THE

LADY'S AND GENTLEMAN'S DIARY,

FOR THE YEAR OF OUR LORD

1858,

Being the second after Bissextile.

DESIGNED PRINCIPALLY FOR THE AMUSEMENT AND INSTRUCTION OF

STUDENTS IN MATHEMATICS:

COMPRISING

MANY USEFUL AND ENTERTAINING PARTICULARS,

INTERESTING TO ALL PERSONS ENGAGED IN THAT DELIGHTFUL PURSUIT.

THE ONE HUNDRED AND FIFTY-FIFTH ANNUAL NUMBER.

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CHRONOLOGICAL NOTES, &c. IN 1858.

Dominical Letter	C	Sundays after Epiphany	3	Easter Day	Apr. 4
Golden Number	16	„ „ Trinity	25	Rogation Sunday	May 9
Epact	15	Septuages. Sund.	Jan. 31	Ascension Day	May 13
Solar Cycle	19	Shrove Sunday	Feb. 14	Whit Sunday	May 23
Number of Direction	14	Lent begins	Feb. 17	Trinity Sunday	May 30
Roman Indiction	1	1st Sund. in Lent	Feb. 21	Mah. year 1275 beg.	Aug. 11
Julian Period	6571	Midlent Sunday	Mar. 14	Jew. „ 5619 „	Sept. 9
Year of the Dionysian	187	Good Friday	Apr. 2	Advent Sunday	Nov. 28

ECLIPSES, &c.

This year there will be FOUR eclipses; Two of SUN, and Two of MOON.

I. *February 27th.—A partial and visible eclipse of the* MOON; magnitude about 4 digits. It begins in the evening at 9h. 10m.; greatest obscuration at 10h. 14m.; ends at 11h. 17m.

$$\left.\begin{array}{l}\text{First}\\ \text{Last}\end{array}\right\}\text{ contact }\left\{\begin{array}{l}154^\circ\\ 104\end{array}\right\}\text{ from the Moon's Vertex towards the West or right hand.}$$

This eclipse will be visible throughout Europe, Asia, and Africa, also to a large western portion of Australia, and on the Atlantic, Indian, and Great Southern Oceans. The moon will likewise rise eclipsed to the more eastern parts of North and South America.

II. *March 15th.—An annular and visible eclipse of the* SUN, of which the central and annular phase will traverse the south of England in a north-east direction, nearly passing through Devizes and Peterborough, and it will be almost total throughout Great Britain. For any place whose north latitude expressed in degrees, is $50^\circ + L$, and west longitude, in minutes of time, is M, the times of beginning and ending may be obtained by means of the following easy formulæ:

$$\left.\begin{array}{llll} & \text{h} & \text{m} \quad\;\; \text{m} & \;\;\text{m} \\ \text{Beginning at} & 11 & 39{\cdot}7 + 1{\cdot}12\,L - 0{\cdot}27\,M \\ \text{Ending at} & 2 & 17{\cdot}3 + 0{\cdot}06\,L - 0{\cdot}24\,M \end{array}\right\}\text{ Greenwich time.}$$

Also for any given meridian so estimated, the latitude where the eclipse will be central and annular $= 52^\circ\, 46'{\cdot}5 - 10'{\cdot}77\, M$.

At Greenwich the eclipse begins in the morning at 11h. 41m.; greatest obscuration (very nearly total, excepting only the upper right hand edge of the solar disc) at 1h. 0m.; eclipse ends at 2h. 17m.

$$\left.\begin{array}{l}\text{First}\\ \text{Last}\end{array}\right\}\text{ contact }\left\{\begin{array}{l}124^\circ\\ 27\end{array}\right\}\text{ from the Sun's Vertex towards the }\left\{\begin{array}{l}\text{West or right hand.}\\ \text{East or left hand.}\end{array}\right.$$

This eclipse will be visible to the whole of Europe, the north-western parts of Africa, the whole of Turkey and Tartary, the northern parts of Persia, the eastern parts of North America, including the West India Islands, Canada, Hudson's Bay, Greenland, Spitzbergen, and an extensive surface of the Atlantic Ocean.

III. *August 24th.—A partial eclipse of the* MOON, but not visible in this country. It will be principally visible in Asia and Australia.

IV. *September 7th.—A total eclipse of the* SUN, also invisible. It will be chiefly visible in South America and the West India Islands.

MERCURY will be visible in the mornings, before the Sun rises, near the eastern horizon, about *February* 7, *June* 7, and *September* 30; and in the evenings, soon after Sunset, near the western horizon, about *April* 20, *August* 18, and *December* 12.

VENUS will be a *Morning Star* until *February* 28; then an *Evening Star* until *December* 13; and afterwards a *Morning Star.*

JUPITER will be an *Evening Star* until *May* 19; then a *Morning Star* until *December* 8; and afterwards an *Evening Star.*

MARS will be in *opposition* to the Sun on *May* 15; but he will not be very favorable for observation on account of his extreme southern declination.

SATURN'S RINGS are visible. The planet will be in *opposition* to the Sun on *January* 14, and very favorable for telescopic observations during the winter months, both at the beginning and the end of the year. The major and minor axes of the rings will appear nearly in the proportion of 3 to 1.

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Nº 155.	JANUARY, 31 DAYS.
Last Quarter	6th, 47m. past Midnight.
New Moon	15th, 32m. past 5 Morn.
First Quarter	22d, 57m. past 4 Aftern.
Full Moon	29th, 11m. past 9 Morn.

SUN ENTERS

19d. 18h. 56m.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉rise	☉set	☉decl.	☽ r.& s.	☽a
1	F	**Circumcision**	8h 9′	3h 59′	23° s 1′	5h a 56′	16
2	S		8 8	4 0	22 56	7 24	17
3	C	**2d Sun. aft. Christmas**	8 8	4 1	50	8 50	18
4	M	♄ rises 4 53 aftern.	8 8	4 2	44	10 9	19
5	Tu		8 8	4 4	37	11 25	20
6	W	**Epiphany**: 12th day	8 7	4 5	30	morn.	21
7	Th		8 7	4 6	23	0 39	22
8	F	*Lucian*	8 7	4 7	15	1 52	23
9	S	♃ sets 2 13 morn.	8 6	4 9	7	3 5	24
10	C	**1st Sun. af. Epiphany**	8 6	4 10	21 58	4 18	25
11	M	Pl. Mond.: Hil. Term	8 5	4 12	49	5 30	26
12	Tu	[beg.	8 4	4 13	39	6 36	27
13	W	*Hilary:* Camb. T. beg.	8 4	4 15	29	7 32	28
14	Th	Oxford Term begins	8 3	4 16	19	8 14	29
15	F		8 2	4 18	8	sets	N
16	S	♂ rises 1 23 morn.	8 1	4 19	20 56	5 a 35	1
17	C	**2d Sun. af. Epiphany**	8 0	4 21	45	6 54	2
18	M	*Prisca*	7 59	4 22	33	8 12	3
19	Tu		7 58	4 24	20	9 31	4
20	W	*Fabian*	7 57	4 26	7	10 51	5
21	Th	*Agnes*	7 56	4 27	19 54	morn.	6
22	F	*Vincent*	7 55	4 29	41	0 13	7
23	S		7 54	4 31	27	1 39	8
24	C	**3d Sun. aft. Epiphany**	7 52	4 33	12	3 8	9
25	M	**Convers. of St. Paul**	7 51	4 34	18 58	4 38	10
26	Tu		7 50	4 36	43	5 56	11
27	W	♀ rises 7 34 morn.	7 49	4 38	27	6 57	12
28	Th	☿ rises 6 24 morn.	7 47	4 40	12	7 41	13
29	F		7 46	4 41	17 56	rises	F
30	S	**K. Chas. I. mart.** 1649	7 44	4 43	39	6 a 20	15
31	C	**Septuagesima Sunday**	7 43	4 45	23	7 42	16

Day.	Length of D.	Day Inc.	D. breaks	Tw. ends	Sun East.	Cl. bef. Sun.	☉Semidiameter
1	7h. 50m.	0 6	6 m 2	6 a 5	4 m 45	3′ 51″	16′ 18″
6	58	14	2	10	49	6 8	18
11	8 7	23	1	15	54	8 14	18
16	18	34	5 59	21	59	10 5	18
21	31	47	55	28	5 5	11 38	17
26	46	1 2	50	35	10	12 52	17

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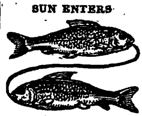
SUN ENTERS



18d. 9h. 37m.

FEBRUARY, 28 DAYS. 1858

Last Quarter ...... 5th, 16m. past 9 Aftern.
New Moon .... 13th, 12m. past 10 Aftern.
First Quarter ...... 20th, 58m. past Midnight.
Full Moon .... 27th, 5m. past 10 Aftern.

D. M.	D. W.	SUNDAYS, HOLIDAYS, &c. *Planets rise and set.*	*Sun rises*	*Sun sets*	*Sun's declin.*	*☾ rises & sets*	*☾'s age*
1	M	Hilary Term ends	7h 41′	4h 47′	17° s 6′	9h a 2′	17
2	Tu	Purif.: Candl.-day	7 40	4 49	16 49	10 18	18
3	W	*Blase*	7 38	4 50	31	11 33	19
4	Th	♄ sets 6 50 morn.	7 37	4 52	13	morn.	20
5	F	*Agatha*	7 35	4 54	15 55	0 47	21
6	S		7 33	4 56	37	2 3	22
7	C	Sexagesima Sunday	7 31	4 58	18	3 16	23
8	M		7 30	5 0	14 59	4 25	24
9	Tu		7 28	5 1	40	5 26	25
10	W	Qu. Victoria mar. 1840	7 26	5 3	21	6 12	26
11	Th		7 24	5 5	1	6 46	27
12	F	♃ sets 0 16 morn.	7 22	5 7	13 41	7 12	28
13	S		7 21	5 9	21	sets	N
14	C	Shrove Sund.: *Valen-*	7 19	5 10	1	5 a 58	1
15	M	[*tine*	7 17	5 12	12 40	7 18	2
16	Tu	Shrove Tuesday	7 15	5 14	19	8 38	3
17	W	Lent beg.: Ash Wed.	7 13	5 16	11 59	10 0	4
18	Th	Camb. Term div. noon	7 11	5 18	37	11 26	5
19	F		7 9	5 20	16	morn.	6
20	S	♂ rises 0 37 morn.	7 7	5 22	10 55	0 54	7
21	C	1st Sunday in Lent	7 5	5 23	33	2 23	8
22	M		7 3	5 25	11	3 45	9
23	Tu		7 1	5 27	9 49	4 50	10
24	W	Ember Week: St.	6 58	5 29	27	5 38	11
25	Th	[Matthias	6 56	5 31	5	6 10	12
26	F	♀ sets 5 24 aftern.	6 54	5 32	8 43	6 32	13
27	S	☿ rises 6 25 morn.	6 52	5 34	20	rises	F
28	C	2d Sunday in Lent	6 50	5 36	7 57	6 a 37	15

Day.	Length of D.	Day Inc.	D. breaks	Tw. ends	Sun East.	Cl. bef. Sun.	☉ Semidiameter
1	9h. 5m.	1 21	5 m 44	6 a 44	5 m 17	13′ 53″	16′ 16″
6	23	39	37	52	23	14 22	15
11	41	57	29	7 0	29	14 31	14
16	10 0	2 16	20	8	34	14 21	13
21	19	35	11	17	40	13 53	12
26	38	54	1	25	45	13 9	11

MARCH, 31 DAYS.

Last Quarter .... 7th, 10m. past 6 Aftern.
New Moon .... 15th, 12m. past 12 Noon.
First Quarter .... 22d, 42m. past 7 Morn.
Full Moon .... 29th, 7m. past 12 Noon.

SUN ENTERS

20d. 9h. 33m.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉ rise	☉ set	☉ decl.	☽ r. & s.	☽ a
1	M	*David:* Least twilight	6h 48'	5h 38'	7° s 35'	7h a 54'	16
2	Tu	*Chad*	6 46	5 39	12	9 10	17
3	W		6 43	5 41	6 49	10 27	18
4	Th	♄ sets 4 55 morn.	6 41	5 43	26	11 43	19
5	F	♃ sets 11 9 aftern.	6 39	5 45	3	morn.	20
6	S		6 37	5 46	5 40	0 59	21
7	ℭ	**3d Sunday in Lent:**	6 35	5 48	16	2 10	22
8	M	[*Perpetua*	6 33	5 50	4 53	3 14	23
9	Tu		6 30	5 51	30	4 7	24
10	W	♂ rises 0 3 morn.	6 28	5 53	6	4 46	25
11	Th		6 26	5 55	3 43	5 14	26
12	F	*Gregory*	6 23	5 57	19	5 36	27
13	S		6 21	5 58	2 55	5 51	28
14	ℭ	**4th, or Midl. Sunday**	6 19	6 0	32	6 4	29
15	M		6 17	6 2	8	sets	N
16	Tu		6 14	6 4	1 44	7 a 42	1
17	W	*St. Patrick* [*K. W. S.*	6 12	6 5	21	9 8	2
18	Th	**Pr. Lou. b.** 1848: *Edw.*	6 10	6 7	0 57	10 38	3
19	F	Equal day and night	6 8	6 9	33	morn.	4
20	S		6 5	6 10	0 s 9	0 8	5
21	ℭ	**5th Sunday in Lent:**	6 3	6 12	0 N 14	1 34	6
22	M	[*Benedict*	6 1	6 14	38	2 44	7
23	Tu	♀ sets 6 44 aftern.	5 58	6 15	1 2	3 37	8
24	W		5 56	6 17	25	4 13	9
25	Th	**Lady-Day**	5 54	6 19	49	4 37	10
26	F	Camb. Term ends	5 51	6 20	2 12	4 54	11
27	S	Oxford Term ends	5 49	6 22	36	5 8	12
28	ℭ	**Palm Sunday**	5 47	6 24	59	5 20	13
29	M		5 45	6 25	3 23	rises	F
30	Tu	☿ sets 6 54 aftern.	5 42	6 27	46	8 a 6	15
31	W		5 40	6 29	4 9	9 23	16

Day.	Length of D.	Day Inc.	D. breaks	Tw. ends	Sun East.	Cl. bef. Sun.	☉ Semidiameter
1	10h. 50m.	3 6	4 m 55	7 a 30	5 m 48	12' 35"	16' 10"
6	11 9	25	44	39	53	11 29	9
11	29	45	32	48	58	10 13	7
16	49	4 5	20	58	6 3	8 50	6
21	12 9	25	7	8 8	8	7 21	5
26	29	45	3 54	18	13	5 49	3

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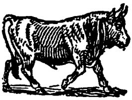
SUN ENTERS



19d. 21h. 40m.

APRIL, 30 DAYS. 1858

Last Quarter......... 6th, 43m. past 1 Aftern.
New Moon .... 13th, 15m. past 11 Aftern.
First Quarter ...... 20th, 26m. past 2 Aftern.
Full Moon .... 28th, 56m. past 2 Morn.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉ rise	☉ set	☉ decl.	☽ r. & s.	☽ a
1	Th	*Maundy Thursday*	5h 38′	6h 30′	4° N 32′	10h a 39′	17
2	F	**Good Friday**	5 36	6 32	56	11 53	18
3	S	*Rich. Bp. of Chich.*	5 33	6 34	5 19	morn.	19
4	ℭ	**East. S.**: *St. Ambrose*	5 31	6 35	41	1 1	20
5	M	**Easter Monday**	5 29	6 37	6 4	1 59	21
6	Tu	**East. Tues.**: Old L.D.	5 27	6 39	27	2 43	22
7	W	**Pr. Leopold b.** 1853	5 24	6 40	50	3 14	23
8	Th		5 22	6 42	7 12	3 38	24
9	F	♄ sets 2 33 morn.	5 20	6 44	34	3 56	25
10	S		5 18	6 45	57	4 10	26
11	ℭ	**1st, or Low Sunday**	5 15	6 47	8 19	4 22	27
12	M		5 13	6 49	41	4 34	28
13	Tu	[and Cam.T.beg.	5 11	6 50	9 3	sets	N
14	W	**Ps. Beat. b.** 1857: Ox.	5 9	6 52	24	8 a 10	1
15	Th	Easter Term begins	5 7	6 54	46	9 44	2
16	F		5 4	6 55	10 7	11 16	3
17	S	♃ sets 9 10 aftern.	5 2	6 57	28	morn.	4
18	ℭ	**2d Sunday aft. Easter**	5 0	6 59	49	0 34	5
19	M	*Alphege*	4 58	7 0	11 10	1 34	6
20	Tu		4 56	7 2	31	2 16	7
21	W	♂ rises 9 43 aftern.	4 54	7 4	51	2 42	8
22	Th		4 52	7 5	12 12	3 3	9
23	F	*St. George*	4 50	7 7	32	3 16	10
24	S		4 48	7 8	52	3 28	11
25	ℭ	**3d S.a.Est.:St.Mk.:**	4 46	7 10	13 11	3 39	12
26	M	[**Ps. Alice b.**	4 44	7 12	31	3 50	13
27	Tu		4 42	7 13	50	4 2	14
28	W	♀ sets 8 40 aftern.	4 40	7 15	14 9	rises	F
29	Th	☿ sets 9 3 aftern.	4 38	7 17	28	9 a 38	16
30	F		4 36	7 18	46	10 48	17

Day.	Length of D.	Day Inc.	D. breaks	Tw. ends	Sun East.	Cl. bef. Sun.	☉ Semidiameter
1	12h. 52m	5 8	3 m 37	8 a 31	6 m 18	3′ 57″	16′ 2″
6	13 12	28	23	42	23	2 28	0
11	31	47	8	54	28	1 4	15 59
16	51	6 7	2 53	9 7	32	0 aft. 13	58
21	14 10	26	37	20	37	1 21	56
26	28	44	21	35	42	2 18	55

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N° 155. MAY, 31 DAYS.

Last Quarter ...... 6th, 40m. past 6 Morn.
New Moon .... 13th, 47m. past 7 Morn.
First Quarter ...... 19th, 20m. past 10 Aftern.
Full Moon .... 27th, 5m. past 6 Aftern.

SUN ENTERS

20d. 21h. 44m.

D.	W.	SUNDAYS, HOLIDAYS, &c.	⊙ rise	⊙ set	⊙ decl.	☾ r. & s.	☾ a
1	S	St. Ph. & Jas.: Pr. Ar.	4h34′	7h20′	15°N 4′	11h a 50′	18
2	C	4th S. a. East. [b. 1850	4 32	7 21	22	morn.	19
3	M	*Inven. of the Cross*	4 30	7 23	40	0 39	20
4	Tu	♄ sets 0 59 morn.	4 29	7 25	58	1 14	21
5	W		4 27	7 26	16 15	1 41	22
6	Th	*John Evan. à P. Lat.*	4 25	7 28	32	2 0	23
7	F		4 23	7 30	49	2 15	24
8	S	Easter Term ends	4 22	7 31	17 5	2 27	25
9	C	Rogation Sunday	4 20	7 33	21	2 39	26
10	M		4 18	7 34	37	2 51	27
11	Tu	♃ sets 8 7 aftern.	4 16	7 36	53	3 3	28
12	W		4 15	7 37	18 8	3 19	29
13	Th	Asc.: Holy Th.: Old	4 13	7 39	23	sets	N
14	F	[May-day	4 12	7 40	38	10 a 11	1
15	S	♂ sets 4 18 morn.	4 10	7 42	52	11 21	2
16	C	Sund. aft. Ascension	4 9	7 43	19 6	morn.	3
17	M		4 7	7 45	20	0 12	4
18	Tu		4 6	7 46	33	0 45	5
19	W	*Dunstan*	4 5	7 48	46	1 8	6
20	Th	♀ sets 9 44 aftern.	4 3	7 49	59	1 24	7
21	F	[T. ends	4 2	7 51	20 11	1 36	8
22	S	Trin. Term beg.: Oxf.	4 1	7 52	23	1 47	9
23	C	Whit Sunday [1819	3 59	7 53	35	1 58	10
24	M	Whit M.: Q. Vict. b.	3 58	7 55	46	2 9	11
25	Tu	Wh. T.: Ps. H. b. 1846	3 57	7 56	57	2 22	12
26	W	Em. W: Ox. T. b.: *Aug.*	3 56	7 57	21 8	2 38	13
27	Th	K. Han. b.: *Ven. Bede:*	3 55	7 59	18	rises	F
28	F	[Cam. T. d. n.	3 54	8 0	28	9 a 41	15
29	S	K. Chas. II. rest. 1660	3 53	8 1	37	10 34	16
30	C	Trinity Sunday	3 52	8 2	47	11 13	17
31	M	☿ rises 3 17 morn.	3 51	8 3	55	11 43	18

Day.	Length of D.	Day Inc.	D. breaks	Tw. ends	Sun East.	Cl. aft. Sun.	⊙ Semidiameter
1	14h. 46m.	7 2	2 m 3	9 a 51	6 m 46	3′ 3″	15′ 54″
6	15 3	19	1 45	10 8	51	3 35	53
11	19	35	24	28	55	3 52	52
16	34	50	1	51	7 0	3 54	51
21	48	8 4	0 28	11 25	4	3 42	50
26	16 1	17	No real Night.		8	3 18	49

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SUN ENTERS

21d. 6h. 13m.

JUNE, 30 DAYS. 1858

Last Quarter .... 4th, 21m. past 8 Aftern.
New Moon .... 11th, 46m. past 2 Aftern.
First Quarter .... 18th, 14m. past 8 Morn.
Full Moon .... 26th, 13m. past 9 Morn.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉ rise	☉ set	☉ decl.	☾ r. & s.	☾ a
1	Tu	*Nicomede*	3h 50'	8h 4'	22° N 4'	morn.	19
2	W		3 50	8 5	12	0h 4'	20
3	Th	*Corpus Christi*	3 49	8 6	19	0 20	21
4	F		3 48	8 7	26	0 34	22
5	S	*Boniface*	3 48	8 8	33	0 45	23
6	C	**1st Sund. aft. Trinity**	3 47	8 9	40	0 56	24
7	M		3 47	8 10	46	1 8	25
8	Tu	♄ sets 10 47 aftern.	3 46	8 11	51	1 23	26
9	W	♃ rises 3 2 morn.	3 46	8 12	57	1 42	27
10	Th		3 45	8 13	23 1	2 8	28
11	F	**St. Barnabas**	3 45	8 14	6	sets	N
12	S	Trinity Term ends	3 45	8 14	10	10 a 1	1
13	C	**2d Sund. aft. Trinity**	3 44	8 15	14	10 43	2
14	M		3 44	8 15	17	11 10	3
15	Tu	♂ sets 1 47 morn.	3 44	8 16	20	11 29	4
16	W		3 44	8 17	22	11 43	5
17	Th	*St. Alban*	3 44	8 17	24	11 55	6
18	F	♀ sets 10 15 aftern.	3 44	8 17	25	morn.	7
19	S	[acc.	3 44	8 18	27	0 6	8
20	C	**3d S. af. Tr.: Qu. Vic.**	3 44	8 18	27	0 17	9
21	M	**Queen Victoria proc.:**	3 44	8 18	28	0 29	10
22	Tu	[Long. d.	3 45	8 19	27	0 44	11
23	W		3 45	8 19	27	1 4	12
24	Th	**Mids.-day: Nat. John**	3 45	8 19	26	1 31	13
25	F	**[Bapt.**	3 45	8 19	25	2 7	14
26	S	☿ rises 2 50 morn.	3 46	8 19	23	rises	F
27	C	**4th Sun. aft. Trinity**	3 46	8 19	21	9 a 46	16
28	M	**Qu. Victoria cor.** 1838	3 47	8 19	18	10 10	17
29	Tu	**St. Peter**	3 47	8 19	15	10 27	18
30	W		3 48	8 18	12	10 40	19

Day.	Length of D.	Day Inc.	D. breaks	Tw. ends	Sun East.	Cl. aft. Sun.	☉ Semidiameter
1	16h. 14m.	8 30			7 m 13	2' 33"	15' 48"
6	23	39	No real Night		16	1 44	47
11	29	45	but constant		19	0 46	47
16	33	49	Day, or Twi-		21	0bef. 4	47
21	34	50	light.		22	1 21	46
26	33	0dec. 1			23	2 25	46

N° 155. JULY, 31 DAYS.

Last Quarter	......	4th, 43m. past	6 Morn.
New Moon	......	10th, 24m. past	9 Aftern.
First Quarter	......	17th, 39m. past	8 Aftern.
Full Moon	......	25th, 3m. past	Midnight.

SUN ENTERS

22d. 17h. 7m.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉ rise	☉ set	☉ decl.	☾ r. & s.	☾ a
1	Th		3h 49'	8h 18'	23° N 8'	10h a 52'	20
2	F	*Visitation V. Mary*	3 49	8 18	4	11 4	21
3	S	Dog-days begin	3 50	8 17	22 59	11 15	22
4	ℭ	5th Sun. af. Tr.: *Trs.*	3 51	8 17	54	11 28	23
5	M	[*St. Martin*	3 52	8 17	49	11 42	24
6	Tu	Old Midsummer-day	3 53	8 16	43	morn.	25
7	W	*Thomas à Becket*	3 53	8 16	37	0 5	26
8	Th	♄ sets 9 0 aftern.	3 54	8 15	30	0 36	27
9	F	Cambridge Term ends	3 55	8 14	23	1 26	28
10	S	Oxford Term ends	3 56	8 14	16	sets	N
11	ℭ	6th Sund. aft. Trinity	3 57	8 13	8	9 a 7	1
12	M		3 58	8 12	0	9 31	2
13	Tu	♃ rises 1 11 morn.	4 0	8 11	21 52	9 47	3
14	W		4 1	8 10	43	10 1	4
15	Th	*St. Swithin*	4 2	8 9	33	10 13	5
16	F		4 3	8 8	24	10 24	6
17	S		4 4	8 7	14	10 36	7
18	ℭ	7th Sun. aft. Trinity	4 5	8 6	4	10 51	8
19	M		4 7	8 5	20 53	11 8	9
20	Tu	*Margaret*	4 8	8 4	42	11 32	10
21	W		4 9	8 3	30	morn.	11
22	Th	*Magdalene*	4 11	8 2	19	0 5	12
23	F	♂ sets 11 19 aftern.	4 12	8 0	7	0 51	13
24	S		4 13	7 59	19 54	1 49	14
25	ℭ	8th S. a. Tr.: St. Jas.	4 15	7 58	41	2 58	15
26	M	*St. Anne*	4 16	7 56	28	rises	F
27	Tu		4 18	7 55	15	8 a 48	17
28	W		4 19	7 53	1	9 0	18
29	Th	♀ sets 9 15 aftern.	4 21	7 52	18 47	9 11	19
30	F	☿ sets 8 41 aftern.	4 22	7 50	33	9 22	20
31	S		4 23	7 49	18	9 33	21

Day.	Length of D.	Day dec.	D.breaks	Tw. ends	Sun East.	Cl. bef. Sun.	☉ Semidiameter
1	16h. 29m.	0 5			7 m 23	3' 26"	15' 46"
6	23	11			22	4 20	46
11	15	19	No real Night.		20	5 7	46
16	5	29			18	5 42	46
21	15 53	41			15	6 4	47
26	40	54	0 m 59	11 a 13	12	6 12	47

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AUGUST, 31 DAYS. 1858

Last Quarter .... 2d, 21m. past 2 Aftern.
New Moon .... 9th, 54m. past 4 Morn.
First Quarter .... 16th, 42m. past 11 Morn.
Full Moon .... 24th, 12m. past 2 Aftern.
Last Quarter .... 31st, 16m. past 8 Aftern.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉ rise	☉ set	☉ decl.	☾ r. & s.	☾ a
1	C	9th Sund. af. Trinity:	4h25′	7h47′	18°N 3′	9ha48′	22
2	M	[Lammas-day	4 26	7 46	17 48	10 8	23
3	Tu	♄ rises 3 43 morn.	4 28	7 44	33	10 33	24
4	W		4 29	7 42	17	11 12	25
5	Th	[*Transfig.*	4 31	7 41	1	morn.	26
6	F	Pr. Alfred born 1844:	4 33	7 39	16 44	0 12	27
7	S	*Name of Jesus*	4 34	7 37	28	1 29	28
8	C	10th Sun. aft. Trinity	4 36	7 35	11	3 0	29
9	M	(Shooting stars)	4 37	7 33	15 53	sets	N
10	Tu	*St. Lawrence*	4 39	7 32	36	8 a 6	1
11	W	Dog-days end	4 40	7 30	18	8 18	2
12	Th	♃ rises 11 29 aftern.	4 42	7 28	0	8 30	3
13	F		4 43	7 26	14 42	8 41	4
14	S	[*B. V. M.*	4 45	7 24	24	8 55	5
15	C	11th S. a. T.: *Assump.*	4 47	7 22	5	9 12	6
16	M		4 48	7 20	13 46	9 34	7
17	Tu	Ds. Kent born 1786	4 50	7 18	27	10 3	8
18	W		4 51	7 16	8	10 44	9
19	Th	♂ sets 10 5 aftern.	4 53	7 14	12 49	11 38	10
20	F	♀ sets 8 24 aftern.	4 55	7 12	29	morn.	11
21	S		4 56	7 10	9	0 44	12
22	C	12th Sun. aft. Trinity	4 58	7 8	11 49	1 56	13
23	M		5 0	7 6	29	3 14	14
24	Tu	St. Bartholomew	5 1	7 4	8	rises	F
25	W		5 3	7 1	10 48	7 a 20	16
26	Th	Pr. Albert born 1819	5 4	6 59	27	7 30	17
27	F	☿ sets 7 19 aftern.	5 6	6 57	6	7 43	18
28	S	*St. Augustine*	5 7	6 55	9 45	7 56	19
29	C	13th S. af. Tr.: *St. Jn.*	5 9	6 53	23	8 13	20
30	M	[*Bapt. beh.*	5 11	6 51	2	8 36	21
31	Tu		5 12	6 48	8 40	9 10	22

Day.	Length of D.	Day dec.	D. breaks	Tw. ends	Sun East.	Cl. bef. Sun.	☉ Semidiameter
1	15h. 22m.	1 12	1 m 30	10 a 42	7 m 6	6′ 2″	15′ 48″
6	6	28	50	21	1	5 38	48
11	14 49	45	2 8	2	6 55	4 59	49
16	32	2 2	24	9 45	49	4 5	50
21	14	20	38	28	43	2 59	51
26	13 55	39	51	12	36	1 40	52

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N° 155. SEPTEMBER, 30 DAYS.

New Moon .... 7th, 14m. past 2 Aftern.
First Quarter ...... 15th, 16m. past 5 Morn.
Full Moon .... 23d, 20m. past 3 Morn.
Last Quarter ...... 30th, 51m. past 1 Morn.

SUN ENTERS

22d. 20h. 24m.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉ rise	☉ set	☉ decl.	☾ r.&s.	☾ a
1	W	*Giles*	5h 14′	6h 46′	8° N 19′	9h a 59′	23
2	TH	London bt. 1666, o.s.	5 15	6 44	7 57	11 8	24
3	F	♄ rises 2 2 morn.	5 17	6 42	35	morn.	25
4	S		5 19	6 40	13	0 31	26
5	ℭ	14th S. af. Trin.: Old	5 20	6 37	6 50	2 3	27
6	M	[Bartholomew	5 22	6 35	28	3 34	28
7	Tu	*Enurchus*	5 23	6 33	6	sets	N
8	W	*Nativity B. V. M.*	5 25	6 30	5 43	6 a 36	1
9	TH		5 27	6 28	20	6 47	2
10	F	♃ rises 9 48 aftern.	5 28	6 26	4 58	7 0	3
11	S		5 30	6 24	35	7 16	4
12	ℭ	15th Sun. aft. Trinity	5 31	6 21	12	7 36	5
13	M		5 33	6 19	3 49	8 2	6
14	Tu	*Holy Cross*	5 35	6 17	26	8 38	7
15	W	Ember Week	5 36	6 14	3	9 26	8
16	TH		5 38	6 12	2 40	10 29	9
17	F	*Lambert*	5 39	6 10	16	11 39	10
18	S	Geo. I. and II. landed	5 41	6 8	1 53	morn.	11
19	ℭ	16th Sun. aft. Trinity	5 42	6 5	30	0 54	12
20	M		5 44	6 3	7	2 11	13
21	Tu	St. Matthew	5 46	6 1	0 43	3 29	14
22	W		5 47	5 58	0 N 20	4 46	15
23	TH	♂ sets 9 1 aftern.	5 49	5 56	0 S 4	rises	F
24	F	♀ sets 7 2 aftern.	5 51	5 54	27	6 a 4	17
25	S	Equal day and night	5 52	5 51	50	6 19	18
26	ℭ	17th Sun. af. Tr.: *St.*	5 54	5 49	1 14	6 42	19
27	M	[*Cyprian*	5 55	5 47	37	7 11	20
28	Tu	☿ rises 4 14 morn.	5 57	5 44	2 1	7 56	21
29	W	Michaelmas-Day	5 59	5 42	24	8 58	22
30	TH	*St. Jerome*	6 0	5 40	47	10 16	23

Day.	Length of D.	Day dec.	D. breaks	Tw. ends	Sun East.	Cl. aft. Sun.	☉ Semidiameter
1	13h. 32m.	3 2	3 m 6	8 a 53	6 m 27	0′ 7″	15′ 54″
6	13	21	18	38	19	1 43	55
11	12 54	40	29	24	11	3 25	56
16	35	59	40	10	4	5 10	57
21	15	4 19	50	7 57	5 56	6 56	59
26	11 55	39	59	44	48	8 40	16 0

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SUN ENTERS

23d. 4h. 42m.

OCTOBER, 31 DAYS. 1858

New Moon .... 7th, 7m. past 2 Morn.
First Quarter ...... 14th, 42m. past Midnight.
Full Moon .... 22d, 19m. past 3 Aftern.
Last Quarter ...... 29th, 33m. past 8 Morn.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉ rise	☉ set	☉ decl.	☾ r. & s.	☾ a
1	F	*Remigius*	6h 2′	5h 37′	3° s 11′	11h a 44′	24
2	S		6 4	5 35	34	morn.	25
3	ℭ	18th Sun. aft. Trinity	6 5	5 33	57	1 12	26
4	M		6 7	5 31	4 20	2 38	27
5	Tu	♄ rises 0 14 morn.	6 9	5 28	44	4 2	28
6	W	*Faith*	6 10	5 26	5 7	5 22	29
7	Th		6 12	5 24	30	sets	N
8	F		6 14	5 22	53	5 a 21	1
9	S	*St. Denys* [T. beg.	6 15	5 20	6 16	5 39	2
10	ℭ	19th S. af. Tr.: Cam.	6 17	5 17	39	6 3	3
11	M	OldMich.-d.: Ox.T.b.	6 19	5 15	7 1	6 35	4
12	Tu	Least twilight	6 20	5 13	24	7 18	5
13	W	*Trs. K. Edw. Conf.*	6 22	5 11	46	8 16	6
14	Th		6 24	5 9	8 9	9 22	7
15	F	♃ rises 7 36 aftern.	6 25	5 6	31	10 35	8
16	S	[*Etheldreda*	6 27	5 4	53	11 50	9
17	ℭ	20th Sun. aft. Trin.:	6 29	5 2	9 15	morn.	10
18	M	St. Luke	6 31	5 0	37	1 7	11
19	Tu		6 32	4 58	59	2 23	12
20	W		6 34	4 56	10 21	3 40	13
21	Th	♂ sets 8 42 aftern.	6 36	4 54	42	4 59	14
22	F	♀ sets 6 11 aftern.	6 38	4 52	11 4	rises	F
23	S		6 39	4 50	25	4 a 45	16
24	ℭ	21st Sun. aft. Trinity	6 41	4 48	46	5 12	17
25	M	*Crispin*	6 43	4 46	12 7	5 52	18
26	Tu		6 45	4 44	27	6 50	19
27	W		6 47	4 42	48	8 5	20
28	Th	St. Simon & St. Jude	6 48	4 40	13 8	9 32	21
29	F		6 50	4 38	28	11 0	22
30	S	☿ rises 6 52 morn.	6 52	4 36	48	morn.	23
31	ℭ	22d Sun. aft. Trinity	6 54	4 34	14 7	0 25	24

Day.	Length of D.	Day dec.	D. breaks	Tw. ends	Sun East.	Cl. aft. Sun.	☉ Semidiameter
1	11h. 35m.	4 59	4 m 8	7 a 31	5 m 40	10′ 19″	16′ 1″
6	16	5 18	17	19	32	11 50	3
11	10 56	38	26	8	25	13 11	4
16	37	57	34	6 57	17	14 21	5
21	18	6 16	43	47	10	15 16	7
26	9 59	35	51	38	4	15 55	8

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N° 155. NOVEMBER, 30 DAYS.

New Moon ...... 5th, 48m. past 4 Aftern.
First Quarter...... 13th, 42m. past 8 Aftern.
Full Moon......... 21st, 25m. past 2 Morn.
Last Quarter ...... 27th, 35m. past 5 Aftern.

SUN ENTERS

22d. 1h. 22m.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉ rise	☉ set	☉ decl.	☾ r. & s.	☾ a
1	M	All Saints	6h 55′	4h 32′	14° s 27′	1h m 47′	25
2	Tu	*All Souls:* Mich. T. b.	6 57	4 30	46	3 6	26
3	W	♄ rises 10 27 aftern.	6 59	4 29	15 5	4 23	27
4	Th	[Will. III. landed	7 1	4 27	23	5 42	28
5	F	Gunpowder Plot: K.	7 3	4 25	42	sets	N
6	S	*Leonard*	7 4	4 23	16 0	4 a 6	1
7	ℭ	23d Sun. aft. Trinity	7 6	4 22	18	4 34	2
8	M		7 8	4 20	35	5 14	3
9	Tu	P. Wales b. 1841: Ld.	7 10	4 18	53	6 6	4
10	W	[Mayor's d.	7 11	4 17	17 10	7 9	5
11	Th	*St. Martin*	7 13	4 15	27	8 19	6
12	F	Cam. T. div. m. (Shoot-	7 15	4 14	43	9 33	7
13	S	*Britius* [ing stars)	7 17	4 12	59	10 47	8
14	ℭ	24th Sun. aft. Trinity	7 18	4 11	18 15	morn.	9
15	M	*Machutus*	7 20	4 9	30	0 1	10
16	Tu	♃ rises 5 21 aftern.	7 22	4 8	46	1 16	11
17	W	*Hugh Bp. of Lincoln*	7 24	4 7	19 0	2 32	12
18	Th		7 25	4 5	15	3 52	13
19	F	♂ sets 8 47 aftern.	7 27	4 4	29	5 18	14
20	S	*Edm. King & Martyr*	7 29	4 3	43	6 48	15
21	ℭ	25th S. af. Tr.: Prs. R.	7 30	4 2	56	rises	F
22	M	*St. Cecilia* [b. 1840	7 32	4 1	20 9	4 a 37	17
23	Tu	*St. Clement*	7 34	4 0	22	5 48	18
24	W	[*Catherine*	7 35	3 59	34	7 13	19
25	Th	Michael. Term ends:	7 37	3 58	46	8 44	20
26	F	♀ sets 5 7 aftern.	7 38	3 57	58	10 13	21
27	S		7 40	3 56	21 9	11 37	22
28	ℭ	Advent Sunday	7 41	3 55	20	morn.	23
29	M	☿ sets 4 35 aftern.	7 43	3 54	30	0 56	24
30	Tu	St. Andrew	7 44	3 53	40	2 14	25

Day.	Length of D.	Day dec.	D. breaks	Tw. ends	Sun East.	Cl. aft. Sun.	☉ Semidiameter
1	9h. 37m.	6 57	5 m 0	6 a 28	4 m 57	16′ 18″	16′ 10″
6	19	7 15	8	20	51	16 14	11
11	2	32	15	13	46	15 49	12
16	8 46	48	23	7	42	15 4	13
21	31	8 3	30	2	39	13 58	14
26	18	16	36	5 59	37	12 32	15

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SUN ENTERS

21d. 14h. 12m.

DECEMBER, 31 DAYS. 1858

New Moon .... 5th, 10m. past 10 Morn.
First Quarter ...... 13th, 29m. past 3 Aftern.
Full Moon .... 20th, 6m. past 1 Aftern.
Last Quarter ...... 27th, 38m. past 5 Morn.

D.	W.	SUNDAYS, HOLIDAYS, &c.	⊙ rise	⊙ set	⊙ decl.	☾ r.&s.	☾ a
1	W		7h 46'	3h 53'	21° s 50'	3h m 30'	26
2	Th	♄ rises 8 34 aftern.	7 47	3 52	59	4 47	27
3	F	♃ rises 4 6 aftern.	7 49	3 51	22 7	6 4	28
4	S		7 50	3 51	16	7 19	29
5	C	2d Sunday in Advent	7 51	3 50	24	sets	N
6	M	*Nicholas*	7 52	3 50	31	3 a 58	1
7	Tu		7 54	3 50	38	4 59	2
8	W	*Conception B. V. M.*	7 55	3 49	44	6 6	3
9	Th		7 56	3 49	51	7 20	4
10	F	♂ sets 8 57 aftern.	7 57	3 49	56	8 32	5
11	S		7 58	3 49	23 1	9 44	6
12	C	3d Sunday in Advent	7 59	3 49	6	10 58	7
13	M	*Lucy*	8 0	3 49	10	morn.	8
14	Tu		8 1	3 49	14	0 11	9
15	W	Ember Week	8 2	3 49	17	1 27	10
16	Th	Cam. T. ends: *O! Sap.*	8 3	3 49	20	2 48	11
17	F	Oxford Term ends	8 4	3 49	23	4 14	12
18	S		8 4	3 49	25	5 44	13
19	C	4th Sunday in Advent	8 5	3 50	26	7 11	14
20	M		8 6	3 50	27	rises	F
21	Tu	St. Thos.: Short. day	8 6	3 51	27	4 a 43	16
22	W		8 7	3 51	28	6 16	17
23	Th	♀ rises 6 39 morn.	8 7	3 52	27	7 49	18
24	F		8 7	3 52	26	9 18	19
25	S	Christmas-Day	8 8	3 53	25	10 42	20
26	C	1st S. af. Christ.: St.	8 8	3 54	23	morn.	21
27	M	St. Jn. Evan. [Steph.	8 8	3 54	21	0 1	22
28	Tu	Innocents	8 9	3 55	18	1 20	23
29	W	☿ sets 4 21 aftern.	8 9	3 56	15	2 37	24
30	Th		8 9	3 57	11	3 54	25
31	F	*Silvester*	8 9	3 58	7	5 9	26

Day.	Length of D.	Day dec.	D. breaks	Tw. ends	Sun East.	Cl. aft. Sun.	⊙ Semidiameter
1	8h. 7m.	8 27	5 m 42	5 a 57	4 m 35	10' 47"	16' 16"
6	7 58	36	47	55	34	8 46	17
11	51	43	52	55	35	6 32	17
16	46	48	56	56	36	4 9	18
21	44	50	59	57	38	1 41	18
26	46	0 inc. 2	6 1	6 0	40	0 bef. 48	18

PRINTED FOR THE COMPANY OF STATIONERS.

POETICAL ANSWERS TO THE PRIZE ENIGMA.

Answer.—Thread.

1. *Acrostic.* *By the* Rev. John Hope, *Stapleton.*

T he coming of Dia each year is a pleasure;
H er mystical lays ever gain on the heart;
R ansacking her stores, we account as a treasure
E ach riddle that's penned with a masterly art:
A ccepting the boons of our mutual friend,
D evoted we still to her interests attend.

2. *To Miss Helen Ogden. By* Mr. Joseph Hutchinson, *near Halifax.*

Fair lady, your prize, with its varied attractions,
Of talent and usefulness, yielded us pleasure;
While musing on those who by praiseworthy actions
Have laid up above incorruptible treasure.
But painfully too, were our feelings excited,
For those who, to earn a poor pittance of bread,
In wretched apartments, sad, sickly, and slighted,
Are early and late at their needle and *thread.*

3. *To the Editor.* *By* Mr. James Herdson, *Tobermory.*

On what a telegraphic *thread,*
Your prizes you suspend!
Last year one reach'd the Isle of Mull!
For which my thanks I send.

4. *By* Clericus.

Readers of Dia saw with gladdened eyes
A lady's offering for their annual prize,
Last year Hope sent a needle, bright and neat,
Miss Helen now finds *thread*, and all's complete.

5. *By* Noah Wilmot, *S——s, near Newcastle-upon-Tyne.*

For sixty years I've Dia read,
But now must cut that pleasing *thread.*

6. *By* Mr. Robert Clemitson, *Morpeth.*

The Queen of Song on Dia's gifted page,
Whose mystic lays our deepest thoughts engage,
With wonted taste most fitly has designed,
That *thread* with "needle" e'er should be combined!
A proper "prize" fair Helen's verse contains,
And thus the Laureate's classic chair regains!
Long may she hold, with dignity and grace,
This just, deserved, and elevated place!

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7. *To * * * about to be married.* *By* Mrs. Furniss, *Mortivals, Takely.*

I've a word of importance to say to a bride—
In wishing you happiness, love, when you wed,
Pray let me advise you, as one that has tried,
Lay in a good stock both of needles and *thread*!

8. *By* Mr. George Starmer, *Heyford.*

Last year friend Hope, Diaria's honoured head,
Produced a needle, and none could refuse it;
And now Miss Ogden kindly brings the *thread*,
For all the ladies that have time to use it.

9. *By* Mr. James Hewitt, *Hexham, Northumberland.*

Och, dear Lady Di, is 't your clothes you've been rending?
That you get such broad hints that you somewhere need mending!
Sure in rags you are flaunting, I very much dread,
Since Hope sends a needle, and Ogden a *thread!*
Or is't proof of hard times, and economy needed?
Then, dear jewel, these hints you can scarce pass unheeded;
And, if in old age you must stitch for your bread,
Success to your needle, good luck to your *thread!*

10. *By* Mr. Joseph Furniss, *Takely, Dunmow (late of Lois Weedon).*

Not to add or to diminish,
Nor indeed to criticise,
The very clever finish
Of last year's ingenious prize;
Yet the lady-laureate, musing,
Peradventure might have said—
"Hope's needle will be useless,
Till I furnish them with *thread*."

11. *By* F. de S., *Nottinghamshire.*

The prize we guessed as soon as read,
Since a needle is usually followed by *thread.*

12. *An Acrostic. By the* Rev. A. Drysdale, *Bishopwearmouth.*

The telegraphic *thread*, the wonder of the age,
Has power to cheer or harrow hopeful hearts;
Reflective minds and souls audacious now engage
Ere long to prove it in earth's furthest parts.
Art and science, warmed by the flame of hope, aspire
Deep through ocean's wastes to draw the mystic wire.

13. *By* Clio, *of Hexham.*

Last year Hope placed before our eyes,
A polished needle for the prize.
Now Ogden takes the Laureate's place,
And *threads* the needle with good grace.

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14. Sonnet to Lady Di. *By* SELIM, *of Fakenham.*

Diaria, patroness of mystic lays,
A humble votary bless, who dares aspire
With feeble hand to tune the golden lyre,
Intent, with glowing heart, to sing thy praise.
What, though around his head no classic bays
Are wreathed, and in his strains, alas! no fire
Homeric gleams; yet still his dear desire
Is to recount how often through the maze
Of enigmatic song he has been led
By train of thought, while onward he has strayed;
As her lov'd Theseus, guided by the *thread*
Which Ariadne gave, his exit made
From labyrinthine depths, and hailed the light
Of open day, which beamed upon his sight.

15. *To the Editor.* *By* Mr. THOMAS PORTER, *Dale Abbey.*

Miss Ogden's *thread* all must admit
Is beautifully fine;
And long I hope her talents rare
On Diary's page will shine.

16. *By* NORTHUMBRIAN, *of St. John's Wood.*

By the charm of the poesy flowing,
To solve the prize theme I was led;
And the last year's prize—needle—I knowing,
Concluded that this must be *thread.*

17. *By* MARY JANE.

Your last year's choicest riddle
This year's most aptly led;
For fitting 'twas the "needle"
Should introduce the *thread.*

18. *By* Mr. JAMES MULCASTER, jun., *Allendale Town.*

Diarian ladies have nothing to dread,
Behold! in succession, a needle and *thread,*

19. *By* B., *of Barum.*

Life, with its troubles, it may well be said,
Is like the prize, a tangled skein of *thread.*

20. *By* Mr. STEPHEN WATSON, *Haydonbridge, Northumberland.*

With eager eyes, all through the prize,
I sought its *thread* to find.
At last I cried, "its *thread* I've spied,"
Since 'tis of *thread* combined.

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GENERAL ANSWERS TO THE ENIGMAS.

1. Maid of honour.	4. Bank.	7. Cap.	10. (Prize.)
2. Air.	5. Oil.	8. Dust.	Thread.
3. Change.	6. Fork.	9. Oak.	

1. *Changes.* *By the* Rev. JOHN HOPE, *Stapleton.*

Come, *maids of honour*ed Dia, hear the song,
The song of changes; something you may find
Perhaps instructive, human things among,
As well as those of more enduring kind.

Changes are frequent—not a single day,
Or single hour but brings them forth to light;
Nay, even moments have their passing sway,
Affecting thousands in their silent flight!

The *air* we breathe remains not still the same,
A thousand *change*s mark its fitful reign,
Sad inroads making on the human frame,
With many ills of which we all complain.

The *bank* of sand upon the winding shore,
On which successive tides exert their force,
Is ever changing 'midst the ceaseless roar,
As wave on wave pursues its onward course.

Changes affect the man on whom the *oil*,
The oil of consecration has been shed,
Though not with *fork* or harrow doomed to toil,
But both to wealth and independence bred:

The student too, arrayed in *cap* and gown,
Who moves conspicuous in the passing crowd,
Whom envied academic honours crown,
And who is justly of his learning proud.

Behold the *dust*—what change it undergoes!
"Where is the dust that has not been alive?"
Ourselves are dust—so Holy Scripture shows—
A truth we daily should in mind revive.

Nor free from changes is the giant *oak*,
Though it for ages crowned with verdure stand;
It falls at last beneath the woodman's stroke,
Or sinks to earth by time's destroying hand.

We all are changing; sad diseases rife,
And years, advancing silently and slow,
Are both employed to cut the *thread* of life,—
Then "dust to dust concludes this scene below."

All here is change, but thanks to gracious Heav'n,
There is a place where change shall be no more,
Where lasting mansions to the just are giv'n,
When once they're wafted to the heav'nly shore.

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O happy station!—saints arrayed in white—
Filled to the brim their cup of endless joy—
Shall find in love and praises pure delight,
Free from the slightest mixture of alloy.

Now take this admonition: seek in time
That change of heart the word of God requires,
That you at last may reach that blessed clime,
To which the faithful Christian mind aspires.

2. *April.* *By* Miss Helen Ogden, *Shaw.*

*C*ome *Ap*ril with thy genial show'rs,
Come like an *honour*'d *maid*en gay,
Adorned with bloom and fragrant flowers,
And chase rude winter's blasts away.

His trumpet notes have echoed loud and long,
Through woodland glen, o'er wild and scar,
Receding now, each little warbler's song
Proclaims his exit near and far.

On sunny *bank*s see little lambkins play,
And breathe the balmy *air* of spring,
As vernal bloom from each unfolding spray
Doth sweet perfume around it fling.

The falling show'rs that on thy steps attend,
Wake dormant nature from repose,
Invigorating all as they descend,
To bloom and blossom as the rose.

While with serener light the midday sun
Enlivens with his transient gleams,
All animated life its course to run,
Or sport beneath his cheering beams.

And though thy nature may be gentle found,
And blessings from thy lap may flow,
What *change*s doth thy onward path surround,
To*o* well thy *f*ic*k*le *r*eign doth show.

And may we not in ebbing life behold
A faint resemblance to thy sway,
Its passing scenes from time to time unfold,
Like thee, an evanescent ray.

The softening showers of grief will fall,
And like thy dark and low'ring skies,
Its clouds con*d*ense *u*pon *t*he heads of all,
As round its pointed arrow flies.

But as the peaceful calm succeeds the gale,
So joy doth sorrow's path surround;
And whispering, tells her happier tale,
And pours forth *oil* in every wound.

'Tis from the keenest pangs of grief and pain,
We pleasure pure can only know;
As from the gentle and refreshing rain
The fruits of earth *are known* to grow.

Then as the earthly race of life we run,
Greet darkest glooms as sunshine bright;
That ere its flimsy *thread* at length be spun,
Exclaim, "Whatever is, is right."

3. *By* Mr. JOSEPH HUTCHINSON, *near Halifax*.

'Tis thirty years—yes, thirty years!
Though short, indeed, the time appears,
Since I essay'd, in mystic song,
To join Diaria's tuneful throng;
And found enroll'd my humble name,
With *maids of honour*, bards of fame.
When with delight and sprightly *air*,
I look'd upon those pages fair.

And still I hail the mystic page
With pleasures though advanc'd in age;
And as in youth its subjects trace,
Though many a *change* has taken place;
And friends are gone whose lyric's here,
From Calder's *banks* were wont to cheer,
While others now their powers engage,
In mental *toil* for Dia's page.

And so 'twill be when we are gone,
*For k*indred minds will still write on,
And other bards new themes create,
To cheer and charm and *cap*tivate;
Alternate frowns and smiles provoke,
On lowly *dust* or lofty *oak*,
Spin *threads* or rhymes with equal care,
And thus preserve these pages fair.

4. *Thoughts on Death. By* Mrs. BAKER, *Vauxhall*.

Dust to dust is often said
On the consecrated ground,
Tears of sorrow oft are shed
By the mourners standing round.

When sad sighs our bosoms heave,
Now *for k*indred thus consign'd,
What impression does it leave
On the agitated mind?

Hark!—I hear the tolling bell,
One is gone,—we know not where;
Hearts bereaved with anguish swell,
Should we not for death prepare?

Oft we mourn departed youth,
Summon'd from this earthly stage;
Surely this momentous truth,
Solemn warning gives to age!

Vain are years of *toil* and strife,
If *cap*ricious Fortune frown,
Castles built in empty *air*,
*Bank*s of sand which crumble down!

Time for one great work is given,
Talents which we *may* not h*ide*;
Time t*o* m*ak*e our peace with Heaven,
Holy Scripture for our guide.

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When the slender *thread* of life
  Breaks and sets the spirit free;
May we *change* this mortal strife
  For a calm eternity,
Where the vile from troub'ling cease,
  And the weary pilgrims rest,
Fill'd with joy and love and peace,
  In the regions of the blest.

5. *The Thunderstorm. By* Mr. JAMES HERDSON, *Tobermory.*

Dark was the night, and fearful was the storm,—
The lightnings *thread* the *air* in *fork*ed form;
What is the structure could es*cap*e the stroke?
To splinters rent, rec*oil'*d the *maid*en *oak:*
The *change* so quick, the forest monarch must
Bend to the *bank,* lie prostrate in the *dust.*

6. *Summer. By* Mr. GEORGE STARMER, *Heyford.*

Bright Sol in Cancer shines with fervid rays,
Curtails the nights and lengthens out the days;
Nature adorned in pompous robes is seen,
Gay as a *maid of honour* or the queen.
T' avoid the heat, the flocks and herds repair,
To shady coverts, from the sultry *air;*
And in the fields or meads, where'er we range,
From winter's piercing cold, oh! what a *change!*
Behold on sunny *bank*s with daisies pied,
The busy ants their winter food provide;
The swains at early morn their t*oil* pursue,
And cut the grass surcharged with pearly dew.
With *fork*s in hand the maidens ted the hay,
And sturdy youths in *cap*s hence bear away,
On heavy cumbrous wain, with ponderous load,
And homeward drawn along the *dust*y road.
In noontide heat the languid group repose,
Beneath the spreading *oak*'s umbrageous boughs;
And while they *th*ere in sh*ad*e supinely lie,
Heed not the sun careering in the sky.

7. *The Coming Spring. By* Mr. JOSEPH FURNISS, *Mortivals, Takely, near Dunmow (late of Lois Weedon).*

"Away on the hills young Spring was seen,
Tipping the buds with virgin green."
CAPERN.

The primrose on the *bank* begins to herald in the spring,
The sturdy *oak* begins to sprout, the birds begin to sing;
The *air* is milder, and a *change* from winter's pinching reign,
Seems coming to revive our hearts and cheer us up again.
And now the little *maids* trip off from many a rustic cot,
In search of early vi'lets in the sheltered sunny spot;
While schoolboys, all unheeding, or the master or the dame,
Toss up their *cap*s for very joy, and then renew their game.

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Ay, lovely spring is coming, and again we soon shall see,
On busy wing in search of flowers, the butterfly and bee;
The roads will soon be *dusty*, and again we soon must wield,
The *fork* and rake for "sowing time" in garden and in field.
And t*oil* itself is pleasure then: a smiling sky above,
And all around sweet melody and twittering of love;
And budding hedges, rising crops, that full of promise peep,
And goodly pastures that supply the browsing kine and sheep;—
All, all are tokens of the love of Him, the great ALL WISE,
Who gives us blessings every day, and every want supplies;—
Of Him, whose promise faithful is to us His creatures frail,
That "seed time, harvest, day and night, and seasons shall not fail."
And while we *thread* our way through life, amid its many snares,
Let us remember there is one who for His children cares;
And ever let our hearts be filled with gratitude, to sing
His praises at all seasons, as we do for lovely spring.

8. *The Old Man's Soliloquy. By* Mr. JAMES BARTHRAM, *Scarborough*.

Down by an old ruin with ivy clung,
An aged man sat, and his head down hung;
Cold winters he'd seen, full ninety and three,
Yet still a stout, hearty, hale man was he;
His forehead was furrow'd, his h*air* was gray,
He thought of the *change*s since life's young day;
Of how in his youth he had sportive hied,
On the flow'ry *bank*s by the river side;
He *ca*ll'd o'er his *p*laymates' names one by one,
A monitor answer'd—"They all are gone."
He thought of *th*e time when he *ready* run,
To meet a fair nymph when his t*oil* was done;
Of moments he'd spent with the charming *maid*,
In calm summer's eve 'neath the *oak*en shade;
His heart it beat light as his mind did trace,
Each love-winning smile and each fond embrace;
The soft, rosy cheeks, where fair ringlets hung,
The musical voice, when a favorite sung;
He ask'd for these youthful charms one by one,
The monitor answer'd—"They all are gone."
He thought of the school in his native vill—
His parents' thatch'd cot by the sloping hill;
The ol*d* r*ust*ic porch, and the rural seat,
The cool woodbine bow'r and the garden neat;
He ask'd *for* all these—the monitor spa*k*e,
"Such—such—is the havoc Old Time doth make;
The friends of thy youth to the grave are gone,
The charms of thy youth unto ruin are run;
Time will not stay, moments hurry fast on;
Yet a little while, and then—all is gone."

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9. *Remembrance.—To Mary.* *By the* CAWKLEY'S LADDIE.

O Mary! dearest Mary! still remembrance clings to thee,
Though months are fled, nay, years are past, since thou wert call'd
from me;
Still oft I think upon the time when we were blithe and young,
When by the cheerful cottage hearth we gaily laugh'd and sung;
When flow'rets op'ning to the day on every *bank* were seen,
And we in sport ran hand in hand upon the village green.

I remember, I remember well, the days when love was new,
When first my heart was *cap*tive ta'en, how strong affection grew;
How we stray'd up the avenue and 'neath the *oak*en shade,
We listen'd to love's tender tales, and scenes around survey'd;
How oft in the cool ev'ning *air* we wander'd on the hill,
And rov'd along the river's side past the lone clacking mill.

I remember, I remember well, I think I am there now,
When first we spoke of wedlock's joys beneath *the* sp*read*ing yew;
When in a loving, fond embrace, I clasp'd thee to my breast,
And when our lips in concert met, how I was truly blest;
Thy welcome smile, thy laughing eye, I never shall forget,
My heart rebounds at thoughts of them, I think I see them yet.

I remember, I remember, how I *toil*'d throughout the day,
When thoughts of thee made labour light, something alike to play
How cheerfully all day I sang, and trimm'd the budding trees;
Guarded at eve the tender shoots from the chill eastern breeze;
And how from blooming borders neat, I choicest blossoms bore,
To grace thy fair and throbbing breast, which now will throb no more.

I remember, I remember, I shall ne'er *for*get the day,
Thy shrun*k*en hand was plac'd in mine as thou on sick-bed lay;
And how thy pale lips falter'd out these words in accents sweet,
"Our hopes on earth are over now—in Heaven may we meet;"
Then fainting on thy pillow sank, no further word could speak,
I watch'd thy *change*d features then, my heart was fit to break.

I remember, I remember well, when thou a corpse was laid,
How friends stood round and wept o'er thee, my pretty Cawkley's *maid*;
Though clos'd that eye, and wan that cheek, that once was bright and fair,
Though pale and silent were thy lips, a smile still rested there;
How when thy body they laid down to moulder in the *dust*,
Fullness of grief my bosom swell'd, 'twas ready for to burst.

O Mary, dearest Mary still, remembrance shall be thine,
Though here on earth 'tis over now, thou never can be mine,
But 'ere 'tis long the time will come, our joys will be complete,
My body resting near to thine, our souls above shall meet;
Where with glad songs in chorus we shall join the blissful choir,
And dwell in love and happiness where nought shall part us more

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10. *To the Editor. By the* Rev. ANDERSON DRYSDALE, *Bishopwearmouth.*

Dear sir, for reply to th 'enigmas this year
Will you please to accept my initials?
Too cryptical 'tis,—I certainly fear
For its fate at the hands of officials.

M, A. and C, B; O, F, with C, D,
O, T, to complete the decade,
Can give us a weed, (we all must agree,)
Too fit for sarcastic tirade.

TOBACCO's that weed whose fumes still ascend
The air ne'ertheless to pollute,
While habit confirmed may eagerly lend
Its arguments meant to confute.

Virginia's plant, which *seven* letters embrace,
Leaves *three* other letters to test,
F, M, and D; now consider the case,
Are Fine, Mild, and Dry, always best?

Th 'enigmas are these:—"*Maid*, *Air*, *Change*, and *Bank*,
Oil, *Fork*, *Cap*, and *Dust*, *Oak* and *Thread*."
Closing my answer, now you I must thank,
For labours that puzzled my head.

11. *The Dark Side of Life. By* MR. JAMES HEWITT, *Hexham, Northumberland.*

'Tis sad to trace the progress of decay,
E'en in inanimate and trivial things,
Which yesterday were born, and bloom to-day,
To-morrow death and black destruction brings!

'Tis sad to see the waning powers of age
Vainly essay to do as they have done;
Whose book of life has reached the final page,
Whose little day beholds its setting sun.

More sad to mark in manhood's May-day pride,
The youth, whose hectic flush too surely shows
That fell disease has sapped the vital tide,
And wastes his lingering strength like vernal snows.

Most sad, perchance, to view th' angelic *maid*,
The pride, the goddess, of some virtuous swain,
With feeble gait, and roses all decayed,
That treads uncheered o'er flower-bespangled plain;

The scented *air* to her no fragrance lends,
Insipid all earth's charms—she longs for *change*;
Now to the sunny *bank* her steps she bends,
And now in woodland glades would wish to range.

Macassar *oil* no more adorns her hair,
The tuning *fork* gives her a jarring sound;
Her once gay *cap*, erst as the lily fair,
Is now disordered, or drawn closely round.

She broods but o'er the grave, and *dust*, and worms,
Or the *oak* coffin which shall her inclose,
When the snapt *thread* has freed from earthly storms,
And low she lies released from friends and foes.

Save when the Christian's hope relights her eye,
And true Religion's fire revives her bloom,
She sees her soul, "restored," beyond the sky
Enraptured rise, triumphant o'er the tomb.

12. *By* Clio, *of Hexham.*

Diarians now, all must allow
Have got a *maid*en queen, ,
A lady f*air*, whose talents rare
Are in her writings seen.

A *change* so great in our estate,
We all delighted hail,
No *cap*tious thought, with envy fraught,
Can e'er in Di prevail.

A friendly *band*, lin*k*ed hand in hand,
They still their course pursue,
And t*oil* and care would gladly share,
With each Diarian true.

For feelings *k*ind pervade each mind,
In*dust*riously employ'd
In conning o'er the mystic lore,
With pleasure unalloy'd.

Proud they evoke Hope's royal *oak*,
And see its branches spread,
Whilst Ogden's muse gives varied hues
Unto her well-spun *thread.*

13. *The Country Maiden's Lament.* *By* Jane, *of Rydale.*

Well-a-day! well-a-day! how can it be?
The laddies a-courting won't come to me,
Though I'm a blithe lassie both fair and free.
O dear! O dear!

I trim up my *cap* to give me an *air*,
But seldom a lover to me comes near,
A *maid* I must die, O! I vastly fear,
O dear! O dear!

My father's been dead now these five long years,
My mother she sits on our old *oak* chairs,
Too feeble *for* wor*k* through old age and cares,
O dear! O dear!

From morn until evening I t*oil* for bread,
Stitching and sewing with needle and *thread*,
For being a poor lassie I can't get wed,
O dear! O dear!

I'm twenty and five on Midsummer day,
Unnoticed my life is passing away,
And not a lone offer, it makes me say,
O dear! O dear!

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There's one I know, not so pretty as I,
And lovers, to win her, many do try,
'Tis *change* in the *bank* that attracts their eye,
O dear! O dear!

There's another I know, if he should hie
And ask me to marry, I'd not deny,
But with him to church I would quickly fly,
O dear! O dear!

I'd wash, and I'd scour, and I'd *dust* his room,
At eve with a smile give him welcome home,
He never from me should have cause to roam.
O dear! O dear!

I'd make him so happy, (however, I'd try,)
I never would cross him—O no! not I,
Then never more should I have cause to cry,
O dear! O dear!

14. *The Scenes of Youth re-visited.* *By* NORTHUMBRIAN, *St. John's Wood.*

I look'd on the valley in *ma*nhood's r*i*pe age,
And I ponder'd o'er youthful scenes there,
And I thought I'd ne'er seen, nor in history's page,
Read of valley so fertile and fair.

O, how balmy its *air*, how serene its repose,
As I wander'd at daylight's decline;
There the wild flowers bloom and their beauties disclose,
On the *bank*s of my sweet native Tyne.

Like *oil* on the waters of strife, to my soul,
*C*ame the peace of t*ha*t valley sere*ne*;
And tears *g*ath*e*r'd fast, which I could not control,
As I wander'd where oft I had been.

On a *fork* of its river I musingly lay,
As I traced out each favorite nook,
Where in boyhood I rambled the long summer day,
The meadow, the woodland, the brook.

O, oft I had seen it when snow-*cap*'d the mountains,
Now, insects were sporting around,
And "the song of its birds an*d* the g*us*h of i*t*s fountains"
Broke sweetly the stillness profound.

Each tower and each tree had a place in my heart,
The willow, the *oak* on the plain;
I sighed, and I felt as if loath to depart,
For soon I must leave them again.

O, often in fancy my fond heart's devotion
Flies away to where boyhood was pass'd,
Where the Tyne murm'ring on *thread*s his way to the ocean;
Those scenes will be dear to the last.

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ANSWERS TO THE REBUSES AND CHARADES.

1. Lap-wing.	5. Tome, mote.	9. Speculation, peculation.
2. Earth, heart, art.	6. P(i)lot, plot.	10. Pinch-beck.
3. Par-rot.	7. Cod-ring-ton.	11. Pagoda, a god.
4. Serge-ant.	8. Sun-day.	12. Con-flag-ration.

1. *The Country. By the* Rev. JOHN HOPE, *Stapleton.*

Speak not of the town with its streets and its alleys,
Its shops and hotels with their varied display;
More cheering to me are the hills and the valleys,
When nature is decked in her fairest array.

The note of the *lapwing* affords me a pleasure;
I joy when the *earth* quits the shadows of night,
The dew-drops imbibing—that bountiful treasure,
Which pours on the summer a flood of delight.

I speak not of regions where *parrots* are flying,
And birds of a plumage most showy appear;
Our own native Britain, all nations outvying,
Is ever to me and to Englishmen dear.

The *sergeant* in town, with his cap and his feather
May marshal his men to attend the parade;
I envy him not; for in clear summer-weather,
A *tome* entertains me, reclined in the shade.

I feel for the *pilot* upon the rough ocean;
I love terra firma when spangled with flowers;
I *Codrington* praise, wish him highest promotion—
A gen'ral approved by "the Western Powers."

In town and in country, how pleasant is *Sunday!*
In a neat country village more pleasant to me;
Its rest nerves the arm for the labour of Monday,
In England, the land of the brave and the free.

Ah, slav'ry begone! thou art harsh and unfeeling;
Thou still hast, alas! too extended a reign;
How horrid in men is the traffic, or dealing!
O Christians, from such *speculation* refrain!

The cotton plantations are verdant and pleasing,
In them lovely nature's assisted by art;
But nature weeps blood when the master is seizing
His slaves, and inflicting the keenest of smart!

As *pinchbeck* with gold in comparison failing,
As higher the moon than the sun ocean swells,
So a church is o'er heathen *pagodas* prevailing,
As far as the Lord ev'ry idol excels.

But I've been digressing to far-distant nations,
And quitting our country, its hills and its vales,
Where *con*tent is my *flag*, and though homely my *rations*,
My pleasures are many, nor gratitude fails.

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2. *To the Rev. John Hope.* *By* Mr. JOSEPH HUTCHINSON, *near Halifax.*

Many thanks, my dear sir, for your kind invitation,
To visit the Border, where landscapes abound;
Replete with the beauties and charms of creation,
And where on the wide heath the *lapwing* is found.

Believe me, 'twould yield an unspeakable pleasure,
No *earth*ly enjoyment more tempting to me,
Than such an excursion, if favour'd with leisure,
Yourself and your mountains and valleys to see.

Your garden, O yes—how delightful to stray in,
Where fragrance exhal'd by each opening flower,
Perfumes the light breezes that fitfully play in
The foliage mantling your "sweet little bower."

And there would we chat on the mystic effusions,
That Dia produces to puzzle and please;
How varied the subjects—and far-fetched allusions;
A *parrot* encaged—and a *sergeant* at ease.

The war with its horrors is over, thank Heaven!
Though *tomes* cannot half its dire evils make known;
And now may each *pilot*'s good service be given,
To commerce and peaceable projects alone.

And who would not rather see *Codrington*'s glory,
Enhanc'd by the capture of slave-ships and crews,
Than by a renewal of battle scenes gory,
Though bravery triumph'd and honour ensues.

But leaving these subjects for others more cheering,
On *Sunday* of course to the church we'd repair,
Where pastor and people together appearing,
Engage in true worship of praises and prayer.

Unlike the poor heathen whose wild *speculations*,
Are placed on an idol of *pinchbeck* or gold;
In splendid *pagoda*, where vain aspirations,
A dark and debased superstition unfold.

Then as we are blest with divine revelation,
May we its Great Author for ever adore;
And wisely prepare for the last *conflagration*,
When nature shall perish and time be no more.

3. *The Postman's Rap.* *By* Miss HELEN OGDEN, *Shaw.*

Give welc*ome t*o the postman's rap, though loud and bold it be,
It may bring tidings of great joy, and fill your heart with glee;
As from his bag he extricates epistles grave and gay,
Penn'd by the hand of love most true, or friendship far away;
Or in the *lapwing*'s plaintive note from *earth*'s remotest bound,
The tidings of some relative, who to his cost has found

That *speculation*'s brightest hopes, in other lands are known
To meet with disappointments sad, as well as in his own;
Or from some *sergeant*, who went forth with *Codrington* to gain
Distinction on the battle-field, amid the heaps of slain;
But who amid the horrid strife, and *conflagration* dire,
Did find a *provi*den*tial* hand to screen him from its ire.

Give welcome *to* the postman's *rap*, and greet him with a smile,
Since from his mult'farious store he may your cares beguile;
Present you with a passport sure to happiness or fame;
Or what perchance more welcome be, some interest to attain;
Or from some h*ap*less wight that's *pinch'd* by poverty and care,
Who without *beck* or bidding may present a suppliant's pray'r;
Or should he in his daily walk the sad announcement bring,
That some belov'd or valu'd one has fallen 'neath Death's sting;
Or that misfortune in her round on you thinks fit to call,
For who is there she visits not upon this earthly ball?
But shrink not at her frown to *day*, to morrow's *sun* may rise
In tenfold splendour on the gloom that now invests the skies.

Give welcome to the postman's rap, and speed him on his way,
Since on another he must call, and that without delay;
Some artless scrawl may rapture give to many an aching heart,
May breathe affection's purest strain undignified by art;
While others dress'd in glittering garb by buoyant hand sent forth,
May gain a smile perhaps from those who deem them little worth;
What gifted mind can there be found to tell what he conveys,
What secret workings of the heart his open hand purveys,
What wants, what wishes, hopes, and fears, on friendship, bus'ness,
trade,
Could they throughout the busy land be openly display'd.
Then give a welcome to his rap, respect his mission still,
What nobler institution's found to serve us at our will.

4. *To the Editor*. *By* Mr. James Herdson, *Tobermory*.

Dear sir, the riddles that you sent,
You never, surely, could have meant
To be mix'd up with rhyme;
Pagoda, *pinchbeck*, *parrot*, *plot*,
Though quite allit'rative, are not
Well suited for to chime.

P'rhaps *Sunday*, *sergeant*, *Codrington*,
May jingle on the shelf with *tome*,—
'Tis but a *speculation*;
Sure as the *lapwing* skims the *earth*,
My rude attempt, so little worth,
Must end in *conflagration*.

5. *By* Mrs. Furniss, *Mortivals*, *Takely*.

To pen a strain that will be worth the ink
Is *mo*re *t*han I can manag*e* well I think;
For, conning o'er the strange mysterious list—
Prospective of how many an ugly twist—
I wonder what on *earth* to write about,
And *speculat*e m*i*sgiving and i*n* doubt.
I nothing know of *lapwing*s but by name;
And as for *parrots*—I'm no spinster-dame,

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And never yet possessed one, but have heard
They talk and scream, and are a gaudy bird.
I've heard and read of *Codrington* the brave,
And all his gallant *sergeants* o'er the wave,
And how they fought, and how, with shell and ball,
The dreadful *conflagration* levelled all.
I know that *pinchbeck* is a metal base,
And shabby-looking for the watch's case;
It suits our boys, however, when they go
To have a holiday and make a show.
I've heard of some old *pilot* sailor, who
Made good men of a wild unruly crew;
And though exposed to many a storm and "lurch,"
They still remembered *Sunday* and the church
Of happy England, in the days of youth,
When they were taught to worship God in truth;
Not idols in the *pagoda*, as do
Poor heathens who the Saviour never knew;
Yet hope and pray we that the Gospel light
Ere long may shine upon their darksome night;
That seeing clear the errors of the past,
They may become the "heirs of grace" at last.

6. *A Nuisance of a Neighbour; or, a little bit of Tittle-tattle for Di.*
By Mr. JAMES HEWITT, *Hexham, Northumberland.*

O, Mrs. B—, did you hear the "norrations?"
With *lapwing*-voice, came screaming Mrs. Lancey;
W'y, how on *earth* can "ivver" ye have patience?
Ye didn't hear! then ye were dead aw fancy.
Lawks, Mrs. *Parrot* says its quite "owdacious,"
An' the o'd *sergeant* says "he's" in a "fransy,"
An' that he meditates among the *tomes*
At midnight, or is catching *motes* and gnomes.

But Mrs. *Pilot* says "he" *plots* at her,
In reading 'loud 'bout *Codrington* an' war,
Because she's a "Peace 'ciety" woman, for
She sees ne honor in a wound or scar;
Because last *Sunday*, as if on the spur,
"He" bolted past her as he never saw 'er;
But this is, may be, only *speculation*,
We shouldn't "jellous" be on ne occasion.

Sometimes he's up a-stuffin' birds an' foxes,
Squirrels, an' fou'marts, banty-cocks, an' things;
An' then he's hammerin' an' makin' boxes
To put them into, till the "heavens" rings;
Or wi' the 'lectrifyin' givin' shocks he 's,
An' showin' what it is bad weather brings;
Folks say that he can tell what makes the thunder,
Has he ne dealin's wi' "o'd Scratch," aw wonder!

O'd Mr. *Pinchbeck* says, wi' thunderin' thumpin',
"He" shook his nice *pagoda* from the shelf,
Then something tumbled down, souse, like a pumpkin,
An' stopt the Yankee clock, an' crackt the "delf;"
W'y aw wad "rayther" live beside a bumpkin,
Than near a learnin' "necrymancin" elf;
Aw tell you what, on some o' "thir" occasions,
He'll set us a' into the *conflagrations*.

W'y then he's "nattlin" an' makin' fiddles,—
An's made a grand un—but aw fear aw'll tire ye—
Anon them wearisome new tunes he "driddles"—
W'y just wi "tellin" on't aw'm a' "pespiry"—
An' sometimes figurin an' makin' riddles,
That's puzzles for a beuk they ca' the Diary—
An "albenack" that he gets doon frae "Lunnun,"
W'y, really, some folks maun be verra cunnin'!

LIST OF POETICAL ANSWERS.

Abigail, Miss, 1, Marine Place, Plymouth, ans. all.
Allanson, Thomas, Surgeon, Rillington, Malton, Yorkshire, ans. all.
Angus, J. C., Horsley Hope, Shotley Bridge, Durham, ans. all.
Awmack, Mrs., Harum, ans. Prize Enigma.
Baker, Mrs., 25, Vauxhall Street, Lambeth, ans. Enigmas.
Barthram, James, Scarborough, ans. all.
B. of Barum, ans. Prize Enigma.
Bowes, Joseph, Ingo, near Kirbymoorside, Yorkshire, ans. all.
Bowman, Thomas, Richmond, Yorkshire, ans. Prize Enigma.
Burdon, Henry, Sutton-on-the-Forest, near York, ans. all.
Buttery, J., Mathematical Master, H. M. Dockyard, Chatham, ans. all.
Carr, M. Rodham, Carr's Villa, Carr's Hill, near Gateshead-on-Tyne, ans. all.
Cawkley's Laddie, ans. all.
Clazey, J. O., ans. all.
Clemitson, Robert, Morpeth, ans. Prize Enigma.
Clericus, ans. all.
Clio, of Hexham, ans. all.
Craiggy, Colin, Crawcrook, ans. all.
Cussons, Richard, Salton Lodge, near Kirbymoorside, Yorkshire, ans. all.
Dargue, Harrison, Carshield, West Allendale, ans. all.
Dawe, Miss M. N., Landulph, Cornwall, ans. all.
Dawson, Sarah, Kirbymoorside, Yorkshire, ans. all.
Dawson, Thomas, Wombleton, near Kirbymoorside, Yorkshire, ans. all.
Densham, Miss, of Jersey, ans. all.
Dodgson, John, jun., Kirby Mills, near Kirbymoorside, Yorkshire, ans. all.
Drysdale, the Rev. Anderson, Bishopwearmouth, ans. all.
Eddy, E. A., St. Just, near Cape Cornwall, ans. Enigmas.
Eddy, Eliza H., St. Just, Cornwall, ans. Prize Enigma.
Eddy, Miss Mary Lawry, St. Just, near Cape Cornwall, ans. Prize Enigma.
Eddy, William H., Truthwall, St. Just, Penwith, Cornwall, ans. all.
Edwards, Thomas, Milthorpe, Northamptonshire, ans. Rebuses, Charades, and Prize Enigma.

Ego, Durham, ans. Enigmas.
Eland, Thomas, 19, Hill Street, Bridge Street, Bolton, ans. Enigmas.
Elizabeth S——, London, ans. all.
Elliott, John, Westcroft, Stanhope, ans. all.
F. de S., Nottinghamshire, ans. Rebuses, Charades, and Prize Enigma.
Fenna, John, Alpraham, ans. all.
Furniss, Mrs., Mortivals, Takely, ans. Prize Enigma.
Furniss, Joseph, Takely, Dunmow (late of Lois Weedon), ans. all.
Grey, John, Castle Eden, Ferry Hill, ans. Prize Enigma.
Grice, George, jun., Wold Newton, near Malton, Yorkshire, ans. all.
Hattam, Thomas, jun., Eddystone Lighthouse, near Plymouth, ans. all.
Herdson, James, Tobermory, Mull, N.B., ans. all.
Hewitt, James, Hexham, Northumberland, ans. all.
Hill, Henry, National Schools, High Wycombe, ans. all.
Hope, the Rev. John, Stapleton Rectory, Carlisle, Cumberland, ans. all.
Hudson, John, Epworth, near Bawtry, ans. all.
Hutchinson, Joseph, near Halifax, ans. all.
Jackson, Thomas, Felling School, Gateshead, ans. all.
Jane, of Rydale, ans. all.
Karcel, R. R., Howpert, ans. all.
Kerr, John C., Harbour Office, Berwick-on-Tweed, ans. Enigma 2, and Rebuses and Charades 2, 3, 7, 8, 10.
Levy, W. H., Shalbourne, near Hungerford, Berkshire, ans. Prize Enigma.
Mary ——s, Miss, West of Cornwall, ans. all.
Mary Jane, ans. Prize Enigma.
Mentor, of Worcester, ans. Enigmas.
Mulcaster, James, jun., Allendale Town, ans. all.
Mulcaster, John Wallis, Allendale, Northumberland, ans. all.
Nattrass, Joseph, Allendale Town, Northumberland, ans. all.
Nodwons, J., Murah, ans. Enigmas.
Northumbrian, St. John's Wood, London, ans. all.
Oats, William, Tregeseal, West Penwith, Cornwall, ans. all.
Ogden, Miss Helen, of Shaw, near Oldham, ans. all.
Otat, C. R., Post Office, Whitby, Yorkshire, ans. all.
P. P., Derby, ans. Prize Enigma.
Perrett, John, Marton, near Kirbymoorside, Yorkshire, ans. all.
Pigg, Edward, Bishopwearmouth, Durham, ans. all.
Porter, Thomas, Cat and Fiddle Lane, Dale Abbey, Derbyshire, ans. Enigmas.
Priestley, F. S., Norton, near Malton, Yorkshire, ans. all.
R. Y. C., of Guernsey, ans. all.
Rowbottom, Samuel, Breadsall, near Derby, ans. Enigmas.
Rutter, Matthew, 65, Lawrence Street, Sunderland, ans. all.
Ryley, Robert, jun., Mickleover, Derbyshire, ans. all.
Selim, of Fakenham, Norfolk, ans. Prize Enigma.
Somerscales, Thomas, "Brig Amphitrite," Greenhithe, River Thames, ans. Prize Enigma.
Standring, John, Epworth, Bawtry, ans. all.
Starmer, George, Heyford, Northamptonshire, ans. all.
T. D. H., Kirby Mills, near Kirbymoorside, Yorkshire, ans. Prize Enigma.
Watson, Stephen, Grammar School, Haydonbridge, Northumberland, ans. all.
White, J., Holley Terrace, Birmingham, ans. all.
White, Thomas, Allendale, ans. all.
Wilmot, Noah, S——s, near Newcastle-upon-Tyne, ans. Prize Enigma.

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NEW ENIGMAS.

I. Enigma (1401); *by* Mr. John Wallis Mulcaster, *Allendale.*

No friend of the ladies, how shall I begin,
To enter in Di and their favours to win;
Ah, never—I'm too much despis'd by the fair,
When I bloom in their cupboards I frankly declare.
I'm seen in the vale, on the hill, and the plain;
In the recess, the garden, I hold my domain;
Or abroad in the meadow, the lawn, and the lea,
On the verdant parterre, I'm richer you'll see;
On the brink of a mountain I heedlessly stand,
I'm exposed to your view in most parts of the land;
I give to the ladies their elegant air;
And yet I'm a foe, of which all are aware:
But if I am viewed in a different light,
If beauty adorn, I afford them delight.
I dwell in the precincts of yonder dark tomb,
Without me all nature would shrink from her bloom;
By my symmetry, ladies, it is that you're won—
But hold, it is time my narration was done.

II. Enigma (1402); *by* Mr. James Barthram, *Scarborough.*

To Creation I run for the date of my birth,
Ere bird, beast, or mortal was known on the earth;
Ere th' garden of Eden by Adam was trod,
I was a free agent and roving abroad.
O'er vast realms of space 'tis my province to stray,
And nature I cheer as I pass on the way;
Yon meadows so fair, and yon blossoming tree,
Would bloom not, nay, bud not, were it not for me;
I gladden the valleys, the woodlands, the grove,
And I am ever welcome wherever I rove.
If I should be banish'd, how dismal the sight,
Adieu—all that gladdens and gives us delight;
E'en beauty and grandeur, they nothing would be,
If 'twas not for services rendered by me.
If I for a twelvemonth entirely were lost,
All things in disorder would strangely be toss'd;
When now for a season abroad I do stray,
As it is my wont, though then short is my stay,
How gloomy the picture, all nature seems sad,
In forest and glade boding voices are heard;
But scenes still revive as again I draw nigh,
And one, ever watchful with vigilant eye,
To give me a welcome when once I appear,
Ne'er fails to salute my approach with a cheer.
In garments of various hues I am seen,
Blue, violet, and yellow, red, orange, and green;

I'm found in the cot; I'm found in the mine;
I'm found on the road; I dart on the line;
I ride on the ocean; I sport in the fen;
I lodge in a box; I am priz'd by all men;
I'm guide to the sailor, of me he's aware;
I dwell in the mind, you'll perhaps find me there.

III. Enigma (1403); *by* Mr. Edward Pigg, *Bishopwearmouth.*

Mortals, awake, behold, in boundless space,
I stand the blessing to the human race;
My virtues mark the best and holiest times,
And yet my name is black with hateful crimes.
When I appear, the bonds of serfs give way
And joy-bells ring my glorious holiday.
Glad music fills a thousand vocal throats,
But I alone can furnish all the notes.
Yet dreadful have I been in days of old,
My name doth horrid mysteries unfold;
Spirits have heard my awful thunders roar,
And my shrill trumpets shriek from shore to shore!
I came with pestilence to sweep the earth,
And heaven was silent when I issued forth.
Black desolation clothed the affrighted land,
When wrath and woe I scattered from my hand;
My shape was horrid to the sons of men,
More hideous far than monsters of the glen.
Vast mountains told the meaning of my form,
When the proud kings I conquered in the storm.
From patience then my ingrate offspring came,
His sheep in thousands bleated forth my name;
From me sprang famine—then I circled round,
And ripened plenty stored the fruitful ground.
Then mighty temples graced the classic shore,
But, ah, my golden lights are now no more;
My valiant knights went forth and peace restored,
My sages taught, and wonders crowned my word!
In Rome I flourished—priests upheld my deeds,
But England cast me off, as waves the weeds.
Still I exist, as works of mercy show,
And in the skies with brightened eyes I glow;
My name is known in every land and creek,
Tell what I am—all *tongues* my secrets speak!

IV. Enigma (1404); *by* Mr. George Starmer, *He*

Ladies and gents, again the muse would try
To pen a strain to grace the page of Di;
But artless in extreme, your piercing eyes
Will quickly strip the hero of disguise.
When youthful tyros at the busy school
Perform their problems by a stated rule,

Me then they have in view, and to obtain
My topmost place each nerve they eager strain;
By perseverance, with the prize in view,
The wished-for success smiles upon a few,
But not on all. At Oxford I am known,
At Cambridge too, where long indeed I've shone;
Advancing higher, I exalt the low,
Hence wealth and honour often I bestow.
Behold the florist, how with skill and care,
His tribes he ranges on the gay parterre!
And them surveys, and, with a master's eye,
In each and all of them can me descry;
The pistils, stamens, and the petals fair,
To him my properties at once declare.
Go to the railway station—there I'm seen,
There with my fellows often I convene,
There I to high and low alike am free,
Nay royalty itself has honoured me.
To human beings also I'm applied:—
O blame me not if some I fill with pride!
If others I depress, and labour hard
In me obtains for them a poor reward!
But praise my name, when I to you declare
That some contented all my blessings share.
Diarian bards, my simple lay is sung,
The chords are mute, the mystic harp unstrung:
The muse's pray'r preferred with breast sincere
Is "that in me you honoured still appear."

V. Enigma (1405); *by* Mrs. Baker, *Vauxhall*.

God spake the word!—This earth from chaos sprung,
And in yon concave, stars unnumber'd hung;
He form'd the greater and the lesser light—
To rule the day—to cheer the gloom of night.
And while the sportsman's bleeding victims die,
In yonder azure firmament am *I*.
But earth is my abode; there I am made
A general useful arbiter of trade.
Justice demands, wherever I appear,
That fair and open be my whole career,
For should I deviate from the upright way
My owner has the penalty to pay.
Then why should man for Mammon's guilty hoard,
Be tempted thus his neighbour to defraud?
Insatiate murder Hindostan beguiles,
While greedy Avarice looks on and smiles!
Bellona fiercely swells the gory flood,
And man remorseless sheds a brother's blood.—

Ah! when shall these dread scenes of horror cease,
And nations be again restor'd to peace?
In statics surely I may claim some praise;
I aid machinery in various ways,
And, firmly keeping honesty in view,
Am trusted as a servant just and true.
Your penetration, gentle bards, is such,
I fear I have already said too much;
Yet one hint more, before I close my rhyme,
Your watch goes well if I but keep my time.

VI. Enigma (1406); *by* Mr. James Herdson, *Tobermory.*

Ladies, my presence no intrusion deem,—
For long ere this I was the poet's theme;
Grant me a niche among your mystic bards,
Hear my plain tale,—allow my own rewards.
By art and nature furnish'd, to adorn,
I add new graces to the human form.
Do you demand my colour?—if you do,
Chamelion-like, I am of changing hue.
But muse, put on the drag, lest you reveal
Too soon my name,—your duty to conceal.
Where can you find so sure a friend as I?
Your confidence you grant,—on me rely;
But trust me not, for danger in me dwells,
And fatal accidents this sad truth tells;
Let not your carriage, as you jaunt abroad,
E'er get upon me in the tortuous road,
For, howe'er valuable I may be priz'd,
There's many a carriage been through me capsized.
Should I attack you as a fell disease,
And on your masticating members seize,—
Ah, cruel fate! then will "the grinders cease,"
And fled is food, and sustenance and ease!
No laughing matter,—yet 'tis said of one,
He will laugh at me, after all is done.
I am not going here, e'en to allege,
That I can vie with Hymen's endless pledge,—
No,—I repudiate any such a thing,
But yet, you'll oft observe me in the ring.
I've a trustworthy partner I must own,
To whom my veriest secrets all are known;
And he protects them too, in all respects,
As the Lord Chancellor his charge protects;
But, in these cut-throat and pick-pocket days,
A base intruder sometimes me betrays.
Now, finally, the flimsy tale to end,—
To navigation I'm a well-known friend;

In me the sportsman, and the soldier bold,
The fatal causes oft of death behold.
Reader,—ere this, no doubt my name you see,—
Th' unguarded muse has given you the key.

VII. Enigma (1407); *by* Mr. Joseph Furniss, *Mortivals, Takely, Dunmow* (*late of Lois Weedon*).

Ladies and gents, though coming in disguise,
'Tis right that I should first apologise;
For where apology is needed, know,
With me indeed it must be doubly so.
'Tis true I may be loving, gentle, kind,
Though quite as often the reverse you'll find:
For sudden, quick, and powerful do I come,
And desolate how many a happy home;
Prostrating mortals—age and youth must bend,
Nay, death itself may often be the end.
The time has been when kings have felt my power,
And dreaded, ah! how much, the coming hour.
What sights and scenes of blood have followed *me*,
What weeping, wailing, and what misery!
And many have condemned me, dating from
The thrilling narrative of "Uncle Tom."
But be it far from me to cause alarm,
Here, in dear Di, at least I shall not harm;
And passing onward—hark! a well-known sound
Makes "creeping schoolboys" scamper o'er the ground;
The urchins hear me, and they take to heel
Lest they not only hear me, but should *feel*.
Their dread am I, but with a task well done
Their commendation in more ways than one.
I am pre-eminent from earliest time,
In oratory, logic, lore, and rhyme;
Bear witness, Dia, of that simple fact,
Where wit and genius together act.
Nor am I stranger to the peasant poor,
Who labours for his bread, and earns no more;
Toiling with sweat on brow and jaded, he
For his poor pittance haply trusts to me.
For want of space, perchance, I may not dwell
On all my various uses, nor to tell
The somewhat "trite old tale" of black and white,
Straight—crooked—zigzagg'd—tortured, left and right;
Master of arts in objects rich and rare,
And crowning effort oft—but I forbear;
Lest after bragging you should see at once,
I'm but the acquisition of a dunce.

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VIII. ENIGMA (1408); *by* Mr. JOSEPH HUTCHINSON, *near Halifax.*

Diarian bards, and ladies of the lyre,
Whose annual strains our hearts and hopes inspire,
Whose ever-varying themes alike engage,
Aspiring youth and hoary-headed sage;
Leave, for awhile at least, your loftier lays,
For reminiscences of early days.
O, happy days of innocence and bliss!
Of pure delight and fond affection's kiss,
When I was present, ever quick to move,
In active service and my value prove;
And who would not in those elysian hours,
Appreciate my charms and own my powers?
But, ah! how short and fleeting earthly joys,
We change the scene and other feelings rise.
For now some victim of mishap demands,
The surgeon's mystic art and skilful hands,
While bandages and plasters serve to trim,
The dislocated joint or fractur'd limb;
And I not less important service lend,
Nor fail the suffering patient to befriend.
Again, you may behold me in the slips,
Where busy artizans construct our ships,
Where England proudly rears her wooden walls,
As commerce flourishes, or danger calls;
And when in port or dock with shatter'd sails,
The batter'd bark is vamp'd for future gales,
I may be seen above the swelling tide,
With those who fearlessly in me confide.
But leaving scenes at home for distant shores,
Where bold adventurers seek Australia's stores,
Or California's glittering mines beguile
The reckless wanderer and reward his toil,
There, while by fortune flush'd or hope inspir'd,
He finds my services at times requir'd;
And whether here or there, you'll often see
That valued treasures are consign'd to me;
Nay more—a hint which you will not contemn,
I'm oft the casket for a priceless gem;
Which must suffice, so take it as a cue,
And till another year, kind friends, adieu!

IX. ENIGMA (1409); *by* Miss HELEN OGDEN, *Shaw.*

Ye honour'd bards of Dia's mystic page,
May I a moment of your time engage,
Just to enrol upon your page of fame
My highly honour'd and distinguish'd name?
When heavenly wisdom form'd in beauty rare,
Each wond'rous tenant of the earth and air,

Each fragrant flow'r that blooms upon the plain,
Each varied native of the roaring main,
His purpose to fulfil, his will to serve,
That man, his noblest creature, may observe,
Th' unerring harmony that crowns the whole,
And reigns triumphantly from pole to pole:
But let him not be here presumptive thought,
To question why his sacred wisdom wrought
A goodly number who, like me, are found
Of little use amid the busy round,
That serve his wants in ev'ry varied way,
Or prove obed'ent to his pow'rful sway;
Amid that class undoubtedly 'tis mine
In stately mien majestically to shine;
Of ancient fame—since sacred records tell
How I before a youthful warrior fell;
How in the court of Babylon 'twas mine,
To show forth clemency the most divine.
Yet I with man in this enlighten'd age,
Am sad and direful conflicts known to wage;
By him subdu'd, and by him captive led,
By him caress'd, and by him daily fed.
But though his kindness I appreciate,
I'm no admirer of the servile state;
For if captivity around me throw
Its galling chain, I'm seldom found to bow;
More bent to roam at liberty, or stray
In independent pride the livelong day;
And though my movements may to earth be bound,
Exalted high amid a group I'm found,
Who long have held a most distinguish'd place,
Assign'd by men of scientific race.
But let me now revert to Albion's isle,
Where freedom wears a most enchanting smile;
With her behold me held in high renown,
In close connexion with her honour'd crown;
Her warlike sons, whose prowess in the field
Has noblest rivals often made to yield,
Exulting boast, amidst the conflict dire,
Of courage dauntless, and heroic fire;
While blazon'd forth throughout the world we see
Them claim a close affinity to me.
But here adieu! let history's page reveal
The tale of war I here would fain conceal.

X. PRIZE ENIGMA (1410); *by the* REV. JOHN HOPE, *Stapleton.*

Respected nymphs of Dia, here awhile,
In mystic garb, I court your pleasing smile.

Unsullied by a spot, I often seem
As pure as virgin Luna's silv'ry beam;
Free from reluctance, still I act my part
In common things, as well as finest art,
My part I say, for I must here record
The freely-urged assistance I afford.
On looking back I cannot now declare,
When first I had in useful works a share;
'Twas not in Eden;—on the sacred page
I never once your searching eyes engage;
Was I unknown in Jewry? Did the bard
Of ancient Greece my fortunes disregard?
The bard whose noble, ever-pleasing lays
Raised fair Penelope to highest praise;
Gave to Andromache, the Trojan dame,
For household duties a distinguished name,
Whilst I, alas! o'erlooked amid the throng,
Had no renown assigned in Ilian song.
But, ladies, now—forgive my honest pride—
Neglect of me has long been laid aside;
I have my honours, but I must confess
My want of total freedom from distress;
A ceaseless course of insult oft is mine,
When pushed with darts I audibly repine;
Not always so,—at times the quickest ear
Can not the slightest murmur from me hear;
When I'm employed in costly dome or cell,
Both life and motion in my bosom swell;
To all the gentler sex I'm most a friend,
Yet I to lordly men assistance lend;
In palaces, in splendid halls I'm found,
In meanest cots, the rural districts round;
But blame, O blame me not, if I deceive
By magic arts, and leave my dupes to grieve!
Now for my form—a threefold one I own,
Such have mythologists Diana's shown;
I am a thoroughfare, extended plane,
But mostly I all passage through restrain;
I'm wholly blind, yet Argus could not boast
Of eyes around his head so large a host!
Be not astonished, if upon my face
At times you curious hieroglyphics trace;
Or kindly sentences impressive stand,
Dictated more by friendship than command:
"Forget me not;" "A tribute of regard;"
"The hand of diligence has due reward:"
I add no more; for now your piercing eyes
Will quickly penetrate my slight disguise.

PRINTED FOR THE COMPANY OF STATIONERS.

NEW CHARADES, REBUSES, &c.

1. Charade; *by the* Rev. John Hope, *Stapleton.*

My *first* is a verb of irregular kind,
And, ladies, it is not a very long word;
My *next* will disclose it, if needful you find
With candour to own that you are not my *third.*

A schoolmaster once did a question propose
To a pupil considered a little remiss;
The boy, who was sitting, exclaimed as he rose,
"My *whole* sir, to answer such questions as this?"

See yonder young lady that's tripping along;
She's said to possess an ingenuous soul;
She appears to advantage her fellows among,
There is not a doubt of her being my *whole.*

2. Charade; *by* Mr. Joseph Hutchinson, *near Halifax.*

My *first* in the forest I'd have you beware,
But fear not its presence aloft in the air;
My *next* you discover in brute and mankind,
If you look for it fairly above or behind;
My *total,* fair ladies, be slow to believe,
Fair looks and fine speeches too often deceive.

3. Rebus; *by* Mrs. Baker, *Vauxhall.*

Short was my life, and brilliant my career;
Behead me, I in lovely green appear;
Behead again, I once was made to save
My chosen inmates from a watery grave.

4. Charade; *by* Mr. James Herdson, *Tobermory.*

My *first* you'll recognise perchance,
The scene of many a rural dance;
My *next* in London streets you'll view,
A slight impediment 'tis true;
In London, too, my *whole* attends,
Is one of Dia's welcome friends.

5. Charade; *by the Same.*

My *first* o'er hill and pasture roves,
'Midst choicest verdure, sweetest flowers;
My *next* proceeds from Flora's groves,
A terror to the evil-doers.
The vict'ries on Crimean field,
Which wing'd our prowess to the pole,
Proclaiming that our arms ne'er yield,
Had ne'er been won without my *whole.*

6. Rebus; *by the* Highland Lassie.

I'm lovely, an' charming, an' sweet, an' a' that,
Yet spitefu' eneugh when opposed,
For I've talons as sharp as the claws of a cat,
Which wad gi'e you mysel' when transposed.

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7. CHARADE; *by* Mr. G. H. BUTLER, *Dalston, London.*

My *first* is the earnest desire of all classes,
Of single young men, and of unmarried lasses,
And for it full many will cheerfully roam,
Many thousands of miles from their own native home;
The stockbroker, gambler, the lawyer, and thief,
Make this of all other achievements the chief.
My *second*'s a nice little word and a letter,
The egotist thinks there was never a better;
My *third* is a name, or a part of one rather,
That perchance may belong to your brother or father;
My *whole* is well known on Di's time-honour'd page,
And beloved by Diarians of every age.

8. CHARADE; *by* Mr. THOMAS BOWMAN, *Richmond, Yorkshire.*

In yon rural vale where the streamlet is flowing,
Reflecting my *first* on its bosom so bright,
My *second* the beauties of nature is showing;
My *whole* cheers the darkness and silence of night.

9. CHARADE; *by* CLERICUS.

Our cooks are capricious, my *first* is half done,
And epicures tell us that "Soyer's the Text,"
Opinions will vary, and therefore with some,
The contention will frequently lead to my *next.*
My *whole*'s a distinction, to the ladies assigned,
An attainment to which they are seldom inclined.

10. CHARADE; *by* Mr. THOMAS ALLANSON, *of Rillington.*

My *first* and *second* you will find,
Are reckon'd primal of their kind;
My first enlightens, but the other
Holds comp'ny with a knavish brother;
May you when in affliction's hour,
Find consolation from my power.

11. CHARADE; *by* Mr. J. NODWONS, *Murah.*

Both night and day my *first* is us'd,
By neither sex is it refused,
'Tis very useful reckon'd;
And decorated, I dare say,
Exposed for sale you often may
Have seen it in my *second.*
My *whole* is what? tell I shall not,
And if you can't, of it you're short.

12. CHARADE; *by* Mr. JAMES HEWITT, *Hexham.*

Oh! if you were my *first*, what a face you'd be making!
Which you might be in figure, as well as reality;
My *next* is the last of you—do not be quaking!
The muse refers not to the pains of mortality.

My *whole* is described, a "fat, unctuous matter,"
Much used by my *last*, as you may sometimes have smelt,
Oft combined with a substance well-known to the batter,
And on such occasions we *both* may be *felt.*

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ANSWERS TO THE QUERIES.

I. QUERY; *by* Mr. JAMES LUGG, *Grampound, Cornwall.*

What was the origin and import of the term "Hue and Cry?"

Answered by Mr. JOHN WHITE, *Holly Terrace, Birmingham.*

Hue and Cry was the old common-law process of pursuing with "horn and with voice" all felons and such as had dangerously wounded another, from hundred to hundred, and county to county; formerly the "*hundred*" was bound to make good all loss occasioned by the robberies therein committed unless the felon were taken.

Though the term has in a great measure fallen into disuse, the process is still recognised by the law of England as a means of arresting felons, without the warrant of a justice of the peace; when "*Hue and Cry*" is raised, all persons are bound to join in the pursuit, and assist in the capture of the felon. A constable who has a warrant against a felon may follow him by "*Hue and Cry*" into a different county from that in which the warrant was granted, without having the warrant backed.

The pursuit of a felon was aided by a description of him in the "Hue and Cry," a gazette established for advertising felons in 1710.

It was also thus answered by Messrs. Thomas Hattam, Hewitt, James Herdson, Thomas Jackson, John Wallis Mulcaster, and William Oats.

II. QUERY; *by* Mr. J. WHITE, *Holly Terrace, Birmingham.*

Can the terms "Case" and "Declension" be legitimately applied to the English language?

Answered by Mr. J. WHITE, *the Proposer.*

The Stoics considered the relation which in discourse a noun hath to a verb in the same member of a sentence with it, under the figure of a right line falling upon a plane. If the line fell perpendicularly, the noun was said to be "in recto casu," or "ἐν ὀρθῇ πτώσει;" that is, in its right or straight (fall) case, by which they meant the nominative; but if the line swerved or *declined* from the perpendicular, then the noun was said to be "in obliquo casu:" this derivation from the perpendicular was termed "declinatio" (declension). Now the right case could be only one, while the oblique cases might be few or many according to the degree of declination or declension.

However inappropriate these terms may appear, grammarians have very good naturedly contented themselves to retain them.

On the principle of imitating the Stoics in names and forms, without a correspondence in nature and idiom, English grammarians have adopted the terms *case* and *declension*.

Now case, according to Dr. Johnson's definition, signifies "the variation of nouns;" that is, the several changes which the noun undergoes in the Latin and Greek tongues (and in some of the modern languages, as the German), in the several numbers are called *cases*, and are designed to express the several views or relations under which the mind considers things with regard to one another; and the variation of the noun for this purpose is called declension. Now we do not vary the noun (properly speaking), therefore neither *case* nor *declension* can be legitimately applied to the English language, although utility may countenance the use of such terms.

Dr. Beattie says, "If cases are to be distinguished by the different significations of the noun, or by the different relations it may bear to the governing word, then we have in our language as many cases almost as there are prepositions, as above a man, about a man, beneath a man, beyond a man, within a man, without a man, &c., shall be cases, as well as, of a man, to a man, and with a man." Instead, therefore, of case and declension, we use prepositions.

Dr. Beard says, that we are justified in declaring, that in English there is no *dative* case, no *accusative* case or *objective* case, no *vocative* case, no *ablative* case; and instead of the nominative case, we may with advantage speak of the *subject*. It thus appears that the only instance in which case appears to be at all applicable, is in the "possessive;" or, more properly, the "Saxon genitive," and even that may in most ways be expressed by the preposition *of*.

"On the whole, therefore, I think it more accordant with fact and good sense, as well as more conducive to intelligibility, to discard the term case altogether."

It was answered in like manner by Mr. Thomas Jackson, Felling School, Gateshead.

Again, by the Rev. JOHN HOPE, *Stapleton.*

I see no reason why this Query should not be answered in the affirmative. I here repeat what I said in answering Query I, last year's Diary: that the case of a noun depends as much on its situation in a sentence as upon change of termination. I mean a noun has different cases according as it is acted upon by other words, though remaining under the same form. This is evident even from the Greek and Latin; for in one Latin declension we find six cases all of the same form, and in the others from three to five. This is surely sufficient to establish the point for which I am contending. I will give an example of case and declension in four different languages, as many as there is room for in the Diary. Taking the word, which is in Hebrew, מֶלֶךְ, (melech); Greek, βασιλευς; Latin, *rex;* and, English, *king;* the declension—taking only the singular number—stands as follows:

Nom.	Melech	βασιλευς	Rex	King
Gen.	Shel-melech	βασιλεος	Regis	King's
Dat.	Le-melech	βασιλει	Regi	To King
Acc.	Eth-melech	βασιλεα	Regem	King
Voc.	Ham-melech	'Ω βασιλευ	O Rex	O King
Abl.	Min-melech	βασιλει	Ab Rege	From King.

Thus it appears that our mode of declension is nearly similar to the Hebrew. The Greeks and Latins—as above alluded—were far from varying the terminations of their nouns universally: thus *gradus*, a step; *gradus*, of a step; *gradus*, O step; *gradus*, steps; *gradus*, acc., steps; *gradus*, O steps. Now, according to the principle adopted by English grammarians—at least by some of them—these would be all considered as one case; but did the Romans consider them so? They did not: they were shown to be different cases by their position in the sentence. According to the principle of most of our grammarians, one might almost prove that the English has only one case, or even no case at all! for the poss. is merely the addition to the noun of the final *s* of *his*, *hers*, *its*, or *theirs*. An attempt at such proof would, however, be absurd. Even Murray admitted three cases of nouns, and he might have gone further, though his objective does well enough for three, and I find no fault with it; yet it is easy to see that, *to a man*, *O man;* and *from*, *by*, *with*, *in a man*, are legitimately the dat., the voc., and the abl. cases of an English substantive.

Third Answer, by the Rev. A. Drysdale, *Bishopwearmouth.*

Giving an affirmative answer to this question, we submit the following observations. The terms *Case* and *Declension*, as used in reference to the ancient languages, may be thus explained. Between case and declension there exists an intimate but not an invariable connection. There are instances in which we have case without declension, as in the singular of Latin neuter nouns in U; but on the other hand, whenever we have declension, it necessarily follows that we have case. Now CASE implies a certain relation between a noun, or pronoun, adjective, or participle, and some other word occurring in the same clause, sentence, or context; and, DECLENSION consists of those terminational differences to which the noun or other declinable part of speech is subjected in particular circumstances. In like manner, English nouns and pronouns have the characteristics of case and declension. These, it must be admitted, have *case*, because they are sometimes employed as subject, sometimes as possessor, and sometimes as object. The nomenclature of the cases is not now the question, but that there *are* cases is proved by that variety of relationship the existence of which cannot surely be denied. Besides case there is also *declension*, that is, modifications of termination. Our language cannot boast of much complexity in this respect, but still its nouns and personal pronouns (especially the latter) undergo variations. Thus, then, we are authorised in applying the said terms to the English language; and it may be added, that if we are not authorised to do so, then it must follow that no language, whether ancient or modern, has either case or declension.

Miss Mary —s, West of Cornwall, and Messrs. Brady, Hattam, Hewitt, and Mulcaster gave answers to the same effect.

III. Query; *by* Mr. James Herdson, *Tobermory.*

Whence was it that money obtained the very common but cant term of "*tin?*"

Answered by Mr. James Herdson, *the Proposer.*

This island has been famous for its tin mines from the remotest period of history; and would not, probably, have been frequented by the ancient Phœnician navigators, if they had not been attracted hither by the great plenty of tin with which it abounds. Several etymologists have endeavoured to show that the name of Britain is derived from a word common to the Syrian and Chaldean languages, denoting *tin;* but on this no great stress can be laid. Tin is, however, one of those metals which were earliest known; and the history of the trade in tin commences with the very earliest records of commercial intercourse with the British isles. In 1684, tin farthings were coined, with a stud of copper in the centre. Others, as well as halfpence of the same metal, were struck by James II, and William and Mary. In 1693 the tin was called in, and copper renewed. Davenant, who wrote upon this subject, informs us that Queen Anne had between 4000 and 5000 tons of tin on hand; a quantity equal to four or five years' consumption. He proposed that 1000 tons of the dead stock should be coined into tin halfpence and farthings. It is probable, therefore, that money obtained the name of *tin* from this origin; a synonymous name for money, the same as "copper," "brass," and "silver."

Similar answers were given by Messrs. Jackson and Mulcaster.

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Again, by Mr. John White, *Holly Terrace, Birmingham.*

It is always difficult to trace words whose application, like this, is so much at variance with its etymology; they often take their origin from among a class of persons who are styled "fast young men," London being particularly fertile in such like terms and phrases.

We might claim for the date of its origin the reign of James II, when in 1685, tin farthings and halfpennies were coined, and even pewter was similarly made use of.

Wright, in his 'Dictionary of Obsolete and Provincial English,' (just published), gives it thus: Tin, (1) money, (2) *preposition*, till; and its origin in Cheshire.

I am, however, not disposed to attach its origin to either of the above inferences, but rather believe it to have originated with a class of *respectable* thieves, as it is a well-known fact they have a dialect peculiar to themselves, particularly in relation to money; as a £5 note is a fineffe or flimsey; a sovereign is a quid, or cooter; a five-shilling piece is a bull; a shilling is a bob; a sixpence is a tanner or tizzey; a fourpenny piece is a joey, &c., and even a £50 note is a monkey; and why may not "Tin" be for money collectively among these *respectables;* but it would be difficult to state precisely when this class first assumed the title of "respectable," and a nomenclature of their own.

It was thus answered by Mr. James Hewitt, of Hexham.

IV. Query; *by the* Rev. John Hope, *Stapleton.*

What conclusions can be drawn from (Gen. i, 2)—"And the earth was without form and void, and darkness was upon the face of the deep?"

Answered by the Rev John Hope, *the Proposer.*

In answering this query, it is necessary to give the original Hebrew,

וְהָאָרֶץ הָיְתָה תֹהוּ וָבֹהוּ וְחֹשֶׁךְ עַל־פְּנֵי תְהוֹם,

that is, Vehaarets hayetha thohu vebohu, vechoshek al-penai thehom. The words *thohu vebohu*, "without form and void," are not often found elsewhere in the Bible. *Thohu* occurs 1 Sam. xii, 21, where it is translated *vain*, to which the word *things* is added; again in Isaiah xxiv, 10, where it is turned *confusion.* The two words appear together Isaiah xxxiv, 11, and stand, in our version, *confusion* and *emptiness.* They are again together, Jer. iv, 23, where they are applied to the earth, as in the passage in question, and translated by the same words. Whether the expression "without form and void" gives an exact notion of the state of things intended to be conveyed, I cannot pretend to determine. Perhaps the idea designed was that of a heterogeneous, confused, and *unformed* mass, in which things were not arranged in their proper places, or in regular order; and "void," or empty, that is without any organized productions. This agrees with the "chaos," or "rudis indigestaque moles" of Ovid, where

"Nulli sua forma manebat,
Obstabatque aliis aliud; quia corpore in uno
Frigida pugnabant calidis, humentia siccis,
Mollia cum duris, sine pondere habentia pondus."

Our conception of this state of things is a dismal one, and it is rendered still more dismal by our being informed that there was no light;—"darkness was upon the face of the deep;" where *thehom*, "deep," is what we call an *abyss*, or great depth, of which water seems to have been the uppermost covering; for the context says, "The Spirit of God moved upon the face of the waters." Now this chaos, this πρωτον θεῶν of the ancient heathens, the rudiments, as it were, of creation, it was the work of HIM who said, "Let there be light, and there was light," to form gradually into this beautiful and life-bearing planet which now affords support and enjoyment to about a thousand millions of human beings, and lower animals innumerable.

Answers agreeing in substance with the above were likewise given by Miss M. N. Dawe, Landulph, Cornwall, the Rev. Thomas Brady of Senica Falls, Ego of Durham, and Messrs. Hattam, Hewitt, Jackson, Mulcaster, Nattrass, Porter, and White.

V. QUERY; *by* MR. WM. GIBSON, *Whittonstall.*

What is the cause of the strong winds which attend showers, especially when they come from the north?

Answered by MR. JOHN WHITE, *Holly Terrace, Birmingham, and* MR. JAMES HEWITT, *Hexham, Northumberland.*

Winds in general are propagated by compression, and any physical cause which produces a compression of the atmosphere from north to south will generate a north wind.

The atmosphere above us usually consists of a mixture of air properly so-called, and water, either in the state of vapour, or in a vesicular state. In either case its sudden conversion into the liquid state, and its consequent precipitation to the earth either in the shape of snow, hail, or rain, leaves the space it occupied in the atmosphere a partial vacuum, and a corresponding rarefaction of the air previously mixed with the vapour ensues; the adjacent strata immediately rush in to establish the equilibrium of pneumatic pressure, causing commotions and rapid movements of the air, and winds are consequently produced.

Second Answer, by MR. JOHN WALLIS MULCASTER, *Allendale, Northumberland.*

A change in the warmth of the weather, says Dr. Derham, is generally followed by a change of the wind; thus, the northerly, and southerly winds, commonly esteemed the causes of cold and warm weather, are really the effects of the cold or warmth of the atmosphere. Again, it is known that cold contributes greatly to rain, apparently by condensing the suspending vapours, and so causing them to descend; thus, very cold months or seasons are generally followed immediately by very rainy ones, and cold summers are always wet ones. Therefore if a north wind convey intense cold, and cold be one of the principal causes of rain, we may safely infer that strong winds will attend showers, and that they will come principally from the north.

Again, by MR. THOMAS HATTAM, *Eddystone Lighthouse, near Plymouth.*

When a condensation of the vapour in the atmosphere suddenly takes place, it gives rise to clouds which speedily conglomerate into rain, and the temperature of the surrounding atmosphere being altered, the colder wind rushes in upon the warmer, and causes a sudden gust, which the sailors designate "a squall."

From my own observations at this and other lighthouse establishments, during the last ten years, I find that these "squalls," or "strong winds that attend showers," almost invariably come from the north-west, and not from the north, as mentioned in the query.

It was thus answered by Ego, of Durham, and Mr. Thomas Jackson, Felling School, Gateshead.

VI. QUERY; *by* CANTAB, M.A., *Sevenoaks.*

What simple rules are there for an amateur astronomer to know where in the heavens to look for *Mercury*, *Venus*, *Jupiter*, and *Saturn?*

Answered by Mr. JOHN WALLIS, *Mulcaster, of Allendale.*

The planets are easily distinguished from the fixed stars, by means of their steady and uniform appearance; for while the latter appear to be incessantly twinkling, the former exhibit a fixed and steady light, but as they are (unlike the stars) continually changing their places, no practical rules can be given to know whereabouts in the heavens to look for them, yet the following observations may be of some assistance.

MERCURY, on account of his proximity to the sun, and the smallness of his apparent magnitude, is seldom seen by the naked eye, being generally hid in the effulgence of the solar rays; but in low latitudes it may at certain periods be seen a little after sunset, and again a little before sunrise, when it will emit a bright whitish light.

VENUS is easily known by her superior brilliancy, being the brightest and most refulgent of all the planets.

MARS is distinguished by his fiery red tinge of colour.

JUPITER is the largest of all the planets, and may be readily distinguished from the fixed stars by means of his splendour and magnitude, appearing to the naked eye almost as brilliant as Venus, but with a larger disc.

SATURN is the most remote planet from the earth of any that are visible to the naked eye; it may be known from the fixed stars by means of its pale and steady light.

Mr. James Hewitt, of Hexham, gives a similar answer, and properly refers to the second page of the Diary for the astronomical information that is most needed.

I. QUERY; *by* Mr. THOMAS HATTAM, jun., *Eddystone Lighthouse.*

Whence, and when was it, that the "Language of Flowers" came into use, and what was the cause of its invention?

II. QUERY; *by the* Rev. JOHN HOPE, *Stapleton.*

Explain critically the words (Gen. i, 2), "The Spirit of God moved upon the face of the waters."

III. QUERY; *by* CANTAB, M.A., *of Sevenoaks.*

What is the exact meaning of the Scripture term, "a contrite heart?"

IV. QUERY; *by* Mr. J. WHITE, *Holly Terrace, Birmingham.*

Give the etymology of the word Cant; and show the justice of its application in Query III. last year.

V. QUERY; *by* Mr. JAMES HERDSON, *Tobermory.*

What is the cause of, and what the cure for, "sneezing?"

VI. QUERY; *by the same.*

What is the origin of the term "bachelor?"

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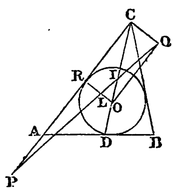
ANSWERS TO THE MATHEMATICAL QUESTIONS PROPOSED LAST YEAR.

I. QUEST. (1914); *by* Mr. ROBERT AMBLER, *Grammar School, Stevenage.*

Given the sum of the three sides, the line bisecting the vertical angle, and the area a given space, or a maximum, to construct the triangle.

Answered by Dr. RUTHERFORD, *of the Royal Military Academy, Woolwich.*

Analysis. Let ACB be the triangle, O the centre of the inscribed circle, COD the line bisecting the vertical angle, and OR a radius perpendicular to AC. Produce CA to P making CP equal to the given semiperimeter of the triangle; through O draw OQ parallel to AC meeting CQ drawn perpendicular to AC in Q, and join PQ intersecting CD in I; then CD will be bisected in I, as may be proved in the following manner. Because CP is equal to the semiperimeter of the triangle, and PR equal to AB; therefore $AC + CB = CP + CR$, and if PQ intersect OR in L, the triangles CIQ and OIL will be equiangular. Now as the lines joining AO and BO bisect the angles CAB and ABC, it is evident that



$$CO : OD :: AC + CB : AB :: CP + CR : PR;$$

and componendo, $CO : CD :: CP + CR : 2CP$...............(1).

Again $CP : CR$ or $OQ :: CQ : OL :: CI : IO$;

and componendo, $CP + CR : CP :: CO : CI$...............(2).

Compounding the ratios in (1) and (2), we get at once $CD = 2CI$, and therefore CD is bisected in I by the line PQ. This property suggests the following neat and simple construction.

Place CP and CQ at right angles to each other, making CP equal to the given semiperimeter, and CQ equal to the radius of the inscribed circle which is given, because the area is given; draw PQ and from C apply to PQ a line CI equal to half the given bisecting line. Produce CI to D making CD equal to twice CI; draw QO parallel to CA meeting CD in O, the centre of the inscribed circle. With centre O and radius CQ describe a circle touching AC, and through C and D draw the tangents CB and AB, completing the triangle ABC. The demonstration is evident from the preceding analysis.

When the area is a maximum, the radius of the inscribed circle is a maximum, and it must therefore coincide with OD; therefore the bisecting line CD will be perpendicular to AB, and the triangle isosceles. In this case, CAD is a right-angled triangle, of which one side CD is given, and the sum of the other two, CA + AD, is given. With these data the triangle ACD is easily constructed, and thence the whole triangle ABC.

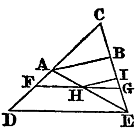
Similar answers to this question were given by Messrs. J. Buttery, T. Buttery, Collins, Dobson, Light, M'Cormick, M'Namara, Rutter, Ryan, Traynor, and Turnbull; and good analytical solutions were also given by Messrs. Brooks, Bull, Collins, Dobson, Hill, Miller, Mulcaster, Robinson, and Watson.

II. QUEST. (1915); *by* Mr. DANIEL WHITE, *of the Educational Institute, Newcastle-upon-Tyne.*

Given AB + BC and AB + AC, together with the angle C, it is required to construct the triangle ABC geometrically.

Answered by Dr. RUTHERFORD.

Let CD and CE include the given angle, and be equal respectively to the given sums AB + AC and AB + BC; draw DE, and parallel thereto draw any line FG. Take EI on EC equal to DF, and apply IH to FG from I, so that HI equal DF or EI; draw EHA meeting CD in A, and then draw AB parallel to HI; then ABC is the triangle required. For since FH is parallel to DE, and AB to HI, we have the three ratios DF to DA, EI to EB, and HI to AB, all equal to the ratio EH to EA; but the antecedents of these three equal ratios are all equal; therefore the consequents DA, EB, and AB are all equal; hence CD is equal to AB + AC, and CE to AB + BC.



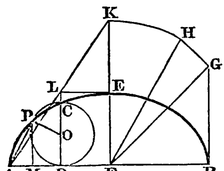
Note.—This solution is similar to that given to this question in Bland's 'Geometrical Problems,' Section 9, p. 307.

It was thus answered by nearly the whole of our correspondents.

III. QUEST. (1916); *by* Mr. STEPHEN WATSON, *Haydonbridge.*

Let AB be the transverse diameter of an ellipse; CD an ordinate, upon which, as a diameter, a circle is described; and AP a tangent to the circle, touching it in P. Determine CD when the point P is on the ellipse.

Answered by Dr. RUTHERFORD, Mr. THOMAS DOBSON, B.A., *Royal Nautical School, Greenwich, and* Messrs. BROOKS, BUTTERY, COLLINS, M'CORMICK, M'NAMARA, RYAN, TRAYNOR, WATSON, *and* TURNBULL.



Let O be the centre of the circle on CD, draw OP, and also PM perpendicular to AB. Take A for the origin of coordinates, and let xy, $x'y'$ be the coordinates of C and P respectively; then since C and P are points in the ellipse, and the tangents AP and AD are equal, we have the following equations and conditions:

$$a^2y^2 = b^2(2ax - x^2) \ldots\ldots(1); \quad a^2y'^2 = b^2(2ax' - x'^2) \ldots\ldots(2);$$
$$(\tfrac{1}{2}y)^2 = (x - x')^2 + (y' - \tfrac{1}{2}y)^2 \ldots\ldots(3); \quad x^2 = x'^2 + y'^2 \ldots\ldots(4).$$

From (3) and (4), we have readily $2x(x - x') = yy'$, and eliminating $x - x'$ from this and (3) we have

$$\left.\begin{array}{l} y' = \dfrac{4x^2y}{4x^2 + y^2}; \quad y - y' = \dfrac{y^3}{4x^2 + y^2}; \\ x' = x\,\dfrac{4x^2 - y^2}{4x^2 + y^2}; \quad x - x' = \dfrac{2xy^2}{4x^2 + y^2}. \end{array}\right\} \ldots\ldots\ldots(5).$$

Now subtracting (2) from (1), we get

$$a^2(y + y')(y - y') = b^2(x - x')\{2a - (x + x')\}$$

or, $\quad a^2y(y + y') \quad = 2b^2x\{2a - (x + x')\}$, by (5).

Writing the values of x' and y' from (5) in this equation, we obtain, by means of (1), the values of x and y, viz.:

$$x = \frac{ab}{(2a^2 + \frac{1}{4}b^2)^{\frac{1}{2}}}, \text{ and } y = \left\{\frac{2b^3\sqrt{(2a^2 + \frac{1}{4}b^2)} - b^4}{2a^2 + \frac{1}{4}b^2}\right\}^{\frac{1}{2}}.$$

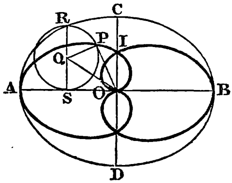
Cor. A very simple geometrical construction may be derived from the value of x. Produce the semiminor axis FE, and take in it the point K, so that $FK^2 = FH^2 = FG^2 + GH^2 = 2FB^2 + GH^2$, where $BG = FB$, and $GH = \frac{1}{2}EF$; draw AK, meeting EL parallel to AB in L, and then draw LCD perpendicular to AB, which determines the position of the ordinate CD; because AD is a fourth proportional to FK, FA, and EF or LD.

Analogous answers were given by *αδα*, Amicus, and Messrs. Hill, M'Namara Miller, and Robinson. Mr. Rutter solved the question geometrically.

IV. QUEST. (1917); *by* Mr. T. M'NAMARA, *Ballina, Ireland.*

From the centre of an ellipse let a tangent be drawn to a circle described on an ordinate to the axis major. Find the equation and area of the curve described by the point of contact.

Answered by Dr. RUTHERFORD.



Let O be the centre of the ellipse ACBD, and RS any ordinate on which as a diameter the circle RPS is described. Draw the tangent OP, and PQ, OQ to the centre Q; then OPQ is a right angle. Let $OP = OS = r$, and angle $AOP = 2\theta$; then $QOS = \theta$; $RS = 2QS = 2r\tan\theta$, and the equation to the ellipse gives

$$4a^2r^2\tan^2\theta + b^2r^2 = a^2b^2, \text{ and } r^2 = \frac{a^2b^2}{b^2 + 4a^2\tan^2\theta}.$$

This is the polar equation to the curve, and if

$$\theta = 0, r = \pm a; \quad \theta = 45^\circ, r = \pm\frac{ab}{\sqrt{(4a^2 + b^2)}}; \quad \theta = 90^\circ, r = 0;$$

therefore the curve passes through A and O, and intersects the minor axis at a distance $OI = \dfrac{ab}{\sqrt{(4a^2+b^2)}}$.

The differential of the area AOP $= \frac{1}{2}r^2 d(2\theta) = r^2 d\theta$

$$= \frac{a^2b^2 d\theta}{b^2 + 4a^2\tan^2\theta} = \frac{a^2b^2 d\tan\theta}{(1+\tan^2\theta)\,(b^2+4a^2\tan^2\theta)}$$

$$= \frac{a^2b^2}{4a^2-b^2}\left\{\frac{4a^2 d\tan\theta}{b^2+4a^2\tan^2\theta} - \frac{d\tan\theta}{1+\tan^2\theta}\right\}.$$

The integral of this is

$$\frac{a^2b}{4a^2-b^2}\left\{2a\tan^{-1}\left(\frac{2a}{b}\tan\theta\right) - b\theta\right\};$$

which, taken between the limits $\theta = 0$, and $\theta = 90°$, gives the area of the curve $APIO = \dfrac{\pi a^2 b}{4a+2b} = \dfrac{a}{4a+2b} \times$ ellipse, and the area of the four branches of the entire curve $= \dfrac{2a}{2a+b} \times$ ellipse.

It was also thus answered by $a\delta a$, Amicus, and Messrs. Brooks, Buttery, Collins, Dobson, Farmar, Hill, Miller, Mulcaster, Robinson, Rutter, Ryan, Traynor, Turnbull, and Watson.

V. QUEST. (1918); *by* Mr. B. Hooke, *London*.

A party of n persons have all different sums of money. The nth person having the largest sum, gives to all the others as much as each had, thereby doubling their respective sums; then the $(n-1)$th person gives to all the others as much as each had, and so on, until each person has made a similar distribution of his money. After which it is found that each had the same sum. Find general expressions for the sums held by each person throughout the several distributions.

Answered by 'A, *$a\delta a$, and* Messrs. Brooks, Bull, Buttery, Collins, Dobson, Eland, M'Cormick, M'Namara, Miller, Mulcaster, Robinson, Rutherford, Rutter, Ryan, Traynor, Turnbull, *and* Watson.

Let x_n denote the sum held by the n^{th} person, and ns the sum held by all of them; then s is the sum held by each finally. Now the n^{th} person, in order to double each of the sums held by the others, must give away $ns - x_n$, and therefore he has remaining $x_n - (ns - x_n)$ $= 2x_n - ns$, and this being doubled $(n-1)$ times before the whole of the distributions have been made, gives

$$s = 2^{n-1}(2x_n - ns) = 2^n x_n - 2^{n-1} ns;$$

$$\therefore x_n = \frac{s}{2^n}\left(1 + 2^{n-1}n\right) = \text{the sum held by the } n^{th} \text{ person.}$$

To avoid fractional sums, let $s = 2^n$; then $x_n = 1 + 2^{n-1}n$.

The last but one of the n persons has $2x_{n-1}$ before he distributes to the others, and after he has given away as much as doubles the sums held by all the others, he has remaining $2^2 x_{n-1} - ns$, and this being doubled $(n-2)$ times, gives

$$s = 2^{n-2}(2^2 x_{n-1} - ns) = 2^n x_{n-1} - 2^{n-2} ns$$

$$\therefore x_{n-1} = \frac{s}{2^n}\left(1 + 2^{n-2} n\right) = 1 + 2^{n-2} n.$$

In this manner, the sum held by each at first, and throughout the distribution, can easily be determined. The sum orginally held by each is the respective term of the series,

$$1+n,\ 1+2n,\ 1+2^2 n \ldots\ldots\ldots\ldots 1+2^{n-1} n.$$

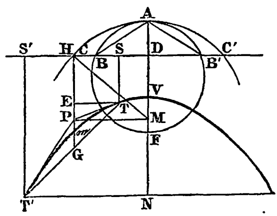
VI. QUEST. (1919); *by* Mr. THOMAS HINDLE, *Tarleton, Lancashire.*

From any point P let two tangents be drawn to a parabola, and from M, the foot of the ordinate PM, as centre, let a circle be described cutting off, from the directrix, a segment CC′ equal to the chord of contact TT′, and meeting the axis produced in A. Then if another circle be described on AF as diameter, through the focus F, and cutting the directrix in B, B′, the area of the triangle ABB′ will be equal to that of the triangle PTT′.

Answered by Dr. RUTHERFORD.

Let V be the vertex of the parabola, and draw TS, T′S′, PH perpendicular to the directrix, and let D be the point of intersection of the axis and directrix. Draw MC, and since the diameter HPG bisects the chord TT′ in G, we have

$$PTT' = 2PTG = PG \cdot HS \ldots\ldots\ldots (1).$$



Let $x'y'$ and $x''y''$ be the coordinates of T and T′ respectively; then

$$y'^2 = 4mx'; \quad y''^2 = 4mx'',$$

and the equations to the tangents PT and PT′ are respectively

$$yy' = 2m(x+x'), \text{and } yy'' = 2m(x+x'').$$

Hence the coordinates of P, the intersection of these tangents, are

$$VM = \frac{y'y''}{4m}; \text{ and } PM = \frac{y'+y''}{2}.$$

$$\text{But } PG = HG - HP = \frac{x'+x''}{2} + m - \left(\frac{y'y''}{4m} + m\right) = \frac{(y'-y'')^2}{8m},$$

$$\text{and } HS = \frac{y'-y''}{2};$$

therefore, by (1)

$$PTT' = PG \cdot HS = \frac{(y'-y'')^3}{16m} \ldots\ldots\ldots (2).$$

Again, the chord

$$TT' = \{(x'-x'')^2 + (y'-y'')^2\}^{\frac{1}{2}} = \frac{y'-y''}{4m}\{(y'+y'')^2 + 16m^2\}^{\frac{1}{2}}$$

$$\therefore MA^2 = MC^2 = MD^2 + CD^2$$

$$= \left(m + \frac{y'y''}{4m}\right)^2 + \frac{(y'-y'')^2}{64m^2}\{(y'+y'')^2 + 16m^2\}$$

$$= \left(m + \frac{y'^2 + y''^2}{8m}\right)^2; \text{ hence } MA = m + \frac{y'^2 + y''^2}{8m}.$$

Consequently, $AD = MA - MD = \frac{(y'-y'')^2}{8m}$, and $DF = 2m$;

$$\therefore BD^2 = AD \,.\, DF = \frac{(y'-y'')^2}{4}, \text{ and } BD = \frac{y'-y''}{2} = HS.$$

Hence the triangle $ABB' = AD.DB = \frac{(y'-y'')^3}{16m} = PTT'$ by (2).

Similar answers were given by 'A, $\alpha\delta\alpha$, and Messrs. Brooks, Buttery, Clazey, Dobson, M'Namara, Ryan, Turnbull, Traynor, and Watson.

Again, by Mr. JOHN JOSHUA ROBINSON, *Portsea; and* Messrs. COLLINS, DOBSON, M'CORMICK, MILLER, *and* RUTTER.

Constructing the figure as stated in the question, draw PH perpendicular to the directrix, meeting the curve in m, and bisecting TT' in G; and TE perpendicular to PH.

Then $Pm = mG$, $DM = mH - mP = mH - mG$,

and $AM^2 = CM^2 = CD^2 + DM^2 = GT^2 + DM^2$
$= 4mH \,.\, mG + (mH - mG)^2 = (mH + mG)^2$.

$\therefore AM = mH + mG$, and $AD = AM - DM = 2mG = PG$.

Moreover, $BD^2 = AD \,.\, DF = 2mG \times 2VF = ET^2$, or $BD = ET$.

Hence, $\triangle\, ABB' = AD \,.\, BD = PG \,.\, ET = \triangle\, PTT'$.

VII. QUEST. (1920); *by* W. P. H., *Harleston, Norfolk.*

On the side BC, (or BC produced) of a plane triangle ABC let points D, E be taken so that BD=CE but measured in opposite directions; similarly on the other sides CA, AB let there be taken CF=AG and AH=BK, so that BD, CF, AH are proportional to the corresponding sides BC, CA, AB. Let AD be drawn to intersect BG, CK in P, Q; BF to intersect CK, AE in R, S; and CH to intersect AE, BG in T, V: then the hexagon PQRSTV will be double the mean proportional between the triangles PRT, QSV.

Answered by Mr. W. J. MILLER, B.A., *Eltham, Kent.*

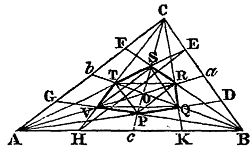
Because BD : BC :: AG : AC, DG is parallel to AB; and hence

AP : PD :: AB : GD :: AC : CG;

Similarly,

AT : TE :: BC : BE :: AC : CG

$\therefore$ AP : PD :: AT : TE; and PT is parallel to BC.



In like manner it may be shown that the other sides of the triangles PRT, QSV, are parallel to those of the triangle ABC.

Again (Bland's 'Geom. Pr.' iii, 33), AVR, BQT, CSP, are *straight* lines, which bisect the sides of the triangles ABC, PRT, QSV, and must, therefore, meet in O, the *centroid* of each of them. Hence the triangles VOP, POQ, QOR, ROS, SOT, TOV, are all equal to one another; and the hexagon PQRSTV is double of the trapezium VRST.

$$\text{Now, } \triangle\,\text{QOV} : \triangle\,\text{VOT} :: \text{QO} : \text{OT} :: \text{VO} : \text{OR}$$
$$:: \triangle\,\text{VOT} : \triangle\,\text{ROT}.$$

Or, taking three times the areas of these triangles,

$$\triangle\,\text{QSV} : \text{VRST} :: \text{VRST} : \triangle\,\text{PRT};$$

that is, the trapezium VRST is a mean proportional between the triangles PRT, QSV; and, consequently, the hexagon PQRSTV is the double of this mean proportional.

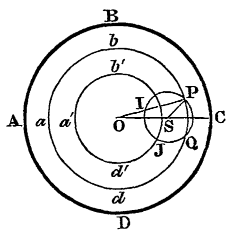
A similar proof extends to the case in which D, E, &c., are taken in the extensions of the sides.

It was nearly thus answered by Amicus, and Messrs. Bull, Collins, Dobson, M'Namara, Mulcaster, Rutter, Ryan, Traynor, and Turnbull; and good analytical solutions were given by $\alpha\delta\alpha$, Dr. Rutherford, and Messrs. Brooks, Buttery, Clazey, Dale, Robinson, and Watson.

VIII. QUEST. (1921); *by* Mr. C. H. Brooks, C.E., *Newcastle-upon-Tyne.*

Two coins each one inch in diameter are thrown horizontally into a circular box two inches in diameter; find the probability of only one of them resting on the box, supposing every possible position of each coin to be equally probable.

General Solution, by $\alpha\delta\alpha$, of Southampton.



Let ABCD be the box, whose radius we shall assume $= k + 1$, while that of either of the coins is unity. Describe two concentric circles with radii k and $k-1$ (abd and $a'b'd'$ in the diagram). The chance that the centre of the first coin shall be between the distances r and $r + dr$ from the centre O of the box, is evidently $d(r^2) : k^2$; and if this coin fall within the circle $a'b'd'$, the chance that the second will rest upon it, and not on the box, is $1 : k^2$. But when the first coin falls with its centre between $a'b'd'$ and abd, the chance is proportional to the area PIJQ. Now if POS $= \phi$, PSC $= \theta$, OPS $= \psi$, then area PIJQ $= \pi + k^2(\phi - \sin\phi\cos\phi) - (\theta - \sin\theta\cos\theta)$, which for convenience we may write, $\pi + k^2\,\Phi - \Theta$, attaching a corresponding meaning to the symbol Ψ. The whole chance therefore of one coin resting entirely on the other is

$$\frac{1}{k^4}\int_0^{(k-1)^2} d(r^2) + \frac{1}{\pi k^4}\int_{(k-1)^2}^{k^2}(\pi + k^2\Phi - \Theta)\,d(r^2)$$

$$= \frac{1}{k^2} + \frac{1}{\pi k^4}\int_{(k-1)^2}^{k^2}(k^2\Phi - \Theta)\,d(r^2)$$

$$= \frac{1}{k^2} + \frac{1}{\pi k^4}\Big[r^2k^2\,\Phi - r^2\Theta\Big]_{(k-1)^2}^{k^2}$$

$$+ \frac{1}{\pi k^4}\int_{(k-1)^2}^{k^2}(2r^2\sin^2\theta\,d\theta - 2r^2k^2\sin^2\phi\,d\phi).$$

Now the triangle OPS gives $r\sin\theta = k\sin\psi$, and $r\sin\phi = \sin\psi$, and also $d\theta - d\phi = d\psi$; thus the last integral becomes immediately

$$k^2\int_{(k-1)^2}^{k^2} 2\sin^2\psi\,d\psi = k^2\Big[\,\Psi\,\Big]_{(k-1)^2}^{k^2}.$$

Thus the expression becomes

$$\frac{1}{k^2} + \frac{1}{\pi k^2}\Big[r^2\,\Phi - \frac{r^2}{k^2}\,\Theta + \Psi\Big]_{(k-1)^2}^{k^2}.$$

The integral vanishes at the lower limit; and at the upper it is easy to see from the figure that

$$\theta = \frac{\pi}{2} + \frac{\phi}{2},\ \ \sin\frac{\phi}{2} = \frac{1}{2k},\ \ r = k,\ \ \psi = \frac{\pi}{2} - \frac{\phi}{2}.$$

Thus the whole chance is

$$\frac{1}{k^2} + \frac{1}{\pi k^2}\Big\{k^2(\phi - \sin\phi\cos\phi) - \phi - \sin\phi\Big\}$$

$$= 4\sin^2\frac{\phi}{2}\Big\{1 - \frac{\phi + \sin\phi}{\pi}\Big\} + \frac{\phi - \sin\phi\cos\phi}{\pi}$$

$$= 2\,(1 - \cos\phi) + \frac{\phi}{\pi}(2\cos\phi - 1) - \frac{\sin\phi\,(2 - \cos\phi)}{\pi}.$$

In the particular case proposed, the chance is $1 - \dfrac{3\sqrt{3}}{4\pi}$.

Note. The chance that both shall touch the box, and the chance that the coins shall not be in contact at all, are the values of the expression

$$\Big(1 - \frac{\phi}{\pi}\Big)(2\cos\phi - 1) + \frac{\sin\phi}{\pi}(2 - \cos\phi)$$

when $\sin\dfrac{\phi}{2} = \dfrac{1}{2k}$ and $\dfrac{1}{k}$ respectively.

Dr. Rutherford gave a still more general solution, supposing the coins to be unequal, and this will be found inserted at page 83. The case proposed in the question was discussed in a similar manner by Mr. Brooks, the proposer, and Messrs. Collins, Dobson, M'Cormick, M'Namara, Robinson, Rutter, Traynor, and Watson.

IX. QUEST. (1922); *by αδα, Southampton.*

From a given paraboloid it is required to cut off a segment by a plane perpendicular to the axis, such that the attraction of the mass of the segment upon a particle in its focus shall be zero.

Answered by αδα, the Proposer.

Let V be the vertex and F the focus of the paraboloid, P any particle of its mass. Let $PFV = \theta$, and let the angle made by the plane PFV with any fixed plane passing through VF be put $= \phi$, also $PF = r$. Then the mass of an indefinitely small element of the mass at P is equal to $rd\theta dr.r\sin\theta d\phi$, and its attraction parallel to $FV = \sin\theta\cos\theta dr\, d\phi\, d\theta$. This expression, integrated from 0 to r and from 0 to 2π, is $2\pi r \sin\theta \cos\theta\ d\theta$; hence the attraction of the solid

$$= 2\pi\int r\cos\theta\sin\theta\, d\theta.$$

If θ_1 be the value of θ required by the problem, and r_1 the corresponding radius vector, we must, from $\theta = 0$ to $\theta = \theta_1$ put

$$r = \frac{2}{1+\cos\theta} \quad \therefore r\sin\theta\cos\theta = 2\sin\theta - \frac{2\sin\theta}{1+\cos\theta};$$

and from θ_1 to π we must put $r\cos\theta\sin\theta = r_1\cos\theta_1\sin\theta$.

Thus the attraction of the solid is equal to

$$4\pi\int_0^{\theta_1}\left(\sin\theta d\theta - \frac{\sin\theta d\theta}{1+\cos\theta}\right) + 4\pi\int_{\theta_1}^{\pi}\frac{\cos\theta_1\sin\theta}{1+\cos\theta_1}d\theta$$

$$= 4\pi\left\{1 + \log\frac{1+\cos\theta_1}{2}\right\} = 4\pi\log\frac{e}{r_1},$$

where e is the base of the Napierian system. Hence, in order that the attraction of the solid may be zero at the focus, the value of r determining the cutting plane must be $= e$, the unit of measure being the distance of the vertex from the focus of the paraboloid.

Again, by Mr. SEPTIMUS TEBAY, B.A., *Rivington.*

Let the equation to the paraboloid be

$$y^2 + z^2 = 4mx = r^2.$$

First let $x < m$; then the attraction of a circular slice on a particle in the focus

$$= 2\pi\rho\left\{1 - \frac{m-x}{\sqrt{(m-x)^2+r^2}}\right\}\delta x = 4\pi\rho\frac{x\delta x}{m+x};$$

where ρ is the density of the paraboloid and δx the thickness of the slice. Hence the attraction of the segment cut off by a plane passing through the focus

$$= 4\pi\rho\int_0^m \frac{xdx}{m+x} = 4\pi\rho m\,(1 - \log_e 2).$$

Next let $x > m$; then the attraction of a slice on a particle at the focus

$$= 2\pi\rho\left\{1 - \frac{x-m}{\sqrt{(x-m)^2 + r^2}}\right\}\delta x = 4\pi\rho m\,\frac{\delta x}{m+x},$$

the integral of which is

$$4\pi\rho m \log_e \frac{m+x}{2m}.$$

$$\therefore 4\pi\rho m\,(1 - \log_e 2) = 4\pi\rho m \log_e \frac{m+x}{2m},$$

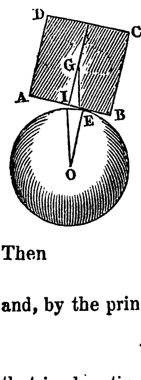
$$\therefore \log_e \frac{m+x}{m} = 1, \quad \frac{m+x}{m} = e, \quad \text{and} \quad x = m\,(e-1).$$

It was answered in like manner by Messrs. Brooks, Collins, Dale, Dobson, M'Namara, Robinson, Rutherford, Rutter, Ryan, Traynor, Turnbull, and Watson.

X. QUEST. (1923); *by* Mr. Thomas Dobson, B.A.

A cube rocks, without sliding, on a fixed sphere, and is acted upon by the gravity of the earth; find the time of a small oscillation, and the condition of stable equilibrium.

Answered by Mr. Dobson, *the Proposer, and* Dr. Rutherford.



Let O be the centre of the fixed sphere, G that of the rocking cube ABCD, and E the point of contact of the cube and sphere at any time t of the motion. Let θ be the angle which OE or GI makes with the vertical at time t, x and y the horizontal and vertical coordinates of G, the centre O being the origin; R and F the mutual actions in directions OE and EA, r the radius of the sphere, a the semiside of the cube, m its mass, and $k^2 = \frac{2}{3}a^2$, the square of the radius of gyration.

Then

$$x = (r+a)\sin\theta - r\theta\cos\theta,$$
$$y = (r+a)\cos\theta + r\theta\sin\theta,$$

and, by the principle of vis viva,

$$\frac{dx^2}{dt^2} + \frac{dy^2}{dt^2} + k^2\frac{d\theta^2}{dt^2} = -2gy + C;$$

that is, denoting by α the initial value of θ,

$$(a^2 + r^2\theta^2 + k^2)\frac{d\theta^2}{dt^2} = 2g\{(r+a)(\cos\alpha - \cos\theta) + r(\alpha\sin\alpha - \theta\sin\theta)\}$$
$$= g\,(r-a)(\alpha^2 - \theta^2) + \ldots\ldots\ldots$$

Hence

$$dt = \frac{r}{\sqrt{g(r-a)}} \cdot \left(\frac{\beta^2 + \theta^2}{\alpha^2 - \theta^2}\right)^{\frac{1}{2}} d\theta, \text{ where } \beta^2 = \frac{a^2 + k^2}{r^2};$$

$$\therefore t = \pi \left\{ 1 + \tfrac{1}{4} \cdot \frac{r^2 \alpha^2}{a^2 + k^2} \right\} \sqrt{\frac{a^2 + k^2}{g(r-a)}}.$$

And if the initial value of α be so small that the term involving α^2 may be omitted, we get, for the time of a small oscillation,

$$t = \pi \sqrt{\left\{ \frac{a^2 + k^2}{g(r-a)} \right\}} = \pi a \sqrt{\left\{ \frac{5}{3g(r-a)} \right\}}.$$

If r is less than a, $r - a$ is negative, and the equilibrium is unstable.

It was similarly answered by $\alpha\delta\alpha$, and Messrs. Brooks, Buttery, Collins, M'Cormick, M'Namara, Robinson, Rutter, Ryan, Tebay, Traynor, and Watson.

XI. QUEST. (1924); *by* PETRARCH.

The fraction $\frac{992}{9801}$ when expanded as a decimal gives in succession all the even numbers of two places of figures to 88 places of decimals.

Answered by $\alpha\delta\alpha$, Southampton.

$$\frac{992}{9801} = \frac{10}{99} + \frac{2}{99^2} = \frac{10}{100-1} + \frac{2}{(100-1)^2}$$

$$= \frac{10}{100} + \frac{10}{100^2} + \frac{10}{100^3} + \frac{10}{100^4} + \ldots\ldots$$

$$+ \frac{2}{100^2} + \frac{4}{100^3} + \frac{6}{100^4} + \ldots\ldots$$

$$= 0{\cdot}10121416\ldots\ldots 9496{\cdot}990103\ldots\ldots$$

After the 88th decimal the *apparent* law changes, and the odd numbers begin to appear.

It was answered in an analogous manner by Messrs. Brooks, Buttery, Collins, Dobson, M'Cormick, M'Namara, Mulcaster, Robinson, Rutherford, Rutter, Ryan, Traynor, and Watson.

Dr. Rutherford adds the following:

Cor. Since $\frac{1}{99} = {\cdot}01010101\ldots\ldots\ldots$, it is obvious that the fraction, when expanded as a decimal, which would give in succession all the odd numbers of two places of figures to 88 places of decimals, is

$$\frac{992}{9801} + \frac{1}{99} = \frac{1091}{9801}.$$

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XII. QUEST. (1925); *by* Mr. MATTHEW COLLINS, *Kilkenny College.*

A uniform chain, 1000 miles long, being suspended vertically, with its lower end just touching the earth's surface, required the velocity of the upper end when it arrives at the earth, taking into account the variation of the force of gravity along the chain, but abstracting from the earth's motion and the friction of the atmosphere.

Answered by Mr. C. H. BROOKS, C.E., *of Newcastle-upon-Tyne.*

Let r denote the radius of the earth, a the extreme height of the top of the chain measured from the earth's centre, and v the common velocity of the particles when this height becomes x, the length of chain then in motion being $x-r$. Then, estimating the mass according to the length of chain, the impressed force of gravity on an element $dx = g\dfrac{r^2}{x^2}dx$; and taking the sum of these over $x-r$, the impressed force on the length $x-r$ is

$$f = gr^2\int_r^x \frac{dx}{x^2} = gr^2\left(\frac{1}{r}-\frac{1}{x}\right) = gr\frac{x-r}{x}.$$

Dividing this by $x-r$, the mass moved, we obtain the accelerating force $\dfrac{dv}{dt} = \dfrac{gr}{x}$.

$$\therefore\ vdv = \frac{gr}{x}\,vdt = -\frac{gr}{x}\,dx,$$

$$\text{and } v^2 = 2gr\int_a^x\left(-\frac{dx}{x}\right) = 2gr\log_e\frac{a}{x}.$$

For calculation we have $r = 3950$ miles, $a = 3950 + 1000 = 4950$ miles, and $g = 32\frac{1}{6}$ feet $= \dfrac{193}{31680}$, so that, when the upper end of the chain arrives at the earth's surface,

$$v^2 = 2gr\log_e\frac{a}{r} = \frac{7900\times193}{31680}\times\log_e\frac{99}{79}$$

$$= \frac{395\times193}{12\times12\times11}\times0{\cdot}225672 = 10{\cdot}8612,$$

and $v = 3{\cdot}2956$ miles per second.

It was similarly answered by Mr. Collins, the proposer, $\alpha\delta\alpha$, and Messrs. Buttery, Dobson, M'Namara, Miller, Robinson, Rutherford, Ryan, Tebay, Traynor, and Watson.

Mr. Tebay also determined the time of descent to be $t = 414\frac{3}{4}$ seconds.

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XIII. QUEST. (1926); *by* Mr. W. H. Levy, *Shalbourne*.

Twenty-eight persons play at dice, each throwing three times with three dice, for a stake of £14. The seventh player having thrown 40, it is required to determine the value of his chance of winning.

Answered by Mr. Levy, *the Proposer*.

Let $\alpha =$ chance of throwing less than 40, and $\beta =$ chance of throwing just 40; then $\alpha + \beta =$ chance of throwing a number not exceeding 40.

Before the seventh player tried his chance its value was the same as that of the subsequent 21 players, who, with himself, made 22 in all at that time. Put this last $= n$. Now, the probability that all the n players will throw numbers not exceeding $40 = (\alpha + \beta)^n$; the probability that some of these will be $40 = (\alpha + \beta)^n - \alpha^n$. Therefore the individual chance of winning with the particular number 40 $= \dfrac{(\alpha + \beta)^n - \alpha^n}{n}$, there being n players. But the seventh player *has* played 40, the chance of which was β; hence the value of his chance of winning on that number after it is played $= \dfrac{(\alpha + \beta)^n - \beta^n}{n\beta} \times$ £14.

Now (Laplace, 'Théorie Analytique des Probabilities,' p. 256, or 'Diary,' 1855, p. 60), we readily compute the values $\alpha = \dfrac{9477316}{10077696}$, and $\beta = \dfrac{205560}{10077696}$. These values, being inserted in the above expression, give the true value of the chance of winning $=$ £4 17*s.* 4*d.*, as required.

The following are the number of ways which can be thrown in three throws with three dice, or in one throw with nine dice:

No. thrown.	No. of ways	No. thrown.	No. of ways.	No. thrown.	No. of ways.
9 or 54	1	17 or 46	12,465	25 or 38	359,469
10 or 53	9	18 or 45	22,825	26 or 37	447,669
11 or 52	45	19 or 44	39,303	27 or 36	536,569
12 or 51	165	20 or 43	63,999	28 or 35	619,569
13 or 50	495	21 or 42	98,979	29 or 34	689,715
14 or 49	1287	22 or 41	145,899	30 or 33	740,619
15 or 48	2994	23 or 40	205,560	31 or 32	767,394
16 or 47	6354	24 or 39	277,464		

Total number of ways = 10,077,696.

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Again, by Dr. RUTHERFORD.

Let $n + 1 = 22 =$ the total number of players, the first six having no chance whatever; then if $\beta =$ the probability of throwing a less number than 40, and $x =$ the probability of throwing just 40, we have evidently, A being the player who turned up 40, $\beta^n =$ the probability that none of the other players will throw 40; and $n\beta^{n-1}x =$ the probability that *one* of them will throw 40, and A is then entitled to half the stakes. Again, $\frac{n(n-1)}{2}\beta^{n-2}x^2 =$ the probability that some two of the other players will throw 40, and then A gets only one third of the stakes, and so on; consequently the probability that A has for winning is the sum of these several probabilities, divided by 1, 2, 3, 4.........$n + 1$, respectively, and it is expressed by

$$\beta^n + \frac{n\beta^{n-1}}{2}x + \frac{n(n-1)}{2\,.\,3}\beta^{n-2}x^2 + \ldots\ldots\ldots\ldots + \frac{x^n}{n+1}.$$

Now $(\beta + x)^n = \beta^n + n\beta^{n-1}x + \frac{n\,(n-1)}{2}\beta^{n-2}x^2 + \ldots\ldots\ldots\ldots + x^n$;

and, multiplying by dx and integrating, this gives

$$\frac{(\beta + x)^{n+1}}{n+1} = x\left\{\beta^n + \frac{n}{2}\beta^{n-1}x + \frac{n(n-1)}{2.3}\beta^{n-2}x^2 + \ldots \frac{x^n}{n+1}\right\} + \text{C}.$$

If $x = 0$, then $\qquad \frac{\beta^{n+1}}{n+1} = 0 + \text{C}$;

$$\therefore \text{ A's total probability} = \frac{(\beta + x)^{n+1} - \beta^{n+1}}{(n+1)\,x};$$

and the value of his expectation is found by multiplying £14 by this expression.

By the usual method of calculating these probabilities, we find that

$$\beta = \frac{9477316}{10077696}, \quad x = \frac{205560}{10077696};$$

$\therefore$ A's expectation = £4 17*s*. 4*d*. nearly.

Analogous solutions were given by Messrs. Brooks, Buttery, Collins, Dobson, M'Namara, Miller, Mulcaster, Robinson, Rutter, Traynor, and Watson.

XIV. QUEST. (1927); *by* Dr. RUTHERFORD, *of the Royal Military Academy, Woolwich.*

Two equal rods AB, AC are freely moveable about a joint at A, and are placed vertically in a given position, with their extremities B, C on a smooth horizontal plane; the extremity B is constrained to move uniformly, whilst the extremity C moves freely along the plane. Determine the path of the joint A, the position when there is no pressure upon it, and also the pressure on the joint when the rods strike the plane.

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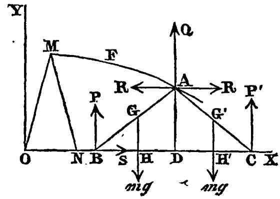
Answered by Dr. Rutherford, *the Proposer; and in like manner by* *αδα, of Southampton.*

Let OX and OY be the rectangular coordinates, OMN being the initial position of the rods; let BAC be the position of the rods at any time t of the motion, MFA the corresponding part of the curve described by the joint at A; and G, G′ the centres of gravity of the rods AB and AC. Draw GH, AD, G′H′ perpendicular to OBC, and let AB $= \text{AC} = 2a$, angle $\text{BAD} = \theta$; P, P′, Q the vertical pressures at B, C, A respectively, and S, R the horizontal pressures at B and A. Let



$m =$ the mass of each rod, $k =$ the radius of gyration about G and G′, $\alpha = \text{MOY}$, the initial value of θ; $\beta =$ the uniform velocity of B in direction BC; $\text{OH} = x$, $\text{OH}' = x'$, $\text{GH} = \text{G}'\text{H}' = y$; then $\text{OB} = t\beta$, $\text{BH} = \text{HD} = \text{DH}' = a \sin\theta$; therefore

$$x = t\beta + a\sin\theta; \quad y = a\cos\theta;$$

$$x' = x + 2a\sin\theta = 3x - 2t\beta.$$

Hence

$$(1) \ldots\ldots \frac{d^2x}{dt^2} = -a\sin\theta\frac{d\theta^2}{dt^2} + a\cos\theta\frac{d^2\theta}{dt^2};$$

$$(2) \ldots\ldots \frac{d^2y}{dt^2} = -a\cos\theta\frac{d\theta^2}{dt^2} - a\sin\theta\frac{d^2\theta}{dt^2};$$

$$(3) \ldots\ldots \frac{d^2x'}{dt^2} = 3\frac{d^2x}{dt^2}.$$

The equations of motion of the two rods are

$$(4) \ldots\ldots m\frac{d^2x}{dt^2} = \text{S} - \text{R}; \quad m\frac{d^2y}{dt^2} = \text{P} - \text{Q} - mg;$$

$$(5) \ldots\ldots m\frac{d^2x'}{dt^2} = \text{R}; \quad m\frac{d^2y}{dt^2} = \text{P}' + \text{Q} - mg:$$

$$(6) \ldots\ldots mk^2\frac{d^2\theta}{dt^2} = (\text{P} + \text{Q})\,a\sin\theta - (\text{R} + \text{S})\,a\cos\theta;$$

$$(7) \ldots\ldots mk^2\frac{d^2\theta}{dt^2} = (\text{P}' - \text{Q})\,a\sin\theta - \text{R}a\cos\theta;$$

and these are the same as if the point B were fixed.

From (3), (4), (5) we have evidently

$$\text{R} = 3(\text{S} - \text{R}), \quad \therefore 4\text{R} = 3\text{S},$$

$$2m\frac{d^2y}{dt^2} = (\text{P} + \text{P}') - 2mg, \quad \text{or} \quad \text{P} + \text{P}' = 2m\left(\frac{d^2y}{dt^2} + g\right).$$

From (6) and (7) we have

$$2mk^2\frac{d^2\theta}{dt^2}=(P+P')a\sin\theta-(2R+S)a\cos\theta;$$

but $$2R+S=\frac{10}{3}R=\frac{10}{3}m\frac{d^2x'}{dt^2}=10m\frac{d^2x}{dt^2}, \text{ by (3)};$$

$$\therefore k^2\frac{d^2\theta}{dt^2}=a\sin\theta\left(\frac{d^2y}{dt^2}+g\right)-5a\cos\theta\frac{d^2x}{dt^2}.$$

Substituting the values of $\frac{d^2x}{dt^2}$ and $\frac{d^2y}{dt^2}$ from (1) and (2), this gives

$$(a^2+k^2)\frac{d^2\theta}{dt^2}=ag\sin\theta-4a^2\cos\theta\left(\cos\theta\frac{d^2\theta}{dt^2}-\sin\theta\frac{d\theta^2}{dt^2}\right)\ \dots\ (8).$$

Multiply by $2d\theta$ and integrate; and we arrive at the following equation, which might have been directly deduced from the conservation of vis viva:

$$(a^2+4a^2\cos^2\theta+k^2)\frac{d\theta^2}{dt^2}=2ag(\cos\alpha-\cos\theta),$$

$$\therefore \frac{d\theta^2}{dt^2}=\frac{3g}{2a}\cdot\frac{\cos\alpha-\cos\theta}{1+3\cos^2\theta}, \text{ since } k^2=\tfrac{1}{3}a^2\dots\dots\dots(9).$$

The angular velocity of the rods at any position during the motion is hence determined.

For the pressure R we have

$$R=m\frac{d^2x'}{dt^2}=3m\frac{d^2x}{dt^2}=3m\left(-a\sin\theta\frac{d\theta^2}{dt^2}+a\cos\theta\frac{d^2\theta}{dt^2}\right);$$

and, substituting the values of $\frac{d\theta^2}{dt^2}$, $\frac{d^2\theta}{dt^2}$ from (8) and (9), we find

$$\left.\begin{aligned} R&=\frac{9}{4}mg\sin\theta\frac{3\cos^3\theta+3\cos\theta-2\cos\alpha}{(1+3\cos^2\theta)^2} \\ \text{Also}\quad Q&=\frac{3}{4}mg\cos\theta\frac{3\cos^3\theta+3\cos\theta-2\cos\alpha}{(1+3\cos^2\theta)^2}\end{aligned}\right\}\dots(10).$$

Therefore, when $R=0$, Q also $=0$, and

$$\cos^3\theta+\cos\theta=\frac{2}{3}\cos\alpha\ \dots\dots\dots\dots\dots\dots(11),$$

from which θ is known when there is no pressure on the joint.

In the preceding expressions let $\theta=\frac{1}{2}\pi$; then

$$\frac{d\theta^2}{dt^2}=\frac{3g}{2a}\cos\alpha,\quad\text{and}\quad R=-\frac{9}{2}mg\cos\alpha\dots\dots(10),$$

which last is the pressure on the hinge when the rods strike the plane, the negative sign implying a *pulling force* at that instant equal to $4\frac{1}{2}$ times the weight of one of the rods multiplied by the cosine of the initial value of θ.

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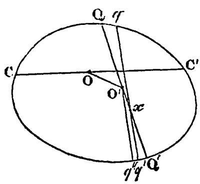
From (9) and the relations $x = t\beta + a\sin\theta$, $y = a\cos\theta$, the differential equation of the trochoidal curve described by G, the centre of gravity of the rod AB, and the same for the joint A, may be readily deduced.

A similar method of solution was followed by Messrs. Brooks, Robinson, and Watson; and the principle of vis viva was employed by Messrs. Collins, Dobson, M'Namara, and Rutter.

XV. or PRIZE QUEST. (1928); *by* PETRARCH.

A rod CC′ of a given length has its two ends in the curve of an ellipse and moved round, having a tracing point P, at the distances c and c' from its ends, tracing a curve. Show that the area contained between the curve and the ellipse $= \pi cc'$, and is therefore independent of the ellipse.

Answered by — MASON, *of Scoulton, Norfolk.*



Let CQC′ be *any* closed curvilineal figure. COC′ the original position of the given chord; O′ the generating point when the chord is in the position QO′Q′, qxq' the next position of the chord, x the ultimate intersection of QQ′ and qq'; $O'Q = OC = c$, $O'Q' = OC' = c'$; θ = the angle through which the rod has revolved from its original position; OO′ a portion of the curve traced out by O; P = the area swept out by O′Q, and Q = that swept out by O′Q′; $xO' = r$.

Then, $$dP = \tfrac{1}{2}(xQ^2 - xO'^2)\,d\theta$$
$$= \tfrac{1}{2}\{(r+c)^2 - r^2\}\,d\theta = \tfrac{1}{2}(2cr + c^2)\,d\theta \ldots\ldots\ldots(1).$$

To find dQ we will suppose the chord to revolve about O′ through the small angle $d\theta$, thus generating the area $Q'O'q'' = \frac{1}{2}c'^2\,d\theta$, and then $O'q''$ to move parallel to itself into the actual position qxq', thus diminishing the area $Q'O'q''$ by the small parallelogram between $O'q''$ and xq', that is, by $c'rd\theta$;

$$\therefore dQ = \tfrac{1}{2}c'^2 d\theta - c'rd\theta = \tfrac{1}{2}(c'^2 - 2c'r)\,d\theta \ldots\ldots\ldots(2).$$

Eliminate r between (1) and (2)

$$\therefore c'.dP + c.dQ = \tfrac{1}{2}(c'c^2 + cc'^2).d\theta;$$
$$\therefore c'.P + c.Q = \tfrac{1}{2}cc'(c + c').\theta.$$

Now when θ has become $= 2\pi$, the chord has returned to its orignal position, and $P\,{}^{\theta = 2\pi}_{\theta = 0}$ = the area contained between the locus of O and the given curve, and $Q\,{}^{\theta = 2\pi}_{\theta = 0}$ = the same area.

$$\therefore (c' + c) \times \text{area between the two curves}$$
$$= \tfrac{1}{2}cc'(c + c').2\pi = cc'(c + c').\pi$$

$\therefore$ the area between the locus of O and the given curve $= cc'.\pi$.

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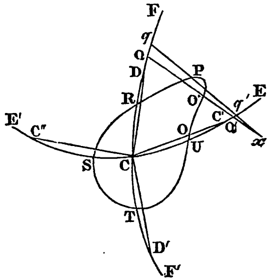
The problem is far more general than the one just proved, as was well known to the author, who proved it for any two curves.



Let E′CE, FCF′ be any two curves, COC′ the original position of the generating chord, O the generating point. $QQ'x$, $qq'x$, two successive positions of the chord; $O'Q = c$, $O'Q' = c'$, $xO' = r$. Then as before

$$dP = \tfrac{1}{2}(2cr + c^2)\,d\theta$$

$$\text{and } dQ = \tfrac{1}{2}(2c'r - c'^2)\,d\theta$$

$$\therefore c'.dP - c.dQ = \tfrac{1}{2}cc'(c+c')d\theta$$

$$\therefore c'.P - c.Q = \tfrac{1}{2}cc'(c+c')\theta,$$

that is, $c'P - c(Q + P - P)$ or

$$(c'+c).P - c.(Q+P) = \frac{cc'}{2}(c+c')\theta.$$

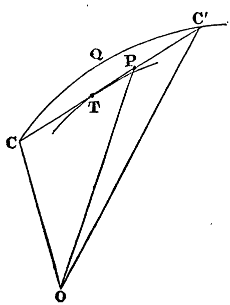
$$\therefore \frac{c'+c}{c}.P - \text{area swept out by } QQ' = \frac{c'}{2}(c+c')\theta.$$

Now as the chord revolves QQ′ describes an area which it then describes back again, and when θ has increased from 0 to 2π the whole area described by QQ′ has been described positively and negatively, and it is $\therefore = 0$. And $P_{\theta=0}^{\theta=2\pi} =$ the oval figure UPRSTU.

$$\therefore \frac{c'+c}{c} \times \text{area of the oval figure} = \frac{c'}{2}(c+c').2\pi$$

$$\therefore \text{the area of the oval} = cc'\pi.$$

Second Solution, by Dr. Rutherford, *and* $\alpha\delta\alpha$, *of Southampton.*



General Solution.—Let CQC′ be a portion of any closed curve whatever; CC′ the constant chord, and P the fixed point in the chord. Let O be the origin of coordinates; x_1y_1, x_2y_2, xy the rectangular coordinates of C, C′, P; $r_1\theta_1$, $r_2\theta_2$, $r\theta$ the polar coordinates; and ϕ the angle of inclination of the chord CC′ to the axis of x; then we have evidently

$$x_1 = x - c\cos\phi;\quad x_2 = x + c'\cos\phi;$$

$$y_1 = y - c\sin\phi;\quad y_2 = y + c'\sin\phi;$$

Hence we have

$$x_1dy_1 - y_1dx_1 = xdy - ydx + c^2d\phi - \lambda cd\phi + \lambda' c \text{ ......(1)};$$
$$x_2dy_2 - y_2dx_2 = xdy - ydx + c'^2d\phi + \lambda c'd\phi - \lambda' c' \text{ ......(2)};$$

where,

$$\lambda = x\cos\phi + y\sin\phi;$$
$$\lambda' = dx\sin\phi - dy\cos\phi.$$

Multiply (1) by c', and (2) by c; then, adding the products, we have

$$c'r_1^2d\theta_1 + cr_2^2d\theta_2 = (c+c')\,r^2d\theta + cc'(c+c')\,d\phi;$$

since $xdy - ydx = r^2d\theta$; $x_1dy_1 - y_1dx_1 = r_1^2d\theta_1$; and so on.

Hence

$$\frac{c'}{c+c'}\cdot\frac{1}{2}\int r_1^2d\theta_1 + \frac{c}{c+c'}\cdot\frac{1}{2}\int r_2^2d\theta_2 = \frac{1}{2}\int r^2d\theta + \tfrac{1}{2}cc'\int d\phi;$$

that is, the area of the original curve = area of new curve + $\pi cc'$; because the chord CC′ moves round through an angle 2π in a closed curve.

This result is independent of the relation between $r_1\theta_1$ and $r_2\theta_2$; and therefore the extremities of the chord CC′ may move on any *two* closed curves; and if C, C′, P denote the areas described by the points C, C′, P respectively, we have obviously

$$\frac{c'\mathrm{C} + c\mathrm{C}'}{c+c'} = \mathrm{P} + \pi cc',$$

or

$$\mathrm{P} = \frac{c'\mathrm{C} + c\mathrm{C}'}{c+c'} - \pi cc'.$$

Third Solution, by Mr. William Hogg, *Mathematical Master, Royal Naval School.*

Let CC′ be any position of the rod, P the tracing point, the segments being CP $= c$, C′P $= c'$; and let T be the point of intersection with a consecutive position, or point of contact with the curve T enveloped by CC′, or to which CC′ is always a tangent. Also let TP $= u$, and the angle which CC′ makes with a fixed axis $= \theta$. Then, if C, T, P, C′ denote the areas of the curves respectively traced by those points, we shall evidently have

$$\mathrm{C} - \mathrm{T} = \tfrac{1}{2}\int \mathrm{TC}^2\,.\,d\theta = \tfrac{1}{2}\int (c-u)^2 d\theta,$$
$$\mathrm{C}' - \mathrm{T} = \tfrac{1}{2}\int \mathrm{TC}'^2\,.\,d\theta = \tfrac{1}{2}\int (c'+u)^2 d\theta,$$
$$\mathrm{P} - \mathrm{T} = \tfrac{1}{2}\int \mathrm{TP}^2\,.\,d\theta = \tfrac{1}{2}\int u^2 d\theta.$$

First, to eliminate the area T, subtract the third equation from each of the two former, and

$$\mathrm{C} - \mathrm{P} = \tfrac{1}{2}\int (c^2 - 2cu)\,d\theta = \tfrac{1}{2}c^2\theta - c\int ud\theta,$$
$$\mathrm{C}' - \mathrm{P} = \tfrac{1}{2}\int (c'^2 + 2c'u)\,d\theta = \tfrac{1}{2}c'^2\theta + c'\int ud\theta.$$

Next, to eliminate $\int u d\theta$ from these equations, multiply the first by c', the second by c, and add; then

$$c'C + cC' - (c + c')\,P = \tfrac{1}{2}\,cc'\,(c + c')\,\theta;$$

$$\therefore\ \frac{c'C + cC'}{c + c'} - P = \tfrac{1}{2}\,cc'\,\theta.$$

When the rod CC' has performed an entire revolution, and returned to the same position, thereby completing the several curves, then $\theta = 2\pi$, and

$$\frac{c'C + cC'}{c + c'} - P = cc'\,\pi.$$

This remarkable general relation is therefore true when the extremities C, C', describe any two closed curves whose areas are C, C'. In the case before us they traverse the same identical curve; $\therefore$ C' = C, and

$$C - P = cc'\,\pi,$$

which is hence true, whatever may be the nature of the directing curve.

A similar answer was given by Astronomicus, and the Rev. Thomas Brady, Senica Falls, New York.

Fourth Solution, by Mr. Septimus Tebay; *and in like manner by* Messrs. Brooks, Collins, Rawson, *and* Robinson.

Let xy, $x'y'$ be the coordinates of C, C'; XY those of P; A = area of the given curve, A' = area of the curve traced out by the point P Then

$$(c + c')\,X = cx' + c'x,$$

$$(c + c')\,Y = cy' + c'y,$$

$$y - y' = \sqrt{(c + c')^2 - (x - x')^2}.$$

Now $A' = \int Y dX$ (between assignable limits)

$$= \int \frac{cy' + c'y}{(c + c')^2} \cdot d\,(cx' + c'x)$$

$$= \frac{(c^2 + c'^2)\,A + cc'\int(y'dx + ydx')}{(c + c')^2}$$

$$= \frac{(c + c')^2\,A - cc'\int(y - y')\,d\,(x - x')}{(c + c')^2}.$$

$$\therefore\ A - A' = \frac{cc'}{(c + c')^2}\int(y - y')\,d\,(x - x')$$

$$= \frac{cc'}{(c + c')^2}\int\sqrt{(c + c')^2 - (x - x')^2}\,.\,d(x - x').$$

Let the curve be a closed one, and of continued curvature. Then if we suppose the rod to move from a position at right angles to the axis of x, till it becomes parallel to it, the limits of $x - x'$ will be 0 and $c + c'$. The above integral taken between these limits is $\frac{1}{4}\pi cc'$; and as this is only $\frac{1}{4}$th of the required result, we have

$$A - A' = \pi cc'.$$

It is plain that if the curve be not of continued curvature, but have one or more points of inflexion, the above result will still be *algebraically* true, provided the curve be a closed one.

A general solution by Mr. W. S. B. Woolhouse, is inserted at page 96.

A sixth solution by Messrs. Amicus, Bills, Buttery, Clazey, Dobson, Fenwick, Mawson, Miller, Pigg, and Watson, was cancelled for want of space.

LIST OF MATHEMATICAL ANSWERS.

'A, ans. 5, 6.
αδα, Southampton, ans. 3 to 12, 14, Prize.
Amicus, of Jersey, ans. 2, 3, 4, 7, Prize.
Astronomicus, Collooney, Ireland, ans. Prize.
Bills, Samuel, Hawton, near Newark-upon-Trent, ans. Prize.
Brady, the Rev. Thomas, Senica Falls, Senica County, State of New York, America, ans. Prize.
Brooks, C. H., C.E., 6, George Street East, Newcastle-upon-Tyne, ans. all the questions.
Bull, Robert, Newport, Isle of Wight, ans. 1, 5, 7.
Buttery, John, Mathematical Master, H.M. Dockyard, Chatham, ans. all the questions.
Buttery, Thomas, Thurcaston, Leicestershire, ans. 1, 2, 5, 7, 8, 11, 13.
Clazey, J. O., ans. 6, 7, Prize.
Collins, Matthew, B.A., Kilkenny College, Senior Moderator in Mathematics and Physics, T. C. D., ans. all the questions.
Dale, James, Leadhills, ans. 7, 9.
Dickinson, Jacob, Allendale Town, Northumberland, ans. 1, 2, 11.
Dobson, Thomas, B.A., Nautical School, Royal Hospital, Greenwich, ans. all the questions.
Drysdale, the Rev. Anderson, Bishopwearmouth, ans. 2.
Eland, Thomas J., 19, Hill Street, Bridge Street, Bolton, ans. 5.
Farmar, William, 50, York Street, Dover, ans. 4.
Fenwick, Stephen, of the Royal Military Academy, Woolwich, ans. Prize.
Flounders, George, Toft Hill Lane, near Bishop Auckland, Durham, ans. 1, 2, 3, 5.
Harle, Jonathan, Allendale Town, Northumberland, ans. 1, 2, 11.
Hattam, Thomas, jun., 1, Marine Place, Plymouth, ans. 7.
Hill, Henry, National Schools, High Wycombe, ans. 1 to 5.
Hogg, William, Royal Naval School, Deptford, ans. Prize.
Levy, W. H., Shalbourne, near Hungerford, Berkshire, ans. 2, 13.
Light, J. F., 1, Spencer Road, Stoke Newington, London, ans. 1, 2.
M'Cormick, Edward, C.E., Grosmount, near Hereford, ans. 1, 2, 3, 5, 6, 8, 10, 11, 13.
Mason, —, of Scoulton, Norfolk, ans. Prize.
Mawson, William, Witton-le-Wear, Durham, ans. Prize.
Miller, W. J., B.A., Eltham, Kent, ans. 1 to 7, 11, 12, 13, Prize.

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M'Namara, T., Ballaghderrin, Ireland, ans. 1 to 14.
Mulcaster, James, jun., Allendale Town, Northumberland, ans. 1 to 7, 9, 11, 12, 14, Prize.
Mulcaster, John Wallis, Allendale, Northumberland, ans. all the questions.
Petrarch, ans. 11, Prize.
Pigg, Edward, Bishopwearmouth, Durham, ans. 11, Prize.
Rawson, Robert, Portsmouth, ans. Prize.
Robinson, John Joshua, of H.M. Dockyard, Portsea, ans. all the questions.
Rutherford, Dr., of the Royal Military Academy, Woolwich, ans. all the questions.
Rutter, Edward, 65, Lawrence Street, Sunderland, ans. all the questions.
Ryan, Laurence, Assistant, National School, Leighlinbridge, Ireland, ans. 1 to 7, 9 to 13.
Short, Nicholas, Allendale, Northumberland, ans. 1, 2, 11.
Stobbart, John, Allendale Town, Northumberland, ans. 1, 2, 11.
Tebay, Septimus, B.A., Head Master of Rivington Grammar School, ans. 9, 10, 12, Prize.
Traynor, James, C.E., Carrickmacross, Ireland, ans. 1 to 13.
Turnbull, John, Bedlington, ans. 1 to 7, 9, 12.
Watson, Stephen, Grammar School, Haydonbridge, Northumberland, ans. all the questions.

It is with deep regret that we have this year to record the decease of Mrs. Long, Vauxhall, on the 29th of September last, aged 66 years, having been previously in an imbecile state for a lengthened period. Mrs. Long was one of the most gifted of our poetical contributors, and her tasteful productions have graced the 'Diary' for a great many years.

The following recent publications are worthy of notice:

A 'Treatise on Mensuration, combining the Elements of the Screw-propeller and Naval Architecture generally,' by Robert Rawson, an able correspondent of the 'Diary.' It is an excellent book, and admirably adapted to the use of schools. The concluding chapter, on Naval Architecture, is a sound and comprehensive digest of what is known on a most important and hitherto neglected subject. (Whittaker and Co.)

'The Philosophy of Education,' by T. Tate, F.R.A.S., formerly a contributor to the 'Diary.' Mr. Tate, who has been indefatigable in his writings, is now author of numerous instructive works, enjoying considerable reputation, and the important subject of Education has engaged a principal share of his valuable attention. (Longmans and Co.)

'The Steam Engine, or the Powers of Flame,' an original poem by T. Baker, formerly a mathematical contributor to the 'Diary.' It consists of ten cantos, which, with the additional notes, pourtray a complete history of the introduction and gradual development of this momentous power. (Hodgson, 22, Portugal Street, Lincoln's Inn.)

*** Our correspondents will please to bear in mind, that the arranging of the matter for the printer is greatly facilitated when they obligingly write out their contributions intended for insertion on one side of the paper only, or so that each distinct answer or subject may admit of an easy separation, without the necessity of having it re-written.—Ed.

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NEW MATHEMATICAL QUESTIONS.

I. QUEST. (1929); *by* Mr. ROBERT AMBLER, *Grammar School, Stevenage.*

Given the base and the two lines trisecting the vertical angle and terminating in the base, to construct the triangle.

II. QUEST. (1930); *by* Mr. SAMUEL BILLS, *of Hawton.*

Let ABC be any plane triangle, and on its three sides let three similar *isosceles* triangles ABC_1, ACB_1, BCA_1 be described; then will the three lines AA_1, BB_1, CC_1 intersect each other in the same point.

III. QUEST. (1931); *by* Mr. W. H. LEVY, *Shalbourne.*

Let k_1, k_2, k_3 be the middle points in the sides of a plane triangle; O the centre, and D, E, F the points of contact on the sides BC, CA, AB of the inscribed circle (r); and let DO, EO, FO be produced to meet Ak_1, Bk_2, Ck_3 in m_1, m_2, m_3; then will

$$\frac{1}{Dm_1}+\frac{1}{Em_2}+\frac{1}{Fm_3}=\frac{2}{r}.$$

IV. QUEST. (1932); *by* Mr. T. M'NAMARA, *Ballaghderrin, Ireland.*

The opposite sides of two triangles circumscribing a given circle are parallel, and thus cut off six triangles: prove that the continued product of the areas of the two triangles into those of the six triangles so cut off is equal to the sixteenth power of the radius of the circle.

V. QUEST. (1933); *by* Mr. STEPHEN WATSON, *Haydonbridge.*

Describe a minimum triangle within a given triangle, such that its sides shall respectively make the same angle with the sides of the given triangle; and show that two such triangles may be inscribed, which are equal and have the same circumscribing circle.

VI. QUEST. (1934); *by* Dr. RUTHERFORD, *of the Royal Military Academy, Woolwich.*

If the triangle DEF be inscribed in the triangle ABC so that each side of the former triangle makes the same angle with the corresponding side of the latter; then the circumferences of the circles circumscribed about the three triangles AEF, BFD, CDE will pass through the same point.

VII. QUEST. (1935); *by* Mr. JOHN BUTTERY, *H.M. Dockyard, Chatham.*

If the depths of immersion of the angular points of a plane triangle below the surface of a fluid be α, β, γ, show that the depth of the centre of pressure below the centre of gravity is

$$\frac{(\alpha-\beta)^2+(\beta-\gamma)^2+(\gamma-\alpha)^2}{12(\alpha+\beta+\gamma)}.$$

VIII. QUEST. (1936); *by* $\alpha\delta\alpha$, *Southampton.*

A pin is dropped at random upon a chessboard: supposing the length of the pin to be equal to the side of each square, what are the respective chances of its falling upon three squares, two squares, or one square only?

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IX. QUEST. (1937); *by* PETRARCH.

If a body be projected with a given velocity from a given height above the horizontal plane, in such a direction that the whole length of the path described shall be a maximum; show that the length of that path is equal to the sum of the two tangents at its extremities and terminated at the directrix of the parabola described.

X. QUEST. (1938); *by* MR. STEPHEN FENWICK, *of the Royal Military Academy, Woolwich.*

A uniform beam AB rests between two smooth planes AC, BC, inclined at angles of 30° and 60° to the horizon: compare the pressure on the plane AC when the beam is in a state of equilibrium, with the pressure on the same when the beam is in a state bordering on motion by the removal of the other plane.

XI. QUEST. (1939); *by* MR. SEPTIMUS TEBAY, B.A., *Rivington.*

A *free* elliptical disc of metal is struck by a given blow perpendicular to its plane. Show that if the blow be applied in the circumference of a concentric ellipse, whose axes are one fourth those of the disc, it will *begin* to revolve about a tangent.

XII. QUEST. (1940); *by* MR. THOMAS DOBSON, B.A., *Greenwich.*

A uniform rod AB (mass M) is free to move about a hinge at A; two strings of equal length and suspending equal weights (P, P) pass over pulleys at C and D, and are fastened to B; also these pulleys are situated on opposite sides of A in the horizontal line CAD, and AB = AC = AD. The rod is held at rest at an inclination (α) to the vertical, and then released. Determine the motion.

XIII. QUEST. (1941); *by* MR. MATTHEW COLLINS, B.A., *Kilkenny College.*

Required the locus of the focus of a varying parabola osculating most closely a given ellipse, or hyperbola, at the various points of the given conic.

XIV. QUEST. (1942); *by* MR. C. H. BROOKS, C.E., *Newcastle-upon-Tyne.*

A sphere slides down another one placed on a horizontal plane; determine the path of the upper one and the point of separation, the surfaces of both spheres and plane being perfectly smooth.

XV. OR PRIZE QUEST. (1943); *by* DR. RUTHERFORD.

A great circle of a given sphere is completely surrounded by n spherical balls, radii ρ_1; another row of n balls, radii ρ_2, lies on the sphere each touching two of the first row, and also touching each other; a third row of n equal balls, radii ρ_3, is similarly placed upon the second row, and so on. It is required to assign a general law determining the magnitude and position of the successive rows.

The several prizes are allotted as follows: 1st, For answering the Prize Enigma, to Mr. Robert Clemitson, of Morpeth, and Clericus, each *ten* Diaries. 2dly, For the general answers to the enigmas, to Mrs. Baker, Vauxhall, London, and the Cawkley's Laddie, each *ten* Diaries. 3dly, For the answers to the rebuses and charades, to Miss Helen Ogden, of Shaw, near Oldham, and Mr. James Hewitt, of Hexham, Northumberland, each *eight* Diaries. 4thly, For answering the Prize Question, to Mr. William Hogg, Royal Naval School, Deptford, and Mr. Septimus Tebay, B.A., Rivington, each *twelve* Diaries. 5thly, For the general mathematical answers, to Mr C. H. Brooks, C.E., Newcastle-upon-Tyne, and Mf. Matthew Collins, B.A., Kilkenny College, each *ten* Diaries. They will please to send (or write, post-paid) for their respective prizes, to Mr. Joseph Greenhill, Stationers' Hall, London.

All letters must, as usual, be directed "TO THE EDITOR OF THE LADY'S AND GENTLEMAN'S DIARY, STATIONERS' HALL, LONDON." They must likewise be post-paid, and arrive *before* May 1st, 1858.

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MATHEMATICAL PAPERS.

NOTES ON D'ALEMBERT'S PRINCIPLE AND SOME OF ITS APPLICATIONS.

By STEPHEN FENWICK, F.R.A.S., *of the Royal Military Academy, Woolwich.*

THE relation which exists between the system of impressed and effective moving forces according to the principle of D'Alembert, in reference to the motion of a rigid body or system of bodies, is well explained and applied in all our works on the higher mechanics; but the relation between the *forces lost*, and the application of that relation when individual elements of the body are considered, are very little noticed in any of our works. In many cases, however, the principle of D'Alembert, under this aspect, seems to be susceptible of a more ready application than when viewed in connexion with the equilibrium of the impressed forces, and the effective moving forces taken with a contrary sign. Its development, moreover, by means of the forces lost, is more in accordance with Newton's law of motion—"action and reaction are equal, and in opposite directions." In its most comprehensive sense, D'Alembert's principle may doubtless be considered as the greatest generalization possible of Newton's law; and when two bodies only are considered, acting upon each other in the line which connects them, it becomes precisely the same as the law of "action and reaction."

My object, then, is to call attention to the application of D'Alembert's principle in reference to the forces lost. It will conduce to a clear exposition of my views, if I first prove D'Alembert's principle in my own way. When I speak of the *proof* of D'Alembert's principle, I am aware that I subject myself to criticism. Mr. Griffin remarks—"The proof of the principle of D'Alembert will rest, like the laws of motion of a molecule, on the verification, by observation of nature, of results obtained from calculations based upon it. (Griffin's 'Treatise on the Motion of a Rigid Body.')

With all deference to the opinion of a mathematician so eminent as Mr. Griffin, I have ventured to use the term *proof*, in preference to that of *statement*, with respect to what I have given on this important principle.

1. The velocity which a force impresses on a particle of a rigid body is obviously different from that which such force would have impressed upon it *had that particle been free*, in consequence of the reaction which the different particles of the body exert upon each

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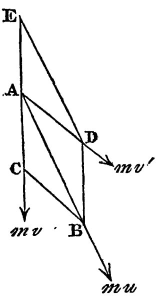
other. The force which is due to the increment of velocity actually produced on the particle, is called the *effective accelerating force* of the particle. We may hence conceive the motion of a rigid body to be decomposed into two, the one of which will *effectively take place*, but the other will be *destroyed* by virtue of the connexion of the elements of the body with each other.

2. By the *impressed forces* are meant those forces which are extraneous to the body, as distinguished from the pressures and other strainings which the different particles exert on each other, called the *molecular forces*, or *forces lost*.

3. Let m be the mass of one of the material particles A of a rigid body in motion under the action of certain forces, $u = \text{AB}$, the increment of velocity which would have been produced on the particle A by the forces acting upon it in the indefinitely small time dt, *had the particle* A *been free*, and $v = \text{AC}$, the increment of velocity which is actually impressed upon it in the same time. The velocity v may be different from u both in magnitude and direction, or it may differ from u only in one of these particulars. Decompose the velocity u into two others, one of which is v and the other $\text{AD} = v'$. Then may mu be regarded as the impressed moving force on the particle A, and mv the effective moving force on the same. But the impressed force mu is equivalent to its two components mv and mv', of which mv alone, as we have seen, is the *effective* moving force. Hence the other component mv' has been destroyed in the time dt, by the connexion of the particle A with the other particles of the system.



If in this manner we find for each element m the force mv' which is destroyed by the remaining elements, and which on that account is called the *force lost*, then we shall have a system of forces which will be in equilibrium. For, by the law of action and reaction, each lost force mv' evidently destroys the action of the other elements, and therefore all these forces will mutually destroy each other, in consequence of the rigid connexion of the elements of the moved body. Hence,

When any material system is in motion under the action of impressed forces, the moving forces lost on the different particles are in equilibrium.

This is one of the forms in which D'Alembert's principle has been given.

4. Again, because mu is the resultant of mv and mv', if mv be supposed to act in an opposite direction, the lost force mv' on A will then be the resultant of the impressed force mu, and the effective one mv applied in an opposite direction. But when the motion of the whole body is contemplated, the forces corresponding to mv', as we have seen, are in equilibrium; hence, in such case, the system of forces mu,

and the system of forces mv when applied in the opposite direction, will also constitute a system of forces in equilibrium. Wherefore,

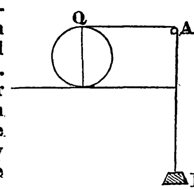
If the effective accelerating forces of the several particles of a rigid body be applied to them in the contrary directions to those in which they act, they will, with the impressed moving forces, satisfy the statical conditions of equilibrium.

This is the other form in which D'Alembert's principle is usually given.

5. In what precedes, the *increments* of the velocities and their corresponding moving forces have alone been considered. By means of the integral calculus, however, we are enabled to deduce, from these incremental velocities, the velocities themselves, and all other circumstances of the motion of bodies, under the action of any forces whatever.

6. Though the force mv' is said to be *lost*, it is lost only in the sense of being counteracted by the connexion of the particle A with the other particles of the system. This force, therefore, may be expressed in the same way as any other force of the system; it is, in fact, the resultant of the impressed force at A, and the effective one taken in the opposite direction.

I proceed now to explain the meaning of the lost force mv' in the solution of physical problems, in which it is assumed that the reader is familiar with the ordinary dynamical equations. The student is recommended to undertake the solution of other examples.



7. A pulley Q is laid flat on a smooth horizontal table, and a string which hangs through a small ring at A, and has a weight P attached to its extremity, is coiled about the pulley. The string is in a vertical plane perpendicular to the axis of the pulley, and passing through its centre of gravity: find the motion of the weight and pulley, the diameter of the pulley being equal to the altitude of the ring above the table.

In a time t from the commencement of the motion, let $x = \text{AP}$, $y = \text{AQ}$, and $\theta =$ the angle through which the pulley has revolved. Also let $m' =$ mass of the pulley, $a =$ its radius, $mk^2 =$ its moment of inertia about its axis, and $m =$ mass of P. Then, by Art. 6, the force lost on P is

$$m\left(g - \frac{d^2x}{dt^2}\right).$$

Now, as this force is lost only in the sense of being counteracted by the connexion of P with the pulley, it is clearly the force exerted by the string on the pulley. But

$$-m'\frac{d^2y}{dt^2}$$

is also the moving force on the pulley in the direction of the string.

Hence, by taking moments about the axis of the pulley, we get the equations

$$ma\left(g-\frac{d^2x}{dt^2}\right)=-m'a\,\frac{d^2y}{dt^2}=m'k^2\,\frac{d^2\theta}{dt^2}\,\ldots\ldots(1).$$

The expression involving y is taken *negative*, because y decreases as the time t increases.

Again, if l be the length of the free part of the string initially, we have, by the geometry,

$$x+y=l+a\theta\,;$$

and therefore $$\frac{d^2x}{dt^2}+\frac{d^2y}{dt^2}=a\,\frac{d^2\theta}{dt^2}\,\ldots\ldots\ldots\ldots\ldots\ldots(2).$$

Eliminating $\frac{d^2y}{dt^2}$ and $\frac{d^2\theta}{dt^2}$ between (1) and (2), we get for the accelerating force on P

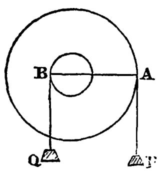
$$\frac{d^2x}{dt^2}=\frac{m(a^2+k^2)g}{m\,(a^2+k^2)+m'k^2}=\frac{3mg}{3m+m'}\ldots\ldots(3).$$

In a similar way the accelerating force on the pulley is found to be

$$\frac{d^2\theta}{dt^2}=\frac{a}{a^2+k^2}\cdot\frac{d^2x}{dt^2}=\frac{2}{a}\cdot\frac{mg}{3m+m'}\ldots\ldots\ldots(4).$$

All other circumstances of the motion are readily deduced from (3) and (4) by direct integration.

8. One weight P draws up another Q on the wheel and axle: find the motion of the weights and the pressure on the axis of rotation.



Let a, b denote the radii of the wheel and axle; m, m' the masses of P and Q; θ the angle through which the wheel and axle have revolved in the time t; Mk^2 the moment of inertia of the wheel and axle together about their common axis; x, y the vertical distances of P and Q below the horizontal plane through the axis at the end of the time t.

Then, because the impressed force at A is $P=mg$, and the effective force at the same point is $m\,\frac{d^2x}{dt^2}$, the force lost on A is (Art. 6)

$$m\left(g-\frac{d^2x}{dt^2}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

But as this force is lost by the connexion of P with the wheel and axle, it is evidently the tension of the string AP.

Similarly, the tension of the string BQ is

$$m'\left(g-\frac{d^2y}{dt^2}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2).$$

Wherefore, taking moments about the axis of the wheel and axle,

$$ma\left(g-\frac{d^2x}{dt^2}\right)-m'b\left(g-\frac{d^2y}{dt^2}\right)=Mk^2\frac{d^2\theta}{dt^2}\text{......}\ (3).$$

Again, if l and l' denote the initial values of x and y, we evidently have, by the geometry,

$$x=l+a\theta,\ y=l'-b\theta.$$

Whence

$$\frac{d^2x}{dt^2}=a\frac{d^2\theta}{dt^2}\ \text{ and }\ \frac{d^2y}{dt^2}=-b\frac{d^2\theta}{dt^2}\ \text{.........}\ (4).$$

Eliminating $\frac{d^2x}{dt^2}$ and $\frac{d^2y}{dt^2}$, between (3) and (4), we get

$$\frac{d^2\theta}{dt^2}=\frac{g(ma-m'b)}{ma^2+m'b^2+Mk^2}.$$

This is the angular accelerating force on the wheel and axle, from which the motion is determined. The pressure on the axis is the sum of the expressions (1) and (2).

It would be easy to amplify in the way of illustration, but enough has been done, it is hoped, to call attention to this interesting subject.

July 27th, 1857.

ON EQUATIONS OF THE FIFTH DEGREE.

By James Cockle, M.A., F.R.A.S., F.C.P.S., *Barrister-at-Law.*

(*The subject resumed from p.* 80 *of the 'Diary' for* 1857.)

80. The given equation being

$$y^5-5Py^3-5Qy^2-5Ry+E=0,$$

and its roots of the form

$$ad+a^2e+a^3f,$$

make

$$P^2+R=S,$$

and assume two quantities u and v, such that

$$u=de^2 \text{ and } v=\frac{e^5}{P}.$$

81. Then (see arts. 62 *et seq.*) we know that

$$u^2+(u-Q)v=0\ \text{..................}(a)$$

$$\frac{Q}{PS}u^2-u+\frac{P^2Q}{S}=v\ \text{..............}(b)$$

and

$$u^5-10P^3v^2u+P^3v^3+P^2(5PQ+E)v^2+P^6v=0\text{...}(c),$$

these three equations being transformations of (31), (32) and (34) respectively. The elimination of u and v furnishes us with the new solvible form alluded to in Art. 72.

82. Dividing (c) by v^2, we have

$$\left(\frac{u^2}{v}\right)^2 u - 10P^3u + P^8v + \frac{P^6}{v} + P^2(5PQ + E) = 0,$$

or

$$\left(\frac{u^2}{v}\right)^2 u - 11P^3u + P^3(u+v) + \&c. = 0,$$

and, substituting for $\frac{u^2}{v}$ and $u + v$ values deducible from (a) and (b) respectively, we obtain

$$u(Q-u)^2 - 11P^3u + \frac{Q}{S}(P^2u^2 + P^5) + \&c. = 0,$$

or

$$u^3 + \left(\frac{P^2Q}{S} - 2Q\right)u^2 + (Q^2 - 11P^3)u + \&c. = 0.$$

83. From this last equation subtract

$$u^3 - Qu^2 + P(P^2 + S)u - P^3Q = 0 \;\ldots\ldots(d),$$

obtained by eliminating v between (a) and (b). The difference gives

$$\left(\frac{P^2Q}{S} - Q\right)u^2 + (Q^2 - 12P^3 - PS)u + N = 0,$$

where

$$N = \frac{P^6}{v} + P^2\left(\frac{P^3Q}{S} + 6PQ + E\right).$$

84. But, from (a), we find

$$\frac{P^6}{v} = P^6\left(\frac{Q-u}{u^2}\right) = -\frac{P^6}{u} + \frac{P^6Q}{u^2},$$

and, multiplying (d) into $\frac{P^3}{Qu}$, transposing, &c., we obtain

$$\frac{P^6}{u} = \frac{P^3}{Q}u^2 - P^3u + \frac{P^4(P^2+S)}{Q} \;\ldots\ldots\ldots(44).$$

Moreover, multiplying this result into $\frac{Q}{u}$, we have

$$\frac{P^6Q}{u^2} = P^3u - P^3Q + P^4(P^2+S)\frac{1}{u},$$

or, substituting for $\frac{1}{u}$ its value deduced from (44) and reducing,

$$\frac{P^6Q}{u^2} = \frac{P(P^2+S)}{Q}u^2 - PSu + \frac{P^2(P^2+S)^2}{Q} - P^3Q.$$

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85. Hence

$$\frac{P^6}{v} = \frac{PS}{Q}u^2 + P(P^2 - S)u + \frac{P^4S}{Q} - P^3Q + \frac{P^2S^2}{Q},$$

and this value, substituted in the N of Art. 83, converts the "difference" into

$$Au^2 + Bu + C = 0 \;\ldots\ldots\ldots\ldots\ldots\ldots\; (e),$$

where

$$A = \frac{P^2Q}{S} + \frac{PS}{Q} - Q,$$

$$B = -11P^3 - 2PS + Q^2,$$

$$C = P^2\left(\frac{P^3Q}{S} + \frac{P^2S}{Q} + 5PQ + \frac{S^2}{Q} + E\right).$$

86. Multiply (d) into A and (e) into u, and subtract the former from the latter product. We find

$$(B + QA)u^2 + \{C - AP(P^2 + S)\}u + AP^3Q = 0,$$

or

$$A'u^2 + B'u + C' = 0 \;\ldots\ldots\ldots\ldots\ldots\ldots\; (f)$$

where

$$A' = -11P^3 + \frac{P^2Q^2}{S} - PS,$$

$$B' = P^2\left(E + 5PQ + \frac{QS}{P}\right),$$

$$C' = P^3\left(\frac{P^2Q^2}{S} + PS - Q^2\right).$$

87. Having first divided (f) by P, substitute Pw for u in both (e) and (f) and reduce. We shall be conducted to a pair of equations which may be represented by

$$\left.\begin{array}{l} lw^2 + mw + n = 0 \\ \lambda w^2 + \mu w + \nu = 0 \end{array}\right\} \;\ldots\ldots\ldots\ldots\ldots\ldots\; (g),$$

where

$$l = A, \quad m = \frac{B}{P}, \quad n = \frac{C}{P^2},$$

$$\lambda = \frac{A'}{P}, \quad \mu = \frac{B'}{P^2}, \quad \nu = \frac{C'}{P^3}.$$

88. The result of the elimination of w from (g) is

$$(\lambda n - l\nu)^2 + (l\mu - \lambda m)(\mu n - m\nu) = 0,$$

a relation which admits of simplification.

89. In fact, if we assume

$$P = z^2, \; Q = qz^3, \; S = sz^4, \; E = ez^5,$$

$$\frac{q}{s} + \frac{s}{q} = t,\ 1 + s = w,\ e + 4q = f,$$

and

$$-11 - 2s = h,$$

that relation becomes

$$z^{24}\{(\alpha c - a\gamma)^2 + (a\beta - \alpha b)(\beta c - b\gamma)\} = 0 \ldots\ldots\ldots (h),$$

where

$$a = t - q,\ b = h + q^2,\ c = f + wt,$$
$$\alpha = h + qt,\ \beta = f + qw,\ \gamma = qa,$$

and the present are to be distinguished from the meanings elsewhere attached to some of the symbols.

90. Confining our attention to the bracketed factors of (h) we have

$$\alpha c - a\gamma = (h + qt)(f + wt) - q(t - q)^2,$$
$$a\beta - \alpha b = (t - q)(f + wq) - (h + qt)(h + q^2),$$
$$\beta c - b\gamma = (f + qw)(f + wt) - q(t - q)(h + q^2),$$

and, if we put (h) under the form

$$z^{24}\{(t - q)f^3 + \phi f^2 + \chi f + \psi\} = 0,$$

we shall find

$$\phi = (t - q)\{w(t + 2q) + q(h + qt)\},$$

and, in fact, that (h) becomes

$$z^{24}(t - q)(f^3 + \delta f^2 + \varepsilon f + \zeta) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (i),$$

where δ, ε and ζ are rational and integral functions of t, q, h, w and a, or of some of those quantities.

91. Having obtained (i) we might transform it into

$$z^{24}(t - q)(e^3 + \eta e^2 + \theta e + \xi) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (j),$$

but (j) may be better obtained by the aid of the following formulæ.

92. Let

$$\frac{\alpha c}{a} - \gamma = \rho \quad \text{and} \quad \beta - \frac{\alpha b}{a} = \sigma,$$

then, comparing (h) and (j), we find

$$a\rho^2 + (\beta c - b\gamma)\sigma = e^3 + \eta e^2 + \theta e + \xi \ldots\ldots\ldots\ldots (k).$$

93. Assuming

$$H = \frac{\alpha}{a} = \frac{h + qt}{t - q},\ K = 4q + wt \text{ and } L = 4 + w,$$

we have

$$\rho = H(e + K) - qa$$
$$\sigma = e + Lq - aH(H - q),$$

and

$$\beta c - b\gamma = (e + K)(e + Lq) - qa^2(H - q),$$

and it will be noticed that

$$K - Lq = w(t - q) \text{ and } h + q^2 = (H - q)(t - q).$$

94. Developing

$$a\{H(e+K)-qa\}^2$$
$$+\{e+Lq-aH(H-q)\}\{(e+K)(e+Lq)-qa^2(H-q)\}$$

we see that the coefficient of e^3 is unity.

95. That of e^2 gives

$$\eta=aH^2+(K+Lq)+Lq-aH(H-q)$$
$$=q(aH+2L)+K=3q+(w+q^2)t.$$

96. That of e gives

$$\theta=a\{2H^2K-2qaH\}+KLq-qa^2(H-q)$$
$$+(K+Lq)\{Lq-aH(H-q)\}$$
$$=K\{\eta+a(Hq-2w)\}$$
$$+a^2\{w^2+q^2-3Hq+wH(H-q)\}$$
$$=(4q+wt)\{(2q^2-w)t-6q\}+w^2(t-q)^2$$
$$+q^2(t-q)^2-3q(h+qt)(t-q)+w(h+qt)(h+q^2)$$
$$=q^4+(s+10)q^3t-(s^2+17s+67)q^2+2sq^2t^2$$
$$+(10-21s-4s^2)qt+4s^3+48s^2+165s+121.$$

97. The absolute term gives

$$\xi=a(HK-qa)^2$$
$$+\{Lq-aH(H-q)\}\{KLq-qa^2(H-q)\}$$
$$=K^3-K^2\eta+K\theta+q^2a^3$$
$$+qa^2(H-q)\{wa+aH(H-q)\}$$
$$=K(K^2-K\eta+\theta)+q^2(t-q)^3$$
$$+q(h+q^2)\{w(t-q)^2+(h+q^2)(h+qt)\}.$$

98. But

$$K^2-K\eta+\theta=K(q-q^2t)+\theta=(s-1)q^2t^2$$
$$+(s+6)q^3t+(11-20s-4s^2)qt+q^4$$
$$-(s^2+17s+63)q^2+wh^2;$$

and if we multiply this expression into K and to the product add

$$q^2t^3-3q^3t^2+3q^4t-q^5$$
$$+w\{(qh+q^3)t^2-2(qh+q^3)qt+(qh+q^3)q^2\}$$
$$+h^2q^2t+2hq^4t+q^6t+h^3q+2h^2q^3+hq^5$$

we find, on reduction,

$$\xi=s^2q^2t^3+(s+12)q^3st^2-(22+26s+4s^2)qst^2$$
$$+q^6t+(4-s)q^4t+(124-90s-26s^2-s^3)q^2t$$
$$+(121+286s+213s^2+52s^3+4s^4)t-(7+s)q^5$$
$$+(wh+2h^2-4s^2-68s-252)q^3+(4wh^2+h^3)q.$$

99. Next, multiplying (k) into qs, we find

$$qse^3 + qs\eta e^2 + qs\theta e + qs\xi =$$
$$qse^3 + \{3q^2s + (1+s+q^2)\,qst\}\,e^2$$
$$+\{2q\,(qst)^2 + (s+10)\,q^3\,(qst) + (10-21s-4s^2)\,q\,(qst)$$
$$+ (4s^3+48s^2+165s+121)\,qs - (s^2+17s+67)\,q^3s\}\,e$$
$$+ (qst)^3 + \{sq^2 + 12q^2 - 22 - 26s - 4s^2\}(qst)^2$$
$$+ \{q^6 + (4-s)\,q^4 + (124-90s-26s^2-s^3)\,q^2$$
$$+ 121 + 286s + 213s^2 + 52s^3 + 4s^4\}qst$$
$$+ (2s^2+7s-21)\,q^4s - (s+7)\,q^6s$$
$$+ (8s^3+60s^2-66s-847)\,q^2s \ldots\ldots\ldots\ldots (l).$$

100. Now form the equation

$$z^{24}qs\{e^3 + \eta e^2 + \theta e + \xi\} = \pi,$$

and by means of the relations of Art. 89 (which furnishes us with

$$qst = q^2 + s^2)$$

eliminate z, q, s, t and e from π. We shall find, after making the necessary substitutions and reductions in (l),

$$\pi = \mathrm{PQSE^3} + \{\mathrm{P^4Q^2 + P^3S^2 + 4P^2Q^2S + PQ^4 + PS^3 + Q^2S^2}\}\mathrm{E^2}$$
$$+\{\mathrm{121P^6QS + 10P^5Q^3 + 175P^4QS^2 - 88P^3Q^3S}$$
$$+ \mathrm{12P^2Q^5 + 27P^2QS^3 - 7PQ^3S^2 + 2Q^5S + 2QS^4}\}\mathrm{E}$$
$$+ \mathrm{121P^9Q^2 + 121P^6S^2 - 561P^7Q^2S}$$
$$+ \mathrm{102P^6Q^4 + 286P^6S^3 + 227P^5Q^2S^2}$$
$$- \mathrm{137P^4Q^4S + 191P^4S^4 + 17P^3Q^6}$$
$$- \mathrm{30P^3Q^2S^3 + 8P^2Q^4S^2 + 26P^2S^5}$$
$$- \mathrm{7PQ^2S^4 - 7PQ^6S + Q^8 + 2Q^4S^3 + S^6}.$$

101. This expression for π is the SYMMETRIC PRODUCT characteristic of equations of the fifth degree.

102. The multiplier ($t-q$) of (i) seems to be an extraneous factor.

103. When π vanishes we have a solvible form of which the quadrinomials of De Moivre and of Euler are particular cases.

104. Thus, let P be zero. We find

$$\pi = \mathrm{Q^2R^2E^2 + 2\,(Q^5R + QR^4)\,E + Q^8 + 2Q^4R^3 + R^6}$$
$$= \{\mathrm{QRE + Q^4 + R^3}\}^2;$$

consequently, when π vanishes, we have

$$\mathrm{QRE + Q^4 + R^3 = 0},$$

a criterion due to Euler. When this condition is satisfied the values of y are all included in the formula

$$\alpha\sqrt[5]{\frac{\mathrm{R}^2}{\mathrm{Q}}} + \alpha^2\sqrt[5]{\frac{\mathrm{Q}^3}{\mathrm{R}}},$$

as is shown by Euler at page 94 of vol. ix of the Petersburgh 'Novi Commentarii.'

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105. When Q and S vanish π vanishes also. These conditions correspond to the well-known form solved by De Moivre in the 'Philosophical Transactions' for 1706-7.

106. The "symmetric product" is pregnant with results. In the particular case in which Q is zero we have

$$\pi = S^2\{P(P^2 + PS)E^2 + (11P^4 + 13P^2S + S^2)^2\},$$

and, consequently, we are conducted not only to the solvible quintic of De Moivre (with which the vanishing of the factor S^2 furnishes us) but also to the solvible quadrinomial form indicated by

$$P(P^2 + S)E^2 + (11P^4 + 13P^2S + S^2)^2 = 0.$$

107. If in this last equation we make

$$S = 0 \text{ and } P = -A^2$$

we may represent the roots of the quintic in y by the formula

$$y = A\left\{\alpha\sqrt[5]{\left(\frac{-\sqrt{11}+\sqrt{15}}{2}\right)} + \alpha^4\sqrt[5]{\left(\frac{-\sqrt{11}-\sqrt{15}}{2}\right)}\right\}.$$

108. Again, if P or $P^2 + S$ be supposed to vanish, the last condition of Art. 106 points to the ordinary binomial form.

(*To be continued.*)

Postscript.—In Art. 47 ('Diary' for 1857, page 75)

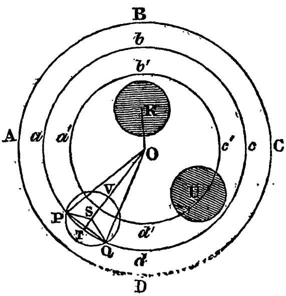
for $B' = -(u+v)$ *read* $B' = -(u^2+v^2)$.

76, Cambridge Terrace, Hyde Park,
30th May, 1857.

GENERAL ANSWER TO QUESTION VIII (1921).

By Dr. Rutherford, *of the Royal Military Academy, Woolwich.*

Two unequal coins are thrown horizontally into a given circular box; find the probability of only the larger coin resting on the box, supposing every possible position of each coin to be equally probable.



Let the radius of the smaller coin be *unity*, the radius of the larger coin $= n$, and that of the box $= m + 1$. Let the circle ABCD represent the magnitude of the bottom of the box, and with radii m, and $m - 1$ describe the concentric circles $abcd$ and $a'b'c'd'$, respectively. Then it is evident, that whilst the larger coin (n) falls within the circle $abcd$, the smaller coin (1) has the entire surface of the larger coin to rest upon, as, for examples, when its centre is at R or U; but when the outer edge of the coin (n) falls between the circumferences *abcd* and ABCD, as when its centre is at S, the smaller coin (1) will have only the area PVQT of the coin (n) to rest upon. We shall consider these cases separately.

Draw OR, OP, OQ, OS, SP, SQ, PQ; let $QOS = \phi$, $QST = \theta$, $OR = r$, and $m - n = k$; then we have area of coin $(n) = n^2\pi$,

area of sector $PSQ = n^2\theta$; area of triangle $PSQ = n^2 \sin\theta \cos\theta$;

area of sector $POQ = m^2\phi$; area of triangle $POQ = m^2 \sin\phi \cos\phi$;

$\therefore$ area $PVQT = \pi n^2 - n^2(\theta - \sin\theta\cos\theta) + m^2(\phi - \sin\phi\cos\phi)$...(1).

Now the chance of the centre R of the coin (n) falling in an elemental concentric ring, whose breadth is dr, is

$$\frac{2\pi r dr}{\pi(m+1-n)^2} = \frac{2rdr}{(m+1-n)^2} = \frac{d(r^2)}{(m+1-n)^2} = \frac{d(r^2)}{(k+1)^2} \dots (2).$$

And the centre of the smaller coin (1) may fall in any part of the circle *abcd*; therefore the chance of its resting on the coin (n) is

$$\frac{\pi n^2}{\pi m^2} = \frac{n^2}{m^2} \dots\dots\dots\dots\dots\dots (3).$$

Therefore, if the outer edge of the coin (n) does not fall beyond the circumference, *abcd*, the probability of the coin (1) resting on the coin (n) is

$$\int \frac{d(r^2)}{(k+1)^2} \times \frac{n^2}{m^2} = \frac{n^2(m-n)^2}{m^2(k+1)^2} = \frac{n^2k^2}{m^2(k+1)^2} \dots\dots\dots (4);$$

between the limits $r = 0$, and $r = m - n = k$.

In the other case, when the outer edge of the coin (n) falls between the circumferences *abcd*, and ABCD, the limits will be $r = m - n = k$, and $r = k + 1$; and the probability of the smaller coin falling upon the larger one is,

$$\int_{k^2}^{(k+1)^2} \frac{d(r^2)}{(k+1)^2} \cdot \frac{PVQT}{\pi m^2} = \frac{1}{\pi m^2 (k+1)^2} \int_{k^2}^{(k+1)^2} PVQT \,.\, d(r^2) \dots (5).$$

But the area PVQT has been determined in (1), and the geometry of the figure gives

$$OQ = m,\ QS = n,\ OS = r,\ QOS = \phi, \text{ and } OSQ = \pi - \theta;$$

hence, $r^2 = m^2 + n^2 - 2mn\cos(\theta - \phi)$,

$$\text{or, } r^2 = m^2 + n^2 - 2mn\cos\theta\cos\phi - 2mn\sin\theta\sin\phi \dots\dots (6),$$

$$n\sin\theta = m\sin\phi \dots\dots\dots\dots\dots\dots (7),$$

$$\therefore n\cos\theta d\theta = m\cos\phi d\phi \dots\dots\dots\dots\dots\dots (8).$$

Now if (6) be multiplied by $d\theta$ and $d\phi$ respectively, and modified by the relations in (7) and (8), we shall obtain very convenient results. Thus:

$$r^2 d\theta = (m^2 + n^2)\, d\theta - 2m^2\cos^2\phi d\phi - 2n^2\sin^2\theta d\theta,$$

$$= (m^2 - n^2)\, d\theta - 2m^2\cos^2\phi d\phi + 2n^2\cos^2\theta d\theta \dots\dots\dots\dots (9).$$

$$r^2 d\phi = (m^2 + n^2)\, d\phi - 2n^2 \cos^2\theta d\theta - 2m^2 \sin^2\phi d\phi$$
$$= -(m^2 - n^2)\, d\phi - 2n^2 \cos^2\theta d\theta + 2m^2 \cos^2\phi d\phi \ldots\ldots\ldots (10).$$

Now if we write in (5) the value of PVQT as found in (1), and integrate, without attending to the limits of r, or the constant multiplier $\frac{1}{\pi m^2 (k+1)^2}$, we have

$$\int \{\pi n^2 - n^2 (\theta - \sin\theta\cos\theta) + m^2 (\phi - \sin\phi\cos\phi)\}\, d(r^2)$$
$$\left.\begin{array}{l} = \pi n^2 r^2 - n^2 r^2 (\theta - \sin\theta\cos\theta) + m^2 r^2 (\phi - \sin\phi\cos\phi) \\ \quad + 2n^2 \int r^2 \sin^2\theta d\theta - 2m^2 \int r^2 \sin^2\phi d\phi \end{array}\right\} \ldots (11).$$

Multiply (9) by $2n^2 \sin^2\theta$, and (10) by $2m^2 \sin^2\phi$, modifying the products by the relation in (7); then we have

$$2n^2 r^2 \sin^2\theta d\theta - 2m^2 r^2 \sin^2\phi d\phi$$
$$= \left\{\begin{array}{l} 2n^2 (m^2 - n^2) \sin^2\theta d\theta + 2m^2 (m^2 - n^2) \sin^2\phi d\phi \\ \quad + n^4 \sin^2 (2\theta)\, d(2\theta) - m^4 \sin^2 (2\phi)\, d(2\phi) \end{array}\right\}$$

Integrating this expression, (11) will become

$$\int \{\pi n^2 - n^2 (\theta - \tfrac{1}{2}\sin 2\theta) + m^2 (\phi - \tfrac{1}{2}\sin 2\phi)\}\, d(r^2)$$
$$= \pi n^2 r^2 + \left\{\begin{array}{l} m^2 r^2 (\phi - \tfrac{1}{2}\sin 2\phi) - n^2 r^2 (\theta - \tfrac{1}{2}\sin 2\theta) + \\ m^2 (m^2 - n^2)(\phi - \tfrac{1}{2}\sin 2\phi) + n^2 (m^2 - n^2)(\theta - \tfrac{1}{2}\sin 2\theta) \\ - m^4 (\phi - \tfrac{1}{4}\sin 4\phi) + n^4 (\theta - \tfrac{1}{4}\sin 4\theta) \end{array}\right\}$$
$$= \pi n^2 r^2 + \left\{\begin{array}{l} m^2 (r^2 - n^2)\phi - n^2 (r^2 - m^2)\theta - \tfrac{1}{2} m^2 (r^2 + m^2 - n^2)\sin 2\phi \\ + \tfrac{1}{2} n^2 (r^2 - m^2 + n^2)\sin 2\theta + \tfrac{1}{4} m^4 \sin 4\phi - \tfrac{1}{4} n^4 \sin 4\theta \end{array}\right\} (12).$$

Now when $r = m - n = k$, the values of ϕ and θ are both zero, and the expression within the brackets vanishes at this limit; and if we take the part $\pi n^2 r^2$ between the specified limits $r = k$ and $r = k + 1$, and add the result to (4), we have by (5),

$$\frac{n^2 k^2}{m^2 (k+1)^2} + \frac{\pi n^2 (2k+1)}{\pi m^2 (k+1)^2} = \frac{n^2 (k+1)^2}{m^2 (k+1)^2} = \frac{n^2}{m^2} \ldots\ldots (13).$$

It only now remains to find the value of the bracketed expression in (12), at the higher limit $r = k + 1$. Referring to the triangle OSQ, we have

$$\cos\phi = \frac{m^2 + (k+1)^2 - n^2}{2m (k+1)} = 1 - \frac{2n - 1}{2m (k+1)} \ldots (14),$$

$$\cos\theta = \frac{m^2 - (k+1)^2 - n^2}{2n (k+1)} = 1 - \frac{2m + 1}{2n (k+1)} \ldots (15);$$

hence, $2\sin^2 \tfrac{1}{2}\phi = \dfrac{2n - 1}{2m (k+1)}$; $2\sin^2 \tfrac{1}{2}\theta = \dfrac{2m + 1}{2n (k+1)}$; ... (16),

$$\therefore \phi = 2\sin^{-1} \left\{\frac{2n - 1}{4m (k+1)}\right\}^{\frac{1}{2}};\ \theta = 2\sin^{-1} \left\{\frac{2m + 1}{4n (k+1)}\right\}^{\frac{1}{2}} \ldots (17).$$

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The expressions in (12) and (13) give the total probability of only the larger coin (n) resting on the box, viz.:

$$\frac{n^2}{m^2}+\frac{1}{\pi m^2(k+1)^2}\{m^2(m+1)(k-n+1)\phi+n^2(n-1)(k+m+1)\theta\}$$
$$+\frac{1}{\pi m^2(k+1)^2}\{-\tfrac{1}{2}m^2(r^2+m^2-n^2)\sin 2\phi+\tfrac{1}{2}n^2(r^2-m^2+n^2)\sin 2\theta\}$$
$$+\frac{1}{\pi m^2(k+1)^2}\{\tfrac{1}{4}m^4\sin 4\phi-\tfrac{1}{4}n^4\sin 4\theta\};$$

where $r=k+1=m-n+1$, and the values of ϕ and θ as in (17). When both coins are equal, $n=1$, and the probability in this case is

$$\frac{1}{m^2}+\frac{1}{\pi(k+1)^2}\{(m^2-1)\phi-(2m^2-1)\sin\phi\cos\phi\}$$
$$+\frac{1}{\pi m^2(k+1)^2}\{\sin\theta\cos\theta+\frac{m^4}{4}\sin 4\phi-\frac{1}{4}\sin 4\theta\};$$

where $\phi=2\sin^{-1}\dfrac{1}{2m}$, and $\theta=2\sin^{-1}\left(\dfrac{2m+1}{4m}\right)^{\frac{1}{2}}=\tfrac{1}{2}(\pi+\phi)$.

Taking the case proposed in the question, we have $m=1$, and the probability is

$$1+\frac{1}{4\pi}(2\sin 2\theta-2\sin 2\phi-\sin 4\theta+\sin 4\phi)$$
$$=1+\frac{1}{4\pi}(2\sin 240^0-2\sin 120^0-\sin 480^0+\sin 240^0)$$
$$=1+\frac{1}{4\pi}(-\sqrt{3}-\sqrt{3}-\tfrac{1}{2}\sqrt{3}-\tfrac{1}{2}\sqrt{3})$$
$$=1-\frac{3\sqrt{3}}{4\pi}.$$

Consequently, the probability of both coins resting on the box is

$$1-\left(1-\frac{3\sqrt{3}}{4\pi}\right)=\frac{3\sqrt{3}}{4\pi}.$$

NOTÆ GEOMETRICÆ.

By T. T. WILKINSON, F.R.A.S., *Cor. Mem. Manch. Lit. and Phil. Society; Mem. Historic Soc. Lancash. and Cheshire, &c.*

1. Although for a considerable period I have been engaged in pursuits not very favorable to mathematical studies, yet I am induced to offer the following as Notes to the Prize Question (1883) in the

'Diary' for 1854-5, and as additions to what has been so kindly claimed for me by the Editor on page 86 of last year's 'Diary.'

2. In the extended solution to which allusion is there made, I noticed that the property became porismatic when *any three points* are taken for the feet of the perpendiculars, and the triangles thence resulting are constructed according to an almost obvious law. All the triangles thus formed are evidently those of *least* perimeter to the *primitives* obtained by bisecting the angles formed by joining the feet of the perpendiculars in each instance, and hence connect themselves immediately with the many beautiful and curious properties known to result from this view of the subject. (See Carnot's 'Géométrie de Position,' pp. 177—238; also the 'Cambridge Mathematical Journal,' vol. i, pp. 157—162, and Potts's 'Euclid and Hints,' pp. 301 and 31 respectively.)

3. But the property admits of a more general enunciation than that given in the Prize Question by projecting the system upon a plane, according to the principles laid down by several authors, but principally by Ferriot, in his 'Application de la Methode des Projections à la recherche de certaines propriétés Géométriques;' for the perpendiculars then become lines drawn conjugate to the opposite sides, the contacts are preserved, and the circles become conics similar and similarly placed.

4. Hence, if ABC be a triangle inscribed in a conic, and if through each vertex there be drawn a transversal respectively conjugate to the opposite side, then

I. *These three transversals will intersect in the same point* O.

II. *The middle points of the lines* OA, OB, OC, *the middle points of the sides* AB, BC, CA, *and the points* a, b, c, *where the transversals meet the sides, are nine points situated in a second conic, similar and similarly placed with respect to the first.*

III. *This second conic is also tangential to the sixteen conics, inscribed and escribed to the triangles* AOB, BOC, COA, *and similar and similarly placed with respect to the two first conics.*

5. I may also add, that the Rev. Joseph Wolstenholme, of Christ College, Cambridge, has published demonstrations of several of the properties of my lemmas in the 'Quarterly Journal of Mathematics,' No. VI, pp. 138—140; both these, however, and many other properties of the *triangle of least perimeter*, were collected by the late Professor Davies, and were published by him under the title of "Symmetrical Properties of Plane Triangles," in the 'Philosophical Magazine' for July, 1827. This paper formed the germ of the "Horæ Geometricæ" in the 'Diary.'

(To be continued.)

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MODERN GEOMETRY.

By JOHN JOSHUA ROBINSON, *of H.M. Dockyard, Portsea.*

(*Continued from page* 92, '*Diary*' *for* 1857.)

My best thanks are due to the Rev. George Salmon, of Trinity College, Dublin, for having called my attention to two errors, which somehow have crept into my former paper. I can only state, in exculpation, that the latter portion of the paper was hastily written, and did not receive as much attention as the subject deserved.

Moreover, as that eminent geometer justly observes in one of his letters to me, "There are many points connected with the 'abridged notation' in which it requires some experience to be secure from error. A good analyst, to whom I sent the equation of the circle through the middle points of the sides of a triangle, gave me up the same equation as you found." Indeed, after having recently devoted a great deal of attention to the subject, I am of opinion that there are many points connected with this notation, as given in works treating on the subject, which would admit of further elucidation. I must again state, as I have already done in my former papers, that Salmon's 'Conics' (especially the third edition, which contains a host of new examples) as a complete treatise is the best in any language. I apprehend, however, that I have perceived where many of the difficulties in applying the method will occur to the learner, and it is not unlikely that I shall hereafter publish a tract on the subject, illustrated by numerous examples.

To return to the errors:—the first occurs on page 89, Art. 35, where the radical axes of inscribed and escribed circles are obtained by subtracting the equations of those circles in trilinear coordinates. A little reflection will at once show that this method is incorrect, since the resulting equations are not, strictly speaking, linear in α, β, γ, and therefore in x and y.

39. The general method of finding the equations of common chords to two curves of the second degree, of which circles are only particular cases, is given in Salmon's 'Conics,' 3d edition, p. 250, and is done somewhat in the following manner—

Let S and S′ represent two conics of the forms

$$(S)\ldots\ldots a_1\alpha^2+a_2\beta^2+a_3\gamma^2+2b_1\alpha\beta+2b_2\alpha\gamma+2b_3\beta\gamma=0\ldots\ldots(1),$$

$$(S')\ldots A_1\alpha^2+A_2\beta^2+A_3\gamma^2+2B_1\alpha\beta+2B_2\alpha\gamma+2B_3\beta\gamma=0\ldots(2).$$

1. Find the *discriminant* of (S); that is, *the condition that equation* (1) *shall break up into two right lines.* Now whenever this is the case

it will be found that the polar of any given point passes through the point of intersection of the two right lines, and is the fourth line of the harmonic pencil, of which the two lines into which the conic breaks up, and the line joining the given point to the point of intersection of these two lines, are the other three. Now if we differentiate (1) in regard to α, β and γ remaining constant in the differentiation, $\frac{dS}{d\alpha}=0$ will represent the *polar* of the point $(\beta\gamma)$. In the same way $\frac{dS}{d\beta}$ will represent the polar of $(\alpha\gamma)$, &c. If we next assume $\frac{dS}{d\alpha}$, $\frac{dS}{d\beta}$ and $\frac{dS}{d\gamma}$ to represent three lines passing through the same point, then $\alpha'\frac{dS}{d\alpha}+\beta'\frac{dS}{d\beta}+\gamma'\frac{dS}{d\gamma}=0$ will also represent a right line which is the polar of the point $(\alpha'\beta'\gamma')$. Hence, differentiating (1) in regard to α, β, γ successively, and putting the resulting equation equal to zero, we have

$$\left(\frac{dS}{d\alpha}\right)\ldots\ldots a_1\alpha+b_1\beta+b_2\gamma=0\ \ldots\ldots\ldots\ldots(3),$$

$$\left(\frac{dS}{d\beta}\right)\ldots\ldots a_2\beta+b_1\alpha+b_3\gamma=0\ \ldots\ldots\ldots\ldots(4),$$

$$\left(\frac{dS}{d\gamma}\right)\ldots\ldots a_3\gamma+b_2\alpha+b_3\beta=0\ \ldots\ldots\ldots\ldots(5).$$

We find the condition that these three lines should pass through the same point by eliminating the ratios $\frac{\beta}{\alpha}$ and $\frac{\gamma}{\alpha}$ between their equations; hence

$$a_1b_3^2+b_1^2a_3+b_2^2a_2-a_1a_2a_3-2b_1b_2b_3=0\ldots\ldots(6)^*$$

2. To find the discriminant of (1) and (2), or of $kS+S'$ (where k is a constant) we have only to write ka_1+A_1 for a_1; ka_2+A_2 for a_2; ka_3+A_3 for a_3, &c. in the equation just found, and we obtain

---

* This equation has heretofore been called the *determinant* of the system of equations (3), (4), (5); but I perceive Mr. Newman (if I remember rightly, for I have not the paper at hand), has, in a recent paper, printed in the 'Transactions of the Royal Society,' proposed to call it the *eliminant*. The latter term I had proposed long before I saw Mr. Newman's paper in print; I think it more appropriate, inasmuch as it is significant of the operation performed on the equations.

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$$k^3(a_1b_3^2+a_2b_2^2+a_3b_1^2-a_1a_2a_3-2b_1b_2b_3)+k^2\{A_1(b_3^2-a_2a_3)$$
$$+A_2(b_2^2-a_1a_3)+A_3(b_1^2-a_1a_2)+2B_1(a_3b_1-b_2b_3)$$
$$+2B_2(a_2b_2-b_1b_3)+2B_3(a_1b_3-b_1b_2)\}$$
$$+k\{a_1(B_3^2-A_2A_3)+a_2(B_2^2-A_1A_3)+a_3(B_1^2-A_1A_2)$$
$$+2b_1(A_3B_1-B_2B_3)$$
$$+2b_2(A_2B_2-B_1B_3)+2b_3(A_1B_3-B_1B_2)\}$$
$$+A_1B_3^2+A_2B_2^2+A_3B_1^2-A_1A_2A_3-2B_1B_2B_3=0\ldots(7).$$

This is generally called the *discriminating cubic*, inasmuch as it will lead to three values of k, which we shall call k_1, k_2, k_3. Then will $k_1S+S'=0$; $k_2S+S'=0$; $k_3S+S'=0$, represent the three pairs of common chords of the two conics.

It will be at once evident that there ought to be *three pairs* of right lines, since if the conics intersect in the points A, B, C, D, we can draw AB and CD, AC and BD, AD and BC. If we represent the discriminant of (S) by ∇, that of (S′) by ∇', then it is clear that the coefficient of k^3 in (7) is ∇, the absolute term, ∇'; the coefficient of k^2 is

$$A_1\frac{d\nabla}{da_1}+A_2\frac{d\nabla}{da_2}+A_3\frac{d\nabla}{da_3}+B_1\frac{d\nabla}{db_1}+B_2\frac{d\nabla}{db_2}+B_3\frac{d\nabla}{db_3}=0,$$

and the coefficient of k is

$$a_1\frac{d\nabla'}{dA_1}+a_2\frac{d\nabla'}{dA_2}+a_3\frac{d\nabla'}{dA_3}+b_1\frac{d\nabla'}{dB_1}+b_2\frac{d\nabla'}{dB_2}+b_3\frac{d\nabla'}{dB_3}=0.$$

For more information on this subject, see Salmon's 'Conics,' p. 250, Art. 296.

40. A circle, as we have before stated, being a particular case of a conic, we know that two circles can, in the most general case, have only one real radical axis, or common chord, and hence intersect in two real points, the other two points of intersection being at infinity; it follows that, "*All circles pass through the same two imaginary points, at infinity*,"* and that, "*Two concentric circles touch each other in two imaginary points at infinity.*"

The above being the general method, it will at once be seen that it behoves us to simplify it for two circles. The following is the plan adopted by the Rev. George Salmon, and communicated to me in a letter, dated Trinity College, Dublin, July 4th, 1857.

41. "The equation to any circle can be obtained by adding to the

* Salmon's 'Conics,' p. 215, Art. 259.

equation of the circumscribed circle the equation of a straight line multiplied by a constant. It must therefore be of the form

$$(8)\ldots(\alpha \sin A + \beta \sin B + \gamma \sin C)(l\alpha + m\beta + n\gamma)$$
$$+ \gamma\beta \sin A + \gamma\alpha \sin B + \alpha\beta \sin C = 0;$$

and if

$$(9)\ldots(\alpha \sin A + \beta \sin B + \gamma \sin C)(l'\alpha + m'\beta + n'\gamma)$$
$$+ \gamma\beta \sin A + \gamma\alpha \sin B + \alpha\beta \sin C = 0$$

represent any other circle, then by subtraction we find

$$\{(l-l')\alpha + (m-m')\beta + (n-n')\gamma\}\{\alpha \sin A + \beta \sin B + \gamma \sin C\} = 0.$$

But $\alpha \sin A + \beta \sin B + \gamma \sin C = 0$ represents a line altogether at an infinite distance, that is, the line joining the two imaginary points of intersection at infinity; therefore

$$(l-l')\alpha + (m-m')\beta + (n-n')\gamma = 0 \ldots\ldots (10)$$

is the equation of the radical axis of (8) and (9)."

This method is readily applicable to equations of the forms (8) and (9), but when the equations are of the form

$$a\alpha^2 + b\beta^2 + c\gamma^2 + 2a_1\beta\gamma + \&c. = 0,$$

we have to find relations among the coefficients a, b, c and those of like powers in any given circle, then proceed to find the radical axis. It will be observed that in finding the radical axis of two circles the coefficients of powers of α, β, γ, above the first, must be made the same in both equations before we subtract, otherwise the resulting equation will not be linear.

42. I have found the following to be the most expeditious method in the case of inscribed and escribed circles.

The equation of the inscribed circle is

$$\alpha^{\frac{1}{2}} \cos\frac{A}{2} + \beta^{\frac{1}{2}} \cos\frac{B}{2} + \gamma^{\frac{1}{2}} \cos\frac{C}{2} = 0 \ldots\ldots\ldots (11),$$

and that of the circle escribed to the side BC, or $\alpha = 0$, is

$$\alpha^{\frac{1}{2}} \cos\frac{A}{2} + \beta^{\frac{1}{2}} \sin\frac{B}{2} + \gamma^{\frac{1}{2}} \sin\frac{C}{2} = 0 \ldots\ldots\ldots (12).$$

From (11) $\quad \alpha^{\frac{1}{2}} \cos\frac{A}{2} = -\left(\beta^{\frac{1}{2}} \cos\frac{B}{2} + \gamma^{\frac{1}{2}} \cos\frac{C}{2}\right),$

" (12) $\quad \alpha^{\frac{1}{2}} \cos\frac{A}{2} = -\left(\beta^{\frac{1}{2}} \sin\frac{B}{2} + \gamma^{\frac{1}{2}} \sin\frac{C}{2}\right).$

Squaring these two equations, we obtain

$$\alpha \cos^2 \frac{A}{2} = \beta \cos^2 \frac{B}{2} + \gamma \cos^2 \frac{C}{2} + 2\sqrt{\beta\gamma} \cos \frac{B}{2} \cos \frac{C}{2} \ldots\ldots (13),$$

$$\alpha \cos^2 \frac{A}{2} = \beta \sin^2 \frac{B}{2} + \gamma \sin^2 \frac{C}{2} + 2\sqrt{\beta\gamma} \sin \frac{B}{2} \sin \frac{C}{2} \ldots\ldots (14).$$

Now, if we eliminate $\sqrt{\beta\gamma}$ between the two latter equations, it is clear that the resulting equation will be linear in α, β, γ, and will therefore represent a right line passing through the points of intersection of (11) and (12). We must, however, bear in mind that, after we have made the coefficients of $\sqrt{\beta\gamma}$ alike in both equations, the α in (13) will have a contrary sign to the α in (14), since the perpendiculars change signs in passing from one side of a line to the other.

The result of such elimination, after some obvious reductions, is

$$\alpha \frac{(\sin B + \sin C) \sin \frac{B+C}{2}}{\sin \frac{B-C}{2}} - \beta \sin B + \gamma \sin C = 0 \ldots\ldots (15);$$

and it is apparent that this is a line through the middle point of BC; that is, through the intersection of $\alpha = 0$ and $\gamma \sin C - \beta \sin B = 0$, and it may easily be shown that it is parallel to the exterior bisector $\beta + \gamma = 0$.

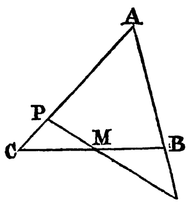
In like manner the equation to the radical axis of the inscribed circle and that escribed to AC or $\beta = 0$ is found to be

$$-\alpha \sin A + \beta \frac{(\sin A + \sin C) \sin \frac{A+C}{2}}{\sin \frac{A-C}{2}} + \gamma \sin C = 0 \ldots\ldots (16),$$

a line through the middle point of AC, or $\beta = 0$ and $\gamma \sin C - \alpha \sin A = 0$.

All the radical axes of the other circles, taken two and two, may be found in the same way.

43. *Note.* In order to verify equation (15), let us take it for granted that this radical axis passes through the middle point of the base BC, and is parallel to the exterior bisector of the angle A, as was shown in last year's 'Diary,' p. 88.



Then to determine its equation we may find the coordinates of P where it intersects AC, and thus find a relation between the perpendiculars let fall from this point on BC and AB. We have now given two points in the line, and hence it must be completely determined.

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Let us first find a formula for a straight line in trilinear coordinates passing through the two points $(\alpha', \beta', \gamma')$ and $(\alpha'', \beta'', \gamma'')$. If α, β, γ be the coordinates of any point in the line, and l, m, n three constants such that when α, β, and γ are respectively multiplied by them and the results added, this sum shall be identically zero; hence

$$l\alpha + m\beta + n\gamma = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (17);$$

and, since $(\alpha', \beta', \gamma')$, $(\alpha'', \beta'', \gamma'')$ lie in the same straight line, we must have

$$l\alpha' + m\beta' + n\gamma' = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (18),$$
$$l\alpha'' + m\beta'' + n\gamma'' = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (19).$$

Finding the values of $\frac{m}{l}$ and $\frac{n}{l}$ from (19) and (18), and substituting in (17), or, in other words, forming the *eliminant* of l, m, n in the three equations, we obtain

$$\alpha(\beta'\gamma'' - \beta''\gamma') + \beta(\gamma'\alpha'' - \gamma''\alpha') + \gamma(\alpha'\beta'' - \alpha''\beta') = 0 \ldots (20),*$$

the relation which must hold true for any point (α, β, γ) in the line; and it is evident that the line is fixed, since its equation is independent of l, m, n.

To find the coordinates of P, take CB for the axis of x, and a line through M at right angles to CB for the axis of y, origin at M. By Cartesian coordinates, we have

For the equation of PM, $y + x \tan \frac{B-C}{2} = 0 \ldots\ldots \ldots\ldots$ (21),

" " AB, $y + x \tan B - \frac{1}{2}a \tan B = 0 \ldots$ (22),

" " AC, $y - x \tan C - \frac{1}{2}a \tan C = 0 \ldots$ (23).

From (21) and (23) we readily find the coordinates of P, or the point of intersection of those lines, viz.

$$x = -\frac{a \tan C}{2\left(\tan C + \tan \frac{B-C}{2}\right)} \text{ and } y = \frac{a \tan C \tan \frac{B-C}{2}}{2\left(\tan C + \tan \frac{B-C}{2}\right)} \ldots (24)$$

If we next find the perpendicular from P (the coordinates of which have just been written down) on AB, whose equation is

$$y + x \tan B - \tfrac{1}{2}a \tan B = 0,$$

we obtain (calling its length γ')

$$\gamma' = -a \frac{\tan C\left(\tan B - \tan \frac{B-C}{2}\right) + \tan B\left(\tan C + \tan \frac{B-C}{2}\right)}{2 \sec B \left(\tan C + \tan \frac{B-C}{2}\right)} \ldots (25);$$

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* Salmon's 'Conics,' p. 56, Art. 59.

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and the value of y, given by (24), is the length of the perpendicular on BC or $\alpha=0$, call this α';

$$\therefore \frac{\gamma'}{\alpha'}=-\frac{\tan C\left(\tan B-\tan\frac{B-C}{2}\right)+\tan B\left(\tan C+\tan\frac{B-C}{2}\right)}{\sec B\tan C\tan\frac{B-C}{2}},$$

which reduces to

$$\frac{\gamma'}{\alpha'}=-\frac{(\sin B+\sin C)\sin\tfrac{1}{2}(B+C)}{\sin C\sin\tfrac{1}{2}(B-C)}.$$

Hence the coordinates of P are

$$\beta'=0 \text{ and } \gamma'=-\alpha'\frac{(\sin B+\sin C)\sin\tfrac{1}{2}(B+C)}{\sin C\sin\tfrac{1}{2}(B-C)} \;\ldots\ldots\; (26),$$

and we know that the coordinates of M, the middle point of BC, are

$$\alpha''=0 \text{ and } \gamma''=\frac{\beta''\sin B}{\sin C} \;\ldots\ldots\; (27);$$

introducing these values of $(\alpha', \beta', \gamma')$, and $(\alpha'', \beta'', \gamma'')$ into equation (20), and striking out the common factor $\alpha'\beta''$, we get

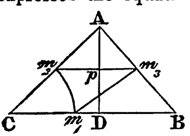
$$\alpha\frac{(\sin B+\sin C)\sin\tfrac{1}{2}(B+C)}{\sin\tfrac{1}{2}(B-C)}-\beta\sin B+\gamma\sin C=0,$$

a result which agrees with equation (15).

44. The second error occurs on page 90 of the same 'Diary,' where it is stated that the equation of the circle through the middle points of the sides of the triangle ABC is

$$(\alpha\sin A-\beta\sin B+\gamma\sin C)(\alpha\sin A+\beta\sin B-\gamma\sin C)\sin A+$$
$$(\beta\sin B+\gamma\sin C-\alpha\sin A)(\alpha\sin A+\beta\sin B-\gamma\sin C)\sin B+$$
$$(\beta\sin B+\gamma\sin C-\alpha\sin A)(\alpha\sin A-\beta\sin B+\gamma\sin C)\sin C=0.$$

This is incorrect, because, though $\alpha\sin A+\gamma\sin C-\beta\sin B$ expresses the equation of the straight line joining the middle point of the base to the middle point of one of the sides, yet it does not express the length of a perpendicular from any point in this line on one of the lines of reference. The perpendicular is, however, readily found; call it $\alpha'=pD$,



for, $\alpha'=\alpha-\frac{1}{2}p$ (if $p=AD$)

$$=\alpha-\frac{\alpha\sin A+\beta\sin B+\gamma\sin C}{2\sin A}{}^{*}$$

$$=\frac{\alpha\sin A-\beta\sin B-\gamma\sin C}{2\sin A} \;\ldots\ldots\ldots\ldots\ldots\; (28).$$

* Because $p\sin A=\alpha\sin A+\beta\sin B+\gamma\sin C$. Salmon's 'Conics,' p. 59, Art. 62.

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In like manner

$$\beta' = \frac{\beta \sin B - \alpha \sin A - \gamma \sin C}{2 \sin B} \quad \ldots\ldots\ldots\ldots\ldots (29)$$

$$\gamma' = \frac{\gamma \sin C - \alpha \sin A - \beta \sin B}{2 \sin C} \quad \ldots\ldots\ldots\ldots\ldots (30).$$

Now the equation of the circle circumscribing the triangle $m_1\ m_2\ m_3$, whose sides are $\alpha' = 0$, $\beta' = 0$, and $\gamma' = 0$, and angles A, B and C is

$$\alpha'\beta' \sin C + \gamma'\alpha' \sin B + \beta'\gamma' \sin A = 0.$$

Introducing the lengths of the perpendiculars given by (28), (29) and (30) into the last equation, we have as the equation to the circle through the middle points of the sides,

$$\frac{(\alpha \sin A - \beta \sin B - \gamma \sin C)(\beta \sin B - \gamma \sin C - \alpha \sin A)}{4 \sin A \sin B} \sin C +$$

$$\frac{(\alpha \sin A - \beta \sin B - \gamma \sin C)(\gamma \sin C - \alpha \sin A - \beta \sin B)}{4 \sin A \sin C} \sin B +$$

$$\frac{(\beta \sin B - \alpha \sin A - \gamma \sin C)(\gamma \sin C - \alpha \sin A - \beta \sin B)}{4 \sin B \sin C} \sin A = 0.$$

After multiplying out and reducing, this may be written in the simple form

$$\alpha^2 \sin 2A + \beta^2 \sin 2B + \gamma^2 \sin 2C - 2\beta\gamma \sin A$$
$$- 2\alpha\gamma \sin B - 2\alpha\beta \sin C = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (31).^*$$

If we suppose $\alpha = 0$ in the last equation, then

$$\beta^2 \sin 2B + \gamma^2 \sin 2C - 2\beta\gamma \sin A = 0,$$

or, $\quad \beta^2 \sin B \cos B + \gamma^2 \sin C \cos C - \beta\gamma \sin (B + C) = 0;$

that is, $\quad (\beta \sin B - \gamma \sin C)(\beta \cos B - \gamma \cos C) = 0;$

hence, $\quad \beta \sin B - \gamma \sin C = 0$ and $\beta \cos B - \gamma \cos C = 0;$

the latter equation shows that the foot of the perpendicular, let fall from the opposite angle on the base, also lies on the same circle; and in like manner this property may be shown to hold true for the other sides.

(To be continued.)

Leicester Cottage, Fratton, Portsea;
July, 1857.

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* Letter from the Rev. George Salmon, dated February 2d, 1857

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GENERAL SOLUTION TO QUEST. XV (1928).

By W. S. B. Woolhouse, *London*.

Let xy denote the rectangular coordinates of the tracing point P; θ the angle which the rod CC′ makes with the axis of x, and C, C′, P the areas of the curves described by those points respectively. Then the coordinates of the extremities C, C′, being $x - c\cos\theta$, $y - c\sin\theta$, and $x + c'\cos\theta$, $y + c'\sin\theta$, we shall have

$$C = \int(y - c\sin\theta)(dx - c\, d\cos\theta) = P + c^2\pi - c\int(y\, d\cos\theta + dx\sin\theta),$$

$$C' = \int(y + c'\sin\theta)(dx + c'd\cos\theta) = P + c'^2\pi + c'\int(y\, d\cos\theta + dx\sin\theta).$$

Hence, eliminating $\int(y\, d\cos\theta + dx\sin\theta)$, we get

$$c'C + cC' = (c + c')P + cc'(c + c')\pi;$$

$$\therefore P = \frac{c'C + cC'}{c + c'} - cc'\pi.$$

In obtaining this remarkable general formula we have assumed the curves to return into themselves and the angle θ to revolve through 2π, but in all other respects the reasoning obviously holds good absolutely. It is not even necessary that the curves described by C, C′ should have any algebraic equation or conform to any law whatever; they may, indeed, be any lines drawn at random.

It will be requisite in all cases to attend to the rule of algebraic signs; and if a point should pass round its curve in a contrary direction, the included area must be regarded as a negative quantity.

Should the rod, instead of revolving, oscillate back to its former position, all the above terms involving π must be cancelled, as the integrations with respect to θ will then vanish, so that in this case

$$P = \frac{c'C + cC'}{c + c'}.$$

Should the extremities C, C′, instead of revolving, oscillate back to their former positions, then $C = 0$, $C' = 0$, and $\therefore P = -cc'\pi$, the negative sign implying that the area P is described in a direction contrary to that in which the rod revolves. Should the rod also oscillate, then $P = 0$, implying that this area will consist of two equal loops, respectively positive and negative. Whether a new loop is positive or negative with respect to a primitive curve will be dependent upon its being turned inwardly or outwardly.

These observations will require to be attended to in applying the general formula to particular cases.

When the extremities C, C′, move on the same identical curve, as in the question, then $C = C'$ and $P = C - cc'\pi$, or $C - P = cc'\pi$.

London, 21st November, 1856.

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