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THE RELATION BETWEEN THE GROWTH OF CHILDREN AND THEIR DEVIATION FROM THE PHYSICAL TYPE OF THEIR SEX AND AGE.


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## THE RELATION BETWEEN THE GROWTH OF CHILDREN AND THEIR DEVIATION FROM THE PHYSICAL TYPE OF THEIR SEX AND AGE.

Wm. Townsend Porter.

Quetelet induced from his measurements of children the law that the weights, heights or other physical dimensions at each age in the period of growth are approximations of a median value, ${ }^{1}$ about which they are grouped in the form of a probability curve, being related to the median value as the individual observations in a series of measurements of the same thing are related to its actual size. Quetelet assumed that the median value of an anthropometric series expressed the physiological type of the series and that each deviation from this value expressed the physiological difference between an individual and the type. Fifty years of research have placed the truth of Quetelet's law beyond all doubt and have not weakened the reasonableness of his assumption, so that both law and hypothesis are rarely questioned and are regarded as a secure base from which to explore the phenomena of growth.

The degree of deviation of the individual measurements from the median value of an anthropometric series is measured by the Probable Deviation, that value which, in the words of Lexis, ${ }^{2}$ is as often exceeded as attained. Hence, if Quetelet's theory is true, the Probable Deviation is a measure of the degree of deviation of individuals from their Physical Type. The Probable Deviation from the median value of a

[^0]See page 62.

series containing many measurements may be calculated by the approximation formula

$$
\begin{equation*}
a= \pm 0.8453 \frac{\Sigma \delta}{n} \tag{1}
\end{equation*}
$$

where $d=$ Probable Deviation,
$\delta=$ Deviation of individual from median value,
$\Sigma{ }^{\circ}=$ Sum of individual deviations,
$n=$ Total number of observations.

The Probable Deviation contains the Error of Observation, as well as the Physiological Difference of the Individual from the Type. The Error of Observation, in a large series of measurements, is always relatively small. Its insignificance can be made clear in several ways. If the height of one boy at any age is measured 1000 times, the Probable Deviation will be much smaller than when the heights of 1000 boys at that age are measured once. Compare, for example, the Probable Deviation from the average height of one boy aged 17 measured 78 times with that of 78 boys aged 17 measured once, the measurements being made under conditions as nearly alike as possible in both instances.

One Boy aged 17 Measured 78 times. Average Height $\quad 176.28 \mathrm{~cm}$. Probable Deviation $\pm 0.24 \mathrm{~cm}$.

78 Boys aged 17
Measured once. 165.13 cm .
$\pm 5.15$ "

In the single boy, the Difference of the Individual from the Type is not present and the Probable Deviation is very small : in the 78 boys, the opposite is true. Yet the difficulty of correct measurement and hence the Error of Observation in each measurement in the two series cannot differ greatly. It follows that by far the greater portion of Probable Deviation is made up of the Physiological Difference of Individual from Type.

Again, the Error of Observation is inversely as the square root of the number of observations and should, were it an important constituent of the Probable Deviation, cause the latter to increase as the number of observations decreased.

Thus, in the following table, comparing the Relative Probable Deviation from the average height standing of boys with the square roots of the number of observations, the Probable Deviation should be much greater at ages 17 and 18 , in which the number of obscrvations is small, than at age 10 or 11 , in in which the observations are much more numerous. A look at the figures shows that the Probable Deviation is very little

| Age at nearest <br> Birthday. | Number of <br> Observations. | Square <br> Root. | Relation of Probable <br> Deviation to Average. |
| :---: | :---: | :---: | :---: |
| 6 | 709 | 26.63 | $3.1 \%$ |
| 7 | 1850 | 43.01 | 3.2 |
| 8 | 2223 | 47.15 | 3.3 |
| 9 | 2205 | 46.95 | 3.0 |
| 10 | 2087 | 45.68 | 3.1 |
| 11 | 1819 | 42.63 | 3.2 |
| 12 | 1653 | 40.67 | 3.2 |
| 13 | 1268 | 35.62 | 3.5 |
| 14 | 925 | 30.42 | 3.8 |
| 15 | 490 | 22.14 | 4.1 |
| 16 | 189 | 13.75 | 3.7 |
| 17 | 78 | 8.85 | 3.1 |
| 18 | 29 | 5.40 | 2.8 |

influenced by variations in the number of observations, within the limits given here. The Probable Deviation may, therefore, without any error of importance, be considered as the Physiological Difference between the Individual and the Type.

Not all observers have taken the Median Value as the Type. The arithmetic mean is frequently employed in Germany, Denmark and elsewhere. In a large series the difference between the two is so small that either may be sately used. The maximum and the mean Median minus Average values for the physical dimensions studied in this paper are as follows:-
median minus average value.

| Dimension. | UNIT OF <br> Measurement. | Maximum. |  | ARITHMETIC Mean. |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Boys. | Girls. | Boys. | Girls. |
| Weight. | Kilogram. | 0.73 | 0.74 | 0.23 | 0.25 |
| Height Standing... | Centimetre | 1.00 | 1.10 | 0.50 | 0.49 |
| Height Sitting..... | Centimetre | 0.94 | 0.99 | 0.44 | 0.67 |
| Span of Arms...... | Centimetre | 1.35 | 1.33 | 0.53 | 0.59 |
| Girth of Chest..... | Centimetre | 0.84 | 0.71 | 0.44 | 0.46 |

There can, therefore, be no objection to the use of the

Average in place of Median Value in the series about to be studied.

The accuracy of the average can be estimated by the formula: ${ }^{1}$

$$
\begin{equation*}
E= \pm \frac{d}{\sqrt{n}} \tag{2}
\end{equation*}
$$

where $E=$ Probable Error of Average,
$d=$ Probable Deviation from Average,
$n=$ Number of Observations.
The values for $E$ are given in Tables No. 6, 7 and 8.
It has already been said that the Physiological Difference between the Individual and the Type is expressed by the Probable Deviation from the Average. According to Geissler and Uhlitzsch, ${ }^{2}$ the interval between Average minus $d$ and Average plus $d$ increases with the age. The values of $d$ found by them in their measurements of height standing are as follows (page 33):

| AGE. | probable devintion $\underset{\text { Boys. }}{(A \pm d)}$ Givls. |  |
| :---: | :---: | :---: |
|  |  |  |
| $6 \frac{1}{3}$ to 7 years | ...3.4488 | 3.5926 |
| " 8 " | ....3.5841 | 3.7362 |
| " 9 | .3.8546 | 3.8293 |
| 9 " 10 " | .. 4.0067 | 3.7785 |
| 10 " 11 " | . 4.2181 | 4.2265 |
| 11 " 12 " | ..... .....4.2434 | 4.4125 |
| 12 " 13 " | .....4.5984 | 4.8013 |
| 13 " 14 " | ...4.7844 | 5.2155 |
| Over 14 " | 5.1479 | 4.8520 |

The authors say concerning this table (page 34) : -
"Hieraus geht hervor, dass diese Intervalle mit dem Alter "im Allgemeinen zunehmen und es dürfte dies auch ganz " natürlich erscheinen, da wohl anzunehmen ist, dass die das "Wachsthum hemmenden bez. fördernden Ursachen bei den " meisten Individuen ziemlich dieselben bleiben, den Unter-

[^1]'s schied also in der Grösse der Individuen immer merklicher " hervortreten lassen. Auch hinsichtlich der Geschlechter ist " ein Unterschied dentlich bemerkbar und zwar zeigen sich " für die Mädchen vom 11 Jahre an grössere Schwankungen "als für die Knaben. Da aber beobachtet worden ist, dass " die Mädchen circa zwei Jahre vor Eintritt der Pubertäts" periode verhältnissmässig rascher wachsen, so dürfte die " Verschiedenheit, mit welcher der Eintritt dieser Zeit erfolgt, " wohl ein Grund mit sein für die grösseren Schwankungen."

It appears from this extract from the valuable work of Geissler and Uhlitzsch that they were very near discovering the law which it is the purpose of this paper to demonstrate and would certainly have done so had they not contented themselves with the Absolute Probable Deviation, in which the real Physiological Difference of Individual from Type lies hidden, and had the material furnished them been sufficiently extensive. The Absolute Probable Deviation is entangled with the size of the individual, and its true value can be known only when this disturbing factor is removed. It is the relation between Probable Deviation and size of individual that must be studied, if the character of the Probable Deviation would be known. But even if Geissler and Uhlitzsch had pursued the method just suggested, the limitations of their material would have prevented them from solving the prohlem completely. For the material given them to analyze extended no further than the fourteenth year, with some observations over, almost wholly missing the period of pre-pubertal acceleration in boys and the early pubertal years in girls and entirely omitting the early pubertal years in boys.

It would seem from their results that the Probable Deviation increases with the age, whereas it shall be presently shown that the Absolute Probable Deviation in height standing as well as in weight, height sitting, span of arms and girth of chest does not increase with age during the whole period of growth. Moreover, the Relative Probable Deviation does not increase during seven of the nine years of boys' growth and five of the nine of girls' growth included in the observations of Geissler and Uhlitzsch, and shows a merely secondary relation
to age during the remaining years. They state further that the amount of the Probable Deviation from age 11 on is greater in girls than in boys, meaning of course that this is true within the limits of their own observations. A wider experience shows that the Absolute Probable Deviation in height standing of girls ceases to be greater than that of boys at age 14 (nearest birthday). Finally, it does not appear from the context that the statement " Da aber beobachtet worden ist, " dass die Mädchen circa zwei Jahren vor Eintritt der Puber"tätsperiode verhältnissmässig rascher wachsen, so dürfte die " Verschiedenheit, mit welcher der Eintritt dieser Zeit erfolgt, "wohl ein Grund mit sein für die grösseren Schwankungen" includes the conception of the relation of the Probable Deviation to the quickness of growth (to be demonstrated below), as distinct from the absolute difference in size of boys and girls at this period.

Before applying to the present material the ideas which the foregoing paragraphs have attempted to state with some precision, it will be interesting to compare the Probable Deviation of Height Standing of German children in the Freiberg district with that of the St. Louis children.

TABLE No. 1.
A Comparison of the Absolute Probable Deviation from the Average Height Standing of School Children in the Freiberg School District (Geissler \& Uhlitzsch) and in the St. Louis Public Schools.

| St. Louis Age at Nearest Birthday. | Boys. |  |  |  | Girls. |  |  | Freiberg AGE. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | St. Louis. |  | Freiberg. |  | St. Louis. | Freiberg. |  |  |
|  | 1 |  | $\pm$ |  | $\pm$ | $\pm$ |  |  |
| 6 | 3.40 |  | 3.4488 |  | 3.42 cm . | 3.5926 |  | $6 \frac{1}{2}$ to 7 yrs . |
| 7 | 3.61 |  | 3.5841 |  | 3.75 " | 3.7362 |  | $7-86$ |
| 8 | 3.89 | " | 3.8546 | 6 | 3.70 " | 3.8293 | ${ }^{6}$ | 8-9 |
| 9 | 3.75 | " | 4.0067 | '6 | 3.03 ' | 3.7785 | " | $9-10$ " |
| 10 | 3.98 | " | 4.2181 | ${ }^{6}$ | 406 " | 4.2265 | ${ }^{\prime}$ | 10-11 ${ }^{6}$ |
| 11 | 423 | " | 4.2434 | " | 4.48 " | 4.4125 | 66 | 11-12 " |
| 12 | 4.47 | " | 4.5984 | " | 5.23 " | 4.8013 | 6 | 12-13 " |
| 13 | 4.98 | " | 4.7844 | 6 | 5.46 " | 5.2155 |  | 13-14 ، |
| 14 | 5.58 | " | 5.1479 |  | 5.15 " | 4.8520 |  | over 14 " |
| 15 | 6.33 | ${ }^{6}$ |  |  | 4.01 " |  |  |  |
| 16 | 5.87 | " |  |  | 4.05 " |  |  |  |
| 17 |  | " |  |  | 3.45 " |  |  |  |
| 18 | 4.98 |  |  |  | 8.39 * |  |  |  |
| 19 |  |  |  |  | 4.04 " |  |  |  |
| 20 |  |  |  |  | 3.14 " |  |  |  |
| 21 |  |  |  |  | 4.27 ، |  |  |  |

The agreement between the series is satisfactory and demonstrates the stability of the method as well as the accuracy with which it has been employed in these particular instances. The coincidence is the more significant because the St. Louis children are taller than the Freiberg children.

TABLE No. 2.
A Comparison of the Average Height Standing of School Children in the Freiberg School District (Geissler and Uhlitzsch) and in the St. Louis Public Schools.

| St. Louis Age at Nearest Birthday. | Boys. |  | Girls. |  | $\begin{gathered} \text { Frbiberg } \\ \text { Age. } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | St. 1.ouis. | Freiberg. | St. Louis. | Freiberg. |  |
| 6 | 108.94 cm . | 108.6 cm . | 107.67 cm . | 107.9 cm. | $6 \frac{1}{2}$ to 7 yrs . |
| 7 | 114.03 " | 112.6 " | 112.95 " | 112.0 " | $7-8$ " |
| 8 | 119.13 " | 117.6 " | 118.36 " | 116.7 " | $8-9$ " |
| 9 | 124.35 " | 122.1 " | 123.67 " | 121.5 6 | 9-10 " |
| 10 | 128.87 " | 126.7 " | 128.43 " | 126.1 " | $110-11$ ، |
| 11 | 133.84 " | 130.6 " | 133.19 " | 131.0 " | 11-12 6 |
| 12 | 138.21 " | 135.5 /6 | 139.11 " | 135.5 * | $12-13$ ' |
| 13 | 142.91 " | 140.1 " | 146.53 " | 141.6 \% | 13-14 ${ }^{6}$ |
| 14 | 148.58 " | 144.1 ، | 150.84 '6 | 145.5 ، | over 14 |
| 15 | 154.90 " |  | 155.04 " |  |  |
| 16 | 160.27 " |  | 157.52 " |  |  |
| 17 | 165.13 " |  | 159.33 " |  |  |
| 18 | 170.41 ' |  | 159.42 ' |  |  |
| 19 |  |  | 158.46 " |  |  |
| 20 |  |  | 159.41 " |  |  |
| 21 |  |  | 159.98 '6 |  |  |

It follows from table No. 1 that The Physiological Difference between individual school children and the Physical Type of their sex and age is essentially the same, where the differences between the children compared are not greater than those existing between the St. Louis and the Freiberg children.
The material now to be discussed is presented in Tables No. 6, 7 and 8 and consists of the Number of Observations, Average, Probable Error of Average, Probable Deviation, Relative Annual Increase of Averago and Relation of Probable Deviation to Average of Weight in indoor clothing, Height standing without shoes, Height Sitting, i. e., height from the crown of the head to the chair on which the child sits erect, Span of Arms, or distance between the tips of the middle fingers when the arms are extended in a plane with the shoulders, and Girth of Chest, obtained by adding the girth of the chest
on a level with the nipples at full inspiration to the girth at full expiration and dividing by 2 , the measurement being made over the boys' shirts and the girls' dresses, the corsets occasionally worn by American school girls being previously removed. The manner of making the measurements is fully described in the author's work On the Growth of St. Louis Children, about to be published by the Academy of Science of St. Louis.

The Absolute Probable Deviation from the Average is given in Table No. 3, extracted from Tables No. 6, 7 and 8. The total deviation of the five dimensions meas-
TABLE No. 3.
The Absolute Probable Deviation from the Average: $d= \pm 0.8453 \frac{\Sigma \delta}{n}$


* Obtained by adding Girth of Chest at full Inspiration to Girth of Chest at full Expiration and dividing by 2.
ured at each age is also stated, in order to secure a more accurate general view. It is seen that the total Absolute Probable Deviation increases continuously from age 6 to age 16 in boys and from age 6 to age 15 in girls (except at age $8)$, after which periods there is a fall. The increase is not uniform, becoming suddenly much greater at age 13 in boys and age 11 in girls, the accelerated increase extending over four years in both sexes and ending as suddenly as it began.

It has been already said that the Absolute Probable Deviation is entangled with the size of the individual. In Table No. 4 this obstacle has been removed and the Physiological Difference of Individual from Type appears. A comparison
TABLE No. 4. The Relation of Probable Deviation (d) to Average (A). $\frac{d}{A}$.

| Dimension. | Sex. | Age at Nearest Birthday and Relation of Probable Deviation to Average. |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| Weight | Boys. Girls. | 7.2 | 7.8 9.0 | $\begin{gathered} 8.2 \\ 8.5 \end{gathered}$ | $\begin{array}{r} \mathbf{8 . 0} \\ 8.9 \end{array}$ | $\begin{gathered} 7.9 \\ 8.4 \end{gathered}$ | $\begin{aligned} & 8.4 \\ & 9.6 \end{aligned}$ | $\begin{array}{r} 7.8 \\ 9.8 \end{array}$ | $\begin{array}{r} 10.6 \\ 11.0 \end{array}$ | $\begin{array}{r} 11.3 \\ 11.0 \end{array}$ | $\begin{array}{r} 11.0 \\ 8.7 \end{array}$ | $\begin{array}{r} 12.0 \\ 8.4 \end{array}$ | $\begin{gathered} 7.9 \\ 7.0 \end{gathered}$ | $\begin{gathered} 6.9 \\ 6.9 \end{gathered}$ | 7.2 | 7.0 |  |
| $\begin{aligned} & \text { Height } \\ & \text { Standing. } \end{aligned}$ | Boys. Girls. | $\begin{array}{r}3.1 \\ 3.2 \\ \hline\end{array}$ | 3.2 3.3 | 3.8 3.1 | 3.0 3.1 | 3.1 3.1 | 3.2 3.2 | 3.2 3.4 | 3.5 <br> 3.8 <br> 8 | $\begin{array}{r}3.8 \\ 3.7 \\ \hline\end{array}$ | 4.1 3.4 | 3.7 2.6 | 3.1 2.2 | $2.8$ | 2.6 | 2.0 | 2.7 |
| Height | Boys. | 4.6 | 4.2 | 3.5 | 3.5 | 3.5 | 3.6 | 3.8 | 3.7 | 4.1 | 4.5 | 4.2 | 4.4 | 3.3 |  |  |  |
| Sitting. | Girls. | 34 | 3.5 | 3.2 | 3.2 | 3.2 | 3.4 | 3.6 | 38 | 4.0 | 3.1 | 2.8 | 2.6 | 2.0 | 2.1 | 2.4 | 2.2 |
| Span of Arms. | Boys. | 3.5 | 3.6 | 3.5 | 3.4 | 3.6 | 3.6 | 3.2 | 3.9 | 4.0 | 4.5 | 4.8 | 8.0 | 2.5 |  |  |  |
| Span of Arms. | Girls. | 3.6 | 3.7 | 3.6 | 3.4 | 3.6 | 3.6 | 3.2 | 3.8 | 3.5 | 2.9 | 2.8 | 2.5 | 2.7 | 2.9 | 2.6 | 27 |
| Girth of Chest* | Boys. Girls. | $\begin{gathered} 3.8 \\ 4.8 \end{gathered}$ | $\begin{array}{r} 3.9 \\ 4.2 \end{array}$ | $\begin{gathered} 3.8 \\ 3.9 \end{gathered}$ | $\begin{gathered} 3.9 \\ 4.0 \end{gathered}$ | $\begin{gathered} 4.1 \\ 4.2 \end{gathered}$ | $\begin{gathered} 3.9 \\ 4.6 \end{gathered}$ | $\begin{array}{r} 4.3 \\ 4.7 \end{array}$ | $\begin{array}{r} 4.4 \\ 5.0 \end{array}$ | $\begin{gathered} 4.9 \\ 4.9 \end{gathered}$ | $4.9$ | $\begin{gathered} 5.8 \\ 4.1 \end{gathered}$ | $\begin{gathered} 3.9 \\ 4.2 \end{gathered}$ | $\begin{gathered} 3.5 \\ 4.0 \end{gathered}$ |  |  |  |
| Total.. | Boys. Girls. | $22.2$ | $22.7$ | $22.3$ | $21.8$ | $22.2$ | $\begin{aligned} & 22.2 \\ & 24.4 \end{aligned}$ | $21.8$ | $26.1$ | $28.1$ | $29.0$ | $30.0$ | $22.3$ | $19.0$ |  |  |  |

of this table with the preceding one is very instructive. The continuous increase observed in the Absolute Probable Deviation has disappeared Indeed the total Deviation at age 12 is less than at age 7. An important increase occurs in the years $13,14,15$ and 16 in boys and $11,12,13$ and 14 in girls. During these four years in each sex, the Physiological Difference of the Individual from the Type is greater than at any other time. Hence this difference does not increase with the age of the Type or with its size. The true relation of the Physiological Difference is made plain by a comparison of Relative Probable Deviation with Relative Annual Increase of Average
'TABLE No. 5.
The Relative Annual Increase of Average.*

| Dimension. | Sex. | Age at nearest Birthday and Relative Annual Increase of Average. |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 6 to 7 | 7 to 8 | 8 to 9 | 9 to 10 | 10 to 11 | 11 to 12 | 12 to 13 | 13 to 14 | 14 to 15 | 15 to 16 | 16 to 17 | 17 to 18 |
| Weight..... ...... | Boys | 9.7 | 9.7 | 9.6 | 8.7 | 9.5 | 8.1 | 9.3 | 10.5 | 14.3 | 11.6 | 7.9 | 5.3 |
|  | Girls. | 10.0 | 9.9 | 9.6 | 9.6 | 9.7 | 11.6 | 14.3 | 9.9 | 10.4 | 7.6 | 4.7 |  |
| Height Standing.. | Boys. | 4.7 | 4.7 | 4.4 | 3.6 | 3.9 | 3.3 | 3.4 | 4.0 | 4.3 | 3.5 | 3.0 | 3.2 |
|  | Girls. | 4.9 | 4.8 | 4.5 | 3.9 | 3.7 | 4.4 | 5.3 | 2.9 | 2.8 | 1.6 | 1.1 | 0.06 |
| Height Sitting.... | Boys | 3.3 | 2.2 | 3.1 | 8.8 | 2.1 | 2.7 | 2.8 | 3.6 | 3.8 | 3.2 | 4.1 | 3.1 |
|  | Girls. | 4.1 | 3.5 | 3.4 | 3.1 | 2.7 | 3.8 | 4.6 | 3.5 | 3.5 | 2.9 | 1.1 | 0.6 |
| Span of Arms..... | Boys. | 5.0 | 4.9 | 4.3 | 4.0 | 3.8 | 4.1 | 3.2 | 4.3 | 4.7 | 3.5 | 2.8 | 4.0 |
|  | Girls. | 5.0 | 5.3 | 4.5 | 4.1 | 4.3 | 4.3 | 5.1 | 3.7 | 2.5 | 1.4 | 3.2 | 0.9 |
| Girth of Chest $\dagger . .$. |  |  |  | 2.8 | 2.6 | 2.5 | 2.3 | 2.7 | 3.8 | 4.5 | 3.5 | 2.7 | 3.8 |
|  | Girls. | 1.9 | 2.3 | 2.8 | 0.8 | 4.5 | 3.8 | 4.3 | 4.0 | 3.6 | 2.7 | 2.0 | 0.1 |
| Total........ | Boys. | 25.4 | 24.1 | 24.2 | 22.7 | 21.8 | 20.5 | 20.9 | 26.2 | 31.6 | 25.3 | $\underline{20.5}$ | 19.4 |
|  | Girls. | 25.9 | 25.8 | 24.8 | 21.5 | 24.9 | 27.9 | 33.6 | 24.0 | 22.8 | 16.2 | 12.1 | 1.7 |
| * Obtained for age 6 to 7 by dividing the increase from age 6 to age 7 by the average at age 6 ; for age 7 rease from age 7 to age 8 by the average at age 7 ; and so on. |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\dagger$ Obtained by adding Girth of chest at full inspiration to Girth of chest at full expiration and dividing by 2. |  |  |  |  |  |  |  |  |  |  |  |  |  |

or quickness of growth, given in Table No. 5. The comparison is made easy by arranging the totals side by side. The

| AGE. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Boys. | Boys. | Girls. | Girls. |
|  | 22.2 |  | 22.1 |  |
| 7. | 22.7 | 25.4 | 23.7 | 25.9 |
| 8 | 22.3 | 24.1 | 22.3 | 25.8 |
| 9. | 21.8 | 24.2 | 22.6 | 24.8 |
| 10. | 22.2 | 22.7 | 22.5 | 21.5 |
| 11. | 22.2 | 21.8 | 24.4 | 24.9 |
| 12. | 21.8 | 20.5 | 24.7 | 27.9 |
| 13. | 26.1 | 20.9 | 27.4 | 33.6 |
| 14. | 28.1 | 26.2 | 27.1 | 24.0 |
| 15 | 29.0 | 31.6 | 22.8 | 22.8 |
| 16. | 20.0 | 25.3 | 20.7 | 16.2 |
| 17 | 22.3 | 20.5 | 18.5 | 12.1 |
| 18...................... | 19.0 | 19.4 | 17.7 | 1.7 |

comparison reveals in both cases a general though not unbroken fall from age 7 to age 13 in boys and 11 in girls, a remarkable increase covering four years in both sexes, and a subsequent great decline. Thus the Relative Deviation coincides in the main with the Quickness of Growth. This relation is shown graphically in Plate I.

The phenomena just described are here seen to advantage. The curve of rate of growth in boys (unbroken red line) falls from age 7 to 12, corresponding to a small fall in Probable Deviation (unbroken black line). At age 12 to 13 , a sudden change appears in both curves. They rise rapidly, reach a high maximum and suddenly fall to their former level. The period from the beginning of the rise to the return to the level of age 12 has the same duration in both curves. The maximum deviation is greater in Relative Anuual Increase than in Relative Probable Deviation, a fact in accord with the greater stability of the latter, and the maximum Annual Increase occurs a little earlier than the maximum Probable Deviation. The period of acceleration is more sharply defined in Annual Increase. At age 18, both curves are still falling. The relation between the Rate of Growth

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Vol. VI. No. 10. Plate I (from Tables No. 4 and 5).

Black: Relative Probable Deviation. Red: Relative Annual Increase of Average.

Unbroken Curves: Boys. Broken Curves: Girls.
and Probable Deviation is even more clearly seen in the girls' curves (broken lines). The rapid rise begins in them at age 10 to 11 , and reaches its maximum at the same age in both curves. The subsequent fall is profound.

The evidence presented above permits the conclusion: The Physiological Difference between the individual childrew in an anthropometric series and the Physical Type of the series is directly related to the Quickness of Growth.
Datá of Height Standing and Weight.

| Age at Nearest <br> Birthday. | Sex. | Height Standing (In Centimetres). |  |  |  |  |  | Weight (In Kilograms). |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \text { Number } \\ & \text { of } \\ & \text { Observa- } \\ & \text { tions. } \end{aligned}$ | Average. <br> A | Probable Error Average. E | Probable Deviation. d | Relative Annual Increase age. | Relation of Probable Devantion to Average. $\frac{d}{A}$ | $\begin{aligned} & \text { Number } \\ & \text { of Ob- } \\ & \text { servations. } \end{aligned}$ | Average. <br> $A$ | Probable Error of Average. $E$ | Probable Deviation. $\qquad$ | Relative Annual Increase age. | Relation <br> Probable <br> Deviation <br> to Average. <br> $\frac{d}{A}$ |
|  |  | 709 | 108.94 | $\frac{1}{0.128}$ | $\frac{ \pm}{3.40}$ | \% | \% | 707 | 19.75 | $\frac{ \pm}{0.054}$ | $\pm$ | \% | $\%_{7.2}$ |
| Six | Girls. | 780 | 107.67 | 0.128 | $\stackrel{3}{342}$ |  | 3.2 | 798 | 18.93 | 0.051 | 1.44 |  | 7.6 |
|  | Boys. | 1850 | 114.03 | 0.084 | 3.61 | $4 . \overline{7}$ | 3.2 | 1814 | 21.67 | 0.039 | 1.68 | 9.7 | 7.8 |
| Seven | Girls. | 1791 | 112.95 | 0.089 | 3.75 | 4.9 | 3.3 | 1714 | 20.82 | 0.045 | 1.88 | 10.0 | 9.0 |
|  | Boys. | 2228 | 119.18 | 0.082 | 3.39 | 4.7 | 3.3 | 2188 | 23.78 | 0.042 | 1.96 | 9.7 | 8.2 |
| Eight............. | Girls. | 2193 | 118.36 | 0.079 | 3.70 | 4.8 | 31 | 2147 | 22.88 | 0.042 | 1.95 | 9.9 | 8.5 |
|  | Boys. | 2205 | 124.35 | 0.080 | 3.75 | 4.4 | 3.0 | 2188 | 26.06 | 0.045 | 2.09 | 9.6 | 8.0 |
| Nine. | Girls. | 2122 | 123.67 | $0.065$ | 3.83 | 4.5 | 3.1 | 2055 | 25.08 | 0.049 | 2.23 | 9.6 | 8.9 |
|  | Boys. | 2087 | 128.87 | 0.087 | 3.98 | 3.6 | 3.1 | 2064 | 28.32 | 0.049 | 2.28 | 8.7 | 7.9 |
| Ten.............. | Girls. | 2053 | 128.43 | 0.089 | 4.06 | 3.9 | 3.1 | 1947 | 27.49 | 0.052 | 2.31 | 9.6 | 8.4 |
|  | Boys. | 1819 | 138.84 | 0.099 | 4.88 | 3.9 | 8.2 | 1743 | 31.00 | 0.062 | 2.60 | 9.5 | 8.4 |
| Eleven | Girls. | 1772 | 133.19 | 0.106 | 4.48 | 3.7 | 3.2 | 1708 | 30.15 | 0.070 | 2.91 | 9.7 | 9.6 |
|  | Boys. | 1653 | 138.21 | 0.116 | 4.47 | 3.3 | 8.2 | 1644 | 33.51 | 0.061 | 2.46 | 8.1 | 7.3 |
| Truelve | Girls. | 1732 | 139.11 | 0.098 | 5.23 | 4.4 | 3.4 | 1676 | 33.66 | 0.681 | 3.31 | 11.6 | 9.8 |
|  | Boys. | 1268 | 142.91 | 0.140 | 4.98 | 3.4 | 3.5 | 1242 | 36.61 | 0.110 | 3.88 | 9.3 | 10.6 |
| Thirtee | Girls. | 1322 | 146.53 | 0.150 | $\stackrel{5}{5} 46$ | 5.3 | 3.8 | 1343 | 38.49 | 0.115 | 4.22 | 143 | 11.0 |
| Fourteen. | Boys. | 925 | 148.58 | 0.188 | 5.58 | 4.0 | 3.8 | 946 | 40.44 | 0.148 | 4.56 | 10.5 | 11.3 |
| Fourteen. | Girls. | 1085 | 150.84 | 0.156 | 5.15 | 2.9 | 3.7 | 1082 | 42.29 | 0.142 | 4.67 | 9.9 | 11.0 |
|  | Boys. | 490 | 154.90 | 0.286 | 6.33 | 4.8 | 4.1 | 498 | 46.22 | 0.227 | 5.06 | 14.3 | 11.0 |
| F | Girls. | 680 | 155.04 | 0.154 | 4.01 | 2.8 | 3.4 | 690 | 46.69 | 0.154 | 4.05 | 10.4 | 8.7 |
|  | Boys. | 189 | 160.27 | 0.427 | 5.87 | 3.5 | 3.7 | 208 | 51.60 | 0.431 | 6.16 | 11.6 | 12.0 |
| Sixteen | Girls. | 420 | 157.52 | 0.197 | 4.05 | 1.6 | 2.6 | 420 | 50.25 | 0.207 | 4.24 | 7.6 | 8.4 |
| Seventeen | Boys. | 78 | 165.13 | 0.592 | 5.15 | 3.0 | 3.1 | 71 | 55.67 | 0.521 | 4.38 | 7.9 | 7.9 |
| Seventeen | Girls. | 206 | 159.33 | 0.241 | 3.45 | 1.1 | 2.2 | 230 | 52.61 | 0.244 | 3.70 | 4.7 | 7.0 |
|  | Boys. | 29 | 170.41 | 0.924 | 4.98 | 3.2 | 2.8 | 17 | 58.64 |  |  | 5.3 |  |
| Eig | Girls. | 164 | 159.42 | 0.265 | 3.39 | 0.06 | 2.1 | 155 | 52.36 | 0.289 | 3.60 |  | 6.9 |
| Nineteen ......... | Girls. | 85 | 158.46 | 0.438 | 4.04 |  | 2.6 | 81 | 52.19 | 0.332 | 3.76 |  | 7.2 |
| Twenty........... | Girls. | 79 | 159.41 | 0.346 | 3.14 |  | 2.0 | 66 | 53.91 , | 0.565 | 3.76 |  | 7.0 |
| Twenty-one . ..... | Girls. | 43 | 159.98 | 0.651 | 4.27 |  | 2.7 |  |  |  |  |  |  |

Data of Helght Sitting and Span of Arms.

| Age at Nearest Birthday. | Sex. | Height Sitling (In Centimetres). |  |  |  |  |  | Span of Arms (In Centimetres). |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \text { Number } \\ & \text { of } \\ & \text { Observa- } \\ & \text { tions. } \end{aligned}$ | Average. <br> A | Probable Error Average. E | Probable Deviation. $a$ | Relative Annual of Average. | Relation of Probable Deviation to A verage. $\frac{d}{A}$ | Number of $\mathrm{Ob}-$ servations. | Average. <br> A | $\begin{gathered} \text { Probable } \\ \text { Error } \\ \text { of } \\ \text { Average. } \\ E \\ \hline \end{gathered}$ |  | Relative Annual Incruase of Average. | Relation of Probable Deviation to Average. $\frac{d}{A}$ |
| Six. | Boys. Giris. | $\begin{gathered} 714 \\ 751 \end{gathered}$ | $\begin{array}{r} 61.31 \\ 59.45 \end{array}$ | $\begin{aligned} & \frac{1}{0.105} \\ & 0.132 \end{aligned}$ | $\frac{1}{2.89}$ 2.08 | \% | $\begin{array}{r} \% \\ 4.6 \\ 3.4 \end{array}$ | $\begin{gathered} 708 \\ 769 \end{gathered}$ | $\begin{array}{r} 108.95 \\ 106.96 \end{array}$ | $\begin{aligned} & \pm \\ & \frac{1}{0.144} \\ & 0.140 \end{aligned}$ | $\begin{gathered} \pm \\ 3.85 \\ 3.87 \end{gathered}$ | \% | $\begin{aligned} & \% \\ & 8.5 \\ & 3.6 \end{aligned}$ |
| Seven.... ........ | Boys. Girls. | $\begin{array}{r} 1858 \\ 1727 \end{array}$ | $\begin{array}{r} 63.32 \\ 61.80 \end{array}$ | $\begin{gathered} 0.061 \\ 0.053 \end{gathered}$ | $\begin{gathered} 2.64 \\ 2.19 \end{gathered}$ | $\begin{aligned} & 3.3 \\ & 4.1 \end{aligned}$ | $\begin{array}{r} 4.2 \\ 3.5 \end{array}$ | $\begin{array}{r} 1862 \\ 1724 \end{array}$ | $\begin{array}{r} 114.42 \\ 112.36 \end{array}$ | $\begin{array}{r} 0.096 \\ 0.107 \end{array}$ | $\begin{array}{r} 4.16 \\ 4.18 \end{array}$ | $\begin{gathered} 5.0 \\ 5.0 \end{gathered}$ | $\begin{array}{r} 3.6 \\ 3.7 \end{array}$ |
| Eight. | boys. Girls. | $\begin{array}{r} 2289 \\ 2120 \end{array}$ | $\begin{array}{r} 64.74 \\ 6397 \end{array}$ | 0.048 0.044 | $\begin{gathered} 2.26 \\ 2.04 \end{gathered}$ | 2.2 | 3.5 3.2 | 2834 2152 | $\begin{array}{r} 120.07 \\ 118.33 \end{array}$ | 0.088 0.092 | $\begin{array}{r} 4.18 \\ 4.28 \end{array}$ | 4.9 5.3 | $\begin{aligned} & 3.5 \\ & 3.6 \end{aligned}$ |
| Nine | Boys. Girls. | $\begin{array}{r} 2238 \\ 2071 \end{array}$ | $\begin{gathered} 66.73 \\ 66.16 \end{gathered}$ | $\begin{gathered} 0.019 \\ 0.046 \end{gathered}$ | $\begin{gathered} 2.34 \\ 2.11 \end{gathered}$ | $\begin{gathered} 3.1 \\ 3.4 \end{gathered}$ | $\begin{array}{r} 3.5 \\ 3.2 \end{array}$ | $\begin{array}{r} 2272 \\ 2065 \end{array}$ | $125.18$ | $\begin{gathered} 0.089 \\ 0.092 \end{gathered}$ | $\begin{gathered} 4.25 \\ 4.18 \end{gathered}$ | 4.3 4.5 | $\begin{aligned} & 3.4 \\ & 3.4 \end{aligned}$ |
| Ten. | Boys. Girls. | $2118$ | $\begin{array}{r} 69.25 \\ 68.19 \end{array}$ | $\begin{array}{r} 0.040 \\ 0.049 \end{array}$ | $2.42$ | $3.8$ | 3.5 3.2 | 2076 2045 | 130.22 128.75 | $\begin{array}{r} 0.103 \\ 0.104 \end{array}$ | $4.70$ | 4.0 4.1 | 3.6 3.6 |
| Eleven. | Boys. Girls. | $\begin{array}{r} 18: 38 \\ 1748 \end{array}$ | $\begin{array}{r} 70.67 \\ 70.03 \end{array}$ | $\begin{array}{r} 0.060 \\ 0.057 \end{array}$ | $\begin{gathered} 2.56 \\ 2.37 \end{gathered}$ | $\begin{aligned} & 2.1 \\ & 2.7 \end{aligned}$ | $3.6$ $3.4$ | 1810 1757 | $\begin{array}{r} 185.18 \\ 134.24 \end{array}$ | $\begin{array}{r} 0.113 \\ 0.116 \end{array}$ | $4.84$ | 3.8 4.3 | $3.6$ $3.6$ |
| Twelve. | Boys. | 1656 | 78.55 | 0.067 | 2.72 | 2.7 | 3.8 | 1664 | 140.60 | 0.112 | 4.57 | 4.1 | 3.2 |
| Prucive. | Girls. | 1707 | 72.67 | 0.063 | 2.61 | 3.8 | 3.6 | 1718 | 140.07 | 0.109 | 4.51 | 4.3 | 3.2 |
| Thirteen | Boys. | 1285 | 74.24 | 0.076 | 2.74 | 2.3 | 3.7 | 1281 | 145.09 | 0.159 | 5.71 | 3.2 | 3.9 |
| Thirteen | Girls. | 1354 | 76.03 | 0.078 | 2.87 | 4.6 | 3.8 | 1368 | 147.19 | 0.150 | 5.55 | 5.1 | 3.8 |
| Fo | Boys. | 933 | 76.84 | 0.103 | 3.15 | 3.6 | 4.1 | 984 | 151.28 | 0.197 | 6.03 | 4.3 | 4.0 |
| F | Girls. | 1065 | 78.68 | 0.095 | 3.11 | 3.5 | 4.0 | 1088 | 152.58 | 0.160 | 5.29 | 3.7 | 3.5 |
| fteen |  | 438 | 79.74 | 0.161 | 3.59 | 3.8 | 4.5 | 495 | 158.48 | 0.321 | 7.15 | 4.7 | 4.5 |
| fteen | Girls. | 674 | 81.42 | 0.098 | 2.54 | 3.5 | 3.1 | 677 | 156.38 | 0.176 | 4.58 | 2.5 | 2.9 |
| Sixteen | Boys. | 193 | 82.28 | 0.250 | 3.48 | 8.2 | 4.2 | 189 | 163.96 | 0.574 | 7.89 | 3.5 | 4.8 |
| Sixteen | Girls. | 411 | 83.76 | 0.116 | 2.36 | 2.9 | 2.8 | 413 | 158.51 | 0.217 | 4.41 | 1.4 | 2.8 |
|  | Boys. | 78 | 85.68 | 0.427 | 8.77 | 4.1 | 4.4 | 75 | 168.56 | 0.581 | 5.03 | 2.8 | 3.0 |
| Sev | Girls. | 202 | 84.66 | 0.153 | 2.17 | 1.1 | 2.6 | 202 | 159.01 | 0.285 | 4.05 | 3.2 | 2.5 |
| Eighteen...... .... | Boys. | 81 | 88.23 | $0.536$ | 2.89 | 3.1 | 3.3 | 32 | 178.31 | 0.761 | 4.31 | 4.0 | 2.5 |
| Eighteen...... .... | Girls. | 167 | 85.20 | $0.133$ | 1.72 | 0.6 | 2.0 | 164 | 160.47 | 0.334 | 4.28 | 0.9 | 2.7 |
| Nineteen . . . . . . . | Girls. | 85 | 84.86 | 0.198 | 1.82 |  | 2.1 | 83 | 158.45 | 0.517 | 4.71 |  | 2.9 |
| Twenty........... | Girls. | 78 | 85.31 | 0.230 | 2.03 |  | 2.4 | 76 | 160.17 | 0.474 | 4.13 |  | 2.6 |
| Twenty-one...... | Girls. | 41 | 85.05 | 0.290 | 1.86 |  | 2.2 | 37 | 161.27 | 0.712 | 4.33 |  | 2.7 |

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TABLE No. 8.
Data of Girth of Chest.

| Age at Nearest Birthday. | Sex. | Girth of Chest* (In Centimetres). |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{gathered} \text { Number } \\ \text { of } \\ \text { Observations. } \end{gathered}$ | Average. $A$ | Probable Error of Average. E | Probable Deviation. <br> $d$ $\qquad$ | Relative Annual Increalse of oferage. | Relation of Probable Deviation to Average. $\frac{d}{A}$ |
| Six. | Boys. Girls. | $\begin{gathered} 677 \\ 741 \end{gathered}$ | $\begin{gathered} 59.05 \\ 58.34 \end{gathered}$ | $\begin{aligned} & \pm \\ & 0.083 \\ & 0.091 \end{aligned}$ | $\begin{aligned} & \pm \\ & \underset{2.22}{ } \\ & \hline 2.48 \end{aligned}$ | \% | $\begin{array}{r} \% \\ 3.8 \\ 43 \\ \hline \end{array}$ |
| Seven. | Boys. Girls. | $\begin{array}{r} 1708 \\ 1631 \end{array}$ | $\begin{array}{r} 60.62 \\ 59.47 \end{array}$ | $\begin{gathered} 0.057 \\ 0.062 \end{gathered}$ | $\begin{array}{r} 2.38 \\ 2.47 \end{array}$ | 2.7 1.9 | $\begin{array}{r} 3.9 \\ 4.2 \end{array}$ |
| Eight.................................. | Boys. Girls. | $\begin{aligned} & 2095 \\ & 2040 \end{aligned}$ | $\begin{array}{r} 62.18 \\ 60.81 \end{array}$ | $\begin{gathered} 0.052 \\ 0.053 \end{gathered}$ | $\begin{aligned} & 2.35 \\ & 2.40 \end{aligned}$ | 2.6 2.3 | $\begin{array}{r}3.8 \\ 3.9 \\ \hline\end{array}$ |
| Nine........................... ........ | Boys. Girls. | $\begin{aligned} & 2120 \\ & 1966 \end{aligned}$ | $\begin{array}{r} \mathbf{6 3 . 9 0} \\ 62.51 \end{array}$ | $\begin{array}{r} 0.055 \\ 0.057 \end{array}$ | $\begin{array}{r} 2.51 \\ 2.53 \\ \hline \end{array}$ | 2.8 2.8 | 3.9 4.0 |
| Ten................... ................ | Boys. Girls. | $\begin{array}{r} 1997 \\ 1893 \end{array}$ | $\begin{array}{r} 65.59 \\ 63.02 \end{array}$ | $\begin{array}{r} 0.061 \\ 0.061 \end{array}$ | $\begin{aligned} & 2.72 \\ & 2.67 \end{aligned}$ | 2.6 0.8 | 4.1 4.2 |
| Eleven................................... | $\begin{aligned} & \text { Boys. } \\ & \text { Girls. } \end{aligned}$ | $\begin{aligned} & 1732 \\ & 1654 \end{aligned}$ | $\begin{array}{r} 67.24 \\ 65.85 \end{array}$ | $\begin{gathered} 0.068 \\ 0.075 \end{gathered}$ | $\begin{aligned} & 2.61 \\ & 3.04 \end{aligned}$ | 2.5 4.5 | 3.9 4.6 |
| Twelve............... .................. | Boys. Girls. | $\begin{gathered} 1565 \\ 1624 \end{gathered}$ | $\begin{array}{r} 68.76 \\ 68.34 \end{array}$ | $\begin{gathered} 0.074 \\ 0.081 \end{gathered}$ | $\begin{aligned} & 2.94 \\ & 3.24 \end{aligned}$ | 2.3 3.8 | 4.3 4.7 |
| Thirteen.. | Boys. Girls. | $\begin{aligned} & 1228 \\ & 1313 \end{aligned}$ | $\begin{array}{r} 70.61 \\ 71.29 \end{array}$ | $\begin{array}{r} 0.089 \\ 0.098 \end{array}$ | $\begin{array}{r} 3.11 \\ 3.54 \\ \hline \end{array}$ | 2.7 4.3 | 4.4 5.0 |
| Fourteen.. | Boys. Girls. | $\begin{aligned} & 9: 5 \\ & 1020 \end{aligned}$ | $\begin{array}{r} 73.27 \\ 74.13 \end{array}$ | $\begin{array}{r} 0.118 \\ 0.114 \end{array}$ | $\begin{aligned} & 8.58 \\ & 3.65 \end{aligned}$ | 3.8 4.0 | 4.9 4.9 |
| Fifteen. . | Boys. Girls. | $\begin{array}{r} 498 \\ 659 \\ \hline \end{array}$ | $\begin{array}{r} 76.56 \\ 76.78 \\ \hline \end{array}$ | $\begin{array}{r} 0.169 \\ 0.143 \end{array}$ | $\begin{aligned} & 3.77 \\ & 3.70 \end{aligned}$ | 4.5 3.6 | 4.9 4.7 |
| Sixteen.... | Boys. Girls. | $\begin{gathered} 205 \\ 397 \end{gathered}$ | $\begin{array}{r} 79.22 \\ 78.85 \end{array}$ | $\begin{array}{r} 0.298 \\ 0.164 \end{array}$ | $\begin{gathered} 4.19 \\ 3.27 \end{gathered}$ | 3.5 2.7 | 5.3 4.1 |
| Seventeen....... | Boys. Girls. | $\begin{array}{r} 80 \\ 206 \end{array}$ | $\begin{array}{r} 81.39 \\ 80.39 \end{array}$ | $\begin{gathered} 0.358 \\ 0.233 \end{gathered}$ | $\begin{array}{r} 3.15 \\ 3.34 \end{array}$ | 2.7 2.0 | 3.9 4.2 |
| Eighteen........ | $\begin{aligned} & \text { Boys. } \\ & \text { Girls. } \end{aligned}$ | $\begin{array}{r} 31 \\ 162 \\ \hline \end{array}$ | $\begin{array}{r} 84.52 \\ 80.45 \\ \hline \end{array}$ | $\begin{gathered} 0.528 \\ 0.254 \\ \hline \end{gathered}$ | $\begin{aligned} & 2.94 \\ & 3.23 \end{aligned}$ | $\begin{array}{r} 3.8 \\ 0.1 \\ \hline \end{array}$ | $\begin{array}{r} 8.5 \\ 4.0 \\ \hline \end{array}$ |

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[^0]:    ${ }^{1}$ Moyenne of Quetelet, see Lettres sur la Théorie des Probabilités, Bruxelles, 1846, page 66; and mean of Sir John Herschel and other English writers, see Edinburg Review, 1850, page 23.
    ${ }^{2}$ Ueber die Theorie der Stabilitäit statistischer Reihen. Hildebrand's Jahrbücher für Nationalökonomie und Statistik. Bd. 32, 1879, S. 60-98.

[^1]:    ${ }^{1}$ Formulas (1) and (2) are contained in L. Stieda's paper: Ueber die Anwendung der Wahrscheinlichkeitsrechnung in der anthropologischen Statistik. Archiv für Anthropologie, Bd. xiv, 1882, S. 167-182.

    2 Arthur Geissler and Richard Uhlitzsch. Die Grössenverhältnisse der Schulkinder im Schulinspectionsbezirk Freiberg. Zeitschrift des königlichen Salchsischen Statistichen Bureaus, xxxiv, Heft 1 and 2, 1888, S. 28-40.

