding section, vessels are only present in the vicinity of the articular extremities of bones, but as the cartilage forming the latter becomes harder in its consistence during the subsequent periods of its development, vessels are gradually introduced upon its surface and into its substance.

Those epiphysal vessels which are subservient to the nutrition of the substance of articular cartilage, will be first treated of, and subsequently those which belong to its free or synovial surface.

The articular arteries in the adult subject, which are so numerous, and which surround so completely the various articulations, terminate by entering the substance of the extremity of the bones, by passing between the articular cartilage and the synovial membrane, and by supplying the latter membrane and the ligaments. In the stage of development which is about to be described, the ends of the bones are not yet ossified; the cartilaginous epiphysal extremities of the bones will therefore be spoken of.

First, the manner in which the branches from the articular arteries gain the interior of the epiphysal cartilage, and their mode of distribution in the subsequent periods of their development, will be demonstrated.

Secondly, the vessels which are situated between the cartilage and the synovial membrane, and which nourish the free surface of the former, will be described.

Of the Blood-vessels in the substance of the Epiphysal Cartilage.—The whole inferior extremity of the os femoris of a fœtus of about five months presents, except at its articular surface, numerous depressions of various depths. The deepest may be regarded as canals, some of which are single, others bifid; they terminate in blind sacs. The direction of some of these canals is towards the centre of the epiphysis, of others towards its point of attachment to the osseous shaft, and of others, those about to be described, towards the articular cartilage, Plate XIII. figs. 1 and 2*. Some of these canals are of a large size, and are frequently considerably dilated at their blind extremities. They do not penetrate into the substance of the articular cartilage. These canals are for the reception of branches of sanguiferous vessels. When the epiphysis is minutely injected, the depressions upon its surface will be found to contain congeries of convoluted blood-vessels, which are more drawn out the deeper the depression, until at length, in the interior of the canals and their divisions, single, and nearly straight vessels are found. These epiphysal vessels have a very peculiar disposition. They consist of an artery having a course more or less straight, which terminates in a dilatation, or in convoluted branches, from which the vein arises, Plate XIII. figs. 3 and 4. From the fact of the presence of these vessels, which converge towards and form convolutions internal to the articular cartilage, it may be inferred that they supply the cells of the latter with a nutrient fluid. As the articular cartilage increases in thickness, and the ossific nucleus which is developed in the epiphysal cartilage becomes larger, these vessels gradually recede from between them, and they leave a considerable mass of non-vascular cartilage between the osseous nucleus and the synovial membrane; all of this appears to be articular cartilage, which is now nourished by the vessels in the interior of the nucleus. See Plate XIII. figs. 6, 7, 8, 9 and 10. The supply of blood-vessels in the cancelli of the osseous nucleus, is remarkably abundant; they are large and are separated from the surrounding cartilage by an extremely delicate lamina of bone, which is principally made up of osseous cells. I am induced to believe that at this stage of development, as in adult age, the fluid passes from the bone into the cartilage and nourishes it. From the difficulty of obtaining a series of specimens, I am unable to state the exact period at which this change in the position of one set of vessels, and the additional function in the other, takes place; but it most probably occurs at different periods in the various articulations, and it is effected in all by the time that they are called upon to encounter concussion, compression, &c. to any extent.

It may be here observed, that the articular cartilage at this early period of life is thicker than in the adult state. Although devoid of canals for the reception of blood-vessels, it presents numerous minute canals, which pervade that portion of it contiguous to the osseous nucleus, and they course from the latter towards the synovial membrane, which however they do not reach. The true nature of these canals could only be examined by powers magnifying between one and two hundred diameters. They are minute and extremely numerous; they divide, subdivide, and communicate with each other and form dilatations. The parietes of these canals present distinct rounded cells, which in some places are arranged in rows and groups. The substance between these tubes is transparent, and contains no corpuscles.

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* To obtain a distinct view of these canals, the epiphysis should be macerated in water for a short time, so as to remove the perichondrium which invests it; this was done to the specimen from which the drawing was taken.

The articular cartilage, above described, is gradually being converted into bone during the whole of life; thus it is thicker in young than in adult subjects, and, as Sir BENJAMIN BRODIE informs me, it is much thinner in old age than in the adult: in fact, it is not very rare to find that the articular cartilage of the head of the os femoris in very old persons has completely disappeared; a change which is probably to be attributed to its entire ossification[*].

Of the Nutrient Vessels of Articular Cartilage during its development, which are situated betwixt it and its Synovial Membrane.—Previous to giving a description of the vessels which are present on the articular cartilage, and which are between it and the synovial membrane investing it, I must here state that I believe that the synovial membrane is extended over the surface of the articular cartilage. The valuable pathological researches of Sir B. BRODIE have induced me to adopt this opinion; in addition to which, in favour of this view, the following accounts of dissections made by HENLE and myself may be cited.

HENLE says, "The epithelium is continued in a thinner layer on the articular surfaces of the cartilage, on which it is separated from the cartilage-corpuscles by a thin layer of cellular tissue[†]."

In a fœtal Calf, towards the latter part of uterine existence, I have removed the synovial membrane from nearly the entire surface of the articular cartilage of the condyle of the femur, to which it was attached by a considerable layer of cellular tissue, in which the blood-vessels that are about to be described were seen to ramify.

These vessels have been alluded to by Dr. W. HUNTER under the name of "*circulus articuli vasculosus*," in the following words[‡]:—"All around the neck of the bone there are a great number of arteries and veins, which ramify into smaller branches, and communicate with one another by frequent anastomoses, like those of the mesentery.

"This might be called the circulus articuli vasculosus, the vascular border of the joint. The small branches divided into still smaller ones upon the adjoining surface, in their progress towards the centre of the cartilage. We are seldom able to trace them into its substance; because they terminate abruptly at the edge of the cartilage, like the vessels of the albuginea oculi when they come to the cornea."

The following is an account by CRUVEILHIER of the injection of these vessels[§]:—"Nous avons fait, MM. BRESCHET, BOGROS et moi, des injections partielles et générales chez de très jeunes sujets; et chez des adultes, des injections partielles avec une solution d'ichthyocolle colorée avec l'indigo. Les synoviales ont été parfaitement injectés; tout autour du cartilage articulaire existe un cercle artériel duquel partent de très petites ramifications qui s'avancent sur ce cartilage dans l'espace d'une

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[*] This appears to be another of the many instances of the disappearance of the animal, and the increased deposit of the earthy constituents of the body in old age.

[†] MÜLLER's Archives, 1838, p. 116.

[‡] Philosophical Transactions, 1743.

[§] Observations sur les cartilages diarthrodiaux, Archives Générales de Médecine, vol. iv. p. 162, 1824.

demi ligne à une ligne, mais qui s'arrêtent toujours dans le point précis, où la synoviale cesse elle même d'être indistincte."

The arteries passing between the synovial membrane and the articular cartilage may be considered as the terminal branches of the articular vessels.

Before they reach the articular cartilage they are but laxly covered by the synovial membrane, but at the border of the cartilage they are firmly bound down to it by the very small quantity of dense cellular tissue existing between them. It is difficult to state generally at what period of fœtal existence the vessels, which have been spoken of in the first stage as forming convolutions around the joints, are prolonged upon its surface, and I can only here give the result of my investigations upon the subjects which I have been able to inject and examine.

I have studied with care the stages by which these vessels are prolonged upon the head of the os femoris at the point where the ligamentum teres is attached.

At between the *third and fourth* months of fœtal life, these vessels are simply a mass of delicate convolutions situated beneath the synovial membrane; at the *fifth* month these convolutions are somewhat unravelled, so as to extend over the surface of the cartilage to the distance of about half a line (Plate XIII. fig. 11.); and at between the *seventh and eighth* months they are drawn out and prolonged to the distance of a line and a half, Plate XIV. fig. 1. At this stage, these vessels consist of arteries of considerable size, which radiate in a straight course from the attachment of the ligamentum teres. They give off but few branches, and, previous to terminating, they divide and subdivide, but do not diminish much in size. They terminate by turning and forming loops with the small veins. Subsequent to the *eighth* month, these vessels begin to recede in their course; and at birth, and the periods subsequent to it, they are again found to be gathered immediately around the point of attachment of the ligamentum teres. After these vessels have receded, the position they occupied at the more early periods may be for some time detected by the white aspect of the cellular tissue between the cartilage and the synovial membrane. In the knee-joint of the human fœtus of five months, these vessels extend to a considerable distance over the surface of the articular cartilage, and in the fœtus at birth, although they have receded considerably, they still occupy the inferior surface of the articular cartilage (Plate XIV. fig. 2.); in adult age they have entirely receded from it.

These vessels in the knee-joint have a different mode of distribution from those above-mentioned in the hip. The arteries take a straight direction towards the centre of the articulation, and in their course they give off small branches, which, in the space between them, form a delicate network, and communicate with small veins. These arteries terminate either by turning in their course and forming broad loops with the venous radicles, or they empty themselves into a single vessel from which the veins arise, Plate XIV. fig. 2.

These vessels form a band which surrounds the circumference of the articular cartilage in all joints, as has been above stated.

In the fœtus, the vessels forming the band are long, while in the young and adult subject they are shorter, having receded to the margin of the cartilage which is not subject to concussion.

In the fœtal subject this band is more broad in some parts than others; but those portions of it which project to an inconsiderable extent over the surface of the cartilage, have, on the opposite part of the circumference, others which project to a considerable extent.

Various Characters of the Synovial Vessels.—These synovial vessels consist of arteries which take a direction towards the centre of the articular cartilage, and of veins which take a retrograde course. The arteries become continuous with the veins in the following ways:—1st, the artery becomes directly continuous with the vein without undergoing any change in its size, forming with the latter a simple loop (Plate XIII. fig. 1.); 2ndly, numerous arteries terminate in a single vessel from which veins arise; (this disposition is evident in the knee-joint, Plate XIV. fig. 2. and Plate XIII. fig. 5.); 3rdly, the artery terminates in largely dilated vessels, from which the veins take their origin, Plate XIII. figs. 3 and 4.

The preceding account of the examination of the vessels of articulations at early periods, shows that a large quantity of blood-vessels exists both at the free and attached surface of articular cartilage during its development. The modes in which these vessels are disposed, the dilated, plexiform, and other characters which they present at the point of communication of the arterial with the venous system, are interesting features in the anatomy of the vascular system, and their presence here must be associated with the large quantity of fluid required for the nutrition of the articular cartilage during development, and which is eliminated from the blood whilst its course is retarded in these vessels*.

C. *Adult Articular Cartilage.*

I have now to speak of the nutrition of articular cartilage when perfectly developed, and subject, in the performance of its functions, to violent concussion, compression, &c.

In reference to the *structure* of adult articular cartilage, it has already been stated that at its attached surface, viz. at the part where it joins the osseous lamella, it presents numerous fine canals, which can be seen only with the higher magnifying powers. These canals are irregular in their distribution; some are merely dilated cavities; frequently several of these cavities are elongated, and arranged serially, running from the attached towards the free surface of the cartilage. At the free or synovial surface, these canals do not exist; the cells of the texture at this part being elongated

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* I have frequently found the sac of the synovial membrane full of colourless size, in joints of which the vessels have been successfully filled with an injection composed of size and vermilion.

and flattened, and having their long diameters parallel to the free surface. These canals contain a transparent fluid, which is seen to ooze from them after a section.

It is most probable that the uninjected vessels observed in sections of cartilage by Meckel, Bichat, and others, were these canals and sinuses.

Into the substance of healthy articular cartilages, I have never been able to trace blood-vessels, and my researches induce me to believe that they do not possess any. Previous to proceeding with my investigations upon this subject, I will give the statements of preceding inquirers with respect to the vascularity of articular cartilage.

Sir B. Brodie, in speaking of articular cartilage, states, "Here is a morbid alteration of structure, the occurrence of which seems to indicate that there must be such a vascular apparatus entering into the formation of cartilage, as enables new materials to be deposited and old materials to be absorbed*."

Bichat recognizes "a vascular system in cartilages, but he was ignorant of the nature of the white fluids which circulate in them†."

M. Beclard. "These cartilages (the articular) have no vessels‡."

M. Boyer. "Leurs (les cartilages articulaires) vaisseaux sanguins sont si petits, qu'il est impossible de les suivre dans leur épaisseur; ces vaisseaux viennent d'un réseau vasculaire qui entoure la partie de l'os comprise entre l'attache du ligament capsulaire, et le bord du cartilage, vers lequel ils s'avancent; ils s'enfoncent entre l'os et le cartilage dans l'épaisseur duquel ils envoient sans doute un grand nombre des ramifications§."

M. Cruveilhier. "The diarthrodial cartilages do not present any trace of organization||."

M. Meckel. "Cartilages do not receive vessels which carry red blood, although in cutting them, distinct vessels are frequently observed in their substance¶."

M. Müller. "The tendons, ligaments and cartilages have blood-vessels, although in small number**."

Since the foregoing quotations were made, I have read with much interest some observations upon the subject by Mr. Liston. I subjoin the following extracts from them. "The possibility of cartilage being acted upon, nourished, absorbed, and repaired by its own vessels, must thus be admitted." Mr. L., however, adds, "This cellular tissue (between the cartilage and bone) is scarcely demonstrable in the healthy condition of parts, any more than is the vascularity of the articular cartilage††."

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* Pathological and Surgical Observations on the Diseases of the Joints. Third Edition, 1834, p. 92.

† Anatomie Générale, Article, *Organization du Systême Cartilagineux.*

‡ Elements of General Anatomy. Knox's Translation, 1830, p. 246.

§ Traité d'Anatomie, vol. i. p. 60. Quatrième Edition.

|| Observations sur les cartilages diarthrodiaux, Archives Générales de Médecine, vol. iv. p. 162.

¶ Manuel d'Anatomie, vol. i. p. 354.

** Elements of Physiology, Translated by Dr. Baly, vol. i. p. 384.

†† Medical and Chirurgical Transactions, vol. xxiii. pp. 94, 95.

In adult life, when the epiphysal cartilage has been ossified, the cancelli of the latter are separated from the articular cartilage by a layer of bone, to which may be given the name of the articular lamella. The nature of this lamella is worthy of particular attention. It is composed of two sets of osseous layers; the one, dense and thick, is continuous with the vertical fibres of the cancelli; the other, delicate and thin, principally composed of osseous corpuscles, is situated at right angles to the latter, and fills up the interspaces of the vertical fibres*. See Plate XV. figs. 2 and 3.

It will be seen that the structure here delineated is admirably calculated to receive shocks, and sustain great forces. The drawing was taken from the inferior extremity of the os femoris, to which the articular extremities of all bones more or less intimately correspond.

Is this articular lamella complete? I have never been able, by the aid of the microscope, to discover any orifices in it, nor have I been able to force mercury through it.

If the articular surface of a bone, which has been minutely injected, be permitted to dry, the adipose substance, a quantity of which is generally found in the interior of the cancelli, in some measure permeates the osseous articular lamella, rendering it transparent, and their contents may then be seen and examined. To facilitate this examination, I have in some instances resorted to the application of varnish, and in others have removed the earthy particles from the bone by the aid of acid. Through the articular lamella numerous vessels of considerable size will be distinctly recognized in the interior of the cancelli. These vessels enter the substance of the bone by the large foramina which are seen at its non-articular surfaces, and they converge towards the articular lamella. With the inner surface of this lamella, they not unfrequently appear to be in contact; and either in contact with it, or near to it, these vessels form dilatations and convolutions, and then take a retrograde course and become continuous with the venous system. The vessels here described are represented as they exist towards the articular lamella in the cuneiform and cuboid bones of the adult human subject, Plate XV. figs. 4 and 5.

It is worthy of remark, that the vessels which are contiguous to the articular lamella are of a considerable size.

The existence of minute orifices in the articular lamella must be acceded to by those writers, who entertain the opinion, that vessels pass from the cancelli into the cartilage. I have been unable, as I have above stated, to detect the presence of these minute orifices, nor have I been able to see near this lamella, the minute vessels which are supposed to pass through them.

I believe that the large vessels which I have already described as forming convolutions and dilatations at the inner surface of the articular lamella, have the function

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* The cribriform appearance of this lamella, seen in cases of extensive disease of the articular cartilage, is produced by the absorption of the thin and the persistence of the dense layer.

of supplying the articular cartilage with a nutrient fluid, and that they do so without entering into its substance. It is necessary that the nutrient fluid brought to the inner surface of this lamella should penetrate its substance. It is most probable that it traverses only the thin layer of the lamella, and not the vertical portions. This thin layer has already been stated to be almost entirely composed of osseous corpuscles, which without doubt assist to convey the fluid from the cancelli into the cartilage.

It appears to me, that not only those vessels which are in immediate contiguity with the articular lamella have the function of nourishing the articular cartilage, but that the large and very numerous vessels which ramify through the substance of the cancellous extremities of bones, and which enter them by large orifices at their non-articular circumference, eliminate into the cancelli a nutrient fluid, which passes through the articular lamella and nourishes the cartilage.

That the nutrition of articular cartilage is actually effected by vessels in the cancelli, may be inferred from their dilatations and convolutions in its vicinity, and from the absence of any other means, as shown by my injections.

In addition to my own preparations, I may refer, in corroboration of the view of the non-vascularity of articular cartilage, to a preparation presented to the Museum of the Royal College of Surgeons by Mr. Swan.

It consists of the bones of the posterior part of the cranium of a Sheep, which, subsequently to death, was allowed to remain suspended with its head downwards, so as to cause the blood to gravitate into it. The vessels forming convolutions in the canals of Havers are much distended with blood of a dark colour. The articular cartilage covering the condyloid processes is, however, perfectly white; not a particle of blood can be discovered in it.

The researches which I have made upon those morbid states of articular cartilage, in which blood-vessels are prolonged into its substance, and upon the manner in which the vessels are introduced into it, confirm the opinions here advanced. I shall defer entering upon them to another opportunity*.

Having proved, I think, that in the healthy state no blood-vessels pass from the interior of bone into the substance of the articular cartilage at its attached surface, I shall now proceed to show that no vessels enter it at its free or synovial surface.

Of the Vessels of the Synovial Membrane which cover the Articular Cartilage, and of the Nutrition by them of the latter.—In a former part of this paper, the vessels appertaining to the free or synovial surface of articular cartilage in the young subject have been described. A few additional observations upon them in the adult state are required here.

During fœtal and infantile life, previous to the period when the articular cartilages are subject to pressure, the synovial vessels extend over certain portions of them, from

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* The investigations of Sir B. Brodie, Mr. Mayo, and more recently of Mr. Liston, leave no doubt that in some diseases to which articular cartilage is subject, blood-vessels are distributed throughout its substance.

which, in childhood and during adult age, owing to the functions of the joints, they are necessarily absent. At the period when the child begins to use the various joints, and subjects them to pressure, these vessels recede; and in adult life they are only found on that margin of articular cartilage which is exempt from the influence of external forces*. The arteries which pass between the articular cartilage and the synovial membrane, like those of the fœtus, may be considered as the termination of the articular arteries. At the point where the reflexed becomes continuous with the articular synovial membrane, it contains large vessels subjacent to it, which are numerous and plexiform. Immediately, however, that they enter the cellular web, between the articular cartilage and synovial membrane, they become enlarged and straight, and pass to a greater or less distance over the border of the articular cartilage, forming loops frequently with considerable dilatations, and becoming finally continuous with the veins. The free surface of adult articular cartilage appears to be nourished by the liquor sanguinis, which exudes from these looped and dilated vessels.

The following are the leading facts which the preceding researches *upon Articular Cartilage* tend to establish:—

1. Epiphysal and articular cartilage are developed and nourished in the early periods of fœtal existence, without the presence of blood-vessels in the substance of the former, or on the surface of the latter.

2. At subsequent periods, canals are formed in the epiphysal cartilage, vessels are prolonged into them, which converge towards the articular cartilage; and also vessels extend over a considerable portion of the free surface of articular cartilage.

3. At later periods, the epiphysal cartilage ossifies, and for a considerable time vessels are placed between the ossified nucleus and the articular cartilage.

4. As age advances, the osseous nucleus increases in size, the blood-vessels disappear from the cartilage which remains unossified, but the nucleus contains large and numerous blood-vessels.

5. Corresponding with the changes just noticed, is the recession of the blood-vessels from the whole of that surface of the articular cartilage which is subject to compression.

6. In adult life, the articular cartilage contains no blood-vessels; but in the cancelli of the bone at its attached surface, are numerous large vessels, from which the cartilage is separated by a delicate lamella of bone; the circumference of its free surface presents numerous dilated blood-vessels.

7. Articular cartilage during the whole of life gradually becomes thinner, by being converted into bone.

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* I shall take another opportunity of discussing the pathological conditions in which, in the adult subject, these vessels extend over nearly the whole of the surface of the articular cartilage; artificial injections of them may be seen in most pathological museums.

2. *Fibro-Cartilage.*

Of the Structure of Fibro-Cartilage.

Anatomists of all ages have recognized the existence of a distinct animal tissue, partaking of the properties of both cartilage and ligament. Respecting the exact nature and composition of fibro-cartilages, the opinions of anatomists have differed considerably.

MECKEL states that in them "the fibrous and cartilaginous tissues form layers which alternate more or less regularly[*]."

BICHAT considered them to be composed "of a fibrous substance more than of a true cartilage."

Dr. TODD describes them as having "a very complicated cellular structure, composed of minute meshes, very irregular in size and shape[†]."

MÜLLER describes them as follows:—"The interarticular cartilages, the intervertebral cartilages, and the cartilages of symphyses are constituted wholly of fibres, and contain none of the corpuscles peculiar to cartilages[‡]."

One of the most recent and able investigators in the branch of anatomy under consideration is MIESCHER, and his opinion agrees with that of MÜLLER; he states that in the interarticular cartilages of the knee, which in infants E. H. WEBER considered to be true cartilage, "he found both in infants and adults only the fibrous structure and no trace of corpuscles[§]."

WEBER states that "the intervertebral substance exhibits no intermixture of cartilaginous substance[‖]."

Fibro-cartilages consist, as their name implies, of both fibrous and cartilaginous tissue; that is to say, I have always found them to be composed of minute fibres, and of the corpuscles characteristic of cartilage.

Fibro-cartilages may with accuracy be divided into two classes, both entering into the structure of joints.

The *first class* exists in those articulations which admit of only a slight degree of motion, and which are deprived of a synovial membrane. Such are the *intervertebral fibro-cartilages*, and the *fibro-cartilages of symphyses*.

Fibro-cartilages of *the second class* are generally called the *interarticular*. They differ from those of the first class in being more or less free, and in having both surfaces covered by synovial membrane; they are situated between the extremities of bones which are covered with articular cartilage.

In both classes the fibrous portion of the tissue is most abundant towards the

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[*] Manuel d'Anatomie, Générale, Déscriptive et Pathologique, vol. i. p. 363.

[†] Encyclopædia of Anatomy and Physiology, vol. ii. p. 261.

[‡] Translation of MÜLLER's Elements of Physiology by BALY, vol. ii. Appendix, p. 4.

[§] MIESCHER, p. 29.

[‖] Ibid.

circumference, and in this portion only are blood-vessels found. In some instances, the centre of the fibro-cartilage is entirely composed of cartilaginous corpuscles. The cartilaginous portion is comparatively more abundant in young than in adult subjects, and in the latter it diminishes as age advances. This diminution of the cartilaginous portion is, doubtless, to be attributed to the gradual conversion of the cartilaginous corpuscles into fibres.

Of the Structure of the First Class of Fibro-Cartilages.—The Intervertebral Substances.—I am induced to enter upon the structure of the fibro-cartilages with some degree of minuteness, in order to remove the vague opinions of its composition which have been hitherto entertained.

I have examined the structure of fibro-cartilage in the intervertebral substance of the Mackerel.

It consists for the most part of large vesicular cavities containing an aqueous humour, in the interior of which are oil-globules and distinct nucleated corpuscles. The tissue in which these cavities are contained is of a fibrous character, though extremely lax.

The intervertebral substance of the Cod (*Gadus Morrhua*) has a composition similar to that of the Mackerel.

In the intervertebral substance of the Porpoise (*Phocæna*) I observed towards its centre, cells of considerable size containing distinct nuclei, as well as nuclei without cells. Fibrous tissue in concentric circles surrounded the central soft portion. Numerous distinct cells, similar to those in the centre, were interspersed between these layers of fibrous tissue. The cells of the central part vary considerably in size and form; some of them are double, each division presenting two central nuclei. These cells are round or oval; the central nucleus which each contains, presents a distinct nucleolus.

In a full-grown young Dog (*Canis Familiaris*) the intervertebral substance of the cervical region presents towards its centre a semifluid gelatinous mass, which is invested by a distinct capsule of fibrous substance, and from which it is easily separable. The gelatinous mass consists of numerous corpuscles of various sizes and in various stages of development. The larger ones present a granulated structure, and appear to contain small corpuscles. The gelatinous mass also contains a small quantity of delicate fibrous tissue, among which the corpuscles above noticed are interspersed. This fibrous tissue is composed of compressed corpuscles which resemble the scales of the epidermis and the epithelium, each presenting a distinct nucleus, and being attached to its fellows by an elongated and attenuated process. Towards the circumference of this central gelatinous mass the fibrous tissue becomes more abundant, in the interspaces of which the corpuscles are seen. The fibres of the fibrous tissue appear to be formed by an elongation and growth of the corpuscles of the cartilage*. The ex-

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* Since I made this dissection the researches of M. Schwann have been published, from which he deduces the opinion that all tissues are developed from cells.

ternal fibrous capsule of the intervertebral substance is firm and dense. Its circumference is almost wholly composed of fibrous tissue arranged in circles, between which circles are rows of cartilaginous corpuscles. Nearer to the centre, the fibrous tissue is found to consist of distinct circular bands, separated from each other by large masses of corpuscles, and more centrally still the fibrous tissue is laxer, and the cells which are scattered through its meshes are larger and more abundant.

I shall now give an account of my examination of the intervertebral substance of the *human subject* at various periods of development.

a. In the human fœtus of the *third month* the intervertebral substance of the second and third lumbar vertebræ is firm and white at its circumference, soft, and of a leaden hue towards its centre. The central part is composed of numerous cells or corpuscles, some being round, others stellated. It also presents large masses of other cells aggregated together, and somewhat of a darker colour. Nearer to the circumference of the intervertebral substance the cells are arranged in distinct lines, and at its circumference they are elongated at each extremity, and so attenuated as to assume the appearance of fibres.

b. In a human fœtus of *seven months* the intervertebral substance is composed of the external fibrous and the central cartilaginous portions. The central portion is almost entirely composed of distinct round cells, which are found, on being traced towards the fibrous part, to become more elongated at their extremities, and to form, as it were, series of fibres. The fibrous portion is made up of circular layers of fibres, between which cells are interspersed.

c. In a human fœtus of *nine months* the central part of the intervertebral substance is soft, and is composed of cells, and of an intercellular substance. The external fibrous portion is distinct, and is interspersed with cells.

d. From the period of birth to adult age, the change undergone by the intervertebral substance, consists in the gradual encroachment of the fibrous portion upon the cartilaginous.

e. *In adult age* the central portion still continues to present corpuscles, although not in so great a number. They are always found interspersed through a gelatinous mass. The external fibrous portion also presents corpuscles between its circles of fibrous tissue.

f. *In old age* the corpuscles of the intervertebral substance are less numerous than in the adult. The fibrous tissue is also more dense and unyielding.

2. The fibro-cartilages, comprising those of the *symphysis pubis,* and of the *sacro-iliac articulation* also, consist of an external fibrous portion, and of a central cartilaginous one, which undergo the same relative changes, as age advances, as the fibro-cartilages just described.

Of the Structure of the Second Class of Fibro-Cartilages.—I have made careful examinations of all the *inter-articular* fibro-cartilages at several periods of their development and growth, and the following is the result of my inquiries concerning their structure. They, like the fibro-cartilages of symphyses, consist of an external fibrous and of a central cartilaginous portion. At the early periods of their development, the cartilaginous is more abundant than the fibrous portion, and it is almost entirely composed of corpuscles. As age advances, the fibrous portion increases in quantity; and towards the later periods of life, the corpuscles of the cartilaginous division are mixed with fibrous tissue.

Of the Vessels of Fibro-Cartilage.

Respecting the vascularity of the fibro-cartilages, Bichat says, "Peu de sang pénètre leur système vasculaire dans l'état ordinaire, mais dans l'inflammation ils sont extrèmement injectés*."

Professor Todd says, "They (the fibro-cartilages) are more vascular than pure cartilage, but in the natural state they admit very few vessels carrying red blood†."

Müller, as quoted above, says "they have blood-vessels, although in a small number."

Of the Vessels of Fibro-Cartilages of the First Class.—1. *Of the inter-vertebral substances.*—I have made numerous injections of the intervertebral substance in Man and animals of various ages, and have found that the external more fibrous portion is pierced by arteries of considerable size; these are guarded from compression by the dense nature of the fibrous tissue through which they pass. They course towards the central cartilaginous portion, into which, however, they do not penetrate, but in its confines they form large convoluted dilatations, from which the recurrent vein arises. The extreme edge of these vascular convolutions presents a line which may be considered as the boundary between the fibrous and cartilaginous portions of the intervertebral fibro-cartilage. See Plate XV. fig. 6.

2. The following is the result of a careful examination of the vessels of the *sacro-iliac fibro-cartilage* in the human subject, at various periods of its development.

a. In the *fifth month* of fœtal existence the vessels of the sacro-iliac fibro-cartilage form a conglomerated mass of large and tortuous arteries and veins contained in the external part of the fibrous portion. Here and there a few more delicate branches diverge from this mass to a slight distance towards the central cartilaginous portion, at the border of which they terminate in dilated extremities.

b. In the fœtus of *six months* the convoluted arteries are prolonged to the circumference of the cartilage, where they divide and subdivide, each branch terminating in a dilatation, which frequently communicates with the one adjacent to it, and from this dilatation or series of dilatations the recurrent veins arise.

c. In the fœtus of *nine months* these vessels are considerably increased in length; they are more distant from each other, and their extremities no longer present the large dilatations.

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* Anatomie Générale. Article, *Organization du Systême Fibro-Cartilagineux.*

† Cyclopædia of Anatomy and Physiology. Article, *Fibro-Cartilage.*

More limited observations upon the state of these vessels in subsequent periods of life, lead to the belief, that, compared to the size of the fibro-cartilage, they gradually become more scanty and small from the period of youth to that of old age.

3. Researches upon the vessels of *the fibro-cartilage of the symphysis pubis* lead me to conclusions similar to those I have just related.

Of the Vessels of the Inter-articular Fibro-Cartilage.—The central part of the inter-articular fibro-cartilages, in the injected specimens that I have examined from the human fœtus as early as the third or fourth month, does not contain any vessels. I possess, however, the inter-articular cartilage of the temporo-maxillary articulation of a fœtal Calf, which is pervaded by blood-vessels throughout its entire substance; a disposition which may take place in all fibro-cartilages at very early periods of their development. Subsequent to these very early periods, however, the central portion, which, like articular cartilage, is subject to concussion and compression, does not contain any blood-vessels.

The inter-articular fibro-cartilages are pierced at their circumference by arteries of considerable size, which converge towards the central cartilaginous portion, into which, however, they do not penetrate, but upon its confines they form dilatations, with which the veins are continuous*, Plate XV. fig. 7.

In some instances, as in the sterno-clavicular fibro-cartilage, the arteries form intricate ramifications and convolutions at the circumference of the cartilage; in other instances, as in the semilunar fibro-cartilages of the knee-joints, they terminate in more simple dilatations, Plate XV. fig. 7.

The vessels of the fibro-cartilages in some parts, as in those of the knee, form a most intricate flexus in the fibrous tissue.

Vessels extend to a short distance on the surface of fibro-cartilages, beneath the synovial membrane, but they are arrested at the part where these structures are subject to pressure, and at this margin they form dilatations similar to the synovial vessels which cover the border of articular cartilages.

The Second Class of Non-vascular Animal Tissues.

Of the Cornea, the Crystalline Lens, and the Vitreous Humour.

The organs enumerated above may be considered as constituting a class of non-vascular organized tissues, inasmuch as each of them is transparent, each forms a part of the eye-ball, and performs a similar function, viz. of transmitting the rays of light to the retina. These three structures are nourished by the penetration into them of a nutrient fluid, which is derived from the numerous blood-vessels which encircle them; although each of them contains corpuscles, they differ from each other in their structure as well as in their relations with the vascular system. Thus the crystalline lens is soft externally, and hard towards its centre, and the

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* For the injection of the blood-vessels of fibro-cartilage, young subjects should be selected.

vitreous humour is semifluid. In the cornea one set of blood-vessels is prolonged upon a part of its free surface; while another is in apposition with the margin which is attached to the sclerotic, and is devoted to the nutrition of the principal part of its substance. In having two sets of blood-vessels, the one devoted to the nutrition of the surface of the tissue, and the other to that of its substance, the cornea has an analogy with cartilage and fibro-cartilage. The crystalline lens receives a supply of nutrient fluid only at its surface, and by means of the ramifications of the arteria centralis retinæ, which are distributed in the capsule which surrounds it. The vitreous humour appears to be nourished by the fluid which it derives from the vessels of the ciliary processes, the latter being received into sulci of the humour. It will be observed, that the vitreous humour has vessels in contact with it only at its surface, and at a small portion only of the latter; it may be inferred that the fluid brought to this part is capable of nourishing the whole vitreous body, on account of the facility which its semifluid character allows of the diffusion of a fluid through its entire mass.

1. *Of the Cornea.*

Structure of the Cornea.—It has been stated, in the Introduction to this paper, that all the non-vascular animal tissues contain the characteristic corpuscles. The cornea does not present any exception to this assertion, although the following account by M. Müller does not appear confirmatory of the accuracy of my opinion: the following are his words:—

"The middle layer, which constitutes the chief substance of the cornea, is formed of an interlacement of bundles of bright fibres, without any intermixture of corpuscles*."

I have frequently made most careful examinations of the substance of the cornea, and I have always found corpuscles present. They are certainly not so abundant here as in some of the above-named tissues. They are better seen in sections made at right angles to the surface of the cornea, than in those parallel with its surface, and they appear to be more evident after the cornea has been immersed in spirits of wine. Some of these cells are rounded, others are oval, and have fine branches radiating from them, similar to the osseous and pigment corpuscles. These cells are surrounded by the bright fibres of which Müller has spoken; these fibres, which compose the larger portion of the cornea, are laxly connected together, so as to have some analogy with cellular tissue; the substance of the cornea being of a loose texture, and easily pervaded by fluids. I believe that the lax texture of the cornea allows of an easy penetration of its substance by the nutrient fluid which circulates around it, and thus there is not an equal necessity for the presence of the corpuscles in this, as in the more dense tissues.

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* Elements of Physiology, by Baly. Appendix, p. 3.

Of the Vessels which nourish the Cornea.

Previous to giving the results of my researches upon the manner in which the cornea is supplied with a nutrient fluid, I shall detail the following opinions of authors upon its vascularity.

M. Boyer. "La cornée reçoit des vaisseaux sanguins, puisqu'elle devient rouge dans les fortes inflammations; mais ces vaisseaux sont si fins, que l'injection ne peut y pénétrer, et que la partie rouge du sang ne s'y introduit qu'en quelques circonstances. On n'a point encore découvert de nerfs dans cette membrane[*]."

M. Cruveilhier. "Les injections les plus fines, passées dans les veines et dans les artères de l'œil, ne démontrent aucun vaisseau dans la cornée[†]."

Mr. Jacob. "The cornea is destitute of red vessels, yet it affords a signal example of a colourless and transparent texture possessing vital powers inferior to no other[‡]."

Mr. Lawrence. "The cornea in its natural structure consists of cartilaginous laminæ and mucous membrane; the cornea is analogous to the articular ends of the bones, in which the articular cartilage is covered by synovial membrane. It (the cornea) agrees with them, (the fibrous structures) in the entire absence of vessels circulating coloured fluids; perfect transparency being essential to its office of transmitting light[§]."

M. Müller. "The existence of vessels in the substance of the cornea is doubtful; they have never been injected. Nevertheless, penetrating ulcers and granulations are formed in the cornea, which can scarcely be conceived to occur without the agency of vessels. I have repeatedly seen in Calves of nearly the full time, vessels in the conjunctiva of the cornea, which contained red blood, and which could with a lens be traced more than a line over the margin of the cornea. Henle has injected these vessels; in the conjunctiva of the cornea they measured $\frac{1}{1319}$th to $\frac{1}{694}$th of an inch, and the finest twigs were not then injected; their trunks, which arose from a circular vessel that ran around the cornea, were even somewhat larger than this. The preparations of these parts I have in my possession. Professor Retzius has, by means of injection, been able to see the same thing in the adult animal.

"All these facts, however, render it very probable, that even the cornea and capsule of the lens, to which vasa serosa have been hitherto ascribed, are really provided with vessels carrying red blood. The vessels of the corneal conjunctiva are certainly less numerous than those of the sclerotic conjunctiva; there is the same difference between these two parts as between that part of the synovial membrane which is free, and that which covers the articular cartilage[‖]."

M. Romer. "Romer of Vienna has described the arteries which ramify from the

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[*] Traité d'Anatomie, quatrième édition, 1815, vol. iv. p. 98.

[†] Anatomie Déscriptive, 1815, vol. iii. p. 462.

[‡] Cyclopædia of Anatomy, vol. ii. p. 177.

[§] On the Diseases of the Eye, 1833, pp. 16, 368.

[‖] Müller's Elements of Physiology by Baly, vol. i. pp. 215, 216.

sclerotic conjunctiva upon the cornea, from injections. The fine twigs of the arteries of the sclerotic conjunctiva unite together around the margin of the cornea into a vascular wreath or circle. From these there arise very numerous branches, which run from the circumference towards the centre of the cornea, and in their course make two or three subdivisions. Their ends bend distinctly inwards, and appear to penetrate the proper substance of the cornea*."

Mr. TYRRELL. "I have been thus satisfied that the vascular organization of the cornea is principally derived from the conjunctiva, and little, if at all, from the sclerotic vessels†."

Mr. WARDROP. "It (the cornea) is also nourished by the same vessels which supply the conjunctiva‡."

There is very considerable difficulty in making a complete injection of the vessels, which have the function of supplying the cornea with a nutritive fluid§.

The vascular system of the cornea resembles that of articular cartilage, in consisting of two sets of vessels, one of which is devoted to the supply of its substance, and which is in contact with its margin where it is attached to the sclerotic; the other supplies its free or mucous surface, the circumference of which it overlaps.

In an eye which is injected with tolerable success, the white sclerotic membrane will be observed to be traversed by two sets of vessels. One of these consists of small and numerous branches, which have a straight direction towards the circumference of the cornea. These are the ultimate branches of the sclerotic arteries which course towards the cornea. The other set of vessels is composed of the large and tortuous trunks which are seen with ease by the naked eye; these are the sclerotic veins, which take a retrograde course to the arteries just alluded to, and become gradually larger as they get more remote from the cornea; these sclerotic veins return the blood devoted to the nutrition of the cornea.

Upon examining *the arteries* with a magnifying glass, they will be found, at the circumference of the cornea, to terminate in two sets of vessels; of these, one is superficial, and consists of delicate branches which pass inwards over the surface of the cornea, between it and its mucous covering, and are analogous to the vessels of joints which pass between the articular cartilage and the synovial membrane. The other set of vessels, in which the sclerotic arteries terminate, are much larger than those just noticed, and are more like the continuation of their trunks; these, at the circumference of the cornea, pass into the substance of the sclerotic, where they come in contact with the attached margin of the cornea.

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[*] Mr. WHARTON JONES, Medical Gazette, vol. xxiii. p. 593.

[†] Medical and Chirurgical Transactions, vol. xxi. p. 17.

[‡] The Morbid Anatomy of the Human Eye.

[§] I have made more than fifty injections of the eye, and I possess only three specimens in which the vessels alluded to are successfully filled.

The *superficial or conjunctival* arteries, upon arising from the sclerotic, take a course parallel with the circumference of the cornea, and are sometimes so long as to receive the name of its circular vessels; from these, branches are given off which pass in a direction towards the centre of the cornea. These, after division and subdivision, form a minute plexus on the border of the latter, and they terminate on its surface from one-eighth to half of a line from its circumference, by becoming continuous with the venous system, Plate XVI. fig. 1.

The modes in which the arteries are continued into *veins*, are various; sometimes they form loops with the venous radicles; sometimes a single arterial branch divides, and both of its divisions take a course retrograde to that of the artery, and they empty themselves into the venous plexuses. This species of vascular arrangement is seen in Plate XVI. fig. 1. Sometimes the artery divides into two or three branches which form loops with a venous radicle. These vessels appear to differ much in their sizes; this difference perhaps depends upon whether, as in the specimen from which the fig. 1. was taken, they only contain a sufficient quantity of colouring matter to enable their course to be traced with distinctness. The small *veins*, which are continuous with the arteries just described, take an opposite course to the latter, and upon reaching the margin of the sclerotic, they empty themselves into the large tortuous veins, which have been noticed above, just as the latter are emerging from the substance of the sclerotic, where it is attached to the cornea.

The *deep or sclerotic arteries* of the cornea are those from which this structure derives its chief nutrition. They are large enough to be considered the continuation of the trunks of the sclerotic arteries; they pass without much diminution of their size towards the point where the sclerotic membrane is continuous with the cornea. At the margin of the latter they suddenly stop and turn back, forming loops; sometimes with, and at other times without dilatations, they become continuous with the veins, Plate XVI. fig. 2. These veins emerge from the substance of the sclerotic, close to the margin of the cornea, at which point they receive the conjunctival veins; and they take a backward course, and form those tortuous veins which have already been noticed, and which are seen by the naked eye.

Such are the investigations which have induced me to state, in the introduction to this second class of tissues, that the cornea in a healthy state does not contain any blood-vessels. The vessels which have the function of supplying this tissue with a nutritive fluid, are those belonging to the two classes which I have described.

In a diseased state, the vessels which, in the healthy condition terminate abruptly at the margin of the cornea, are prolonged through its entire substance; while those which, when healthy, extend over the surface of the cornea from the one-eighth to half of a line, in disease form a band of considerable extent. I shall defer entering upon these pathological conditions to a future opportunity.

2. *The Crystalline Lens.*

Of the Structure of the Crystalline Lens.—According to the researches of M. Schwann, the crystalline lens, in the earliest periods of its development, is entirely composed of cells; and it is the opinion of Valentin and himself, that these cells are converted into the fibres of the lens. The dentations of the fibres of the lens are compared by M. Schwann to the sinuosities of a not uncommon form of vegetable cells*.

In the examinations that I have made of the crystalline lens, I have not only found cells interspersed among its fibres, but have frequently seen the fibres themselves, composing the external part of the lens, made up of these cells; and in other instances they occupy the margin of the fibres only.

Of the Blood-vessels which nourish the Crystalline Lens.—The crystalline lens, in a healthy state, has never been seen to contain blood-vessels. Müller, in speaking of the mode of development of this organ, says, "The matrix of the crystalline lens is its capsule, which seems to secrete the layers of the crystalline from its inner surface." The presence of vessels in its substance would certainly interfere with the functions of the crystalline lens; for, as I have said, perfect transparency is essential to its office of transmitting light. In the anterior capsule, according to Müller, "the vessels are extremely difficult to inject;" he however states that "in inflamed eyes they are distinct, both on the anterior and posterior walls of the capsule." The presence of blood-vessels in the anterior capsule of the lens would as effectually derange the functions of the eye, as if they were in the substance of the cornea, or in the lens itself.

I have not only been unable to trace vessels into the anterior capsule, but I hope to prove that in the healthy state no vessels do enter it. The posterior capsule of the lens is, however, injected with facility, and contains large and numerous ramifications of blood-vessels; I ascribe to them the function of supplying the crystalline lens with a nutrient fluid. These vessels arise from the arteria centralis retinæ; the latter, having traversed the centre of the vitreous humour, expands upon the capsule, and forms the ramifications just noticed. Now in some injections which I have made of the eyes of a human fœtus, of the sixth or seventh month, these vessels were not confined to the posterior surface of the capsule; they pass round its border and extend upon its anterior face to the extent of one quarter of a line. I have not been able to make a perfect injection of the vessels of the capsule of the lens in ages antecedent to the fifth or sixth month of the fœtal life, and therefore am unable to say whether in the very early periods of development, the anterior capsule, like the membrana pupillaris, is entirely traversed by vessels; the crystalline lens would, under such circumstances, be completely surrounded by blood-vessels.

The branches of the arteria centralis retinæ in the early periods of life, as noticed

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* British and Foreign Medical Review, vol. ix. p. 512.

above, extend upon the anterior surface of the capsule. Immediately they reach the latter they become straight, run parallel with each other, and are directed towards the centre of the anterior surface for the distance of a quarter of a line, when they stop in their course, and form looped dilatations, which give origin to small veins, Plate XVI. fig. 6. It is most probable that these vessels recede at subsequent periods of development, so as to leave the whole of the anterior surface of the capsule capable of being permeated by the rays of light*. These vessels, in a diseased state, are sometimes prolonged into the whole of the anterior capsule, (or to speak with more propriety, the anterior half of the capsule,) where, in morbid specimens, they have been injected by Schroeder Van der Kolk. The capsule of the lens is thus pervaded by large and numerous ramifications of blood-vessels, which I believe pour out upon its inner or lenticular surface a nutrient fluid; this *fluid will immediately come in contact with* the mass of delicate cells described by Schwann as situated between the lens and the capsule.

The mode of nutrition of the crystalline lens may be explained, by supposing that the nutrient fluid is received by the cells just alluded to, and conducted to the lens (perhaps has its characters changed in its course by the metabolic functions ascribed to them by M. Schwann) through which it is diffused. It has been stated, that the presence of blood-vessels in the cornea, the anterior half of the capsule of the crystalline lens, and in the substance of the lens itself, appears to be incompatible with their function of transmitting the rays of light to the retina. Nevertheless large vessels ramify on the posterior half of the capsule. The knowledge of the existence of this arrangement of vessels led me to perform some experiments with lenses, from which I have deduced the fact, that objects (radiating lines, for instance) situated on the *anterior* surface of the crystalline lens, produce an indistinctness in the image which is formed upon the retina; whereas, when these lines exist upon the *posterior* surface of the lens, the image is perfectly clear.

3. *The Vitreous Body or Humour.*

Structure.—The vitreous humour is composed of cellular cavities which are filled with a transparent fluid. The membrane of which the walls of these cells are composed,—the tunic of the vitreous humour,—is very delicate, and is interspersed with corpuscles.

The Vessels of the Vitreous Humour.

Many anatomists† have stated that the arteria centralis retinæ, in its course through the vitreous humour, gives off minute branches into the substance of the latter; but these branches have never been described; on the contrary, Müller, who has paid

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* I have hitherto not succeeded in making a complete injection of these vessels in the adult subject.

† Harrison, Boyer, Cruveilhier, &c.

especial attention to this subject, and has examined injections of inflamed eyes, says, "I have not seen any injected vessels in the vitreous body;" and adds, "I do not despair of seeing this part also injected*."

Since the above was written, I have met with the following account of an injection of the membrane surrounding the vitreous body:—"Mr. Dalrymple has succeeded in injecting a number of minute ramifications, of very delicate vessels, on the periphery of the membrane (of the vitreous body) derived from a branch or branches of the central artery, which passed by the spot termed the foramen of Söemmering; he kindly permitted me to inspect the preparation, which was most satisfactory, but which I much regret has been since destroyed by accident†."

My researches induce me to believe that the vessels of the vascular ciliary processes of the choroid membrane, to which no specific function has hitherto been given, have the function of nourishing the vitreous body. My opinion is confirmed by that of Müller, who says, "The zonula Zinni, or corona ciliaris, appears from Henle's and Schroeder's injections, to be a vascular organ, and to be of great importance for the nourishment of the transparent humours‡."

I have succeeded in making very minute injections of the vessels of the ciliary processes, the disposition of which is remarkably beautiful; they have been most accurately delineated and described by Zinn. I have particularly to allude to the immense quantity of blood that their large size allows to be continually circulating through them, and to their plexiform character, which is productive of a slow circulation of the fluid they carry. At the free border of each ciliary process is a large single vessel, which is received into the base of the sulcus of the vitreous humour, and is in immediate contact with it. When it is remembered that the investing membrane of the vitreous humour is very delicate, and that it is made up of cells or corpuscles, and that it has in immediate contact with its free surface these large and numerous vessels, they may perhaps with great reason be considered as the organs of nutrition of the vitreous humour, by eliminating a nutrient fluid which penetrates into, and is diffused through, the substance of the latter.

The Third Class of Non-vascular Animal Tissues.

Of the Epithelium, the Epidermis, Nails and Claws, Hoofs of various kinds, Hair and Bristles, Feathers, Horn and Teeth.

The above structures are placed together as forming a distinct class of extravascular tissues, on account of their being all developed upon the surface of the chorion, and very analogous to each other in their structure, their mode of growth and

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[*] Müller's Elements of Physiology by Baly, vol. i. p. 216.

[†] A practical work on the Diseases of the Eye, by F. Tyrrell, p. 97.

[‡] *Loc. cit.*, p. 219.

nutrition. Each of them is in contact, at its attached surface, with numerous and large branches of the vascular system, and, with the exception of the teeth, each is almost entirely composed of corpuscles or cells, which are of a somewhat circular form, where they are near to the vascular chorion, and are compressed and flattened where they are further removed from it. These tissues grow, by the addition to them, at their point of attachment with the chorion, of new cells, and from the increase in size of those already developed*.

The Epithelium is composed of corpuscles which are round where they are in contact with the vascular chorion, and of others which are flat and in the state of scales situated at its free surface. Immediately subjacent to the epithelium the chorion presents ramifications of blood-vessels which nourish it, and which have different characters in different portions of the mucous membrane, according to the various functions which they have to perform.

The chorion of the *Integuments* of the human subject, where it is subjacent to the thin and delicate cuticle, presents a minute and intricate network of blood-vessels. The arteries divide, subdivide, and form a network, from which the veins arise. The vessels of the chorion, however, are differently arranged, where it is covered by thick and dense cuticle. Thus, it is well known, that at the palms of the hands, the anterior surface of the extremities of the fingers, the posterior part of the heel and at the sole of the foot, the thick *Epidermis* forming corns, &c., the arteries of the chorion are observed to terminate in numerous dilated loops, a disposition which bears a close analogy to certain synovial vessels.

The vessels of the chorion have the function, not only of secreting the perspiration, but of developing and nourishing the ducts through which the latter fluid is excreted†.

In parts where the *Epidermis* undergoes a still greater degree of condensation, and is found in larger masses than in parts already alluded to, the vascular system of the chorion presents still greater dilatations, and more complicated arrangements. Such parts are the nails and claws, hoofs, hair and bristles, feathers and horn.

The Nails.—Where the nails are in cohesion with the vascular chorion, they are more or less soft; and when portions of them at this part are examined by the microscope, they are seen to consist of corpuscles somewhat compressed and connected to-

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* In the first edition of his Elements of Physiology, page 384, MÜLLER enters upon an examination of the mode of growth "of unorganized non-vascular parts." These he divides into three classes:—1. The horny tissues; 2. The teeth; 3. The crystalline lens. In the second edition, p. 416, having omitted the word unorganized, he adds, "These parts, however, though not vascular, and though formed in the manner which we have described, have nevertheless a definite structure or organization, and afford confirmation of our previous remarks, that the vessels are merely destined to pour out the materials for nutrition, and that the formation of the elementary parts of each tissue takes place in the matter effused independently of any action of the vessels."

† The walls of the perspiratory ducts of the cuticle appear to me to be formed of a single filament (itself composed of corpuscles) spirally arranged, having thus an analogy with the spiral ducts of plants.

gether by a gelatinous substance. The harder part of the nail consists of compressed and transparent corpuscles.

Although the nails do not contain any blood-vessels, they change their colour and become friable under certain conditions of the circulation, thus showing that their component cells have the power of circulating through them the fluid which is brought to them by the blood-vessels, at their attached surface.

The nails are in contact with the vascular chorion at two points; their attached margin is inserted into a groove of the chorion, and the attached surface lies upon that portion which covers the dorsum of the terminal phalanges.

The vessels which have the office of supplying the nail with a nutrient fluid are large and numerous. The arteries take their origin from the digital trunks, and they converge towards the dorsal surface of the terminal phalanx, on which they ramify, and where they may be considered to terminate in two sets of vessels; one of which is devoted to the supply of the ungual groove, the other to the ungual surface of the phalanx. The ungual groove is very vascular; it presents the ramifications of arteries, which, after division and subdivision, form large plexuses in the margin of that part of the groove which overlaps the nails, and the arteries terminate in loops of considerable size. The vessels of the ungual surface are of a considerable size; they form large convolutions; these give off branches, which, with others from the interior and lateral part of the phalanx, form an intricate capillary network, which is in immediate opposition with the attached surface of the nail.

The Claws and Horns of quadrupeds, and the claws and beaks of birds, have a structure very analogous to the nails of the human subject. They are in contact, at their attached surface, with large vessels. In some instances the bone upon which they rest is perforated by foramina, the chorion subjacent to them, and between them and the bone, is very attenuated, and they appear to be nourished also by the vessels contained in the bone itself.

Hoofs can be considered only as condensations of the cuticle. In fœtal Calves, at an early period of their existence, the hoof is not thicker than the cuticle of the heel of the adult human subject, to which it is analogous in its structure. In adult age that part of the hoof which is in connection with the chorion retains its analogy with the cuticle, but at its free surface it becomes hard and somewhat friable. It is impossible for me here to enter into the details of the structural varieties of the hoofs of animals, so as to point out any peculiarities in their composition which shall assist in diffusing the nutrient fluid through the whole of their substance, to endow it with the elasticity essential to the due performance of its functions. I will content myself with stating, that it appears to me that the use of the elongated tubes containing white soft matter, and which pervade the substance of the hoof of the Horse, is to convey through its substance the fluid secreted at its attached surface.

The chorion subjacent to the hoofs of animals presents two different characters, the one being that of numerous fine villosities, the other of compressed lamellæ; these are revived into depressions on the surface of the hoof, and are composed of vessels which terminate in loops, possessing frequently considerable dilatations.

Hair and bristles, of various kinds, in the neighbourhood of the vascular chorion, are composed of roundish corpuscles loosely connected; more remote from the chorion, their substance is harder; the corpuscles are flattened, and they appear to possess the characters of horn. The chorion, with which the hair is connected, consists of a papilla, which is inserted into the interior of the base of the hair, and of a sheath or capsule which surrounds it; both the papilla and capsule are supplied with large and numerous blood-vessels, which form loops and dilatations.

Feathers.—In his elaborate article "Aves," in the Cyclopædia of Anatomy and Physiology, Professor OWEN, after quoting the interesting observations of himself, Sir W. JARDINE, Mr. BLYTH and Mr. YARRELL, on the changes that take place in the colour of the plumage of birds subsequent to their complete development, says, "Notwithstanding the extra-vascular nature of feathers, they are subject to influences apparently of a vital nature, which occasion a change of colour in them after they are completely formed."

Feathers near to the vascular chorion consist of corpuscles more or less compressed; further removed from the chorion they are highly compressed, and present a similarity to the hair. The chorion to which the feathers are attached presents a pulp and capsule, which have a disposition of vessels analogous to those of the hair.

The Teeth are now considered to be permeated by an infinity of fine tubes, which are supposed to have the function of conducting from the surface of the vascular pulp, a nutrient fluid, which is distributed over the substance of the tooth. In this way may be explained the manner in which the teeth change their colour during diseases, being impregnated by the various fluids circulating in the system*.

From the preceding observations by myself, and from those made by the physiologists whose writings I have quoted, I think it is established, as a general law in Animal Physiology, that certain tissues are capable of being nourished without the presence of blood-vessels within them: it has been shown that all these tissues are surrounded by large blood-vessels, which appear to have no other function than of supplying to them a nutrient fluid; and the way in which this nutrient fluid is conveyed into the substance of these tissues has been also pointed out.

* Recent investigators have thrown so much light upon the structure and the mode of growth of the *Epidermoid Class* of non-vascular tissues, that (as is apparent) I have added but few new facts in this department of my researches.

The analogy between the extra-vascular animal and the vegetable tissues is manifest.

The application of the above-named law to the study of *Surgery*, in reference to the causes of the prolongation of vessels into the extra-vascular tissues, and to the measures to be adopted for the prevention and cure of those diseases which are dependent thereon; and to *Pathology*, in the investigation of the nature of morbid structures, particularly of those classes which contain no vessels,—will, I feel certain, be productive of interest and great advantage.

In conclusion, I have to thank my estimable friend Mr. Edward Dickinson, for the many acts of disinterested kindness he has conferred upon me, which have tended much to supply matter to this paper.

Every one will see how much I owe to the skill of Mrs. Holmes.

My brother, Mr. George Toynbee, by the valuable assistance he has afforded me in editing and arranging this paper, has added another to the many deep obligations which I owe him. For his good counsel, from the time I entered the profession to the present day, I am more indebted than words can express.

Explanation of the Plates.

PLATE XIII.

Fig. 1. The anterior surface of the inferior extremity of the femur of a human fœtus of about four or five months. It has been macerated for some time in water, and the cartilage of the epiphysis is somewhat transparent, so that a canal for the epiphysal blood-vessels is seen in its substance.
- A. The orifice of a canal.
- B. The synovial surface of the cartilage.

Fig. 2. A section of a similar extremity of the femur of a human fœtus somewhat older.
- A. The osseous cylinder of the femur.
- B. The cartilaginous epiphysis.
- C. Canals for the blood-vessels.
- D. The synovial surface of the cartilage.

Fig. 3. The blood-vessels which are contained in the canals of the epiphysis.
- A. The artery.
- B. The dilatations with which the arteries and veins are continuous.
- C. The veins.

Fig. 4. The blood-vessels contained in a longer epiphysal canal.
- A. The artery.
- B. The intermediate dilatation.
- C. The vein.

Fig. 5. A plexus of blood-vessels situated between the synovial membrane and the circumference of the articular cartilage; from the same specimen as fig. 3.

Figs. 6, 7, 8, 9 and 10. Diagrams of the inferior extremity of the os femoris, to show the relations of the blood-vessels with the articular cartilage, and the osseous nucleus of the epiphysis, during the different periods of their development.

Fig. 11. Vessels situated between the attached synovial membrane and the articular cartilage, at the point where the ligamentum teres is inserted in the head of the os femoris of the human subject, between the third and fourth months of fœtal life. (Magnified thirty diameters.)

A. The surface of the articular cartilage.
B. The vessels between the articular cartilage and the synovial membrane.
C. The surface to which the ligamentum teres was attached.
D. The vein.
E. The artery.

PLATE XIV.

Fig. 1. Vessels similarly situated to those of fig. 11. Plate XIII. but between the seventh and eighth months of fœtal life. (Magnified thirty diameters.)

A. The surface of the articular cartilage.
B. The vessels between the articular cartilage and the synovial membrane.
C. The surface to which the ligamentum teres was attached.

Fig. 2. Vessels situated between the synovial membrane and the articular cartilage on the circumference of the condyle of the os femoris of the human fœtus, at the period of birth. In one part the arteries are observed to terminate in a single vessel from which the veins take their origin; in another part the arteries communicate with the veins by means of loops. (Magnified thirty diameters.)

A. The surface of the articular cartilage.
B. The vessels between the attached synovial membrane and the articular cartilage.
C. The reflexed synovial membrane.
D. The lateral surface of the condyle of the femur.

PLATE XV.

Fig. 1. Vessels situated between the attached synovial membrane and the articular cartilage at the circumference of the head of the metatarsal bone of the human subject.

Fig. 2. A vertical section of a portion of the inferior extremity of the os femoris. Natural size.

A. The horizontal lamella, to which in the recent state the articular cartilage is firmly attached, and which separates the latter from the cancelli of the bone.

B. The vertical fibres of the cancelli; these are implanted into the upper surface of the articular lamella.

Fig. 3. A portion of the inferior extremity of the os femoris slightly magnified.

A. The articular lamella.

B. The cancelli of the bone.

Fig. 4. Displays the manner in which the blood-vessels are disposed in the cells of the cuboid bone, internal to the articular lamella. This lamella has been rendered transparent, and the vessels are seen internal to it.

Fig. 5. A vertical section of a cuneiform bone of the human foot, and of the articular cartilage which is implanted upon it.

A. The interior of the bone.

B. The articular lamella.

C. The articular cartilage.

Fig. 6. The blood-vessels of the intervertebral substance; the arteries traverse the fibrous portion, and terminate at the circumference of the central cartilage in large dilatations.

Fig. 7. The semilunar fibro-cartilage of the knee-joint from a young human subject.

A. The central part not containing blood-vessels.

B. The external fibrous and vascular portion.

PLATE XVI.

Fig. 1. Represents the mode of distribution of the blood-vessels on the free surface of the circumference of the cornea. (Magnified thirty diameters.)

A. A portion of the cornea.

B. The blood-vessels situated on the surface of the cornea, being covered by the membrana conjunctiva.

C. A portion of the membrana sclerotica.

Fig. 2. Represents the blood-vessels which are situated in the substance of the membrana sclerotica where the cornea is attached to it. (Magnified thirty diameters.)

A. The cornea.

B. The sclerotic vessels.

Fig. 3. A highly magnified view of the blood-vessels in the membrana sclerotica at the circumference of the cornea.

A. The cornea.

B. The membrana sclerotica.

Fig. 4. A plexus of blood-vessels situated on the surface of the circumference of the cornea, between the latter and the membrana conjunctiva.

Fig. 5. A plexus of vessels similarly situated to that of fig. 4.

Fig. 6. Displays the manner in which the blood-vessels that ramify on the posterior part of the capsule of the crystalline lens are disposed at the circumference of the anterior surface of the capsule. From a human fœtus of between four and five months.

A. The anterior surface of the capsule of the crystalline lens.

B. Blood-vessels situated on the anterior surface of the capsule at its circumference.

[In the above Plates, the arteries, dilatations and veins, are distinguished from each other by the difference in the mode of engraving.]

XIV. *Supplementary Note to a Paper entitled "Researches in Embryology. Third Series: A Contribution to the Physiology of Cells."* *By* MARTIN BARRY, *M.D., F.R.SS. L. and E.*

Received December 6,—Read December 10, 1840.

IN the paper above mentioned it was shown, that after the ovum of the Rabbit has entered the Fallopian tube, cells are found collected around its thick transparent membrane or "zona pellucida;" which cells, by coalescing, form a thinner membrane—the incipient chorion[†].

I have now to add, that the formation of this thinner membrane does not exhaust the whole layer of these cells; but that a stratum of them is found remaining on, and entirely surrounding the "zona," after the thinner membrane has risen from it (Plate XIX. fig. 254.). The fluid space also, between the "zona" and the thinner membrane, presents a large number of cells or discoid objects, each of which contains a brilliantly pellucid and highly refracting globule. In some parts, several of these discs, closely joined together, have the appearance of shreds of membrane. In others, there are found pellucid globules, some of which are exceedingly minute.

The discs now mentioned collect at the periphery, for the thickening of the chorion. They seem to proceed from the region of the "zona;" and probably have their origin in the cells by which the latter is surrounded. If so, we cannot suppose them to arise in any other way than that which, according to my observations, appears to be the universal mode of reproduction; namely, by division of the nuclei of the parent cells. Nor can we suppose that minuteness is any hindrance to their subsequent increase by the same means.

In the accompanying figure (Plate XIX. fig. 254.), the parts forming the immediate subject of this note are delineated only on one side. The other parts are represented in outline.

[†] Philosophical Transactions, 1840, Part II. p. 529, par. 369 to 373.

XV. *On the Chorda Dorsalis.* *By* MARTIN BARRY, *M.D., F.R.SS. L. and E.*

Received January 3,—Read January 7, 1841.

FOR the discovery of the remarkable structure in the embryo of some of the Vertebrata denominated the chorda dorsalis, we are indebted to Professor BAER. This naturalist considered it "the axis around which the first parts of the fœtus form," and "the true *virga mensoria* for the whole body and all the chief systems[†]." REICHERT supposes it to be that embryonic structure which serves as a "support and stay" for parts developed in two halves[‡].

In the course of my researches on the mammiferous ovum, an object was noticed which seems to correspond in appearance to the incipient chorda now referred to. I am desirous of drawing attention to this similarity in appearance, but more particularly to point out some important differences between my own observations and those of others, as to the mode of origin of the objects in question, and their relation to surrounding parts. For, should it be found that these objects are the same, my belief is that even the most recent views on the incipient growth of the embryo must undergo a change.

Before entering upon the comparison, I must ask the reader to place before him my Second Series of Researches in Embryology[§], Plates VI. VII. and VIII.; to certain of the figures in which I will now briefly refer.

The object *bb*, figs. 113 to 116, is the rudimental embryo of the Rabbit in its earliest stage. It is a finely granular, hollow sphere—the nucleus of a cell. Its centre, the nucleolus, is brilliantly pellucid.

The changes which this nucleus undergoes are two-fold. The one consists in the formation of a pointed process, as in fig. 121 D, which is sometimes, and perhaps generally, curved; its concave side being directed towards the centre of the ovum. In this state the rudimental embryo appears to correspond to what has been supposed to be the "*primitive* trace" in the ovum of the Bird. The other change which this nucleus undergoes consists in the origin of fresh substance in its pellucid centre, and the expansion of its peripheral portion into cells. Some of these cells occupy the area pellucida: others, coalescing, form a little sac (fig. 117. $bb^{2\prime}$.), which, expanding and receiving the yelk into its cavity, assumes the form of a network (fig. 132.), and lines the remainder of the ovum (fig. 119. $bb^{2\prime}$.).

[†] Ueber Entwickelungsgeschichte der Thiere. Beobachtung und Reflexion, 1828. Taf. I. fig. II. III. a. p. 15.

[‡] Das Entwickelungsleben, 1840, p. 108.—The chorda dorsalis has received other names; viz. Chorda vertebralis, Rückensaite, Spinalsaite, Wirbelsaite.

[§] Philosophical Transactions, 1839, Part II. p. 307.

This two-fold change is but the commencement of a series of metamorphoses, by which the minute object *bb* of fig. 113, is converted into the objects bb^1, bb^2, and $bb^{2'}$, collectively, of fig. 122. For it is to be remarked that there is a continual origin of fresh substance in the centre; by which means previously formed parts are pushed further out. These parts consist of cells arranged in layers (fig. 122.), each cell being the seat of a process essentially the same (fig. 149.). As the cells are pushed into a more external situation, they expand and become pale; while in the centre the continued origin of new and unexpanded cells, presents the appearance of a row of globules, nearly black (fig. 122.). The larger end (fore-end) of this row of globules is spherical, and, as we have seen, has a minute pellucid centre. It is this row of globules, in the mammiferous ovum, which I think must correspond to Baer's incipient chorda dorsalis of the Bird, described by him in the following passage.

"The chorda dorsalis," he says, "originally consists of a simple row of dark globules, which towards the fore-end are more closely pressed together, and towards the hinder end more separated.***It becomes thicker and darker, from an increase in the number of its globules. The fore-end is at a very early period developed into a round, much thicker knob; and hence the whole chorda resembles a very fine needle with a minute head[†]."

Although the object thus described by Baer, as seen in the ovum of the Bird, presents a sufficiently close resemblance to that which I had delineated and have just described in the mammiferous ovum, to warrant the belief that both objects are the same, yet there are some important differences between the two descriptions.—Von Baer speaks of the "fore-end" of his incipient chorda as becoming "at a very early period *developed into* a round, much thicker knob." With me, on the contrary, the linear portion *proceeds from* the round and thicker knob.—Again, he makes no mention of the remarkable pellucid cavity, contained within this round and thicker knob—a part of prime importance, if it be, as my observations show, the main centre for the origin of new substance. Whether these differences between the objects of our respective descriptions really exist (supposing them to be the same objects), or whether, if the embryo of the Bird were examined in a sufficiently early stage, it would not be found to begin in the way I have described as producing the embryo of the Mammal, future observation must determine.

Von Baer farther says that his row of dark globules is "surrounded by a pellucid border:" and that "the border is seen from all sides," being therefore "a sheath for the chorda. Originally the chorda and its sheath are one[†]." How far these particulars accord with my observations, will be seen on reference to figs. 121 to 123.

Von Baer considered the chorda to arise simultaneously with his "laminæ dorsales;" and this by "a separation of the [supposed] primitive trace into two lateral halves (the laminæ dorsales) and a middle streak (the chorda)[‡]." The separation here mentioned, I can regard as no other than the pushing out of the lateral portions of

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[†] *L. c.*, pp. 15, 16.

[‡] *L. c.*, p. 15.

the so-called "primitive trace," by the formation of fresh substance in the interior (compare with my fig. 122.). If so, the formation of BAER's incipient chorda is not simultaneous with, but subsequent to, that of his "laminæ dorsales." (The "laminæ dorsales" of BAER correspond to REICHERT's "central nervous system.")

VON BAER describes the chorda as "the axis, around which the first parts of the fœtus form†." The dark pin-like object we have been considering in the mammiferous ovum, presents merely one of the many layers of incipient cells into which a nucleus becomes resolved.

Notwithstanding these differences, however, facts mentioned by authors regarding the chorda dorsalis at later periods I think afford evidence of the identity of the two objects in question, as will be seen by what follows.

RATHKE states that in osseous fishes the chorda dorsalis is inclosed in a membranous sheath; and that from this sheath there grow in pairs a great number of minute filaments, which are the incipient crura of the arches of the vertebræ. The foundations of the bodies of the vertebræ appear, from the description given by this author, to consist of minute tables or traces, which proceed out of the vertebral arches; and he adds, that "the thickening and ossification of the vertebræ take place at the expense of the inclosed nucleus of the chorda‡."

REICHERT, in his researches on the development of the Batrachian Reptile and the Bird, found the chorda to decline more and more as the vertebral system advanced in its formation§. "The chorda," says this author, "is reduced in proportion as ossification proceeds, until for the most part only its remains are to be found between the vertebræ‖."

It is known that the chorda dorsalis in a comparatively advanced state is composed of cells. For this discovery we are indebted to Professor J. MÜLLER. SCHWANN has since found the characteristic nucleus in its cells¶. "If we closely examine," says this observer, "the outer rind of the chorda in *Pelobates fuscus*—covered as this rind is with scattered grains—we find these grains closely to resemble the nuclei of cells; only that they are about half the size. They are besides oval, and furnished with a nucleolus. This rind is not distinctly separated from the proper tissue of the chorda dorsalis: and as the cells of the latter rapidly diminish in size towards the rind, I believe these grains of the rind to be the cytoblasts of flattened down cells, which form the rind††."

I think there is a great deal in what has now been quoted from these authors, that

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† *L. c.*, p. 15.

‡ BURDACH's Physiologie als Erfahrungswissenschaft, 1837, Band II. pp. 279–281.

§ *L. c.*, p. 68.

‖ *L. c.*, p. 71.

¶ Mikroskopische Untersuchungen über die Uebereinstimmungen in der Struktur und dem Wachsthum der Thiere und Pflanzen, 1838, 1839, Tab. I. fig. 4.

†† *L. c.*, p. 12. It is deserving of notice that SCHWANN conjectures the chorda dorsalis to contain no vessels.

shows a "growing from" the sheath, as it is called, of the chorda,—"at the expense of its inclosed nucleus,"—or as the chorda is "found to decline," even in these the last stages of this structure. But if we examine it at periods anterior to these, I think we shall find still stronger grounds for believing, that the chorda dorsalis of authors, in an early state, corresponds to the pin-like object in the mammiferous ovum which I have been comparing with it.

Reichert seems to have been led to form the opinion above-mentioned, that the chorda serves as a "support and stay" for parts developed in two halves, by the following observations; namely, that the chorda becomes visible as a single structure at the same time as the foundations, in two halves, of the central nervous system; and that the central nervous system on the one hand, and his membrana intermedia on the other, are so intimately connected by means of the chorda, that it is not possible to separate them. He even states, that the chorda *passes into* the foundation of the embryo.

Farther, Reichert says, "There are *developed on* the chorda dorsalis the original halves of the central nervous system, with the higher organs of sense, separating as these do from the central nervous system[†]." He states that, with a union of the two halves of the central nervous system—such union taking place first at the fore-end—there is observed a decline in the corresponding part of the chorda. On the subject of the membrana intermedia, the same author remarks, that it is found between the central nervous system and the mucous membrane; and that it (the membrana intermedia) "is the common original foundation of all structures, systems, and organs, which are the means of operation for the two central organs of animal life. Hence from it [the membrana intermedia] there are developed the vertebral system, the dermal system, the circulating system, and finally, all the structures which support the mucous membrane, and which," says he, "I comprehend under the name of system of the intestinal membrane[‡]."

It would thus seem, according to Reichert, that there is little in the embryo which is not developed out of either the central nervous system, or the membrana intermedia. But these are the very parts which the same observer found so intimately connected by means of the chorda, that it was not possible to separate them. And it appears to be these same parts called by Reichert "the foundation of the embryo," *into which he says "the chorda passes."*

Taking then the observations of Baer, Rathke, and Reichert, in connection with my own, I venture to believe, that it is not enough to say, with Baer, that the chorda dorsalis is the axis around which the first parts of the fœtus form; nor, with Rei-

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[†] *L. c.*, pp. 58, 59.

[‡] *L. c.*, p. 107. "Da die ferneren Doppel-gebilde," says Reichert, "sämmtlich von der *membrana intermedia* ausgehen, so tritt die Wirbelsaite [chorda dorsalis] in die innigste Beziehung zu der Letzteren."

CHERT, to denominate it a support and stay for surrounding parts: but that it is the continually renewed central portion of the nucleus of a cell, it being out of this nucleus that the embryo arises:—in other words, that growth at the earliest periods consists, not in *external* additions, but in the continual origin of new substance *in the centre*, by which means previously formed substance is pushed farther out. The nucleus of every cell, also, of which the embryo is composed, seems to be the seat of a like process; that is, a subordinate point for the origin of new substance.

The origin of the embryo from the nucleus of a cell, may assist to solve a question on which, I believe, physiologists are not agreed. "The primitive trace," says VALENTIN, "as well as the***chorda dorsalis, has been made use of by BAER and BURDACH against SERRES, BOURDON, and others, to show that the first rudiments of the parts are not two halves, but a whole, which subsequently splits into two oppositely situated halves. Such positions, however," VALENTIN adds, "are altogether more adapted for metaphysical acumen, and cannot, and never will, be settled by experience and observation; since by this we do not learn the act of arising itself,—it shows no more than an arisen turn in the formation.***Such problems must remain far from the province of the observing part of anatomy and physiology[†]."

I am here compelled to express a different opinion from that of Professor VALENTIN. The subject in question seems to me very properly to belong to the province of observation. But then it is essential that observation should be directed to a period earlier than that with which physiologists have usually begun. By this means it seems possible actually to observe, that if the nucleus of a cell is a single object, the first rudiments of the embryo are not two halves.

Unless the condition just mentioned be fulfilled, namely, an investigation of the earliest periods, it is in vain that we attempt to learn what it is of which the rudiments of the embryo are composed. It appears to have been because of the non-fulfilment of this condition, that physiologists supposed their "*primitive* trace" to arise in the substance of a membrane. And to the same cause seems referable the opinion recently advanced by REICHERT, that the first traces of the new being are derived from cells of the yelk[‡].

[†] Handbuch der Entwickelungsgeschichte des Menschen mit vergleichender Rücksicht der Entwickelung der Säugethiere und Vögel, 1835, p. 156.

[‡] REICHERT very properly denies that the embryo arises in the substance of a membrane; and it is gratifying to me to find an observation previously published by myself confirmed by this talented investigator. Dr. REICHERT, however, does not seem to be aware that he had been anticipated in this discovery, as he will perceive on reference to the "Proceedings" of the Royal Society, April 18, 1839, published at that time, and copied into several periodicals, among which may be mentioned VON FRORIEP's "Notizen," No. 228, July 1839, p. 116.

XVI. *On the Corpuscles of the Blood.—Part II. By* Martin Barry, *M.D., F.R.SS. L. and E.*

Received January 13,—Read January 14, 1841.

SOME time since, I laid before the Royal Society a few facts, which had incidentally fallen under my notice, connected with the red particles or corpuscles of the blood[†]. Those facts were of a character which led me to expect that the farther prosecution of the subject might be rewarded by the discovery of others; although, in a field of physiological research so often traversed from the days of Malpighi and Leeuwenhoek down to the present time, I could hardly expect to succeed in making any addition to the exact description given by some able writers of the present day, regarding the appearance of these corpuscles. But some ideas suggested during the examination of objects figured in the memoir just referred to, induced me to make the blood-corpuscles the subject of direct inquiry with reference to their mode of origin, and certain changes which they undergo.

It will be remarkable if the mammiferous ovum, which, because of its minuteness and the supposed difficulty of obtaining it, had been generally considered beyond the reach of satisfactory observation, should now become the means of studying, not merely other ova, but certain processes by which nourishment is communicated, and the growth of the body effected at all future periods of life. Such, however, I think will really be the case.

For the sake of avoiding a great deal of repetition, I shall at present confine myself very much to the general results; referring, for a particular description of the figures which accompany this memoir, to the minute explanation of them separately given (par. 87.). I may here state that, instead of perplexing the reader with measurements expressed in arithmetical figures, I have thought it preferable in all the drawings to represent the lines of the micrometer on which the objects lay; which lines are more particularly referred to at the foot of Plate XVII. I have taken the utmost care to represent faithfully the appearances presented by the objects examined; and an inspection of them will show that the undertaking occupied no small portion of time. Some of the differences between the drawings now presented and those of previous observers, are, I think, to be attributed to my having used a greater degree of care in the removal, by the usual means, of a portion of the colouring matter from the corpuscle, where required (par. 72.). In other instances, however,

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[†] Philosophical Transactions, 1840, Part II. p. 595.

where none of the colouring matter was removed, I am unable to explain the cause of the differences in our observations.

53. A very superficial examination of the drawings, will suffice to discover the principal differences here referred to. It will be seen that the nucleus of the blood-corpuscle—usually considered, I believe, as a single object—is represented by me as composed, in some instances, of two, three, or even many parts; these having a constant and determinate form. The substance surrounding the nucleus is, I think, even by the latest observers, regarded simply as "the red colouring matter,"—forming the contents of what has been denominated the blood-cell. I find it not uncommon to be able to distinguish in this substance a great number of discs or cell-like objects; and, what is equally remarkable, to discern (in certain states of the corpuscle) an orifice in the delicate membrane by which this substance is surrounded. But it is not so much these facts to which I ask attention, as certain conclusions which they, with others, have enabled me to form.

54. In another memoir, also presented to the Royal Society[†], I was under the necessity of expressing opinions no less differing from those of very distinguished physiologists, on a subject of the first importance; affecting, as it does, every tissue of which organized beings are composed—the physiology of "cells." SCHLEIDEN had stated, that, after giving origin to the membrane of its cell, the nucleus has performed its chief office; and that, when it does not continue at the surface of the cell, the nucleus is "cast off as a useless member, and absorbed[‡]." SCHWANN had said, that, with the exception of fat-globules having in some instances, in fat-cells, been seen to arise out of it, he had never observed anything whatever to be produced by the nucleus of the cell[§]. My observations, on the contrary, showed the nucleus to have a higher office to perform than that of giving origin simply to the membrane of a cell; that, instead of being "cast off as useless, and absorbed," the nucleus is the source of new substance,—a centre for the origin, not only of the transitory contents of its own cell ("Zelleninhalt" of German authors), but also of the two or three principal and last formed cells destined to succeed that cell; and, in fact, that by far the greater portion of the nucleus, instead of existing anterior to the formation of the cell, arises within its cavity.—A separation of the nucleus into two or three parts, where previously observed—namely, in the globule of pus and mucus—had been attributed by GUTERBOCK, HENLE, and others, to a chemical reagent used in the examination—acetic acid[||]. But, we saw that neither acetic acid nor any other foreign

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[†] Researches in Embryology. Third Series: A Contribution to the Physiology of Cells. Philosophical Transactions, 1840, Part II. p. 529.

[‡] SCHLEIDEN, Beiträge zur Phytogenesis. MÜLLER's Archiv, 1838, Heft II. p. 146.

[§] Mikroskopische Untersuchungen über die Uebereinstimmung in der Struktur und dem Wachsthum der Thiere und Pflanzen, 1838–9, p. 54.

[||] HENLE, HUFELAND's Journal, Mai 1838, p. 62, pl. i. fig. 9–12.

substance is required to produce the separation in question; this separation being natural, apparently common to nuclei in general, and forming part of the process by which cells are reproduced. We found that young cells originate through division of the nucleus of the parent cell, instead of arising as a sort of "product of crystallization, in the fluid cytoblastema" of the parent cell, as since supposed by Reichert[†]. The so-called nucleolus had been described as a distinct object, existing before the nucleus. But, we saw the nucleolus to be merely one of a series of appearances arising in succession, the one within the other, at a certain part of the periphery of the nucleus; and continuing to arise even after the formation of the cell.

55. My later investigations, I have now the satisfaction of stating, confirm the opinions just referred to, as well as others which I formerly expressed, but which are not recapitulated here; and the object of the present paper is to show, that they admit of being extended to the corpuscles of the blood[‡].

56. The observations recorded in this memoir are based on an examination of the blood in all classes of vertebrated animals, in the embryo of Mammalia as well as that of Birds, and in some of the Invertebrata.

57. In certain states of the corpuscle of the blood its nucleus is single, like the central portion of the germinal spot in the germinal vesicle of the ovum. Such is the case at α in Plate XVIII. fig. 51. and at η in fig. 54. from Fishes, at α in figs. 49 and 50. from two of the Reptilia, at β in figs. 43 and 45. from the Bird, and in fig. 32. from the Mammal. In all the blood-corpuscles now referred to—which it will be seen are those of the four classes of vertebrated animals, including the two divisions of both Reptiles and Fishes,—the nucleus has a cavity or a depression; at the margin of which there occurs a high refraction of light. The appearance thus produced, seems to represent one of the states of the "nucleolus" of authors, in other cells.

58. The centre of the germinal spot undoubtedly communicates at a certain period with the exterior of the germinal vesicle; for strong presumptive evidence was brought forward, showing the introduction of some substance into the centre of this spot from the exterior of its vesicle. And, besides, I mentioned that in a certain

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[†] Das Entwickelungsleben im Wirbelthier-reich, 1840, p. 2.

[‡] They probably admit of being extended much farther. I cannot, indeed, but recognize in a paper by Professor Valentin, not merely dissent (in some instances at least) from the view above referred to,—that the nucleus is cast off as "*useless*,"—but also facts which go to corroborate some of my own observations; however little the conclusions at which I arrived, resemble those of the author now mentioned. See Müller's Archiv, 1840, Heft II. pp. 229, 230, and 235. The recent work of Reichert also, before referred to, contains observations respecting which I would make the same remark. This author, however, adopts the mistaken view, that the nucleus of the cell is absorbed. For, considering the germinal vesicle as a nucleus, he *therefore* supposes it to disappear (*l. c.*, pp. 102, 103). The "finely granular" and "globular precipitate" of Reichert, appearing around the nucleus, evidently arose from decomposition of the free portion of the nucleus (See my Third Series, *l. c.*, par. 391): to which cause seems referable the reduction he observed in the size of this object (*l. c.*, p. 91).

condition of the ovum, I had not been able to discern a continuation of the membrane of the germinal vesicle over the central portion of the spot. So likewise the corpuscle of the blood in certain states exhibits an orifice, by means of which there is a communication between the exterior of the corpuscle and the cavity in its nucleus.

59. The free portion of the germinal spot resolves itself into discoid objects—the foundations of cells; with which the germinal vesicle becomes filled. In like manner, the blood-corpuscle is seen filled with corpuscles of minuter size; presenting an appearance which I can attribute to no other than the same process. This is shown in the blood-corpuscle of the Fish at γ, fig. 52, at α and β, fig. 53, and at ε and ζ, fig. 54—in that of the Reptile, ε, figs. 49 and 50—in that of the Bird, fig. 36 β, fig. 41 β, fig. 43 β, γ, fig. 45, and fig. 47.

60. The foundation of the new being in the mammiferous ovum, arising in the centre of the altered germinal spot—which centre, from this and other facts, I denominated the point of fecundation—we saw to consist of two discs; each of these discs giving origin to a cell. So also, in more than half the numerous figures accompanying this memoir, we find the same appearances in blood-corpuscles, including those of Man. (In certain states, the nucleus of the blood-corpuscle is composed of three or more discs, instead of two. Figs. 33, 34 γ, 41 β, 43 δ, 45 θ, ι, 52 γ, δ, 53 α, 54 ζ. Compare with the nucleus of a yelk-globule from an immature ovum of the Tiger in my Second Series on the Embryo†).

61. The two cells in question, in the ovum, at a certain period are set free; each being destined to give origin to others. Such also is the case with the corpuscles of the blood. See figs. 29, 30, 39, 41 γ.

62. The foregoing analogies are not confined to vertebrated animals; for I have met with some of them, at least, in three classes of the Invertebrata. See figs. 57, 61 and 62.

63. Such facts as these appear to warrant the conclusion, that the corpuscles of the blood are generated by a process essentially the same as that described in one of my former memoirs, as giving origin to those cells which are the immediate successors of the germinal vesicle, or original parent cell.

64. If the blood-corpuscles of Man in fig. 23 be minutely examined in the order in which they are alphabetically lettered, and in connection with the description separately given (par. 87),—and if they be then compared with a succession of appearances delineated in my former paper on the Corpuscles of the Blood‡,—I think it will be obvious that those appearances arose from the blood-corpuscle resolving itself into minuter objects, by a continuation of the same process as that through which, as we have just seen, it is itself produced.

65. Eighty years have elapsed since the ingenious Father Di Torre, after presenting to the Royal Society some rude but powerful lenses of his own preparing, afforded

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† Philosophical Transactions Part II. 1839, Plate V. fig. 87.

‡ *L. c.*, p. 595. Plate XXIX. fig. 2.

Sir F. H. E. Stiles opportunities for viewing human blood by means of lenses of the same kind; when the latter communicated to the Society his observations, or rather the observations of Di Torre, as confirmed by himself[†]. So little importance is attached, in the present day, to the observations of Di Torre, that in historical accounts of the blood-corpuscles, I believe it is usual to pass them by, as not deserving of notice: probably from an idea that his optical instruments were too imperfect. But I shall not hesitate to here transcribe a passage from those observations of Di Torre, although far from vouching for their entire accuracy.

66. "When," says the writer, "any of the globules [blood-corpuscles] happened to move with the serum in the most perfect focus,*** I could with great clearness distinguish the exterior and interior circumference of the ring, of which each globule [corpuscle] consisted; the interior one being bounded by a black line or shade next the perforation, exactly resembling that which bounded the exterior one,***. In such globules [corpuscles] I could easily observe the ring to be articulated, the transverse lines at the joints being very distinguishable. The figures of the articulations were various; in some they were roundish, so that the ring appeared like a bead necklace; in others, cylindrical, and of some length. The number of which the whole was composed, seemed uncertain, varying from two or three to six or seven; many of the rings were broke, either by some confinement of the talks [talc], or by beating against each other, which I saw them continually do; and by these accidents the joints of the rings were detached, and wandered about separately in great numbers; and indeed they appeared separable with as much ease as if they had been united by mere contact only. Some of the rings were broke into semicircles, others into greater or less portions, and others again divided into their constituent articulations, which in some places floated about single, and in others formed by their mutual attraction a lateral union, like the pipes of an organ. I must observe also, that these separated parts seemed to be hollow and transparent, and like inflated bladders, would easily yield, and change their figure, stretching or contracting themselves from round to oval and cylindrical, and *vice versâ*, as any lateral pressure in crouding [crowding] them along with the serum brought a constraint upon them. *** Although the articulation was not distinguishable in every globule, I think it was so in the greater part of them; and it is natural to imagine that the rest were articulated likewise, though they might not pass at the proper distance for its being distinguished[‡]."

67. Such were the observations placed on record by the Royal Society seventy-six years ago, which I had not read until fig. 23. of the present paper was nearly finished. As already stated, I do not believe they were accurate in all respects. But if the figure just mentioned be again referred to, it will be found that observations made by myself in 1840 and 1841, with an excellent instrument, while they very widely differ from those of late observers, confirm to a degree that is astonishing, some of the

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[†] Philosophical Transactions, 1765, p. 252.

[‡] *L. c.*, pp. 254, 255.

observations which had been made by Di Torre, with his rude and smoked lenses, in 1761. No one can feel more conscious than I do, how much we owe to Hewson for his researches on the corpuscles of the blood: but I think he erred in dismissing as valueless the remarks of the Italian observer.

68. I would add, once more referring to fig. 23, that the arrangement of the contents of the blood-corpuscle in the human subject, will be seen to coincide, in some instances, with its well-known usually biconcave form (pars. 73 to 76).

69. The form and internal state of the blood-corpuscle found in the adult of certain animals, very much resembles that existing only in the fœtal life of others. This will be obvious on an inspection of the drawings. That such resemblance is referable to uniformity in the mode of evolution, results from our whole experience on the reproduction of cells, as I explain it, and from facts above mentioned, showing that blood-corpuscles are generated by a like process; the difference between the condition of these objects in the embryo and in the adult of the same animal, being apparently referable to a difference in the degree of their development as cells.

70. An incidental observation may perhaps be mentioned here. The brain in the embryo of the Ox, at certain periods, appears to consist almost entirely of objects such as those in figs. 59 and 60; which, it will be seen, are composed of discs. Before the addition of acetic acid, such objects in many instances are surrounded by a halo, which seems to represent the membrane of a minute cell. If the objects now mentioned be compared with some of the nuclei of blood-corpuscles which I have figured, it will be found that there is a remarkable similarity between them.

71. On a review of the facts stated in the course of the present communication, the opinion I have been led to form, is that the mode of evolution of the minute mammiferous ovum is deserving of close attention in connection with some of the processes by which nourishment is communicated, and the growth of the body effected at all future periods of life.

Appendix.

The following additional remarks were kept out of the memoir itself, for the purpose of rendering it as simple and brief as possible.

72. The chemical reagent principally used in the investigations forming the subject of this paper, has been dilute acetic acid of the strength of distilled vinegar. I have found it an advantage to add this acid in a quantity so very small, as to be enabled to witness its gradual effect upon the contents of the blood-corpuscle. It has been by this means, in part, that I have discerned the interior of this object to be, to my astonishment, so different from that represented by previous observers.

73. Professor **Müller** remarks, "It is quite impossible to imagine the cause of the different forms of the red particles in the different classes of vertebrated animals. There are no similar elementary forms in the whole body[†]."

74. It will be obvious that I cannot agree with the eminent physiologist now mentioned, in the latter part of the quotation; the main object of this memoir having been to show that the blood-corpuscles are generated by a process of the same kind as that which I have elsewhere described. But I wish to ask attention in this place to the former part of the passage just quoted from Professor **Müller**,—as to the possibility of imagining the cause of the different forms of the red particles in the different classes of vertebrated animals; for, perhaps some of the accompanying delineations may assist us even here.

75. Dr. **Young** denied the existence of a nucleus in the corpuscle of human blood: **Hewson** believed it to be globular: **Müller** remarks, "And as this nucleus [of the blood-corpuscle] has under the microscope exactly the same appearance in the red particles of Birds and Fishes as in those of Amphibia, it would be expected to exist in those of Mammalia also. And although, on account of the minuteness of these bodies in Mammalia, it is more difficult to demonstrate the nucleus in them, I have with an excellent (**Fraunhofer**) microscope really seen it, and distinctly. Even in the red particles of human blood, I have seen a minute, round, accurately defined nucleus, which had a more yellowish and shining aspect than the transparent part around it. The existence of the nucleus can also be demonstrated by the action of acetic acid, though much less distinctly than in the case of frog's blood[‡]."

76. To me the appearances presented by the nucleus of the corpuscle in human blood, after the addition of acetic acid, were such as those represented in fig. 23: some of which, I think, are really sufficient to explain its usually biconcave form, and the difference in this respect between the blood-corpuscle of the Mammal and that of other Vertebrata.

77. I am very much inclined to believe, that in the many instances in which authors on "cells" have described and figured more than one nucleolus in a nucleus, there has been either an incipient division of the nucleus into discs, or the nucleus has consisted of two or more discs: the nucleoli of those authors having been the minute and highly refracting cavities or depressions in the discs[§]. If this has really been the case, it affords additional evidence, I think, that reproduction of cells by the process I have described—namely, division of the nucleus of a parent cell—is universal; so numerous have been the instances in question. I may refer to the figures given

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[†] Elements of Physiology, translated by Dr. **William Baly**, p. 146.

[‡] *L. c.*, pp. 100, 101.

[§] The observer is very apt not to perceive that a nucleus is composed of more than one disc, because of the refraction of light being very small at the part where the discs lie one against the other.

by SCHWANN[†], who examined nearly every animal tissue, and to those of SCHLEIDEN[‡], whose observations have been so extended on the cells of plants. I think, indeed, that many of the figures of SCHWANN afford evidence of the division in question having taken place. It is to be recognized in his delineations of the cells of cartilage[§], cellular tissue[||], the middle coat of the aorta[¶], muscle[††], tendon[‡‡], the feather[§§], &c. The same remark is applicable to a figure given by REICHERT of ciliated epithelium-cells[||||]. Dr. HENLE found that in the layers of his "Pflasterepithelium" cells, the nucleus, very distinct in the lower cells, had almost disappeared in those situated at the upper part. From this observation, and from the presence of two nucleoli in some of the nuclei figured by this observer[¶¶], as well as from the nucleus becoming "more granular[†††]," I think it extremely probable that these cells also (including those of the epidermis) are reproduced by the process just referred to—division of the nucleus; additions being no doubt continually made at the lower part of the layer, by which cells previously there are pushed further out. Dr. HENLE mentions that in the lowest of the cells of "Pflasterepithelium," the nucleus is "reddish yellow, and has a remote resemblance to blood-corpuscles[‡‡‡]." This is very interesting in connection with an observation recorded in my former paper on the Corpuscles of the Blood. I stated that epithelium-cells had often presented appearances which almost suggested the idea that these too were changed corpuscles of the blood: and that it was not easy to draw a line, by which the two could be distinguished, either in colour, form, or general appearance[§§§]. The cells here referred to were probably the "cylinder," or rather the subsequently "flimmerepithelium" cells of HENLE[|||||].

78. I showed on a former occasion that the chorion arises by a coalescence of minute cells[¶¶¶]; and we have since seen the thickening of this membrane to be effected by the same means[††††]. The cell which (in the first of my communications to the Society) I called the ovisac, appears to arise in the same manner, as well as the membrane *e* of my papers on the Embryo, for it really does appear, as I formerly suggested[‡‡‡‡], that the substance surrounding the germinal vesicle in the mammiferous ovum, usually called the yelk, may be termed a great "cytoblast."

[†] *L. c.*

[‡] *L. c.*

[§] *L. c.*, Tab. I. figs. 8, 9. Tab. III. fig. 1.

[||] Ibid. Tab. III. figs. 6. 9.

[¶] Ibid. Tab. III. fig. 12.

[††] Ibid. Tab. III. fig. 13.

[‡‡] Ibid. Tab. III. fig. 11.

[§§] Ibid. Tab. II. fig. 11 *a*.

[||||] *L. c.*, Tab. I. fig. 4.

[¶¶] *L. c.*, Tab. I.

[†††] Ibid. p. 7.

[‡‡‡] *L. c.*, p. 6.

[§§§] *L. c.*, par. 49. Plate XXIX. figs. 11, 12.

[|||||] See my Researches in Embryology, Third Series, *l. c.*, Plate XXVIII. fig. 251, which presents apparently more advanced stages of the objects delineated in the figures referred to in the preceding note.

[¶¶¶] Derived from corpuscles of the blood. Researches in Embryology, Third Series, *l. c.*, par. 369–373. On the Corpuscles of the Blood, *l. c.*, par. 20–26.

[††††] Supplementary Note to a paper entitled "Researches in Embryology. Third Series: A Contribution to the Physiology of Cells," in the present volume, p. 193.

[‡‡‡‡] Third Series, *l. c.*, par. 350. *Note.*

79. Now this mode of origin of cells of comparatively large size, may perhaps assist us in the consideration—how are the minuter ones produced? I formerly stated[†] that it was not easy to point out where the disc terminates, and the cell begins; and that I did not recollect to have observed any of the discoid objects in the transition state in question, in which there was not an appearance in the most superficial part denoting decomposition. I added, that so uniformly had this been met with that I was ready to suppose the formation of the cell-membrane to be connected with such decomposition, or perhaps dependent on it. From later observations, I am more and more disposed to think that this really is the case; and that exceedingly minute discs, into which the outer portion of a larger disc has been resolved, coalesce to form the membrane of the cell. Thus in fig. 54. the outer portion of discs such as those at α, seemed at β to have resolved itself into minuter discs, which at γ were more advanced, and at δ had coalesced to form the membrane of the corpuscle or blood-cell.

80. It will be seen from what I have in this memoir, and elsewhere, stated, that the disc ("nucleus" of authors) is the most primitive object we are acquainted with. It will also be perceived that the cavity or depression ("nucleolus") in the disc, is the situation of the future orifice, communicating with the exterior of the cell. The nucleus of the cell seems to be reproduced by the fissiparous mode.

81. The objects into which the nucleus of the mammiferous blood-corpuscle separates (fig. 23.), are no doubt the source from whence proceeds the substance for the origin and thickening of the chorion, and the formation of the muscular fibril[‡].

82. The process by which the nucleus of the cell finally divides into several parts, seems to consist in the originally single and pellucid cavity or depression in the nucleus, gradually becoming finely granular (see the explanation of fig. 43.); and by degrees separating into several cavities. An idea of this change may perhaps be obtained from the condition of the nucleus in the blood-corpuscle ζ, fig. 45.

83. The nuclei which several observers have found lying among the fibres of various tissues, have been considered by them as the "remains of cells." This may have been the case; but so far from thinking, with those observers, that the nuclei in question were "destined to be absorbed," I am disposed to consider that they were the sources from which there would have arisen new cells. See, for instance, delineations by SCHWANN of nuclei lying among fibres from a feather (*l. c.*, Tab. II. fig. 13.). (And I may here take the opportunity of remarking, that I apprehend the fibres in

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[†] Third Series. Additional observations, par. 439. *Note.*

[‡] See my Third Series on the Embryo, *l. c.*, and also the figure accompanying a Supplementary Note to that Third Series, in the present volume, p. 193; as well as my first paper on the Corpuscles of the Blood, *l. c.*

the more advanced of the objects in this figure, to have arisen from the coalescence of discs such as those represented in the object presenting an earlier state: a view which SCHWANN does not appear to have taken.)

84. I have observed, here and there, in blood taken from the liver in the fœtus of the Ox, large cells such as those represented in outline in fig. 31. (See the explanation of the Plates.) The nuclei of these cells presented a very remarkable appearance. They were composed of transparent discs, having, as viewed in the microscope, that peculiar yellow colour, which seems to characterize the corpuscle of the blood; and which when these corpuscles are accumulated, presents to the naked eye the well-known red. These discs, perhaps in some instances about twelve in number in each nucleus, were elliptical, and exhibited a cavity or depression. They were distinctly unconnected, though in contact, with one another; and seemed on the point of being separated. The figure represents one cell from which they were escaping.

85. Future observation must determine what these discs really are, I venture to believe it very possible that they represent a state of the corpuscles of the blood.

86. On a former occasion[†], I showed that the blood-corpuscles in the embryo are not formed, as supposed by some observers, out of granules of the yelk. The facts recorded in the foregoing memoir leave little doubt, I think, that these corpuscles,—not only in the embryo, but at all periods of life,—are descendants of the two cells constituting the foundation of the new being in the ovum. If so, it is not requisite to seek the origin of these corpuscles in the organized parenchymatous substance of the body, or in the globules of the chyle; "the only two sources," it has been said, in which it was possible for them to arise. Authors on "cells," regarding the liquor sanguinis as the "cytoblastema," appear inclined to consider the corpuscles of the blood as arising in it, independently of previously existing corpuscles.

87. EXPLANATION OF THE PLATES.

PLATE XVII.

Fig. 23. Man. Blood-corpuscles, after the addition of acetic acid. α. The nucleus consists of two adherent discs. β. The two discs have separated, and increased in size. γ. They exhibit symptoms of division. δ. The discs are four in number; but two of them remain attached. ε, ε. There exist three separate discs, with indications of a division of some of these discs. ζ. The number of discs visible is five. Three of the objects are in this state. η. The nucleus is indistinctly seen, from the surrounding discs and red colouring matter having been imperfectly dissolved.

[†] On the Corpuscles of the Blood, *l. c.*, par. 11.

Fig. 24. Ox (*Bos Taurus*, Linn.); embryo of ¾ths of an inch in length. Corpuscles of blood taken from the back part of the head. *α*. The nucleus consists in two instances of two, in another instance of three discs, which are in close approximation. *β*. The discs have separated, much increased in size, and assumed a cell-like appearance. *γ*. The corpuscle exhibits an orifice.

Fig. 25. Blood-corpuscles from the same embryo, after remaining twenty-four hours between two plates of glass. They had begun to collapse; a change which seems to commence by a falling in of that part of the membrane where the nucleus lies, and where an orifice in the membrane is in some states to be discerned.

Fig. 26. Blood-corpuscle from the liver of the same embryo. The nucleus seemed to consist of two portions.

Fig. 27. Blood-corpuscles from the liver of the same embryo, after the addition of acetic acid. The latter has made them globular. In all, the nucleus consists of two discs. In the lower one, large discs or incipient cells are represented surrounding the nucleus.

Fig. 28. Ox (*Bos Taurus*, Linn.); embryo of about one inch in length. Outline of corpuscles in blood taken from the back part of the head. These were not very flat, especially the smaller ones, which were much more numerous than the large. *α*, *α*. The corpuscle presents an orifice. *β*. A nucleus is visible, consisting of two closely adherent discs.

Fig. 29. Blood-corpuscles, chiefly in outline, from the same part of the same embryo, after the addition of acetic acid; which has rendered most of them spherical in form. After some time, they became shrivelled in appearance. *α*. Ruptured corpuscle. *β*. Corpuscle from which a globular object (presenting on one side the membrane of a minute cell?) is escaping. *γ*. Object nearly resembling a mature blood-corpuscle, surrounded at a little distance by a membrane. *δ*. A similar object on its edge, but without a surrounding membrane. It presents an orifice on one of its broad surfaces.

Fig. 30. Ox (*Bos Taurus*, Linn.); embryo of 1¼ inch in length. Outline of blood-corpuscles from the liver, after the addition of acetic acid. *α*. Corpuscle containing two globules, composed of discs. *β*. Corpuscle discharging a globule of the same kind; the membrane of a minute cell rising from this globule. *γ*. A similar compound globule, probably recently discharged from a corpuscle, and now eccentric in a minuter cell.

Fig. 31. Ox (*Bos Taurus*, Linn.); embryo of 1¾ inch in length. Outline of corpuscles in blood from the back part of the head, after the addition of a very minute quantity of acetic acid. In the larger ones, the nucleus

became well circumscribed through this addition. These corpuscles exhibited a pellucid orifice.

Fig. 32. Outline of a corpuscle from the same embryo; the nucleus consisting of a single disc.

Fig. 33. Sheep (*Ovis aries*, LINN.); embryo of 2½ inches in length. Blood-corpuscles after the addition of acetic acid.

Fig. 34. Blood-corpuscles from the liver of the Ox-embryo from which figs. 31 and 32 were taken. Acetic acid had been added. α. Corpuscles resembling those in fig. 27. β, β. Escaped nuclei; each consisting of two discs. γ, γ. Corpuscles resembling those in fig. 33; but their compound nuclei separating into discs. δ. Corpuscle exhibiting an orifice in its membrane; the sides of the orifice presenting a finely granular substance. ε. Corpuscle containing three compound globules similar to those in fig. 30; but much more minute. Several of the corpuscles in this figure (fig. 34.) were seen to be filled with enlarged discs or incipient cells. A few of these have been represented in outline.

Fig. 35. Outline of large cells found with those in the preceding figure. Their membranes appeared shrivelled. These cells were filled with altered discs, and more or less incipient cells. Each of their nuclei consisted of many discs, of a deep yellow (reddish) colour. (Such cells were found in blood from another embryo of the Ox, measuring in length 1¼ inch.)

PLATE XVIII.

Fig. 36. Sparrow (*Fringilla domestica*, LINN.). Blood-corpuscles, chiefly in outline. It will be observed that two of these are round. Like the rest, however, these two were flattened. Round ones were seen only here and there. α. A group of young corpuscles of the blood. Their colour was the same as that of the other corpuscles in this figure; but they were less flattened. The pellucid space represented in some of these objects (α), indicates the situation of the future nucleus (see the explanation of fig. 43.). β. A blood-corpuscle in which there were observed discs or young cells around the nucleus. The nucleus has not been figured.

Fig. 37. Sparrow (*Fringilla domestica*, LINN.). Blood-corpuscles with their nuclei —both in outline—as seen after the addition of a *minute* quantity of acetic acid. The corpuscles on the left hand, in the figure, had undergone only a partial change in their form from the acetic acid; while those on the right (lying in another part of the field of view) had become globular from this acid. α, α, α. The nucleus has changed its position;

having become in two instances oblique, in another instance situated on one side. β. It is dividing into two parts. γ. This division is complete. δ. From the direction of the nucleus, only one of its extremities is seen. The corpuscle in this instance had lost its flattened form, but not yet become globular.

Fig. 38. Sparrow (*Fringilla domestica*, Linn.). Nuclei of blood-corpuscles, after the removal of the surrounding substance by acetic acid.

Fig. 39. Sparrow (*Fringilla domestica*. Linn.). Two blood-corpuscles filled with discs, or young corpuscles; and two young corpuscles in nearly the same state, but no longer contained within a parent corpuscle (cell). Acetic acid had been added in *minute* quantity.

Fig. 40. Common Fowl (*Phasianus Gallus*, Linn.) in an egg incubated eighty hours. Outline of blood-corpuscles. An orifice is visible in some, and not in others.

Fig. 41. Blood-corpuscles from the same egg, after the addition of acetic acid. They are represented partly in outline. α. The nucleus consists of two discs. β. It is composed of several. γ. Globular corpuscle filled with young corpuscles.

Fig. 42. Common Fowl (*Phasianus Gallus*, Linn.), in an egg incubated eighty-five hours. Outline of blood-corpuscles. (Discs were indistinctly visible in the interior, even before the addition of any acetic acid.)

Fig. 43. From the same egg. Outline of blood-corpuscles, after the addition of acetic acid. α. The corpuscle has an elongated orifice. β. The corpuscle is filled with minute cells. Its finely granular nucleus has a pellucid cavity, communicating with the exterior of the corpuscle (compare this object with the germinal vesicle, Phil. Trans. 1840, Part II. Plate XXII. fig. 159.). This orifice is originally larger. It becomes reduced in size with the appearance of the finely granular substance: the latter preceding the formation of the discs, into which the nucleus is resolved. γ. The nucleus consists of two discs. δ. The nucleus is composed of several discs.

Fig. 44. Common Fowl (*Phasianus Gallus*, Linn.), in an egg incubated ninety-two hours. Outline of corpuscles of the blood.

Fig. 45. From the same egg. Blood-corpuscles, chiefly in outline, after the addition of acetic acid. These were globular, or nearly so, excepting two, which were elliptical. The latter form was not frequent after acetic acid had been added. All the corpuscles in this figure were seen to be filled with discs or incipient cells. α. Corpuscle with a large orifice. β. The nucleus finely granular, with a minute cavity (see the description of fig. 43.). γ. The nucleus as in β; concentric layers of discs or incipient cells around it. δ. The orifice elongated, with an accumula-

tion of finely granular substance beneath it, having a minute central and pellucid cavity (see the description of fig. 43.). ε. The nucleus consists of two discs. ζ. One of the two discs of which the nucleus is composed seems preparing to divide into minuter discs. η. The nucleus consists of three discs. θ. The nucleus is composed of several discs, surrounding a pellucid cavity. ι. The nucleus consists of many discs. κ. Globular corpuscle filled with discs, apparently young corpuscles.

Fig. 46. Common Fowl (*Phasianus Gallus,* Linn.), in an egg incubated 166 hours. Outline of corpuscles of the blood.

Fig. 47. From the same egg. Two blood-corpuscles after the addition of acetic acid. The drawings of the discs, or incipient cells, around the nucleus in these are more finished than those of the corresponding parts in many of the other figures. α. The nucleus consists of two discs. β. It is composed of three objects, each of which seems to be dividing into two discs.

Fig. 48. Turtle. Blood-corpuscles after the addition of acetic acid; some of them in outline. α. The nucleus dividing into two discs. β, β. The nucleus consists of two closely adherent discs. γ. The nucleus is composed of two discs; the one lying across the other.

Fig. 49. Turtle. Blood-corpuscles after the addition of acetic acid; two of them in outline. α. The nucleus is a single disc. In one of these corpuscles there are represented discs or incipient cells. β. Corpuscle beginning to collapse.

Fig. 50. Frog (*Rana temporaria,* Linn.). Blood-corpuscles after the addition of acetic acid; one of them in outline only. α. The nucleus is a single disc. β. The nucleus is dividing into two discs. γ. The nucleus consists of two distinct discs. δ. The nucleus is dividing into three discs. ε. The corpuscle contains many incipient cells. (The nucleus has not been represented in ε.)

Fig. 51. Thornback (*Raia clavata,* Linn.). Blood-corpuscles, after the addition of acetic acid; one of them in outline. Some remained elliptical. α. The nucleus is a single disc. β. The nucleus is dividing into two discs. γ. The nucleus consists of two distinct discs. δ. Minute corpuscle filled with minuter cells; its nucleus consisting of two closely adherent discs. ε. Corpuscle, the nucleus of which consists of two compound globules. Compare with fig. 30 α, and fig. 34 ε.

Fig. 52. Skate (*Raia batis,* Linn.). Blood-corpuscles, after the addition of acetic acid. The larger ones are in outline. (The greater number of the corpuscles seen were of the second and third size.) The corpuscles did not become globular on the addition of acetic acid. α. The nucleus is a single disc, but it is beginning to divide into two. At β this division

has proceeded further. At γ, γ the nucleus is dividing into three parts. In one of the corpuscles γ, are seen, in outline, minute cells. δ, δ. The nucleus consists, in one corpuscle of two, and in another of three, distinct discs.

Fig. 53. Cod (*Gadus Morrhua*, Linn.). Blood-corpuscles after the addition of a *minute* quantity of acetic acid. They are represented for the most part in outline. The sizes and forms most frequent were such as those of the larger corpuscles on the left side in the figure. α. Discs or cell-like objects are contained within the corpuscle, around the nucleus. This was seen to be the case in corpuscles generally throughout the field of view. The nucleus consisted in general of two or more discs. β. Globular corpuscle filled with discs, apparently young blood-corpuscles.

Fig. 54. Cod (*Gadus Morrhua*, Linn.). Blood-corpuscles after the addition of a *minute* quantity of acetic acid. Some of them are in outline only. The objects α, β, γ, δ appeared to be young blood-corpuscles; of which α represents the least advanced, and δ the most forward state (par. 79.). Discs or young cells were seen around the nucleus in all the larger corpuscles of this figure; and indeed in the majority of such of the corpuscles from this individual, as were minutely examined. They were visible in a great number of instances without any addition having been made. The discs in the corpuscle ζ are larger than those in ε; while the nucleus is smaller. At η, the crenate circle represents the outline of a layer of discs or young cells, which surrounded the nucleus. The space external to this layer, seemed to be occupied by red colouring matter.

PLATE XIX.

Fig. 55. Frog (*Rana temporaria*, Linn.). Nuclei of blood-corpuscles, and other objects, observed in the blood, after the addition of acetic acid. α Resembles the compound globules in figs. 30, 33, ε of fig. 34, and ε of fig. 51.

Fig. 56. Nuclei of blood-corpuscles (and other objects?), observed in blood of the same animal, several days dead. These nuclei were no longer contained in cells.

Fig. 57. Oyster (*Ostrea edulis*, Linn.). Objects found in the blood. α. Cell, the nucleus of which consists of two closely adherent discs. Compare this cell with many of the blood-corpuscles from vertebrated animals in Plates XVII. and XVIII. β, β. Globules composed of three or more discs. γ. Globules circumscribed by a membrane-like lamina, and composed of discs resembling those in figs. 30, 33, and ε of fig. 34. from

the Mammal, and ε of fig. 51. from the Fish. δ. Globules composed of discs surrounding a pellucid cavity. Compare with the nucleus of θ in fig. 45. from the Bird. ε, ε, ε. Cup-like objects composed of discs; one of them in outline. ζ. One of the same objects apparently in a more incipient state. Compare the objects ε and ζ with those in fig. 58. from the Turtle.

Fig. 58. Turtle. Cup-like objects observed in the blood, after the addition of acetic acid. Five of them are in outline. Compare these objects with ε of fig. 57. from the Oyster.

Fig. 59. Ox (*Bos Taurus*, Linn.). Embryo of about one inch in length. Globules and other objects from the brain. They are composed of discs. Compare these objects with some of the nuclei of blood-corpuscles in Plates I. and II. Acetic acid had been added.

Fig. 60. Objects from the Brain of another embryo of the same animal, measuring 1¾ inch in length, after the addition of acetic acid. Previously such globules in many instances seemed to be surrounded by the membrane of a minute cell. Compare as in the description of fig. 59.

Fig. 61. Lobster (*Cancer marinus*, Linn.). Corpuscles (partly in outline) observed in the blood, after the addition,—α. of alcohol,—β. of acetic acid.

Fig. 62. Leech (*Hirudo medicinalis*, Linn.). Corpuscles of the blood. Some were observed larger than any of those represented in the figure.

XVII. *On the Corpuscles of the Blood.—Part III.* *By* Martin Barry, *M.D., F.R.SS. L.* and *E.*

Received June 17,—Read June 17, 1841.

NOTWITHSTANDING the great length of time during which the blood has been the subject of physiological research, an eminent anatomist, so late as the year 1838, remarks, that "we have no clear conception of the mode in which the floating corpuscles of the blood conduce to nourishment[†]." That Professor Weber was not mistaken in coming to such a conclusion, I think will be admitted by every one who takes the pains to consult the records of discovery in this most interesting field of observation.

I am not aware that, since the period just mentioned, any additional facts have been published, relating to "the mode in which the floating corpuscles of the blood conduce to nourishment," unless my own communications, already presented to the Society[‡], are to be so regarded,—those communications having reference to the mode of propagation of the floating blood-corpuscle, and to its conversion into two or three kinds of tissues.

The object of the present memoir is to bring together a large number of observations, made by myself, showing that every structure I have examined arises out of corpuscles having the same appearance as corpuscles of the blood. I may here mention, that the tissues submitted to actual observation, with the result just mentioned, will be found to include the cellular, nervous, and muscular; besides cartilage, the coats of blood-vessels, several membranes, the tables, cells, and cylinders of the epithelium, the pigmentum nigrum, the ciliary processes, the crystalline lens itself, and even the spermatozoon and the ovum. And among the vast number of observations made, I have not been able, with the greatest care, to detect a single fact inconsistent with the conclusion above announced. If that conclusion—which regards the *formation* of the tissues—be correct, it may, I think, assist us in considering "the mode in which the floating corpuscles of the blood conduce to nourishment" *during life*.

For the detail of these observations, I shall rely principally upon the drawings, and the minute explanation of them separately given. The perusal of that explanation will, I conceive, be necessary for the understanding of some general remarks I shall

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[†] E. H. Weber, in Müller's Archiv, 1838, p. 463.

[‡] On the Corpuscles of the Blood. Philosophical Transactions, 1840, Part II. p. 595: and Part II. on the same subject, in the present volume, p. 201.

have to offer: and I can scarcely expect the reader to admit the conclusions drawn, until he is in possession of the unequivocal evidence, to be derived in no other way than by a close examination of the Plates, in connection with the explanation just referred to.

I may here mention, that it is not my object in this memoir, to trace the tissues investigated into a perfectly formed state; but simply to present such of their earliest stages, as show them to be derived from objects having the same appearance as corpuscles of the blood. In so doing, I shall have to mention a variety of facts, which, though met with incidentally, and recorded without remark, may not be considered destitute of physiological interest.

88. It is important that any one disposed to repeat the following observations, should, before entering upon them, carefully notice the colour,—the transparent yellow colour,—of the corpuscles of the blood, viewed singly, with a high magnifying power (as, for example, the corpuscles in blood obtained by a puncture of the finger); so that when the same colour is met with elsewhere, he may recognize it. The yellow of the magnified corpuscle as thus singly viewed, is obviously that which gives to the mass of blood—seen with the naked eye, and by reflected light—its well-known red. The reader will bear this in mind, when colour is spoken of in the following memoir.

89. Some of the observations I am about to communicate, will be found at variance with those of other investigators in general anatomy, not excepting even the most recent. I have not room, within the limits of a paper, to introduce the opinions of the authors referred to.

90. On former occasions, I have mentioned a certain minute structure, under the denomination *disc*. As the same term will be constantly employed in this memoir, it is better, once for all, to define it, as a flat, elliptical or circular body; usually having a concavity in the middle of the flat surface (fig. 141 δ, δ, ε.). Frequently, however, these minute bodies lose their elliptical or circular form, and assume, as if by pressure, a polyhedral shape; and, in certain states, the concavity becomes an orifice. There are also conditions in which a minute projection is presented at this part. Numerous examples of discs are to be found in almost every figure which accompanies the paper.

91. The *disc* seems in many instances to correspond to the "cytoblast" of Schleiden; though his description of the "cytoblast" differs in some material points from the definition I have given of the disc[†]: and our experience of the destination of these objects is no less different. In the proper place it will be shown that the disc has been described by some as a peculiar object, found in two or three kinds of globules.

92. By *division into discs*, an expression frequently made use of in this paper, I do not mean simple separation. For, from the analogy which seems to exist between the mode of propagation of the blood-corpuscle, and that of the cells which are the immediate successors of the germinal vesicle in the ovum, there cannot be a doubt

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[†] See Third Series on Embryology, Philosophical Transactions, 1840, Part II. pars. 385, 425.

but that, in all instances, and whatever the minuteness of the object, the division in question takes place by a similar process,—a process which we found to be elaborate in the extreme.

93. The expression, *having the same appearance as the corpuscles of the blood*, is constantly made use of in the following pages; though most of the corpuscles to which it is applied would have been at once denominated corpuscles of the blood, but for a reason given in the concluding portion of the memoir (par. 196.).

94. Besides the substances above mentioned, I have examined the pus and mucus globules, which I shall first describe.

The Globules of Pus derived from Corpuscles of the Blood:—Mucus-globules compared with them.

95. "The nucleus of the mucus-corpuscle," says SCHWANN, "has the peculiarity, discovered by GÜTERBOCK, of becoming separated into two or three corpuscles of minuter size by acetic acid; while the surrounding part is gradually dissolved by this reagent. VOGEL supposes this property to belong only to pus-corpuscles, and to the corpuscles of unhealthy mucus. HENLE, however, informs me," continues SCHWANN, "that the same peculiarity *** is found in the true mucus-corpuscles, present in healthy mucus[†]." Respecting pus-corpuscles, SCHWANN remarks, "They share with them [corpuscles of mucus] the peculiar relation towards acetic acid[‡]."

96. My own observations on this subject are the following. In Plate XX. fig. 64 β. are pus-globules, to which no addition whatever had been made; and fig. 63 ζ presents one of these globules, as viewed after the addition of dilute spirit. Now as the objects in both figures exhibit the division of the "nucleus," just spoken of, without the addition of acetic acid, I am compelled to form an opinion opposed to that of the authors just referred to, and to maintain that in the pus-globule acetic acid is not required to produce such division.

97. But facts recorded in several of my former memoirs appear to have been sufficient for showing that neither acetic acid nor any other foreign substance is required to produce division of what has been called the nucleus of the pus-globule; such division being part of the process by which cells are reproduced, and apparently universal in its operation. The present memoir, also, will be found full of facts, showing that this really is the case.

98. As to the mode of origin of the objects in question, SCHWANN remarks, "The pus-corpuscles are thus probably peculiar cells, forming in the pus-serum, that is in the cytoblastema, which in inflammation exudes in greater quantity and unusual mixture, in the same manner as the mucus-corpuscles form in mucus, and as all cells

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[†] Mikroskopische Untersuchungen über die Uebereinstimmung in der Struktur und dem Wachsthum der Thiere und Pflanzen, 1839, pp. 77, 78.

[‡] *L. c.*, pp. 78, 79.

form in their cytoblastema. Their formation appears, according to the observations of H. Wood, to take place first at the surface of the granulations[†]."

99. The latest published view I am acquainted with, as to the mode of origin of pus and mucus-globules, is that of Dr. Mandl[‡], with whom it appears to have been a principal object to point out the relation between these globules and the blood. This author considers the globules of pus and mucus as identical; but that they can by no means be regarded as altered corpuscles of the blood; for he supposes that the latter, by contact with pus, are dissolved. As to the nature of the globules of pus and mucus, he states them to be "fibrinous globules," such as those which he was the first to describe. He thinks that the blood passes through the walls of its vessels, with all its elements except the corpuscles, which cannot be thus transuded; and that the liquor sanguinis which contains the fibrin in solution, thus placed out of the circulation, gives origin to a coagulation of the fibrin; and, as the serum itself transudes only drop by drop, it is in drops that the fibrin coagulates; thus forming the corpuscles known by the name of globules of pus, mucus, &c.[§].

100. The following are my own observations on this subject; altogether differing, it will be seen, from those just referred to.

101. Fig. 63. represents objects seen in fluid of a blood-red colour[||], from an abscess in the human subject, to which fluid no addition had been made. The field of view was occupied by myriads of young blood-corpuscles, among which were seen all the states represented in this figure. Proceeding through the figure in the order in which the objects are alphabetically lettered, we find the corpuscle of the blood—known to be discoid in its form—gradually assuming an orange shape, and finally becoming globular. Then, however, it is not the entire corpuscle, but the enlarged nucleus of the same. In the situation of the depression in the original blood-disc, there is now seen a pellucid orifice (α). This orifice appears to be at first round. It either continues of this form,—or it elongates,—or becomes triangular (β). A reddish substance, stretching across the orifice, divides it, when elongated into two, and when triangular into three orifices (γ). The intervening substance seems to become more consistent (δ); many minuter orifices come into view,—apparently the pellucid centres of as many discs, the outlines of which discs are hidden by red colouring matter (ε); and, finally, on the extreme right in the figure, we have the formed pus-globule (ζ).

102. From this observation, with others, I am induced to believe that the pus-globule is the altered nucleus of a corpuscle of the blood.

103. The microscopical appearance of pus-globules is pretty generally known. Dr. Vogel gave me drawings of them, accompanied by a description, from his own observations, in 1837; and I have not seen them more accurately delineated or described by any subsequent observer. "They are uneven at the surface," says Dr.

[†] Schwann, *l. c.*, p. 79. [‡] Gazette Médicale de Paris, 4 Juillet, 1840. [§] *L. c.*, p. 419.

[||] Furnished by my friend William Marten Cooke, M.B.

VOGEL, "being covered with minute granules; and with the addition of acetic acid they undergo a peculiar change. There then appear simple granules, generally two or three in each pus-globule (rarely one only or four), which are oval or roundish, having for the most part a somewhat projecting margin, giving them almost a basin-shaped appearance. They are surrounded by a delicate halo or covering, which, with the continued influence of acetic acid, entirely disappears, so that the isolated granules remain alone[†]." The numerous observations of GULLIVER on pus, are deserving of attention[‡]. This author, however, describes the "molecules" (the term he uses) as spherical, and as "*centrically* inclosed in an external part;" and he does not mention the depression which each of these "molecules" presents.

104. My own views of the pus-globule are in some respects peculiar. The formation of this object out of the nucleus of the blood-corpuscle, appears to me to be referable to the same process, essentially, as that by means of which the germinal spot comes to fill the germinal vesicle, in the ovum. This process having been particularly described in a former memoir[§], I need not refer to it in detail here. Its effects are seen progressing in fig. 63, and they have been rendered more obvious by acetic acid, and by alcohol in fig. 64. In the latter figure, an originally single nucleus consists, for the most part of two, in one instance of three, layers of discs or incipient cells; the highly refracting (and probably most essential) portion being, as in all other nuclei, the part formed last.—That the term *nucleus* is here not inappropriate, as applied to the pus-globule, will I presume be admitted, should others, before adding any foreign substance, find the pus-globule to be contained within a cell, as frequently as I have noticed this to be the case. If so, however, the highly refracting discs form together, not the "nucleus," as they have been called, but the nucleolus.—It will be seen that I differ from previous observers, in considering the *outer* portion of the pus-globule to be composed of discs or incipient cells. But this is only one of many differences between the observations here recorded, and those of other investigators.

105. I have already stated that the object called by me the *disc*, had in several instances been described by some as a peculiar structure found in two or three kinds of globules. The term *disc* is synonymous with the "basin-shaped granules" of VOGEL, and the "spherical molecules" of GULLIVER, seen in the globule of pus; the same object having been observed also by GÜTERBOCK, HENLE, and others who have investigated the structure of the pus-globule.

106. The condition of portions of a capillary network, which I have found in pus, —a minute fragment of which, in outline, is represented in fig. 65,—confirms the

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[†] Dr. VOGEL has since published a work entitled "Physiologisch-pathologische Untersuchungen über Eiter, Eiterung, &c." Erlangen, 1838, which, however, I have not seen.

[‡] London Medical Gazette, 1839, 1840. Medico-Chirurgical Transactions, 1840.

[§] Researches in Embryology: Third Series. Philosophical Transactions, 1840, Part II. par. 385.

idea recorded in my first communication on the corpuscles of the blood[†], namely, that the appearance of these corpuscles in inflammation is the same as that which they present in vital turgescence of the vessels; an appearance which seems to be referable to changes effected by a process of the same kind as that above referred to. In such portions of capillary network, the corpuscles are found to have assumed the same colour as the pus-globules. The fragment in question contained what seemed to be a pus-globule at a certain part; and very possibly the other corpuscles in this vessel had been destined to furnish globules of pus. The occurrence, however, of pus-globules in the blood-vessels, I am aware is by no means new.

107. "According to Henle," says Schwann, "the corpuscles of pus are not distinguishable from those of mucus[‡]." Dr. Mandl's opinion, that these objects are identical, has been already mentioned. If then pus and mucus-globules are "identical," or "not distinguishable," their source is not likely to be very different: and pus-globules we have just seen to be derived from corpuscles of the blood. I would also compare figs. 68 to 71. (mucus), with figs. 63 and 64. (pus) in this memoir, as well as with fig. 23. (blood-corpuscles) in my last paper[§]. See also figs. 72 and 73. in the present communication. The blackish mucus from which these last were taken, presented, not merely corpuscles having the same appearance as altered blood-corpuscles, but such as resembled young corpuscles of the blood themselves, of the characteristic colour, and in an unaltered state.

108. Whether the highly refracting globules in the cells fig. 103. were those of fat, I do not know; but their appearance suggested the idea that this was the case; and the cells containing these globules were certainly altered corpuscles of the blood. For a particular description of these cells, I refer to the explanation of the figures. It may, however, be added here, that each of the globules resembling fat seemed to occupy the *central* part of what had previously been discs.

109. There are but few of the figures accompanying this memoir, which do not directly or indirectly confirm the observations recorded in my last. Among those affording direct confirmation of the same, may be mentioned figs. 75, 76, 77, 78, 79, 80, 81, 82, 83, 84. Some of the blood-corpuscles in these figures exhibit discs, existing both within and around the nucleus of the blood-corpuscle while in circulation. Others (figs. 67 α. 76. 78.) seem to represent conditions of the "lymph-globules," or "corpuscles of the second form" of authors.

110. The only views I am acquainted with, regarding the place which blood-corpuscles should be considered to occupy as "cells," are those of Schwann, and Valentin. The former considers this corpuscle as a nucleated cell; while the latter

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[†] *L. c.*, par. 48. fig. 20.

[‡] *L. c.*, p. 80.

[§] On the Corpuscles of the Blood, Part II. *l. c.*, p. 201.

maintains that it is a nucleus,—the object usually termed the nucleus, being really the nucleolus. From my own observations, it appears that, paradoxical as it may seem, both these views admit of being established. The fact is, that at an early period the corpuscle of the blood is a mere disc, having a cavity or depression representing the "nucleolus." At this period the corpuscle may be called a nucleus; though it does not seem desirable to use this term, before the formation of the cell. Subsequently, the outer portion of the corpuscle or disc becomes transformed into minuter discs, which coalesce to form a membrane; and the inner portion is now the nucleus, the entire corpuscle being a cell.

111. At α of fig. 144. is a corpuscle resembling a young corpuscle of the blood, in which the central part was red, and the outer part pale. At α, fig. 95, is a blood-corpuscle, the discs in the altered nucleus of which, generally speaking, had a deeper red, the nearer they were to the centre of the nucleus. These may serve as examples, showing that there is a continual re-appearance of red colouring matter, and that it comes into view around the orifice in the nucleus.

Epithelium-tables, cells, and cylinders compared with Corpuscles of the Blood.

112. On examining the tail in a great number of Tadpoles,—the larva of the large Toad found in Jersey,—these Tadpoles measuring from $4'''$ to $6'''$ (Paris lines) in length, I found the central part of the epithelium-tables to present an appearance so varied, that it would have been vain to expect that, by figuring those of any one part, an idea could be given of their general appearance. Nor can I hope to have yet done so; but it seemed desirable to sketch a few of the states noticed, without regarding the relations of the tables to one another, and without confining myself, in the delineation, to the same individual.

113. Before referring more particularly to these, I will just express my surprise at its having been possible for any one to examine the tail of a single minute Tadpole, without recognizing in its epithelium, corpuscles having the same appearance as corpuscles of the blood. And I see no way of reconciling the acuteness of observers with the absence of this recognition, except by supposing the Tadpoles examined to have attained a far greater size than those examined by myself—measuring chiefly from $4'''$ to $5'''$. In those of the lengths now mentioned, I find it impossible to discern an essential difference in appearance between the objects entering into the formation of the edge of the tail, and blood-corpuscles circulating in this larva (par. 196.). The comparison is very easy, if made after the corpuscles are at rest within their vessels. It frequently happens too, that the placing of a piece of glass or mica upon the tail, lacerates the edge, and separates epithelium-tables from it (fig. 86. α, β.); when it is seen that these tables, in their form, size, colour, and internal state, have the same appearance as the blood-corpuscles which were circulating just before, in the same field of view.

114. The nucleus in some of the epithelium-tables fig. 86, bore a curious resem-

blance to certain states of the germinal vesicle figured in one of my former papers[†]. Other conditions of the nucleus in some of the same epithelium-tables, equally resemble the appearance of the discs or incipient cells succeeding that vesicle in the ovum[‡]: and in fig. 88. we have remarkable evidence of the appearances presented by the contents of the ovum, and those of the epithelium-tables, being both referable to the operation of the same process. This remark applies equally to blood-corpuscles, apparently destined to enter into the formation of the epithelium, in fig. 95.

115. The resemblance between corpuscles of the epithelium, and those of the blood, is not confined to the part we have been considering. All the epithelium-cells which have fallen under my notice present it more or less decidedly. Even those collected from the surface of the tongue (fig. 92.), while they exhibit the greatest irregularity in size and form, and considerable variety in the appearance of the interior, present the same sort of discs, and the same division into minuter discs; many of them being also tinged with the same red colouring matter as is perpetuated or reappears in the discs of other parts, more closely resembling corpuscles of the blood. The nucleus also of the epithelium-cell from the tongue, is composed of discs; and I have seen it remarkably tinged with red. Among the cells from a furred tongue, there were seen heaps of discs of a blood-red colour. The membrane of the epithelium-cell—where a cell exists, as in fig. 92.—is obviously formed by the same coalescence of discs, as that which we shall find to give origin to other membranes, —for instance, to the membrane of the ovisac, figs. 170, 171, 172, *h*.

116. The formation of what have been denominated epithelium-cylinders, I have not had the opportunity of particularly following; but may offer a few incidental observations. In fig. 94 *α*. is the outline of a cell which was red throughout, and filled with young epithelium-cells. The latter presented traces of division into objects still more minute; and this division at one part had really taken place. Traces of division were observed also in the objects figs. 95. 99. and 100; the blood-red colour of many of which, passed at one extremity nearly into black. Should these have been, as I believe they were, incipient epithelium-cylinders, the observation, so far from having realized the following conjecture of Professor Valentin, will stand in direct opposition to it. "It appears," says he, "judging at least from the nuclei, as though cilia-cylinders arose through the coalescence of two adjacent cells, and the disappearance of the partitions. There are often seen, at least, in one and the same cylinder, two nuclei, one on the other, or in part covering one another[§]." Were not the two nuclei, seen by Valentin, produced—like those in the figures, last referred to—by division of a previously single nucleus? If so, it would seem that cilia-cylinders arise, not by coalescence, but by division, like some of the Vorticellæ; which they resemble also in the position of the cilia.

[†] Compare with *c*, figs. 159, 160, 162, 169, in my Third Series on Embryology. Philosophical Transactions, 1840, Part II.

[‡] Compare with fig. 195 in my Third Series on Embryology, *l. c.*

[§] Müller's Archiv, 1840, Heft II. p. 205. *Note.*

The Elements of the Pigmentum nigrum, and those of the Ciliary Processes, compared with Corpuscles of the Blood.

117. The general appearance of black pigment, as seen, in a formed state, in the choroid coat of the eye, is well known. And that variety of this substance met with in the tail of the Tadpole, has been accurately represented by Professor Schwann, in a certain stage[†]. As to the mode of origin, however, of this substance, in either of the localities just mentioned, I am not aware that we possess any published information: a remark equally applicable to the blackish substance found in mucus from the air-passages. Perhaps the following observations, therefore, may be useful.

118. In Tadpoles of $4\frac{1}{2}'''$ to $5'''$, I find the blood-corpuscles to contain, situated on the nucleus, certain red globules (fig. 75. α.), appearing to transform themselves into discs (β) of the same red colour. Now in the epithelium-tables above mentioned, as so much resembling the blood-corpuscle, the peripheral part is composed of red discs. The red discs of several tables are necessarily in contact. They coalesce, and present the appearance delineated in fig. 89. Each line of discs in this figure, it will be seen, is made up of those derived from two adjacent tables. These discs, quite red in the figure now referred to, subsequently divide into extremely minute, blackish objects, which adhere together, and form partitions between the central portions of the tables. In a state more advanced, it is not easy to discern this partition-like appearance, nor to connect the very irregular forms into which the partitions are distorted, with their original figures of six sides. At γ in the figure last mentioned, is a stage more advanced than that at β; but the objects were still seen to be composed of discs. They also presented a trace of the partition-like appearance, and were still red. In fig. 91, are stages of these objects yet more advanced (but from another part), and apparently corresponding to those figured by Schwann from the tail of the Tadpole. In the latter stages they are known under the name of pigment ramifications,—of which I think the observations now detailed, may assist to show the mode of origin.

119. The large object connected with the pigment ramifications (figs. 90, 91.) appears to be a centre for the reproduction of epithelium-tables; for those in fig. 88. are corresponding objects, in which this reproduction is very obvious—the pellucid, germinal vesicle-like nucleus on one side of the object figs. 90 and 91, being, more particularly, the centre from which the reproduction proceeds.

120. But one of the figures just referred to (fig. 91.) was taken from the pigment of the eye; which seems to be produced in a manner precisely such as that just described, as giving origin to similar appearances in the tail. This will be obvious, I think, if the object on the right hand in fig. 93. (eye) be compared with fig. 89. (tail). And each of these presents a centre for the origin of new substance (the peripheral discs), like the centres just referred to. (Fig. 93. will be found fully described in the

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[†] *L. c.*, Tab. II. figs. 8 and 9.

explanation of the Plates; with the mode of origin of the pigment in the choroid. The discs of the bright red δ and ε, in this figure, undergo division, and are given off, to enter into the formation of the darker and blackish ζ. It appears that what is seen of ζ in the figure, had been formed by portions previously given off in this manner. So that here, in the pigment of the eye also, we find centres (δ. ε.) for the origin of new substance). It is important to add, that all the objects now mentioned, had the appearance of altered corpuscles of the blood.

121. The black substance found in mucus from the air-passages, arises in a manner somewhat similar; presenting itself at the outer part of corpuscles quite as red as corpuscles of the blood. See fig. 72.

122. Portions of ciliary processes, as seen in the eye of a Tadpole of $5\frac{1}{2}'''$, are represented in fig. 101. They are in outline only; for the parts composing them, from their colour, form, size, and general appearance, so much resembled slightly altered blood-corpuscles, that it did not appear requisite to make elaborate drawings of them. It is, besides, extremely easy to repeat, and therefore to confirm, this observation, or show it to have been erroneous.

The Primitive Discs exhibit an inherent contractile power.

123. This was manifested by the elongated discs of the epithelium-cylinder fig. 98. α, β; and by the isolated disc, two appearances of which are represented at δ in the same figure. This isolated disc was observed for a considerable time to change its form and place. Some of the discs composing the paler part of the cylinder α, β, were, for about twenty minutes, seen to be in motion; and there was thus produced a very slow revolution of the entire object on its axis, in the direction of the arrow. (In connection with this subject, it may be mentioned, that the object in fig. 139 which is more finished in delineation than the rest, and constituted the centre of the forming crystalline, on being viewed repeatedly for a considerable time, was found to vary in its appearance; a phenomenon which seemed to arise from the discs changing their position. This object, as will be found stated elsewhere (par. 181.), had all the redness of a corpuscle of the blood.)

The Nuclei of Blood-corpuscles furnish themselves with Cilia, revolve, and perform Locomotion.

124. Ciliated corpuscles are seen in figs. 105 and 104. Those in the first of these two figures were observed in the blood of a rabbit, taken from vessels in the immediate neighbourhood of a Graafian vesicle, which, from its size and vascularity, had evidently been destined to expel an ovum. The corpuscles in the latter figure were noticed in a substance from the eye of a fœtal calf of $5\frac{1}{4}$ inches, more particularly described in the explanation of the Plates. The rabbit had been killed eighteen

hours; and two days had elapsed after the fœtal calf had been taken from the body of its mother, before I made the observation now recorded.

125. The corpuscles in question had thrown off their membrane (if they ever had one,—for they seemed altered *young* corpuscles, or mere discs). It was therefore the nucleus of the corpuscle, which had become endowed with cilia; but this nucleus was quite red. The cilia presented the appearance of acuminated processes from the discs composing the corpuscles. It seemed to be by means of these, that the corpuscles in some instances slowly revolved, and in others changed their place. One of the corpuscles (β. fig. 104.) appeared to shoot forth a process, the cilium, which then as quickly disappeared, as if drawn in. Cilia could not be discerned on the corpuscle β, fig. 105. Yet one of its discs seemed to be in (incipient?) motion. We have elsewhere seen, however, that discs exhibit motion independently of cilia (par. 123.).

126. Besides the above, I have observed young blood-corpuscles (fig. 106.) performing constant and considerable locomotion; which, in some instances, was by no means slow. The motion of some resembled that known under the name of molecular, and others of these moving blood-corpuscles were so minute as to be comparable to molecules enlarged (pars. 198, 199.).

127. In a former communication[†] I described some most curious motions, or rather changes in form, observed under particular circumstances in corpuscles of the blood, and, in a note, added to that communication, stated that I had been induced to believe that those changes in form were referable to contiguous cilia. I ought now to state that subsequent observation enables me to say that these changes in form arise from some inherent power, distinct from the motions occasioned by cilia.

Molecular Motions discernible within Corpuscles of the Blood.

128. Minute red points around the nucleus within the blood-corpuscle, have been already mentioned, as observed in the Tadpole (fig. 75. α). Such points or globules exhibited molecular motions. The discs, formed apparently out of such objects, as in the corpuscle β of the same figure, did not exhibit motions, but were pressed together into polyhedral forms. In fig. 124. α, and fig. 123, globules such as those just mentioned appeared to be undergoing division. They at least presented the appearance of a rose, the same as that so often seen elsewhere. Molecular motions were exhibited by the globules contained within the cell fig. 102. I refer to the explanation of the Plates for an account of the circumstances under which these motions were observed; and have only to add, that the colour of the entire cell was precisely the same as that of the blood-corpuscles along with which it was observed.

129. It is known, through the researches of **Henle**, that the rete Malpighii consists of round cells furnished with a nucleus; which cells this observer, and subse-

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[†] On the Corpuscles of the Blood, Phil. Trans., 1840, Part II. pars. 12—18.

quently SCHWANN, succeeded in tracing into the cells of the epidermis. The latter supposes that a cell-formation takes place immediately on the surface of the cutis[†]. But here I cannot help referring to the process of division we have seen reproducing the epithelium-tables of the Tadpole; a process which it seems probable, from other of my observations, is universal, including, therefore, even the epidermis.

The Elements of Blood-vessels compared with Corpuscles of the Blood.

130. Capillaries in the course of formation out of corpuscles having the same appearance as corpuscles of the blood, are represented in fig. 107. These corpuscles, still red (α), apply themselves together so as to form an object resembling a necklace composed of elliptical beads; and having coalesced, and become pale (β), and the membranous partitions (at the extremities of the beads) having disappeared, they constitute a tube. The membrane of this tube is formed in the same manner as, according to my observations, the membrane of the ovisac, the chorion, and other membranes;—namely, by the coalescence of discs, as at α in the upper part of the above figure. At certain parts, the corpuscles apply themselves in such a manner as to form a branched vessel. An example of this is to be found in the figure; which also shows the corpuscles, coalescing in this instance for the formation of capillaries, to have been of very minute size. They had the same appearance as *young* corpuscles of the blood. At δ is a round space. This space was colourless, and brilliantly pellucid. And I am by no means sure that it was not an orifice left in the membrane of the tube (pars. 152, 162, 178.).

131. If the vessels have their origin in corpuscles of the blood, it is scarcely needful to inquire from what source they derive the materials for their thickening also, and for the formation of the several coats of which they become composed. It may be mentioned, that some of the pale discs in fig. 142.—evidently derived from corpuscles of the blood—seemed, by coalescence, to be entering into the formation of the membrane of the vessel. But here also, as appears to be the case in every other tissue, the original corpuscles (fig. 107.) no doubt contain within themselves the means of perpetuation.

132. While the investigations forming the subject of this memoir were in progress, I received a letter from Dr. W. B. CARPENTER, of which the following is an extract. It requires no comment, except the expression of my concurrence, as furnished by the foregoing observations:—"Having been just lecturing on the blood and the process by which it becomes organized, I think it well to mention to you some views which have occurred to my mind in reference to your observations on the conversion of blood-corpuscles into tissue[‡]. When *lymph* is thrown out upon an inflamed membrane, and is in process of becoming organized, it has been long known that the

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[†] SCHWANN, *l. c.*, p. 86.

[‡] Dr. CARPENTER here refers to my first paper on the Corpuscles of the Blood, Philosophical Transactions, 1840, Part II.

membrane beneath is usually much softened, and that its vessels seem to have a tendency to prolong themselves into lymph. Some have asserted that vessels and *red blood originate* in the lymph; but this has always appeared to me very doubtful. Now if your views be correct, I should see no difficulty in understanding that the vessels of the subjacent membrane prolong themselves, not by any *vis a tergo* (as some have supposed), but by the development of the blood-corpuscles stagnated in them first into cells and then into tubes, which convey blood into the coagulum; and this blood again contains the elements of a further ramification of capillaries, which will go on being formed in this manner, until a complete network is produced. It may be objected to this view, that coagulated *blood* ought to become organized more readily than *lymph*—which experience shows that it does not. But it may be replied that blood *mechanically* effused is in a very different state of vitality from the coagulable lymph or liquor sanguinis poured out on the surface of an inflamed membrane; and that these changes of the corpuscles do not take place in those of the clot or coagulum itself, but in those of the vessels in the living tissue beneath. It is very interesting to find a theory harmonizing with previous observations, which is I believe the case in this instance. Rows of corpuscles, proceeding from the red points of the subjacent membrane, have been seen in coagulable lymph; but their *import* was not known as I believe it now may be†."

The Elements of Cellular Tissue compared with Corpuscles of the Blood.

133. There is no tissue the elements of which resemble altered blood-corpuscles more than those of the cellular. And there is certainly no tissue which it is of more importance to trace back to such objects, than this; because of its presence almost everywhere in the body, either as cellular tissue, or in the more condensed state of the parts, into the formation of which it enters. This remark is of course applicable in a pathological, as well as in a physiological point of view.

134. On reference to figs. 109. 111. 112, it will be seen that the first changes visible in the formation of this tissue, are in no small degree like those presented by the incipient pus-globule (par. 101.). And this similarity we shall meet with again and again, as we examine the elements of other tissues. The corpuscle at α, fig. 109, presenting the same appearance as a slightly altered corpuscle of the blood, has one large orifice; in those at β there are two or more orifices of smaller size; and subsequently a number of bright points come into view. These points seem to be the cavities or depressions in as many discs, between which discs red colouring matter still lies, and conceals their margins. The bright points, therefore, *seem* as if contained in a mass of red colouring matter. The form of the corpuscle sometimes undergoes a considerable change, before the margins of the discs are distinctly seen, as in fig. 109. γ; where it presented the appearance of an elongated, flattened mass. The discs, however, are soon distinct (fig. 112. β, fig. 111.); and the explanation of

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† Dated Kingsdown, Bristol, Feb. 8, 1841.

the Plates will show that here, as in figures of the elements of other tissues, the transition from a corpuscle having the same appearance as a corpuscle of the blood, into a mass of discs, was quite unequivocal, in colour and in form, as well as in the gradual appearance of the discs. It is important to remark, that the mass of discs presents, not the entire corpuscle, but its nucleus enlarged.

135. This mass, already elliptical (fig. 109. γ), becomes pointed, and usually at both ends (figs. 110. 111, fig. 112. γ). The pointed extremity is then elongated into a filament (see the same figures); and apparently by the following means. The comparatively large discs into which the corpuscle has divided, undergo division and subdivision themselves, until discs are produced of extreme minuteness. These coalesce to form the filament; which has in its interior the means of perpetuation, through discs, into threads still finer. (This seems to be the general mode of production and thickening of membranes and fibres.) The elongation of the mass of discs into filaments, takes place sometimes in more than two directions. The whole mass of discs does not at once pass into filaments; but a part, enlarging, becomes a special centre, apparently for the origin of new substance (fig. 112. γ); as we saw in the epithelium, and pigmentum nigrum (pars. 119, 120.). But every disc, large enough to be discerned and traced, seems to exhibit a reproducing property.

136. The figures present examples of the elements of cellular tissue thus formed, from various parts, and, among others, from the thigh (fig. 109.), the neck (fig. 111.), and the axilla (figs. 112, 113.); as well as a drawing of this tissue, entering into the formation of the sheath of the spinal chord (fig. 116.). The fibres of cellular tissue forming this sheath seemed to be interlaced.

The Elements of the Corpus luteum derived from Corpuscles of the Blood.

137. In a former memoir[†], I described the Graafian vesicle as formed by the addition of a covering to the previously existing ovisac. The covering was stated to consist of a kind of dense cellular tissue, susceptible of becoming highly vascular. In a later communication, I came to the conclusion, that the covering of the ovisac becomes the corpus luteum[‡].

138. Confirming these observations, I have now to make the following addition. On examining the vessels entering into the formation of the covering of the ovisac, and rendering it highly vascular, in a rabbit killed three hours post coitum, I found them filled with blood-corpuscles in an altered state. The central part was greatly enlarged, colourless, and brilliantly pellucid; and the contour was lost by the slightest pressure, so that the red colouring matter of adjacent corpuscles had the appearance of being blended into a mass, filling up the interstices between their round, colourless, and brilliant centres. This condition of the blood-corpuscles within the vessels, too much resembled that which I had met with in other parts, formed out of corpuscles

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[†] Researches in Embryology: First Series. Philosophical Transactions, 1838, Part II. par. 24.

[‡] Second Series. Philosophical Transactions, 1839, Part II. par. 156.

having the same appearance as corpuscles of the blood, not to induce the belief, that it is the blood-corpuscles entering into the formation of the covering of the ovisac, which form the corpus luteum.

139. As long since as in the autumn of 1838, my friend Dr. HODGKIN mentioned to me a view entertained by him, that the blood sent to the enlarged Graafian vesicle, becomes the corpus luteum. This requires from me no other comment than that afforded by the observation now recorded.

The Elements of Cartilage and other Tissues compared with Corpuscles of the Blood.

140. The early stages in the formation of cartilage, which happen to have fallen under my notice, were observed in the foundation of a bone in one of the lower extremities of a fœtal chick, on the tenth day of incubation (figs. 119, 120.); in the foundation of a spinal vertebra from the same chick (fig. 122.); in that of one of the cranial vertebræ from a Tadpole of 5‴ (fig. 117.); in the foundation of the orbit in another Tadpole of the same size (fig. 121.),—this cartilage having been seen in a different state in a Tadpole of 6‴ (fig. 118.); and also in an incipient spinal vertebra from one of these larvæ. In these observations, a transition out of corpuscles having the same appearance as corpuscles of the blood, was either directly witnessed, or to be inferred. Such corpuscles, generally speaking, are not represented in the figures, because the cartilage was too far advanced. Yet in fig. 117, on the left side, there may be seen two remarkable corpuscles of this kind. The eye becomes so accustomed to the altered nucleus of the corpuscle, having the same appearance as the corpuscle of the blood, that it is scarcely requisite to see that corpuscle in an entire state. In these observations, however, there was not wanting the characteristic red. In fig. 119, for instance, this colour was recognizable after the nuclei had begun to arrange themselves into fibres; in fig. 122. the young corpuscles *α*, situated in the incipient cartilage of a vertebra, were blood-red; in fig. 117, on the left side, as already mentioned, is the outline of two elliptical corpuscles having the same colour and general appearance as the blood-corpuscles circulating in the vessels of the same larva (the Tadpole); and the red colour was noticed also in the objects of fig. 121.

141. It may here be remarked, that the corpuscles which, in the incipient cartilage fig. 117, had the same appearance as the blood-corpuscles of the animal, were situated in the most superficial part, as though *added* to corpuscles previously there. This was the case, also, with the young and red corpuscles *α* in fig. 122. And we shall hereafter find the same superficial situation to be occupied by the newest corpuscles entering into the formation of the optic nerve (par. 151.).

142. In examining the foundation of cartilage, the observer is struck with the regularity of distance between the nuclei of the corpuscles; this being referable, apparently, to the presence originally of the entire corpuscles (fig. 117.), the outer part of which afterwards disappears (fig. 121.); and a dense mass is formed, by continual additions from the nuclei, as centres for the origin of new substance. It seems to

have been on cartilage in this more advanced state, that observations heretofore have been principally made. And the figures given by Schwann of cartilage[†], show, I think, the continued existence of centres of this kind.

143. A suggestion offered in one of my former memoirs[‡], seems to have been realized by the observations recorded in this paper; for the elements of cartilage are certainly the seat of changes essentially the same as those we formerly witnessed in the cells succeeding the germinal vesicle in the ovum.

144. We also find that cartilage exhibits centres, such as those we have seen to exist in the epithelium (par. 119.),—in the black pigment of the eye (par. 120.),—in cellular tissue (par. 135.),—as are to be recognized, I think, in the "Ganglion-globules,"—and the most remarkable of which centres is the ovum. But in these special centres there is to be witnessed no other than the same process, as that which operates in the nucleus of every cell.

145. Fig. 116½. represents the outline of very large cells, filled with other cells. The large cells had the appearance of altered blood-corpuscles; the colour being also pale red. The nuclei were for the most part blood-red, and not distinguishable from young corpuscles of the blood (fig. 106.), lying near them. From the great length of the object formed by the large cells just mentioned, and from its general appearance, I am disposed to regard it as the foundation of the cartilage destined to form a bone; but the object, though consisting of large cells, was so minute, that an opinion here must be cautiously expressed.

146. The extremity of the incipient beak, as well as that of a claw, from a fœtal duck, on the 13th day of incubation, were found to be in a state of the same kind as the forming cartilage in fig. 119; and there was observed a transition out of corpuscles having the same appearance as corpuscles of the blood.

147. A like origin has seemed to be presented by the elements of feathers in the chick: but my observations here have been but few.

148. Though rather out of place, it may be mentioned, that in the Tadpole which yielded the drawing fig. 121, I saw what appeared to be the foundation of a ligament, also having an origin in corpuscles which presented the same appearance as corpuscles of the blood, and seemed to be arranging themselves into a fibre.

The Elements of Nervous Tissue compared with Corpuscles of the Blood.

149. The optic nerve in the Tadpole of $4\frac{1}{2}'''$ to $5'''$, seen as it is entering the eyeball, is very easily found, and there is not much danger of any other object being mistaken for it. My observations on the optic nerve, at this early period, have been very numerous, and, it may be added, highly satisfactory.

[†] *L. c.*, Tab. I. figs. 5–9. Tab. III. figs. 1, 2.

[‡] Researches in Embryology: Third Series, *l. c.*, par. 393.

150. Figs. 123, 124 and 125, present portions of this nerve in a more or less incipient state. The objects represented, chiefly in outline, in fig. 123, observed at a part of the nerve near its entrance into the eye-ball, were all corpuscles having the same appearance essentially as corpuscles circulating in the blood. I refer for a particular description of these to the explanation of the Plates. The nerve did not present any fibres; and so incipient was it, that at the part represented in the figure, the corpuscles had their original elliptical form. In another instance, where the nerve was rather more advanced, the globules surrounding the nucleus (see the corpuscle more finished in delineation than the rest in fig. 123.), had very much disappeared, and the corpuscles were pressed into polyhedral forms.

151. In a former page, I mentioned that the foundation of cartilage had presented an appearance, suggesting, from the more advanced state of the interior, that corpuscles had been *added* at the outer part. Such an appearance is even more remarkable in the optic nerve, as shown in fig. 124; all the objects in which (presenting, it will be seen, very different states) were observed in the same nerve. I have stated that the corpuscles in fig. 123. had *essentially* the same appearance as that of corpuscles of the blood. Respecting those at α, in fig. 124, it may be said that their appearance was *precisely* such as that of corpuscles circulating in the blood of the same larva. Yet the interior of the nerve in this instance was in a more forward state than that of the one before referred to (fig. 123.); the other objects in fig. 124. having been composed of discs (some of them forming necklace-like objects, or incipient fibres), derived from the nuclei of corpuscles, such as those just mentioned (α). These discs were all red; the colour being paler in the more advanced. (See the figure and the minute explanation of it.)

152. Portions of this nerve, in a more advanced state, are seen in fig. 125. The discs at α, into which the nuclei of the corpuscles had divided, were arranging themselves in something like lines. They presented minuter discs in their interior, as in the part represented less in outline than the rest. The tube β, yet more advanced, was forming out of discs, which were coalescing at its periphery. These discs, corresponding apparently to the minutest of those at α, again, presented other discs in their interior. Pellucid points, apparently orifices, were seen here and there. They seemed to communicate with the exterior of the tube (pars. 130, 162, 178.). These pellucid points correspond to the depressions in the original discs. As in muscle, there are doubtless in the interior of an object such as β, the elements of new substance,—the essential portion of the nerve.

153. In these researches, a considerable share of my attention has been devoted to the elements of the retina; which afforded ample proof of their origin in corpuscles, having the same appearance as corpuscles of the blood. Some of these elements are represented in figs. 126 to 131; and it will be seen that they are of the same character, whether taken from the fœtal Calf, fig. 130, or from the Tadpole, fig. 131; consisting, in both, of round, flattish masses of discs.

154. Here also, corpuscles having the same appearance as young corpuscles of the blood, become orange-shaped, present two or more bright points with red colouring matter between them (see the figures); and, as the colouring matter disappears, admit of being traced into the masses of discs just mentioned; these being the altered nuclei of the corpuscles in question. We thus find in the elements of the retina, as well as in those of cellular tissue (par. 134.), the same appearances as those presented in the formation of pus-globules out of corpuscles of the blood.

155. After the corpuscle, having the same appearance as corpuscles of the blood, has become a mass of discs, the discs may undergo division and subdivision to an inconceivable extent, and to which we can set no bounds, as regards either the number or the minuteness of the resulting objects. It will be observed, from fig. 130, that cells are formed; their membranes arising—as elsewhere, according to my observations—from the coalescence of minute discs. These cells will be found minutely described in the explanation separately given. It is not my purpose in this memoir to follow their subsequent progress.

156. I have frequently met with the rudiment of the spinal chord,—sometimes in fragments, sometimes almost entire,—as it exists in the tail of the Tadpole; this larva measuring $5'''$ or $5\frac{1}{2}'''$. Its newest part I found composed entirely of corpuscles having essentially the same appearance as corpuscles of the blood, and being in a state resembling (but somewhat more advanced than) that which they presented in the optic nerve fig. 124 α, and also arranged in lines. In fig. 132. is an outline of some of the corpuscles which I observed constituting this structure in a Tadpole of $5\frac{1}{2}'''$. The spinal chord has uniformly appeared to be less advanced than the cellular tissue which seemed to be entering into the formation of its sheath (par. 136.).

157. Corpuscles from the cortical substance of the brain in a fœtal Calf of $5\frac{1}{4}$ inches, are seen in fig. 133: and fig. 134. represents some of those observed in the medullary part of this organ, in the same subject.

158. Objects such as those in the first of these two figures, were observed in large number; and often seen to be flattish in their form. They were all more or less red,—some blood-red; all were either discs, or composed of discs,—being either themselves corpuscles, having the same appearance as corpuscles of the blood, or immediately derived from such corpuscles. Here, as often observed in other parts in the course of formation, the corpuscles were found to be of a minute size; being apparently young corpuscles. They presented the same gradual transition into the elements of the brain, as we have seen in other parts, regarding a change in form, colour, and division into discs; besides the previous appearance of an orifice (β), in the situation of the depression presented by the corpuscle when discoid in its form. The gradual formation of a membrane too, by coalescence of the outer discs, was witnessed here (η). These details will be found more fully given in the explanation of the Plates. And perhaps there is no figure more instructive, as regards the changes

presented by the corpuscle or disc,—its passage into *layers* of discs,—the comparative age of these,—the reappearance of red colouring matter,—and the *secondary* nature of the "cell," than the figure now before us—fig. 133. To the description of this figure, in connection with that of fig. 134, I particularly refer.

159. Very much the same general remarks apply also to the elements of the medullary portion of the brain (fig. 134.).

The Elements of Muscular Tissue compared with Corpuscles of the Blood.

160. Some of the earliest appearances presented by muscle, in its formation out of blood-corpuscles, were given in one of my former communications to the Society[†]. The muscles then examined, being involuntary, did not afford so interesting a field for observation, as those the development of which has since had my attention,—the voluntary muscles; nor did I then prosecute the investigation farther than as opportunity was incidentally afforded. Nevertheless, it will be seen that the facts I have now to mention, accord with those stated in that memoir.

161. Corpuscles more or less blood-red, and having the appearance of young blood-corpuscles, apply themselves to one another in the manner represented in fig. 135, and also in the memoir just referred to; by which a necklace-like object is produced. These corpuscles are cells, which I have found filled with discs (β). The appearance of these discs, together with that of the nucleus and nucleolus, resembled in a remarkable manner the corresponding parts in certain states of the germinal vesicle and germinal spot[‡]. In no instance have I more clearly seen the nature of the so-called "nucleolus" of authors; and that at this part there exists an orifice, communicating with the exterior of the cell (fig. 135. β). By degrees, the appearance of a cylinder is produced (fig. 136.), which becomes more perfect, as the partitions between the cells, just mentioned, disappear—fig. 137. In this figure, nuclei are still seen, having a parietal situation; these being, as SCHWANN supposed, the nuclei of the original cells (fig. 135.). It must not, however, be supposed that no change takes place in the nuclei; or that the nuclei seen in such a stage as that in fig. 135, are *identical* with those presented in the later state, fig. 137. It appears to me that the nuclei in the later state, merely occupy the same place as those in the earlier. For here, in the muscle-cylinder,—or as SCHWANN has termed it, the "secondary" cell,—the same process is in operation as, according to my observations, is seen everywhere else in what have been called the primitive cells; namely, a continual change in the nucleus. A part continually passes off in the form of discs, which are contained within the cylinder; while a fresh supply of discs is continually coming into view around an orifice, connecting apparently the interior and exterior of the muscle-cylinder, or "secondary cell." This *orifice*, however, appears to be identical with that existing in the original cells, fig. 135.

[†] On the Corpuscles of the Blood, Philosophical Transactions, 1840, Part II. p. 595.

[‡] Compare β of fig. 135, with *c* of fig. 160, in my Third Series on Embryology, *l. c.*

162. It is to Valentin and Schwann that we are indebted for the observation, that the muscle-cylinder is formed of nucleated cells. But the other facts now mentioned[†], I believe are new; namely, the more or less blood-red colour of the original cells (fig. 135.), the presence of discs in their interior, the derivation of these discs from the nucleus, the existence of an orifice at a certain part of the nucleus, and a continuation of these appearances in the muscle-cylinder, or "secondary[‡]" cell. We may hereafter see reason for thinking it not unimportant, that the contents of the "primitive" cell, and those of the "secondary" cylinder, should have their origin in the nucleus; as well as to connect this fact with the existence of the orifice in question. The muscle-cylinder thus contains centres such as we have seen elsewhere[‡] (pars. 130, 152, 178.).

163. The discs filling the muscle-cylinder, are the seat of elaborate changes; as is to be inferred, I think, from the very remarkable appearances they afterwards present. These appearances will form the subject of a future memoir, which it is my intention to offer to the Society; for I feel it incumbent upon me, because due to the Society, to show that certain conjectures in one of my former papers, printed in the Philosophical Transactions, have been fully realized.

The Elements of the Crystalline Lens compared with Corpuscles of the Blood.

164. If there is any structure which it would at first sight seem impossible to trace back to corpuscles having the same appearance as corpuscles of the blood, that structure is surely the crystalline: and none but the most conclusive evidence will suffice, to prove that this admits of being done. Whether the observations I have to offer are of this character, the future must determine. I can only say that they were repeatedly confirmed.

165. I am enabled to verify Professor Schwann's description of the pale cells of the crystalline; except as regards the nucleus, nucleolus, and contents of the cell. (But in these respects my views are peculiar, with reference to "cells" in general: a remark applying also to the mode of formation of the membrane of the "cell," to the manner in which cells are reproduced, and to the secondary nature of the "cell" (par. 173.)). My observations also corroborate his view, that the foundation of the fibres of the lens consists of cells. But they do not enable me to adopt the

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[†] Though merely an extension of those I formerly communicated with reference to the ovum.

[‡] In later stages there exist other centres for the origin of new substance within the cylinder; and chiefly in its central part. Some of these have been figured by Schwann (*l. c.*, Tab. IV. figs. 1, 2.), and especially by Bowman (Philosophical Transactions, 1840, Part II. Pl. XVIII.), who denominates them the "corpuscles of the primitive fasciculi." I recommend particular attention to the observations of the last-named author, on these corpuscles (*l. c.*, pp. 484, 485.). He considers it "not impossible that" *** "there may be during development and subsequently, a farther and successive deposit of corpuscles, from which both growth and nutrition may take their source" (*l. c.*, p. 485.). In this I concur; and may add, that the process we saw in operation in the germinal spot, and, as above described, in muscle,—is no doubt applicable to each of the corpuscles observed by Bowman; which in my opinion are continually reproduced by division.

opinion of this eminent observer, that the fibres are *elongated* cells. The mode in which the fibres seem to me to form out of cells, will be hereafter mentioned. The first, and more important question is, what is the material furnishing the cells, and therefore the fibres of the lens? It is here needful to make a few preliminary remarks.

166. Slices cut from the surface of the lens in the fœtal Calf and Sheep, have presented to me in the microscope, not only red blood, but young blood-corpuscles, as well as the parent-corpuscles from which they were escaping. Seldom have I had so favourable an opportunity for observing the manner in which the blood-corpuscle is perpetuated (fig. 141. δ δ), and for making the observations I have now to mention,—that the young blood-corpuscle has a peculiarly bright colour, and—though in Mammalia—an elliptical form. The young corpuscle soon becomes round, though at first it continues flat; the latter no doubt answering an important end connected with its motion in the vessels.

167. Fig. 141. affords a proof that the diameter of the corpuscles usually seen in circulation, is no criterion by which to judge whether red blood can make its way into parts generally considered colourless. For, at α, a portion of vessel is filled with corpuscles, having a diameter not much more than half of that usually ascribed to them: the fact being that, besides the minuteness of the objects into which blood-corpuscles may divide, the form of the corpuscles is susceptible of every change; and minute indeed must be the orifice they could not be made to enter, so attenuated is the shape they sometimes assume. It may be also added, that I have observed a capillary forming, the diameter of which measured only $\frac{1}{700}'''$, which indeed was about the size of some of those in fig. 141: and I have seen objects having precisely the appearance of blood-corpuscles, of less than $\frac{1}{1000}'''$. Mere points too, are found, having the red colour of the blood, and we have already seen that minute objects exhibiting molecular motions, may be observed within corpuscles of the blood (par. 128.); as well as that young corpuscles of the blood are met with, exhibiting motions comparable to that called "molecular," and so minute indeed as to have the appearance of "molecules" enlarged (par. 126.).

168. Corpuscles having the same appearance as young blood-corpuscles, are also seen within tubes—fig. 144. And this figure shows that, while the red colouring matter was still present in some, it had disappeared in others,—the latter being represented in outline only.—Other states of the corpuscles are observed, also within tubes, as in fig. 149. They here exhibited the same division into discs which we have found to take place elsewhere: and while some of the corpuscles undergoing this division were blood-red, others were becoming pale. A farther change is met with: the masses of discs, derived from such corpuscles, seem to be arranging themselves into lines, as if to form fibres (fig. 150.).

169. I have seen red capillary vessels on the posterior surface of the lens. Here the vessels radiated from central trunks, *towards the margin* of the lens. One of

these vessels is seen in fig. 142. It was a branch, proceeding from a trunk more than three times its diameter.

170. In delicate, flat, and branchless tubes, from the edge of the lens—where red tubes have presented to me a parallel direction, from behind forwards—I have found corpuscles, having the same appearance as corpuscles of the blood, arranged with curious regularity; their flat surfaces being in contact with one another (fig. 145.). These corpuscles were blood-red, and red colouring matter was seen between or around them. For other particulars respecting these corpuscles, I refer to the explanation separately given, and to figs. 146, 147, 148; here mentioning only the brightly pellucid object in the situation of the original cavity or depression in the corpuscles. I may return to the subject of the corpuscles in these figures, in a future communication.

171. In figs. 151, 152, are portions of tubes, also from the edge of the lens. These figures are almost entirely in outline. Tubes are very frequently met with, having their contents (which resemble altered blood-corpuscles) in this state. The pellucid object arising in the centre of the corpuscle, is originally round (fig. 151.). It appears that these pellucid objects enlarge, and then coalesce with one another; the red colouring matter at the same time disappearing. By this means transparent, colourless, and very bright spaces are produced; sometimes so large as to occupy a very considerable, and even the principal, portion of the tube. But here again I must refer to the explanation of the figures; remarking only, that there was visible in some of the pellucid spaces (in fig. 152.) a minute, highly refracting object (δ), apparently an orifice communicating with the exterior of the tube (δ δ). Tubes with contents such as those in fig. 152, vary much in their diameter at different parts. I met with a tube of this kind, which, at one part measured $\frac{1}{100}'''$, then decreased to $\frac{1}{200}'''$, and then again enlarged to $\frac{1}{100}'''$: and all this variation in the diameter of the tube, was observed within $\frac{1}{10}'''$ of its length. I have frequently noticed corpuscles, having the appearance of altered corpuscles of the blood, some of which had perhaps escaped from such tubes as those last referred to. Some of these corpuscles are seen at θ and ι, fig. 142, and in fig. 143.

172. In figs. 153, 154, 155, 156, are pale cells, such as those constituting the foundation of the crystalline. As was observed by SCHWANN, there is a nucleus in many of these cells; but, as he likewise saw, there are very many in which no nucleus is found. On very closely examining the cells which had no nucleus, I have thought many of them filled with discs (fig. 153.). These discs (if I was not mistaken in the observation) had no doubt resulted from a division of the nucleus of the cell. But the cells are so very pale, that often nothing can be discerned in their interior. In figs. 156 and 159, a nucleus is seen to have divided into two parts; each of which was a disc, and presented an indication of subdivision into other discs.

173. Besides confirming, in this way, my opinion as to the mode of origin of the

contents of "cells" in general, an attentive examination of these pale cells of the crystalline has also strengthened a view mentioned in my last memoir[†],—that, however minute the "cell," and wherever it exists, it is by the coalescence of discs that its membrane is formed. Thus in fig. 156. α, β, the outer discs are seen entering into the formation of the membrane of the cell; the remainder being now the nucleus of the cell. At β, fig. 154, the cell-membrane is formed. We thus seem to have even here a mode of origin of the membrane of the cell, essentially the same as that which, in a former paper, I showed to form the chorion[‡],—as will be hereafter pointed out in the membrane of the ovisac,—and as seems to be the mode of production of other, and perhaps all, membranes, as well as fibrous tissues.

174. Other facts observable in an examination of the pale cells of the lens, are equally confirmatory of my views regarding "cells" in general. The nucleus is very distinctly seen to be composed of discs (figs. 154, 155, 156.), into which it divides; and at a certain part, there is sometimes seen a highly refracting object, corresponding apparently to the nucleolus of authors, but, as I believe, being really an orifice communicating with the exterior of the cell.

175. I have already mentioned my inability to confirm the idea of Professor Schwann, who supposed it to be by *elongation,* that these pale cells form the fibres of the lens. It is true he makes the addition, "I have several times observed the arrangement, one upon another, of the nuclei of cells; but do not know what it indicates. It is also very possible that the coalescence of cells may take place to form a fibre; but hitherto I have no decisive observations[§]." It affords me satisfaction to find that, while bringing forward facts opposed to the views of this excellent observer, I am not without an admission—very important as coming from him—of the possibility that the fibres may arise by another mode.

176. It remains to add, that the appearance now and then, in very large numbers, and in parallel lines, of objects such as those in figs. 157, 158, 159, has led me to believe that the fibre of the lens is formed by the *coalescence of the cells in question, previously arranged in a line.*

177. These fibres in many instances originally present the nucleus in their cells (see the figures). Sometimes the so-called nucleolus—really an orifice—is also present (figs. 157, 158. α), communicating with the exterior of the cell. In another instance (fig. 157. δ), I saw the cell filled with discs, there being no nucleus in this cell.

178. The fibres are originally of a necklace form; their bead-like segments being often of unequal magnitude (see the figures). When this is the case, deficiency in size is sometimes compensated by the number of the cells (fig. 158. β). It is probable that in other instances an equal diameter throughout the fibre, is obtained by elongation of the larger cell. Sometimes the cells are very large (fig. 159.). The bead-like segments coalesce; the intervening membranous partitions disappear; and thus

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[†] On the Corpuscles of the Blood, Part II. *l. c.*, par. 79.

[‡] Researches in Embryology: Third Series, *l. c.*

[§] *L. c.*, p. 102.

a cylinder is formed. In the parietes of this cylinder I have noticed pellucid objects, possibly denoting the continued existence of the orifices above mentioned, as present in the nuclei. If such has been the case, it would appear that there exist communications between the interior and the exterior of the cylinder (pars. 130, 152, 162.). At all events we cannot doubt but that here, as in muscle (par. 161.), the nucleus of the original cell is the source from whence the contents of the cylinder immediately proceed: and that here, therefore, as elsewhere, we have special centres for the origin of new substance.

179. The main question, however, continues,—are the cells we have been considering, derived from corpuscles having the same appearance as corpuscles of the blood? At the surface of the lens, we saw an abundance of young corpuscles forming for some special purpose, and corpuscles, having a similar appearance, exhibiting very remarkable internal changes. Other changes are represented in fig. 155; the objects in which figure, found at the edge of the lens, had the same appearance as altered corpuscles of the blood. The two in outline (α) exhibited the characteristic red, as well as the general appearance of blood-corpuscles, in an unaltered state; excepting only, that in their irregular contour, there were some indications of a future division into discs. γ was a flattish corpuscle, composed of discs, and still blood-red. The objects δ, δ resembled γ, but were larger. The nuclei of the cells ε, ε, ζ, η, were quite red enough to show their origin to have been in corpuscles having the same appearance as corpuscles of the blood. All the objects in this figure thus presented red colouring matter,—which in α, β, γ, δ pervaded the entire object, but in ε, ζ, η was confined to the nucleus of the cell. The more or less incipient cells of fig. 156, were obviously altered corpuscles having the same appearance as corpuscles of the blood. And the remark just made, respecting colour, applies to the nuclei observed in the other cells above referred to as the elements of the crystalline (fig. 154.), as well as to the nuclei of those cells which we have seen arranging themselves to form its fibres (figs. 157, 158, 159.).

180. These are the principal observations, which have induced me to believe the crystalline lens to be derived from corpuscles of the blood. And, upon the whole, I should find it difficult to point out a line, separating the one from what are allowed to be the elements of the other.

181. It may be added, that among the figures there will be found three outline sketches (figs. 138, 139, 140.), representing appearances incidentally observed, in the course of my examinations of the Tadpole. The first of these, in a Tadpole of 5‴, seemed the foundation of the crystalline, as it lay, surrounded by black pigment, and imbedded in the vitreous humour. It had the appearance of an altered and prodigiously enlarged corpuscle of the blood; being throughout red, except at the anterior part, α (where it was colourless and pellucid), and being filled with discs, resembling those arising in the interior of corpuscles of the blood. At the part marked β, these discs were largest, and of the deepest red. This appearance, together with those

exhibited in figs. 139, 140, observed in similar situations in others of these larvæ (for a particular account of which I refer to the explanation of the Plates), induce me farther to believe that the crystalline may have its origin in a *single* corpuscle, having the same appearance as a corpuscle of the blood.

The Elements of the Spermatozoon and those of the Ovum compared with Corpuscles of the Blood.

182. I had made the principal part of the foregoing observations,—when two others followed, which had not been at all anticipated. For, although the facts observed had led me pretty nearly to the conclusion, that every tissue in the body has its origin in corpuscles having the same appearance as corpuscles of the blood,—yet the thought, I confess, had not occurred, that the spermatozoon and the ovum might be immediately derived from the same source. It was not until the red colouring matter was noticed, by which I recognized corpuscles having the same appearance as altered blood-corpuscles, in some seminal fluid under examination, from the testis of a Bird, that the idea suggested itself, and led to a farther examination.

183. The experience gained during the long investigations, the principal results of which have been mentioned in the preceding pages, now make it easy for me to see that the "granules" which previous observers had noticed in this fluid, were masses of discs, or rather, cells filled with discs,—the altered nuclei of corpuscles having the same appearance as corpuscles of the blood. And on examining the ovary, I became equally convinced, that the object figured by myself in the Royal Society's Transactions three years since, as the ovum in a rudimental form,—while it admitted of delineation just as I then represented it,—was also derived from the same source.

184. I need scarcely mention the satisfaction afforded by these two additional observations; not only on account of their being in themselves in the highest degree interesting, and, as it appeared to me, important,—but because of the confirmation I of course believed them to give to all the rest.

185. In passing into the granular mass, or more properly, into the cell filled with discs (fig. 160.), where the spermatozoa seem to form, the nucleus of the corpuscle in question presented to me appearances in some respects similar to those which I had met with in tracing it into tissues. But I was perhaps more struck with the depth of the red colour, in the more advanced elements of spermatozoa that fell under my notice. The object fig. 161, for instance, was of a deep red. I have seen these seminal "granules" in some of the Mammalia; certain of them appearing to contain incompletely formed spermatozoa.

186. The view, however, just propounded implies another; which, so far as I know, is also new. The so-called spermatozoon appears to me to be composed of a few coalesced discs. Such has appeared to be its condition in the Rabbit, and in certain Birds. Of course the different forms of spermatozoa in different animals, sug-

gest variety in their particular mode of origin. But I have no reason to doubt that the foundation of spermatozoa in general consists of the objects I have denominated discs.

187. The corkscrew-like spermatozoon of certain Birds has presented to me appearances, which it seemed worth while to delineate (figs. 162, 163.); and I recommend that the description of these figures should be referred to; leaving it, however, for the future to determine, whether the curious division of discs in a longitudinal direction, there suggested, really takes place.

188. In a former page, I referred to a parent corpuscle (fig. 94. α) having the same appearance as the corpuscle of the blood, filled with young corpuscles, which had been destined to form epithelium-cells. I have now to mention that young corpuscles are met with, while still within their parent cell, manifesting a very different destination. The cell in fig. 164. is a parent corpuscle having the appearance of an enlarged corpuscle of the blood; and each of the contained objects is a young corpuscle. But each of these young corpuscles is also a rudimental ovum. (Compare with the ovum in my First Series on Embryology, *l. c.*, Plate V. fig. 19.) The objects also in fig. 165, though mere *discs* having the same appearance as blood-discs, are rudimental ova. Those in figs. 166, 167, 168, 169,—presenting the ovum in states somewhat more advanced,—are also altered corpuscles having the same appearance as corpuscles of the blood. Even the most forward of the objects now referred to, presented red colouring matter quite sufficient to show, to an eye accustomed to these investigations, from whence they arose. But besides this, they admitted of being traced back into corpuscles having the same appearance as corpuscles of the blood (figs. 164, 165.). And all the objects in question, it may be added, were from the ovary of the same individual Bird.

189. The essential part of the ovum in these figures, is that marked *c*. It is the germinal vesicle: very much in advance, it will be observed, of other parts, in the degree of its development (see its large size in fig. 169. γ); being, as I originally said, the first part of the ovum which is formed. Its progress admits of being traced, from merely a pellucid space—the centre of a corpuscle having the same appearance as a blood-corpuscle, fig. 165. α—through the discoid form, as in the centre of the corpuscle β—to the state of an incipient cell, as in γ of this, and in some of the other figures. I have seen the future germinal vesicle, as a disc, measuring in diameter no more than $\frac{1}{700}$th of a Paris line (figs. 164, 165.).

190. The most essential part of the germinal vesicle, is seen in many of the figures which represent the latter. While the future germinal vesicle is a mere disc, its most essential part is the depression in the centre of the surface of this disc: and when the future germinal vesicle has really become somewhat vesicular in form, there is seen an orifice in the same situation (figs. 165 γ, 166, 167.). This orifice denotes the situation of the essential part of the germinal vesicle,—the future germinal spot.

We thus find that in the minutest details, such as those which I have elsewhere described, the germinal spot and vesicle are formed like other nuclei and cells.

191. I long found it impossible to understand in what way the membrane of the ovisac, as I termed it, was formed around the mass of peculiar granules having the germinal vesicle in its interior[†]; and equally difficult to conceive the mode of origin of the discus vitellinus, the true yelk, and the membrane of the yelk. It now appears to me that all these (as well as the germinal vesicle and spot) originate in a corpuscle having the same appearance as a corpuscle of the blood. That part of this corpuscle which surrounds the germinal vesicle (see the figures), becomes the source of the peculiar granules (as I formerly termed them) by which this vesicle is surrounded. These peculiar granules—the objects subsequently contained within the ovisac—are not originally cells: so that the term granule proves to have been not inappropriate. The fact is, they are *discs*, having the same form and general appearance as discs in other situations (only being very red); and by a change the same as that we have so often seen elsewhere, assume a cell-like appearance, and are reproduced in the same manner as other discs. Hence, in advanced states of the ovisac, the quantity of these objects becomes very large: and, as we formerly had occasion to observe, they arrange themselves into structures, some of which—the retinacula—enter into the formation of the mechanism regulating the expulsion of the ovum[‡].

192. But my purpose in referring to these discs so fully in this place, is to be enabled to make the following addition: namely, that they also give origin to the membrane of the ovisac, the discus vitellinus as well as the true yelk, and the membrane of the yelk. And as the discs in question are derived from a corpuscle having the same appearance as a corpuscle of the blood, so therefore are all these objects.

193. Delineations are given of the membrane of the ovisac thus forming. In figs. 170 and 171. the discs (*g*), quite red and large where in the neighbourhood of the germinal vesicle (*c*), were undergoing division; the young and paler discs passing outwards, increasing in size, dividing in their turn, then enlarging, and coalescing to form the membrane (*h*) of the ovisac. Traces of these coalesced discs, as well as a tinge of red, are to be discerned even after the ovisac has attained a considerable size; producing the appearance which, when first describing this membrane, I compared to that of plaits or folds.

194. I do not think that there is any essential difference between the discus vitellinus and what is called the true yelk: at least it would seem difficult to draw a line between the two. Where it is required to provide before-hand a stock of substance for fœtal use,—as in many of the Ovipara,—the discs *g* are made to form the yelk in

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[†] First Series on Embryology, *l. c.*, pars. 14, 23.

[‡] It may now be added, that the delicate membrane I described as sometimes seen investing the retinacula, and reflected from their branches to the membrana granulosa, seems to be formed by the coalescence of some of these discs.

large quantity. But in viviparous animals, these discs appear to produce little more than an object corresponding to that which in the Bird, for instance, has the form of the discus vitellinus. Fig. 173. presents these discs (*g*) rapidly dividing for the formation of the yelk in the Bird.—I refer to my First Series on the Embryo (*l. c.*, Plate V. fig. 25. *e*) for the earliest appearance of the membrane of the yelk.

195. The very minute ovisacs, myriads of which I described as found in the walls of the Graafian vesicle, seem to owe to the following circumstance their parasitic situation. The covering acquired by the ovisac consists of cellular tissue. Cellular tissue is formed out of corpuscles having the same appearance as corpuscles of the blood. But each of the minute ovisacs was once a young corpuscle of the same kind. And it appears that, while some of these corpuscles enter into the formation of the cellular tissue investing a large ovisac, other corpuscles are developed into smaller ovisacs, which therefore are found in the interstices of that tissue.

196. I have already stated that the individual discs exhibit a reproducing property. This is the case as well with those discs that have begun to enter into the formation of a structure, as with those not yet appropriated,—that is, still in circulation. With regard to the former it is to be remarked, that, in many parts, red colouring matter is reproduced along with the new discs (par. 111.), giving them quite as deep a colour as the floating corpuscles themselves; which in other respects also they resemble. Hence my general employment of the expression, *having the same appearance as* corpuscles of the blood: for it was impossible to distinguish those corpuscles which had been themselves extravasated; and I presume that, generally speaking, the term "corpuscles *of the blood*" would have been inapplicable to others, though immediately descended from them.

197. *Recapitulation.*

1. The nucleus of the corpuscle of the blood admits of being traced into the pus-globule.

2. The various structures arise out of corpuscles having the same appearance, form, and size as corpuscles of the blood.

3. The corpuscles having this appearance, and giving origin to structures, are propagated by division of their nuclei.

4. The corpuscles of the blood, also, are propagated by division of their nuclei.

5. The minuteness of the young blood-corpuscles is sometimes extreme; and they are to be found in parts usually considered not permeable by red blood.

Postscript (June 23, 1841).

198. Blood found in the heart (immediately after death by bleeding) often presents incessant alterations in the position of its corpuscles. When one of the corpuscles is

examined very attentively, it is seen to change its form; and I am disposed to think it is this change of form that produces the alterations in position. The changes of form are slight, as compared with those referred to in par. 127, and are not seen without close attention. The motions resemble that called molecular, and in the minutest corpuscles, which are mere points, nothing besides molecular motion can be discerned. It may be a question, whether molecular motion differs in its nature from the motion of the larger corpuscles just referred to. The division of the blood-corpuscles into corpuscles of minuter size, though apparent in blood from either side of the heart, has seemed more general in that from the left side; which is perhaps deserving of notice in connection with the subject of respiration.

199. (*October*, 1841.) As explanatory of the foregoing paragraph, I may perhaps be permitted to mention some appearances that have since fallen under my notice. It is necessary, however, to apprise the reader that the remarkable appearances about to be detailed are not all of them to be always found; and indeed that sometimes most if not all of them are absent. To what circumstances their presence or their absence is to be attributed, it would be premature at this time to speculate, without facts that are still wanted regarding the action of the atmospheric air upon the blood-corpuscles in their passage through the lungs; and some unknown change which, from observations not yet completed, I am inclined to think these corpuscles undergo in that portion of the circulating fluid which passes through the liver. Neither am I in possession of facts connected with the process of digestion, which may possibly influence the reproduction of the blood-corpuscle; supposing this reproduction to be effected in the way suggested by the observations of the foregoing paper, and my previous memoirs. And here I am reminded to mention, that some of the animals from which the blood in question (after death) was taken, had not received food for many hours previously. Such were the sheep at the London slaughter-houses. In these instances, therefore, the minute objects about to be described cannot be referred to the formation of chyle, and its addition to the mass of blood.

Without attaching undue importance to the circumstance, it should be stated that the blood presenting the corpuscles about to be described, was taken from the left auricle, and therefore had recently passed the lungs. In the first place, I have at different times found that there exist corpuscles much larger than the rest, and a few of them of prodigious size ($\frac{1}{50}'''$). They are, as I have seen them, always *very pale, and sometimes even colourless.* They are obviously membranous at the surface. You sometimes see them ruptured, and partially discharged of their contents. In this state, they frequently appear shrivelled. When not ruptured they are distinctly filled with young corpuscles. Secondly. The young corpuscles after liberation from the parent corpuscle, and sometimes before that change, are seen to have acquired red colouring matter. When so liberated and individualized, they present a star-like form resembling that in figs. 104 and 105. This form I have in some instances noticed them to have while within the parent corpuscle. In a good light, these star-like corpuscles are evidently seen to be compound objects, consisting of about half a dozen segments. Each of these segments sends out a cilium; and it is by this means that the appearance of a star is produced. It is often possible to perceive these star-like corpuscles effecting alterations in their form, slowly revolving, and performing locomotion. Thirdly. Besides the two orders of corpuscles now described I have seen a third, of minuter size, apparently arising by division of those last mentioned; of which, therefore, they must have been the separated segments. Their motions were extremely vivid, and of such a kind as might be produced by the rapid vibrations of a tail-like appendage or cilium. Such cilium each of them would possess if they arise, as I think they do, by division of the ciliated corpuscles above described. Fourthly. Many corpuscles also, about the size of those last mentioned, are to be seen in the same fluid, of a star-like form. Lastly. There are others, so immeasurably small as to appear as mere points. They have precisely the same colour as corpuscles of larger size, and exhibit most vivid motions.—Some of the states now described, I have seen in the blood of the Common Mussel.

200. Explanation of the Plates.

PLATE XX.

Fig. 63. Man. Corpuscles found in fluid having nearly the colour of blood, taken from an abscess. This figure shows the young corpuscle of the blood becoming changed into the pus-globule. See par. 101. for the details of this change. In the objects presented by this figure, proceeding from left to right, a gradual enlargement was observed, as well as a change from the flattened to the globular form; and those on the extreme right had lost, in some degree, the peculiar tint characterizing the corpuscle of the blood. (The more or less altered blood-corpuscles in the fluid from which this figure was taken, were observed to arrange themselves into rouleaux, like those in blood unaltered.)

Fig. 64. Man. Globules and cells from well-formed pus, after the addition of (α) acetic acid, and (β) dilute spirit.—α. The largest globule was still surrounded by, and eccentric in, an elliptical cell, which is shown in outline. A corresponding cell had perhaps disappeared from around the other globules; for many such cells were seen in this pus, before any addition had been made. When a cell is present, the pus-globule is the nucleus; and its eccentric, highly refracting portion, is the nucleolus. The pus-globule is composed of elliptical discs; those forming the nucleolus having a high refracting power, after, as in this instance, the addition of acetic acid. The nucleolus consists of either a

single disc, or two or three discs. Where the discs of the nucleolus lie one on the other, the degree of refraction is very small. β. Similar objects, as viewed after the addition of dilute spirit. Each of the two lower ones has the appearance of being circumscribed by a membrane, and is contained within a minute cell. The upper object presents three layers of discs; the outer layer being pale.

Fig. 65. Man. Fragment of a capillary vessel, found in the pus of the preceding figure. It is filled with altered corpuscles of the blood. The largest corpuscle presented an appearance resembling that of the pus-globule. (Acetic acid.)

Fig. 66. Man. Altered blood-corpuscles, observed in fluid from abscesses.

Fig. 67. Man. Altered blood-corpuscles, observed in fluid from the intestinal canal, where the existence of pus was suspected (compare with fig. 63.). α Corresponds, apparently, to the "lymph-globule" of authors.

Fig. 68. Man. Globules and cells in mucus from the air-passages of a healthy person. This mucus had in some parts a slight tinge of yellow. (Acetic acid.)

Fig. 69. Man. Globules and cells in mucus from the air-passages of a healthy person. This mucus was a tenacious, colourless, pellucid fluid. (Acetic acid.)

Fig. 70. Man. Cells in mucus from the Schneiderian membrane. The discs blood-red. (Acetic acid.)

Fig. 71. Man. A later stage of apparently corresponding objects, taken from the same part. (Acetic acid.)

Fig. 72. Man. From blackish mucus of the air-passages. α. Corpuscle having the same appearance as a young corpuscle of the blood; blood-red. β. Mass of blood-red discs; which had the appearance of an altered corpuscle of the blood. Such also was the appearance of γ, δ, ε, ζ. γ In outline: colour blood-red. This object was elliptical, and presented an orifice at one end; the orifice occupying the situation of the depression existing when the corpuscle had a discoid form. The object γ was composed of discs. δ, ε. Similar objects; but presenting numerous black points. Some of these black points are seen to have been in the centres of the discs, and others between the discs. The orifice was less distinct in ε; but there was a pellucid part in the same situation. ζ. Outline of a later state of the last-mentioned object.

Fig. 73. From the same mucus. α. Corpuscles, having the same appearance as young corpuscles of the blood. They were of the characteristic colour of blood-corpuscles, and exceedingly minute, but not the minutest seen; mere points having been observed in large number, of the same colour, and apparently derived from the same source. β. Outline of

similar corpuscles, arranging themselves into fibres, and then exhibiting indications of division into discs still more minute. γ. Outline of corpuscles of the same kind, arranging themselves into a cellular tissue-like object, and at the same time undergoing division into discs. β, γ. Blood-red.

Fig. 74. Tadpole of the large Toad of Jersey; about 6‴. α. Young blood-corpuscles; some of them exhibiting an orifice. Colour, pale red. β. Young blood-corpuscles of a deeper red.

Fig. 75. Tadpole, about 6‴. Blood-corpuscles; partly in outline. α. The minute red points around the nucleus in this corpuscle exhibited molecular motions. β. Discs are seen in the situation occupied by the minute red points in α; these discs pressed into polyhedral forms.

Fig. 76. Pale objects, composed of discs, from the blood of the same Tadpole. They correspond apparently to the "lymph-globules," or "corpuscles of the second form," of authors. α. The discs are numerous and minute. β. Discs fewer and larger. γ. The object in two portions,—perhaps a nucleus (eccentric), and an incipient cell; the nucleus composed of pale discs,—the surrounding part reddish. Discs were distinctly visible in this surrounding part also.

Fig. 77. Water Newt. Outline of blood-corpuscles, some of which were observed to contain blood-corpuscles, besides their nucleus.

Fig. 78. From the same. Objects found with blood-corpuscles,—all in outline except one. The latter, much enlarged, is seen to have been filled with, or made up of, discs; which was the case with all the rest. These objects appear to correspond to the "lymph-globules" of authors.

Fig. 79. From the same. Outline of a blood-corpuscle, and of the discs contained within it. The central object is the nucleus. (Acetic acid.)

Fig. 80. From the same. Outline of blood-corpuscles and their nuclei, as seen after the addition of an aqueous solution of nitrate of silver. The interior of one of the nuclei is shown. It was filled with discs, as were all the others.

Fig. 81. From the same. Outline of two blood-corpuscles, as viewed after the addition of a solution of corrosive sublimate. The interior of the nuclei is shown. α. In the nucleus were discs of about equal size. β. The nucleus presented discs at its central part; around which there was a space, apparently occupied by discs in a more advanced state and larger.

Fig. 82. Green Lizard. Corpuscles of the blood. α. Outline of one of these. β. Two corpuscles, the nuclei of which were composed of discs. γ. Corpuscle filled with discs. δ. Disc of the same colour as the corpuscles of the blood: perhaps a young corpuscle.

Fig. 83. Outline of blood-corpuscles from the egg of the Duck, incubated about 12 days.

Fig. 84. Blood-corpuscles, chiefly in outline, from the same egg. They are apparently in a stage more advanced than those in the preceding figure, with which, however, they were mixed. A nucleus, composed of discs, has been represented in some of these; and in one of the corpuscles, discs are seen surrounding the nucleus.

PLATE XXI.

Fig. 85. Tadpole, about 5‴. From the tail. Outline of epithelium-tables, having essentially the same appearance as blood-corpuscles circulating in the vessels of this larva. The peripheral portion of the six-sided tables resembled that of the object fig. 89.

Fig. 86. Tadpoles, about 5‴. Epithelium-tables, from the tail. α, β. Removed from the lacerated edge of the tail. α Is in outline. The form and size are here seen to have been very much the same as those of the blood-corpuscle in this larva. The same remark applies to the interior of this object, which is not shown in the figure. The envelope was also membranous. β. The germinal vesicle-like nucleus eccentric, and scarcely coloured: the discs on the nucleus, blood-red. The other objects in this figure were seen *in situ.* γ Resembled β, but it was membranous at the surface. δ, δ, ε, ζ, Resembled the ovum in their interior, and were blood-red at the surface.

Fig. 87. Tadpole, about 5‴. From the tail. Outline of the appearance presented by an epithelium-table undergoing division. (Dilute spirit.)

Fig. 88. Tadpole, 6‴. From the tail. Outline of three stages in the reproduction of epithelium-tables, having a situation corresponding to that of the large objects (centres) connected with pigment ramifications in figs. 90 and 91. The object on the left hand (in fig. 88.) represents the earliest, and that on the right the most advanced, of these three stages. These tables propagate by division, like every other disc. Each of the objects in this figure consisted of two parts; of which α was dark,—the other, β, colourless and pellucid. α. Oldest and largest tables; β, newest and smallest tables,—mere discs. In the centre of β, in the largest object, there was seen a part still more pellucid. β Is not in the centre of the object, but on one side. This corresponds with the situation of the most essential part in all other discs. In some instances, the number of parts into which objects such as those in the present figure were dividing, was observed to be four; this having been a stage still earlier.

Fig. 89. Tadpole, about 5‴. Peripheral parts of several epithelium-tables, consisting of red discs, which (red discs) enter into the formation of the so-called ramifications of pigment (par. 118.). The central portion of the epithelium-tables here seen, appeared to be dividing into four parts, which, with a pale surrounding substance, are represented in outline only. From the tail.

Fig. 90. Tadpole, about 5‴. A later stage of partitions (pigment ramifications) such as those in fig. 89, together with an object of a deep red colour, and an ovum-like interior. It is a centre for the origin of new epithelium-tables. From the tail.

Fig. 91. Tadpole, about $5\frac{1}{2}$‴. Appearance presented by portions of the choroid, arisen in the manner described in par. 118, as that in which the ramifications of black pigment in the epithelium of the tail have their origin. One of the objects is in outline. The so-called ramifications are really partitions ; but not represented as such in the figure, which is intended to show no more than their appearance on a superficial view.

Fig. 92. Man. Objects from the surface of a furred tongue. α, β. Outline of epithelium-cells. Their contents, more or less red discs, which were reddest the nearer they were to the nucleus, excepting that in α the very red nucleus was surrounded by a more pellucid space, apparently occupied by larger discs. In β, the nucleus was very large ; its pellucid nucleolus measuring as much as the nucleus of α. The nucleolus of β seemed to contain pale discs. γ. Outline of a mass of pale blood-red discs.

Fig. 93. Tadpoles, about 5‴. Pigment of the eye, forming out of objects resembling corpuscles of the blood. The part marked ζ is that which had become the blackest. α. Four young blood-corpuscles, observed, along with others of the same kind, in the choroid. They had probably arisen in a manner analogous to that giving origin to the objects at α fig. 94. Three of these young corpuscles are in outline. They were composed of discs ; and in the finished one, a pellucid nucleus was visible on one side. They were blood-red. β, γ. From the same part. β For the most part bright red, but approaching black in some parts. A nucleus visible in each,—distinctly in β, obscurely in γ. An orifice on one side in the nucleus of β. δ. An object somewhat resembling β. The nucleus excepted, it was blood-red. ε. Part of an object of the same kind as δ. ζ. Partitions between spaces, such as those occupied by δ and ε ; at first sight appearing black, but, when more closely examined, found to be of a reddish colour. The discs of the bright red δ and ε undergo division, and are given off, to enter into the formation of the darker and blackish ζ. It appears that what is seen of ζ in the figure,

had been formed by portions previously given off in this manner from the centre δ. The object now described, on the right hand in the figure, was lying on the crystalline. I have seen similar objects in the choroid. The mode of origin of the partitions ζ, just described, is precisely analogous to that of the partitions in fig. 89, from the tail.

Fig. 94. Tadpole, about $6'''$. α. Outline of a cell, filled with young epithelium-cells. This cell was red throughout. The cells which it contained, presented indications of further division, as seen by the one somewhat more finished, in delineation, than the rest. And at the left side, one of these young cells was broken down into cells or discoid objects of extreme minuteness. β. Two epithelium-cells, of a blood-red colour, except at the lower part, where they had become quite black. (Acetic acid.)

Fig. 95. Tadpole, about $6'''$. Blood-corpuscles, apparently forming epithelium. All of these contained discs or incipient cells, which, generally speaking, had a deeper red, the nearer they were to the centre of the enlarging nucleus. The nucleus increased so as nearly to fill its corpuscle. In α, it consisted of a central portion, composed of four cell-like objects; and of two parts, concentrically arranged around these. β. The corpuscle is in outline. Four cell-like objects were seen within it, each of which was filled with discs. (Acetic acid.)

Fig. 96. Tadpole, $5'''$. Epithelium-cell, having very much the appearance of an altered corpuscle of the blood. At β, it was nearly black; at most other parts, blood-red. Discs were seen, with more or less distinctness, at nearly all parts. At α, the surface was membranous; and internal to this portion of membrane, the substance was of a pale red, and free from the very highly refracting globules so numerous elsewhere in this object. At a certain part, is seen the nucleus; pale in colour, and in some degree pellucid. It presented discs.

Fig. 97. Tadpole, $5'''$. An object entering into the formation of the epithelium, and for the most part blood-red. The eccentric nucleus exhibited a nucleolus, having apparently a peripheral situation. This nucleolus seemed to be an orifice, possibly communicating with the exterior. (Tincture of iodine.)

Fig. 98. Tadpole, about $5'''$. Incipient epithelium-cylinders. Colour red, passing at the large extremity almost into black. Some of the discs composing the object α, were in motion, effecting changes in their form: and the object very slowly revolved on its axis, in the direction of the arrow. β. The same object, as viewed about half an hour after it presented the appearance at α. Its form was different. This may have arisen partly from a change in its direction,—the part which is pointed in β, having possibly been, in the condition α, directed towards the

observer. But it appeared that some alteration had really taken place in the form. A further change was noticed. The discs of the dark part in the condition α, were very indistinct. In the state β, they were well defined, and presented the appearance of little cylinders: which appearance was noticed also in the paler part of β. Motion not observed in the condition β. γ. State rather more advanced. The nucleus presented an orifice, having the appearance of a rent or fissure. The red colour was very deep. δ. Two of the appearances presented by a minute and isolated disc which was in motion,—changing its form and place.

Fig. 99. Tadpole, $5'''$. An object, which was apparently an epithelium-cylinder, about to undergo longitudinal division. It contained two nuclei; one in a more advanced state than the other. Each of these was composed of discs, situated in concentric order around a space representing the nucleolus. This object was blood-red. (Tincture of iodine.)

Fig. 100. Tadpole, about $5'''$. More advanced state of a corresponding object. The two nuclei pale. The surrounding part blood-red, passing nearly into black at some parts, especially at the lower end.

Fig. 101. Tadpole, about $5'''$. Outline of portions of ciliary processes; the parts composing them having the same appearance as corpuscles of the blood.

Fig. 102. Duck. α. A cell observed lying among blood-corpuscles taken from the surface of the yelk, in an egg incubated five days. It had the same colour as the corpuscles of the blood. The membrane of this cell was of extreme delicacy. It contained a pale yellow substance, in which were globules, yellowish in colour, highly refracting light, and in vivid molecular motion. These motions were not observed when the cell was first seen; at which time, also, the contained globules were in closer approximation than the figure shows. Possibly the cell was ruptured while under examination. β. Similar globules, myriads of which were seen loose in the same field of view. Their molecular motion was most vivid, and attended with constant and considerable change of place.

Fig. 103. Tadpole, $4\frac{1}{2}'''$. The larger object is a blood-corpuscle, and contents, for the most part in outline, very much enlarged. The interior presented globules resembling those of fat. These globules appeared to occupy the situation of the central part of what had previously been discs. Compare the disc α, and its centre, with apparently corresponding objects at β. The corpuscle exhibited redness at all parts, but chiefly on the side γ,—where, here and there, it was very dark,—though this appearance has not been at all represented in the figure. Something

like a nucleus was visible at δ. The smaller object is a blood-corpuscle of nearly the size usually met with. It also is, for the most part, in outline. It appeared to be an early state of such an object as the larger one. δ. Situation of the nucleus.

PLATE XXII.

Fig. 104. Ox (*Bos Taurus*, Linn.); fœtus of 5¼ inches. Nuclei of blood-corpuscles furnished with cilia, and changing their place. Colour blood-red. Two of them in outline. α. Observed in substance cut with scissors from the crystalline lens, while the lens was still imbedded in the vitreous humour; so that a portion of each may have been placed in the microscope. Two days had elapsed since the fœtus was taken from the body of its mother. β. Seen along with a portion of the retina and black pigment, from the other eye. The discs of this corpuscle appeared to shoot forth a process—the cilium—which then disappeared, as if drawn in. This corpuscle, as well as those at α, crawled about like an insect; but very slowly.

Fig. 105. Rabbit (*Lepus Cuniculus*, Linn.); killed two hours, *post coitum*. Blood-corpuscles and nuclei of blood-corpuscles observed in fluid taken from vessels in the immediate neighbourhood of a Graafian vesicle, which, from its size and vascularity, had evidently been destined to expel an ovum. α. A group of young blood-corpuscles. β. Outline of the nucleus of a corpuscle, one of the projections (altered discs) in which, appeared to be in motion. γ, γ, γ, γ. Four ciliated corpuscles,—or rather, ciliated nuclei of corpuscles,—of the blood. The cilia seem to be the filamentous extremities of discoid objects, into which the nucleus of the blood-corpuscle becomes divided. Objects such as those at γ, were seen very gradually to change their place; and others, of similar forms, were noticed to revolve; both of these effects seeming referable to their cilia. Examined eighteen hours after death. Among the corpuscles in this figure (and therefore apparently from the interior of a blood-vessel), were many objects of immeasurable minuteness, exhibiting molecular motions. These minuter objects had precisely the red colour of corpuscles of the blood, in which they probably had their origin (see par. 167.). δ. Outline of the nucleus of a blood-corpuscle, composed of discs not terminating in cilia, like those of γ.

Fig. 106. Common Fowl (*Phasianus Gallus*, Linn.); chick *in ovo*. Young blood-corpuscles observed in the immediate neighbourhood of the object fig. 116½. One of these is in outline. They were blood-red. These young blood-corpuscles, with others of the same kind, were in constant

motion; effecting a change of place (see par. 126.). Examined twenty-four hours after death.

Fig. 107. Ox; fœtus of eighteen inches. From the retina. Capillary vessels, forming out of corpuscles, having the same appearance as young corpuscles of the blood. These corpuscles, while still red (α), apply themselves together so as to form a necklace-like object, composed of elliptical beads: and having coalesced, and become pale (β), and the membranous partitions having disappeared, they form a tube. The corpuscles seem to apply themselves at certain parts in such a manner as to form a branched vessel. Resolving themselves (here, as well as when forming other tissues) into discs, the corpuscles contain within themselves the elements of new corpuscles. See α in the upper part of the figure. γ. Group of blood-corpuscles, and what seemed parts of blood-corpuscles (chiefly in outline), which may serve as a specimen of the many forms of these corpuscles, observed along with the foregoing. The central corpuscle in this group, resembles one of those marked β in fig. 63; where the nucleus of the blood-corpuscle is forming the pus-globule. (Blood-vessels seen forming in the same manner, and out of similar objects, in the retina of a chick; the egg incubated ten days.)

Fig. 108. Ox; fœtus of about seven inches. Outline of blood-corpuscles, observed among fibres from a muscle of the thigh. Most of these corpuscles were orange-shaped or globular, and exhibited an orifice. Some of them presented discs in their interior. The minutest objects in this figure were of the same colour as the larger ones,—blood-red.

Fig. 109. Ox; fœtus of about seven inches. Corpuscles having the same appearance as blood-corpuscles, observed among fibres from a muscle of the thigh. They appeared to be passing into the elements of cellular tissue. α. The corpuscle has one large orifice. β, β, β, β, β. It exhibits two or more small orifices. γ. The corpuscle has passed into a mass of discs; this mass being, not cylindrical, but flattened. In such objects as β and γ, the outline of discs is hidden, apparently, by red colouring matter.

Fig. 110. From the same. Altered corpuscle, having the same appearance as a corpuscle of the blood. It had become a mass of discs resembling those marked γ in fig. 109, but terminated at each extremity in a fibre; thus presenting a more advanced stage in the formation of cellular tissue.

Fig. 111. Ox; fœtus of about five inches. Altered corpuscles having the same appearance as blood-corpuscles, passing into cellular tissue. The filaments into which the corpuscles are prolonged, consist of coalesced discs. Colour quite red. Taken from under the superficial muscles of the neck.

Fig. 112. Ox; fœtus of five inches. From the axilla. Three altered corpuscles having the same appearance as corpuscles of the blood. *α*. Many bright points seen (par. 134.). *β*. Discs now visible. *γ*. Some of the discs have enlarged, while others have divided into smaller discs, which are coalescing, and thus entering into the formation of the filaments of cellular tissue, with the elements in themselves of further perpetuation. This corpuscle (*γ*) was bent, from its position in relation to *β*. (Tartaric acid.)

Fig. 113. Ox; fœtus of five inches. Altered corpuscle having the same appearance as a blood-corpuscle, passing into cellular tissue. (Transition unequivocal.) From the axilla. (Citric acid.)

Fig. 114. Rabbit; killed two hours *post coitum*. From the fimbriated extremity of the Fallopian tube. Cellular tissue, forming out of altered corpuscles having the same appearance as corpuscles of the blood.

Fig. 115. Tadpole, about 6'''. Outline of altered corpuscles having the same appearance as corpuscles of the blood. They are composed of discs, entering into the formation of cellular tissue. All the objects were pale red; and the transition out of corpuscles having the same appearance as unaltered blood-corpuscles was observed.

Fig. 116. Tadpole, 5'''. Outline of incipient cellular tissue, composed of discs. It was forming out of corpuscles having the same appearance as corpuscles of the blood; and still red. This cellular tissue lay around the spinal chord, at a part where the latter had a diameter of $\frac{1}{20}'''$; being apparently in the course of forming the sheath of the spinal chord. *α*. The outer part of this cellular tissue. *β*. A part of the sheath more internal; consisting of minuter discs. (Acetic acid.)

Fig. 116½. Common Fowl; chick *in ovo*. Outline of what seemed to be the foundation of two portions of cartilage in the wing, in an extremely early stage (par. 145.). These were composed of very large cells, filled with other cells. Colour pale red. The large cells had the appearance of altered blood-corpuscles. *α*. Some of the inner cells are seen; *β*, none but the outer cells. The nuclei of the inner cells in *α* were blood-red, and not distinguishable from the young blood-corpuscles fig. 106, lying near. (Some of the nuclei of the inner cells in *β* presented the same appearance; while others were larger and paler.)

Fig. 117. Tadpole, 5'''. Portion of the foundation of a cranial vertebra, composed of corpuscles having the appearance of more or less altered corpuscles of the blood. The round objects are the *nuclei* of the corpuscles. Two of the entire (elliptical) corpuscles are seen of an unaltered shape. All in outline except two of the nuclei. All the nuclei filled with discs.

Fig. 118. Tadpole, 6'''. Cells entering into the formation of cartilage, apparently

part of the foundation of the orbit. They are in outline only; the nuclei rather more finished. The latter presented discs. α. The cell contained two nuclei, besides discoid objects. The former and a few of the latter are represented in the figure. These discoid objects probably resulted from decomposition of the outer part of one of the nuclei,—thus rendered smaller than the other. (The contents of the other cells are not represented in the figure.) (Essentially the same state observed in the foundation of a vertebra, in a Tadpole of $7'''$.)

Fig. 119. Common Fowl; chick *in ovo*, on the tenth day of incubation. Elements of cartilage. (Foundation of a bone—diameter $\frac{1}{7}'''$—in one of the lower extremities.) Partly in outline. Red colouring matter still present in the objects β; which, however, were paler than the objects α. Those at β formed a dense mass. (Portions of incipient vertebræ seen in a similar state from the same chick (fig. 122.); substance also from the extremity of a duck's bill, and from its claw (thirteenth day of incubation), observed to be in essentially the same state, and with a corresponding transition out of corpuscles, having the same appearance as corpuscles of the blood.)

Fig. 120. From the same object as fig. 119. Appearance, chiefly in outline, of the marginal portion.

Fig. 121. Tadpole, $5'''$. Portion of the cartilaginous foundation of one of the orbits; composed of the nuclei of corpuscles, resembling corpuscles of the blood. Colour red.

Fig. 122. Objects from the foundation of a transverse process of one of the vertebræ in the chick from which fig. 119. was taken. α. Blood-red corpuscles composed of two or three discs. They resembled altered young blood-corpuscles. These appeared to be entering into the formation of the cortical portion of the transverse process, in which objects such as those at β occupied a less superficial place.

PLATE XXIII.

Fig. 123. Tadpole, $4\frac{1}{2}'''$. Outline of corpuscles having essentially the same appearance as blood-corpuscles, collected to form the optic nerve. Taken from a part of the nerve, just before its entrance into the eye-ball, which part measured in diameter about $\frac{1}{20}'''$. One of the corpuscles is not merely in outline; the peripheral part being represented in a more finished state. The central portion however—nucleus—is not shown. At the periphery are seen minute, highly refracting, red discs, dividing into minuter discs. The nerve did not present any fibres, the corpuscles being merely in contact with one another. In another instance, where

the part was more advanced, the surrounding globules had very much disappeared; and the corpuscles were in polyhedral forms.

Fig. 124. Tadpole, about 5‴. Corpuscles having precisely the same appearance as blood-corpuscles, passing into the elements of the optic nerve. Taken from various parts of a fasciculus of incipient fibres in the immediate neighbourhood of the eye-ball, which the fasciculus was seen to enter. α. The corpuscles in question arranging themselves in a line. These are all in outline except one. Immediately within their membrane was red colouring matter. This surrounded a pale globular object (nucleus), in which were discs. In the red colouring matter, there were seen very minute globules, having a high refracting power. These were of a brilliant red colour. Many of them appeared to be undergoing division. They presented the appearance of a rose. These globules occupy the situation of such as are seen, in many instances, to exhibit molecular motions in corpuscles of the blood.—All the other objects in the figure were composed of discs; and all were red, the colour becoming paler in the more advanced.

Fig. 125. Tadpole, about 7‴. Portions of the optic nerve, forming out of corpuscles having the same appearance as corpuscles of the blood. Quite red. Almost entirely in outline. α. The discs into which the nuclei of the said corpuscles had passed, were arranging themselves in something like lines; and they presented discs in their interior, as in the more finished disc of this object. β. From a part of the same nerve, more advanced. This object is a tube, at the periphery of which there were seen discs coalescing. These discs presented discs in their interior, corresponding apparently to the minutest of those in the object α. Pellucid points, apparently orifices, were seen here and there; and they seemed to communicate with the exterior of the incipient tube. (Dilute spirit.)

Fig. 126. Ox; fœtus of eighteen inches. From the retina. Two corpuscles having the same appearance as corpuscles of the blood. α. But little changed. β. Orange-shaped; a large orifice in the situation of the original depression (which existed when the corpuscle was a disc). Two pellucid points on one side (par. 154.). (Dilute spirit.)

Fig. 127. Ox; fœtus of ten inches. From the retina. Two altered corpuscles having the same appearance as corpuscles of the blood. α. The orifice is very large; but single. β. There exist two orifices (par. 154.).

Fig. 128. Sheep (*Ovis Aries*, Linn.); fœtus of eight inches. Corpuscles having the same appearance as corpuscles of the blood, passing into globules for the formation of the retina. See the description of fig. 130; the letters denoting similar objects in both figures.

Fig. 129. Ox; fœtus of eighteen or twenty inches. Objects from the retina, as seen after the addition of acetic acid. (See the description of the following figure.)

Fig. 130. From the same fœtus. Elements of the retina, formed of corpuscles, having the same appearance as corpuscles of the blood. *α*, *β*, *γ*, Are such corpuscles, passing into globules. *α*. They have undergone but little change. *β*. Two or more pellucid points are seen in each corpuscle (par. 154.). *γ*. The globule is now seen to be composed of discs. *δ*. A cell, the nucleus of which consists of several discs. *ε*. Cell rather larger than that at *δ*. Its nucleus, which had a similar appearance, is in outline. *ζ*. Cell, the nucleus of which was surrounded by a space more pellucid than the rest of the cavity of the cell. The whole in outline. *η*. Cell, in the nucleus of which are seen five discs. The cavity or depression in one of these was very large; and probably indicated the situation of the orifice in the membrane of the cell. *θ*. Very large cell. Its nucleus consisted of two parts. One of these was dark, and globular in form. The other part surrounded that just mentioned, and consisted of many globules. Each of the globules (in both parts) was composed of discs, which were circumscribed by a delicate membrane. The whole nucleus, also, was similarly circumscribed. *ι*. Spindle-shaped object, composed of discs, possibly representing an altered state of a globule such as *γ*,—the first change being seen at *κ*. When passing into the elements of the retina, corpuscles having the same appearance as corpuscles of the blood seem to continue longer of a flattened form than is usual elsewhere; *γ*, for instance, being rather orange-shaped than globular. All the nuclei (*δ*, *ε*, *ζ*, *η*) were circumscribed by a delicate membrane.

Fig. 131. Tadpole, $4\frac{1}{2}'''$. Objects derived from corpuscles having the same appearance as corpuscles of the blood, entering into the formation of the elements of the retina. (The transition observed, though not represented in the figure.)

Fig. 132. Tadpole, about $5\frac{1}{2}'''$. Outline of corpuscles having the same appearance as slightly altered corpuscles of the blood; as seen entering into the formation of the spinal chord. In one of these corpuscles, are delineated the objects which were seen surrounding the nucleus. (Dilute spirit.)

Fig. 133. Ox; fœtus of $5\frac{1}{4}$ inches. From the cortical portion of the brain, in which objects such as those in the figure were observed in large numbers,—often seen to be flattish in their form. They were all more or less red, some blood-red, the difference being in some degree represented by the shading; all were either discs themselves, or composed of discs; and

all were either corpuscles themselves, having the same appearance as blood-corpuscles, or immediately derived from such corpuscles. The less advanced of these objects had the same appearance as young blood-corpuscles; these passing into the objects more forward. α, β, γ, δ, Presented exactly the same appearance as young corpuscles of the blood. α Becoming orange-shaped,—blood-red; β, nearly globular,—an orifice in the situation of the original depression presented by the corpuscle when discoid in form: paler, yet blood-red. γ. Discs visible, yet indistinctly, from the presence of red colouring matter. Dark blood-red. δ. Paler, yet still blood-red. ε Resembles δ, but is more advanced,—being paler, and the discs being more distinct. ζ. Two sets of discs, an inner and an outer; the inner, deep red,—the outer, pale red. This seems to correspond to ε more advanced; the inner discs of ζ occupying the situation of the pellucid centre of ε. η, η. Three concentric parts,—namely, first, an outer part, consisting of pale and nearly colourless discs—this being the oldest part of the corpuscle;—secondly, a middle part, composed of blood-red discs (obscured in one, distinct in the other),—this part being the next in age;—thirdly, a central, round, blood-red object, pellucid in its middle, which is the newest part. θ. A mass of blood-red discs.

Fig. 134. Objects from the medullary portion of the brain in the same calf; observed along with such as those in the preceding figure. The same general remark as that in the explanation of fig. 133, respecting discs, form, colour, and origin in corpuscles having the same appearance as blood-corpuscles, is applicable here. α, β. Young blood-corpuscles; both blood-red. β Is passing into discs. γ. Mass of quite red discs. δ. The outer discs almost colourless; the middle set, as well as the central part, still red. Compare with η, η of fig. 133, of which δ in the present figure seems to be a more advanced state. The pellucid object in its central part, more defined. (Compare with the remarks in the explanation of fig. 43 β in Part II.) ε. Discoid, and very pale; yet reddish. These objects were present in large numbers, and contained discs themselves. They resemble in some degree the corpuscles from which they are derived. Not generally seen in rows.

Fig. 135. Common Fowl; chick *in ovo*. From the leg. Cells having the appearance of altered young corpuscles of the blood. Quite as red as blood. α. Outline of some of these arranging themselves to form muscle. β. Another, exhibiting the internal state. It was filled with discs;—outer part of the nucleus, a layer of very minute discs. The inner part of the nucleus consisted of finely granular substance. Centre, an orifice (par. 161.). (Dilute spirit.)

Fig. 136. Same chick. Outline of corresponding cells, from the same part, and in a more advanced state. The nuclei presented indications of division into discs (par. 161.). (Dilute spirit.)

Fig. 137. From the leg of the same chick. Outline of a muscle-cylinder, containing discs, a few of which are represented in the figure. The partitions between the original cells which form the cylinder, have disappeared. The nuclei were parietal, and composed of discs. The nucleus finished in delineation, exhibits these discs in concentric layers, around an orifice, communicating with the exterior of the cylinder (par. 161.). (Dilute spirit.)

Fig. 138. Tadpole, $5'''$. Outline of part of the foundation of the crystalline lens, as it lay surrounded by black pigment, and imbedded in the vitreous humour. It had the appearance of an altered and greatly enlarged blood-corpuscle; being throughout red,—except at α (the anterior part), where it was pellucid,—and being filled with discs resembling those arising in the interior of a corpuscle of the blood. These discs were largest and darkest at the part β.

Fig. 139. Tadpole, about $5'''$. Outline of cells forming a portion of the crystalline lens. They were very pale, yet tinged with red. Red colouring matter was seen between the cells. The interior of all these cells presented discs; and here also was observed red colouring matter. The cell, the interior of which has been delineated in the figure, was brighter than the rest, and its discs were more defined. On being viewed repeatedly, for a considerable time, the appearance of the interior of this cell was found to vary, *apparently from the discs changing their position.* This cell possibly corresponded to the central cell in fig. 140.—(Surrounding the incipient lens, there was forming a membrane,—perhaps the membrane of the lens. It seemed forming by the coalescence of objects, redder and less pale than those in the present figure; and in just the same manner as the membrane of the ovisac (fig. 172.). The crystalline and its membrane may perhaps arise, like the ovum and its ovisac, from a *single* corpuscle having the same appearance as a corpuscle of the blood (par. 181.)).

Fig. 140. Tadpole, about $5'''$. Appearance of a part, probably the centre, of the crystalline lens; the lens measuring in diameter $\frac{1}{20}'''$. The object α was dark red; the branched substance around it, very pale, yet reddish. The latter extended, in the same branched form, to the edge of the lens.

Fig. 141. Sheep; fœtus of eight inches. Objects observed in a portion of the crystalline lens, cut with scissors from the surface. The field of view presented a large number of capillary vessels, having a diameter of $\frac{1}{600}'''$

to $\frac{1}{100}'''$, and filled with red blood. One of the minutest is seen at α. The corpuscles in it were pressed into irregular forms. α, α. Some of the same vessels, in outline. β. Corpuscle of the blood, filled with young corpuscles; these being mere discs, elliptical, and differing from other discs only in being redder. γ. Outline of two of the same parent-corpuscles, which were filled, in like manner, with young corpuscles; their membranes shrivelled. δ. Parent-corpuscles of the blood, ruptured, and from which young corpuscles were escaping. Some of the latter were still elliptical; while others had become round, but continued flat. ε. Young blood-corpuscles, larger than those at δ.

PLATE XXIV.

Fig. 142. Ox; fœtus of ten inches. From the posterior surface of the crystalline lens. Vessels here radiated from central trunks, towards the margin of the lens. On the left hand in the figure, are three corpuscles having the appearance of much altered corpuscles of the blood. A few blood-corpuscles have been represented in the portion of vessel figured. It contained many not here shown. Most of the corpuscles figured, as well as the vessel itself, are in outline. α. Young blood-corpuscles, still elliptical, mere discs, and red. β. Blood-corpuscle in nearly the usual state. γ. Form altered, but the corpuscle still quite red. δ. Blood-corpuscles dividing into discs, and of a paler red. ε. This division of the blood-corpuscle into discs, has proceeded farther; and the effects are seen of the same process as that which forms cellular tissue (par. 134.). ζ. Pale discs, derived from blood-corpuscles. η. Corpuscle having the appearance of an altered corpuscle of the blood. It consisted for the most part of pale red discs. Two of the discs presented by this corpuscle, were of a deeper red, and had a darker outline. These occupied the situation of the original orifice, and afford a remarkable instance of the identity of the process effecting changes in corpuscles, having the same appearance as blood-corpuscles, with that producing the first alterations, *post coitum*, in the mammiferous ovum; and also of the similarity between the changes producing pus-globules, and some other objects. θ. Corpuscle, having the appearance of a blood-corpuscle very much enlarged. It was colourless and brilliantly pellucid in the situation of the original orifice; and at the part surrounding this, the corpuscle was blood-red, presenting indications of the formation there of discs. ι. Similar state of a corpuscle of smaller size.

Fig. 143. Common Fowl; chick *in ovo*. Corpuscle in a state analogous to that of β. fig. 155. and of θ and ι, fig. 142; and also having the appearance of an altered blood-corpuscle. It was observed, along with many others, having a similar condition, in the neighbourhood of the crystalline lens. Shreds also were seen, composed of corpuscles in the same state.

Fig. 144. Ox; fœtus of eighteen inches. From the edge of the crystalline. Portion of a tube filled with corpuscles having the same appearance as young corpuscles of the blood. A few of these are represented in the figure. Where shaded, the corpuscles were red; where the corpuscles are in outline, the red colouring matter had disappeared. They were mere discs. α. The corpuscle was red in the centre (the situation of the original cavity or depression), and colourless in the surrounding part.

Fig. 145. Sheep; fœtus of six inches. From the edge of the crystalline lens. Portion of a flattish tube, filled with corpuscles having the same appearance as corpuscles of the blood. These corpuscles, most of which are represented in outline only, were arranged with great regularity; their flat surfaces being in contact with one another. They were blood-red, and red colouring matter was seen between or around them.

Fig. 146. Another part of the same tube, filled in a like manner, but much smaller in diameter, and exhibiting a space unoccupied by corpuscles. The membrane of the tube very delicate.

Fig. 147. Another portion of the same tube. The corpuscles exhibited a brilliantly pellucid object in the situation of their original cavity or depression; their external part being still red.

Fig. 148. Twisted portion of a tube of the same kind, and from a similar locality in the crystalline lens of the same fœtus. The corpuscles were red, and in a state which in other respects, also, was similar to that of the corpuscles in fig. 147.

Fig. 149. Ox; fœtus of eighteen inches. From the edge of the crystalline lens. Portion of a tube filled with corpuscles having the appearance of blood-corpuscles in a more or less altered state. These are in outline only. Some were round; others elliptical. Most of them exhibited traces of division into discs, which in two instances have been represented in the figure. Here and there, these corpuscles were still blood-red; but the most advanced of them had become pale.

Fig. 150. Ox; fœtus of eighteen inches. From the edge of the crystalline lens. Portion of a tube filled with corpuscles having the appearance of more or less altered blood-corpuscles. Those on the side α were blood-red; while those on the side β had become pale. The latter were also elongated, and dividing into discs. In some parts, the corpuscles seemed to be arranging themselves in lines, as if to form fibres.

Fig. 151. Sheep; fœtus of six inches. From the edge of the crystalline lens. Portion of the tube filled with corpuscles having the appearance of altered and enlarged blood-corpuscles. In outline. These corpuscles exhibited a brilliantly pellucid object in the situation of the original cavity or depression. In some instances, this pellucid object seemed double. The surrounding part of the corpuscles was blood-red.

Fig. 152. Ox; fœtus of eighteen inches. From the edge of the crystalline lens. Portion of a tube filled with corpuscles having the appearance of blood-corpuscles in greatly enlarged and altered states. (See the description of fig. 142.) α, β. Several of the many pellucid spaces, occupying the situation, apparently, of the original cavity or depression in the corpuscles. Some of these spaces were very minute; and others, β, so large, that they seemed to have resulted from the coalescence of several smaller ones. By degrees, indeed, the tube may thus become colourless in its whole diameter; the red colouring matter, γ (which is situated between the pellucid spaces), having disappeared. The figure, which is chiefly in outline, represents the red colouring matter only at the upper part (γ); where a trace is seen of the division between the corpuscles. δ. There is visible in some of the pellucid spaces, a minute, highly refracting object, apparently an orifice, which at δ, δ, seemed to communicate with the exterior of the tube. ε. Outline of corpuscles having the same appearance as young corpuscles of the blood, observed with the foregoing. One of these, it will be seen, did not exceed $\frac{1}{800}'''$ in length.

Fig. 153. Sheep; fœtus of six inches. From the crystalline lens. Outline of pale cells. One of them was seen to be filled with discs, which also are represented in outline.

Fig. 154. Ox; fœtus of ten inches. From the crystalline lens. Pale cells, for the most part in very early stages of formation, and exceedingly minute. The cells are in outline, except their nuclei, a few only of which (nuclei) have been delineated. In many of the cells, however, a nucleus was not discerned. And this indeed was in most instances the case. α. Objects composed of discs. The outer portion (exceedingly minute discs) of such objects appears to separate from the rest, to form the membrane of a cell; the remainder being the nucleus of the cell. β. The cell-membrane formed. γ. Nucleated cells more advanced. The other cells in this figure are in outline. Some of them, δ, are arranged in rows; which was observed in many instances to be the case. Cells even of the minuteness of those in this figure, often seem to contain discoid objects; especially if the nucleus has disappeared.

Fig. 155. Ox; fœtus of eighteen inches. From the edge of the crystalline lens.

Corpuscles, having the same appearance as corpuscles of the blood, passing into pale cells,—the elements of the crystalline. α. Outline of two of these corpuscles, which were but little changed. Their contour was irregular. Corpuscles in such states, when required to pass through a minute space, undergo remarkable alterations in their form, and are seen to be susceptible of extreme attenuation. β. Composed of discs. Two brilliantly pellucid orifices are seen on one side; this being the situation of the originally single orifice in the corpuscle or disc, before its division into other discs. This is the situation, also, of the nucleus of the future cell, if formed. γ. Flattish corpuscle, composed of discs, and blood-red. δ, δ. Corpuscles resembling γ, but larger, and in little more than outline. ε, ε. Nucleated cells. The nuclei are dividing into discs. ζ. An orifice in the nucleus, apparently communicating with the exterior of the cell. This orifice would have been the situation of the future nucleus, after the division of the present one into discs; which discs were already formed, but not separated. η. Besides a nucleus, consisting of discs, the cell exhibits discs in its cavity. Only two of these are represented in the drawing. All the objects in this figure presented red colouring matter; which in α, β, γ, δ pervaded the entire object; but in ε, ζ, and η, was confined to the nucleus.

Fig. 156. Ox; fœtus of ten inches. From the crystalline lens. Outline of cells, the elements of the crystalline, for the most part exceedingly pale. They were altered corpuscles having the same appearance as corpuscles of the blood; and presented states which, in general (not in every instance), were more advanced than those in fig. 155. α. The contour irregular, from the incomplete formation of the membrane, which discs were coalescing to produce. The dark round object is the future nucleus. In it is an orifice, communicating with the still incipient cell. This orifice indicates the situation of a future nucleus, to form as the existing one divides into discs. β. A state resembling α, but the nucleus already composed of discs. γ. The nucleus has divided into two portions, each of which consists of incipient discs. δ, δ. A nucleus not visible. But the interior of the cell presented discs; too indistinctly, however, to admit of delineation. Such was the case also with the cell γ.

PLATE XXV.

Fig. 157. Ox; fœtus of ten inches. From the crystalline lens. Incipient fibres, forming by the coalescence of cells of the same kind as some of those in figs. 153, 154, 155, 156; which are derived from corpuscles having

the same appearance as corpuscles of the blood. These incipient fibres are in outline. α. An orifice in the nucleus, probably communicating with the exterior of the cell. One part of the fibre β still presented a bead-like appearance. In another part, the cells had fully coalesced into a cylinder. The latter was the case throughout the fibres γ. In some of these latter fibres, peripheral objects were still seen (par. 178.). δ. A cell filled with discs; a nucleus not being observed in this cell.

Fig. 158. From the same crystalline lens. Outline of incipient fibres. See the description of the preceding figure. α. An orifice in the nucleus, communicating with the exterior of the cell. Nuclei were not observed in any of the cells at the upper part of this figure; the cause being probably that these cells were more advanced than those in the lower line, and that their nuclei had been divided into the contents of the cells. β. Deficiency in the size of the cells, made up for by the number present in the diameter of the fibre.

Fig. 159. From the crystalline lens of the opposite eye in the same fœtus. Cells such as those in the two preceding figures,—but, generally speaking, larger—arranging themselves to form a fibre of the crystalline. Some of these cells presented a nucleus. In others, no nucleus was seen. The nucleus, as elsewhere, appeared to be composed of discs. The nucleus on the left consisted of two discs; and each of these presented indications of a subdivision into minuter discs. Two of the cells entering into the formation of the fibre in this figure, were very minute compared with the rest.

Fig. 160. Stone-chat (*Motacilla rubicola*, Linn.). From the testis, after maceration for a night in water. Altered corpuscles having the appearance of blood-corpuscles. They were blood-red, consisted of discs, and were apparently progressing to form fasciculi of spermatozoa. The smallest is in outline.

Fig. 161. Yellow Bunting (*Emberiza citrinella*, Linn.). From the testis. Altered corpuscle having the appearance of a blood-corpuscle, composed of discs which were deep red; and apparently in the course of forming a fasciculus of spermatozoa.

Fig. 161½. Rabbit; killed two hours *post coitum*. From the vagina. α. Several spermatozoa, apparently composed of discs, the number of which discs appeared greater in some than in others. β. An object composed of discs. On the left, this object presented a part much darker than the rest, apparently an orifice, in which the discs were seen with great distinctness. γ. Outline of an object of the same kind, but smaller. The darker part obviously communicated with the exterior by an orifice, γ γ, in which the discs were exceedingly distinct, as well as at the part circumscribed by a dotted line; the latter part being continued from the orifice (γ γ,) to the centre of the object γ, and brought into view by depressing the instrument. δ. A larger object, being either an advanced state of β and γ, or else containing several objects such as these. δ Seemed to be composed of discs, among which were seen some with caudal appendages,—probably spermatozoa. δ Was circumscribed by a membrane. (Acetic acid.)

Fig. 162. Stone-chat. Objects from the testis; namely, α, altered corpuscles having the same appearance as corpuscles of the blood,—and β, γ, δ, portions of spermatozoa, forming out of such altered corpuscles. Red colouring matter visible in all. The corpuscles composed of discs. β. Part of two spermatozoa, in the course of formation out of discs. γ. Discs which appeared to have coalesced at their extremities and sides. At the upper part, and on the left side, of this object, is seen a partially formed spermatozoon. It would thus seem that the spermatozoa are here formed by division of the discs; and this mode of origin is no doubt connected with their spiral form. δ. Portion of a spermatozoon completely formed.

Fig. 163. From the testis of the same Bird, after a night's maceration in water. α. The large corkscrew-like extremity of a spermatozoon. β. Outline of two similar objects, the position of which, in relation to each other, was such as to suggest the idea that the division of the discs, for the formation of the spermatozoa, was either incomplete, or so recent that the spermatozoa were still imperfect; and that their position had not changed. The objects in this figure were blood-red.

In the figures of the ovum (figs. 164 to 173.) the letter c denotes the germinal vesicle, g the discs of the ovisac, and h the membrane of the ovisac[†].

Fig. 164. Wryneck (*Yunx Torquilla*, Linn.). Corpuscle having the same appearance as a blood-corpuscle, very much enlarged, and filled with young corpuscles. Each of these young corpuscles is an ovum (including the ovisac) in a rudimental form. These objects are for the most part in outline; but three of them are in a more finished state. g. Discs, into which a portion of the young corpuscle has divided. In some parts these discs were quite red; in others pale. c. Germinal vesicle, in the situation of the original depression existing in the young corpuscle, when the latter was a disc. In the young corpuscle on the right, c is still a mere disc, having a diameter of $\frac{1}{700}'''$. At a certain part, the membrane of c presents an orifice, indicating the situation of the future germinal spot. (Acetic acid.)

Fig. 165. From the same ovary. Corpuscles having the same appearance as young

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[†] The same letters are here used, and they denote the same objects, as in my memoirs on Embryology, *ll. c.*

blood-corpuscles, forming ova. They differed from the objects of the preceding figure in retaining more the appearance of *discs*; the transition from which is here very obvious,—α having apparently undergone no further perceptible change than that of becoming round. β. The outer portion now consists of discs,—the inner part being itself a disc, the future germinal vesicle. γ. A more advanced state; the part *c*, corresponding to the central disc of β, being now the germinal vesicle. In the object not marked, two discs seemed to occupy the situation of *c* in γ. (Acetic acid.)

Fig. 166. From the same ovary. Part of a group of corpuscles having the same appearance as young blood-corpuscles, just escaped from a parent corpuscle, such as that in fig. 164. They were blood-red, and appeared to be membranous at the surface. These objects are rudimental ova. The letters as above. A distinct orifice in *c*. (Acetic acid.)

Fig. 167. From the same ovary. Part of a group of objects of the same kind, colour, origin, and general appearance, except that they were not membranous at the surface. One of the germinal vesicles, *c*, is larger than the rest; and the orifice in it is of considerable size. (Acetic acid.)

Fig. 168. From the same ovary. State of the rudimental ovum (altered corpuscle having the same appearance as a blood-corpuscle) rather more advanced. The germinal vesicle (*c*) was larger, and the discs *g* were dividing into minuter discs. These discs were quite red. (In one instance, in which the size of the rudimental ovum was about the same, the discs *g* had begun to coalesce for the formation of the membrane of the ovisac.) (Acetic acid.)

Fig. 169. From the ovary of the same Bird. Conditions which, excepting α, are still more advanced. α. Still blood-red. β. In outline. Redness diminished. Globules between the discs. γ. Chiefly in outline. Each of the discs presented a cell-like object in the situation of its original depression. δ. Blood-red. The discs *g* dividing into smaller discs. The germinal vesicle (*c*) obscurely seen in the interior.

Fig. 170. From the ovary of the same Bird. Portion of an ovisac, elliptical in form, and $\frac{1}{40}'''$ in length. Partly in outline. *h*. Membrane of the ovisac, forming out of the discs *g*, which for this purpose are dividing into minuter discs. *g*. Quite red, and becoming pale in *h*. *c*. Portion of the germinal vesicle.

Fig. 171. From the same ovary. Similar objects in a more advanced state. Length of the ovisac $\frac{1}{20}'''$. The discs *g*, within the ovisac, were smaller than those in fig. 170; having undergone division. Where lying around the germinal vesicle (*c*), they were smallest, had a high refracting power, and were quite red. Proceeding from this situation towards the mem-

brane, *h*, of the ovisac, we find them increasing in size, losing their high refracting power, and becoming pale. Where entering into the formation of the membrane *h*, they were largest, and undergoing a further division. Even the outermost of these discs *g*—those coalescing to form *h*—though very pale, presented a tinge of red.

Fig. 172. From the same ovary. Portion of the membrane of an ovisac, which was elliptical, and $\frac{1}{3}'''$ in length, as it lay crushed under a piece of glass. It still presented a pale tinge of red, as well as traces of some of the discs of which this membrane is composed.

Fig. 173. Canary bird (*Fringilla Canaria*, Linn.). Part of an ovisac ($\frac{1}{30}'''$ in diameter), with its contents; the whole derived from a corpuscle having the same appearance as a corpuscle of the blood, and the whole still more or less red. Respecting *g* and *h*, see the explanation of the preceding figures. (The figure does not represent the membrane of the yelk,) *c*. Germinal vesicle, containing reddish discs; the minuter of which surround a central disc, which is the newest solid part. In the centre of the latter there is a dark point, representing (by refraction) a fluid space,—the situation of the future germinal spot.

XVIII. *On the Nervous Ganglia of the Uterus.* *By* ROBERT LEE, *M.D., F.R.S.*

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IN a communication to this Society which was read on the 12th December, 1839, I described four great plexuses under the peritoneum of the gravid uterus, which had an extensive connection with the hypogastric and spermatic nerves. From their form, colour, and general distribution, and their resemblance to ganglionic plexuses of nerves, and from their branches actually coalescing with those of the hypogastric and spermatic nerves, I was induced to believe, on first discovering them, that they were nervous ganglionic plexuses, and constituted the special nervous system of the uterus.

Subsequent dissections of the unimpregnated uterus, and of the gravid uterus in the third, fourth, sixth, seventh and ninth months of pregnancy, have enabled me not only to confirm the accuracy of my former observations, but to discover the important fact, that there are many large ganglia on the uterine nerves, and on those of the vagina and bladder, which enlarge with the coats, blood-vessels, nerves, and absorbents of the uterus during pregnancy, and which return after parturition to their original condition before conception takes place.

The uterus and its appendages are wholly supplied with nerves from the great sympathetic and sacral nerves. At the bifurcation of the aorta, the right and left cords of the great sympathetic nerve unite upon the anterior part of the aorta, and form the aortic plexus. This plexus divides into the right and left hypogastric nerves, which soon subdivide into a number of branches to form the right and left hypogastric plexus. Each of these plexuses, having the trunk of the hypogastric nerve continued through its centre, after giving off branches to the ureter, peritoneum, rectum, and trunks of the uterine blood-vessels, descends to the side of the cervix, and there terminates in a great ganglion, which, from its situation and relations, may be called the hypogastric ganglion, or utero-cervical ganglion.

This ganglion is situated by the side of the neck of the uterus, behind the ureter, where it is passing to the bladder. In the unimpregnated state it is usually of an irregular, triangular, or oblong shape, with several lobes or processes projecting from it where the nerves enter, or are given off from it. In the long diameter it usually measures from half an inch to three-quarters of an inch, varying in dimensions with the size of the nerves with which it is connected. The hypogastric ganglion always consists of cineritious and white matter like other ganglia, and gray and white nerves issue from it, which proceed to the rectum, bladder, uterus and vagina. It is covered with the trunks of the vaginal and vesical arteries and veins, and the ganglion has an artery of considerable size, which enters it near the centre, and divides into branches which accompany the nerves given off from its inner surface, and from its anterior and inferior borders. The hypogastric nerve, after separating into a plexus, enters its upper edge, and branches from the third and other sacral nerves its posterior border, and the whole of its outer surface. None of the branches of the sacral nerves pass over the ganglion to the bladder, though some of them enter its anterior edge where the vesical nerves are given off.

From the inner and posterior surface of each hypogastric ganglion, numerous large nerves are given off, which go backward to anastomose with the hæmorrhoidal nerves, which accompany the arteries to the rectum, and pass with them between the muscular fasciculi of the organ. An extensive connection is thus established between the two hypogastric ganglia and the nerves of the rectum, and many large broad nerves pass off from the posterior and inferior part of these ganglia to ramify on the sides of the vagina, and between the vagina and rectum.

From the inferior border of each hypogastric ganglion several fasciculi of small nerves are sent off, which pass down on the sides of the vagina, and enter several large flat ganglia about midway between the os uteri and ostium vaginæ. From these vaginal ganglia, innumerable filaments of nerves, on which small ganglia are formed, extend downwards to the sphincter, where they are lost in a white dense membranous expansion, from which they cannot be separated without laceration. From this great web of ganglia and nerves on the sides of the vagina, by which it is completely covered, numerous branches are sent to the sides of the bladder, which enter it around the ureter. All these nerves of the vagina are accompanied with arteries, and they often form complete rings of nerve around the trunks of the great veins.

From the anterior margin of each hypogastric ganglion, large white and gray nerves are sent off, some of which pass on the outside, and others on the inside of the ureter, and these branches meet in front of the ureter in a ganglion, which may be termed the *middle* vesical ganglion. There are other two ganglia formed on these nerves, one between the uterus and ureter, and the other between the ureter and vagina. These may be called the internal and external vesical ganglia. The ureter is thus inclosed within a great ring of nerve, which resembles the œsophageal ganglion in some of the Invertebrata. The trunks of the uterine artery and vein are likewise encircled by a great collar of nervous matter, between which and the hypogastric ganglion, several large and some small branches pass.

The internal vesical ganglion, which usually has a flattened or long bulbous shape, is formed entirely upon the nerves which pass from the hypogastric plexus and ganglion, and run between the uterus and the ureter. It has an artery which passes through its centre. It first gives off a large branch to the ring of nerve or ganglion which surrounds the uterine blood-vessels; it then sends branches to the anterior part of the cervix uteri, and afterwards a great number of small filaments to the muscular coat of the bladder behind, where it is in contact with the uterus. The internal vesical ganglion then sends forward a large branch which terminates in the middle vesical ganglion.

This ganglion sends off a great number of large nerves to the bladder. Some of these accompany the arteries, and can be seen ramifying with them upon the whole of the superior part of the organ, even to the fundus. Filaments of these nerves, scarcely visible to the naked eye, are seen ramifying upon the bundles of muscular fibres, occasionally forming loops, and inclosing them, or passing down between them to the strata of fibres below. Some of the smaller branches of the middle vesical ganglion do not accompany the arteries, but are distributed at once to the parts of the bladder around the ureter.

The external vesical ganglion is formed entirely upon the nerves which proceed from the hypogastric ganglion, and pass on the outside of the ureter. This is a small thin ganglion, the branches of which are sent immediately into the muscular coat of the bladder. It usually sends down a long branch to anastomose with the nerves and ganglia situated on the side of the vagina.

From the inner surface of each hypogastric ganglion numerous small, white, soft nerves pass to the uterus, some of which ramify upon the muscular coat about the cervix, and others spread out under the peritoneum, to coalesce with the great ganglia and plexuses situated on the posterior and anterior surfaces of the organ. Large branches also go off from the inner surface of the hypogastric ganglion to the nerves surrounding the blood-vessels of the uterus, which they accompany in all their ramifications throughout its muscular coat. Other branches of nerves pass down from the ganglion between the vagina and bladder. Soon after conception the blood-vessels of the nervous ganglia and plexuses now described enlarge, and the ganglia and plexuses themselves expand with the uterus. The long diameter of the hypogastric ganglion at the end of the ninth month measures about an inch and a half.

I have published a full description, with illustrations of the great ganglionic nerves surrounding and accompanying the blood-vessels, and of the ganglia and plexuses, situated on the body of the uterus*. The appearances presented in the fourth month of pregnancy by the hypogastric ganglia, and the ganglia and nerves of the rectum, bladder, vagina and uterus, and also the great plexuses of nerves situated on the anterior surface of the uterus, are seen in the Plates which accompany this paper.

From an examination with the microscope of portions of the plexuses under the peritoneum of a gravid uterus of nine months, which had long been immersed in rectified spirit, Professor Owen and Mr. Kiernan were led to conclude that they were not nervous plexuses, but bands of elastic tissue.

"The tissue of the broad, white, reticularly inter-communicating bands of fibrous matter resembling nerves of the uterus," observes Professor Owen, "consisted of minute fibres, which were solid, smooth, equal-sized, cylindrical and nearly transparent, irregularly interblended in their course; their diameter does not exceed $\frac{1}{10,000}$th of a line. These bands correspond in structure with the fibrous modification of cellular

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* The Anatomy of the Nerves of the Uterus. London, 1841. Fol.

tissue. The component fibres did not form tubes, nor were their interspaces filled with the primitive granules or cells of the nervous tissue.

"In the nerves of the spinal system, the primitive fibres of the neurilema, which closely resemble those of the ordinary cellular and fibrous tissues, are arranged in the form of tubes, and can be distinguished into cylinder and contents. The same structure, on a minute scale, exists, according to VALENTIN*, in the sympathetic nerves; but according to the observations of REMAK and SCHWANN†, the component fibres form solid bands, and are of a more transparent character than in the spinal nerves, but marked occasionally with swellings, and having granules in the interspaces.

"I consider that the difference between the nerves of the sympathetic and the fibrous cellular tissue to consist, as regards their microscopic character, in the greater proportion of the granules or cells in the interspaces of the fine, reticularly interwoven, component fibres of the nervous band; and this difference I believe to exist between the two nerves of the sympathetic system and the white bands of fibrous matter which connect the peritoneum with the muscular substance of the womb, and which resemble a plexus of nerves."

The tubular structure of the ganglionic plexuses on the body of the uterus has since been observed by Mr. DALRYMPLE, and the perfect resemblance of the uterine nerves to those of the stomach and intestines demonstrated. The following letter contains an account of Mr. DALRYMPLE's microscopical examination of the uterine nerves.

6 Holles Street, April 21, 1841.

MY DEAR SIR,

After having seen and very carefully examined, some weeks since, your very beautiful preparations of the nerves of the impregnated uterus, and after having felt convinced by their continuity, colour, texture, and mode of distribution, that they really were nerves, I was a good deal surprised to hear from you, and others, that their dentity had been doubted. I was aware that it would have been worse than useless to have asked you for a portion of such suspected cords to submit to the microscope, knowing that they had been very many months immersed in strong alcohol. It would neither have been fair to you, nor satisfactory to me, to have made such an attempt at solving the question.

Being anxious, however, to satisfy myself upon the subject, I obtained an uterus (unimpregnated), and while it was quite recent, I traced several nerves, which I recognised, from their situation round the ureter, and upon the body of the uterus, to be similar to some you had previously pointed out to me. These filaments I submitted to the microscope, and used a very beautiful eighth-of-an-inch object-glass made by Ross. I found that it was impossible, with the most careful dissection, to detach any filament of nerve without including a quantity of cellular and elastic tissue; so that although the tubular portion, indicating the nerve, was distinct, yet it

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* Repertorium, iii. p. 76.

† Mikroskopische Untersuch., p. 179.

was surrounded by innumerable extremely minute threads, coiled and contorted, such as one finds the component of elastic tissue, and the ultimate element of cellular membrane.

Under slight pressure, however, the tube was plainly discernible, containing granular matter, not uniformly distributed, but collected in minute masses, at intervals. Small blood-vessels were also seen here and there, with blood-discs within them, which served to indicate the difference between the nervous and vascular tubes, and thus to avoid the possibility of error.

Being, however, aware that some of the most distinguished foreign microscopical anatomists had differed as to what was the real characteristic of nerves of the sympathetic system, I should not have troubled you with this communication had I stopped here.

Feeling, from this discordance of opinion, that there was no absolute test, or at least one which was not open to cavil, I thought to try a comparison of the uterine nerves with those that undeniably belonged to the ganglionic system. I traced, therefore, some nerves upon the surface of the stomach, up to the great ganglion that gave them origin; and I selected some also from the small intestine. These I submitted to the same microscopical power, and under the same circumstances of light, and pressure, and medium.

In all of these I observed the tubular part filled with granular matter, and similarly collected in minute masses.

I also observed that each tube was surrounded by the minute serpentine threads before described. In fact so closely did they agree, in every particular, with the appearances presented by the uterine nerves, that it would have been impossible to distinguish the one from the other.

Thus by comparing the unknown with the known, despite the want of any absolute test, I feel perfectly satisfied of the true nervous character of the very beautiful plexuses you have so patiently and with so much labour developed.

Admitting then this intricate structure to be really nervous, it is a matter of no marvel that they increase in size during pregnancy. It would indeed be wonderful if the nerves alone remained stationary, while the muscular and cellular, the serous and mucous, and the vascular tissues increased, as it is notorious those structures of the uterus do during the period of child-bearing.

If, as is also indisputable, nerves shrink and atrophy when the function of an organ they supplied is lost or destroyed, is it singular that the uterine nerves should increase, when that organ rouses itself from inaction, to one of the most extraordinary exemplifications of temporary functional vigour that the animal œconomy can anywhere exhibit? Pardon me this prolixity, and believe me,

My dear Sir,
Yours very faithfully,
John Dalrymple.

Dr. Robert Lee, F.R.S.

Explanation of the Plates.

PLATE XXVI.

Exhibits a posterior and lateral view of the gravid uterus in the fourth month of pregnancy, of the vagina, rectum and bladder, with their ganglia and nerves.

A. The fundus and body of the uterus covered with peritoneum.

B. The vagina.

C. The bladder.

D. The rectum.

E, F. The ovaria.

G. The great sympathetic nerve where it divides into the two hypogastric nerves and plexuses. The arteries and veins of the great sympathetic are all injected in the preparation from which the drawing has been made. A little above the bifurcation of the great sympathetic nerve, there is a deposit of cineritious matter in its substance, and the nerve itself is enlarged as high as the kidneys.

H. The right and left hypogastric nerves and plexuses. The artery of the right is injected, and accompanies the nerve to the great ganglion at the cervix in which it terminates.

I. The left hypogastric or great utero-cervical ganglion, with an artery passing into it near the centre.

J. The third and other sacral nerves, sending numerous large branches into the posterior border of the ganglion, and the whole of its outer surface.

K. The hæmorrhoidal nerves accompanying the arteries to the rectum, and sending numerous branches to anastomose with nerves sent off from the posterior edge of the ganglion.

L. Branches of nerves with ganglia sent off from the left hypogastric nerve, which pass down on the inside of the ureter to the trunks of the uterine artery and veins, and enter ganglia which surround these blood-vessels.

M. The left ureter, with a nerve accompanying it, which passes into the vesical ganglion, situated on the anterior part of the ureter.

N. Rings of nerve, surrounding the uterine blood-vessels.

O. The middle vesical ganglion, into which large nerves enter, which are sent off from the anterior border of the left hypogastric ganglion, and pass on the outside of the ureter.

P. Broad, flat ganglia formed on the great plexus of nerves which covers the upper part of the vagina.

Q. The orifices of the divided veins of the vagina, which are completely encircled with ganglionic plexuses of nerves.

R. Filaments of vaginal nerves passing under the sphincter.

S. Large nerves covering the posterior wall of the vagina, and anastomosing with the hæmorrhoidal nerves.

PLATE XXVII.

Exhibits an anterior and lateral view of the gravid uterus in the fourth month, and of the vagina and bladder.

A. The right hypogastric nerve.

B. The sacral nerves.

C. The right hypogastric ganglion.

D. Nerves from the hypogastric nerve to the ganglia on the blood-vessels of the uterus.

E. Ganglia surrounding the uterine artery and veins.

F. Ganglionic plexus, under the peritoneum on the fore-part of the uterus.

G. Filaments from this plexus passing out with the round ligament.

H. The round ligament.

I. The right ureter and trunk of the vaginal and vesical veins surrounded with nerves.

J. Ganglia and nerves of the vagina.

K. Nerves passing between the vagina and rectum.

L. Ganglia and nerves of the bladder.

M. Vaginal nerves passing into the bladder around the ureter.

N. Blood-vessels and nerves of upper part of the bladder.

O. Plexus of nerves under the peritoneum on the left side of the uterus, the blood-vessels of which have not been injected.

P. Filaments from this plexus passing out with the round ligaments.

Q. The peritoneum of the anterior part of the body and cervix of the uterus reflected upwards, to expose the ganglionic plexuses situated below.

XIX. *On a Cycle of Eighteen Years in the Mean Annual Height of the Barometer in the Climate of London, and on a constant variation of the Barometrical Mean according to the Moon's Declination. By* LUKE HOWARD, *Esq., F.R.S.*

Received February 4,—Read March 11, 1841.

I HAVE already treated this subject, partially and in detail, in the 'Climate of London*.' The further and full development of it in that way will be found an undertaking more of labour than of difficulty, the materials being already provided for doing this through a lunar cycle of eighteen years; but I am enabled, by means of these, to present to the Royal Society some general results, which will prove interesting, and probably important to the science to which they belong.

The like method has been adopted in this paper as in my two former, read before the Society, on the connexion of the barometric variation with the *Lunar Phases* and *Apsides.* I have excluded, by appropriate averages, those effects of the lunar influence which belong not to the subject immediately before us. These, however, will require, whensoever we may think it time to form a theory, to be examined conjointly with the present and every other of the elements of this intricate subject.

TABLE I.

Barometrical Averages on successive Solar Years, from 1815 to 1832, constructed to show the Moon's influence on the Mean Heights, varying according to her Declination: for the manner of forming which, see the remainder of this paper.

Year.	Days' observations.	Annual mean.	Moon in or near the equator.	Moon at or near her greatest north declination.	Moon in or near the equator.	Moon at or near her greatest south declination.	Averages on nine years.
		in.	in.	in.	in.	in.	in.
1815	370	29·766	29·8391	29·7819	29·7947	29·8880	
1816	368	29·648	29·7883	29·7128	29·7046	29·8357	
1817	362	29·733	29·7908	29·8590	29·8420	29·7499	
1818	369	29·826	29·8116	29·8649	29·8363	29·8348	
1819	361	29·831	29·7930	29·8168	29·9287	29·7106	
1820	369	29·839	29·8014	29·8020	29·9363	29·8622	
1821	362	29·805	29·8206	29·9085	29·7880	29·7044	
1822	362	29·889	29·8543	29·8472	29·9354	29·9426	
1823	369	29·763	29·8040	29·8436	29·6741	29·7203	
1824	368	29·878	29·9788	29·9126	29·9129	29·7546	29·8111
1825	362	29·987	30·0823	30·0285	29·8932	29·9933	29·8235
1826	369	30·033	30·0899	30·0213	29·9959	29·9910	29·8501
1827	362	29·938	29·9374	29·8875	29·9218	29·9829	29·8723
1828	363	29·814	29·8590	29·7832	29·7990	29·8608	29·8848
1829	363	29·688	29·6838	29·6563	29·6857	29·7002	29·8829
1830	368	29·671	29·7404	29·6902	29·6604	29·6900	29·8661
1831	362	29·653	29·6351	29·6310	29·6700	29·5968	29·8512
1832	363	29·702	29·6480	29·8210	29·7293	29·6830	29·8250

The averages on successive periods of nine years in the last column exhibit the barometrical mean, increasing and decreasing, as follows:—29·8111 + 0124 + 0266 + 0222 + 0125 = 29·8848 — 0019 — 0168 — 0149 — 0262 = 29·8250 inch. Then, to complete the cycle, 29·8250 — 0139 = 29·8111 inch.

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* Vol. i. p. 172. 2nd Edition.

Table II.

Barometrical Averages on successive Cycles of nine Solar Years, classed according to the Moon's place in Declination.

Periods taken.	1. Moon at or near equator, and going north.	2. Moon at or near her greatest north declination.	3. Moon at or near equator, and going south.	4. Moon at or near her greatest south declination.	5. Averages on whole periods of nine years.	6. Averages on the four results preceding.
	in.	in.	in.	in.	in.	in.
1815–23	29·8114	29·8263	29·8267	29·8054	29·8173	29·8174
1816–24	29·8270	29·8408	29·8398	29·7794	29·8059	29·8218
1817–25	29·8596	29·8759	29·8608	29·8081	29·8366	29·8511
1818–26	29·8929	29·8939	29·8779	29·8349	29·8577	29·8749
1819–27	29·9069	29·8964	29·8873	29·8513	29·8696	29·8856
1820–28	29·9142	29·8927	29·8729	29·8680	29·8691	29·8870
1821–29	29·9011	29·8765	29·8451	29·8500	29·8518	29·8682
1822–30	29·8922	29·8523	29·8304	29·8484	29·8372	29·8560
1823–31	29·8678	29·8282	29·8014	29·8100	29·8123	29·8269
1824–32	29·8505	29·8257	29·8076	29·8058	29·8076	29·8225
Mean by the columns.	29·8724	29·8608	29·8450	29·8261	29·8365	29·8511

The averages presented at the foot of columns 1 to 4, show a decrease in the barometrical mean, consequent on the moon's varying positions in declination, which may be thus stated: 29·8724 in. on equator, *minus* by north place, ·0116 in.; again, *minus* by passage of equator south, ·0158 in.; again, *minus* by south place, ·0189 in.; lastly, *plus* by return north over equator, ·0463 in.

The averages in columns 5 and 6 exhibit the barometrical mean, increasing and decreasing with great regularity, during the course of a lunar cycle of eighteen years.

The averages which form the two Tables before us were obtained in the following manner:—

1. The *year* was divided, by an ephemeris, into periods of lunar declination, the whole set in each case including not less than 361, nor more than 370 days.

2. These periods of declination were subdivided into *weeks* (or spaces of from six to eight days, generally *seven*) with the moon's extreme north, her extreme south, and her respective positions on the equator, coming and going, placed as nearly as might be *in the midst of the space* on which the average was taken—to wit, the average of the *medium heights* of the barometer for each twenty-four hours of the space.

3. These weekly averages, obtained generally from the curves inscribed by the barometer, on the face of a clock by Cumming, in my possession, were then placed under their respective heads of the four positions of the moon above-mentioned.

4. They were then laid together for the whole year, or for the number of days necessarily so accounted, which numbers make an average of 365⅓ days to the year.

5. Averages were, lastly, taken under the respective heads of north, south, &c. on successive *periods of nine years*, as 1815–23, 1816–24, &c., the series beginning 23rd December 1814, and ending 19th December 1832. These results occupy the four leading columns of the second Table; the preceding are in Table I.

6. The leading column in Table I. contains a set of *annual barometrical means* taken (with the exception of the last) from those I have already published in the 'Climate of London.' These are calculated from the Tables for each month in the ordinary way, and not on the solar years. I have given them as they stand in that work, though in the years from 1815 to 1817 they ought possibly to be higher by a tenth of an inch, from the too high placing the scale in those years; but this (with other like inaccuracies which may be hereafter found and rectified) I do not consider as affecting much *the proportions found among the results in any given year.* In calculating the set of averages on periods of nine years, placed in the last column of this Table, I have, however, to prevent discrepancies, *added* this tenth of an inch upon each of the three years.

7. The fifth column of Table II. contains the barometrical mean, calculated upon the whole period of solar years, which, in the four preceding columns, are averaged under the respective lunar positions of north, south, &c. The sixth column of this Table shows a mean founded on a direct average of the four results placed under these heads. I have noticed some features of the variation at the foot of the Tables. I shall proceed now to state some general results, of course as to the barometer alone. The effects on the mean temperature and rain must for the present be left unnoticed.

The barometrical mean in our climate is depressed (on an average of years) by the moon's position in south declination.

In every one of these averages upon periods of nine years, in Table II., the mean under *south* is lower than that under *north* declination; the difference being in some cases between six and seven hundredths of an inch: and it is larger on the averages in the fore-part than on those in the latter part of the series.

The mean under *south* declination is also *lower than either of the other three*; with exception of the four latter averages, in which it exceeds a little that of the position "going south."

This depression is gradual: it commences with the moon in full *north* declination, and proceeds through her remaining positions to the time when she again crosses the equator to return north; at which season the whole weight that had been abstracted is suddenly restored—this of course must be understood of the small differences in the mean here treated. There will be found, in the observations employed, an abundance of particular cases of variation which contradict such a rule, but the *compensations,* it appears, cover these in its favour.

We have here, I think, evidence of a great *tidal wave* or swell in the atmosphere, caused by the moon's attraction, preceding her in her approach to us, and following slowly as she departs from these latitudes. Were the atmosphere a calm fluid ocean of air of uniform temperature, this tide would be manifested with as great regularity as are those of the ocean of waters. But the currents, uniformly kept up by the sun's varying influence, effectually prevent this, and so complicate the problem.

There is also manifest in the lunar influence *a gradation of effects,* which is here shown, as it is found to operate *through a cycle of eighteen years*. In these, the mean weight of our atmosphere increases through the fore-part of the period; and, having kept for a year at the maximum it has attained, decreases again through the remaining years to a minimum; about which there seems to be some fluctuation, before the mean begins to rise again.

This result is brought out in different ways by all the averages upon *years*; and it pervades, though with less of uniformity, those upon the quarter periods or *weeks* of declination. The study of these, *with a view to theory*, rude and imperfect as they are, may become, I would willingly hope, an occupation for those more capable and better prepared than myself to grapple with the subject.

L. H.

February 3, 1841.

XX. *Computation of the Ratio of the Diameter of a Circle to its circumference to 208 places of figures.* *By* WILLIAM RUTHERFORD, *Esq., of the Royal Military Academy.* *Communicated by* S. HUNTER CHRISTIE, *Esq., M.A., Sec. R.S. &c. &c.*

Received April 16,—Read May 6, 1841.

BEFORE the time of MACHIN, the approximation to the ratio of the circumference of a circle to its diameter had been carried as far as seventy-two places of decimals by ABRAHAM SHARP, by means of the series

$$\frac{\pi}{6} = \frac{1}{3}\sqrt{3}\left\{1 - \frac{1}{3}\cdot\frac{1}{3^2} + \frac{1}{5}\cdot\frac{1}{3^4} - \frac{1}{7}\cdot\frac{1}{3^6}, \&c.\right\}.$$

By employing the series arising from the formula,

$$\frac{\pi}{4} = 4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239},$$

MACHIN extended the approximation to 100 places. By the same means M. DE LAGNY carried this approximation to 127 places; and in an Oxford manuscript it is extended to 152 places, which, as far as I am aware, is the greatest extent to which the approximation has ever been pushed.

The processes employed in these approximations may be greatly simplified by replacing $\tan^{-1}\frac{1}{239}$ by $\tan^{-1}\frac{1}{70} - \tan^{-1}\frac{1}{99}$, inasmuch as the calculation of the terms of the series involving inverse powers of 70 and 99 may be effected by arithmetical processes of very great facility. By employing the synthetic process of division, the division by 99 (100 − 1) becomes even more simple than that by 9 or 11, since it is effected by adding together two numbers each less than 10.

By means of the formula

$$\frac{\pi}{4} = 4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99}*,$$

I have computed the value of $\frac{\pi}{4}$, and thence that of π to 208 places of decimals. Previously to entering upon the calculations I considered whether it would be simpler

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* When the calculations for determining the value of π were presented to the Royal Society, it was presumed that the formula $\frac{\pi}{4} = 4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99}$ had not before been investigated. I have since found that EULER, in an article entitled "De progressionibus arcuum circularium quorum tangentes secundum certam legem procedunt," obtained the very same formula.—*Novi Commentarii Petropol.*, tom. ix. 1764.

to combine the respective terms of the three series, or to combine only the terms of the two series arising from $\tan^{-1}\frac{1}{70}$ and $\tan^{-1}\frac{1}{99}$, computing the value of $4\tan^{-1}\frac{1}{5}$ separately. The reciprocals of the powers of 5 being terminate decimals, and the results of the several terms in the series for $\tan^{-1}\frac{1}{5}$ circulating in small periods, induced me to compute its value apart from that of the other two.

It is unnecessary to give a lengthened description of the mode of computation, and I shall only briefly state the method of constructing the auxiliary tables which I employed.

The first of these Tables marked (A), comprises the reciprocals of the successive odd powers of 5, and is formed by dividing continuously by 25, or multiplying successively by ·04. Tables (B) and (C) contain the reciprocals of the successive powers of 70 and 99 respectively, the former being obtained by continuous divisions by 70, and the latter by dividing continuously by 99, by the very simple process already adverted to. Tables (D) and (E) contain the values of the several terms of the first series, viz.

$$\frac{1}{5}-\frac{1}{3}\cdot\frac{1}{5^3}+\frac{1}{5}\cdot\frac{1}{5^5}-\frac{1}{7}\cdot\frac{1}{5^7}+\frac{1}{9}\cdot\frac{1}{5^9}-\frac{1}{11}\cdot\frac{1}{5^{11}}+\ldots\ldots.$$

the former table comprising the values of the positive terms, and the latter those of the negative terms, both being derived from the subsidiary table (A). Table (F) is formed in like manner from the subsidiary tables (B) and (C), Part I. comprising the negative terms, and Part II. the positive terms of the series

$$\left(\frac{1}{70}-\frac{1}{99}\right)-\frac{1}{3}\left(\frac{1}{70^3}-\frac{1}{99^3}\right)+\frac{1}{5}\left(\frac{1}{70^5}-\frac{1}{99^5}\right)-\frac{1}{7}\left(\frac{1}{70^7}-\frac{1}{99^7}\right)+\ldots.$$

And for the readier verification of the summation of the several columns in Tables (D), (E), (F), the sum of each column is written in full in a diagonal position, preserving the local values of the several figures in each sum, from which by a second summation the total sum is finally obtained. The excess of the sum of the positive terms in Table (D) above that of the negative terms in Table (E) is then multiplied by 4 to obtain the value of $4\tan^{-1}\frac{1}{5}$, and the excess of the sum of the positive terms in Table (F) above that of the negative terms gives the value of $\tan^{-1}\frac{1}{70}-\tan^{-1}\frac{1}{99}$. This value is then transferred to Table (D), and subtracted from that of $4\tan^{-1}\frac{1}{5}$, which gives the value of $\frac{\pi}{4}$, and thence, finally, by multiplying by 4, the value of π, or the ratio of the diameter of a circle to the circumference to 208 places of decimals.

In conclusion I have only to remark, that the computations have been very carefully conducted, and that almost every part of the work has been verified by myself or the independent computations of others. The value of π which I have thus obtained from the formula

$$\frac{\pi}{4} = 4 \tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99}, \text{ is}$$

$\pi = 3{\cdot}14159$ 26535 89793 23846 26433 83279 50288 41971
69399 37510 58209 74944 59230 78164 06286 20899
86280 34825 34211 70679 82148 08651 32$\overset{*}{8}$23 06647
09384 46095 50582 23172 53594 08128 48473 78139
20386 33830 21574 73996 00825 93125 91294 01832
80651 744

which, it is presumed, is accurate to the last, or 208th place of decimals inclusive, the computations having been carried as far as 210 places of figures.

Royal Military Academy,
April 16, 1841.

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[*] The suspicion about the figure in the 113th place of decimals is now completely removed. I find it to be 8, instead of 7 as it is frequently printed, agreeing with the result as given by Vega, and also with that in the Oxford Manuscript.

XXI. *Researches in the Theory of Machines.* *By the Rev.* Henry Moseley, *M.A., F.R.S., Prof. Natural Philosophy and Astronomy, King's College, London.*

Received June 10,—Read June 18, 1841.

THE work of a mechanical agent may be defined as the union of a continual pressure with a continual motion. The work of overcoming a pressure of one pound through a space of one foot, is in this country taken as the unit in terms of which any other amount of work is estimated*. The work of any pressure operating through any space is evidently measured in terms of such units, by multiplying the number of pounds in the pressure by the number of feet in the space, if the direction of the pressure be continually that in which the space is described. If not, it follows, by a simple geometrical deduction, that it is measured by the product of the number of pounds in the pressure, by the number of feet in the projection of the space described†, upon the direction of the pressure; that is, by the product of the pressure by its virtual velocity. Thus then we conclude, at once, by the principle of virtual velocities, that if a machine work under a constant equilibrium of the pressures applied to it, or if it work uniformly, then is the aggregate work of those pressures which tend to accelerate its motion, equal to the aggregate work of those which tend to retard it; and, by the principle of vis viva, that if the machine do not work under an equilibrium of the forces impressed upon it, then is the aggregate work of those which tend to accelerate the motion of the machine, greater or less than the aggregate work of those which tend to retard its motion by one-half the aggregate of the vires vivæ acquired or lost by the moving parts of the system, whilst the work is being done upon it. In no respect have the labours of the illustrious President of the Academy of Sciences more contributed to the development of the theory of

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* The sense in which the term *work* is here used, will be recognised to be that in which "dynamical effect," "efficiency," "work done," "labouring force," &c. have been understood by different English writers, and "moment d'activité," "quantité d'action," "puissance mécanique," "travail," by the French. Among the latter this variety of terms has at length given place to the most intelligible and the simplest of them, "travail." The English word *work* is the obvious translation of "*travail*," and the use of it appears to be recommended by the same considerations. M. Dupin has proposed the application of the term "dyname" to a unit of work. The author of this paper has gladly sheltered himself from the charge of adding to the vocabulary of scientific words by assuming the term itself, "*unit of work*," to represent concisely and conveniently enough, without translation, the idea which is attached to it.

† If the direction of the pressure remain always parallel to itself, the space described may be any finite space; if it do not, the space is understood to be so small, that the direction of the pressure may be supposed to remain parallel to itself whilst that space is described.

machines, than in the application which he has so successfully made to it of this principle of vis viva[*]. In the elementary discussion, however, of this principle, which is given by M. PONCELET in the Introduction to his *Mécanique Industrielle,* he has revived the term vis inertiæ (vis inertiæ, vis insita (NEWTON)), and associating with it the definitive idea of a force of resistance opposed to the acceleration or the retardation of a body's motion, he has shown (Arts. 66. and 122.) the work expended in overcoming this resistance through any space, to be measured by one-half the vis viva accumulated through the space; so that throwing into the consideration of the forces under which a machine works, the vires inertiæ of its moving elements, and observing that one-half of their aggregate vis viva is equal to the aggregate work of their vires inertiæ, it follows by the principle of virtual velocities, that the difference between the aggregate work of those forces impressed upon a machine which tend to accelerate its motion, and the aggregate work of those which tend to retard the motion, is equal to the aggregate work of the vires inertiæ of the moving parts of the machine: under which form the principle of vis viva resolves itself into the principle of virtual velocities. So many difficulties, however, oppose themselves to the introduction of the term vis inertiæ, associated with the definitive idea of an opposing force, into the discussion of questions of mechanics, and especially of practical and elementary mechanics, that it has appeared to the author of this paper desirable to avoid it. It is with this view, that in the researches which form the subject of the paper now submitted to the Society, a new interpretation is given to that function of the velocity of a moving body which is known as its vis viva; one-half that function being interpreted to represent the number of units of work accumulated in the body so long as its motion is continued, and which number of units of work it is capable of reproducing upon any resistance which may be opposed to its motion, and bring it to rest. A very simple investigation will establish the truth of this interpretation of the analytical formula represented by the term *vis viva.* Let a body whose weight is W be conceived to descend freely by gravity through a height H, and to acquire a velocity V. It will have become capable, by reason of its motion, of overcoming a certain pressure through a certain space, that is, of yielding a certain amount of work, which amount of work may be conceived to be accumulated in it. The amount of the work which it has become capable of yielding, is manifestly that which would raise another body of the same weight W, to the same vertical height H[†]; or it is equivalent to a number of units of work represented by W H, or (since $V^2 = 2gH$) by $\frac{1}{2}\frac{W}{g}\cdot V^2$, that is, by one-half the vis viva. Thus the work accumulated in a body moving with the velocity V, is represented by half the vis viva, when that velocity is acquired by the action of gravity. Now the work accumulated in a body moving

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[*] See PONCELET, Mécanique Industrielle, troisième partie.

[†] If a mechanical contrivance could be so interposed as to receive the whole of the work of the descending weight, and communicate it to an equal ascending weight, this last would manifestly be projected upwards with the same velocity with which the first reached the ground, and would therefore ascend to the same height.

with this velocity V, is manifestly the same under whatever circumstances that velocity may have been acquired; the effects which a body having a given weight, and moving with a given velocity, is capable of producing (the work which it is capable of yielding) being manifestly independent of the causes from the operation of which that velocity has resulted. Since then the work which a body is capable of yielding, when its velocity has been acquired by the free action of gravity, is represented by that function of its velocity which we call one-half its vis viva, it is represented by the same function when that velocity has been acquired by the action of any other force, or under any other circumstances whatever; and if the work which it is capable of yielding upon any resistance opposed to its motion be said to be accumulated in it before it encounters that resistance, then under all circumstances is the accumulated work of a moving body represented by one-half its vis viva. Giving to the term vis viva this new interpretation, the principle of vis viva, as applied to machines, may be enunciated thus:—"The difference between the aggregate work done upon the machine during any time by those forces which tend to accelerate the motion, and the aggregate work during the same time of those which tend to retard the motion, is equal to the aggregate number of units of work accumulated in the moving parts of the machine during that time if the former aggregate exceed the latter, and lost from them during that time if the former aggregate fall short of the latter." Thus, then, if the aggregate work of the forces which tend to accelerate the motion of a machine exceeds that of the forces which tend to retard it, then is the surplus work (that done upon the driving points, above that expended upon the working points) continually accumulated in the moving elements of the machine, and their motion is thereby continually accelerated. And if the former aggregate be less than the latter, then is the deficiency supplied from the work already accumulated in the moving elements, or it is lost by them, so that their motion is in this case continually retarded.

2. The moving power divides itself whilst it operates in a machine, first, into that which overcomes the prejudicial resistances of the machine, or those which are opposed by friction and other causes, uselessly absorbing the work in its transmission. Secondly, into that which accelerates the motion of the various moving parts of the machine, and which accumulates in them so long as the work done by the moving power upon it exceeds that expended upon the various resistances opposed to the motion of the machine. Thirdly, into that which overcomes the useful resistances, or those which are opposed to the motion of the machine at the working point, or points, by the useful work which is done by it. Now the aggregate number of units of useful work yielded by any machine at its working points, is less than the number received upon the machine directly from the moving power, by the number of units expended upon the prejudicial resistances, and by the number of units accumulated in the moving parts of the machine whilst the work is being done. For if ΣU_1 represent the number of units of work received upon the machine immediately from

the operation of the moving power, Σu the whole number of such units absorbed in overcoming the prejudicial resistances opposed to the working of the machine, ΣU_2 the whole useful work of the machine (or that done in producing its useful effect), and $\frac{1}{2g} \Sigma w (v_2^2 - v_1^2)$ one half the aggregate difference of the vires vivæ of the various moving parts of the machine at the commencement and termination of the period during which the work is estimated, then, by the principle of vis viva,

$$\Sigma U_1 = \Sigma U_2 + \Sigma u + \frac{1}{2g} \Sigma w (v_2^2 - v_1^2), \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

in which v_1 and v_2 represent the velocities, at the commencement and termination of the period, during which the work is estimated, of that moving element of the machine whose weight is w. But one-half the aggregate difference of the vires vivæ of the moving elements, represents the work accumulated in them during the period in respect to which the work is estimated.

3. At every period of the motion of a machine, there obtains a relation between the motion of each one of its elements, and that of every other element, so that the velocity of every other moving element of the machine may at any time be expressed by an algebraical function of the velocity of that one element, and the space traversed by it from a given period of the motion, the constants entering into which function are determined by the forms, dimensions, and combination of the elements of the machine*. If any one such element be made to move uniformly, the other elements will either move uniformly or with a periodical motion, or some of them uniformly, and others with a periodical motion. In the first case it is evident that the motion of every element will bear a given constant ratio to that of every other. In the second case, that it will bear to it a ratio which will become the same at the expiration of each given period; it is evident moreover that this given ratio between the velocities of the moving elements, will obtain constantly or periodically under a variable as well as a constant motion of the first element of the machine. Suppose the work to be estimated during a period which is a common multiple of the periods or cycles of the different moving elements. Let V_1 represent the velocity of the moving point, or first element of the machine at the commencement of this cycle or period, which is a common multiple of all the other periods, and V_2 that at its termination, and v_1 and v_2 the velocities of any other element at the commencement and termination of the same cycle or period; then $\lambda . V_1 = v_1$, $\lambda . V_2 = v_2$, where λ represents a constant quantity given in terms of the forms, dimensions, and combination of the intervening elements of the machine. The same being true of every other element, it follows that

$$\Sigma w v_1^2 = V_1^2 . \Sigma w \lambda^2, \; \Sigma w v_2^2 = V_2^2 \Sigma w \lambda^2;$$

$$\therefore \frac{1}{2} \Sigma w (v_1^2 - v_2^2) = \frac{1}{2} (V_1^2 - V_2^2) . \Sigma w \lambda^2.$$

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* Professor Willis has determined the form of this function in respect to each of the principal elements of complex machinery, in his work recently published, entitled 'The Principles of Mechanism.'

Substituting this value in the preceding equation (1.),

$$\Sigma U_1 = \Sigma U_2 + \Sigma u + \frac{1}{2g}(V_1^2 - V_2^2)\Sigma w \lambda^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

This equation expressing a relation between the work ΣU_1 done upon the moving point of a machine, and that ΣU_2 yielded at its working points, it is proposed to call the *Modulus* of the machine.

If the velocity V_1 of the moving point be constant, or if it return to the same value at the expiration of each period, then

$$V_1 = V_2, \text{ and } \Sigma U_1 = \Sigma U_2 + \Sigma u.$$

This may be called the modulus of uniform or periodical, and the other that of variable motion. The modulus is thus in respect to any machine, the particular form applicable to that machine of the above equation, and being dependent for its amount upon the amount of work Σu expended upon the friction, and other prejudicial resistances opposed to the motion of the various elements of the machine, it measures in respect to each such machine, the loss of the work due to these causes, and therefore constitutes a true standard for comparing the expenditure of moving power necessary to the production of the same effects by different machines, and (*cæteris paribus*) a true measure of the working qualities of such machines. It has been the principal object of the researches which the author proposes to submit to the Society, in this and a subsequent paper, to develope these properties of the modulus under a general form, to determine the particular moduli of some of those elements which enter most commonly into the composition of machinery, and to deduce the moduli of various compound machines, by a general method, from the moduli of their component elements.

4. Solving equation (2.) in respect to V_2, we obtain

$$V_2^2 - V_1^2 = 2g\left\{\frac{\Sigma U_1 - \Sigma U_2 - \Sigma u}{\Sigma w \lambda^2}\right\}.$$

It is evident from this equation, that any inequality between the work ΣU_1 done upon the moving point, and that $\Sigma U_2 + \Sigma u$ yielded upon the work done, and upon the prejudicial resistances, produces a greater or less variation in the velocity of the machine, according as the quantity represented by $\Sigma w \lambda^2$ is greater or less.

It is proposed to call this quantity, which has a different value under every different mechanical combination, and which is here, it is believed, first introduced into the discussion of the theory of machines, the *coefficient of equable motion*. Being determined in respect to any machine, it measures (every other consideration being excepted) the greater or less steadiness of the motion, which is maintained by that machine under a given variation of the power which impels it.

5. *General form of the Modulus of a Machine.*

Let P_1 represent the pressure upon the moving point of a machine, and $P_2 P_3 \ldots . P_n$

the pressures upon its different working points, and let that relation which obtains at any period of the motion between the moving pressure P_1 and the working pressures P_2, P_3, &c., when in the state bordering upon motion, and subject to the various prejudicial resistances under which the machine works, be represented by

$$P_1 = F(P_2, P_3, \&c.) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.)$$

Let s_1, s_2, s_3, &c. represent the spaces described in the same exceedingly small time by the points of application of P_1, P_2, &c., if these points move in the directions in which those pressures severally act, and if not let them represent the projections of these spaces on the directions of the pressures. Then are these spaces, s_2, s_3, &c., evidently related to the space s_1 by equations of the form

$$\mu_2 s_2 = s_1, \quad \mu_3 s_3 = s_1, \quad \mu_4 s_4 = s_1, \ \&c. \ \&c.,$$

where μ_2, μ_3, μ_4, &c. are certain constant quantities determined by the forms and dimensions of the moving elements of the machine and their combination, or certain functions of these and of the space s_1 which the moving point has described from the commencement of any given period of its motion. Let now u_1 represent the work of the pressure P_1 through the space s_1, u_2 that of P_2 through s_2, &c.

$$\therefore u_1 = P_1 s_1, \quad u_2 = P_2 s_2, \quad u_3 = P_3 s_3, \ \&c.$$

$$\therefore P_1 = \frac{u_1}{s_1}, \quad P_2 = \frac{\mu_2 u_2}{s_1}, \quad P_3 = \frac{\mu_3 u_3}{s_3}, \ \&c.$$

$$\therefore \frac{u_1}{s_1} = F\left(\frac{\mu_2 u_2}{s_1}, \frac{\mu_3 u_3}{s_1}, \&c.\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

Which equation,—expressing a relation between the work u_1 at the driving point, through a small increment s_1 of the space S_1 described by that point, and the work u_2, u_3, &c. yielded during the same period at the several working points—is the modulus of the machine in respect to an exceeding small motion of its elements.

If the pressures P_1, P_2, &c. remain constant during any given period of the operation of the machine, and act continually in the same directions, it is evident that the above reasoning obtains whatever may be space s_1 through which the work u_1 is done; so that the exceeding small quantities u_1, u_2, &c. s_1 may in this case be replaced by the finite quantities U_1, U_2, &c. S_1*; S_1 representing any finite space through which the work U_1 is done at the driving point, whilst the work U_2, U_3, &c. is yielded at the working points of the machine.

If the pressures P_1, P_2, P_3, &c. be variable during any given period of the continuous operation of the machine, as it respects their several amounts, or their directions, or as to both these elements, then are they (in every case presented in the operation of machinery, simply and without the interposition of any voluntary agent) functions of the spaces S_1, S_2, S_3, &c. traversed by their points of application, and therefore of the

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* If the direction of the pressure P_1 be other than that in which its point of application is made to move, S_1 must be taken to represent the projection of the space described by that point on the direction of the force.

space S_1 traversed by the point of application of the moving power; so that, representing P_1 by its value $\frac{u_1}{s_1}$, we have by equation 3,

$$\frac{u_1}{s_1} = F(P_2, P_3, \&c.),$$

where the second member is a function of S_1. Now if the direction in which the point of application of P_1 is made to move do not coincide with the direction in which that force acts, being inclined to it in any position at an angle θ, then, since s_1 represents in this case the projection of the increment ΔS_1 of the space described by the point of application of P_1 on the direction of that force, we have $s_1 = \Delta S_1 \cos \theta$; observing, therefore, that u_1 is the increment of U_1, and representing it by ΔU_1, we have

$$\frac{u_1}{s_1} = \frac{\Delta U_1}{\Delta S_1} \cdot \frac{1}{\cos \theta} = F(P_2, P_3 . \&c.),$$

and passing to the limit

$$\frac{dU_1}{dS_1} = \cos \theta . F(P_2, P_3, \&c.).$$

$$\therefore U_1 = \int \cos \theta . F(P_2, P_3, \&c.)\, dS_1 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (5.)$$

where θ and $F(P_2, P_3, \&c.)$ are functions of S_1.

The work U_1 done through a given space S_1 at the driving point under the pressures P_2, P_3, &c., at the working points of the machine, is determined by this equation in terms of S_1. Now the pressure P_2 is given in terms of the work U_2 done by it, and the distance S_2 through which it is done; and S_2 is given in terms of S_1; so that P_2 is given in terms of U_2 and S_1. In like manner P_3 is given in terms of U_3 and S_1; and so of the rest. If, therefore, we substitute for P_2, P_3, &c. in the above equation their values thus determined, we shall obtain a relation between U_1, U_2, U_3, &c. and S_1, which is the modulus required.

6. There exists in every case a relation between the quantities μ_2, μ_3, &c., which will be found useful in determining the moduli of a large class of machines. Let $P_1^{(0)}$ be taken to represent that value of P_1 which would be necessary to give motion to the machine if there were no prejudicial resistances opposed to the motion of its parts; and let $F^{(0)}(P_2, P_3, \&c.)$ represent the corresponding value of $F(P_1, P_2, \&c.)$,

$$\therefore P_1^{(0)} = F^{(0)}(P_2, P_3, \&c.).$$

Also by the principle of virtual velocities, since $P_1^{(0)}$, P_2, P_3, &c. are pressures in equilibrium, we have

$$P_1^{(0)} . s_1 = P_2 . s_2 + P_3 . s_3 + \ldots ;$$

substituting for s_2, s_3, &c., their values $\frac{s_1}{\mu_2}$, $\frac{s_1}{\mu_3}$, &c., and dividing by s_1,

$$\frac{P_2}{\mu_2} + \frac{P_3}{\mu_3} + \&c. = F^{(0)}(P_2, P_3, \&c.) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6.)$$

In that large class of machines which present but one moving and one working point, the relation between P_1 and P_2 (equation 3.) will be found to present itself under the form

$$P_1 = a P_2 + E; \quad \ldots \ldots \ldots \ldots \quad (7.)$$

where a is a function of the prejudicial resistances assuming a finite value, which may be represented by $a^{(0)}$, when these resistances vanish; and where E is a function of P_2 and also of the prejudicial resistances, which vanishes with them. In this case, therefore,

$$P_1^{(0)} = F^{(0)} (P_2, P_3, \&c.) = a^{(0)} P_2;$$

and by equation 6,

$$\frac{P_2}{\mu_2} = a^{(0)} P_2 \quad \therefore \mu_2 = \frac{1}{a^{(0)}},$$

also

$$P_1 = F (P_2, P_3, \&c.) = a P_2 + E;$$

therefore, by equation 4,

$$\frac{u_1}{s_1} = a \frac{\mu_2 u_2}{s_1} + E;$$

substituting for μ_2 its value $\frac{1}{a^{(0)}}$,

$$\frac{u_1}{s_1} = \frac{a}{a^{(0)}} \cdot \frac{u_2}{s_1} + E, \quad \ldots \ldots \ldots \ldots \quad (8.)$$

by which equation the modulus of the machine, in respect to an exceedingly small motion of its parts, is determined in terms of the relation expressed by equation 7, between the moving and working pressures P_1 and P_2 in the state bordering upon motion. Assuming the moving pressure to be applied in the direction of the motion of the moving point, observing that s_1, u_1, u_2 are the increments of S_1, U_1, U_2, and passing to the limit, we have by equation (8.),

$$\frac{d U_1}{d S_1} = \frac{a}{a^{(0)}} \cdot \frac{d U_2}{d S_1} + E.$$

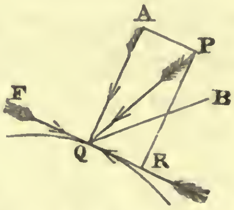
$$\therefore U_1 = \frac{a}{a^{(0)}} \cdot U_2 + \int E \, d S_1, \quad \ldots \ldots \ldots \ldots \quad (9.)$$

which is the modulus of the machine. If the working pressure be *constant*, both as to its amount and its direction, E is constant, and the modulus becomes

$$U_1 = \frac{a}{a^{(0)}} \cdot U_2 + E \, . \, S_1. \quad \ldots \ldots \ldots \quad (10.)$$

7. It remains now to consider on what general principles the relation expressed by equation 3. between the moving and the working pressures in their state bordering upon motion, may in each particular case be determined. Amongst these pressures there is, in every machine, included the resistance of one or more surfaces. Did no friction result from the pressure of the surfaces of bodies upon one another, their mutual resistance would be exerted in the direction of the common normal to their point

of contact. We know, however, by daily experience, that the resistance of no two surfaces is limited to this single direction. Friction presents itself wherever the resistance of the surfaces of solid bodies is exerted, and is, in fact, but the resolved part of that resistance in a tangent plane to the surfaces at their point of contact. And from the laws which have been proved by experiment to obtain approximately in respect to it, it follows that within the surface of a certain cone, called the cone of resistance, whose apex is at the point of contact of the surfaces, whose axis coincides with the normal, and whose angle is twice that which has for its tangent the coefficient of friction, every direction that can be taken is one in which the mutual resistances of the surfaces of contact is exerted as perfectly as in the normal direction; in fact, that any pressure (less than that which produces abrasion) being applied to the surface of an immoveable solid body by the intervention of another body moveable upon it, is sustained by the resistance of the surfaces of contact, whatever be its direction, provided only the angle which that direction makes with the perpendicular to the surfaces of contact do not exceed a certain angle, called the limiting angle of resistance of those surfaces. This is true, however great the pressure may be, within the limits of abrasion. Also, if the inclination of the pressure to the perpendicular exceed the limiting angle of resistance, then this pressure will not be sustained by the resistance of the surfaces of contact; and this is true however small the pressure may be.



Let P Q represent the direction in which the surfaces of two bodies are pressed together at Q; and let Q A be a perpendicular, or *normal* to the surfaces of contact at that point; then will the pressure P Q be sustained by the resistance of the surfaces, however *great* it may be, provided its direction lie within a certain given angle, A Q B, called the limiting angle of resistance; and it will not be sustained however small it may be, provided its direction lie without that angle. For let this pressure be represented in magnitude by P Q, and let it be resolved into two others, A Q and R Q, of which A Q is that by which it presses the surfaces together perpendicularly, and R Q that by which it tends to cause them to slide upon one another; if therefore the friction F produced by the first of these pressures exceed the second pressure R Q, then the one body will not be made to slip upon the other by this pressure P Q, however great it may be; but if the friction F, produced by the perpendicular pressure A Q, be less than the pressure R Q, then the one body will be made to slip upon the other, however small P Q may be. Let the pressure in the direction of P Q be represented by P, and the angle A Q P by θ, the perpendicular pressure in A Q is then represented by $P \cos \theta$, and therefore the friction of the surfaces of contact by $f P \cos \theta$, f representing the coefficient of friction. Moreover, the resolved pressure in the direction R Q is represented by $P \sin \theta$. The pressure P will, therefore, be sustained by the friction of the surfaces of contact, or not, according as

$$P \sin \theta \text{ is less or greater than } f P \cos \theta;$$

or dividing both sides of this inequality by P cos θ, according as

$$\tan \theta \text{ is less or greater than } f.$$

Let now the angle A Q B equal that angle whose tangent is f, and let it be represented by ϕ, so that $\tan \phi = f$.

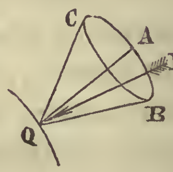
Substituting this value of f in the last inequality, it appears that the pressure P will be sustained by the friction of the surfaces of contact, or not, according as

$$\tan \theta \text{ is less or greater than } \tan \phi,$$

that is, according as

$$\theta \text{ is less or greater than } \phi,$$

or according as A Q P is less or greater than A Q B.



If the angle A Q B be conceived to revolve about the axis A Q, so that B Q may generate the surface of a cone B Q C, then does this cone evidently possess the properties assigned to the cone of resistance in the commencement of this section.

If the direction of the pressure P coincide with the surface of the cone, it will be sustained by the friction of the surfaces of contact, but the body to which it is applied will be upon the point of slipping on the other. The state of the equilibrium is then said to be that *bordering upon motion.*

If the pressure P admit of being applied only in a given plane, there are but two such states corresponding to those directions of P which coincide with the two intersections of the plane with the surface of the cone; these are the superior and the inferior states bordering upon motion.

Thus, then, it follows, conversely, that "when any pressure applied to a body moveable upon another which is fixed, is sustained by the resistance of the surfaces of contact of the bodies, and is in either state of the equilibrium bordering upon motion, then is the direction of that pressure, and therefore of the opposite resistance of the surface inclined to the normal at a given angle, that called the limiting angle of resistance*."

8. If any number of pressures P_1, P_2, P_3, &c. applied in the same plane to a body moveable about a cylindrical axis, be in the state bordering upon motion, then is the direction of the resistance of the axis inclined to its radius, at the point where it intersects its circumference, at an angle equal to the limiting angle of resistance. For let R represent the resultant of P_1, P_2, &c.; then, since these forces are supposed to be upon the point of causing the axis of the body to turn upon its bearings, their resultant would, if made to replace them, be also upon the point of causing the axis to turn on its bearings. Hence it follows that the direction of this resultant R cannot be through the centre C of the axis; for if it were, then the axis would be pressed by it in the

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* The principle here stated was first published in the Cambridge Philosophical Transactions, vol. 5, by the author of this paper.

direction of a radius, that is, perpendicularly upon its bearings, and could not be made to turn upon them by that pressure, or to be upon the point of turning upon them. The direction of R must then be on one side of C, so as to press the axis upon its bearings in a direction R L, inclined to the perpendicular C L (at the point L where it intersects the circumference of the axis,) at a certain angle, R L C. Moreover, it is evident (by the last article) that since this force R pressing the axis upon its bearings at L is upon the point of causing it to slip upon them, this inclination R L C of R to the perpendicular C L is equal to the limiting angle of resistance of the axis and its bearings*. Now the resistance of the axis is evidently equal and opposite to the resultant R of all the forces P_1, P_2, &c. impressed upon the body. The resistance acts, therefore, in the direction L R, and is inclined to C L at an angle equal to the limiting angle of resistance.

If the radius C L of the axis be represented by ϱ, and the limiting angle of resistance C L R by φ, then is the perpendicular C m upon the resistance R from the centre C of the axis represented by $\varrho \sin \varphi$, so that the moment of R about that point is represented by $R \varrho \sin \varphi$.

9. The conditions of the equilibrium of any number of pressures in the same plane, applied to a body moveable about a cylindrical axis in the state bordering upon motion.

Let P_1, P_2, P_3, &c. represent these pressures, and R their resultant. Also let a_1, a_2, a_3 represent the perpendiculars let fall upon them severally from the centre of the axis, those perpendiculars being taken with the positive signs whose corresponding pressures tend to turn the system in the same direction as the pressure P_1, and those negatively which tend to turn it in the opposite direction. Also let λ represent the perpendicular distance of the direction of the resultant R from the centre of the axis, then, since R is equal and opposite to the resistance of the axis, and that this resistance and the pressures P_1, P_2, P_3, &c. are pressures in equilibrium, we have by the principle of the equality of moments,

$$P_1 a_1 + P_2 a_2 + P_3 a_3 + \&c. = \lambda R.$$

Representing, therefore, the inclinations of the directions of the pressures P_1, P_2, P_3, &c. to one another by $\iota_{1.2}$, $\iota_{1.3}$, $\iota_{2.3}$†, &c. &c., and substituting for the value of R‡,

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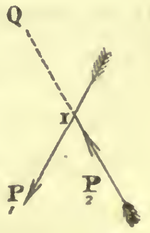
* The side of C on which R L falls, is manifestly determined by the direction towards which the motion is about to take place. In this case it is supposed about to take place towards the left. If it had been to the right, the direction of R would have been on the opposite side of C.

† The inclination $\iota_{1.2}$ of the directions of any two pressures in the above expression, is taken on the supposition that both the pressures act *from*, or both *towards* the point in which they intersect, and not one *towards* and the other *from* that point; so that in the case represented in the accompanying figure the inclination $\iota_{1.2}$ of the pressures P_1 and P_2 represented by the arrows, is not the angle P_1 I P_2, but the angle P_1 I Q, since I Q and I P_1 are directions of these pressures, both tending *from* their point of intersection; whilst the directions of P_2 I and I P_1 are one of them *towards* that point, and the other *from* it.



‡ Poisson, Mécanique, Art. 33.

$$P_1 a_1 + P_2 a_2 + P_3 a_3 + \ldots\ldots = \lambda \left\{ \begin{array}{l} P_1^2 + P_2^2 + P_3^2 + \ldots\ldots \\ \quad + 2\,P_1 P_2 \cos \iota_{1.2} + 2\,P_1 P_3 \cos \iota_{1.3} + \ldots\ldots \\ \quad + 2\,P_2 P_3 \cos \iota_{1.3} + 2\,P_2 P_4 \cos \iota_{2.4} + \ldots\ldots \\ \quad + \text{\&c. \&c.} \end{array} \right\}^{\frac{1}{2}}.$$

$$\therefore P_1 = - \frac{P_2 a_2 + P_3 a_3 + \ldots}{a_1} + \frac{\lambda}{a_1} \left\{ \begin{array}{l} P_1^2 + 2\,P_1 (P_2 \cos \iota_{1.2} + P_3 \cos \iota_{1.3} + \ldots\ldots) \\ \quad + P_2^2 + P_3^2 + P_4^2 + \ldots\ldots \\ \quad + 2\,P_2 P_3 + 2\,P_2 P_4 + \ldots\ldots \\ \quad + \text{\&c. \&c.} \end{array} \right\}^{\frac{1}{2}}$$

If the value of P_1 involved in this equation be expanded by LAGRANGE's theorem*, in a series ascending by powers of λ, and terms involving powers above the first be omitted, we shall obtain the following value of that quantity:—

$$P_1 = - \frac{P_2 a_2 + P_3 a_3 + \ldots}{a_1} + \left(\frac{\lambda}{a_1}\right) \left\{ \begin{array}{l} \frac{1}{a_1^2} (P_2 a_2 + P_3 a_3 + P_4 a_4 + \ldots\ldots\ldots)^2 \\ \quad - \frac{2}{a_1} (P_2 a_2 + P_3 a_3 + P_4 a_4 + \ldots\ldots\ldots) \cdot \\ \qquad (P_2 \cos \iota_{1.2} + P_3 \cos \iota_{1.3} + P_4 \cos \iota_{1.4} + \ldots\ldots) \\ \quad + P_2^2 + P_3^2 + P_4^2 + \ldots\ldots\ldots \\ \quad + 2\,P_2 P_3 \cos \iota_{2.3} + 2\,P_2 P_4 \cos \iota_{2.4} \\ \quad + 2\,P_3 P_4 \cos \iota_{3.4} + \ldots\ldots \end{array} \right\}^{\frac{1}{2}};$$

or reducing,

$$P_1 = - \frac{P_2 a_2 + P_3 a_3 + \ldots}{a_1} + \frac{\lambda}{a_1^2} \left\{ \begin{array}{l} P_2^2 (a_1^2 - 2\,a_1 a_2 \cos \iota_{1.2} + a_2^2) \\ + P_3^2 (a_1^2 - 2\,a_1 a_3 \cos \iota_{1.3} + a_3^2) \\ + \text{\&c. \&c.} \\ + 2 P_2 P_3 \{a_2 a_3 - a_1 (a_1 \cos \iota_{2.3} + a_2 \cos \iota_{1.3} + a_3 \cos \iota_{1.2})\} \\ + 2 P_2 P_4 \{a_2 a_4 - a_1 (a_1 \cos \iota_{2.4} + a_2 \cos \iota_{1.4} + a_4 \cos \iota_{1.2})\} \\ + \text{\&c. \&c.} \end{array} \right\}^{\frac{1}{2}}.$$

Now $a_1^2 - 2\,a_1 a_2 \cos \iota_{1.2} + a_2^2$ represents the square of the line joining the feet of the perpendiculars a_1 and a_2 let fall from the centre of the axis upon P_1 and P_2; similarly $a_1^2 - 2\,a_1 a_3 \cos \iota_{1.3} + a_3^2$ represents the square of the line joining the feet of the perpendicular let fall upon P_1 and P_3, and so of the rest. Let these lines be represented by $L_{1.2}$, $L_{1.3}$, $L_{1.4}$, &c., and let the different values of the function

$$\{a_2 a_3 - a_1 (a_1 \cos \iota_{2.3} + a_2 \cos \iota_{1.3} + a_3 \cos \iota_{1.2})\}$$

be represented by $M_{2.3}$, $M_{2.4}$, $M_{3.4}$, &c.,

$$\therefore P_1 = - \frac{P_2 a_2 + P_3 a_3 + \ldots}{a_1} + \frac{\lambda}{a_1^2} \left\{ \begin{array}{l} P_2^2 L_{1.2}^2 + P_3^2 L_{1.3}^2 + P_4^2 L_{1.4}^2 + \ldots \\ \quad + 2\,P_2 P_3 M_{2.3} + 2\,P_2 P_4 M_{2.4} + \ldots \end{array} \right\}^{\frac{1}{2}} \quad . \quad . \quad (11.)$$

10. The conditions of the equilibrium of three pressures P_1, P_2, P_3 in the same

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* This expansion may be effected by squaring both sides of the equation, solving the quadratic in respect to P_1, neglecting powers of λ above the first, and reducing; this method is however exceedingly laborious.

plane applied to a body moveable about a fixed axis, the direction of one of them P_3 passing through the centre of the axis, and the system being in the state bordering upon motion by the preponderance of P_1*.

Let $\iota_{1.2}\, \iota_{1.3}\, \iota_{2.3}$ be taken, as in the preceding section, to represent the inclinations of the directions of the pressures P_1, P_2, P_3 to one another, and a_1, a_2 the perpendiculars let fall from the centre of the axis upon P_1, P_2; and λ the perpendicular let fall from the same point upon the resultant R of P_1, P_2, P_3. Then since R is equal and opposite to the *resistance* of the axis (section 8.), and that P_3 acts through the centre of the axis, and P_1 and P_2 act to turn the system in opposite directions about that centre,

$$P_1\, a_1 - P_2\, a_2 = \lambda\, R.$$

Substituting for R its value†,

$$P_1\, a_1 - P_2\, a_2 = \lambda\, \{P_1^2 + P_2^2 + P_3^2 + 2\, P_1\, P_2 \cos \iota_{1.2} + 2\, P_1\, P_3 \cos \iota_{1.3} + 2\, P_2\, P_3 \cos \iota_{2.3}\}^{\frac{1}{2}};$$

squaring both sides of this equation and transposing,

$$P_1^2\, (a_1^2 - \lambda^2) - 2\, P_1\, \{P_2\, a_1\, a_2 + \lambda^2\, (P_2 \cos \iota_{1.2} + P_3 \cos \iota_{1.3})\}$$
$$= -\, P_2^2\, a_2^2 + \lambda^2\, \{P_2^2 + P_3^2 + 2\, P_2\, P_3 \cos \iota_{2.3}\};$$

solving this quadratic in respect to P_1, and omitting terms which involve powers of λ above the first,

$$P_1\, a_1^2 = P_2\, a_1\, a_2 + \lambda\, \{P_2^2\, (a_1^2 + 2\, a_1\, a_2 \cos \iota_{1.2} + a_2^2) + P_3^2\, a_1^2$$
$$+ 2\, P_2\, P_3\, a_1\, (a_2 \cos \iota_{1.3} + a_1 \cos \iota_{2.3})\}^{\frac{1}{2}};$$

or representing the line which joins the feet of the perpendiculars a_1 and a_2 by L, and the function $a_1\, (a_2 \cos \iota_{1.3} + a_1 \cos \iota_{2.3})$ by M,

$$P_1 = P_2 \left(\frac{a_2}{a_1}\right) + \frac{\lambda}{a_1^2}\, \{P_2^2\, L^2 + P_3^2\, a_1^2 + 2\, P_2\, P_3\, M\}^{\frac{1}{2}} \quad \ldots \quad (12.)$$

If P_3 be so small as compared with P_2, that in the expansion of the irrational quantity, terms involving powers of $\frac{P_3}{P_2}$ above the first may be neglected, the above equation will become by reduction,

$$P_1 = \left(\frac{a_2}{a_1}\right) \left\{1 + \frac{L\, \lambda}{a_1\, a_2}\right\} P_2 + \frac{M\, \lambda}{a_1^2\, L}\, P_3 \quad \ldots \quad (13.)$$

If in the expressions represented by $L_{1.2}$ and $M_{2.3}$ (section 9.) we make $a_3 = 0$, give to a_2 the negative sign (since the forces P_1 and P_2 tend to turn the system in opposite directions about the axis), and observe that, since P_2 receives an opposite direction, $\cos \iota_{2.3}$ becomes negative‡, these expressions will become identical with those represented by L and M in the preceding equation (12.), and that equation will

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[*] This problem is here investigated by an *independent* method as a verification of the theorem established in the preceding article, and as an application of it to a case of frequent occurrence in machinery.

[†] Poisson, Mécanique, Art. 33.

[‡] See note, p. 295.

have become identical with equation (11.), and will have supplied a verification of that equation.

If the body to which the pressures P_1, P_2, P_3 are applied have its centre of gravity in the centre of the axis about which it revolves, as is commonly the case in machines, then may its weight be supposed to act through the centre of its axis, and to be represented by P_3 in the preceding formula, so that, by that formula there is represented the relation between any two pressures P_1 and P_2 applied to such a body moveable about a fixed axis, the friction of that axis and the weight of the body being taken into account.

11. The modulus of a simple machine to which are applied one moving and one working pressure, which is moveable about a fixed axis, and has its centre of gravity in the centre of that axis, the weight of the machine being taken into account.

Let P_1 and P_2 represent the moving and working pressures on the machine, and P_3 its weight, then is the relation between these pressures in the state bordering upon motion determined by equation (12.), in which λ represents the perpendicular upon the direction of the resistance of the axis, and is therefore equal (section 8.) to $\varrho \sin \varphi$, if ϱ represents the radius of the axis, and φ the limiting angle of resistance. By the substitution of this value of λ, equation (12.) becomes

$$P_1 = P_2 \left(\frac{a_2}{a_1}\right) + \frac{\varrho \sin \phi}{a_1^2} \left\{ P_2^2 L^2 + 2 P_2 P_3 M + P_3^2 a_1^2 \right\}^{\frac{1}{2}} \quad . \quad . \quad . \quad (14.)$$

Now it is evident that this equation is of the form assumed in equation 7, section 6, the term involving the irrational quantity being represented by E (in equation 7.), and the coefficient of P_2, $\frac{a_2}{a_1}$, by a. The value of $\frac{a_2}{a_1}$ is evidently in this case independent of the prejudicial resistances, so that $a_{(0)} = \frac{a_2}{a_1}$, and $\frac{a}{a_{(0)}} = 1$. Assuming, therefore, the direction of the moving pressure P_1 to be the same with that in which its point of application is made to move, representing by θ the angle through which that point has at any time revolved, and observing that $\frac{d S_1}{d \theta} = a_1$, we have by equation 9,

$$U_1 = U_2 + \frac{\rho \sin \phi}{a_1} \int_0^\theta \{P_2^2 L^2 + 2 P_2 P_3 M + P_3 a_1^2\}^{\frac{1}{2}} d\theta, \quad . \quad . \quad . \quad . \quad (15.)$$

which is the modulus of the machine, and in which the term δ, involving the integral, represents the work lost by friction whilst the angle θ is described about the axis.

If the directions of the pressures P_1 and P_2 remain the same during the revolution of the body, and the working pressure P_2 be constant, then is the irrational quantity in the above expression constant, and the term involving the integral becomes by integration,

$$\frac{\rho \sin \phi}{a_1} \left\{ P_2^2 L^2 + 2 P_2 P_3 M + P_3^2 a_1^2 \right\}^{\frac{1}{2}} . \theta, \text{ or } \frac{\rho \sin \phi}{a_1 a_2} \left\{ P_2^2 L^2 + 2 P_2 P_3 M + P_3^2 a_1^2 \right\}^{\frac{1}{2}} . S_2,$$

(observing that $\theta\, a_2 = S_2$), or bringing S_2 under the radical sign,

$$\frac{\rho \sin \phi}{a_1\, a_2} \left\{ P_2^2\, S_2^2\, L^2 + 2\, P_2\, S_2^2\, P_3\, M + P_3^2\, S_2^2\, a_1^2 \right\}^{\frac{1}{2}},$$

or

$$\frac{\rho \sin \phi}{a_1\, a_2} \left\{ U_2^2\, L^2 + 2\, U_2\, S_2\, P_3\, M + P_3^2\, S_2^2\, a_1^2 \right\}^{\frac{1}{2}};$$

so that in this case of a constant direction of the moving pressure, and a constant amount and direction of the working pressure, the modulus becomes,

$$U_1 = U_2 + \frac{\rho \sin \phi}{a_1\, a_2} \left\{ U_2^2\, L^2 + 2\, U_2\, S_2\, P_3\, M + P_3^2\, S_2^2\, a_1^2 \right\}^{\frac{1}{2}}; \quad . \ . \ . \quad (16.)$$

and the work lost by friction whilst the space S_2 is described by the working point, is represented by the term involving the irrational quantity in this equation.

12. A machine working about an axis of given dimensions under two pressures, P_1 and P_2, the direction and amount of one of which P_2 are given, it is required to determine that constant direction in which the other pressure P_1 must be applied, so that the machine may be worked with the greatest economy of power.

It has been shown in the last section that the work lost by friction is represented, in the case here supposed, by the formula

$$\frac{\rho \sin \phi}{a_1\, a_2} \left\{ U_2^2\, L^2 + 2\, U_2\, S_2\, P_3\, M + P_3^2\, S_2^2\, a_1^2 \right\}^{\frac{1}{2}} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (17.)$$

The machine is evidently worked then with the greatest economy of power to yield a given amount of work, U_2, when this function is a minimum. Substituting for L^2 its value

$$a_1^2 + 2\, a_1\, a_2 \cos \iota_{1.2} + a_2^2,$$

and for M its value

$$a_1 \{ a_2 \cos \iota_{1.3} + a_1 \cos \iota_{2.3} \} \text{ (section 10.)},$$

it becomes

$$\frac{\rho \sin \phi}{a_1\, a_2} \left\{ U_2^2 (a_1^2 + 2\, a_1 a_2 \cos \iota_{1.2} + a_2^2) + 2\, U_2 P_3 S_2 a_1 (a_2 \cos \iota_{1.3} + a_1 \cos \iota_{2.3}) + P_3^2 S_2^2 a_1^2 \right\}^{\frac{1}{2}} \ (18.)$$

Now let us suppose that the perpendicular distance a_2 from the centre of the axis at which the work is done, and the inclination $\iota_{2.3}$ of its direction to the vertical, are both given, as also the space S_2 through which it is done, so that the work is given in every respect; let also the perpendicular distance a_1 at which the power is applied, be given; it is required to determine that inclination $\iota_{1.2}$ of the power to the work which will under these circumstances give to the above function its minimum value, and which is, therefore, consistent with the most economical working of the machine.

Collecting all the terms in the function (18.) which contain (on the above suppositions) only constant quantities, and representing their sum

$$U_2^2 (a_1^2 + a_2^2) + 2\, P_3\, S_2\, a_1^2 (U_2 \cos \iota_{2.3} + P_3\, S_2)$$

by C^2, it becomes

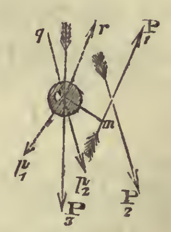
$$\frac{\rho \sin \phi}{a_1\, a_2} \left\{ 2\, a_1\, a_2\, U_2 (U_2 \cos \iota_{1.2} + P_3\, S_2 \cos \iota_{1.3}) + C^2 \right\}^{\frac{1}{2}}.$$

Now C^2 being essentially positive, this quantity is a minimum when

$$2\,a_1\,a_2\,U_2\,(U_2 \cos \iota_{1.2} + P_3\,S_2 \cos \iota_{1.3})$$

is a minimum; or, observing that $U_2 = P_2\,S_2$, and dividing by the constant factor $2\,a_1\,a_2\,U_2\,S_2$; when

$$P_2 \cos \iota_{1.2} + P_3 \cos \iota_{1.3} \quad \text{is a minimum.}$$



From the centre of the axis C let lines C p_1 C p_2 be drawn parallel to the directions of the pressures P_1, P_2 respectively; and whilst C p_2 and C P_3 retain their positions, let the angle p_1 C P_3 or $\iota_{1.3}$ be conceived to increase until P_1 attains a position in which the condition $P_2 \cos \iota_{1.2} + P_3 \cos \iota_{1.3} =$ a minimum is satisfied. Now

$$p_1\,C\,P_3 = p_1\,C\,p_2 - p_2\,C\,P_3, \text{ or } \iota_{1.3} = \iota_{1.2} - \iota_{2.3};$$

substituting which value of $\iota_{1.3}$, this condition becomes

$$P_2 \cos \iota_{1.2} + P_3 \cos (\iota_{1.2} - \iota_{2.3}) = \text{a minimum},$$

or

$$P_2 \cos \iota_{1.2} + P_3 \cos \iota_{1.2} \cos \iota_{2.3} + P_3 \sin \iota_{1.2} \sin \iota_{2.3} = \text{a minimum},$$

or

$$(P_2 + P_3 \cos \iota_{2.3}) \cos \iota_{1.2} + P_3 \sin \iota_{2.3} \sin \iota_{1.2} = \text{a minimum}.$$

Let now $\dfrac{P_3 \sin \iota_{2.3}}{P_2 + P_3 \cos \iota_{2.3}} = \tan \gamma$,

so that

$$P_3 \sin \iota_{2.3} = (P_2 + P_3 \cos \iota_{2.3}) \tan \gamma$$

$$\therefore \; (P_2 + P_3 \cos \iota_{2.3}) \cos \iota_{1.2} + (P_2 + P_3 \cos \iota_{2.3}) \tan \gamma \sin \iota_{1.2} = \text{a minimum},$$

or dividing by the constant quantity $(P_2 + P_3 \cos \iota_{2.3})$, and multiplying by $\cos \gamma$,

$$\cos \iota_{1.2} \cos \gamma + \sin \iota_{1.2} \sin \gamma = \cos (\iota_{1.2} - \gamma) = \text{a minimum}.$$

$$\therefore \; \iota_{1.2} - \gamma = \pi.$$

$$\therefore \; \iota_{1.2} = \pi + \tan^{-1} \left\{ \frac{P_3 \sin \iota_{2.3}}{P_2 + P_3 \cos \iota_{2.3}} \right\} \quad \ldots\ldots\ldots\ldots \quad (19.)$$

To satisfy the conditions of a minimum, the angle p_1 C p_2 must therefore be increased until it exceeds 180° by that angle γ whose tangent is represented by

$$\frac{P_3 \sin \iota_{2.3}}{P_2 + P_3 \cos \iota_{2.3}}.$$

To determine the actual direction of P_1, produce then p_2 C to q, make the angle q C r equal to γ; and draw C m perpendicular to C r, and equal to the given perpendicular distance a_1 of the direction of P_1 from the centre of the axis. If $m\,P_1$ be then drawn through the point m parallel to C r, it will be in the required direction of P_1; so that being applied in this direction, the moving pressure P_1 will work the machine with a greater economy of power than when applied in any other direction round the axis.

It is evident that since the value of the angle $\iota_{1.2}$ or p_2 C p_1, which satisfies the condition of the greatest economy of power, or of the least resistance, is essentially greater than two right angles, P_1 and P_2 must, to satisfy that condition, both be applied on the same side of the axis. It is then a condition necessary to the most economical working of any machine (whatever may be its weight) which is moveable about a cylindrical axis under two given pressures, that the moving pressure should be applied on that side of the axis of the machine on which the resistance is overcome, or the work done. It is a further condition of the greatest economy of power in such a machine, that the direction in which the moving pressure is applied should be inclined to the vertical at an angle $\iota_{1.3}$ determined by the formula

$$\iota_{1.3} = \pi - \iota_{2.3} + \tan^{-1}\left\{\frac{P_3 \sin \iota_{2.3}}{P_2 + P_3 \cos \iota_{2.3}}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20.)$$

When $\iota_{2.3} = 0$, or when the work is done in a vertical direction, $\iota_{1.3} = \pi$, whence it follows that the moving power also must in this case be applied in a vertical direction, and on the same side of the axis as the work. When $\iota_{2.3} = \frac{\pi}{2}$, or when the work is done horizontally, $\tan \gamma = \frac{P_3}{P_2}$;

$$\therefore \ \iota_{1.2} = \pi + \tan^{-1}\left(\frac{P_3}{P_2}\right).$$

The moving power must therefore in this case be applied on the same side of the axis as the work, and at an inclination to the horizon whose tangent equals the fraction obtained by dividing the weight of the machine by the working pressure.

Since the angle $\iota_{1.2}$ is greater than π and less than $\frac{3\pi}{2}$, therefore $\cos \iota_{1.2}$ is negative; and, for a like reason, $\cos \iota_{1.3}$ is also in certain cases negative. Whence it is apparent that the function (18.) admits of a minimum value under certain conditions, not only in respect to the inclination of the moving pressure, but in respect to the distance a_1 of its direction from the centre of the axis. If we suppose the space S_1 through which the power acts whilst the given amount of work U_2 is done, to be given, and substitute in that function for the product $S_2 a_1$ its value $S_1 a_2$, and then assume the differential coefficient of the function in respect to a_1 to vanish, we shall obtain by reduction,

$$a_1 = -a_2 \cdot \frac{U_2^2 + 2 U_2 P_3 S_1 \cos \iota_{1.3} + P_3^2 S_1^2}{U_2^2 \cos \iota_{1.2} + U_2 P_3 S_1 \cos \iota_{2.3}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21.)$$

If we proceed in like manner, assuming the space S_2 instead of S_1 to be constant, and substituting in the function (18.) for $S_1 a_2$ its value $S_2 a_1$, we shall obtain by reduction,

$$a_1 = -\frac{P_2 a_2}{P_2 \cos \iota_{1.2} + P_3 \cos \iota_{2.3}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22.)$$

It is easily seen that, if, when the values of $\iota_{1.2}$ and $\iota_{1.3}$ determined by equations 19 and 20. are substituted in these equations, the resulting values of a_1 are positive, they correspond, in the two cases, to minimum values of the function (18.), and determine completely the conditions of the greatest economy of power in the machine, in respect to the direction of the moving pressure applied to it.

13. *The Modulus of the Pulley.*

Let P_1 and P_2 be taken to represent the *moving* and *working* (or the preponderating and yielding) tensions upon the two parts of the cord passing over a pulley; let W represent its weight, a its radius measuring to the centre of the cord, ϱ the radius of its axis, and φ the limiting angle of resistance between the axis and its bearings. Then if the cord were without rigidity, we should have by equation (13.), observing that $a_1 = a_2 = a$, and substituting W for P_3, and $\varrho \sin \varphi$ for λ,

$$P_1 = \left\{1 + \frac{L\rho}{a_2} \sin \varphi\right\} P_2 + \frac{M\rho}{L a^2} \,.\, W \sin \varphi.$$

But by the experiments of Coulomb (as reduced by M. Poncelet)*, it appears that the effect of the rigidity of the cord is the same as though it increased the tension P_2 so as to become $P_2\left(1 + \frac{E}{a}\right) + \frac{D}{a}$, where E and D are certain constants given in terms of the diameter of the rope. Taking into account the effect of this rigidity, the relation between P_1 and P_2 becomes therefore

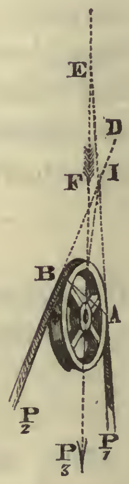
$$P_1 = \left\{1 + \frac{L\rho}{a^2} \sin \varphi\right\} \left\{P_2\left(1 + \frac{E}{a}\right) + \frac{D}{a}\right\} + \frac{M\rho}{L a^2} W \sin \varphi,$$

whence by reduction we have

$$P_1 = \left(1 + \frac{E}{a}\right)\left\{1 + \frac{L\rho}{a^2} \sin \varphi\right\} P_2 + \frac{D}{a}\left\{1 + \left(\frac{L}{a^2} + \frac{M W}{L D a}\right) \varrho \sin \varphi\right\}, \quad . \quad (23.)$$

where L represents the chord of the arc embraced by the string, and M the quantity $a^2 (\cos \iota_{1.3} + \cos \iota_{2.3})$, $\iota_{1.3}$ and $\iota_{2.3}$ being the inclinations of the two parts of the string to the vertical (section 10.).

Let the accompanying figure be taken to represent the pulley with the cord passing over it, and E P_3 the direction of the weight of the pulley, supposed to act through the centre of its axis, then are the angles $\iota_{1.3}$ and $\iota_{2.3}$ represented by P_1 E P_3, and P_2 F P_3, or their *supplements*, according as the pressures P_1 and P_2 respectively act *downwards*, as shown in the figure, or *upwards*†; so that if *both* these pressures act upwards, then the cosines of both angles become negative, and the value of M is negative; whilst if *one* only acts upwards, then one term only of the value of M assumes a negative value. Let the inclination A I B of the two parts of the string be represented by $2\,\iota$, then $L = A\,B = 2\,a \cos \iota$. Substituting this value for L, and also its value $a^2 (\cos \iota_{1.3} + \cos \iota_{2.3})$ for M, and omitting terms which involve products of the exceedingly small quantities $\frac{D}{a}$, $\frac{E}{a}$ and $\frac{\rho}{a} \sin \varphi$, we have



$$P_1 = \left\{1 + \frac{E}{a} + \frac{2\rho}{a} \cos \iota \sin \varphi\right\} P_2 + \frac{D}{a} + \frac{W \rho (\cos \iota_{1.3} + \cos \iota_{2.3}) \sin \phi}{2\, a \cos \iota}.$$

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* See Poncelet, Mécanique Industrielle, 128.

† See Note, Section 9.

Whence* we obtain for the *modulus* of the pulley,

$$U_1 = \left\{ 1 + \frac{E}{a} + \frac{2\rho}{a} \cos \iota \sin \varphi \right\} U_2 + \left\{ \frac{D}{a} + \frac{W\rho(\cos \iota_{1.3} + \cos \iota_{2.3}) \sin \phi}{2\, a \cos \iota} \right\} S_1. \quad . \quad (24.)$$

If both the strings be inclined at equal angles to the vertical, on opposite sides of it, or if $\iota_{1.3} = \iota_{2.3} = \iota$, so that $\cos \iota_{1.3} + \cos \iota_{2.3} = 2 \cos \iota$, the modulus becomes

$$U_1 = \left\{ 1 + \frac{E}{a} + \frac{2\rho}{a} \cos \iota \sin \varphi \right\} U_2 + \left\{ \frac{D}{a} + \frac{W\rho}{a} \sin \varphi \right\} S_1. \quad . \quad . \quad . \quad . \quad (25.)$$

If one part of the cord passing over a pulley have a horizontal, and the other a vertical direction, as, for instance, when it passes into the *shaft* of a mine over the sheaf or wheel which overhangs its mouth, then one of the angles $\iota_{1.3}, \iota_{2.3}$ (equation 24.) becomes $\frac{\pi}{2}$, and the other 0 or π, according as the tension of the vertical part of the cord is upwards or downwards, so that $\cos \iota_{1.3} + \cos \iota_{2.3} = \pm 1$, the sign $\pm$ being taken according as the tension on the vertical branch of the cord is upwards or downwards: moreover in this case $\iota = \frac{\pi}{4}$, and $\cos \iota = \frac{1}{\sqrt{2}}$, therefore by equation (24.),

$$U_1 = \left\{ 1 + \frac{E}{a} + \frac{\rho\sqrt{2}}{a} \sin \varphi \right\} U_2 + \frac{1}{a} \left\{ D \pm \frac{W\rho}{\sqrt{2}} \sin \varphi \right\} S_1. \quad . \quad . \quad . \quad . \quad (26.)$$

If the two parts of the cord passing over the pulley be parallel, and their common inclination to the vertical be represented by ι, so that $\iota_{1.3} = \iota_{2.3} = \iota$; then, since in this case $L = 2\, a$, we have by equation (23.), neglecting terms of more than one dimension in $\frac{E}{a}$ and $\frac{\rho}{a}$,

$$U_1 = \left\{ 1 + \frac{E}{a} + \frac{2\rho}{a} \sin \varphi \right\} U_2 + \frac{D}{a} \left\{ 1 + \left(\frac{2}{a} + \frac{W \cos \iota}{D} \right) \varrho \sin \varphi \right\}, \quad . \quad . \quad (27.)$$

in which equation, ι is to be taken greater or less than $\frac{\pi}{2}$, and therefore the sign of $\cos \iota$ is to be taken positively or negatively, according as the tensions on the cords act downwards or upwards. If the tensions are vertical, $\iota = 0$ or π, according as they act upwards or downwards, so that $\cos \iota = \pm 1$. If the parallel tensions are *horizontal,* then $\iota = \frac{\pi}{2}$, and the terms involving $\cos \iota$ in the above equations *vanish.*

If both parts of the cord passing over a pulley be in the same horizontal straight line, so that the pulley sustains no pressure resulting from the tension of the cord, but only bears its *weight,* then $\iota = \frac{\pi}{2}$, and the term involving $\cos \iota$ in equation (25.) vanishes.

It is, however, to be observed, that the weight bearing upon the axis of the pulley, is the weight of the pulley increased by the weight of the cord which it is made to support; so that if the *length* of cord supported by the pulley be represented by s, and the weight of each unit of length by μ, then is

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* See Section 6, Equation 10.

the *weight* sustained by the axis of each pulley represented by $W + \mu s$. Substituting this value for W and assuming $\cos \iota = 0$ in equation (25.), we have for the modulus of the pulley in this case,

$$U_1 = \left(1 + \frac{E}{a}\right) U_2 + \frac{1}{a}\left\{D + (W + \mu s)\,\varrho \sin \varphi\right\} S_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (28.)$$

In which equation it is supposed that although the direction of the rope on either side of each pulley is so nearly horizontal that $\cos \iota$ may be considered evanescent, yet the rope *does* so far *bend* itself over each pulley, as that its surface may adapt itself to the curved surface of the pulley, and thereby produce the whole of that resistance which is due to the rigidity of the cord.

Let it now be supposed that there is a system of n equal pulleys, or sheaves of the same dimensions, placed at equal distances in the same horizontal straight line, and sustaining each the same length s of rope.

Let U_1 represent the work done upon the cord, through the space S_1, by the moving power, or before it has passed over the first pulley of the series; U_1 the work done upon it after it has passed over the first pulley; U_2 after it has passed over the second, &c.; and U_n after it has passed over the nth pulley or sheaf; then

$$U_1 = \left(1 + \frac{E}{a}\right) U_2 + \frac{1}{a}\left\{D + (W + \mu s)\,\varrho \sin \varphi\right\} S_1;$$

$$U_2 = \left(1 + \frac{E}{a}\right) U_3 + \frac{1}{a}\left\{D + (W + \mu s)\,\varrho \sin \varphi\right\} S_1, \text{ \&c. \&c.};$$

$$U_n = \left(1 + \frac{E}{a}\right) U_{n-1} + \frac{1}{a}\left\{D + (W + \mu s)\,\varrho \sin \varphi\right\} S_1.$$

Eliminating the $n - 1$ quantities $U_2\ U_3 \ldots U_{n\ 1}$ between these n equations, and neglecting terms involving powers of $\frac{E}{a}, \frac{D}{a}, \frac{\rho}{a} \sin \varphi$ above the first, we have

$$U_1 = \left(1 + \frac{nE}{a}\right) U_n + \frac{n}{a}\left\{D + (W + \mu s)\,\varrho \sin \varphi\right\} S_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (29.)$$

Let us now suppose that the rope, after passing horizontally over n equal pulleys, the radius of each of which is represented by a, and its weight by W, as in the preceding case, assumes at length a vertical direction, passing over a pulley or sheaf of different dimensions, whose radius is represented by a_1, that of its axis by ϱ_1, and its weight by W_1; as for instance, when the rope of a mine descends into the shaft after having traversed the space between it and the engine, supported upon pulleys.

Let U_2 represent the work done upon the rope through the space S_1 after it has assumed the vertical direction or passed into the shaft, and let U_n represent, as before, the work done upon it after it has passed over the n horizontal pulleys, and before it passes over that which overhangs the shaft. Then by equation (26.),

$$U_n = \left\{1 + \frac{E}{a_1} + \frac{\rho_1 \sqrt{2}}{a_1} \sin \varphi\right\} U_2 + \frac{1}{a_1}\left\{D + \frac{W_1 \rho_1}{\sqrt{2}} \sin \varphi\right\} S_1.$$

Eliminating the value of U_n between this equation and equation (29.), and neglecting dimensions above the first in $\frac{E}{a}$, &c., we have

$$U_1 = \Big\{ 1 + E\left(\frac{1}{a_1} + \frac{n}{a}\right) + \frac{\rho_1 \sqrt{2}}{a_1} \sin \varphi \Big\} U_2 + \Big\{ D\left(\frac{1}{a_1} + \frac{n}{a}\right)$$

$$+ \left\{ \frac{W_1 \rho_1}{a_1 \sqrt{2}} + \frac{(n W + w)\rho}{a} \right\} \sin \varphi \Big\} S_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30.)$$

where w represents the whole weight $n \mu s$ of the rope supported horizontally by the pulleys. In this, as in the preceding case, it is assumed that although the rope is so nearly in the same straight line on either side of each pulley that $\cos \iota$ may be considered evanescent, yet it does so far bend as to adapt itself to the circumference of each, and thereby produce the whole of that resistance which is due to its rigidity.

HENRY MOSELEY.

King's College,
June 9, 1841.

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TO THE

PHILOSOPHICAL TRANSACTIONS

FOR THE YEAR 1841.

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METEOROLOGICAL JOURNAL FOR JANUARY AND FEBRUARY, 1841.

1841.	9 o'clock, A.M.			3 o'clock, P.M.			Dew Point at 9 A.M., deg. Fahr.	Diff. of Wet and Dry Bulb Ther.	External Thermometers.				Rain in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	REMARKS.
	Barometer uncorrected.		Att. Ther.	Barometer uncorrected.		Att. Ther.			Fahrenheit.		Self-registering				
	Flint Glass.	Crown Glass.		Flint Glass.	Crown Glass.				9 A.M.	3 P.M.	Lowest	Highest			
JANUARY															
F 1	29.962	29.954	37.8	29.910	29.904	39.2	34	01.4	40.3	44.4	37.5	44.2	.016	W	Overcast—deposition—brisk wind throughout the day, as also evening.
S 2	30.096	30.088	40.0	30.108	30.100	40.7	35	01.8	39.7	42.2	40.0	45.5		W	Fine—light clouds—brisk wind throughout the day. Ev. Overcast.
* ⊙ 3	29.304	29.300	40.5	29.168	29.164	40.5	35	01.0	35.8	36.4	36.0	44.8	.166	S	A.M. Overcast—light wind and snow—very high wind throughout the night.* P.M. Lt. clds. & wind. Ev. Fine & starlight—sharp frost.
M 4	29.002	28.998	36.9	29.050	29.042	36.3	32	*frozen*	30.4	35.7	29.8	38.7	.061	S	A.M. Thick fog—white frost. P.M. Overcast—light snow and wind. Evening, Overcast—light snow and frost.
T 5	29.308	29.300	35.0	29.322	29.314	35.4	29	01.3	31.8	33.3	30.0	36.3	.036	NW	A.M. Fine—light clouds—brisk wind. P.M. Overcast—light snow—brisk wind. Evening, Continued snow.
W 6	29.528	29.520	33.2	29.568	29.560	33.0	27	*frozen*	27.7	29.3	28.3	34.4		N	A.M. Ovct.—sharp frost. P.M. Fine—lt. clds. Ev. Ovct.—sharp frost.
○ T 7	29.696	29.688	29.7	29.704	29.696	29.3	21	*ditto*	19.7	27.2	20.7	29.8		NW	A.M. Fine—light clouds—sharp frost. P.M. Thick haze—sharp frost. Evening, Cloudy and frosty.
F 8	29.884	29.878	27.7	29.872	29.864	27.3	24	*ditto*	19.7	21.8	19.5	27.3		SSW	A.M. Thick fog—light snow—sharp frost. P.M. Overcast—sharp frost. Evening, Moonlight—sharp frost.
S 9	29.706	29.698	24.2	29.594	29.586	26.6	18	*ditto*	21.2	30.3	14.9	21.4		SW	A.M. Light fog—sharp frost. P.M. Fine—light clouds and wind—slight thaw. Ev. Overcast—lt. snow—sharp frost.
⊙ 10	29.230	29.224	28.2	29.128	29.122	29.8	25	*ditto*	32.4	37.0	22.2	32.8		SE	A.M. Ovct.—snow during night. P.M. Ovct.—thaw. Ev. Ovct.—rapid thaw.
M 11	28.864	28.858	31.9	28.850	28.844	33.8	28	01.0	34.3	35.5	32.8	37.7	.205	S	A.M. Overcast—continued thaw—light wind. P.M. Fine—light clouds. Evening, Cloudy.
T 12	29.366	29.358	34.8	29.504	29.498	35.8	30	Broken by frost	35.2	36.8	33.0	37.2		W	A.M. Lightly overcast—light wind. P.M. Fine—light clouds. Evening, Frosty—light fog.
W 13	29.550	29.546	35.0	29.334	29.326	35.9	32	01.2	35.2	33.3	32.8	38.0		S	A.M. Light fog—rain and wind. P.M. Overcast—lt. snow. Ev. Thaw.
T 14	29.370	29.364	35.8	29.250	29.246	36.2	32	01.7	35.2	33.3	34.0	39.2	.352	NE	Overcast, with occasional falls of snow, with brisk wind throughout the day. Evening, Continued snow—brisk wind.
F 15	29.578	29.570	36.6	29.664	29.656	37.8	33	00.5	34.8	35.5	33.3	37.2	.700	NW	Fine—light clouds & wind throughout the day. Ev. Overcast—thaw.
S 16	29.654	29.648	56.9	29.462	29.456	38.2	35	00.9	37.4	44.6	33.6	38.2	.088	E	Overcast—light rain and wind nearly the whole of the day, as also the evening.
⊙ 17	29.632	29.626	43.0	29.616	29.608	44.6	41	01.4	47.4	51.3	37.0	51.8	.166	SE	A.M. Fine—light clouds and wind. P.M. Cloudy—light wind. Evening, The same.
M 18	29.736	29.728	45.7	29.714	29.706	46.6	42	01.4	46.8	47.7	46.7	52.3		S	A.M. Overcast—slight rain and wind. P.M. Cloudy—light wind. Evening, Overcast—slight rain.
T 19	29.812	29.804	42.7	29.886	29.878	42.6	37	01.2	35.5	36.7	35.0	48.7	.227	NNW	A.M. Ovct.—lt. fog & wind. P.M. Ovct.—fine rain. Ev. Light snow.
W 20	30.050	30.044	39.2	30.106	30.098	39.0	32	02.1	34.3	33.7	32.6	37.8	.022	NW	A.M. Cloudy—brisk wind. P.M. Fine—light clouds. Evening, Fine and starlight.
T 21	30.410	30.402	35.6	30.384	30.376	36.6	29	*frozen*	29.8	38.8	28.4	34.7		W	A.M. Lightly overcast—sharp frost. P.M. Fine—light clouds. Evening, Fine and starlight.
● F 22	30.396	30.388	37.2	30.256	30.250	38.6	34	02.0	34.5	40.5	30.2	39.4		S	A.M. Fine—light clouds and wind. P.M. Cloudy—light wind. Evening, Overcast—light rain.
S 23	30.084	30.076	40.0	30.116	30.108	41.0	37	01.7	39.3	41.0	34.3	44.7	.100	NW	Fine—light clouds and wind throughout the day. Ev. Overcast.
⊙ 24	29.796	29.790	39.6	29.860	29.854	39.5	33	02.4	36.7	35.0	36.0	42.0		NW	A.M. Fine—light clouds and wind—shortly before 12 o'clock, snow storm, with h. wind. P.M. Ovct.—h. wind. Ev. Fine & starlight.
M 25	30.350	30.342	36.6	30.274	30.266	37.2	31	01.9	31.5	37.5	31.0	38.0	.033	SW	Fine—light clouds and wind throughout the day. Evening, Overcast.
T 26	30.000	29.992	38.7	29.996	29.988	40.6	35	01.4	42.7	47.3	31.0	43.6		WSW	A.M. Overcast—deposition—light wind. P.M. Fine—light clouds. Evening, Overcast.
W 27	30.050	30.044	42.2	30.154	30.148	44.5	40	01.5	47.3	49.3	43.0	48.6		SSW	A.M. Light fog and wind. P.M. Fine—lt. clouds. Ev. Fine & starlight.
T 28	30.322	30.314	41.9	30.282	30.274	42.8	36	01.9	37.8	43.8	36.0	52.8		SW	A.M. Light fog and wind. P.M. Cloudy—light wind. Evening, Fine and starlight.
F 29	30.202	30.196	42.2	30.204	30.198	43.8	37	01.8	37.7	43.4	36.0	44.8		NW	A.M. Fine—light clouds and wind. P.M. Cloudy—light wind. Evening, Cloudy.
S 30	30.282	30.274	42.6	30.256	30.248	44.3	37	02.1	40.2	40.3	37.9	44.2		S	A.M. Light fog & wind. P.M. Overcast—light rain. Ev. The same.
⊙ 31	30.210	30.202	42.7	30.204	30.198	42.8	38	01.4	40.8	34.7	40.2	41.7	.052	S	Overcast—light rain, snow, and fog, nearly the whole of the day. Evening, Overcast—light rain, snow, and wind.
MEAN.	29.788	29.781	37.9	29.769	29.761	38.1	33	01.5	35.3	38.0	32.7	40.3	Sum. 2.224 *Rain and melted snow.]*		Mean Barometer corrected .... 9 A.M. 3 P.M. F. 29.767 .. 29.748 C. 29.759 .. 29.739
FEBRUARY															
M 1	30.358	30.350	37.0	30.284	30.276	36.9	30	*Frozen.*	30.7	31.7	29.7	42.0	.116	N	A.M. Overcast—light snow and wind—sharp frost. P.M. Fine—light clouds. Evening, Fine and starlight—sharp frost.
T 2	30.072	30.064	32.4	29.946	29.940	33.4	24		27.7	28.7	25.8	32.4		N	Overcast—light snow & brisk wind—sharp frost throughout the day, as also the evening.
W 3	30.000	29.994	29.0	29.998	29.992	29.6	20		22.2	24.8	21.6	29.7		N	A.M. Ovct.—sharp frost—lt. wind. P.M. Fine—lt. clds.—sharp frost. Ev. Overcast—sharp frost.
T 4	29.760	29.752	27.3	29.666	29.660	28.2	21		25.7	26.7	22.2	25.7		E	Overcast—sharp frost—brisk wind throughout the day. Evening, Fine and moonlight—sharp frost.
F 5	29.682	29.674	29.2	29.694	29.688	31.7	22		25.7	28.5	24.6	30.0		NE	Fine—lt. clds.—sharp frost—brisk wind throughout the day. Evening, Overcast—sharp frost.
○ S 6	29.648	29.642	28.2	29.590	29.586	29.7	24		27.3	27.8	25.2	29.8		NE	Overcast—high wind throughout the day, as also the night. Ev. The like, with sharp frost.
⊙ 7	29.480	29.472	26.8	29.418	29.414	26.6	20		23.8	24.9	23.8	28.8		NE	Overcast—high wind throughout the day, as also throughout the night. Evening, The like, with sharp frost.
M 8	29.372	29.364	27.0	29.378	29.372	28.6	23		26.7	28.7	24.0	26.7		N	A.M. Overcast—brisk wind—sharp frost. P.M. Overcast—light rain, which froze in falling. Evening, Overcast—sharp frost.
T 9	29.636	29.630	28.9	29.774	29.766	29.8	24		27.4	29.7	27.2	29.3		N	Overcast—brisk wind—sharp frost throughout the day—snow the early part of A.M. Evening, Overcast—sharp frost.
W 10	30.014	30.006	29.7	29.980	29.974	30.2	25		28.3	29.8	27.0	30.2		SE	Ovct.—lt. wind—sharp frost throughout the day, as also the evening.
T 11	29.850	29.844	31.8	29.744	29.736	34.2	28	00.5	36.7	38.7	27.0	37.2		SE	Overcast—rapid thaw throughout the day. Evening, Light rain.
F 12	29.672	29.666	39.2	29.726	29.718	41.3	36	00.6	41.7	47.7	35.6	42.5		S	Light fog and wind with deposition throughout the day, as also the evening.
S 13	29.672	29.664	42.2	29.516	29.508	44.3	39	01.4	44.8	45.7	41.4	49.0		SSE	Overcast—light wind, with occasional light rain throughout the day. Evening, The like.
⊙ 14	29.180	29.174	44.8	29.220	29.214	47.0	42	01.2	47.3	50.3	45.0	48.3	.022	S	Overcast—high wind throughout the day, as also very high wind throughout the night. Evening, The like.
M 15	29.274	29.268	45.6	29.140	29.134	47.0	42	01.5	43.4	47.7	42.4	51.4		SSE	A.M. Overcast—lt. wind. P.M. Light rain & wind. Ev. The same.
T 16	29.076	29.070	46.9	29.082	29.076	48.3	43	01.3	45.7	48.3	42.5	49.2	.091	E	A.M. Overcast—light wind. P.M. Fine—light clouds and wind. Evening, Cloudy.
W 17	29.268	29.260	45.7	29.452	29.446	45.7	41	01.1	40.3	42.7	40.8	50.0	.125	NW	Overcast—light rain and wind nearly the whole of the day. Ev. Overcast—light wind.
T 18	29.642	29.638	46.0	29.530	29.524	48.0	41	01.6	43.3	50.3	40.7	45.4		SE	Fine—light clouds and wind throughout the day. Ev. Overcast.
F 19	29.672	29.664	49.3	29.726	29.718	50.8	46	01.4	46.8	48.8	43.0	51.4	.088	S	Overcast—slight rain & wind throughout the day, as also the evening.
S 20	29.846	29.840	48.7	29.870	29.862	49.7	45	02.2	46.7	50.4	40.4	50.0	.033	S	A.M. Fine—light clouds & wind. P.M. Overcast—light brisk wind. Evening, Light rain.
● ⊙ 21	30.198	30.192	48.3	30.246	30.240	49.4	44	02.1	45.3	49.2	44.7	52.2	.063	NW	Lightly overcast—light wind throughout the day. Evening, Starlight—light fog.
M 22	30.390	30.382	46.5	30.356	30.350	46.6	42	01.1	40.3	42.7	40.2	50.2		NW	A.M. Light fog and wind. P.M. Overcast—lt. wind. Ev. The like.
T 23	30.356	30.350	45.3	30.316	30.308	45.3	41	01.7	38.6	38.7	38.6	43.7		NE	Overcast—brisk wind throughout the day. Evening, The same.
W 24	30.328	30.320	41.6	30.324	30.318	42.3	36	01.6	34.7	37.0	33.8	41.3		N	Overcast—lt. brisk wind throughout the day. Ev. Very slight rain.
T 25	30.336	30.330	41.3	30.228	30.220	42.3	37	02.5	38.8	42.7	33.8	41.5		N	A.M. Overcast—light rain and wind. P.M. Cloudy—light wind. Evening, Overcast—light rain.
F 26	29.378	29.372	44.7	29.780	29.772	45.7	42	02.2	43.7	43.2	38.7	44.5	.108	WNW	Overcast—brisk wind throughout the day. Ev. Light rain and wind.
S 27	29.756	29.750	42.9	29.820	29.812	44.3	37	02.6	39.7	42.7	37.0	47.7	.100	NW	Overcast—light brisk wind throughout the day. Ev. Light rain.
⊙ 28	29.880	29.874	41.8	29.812	29.804	42.7	34	02.8	36.9	41.5	35.0	43.3	.044	WNW	Fine—light clouds and wind throughout the day. Ev. Overcast.
MEAN.	29.778	29.772	38.9	29.772	29.765	40.0	34	01.6	36.4	38.9	34.0	40.8	Sum. .790		Mean Barometer corrected .... 9 A.M. 3 P.M. F. 29.755 .. 29.745 C. 29.748 .. 29.773

* On Sunday, the 3rd of January, a few minutes before 7 A.M., the wind increased much, accompanied by a very heavy shower of hail, snow, and rain, with loud thunder, and very vivid lightning. The lightning had a very purplish appearance. The storm continued for the space of twelve to fourteen minutes, after which it became perfectly calm. At A.M. the barometer was much depressed, having fallen nearly an inch from the previous afternoon. It stood thus on the 2nd, at 3 P.M., F. 30.108, C. 30.100; and on the 3rd, at A.M. F. 29.324, C. 29.318; and it continued falling until the following day, when at 3 P.M. it began to rise.—J. D. R.

METEOROLOGICAL JOURNAL FOR MARCH AND APRIL, 1841.

1841.	9 o'clock, A.M.			3 o'clock, P.M.			Dew Point at 9 A.M., deg. Fahr.	Diff. of Wet and Dry Bulb Thermometer.	External Thermometers.				Rain in Inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	REMARKS.
	Barometer uncorrected.		Att. Ther.	Barometer uncorrected.		Att. Ther.			Fahrenheit.		Self-registering				
	Flint Glass.	Crown Glass.		Flint Glass.	Crown Glass.				9 A.M.	3 P.M.	Lowest	Highest			
MARCH															
M 1	29.614	29.606	41.9	29.644	29.636	42.6	.36	02.3	39.3	41.7	36.9	42.6		SE	Cloudy—light wind throughout the day. Ev. Overcast—light rain.
T 2	29.688	29.680	41.9	29.542	29.536	43.6	37	02.5	37.4	42.4	35.0	43.2	.025	SW	Overcast—lt. rain—brisk wind throughout the day. Ev. The same.
W 3	29.324	29.318	43.7	29.428	29.422	44.8	39	02.2	42.4	47.3	37.3	46.7	.283	NW	A.M. Fine—light clouds and wind. P.M. Cloudy, with occasional slight rain. Evening, Overcast.
T 4	29.912	29.904	42.3	29.874	29.868	43.8	36	02.7	36.8	44.7	35.2	48.3		NW	Fine—light clouds and wind throughout the day. Evening, Overcast—light rain.
F 5	29.876	29.870	44.9	29.616	29.610	46.6	40	02.4	42.0	45.3	36.2	45.4	.083	S	A.M. Fine—light clouds and wind. P.M. Overcast—light rain—high wind. Evening, The same.
S 6	29.764	29.758	45.7	29.900	29.896	47.0	41	03.4	43.2	50.4	40.5	50.3	.158	NW	Fine—light clouds and wind throughout the day. Ev. Cloudy.
○☉ 7	30.080	30.072	48.7	30.140	30.136	50.2	43	03.0	51.7	57.8	42.6	52.5		W	Fine—lt. clouds & wind throughout the day. Ev. Fine & moonlight.
M 8	30.414	30.406	50.0	30.406	30.400	52.7	45	01.9	50.5	58.8	49.6	59.2		SW	A.M. Light fog, with very slight rain. P.M. Fine—light clouds. Evening, Fine and moonlight.
T 9	30.444	30.438	51.0	30.418	30.410	52.4	46	01.7	44.5	55.6	43.4	60.3		SSW	A.M. Light fog. P.M. Fine—light clouds. Ev. Fine and moonlight.
W 10	30.490	30.484	50.6	30.454	30.448	52.3	45	02.5	46.7	57.6	43.3	56.5		S	A.M. Light fog. P.M. Cloudy—light wind. Ev. Fine & moonlight.
T 11	30.512	30.504	49.4	30.450	30.442	51.3	44	02.5	45.2	58.3	40.0	58.7		S	A.M. Light fog. P.M. Fine & cloudless. Ev. Fine and moonlight.
F 12	30.400	30.394	51.2	30.346	30.338	52.9	46	02.2	46.6	59.7	42.2	59.3		E	A.M. Light fog. P.M. Fine and cloudless. Evening, Fine and starlight—light fog.
S 13	30.398	30.392	51.9	30.390	30.382	53.0	47	02.5	46.6	55.3	44.6	61.0		N	A.M. Fine—light clouds and wind. P.M. Fine and cloudless. Ev. Overcast—light fog.
☉14	30.388	30.380	47.7	30.296	30.290	50.0	45	01.3	40.2	52.4	37.7	56.7		E	A.M. Overcast—lt. fog. P.M. Fine—lt. clouds. Ev. Fine & starlight.
M 15	30.176	30.170	48.5	30.098	30.092	52.0	40	02.0	42.8	59.7	39.6	53.7		NE	A.M. Light fog. P.M. Fine & cloudless. Ev. Fine and starlight.
T 16	29.942	29.936	50.6	29.818	29.810	52.7	47	03.9	49.7	60.2	41.6	60.6		S	A.M. Fine—lt. fog. P.M. Fine & cloudless. Ev. Fine & starlight.
W 17	29.630	29.624	52.2	29.604	29.598	54.0	49	02.9	51.3	54.7	49.2	61.7		S	A.M. Overcast—slight rain and wind. P.M. Fine—light clouds and wind. Evening, Fine and starlight.
T 18	29.444	29.438	51.6	29.512	29.506	53.8	48	03.5	51.7	53.0	46.8	56.7		E var.	A.M. Overcast—high wind—slight rain early. P.M. Cloudy—high wind. Evening, Fine and starlight—brisk wind.
F 19	29.628	29.622	51.0	29.636	29.630	52.5	48	04.1	49.7	51.7	44.2	56.0		SSW var.	A.M. Cloudy—high wind, same throughout the night. P.M. Fine—lt. clds.—brisk wind. Ev. Fine & starlight—brisk wind.
S 20	30.650	29.644	53.0	29.640	29.632	51.7	46	04.6	50.3	47.7	44.3	54.0	.022	E	A.M. Fine—lt. clouds—brisk wind. P.M. Ovct.—lt. rain—brisk wind. Ev. ditto.
☉21	29.672	29.666	53.2	29.628	29.622	52.5	46	03.5	49.5	52.5	43.6	56.4	.133	S	A.M. Fine—light clouds and wind. P.M. Cloudy—brisk wind. Evening, Overcast—brisk wind.
M 22	29.378	29.372	52.2	29.456	29.448	45.5	50	02.0	51.9	55.5	48.7	54.3	.036	S var.	A.M. Ovct.—very heavy gale, also throughout the night. P.M. Fine—lt.clds.—h.wind. Ev. Fine & starlt. 6½ P.M. hail & rain—rainbow.
● T 23	29.878	29.870	54.3	30.030	30.022	53.9	48	03.5	49.3	56.5	43.2	56.6	.025	SSW	A.M. Cloudy—brisk wind. P.M. Fine—light clouds & wind. Ev. Fine and starlight.
W 24	30.270	30.262	52.0	29.258	30.250	53.6	46	03.5	51.4	54.3	46.6	58.3		SE	Cloudy—light wind throughout the day. Evening, Fine & starlight.
T 25	30.160	30.152	54.2	30.044	30.036	53.9	45	03.5	51.8	57.3	44.3	55.7		SSE	Fine—lt. clouds & haze throughout the day. Ev. Fine and starlight.
F 26	29.728	29.722	55.0	29.596	29.590	56.4	49	04.8	55.5	62.3	47.0	59.0		S	Cloudy—light wind throughout the day. Ev. Early part cloudy—slight rain; afterwards fine and starlight.
S 27	29.778	29.772	55.6	29.792	29.784	55.2	48	02.6	51.5	54.6	45.0	64.2	.036	S	A.M. Fine—lt. clds. P.M. Fine—lt. clds. with occasional showers. Ev. ½ to 6, shower, with rainbow—after, fine and starlight.
☉28	29.970	29.966	57.9	29.930	29.924	54.9	44	02.7	45.0	54.5	41.2	59.7	.105	SSW	A.M. Fine—lt. clouds and wind. P.M. Overcast, as also the evening.
M 29	29.938	29.932	52.3	29.874	29.866	55.0	48	03.0	51.2	54.3	44.8	56.6		S	Cloudy—lt. wind throughout the day. Evening, Overcast—lt. rain.
T 30	29.912	29.904	55.9	29.950	29.944	54.4	48	03.3	48.2	53.8	43.0	57.0	.072	W	A.M. Fine—light clouds and wind. P.M. Cloudy—light wind. Evening, Overcast—very high wind.
W 31	29.678	29.672	57.7	29.594	29.588	53.7	46	04.1	49.0	52.7	44.7	55.0	.069	S var.	A.M. High wind, and rain early, after which fine—light clouds. P.M. Cloudy—high wind. Ev. Overcast—light rain.
MEAN.	29.940	29.933	50.6	29.915	29.908	51.3	45	02.9	47.1	53.6	42.7	55.4	Sum. 1.047		Mean Barometer corrected .... 9 A.M. 3 P.M. F. 29.885 .. 29.859 C. 29.877 .. 29.850
APRIL															
T 1	29.642	29.636	50.6	29.654	29.648	51.7	45	03.2	46.3	50.7	43.7	54.7	.061	W	Overcast—light wind throughout the day. Evening, Light rain.
F 2	29.662	29.656	58.2	29.680	29.674	53.3	46	03.6	47.7	52.2	44.6	51.7	.058	NW	Fine—lt. clouds & wind throughout the day. Ev. Fine and starlight.
S 3	29.728	29.720	51.4	29.638	29.630	51.8	44	02.7	45.0	51.5	38.2	53.6		SW	A.M. Fine—lt. fog. P.M. Fine—lt. clouds. Ev. Fine & moonlight.
☉ 4	29.676	29.670	59.8	29.552	29.548	51.6	41	03.0	43.7	52.3	38.3	53.6		S	A.M. Fine—light clouds and wind. P.M. Overcast—brisk wind. Evening, Light rain.
M 5	29.404	29.398	49.2	29.432	29.426	51.9	43	03.3	47.0	51.7	43.0	56.0	.102	SE	A.M. Lightly overcast—light showers and wind. P.M. Fine—light clouds. Evening, Cloudy.
○ T 6	29.710	29.702	49.9	29.736	29.728	51.5	43	02.8	46.4	49.4	43.2	54.2		N	Cloudy—light wind throughout the day. Evening, Overcast—lt. rain.
W 7	29.806	29.798	48.6	29.792	29.786	51.0	40	02.9	43.3	50.8	40.3	53.8	.036	E	Cloudy—lt. wind throughout the day. Evening, Overcast—lt. rain.
T 8	29.806	29.800	52.4	29.758	29.750	51.4	45	03.5	47.0	50.7	42.4	54.0	.033	NW	Fine—light clouds, with occasional light showers throughout the day. Evening, Overcast—light rain.
F 9	29.910	29.902	50.0	29.884	29.876	51.3	44	03.2	48.0	52.3	41.7	52.5	.125	NW	Fine—lt. clouds and wind throughout the day. Ev. Fine & starlight.
S 10	30.082	30.074	48.5	30.040	30.034	48.4	41	03.7	44.7	45.7	40.3	53.7		NW	Cloudy—light wind throughout the day. Ev. Fine & starlight.
☉11	29.954	29.948	46.0	29.944	29.938	47.5	38	02.7	40.8	44.5	38.6	46.7		N	Lightly overcast—light wind, with occasional showers throughout the day. 5½ P.M. rainbow. Ev. Fine and starlight.
M 12	29.912	29.904	45.8	29.924	29.916	46.6	35	03.0	41.2	44.3	36.7	47.2	.025	NNW	Fine—light clouds—brisk wind throughout the day. Evening, Overcast—very slight rain.
T 13	30.100	30.092	45.7	30.118	30.110	48.3	38	03.0	45.3	49.8	37.7	46.6		S	A.M. Fine—light clouds and wind. P.M. Cloudy—light wind. Ev. Overcast—light rain.
W 14	30.110	30.102	48.3	30.034	30.026	50.3	41	02.7	49.8	54.8	45.0	53.0	.080	S	Overcast—light wind throughout the day. Ev. Overcast—very slight rain.
T 15	29.916	29.908	55.9	29.818	29.810	51.6	43	04.0	48.7	47.3	42.4	56.7		S	A.M. Fine—light clouds and wind, with occasional showers. P.M. Overcast—hail and rain. Evening, Light rain.
F 16	29.742	29.736	54.7	29.728	29.722	50.2	41	02.6	45.7	49.2	37.7	55.3	.063	SSW	A.M. Fine—lt. clouds & wind. P.M. Overcast—lt. wind. Ev. The like.
S 17	29.914	29.906	53.2	29.934	29.926	50.3	41	03.9	46.4	53.2	42.4	54.6		W	Thick haze—light wind throughout the day. Ev. Fine and starlight.
☉18	29.972	29.964	49.0	29.942	29.938	51.5	43	03.5	49.4	57.5	42.8	54.6		SW	A.M. Lightly overcast—light wind. P.M. Fine—light clouds and wind. Evening, Overcast.
M 19	29.908	29.900	54.0	29.910	29.902	53.5	42	03.2	48.7	54.3	44.8	59.2	.166	NW	Fine—light clouds and wind throughout the day—heavy rain early. Evening, Fine and starlight.
T 20	29.800	29.794	50.9	29.690	29.682	52.4	39	03.7	46.8	52.0	42.2	56.8		W	A.M. Light fog and wind. P.M. Overcast. Evening, The like.
● W 21	29.852	29.848	53.3	29.860	29.852	51.9	45	04.1	47.6	51.7	41.7	52.8		NNE	Fine—lt. clouds & wind throughout the day. Ev. Fine and starlight.
T 22	29.882	29.874	48.4	29.806	29.800	50.3	39	04.3	45.3	49.8	41.0	53.2		NE	Cloudy—lt. wind throughout the day. Ev. Overcast—lt. rain & wind.
F 23	29.424	29.416	48.3	29.534	29.528	49.9	44	00.2	43.7	46.7	42.3	51.6	.500	NW	A.M. Overcast—light steady rain, as also throughout the night. P.M. Overcast. Evening, Fine and starlight.
S 24	29.712	29.706	60.3	29.642	29.636	52.8	43	03.7	48.5	54.3	40.2	55.0	.333	S var.	A.M. Cloudy—light brisk wind. P.M. Fine—light clouds—brisk wind. Evening, Overcast—light rain—high wind.
☉25	29.688	29.682	72.9	29.700	29.696	54.5	46	03.8	50.5	54.5	43.9	65.7	.072	S	A.M. Fine—light clouds—high wind—very high wind during the night. P.M. Overcast—brisk wind. Ev. The like.
M 26	30.040	30.032	54.5	30.116	30.108	57.7	51	04.2	57.7	63.2	50.2	59.3		S	Cloudy—light wind throughout the day, as also the evening.
T 27	30.140	30.136	71.6	30.136	30.128	61.8	56	05.4	63.3	71.0	56.5	72.2		S	Fine—lt. clouds & wind throughout the day. Ev. Fine and starlight.
W 28	30.200	30.192	62.3	30.146	30.138	62.8	58	04.4	63.3	70.6	57.0	72.4	.022	N	Fine—light clouds and wind throughout the day—rain early. Evening, Fine and starlight—light clouds.
T 29	30.152	30.144	59.7	30.142	30.134	62.2	55	01.3	52.3	67.0	50.6	71.6	.072	N	A.M. Cloudy, with occasional showers. P.M. Fine and cloudless. Evening, Fine and starlight.
W 30	30.226	30.218	65.3	30.162	30.154	61.3	47	07.0	58.4	65.3	46.6	69.2		NE	Fine—lt. clouds—brisk wind throughout the day. Ev. Fine & starlight.
MEAN.	29.869	29.862	54.0	29.848	29.841	52.7	44	03.4	48.4	53.6	43.2	56.4	Sum. 1.748		Mean Barometer corrected .... 9 A.M. 3 M.P. F. 29.805 .. 29.787 C. 29.797 .. 29.779

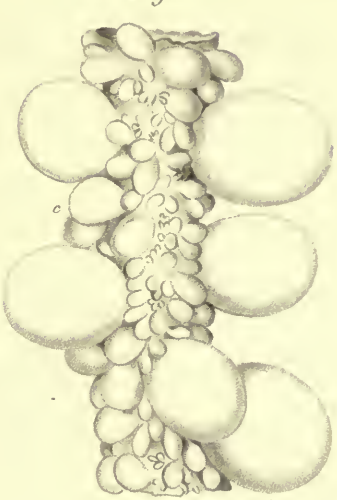
METEOROLOGICAL JOURNAL FOR MAY AND JUNE, 1841.

1841.	9 o'clock, A.M.			3 o'clock, P.M.			Dew Point at 9 A.M., deg. Fahr.	Diff. of Wet and Dry Bulb Thermometer.	External Thermometers.				Rain in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	REMARKS.
	Barometer uncorrected.		Att. Ther.	Barometer uncorrected.		Att. Ther.			Fahrenheit.		Self-registering				
	Flint Glass.	Crown Glass.		Flint Glass.	Crown Glass.				9 A.M.	3 P.M.	Lowest	Highest			
S 1	30.040	30.032	65.9	29.930	29.922	61.0	52	04.9	57.8	68.3	47.0	67.2		NE	Fine—nearly cloudless, with light breeze throughout the day. Ev. Fine and starlight—light fog.
☉ 2	29.720	29.712	68.3	29.684	29.680	63.5	57	05.1	61.7	61.0	53.2	70.4		S	A.M. Fine—light clouds and wind. P.M. Overcast—rain and wind, with distant thunder. Evening, Overcast.
M 3	29.772	29.764	57.9	29.816	29.808	57.6	52	02.0	47.7	46.7	47.2	71.5	.375	N	Overcast—light rain and wind throughout the day. Ev. The same.
T 4	29.628	29.620	54.0	29.552	29.546	58.4	49	03.8	53.7	64.4	45.3	54.8	.250	E	A.M. Cloudy—lt. wind. P.M. Fine—lt. clouds & wind. Ev. Overcast.
☽ W 5	29.464	29.456	58.5	29.562	29.554	60.0	55	04.1	55.4	58.4	52.0	66.0	.719	SE	A.M. Cloudy, high wind & occasional showers—heavy rain during the night. P.M. Fine—lt. clouds—high wind. Ev. Cloudy—high wind.
T 6	29.556	29.550	58.2	29.680	29.672	60.8	55	03.9	57.8	61.7	53.0	61.3	.077	S	A.M. Cloudy, brisk wind, with showers. P.M. Fine—light clouds and wind. Evening, Fine and starlight.
F 7	29.746	29.740	64.0	29.644	29.636	62.0	56	04.8	59.2	65.3	49.7	64.6	.019	S	Cloudy—lt. brisk wind throughout the day. Ev. The like, with showers.
S 8	29.548	29.540	69.8	29.558	29.550	61.7	54	04.8	57.5	56.4	52.5	66.6	.116	S	Cloudy—brisk wind, with occasional showers throughout the day. Evening, Overcast—light rain.
☉ 9	30.094	30.086	74.5	30.128	30.120	61.2	52	02.4	56.3	60.8	47.3	72.2	.044	S	Fine—light clouds throughout the day. Evening, Overcast.
M 10	30.180	30.172	60.2	30.208	30.200	61.2	53	05.1	60.2	61.7	53.0	63.3		S	Cloudy—light wind throughout the day. Evening, Overcast.
T 11	30.094	30.086	60.8	30.040	30.032	62.7	55	06.2	62.2	69.3	51.6	63.8		S	A.M. Cloudy—light wind. P.M. Fine—light clouds and wind. Evening, Overcast—slight rain.
W 12	30.190	30.182	65.3	30.228	30.220	62.9	56	04.3	57.0	61.7	53.0	70.7	.019	NNW	Cloudy—light wind throughout the day. Evening, Overcast.
T 13	30.388	30.380	69.0	30.362	30.354	61.6	52	04.0	55.7	61.3	47.5	63.4		N	Fine—lt. clouds & wind throughout the day. Ev. Fine & starlight.
F 14	30.424	30.418	69.6	30.342	30.334	61.2	49	04.7	53.7	60.7	46.3	63.6		NE	Fine—light clouds & wind throughout the day. Ev. Fine & starlight.
MAY S 15	30.244	30.238	69.8	30.140	30.132	62.3	52	05.4	59.7	67.7	48.7	69.4		S	A.M. Fine & cloudless. P.M. Fine—lt. clouds. Ev. Fine & starlight.
☉ 16	29.968	29.964	66.0	29.842	29.838	62.0	50	02.9	53.7	68.4	49.0	66.0		SSW	A.M. Overcast. P.M. Fine—lt. clouds & wind. Ev. Fine & starlight.
M 17	29.664	29.656	60.6	29.598	29.592	62.8	55	04.8	58.3	59.7	52.3	70.0		S	Cloudy—high wind throughout the day. Evening, Fine & starlight.
T 18	29.610	29.602	68.0	29.594	29.588	62.6	54	05.0	57.8	61.2	49.4	63.0		S	Fine—light clouds and wind throughout the day. At ¼ before 7 P.M. rainbow, with slight rain—the rest of the evening fine & starlight.
W 19	29.406	29.398	59.3	29.284	29.276	59.2	53	04.2	55.3	54.5	51.3	65.7		S	Overcast—slight rain—high wind nearly the whole of the day. Evening, Light clouds—high wind, with occasional rain.
● T 20	29.328	29.320	71.3	29.444	29.438	60.9	53	05.5	58.2	59.6	50.7	59.6	.075	W	A.M. Cldy.—high wind throughout the night. P.M. Fine—lt. clouds—brisk wind, with rain. Ev. Fine & starlight.
F 21	29.702	29.698	70.2	29.628	29.620	62.2	52	06.0	58.7	60.8	48.4	67.7	.044	ENE	A.M. Fine—lt. clouds & wind. P.M. Cloudy—lt. wind. Ev. Ovct.—lt. steady rain.
S 22	29.672	29.664	60.4	29.760	29.752	61.2	54	04.8	59.7	62.3	56.5	65.8	.133	N	Overcast—light wind, with occasional rain throughout the day. Evening, Fine and starlight.
☉ 23	30.040	30.032	59.7	30.118	30.114	63.0	54	04.0	58.3	68.7	53.7	64.2		S	A.M. Light fog. P.M. Fine—light clouds. Ev. Fine and starlight.
M 24	30.242	30.234	65.2	30.192	30.184	65.2	55	07.1	63.5	68.3	53.6	70.3		NE	Fine—light clouds and wind throughout the day. Evening, Cloudy, with lightning.
T 25	30.220	30.214	77.4	30.196	30.188	68.3	61	05.8	63.5	68.8	55.8	70.6		N	Fine—light clouds and wind throughout the day. Ev. Overcast.
W 26	30.146	30.140	75.6	30.096	30.088	68.4	62	05.4	67.7	74.3	55.0	70.0		NE	Fine—light clouds, with light brisk wind throughout the day. Ev. Fine and starlight.
T 27	29.960	29.954	84.8	29.892	29.884	72.2	66	06.4	71.7	76.8	62.0	79.4		NE	Fine & cloudless—lt. breeze throughout the day. Ev. Cloudy—very vivid lightning, accompanied with heavy rain and thunder.
F 28	30.000	29.994	73.6	30.030	30.022	72.6	65	05.9	70.3	74.4	64.8	79.0	.116	SSE	Fine—lt. clouds and breeze throughout the day. Ev. Fine & starlight.
S 29	30.186	30.178	81.9	30.150	30.144	72.0	55	07.1	65.7	68.8	57.7	78.3		NW	A.M. Fine—light clouds and breeze. P.M. Cloudy—light breeze. Ev. Fine and starlight.
☉ 30	30.078	30.072	67.0	30.052	30.046	70.0	59	03.6	61.5	70.0	58.0	71.9		NW	A.M. Overcast—lt. fog. P.M. Heavy clouds. Ev. Fine and starlight.
M 31	30.086	30.080	76.3	30.064	30.056	71.8	62	06.0	66.7	72.8	57.3	78.6		S	A.M. Fine & cloudless. P.M. Fine—lt. clouds. Ev. Fine & starlight.
MEAN.	29.916	20.909	67.2	29.897	29.890	63.6	55	04.8	59.6	64.4	52.4	68.0	Sum. 1.987		Mean Barometer corrected.............. 9 A.M. 3 P.M. F. 29.817 .. 29.808 C. 29.809 .. 29.799
T 1	30.214	30.206	78.3	30.200	30.192	71.0	61	05.1	64.3	69.2	56.2	80.5		S	Cloudy throughout the day. Evening, Fine and starlight.
W 2	30.272	30.264	83.7	30.226	30.214	71.0	62	05.9	65.0	72.3	55.3	82.3		S	Cloudy—light breeze throughout the day. Evening, The same.
T 3	30.250	30.244	75.7	30.220	30.212	70.4	59	05.1	63.0	69.7	56.5	73.4		N	Fine—light clouds throughout the day. Evening, Fine & starlight.
☽ F 4	30.440	30.432	79.8	30.378	30.370	68.9	53	09.5	62.7	66.8	52.2	73.6		W	A.M. Fine—light clouds and breeze. P.M. Fine—nearly cloudless. Evening, Fine—light clouds.
S 5	30.250	30.242	71.8	30.142	30.134	68.4	54	07.3	62.2	67.8	52.0	73.6		W	A.M. Cloudy—light breeze. P.M. Fine and cloudless. Evening, Overcast—light rain.
☉ 6	30.036	30.030	72.7	30.014	30.008	66.0	54	05.3	55.3	56.5	51.3	69.0	.019	WNW	A.M. Fine—light clouds and wind. P.M. Heavy clouds—lt. wind. Evening, Cloudy.
M 7	29.964	29.956	63.7	29.976	29.968	61.2	50	03.7	51.3	51.3	45.3	60.0		NW	Cloudy—brisk wind throughout the day. Ev. Ovct.—very slight rain.
T 8	29.996	29.988	57.9	30.002	29.994	59.0	51	02.6	51.0	54.8	47.6	56.4		NW	Cloudy—light wind throughout the day. Ev. Overcast—brisk wind.
W 9	29.970	29.962	55.4	29.976	29.968	58.0	47	02.8	50.3	56.2	47.4	57.2		NNW	Overcast—light wind throughout the day. Ev. Fine and starlight.
T 10	29.838	29.830	56.0	29.746	29.738	58.7	47	03.0	52.3	56.8	47.8	57.6		NW	A.M. Overcast—light wind. P.M. Fine—light clouds and wind. Evening, Fine and starlight.
F 11	29.722	29.714	57.0	29.764	29.756	57.7	50	02.2	50.5	52.8	49.3	68.7		NW	Dark heavy clouds—brisk wind throughout the day. Evening, Overcast—brisk wind.
S 12	29.850	29.842	55.2	29.934	29.926	56.9	47	02.0	50.7	54.0	46.8	54.7		N	Overcast—brisk wind throughout the day. Evening, The same.
JUNE ☉ 13	30.064	30.056	54.2	30.066	30.060	57.0	47	02.0	50.2	58.2	47.0	55.2		NW	A.M. Cloudy—slight rain and wind. P.M. Overcast. Evening, Fine and starlight.
M 14	30.100	30.094	67.6	30.036	30.028	60.8	48	03.2	57.3	67.7	46.7	72.6		W	Fine—light clouds and wind throughout the day. Evening, Fine and starlight.
T 15	29.940	29.932	59.3	30.008	30.000	61.3	53	02.5	57.4	62.3	55.6	66.0		W	Cloudy—brisk wind throughout the day. Evening, The same.
W 16	30.274	30.270	73.7	30.224	30.216	63.3	50	02.1	60.5	66.8	47.7	67.8		S	A.M. Fine—light clouds and breeze. P.M. Lightly cloudy. Ev. Fine and starlight.
T 17	30.136	30.128	70.2	30.044	30.036	64.3	56	01.2	61.7	66.5	51.5	80.3		S	A.M. Cloudy—light breeze. P.M. Fine—light clouds. Ev. Fine and starlight.
F 18	29.822	29.816	68.8	29.720	29.712	66.6	56	02.1	64.3	72.8	53.6	76.4		NNW	Fine—lt. clouds & breeze throughout the day. Ev. Cloudy—slight rain—distant thunder.
● S 19	29.650	29.644	63.5	29.674	29.666	66.7	59	02.7	60.7	65.3	58.2	75.0	.111	W	A.M. Cloudy—light breeze—very heavy rain in the night. P.M. Cloudy—light breeze. Evening, Fine and starlight.
☉ 20	29.842	29.838	87.0	29.796	29.792	67.5	57	04.3	64.3	62.6	51.2	87.0	.027	S	A.M. Fine—light clouds and breeze. P.M. Heavy clouds—brisk wind. Evening, Early part, dark clouds—after, fine & starlight.
M 21	29.848	29.842	73.7	29.964	29.956	65.8	57	06.8	64.2	64.0	56.3	71.0		S var.	A.M. Dark heavy clouds—high wind. P.M. Overcast—heavy rain—brisk wind. Ev. Fine & starlight.
T 22	30.118	30.112	71.7	30.076	30.068	66.4	56	06.0	61.7	66.7	53.7	70.2	.366	S	A.M. Fine—lt. clouds & wind, with slight rain. P.M. Fine—lt. clds.—brisk wind. Ev. Fine & starlt.
W 23	30.030	30.026	73.4	29.826	29.818	66.3	56	06.5	62.0	65.7	53.4	69.7		S	Fine—lt. clouds & breeze throughout the day. Ev. Fine & starlight.
T 24	29.800	29.794	69.7	29.750	29.742	65.6	55	07.1	63.0	57.0	53.8	69.4		E	A.M. Fine—lt. clds. & br. P.M. Ovct.—hvy. shwr. Ev. Ovct.—lt. rain.
F 25	29.542	29.536	62.0	29.560	29.552	64.9	58	02.0	55.7	63.8	56.2	68.4	.525	E	A.M. Ovct.—lt. wind, with showers. P.M. Fine—lt. clds. & wind. Ev. Overcast.
S 26	29.686	29.680	70.2	29.758	29.750	67.0	59	07.8	64.0	65.7	56.2	78.0	.133	S	A.M. Fine—lt. clds.—high wind—high wind during night. P.M. Cldy. with showers. Ev. Fine & starlight.
☉ 27	30.012	30.004	65.0	30.120	30.114	66.0	59	06.2	62.5	65.0	56.4	67.6		W	A.M. Overcast—lt. wind, with heavy showers. P.M. Cloudy, with showers—distant thunder. Evening, Cloudy.
M 28	30.012	30.004	62.3	29.950	29.942	63.7	58	04.3	58.3	59.4	55.3	68.7	.430	S	Cloudy—light showers—high wind throughout the day. Evening, Overcast—steady rain—brisk wind.
T 29	29.820	29.814	69.8	29.822	29.814	64.8	57	06.0	61.3	61.8	52.8	67.4	.675	S	A.M. Fine—light clouds, with showers—heavy rain, high wind in the night. P.M. Fine—lt. clouds & wind. Ev. Cloudy—slight rain.
W 30	30.046	30.040	74.0	30.090	30.082	65.4	56	05.5	59.7	63.7	52.7	73.6	.022	W	Fine—light clouds and breeze throughout the day. Ev. Fine & starlight, with a few clouds.
MEAN	29.985	29.978	68.1	29.969	29.961	64.4	54	04.4	58.9	62.8	52.2	69.7	Sum. 2.308		Mean Barometer corrected .... 9 A.M. 3 P.M. F. 29.883 .. 29.877 C. 29.875 .. 29.868

Organs of Reproduction in Myriapoda. (Julus.)

Phil. Trans. MDCCCXLI. Plate III.

Fig. 5.

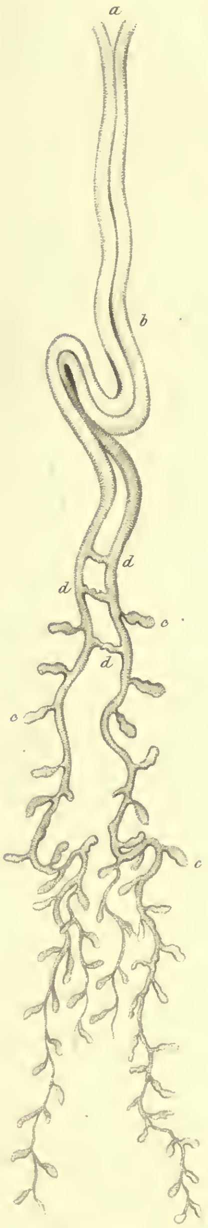


magnified about 30 diameters.

Fig. 7

Fig. 8

Fig. 1.



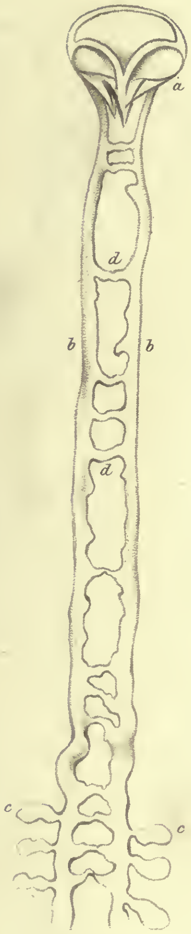
6 diameters

Fig. 4

6 diameters.

Fig. 9.

Fig. 3.



4 diameters.

Fig. 1

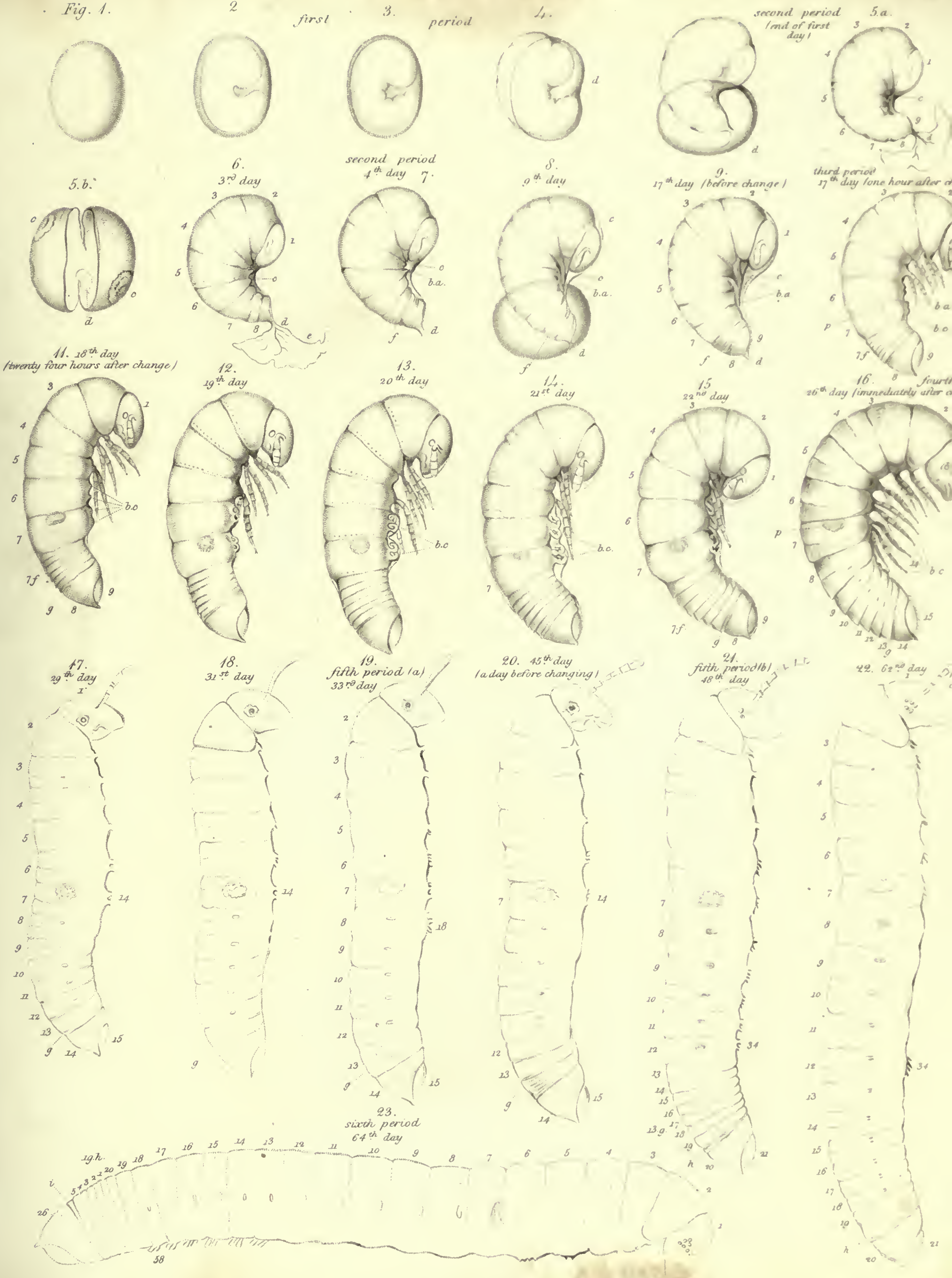
Fig. 6.

Fig.

Fig. 2.

G. Newport, del.

J. Basire.



G. Newport. del.

All the figures magnified about 25 diameters.

Phil. Trans. MDCCCXLI. Plate XIII.

Fig. 1.

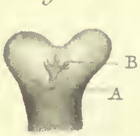


Fig. 2.

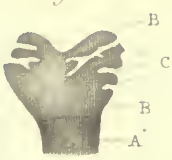


Fig. 3.

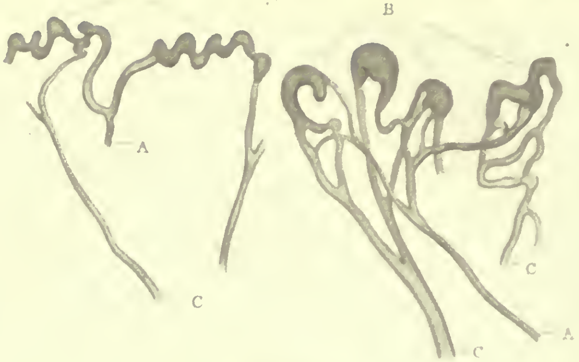


Fig. 4.

Fig. 5.

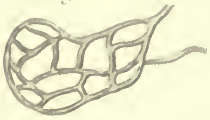


Fig. 6.

Fig. 7.

Fig. 8.

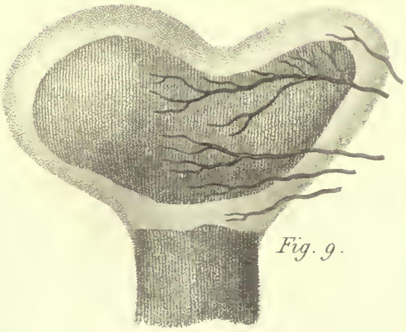


Fig. 9.

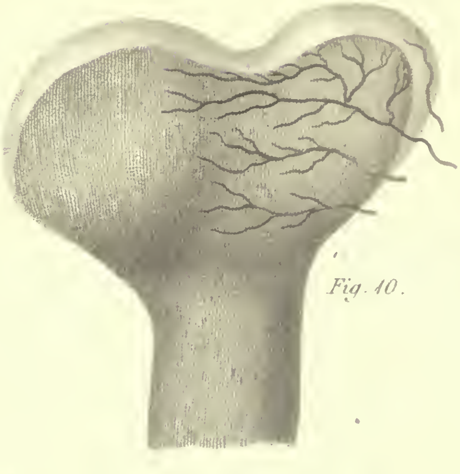


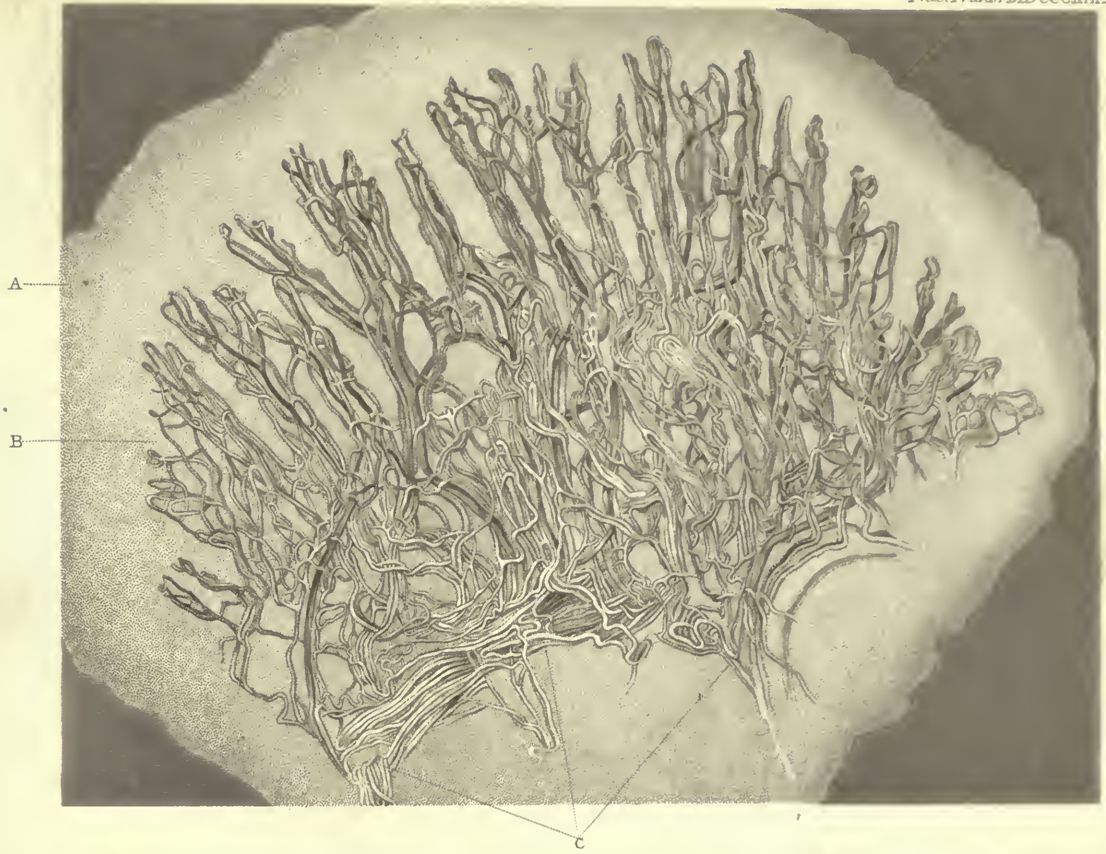
Fig. 10.

Fig. 11.

H. Holmes, del.

J. Basire sc.

Fig. 1



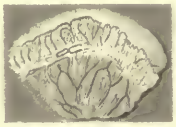


Fig. 2.

H. Holmes, del.

J. Basire sc

Fig. 1.

Fig. 2.

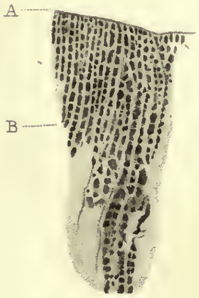


Fig. 3.

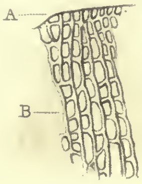


Fig. 4.

Fig. 6.

Fig. 5.

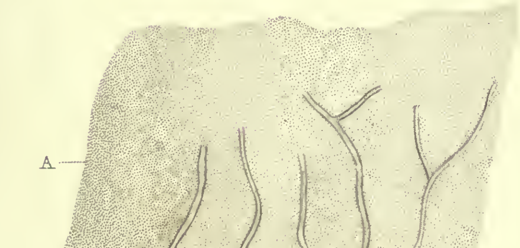
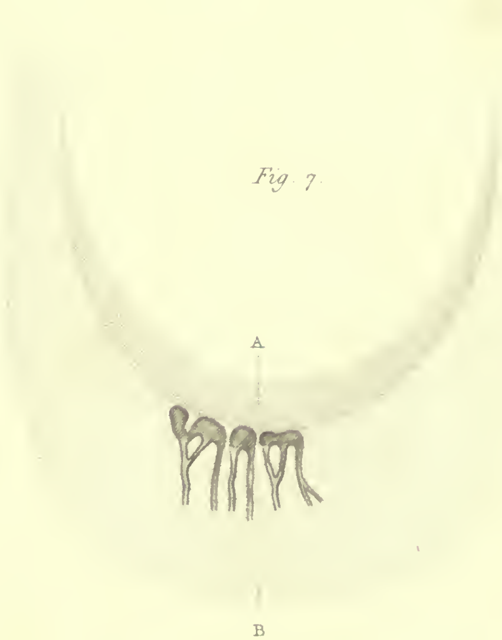


Fig. 7.



H. Holmes del.

Fig. 1.

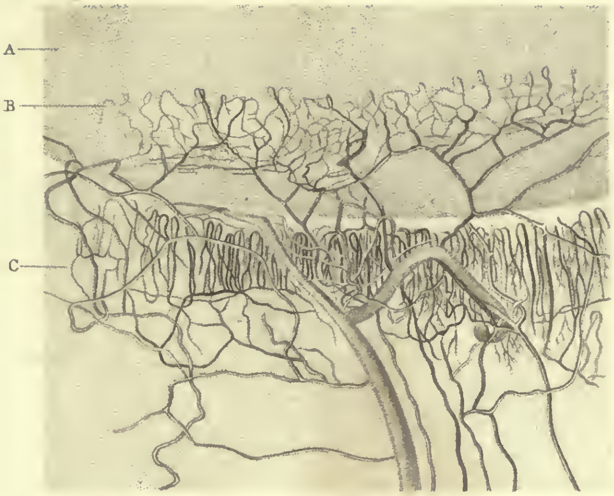


Fig. 2.

Fig. 3.

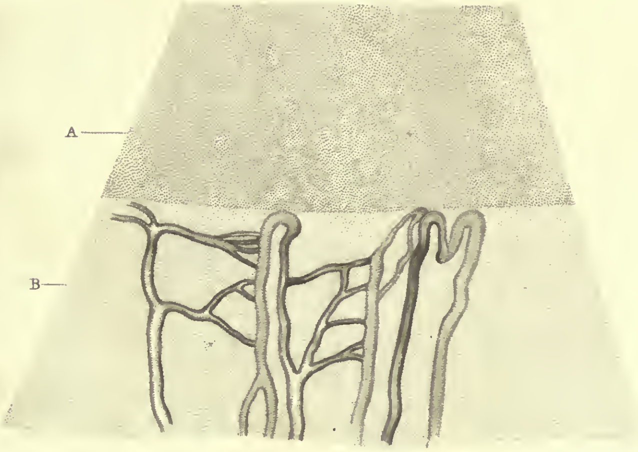
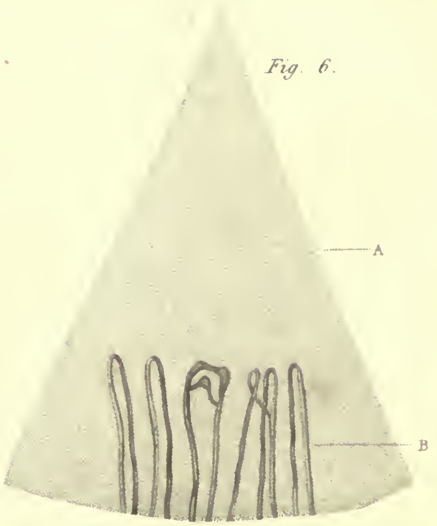


Fig. 4.

Fig. 5.

Fig. 6.



H. Holmes, del

J. Basire, sc

Corpuscles of the Blood.

24

Fœtal Ox. ¾ Inch.

23

Man. — Acetic Acid.

27

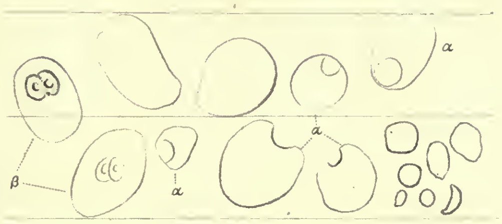
Fœtal Ox, ⅞ Inch
Acetic Acid

25

Fœtal Ox. ¾ Inch.
Incipient Collapse.

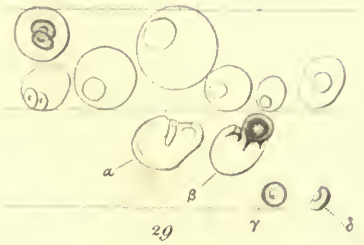
26

Fœtal Ox, ¾ Inch.



28

Fœtal Ox, 1 Inch.



29

Fœtal Ox. 1 Inch — Acetic Acid.

30

Fœtal Ox, 1¼ Inch.
Acetic Acid.

31

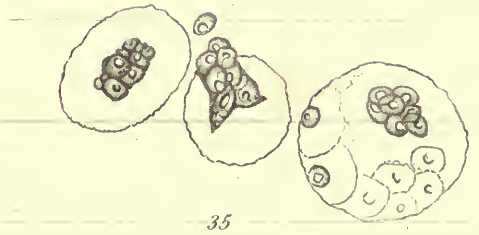
Fœtal Ox, 1¾ Inch.
Acetic Acid.

32

Fœtal Ox,
1¾ Inch

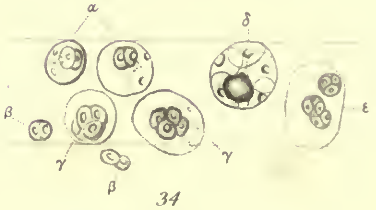
33

Fœtal Sheep 2½ Inches.
Acetic Acid.



35

Fœtal Ox, 1¾ Inch. — Liver.
Acetic Acid



34

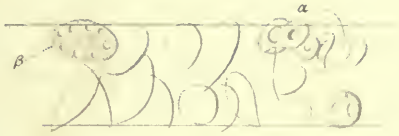
Fœtal Ox, 1¾ Inch.
Acetic Acid.

All the objects are seen of their relative sizes, being alike magnified 600 diameters. Their actual sizes may be determined by reference to the spaces they occupy between the horizontal lines, which are $\frac{1}{100}$th of a Paris line apart in the micrometer itself.
Thus in Fig. 29. the actual diameter of γ is $\frac{1}{300}'''$ (Paris Line.)

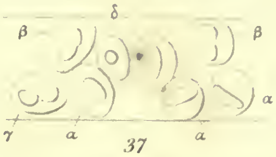
Ad Nat. del. Martin Barry.

Basire sculp.

Corpuscles of the Blood.



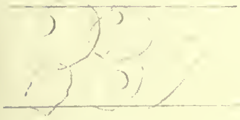
36
Sparrow.



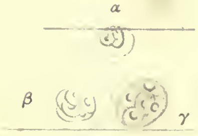
37
Sparrow.
Acetic Acid.

38
Sparrow.
Acetic Acid.

39
Sparrow.
Acetic Acid.



40
Chick in ovo.
80 Hours.

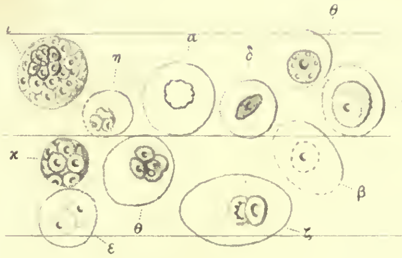


41
Chick in ovo.
80 Hours. Acetic Acid.

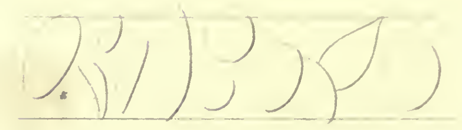
42
Chick in ovo.
85 Hours.

43
Chick in ovo.
85 Hours. Acetic Acid.

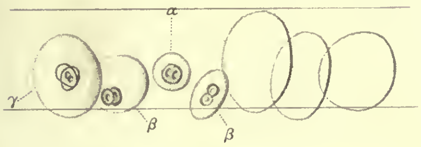
44
Chick in ovo.
92 Hours.



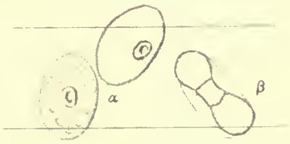
45
Chick in ovo.
92 Hours. Acetic Acid.



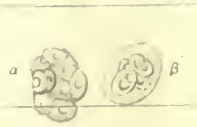
46
Chick in ovo.
166 Hours.



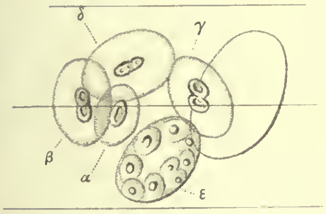
48
Turtle.
Acetic Acid.



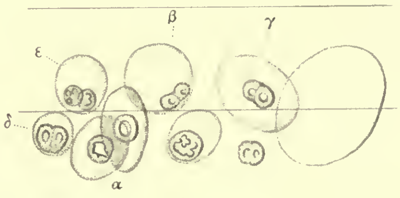
49
Turtle.
Acetic Acid.



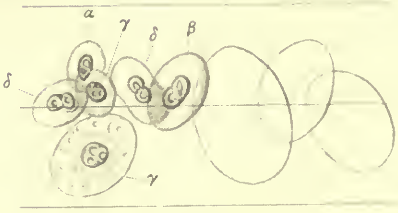
47
Chick in ovo.
166 Hours. Acetic Acid.



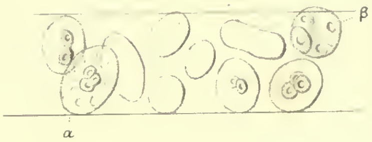
50
Frog.
Acetic Acid.



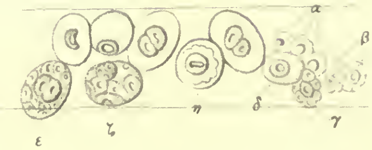
51
Thornback.
Acetic Acid.



52
Skate.
Acetic Acid.



53
Cod.
Acetic Acid.



54
Cod.
Acetic Acid.

All the Objects are magnified 600 Diameters.

(The horizontal Lines are described at the foot of Plate XVII.)

Ad Nat. del. Martin Barry.

Phil. Trans. MDCCCXLI. Plate XIX. p. 216.

Corpuscles of the Blood.

55

Frog.
Acetic Acid.

56

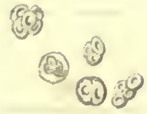
Frog.

57

Oyster.

58.

Turtle.
Acetic Acid.



59

60

Objects from the Brain, in the Fœtus of the Ox.
Acetic Acid.

α
Dilute Spirit.

61
Lobster.

β
Acetic Acid.

62
Leech.

All the Objects are magnified 600 Diameters.

(The horizontal Lines are described at the foot of Plate XVII.)

Embryology. (see p. 293.)

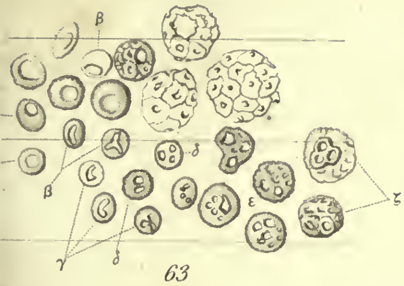
254

bs The Germ.
f "Zona pellucida".
f′ Fluid imbibed by the Chorion.
cho Objects giving origin to the Chorion.

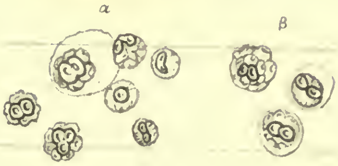
Ad Nat. del. Martin Barry.

J. Basire sc.

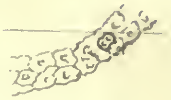
Corpuscles of the Blood.



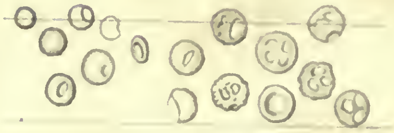
63



64

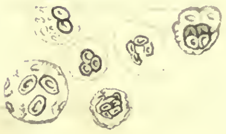


65

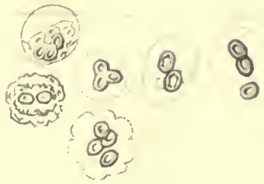


66

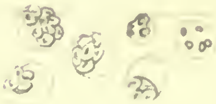
67



68



69



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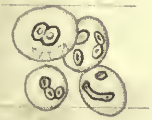
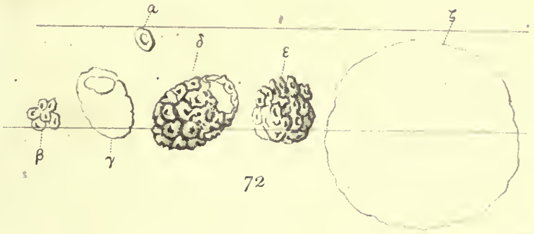


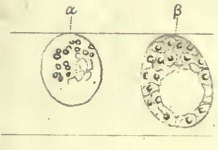
71



72

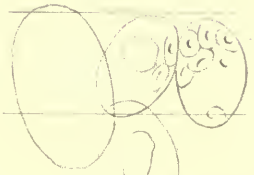
73

74
Tadpole 6‴

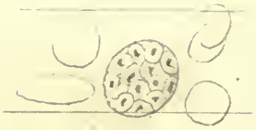


75
Tadpole 6‴

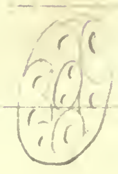
76
Tadpole 6‴



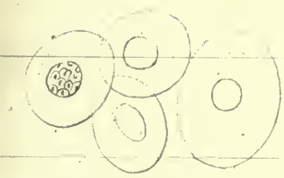
77
Newt.



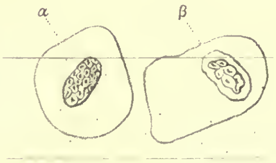
78
Newt.



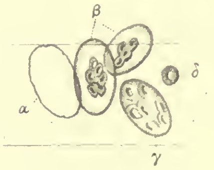
79
Newt.
Acetic Acid.



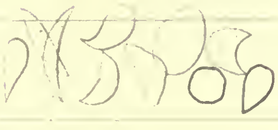
80
Newt.
Nitrate of Silver.



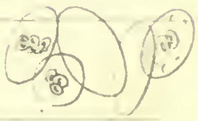
81
Newt.
Corrosive Sublimate.



82
Green Lizard.



83
Fœtal Duck.



84
Fœtal Duck.

All the Objects are magnified 600 Diameters.—(The horizontal Lines are described at the foot of Plate XVII.)

Figs. 63 to 67. Nucleus of the Blood-corpuscle passing into the Pus-globule.

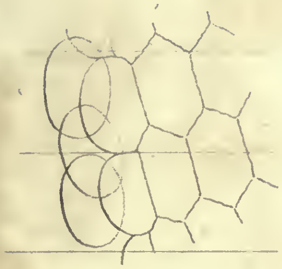
68 to 73. Globules, Cells, &c. of Mucus:—derived from Corpuscles having the same appearance as Corpuscles of the Blood.

74 to 84. Blood-corpuscles, and Objects found along with them.

Ad Nat. del. Martin Barry.

J. Basire sculp.

Corpuscles of the Blood.



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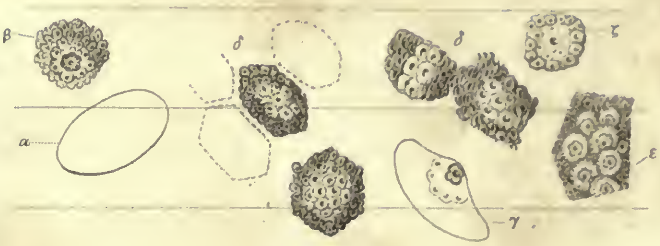
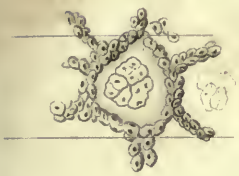
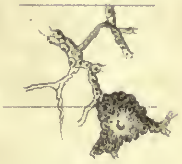


86

87

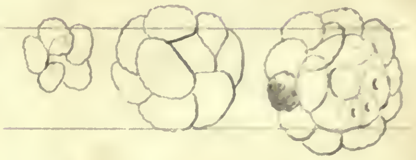


89



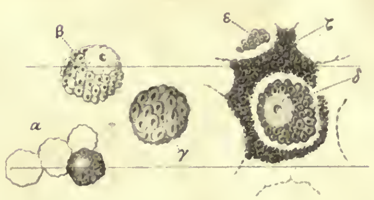
90

91

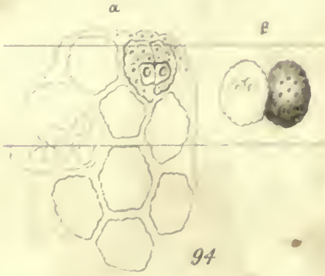


88

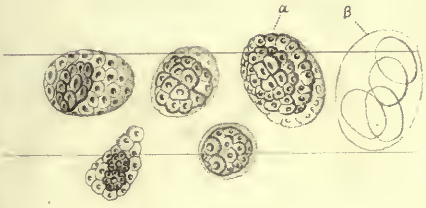
92



93

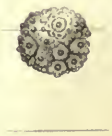


94



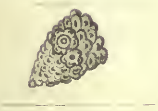
95

96



97

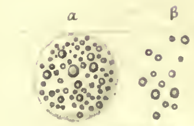
98



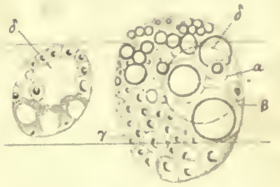
99

100

101



102



103

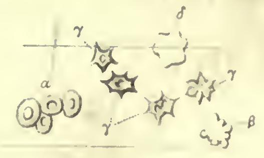
All the Objects are magnified 600 Diameters. — *(The horizontal Lines are described at the foot of Plate XVII.)*

Fig. 85 to 94. *Epithelium-tables, cells, and cylinders — and Pigmentum nigrum* { *Derived from Corpuscles having the same appearance as Corpuscles of the Blood.*
95. *Blood-corpuscles, altered.*
96 to 100. *Epithelium cylinders*
101. *Ciliary Processes.*
102, 103. *Miscellaneous Objects.* } *Ditto.*

Ad Nat. del. Martin Barry.
J. Basire sc

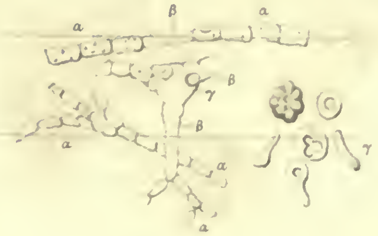
Corpuscles of the Blood.

104



105

106



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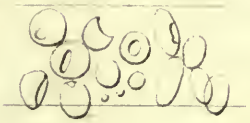
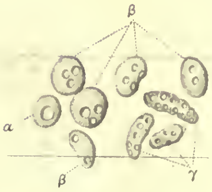
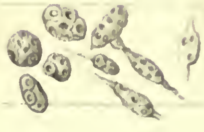


108

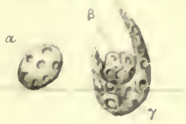


109

110



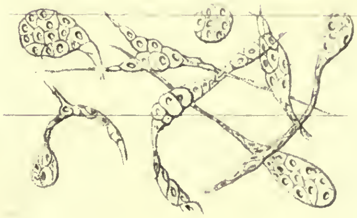
111



112

113

114



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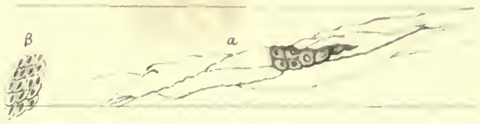
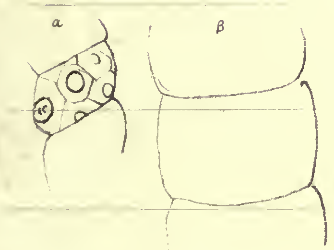
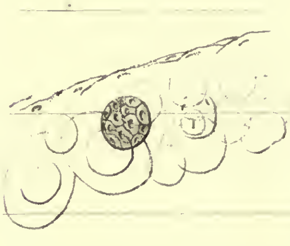


116



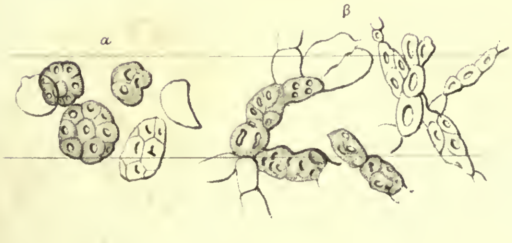
116½



117

118

121



119

120

122

All the Objects are magnified 600 Diameters.—(The horizontal Lines are described at the foot of Plate XVII.)

Fig^s. 104 to 106. Ciliated Nuclei of Blood-corpuscles, Young Blood-corpuscles, &c.

107. Elements of Capillaries, &c.
108 to 116. Elements of Cellular Tissue.
116½ to 122. Elements of Cartilage.

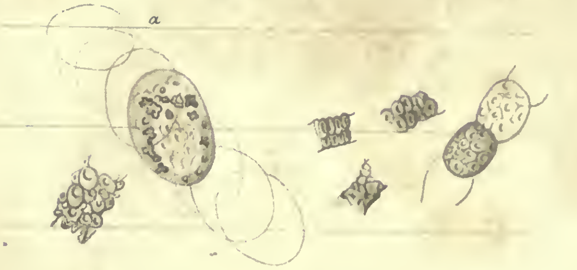
Derived from Corpuscles having the same appearance as Corpuscles of the Blood.

Ad Nat. del. Martin Barry.

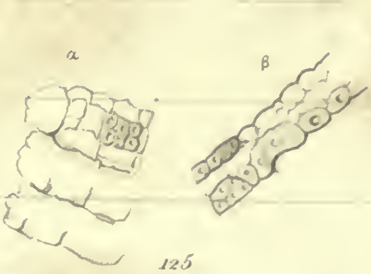
J. Basire sculp.

Corpuscles of the Blood.


123



124



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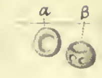

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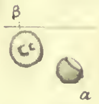

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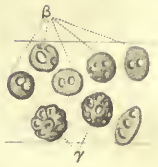

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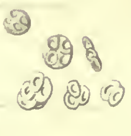
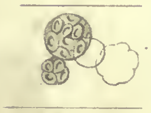

129

130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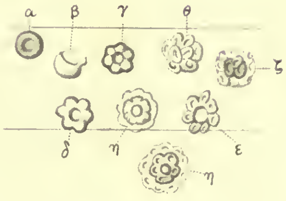

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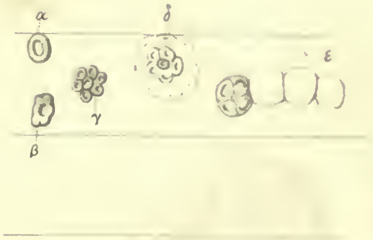

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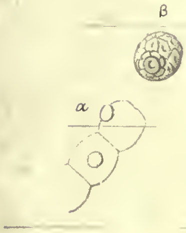

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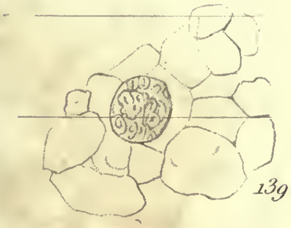
136


137

138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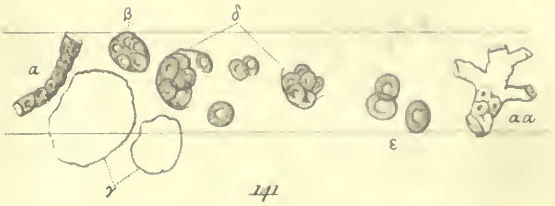


141

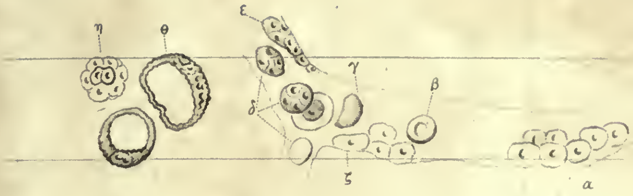
All the Objects are magnified 600 Diameters. (*The horizontal Lines are described at the foot of Plate XVII.*)

Fig.s 123 to 125. Elements of the Optic Nerve.
126 to 131 .... Retina.
132 .... Spinal Chord.
133, 134 .... Brain.
135 to 137 .... Muscle.
138 to 141 .... Crystalline Lens.

Derived from Corpuscles having the same appearance as Corpuscles of the Blood.

Ad Nat. del. Martin Barry.
J. Basire sculp.

Corpuscles of the Blood.



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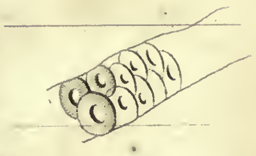
143

144



145

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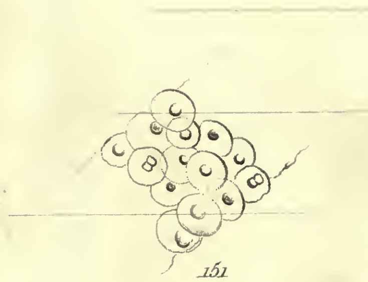
148

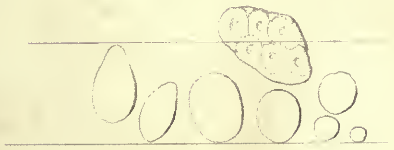
149

150



151

152



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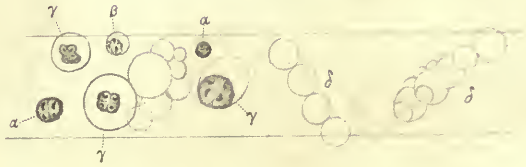

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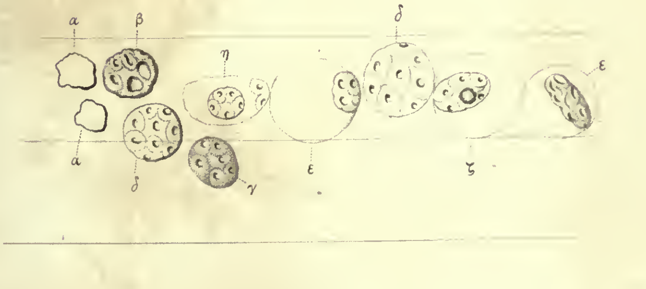

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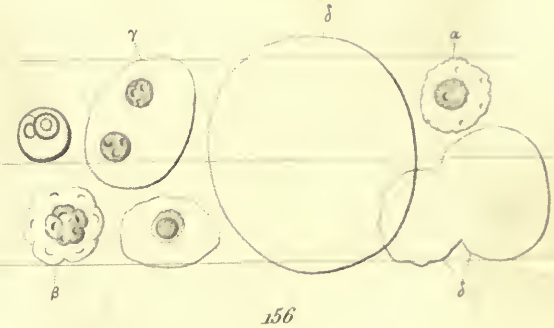


156

All the Objects are magnified 600 Diameters.—(The horizontal Lines are described at the foot of Plate XVII.)

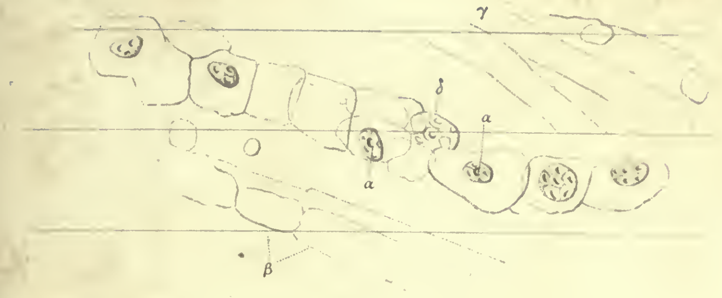
Elements of the Crystalline Lens, derived from Corpuscles

having the same appearance as Corpuscles of the Blood.

Ad Nat. del. Martin Barry.

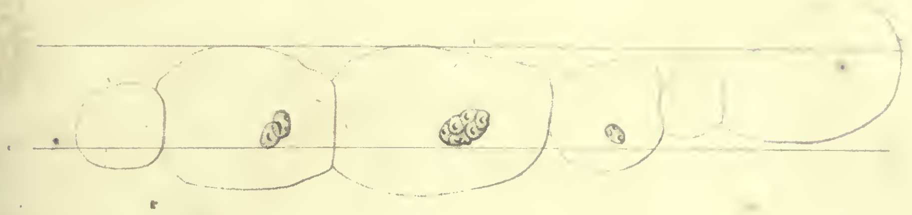
J. Basire sculp.

Corpuscles of the Blood.

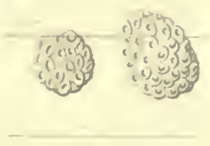


157

158



159



160

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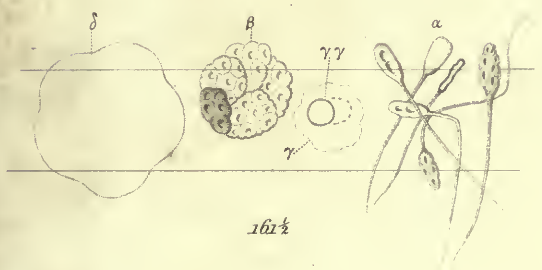
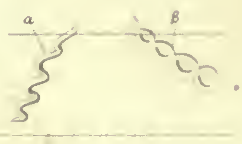
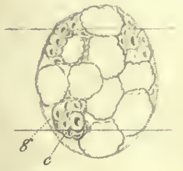


161½

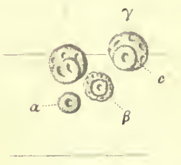
162



163



164



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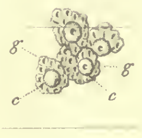
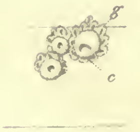
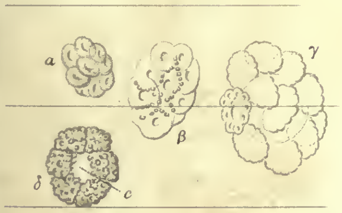


166



167

168



169

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173

All the Objects are magnified 600 Diameters. (The horizontal Lines are described at the foot of Plate XVII.)

Figs. 157 to 159. Elements of the Crystalline Lens.
160 to 163. Elements of the Spermatozoa.
164 to 173. Elements of the Ovum.

Derived from Corpuscles having the same appearance as Corpuscles of the Blood.

Ad. Nat. del. Martin Barry.

J. Basire sculp.

Joseph Perry. del.

¾ the Natural Size.

J. Basire sculp.