ELASTIC SECOND-ORDER COMPUTER ANALYSIS OF BEAM-COLUMNS AND FRAMES

To my wife Virginia

and my children

Mervin and Erica

ELASTIC SECOND-ORDER COMPUTER ANALYSIS OF BEAM-COLUMNS AND FRAMES

by

HERMOGENES REYES MUÑOZ, B.S.

THESIS

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

MASTER OF SCIENCE IN ENGINEERING

THE UNIVERSITY OF TEXAS AT AUSTIN

AUGUST 1991

ACKNOWLEDGMENTS

The author extends his sincere gratitude to Dr. Michael D. Engelhardt for his guidance and supervision of this thesis, and his moral support to my academic endeavors at The University of Texas at Austin.

The author also wishes to express his sincere appreciation to Dr. Dan L. Wheat for his support, and his genuine effort to read the thesis.

Special thanks is also extended to Dr. James O. Jirsa for his help in achieving my Educational Plan at the University of Texas at Austin.

And most importantly, my sincere thanks to the United States Navy for sponsoring my graduate education, and to the personnel of the Naval Reserve Officers' Training Corps Unit at the University of Texas at Austin for their administrative support.

Finally, the author is indebted to his wife and children for their love, understanding, and moral support, and in return this thesis is dedicated to them.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

1 INTRODUCTION

1.1 GENERAL

Today, many computer programs in structural analysis are readily available and each is designed for a specific use. Just to name a few are SAP90, ETABS, SAFE, M-STRUDL, MICROFEAP, MULTIFRAME, RISA-2D, etc... and many more. These programs are generally used for structural analysis and design. Most of these programs have graphics capability which provides visual presentation on the behavior and response of the structure.

While most of these programs are developed using sophisticated computer methods in elastic structural analysis, few are designed to perform elastic second-order analysis. Its use can be of considerable importance in the design of various structural systems, particularly frame type structures.

In this report, the development and implementation of computerized elastic second-order analysis of frames is presented. The technique is illustrated by several examples.

1.2 OBJECTIVES

The primary objective of this report is to develop and implement into a computer program the concepts of elastic second-order analysis. An additional objective is to extend these concepts to perform computerized elastic stability analysis.

In the current LRFD Specification for steel buildings [1] (1st Ed.,AISC 1986), Chapter C, Section C1 states that, " second order (P-Delta) effects shall be considered in the design of frames. " Chapter H, Section 2a, of the Manual, which deals with beam-column design, specifies three (3) ways to determine the required flexural strength M_u of a member:

a. M_u may be determined using second-order elastic analysis using factored loads

b. M_u may be determined using plastic analysis that satisfies the requirements of Sections C1 and C2 of Chapter C of the Manual. Section C1 deals with the second-order analysis while Section C2 deals with frame stability.

c. M_u may be determined using linear-elastic analysis (first-order) combined with approximate amplification factors.

The LRFD Specification therefore requires explicit consideration of second-order effects and permits either exact or approximate methods of analysis.

Typical current design practice uses elastic first-order analysis with approximate amplification factors to determine M_u. This procedure may be tedious and time consuming, even if computers are being used. In addition, the approximate amplification factors provided in the LRFD Specification, are generally applicable only to simple, rectangular framing. The approximate methods of determining the amplification factors for complex irregular framing are not yet well developed.

It is the specific purpose of this report to develop an automated elastic-second order analysis computer program that will meet the requirements of Section 2a, Chapter H and Chapter C of the LRFD Manual. The use of an automated elastic second-order analysis program will remove the burden of hand calculations associated with the approximate methods. It will also permit accurate elastic second-order analysis of planar frames of arbitrary geometry.

1.3 SCOPE

1.3-1 Methods of Analysis

The elastic second-order analysis of a beam-column originated from the "Buckling of Columns" which was first introduced in 1729 by Van Musschenbroek [2]. In 1744, Euler introduced the first accurate stability analysis of a centrally loaded elastic column [3]. Other members that were the subject of subsequent study included eccentrically loaded columns, columns with end moments, and laterally loaded columns. These types of columns were treated as "beam-columns".

There are two basic methods commonly used in performing computerized elastic second-order analysis of beam-columns and frames. These are called the stability function approach and the geometric stiffness approach. The latter is an approximate approach, and will provide accurate solutions for a limited class of problems. The stability function approach, on the other hand, is directly based on the governing differential equations of the elastic beam-column, and provides exact solutions. Further, in order to perform accurate elastic stability analysis requires the use of stability functions. This approach is used in the development of the Elastic

Second-order Computer Analysis Program [E S C A P] described in this report.

Timoshenko and Gere[4], Chen and Atsuta[2], Chen and Lui[5], provide solutions to various cases of elastic beam-columns. Salmon and Johnson[6] give an extensive list of references on the subject of beam-columns.

1.3-2 Assumptions

The development of elastic second-order analysis is based on the following assumptions[2, 3, 7] :

a. The material is linearly elastic.

b. Rotations are small, so that the following approximations can be made :

$$i.\quad \sin\theta = \theta$$

$$ii.\quad \cos\theta = 1$$

$$iii.\quad \theta^2 = 0$$

c. The exact expression for the curvature of a line is :

$$\phi = \frac{1}{\rho} = \frac{y''}{[1 + (y')^2]^{3/2}} \qquad Eq.\ (1)$$

where ρ = radius of curvature

ϕ = curvature

y = deflection

As a consequence of the small rotation assumption (y' = rotation), the usual relationship between curvature and deflection results :

$$\phi = y'' \qquad Eq.\ (1a)$$

d. The beam-column is prismatic.

e. Shear deformation is neglected.

f. The frame members are two-dimensional and subjected to in-plane forces and deformations only.

1.3-3 Organization of the Report

The report consists of six chapters. The first chapter is the Introduction. Chapter 2 deals with the basic theory of beam-columns. The concept of elastic second-order is explored at some depth to give an appreciation of the subject. Its relative importance to the elastic stability problem is briefly discussed and later covered in Chapter 4.

Chapter 3 presents the methodology that is implemented in the development of the computer program [E S C A P]. It explains how an ordinary first-order linear-elastic analysis program using the direct-stiffness method [8] is modified to include second-order effects through the implementation of stability functions. The derivation of the beam-column stiffness matrix is presented using second-order differential equations. In Section 3.4-2, a method for determining the maximum elastic second-order moment within a member is presented.

In Chapter 4, the reader is introduced to the concept of elastic stability analysis of beam-columns and frames. The use of Cholesky's decomposition of the stiffness matrix in determining the elastic buckling load of frames and beam-columns is introduced and discussed in

detail.

Chapter 5 presents several examples of beam-columns and frames. The computer solutions are compared to closed-form or published solutions. Some of the computer solutions to the sample problems are listed in Appendix B. Also, a Users Manual for the computer program is listed in Appendix A.

And finally, Chapter 6 provides a conclusion on the results of the analysis of the computer program [E S C A P]. The importance of elastic second-order analysis and elastic stability analysis of frames are also summarized. A list of selected references is also provided.

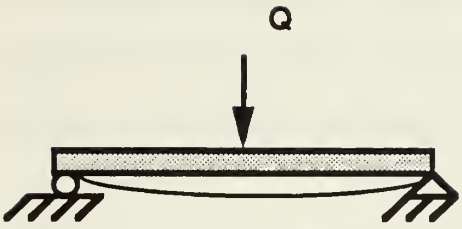
2 BACKGROUND ON BEAM-COLUMNS

The term beam-column refers to a structural member that is subjected to both axial compression (or tension) force and bending moment. It is a structural element simplifying to a beam or a column as special cases. A column is a structural element where axial compression predominates, while a beam is a structural element where bending predominates. The interaction of the axial force and bending moment may result to a nonlinear behavior of a member in terms of :

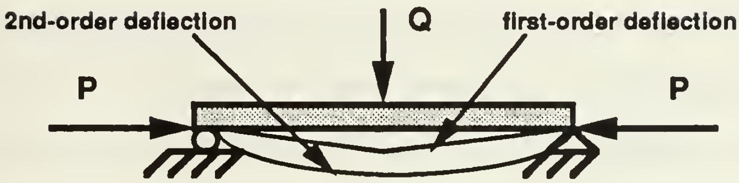
a. the axial force and the resulting second-order deflections, and ;

b. the axial force and the resulting second-order moments.

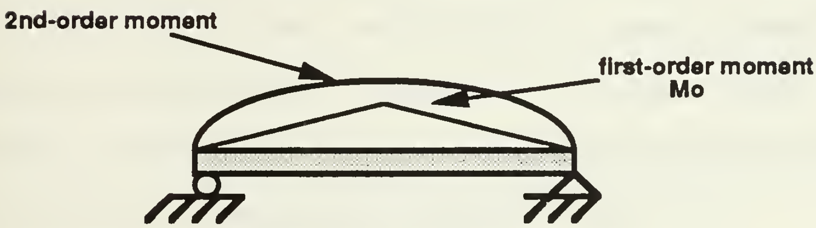
The elastic analysis of beam-columns must consider the effect of change in geometry of the structural element, i.e, second-order effects. As shown in Fig. 1, the axial force will magnify the deflection and moment in the beam. When the axial force approaches a critical value, the deflection and moment of the beam-



(a) Deflection due to Q



(b) Deflection amplified
by axial force P



(c) Moment amplified
by axial force P

Fig. 1 Effects of Axial Force
In a Beam-Column

column increase without bound. This critical value is the elastic buckling load and can be determined by performing elastic stability analysis of the member or structure.

Most structures are composed of many structural elements, where each element serves a unique purpose, and the combination meets the functional objectives of the structure. Because these elements are interconnected, they interact under a given loading pattern. When both axial forces and moments are present in the members, they respond as beam-columns and the deformations resulting from the bending moments induced by the externally applied loads may be amplified by the presence of compressive axial loads. However, when the axial force is tension, the effect on the resulting deformations is opposite of the compressive axial force.

Since nearly all the members of a structure are subjected to both bending moments and axial forces, the interactive effect of these forces may be significant in the analysis of beam-columns. When the magnitude of the internal stresses and deflections produced by the axial force in a member is relatively small compared to the bending effect, it may be practical to analyze and design the member as a beam element. On the other hand, if the

bending effect is relatively small, analysis and design of the member as a column is appropriate.

The presence of a compressive axial force reduces the force needed to cause a unit rotation or a unit translation in the transverse direction at one end of a member. Conversely, the required forces increase when the axial force is tensile. In other words, the presence of axial force affects the flexural stiffness of a member or frame. The case in which the axial force is compressive is usually of greater interest, since it leads to the possibility of member or frame instability. Using second-order analysis techniques, the instability can be predicted as the axial compressive force approaches a critical value, which reduces the flexural stiffness to zero. Second-order analysis is therefore central to the solution of stability problems.

3 ELASTIC SECOND-ORDER ANALYSIS

3.1 GENERAL

Elastic second-order analysis of members and frames can be accomplished by developing and solving the second-order differential equations of equilibrium. Classical closed form solutions for single members and simple frames are available [2,4,5,6]. However, for analysis of large frames of arbitrary geometry, closed-form solutions are difficult and cumbersome to obtain. However, with the advent of computers, automated second-order analysis for such cases is possible.

The direct stiffness method is commonly used for computerized first-order structural analysis. In this Chapter, this method will be used in extending the first-order analysis to a second-order analysis described in the computer program [E S C A P] using stability functions.

3.2 METHODOLOGY

As previously noted, the presence of large axial force (i.e, P / P_e is large, where P_e is the elastic buckling load), affects the geometry and stiffness of a beam-column element. If the axial force P is known, the maximum deflection and bending moment can be calculated using second-order formulations. However, the member or frame is initially indeterminate and the axial forces are unknown. The axial forces are related to the joint displacements and must be first calculated because the axial forces influence the members' fixed-end actions, which are the elements of the load vector used in determining the joint displacements. Additionally, the axial forces also influence the stiffness matrix of the member or frame. One way of determining the initial axial forces is to perform a first-order linear-elastic analysis (first cycle) using the direct-stiffness method. This will give the first-order shears, moments, deflections and axial forces.

In the second cycle, the previously computed axial forces are used to modify member stiffness and fixed-end actions. Then new values of joint deflections and

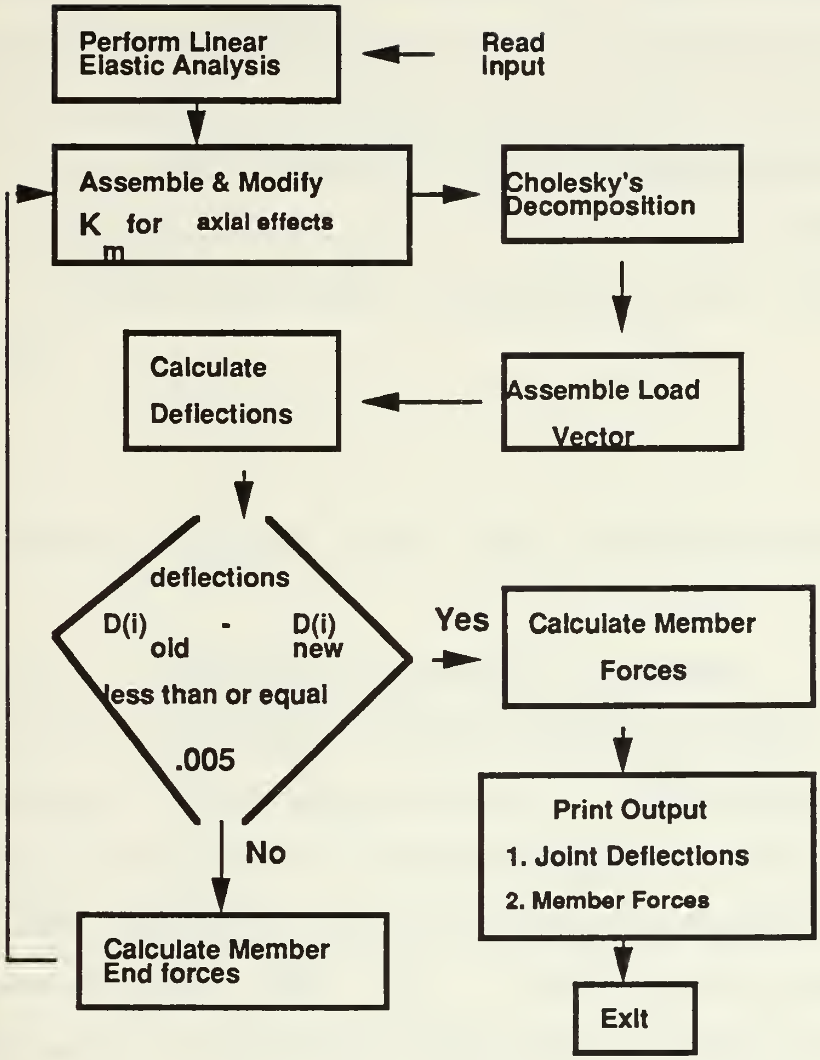


Fig. 2 - Flowchart
Elastic Second-Order Analysis

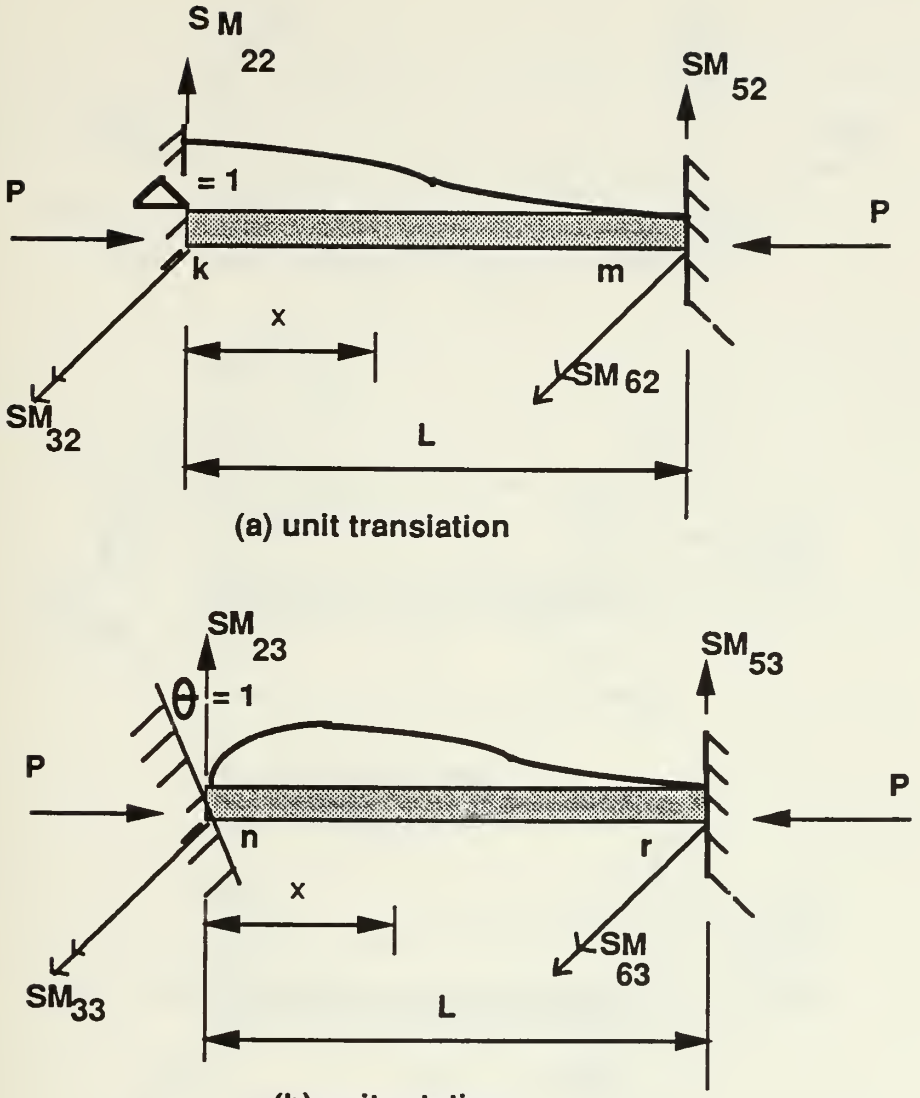
member-end actions are calculated. This process is repeated until the differences between the deflections in successive cycles are within the acceptable tolerance. This is the basic algorithm used in implementing the computer program [E S C A P]. The flowchart shown in Fig. 2 outlines the basic algorithm of the computer program.

3.3 DERIVATION OF THE SECOND-ORDER STIFFNESS MATRIX

3.3 - 1 Translation Stability Functions

To begin our derivation of the stiffness matrix of a beam-column, let us consider Fig. 3 . The beam-column element is subjected to a unit translation (Fig. 3a) and a unit rotation (Fig. 3b) under the presence of a compressive axial load P . The elastic stiffness coefficients for these two beam-column elements will be derived from second-order differential equations of equilibrium.

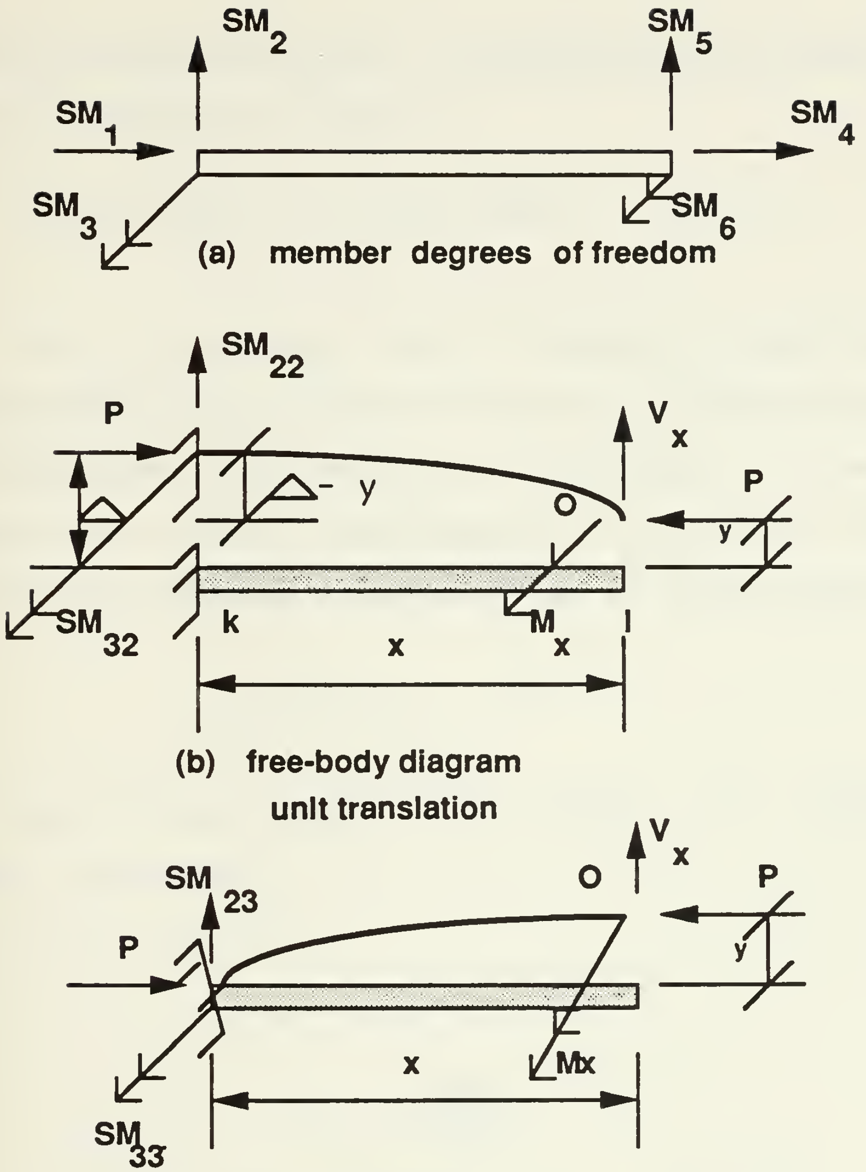
The degrees of freedom of a beam-column element are shown in Fig. 4a. A free-body diagram of a section of the beam-column element cut at distance x from the left support is shown in Fig. 4b. The sum of external



(a) unit translation

(b) unit rotation

Fig. 3 Stiffness Coefficients of a Beam-Column



(a) member degrees of freedom

(b) free-body diagram
unit translation

(c) free-body diagram
unit rotation

Fig. 4 Unit Translation - Unit Rotation
of a Beam-Column

moments at point O (Fig. 4b), assuming counter-clockwise moment is positive is given by

$$M_{ext} = -P(\Delta - y) + SM_{32} - SM_{22}x \,, \qquad Eq.\ (2)$$

where SM $_{ij}$ = is the member stiffness coefficient corresponding to the degree of freedom i induced by a unit displacement corresponding to the degree of freedom j while the other displacements are kept to zero. Applying the differential equation of an elastic curve

$$M = -EIy'' \,, \qquad Eq.\ (3)$$

then the interior moment must balance the exterior moments, resulting in :

$$M = -EI\frac{d^2y}{dx^2} =$$

$$-P(\Delta - Y) + SM_{32} - SM_{22}X \qquad Eq.\ (4)$$

$$M = EI\frac{d^2y}{dx^2} = P(\Delta - Y) + SM_{22}x - SM_{32}$$

$$Eq.\ (5)$$

Rearranging the terms of Eq. (5) yields

$$\frac{d^2y}{dx^2} + k^2y = k^2\Delta + \frac{(SM_{22}X - SM_{32})}{EI} \qquad Eq.\ (6)$$

$$\text{where } k^2 = \frac{P}{EI} \qquad Eq.\ (7)$$

The complementary solution of Eq. (6) is given by

$$y_c = A \sin kx + B \cos kx \qquad Eq.\ (8)$$

and the particular solution of Eq. (6) has the form

$$y_p = Cx + D\ . \qquad Eq.\ (9)$$

Differentiating Eq. (9) twice with respect to x and substituting the first and second derivatives into Eq. (6) yields Eq. (10). The constants C and D are determined by comparing coefficients of the terms on the left hand side of Eq. (10) to the terms on the right

hand side as shown below

$$0 + k^2(Cx + D) =$$

$$k^2\Delta + \frac{SM_{22}x}{EI} - \frac{SM_{32}}{EI}, \qquad Eq.\ (10)$$

$$where\ C = \frac{SM_{22}}{k^2EI}\ ;\ D = \Delta - \frac{SM_{32}}{k^2EI}$$

The general solution of Eq. (6) is the sum of its particular solution and complementary solution. Inserting the values of C and D into Eq. (9), the general solution has the form

$$y = A\sin kx + B\cos kx + \Delta +$$

$$\frac{(SM_{22}x - SM_{32})}{k^2EI} \qquad Eq.\ (11)$$

To solve for the constants A and B, apply the boundary conditions at joint k of Fig. 4b . At $x = 0$, $y = \Delta$; then

$$B = \frac{SM_{32}}{k^2 EI} \ . \qquad Eq. (12)$$

At $x = 0$, $dy/dx = 0$; then

$$A = -\frac{SM_{22}}{k^3 EI} \ . \qquad Eq. (13)$$

The stiffness coefficients SM_{22} and SM_{32} are determined by applying the boundary conditions at joint m of Fig. 3a. When $x = L$, $y = 0$; then

$$A \sin kL + B \cos kL + \Delta + \frac{(SM_{22} L - SM_{32})}{k^2 EI} = 0$$

Eq. (14)

When $x = L$, $dy/dx = 0$; then

$$Ak \cos kL - Bk \sin kL + \frac{SM_{22}}{k^2 EI} = 0 \qquad Eq. (15)$$

Inserting the values of A and B into Eqs. (14) & (15) yields Eqs. (16a) & (16b) :

$$-\frac{SM_{22}}{k^3EI}\sin kL + \frac{SM_{32}}{k^2EI}\cos kL + \Delta +$$

$$\frac{(SM_{22}L - SM_{32})}{k^2EI} = 0 \qquad Eq.\ (16a)$$

$$-\frac{SM_{22}}{k^2EI}\cos kL - \frac{SM_{32}}{kEI}\sin kL$$

$$+\frac{SM_{22}}{k^2EI} = 0\ . \qquad Eq.\ (16b)$$

Simplifying Eqs. (16a) & (16b) yields

$$SM_{22}(kL - \sin kL) +$$

$$SM_{32}\ k(\cos kL - 1) = -k^3EI\Delta \qquad Eq.\ (17)$$

$$SM_{22}\ (1 - \cos kL) - SM_{32}\sin kL = 0 \qquad Eq.\ (18)$$

Solving Eqs. (17) & (18) for SM_{22} yields

$$SM_{22}\ [kL \sin kL - \sin^2 kL +$$
$$(\cos kL - 1 - \cos^2 kL + \cos kL)]$$
$$= -k^3 EI \sin kL\ \Delta \qquad Eq.\ (19)$$

Simplifying Eq. (19) yields

$$SM_{22}\ [2 - 2 \cos kL - kL \sin kL] =$$
$$k^3 EI \sin kL\ \Delta \qquad Eq.\ (20)$$

Multiply Eq. (20) by $12L^3$ / $12L^3$ and introduce the term

$$\phi_c = (2 - 2 \cos kL - kL \sin kL) \qquad Eq.\ (21)$$

the stiffness coefficient SM_{22} is

$$SM_{22} = \left(\frac{2EI}{L^3}\right)\left(\frac{6\ kL^3 \sin kL}{12\ \phi_c}\right) \qquad Eq.\ (22)$$

$$SM_{22} = \frac{2\ EI}{L^3}(6\ s_1)\ \Delta \qquad Eq.\ (23)$$

where S_1 is referred to as a stability function and is equal to

$$s_1 = \frac{(kL)^3 \sin kL}{12\ \phi_c} \qquad Eq.\ (24)$$

Substitute Eq. (22) into Eq. (17) and solve for SM_{32}

$$SM_{32} = \frac{2\ EI}{L^3}\ (3L\ s_2)\ \Delta \qquad Eq.\ (25)$$

The other translational stability function S_2 is

$$s_2 = \frac{(kL)^2\ (1 - \cos kL)}{6\ \phi_c} \qquad Eq.\ (26)$$

3.3 - 2 Rotation Stability Functions

Consider now the beam-column element in Fig. 3b which is subjected to a compressive axial force P and a unit rotation at joint n . A free-body diagram is taken at a distance x from the left support as shown in Fig. 4c . Summing the external moments about point O, gives

$$M_{ext} = Py - SM_{23}x + SM_{33} \qquad Eq. (27)$$

Applying the differential equation of an elastic curve {Eq. (2)} and using equilibrium, the external moments {Eq. (27)}, must be equal to the internal moment. Rearranging the terms yields

$$EI\frac{d^2y}{dx^2} + Py = SM_{23}x - SM_{33} \qquad Eq. (28)$$

The complementary solution of Eq. (28) is the same as Eq. (8). Using similar procedures described in Section 3.2-1, the particular solution of Eq. (28) has the form

$$y_p = \frac{SM_{23}}{k_2EI}x - \frac{SM_{33}}{k^2EI} \qquad Eq.\,(29)$$

The general solution of Eq. (28) is the sum of the complementary solution {Eq. (8)} and the particular solution {Eq. (29)} , and has the form

$$y = A\sin kx + B\cos kx + \frac{SM_{23}}{k^2EI}x - \frac{SM_{33}}{k^2EI} \qquad Eq.\,(30)$$

$$y' = Ak\cos kx - Bk\sin kx + \frac{SM_{23}}{k^2EI} \qquad Eq.\,(31)$$

The constants A and B of Eq. (30) are determined by applying the boundary conditions at joint n of Fig. 3b:

$$at\ x = 0\ ;\ y = 0\quad then\quad B = \frac{SM_{33}}{k^2 EI} \qquad Eq.\ (32)$$

$$at\ x\ 0\ ;\ y' = \theta\quad then\quad A = \frac{\theta}{k} - \frac{SM_{23}}{k^3 EI} \qquad Eq.\ (33)$$

Inserting the values of A and B to Eq. (31) yields

$$y' = \left(\frac{\theta}{k} - \frac{SM_{23}}{k^3EI}\right) k \cos kx - \frac{SM_{33}}{k^2EI} k \sin kx +$$

$$\frac{SM_{23}}{k^2EI} \qquad Eq.\ (34)$$

Applying the boundary conditions (joint r of Fig. 3b) to Eq. (34) at x = L , y′ = 0 ; then

$$\left(\frac{\theta}{k} - \frac{SM_{23}}{k^3EI}\right) k \cos kL - \frac{SM_{33}}{k^2EI} k \sin kL +$$

$$\left(\frac{SM_{23}}{k^2EI}\right) = 0 \qquad Eq. (35)$$

At x = L , y = 0 ; then

$$\left(\frac{\theta}{k} - \frac{SM_{23}}{k^3EI}\right) \sin kL + \frac{SM_{33}}{k^2EI} \cos kL +$$

$$\left(\frac{SM_{23}}{k^2EI}\right) L - \frac{SM_{33}}{k^2EI} = 0 \qquad Eq. (36)$$

Solving Eqs. (35) and (36) simultaneously yields

$$SM_{33} = \frac{EIk}{\phi_c}[\sin kL - kL \cos kL] * \theta$$

Eq. (37)

where $\phi_c = [\ 2 - 2 \cos kL - kL\ ]$ *Eq.* (21)

Multiplying Eq. (37) by $4L^3$ / $4L^3$ and simplifying the terms :

$$SM_{33} = \frac{2EI}{L^3} * (2L^2) * (s_3)\ \theta \qquad Eq.(38)$$

where

$$s_3 = \frac{kL\ (\sin kL - kL \cos kL)}{4\ \phi_c} \qquad Eq.\ (39)$$

Substitute Eq. (37) into Eq. (36) yields

$$SM_{23} = \frac{EIk^2}{\phi_c}\ (\ 1 - \cos kL\ )\ \theta \qquad Eq.(40)$$

Equations (25) and (40) are equivalent and satisfy Betti's Law and Maxwell's Law [11]. This indicates that the second-order stiffness matrix is symmetric. The remaining rotation stability function is calculated by summing moments at joint r of Fig. 3b assuming counter-clockwise moment is positive. By equilibrium we have

$$SM_{23}\, L - P(0) + SM_{33} + SM_{63} = 0 \qquad Eq.\,(41)$$

Substituting Eqs. (37) & (40) into Eq. (41) yields

$$SM_{63} = \frac{EIk^2\theta}{\phi_c}[L - L\cos kL] -$$

$$\frac{EIk\theta}{\phi_c}[\sin kL - kL\cos kL]$$

$$= \frac{EIk\theta}{\phi_c}\ [kL - \sin kL\ ] \qquad Eq.\,(42)$$

Introducing the term $2L^3/2L^3$ in Eq. (42) and simplify to Eq. (43), which is the coefficient of stiffness at joint r caused by a unit rotation at joint n , Fig. 3b :

$$SM_{63} = \frac{2EI}{L^3} * 2L^2 \,(s_4) * \theta \qquad Eq.(43)$$

where

$$s_4 = \frac{kL\,(kL - \sin kL)}{2\,\Phi_c} \qquad Eq.\ (44)$$

For negative values of axial force P (tension) , the stability functions are derived in similar fashion applying the same concept of elementary beam analysis. To illustrate, let us investigate Eq. (4). Assuming that axial tensile force is negative, then the homogeneous second-order differential equation has the form of Eqs. (45) & (46) :

$$EIy'' - Py = 0 \qquad Eq.\ (45)$$

$$y'' - k^2y = 0 \qquad Eq.\ (46)$$

$$\text{where } k^2 = \frac{P}{EI} \qquad Eq.\ (7)$$

The complimentary solution of Eq. (46) is

$$y_c = A \sinh kx + B \cosh kx \qquad Eq.\ (47)$$

$$y_c = A \sin kx + B \cos kx \qquad Eq.\ (8)$$

Equations (8) and (47) are similar in form except the former contains the trigonometric functions while the latter has the hyperbolic functions. Using the complimentary solution of Eq. (47), the stability functions for an axial force in tension are calculated following the same procedures as when the axial load is in compression. The four stability functions are listed in TABLE I for axial forces in compression, tension, and zero [12].

TABLE - I STABILITY FUNCTIONS OF A BEAM-COLUMN

Stability Functions for a Beam Subjected to Axial Forces

Direction of Axial Force

Function	Compression	Tension
s_1	$\dfrac{(kL)^3 \sin kL}{12\ \phi_c}$	$\dfrac{(kL)^3 \sinh kL}{12\ \phi_t}$
s_2	$\dfrac{(kL)^2 (1 - \cos kL)}{6\ \phi_c}$	$\dfrac{(kL)^2 (\cosh kl - 1)}{6\ \phi_t}$
s_3	$\dfrac{kL\ (\sin kL - kl \cos kL}{4\ \phi_c}$	$\dfrac{kL\ (\sinh kL - kl \cosh kL)}{4\ \phi_t}$
s_4	$\dfrac{kL\ (kL - \sin kL)}{2\ \phi_c}$	$\dfrac{kL\ (\sinh kL - kL)}{2\ \phi_t}$

$$\phi_c = 2 - 2 \cos kL - kL \sin kL$$

$$\phi_t = 2 - 2 \cosh kL + kL \sinh kL$$

Note: When axial force is zero :

$$s_1 = s_2 = s_3 = s_4 = 1$$

The member stiffness matrix with the stability functions is represented by Eq. (48), where [K_m] is the modified stiffness matrix and includes the effects of axial forces in tension or compression.

$$[K_m] = \frac{E}{L}\begin{bmatrix} A & 0 & 0 & -A & 0 & 0 \\ 0 & \frac{12I}{L^2}s_1 & \frac{6I}{L}s_2 & 0 & -\frac{12I}{L^2}s_1 & \frac{6I}{L}s_2 \\ 0 & \frac{6I}{L}s_2 & 4Is_3 & 0 & -\frac{6I}{L}s_2 & 2Is_4 \\ -A & 0 & 0 & A & 0 & 0 \\ 0 & -\frac{12I}{L^2}s_1 & -\frac{6I}{L}s_2 & 0 & \frac{12I}{L^2}s_1 & -\frac{6I}{L}s_2 \\ 0 & \frac{6I}{L}s_2 & 2Is_4 & 0 & -\frac{6I}{L}s_2 & 4Is_3 \end{bmatrix}$$

Eq. (48)

When the axial force is zero, the stability functions are reduced to unity and Eq. (48) is reduced to Eq. (49) :

$$[K_e] = \frac{E}{L}\begin{bmatrix} A & 0 & 0 & -A & 0 & 0 \\ 0 & \frac{12I}{L^2} & \frac{6I}{L} & 0 & -\frac{12I}{L^2} & \frac{6I}{L} \\ 0 & \frac{6I}{L} & 4I & 0 & -\frac{6I}{L} & 2I \\ -A & 0 & 0 & A & 0 & 0 \\ 0 & -\frac{12I}{L^2} & -\frac{6I}{L} & 0 & \frac{12I}{L^2} & -\frac{6I}{L} \\ 0 & \frac{6I}{L} & 2I & 0 & -\frac{6I}{L} & 4I \end{bmatrix}$$

Eq. (49)

Equation (49) is the first-order elastic stiffness matrix.

3.4 LOAD FUNCTIONS AND MAXIMUM MOMENTS

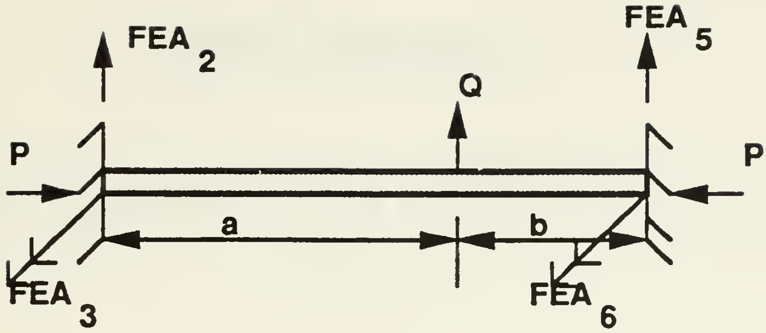
3.4 - 1 Load Functions

The fixed-end actions for a beam-column produced by applied lateral loads depends upon the magnitude of the axial force acting on the member as well as upon the

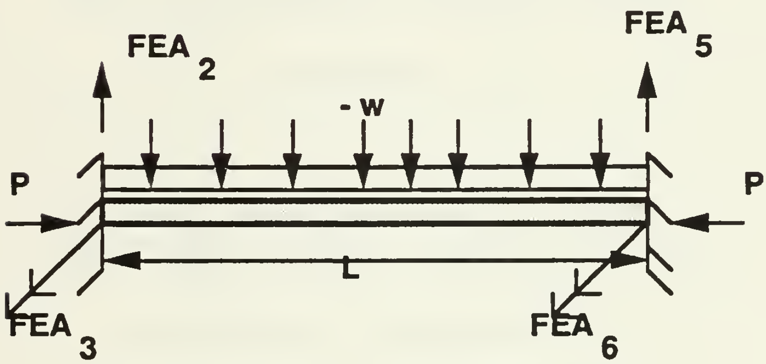
distribution and intensity of the lateral loads. These fixed-end actions or joint loads are used in determining the joint deflections and member forces in the structure. The fixed-end actions form the elements of the load vector in the direct stiffness method.

The two types of member loading (Fig. 5) implemented in the computer program are: (1) uniform load distributed over the full length of the member, and (2) concentrated loads applied anywhere along the span. The procedures used in deriving these fixed-end actions are similar to those described in Section 3.3 and will not be repeated here. A complete derivation of the fixed-end actions for the two types of loading is presented in detail in reference [14]. Timoshenko and Gere [4] present solutions to specific cases of beam-columns with various constraints at the ends. This includes a member simply supported on one end and built in at the other end, as shown in Example 5.2-3, Section 5.2 . Ghali and Neville [9] present a different procedure in determining the fixed-end actions of a beam-column using matrix methods.

The fixed-end actions of a beam-column for the two types of member loading mentioned above are shown in figures Fig. 6 through 8 [10, 12, 14].



(a) Concentrated Load



(b) Uniform Load

Fig. 5 Types of Loading

<u>Axial Compression</u>

$$FEA_2 = \frac{Q}{\phi_c}\left(\cos kL - \cos ka + \cos kb + kb \sin kL - 1\right)$$

$$FEA_3 = \frac{Q}{k\phi_c}\left(\sin kL - \sin ka - \sin kb - kb \cos kL + kL \cos kb - ka\right)$$

$$FEA_5 = \frac{Q}{\phi_c}\left(\cos kL - \cos kb + \cos ka + ka \sin kL - 1\right)$$

$$FEA_6 = \frac{Q}{k\phi_c}\left(-\sin kL + \sin kb + \sin ka + ka \cos kL - kL \cos ka + kb\right)$$

$$\phi_c = 2 - 2\cos kL - kL \sin kL$$

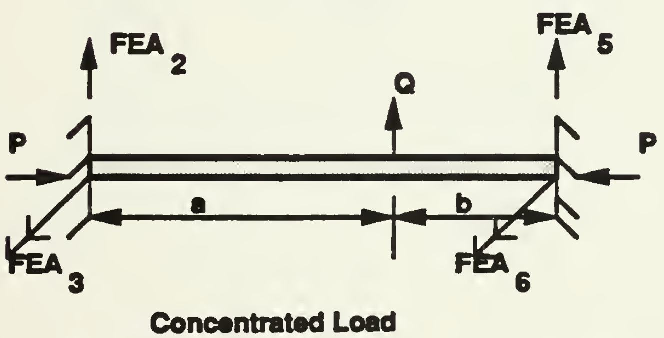


Fig. 6 - Fixed-End Actions for a Concentrated Load

Axial Tension

$$FEA_2 = \frac{Q}{\Phi_t} (\cosh kL - \cosh ka + \cosh kb - kb \sinh kL - 1)$$

$$FEA_3 = \frac{Q}{k\Phi_t} (\sinh kL - \sinh ka - \sinh kb - kb \cosh kL + kL \cosh kb - ka)$$

$$FEA_5 = \frac{Q}{\Phi_t} (\cosh kL - \cosh kb + \cosh ka - ka \sinh kL - 1)$$

$$FEA_6 = \frac{Q}{k\Phi_t} (-\sinh kL + \sinh kb + \sinh ka + ka \cosh kL - kL \cosh ka + kb)$$

$$\Phi_t = 2 - 2 \cosh kL + kL \sinh kL$$

$$k = \sqrt{\frac{P}{EI}}$$

Fig. 7 - Fixed-End Actions for a Concentrated Load

Axial Compression

$$FEA_2 = FEA_5 = -\frac{wL}{2}$$

$$FEA_3 = -\frac{wL^2}{12}\left[\frac{12}{(kL)^2} + \frac{6\sin kL}{kL(\cos kL - 1)}\right]$$

$$FEA_6 = -FEA_3$$

Axial Tension

$$FEA_2 = FEA_5 = -\frac{wL}{2}$$

$$FEA_3 = -\frac{wL^2}{12}\left[\frac{12}{(kL)^2} - \frac{6\sinh kL}{kL(\cosh kL - 1)}\right]$$

$$FEA_6 = -FEA_3$$

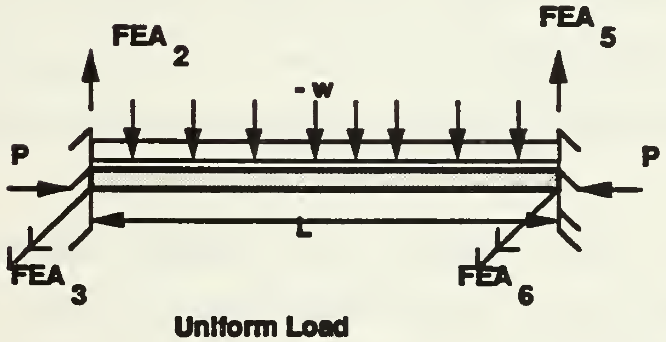


Fig. 8 - Fixed-End Actions for a Uniform Load

3.4 - 2 Maximum Elastic Second-Order Moments

It is important for a designer to know the location and magnitude of the maximum moment within a member. Some structural analysis programs only calculate the member end actions (i.e. shears,moments and axial forces) and the joint deflections. In some cases, the maximum moments occur at the joints, when the axial forces are small and no member lateral loads are present. However, when the axial forces acting on the members become large, the maximum moment may occur within the span as shown in Fig. 9 , even if no lateral loads are present.

In this section, the derivation of the maximum moment is limited to the case of a uniform distributed load. The formulation for this case is straight forward, since an equation for bending moment can be expressed as a continuous function along the length of the member. The bending moment at a concentrated load point can be easily obtained by introducing a node at the location of the load.

The maximum moment in a member with a uniform distributed load is calculated based on the procedure suggested by White and Hajjar [13] . An isolated

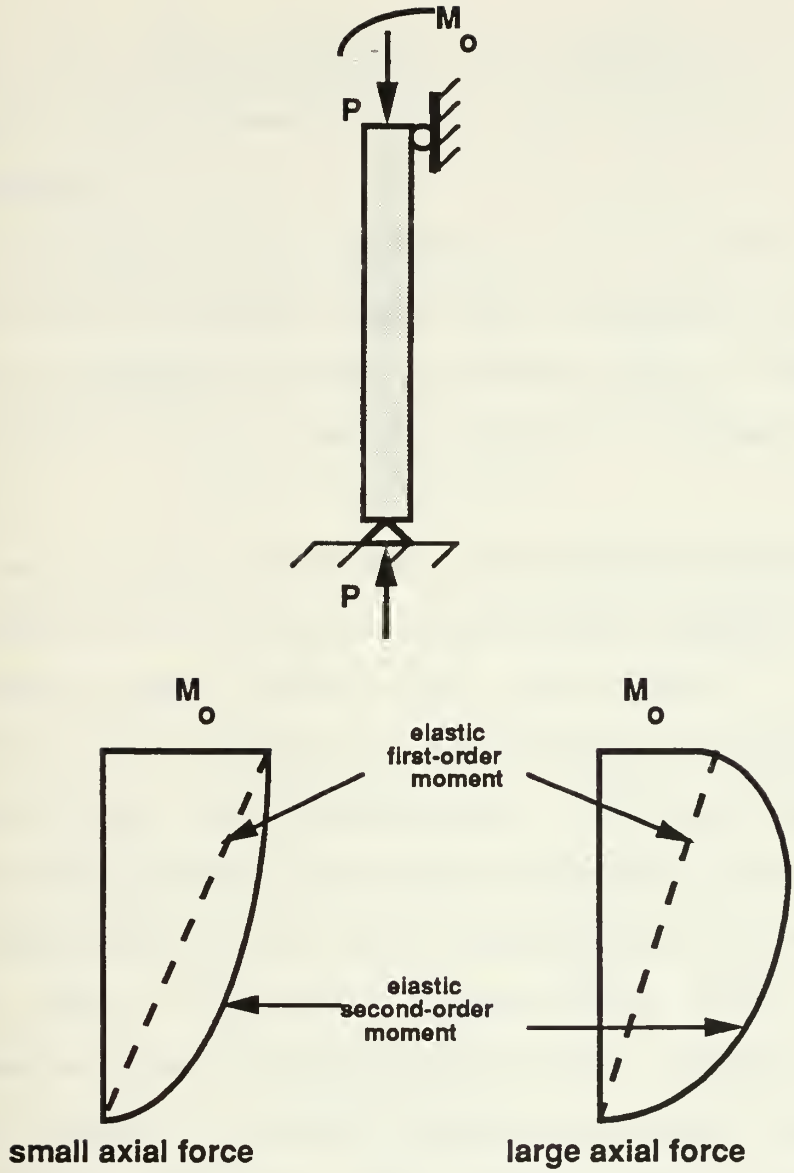


Fig. 9 Maximum Elastic

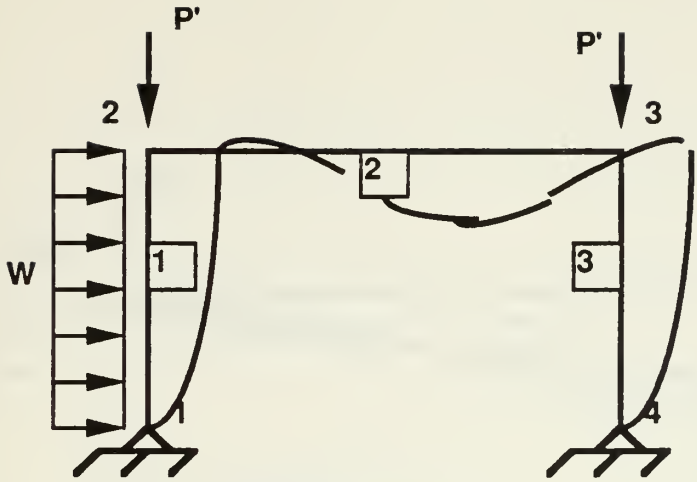
Second-Order Moment

member of a frame structure is treated as a simply supported beam-column that has joint rotation and no translation.

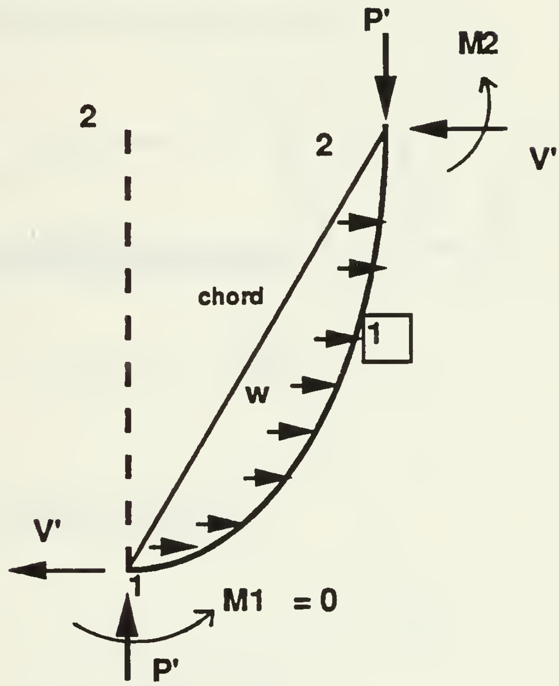
To illustrate this procedure, Fig. 10 shows a rigid frame with its deformed shape when subjected to the two types of loading described in Section 3.4-1. Member 1 is isolated and the free-body diagram is shown in Fig. 10b .

Let us assume the member end actions are known from the results of the elastic second-order analysis. When we connect joints 1 and 2 (Fig. 11a), the member's chord location is established and calculated using the new joint coordinates . If the member end forces and any other loads applied directly to the member are transformed to local axes oriented along the member's chord, White and Hajjar suggest that there is no difference between the behavior of this isolated member (Fig. 10 and 11) and the behavior of a simply supported beam-column with the same applied loads (Fig. 15, Example 5.2-1).

To derive the location of the maximum elastic second-order moment, we shall use the differential equation of an elastic curve { Eq. (3) } about the transformed local axes of Fig. 11b .

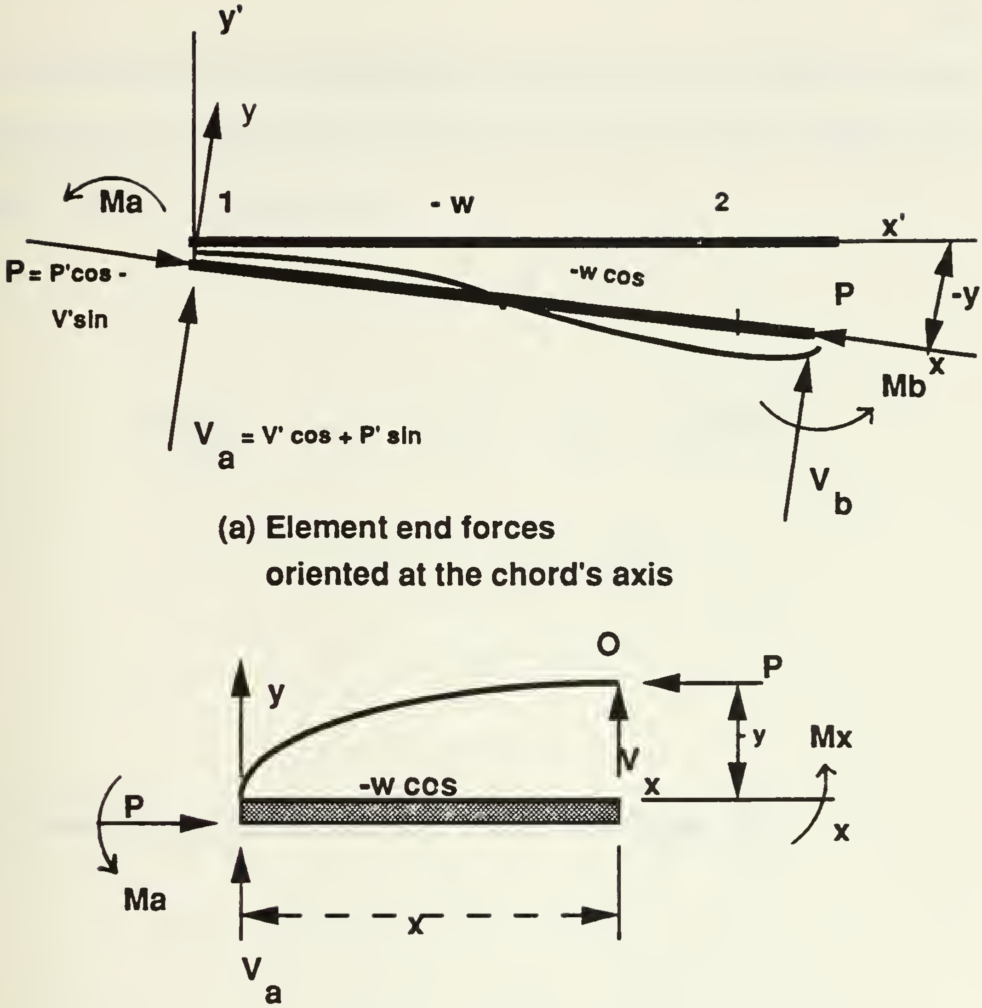


(a) Frame Geometry
Loading, and Deflected Shape



(b) Member 1
Loading and
Deflected Shape

Fig. 10 Derivation of Maximum
Elastic Second-Order Moment



(a) Element end forces oriented at the chord's axis

(b) free-body diagram

Fig. 11 Transformed Axis of a Beam-Column

The second-order differential equation of a beam-column subjected to a uniformly distributed load with member end forces known is given by

$$EI\frac{d^2y}{dx^2} = -[M_a - P(-y) - V_a x + \frac{(-w)x^2}{2}]$$

Eq. (50)

The general solution of Eq. (50) is given by

$$y = A\sin kx + B\cos kx + \frac{1}{k^2 EI}[\frac{wx^2}{2}$$

$$+ V_a x - \frac{w}{k^2} - M_a] \qquad \text{Eq. (51)}$$

Differentiating Eq. (51) twice and applying the differential equation of an elastic curve { Eq. (3) } yields

$$y'' = -A\,k^2 \sin kx - B\,k^2 \cos kx + \frac{w}{k^2EI}$$

Eq. (52)

$$\begin{aligned} M_x &= -EIy'' \\ &= EIk^2\,[A \sin kx + B \cos kx] - \frac{w}{k^2} \end{aligned}$$

Eq. (53)

When $x = 0\,;\ M_x = M_a$

then $B = \frac{1}{P}\,[M_a + \frac{w}{k^2}]$. Eq. (54)

When $x = L\,;\ M_x = M_b$

then $A = \dfrac{M_b - M_a \cos kL}{P \sin kL} + \dfrac{w}{Pk^2} \tan \dfrac{kL}{2}$ Eq. (55)

where $P = k^2 EI$

Differentiating Eq. (53) once with respect to x gives the expression of shear force anywhere along the span and has the form

$$V_x = M_x' = [A \cos kx - B \sin kx]\, EIk^2 \qquad Eq.\ (56)$$

When the shear force is equal to zero, the moment is at local maximum. Inserting the values of A and B into Eq. (56), the location of the maximum moment anywhere along the span of a beam-column with a uniform distributed is determined using Eq. (57):

$$\tan kx = \left[\frac{\frac{w}{k^2}\tan\frac{kL}{2} + \frac{M_b - M_a \cos kL}{\sin kL}}{M_a + \frac{w}{k^2}}\right] \qquad Eq.\ (57)$$

where x is measured from the left support. All the applied loads must be transformed in the local axes when using Eq. (57).

When the end moments in Fig. 11, are equal to zero, the constants A and B are given by Eq. (58). The moment anywhere along the member is then given by Eq. (59) :

$$B = \frac{w}{Pk^2} \quad ; A = \frac{w}{Pk^2} \tan \frac{kL}{2} \qquad Eq.\ (58)$$

$$M_x = \frac{w}{k^2} [\tan \frac{kL}{2} \sin kx + \cos kx - 1] \qquad Eq.\ (59)$$

The maximum moment for a simply-supported beam-column with uniform distributed load is located at the midspan. Thus ,

$$\tan kx = \tan \frac{kL}{2} ; \text{ where } x = \frac{L}{2} . \qquad Eq.\ (60)$$

Inserting the value of x into Eq. (59) yields

$$M_x = \frac{w}{k^2} [\tan \frac{kL}{2} \sin \frac{kL}{2} + \cos \frac{kL}{2} -1] \qquad Eq.\ (61)$$

Introducing the term u, Eq. (61) becomes

$$M_x = \frac{w}{k^2}\,[\tan u \sin u + \cos u - 1] \qquad Eq.\ (62)$$

$$\text{where} \quad u = \frac{kL}{2}$$

Applying trigonometric identities and simplifying the terms in Eq. (62) yields

$$M_x = \frac{wL^2}{8}\,[\frac{2\,(\sec u - 1)}{u^2}]\ . \qquad Eq.\ (63)$$

Equation (63) is the general expression of the maximum moment at midspan of a simply-supported beam-column with uniform distributed load. The first-order moment ($wL^2 / 8$) is amplified by the terms inside the brackets. This term is called the moment amplification factor (MAF).

Let us now investigate what happens when the lateral load $w = 0$ and M_a, M_b are not equal to zero.

Applying the boundary conditions at x = 0 and x = L, from Eq. (51), the constants A and B are determined. When x = 0 , y = 0 , and w = 0 ; then

$$B = \frac{M_a}{P} \; . \qquad Eq. \; (64)$$

When x = L , y = 0 , and w = 0 ; then

$$A = \frac{M_a}{P} \tan \frac{kL}{2} - \frac{M_b + M_a}{P \sin kL} \qquad Eq. \; (65)$$

$$where \; V_a L = \frac{(M_a + M_b) \, L}{L} \qquad Eq. \; (66)$$

Inserting the value of the trigonometric identity { Eq. (67) } into Eq. (65),

$$\tan \frac{kL}{2} = \frac{(1 - \cos kL)}{\sin kL} \qquad Eq. \; (67)$$

Then the constant A is given by Eq. (68).

$$A = -\frac{(M_a \cos kL + M_b)}{P \sin kL} \qquad Eq.\ (68)$$

Inserting the values of A and B into Eq. (53) when w is equal 0 , yields

$$\tan kx = \frac{A}{B} = -\frac{(M_b + M_a \cos\ kL)}{M_a \sin kL} \qquad Eq.\ (69)$$

$$M_x = -\left[\frac{(M_b + M_a \cos kL)}{\sin kL}\right] \sin kx + M_a \cos kx$$

$$Eq.\ (70)$$

Thus, Eq. (70) is the moment at any location along a beam-column when subjected to end moments M_a, M_b and an axial force P. Inserting the value of kx from Eq. (69) into Eq. (70), the maximum moment is given by Eq. (71). The minus sign that appears in Eq. (71) simply indicates that the moment M_{max} causes tension on the top fiber of the cross section [5].

$$M_{max} = -M_b\left[\sqrt{\frac{(M_a/M_b)^2 + 2\ (M_a/M_b)\cos kL + 1}{\sin^2 kL}}\right]$$

Eq. (71)

When end moments are present in the member, there is always a possibility that the maximum moment will occur at the end of the span. Therefore, the value of x in Eq. (69) must always be evaluated. When x is greater than the span length or less than zero, the maximum moment occurs at the end of the member and it is equal to the larger of the two end moments.

Equations (51), (53), (54), (55) and (57) are used in the computer program to calculate the location of the maximum moment of a member with uniform load and subjected to end-moments. These equations have been developed for the case where the axial force is in compression. For axial force in tension, similar equations are used, except hyperbolic functions replace the corresponding trigonometric functions. The maximum moment is then compared with the end moments of the member, and the larger of the three controls. The program also calculates the deflection at the point of maximum moment. If the maximum moment occurs at one

end of the member, then the computer program prints out the deflection at that joint. For deflections within a member, the computer program only calculates the displacements relative to the chord member. (See Example 5.2-3)

4 ELASTIC BUCKLING LOAD

4.1 GENERAL

This chapter deals with the elastic stability analysis of beam-columns and frames. The same stability functions and stiffness matrix developed in Chapter 3 are used in determining the elastic critical loads. A load factor increment is applied successively to the external loads until the stiffness matrix of the structure has become singular. The decomposition of the stiffness matrix using Cholesky's method will also be presented. Several examples are examined and presented in Chapter 5.

4.2 COMPUTER ANALYSIS OF THE ELASTIC STABILITY OF PLANE FRAMES

In Section 3.3, the stability functions of the beam-column were derived and used to modify the member stiffness matrix {Eq. (48)} to include the effects of axial forces. The joint displacements and member end forces are calculated iteratively.

The computer program was modified to calculate the elastic critical load using load increments. The elastic critical load is determined by performing successive computations until the stiffness matrix of the structure becomes singular.

In the computer program, the structure's stiffness matrix is decomposed using Cholesky's method [8, 14]. The stiffness matrix [K_m] {Eq. (48)} is factored into an upper and lower triangular matrices.

In general, structural systems generate stiffness matrices that are symmetrical, square, and with positive diagonal elements. They display the unique property of positive-definiteness. If this is the case, then Cholesky's method is valid and the stiffness matrix can be expressed as $\{ K_m \} = \{U^t\} * \{ U \}$. The matrix { U } represents the upper diagonal elements of the structure's stiffness matrix {K_m} after the decomposition process. The elements of the lower triangular matrix are represented by the matrix { U^t }, which is the transpose of the matrix { U }. Eq. (72) shows the elements of the structure's stiffness matrix { K_m } and the elements of the upper and lower triangular matrices after the decomposition process.

$$
\begin{bmatrix}
k_{11} & k_{12} & k_{13} & k_{14} & \cdots & k_{1n} \\
k_{21} & k_{22} & k_{23} & k_{24} & \cdots & k_{2n} \\
k_{31} & k_{32} & k_{33} & k_{34} & \cdots & k_{3n} \\
\cdot & & & & & \\
\cdot & & & & & \\
k_{n1} & k_{n2} & \cdot & \cdot & \cdots & k_{nn}
\end{bmatrix} =
$$

$$
\begin{bmatrix}
u_{11} & 0 & 0 & \cdots & 0 \\
u_{12} & u_{22} & 0 & \cdots & 0 \\
u_{13} & u_{23} & u_{33} & \cdots & 0 \\
\cdot & & & & \\
\cdot & & & & \\
u_{1n} & u_{2n} & u_{3n} & \cdots & u_{nn}
\end{bmatrix} *
\begin{bmatrix}
u_{11} & u_{12} & u_{13} & u_{14} & \cdots & u_{1n} \\
0 & u_{22} & u_{23} & u_{24} & \cdots & u_{2n} \\
0 & 0 & u_{33} & u_{34} & \cdots & u_{3n} \\
\cdot & & & & & \\
\cdot & & & & & \\
0 & 0 & 0 & 0 & \cdots & u_{nn}
\end{bmatrix}
$$

Eq. (72)

Performing the indicated multiplication of { U } and { U^t } and equating the product to the elements in { K_m } yields Eqs. (73) & (74) shown in Fig. 12. If during decomposition one of the diagonal elements u_{ii} approaches zero or is less than zero, the structure has reached the elastic buckling load.

$$k_{11} = u_{11}^2 \quad \rightarrow \quad u_{11} = \sqrt{k_{11}}$$

$$k_{12} = u_{11}u_{12} \quad \rightarrow \quad u_{12} = \frac{k_{12}}{u_{11}}$$

$$k_{13} = u_{11}u_{13} \quad \rightarrow \quad u_{13} = \frac{k_{13}}{u_{11}}$$

$$k_{22} = u_{12}^2 + u_{22}^2 \quad \rightarrow \quad u_{22} = \frac{\sqrt{k_{22} - u_{12}{}^2}}{1}$$

$$k_{23} = u_{12}u_{13} + u_{22}u_{23}$$

$$\rightarrow u_{23} = \frac{(k_{23} - u_{12}U_{13})}{u_{22}}$$

$$k_{33} = u_{13}^2 + u_{23}^2 + u_{33}^2 \quad \rightarrow u_{33} = \sqrt{k_{33} - (u_{13} + u_{23}^2)}$$

$$k_{34} = u_{13}u_{14} + u_{23}u_{24} + u_{33}u_{34}$$

$$\rightarrow u_{34} = \frac{k_{34} - (u_{13}u_{14} + u_{23}u_{24})}{u_{33}}$$

$$\text{where } u_{ii} = \sqrt{k_{ii} - \sum_{a-1}^{a=i-1} u_{ai}^2} \qquad Eq.\ (73)$$

are diagonal elements of $[U]$

k_{ii} = *diagonal elements of* $[K]$

$$u_{ij} = \frac{k_{ij} - \sum_{a=1}^{a=i-1} u_{ai}\, u_{aj}}{u_{ii}} \qquad Eq.\ (74)$$

$i \geq j$ *(lower diagonal elements)*

$i < j$ *(upper diagonal elements)*

Fig. - 12 DECOMPOSITION OF MATRIX [K_s]

To better understand the concept, a flowchart is shown in Fig. 13. In the flowchart, the process starts with the calculation of the first-order axial forces. Then these axial forces are used to modify the structure's stiffness matrix [K_m]. The subroutine DECOMP decomposes the structure's stiffness matrix using the procedure described in Fig. 12. The program then checks the diagonal elements u_{ii} for value less than or equal to zero. If all the diagonal elements are greater than zero, a new load factor (LF) is calculated and the process continues.

If any one of the diagonal elements is less than zero, the program stores the load factor LF, calculates the joint displacements, prints the output and then terminates the process. The output contains the load factor LF and the joint displacements. To obtain an accurate buckling load, small load increments must be used.

The buckling mode shape is determined by converting the calculated joint displacements into relative joint displacements. The joint displacements at buckling are expressed in terms of relative displacements.

This approach of determining the elastic buckling load is simple to implement in the computer program,

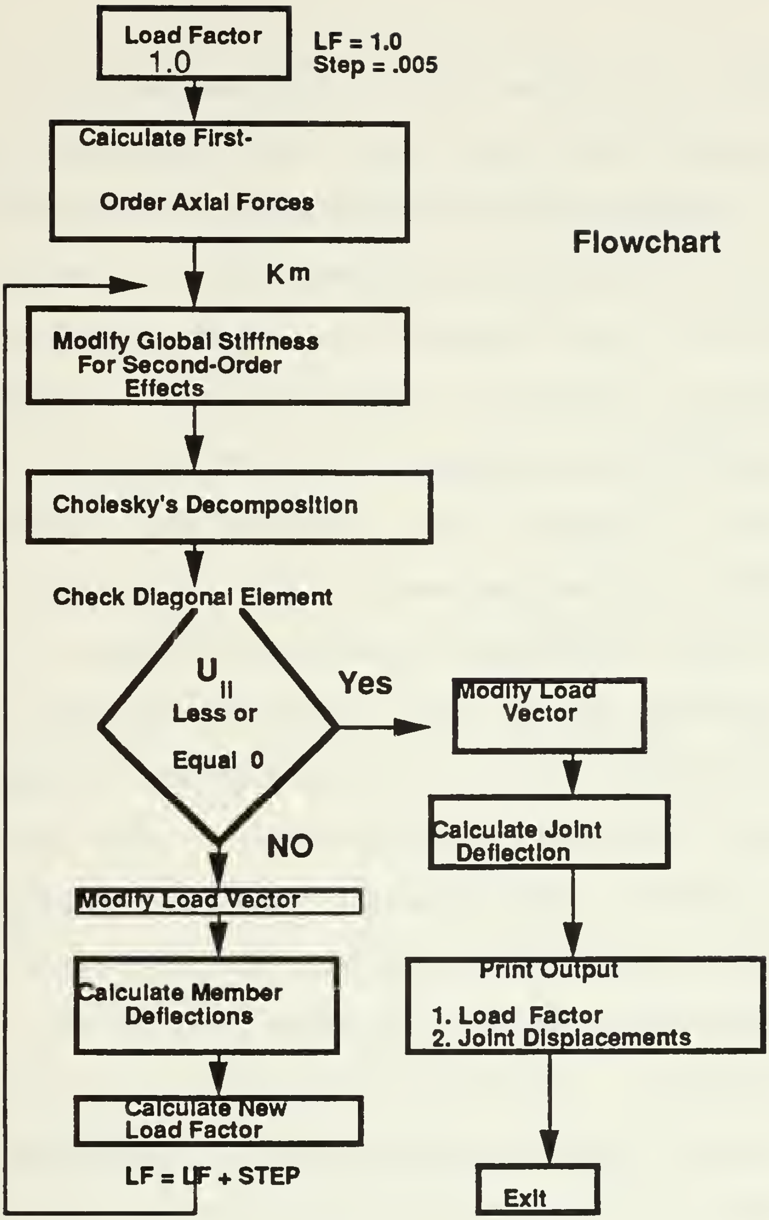


Fig. 13 Elastic Stability

Analysis of a Frame and Beam-Column

since the decomposition of the matrix is routinely performed during the first and second-order analyses.

A flowchart showing the subroutine DECOMP [8] is shown in Fig. 14. The decomposed structure stiffness matrix is also used in solving for the unknown displacements. In the flowchart, there are two returns. One occurs when the value of I becomes greater than N and the processing is complete. The second is when the decomposition fails. The latter serves as a safeguard against an undefined arithmetic operation and it also indicates that the critical load of the structure has been reached. The undefined arithmetic operation will result to an error in determining the critical load if at the beginning of the linear-elastic (first-order) analysis, the structure stiffness matrix is not positive-definite. In order to safeguard against this error, one should perform a first-order analysis. If the error message "WARNING !!! DECOMPOSITION FAILED" shows, then the stiffness matrix is "singular." The input file should be checked for incorrect data of joint coordinates and member connectivity, which are the common sources of error.

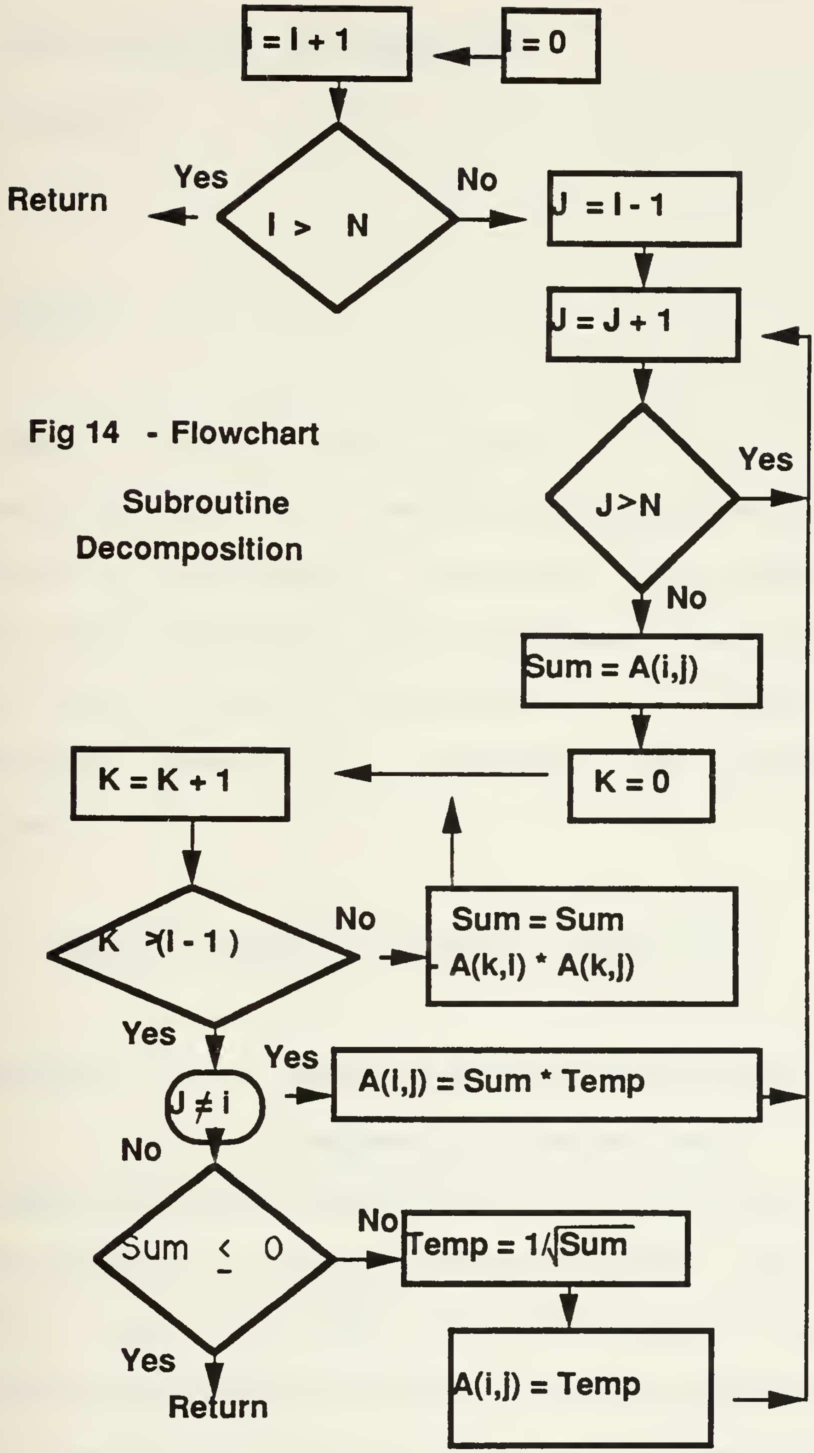


Fig 14 - Flowchart

Subroutine Decomposition

5 VERIFICATION OF COMPUTER PROGRAM

5.1 GENERAL

In order to verify the program [E S C A P], the following Sections will compare closed-form or published solutions to the computer solutions. The second-order moments and deflections are calculated first using the closed-form solution of the differential equation. Then a computer solution is generated and results are compared.

5.2 SAMPLE PROBLEMS OF BEAM-COLUMNS

EXAMPLE 5.2 - 1 Simply Supported Beam-Column with Uniform Distributed Load.

The simply supported beam-column in Fig. 15 has a uniform load <u>w</u> of 1 kip/ft acting downward. The span L = 500 in. and I = 1000 in^4. Calculate the maximum deflection and moment at midspan based on second-order elastic analysis. Determine the elastic buckling load

and the buckled shape of the member. Assume shear strain is negligible and neglect the weight of the beam.

The maximum deflection and moment occur at midspan. The exact closed-form solutions are given by Timoshenko and Gere[4], and Chen and Lui [5]. The general solutions are given by Eqs. (75) & (76).

$$y_{max} = y_o \left[\frac{12(2\sec u - 2 - u^2)}{5u^4} \right] \qquad Eq.\ (75)$$

$$\text{where } u = \frac{kL}{2} \ ; \ k = \sqrt{\frac{P}{EI}} \ ; \ y_o = \frac{5wL^4}{384EI}$$

$$M_{max} = M_o \left[\frac{2(\sec u - 1)}{u^2} \right] \qquad Eq.\ (76)$$

$$\text{where } M_o = \frac{wL^2}{8} \ .$$

Example 5.2-1

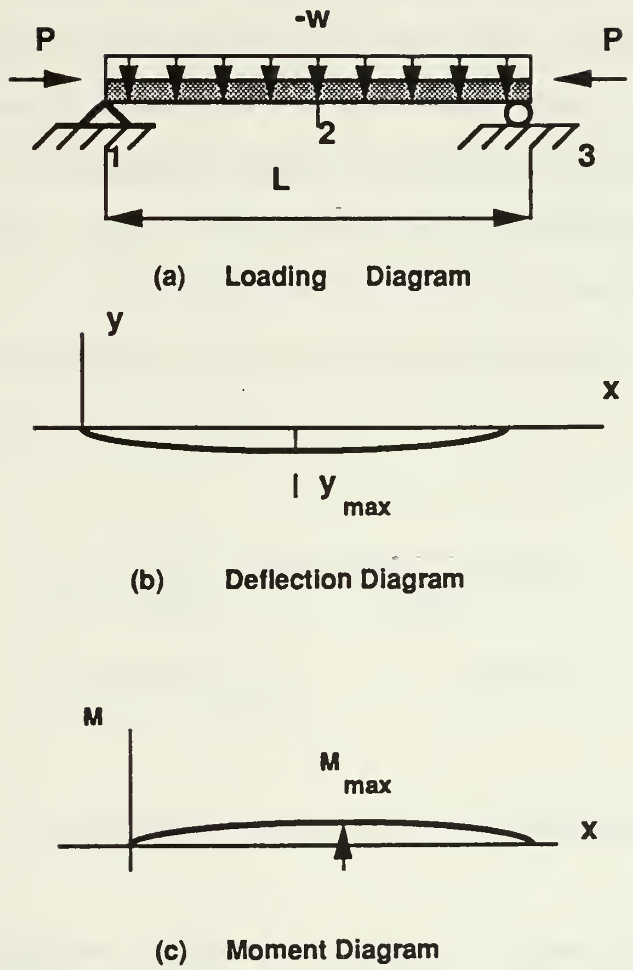


Fig. 15 Simply Supported
Beam-Column with uniform Load

The computer results are compared with these solutions in TABLES II and III. These tables show the computer solution agrees very closely to the exact solution. The amplification factors (AF) reported in these tables are the ratio of the second-order to the first-order moments or deflections. Note that second-order effects are quite significant at high values of axial compressive force.

The theoretical elastic critical load for the beam-column in Fig. 15 is given by ;

$$P_e = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 * 30000 * 1000}{500^2}$$

$$= 1184.35\ kips. \qquad Eq.\ (77)$$

The symbol P_e is used to represent the elastic critical load.

The computer solution reports a load factor LF = 2.37, with an applied axial force of 500 kips. The computed elastic critical load is therefore 500 * 2.37 = 1185 kips, versus the exact 1184.35 kips. The computer program used a load factor increment of .005 .

The computed relative joint displacements for the buckled member are listed in TABLE IV. The buckled shape

is plotted in Fig. 15b. The buckled shape from the computer solution agrees with that predicted by the exact solution.

The values of P / P_e versus the moment and deflection amplification factors (MAF,DAF) are plotted in Fig. 16. From this figure, the following observations are made :

- the moment and deflection amplification factors are non-linear with respect to P,

- the amplification factors are very small for small values of P / P_e ,on the order of .05 to .10,

- AF increases without bound as P approaches P_e.

These observations are typical for second-order elastic response.

Table II- MAXIMUM BENDING MOMENTS FOR A SIMPLY SUPPORTED BEAM-COLUMN

Axial Load (kips)	Maximum Bending Moments (at midspan) Exact solution (kip-in)	Computer solution (kip-in)	MAF
100	2851.0	2851.1	1.09
200	3148.3	3148.5	1.21
300	3513.1	3513.2	1.35
400	3971.0	3971.2	1.53
500	4563.0	4563.2	1.75

Table III - MAXIMUM DEFLECTIONS FOR A SIMPLY SUPPORTED BEAM-COLUMN

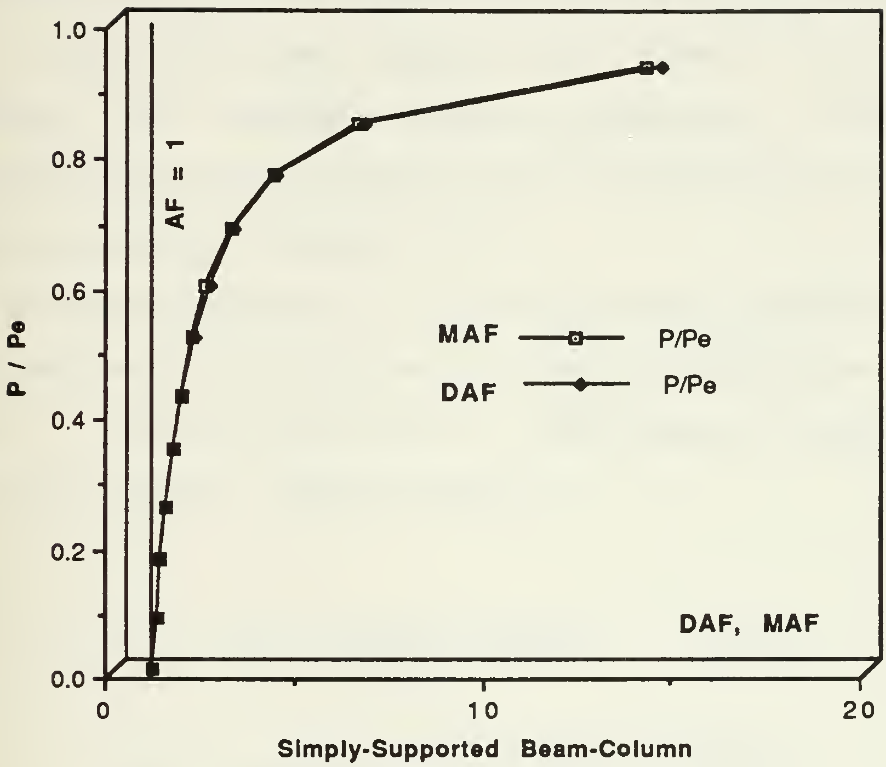
Axial Load (kips)	Maximum Deflections (at midspan) Exact solution (in.)	Computer solution (in.)	DAF
100	- 2.46965	- 2.4698	1.09
200	- 2.72135	- 2.7215	1.20
300	- 3.02999	- 3.0301	1.34
400	- 3.41735	- 3.4175	1.51
500	- 3.91793	- 3.9181	1.73

Table - IV RELATIVE DISPLACEMENTS AT INSTABILITY OF SIMPLY SUPPORTED BEAM-COLUMN

JOINT	RELATIVE DISPLACEMENTS D(X)	D(Y)	ROTATION
1	0	0	- 1
2	0	- 1	0
3	0	0	1

Fig. 16

Load vs. Deflection/Moment Curve



EXAMPLE 5.2 - 2 FIXED-FIXED BEAM-COLUMN WITH UNIFORM DISTRIBUTED LOAD

The beam-column shown in Fig. 17 is similar to Example 5.2-1 except the end supports are fixed. We will determine the maximum moment, deflection, elastic critical load and the buckled shape by following the same steps described in Example 5.2-1.

The exact solution to the differential equation of this particular beam-column is found in Timoshenko and Gere [4], and Chen and Lui [5]. The general solutions are given by Eqs. (78) and (79):

$$y_{max} = y_o \left[\frac{12(2 - 2\cos u - u\sin u}{u^2 \sin u} \right] \qquad Eq. (78)$$

$$\text{where} \quad y_{max} \text{ occurs at } x = \frac{L}{2} \; ; y_o = \frac{wL^4}{384EI}$$

$$M_{max} = -\frac{wL^2}{12} \left[\frac{3(\tan u - u)}{u^2 \tan u} \right] \qquad Eq. (79)$$

Using w = 1 kip/ft, L = 500 in, and I = 1000 in^4 as in Example 5.2-1, the deflection at midspan and moment at the end supports are calculated and compared to the computer solutions in TABLES V and VI. Again, the results are in agreement.

Example 5.2-2

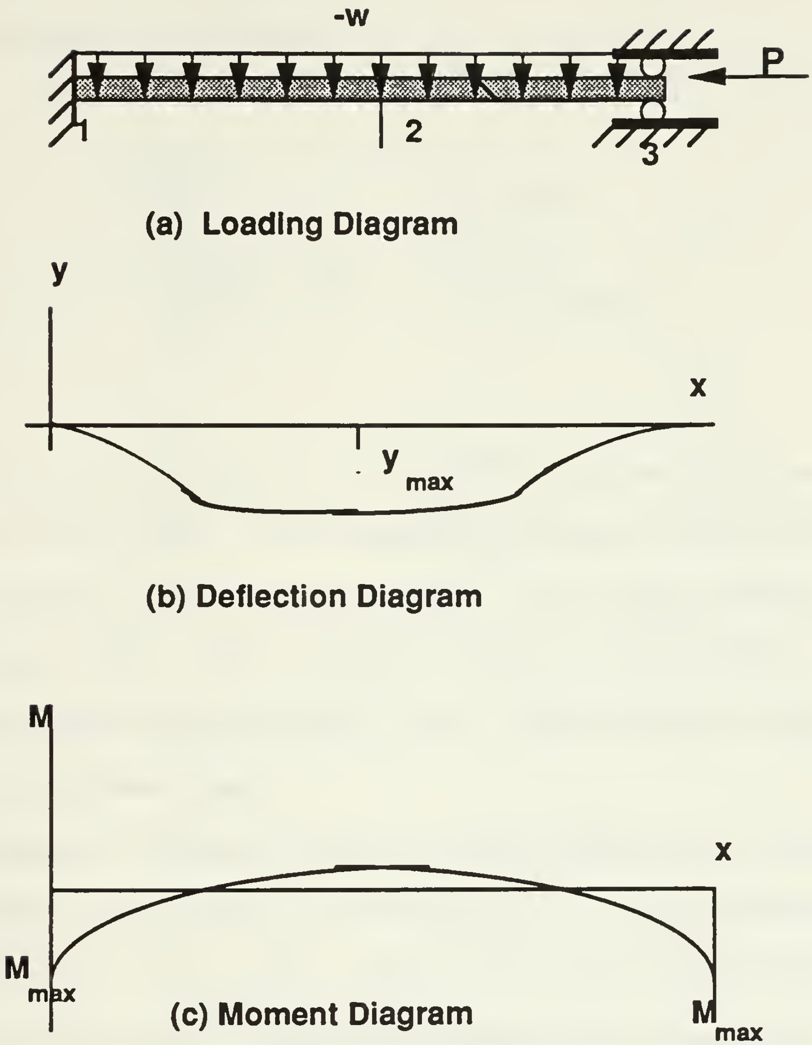


Fig. 17 Fixed-Fixed Beam-Column with Uniform Load

The theoretical elastic critical load for a fixed-fixed beam-column is given by :

$$P_e = \frac{4\pi^2 EI}{L^2} \qquad Eq.\ (80)$$

$$= \frac{4 * \pi^2 * 30000 * 1000}{500^2} = 4737.41\ kips$$

The computed load factor LF = 9.4752 with an applied axial load of 500 kips. The computed elastic critical load is 9.4752 * 500 = 4737.6 kips. The load factor increment was .005. The elastic critical load for a fixed-fixed beam-column is four (4x) times larger than the simply-supported beam.

The computed relative joint displacements for the buckled member are listed in TABLE VII. The buckled shape is plotted in Fig. 17b. The buckled shape from the computer solution agrees with that predicted by the exact solution.

The relationships between P / P_e and the amplification factors are similar to those in Example 5.2-1.

Table - V MAXIMUM BENDING MOMENTS
FIXED-FIXED BEAM-COLUMN

Axial Load (kips)	Maximum Bending Moments (at end supports) Exact Solution (kip-in)	Computer Solution (kip-in)	MAF
100	1760.6	1760.7	1.01
200	1786.3	1786.3	1.03
300	1813.0	1813.0	1.04
400	1841.0	1841.0	1.06
500	1870.0	1870.0	1.08

Table - VI MAXIMUM DEFLECTIONS FIXED-FIXED BEAM-COLUMN

Axial Load (kips)	Maximum Deflections (at midspan) Exact Solution (in.)	Computer Solution (in.)	DAF
100	- .46172	- .46174	1.02
200	- .47176	- .47178	1.04
300	- .48226	- .48228	1.07
400	- .49324	- .49326	1.09
500	- .50474	- .50476	1.12

Table - VII RELATIVE DISPLACEMENTS AT INSTABILITY OF FIXED-FIXED BEAM-COLUMN

JOINT	RELATIVE DISPLACEMENTS D(X)	D(Y)	ROTATION
1	0	0	0
2	0	- 1	0
3	0	0	0

EXAMPLE 5.2 - 3 HINGED-FIXED BEAM-COLUMN WITH UNIFORM DISTRIBUTED LOAD

This problem is similar to the previous two examples, except this beam-column is restrained against rotation at joint _B_ and free to rotate at joints _A and C_ (Fig. 18) . The maximum moment, deflection at the location of the maximum moment, elastic critical load, and the buckled shape of the member will be determined by following the same procedures described in the previous examples.

The exact solution is determined by superposition. The general solution is a combination of a simply-supported beam-column with a uniform load (Example 5.2-1 and Fig. 19a) and one with an end moment acting at joint B without transverse load (Fig. 19b). For the latter case the general solutions are given by Timoshenko and Gere [4] :

$$y = \frac{M_b}{P}\left(\frac{\sin kx}{\sin kL} - \frac{x}{L}\right) \qquad Eq.\ (81)$$

$$M_x = -M_b\frac{\sin kx}{\sin kL} \qquad Eq.\ (82)$$

Example 5.2-3

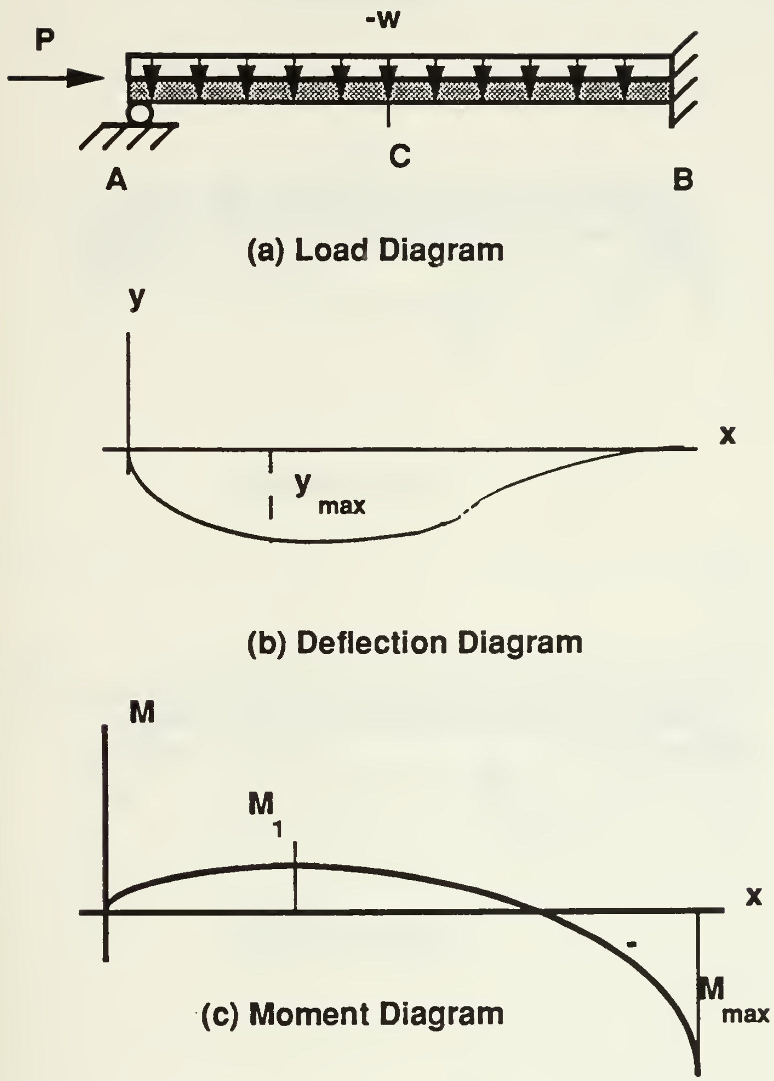
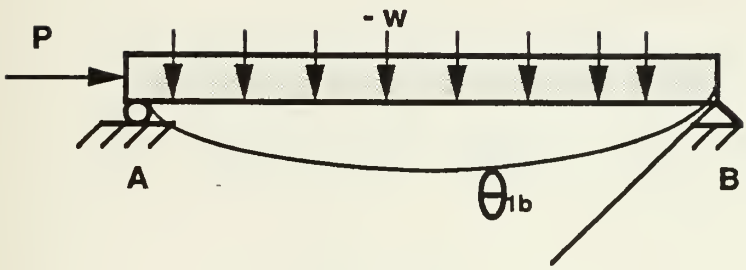
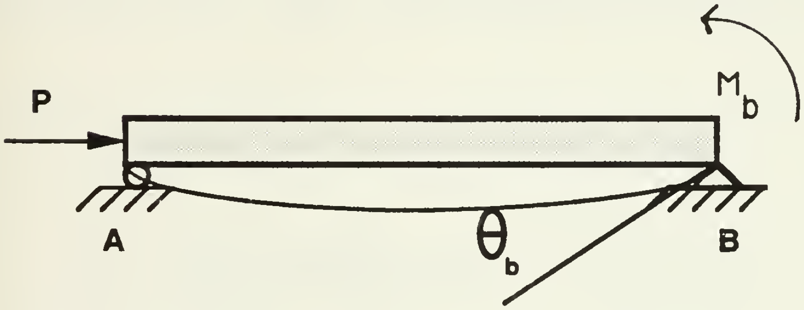


Fig. 18 Hinged-Fixed Beam-Column with Uniform Load

Example 5.2-3



(a) uniform load



(b) end moment

Fig. 19 Superposition of Lateral Load and Moment

The rotation at joint B for the case in Fig. 19b is :

$$\theta_b = \frac{M_b L}{3EI} \frac{3}{2u} \left(\frac{1}{2u} - \frac{1}{\tan u} \right) \qquad Eq.\ (83)$$

For the casein Fig. 19a, the rotation at joint B is :

$$\theta_{1b} = \frac{wL^3}{24EI} \frac{3(\tan u - u)}{u^3} \qquad Eq.\ (84)$$

By imposing the rotational boundary condition at B, such that $\theta_b + \theta_{1b} = 0$, the moment M_b is obtained.
The moments and the deflections at the location of the maximum moments for members AC and CB are calculated and compared to the computer solution in TABLES VIII thru X.

The solutions are in agreement.

The theoretical elastic critical load is [4] :

$$P_e = \frac{20.191 EI}{(L)^2} = \frac{20.191*30000*1000}{(500)^2}$$

Eq. (85)

$$P_e = 2423\ kips$$

The computed load factor LF = 4.8501 for an applied load of 500 kips. The computed elastic critical load is 4.8501 x 500 = 2425.05 kips. The load factor increment was .005. The resulting load increment was .005 x 500 kips = 2.5 kips. The computed elastic critical load is within this tolerance. Closer agreement is possible by using a smaller load factor increment.

The computed relative joint displacements for the buckled member are listed in TABLE XI. The buckled shape is plotted in Fig. 18b. The buckled shape from the computer solution agrees with that predicted by the exact solution.

Table - VIII MAXIMUM BENDING MOMENTS
HINGED-FIXED BEAM-COLUMN

Axial Load (kips)	Maximum Bending Moments (at Joint B of member CB) Exact Solution (kip-in)	Computer Solution (kip-in)	MAF
100	- 2679.4	- 2679.4	1.03
200	- 2761.0	- 2761.0	1.06
300	- 2849.9	- 2849.9	1.09
400	- 2947.0	- 2947.0	1.13
500	- 3053.7	- 3053.7	1.17

The deflections at the support are 0.

Table - IX MAXIMUM MOMENTS FOR MEMBER AC
HINGED - FIXED BEAM-COLUMN

Maximum Bending Moments
(for member AC at distance x from left support)

Axial Load (kips)	Distance X (in.)	Exact Solution (kip-in)	Computer Solution (kip-in)
100	187.11	1533.2	1533.3
200	186.72	1608.0	1608.1
300	186.32	1690.0	1690.0
400	185.91	1780.2	1780.3
500	185.49	1880.0	1880.1

Table - X DEFLECTIONS AT MEMBER AC
HINGED-FIXED BEAM-COLUMN

Deflections
(for member AC at distance x from left support)

Axial Load (kips)	Distance X (in.)	Exact Solution (in)	Computer Solution (in)	DAF
100	187.11	- .96546	- .96636	1.04
200	186.72	- 1.0083	- 1.0092	1.09
300	186.32	- 1.0552	- 1.0561	1.14
400	185.91	- 1.1073	- 1.1076	1.19
500	185.49	- 1.1646	- 1.1644	1.25

Table - XI RELATIVE DISPLACEMENTS AT INSTABILITY
HINGED-FIXED BEAM-COLUMN

	RELATIVE DISPLACEMENTS		
JOINT	D(X)	D(Y)	ROTATION
1	0	0	- 1
2	0	- 1	.33428
3	0	0	0

EXAMPLE 5.2-4 SECOND-ORDER ANALYSIS OF A FRAME

In previous examples, the given structures consisted of a single-element. In this example, a frame composed of several elements will be analyzed.

Since the closed-form solutions for frames are difficult to derive, the published solution for the given frame is used to verify the accuracy of the computer solution. The effect of axial forces on the behavior of a rigid frame are illustrated by means moment diagrams.

The rigid jointed plane frame is loaded as shown in Fig. 20 (Beaufait et al.)[14] EI is constant for all members. The deflections and moments at the joints considering the effects of axial forces are required. A summary of the published and computer solutions is shown in TABLES XII & XIII. The tables show that the solutions are in good agreement. The first and second-order moments are plotted in Fig. 21. From this figure, the following observations are made :

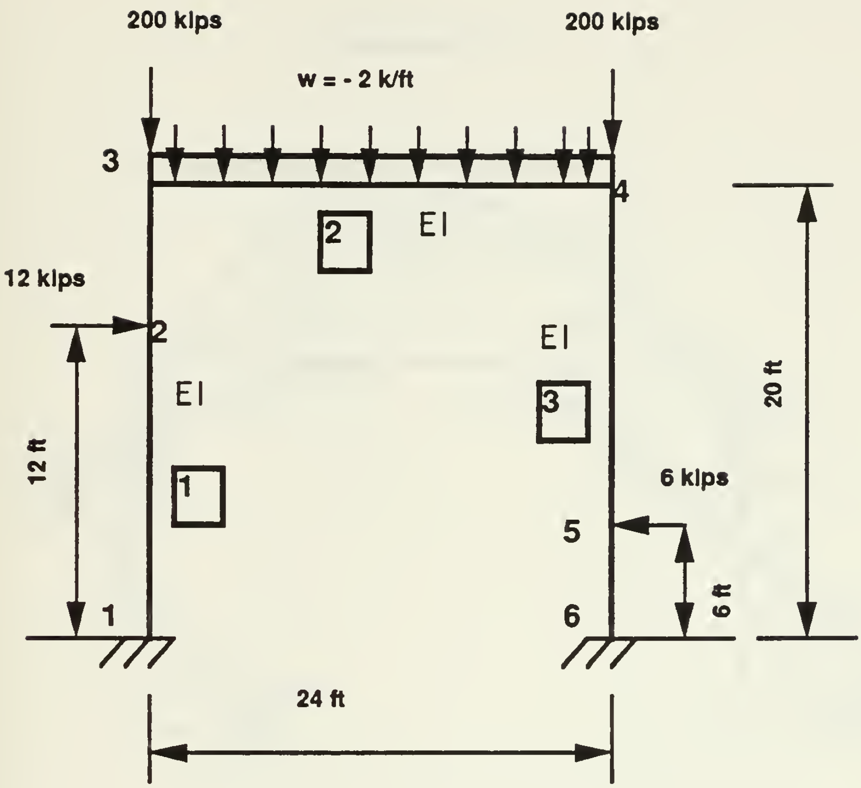
- the largest amplification factors occur at the support joints 1 and 4 , and are on the order of 23% to 25% .

- for each member of the frame, the maximum moment is located at the end joint other than the support

joints. The amplification factors at the points of maximum moments are less than 10% .

In both examples, Example 5.2-3 and this example, the maximum moment do occur at the member ends.

Example 5.2-4



$I = 250\ in^4$

$E = 30{,}000\ ksi$

Fig. 20 Rigid Jointed Frame

Geometry and Loading Condition[9]

Example 5.2-4

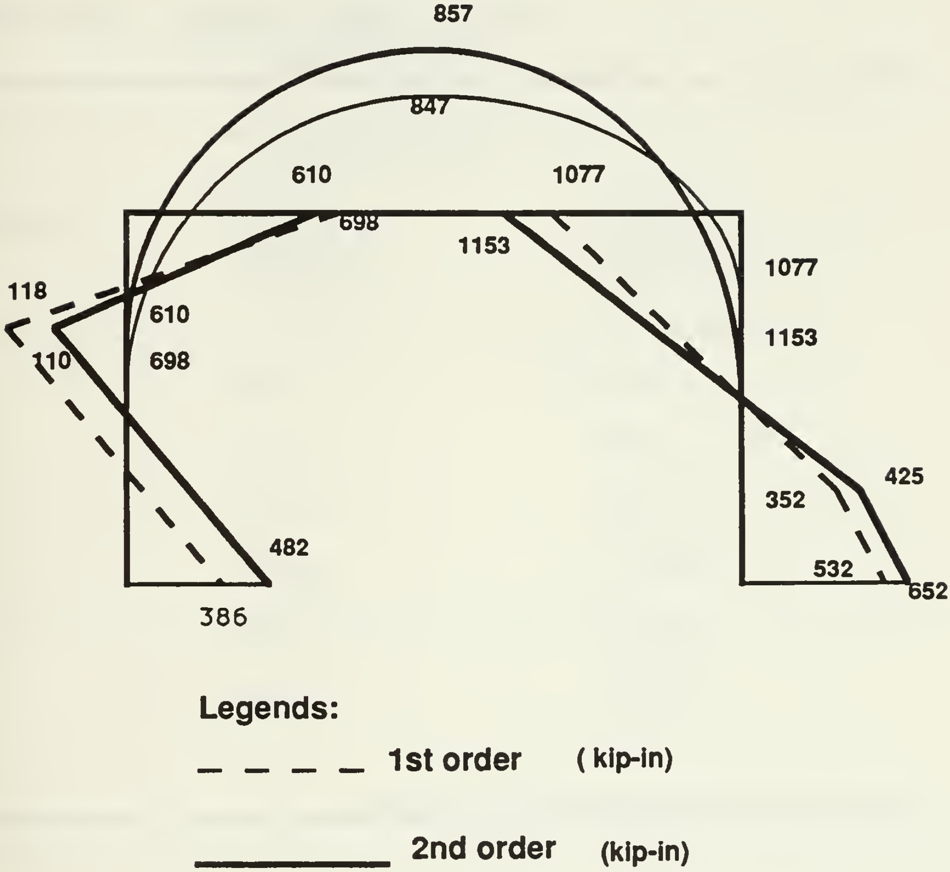


Fig. 21 Moment Diagrams

First and Second Order

Table - XII SECOND-ORDER MOMENTS OF A PLANE FRAME

Joint	Second-Order Moments Plane Frame Published Solution (kip-in)	Computer Solution (kip-in)
1	480	482
2	110	110
3	- 610	- 609
4	1153	1152
5	425	425
6	653	652

Table - XIII SECOND-ORDER DEFLECTIONS OF A PLANE FRAME

Second-Order Deflections
Plane Frame

Joint	Published Solution D (X)	Published Solution Rotation	Computer Solution D(X)	Computer Solution Rotation
1	0	0	0	0
3	.8483	- .007	.8526	- .007
4	.8473	.0035	.8495	.0035
6	0	0	0	0

EXAMPLE 5.2 - 5 ELASTIC BUCKLING LOAD OF A FRAME

In this example, the elastic critical load of a frame will be computed using the method described in Chapter 4. The results are compared to the published solution taken from Ghali and Neville [9]. The frame shown in Fig. 22 has a load Q and Q/2 applied at joints 2 and 3, respectively. Member 2 has a flexural rigidity equal to EI , while members 1 and 3 have a flexural rigidity equal to .75EI . Assuming that the loads are applied simultaneously, calculate the elastic critical load Q_e which causes buckling of the frame. Assume also that the members are made of structural steel with the following cross-sectional and material properties:

$$I_x = 533 \text{ in.}^4; \; A = 19.10 \text{ in.}^2 \; ; \; E = 29{,}000 \text{ ksi.}$$

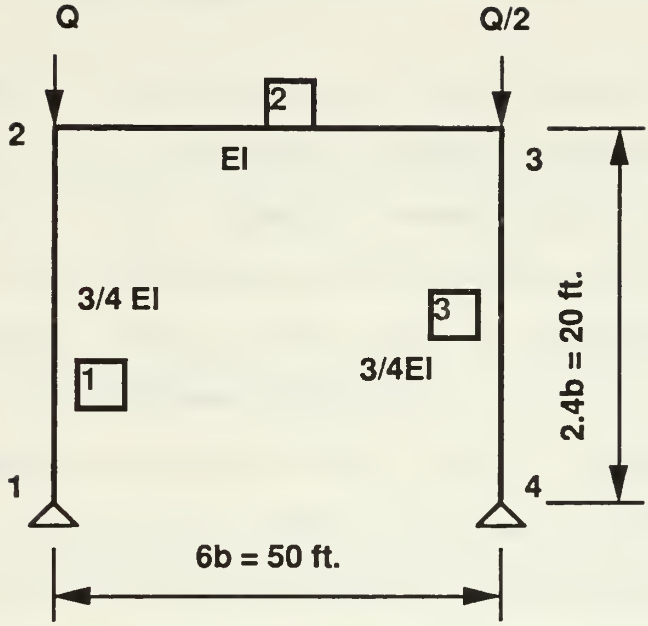
The elastic critical load is determined by implementing the procedures described in Section 4.3. Using an applied load Q = 100 kips, the computer solution gives a load factor LF = 3.94. The computed elastic critical load is therefore 3.94 x 100 = 394 kips. The published solution is $.254 \; EI / b^2 = 392.6$ kips. The error between the two solutions is very small (.36 %).

The computed elastic critical load approaches the

exact solution when the load factor increment is reduced. The buckled shape from the computer solution agrees with that predicted by the exact solution.

For the five examples presented, the computer solutions are in excellent agreement with the exact solutions.

Example 5.2-5



$Q_{cr} = .254\ EI/b^2$ $\qquad I = 533\ in^4$

$A = 19.10\ in^2$ $\qquad Q = 100\ kips$

Fig. 22 Portal Frame

Geometry and Loading Condition

EXAMPLE 5.2-6 ELASTIC SECOND-ORDER ANALYSIS OF A THREE-STORY FRAME

In this example, the capability of the computer program [E S C A P] analyzing larger structures is demonstrated. The elastic second-order moments, deflections, and the amplification factors of a three-story structural steel frame will be determined and compared to the published solution.

The three-story frame in Fig. 23 [15], is subjected to a combined gravity and lateral loading as shown. The computed elastic second-order moments and deflections are shown in TABLES IV and XV. TABLE IV shows that the two solutions are in agreement.

The maximum lateral displacement located at the top of the structure (joint 7), is amplified about a factor of 1.03 . This lateral displacement becomes a very important design consideration in cases of taller buildings. The lateral displacement at joint 7 represents a story drift of .003H, where H is the story height of the building. The acceptable lateral displacements for tall buildings are in the range of 0.002 to 0.004 of the height of the building [16].

Example 5.2-6

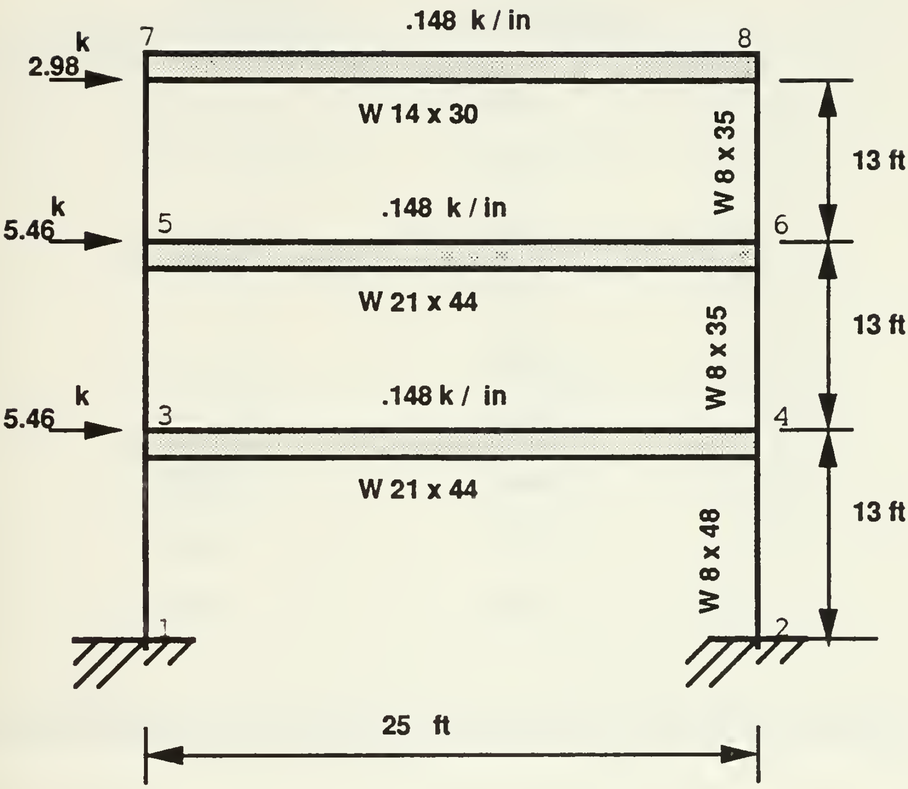


Fig. 23 Three-Story Frame

Geometry and Loading

Table-XIV MAXIMUM SECOND-ORDER MOMENTS - EXAMPLE 5.2-6

MEMBER	Published Solution (kip-in)	Computer Solution (kip-in)	MAF
1	------	442	1.04
2	------	- 26	0.72
3	------	- 333	1.00
4	------	- 1518	1.02
5	------	- 1199	1.01
6	------	- 579	1.01
7	845	846	1.02
8	695	695	1.01
9	579	579	1.01

Table-XV FIRST and SECOND-ORDER LATERAL DISPLACEMENTS EXAMPLE 5.2-6

JOINT	First-Order Solution (in)	2nd-Order Solution (in)	DAF
1	0	0	0
2	0	0	0
3	0.5419	0.5603	1.03
4	0.5406	0.5590	1.03
5	1.1093	1.1470	1.03
6	1.1080	1.1460	1.03
7	1.3755	1.4190	1.03
8	1.3674	1.4100	1.03

6 CONCLUSIONS

In the analysis of structures, neglecting the second-order effects may overestimate the strength and stiffness of a member or frame. The elastic forces generated within a member or frame can be more accurately predicted with the use of an elastic second-order analysis.

The second-order effects (amplified moments and deflections) are of increasing importance as lighter, and more flexible structures are constructed. The use of higher strength materials and less rigid non-structural elements is producing more flexible structures where second-order effects are of greater importance.

The behavior of multistory frames subjected to either gravity loads only or under combined gravity and lateral loading cannot be accurately predicted by a first-order elastic analysis, when the compressive axial load level is significant [17]. The lateral displacements may be rather large and may not be within the acceptable range of lateral displacements for tall buildings. An elastic second-order analysis such as [E S C A P] can be used to provide an accurate

estimate of the lateral displacements and the elastic internal forces generated in the structure. If the lateral displacements become too large, they may become noticeable and disturbing to the building occupants [16].

When second-order effects are included in the analysis, a more rational design of individual members may be possible. The beam-column interaction equation in LRFD is based on the use of elastic second-order moments. LRFD permits direct application of computer generated elastic second-order solutions, such as those generated by the computer program [E S C A P].

Structures with very slender members may fail by elastic instability. For such structures, elastic stability analysis is directly applicable when determining the elastic buckling load.

Typical members or frames in structural steel buildings will fail by inelastic instability, rather than elastic instability. In such cases, elastic frame stability analysis is not directly applicable for predicting the structural strength. However, the LRFD beam-column interaction equations, which predict frame instability, require the use of effective length factors. An elastic frame stability analysis provides a powerful

tool to determine the effective length factors.

The program [E S C A P] was written to provide exact elastic second-order analysis based on the use of stability functions. The program was verified by comparison with published closed-form solutions.

The program [E S C A P] also permits elastic stability analysis of members and plane frames. Instability is determined by increasing the applied load increments until the elastic second-order stiffness matrix becomes singular. Rather than checking the singularity by computing the determinant, singularity was detected by checking for a zero or negative diagonal element using Cholesky's decomposition of the stiffness matrix. This provides a simple technique to incorporate stability analysis into an elastic second-order program.

The accuracy of the program [E S C A P] in computing elastic stability loads was verified by comparison with known closed-form solutions. The solutions are in excellent agreement.

The future of structural design practice will include second-order effects in the computer analysis and design of structures. Structural analysis programs like [E S C A P] will become routine.

APPENDIX

APPENDIX A

USERS MANUAL

ELASTIC SECOND-ORDER COMPUTER ANALYSIS PROGRAM

A1 INTRODUCTION

The Elastic Second-Order Computer Analysis Program [E S C A P] was developed to provide designers a tool in the analysis and design of beam-columns and frames. The program [E S C A P] has the capability of performing first-order linear elastic analysis as well as elastic second-order analysis. The program was also designed to perform elastic stability analysis of frames and beam-columns. The program can be used to perform structural analysis on any two-dimensional structures subjected to in-plane bending.

The program can be loaded in any IBM PC or compatible with a hard-disk or floppy. It is written in single-precision FORTRAN with the IBM Professional Fortran Compiler.

A2 CAPABILITY

The program E S C A P has the following capabilities :

a. Performs first-order linear analysis of plane frames, beams, columns, plane trusses and beam-columns.

b. Performs elastic second-order analysis.

c. Performs elastic stability analysis.

d. Determines the location and magnitude of the maximum elastic second-order moment within each member.

e. Has the capability of analyzing multiple load patterns.

A3 PROGRAM PARAMETERS

The following parameters were used in the computer program [E S C A P] :

PID - Problem Identification. This is the name of the input file with a brief description of the problem. It consists of 12 characters per line with a maximum of three (3) lines provided in the input file.

TITLE - is a heading that describes the input parameters in a read statement.

NJ - number of joints; Maximum of 40. The user has an option to increase the size by increasing the dimension of the joint coordinates X ( ) and Y ( ).

NDELTA - is a code that determines the type of analysis.

0 : 1st order analysis

1 : 2nd order analysis

NCYCLE - it is the maximum number of iterations. The program will terminate if after NCYCLE the second-order solution has not converged, and will print the output of the last computed values of member forces and joint deflections. It normally takes between 4 to 10 iterations before the solution converges.

EPS1 - is the tolerance factor used to test convergence.(recommend .005)

STEP - is the load factor increment used in the elastic stability analysis.(recommend .005)

LB - is the code that determines if buckling load analysis is desired.

0 : NO

1 : YES

X (J) - is the x - coordinate of a joint J, in feet.

Y (K) - is the y - coordinate of a joint K in feet.

MCL - is the joint release code.

0 : joint is free

1 : joint is fixed.

NDOF - is the total number of degrees of freedom.

(equal to 3 * NJ)

NRJ - number of restrained joints.

S (I,J) - is the stiffness matrix of the structure or a member

NRL - number of restrained degrees of freedom (DOF's) that are fixed and have 0 displacements.

MNDOF = NDOF - NRL , is the total number of degrees of freedom that are free to rotate and translate.

NM - is the number of members. Maximum is 80. The user has the option to increase the size by changing the dimension of the other parameters in the main program.

MPC - Matrix Print Code for the stiffness matrix.

0 : suppress output

1 : activate output

GX - is the unit weight of the material; 490 pcf for structural steel.

IHINGE (I) - is a joint code that describes the type of restraints imposed at the joints. It is used to

modify the stiffness of a member that is pinned at one end.

For members without supports :

0 : pinned at start joint and rigid at the end joint

1 : rigid joints at both ends

2 : joints pinned at both ends

For members with support joints (hinged or fixed):

1 : if the beginning joint is a support joint

0 : if the beginning joint is pinned and not a support joint

JFIX (J) , KFIX (K) = are joint codes used to modify the load vector and the force vector for a pin joint.

1 : for a fixed support or rigid joint

0 : for a hinged support or pinned joint

JM (I) - beginning joint of a member

KM (I) - end joint of a member

AM (I) - area of cross-section, sq. in.

ZM (I) - moment of inertia; in^4

EM (I) - modulus of elasticity of the material

Example : Enter 29 (ksi) for steel

SPAN - member length in inches

SM (I) - member length in feet

CX (I) - cosine

CY (I) - sine

WT - total weight of structure in tons.

INODE ( ) - is a degree of freedom (DOF) at the ends of a member :

(1) - DOF at joint JM (I) along the x - axis

(2) - DOF at joint JM (I) along the y - axis

(3) - rotation at joint JM (I)

(4) - DOF at joint KM (I) along the x - axis

(5) - DOF at joint KM (I) along the y - axis

(6) - rotation at joint KM (I)

Q (I,J) - stiffness matrix for the restrained DOF's

NLP - number of loading patterns

LP - load pattern

NLJ - number of loaded joints

WX - joint load in x - direction (kips)

WY - joint load in y - direction (kips)

WM - joint moment (kip-ft)

W ( ) - sum of joint loads in x , y or moments

FEA ( ) - fixed-end actions or forces in members due to externally applied loads.

MEA ( ) - member end actions after the analysis

NMC - number of members with concentrated loads

NCL - number of concentrated loads on a member (limited to 4). The user can increase the size by changing the dimension of LCN in the main program. The location of loading is expressed as a fraction of the span measured from the left support.

FX - concentrated load in x - direction (kips)

FY - concentrated load in y - direction (kips)

MO - moment in z - direction (kip-ft)

NMU - number of members with uniform loads

NUL - number of uniform loads on a member

SUMWY ( ) - sum of transformed uniform distributed loads on the member along the y - axis .

MWC - member weight code.

0 : exclude members weight in the analysis

1 : include member weight in the analysis

R ( ) - reaction vector

G ( ) - modified fixed end actions

RAL ( ) - identifies the restrained DOF

D ( ) - deflection vector

XNEW ( ) - new x-coordinates

YNEW ( ) - new y- coordinates

D1, D2, D3, D4, D5, D6 - deflections at member's local axis

FIX ( ) - calculated fixed-end actions during second-order iteration analysis

ST ( ) - modified stiffness matrix for second-order effects.

LF - load factor use in load buckling analysis. It is equal to 1 at the start of the analysis.

ITER - counts number of times the load factor increment STEP was added to LF before the buckling load is achieved.

QT - restrained stiffness matrix for the second-order analysis

P - is the axial load or force

XKL = sqrt (P / EI)

VA, VB - vertical reactions considering axial load effects

MA, MB - moment reactions considering axial load effects

COMP - stability functions for axial compression

TENS - stability functions for axial tension

FLEXMAX - maximum moment anywhere in the span

YMAX - deflection of FLEXMAX. (Note: this may or may not be the maximum deflection)

XMAX - location of FLEXMAX and YMAX

A4 LIST OF SUBROUTINES

a. E S C A P is the main program. It reads the input file and performs the first-order analysis.

b. NODE is a subroutine that generates the degrees of freedom for each node.

c. STIFF calculates and assembles the global first-order stiffness matrix.

d. MATPRT prints the global stiffness matrix.

e. DECOMP is the CHOLESKY's matrix decomposition program. It decomposes the global stiffness matrix and stores the matrix in S (I,J) or ST (I,J).

f. LOAD modifies the fixed-end actions FEA for a member that is pinned at the beginning node or both.

g. SOLVER solves the equations for unknown deflections and stores them at vector D ( ).

h. MFORCE calculates the first-order member end actions in the local axis and stores them at vector MEA

i. PNDELTA performs the second-order analysis.

j. XDELTA generates the global second-order stiffness matrix and stores the matrix at vector ST .

k. DFORCE solves the members end actions (MEA) to include second-order effects.

l. CALMAX determines the location of the maximum

moment for a uniformly loaded member with end moments. It also calculates the elastic second-order moments and deflections.

A5 INPUT DATA PREPARATION

An editor (such as PE FORTRAN) is used to prepare the input file. The parameters of every read statements are entered in separate lines and are format free. A sample copy of the input file is shown in Appendix B.

PROCEDURES:

A. READ PID - in alphanumeric characters and consists of 12 characters per line with a maximum of 3 lines.

B. READ TITLE - Enter > number of joints,NDELTA,NCYCLE, EPS1,STEP,LB

C. READ NJ, NDELTA, NCYCLE, EPS1

D. READ STEP, LB

E. READ TITLE - Enter> Joint Coordinates

F. READ J, X (J), Y (J)

J : is the joint or node

G. READ TITLE - Enter> Number of restrained joints

H. READ NRJ

I. READ TITLE - Enter> Boundary Conditions (Note:

these are the boundery conditions of the restrained joints in procedure G & H)

J. READ J, ,MCL (1), MCL (2) , MCL (3)

J : is the joint or node

MCL (1) : DOF in X - DIRECTION

MCL (2) : DOF in Y - DIRECTION

MCL (3) : DOF in ROTATION

K. READ TITLE - Enter> Number of members,print code

L. READ NM,MPC

M. READ TITLE - Enter> Member Data

N. READ I,J,K,A,Z,E,GX

O. READ IHINGE (I),JFIX (J), KFIX (K)

I : is the member

J : beginning node

K : ending node

A : area of cross-section (in^2)

Z : moment of inertia (in^4)

E : section modulus (KSI/1000)

GX : unit weight of member (LBS/FT^3)

P. READ TITLE - Enter> Number of Load Patterns

Q. READ LP

R. READ TITLE - Enter> Number of loaded joints

S. READ NLJ ; if NLJ is equal to 0 go to

step V .

T. READ TITLE - Enter> Loaded joints

U. READ J,WX,WY,WM

J : is the joint or node

V. READ TITLE - Enter>Number of members with concentrated loads

W. READ NMC ; if NMC is equal to 0 , go to step AA

X. READ TITLE - Enter> Members concentrated loads

Y. READ M, NCL

Z. READ K, A, FX, FY, MO

K : load orientation

0 : member axis

1 : structure or global axis

A : location of loading (fraction of the span)

AA. READ TITLE - Enter> Number of members with distributed loads

BB. READ NMU ; if NMU is equal to 0 go to step FF

CC. READ TITLE - Enter> Members uniform loads

DD. READ M,NUL

EE. READ K,FX,FY,MO

M : name of member

K : load orientation

0 : local or member axis

1 : global or structure axis

MO : is expressed in kip-ft / ft

FF. READ TITLE - Enter> Member weight code

GG. READ MWC

0 : exclude member weight

1 : include member weight

A6 OUTPUT DATA

The computer output shows the joint coordinates, members data, joint loading, and the member loading in a load pattern. It also prints the joint deflections, member end actions and the reaction forces. The location of the maximum moment, its magnitude and deflection are also shown. The load factor LF is printed and used in determining the actual buckling load, and is obtained by multiplying the load factor LF with the member's load or joint load. A sample output data is shown in Appendix B.

A7 RUNNING THE PROGRAM

The following steps describe how to run the Elastic Second-Order Computer Analysis Program [E S C A P]:

1. Name the input file described in Section A6 above. The input file is named when saving the input data.

2. Go to the Fortran Directory.

3. To run the program :

   ENTER > XFRAME

   RETURN

4. The program will prompt>

   What is the name of the input file?

5. ENTER > Name of input file. Make sure the input file is saved in the Fortran directory.

6. RETURN

7. Execution of the program is completed when the message <u>Execution Terminated : 0</u> shows in the screen.

8. The output file is automatically sent to the file named <u>SOLOUT</u> in the Fortran directory.

APPENDIX B

COMPUTER INPUT AND OUTPUT DATA

For Example 5.2-4

```
        INPUT FILE - SAMPLE.IN
            RIGID FRAME
           EXAMPLE 5.2-4
NUMBER OF JOINTS,NDELTA,NCYCLE,EPS1,STEP,LB
6,1,1000,.005
.005,0
JOINT COORDINATES
1,0,0
2,0,12
3,0,20
4,24,20
5,24,6
6,24,0
NUMBER OF RESTRAINED JOINTS
2
BOUNDERY CONDITIONS
1,1,1,1
6,1,1,1
NM,MPC,MCODE
5,0,1
MEMBER DATA
1,1,2,13.3,250,30,490
1,1,1
2,2,3,13.3,250,30,490
1,1,1
3,3,4,13.3,250,30,490
1,1,1
4,4,5,13.3,250,30,490
1,1,1
5,5,6,13.3,250,30,490
1,1,1
LOAD PATTERNS
1
NUMBER OF LOADED JOINTS
4
LOADED JOINTS
2,12,0,0
3,0,-200,0
4,0,-200,0
5,-6,0,0
NUMBER OF MEMBERS W/ CONCENTRATED LOADS
0
NUMBER OF MEMBERS WITH DISTRIBUTED LOADS
1
LOADED MEMBER
3,1
1,0,-2,0
```

MEMBERS WEIGHT CODE
0

" OUTPUT DATA "

ELASTIC SECOND-ORDER COMPUTER
ANALYSIS OF BEAM-COLUMNS AND FRAMES
PLANE FRAME ANALYSIS BY STIFFNESS METHOD

* * * * * * * * * * * * * * * * *
* * * G E N E M U N O Z * * *
* * * * * * * * * * * * * * * * *

* * * * * * * * * * *
* J O I N T D A T A *
* * * * * * * * * * *

JOINT	COORDINATES X (FEET)	Y (FEET)	RESTRAINT CONDITION X	Y	R
1	0.000	0.000	R		
				R	
					R
2	0.000	12.000			
3	0.000	20.000			
4	24.000	20.000			
5	24.000	6.000			
6	24.000	0.000	R		
				R	
					R

* * * * * * * * * * * *
* M E M B E R D A T A *
* * * * * * * * * * * *

MEMBER	LENGTH (FEET)	CROSS-SECTION PROPERTIES AREA (INS**2)	I(ZZ) (INS**4)	MODULUS OF ELASTICITY (KSI/1000)
1	12.000	13.300	250.00	30.000
2	8.000	13.300	250.00	30.000
3	24.000	13.300	250.00	30.000
4	14.000	13.300	250.00	30.000
5	6.000	13.300	250.00	30.000

TOTAL WEIGHT OF FRAME: 1.448 TONS

* * * * * * * * * * * * * * * * * * * *
* * * F I R S T O R D E R * * * * * *
* * * * A N A L Y S I S * * * * * *
* * * * * * * * * * * * * * * * * * * *

LOADING PATTERN 1 OF 1 LOADING PATTERNS

* * * * * * * * * * * * * * *
* J O I N T L O A D I N G *
* * * * * * * * * * * * * * *

JOINT		LOADING	
	F(X) (KIPS)	F(Y) (KIPS)	COUPLE (KIP-FEET)
2	12.000	0.000	0.000
3	0.000	-200.000	0.000
4	0.000	-200.000	0.000
5	-6.000	0.000	0.000

* * * * * * * * * * * * * * * *
* M E M B E R L O A D I N G *
* * * * * * * * * * * * * * * *

NO MEMBERS WITH CONCENTRATED LOADS IN PATTERN

MEMBER		UNIFORMLY	DISTRIBUTED	LOADING
	ORIENTATION (AXES)	W(X) (KIPS/FT)	W(Y) (KIPS/FT)	M(O) (K-FT/FT)
3	S	0.000	-2.000	0.000

* *
* MEMBER WEIGHTS NOT INCLUDED IN ANALYSIS *
* *

LOADING PATTERN 1 OF 1 LOADING PATTERNS

* * * * * * * * * * * * * * * * * * *
* J O I N T D E F L E C T I O N S *
* * * * * * * * * * * * * * * * * * *

JOINT	D(X) (INCHES)	D(Y) (INCHES)	ROTATION (RADS)
	D E F L E C T I O N S		
1	0.0000E-01	0.0000E-01	0.0000E-01
2	3.0288E-01	-8.0369E-02	-2.5911E-03
3	6.4644E-01	-1.3395E-01	-6.3060E-03
4	6.4031E-01	-1.3553E-01	3.8776E-03
5	1.6280E-01	-4.0658E-02	-4.2348E-03
6	0.0000E-01	0.0000E-01	0.0000E-01

LOADING PATTERN 1 OF 1 LOADING PATTERNS

* * * * * * * * * * * * * * * * * * * *
* M E M B E R E N D A C T I O N S *
* * * * * * * * * * * * * * * * * * * *

MEMBER	END JOINTS	X-FORCE (KIPS)	Y-FORCE (KIPS)	MOMENT (K-IN)
1	1	2.2269E+02	3.5060E+00	3.8739E+02
	2	-2.2269E+02	-3.5060E+00	1.1748E+02
2	2	2.2269E+02	-8.4940E+00	-1.1748E+02
	3	-2.2269E+02	8.4940E+00	-6.9794E+02
3	3	8.4941E+00	2.2688E+01	6.9794E+02
	4	-8.4941E+00	2.5312E+01	-1.0757E+03

4	4	2.2531E+02	8.4941E+00	1.0757E+03
	5	-2.2531E+02	-8.4941E+00	3.5134E+02
5	5	2.2531E+02	2.4941E+00	-3.5134E+02
	6	-2.2531E+02	-2.4941E+00	5.3091E+02

LOADING PATTERN 1 OF 1 LOADING PATTERNS

* * * * * * * * * * * * * * * * *
* R E A C T I V E F O R C E S *
* * * * * * * * * * * * * * * * *

JOINT	R E A C T I O N S R(X) (KIPS)	R(Y) (KIPS)	MOMENT (KIP-INCH)
1	-3.5060E+00	2.2269E+02	3.8739E+02
6	-2.4941E+00	2.2531E+02	5.3091E+02

```
* * * * * * * * * * * * * * * * *
*  S E C O N D   O R D E R  * * *
* *   A N A L Y S I S       * * *
* * * * * * * * * * * * * * * * *
```

LOADING PATTERN 1 OF 1 LOADING PATTERNS

```
* * * * * * * * *   * * * * * * * * *
* J O I N T   D E F L E C T I O N S *
* * * * * * * * * * * * * * * * * * *
```

JOINT	D(X) (INCHES)	D(Y) (INCHES)	ROTATION (RADS)
1	0.0000E-01	0.0000E-01	0.0000E-01
2	4.0855E-01	-8.0162E-02	-3.7614E-03
3	8.5261E-01	-1.3360E-01	-7.0316E-03
4	8.4646E-01	-1.3587E-01	3.5363E-03
5	2.0170E-01	-4.0761E-02	-5.2386E-03
6	0.0000E-01	0.0000E-01	0.0000E-01

CONVERGENCE OCCURED AT ITER = 1

LOADING PATTERN 1 OF 1 LOADING PATTERNS

* * * * * * * * * * * * * * * * * * * *
* M E M B E R E N D A C T I O N S *
* * * * * * * * * * * * * * * * * * * *

MEMBER	END JOINTS	X-FORCE (KIPS)	Y-FORCE (KIPS)	MOMENT (K-IN)
1	1	2.2212E+02	3.4786E+00	4.8161E+02
	2	-2.2212E+02	-3.4786E+00	1.1006E+02
2	2	2.2212E+02	-8.5214E+00	-1.1006E+02
	3	-2.2212E+02	8.5214E+00	-6.0936E+02
3	3	8.5212E+00	2.2115E+01	6.0936E+02
	4	-8.5212E+00	2.5885E+01	-1.1522E+03
4	4	2.2588E+02	8.5213E+00	1.1522E+03
	5	-2.2588E+02	-8.5213E+00	4.2502E+02
5	5	2.2588E+02	2.5213E+00	-4.2502E+02
	6	-2.2588E+02	-2.5213E+00	6.5212E+02

LOADING PATTERN 1 OF 1 LOADING PATTERNS

* * * * * * * * * * * * * * * * *
* R E A C T I V E F O R C E S *
* * * * * * * * * * * * * * * * *

JOINT	R E A C T I O N S R(X) (KIPS)	R(Y) (KIPS)	MOMENT (KIP-INCH)
1	-3.4786E+00		
		2.2212E+02	
			4.8161E+02
6	-2.5213E+00		
		2.2588E+02	
			6.5211E+02

```
LOADING PATTERN  1 OF  1 LOADING PATTERNS

        * * * * * * * * * * * * * * * * *
        * M A X I M U M   M O M E N T S *
        * * * * * * * * * * * * * * * * *

MEMBER             M O M E N T S
           XMAX          YMAX          MOMENT
          (INCH)        (INCH)       (KIP-INCH)

 1      0.0000E-01
                     0.0000E-01
                                  4.8161E+02
 2       9.5948E+01
                    -1.3360E-01
                                 -6.0936E+02
 3      2.8799E+02
                   -1.3587E-01
                                 -1.1522E+03
 4      0.0000E-01
                   -1.3587E-01
                                  1.1522E+03
 5      7.1960E+01
                    0.0000E-01
                                  6.5212E+02

ANALYSIS COMPLETED
```

REFERENCES

[1] Load and Resistance Factor Design Specification for Structural Steel, First Edition, American Institute Of Steel Construction, Inc., 400 North Michigan Avenue, Chicago, Illinois 60611, 1986

[2] Chen, Wai-Fah and Atsuta, Toshio, Theory of Beam-Columns, Volume 1 : In-Plane Behavior and Design, McGraw-Hill Book Company, Inc., New York, 1976, p. 1-170

[3] Chajes, Alexander, Principles of Structural Stability, Prentice-Hall, Inc., Englewood Cliffs New Jersey, 1974, p.1-74, 133-143, 145-194

[4] Stephen P. Timoshenko and James M. Gere, Theory of Elastic Stability, McGraw-Hill Book Company, Inc., New York, 1961, p. 1-45

[5] Chen, W.F, and Lui, E.M, Structural Stability, Theory and Implementation, Elsevier Science Publishing Co., Inc., 52 Vanderbilt Avenue, New York, NY 10017, 1987, p.147-192

[6] Salmon, Charles G. and Johnson, John E, Steel Structures, Design and Behavior Emphasizing Load and Resistance Factor Design, Third Edition Harper & Row, Publishers, Inc., 10 East 53rd Street,New York, NY 10022, 1990, p.829-832

[7] Iyengar, N.G.R, Structural Stability of Columns and Plates, Ellis Horwood Limited, Halsted Press John Wiley & Sons, New York, 1988, p.1-65, 110-137

[8] Crawley, Stanley W., Dillon, Robert M., and Carter, Winfred O, Steel Buildings , Analysis and Design, Third Edition, John Wiley & Sons, New York, 1984 p.590-598, 615-637

[9] Ghali, A. and Neville, A.M, Structural Analysis - A Unified Classical and Matrix Approach, Chapman and Hall Ltd.,11 New Fetter Lane, London ECP4P 4EE, 1979, p. 356- 397

[10] Harrison, H.B, Computer Methods in Structural Analysis, Prentice-Hall, Inc.,Englewood Cliffs, New Jersey, 1973, p. 235-276

[11] West, Harry, Analysis of Structures, An Integration of Classical and Modern Methods, John Wiley & Sons, Inc., New York, 1980, p.64

[12] Weaver, William Jr., and Gere, James M., Matrix Analysis of Framed Structures, Third Edition, Van Nostrand Reinhold, New York, 1990, p.428-433

[13] White, Donald W., and Hajjar, Jerome F, Proceedings: "Application of Second-Order Elastic Analysis in LRFD", AISC National Steel Construction Conference, Kansas City, 1990, p. 11- 3, 11-22

[14] Beaufait, Fred W., Rowan, William Jr. H, Hoadley, Peter G., and Hackett, Robert M, Computer Methods of Structural Analysis, Prentice-Hall, Inc., Englewood, New Jersey, 1970, p.36-41, 342-386

[15] Lui, Eric M., A Practical P-Delta Analysis Method for Type FR and PR Frames, Engineering Journal, American Institute of Steel Construction, Third Quarter, 1988, p. 85 - 98.

[16] Fleming, John F., Computer Analysis of Structural Structural Systems, McGraw-Hill Book Company, New York, 1989, pp. 528-541

[17] Wood, Brian R., Beaulieu, Dennis, and Adams, Peter F., Column Design By P-DELTA Method, Journal of the Structural Division, February 1976, pp. 411-427

VITA

Hermogenes Reyes Munoz was born in San Fernando, Pampanga, Republic of the Philippines, on April 28, 1953, the son of Alberto Munoz and Encarnacion Reyes Munoz. After completing his work at Jose Abad Santos High School, San Fernando, Pampanga, Philippines, he entered the University of Assumption, San Fernando, Pampanga, Philippines. On January 27, 1975, he enlisted in the United States Navy, and has been serving since then. He received the degree of Bachelor of Science from the University of Mississippi in December, 1982. In January, 1990, he entered The Graduate School of The University of Texas.

Permanent Address: 266 Clydsdale Drive
Vallejo, California 94592

This thesis was typed by the author.