Edmund B Walcutt
Union College
Sept. 1843.

[illegible]
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Saturn & his rings

Moon 5 days old.

Daggett. Hinman & Co. Sc.

Jupiter and his Moons

TELESCOPIC VIEW OF THE FULL MOON

E N B Walsworth.

1842

AN

INTRODUCTION

TO

ASTRONOMY;

DESIGNED AS A

TEXT BOOK

FOR THE

STUDENTS OF YALE COLLEGE.

BY DENISON OLMSTED, A. M.

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PREFACE.

Nearly all who have written Treatises on Astronomy, designed for young learners, appear to have erred in one of two ways; they have either disregarded demonstrative evidence, and relied on mere popular illustration, or they have exhibited the elements of the science in naked mathematical formulæ. The former are usually diffuse and superficial; the latter, technical and abstruse.

In the following Treatise, we have endeavored to unite the advantages of both methods. We have sought, first, to establish the great principles of astronomy on a mathematical basis; and, secondly, to render the study interesting and intelligible to the learner, by easy and familiar illustrations. We would not encourage any one to believe that he can enjoy a full view of the grand edifice of astronomy, while its noble foundations are hidden from his sight; nor would we assure him that he can contemplate the structure in its true magnificence, while its basement alone is within his field of vision. We would, therefore, that the student of astronomy should confine his attention neither to the exterior of the building, nor to the mere analytic investigation of its structure. We would desire that he should not only study it in models and diagrams, and mathematical formulæ, but should at the same time acquire a love of nature herself, and cultivate the habit of raising his views to the grand originals. Nor is the effort to form a clear conception of the motions and dimensions of the heavenly bodies, less favorable to the improvement of the intellectual powers, than the study of pure geometry.

But it is evidently possible to follow out all the intricacies of an analytical process, and to arrive at a full conviction of the great truths of astronomy, and yet know very little of nature. According to our experience, however, but few students in the course of a liberal education will feel satisfied with this. They do not need so much to be convinced that the assertions of astronomers are true, as they desire to know what the truths are, and how they were ascertained; and they will derive from the study of astronomy little of that moral and intellectual elevation which they had anticipated, unless they learn to look upon the heavens with new views, and a clear comprehension of their wonderful mechanism.

Much of the difficulty that usually attends the early progress of the astronomical student, arises from his being too soon introduced to the most perplexing part of the whole subject,—the planetary motions. In this work, the consideration of these is for the most part postponed until the learner has become familiar with the artificial circles of the sphere, and conversant with the celestial bodies. We then first take the most simple view possible of the planetary motions by contemplating them as they really are in nature, and afterwards proceed to the more difficult inquiry, why they appear as they do.

Although we have found it convenient to defer the consideration of the fixed stars to a late period, yet we would earnestly recommend to the student to begin to learn the constellations, and the stars of the first magnitude at least, as soon as he enters upon the study of astronomy. A few evenings spent in this way, assisted, where it is practicable, by a friend already conversant with the stars, will inspire a higher degree of enthusiasm for the science, and render its explanations more easily understood.

It is recommended to the learner to make a free use of the *Analysis*, especially in reviewing the ground already traversed. If, by repeated recurrence to these heads, he associates with each a train of ideas, carrying along with him, as he advances, all the particulars indicated in these *hints*, he will secure to them an indelible place in his memory.

With such aids at hand, as Newton, La Place, and Delambre, to expound the laws of astronomy, and such popular writers as Ferguson, Biot, and Francoeur, to supply familiar illustrations of those laws, it might seem an easy task to prepare a work like the present; but a text book made up of extracts from these authors, would be ill suited to the wants of our students. We have deemed it better therefore, first, to acquire the clearest views we were able of the truths to be unfolded, both from an extensive perusal of standard authors, and from diligent reflection, and then to endeavor to transfuse our own impressions into the mind of the learner. Writers of profound attainments in astronomy, and of the highest reputation, have often failed in the preparation of elementary works, because they lacked one qualification—the experience of the teacher. Familiar as they were with the truths of the science, but unaccustomed to hold communion with young pupils, they were incapable of apprehending the difficulty and the slowness with which these truths make their entrance into the mind for the first time. Even when they attempt to feel their way into young minds, by assuming the garb of the instructor, and employing popular illustrations, they often betray their want of the experience and art of the professional teacher. In this number perhaps we may place Sir John Herschel, whose work on the Elements of Astronomy, no sooner made its appearance in this country, than it was seized upon with avidity by many of the professors of our colleges, and among others by the author of this treatise. But with great deference to his exalted rank in the scientific world, and with the highest respect for his profound and varied attainments, we feel compelled to say, after several years trial, that his work is wholly unsuited to the case of the college student, and, indeed, but ill adapted to the wants of the general reader.

Although the mathematical complexion of some parts of this work, may seem repulsive to the general reader, yet the propositions demonstrated are generally also illustrated in a popular way, so that they may be understood by those who are unable or unwilling to toil through the demonstrations. By occasionally omitting these, such may find this treatise well suited to give them a concise and comprehensive view of the latest discoveries of astronomy; and we hope it may prove no less worthy of the library of the scholar, than of the lecture room of the student.

Yale College, May, 1839.

ANALYSIS.

DESIGNED AS A BASIS FOR REVIEW AND EXAMINATION.

Chapter X.—SUPERIOR PLANETS, MARS, JUPITER, SATURN, AND URANUS.

Part III.—OF THE FIXED STARS AND SYSTEM OF THE WORLD.

Chapter I.—FIXED STARS—CONSTELLATIONS.

Chapter II.—CLUSTERS OF STARS—NEBULÆ—VARIABLE STARS—TEMPORARY STARS—DOUBLE STARS.

ERRATA.—Art. 84, line 6, for *subtracted* read *added*.
153, " 25, for *eastward* read *westward*.
218, " 18, for *or* read *on*.

On page 247, the place of *Vindemiatrix* is incorrectly stated. It should be, 15° east of Denebola, and 20° north of Spica, in the arm of *Virgo*.

☞ *Diagrams for public recitations.*

As many of the figures of this work are too complicated to be drawn on the black board at each recitation, we have found it very convenient to provide a set of permanent cards of paste-board, on which the diagrams are inscribed on so large a scale, as to be distinctly visible in all parts of the lecture room. The letters may be either made with a pen, or better procured of the printer, and pasted on.

The cards are made by the bookbinder, and consist of a thick paper board about 18 by 14 inches, on each side of which a white sheet is pasted, with a neat finish around the edges. A loop attached to the top is convenient for hanging the card on a nail.

Cards of this description, containing diagrams for the whole course of mathematical and philosophical recitations, have been provided in Yale College, and are found a valuable part of our apparatus of instruction.

Similar sets, drawn with neatness and elegance, by persons accustomed to make them for Yale College, embracing all the figures of the present work, may be obtained by public Institutions on application either to the author or the publishers, at an expense of from twenty five to thirty dollars.

INTRODUCTION TO ASTRONOMY.

PRELIMINARY OBSERVATIONS.

1. Astronomy *is that science which treats of the heavenly bodies.* More particularly, its object is to teach what is known respecting the Sun, Moon, Planets, Comets, and Fixed Stars; and also to explain the methods by which this knowledge is acquired. Astronomy is sometimes divided into Descriptive, Physical, and Practical. Descriptive Astronomy respects *facts;* Physical Astronomy, *causes;* Practical Astronomy, the *means of investigating the facts,* whether by instruments, or by calculation. It is the province of Descriptive Astronomy to observe, classify, and record, all the phenomena of the heavenly bodies, whether pertaining to those bodies individually, or resulting from their motions and mutual relations. It is the part of Physical Astronomy to explain the causes of these phenomena, by investigating and applying the general laws on which they depend; especially by tracing out all the consequences of the law of universal gravitation. Practical Astronomy lends its aid to both the other departments.

2. Astronomy is the most ancient of all the sciences. At a period of very high antiquity, it was cultivated in Egypt, in Chaldea, in China, and in India. Such knowledge of the heavenly bodies as could be acquired by close and long continued observation, without the aid of instruments, was diligently amassed; and tables of the celestial motions were constructed, which could be used in predicting eclipses, and other astronomical phenomena.

About 500 years before the Christian era, *Pythagoras*, of Greece, taught astronomy at the celebrated school at Crotona, and exhibited more correct views of the nature of the celestial motions, than were entertained by any other astronomer of the ancient world. His views, however, were not generally adopted, but lay neglected for nearly 2000 years, when they were revived and established by Copernicus and Galileo. The most celebrated astronomical school of antiquity, was at Alexandria in Egypt, which was established and sustained by the Ptolemies, (Egyptian princes,) about 300 years before the Christian era. The employment of instruments for measuring angles, and bringing in trigonometrical calculations to aid the naked powers of observation, gave to the Alexandrian astronomers great advantages over all their predecessors. The most able astronomer of the Alexandrian school was *Hipparchus*, who was distinguished above all the ancients for the accuracy of his astronomical measurements and determinations. The knowledge of astronomy possessed by the Alexandrian school, and recorded in the *Almagest*, or great work of *Ptolemy*, constituted the chief of what was known of our science during the middle ages, until the fifteenth and sixteenth centuries, when the labors of *Copernicus* of Prussia, *Tycho Brahe* of Denmark, *Kepler* of Germany, and *Galileo* of Italy, laid the solid foundations of modern astronomy. Copernicus expounded the true theory of the celestial motions; Tycho Brahe carried the use of instruments and the art of astronomical observation to a far higher degree of accuracy than had ever been done before; Kepler discovered the great laws of physical astronomy; and Galileo, having first enjoyed the aid of the telescope, made innumerable discoveries in the solar system. Near the beginning of the eighteenth century, *Sir Isaac Newton* discovered, in the law of universal gravitation, the great principle that governs the celestial motions; and recently, *La Place* has more fully completed what Newton begun, having followed out all the consequences of the laws of universal gravitation, in his great work, the Mecanique Celeste.

3. Among the ancients, astronomy was studied chiefly as subsidiary to astrology. *Astrology was the art of divining future events by the stars.* It was of two kinds, natural and judicial. *Natural Astrology*, aimed at predicting remarkable occurrences in the natural world, as earthquakes, volcanoes, tempests, and pestilential diseases. *Judicial Astrology*, aimed at foretelling the fates of individuals, or of empires.

4. Astronomers of every age, have been distinguished for their persevering industry, and their great love of accuracy. They have uniformly aspired to an exactness in their inquiries, far beyond what is aimed at in most geographical investigations, satisfied with nothing short of numerical accuracy wherever this is attainable; and years of toilsome observation, or laborious calculation, have been spent with the hope of attaining a few seconds nearer to the truth. Moreover, a severe but delightful labor is imposed on all, who would arrive at a clear and satisfactory knowledge of the subject of astronomy. Diagrams, artificial globes, orreries, and familiar comparisons and illustrations, proposed by the author or the instructor, may afford essential aid to the learner, but nothing can convey to him a perfect comprehension of the celestial motions, without much diligent study and reflection.

5. In expounding the doctrines of astronomy, we do not, as in Geometry, claim that every thing shall be proved as soon as asserted. We may first put the learner in possession of the leading facts of the science, and afterwards explain to him the methods by which those facts were discovered, and by which they may be verified; we may assume the principles of the true system of the world, and employ those principles in the explanation of many subordinate phenomena, while we reserve the discussion of the merits of the system itself, until the learner is extensively acquainted with astronomical facts, and therefore better able to appreciate the evidence by which the system is established.

6. The *Copernican system* is that which is held to be the true system of the world. It maintains (1,) That the *apparent* diurnal revolution of the heavenly bodies, from east to west, is owing to the *real* revolution of the earth on its own axis from west to east, in the same time; and (2,) That the sun is the center around which the earth and planets all revolve from west to east, contrary to the opinion that the earth is the center of motion of the sun and planets.

7. We shall treat, first, of the Earth in its astronomical relations; secondly, of the Solar System; thirdly, of the Fixed Stars; and fourthly, of Astronomical Observations and Calculations.

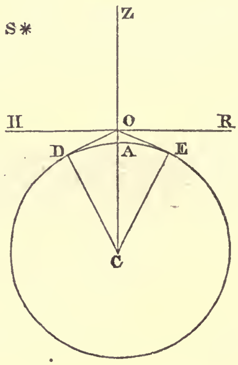
PART I.—OF THE EARTH.

CHAPTER I.

OF THE FIGURE AND DIMENSIONS OF THE EARTH, AND THE DOCTRINE OF THE SPHERE.

8. *The figure of the earth is nearly globular.* This fact is known, first, by the circular form of its shadow cast upon the moon in a lunar eclipse; secondly, from analogy, each of the other planets being seen to be spherical; thirdly, by our seeing the tops of distant objects while the other parts are invisible, as the topmast of a ship, while either leaving or approaching the shore, or the lantern of a light-house, which when first descried at a distance at sea, appears to glimmer upon the very surface of the water; fourthly, by the depression or *dip of the horizon* when the spectator is on an eminence; and, finally, by actual observations and measurements, made for the express purpose of ascertaining the figure of the earth, by means of which astronomers are enabled to compute the distances from the center of the earth of various places on its surface, which distances are found to be nearly equal.

Fig. 1.



9. The *Dip of the Horizon*, is the apparent angular depression of the horizon, to a spectator elevated above the general level of the earth. The eye thus situated takes in more than a celestial hemisphere, the excess being the measure of the dip.

Thus, in Fig. 1, let AO represent the height of a mountain, ZO the direction of the plumb line, HOR a line touching the earth at the point O, and at right angles to the plumb line, C the center of the earth, DAE the portion of the earth's surface seen from O; OD, OE, lines drawn from the place of the spectator to the most distant parts of the horizon, and CD a radius of the earth. The dip of the horizon is the angle HOD or ROE. Now the angle made between the direction of the plumb line and that of the extreme line of the horizon or the surface of the sea, namely, the angle ZOD, can be easily measured; and subtracting the right angle ZOH from ZOD, the remainder is the dip of the horizon, from which the length of the line OD may be calculated, the height of the spectator, that is, the line OA, being known. This length, to whatever point of the horizon the line is drawn, is always found to be the same; and hence it is inferred, that the boundary which limits the view on all sides, is a circle. Moreover, at whatever elevation the dip of the horizon is taken, in any part of the earth, the space seen by the spectator is always circular. Hence the surface of the earth is spherical.

10. The earth being a sphere, the dip of the horizon HOD = OCD. Therefore, to find the dip of the horizon corresponding to any given height AO, (the diameter of the earth being known,) we have in the triangle OCD, the right angle at D, and the two sides CD, CO, to find the angle OCD. Therefore,

CO : rad. : : CD : cos. OCD. Learning the dip corresponding to different altitudes, by giving to the line AO different values, we may arrange the results in a table.

Table showing the Dip of the Horizon at different elevations, from 1 *foot to* 100 *feet.*

Feet.	′ ″	Feet.	′ ″	Feet.	′ ″
1	0.59	13	3.33	26	5.01
2	1.24	14	3.41	28	5.13
3	1.42	15	3.49	30	5.23
4	1.58	16	3.56	35	5.49
5	2.12	17	4.03	40	6.14
6	2.25	18	4.11	45	6.36
7	2.36	19	4.17	50	6.58
8	2.47	20	4.24	60	7.37
9	2.57	21	4.31	70	8.14
10	3.07	22	4.37	80	8.48
11	3.16	23	4.43	90	9.20
12	3.25	24	4.49	100	9.51

Such a table is of use in estimating the altitude of a body above the horizon, when the instrument (as usually happens) is more or less elevated above the general level of the earth. For if it be a star whose altitude above the horizon is required, the instrument being situated at O, (Fig. 1,) the inquiry is how far the star is elevated above the level HOR, but the angle taken is that above the visible horizon OD. The dip, therefore, or the angle HOD, corresponding to the height of the point O, must be subtracted, to obtain the true altitude. On the Peak of Teneriffe, Humboldt observed the surface of the sea to be depressed on all sides nearly 2 degrees. The sun arose to him 12 minutes sooner than to an inhabitant of the plain; and from the plain, the top of the mountain appeared enlightened 12 minutes before the rising or after the setting of the sun.

11. The foregoing considerations show that the form of the earth is spherical; but more exact determinations prove, that the earth, though nearly globular, is not exactly so: its diameter from the north to the south pole is about 26 miles less than through the equator, giving to the earth the form of an oblate spheroid,* or a flattened sphere resembling an orange. We shall reserve the explanations of the methods by which this fact is established, until the learner is better prepared than at present to understand them.

12. The mean or average *diameter of the earth,* is 7912.4 miles, a measure which the learner should fix in his memory as a standard of comparison in astronomy, and of which he should endeavor to form the most adequate conception in his power. The circumference of the earth is about 24,000 miles (23857.5).† Although the surface of the earth is uneven, sometimes rising in high mountains, and sometimes descending in deep valleys, yet these elevations and depressions are so small in comparison with the immense

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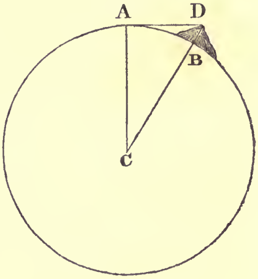
* An *oblate spheroid* is the solid described by the revolution of an ellipse about its shorter axis.

† It will generally be sufficient to treasure up in the memory the *round number;* but sometimes, in astronomical calculations, the more exact number may be required, and it is therefore inserted.

volume of the globe, as hardly to occasion any sensible deviation from a surface uniformly curvilinear. The irregularities of the earth's surface, in this view, are no greater than the rough points on the rind of an orange, which do not perceptibly interrupt its continuity; for the highest mountain on the globe is only about five miles above the general level; and the deepest mine hitherto opened is only about half a mile.* Now $\frac{5}{7912}=\frac{1}{1582}$, or about one sixteen hundredth part of the whole diameter, an inequality which, in an artificial globe of eighteen inches diameter, amounts to only the eighty eighth part of an inch.

Fig. 2.



13. The diameter of the earth, *considered as a perfect sphere*, may be determined by means of observations on a mountain of known elevation, seen in the horizon from the sea. Let BD (Fig. 2,) be a mountain of known height a, whose top is seen in the horizon by a spectator at A, b miles from it. Let x denote the radius of the earth. Then $x^2+b^2=(x+a)^2=x^2+2ax+a^2$. Hence, $2ax=b^2-a^2$, and $x=\frac{b^2-a^2}{2a}$. For example, suppose the height of the mountain is just one mile; then it will be found, by observation, to be visible on the horizon at the distance of 89 miles $=b$. Hence, $\frac{b^2-a^2}{2a}=\frac{(89)^2-1}{2}=\frac{7921-1}{2}=3960=$ radius of the earth, and $7920=$ the earth's diameter.

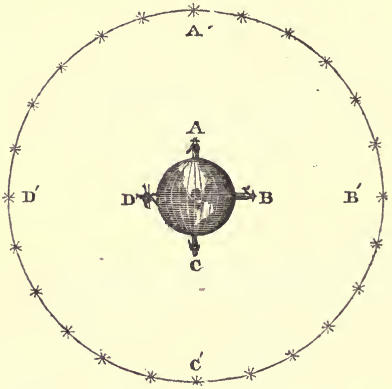
14. Another method, and the most ancient, is to ascertain the distance on the surface of the earth, corresponding to a degree of latitude. Let us select two convenient places, one lying directly north of the other, whose difference of latitude is known. Suppose this difference to be $1^\circ\ 30'$, and the distance between the two places, as measured by a chain, to be 104 miles. Then, since there are 360 degrees of latitude in the entire circumference, $1^\circ\ 30' : 104 :: 360^\circ : 24960$. And $\frac{24960}{3.1416}=7944$.

* Sir John Herschel.

The foregoing *approximations* are sufficient to show that the earth is about 8,000 miles in diameter.

15. The greatest difficulty in the way of acquiring correct views in astronomy, arises from the erroneous notions that preoccupy the mind. To divest himself of these, the learner should conceive of the earth as a huge globe occupying a small portion of space, and encircled on all sides with the starry sphere. He should free his mind from its habitual proneness to consider one part of space as naturally *up* and another *down*, and view himself as subject to a force which binds him to the earth as truly as though he were fastened to it by some invisible cords or wires, as the needle attaches itself to all sides of a spherical loadstone. He

Fig. 3.



should dwell on this point until it appears to him as truly *up* in the direction of BB′, CC′, DD′, (Fig. 3,) when he is at B, C, and D, respectively, as in the direction AA′ when he is at A.

DOCTRINE OF THE SPHERE.

16. The definitions of the different lines, points, and circles, which are used in astronomy, and the propositions founded upon them, compose the *Doctrine of the Sphere.**

* It is presumed that many of those who read this work, will have studied Spherical Geometry; but it is so important to the student of astronomy to have a clear idea of the circles of the sphere, that it is thought best to introduce them here.

17. A section of a sphere by a plane cutting it in any manner, is a circle. *Great circles* are those which pass through the center of the sphere, and divide it into two equal hemispheres: *Small circles*, are such as do not pass through the center, but divide the sphere into two unequal parts. Every circle, whether great or small, is divided into 360 equal parts called *degrees.* A degree, therefore, is not any fixed or definite quantity, but only a certain aliquot part of any circle.

18. The *axis* of a circle, is a straight line passing through its center at right angles to its plane.

19. The *pole* of a great circle, is the point on the sphere where its axis cuts through the sphere. Every great circle has two poles, each of which is every where 90° from the great circle. For, the measure of an angle at the center of a sphere, is the arc of a great circle intercepted between the two lines that contain the angle; and, since the angle made by the axis and any radius of the circle is a right angle, consequently its measure on the sphere, namely, the distance from the pole to the circumference of the circle, must be 90°. If two great circles cut each other at right angles, the poles of each circle lie in the circumference of the other circle. For each circle passes through the axis of the other.

20. All great circles of the sphere cut each other in two points diametrically opposite, and consequently, their points of section are 180° apart. For the line of common section, is a diameter in both circles, and therefore bisects both.

21. A great circle which passes through the pole of another great circle, cuts the latter at right angles. For, since it passes through the pole and the center of the circle, it must pass through the axis; which being at right angles to the plane of the circle, every plane which passes through it is at right angles to the same plane.

The great circle which passes through the pole of another great circle and is at right angles to it, is called a *secondary* to that circle.

22. The angle made by two great circles on the surface of the sphere, is measured by the arc of another great circle, of which the angular point is the pole, being the arc of that great circle intercepted between those two circles. For this arc is the measure of the angle formed at the center of the sphere by two radii, drawn at right angles to the line of common section of the two circles, one in one plane and the other in the other, which angle is therefore that of the inclination of those planes.

23. In order to fix the position of any plane, either on the surface of the earth or in the heavens, both the earth and the heavens are conceived to be divided into separate portions by circles, which are imagined to cut through them in various ways. The earth thus intersected is called the *terrestrial*, and the heavens the *celestial* sphere. The learner will remark, that these circles have no existence in nature, but are mere landmarks, artificially contrived for convenience of reference. On account of the immense distance of the heavenly bodies, they appear to us, wherever we are placed, to be fixed in the same concave surface, or celestial vault. The great circles of the globe, extended every way to meet the concave surface of the heavens, become circles of the celestial sphere.

24. The *Horizon* is the great circle which divides the earth into upper and lower hemispheres, and separates the visible heavens from the invisible. This is the *rational* horizon. The *sensible* horizon, is a circle touching the earth at the place of the spectator, and is bounded by the line in which the earth and skies seem to meet. The sensible horizon is parallel to the rational, but is distant from it by the semi-diameter of the earth, or nearly 4,000 miles. Still, so vast is the distance of the starry sphere, that both these planes appear to cut that sphere in the same line; so that we see the same hemisphere of stars that we should see if the upper half of the earth were removed, and we stood on the rational horizon.

25. The poles of the horizon are the zenith and nadir. The *Zenith* is the point directly over our head, and the *Nadir* that di-

rectly under our feet. The plumb line is in the axis of the horizon, and consequently directed towards its poles.

Every place on the surface of the earth has its own horizon; and the traveller has a new horizon at every step, always extending 90 degrees from him in all directions.

26. *Vertical circles* are those which pass through the poles of the horizon, perpendicular to it.

The *Meridian* is that vertical circle which passes through the north and south points.

The *Prime Vertical*, is that vertical circle which passes through the east and west points.

27. As in geometry, we determine the position of any point by means of rectangular coordinates, or perpendiculars drawn from the point to planes at right angles to each other, so in astronomy we ascertain the place of a body, as a fixed star, by taking its angular distance from two great circles, one of which is perpendicular to the other. The horizon and the meridian, or the horizon and the prime vertical, are coordinate circles used for such measurements.

The *Altitude* of a body, is its elevation above the horizon, measured on a vertical circle.

The *Azimuth* of a body, is its distance measured on the horizon from the meridian to a vertical circle passing through the body.

The *Amplitude* of a body, is its distance on the horizon, from the prime vertical, to a vertical circle passing through the body.

Azimuth is reckoned 90° from either the north or south point; and amplitude 90° from either the east or west point. Azimuth and amplitude are mutually complements of each other. When a point is *on* the horizon, it is only necessary to count the number of degrees of the horizon between that point and the meridian, in order to find its azimuth; but if the point is *above* the horizon, then its azimuth is estimated by passing a vertical circle through it, and reckoning the azimuth from the point where this circle cuts the horizon.

The *Zenith Distance* of a body is measured on a vertical circle, passing through that body. It is the complement of the altitude.

28. The *Axis of the Earth* is the diameter, on which the earth is conceived to turn in its diurnal revolution. The same line continued until it meets the starry concave, constitutes the *axis of the celestial sphere.*

The *Poles of the Earth* are the extremities of the earth's axis: *the Poles of the Heavens*, the extremities of the celestial axis.

29. The *Equator* is a great circle cutting the axis of the earth at right angles. Hence the axis of the earth is the axis of the equator, and its poles are the poles of the equator. The intersection of the plane of the equator with the surface of the earth, constitutes the *terrestrial*, and with the concave sphere of the heavens, the *celestial* equator. The latter, by way of distinction, is sometimes denominated the *equinoctial.*

30. The secondaries to the equator, that is, the great circles passing through the poles of the equator, are called *Meridians*, because that secondary which passes through the zenith of any place is the meridian of that place, and is at right angles both to the equator and the horizon, passing as it does through the poles of both. These secondaries are also called *Hour Circles*, because the arcs of the equator intercepted between them are used as measures of time.

31. The *Latitude* of a place on the earth, is its distance from the equator north or south. The *Polar Distance*, or angular distance from the nearest pole, is the complement of the latitude.

32. The *Longitude* of a place is its distance from some standard meridian, either east or west, measured on the equator. The meridian usually taken as the standard, is that of the Observatory of Greenwich, in London. If a place is directly *on* the equator, we have only to inquire how many degrees of the equator there are between that place and the point where the meridian of Greenwich cuts the equator. If the place is north or south of the equator, then its longitude is the arc of the equator intercepted between the meridian which passes through the place, and the meridian of Greenwich.

33. The *Ecliptic* is a great circle in which the earth performs its annual revolution around the sun. It passes through the center of the earth and the center of the sun. It is found by observation that the earth does not lie with its axis at right angles to the plane of the ecliptic, but that it is turned about 23½ degrees out of a perpendicular direction, making an angle with the plane itself of 66½°. The equator, therefore, must be turned the same distance out of a coincidence with the ecliptic, the two circles making an angle with each other of 23½°. It is particularly important for the learner to form correct ideas of the ecliptic, and of its relations to the equator, since to these two circles a great number of astronomical measurements and phenomena are referred.

34. The *Equinoctial Points*, or *Equinoxes*,* are the intersections of the ecliptic and equator. The time when the sun crosses the equator in returning northward is called the *vernal*, and in going southward, the *autumnal* equinox. The vernal equinox occurs about the 21st of March, and the autumnal the 22d of September.

35. The *Solstitial Points* are the two points of the ecliptic most distant from the equator. The times when the sun comes to them are called *solstices*. The summer solstice occurs about the 22d of June, and the winter solstice about the 22d of December.

The ecliptic is divided into twelve equal parts of 30° each, called *signs*, which, beginning at the vernal equinox, succeed each other in the following order:

Northern.		*Southern.*	
1. Aries	♈	7. Libra	♎
2. Taurus	♉	8. Scorpio	♏
3. Gemini	♊	9. Sagittarius	♐
4. Cancer	♋	10. Capricornus	♑
5. Leo	♌	11. Aquarius	♒
6. Virgo	♍	12. Pisces	♓

* The term Equinoxes strictly denotes the *times* when the sun arrives at the equinoctial points, but it is also frequently used to denote those points themselves.

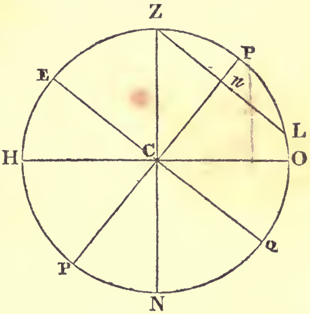
The mode of reckoning on the ecliptic, is by signs, degrees, minutes, and seconds. The sign is denoted either by its name or its number. Thus 100° may be expressed either as the 10th degree of Cancer, or as 3^s 10°.

36. Of the various meridians, two are distinguished by the name of *Colures.* The *Equinoctial Colure,* is the meridian which passes through the equinoctial points. From this meridian, right ascension and celestial longitude are reckoned, as longitude on the earth is reckoned from the meridian of Greenwich. The *Solstitial Colure,* is the meridian which passes through the solstitial points. As the solstitial points are 90° from the equinoctial points, so the solstitial colure is 90° from the equinoctial colure. It is also at right angles, or a secondary, to both the ecliptic and equator. For, like every other meridian, it is of course perpendicular to the equator, passing through its poles. Moreover, the equinox, being a point both in the equator and in the ecliptic, is 90° from the solstice, from the pole of the equator, and from the pole of the ecliptic. Hence the solstitial colure, which passes through the solstice and the pole of the equator, passes also through the pole of the ecliptic, being the great circle of which the equinox itself is the pole. Consequently, the solstitial colure is a secondary to both the equator and the ecliptic. (See Arts. 19, 20, 21.)

37. The position of a celestial body is referred to the equator by its right ascension and declination. (See Art. 27.) *Right Ascension,* is the angular distance from the vernal equinox, measured on the equator. If a star is situated *on* the equator, then its right ascension is the number of degrees of the equator between the star and the vernal equinox. But if the star is north or south of the equator, then its right ascension is the arc of the equator intercepted between the vernal equinox and that secondary to the equator which passes through the star. *Declination* is the distance of a body from the equator, measured on a secondary to the latter. Therefore, right ascension and declination correspond to terrestrial longitude and latitude, right ascension being reckoned from the equinoctial colure, in the same manner as longitude is

reckoned from the meridian of Greenwich. On the other hand, celestial longitude and latitude are referred, not to the equator, but to the ecliptic. *Celestial Longitude*, is the distance of a body from the vernal equinox reckoned on the ecliptic. *Celestial Latitude*, is distance from the ecliptic measured on a secondary to the latter. Or, more briefly, Longitude is distance *on* the ecliptic; Latitude, distance *from* the ecliptic. The *North Polar Distance* of a star, is the complement of its declination.

Fig. 4.



38. *Parallels of Latitude* are small circles parallel to the equator. They constantly diminish in size as we go from the equator to the pole, the radius being always equal to the cosine of the latitude. In fig. 4, let HO be the horizon, EQ the equator, PP the axis of the earth, ZN the prime vertical, and ZL a parallel of latitude of any place Z. Then ZE is the latitude, (Art. 31.) and ZP the complement of the latitude; but Z*n* the radius of the parallel of latitude ZL, is the sine of ZP, and therefore the cosine of the latitude.

39. The *Tropics* are the parallels of latitude that pass through the solstices. The northern tropic is called the tropic of Cancer; the southern, the tropic of Capricorn.

40. The *Polar Circles* are the parallels of latitude that pass through the poles of the ecliptic, at the distance of 23½ degrees from the pole of the earth. (Art. 33.)

41. The earth is divided into five zones. That portion of the earth which lies between the tropics, is called the *Torrid Zone;* that between the tropics and polar circles, the *Temperate Zones;* and that between the polar circles and the poles, the *Frigid Zones.*

42. The *Zodiac* is the part of the celestial sphere which lies about 8 degrees on each side of the ecliptic. This portion of the heavens is thus marked off by itself, because all the planets move within it.

43. *The elevation of the pole is equal to the latitude of the place.* The arc PE (Fig. 4.) $= ZO \therefore PO = ZE$ which equals the latitude.

44. *The elevation of the equator is equal to the complement of the latitude.*

$ZH = 90°$. But $ZE = \text{Lat.} \therefore EH = 90 - \text{Lat.}$

45. *The distance of any place from the pole (or the polar distance) equals the complement of the latitude.*

$EP = 90°$. But $EZ = \text{Lat.} \therefore ZP = 90 - \text{Lat.}$

CHAPTER II.

DIURNAL REVOLUTION—ARTIFICIAL GLOBES—ASTRONOMICAL PROBLEMS.

46. The apparent diurnal revolution of the heavenly bodies from east to west, is owing to the actual revolution of the earth on its own axis from west to east. If we conceive of a radius of the earth's equator extended until it meets the concave sphere of the heavens, then as the earth revolves, the extremity of this line would trace out a curve on the face of the sky, namely, the celestial equator. In curves parallel to this, called the *circles of diurnal revolution*, the heavenly bodies actually *appear* to move, every star having its own peculiar circle. After the learner has first rendered familar the real motions of the earth from west to east, he may then, without danger of misconception, adopt the common language, that all the heavenly bodies revolve around the earth once a day from east to west, in circles parallel to the equator and to each other.

47. The time occupied by a star in passing from any point in the meridian until it comes round to the same point again, is called a *sidereal day*, and measures the period of the earth's revolution on its axis. If we watch the returns of the same star from day to

day, we shall find the intervals exactly equal to one another; that is, *the sidereal days are all equal.** Whatever star we select for the observation, the same result will be obtained. The stars, therefore, always keep the same relative position, and have a common movement round the earth,—a consequence that naturally flows from the hypothesis, that their *apparent* motion is all produced by a single *real* motion, namely, that of the earth. The sun, moon, and planets, as well the fixed stars, revolve in like manner, but their returns to the meridian are not, like those of the fixed stars, at exactly equal intervals.

48. The *appearances* of the diurnal motions of the heavenly bodies are different in different parts of the earth, since every place has its own horizon, (Art. 15,) and different horizons are variously inclined to each other. Let us suppose the spectator viewing the diurnal revolutions from several different positions on the earth.

49. On the *equator*, his horizon would pass through both poles; for the horizon cuts the celestial vault at 90 degrees in every direction from the zenith of the spectator; but the pole is likewise 90 degrees from his zenith, and consequently, the pole must be in the horizon. The celestial equator would coincide with the Prime Vertical, being a great circle passing through the east and west points. Since all the diurnal circles are parallel to the equator, consequently, they would all, like the equator, be perpendicular to the horizon. Such a view of the heavenly bodies, is called a right sphere; or,

A RIGHT SPHERE *is one in which all the daily revolutions of the stars, are in circles perpendicular to the horizon.*

A right sphere is seen only at the equator. Any star situated in the celestial equator, would appear to rise directly in the east, at noon to be in the zenith of the spectator, and to set directly in the west; in proportion as stars are at a greater distance from the equator towards the pole, they describe smaller and smaller circles, until, near the pole, their motion is hardly perceptible. Every star remains an equal time above and below the horizon; and since the

* Allowance is here supposed to be made for the effects of precession, &c.

times of their revolutions are equal, the velocities are as the lengths of the circles they describe. Consequently, as the stars are more remote from the equator towards the pole, their motions become slower, until, at the pole, the north star appears stationary.

50. If the spectator advances one degree towards the north pole, his horizon reaches one degree beyond the pole of the earth, and cuts the starry sphere one degree below the pole of the heavens, or below the north star, if that be taken as the place of the pole. As he moves onward towards the pole, his horizon continually reaches farther and farther beyond it, until when he comes to the pole of the earth, and under the pole of the heavens, his horizon reaches on all sides to the equator and coincides with it. Moreover, since all the circles of daily motion are parallel to the equator, they become, to the spectator at the pole, parallel to the horizon. This is what constitutes a parallel sphere. Or,

A PARALLEL SPHERE *is that in which all the circles of daily motion are parallel to the horizon.*

51. To render this view of the heavens familiar, the learner should follow round in his mind a number of separate stars, one near the horizon, one a few degrees above it, and a third near the zenith. To one who stood upon the north pole, the stars of the northern hemisphere would all be perpetually in view when not obscured by clouds or lost in the sun's light, and none of those of the southern hemisphere would ever be seen. The sun would be constantly above the horizon for six months in the year, and the remaining six constantly out of sight. That is, at the pole the days and nights are each six months long. The phenomena at the south pole are similar to those at the north.

52. A perfect parallel sphere can never be seen except at one of the poles,—a point which has never been actually reached by man ; yet the British discovery ships penetrated within a few degrees of the north pole, and of course enjoyed the view of a sphere nearly parallel.

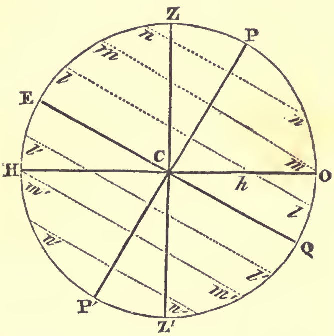
53. As the circles of daily motion are parallel to the horizon of the pole, and perpendicular to that of the equator, so at all places

between the two, the diurnal motions are oblique to the horizon. This aspect of the heavens constitutes an oblique sphere, which is thus defined:

An Oblique Sphere *is that in which the circles of daily motion are oblique to the horizon.*

Suppose for example the spectator is at the latitude of fifty degrees. His horizon reaches 50° beyond the pole of the earth, and gives the same apparent elevation to the pole of the heavens. It cuts the equator, and all the circles of daily motion, at an angle of 40°, being always equal to the co-altitude of the pole. Thus, let HO (Fig. 5,) represent the horizon, EQ the equator, and PP′ the axis of the earth. Also, *ll*, *mm*, &c. parallels of latitude. Then the horizon of a spectator at Z, in latitude 50° reaches to 50° beyond the pole (Art. 50); and the angle ECH, is 40°. As we advance still farther north the elevation of the diurnal circles grows less and less, and consequently the motions of the heavenly bodies more and more oblique, until finally, at the pole, where the latitude is 90°, the angle of elevation of the equator vanishes, and the horizon and equator coincide with each other, as before stated.

Fig. 5.



54. The circle of perpetual apparition, *is the boundary of that space around the elevated pole, where the stars never set.* Its distance from the pole is equal to the latitude of the place. For, since the altitude of the pole is equal to the latitude, a star whose polar distance is just equal to the latitude, will when at its lowest point only just reach the horizon; and all the stars nearer the pole than this will evidently not descend so far as the horizon.

Thus, *mm* (Fig. 5,) is the circle of perpetual apparation, between which and the north pole, the stars never set, and its distance from the pole OP is evidently equal to the elevation of the pole, and of course to the latitude.

55. In the opposite hemisphere, a similar part of the sphere adjacent to the depressed pole never rises. Hence,

The CIRCLE OF PERPETUAL OCCULTATION, *is the boundary of that space around the depressed pole, within which the stars never rise.* Thus, $m' m'$ (Fig. 5,) is the circle of perpetual occultation, between which and the south pole, the stars never rise.

56. In an oblique sphere, the horizon cuts the circles of daily motion unequally. Towards the elevated pole, more than half the circle is above the horizon, and a greater and greater portion as the distance from the equator is increased, until finally, within the circle of perpetual apparition, the whole circle is above the horizon. Just the opposite takes place in the hemisphere next the depressed pole. Accordingly, when the sun is in the equator, as the equator and horizon, like all other great circles of the sphere, bisect each other, the days and nights are equal all over the globe. But when the sun is north of the equator, the days become longer than the nights, but shorter when the sun is south of the equator. Moreover, the higher the latitude, the greater is the inequality in the lengths of the days and nights. All these points will be readily understood by inspecting figure 5.

57. Most of the phenomena of the diurnal revolution can be explained, either on the supposition that the celestial sphere actually all turns around the earth once in 24 hours, or that this motion of the heavens is merely apparent, arising from the revolution of the earth on its axis in the opposite direction,—a motion of which we are insensible, as we sometimes lose the consciousness of our own motion in a ship or a steam boat, and observe all external objects to be receding from us with a common motion. Proofs entirely conclusive and satisfactory, establish the fact, that it is the earth and not the celestial sphere that turns; but these proofs are drawn from various sources, and the student is not prepared to appreciate their value, or even to understand some of them, until he has made considerable proficiency in the study of astronomy, and become familiar with a great variety of astronomical phenomena. To such a period of our course of instruction, we therefore postpone the discussion of the hypothesis of the earth's rotation on its axis.

58. While we retain the same place on the earth, the diurnal revolution occasions no change in our horizon, but our horizon goes round as well as ourselves. Let us first take our station on the equator at sunrise; our horizon now passes through both the poles, and through the sun, which we are to conceive of as at a great distance from the earth, and therefore as cut, not by the terrestrial but by the celestial horizon. As the earth turns, the horizon dips more and more below the sun, at the rate of 15 degrees for every hour, and, as in the case of the polar star, the sun appears to rise at the same rate. In six hours, therefore, it is depressed 90 degrees below the sun, which brings us directly under the sun, which, for our present purpose, we may consider as having all the while maintained the same fixed position in space. The earth continues to turn, and in six hours more, it completely reverses the position of our horizon, so that the western part of the horizon which at sunrise was diametrically opposite to the sun now cuts the sun, and soon afterwards it rises above the level of the sun, and the sun sets. During the next twelve hours, the sun continues on the invisible side of the sphere, until the horizon returns to the position from which it started, and a new day begins.

59. Let us next contemplate the similar phenomena at the *poles*. Here the horizon, coinciding as it does with the equator, would cut the sun through its center, and the sun would appear to revolve along the surface of the sea, one half above and the other half below the horizon. This supposes the sun in its annual revolution to be at one of the equinoxes. When the sun is north of the equator, it revolves continually round in a circle which, during a single revolution, appears parallel to the equator, and it is constantly day; and when the sun is south of the equator, it is, for the same reason, continual night.

60. We have endeavored to conceive of the manner in which the apparent diurnal movements of the sun are *really* produced at two stations, namely, in the right sphere, and in the parallel sphere. These two cases being clearly understood, there will be little difficulty in applying a similar explanation to an oblique sphere.

ARTIFICIAL GLOBES.

61. Artificial globes are of two kinds, terrestrial and celestial. The first exhibits a miniature representation of the earth; the second, of the visible heavens; and both show the various circles by which the two spheres are respectively traversed. Since all globes are similar solid figures, a small globe, imagined to be situated at the center of the earth or of the celestial vault, may represent all the visible objects and artificial divisions of either sphere, and with great accuracy and just proportions, though on a scale greatly reduced. The study of artificial globes, therefore, cannot be too strongly recommended to the student of astronomy.*

62. An artificial globe is encompassed from north to south by a strong brass ring to represent the meridian of the place. This ring is made fast to the two poles and thus supports the globe, while it is itself supported in a vertical position by means of a frame, the ring being usually let into a socket in which it may be easily slid, so as to give any required elevation to the pole. The brass meridian is graduated each way from the equator to the pole 90°, to measure degrees of latitude or declination, according as the distance from the equator refers to a point on the earth or in the heavens. The horizon is represented by a broad zone, made broad for the convenience of carrying on it a circle of azimuth, another of amplitude, and a wide space on which are delineated the signs of the ecliptic, and the sun's place for every day in the year; not because these points have any special connexion with the horizon, but because this broad surface furnishes a convenient place for recording them.

63. *Hour Circles* are represented on the terrestrial globe by great circles drawn through the pole of the equator; but, on the celestial globe, corresponding circles pass through the poles of the ecliptic, constituting *circles of latitude*, while the brass meridian,

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* It were desirable, indeed, that every student of the science should have the celestial globe at least, constantly before him. One of a small size, as eight or nine inches, will answer the purpose, although globes of these dimensions can not usually be relied on for nice measurements.

being a secondary to the equinoctial, becomes an hour circle of any star which, by turning the globe, is brought under it.

64. The *Hour Index* is a small circle described around the pole of the equator, on which are marked the hours of the day. As this circle turns along with the globe, it makes a complete revolution in the same time with the equator; or, for any less period, the same number of degrees of this circle and of the equator pass under the meridian. Hence the hour index measures arcs of right ascension.

65. The *Quadrant of Altitude* is a flexible strip of brass, graduated into ninety equal parts, corresponding in length to degrees on the globe, so that when applied to the globe and bent so as closely to fit its surface, it measures the angular distance between any two points. When the zero, or the point where the graduation begins, is laid on the pole of any great circle, the 90th degree will reach to the circumference of that circle, and being therefore a great circle passing through the pole of another great circle, it becomes a secondary to the latter. (Art. 21.) Thus the quadrant of altitude may be used as a secondary to any great circle on the sphere; but it is used chiefly as a secondary to the horizon, the point marked 90° being screwed fast to the pole of the horizon, that is, the zenith, and the other end, marked 0, being slid along between the surface of the sphere and the wooden horizon. It thus becomes a vertical circle, *on* which to measure the altitude of any star through which it passes, or *from* which to measure the azimuth of the star, which is the arc of the horizon intercepted between the meridian and the quadrant of altitude passing through the star, (Art. 27.)

66. To *rectify the globe for any place*, the north pole must be elevated to the latitude of the place (Art. 43); then the equator and all the diurnal circles will have their due inclination in respect to the horizon; and, on turning the globe, every point on either globe will revolve as the same point does in nature; and the relative situations of all places will be the same as on the native spheres.

PROBLEMS ON THE TERRESTRIAL GLOBE.

67. *To find the Latitude and Longitude of a place:* Turn the globe so as to bring the place to the brass meridian; then the degree and minute on the meridian directly over the place will indicate its latitude, and the point of the equator under the meridian, will show its longitude.

Ex. What is the Latitude and Longitude of the city of New York?

68. *To find a place having its latitude and longitude given:* Bring to the brass meridian the point of the equator corresponding to the longitude, and then at the degree of the meridian denoting the latitude, the place will be found.

Ex. What place on the globe is in Latitude 39 N. and Longitude 77 W.?

69. *To find the bearing and distance of two places:* Rectify the globe for one of the places (Art. 66); screw the quadrant of altitude to the zenith,* and let it pass through the other place. Then the azimuth will give the bearing of the second place from the first, and the number of degrees on the quadrant of altitude, multiplied by 69, (the number of miles in a degree,) will give the distance between the two places.

Ex. What is the bearing of New Orleans from New York, and what is the distance between these places?

70. *To determine the difference of time in different places:* Bring the place that lies eastward of the other to the meridian, and set the hour index at XII. Turn the globe eastward until the other place comes to the meridian, then the index will point to the hour required.

Ex. When it is noon at New York, what time is it at London?

71. *The hour being given at any place, to tell what hour it is in any other part of the world:* Bring the given place to the meridian, and set the hour index to the given time; then turn the

* The zenith will of course be the point of the meridian over the place.

globe, until the other place comes under the meridian, and the index will point to the required hour.

Ex. What time is it at Canton, in China, when it is 9 o'clock A. M. at New York ?

72. *To find the antœci,* the periœci,† and the antipodes‡ of any place:* Bring the given place to the meridian ; then in the opposite hemisphere, in the same degree of latitude, will be found the antœci. The same place remaining under the meridian, set the index to XII, and turn the globe until the other XII is under the index; then the periœci will be on the meridian under the same degree of latitude with the given plane, and the antipodes will be under the meridian, in the same latitude, in the opposite hemisphere.

Ex. Find the antœci, the periœci, and the antipodes of the citizens of New York.

The antœci have the same hour of the day, but different seasons of the year ; the periœci have the same seasons, but opposite hours ; and the antipodes have both opposite hours and opposite seasons.

73. *To rectify the globe for the sun's place:* On the wooden horizon, find the day of the month, and against it is given the the sun's place in the ecliptic, expressed by signs and degrees.§ Look for the same sign and degree on the ecliptic, bring that point to the meridian and set the hour index to XII. To all places under the meridian it will then be noon.

Ex. Rectify the globe for the sun's place on the 1st of September.

74. *The latitude of the place being given, to find the time of the sun's rising and setting on any given day at that place:* Having rectified the globe for the latitude, (Art. 66,) bring the sun's place in the ecliptic to the graduated edge of the meridian, and set the hour index to XII ; then turn the globe so as to bring the sun to the eastern and then to the western horizon, and the hour index will show the times of rising and setting respectively.

* αντι οικος. † περι οικος. ‡ αντι πȣς.

§ The larger globes have the day of the month marked against the corresponding sign on the ecliptic itself.

Ex. At what time will the sun rise and set at New Haven, Lat. 41° 18′ on the 10th of July?

PROBLEMS ON THE CELESTIAL GLOBE.

75. *To find the Declination and Right Ascension of a heavenly body:* Bring the place of the body (whether the sun or a star) to the meridian. Then the degree and minute standing over it will show its declination, and the point of the equinoctial under the meridian will give its right ascension. It will be remarked, that the declination and right ascension are found in the same manner as latitude and longitude on the terrestrial globe. Right ascension is expressed either in degrees or in hours; both being reckoned from the vernal equinox, (Art. 37.)

Ex. What is the declination and right ascension of the bright star Lyra?—also of the sun on the 5th of June?

76. *To represent the appearance of the heavens at any time:* Rectify the globe for the latitude, bring the sun's place in the ecliptic to the meridian, and set the hour index to XII; then turn the globe westward until the index points to the given hour, and the constellations would then have the same appearance to an eye situated at the center of the globe, as they have at that moment in the sky.

Ex. Required the aspect of the stars at New Haven, Lat. 41° 18′, at 10 o'clock, on the evening of December 5th.

77. *To find the altitude and azimuth of any star:* Rectify the globe for the latitude, and let the quadrant of altitude be screwed to the zenith, and be made to pass through the star. The arc on the quadrant, from the horizon to the star, will denote its altitude, and the arc of the horizon from the meridian to the quadrant, will be its azimuth.

Ex. What is the altitude and azimuth of Sirius (the brightest of the fixed stars) on the 25th of December at 10 o'clock in the evening, in Lat. 41°?

78. *To find the angular distance of two stars from each other:* Apply the zero mark of the quadrant of altitude to one of the stars, and the point of the quadrant which falls on the other star, will show the angular distance between the two.

Ex. What is the distance between the two largest stars of the Great Bear.*

79. *To find the sun's meridian altitude, the latitude and day of the month being given:* Having rectified the globe for the latitude, (Art. 66,) bring the sun's place in the ecliptic to the meridian, and count the number of degrees and minutes between that point of the meridian and the zenith. The complement of this arc will be the sun's meridian altitude.

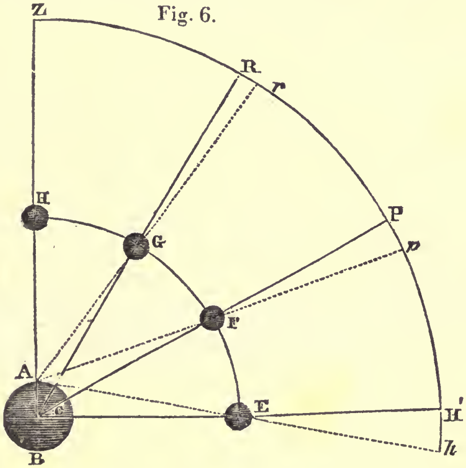
Ex. What is the sun's meridian altitude at noon on the 1st of August, in Lat. 41° 18′?

CHAPTER III.

OF PARALLAX, REFRACTION, AND TWILIGHT.

80. Parallax *is the apparent change of place which bodies undergo by being viewed from different points.* Thus in figure 6, let A represent the earth, CH′ the horizon. H′Z a quadrant of

Fig. 6.



* These two stars are sometimes called "the Pointers," from the line which passes through them being always nearly in the direction of the north star. The angular distance between them is about 5°, and may be learned as a standard for reference in estimating by the eye, the distance between any two points on the celestial vault.

a great circle of the heavens, extending from the horizon to the zenith; and let E, F, G, H, be successive positions of the moon at different elevations, from the horizon to the meridian. Now a spectator on the surface of the earth at A, would refer the place of E to h, whereas, if seen from the center of the earth, it would appear at H′. The arc H′h is called the parallactic arc, and the angle H′Eh, or its equal AEC, is the angle of parallax. The same is true of the angles at F, G, and H, respectively.

81. Since then a heavenly body is liable to be referred to different points on the celestial vault, when seen from different parts of the earth, and thus some confusion occasioned in the determination of points on the celestial sphere, astronomers have agreed to consider the true place of a celestial object to be that where it would appear if seen from the center of the earth. The doctrine of parallax teaches how to reduce observations made at any place on the surface of the earth, to such as they would be if made from the center.

82. The angle AEC is called the horizontal parallax, which may be thus defined. *Horizontal Parallax*, is the change of position which a celestial body, appearing in the horizon as seen from the surface of the earth, would assume if viewed from the earth's center. It is the angle subtended by the semi-diameter of the earth, as viewed from the body itself. If we consider any one of the triangles represented in the figure, ACG for example,

$$\text{Sin. AGC} : \text{Sin. GAZ} :: \text{AC} : \text{CG} \therefore$$

$$\text{Sin. Parallax} = \frac{\text{Sin. GAZ} \times \text{AC}}{\text{CG}} \propto \frac{\text{Sin. GAZ}}{\text{CG}}.$$

Hence the sine of the angle of parallax, or (since the angle of parallax is always very small*) *the parallax itself varies as the sine of the zenith distance of the body directly, and the distance of the body from the center of the earth inversely.* Parallax, therefore, increases as a body approaches the horizon, (but increasing

* The moon, on account of its nearness to the earth, has the greatest horizontal parallax of any of the heavenly bodies; yet this is less than 1° (being 57′) while the greatest parallax of any of the planets does not exceed 30″. The difference in an arc of 1°, between the length of the arc and the sine, is only 0.″18.

with the sines, it increases much slower than in the simple ratio of the distance from the zenith,) and diminishes, as the distance from the spectator increases. Again, since the parallax AGC is as the sine of the zenith distance, let P represent the horizontal parallax, and P′ the parallax at any altitude; then,

$$P' : P :: \text{sin. zenith dist.} : \text{sin. } 90^\circ \therefore P = \frac{P'}{\text{sin. zen. dist.}}.$$

Hence, the horizontal parallax of a body equals its parallax at any altitude, divided by the sine of its distance from the zenith.

83. From observations, therefore, on the parallax of a body at any elevation, we are enabled to find the angle subtended by the semi-diameter of the earth as seen from the body. Or if the horizontal parallax is given, the parallax at any altitude may be found, for

$$P' = P \times \text{sin. zenith distance.}$$

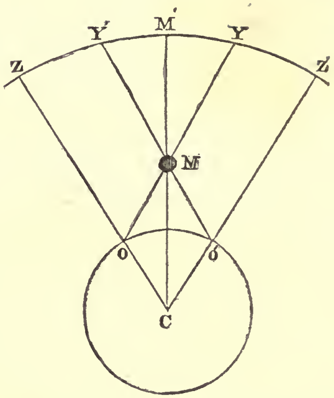
Hence, in the zenith the parallax is nothing, and is at its maximum in the horizon.

84. It is evident from the figure, that the effect of parallax upon the place of a celestial body is *to depress it.* Thus, in consequence of parallax, E is depressed by the arc $H'h$; F by the arc Pp; G by the arc Rr; while H sustains no change. Hence, in all calculations respecting the altitude of the sun, moon, or planets, the amount of parallax is to be subtracted; the stars, as we shall see hereafter, have no sensible parallax. As the depression which arises from parallax is in the direction of a vertical circle, when the body is on the meridian, the body has only a parallax in declination; but in other situations, there is at the same time a parallax in declination and right ascension; for its direction being *oblique* to the equinoctial, it can be resolved into two parts, one of which (the declination) is perpendicular, and the other (the right ascension) is parallel to the equinoctial.

85. *The mode of determining the horizontal parallax*, is as follows:

Let O, O′, (Fig. 7,) be two places on the earth, situated under the same meridian, at a great distance from each other; one place, for example, at the Cape of Good Hope, and the other in the north

Fig. 7.



of Europe. The latitude of each place being known, the arc of the meridian OO′ is known, and the angle OCO′ also is known. Let the celestial body M, (the moon for example,) be observed simultaneously at O and O′, and its zenith distance at each place accurately taken, namely, ZY and Z′Y′; then the angles ZOM and Z′O′M, and of course their supplements COM, CO′M are found. Then in the quadrilateral figure COMO′, we have all the angles and the two radii, CO, CO′, whence the side CM may be easily found. But, CM : CO : : sin. ZOM : sin. CMO = sine of the angle of parallax; or (since the arc is very small) equals the parallax P′. But when M as seen from O is in the horizon, ZOM becomes a right angle, and its sine equal to radius. Then,

$$\text{CM} : \text{CO} :: 1 : \text{P} = \text{horizontal parallax} = \frac{\text{CO}}{\text{CM}}.$$

On this principle, the horizontal parallax of the moon was determined by La Caille and La Lande, two French astronomers, one stationed at the Cape of Good Hope, the other at Berlin; and and in a similar way the parallax of Mars was ascertained, by observations made simultaneously at the Cape of Good Hope and at Stockholm.

86. On account of the great distance of the sun, his horizontal parallax, which is only 8″.6, cannot be accurately ascertained by this method. It can, however, be determined by means of the transits of Venus, a process to be described hereafter.

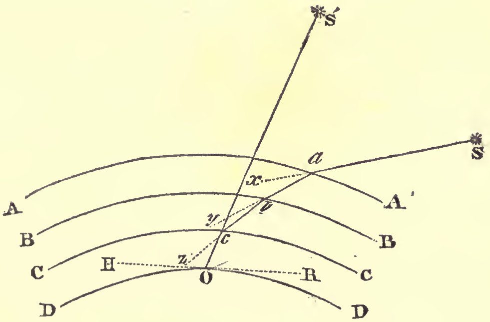
87. The determination of the horizontal parallax of a celestial body is an element of great importance, since it furnishes the means of estimating the distance of the body from the center of the earth. Thus, if the angle AEC (Fig. 6,) be found, the radius of the earth AC being known, we have in the right angled triangle AEC, the side AC and all the angles, to find the hypo-

thenuse CE, which is the distance of the moon from the center of the earth.

REFRACTION.

88. While parallax depresses the celestial bodies subject to it, *refraction elevates them;* and it affects alike the most distant as well as nearer bodies, being occasioned by the change of direction which light undergoes in passing through the atmosphere. Let us conceive of the atmosphere as made up of a great number of concentric strata, as AA, BB, CC, and DD, (Fig. 8,)

Fig. 8.



increasing rapidly in density (as is known to be the fact) in approaching near to the surface of the earth. Let S be a star, from which a ray of light S*a* enters the atmosphere at *a*, where, being turned towards the radius of the convex surface, it would change its direction into the line *ab*, and again into *bc*, and *c*O, reaching the eye at O. Now, since an object always appears in the direction in which the light finally strikes the eye, the star would be seen in the direction of the last ray *c*O, and consequently, the star would apparently change its place, in consequence of refraction, from S to S′, being elevated out of its true position. Moreover, since on account of the continual increase of density in descending through the atmosphere, the light would be continually turned out of its course more and more, it would therefore move, not in the polygon represented in the figure, but in a corresponding curve, whose curvature is rapidly increased near the surface of the earth.

89. When a body is in the zenith, since a ray of light from it enters the atmosphere at right angles to the refracting medium, it suffers no refraction. Consequently, the position of the heavenly bodies, when in the zenith, is not changed by refraction, while near the horizon, when a ray of light strikes the medium very obliquely, and traverses the atmosphere through its densest part, the refraction is greatest. The following numbers, taken at different altitudes, will show how rapidly refraction diminishes from the horizon upwards. The amount of refraction at the horizon is 34′ 00″. At different elevations it is as follows.

Elevation.	Refraction.	Elevation.	Refraction.
0° 10′	32′ 00″	30°	1′ 40″
0° 30′	30′ 00″	40°	1′ 09″
1° 00′	24′ 25″	45°	0′ 58″
5° 00′	10′ 00″	60°	0′ 33″
10° 00′	5′ 20″	80°	0′ 10″
20° 00′	2′ 39″	90°	0′ 00″

From this table it appears, that while refraction at the horizon is 34 minutes, at so small an elevation as only 10 minutes above the horizon it loses 2 minutes, more than the entire change from the elevation of 30° to the zenith. From the horizon to 1° above, the refraction is diminished nearly 10 minutes. The amount at the horizon, at 45°, and at 90°, respectively, is 34′, 58″, and 0. In finding the altitude of a heavenly body, the effect of parallax must be added, but that of refraction subtracted.

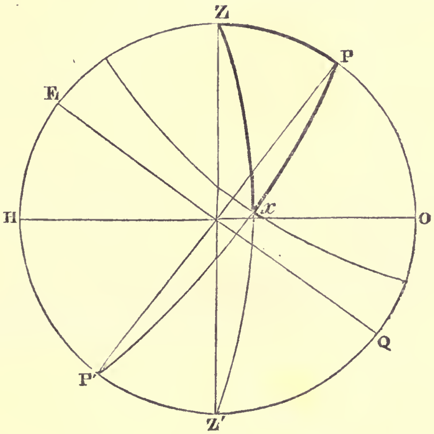
90. Let us now learn the method, by which the amount of refraction at different elevations is ascertained. To take the simplest case, we will suppose ourselves in a high latitude, where some of the stars within the circle of perpetual apparition pass through the zenith of the place. We measure the distance of such a star from the pole when on the meridian above the pole, that is, in the zenith, where it is not at all affected by refraction, and again its distance from the pole in its lower culmination. Were it not for refraction, these two polar distances would be equal, since, in the diurnal revolution of a star, it is in fact always at the same distance from the pole; but, on account of refraction, the lower distance will be less than the upper, and the difference

between the two will show the amount of refraction at the lower culmination, the latitude of the place being known.

Example. At Paris, latitude 48° 50′, a star was observed to pass the meridian 6′ north of the zenith, and consequently, 41° 4′, from the pole.* It should have passed the meridian at the same distance below the pole, but the distance was found to be only 40° 57′ 35″. Hence, 41° 4′ – 40° 57′ 35″ = 6′ 25″ is the refraction due to that altitude, that is, at the altitude of 7° 46′. By taking similar observations in various places situated in high latitudes, the amount of refraction might be ascertained for a number of different altitudes, and thus the law of increase in refraction as we proceed from the zenith towards the horizon, might be ascertained.

91. Another method of finding the refraction at different altitudes, is as follows. Take the altitude of the sun or a star, whose right ascension and declination are known, and note the time by the clock. Observe also when it crosses the meridian, and the difference of time between the two observations will give the hour angle ZPx, (Fig. 9.) In this triangle ZPx we also know PZ the

Fig. 9.



co-latitude and Px the co-declination. Hence we can find the co-altitude Zx, and of course the true altitude. Compare the alti-

* For the polar distance of the place = 90 – 48° 50′ = 41° 10′; and 41° 10′ – 6′ = 41° 4′.

tude thus found with that before determined by observation, and the difference will be the refraction due to the apparent altitude.

Ex. On May 1, 1738, at 5h. 20m. in the morning, Cassini observed the altitude of the sun's center at Paris to be 5° 0′ 14″. The latitude of Paris being 48° 50′ 10″, and the sun's declination at that time being 15° 0′ 25″: *Required the refraction.*

By spherical trigonometry, Zx will be found equal to 85° 10′ 8″; consequently, the true altitude was 4° 49′ 52″. Now to 5° 0′ 14″, the apparent altitude, 9″ must be added for parallax, and we have 5° 0′ 23″ the apparent altitude corrected for parallax. Hence, 5° 0′ 23″ − 4° 49′ 52″ = 10′ 31″, the refraction at the apparent altitude 5° 0′ 14″.*

92. By these and similar methods, we could easily determine the refraction due to any elevation above the horizon, provided the refracting medium (the atmosphere) were always uniform. But this is not the fact: the refracting power of the atmosphere is altered by changes in density and temperature.† Hence in delicate observations, it is necessary to take into the account the state of the barometer and of the thermometer, the influence of the variations of each having been very carefully investigated, and rules having been given accordingly. With every precaution to insure accuracy, on account of the variable character of the refracting medium, the tables are not considered as entirely accurate to a greater distance from the zenith than 74°; but almost all astronomical observations are made at a greater altitude than this.

93. Since the whole amount of refraction near the horizon exceeds 33′, and the diameters of the sun and moon are severally less than this, these luminaries are in view both before they have actually risen and after they have set.

94. The rapid increase of refraction near the horizon, is strikingly evinced by the *oval* figure which the sun assumes when near the horizon, and which is seen to the greatest advantage

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* Gregory's Ast. p. 65.

† It is said that the effects of *humidity* are insensible; for the most accurate experiments seem to prove that watery vapor diminishes the density of air in the same ratio as its own refractive power is greater than that of air. (New Encyc. Brit. III, 762.)

when light clouds enable us to view the solar disk. Were all parts of the sun equally raised by refraction, there would be no change of figure ; but since the lower side is more refracted than the upper, the effect is to shorten the vertical diameter and thus to give the disk an oval form. This effect is particularly remarkable when the sun, at his rising or setting, is observed from the top of a mountain, or at an elevation near the sea shore ; for in such situations the rays of light make a greater angle than ordinary with a perpendicular to the refracting medium, and the amount of refraction is proportionally greater. In some cases of this kind, the shortening of the vertical diameter of the sun has been observed to amount to 6′, or about one fifth of the whole.

95. The apparent *enlargement of the sun and moon in the horizon*, arises from an optical illusion. These bodies in fact are not seen under so great an angle when in the horizon, as when on the meridian, for they are nearer to us in the latter case than in the former. The distance of the sun is indeed so great that it makes very little difference in his apparent diameter, whether he is viewed in the horizon or on the meridian ; but with the moon the case is otherwise ; its angular diameter, when measured with instruments, is perceptibly larger at the time of its culmination. Why then do the sun and moon appear so much larger when near the horizon ? It is owing to that general law, explained in optics, by which we judge of the magnitudes of distant objects, not merely by the angle they subtend at the eye, but also by our impressions respecting their distance, allowing, under a given angle, a greater magnitude as we imagine the distance of a body to be greater. Now, on account of the numerous objects usually in sight between us and the sun, when on the horizon, he appears much farther removed from us than when on the meridian, and we assign to him a proportionally greater magnitude. If we view the sun, in the two positions, through smoked glass, no such difference of size is observed, for here no objects are seen but the sun himself.

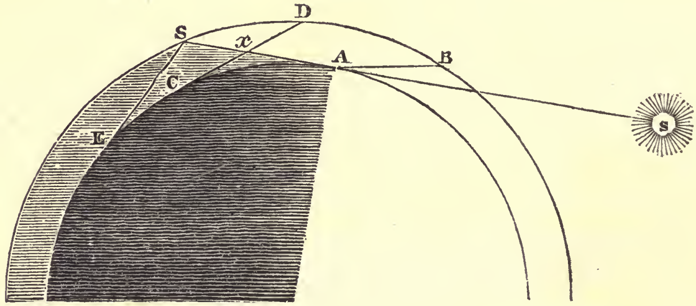
96. The extraordinary enlargement of the sun or moon, particularly the latter, when seen at its rising through a grove of trees,

depends on a different principle. Through the various openings between the trees, we see different images of the sun, a great number of which overlapping each other, swell the dimensions of the moon, under the most favorable circumstances, to a very unusual size.

TWILIGHT.

97. *Twilight* also is another phenomenon depending upon the agency of the earth's atmosphere. It is due partly to refraction and partly to reflexion, but mostly to the latter. While the sun is within 18° of the horizon, before it rises or after it sets, some portion of its light is conveyed to us by means of numerous reflections from the atmosphere. Let AB (Fig. 10,) be the horizon

Fig. 10.



of the spectator at A, and let SS be a ray of light from the sun when it is two or three degrees below the horizon. Then to the observer at A, the segment of the atmosphere ABS would be illuminated. To a spectator at C, whose horizon was CD, the small segment SDx would be the twilight; while, at E, the twilight would disappear altogether.

98. At the equator, where the circles of daily motion are perpendicular to the horizon, the sun descends through 18° in an hour and twelve minutes ($\frac{18}{15}=1\frac{1}{5}$h.), and the light of day therefore declines rapidly and as rapidly advances after daybreak in the morning. At the pole, a constant twilight is enjoyed while the sun is within 18° of the horizon, occupying nearly two thirds of the half year when the direct light of the sun is withdrawn, so that the progress from continual day to constant night is exceedingly

gradual. To the inhabitants of an oblique sphere, the twilight is longer in proportion as the place is nearer the elevated pole.

99. Were it not for the power the atmosphere has of dispersing the solar light, and scattering it in various directions, no objects would be visible to us out of direct sunshine; every shadow of a passing cloud would be pitchy darkness; the stars would be visible all day, and every apartment into which the sun had not direct admission, would be involved in the obscurity of night. This scattering action of the atmosphere on the solar light, is greatly increased by the irregularity of temperature caused by the sun, which throws the atmosphere into a constant state of undulation, and by thus bringing together masses of air of different temperatures, produces partial reflections and refractions at their common boundaries, by which means much light is turned aside from the direct course, and diverted to the purposes of general illumination. In the upper regions of the atmosphere, as on the tops of very high mountains, where the air is too much rarefied to reflect much light, the sky assumes a black appearance, and stars become visible in the day time.

CHAPTER IV.

OF TIME.

100. Time *is a measured portion of indefinite duration.*

The great standard of time is the period of the revolution of the earth on its axis, which, by the most exact observations, is found to be always the same. The time of the earth's revolution on its axis is called a *sidereal day*, and is determined by the revolution of a star from the instant it crosses the meridian, until it comes round to the meridian again. This interval being called a sidereal day, it is divided into 24 *sidereal hours*. Observations taken upon numerous stars, in different ages of the world, show that they all perform their diurnal revolutions in the same time, and that their motion during any part of the revolution is perfectly uniform.

101. *Solar time* is reckoned by the apparent revolution of the sun, from the meridian round to the same meridian again. Were the sun stationary in the heavens, like a fixed star, the time of its apparent revolution would be equal to the revolution of the earth on its axis, and the solar and the sidereal days would be equal. But since the sun passes from west to east, through 360° in 365¼ days, it moves eastward nearly 1° a day, (59′ 8″.3). While, therefore, the earth is turning round on its axis, the sun is moving in the same direction, so that when we have come round under the same celestial meridian from which we started, we do not find the sun there, but he has moved eastward nearly a degree, and the earth must perform so much more than one complete revolution, in order to come under the sun again. Now since a place on the earth gains 359° in 24 hours, how long will it take to gain 1°?

$$359 : 24 :: 1 : \frac{24}{359} = 4^{m} \text{ nearly.}$$

Hence the solar day is about 4 minutes longer than the sidereal; and if we were to reckon the sidereal day 24 hours, we should reckon the solar day 24h. 4m. To suit the purposes of society at large, however, it is found most convenient to reckon the solar day 24 hours, and to throw the fraction into the sidereal day. Then,

24h. 4m. : 24 : : 24 : 23h. 56m. nearly (23h. 56^{m} $4^{s}.09$)=the length of a sidereal day.

102. The solar days, however, do not always differ from the sidereal by precisely the same fraction, since the increments of right ascension, (Art. 37,) which measure the difference between a sidereal and a solar day, are not equal to each other. *Apparent time*, is time reckoned by the revolutions of the sun from the meridian to the meridian again. These intervals being unequal, of course the apparent solar days are unequal to each other.

103. *Mean time*, is time reckoned by the *average* length of all the solar days throughout the year. This is the period which constitutes the civil day of 24 hours, beginning when the sun is on

the lower meridian, namely, at 12 o'clock at night, and counted by 12 hours from the lower to the upper culmination, and from the upper to the lower. The *astronomical* day is the apparent solar day counted through the whole 24 hours, instead of by periods of 12 hours each, and begins at noon. Thus 10 days and 14 hours of astronomical time, would be 11 days and 2 hours of apparent time.

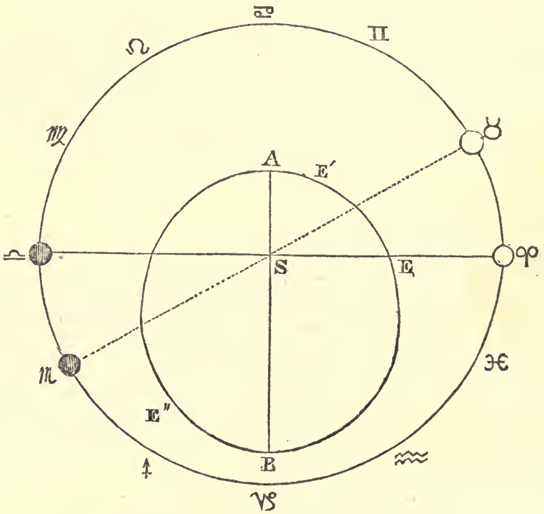
104. Clocks are usually regulated so as to indicate mean solar time; yet as this is an artificial period, not marked off, like the sidereal day, by any natural event, it is necessary to know how much is to be added to or subtracted from the apparent solar time, in order to give the corresponding mean time. The interval by which apparent time differs from mean time, is called the *equation of time.* If a clock were constructed (as it may be) so as to keep exactly with the sun, going faster or slower according as the increments of right ascension were greater or smaller, and another clock were regulated to mean time, then the difference of the two clocks, at any period, would be the equation of time for that moment. If the apparent clock were *faster* than the mean, then the equation of time must be subtracted; but if the apparent clock were *slower* than the mean, then the equation of time must be added, to give the mean time. The two clocks would differ most about the 3d of November, when the apparent time is $16\frac{1}{4}^{m}$ greater than the mean ($16^{m}\ 16^{s}.7$). But, since apparent time is sometimes greater and sometimes less than mean time, the two must obviously be sometimes equal to each other. This is in fact the case four times a year, namely, April 15th, June 15th, September 1st, and December 22d. These epochs, however, do not remain constant; for, on account of the change in the position of the perihelion, or the point where the earth is nearest the sun, (which shifts its place from west to east about 12″ a year,) the period when the sun's motions are most rapid, as well as that when they are slowest, will occur at different parts of the year. The change is indeed exceedingly small in a single year; but in the progress of ages, the time of year when the sun's motion in its orbit is most accelerated, will not be, as at present, on the first of January, but may fall on the first of March, June, or

any other day of the year, and the amount of the equation of time is obviously affected by the sun's distance from its perihelion, since the sun moves most rapidly when nearest the perihelion, and slowest when farthest from that point.

105. *The inequality of the solar days depends on two causes, the unequal motion of the earth in its orbit, and the inclination of the equator to the ecliptic.*

First, on account of the eccentricity* of the earth's orbit, the earth actually moves faster from the autumnal to the vernal equinox, than from the vernal to the autumnal, the difference of the two periods being about eight days (7d. 17h. 17m.) Thus, let AEB (Fig. 11,) represent the earth's orbit, S being the place of

Fig. 11.



the sun, A the *perihelion*, or nearest distance of the earth from the sun, B the *aphelion*, or greatest distance, and E, E′, E″, positions of the earth in different points of its orbit. The place of the earth among the signs is the part of the heavens to which it

* The exact figure of the earth's orbit will be more particularly shown hereafter. All that the student requires to know, in order to understand the present subject, is that the earth's orbit is an ellipse, and that the earth's *real* motion, and consequently the sun's *apparent* motion, is greater in proportion as the earth is nearer the sun.

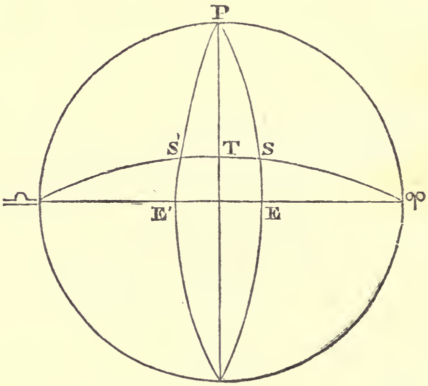
would be referred if seen from the sun; and the place of the sun is the part of the heavens to which it is referred as seen from the earth. Thus, when the earth is at E, it is said to be in Aries; and as it moves from E through E′ to A, its path in the heavens is through Aries, Taurus, Gemini, &c. Meanwhile the sun takes its place successively in Libra, Scorpio, Sagittarius, &c. Now, as will be shown more fully hereafter, the earth moves faster when proceeding from Aries through its perihelion to Libra, than from Libra through its aphelion to Aries, and, consequently, describes the half of its apparent orbit in the heavens, ♈, ♋, ♎, sooner than the half ♎, ♑, ♈. The line of the apsides, that is, the major axis of the ellipse, is so situated at present, that the perihelion is in the sign Leo, nearly 100° (99° 30′ 5″) from the vernal equinox. The earth passes through it about the first of January, and then its velocity is the greatest in the whole year, being always greater as the distance is less, the angular velocity being inversely as the square of the distance, as will be shown by and by.

106. But differences of time are not reckoned on the ecliptic, but on the equinoctial; for the ecliptic being oblique to the meridian in the diurnal motion, and cutting it at different angles at different times, equal portions will not pass under the meridian in equal times, and therefore such portions could not be employed, as they are in the equinoctial, as measures of time. If therefore the sun moved uniformly in his orbit, so as to make the daily increments of longitude equal, still the corresponding arcs of right ascension, which determine the lengths of the solar day, would be unequal. Let us start from the equinox, from which both longitude and right ascension are reckoned, the former on the ecliptic, the latter on the equinoctial. Suppose the sun has described 70° of longitude; then to ascertain the corresponding arc of right ascension, we let a meridian pass through the sun: the point where it cuts the equator gives the sun's right ascension. Now since the ecliptic makes an acute angle with the meridian, while the equinoctial makes a right angle with it, consequently the arc of longitude is greater than the arc of right ascension. The difference, however, grows constantly less and less as we approach the tropic, as the

angle made between the ecliptic and the meridian constantly increases, until, when we reach the tropic, the meridian is at right angles to both circles, and the longitude and right ascension each equals 90°, and they are of course equal to each other. Beyond this, from the tropic to the other equinox, the arc of the ecliptic intercepted between the meridian and the autumnal equinox being greater than the corresponding arc of the equinoctial, of course its supplement, which measures the longitude, is less than the supplement of the corresponding arc of the equator which measures the right ascension. At the autumnal equinox again, the right ascension and longitude become equal. In a similar manner we might show that the *daily increments* of longitude and right ascension are unequal.

In order to illustrate the foregoing points, let ♈♎ (Fig. 12,) represent the equator, ♈ T ♎ the ecliptic, and PSE, PS′E′, two meridians meeting the sun in S and S′. Then in the trian-

Fig. 12.



gle ♈ES, the arc of longitude ♈S, is greater than ♈E, the corresponding arc of right ascension; but towards the tropic the difference between the two arcs evidently grows less and less, until at T the arcs become equal, each being 90°. But, beyond the tropic, since ♈E′♎, ♈S′♎, are equal to each other, each being equal to 180°, and since S′♎ is greater than E′♎, therefore ♈S′ must be less than ♈E′.

107. As the *whole arc* of right ascension reckoned from the first of Aries, does not keep uniform pace with the corresponding

arc of longitude, so the *daily increments* of right ascension differ from those of longitude. If we suppose in the quadrant ♈T, points taken to mark the progress of the sun from day to day, and let meridians like PSE pass through these points, the arc of the ecliptic embraced between the meridians will be the daily increments of longitude, while the corresponding parts of the equinoctial will be the daily increments of right ascension. Near ♈, the oblique direction in which the ecliptic cuts the meridian, will make the daily increments of longitude exceed those of right ascension; but this advantage is diminished as we approach the tropic, where the ecliptic becomes less oblique, and finally parallel to the equinoctial; while the convergence of the meridians contributes still farther to lessen the ratios of the daily increments of longitude to those of right ascension. Hence, at first, the diurnal arcs of right ascension are less than those of longitude, but afterwards greater; and they continue greater for a similar distance beyond the tropic.

108. From the foregoing considerations it appears, that the diurnal arcs of right ascension, by which the difference between the sidereal and the solar days is measured, are unequal, on account both of the unequal motion of the sun in his orbit, and of the inclination of his orbit to the equinoctial.

109. As astronomical time commences when the *sun* is on the meridian, so sidereal time commences when the vernal equinox is on the meridian, and is also counted from 0 to 24 hours. By 3 o'clock, for instance, of sidereal time, we mean that it is three hours since the vernal equinox crossed the meridian; as we say it is 3 o'clock of astronomical or of civil time, when it is three hours since the sun crossed the meridian.

THE CALENDAR.

110. The *astronomical year* is the time in which the sun makes one revolution in the ecliptic, and consists of 365d. 5h. 48m. $51^{s}.60$. The *civil year* consists of 365 days. The difference is nearly 6 hours, making one day in four years.

111. The most ancient nations determined the number of days in the year by means of the *stylus*, a perpendicular rod which

cast its shadow on a smooth plane, bearing a meridian line. The time when the shadow was shortest, would indicate the day of the summer solstice ; and the number of days which elapsed until the shadow returned to the same length again, would show the number of days in the year. This was found to be 365 whole days, and accordingly this period was adopted for the civil year. Such a difference, however, between the civil and astronomical years, at length threw all dates into confusion. For, if at first the summer solstice happened on the 21st of June, at the end of four years, the sun would not have reached the solstice until the 22d of June, that is, it would have been behind its time. At the end of the next four years the solstice would fall on the 23d; and in process of time it would fall successively on every day of the year. The same would be true of any other fixed date. Julius Cæsar made the first correction of the calendar, by introducing an intercalary day every fourth year, making February to consist of 29 instead of 28 days, and of course the whole year to consist of 366 days. This fourth year was denominated *Bissextile.** It is also called Leap Year.

112. But the true correction was not 6 hours, but 5h. 49m. ; hence the intercalation was too great by 11 minutes. This small fraction would amount in 100 years to $\frac{3}{4}$ of a day, and in 1000 years to more than 7 days. From the year 325 to 1582, it had in fact amounted to about 10 days; for it was known that in 325, the vernal equinox fell on the 21st of March, whereas, in 1582 it fell on the 11th. In order to restore the equinox to the same date, Pope Gregory XIII decreed, that the year should be brought forward 10 days, by reckoning the 5th of October the 15th. In order to prevent the calendar from falling into confusion afterwards, the following rule was adopted :

Every year whose number is not divisible by 4 *without a remainder, consists of* 365 *days ; every year which is so divisible, but is not divisible by* 100, *of* 366 ; *every year divisible by* 100 *but not by* 400, *again of* 365 ; *and every year divisible by* 400, *of* 366.

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* The *sextus dies ante Kalendas* being reckoned twice, (Bis).

Thus the year 1838, not being divisible by 4, contains 365 days, while 1836 and 1840 are leap years. Yet to make every fourth year consist of 366 days would increase it too much by about $\frac{3}{4}$ of a day in 100 years; therefore every hundredth year has only 365 days. Thus 1800, although divisible by 4 was not a leap year, but a common year. But we have allowed a *whole* day in a hundred years, whereas we ought to have allowed only *three fourths* of a day. Hence, in 400 years we should allow a day too much, and therefore we let the 400th year remain a leap year. This rule involves an error of less than a day in 4237 years.* If the rule were extended by making every year divisible by 4,000 (which would now consist of 366 days) to consist of 365 days, the error would not be more than one day in 100,000 years.†

113. This reformation of the calendar was not adopted in England until 1752, by which time the error in the Julian calendar amounted to about 11 days. The year was brought forward, by reckoning the 3d of September the 14th. Previous to that time the year began the 25th of March; but it was now made to begin on the 1st of January, thus shortening the preceding year, 1751, one quarter.‡

114. As in the year 1582, the error in the Julian calendar amounted to 10 days, and increased by $\frac{3}{4}$ of a day in a century, at present the correction is 12 days; and the number of the year will differ by one with respect to dates between the 1st of January and the 25th of March.

Examples. General Washington was born Feb. 11, 1731, old style; to what date does this correspond in new style?

As the date is the earlier part of the 18th century, the correction is 11 days, which makes the birth day fall on the 22d of February; and since the year 1731 closed the 25th of March, while according to new style 1732 would have commenced on

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* Woodhouse, p. 874. † Herschel's Ast. p. 384.

‡ Russia, and the Greek Church generally, adhere to the old style. In order to make the Russian dates correspond to ours, we must add to them 12 days. France and other Catholic countries, adopted the Gregorian calendar soon after it was promulgated.

the preceding 1st of January; therefore, the time required is Feb. 22, 1732. It is usual, in such cases, to write both years, thus: Feb. 11, 1731–2, O. S.

2. A great eclipse of the sun happened May 15th, 1836; to what date would this time correspond in old style?

Ans. May 3d.

115. *The common year begins and ends on the same day of the week; but leap year ends one day later in the week than it began.*

For $52 \times 7 = 364$ days; if therefore the year begins on Tuesday, for example, 364 days would complete 52 weeks, and one day would be left to begin another week, and the following year would begin on Wednesday. Hence, any day of the month is one day later in the week than the corresponding day of the preceding year. Thus, if the 16th of November, 1838, falls on Friday, the 16th of November, 1837, fell on Thursday, and will fall in 1839 on Saturday. But if leap year begins on Sunday, it ends on Monday, and the following year begins on Tuesday; while any given day of the month is two days later in the week than the corresponding date of the preceding year.

116. Fortunately for astronomy, the confusion of dates involved in different calendars affects recorded observations but little. Remarkable eclipses, for example, can be calculated back for several thousand years, without any danger of mistaking the day of their occurrence; and whenever any such eclipse is so interwoven with the account given by an ancient author of some historical event, as to indicate precisely the interval of time between the eclipse and the event, and at the same time completely to identify the eclipse, that date is recovered and fixed forever.*

* An elaborate view of the Calendar may be found in Delambre's Astronomy, t. III. A useful table for finding the day of the week of any given date, is inserted in the American Almanac for 1832, p. 72.

CHAPTER V.

OF ASTRONOMICAL INSTRUMENTS AND PROBLEMS—FIGURE AND DENSITY OF THE EARTH.

117. The most ancient astronomers employed no instruments for measuring angles, but acquired their knowledge of the heavenly bodies by long continued and most attentive inspection with the naked eye. In the Alexandrian school, about 300 years before the Christian era, instruments began to be freely used, and thenceforward trigonometry lent a powerful aid to the science of astronomy. Tycho Brahe, in the 16th century, formed a new era in practical astronomy, and carried the measurement of angles to 10″,—a degree of accuracy truly wonderful, considering that he had not the advantage of the telescope. By the application of the telescope to astronomical instruments, a far better defined view of objects was acquired, and a far greater degree of refinement was attainable. The astronomers royal of Great Britain perfected the art of observation, bringing the measurement of angles to 1″, and the estimation of differences of time to $\frac{1}{5}$ of a second. Beyond this degree of refinement it is supposed that we cannot advance, since unavoidable errors arising from the uncertainties of refraction, and the necessary imperfection of instruments, forbid us to hope for a more accurate determination than this. But a little reflection will show us, that 1″ on the limb of an astronomical instrument, must be a space exceedingly small. Suppose the circle, on which the angle is measured, be one foot in diameter. Then $\frac{12 \times 3.14159}{360} = \frac{1}{10}$ inch = space occupied by 1°. Hence $\frac{1}{10 \times 60} = \frac{1}{600}$ = space of 1′, and $\frac{1}{36000}$ = space of 1″. Such minute angles can be measured only by large circles. If, for example, a circle is 20 feet in diameter, a degree on its periphery would occupy a space 20 times as large as a degree on a circle of 1 foot. A degree therefore of the limb of such an instrument would occupy a space of 2 inches: one minute, $\frac{1}{30}$ of an inch; and one second, $\frac{1}{1800}$ of an inch.

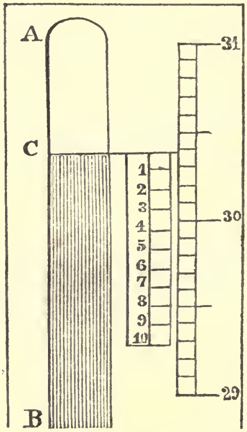
118. But the actual divisions on the limb of an astronomical instrument never extend to seconds: in the smaller instruments they reach only to 10′, and on the largest rarely lower than 1′. The subdivisions of these spaces is carried on by means of the Vernier, which may be thus defined:

A VERNIER *is a contrivance attached to the graduated limb of an instrument, for the purpose of measuring aliquot parts of the smallest spaces, into which the instrument is divided.*

The vernier is usually a narrow zone of metal, which is made to slide on the graduated limb. Its divisions correspond to those on the limb, except that they are a little larger,* one tenth, for example, so that ten divisions on the vernier would equal eleven on the limb. Suppose now that our instrument is graduated to degrees only, but the altitude of a certain star is found to be 40° and *a fraction*, or $40°+x$. In order to estimate the amount of this fraction, we bring the zero point of the vernier to coincide with the point which indicates the exact altitude, or $40°+x$. We then look along the vernier until we find where one of its divisions coincides with one of the divisions of the limb. Let this be at the fourth division of the vernier. In four divisions, therefore, the vernier has gained upon the divisions of the limb, a space equal to x; and since, in the case supposed, it gains $\frac{1}{10}$ of a degree, or 6′ at each division, the entire gain is 24′, and the arc in question is 40° 24′.

119. As the vernier is used in the barometer, where its application is more easily seen than in astronomical instruments, while the principle is the same in both cases, let us see how it is applied to measure the exact height of a column of mercury. Let AB (Fig. 13,) represent the upper part of a barometer, the level of the mercury being at C, namely, at 30.3 inches, and nearly another tenth. The vernier being brought (by a screw which is usually attached to it) to coincide with the surface of the mercury, we look along down the

Fig. 13.



* In the more modern instruments the divisions of the vernier are smaller than those of the limb.

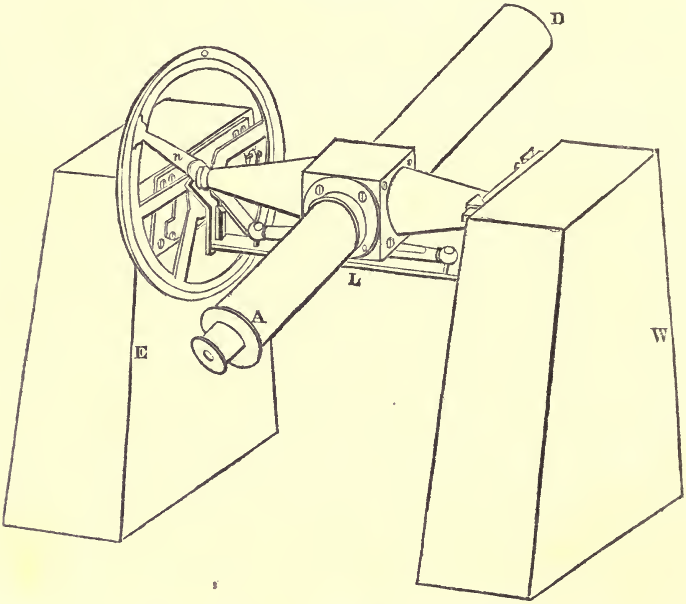
scale, until we find that the coincidence is at the 8th division of the vernier. Now as the vernier gains $\frac{1}{10}$ of $\frac{1}{10}=\frac{1}{100}$ of an inch at each division upward, it of course gains $\frac{8}{100}$ in eight divisions. The fractional quantity, therefore, is .08 of an inch, and the height of the mercury is 30.38. If the divisions of the vernier were such, that each gained $\frac{1}{60}$ (when 60 on the vernier would equal 61 on the limb) on a limb divided into degrees, we could at once take off minutes; and were the limb graduated to minutes, we could in a similar way read off seconds.

120. The instruments most used for astronomical observations, are the Transit Instrument, the Astronomical Clock, the Mural Circle, and the Sextant. A large portion of all the observations made in an astronomical observatory, are taken *on the meridian.* When a heavenly body is on the meridian, being at its highest point above the horizon, it is then least affected by refraction and parallax; its zenith distance (from which its altitude and declination are easily derived) is readily estimated; and its right ascension may be very conveniently and accurately determined by means of the astronomical clock. Having the right ascension and declination of a heavenly body, various other particulars respecting its position may be found, as we shall see hereafter, by the aid of spherical trigonometry. Let us then first turn our attention to the instruments employed for determining the right ascension and declination. They are the Transit Instrument, the Astronomical Clock, and the Mural Circle.

121. The *Transit Instrument* is a telescope, which is fixed permanently in the meridian, and moves only in that plane. It rests on a horizontal axis, which consists of two hollow cones applied base to base, a form uniting lightness and strength. The two ends of the axis rest on two firm supports, as pillars of stone, for example, so connected with the building as to be as free as possible from all agitation. In figure 14, AD represents the telescope, E, W, massive stone pillars supporting the horizontal axis, beneath which is seen a spirit level, (which is used to bring the axis to a horizontal position,) and n a vertical circle graduated into degrees and minutes. This circle serves the purpose of pla-

cing the instrument at any required altitude or distance from the zenith, and of course for determining altitudes and zenith distances.

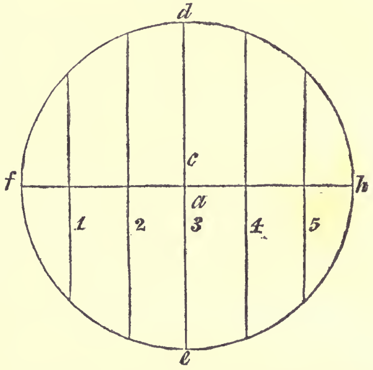
Fig. 14.



122. Various methods are described in works on practical astronomy, for placing the Transit Instrument accurately in the meridian. The following method by observations on the pole star, may serve as an example. If the instrument be directed towards the north star, and so adjusted that the star Alioth (the first in the tail of the Great Bear) and the pole star are both in the same vertical circle, the former below the pole and the latter above it, the instrument will be *nearly* in the plane of the meridian. To adjust it more exactly, compare the time occupied by the pole star in passing from its upper to its lower culmination, with the time of passing from its lower to its upper culmination. These two intervals ought to be precisely equal; and if they are so, the instrument is truly placed in the meridian; but if they are not equal, the position of the instrument must be shifted until they become exactly equal.

123. The *line of collimation* of a telescope, is a line joining the center of the object glass with the center of the eye glass. When the transit instrument is properly adjusted, this line, as the instrument is turned on its axis, moves in the plane of the meridian. Having, by means of the vertical circle n, set the instrument at the known altitude or zenith distance of any star, upon which we wish to make observations, we wait until the star enters the field of the telescope, and note the exact instant when it crosses the vertical wire in the center of the field, which wire marks the true plane of the meridian. Usually, however, there are placed in the focus of the eye glass five parallel wires or threads, two on each side of the central wire, and all at equal distances from each other, as is represented in the following diagram. The time of arriving at each of the wires being noted, and all the times added together and divided by the number of observations, the result gives the instant of crossing the central wire.

Fig. 15.



124. The *Astronomical Clock* is the constant companion of the Transit Instrument. This clock is so regulated as to keep exact pace with the stars, and of course with the revolution of the earth on its axis; that is, it is regulated to sidereal time. It measures the progress of a star, indicating an hour for every 15°, and 24 hours for the whole period of the revolution of the star. Sidereal time, it will be recollected, commences when the vernal equinox is on the meridian, just as solar time commences when the sun is on the meridian. Hence, the hour by the sidereal clock has no correspondence with the hour of the day, but simply indicates how long it is since the equinoctial point crossed the meridian. For example, the clock of an observatory points to 3h. 20m.; this may be in the morning, at noon, or any other time of the day, since it merely shows that it is 3h. 20m. since the equinox was on the meridian. Hence, when a star is on the meridian, the clock itself shows its right ascension; and the

interval of time between the arrival of any two stars upon the meridian, is the measure of their difference of right ascension.

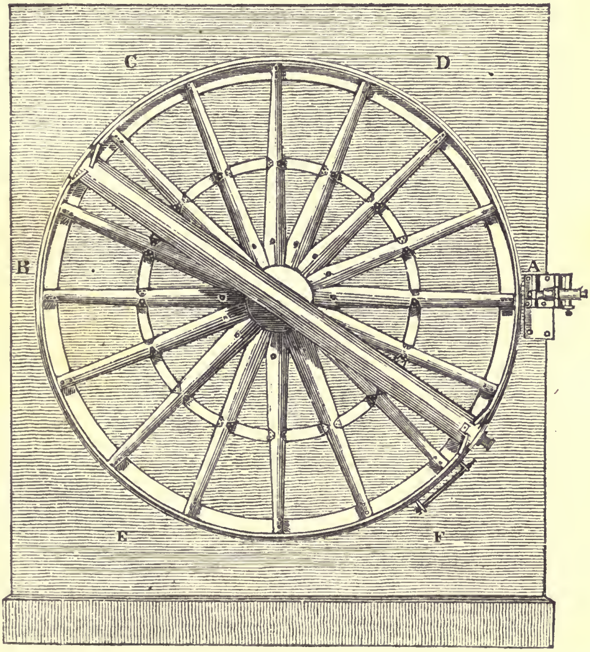
125. Astronomical clocks are made of the best workmanship, with a compensation pendulum, and every other advantage which can promote their regularity. The Transit Instrument itself, when once accurately placed in the meridian, affords the means of testing the correctness of the clock, since one revolution of a star from the meridian to the meridian again, ought to correspond to exactly 24 hours by the clock, and to continue the same from day to day; and the right ascension of various stars as they cross the meridian, ought to be such by the clock as they are given in the tables, where they are stated according to the most accurate determinations of astronomers. Or by taking the difference of right ascension of any two stars on successive days, it will be seen whether the going of the clock is uniform for that part of the day; and by taking the right ascension of different pairs of stars, we may learn the rate of the clock at various parts of the day. We thus learn, not only whether the clock accurately measures the length of the sidereal day, but also whether it goes uniformly from hour to hour.

Although astronomical clocks have been brought to a great degree of perfection, so as to vary hardly a second for many months, yet none are absolutely perfect, and most are so far from it as to require to be corrected by means of the Transit Instrument every few days. Indeed, for the nicest observations, it is usual not to attempt to bring the clock to an absolute state of correctness, but after bringing it as near to such a state as can conveniently be done, to ascertain how much it gains or loses in a day; that is, to ascertain its *rate* of going, and to make allowance accordingly.

126. The vertical circle (n, Fig. 14,) usually connected with the Transit Instrument, affords the means of measuring arcs on the meridian, as meridian altitudes, zenith distances, and declinations; but as the circle must necessarily be small, and therefore incapable of measuring very minute angles, the *Mural Circle* is usually employed for measuring arcs of the meridian.

The Mural Circle is a graduated circle, usually of very large size, fixed permanently in the plane of the meridian, and attached firmly to a perpendicular *wall.* It is made of large size, sometimes 20 feet in diameter, in order that very small angles may be measured on its limb; and it is attached to a massive wall of solid masonry in order to insure perfect steadiness, a point the more difficult to attain in proportion as the instrument is heavier. The annexed diagram represents a Mural Circle fixed to its wall and ready for observations. It will be seen that every expedient is employed to give the instrument firmness of parts

Fig. 16.



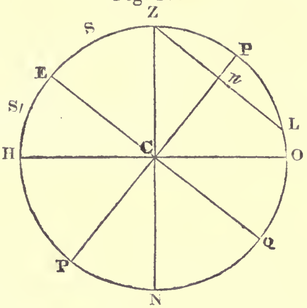
and steadiness of position. Its radii are composed of hollow cones, uniting lightness and strength, and its telescope revolves on a large horizontal axis, fixed as firmly as possible in a solid wall. The graduations are made on the outer rim of the instrument, and are read off by six microscopes attached to the wall, one of which is represented at A, and the places of the five others are marked by the letters B, C, D, E, F. Six are used in order that by taking the mean of such a number of readings, a higher degree of accuracy may be insured, than could be attained by a single reading. In a circle of six feet diameter, like that represented in the figure, the divisions may be easily carried to five minutes each. The microscope (which is of the variety called *compound microscope*) forms an enlarged image of each of these divisions in the focus of the eye glass. In the focus is also placed a delicate wire, which may be moved by means of a screw in a direction parallel to the divisions of the limb, and which is so adjusted to the screw as to move over the whole magnified space of five minutes by five revolutions of the screw. Of course one revolution of the screw measures one minute. Moreover, if the screw itself is made to carry an index attached to its axis and revolving with it over a disk graduated into sixty equal parts, then the space measured by moving the index over one of these parts, will be one second.

We have been thus minute in the description of this instrument, in order to give the learner some idea of the vast labor and great patience demanded of practical astronomers, in order to obtain measurements of such extreme accuracy as those to which they aspire.

On account of the great dimensions of this circle, and the expense attending it, as well as the difficulty of supporting it firmly, sometimes only one fourth of it is employed, constituting the *Mural Quadrant.* This instrument has the disadvantage, however, of being applicable to only one hemisphere at a time, either the northern or the southern, according as it is fixed to the eastern or the western side of the wall.

127. We have before shown (Art. 124,) the method of finding the right ascension of a star by means of the Transit Instrument and the clock. The declination may be obtained by means of the mural circle in several different ways, our object being always to find the distance of the star, when on the meridian, from the equator (Art. 37.) First, the declination may be found from the *meridian altitude.* Let S (Fig. 17,) be the place of a star when on the meridian. Then its meridian altitude will be SH, which

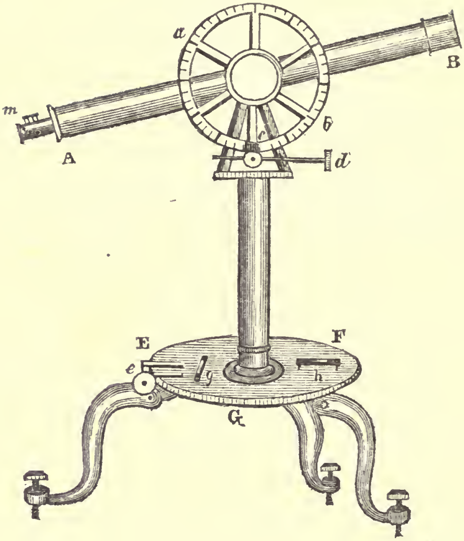
Fig. 17.



will best be found by taking its zenith distance ZS, of which it is the complement. From SH, subtract EH, the elevation of the equator, which equals the colatitude of the place of observation, (Art. 44,) and the remainder SE is the declination. Or if the star is nearer the horizon than the equator is, as at S′, subtract its meridian altitude from the colatitude, for the declination. Secondly, the declination may be found from the *north polar distance*, of which it is the complement. Thus from P to E is 90°. Therefore, PE−PS=90°−PS=SE=the declination. The height of the pole P is always known when the latitude of the place is known, being equal to the latitude.

128. The astronomical instruments already described are adapted to taking observations on the meridian only; but we sometimes require to know the *altitude* of a celestial body when it is not on the meridian, and its *azimuth*, or distance from the meridian measured on the horizon; and also the *angular distance* between two points on any part of the celestial sphere. An instrument especially designed to measure altitudes and azimuths, is called an *Altitude and Azimuth Instrument*, whatever may be its particular form. When a point is on the horizon its distance from the meridian, or its azimuth, may be taken by the common surveyor's compass, the direction of the meridian being determined by the needle; but when the object, as a star, is not on the horizon, its azimuth, it must be remembered, is the arc of the horizon from the meridian to a vertical circle passing through the star (Art. 27); at whatever different altitudes, therefore, two stars may be, and however the plane which passes through them may be inclined to the horizon, still it is their angular distance measured *on the horizon* which determines their difference of azimuth. Figure 18 represents an Altitude and Azimuth Instrument, several of the usual appendages and subordinate contrivances being omitted for the sake of distinctness and simplicity. Here *abc* is the vertical or altitude circle, and EFG the horizontal

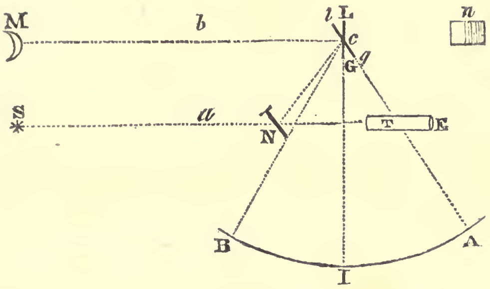
Fig. 18.



or azimuth circle ; AB is a telescope mounted on a horizontal axis and capable of two motions, one in altitude parallel to the circle *abc*, and the other in azimuth parallel to EFG. Hence it can be easily brought to bear upon any object. At *m*, under the eye glass of the telescope, is a small mirror placed at an angle of 45° with the axis of the telescope, by means of which the image of the object is reflected upwards, so as to be conveniently presented to the eye of the observer. At *d* is represented a *tangent screw*, by which a slow motion is given to the telescope at *c*. At *h* and *g* are seen two *spirit levels*, at right angles to each other, which show when the azimuth circle is truly horizontal. The instrument is supported on a *tripod*, for the sake of greater steadiness, each foot being furnished with a screw for levelling.

129. The *Sextant* is one of the most useful instruments, both to the astronomer and the navigator, and will therefore merit particular attention. In figure 19, ABL represents the plane of the instrument, LG and N, two small mirrors, and T, a small telescope. The line LGI represents a movable arm, or radius, which carries an index at I. The radius turns on a pivot in the center of LG,

Fig. 19.



and the index moves on a graduated arc BA. LG is called the *Index Glass*, and N the *Horizon Glass*. The under part only of the horizon glass is coated with quicksilver, the upper part being left transparent, as in *n*; so that while one object is seen through the upper part of *n* by direct vision, another may be seen through the lower part by reflexion from the two mirrors. The instrument is so contrived, that when the index is moved up to A, where the zero point is placed, or the graduation begins, the two reflectors LG and N are exactly parallel to each other, the index glass being then in the position *lg*. In this position of the mirrors, if the eye at E look through the telescope, T, so pointed as to see the star S through the transparent part of the horizon glass, it will see the same star, in the same place reflected from the silvered part; for the star (or any similar object) is at such a distance that the rays of light which strike upon the index glass LG are parallel to those which enter the eye directly. Therefore the angle of incidence *bc*N being equal to the angle of reflexion at *c*NE, the ray *b* will be made, by reflexion, to coincide with the ray *a*, and exhibit the object at the same place. Now, suppose it were the object to measure the angular distance between two bodies, as the moon and a star, and let the star be at S and the moon at M. The telescope being still directed to S, turn the index arm LI from A towards B until the image of the moon is brought down to S, its lower limb just touching S. By a principle in optics, the angular distance which the image of the moon passes over, is twice that of the mirror LG. But the mirror has passed over the graduated arc AI; therefore double that arc is the angular distance between the star and the moon's *lower* limb. If we then bring the *upper* limb into contact with the star, the sum of both observations, divided by 2, will give the angular distance between the star and the moon's *center*. As each degree on the limb AB measures two degrees of angular distance, hence the divisions for single degrees are in fact only half a degree asunder; and a sextant, or the sixth part of the circle, measures an angular distance of 120°. The upper and lower points in the disk of the sun or of the moon may be considered as two separate objects, whose distance from each other may be taken in a similar manner, and thus their apparent diameters at any time be determined. We may select our points of observation either in a vertical, or in a horizontal direction.

130. If we make a star, or the limb of the sun or moon, one of the objects, and the point in the horizon directly beneath, the other, we thus obtain the *altitude* of the object. In this observation, the horizon is viewed through the transparent part of the horizon glass. At sea, where the horizon is usually well defined, the horizon itself may be used for taking altitudes; but on land, inequalities of the earth's surface, oblige us to have recourse to an *artificial horizon*. This in its simple state, is a basin of either water or quicksilver. By this means we see the image of the sun (or other body) just as far below the horizon as it is in reality above it. Hence, if we turn the index glass until the limb of the sun, as reflected from it, is brought into contact with the image seen in the artificial horizon, we obtain double the altitude.*

The sextant must be held in such a manner, that its plane shall pass through the plane of the two objects. It must be held therefore in a vertical plane in taking altitudes, and in a horizontal plane in taking the horizontal diameters of the sun and moon. Holding the instrument in the true plane of the two bodies, whose angular distance is measured, is indeed the most difficult part of the operation.

The peculiar value of the sextant consists in this, that the observations taken with it are not affected by any motion in the

* Woodhouse's Ast. p. 774.

observer; hence it is the chief instrument used for angular measurements at sea.

131. *Examples illustrating the use of the Sextant.*

Ex. 1. Alt. ☉'s lower limb, . .	49° 10′ 00″
☉'s semi-diameter, . .	0 15 51
	49° 25′ 51″
Subtract Refraction, . .	00 00 49
	49° 25′ 02″
Add Parallax, . . .	00 00 06
True altitude ☉'s center, .	49° 25′ 08″

Ex. 2. *With the Artificial Horizon.*

Altitude of ☉'s upper limb above the image in the artificial horizon, 100° 2′ 47″.

True altitude, ...	50° 01′ 23.″5
☉'s semi-diameter, . . .	00 15 50.
	49° 45′ 33.″5
Refraction, ...	00 00 48.
	49° 44′ 45.″5
Parallax, ...	00 00 05.
True altitude of ☉'s center, . .	49° 44′ 50.″5

ASTRONOMICAL PROBLEMS.*

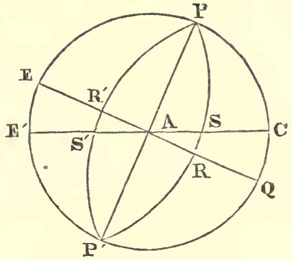
132. *Given the sun's Right Ascension and Declination, to find his Longitude and the Obliquity of the Ecliptic.*

Let PCP′ (Fig. 20,) represent the celestial meridian that passes through the first of Cancer and Capricorn, (the solstitial colure,) PP′ the axis of the sphere, EQ the equator, E′C the ecliptic, and PSP′ the declination circle (Art. 37,) passing through the sun S; then ARS is a right angle, and in the right angled spherical

* Young's Spherical Trigonometry, p. 136. Vince's Complete System, Vol. I.

triangle ARS, are given the right ascension AR (Art. 37,) and the declination RS, to find the longitude AS (Art. 37,) and the obliquity SAR.

Fig. 20.



As longitude and right ascension are measured from A, the first point of Aries, in the direction AS of the signs, quite round the globe, when, of the four quantities mentioned in the problem, the obliquity and the declination are given to find the others, we must know whether the sun is north, or whether it is south of the equator, the longitude being in the one case AS, and in the other, instead of AS′, it is 360 – AS′, that is, the supplement of AS′. We must also know on which side of the tropic the sun is, for the sun in passing from one of the tropics to the equinox, passes through the same degrees of declination as it had gone through in ascending from the other equinox to the tropic, although its longitude and right ascension go on continually increasing. From the 21st of March to the 21st of June, while describing the first quadrant from the vernal equinox, the declination is north and increasing; north but decreasing, in the second quadrant, until the 23d of September; south and increasing in the third quadrant, until the 21st of December; and finally, in the fourth quadrant, south but decreasing until the 21st of March.

Ex. 1. On the 17th of May, the sun's Right Ascension was 53° 38′, and his Declination 19° 15′ 57″: required his Longitude and the Obliquity of the Ecliptic.

Applying *Napier's rule** to the right angled triangle, ARS, we have

1. Rad. cos. AS = cos. AR cos. RS.

2. Rad. sin. AR = tan. RS cot. A.·.cot. A $= \dfrac{\text{rad. sin. AR}}{\text{tan. RS}}$.

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* The student is supposed to be acquainted with Spherical Trigonometry; but to refresh his memory, we may insert a remark or two.

It will be recollected that in Napier's rule for the solution of a right angled spherical triangle, by means of the Five Circular Parts, we proceed as follows.

In a right angled spherical triangle we are to recognize but five parts, viz. the three sides and the two oblique angles. If we take any one of these as a *middle*

Hence the computation for AS and A is as follows:

For the Longitude AS.		*For the Obliquity* A.	
cos. AR 53° 38′ 00″	9.7730185	sin. AR	9.9059247
cos. RS 19 15 57	9.9749710	tan. RS, ar. com.	0.4565209
cos. AS 55 57 43	9.7479895	cot. A 23° 27′ 50½″	10.3624456

Ex. 2. On the 31st of March, 1816, the sun's Declination was observed at Greenwich to be 4° 13′ 31½″: required his Right Ascension, the obliquity of the ecliptic being 23° 27′ 51″.

Ans. 9° 47′ 59″.

Ex. 3. What was the sun's Longitude on the 28th of November, 1810, when his Declination was 21° 16′ 4″, and his Right Ascension, in time, 16h. 14m. 58.4s.?

Ans. 245° 39′ 10″.

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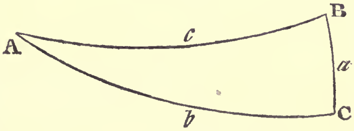
part, the two which lie next to it on each side will be *adjacent parts*. Thus, (in Fig. 21,) taking A for a middle part, b and c will be the adjacent parts; if we take c for the middle part, A and B will be the adjacent parts; if we take B for the middle part, c and a will be the adjacent parts; but if we take a for the middle part, then as the angle C is not considered as one of the circular parts, B and b are the adjacent parts; and, lastly, if b is the middle part, then the adjacent parts are A and a. The two parts immediately beyond the adjacent parts on each side, still disregarding the right angle, are called the *opposite parts;* thus if A is the middle part, the opposite parts are a and B. Napier's rule is as follows:

Fig. 21.



Radius into the sine of the middle part, equals the product of the tangents of the adjacent extremes, or of the cosines of the opposite extremes.

(The corresponding vowels are marked to aid the memory.) This rule is modified by using the *complements* of the two angles and the hypothenuse instead of the parts themselves. Thus instead of rad.$\times$sin. A, we say rad.$\times$cos. A, when A is the middle part; and rad.$\times$cos. AB, when the hypothenuse is the middle part.

Examples. 1. In the right angled triangle ABC, are given the two perpendicular sides, viz. a=48° 24′ 10″, b=59° 38′ 27″, to find the hypothenuse c. The hypothenuse being made the middle part, the other sides become the opposite parts, being separated from the middle part by the angles A and B. Hence, rad. cos. c= cos. a cos. b $\therefore$ cos. $c=\frac{\text{cos. } a \text{ cos. } b}{\text{rad.}}$=70° 26′ 29″.

2. In the spherical triangle, right angled at C, are given two perpendicular sides, viz. a=116° 30′ 43″, b=29° 41′ 32″, to find the angle A.

Here, the required angle is *adjacent* to one of the given parts, viz. b, which make the middle part. Then,

$$\text{Rad.}\times\text{sin. } b=\text{cot. A tan. } a \therefore \text{cot. A}=\frac{\text{rad.}\times\text{sin. } b}{\text{tan. } a}=76^\circ\ 7'\ 14''.$$

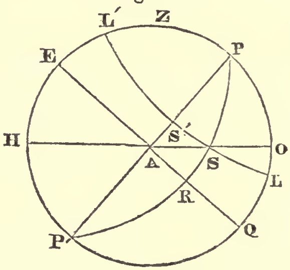
Ex. 4. The sun's Longitude being 8s. 7° 40′ 56″, and the Obliquity 23° 27′ 42¼″, what was the Right Ascension in time?

Ans. 16h. 23m. 34s.

133. *Given the sun's Declination to find the time of his Rising and Setting at any place whose latitude is known.*

Let PEP′ (Fig. 22,) represent the meridian of the place, Z being the zenith, and HO the horizon; and let LL′ be the apparent path of the sun on the proposed day, cutting the horizon in S. Then the arc EZ will be the latitude of the place, and consequently EH, or its equal QO, will be the co-latitude, and this measures the angle OAQ; also RS will be the sun's declination, and AR expressed in time will be the time of rising before 6 o'clock. For it is evident that it will be sunrise when the sun arrives at the horizon at S; but PP′ being an hour circle whose plane is perpendicular to the meridian, (and of course projected into a straight line on the plane of projection,) the time the sun is passing from S to S′ taken from the time of describing S′L, which is six hours, must be the time from midnight to sunrise. But the time of describing SS′ is measured on the corresponding arc of the equinoctial AR.

Fig. 22.



In the right angled triangle ARS, we have the declination RS, and the angle A to find AR. Therefore,

$$\text{Rad.} \times \text{sin. AR} = \text{cot. A} \times \text{tan. RS}.$$

Ex. 1. Required the time of sunrise at latitude 52° 13′ N. when the sun's declination is 23° 28′.

Rad.		10.
Cot. A or tan.	52° 13′	10.1105786
Tan. RS =	23° 28′	9.6376106
Sin.	34° 03′ 21¼″	9.7481892
	4*	
	2h. 16′ 13″ 25‴	
	6	

3h. 43′ 46″ 35‴ = the time after midnight, and of course the time of rising.

* Degrees are converted into hours by multiplying by 4 and dividing by 60.

Ex. 2. Required the time of sunrise at latitude 57° 2′ 54″ N. when the sun's declination is 23° 28′.

Ans. 3h. 11m. 49s.

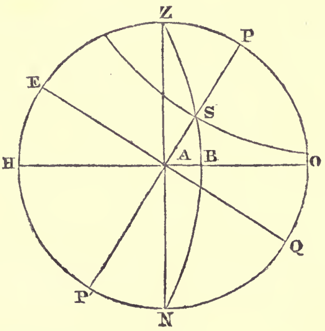
Ex. 3. How long is the sun above the horizon in latitude 58° 12′ N. when his declination is 18° 40′ S. ?

Ans. 7h. 35m. 52s.

134. *Given the Latitude of the place, and the Declination of a heavenly body, to determine its Altitude and Azimuth when on the six o'clock hour circle.*

Let HZO (Fig. 23,) be the meridian of the place, Z the zenith, HO the horizon, S the place of the object on the 6 o'clock hour circle PSP′, which of course cuts the equator in the east and west points, and ZSB the vertical circle passing through the sun. Then in the right angled triangle SBA, the given quantities are AS, which is the declination, and the arc OP or angle SAB, the latitude of the place, to find the altitude BS, and the azimuth BO, or the amplitude AB, which is its complement.

Fig. 23.



Ex. 1. What was the altitude and azimuth of Arcturus, when upon the six o'clock hour circle of Greenwich, lat. 51° 28′ 40″ N. on the first of April, 1822; its declination being 20° 6′ 50″ N. ?

For the Altitude.		*For the Azimuth.*	
Rad. sin. BS = sin. AS sin. A		Rad. cos. A = cot. BO cot. AS	
Rad. . .	10.	Cot. 20° 06′ 50″	10.4362545
Sin. 20° 06′ 50″	9.5364162	Cos. 51 28 40	9.7943612
Sin. 51 28 40	9.8934103	Rad. . .	10.
Sin. 15 36 27	9.4298265	Cot. 77° 09′ 04″	9.3581067

Ex. 2. At latitude 62° 12′ N. the altitude of the sun at 6 o'clock in the morning was found to be 18° 20′ 23″: required his declination and azimuth.

Ans. Dec. 20° 50′ 12″ N. Az. 79° 56′ 11″.

135. *The Latitudes and Longitudes of two celestial objects being given, to find their Distance apart.*

Let P (Fig. 24,) represent the pole of the ecliptic, and PS, PS′, two arcs of celestial latitude (Art. 37,) drawn to the two objects SS′; then will these arcs represent the co-latitudes, the angle P will be the difference of longitude, and the arc SS′ will be the distance sought. Here we have the two sides and included angle given to find the third side. By Napier's Rules for the solution of oblique angled spherical triangles, (see Spherical Trigonometry,) the sum and difference of the two angles opposite the given sides may be found, and thence the angles themselves. The required side may then be found by the theorem, that the sines of the sides are as the sines of their opposite angles.* The computation is omitted here on account of its great length. If P be the pole of the *equator* instead of the ecliptic, then PS and PS′ will represent arcs of co-declination, and the angle P will denote difference of right ascension. From these data, also, we may therefore derive the distance between any two stars. Or, finally, if P be the pole of the *horizon*, the angle at P will denote difference of azimuth, and the sides PS, PS′, zenith distances, from which the side SS′ may likewise be determined.

Fig. 24.

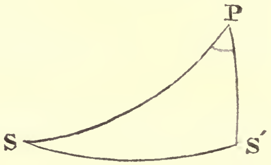


FIGURE AND DENSITY OF THE EARTH.

136. We have already shown, (Art. 8,) that the figure of the earth is *nearly* globular; but since the semi-diameter of the earth is taken as the base line in determining the parallax of the heavenly bodies, and lies therefore at the foundation of all astronomical measurements, it is very important that it should be ascertained with the greatest possible exactness. Having now learned the use of astronomical instruments, and the method of measuring arcs on the celestial sphere, we are prepared to understand the methods employed to determine the exact figure of the earth. This element is indeed ascertained in four different ways, each of which is independent of all the rest, namely, by investigating the effects of the *centrifugal force* arising from the revolution of

* More concise formulæ for the solution of this case may be found in Young's Trigonometry, p. 99.—Francoeur's Uranography, Art. 330.—Dr. Bowditch's Practical Navigator, p. 436.

the earth on its axis—by measuring *arcs of the meridian*—by experiments with the *pendulum*—and by the unequal action of the earth on the *moon*, arising from the redundance of matter about the equatorial regions. We will briefly consider each of these methods.

137. First, *the known effects of the centrifugal force, would give to the earth a spheroidal figure, elevated in the equatorial, and flattened in the polar regions.*

Had the earth been originally constituted (as geologists suppose) of yielding materials, either fluid or semi-fluid, so that its particles could obey their mutual attraction, while the body remained at rest it would spontaneously assume the figure of a perfect sphere ; as soon, however, as it began to revolve on its axis, the greater velocity of the equatorial regions would give to them a greater centrifugal force, and cause the body to swell out into the form of an oblate spheroid. Even had the solid part of the earth consisted of unyielding materials and been created a perfect sphere, still the waters that covered it would have receded from the polar and have been accumulated in the equatorial regions, leaving bare extensive regions on the one side, and ascending to a mountainous elevation on the other.

On estimating, from the known dimensions of the earth and the velocity of its rotation, the amount of the centrifugal force in different latitudes, and the figure of equilibrium which would result, Newton inferred that the earth must have the form of an oblate spheroid before the fact had been established by observation ; and he assigned nearly the true ratio of the polar and equatorial diameters.

138. Secondly, *the spheroidal figure of the earth is proved, by actually measuring the length of a degree on the meridian in different latitudes.*

Were the earth a perfect sphere, the section of it made by a plane passing through its center in any direction would be a perfect circle, whose curvature would be equal in all parts ; but if we find by actual observation, that the curvature of the section is not uniform, we infer a corresponding departure in the earth from the figure of a perfect sphere. This task of measuring portions of the meridian, has been executed in different countries by means of a system of triangles with astonishing accuracy.* The result is, that the length of a degree increases as we proceed from the equator towards the pole, as may be seen from the following table:

Places of observation.	Latitude.	Length of a degree in miles.
Peru,	00° 00′ 00″	68.732
Pennsylvania,	39 12 00	68.896
Italy,	43 01 00	68.998
France,	46 12 00	69.054
England,	51 29 54½	69.146
Sweden,	66 20 10	69.292

Combining the results of various estimates, the dimensions of the terrestrial spheroid are found to be as follows:

Equatorial diameter, . . . 7925.648

Polar diameter, ... 7899.170

Mean diameter, ... 7912.409

The difference between the greatest and least, is $26.478 = \frac{1}{299}$ of the greatest. This fraction $(\frac{1}{299})$ is denominated the *ellipticity* of the earth, being the excess of the transverse over the conjugate axis, on the supposition that the section of the earth coinciding with the meridian, is an ellipse; and that such is the case, is proved by the fact that calculations on this hypothesis, of the lengths of arcs of the meridian in different latitudes, agree with the lengths obtained by actual measurement.

139. Thirdly, *the figure of the earth is shown to be spheroidal, by observations with the pendulum.*

The use of the pendulum in determining the figure of the earth, is founded upon the principle that *the number of vibrations performed by the same pendulum, when acted on by different forces, varies as the square root of the forces.*† Hence, by carrying a pendulum to different parts of the earth, and counting the number of vibrations it performs in a given time, we obtain the relative forces of gravity at those places, and this leads to a knowledge of the relative distance of each place from the center of the earth, and finally, to the ratio between the equatorial and the polar diameters.

* See Day's Trigonometry.

† Mechanics, Art. 183.

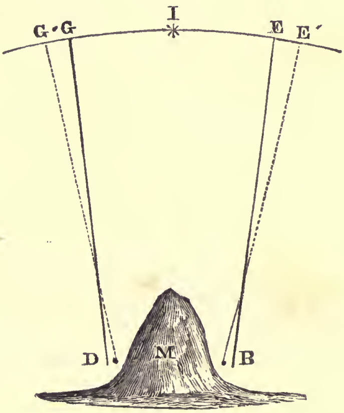
140. Fourthly, *that the earth is of a spheroidal figure, is inferred from the motions of the moon.*

These are found to be affected by the excess of matter about the equatorial regions, producing certain irregularities in the lunar motions, the amount of which becomes a measure of the excess itself, and hence affords the means of determining the earth's ellipticity. This calculation has been made by the most profound mathematicians, and the figure deduced from this source corresponds very nearly to that derived from the several other independent methods.

We thus have the shape of the earth established upon the most satisfactory evidence, and are furnished with a starting point from which to determine various measurements among the heavenly bodies.

141. The *density* of the earth compared with water, that is, its specific gravity, is $5\frac{1}{2}$.* The density was first estimated by Dr. Hutton, from observations made by Dr. Maskelyne, Astronomer Royal, on Schehallien, a mountain of Scotland, in the year 1774. Thus, let M (Fig. 25,) represent the mountain, D, B, two stations on opposite sides of the mountain, and I a star; and let IE and IG be the zenith distances as determined by the differences of latitudes of the two stations. But the apparent zenith distances as determined by the plumb line are IE′ and IG′. The deviation towards the mountain on each side exceeded 7″.† The attraction of the mountain being observed on both sides of it, and its mass being computed from a number of sections taken in all directions, these data, when compared with the known attraction and magnitude of the earth, led to a knowledge of its mean density. According to Dr. Hutton, this is to that of water as 9 to 2;

Fig. 25.



* Bailly, Ast. Tables, p. 21.

† Robison's Phys. Ast.

but later and more accurate estimates have made the specific gravity of the earth as stated above. But this density is nearly double the average density of the materials that compose the exterior crust of the earth, showing a great increase of density towards the center.

The density of the earth is an important element, as we shall find that it helps us to a knowledge of the density of each of the other members of the solar system.

PART II.—OF THE SOLAR SYSTEM.

142. Having considered the Earth, in its astronomical relations, and the Doctrine of the Sphere, we proceed now to a survey of the Solar System, and shall treat successively of the Sun, Moon, Planets, and Comets.

CHAPTER I.

OF THE SUN—SOLAR SPOTS—ZODIACAL LIGHT.

143. The *figure* which the sun presents to us is that of a perfect circle, whereas most of the planets exhibit a disk more or less elliptical, indicating that the true shape of the body is an oblate spheroid. So great, however, is the distance of the sun, that a line 400 miles long would subtend an angle of only 1″ at the eye, and would therefore be the least space that could be measured. Hence, were the difference between two conjugate diameters of the sun any quantity less than this, we could not determine by actual measurement that it existed at all. Still we learn from theoretical considerations, founded upon the known effects of centrifugal force, arising from the sun's revolution on his axis, that his figure is not a perfect sphere, but is slightly spheroidal.*

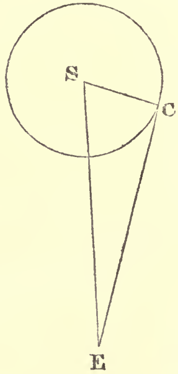
144. The *distance of the sun from the earth*, is nearly 95,000,000 miles. For, its horizontal parallax being 8.″6, (Art. 86,) and the semi-diameter of the earth 3956 miles,

Sin. 8.″6 : 3956 : : Rad. : 95,000,000 nearly. In order to form some faint conception at least of this vast distance, let us reflect that a railway car, moving at the rate of 20 miles per hour, would require more than 500 years to reach the sun.

* See Mecanique Celeste, III, 165. Delambre, t. I, p. 483.

145. The apparent *diameter* of the sun may be found either by the Sextant, (Art. 129,) by an instrument called the *Heliometer*, specially designed for measuring its angular breadth, or by the time it occupies in crossing the meridian. If, for example, it occupied 4^m, its angular diameter would be 1°. It in fact occupies a little more than 2^m, and hence its apparent diameter is a little more than half a degree, ($32'\ 3''$). Having the distance and angular diameter, we can easily find its *linear* diameter. Let E (Fig. 26,) be the earth, S the sun, ES a line drawn to the center of the disk, and EC a line drawn touching the disk at C. Join SC; then

Fig. 26.



Rad. : ES (95,000,000) : : sin. $16'\ 1''.5$: 442840 = semi-diameter, and 885680 = diameter. And $\frac{885680}{7912} = 112$ nearly; that is, it would require one hundred and twelve bodies like the earth, if laid side by side, to reach across the diameter of the sun; and a ship sailing at the rate of ten knots an hour, would require more than ten years to sail across the solar disk. Since spheres are to each other as the cubes of their diameters,

$1^3 : 112^3 :: 1 : 1{,}400{,}000$ nearly; that is, the sun is about 1,400,000 times as large as the earth. The distance of the moon from the earth being 237,000 miles, were the center of the sun made to coincide with the center of the earth, the sun would extend every way from the earth more than twice as far as the moon.

146. In *density*, the sun is only one fourth that of the earth, being but a little heavier than water (Art. 141); and since the quantity of matter, or *mass* of a body, is proportioned to its magnitude and density, hence, $1{,}400{,}000 \times \frac{1}{4} = 350{,}000$, that is, the quantity of matter in the sun is three hundred and fifty thousand (or, more accurately, 354,936) times as great as in the earth. Now the weight of bodies (which is a measure of the force of gravity) varies directly as the quantity of matter, and inversely as the square of the distance. A body, therefore, would weigh 350,000 times as much on the surface of the sun as on the earth, if the

distance of the center of force were the same in both cases; but since the attraction of a sphere is the same as though all the matter were collected in the center, consequently, the weight of a body, so far as it depends on its distance from the center of force, would be the square of 112 times less at the sun than at the earth. Or, putting W for the weight at the earth, and W′ for the weight at the sun, then

$$W : W' :: \frac{1}{1^2} : \frac{350000}{(112)^2} = 27.9 \text{ lbs.}$$

Hence a body would weigh nearly 28 times as much at the sun as at the earth. A man weighing 200 lbs. would, if transported to the surface of the sun, weigh 5,580 lbs., or nearly $2\frac{1}{2}$ tons. To lift one's limbs, would, in such a case, be beyond the ordinary power of the muscles. At the surface of the earth, a body falls through $16\frac{1}{12}$ feet in a second; and since the spaces are as the velocities, the times being equal, and the velocities as the forces, therefore a body would fall at the sun in one second, through $16\frac{1}{12} \times 27\frac{9}{10} = 448.7$ feet.

SOLAR SPOTS.

147. The surface of the sun, when viewed with a telescope, usually exhibits dark spots, which vary much, at different times, in number, figure, and extent. One hundred or more, assembled in several distinct groups, are sometimes visible at once on the solar disk. The greatest part of the solar spots are commonly very small, but occasionally a spot of enormous size is seen occupying an extent of 50,000 miles in diameter. They are sometimes even visible to the naked eye, when the sun is viewed through colored glass, or, when near the horizon, it is seen through light clouds or vapors. When it is recollected that 1″ of the solar disk implies an extent of 400 miles, (Art. 143,) it is evident that a space large enough to be seen by the naked eye, must cover a very large extent.

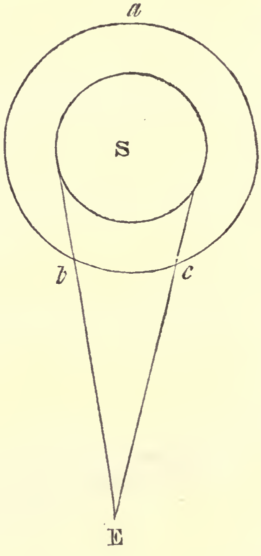
A solar spot usually consists of two parts, the *nucleus* and the *umbra*, (Fig. 27.) The nucleus is black, of a very irregular shape, and is subject to great and sudden changes, both in form and size. Spots have sometimes seemed to burst asunder, and to project fragments in different directions. The umbra is a wide margin of lighter

Fig. 27.

shade, and is often of greater extent than the nucleus. The spots are usually confined to a zone extending across the central regions of the sun, not exceeding 60° in breadth. When the spots are observed from day to day, they are seen *to move across the disk of the sun*, occupying about two weeks in passing from one limb to the other. After an absence of about the same period, the spot returns, having taken 27d. 7h. 37m. in the entire revolution.

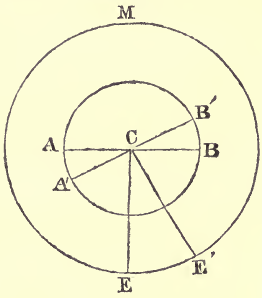
148. The spots must be nearly or quite *in contact with the body of the sun.* Were they at any considerable distance from it, the time during which they would be seen on the solar disk, would be less than that occupied in the remainder of the revolution. Thus, let S (Fig. 28,) be the sun, E the earth, and *abc* the path of the body, revolving about the sun. Unless the spot were nearly or quite in contact with the body of the sun, being projected upon his disk only while passing from *b* to *c*, and being invisible while describing the arc *cab*, it would of course be out of sight longer than in sight, whereas the two periods are found to be equal. Moreover, the lines which all the solar spots describe on the disk of the sun, are found to be parallel to each other, like the circles of diurnal revolution around the earth, and hence it is inferred that they arise from a similar cause, namely, *the revolution of the sun on his axis*, a fact which is thus made known to us.

Fig. 28.



But although the spots occupy about 27¼ days in passing from one limb of the sun around to the same limb again, yet this is not the period of the sun's revolution on his axis, but exceeds it by nearly two days. For, let AA′B (Fig. 29,) represent the sun, and EE′M the orbit of the earth. Thus, when the earth is at E, the visible disk of the sun will be AA′B; and if the earth remained stationary at E, the time occupied by a spot after leaving A until it returned to A, would be just equal to the time of the sun's revolution on his axis. But during the 27½ days in which the spot has been performing its apparent revolution, the earth has been advancing in his orbit from E to E′, where the visible disk of the sun is A′B′. Consequently, before the spot can appear again on the limb from which it set out, it must describe so much more than an entire revolution as equals the arc AA′, which equals the arc EE′. Hence,

Fig. 29.



365d. 5h. 48m.+27d. 7h. 37m. : 365d. 5h. 48m. : : 27d. 7h. 37m. : 25d. 9h. 56m. = the time of the sun's revolution on his axis.

149. If the path which the spots appear to describe by the revolution of the sun on his axis left each a visible trace on his surface, they would form, like the circles of diurnal revolution on the earth, so many parallel rings, of which that which passed through the center would constitute the solar equator, while those on each side of this great circle would be small circles, corresponding to parallels of latitude on the earth. Let us conceive of an artificial sphere to represent the sun, having such rings plainly marked on its surface. Let this sphere be placed at some distance from the eye, with its axis perpendicular to the axis of vision, in which case the equator would coincide with the line of vision, and its edge be presented to the eye. It would therefore be projected into a straight line. The same would be the case with all the smaller rings, the distance being supposed such that the rays of light come from them all to the eye nearly parallel. Now let the axis, instead of being perpendicular to the line of vision, be inclined to that line, then all the rings being seen obliquely would be projected into ellipses. If, however, while the sphere remained in a fixed position, the eye were carried around it,

(being always in the same plane,) twice during the circuit it would be in the plane of the equator, and project this and all the smaller circles into straight lines; and twice, at points 90° distant from the foregoing positions, the eye would be at a distance from the planes of the rings equal to the inclination of the equator of the sphere to the line of vision. Here it would project the rings into wider ellipses than at other points; and the ellipses would become more and more acute as the eye departed from either of these points, until they vanished again into straight lines.

150. It is in a similar manner that the eye views the paths described by the spots on the sun. If the sun revolved on an axis perpendicular to the plane of the earth's orbit, the eye being situated in the plane of revolution, and at such a distance from the sun that the light comes to the eye from all parts of the solar disk nearly parallel, the paths described by the spots would be projected into straight lines, and each would describe a straight line across the solar disk, parallel to the axis of vision. But the axis of the sun is inclined to the ecliptic about $7\frac{1}{4}°$ from a perpendicular, so that usually all the circles described by the spots are projected into ellipses. The breadth of these, however, will vary as the eye, in the annual revolution, is carried around the sun, and when the eye comes into the plane of the rings, as it does twice a year, they are projected into straight lines, and for a short time a spot seems moving in a straight line inclined to the plane of vision $7\frac{1}{4}°$. The two points where the sun's equator cuts the ecliptic are called the sun's *nodes.* The longitudes of the nodes are 80° 7′ and 260° 7′, and the earth passes through them about the 12th of December, and the 11th of June. It is at these times that the spots appear to describe straight lines. We have mentioned the various changes in the apparent paths of the solar spots, which arise from the inclination of the sun's axis to the plane of the ecliptic; but it was in fact by first observing these changes, and proceeding in the reverse order from that which we have pursued, that astronomers ascertained that the sun revolves on his axis, and that this axis is inclined to the ecliptic $82\frac{3}{4}°$.

151. With regard to the *cause* of the solar spots, various hypotheses have been proposed, none of which is entirely satisfactory. That which ascribes their origin to *volcanic action*, appears to us the most reasonable.*

Besides the dark spots on the sun, there are also seen, in different parts, places that are brighter than the neighboring portions of the disk. These are called *faculæ*. Other inequalities are observable in powerful telescopes, all indicating that the surface of the sun is in a state of constant and powerful agitation.

ZODIACAL LIGHT.

152. The Zodiacal Light is a faint light resembling the tail of a comet, and is seen at certain seasons of the year following the course of the sun after evening twilight, or preceding his approach in the morning sky. Figure 30 represents its appearance as seen in the evening in March, 1836. The following are the leading facts respecting it.

Fig. 30.

1. *Its form is that of a luminous pyramid*, having its base towards the sun. It reaches to an immense distance from the sun, sometimes even beyond the orbit of the earth. It is brighter in the parts nearer the sun than in those that are more remote, and terminates in an obtuse apex, its light fading away by insensible gradations, until it becomes too feeble for distinct vision. Hence its limits are at the same time, fixed at different distances from the sun by different observers, according to their respective powers of vision.

2. *Its aspects vary very much with the different seasons of the year*. About the first of October, in our climate (Lat. 41° 18′) it becomes visible before the dawn of day, rising along north of

* In the system of instruction in Yale College, subjects of this kind are discussed in a course of astronomical lectures, addressed to the class after they have finished the perusal of the text-book.

the ecliptic, and terminating above the nebula of Cancer. About the middle of November, its vertex is in the constellation Leo. At this time no traces of it are seen in the west after sunset, but about the first of December it becomes faintly visible in the west, crossing the Milky Way near the horizon, and reaching from the sun to the head of Capricornus, forming, as its brightness increases, a counterpart to the Milky Way, between which on the right, and the Zodiacal Light on the left, lies a triangular space embracing the Dolphin. Through the month of December, the Zodiacal Light is seen on both sides of the sun, namely, before the morning and after the evening twilight, sometimes extending 50° westward, and 70° eastward of the sun at the same time. After it begins to appear in the western sky, it increases rapidly from night to night, both in length and brightness, and withdraws itself from the morning sky, where it is scarcely seen after the month of December, until the next October.

3. *The Zodiacal Light moves through the heavens in the order of the signs.* It moves with unequal velocity, being sometimes stationary and sometimes retrograde, while at other times it advances much faster than the sun. In February and March, it is very conspicuous in the west, reaching to the Pleiades and beyond; but in April it becomes more faint, and nearly or quite disappears during the month of May. It is scarcely seen in this latitude during the summer months.

4. *It is remarkably conspicuous at certain periods of a few years, and then for a long interval almost disappears.*

5. *The Zodiacal Light was formerly held to be the atmosphere of the sun.** But La Place has shown that the solar atmosphere could never reach so far from the sun as this light is seen to extend.† It has been supposed by others to be a nebulous body revolving around the sun. The idea has been suggested, that the extraordinary *Meteoric Showers*, which at different periods visit the earth, especially in the month of November, may be derived from this body.‡

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* Mairan, Memoirs French Academy, for 1733. † Mec. Celeste, III, 525.

‡ The "origin of Meteoric Showers" is deemed to be a subject not yet sufficiently matured to have place in an elementary work like the present; but the author begs leave to refer the reader to some observations of his on this subject in different volumes of the American Journal of Science.

CHAPTER II.

OF THE APPARENT ANNUAL MOTION OF THE SUN—SEASONS—FIGURE OF THE EARTH'S ORBIT.

153. The revolution of the earth around the sun once a year, produces an apparent motion of the sun around the earth in the same period. When bodies are at such a distance from each other as the earth and the sun, a spectator on either would project the other body upon the concave sphere of the heavens, always seeing it on the opposite side of a great circle, 180° from himself. Thus when the earth arrives at Libra (Fig. 11,) we see the sun in the opposite sign Aries. When the earth moves from Libra to Scorpio, as we are unconscious of our own motion, the sun it is that appears to move from Aries to Taurus, being always seen in the heavens, where a line drawn from the eye of the spectator through the body meets the concave sphere of the heavens. Hence the line of projection carries the sun forward on one side of the ecliptic, at the same rate as the earth moves on the opposite side; and therefore, although we are unconscious of our own motion, we can read it from day to day in the motions of the sun. If we could see the stars at the same time with the sun, we could actually observe from day to day the sun's progress through them, as we observe the progress of the moon at night; only the sun's rate of motion would be nearly fourteen times slower than that of the moon. Although we do not see the stars when the sun is present, yet after the sun is set, we can observe that it makes daily progress eastward, as is apparent from the constellations of the Zodiac occupying, successively, the western sky after sunset, proving that either all the stars have a common motion eastward independent of their diurnal motion, or that the sun has a motion past them, from west to east. We shall see hereafter abundant evidence to prove, that this change in the relative position of the sun and stars, is owing to a change in the apparent place of the sun, and not to any change in the stars.

154. Although the apparent revolution of the sun is in a direction opposite to the real motion of the earth, as regards absolute space, yet both are nevertheless from west to east, since these terms do not refer to any directions in absolute space, but to the order in which certain constellations (the constellations of the Zodiac) succeed one another. The earth itself, on opposite sides of its orbit, does in fact move towards directly opposite points of space; but it is all the while pursuing its course in the order of the signs. In the same manner, although the earth turns on its axis from west to east, yet any place on the surface of the earth is moving in a direction in space exactly opposite to its direction twelve hours before. If the sun left a visible trace on the face of the sky, the ecliptic would of course be distinctly marked on the celestial sphere as it is on an artificial globe; and were the equator delineated in a similar manner, (by any method like that supposed in Art. 46,) we should then see at a glance the relative position of these two circles, the points where they intersect one another constituting the equinoxes, the points where they are at the greatest distance asunder, or the solstices, and various other particulars, which, for want of such visible traces, we are now obliged to search for by indirect and circuitous methods. It will even aid the learner to have constantly before his mental vision, an imaginary delineation of these two important circles on the face of the sky.

155. *The method of ascertaining the nature and position of the earth's orbit, is by observations on the sun's Declination and Right Ascension.*

The exact declination of the sun at any time is determined from his *meridian altitude*, or *zenith distance*, the latitude of the place of observation being known, (Art. 37.) The instant the center of the sun is on the meridian, (which instant is given by the transit instrument,) we take the distance of his upper and that of his lower limb from the zenith: half the sum of the two observations corrected for refraction, gives the zenith distance of the center. This result is diminished for parallax, (Art. 84,) and we obtain the zenith distance as it would be if seen from the center of the earth. The zenith distance being known, the declination is readily found, by subtracting that distance from the latitude. By thus taking the sun's declination for every day of the year at noon, and comparing the results, we learn its motion *to and from the equator.*

156. To obtain the motion in right ascension, we observe, with a transit instrument, the instant when the center of the sun is on the meridian. Our sidereal clock gives us the right ascension in time (Art. 124,) which we may easily, if we choose, convert into degrees and minutes, although it is more common to express right ascension by hours, minutes, and seconds. The differences of right ascension from day to day throughout the year, give us the sun's annual motion *parallel to the equator.* From the daily records of these two motions, at right angles to each other, arranged in a table,* it is easy to trace out the path of the sun on the artificial globe; or to calculate it with the greatest precision by means of spherical triangles, since the declination and right ascension constitute two sides of a right angled spherical triangle, the corresponding arc of the ecliptic, that is, the longitude, being the third side, (Art. 132.) By inspecting a table of observations, we shall find that the declination attains its greatest value on the 22d of December, when it is 23° 27′ 54″ south; that from this period it diminishes daily and becomes nothing on the 21st of March; that it then increases towards the north, and reaches a similar maximum at the northern tropic about the 22d of June; and, finally, that it returns again to the southern tropic by gradations similar to those which marked its northward progress. Our table of observations also shows us, that the daily differences of declination are very unequal; that, for several days, when the sun is near either tropic, its declination scarcely varies at all; while near the equator, the variations from day to day are very rapid,—a fact which is easily understood, when we reflect, that at the solstices the equator and the ecliptic are parallel to each other,† both being at right angles to the meridian; while at the

* Such a table may be found in Biot's Astronomy, in Delambre, and in most collections of Astronomical Tables.

† Or, more properly, the *tangents* of the two circles (which denote the directions of the curves at those points) are parallel.

equinoxes, the ecliptic departs most rapidly from the direction of the equator.

On examining, in like manner, a table of observations of the right ascension, we find that the daily differences of right ascension are likewise unequal; that the mean of them all is 3′ 56″, or 236″, but that they have varied between 215″ and 266″. On examining, moreover, the right ascension at each of the equinoxes, we find that the two records differ by 180°; which proves that the path of the sun is a great circle, since no other would bisect the equinoctial as this does.

157. *The obliquity of the ecliptic is equal to the sun's greatest declination.* For, by article 22, the inclination of any two great circles is equal to their greatest distance asunder, as measured on the sphere. The obliquity of the ecliptic may be determined from the sun's meridian altitude, or zenith distance, on the day of the solstice. The exact instant of the solstice, however, will not of course occur when the sun is on the meridian, but may happen at some other meridian; still, the changes of declination near the solstice are so exceedingly small, that no material error will result from this source. The obliquity may also be found, without knowing the latitude, by observing the greatest and least meridian altitudes of the sun, and taking half the difference. This is the method practiced in ancient times by Hipparchus, (Art. 2.) On comparing observations made at different periods for more than two thousand years, it is found, that the obliquity of the ecliptic is not constant, but that it undergoes a slight diminution from age to age, amounting to 52″ in a century, or about half a second annually. We might apprehend that by successive approaches to each other the equator and ecliptic would finally coincide; but astronomers have found by a most profound investigation, founded on the principles of universal gravitation, that this variation is confined within certain narrow limits, and that the obliquity, after diminishing for some thousands of years, will then increase for a similar period, and will thus vibrate for ever about a mean value.

158. *The dimensions of the earth's orbit, when compared with its own magnitude, are immense.*

Since the distance of the earth from the sun is 95,000,000 miles, and the length of the entire orbit nearly 600,000,000 miles, it will be found, on calculation, that the earth moves 1,640,000 miles per day, 68,000 miles per hour, 1,100 miles per minute, and nearly 19 miles every second, a velocity nearly sixty times as great as the maximum velocity of a cannon ball. A place on the earth's equator turns, in the diurnal revolution, at the rate of about 1,000 miles an hour and $\frac{5}{18}$ of a mile per second. The motion around the sun, therefore, is nearly 70 times as swift as the greatest motion around the axis.

THE SEASONS.

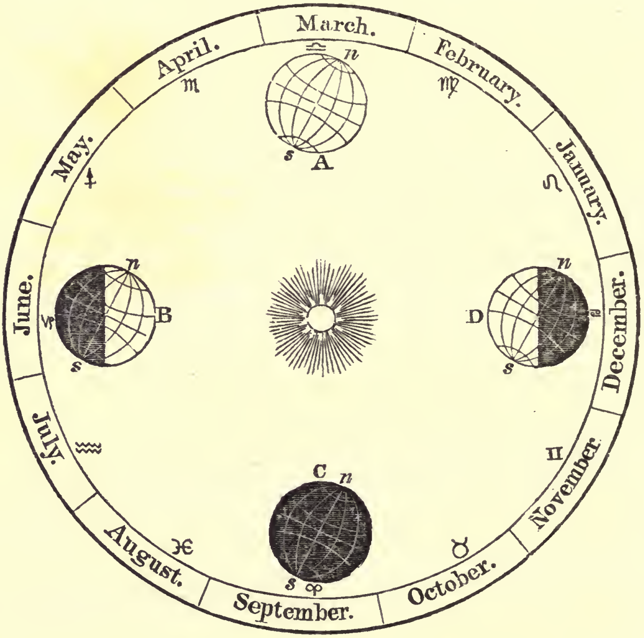
159. *The change of seasons depends on two causes,* (1) *the obliquity of the ecliptic, and* (2) *the earth's axis always remaining parallel to itself.* Had the earth's axis been perpendicular to the plane of its orbit, the equator would have coincided with the ecliptic, and the sun would have constantly appeared in the equator. To the inhabitants of the equatorial regions, the sun would always have appeared to move in the prime vertical; and to the inhabitants of either pole, he would always have been in the horizon. But the axis being turned out of a perpendicular direction 23° 28′, the equator is turned the same distance out of the ecliptic; and since the equator and ecliptic are two great circles which cut each other in two opposite points, the sun, while performing his circuit in the ecliptic, must evidently be once a year in each of those points, and must depart from the equator of the heavens to a distance on either side equal to the inclination of the two circles, that is, 23° 28′. (Art. 22.)

160. The earth being a globe, the sun constantly enlightens the half next to him,* while the other half is in darkness. The boundary between the enlightened and the unenlightened part, is called *the circle of illumination.* When the earth is at one of the equinoxes, the sun is at the other, and the circle of illumina-

* In fact, the sun enlightens a little more than half the earth, since on account of his vast magnitude the tangents drawn from opposite sides of the sun to opposite sides of the earth, converge to a point behind the earth, as will be seen by and by in the representation of eclipses.

tion passes through both the poles. When the earth reaches one of the tropics, the sun being at the other, the circle of illumination cuts the earth, so as to pass 23° 28′ beyond the nearer, and the same distance short of the remoter pole. These results would not be uniform, were not the earth's axis always to remain parallel to itself. The following figure will illustrate the foregoing statements.

Fig. 31.



Let ABCD represent the earth's place in different parts of its orbit, having the sun in the center. Let A, C, be the position of the earth at the equinoxes, and B, D, its positions at the tropics, the axis *ns* being always parallel to itself.* At A and C the sun shines on both *n* and *s*; and now let the globe be turned round on its axis, and the learner will easily conceive that the sun will appear to describe the equator, which being bisected by the hori-

* The learner will remark that the hemisphere towards *n* is above, and that towards *s* is below the plane of the paper. It is important to form a just conception of the position of the axis with respect to the plane of its orbit.

zon of every place, of course the day and night will be equal in all parts of the globe.* Again, at B when the earth is at the southern tropic, the sun shines $23\frac{1}{2}^{\circ}$ beyond the north pole n, and falls the same distance short of the south pole s. The case is exactly reversed when the earth is at the northern tropic and the sun at the southern. While the earth is at one of the tropics, at B for example, let us conceive of it as turning on its axis, and we shall readily see that all that part of the earth which lies within the north polar circle will enjoy continual day, while that within the south polar circle will have continual night, and that all other places will have their days longer as they are nearer to the enlightened pole, and shorter as they are nearer to the unenlightened pole. This figure likewise shows the successive positions of the earth at different periods of the year, with respect to the signs, and what months correspond to particular signs. Thus the earth enters Libra and the sun Aries on the 21st of March, and on the 21st of June the earth is just entering Capricorn and the sun Cancer.

161. Had the axis of the earth been perpendicular to the plane of the ecliptic, then the sun would always have appeared to move in the equator, the days would every where have been equal to the nights, and there could have been no change of seasons. On the other hand, had the inclination of the ecliptic to the equator been much greater than it is, the vicissitudes of the seasons would have been proportionally greater than at present. Suppose, for instance, the equator had been at right angles to the ecliptic, in which case the poles of the earth would have been situated in the ecliptic itself; then in different parts of the earth the appearances would have been as follows. To a spectator on the equator, the sun as he left the vernal equinox would every day perform his diurnal revolution in a smaller and smaller circle, until he reached the north pole, when he would halt for a moment and then wheel about and return to the equator in the reverse order. The progress of the sun through the southern signs, to the south pole, would be similar to that already described. Such would be the

* At the pole, the solar disk, at the time of the equinox, appears bisected by the horizon.

appearances to an inhabitant of the equatorial regions. To a spectator living in an oblique sphere, in our own latitude for example, the sun while north of the equator would advance continually northward, making his diurnal circuits in parallels farther and farther distant from the equator, until he reached the circle of perpetual apparition, after which he would climb by a spiral course to the north star, and then as rapidly return to the equator. By a similar progress southward, the sun would at length pass the circle of perpetual occultation, and for some time (which would be longer or shorter according to the latitude of the place of observation) there would be continual night.

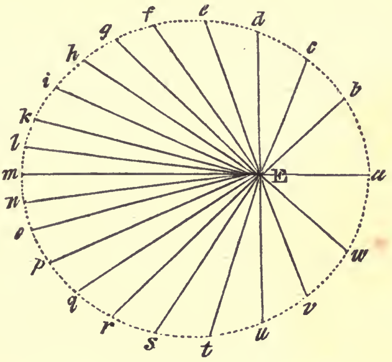
The great vicissitudes of heat and cold which would attend such a motion of the sun, would be wholly incompatible with the existence of either the animal or the vegetable kingdoms, and all terrestrial nature would be doomed to perpetual sterility and desolation. The happy provision which the Creator has made against such extreme vicissitudes, by confining the changes of the seasons within such narrow bounds, conspires with many other express arrangements in the economy of nature to secure the safety and comfort of the human race.

FIGURE OF THE EARTH'S ORBIT.

162. Thus far we have taken the earth's orbit as a great circle, such being the projection of it on the celestial sphere; but we now proceed to investigate its actual figure.

Were the earth's path a circle, having the sun in the center, the sun would always appear to be at the same distance from us; that is, the radius of its orbit, or *radius vector*, the name given to a line drawn from the center of the sun to the orbit of any planet, would always be of the same length. But the earth's distance from the sun is constantly varying, which shows that its orbit is not a circle. We learn the true figure of the orbit, by ascertaining the *relative distances* of the earth from the sun at various periods of the year. These all being laid down in a diagram, according to their respective lengths, the extremities, on being connected, give us our first idea of the shape of the orbit, which appears of an oval form, and at least *resembles* an ellipse; and, on further trial, we find that it has the properties of an ellipse. Thus, let E (Fig. 32,) be the place of the earth, and *a*, *b*, *c*, &c. successive positions of the sun; the *relative* lengths of the lines E*a*, E*b*, &c. being known: on connecting the points, *a*, *b*, *c*, &c. the resulting figure indicates the true shape of the earth's orbit.

Fig. 32.



163. These relative distances are found in two different ways; first, *by changes in the sun's apparent diameter*, and, secondly, *by variations in his angular velocity.* Were the variations in the sun's horizontal parallax considerable, as is the case with the moon's, this might be made the measure of the relative distances, for the parallax varies inversely as the distance, (Art. 82); but the whole horizontal parallax of the sun is only 9″, and its variations are too slight and delicate, and too difficult to be found, to serve as a criterion of the changes in the sun's distance from the earth. But the changes in the *sun's apparent diameter*, are much more sensible, and furnish a better method of measuring the relative distances of the earth from the sun. By a principle in optics, the apparent diameter of an object, at different distances from the spectator, is inversely as the distance.* Hence, the apparent diameters of the sun, taken at different periods of the year, become measures of the different lengths of the radius vector.

* More exactly, the *tangent* of the apparent diameter is inversely as the distance; but in small angles like those concerned in the present inquiry, the angle itself may be taken for the tangent.

164. The point where the earth, or any planet, in its revolution, is nearest the sun, is called its *perihelion :* the point where it is farthest from the sun, its *aphelion.* The place of the earth's perihelion is known, since there the apparent magnitude of the sun is greatest; and when the sun's magnitude is least, the earth is known to be at its aphelion. The sun's apparent diameter when greatest is 32′ 35.″6; and when least, 31′ 31″; hence the radius vector at the aphelion : rad. vector at the perihelion : : 32.5933 : 31.5167 : : 1.034 : 1. Half the difference of the two is equal to the distance of the focus of the ellipse from the center, a quantity which is always taken as the measure of the *eccentricity* of a planetary orbit.

165. The differences of *angular velocity* in the sun in the different parts of his apparent revolution, are still more remarkable. At the perihelion, the sun moves in twenty four hours over an arc of 61′, while at the aphelion he describes in the same time an arc of only 57′, these being the daily increments of longitude in those two points respectively. If the apparent motions of the sun depended alone on our different distances from him, the angular velocity would vary inversely as the distance, and the ratio expressed by these two numbers would be the same as that of the two numbers which denote the differences of apparent diameter in these two points. That is, $\frac{61}{57}$ ($=1.07$) would equal $\frac{32.5933}{31.5167}$ ($=1.034$); but the first fraction is equal to the square of the second, for $1.07=\overline{1.034}^2$. Hence,

The sun's angular velocities at the perihelion and the aphelion, are to each other inversely as the squares of the distances.

The angular velocities, therefore, which can be measured very accurately by the daily differences of right ascension and declination (Art. 132,) converted into corresponding longitudes, enable us to determine the different distances of the earth from the sun at various points in the orbit.

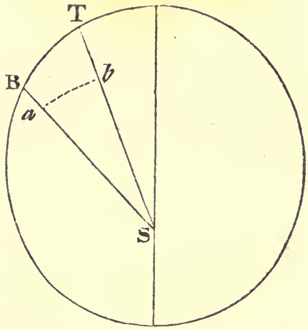
166. Since the arcs described by the earth in any small times, as in single days, are inversely as the squares of the distances, consequently, *the distances are as the square roots of the arcs.* Upon

this principle, the relative distances of the earth from the sun, in every point of its revolution, may be easily calculated. Thus, we have seen that the arcs described by the sun in one day at the perihelion and aphelion are as 61 to 57. Hence the distances of the earth from the sun at those two points are as $\sqrt{57}$ to $\sqrt{61}$, or as 1 to 1.034. From twenty four observations made with the greatest care by Dr. Maskelyne at the Royal Observatory of Greenwich, the following distances of the earth from the sun are determined for each month in the year.

Time of Observation.		*Distances.*	*Time of Observation.*		*Distances.*
January	12–13,	0.98448	July	18–19,	1.01658
February	17–18,	0.98950	August	26–27,	1.01042
March	14–15,	0.99622	September	22–23,	1.00283
April	28–29,	1.00800	October	24–25,	0.99303
May	15–16,	1.01234	November	18–20,	0.98746
June	17–18,	1.01654	December	17–18,	0.98415

167. The angular velocity being inversely as the square of the distance in all parts of the solar orbit, it follows that *the product of the angle described in any given time, by the square of the distance, is always the same constant quantity.* For if of two factors, $A \times B$, A is increased as B is diminished, the product of A and B is always the same. If, therefore, from the sun S (Fig. 33,) two radii be drawn to T, B, the extremities of the arc described in one day, then $ST^2 \times TB$ gives the same product in all parts of the orbit.*

Fig. 33.



168. *The radius vector of the solar orbit describes equal spaces in equal times, and in unequal times, spaces proportional to the times.*

Let TB (Fig. 33,) be the arc described by the sun in one day; then, $$\text{Sector } TSB = \tfrac{1}{2} SB \times TB.$$

* TB, as seen from the earth, would be projected into a circular arc, equal to the measure of the angle at S.

Taking Sb as any radius, describe the circular arc ab, which is the measure of the angle at S. Now,

$$Sb : ab :: SB : BT = SB \times \frac{ab}{Sb};$$ and substituting this value of BT in the above equation, we have $TSB = \frac{1}{2}SB \times SB \times \frac{ab}{Sb} = \frac{1}{2}SB^2 \times \frac{ab}{Sb}$. But S$b$ is constant, and the product of $SB^2 \times ab$ is likewise constant; therefore the sector is always equal to a constant quantity, and therefore the radius vector passes over equal spaces in equal times.*

The sun's orbit may be accurately represented by taking some point as the perihelion, drawing the radius vector to that point, and, considering this line as unity, drawing other radii making angles with each other such that the included areas shall be proportional to the times, and of a length required by the distance of each point as given in the table (Art. 166.) On connecting these radii, we shall thus see at once how little the earth's orbit departs from a perfect circle. Small as the difference appears between the greatest and least distances, yet it amounts to nearly $\frac{1}{29}$ of the perihelion distance, a quantity no less than 3,000,000 of miles.

169. The foregoing method of determining the figure of the earth's orbit is founded on *observation;* but this figure is subject to numerous irregularities, the nature of which cannot be clearly understood without a knowledge of the leading principles of Universal Gravitation. An acquaintance with these will also be indispensable to our understanding the causes of the numerous irregularities which (as will hereafter appear) attend the motions of the moon and planets. To the laws of universal gravitation, therefore, let us next apply our attention.

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* Francœur, Uran., p. 62.

CHAPTER III.

OF UNIVERSAL GRAVITATION.

170. Universal Gravitation, *is that influence by which every body in the universe, whether great or small, tends towards every other, with a force which is directly as the quantity of matter, and inversely as the square of the distance.*

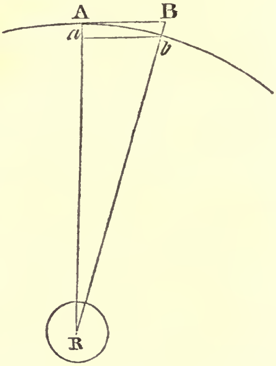
As this force acts as though bodies were drawn towards each other by a mutual attraction, the force is denominated *attraction;* but it must be borne in mind, that this term is figurative, and implies nothing respecting the nature of the force.

The *existence* of such a force in nature was distinctly asserted by several astronomers previous to the time of Sir Isaac Newton, but its *laws* were first promulgated by this wonderful man in his Principia, in the year 1687. It is related, that while sitting in a garden, and musing on the cause of the falling of an apple, he reasoned thus:* that, since bodies far removed from the earth fall towards it, as from the tops of towers, and the highest mountains, why may not the same influence extend even to the moon; and if so, may not this be the reason why the moon is made to revolve around the earth, as would be the case with a cannon ball were it projected horizontally near the earth with a certain velocity. According to the first law of motion, the moon, if not continually drawn or impelled towards the earth by some force, would not revolve around it, but would proceed on in a straight line. But going around the earth as she does, in an orbit that is nearly circular, she must be urged towards the earth by some force, which, in a given time, may be represented by the versed sine of the arc described in that time. For let the earth (Fig. 34,) be at E, and let the arc described by the moon in one second of time be A*b*. Were the moon influenced by no extraneous force, to turn her aside, she would have described, not the arc A*b*, but the straight line AB, and would have been found at

* Pemberton's View of Newton's Philosophy.

the end of the given time at B instead of *b*. She therefore departs from the line in which she tends naturally to move, by the line B*b*, which in small angles may be taken as equal to the versed sine A*a*. This deviation from the tangent must be owing to *some* extraneous force. Does this force correspond to what the force of gravity exerted by the earth, would be at the distance of the moon? Now we know the distance of the moon from the earth, and of course the circumference of her orbit. We also know the time of her revolution around the earth. Hence we may estimate the length of the arc A*b* described in one second; and knowing the arc we can calculate its versed sine. For the moon being 60 times as far from the center of the earth, as the surface of the earth is from the center, consequently, since the force of gravity decreases as the square of the distance increases, the space through which the moon would fall by the force of the earth's attraction alone, would be $\frac{16\frac{1}{12}}{60^2}=.05$ inches. On calculating the value of the versed sine of the arc described in one second, it proves to be the same. Hence gravity, and no other force than gravity, causes the moon to circulate around the sun.

Fig. 34.

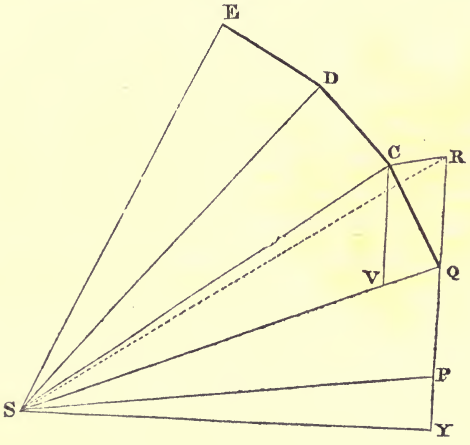


171. By this process it was discovered that the law of gravitation extends to the moon. By subsequent inquiries it was found to extend in like manner to all the planets, and to every member of the solar system; and, finally, recent investigations have shown that it extends to the fixed stars. The law of gravitation, therefore, is now established as the grand principle which governs all the motions of the heavenly bodies. Hence, nothing can be more deserving of the attention of the student, than the development of the results of this universal law. A few of them only are all that can be exhibited in a work like the present: their full development must be sought for in such great works as the Mecanique Celeste of La Place.

172. *If a body revolves about an immovable center of force, and is constantly attracted to it, it will always move in the same plane, and describe areas about the center proportional to the times.**

Let S (Fig. 35,) be the center of force, and suppose a body to be projected at P in the direction PQR, and take PQ=QR; then by the first law of motion, the body would move uniformly in the direction PQR, and describe PQ, QR, in the same time, if no other force acted upon it. But when the body comes to Q,

Fig. 35.



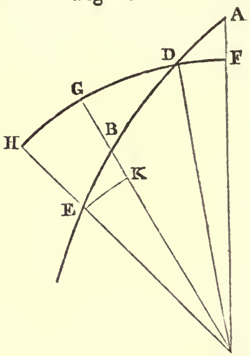
let a single impulse act at S, sufficient to draw the body through QV, in the time it would have described QR; and complete the parallelogram VQRC, and the body in the same time will describe QC; therefore, PQ, QC, are described in the same time. But the triangle SCQ=SRQ=SPQ; that is, *equal areas are described in equal times.* For the same reason, if a single impulse act at C, D, E, &c. at equal intervals of time, the several areas SPQ, SQC, SCD, SDE, &c. will all be equal to each other. Now this demonstration is independent of any particular dimensions in the several triangles, and consequently, holds good when they are taken indefinitely small, in which case we may consider the force as acting, not by separate impulses, but *constantly*, causing the body to describe a curve around S. And as no force acts

* The learner will remark that what has been before proved (Art. 168,) respecting the radius vector of the earth, is here shown to hold good with respect to *every* body which revolves around a center of force; and the same is true of several other propositions demonstrated in this chapter.

out of the plane SPQ, the whole curve must lie in that plane; that is, *the body moves always in the same plane.*

173. *If a body describes a curve around a center towards which it tends by any force, the angular velocity of the body around that center is reciprocally as the square of the distance from it.**

Fig. 36.



Let ABE (Fig. 36,) be any curve described about the center S; draw SA, SB, to any two points of the curve A and B; and let AD, BE, be described in indefinitely small equal times. Join SD and SE, and with the center S and distance SD, describe a circle meeting SA, SB, SE, in F, G, H; and with the center S and distance SE describe a circle meeting SB in K.

Because AD and BE are described in equal times, the triangles ASD, BSE, are equal. Hence, (Euc. 15. 6.)

$$DF : EK :: KS : FS :: BS : AS† :: BS^2 : BS \times AS \ (1)$$
$$GH : EK :: SH : SE :: SF : SE :: SA : SB :: SA^2 : BS \times AS (2)$$

Hence, (1) $DF : BS^2 :: EK : BS \times AS$

(2) $GH : AS^2 :: EK : BS \times AS$

$\therefore DF : GH :: BS^2 : AS^2$.

But DF and GH measure the respective angular velocities at A and B, while AS and BS represent the distance at the same points. Therefore, the angular velocities are reciprocally as the squares of the distances.‡

174. *In the same curve, the velocity, at any point of the curve, varies inversely as the perpendicular drawn from the center of force to the tangent at that point.*

Draw SY (Fig. 35,) perpendicular to QP produced; then the area $SPQ = \frac{1}{2}PQ \times SY$, which varies as $PQ \times SY \therefore PQ \propto \frac{\text{area } SPQ}{SY}$. But $PQ \propto V$, the velocity at $P \therefore V \propto \frac{\text{area } SPQ}{SY}$. Now

* It will be remarked that this is a *general* proposition, of which article 165 affords a particular example.

† For, when the triangles are indefinitely diminished, these two ratios are ultimately equal.

‡ Stewart's Phys. and Math. Essays.

in the curve described from P, with a constant force, SY becomes a perpendicular to the tangent to the curve. But by article 172, the area described in a given time is constant. Therefore SPQ is constant, and $V \propto \frac{1}{SY}$; that is, the velocity varies inversely as the perpendicular upon the tangent. Hence, the velocity of a revolving body increases as it approaches the center of force.

175. *If equal areas be described about a center in equal times, the force must tend towards that center.*

Let SPQ (Fig. 35,) = SQC; now SPQ = SQR ∴ SQC = SQR ∴ CR is parallel to QS. Complete the parallelogram QRCV, and by the supposition the body describes QC, in consequence of the impulse at Q, and it would have described QR if no such impulse had acted; therefore QV must represent that motion impressed at Q, which, in conjunction with the motion QR, can make a body describe QC, and QV is directed to S.

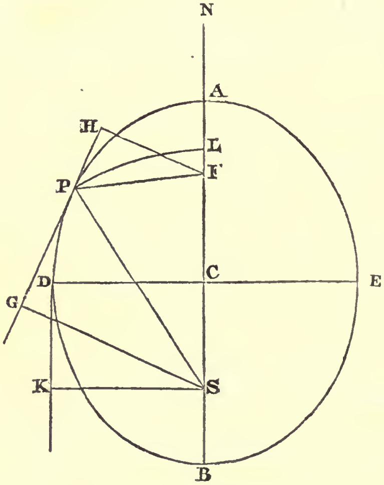
176. Now it appears from article 168, that it is a *fact*, derived from observation, that the earth's radius vector describes equal areas in equal times; and by similar observations, the same is found to be true of each of the primary planets about the sun, and of each of the satellites about its primary. Hence, it is inferred, that the primary planets all gravitate towards the sun, and that the secondary planets all gravitate towards their respective primaries.

It has farther been established by observation, (Art. 162,) that the planetary orbits are *ellipses;* and hence the application of the principles of gravitation, so far as respects the sun and planets may be confined to the consideration of the motion of a body in an *elliptical orbit.*

177. *The distance of any planet from the sun at any point in its orbit, is to its distance from the superior focus, as the square of its velocity at its mean distance from the sun, is to the square of its velocity at the given point.*

Let ADBE (Fig. 37,) be the orbit of a planet, S the focus in which the sun is placed, AB the transverse and DE the conjugate axis, C the center, and F the superior focus. Let the planet be any where at P; and draw a tangent to the orbit at P, on which from the foci let fall the perpendiculars SG, FH. Draw also DK touching the orbit in D, and let SK be perpendicular to it. Let

Fig. 37.



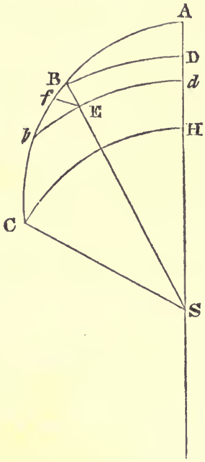
the velocity of the planet when at the mean distance at D=C, and when at P=V. Join SP, FP. Then (Art. 174,) the velocity at D is to the velocity at P, as SG to SK; that is,

$$C : V :: SG : DC, \text{ or } V = C\frac{DC}{SG}.$$

But because the triangles SGP, FHP, are equiangular, having right angles at G and H, and also the angles SPG, FPH equal, from the nature of the ellipse, SP : PF :: SG : FH, and therefore $SP : PF :: SG^2 : (FH \times SG =)CD^2$. Hence, $\frac{CD^2}{SG^2} = \frac{PF}{SP}$. Now, as was found above, $V = C\frac{DC}{SG}$. Therefore, $V^2 = C^2\frac{DC^2}{SG^2}$; and $V^2 = C^2\frac{PF}{SP}$, and $V^2 \times SP = C^2 \times PF$ and $SP : PF :: C^2 : V^2$. Hence, the distance of any planet, &c.

178. *If of two bodies gravitating to the same center, one descends in a straight line, and the other revolves in a curve; then, if the velocities of these bodies are equal in any one case, when they are equally distant from the center, they will always be equal when they are equally distant from it.*

Fig. 38.



Let ABC (Fig. 38,) be a curve which a body describes about a center S to which it gravitates, while another body descends in a straight line AS to that center. Let BC be any arc of the curve ABC, and let BD, CH, be arcs of circles described from the center S, intersecting the line AS in D and H. From the center S describe the arc bd, indefinitely near to BD, and draw Ef perpendicular to Bb. Then, because the distances SD and SB are equal, the forces of gravity at D and B are also equal. Let these forces be expressed by the equal lines Dd and BE; and let the force BE be resolved into the forces Ef and Bf. The force Ef, acting at right angles to the path of the body, will not affect its velocity in that path, but will only draw it aside from a rectilinear course and make it proceed in the curve BbC. But the other force Bf, acting in the direction of the course of the body, will be wholly employed in accelerating it. And because B and b are indefinitely near to each other, and likewise D and d, the accelerating force from B to b and from D to d, may be considered as acting uniformly. Therefore, the accelerations of the bodies in D and B, produced in equal times, are as the lines Dd, Bf; and hence, putting d for the velocity at d, and f for the velocity at f,

$$d : f :: \mathrm{D}d \text{ or BE} : \mathrm{B}f . (1)$$

And because the angle at E is a right angle,

$$\mathrm{BE}^2 = \mathrm{B}b \times \mathrm{B}f \therefore \mathrm{BE} = \sqrt{\mathrm{B}b} \times \sqrt{\mathrm{B}f} \therefore \mathrm{BE} \times \sqrt{\mathrm{B}f} = \sqrt{\mathrm{B}b} \times \mathrm{B}f.$$

Hence, $\mathrm{BE} : \mathrm{B}f :: \sqrt{\mathrm{B}b} : \sqrt{\mathrm{B}f} . (2)$

And, (1) and (2), $d : f :: \sqrt{\mathrm{B}b} : \sqrt{\mathrm{B}f} . (3)$

But, putting b for the velocity at b, and observing that, in falling bodies, the velocities are as the square roots of the spaces,

$$b : f :: \sqrt{\mathrm{B}b} : \sqrt{\mathrm{B}f} . (4)$$

Therefore, (3) and (4), $b : f :: d : f \therefore b = d$; that is, the velocity at b equals the velocity at d. And, since the same reasoning holds for successive points that may be taken at equal distances from B and D, therefore, if of two bodies, &c.*

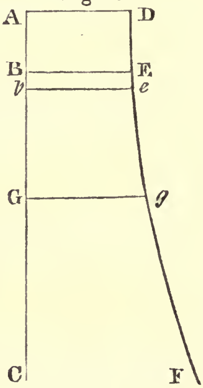
* Principia, Lib. I, Pr. 40. Stewart's Math. and Phys. Essays, Pr. 13.

179. *The law according to which the planets gravitate is such, that any body under the influence of the same force, and falling direct to the sun, will have its velocity at any point equal to a constant velocity multiplied into the square root of the distance it has fallen through, divided by the square root of the distance between the body and the sun's center.*

Suppose a planet to revolve in the elliptical orbit APB (Fig. 37); at A, the higher apsis, the velocity $V=C\left(\frac{AF}{AS}\right)^{\frac{1}{2}}$, (Art. 177); or if AN, in the axis produced $=AF$, $V=C\left(\frac{AN}{AS}\right)^{\frac{1}{2}}$. Let a body at A begin to descend towards S with this velocity, then if $SL=SP$, the velocity of the planet at P will be the same as that of the falling body at L, (Art. 178.) But the velocity of the planet at P is $C\left(\frac{PF}{PS}\right)^{\frac{1}{2}}=C\left(\frac{NL}{SL}\right)^{\frac{1}{2}}$.* But this velocity is equal to the constant velocity expressed by C, multiplied into the square root of NL, the distance fallen through,† divided by the square root of LS, the distance between the body and the sun's center.‡

180. *The force with which any planet gravitates to the sun, is inversely as the square of its distance from the sun's center.*

Fig. 39.



Let C (Fig. 39,) be the center to which the falling body gravitates, A the point from which it begins to fall, and let its velocity at any point B, be to its velocity in the point G, which bisects AC, as $\left(\frac{AB}{BC}\right)^{\frac{1}{2}} : 1$.§ Let DEF be a curve such that if AD be an ordinate or a perpendicular to AC, meeting the curve in D, and BE any other ordinate, AD is to BE as the force at A to the force at B, then will twice the area ABED be equal to the square of the velocity which the body

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* For $SN=AB=SP+PF=SP+NL \therefore PF=NL$.

† For at N, LN becomes 0, and consequently the velocity 0; hence, the body begins to fall at N, and at any other point, LN is the distance fallen through.

‡ Playfair, *Phys. Ast.*

§ For, denoting the velocity at B by V, and the velocity at G by V′,

$$V : V' :: C\left(\frac{AB}{BC}\right)^{\frac{1}{2}} : C\left(\frac{AG}{GC}\right)^{\frac{1}{2}} :: \left(\frac{AB}{BC}\right)^{\frac{1}{2}} : \left(\frac{AG}{GC}\right)^{\frac{1}{2}} :: \left(\frac{AB}{BC}\right)^{\frac{1}{2}} : 1.$$

has acquired in B.* If therefore the velocity at B be V, that at the middle point G being c, $V=c\left(\frac{AB}{BC}\right)^{\frac{1}{2}}$ by hypothesis, and therefore $2ABED=c^2.\frac{AB}{BC}$; and since $AB=AC-BC$, $2ABED=c^2.\frac{AC-BC}{BC}=c^2\left(\frac{AC}{BC}-1\right)$. For the same reason, if be be drawn indefinitely near to BE, $2AbeD=c^2\left(\frac{AC}{bC}-1\right)$, and therefore the difference of these areas, or $2BbeE$, that is, $2EB\times Bb=c^2\left(\frac{AC}{bC}-\frac{AC}{BC}\right)=c^2\frac{AC(BC-bC)}{BC\times bC}=c^2.\frac{AC\times Bb}{BC^2}$†. Wherefore, dividing by Bb, $2EB=c^2.\frac{AC}{BC^2}$; or $EB=c^2\frac{AG}{BC^2}$; now c^2 and AG are constant quantities, therefore EB varies inversely as BC^2. But EB represents the force of gravity at B, and BC the distance from the sun. Therefore, the force of gravity of a planet in different parts of its orbit, is inversely as the square of its distance from the sun.

181. The line CG is the same with the mean distance of the planet, in an orbit of which AC is the length of the transverse axis; and if the gravitation at that distance $=F$, and the mean distance itself $=a$, then since $EB=c^2\frac{AG}{BC^2}$, $F=c^2\times\frac{a}{a^2}=\frac{c^2}{a}$, or $aF=c^2$.

182. *The squares of the times of revolution of any two planets, are as the cubes of their mean distances from the sun.*

If a be the mean distance, or the semi-transverse axis, b the semi-conjugate, then $\pi ab=$ area of the orbit.‡ But as c is the velocity at the mean distance, or the elliptic arch which the planet moves over in a second when it is at D, (Fig. 37,) the vertex of the

* This principle is demonstrated by the aid of Fluxions as follows:

By construction, BE is proportional to the force at $B=\frac{dv}{dt}$, v being the velocity which the moving body has acquired at B, and t the time of the descent from A to B. Now Bb is the momentary increment of BA the space, and therefore $=vdt$; therefore $BE\times Bb=vdv$. And $2BE\times Bb=2vdv$. But $BE\times Bb$ is the momentary increment of the area ABED, and $2vdv$ is the momentary increment of v^2; therefore the square of the velocity of the moving body, and twice the area of ABED, increase at the same rate, and begin to exist at the same time; therefore they are equal. (See Playfair's Outlines, *Mechanics*, Art. 96.)

† bC being ultimately equal to BC. ‡ Day's Mensuration.

conjugate axis, therefore $\frac{1}{2}bc$ is the area described in that second by the radius vector; and since the area is the same for every second of the planet's revolution (Art. 172), therefore the area of the orbit divided by $\frac{1}{2}bc$ will give the number of seconds in which the revolution is completed, which $=\frac{\pi ab}{\frac{1}{2}bc}=\frac{2\pi a}{c}$; or, since $c^2=a\text{F}$, (Art. 181,) the time of a revolution $=\frac{2\pi a}{\sqrt{a\text{F}}}=2\pi\sqrt{\frac{a}{\text{F}}}$. Hence, let t, t', be the times of revolutions for two different planets, of which the mean distances are a, a', and the force of gravity at those distances F, F′. Then $t : t' :: 2\pi\sqrt{\frac{a}{\text{F}}} : 2\pi\sqrt{\frac{a'}{\text{F}'}} :: \sqrt{\frac{a}{\text{F}}} : \sqrt{\frac{a}{\text{F}'}} \therefore t^2 : t'^2 :: \frac{a}{\text{F}} : \frac{a'}{\text{F}'}$. But (Art. 180,) $\text{F} : \text{F}' :: a'^2 : a^2 \therefore t^2 : t'^2 :: \frac{a}{a'^2} : \frac{a'}{a^2}$, or $t^2 : t'^2 :: a^3 : a'^3$. That is, the squares of the times are as the cubes of the mean distances; or, since the major axes of the orbits are double the mean distances, the squares of the times are as the cubes of the major axes.

183. This is one of *Kepler's three great Laws*, which, taken in connexion, are as follows:

1. *The orbits of all the planets are ellipses, the sun occupying the common focus.* (Art. 176.)

2. *The radius vector of any planet describes areas proportional to the times.* (Art. 172.)

3. *The squares of the periodical times are as the cubes of the major axis of the orbit.* (Art. 182.)

These great and fundamental principles of the planetary motions, were discovered by the illustrious Kepler by long and assiduous study of the observations made by Tycho Brahe, and hence he has been called the *legislator of the skies.* They, therefore, became known as *facts*, before they were demonstrated mathematically. The glory of this achievement was reserved for Newton, who proved that they were necessary results of the law of universal gravitation.

MOTION IN AN ELLIPTICAL ORBIT.

184. Having now acquired some knowledge of the law of universal gravitation, let us next endeavor to gain a just conception

of the forces by which the planets are made to revolve in their orbits about the sun. In obedience to the first law of motion, every moving body *tends* to move in a straight line; and were not the planets deflected continually towards the sun by the force of attraction, these bodies as well as others would move forward in a rectilineal direction. We call the force by which they tend to such a direction the *projectile force,* because its effects are the same as though the body were originally projected from a certain point in a certain direction. It is an interesting problem for mechanics to solve, what was the nature of the impulse originally given to the earth, in order to impress upon it its two motions, the one around its own axis, the other around the sun? If struck in the direction of its center of gravity it might receive a forward motion, but no rotation on its axis. It must, therefore, have been impelled by a force, whose direction did not pass through its center of gravity. Bernouilli, a celebrated mathematician, has calculated that the impulse must have been given very nearly in the direction of the center, the point of projection being only the 165th part of the earth's radius from the center.* This impulse alone would cause the earth to move in a right line: gravitation towards the sun causes it to describe an orbit. Thus a top spinning on a smooth plane, as that of glass or ice, if impelled in a direction not coinciding with that of the center of gravity, may be made to imitate the two motions of the earth, especially if the experiment is tried in a concave surface like that of a large bowl. The resistance occasioned by the surface on which the top moves, and that of the air, will generally destroy the force of projection and cause the top to revolve in a smaller and smaller orbit; but the earth meets with no such resistance, and therefore makes both her days and years of the same length from age to age. A body, therefore, revolving in an orbit about a center of attraction, is constantly under the influence of two forces,—the *projectile* force, which tends to carry it forward in a straight line which is a tangent to its orbit, and the *centripetal* force, by which it tends towards the center.

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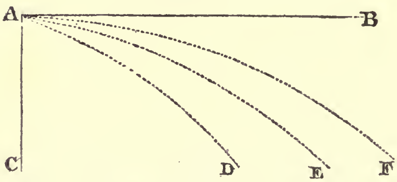
* Francœur, Uran. p. 49.

185. The most simple example we have of the combined action of these two forces is the motion of a missile thrown from the hand, or of a ball fired from a cannon. It is well known that the particular form of the curve described by the projectile, in either case, will depend upon the velocity with which it is thrown. In each case the body will begin to move in the line of direction in which it is projected, but it will soon be deflected from that line towards the earth. It will however continue nearer to the line of projection as the velocity of projection is greater. Thus let AB (Fig. 40,) perpendicular to AC represent the line of projection. The body will, in every case, commence

Fig. 40.

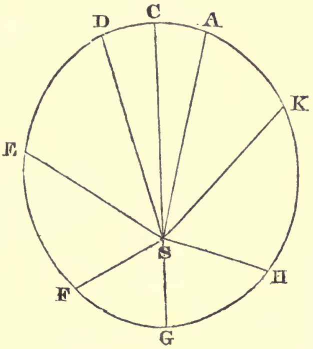


its motion in the line AB, which will therefore be the tangent to the curve it describes; but if it be thrown with a small velocity, it will soon depart from the tangent, describing the line AD; with a greater velocity it will describe a curve nearer to the tangent, as AE; and with a still greater velocity it will describe the curve AF.

186. In figure 41, suppose the planet to have passed the point C with so small a velocity, that the attraction of the sun bends its path very much, and causes it immediately to begin to approach towards the sun; the sun's attraction will increase its velocity as it moves through D, E, and F. For the sun's attractive force on the planet, when at D, is acting in the direction DS, and, on account of the small inclination of DE to DS, the force acting in the line DS helps the planet forward in the path DE, and thus increases its velocity. In like manner the velocity of the planet will be continually increasing as it passes through D, E, and F; and though the attractive force, on account of the planet's nearness, is so much increased, and tends, therefore, to make the orbit more curved, yet the velocity is also so much increased that the orbit is not more curved than before. The same

increase of velocity occasioned by the planet's approach to the sun, produces a greater increase of centrifugal force which carries it off again. We may see also why, when the planet has reached the most distant parts of its orbit, it does not entirely fly off, and never returns to the sun. For when the planet passes along H, K, A, the sun's attraction retards the planet, just as gravity retards a ball rolled up hill; and when it has reached C, its velocity is very small, and the attraction at the center of force causes a great deflection from the tangent, sufficient to give its orbit a great curvature, and the planet turns about, returns to the sun, and goes over the same orbit again.* As the planet recedes from the sun, its centrifugal force diminishes faster than the force of gravity, so that the latter finally preponderates.†

Fig. 41.



187. We may imitate the motion of a body in its orbit by suspending a small ball from the ceiling by a long string. The ball being drawn out of its place of rest, (which is directly under the point of suspension,) it will tend constantly towards the same place by a force which corresponds to the force of attraction of a central body. If an assistant stands under the point of suspension, his head occupying the place of the ball when at rest, the ball may be made to revolve about his head as the earth or any planet revolves about the sun. By projecting the ball in different directions, and with different degrees of velocity, we may make it describe different orbits, exemplifying principles which have been explained in the foregoing propositions.

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* Airy.

† The centrifugal force varies inversely as the *cube* of the distance, while the force of gravity is inversely as the *square*. The centrifugal force, therefore, increases faster than the force of gravity as a body is approaching the sun, and decreases faster as the body recedes from the sun. (See M. Stewart's Phys. and Math. Tracts, Prop. 8.)

CHAPTER IV.

PRECESSION OF THE EQUINOXES—NUTATION—ABERRATION—MOTION OF THE APSIDES—MEAN AND TRUE PLACES OF THE SUN.

188. THE PRECESSION OF THE EQUINOXES, *is a slow but continual shifting of the equinoctial points from east to west.*

Suppose that we mark the exact place in the heavens where, during the present year, the sun crosses the equator, and that this point is close to a certain star; next year the sun will cross the equator a little way westward of that star, and so every year a little farther westward, until, in a long course of ages, the place of the equinox will occupy successively every part of the ecliptic, until we come round to the same star again. As, therefore, the sun, revolving from west to east in his apparent orbit, comes round towards the point where it left the equinox, it meets the equinox before it reaches that point. The appearance is as though the equinox *goes forward* to meet the sun, and hence the phenomenon is called the *Precession of the Equinoxes*, and the fact is expressed by saying that the equinoxes retrograde on the ecliptic, until the line of the equinoxes makes a complete revolution from east to west. The equator is conceived as *sliding* westward on the ecliptic, always preserving the same inclination to it, as a ring placed at a small angle with another of nearly the same size, which remains fixed, may be slid quite around it, giving a corresponding motion to the two points of intersection. It must be observed, however, that this mode of conceiving of the precession of the equinoxes is purely imaginary, and is employed merely for the convenience of representation.

189. The *amount of precession* annually is 50.″1; whence, since there are 3600″ in a degree, and 360° in the whole circumference, and consequently, 1296000″, this sum divided by 50.1 gives 25868 years for the period of a complete revolution of the equinoxes.

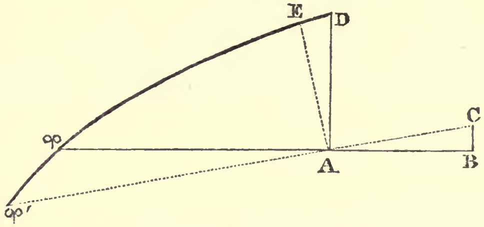
190. Suppose now we fix to the center of each of the two rings (Art. 188,) a wire representing its axis, one corresponding to the axis of the ecliptic, the other to that of the equator, the extremity of each being the pole of its circle. As the ring denoting the equator turns round on the ecliptic, which with its axis remains fixed, it is easy to conceive that the axis of the equator revolves around that of the ecliptic, and the pole of the equator around the pole of the ecliptic, and constantly at a distance equal to the inclination of the two circles. To transfer our conceptions to the celestial sphere, we may easily see that the axis of the diurnal sphere, (that of the earth produced, Art. 28,) would not have its pole constantly in the same place among the stars, but that this pole would perform a slow revolution around the pole of the ecliptic from east to west, completing the circuit in about 26,000 years. Hence the star which we now call the pole star, has not always enjoyed that distinction, nor will it always enjoy it hereafter. When the earliest catalogues of the stars were made, this star was 12° from the pole. It is now 1° 24′, and will approach still nearer; or, to speak more accurately, the pole will come still nearer to this star, after which it will leave it, and successively pass by others. In about 13,000 years, the bright star Lyra, which lies on the circle of revolution opposite to the present pole star, will be within 5° of the pole, and will constitute the Pole Star. As Lyra now passes near our zenith, the learner might suppose that the change of position of the pole among the stars, would be attended with a change of altitude of the north pole above the horizon. This mistaken idea is one of the many misapprehensions which result from the habit of considering the horizon as a fixed circle in space. However the pole might shift its position in space, we should still be at the same distance from it, and our horizon would always reach the same distance beyond it.

191. *The precession of the equinoxes is an effect of the spheroidal figure of the earth, and arises from the attraction of the sun and moon upon the excess of matter about the earth's equator.*

Were the earth a perfect sphere the attractions of the sun and moon upon the earth would be in equilibrium among themselves.

But if a globe were cut out of the earth, (taking half the polar diameter for radius,) it would leave a protuberant mass of matter in the equatorial regions, which may be considered as all collected into a ring resting on the earth. The sun being in the ecliptic, while the plane of this ring is inclined to the ecliptic 23° 28′, of course the action of the sun is oblique to the ring, and may be resolved into two forces, one in the plane of the equator, and the other perpendicular to it. The latter only can act as a disturbing force, and tending as it does to draw down the ring to the ecliptic, the ring would turn upon the line of the equinoxes as upon a hinge, and, dragging the earth along with it, the equator would ultimately coincide with the ecliptic were it not for the revolution of the earth upon its axis. This may be better understood by the aid of a diagram. Let ♈AB (Fig. 42,) represent the equator,

Fig. 42.



♈ED the ecliptic, and AD the solstitial colure. Let AB be the movement of rotation for a very short time, being of course in the order of the signs and in the direction of the equator. Let BC be the movement produced by the disturbing force of the sun in the same time. The point A will describe the diagonal AC, the equator will take the inclined situation CA♈′; the equinoctial point will retrograde from ♈ to ♈′; the colure AD will take the position AE, while the inclination of the two planes, that is, the obliquity of the ecliptic, will remain nearly the same.*

192. The moon conspires with the sun in producing the precession of the equinoxes, its effect, on account of its nearness to the earth, being more than double that of the sun, or as 7 to 3. The planets likewise by their attraction, produce a small effect upon the equatorial ring, but the result is slightly to diminish the

* Delambre, t. 3, p. 145. Playfair's Outlines, 2, 308.

amount of precession. The whole effect of the sun and moon being 50.″41, that of the planets is 0.31, leaving the actual amount of precession 50.″1.*

193. *The time occupied by the sun in passing from the equinoctial point round to the same point again, is called the* TROPICAL YEAR. As the sun does not perform a complete revolution in this interval but falls short of it 50.″1, the tropical year is shorter than the sidereal by 20m. 20s. in mean solar time, this being time of describing an arc of 50.″1 in the annual revolution.† The changes produced by the precession of the equinoxes in the apparent places of the circumpolar stars, have led to some interesting results in *chronology.* In consequence of the retrograde motion of the equinoctial points, the *signs* of the ecliptic (Art. 35,) do not correspond at present to the *constellations* which bear the same names, but lie about one whole sign or 30° westward of them. Thus, that division of the ecliptic which is called the sign Taurus, lies in the constellation Aries, and the sign Gemini in the constellation Taurus. Undoubtedly however when the ecliptic was thus first divided, and the divisions named, the several constellations lay in the respective divisions which bear their names. How long is it, then, since our zodiac was formed?

$$50.''1 : 1 \text{ year} :: 30^\circ (=108000'') : 2155.6 \text{ years.}$$

The result indicates that the present divisions of the zodiac, were made soon after the establishment of the Alexandrian school of astronomy. (Art. 2.)

NUTATION.

194. NUTATION *is a vibratory motion of the earth's axis, arising from periodical fluctuations in the obliquity of the ecliptic.*

If the sun and moon moved in the plane of the equator, there would be no precession, and the effect of their action in producing it varies with their distance from that plane. Twice a year, therefore, namely, at the equinoxes the effect of the sun is nothing; while at the solstices the effect of the sun is a maximum. On this account, the obliquity of the ecliptic is subject to a semiannual variation, since the sun's force which tends to produce a

* Francoeur, Uran. 162.

† 59′ 8.″3 : 24h. :: 50.″1 : 20m. 20s.

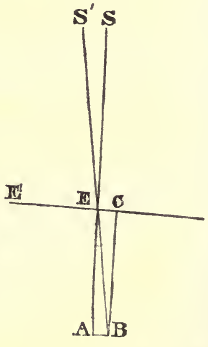
change in the obliquity is variable, while the diurnal motion of the earth which prevents the change from taking place, is constant. Hence the plane of the equator is subject to an irregular motion which is called the *Solar Nutation.* The name is derived from the oscillatory motion communicated by it to the earth's axis, while the pole of the equator is performing its revolution around the pole of the ecliptic (Art. 190.) The effect of the sun however is far less than that of the moon. By the nutation alone the pole of the earth would perform a revolution in a very small ellipse, only 18″ in diameter, the center being in the circle which the pole describes around the pole of the ecliptic; but the combined effects of precession and nutation convert the circumference of this circle into a wavy line. The motion of the equator occasioned by nutation, causes it alternately to approach to and recede from the stars, and thus to change their declinations. The solar nutation, depending on the position of the sun with respect to the equinoxes, passes through all its variations annually; but the lunar nutation depending on the position of the moon with respect to her nodes, varies through a period of about 18½ years.

ABERRATION.

195. ABERRATION *is an apparent change of place in the stars, occasioned by the joint effects of the motion of the earth in its orbit, and the progressive motion of light.*

Let EE′ (Fig. 43,) represent a part of the earth's orbit, and SE a ray of light from the star S. Take EC and EA proportional to the velocity of each respectively; complete the parallelogram, and draw the diagonal EB. Since an object always appears in the direction in which a ray of light coming from it, meets the eye, the combination of the two motions produces an impression on the eye exactly similar to that which would have been produced if the eye had remained at rest in the point E, and the particle of light had come down to it in the direction ES′; the star, therefore, whose place is at S, will appear to the spectator at E

Fig. 43.



to be situated at S′. The difference between its true and its apparent place, that is, the angle SES′ is the aberration, the magnitude of which is obtained from the known ratio of EA to EC, or the velocity of light to that of the earth in its orbit.

The velocity of light is 192,000 miles per second, while that of the earth in its orbit is about 19 miles per second. Representing the velocity of light by the line EA, and that of the earth by AB, then,

192,000 : 19 : : Rad. : tan. 20.″5 = the angle at E, which is the amount of aberration when the direction of the ray of light is perpendicular to the earth's motion.

The effect of aberration upon the places of the fixed stars is to carry their apparent places a little forward of their real places in the direction of the earth's motion. The effect upon each particular star will be to make it describe a small ellipse in the heavens, having for its center the point in which the star would be seen if the earth were at rest.

MOTION OF THE APSIDES.

196. The two points of the ecliptic where the earth is at the greatest and least distances from the sun respectively, do not always maintain the same places among the signs, but gradually shift their positions from west to east. If we accurately observe the place among the stars, where the earth is at the time of its perihelion the present year, we shall find that it will not be precisely at that point the next year when it arrives at its perihelion, but about 12″ (11.″66) to the east of it. And since the equinox itself, from which longitude is reckoned, moves in the opposite direction 50.″1 annually, the longitude of the perihelion increases every year 61.″76, or a little more than one minute. This fact is expressed by saying that the line of the apsides of the earth's orbit has a slow motion from west to east, and completes one entire revolution in its own plane in about 100,000 years (111,149.)

The mean longitude of the perihelion at the commencement of the present century was 99° 30′ 5″, and of course in the ninth degree of Cancer, a little past the winter solstice. In the year 1248, the perihelion was at the place of this solstice; and since the increase of longitude is 61.″76 a year, hence,

$61.''76 : 1 :: 90° : 5246$ = the time occupied in passing from the first of Aries to the solstice. Hence, $5246 - 1248 = 3998$, which is the time before the Christian era, when the perigee was at the first of Aries. But this differs only 6 years from the time of the creation of the world, which is fixed by chronologists at 4004 years A. C. At the period of the creation, therefore, the line of the apsides of the earth's orbit, coincided with the line of the equinoxes.

197. The angular distance of a body from its aphelion is called its *Anomaly ;* and the interval between the sun's passing the point of the ecliptic corresponding to the earth's aphelion, and returning to the same point again, is called the *anomalistic year.* This period must be a little longer than the sidereal year, since, in order to complete the anomalistic revolution, the sun must traverse an arc of $11.''66$ in addition to 360°.

Now $360° : 365.256 :: 11.''66 : 4m.\ 44s.$

198. Since the points of the annual orbit, where the sun is at the greatest and least distances from the earth, change their position with respect to the solstices, a slow change is occasioned in the duration of the respective seasons. For, let the perihelion correspond to the place of the winter solstice, as was the case in the year 1248 ; then as the sun moves more rapidly in that part of his orbit, the winter months will be shorter than the summer. But, again, let the perihelion be at the summer solstice, as it will be in the year 6485* ; then the sun will move most rapidly through the summer months, and the winters will be longer than the summers. At present the perihelion is so near the winter solstice, that, the year being divided into two equal parts by the equinoxes, the six winter months are passed over between seven and eight days sooner than the summer months.

MEAN AND TRUE PLACES OF THE SUN.

199. *The Mean Motion of any body revolving in an orbit, is that which it would have if, in the same time, it revolved uniformly in a circle.*

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* Biot.

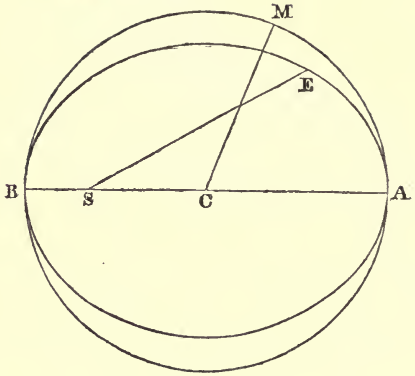
In surveying an irregular field, it is common first to strike out some regular figure, as a square or a parallelogram, by running long lines, and disregarding many small irregularities in the boundaries of the field. By this process, we obtain an approximation to the contents of the field, although we have perhaps thrown out several small portions which belong to it, and included a number of others which do not belong to it. These being separately estimated and added to or subtracted from our first computation, we obtain the true area of the field. In a similar manner, we proceed in finding the place of a heavenly body, which moves in an orbit more or less irregular. Thus we estimate the sun's distance from the vernal equinox for every day of the year at noon, on the supposition that he moves uniformly in a circular orbit: this is the sun's *mean longitude.* We then apply to this result various corrections for the irregularity of the sun's motions, and thus obtain the *true longitude.*

200. The corrections applied to the mean motions of a heavenly body, in order to obtain its true place, are called *Equations.* Thus the elliptical form of the earth's orbit, the precession of the equinoxes, and the nutation of the earth's axis, severally affect the place of the sun in his apparent orbit, for which equations are applied. In a collection of Astronomical Tables, a large part of the whole are devoted to this object. They give us the amount of the corrections to be applied under all the circumstances and constantly varying relations in which the sun, moon, and earth are situated with respect to each other. The angular distance of the earth or any planet from its aphelion, on the supposition that it moves uniformly in a circle, is called its *Mean Anomaly:* its actual distance at the same moment in its orbit is called its *True Anomaly.**

Thus in figure 44, let AEB represent the orbit of the earth having the sun in one of the foci at S. Upon AB describe the circle AMB. Let E be the place of the earth in its orbit, and M the corresponding place in the circle; then the angle MCA is the mean, and ESA the true anomaly. The difference between the

* In some astronomical treatises, the anomaly is reckoned from the perihelion.

Fig. 44.



mean and true anomaly, MCA−ESA, is called the *Equation of the Center*, being that correction which depends on the elliptical form of the orbit, or on the distance of the center of attraction from the center of the figure, that is, on the eccentricity of the orbit. It is much the greatest of all the corrections used in finding the sun's true longitude, amounting, at its maximum, to nearly two degrees ($1^\circ\ 55'\ 26.''8$.)

CHAPTER V.

OF THE MOON—LUNAR GEOGRAPHY—PHASES OF THE MOON—HER REVOLUTIONS.

201. Next to the Sun, the Moon naturally claims our attention. The Moon is an attendant or satellite to the earth, around which she revolves at the distance of nearly 240,000 miles. Her mean horizontal parallax being $57'\ 09''$, consequently, sin. $57'\ 09''$: semi-diameter of the earth (3956.2) : : rad. : 238,545. (Art. 87.)

The moon's *apparent diameter* is $31'\ 7''$, and her *real diameter* 2160 miles. For,

Rad. : 238,545 : : sin. $31'\ 7''$: 2160. (See Fig. 26, p. 70.)

And, since spheres are as the cubes of the diameters, their *volume* of the moon is $\frac{1}{49}$ that of the earth. Her *density* is nearly $\frac{2}{3}$ (.615) the density of the earth, and her *mass* $(=\frac{1}{49}\times.615)$ is about $\frac{1}{80}$.

202. The moon shines by *reflected light* borrowed from the sun, and when full, exhibits a disk of silvery brightness, diversified by extensive portions partially shaded. By the aid of the telescope, we see undoubted signs af a varied surface, composed of extensive tracts of level country, and numerous mountains and valleys.

203. The line which separates the enlightened from the dark portions of the moon's disk, is called the *Terminator*. (See Fig. 2. *Frontispiece.*) As the terminator traverses the disk from new to full moon, it appears through the telescope exceedingly broken in some parts, but smooth in others, indicating that portions of the lunar surface are uneven while others are level. The broken regions appear brighter than the smooth tracts. The latter have been taken for seas, but it is supposed with more probability that they are extensive plains, since they are still too uneven for the perfect level assumed by bodies of water. That there are *mountains* in the moon, is known by several distinct indications. First, when the moon is increasing, certain spots are illuminated sooner than the neighboring places, appearing like bright points beyond the terminator, within the dark path of the disk. (See Fig. 2. *Frontispiece.*) Secondly, after the terminator has passed over them, they project shadows upon the illuminated part of the disk, always opposite to the sun, corresponding in shape to the form of the mountain, and undergoing changes in length from night to night, according as the sun shines upon that part of the moon more or less obliquely. Many individual mountains rise to a great height in the midst of plains, and there are several very remarkable mountainous groups, extending from a common center in long chains.

204. That there are also *valleys* in the moon, is equally evident. The valleys are known to be truly such, particularly by the man-

ner in which the light of the sun falls upon them, illuminating the part opposite to the sun while the part adjacent is dark, as is the case when the light of a lamp shines obliquely into a china cup. These valleys are often remarkably regular, and some of them almost perfect circles. In several instances, a circular chain of mountains surrounds an extensive valley, which appears nearly level, except that a sharp mountain sometimes rises from the center. The best time for observing these appearances is near the first quarter of the moon, when half the disk is enlightened;* but in studying the lunar geography, it is expedient to observe the moon every evening from new to full, or rather through her entire series of changes.

205. The various places on the moon's disk have received appropriate *names*. The dusky regions, being formerly supposed to be seas, were named accordingly; and other remarkable places have each two names, one derived from some well known spot on the earth, and the other from some distinguished personage. Thus the same bright spot on the surface of the moon is called *Mount Sinai* or *Tycho*, and another, *Mount Etna* or *Copernicus*. The names of individuals, however, are more used than the others. The frontispiece exhibits the telescopic appearance of the full moon. A few of the most remarkable points have the following names, corresponding to the numbers and letters on the map. (See *Frontispiece.*)

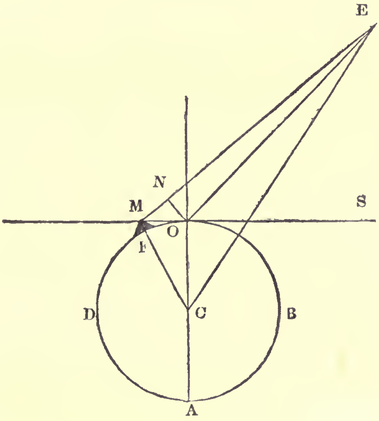
1. Tycho,	A. Mare Humorum,
2. Kepler,	B. Mare Nubium,
3. Copernicus,	C. Mare Imbrium,
4. Aristarchus,	D. Mare Nectaris,
5. Helicon,	E. Mare Tranquilitatis,
6. Eratosthenes,	F. Mare Serenitatis,
7. Plato,	G. Mare Fecunditatis,
8. Archimedes,	H. Mare Crisium.
9. Eudoxus,	
10. Aristotle,	

* It is earnestly recommended to the student of astronomy, to examine the moon repeatedly with the best telescope he can command, using low powers at first, for the sake of a better light.

206. *The method of estimating the height of lunar mountains is as follows.*

Let ABO (Fig. 45,) be the illuminated hemisphere of the moon, SO a solar ray touching the moon in O, a point in the circle which separates the enlightened from the dark part of the moon. All the part ODA will be in darkness; but if this part contains a mountain MF, so elevated that its summit M reaches to the solar ray SOM, the point M will be enlightened. Let E be the place of the observer on the earth, the moon being at any elongation from

Fig. 45.



the sun, as measured by the angle EOS. Draw the lines EM, EO, and CM, C being the center of the moon; and let FM be the height of the mountain. Draw ON perpendicular to EM. The line EO being known, and the angle OEM being measured with a micrometer, the value of ON, the projection of the line OM, becomes known. Now $OM=\frac{ON}{\cos. MON}$; and since OEN is a very small angle, EON may be considered as a right angle; consequently, $MON=MOE-90$. Therefore $OM=\frac{ON}{\cos.(MOE-90)}$ $=\frac{ON}{\sin. MOE}=\frac{ON}{\sin. EOS}$. That is, the distance between the summit of the mountain and the illuminated part of the moon's disk, is equal to the projected distance as measured by the micrometer, divided by the sine of the moon's elongation from the sun.

Suppose the distance $OM = nCO$, where n represents the fraction the part OM is of CO. Then, $CM^2 = CO^2 + OM^2 = CO^2 + n^2CO^2 = CO^2(1+n^2) \therefore CM = CO(1+n^2)^{\frac{1}{2}} \therefore CM - CO$ or $FM = CO(\sqrt{1+n^2} - 1) = \frac{n^2}{2}CO$, neglecting the higher powers of n, which would be of too little value to be worth taking into the account. The value of n has been found in one case equal to $\frac{1}{13}$ which gives the height of the mountain equal to $\frac{1}{338}$ the semi-diameter of the moon, that is, $3\frac{1}{5}$ miles.

When the moon is exactly at quadrature, then EOM becomes a right angle, and the value of OM is obtained directly from actual measurement ; and having CO and OM, we easily obtain CM and of course FM.

207. Schroeter, a German astronomer, estimated the heights of the lunar mountains by observations on their *shadows*. He made them in some cases as high as $\frac{1}{214}$ of the semi-diameter of the moon, that is, about 5 miles. The same astronomer also estimates the depths of some of the lunar valleys at more than four miles. Hence it is inferred that the moon's surface is more broken and irregular than that of the earth, its mountains being higher and its valleys deeper in proportion to the size of the moon than those of the earth.

208. Dr. Herschel is supposed also to have obtained decisive evidence of the existence of *volcanoes* in the moon, not only from the light afforded by their fires, but also from the formation of new mountains by the accumulation of matter where fires had been seen to exist, and which remained after the fires were extinct.

209. Some indications of an *atmosphere* about the moon have been obtained, the most decisive of which are derived from appearances of twilight, a phenomenon that implies the presence of an atmosphere. Similar indications have been detected, it is supposed, in eclipses of the sun, denoting a transparent refracting medium encompassing the moon.

210. The improbability of our ever identifying *artificial structures* in the moon, may be inferred from the fact that a line one

mile in length in the moon subtends an angle at the eye of only about one second. If, therefore, works of art were to have a sufficient horizontal extent to become visible, they can hardly be supposed to attain the necessary elevation, when we reflect that the height of the great pyramid of Egypt is less than the sixth part of a mile.

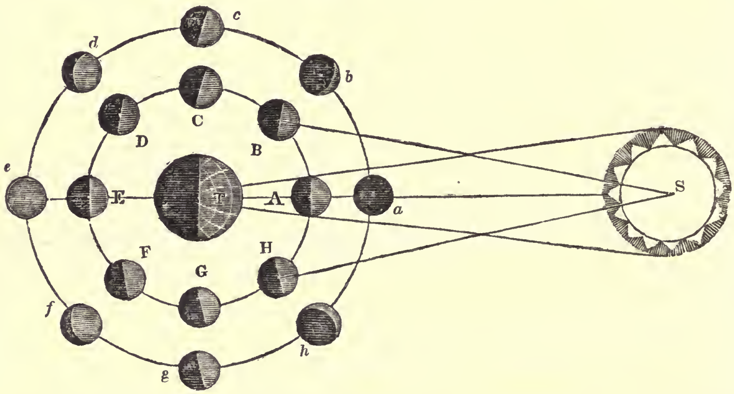
PHASES OF THE MOON.

211. The changes of the moon, commonly called her Phases, arise from different portions of her illuminated side being turned towards the earth at different times. When the moon is first seen after the setting sun, her form is that of a bright crescent, on the side of the disk next to the sun, while the other portions of the disk shine with a feeble light, reflected to the moon from the earth. Every night we observe the moon to be farther and farther eastward of the sun, and at the same time the crescent enlarges, until, when the moon has reached an elongation from the sun of 90°, half her visible disk is enlightened, and she is said to be in her *first quarter.* The terminator, or line which separates the illuminated from the dark part of the moon, is convex towards the sun from the new moon to the first quarter, and the moon is said to be *horned.* The extremities of the crescent are called *cusps.* At the first quarter, the terminator becomes a straight line, coinciding with a diameter of the disk; but after passing this point, the terminator becomes concave towards the sun, bounding that side of the moon by an elliptical curve, when the moon is said to be *gibbous.* When the moon arrives at the distance of 180° from the sun, the entire circle is illuminated, and the moon is full. She is then *in opposition* to the sun, rising about the time the sun sets. For a week after the full, the moon appears gibbous again, until, having arrived within 90° of the sun, she resumes the same form as at the first quarter, being then at her *third quarter.* From this time until new moon, she exhibits again the form of a crescent before the rising sun, until, approaching her *conjunction* with the sun, her narrow thread of light is lost in the solar blaze; and finally, at the moment of passing the sun, the dark side is wholly turned towards us and for some time we lose sight of the moon.

The two points in the orbit corresponding to new and full moon respectively, are called by the common name of *syzigies ;* those which are 90° from the sun are called *quadratures ;* and the points half way between the syzigies and quadratures are called *octants.* The circle which divides the enlightened from the unenlightened hemisphere of the moon, is called the *circle of illumination :* that which divides the hemisphere that is turned towards us from the hemisphere that is turned from us, is called the *circle of the disk.*

212. As the moon is an opake body of a spherical figure, and borrows her light from the sun, it is obvious that that half only which is towards the sun can be illuminated. More or less of this side is turned towards the earth, according as the moon is at a greater or less elongation from the sun. The reason of the different phases will be best understood from a diagram. Therefore let T (Fig. 46,) represent the earth, and S the sun. Let A, B, C, &c. be successive positions of the moon. At A the entire

Fig. 46.



dark side of the moon being turned towards the earth, the disk would be wholly invisible. At B, the circle of the disk cuts off a small part of the enlightened hemisphere, which appears in the heavens at *b*, under the form of a crescent. At C, the first quarter, the circle of the disk cuts off half the enlightened hemisphere, and the moon appears *dichotomized* at *c*. In like manner it will be seen that the appearances presented at D, E, F, &c. must be those represented at *d*, *e*, *f*.

REVOLUTIONS OF THE MOON.

213. *The moon revolves around the earth from west to east, making the entire circuit of the heavens in about $27\frac{1}{4}$ days.*

The precise law of the moon's motions in her revolution around the earth, is ascertained, as in the case of the sun, (Art. 155,) by daily observations on her meridian altitude and right ascension. Thence are deduced by calculation her latitude and longitude, from which we find, that the moon describes on the celestial sphere a great circle of which the earth is the center.

The period of the moon's revolution from any point in the heavens round to the same point again, is called a *month.* A *sidereal month* is the time of the moon's passing from any star, until it returns to the same star again. A *synodical month** is the time from one conjunction or new moon to another. The synodical month is about $29\frac{1}{2}$ days, or more exactly, 29d. 12h. 44m. $2^s.8 = 29.53$ days. The sidereal month is about two days shorter, being 27d. 7h. 43m. $11^s.5 = 27.32$ days. As the sun and moon are both revolving in the same direction, and the sun is moving nearly a degree a day, during the 27 days of the moon's revolution, the sun must have moved 27°. Now since the moon passes over 360° in 27.32 days, her daily motion must be 13° 17′. It must therefore evidently take about two days for the moon to overtake the sun. The difference between these two periods may, however, be determined with great exactness. The middle of an eclipse of the sun marks the exact moment of conjunction or new moon; and by dividing the interval between any two solar eclipses by the number of revolutions of the moon, or *lunations*, we obtain the precise period of the synodical month. Suppose, for example, two eclipses occur at an interval of 1,000 lunations; then the whole number of days and parts of a day that compose the interval divided by 1,000 will give the exact time of one lunation.† The time of the synodical month being ascertained, the exact period of the sidereal month may be

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* ϛυν and οδος, implying that the two bodies *come together.*

† It might at first view seem necessary to know the period of one lunation before we could know the number of lunations in any given interval. This period is known very nearly from the interval between one new moon and another.

derived from it. For the arc which the moon describes in order to come into conjunction with the sun, exceeds 360° by the space which the sun has passed over since the preceding conjunction, that is, 29.53 days. Therefore,

365.24 : 360° : : 29.53 : 29°.1 = arc which the moon must describe more than 360° in order to overtake the sun. Hence,

13° 17′ : 1d. : : 29°.1 : 2.21d. = difference between the sidereal and synodical months; and 29.53 − 2.21 = 27.32, the time of the sidereal revolution.

214. *The moon's orbit is inclined to the ecliptic in an angle of about* 5° (5° 8′ 48″). It crosses the ecliptic in two opposite points called her *nodes*. The amount of inclination is ascertained by observations on the moon's latitude when at a maximum, that being of course the greatest distance from the ecliptic, and therefore equal to the inclination of the two circles.

215. The moon, at the same age, crosses the meridian at different altitudes at different seasons of the year. The full moon, for example, will appear much farther in the south when on the meridian at one period of the year than at another. The reason of this may be explained as follows. When the sun is in the part of the ecliptic south of the equator, the earth and of course the moon, which always keeps near to the earth, is in the part north of the equator. At such times, therefore, the new moons being projected near the sun, will have great southern declination, as is the case during the winter months; but, in the summer, when the sun is towards the northern tropic and the earth towards the southern, the new moons run high and the full moons low. This arrangement gives us a great advantage in respect to the amount of light received from the moon; since the full moon is longest above the horizon during the long nights of winter, when her presence is most needed. This circumstance is especially favorable to the inhabitants of the polar regions, the moon, when full, traversing that part of her orbit which lies north of the equator, and of course above the horizon of the north pole, and traversing the portion that lies south of the equator, and below the polar horizon, when new. During the polar winter, therefore, the moon, from the first

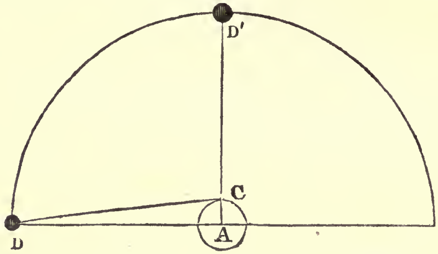
to the last quarter, is commonly above the horizon, while the sun is absent; whereas, during summer, while the sun is present, the moon is above the horizon while describing her first and last quadrants.

216. About the time of the autumnal equinox, the moon when near the full, rises about sunset for a number of nights in succession; and as this is, in England, the period of harvest, the phenomenon is called the *Harvest Moon*. To understand the reason of this, since the moon is never far from the ecliptic, we will suppose her progress to be in the ecliptic. If the moon moved in the equator, then, since this great circle is at right angles to the axis of the earth, all parts of it, as the earth revolves, cut the horizon at the same constant angle. But the moon's orbit, or the ecliptic, which is here taken to represent it, being oblique to the equator, cuts the horizon at different angles in different parts, as will easily be seen by reference to an artificial globe. When the first of Aries, or vernal equinox, is in the eastern horizon, it will be seen that the ecliptic, (and consequently the moon's orbit,) makes its least angle with the horizon. Now, at the autumnal equinox, the sun being in Libra, the moon at the full is in Aries, and rises when the sun sets. On the following evening, although she has advanced in her orbit about 13°, (Art. 213,) yet her progress being oblique to the horizon, and at a small angle with it, she will be found at this time but a little way below the horizon, compared with the point where she was at sunset the preceding evening. She therefore rises but little later, and so for a week only a little later each evening than she did the preceding night.

217. *The moon is about $\frac{1}{60}$ nearer to us when near the zenith than when in the horizon.*

The horizontal distance CD (Fig. 47,) is nearly equal to AD = AD′, which is greater than CD′ by AC, the semi-diameter of the earth = $\frac{1}{60}$ the distance of the moon. Accordingly, the apparent diameter of the moon, when actually measured, is about 30″ (which equals about $\frac{1}{60}$ of the whole) greater when in the zenith than in the horizon. The apparent enlargement of the full moon when rising, is owing to the same causes as that of the sun, as explained in article 96.

Fig. 47.



218. *The moon turns on its axis in the same time in which it revolves around the earth.*

This is known by the moon's always keeping nearly the same face towards us, as is indicated by the telescope, which could not happen unless her revolution on her axis kept pace with her motion in her orbit. Thus, it will be seen by inspecting figure 31, that the earth turns different faces towards the sun at different times; and if a ball having one hemisphere white and the other black be carried around a lamp, it will easily be seen that it cannot present the same face constantly towards the lamp unless it turns once on its axis while performing its revolution. The same thing will be observed when a man walks around a tree, keeping his face constantly towards it. Since however the motion of the moon on its axis is uniform, while the motion in its orbit is unequal, the moon does in fact reveal to us a little sometimes of one side and sometimes of the other. Thus when the ball above mentioned is placed before the eye with its light side towards us, or carrying it round, if it is moved faster than it is turned on its axis, a portion of the dark hemisphere is brought into view on one side; or if it is moved forward slower than it is turned on its axis, a portion of the dark hemisphere comes into view on the other side.

219. These appearances are called the moon's *librations in longitude.* The moon has also a *libration in latitude,* so called, because in one part of her revolution, more of the region around one of the poles comes into view, and in another part of the rev-

olution, more of the region around the other pole; which gives the appearance of a tilting motion to the moon's axis. This has nearly the same cause with that which occasions our change of seasons. The moon's axis being inclined to the plane of her orbit, and always remaining parallel to itself, the circle which divides the visible from the invisible part of the moon, will pass in such a way as to throw sometimes more of one pole into view, and sometimes more of the other, as would be the case with the earth if seen from the sun. (See Fig. 31.)

The moon exhibits another phenomenon of this kind called her *diurnal libration*, depending on the daily rotation of the spectator. She turns the same face towards the center of the earth only, whereas we view her from the surface. When she is on the meridian, we see her disk nearly as though we viewed it from the center of the earth, and hence in this situation it is subject to little change; but when near the horizon, our circle of vision takes in more of the upper limb than would be presented to a spectator at the center of the earth. Hence, from this cause, we see a portion of one limb while the moon is rising, which is gradually lost sight of, and we see a portion of the opposite limb as the moon declines towards the west. It will be remarked that neither of the foregoing changes implies any actual motion in the moon, but that each arises from a change of position in the spectator.

220. An inhabitant of the moon would have but one day and one night during the whole lunar month of 29½ days. One of its days, therefore, is equal to nearly 15 of ours. So protracted an exposure to the sun's rays, especially in the equatorial regions of the moon, must occasion an excessive accumulation of heat; and so long an absence of the sun must occasion a corresponding degree of cold. Each day would be a wearisome summer; each night a severe winter.* A spectator on the side of the moon which is opposite to us would never see the earth; but one on the side next to us would see the earth presenting a gradual succession of changes during his long night of 360 hours. Soon after the earth's conjunction with the sun, he would have the light of the

* Francœur, Uranog. p. 91.

earth reflected to him, presenting at first a crescent, but enlarging, as the earth approaches its opposition, to a great orb, 13 times as large as the full moon appears to us, and affording nearly 13 times as much light. Our seas, our plains, our mountains, our volcanoes, and our clouds, would produce very diversified appearances, as would the various parts of the earth brought successively into view by its diurnal rotation. The earth while in view to an inhabitant of the moon, would *remain immovably fixed in the same part of the heavens.* For being unconscious of his own motion around the earth, as we are of our motion around the sun, the earth would seem to revolve around his own planet from west to east; but, meanwhile, his rotation along with the moon on her axis, would cause the earth to have an apparent motion westward at the same rate. The two motions, therefore, would exactly balance each other, and the earth would appear all the while at rest. The earth is full to the moon when the latter is new to us; and universally the two phases are complementary to each other.*

221. It has been observed already, (Art. 214,) that the moon's orbit crosses the ecliptic in two opposite points called the *nodes.* That which the moon crosses from south to north, is called the *ascending node*; that which the moon crosses from north to south, the *descending node.*

From the manner in which the figure representing the earth's orbit and that of the moon, is commonly drawn, the learner is sometimes puzzled to see how the orbit of the moon can cut the ecliptic in two points directly opposite to each other. But he must reflect that the lunar orbit cuts the *plane* of the ecliptic and not the earth's path in that plane, although these respective points are projected upon that path in the heavens.

222. We have thus far contemplated the revolution of the moon around the earth as though the earth were at rest. But, in order to have just ideas respecting the moon's motions, we must recollect that the moon likewise revolves along with the earth around the sun. It is sometimes said that the earth *carries* the moon along with her in her annual revolution. This language may

* Francœur, p. 92.

convey an erroneous idea ; for the moon, as well as the earth, revolves around the sun under the influence of two forces, and would continue her motion around the sun, were the earth removed out of the way. Indeed, the moon is attracted towards the sun $2\frac{1}{5}$ times more than towards the earth,* and would abandon the earth were not the latter also carried along with her by the same forces. So far as the sun acts equally on both bodies, their motion with respect to each other would not be disturbed. Because the gravity of the moon towards the sun is found to be greater, at the conjunction, than her gravity towards the earth, some have apprehended that, if the doctrine of universal gravitation is true, the moon ought necessarily to abandon the earth. In order to understand the reason why it does not do thus, we must reflect, that when a body is revolving in its orbit under the action of the projectile force and gravity, whatever diminishes the force of gravity while that of projection remains the same, causes the body to recede from the center ; and whatever increases the amount of gravity carries the body towards the center. Now, when the moon is in conjunction, her gravity towards the earth acts in opposition to that towards the sun, while her velocity remains too great to carry her, with what force remains, in a circle about the sun, and she therefore recedes from the sun, and commences her revolution around the earth. On arriving at the opposition, the gravity of the earth conspires with that of the sun, and the moon's projectile force being less than that required to make her revolve in a circular orbit, when attracted towards the sun by the sum of these forces, she accordingly begins to approach the sun and descends again to the conjunction.†

223. The attraction of the sun, however, being every where greater than that of the earth, the actual path of the moon around

* It is shown by writers on Mechanics, that the forces with which bodies revolving in circular orbits tend towards their centers, are as the radii of their orbits divided by the squares of their periodical times. Hence, supposing the orbits of the earth and the moon to be circular, (and their slight eccentricity will not much affect the result,) we have

$$G : G' :: \frac{400}{(365.25)^2} : \frac{1}{(27.32)^2} :: 2.2 : 1.$$

† M'Laurin's Discoveries of Newton, B. IV, ch. 5.

the sun is every where concave towards the latter. Still the elliptical path of the moon around the earth, is to be conceived of in the same way as though both bodies were at rest with respect to the sun. Thus, while a steamboat is passing swiftly around an island, and a man is walking slowly around a post in the cabin, the line which he describes in space between the forward motion of the boat and his circular motion around the post, may be every where concave towards the island, while his path around the post will still be the same as though both were at rest. A nail in the rim of a coach wheel, will turn around the axis of the wheel, when the coach has a forward motion in the same manner as when the coach is at rest, although the line actually described by the nail will be the resultant of both motions, and very different from either.

CHAPTER VI.

LUNAR IRREGULARITIES.

224. We have hitherto regarded the moon as describing a great circle on the face of the sky, such being the visible orbit as seen by projection. But, on more exact investigation, it is found that her orbit is not a circle, and that her motions are subject to very numerous irregularities. These will be best understood in connection with the causes on which they depend. The law of universal gravitation has been applied with wonderful success to their investigation, and its results have conspired with those of long continued observation, to furnish the means of ascertaining with great exactness the place of the moon in the heavens at any given instant of time, past or future, and thus to enable astronomers to determine longitudes, to calculate eclipses, and to solve various other problems of the highest interest. A complete understanding of all the irregularities of the moon's motions, must be sought for in more extensive treatises of astronomy than the present; but some general acquaintance with the subject, clear

and intelligible as far as it goes, may be acquired by first gaining a distinct idea of the mutual actions of the sun, the moon, and the earth.

225. *The irregularities of the moon's motions, are due chiefly to the disturbing influence of the sun, which operates in two ways; first, by acting unequally on the earth and moon, and, secondly, by acting obliquely on the moon, on account of the inclination of her orbit to the ecliptic.**

If the sun acted equally on the earth and moon, and always in parallel lines, this action would serve only to restrain them in their annual motions round the sun, and would not affect their actions on each other, or their motions about their common center of gravity. In that case, if they were allowed to fall directly towards the sun, they would fall equally, and their respective situations would not be affected by their descending equally towards it. We might then conceive them as in a plane, every part of which being equally acted on by the sun, the whole plane would descend towards the sun, but the respective motions of the earth and the moon in this plane, would be the same as if it were quiescent. Supposing then this plane and all in it, to have an annual motion imprinted on it, it would move regularly round the sun, while the earth and moon would move in it with respect to each other, as if the plane were at rest, without any irregularities. But because the moon is nearer the sun in one half of her orbit than the earth is, and in the other half of her orbit is at a greater distance than the earth from the sun, while the power of gravity is always greater at a less distance; it follows, that in one half of her orbit the moon is more attracted than the earth towards the sun, and in the other half less attracted than the earth. The *excess* of the attraction, in the first case, and the *defect* in the second constitutes a disturbing force, to which we may add another, namely, that arising from the *oblique action* of the solar force, since this action is not directed in parallel lines, but in lines that meet in the center of the sun.

* M'Laurin's Discoveries of Newton, B. IV, ch. 4. La Place's Syst. du Monde, B. IV, ch. 5.

226. To see the effects of this process, let us suppose that the projectile motions of the earth and moon were destroyed, and that they were allowed to fall freely towards the sun. If the moon was in conjunction with the sun, or in that part of her orbit which is nearest to him, the moon would be more attracted than the earth, and fall with greater velocity towards the sun; so that the distance of the moon from the earth would be increased in the fall. If the moon was in opposition, or in the part of her orbit which is farthest from the sun, she would be less attracted than the earth by the sun, and would fall with a less velocity towards the sun, and would be left behind; so that the distance of the moon from the earth would be increased in this case also. If the moon was in one of the quarters, then the earth and moon being both attracted towards the center of the sun, they would both descend directly towards that center, and by approaching it, they would necessarily at the same time approach each other, and in this case their distance from each other would be diminished. Now whenever the action of the sun would increase their distance, if they were allowed to fall towards the sun, then the sun's action, by endeavoring to separate them, diminishes their gravity to each other; whenever the sun's action would diminish the distance, then it increases their mutual gravitation. Hence, in the conjunction and opposition, that is, *in the syzigies*, their gravity towards each other is diminished by the action of the sun, while in the quadratures it is increased. But it must be remembered that it is not the total action of the sun on them that disturbs their motions, but only that part of it which tends at one time to separate them, and at another time to bring them nearer together. The other and far greater part, has no other effect than to retain them in their annual course around the sun.

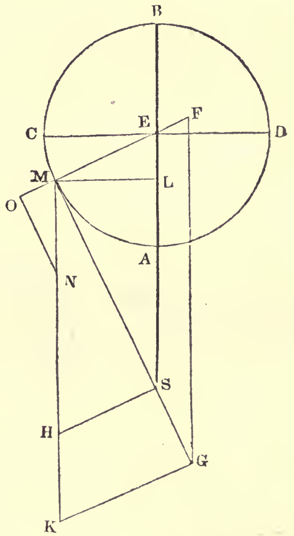
227. Suppose the moon setting out from the quarter that precedes the conjunctions with a velocity that would make her describe an exact circle round the earth, if the sun's action had no effect on her: since her gravity is increased by that action, she must descend towards the earth and move within that circle. Her orbit then would be more curved than it otherwise would have been; because the addition to her gravity will make her

fall farther at the end of an arc below the tangent drawn at the other end of it. Her motion will be thus accelerated, and it will continue to be accelerated until she arrives at the ensuing conjunction, because the direction of the sun's action upon her, during that time, makes an acute angle with the direction of her motion. (See Fig. 41.) At the conjunction, her gravity towards the earth being diminished by the action of the sun, her orbit will then be less curved, and she will be carried farther from the earth as she moves to the next quarter; and because the action of the sun makes there an obtuse angle with the direction of her motion, she will be retarded in the same degree as she was accelerated before.

228. After this *general* explanation of the mode in which the sun acts as a disturbing force on the motions of the moon, the learner will be prepared to understand the mathematical development of the same doctrine.

Therefore, let ADBC (Fig. 48,) be the orbit, nearly circular, in which the moon M revolves in the direction CADB, round the earth E. Let S be the sun, and let SE the radius of the earth's orbit, be taken to represent the force with which the earth gravitates to the sun. Then (Art. 180,) $\frac{1}{SE^2} : \frac{1}{SM^2} :: SE : \frac{SE^3}{SM^2}$ = the force by which the sun draws the moon in the direction MS. Take $MG = \frac{SE^3}{SM^2}$, and let the parallelogram KF be described, having MG for its diagonal, and having its sides parallel to EM and ES. The force MG may be resolved into two, MF and MK, of which MF, directed towards E, the center of the earth, increases the gravity of the moon to the earth, and does not hinder the areas described by the radius vector from being proportional to the

Fig. 48.



times. The other force MK draws the moon in the direction of the line joining the centers of the sun and earth. It is, however, only the excess of this force, above the force represented by SE, or that which draws the earth to the sun, which disturbs the relative position of the moon and earth. This is evident, for if KM were just equal to ES, no disturbance of the moon relative to the sun could arise from it. If then ES be taken from MK, the difference HK is the whole force in the direction parallel to SE, by which the sun disturbs the relative position of the moon and earth. Now, if in MK, MN be taken equal to HK, and if NO be drawn perpendicular to the radius vector EM produced, the force MN may be resolved into two, MO and ON, the first lessening the gravity of the moon to the earth; and the second, being parallel to the tangent of the moon's orbit in M, accelerates the moon's motion from C to A, and retards it from A to D, and so alternately in the other two quadrants. Thus the whole solar force directed to the center of the earth, is composed of the two parts MF and MO, which are sometimes opposed to one another, but which never affect the uniform description of the areas about E. Near the quadratures the force MO vanishes, and the force MF, which increases the gravity of the moon to the earth, coincides with CE or DE. As the moon approaches the conjunction at A, the force MO prevails over MF, and lessens the gravity of the moon to the earth. In the opposite point of the orbit, when the moon is in opposition at B, the force with which the sun draws the moon is less than that with which the sun draws the earth, so that the effect of the solar force is to separate the moon and earth, or to increase their distance; that is, it is the same as if, conceiving the earth not to be acted on, the sun's force drew the moon in the direction from E to B. This force is negative, therefore, in respect to the force at A, and the effect in both cases is to draw the moon from the earth in a direction perpendicular to the line of the quadratures. Hence, the general result is, that by the disturbing force of the sun, *the gravity to the earth is increased at the quadratures, and diminished at the syzigies.* It is found by calculation that the average amount of this disturbing force is $\frac{1}{583}$ of the moon's gravity to the earth.*

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* Playfair.

229. With these general principles in view, we may now proceed to investigate the figure of the moon's orbit, and the irregularities to which the motions of this body are subject.

230. *The figure of the moon's orbit is an ellipse, having the earth in one of the foci.*

The elliptical figure of the moon's orbit, is revealed to us by observations on her changes in apparent diameter, and in her horizontal parallax. First, we may measure from day to day the apparent diameter of the moon. Its variations being inversely as the distances (Art. 163,) they give us at once the *relative* distance of each point of observation from the focus. Secondly, the variations on the moon's horizontal parallax, which also are inversely as the distances, (Art. 82,) lead to the same results. Observations on the angular velocities, combined with the changes in the lengths of the radius vector, afford the means of laying down a plot of the lunar orbit, as in the case of the sun, represented in figure 32. The orbit is shown to be nearly an ellipse, because it is found to have the properties of an ellipse.

The moon's greatest and least apparent diameters are respectively 33′.518 and 29′.365, while her corresponding changes of parallax are 61′.4 and 53′.8. The two ratios ought to be equal, and we shall find such to be the fact very nearly; for,

$$61.4 : 53.8 :: 33.518 : 29.369.$$

The greatest and least distances of the moon from the earth, derived from the parallaxes, are 63.8419 and 55.9164, or nearly 64 and 56, the radius of the earth being taken for unity. Hence, taking the arithmetical mean, which is 59.879, we find that the mean distance of the moon from the earth is very nearly 60 times the radius of the earth. The point in the moon's orbit nearest the earth, is called her *perigee;* the point farthest from the earth, her *apogee.*

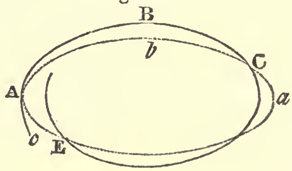
The greatest and least apparent diameters of the *sun* are respectively 32.583, and 31.517, which numbers express also the ratio of the greatest and least distances of the earth from the sun. By comparing this ratio with that of the distances of the moon, it will be seen that the latter vary much more than the former, and consequently that the lunar orbit is much more eccentric than the solar. The eccentricity of the moon's orbit is in fact 0.0548, (the semi-major axis being as usual taken for unity) $=\frac{1}{18}$ of its mean distance from the earth, while that of the earth is only $.01685=\frac{1}{59}$ of its mean distance from the sun.

231. *The moon's nodes constantly shift their positions in the ecliptic from east to west, at the rate of* 19° 35′ *per annum, returning to the same points in* 18.6 *years.*

Suppose the great circle of the ecliptic marked out on the face of the sky in a distinct line, and let us observe, at any given time, the exact point where the moon crosses this line, which we will suppose to be close to a certain star; then, on its next return to that part of the heavens, we shall find that it crosses the ecliptic sensibly to the westward of that star, and so on, farther and farther to the westward every time it crosses the ecliptic at either node. This fact is expressed by saying that *the nodes retrograde on the ecliptic*, and that the line which joins them, or the line of the nodes, revolves from east to west.

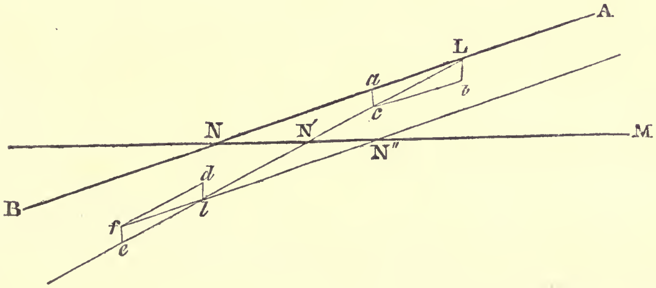
232. This shifting of the moon's nodes implies that the lunar orbit is not a curve returning into itself, but that it more resembles a spiral like the curve represented in figure 49, where *abc* represents the ecliptic, and ABC the lunar orbit, having its nodes at C and E, instead of A and *a*; consequently, the nodes shift backwards through the arcs *a*C and AE. The manner in which this effect is produced may be thus explained. That part of the solar force which is parallel to the line joining the centers of the sun and earth, (See Fig. 48,) is not in the plane of the moon's orbit, (since this is inclined to the ecliptic about 5°,) except when the sun itself is in that plane, or when the line of the nodes being produced, passes through the sun. In all other cases it is oblique to the plane of the orbit, and may be resolved into two forces, one of which is at right angles to that plane, and is directed towards the ecliptic. This force of course draws the moon continually towards the ecliptic, or produces a continual deflection of the moon from the plane of her own orbit towards that of the

Fig. 49.



earth. Hence the moon meets the plane of the ecliptic sooner than it would have done if that force had not acted. At every half revolution, therefore, the point in which the moon meets the ecliptic, shifts in a direction contrary to that of the moon's motion, or contrary to the order of the signs. If the earth and sun were at rest, the effect of the deflecting force just described, would be to produce a retrograde motion of the line of the nodes till that line was brought to pass through the sun, and of consequence, the plane of the moon's orbit to do the same, after which they would both remain in their position, there being no longer any force tending to produce change in either. But the motion of the earth carries the line of the nodes out of this position, and produces, by that means, its continual retrogradation. The same force produces a small variation in the inclination of the moon's orbit, giving it an alternate increase and decrease within very narrow limits.* These points will be easily understood by the aid of a diagram. Therefore, let MN (Fig. 50,) be the ecliptic, ANB the orbit of the moon, the moon being in L, and N its descending node. Let the disturbing force of the sun which tends to

Fig. 50.



bring it down to the ecliptic be represented by L*b*, and its velocity in its orbit by L*a*. Under the action of these two forces, the moon will describe the diagonal L*c* of the parallelogram *ba*, and its orbit will be changed from AN to LN′; the node N passes to N′; and the exterior angle at N′ of the triangle LNN′ being greater than the interior and opposite angle at N, the inclination of the orbit is increased at the node. After the moon has passed the ecliptic to the south side to *l*, the disturbing force *ld* produces a

* Playfair.

new change of the orbit $N'le$ to $N''lf$, and the inclination is diminished as at N''. In general, while the moon is receding from one of the nodes, its inclination is diminishing; while it is approaching a node, the inclination is increasing.*

233. The period occupied by the sun in passing from one of the moon's nodes until it comes round to the same node again, is called *the synodical revolution of the node.* This period is shorter than the sidereal year, being only about 346½ days. For since the node shifts its place to the westward 19° 35′ per annum, the sun, in his annual revolution, comes to it so much before he completes his entire circuit; and since the sun moves about a degree a day, the synodical revolution of the node is $365-19=346$, or more exactly, 346.619851. The time from one new moon, or from one full moon, to another, is 29.5305887 days. Now 19 synodical revolutions of the nodes contain very nearly 223 of these periods.

$$\text{For } 346.619851 \times 19 = 6585.78,$$
$$\text{And } 29.5305887 \times 223 = 6585.32.$$

Hence, if the sun and moon were to leave the moon's node together, after the sun had been round to the same node 19 times, the moon would have performed very nearly 223 synodical revolutions, and would, therefore, at the end of this period meet at the same node, to repeat the same circuit. And since eclipses of the sun and moon depend upon the relative position of the sun, the moon, and node, these phenomena are repeated in nearly the same order, in each of those periods. Hence, this period, consisting of about 18 years and 10 days, under the name of the *Saros*, was used by the Chaldeans and other ancient nations in predicting eclipses.

234. The *Metonic Cycle* is not the same with the Saros, but consists of 19 tropical years. During this period the moon makes very nearly 235 synodical revolutions, and hence the *new and full moons*, if reckoned by periods of 19 years, recur at the same dates. If, for example, a new moon fell on the fiftieth day of one cycle, it would also fall on the fiftieth day of each succeed-

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* Francœur, Uranog. p. 158.—Robison's Phys. Astronomy, Art. 264.

ing cycle; and, since the regulation of games, feasts, and fasts, has been made very extensively according to new or full moons, hence this lunar cycle has been much used both in ancient and modern times. The Athenians adopted it 433 years before the Christian era, for the regulation of their calendar, and had it inscribed in letters of gold on the walls of the temple of Minerva. Hence the term *Golden Number*, which denotes the year of the lunar cycle.

235. *The line of the apsides of the moon's orbit revolves from west to east through her whole orbit in about nine years.*

If, in any revolution of the moon, we should accurately mark the place in the heavens where the moon comes to its perigee, (Art. 230,) we should find, that at the next revolution, it would come to its perigee at a point a little farther eastward than before, and so on at every revolution, until, after 9 years, it would come to its perigee at nearly the same point as at first. This fact is expressed by saying that the perigee, and of course the apogee, revolves, and that the line which joins these two points, or the line of the apsides, also revolves.

The place of the perigee may be found by observing when the moon has the greatest apparent diameter. But as the magnitude of the moon varies slowly at this point, a better method of ascertaining the position of the apsides, is to take two points in the orbit where the variations in apparent diameter are most rapid, and to find where they are equal on opposite sides of the orbit. The middle point between the two will give the place of the perigee.

The angular distance of the moon from her perigee in any part of her revolution, is called the *Moon's Anomaly.*

236. The change of place in the apsides of the moon's orbit, like the shifting of the nodes, is caused by the disturbing influence of the sun. If when the moon sets out from its perigee, it were urged by no other force than that of projection, combined with its gravitation towards the earth, it would describe a symmetrical curve (Art. 186,) coming to its apogee at the distance of 180°. But as the mean disturbing force in the direction of the radius

vector tends, on the whole, to diminish the gravitation of the moon to the earth, the portion of the path described in an instant will be less deflected from her tangent, or less curved, than if this force did not exist. Hence the path of the moon will not intersect the radius vector at right angles; that is, she will not arrive at her apogee until after passing more than 180° from her perigee, by which means these points will constantly shift their positions from west to east.* The motion of the apsides is found to be 3° 1′ 20″ for every sidereal revolution of the moon.

237. On account of the greater eccentricity of the moon's orbit above that of the sun, the *Equation of the Center*, or that correction which is applied to the moon's mean anomaly to find her true anomaly (Art. 200,) is much greater than that of the sun, being when greatest more than six degrees, (6° 17′ 12″.7,) while that of the sun is less than two degrees, (1° 55′ 26″.8.)

238. Next to the equation of the center, the greatest correction to be applied to the moon's longitude, is that which belongs to the *Evection*. The evection is *a change of form in the lunar orbit*, by which its eccentricity is sometimes increased, and sometimes diminished. It depends on the position of the line of the apsides with respect to the line of the syzigies.

This irregularity, and its connexion with the place of the perigee with respect to the place of conjunction or opposition, was known *as a fact* to the ancient astronomers, Hipparchus and Ptolemy; but its *cause* was first explained by Newton in conformity with the law of universal gravitation. It was found, by observation, that the equation of the center itself was subject to a periodical variation, being greater than its mean, and greatest of all when the conjunction or opposition takes place at the perigee or apogee, and least of all when the conjunction or opposition takes place at a point half way between the perigee and apogee; or, in the more common language of astronomers, the equation of the center is increased when the line of the apsides is in syzigy, and diminished when that line is in quadrature. If, for example, when the line of the apsides is in syzigy, we compute the moon's

* Playfair.

place by deducting the equation of the center from the mean anomaly (See Art. 200,) seven days after conjunction, the computed longitude will be greater than that determined by actual observation, by about 80 minutes; but if the same estimate is made when the line of the apsides is in quadrature, the computed longitude will be less than the observed, by the same quantity. These results plainly show a connexion between the velocity of the moon's motions and the position of the line of the apsides with respect to the line of the syzigies.

239. Now any cause which, at the perigee, should have the effect to increase the moon's gravitation towards the earth beyond its mean, and, at the apogee, to diminish the moon's gravitation towards the earth, would augment the difference between the gravitation at the perigee and at the apogee, and consequently increase the eccentricity of the orbit. Again, any cause which at the perigee should have the effect to lessen the moon's gravitation towards the earth, and, at the apogee, to increase it, would lessen the difference between the two, and consequently diminish the eccentricity of the orbit, or bring it nearer to a circle. Let us see if the disturbing force of the sun produces these effects. The sun's disturbing force, as we have seen in article 228, admits of two resolutions, one in the direction of the radius vector, (OM, Fig. 48,) the other (ON) in the direction of a tangent to the orbit. First, let AB be the line of the apsides in syzigy, A being the place of the perigee. The sun's disturbing force OM is greatest in the direction of the line of the syzigies; yet being proportional to the distance of the moon from the earth, it is at a minimum when acting at the perigee, and at a maximum when acting at the apogee. Hence its effect is to draw away the moon from the earth less than usual at the perigee, and of course to make her gravitation towards the earth greater than usual, while at the apogee its effect is to diminish the tendency of the moon to the earth more than usual, and thus to increase the disproportion between the two distances of the moon from the focus at these two points, and of course to increase the eccentricity of the orbit. The moon, therefore, if moving towards the perigee, is brought to the line of the apsides in a point between its mean place and

the earth ; or if moving towards the apogee, she reaches the line of the apsides in a point more remote from the earth than its mean place.

Secondly, let CD be the line of the apsides, in quadrature, C being the place of the perigee. The effect of the sun's disturbing force is to increase the tendency of the moon towards the earth when in quadrature. If, however, the moon is then at her perigee, such increase will be less than usual, and if at her apogee, it will be more than usual ; hence its effect will be to lessen the disproportion between the two distances of the moon from the forces at these two points ; and of course to diminish the eccentricity of the orbit. The moon, therefore, if moving towards the perigee, meets the line of the apsides in a point more remote from the earth than the mean place of the perigee ; and if moving towards the apogee, in a point between the earth and the mean place of the apogee.*

240. A third inequality in the lunar motions, is the *Variation.* By comparing the moon's place as computed from her mean motion corrected for the equation of the center and for evection, with her place as determined by observation, Tycho Brahe discovered that the computed and observed places did not always agree. They agreed only in the syzigies, and disagreed most at a point half way between, that is, at the *octants.* Here, at the maximum, it amounted to more than half a degree (35′ 41.″6.) It appeared evident from examining the daily observations while the moon is performing her revolution around the earth, that this inequality is connected with the angular distance of the moon from the sun, and in every part of the orbit could be correctly expressed by multiplying the maximum value as given above into the sine of twice the angular distance between the sun and the moon. It is, therefore, 0 at the conjunctions and quadratures, and greatest at the octants. Tycho Brahe knew the *fact:* Newton investigated the *cause.*

It appears by article 228, that the sun's disturbing force can be resolved into two parts,—one in the direction of the radius vector, the other at right angles to it in the direction of a tangent to the

* Woodhouse's Ast. p. 680.

moon's orbit. The former, as already explained, produces the Evection: the latter produces the Variation. This latter force will accelerate the moon's velocity, in every point of the quadrant which the moon describes in moving from quadrature to conjunction, or from C to A, (Fig. p. 127,) but at an unequal rate, the acceleration being greatest at the octant, and nothing but at the quadrature and the conjunction; and when the moon is past conjunction, the tangential force will change its direction and retard the moon's motion. All these points will be understood by inspection of figure 48.

241. A fourth lunar inequality is the *Annual Equation.* This depends on the distance of the earth (and of course the moon) from the sun. Since the disturbing influence of the sun has a greater effect in proportion as the sun is nearer,* consequently all the inequalities depending on this influence must vary at different seasons of the year. Hence, the amount of this effect due to any particular time of the year is called the Annual Equation.

242. The foregoing are the largest of the inequalities of the moon's motions, and may serve as *specimens* of the corrections that are to be applied to the mean place of the moon in order to find her true place. These were first discovered by actual observation; but a far greater number, though most of them are exceedingly minute, have been made known by the investigations of Physical Astronomy, in following out all the consequences of universal gravitation. In the best tables, about 30 equations are applied to the mean motions of the moon. That is, we first compute the place of the moon on the supposition that she moves uniformly in a circle. This gives us her *mean* place. We then proceed, by the aid of the Lunar Tables, to apply the different corrections, such as the equation of the center, evection, variation, the annual equation, and so on, to the number of 28. Numerous as these corrections appear, yet La Place informs us, that the whole number belonging to the moon's longitude is no less than 60; and that to give the tables all the requisite degree of precision, additional investigations will be necessary, as extensive at least as

* Varying reciprocally as the *cube* of the sun's distance from the earth.

those already made.* The best tables in use in the time of Tycho Brahe, gave the moon's place only by a distant approximation. The tables in use in the time of Newton, (Halley's tables,) approximated within 7 minutes. Tables at present in use give the moon's place to 5 seconds. These additional degrees of accuracy have been attained only by immense labor, and by the united efforts of Physical Astronomy, and the most refined observations.

243. The inequalities of the moon's motions are divided into periodical and secular. *Periodical* inequalities are those which are completed in comparatively short periods, like evection and variation: *Secular* inequalities are those which are completed only in very long periods, such as centuries or ages. Hence the corresponding terms *periodical equations*, and *secular equations.* As an example of a secular inequality, we may mention the *acceleration of the moon's mean motion.* It is discovered, that the moon actually revolves around the earth in less time now than she did in ancient times. The difference however is exceedingly small, being only about 10″ in a century, but increases from century to century as the square of the number of centuries. This remarkable fact was discovered by Dr. Halley.† In a lunar eclipse the moon's longitude differs from that of the sun, at the middle of the eclipse, by exactly 180°; and since the sun's longitude at any given time of the year is known, if we can learn the day and hour when an eclipse occurs, we shall of course know the longitude of the sun and moon. Now in the year 721 before the Christian era, on a specified day and hour, Ptolemy records a lunar eclipse to have happened, and to have been observed by the Chaldeans. The moon's longitude, therefore, for that time is known; and as we know the mean motions of the moon at present, starting from that epoch, and computing, as may easily be done, the place which the moon ought to occupy at present at any given time, she is found to be actually nearly a degree and a half in advance of that place. Moreover, the same conclusion is derived from a comparison of the Chaldean observations with those made by an Arabian astronomer of the tenth century.

* Syst. du Monde, l. iv, c. 5.

† Astronomer Royal of Great Britain, and cotemporary with Sir Isaac Newton

This phenomenon at first led astronomers to apprehend that the moon encountered a resisting medium, which, by destroying at every revolution a small portion of her projectile force, would have the effect to bring her nearer and nearer to the earth and thus to augment her velocity. But in 1786, La Place demonstrated that this acceleration is one of the legitimate effects of the sun's disturbing force, and is so connected with changes in the eccentricity of the earth's orbit, that the moon will continue to be accelerated while that eccentricity diminishes, but when the eccentricity has reached its minimum (as it will do after many ages) and begins to increase, then the moon's motion will begin to be retarded, and thus her mean motions will oscillate forever about a mean value.

244. The lunar inequalities which have been considered are such only as effect the moon's longitude; but the sun's disturbing force also causes inequalities in the moon's *latitude* and *parallax.* Those of latitude alone require no less than twelve equations. Since the moon revolves in an orbit inclined to the ecliptic, it is easy to see that the oblique action of the sun must admit of a resolution into two forces, one of which being perpendicular to the moon's orbit must effect changes in her latitude. Since also several of the inequalities already noticed involve changes in the length of the radius vector, it is obvious that the moon's parallax must be subject to corresponding perturbations.

CHAPTER VII.

ECLIPSES.

245. An *eclipse of the moon* happens, when the moon in its revolution about the earth, falls into the earth's shadow. An *eclipse of the sun* happens, when the moon, coming between the earth and the sun, covers either a part or the whole of the solar disk. An eclipse of the sun can occur only at the time of con-

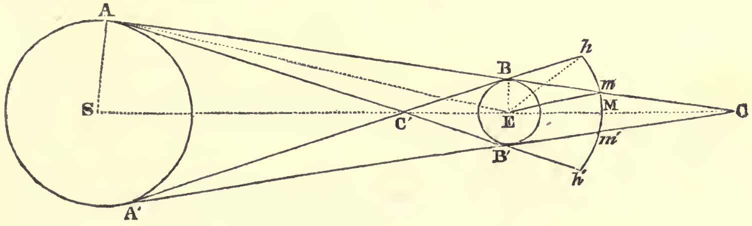
junction, or new moon; and an eclipse of the moon, only at the time of opposition, or full moon. Were the moon's orbit in the same plane with that of the earth, or did it coincide with the ecliptic, then an eclipse of the sun would take place at every conjunction, and an eclipse of the moon at every opposition; for as the sun and earth both lie in the ecliptic, the shadow of the earth must also extend in the same plane, being of course always directly opposite to the sun; and since, as we shall soon see, the length of this shadow is much greater than the distance of the moon from the earth, the moon, if it revolved in the plane of the ecliptic, must pass through the shadow at every full moon. For similar reasons, the moon would occasion an eclipse of the sun, partial or total, in some portions of the earth at every new moon. But the lunar orbit is inclined to the ecliptic about 5°, so that the center of the moon, when she is farthest from her node, is 5° from the axis of the earth's shadow (which is always in the ecliptic;) and, as we shall show presently, the greatest distance to which the shadow extends on each side of the ecliptic, that is, the greatest semi-diameter of the shadow, where the moon passes through it, is only about $\frac{3}{4}$ of a degree, while the semi-diameter of the moon's disk is only about $\frac{1}{4}$ of a degree; hence the two semi-diameters, namely, that of the moon and the earth's shadow, cannot overlap one another, unless, at the time of new or full moon, the sun is at or very near the moon's node. In the course of the sun's apparent revolution around the earth once a year, he is successively in every part of the ecliptic; consequently, the conjunctions and oppositions of the sun and moon may occur at any part of the ecliptic, either when the sun is at the moon's node, (or when he is passing that point of the celestial vault on which the moon's node is projected as seen from the earth;) or they may occur when the sun is 90° from the moon's node, where the lunar and solar orbits are at the greatest distance from each other; or, finally, they may occur at any intermediate point. Now the sun, in his annual revolution, passes each of the moon's nodes on opposite sides of the ecliptic, and of course at opposite seasons of the year; so that, for any given year, the eclipses commonly happen in two opposite months, as January and July, February and August, May and December. These, therefore, are called *Node Months.*

246. If the sun were of the same size with the earth, the shadow of the earth would be cylindrical and infinite in length, since the tangents drawn from the sun to the earth (which form the boundaries of the shadow) would be parallel to each other; but as the sun is a vastly larger body than the earth, the tangents converge and meet in a point at some distance behind the earth, forming a cone of which the earth is the base, and whose vertex (and of course its axis) lies in the ecliptic. A little reflection will also show us, that the form and dimensions of the shadow must be affected by several circumstances; that the shadow must be of the greatest length and breadth when the sun is farthest from the earth; that its figure will be slightly modified by the spheroidal figure of the earth; and that the moon, being, at the time of its opposition, sometimes nearer to the earth, and sometimes farther from it, will accordingly traverse it at points where its breadth varies more or less.

247. *The semi-angle of the cone of the earth's shadow, is equal to the sun's apparent semi-diameter, minus his horizontal parallax.*

Let AS (Fig. 51,) be the semi-diameter of the sun, BE that of the earth, and EC the axis of the earth's shadow. Then the semi-angle of the cone of the earth's shadow ECB=AES−EAB,

Fig. 51.



of which AES is the sun's semi-diameter and EAB his horizontal parallax; and as both these quantities are known, hence the angle at the vertex of the shadow becomes known. Putting δ for the sun's semi-diameter, and p for his horizontal parallax, we have the semi-angle of the earth's shadow ECB$=\delta-p$.

248. *At the mean distance of the earth from the sun, the length of the earth's shadow is about* 860,000 *miles, or more than three times the distance of the moon from the earth.*

In the right angled triangle ECB, the angle ECB being known, and the side EB, we can find the side EC. For sin. $\delta - p$: EB : : R : EC $= \frac{\text{EB}}{\sin.\ \delta - p}$. This value will vary with the sun's semi-diameter, being greater as that is less. Its mean value being $16'\ 1''.5$ and the sun's horizontal parallax being $8''.6$, $\delta - p = 15'\ 52''.9$, and EB$=3956.2$. Hence,

$$\text{Sin. } 15'\ 53'' : \text{Rad.} :: 3956.2 : 856{,}275.$$

Since the distance of the moon from the earth is 238.545 miles, the shadow extends about 3.6 times as far as the moon, and consequently, the moon passes the shadow towards its broadest part, where its breadth is much more than sufficient to cover the moon's disk.

249. *The average breadth of the earth's shadow where it eclipses the moon, is almost three times the moon's diameter.*

Let mm' (Fig. 51,) represent a section of the earth's shadow where the moon passes through it, M being the center of the circular section. Then the angle MEm will be the angular breadth of half the shadow. But,

MEm=BmE$-$BCE; that is, since BmE is the moon's horizontal parallax (Art. 82,) and BCE equals the sun's semi-diameter minus his horizontal parallax ($\delta - p$), therefore, putting P for the moon's horizontal parallax, we have

MEm $= \text{P} - (\delta - p) = \text{P} + p - \delta$; that is, since $\text{P} = 57'\ 1''$ and $\delta - p = 15'\ 52''.9$, therefore, $57'\ 1'' - 15'\ 52''.9 = 41'\ 8''.1$, which is nearly three times $15'\ 33''$, the semi-diameter of the moon. Thus, it is seen how, by the aid of geometry, we learn to estimate various particulars respecting the earth's shadow, by means of known data derived from observation.

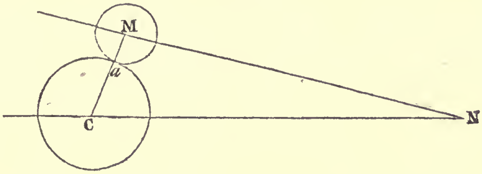
250. The distance of the moon from her node when she just touches the shadow of the earth, in a lunar eclipse, is called the *Lunar Ecliptic Limit;* and her distance from the node in a solar eclipse, when the moon just touches the solar disk, is called

the *Solar Ecliptic Limit.* The Limits are respectively the farthest possible distances from the node at which eclipses can take place.

251. *The Lunar Ecliptic Limit is nearly* 12 *degrees.*

Let CN (Fig. 52,) be the sun's path, MN the moon's, and N the node. Let C*a* be the semi-diameter of the earth's shadow, and M*a* the semi-diameter of the moon. Since C*a* and M*a* are known

Fig. 52.



quantities, their sum CM is also known. The angle at N is known, being the inclination of the lunar orbit to the ecliptic. Hence, in the spherical triangle MCN, right angled at M,* by Napier's theorem, (p. 60.)

$$\text{Rad.} \times \sin.\ \text{CM} = \sin.\ \text{CN} \times \sin.\ \text{MNC}.$$

The greatest apparent semi-diameter of the earth's shadow where the moon crosses it, computed by article 249, is 45′ 52″, and the moon's greatest apparent semi-diameter, is 16′ 45″.5, which together, give MC equal to 62′ 37″.5. Taking the inclination of the moon's orbit, or the angle MNC (what it generally is in these circumstances) at 5° 17′, and we have Rad. $\times$ sin. 62′ 37″.5 $=$ sin. CN $\times$ sin. 5° 17′, or sin. CN $= \dfrac{\text{Rad} \times \sin. 62' \, 37''.5}{\sin.\ 5^\circ \, 17'}$ $= 11^\circ \, 25' \, 40''$.† This is the greatest distance of the moon from her node (in longitude) at which an eclipse of the moon can take place. By varying the value of CM, corresponding to variations in the distances of the sun and moon from the earth, it is found that if NC is less than 9°, there *must* be an eclipse; but between this and the limit, the case is doubtful.

When the moon's disk only comes in contact with the earth's shadow, as in figure 52, the phenomenon is called an *appulse;*

* The line CM is to be regarded as the *projection* of the line which connects the centers of the moon and section of the earth's shadow, as seen from the earth.

† Woodhouse's Astronomy, p. 718.

when only a part of the disk enters the shadow, the eclipse is said to be *partial*, and *total* if the whole of the disk enters the shadow. The eclipse is called *central* when the moon's center coincides with the axis of the shadow, which happens when the moon at the time of the eclipse is exactly at her node.

252. Before the moon enters the earth's shadow, the earth begins to intercept from it portions of the sun's light, gradually increasing until the moon reaches the shadow. This partial light is called the moon's *Penumbra*. Its limits are ascertained by drawing the tangents AC′B′, and A′C′B. Throughout the space included between these tangents more or less of the sun's light is intercepted from the moon by the interposition of the earth ; for it is evident, that as the moon moves towards the shadow, she would gradually lose the view of the sun, until, on entering the shadow, the sun would be entirely hidden from her.

253. *The semi-angle of the Penumbra equals the sun's semi-diameter and horizontal parallax, or* $\delta+p$.

The angle hC′M (Fig. 51,)$=$AC′S$=$AES$+$B′AE. But AES is the sun's semi-diameter, and B′AE is the sun's horizontal parallax, both of which quantities are known.

254. *The semi-angle of a section of the Penumbra, where the moon crosses it, equals the moon's horizontal parallax, plus the sun's semi-diameter.*

The angle hEM (Fig. 51,)$=$EhC′$+$EC′h. But EhC′$=$P, the moon's horizontal parallax, and EC′H$=\delta+p$ (Art. 252,) $\therefore$ hEM $=\text{P}+p+\delta$, all which are likewise known quantities.

Hence, by means of these few elements, which are known from observation, we ascend to a complete knowledge of all the particulars necessary to be known respecting the moon's penumbra.

255. In the preceding investigations, we have supposed that the cone of the earth's shadow is formed by lines drawn from the sun, and touching the earth's surface. But the apparent diameter of the shadow is found by observation to be somewhat greater than would result from this hypothesis. The fact is ac-

counted for by supposing that a portion of the solar rays which graze the earth's surface are absorbed and extinguished by the lower strata of the atmosphere. This amounts to the same thing as though the earth were larger than it is, in which case the moon's horizontal parallax would be increased; and accordingly, in order that theory and observation may coincide, it is found necessary to increase the parallax by $\frac{1}{60}$.

256. In a total eclipse of the moon, its disk is still visible, shining with a dull red light. This light cannot be derived directly from the sun, since the view of the sun is completely hidden from the moon; nor by reflexion from the earth, since the illuminated side of the earth is wholly turned from the moon; but it is owing to refraction by the earth's atmosphere, by which a few scattered rays of the sun are bent round into the earth's shadow and conveyed to the moon, sufficient in number to afford the feeble light in question.

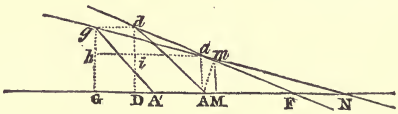
257. In *calculating an eclipse of the moon*, we first learn from the tables in what month the sun, at the time of full moon in that month, is near the moon's node, or within the lunar ecliptic limit. This it must evidently be easy to determine, since the tables enable us to find both the longitudes of the nodes, and the longitudes of the sun and moon, for every day of the year. Consequently, we can find when the sun has nearly the same longitude as one of the nodes, and also the precise moment when the longitude of the moon is 180° from that of the sun, for this is the time of opposition, or of the middle of the eclipse. Having the time of the middle of the eclipse, and the breadth of the shadow, (Art. 249,) and knowing, from the tables, the rate at which the moon moves per hour faster than the shadow, we can find how long it will take her to traverse half the breadth of the shadow; and this time subtracted from the time of the middle of the eclipse, will give the beginning, and added to the time of the middle will give the end of the eclipse. Or if instead of the breadth of the shadow, we employ the breadth of the penumbra (Art. 253,) we may find, in the same manner, when the moon enters and when she leaves the penumbra. We see,

therefore, how by having a few things known by observation, such as the sun and moon's semi-diameters, and their horizontal parallaxes, we rise, by the aid of trigonometry, to the knowledge of various particulars respecting the length and breadth of the shadow and of the penumbra. These being known, we next have recourse to the tables which contain all the necessary particulars respecting the motions of the sun and moon, together with equations or corrections, to be applied for all their irregularities. Hence it is comparatively an easy task to calculate with great accuracy an eclipse of the moon.

258. Let us then see how we may find the exact time of the beginning, end, duration, and magnitude, of a lunar eclipse.

Let NG (Fig. 53,) be the ecliptic, and N*ag* the moon's orbit, the sun being in A when the moon is in opposition at *a*; let N be the ascending node, and A*a* the moon's latitude at the instant

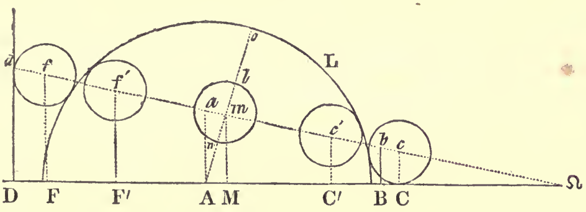
Fig. 53.



of opposition. An hour afterwards the sun will have passed to A′, and the moon to *g*, when the difference of longitude of the two bodise will be *g*A′. Then *gh* is the moon's hourly motion in latitude, and *ah* her hourly motion in longitude. As the character and form of the eclipse will depend solely upon the distances between the centers of the sun and moon, that is, upon the line *g*A′, instead of considering the two bodies as both in motion, we may suppose the sun at rest in A, and the moon as advancing with a motion equal to the difference between its rate and that of the sun, a supposition which will simplify the calculation. Therefore, draw *gd* parallel and equal to A′A, join *d*A, and this line being equal to *g*A′, the two bodies will be in the same relative situation as if the sun were at A′ and the moon at *g*. Join *da* and produce the line *da* both ways, cutting the ecliptic in F; then *da*F will be the moon's *Relative Orbit.* Hence $ai = ah - AA'$ = the difference of the hourly motions of the sun and moon, that is, the moon's *relative motion* in longitude, and di = the moon's hourly motion in latitude.

Draw CD (Fig. 54,) to represent the ecliptic, and let A be the place of the sun. As the tables give the computation of the moon's latitude at every instant, consequently, we may take from them the latitude corresponding to the instant of opposition, and to one hour later; and we may take also the sun's and moon's hourly motions in longitude. Take AD, AB, each equal to the

Fig. 54.



relative motion, and Aa = the latitude in opposition, Dd = the latitude one hour afterwards; join da and produce the line da both ways, and it will represent the moon's relative orbit. Draw Bb at right angles to CD and it will be the latitude an hour before opposition. At the time of the eclipse, the apparent distance of the center of the shadow from the moon is very small; consequently, CD, cd, Dd, &c. may be regarded as straight lines. During the short interval between the beginning and end of an eclipse, the motion of the sun, and consequently that of the center of the shadow, may likewise be regarded as uniform.

259. The various particulars that enter into the calculation of an eclipse are called its *Elements;* and as our object is here merely *to explain the method* of calculating an eclipse of the moon, (reserving the actual computation for the fourth part of this work,) we may take the elements at their mean value. Thus, we will consider cd as inclined to CD 5° 9′, the moon's horizontal parallax as 58′, its semi-diameter as 16′, and that of the earth's shadow as 42′. The line Am perpendicular to cd gives the point m for the place of the moon at the middle of the eclipse, for this line bisects the chord, which represents the path of the moon through the shadow; and mM, perpendicular to CD, gives AM for the time of the middle of the eclipse before opposition, the number of minutes before opposition being the same part of an hour that

AM is of AB.* From the center A, with a radius equal to that of the earth's shadow (42′) describe the semi-circle BLF, and it will represent the projection of the shadow traversed by the moon. With a radius equal to the semi-diameter of the shadow and that of the moon ($=42'+16'=58'$) and with the center A, mark the two points c and f on the relative orbit, and they will be the places of the center of the moon at the beginning and end of the eclipse. The perpendiculars cC, fF, give the times AC and AF of the commencement and the end of the eclipse, and CM, or MF gives half the duration. From the centers c and f with a radius equal to the semi-diameter of the moon (16′) describe circles, and they will each touch the shadow, (Euc. 3.12.) indicating the position of the moon at the beginning and end of the eclipse. If the same circle described from m is wholly within the shadow, the eclipse will be *total;* if it is only partly within the shadow, the eclipse will be *partial.* With the center A, and radius equal to the semi-diameter of the shadow minus that of the moon ($42'-16'=26'$) mark the two points c', f', which will give the places of the center of the moon, at the beginning and end of total darkness, and MC′, MF′ will give the corresponding times before and after the middle of the eclipse. Their sum will be the duration of total darkness.

260. If the foregoing projection be accurately made from a scale, the required particulars of the eclipse may be ascertained by measuring on the same scale, the lines which respectively represent them; and we should thus obtain a near approximation to the elements of the eclipse. A more accurate determination of these elements may, however, be obtained by actual calculation. The general principles of the calculation will be readily understood.

First, knowing ai, (Fig. 53,) the moon's relative longitude, and di, her latitude, we find the angle dai, which is the inclination of the moon's relative orbit. But $dai=a\text{A}m$; and, in the triangle $a\text{A}m$, we have the angle at A, and the side Aa, being the moon's latitude at the time of opposition, which is given by the tables. Hence we can find the side Am. In the triangle AmM,

* The situation of the moon when at m is called *orbit opposition;* and her situation when at a, *ecliptic opposition.*

(Fig. 54,) having the side Am and the angle AmM ($=a$Am) we can find AM = the arc of relative longitude described by the moon from the time of the middle of the eclipse to the time of opposition; and knowing the moon's hourly motion in longitude, we can convert AM into time, and this subtracted from the time of opposition gives us the *time of the middle of the eclipse.*

Secondly, since we know the length of the line Ac* and can easily find the angle cAC, we can thus obtain the side AC; and AC − AM = MC, which arc, converted into time by comparing it with the moon's hourly motion in longitude, gives us, when subtracted from the time of the middle of the eclipse, *the time of the beginning of the eclipse,* or when added to that of the middle, *the time of the end of the eclipse.* The sum of the two equals the *whole duration.*

Thirdly, by a similar method we calculate the value of MC′, which converted into time, and subtracted from the time of the middle of the eclipse, gives the *commencement of total darkness,* or when added gives the *end of total darkness.* Their sum is the *duration of total darkness.*

Fourthly, the *quantity of the eclipse* is determined by supposing the diameter of the moon divided into twelve equal parts called *Digits,* and finding how many such parts lie within the shadow, at the time when the centers of the moon and the shadow are nearest to each other. Even when the moon lies wholly within the shadow, the quantity of the eclipse is still expressed by the number of digits contained in that part of the line which joins the center of the shadow and the center of the moon, which is intercepted between the edge of the shadow and the inner edge of the moon. Thus in figure 54, the number of digits eclipsed, equals $\frac{no}{\frac{1}{12}nl}=\frac{\mathrm{A}o-\mathrm{A}n}{\frac{1}{12}nl}=\frac{\mathrm{A}o-(\mathrm{A}m-nm)}{\frac{1}{12}nl}$, an expression containing only known quantities.

261. The foregoing will serve as an explanation of the *general principles,* on which proceeds the calculation of a lunar eclipse. The actual methods practiced employ many expedients to facilitate the process, and to insure the greatest possible accuracy, the

* This line is not represented in the figure, but may be easily imagined.

nature of which will be explained and exemplified in the fourth part of this work.

262. The leading particulars respecting an ECLIPSE OF THE SUN, are ascertained very nearly like those of a lunar eclipse. The shadow of the moon travels over a portion of the earth, as the shadow of a small cloud, seen from an eminence in a clear day, rides along over hills and plains. Let us imagine ourselves standing on the moon; then we shall see the earth partially eclipsed by the shadow of the moon, in the same manner as we now see the moon eclipsed by the earth's shadow; and we might proceed to find the length of the shadow, its breadth where it eclipses the earth, the breadth of the penumbra, and its duration and quantity, in the same way as we have ascertained these particulars for an eclipse of the moon.

But, although the general characters of a solar eclipse might be investigated on these principles, so far as respects the earth at large, yet as the appearances of the same eclipse of the sun are very different at different places on the earth's surface, it is necessary to calculate its peculiar aspects for each place separately, a circumstance which makes the calculation of a solar eclipse much more complicated and tedious than of an eclipse of the moon. The moon, when she enters the shadow of the earth, is deprived of the light of the part immersed, and that part appears black alike to all places when the moon is above the horizon. But it is not so with a solar eclipse. We do not see this by the shadow cast on the earth, as we should do if we stood on the moon, but by the interposition of the moon between us and the sun; and the sun may be hidden from one observer while he is in full view of another only a few miles distant. Thus, a small insulated cloud sailing in a clear sky, will, for a few moments, hide the sun from us, and from a certain space near us, while all the region around is illuminated.

263. We have compared the motion of the moon's shadow over the surface of the earth to that of a cloud; but its *velocity* is incomparably greater. The mean motion of the moon around the earth is about 33′ per hour; but 33′ of the moon's orbit is 2280

miles, and the shadow moves of course at the same rate, or 2280 miles per hour, traversing the entire disk of the earth in less than four hours. This is the velocity of the shadow when it passes *perpendicularly* over the earth; when the direction of the axis of the shadow is oblique to the earth's surface the velocity is increased in proportion of radius to the sine of obliquity; for having a greater space to pass over in the same time, its velocity must of course be greater. When the conjunction takes place exactly at the node, the axis of the moon's shadow lies in the ecliptic, and the shadow traverses the earth perpendicularly to its surface; but when the conjunction occurs on either side of the node, but within the solar ecliptic limits, the shadow falls obliquely on the earth. The moon's shadow being a cone, the oblique section of it made by the earth, is an ellipse.

Let us endeavor to form a just conception of the manner in which these three bodies, the sun, the earth, and the moon, are situated with respect to each other at the time of a solar eclipse. First, suppose the conjunction to take place at the node. Then the straight line which connects the centers of the sun and the earth, also passes through the center of the moon, and coincides with the axis of its shadow; and, since the earth is bisected by the plane of the ecliptic, the shadow would traverse the earth in the direction of the terrestrial ecliptic, from west to east, passing over the middle regions of the earth. Here the diurnal motion of the earth being in the same direction with the shadow, but with a less velocity, the shadow will appear to move with a speed equal only to the difference between the two. Secondly, suppose the moon is on the north side of the ecliptic at the time of conjunction, and moving towards her descending node, and that the conjunction takes place just within the solar ecliptic limit, say 16° from the node. The shadow will now not fall in the plane of the ecliptic, but a little northward of it, so as just to graze the earth near the pole of the ecliptic. The nearer the conjunction comes to the node, the farther the shadow will fall from the pole of the ecliptic towards the equatorial regions. In certain cases, the shadow strikes beyond the pole of the earth; and then its easterly motion being opposite to the diurnal motion of the places which it traverses, consequently its velocity is greatly increased, being equal to the sum of both.

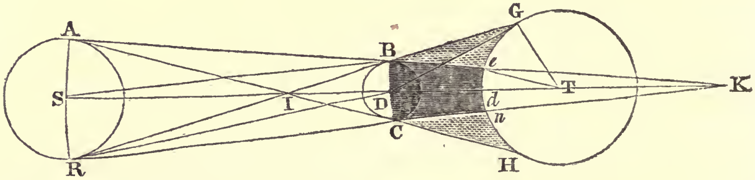
264. After these general considerations, we will now examine more particularly the method of investigating the elements of a solar eclipse.

The *length of the moon's shadow*, is the first object of inquiry. The moon as well as the earth, is at different distances from the sun at different times, and hence the length of her shadow varies, being always greatest when she is farthest from the sun. Also, since her distance from the earth varies, the section of the moon's shadow made by the earth, is greater in proportion as the moon is nearer the earth. The greatest eclipses of the sun, therefore, happen when the sun is in apogee,* and the moon in perigee.

265. *When the moon is at her mean distance from the earth, and from the sun, her shadow nearly reaches the earth's surface.*

Let S (Fig. 55,) represent the sun, D the moon, and T the earth. Then, the semi-angle of the cone of the moon's shadow, DKR, will, as in the case of the earth, (Art. 247,) equal SDR — DRK, of which SDR is the sun's apparent semi-diameter, as seen from the moon, and DRK, is the sun's horizontal parallax at the moon. Since, on account of the great distance of the

Fig. 55.



sun, compared with that of the moon, the semi-diameter of the sun as seen from the moon, must evidently be very nearly the same as when seen from the earth, and since on account of the minuteness of the moon's semi-diameter when seen from the sun, the sun's horizontal parallax at the moon must be very small, we might, without much error, take the sun's apparent semi-diameter from the earth, as equal to the semi-angle of the cone of the moon's shadow; but, for the sake of greater accuracy, let us estimate the value of the sun's semi-diameter and horizontal parallax at the moon.

* The sun is said to be in apogee, when the earth is in aphelion.

Now, SDR : STR :: ST : SD* :: 400 : 399 ; hence SDR $=\frac{400}{399}$ STR $=1.0025$ STR ; and the sun's mean semi-diameter STR being 16.025, hence SDR $=1.0025\times16.025=16.065=$ $16'\ 3''.9$.

Again, since parallax is inversely as the distance, the sun's horizontal parallax at the moon, is on account of her being nearer the sun $\frac{1}{400}$ greater than at the earth; but on account of her inferior size it is $\frac{7912}{2160}$ less than at the earth. Hence, increasing the sun's horizontal parallax at the earth by the former fraction, and diminishing it by the latter, we have $\frac{400}{399}\times\frac{2160}{7912}\times9''=2''.5=$ sun's horizontal parallax at the moon. Therefore, the semi-angle of the cone of the moon's shadow, which, as appears above, equals SDR $-$ DRK, equals $16'\ 3''.9-2''.5=16'\ 1''.4$, which so nearly equals the sun's apparent semi-diameter, as seen from the earth, that we may adopt the latter as the value of the semi-angle of the shadow. Hence, sin. $16'\ 1''.5$: 1080(BD) :: Rad. : DK $=$ 231690. But the mean distance of the moon from the surface of the earth is $238545-3956=234589$, which exceeds a little the mean length of the shadow as above.

But when the moon is nearest the earth her distance from the center of the earth is only 221148 miles; and when the earth is farthest from the sun, the sun's apparent semi-diameter is only $15'\ 45''.5$. By employing this number in the foregoing estimate, we shall find the length of the shadow 235630 miles; and $235630-221148=14482$, the distance which the moon's shadow may reach beyond the center of the earth.

266. *The diameter of the moon's shadow where it traverses the earth, is, at its maximum, about* 170 *miles.*†

In the triangle eTK, the angle at K $=15'\ 45''.5$ (Art. 265,) the side T$e=3956$, and TK $=14482$.

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* The apparent magnitude of an object being reciprocally as its distance from the eye. See Note, p. 85.

† This supposes the conjunction to take place at the node, and the shadow to strike the earth perpendicularly to its surface; where it strikes it obliquely, the section may be greater than this.

Or, 3956 : 14482: :sin. 15′ 45.″5 : sin. 57′ 41″.5.
And 57′ 41.″5+15′ 45.″5=1° 13′ 27″=dTe, on the arc de.
And 2de=2° 26′ 54″=en.
Hence 360 : 2.45 (=2° 26′ 54″): :24899* : 170 (nearly).

267. *The greatest portion of the earth's surface ever covered by the moon's penumbra, is about* 4393 *miles.*

The semi-angle of the penumbra BID=BSD+SBR, of which BSD the sun's horizontal parallax at the moon=2″.5, and SBR the sun's apparent semi-diameter=16′ 3″.9, and hence BID is known. The moon's apparent semi-diameter BGD=16′ 45″.5. Therefore GDT is known, as likewise DT and TG. Hence the angle GTd may be found, and the arc dG and its double GH, which equals the angular breadth of the penumbra. It may be converted into miles by stating a proportion as in article 266. On making the calculation it will be found to be 4393 miles.

268. The apparent diameter of the moon is sometimes larger than that of the sun, sometimes smaller, and sometimes exactly equal to it. Suppose an observer placed on the right line which joins the centers of the sun and moon; if the apparent diameter of the moon is greater than that of the sun, the eclipse will be total. If the two diameters are equal, the moon's shadow just reaches the earth, and the sun is hidden but for a moment from the view of spectators situated in the line which the vertex of the shadow describes on the surface of the earth. But if, as happens when the moon comes to her conjunction in that part of her orbit which is towards her apogee, the moon's diameter is less than the sun's, then the observer will see a ring of the sun encircle the moon, constituting an *Annular Eclipse.*

269. Eclipses of the sun are modified by the elevation of the moon above the horizon, since its apparent diameter is augmented as its altitude is increased, (Art. 217.) This effect, combined with that of parallax, may so increase or diminish the apparent distance between the centers of the sun and moon, that from this

* The equatorial circumference.

cause alone, of two observers at a distance from each other, one might see an eclipse which was not visible to the other.* If the horizontal diameter of the moon differs but little from the apparent diameter of the sun, the case might occur where the eclipse would be annular over the places where it was observed morning and evening, but total where it was observed at mid-day.

The earth in its diurnal revolution and the moon's shadow both move from west to east, but the shadow moves faster than the earth ; hence the moon overtakes the sun on its western limb and crosses it from west to east. The excess of the apparent diameter of the moon above that of the sun in a total eclipse is so small, that total darkness seldom continues longer than four minutes, and can never continue so long as eight minutes. An annular eclipse may last 12m. 24s.

Since the sun's ecliptic limits are more than 17° and the moon's less than 12°, eclipses of the sun are more frequent than those of the moon. Yet lunar eclipses being visible to every part of the terrestrial hemisphere opposite to the sun, while those of the sun are visible only to the small portion of the hemisphere on which the moon's shadow falls, it happens that for any particular place on the earth, lunar eclipses are more frequently visible than solar. In any year, the number of eclipses of both luminaries cannot be less than two nor more than seven : the most usual number is four, and it is very rare to have more than six. A total eclipse of the moon frequently happens at the next full moon after an eclipse of the sun. For since, in an eclipse of the sun, the sun is at or near one of the moon's nodes, the earth's shadow must be at or near the other node, and may not have passed so far from the node as the lunar ecliptic limits, before the moon overtakes it.

270. It has been observed already, that were the spectator on the moon instead of on the earth, he would see the earth eclipsed by the moon, and the calculation of the eclipse would be very similar to that of a lunar eclipse ; but to an observer on the earth the eclipse does not of course begin when the earth first enters the moon's shadow, and it is necessary to determine not only

* Biot, Ast. Phys. p. 401.

what portion of the earth's surface will be covered by the moon's shadow, but likewise the path described by its center relative to various places on the surface of the earth. This is known when the latitude and longitude of the center of the shadow on the earth, is determined for each instant. The latitude and longitude of the moon are found on the supposition that the spectator views it from the center of the earth, whereas his position on the surface changes, in consequence of parallax, both the latitude and longitude, and the amount of these changes must be accurately estimated, before the appearance of the eclipse at any particular place can be fully determined.

The details of the method of calculating a solar eclipse cannot be understood in any way so well, as by actually performing the process according to a given example. Such details therefore are reserved for a subsequent part of this work.

271. In total eclipses of the sun, there has sometimes been observed a remarkable radiance of light from the margin of the sun. This has been ascribed to an illumination of the solar atmosphere, but it is with more probability owing to the zodiacal light (Art. 152,) which at that time is projected around the sun, and which is of such dimensions as to extend far beyond the solar orb.*

A total eclipse of the sun is one of the most sublime and impressive phenomena of nature. Among barbarous nations it is ever contemplated with fear and astonishment, while among cultivated nations it is recognized, from the exactness with which the time of occurrence and the various appearances answer to the prediction, as affording one of the proudest triumphs of astronomy. By astronomers themselves it is of course viewed with the highest interest, not only as verifying their calculations, but as contributing to establish beyond all doubt the certainty of those grand laws, the truth of which is involved in the result. During the eclipse of June, 1806, which was one of the most remarkable on record, the time of total darkness, as seen by the author of this work, was about mid-day. The sky was entirely cloudless, but

* See an excellent description and delineation of this appearance as it was exhibited in the eclipse of 1806, in the Transactions of the Albany Institute, by the late Chancellor De Witt.

as the period of total obscuration approached, a gloom pervaded all nature. When the sun was wholly lost sight of, planets and stars came into view; a fearful pall hung upon the sky, unlike both to night and to twilight; and, the temperature of the air rapidly declining, a sudden chill came over the earth. Even the animal tribes exhibited tokens of fear and agitation.

From 1831 to 1838 was a period remarkable for great eclipses of the sun, in which time there were no less than five of the most remarkable character. The next total eclipse of the sun, visible in the United States, will occur on the 7th of August, 1869.

CHAPTER VIII.

LONGITUDE—TIDES.

272. As eclipses of the sun afford one of the most approved methods of finding the longitudes of places, our attention is naturally turned next towards that subject.

The ancients studied astronomy in order that they might read their destinies in the stars: the moderns, that they may securely navigate the ocean. A large portion of the refined labors of modern astronomy, has been directed towards perfecting the astronomical tables with the view of finding the longitude at sea,—an object manifestly worthy of the highest efforts of science, considering the vast amount of property and of human life involved in navigation.

273. *The difference of longitude between two places, may be found by any method by which we can ascertain the difference of their local times, at the same instant of absolute time.*

As the earth turns on its axis from west to east, any place that lies eastward of another will come sooner under the sun, or will have the sun earlier on the meridian, and consequently, in respect to the hour of the day, will be in advance of the other at the rate of one hour for every 15°, or four minutes of time for each degree. Thus, to a place 15° east of Greenwich, it is 1 o'clock, P. M. when it is noon at Greenwich ; and to a place 15° west of that meridian, it is 11 o'clock, A. M. at the same instant. Hence, the difference of time at any two places, indicates their difference of longitude.

274. The easiest method of finding the longitude is by means of an accurate time piece, or *chronometer*. Let us set out from London with a chronometer accurately adjusted to Greenwich time, and travel eastward to a certain place, where the time is accurately kept, or may be ascertained by observation. We find, for example, that it is 1 o'clock by our chronometer, when it is 2 o'clock and 30 minutes at the place of observation. Hence, the longitude is $15 \times 1.5 = 22\frac{1}{2}°$ E. Had we travelled westward until our chronometer was an hour and a half in advance of the time at the place of observation, (that is, so much later in the day,) our longitude would have been $22\frac{1}{2}°$ W. But it would not be necessary to repair to London in order to set our chronometer to Greenwich time. This might be done at any observatory, or any place whose longitude had been accurately determined. For example, the time at New York is 4h. 56m. $4^s.5$ behind that of Greenwich. If, therefore, we set our chronometer so much before the true time at New York, it will indicate the time at Greenwich. Moreover, on arriving at different places, any where on the earth, whose longitude is accurately known, we may learn whether our chronometer keeps accurate time or not, and if not, the amount of its error. Chronometers have been constructed of such an astonishing degree of accuracy, as to deviate but a few seconds in a voyage from London to Baffin's Bay and back, during an absence of several years. But chronometers which are sufficiently accurate to be depended on for long voyages, are too expensive for general use, and the means of verifying their accuracy are not sufficiently easy. Moreover, chronometers, by being transported from one place to another, change their daily rate, or depart from that mean rate which they preserve at a fixed station. A chronometer, therefore, cannot be relied on for determining the longitudes of places where the greatest degree of accuracy is re-

quired, especially where the instrument is conveyed over land, although the uncertainty attendant on one instrument may be nearly obviated by employing several and taking their mean results.*

275. *Eclipses of the sun and moon* are sometimes used for determining the longitude. The exact instant of immersion or of emersion, or any other definite moment of the eclipse which presents itself to two distant observers, affords the means of comparing their difference of time, and hence of determining their difference of longitude. Since the entrance of the moon into the earth's shadow, in a lunar eclipse, is seen at the same instant of absolute time at all places where the eclipse is visible, (Art. 262,) this observation would be a very suitable one for finding the longitude were it not that, on account of the increasing darkness of the penumbra near the boundaries of the shadow, it is difficult to determine the precise instant when the moon enters the shadow. By taking observations on the immersions of known spots on the lunar disk, a mean result may be obtained which will give the longitude with tolerable accuracy. In an eclipse of the sun, the instants of immersion and emersion may be observed with greater accuracy, although, since these do not take place at the same instant of absolute time, the calculation of the longitude from observations on a solar eclipse are complicated and laborious.

A method very similar to the foregoing, by observations on eclipses of Jupiter's satellites, and on occultations of stars, will be mentioned hereafter.

276. *The Lunar method of finding the longitude*, at sea, is in many respects preferable to every other. It consists in measuring (with a sextant) the angular distance between the moon and the sun, or between the moon and a star, and then turning to the Nautical Almanac,† and finding what time it was at Greenwich when

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* Woodhouse, p. 838.

† The *Nautical Almanac*, is a book published annually by the British Board of Longitude, containing various tables and astronomical information for the use of navigators. The *American Almanac* also contains a variety of astronomical information, peculiarly interesting to the people of the United States, in connexion with a vast amount of statistical matter. It is well deserving a place in the library of the student.

that distance was the same. The moon moves so rapidly, that this distance will not be the same except at very nearly the same instant of absolute time. For example, at 9 o'clock, A. M., at a certain place, we find the angular distance of the moon and the sun to be 72°; and, on looking into the Nautical Almanac, we find that at the time when this distance was the same for the meridian of Greenwich was 1 o'clock, P. M.; hence we infer that the longitude of the place is four hours, or 60° west.

The Nautical Almanac contains the true angular distance of of the moon from the sun, from the four large planets, (Venus, Mars, Jupiter, and Saturn,) and from nine bright fixed stars, for the beginning of every third hour of mean time for the meridian of Greenwich; and the mean time corresponding to any intermediate hour, may be found by proportional parts.*

277. It would be a very simple operation to determine the longitude by Lunar Distances, if the process as described in the preceding article were all that is necessary; but the various circumstances of parallax, refraction, and dip of the horizon, would differ more or less at the two places, even were the bodies whose distances were taken in view from both, which is not necessarily the case. The observations, therefore, at each meridian, require to be reduced to the center of the earth, being cleared of the effects of parallax and refraction. Hence, three observers are necessary in order to take a lunar distance in the most exact manner, viz. two to measure the altitudes of the two bodies respectively, at the same time that the third takes the angular distance between them. The altitudes of the two luminaries at the time of observation must be known, in order to estimate the effects of parallax and refraction.

278. Although the lunar method of finding the longitude at sea has many advantages over the other methods in use, yet it has also its disadvantages. One is, the great exactness requisite in observing the distance of the moon from the sun or star, as a small error in the distance makes a considerable error in the longitude. The moon moves at the rate of about a degree in two

* See Bowditch's Navigator, Tenth Ed. p, 226.

hours, or one minute of space in two minutes of time. Therefore, if we make an error of one minute in observing the distance, we make an error of two minutes in time, or 30 miles of longitude at the equator. A single observation with the best sextants, may be liable to an error of more than half a minute; but the accuracy of the result may be much increased by a mean of several observations taken to the east and west of the moon. The imperfection of lunar tables was until recently considered as an objection to this method. Until within a few years, the best lunar tables were frequently erroneous to the amount of one minute, occasioning an error of 30 miles. The error of the best tables now in use will rarely exceed 7 or 8 seconds.*

TIDES.

279. The tides are an alternate rising and falling of the waters of the ocean, at regular intervals. They have a maximum and a minimum twice a day, twice a month, and twice a year. Of the daily tide, the maximum is called *High tide*, and the minimum *Low tide*. The maximum for the month is called *Spring tide*, and the minimum *Neap tide*. The rising of the tide is called *Flood* and its falling *Ebb* tide.

Similar tides, whether high or low, occur on opposite sides of the earth at once. Thus at the same time it is high tide at any given place, it is also high tide on the inferior meridian, and the same is true of the low tides.

The interval between two successive high tides is 12h. 25m.; or, if the same tide be considered as returning to the meridian, after having gone around the globe, its return is about 50 minutes later than it occurred on the preceding day. In this respect, as well as in various others, it corresponds very nearly to the motions of the moon.

The average height for the whole globe is about 2½ feet; or, if the earth were covered uniformly with a stratum of water, the difference between the two diameters of the oval would be 5 feet, or more exactly 5 feet and 8 inches; but its natural height at various places is very various, sometimes rising to 60 or 70 feet,

* Brinkley's Elements of Astronomy, p. 241.

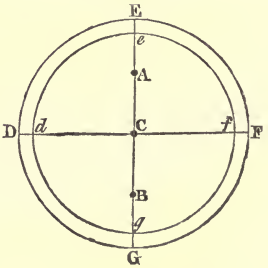
and sometimes being scarcely perceptible. At the same place also, the phenomena of the tides are very different at different times.

Inland lakes and seas, even those of the largest class, as Lake Superior, or the Caspian, have no perceptible tide.

280. *Tides are caused by the unequal attraction of the sun and moon upon different parts of the earth.*

Suppose the projectile force by which the earth is carried forward in her orbit, to be suspended, and the earth to fall towards one of these bodies, the moon, for example, in consequence of their mutual attraction. Then, if all parts of the earth fell equally towards the moon, no derangement of its different parts would result, any more than of the particles of a drop of water in its descent to the ground. But if one part fell faster than another, the different portions would evidently be separated from each other. Now this is precisely what takes place with respect to the earth in its fall towards the moon. The portions of the earth in the hemisphere next to the moon, on account of being nearer to the center of attraction, fall faster than those in the opposite hemisphere, and consequently leave them behind. The solid earth, on account of its cohesion, cannot obey this impulse, since all its different portions constitute one mass, which is acted on in the same manner as though it were all collected in the center; but the waters on the surface, moving freely under this impulse, endeavor to desert the solid mass and fall towards the moon. For a similar reason the waters in the opposite hemisphere falling less towards the moon than the solid earth are left behind, or appear to rise from the center of the earth.

Fig. 56.



281. Let DEFG (Fig. 56,) represent the globe; and, for the sake of illustrating the principle, we will suppose the waters entirely to cover the globe at a uniform depth. Let *defg* represent the solid globe, and the circular ring exterior to it, the covering of waters. Let C be the center of gravity of the solid mass, A that of the hemisphere next to the moon, and B

that of the remoter hemisphere. Now the force of attraction exerted by the moon, acts in the same manner as though the solid mass were all concentrated in C, and the waters of each hemisphere at A and B respectively; and (the moon being supposed above E) it is evident that A will tend to leave C, and C to leave B behind. The same must evidently be true of the respective portions of matter, of which these points are the centers of gravity. The waters of the globe will thus be reduced to an oval shape, being elongated in the direction of that meridian which is under the moon, and flattened in the intermediate parts, and most of all at points ninety degrees distant from that meridian.

Were it not, therefore, for impediments which prevent the force from producing its full effects, we might expect to see the great *tide wave*, as the elevated crest is called, always directly beneath the moon, attending it regularly around the globe. But the inertia of the waters prevents their instantly obeying the moon's attraction, and the friction of the waters on the bottom of the ocean, still farther retards its progress. It is not therefore until several hours (differing at different places) after the moon has passed the meridian of a place, that it is high tide at that place.

282. The *sun* has a similar action to the moon, but only *one third* as great. On account of the great mass of the sun compared with that of the moon, we might suppose that his action in raising the tides would be greater than that of the moon; but the nearness of the moon to the earth more than compensates for the sun's greater quantity of matter. Let us, however, form a just conception of the advantage which the moon derives from her proximity. It is not that her actual amount of attraction is thus rendered greater than that of the sun; but it is that her attraction for the *different parts* of the earth is very unequal, while that of the sun is nearly uniform. It is the *inequality* of this action, and not the absolute force, that produces the tides. The diameter of the earth is $\frac{1}{30}$ of the distance of the moon, while it is less than $\frac{1}{10000}$ of the distance of the sun.

283. Having now learned the general cause of the tides, we will next attend to the explanation of *particular phenomena*.

The *Spring tides*, or those which rise to an unusual height twice a month, are produced by the sun and moon's acting together; and the *Neap tides*, or those which are unusually low twice a month, are produced by the sun and moon's acting in opposition to each other. The Spring tides occur at the syzigies: the Neap tides at the quadratures. At the time of new moon, the sun and moon both being on the same side of the earth, and acting upon it in the same line, their actions conspire, and the sun may be considered as adding so much to the force of the moon. We have already explained how the moon contributes to raise a tide on the opposite side of the earth. But the sun as well as the moon raises its own tide-wave, which, at new moon, coincides with the lunar tide-wave. At full moon, also, the two luminaries conspire in the same way to raise the tide; for we must recollect that each body contributes to raise the tide on the opposite side of the earth as well as on the side nearest to it. At both the conjunctions and oppositions, therefore, that is, at the syzigies, we have unusually high tides. But here also the maximum effect is not at the moment of the syzigies, but 36 hours afterwards.

At the quadratures, the solar wave is lowest when the lunar wave is highest; hence the low tide produced by the sun is subtracted from high water and produces the Neap tides. Moreover, at the quadratures the solar wave is highest when the lunar wave is lowest, and hence is to be added to the height of low water at the time of Neap tides. Hence the difference between high and low water is only about half as great at Neap tide as at Spring tide.

284. The power of the moon or of the sun to raise the tide is found by the doctrine of universal gravitation to be *inversely as the cube of the distance.** The variations of distance in the sun are not great enough to influence the tides very materially, but the variations in the moon's distances have a striking effect. The tides which happen when the moon is in perigee, are considerably greater than when she is in apogee; and if she happens to be in perigee at the time of the syzigies, the Spring tide is unusually high. When this happens at the equinoxes, the highest tides of the year are produced.

* La Place, Syst. du Monde, l. IV, c. x.

285. The *declinations of the sun and moon* have a considerable influence on the height of the tide. When the moon for example, has no declination, or is in the equator, as in figure 57,* the rotation of the earth on its axis NS will make the two tides exactly equal on every part of the earth. Thus a place which is carried through the parallel TT′ will have the height of one tide T2 and the other tide T′3. The tides are in this case greatest at the equator, and diminish gradually to the poles, where it will be low water during the whole day. When the moon is on the north side of the equator, as in figure 58, at her greatest northern

Fig. 57.

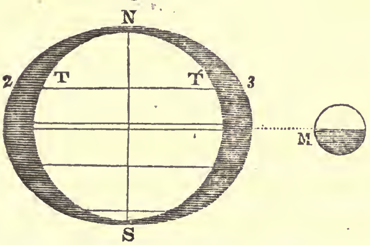
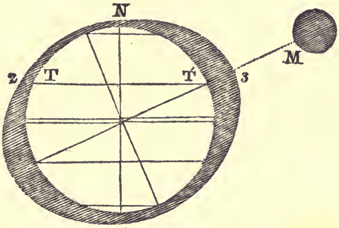


Fig. 58.



declination, a place describing the parallel TT′ will have T′3 for the height of the tide when the moon is on the superior meridian, and T2 for the height when the moon is on the inferior meridian. Therefore, all places north of the equator will have the highest tide when the moon is above the horizon, and the lowest when she is below it; the difference of the tides diminishing towards the equator, where they are equal. In like manner, places south of the equator have the highest tides when the moon is below the horizon, and the lowest when she is above it. When the moon is at her greatest declination, the highest tides will take place towards the tropics. The circumstances are all reversed when the moon is south of the equator.†

* Diagrams like these are apt to mislead the learner, by exhibiting the protuberance occasioned by the tides as much greater than the reality. We must recollect that it amounts, at the highest, to only a very few feet in eight thousand miles. Were the diagram, therefore, drawn in just proportions, the alterations of figure produced by the tides would be wholly insensible.

† Edinb. Encyc. Art. *Astronomy*, p. 623.

286. The motion of the tide-wave, it should be remarked, is not a *progressive* motion, but a mere undulation, and is to be carefully distinguished from the currents to which it gives rise. If the ocean completely covered the earth, the sun and moon being in the equator, the tide-wave would travel at the same rate as the earth on its axis. Indeed, the correct way of cŏnceiving of the tide-wave, is to consider the moon at rest, and the earth in its rotation from west to east as bringing successive portions of water under the moon, which portions being elevated successively at the same rate as the earth revolves on its axis, have a relative motion westward in the same degree.

287. The tides of *rivers, narrow bays, and shores far from the main body of the ocean,* are not produced in those places by the direct action of the sun and moon, but are subordinate waves propagated from the great tide-wave.

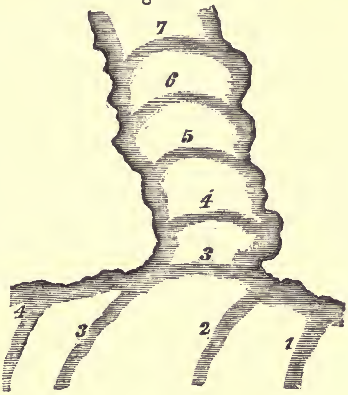
Lines drawn through all the adjacent parts of any tract of water, which have high water at the same time, are called *cotidal lines.** We may, for instance, draw a line through all places in the Atlantic Ocean which have high tide in a given day at 1 o'clock, and another through all places which have high tide at 2 o'clock. The cotidal line for any hour may be considered as representing the summit or ridge of the tide-wave at that time; and could the spectator, detached from the earth, perceive the summit of the wave, he would see it travelling round the earth in the open ocean once in twenty four hours, followed by another twelve hours distant, and both sending branches into rivers, bays, and other openings into the main land. These latter are called *Derivative* tides, while those raised directly by the action of the sun and moon, are called *Primitive* tides.

288. The velocity with which the wave moves will depend on various circumstances, but principally on the depth, and probably on the regularity of the channel. If the depth be nearly uniform, the cotidal lines will be nearly straight and parallel. But if some parts of the channel are deep while others are shallow, the tide will be detained by the greater friction of the shal-

* Whewell, Phil. Transaction for 1833, p. 148.

low places, and the cotidal lines will be irregular. The direction also of the derivative tide, may be totally different from that of the primitive. Thus, (Fig. 59,) if the great tide-wave, moving from east to west, be represented by the lines 1, 2, 3, 4, the derivative tide which is propagated up a river or bay, will be represented by the cotidal lines 3, 4, 5, 6, 7. Advancing faster in the channel than next the banks, the tides will lag behind towards the shores, and the cotidal lines will take the form of curves as represented in the diagram.

Fig. 59.



289. On account of the retarding influence of shoals, and an uneven, indented coast, the tide-wave travels more slowly along the shores of an island than in the neighboring sea, assuming convex figures at a little distance from the island and on opposite sides of it. These convex lines sometimes meet and become blended in such a manner as to create singular anomalies in a sea much broken by islands, as well as on coasts indented with numerous bays and rivers.* Peculiar phenomena are also produced, when the tide flows in at opposite extremities of a reef or island, as into the two opposite ends of Long Island Sound. In certain cases a tide-wave is forced into a narrow arm of the sea, and produces very remarkable tides. The tides of the Bay of Fundy (the highest in the world) sometimes rise to the height of 60 or 70 feet; and the tides of the river Severn, near Bristol in England, rise to the height of 40 feet.

290. The *Unit of Altitude* of any place, is the height of the maximum tide after the syzigies, (Art. 283,) being usually about 36 hours after the new or full moon. But as the amount of this tide would be affected by the distance of the sun and moon from

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* See an excellent representation and description of these different phenomena by Professor Whewell, Phil. Trans. 1833, p. 153.

the earth, (Art. 284,) and by their declinations, (Art. 285,) these distances are taken at their mean value, and the luminaries are supposed to be in the equator; the observations being so reduced as to conform to these circumstances. The unit of altitude can be ascertained by observation only. The actual rise of the tide depends much on the strength and direction of the wind. When high winds conspire with a high flood tide, as is frequently the case near the equinoxes, the tide often rises to a very unusual height. We subjoin from the American Almanac a few examples of the unit of altitude for different places.

	Feet.
Cumberland, head of the Bay of Fundy,	71
Boston, ...	11¼
New Haven, ...	8
New York, ...	5
Charleston, S. C., . . .	6

291. The *Establishment* of any port is the mean interval between noon and the time of high water, on the day of new or full moon. As the interval for any given place is always nearly the same, it becomes a criterion of the retardation of the tides at that place. On account of the importance to navigation of a correct knowledge of the tides, the British Board of Admiralty, at the suggestion of the Royal Society, recently issued orders to their agents in various important naval stations, to have accurate observations made on the tides, with the view of ascertaining the establishment and various other particulars respecting each station;* and the government of the United States is prosecuting similar investigations respecting our own ports.

292. According to Professor Whewell,† the tides on the coast of North America are derived from the great tide-wave of the South Atlantic, which runs steadily northward along the coast to the mouth of the Bay of Fundy, where it meets the northern tide-wave flowing in the opposite direction. Hence he accounts for the high tides of the Bay of Fundy.

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* Lubbock, Report on the Tides, 1833. † Phil. Trans. 1833, p. 172.

293. The largest *lakes* and *inland seas* have no perceptible tides. This is asserted by all writers respecting the Caspian and Euxine, and the same is found to be true of the largest of the North American lakes, Lake Superior.*

Although these several tracts of water appear large when taken by themselves, yet they occupy but small portions of the surface of the globe, as will appear evident from the delineation of them on an artificial globe. Now we must recollect that the primitive tides are produced by the *unequal* action of the sun and moon upon the different parts of the earth; and that it is only at points whose distance from each other bears a considerable ratio to the whole distance of the sun or the moon, that the inequality of action becomes manifest. The space required is larger than either of these tracts of water. It is obvious also that they have no opportunity to be subject to a derivative tide.

294. To apply the theory of universal gravitation to all the varying circumstances that influence the tides, becomes a matter of such intricacy, that La Place pronounces "the problem of the tides" the most difficult problem of celestial mechanics.

295. The *Atmosphere* that envelops the earth, must evidently be subject to the action of the same forces as the covering of waters, and hence we might expect a rise and fall of the barometer, indicating an atmospheric tide corresponding to the tide of the ocean. La Place has calculated the amount of this aerial tide. It is too inconsiderable to be detected by changes in the barometer, unless by the most refined observations. Hence it is concluded, that the fluctuations produced by this cause are too slight to affect meteorological phenomena in any appreciable degree.†

* See Experiments of Gov. Cass, Am. Jour. Science.

† Bowditch's La Place, II, 797.

CHAPTER IX.

OF THE PLANETS—THE INFERIOR PLANETS, MERCURY AND VENUS.

296. The name planet signifies a *wanderer*,* and is applied to this class of bodies because they shift their positions in the heavens, whereas the fixed stars constantly maintain the same places with respect to each other. The planets known from a high antiquity, are Mercury, Venus, Earth, Mars, Jupiter, and Saturn. To these, in 1781, was added Uranus,† (or *Herschel*, as it is sometimes called from the name of its discoverer,) and, as late as the commencement of the present century, four more were added, namely, Ceres, Pallas, Juno, and Vesta. These bodies are designated by the following characters:

1.	Mercury	☿	7.	Ceres	⚳
2.	Venus	♀	8.	Pallas	⚴
3.	Earth	⊕	9.	Jupiter	♃
4.	Mars	♂	10.	Saturn	♄
5.	Vesta	⚶	11.	Uranus	♅
6.	Juno	⚵			

The foregoing are called the *primary* planets. Several of these have one or more attendants, or satellites, which revolve around them, as they revolve around the sun. The earth has one satellite, namely, the moon; Jupiter has four; Saturn, seven; and Uranus, six. These bodies also are planets, but in distinction from the others they are called *secondary* planets. Hence, the whole number of planets are 29, viz. 11 primary, and 18 secondary planets.

297. With the exception of the four new planets, these bodies have their orbits very nearly in the same plane, and are never seen far from the ecliptic. Mercury, whose orbit is most inclined of all, never departs farther from the ecliptic than about 7°, while

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* From the Greek, *πλανητης*. † From *Ουρανος*.

most of the other planets pursue very nearly the same path with the earth, in their annual revolution around the sun. The new planets, however, make wider excursions from the plane of the ecliptic, amounting, in the case of Pallas, to 34½°.

298. Mercury and Venus are called *inferior* planets, because they have their orbits nearer to the sun than that of the earth; while all the others, being more distant from the sun than the earth, are called *superior* planets. The planets present great diversities among themselves in respect to distance from the sun, magnitude, time of revolution, and density. They differ also in regard to satellites, of which, as we have seen, three have respectively four, six, and seven, while more than half have none at all. It will aid the memory, and render our view of the planetary system more clear and comprehensive, if we classify, as far as possible, the various particulars comprehended under the foregoing heads.

299. DISTANCES FROM THE SUN.*

1. Mercury,	37,000,000	0.3870981
2. Venus,	68,000,000	0.7233316
3. Earth,	95,000,000	1.0000000
4. Mars,	142,000,000	1.5236923
5. Vesta,	225,000,000	2.3678700
6. Juno,	261,000,000	2.6690090
7. Ceres,		2.7672450
8. Pallas,		2.7728860
9. Jupiter,	485,000,000	5.2027760
10. Saturn,	890,000,000	9.5387861
11. Uranus,	1800,000,000	19.1823900

The *dimensions* of the planetary system are seen from this table to be vast, comprehending a circular space thirty six hundred millions of miles in diameter. A railway car, travelling constantly

* The distance in miles, as expressed in the first column, in round numbers, is to be treasured up in the memory, while the second column expresses the relative distances, that of the earth being 1, from which a more exact determination may be made, when required, the earth's distance being taken at 94,885,491. (Baily.)

at the rate of 20 miles an hour, would require more than 20,000 years to cross the orbit of Uranus.

It may aid the memory to remark, that in regard to the planets nearest the sun, the distances increase in an arithmetical ratio, while those most remote increase in a geometrical ratio. Thus, if we add 30 to the distance of Mercury, it gives us nearly that of Venus; 30 more gives that of the Earth; while Saturn is nearly twice the distance of Jupiter, and Uranus twice the distance of Saturn. Between the orbits of Mars and Jupiter, a great chasm appeared, which broke the continuity of the series; but the discovery of the new planets has filled the void. A more exact law of the series was discovered a few years since by Mr. Bode of Berlin. It is as follows: if we represent the distance of Mercury by 4, and increase each term by the product of 3 into a certain power of 2, we shall obtain the distances of each of the planets in succession. Thus,

Mercury, . . .	4	$= 4$
Venus, . . .	$4+3.2^0$	$= 7$
Earth, . . .	$4+3.2^1$	$= 10$
Mars, . . .	$4+3.2^2$	$= 16$
Ceres, . . .	$4+3.2^3$	$= 28$
Jupiter, . . .	$4+3.2^4$	$= 52$
Saturn, . . .	$4+3.2^5$	$=100$
Uranus, . . .	$4+3.2^6$	$=196$

For example, by this law, the distances of the Earth and Jupiter are to each other as 10 to 52. Their actual distances as given in the table (Art. 299,) are as 1 to 5.202776; but 1 : 5.202776 : : 10 : 52 nearly.

The mean distances of the planets from the sun, may also be determined by means of Kepler's law, that the squares of the periodical times are as the cubes of the distances, (Art. 192.) Thus the earth's distance being previously ascertained by means of the sun's horizontal parallax, (Art. 87,) and the period of any other planet, as Jupiter, being learned from observation, we say as $\overline{365.256}^2 : \overline{4332.585}^2$* : : $1^3 : 5.202^3$. But 5.202 is the number, which, according to the table (Art. 299,) expresses the distance of Jupiter from the sun.

* This is the number of days in one revolution of Jupiter.

300. MAGNITUDES.

	Diam. in Miles.	Mean apparent Diam.	Volume.
Mercury, . . .	3140	6″.9	$\frac{1}{17}$
Venus, . . .	7700	16″.9	$\frac{9}{10}$
Earth, . . .	7912		1
Mars, . . .	4200	6″.3	$\frac{1}{7}$
Ceres, . . .	160	0″.5	
Jupiter, . . .	89000	36″.7	1281
Saturn, . . .	79000	16″.2	995
Uranus, . . .	35000	4″.0	80

We remark here a great diversity in regard to magnitude, a diversity which does not appear to be subject to any definite law. While Venus, an inferior planet, is $\frac{9}{10}$ as large as the earth, Mars, a superior planet is only $\frac{1}{7}$, while Jupiter is 1281 times as large. Although several of the planets, when nearest to us, appear brilliant and large when compared with the fixed stars, yet the angle which they subtend is very small, that of Venus, the greatest of all, never exceeding about 1′, or more exactly 61″.2, and that of Jupiter being when greatest only about $\frac{3}{4}$ of a minute.

The distance of one of the near planets, as Venus or Mars, may be determined from its parallax; and the distance being known, its real diameter can be estimated from its apparent diameter, in the same manner as we estimate the diameter of the sun. (Art. 145.)

301. PERIODIC TIMES.

	Revolution in its orbit.				Mean daily motion.
Mercury,	3 months,	or	88	days,	4° 5′ 32″.6
Venus,	7½ "	"	224	"	1° 36′ 7″.8
Earth,	1 year,	"	365	"	0° 59′ 8″.3
Mars,	2 "	"	687	"	0° 31′ 26″.7
Ceres,	4 "	"	1681	"	0° 12′ 50″.9
Jupiter,	12 "	"	4332	"	0° 4′ 59″.3
Saturn,	29 "	"	10759	"	0° 2′ 0″.6
Uranus,	84 "	"	30686	"	0° 0′ 42″.4

From this view, it appears that the planets nearest the sun move most rapidly. Thus Mercury performs nearly 350 revolutions while Uranus performs one. This is evidently not owing merely to the greater dimensions of the orbit of Uranus, for the length of its orbit is not 50 times that of the orbit of Mercury, while the time employed in describing it is 350 times that of Mercury. Indeed this ought to follow from Kepler's law that the squares of the periodical times are as the *cubes* of the distances, from which it is manifest that the times of revolution increase faster than the dimensions of the orbit. Accordingly, the apparent progress of the most distant planets is exceedingly slow, the daily rate of Uranus being only 42″.4 per day; so that for weeks and months, and even years, this planet but slightly changes its place among the stars.

THE INFERIOR PLANETS, MERCURY AND VENUS.

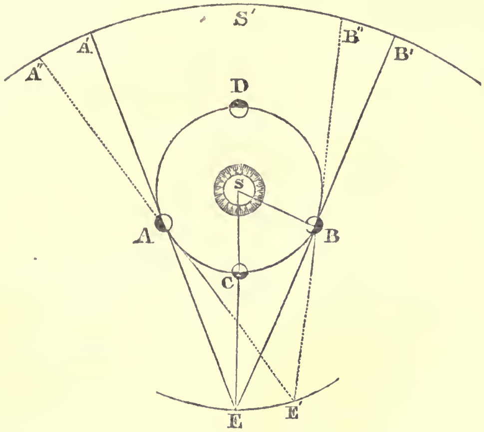
302. The inferior planets, Mercury and Venus, having their orbits so far within that of the earth, appear to us as attendants upon the sun. Mercury never appears farther from the sun than 29° (28° 48′) and seldom so far; and Venus never more than about 47° (47° 12′). Both planets, therefore, appear either in the west soon after sunset, or in the east a little before sunrise. In high latitudes, where the twilight is prolonged, Mercury can seldom be seen with the naked eye, and then only at the periods of its greatest elongation.* The reason of this will readily appear from the following diagram.

Let S (Fig. 60,) represent the sun, ADB the orbit of Mercury, and E the place of the Earth. Each of the planets is seen at its greatest elongation, when a line, EA or EB in the figure, is a tangent to its orbit. Then the sun being at S′ in the heavens, the planet will be seen at A′ and B′, when at its greatest elongations, and will appear no further from the sun than the arc S′A′ or S′B′ respectively.

303. A planet is said to be in *conjunction* with the sun, when it is seen in the same part of the heavens with the sun, or when it has the same longitude. Mercury and Venus have each two

* Copernicus is said to have lamented on his death-bed that he had never been able to obtain a sight of Mercury, and Delambre saw it but twice.

Fig. 60.



conjunctions, the inferior, and the superior. The *inferior conjunction* is its position when in conjunction on the same side of the sun with the earth, as at C in the figure: the *superior conjunction* is its position when on the side of the sun most distant from the earth, as at D.

304. The period occupied by a planet between two successive conjunctions with the earth, is called its *synodical revolution.* Both the planet and the earth being in motion, the time of the synodical revolution exceeds that of the sidereal revolution of Mercury or Venus; for when the planet comes round to the place where it before overtook the earth, it does not find the earth at that point, but far in advance of it. Thus, let Mercury come into inferior conjunction with the earth at C, (Fig. 60.) In about 88 days, the planet will come round to the same point again; but meanwhile the earth has moved forward through the arc EE′, and will continue to move while the planet is moving more rapidly to overtake her, the case being analogous to that of the hour and second hand of a clock.

Having the *sidereal* period of a planet, (which may always be accurately determined by observation,) we may ascertain its *synodical* period as follows. Let T denote the sidereal period of the earth, and T′ that of the planet. Since, in the time T the earth describes a complete revolution, $T : T' :: 1 : \frac{T'}{T}$ = the part of the circumference described by the earth in the time T′. But during the same time the planet describes a whole circumference. Therefore, $1 - \frac{T'}{T}$ is what the planet gains on the earth in one revolution. In order to a new conjunction the planet must gain an entire circumference; therefore, denoting the synodical period by S, the gain in one revolution will be to the time in which it is acquired, as a whole circumference is to the time in which that is gained, which is the synodical period. That is,

$$1 - \frac{T'}{T} : T' :: 1 : S = \frac{TT'}{T - T'}.$$

From this formula we may find the synodical period of Mercury or Venus by substituting the numbers denoted by the letters. Thus, $\frac{365.256 \times 87.969}{277.287} = 115.877$, which is the synodical period of Mercury.

By a similar computation, the synodical revolution of Venus will be found to be about 584 days.

305. *The motion of an inferior planet is direct in passing through its superior conjunction, and retrograde in passing through its inferior conjunction.* Thus Venus, while going from B through D to A, (Fig. 60,) moves in the order of the signs, or from west to east, and would appear to traverse the celestial vault B′S′A′ from right to left; but in passing from A through C to B, her course would be retrograde, returning on the same arc from left to right. *If the earth were at rest,* therefore, (and the sun, of course, at rest,) the inferior planets would appear to oscillate backwards and forwards across the sun. But, it must be recollected, that the earth is moving in the same direction with the planet, as respects the signs, but with a slower motion. This modifies the motions of the planet, accelerating it in the superior and retarding it in the inferior conjunctions. Thus in figure 60, Venus while moving through BDA would seem to move in the heavens from B′ to A′ were the earth at rest; but meanwhile the earth changes its position from E to E′, by which means the planet is not seen

at A′ but at A″, being accelerated by the arc A′A″ in consequence of the earth's motion. On the other hand, when the planet is passing through its inferior conjunction ACB, it appears to move backwards in the heavens from A′ to B′ if the earth is at rest, but from A′ to B″ if the earth has in the mean time moved from E to E′, being retarded by the arc B′B″. Although the motions of the earth have the effect to accelerate the planet in the superior conjunction, and to retard it in the inferior, yet, on account of the greater distance, the apparent motion of the planet is much slower in the superior than in the inferior conjunction.

306. *When passing from the superior to the inferior conjunction, or from the inferior to the superior conjunction, through the greatest elongations, the inferior planets are stationary.*

If the earth were at rest, the stationary points would be at the greatest elongations as at A and B, for then the planet would be moving directly towards or from the earth, and would be seen for some time in the same place in the heavens; but the earth itself is moving nearly at right angles to the line of the planet's motion, that is, the line which is drawn from the earth to the planet through the point of greatest elongation; hence a direct motion is given to the planet by this cause. When the planet, however, has passed this line, by its superior velocity it soon overcomes this tendency of the earth to give it a relative motion eastward, and becomes retrograde as it approaches the inferior conjunction. Its stationary point obviously lies between its place of greatest elongation, and the place where its motion becomes retrograde. Mercury is stationary at an elongation of from 15° to 20° from the sun; and Venus at about 29°.*

307. *Mercury and Venus exhibit to the telescope phases similar to those of the moon.*

When on the side of their inferior conjunctions, these planets appear horned, like the moon in her first and last quarters; and when on the side of their superior conjunctions they appear gibbous. At the moment of superior conjunction, the whole enlightened orb of the planet is turned towards the earth, and the

* Herschel, p. 242.—Woodhouse, 557.

appearance would be that of the full moon, but the planet is too near the sun to be commonly visible.

These different phases show that these bodies are opake, and shine only as they reflect to us the light of the sun; and the same remark applies to all the planets.

308. *The distance of an inferior planet from the sun, may be found by observations at the time of its greatest elongation.*

Thus if E be the place of the earth, and B that of Venus at the time of her greatest elongation, the angle SBE will be known, being a right angle. Also the angle SEB is known from observation. Hence the *ratio* of SB to SE becomes known; or, since SE is given, being the distance of the earth from the sun, SB the radius of the orbit of the planet is determined. If the orbits were both circles, this method would be very exact; but being elliptical, we obtain the mean value of the radius SB by observing its greatest elongation in different parts of its orbit.*

309. *The orbit of Mercury is the most eccentric, and the most inclined of all the planets;† while that of Venus varies but little from a circle, and lies much nearer to the ecliptic.*

The eccentricity of the orbit of Mercury is nearly $\frac{1}{5}$ its semi-major axis, while that of Venus is only $\frac{1}{145}$; the inclination of Mercury's orbit is 7°, while that of Venus is less than $3\frac{1}{2}$°‡. Mercury, on account of his different distances from the earth, varies much in his apparent diameter, which is only 5″ in the apogee, but 12″ in the perigee. The inclination of his orbit to his equator being very great, the changes of his seasons must be proportionally great.

310. The most favorable time for determining the sidereal revolution of a planet, is when its conjunction takes place at one of its nodes; for then the sun, the earth, and the planet, being in the same straight line, it is referred to its true place in the heavens, whereas, in other positions, its apparent place is more or less affected by perspective.

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* Herschel, p. 239.

† The new planets are of course excepted. ‡ Baily's Tables.

311. *An inferior planet is brightest at a certain point between its greatest elongation and inferior conjunction.*

Its maximum brilliancy would happen at the inferior conjunction, (being then nearest to us,) if it shined by its own light; but in that position, its dark side is turned towards us. Still, its maximum cannot be when most of the illuminated side is towards us; for then, being at the superior conjunction, it is at its greatest distance from us. The maximum must therefore be somewhere between the two. Venus gives her greatest light when about 40° from the sun.

312. *Mercury and Venus both revolve on their axes, in nearly the same time with the earth.*

The diurnal period of Mercury is 24h. 5m. 28s., and that of Venus 23h. 21m. 7s. The revolutions on their axes have been determined by means of some spot or mark seen by the telescope, as the revolution of the sun on his axis is ascertained by means of his spots.

313. Venus is regarded as the most beautiful of the planets, and is well known as the *morning and evening star*. The most ancient nations did not indeed recognize the evening and morning star as one and the same body, but supposed they were different planets, and accordingly gave them different names, calling the morning star Lucifer, and the evening star Hesperus. At her period of greatest splendor, Venus casts a shadow, and is sometimes visible in broad daylight. Her light is then estimated as equal to that of twenty stars of the first magnitude.* At her period of greatest elongation, Venus is visible from three to four hours after the setting or before the rising of the sun.

314. *Every eight years, Venus forms her conjunctions with the sun in the same part of the heavens.*

For, since the synodical period of Venus is 584 days, and her sidereal period 224.7,

$224.7 : 360° :: 584 : 935.6$ = the arc of longitude described by Venus between the first and second conjunctions. Deducting

* Francœur, Uranography, p. 125.

720°, or two entire circumferences, the remainder, 215°.6, shows how far the place of the second conjunction is in advance of the first. Hence, in five synodical revolutions, or 2920 days, the same point must have advanced $215°.6 \times 5 = 1078°$, which is nearly three entire circumferences, so that at the end of five synodical revolutions, occupying 2920 days, or 8 years, the conjunction of Venus takes place nearly in the same place in the heavens as at first.

Whatever appearances of this planet, therefore, arise from its positions with respect to the earth and the sun, they are repeated every eight years in nearly the same form.

TRANSITS OF THE INFERIOR PLANETS.

315. *The Transit of Mercury or Venus, is its passage across the sun's disk, as the moon passes over it in a solar eclipse.*

As a transit takes place only when the planet is in inferior conjunction, at which time her motion is retrograde (Art. 305,) it is always from left to right, and the planet is seen projected on the solar disk in a black round spot. Were the orbits of the inferior planets coincident with the plane of the earth's orbit a transit would occur to some part of the earth at every inferior conjunction. But the orbit of Venus makes an angle of $3\frac{1}{2}°$ with the ecliptic, and Mercury an angle of 7°; and, moreover, the apparent diameter of each of these bodies is very small, both of which circumstances conspire to render a transit a comparatively rare occurrence, since it can happen only when the sun, at the time of an inferior conjunction, chances to be at or extremely near the planet's node. The nodes of Mercury lie in longitude 46° and 226°, points which the sun passes through in May and November. It is only in these months, therefore, that transits of Mercury can occur. For a similar reason, those of Venus occur only in June and December. Since, however, the nodes of both planets have a small retrograde motion, the months in which transits occur will change in the course of ages.

316. The *intervals between successive transits,* may be found in the following manner. The formula which gives the synodical

period (Art. 304,) is $S=\frac{T\times T'}{T-T'}$, where S denotes the period, T the sidereal revolution of the earth, and T′ that of the planet. If we now represent by m the number of synodical revolutions of the earth in the required period, and by n the number of revolutions of the planet in the same time, then, since the number of revolutions in each case is inversely as the time of one, we have, $T : \frac{T\times T'}{T-T} :: n : m \therefore mT=\frac{nTT'}{T-T} \therefore \frac{m}{n}=\frac{T'}{T-T'}$. But m is the number of revolutions which the earth must perform in order that the two bodies, after having once met at the planet's nodes, may meet again at the same place. In the case of Mercury, whose sidereal period is 87.969 days, while that of the earth is 365.256 days, $\frac{m}{n}=\frac{87969}{277287}$; that is, after the earth has revolved 87969 times, (or after this number of years,) Mercury will have revolved just 277287, and the two bodies will be together again at the place where they started. But as periods of such enormous length do not fall within the observation of man, let us search for smaller numbers having nearly the same ratio. Now,

$$87969 : 365256 :: 1 : 4\tfrac{1}{6} \text{ (nearly.)}$$

This shows that in one year Mercury will have made 4 revolutions and $\frac{1}{6}$ of another; so that, when the sun returns to the same node, Mercury will be more than 60° in advance of it; consequently, no transit can take place after an interval of one year. But, by making trial of 2, 3, 4, &c. years, we shall find a nearer approximation at the end of 6 years; for,

$87969 : 365265 :: 6 : 25-\frac{1}{11}$. In 6 years, therefore, Mercury will fall short of reaching the node by only $\frac{1}{11}$ of a revolution, or about 33°. In 13 years the chance of meeting will be much greater, for in this period the earth will have made 13 and Mercury 54 revolutions. The numbers 33 and 137, 46 and 191, afford a still nearer approximation.*

317. In a similar manner, transits of Venus are probable after 8, 227, 235, and 243 years. Since Venus returns to her conjunction at nearly the same point of her orbit, after 8 years, (Art. 314,)

* This series may readily be obtained by the method of *Continued Fractions.* See Davies's Bourdon's Algebra.

it frequently happens that a transit takes place after an interval of 8 years. But at that time Venus is so far from her node, that her latitude amounts to from 20′ to 24′. Still she may possibly come within the sun's disk as she passes by him; for suppose at the preceding transit her latitude was 10′ on one side of the node and is now 10′ on the other side, this being less than the sun's semi-diameter, a transit may occur 8 years after another. Thus transits of Venus took place in 1761 and 1769. But in 16 years the latitude changes from 40′ to 48′, and Venus could not reach any part of the solar disk in her inferior conjunction.

From the above series we should infer that another transit could not take place under 227 years; but since there are two nodes, the chance is doubled, so that a transit may occur at the other node in half that interval, or in about 113 years. If, at the occurrence of the first transit, Venus had passed her node, the next transit at the other node will happen 8 years before the 113 are completed; or if she had not reached the node, it will happen 8 years later. Hence, after two transits have occurred within 8 years, another cannot be expected before 105, 113, or 121 years. Thus, the next transit will happen in 1874=1769+105; also in 1782=1874+8.

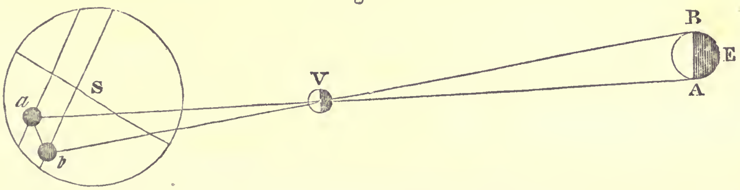
318. The great interest attached by astronomers to a transit of Venus, arises from its furnishing the most accurate means in our power of determining the *sun's horizontal parallax*,—an element of great importance, since it leads us to a knowledge of the distance of the earth from the sun, and, consequently, by the application of Kepler's law, (Art. 183,) to the distances of all the other planets. Hence, in 1769, great efforts were made throughout the civilized world, under the patronage of different governments, to observe this phenomenon under circumstances the most favorable for determining the parallax of the sun.

The method of finding the parallax of a heavenly body described in article 85, cannot be relied on to a greater degree of accuracy than 4″. In the case of the moon, whose greatest parallax amounts to about 1°, this deviation from absolute accuracy is not material; but it amounts to nearly half the entire parallax of the sun.

319. If the sun and Venus were equally distant from us, they would be equally affected by parallax as viewed by spectators in different parts of the earth, and hence their *relative* situation would not be altered by it; but since Venus, at the inferior conjunction, is only about one third as far off as the sun, her parallax is proportionally greater, and therefore spectators at distant points will see Venus projected on different parts of the solar disk, and as the planet traverses the disk, she will appear to describe chords of different lengths, by means of which the duration of the transit may be estimated at different places. The difference in the duration of the transit does not amount to many minutes; but to make it as large as possible very distant places are selected for observation. Thus in the transit of 1769, among the places selected, two of the most favorable were Wardhuz in Lapland, and Oteheite, one of the South Sea Islands.

The principle on which the sun's horizontal parallax is estimated from the transit of Venus, may be illustrated as follows: Let E (Fig. 61,) be the earth, V Venus, and S the sun. Suppose A, B, two spectators at opposite extremities of that diameter of the earth which is perpendicular to the ecliptic. The spectator at A will see Venus on the sun's disk at *a*, and the spectator at

Fig. 61.



B will see Venus at *b*; and since AV and BV may be considered as equal to each other, as also V*b* and V*a*, therefore the triangles AVB and V*ab* are similar to each other, and AV : V*a* : : AB : *ab*. But the ratio of AV to V*a* is known, (Art. 308); hence, the ratio of AB to *ab* is known, and when the angular value of *ab* as seen from the earth, is found, that of AB becomes known, as seen from the sun;* and half AB, or the semi-diameter of the earth as seen

* If, for example, *ab* is $2\frac{1}{2}$ times AB, (which is nearly the fact,) then if AB were on the sun instead of on the earth, it would subtend an angle at the eye equal to $\frac{1}{2.5}$ of *ab*. But if viewed from the sun, the distance being the same, its apparent diameter must be the same.

from the sun, is the sun's horizontal parallax. To find the apparent diameter of *ab*, we have only to find the breadth of the space between the two chords. Now, we can ascertain the value of each chord by the time occupied in describing it, since the motions of Venus and those of the sun are accurately known from the tables. Each chord being double the sine of half the arc cut off by it, therefore the sine of half the arc and of course the versed sine becomes known, and the difference of the two versed sines is the breadth of the zone in question. There are many circumstances to be taken into the account in estimating, from observations of this kind, the sun's horizontal parallax; but the foregoing explanation may be sufficient to give the learner an idea of the *general principles* of this method. The appearance of Venus on the sun's disk, being that of a well defined black spot, and the exactness with which the moment of external or internal contact may be determined, are circumstances favorable to the exactness of the result; and astronomers repose so much confidence in the estimation of the sun's horizontal parallax as derived from the observations on the transit of 1769, that this important element is thought to be ascertained within $\frac{1}{10}$ of a second. The general result of all these observations give the sun's horizontal parallax 8.″6, or more exactly, 8.″5776.*

320. During the transits of Venus over the sun's disk in 1761 and 1769, a sort of penumbral light was observed around the planet by several astronomers, which was thought to indicate an *atmosphere*. This appearance was particularly observable while the planet was coming on and going off the solar disk. The total immersion and emersion were not instantaneous; but as two drops of water when about to separate, form a ligament between them, so there was a dark shade stretched out between Venus and the sun, and when the ligament broke, the planet seemed to have got about an eighth part of her diameter from the limb of the sun.† The various accounts of the two transits abound with remarks like these, which indicate the existence of an atmosphere about Venus of nearly the density and extent of the earth's at-

* Delambre, t. 2. Vince's Complete Syst. Vol. 1. Woodhouse, p. 754. Herschel, p. 243.

† Edinb. Encyc. Art. *Astronomy*.

mosphere. Similar proofs of the existence of an atmosphere around this planet, are derived from appearances of twilight.

The elder astronomers imagined they had discovered a *satellite* accompanying Venus in her transit. If Venus had in reality any satellite, the fact would be obvious at her transits, as the satellite would be projected near the primary on the sun's disk; but later astronomers have searched in vain for any appearances of the kind, and the inference is that former astronomers were deceived by some optical illusion.

Astronomers have detected very high *mountains* on Venus, sometimes reaching to the elevation of 22 miles; and it is remarkable that the highest mountains in Venus, in Mercury, in the moon, and in the earth, are always in the southern hemisphere.

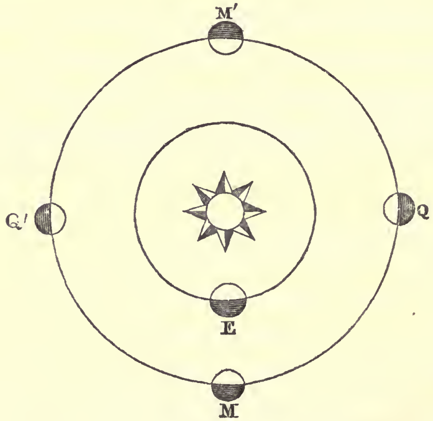
CHAPTER X.

OF THE SUPERIOR PLANETS—MARS, JUPITER, SATURN, AND URANUS.

321. THE Superior planets are distinguished from the Inferior, by being seen at all distances from the sun from 0° to 180°. Having their orbits exterior to that of the earth, they of course never come between us and the sun, that is, they never have any inferior conjunction like Mercury and Venus, but they are sometimes seen in superior conjunction, and sometimes in opposition. Nor do they, like the inferior planets, exhibit to the telescope different phases, but, with a single exception, they always present the side that is turned towards the earth fully enlightened. This is owing to their great distance from the earth; for were the spectator to stand upon the sun he would of course always have the illuminated side of each of the planets turned towards him; but, so distant are all the superior planets except Mars, that they are viewed by us very nearly in the same manner as they would be if we actually stood on the sun.

322. Mars is a small planet, his diameter being only about half that of the earth, or 4100 miles. He also, at times, comes nearer to the earth than any other planet except Venus. His *mean* distance from the sun is 142,000,000 miles; but his orbit is so eccentric that his distance varies much in different parts of his revolution. Mars is always very near the ecliptic, never varying from it 2°. He is distinguished from all the planets by his deep red color, and fiery aspect; but his brightness and apparent magnitude vary much at different times, being sometimes nearer to us than at others, by the whole diameter of the earth's orbit, that is, by about 190,000,000 of miles. When Mars is on the same side of the sun with the earth, or at his opposition, he comes within 47,000,000 miles of the earth, and rising about the time the sun sets surprises us by his magnitude and splendor; but when he passes to the other side of the sun to his superior conjunction, he dwindles to the appearance of a small star, being then 237,000,000 miles from us. Thus, let M (Fig. 62,) represent Mars in opposition, and M′ in the superior conjunction. It is obvious that in the former situation, the planet must be nearer to the earth than in the latter by the whole diameter of the earth's orbit.

Fig. 62.



323. Mars is the only one of the superior planets which exhibits phases. When he is towards the quadratures at Q or Q′, it is evident from the figure that only a part of the circle of illumina-

tion is turned towards the earth, such a portion of the remoter part of it being concealed from our view as to render the form more or less gibbous.

324. When viewed with a powerful telescope, the surface of Mars appears diversified with numerous varieties of light and shade. The region around the poles is marked by white spots, which vary their appearance with the changes of seasons in the planet. Hence Dr. Herschel conjectured that they were owing to ice and snow, which alternately accumulates and melts, according to the position of each pole with respect to the sun.* It has been common to ascribe the ruddy light of this planet to an extensive and dense atmosphere, which was said to be distinctly indicated, by the gradual diminution of light observed in a star as it approached very near to the planet in undergoing an occultation; but more recent observations afford no such evidence of an atmosphere.†

By observations on the spots we learn that Mars revolves on his axis in very nearly the same time with the earth, (24h. 39m. $21^s.3$); and that the inclination of his axis to that of the ecliptic is also nearly the same, being 30° 18′ 10″.8.

325. As the diurnal rotation of Mars is nearly the same as that of the earth, we might expect a similar flattening at the poles, giving to the planet a spheroidal figure. Indeed the compression or ellipticity of Mars greatly exceeds that of the earth, being no less than $\frac{1}{14}$ of the equatorial diameter, while that of the earth is only $\frac{1}{300}$, (Art. 138.) This remarkable flattening of the poles of Mars has been supposed to arise from a great variation of density in the planet in different parts.‡

326. Jupiter is distinguished from all the other planets by his vast *magnitude.* His diameter is 86,000 miles, and his volume 1280 times that of the earth. His figure is strikingly spheroidal, the equatorial being larger than the polar diameter in the proportion of 107 to 100. (See Frontispiece, Fig. 4.) Such a figure might

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* Phil. Trans. 1784. † Sir James South, Phil. Trans. 1833.

‡ Ed. Encyc. Art. *Astronomy.*

naturally be expected from the rapidity of his diurnal rotation, which is accomplished in about 10 hours. A place on the equator of Jupiter must turn 27 times as fast as on the terrestrial equator. The distance of Jupiter from the sun is nearly 490,000,000 miles, and his revolution around the sun occupies nearly 12 years.

327. The view of Jupiter through a good telescope, is one of the most magnificent and interesting spectacles in astronomy. The disk expands into a large and bright orb like the full moon ; the spheroidal figure which theory assigns to revolving spheres, is here palpably exhibited to the eye ; across the disk, arranged in parallel stripes, are discerned several dusky bands, called *belts ;* and four bright satellites, always in attendance, but ever varying their positions, compose a splendid retinue. Indeed, astronomers gaze with peculiar interest on Jupiter and his moons as affording a miniature representation of the whole solar system, repeating on a smaller scale, the same revolutions, and exemplifying, in a manner more within the compass of our observation, the same laws as regulate the entire assemblage of sun and planets. (See Fig. 63.)

328. The *Belts of Jupiter*, are variable in their number and dimensions. With the smaller telescopes, only one or two are seen across the equatorial regions; but with more powerful instruments, the number is increased, covering a great part of the whole disk. Different opinions have been entertained by astronomers respecting the cause of the belts; but they have generally been regarded as clouds formed in the atmosphere of the planet, agitated by winds as is indicated by their frequent changes, and made to assume the form of belts parallel to the equator by currents that circulate around the planet like the trade winds and other currents that circulate around our globe.* Sir John Herschel supposes that the belts are not ranges of clouds, but portions of the planet itself brought into view by the removal of clouds and mists that exist in the atmosphere of the planet through which are openings made by currents circulating around Jupiter.†

329. The *Satellites of Jupiter* may be seen with a telescope of very moderate powers. Even a common spy glass will enable

* Ed. Encyc. Art. *Astronomy.* † Herschel's Astron. p. 266.

us to discern them. Indeed one or two of them have been occasionally seen with the naked eye. In the largest telescopes, they severally appear as bright as Sirius. With such an instrument the view of Jupiter with his moons and belts is truly a magnificent spectacle, a world within itself. As the orbits of the satellites do not deviate far from the plane of the ecliptic, and but little from the equator of the planet, they are usually seen in nearly a straight line with each other extending across the central part of the disk. (See Frontispiece.)

330. Jupiter's satellites are distinguished from one another by the denominations of *first*, *second*, *third*, and *fourth*, according to their relative distances from Jupiter, the first being that which is nearest to him. Their apparent motion is oscillatory, like that of a pendulum, going alternately from their greatest elongation on one side to their greatest elongation on the other, sometimes in a straight line, and sometimes in an elliptical curve, according to the different points of view in which we observe them from the earth. They are sometimes stationary; their motion is alternately direct and retrograde; and, in short, they exhibit in miniature all the phenomena of the planetary system. Various particulars of the system are exhibited in the following table. The distances are given in radii of the primary.

Satellite.	Diameter.	Mean Distance.	Sidereal Revolution.		
1	2508	6.04853	1d.	18h.	28m.
2	2068	9.62347	3	13	14
3	3377	15.35024	7	3	43
4	2890	26.99835	16	16	32

Hence it appears, first, that Jupiter's satellites are all somewhat larger than the moon, but that the second satellite is the smallest, and the third the largest of the whole, but the diameter of the latter is only about $\frac{1}{26}$ part of that of the primary; secondly, that the distance of the innermost satellite from the planet is three times his diameter, while that of the outermost satellite is nearly fourteen times his diameter; thirdly, that the first satellite completes its revolution around the primary in one day and three fourths, while the fourth satellite requires nearly sixteen and three fourths days.

331. The orbits of the satellites are nearly or quite circular, and deviate but little from the plane of the planet's equator, and of course are but slightly inclined to the plane of his orbit. They are, therefore, in a similar situation with respect to Jupiter as the moon would be with respect to the earth if her orbit nearly coincided with the ecliptic, in which case she would undergo an eclipse at every opposition.

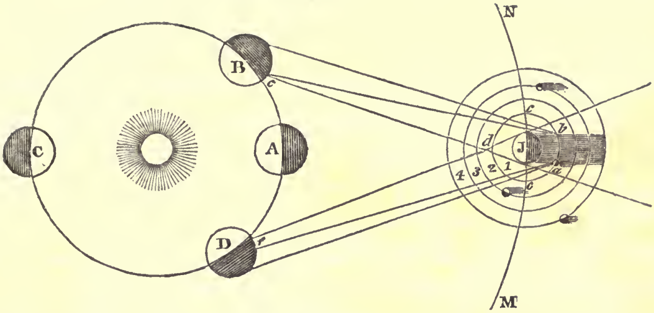
332. The eclipses of Jupiter's satellites, in their general conception, are perfectly analogous to those of the moon, but in their detail they differ in several particulars. Owing to the much greater distance of Jupiter from the sun, and its greater magnitude, the cone of its shadow is much longer and larger than that of the earth, (Art. 246.) On this account, as well as on account of the little inclination of their orbits to that of their primary, the three inner satellites of Jupiter pass through the shadow, and are totally eclipsed at every revolution. The fourth satellite, owing to the greater inclination of its orbit, sometimes though rarely escapes eclipse, and sometimes merely grazes the limits of the shadow or suffers a partial eclipse.* These eclipses, moreover, are not seen, as is the case with those of the moon, from the center of their motion, but from a remote station, and one whose situation with respect to the line of the shadow is variable. This, of course, makes no difference in the *times* of the eclipses, but a very great one in their visibility, and in their apparent situations with respect to the planet at the moment of their entering or quitting the shadow.

333. The eclipses of Jupiter's satellites present some curious phenomena, which will be understood from the following diagram.

Let A, B, C, D, (Fig. 63,) represent the earth in different parts of its orbit; J, Jupiter in his orbit MN, surrounded by his four satellites, the orbits of which are marked 1, 2, 3, 4. At *a* the first satellite enters the shadow of the planet, and emerges from it at *b*, and advances to its greatest elongation at *c*. Since the shadow is always opposite to the sun, only the *immersion* of a satellite will be visible to the earth while the earth is somewhere between

* Sir J. Herschel, Ast. p. 276.

Fig. 63.



C and A, that is, while the earth is passing from the position where it has the planet in superior conjunction, to that where it has the planet in opposition; for while the earth is in this situation, the planet conceals from its view the *emersion*, as is evident from the direction of the visual ray *fd*. For a similar reason the emersion only is visible while the earth passes from A to C, or from the opposition to the superior conjunction. In other words, when the earth is to the *westward* of Jupiter, only the immersions of a satellite are visible; when the earth is to the *eastward* of Jupiter, only the emersions are visible. This, however, is strictly true only of the first satellite; for the third and fourth, and sometimes even the second, owing to their greater distances from Jupiter, occasionally disappear and reappear on the same side of the disk. The reason why they should reappear on the *same* side of the disk, will be understood from the figure. Conceive the whole system of Jupiter and his satellites as projected on the more distant concave sphere, by lines drawn, like *fd*, from the observer on the earth through the planet and each of the satellites; then it is evident that the remoter parts of the shadow where the interior satellites traverse it, will fall to the westward of the planet, and of course these satellites as they emerge from the shadow will be projected to a point on the same side of the disk as the point of their emersion. The same mode of reasoning will show that when the earth is to the eastward of the planet, the immersions and emersions of the outermost satellites will be both seen on the east side of the disk. When the earth is in either of the positions C or A, that is, at the superior conjunction or opposition of the planet, both the immersions and emersions take place behind the planet, and the eclipses occur close to the disk.

334. When one of the satellites is passing between Jupiter and the sun, it casts a shadow upon its primary, which is seen by the telescope travelling across the disk of Jupiter, as the shadow of the moon would be seen to traverse the earth by a spectator favorably situated in space. When the earth is to the westward of Jupiter, as at D, the shadow reaches the disk of the planet, or is seen on the disk, before the satellite itself reaches it. For the satellite will not enter on the disk, until it comes up to the line *fd* at *d*, a point which it reaches later than its shadow reaches the same line. After the earth has passed the opposition, as at B, then the satellite will reach the visual ray *cd* at *d* sooner than the shadow, and of course be sooner projected on the disk. In the transits of Jupiter's satellites, which with very powerful telescopes may be observed with great precision, the satellite itself is sometimes seen on the disk as a *bright* spot, if it chances to be projected upon one of the belts. Occasionally, also, it is seen as a *dark* spot, of smaller dimensions than the shadow. This curious fact has led to the conclusion, that certain of the satellites have sometimes on their own bodies or in their atmospheres, obscure spots of great extent.*

335. A very singular relation subsists between the mean motions of the three first satellites of Jupiter. If the mean angular velocity of the first satellite be added to twice that of the third, the sum will be equal to three times that of the second. Hence, if from the mean longitude of the first plus twice the mean longitude of the second, be subtracted three times the mean longitude of the third, the remainder will always be the same constant quantity, which observation shows to be equal to 180°; so that the situation of any two of them being given, that of the third may be found. One curious consequence of this relation is, that these three satellites cannot be all eclipsed at once; for when the second and third lie in the same direction from the center, the first must lie on the opposite side; and when the first is eclipsed, the other two must lie between the sun and planet, throwing their shadows on the disk.† It will be remarked, that these phe-

* Sir J. Herschel. † Ibid.

nomena are such as would present themselves to a spectator on Jupiter, and not to a spectator on the earth.

336. The eclipses of Jupiter's satellites have been studied with great attention by astronomers, on account of their affording one of the easiest methods of determining the *longitude.* On this subject Sir J. Herschel remarks:* The discovery of Jupiter's satellites by Galileo, which was one of the first fruits of the invention of the telescope, forms one of the most memorable epochs in the history of astronomy. The first astronomical solution of the great problem of "the longitude,"—the most important problem for the interests of mankind that has ever been brought under the dominion of strict scientific principles, dates immediately from their discovery. The final and conclusive establishment of the Copernican system of astronomy, may also be considered as referable to the discovery and study of this exquisite miniature system, in which the laws of the planetary motions, as ascertained by Kepler, and especially that which connects their periods and distances, were speedily traced, and found to be satisfactorily maintained.

337. The entrance of one of Jupiter's satellites into the shadow of the primary being seen like the entrance of the moon into the earth's shadow, at the same moment of absolute time, at all places where the planet is visible, and being wholly independent of parallax; being, moreover, predicted beforehand with great accuracy for the instant of its occurrence at Greenwich, and given in the Nautical Almanac, this would seem to be one of those events (Art. 273,) which are peculiarly adapted for finding the longitude. It must be remarked, however, that the extinction of light in the satellite at its immersion, and the recovery of its light at its emersion, are not instantaneous but gradual; for the satellite, like the moon, occupies some time in entering into the shadow or in emerging from it, which occasions a progressive diminution or increase of light. The better the light afforded by the telescope with which the observation is made, the later the satellite will be seen at its immersion, and the sooner

* Elements of Ast. p. 279.

at its emersion.* In noting the eclipses even of the first satellite, the time must be considered as uncertain to the amount of 20 or 30 seconds; and those of the other satellites involve still greater uncertainty. Two observers, in the same room, observing with different telescopes the same eclipse, will frequently disagree in noting its time to the amount of 15 or 20 seconds; and the difference will be always the same way.†

Better methods, therefore, of finding the longitude are now employed, although the facility with which the necessary observations can be made, and the little calculation required, still render this method eligible in many cases where extreme accuracy is not important. As a telescope is essential for observing an eclipse of one of the satellites, it is obvious that this method cannot be practiced at sea.

338. The grand discovery of the *progressive motion of light*, was first made by observations on the eclipses of Jupiter's satellites. In the year 1675, it was remarked by Roemer, a Danish astronomer, on comparing together observations of these eclipses during many successive years, that they take place sooner by about sixteen minutes (16m. $26^s.6$) when the earth is on the same side of the sun with the planet, than when she is on the opposite side. This difference he ascribed to the progressive motion of light, which takes that time to pass through the diameter of the earth's orbit, making the velocity of light about 192,000 miles per second. So great a velocity startled astronomers at first, and produced some degree of distrust of this explanation of the phenomenon; but the subsequent discovery of the aberration of light (Art. 195,) led to an independent estimation of the velocity of light with almost precisely the same result.

339. Saturn comes next in the series as we recede from the sun, and has, like Jupiter, a system within itself, on a scale of great magnificence. In size it is, next to Jupiter, the largest of the planets, being 79,000 miles in diameter, or about 1,000 times as large as the earth. It has likewise belts on its surface and is

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* This is the reason why observers are directed in the Nautical Almanac to use telescopes of a *certain power*.

† Woodhouse, p. 840.

attended by seven satellites. But a still more wonderful appendage is its *Ring*, a broad wheel encompassing the planet at a great distance from it. We have already intimated that Saturn's system is on a grand scale. As, however, Saturn is distant from us nearly 900,000,000 miles, we are unable to obtain the same clear and striking views of his phenomena as we do of the phenomena of Jupiter, although they really present a more wonderful mechanism.

340. Saturn's ring, when viewed with telescopes of a high power, is found to consist of two concentric rings,* separated from each other by a dark space. (See Frontispiece.) Although this division of the rings appears to us, on account of our immense distance, as only a fine line, yet it is in reality an interval of not less than about 1800 miles. The dimensions of the whole system are in round numbers, as follows :†

	Miles.
Diameter of the planet, ...	79,000
From the surface of the planet to the inner ring,	20,000
Breadth of the inner ring, . . .	17,000
Interval between the rings, . . .	1,800
Breadth of the outer ring, . . .	10,500
Extreme dimensions from outside to outside,	176,000

The figure represents Saturn as it appears to a powerful telescope, surrounded by its rings, and having its body striped with dark belts, somewhat similar but broader and less strongly marked than those of Jupiter, and owing doubtless to a similar cause. That the ring is a solid opake substance, is shown by its throwing its shadow on the body of the planet on the side nearest the sun and on the other side receiving that of the body. From the parallelism of the belts with the plane of the ring, it may be conjectured that the axis of rotation of the planet is perpendicular to that plane; and this conjecture is confirmed by the occasional appearance of extensive dusky spots on its surface, which when watched indicate a rotation parallel to the ring in 10h. 29m. 17s. This motion, it will be remarked, is nearly the

* It is said that several additional divisions of the ring have been detected.—(Kater, Ast. Trans. iv. 383.)

† Prof. Struve, Mem. Ast. Soc., 3. 301.

same with the diurnal motion of Jupiter, subjecting places on the equator of the planet to a very swift revolution, and occasioning a high degree of compression at the poles, the equatorial being to the polar diameter in the high ratio of 11 to 10. But it is remarkable that the globe of Saturn appears to be flattened at the equator as well as at the poles. The polar compression extends to a great distance over the surface of the planet, and the greatest diameter is that of the parallel of 43° of latitude. The disk of Saturn, therefore, resembles a square of which the four corners have been rounded off.* It requires a telescope of high magnifying powers and a strong light to give a full and striking view of Saturn with his rings and belts and satellites; for we must bear in mind that at the distance of Saturn one second of angular measurement corresponds to 4,000 miles, a space equal to the semi-diameter of our globe. But with a telescope of moderate powers, the leading phenomena of the ring itself may be observed.

341. *Saturn's ring, in its revolution around the sun, always remains parallel to itself.*

If we hold opposite to the eye a circular ring or disk like a piece of coin, it will appear as a complete circle when it is at right angles to the axis of vision, but when oblique to that axis it will be projected into an ellipse more and more flattened as its obliquity is increased, until, when its plane coincides with the axis of vision, it is projected into a straight line. Let us place on the table a lamp to represent the sun, and holding the ring at a certain distance inclined a little towards the lamp, let us carry it round the lamp always keeping it parallel to itself. During its revolution it will twice present its edge to the lamp at opposite points, and twice at places 90° distant from those points, it will present its broadest face towards the lamp. At intermediate points, it will exhibit an ellipse more or less open, according as it is nearer one or the other of the preceding positions. It will be seen also that in one half of the revolution the lamp shines on one side of the ring, and in the other half of the revolution on the other side. Such would be the successive appearances of Saturn's ring to a spectator on the sun; and since the earth is,

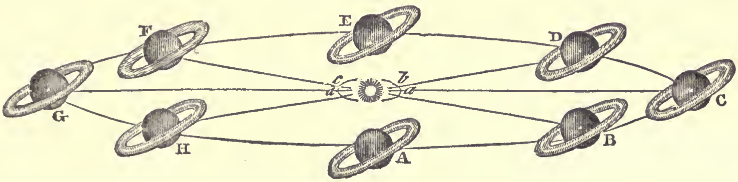
* Sir W. Herschel, Phil. Tr. 1806, Part II.

in respect to so distant a body as Saturn, very near the sun, those appearances are presented to us in nearly the same manner as though we viewed them from the sun. Accordingly, we sometimes see Saturn's ring under the form of a broad ellipse, which grows continually more and more acute until it passes into a line, and we either lose sight of it altogether, or with the aid of the most powerful telescopes, we see it as a fine thread of light drawn across the disk and projecting out from it on each side. As the whole revolution occupies 30 years, and the edge is presented to the sun twice in the revolution, this last phenomenon, namely, the disappearance of the ring, takes place every 15 years.

342. The learner may perhaps gain a clearer idea of the foregoing appearances from the following diagram:

Let A, B, C, &c. represent successive positions of Saturn and his ring in different parts of his orbit, while *abc* represents the orbit of the earth.* Were the ring when at C and G perpendicular to the line CG, it would be seen by a spectator situated at *a* or *d* a perfect circle, but being inclined to the line of vision 28° 4′, it is projected into an ellipse. This ellipse contracts in breadth

Fig. 64.



as the ring passes towards its nodes at A and E, where it dwindles into a straight line. From E to G the ring opens again, becomes broadest at G, and again contracts till it becomes a straight line at A, and from this point expands till it recovers its original breadth at C. These successive appearances are all exhibited to a telescope of moderate powers. The ring is extremely *thin*, since the smallest satellite, when projected on it, more than covers it. The thickness is estimated at 100 miles.

* It may be remarked by the learner, that these orbits are made so elliptical, not to represent the eccentricity of either the earth's or Saturn's orbit, but merely as the projection of circles seen very obliquely.

343. *Saturn's ring shines wholly by reflected light derived from the sun.* This is evident from the fact, that that side only which is turned towards the sun is enlightened; and it is remarkable, that the illumination of the ring is greater than that of the planet itself, but the outer ring is less bright than the inner. Although, as we have already remarked, we view Saturn's ring nearly as though we saw it from the sun, yet the plane of the ring produced may pass between the earth and the sun, in which case also the ring becomes invisible, the illuminated side being wholly turned from us. Thus, when the ring is approaching its node at E, a spectator at *b* would have the dark side of the ring presented to him. The ring was invisible in 1833, and will be invisible again in 1847. At present (1839) it is the northern side of the ring that is seen, but in 1855 the southern side will come into view.

It appears, therefore, that there are three causes for the disappearance of Saturn's ring; first, when the edge of the ring is presented to the sun; secondly, when the edge is presented to the earth; and thirdly, when the unilluminated side is towards the earth.

344. *Saturn's ring revolves in its own plane in about* 10½ *hours*, (10h. 32m. 15s.4). La Place inferred this from the doctrine of universal gravitation. He proved that such a rotation was necessary, otherwise the matter of which the ring is composed would be precipitated upon its primary. He showed that in order to sustain itself, its period of rotation must be equal to the time of revolution of a satellite, circulating around Saturn at a distance from it equal to that of the middle of the ring, which period would be about 10½ hours. By means of spots in the ring Dr. Herschel followed the ring in its rotation, and actually found its period to be the same as assigned by La Place,—a coincidence which beautifully exemplifies the harmony of truth.*

345. Although the rings are very nearly *concentric*, yet recent measurements of extreme delicacy have demonstrated, that the coincidence is not mathematically exact, but that the center of gravity of the rings describes around that of the body a very

* Systeme du Monde, l. iv. c. 8.

minute orbit. This fact, unimportant as it may seem, is of the utmost consequence to the stability of the system of rings. Supposing them mathematically perfect in their circular form, and exactly concentric with the planet, it is demonstrable that they would form (in spite of their centrifugal force) a system in a state of *unstable equilibrium*, which the slightest external power would subvert—not by causing a rupture in the substance of the rings—but by precipitating them *unbroken* on the surface of the planet.* The ring may be supposed of an unequal breadth in its different parts, and as consisting of irregular solids, whose common center of gravity does not coincide with the center of the figure. Were it not for this distribution of matter, its equilibrium would be destroyed by the slightest force, such as the attraction of a satellite, and the ring would finally precipitate itself upon the planet.†

As the smallest difference of velocity between the planet and its rings must infallibly precipitate the rings upon the planet, never more to separate, it follows either that their motions in their common orbit round the sun, must have been adjusted to each other by an external power, with the minutest precision, or that the rings must have been formed about the planet while subject to their common orbitual motion, and under the full and free influence of all the acting forces.

The rings of Saturn must present a magnificent spectacle from those regions of the planet which lie on their enlightened sides, appearing as vast arches spanning the sky from horizon to horizon, and holding an invariable situation among the stars. On the other hand, in the region beneath the dark side, a solar eclipse of 15 years in duration, under their shadow, must afford (to our ideas) an inhospitable abode to animated beings, but ill compensated by the full light of its satellites. But we shall do wrong to judge of the fitness or unfitness of their condition from what we see around us, when, perhaps, the very combinations which convey to our minds only images of horror, may be in reality theatres of the most striking and glorious displays of beneficent contrivance.‡

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* Sir J. Herschel. † La Place. ‡ Sir J. Herschel.

346. Saturn is attended by *seven satellites.* Although bodies of considerable size, their great distance prevents their being visible to any telescopes but such as afford a strong light and high magnifying powers. The outermost satellite is distant from the planet more than 30 times the planet's diameter, and is by far the largest of the whole. It is the only one of the series whose theory has been investigated further than suffices to verify Kepler's law of the periodic times, which is found to hold good here as well as in the system of Jupiter. It exhibits, like the satellites of Jupiter, periodic variations of light, which prove its revolution on its axis in the time of a sidereal revolution about Saturn. The next satellite in order, proceeding inwards, is tolerably conspicuous; the three next are very minute, and require pretty powerful telescopes to see them; while the two interior satellites, which just skirt the edge of the ring, and move exactly in its plane, have never been discovered but with the most powerful telescopes which human art has yet constructed, and then only under peculiar circumstances. At the time of the disappearance of the rings (to ordinary telescopes) they were seen by Sir William Herschel with his great telescope, projected along the edge of the ring, and threading like beads the thin fibre of light to which the ring is then reduced. Owing to the obliquity of the ring, and of the orbits of the satellites to that of their primary, there are no eclipses of the satellites, the two interior ones excepted, until near the time when the ring is seen edgewise.*

347. Uranus is the remotest planet belonging to our system, and is rarely visible except to the telescope. Although his diameter is more than four times that of the earth, (35,112 miles,) yet his distance from the sun is likewise nineteen times as great as the earth's distance, or about 1,800,000,000 miles. His revolution around the sun occupies nearly 84 years, so that his position in the heavens for several years in succession is nearly stationary. His path lies very nearly in the ecliptic, being inclined to it less than one degree, (46′ 28″.44.)

* Sir J. Herschel.

The sun himself when seen from Uranus dwindles almost to a star, subtending as it does an angle of only 1′ 40″; so that the surface of the sun would appear there 400 times less than it does to us.

This planet was discovered by Sir William Herschel on the 13th of March, 1781. His attention was attracted to it by the largeness of its disk in the telescope; and finding that it shifted its place among the stars, he at first took it for a comet, but soon perceived that its orbit was not eccentric like the orbits of comets, but nearly circular like those of the planets. It was then recognized as a new member of the planetary system, a conclusion which has been justified by all succeeding observations.

348. Uranus is attended by *six satellites*. So minute objects are they that they can be seen only by powerful telescopes. Indeed the existence of more than two is still considered as somewhat doubtful. These two, however, offer remarkable, and indeed quite unexpected and unexampled peculiarities. Contrary to the unbroken analogy of the whole planetary system, *the planes of their orbits are nearly perpendicular to the ecliptic*, being inclined no less than 78° 58′ to that plane, and in these orbits *their motions are retrograde;* that is, instead of advancing from west to east around their primary, as is the case with all the other planets and satellites, they move in the opposite direction.* With this exception, all the motions of the planets, whether around their own axes, or around the sun, are from west to east.

OF THE NEW PLANETS, CERES, PALLAS, JUNO, AND VESTA.

349. The commencement of the present century was rendered memorable in the annals of astronomy, by the discovery of four new planets between Mars and Jupiter. Kepler, from some analogy which he found to subsist among the distances of the planets from the sun, had long before suspected the existence of one at this distance; and his conjecture was rendered more probable by the discovery of Uranus, which follows the analogy of the other planets. So strongly, indeed, were astronomers im-

* Sir J. Herschel.

pressed with the idea that a planet would be found between Mars and Jupiter, that, in the hope of discovering it, an association was formed on the continent of Europe of twenty four observers, who divided the sky into as many zones, one of which was allotted to each member of the association. The discovery of the first of these bodies was however made accidentally by Piazzi, an astronomer of Palermo, on the first of January, 1801. It was shortly afterwards lost sight of on account of its proximity to the sun, and was not seen again until the close of the year, when it was re-discovered in Germany. Piazzi called it *Ceres* in honor of the tutelary goddess of Sicily, and her emblem, the sickle ⚳, has been adopted as its appropriate symbol.

The difficulty of finding Ceres induced Dr. Olbers, of Bremen, to examine with particular care all the small stars that lie near her path, as seen from the earth; and while prosecuting these observations, in March, 1802, he discovered another similar body, very nearly at the same distance from the sun, and resembling the former in many other particulars. The discoverer gave to this second planet the name of *Pallas*, choosing for its symbol the lance ⚴, the characteristic of Minerva.

350. The most surprising circumstance connected with the discovery of Pallas, was the existence of two planets at nearly the same distance from the sun, and apparently having a common node. On account of this singularity, Dr. Olbers was led to conjecture that Ceres and Pallas are only fragments of a larger planet, which had formerly circulated at the same distance, and been shattered by some internal convulsion. La Grange, a mathematician of the first eminence, investigated the forces that would be necessary to detach a fragment from a planet with a velocity that would cause it to describe such orbits as these bodies are found to have. The hypothesis suggested the probability that there might be other fragments, whose orbits, however they might differ in eccentricity and inclination, might be expected to cross the ecliptic at a common point, or to have the same node. Dr. Olbers, therefore, proposed to examine carefully every month the two opposite parts of the heavens in which the orbits of Ceres and Pallas intersect one another, with a view to the discovery of other planets, which

might be sought for in those parts with greater chance of success than in a wider zone, embracing the entire limits of these orbits. Accordingly, in 1804, near one of the nodes of Ceres and Pallas, a third planet was discovered. This was called *Juno*, and the character ⚵ was adopted for its symbol, representing the starry sceptre of the queen of Olympus. Pursuing the same researches, in 1807, a fourth planet was discovered, to which was given the name of *Vesta*, and for its symbol the character ⚶ was chosen, an altar surmounted with a censer holding the sacred fire.

After this historical sketch, it will be sufficient to classify under a few heads the most interesting particulars relating to the New Planets.

351. The *average distance* of these bodies from the sun is 261,000,000 miles; and it is remarkable that their orbits are very near together. Taking the distance of the earth from the sun for unity, their respective distances are 2.77, 2.77, 2.67, 2.37.

As they are found to be governed, like the other members of the solar system, by Kepler's law, that regulates the distances and times of revolution, their *periodical times* are of course pretty nearly equal, averaging about 4½ years.

In respect to the *inclination of their orbits*, there is considerable diversity. The orbit of Vesta is inclined to the ecliptic only about 7°, while that of Pallas is more than 34°. They all therefore have a higher inclination than the orbits of the old planets, and of course make excursions from the ecliptic beyond the limits of the Zodiac.

The *eccentricity of their orbits* is also, in general, greater than that of the old planets; and the eccentricities of the orbits of Pallas and Juno exceed that of the orbit of Mercury.

Their *small size* constitutes one of their most remarkable peculiarities. The difficulty of estimating the apparent diameter of bodies at once so very small and so far off, would lead us to expect different results in the actual estimates. Accordingly, while Dr. Herschel estimates the diameter of Pallas at only 80 miles, Schroeter places it as high as 2,000 miles, or about the size of the moon. The volume of Vesta is estimated at only one fifteen thousandth part of the earth's, and her surface is only about equal to that of the kingdom of Spain.* These little bodies are surrounded by *atmospheres* of great extent, some of which are uncommonly luminous, and others appear to consist of nebulous matter. These planets in general shine with a more vivid light than might be expected from their great distance and diminutive size.

CHAPTER XI.

MOTIONS OF THE PLANETARY SYSTEM.

352. We have waited until the learner may be supposed to be familiar with the contemplation of the heavenly bodies, individually, before inviting his attention to a systematic view of the planets, and of their motions around the sun. The time has now arrived for entering more advantageously upon this subject, than could have been done at an earlier period.

There are two methods of arriving at a knowledge of the motions of the heavenly bodies. One is to begin with the *apparent*, and from these to deduce the *real* motions; the other is, to begin with considering things as they really are in nature, and then to inquire why they appear as they do. The latter of these methods is by far the more eligible; it is much easier than the other, and proceeding from the less difficult to that which is more difficult, from motions that are very simple to such as are complicated, it finally puts the learner in possession of the whole machinery of the heavens. We shall, in the first place, therefore, endeavor to introduce the learner to an acquaintance with the simplest motions of the planetary system, and afterwards to conduct him gradually through such as are more complicated and difficult.

353. Let us first of all endeavor to acquire an adequate idea of *absolute space*, such as existed before the creation of the world.

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* New Encyc. Brit. Art. *Astronomy.*

We shall find it no easy matter to form a correct notion of infinite space; but let us fix our attention, for some time, upon extension alone, devoid of every thing material, without light or life, and without bounds. Of such a space we could not predicate the ideas of up or down, east, west, north, or south, but all reference to our own horizon (which habit is the most difficult of all to eradicate from the mind) must be completely set aside. Into such a void we would introduce the SUN. We would contemplate this body alone, in the midst of boundless space, and continue to fix the attention upon this object, until we had fully settled its relations to the surrounding void. The ideas of up and down would now present themselves, but as yet there would be nothing to suggest any notion of the cardinal points. We suppose ourselves next to be placed on the surface of the sun, and the firmament of stars to be lighted up. The slow revolution of the sun on his axis, would be indicated by a corresponding movement of the stars in the opposite direction; and in a period equal to more than 27 of our days, the spectator would see the heavens perform a complete revolution around the sun, as he now sees them revolve around the earth once in 24 hours. The point of the firmament where no motion appeared, would indicate the position of one of the poles, which being called North, the other cardinal points would be immediately suggested.

Thus prepared, we may now enter upon the consideration of the *planetary motions.*

354. Standing on the sun, we see all the planets moving slowly around the celestial sphere, nearly in the same great high way, and in the same direction from west to east. They move, however, with very unequal velocities. Mercury makes very perceptible progress from night to night, like the moon revolving about the earth, his daily progress eastward being about one third as great as that of the moon, since he completes his entire revolution in about three months. If we watch the course of this planet from night to night, we observe it, in its revolution, to cross the ecliptic in two opposite points of the heavens, and wander about 7° from that plane at its greatest distance from it. Knowing the position of the orbit of Mercury with respect to

the ecliptic, we may now, in imagination, represent that orbit by a great circle passing through the center of the planet and the center of the sun, and cutting the plane of the ecliptic in two opposite points at an angle of 7°. We may imagine the intersection of these two great circles, with the celestial vault to be marked out in plain and palpable lines on the face of the sky; but we must bear in mind that these orbits are mere *mathematical planes*, having no permanent existence in nature, any more than the path of an eagle flying through the sky; and if we conceive of their orbits as marked on the celestial vault, we must be careful to attach to the representation the same notion as to a thread or wire carried round to trace out the course pursued by a horse in a race-ground.*

The *planes* of both the ecliptic and the orbit of Mercury, may be conceived of as indefinitely extended to a great distance until they meet the sphere of the stars; but the *lines* which the earth and Mercury describe in those planes, that is, their orbits, may be conceived of as comparatively *near to the sun*. Could we now for a moment be permitted to imagine that the planes of the ecliptic, and of the orbit of Mercury, were made of thin plates of glass, and that the paths of the respective planets were marked out on their planes in distinct lines, we should perceive the orbit of the earth to be almost a perfect circle, while that of Mercury would appear distinctly elliptical. But having once made use of a palpable surface and visible lines to aid us in giving position and figure to the planetary orbits, let us now throw aside these devices, and hereafter conceive of these planes and orbits as they are in nature, and learn to refer a body to a mere *mathematical* plane, and to trace its path in that plane through absolute space.

355. A clear understanding of the motions of Mercury and of the relation of its orbit to the plane of the ecliptic, will render it easy to understand the same particulars in regard to each of the

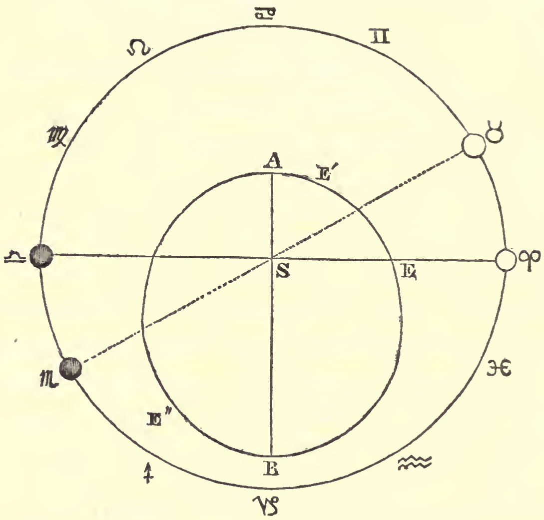
* It would seem superfluous to caution the reader on so plain a point, did not the experience of the instructor constantly show that young learners, from the habit of seeing the celestial motions represented in orreries and diagrams, almost always fall into the absurd notion of considering the orbits of the planets as having a distinct and independent existence.

other planets. Standing on the sun we should see each of the planets pursuing a similar course to that of Mercury, all moving from west to east, with motions differing from each other chiefly in two respects, namely, in their velocities, and in the distances to which they ever recede from the ecliptic.

The earth revolves about the sun very much like Venus, and to a spectator on the sun, the motions of these two planets would exhibit much the same appearances. We have supposed the observer to select the plane of the earth's orbit as his standard of reference, and to see how each of the other orbits is related to it; but such a selection of the ecliptic is entirely arbitrary; the spectator on the sun, who views the motions of the planets as they actually exist in nature, would make no such distinction between the different orbits, but merely inquire how they were mutually related to each other. Taking, however, the ecliptic as the plane to which all the others are referred, we do not, as in the case of the other planets, inquire how its plane is *inclined*, nor what are its *nodes*, since it has neither inclination nor node.

356. Such, in general, are the *real* motions of the planets, and such the appearances which the planetary system would exhibit to a spectator at the center of motion. But in order to represent correctly the positions of the planetary orbits, at any given time, *three* things must be regarded,—the *Inclination* of the orbit to the ecliptic—the position of *the line of the Nodes*—and the position of *the line of the Apsides.* In our common diagrams, the orbits are incorrectly represented, being all in the same plane, as in the following diagram, where AEB (Fig. 65,) represents the orbit of Mercury as lying in the same plane with the ecliptic. To exhibit its position justly (AB being taken as the line of the nodes) it should be elevated on one side about 7° and depressed by the same number of degrees on the other side, turning on the line AB as on a hinge. But even then the representation may be incorrect in other respects, for we have taken it for granted that the line of the nodes coincides with the line of the apsides, or that the orbit of Mercury cuts the ecliptic in the line AB. Whereas, it may lie in any given position with respect to the line of the apsides depending on the longitude of the nodes. If, for exam-

Fig. 65.



ple, the line of the nodes had chanced to pass through Taurus and Scorpio instead of Cancer and Capricorn, then it would have been represented by the line ♉ ♏ instead of ♋♑, and the plane when elevated or depressed with respect to the plane of the equator, would be turned on this line in our figure.* Moreover, our diagram represents the line of the apsides as passing through Cancer and Capricorn, whereas it may have any other position among the signs, according to the longitudes of the perigee and apogee.

357. The attempt to exhibit the motions of the solar system, and the positions of the planetary orbits by means of diagrams, or even orreries, is usually a failure. The student who relies exclusively on such aids as these, will acquire ideas on this subject that are both inadequate and erroneous. They may aid reflection, but can never supply its place. The impossibility of representing things in their just proportions will be evident when we reflect, that to do this, if, in an orrery, we make Mercury as large as a cherry, we should require to represent the sun by a globe six

* The learner will find it useful to construct such representations of the mutual relations of the planetary orbits of paste board.

feet in diameter. If we preserve the same proportions in regard to distance, we must place Mercury 250 feet, and Uranus 12,500 feet, or more than two miles from the sun. The mind of the student of astronomy must, therefore, raise itself from such imperfect representations of celestial phenomena as are afforded by artificial mechanism, and, transferring his contemplations to the celestial regions themselves, he must conceive of the sun and planets as bodies that bear an insignificant ratio to the immense spaces in which they circulate, resembling more a few little birds flying in the open sky, than they do the crowded machinery of an orrery.

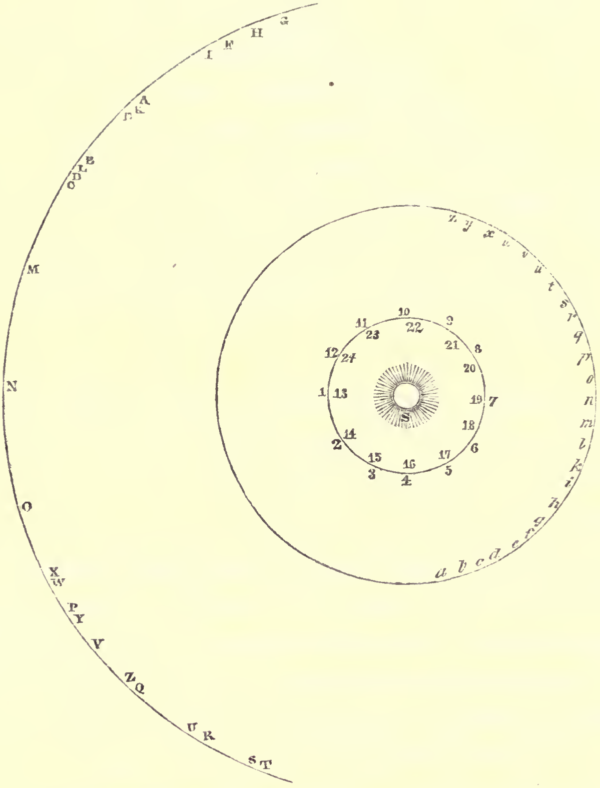
358. Having acquired as correct an idea as we are able of the planetary system, and of the positions of the orbits with respect to the ecliptic, let us next inquire into the nature and causes of the *apparent motions.*

The apparent motions of the planets are exceedingly unlike the real motions, a fact which is owing to two causes; first, *we view them out of the center of their orbits;* secondly, *we are ourselves in motion.* From the first cause, the apparent places of the planets are greatly changed by perspective; and from the second cause, we attribute to the planets changes of place which arise from our own motions of which we are unconscious.

359. The situation of a heavenly body as seen from the center of the sun, is called its *heliocentric* place; as seen from the center of the earth, its *geocentric* place. The geocentric motions of the planets must, according to what has just been said, be far more irregular and complicated than the heliocentric, as will be evident from the following diagram, which represents the geocentric motions of Mercury for two entire revolutions, embracing a period of nearly six moths.

Let S (Fig. 66,) represent the sun, 1, 2, 3, &c. the orbit of Mercury, *a*, *b*, *c*, &c. that of the earth, and GT the concave sphere of the heavens. The orbit of Mercury is divided into 12 equal parts, each of which he describes in 7⅓ days, and a portion of the earth's orbit described by that body in the time that Mercury describes the two complete revolutions, is divided into 24 equal

Fig. 66.



parts. Let us now suppose that Mercury is at the point 1 in his orbit, when the earth is at the point *a*; Mercury will then appear in the heavens at A. In $7\frac{1}{3}$ days Mercury will have reached 2, while the earth has reached *b*, when Mercury will appear at B. By laying a ruler on the point *c* and 3, *d* and 4, and so on, in the order of the alphabet, the successive apparent places of Mercury in the heavens will be obtained.

From A to C, the apparent motion is direct, or in the order of the signs; from C to G it is retrograde; at G it is stationary awhile, and then direct through the whole arc GT. At T the planet is again stationary, and afterwards retrograde along the arc TX.

360. Venus exhibits a variety of motions similar to those of Mercury, except that the changes do not succeed each other so

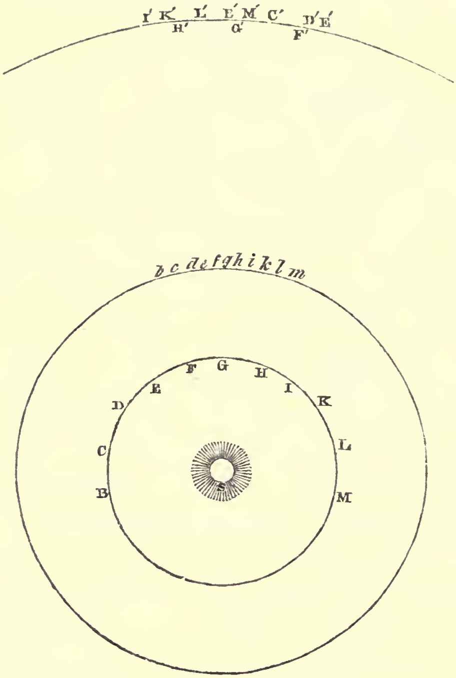
rapidly, since her period of revolution approaches much more nearly to that of the earth.

261. The apparent motions of the *superior* planets, are, like those of Mercury and Venus, alternately direct, stationary, and retrograde. In this case, however, the earth moves faster than the planet, and the planet has its opposition but no inferior conjunction, whereas an inferior planet has its inferior conjunction, but no opposition. These differences render the apparent motions of the superior planets in some respects unlike those of Mercury and Venus. When a superior planet is in conjunction, its motion is direct, because, as in the case of Venus in her *superior* conjunction, (See Fig. 60,) the only effect of the earth's motion is to accelerate it; but when the planet is in opposition, the earth is moving past it with a greater velocity, and makes the planet seem to move backwards, like the apparent backward motion of a vessel when we overtake it and pass rapidly by it in a steamboat.

362. But the various motions of a superior planet will be best understood from a diagram. Hence, let S (Fig. 67,) be the sun; B, C, D, E, the orbit of the earth; *b*, *c*, *d*, &c. the orbit of a superior planet, as Jupiter for example; and I′E′ a portion of the concave sphere of the heavens. Let *bm* be the arc described by Jupiter in the time the earth describes the arc BM; let *bc*, *cd*, and *de*, &c. be described by Jupiter while the earth describes BC, CD, and DE. Now when the earth is at B and Jupiter at *b*, he will appear in the heavens at B′. When the earth reaches C, the planet reaches *c* and will be seen at C′, his motions having been direct from west to east. While the earth moves from C to D and from D to E, Jupiter has moved from *c* to *d*, and from *d* to *e*, and will appear to have advanced among the stars from C′ to D′, and from D′ to E′, his motion being still direct, but slower than before, as he has passed over only the space D′E′ in the same time that he before moved through the greater spaces B′C′ and C′D′.

During the motion of the earth from E to F, and of Jupiter from *e* to *f*, the earth passes by Jupiter; and not being conscious of our own motion, Jupiter seems to us to have moved backward

Fig. 67.



from E′ to F′. At E′ where the direct motion was changed to a retrograde, he would appear to be stationary. Upon the arrival of the earth at G, and of Jupiter at *g*, in opposition to the sun, Jupiter will appear at G′, having moved with apparently great velocity over a large space F′G′. While the earth passes from G to H, and from H to I, and Jupiter from *g* to *h*, and from *h* to *i*, he will appear to have moved from G′ to I′. At I′ he will again appear stationary in the heavens; but when he advances from *i* to *k* in the time the earth moves from I to K, he has described the arch I′K′, and has therefore resumed his direct motion from west to east. While the earth moves from K to L and from L to M, and Jupiter through the corresponding spaces *kl* and *lm*, the planet will appear still to continue his direct motion from K′ to L′ and from L′ to M′ in the heavens.

Thus, during a period of six months, while the earth is performing one half of her annual circuit, Jupiter has a diversity of motions, all performed within a small portion of the heavens.

CHAPTER XII.

DETERMINATION OF THE PLANETARY ORBITS—KEPLER'S DISCOVERIES—ELEMENTS OF THE ORBIT OF A PLANET—QUANTITY OF MATTER IN THE SUN AND PLANETS—STABILITY OF THE SOLAR SYSTEM.

363. In chapter II, we have shown that the figure of the earth's orbit is an ellipse, having the sun in one of its foci, and that the earth's radius vector describes equal areas in equal times ; and in Chapter III, we have remarked that these are only particular examples under the law of Universal Gravitation, as is also the additional fact, that the squares of the periodical times of the planets are as the cubes of the major axes of their orbits. We may now learn, more particularly, the process by which the illustrious Kepler was conducted to the discovery of these grand laws of the planetary system.

364. Ptolemy, while he held that the orbits of the planets were perfect circles in which the planets revolved uniformly about the earth, was nevertheless obliged to suppose that the earth was situated out of the center of the circles, and that at the same distance on the other side of the center was situated the point (*punctum æquans*) about which the angular motion of the body was equable and uniform. In regard to the orbit of the sun, however, the earth was held to occupy the exact center. On nearly the same suppositions, Tycho Brahe had made a great number of very accurate observations on the planetary motions, which served Kepler as standards of comparison for results, which he deduced from calculations founded on the application of geometrical reasoning to hypotheses of his own.

Kepler first applied himself to investigate the orbit of Mars, the motions of which planet appeared more irregular than those of any other, except Mercury, which, being seldom seen, had then been very little studied. According to the views of Ptolemy and Tycho, he at first supposed the orbit to be circular, and the planet to move uniformly about a point at a certain distance from the sun. He made seventy suppositions before he obtained one that agreed with observation, the calculation of which was extremely long and tedious, occupying him more than five years.* The supposition of an equable motion in a circle, however varied, could not be made to conform to the observations of Tycho, whereas the supposition that the orbit was of an oval figure, depressed at the sides, but coinciding with a circle at the perihelion, agreed very nearly with observation. Such a figure naturally suggested the idea of an ellipse, and reasoning on the known properties of the ellipse, and comparing the results of calculation with actual observation, the agreement was such as to leave no doubt that the orbit of Mars is an ellipse, having the sun in one of the foci. He immediately conjectured that the same is true of the orbits of all the other planets, and a similar comparison of this hypothesis with observation, confirmed its truth. Hence he established the first great law, *that the planets revolve about the sun in ellipses, having the sun in one of the foci.*

365. Kepler also discovered from observation, that the velocities of the planets when in their apsides, are inversely as their distances from the sun, whence it follows that they describe, in these points, equal areas about the sun in equal times. Although he could not prove, from observation, that the same was true in every point of the orbit, yet he had no doubt that it was so. Therefore, assuming this principle as true, and hence deducing the equation of the center, (Art. 200,) he found the result to agree with observation, and therefore concluded in general, *that the planets describe about the sun equal areas in equal times.*

366. Having, in his researches that led to the discovery of the first of the above laws, found the relative mean distances of the planets from the sun, and knowing their periodic times, Kepler next endeavored to ascertain if there was any relation between

* Si te hujus laboriosæ methodi pertæsum fuerit, jure mei te misereat, qui eam ad minimum septuagies ivi cum plurima temporis jactura; et mirari desines hunc quintum jam annum abire, ex quo Martem aggressus sum.

them, having a strong passion for finding analogies in nature. He saw that the more distant a planet was from the sun, the slower it moved; so that the periodic times of the more distant planets would be increased on two accounts, first, because they move over a greater space, and secondly, because their motions in their orbits is actually slower than the motions of the planets nearer the sun. Saturn, for example, is 9½ times further from the sun than the earth is, and the circle described by Saturn is greater than that of the earth in the same ratio; and since the earth revolves around the sun in one year, were their velocities equal, the periodic time of Saturn would be 9½ years, whereas it is nearly 30 years. Hence it was evident, that the periodic times of the planets increase in a greater ratio than their distances, but in a less ratio than the squares of their distances, for on that supposition the periodic time of Saturn would be about 90¼ years. Kepler then took the squares of the times and compared them with the cubes of the distances, and found an exact agreement between them. Thus he discovered the famous law, *that the squares of the periodic times of all the planets, are as the cubes of their mean distances from the sun.**

This law is strictly true only in relation to planets whose quantity of matter in comparison with that of the central body is inappreciable. When this is not the case, the periodic time is shortened in the ratio of the square root of the sun's mass divided by the sun's plus the planet's mass $\left(\frac{M}{M+m}\right)^{\frac{1}{2}}$. The mass of most of the planets is so small compared with the sun's, that this modification of the law is unnecessary except where extreme accuracy is required.

ELEMENTS OF THE PLANETARY ORBITS.

367. The particulars necessary to be known in order to determine the precise situation of a planet at any instant, are called the *Elements of its Orbit.* They are seven in number, of which the first two determine the absolute situation of the orbit, and the

* Vince's Complete System, I, 98.

other five relate to the motion of the planet in its orbit. These elements are,

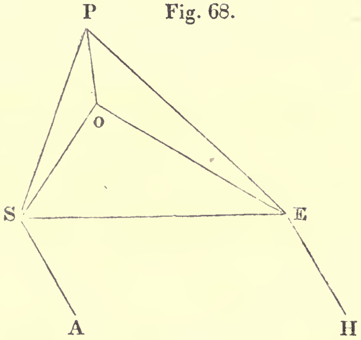
(1.) *The position of the line of the nodes.*
(2.) *The inclination to the ecliptic.*
(3.) *The periodic time.*
(4.) *The mean distance from the sun, or semi-axis major.*
(5.) *The eccentricity.*
(6.) *The place of the perihelion.*
(7.) *The place of the planet in its orbit at a particular epoch.*

368. It may at first view be supposed that we can proceed to find the elements of the orbit of a planet in the same manner as we did those of the solar or lunar orbit, namely, by observations on the right ascension and declination of the body, converted into latitudes and longitudes by means of spherical trigonometry, (See page 59.) But in the case of the moon, we are situated in the center of her motions, and the apparent coincide with the real motions; and, in respect to the sun, our observations on his *apparent* motions give us the earth's *real* motions, allowing 180° difference in longitude. But, as we have already seen, the motions of the planets appear exceedingly different to us, from what they would if seen from the center of their motions. It is necessary therefore to deduce from observations made on the earth the corresponding results as they would be if viewed from the center of the sun; that is, in the language of astronomers, having the *geocentric* place of a planet, it is required to find its *heliocentric* place.

369. The first steps in this process are the same as in the case of the sun and moon. That is, for the purpose of finding the right ascension and declination, the planet is observed on the meridian with the Transit Instrument and Mural circle, (See Arts. 155 and 230,) and from these observations, the planet's *geocentric* longitude and latitude are computed by spherical trigonometry. The distance of the planet from the sun is known nearly by Kepler's law. From these data it is required to find the heliocentric longitude and latitude.

Let S and E (Fig. 68,) be the sun and earth, P the planet, PO a line drawn from P perpendicular to the ecliptic, SA the direction of Aries, and EH parallel to SA, and therefore (on account of the immense distance of the fixed stars) also in the direction of Aries. Then OEH, being the apparent distance of the planet from Aries in the direction of the ecliptic, is the geocentric longitude, and OEP, being the apparent distance of the planet from the ecliptic taken on a secondary to the ecliptic, is the geocentric latitude. It is obvious also that the angles OSA and PSO are

Fig. 68.



the heliocentric longitude and latitude. The planet's angular distance from the sun, PES, is also known from observation. Hence, in the triangle SEP, we know SP and SE and the angle SEP, from which we can find PE; and knowing PE and the angle PEO, we can find OE, since OEP is a right angled triangle. Hence in the triangle SEO, ES and EO, and the angle SEO (=OEH−SEH=difference of longitude of the planet and the sun) are known, and hence we can obtain OSE, (Art. 135,) which added to the sun's longitude ESA, gives us OSA the planet's *heliocentric longitude.*

Also, because PS : Rad. : : OP : Sin. PSO

$\therefore$ PS × Sin. PSO = OP × Rad.

But EP : Rad. : : OP : Sin. OEP

$\therefore$ EP × Sin. OEP = OP × Rad.

$\therefore$ PS × Sin. PSO = EP × Sin. OEP

$\therefore$ PS : EP : : Sin. OEP : Sin. PSO.

The first three terms of this proposition being known, the last is found which is the *heliocentric latitude.**

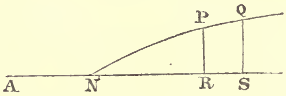
* Brinkley's Elements of Astronomy, p. 164.

370. Having now learned how observations made at the earth may be converted into corresponding observations made at the sun, we may proceed to explain the mode of finding the several elements before enumerated; although our limits will not permit us to enter further into detail on this subject, than to explain the leading principles on which each of these elements is determined.*

371. First, to determine the *position of the Nodes*, and the *Inclination of the Orbit.*

These two elements, whieh determine the orbit, (Art. 368,) may be derived from two heliocentric longitudes and latitudes. Let AR and AS (Fig. 69,) be two heliocentric longitudes, PR and QS the heliocentric latitudes, and N the ascending node. Then, by Napier's theorem, (Art. 132.)

Fig. 69.



$$\frac{\text{Sin. NR}(=\text{AR}-\text{AN})}{\text{tan. PR}}=\text{cot. PNR}=\frac{\text{sin. NS}\ (=\text{AS}-\text{AN})}{\text{tan. QS}}$$

$$\therefore \frac{\text{Sin. AR}\times\text{cos. AN}-\text{cos. AR}\times\text{sin. AN}\dagger}{\text{tan. PR}}=$$

$$\frac{\text{sin. AS}\times\text{cos. AN}-\text{cos. AS}\times\text{sin. AN}}{\text{tan. QS}}.$$

$$\text{But tan. AN}=\frac{\text{sin. AN}}{\text{cos. AN}}=\frac{\text{Sin. AR}\times\text{tan. QS}-\text{sin. AS}\times\text{tan. PR}}{\text{Cos. AR}\times\text{tan. QS}-\text{cos. AS}\times\text{tan. PR}}.$$

But AN is the *longitude of the ascending node;* and its value is found in terms of the heliocentric longitudes and latitudes previously determined, (Art. 369.)

Again, since AN is found, we may deduce from the first equation above the value of PNR, which is the *inclination of the orbit.*‡

372. Secondly, to find the *Periodic Time.*

This element is learned, by marking the interval that passes from the time when a planet is in one of the nodes until it re-

* Most of these elements admit of being determined in several different ways, an explanation of which may be found in the larger works on Astronomy, as Vince's Complete System, Vol. 1. Gregory's Ast. p. 212. Woodhouse, p. 562.

† Day's Trig. Art. 208.

‡ Brinkley, p. 166.

turns to the same node. We may know when a planet is at the node because then its latitude is nothing. If, from a series of observations on the right ascension and declination of a planet, we deduce the latitudes, and find that one of the observations gives the latitude 0, we infer that the planet was at that moment at the node. But if, as commonly happens, no observation gives exactly 0, then we take two latitudes that are nearest to 0, but on opposite sides of the ecliptic, one south and the other north, and as the sum of the arcs of latitude is to the whole interval, so is one of the arcs to the corresponding time in which it was described, which time being added to the first observation, or subtracted from the second, will give the precise moment when the planet was at the node.

By repeated observations it is found, that the nodes of the planets have a very slow retrograde motion.

373. If the orbit of a planet cut the ecliptic at right angles, then small differences of latitude would be appreciable; but in fact the planetary orbits are in general but little inclined to the ecliptic, and some of them lie almost in the same plane with it. Hence arises a difficulty in ascertaining the exact time when a planet reaches its node. Among the most valuable observations for determining the elements of a planet's orbit, are those made when a superior planet is in or near *its opposition* to the sun, for then the heliocentric and geocentric longitudes are the same. When a number of oppositions are observed, the planet's motion in longitude as would be observed from the sun will be known. The inferior planets, also, when in superior conjunction, have their geocentric and heliocentric longitudes the same. When in inferior conjunction, these longitudes differ 180°; but the inferior planets can seldom be observed in superior conjunction, on account of their proximity to the sun, nor in inferior conjunction except in their transits, which occur too rarely to admit of observations sufficiently numerous. Therefore, we cannot so readily ascertain by simple observation, the motions of the inferior planets seen from the sun, as we can those of the superior.*

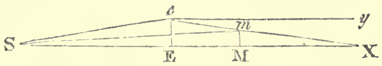
* Brinkley, p. 167.

Hence, in order to obtain accurately the periodic time of a planet, we find the interval elapsed between two oppositions separated by a long interval, when the planet was nearly in the same part of the Zodiac. From the periodic time, as determined approximately by other methods, it may be found when the planet has the same heliocentric longitude as at the first observation. Hence the time of a complete number of revolutions will be known, and thence the time of one revolution. The greater the interval of time between the two oppositions, the more accurately the periodic time will be obtained, because the errors of observation will be divided between a great number of periods; therefore by using very accurate observations, much precision may be attained. For example, the planet Saturn was observed in the year 228 B. C. March 2, (according to our reckoning of time,) to be near a certain star called γ Virginis, and it was at the same time nearly in opposition to the sun. The same planet was again observed in opposition to the sun, and having nearly the same longitude in Feb. 1714. The exact difference between these dates was 1943y. 118d. 21h. 15m. It is known from other sources, that the time of a revolution is 29½ years nearly, and hence it was found that in the above period there were 66 revolutions of Saturn; and dividing the interval by this number, we obtain 29.444 years, which is nearly the periodic time of Saturn according to the most accurate determination.

374. Thirdly, to determine *the distance from the sun, and major axes of the planetary orbits.*

The distance of the earth from the sun being known, the *mean* distance of any planet (its periodic time being known) may be found by Kepler's law, that the squares of the periodic times are as the cubes of the distances. The method of finding the distance of an *inferior* planet from the sun by observations at the greatest elongation, has been already explained, (See Art. 308.) The distance of a *superior* planet may be found from observations on its retrograde motion at the time of opposition. The periodic times of two planets being known, we of course know their mean angular velocities, which are inversely as the times. Therefore, let E*e* (Fig. 70,) be a very small portion of the earth's orbit, and M*m* a corresponding portion of that of a superior planet, described on the day of opposition, about the sun S, on which day the three bodies lie in one straight line SEMX. Then the angle ES*e* and MS*m*, representing the respective angular veloci-

Fig. 70.



ties of the two bodies are known. Now if *em* be joined, and prolonged to meet SM continued in X, the angle *e*X*e*, which is equal to the alternate angle X*ey*, being equal to the retrogradation of the planet in the same time (being known from observation) is also given. E*e*, therefore, and the angle EX*e* being given in the right angled triangle E*e*X, the side EX is easily calculated, and thus SX becomes known. Consequently, in the triangle S*m*X, we have given the side SX, and the two angles *m*SX and *m*XS, whence the other sides S*m* and *m*X are easily determined. Now S*m* is the radius of the orbit of the superior planet required, which in this calculation is supposed circular as well as that of the earth,—a supposition not exact, but sufficiently so to afford a satisfactory approximation to the dimensions of its orbit, and which, if the process be often repeated, in every variety of situation at which the opposition can occur, will ultimately afford an average or mean value of its distance fully to be depended on.*

375. *The transverse or major axes of the planetary orbits remain always the same.* Amidst all the perturbations to which other elements of the orbit are subject, the line of the apsides is of the same invariable length. It is no matter in what *direction* the planet may be moving at that moment. Various circumstances will influence the eccentricity and the position of the ellipse, but none of them affects its length.

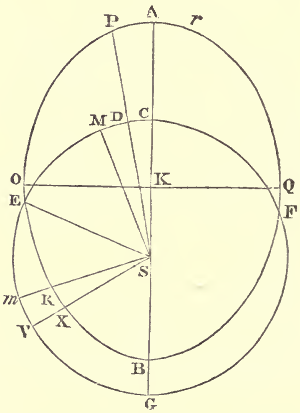
376. Fourthly, to determine *the place of the perihelion—the epoch of passing the perihelion—and the eccentricity.*

* Sir J. Herschel.

There are various methods of finding the eccentricity of a planet's orbit and the place of the perihelion, and of course the position of the line of the apsides. One is derived from the *greatest equation of the center*, (Art. 200.) The greatest equation is the greatest difference that occurs between the mean and the true motion of a body revolving in an ellipse. It will be necessary first to explain the manner in which the greatest equation is found.

Let AEBF (Fig. 71,) be the orbit of the planet, having the sun in the focus at S. In an ellipse, the square root of the product of the semi axes gives the radius of a circle of the same area as the ellipse.* Therefore with the center S, at the distance $SE = \sqrt{AK \times OK}$, describe the circle CEGF, then will the area of this circle be equal to that of the ellipse. At the same time that a planet departs from A the aphelion, a body begins to move with a uniform motion from C through the periphery CEGF, and performs a whole revolution in the same period that the planet describes the ellipse; the motion of this body will represent the equal or mean motion of the earth, and it will describe around S areas or sectors of circles which are proportional to the times, and equal to the elliptic areas described in the same time by the planet. Let the equal motion, or the angle about S proportional to the time, be CSM, and take ASP equal to the sector CSM; then the place of the planet will be P; MSC will be the *mean anomaly*, (Art. 200,) DSC the *true anomaly*, and MSD the *equation of the center*. Since the sectors CSM and ASP are equal, and the part CSD is common to both, PACD and SDM are equal; and since the areas of circular sectors are proportional to their arcs, the equation of the center is greatest when the area ACPD is greatest, that is, at the point E where the ellipse and circle intersect one another. For when the planet descends further, to R for instance, the equation becomes

Fig. 71.



* Day's Mensuration.

proportional to the difference of the areas ACE and mER, or to the area GBRm, V being the situation of the body moving equably; for the sector CSV will be equal to the elliptic area ASR, and taking away the common spaces ACE, REm=the sector VSm=the equation. At the points E and F, where the circle and ellipse intersect, the radius vector of the earth and the radius of the circle of equable motion are equal, and of course those radii then describe equal areas in equal times; hence, when the real motion of the earth is equal to the mean motion, the equation of the center is greatest.* The *mean motion* for any given time is easily found; for the periodic time : 360 : : the given time : the number of degrees for that time. Observation shows when the *actual motion* of the planet is the same with this.

377. Now the equation of the center is greatest twice in the revolution, on opposite sides of the orbit, as at E and F, which points lie at equal distances from the apsides; and since the whole arc EAF or EBF is known from the time occupied in describing it, therefore, by bisecting this arc, we find the points A and B, the *aphelion* and *perihelion*, and consequently the position of *the line of the apsides.* The time of describing the area EBF being known, by bisecting this interval, we obtain the moment of passing the perihelion, which gives us *the place of the planet in its orbit at a particular epoch.*

The amount of the greatest equation obviously depends on the eccentricity of the orbit, since it arises wholly from the departure of the ellipse from the figure of a perfect circle; hence, the greatest equation affords the means of determining the eccentricity itself. In orbits of small eccentricity, as is the case with most of the planetary orbits, it is found that the arc which measures the greatest equation is very nearly equal to the distance between the foci,† which always equals twice the eccentricity, the eccentricity being the distance from the center to the focus. Consequently, $57° \; 17' \; 44''.8$‡ : rad. : : half the greatest equation : *the eccentricity.*

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* Gregory's Astronomy, p. 197. † Vince's Complete System, I, 113.

‡ The value of an arc equal to radius; for 3.14159 : 1 : : 180 : $57° \; 17' \; 44''.8$.

The foregoing explanations of the methods of finding the elements of the orbits, will serve in general to show the learner how these particulars are or may be ascertained, yet the methods actually employed are usually more refined and intricate than these. In astronomy scarcely an element is presented simple and unmixed with others. Its value when first disengaged, must partake of the uncertainty to which the other elements are subject ; and can be supposed to be settled to a tolerable degree of correctness, only after multiplied observations and many revisions.*

So arduous has been the task of finding the elements of the planetary orbits.

QUANTITY OF MATTER IN THE SUN AND PLANETS.

378. It would seem at first view very improbable, that an inhabitant of this earth would be able to weigh the sun and planets, and estimate the exact quantity of matter which they severally contain. But the principles of Universal Gravitation conduct us to this result, by a process remarkable for its simplicity. By comparing the relations of a few elements that are known to us, we ascend to the knowledge of such as appeared beyond the pale of human investigation. *We learn the quantity of matter in a body by the force of gravity it exerts.* Let us see how this force is ascertained.

379. *The quantities of matter in two bodies, may be found in terms of the distances and periodic times of two bodies revolving around them respectively, being as the cubes of the distances divided by the squares of the periodic times.*

The force of gravity G in a body whose quantity of matter is M and distance D, varies directly as the quantity of matter, and inversely as the square of the distance ; that is, $G \propto \frac{M}{D^2}$. But it is shown by writers on Central Forces, that the force of gravity also varies as the distance divided by the square of the periodic time, or $G \propto \frac{D}{P^2}$. Therefore, $\frac{M}{D^2} \propto \frac{D}{P^2}$, and $M \propto \frac{D^3}{P^2}$. Thus we may find the respective quantities of matter in the earth and the

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* Woodhouse, p. 579.

sun, by comparing the distance and periodic time of the moon, revolving around the earth, with the distance and periodic time of the earth revolving around the sun. For the cube of the moon's distance from the earth divided by the square of her periodic time, is to the cube of the earth's distance from the sun divided by the square of her periodic time, as the quantity of matter in the earth is to that in the sun. That is, $\frac{238545^3}{27.32^2} : \frac{95,000,000^3}{365.256^2} :: 1 : 353,385$. The most exact determination of this ratio, gives for the mass of the sun 354,936 times that of the earth. Hence it appears that the sun contains more than three hundred and fifty four thousand times as much matter as the earth. Indeed the sun contains eight hundred times as much matter as all the planets.

Another view may be taken of this subject which leads to the same result. Knowing the velocity of the earth in its orbit, we may calculate its *centrifugal force*. Now this force is counterbalanced, and the earth retained in its orbit, by the attraction of the sun, which is proportional to the quantity of matter in the sun. Therefore we have only to see what amount of matter is required in order to balance the earth's centrifugal force. Is is found that the earth itself or a body as heavy as the earth acting at the distance of the sun, would be wholly incompetent to produce this effect, but that in fact it would take more than three hundred and fifty four thousand such bodies to do it.

380. The mass of each of the other planets *that have satellites* may be found, by comparing the periodic time of one of its satellites with its own periodic times around the sun. By this means we learn the ratio of its quantity of matter to that of the sun. The masses of those planets which have no satellites, as Venus or Mars, have been determined, by estimating the force of attraction which they exert in disturbing the motions of other bodies. Thus, the effect of the moon in raising the tides, leads to a knowledge of the quantity of matter in the moon; and the effect of Venus in disturbing the motions of the earth, indicates her quantity of matter.*

* These estimates are made by the most profound investigations in La Place's Mecanique Celeste, Vol. III.

381. The quantity of matter in bodies varies as their magnitudes and densities conjointly. Hence, their *densities* vary as their masses divided by their magnitudes; and since we know the magnitudes of the planets, and can compute as above their masses, we can thus learn their densities, which, when reduced to a common standard, give us their specific gravities, or show us how much heavier they are than water. Worlds therefore are weighed with almost as much ease as a pebble or an article of merchandize.

The *densities and specific gravities* of the sun, moon, and planets, are estimated as follows:*

	Density.	Specific Gravity.
Sun, . . .	0.2543	1.40†
Moon, . . .	0.6150	3.37
Mercury, . . .	2.7820	15.24
Venus, . . .	0.9434	5.17
Earth, . . .	1.0000	5.48
Mars, . . .	0.1293	0.71
Jupiter, . . .	0.2589	1.42
Saturn, . . .	0.1016	0.56
Uranus, . . .	0.2797	1.53

From this table it appears, that the sun consists of matter but little heavier than water; but that the moon is more than three times as heavy as water, though less dense than the earth. It also appears that the planets near the sun are, as a general fact, more dense than those more remote, Mercury being as heavy as the heaviest metals except two or three, while Saturn is as light as a cork. The decrease of density however is not entirely regular, since Venus is a little lighter than the earth, while Jupiter is heavier than Mars, and Uranus than Saturn.

382. The *perturbations* occasioned in the motions of the planets by their action on each other are very numerous, since every body in the system exerts an attraction on every other, in con-

* Francœur.

† The earth being taken, according to Bailly, at 5.48, the specific gravities of the other bodies (which are found by multiplying the density of each by the specific gravity of the earth) are here stated somewhat higher than they are given in most works.

formity with the law of Universal Gravitation. Venus and Mars, approaching as they do at times comparatively near to the earth, sensibly disturb its motions, and the satellites of the remoter planets greatly disturb each other's movements.

STABILITY OF THE SOLAR SYSTEM.

383. The derangement which the planets produce in the motion of one of their number will be very small in the course of one revolution; but this gives us no security that the derangement may not become very large in the course of many revolutions. The cause acts perpetually, and it has the whole extent of time to work in. Is it not easily conceivable then that in the lapse of ages, the derangements of the motions of the planets may accumulate, the orbits may change their form, and their mutual distances may be much increased or diminished? Is it not possible that these changes may go on without limit, and end in the complete subversion and ruin of the system? If, for instance, the result of this mutual gravitation should be to increase considerably the eccentricity of the earth's orbit, or to make the moon approach continually nearer and nearer to the earth at every revolution, it is easy to see that in the one case, our year would change its character, producing a far greater irregularity in the distribution of the solar heat: in the other, our satellite must fall to the earth, occasioning a dreadful catastrophe. If the positions of the planetary orbits with respect to that of the earth, were to change much, the planets might sometimes come very near us, and thus increase the effect of their attraction beyond calculable limits. Under such circumstances we might have years of unequal length, and seasons of capricious temperature; planets and moons of portentous size and aspect glaring and disappearing at uncertain intervals; tides like deluges sweeping over whole continents; and, perhaps, the collision of two of the planets, and the consequent destruction of all organization on both of them. The fact really is, that changes are taking place in the motions of the heavenly bodies, which have gone on progressively from the first dawn of science. The eccentricity of the earth's orbit has been diminishing from the earliest observations to our times. The moon has been moving quicker from the time

of the first recorded eclipses, and is now in advance by about four times her own breadth, of what her own place would have been if it had not been affected by this acceleration. The obliquity of the ecliptic also, is in a state of diminution, and is now about two fifths of a degree less than it was in the time of Aristotle.*

384. But amid so many seeming causes of irregularity, and ruin, it is worthy of grateful notice, that effectual provision is made for the *stability of the solar system.* The full confirmation of this fact, is among the grand results of Physical Astronomy. Newton did not undertake to demonstrate either the stability or instability of the system. The decision of this point required a great number of preparatory steps and simplifications, and such progress in the invention and improvement of mathematical methods as occupied the best mathematicians of Europe for the greater part of the last century. Towards the end of that time, it was shown by La Grange and La Place, that the arrangements of the solar system are stable; that, in the long run, the orbits and motions remain unchanged; and that the changes in the orbits, which take place in shorter periods, never trangress certain very moderate limits. Each orbit undergoes deviations on this side and on that side of its average state; but these deviations are never very great, and it finally recovers from them, so that the average is preserved. The planets produce perpetual perturbations in each other's motions, but these perturbations are not indefinitely progressive, but periodical, reaching a maximum value and then diminishing. The periods which this restoration requires are for the most part enormous,—not less than thousands, and in some instances millions of years. Indeed some of these apparent derangements, have been going on in the same direction from the creation of the world. But the restoration is in the sequel as complete as the derangement; and in the mean time the disturbance never attains a sufficient amount seriously to affect the stability of the system.* I have succeeded in demonstrating (says La Place) that, whatever be the masses of the planets, in consequence of the fact that they all move in the same direction,

* Whewell, in the Bridgewater Treatises, p. 128.

in orbits of small eccentricity, and but slightly inclined to each other, their secular irregularities are periodical and included within narrow limits; so that the planetary system will only oscillate about a mean state, and will never deviate from it except by a very small quantity. The ellipses of the planets have been and always will be nearly circular. The ecliptic will never coincide with the equator; and the entire extent of the variation in its inclination, cannot exceed three degrees.

385. To these observations of La Place, Professor Whewell* adds the following on the importance, to the stability of the solar system, of the fact that those planets which have *great masses* have orbits of *small eccentricity.* The planets Mercury and Mars, which have much the largest eccentricity among the old planets, are those of which the masses are much the smallest. The mass of Jupiter is more than two thousand times that of either of these planets. If the orbit of Jupiter were as eccentric as that of Mercury, all the security for the stability of the system, which analysis has yet pointed out, would disappear. The earth and the smaller planets might in that case change their nearly circular orbits into very long ellipses, and thus might fall into the sun, or fly off into remote space. It is further remarkable that in the newly discovered planets, of which the orbits are still more eccentric than that of Mercury, the masses are still smaller, so that the same provision is established in this case also.

CHAPTER XIII.

OF COMETS.

386. A Comet,† when perfectly formed, consists of three parts, the Nucleus, the Envelope, and the Tail. The *Nucleus*, or body of the comet, is generally distinguished by its forming a bright point in the center of the head, conveying the idea of a solid, or

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* Bridgewater Treatises, p. 131. See also Playfair's Outlines, 2, 290.

† κόμη *coma*, from the *bearded* appearance of comets.

at least of a very dense portion of matter. Though it is usually exceedingly small when compared with the other parts of the comet, yet it sometimes subtends an angle capable of being measured by the telescope. The *Envelope*, (sometimes called the *coma*) is a dense nebulous covering, which frequently renders the edge of the nucleus so indistinct, that it is extremely difficult to ascertain its diameter with any degree of precision. Many comets have no nucleus, but present only a nebulous mass extremely attenuated on the confines, but gradually increasing in density towards the center. Indeed there is a regular gradation of comets, from such as are composed merely of a gaseous or vapory medium, to those which have a well defined nucleus. In some instances on record, astronomers have detected with their telescopes small stars through the densest part of a comet.

The *Tail* is regarded as an expansion or prolongation of the coma; and, presenting as it sometimes does, a train of appalling magnitude, and of a pale, disastrous light, it confers on this class of bodies their peculiar celebrity.

387. The *number* of comets belonging to the solar system, is probably very great. Many, no doubt, escape observation by being above the horizon in the day time. Seneca mentions, that during a total eclipse of the sun, which happened 60 years before the Christian era, a large and splendid comet suddenly made its appearance, being very near the sun. The elements of at least 130 have been computed, and arranged in a table for future comparison. Of these *six* are particularly remarkable, viz. the comets of 1680, 1770, and 1811; and those which bear the names of Halley, Biela and Encke. The comet of 1680, was remarkable not only for its astonishing size and splendor, and its near approach to the sun, but is celebrated for having submitted itself to the observations of Sir Isaac Newton, and for having enjoyed the signal honor of being the first comet whose elements were determined on the sure basis of mathematics. The comet of 1770, is memorable for the changes its orbit has undergone by the action of Jupiter, as will be more particularly related in the sequel. The comet of 1811 was the most remarkable in its appearance of all that have been seen in the present century. Halley's

comet (the same which re-appeared in 1735) is distinguished as that whose return was first successfully predicted, and whose orbit is best determined; and Biela's and Encke's comets are well known, for their short periods of revolution, which subject them frequently to the view of astronomers.

388. In *magnitude* and *brightness* comets exhibit a great diversity. History informs us of comets so bright as to be distinctly visible in the day time, even at noon and in the brightest sunshine. Such was the comet seen at Rome a little before the assassination of Julius Cæsar. The comet of 1680 covered an arc of the heavens of 97°, and its length was estimated at 123,000,000 miles.* That of 1811, had a nucleus of only 428 miles in diameter, but a tail 132,000,000 miles long.† Had it been coiled around the earth like a serpent, it would have reached round more than 5,000 times. Other comets are of exceedingly small dimensions, the nucleus being estimated at only 25 miles; and some which are destitute of any perceptible nucleus, appear to the largest telescopes, even when nearest to us, only as a small speck of fog, or as a tuft of down. The majority of comets can be seen only by the aid of the telescope.

The same comet, indeed, has often very different aspects, at its different returns. Halley's comet in 1305 was described by the historians of that age, as *cometa horrendæ magnitudinis;* in 1456 its tail reached from the horizon to the zenith, and inspired such terror, that by a decree of the Pope of Rome, public prayers were offered up at noon-day in all the Catholic churches to deprecate the wrath of heaven, while in 1682, its tail was only 30° in length, and in 1759 it was visible only to the telescope, until after it had passed its perihelion. At its recent return in 1835, the greatest length of the tail was about 12°.‡ These changes in the appearances of the same comet are partly owing to the different positions of the earth with respect to them, being sometimes much nearer to them when they cross its track than at others; also one spectator so situated as to see the coma at a higher

* Arago. † Milne's Prize Essay on Comets.

‡ But might be seen much longer by *indirect* vision. (*Prof. Joslin*, Am. Jour. Science, 31, 328.)

angle of elevation or in a purer sky than another, will see the train longer than it appears to another less favorably situated; but the extent of the changes are such as indicate also a real change in their magnitude and brightness.

389. The *periods* of comets in their revolutions around the sun, are equally various. Encke's comet, which has the shortest known period, completes its revolution in $3\frac{1}{3}$ years, or more accurately, in 1208 days; while that of 1811 is estimated to have a period of 3383 years.*

390. The *distances* to which different comets recede from the sun, are also very various. While Encke's comet performs its entire revolution within the orbit of Jupiter, Halley's comet recedes from the sun to twice the distance of Uranus, or nearly 3600,000,000 miles. Some comets, indeed, are thought to go to a much greater distance from the sun than this, while some even are supposed to pass into parabolic or hyperbolic orbits, and never to return.

391. Comets shine *by reflecting the light of the sun.* In one or two instances they have exhibited distinct *phases*,† although the nebulous matter with which the nucleus is surrounded, would commonly prevent such phases from being distinctly visible, even when they would otherwise be apparent. Moreover, certain qualities of polarized light enable the optician to decide whether the light of a given body is direct or reflected; and M. Arago, of Paris, by experiments of this kind on the light of the comet of 1819, ascertained it to be reflected light.‡

392. The tail of a comet usually increases very much as it approaches the sun; and frequently does not reach its maximum until after the perihelion passage. In receding from the sun, the tail again contracts, and nearly or quite disappears before the body of the comet is entirely out of sight. The tail is frequently divided into two portions, the central parts, in the direction of the axis, being less bright than the marginal parts. In 1744, a comet appeared which had six tails, spread out like a fan.

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* Milne. † Delambre, t. 3, p. 400. ‡ Francœur, 181.

The tails of comets extend in a direct line from the sun, although they are usually more or less curved, like a long quill or feather, being convex on the side next to the direction in which they are moving; a figure which may result from the less velocity of the portions most remote from the sun. Expansions of the Envelope have also been at times observed on the side next the sun,* but these seldom attain any considerable length.

393. The *quantity of matter* in comets is exceedingly small. Their tails consist of matter of such tenuity that the smallest stars are visible through them. They can only be regarded as great masses of thin vapor, susceptible of being penetrated through their whole substance by the sunbeams, and reflecting them alike from their interior parts and from their surfaces. It appears, perhaps, incredible that so thin a substance should be visible by reflected light, and some astronomers have held that the matter of comets is self-luminous; but it requires but very little light to render an object visible in the night, and a light vapor may be visible when illuminated throughout an immense stratum, which could not be seen if spread over the face of the sky like a thin cloud. From the extremely small quantity of matter of these bodies, compared with the vast spaces they cover, Newton calculated that if all the matter constituting the largest tail of a comet, were to be compressed to the same density with atmospheric air, it would occupy no more than a cubic inch.† This is incredible, but still the highest clouds that float in our atmosphere, must be looked upon as dense and massive bodies, compared with the filmy and all but spiritual texture of a comet.‡

394. The small quantity of matter in comets is proved by the fact that *they have sometimes passed very near to some of the planets without disturbing their motions in any appreciable degree.* Thus the comet of 1770, in its way to the sun, got entangled among the satellites of Jupiter, and remained near them

* See Dr. Joslin's remarks on Halley's comet, Amer. Jour. Science, Vol. 31.
† Principia, iii, 41. ‡ Sir J. Herschel.

four months, yet it did not perceptibly change their motions. The same comet also came very near the earth; so near, that, had its mass been equal to that of the earth, it would have caused the earth to revolve in an orbit so much larger than at present, as to have increased the length of the year 2h, 47m.* Yet it produced no sensible effect on the length of the year, and therefore its mass, as is shown by La Place, could not have exceeded $\frac{1}{5000}$ of that of the earth, and might have been less than this to any extent. It may indeed be asked, what proof we have that comets have any matter, and are not mere reflexions of light. The answer is that, although they are not able by their own force of attraction to disturb the motions of the planets, yet they are themselves exceedingly disturbed by the action of the planets, and in exact conformity with the laws of universal gravitation. A delicate compass may be greatly agitated by the vicinity of a mass of iron, while the iron is not sensibly affected by the attraction of the needle.

By approaching very near to a large planet, a comet may have its orbit entirely changed. This fact is strikingly exemplified in the history of the comet of 1770. At its appearance in 1770, its orbit was found to be an ellipse, requiring for a complete revolution only 5½ years; and the wonder was, that it had not been seen before, since it was a very large and bright comet. Astronomers suspected that its path had been changed, and that it had been recently compelled to move in this short ellipse, by the disturbing force of Jupiter and his satellites. The French Institute, therefore, offered a high prize for the most complete investigation of the elements of this comet, taking into account any circumstances which could possibly have produced an alteration in its course. By tracing back the movements of this comet for some years previous to 1770, it was found that, at the beginning of 1767, it had entered considerably within the sphere of Jupiter's attraction. Calculating the amount of this attraction from the known proximity of the two bodies, it was found what must have been its orbit previous to the time when it became subject to the disturbing action of Jupiter. The

* La Place.

result showed that it then moved in an ellipse of greater extent, having a period of 50 years, and having its *perihelion* instead of its aphelion near Jupiter. It was therefore evident why, as long as it continued to circulate in an orbit so far from the center of the system, it was never visible from the earth. In January 1767, Jupiter and the comet happened to be very near one another, and as both were moving in the same direction, and nearly in the same plane, they remained in the neighborhood of each other for several months, the planet being between the comet and the sun. The consequence was, that the comet's orbit was changed into a smaller ellipse, in which its revolution was accomplished in $5\frac{1}{2}$ years. But as it was approaching the sun in 1779, it happened again to fall in with Jupiter. It was in the month of June, that the attraction of the planet began to have a sensible effect; and it was not until the month of October following that they were finally separated.

At the time of their nearest approach, in August, Jupiter was distant from the comet only $\frac{1}{491}$ of its distance from the sun, and exerted an attraction upon it 225 times greater than that of the sun. By reason of this powerful attraction, Jupiter being farther from the sun than the comet, the latter was drawn out into a new orbit, which even at its perihelion came no nearer to the sun than the planet Ceres. In this third orbit, the comet requires about 20 years to accomplish its revolution; and being at so great a distance from the earth, it is invisible, and will forever remain so unless, in the course of ages, it may undergo new perturbations, and move again in some smaller orbit as before.*

ORBITS AND MOTIONS OF COMETS.

395. The planets, as we have seen, (with the exception of the four new ones, which seem to be an intermediate class of bodies between planets and comets,) move in orbits which are nearly circular, and all very near to the plane of the ecliptic, and all move in the same direction from west to east. But the orbits of comets are far more excentric than those of the planets; they are

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* Milne.

inclined to the ecliptic at various angles, being sometimes even nearly perpendicular to it; and the motions of comets are sometimes retrograde.

396. The *Elements* of a comet are five, viz. (1) The *perihelion distance;* (2) *longitude of the perihelion;* (3) *longitude of the node;* (4) *inclination of the orbit;* (5) *time of the perihelion passage.*

The investigation of these elements is a problem extremely intricate, requiring for its solution, a skilful and laborious application of the most refined analysis. Newton himself, pronounced it *Problema longe difficilimum;* and with all the advantages of the most improved state of science, the determination of a comet's orbit is considered one of the most complicated problems in astronomy. This difficulty arises from several circumstances peculiar to comets. In the *first* place, from the elongated form of the orbits which these bodies describe, it is during only a very small portion of their course, that they are visible from the earth, and the observations made in that short period, cannot afterwards be verified on more convenient occasions; whereas in the case of the planets, whose orbits are nearly circular, and whose movements may be followed uninterruptedly throughout a complete revolution, no such impediments to the determination of their orbits occur. In the *second* place, there are many comets which move in a direction opposite to the order of the signs in the zodiac, and sometimes nearly perpendicular to the plane of the ecliptic; so that their apparent course through the heavens is rendered extremely complicated, on account of the contrary motion of the earth. In the *third* place, as there may be a multitude of elliptic orbits, whose perihelion distances are equal, it is obvious that, in the case of very eccentric orbits, the slightest change in the position of the curve near the vertex, where alone the comet can be observed, must occasion a very sensible difference in the length of the orbit; and therefore, though a small error produces no perceptible discrepancy between the observed and the calculated course, while the comet remains visible from the earth, its effect when diffused over the whole extent of the orbit, may acquire a most material or even a fatal importance.

On account of these circumstances, it is found exceedingly difficult to lay down the path which a comet actually follows through the whole system, and least of all, possible to ascertain with accuracy, the length of the major axis of the ellipse, and consequently the periodical revolution.* An error of only a few seconds may cause a difference of many hundred years. In this manner, though Bessel determined the revolution of the comet of 1769 to be 2089 years, it was found that an error of no more than 5″ in observation, would alter the period either to 2678 years, or to 1692 years. Some astronomers, in calculating the orbit of the great comet of 1680, have found the length of its greater axis 426 times the earth's distance from the sun, and consequently its period 8792 years; whilst others estimate the greater axis 430 times the comet's distance, which alters the period to 8916 years. Newton and Halley, however, judged that this comet accomplished its revolution in only 570 years.

397. Disheartened by the difficulty of attaining to any precision in that circumstance, by which an elliptic orbit is characterized, and, moreover, taking into account the laborious calculations necessary for its investigation, astronomers usually satisfy themselves with ascertaining the elements of a comet on the supposition of its describing a parabola; and, as this is a curve whose axis is infinite, the procedure is greatly simplified by leaving entirely out of consideration, the periodical revolution. It is true that a parabola may not represent with mathematical strictness the course which a comet actually follows; but as a parabola is the intermediate curve between the hyperbola and ellipse, it is found that this method, which is so much more convenient for computation, also accords sufficiently with observations, except in cases when the ellipse is a comparatively short one, as that of Encke's comet, for example.

398. The elements of a comet, with the exception of its periodic time, are calculated in a manner similar to those of the plan-

* For when we know the length of the major axis, we can find the periodic time by Kepler's law, which applies as well to comets as to planets.

ets. Three good observations on the right ascension and declination of the comet (which are usually found by ascertaining its position with respect to certain stars, whose right ascensions and declinations are accurately known) afford the means of calculating these elements.

The appearance of the same comet at different periods of its return are so various, (Art. 388,) that we can never pronounce a given comet to be the same with one that has appeared before, from any peculiarities in its physical aspect. The identity of a comet with one already on record, is determined by the identity of the elements. It was by this means that Halley first established the identity of the comet which bears his name, with one that had appeared at several preceding ages of the world, of which so many particulars were left on record, as to enable him to calculate the elements at each period. These were as in the following table.

Time of appear.	Inclin. of the orbit.	Long. of the Node.	Long. of Per.	Per. Dist.	Course.
1456	17° 56′	48° 30′	301° 00	0.58	Retrograde.
1531	17 56	49 25	301 39	0.57	Retrograde.
1607	17 02	50 21	302 16	0.58	Retrograde.
1682	17 42	50 48	301 36	0.58	Retrograde.

On comparing these elements, no doubt could be entertained that they belonged to one and the same body; and since the interval between the successive returns was seen to be 75 or 76 years, Halley ventured to predict that it would again return in 1758. Accordingly, the astronomers who lived at that period, looked for its return with the greatest interest. It was found however, that on its way towards the sun it would pass very near to Jupiter and Saturn, and by their action on it, it would be retarded for a long time. Clairaut, a distinguished French mathematician, undertook the laborious task of estimating the exact amount of this retardation, and found it to be no less than 618 days, namely, 100 days by the action of Jupiter, and 518 days by that of Saturn. This would delay its appearance until early in the year 1759, and Clairaut fixed its arrival at the perihelion within a month of April 13th. It came to the perihelion on the 12th of March.

399. The return of Halley's comet in 1835, was looked for with no less interest than in 1759. Several of the most accurate mathematicians of the age had calculated its elements with inconceivable labor. Their zeal was rewarded by the appearance of the expected visitant at the time and place assigned; it traversed the northern sky presenting the very appearances, in most respects, that had been anticipated; and came to its perihelion on the 16th of November, within two days of the time prescribed by Pontecoulant, a French mathematician who had, it appeared, made the most successful calculation.* On its previous return, it was deemed an extraordinary achievement to have brought the prediction within a month of the actual time.

Many circumstances conspired to render this return of Halley's comet an astronomical event of transcendent interest. Of all the celestial bodies, its history was the most remarkable; it afforded most triumphant evidence of the truth of the doctrine of universal gravitation, and of course of the received laws of astronomy; and it inspired new confidence in the power of that instrument, (the Calculus,) by means of which its elements had been investigated.

400. Encke's comet, by its frequent returns, affords peculiar facilities for ascertaining the laws of its revolution; and it has kept the appointments made for it, with great exactness. On its late return (1839) it exhibited to the telescope a globular mass of nebulous matter, resembling fog, and moved towards its perihelion with great rapidity.

But what has made Encke's comet particularly famous, is its having first revealed to us the existence of a *Resisting Medium* in the planetary spaces. It has long been a question whether the earth and planets revolve in a perfect void, or whether a fluid of extreme rarity may not be diffused through space. A perfect vacuum was deemed most probable, because no such effects on the motions of the planets could be detected as indicated that they encountered a resisting medium. But a feather or a lock of cotton propelled with great velocity, might render obvious the

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* See Professor Loomis's Observations on Halley's Comet, *Amer. Jour. Science,* 30. 209.

resistance of a medium which would not be perceptible in the motions of a cannon ball. Accordingly, Encke's comet is thought to have plainly suffered a retardation from encountering a resisting medium in the planetary regions. The effect of this resistance, from the first discovery of the comet to the present time, has been to diminish the time of its revolution about two days. Such a resistance by destroying a part of the projectile force, would cause the comet to approach nearer to the sun, and thus to have its periodic time shortened. The ultimate effect of this cause will be to bring the comet nearer to the sun at every revolution, until it finally falls into that luminary, although many thousand years will be required to produce this catastrophe.* It is conceivable, indeed, that the effects of such a resistance may be counteracted by the attraction of one or more of the planets near which it may pass in its successive returns to the sun.

401. It is peculiarly interesting to see a portion of matter of a tenuity exceeding the thinnest fog, pursuing its path in space, in obedience to the same laws as those which regulate such large and heavy bodies as Jupiter or Saturn. In a perfect void, a speck of fog if propelled by a suitable projectile force would revolve around the sun, and hold on its way through the widest orbit, with as sure and steady a pace as the heaviest and largest bodies in the system.

402. Of the *physical nature* of comets, little is understood. It is usual to account for the variations which their tails undergo by referring them to the agencies of heat and cold. The intense heat to which they are subject in approaching so near the sun as some of them do, is alleged as a sufficient reason for the great expansion of thin nebulous atmospheres forming their tails; and the inconceivable cold to which they are subject in receding to such a distance from the sun, is supposed to account for the condensation of the same matter until it returns to its original dimensions. Thus the great comet of 1680 at its perihelion ap-

* See Professor Whewell's Observations on this subject in Bridgewater Treatises, Ch. viii.

proached 166 times nearer the sun than the earth, being only 130,000 miles from the surface of the sun.* The heat which it must have received, was estimated to be equal to 28,000 times that which the earth receives in the same time and 2000 times hotter than red hot iron. This temperature would be sufficient to volatalize the most obdurate substances, and to expand the vapor to vast dimensions ; and the opposite effects of the extreme cold to which it would be subject in the regions remote from the sun, would be adequate to condense it into its former volume.

This explanation however, does not account for the direction of the tail, extending as it usually does, only in a line opposite to the sun. Some writers therefore, as Delambre, suppose that the nebulous matter of the comet after being expanded to such a volume, that the particles are no longer attracted to the nucleus unless by the slightest conceivable force, are carried off in a direction from the sun, by the impulse of the solar rays themselves.† But to assign such a power of communicating motion to the sun's rays while they have never been proved to have any momentum, is unphilosophical; and we are compelled to place the phenomena of comets' tails among the points of astronomy yet to be explained.

403. Since those comets which have their perihelion very near the sun, like the comet of 1680, cross the orbits of all the planets, *the possibility that one of them may strike the earth*, has frequently been suggested. Still it may quiet our apprehensions on this subject, to reflect on the vast extent of the planetary spaces, in which these bodies are not crowded together as we see them erroneously represented in orreries and diagrams, but are sparsely scattered at immense distances from each other. They are like insects flying in the expanse of heaven. If a comet's tail lay with its axis in the plane of the ecliptic when it was near the sun, we can imagine that the tail might sweep over the earth ; but the tail may be situated at any angle with the ecliptic as well as in the same plane with it, and the chances that it will not be in the same plane, are almost infinite. It is also

* See Principia, Lib. III, 41.

† Delambre's Astronomy, t. 3, p. 401.

extremely improbable that a comet will cross the plane of the ecliptic precisely at the earth's path in that plane, since it may as probably cross it at any other point, nearer or more remote from the sun. Still some comets have occasionally approached near to the earth. Thus Biela's comet in returning to the sun in 1832, crossed the ecliptic very near to the earth's track, and had the earth been then at that point of its orbit, it might have passed through a portion of the nebulous atmosphere of the comet. The earth was within a month of reaching that point. This might at first view seem to involve some hazard; yet we must consider that a month short implied a distance of nearly 50,000,000 miles. La Place has assigned the consequences that would ensue in case of a direct collision between the earth and a comet;* but terrible as he has represented them on the supposition that the nucleus of the comet is a solid body, yet considering a comet (as most of them doubtless are) as a mass of exceedingly light nebulous matter, it is not probable, even were the earth to make its way directly through a comet, that a particle of the comet would reach the earth. The portions encountered by the earth, would be arrested by the atmosphere, and probably inflamed; and they would perhaps exhibit on a more magnificent scale than was ever before observed, the phenomena of shooting stars, or meteoric showers.

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* Syst. du Monde, l. iv, c. 4.

PART III.—OF THE FIXED STARS AND SYSTEM OF THE WORLD.

CHAPTER I.

OF THE FIXED STARS—CONSTELLATIONS.

404. THE FIXED STARS are so called, because, to common observation, they always maintain the same situations with respect to one another.

The stars are classed, by their apparent *magnitudes*. The whole number of magnitudes recorded are *sixteen*, of which the first six only are visible to the naked eye ; the rest are *telescopic stars*. These magnitudes are not determined by any very definite scale, but are merely ranked according to their relative degrees of brightness, and this is left in a great measure to the decision of the eye alone, although it would appear easy to measure the comparative degree of light in a star by a photometer, and upon such measurement to ground a more scientific classification of the stars. The brightest stars to the number of 15 or 20 are considered as stars of the *first* magnitude ; the 50 or 60 next brightest, of the *second* magnitude; the next 200 of the *third* magnitude ; and thus the number of each class increases rapidly as we descend the scale, so that no less than fifteen or twenty thousand are included within the first seven magnitudes.

405. The stars have been grouped in *Constellations* from the most remote antiquity : a few, as Orion, Bootes, and Ursa Major, are mentioned in the most ancient writings under the same names as they bear at present. The names of the constellations are sometimes founded on a supposed resemblance to the objects to which the names belong; as the Swan and the Scorpion were evidently so denominated from their likeness to those animals;

but in most cases it is impossible for us to find any reason for designating a constellation by the figure of the animal or the hero which is employed to represent it. These representations were probably once blended with the fables of pagan mythology. The same figures, absurd as they appear, are still retained for the convenience of reference; since it is easy to find any particular star, by specifying the part of the figure to which it belongs, as when we say a star is in the neck of Taurus, in the knee of Hercules, or in the tail of the Great Bear. This method furnishes a general clue to its position; but the stars belonging to any constellation are distinguished according their apparent magnitudes as follows:—first, by the Greek letters, Alpha, Beta, Gamma, &c. Thus α *Orionis*, denotes the largest star in Orion, β *Andromedæ*, the second star in Andromeda, and γ *Leonis*, the third brightest star in the Lion. Where the number of the Greek letters is insufficient to include all the stars in a constellation, recourse is had to the letters of the Roman alphabet, a, b, c, &c.; and, in cases where these are exhausted, the final resort is to numbers. This is evidently necessary, since the largest constellations contain many hundreds or even thousands of stars. *Catalogues* of particular stars have also been published by different astronomers, each author numbering the individual stars embraced in his list, according to the places they respectively occupy in the catalogue. These references to particular catalogues are sometimes entered on large celestial globes. Thus we meet with a star marked 84 H., meaning that this is its number in Herschel's catalogue, or 140 M. denoting the place the star occupies in the catalogue of Mayer.

406. The earliest catalogue of the stars was made by Hipparchus of the Alexandrian School, about 140 years before the Christian era. A new star appearing in the firmament, he was induced to count the stars and to record their positions, in order that posterity might be able to judge of the permanency of the constellations. His catalogue contains all that were conspicuous to the naked eye in the latitude of Alexandria, being 1022. Most persons unacquainted with the actual number of the stars which compose the visible firmament, would suppose it to be

much greater than this; but it is found that the catalogue of Hipparchus, embraces nearly all that can now be seen in the same latitude, and that on the equator, when the spectator has the northern and southern hemispheres both in view, the number of stars that can be counted does not exceed 3000. A careless view of the firmament in a clear night, gives us the impression of an infinite multitude of stars; but when we begin to count them, they appear much more sparsely distributed than we supposed, and large portions of the sky appear almost destitute of stars.

By the aid of the telescope, new fields of stars present themselves of boundless extent; the number continually augmenting as the powers of the telescope are increased. Lalande, in his Histoire Celesté, has registered the positions of no less than 50,000; and the whole number visible in the largest telescopes amount to many millions.

407. It is strongly recommended to the learner to acquaint himself with the leading constellations at least, and with a few of the most remarkable individual stars. The task of learning them is comparatively easy, and hardly any kind of knowledge, attained with so little labor, so amply rewards the possessor. It will generally be advisable, at the outset, to get some one already acquainted with the stars, to point out a few of the most conspicuous constellations, those of the Zodiac for example; the learner may then resort to a celestial globe,* and fill up the outline by tracing out the principal stars in each constellation as there laid down. By adding one new constellation to his list every night, and reviewing those already acquired, he will soon become familiar with the stars, and will greatly augment his interest and improve his intelligence in celestial observations and practical astronomy.

CONSTELLATIONS.

408. We will point out particular marks by which the leading constellations may be recognized, leaving it to the learner, af-

* For the method of rectifying the globe so as to represent the appearance of the heavens on any particular evening, see page 26, Prob. 76.

ter he has found a constellation, to trace out additional members of it by the aid of the celestial globe, or by maps of the stars. Let us begin with the *Constellations of the Zodiac*, which succeeding each other as they do in a known order, are most easily found.

ARIES (The RAM) is a small constellation, known by two bright stars which form his head, α and β *Arietis*. These two stars are three degrees* apart, and directly south of β at the distance of one degree, is a smaller star, γ *Arietis*. It has been already intimated (Art. 193) that the vernal equinox probably was near the head of Aries, when the signs of the Zodiac received the present names.

TAURUS (The BULL) will be readily found by the seven stars or *Pleiades*, which lie in his neck. The largest star in Taurus is *Aldebaran*, in the Bull's eye, a star of the first magnitude, of a reddish color somewhat resembling the planet Mars. Aldebaran and four other stars in the face of Taurus, compose the *Hyades*.

GEMINI (The TWINS) is known by two very bright stars, Castor and Pollux, four degrees asunder. Castor (the northern) is of the first, and Pollux of the second magnitude.

CANCER (The CRAB). There are no large stars in this constellation, and it is regarded as less remarkable than any other in the Zodiac. It contains however an interesting group of small stars, called *Præsepe* or the Nebula of Cancer, which resembles a comet, and is often mistaken for one, by persons unacquainted with the stars. With a telescope of very moderate powers this nebula is converted into a beautiful assemblage of exceedingly bright stars.

LEO (The LION) is a very large constellation, and has many interesting members. *Regulus* (α *Leonis*) is a star of the first magnitude, which lies directly in the ecliptic, and is much used in astronomical observations. North of Regulus lies a semi-circle of bright stars, forming a *sickle* of which Regulus is the handle. *Denebola*, a star of the second magnitude, is in the Lion's tail, 20° northeast of Regulus.

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* These measures are not intended to be stated with exactness, but only with such a degree of accuracy as may serve for a general guide.

VIRGO (The VIRGIN) extends a considerable way from west to east, but contains only a few bright stars. *Spica*, however is a star of the first magnitude, and lies very near the place of the autumnal equinox. Four degrees eastward of Spica, and six degrees south of Denebola, is *Vindemiatrix*, in the head of Virgo, a star of the third magnitude.

LIBRA (The BALANCE) is distinguished by three large stars, of which the two brightest constitute the beam of the balance, and the smallest forms the top or handle.

SCORPIO (The SCORPION) is one of the finest of the constellations. His head is formed of five bright stars arranged in the arc of a circle, which is crossed in the center by the ecliptic nearly at right angles, near the brightest of the five, β *Scorpionis*. Four degrees southeast of this, is a remarkable star of the first magnitude, of a reddish color, called *Cor Scorpionis*, or *Antares*. South of this a succession of bright stars sweep round towards the east, terminating in several small stars, forming the tail of the Scorpion.

SAGITTARIUS (The ARCHER). Northeast of the tail of the Scorpion, are three stars in the arc of a circle which constitute the *bow* of the Archer, the central star being the brightest, directly west of which is a bright star which forms the *arrow*.

CAPRICORNUS (The GOAT) lies northeast of Sagittarius, and is known by two bright stars, two degrees apart, which form the head.

AQUARIUS (The WATER BEARER) is recognized by two stars in a line with α *Capricorni*, forming the shoulders of the figure. These two stars are 10° apart, and 3° southeast is a third star, which together with the other two, makes an acute triangle, of which the westernmost is the vertex.

PISCES (The FISHES) lie between Aquarius and Aries. They are not distinguished by any large stars, but are connected by a series of small stars, that form a crooked line between them. *Piscis Australis*, the Southern Fish, lies directly below Aquarius, and is known by a single bright star far in the south, having a declination of 30°. The name of this star is *Fomalhaut*, and is much used in astronomical measurements.

409. The Constellations of the Zodiac, being first well learned, so as to be readily recognized, will facilitate the learning of others that lie north and south of them. Let us therefore next review the principal *Northern Constellations*, beginning north of Aries and proceeding from west to east.

ANDROMEDA, is characterised by three stars of the second magnitude, situated in a straight line, extending from west to east. The middle star is about 17° north of β *Arietis*. It is in the girdle of Andromeda, and is named *Mirach*. The other two lie at about equal distances, 14° west and east of Mirach. The western star, in the head of Andromeda, lies in the Equinoctial Colure. The eastern star, *Alamak*, is situated in the foot.

PERSEUS lies directly north of the Pleiades, and contains several bright stars. About 18° from the Pleiades is *Algol*, a star of the second magnitude, in the Head of Medusa, which forms a part of the figure ; and 8° north of Algol is *Algenib*, of the same magnitude in the breast of Perseus. Between Algenib and the Pleiades are three bright stars, at nearly equal intervals, which compose the right leg of Perseus.

AURIGA (the WAGONER) lies directly east of Perseus, and extends nearly parallel to that constellation from north to south. *Capella* a very white and beautiful star of the first magnitude, distinguishes this constellation. The feet of Auriga are near the Bull's Horns.

The LYNX comes next, but presents nothing particularly interesting, containing no stars above the fourth magnitude.

LEO MINOR consists of a collection of small stars north of the sickle in Leo, and south of the Great Bear. Its largest star is only of the third magnitude.

COMA BERENICES is a cluster of small stars, north of Denebola, in the tail of the Lion, and of the head of Virgo. About 10° directly north of Berenice's Hair, is a single bright star called *Cor Caroli*, or Charles's Heart.

BOOTES, which comes next, is easily found by means of *Arcturus*, a star of the first magnitude, of a reddish color, which is situated near the knee of the figure. Arcturus is accompanied by three small stars forming a triangle a little to the southeast. Two bright stars γ and δ *Bootis*, form the shoulders, and β of the third magnitude is in the head of the figure.

Corona Borealis (The Crown) which is situated to the N. E. of Bootes, is very easily recognized, composed as it is of a semicircle of bright stars. In the center of the bright crown, is a star of the second magnitude, called *gemma;* the remaining stars are all much smaller.

Hercules, lying between the Crown on the west and the Lyre on the east, is very thick set with stars, most of which are quite small. The Constellation covers a great extent of the sky, especially from N. to S., the head terminating within 15° of the equator, and marked by a star of the third magnitude, called *Rasalgethi*, which is the largest in the Constellation.

Ophiucus is situated directly south of Hercules, extending some distance on both sides of the equator, the feet resting on the Scorpion. The head terminates near the head of Hercules, and like that, is marked by a bright star within 5° of *α Herculis.* Ophiucus is represented as holding in his hands the Serpent, the head of which, consisting of three bright stars, is situated a little south of the Crown. The folds of the serpent will be easily followed by a succession of bright stars which extend a great way to the east.

Aquila (The Eagle) is conspicuous for three bright stars in its neck, of which the central one, *Altair*, is a very brilliant white star of the first magnitude. *Antinous* lies directly south of the Eagle, and north of the head of Capricornus.

Delphinus (The Dolphin) is a small but beautiful Constellation, a few degrees east of the Eagle, and is characterized by four bright stars near to one another, forming a small rhombic square. Another star of the same magnitude 5° south, makes the tail.

Pegasus lies between Aquarius on the south and Andromeda on the north. It contains but few large stars. A very regular square of bright stars is composed of *α Andromedæ*, and the three largest stars in Pegasus, namely, *Scheat*, *Markab*, and *Algenib.* The sides composing this square are each about 15°. Algenib is situated in the equinoctial colure.

410. We may now review the *Constellations which surround the North Pole*, within the circle of perpetual apparition. (Art. 54.)

URSA MINOR (The LITTLE BEAR) lies nearest the pole. The Pole-star, *Polaris*, is in the extremity of the tail, and is of the third magnitude. Three stars in a straight line 4° or 5° apart, commencing with the Pole-star, lead to a trapezium of four stars, and the whole seven form together a *dipper*, the trapezium being the body, and the three stars the handle.

URSA MAJOR (The GREAT BEAR) is situated between the pole and the Lesser Lion, and is usually recognized by the figure of a larger and more perfect dipper, which constitutes the hinder part of the animal. This has also seven stars, four in the body of the dipper, and three in the handle. All these are stars of much celebrity. The two in the western side of the dipper, α and β, are called *Pointers*, on account of their always being in a right line with the Pole-star, and therefore affording an easy mode of finding that. The first star in the tail, next the body, is named *Alioth*, and the second *Mizar*. The head of the Great Bear lies far to the westward of the Pointers, and is composed of numerous small stars ; and the feet are severally composed of two small stars very near to each other.

DRACO (The DRAGON) winds round between the Great and Little Bear; and commencing with the tail, between the Pointers and the Pole-star, it is easily traced by a succession of bright stars extending from west to east, passing under Ursa Minor, it returns westward, and terminates in a triangle which forms the head of Draco, near the feet of Hercules, northwest of Lyra.

CEPHEUS lies eastward of the breast of the Dragon, but has no stars above the third magnitude.

CASSIOPEIA is known by the figure of a *chair*, composed of four stars which form the legs, and two which form the back. This Constellation lies between Perseus and Cepheus, in the Milky Way.

CYGNUS (The SWAN) is situated also in the Milky Way, some distance southwest of Cassiopeia, towards the Eagle. Three bright stars, which lie along the Milky Way, form the body and neck of the Swan, and two others in a line with the middle one of the three, one above and one below, constitute the wings. This Constellation is among the few, that exhibit some resemblance to the animals whose names they bear.

LYRA (The LYRE) is directly west of the Swan, and is easily distinguished by a beautiful white star of the first magnitude, *α Lyræ.*

411. The *Southern Constellations* are comparatively few in number. We shall notice only the Whale, Orion, the Greater and Lesser Dog, Hydra, and the Crow.

CETUS (The WHALE) is distinguished rather for its extent than its brilliancy, reaching as it does through 40° of longitude, while none of its stars except one, are above the third magnitude. *Menkar* (*α* Ceti) in the mouth, is a star of the second magnitude, and several other bright stars directly south of Aries, mark the head and neck of the Whale. *Mira* (*o Ceti*) in the neck of the Whale, is a variable star.

ORION is one of the largest and most beautiful of the constellations, lying southeast of Taurus. A cluster of small stars form the head; two large stars, *Betalgeus* of the first and *Bellatrix* of the second magnitude, make the shoulders; three more bright stars compose the buckler, and three the sword; and *Rigel*, another star of the first magnitude, makes one of the feet. In this Constellation there are 70 stars plainly visible to the naked eye, including two of the first magnitude, four of the second, and three of the third.

CANIS MAJOR lies S. E. of Orion, and is distinguished chiefly by its containing the largest of the fixed stars, *Sirius.*

CANIS MINOR a little north of the equator, between Canis Major and Gemini, is a small Constellation, consisting chiefly of two stars, of which *Procyon* is of the first magnitude.

HYDRA has its head near Procyon, consisting of a number of stars of ordinary brightness. About 15° S. E. of the head, is a star of the second magnitude, forming the heart, *(Cor Hydræ)*; and eastward of this, is a long succession of stars of the fourth and fifth magnitudes composing the body and the tail, and reaching as far as a few degrees south of Spica Virginis.

CORVUS (The CROW) is represented as standing on the tail of Hydra. It consists of small stars, only three of which are as large as the third magnitude.

412. The foregoing brief sketch is designed merely to aid the student in *finding* the principal constellations and the largest fixed stars. When we have once learned to recognize a constellation by some characteristic marks, we may afterwards fill up the outline by the aid of a celestial globe or a map of the stars. It will be of little avail however, merely to commit this sketch to memory; but it will be very useful for the student at once to render himself familiar with it, from the actual specimens which every clear evening presents to his view.

CHAPTER II.

CLUSTERS OF STARS—NEBULÆ—VARIABLE STARS—TEMPORARY STARS—DOUBLE STARS.

413. In various parts of the firmament are seen large groups or *clusters*, which, either by the naked eye, or by the aid of the smallest telescope, are perceived to consist of a great number of small stars. Such are the Pleiades, Coma Berenices, and Præsepe or the Bee-hive, in Cancer. The Pleiades, or *Seven Stars*, as they are called, in the neck of Taurus, is the most conspicuous cluster. When we look *directly* at this group, we cannot distinguish more than six stars, but by turning the eye *sideways** upon it, we discover that there are many more. Telescopes show 50 or 60 stars crowded together and apparently insulated from the other parts of the heavens.† Coma Berenices has fewer stars, but they are of a larger class than those which compose the Pleiades. The Bee Hive or Nebula of Cancer as it is called, is one of the finest objects of this kind for a small telescope, being by its aid converted into a rich congeries of shining points. The head of Orion affords an example of another cluster, though less remarkable than the others.

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* Indirect vision is far more delicate than direct. Thus we can see the Zodiacal Light or a Comet's Tail, much more distinctly and better defined, if we fix one eye on a part of the heavens at some distance, and turn the other eye obliquely upon the object.

† Sir J. Herschel.

414. *Nebulæ* are those faint misty appearances which resemble comets, or a small speck of fog. The Galaxy or Milky Way presents a continued succession of large nebulæ. A very remarkable Nebula, visible to the naked eye, is seen in the girdle of Andromeda. No powers of the telescope have been able to resolve this into separate stars. Its dimensions are astonishingly great. In diameter it is about 15′. The telescope reveals to us innumerable objects of this kind. Sir William Herschel has given catalogues of 2000 Nebulæ, and has shown that the nebulous matter is distributed through the immensity of space in quantities inconceivably great, and in separate parcels of all shapes and sizes, and of all degrees of brightness between a mere milky appearance and the condensed light of a fixed star. Finding that the gradations between the two extremes were tolerably regular, he thought it probable that the nebulæ form the materials out of which nature elaborates suns and systems; and he conceived that, in virtue of a central gravitation, each parcel of nebulous matter becomes more and more condensed, and assumes a rounder form. He infers from the eccentricity of its shape, and the effects of the mutual gravitation of its particles, that it acquires gradually a rotary motion; that the condensation goes on increasing until the mass acquires consistency and solidity, and all the other characters of a comet or a planet; that by a still further process of condensation, the body becomes a real star, self-shining; and that thus the waste of the celestial bodies, by the perpetual diffusion of their light, is continually compensated and restored by new formations of such bodies, to replenish forever the universe with planets and stars.*

415. These opinions are recited here rather out of respect to their notoriety and celebrity, than because we suppose them to be founded on any better evidence than conjecture. The Philosophical Transactions for many years, both before and after the commencement of the present century, abound with both the observations and speculations of Sir William Herschel. The former are deserving of all praise; the latter of very little confidence.

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* Phil. Trans. 1811.

Changes, however, are going on in some of the nebulæ, which plainly show that they are not, like planets and stars, fixed and permanent creations. Thus the great nebula in the girdle of Andromeda, has very much altered its structure since it first became an object of telescopic observation.* Many of the nebulæ are of a globular form, (Fig. 72, *a*) but frequently they present the appear-

(Fig. 72, *a*.) (Fig. 72, *b*.)

ance of a rapid increase of numbers towards the center, (Fig. 72, *b*) the exterior boundary being irregular, and the central parts more nearly spherical.

416. The Nebula in the sword of Orion is particularly celebrated, being very large and of a peculiarly interesting appearance. According to Sir John Herschel, its nebulous character is very different from what might be supposed to arise from the assemblage of an immense collection of small stars. It is formed of little flocculent masses like wisps of clouds; and such wisps seem to adhere to many small stars at its outskirts, and especially to one considerable star which it envelops with a nebulous atmosphere of considerable extent and singular figure.

Descriptions, however, can convey but a very imperfect idea of this wonderful class of astronomical objects, and we would therefore urge the learner studiously to avail himself of the first opportunity he may have to view them through a large telescope, especially the Nebula of Andromeda and of Orion.

417. *Nebulous Stars* are such as exhibit a sharp and brilliant star surrounded by a disk or atmosphere of nebulous matter. These atmospheres in some cases present a circular, in others an

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* Astron. Trans. II, 495.

oval figure; and in some instances, the nebula consists of a long, narrow spindle-shaped ray, tapering away at both ends to points.

Annular Nebulæ also exist, but are among the rarest objects in the heavens. The most conspicuous of this class, is to be found exactly half way between the stars β and γ Lyræ, and may be seen with a telescope of moderate power.*

Planetary Nebulæ constitute another variety, and are very remarkable objects. They have, as their name imports, exactly the appearance of planets. Whatever may be their nature, they must be of enormous magnitude. One of them is to be found in the parallel of γ Aquarii, and about 5m. preceding that star. Its apparent diameter is about 20″. Another in the Constellation Andromeda, presents a visible disk of 12″, perfectly defined and round. Granting these objects to be equally distant from us with the stars, their real dimensions must be such as, on the lowest computation, would fill the orbit of Uranus. It is no less evident that, if they be solid bodies, of a solar nature, the intrinsic splendor of their surfaces must be almost infinitely inferior to that of the sun. A circular portion of the sun's disk, subtending an angle of 20″, would give a light equal to 100 full moons; while the objects in question are hardly, if at all, discernible with the naked eye.†

418. The *Galaxy* or *Milky Way* is itself supposed by some to be a nebula of which the sun forms a component part; and hence it appears so much greater than other nebulæ only in consequence of our situation with respect to it, and its greater proximity to our system. So crowded are the stars in some parts of this zone, that Sir William Herschel, by counting the stars in a single field of his telescope, estimated that 50,000 had passed under his review in a zone two degrees in breadth during a single hour's observation. Notwithstanding the apparent contiguity of the stars which crowd the galaxy, it is certain that their mutual distances must be inconceivably great.

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* A list of 288 bright nebulæ, with references to well known stars, near which they are situated, is given in the Edinburg Encyclopœdia, Art. *Astronomy*, p. 781. It is convenient for finding any required nebula.

† Sir J. Herschel.

419. Variable Stars are those which undergo a periodical change of brightness. One of the most remarkable is the star *Mira* in the Whale, (*o Ceti*). It appears once in 11 months, remains at its greatest brightness about a fortnight, being then, on some occasions, equal to a star of the second magnitude. It then decreases about three months, until it becomes completely invisible, and remains so about five months, when it again becomes visible, and continues increasing during the remaining three months of its period.

Another very remarkable variable star is *Algol* (β Persei). It is usually visible as a star of the second magnitude, and continues such for 2d. 14h. when it suddenly begins to diminish in splendor, and in about 3½ hours is reduced to the fourth magnitude. It then begins again to increase, and in 3½ hours more, is restored to its usual brightness, going through all its changes in less than three days. This remarkable law of variation appears strongly to suggest the revolution round it of some opake body, which, when interposed between us and Algol, cuts off a large portion of its light. It is (says Sir J. Herschel) an indication of a high degree of activity in regions where, but for such evidences, we might conclude all lifeless. Our sun requires almost nine times this period to perform a revolution on its axis. On the other hand, the periodic time of an opake revolving body, sufficiently large, which would produce a similar temporary obscuration of the sun, seen from a fixed star, would be less than fourteen hours.

The duration of these periods is extremely various. While that of β Persei above mentioned, is less than three days, others are more than a year, and others many years.

420. Temporary Stars are new stars which have appeared suddenly in the firmament, and after a certain interval, as suddenly disappeared and returned no more.

It was the appearance of a new star of this kind 125 years before the Christian era, that prompted Hipparchus to draw up a catalogue of the stars, the first on record. Such also was the star which suddenly shone out A. D. 389, in the Eagle, as bright as Venus, and after remaining three weeks disappeared entirely. At other periods, at distant intervals, similar phenomena have presented themselves. Thus the appearance of a star in 1572, was so sudden, that Tycho Brahe returning home one day was surprized to find a collection of country people gazing at a star which he was sure did not exist half an hour before. It was then as bright as Sirius, and continued to increase until it surpassed Jupiter when brightest, and was visible at mid-day. In a month it began to diminish, and in three months afterwards it had entirely disappeared.

It has been supposed by some that in a few instances, the same star has returned, constituting one of the periodical or variable stars of a long period.

Moreover, on a careful re-examination of the heavens, and a comparison of catalogues, many stars are now found to be missing.*

421. Double Stars are those which appear single to the naked eye, but are resolved into two by the telescope; or, if not visible to the naked eye, are seen in the telescope so close together as to be recognized as objects of this class. Sometimes three or more stars are found in this near connexion, constituting *triple* or *multiple* stars. Castor, for example, when seen by the naked eye, appears as a single star, but in a telescope even of moderate powers, it is resolved into two stars of between the third and fourth magnitudes, within 5″ of each other. These two stars are nearly of equal size, but more commonly one is exceedingly small in comparison with the other, resembling a satellite near its primary, although in distance, in light, and in other characteristics, each has all the attributes of a star, and the combination therefore cannot be that of a planet with a satellite. In most instances, also, the distance detween these objects is much less than 5″, and in many cases it is less than 1″. The extreme closeness, together with the exceeding minuteness of most of the double stars, requires the best telescopes united with the most acute powers of observation. Indeed, certain of these objects are regarded as the severest *tests* both of the excellence of the instruments, and of the skill of the observer.

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* Sir J. Herschel.

422. Many of the double stars exhibit the curious and beautiful phenomena of contrasted or *complementary colors.** In such instances, the larger star is usually of a ruddy or orange hue, while the smaller one appears blue or green, probably in virtue of that general law of optics, which provides that when the retina is excited by any bright colored light, feebler lights which seen alone would produce no sensation but of whiteness, appear colored, with the tint complementary to that of the brighter. Thus a yellow color predominating in the light of the brighter star, that of the less bright one in the same field of view will appear blue; while, if the tint of the brighter star verges to crimson, that of the other will exhibit a tendency to green, or even under favorable circumstances, will appear as a vivid green. The former contrast is beautifully exhibited by ι Cancri, the latter by γ Andromedæ, both fine double stars. If, however, the colored star is much the less bright of the two, it will not materially affect the other. Thus for instance, η Cassiopeiæ exhibits the beautiful combination of a large white star, and a small one of a rich ruddy purple. It is by no means, however, intended to say, that in all such cases, one of the colors is the mere effect of contrast, and it may be easier suggested in words, than conceived in imagination, what variety of illumination *two suns*, a red and green, or a yellow and a blue sun, must afford a planet circulating about either; and what charming contrasts and "grateful vicissitudes," a red and green day for instance, alternating with a white one and with darkness, might arise from the presence or absence of one or other, or both above the horizon. Insulated stars of a red color, almost as deep as that of blood, occur in many parts of the heavens, but no green or blue star, of any decided hue, has, we believe, ever been noticed, unassociated with a companion brighter than itself.†

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* Complementary colors are such as together make white light. If all the colors of the spectrum be laid down on a circular ring, each occupying its proportionate space, any two colors on the opposite sides of the zone, are complementary to each other, and when *of the same degree of intensity*, they compose white light. Brewster's Optics, Part III, c. 26.

† Sir J. Herschel.

423. Our knowledge of the double stars almost commenced with Sir William Herschel, about the year 1780. At the time he began his search for them, he was acquainted with only *four*. Within five years, he discovered nearly 700 double stars.* In his memoirs published in the Philosophical Transactions,† he gave most accurate measurements of the distances between the two stars, and of the angle which a line joining the two, formed with the parallel of declination.‡ These data would enable him, or at least posterity, to judge whether these minute bodies ever change their position with respect to each other.

Since 1821, these researches have been prosecuted with great zeal and industry by Sir James South and Sir John Herschel in England, and by Professor Struve at Dorpat in Russia, and the whole number of double stars now known, amounts to several thousands.§ Two circumstances add a high degree of interest to the phenomena of the double stars,—the first is, that a few of them at least are found to have a revolution around each other, and the second, that they are supposed to afford the means of obtaining the parallax of the fixed stars. Of these topics we shall treat in the next chapter.

CHAPTER III.

MOTIONS OF THE FIXED STARS—DISTANCES—NATURE.

424. In 1803, Sir William Herschel first determined and announced to the world, that there exist among the stars, separate systems, composed of two stars revolving about each other in regular orbits. These he denominated *Binary Stars*, to distinguish them from other double stars where no such motion is detected, and whose proximity to each other may possibly arise from casual juxta-position, or from one being in the range of the

* During his life he observed in all, 2400 double stars.

† Phil. Trans. 1782—1785.

‡ Baily *Astron. Trans.* II, 542.

§ The Catalogue of Struve, contains 3063.

other. Between fifty and sixty instances of changes to a greater or less amount of the relative position of double stars, are mentioned by Sir William Herschel; and a few of them had changed their places so much within 25 years, and in such order, as to lead him to the conclusion that they performed revolutions, one around the other, in regular orbits.

425. These conclusions have been fully confirmed by later observers, so that it is now considered as fully established, that there exist among the fixed stars, binary systems, in which two stars perform to each other the office of sun and planet, and that the periods of revolution of more than one such pair have been ascertained with something approaching to exactness. Immersions and emersions of stars behind each other have been observed, and real motions among them detected rapid enough to become sensible and measurable in very short intervals of time.* The following table exhibits the present state of our knowledge on this subject.

Names.	Period in years.	Major axis of the orbit.	Eccentricity.
η Coronæ,	43.40	———	———
ζ Cancri,	55.00	———	———
ξ Ursæ Majoris,	58.26	7″.714	0.4164
70 Ophiuchi,	80.34	8.784	0.4667
Castor,	252.66	16.172	0.7582
σ Coronæ,	286.00	7.358	0.6112
61 Cygni,	452.00	30.860	———
γ Virginis,	628.90	24.000	0.8335
γ Leonis,	1200.00	———	———

From this table it appears (1) that the periods of the double stars are very various, ranging, in the case of those already ascertained, from forty three years to one thousand; (2) that their orbits are very small ellipses more eccentric than those of the planets, the greatest of which (that of Mercury) having an eccentricity of only about .2 of the major axis.

The most remarkable of the binary stars is γ Virginis, on account not only of the length of its period, but also of the great diminution

* Ast. Trans. II, 544.

of apparent distance, and rapid increase of angular motion about each other of the individuals composing it. It is a bright star of the fourth magnitude, and its component stars are almost exactly equal. It has been known to consist of two stars since the beginning of the eighteenth century, their distance being then between six and seven seconds; so that any tolerably good telescope would resolve it. Since that time, they have been constantly approaching, and are at present hardly more than a single second asunder; so that no telescope that is not of a very superior quality, is competent to show them otherwise than as a single star, somewhat lengthened in one direction. It fortunately happens that Bradley (Astronomer Royal) in 1718, noticed, and recorded in the margin of one of his observation books, the apparent direction of their line of junction, as being parallel to that of two remarkable stars α and δ of the same constellation, as seen by the naked eye,—a remark which has been of signal service in the investigation of their orbit. It is found that it passed its perihelion, August 18th, 1834, and that in the interval from 1839 to 1841, this star will have completed a full revolution from the epoch of the first measurement of its position in 1781; and the regularity with which it has maintained its motion, is said to have been exceedingly beautiful.*

426. The revolutions of the binary stars have assured us of that most interesting fact, that *the law of gravitation extends to the fixed stars.* Before these discoveries, we could not decide except by a feeble analogy that this law transcended the bounds of the solar system. Indeed, our belief of the fact rested more upon our idea of unity of design in all the works of the Creator, than upon any certain proof; but the revolution of one star around another in obedience to forces which must be similar to those that govern the solar system, establishes the grand conclusion, that the law of gravitation is truly the law of the material universe.

We have the same evidence (says Sir John Herschel) of the revolutions of the binary stars about each other, that we have of those of Saturn and Uranus about the sun; and the correspond-

* Astron. Trans. v, 35.

ence between their calculated and observed places in such elongated ellipses, must be admitted to carry with it a proof of the prevalence of the Newtonian law of gravity in their systems, of the very same nature and cogency as that of the calculated and observed places of comets round the center of our own system.

But (he adds) it is not with the revolutions of bodies of a planetary or cometary nature round a solar center that we are now concerned; it is with that of sun around sun, each, perhaps, accompanied with its train of planets and their satellites, closely shrouded from our view by the splendor of their respective suns, and crowded into a space, bearing hardly a greater proportion to the enormous interval which separates them, than the distances of the satellites of our planets from their primaries, bear to their distances from the sun itself.

427. *Some of the fixed stars appear to have a real motion in space.*

The *apparent* change of place in the stars arising from the precession of the equinoxes, the nutation of the earth's axis, the diminution of the obliquity of the ecliptic, and the aberration of light, have been already mentioned; but after all these corrections are made, changes of place still occur, which cannot result from any changes in the earth, but must arise from changes in the stars themselves. Such motions are called the *proper motions* of the stars. Nearly 2000 years ago, Hipparchus and Ptolemy made the most accurate determinations in their power of the relative situations of the stars, and their observations have been transmitted to us in Ptolemy's Almagest; from which it appears that the stars retain at least very *nearly* the same places now as they did at that period. Still the more accurate methods of modern Astronomers, have brought to light minute changes in the places of certain stars which force upon us the conclusion, *either that our solar system causes an apparent displacement of certain stars, by a motion of its own in space, or that they have themselves a proper motion.* Possibly, indeed, both these causes may operate.

428. If the sun, and of course the earth which accompanies him, is actually in motion, the fact may become manifest from

the apparent approach of the stars in the region which he is leaving, and the recession of those which lie in the part of the heavens towards which he is travelling. Were two groves of trees situated on a plain at some distance apart, and we should go from one to the other, the trees before us would gradually appear farther and farther asunder, while those we left behind would appear to approach each other. Some years since, Sir William Herschel supposed he had detected changes of this kind among two sets of stars in opposite points of the heavens, and announced that the solar system was in motion towards a point in the constellation Hercules;* but other astronomers have not found the changes in question such as would correspond to this motion, or to any motion of the sun; and while it is a matter of general belief that the sun has a motion in space, the fact is not considered as yet entirely proved.

429. In most cases where a proper motion in certain stars has been suspected, its annual amount has been so small, that many years are required to assure us, that the effect is not owing to some other cause than a real progressive motion in the stars themselves; but in a few instances the fact is too obvious to admit of any doubt. Thus the two stars 61 Cygni, which are nearly equal, have remained constantly at the same, or nearly at the same distance of 15″ for at least fifty years past. Meanwhile they have shifted their local situation in the heavens, 4′ 23″, the annual proper motion of each star being 5.″3, by which quantity this system is every year carried along in some unknown path, by a motion which for many centuries must be regarded as uniform and rectilinear. A greater proportion of the double stars than of any other indicate proper motions, especially the binary stars or those which have a revolution around each other. Among stars not double, and no way differing from the rest in any other obvious particular, μ Cassiopeiæ has the greatest proper motion of any yet ascertained, amounting to nearly 4″ annually.

DISTANCES OF THE FIXED STARS.

430. *We cannot ascertain the actual distance of any of the fixed stars, but can certainly determine that the nearest star is*

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* Phil. Trans. 1783, 1805, and 1806.

more than (20,000,000,000,000,) *twenty billions of miles from the earth.*

For all measurements relating to the distances of the sun and planets, the *radius of the earth* furnishes the base line (Art. 87). The length of this line being known, and the horizontal parallax of the body, whose distance is sought, we readily obtain the distance by the solution of a right angled triangle. But any star viewed from the opposite sides of the earth, would appear from both stations, to occupy precisely the same situation in the celestial sphere, and of course it would exhibit no horizontal parallax.

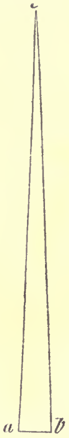
But astronomers have endeavored to find a parallax in some of the fixed stars by taking the *diameter of the earth's orbit* as a base line. Yet even a change of position amounting to 190 millions of miles, proves insufficient to alter the place of a single star, from which it is concluded that the stars have not even any *annual parallax;* that is, the angle subtended by the semidiameter of the earth's orbit, at the nearest fixed star is insensible. The errors to which instrumental measurements are subject, arising from the defects of the instruments themselves, from refraction, and from various other sources of inaccuracy, are such, that the angular determinations of arcs of the heavens cannot be relied on to less than 1″. But the change of place in any star when viewed at opposite extremities of the earth's orbit, is less than 1″, and therefore cannot be appreciated by direct measurement. It follows, that, when viewed from the nearest star, the diameter of the earth's orbit would be insensible: the spider line of the telescope would more than cover it.

431. Taking, however, the annual parallax of a fixed star at 1″, let $a\ b$ (Fig. 73) represent the radius of the earth's orbit and c a fixed star, the angle at c being 1″ and the angle at b a right angle; then,

$$\text{Sin. } 1'' : \text{Rad.} :: 1 : 200{,}000, \text{ nearly.}$$

Hence the hypothenuse of a triangle whose vertical angle is 1″ is about 200,000 times the base; consequently the distance of the nearest fixed star *must exceed* $95000000 \times 200000 = 190000000 \times 100000$, or one hundred thousand times one hundred and ninety millions of miles. Of a distance so vast we can form no adequate conceptions, and even seek to measure it only by the time that light, (which moves more than 192,000 miles per second and passes from the sun to the earth in 8m. 7sec.,) would take to traverse it, which is found to be more than three and a half years.

Fig. 73.



If these conclusions are drawn with respect to the largest of the fixed stars, which we suppose to be vastly nearer to us than those of the smallest magnitude, the idea of distance swells upon us when we attempt to estimate the remoteness of the latter. As it is uncertain, however, whether the difference in the apparent magnitudes of the stars is owing to a real difference or merely to their being at various distances from the eye, more or less uncertainty must attend all efforts to determine the relative distances of the stars ; but astronomers generally believe that the lower orders of stars are vastly more distant from us than the higher. Of some stars it is said, that thousands of years would be required for their light to travel down to us.

432. We have said that the stars have no annual parallax ; yet it may be observed that astronomers are not exactly agreed on this point. Dr. Brinkley, a late eminent Irish astronomer, supposed that he had detected an annual parallax in α Lyræ amounting to $1''.13$ and in α Aquilæ of $1''.42$.* These results were controverted by Mr. Pond of the Royal Observatory of Greenwich ; and Mr. Struve of Dorpat has shown that in a number of cases, the parallax is *negative*, that is in a direction opposite to that which would arise from the motion of the earth. Hence it is considered doubtful whether in all cases of an apparent parallax, the effect is not wholly due to errors of observation.

433. Indirect methods have been proposed for ascertaining the parallax of the fixed stars by means of observations on the *double stars.* If the two stars composing a double star are at different distances from us, parallax would affect them unequally, and change their relative position with respect to each other ; and

* Phil. Trans. 1821.

since the ordinary sources of error arising from the imperfection of instruments, from precession, nutation, aberration, and refraction, would be avoided, (since they would affect both objects alike, and therefore would not disturb their relative positions,) measurements taken with the micrometer of changes much less than 1″ may be relied on. Sir John Herschel proposes a method* by which changes may be determined which amount to only $\frac{1}{40}$ of a second.†

434. The immense distance of the fixed stars is inferred also from the fact that the largest telescopes do not increase their apparent magnitude. They are still points, when viewed with the highest magnifiers, although they sometimes present a spurious disk, which is owing to irradiation.‡

NATURE OF THE STARS.

435. *The stars are bodies greater than our earth.* If this were not the case they could not be visible at such an immense distance. Dr. Wollaston, a distinguished English philosopher, attempted to estimate the magnitudes of certain of the fixed stars from the light which they afford. By means of an accurate photometer (an instrument for measuring the relative intensities of light) he compared the light of Sirius with that of the sun. He next inquired how far the sun must be removed from us in order to appear no brighter than Sirius. He found the distance to be 141,400 times its present distance. But Sirius is more than 200,000 times as far off as the sun (Art. 431). Hence he inferred that, upon the lowest computation, Sirius must actually

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* Phil. Trans. 1826.

† Very recent intelligence informs us, that Professor Bessel of Königsberg, has obtained decisive evidence of an annual parallax in 61 Cygni, amounting to 0.″3136. This makes the distance of that star, equal to 657700 times 95 millions of miles,—a distance which it would take light 10.3 years to traverse.

‡ Irradiation is an enlargement of objects beyond their proper bounds, in consequence of the vivid impression of light on the eye. It is supposed to increase the apparent diameters of the sun and moon from three to four seconds, and to create an appearance of a disk in a fixed star which, when this cause is removed, is seen as a mere point. See Richardson, Astr. Trans. v, 1.

give out twice as much light as the sun; or that, in point of splendor, Sirius must be at least equal to two suns. Indeed, he has rendered it probable that the light of Sirius is equal to fourteen suns.

436. *The fixed stars are suns.* We have already seen that they are large bodies; that they are immensely farther off than the farthest planet; that they shine by their own light; in short, that their appearance is, in all respects, the same as the sun would exhibit if removed to the region of the stars. Hence we infer that they are bodies of the same kind with the sun.

437. We are justified therefore by a sound analogy, in concluding that the stars were made for the same end as the sun, namely, as the centers of attraction to other planetary worlds, to which they severally dispense light and heat. Although the starry heavens present, in a clear night, a spectacle of ineffable grandeur and beauty, yet it must be admitted that the chief purpose of the stars, could not have been to adorn the night, since by far the greatest part of them are wholly invisible to the naked eye; nor as landmarks to the navigator, for only a very small proportion of them are adapted to this purpose; nor, finally, to influence the earth by their attractions, since their distance renders such an effect entirely insensible. If they are suns, and if they exert no important agencies upon our world, but are bodies evidently adapted to the same purpose as our sun, then it is as rational to suppose that they were made to give light and heat, as that the eye was made for seeing and the ear for hearing. It is obvious to inquire next, to what they dispense these gifts if not to planetary worlds; and why to planetary worlds, if not for the use of percipient beings? We are thus led, almost inevitably, to the idea of a *Plurality of Worlds;* and the conclusion is forced upon us, that the spot which the Creator has assigned to us is but a humble province of his boundless empire.*

* See this argument, in its full extent, in *Dick's Celestial Scenery.*

CHAPTER III.

OF THE SYSTEM OF THE WORLD.

438. *The arrangement of all the bodies that compose the material universe, and their relations to each other, constitute the System of the World.*

It is otherwise call the Mechanism of the Heavens; and indeed in the System of the world, we figure to ourselves a machine, all the parts of which have a mutual dependence, and conspire to one great end. "The machines that are first invented (says Adam Smith) to perform any particular movement, are always the most complex; and succeding artists generally discover that with fewer wheels and with fewer principles of motion than had originally been employed, the same effects may be more easily produced. The first systems, in the same manner, are always the most complex; and a particular connecting chain or principle is generally thought necessary to unite every two seemingly disjointed appearances; but it often happens, that *one great connecting principle* is afterwards found to be sufficient, to bind together all the discordant phenomena that occur in a whole species of things." This remark is strikingly applicable to the origin and progress of systems of astronomy.

439. From the visionary notions which are generally understood to have been entertained on this subject by the ancients, we are apt to imagine that they knew less than they actnally did of the truths of astronomy. But Pythagoras, who lived 500 years before the Christian era, was acquainted with many important facts in our science, and entertained many opinions respecting the system of the world which are now held to be true. Among other things well known to Pythagoras were the following:

1. The principal *Constellations.* These had begun to be formed in the earliest ages of the world. Several of them bearing the same names as at present are mentioned in the writings of

Hesiod and Homer; and the "sweet influences of the Pleiades" and the "bands of Orion," are beautifully alluded to in the book of Job.

2. *Eclipses.* Pythagoras knew both the causes of eclipses and how to predict them;* not indeed in the accurate manner now employed, but by means of the Saros (Art. 233).

3. Pythagoras had divined the true system of the world, holding that the sun and not the earth, (as was generally held by the ancients, even for many ages after Pythagoras,) is the center around which all the planets revolve, and that the stars are so many suns, each the center of a system like our own.† Among lesser things, he knew that the earth is round; that its surface is naturally divided into five Zones; and that the ecliptic is inclined to the equator. He also held that the earth revolves daily on its axis, and yearly around the sun; that the galaxy is an assemblage of small stars; and that it is the same luminary, namely, Venus, that constitutes both the morning and the evening star, whereas all the ancients before him had supposed that each was a separate planet, and accordingly the morning star was called Lucifer, and the evening star Hesperus.‡ He held also that the planets were inhabited, and even went so far as to calculate the size of some of the animals in the moon.§ Pythagoras was so great an enthusiast in music, that he not only assigned to it a conspicuous place in his system of education, but even supposed the heavenly bodies themselves to be arranged at distances corresponding to the diatonic scale, and imagined them to pursue their sublime march to notes created by their own harmonious movements, called the "music of the spheres;" but he maintained that this celestial concert, though loud and grand, is not audible to the feeble organs of man, but only to the gods.

440. With few exceptions, however, the opinions of Pythagoras on the System of the World, were founded in truth. Yet they were rejected by Aristotle and by most succeeding astronomers down to the time of Copernicus, and in their place was

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* Long's Astronomy, 2, 671.

† Library of Useful Knowledge, *History of Astronomy.*

‡ Long's Ast. 2. 673. § Ed. Encyclopædia.

substituted the doctrine of *Crystalline Spheres*, first taught by Eudoxus. According to this system, the heavenly bodies are set like gems in hollow solid orbs, composed of crystal so pellucid that no anterior orb obstructs in the least the view of any of the orbs that lie behind it. The sun and the planets have each its separate orb; but the fixed stars are all set in the same grand orb; and beyond this is another still, the *Primum Mobile*, which revolves daily from east to west, and carries along with it all the other orbs. Above the whole, spreads the *Grand Empyrean*, or third heavens, the abode of perpetual serenity.*

To account for the planetary motions, it was supposed that each of the planetary orbs as well as that of the sun, has a motion of its own eastward, while it partakes of the common diurnal motion of the starry sphere. Aristotle taught that these motions are effected by a tutelary genius of each planet, residing in it, and directing its motions, as the mind of man directs his motions.

441. On coming down to the time of Hipparchus, who flourished about 150 years before the Christian era, we meet with astronomers who acquired far more accurate knowledge of the celestial motions. Hipparchus was in possession of instruments for measuring angles, and knew how to resolve spherical triangles. He ascertained the length of the year within 6m. of the truth. He discovered the eccentricity of the solar orb, (although he supposed the sun actually to move uniformly in a circle, but the earth to be placed out of the center,) and the positions of the sun's apogee and perigee. He formed very accurate estimates of the obliquity of the ecliptic and of the precession of the equinoxes. He computed the exact period of the synodic revolution of the moon, and the inclination of the lunar orbit; discovered the motion of her node and of her line of apsides; and made the first attempts to ascertain the horizontal parallaxes of the sun and moon.

Such was the state of astronomical knowledge when Ptolemy wrote the Almagest, in which he has transmitted to us an encyclopædia of the astronomy of the ancients.

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* Long's Ast. 2. 640—Robinson's Mech. Phil. 2. 83—Gregory's Ast. 132—Playfair's Dissertation, 118.

442. The systems of the world which have been most celebrated are three—the Ptolemaic, the Tychonic, and the Copernican. We shall conclude this part of our work with a concise statement and discussion of each of these systems of the Mechanism of the Heavens.

THE PTOLEMAIC SYSTEM.

443. The doctrines of the Ptolemaic System were not originated by Ptolemy, but being digested by him out of materials furnished by various hands, it has come down to us under the sanction of his name.

According to this system, the earth is the center of the universe, and all the heavenly bodies daily revolve around it from east to west. In order to explain the planetary motions, Ptolemy had recourse to *deferents* and *epicycles*,—an explanation devised by Apollonius one of the greatest geometers of antiquity.* He conceived that, in the circumference of a circle, having the earth for its center, there moves the center of another circle, in the circumference of which the planet actually revolves. The circle surrounding the earth was called the *deferent*, while the smaller circle whose center was always in the periphery of the deferent, was called the *epicycle*. The motion in each was supposed to be uniform. Lastly, it was conceived that the motion of the center of the epicycle in the circumference of the deferent, and of the planet in that of the epicycle, are in opposite directions, the first being towards the east, and the second towards the west.

444. But these views will be better understood from a diagram. Therefore, let ABC (Fig. 74,) represent the *deferent*, E being the earth a little out of the center. Let *abc* represent the *epicycle*, having its center at *v*, on the periphery of the deferent. Conceive the circumference of the deferent to be carried about the earth every twenty four hours in the order of the letters; and at the same time, let the center *v* of the epicycle *abcd*, have a slow motion in the opposite direction, and let a body revolve in this circle in

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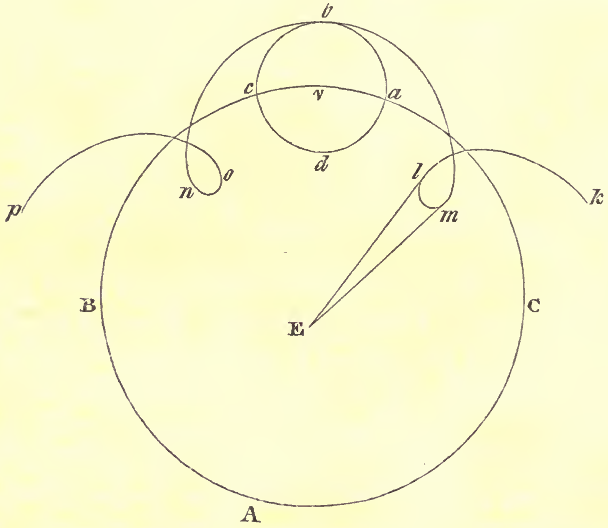
* Playfair, Dissertation Second, 119.

the direction *abcd*. Then it will be seen that the body would actually describe the looped curves *klmnop*; that it would appear

(Fig. 74.)



stationary at *l* and *m*, and at *n* and *o*; that its motion would be direct from *k* to *l*, and then retrograde from *l* to *m*; direct again from *m* to *n*, and retrograde from *n* to *o*.

445. Such a deferent and epicycle may be devised for each planet as will fully explain all its ordinary motions; but it is inconsistent with the *phases* of Mercury and Venus, which being between us and the sun on both sides of the epicycle, would present their dark sides towards us in both these positions, whereas at one of the conjunctions they are seen to shine with full face.* It is moreover absurd to speak of a geometrical center which has no bodily existence, moving around the earth on the circumference of another circle; and hence some suppose that the ancients merely assumed this hypothesis as affording a convenient geometrical representation of the phenomena,—a diagram simply, without conceiving the system to have any real existence in nature.

* Vince's Complete System, I, 96.

446. The *objections* to the Ptolemaic system, in general, are the following: First, it is a mere hypothesis, having no evidence in its favor, except that it explains the phenomena. This evidence is insufficient of itself, since it frequently happens that each of two hypotheses, directly opposite to each other, will explain all the known phenomena. But the Ptolemaic system does not even do this, as it is inconsistent with the phases of Mercury and Venus, as already observed. Secondly, now that we are acquainted with the distances of the remoter planets, and especially of the fixed stars, the *swiftness of motion* implied in a daily revolution of the starry firmament around the earth, renders such a motion wholly incredible. Thirdly, the *centrifugal force* that would be generated in these bodies, especially in the sun, renders it impossible that they can continue to revolve around the earth as a center.

These reasons are sufficient to show the absurdities of the Ptolemaic System of the World.

THE TYCHONIC SYSTEM.

447. Tycho Brahe, like Ptolemy, placed the earth in the center of the universe, and accounted for the diurnal motions in the same manner as Ptolemy had done, namely, by an actual revolution of the whole host of heaven around the earth every twenty four hours. But he rejected the scheme of deferents and epicycles, and held that the moon revolves about the earth as the center of her motions; that the sun, and not the earth, is the center of the planetary motions; and that the sun accompanied by the planets moves around the earth once a year, somewhat in the manner that we now conceive of Jupiter and his satellites as revolving around the sun.

448. The system of Tycho serves to explain all the common phenomena of the planetary motions, but it is encumbered with the same objections as those that have been mentioned as resting against the Ptolemaic system, namely, that it is a mere hypothesis; that it implies an incredible swiftness in the diurnal motions; and that it is inconsistent with the known laws of universal grav-

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itation. But if the heavens do not revolve, the earth must, and this brings us to the system of Copernicus.

THE COPERNICAN SYSTEM.

449. Copernicus was born at Thorn in Prussia in 1473. The system that bears his name was the fruit of forty years of intense study and meditation upon the celestial motions. As already mentioned, (Art. 6,) it maintains (1) That the *apparent* diurnal motions of the heavenly bodies, from east to west is owing to the *real* revolution of the earth on its own axis from west to east; and (2) That the sun is the center around which the earth and planets all revolve from west to east. It rests on the following arguments:

450. First, *the earth revolves on its own axis.*

1. Because this supposition is vastly more *simple.*

2. It is agreeable to *analogy*, since all the other planets that afford any means of determining the question, are seen to revolve on their axes.

3. The *spheroidal figure* of the earth, is the figure of equilibrium, that results from a revolution on its axis.

4. The *diminished weight* of bodies at the equator, indicates a centrifugal force arising from such a revolution.

5. Bodies let fall from a high eminence, fall *eastward of their base,* indicating that when farther from the center of the earth they were subject to a greater velocity, which in consequence of their inertia, they do not entirely lose in descending to the lower level.*

451. Secondly, *the planets, including the earth, revolve about the sun.*

1. The *phases* of Mercury and Venus are precisely such, as would result from their circulating around the sun in orbits within that of the earth; but they are never seen in opposition, as they would be if they circulated around the earth.

2. The superior planets do indeed revolve around the earth; but they also revolve around the sun, as is evident from their

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* Biot.

phases and from the known dimensions of their orbits; and that the sun and not the earth, is the *center* of their motions, is inferred from the greater symmetry of their motions as referred to the sun than as referred to the earth, and especially from the laws of gravitation which forbid our supposing that bodies so much larger than the earth, as some of these bodies are, can circulate permanently around the earth, the latter remaining all the while at rest.

3. The annual motion of *the earth* itself is indicated also by the most conclusive arguments. For, first, since all the planets with their satellites, and the comets, revolve about the sun, *analogy* leads us to infer the same respecting the earth and its satellite. Secondly, The motions of the satellites, as those of Jupiter and Saturn, indicate that it is a law of the solar system that the smaller bodies revolve about the larger. Thirdly, on the supposition that the earth performs an annual revolution around the sun, it is embraced along with the planets, in Kepler's law, that the squares of the times are as the cubes of the distances; otherwise, it forms an exception, and the only known exception to this law. Lastly, the aberration of light affords a sensible proof of the motion of the earth, since that phenomenon indicates both a progressive motion of light, and a motion of the earth from west to east. (Art. 195).

452. It only remains to inquire, whether there subsist higher orders of relations between the stars themselves.

The revolutions of the *binary stars* (Art. 424) afford conclusive evidence of at least subordinate systems of suns, governed by the same laws as those which regulate the motions of the solar system. The *nebulæ* also compose peculiar systems, in which the members are evidently bound together by some common relation.

In these marks of organization,—of stars associated together in clusters,—of sun revolving around sun,—and of nebulæ disposed in regular figures, we recognize different members of some grand system, links in one great chain that binds together all parts of the universe; as we see Jupiter and his satellites combined in one subordinate system, and Saturn and his satellites in another, —each a vast kingdom, and both uniting with a number of other individual parts to compose an empire still more vast.

453. This fact being now established, that the stars are immense bodies like the sun, and that they are subject to the laws of gravitation, we cannot conceive how they can be preserved from falling into final disorder and ruin, unless they move in harmonious concert like the members of the solar system. Otherwise, those that are situated on the confines of creation, being retained by no forces from without, while they are subject to the attraction of all the bodies within, must leave their stations, and move inward with accelerated velocity, and thus all the bodies in the universe would at length fall together in the common center of gravity. The immense distance at which the stars are placed from each other, would indeed delay such a catastrophe; but such must be the ultimate tendency of the material world, unless sustained in one harmonious system by nicely adjusted motions.* To leave entirely out of view our confidence in the wisdom and preserving goodness of the Creator, and reasoning merely from what we know of the stability of the solar system, we should be justified in inferring, that other worlds are not subject to forces which operate only to hasten their decay, and to involve them in final ruin.

We conclude, therefore, that the material universe is one great system; that the combination of planets with their satellites constitutes the first or lowest order of worlds; that next to these planets are linked to suns; that these are bound to other suns, composing a still higher order in the scale of being; and, finally, that all the different systems of worlds, move around their common center of gravity.

* Robison's Physical Astronomy.

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