PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

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CONTENTS.

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Part I] VOL. XLI. [Pp. 1-96

CONTENTS.

[*Continued on page* iv *of Cover*.

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PROCEEDINGS

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VOL. XLI. 1920–21.

I.—Size, A Neglected Factor in Stelar Morphology. Opening Address by Professor F. O. Bower, M.A., D.Sc., LL.D., F.R.S., F.L.S., President.

(Read October 25, 1920. MS. received November 1, 1920.)

THE principle of similar structures was first enunciated by Galileo. Applying it mechanically, it appears that the strength of a structure varies as the square of the linear dimensions, and the weight as the cube. This principle and its mechanical applications have been widely illustrated by reference to the bodies of animals, and many of their peculiarities are necessary consequences of its effect in the course of their evolution. For instance, the columnar legs of the elephant or of the moa are held to be the inevitable sequel to the large size and consequent weight of those animals, while the thin arched legs of insects are only possible where the body itself is small and light. Such questions have been adequately dealt with in D'Arcy Thompson's book on *Growth and Form*. Botanists have, however, been slower in applying the principle to the study of plants. It is true that the question of the practicable limit of size of trees has long ago been discussed from this point of view, and it is recognised that a change either of material or of method of construction would be necessary for effective growth beyond the limits already reached by some of the largest of them. In fact, that about 300 feet is the extreme height that can be self-supporting with the usual type of construction of the trunk. But the principle is also applicable to other points of construction, such as the size and constitution of individual cells, and even to the forms of chloroplast: as well as to various problems of distribution of tissue-masses, and their relation to the physiological functions which they perform. So far, however, the principle of similar structures has not been applied in botanical science as freely as its importance would justify. An attempt will be made in this address to show how the principle has affected the internal morphology of the Vascular System of Plants. The most marked illustrations will be taken from the Ferns, a class of plants in which the vascular system has always attracted attention by reason of the complexity and peculiarity of its details. But further evidence will be brought to show that it has its application in other types of plants as well.

The stems and roots of most plants are approximately cylindrical. The same is the case as a rule for their conducting tracts also. The cylinder is one of those solid forms in which the proportion of external surface to bulk is exceptionally low. Any deviation from the cylindrical form, either by external projections or by involutions, necessarily leads to increase in the proportion of surface to bulk. The surface varies only as the square of the linear dimensions, but the bulk as the cube. It follows therefore that in carrying out any of those physiological functions of a living organism which depend upon surface, as do all those of the acquisition and interchange of material, the actual size of the part which exercises that function is a matter of the greatest moment. It may be assumed that, if other things be equal, such as the structure and quality of the tissues that form the surfaces in question, the rate of interchange by diffusion of soluble gases or salts through a tissue-surface will be directly proportional to the area of the diffusing surface. If that be so, then for each such function there will be a limit of size beyond which its exercise with sufficient rapidity will become impossible if the form be maintained, or if the quality of the surface-tissue through which the transit occurs remains the same. This suggests that the larger the plant is the more dependent it will be upon its form and detailed structure, not only for its stability, but also for the performance of its functions of absorption and transit of liquids and gases. This will apply not only to the external surface, but also to those internal surfaces which limit one tissue-tract from another. Not only the outer surfaces, but also the limiting surfaces of the internal tissue-tracts should then be carefully examined, both as to area and as to their detailed structure.

In point of fact stems and roots are only approximately cylindrical. Fluctuations of size either by increase or by decrease are common. But the most general and the most important of them all is that primary increase of dimensions which is found in the stems of most plants as they pass from the juvenile to the adult state. For the moment only the *primary* increase is meant: all secondary or cambial increase may be ruled out of this discussion, however interesting its problems may be. Here the intention is to concentrate upon those problems which any land-living organism having no cambial increase must face as it passes from the juvenile to the adult state. It is the facts of ontogenetic development in plants without secondary growth which provide the most cogent evidence of the effect of increase in size upon internal structure. Good illustrations are provided by the Filicales. Here the first leaves are small: the later leaves are successively larger. The stem which bears them is relatively small at its base, but in proportion as larger leaves are formed the supporting stem becomes progressively larger, till the adult size is reached. The same is the case for the stele that lies within (fig. 1). It is small at the base, and approximately cylindrical; but passing upwards its transverse section gradually increases, till finally in most ferns it takes one of those complicated forms that are so characteristic of the class (fig. 2). The form of the stem at its base, and of the stele within, is then not a cylinder but a gradually enlarging cone. Consequently problems depending on the proportion of surface to bulk, whether of the stem as a whole or of the stele which it contains, will be progressively changing in each

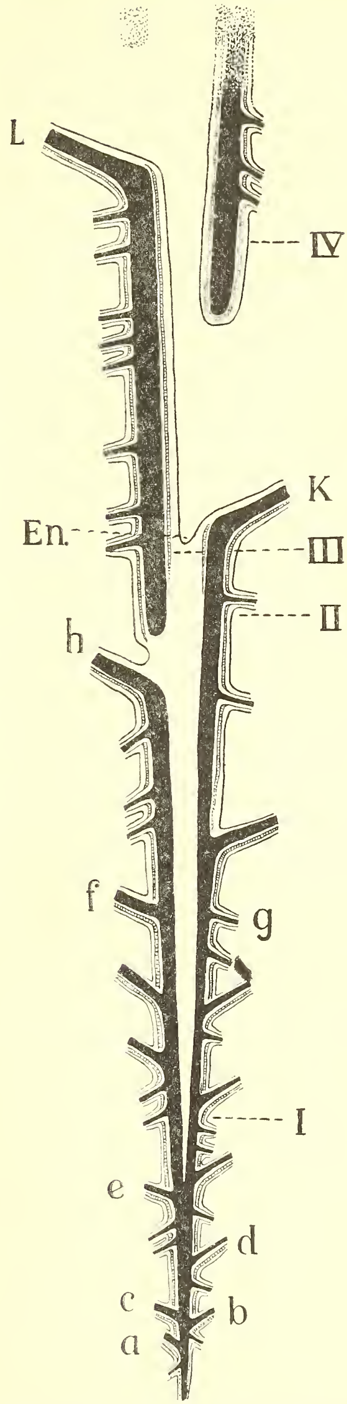


Fig. 1.—Plan of stelar construction of a juvenile plant of *Gleichenia pectinata*, after Dr J. M. Thompson, showing in median section the way in which the stele enlarges conically upwards, and widens into a solenostele, with leaf-gaps. *a–L*=the insertions of the successive leaves. *En*=endodermis.

successive transverse zone from the juvenile to the adult region. It may be anticipated that at some point of size a critical proportion of surface to bulk will be reached, where the interchanges between stele and cortex will demand some alteration of structure if they are to be satisfactorily carried out.

Conversely, however, an axis or root may diminish progressively in bulk from the base upwards. In a fern that has been starved by unfavourable culture the size of its stem is less than in the region developed under normal conditions, and the internal tissues follow suit, with simplification of structure. Certain strut roots of palms develop thinner after entering the ground than above it. These should then show progressively converse structural changes, if the reasoning be correct. For them the problem, so far as it depends upon size, would be progressively simplified, and the evidence of this might be expected to appear in their structure. It will be shown later that this expectation is justified by the facts.

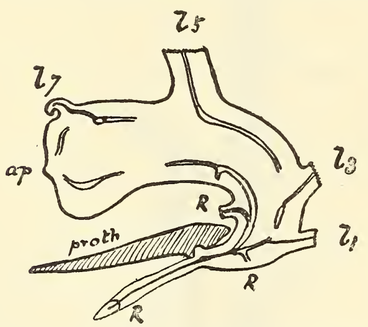


FIG. 2.—Median longitudinal section through the prothallus and embryo of *Polypodium vulgare*. × 6. Leaves l_1, l_3, etc. R=roots. ap=apex of stem. The drawing shows the widely expanding conical stem—small at the base, where it is protostelic; larger above, where it is dictyostelic.

An illustration already familiar to botanists of the way in which form may be modified so as to secure an increase of surface-area, and so to facilitate transit through that surface, is seen in the case of submerged leaves. In *Hottonia*, *Potamogeton pectinatus*, *Ranunculus aquatilis*, and *Cabomba*, etc., the submerged leaves are cut into narrow segments, differing thus in marked degree from the undivided blades of the aerial leaves of allied plants. In the cases of *Cabomba* and *R. aquatilis* (*heterophyllus*) the difference appears even in the leaves of the same plant. On the other hand, in *Ouvirandra* the submerged leaves are perforated by many oblong holes. The biological reason for these peculiarities is to be sought in the fact that, by being thus subdivided or perforated, they expose a relatively large absorbent surface to the water, out of which they abstract the materials for their food. In particular, oxygen and carbon-dioxide are exchanged with the water through the epidermis, which has here no stomata, so that their external surface is the only available surface for the purpose. In this respect their structural difference from the entire and slightly lobed aerial leaves which bear stomata makes an increase of absorptive surface all the more necessary.

This example of a readily intelligible case, involving elaboration of external form and increase of the surface thereby, is held as an important parallel to certain cases of surface-adjustment of internal tissues which are to be described later.

In the young stems of Vascular Plants generally, and in those of the Ferns in particular, the conducting tracts are strictly delimited from the surrounding tissues by endodermis. The same is the case also for roots.

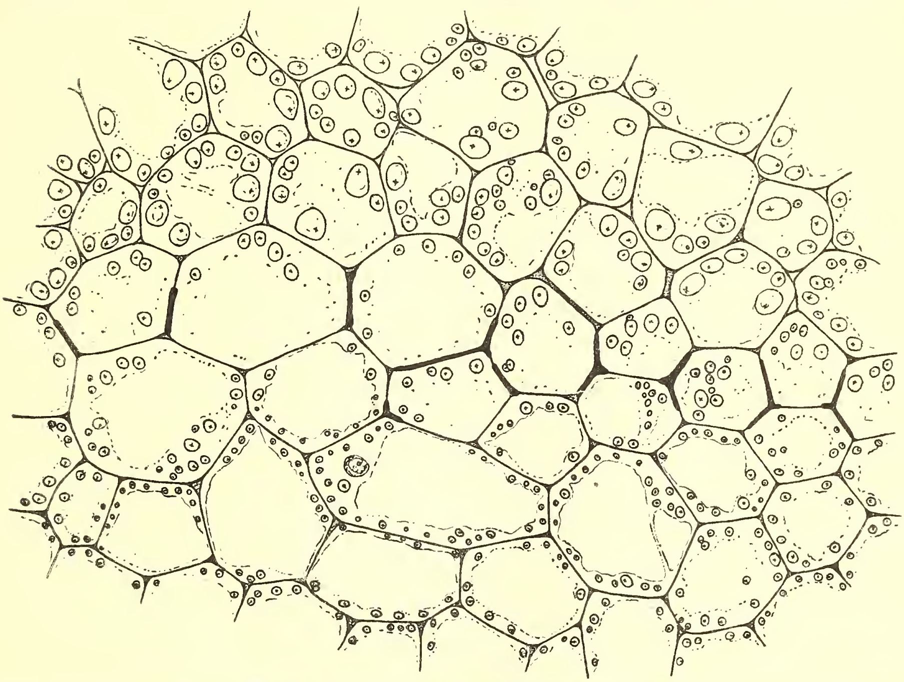


FIG. 3.—*Helminthostachys zeylanica:* part of transverse section of root (Gwynne-Vaughan collection, slide 589 ; × 66). The endodermis, recognised by the characteristic structure of its radial walls, marks a boundary between the outside cortex, with large starch-grains (here above), and the inner conjunctive parenchyma (here below), with small grains. Drawn by Dr J. M. Thompson.

This endodermis forms not only a morphological, but also a physiological boundary that is without any gap or imperfection. Its physiological importance consists in the fact that the structure of the endodermis places the contents of the conducting tract under strictly protoplasmic control. All the lateral walls of its cells are so specialised in substance that, instead of being permeable like ordinary cellulose walls, they are impervious to fluids. Thus all possible leakage is stopped, and the only channel of transit for substances into or out of the stele is under the control of the living protoplasts of the endodermal cells. This control applies not only to salts, sugars, and other similar soluble substances, but also to gases. Since in young and primitive plants the mantle is unbroken by intercellular spaces, even the respiration of the living cells within the barrier can only be conducted by interchange of gases passed in solution through the cells of the endodermis. These structural facts, which can be verified by sections of the stem of any young fern-plant, or of any root, form the foundation of a theory which may account for some of the most extraordinary vascular developments seen in plants.

Evidence of the effectiveness of the endodermis as a physiological

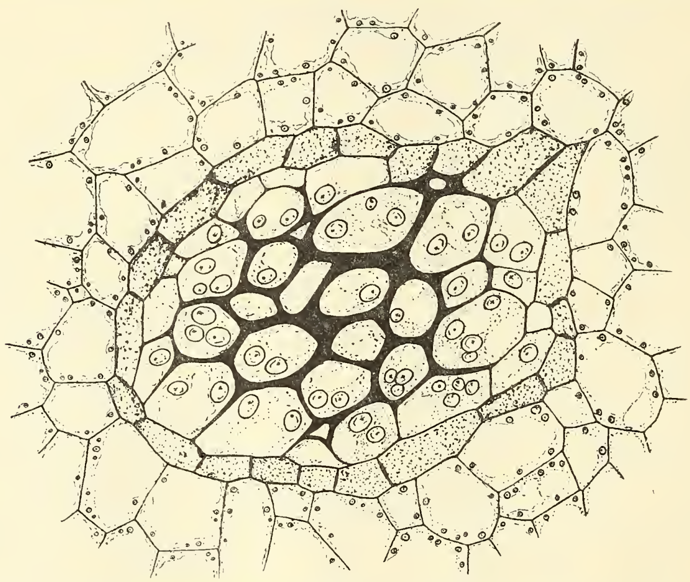


Fig. 4.—Part of transverse section of stem of *Acrostichum aureum*, showing the centre of the solenostele, with endodermis surrounding a small central pith with large starch-grains; outside the endodermis is conjunctive parenchyma with small starch-grains. Drawn by Dr J. M. Thompson. (×66.)

barrier is afforded by comparison of the cell-contents outside and inside it; sometimes starch is absent from the stelar tissues but present in the surrounding cortex. Marked cases may commonly be found of difference in size of the starch-grains on either side of the barrier. This is seen in the storage-rhizomes of *Pteridium*, and fig. 3 shows it in the case of the rhizome of *Helminthostachys*. In the young stem of *Acrostichum aureum* the difference is still more striking (fig. 4). Such facts indicate that the endodermis controls the passage of soluble sugar. It has been shown by de Lavison (*Rév. gén. de Bot.*, 1910, p. 225), and by Priestley (*New Phyt.*, 1920, p. 192), that it is an effective barrier to the passage of such soluble substances as are incapable of penetrating the protoplasm, but whose passage through the walls can be followed by their colour, or by staining reactions. Such evidence points to the endodermis as a selective screen, or even an effective barrier to physiological transit between the outer tissues and the conducting system. Hence the constant diminution of the proportion of surface to bulk as the stele increases in size becomes a matter of the utmost importance. A conical increase in size of the stele is illustrated in all ferns, as well as in other plants. It starts from the minute stele of the sporeling, and expands as a support for the successively larger leaves of the established plant (figs. 1, 2). Often the increase is rapid, especially in ferns with short internodes. For each plant which thus enlarges its stele in conical form, a limit must ultimately be reached where the facility for interchange through the endodermis will not suffice for the needs of the tissues within. This facility for interchange will then become a "limiting factor." Either some means of increasing the surface area of the stele, and so of increasing the means of transit, must be supplied, or the conical enlargement of the stele must be checked, and the later regions of the stele will be cylindrical. The increase cannot be continued indefinitely in the form of a cone. But on the other hand, any deviation from the simple conical form, by involution of surface or by excrescence, will give an increase of the proportion of surface to bulk, and thus tend to overcome the difficulty. We may now proceed to see how these demands following on increase in size have been met in the stems of ferns.

It is generally admitted that the *protostele* is the most primitive stelar type. It is present in the juvenile stage of all ferns, and it is permanently retained in the adult stems of some of them. It consists of a central core of xylem often composed only of tracheides, as in *Botryopteris cylindrica* (fig. 5). This is surrounded by a band of phloem, followed by the pericycle, and finally the stele is delimited externally by the continuous sheath of the endodermis. No intercellular spaces have been found in the protostele, and the endodermis serves as a complete gas-barrier limiting the ventilating system of the cortex internally. Thus constructed, the stele receives the trace of each successive leaf, and it is important to note that its entry is effected without any break of continuity of the endodermal envelope, which thus forms a gas-tight barrier surrounding the whole vascular system. The protostelic structure is retained in the adult stems of *Botryopteris*, *Gleichenia*, *Lygodium*, and *Cheiropleuria*. It is also characteristic of the stems of the Hymenophyllaceæ, which with others are relatively primitive types, having stems of moderate dimensions.

In the ferns named the stele is often minute, and never actually large. In *Botryopteris cylindrica* it is about ·5 mm.; in *Lygodium* 1 mm. to 2 mm.; in *Cheiropleuria* about 1 mm.; in *Trichomanes scandens*, one of the larger

Hymenophyllaceæ, it is ·5 mm. in diameter. In all of them its form is conical at first, but after reaching a certain size it retains that size through life, as a cylinder traversing the cylindrical rhizome. The limiting factors have come into play, one of which is the proportion of surface of the stele to its bulk. When the stele attains larger dimensions, as it did in certain fossils while still retaining its protostelic state, is is seen to have undergone a modification of form. For instance, in *Ankyropteris Grayi* (fig. 6, ii), which is 2–3 mm. in diameter, it is corrugated, the insertions of the leaf-

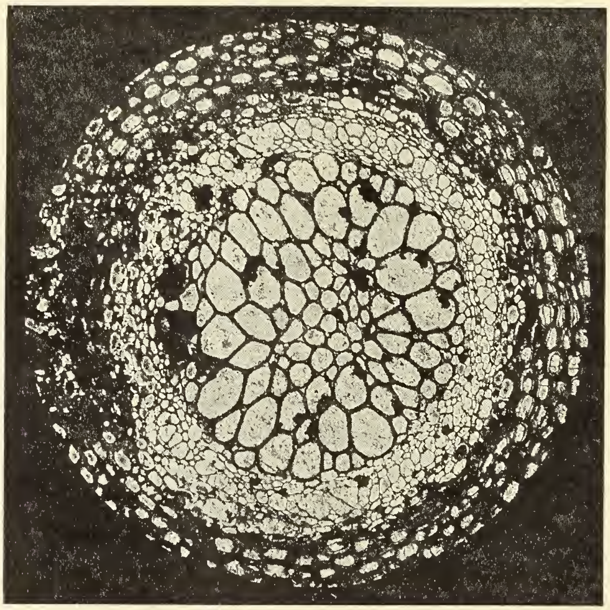


Fig. 5.—Transverse section of a stem of *Botryopteris cylindrica*, showing a protostele with a solid central core of xylem, and peripheral phloem. The endodermis is not clearly shown in this fossil Fern.

traces projecting, and the surfaces between being hollowed. Moreover, the curvatures of the hollows are deeper in the larger than in the smaller specimens (fig. 6, iii). A still more extreme case of this is seen in the stele of *Asterochlœna laxa*, which may be as much as 15·5 mm. in diameter (fig. 6, iv). Here the stele is thrown into deep involutions of the surface. It is obvious that this will give a very greatly increased proportion of surface to bulk. It seems natural to conclude in such cases that the more elaborate form of the stele has made the larger size possible, by overcoming the limiting factor. But notwithstanding the complicated outline, and the well-known differentiation of the xylem of these fossils, their steles are still of the nature of protosteles: their non-medullated structure is maintained.

In other primitive ferns, as a larger size of the stele is attained in the growing plant a change of internal structure appears, leading to *medullation.* Since the leaf-traces are inserted peripherally, it is in the outer xylem that the water-transit will be most active. As the stele enlarges, the water in the central region will tend to stagnate, and thin-walled cells

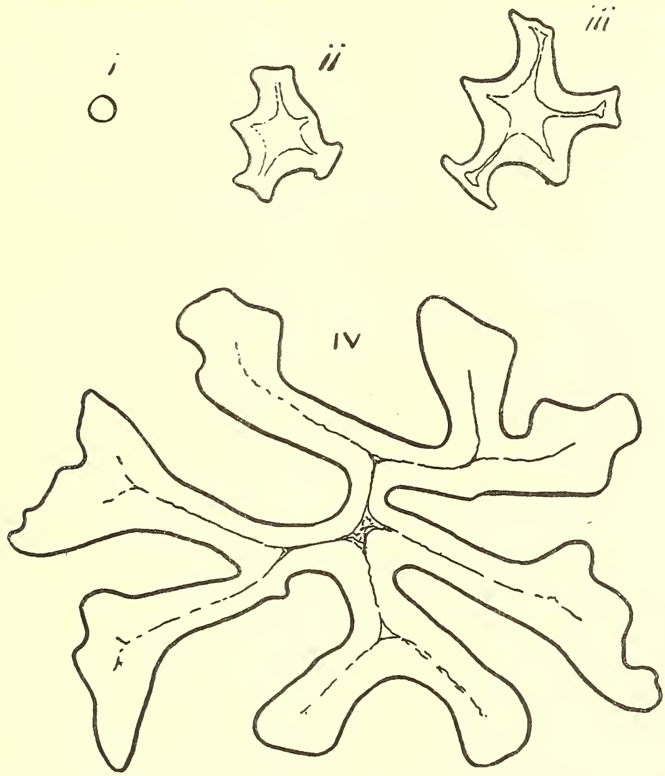


Fig. 6.—Outlines of xylem of steles, all drawn to the same scale (× 5), to show approximately relative size.

i. *Botryopteris cylindrica*, diameter ·05 mm.
ii. *Ankyropteris Grayi*, diameter 2·0 mm.
iii. *Ankyropteris Grayi*, diameter 2·5 mm.
iv. *Asterochlœna laxa*, diameter 12·0 mm.

The elaborateness of outline increases with the size.

will serve for its storage as well as thick-walled tracheides would do. This is probably the *rationale* of the conditions of "mixed pith," and of the formation of a parenchymatous medulla. Medullation in one form or another is common to the great majority of ferns. Its intra-stelar origin has been followed most convincingly for upright stems in the stratigraphical sequence of the fossil Osmundaceæ, described in our *Transactions* by Kidston and Gwynne-Vaughan. It has also been demonstrated in *Gleichenia pectinata* and other ferns by Dr Thompson (*Trans. R.S.E.*, vol. lii, pt. iv, p. 715). The parenchymatous pith once established may serve not only as a place of storage for water, but also for plastic substances. In the living Osmundaceæ it contains large quantities of starch. As usual in storage tissues, intercellular spaces are present, though these are absent from the simplest steles. Here they form an internal ventilating system, quite separate from that of the cortex, excepting that in some species there is communication at the point of dichotomy of the stem. A careful examination of *Osmunda regalis* and of *Todea barbara* shows no connection at the xylic gaps between the outer and inner ventilating systems. The endodermis surrounds the stele completely as well as each leaf-trace. Consequently the inner ventilating system is here as isolated and self-contained as is the intercellular system of a submerged plant. Where the

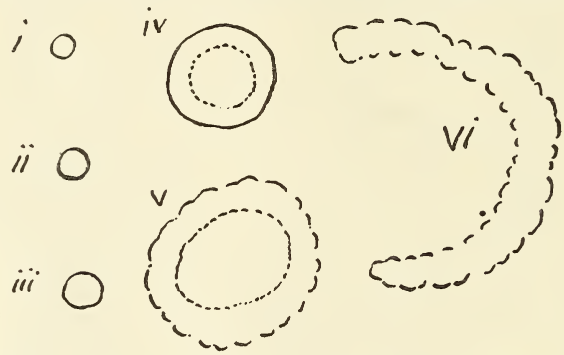


Fig. 7.—Traces of the actual sizes of steles of living and fossil Osmundaceæ, all to the same scale, *i.e.* approx. nat. size.

i. *Todea barbara* (3 mm.).
ii. *Osmunda cinnamomea* (4 mm.).
iii. *Osmunda regalis* (5 mm.).
iv. *Thamnopteris schlechtendalii* (13 mm.).
v. *Osmundites skidegatensis* (25 mm.).
vi. *Osmundites Carnieri* (35 mm.).

size is small this condition is possible. It is so in *T. barbara* with a stele 3 mm. in diameter, or in *O. regalis* (5 mm.). In such instances the proportion of surface to bulk of the small stele is relatively high (fig. 7, i, ii, iii); but the case is different for the large steles of *Osmundites skidegatensis* (25 mm. in diam.), or of *O. Carnieri* (33 mm. in diam.), and in them the problem is solved by breaking down the barrier. In the former fossil each leaf-trace at its departure interrupts the whole vascular ring, and the pith is continuous with the cortex through each leaf-gap. Moreover, no layer resembling an endodermis can be distinguished, so that it is practically impossible to set a definite limit to the stele. In *O. Carnieri*, though a line of delimitation appears which is believed to be endodermis, it is discontinuous at irregular intervals, and the ventilated cortex is directly related to the greatly distended pith (fig. 8). Thus in both of these large fossil stems a concomitant, and it probably was even a necessary condition of their large size, was this interruption of the endodermal barrier and the completion of a common ventilating system for cortex and pith. The "limiting factor" was met by interruption. This is in point of fact a more effective device than that seen in the enlarged protosteles, which maintained their endodermis but enlarged its area.

Lang has shown in the living Ophioglossaceæ a method of resolving the difficulty similar in effect to that of the Osmundaceæ. In *Botrychium* and *Helminthostachys* the young plant has a complete endodermal barrier, as in other ferns, shutting off the vascular system from the surrounding cortex. But as the plant advances, the conical stele enlarges, and a pith is

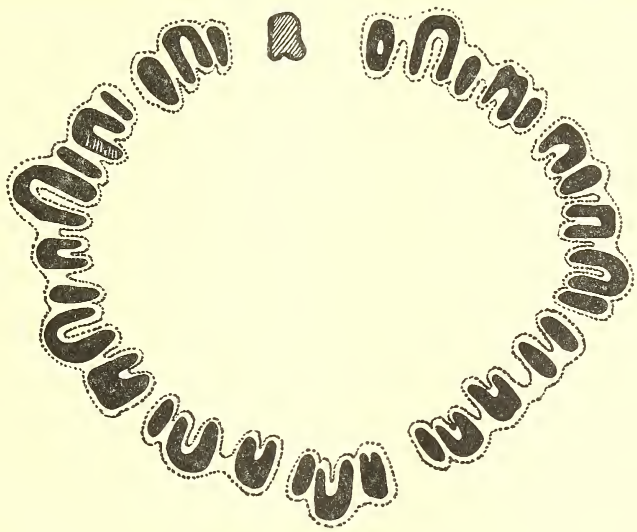


Fig. 8.—*Osmundites Carnieri*, Schuster. Arrangement of meristeles. The endodermis is shown by dotted lines. After Kidston and Gwynne-Vaughan.

formed which serves for storage, and contains starch. Intercellular spaces appear in it, but the internal ventilation-system is at first wholly shut off from the cortical, and it remains so till the plant is well advanced. As the stem enlarges free communication is established by foliar gaps, which naturally open outwards to the cortex, but here they are always open inwards also to the pith. This has the disadvantage of laying open the conducting tract, and destroying the completeness of endodermal control; but it resolves the difficulty of communication between the outer and the inner tissues, which becomes more acute as the stem enlarges. The advantage gained is probably greater than the disadvantage that follows.

What is thus seen in less complete form in *Botrychium* and *Helminthostachys* is carried much further by the genus *Ophioglossum*. Here the endodermis is discarded early. The stele dilates with a distended pith, which communicates directly with the cortex through very wide leaf-gaps. The extreme condition is seen in the tuberous stocks of *O. palmatum*, which may be as much as 2 cm. in diameter. This seemingly reckless discarding of the protective endodermis goes along with a leathery foliage. Such plants have only a sluggish circulation of fluids, and the protection of their conducting tracts seems less vital for them than the establishment of free gaseous and other interchange between the tissues of their sappy stocks.

The parallel condition seen in the Marattiaceæ indicates the truth of this. The massive stocks of these ferns are also soft and sappy, and their leaves are as a rule leathery and thick. They dispense early in their ontogeny with the endodermis, and the stele at once breaks up into small parts which are widely scattered through the transverse section. The stem grows to a large size, with no limit between the distended pith and the cortex. Consequently after the brief juvenile stage is past no question of proportion of surface to bulk arises. But, on the other hand, by discarding the endodermis the conducting tracts have lost that protoplasmic control which the endodermis gives. This state may serve for semi-xerophytic plants, such as the Ophioglossaceæ and Marattiaceæ, with sappy stocks and leathery leaves, and sluggish fluid-transit. But it would not serve for plants where fluid-transit requires to be rapid, and in particular for those with delicate leaf-structure.

The Leptosporangiatæ, which are mostly delicate hygrophytes, comprise the vast majority of living species of ferns. They have taken a quite different course of structural development, in which the endodermal barrier is strictly maintained in its complete form, while intercellular spaces are as a rule absent from their vascular tracts. They show in their peculiar vascular structure to what shifts a plant is put as it increases in size by primary and not by cambial activity, maintaining meanwhile its vascular system under complete protoplasmic control. All of them start from the protostelic state. It appears from comparison along phyletic lines parallel but yet distinct, that a transition has taken place from the protostele to a disintegrated stelar structure. The successive steps of this may be seen with varying degrees of clearness of detail in the successive stages of the individual life. These steps appear as the stele enlarges. According to the reasoning already brought forward, it is on enlargement that the problem of proportion of surface to bulk of the stele becomes insistent. The modifications of form of the stele seen in the advanced Leptosporangiate Ferns may be held as the means of its solution. The critical point in the individual development of the solenostelic type is where the transition is effected from the protostele. Medullation may precede this, as it does in *Gleichenia pectinata.* In others no previous medullation may be seen. In either case a condition of physiological success appears in the Leptosporangiate Ferns to be the continuity of the sheath, so as to allow no leakage. In the Eusporangiates, as already explained, this appears to be less important.

The chief steps in the advancing complexity of the vascular system of

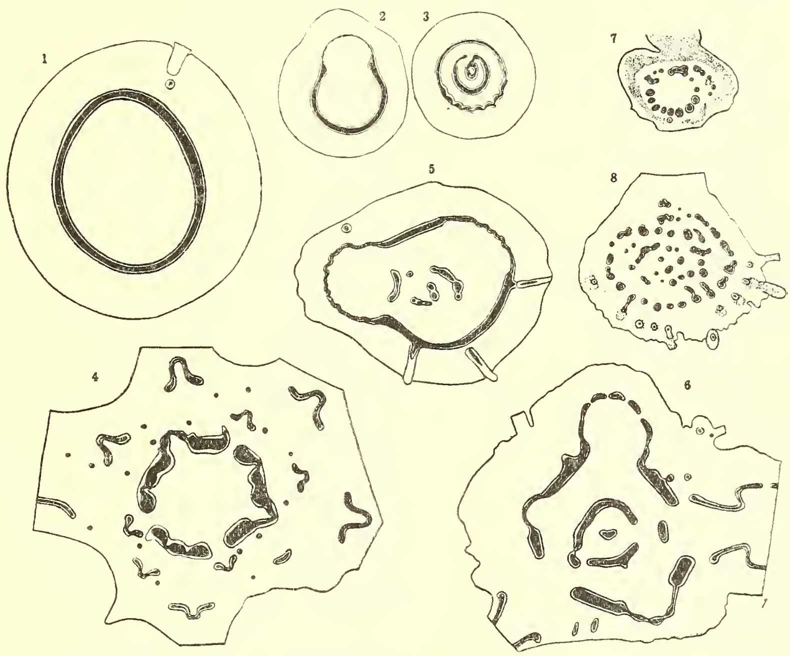


FIG. 9.—Series of solenostelic and dictyostelic stems of Ferns, all drawn to same scale. (× 2.)

1, *Metaxya;* 2, *Dipteris conjugata;* 3, *Matonia pectinata;* 4, *Plagiogyria pycnophylla;* 5, *Thyrsopteris elegans;* 6, *Saccoloma elegans;* 7, *Platycerium alcicorne;* 8, *Platycerium æthiopicum.*

These drawings show that the disintegration of the stele does not depend on absolute size alone.

the Leptosporangiate Ferns are known as *solenostely, polycycly, perforation,* and *dictyostely.* Such advances may be traced either in the ontogeny, or in the race by comparison of distinct species or genera. They all result in increase of surface in proportion to bulk of the stelar tissue, and in all of them the endodermal barriers are strictly maintained, while intercellular spaces are consistently absent from the vascular tissues. They all follow on a very considerable increase in size of the system as a whole and are believed to be causally related to it. In *solenostely* the solid protostele is replaced by a hollow tube lined within and without by continuous endodermis (fig. 9: 1, 2). At each leaf-insertion a foliar gap leads from the cortex through the tube to the inner pith, giving ready communication by intercellular spaces, and by continuous tracts of living cells between the cortex and the pith. But the lips of the gap are entirely sheathed by endodermis, so that the barrier between vascular and non-vascular tissue is maintained intact: and it is continued without any leak or imperfection to the base of the plant. The passage from the protostele to the solenostele in the individual plant is marked by a great increase of the transverse section of the stele (fig. 11). Clearly the proportion of

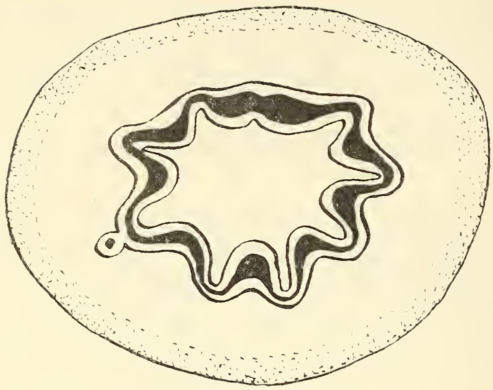


Fig. 10.—*Histiopteris incisa* (Thunbg.) J. Sm. Transverse section of internode of rhizome (×10) showing corrugation of solenostele. (Gwynne-Vaughan collection, slide 1163, by Tansley.)

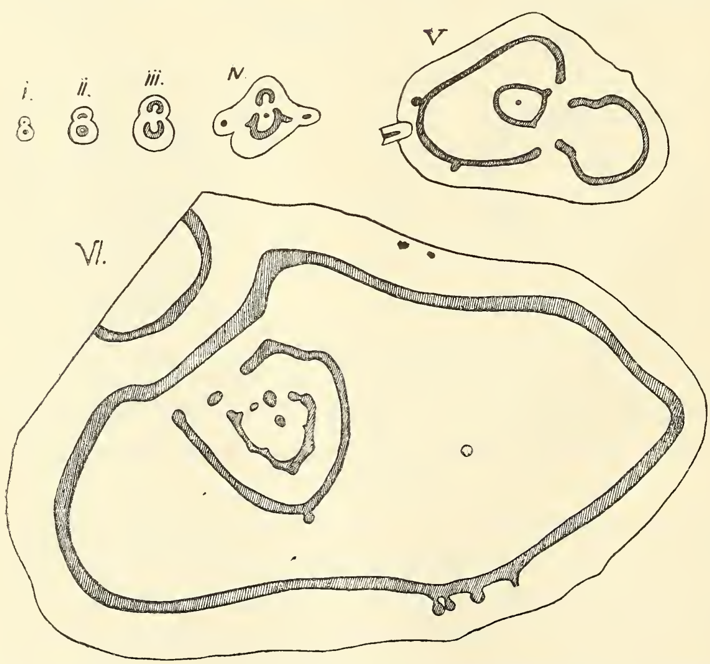


Fig. 11.—Series of transverse sections of the stem of *Pæsia podophylla*, all drawn to the same scale, showing the great increase of stelar complexity as the conical stem expands upwards. (×4.)

surface to bulk of the stele is greatly increased by the tubular form. But sometimes a still further increase is secured by corrugation of the tube, as in *Histiopteris incisa* (fig. 10). A still more remarkable way in which this end is attained is by the further advance to *polycycly*, which is seen in beautiful examples in *Matonia* (fig. 9 : 3), or *Pæsia podophylla* (fig. 11). Three or even four concentric cycles of vascular tissue have been observed, while open communication is maintained at or near to the nodes between the outer and inner cycles, as well as between the tissues that embed them. The proportion of surface to bulk is still further advanced by such means as these, which are exemplified in numerous cases of ferns not otherwise resembling one another. From this it may be concluded that a general cause has been at work, which has affected the development in distinct phyletic lines.

The same ends as are gained by solenostely are still further promoted by the appearance of *perforations* in the vascular tube. These are often very numerous, and are specially found in ferns of advanced type, such as *Davallia, Platycerium* (fig. 9 : 7, 8), *Polypodium*, or *Stenochlœna* (fig. 12). Each perforation is entirely lined by endodermis, which still shuts in the vascular tissue completely, while interchange between the tissues within and without the tube is promoted. The perforated stele may be compared structurally and physiologically with the perforated leaf of *Ouvirandra*: the problem of surface-interchange has been solved in both cases by increased surface. The attenuated network of vascular tissue which after perforation represents the solenostele is characteristic of the most recent and advanced ferns, and their prevalence in genera and species now living is a testimony to the physiological success which perforation brings.

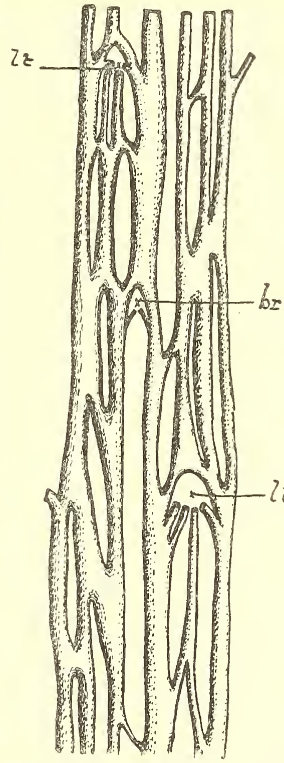
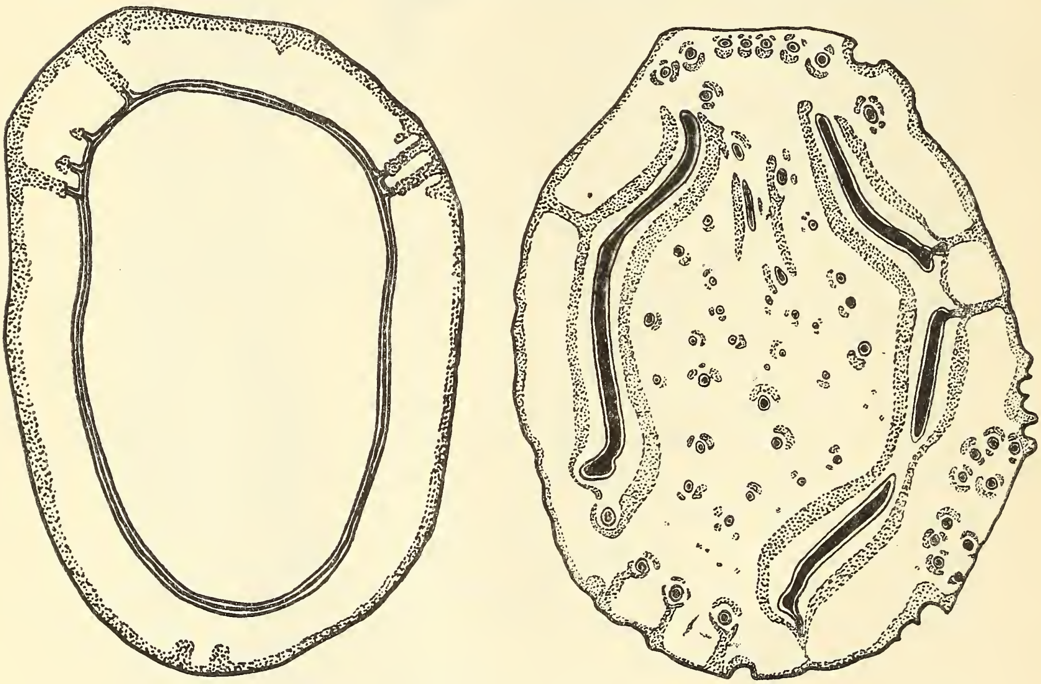


Fig. 12.—*Stenochlœna tenuifolia* (Desv.) Moore, after Mettenius. Stelar system flattened into a single plane, showing perforations.
l.t. = leaf-trace.
br. = vascular supply to a branch.

Another step, distinct in time and manner of its origin from perforation though resembling it in its effect, is the overlapping of foliar gaps in short stems with crowded leaves. The result is what is described as *dictyostely*. If a foliar gap underlies each leaf-insertion, and the leaves are crowded on the axis, any transverse section will cut through more than one of them, and in a transverse section the vascular ring will appear divided into a number of isolated tracts. This is seen in the Male Fern or in large Tree Ferns (fig. 13, B). It is frequently combined with perforation. Though the perforations and the leaf-gaps are essentially different in their real origin, the physiological effect of them is alike. By either means the endodermal surface is increased, and interchange facilitated; while the complete investment of the vascular tracts with endodermis is maintained. The final effect of these several factors, separately or in combination, is to break up the vascular tracts as seen in transverse section into relatively small circular or oval masses, each with a relatively large proportion of surface to bulk. The physiological



A. B.

Fig. 13.—Transverse sections of stems, drawn to the same scale, showing that stelar complication does not depend directly upon size alone. (×2.)

A = *Cibotium Barometz.* B = *Hemitelia setosa.*

difficulty following on increase of size is thus fully met in Leptosporangiate Ferns (fig. 9: 4, 6, 8).

If, as the anatomy of the ferns seems to suggest, actual *Size* is one of the factors determining the form which the stelar tissues take, and that increase beyond certain dimensions leads to those peculiarities which are seen in them, and particularly to the breaking up of the stele into meristeles, then tuberous development should lead to such disintegration. More especially should the change be apparent where the normal part shows a relatively simple stelar structure. A good example of this is seen in the tubers borne upon the protostelic stolons of *Nephrolepis* (fig.

14, A). It has been shown by Lachmann (*Thesis*, Paris, 1889), and by Sahni (*New Phyt.*, vol. xv, p. 72, 1916) that the protostele of the stolon expands at the base of the distended tuber. As seen in transverse section it first acquires a central mass of phloem, followed successively by pericycle, endodermis, and ground-parenchyma. In fact it becomes solenostelic. As the base of the tuber expands the ring breaks up by irregular perforations, as it does in leafy shoots of many Leptosporangiate Ferns. But here there are only perforations: since no leaves are borne on the stolon there are naturally no foliar gaps. A network of meristeles is thus

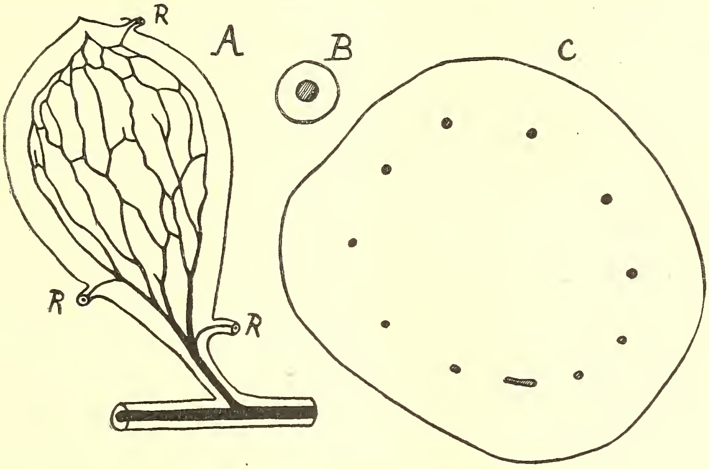


FIG. 14.—*Nephrolepis cordifolia.*

A, Stolon bearing a tuber, in which the protostele breaks up into a cylindrical network, contracting again at the apex. After Sahni.
B, Transverse section of protostelic stolon. (×5.)
C, Transverse section of tuber (also ×5) showing ring of meristeles each limited by endodermis.
Diameter of stolon, 1·6 mm. Diameter of tuber, 11·0 mm.

formed, each limited by a complete endodermis, and arranged in an expanded ring (fig. 14, C). At the distal end, where the tuber contracts again, the network narrows down through stages of condensation the reverse of the previous disintegration. In a given case the diameter of the stolon was 1·6 mm., and of its protostele ·6 mm. The diameter of the tuber was 1·1 cm., and of its ring of meristeles ·74 cm.: that is, nearly fifteen times that of the original protostele. It thus appears that, while complete endodermal control is maintained, when the stolon of *Nephrolepis* dilates into a tuber the same features of stelar expansion appear as in the conically enlarging axis of many Leptosporangiate Ferns. This suggests that the increase in size in both cases determines the structural change, while the reversal of that change which follows on the apical contraction of the tuber strongly supports that conclusion.

A somewhat similar result is found in the tubers formed on the rhizomes of certain species of *Equisetum*. Each tuber results from the dilatation of an internode of the rhizome to a size beyond the normal, and this is found to be accompanied by a disintegration of the stele. *E. arvense* is one of those species in which the stele is strictly circumscribed by an unbroken external endodermis. In the internode of the rhizome the stele is relatively compact, and in an average case it measures about 1 mm. in diameter. The individual vascular strands may vary in number. In fig. 15, A, six are seen. In the distended tuber the strands appear widely apart, each surrounded by its own endodermal sheath, which closes round it as the strands separate at the base of the tuber. The ring which they form may be 3 mm. or more in diameter, fully three times that of the original stele (fig. 15, B). As in Leptosporangiate Ferns, the stele has divided into meristeles, and as in them the disintegration accompanies an increase in bulk, which may be held to be one of the factors, or perhaps the chief factor in determining it. As the next node is approached the tuber contracts, and the stele is reconstituted from its meristeles. The significance of these two examples in supporting the theory cannot be mistaken.

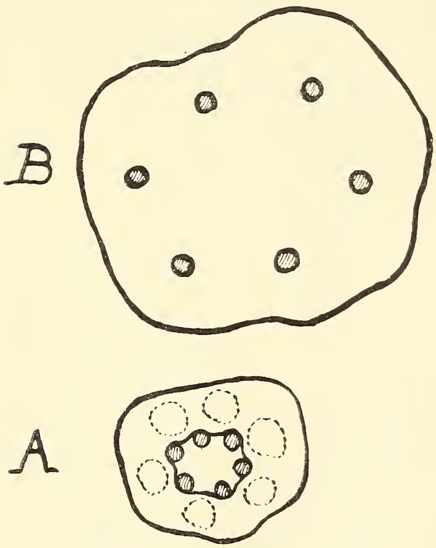


FIG. 15.—*Equisetum arvense.*

A, Transverse section of internode of rhizome: stele with encircling endodermis, 1 mm. in diameter.
B, Transverse section of tuber, to the same scale, showing ring of meristeles, each with its own endodermal sheath, 3 mm. in diameter. (×10.)

In the petiole of the fern-leaf the leaf-trace has as a rule the form of an arc concave on the adaxial side. In many relatively primitive ferns the leaf-trace is undivided. Being thus a continuous curved tract limited by endodermis, it may form in large leaves a formidable obstacle to communication from the outer surface inwards, and especially at the leaf-base, where naturally interchange would be active and the petiole is at its largest. In many ferns the leaf-trace is broken up at, or near to, the base into separate strands. There is often a median slit dividing the trace into equal halves, as in the type of *Asplenium*, *Athyrium*, or *Gymnogramme* (fig. 16). But often the subdivision may be carried further, as in the case of many Cyatheoid Ferns (fig. 17: 6). Sometimes the slits close again upwards and downwards, as is very well seen in *Plagiogyria* *semicordata.* The slits are in fact perforations of essentially the same nature as those which occur in the axis, and their function is the same. They are most prominently seen in leaves of large size, and especially where the margins of the curve approach one another, as in *Thyrsopteris* or *Alsophila* (fig. 17: 5, 6). It may, then, be concluded that in ferns of advanced type the leaf-trace is subject to perforation in essentially the same way, and with the same physiological effect, as is the stele of their axes.

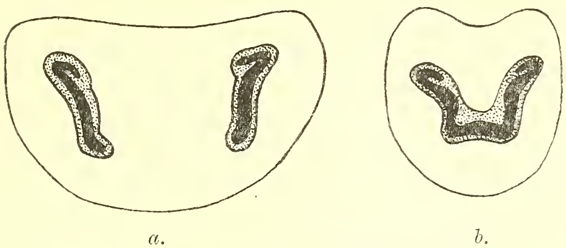


Fig. 16.—Transverse section of petiole of *Athyrium filix fœmina.*
(*a*) Near base; (*b*) higher up. After Luerssen. (×7.)

The stele in the root of ferns is always small, so that no difficult problem of proportion of surface to bulk arises. The general tendency to concentrate the vascular tissues at the centre of the transverse section of the root, which is so general in the Pteridophytes at large, leads to the same result. Even in the large roots of the Marattiaceæ there is no need for any disruption of the compact stele.

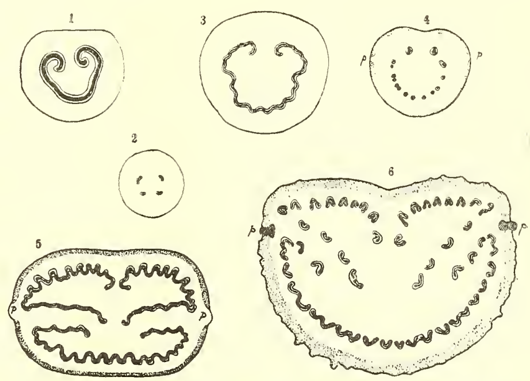


Fig. 17.—Transverse sections of petioles, all drawn to the same scale. (×2.)
1, *Dipteris conjugata*; 2, *Dipteris Lobbiana*; 3, *Metaxya*; 4, *Phlebodium aureum*; 5, *Thyrsopteris*; 6, *Alsophila australis.*
These show that while greater size leads to vascular disintegration, there is no definite proportion.

The triumph of the Leptosporangiate Ferns which show disintegration of the stele in stem and leaf in all their more advanced types, is witnessed by their 6000 living species spread widely over the face of the earth. That triumph has been won by a compromise, effected without cambial increase in the enlarging stem. The conducting stele has enlarged with the conically enlarging shoot: it has maintained its endodermal barriers complete, and has met the difficulty of physiological interchange consequent on that enlargement by various steps of moulding and disintegration of the stele. This has given those conducting tracts the requisite proportion of surface to bulk even for stems as large as those of the Tree Ferns.

The analogy between what is seen in ferns and the "polystelic" state of certain of the larger species of *Selaginella* suggests that they should also be examined from the physiological-anatomical point of view. The similarity between the vascular arrangements in *S. lævigata* and that in solenostelic ferns seems to indicate that there is some common underlying cause which has brought such likeness into existence. It is suggested that the need for physiological interchange over the surface of an enlarged stele has been the determining factor in both cases, though the need for a larger surface seems less apparent in *Selaginella* with its peculiar structure of the endodermis than elsewhere. The Medullosæ with their recurrent "polystely" also provoke comparison. Scott (*Studies*, vol. ii, p. 444) remarks that their polystely has no relation to leaf-gaps, and he suggests that "the breaking up of the original stele depended on some other cause." Does it not appear probable that the cause may have been connected with questions of proportion of surface to bulk in the original steles of these large stems, before cambial increase set in? With the knowledge of their primary steles and of their sheaths so incomplete as it is, one cannot do more than suggest that this is an aspect worthy of consideration, especially as it has been found to illuminate the cognate structure in ferns.

Having thus seen how firmly established the disintegrated stelar state is among Pteridophytes, the question arises whether Flowering Plants show any similar modifications of stelar structure with enlarging size. Most of the Flowering Plants have met their problem of enlargement in other ways, as will be noted later. But the large prop-roots of certain palms have advanced structurally along lines that show a close analogy with what has been seen in the ferns. Such roots are often thick. They have normally a cylindrical stele delimited by an endodermis that is sclerotic. There is a cortical system of large intercellular spaces, and an intra-stelar ventilating system that is often also large. These are separated by the barrier of endodermis. Consequently the same problem arises with increasing size as in the stems of ferns. Modifications in the direction of disintegration, or of "polystely" as it is sometimes called, have been produced by involution and finally by disruption of the endodermis, in *Areca catechu*, *Archontophœnix cunninghami*, *Dictyosperma aurea*, *Verschaffeltia splendida*, and other palms. The departure from type appears in roots about 1 cm. in diameter, and it is still more marked in those of larger size. In roots about 1 cm. in diameter the stele takes a fluted form. In larger roots the involutions are deeper, and the endodermis and ring of vascular tissues within it are interrupted by bands of parenchyma with continuous intercellular spaces (fig. 18). In roots of the largest size, as occasionally in *Areca*, but more markedly in *Verschaffeltia*, the separate vascular tracts may be rounded off, and each completely surrounded by its own endodermis (fig. 19). The stele is thus disintegrated into a number of meristeles, and the analogy with the ferns is significant. But all large Angiospermic roots do not behave in this way. A prominent exception is seen in *Pandanus*, the large prop-roots of which may be as much as 5 cm. in diameter, and yet show no stelar disintegration. Thus size does not always prescribe it. These references by no means exhaust the examples which might be quoted from the higher vascular plants, in which distension, with or without a definite cambial zone, results in a distended mass of tissue. In such cases the texture is often sappy, and the vascular system broken up into numerous isolated strands. This is seen in *Welwitschia*, *Cycas*, *Gunnera*, and *Nymphæa*. It is worthy of remark that these plants, which resemble the Marattiaceæ in their general construction of the stem though not in its details, are all either actually xerophytes, or they live under circumstances in which a rapid fluid-transit is not necessary.

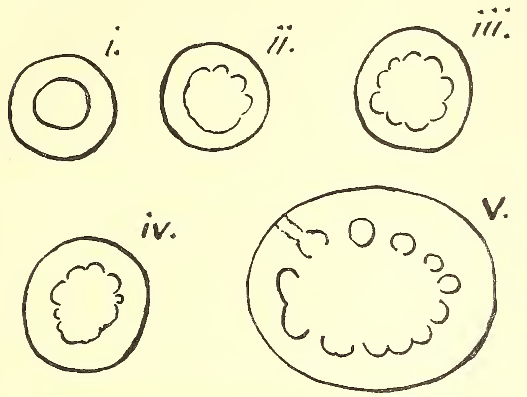


Fig. 18.—Transverse sections of root of *Areca*.
i–iv, Successive sections from the same root: i is 15 mm. from apex; ii at 77 mm.; iii at 115 mm.; iv at 150 mm. v is a section from another, larger root. After Cormack.

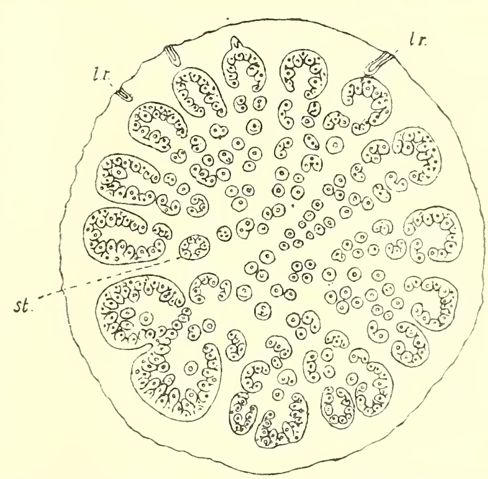


Fig. 19.—Transverse section of a large strut-root of *Verschaffeltia splendida*, showing high state of stelar disintegration.

st. = a completely cylindrical meristele.
l.r. = lateral roots. (×2). After Cormack.

Nevertheless, that actual size is causally related to these abnormalities in the roots of palms is shown by the fact that when the root diminishes progressively the abnormality ceases. It was demonstrated by Cormack (*Trans. Linn. Soc.*, vol. v, pt. 6, 1896) that there is a continuous transition to normal monostelic structure in the younger and thinner parts of roots that are fluted in their older and thicker region (fig. 18). He was led to conclude that this difference of structure is due to a progressive change in the mode of differentiation of the apical meristem. It is in fact here, as it is also in ferns, a question of procambial destination. But how it is, and why it is, that the meristem of a part beyond a certain size should in fern-stems and in palm-roots differentiate its procambium so as to produce a disintegrated stele, and the same root below that size so as to produce an integral stele: and why the latter type of meristem should pass into the former, as in the ontogeny of the Leptosporangiate Ferns: or the former into the latter, as in certain individual roots of palms, must for the present remain an open question. It is in fact as great a mystery as is the power of the root-tip to receive the stimulus of gravity, and to pass it on to the point of reaction. The behaviour of meristems remains now, as it has always been, the greatest enigma of the plant-body, and not the least of the questions which it raises is this: How does the meristem forecast in its embryonic tissues those proportions of surface to bulk which will be necessary when the tissues still embryonic shall have matured to their full size? In this the biologist of the present day may see a new application for the old word "Prolepsis."

It may be asked, if certain plants show these modifications of structure according to size, why are they not more common among the higher plants, many of which seem to be of such size as to justify, or even to demand them? The reply is that in the stems and roots of very many seed-bearing plants, the difficulty of a delimiting endodermis does not arise, except in the young state, and while they are relatively small. In many small Monocotyledon-stems, such as the creeping rhizomes of *Convallaria majalis*, a well-marked endodermis may exist. Even in this plant Schwendener has recorded a "perforation" similar in nature to those seen in Ferns. But in plants of larger size where the stele is dilated so as to take up almost the whole of the transverse section, as is the case in the palm-type generally, the endodermis is inconspicuous, and does not appear as an efficient barrier. These plants have in fact adopted a condition which for practical purposes is like that of the Marattiaceae, though it has been reached by a different developmental method. Isolated vascular strands traverse the parenchyma of the distended stele, and neither they nor the stele have an efficient endodermal barrier.

Those plants which possess secondary thickening, such as the Dicotyledons and Gymnosperms, stand in a different position. Frequently they show in their young stems and roots a well-marked endodermis. But before the stem or root enlarges to that size which appears to be critical in so many plants (that is about 1 cm. in diameter), the secondary cambial increase within has already so stretched and disorganised the endodermis that it can no longer act as a physiological barrier. It would be worth while to examine it carefully as to the manner of its functional change, from the point of view here suggested, and to trace in detail the disruption or disorganisation of the sheath. But such observations would not be easy. It is only in cases where the endodermis is marked by special structure that this can be readily seen. An example has been worked out by Miss Mann in the large roots of *Dracæna*, which show secondary thickening. The effect of internal expansion of tissues is shown in bursting asunder the thickened cells, and communication is thus established between the cortical and stelar systems by tracts of ventilated parenchyma. Once the endodermal barrier is interrupted, communication is continued radially inwards along the medullary rays of Dicotyledons and Gymnosperms, as Strasburger showed long ago. It is in some such way as this that the problem is solved for trunks and roots of the largest size The same holds also for the dendroid stems of Monocotyledons, such as *Dracæna* and *Cordyline*. In them tangential sections show that intercellular spaces run radially inwards in the hardened tissues that lie between the secondary strands.

The success with which the flowering plants have thus met the requirements of increasing size stand in strong antithesis to the difficulties with which the ferns have had to contend. Doubtless the strictly enclosed conducting system of ferns is a more effective means of transport for plants so dependent as they are upon a constant water-supply. But the problems which increasing size has raised in them could only be solved by extraordinary modifications of structure: and that seems to be the physiological explanation of those remarkable vascular systems which they show, often in clearly marked succession in their individual development. The ontogenetic evidence is in fact more weighty than that from comparison.

Those who pursue sciences of exact measurement may expect me to give precise statements as to the actual size of stele which will be possible with a certain structure, and of the exact dimensions at which the "limiting factor" will become operative: that at which either a new form of stele must be adopted, or a change be made in the visible structure of the endodermis, so as to alter its permeability. At present it is not possible to put forward measurements of that nature. The difficulty is illustrated by the drawings, all to the same scale, in figs. 9 and 17 ; and especially by the cases of *Dipteris* and *Matonia*, two genera of allied ferns, with many analogies of structure. They are both rhizomatous and solenostelic: transverse sections of them drawn to the same scale are shown in fig. 9 : 2, 3. *Dipteris conjugata* is the larger, and shows only a simple solenostele: *Matonia pectinata* is the smaller, and shows three concentric rings. A further example is seen in fig. 13, A, B, where in stems of equal size that of *Cibotium* is a simple solenostele, that of *Hemitelia* is a dictyostele with medullary strands. A comparison of drawings illustrating the origin of the solenostele in individual plants of *Gleichenia pectinata*, *Loxsoma*, and *Histiopteris incisa*, shows that no absolute size-limit rules for them, though in each individual a great increase of size accompanies the structural change. There need be no surprise that this difficulty should arise, for it is common knowledge that the characteristics of plants, and ultimately of their protoplasts, differ. One is more resistant to temperature, or drought, or insolation than another. The incidence of the "limiting factor" will depend upon the specific permeability of the endodermis, and particularly of its protoplasts. It seems probable that this may vary from plant to plant without any visible difference of structure, just as much as other characteristics of the protoplasts do. It is only when the specific permeability of the endodermis has been measured for any plant that we can reasonably expect to be able to state in terms of exact measurement where for it the incidence of the limiting factor may come. With our present knowledge it is only a rough suggestion of the existence of a critical point that can be indicated. But this gives a sufficient ground for recognising the underlying principle of similar structures as applicable to vascular tissues. It appears to have determined certain of their peculiarities, which have hitherto appeared as strange and unexplained phenomena. It is believed that by directing attention to the sheaths surrounding the vascular tracts, their presence or absence, their structure and permeability, and their relations however roughly to absolute size, a better understanding of the vascular systems of plants, and of the ferns in particular, will be obtained than by the most carefully drawn comparisons of mere formal anatomy. Size must then be considered not only in its bearing on external form and the strength of materials, but also as it may tend to modify, or even in some cases to rule decisively the structure and disposition of internal tissues.

As far as I am aware only one definite attempt has hitherto been made to correlate size with internal structure in the massive plant-body. Professor Compton, in a very remarkable comparative study of the seedling structure in the Leguminosæ (*Linn. Journ.*, vol. xli, p. i, 1912), has shown that in determining the level of transition from root-structure to stem-structure in the hypocotyl, the diameter of the axis is a most important factor. He found that low transitions are characteristic of massive hypocotyls, high transitions of those which are slender. Here again it was not possible to put forward exact measurements, but the principle emerges from the average taken from a large number of seedlings of different species. The correlation in that case was, however, between size and the readjustment of intra-stelar tissues: the form of the stele itself was throughout approximately cylindrical. Questions of permeability of the endodermal barrier will hardly have affected them. That size may thus be correlated on the one hand with the internal disposition of the stelar tissues, and on the other with the actual conformation of the stele itself, shows how important it may be in determining features which are habitually used in comparison. The effect should be to impose caution in drawing phylogenetic or classificatory conclusions from characters so pliable: for at once the door is opened for frequent homoplasy in stocks phyletically quite distinct from one another. Thus the recognition of the principle of similar structures and its consequences, while it involves interesting points in physiology and anatomy, may react finally even upon classification.

(*Issued separately December* 30, 1920.)

II—On the Equations of Motion of a Single Particle.

By J. H. M. Wedderburn.

(MS. received August 9, 1920. Read November 1, 1920.)

§ 1. When solved for the second derivatives, the Lagrangian equations of motion for a system in which there are no extraneous forces have the form *

$$\frac{d^2x_k}{dt^2}+\sum_{ij}\left\{ {i\ j \atop k} \right\}\frac{dx_i}{dt}\frac{dx_j}{dt}=0, \quad (k=1, 2, \ldots n),$$

or, disregarding the parameter t,

$$(1) \quad . \quad . \quad d^2x_k+\sum_{ij}\left\{ {i\ j \atop k} \right\}dx_idx_k=0, \quad (k=1, 2, \ldots n),$$

$\left\{ {i\ j \atop k} \right\}$ being the second Christoffel symbol † of the matrix associated with the kinetic energy. If there are extraneous forces F_k and, denoting t by x_{n+1}, we add $d^2x_{n+1}=0$ to the set of equations, the equations of motion are

$$(2) \quad d^2x_k+\sum_{ij}\left\{ {i\ j \atop k} \right\}dx_idx_j-F_kdx_{n+1}{}^2=0, \quad (k=1, 2, \ldots n), \quad d^2x_{n+1}=0.$$

These equations have a general similarity to (1) with the number of variables increased by one, and would in fact have exactly the same mathematical form if there existed a matrix ψ for which

$$(3) \quad \left\{ {i\ j \atop k} \right\}'=\left\{ {i\ j \atop k} \right\}, \quad \left\{ {i\ n+1 \atop k} \right\}'=0, \quad \left\{ {n+1\ n+1 \atop k} \right\}'=-F_k, \quad (i, j, k=1, 2, \ldots n),$$
$$\left\{ {i\ j \atop n+1} \right\}'=0, \quad (i, j=1, 2, \ldots n+1),$$

where the dashes indicate symbols belonging to the new matrix ψ. It is readily seen, however, that these relations cannot hold in general, and it is the principal object of this note to investigate the circumstances under which they can be approximately satisfied. If ordinary dynamics can be modified so that this is so, and the time ‡ variable is regarded as a special case of the space variables, it will follow that the equations of motion in the modified

* *Cf.* Whittaker, *Analytical Dynamics*, p. 39 ; or Wright, *Invariants of Quadratic Differential Forms*, p. 83.

† The definition and properties of these symbols may be found in Wright, *loc. cit.*, p. 10.

‡ If there is more than one particle in question, it may be necessary to introduce more than one time variable. In many ways it is best to consider these as strictly space variables and to assume that particles in our universe are moving in the direction of the fourth space direction so nearly uniformly in straight lines that the distances measured in this direction are, to a first approximation, proportional to the time.

system will have the same form independently of the system of coordinates used, and all forces will appear as geometrical constraints.

§ 2. For the present we shall consider only the motion of a single particle whose position is defined by rectangular coordinates x_1, x_2, x_3, the time variable being denoted by x_4 and the potential function by V. The Newtonian equations of motion are then

$$(4) \quad . \quad . \quad . \quad d^2x_k = \frac{\partial V}{\partial x_k}dx_4^2, \quad (k=1, 2, 3), \quad d^2x_4 = 0,$$

and our problem is to determine a symmetric matrix $\psi = (a_{rs})$ such that the equations

$$(5) \quad . \quad . \quad . \quad d^2x_k + \sum_{ij}\left\{\begin{matrix} i\ j \\ k \end{matrix}\right\}dx_i dx_j = 0, \quad (k=1, 2, 3, 4)$$

or the equivalent set

$$(5') \quad . \quad . \quad \sum_s a_{ks}d^2x_s + \sum_{ij}\begin{bmatrix} i\ j \\ k \end{bmatrix}dx_i dx_j = 0, \quad (k=1, 2, 3, 4)$$

are approximately satisfied when (4) is satisfied, and *vice versa*. To determine the conditions for this, we substitute the values of d^2x_k from (4) in the first three equations of (5′), thus obtaining

$$(6) \quad . \quad \sum_{s=1}^{3} a_{ks}\frac{\partial V}{\partial x_s}dx_4^2 = -\tfrac{1}{2}\sum_{ij}\left(\frac{\partial a_{ik}}{\partial x_j} + \frac{\partial a_{jk}}{\partial x_i} - \frac{\partial a_{ij}}{\partial x_k}\right)dx_i dx \quad (k=1, 2, 3)$$

so that the term under the Σ on the right is zero except when i and j are both 4. It follows immediately that the part of the matrix whose subscripts do not exceed three, *i.e.* the part which refers to the subspace x_1, x_2, x_3, is independent of x_1, x_2, x_3; and therefore, since ψ is symmetric, there is a real orthogonal transformation, with coefficients independent of x_1, x_2, x_3, which reduces this part of ψ to the main diagonal. We may therefore assume without loss of generality that $a_{ij}=0$, $(i, j=1, 2, 3;\ i \neq j)$ and $\partial a_{ii}/\partial x_j = 0$ $(i, j=1, 2, 3)$. Further, since $\begin{bmatrix} i\ 4 \\ k \end{bmatrix} = 0$ $(i, k \neq 4)$, we have

$$\frac{\partial a_{k4}}{\partial x_j} + \frac{\partial a_{ik}}{\partial x_4} - \frac{\partial a_{i4}}{\partial x_k} = 0,$$

and similarly

$$\frac{\partial a_{i4}}{\partial x_k} + \frac{\partial a_{ik}}{\partial x_4} - \frac{\partial a_{k4}}{\partial x_i} = 0,$$

so that $\partial a_{ik}/\partial x_4 = 0$ (showing that the coefficients of the orthogonal transformation used above are also independent of x_4) and

$$\frac{\partial a_{i4}}{\partial x_k} = \frac{\partial a_{k4}}{\partial x_i}, \quad (i, k \neq 4)$$

so that we may set

$$(7) \quad . \quad . \quad . \quad . \quad . \quad . \quad a_{i4} = \frac{\partial A}{\partial x_i} \quad (i=1, 2, 3).$$

Finally, comparing coefficients of dx_4^2 in (6) and using the relations just derived, we have

$$(8) \quad . \quad . \quad a_{kk}\frac{\partial V}{\partial x_k} = -\frac{\partial a_{k4}}{\partial x_4} + \frac{1}{2}\frac{\partial a_{44}}{\partial x_k} = \frac{\partial}{\partial x_k}\left(\frac{1}{2}a_{44} - \frac{\partial A}{\partial x_4}\right) \quad (k=1, 2, 3).$$

If a_{11}, a_{22}, a_{33} are all different, we infer that V has the form $g_1(x_1)+g_2(x_2)+g_3(x_3)$, or if say $a_{11}=a_{22}\neq a_{33}$, it has the form $g_1(x_1, x_2)+g_3(x_3)$; but since the x's are ordinary rectangular coordinates, this form does not agree to a first approximation with the Newtonian potential, so that when we are dealing with Newtonian dynamics we must have $a_{11}=a_{22}=a_{33}$ and, being constant, we may set each equal to unity. This gives immediately

$$(9) \quad . \quad . \quad . \quad . \quad . \quad a_{44} = 2V + 2\frac{\partial A}{\partial x_4},$$

the arbitrary function of x_4 introduced by the integration being included in A. The fourth equation of (5′) then becomes

$$\sum_1^3 \frac{\partial A}{\partial x_i}d^2x_i + 2\left(V + \frac{\partial A}{\partial x_4}\right)d^2x_4 = -\sum_1^3 \frac{\partial^2 A}{\partial x_i \partial x_j}dx_i dx_j - 2\sum_1^3\left(\frac{\partial V}{\partial x_i} + \frac{\partial^2 A}{\partial x_i \partial x_4}\right)dx_i dx_4 - \left(\frac{\partial V}{\partial x_4} + \frac{\partial^2 A}{\partial x_4^2}\right)dx_4^2,$$

so that in order that d^2x_4 may be a second order quantity it is sufficient to assume that $V+\partial A/\partial x_4$ is large compared with the other coefficients in the equation, a condition that can be readily attained, since $V+\partial A/\partial x_4$ contains an arbitrary function of x_4 which can have terms not entering into any other coefficient.

We assume, therefore, that ψ has the form

$$(10) \quad . \quad . \quad . \quad \psi = \begin{vmatrix} 1 & 0 & 0 & A_1 \\ 0 & 1 & 0 & A_2 \\ 0 & 0 & 1 & A_3 \\ A_1 & A_2 & A_3 & 2V+2A_4 \end{vmatrix}$$

where we have set A_i for $\partial A/\partial x_i$. The corresponding quadratic form is

$$(11) \quad . \quad . \quad . \quad . \quad . \quad \sum dx_i^2 + 2V dx_4^2 + 2dA\,dx_4,$$

and the corresponding Lagrangian equations can be readily reduced to

$$(12) \quad . \quad . \quad . \quad \begin{cases} d^2x_k + A_k d^2x_4 - V_k dx_4^2 = 0 \quad (k=1, 2, 3), \\ (2V + A_4)d^2x_4 + d^2A + 2dV dx_4 - V_4 dx_4^2 = 0, \end{cases}$$

the form (11) equated to a constant being, as is well known, a first integral of (12).*

* This seems the most natural way of introducing the form (11) into the dynamical system. It suggests that it is natural to consider this form as defining time rather than distance, thus leaving open the possibility of using a different form to define the geometry of space.

When the potential is independent of the time x_4, and $\partial A/\partial x_4$ is constant, equal to $-a$, say, a first integral of the last equation of (12) is

$$(2V - a)dx_4 + dA = \text{const.},$$

or

$$2(V - a)dx_4 + \sum \frac{\partial A}{\partial x_i} dx_i = \text{const.}$$

§ 3. An important case of the equations of the preceding paragraph arises when (i) the potential V is a function of r alone, and (ii) $A = -ax_4$, so that $A_4 = -a$, a constant.

When (ii) holds and also $V_4 = 0$, equations (11) and (12) become, on introducing a parameter τ,

$$\text{(13)} \quad \sum_1^3 \left(\frac{dx_i}{\partial \tau}\right)^2 + 2(V - a)\left(\frac{dx_4}{\partial \tau}\right)^2 = H, \quad \text{(H constant)}$$

$$\text{(14)} \quad \frac{d^2 x_i}{d\tau^2} - V_i \left(\frac{dx_4}{d\tau}\right)^2 = 0, \quad (i = 1, 2, 3)$$

$$2(V - a)\frac{d^2 x_4}{d\tau^2} + 2\frac{dV}{d\tau}\frac{dx_4}{d\tau} = 0,$$

whence

$$\text{(14')} \quad \frac{dx_4}{d\tau} = \frac{\kappa}{V - a}, \quad (\kappa \text{ constant})$$

so that, setting $U = -\kappa^2/(V - a)$,

$$\text{(15)} \quad \frac{d^2 x_i}{d\tau^2} = U_i \quad (i = 1, 2, 3), \quad \frac{dx_4}{d\tau} = -\frac{U}{\kappa}.$$

$$\text{(15')} \quad \sum_1^3 \left(\frac{dx_i}{d\tau}\right)^2 - 2U = H.$$

If now V, and therefore U, is a function of r alone, it follows by the usual methods that the motion is in a plane, and, choosing spherical polar co-ordinates so that this plane is $\phi = 0$, the equations of motion become

$$\frac{d^2 r}{d\tau^2} - r\left(\frac{d\theta}{d\tau}\right)^2 = \frac{\kappa^2}{(V - a)^2}\frac{dV}{dr} = \frac{dU}{dr}, \quad r^2 \frac{d\theta}{d\tau} = h,$$

where h is constant, or setting $u = 1/r$,

$$\text{(16)} \quad \frac{d^2 u}{d\theta^2} + u = -\frac{\frac{dU}{dr}}{h^2 u^2} = \frac{1}{h^2}\frac{dU}{du}, \quad \frac{d\theta}{d\tau} = hu^2.$$

If we now assume that V has the Newtonian form m/r, and set $\kappa = -a$ so that x_4 equals τ when r is large, then the equation for u becomes on neglecting terms in $1/a^2$,

$$\text{(16')} \quad \frac{d^2 u}{d\theta^2} + \left(1 - \frac{2m^2}{h^2 a}\right)u = \frac{m}{h^2},$$

which gives

(17) . . . $$u=\frac{m}{h^2-2m^2a^{-1}}+a\cos\sqrt{1-\frac{2m^2}{h^2a}}(\theta-\theta_0),$$

so that the orbit has the approximate period $2\pi\left(1+\frac{m^2}{h^2a}\right)$ in θ. To agree with Einstein's result this would require $a=c^2/3$, and then when r is large the quadratic (11) would approach $\Sigma dx_i^2-\frac{2}{3}c^2dx_4^2$. The Newtonian form of V therefore leads to results which clash with the classical form of the relativity theory.

The usual form of the quadratic is easily obtained by assuming that the Newtonian form of V is merely the first term in the expansion of V in negative powers of c^2; but the method of the next paragraph is more satisfactory.

§ 4. The Newtonian potential is a solution of Laplace's equation. If this equation is regarded as associated with the form $\Sigma dx_i^2-c^2dx_4^2$, V being independent of x_4, the analogous equation associated with the form $\Sigma a_{rs}dx_rdx_s$ is, as is well known,*

(17) . . . . $$\Delta_2 V=0,$$

where $\Delta_2 V$ is the Beltrami parameter $\Sigma a^{rs}V_{rs}$, (a^{rs}) being the matrix reciprocal to (a_{rs}), and

(18) . . . . . $$V_{rs}\equiv\frac{\partial^2 V}{\partial x_r\partial x_s}-\sum_j\begin{Bmatrix}r\ s\\ j\end{Bmatrix}\frac{\partial V}{\partial x_j}$$

being the covariant derivative.

In the case considered in §3, (17) becomes

$$\frac{d^2V}{dr^2}+\frac{2}{r}\frac{dV}{dr}+\frac{1}{2(V-a)}\left(\frac{dV}{dr}\right)^2=0,$$

which leads to

(19) . . . . . $$V-a=-\left(\alpha-\frac{\beta}{r}\right)^{\frac{2}{3}}.$$

Setting $\kappa=-\alpha^{\frac{2}{3}}=-\frac{1}{2}c^2$, $2\beta=3m\alpha^{\frac{1}{2}}$, this gives in place of (16′)

(20) . . . . . $$\frac{d^2u}{d\theta^2}+\left(1-\frac{5m^2}{c^2h^2}\right)u=\frac{m}{h^2},$$

which leads to $\frac{5}{6}$ of Einstein's value † for the motion of the perihelion of Mercury, a result which is slightly closer to the observed value. The corresponding form does not, however, lead to the correct deviation of a ray of light in a gravitational field.

There are other methods of deriving an equation analogous to Laplace's

* *Cf.* Wright, *loc. cit.*, p. 53.

† The Einstein value is obtained by setting

$$V-a=-\tfrac{1}{2}c^2\left(1-\frac{4m}{c^2r}\right)^{\frac{1}{2}}.$$

which lead to results different from those just obtained.* It will be seen below that in certain important cases, in addition to the case discussed above, a change of variable enables us to bring (11) into the form

$$ds_0^2 - 2(a-\mathrm{V})dy_4^2,$$

where ds_0^2 depends only on x_1, x_2, x_3, and $\partial y_4/\partial x_4 = 1$. Comparing this form with $ds_0^2 - c^2dt^2$, it is natural to interpret y_4 as t and to regard $2(a-\mathrm{V})$ as the square of a variable velocity of light. From this point of view, then, we should expect $\sqrt{2(a-\mathrm{V})}$ to satisfy the generalised Laplace equation rather than V itself, especially as when a is large the value of V so determined will agree with the Newtonian potential to a first approximation. In the case discussed in the preceding paragraph this second method of determining V leads without approximation to the Newtonian form.

§ 5. From the dynamical point of view, there does not appear to be any reason for imposing any particular form of restriction on V and A in addition to the general restrictions of § 1 and § 2, which are sufficient to ensure that the equations of motions are equivalent to the Newtonian equations to a first approximation. In view of the particular case partially developed in § 3, it is, however, natural to investigate the case in which A depends on V alone except for an additive term of the form $-ax_4$ (a constant) introduced in the integration of (8). This makes the equations of the system depend solely on the potential. We shall not impose this condition in its entirety at first, but will introduce it as required in the course of the discussion.

We shall first determine conditions under which it is possible so to transform the variables as to remove the term $d\mathrm{A}dx_4$ in (11). Setting

$$y_i = x_i \quad (i=1, 2, 3), \qquad y_4 = g(x_1, x_2, x_3, x_4),$$
$$x_i = y_i \quad (i=1, 2, 3), \qquad x_4 = f(y_1, y_2, y_3, y_4),$$

in (11), and equating the coefficient of dy_idy_4 to zero, we get

$$(21) \quad . \quad . \quad . \quad \frac{\partial g}{\partial x_4}\frac{\partial \mathrm{A}}{\partial x_i} - 2(\mathrm{V}+\mathrm{A}_4)\frac{\partial g}{\partial x_i} = 0 \quad (i=1, 2, 3).$$

If A is a function of V and x_4 alone and V is independent of x_4, this leads to

$$(22) \quad . \quad . \quad . \quad . \quad \frac{\partial g}{\partial x_4}\frac{\partial \mathrm{A}}{\partial \mathrm{V}} - 2\left(\frac{\partial \mathrm{A}}{\partial x_4} + \mathrm{V}\right)\frac{\partial g}{\partial \mathrm{V}} = 0,$$

any solution of which will give the required result. If $\partial\mathrm{A}/\partial\mathrm{V}$ and $\partial\mathrm{A}/\partial x_4$ are functions of V alone,† *i.e.* $\mathrm{A} = -ax_4 + f(\mathrm{V})$, the solution of (22) is

$$(23) \quad . \quad . \quad . \quad . \quad y_4 = \mathrm{F}\left(x_4 + \tfrac{1}{2}\int\frac{\partial \mathrm{A}}{\partial \mathrm{V}}\frac{d\mathrm{V}}{(\mathrm{V}-a)}\right),$$

* *Cf.* Levi-Civita, "Statica Einsteiniana," *Rom. Acc. L. Rend.*, xxvi (1917), p. 458, where the connection with Einstein's equations is discussed.

† There are, of course, other cases in which an explicit solution of (21) can be obtained, *e.g.* when $2\mathrm{V}+\mathrm{A}$ is a function of A and x_4 alone, or if A has the form $\mathrm{A}_1(\mathrm{V}) + x_4\mathrm{A}_2(\mathrm{V})$.

where F is an arbitrary function, the most important case being

$$(23') \quad . \quad . \quad . \quad . \quad . \quad y_4 = x_4 + \tfrac{1}{2}\int \frac{\partial A}{\partial V}\frac{dV}{(V-a)}.$$

When (23′) is used, the quadratic (11) becomes

$$(24) \quad . \quad . \quad . \quad \sum dx_i^2 - \left(\frac{\partial A}{\partial V}\right)^2 \frac{dV^2}{2(V-a)} + 2(V-a)dy_4^2.$$

We shall now assume that $V_1 \equiv V - a = constant$ is one of a triply orthogonal system of surfaces. Making a slight change in notation, we denote the corresponding curvilinear coordinates by $y_1 \equiv V_1$, y_2, y_3, and set the square of the element of distance in the space x_1, x_2, x_3, equal to $H_1dy_1^2 + H_2dy_2^2 + H_3dy_3^2$, so that (24) becomes

$$(25) \quad . \quad . \quad . \quad \left(H_1 - \frac{A_1^2}{2V_1}\right)dy_1^2 + H_2dy_2^2 + H_3dy_3^2 + 2V_1dy_4^2,$$

where $A_1 \equiv \partial A/\partial y_1$, or say

$$(25') \quad . \quad . \quad . \quad . \quad a_1dy_1^2 + a_2dy_2^2 + a_3dy_3^2 + a_4dy_4^2.$$

As a further restriction on V, we now assume that it, and therefore also V_1, satisfies the Laplace equation which is covariant relatively to (25), namely

$$(26) \quad . \quad . \quad \Delta_2 W \equiv \sum_1^4 \left(\frac{1}{a_i}\frac{\partial^2 W}{\partial y_i^2} - \frac{1}{2a_i^2}\frac{\partial a_i}{\partial y_i}\frac{\partial W}{\partial y_i} + \frac{1}{2a_i}\sum_{ji}\frac{1}{a_j}\frac{\partial a_i}{\partial y_j}\frac{\partial W}{\partial y_j}\right) = 0,$$

or, if W is a function of y_1 only,

$$2\frac{\partial^2 W}{\partial y_1^2} + \frac{\partial \log(a_2a_3a_4/a_1)}{\partial y_1}\frac{\partial W}{\partial y_1} = 0.$$

In particular, if $W = V_1 \equiv y_1$,

$$\frac{\partial}{\partial y_1}\left(\frac{a_2a_3a_4}{a_1}\right) = 0, \quad \text{or} \quad \frac{a_2a_3a_4}{a_1} = 2f(y_2, y_3),$$

where f is an arbitrary function of y_2 and y_3, y_4 being absent since the a's are independent of y_4. Inserting the values of the a's from (25), we get

$$(27) \quad . \quad . \quad . \quad . \quad . \quad \frac{H_2H_3V_1}{H_1 - \dfrac{A_1^2}{2V_1}} = f(y_2, y_3),$$

so that H_2H_3 is the product of a function of V_1 alone and a function of y_2 and y_3 alone.

If now V_1 is a function of $r = \sqrt{x_1^2 + x_2^2 + x_3^2}$, we may use spherical polar co-ordinates with $y_2 = \theta$ and $y_3 = \phi$, so that $H_1 = 1/\left(\frac{dV_1}{dr}\right)^2$, $H_2 = r^2$ and $H_3 = r^2 \sin^2\theta$. (27) then becomes

$$H_1 - \frac{1}{2V_1}\left(\frac{\partial A}{\partial V_1}\right)^2 = \gamma r^4 V_1, \quad (\gamma \text{ constant})$$

and in place of (25) we get

$$\gamma r^4 V_1\left(\frac{dV}{dr}\right)^2 dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 + 2V_1 dy_4^2. \tag{29}$$

The values of A and y_4 are given by

$$\left\{\begin{aligned} A &= -ax_4 + \int\left(\frac{2V_1}{\left(\frac{\partial V_1}{\partial r}\right)^2} - 2\gamma r^4 V_1^2\right)^{\frac{1}{2}} dV_1 \\ y_4 &= x_4 + \tfrac{1}{2}\int\left(\frac{2V_1}{\left(\frac{\partial V_1}{\partial r}\right)^2} - 2\gamma r^4 V_1^2\right)^{\frac{1}{2}} \frac{dV_1}{V_1}. \end{aligned}\right. \tag{30}$$

Here V_1 is completely undetermined except that (i) it is a function of r alone, (ii) it agrees with the Newtonian potential to a first approximation apart from the additive constant a.

If, in addition, we impose Kottler's * condition that the determinant of the form (29) equals $-c^2$ when expressed in rectangular coordinates, we readily deduce that $V_1 = -\sqrt{(a+\beta/r)}$, a being constant and $\beta = -2c/\sqrt{-2\gamma}$, and then (29) becomes

$$-\frac{c^2}{2V_1}dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2 + 2V_1 dy_4^2, \tag{31}$$

which is equivalent to the Einstein-Schwarzschild form.

A similar discussion can be easily made if the second form of Laplace's equation is adopted, the result being that V has the form given in (19) and that (29) is replaced by

$$\frac{r^4}{m^2}\left(\frac{dV}{dr}\right)^2 dr^2 + r^2\theta d^2 + r^2\sin^2\theta d\phi^2 + 2V_1 dy_4^2.$$

* F. Kottler, *Ann. d. Phys.*, lvi (1918), p. 401.

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III.—Æther and the Quantum Theory. By H. Stanley Allen, M.A., D.Sc.

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§ 1. Extreme supporters of the principle of relativity find no place for an æther. Thus Lord Haldane in discussing the theory of Einstein writes: "Space, as a physical thing with unvarying geometrical properties, is to be banished, just for the same sort of reasons as the æther was banished before it. Only observable things are to be recognised as real in the new system of modern physicists." On the other hand, many followers of Faraday and Maxwell have regarded æther as the primary real substance; all mass, momentum, or energy being mass, momentum, or energy of the æther. Without entering into a discussion of the significance of physical "reality," we may, with most physicists, agree to use the concept of æther as giving a model, inadequate though it is at present, for the interpretation of physical phenomena.

§ 2. During the present century it has been recognised that certain classes of observations cannot be explained on the basis of Newtonian mechanics, and a new theory has been developed which has proved extraordinarily fertile. "The quantum theory is believed to have disclosed in nature a certain atomicity of a kind unsuspected by the older mechanics." The theory centres round the idea of spasmodic interchanges, losses or gains of energy of amount $h\nu$, where h is Planck's constant, and ν is a frequency (number of oscillations per second). According to Poincaré, the hypothesis of quanta is the only one leading to the law of Planck which represents the distribution of energy between different wave-lengths in "black-body" radiation. In his invaluable report on Radiation and the Quantum Theory, Jeans has pointed out that if the æther is part of the dynamical system, then the energy in the æther must be treated as part of the energy of the system, and to arrive at Planck's formula it would appear to be necessary to suppose that the vibrations in the æther themselves gain or lose energy by whole quanta. The assumptions underlying the quantum theory have been stated in somewhat different forms by various theoretical physicists. For our present purpose we may adopt the hypotheses proposed by W. Wilson,* as these have been employed

* W. Wilson, *Phil. Mag.*, vol. xxix, p. 795 (1915); vol. xxxi, p. 156 (1916).

by Sommerfeld * with marked success in explaining the fine structure of spectral lines of hydrogen and helium. During certain intervals each dynamical system behaves as a conservative one, and between these intervals are relatively very short ones during which definite amounts of energy may be emitted or absorbed. The motion of a system in the intervals between such discontinuous energy exchanges is determined by Hamiltonian dynamics as applied to conservative systems. Let $q_1, q_2, \ldots$ $p_1, p_2, \ldots$ be the Hamiltonian positional and impulse coordinates of a system in one of its steady states. The kinetic energy, T, can be expressed as a quadratic function of the form

$$\begin{aligned} T &= \tfrac{1}{2}A_1\dot{q}_1^2 + \tfrac{1}{2}A_2\dot{q}_2^2 + \ldots \\ &= T_1 + T_2 + \ldots \end{aligned}$$

Further, $2T_1 = \dot{q}_1 p_1$, $2T_2 = \dot{q}_2 p_2$, etc.; p_1, p_2 being Hamiltonian "moments" corresponding to the canonical coordinates q_1, q_2. Wilson's hypothesis is that the discontinuous energy exchanges always occur in such a way that the steady motions satisfy equations of the form

$$\int p_1 dq_1 = n_1 h \quad \text{or} \quad 2\int T_1 dt = n_1 h,$$

the integration being extended over the period corresponding to the q considered, and n being a positive integer (including zero).

§ 3. Planck's constant h, which has the value $6{\cdot}558 \times 10^{-27}$ erg sec., may be regarded as a quantum of action. It is, however, simpler to look upon this universal constant as an angular momentum, a view suggested by J. W. Nicholson † in June 1912, and employed by Bohr ‡ in his theory of the origin of spectral lines. S. B. M'Laren § identified this natural unit of angular momentum with the angular momentum of the magneton. "Rejecting entirely the idea of magnetic or electric substance, the magneton may be regarded as an inner limiting surface of the æther, formed like an anchor-ring. The tubes of electric induction which terminate on its surface give it an electric charge, the magnetic tubes linked through its aperture make it a permanent magnet." For a magneton of any shape of cross-section the angular momentum, according to M'Laren, is proportional to the product of the number of tubes of electric induction, N_e, and the number of tubes of magnetic induction, N_m. In a paper communicated to the *Philosophical Magazine* I have shown

* Sommerfeld, *Ann. d. Physik*, vol. li, pp. 1, 125 (1916).
† Nicholson, *Monthly Notices, R.A.S.*, June 1912.
‡ Bohr, *Phil. Mag.*, vol. xxvi, pp. 1, 476 (1913).
§ M'Laren, *Nature*, vol. xcii, p. 165 (1913).

that this result may be proved in a very general way, and that in ordinary units we have

$$\text{Angular momentum} = \frac{1}{2\pi} N_e N_m.$$

It is to be noted that this expression is quite independent of the quantum hypothesis. If we now apply the quantum theory to this case we have

$$\int_0^{2\pi} p d\phi = nh,$$

or as p, the angular momentum, is constant

$$2\pi p = nh.$$

Identifying these two expressions for the angular momentum, we obtain

$$N_e N_m = nh.$$

If we regard the charge of the magneton as equal to ϵ, the electron charge, the relation may be written

$$N_m = n\left(\frac{h}{\epsilon}\right),$$

or the number of magnetic tubes passing through the aperture of the magneton is directly proportional to an integer n.

§ 4. It seems probable that this result may be applied not only to M'Laren's magneton, but also to the case of a classical electron circulating in a closed orbit. Such an extension has in fact been suggested in an interesting, but not altogether convincing, paper by A. L. Bernoulli.* This author has given an electrodynamic interpretation of Planck's constant by introducing a principle which he terms the "Principle of the Universal Flux of Induction," defined as follows:—"If electrons are moving in identical closed trajectories in a molecular magnetic field, the number of lines of force cut by the radii vectores at each revolution is one and the same universal constant." In other words, all the electron-resonators are traversed by a like tube of magnetic force. The product of the induction flux and the charge is equal to Planck's constant.

In the paragraph immediately following I have attempted to give a more general proof of this principle, with the object of avoiding as far as possible particular assumptions as to the character of the electrical distribution.

§ 5. Consider a system composed of any number of point charges e_1, e_2, . . . rotating with angular velocity ω about a common axis. These will be the starting-points of electrostatic tubes rotating about the same axis.

* Bernoulli, *Archives des Sciences*, vol. xlii, p. 24 (1916).

Such an electrical system might correspond to an atom in one of its steady states. The equivalent mass per unit volume of a tube is $4\pi\mu D^2 \sin^2\theta$, where D is the electric polarisation or displacement, and θ is the angle between the direction of the tube and its velocity. Hence the angular momentum for unit volume of the tube is $4\pi\mu D^2 \sin^2\theta r^2\omega$. The moving Faraday tubes are accompanied by a magnetic field, at right angles to their length and to their direction of motion, given by $H=4\pi D \sin\theta r\omega$. Hence $D\sin\theta r = H/4\pi\omega$, and the angular momentum for unit volume of the tube may be written $4\pi\mu\omega\left(\frac{H}{4\pi\omega}\right)^2 = \frac{2}{\omega}\times\frac{\mu H^2}{8\pi}$. Hence the total angular momentum of the system takes the form $\frac{2}{\omega}\sum\frac{\mu H^2}{8\pi}$, the summation extending over the whole space occupied by the magnetic tubes.

If the frequency of rotation be sufficiently high, the movement of the charges e_1, e_2, . . . may be regarded as equivalent to currents i_1, i_2, . . . where $i_1=\frac{e_1\omega}{2\pi}$, $i_2=\frac{e_2\omega}{2\pi}$, . . .

The sum $\sum\frac{\mu H^2}{8\pi}$, which represents the electrokinetic energy, may be expressed in the form $\frac{1}{2}L_1 i_1^2 + \ . \ . + M_{12}i_1 i_2 + \ . \ . \ .$, where L_1 is the self-inductance for the circuit i_1, M_{12} the mutual inductance for the circuits i_1, i_2, etc.

Hence the total angular momentum

$$=\frac{L_1 i_1^2}{\omega}+\ .\ .\ +\frac{2M_{12}i_1 i_2}{\omega}$$

$$=\sum\frac{i_1}{\omega}(L_1 i_1 + M_{12}i_2 + \ .\ .)$$

$$=\sum\frac{e_1}{2\pi}N_1,$$

where $N_1 = L_1 i_1 + M_{12} i_2 + \ . \ .$, and denotes the total number of magnetic tubes passing through the circuit i_1.

In this case the application of the quantum theory to the steady state gives the result $\int_0^{2\pi} p\,d\phi = nh$ (n an integer), since the integration is to be extended over the full period, which is common to all the rotating charges. Hence

$$2\pi p = nh.$$

Identifying p with the above expression for the angular momentum we find

$$\sum e_1 N_1 = nh.$$

According to the electron theory, the charges e_1, . . . must be integral multiples of the fundamental charge ϵ, so that $e_1 = k_1\epsilon$, where k_1 is an integer, which may be either positive or negative. Therefore

$$\sum k_1 N_1 = n(h/\epsilon).$$

The simplest interpretation to give to this result is to regard the quantity h/ϵ as defining a unit magnetic tube; the equation then expresses the fact that in the steady state of such a system the number of magnetic tubes is an integral number of such unit tubes.

If we now generalise this result we may say that as, according to the quantum theory, the steady motions of any dynamical system satisfy equations of the form

$$\int pdq = nh,$$

such a steady motion is always associated with an integral number of unit magnetic tubes. In fact we postulate an atomicity not merely of electric charges, but of magnetic tubes.

§ 6. The unit magnetic tube is determined by the ratio of Planck's constant, h, to the fundamental electron charge, ϵ. Taking $h = 6{\cdot}558 \times 10^{-27}$ erg sec. and $\epsilon = 4{\cdot}774 \times 10^{-10}$ E.S.U.,

$$\text{Unit magnetic tube} = \frac{h}{\epsilon} = \frac{6{\cdot}558 \times 10^{-27}}{4{\cdot}774 \times 10^{-10}} = 1{\cdot}374 \times 10^{-17} \text{ E.S.U.}$$

Assuming for the velocity of light the value $c = 2{\cdot}999 \times 10^{10}$ cm./sec., we find in electromagnetic measure one unit magnetic tube $= 4{\cdot}120 \times 10^{-7}$ C.G.S.

Thus one C.G.S. line of magnetic force (one "Maxwell") contains $2{\cdot}43 \times 10^6$ unit tubes, or "quantum tubes" as they might be termed.

§ 7. Since there is a strong presumption that things which can be counted must have a physical existence, we are led to regard these discrete quantum tubes of magnetic force as physical realities. In view of the fact that one C.G.S. magnetic line is a bundle of nearly two and a half million quantum tubes, it is scarcely surprising that the smaller tubes have not revealed their existence as such in ordinary magnetic experiments. There is, after all, nothing novel in the suggestion of the actual existence of magnetic lines of force. Students of Faraday's *Experimental Researches* will find that in vol. iii he insists again and again on this point of view. Starting from the consideration of such lines in the abstract "as expressing accurately the nature, condition, direction, and amount of the force in any given region either within or outside of the bar [magnet]," he passes on to the inquiry "of the possible and probable *physical existence* of such lines" (p. 438). If the existence of *curved* lines of magnetic force be conceded, then the physical nature of the lines must be granted also. The question of the existence of physical lines of force he regards as "both important, and likely to be answered ultimately in the affirmative" (p. 437). Again, in the celebrated letter published under the title "Thoughts on ray-vibrations" (p. 447) he writes: "The view which I am so bold as to put forth considers, therefore, radiation as a high species of vibration in the lines of force which are known to connect particles and also masses of matter together. It endeavours to dismiss the æther, but not the vibration." "It may be asked, what lines of force are there in nature which are fitted to convey such an action and supply for the vibrating theory the place of the æther?" "The lines of weight or gravitating force are, certainly, extensive enough to answer in this respect any demand made upon them by radiant phænomena; and so, probably, are the lines of magnetic force. . . ."

As is well known, Clerk Maxwell developed and gave mathematical expression to the views of Faraday, and one of his earliest papers deals with the illustration of Faraday's lines of force by the lines of flow of a liquid (1856). As early as 1857 Faraday was speculating whether the velocity of propagation of magnetic action is of the same order as that of light, but Maxwell's memoir, entitled "A Dynamical Theory of the Electromagnetic Field," in which the electromagnetic theory of light was formulated, was not completed till 1864.

Sir Joseph Thomson has developed extensively the theory of moving tubes of force, but unlike Faraday he regarded magnetism as a secondary effect, ascribing magnetic fields, not to the presence of magnetic tubes, but to the motion of electric tubes. The conception of lines of electric force stretching from positive to negative charges presents certain difficulties, as in the case where magnetic fields occur without any manifestation of electric force. In order to account for such facts it may be supposed that two sets of Faraday tubes exist starting from positive and negative charges respectively. In a steady magnetic field the positive and negative tubes may be thought of as moving in opposite directions with equal velocities. Such difficulties have led many physicists to reject the idea of the discrete existence of electrostatic tubes, in spite of the fact that "they give a natural and simple explanation of the electrokinetic momentum in the æther" (Jeans).

The results of the present paper suggest that the magnetic tubes may be more fundamental than the electrostatic.

§ 8. Faraday's proposal to "dismiss the æther" seems to have reference to the particular view of the æther then prevalent. His own hypotheses as to the existence of lines of force may be regarded as leading to a new mode of representing the luminiferous medium. During the latter half of the nineteenth century many attempts were made, especially by British physicists, to construct mechanical models to illustrate the properties of the æther. These are described in Sir Oliver Lodge's *Modern Views of Electricity* (1907), and in Professor Whittaker's *History of the Theories of Æther and Electricity* (1910). The rotatory character of magnetism, suggested to W. Thomson (Kelvin) by a study of the magnetic rotation of the plane of polarisation of light, was adopted by Maxwell,* and elaborated in a series of papers on "Physical Lines of Force," in which a theory of molecular vortices was applied to magnetic phenomena. He assumed the existence in the magnetic field of vortices or eddies, having their axes in the direction of the lines of force, and having their direction of rotation determined by that of the lines of force. The vortices are separated from each other by a single layer of round particles, which are in rolling contact with both the vortices which they separate, and, acting as "idle wheels," allow neighbouring vortices to revolve in the same direction. This conception, says Maxwell (p. 346), "may appear somewhat awkward. . . . It is, however, a mode of connection which is mechanically conceivable and easily investigated, and it serves to bring out the actual mechanical connections between the known electromagnetic phenomena; so that I venture to say that anyone who understands the provisional and temporary character of this hypothesis, will find himself rather helped than hindered by it in his search after the true interpretation of the phenomena."

The theory of vortex motion has been applied in constructing models of the æther by W. Thomson (Kelvin),† FitzGerald,‡ and Hicks.§ A fluid in which portions in rotational and irrotational motion are finely mixed together has been termed a vortex-sponge. Laminar disturbances are propagated through such a vortex-sponge in the same manner as waves of distortion in a homogeneous elastic solid. Kelvin came to the conclusion later that hollow vortices are better adapted than ordinary vortex filaments for the construction of such models.

A model of another type is found in Kelvin's || gyrostatic model of the luminiferous working of the æther. This type has been developed and its limitations pointed out by Larmor.¶ In a lecture delivered before the

* Maxwell, *Phil. Mag.*, vol. xxi, pp. 161, 281, 338 (1861).

† Kelvin, *Brit. Assoc. Report*, p. 473 (1880); *Math. and Phys. Papers*, vol. iv, p. 308.

‡ FitzGerald, *Scient. Proc. Roy. Dublin Soc.* (1885).

§ Hicks, *Brit. Assoc. Report*, p. 595 (1895).

|| Kelvin, *Math. and Phys. Papers*, vol. iii, p. 466.

¶ Larmor, *Æther and Matter*, Appendix E.

International Congress of Mathematicians at Strasbourg on September 23, 1920, Larmor said: "The vortex theory and the elastic solid æther theory had had their day, but there was no reason at present why we should not admit the existence of an æther—a new æther the properties of which were so different from those of ordinary matter that they could be expressed only in terms of non-Euclidean space. The alternative was complete abstraction, the absence of a basis on which to found our theories." *

The difficulties to be faced on the older view of the æther in connection with the principle of relativity have been clearly explained by N. R. Campbell in his books on *Modern Electrical Theory*.† "Our original idea of the æther was 'the body in which radiant energy is localised'; if, in that statement, we put 'bodies' for 'body,' all our difficulties vanish; if the energy from different sources is localised in different bodies, then it is obvious that the velocity of an observer relative to the body in which the energy of one source is localised may be different from that relative to the body in which the energy from another source is localised. If we speak of 'æthers' and not 'the æther,' all our experiments prove is that the particular æther with which we are concerned in any case is that which is at rest relatively to the source and may be regarded as forming part of it." One way of visualising such a conception is by employing the electrostatic tubes of Faraday in the manner sketched by J. J. Thomson. The results of the present paper suggest that the magnetic tubes have at least an equal claim to be considered as constituting "the æthers."

§ 9. Assuming the existence of discrete magnetic tubes in accordance with the hypothesis of Faraday and the indications of the quantum theory, we have to consider the possible nature of the tubes. The method by which the magnitude of the unit tube has been arrived at suggests unmistakably that the distinguishing feature of the tube is spin about an axis in the direction of its length. This is essentially the view developed by Maxwell in his theory of molecular vortices, but it is not necessary to adopt the particular mechanism by means of which neighbouring vortices were enabled to rotate in the same direction. A classical electron moving round the periphery of the spinning tube is, however, not unlike one of the "idle wheels" imagined by Maxwell. This mode of representation resembles that suggested by Larmor in the Physical Society Discussion on the Ring Electron (1918)—"One or more electrons constrained to move round a channel would be like an amperean current. It is not unlikely

* *Nature*, vol. cvi, p. 196 (1920).

† *Modern Electrical Theory*, First Edition, ch. xiv; Second Edition, Appendix I.

that constraint of this kind will have to be introduced into molecular models to give an account of paramagnetism and ferromagnetism—namely, structure in space or atom involving channels more or less definite for the electrons to circulate in." It would seem that such a view is consistent with Bohr's theory of stationary orbits and with the later developments of that theory by Sommerfeld and others in connection with the fine structure of spectral lines. In these applications of the quantum theory it is not merely the size of the orbit, but also its shape and its position in space which must be determined by the quantum principle, and can assume certain prescribed values only. Consequently, as Sommerfeld puts it, the stationary paths of the electrons in the atom, and also in the molecule, form not a continuum but a network. The "phase-space" is interwoven by the curves of the stationary paths as by the meshes of a net. The size of the mesh is determined by Planck's h.

On the present hypothesis the network is formed by the magnetic tubes, between and around which the electrons are able to circulate. It may prove necessary to proceed a step further and suppose that the tubes can be divided up in the direction of their length so as to form "æther cells," but such a subdivision need not at present be discussed, beyond pointing out that in considering how the theory of molecular vortices could be applied to statical electricity Maxwell * assumed the rotating matter to be the substance of certain cells, such substance possessing elasticity of figure. On this hypothesis he was able to explain electrostatic phenomena, and showed that the velocity of transverse undulations in a medium so constituted was in almost exact agreement with the velocity of light, so that "we can scarcely avoid the inference that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena."

§ 10. It is commonly assumed that an explanation of magnetic phenomena can be obtained from the theory of the classical electron on the basis of ordinary mechanical laws. That this supposition is incorrect was shown by W. Voigt,† who proved that, assuming the electronic motions are undamped, changes in the velocities of the electrons in consequence of the formation of magnetic fields will not give rise to magnetic phenomena. This result, recognised by Bohr ‡ in his application of the quantum theory to an atomic model, received confirmation in the work of M'Laren,§ who

* Maxwell, *Phil. Mag.*, vol. xxiii, p. 12 (1862). It should be noted that this paper is two years earlier than that which gives "A Dynamical Theory of the Electromagnetic Field."

† Voigt, *Annalen d. Physik*, vol. ix, p. 115 (1902).

‡ Bohr, *Studier over Metallernes Elektrontheori*, p. 106.

§ M'Laren, *Phil. Mag.*, vol. xxv, p. 43 (1913).

proved that the classical dynamics cannot explain magnetism. No one, in fact, has been able to explain how an electron orbit can be tilted under the influence of a magnetic field.

Sir Alfred Ewing has directed attention recently to the resemblance between the form of the curve showing the relation between specific heat and temperature deduced from Planck's theory and that representing the magnetisation of a specimen of iron. The resemblance in mathematical form between Planck's equation and that of Langevin for the intensity of magnetisation of a paramagnetic gas had previously attracted the attention of Benedicks.* These similarities suggest that the same type of action underlies the quantum theory as the various physical phenomena here referred to.

All these results seem to be in harmony with the view put forward in the present paper, that the "quantum" is itself essentially magnetic. It must not, however, be supposed that the view "reconciles" the quantum theory and classical dynamics. My object is rather to seek to understand more clearly the nature of the quantum, whilst accepting the conclusion that some modification of the old theories is inevitable.

* Benedicks, *Annalen d. Physik*, vol. xlii, p. 133 (1913).

(*Issued separately January* 24, 1921.)

IV.—On the Transverse Galvanomagnetic and Thermomagnetic Effects in several Metals. By F. Unwin, M.Sc. *Communicated by* Professor F. G. Baily.

(MS. received November 10, 1920. Read January 10, 1921.)

Introduction.

In a previous paper * the author has described experiments to determine the Hall effect and the two transverse thermomagnetic effects in several metals. The fourth transverse effect—namely, the galvanomagnetic temperature effect—was not measured at that time, as the measuring arrangement was not sufficiently sensitive for this purpose.

In the present state of the electron theory of metals the ratios of these transverse effects to each other is of very considerable interest, and hence it is desirable to have determinations of all four transverse effects. Determinations have been made by other workers, but, so far as the author is aware, only in the case of a few metals. The present paper records the determinations for eight different metals—iron, nickel, cobalt, silver, copper, zinc, cadmium, aluminium.

Description of Effects and Definition of Coefficients.

Galvanomagnetic Effects.

Suppose a strip of metal, carrying an electric current, to be placed between the poles of an electromagnet. When the magnet is excited it is found that there are set up between the edges of the plate a difference of potential and a difference of temperature. These constitute respectively the Hall Effect and the Galvanomagnetic Temperature Effect. The coefficients R and P are the values of these effects in a plate 1 cm. wide placed in magnetic field of unit strength when the potential gradient along the plate is 1 c.g.s. unit per cm.

Thermomagnetic Effects.

If with the above arrangement a heat current is sent along the plate instead of the electric current, it is again found that a transverse potential difference and a temperature difference are set up.

These constitute the Thermomagnetic "Potential Effect" and "Temperature Effect" respectively, and the coefficients Q and S are the magnitudes of these effects under the standard conditions named, the temperature gradient along the plate being one degree Centigrade per cm.

* *Proc. Roy. Soc. Edin.*, vol. xxxiv, p. 208 (1914).

Signs of the Effects.

The diagram (fig. 1) indicates the directions of the effects corresponding

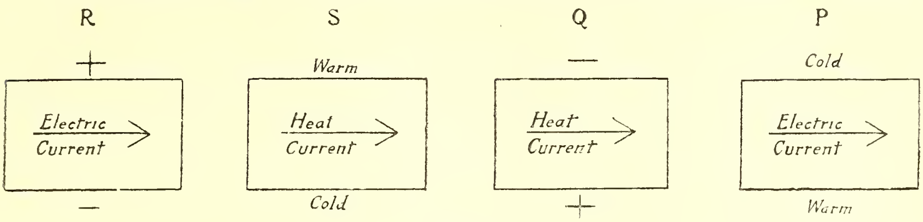

Fig. 1.

to positive values of the coefficients; it being understood that the magnetic field is acting downwards perpendicular to plane of paper.

Apparatus.

The apparatus and measuring arrangements have been described in detail in the author's earlier paper, and the description need not be repeated here. In dealing with the galvanomagnetic temperature effect it was found necessary to introduce a modification in order to make the measurement possible. This effect consists in a temperature difference set up between the two edges of a small plate of metal, carrying an electric current, when

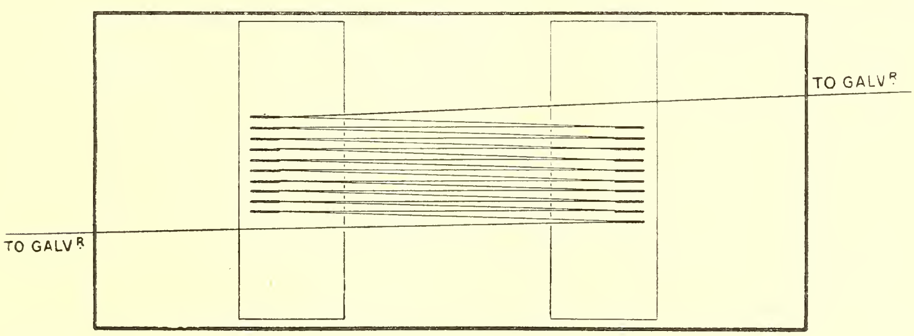

Fig. 2.

it is placed in a magnetic field whose direction is perpendicular to the plane of the plate. This temperature difference is extremely small, and cannot be measured in the usual way by thermocouples attached directly to the plate, since with this arrangement the deflections of the galvanometers in circuit with the couples are due mainly, not to the temperatures of the junctions, but to electromotive forces set up by the current in the plate. Under these circumstances it was useless to increase the sensibility of the galvanometers.

The difficulty was overcome by using, in place of the two couples, a small thermopile of ten pairs of junctions. This was built up (see fig. 2) on a support, and arranged so that one set of junctions could be pressed against one edge of the plate and the other set against the other edge, a very thin sheet of mica being placed between the junctions and the plate to prevent any direct action of the electric current.

This sheet of mica, of course, prevents perfect thermal contact between the junctions and the plate, and a large correction is necessary on this account. The values of the galvanomagnetic temperature effect have therefore not the same degree of accuracy as the other transverse effects. Nevertheless they are sufficiently accurate to allow interesting conclusions to be drawn in connection with the electron theory.

The thermopile readings were corrected in the following manner :—

The value of the coefficient S was calculated from readings taken with the thermocouples before the thermopile was placed on the plate. It was also calculated from observations made with the thermopile. The ratio of these two values of S was taken as the correcting factor for the thermopile readings in the determination of P.

Results.

The values of the four coefficients, R, S, Q, P, for the eight metals tested, and the values of the ratios, $\frac{R}{S}, \frac{Q}{P}, \frac{R}{P}, \frac{Q}{S}$, are given in the following tables.

	$R \times 10^7$.	$S \times 10^7$.	$Q \times 10^4$.	$P \times 10^{13}$.
Iron	+6·1	+5·2	+ 9·5	+30
Nickel . . .	−3·3	−2·5	−10·0	−34
Cobalt . . .	+2·5	+1·1	− 7·8	−22
Silver . . .	−3·2	−2·7	+ 1·8	+ 6·3
Copper . . .	−2·8	−2·1	+ 1·9	+ 8·2
Zinc	+1·2	+1·1	+ 0·73	+ 4·2
Cadmium . . .	+1·2	+0·89	+ 1·2	+ 3·9
Aluminium . .	−1·0	−0·62	− 0·42	− 2·8

	$\frac{R}{S}$.	$\frac{Q}{P} \times 10^{-8}$.	$\frac{R}{P} \times 10^{-5}$.	$\frac{Q}{S} \times 10^{-3}$.
Iron	+1·2	+3·2	+2·0	+1·8
Nickel . . .	+1·3	+2·9	+0·97	+4·0
Cobalt . . .	+2·2	+3·4	−1·1	−7·3
Silver . . .	+1·2	+3·0	−5·1	−0·68
Copper . . .	+1·3	+2·3	−3·4	−0·95
Zinc	+1·1	+1·7	+2·9	+0·68
Cadmium . . .	+1·3	+3·0	+3·0	+1·3
Aluminium . .	+1·6	+1·5	+3·6	+0·68

Discussion of Results.

An inspection of the table shows that the four effects have not always a common sign, but that in some cases two of the effects are positive and two negative.

Zahn * put forward the suggestion that, while R and S may have different signs from Q and P, it will always be found that R has the same sign as S, and Q the same sign as P,† so that the ratios $\frac{R}{S}$ and $\frac{P}{Q}$ are always positive.

Zahn appears to have drawn this conclusion from slender experimental evidence, but its truth is borne out by the observations recorded in this paper.

It will be observed that the values of $\frac{R}{S}$ and $\frac{Q}{P}$ do not vary very widely from metal to metal, although the individual effects vary both in magnitude and sign. The ordinary electron theory of conduction in metals, taken in conjunction with J. J. Thomson's suggestion of a local magnetic field in the immediate neighbourhood of a molecule, is sufficient to account for the variation in the sign of the effects from metal to metal, but it cannot account for the difference between the signs of R and S, and those of Q and P, in the same metal.

For this some other modification of the theory must be sought.

G. H. Livens ‡ has recently developed a theory of conduction in metals in which he arrives at the four expressions quoted below:

$$R = \frac{\Gamma\left(\frac{3}{2}+\frac{4}{s}\right)}{\Gamma\left(2+\frac{2}{s}\right)} \cdot \frac{e l_m q^{\left(\frac{1}{2}-\frac{2}{s}\right)}}{mc},$$

$$S = \frac{7s^2+8s+16}{8s(s+1)} \times \frac{\Gamma\left(\frac{3}{2}+\frac{4}{s}\right)}{\Gamma\left(2+\frac{2}{s}\right)} \cdot \frac{e l_m q^{\left(\frac{1}{2}-\frac{2}{s}\right)}}{mc},$$

$$Q = -\left(\frac{2}{s}-\frac{1}{2}\right)\frac{\Gamma\left(\frac{3}{2}+\frac{4}{s}\right)}{\Gamma\left(2+\frac{2}{s}\right)} \cdot \frac{\mathsf{R} l_m q^{\left(\frac{1}{2}-\frac{2}{s}\right)}}{mc},$$

$$P = -\frac{e^2 l_m}{mc\mathsf{R}} \cdot \left(\frac{\frac{2}{s}-\frac{1}{2}}{2+\frac{2}{s}}\right) \cdot \frac{\Gamma\left(\frac{3}{2}+\frac{4}{s}\right)}{\Gamma\left(2+\frac{2}{s}\right)} \cdot q^{\left(\frac{1}{2}-\frac{2}{s}\right)},$$

* *Ann. d. Phys.*, vol. xiv, p. 886 (1904).

† Zahn's actual statement is that Q and P have always opposite signs, but the definition of the positive direction of P used by the author is opposite to that used by Zahn.

‡ *Phil. Mag.*, vol. xxx, p. 526 (1915).

where $q = \frac{3}{2u_m^2}$, u_m^2 being the mean square velocity of the electrons; and s is given by the relation

$$\frac{m}{2}\left(\frac{\mu}{r}\right)^s = \text{potential energy relative to atom at distance } r.$$

It will be seen that the signs of Q and P vary according as s is greater or less than 4, while the signs of R and S are always the same; so that, taken in conjunction with J. J. Thomson's suggestion, this statement of the theory is capable of accounting for at least the signs of the four effects.

The four expressions given above are rather complex, but by taking ratios we obtain the values:

$$\frac{\mathrm{R}}{\mathrm{P}} = -\frac{\mathsf{R}}{e} \cdot \frac{\left(2+\frac{2}{s}\right)}{\left(\frac{2}{s}-\frac{1}{2}\right)}.$$

$$\frac{\mathrm{Q}}{\mathrm{S}} = -\frac{\left(\frac{2}{s}-\frac{1}{2}\right)}{\frac{7s^2+8s+16}{8s(s+1)}} \cdot \frac{\mathsf{R}}{e}.$$

These are comparatively simple, and the only unknown is s, which can be calculated from the ratios found by experiment.

The following table gives the values of s for the different metals, calculated from the observed values of $\frac{\mathrm{R}}{\mathrm{P}}$ and $\frac{\mathrm{Q}}{\mathrm{S}}$ respectively.

	Values of s.	
	From $\frac{\mathrm{R}}{\mathrm{P}}$.	From $\frac{\mathrm{Q}}{\mathrm{S}}$.
Iron . . .	5·7	5·7
Nickel . . .	10·0	9·7
Cobalt . . .	2·4	1·8
Silver . . .	3·6	3·6
Copper . . .	3·3	3·5
Zinc . . .	5·1	4·6
Cadmium . .	5·1	5·2
Aluminium . .	4·8	4·6

The two values of s agree reasonably well except in the case of cobalt.*

The value of s for silver differs widely from the value for nickel, and

* The cobalt plate was prepared with some difficulty by electrolysis, and was not so satisfactory as the plates of the other metals used. On this account the results for cobalt cannot be considered as having the same accuracy as those for the other metals.

consequently the term $q^{\left(\frac{1}{2}-\frac{2}{s}\right)}$, which enters into the expression for the Hall effect, will differ greatly in value from the one metal to the other. This is difficult to reconcile with the experimental fact that the Hall effect has about the same value in both metals.

Summary.

The four transverse galvanomagnetic and thermomagnetic effects were determined in the case of iron, nickel, cobalt, silver, copper, zinc, cadmium, aluminium.

The results were considered in the light of the electron theory of conduction in metals as formulated by G. H. Livens, and were found to be in fair accord with the theory as regards the *ratios* of the effects, but they could not be reconciled with the expressions given for the effects themselves.

(*Issued separately March* 17, 1921.)

V.—Observations on the Interruption of the Endodermis in a Secondarily Thickened Root of Dracæna fruticosa, Koch. By Annette G. Mann, B.Sc. *Communicated by* Professor F. O. Bower, F.R.S.

(MS. received October 25, 1920. Read December 6, 1920.)

It is generally accepted that the position and development of any one tissue in a plant is to be studied from the point of view of function: nevertheless the physiological need for it may alter as the plant develops. The endodermis is an illustration of this. Its primary function (1) is to restrict the passage of water and soluble substances to certain definite channels: it follows that its best development is in those parts nearest to the source of supply, *e.g.* in roots and young stems. The cell-walls are at first relatively thin, with the characteristic suberised strip on the radial and transverse walls. Its cells are in uninterrupted contact one with another, there being no intercellular spaces. This leads incidentally to its being also a barrier to gaseous interchange. The ventilating system of the cortex is limited by it internally, and cut off from such intercellular spaces as may lie within.

A secondary function which is mechanical is often assumed by it, as shown by Schwendener (2). In older roots the cell-walls, particularly the radial and inner tangential walls, become indurated and resistant. When in this latter condition the endodermis forms a rigid structure enclosing the stele, and forming a very effective barrier between stele and cortex.

It has been shown by Professor Bower (3) that size is a factor which has to be considered in this connection. Since the surface of a cylinder varies as the square, and the bulk as the cube of the linear dimensions, the larger the cylinder is, the greater will be the difficulty of adequate transit through the barrier, and a limit may be expected when the barrier must be extended or be interrupted in some way or another, otherwise the check on interchange through the endodermis would become a serious risk. In roots of Dicotyledons the cortex and endodermis peel off as secondary growth occurs. In thickening stems it disappears, though the steps by which this is carried out have never been accurately followed. But the roots of some Monocotyledons, in particular *Dracæna,* offer a favourable opportunity for tracing the disruption. *Dracæna,* which has a well-marked and indurated endodermis, has also, in some of its largest roots, a process of cambial increase similar in its nature to that seen in the stem; accordingly it was selected as an object likely to throw light on the method of breaking down the barrier in an enlarging part.

The following observations were made on transverse hand sections cut from two roots of a plant of *Dracæna fruticosa* grown in Glasgow Botanic Gardens.

The first root examined was about 5 inches in length, and the first

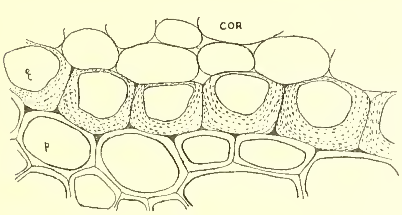

Fig. 1.

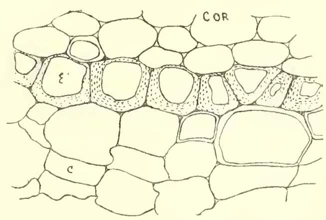

Fig. 2.

section was cut at a distance of $1\frac{1}{2}$ inches from the apex, and section-cutting was continued upwards towards the junction of root and stem. The first sections showed the root to be about $\frac{1}{4}$ inch in its greatest diameter, and the endodermis was a complete, almost regular ring, with all the cells typically thickened on the inner tangential and the two radial walls, the primary stele being undisturbed (fig. 1). The upward sequence

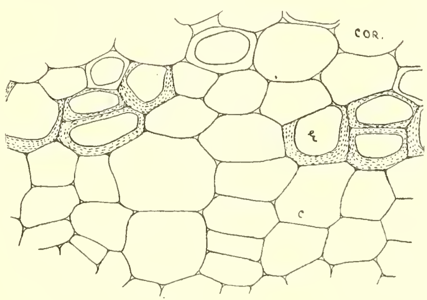

Fig. 3.

of sections from the apex showed a gradual beginning of growth and division of the cells of the pericycle within the endodermis to form a cambium (fig. 2), and, with the slight increase in girth of the root thus brought about, the endodermal cells became somewhat distorted in shape, and the ring itself irregular; then, finally, the sequence of endodermal cells was interrupted (figs. 3, 4, 5). The development of this inner or *internal* cambium began first at one part only of the sections, and was most active in the middle of this area, where the cambial cells divided quickly, and often showed four to six radial rows of cells. Above this area of active growth and division there occurred interruption and separation of the cells in the endodermal ring by the intrusion and growth between the endodermal cells of one or more parenchymatous cells either from the pericycle or from the cortical cells just outside the endodermis (see cells *x*, *x*, fig. 4). In either case there arose bending of the endodermal ring, and displacement of the cells as shown in fig. 5. The next few sections showed a rather quick disappearance of cambial activity until there was practically

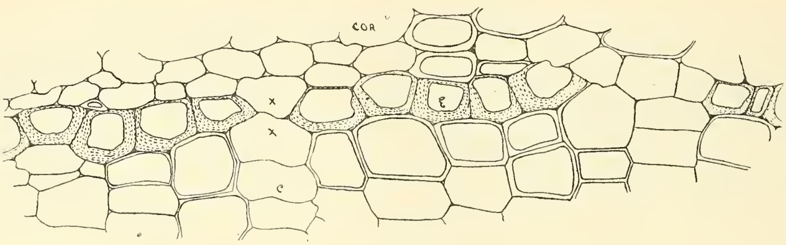

Fig. 4.

no division of pericyclic cells, and the endodermis was again a complete ring of typically thickened cells. Internal cambium then began again with a quick division of its cells, resulting in a considerable amount of separation of the cells of the endodermis by intrusive parenchyma. Again occurred a falling off of cambial activity, and a consequent linking up of the endodermal cells into a more complete ring. There were now $1\frac{3}{4}$ inches

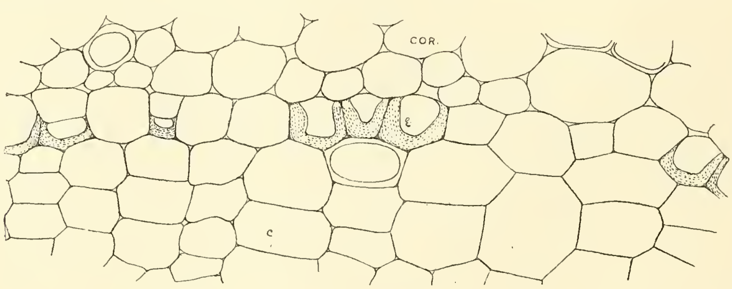

Fig. 5.

of root left uncut, and sections were continued at about $1\frac{1}{4}$ inches from its junction with the stem. The first sections cut here showed still a very small amount of internal cambium, and the endodermis almost complete; but, in addition, there was a slight division of certain cortical cells just outside the endodermis. These formed small isolated patches of outer or *external* cambium made up of ten to twenty cells in two radial rows. The succeeding sections showed the endodermis complete and no cambial development on the inside, but there was increasing cambial development on the outside (fig. 6). The cells often divided by obliquely tangential walls, therefore did not appear at first in even typical radial rows. The external cambium spread gradually round the sections, the cells dividing by periclinal walls, and their arrangement becoming regular and broader, while the endodermis remained undisturbed. A few sections further up showed cambium appearing in parts of the pericycle also, and from this point upwards the internal cambium caused breaking of the endodermal ring in the manner already described and figured. For a considerable number of sections the pericyclic cambium showed only a slight development of secondary tissue, and the external cambium none at all, but about $\frac{3}{4}$ of an inch from the attachment of the root on the stem, there appeared the beginnings of two small secondary bundles—typical xylem elements surrounding central phloem—on the *outside* of the endodermis, which was continuous below them. The following sections showed rapid development of secondary tissue both from external and internal cambium. In consequence of the activity of the latter, single cells or groups of two or more endodermal cells were forced to the outside of the secondary tissue, or were left lying between the bundles in the midst of this tissue. This follows from the division and growth of the cambial cells on either side of an endodermal group being quicker than that of the cells immediately beneath it. The last sections made, about $\frac{1}{4}$ inch from the end of the root, showed internal cambium and development of secondary tissue to a more or less degree practically all round, but the thickening was much more strongly developed on that side of the root which showed both internal and external cambial activity. The endodermis appeared almost completely broken up into patches of cells, and even into isolated cells (Photo II).

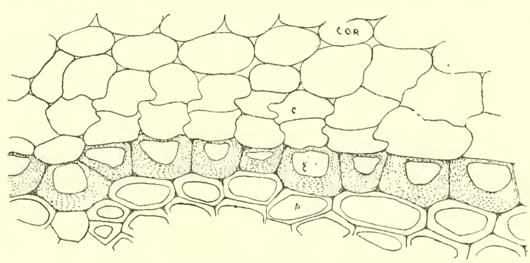

Fig. 6.

The second root examined was of slightly greater diameter than the first throughout all the series of sections, and its tissues had become much more sclerotic and woody, even the pericycle being comparatively thick-walled. The only thin-walled cells apparent were those of the cortex, and of the cambium to the outside of the secondary tissue. The development of cambium first began here in the cortex just outside the endodermis, and spread round, giving rise to secondary bundles to a greater or lesser degree in several isolated places, the endodermis being complete below the bundles (fig. 7). When a considerable amount of *external* secondary tissue had been developed, two or more cells in isolated parts of the pericycle, in conjunction with several tracheids just below, were found to have enlarged

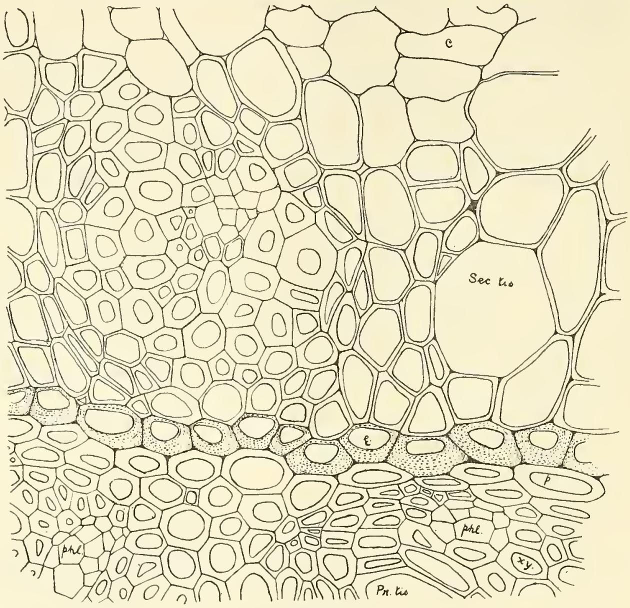

Fig. 7.

and extended outwards towards the endodermis. This enlarging of pericyclic cells appeared in a number of places, the endodermis becoming curved and broken as one cut further up from the root apex. The pressure of these thick-walled cells must have caused first that curving and then that splitting apart of the endodermal cells which is shown in fig. 8 and Photo I. The changes in the endodermal ring due to inside pressure occurred where there was greatest development of secondary tissue outside it, the cells of the secondary conjunctive tissue appearing to assist in the process of separation of its cells (see fig. 8, cells x_1, x_2). When the endodermis had been ruptured and the isolated parts pushed slightly outwards (fig. 9), the pericyclic

Photo I.

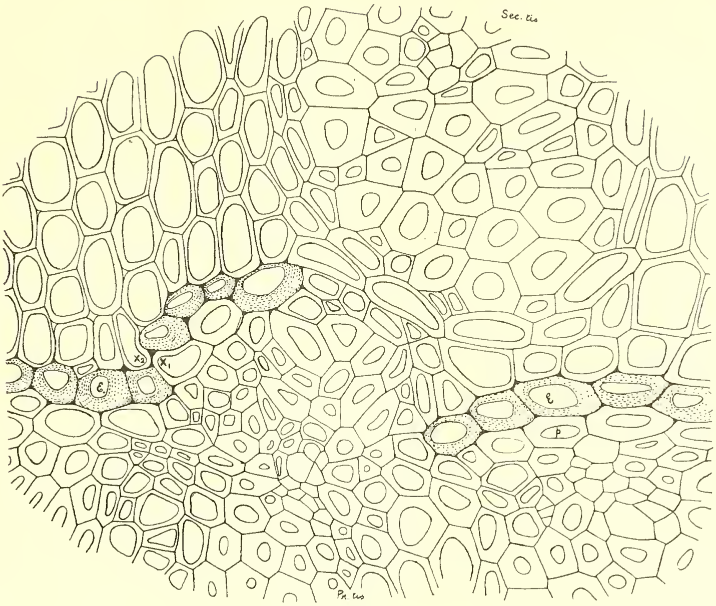

Fig. 8.

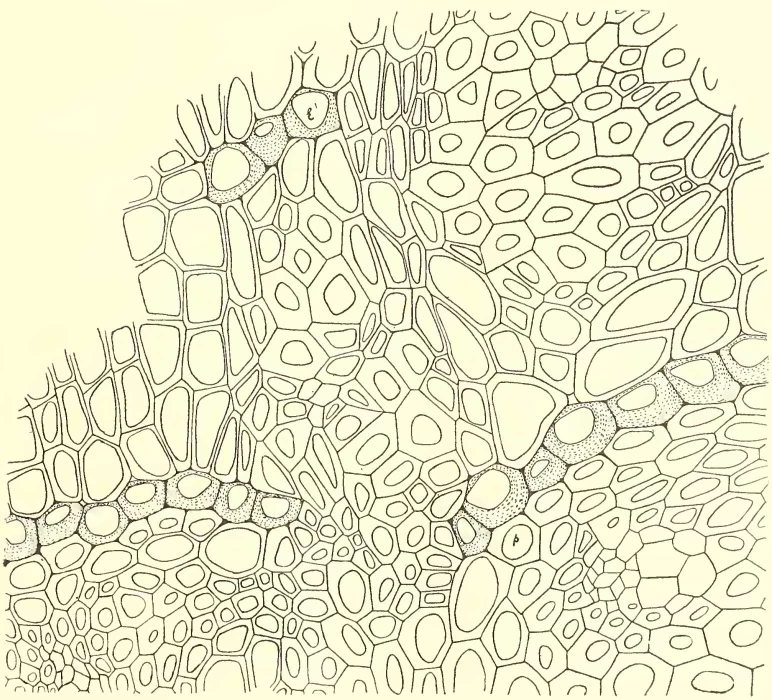

Fig. 9.

cells below the curves, and also elsewhere in the pericycle, where there was previously no external cambium, divided up rapidly by periclinal walls. They thus formed a typical cambium from which were developed secondary bundles and conjunctive tissue. Such developments have already been noted by Haberlandt (1) and verified by Miss Spratt (4) as being of the nature of fibrous tracheids. As secondary growth from

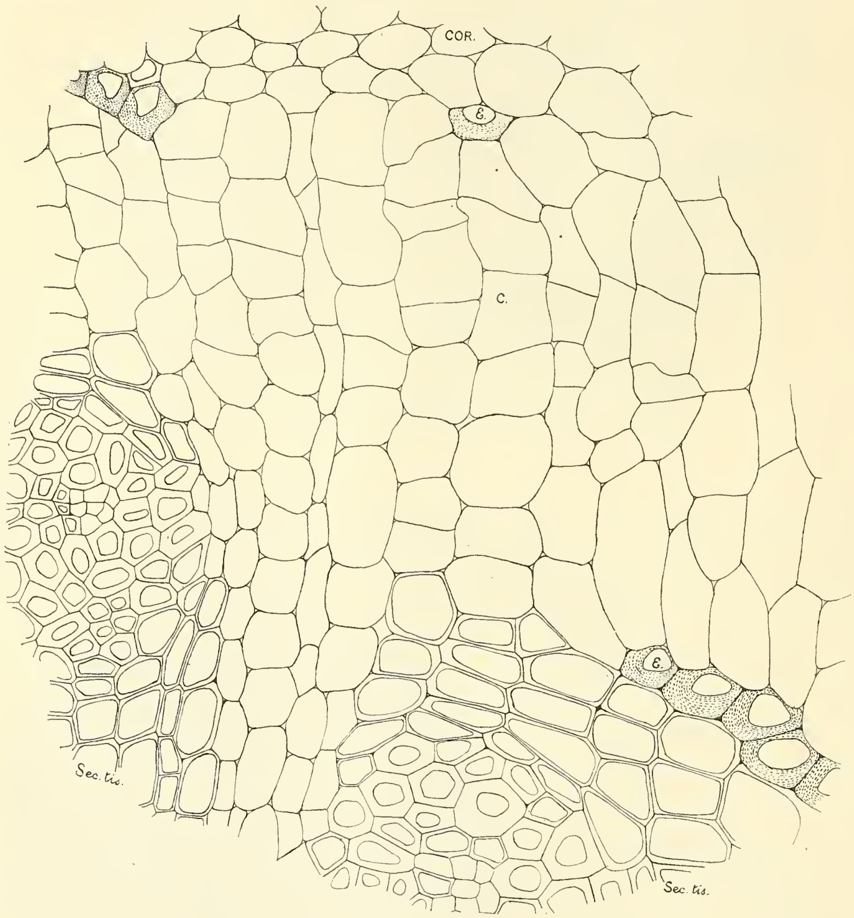

Fig. 10.

the internal cambium spread round and became more active, the secondary tissue formed from it joined up through the gaps in the endodermis with that formed by the external cambium. The endodermal ring now appeared much broken, many of the cells being carried up on the outside of the internal cambium, and left lying in little groups of a few cells each in the midst of the thick-walled conjunctive tissue between the secondary bundles (fig. 10). The last sections cut from root 2 showed a very similar appearance to those of root 1 at the same stage in secondary growth, *i.e.* they showed endodermal strands lying between the secondary bundles, and also on the extreme outside of the secondary tissue where internal cambial activity was very marked (Photo II).

In both roots the pericyclic cambium, once it was fairly established, was most active, whether or no it was developed before the cortical cambium, since endodermal cells were always found external to the cambium outside the greatest development of secondary tissue. There appears to be no rule whether the cambium shall appear first in the pericycle or in the cortex just outside the endodermis. Both the roots examined show that the formation of external cambium does not, as Miss Spratt suggests, necessarily follow after the formation of pericyclic cambium, but may precede it. In that case the endodermal ring still continues to form a serious barrier to ventilation, though there is abundant evidence of intercellular spaces in the secondary tissue. Where the cambium originates first from the inside the parenchyma cells penetrating the endodermis show air-spaces between them (see figs. 3, 4, 5), so that the endodermal barrier to ventilation is broken. But I imagine the primary cause of the interruption is not to allow of greater ventilation of the inner tissue—though this of course follows—but rather is it an adaptation to allow of a greater increase in girth of the root for the formation of new vascular tissues, and for a quicker interchange between the outer and inner tissues of the enlarging root. If greater ventilation was the primary object, then internal cambium would, of necessity, be developed first.

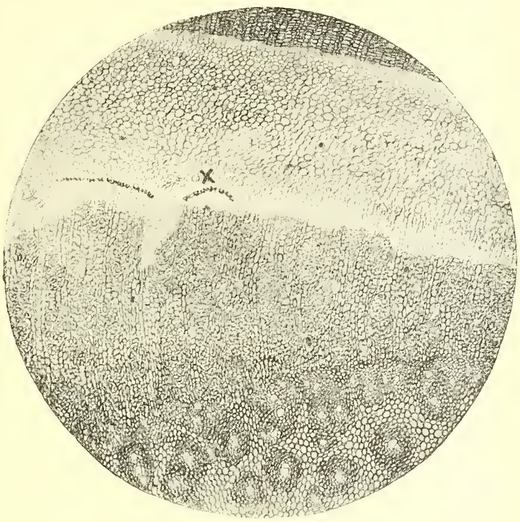

Photo II.

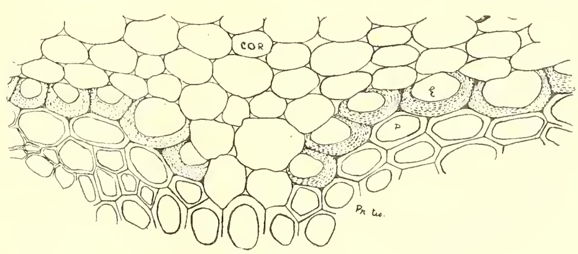

Fig. 11.

Professor Scott and Mr Brebner (5) have described a somewhat similar behaviour of endodermis in the secondarily thickened roots of other species of *Dracœna*, and the phenomenon of the penetration of parenchyma cells between thicker-walled cells, with the consequent splitting apart of the latter, has been shown by others to occur elsewhere. Schwendener (2) figured a cross-section of a root of *Convallaria majalis* showing the interruption of the endodermis by thin-walled parenchyma; and Miss A. M. Clark (6) has described an ingrowing of parenchyma among secondary xylem elements in the stem of *Kendrickia Walkeri*. In fig. 11 I have demonstrated an example of the intrusion of parenchyma from the cortex which seems to correspond in its nature and origin to the case of *Convallaria* mentioned by Schwendener. This intrusion into the endodermis occurred before the appearance of cambium at that point, though secondary thickening was active at the opposite side of the root.

In conclusion, I wish to acknowledge my indebtedness and grateful thanks to Professor Bower for so kindly supervising the work, and to Dr J. M. Thompson for valuable criticism.

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(2) Schwendener, S., *Die Schutzscheiden und ihre Verstärkungen*, 1882.

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(4) Spratt, A. V., Miss, "Some Anomalies in Monocotyledon Roots," *Annals of Botany*, 1920.

(5) Scott and Brebner, "The Secondary Tissues in certain Monocotyledons," *Annals of Botany*, 1893.

(6) Clark, A. M., Miss, "Secondary Thickening in *Kendrickia Walkeri*," *Annals of Botany*, 1907.

ILLUSTRATIONS.

E. = endodermis : p. = pericycle : c. = cambium : phl. = phloem : xy. = xylem : Pr. tis. = primary tissue : Sec. tis. = secondary tissue : cor. = cortex.

Text-Figures.

Fig. 1. Endodermis complete without cambium on either side. × 250.

Fig. 2. Endodermis complete with pericyclic cambium. × 250.

Fig. 3. Separation of endodermal cells by internal cambial activity. Intercellular spaces are numerous in the thin-walled tissue. × 250.

Figs. 4 and 5. Steps in the breaking up and displacement of endodermis by intrusion of parenchyma cells. × 250.

Fig. 6. Endodermis complete, and beginning of external cambium. × 250.

Fig. 7. Endodermis complete, and formation of secondary tissue from external cambium. × 250.

Fig. 8. Endodermis curved and continuity broken by pressure of tissues within. Cells x_1 x_2 show the beginning of separation of endodermal cells. × 250.

Fig. 9. Three endodermal cells are shown pushed out from the ring. The bundle just within was formed from internal cambium ; tissues to right and left from external cambium. × 250.

Fig. 10. Endodermal cells pushed right out to cortex by the activity of internal cambium. Those cells on right have been left behind among secondary elements. × 200.

Fig. 11. Separation of endodermis by intrusion of cortex. × 250.

Photographs.

I. Curving and splitting of endodermis (x) due to inside pressure. × 24.

II. Endodermal cells (x) pushed out to cortex by internal cambial activity. × 24.

(*Issued separately March* 17, 1921.)

VI.—On Fechner's Law and the Self-Luminosity of the Eye. By Professor William Peddie, D.Sc.

(MS. received November 22, 1920. Read November 22, 1920.)

(Abstract.)

Fechner's law states that change of visual perceptivity is proportional to the fractional change in the intensity of the light. At weak intensities a term, regarded as constant, has to be added to the intensity of the external light on account of the self-luminosity of the eye. By integration over the whole stimulated part of the retina, Helmholtz obtained an expression for the perceptivity which agreed with observation in so far as the general nature of the relation between perceptivity and external stimulus is concerned. Close correspondence can be obtained by assuming that the self-luminosity term in Fechner's expression is itself a simple function of the external stimulus, rising rapidly to a maximum and thereafter slowly falling to a steady value.

VII.—The Relation of the Soil Colloids to the Thermal Conductivity of the Soil. By Capt. T. Bedford Franklin, B.A. (Cantab.).

(MS. received December 28, 1920. Read February 7, 1921.)

SUMMARY.

I. Introduction.

Early investigators regarded the soil as an inert framework of soil grains of various sizes covered with a continuous film of water, and the properties which in theory it should possess under such a hypothesis were found not to accord too well with the results of experiment. But when the existence of soil colloids was understood, the differences between theory and experiment tended to disappear one by one, as shown by the work of Bouyoncos in America in 1915 and of Keen in England in 1914, 1919, and 1920.*

These and other investigators have shown that it is essential to take into account the colloidal properties of the soil before its physical properties can be understood properly, and in doing so have cleared up many of the points of difference between experiment and the old theory which was based on a hypothesis that disregarded these soil colloids.

In a previous paper on soil temperature I have discussed the effect of

* "The Effect of Temperature on the most important Physical Processes in Soils." G. J. Bouyoncos, *Technical Bulletin No.* 22, *Michigan Experimental Station*, 1915.

"The Evaporation of Water from Soil," B. A. Keen, *Journal of Agricultural Science*, vol. vi, part iv, Dec. 1914.

"A Quantitative Relation between Soil and the Soil Solution," B. A. Keen, *ibid.*, vol. ix, part iv, Oct. 1919.

"The Relations existing between the Soil and its Water Content," B. A. Keen, *ibid.*, vol. x, part i, Jan. 1920.

rain, snow, frost, a dry surface mulch, etc., on the soil temperatures at various depths; * in this present paper I propose to show that soil temperatures are also affected by soil colloids, since the properties of these colloids bring about changes in the conductivity of the soil.

II. Scope of the Investigation and Effect of Change of Period on Values of $\frac{R_4}{R_0}$.

For this investigation it was essential that variations in the value of $\frac{R_4}{R_0}$ due to any cause other than temperature should be eliminated. To this end hourly readings were made from minimum to maximum only, and no observations were taken on days of precipitation or when the ground was frozen or covered with a dry surface mulch: moreover, to ensure the more or less uniform water content of the soil, observations in summer were made on those days only when rain has fallen in the previous twenty-four hours, so that any observed variations in the values of $\frac{R_4}{R_0}$ were due apparently either to variations in the length of the interval from minimum to maximum or to temperature changes in the soil.

It was assumed that the temperature curves from minimum to maximum could be regarded as half a sine curve of period double the interval from minimum to maximum; we were thus able to use the formula $\frac{R_4}{R_0}=e^{-\frac{10}{h}\sqrt{\frac{\pi}{T}}}$, where $\frac{R_4}{R_0}$ is the ratio of the ranges of temperature at the 4-inch depth and the surface from minimum to maximum, h^2 is the diffusivity of the soil, and T is the period, *i.e.* double the interval from minimum to maximum, reckoned in seconds.†

From chosen observations in June 1920, when the mean surface temperature was 10° C. and the period 24 hours, $\frac{R_4}{R_0}$ in sand and clay loam were found to be ·52 and ·42 respectively.

Thus $\frac{R_4}{R_0}=\cdot 52=e^{-\frac{10}{h}\times \cdot 006}$ for sand, giving a value of ·09 for h in sand; in clay loam the corresponding value for h was ·07.

Assuming that for the rest of the observations these values for h remained constant for the two soils, and this seemed probable since

* "The Effect of Weather Changes on Soil Temperature," T. B. Franklin, *Proc. Roy. Soc. Edin.*, vol. xl, part i, No. 8, 1920.

† *Mathematical Theory of Heat Conduction*, Ingersoll and Zobel.

observations were made only when the soils had more or less the same water content, we could find the values of $\frac{R_4}{R_0}$ for each soil corresponding to various values of the period T.

These values are given in Table I alongside some observed values of $\frac{R_4}{R_0}$ in clay loam for those periods.

TABLE I.—CHANGE OF $\frac{R_4}{R_0}$ WITH CHANGE OF PERIOD.

Interval Min. to Max. in Hrs.	Period T in Hrs.	$\frac{R_4}{R_0}$ Sand.	$\frac{R_4}{R_0}$ Clay Loam.	Observed Value of $\frac{R_4}{R_0}$ in Clay Loam for that Period.	Mean Surface Temperature for Period.
6	12	·41	·30	·26	2·5° C.
8	16	·44	·33	·30	5·0° C.
9	18	·46	·35	·32	3·0° C.
10	20	·475	·375	...	...
12	24	·52	·42	·42	10·0° C.
24	48	...	·54	·50 ·52	5·0° C. 5·0° C.
36	72	...	·60	·60 ·61	8·5° C. 9·0° C.

It will be seen that good agreement was obtained between the calculated and observed values of $\frac{R_4}{R_0}$ in clay loam when the mean surface temperature during the period was about 10° C.—the temperature at which the standard observation was made,—but that $\frac{R_4}{R_0}$ fell considerably below the calculated value when the mean surface temperature was below 10° C.

This suggested plotting the values of $\frac{R_4}{R_0}$ at various temperatures but the same period against the corresponding mean surface temperatures; the results for sand and clay loam are given in the next section.

III. EVIDENCES OF CHANGES IN VALUES OF $\frac{R_4}{R_0}$ WITH CHANGES IN MEAN SURFACE TEMPERATURE OF SOIL.

(a) *In Sand.*

Owing to the strict conditions under which observations were made for this investigation, only about sixty values of $\frac{R_4}{R_0}$ in each soil were recorded as being sufficiently trustworthy out of the whole year 1920; of these 32 were arranged according to their length of period:—

11 for a period of	12 hours—Min. to Max.		6 hours.	
14 „	18 „	„	9 „	
7 „	22 „	„	11 „	

These values for the two soils are shown plotted according to their length of period against the mean surface temperature in figs. 1 and 2; in both figures it will be noticed that the distribution of the plotted values of $\frac{R_4}{R_0}$ at low mean surface temperatures is wider than at high temperatures: this is due to the difficulty of computing exactly the value of $\frac{R_4}{R_0}$ in winter when both R_4 and R_0 are small, and the least error in either makes a considerable difference in the value of $\frac{R_4}{R_0}$.

The vertical dotted lines on the graph are the lines on which the values of $\frac{R_4}{R_0}$ in sand should lie at periods of 12, 18, 22 hours respectively if there was no variation in value of $\frac{R_4}{R_0}$ with a change of mean surface temperature.

The plotted points do fall so nearly on these lines that it seems justifiable to assume *that there is no change in conductivity of sand with change of temperature.*

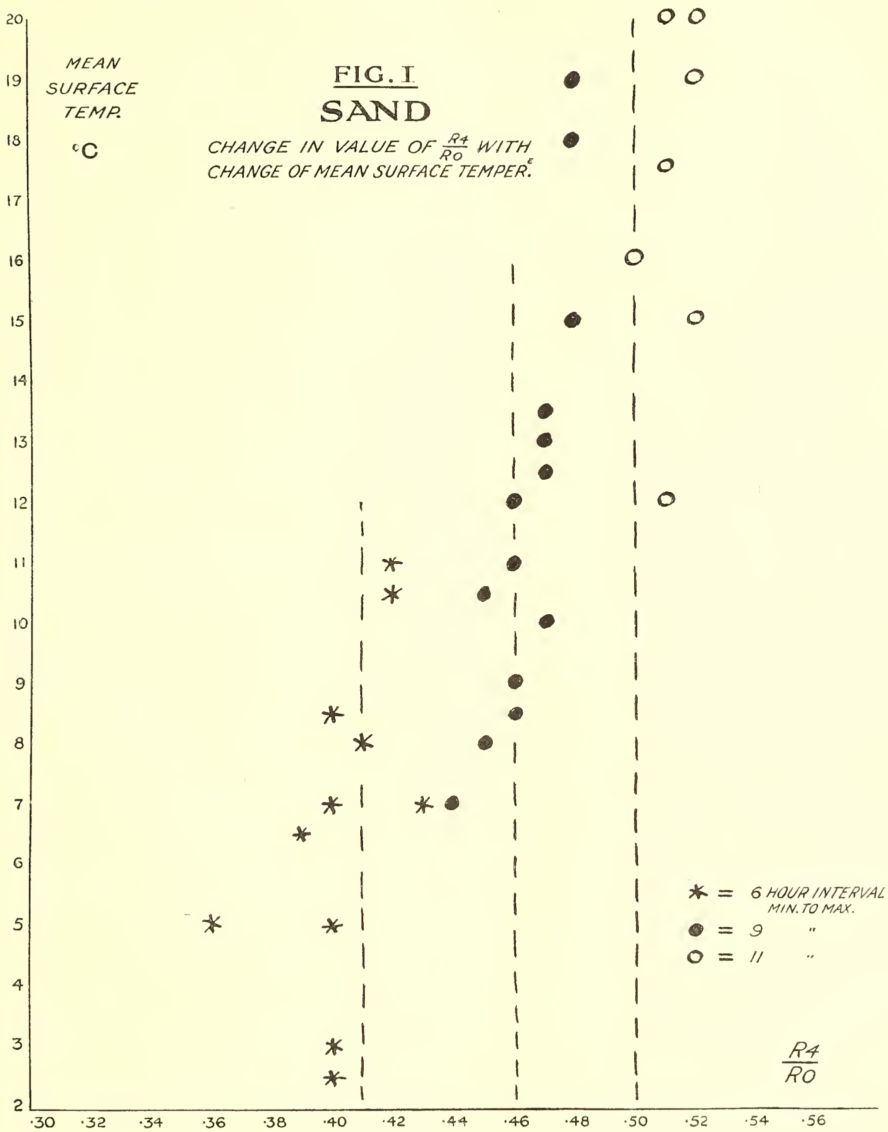
(*b*) *In Clay Loam.*

Here we have quite a different picture, as the groups of points for $\frac{R_4}{R_0}$ lie diagonally and not vertically as in sand, and it seems *that there is a distinct change of conductivity in clay loam with a change in the mean surface temperature.* The chief difference between clay loam and sand—apart from the size of the soil grains—is that clay loam contains organic matter and soil colloids, while sand contains neither; therefore it would appear that the change in conductivity of clay loam with temperature is due to one or both of these constituents.

(*c*) $\frac{R_3}{R_0}$ *in Clay Loam, Ignited Clay Loam, and Sand.*

During the autumn of 1920 I compared the values of $\frac{R_3}{R_0}$ in clay loam, ignited clay loam, and sand; the interval from minimum to maximum was about 8 hours—period 16 hours—for most of the time during which the observations were made.

The expected values for $\frac{R_3}{R_0}$ for a mean surface temperature of 10° C. would be ·45 in clay loam, and ·55 in sand: it happened that the ignited clay



loam had a value of $\frac{R_3}{R_0}$ equal to ·55 also, and the curves for it and sand have run together faithfully day by day for over two months, during which the mean surface temperature has fluctuated between 9° C. and just above 0° C.

The values of $\frac{R_3}{R_0}$ in ignited clay loam and sand have only varied between ·54 and ·57 during this time, while for clay loam the variation is from

·40 to ·48, according as the mean surface temperature was near 0° C. or 9° C.

Now, ignition destroys the organic matter and soil colloids, so that the similar small fluctuations of $\frac{R_3}{R_0}$ in ignited clay loam and sand—neither of which contain organic matter or soil colloids—compared with the larger fluctuations of $\frac{R_3}{R_0}$ in clay loam, which is rich in colloids, is a point of great significance in the present investigation.

IV. Conclusions.

The addition of organic matter to a soil reduces the conductivity of that soil. Thus Bouyoncos found that sand with the addition of 3·32 per cent. organic matter was a better conductor than sand with 6·95 per cent. organic matter; moreover, in the soils he tested, the conductivity, both dry and wet, was in the inverse ratio to the organic matter present.*

Therefore the change in conductivity of clay loam cannot be due to the organic matter present, but must be caused by the colloids present in that soil. This change of conductivity is probably brought about by the colloidal films surrounding the soil grains swelling with the rise in temperature, and so automatically compacting the soil, and reducing the transfer resistance to heat between the particles within the soil.

* "An Investigation on Soil Temperature and some of the most important factors influencing it," G. J. Bouyoncos, *Technical Bulletin* No. 17, 1913, *Michigan Experimental Station.*

(*Issued separately May* 9, 1921.)

VIII.—On a Graphical Method of determining Shear Influence Lines and Diagrams of Maximum Shearing Force for a Beam subjected to a Series of Concentrated Rolling Loads.

By **Alex. R. Horne**, B.Sc. (Lond.), Professor of Engineering, Robert Gordon's Technical College, Aberdeen.

(MS. received November 30, 1920. Read March 7, 1921.)

The shear influence line is a line the ordinates of which give the values of the shearing forces at any one point in a beam or bridge as a load, or a series of loads, pass over it. There is thus, for any one beam, an influence line for every point in it.

These influence lines are of great value in the design of structures, such as bridges and arches, where it is necessary to determine the greatest maximum and minimum shearing forces which occur at every point in them.

The methods generally used to obtain these lines prove laborious in practice, especially when there are, as is often the case, several loads, such as the wheel loads of a locomotive. The ordinates of each influence line are generally determined by calculation, when it becomes necessary to estimate the shearing forces for many positions of the loading. Alternatively, a graphical method, which requires the construction of funicular polygons, and which affords only approximate results, is resorted to. This latter method is inconvenient when the load length exceeds the span, as is often the case in practice.

In this paper, a simple graphical method of constructing an influence line is explained; and the system is extended to provide a ready means of drawing the influence lines for as many points in the beam as may be desired. From these a diagram of maximum positive and negative shears can be constructed. No calculation whatever is required, and the method is an exact one. Moreover, the system is not limited to the case where the total length of the load does not exceed the span of the beam.

Let a series of loads, W_1, W_2, W_3 (fig. 1) cross a beam AB, of span L, moving towards the right. When the leading load, W_1, is over the right abutment B, the bending moments at A, due to W_1, W_2, W_3, are M_1, M_2, M_3 respectively. Let the total bending moment at A, due to these loads, be represented by cd to a scale of $1'' = m$ units.

If the beam is freely supported at A, the resultant bending moment there is zero. It follows that, if R is the reaction of the support at B,

$$RL = M_1 + M_2 + M_3 = cd \text{ (in inches)} \times m,$$

hence $$R = cd \times \frac{m}{L};$$

therefore cd represents the reaction R to a scale of $1'' = \frac{m}{L} = n$ units.

If, now, the loads move to the *left* by a distance l_1, it is easy to show that R is represented by c_1d_1 = the depth of the diagram at a distance l_1 to the *right* of A.

Again, if the loads move a distance l_2 to the *right*, the sum of the moments at A due to all the loads will be represented by ed_2, where $ce = l_2$. But since W_1 is now off the beam, the bending moment, ec_2, due to it is ineffective. The true bending moment is now c_2d_2; and, from what has

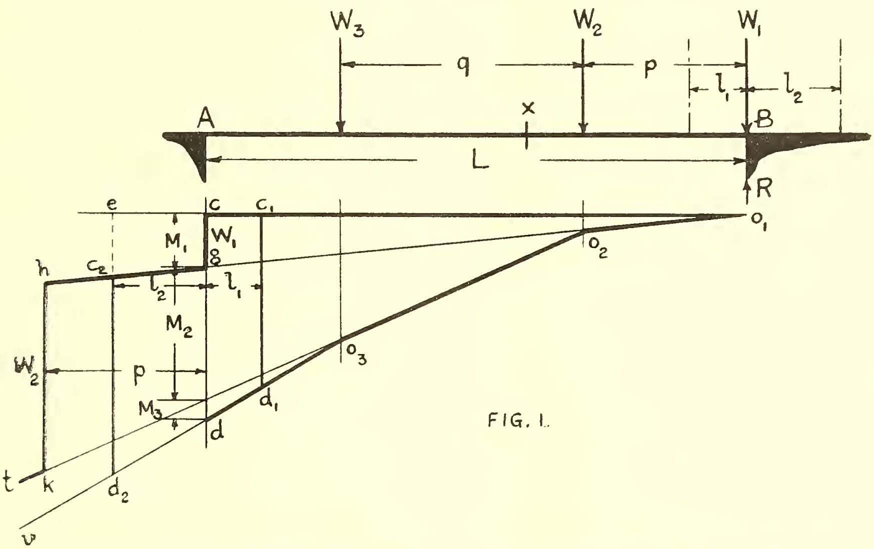


FIG. 1.

gone before, it follows that the new value of R is represented by c_2d_2 for this position of the loading.

When W_1 is at an infinitely small distance to the left of B, R = cd; but, immediately it passes off the beam, R is reduced by W_1 or cg. Similarly, just when W_2 leaves the beam at B, the reaction is suddenly reduced by W_2. This reduction is shown by $hk(=W_2)$, which is drawn at a distance p from A, where p is the spacing between W_1 and W_2. A similar treatment is adopted for the adjustment of the reaction R when W_3 passes B. This is not shown on the diagram.

Generally, the vertical intercept between the lines o_1cghkt—conveniently termed the "control" line, and the line $o_1o_2o_3v$, which may be referred to as the "moment" line, at a distance l, where l is the motion of the loads, measured horizontally from A in a direction *opposite to* that in which the loads have moved from the position where W_1 is over B, gives the then value of R.

Consider now the general case, where the load length exceeds the span (fig. 2). The "control" line ab_4 is drawn exactly as has been described, where $b_1a_1 = W_1$, $b_2a_2 = W_2$, $b_3a_3 = W_3$, and $b_4a_4 = W_1$, and the horizontal distances between these lines are equal to the respective spacings of the loads. The "moment" line aa_4, which forms the base line of the influence line, is constructed in the manner already explained. The vertical ordinates between these two lines give the values of the reactions R as the loads move from the position where W_1 is at A until W_4 reaches B.

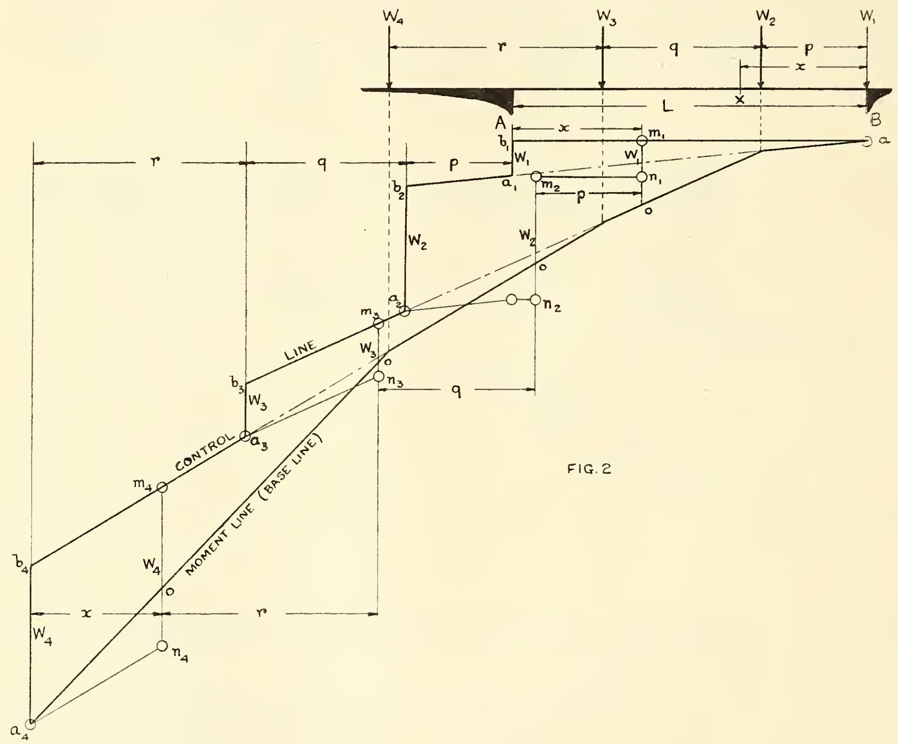


FIG. 2

The ordinate under A has two values, differing by W_1, according as W_1 is just to the left, or just to the right, of B.

Considering the point X on the beam, distant x to the *left* of B. When W_1 is immediately to the left of X (and the other loads are in corresponding positions), the positive shear at B is equal to the reaction m_1o, where m_1 is distant x to the *right* of A. The shearing force is considered positive when the portion of the beam to the right of X tends to move upwards relatively to the portion on the left of X, and *vice versa*.

Immediately the loads move so that W_1 is between X and B, the positive shear at $X = R - W_1$, R varying as the loads proceed to the right. When W_1 has moved a distance p to the right of X, W_2 comes over X, and the shear at X is now either m_2o or $-n_2o$ according as W_2 is just to the left or just to the right of X. Clearly $m_2n_1=p$, the distance between W_1 and W_2; and $m_2n_2=W_2$.

Similarly, if $n_2a_2m_3$ is drawn everywhere parallel to the control line $m_1b_1a_1b_2a_2m_3$, the horizontal distance from n_2 to n_3 being equal to q—the distance between W_2 and W_3,—the ordinate between the line $n_2a_2m_3$ and the moment, or base, line at the appropriate point gives the shearing force at X. Finally, m_3n_3 is made equal to W_3; $n_3a_3m_4$ is drawn parallel to the control line for a distance r—the spacing between W_3 and W_4; m_4n_4 is set down equal to W_4, and n_4 is joined to a_4.

The line am_1n_1 . . . $m_4n_4a_4$ is the influence line for the point X, and is easily traced on the diagram by the small circles, to the base line $aooa_4$.

Where the influence line falls above the base line the shear at X is positive; while, when it falls below the base line, the shear at X is negative.

By an extension of the diagram of fig. 2, a simple means is afforded by which to obtain the appropriate influence line for any point in the beam. The process is indicated in fig. 3, and is as follows:—

Place the loads so that W_1 is over the right abutment. Produce the load lines through W_1, W_2, W_3, and W_4 vertically downwards (dotted lines), and draw vertical (chain-dotted) lines, similarly spaced, from A.

Draw $ab_1=L$, the span. Set down $b_1a_1=W_1$ at A. In the direction aa_1 draw ac and a_1b_2 in the spaces p. Set down $b_2a_2=W_2$.

In the direction ca_2 draw cd and a_2b_3 in the spaces q. Set down $b_3a_3=W_3$.

In similar manner, draw de and a_3b_4 in the spaces r. Set down $b_4a_4=W_4$, and join e to a_4.

The line $acdea_4$ is the base line of all the influence lines, and $ab_1a_1b_2a_2$ . . . b_4a_4 is the control line.

From a_1, a_2, a_3, and a_4 draw lines a_1h_1, a_2h_2 . . . a_4h_4 everywhere parallel to the control line above, to meet the verticals through W_1, W_2, W_3, and W_4 respectively, and so obtain a series of parallel-sided figures, each of horizontal length L.

To draw the influence line relative to the point X in the beam, distant x from B, draw verticals m_1n_1, m_2n_2 . . . m_4n_4 distant x from the left-hand ends of the parallel-sided figures. The resulting influence line is indicated by the circles, to the base line $acdea_4$.

In the design of structures, only the greatest maximum positive and negative shears at each point in the span may be required. An inspection of the diagram shows that these occur on the lines m_1n_1, m_2n_2 . . . m_4n_4, and the inclined portions of the influence lines are not necessary in such cases.

In the case dealt with, the several maximum positive shears are om_1, on_1, om_2, om_3, and om_4, of which om_4 is the greatest; while the maximum negative shears are on_2, on_3, and on_4, the last being numerically the greatest.

By dividing the span into a number of equal parts, say ten, and the

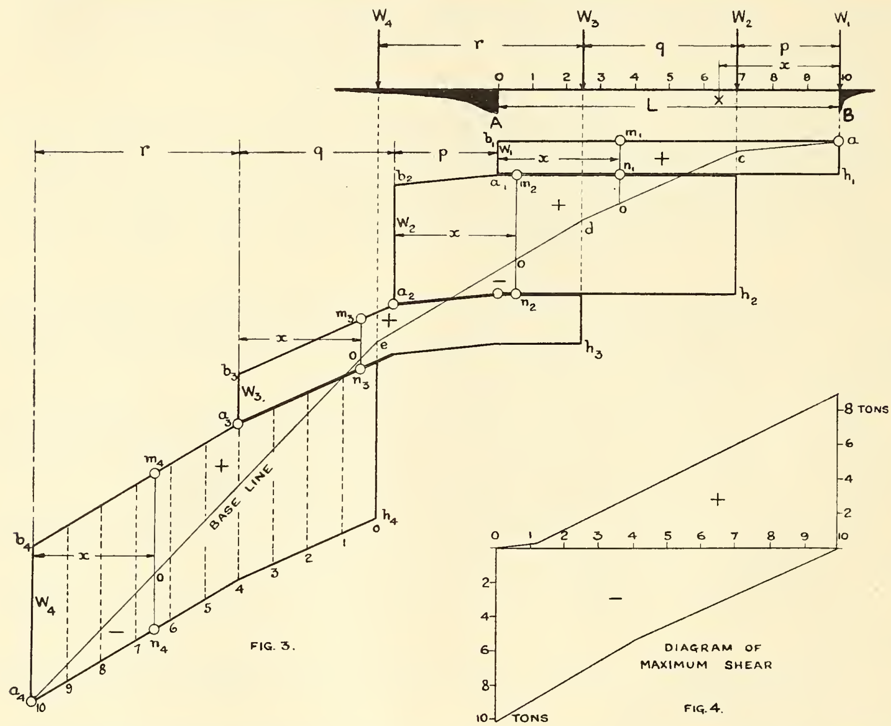


FIG. 3.

FIG. 4.

four parallel-sided figures into an equal number of parts, the greatest positive and negative shears for each of the several points in the beam may be determined by inspection. From these a diagram of maximum shears (fig. 4) can be drawn. In practice, the maxima values can be obtained by means of tracing paper.

To save confusion, only one of the four diagrams has been so divided. It is to be observed that the ordinates of the figures—1 to 10—are numbered in the reverse direction to the points on the beam.

The diagrams, figs. 3 and 4, have been drawn to scale for the case where $W_1=2$, $W_2=7$, $W_3=3$, and $W_4=9$ tons; $p=3$, $q=4{\cdot}5$ and $r=6$ feet. A scale of shearing force is attached to the diagram in fig. 4.

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IX.—The Confluent Hypergeometric Functions of Two Variables. By Pierre Humbert. *Communicated by* Professor E. T. Whittaker, F.R.S.

(MS. received December 6, 1920. Read January 10, 1921.)

Introduction.

This memoir is devoted to the study of certain new functions, which may be regarded as limiting cases of the "hypergeometric functions of two variables" discovered by Appell.* The relation which the new functions bear to Appell's functions is, in fact, analogous to that which the "confluent hypergeometric functions,"†

$$\Phi(\alpha, \gamma, x) = 1 + \frac{\alpha x}{1 . \gamma} + \frac{\alpha(\alpha+1)}{2 ! \gamma(\gamma+1)} x^2 + \ldots$$

and

$$B(\gamma, x) = 1 + \frac{x}{1 . \gamma} + \frac{x^2}{2 ! \gamma(\gamma+1)} + \ldots,$$

bear to the ordinary hypergeometric function.

There are four hypergeometric series of two variables. If we denote the product

$$\lambda(\lambda+1) \ldots (\lambda+n-1),$$

where λ is arbitrary and n a positive integer, by the symbol

$$(\lambda, n),$$

with the convention $(\lambda, 0) = 1$, these functions are as follows:—

$$F_1(\alpha ; \beta, \beta' ; \gamma ; x, y) = \sum_{m=0}^{m=+\infty} \sum_{n=0}^{n=+\infty} \frac{(\alpha, m+n)(\beta, m)(\beta', n)}{(\gamma, m+n)} \frac{x^m y^n}{m ! n !}$$

$$F_2(\alpha ; \beta, \beta' ; \gamma, \gamma' ; x, y) = \sum \sum \frac{(\alpha, m+n)(\beta, m)(\beta', n)}{(\gamma, m)(\gamma', n)} \frac{x^m y^n}{m ! n !}$$

$$F_3(\alpha, \alpha', \beta, \beta' ; \gamma ; x, y) = \sum \sum \frac{(\alpha, m)(\alpha', n)(\beta, m)(\beta', n)}{(\gamma, m+n)} \frac{x^m y^n}{m ! n !}$$

$$F_4(\alpha, \beta ; \gamma, \gamma' ; x, y) = \sum \sum \frac{(\alpha, m+n)(\beta, m+n)}{(\gamma, m)(\gamma', n)} \frac{x^m y^n}{m ! n !}.$$

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* *J. math. pures appl.*, 1882, p. 173 ; 1884, p. 407.

† For an account of the confluent hypergeometric functions, see chapter xvi of Whittaker and Watson's *Modern Analysis*.

These functions satisfy partial differential equations, and can be expressed as definite integrals. Appell has given some applications of them to certain problems of celestial mechanics, and expressed in terms of them the polynomials of Hermite and Didon and some more general polynomials.

An interesting advance in the theory has been made recently by Appell, who has shown that the polynomials $V_{m,n}$ of Hermite, which are particular cases of the function F_2, are solutions of the potential equation in hyperspherical co-ordinates, and can be considered as hyperspherical harmonic functions on the hypersphere

$$x^2+y^2+z^2+t^2=1.$$

PART I.

DEFINITION AND PROPERTIES OF THE FUNCTIONS.

Chapter I.

FORMATION OF THE CONFLUENT HYPERGEOMETRIC SERIES.

The confluent hypergeometric functions of two variables may be formed by confluence from Appell's functions in the following way:—

First, in Appell's function $F_1(a;\beta,\beta';\gamma,x,y)$, make $\beta'\rightarrow\infty$, at the same time dividing y by β': we thus obtain the first of our confluent functions,

$$\Phi_1(a;\beta;\gamma;x,y)=\sum\sum\frac{(a,m+n)(\beta,m)}{(\gamma,m+n)}\frac{x^m y^n}{m!\,n!}.$$

A second function can be obtained from F_1 by dividing x and y by a, and causing a to tend to infinity: we are thus led to the function

$$\Phi_2(\beta,\beta';\gamma;x,y)=\sum\sum\frac{(\beta,m)(\beta',n)}{(\gamma,m+n)}\frac{x^m y^n}{m!\,n!}.$$

A third new function can be derived from F_1 by making the two parameters a and β' infinite, after replacing x and y by $\dfrac{x}{a}$ and $\dfrac{y}{a\beta'}$: this gives the function

$$\Phi_3(\beta;\gamma;x,y)=\sum\sum\frac{(\beta,m)}{(\gamma,m+n)}\frac{x^m y^n}{m!\,n!}.$$

Taking next Appell's series F_2, we apply to it the same process, and obtain two new functions, the first one by dividing y by β', and making β' infinite; and the other one by dividing x by β, y by β', and making both β and β' tend to infinity. These two functions will be denoted by the symbol Ψ: their expressions are

$$\Psi_1(\alpha\,;\,\beta\,;\,\gamma,\gamma'\,;\,x,y)=\sum\sum\frac{(\alpha,m+n)(\beta,m)}{(\gamma,m)(\gamma',n)}\frac{x^m y^n}{m!\,n!}$$

$$\Psi_2(\alpha\,;\,\gamma,\gamma'\,;\,x,y)=\sum\sum\frac{(\alpha,m+n)}{(\gamma,m)(\gamma',n)}\frac{x^m y^n}{m!\,n!}.$$

From the function

$$F_3(\alpha,\alpha',\beta,\beta'\,;\,\gamma\,;\,x,y)=\sum\sum\frac{(\alpha,m)(\alpha',n)(\beta,m)(\beta',n)}{(\gamma,m+n)}\frac{x^m y^n}{m!\,n!}$$

may be obtained in like manner a new function by dividing y by β', and making $\beta'\to\infty$. This is the function

$$\Xi_1(\alpha,\alpha'\,;\,\beta\,;\,\gamma\,;\,x,y)=\sum\sum\frac{(\alpha,m)(\alpha',n)(\beta,m)}{(\gamma,m+n)}\frac{x^m y^n}{m!\,n!}.$$

Similarly, replacing y by $y/\alpha'\beta'$, and making α' and β' infinite, we obtain a function

$$\Xi_2(\alpha,\beta\,;\,\gamma\,;\,x,y)=\sum\sum\frac{(\alpha,m)(\beta,m)}{(\gamma,m+n)}\frac{x^m y^m}{m!\,n!}.$$

Chapter II.

Various expansions for the functions; relations between them.

The seven confluent functions which we have introduced, and defined by double power-series, may also be represented by simple power-series in x, or in y, by performing the process of confluence on the similar expressions given by Appell for the four F functions. We thus find

$$\Phi_1(\alpha\,;\,\beta\,;\,\gamma\,;\,x,y)=\sum_{m=0}^{\infty}\frac{(\alpha,m)(\beta,m)}{(\gamma,m)}\Phi(\alpha+m,\gamma+m,y)\frac{x^m}{m!}$$
$$=\sum_{m=0}^{\infty}\frac{(\alpha,m)}{(\gamma,m)}F(\alpha+m,\beta;\gamma+m\,;\,x)\frac{y^m}{m!}$$

and similar formulæ for the other confluent functions.

We shall next consider formulæ derived from the definite-integral values of the F functions, such as

$$F_2=\frac{\Gamma(\gamma)\Gamma(\gamma')}{\Gamma(\beta)\Gamma(\beta')\Gamma(\gamma-\beta)\Gamma(\gamma'-\beta')}\int_0^1\int_0^1 u^{\beta-1}v^{\beta'-1}(1-u)^{\gamma-\beta-1}(1-v)^{\gamma'-\beta'-1}(1-ux-vy)^{-\alpha}du\,dv.$$

We have

$$(1-ux-vy)^{-\alpha}=(-1)^{-\alpha}[1-(1-ux)-(1-vy)]^{-\alpha}$$
$$=(-1)^{-\alpha}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(\alpha,m+n)}{m!\,n!}(1-ux)^m(1-vy)^n$$

so

$$F_2 = \frac{(-1)^{-a}\Gamma(\gamma)\Gamma(\gamma')}{\Gamma(\beta)\Gamma(\beta')\Gamma(\gamma-\beta)\Gamma(\gamma'-\beta')}\sum\sum\frac{(a,\ m+n)}{m\,!\ n\,!}\int^1 u^{\beta-1}(1-u)^{\gamma-\beta-1}(1-ux)^m du$$

$$\int_0^1 v^{\beta'-1}(1-v)^{\gamma'-\beta'-1}(1-vy)^n dv$$

$$=(-1)^{-a}\sum\sum\frac{(a,\ m+n)}{m\,!\ n\,!}F(-m,\ \beta\ ;\ \gamma\ ;\ x)F(-n,\ \beta'\ ;\ \gamma'\ ;\ y),$$

a formula to which we can apply the ordinary process of confluence, obtaining the interesting expansions for the Ψ functions :

$$\Psi_1(a,\ \beta\ ;\ \gamma,\ \gamma'\ ;\ x,\ y)=(-1)^{-a}\sum\sum\frac{(a,\ m+n)}{m\,!\ n\,!}F(-m,\ \beta\ ;\ \gamma\ ;\ x)\Phi(-n,\ \gamma',\ y)$$

$$\Psi_2(a\ ;\ \gamma,\ \gamma'\ ;\ x,\ y)=(-1)^{-a}\sum\sum\frac{(a,\ m+n)}{m\,!\ n\,!}\Phi(-m,\ \gamma,\ x)\Phi(-n,\ \gamma',\ y).$$

By reasoning of a similar type we find

$$F_1(a\ ;\ \beta,\ \beta'\ ;\ \gamma\ ;\ x,\ y)=\sum(-1)^m\frac{(a,\ m)(\beta,\ m)(\beta',\ m)}{(\gamma-\beta',\ m)(\gamma,\ m)}\frac{x^m}{m\,!}F_2(a+m\ ;\ \beta+m\ ;\ \beta'+m\ ;$$

$$\gamma-\beta'+m,\ \gamma+m\ ;\ x,\ y)$$

or

$$F_1=\sum(-1)^m\frac{(a,\ m)(\beta,\ m)(\beta',\ m)}{(\gamma-\beta,\ m)(\gamma,\ m)}\frac{y^m}{m\,!}F_2(a+m\ ;\ \beta+m,\ \beta'+m\ ;\ \gamma+m,\ \gamma-\beta+m\ ;\ x,\ y).$$

From these we obtain by confluence

$$\Phi_1(a\ ;\ \beta\ ;\ \gamma\ ;\ x,\ y)=\sum(-1)^m\frac{(a,\ m)(\beta,\ m)}{(\gamma-\beta,\ m)(\gamma,\ m)}\frac{y^m}{m\,!}\Psi_1(a+m,\ \beta+m\ ;\ \gamma+m,\ \gamma-\beta+m\ ;\ x,\ y)$$

$$\Phi_3(\beta,\ \gamma\ ;\ x,\ y)=\sum(-1)^m\frac{(\beta,\ m)}{(\gamma-\beta,\ m)(\gamma,\ m)}\frac{y^m}{m\,!}\Phi(\beta+m,\ \gamma+m,\ x)B(\gamma-\beta+m,\ y),$$

which may be transformed into

$$\Phi_3(\beta,\ \gamma\ ;\ x,\ y)=e^x\sum(-1)^m\frac{(\beta,\ m)}{(\gamma-\beta,\ m)(\gamma,\ m)}\frac{y^m}{m\,!}\Phi(\gamma-a,\ \gamma+m,\ -x)B(\gamma-\beta+m,\ y)\ ;$$

lastly, from F_3 we obtain in a similar way

$$\Xi_1(a,\ a'\ ;\ \beta\ ;\ \gamma\ ;\ x,\ y)=\sum(-1)^m\frac{(a,\ m)(\beta,\ m)(a',\ m)}{(\gamma,\ m)(\gamma-a',\ m)}\frac{x^m}{m\,!}F(a+m,\ \beta+m,\ \gamma-a'+m,x)$$

$$\Phi(a'+m,\ \gamma+m,\ y)$$

$$\Phi_2(a,\ a'\ ;\ \gamma\ ;\ x,\ y)=\sum(-1)^m\frac{(a,\ m)(a',\ m)}{(\gamma,\ m)(\gamma-a',\ m)}\frac{x^m}{m\,!}\Phi(a+m,\ \gamma-a'+m,\ x)$$

$$\Phi(a'+m,\ \gamma+m,\ y)$$

$$\Xi_2(a,\ \beta,\ \gamma,\ x,\ y)=\sum(-1)^m\frac{(a,\ m)}{(\gamma,\ m)(\gamma-a,\ m)}\frac{y^m}{m\,!}F(a+m,\ \beta\ ;\ \gamma+m\ ;\ x)$$

$$B(\gamma-a+m,\ y).$$

Another type of expansion may be obtained as follows: in the formula

$$F_1=\frac{\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\beta')\Gamma(\gamma-\beta-\beta')}\int_0^1\int_0^1 t^{\beta-1}v^{\beta'-1}(1-t)^{\gamma-\beta-\beta'-1}(1-v)^{\gamma-\beta'-1}(1-tx-vy+vtx)^{-a}dtdv$$

take

$$(1-tx-vy+vtx)^{-\alpha}=(-1)^{-\alpha}[1-(1-tx)-(1-vy)-vtx]^{-\alpha}$$

$$=(-1)^{-\alpha}\sum_m\sum_n\sum_p\frac{(\alpha,\ m+n+p)}{m!\,n!\,p!}(1-tx)^m(1-vy)^n v^p t^p x^p,$$

whence

$$\mathrm{F}_1=(-1)^{-\alpha}\sum\sum\sum x^p\frac{(\alpha,\ m+n+p)(\beta,\ p)(\beta',\ p)}{m!\,n!\,p!\,(\gamma,\ p)(\gamma-\beta',\ p)}\mathrm{F}(-m,\ \beta+p\,;\ \gamma-\beta'+p\,;\ x)$$
$$\mathrm{F}(-n,\ \beta'+p,\ \gamma+p,\ y)$$

and

$$\Phi_1(\alpha,\ \beta,\ \gamma,\ x,\ y)=(-1)^{-\alpha}\sum\sum\sum y^p\frac{(\alpha,\ m+n+p)(\beta,\ p)}{m!\,n!\,p!\,(\gamma,\ p)(\gamma-\beta,\ p)}\mathrm{F}(-m,\ \beta+p\,;\gamma+p,\ x)$$
$$\Phi(-n,\ \gamma-\beta+p,\ y).$$

All these expansions show the intimate connection between these functions and the similar one-variable functions.

It is easy to show also that an important relation exists between Φ_1 and Ξ_1, and that, in fact, they always reduce to one another. Let us start from the expansion which we gave for Φ_1, in ascending powers of x; then, using the relation

$$\Phi(\alpha+m,\ \gamma+m,\ y)=e^y\Phi(\gamma-\alpha,\ \gamma+m,\ -y),$$

we can write

$$\Phi_1(\alpha\,;\ \beta\,;\ \gamma\,;\ x,\ y)=e^y\sum\frac{(\alpha,\ m)(\beta,\ m)}{(\gamma,\ m)}\Phi(\gamma-\alpha\,;\ \gamma+m,\ -y)\frac{x^m}{m!},$$

and, comparing with the expansion for Ξ_1,

$$\Phi_1(\alpha\,;\ \beta\,;\ \gamma\,;\ x,\ y)=e^y\Xi_1(\alpha,\ \gamma-\alpha\,;\ \beta\,;\ \gamma\,;\ x,\ y),$$

which is the relation in question.

Chapter III.

DIFFERENTIAL EQUATIONS SATISFIED BY THE FUNCTIONS.

The seven confluent functions satisfy partial differential equations of rather simple forms, which it is easy to obtain, by confluence, from the four systems of equations found by Appell for the F functions.

Writing

$$\frac{\partial z}{\partial x}=p,\quad \frac{\partial z}{\partial y}=q,\quad \frac{\partial^2 z}{\partial x^2}=r,\ \text{etc.},$$

we find that the system for the function Φ_1 is

$$\begin{cases} x(1-x)r+y(1-x)s+[\gamma-(\alpha+\beta+1)x]p-\beta yq-\alpha\beta z=0\\ yt+xs+(\gamma-y)q-px-\alpha z=0,\end{cases}$$

that for Ψ_1 is

$$\begin{cases} x(1-x)r - xys + [\gamma - (\alpha+\beta+1)x]p - \beta yq - \alpha\beta z = 0 \\ yt + (\gamma' - y)q - px - \alpha z = 0, \end{cases}$$

and similarly for the other functions.

Each of the systems is of the type

$$\begin{cases} r = a_1 s + a_2 p + a_3 q + a_4 z \\ t = b_1 s + b_2 p + b_3 q + b_4 z \end{cases}$$

(the a's and b's being functions of x and y), of which a general theory has been given by Appell, with the aid of certain propositions established by Bouquet. When the expression

$$1 - a_1 b_1$$

is different from zero (which is the case for the Ψ and Ξ systems), the general solution of the system is a linear function of four independent solutions

$$z = C_1 z_1 + C_2 z_2 + C_3 z_3 + C_4 z_4,$$

and if

$$1 - a_1 b_1 = 0$$

(which occurs for the Φ systems), it is a linear function of three independent solutions

$$z = C_1 z_1 + C_2 z_2 + C_3 z_3 .$$

We may observe that the system satisfied by the function

$$\Psi_1(\alpha\,;\beta\,;\gamma,\gamma'\,;x,y)$$

admits also the independent solutions

$$x^{1-\gamma}\Psi_1(\alpha+1-\gamma\,;\beta+1-\gamma\,;2-\gamma,\gamma'\,;x,y)$$
$$y^{1-\gamma'}\Psi_1(\alpha+1-\gamma'\,;\beta\,;\gamma,2-\gamma'\,;x,y)$$
$$x^{1-\gamma}y^{1-\gamma'}\Psi_1(\alpha+2-\gamma-\gamma'\,;\beta+1-\gamma\,;2-\gamma,2-\gamma'\,;x,y).$$

A similar result may be obtained for the function Ψ_2, so that the general solution of the two Ψ systems may readily be expressed in terms of the Ψ functions themselves.

Chapter IV.

some special properties of the Φ and Ξ functions.

We shall next give a few formulæ illustrative of the properties of the Φ and Ξ functions: the function Ψ_2, which has a special importance, will be considered in the next chapter.

The Φ_1 function admits recurrence formulæ analogous to the well-known relations between contiguous hypergeometric functions of one variable; thus

$$\begin{cases}\dfrac{\beta x}{\gamma}\Phi_1(\alpha+1\,;\,\beta+1\,;\,\gamma+1\,;\,x,y)+\dfrac{y}{\gamma}\Phi_1(\alpha+1\,;\,\beta\,;\,\gamma+1\,;\,x,y) \\ \qquad\qquad =\Phi_1(\alpha+1\,;\,\beta\,;\,\gamma\,;\,x,y)-\Phi_1(\alpha\,;\,\beta\,;\,\gamma\,;\,x,y)\end{cases}$$

$$\Phi_1(\alpha\,;\,\beta+1\,;\,\gamma\,;\,x,y)=\Phi_1(\alpha\,;\,\beta\,;\,\gamma\,;\,x,y)+\frac{\alpha x}{\gamma}\Phi_1(\alpha+1\,;\,\beta+1\,;\,\gamma+1\,;\,x,y).$$

The relation

$$\frac{\partial}{\partial x}\Phi_1(\alpha\,;\,\beta\,;\,\gamma\,;\,x,y)=\frac{\alpha\beta}{\gamma}\Phi_1(\alpha+1\,;\,\beta+1\,;\,\gamma+1\,;\,x,y)$$

shows that the derivates of the Φ_1 function are expressible in terms of the function itself.

Similar formulæ may be obtained for Φ_2.

Φ_1 may be expressed by a simple definite integral

$$\Phi_1(\alpha\,;\,\beta\,;\,\gamma\,;\,x,y)=\frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\gamma-\alpha)}\int_0^1 u^{\alpha-1}(1-u)^{\gamma-\alpha-1}(1-ux)^{-\beta}e^{uy}du,$$

while Φ_2 may be expressed by the double integral

$$\Phi_2(\beta,\beta'\,;\,\gamma\,;\,x,y)=\frac{\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\beta')\Gamma(\gamma-\beta-\beta')}\iint u^{\beta-1}v^{\beta'-1}(1-u-v)^{\gamma-\beta-\beta'-1}e^{ux+vy}dudv$$

$$(u\geqslant 0,\ v\geqslant 0,\ 1-u-v\geqslant 0).$$

Formulæ of the same type may be obtained for the Ξ functions: thus we have

$$\Xi_1(\alpha,\alpha',\beta\,;\,\gamma\,;\,x,y)=\frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\alpha')\Gamma(\gamma-\alpha-\alpha')}\iint u^{\alpha-1}v^{\alpha'-1}(1-ux)^{-\beta}e^{vy}(1-u-v)^{\gamma-\alpha-\alpha'-1}dudv,$$

the field being the same as above.

Chapter V.

THE FUNCTION Ψ_2 AND ITS TRANSFORMATIONS.

The function Ψ_2 proves to be the most interesting of the seven, as its properties afford a very direct generalisation of the one-variable confluent hypergeometric function. To render this fact more conspicuous, we shall substitute for Ψ_2 a new function, just as Whittaker * studied, instead of Φ, his functions M or W.

We therefore make the following change of parameters:

$$\alpha=\mu+\nu-k+1$$
$$\gamma=2\mu+1$$
$$\gamma'=2\nu+1,$$

* *Bull. Amer. Math. Soc.*, iv, p. 125.

and we define the function

$$M_{k,\mu,\nu}(x, y) = x^{\mu+\frac{1}{2}}y^{\nu+\frac{1}{2}}e^{-\frac{x+y}{2}}\Psi_2(\mu+\nu-k+1\,;\ 2\mu+1,\ 2\nu+1\,;\ x,\ y).$$

The development in ascending powers of x and y is then

$$M_{k,\mu,\nu} = x^{\mu+\frac{1}{2}}y^{\nu+\frac{1}{2}}e^{-\frac{x+y}{2}}\sum_m\sum_n \frac{(\mu+\nu-k+1,\ m+n)}{(2\mu+1,\ m)(2\nu+1,\ n)}\frac{x^m y^n}{m!\ n!}.$$

We see that this function exists only when μ and ν are both different from the half of a negative integer; a similar feature occurs with the one-variable confluent hypergeometric function

$$M_{k,\mu}(x) = x^{\mu+\frac{1}{2}}e^{-\frac{x}{2}}\lim_{\rho=\infty} F\left(\mu-k+\tfrac{1}{2},\ \rho\,;\ 2\mu+1\,;\ \frac{x}{\rho}\right)$$

which disappears if 2μ is a negative integer. If, however, we suppose ν, for instance, to become equal to $-\frac{1}{2}$, and *simultaneously* y to become equal to zero, with the condition that the fraction $\frac{y}{2\nu+1}$ tends to zero, the function becomes, as it is easy to verify by considering the above expansion, equal to

$$x^{\mu+\frac{1}{2}}e^{-\frac{x}{2}}\sum_m \frac{(\mu-k+\frac{1}{2},\ m)}{(2\mu+1,\ m)}\frac{x^m}{m!}$$

or precisely $M_{k,\mu}(x)$. We then have the most important relation

$$M_{k,\mu,-\frac{1}{2}}(x,\ 0) = M_{k,\mu}(x),$$

provided that $\lim \frac{y}{2\nu+1} = 0$; to which can be added the similar one

$$M_{k,-\frac{1}{2},\nu}(0,\ y) = M_{k,\nu}(y),$$

provided that $\lim \frac{x}{2\mu+1} = 0$.

It is easy to form the system of partial differential equations satisfied by $M_{k,\mu,\nu}$: it is

$$(\mathrm{S})\qquad \begin{cases} x^2r - xyq + z\left(-\frac{x^2}{4} - \frac{xy}{2} + kx + \frac{1}{4} - \mu^2\right) = 0 \\ y^2t - xyp + z\left(-\frac{y^2}{4} - \frac{xy}{2} + ky + \frac{1}{4} - \nu^2\right) = 0. \end{cases}$$

If in this system we take $y=0$ and $\nu=-\frac{1}{2}$, the second equation vanishes, and the first one becomes

$$x^2\frac{d^2z}{dx^2} + z\left(-\frac{x^2}{4} + kx + \frac{1}{4} - \mu^2\right) = 0,$$

which is precisely the confluent hypergeometric equation of one variable, in Whittaker's form; and we obtain a similar result by taking $x=0$ and $\mu=-\frac{1}{2}$.

We shall, in general, denote by $W_{k,\mu,\nu}(x, y)$ a solution of this system with the condition that it reduces to $W_{k,\mu}(x)$ for $y = 2\nu+1 = 0$, and to $W_{k,\nu}(y)$ for $x = 2\mu+1 = 0$.

The general solution of the system S is readily found to be of the form

$$C_1 M_{k,\mu,\nu}(x, y) + C_2 M_{k,-\mu,\nu}(x, y) + C_3 M_{k,\mu,-\nu}(x, y) + C_4 M_{k,-\mu,-\nu}(x, y),$$

the C's being arbitrary constants.

Numerous recurrence formulæ may be written for the M function. We shall only give the following as an example:

$$x\frac{\partial}{\partial x} M_{k,\mu,\nu} = \left(\mu + \tfrac{1}{2} - \frac{x}{2}\right) M_{k,\mu,\nu} + x^{\frac{1}{2}} \frac{\mu+\nu-k+2}{2\mu+1} M_{k-\frac{1}{2},\mu+\frac{1}{2},\nu}.$$

The expansions for Ψ_2 furnish analogous results. We thus obtain, bearing in mind the definition of the one-variable M function and its relation with Φ,

$$M_{k,\mu,\nu}(x, y) = e^{-\frac{x}{2}} \sum_m x^{m+\mu+\frac{1}{2}} \frac{(\mu+\nu-k+1, m)}{(2\mu+1, m) m!} M_{k-\mu-m-\frac{1}{2},\nu}(y)$$

and

$$M_{k,\mu,\nu}(x, y) = e^{-\frac{y}{2}} \sum_m y^{m+\nu+\frac{1}{2}} \frac{(\mu+\nu-k+1, m)}{(2\nu+1, m)\, m!} M_{k-\nu-m-\frac{1}{2},\mu}(x).$$

By transforming the formula

$$\Psi_2(\alpha; \gamma, \gamma'; x, y) = (-1)^{\alpha} \sum_m \sum_n \frac{(\alpha, m+n)}{m!\, n!} \Phi(-m, \gamma, x) \Phi(-n, \gamma', y)$$

we obtain

$$M_{k,\mu,\nu}(x, y) = (-1)^{k-\mu-\nu-1} \sum_m \sum_n \frac{(\mu+\nu-k+1, m+n)}{m!\, n!} M_{m+\mu+\frac{1}{2},\mu}(x) M_{n+\nu+\frac{1}{2},\nu}(y).$$

Let us consider the one-variable M functions which occur under the symbol of summation. The expansion of the first one is

$$M_{m+\mu+\frac{1}{2},\mu}(x) = x^{\mu+\frac{1}{2}} e^{-\frac{x}{2}} \sum_{p=0}^{p=\infty} \frac{(-m, p)}{(2\mu+1, p)} \frac{x^p}{p!};$$

but, as m is an integer, the product $(-m, p)$ vanishes whenever p is greater than m, so that the sum represents not an infinite series, but a polynomial of degree m in x,

$$P(x) = \sum_{p=0}^{p=m} \frac{(-m, p)}{(2\mu+1, p)} \frac{x^p}{p!}.$$

The question is now, what is this polynomial? Let us write

$$P(x) = \sum_{q=0}^{q=m} \frac{(-m, m-q)}{(2\mu+1, m-q)} \frac{x^{m-q}}{(m-q)!}$$

$$= \Gamma(2\mu+1) m! \sum_{q=0}^{q=m} (-1)^{m-q} \frac{x^{m-q}}{q!\,(m-q)!\,\Gamma(2\mu+m-q)}$$

or

$$P(x) = (-1)^m m!\,\Gamma(2\mu+1)\left[\frac{x^m}{0!\,m!\,\Gamma(2\mu+m)} - \frac{x^{m-1}}{1!\,(m-1)!\,\Gamma(2\mu+m-1)} + \ldots\right]$$

and the expression between brackets is the polynomial of degree m considered by Sonine * in his researches on the Bessel functions; it is here $T^m_{2\mu}(x)$, the definition of the polynomial T^β_α being the expansion

$$\frac{e^{\frac{tx}{1+t}}}{(1+t)^{\alpha+1}} = \sum_{\beta=0}^{\beta=\infty} \Gamma(\alpha+\beta+1)t^\beta T^\beta_\alpha(x),$$

and its expression, as given by Sonine, being precisely

$$T^\beta_\alpha(x) = \frac{x^\beta}{\beta!\,0!\,\Gamma(\alpha+\beta)} - \frac{x^{\beta-1}}{(\beta-1)!\,1!\,\Gamma(\alpha+\beta-1)} + \frac{x^{\beta-2}}{(\beta-2)!\,2!\,\Gamma(\alpha+\beta-2)} - \ldots$$

We can then write the following expression:

$$M_{m+\mu+\frac{1}{2},\,\mu}(x) = (-1)^m m!\,\Gamma(2\mu+1)x^{\mu+\frac{1}{2}}e^{-\frac{x}{2}}T^m_{2\mu}(x),$$

a result which can be verified by using the expression of the T polynomial in terms of the $W_{k,m}$ function, as given by Whittaker.†

We then obtain at once the very simple and remarkable expansion

$$M_{k,\mu,\nu}(x, y) = x^{\mu+\frac{1}{2}}y^{\nu+\frac{1}{2}}e^{-\frac{x+y}{2}}\Gamma(2\mu+1)\Gamma(2\nu+1)\sum_m\sum_n(-1)^{k-\mu-\nu-m-n-1}$$
$$(\mu+\nu-k+1,\ m+n)T^m_{2\mu}(x)T^n_{2\nu}(y).$$

Some interesting consequences, concerning certain particular cases of the M function, can be deduced from this formula.

If we suppose, in the first place, μ and ν to be of the form $\frac{a}{2}+\frac{1}{4}$, $\frac{b}{2}+\frac{1}{4}$, where a and b are integers, we have to consider in the expansion polynomials of the type

$$T^m_{a+\frac{1}{2}}(x),$$

for which we readily find the simple expression

$$T^m_{a+\frac{1}{2}}(x) = \frac{d^{a+1}}{dx^{a+1}}T^{a+m+1}_{-\frac{1}{2}}(x).$$

But we can observe with Sonine that, if λ is an integer,

$$T^\lambda_{-\frac{1}{2}}(x) = U_{2\lambda}(\sqrt{x})$$

where U is Hermite's polynomial,

$$U_{2\lambda}(z) = e^{z^2}\frac{d^{2\lambda}}{dz^{2\lambda}}e^{-z^2},$$

* *Math. Ann.*, xvi (1880), p. 41.
† *Modern Analysis*, 3rd edition (1920), p. 352.

so that we can write, for an M function of the aforesaid type,

$$M_{k,\frac{a}{2}+\frac{1}{4},\frac{b}{2}+\frac{1}{4}}(x, y) = x^{\frac{a}{2}+\frac{3}{4}} y^{\frac{b}{2}+\frac{3}{4}} e^{-\frac{x+y}{2}} \Gamma(a+\tfrac{3}{2})\Gamma(b+\tfrac{3}{2}) \sum\sum(-1)^{k-\frac{a+b+3}{2}-m-n}$$
$$\times\left(\frac{a+b+3}{2}-k, m+n\right) \times \frac{d^{a+1}}{dx^{a+1}} U_{2(a+m+1)}(\sqrt{x}) \times \frac{d^{b+1}}{dy^{b+1}} U_{2(b+n+1)}(\sqrt{y}).$$

As any differential coefficient of the U polynomials can be expressed in terms of the U themselves, we can express any function M where μ and ν are of the type $\frac{a}{2}+\frac{1}{4}, \frac{b}{2}+\frac{1}{4}$ in terms of Hermite's polynomials.

Let us take, in particular, $a=b=-1$; we have at once, with a change of variables,

$$M_{k,-\frac{1}{4},-\frac{1}{4}}\left(\frac{x^2}{2}, \frac{y^2}{2}\right) = \pi\sqrt{\frac{xy}{2}}(-1)^{k+\frac{1}{2}} e^{-\frac{x^2+y^2}{4}} \times \sum\sum(-1)^{m+n}(\tfrac{1}{2}-k, m+n) U_{2m}\left(\frac{x}{\sqrt{2}}\right) U_{2n}\left(\frac{y}{\sqrt{2}}\right).$$

This formula connects the special M function with the parabolic-cylinder functions.

Chapter VI.

CONNECTION BETWEEN CERTAIN KNOWN FUNCTIONS AND THE CONFLUENT HYPERGEOMETRIC FUNCTIONS.

Several functions of two variables introduced by different authors can be connected with some of the seven confluent functions of two variables. Of this we shall give three examples.

1. *The Two-variable Polynomials $A_{m,n}$ of Appell.*—It is a well-known fact that limiting cases of a great number of one-variable polynomials are expressible by the $W_{k,m}$ function or by Bessel functions. For instance, as anyone knows, for Legendre functions we have

$$\lim_{n=\infty}\left[n^{-m} P_n^m\left(1-\frac{x^2}{2n^2}\right)\right] = J_m(x).$$

We can establish a similar property for certain two-variable polynomials.

Let us consider the two-variable polynomials discussed by Appell,* and defined by

$$A_{m,n}(x, y) = x^{1-\gamma} y^{1-\gamma'} (1-x-y)^{\gamma+\gamma'-\delta+m+n} \frac{\partial^{m+n}}{\partial x^m \partial y^n}\left[x^{m+\gamma-1} y^{n+\gamma'-1} (1-x-y)^{\delta-\gamma-\gamma'}\right].$$

As shown by Appell himself, they can be written under the form

$$A_{m,n} = (\gamma, m)(\gamma', n)(1-x-y)^{m+n} F_2\left(\gamma+\gamma'-\delta; -m, -n; \gamma, \gamma'; \frac{x}{x+y-1}, \frac{x}{x+y-1}\right).$$

* *Archiv Math. Phys.*, lxvi, 1881, p. 238.

Let us now divide x by m and y by n, causing m and n to tend simultaneously to infinity: we observe then that

$$\lim_{m,\,n=\infty} \frac{A_{m,\,n}\left(\frac{x}{m},\,\frac{y}{n}\right)}{(\gamma,\,m)(\gamma',\,n)} = e^{-2(x+y)}\Psi_2(\gamma+\gamma'-\delta\,;\ \gamma,\,\gamma'\,;\ x,\,y),$$

showing a connection between $A_{m,\,n}$ and Ψ_2 of the same nature as the connection between Legendre and Bessel functions.

2. *The Two-variable Polynomials $H_{m,\,n}$ of Hermite.*—These functions, introduced by Hermite,* arise from the derivation of an exponential where the exponent is a quadratic form of x and y; their definition is

$$H_{m,\,n}(x,\,y) = (-1)^{m+n} e^{\frac{1}{2}\phi(x,\,y)} \frac{\partial^{m+n}}{\partial x^m \partial y^n} e^{-\frac{1}{2}\phi(x,\,y)}$$

where

$$\phi(x,\,y) = ax^2 + 2bxy + cy^2.$$

It may be shown that this polynomial depends essentially on the function

$$W_{\frac{m+n-1}{2},\,-\frac{1}{4},\,-\frac{1}{4}}.$$

3. *The Two-variable Bessel Function of order Zero.*—Several results have been published lately on the subject of new functions of two variables possessing certain properties analogous to Bessel functions.† These two-variable Bessel functions are defined by the expansion

$$e^{\frac{x}{2}\left(u-\frac{1}{u}\right)+\frac{y}{2}\left(u^2-\frac{1}{u^2}\right)} = \sum_n J_n(x,\,y)u^n,$$

or by the integral

$$J_n(x,\,y) = \frac{1}{\pi}\int_0^\pi \cos\,(nu - x\sin u - y\sin 2u)du.$$

It may be shown that the simplest of these functions, $J_0(x,\,y)$, satisfies the same differential equation as our solution

$$e^{-iy}\Phi_3(\tfrac{1}{2},\,1,\,2iy,\,-\tfrac{1}{4}x^2).$$

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* *Œuvres*, ii, p. 293.

† *Cf.* a paper by Jekhowsky, with a bibliography of the subject, in *Bull. Astron.*, t. xxxv, 1918.

PART II.

THE CONFLUENT HYPERGEOMETRIC FUNCTIONS AND THE POTENTIAL.

VERY important connections exist between the confluent hypergeometric functions of two variables and the theory of potential in hyperspace, a fact which generalises in an interesting way the well-known relations between the hypergeometric functions of one variable, or their confluent forms, and the potential in three-dimensional space. We shall now develop some of these propositions.

CHAPTER I.

THE FUNCTIONS OF THE PARABOLIC HYPERCYLINDER.

Let us consider a four-dimensional space, where the Cartesian co-ordinates will be denoted by x, y, z and t, Laplace's equation in this system being

$$\Delta U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial t^2} = 0.$$

Let us now make the change of variables

$$\begin{aligned} x &= uv \cos \phi \\ y &= uv \sin \phi \\ z &= \frac{u^2 - v^2}{2} \\ t &= t. \end{aligned}$$

The hypersurfaces thus introduced are rather simple ones: it is at once obvious that $t = \text{const.}$ and $\phi = \text{const.}$ are hyperplanes; as for $u = \text{const.}$, we obtain the corresponding hypersurface by eliminating v and ϕ between the first three equations, obtaining

$$x^2 + y^2 = u^2(u^2 - 2z) \quad . \quad . \quad . \quad . \quad . \quad (A)$$

so that it is an hypercylinder, with its generatrices parallel to the t-axis, its basis in the xyz space being the quadric (A), which is a paraboloïd of revolution. We shall say, therefore, that this hypersurface $u = \text{const.}$ is a parabolic-hypercylinder. The hypersurface $v = \text{const.}$ is of the same nature.

If now we transform Laplace's equation, we obtain

$$uv\left(\frac{\partial^2 U}{\partial u^2} + \frac{\partial^2 U}{\partial v^2}\right) + v\frac{\partial U}{\partial u} + u\frac{\partial U}{\partial v} + \frac{u^2 + v^2}{uv}\frac{\partial^2 U}{\partial \phi^2} + uv(u^2 + v^2)\frac{\partial^2 U}{\partial t^2} = 0.$$

A solution may be obtained by taking

$$U = U_1(u, v)\cos m\phi \cos nt;$$

the function U_1 verifies then the equation

$$\frac{\partial^2 U_1}{\partial u^2} + \frac{\partial^2 U_1}{\partial v^2} + \frac{1}{u}\frac{\partial U_1}{\partial u} + \frac{1}{v}\frac{\partial U_1}{\partial v} - m^2\left(\frac{1}{u^2} + \frac{1}{v^2}\right)U_1 - n^2(u^2 + v^2)U_1 = 0 \quad . \quad (B)$$

which we shall name the parabolic hypercylinder equation, the two-variable function $U_1(u, v)$ being a *function of the parabolic hypercylinder.*

It is obvious that equation (B) may be solved by the product of a function of u alone by a function of v alone; but solutions of that kind we shall consider as degenerate, and therefore reject them.

A real two-variable solution, however, is easily found by considering the $W_{k, \mu, \nu}$ system. If in this general system we take

$$k = 0$$
$$\mu = \nu = \frac{m}{2}$$

and replace x and y by $\frac{nx^2}{2}, \frac{ny^2}{2}$, we shall see that this function verifies

$$\begin{cases} r - \frac{p}{x} - nyq + z\left(-\frac{n^2x^2}{4} - \frac{n^2y^2}{2} + \frac{1-m^2}{x^2}\right) = 0 \\ t - \frac{q}{y} - nxp + z\left(-\frac{n^2y^2}{4} - \frac{n^2x^2}{2} + \frac{1-m^2}{y^2}\right) = 0; \end{cases}$$

and that, if z_1 is a solution of this system, the function

$$z = \frac{1}{xy}e^{-\frac{n}{4}(x^2+y^2)}z_1$$

verifies

$$\begin{cases} r + p\left(nx + \frac{1}{x}\right) - nyq + z\left(-n^2y^2 - \frac{m^2}{x^2}\right) = 0 \\ t + q\left(ny + \frac{1}{y}\right) - nxp + z\left(-n^2x^2 - \frac{m^2}{y^2}\right) = 0, \end{cases}$$

and therefore the single equation obtained by addition of these, *i.e.*

$$r + t + \frac{p}{x} + \frac{q}{y} - m^2\left(\frac{1}{x^2} + \frac{1}{y^2}\right)z - n^2(x^2 + y^2)z = 0,$$

which is precisely equation (B). We have then this remarkable result: the function

$$U_1(u, v) = \frac{1}{uv}e^{-\frac{n}{4}(u^2+v^2)}W_{0, \frac{m}{2}, \frac{m}{2}}\left(\frac{nu^2}{2}, \frac{nv^2}{2}\right)$$

is a function of the parabolic hypercylinder, and Laplace's product in parabolic hypercylindrical co-ordinates may be taken equal to

$$U = \frac{\cos m\phi \cos nt}{uv}e^{-\frac{n}{4}(u^2+v^2)}W_{0, \frac{m}{2}, \frac{m}{2}}\left(\frac{nu^2}{2}, \frac{nv^2}{2}\right).$$

Chapter II.

THE FUNCTIONS OF THE HYPERPARABOLOÏD OF REVOLUTION.

Let us now, in like manner, consider the change of variables

$$x = uv \sin\phi \cos\theta$$
$$y = uv \sin\phi \sin\theta$$
$$z = uv \cos\phi$$
$$t = \frac{u^2 - v^2}{2}.$$

The hypersurfaces $\phi = \text{const.}$ and $\theta = \text{const.}$ are hyperplanes; the equation of the hypersurface $u = \text{const.}$, obtained by elimination of v, ϕ, and θ, is

$$x^2 + y^2 + z^2 = u^2(u^2 - 2t).$$

It represents therefore an hypersurface of the second order, obtained by rotation about the t-axis of the quadric

$$x^2 + y^2 = u^2(u^2 - 2t),$$

which is, in the xyt space, a paraboloïd of revolution about the t-axis. We shall term this hypersurface an hyperparaboloïd of revolution. An hypersurface of the same type corresponds to $v = \text{const.}$

Laplace's equation is now

$$\frac{\partial}{\partial u}\left[u^2v^2 \sin\phi \frac{\partial U}{\partial u}\right] + \frac{\partial}{\partial v}\left[u^2v^2 \sin\phi \frac{\partial U}{\partial v}\right] + \frac{\partial}{\partial\phi}\left[(u^2+v^2)\sin\phi \frac{\partial U}{\partial\phi}\right] + \frac{\partial}{\partial\theta}\left[\frac{u^2+v^2}{\sin\phi}\frac{\partial U}{\partial\theta}\right] = 0.$$

We easily get rid of the θ variable by taking

$$U = U_1(u, v, \phi) \cos m\theta,$$

which gives

$$\frac{\partial}{\partial u}\left(u^2v^2\frac{\partial U_1}{\partial u}\right) + \frac{\partial}{\partial v}\left(u^2v^2\frac{\partial U_1}{\partial v}\right) + (u^2+v^2)\left[\frac{1}{\sin\phi}\frac{\partial}{\partial\phi}\left(\sin\phi\frac{\partial U_1}{\partial\phi}\right) - \frac{m^2 U_1}{\sin^2\phi}\right] = 0.$$

But it is also obvious to take

$$U_1 = U_2(u, v) P_n^m(\cos\phi),$$

where P_n^m is the associated Legendre function, which satisfies

$$\frac{1}{\sin\phi}\frac{d}{d\phi}\left(\sin\phi\frac{dP}{d\phi}\right) + \left[n(n+1) - \frac{m^2}{\sin^2\phi}\right]P = 0,$$

and we obtain an equation in u and v only,

$$\frac{\partial^2 U_2}{\partial u^2} + \frac{\partial^2 U_2}{\partial v^2} + \frac{2}{u}\frac{\partial U_2}{\partial u} + \frac{2}{v}\frac{\partial U_2}{\partial v} - n(n+1)\left(\frac{1}{u^2} + \frac{1}{v^2}\right)U_2 = 0 \quad . \quad . \quad \text{(C)}$$

a solution of which, really depending on two variables, will be a *function of the hyperparaboloïd of revolution.*

Introducing again the $W_{k,\mu,\nu}$ function, we shall form the system satisfied by

$$z = x^{-\frac{3}{2}} y^{-\frac{3}{2}} e^{\frac{1}{4}(y^2 - x^2)} W_{k,\mu,\mu}\left(\frac{x^2}{2}, -\frac{y^2}{2}\right).$$

There is no need to give the complete calculation: let us say only that, adding together the equations thus obtained, we find that z is a solution of

$$r + t + \frac{2}{x}p + \frac{2}{y}q + z\left(\frac{1}{x^2} + \frac{1}{y^2}\right)\left(\tfrac{1}{4} - 4\mu^2\right) = 0,$$

the parameter k having disappeared.

If we take therefore $4\mu^2 - \frac{1}{4} = n(n+1)$, or $\mu = \frac{n}{2} + \frac{1}{4}$, we obtain complete identity with equation (C), so that

$$U_2(u, v) = u^{-\frac{3}{2}} v^{-\frac{3}{2}} e^{\frac{1}{4}(v^2 - u^2)} W_{k, \frac{n}{2}+\frac{1}{4}, \frac{n}{2}+\frac{1}{4}}\left(\frac{u^2}{2}, -\frac{v^2}{2}\right)$$

is a function of the hyperparaboloïd of revolution.

We shall now make several remarks touching these functions.

1. The confluent function which appears in the question is of a type we studied in Part I, Chapter V, where the two last parameters μ and ν are of the form $\frac{a}{2} + \frac{1}{4}$, a being an integer. Referring to a property established there, we can say that the function of the hyperparaboloïd of revolution is expressible in terms of the parabolic-cylinder functions.

2. Let us consider the change of variable

$$x = \frac{u^2 - v^2}{2} \sin\phi \cos\theta$$

$$y = \frac{u^2 - v^2}{2} \sin\phi \sin\theta$$

$$z = \frac{u^2 - v^2}{2} \cos\phi$$

$$t = uv.$$

The hypersurface $u = \text{const.}$,

$$4(x^2 + y^2 + z^2) = \left(u^2 - \frac{t^2}{u^2}\right)^2$$

is obtained through rotation about the t-axis of the surface

$$4(x^2 + y^2) = \left(u^2 - \frac{t^2}{u^2}\right)^2,$$

which is of the fourth order in the xyt space, and itself the result of the rotation about the t-axis of the parabola

$$2x = u^2 - \frac{t^2}{u^2}.$$

Laplace's equation, written in this system, is easily solved by the change of variables

$$u=\frac{u'+v'}{\sqrt{2}}$$

$$v=\frac{u'-v'}{\sqrt{2}},$$

which reduces it to the equation for the hyperparaboloïd of revolution. A solution is then

$$W_{k,\frac{n}{2}+\frac{1}{4},\frac{n}{2}+\frac{1}{4}}\left(\frac{u'^2}{2}, -\frac{v'^2}{2}\right),$$

multiplied by a convenient factor, or

$$W_{k,\frac{n}{2}+\frac{1}{4},\frac{n}{2}+\frac{1}{4}}[(u+v)^2, -(u-v)^2].$$

A similar remark may be made regarding the change of variables

$$x=\frac{u^2-v^2}{2}\sin\phi$$

$$y=\frac{u^2-v^2}{2}\cos\phi$$

$$z=uv$$

$$t=t,$$

where Laplace's product is readily reduced to the parabolic hypercylinder function.

3. We shall show presently that there exists, between the hyperspherical zonal functions and the functions of the hyperparaboloïd of revolution, a connection similar to that between Legendre polynomials and the parabolic cylinder functions.

Let us first investigate the relation between the one-variable functions, and for that purpose give a preliminary definition.

If between two differential, linear and homogeneous, equations, E and D, there exists a relation such that, deriving equation D n times with respect to the independent variable x, we obtain equation E, we shall say that D is the *equation of Didon* of E for the order n. Similarly, if between two linear and homogeneous two-variable systems, S and Δ, there exists a relation such that, deriving both equations of Δ m times with respect to x and n times with respect to y, we find the system S, Δ will be called the *system of Didon* of S for orders m and n. This definition, which we proposed some time ago,* has its origin in the fact that F. Didon made an extensive and successful use of the transformation in question to solve differential equations occurring in the theory of two-variable polynomials.

* *Nouv. Ann. de Math.*, décembre 1919.

Let us consider now the differential equation for Legendre polynomials,

$$(x^2-1)y''+2xy'-n(n+1)y=0 \quad . \quad . \quad . \quad . \quad (E_1)$$

and its equation of Didon for the order n, which is

$$(x^2-1)z''-2(n-1)xz'-2nz=0 \quad . \quad . \quad . \quad . \quad (D_1)$$

In this last equation, let us replace x by $\frac{x}{\sqrt{n}}$, and cause n to tend to infinity. We obtain

$$z''+2xz'+2z=0 \quad . \quad . \quad . \quad . \quad . \quad (D_2)$$

which equation is itself the equation of Didon for

$$y''+2xy'+2(n+2)y=0 \quad . \quad . \quad . \quad . \quad . \quad (E_2)$$

But (E_2) is verified by

$$y=x^{-\frac{1}{2}}e^{-\frac{x^2}{2}}W_{-\frac{n}{2}-\frac{3}{2},\,-\frac{1}{4}}(-x^2),$$

where the W function is of the type of the parabolic-cylinder functions. The connection between it and Legendre function is therefore established, through their equations of Didon.

Let us come now to the field of two variables, and consider the polynomials $V_{m,n}(x, y)$ studied by Hermite, which, as we mentioned in our Introduction, Appell showed to be hyperspherical zonal functions. The $V_{m,n}$ function satisfies the system

$$(S_1)\quad \begin{cases}(1-x^2)r-xys-(n+3)xp+myq+m(m+n+2)z=0\\(1-y^2)t-xys-(m+3)yq+nxp+n(m+n+2)\,z=0,\end{cases}$$

of which the system of Didon for the orders m and n is

$$(\Delta_1)\quad \begin{cases}(1-x^2)r-xys+(N-3)px+Nqy+2Nz=0\\(1-y^2)t-xys+(N-3)qy+Npx+2Nz=0,\end{cases}$$

where $N=m+n$. Replacing in it x and y by $\frac{x}{\sqrt{N}}$ and $\frac{y}{\sqrt{N}}$, and making then N infinite, we obtain the system

$$(\Delta_2)\quad \begin{cases}r+px+qy+2z=0\\t+px+qy+2z=0,\end{cases}$$

itself the system of Didon of

$$(S_2)\quad \begin{cases}r+px+qy+(m+n+2)z=0\\t+px+qy+(m+n+2)z=0.\end{cases}$$

But a solution of (S_2) is

$$z=x^{-\frac{1}{2}}y^{-\frac{1}{2}}e^{-\frac{1}{4}(x^2+y^2)}W_{-\frac{m+n+1}{2},\,-\frac{1}{4},\,-\frac{1}{4}}\left(-\frac{x^2}{2},\,-\frac{y^2}{2}\right),$$

where W is of the hyperparaboloïd type, the connection between this function and the hyperspherical polynomials being exactly the same as that between the two one-variable functions considered.

Chapter III.

HYPERCYLINDRICAL FUNCTIONS.

Another example of the use of one of the confluent hypergeometric functions for solving Laplace's equation is given by the introduction of hypercylindrical co-ordinates. We shall first consider the problem in four-dimensional space, and afterwards generalise it.

The change of variables

$$\begin{aligned} x &= \rho \sin\theta \sin\phi \\ y &= \rho \sin\theta \cos\phi \\ z &= \rho \cos\theta \\ t &= t \end{aligned}$$

defines what may be called hypercylindrical co-ordinates, the hypersurface $\rho = \text{const.}$ being an hypercylinder with its generatrices parallel to the t-axis, and with the sphere

$$x^2 + y^2 + z^2 = \rho^2$$

as a basis in the xyz space. It may therefore be termed a spherical hypercylinder. Laplace's equation is then

$$\Delta U = \frac{\partial^2 U}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 U}{\partial \theta^2} + \frac{1}{\rho^2 \sin^2\theta}\frac{\partial^2 U}{\partial \phi^2} + \frac{\partial^2 U}{\partial t^2} + \frac{2}{\rho}\frac{\partial U}{\partial \rho} + \frac{\cot\theta}{\rho^2}\frac{\partial U}{\partial \theta} = 0.$$

A solution may be obtained by taking

$$U = e^{\mu t} \cos \nu\phi \, U_1(\rho, \theta),$$

the two-variable function U_1, which we shall call a function of the spherical hypercylinder, or more briefly hypercylindrical function, being a solution of

$$\frac{\partial^2 U_1}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 U_1}{\partial \theta^2} + \frac{2}{\rho}\frac{\partial U_1}{\partial \rho} + \frac{\cot\theta}{\rho^2}\frac{\partial U_1}{\partial \theta} + \mu^2 U_1 - \frac{\nu^2}{\rho^2 \sin^2\theta} U_1 = 0\,;$$

or, by putting $\cos\theta = \omega$,

$$\text{(A)} \qquad \rho^2 \frac{\partial^2 U_1}{\partial \rho^2} + (1-\omega^2)\frac{\partial^2 U_1}{\partial \omega^2} + 2\rho\frac{\partial U_1}{\partial \rho} - 2\omega\frac{\partial U_1}{\partial \omega} + \mu^2\rho^2 U_1 - \frac{\nu^2}{1-\omega^2} U_1 = 0.$$

Let us denote by $U_1'(\rho, \omega)$ a solution of the same equation in which we gave to ν the value zero. The product

$$U' = e^{\mu t} U_1'(\rho, \omega) = e^{\mu t} U_1'(\rho, \theta)$$

is a solution, independent of ϕ, of Laplace's equation, and therefore a zonal harmonic function. We shall say therefore that the function U_1', solution of

$$\text{(B)} \qquad \rho^2\frac{\partial^2 U_1'}{\partial\rho^2}+(1-\omega^2)\frac{\partial^2 U_1'}{\partial\omega^2}+2\rho\frac{\partial U_1'}{\partial\rho}-2\omega\frac{\partial U_1'}{\partial\omega}+\mu^2\rho^2 U_1'=0,$$

is an hypercylindrical zonal function.

Such a function is readily found by considering our confluent function

$$z=\Xi_2(\alpha,\ \beta,\ \gamma,\ x,\ y),$$

which, as we said, satisfies the system

$$\begin{cases} x(1-x)r+ys+[\gamma-(\alpha+\beta+1)x]p-\alpha\beta z=0 \\ yt+xs+\gamma q-z=0, \end{cases}$$

and therefore the single equation, obtained through elimination of s,

$$x^2(x-1)r+y^2t+\gamma qy-[\gamma-(\alpha+\beta+1)x]px+\alpha\beta zx-zy=0.$$

Let us now, in this last equation, make the change of variable

$$x=\frac{1}{\xi^2},\quad y=\lambda\eta^2,$$

and substitute for z a new function ζ defined by

$$z=\xi\zeta.$$

Let us then give to the parameters the special values

$$\alpha=1,\quad \beta=\tfrac{1}{2},\quad \gamma=\tfrac{3}{2}.$$

We find

$$(1-\xi^2)\frac{\partial^2\zeta}{\partial\xi^2}+\eta^2\frac{\partial^2\zeta}{\partial\eta^2}+2\eta\frac{\partial\zeta}{\partial\eta}-2\xi\frac{\partial\zeta}{\partial\xi}-4\lambda\eta^2\zeta=0,$$

an equation which becomes identical with (B) if we take $\lambda=-\frac{\mu^2}{4}$; so that the function

$$U_1'(\rho,\ \omega)=\frac{1}{\omega}\Xi_2\left(1,\ \tfrac{1}{2},\ \tfrac{3}{2};\ \frac{1}{\omega^2};\ -\frac{\mu^2\rho^2}{4}\right)$$

or

$$U_1'(\rho,\ \theta)=\frac{1}{\cos\theta}\Xi_2\left(1,\ \tfrac{1}{2};\ \tfrac{3}{2};\ 1+\tan^2\theta,\ -\frac{\mu^2\rho^2}{4}\right)$$

is a zonal hypercylindrical function.

The research of a complete hypercylindrical function is now exceedingly simple, if we make the following remark: if U_1' is a solution of (B), a solution of (A) will be

$$U_1(\rho,\ \omega)=(1-\omega^2)^{\frac{\nu}{2}}\frac{\partial^\nu}{\partial\omega^\nu}U_1'$$

(just as $P_n^m(x)=(1-x^2)^{\frac{m}{2}}\frac{d^m}{dx^m}P_n(x)$ is a solution of the associated Legendre equation, and a complete spherical function, while P_n is a zonal one).

Therefore

$$U_1 = \sin^\nu \theta \frac{\partial^\nu}{\partial(\cos\theta)^\nu}\left[\Xi_2\left(1, \tfrac{1}{2}; \tfrac{3}{2}; 1+\tan^2\theta, -\frac{\mu^2\rho^2}{4}\right)\frac{1}{\cos\theta}\right]$$

is a complete hypercylindrical function. But the differential which occurs in this expression is easily reduced. Let us, for brevity, put

$$-\frac{\mu^2\rho^2}{4} = u.$$

We have, considering the expansion of Ξ_2,

$$U_1' = \sum\sum \frac{(\frac{1}{2}, m)}{(\frac{3}{2}, m+n)} \frac{1}{\omega^{2m+1}} \frac{u^n}{n!}$$

and

$$\frac{\partial^\nu U_1'}{\partial\omega^\nu} = (-1)^\nu \sum\sum \frac{(\frac{1}{2}, m)(2m+1, \nu)}{(\frac{3}{2}, m+n)} \frac{1}{\omega^{2m+\nu+1}} \frac{u^n}{n!},$$

and a rather simple transformation shows that

$$(\tfrac{1}{2}, m)(2m+1, \nu) = \frac{\left(\frac{\nu}{2}+1, m\right)\left(\frac{\nu}{2}+\frac{1}{2}, m\right)}{m!}\nu!,$$

so that we have

$$\frac{\partial^\nu U_1'}{\partial\omega^\nu} = (-1)^\nu \frac{\nu!}{\omega^{\nu+1}} \sum\sum \frac{\left(\frac{\nu}{2}+1, m\right)\left(\frac{\nu}{2}+\frac{1}{2}, m\right)}{(\frac{3}{2}, m+n)m!} \frac{1}{\omega^{2m}} \frac{u^n}{n!}$$

$$= (-1)^\nu \frac{\nu!}{\omega^{\nu+1}} \Xi_2\left(\frac{\nu}{2}+1, \frac{\nu}{2}+\frac{1}{2}; \frac{3}{2}; \frac{1}{\omega^2}, u\right).$$

The function

$$U_1(\rho, \theta) = \frac{\tan_\nu \theta}{\cos\theta}\Xi_2\left(\frac{\nu}{2}+1, \frac{\nu}{2}+\frac{1}{2}; \frac{3}{2}; 1+\tan^2\theta, -\frac{\mu^2\rho^2}{4}\right)$$

is therefore an hypercylindrical function, Laplace's product for hypercylindrical co-ordinates being

$$U = e^{\mu t}\cos\nu\phi \frac{\tan^\nu\theta}{\cos\theta}\Xi_2\left(\frac{\nu}{2}+1, \frac{\nu}{2}+\frac{1}{2}; \frac{3}{2}; 1+\tan^2\theta, -\frac{\mu^2\rho^2}{4}\right).$$

The same problem in $(n+2)$-dimensional space leads us again to the Ξ_2 function. If the Cartesian co-ordinates are denoted by $x_1, x_2, \ldots, x_{n+2}$, hypercylindrical co-ordinates will be defined by

$$\begin{aligned}
x_1 &= \rho\sin\theta\sin\phi_1\sin\phi_2 \ldots \sin\phi_{n-2}\sin\phi_{n-1} \\
x_2 &= \rho\sin\theta\sin\phi_1\sin\phi_2 \ldots \sin\phi_{n-2}\cos\phi_{n-1} \\
x_3 &= \rho\sin\theta\sin\phi_1\sin\phi_2 \ldots \cos\phi_{n-2} \\
&\ldots\ldots\ldots\ldots \\
x_n &= \rho\sin\theta\cos\phi_1 \\
x_{n+1} &= \rho\cos\theta \\
x_{n+2} &= t
\end{aligned}$$

and Laplace's equation is

$$0=\Delta U = \frac{\partial^2 U}{\partial x_1^2}+\frac{\partial^2 U}{\partial x_2^2}+\ldots+\frac{\partial^2 U}{\partial x_{n+2}^2}$$

$$= \frac{\partial}{\partial\rho}\left[\rho^n \sin^{n-1}\theta \sin^{n-2}\phi_1 \ldots \sin\phi_{n-2}\frac{\partial U}{\partial\rho}\right]$$

$$+\frac{\partial}{\partial\theta}\left[\rho^{n-2}\sin^{n-3}\theta\sin^{n-2}\phi_1 \ldots \sin\phi_{n-2}\frac{\partial U}{\partial\theta}\right]$$

$$+\frac{\partial}{\partial t}\left[\rho^n\sin^{n-1}\theta\sin^{n-2}\phi_1 \ldots \sin\phi_{n-2}\frac{\partial U}{\partial t}\right]$$

$$+\sum_{i=1}^{i=n-1}\frac{\partial}{\partial\phi_i}\left[\rho^{n-2}\sin^{n-3}\theta\sin^{n-4}\phi_1 \ldots \sin^{n-i-2}\phi_{i-1}\sin^{n-i-1}\phi_i\sin^{n-i-2}\phi_{i+1}\ldots\sin\phi_{n-2}\frac{\partial U}{\partial\phi_i}\right].$$

We shall try to solve it by a function of the form

$$U=f_0(t)f_1(\phi_{n-1}) \ldots f_{n-1}(\phi_1)U_1(\rho,\theta).$$

We readily observe that we may take

$$f_0=e^{kt}$$

and

$$f_1=\cos m_1\phi_{n-1},$$

where m_1 is an arbitrary integer.

The terms containing ϕ_{n-2} can then be dissociated from the rest, and we find for f_2 the equation

$$\frac{d^2f_2}{d\phi_{n-2}^2}+\cot\phi_{n-2}\frac{df_2}{d\phi_{n-2}}-m_1^2\frac{f_2}{\sin^2\phi_{n-2}}+Ff_2=0,$$

F being a certain function of $\phi_{n-3}, \ldots, \phi_1, \rho, \theta$. We shall therefore introduce the associated Legendre function and take

$$f_2=P_{m_2}^{m_1}(\cos\phi_{n-2}),$$

m_2 being an arbitrary integer, and the equation for f_3 will then be

$$\frac{d^2f_3}{d\phi_{n-3}^2}+2\cot\phi_{n-3}\frac{df_3}{d\phi_{n-3}}-\frac{m_2(m_2-1)}{\sin^2\phi_{n-3}}f_3+F_1f_3=0,$$

the law of formation of the successive equations appearing clearly enough.

Let us now consider the polynomial $C_\lambda^\nu(2)$ of Gegenbauer, which is the coefficient of α^λ in the expansion, in ascending powers of α, of

$$(1-2\alpha z+\alpha^2)^{-\nu},$$

and let us denote the function

$$(1-z^2)^{\frac{\mu}{2}}\frac{d^\mu}{dz^\mu}C_\lambda^\nu(z),$$

where μ is an integer, by the symbol $C_{\lambda,\mu}^\nu(z)$. This function $C_{\lambda,\mu}^\nu$ plays with respect to C^{ν}_{λ} the same part as P^{m}_{n} to P_n; and we have in particular

$$C^{\frac{1}{2}}_{\lambda,\mu} = P^{\mu}_{\lambda}.$$

It is easily seen that the function $C^{\nu}_{\lambda;\mu}(\cos\phi)$ satisfies the differential equation

$$\frac{d^2u}{d\phi^2} + 2\nu\cot\phi\frac{du}{d\phi} + u\left[\lambda(\lambda+2\nu) - \frac{\mu(\mu+2\nu-1)}{\sin^2\phi}\right] = 0,$$

so that there appears a way of solving our above equations by taking, in general,

$$f_i(\phi_{n-i}) = C^{\frac{i-1}{2}}_{m_i,\, m_{i-1}}(\cos\phi_{n-i}).$$

The last equation, which contains only θ and ρ, is found to be

$$0 = \rho^2\frac{\partial^2 U_1}{\partial\rho^2} + n\rho\frac{\partial U_1}{\partial\rho} + \frac{\partial^2 U_1}{\partial\theta^2} + (n-1)\cot\theta\frac{\partial U_1}{\partial\theta} + U_1\left[k^2\rho^2 + \frac{m_{n-1}(m_{n-1}+n-2)}{\sin^2\theta}\right].$$

We can then easily verify, as we did in the case of four dimensions, that a solution of it is

$$U_1 = \frac{\tan^{m_{n-1}}\theta}{\cos^{n-1}\theta}\Xi_2\left(\frac{m_{n-1}+n}{2}, \frac{m_{n-1}+n-1}{2}; \frac{n+1}{2}; 1+\tan^2\theta, -\frac{k^2\rho^2}{4}\right),$$

so that Laplace's product in hypercylindrical co-ordinates and $(n+2)$-dimensional space is

$$U = e^{kt}\cos m_1\phi_{n-1}C^{\frac{1}{2}}_{m_2,\,m_1}(\cos\phi_{n-2})C^{1}_{m_3,\,m_2}(\cos\phi_{n-3}) \;.\;.\;.\; C^{\frac{n}{2}-1}_{m_{n-1},\,m_{n-2}}(\cos\phi_1)\frac{\tan^{m_{n-1}}\theta}{\cos^{n-1}\theta}$$

$$\times\,\Xi_2\left(\frac{m_{n-1}+n}{2}, \frac{m_{n-1}+n-1}{2}; \frac{n+1}{2}; 1+\tan^2\theta, -\frac{k^2\rho^2}{4}\right).$$

If we seek a solution independent of the ϕ's, which will therefore be a zonal function, we equate all the m's to zero, and obtain

$$U_1'(\rho,\theta) = \frac{1}{\cos^{n-1}\theta}\Xi_2\left(\frac{n}{2}, \frac{n-1}{2}; \frac{n+1}{2}; 1+\tan^2\theta, -\frac{k^2\rho^2}{4}\right).$$

Let us take in particular $n=1$: we are in three-dimensional space, and we obtain a cylindrical zonal function which is here

$$U_1'(\rho,\theta) = \Xi_2\left(\tfrac{1}{2}, 0; 1; 1+\tan^2\theta, -\frac{k^2\rho^2}{4}\right);$$

but this is equal, through the expansion of Ξ_2, to

$$\sum_n \frac{\left(-\frac{k^2\rho^2}{4}\right)^n}{(1,n)n!}$$

which is precisely Bessel function $J_0(k\rho)$; an obvious result which, however, affords a good confirmation of the preceding theories.

These examples are sufficient to show the importance of the confluent hypergeometric functions of two variables in four-dimensional potential theory. We present them together in the following scheme, adding Appell's previous results on Hermite's two-variable polynomials, and the corresponding relations between one-variable functions and the three-dimensional potential theory: the scheme explains itself.

Solutions of Laplace's Equation by Hypergeometric Functions.

(a) 3 Dimensions—One Variable.		
1. Sphere	Legendre functions	particular case of the hypergeometric function F.
2. Parabolic cylinder	Weber functions	particular case of the confluent hypergeometric function Φ.
3. Circular cylinder	Bessel functions	confluent hypergeometric function B.
(b) 4 Dimensions—Two Variables.		
1. Hypersphere	Hermite polynomials	particular case of the hypergeometric function F_2.
2. Parabolic hypercylinder 3. Hyperparaboloïd	Function $W_{k, \mu, \nu}$	particular case of the confluent hypergeometric function Ψ_2, deduced from F_2.
4. Spherical hypercylinder	Function Ξ_2	confluent hypergeometric function, deduced from F_3.

(*Issued separately May* 9, 1921.)

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Cells, Liver,—Intra-cellular Canaliculi in.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

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CONTENTS.

[*Continued on page iv of Cover.*

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[*Continued on next page.*

ERRATA in the Abstract of Sir J. A. Ewing's Address on Molecular Energy in Gases, *Proc. R.S.E.*, vol. xl, pp. 102–111, for May 3, 1920.

Page 105, line 20, *for* $\frac{5}{3}$ *read* $\frac{5}{2}$ (*bis*), and *for* $\frac{7}{3}$ *read* $\frac{7}{2}$.
Page 106, line 28, *for* 10^{-23} *read* 10^{23}.

X.—An Experimental Analysis of the Losses by Evaporation of Liquid Air contained in Vacuum Flasks. By Professor Henry Briggs, D.Sc.

(MS. received March 1, 1921. Read March 21, 1921.)

The experiments here described were made on behalf of the Oxygen Research Committee of the Scientific and Industrial Research Department, and the paper is given by permission of the Department. The fullest acknowledgment is due to Dr J. A. Harker, F.R.S., and to his co-workers, Professor G. W. Todd and Mr S. H. Groom, who have, in a series of able memoirs presented to the Oxygen Committee, analysed the nature of the heat-transfer from the outer atmosphere to the interior of metal vacuum bottles; but for their memoirs the writer's experiments would not have suggested themselves.

The Dewar Vacuum Vessel.

The Dewar vacuum flask, which enables low boiling-point liquids to be stored and transported, has been the principal means of rendering possible the great expansion now proceeding in the scientific and commercial uses of liquid air and liquid oxygen. These liquids are being increasingly employed as laboratory reagents, and are being put to service in mine rescue apparatus, for blasting, in aviation and therapeutics, and in evacuation plant.

Vacuum vessels are made in glass, silica ware, porcelain, and metal; but for carrying and handling the liquids in bulk, only the last kind is at present of much value. The glass vessels devised by Dewar in the course of his researches on liquefied gases, and made by him in many forms, are too well known to need description.*

Dewar described the metallic vacuum vessel in 1906,† and not long after that date its manufacture was taken up in Germany, whence, before the war, all the flasks required at British mine rescue stations were obtained. During the latter part of the war these bottles became necessary for the Services, and as a result of the exertions of makers and of the

* Sir James Dewar, "Liquid Atmospheric Air," *Proc. Roy. Inst.*, xiv (1893), p. 1; "The Coming of Age of the Vacuum Flask," *ibid.*, xxi, p. 240.

† *Ibid.*, "Studies on Charcoal and Liquid Air," xviii, p. 439.

guidance of Government research officers they are being successfully produced in this country.

The most convenient size of liquid air or liquid oxygen *container*, or storage and transport flask, is designed to carry 50 lbs. of the fluid. A metal container of this capacity and of proportions usual at the present time is shown in sectional elevation in fig. 1. It consists of inner and outer spheres, *A* and *B*, respectively 14 and 15 ins. in diameter, the inner one being suspended by a thin, narrow neck, *C*, of low-conductivity alloy which is soldered at its upper end into a metal plug, *E*. The space between the globes is evacuated through a lead pipe, *G*, which, when the operation is complete, is squeezed flat and sealed off by means of a flame; the pipe is finally protected (as is shown) by a metal cap containing bitumen or wax in which the end of the tube is embedded. The care needed in making these bottles will to some extent be realised from the fact that a high vacuum requires to be maintained through the agency of seven soldered joints. A dish-shaped metal spinning, *F*, is attached to the lower half of the inner globe, and holds activated charcoal. This important addition is due to Dewar; without it, even a well-made and well-evacuated metal flask would not long hold a high vacuum. In the container illustrated, the charcoal is connected to the vacuum space by means of one or more openings in the dish, the openings being covered by fine gauze. At liquid-air temperature the charcoal's power of adsorption is very strong; it draws into and retains in its own capillaries and inter-molecular passages most of the residual gas in the vacuum space, and therefore automatically preserves the high degree of vacuum needed.

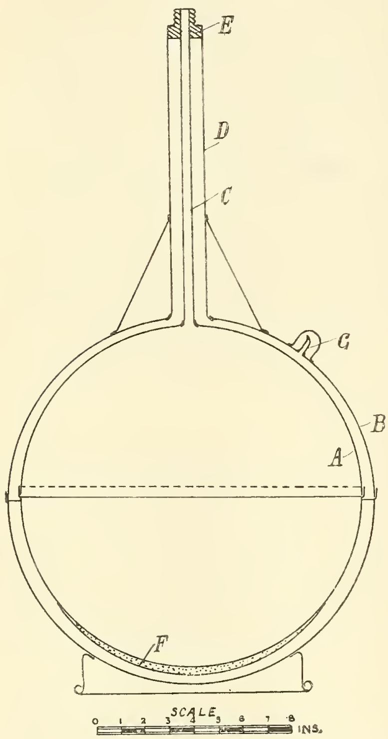

Fig. 1.

The inner surface of the outer globe and the outer surfaces of the inner globe and charcoal dish are highly polished, to reduce as far as possible the heat transferred across the vacuum space by radiation.

Most of the containers made in this country are constructed of copper—

the metal which, with the sole exception of silver, has the lowest emissivity. The German-built vessels now in this country are of brass, in the manufacture of which alloy, in its finest grade, the Germans are unrivalled. Dewar, before the war, built satisfactory vacuum flasks of nickel.

A first-class 50-lb. container loses, by tranquil evaporation, about 2·5 lbs. of liquid air per day. The average loss of these vessels is probably about 3·5 lbs. per day.

Causes of Evaporative Loss.

Leaving out of account certain minor and negligible causes of heat-transfer, there may be said to be three ways by which heat from the outside atmosphere may reach liquid air stored in a vacuum flask. Stated in their order of importance for a good flask, these are:

(1) By radiation from the relatively warm outer vessel to the cold inner vessel;

(2) By conduction across the vacuum space; and

(3) By conduction down the neck of the flask.

When the vacuum is failing, conduction across the vacuum becomes responsible for a greater heat transfer than radiation.

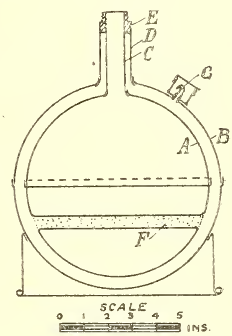

Fig. 2.

Let θ_1 be the absolute temperature of the hotter outer globe, and θ_2 that of the colder inner globe. Let us deal with the three processes of heat-transfer in the order given above.

(1) *Radiation.*—We are here concerned with heat passing from a relatively hot spherical surface (whose temperature was caused to assume different values in the course of the experiments) to a similar cold surface whose temperature remains constant, the latter being the boiling-point of the liquid the vessel contains. Variation in the temperature of the hotter surface involves variation in the dominant wave-length of the radiation, and it is necessary to inquire whether this does not, in its turn, involve a change in emissivity.

The dominant wave-length, in microns, λ_m, and the absolute temperature of the hotter surface, θ_1, are connected by the expression $\lambda_m\theta_1 = 2950$. In the case of the three-litre gilding-metal flask (fig. 2), whose evaporation rates were determined (see below) at external temperatures of 10°, 44°, 70°, and 100°C., λ_m assumes the values 10·4, 9·4, 8·6, and 7·9 respectively. The gilding metal contained 95 per cent. of copper; and although it will eventually be seen that its emissivity is considerably higher than that of the polished pure metal, it is justifiable to assume that the *variation* in emissivity, over the range of λ_m in question, will be similar to that of copper. The data available indicate that the emissivity of copper is very nearly constant between $\lambda_m = 10{\cdot}4\mu$ and $\lambda_m = 7{\cdot}9\mu$; and that, if it be taken as 0·016, it will be correct over the whole range to the third place of decimals. The emissivity of gilding metal has accordingly been regarded as constant for the temperatures used in the tests described below.

The first of the analyses here attempted is based upon figures given by Dewar for a glass flask in which the inner vessel was "silvered" with mercury, the outer one being untreated. The evaporation loss of the flask was ascertained over a temperature-range (of θ_1) extending between 158° abs. and 338° abs. Information does not appear to be available regarding the change of emissivity of glass and mercury surfaces over such a wide range of λ_m as is here involved, and it has been necessary to assume the emissivity of these surfaces to be constant between the stated extremes of temperature. Owing to the uncertainty resulting from the incompleteness of the physical data, the results obtained for the glass flask must be regarded as rough approximations. When emissivity is constant, the heat radiation may be expressed as

$$L_R = a(\theta_1^4 - \theta_2^4) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which a is a coefficient depending upon the emissivity of the surfaces and upon their dimensions. The unit may be calories per second, or, as is here more convenient, grams of liquid air evaporated per hour.

(2) *Conduction across the Vacuum Space.*—With the highly refined vacua with which we are concerned, the mean free path of the gas molecules is greater than the distance (about 1 cm.) between the hot and cold surfaces; hence, if conductivity had been independent of temperature, the heat carried by conduction across the vacuum would be proportional to $(\theta_1 - \theta_2)$. In a gas, however, the conductivity varies as the square root of the absolute temperature, and, as the mean temperature across the vacuum space is $0{\cdot}5(\theta_1 + \theta_2)$, the expression representing the evaporation loss due to this cause is, in grams of liquid air evaporated per hour:

$$L_c = b(\theta_1 - \theta_2)\sqrt{\theta_1 + \theta_2} \quad . \quad . \quad . \quad . \quad (2)$$

where b is constant for a given bottle.

(3) *Neck Conduction.*—The amount of liquid evaporated because of heat conducted along the inner neck, *C*, of the flask cannot be evaluated even roughly by direct computation based upon the dimensions of the neck and the conductivity of the metal, owing to the fact that the tube forms the

channel of discharge of the cold gas leaving the vessel. Much of, and in many cases all of, the heat flowing down the neck-tube enters the uprising current of air in that tube, and never reaches the liquid in the flask. The evaporation loss due to neck conduction thus depends upon the rate of discharge of gas from the flask as well as upon the dimensions of the neck, and in a poor flask with a high total evaporation loss the neck-loss will be actually as well as relatively less than in a good flask of the same dimensions.

Other things being equal, the evaporation rate of a large vessel, though proportionately less, is, in grams per hour, actually more than in a small vessel. It therefore follows that as the size of the flask is increased the neck may be shortened, or alternatively, made stouter, without the evaporation rate due to neck conduction being affected. Heat-transfer along the neck is studied experimentally in a later part of the paper.

In the cases examined, the temperature of the gas issuing from the mouths of metal vacuum flasks containing liquid air lay between $-4°$ C. and $-30°$ C. As the inner globe is made of so good a conductor as copper, gilding metal, or brass, its temperature may be regarded as uniform at all points of the sphere, that temperature being the boiling-point of the liquid. It is therefore apparent that the heat transferred to the inner globe by radiation and conduction across the vacuum space is all absorbed in giving latent heat to the gas boiled off, and that the neck alone is responsible for heating the evaporated gas from the boiling temperature to that at which it discharges into the outer air.

Evaluation of the Effects of Radiation and Conduction for a Glass Flask.

The writer's method of analysing the tranquil evaporation-rate of a vessel holding liquid air or oxygen, so as to apportion the amount of loss due to the three several causes set forth above, is indicated by the present example, which consists of a simplification of the general problem in that the transference of heat down the glass neck, and in opposition to the upward flow of cold gaseous oxygen, must have been altogether negligible.

Dewar filled a glass vacuum flask with liquid oxygen (boiling-point, $-185°$ C.) and measured its evaporation rate when the flask was immersed in liquids maintained at different temperatures, with the results stated in Table I.* At that time (1893) the vacuum was obtained by washing out

* Sir James Dewar, "Liquid Atmospheric Air," *Proc. Roy. Inst.*, xiv (1893), p. 1.

the space between the inner and outer vessels with mercury vapour, and then exhausting. Some condensation of the residual vapour took place when liquid oxygen was introduced, causing the formation of a mercury mirror on the surface of the inner vessel.

Table I.—Glass Flask: Evaporation Losses at Different External Temperatures.

Temperature of Outer Vessel.	Absolute Temperature of Outer Vessel. θ_1.	Absolute Temperature of Inner Vessel. θ_2.	Evaporation, c.cs. of Gas per minute. L.
−115° C.	156°	91°	60
− 78° C.	195°	91°	120
+ 6° C.	279°	91°	300
+ 65° C.	338°	91°	600

Neck-loss being inconsiderable, the total evaporation loss, L, is the sum of the losses due to radiation and conduction across the vacuum; or, from equations (1) and (2):—

$$L = a(\theta_1^4 - \theta_2^4) + b(\theta_1 - \theta_2)\sqrt{\theta_1 + \theta_2} \quad . \quad . \quad . \quad (3)$$

The values of the table are in reasonable agreement with the equation:—

$$L = 3{\cdot}01(\theta_1^4 - \theta_2^4)10^{-8} + 0{\cdot}041(\theta_1 - \theta_2)\sqrt{\theta_1 + \theta_2} \quad . \quad . \quad (4)$$

The first term on the right-hand side of the equation expresses the loss due to radiation, and the second that due to conduction. At 15° C. radiation was responsible for the evaporation of 236 c.cs., and conduction across the vacuum for 138 c.cs. of gas per minute. Had the flask held liquid hydrogen instead of liquid oxygen, the losses at an external temperature of 15° C. would have been 238 c.cs. and 187 c.cs. per minute respectively.

Estimation of the Three Causes of Evaporation Loss for a Small Metal Flask.

The vessel is illustrated in fig. 2. It is a *vaporiser* flask, *i.e.* one whose function it is, by the aid of certain fittings, to evaporate liquid air at set rates. The fittings are not shown; they were not attached during these tests. The capacity of the flask is 3 litres (about 7 lbs.); it is made of gilding metal (95 per cent. copper; 5 per cent. tin), and the inner neck, *C*, is of cupronickel, an alloy having one-seventh the conductivity of copper.

The charcoal (plumstone) is in this instance held in a copper tube, *F*, passing through the inner globe, the ends of the tube being covered with gauzes. The spheres are respectively $8\frac{3}{8}$ ins. and $7\frac{1}{2}$ ins. in diameter, and their surfaces are 220 and 177 sq. ins. or 1419 and 1152 sq. cms. in area. The neck is unusually short and wide in comparison to the size of the bottle, being $3\frac{1}{4}$ ins. long, of which only $2\frac{3}{8}$ ins. (6 cms.) are surrounded by vacuum. The bore of the neck is $\frac{5}{8}$ in. and the metal 0·024 in. in thickness; there is 0·047 sq. in. or 0·303 sq. cm. of metal in a cross-section of the tube.

Before any of the following observations were made, the flask held liquid air for twelve hours. It was never allowed to boil empty during the fortnight over which the test extended. The same weight of liquid air (5 lbs.) was put in, and about 2 lbs. of air were allowed to evaporate on each "run." The exact losses were ascertained by weighing. The lowest of the external temperatures recorded below was obtained by standing the flask in a cellar, and the others by immersing it, up to the base of the neck-screw, in a water-bath kept at constant temperature. The average composition of the liquid air was 50 per cent. oxygen, 50 per cent. nitrogen, and was found from samples of the liquid drawn from the flask at the beginning and end of a "run." This mixture boils at $-191°$ C. The ascertained losses at different external temperatures are given in Table II.

Temperature measurements of the air passing up the neck were made during the first few tests in order to find out if neck-loss was serious. They showed that, (*a*) with the short neck in question, this source of loss could not be neglected, and (*b*) the neck-loss was proportionately less important as the outside temperature rose.

TABLE II.—METAL FLASK: EVAPORATION LOSSES AT DIFFERENT EXTERNAL TEMPERATURES.

Period of Test. Hours.	Temperature of Outer Globe.	Absolute Temperature, Outer Globe. θ_1.	Absolute Temperature, Inner Globe. θ_2.	Evaporation Loss, grams per hour.
(*a*) 21 (*b*) $16\frac{1}{2}$ (*c*) 14 (*d*) $11\frac{1}{2}$	10° C.	283°	82°	42·5
(*a*) $13\frac{1}{2}$ (*b*) $13\frac{1}{2}$	44° C.	317°	82°	61·3
13	70° C.	343°	82°	72·0
$9\frac{1}{2}$	100° C.	373°	82°	97·6

To enable neck-loss to be evaluated independently of the other modes of heat-transfer, an inner tube of cupronickel of the same length as the neck was slipped inside the latter, and the evaporation rates were again determined at the selected temperatures.* By inserting this extra tube, the sectional area was increased from 0·303 sq. cm. to 0·856 sq. cm. of metal. The enhanced rates of evaporation resulting therefrom are set forth in Table III.

TABLE III.—METAL FLASK: EVAPORATION LOSSES WHEN ADDITIONAL TUBE WAS INSERTED IN NECK.

Period of Test. Hours.	Absolute Temperature of Outer Globe. θ_1.	Absolute Temperature of Inner Globe. θ_2.	Evaporation Loss, grams per hour.
14	283°	82°	62·5
10	317°	82°	89·4
8½	343°	82°	99·7
7	373°	82°	124·1

It will be observed that when the outside temperature was 10° C. (283° abs.) the extra tube added 20 grams per hour to the rate of evaporation. By simple proportion, the neck-loss at this temperature when the additional tube was absent was 11 grams per hour. The equivalent losses at the other stated temperatures (Table IV) were obtained in the same manner.

To make sure that the flask was not deteriorating under the treatment it was receiving, frequent check determinations were made of the normal evaporation rate at 10° C. There was no sign of deterioration.

TABLE IV.—METAL FLASK: NECK LOSSES CALCULATED FROM FOREGOING TABLES.

Absolute Temperature, Outer Globe. θ_1.	Increase of Evaporation due to insertion of Extra Tube, 0·553 sq. cm. section; grams per hour.	Evaporation per sq. cm. of Neck Section; grams per hour.	Actual Neck-Loss, grams per hour.
283°	20·0	36·2	11·0
285° (25° C.)	...	...	11·9 †
317°	28·1	50·9	15·4
343°	27·7	50·1	15·2
373°	26·5	48·0	14·6

† Interpolated.

* This method of determining neck-loss is described in "Grundlagen zum Bau von Transportgefässen für verflüssigte Gase," by F. Banneitz, G. Rhein, and B. Kurze, *Annalen der Physik*, vol. lxi (1920), p. 113.

The last table shows that as θ_1 increased, the neck-loss rose to a maximum and then fell. The fall was due to the fact that the stream of cold air passing up the neck increased at a more rapid rate than did the passage of heat down the metal of the neck.

By subtracting the ascertained neck-losses (Table IV) from the total losses (Table II) the rates of evaporation (L) due to radiation *plus* conduction across the vacuum were obtained (Table V):—

TABLE V.—METAL FLASK: LOSSES DUE TO CONDUCTION PLUS RADIATION ACROSS THE VACUUM; NECK-LOSSES ELIMINATED.

Absolute Temperature, Outer Globe. θ_1.	Absolute Temperature, Inner Globe. θ_2.	Radiation, plus Conduction, grams per hour. L.
283°	82°	31·5
317°	82°	45·9
343°	82°	56·8
373°	82°	83·0

With these values equation (3) takes the form:—

$$L = 8{\cdot}228(\theta_1^4 - \theta_2^4)10^{-9} + 0{\cdot}00284(\theta_1 - \theta_2)\sqrt{\theta_1 + \theta_2} \quad . \quad . \quad (5)$$

Since, as before, the first term of the right-hand side of this equation determines radiation and the second term determines conduction, the complete analysis of the loss by evaporation is now possible. It is given in Table VI, the values for neck-conduction being copied from Table IV. The degree of agreement between the results derived from equation (5) and those obtained by experiment may be gathered by comparing the last columns of Tables II and VI.

TABLE VI.—METAL FLASK: LOSSES DUE TO CONDUCTION, RADIATION AND NECK, SEVERALLY STATED.

Temperature, Outer Globe.	Conduction across Vacuum, grams per hour.	Radiation, grams per hour.	Neck Conduction, grams per hour.	Total Loss, grams per hour.
10° C.	10·9	20·6	11·0	42·5
15°	11·3	22·1	11·9	45·3
44°	13·3	32·5	15·4	61·2
70°	15·3	44·5	15·2	75·0
100°	17·6	62·4	14·6	94·6

The radiation loss at ordinary external temperatures is thus, in this flask, about twice that due to conduction of heat across the vacuous space.

Pressure in the Vacuum Space of the 3-Litre Flask, and Emissivity of the Metal Surfaces.

The results stated in Table VI allow the emissivity of the reflecting surfaces of the flask and the degree of tenuity of the vacuum to be obtained.

Before emissivity can be calculated it is necessary to inquire into the manner of heat-exchange, by radiation, of two reflecting surfaces facing each other. The following demonstration is due to J. A. Harker:—

Let the emissivity of either of the two similar surfaces be E. Suppose E units of heat to be emitted by the hotter surface; of these E^2 will be absorbed and $E(1-E)$ will be reflected by the second or cooler surface. The first surface will then reflect $E(1-E)^2$ units, of which the second surface will absorb $E^2(1-E)^2$. Proceeding thus, it appears that when the hotter surface emits E units the cooler surface gains an amount which is the sum of an infinite G.P. whose first term is E^2 and whose common ratio is $(1-E)^2$, and that summation is $\frac{E}{2-E}$.

Applying Harker's result to Stefan's Law; taking the constant of that law as $1·385 \times 10^{-12}$ (calorie units); taking the latent heat of a half-and-half mixture of liquid nitrogen and liquid oxygen as 50·2 cals. per grm.; and making use of the dimensions of the flask, the emissivity of the gilding-metal surfaces was found to be 0·050. That for pure copper polished to the highest degree is 0·016. The fact that the makers of the flask have only succeeded in getting an emissivity amounting to thrice that of copper is important. In studying to reduce the tranquil evaporation rate in these particular flasks it is evidently on the radiation loss that most attention should be focussed. A small increase upon the value for the emissivity of copper might have been expected owing to the gilding metal containing tin, and a further slight increase is no doubt due to the smear of solder running equatorially round the inner sphere; but the main reason for the enhanced radiation is probably that water vapour condensed on the inner sphere and spoilt the surface. It is a most difficult matter, with the method of evacuation used at present for soft-soldered metal flasks, to rid the charcoal and vacuum space entirely of water vapour.

With pressures as low as those in the vacuum spaces of liquid-air bottles, the heat-transfer by conduction across such a space is proportional to the difference of temperature, $(\theta_1-\theta_2)$, to the pressure, p, and to the area of the surface, A, and is independent of the distance.

That is to say:—

$$\text{Heat transferred by conduction across the vacuum} = c(\theta_1-\theta_2)pA \quad . \quad (6)$$

It is also known, when p is measured in mms. of mercury and A in sq. cms., that c takes a value for air of approximately 2×10^{-5} at 30° C. To apply equation (6), this figure has first to be adjusted to the average temperature of $0{\cdot}5\ (\theta_1 + \theta_2)$; Table VI has to be consulted for the conduction-loss at any given value of θ_1; and the latent heat of the liquid mixture in the flask and the dimensions of the flask have to be taken into account. The pressure in the vacuum space, p, can then be computed. It was found in this case to be 0·00038 mm. mercury. Considering that the flask had been evacuated twenty months when the tests were made, this degree of tenuity may be regarded as satisfactory.

Effect of Surrounding the 3-Litre Flask by an Insulating Medium.

As radiation proved to be the most important cause of evaporation, and as, in radiation between two given surfaces, the temperature of the hotter surface is the all-important factor, it appeared probable that evaporation would be appreciably reduced by insulating the flask. The neck was extended by soldering to it a length of brass tube, and the whole of the flask, excepting a short part of that tube, was encased in slag-wool in a sheet-metal canister. The immediate effect was an increase in the rate of evaporation; but after the slag-wool had been given time to cool down, its influence became beneficial. Eventually the rate of evaporation settled to a value that was 82·5 per cent. of the rate given at the same external temperature (8° C.), when the flask was uninsulated. The increased bulk and clumsiness of the insulated flask outweighed, from the practical point of view, any advantage gained.

The Neck-Loss of Liquid-Air Containers.

Measurements were made to ascertain the temperature-gradients along the necks of four containers for the purpose of finding whether the relatively great length of the necks (see fig. 1) was necessary. Temperatures were taken by means of a Foster pyrometer designed for low-temperature observation. The wires of the thermo-element were 1·5 mm. in diameter. The thermometric scale of the galvanometer was tested at room temperature, and at that of boiling liquid air, to ascertain the proportional correction to apply to readings. In taking a reading the thermo-junction was lowered to the desired point in the neck and allowed to stay there until the galvanometer needle became stationary. Inasmuch as the thermo-couple was in the up-flowing current of cold air, and not necessarily in effectual contact with the neck-tube at the point, the temperatures obtained were not strictly those of the metal; the difference, however, was probably not more than a few degrees, and was greatest where it mattered least—namely, near the mouths of the flasks.

The results are set forth graphically in fig. 3.

Three of the bottles were 50-lb. containers of dimensions substantially those indicated by fig. 1. The fourth was a German-built vessel—the largest vacuum flask in the country—capable of holding over 300 lbs. of liquid air.

The first container examined (see curve labelled "German 50-lbs.," fig. 3) was a brass vessel built and evacuated in Germany before the war. After many years of continuous service at the Newcastle mine rescue station its vacuum was showing signs of breakdown; its evaporation rate at the time of the test was 1·85 litres of gas per minute at 18° C., which is equivalent to a daily loss of over 7 lbs. The top of the outer neck was thickly coated with ice. The inner neck was of German silver, $\frac{3}{8}$-in. bore, the thickness of the metal being 0·02 in. As fig. 3 indicates, the neck enters the vacuum space $3\frac{9}{16}$ ins. from the mouth of the flask. The temperature $\frac{7}{16}$ in. below that point was as low as $-172°$ C., and at all parts below $4\frac{7}{16}$ ins. from that point the temperature was the same as that of the liquid in the flask. Under the conditions then obtaining, therefore, the loss due to neck-conduction was nil, and if the neck had been shorter by 5 ins. it would still have been nil.

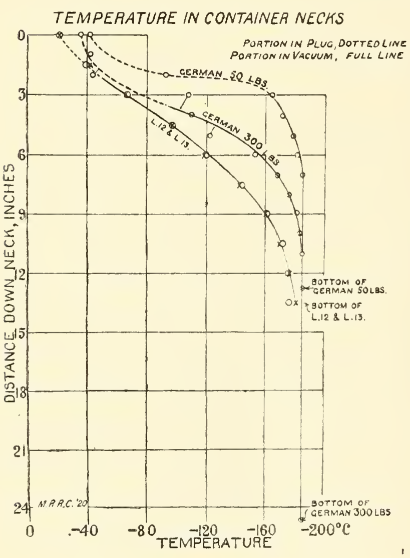

Fig. 3.

The second and third containers examined were made in this country, and were in satisfactory condition. They were of copper, with German-silver necks $13\frac{1}{2}$ ins. long and $\frac{1}{2}$-in. bore. Their rates of gaseous discharge (measured at 16° C.) were respectively 1·15 and 1·03 litres per minute at the time of the tests. The curve labelled "L. 12 and L. 13," fig. 3, expressed the fall of temperature down the neck for both these flasks. The temperatures at the bottom of the necks were, in these cases, appreciably above the temperature of the liquid, a fact which seems to indicate that the evaporated gas gained a little heat from the upper part of the inner globe before it reached the neck in its upward path. With these flasks the gradient near the base of the neck, though slow, was not zero: a certain amount of heat, therefore, reached the inner sphere by the neck. The graph indicates, however, that the transference of heat due to this cause was here equivalent to that which would have been yielded by a neck of the same sectional dimensions about 64 ins. long in which the gradient was uniform from top to bottom.

The fourth container, as already stated, was a very large German vessel; at the time of testing, it was found to be discharging gas at the rate of 3·3 litres per minute, measured at 18° C. It was constructed of brass, with a German-silver neck $24\frac{3}{4}$ ins. long and $\frac{9}{16}$-in. bore. The thickness of the metal of the neck could not be determined; the plug (*E*, fig. 1) probably extended to a depth of about $3\frac{1}{2}$ ins. from the mouth of the bottle. The measurements (see fig. 3, curve marked "German 300-lbs.") show that the passage of heat to the liquid *via* the neck was zero, and that the rate of evaporation would not have been affected in the least had the neck been half the length.

In general, the results indicate that container necks may be considerably shortened, or both shortened and thickened, without appreciably increasing the rate of loss of the flasks. Such an alteration in construction will strengthen the flask and save weight, and, while preserving sufficient flexibility in the neck to allow of the spheres touching during the act of pouring, the excess loss during transportation—which is principally due to the continual bumping together of the cold and hot globes—will be lessened. Underground transport (a matter that the present writer has especially in view) will be facilitated by the reduced height of the bottle.

Results obtained with Short-necked Containers.

Believing at that time that container necks could be shortened and strengthened without any serious effect upon the evaporation rate, the writer designed, in 1918, a 50-lb. container of which twelve were made and ten proved sound. Though there were in these bottles a number of variations upon the standard design illustrated by fig. 1, only two of them could have any influence upon the rate of evaporation. The first of these—namely, the provision of a loose, insulated cap to fit over the mouth of the bottle—was found to have only a slight effect; the cap, when in place, only reduced the loss by a few ounces per diem, a fact which itself demonstrated the relatively small heat-inflow by the neck. The second variation was in the size of the neck, which in each of these special containers was of cupronickel (an alloy having about thrice the conductivity of German silver); the bore was $\frac{1}{2}$ in., the thickness of the metal $\frac{1}{16}$ in., and the length $9\frac{1}{4}$ ins. Had it been possible to disregard the influence of the up-flowing stream of cold air along the neck—that is to say, had neck-loss been merely a question of the conductivity, sectional area, and length of the tube for a given temperature difference—the neck-loss of each of these modified containers would have been 17·5 times that of the German 50-lbs. container referred to above. Actually the rates of evaporation of these more robust flasks showed little if any increase upon those of an equal number of containers of the usual design, selected at random. Their daily evaporation losses proved to be respectively 3·43, 3·75, 4·00, 2·75, 3·31, 3·31, 3·62, 3·69, 3·75, and 4·00 lbs.

Acknowledgment is due to Messrs J. Mallinson, B.Sc., W. Cooper, M.A., B.Sc., and J. J. Brodie, Government research workers, for their help in the experiments here discussed. In the tests upon the 3-litre flask, weighings and temperature observations had to be taken at all hours of the night and day for a fortnight.

Summary.

(1) J. A. Harker and his co-workers having shown that of all the possible causes of heat-inflow to liquid air in a vacuum flask only three, viz. radiation, conduction across the vacuum space, and conduction along the neck of the flask, are of importance, the writer illustrates by two instances an experimental method enabling these sources of heat-transfer to be separately assessed.

(2) In the two selected instances radiation proves to be the main method of heat-transfer.

(3) In the second example (a 3-litre metal flask) the analysis is carried further, and the emissivity of the surfaces and pressure in the vacuous space are determined. The reasons for the relatively high emissivity are discussed.

(4) Pyrometer measurements in the necks of long-necked, large metal storage flasks (containers) show the loss due to neck-conduction to be either zero or entirely negligible. The results indicate that the necks of such vessels may be shortened with advantage.

(5) The tranquil evaporation rates are given of ten metal containers having relatively short, stout necks, in proof of the foregoing conclusion.

(*Issued separately June* 20, 1921.)

XI.—Note on a Continuant of Cayley's of the Year 1874. By Sir Thomas Muir, F.R.S.

(MS. received March 8, 1921. Read March 21, 1921.)

(1) In a "Note sur une formule d'intégration indéfinie" of the year 1874,* Cayley has occasion to use a determinant of a very peculiar structure, whose value when of the $(n+1)^{\text{th}}$ order is

$$\{[a]x^2+[b]y^2\}^n,$$

$[a]^r$ in the development of this standing for

$$a(a-1)\ .\ .\ .\ .\ (a-r+1).$$

The first three instances are

$$\begin{vmatrix} 1 & ax-by \\ y & a(x^2+xy) \end{vmatrix} = ax^2+by^2,$$

$$\begin{vmatrix} 1 & (a-1)x-by & 1 \\ 2y & (a-1)(x^2+xy) & (a+1)x-(b-1)y \\ y^2 & . & a(x^2+xy) \end{vmatrix} = a(a-1)x^4 + 2abx^2y^2 + b(b-1)y^4,$$

$$\begin{vmatrix} 1 & (a-2)x-by & 1 & . \\ 3y & (a-2)(x^2+xy) & ax-(b-1)y & 2 \\ 3y^2 & . & (a-1)(x^2+xy) & (a+2)x-(b-2)y \\ y^3 & . & . & a(x^2+xy) \end{vmatrix}$$
$$= a(a-1)(a-2)x^6 + 3a(a-1)bx^4y^2 + 3ab(b-1)x^2y^4 + b(b-1)(b-2)y^6,$$

where, it is as well to note, the coefficients of x^2+xy in the main diagonal increase by 1 at each step, and the coefficients of x in the adjacent minor diagonal increase by 2. Viewed as an equivalent for a power of $[a]x^2+[b]y^2$ the form of the determinant is far from attractive, its order being higher than seems natural, and there being nothing in it to show that it is unaltered by the change of x into $-x$, of y into $-y$, or by the simultaneous interchange of a with b and x with y.

(2) Resembling it, but more pleasing in form, is another determinant, also noted by Cayley,† the first three instances of which are

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* *Comptes rendus . . . Acad. des Sci.* (Paris), lxxviii, pp. 1624–1629: or *Coll. Math. Papers*, ix, pp. 500–503.

† The reference for this I cannot at present find.

$$\begin{vmatrix} (a-1)x+by & 1 \\ (a-1)(x^2-xy) & (a+1)x+(b-1)y \end{vmatrix} = \{[a]x+[b]y\}^2$$

$$\begin{vmatrix} (a-2)x+by & 1 & . \\ (a-2)(x^2-xy) & ax+(b-1)y & 2 \\ . & (a-1)(x^2-xy) & (a+2)x+(b-2)y \end{vmatrix} = \{[a]x+[b]y\}^3$$

$$\begin{vmatrix} (a-3)x+by & 1 & . & . \\ (a-3)(x^2-xy) & (a-1)x+(b-1)y & 2 & . \\ . & (a-2)(x^2-xy) & (a+1)x+(b-2)y & 3 \\ . & . & (a-1)(x^2-xy) & (a+3)x+(b-3)y \end{vmatrix} = \{[a]x+[b]y\}^4.$$

Here the determinant is a pure continuant, and its order seems natural, so that the only one of our critical queries remaining is that in regard to the interchange of a, x with b, y.

(3) Writing x^2 for x and y^2 for y in the second series of determinants we obtain equivalents for the determinants of the first series, for example,

$$\begin{vmatrix} 1 & (a-1)x-by & 1 \\ 2y & (a-1)(x^2+xy) & (a+1)x-(b-1)y \\ y^2 & . & a(x^2+xy) \end{vmatrix} = \begin{vmatrix} (a-1)x^2+by^2 & 1 \\ (a-1)(x^4-x^2y^2) & (a+1)x^2+(b-1)y^2 \end{vmatrix},$$

so that we have two different determinant expressions for

$$\{[a]x^2+[b]y^2\}^n,$$

and a not very simple-looking problem facing us in transformation.

(4) Further, the n^{th} member of the second series with the sign of y changed is the cofactor of the element y^n in the $(n+1)^{\text{th}}$ member of the first series; and, following this up, we obtain the curious expansion-theorem

$$\begin{aligned}(-1)^n\{[a]x^2+[b]y^2\}^n = \quad & y^n\{[a]x-[b]y\}^n \\ & -(n)_1y^{n-1}\{[a]x-[b]y\}^{n-1}\,.\,a(x^2+xy) \\ & +(n)_2y^{n-2}\{[a]x-[b]y\}^{n-2}\,.\,[a]^2(x^2+xy)^2 \\ & -(n)_3y^{n-3}\{[a]x-[b]y\}^{n-3}\,.\,[a]^3(x^2+xy)^3 \\ & +\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\end{aligned}$$

(5) Before referring further to these forms of Cayley's it is desired to draw attention to a new form, which, besides being interesting in itself, adds considerably to the interest of the others. It arises as follows:—Putting

$$\begin{array}{llll} \beta_1 \text{ for } ax+by & \text{and} & \pi_1 & \text{for } [a]x+[b]y, \\ \beta_2 \text{ for } ax^2+by^2 & & 2!\,\pi_2 & \text{for } \{[a]x+[b]y\}^2, \\ \beta_3 \text{ for } ax^3+by^3 & & 3!\,\pi_3 & \text{for } \{[a]x+[b]y\}^3, \\ \;.\;.\;.\;.\;. & & \;.\;.\;.\;.\;. & \end{array}$$

we find that

$$\begin{aligned} \beta_1 - \pi_1 &= 0 \\ \beta_2 - \beta_1\pi_1 + 2\pi_2 &= 0 \\ \beta_3 - \beta_2\pi_1 + \beta_1\pi_2 - 3\pi_3 &= 0 \\ \beta_4 - \beta_3\pi_1 + \beta_2\pi_2 - \beta_1\pi_3 + 4\pi_4 &= 0 \\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot &\quad\cdot\quad\cdot \end{aligned}$$

and on solving for the π's there is readily obtained

$$\pi_r = \frac{1}{r\,!}\begin{vmatrix} \beta_1 & 1 & \cdot & \cdot & \cdots \\ \beta_2 & \beta_1 & 2 & \cdot & \cdots \\ \beta_3 & \beta_2 & \beta_1 & 3 & \cdots \\ \beta_4 & \beta_3 & \beta_2 & \beta_1 & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdots \end{vmatrix}_r$$

whence

$$\{[a]x + [b]y\}^r = \begin{vmatrix} ax+by & 1 & \cdot & \cdot & \cdots \\ ax^2+by^2 & ax+by & 2 & \cdot & \cdots \\ ax^3+by^3 & ax^2+by^2 & ax+by & 3 & \cdots \\ ax^4+by^4 & ax^3+by^3 & ax^2+by^2 & ax+by & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdots \end{vmatrix}_r$$

—the new result referred to.

(6) The most interesting point about this is that the relation on which it depends,

$$\beta_r - \beta_{r-1}\pi_1 + \beta_{r-2}\pi_2 - \cdots\cdot + (-1)^r r\pi_r = 0,$$

is exactly Newton's relation between the s's and the c's of a number of quantities, that is to say, where s_r is the sum of the r^{th} powers of the quantities and c_r the sum of the r^{th} combinations of them.

(7) Turning now to Cayley's second form we shall first show how there may be derived from it a better, especially as regards the interchange of a, x with b, y. Using the fact that any factor of an element of a continuant may be removed and attached to the conjugate element, we obtain in the case of the fourth order

$$\begin{vmatrix} (a-3)x+by & x & \cdot & \cdot \\ (a-3)(x-y) & (a-1)x+(b-1)y & 2x & \cdot \\ \cdot & (a-2)(x-y) & (a+1)x+(b-2)y & 3x \\ \cdot & \cdot & (a-1)(x-y) & (a+3)x+(b-3)y \end{vmatrix},$$

which for shortness' sake we may denote by

$$K_4(a-3, x, b, y),$$

the $a-3$ under the functional symbol being written instead of a to recall the element in the $(1, 1)^{\text{th}}$ place. If in this we diminish the second row by the first, the third by the new second, and so on, we obtain

$$\begin{vmatrix} (a-3)x+by & x & \cdot & \cdot \\ (3-a-b)y & (a-2)x+(b-1)y & 2x & \cdot \\ -(3-a-b)y & (3-a-b)y & (a-1)x+(b-2)y & 3x \\ (3-a-b)y & -(3-a-b)y & (3-a-b)y & ax+(b-3)y \end{vmatrix},$$

where now the diagonal term is invariant to the interchange of a, x with b, y, and so likewise the coefficient of y in the elements outside the diagonal. The latter coefficient in the case of the n^{th} order is $n-1-(a+b)$, and may be conveniently replaced by a single letter.

(8) If we try to simplify the form of § 7 by increasing the first column by the second, the second by the third, and so on, the diagonal elements return to their original awkward law of formation, the result being

$$\begin{vmatrix} (a-2)x+by & x & \cdot & \cdot \\ (a-2)(x-y) & ax+(b-1)y & 2x & \cdot \\ \cdot & (a-1)(x-y) & (a+2)x+(b-2)y & 3x \\ \cdot & \cdot & a(x-y) & ax+(b-3)y \end{vmatrix}.$$

This, however, is not valueless, when we observe that it would be $K_4(a-2, x, b, y)$ save for the occurrence of ax in its last element instead of $(a+4)x$. Changing therefore ax into $(a+4)x-4x$, we have the quasi recurrence-formula

$$K_4(a-3, x, b, y) = K_4(a-2, x, b, y) - 4xK_3(a-2, x, b, y),$$

and generally

$$K_n(a-n+1, x, b, y) = K_n(a-n+2, x, b, y) - nxK_{n-1}(a-n+2, x, b, y).$$

As it is only the first of the four variables that changes in it, the equality may be viewed as giving the increment of K_n due to a receiving the increment 1.

(9) There is a generalisation of the foregoing which, merely for its own sake, deserves to be noted in passing. *If the* n-*line continuant whose diagonals are*

$$\begin{matrix} & b & & 2b & & 3b & & \cdots \\ a & & a+b+z & & a+2b+2z & & a+3b+3z & \cdots \\ & c & & c+z & & c+2z & & \cdots \end{matrix}$$

be denoted by

$$H_n\begin{pmatrix} & b & b \\ a & & \\ & c & z \end{pmatrix},$$

then

$$H_n\begin{pmatrix} & b & b \\ a & & \\ & c & z \end{pmatrix} = H_n\begin{pmatrix} & b & b \\ a+b & & \\ & c+z & z \end{pmatrix} - nbH_{n-1}\begin{pmatrix} & b & b \\ a+b & & \\ & c+z & z \end{pmatrix}.$$

The mode of proof is quite similar to that of § 8.

(10) Following on § 8 we now show that the "recurrence-formula" of $\{[a]x+[b]y\}^n$ is the same as the corresponding recurrence-formula found for the continuant. The increment of the former expression, due to a becoming $a+1$, is

$$\sum_{r=0}^{r=n}(n)_r[a+1]^{n-r}b^r x^{n-r}y^r - \sum_{r-0}^{r=n}(n)_r[a]^{n-r}b^r x^{n-}\,y^r,$$

and this

$$= \sum_{r=0}^{r=n}(n)_r \,.\, \{[a+1]^{n-r} - [a]^{n-r}\} \,.\, b^r x^{n-r}y^r$$

$$= \sum_{r=0}^{r=n}(n)_r \,.\, (n-r)[a]^{n-r-1} \,.\, b^r x^{n-r}y^r$$

$$= \sum_{r=0}^{r=n}(n-1)_r n \,.\, [a]^{n-r-1} \,.\, b^r x^{n-r}y^r$$

$$= \sum_{r=0}^{r=n}(nx \,.\, (n-1)_r[a]^{n-r-1}b^r x^{n-r-1}y^r$$

$$= nx \,.\, \{[a]x+[b]y\}^{n-1},$$

as desired. The proof, however, for the case where n is 4 would probably be very little less convincing.

(11) And now looking closer into Cayley's continuant, and examining specially the recurrence-formula of it as got by expanding in terms of the elements of the last column and their complementaries, namely, the formula

$$K_{n+1}(a-n,x,b,y) = \{(a+n)x+(b-n)y\}.K_n(a-n,x,b,y) - n(a-1)(x^2-xy).K_{n-1}(a-n,x,b,y),$$

we see that the algebraical equality which we have got to verify in order to establish the truth of Cayley's result is

$$\{[a]x+[b]y\}^{n+1} = \{(a+n)x+(b-n)y\}\{[a-1]x+[b]y\}^n - n(a-1)(x^2-xy)\{[a-2]x+[b]y\}^{n-1}.$$

This can of course be done, by proving that the coefficient of x^r on the left is identical with the coefficient on the right; but the doing of it forces on our attention how complexity has been unnecessarily introduced into the affair. The evil centres in the fact that each determinant of the series is not the first primary minor of the next determinant; or—confining ourselves to a portion of this—that while the $(1, 1)^{\text{th}}$ element of the 1-line continuant is of necessity $ax+by$, the corresponding element of the 2-line continuant is $(a-1)x+by$, of the 3-line continuant $(a-2)x+by$, and so on. The above algebraical equality is thus an equality connecting

$$\{[a]x+[b]y\}^{n+1}, \quad \{[a-1]x+[b]y\}^n, \quad \{[a-2]x+[b]y\}^{n-1},$$

whereas what is wanted is an equality connecting

$$\{[a]x+[b]y\}^{n+1}, \quad \{[a]x+[b]y\}^n, \quad \{[a]x+[b]y\}^{n-1}.$$

(12) This desired equality I have found to be

$$\{[a]x+[b]y\}^{n+1} = \{(a-n)x+(b-n)y\}\{[a]x+[b]y\}^n - nxy(a+b-n+1)\{[a]x+[b]y\}^{n-1};$$

so that if we put F_n for $\{[a]x+[b]y\}^n$ we have

$$\begin{aligned}
&F_1-\{ax+by\} &&=0\\
&F_2-\{(a-1)x+(b-1)y\}F_1+xy(a+b) &&=0\\
&F_3-\{(a-2)x+(b-2)y\}F_2+2xy(a+b-1)F_1 &&=0\\
&F_4-\{(a-3)x+(b-3)y\}F_3+3xy(a+b-2)F_2 &&=0\\
&\cdots\cdots\cdots\cdots\cdots
\end{aligned}$$

and thence

$$\{[a]x+[b]y\}^n = \begin{vmatrix} ax+by & xy & . & . & \cdots \\ a+b & (a-1)x+(b-1)y & 2xy & . & \cdots \\ . & a+b-1 & (a-2)x+(b-2)y & 3xy & \cdots \\ . & . & a+b-2 & (a-3)x+(b-3)y & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{vmatrix}_n$$

which, besides being ideally simple in its law of formation, remains invariable to any and all of the above-mentioned changes in a, x, b, y.

RONDEBOSCH, S.A.,
16th February 1921.

(*Issued separately August* 23, 1921.)

XII.—On the Old Red Sandstone Plants showing Structure, from the Rhynie Chert Bed, Aberdeenshire. Part IV. Restorations of the Vascular Cryptogams, and discussion of their bearing on the General Morphology of the Pteridophyta and the Origin of the Organisation of Land-Plants. Part V. The Thallophyta occurring in the Peat Bed; the Succession of the Plants throughout a Vertical Section of the Bed, and the Conditions of Accumulation and Preservation of the Deposit. By Dr R. Kidston, LL.D., D.Sc., F.R.S., and Professor W. H. Lang, D.Sc., F.R.S.

(Read May 2, 1921. MS. of Part IV received May 2, 1921, and Part V June 28, 1921.)

(Abstract. Papers published in full in *Transactions*, Vol. LII.)

Part IV.—This paper concludes the authors' account of the Vascular Cryptogams found in the Rhynie deposit. Restorations of the four plants, *Rhynia Gwynne-Vaughani*, *R. major*, *Hornea Lignieri*, and *Asteroxylon Mackiei*, are given. A few additional features, supplementary to the descriptions of these plants in the preceding papers of the series, are described and illustrated. The hemispherical projections of *Rhynia Gwynne-Vaughani* are shown to have originated underneath stomata. A comparison is made between them and certain intumescences in existing plants. Areas of necrosis and marked wound-reactions of the tissues around them are described for both species of *Rhynia*. The apex of a stem of *R. major* is figured. For *Asteroxylon* additional figures are given of a large rhizome, of the leaf-arrangement and immature structure of the stem in the region of a shoot-apex, and of the longitudinal markings on the epidermal cells resembling those found in *Rhynia Gwynne-Vaughani*. The discussion summarises the authors' views on the main bearings of the facts described in Parts 1–4 on various problems in plant morphology.

Part V.—The Thallophyta occurring in the peat bed; the succession of the plants throughout a vertical section of the bed, and the conditions of accumulation and preservation of the deposit.

In this concluding part of this series of papers the Thallophyta found in the silicified peat are described. The most abundant are Fungi, represented by hyphæ of the mycelium and vesicles or resting-spores borne on this. With the exception of one specimen, the hyphæ were non-septate and

the fungi are regarded as belonging to the Phycomycetes. A number of form-types are described and illustrated by photographs. Some of the most distinct of these forms are given specific names in the comprehensive genus *Palæomyces.* The species distinguished are *Palæomyces Gordoni, P. Gordoni* var. *major, P. Asteroxyli, P. Horneæ, P. vestita, P. Simpsoni, P. agglomerata.* The possibility of there being a symbiotic (mycorrhizal) relation between certain fungi and the Vascular Cryptogams is discussed; there is no conclusive evidence in favour of this, but the question is left to some extent open. The majority of the fungi in the Rhynie peat were certainly living as saprophytes.

Bacteria were doubtless present in abundance, but are difficult to distinguish in the granular matrix. The most remarkable representative of the Schizophyta is a filamentous organism with the small protoplasts preserved. It is named *Archæothrix oscillatoriformis* and is compared with *Beggiatoa* and *Oscillatoria* among existing plants.

Scattered remains of a remarkable Alga, the vegetative structure of which presents a number of resemblances to existing Characeæ, are described under the name *Algites* (*Palæonitella*) *Cranii.* Two fragments belonging to an organism with the characteristic structure of *Nematophyton* are described as *N. Taiti.* The occurrence of this plant in such a deposit is noteworthy, and the small specimens are of importance in showing the structure of the peripheral region that has not been preserved in specimens previously described.

The succession of the plants throughout a section of the Chert Bed as exposed *in situ* is followed, and the conditions of formation of the Rhynie deposit discussed. On grounds mainly of resemblances presented by *Asteroxylon* to *Thursophyton* (*Lycopodites*) *Milleri*, the view is expressed that the Rhynie Chert band is probably of Middle Old Red Sandstone age.

(*Issued separately August* 23, 1921.)

XIII.—The Adsorption of Gas under Pressure. By Henry Briggs, D.Sc., Ph.D., A.R.S.M., and William Cooper, M.A., B.Sc.

(MS. received June 15, 1921. Read July 4, 1921.)

I. Introduction.

In 1917 Messrs F. C. Short, B.Sc., and F. W. Moore, of Walsall, applied for a patent for a method of storing gas, under compression, in cylinders or containers filled with charcoal which had been impregnated with a metal (*e.g.* iron, nickel, palladium) in a very fine state of division. The immediate intention of the inventors was to provide a compact method of storage of coal-gas serving as fuel for internal combustion engines. They realised that by filling a cylinder with impregnated charcoal its gas-capacity would be augmented. Independently, in 1919, one of us began experiments to ascertain if it were feasible to increase the capacity of a nitrogen cylinder, without increasing the pressure, by loading the cylinder with an unimpregnated activated charcoal before compressing the gas into it, and the results given below indicate that such an increase is in some degree possible. It is unfortunate that experiments with the gas in which we were principally interested, namely, oxygen, had to be limited to colloidal silica, this being the only non-inflammable adsorbent available; for had oxygen been compressed into charcoal there would have been grave risk of explosion. We have recently learnt that similar experiments have been carried out on coal by Mr J. I. Graham at the Doncaster Coalowners' Research Laboratory, though at the time of writing his results have not been published. The only published investigation known to us on the effect upon gaseous adsorption of pressures higher than atmospheric is that of Sir James Dewar,* who ascertained the volume of hydrogen taken in by a mass of 6·7 grams of cocoanut charcoal at $-185°$ C. He found that the charcoal adsorbed a volume which increased from 620 c.c. at atmospheric pressure to 1050 c.c. at 10 atmospheres, and that between 10 and 25 atmospheres no further gas was taken up.

The use of a porous medium, such as kieselguhr or asbestos wool, in acetylene cylinders is well known. Its function is to absorb the acetone which, in those cylinders, is used to dissolve the gas. The solution of acetylene in acetone is, however, a phenomenon of an entirely different kind to the adsorption of, say, nitrogen by charcoal.

* "Studies on Charcoal and Liquid Air," *Proc. Roy. Inst.*, xviii (1906), p. 433.

II. Method of Experiment.

Measurements were carried out at 15° C. and at pressures sometimes reaching 100 atmospheres. The gases mainly used were nitrogen, hydrogen, and oxygen; a few special tests were carried out with firedamp and carbon dioxide. The nitrogen and oxygen employed were the commercial gases supplied by the British Oxygen Company; they usually contained about 2 per cent. of impurity. Hydrogen and, on some occasions, CO_2 were made in the laboratory. The firedamp was obtained in compressed form from South Wales, and contained about 98 per cent. CH_4. Tests were carried out with (*a*) activated cocoanut charcoal, (*b*) activated birch charcoal, (*c*) German impregnated charcoal, (*d*) common wood charcoal, and (*e*) activated colloidal silica made in the laboratory from the hydrogel by the method described in a paper recently given before the Royal Society, London.* In addition, a few special tests, dealt with in the next section, were made upon anthracite.

The method consisted in filling a small steel cylinder with the substance to be tested, the material being well shaken down within. Before loading the cylinder the charcoal or silica was dried in thin layers in a gas furnace; it was inserted hot. The cylinder was fitted with a gas-cock screwed and soldered into place. After it had cooled it was charged with dry gas to the desired pressure, and a sufficient time allowed to elapse to allow the heat of adsorption to dissipate and the pressure to stabilise. The cylinder was then provided with a throttle-valve and pressure-gauge and placed in a water-bath. The volume contained by it was determined by allowing the gas to flow out very slowly through a meter. After the discharge of 1 litre, or, in some cases, 5 litres of gas, the throttle was closed, and a pressure-reading taken after stability had again been attained. The operation was repeated until no more gas was discharged. The pressure-gauges were tested from time to time against a large gauge which had been calibrated at the National Physical Laboratory. The meter was tested against displacement.

III. General Results.

The experimental method described ascertained the volume, at 15° C., discharged between any given pressure and atmospheric pressure. When constructing the graphs it was necessary to add to that volume the amount of gas retained by the adsorbent at atmospheric pressure and 15° C. This

* H. Briggs, "The Adsorption of Gas by Charcoal, Silica, and other Substances," *Proc. Roy. Soc.*, A, 1921 (in course of publication).

additional quantity was separately determined for nitrogen and cocoanut charcoal and nitrogen and silica by methods which have previously been described (Briggs, *loc. cit.*). For each of the other gases and substances employed the addition in question was made by extrapolation from the graph itself.

Fig. 1 records the results obtained with activated cocoanut charcoal, which, of all the substances tried, gave the greatest adsorption under pressure. Curves *A* and *B* respectively indicate for various absolute

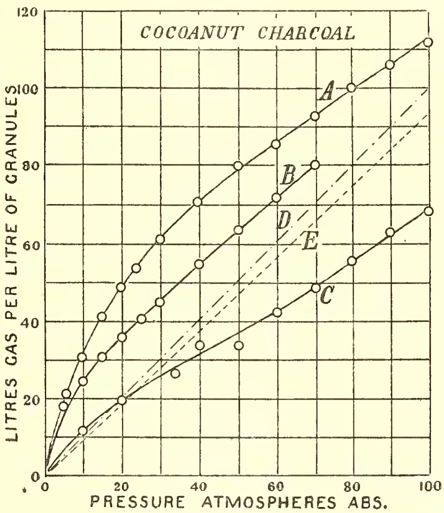

Fig. 1.

A, Adsorption isotherm for nitrogen.
B, Adsorption isotherm for hydrogen.
C, Adsorption isotherm for nitrogen with damp charcoal.
D, Simple compression according to Boyle's law.
E, Simple compression for hydrogen.

pressures (abscissæ) the total volumes (ordinates) of nitrogen and hydrogen contained by a gross volume * of 1 litre of charcoal. Gas volumes are expressed at N.P. and 15° C. To show the great influence of damp, a test was carried out on nitrogen, using cocoanut charcoal which had been for many months exposed to the air and which was found to hold 25 per cent. by weight of moisture. The results of that experiment are set forth by curve *C*. The straight lines *D* and *E* are included for the sake of comparison; the former expresses for 1 litre of open space the *pv* relation according to Boyle's law, and the latter expresses that relation for hydrogen.

* Gross volume includes the interstitial spaces, or voids, between the granules.

It is clear from fig. 1 that the gas-capacity of a cylinder intended to hold nitrogen under compression may be increased by filling the cylinder with dry cocoanut charcoal. The advantage gained from the charcoal is especially marked for pressures below 50 atmospheres. When blown off from 35 atmos. abs. to 1 atmos. abs., for example, a cylinder containing cocoanut charcoal would discharge 66 per cent. more nitrogen than the same cylinder containing no adsorbent. Put in another way, the results indicate that a cylinder of 1 cubic foot water-capacity filled with dry cocoanut charcoal and charged with nitrogen at 21 atmospheres would hold

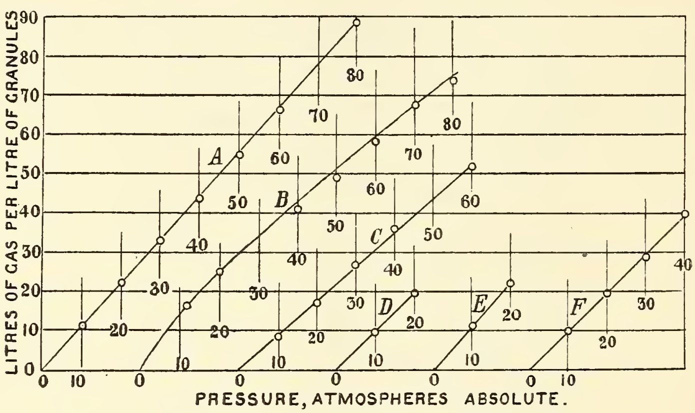

Fig. 2.

A, Oxygen in colloidal silica.
B, Nitrogen in common wood charcoal.
C, Nitrogen in birch charcoal.
D, Hydrogen in colloidal silica.
E, Hydrogen in German impregnated charcoal.
F, Nitrogen in German impregnated charcoal.

a total volume of 50 cubic feet of gas, of which 43·5 cubic feet would be discharged on the pressure being released to atmospheric. Had the cylinder not held charcoal it would require to have been charged to 44·5 atmos. abs. to yield the same volume. The increase of available volume could probably be raised from 66 to about 80 per cent. at 35 atmospheres by ramming the charcoal forcibly into the cylinder. A similar but smaller difference is observable for hydrogen (compare curves *B* and *E*, fig. 1).

As gaseous adsorption is much improved by cold, it follows that a cylinder containing charcoal could be charged with a given mass of gas at low temperature (obtained, say, from a Claude or Linde plant) with a less expenditure of energy than if charged at ordinary temperature.

The influence of moisture (compare curves *A* and *C*, fig. 1) points to the need for dryness in both adsorbent and gas if this method of gas storage be adopted.

Most commercial gas-cylinders are charged to 120 atmospheres; at that pressure the advantage to be gained by the charcoal is small.

In fig. 2 a number of other pressure-volume isotherms are given, the ordinates in each case being the volumes (at N.P. and 15° C.) taken up by 1 litre of the granules at 15° C. These, with the exception of *B* (nitrogen and common wood charcoal), are straight lines having much the same slope. The slope, moreover, is not far from 45°, which is that of the *pv* relation according to Boyle's law. In some of the cases illustrated, for example

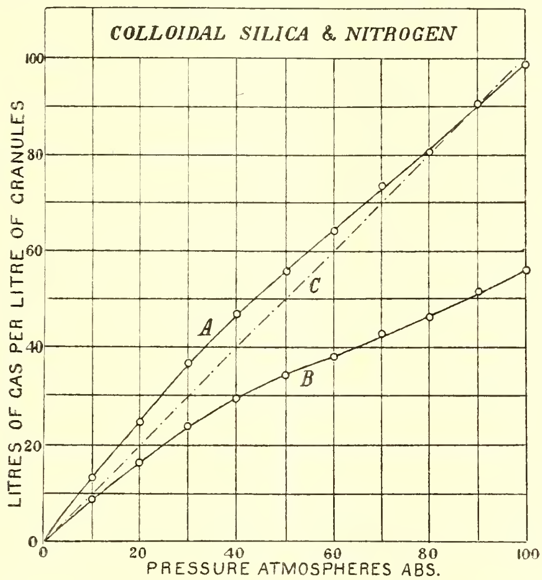

Fig. 3.

A, Including the gas compressed in the interstitial spaces.
B, Corrected for the gas in the interstitial spaces.
C, Simple compression according to Boyle's law.

A (oxygen and colloidal silica), the cylinder held slightly more than it would hold without adsorbent; while in other cases, such as *C* (nitrogen and birch charcoal), it held slightly less. In all these instances, then, the adsorption has been considerable; in some it obeys Henry's law, while in others it departs from that law. Curve *A*, fig. 3 (colloidal silica and nitrogen), expresses a relationship differing from Henry's law; it is analysed below.

Tests made with firedamp compressed into anthracite threw fresh light upon the problem of sudden outbursts of gas in collieries, and that subject has been dealt with recently by one of us in a paper read before the Institution of Mining Engineers.* These outbursts, which occasionally

* H. Briggs, "Characteristics of Sudden Outbursts of Gas in Mines," *Trans. Inst. Min. Engs.*, vol. lxi.

happen in this country, are characterised by the almost instantaneous discharge into the mine workings of thousands, and in some cases millions, of cubic feet of firedamp, and along with the gas hundreds of tons of coal are displaced. The coal associated with an outburst is soft, sooty, and disintegrated. By taking some anthracite ejected at an outburst in South Wales, drying it, filling it into a gas-cylinder, and pumping in pit firedamp, it was found that 1 cubic foot of the coal was able to take up at 6 or 8 atmospheres pressure, and hold in a condition ready for almost instantaneous release, a quantity of firedamp considerably greater than could be contained in 1 cubic foot of open space charged with gas at the same pressure. The sudden outburst of gas constitutes, in fact, a problem in gaseous adsorption; the gas is held in the coal in a state available for discharge when the pressure is released.

IV. Theoretical Considerations.

Dr A. M. Williams has shown by an able theoretical analysis * that the most probable form for an adsorption isotherm at low concentrations is

$$\log_e\left(\frac{v}{p}\right) = A_0 - A_1 v \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where p is the absolute pressure and v the mass of gas adsorbed at that pressure by a given mass of the substance. At present it is convenient to express v in litres of gas expanded to N.P. and 15° C. A_0 and A_1 are coefficients. Williams found the above equation to represent very closely the connection between v and p as observed by several previous workers who had experimentally determined that connection for various gases at pressures below atmospheric. Our own results go to show that the equation (with one correcting factor) also holds at high pressure for, at any rate, gases above their critical temperatures, though, at those pressures, the degree of concentration is considerable.

To enable this matter to be studied, it was necessary to eliminate the effect of the interstitial spaces between the granules. We had previously found the interstitial space of cocoanut charcoal (well shaken down) to be 40 per cent., and that of silica to be 43 per cent. of the gross volume (Briggs, *loc. cit.*), and these values were used in making the correction in question. In fig. 3, *A* is the original curve plotted from the experimental data and *B* is the curve resulting after the volumes compressed into the interstitial space had been deducted. Now it will be seen that *B* has a double flexure, the gradient first decreasing as pressure rises and then, at the high

* *Proc. Roy. Soc. Edin.*, xxxix (1918–19), p. 48 ; *Proc. Roy. Soc.*, A, xcvi (1919), p. 287.

pressures, slightly increasing again. If the pressure is carried high enough, this effect is found to be common to all the pressure-volume relations we have examined, provided the relationship is of the non-linear type. At 15° C. and with the gases concerned the possibility of a change of phase at the higher pressures is nil, and another explanation has to be sought for the effect.

The internal gaseous volume of an adsorbent granule is made up partly of openings of molar dimensions and partly of much larger canals or capillaries, some of which may be visible under the microscope. With these gases of low critical temperature, even under pressures of about 100 atmospheres, the adsorbed film cannot occupy the whole of the internal gaseous volume, some of which must therefore be occupied by gas, not adsorbed, but approximately obeying Boyle's law. Thus it appears necessary to modify equation (1) so as to include a term to cover the effect of compression upon unadsorbed gas existing in the capillaries. Let v be the volume (expressed at N.P. and 15° C.) taken up by a gross litre of the granules, the volume compressed in the interstitial spaces not being included. Let v_1 be the volume taken up by true adsorption upon the solid surfaces, and v_2 be the volume contained in the capillaries under simple compression.

Then from equation (1)

$$\log_e\left(\frac{v_1}{p}\right) = A_0 - A_1 v_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and

$$v = v_1 + v_2,$$

but, by Boyle's law, $v_2 = kp$; therefore

$$v = v_1 + kp \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Equation (3) thus expresses the extent of the modification of Williams' rule needed to take into account the existence, in the granules, of gas under simple compression.*

The curve for nitrogen in silica (*B*, fig. 3) was found to agree remarkably well with the equations

$$\log_e\left(\frac{v_1}{p}\right) = -\cdot092 - \cdot0155 v_1 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$v = v_1 + \cdot10p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For cocoanut charcoal with nitrogen the equations are

$$\log_e\left(\frac{v_1}{p}\right) = 2\cdot43 - \cdot045 v_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$v = v_1 + \cdot10p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

* Williams has made use of a similar correction to allow for the volume occupied by the adsorbed layer, *ibid.*, p. 306.

though in this case the agreement is not so good. For hydrogen in cocoanut charcoal the following equations satisfy the data very exactly:—

$$\log_e\left(\frac{v_1}{p}\right) = 1{\cdot}92 - {\cdot}076v_1 \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$v = v_1 + {\cdot}25p \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The argument does not take into account the probable thickening of the adsorbed layer as the pressure rises; if this effect were allowed for, k would become a variable depending upon the pressure. With gases below their critical temperature the effect in question will be considerable, but for gases such as hydrogen and nitrogen at 15° C. the proportional influence of such a correction must be small, and it has been disregarded.

It will be seen (equations (4) to (7)) that the amount of the internal gaseous volume unoccupied by adsorbed films is, with nitrogen, about the same for silica and cocoanut charcoal. As the specific attraction between silica and nitrogen is, according to available evidence, less than that subsisting between cocoanut charcoal and nitrogen, the thickness of the adsorbed film will be less with the former than with the latter substance; therefore it is probable that the average section of the internal passages is smaller in our colloidal silica than in the charcoal. This may in part be due to the absence in the silica of the relatively very large (microscopic) openings present in charcoal. The higher value of k in equation (9) points to the conclusion that, with cocoanut charcoal, the surface film of hydrogen is thinner than that of nitrogen.

The straight-line relationships, exemplified by all but one of the graphs of fig. 2, should, we think, not be regarded as exceptions to the general law expressed by equations (2) and (3), but rather as special cases in which A_1 is of negligible influence; for if the term A_1v_1 is inappreciable, the equations reduce to the straight line

$$v = p(e^{A_0} + k). \quad . \quad . \quad . \quad . \quad . \quad (10)$$

We propose to reserve for the present the consideration of adsorption under pressure of a gas, such as carbon dioxide, whose critical temperature is above 15° C.—in this instance the temperature at which the experiments were carried out. It is sufficient to say that the double flexure of the curve, referred to above, is much more marked with such a gas, and, as might be expected, the slope of the curve increases rapidly as the pressure of liquefaction is approached.

We wish to express our thanks to the Department of Scientific and Industrial Research for permission to publish this paper.

Summary.

1. Experiments made at 15° C. with various adsorbents and gases (chiefly nitrogen and hydrogen) show that a cylinder filled with adsorbent granules has a capacity for dry gas under a given pressure which is generally greater than its capacity when containing no adsorbent. For example, a cylinder charged with nitrogen at 35 atmospheres has its capacity increased by 66 per cent. by filling it with cocoanut charcoal.

2. Sudden outbursts of firedamp in coal-mines are the result of releasing immense quantities of gas adsorbed under pressure in coal.

3. The logarithmic relation derived by Williams is shown to apply to the adsorption isotherms of gases above their critical temperature up to pressures of 100 atmospheres, providing a correction be applied for the gas in the capillaries which is not adsorbed, but which exists under simple compression.

(*Issued separately August* 23, 1921.)

XIV.—Utilisation of Solid Caustic Soda in the Absorption of Carbon Dioxide. By Elizabeth Gilchrist, M.A., B.Sc., A.I.C. *Communicated by* Professor Henry Briggs, D.Sc., Ph.D.

(MS. received June 17, 1921. Read July 4, 1921.)

In the course of research on Mine Rescue Apparatus under the Department of Scientific and Industrial Research it was found necessary to investigate the conditions of utilisation of solid caustic soda for absorbing carbon dioxide, and particularly the effects of variations of temperature and water vapour upon the reaction.

Though the experiments relate especially to the conditions prevalent in breathing apparatus, it is believed that they have sufficient general interest to warrant their description in a separate paper. The tests were carried out by Mr D. Penman, B.Sc., and the writer, under the direction of Professor Henry Briggs, and the results are published by permission of the Research Department.

Preliminary Experiments.

Preliminary comparative experiments were carried out in which air containing carbon dioxide was passed through U-tubes holding caustic soda granules. These exploratory tests showed that:—

1. The temperature at which the caustic soda is maintained had a most important effect on its power of absorption, and that the absorption fell off almost to zero if that temperature was kept at or below 0° C.

2. The size of particle had a great effect on the absorptive efficiency, small particles being more effectual than large particles. This result doubtless follows from the greater area of surface exposed by the small granules.

3. When the soda particles act efficiently they swell considerably, and in so doing tend to fill up the interstices between them, thus causing choking unless measures are taken to prevent it.

Apparatus and Methods.

In view of the results of the preliminary inquiry, it was found to be necessary to carry out all subsequent experiments with granules of caustic soda which had been sized. The size adopted was that of from $\frac{1}{4}$-inch to $\frac{1}{5}$-inch diameter. Thirty grams of granules were employed in each experiment, the caustic being weighed out in a weighing-bottle with a variation not exceeding ±·005 gram; the soda was then transferred, immediately before starting the test, to the apparatus described below, and sealed in. The air current caused to flow over the caustic granules was made to contain a uniform 4 per cent. of carbon dioxide; it thus resembled in composition the air expired from the lungs of the wearer of a rescue apparatus. Its rate of flow was 3·5 litres per minute. The supply of air to be purified was obtained from a Briggs gas-testing tank.*

During any such test as those to be described, the soda becomes progressively less active as the test proceeds, and towards the end, though often a considerable percentage of hydrate remains, the caustic has virtually ceased to extract carbon dioxide from the air. A period of test of forty minutes was adopted throughout.

A special apparatus was designed for the tests. It consisted of a strong tin bath (fig. 1) of rectangular shape, supported on wrought-iron legs high enough to permit of an ordinary gas bunsen being placed under it. Surrounded by the water in the bath, J, were two air-tight compartments, B and E. At one end of the partition separating B and E was an opening, C, ½ inch wide and 2 inches long. The lower compartment, B, was kept either half full of water or empty, as desired, and into the upper compartment, E, was slipped a tray containing the caustic. The gas mixture entered the compartment B at the point X; rose through the opening C into the upper compartment, and flowed over the surface of the caustic to an outlet, F, where samples could be taken. The end, H, of the upper compartment was detachable, and after the tray containing the caustic had been inserted the end was sealed on.

The tray on which the caustic was spread was 11 inches long by 3 inches wide, and consisted of a tin base on which was fixed wire gauze crimped to form six V-shaped troughs. The caustic was spread uniformly over the surface of the gauze along the troughs. This form of support for the soda has been found most effectual in rescue apparatus.† It is to be noted that all temperatures given are those of the surrounding bath; not necessarily of the caustic particles. The caustic, however, rested on metal, and, as the vessel was entirely of metal, the conduction of heat was fairly effective. For temperatures between 0° and 100° a bath of water was used. Below zero a mixture of ice and salt, and above 100° paraffin wax was employed.

During the experiments samples of the ingoing and outgoing gaseous

* *Colliery Guardian*, December 15, 1911.

† *Second Report*, *Mine Rescue Apparatus Research Committee*, 1920, p. 46.

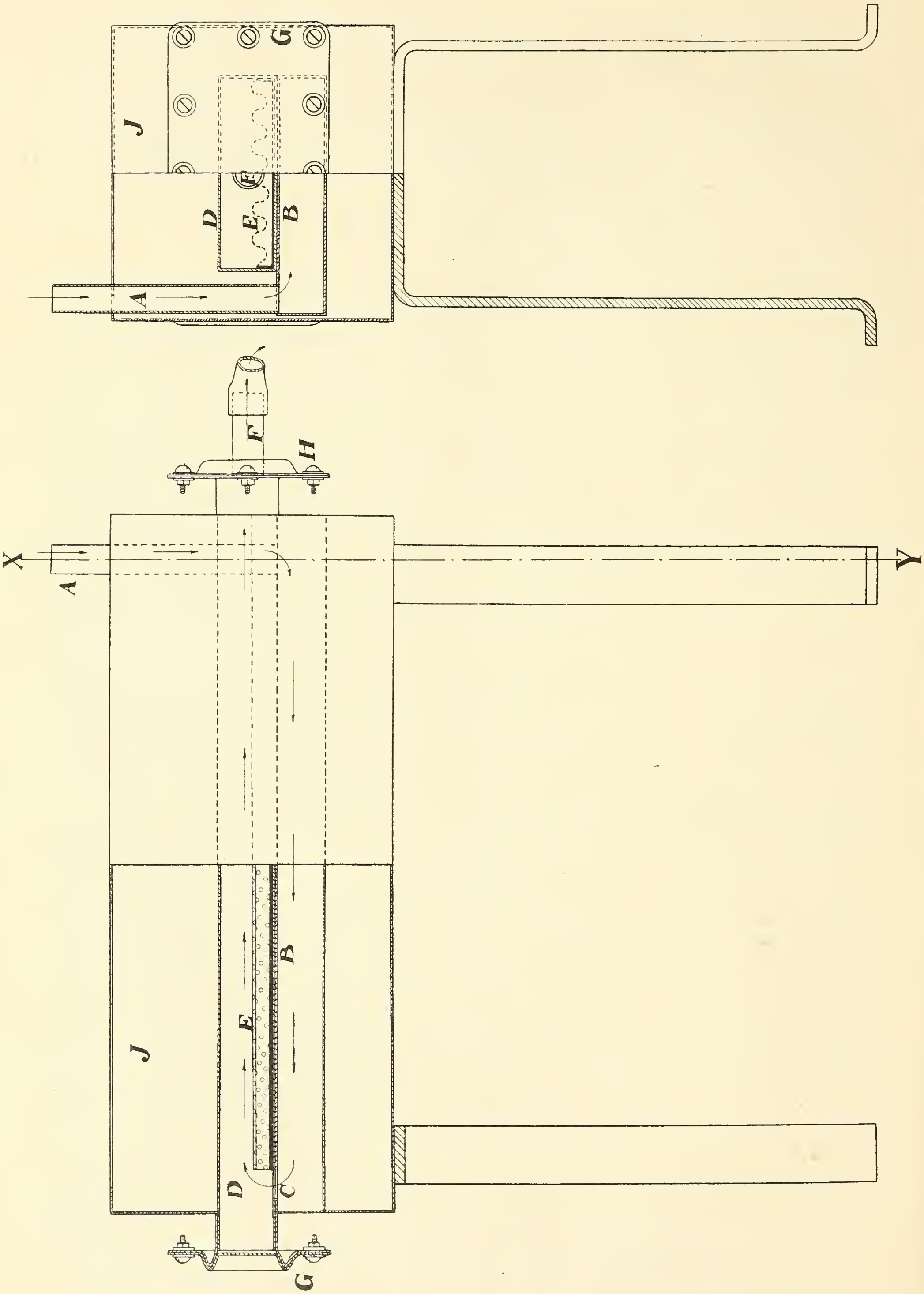

Fig. 1.

mixtures were taken every five minutes and were analysed in a portable Haldane gas-analysis apparatus.*

Four series of tests were carried out. For the first series the gas mixture was dried before it entered the caustic soda compartment. In the second series the gaseous mixture was saturated at room temperature, 12° C., before coming into contact with the caustic; and the third series was similarly saturated at blood temperature, 37° C.; while in the fourth series the mixture was allowed to saturate itself with moisture at the temperature at which the bath was maintained. In each of the first three series the weight of moisture carried by the gas was constant throughout the tests, while in the last it varied in accordance with the temperature of the test.

I. *Gaseous Mixture dried before passing over Caustic Soda.*

The 4 per cent. mixture was dried by passing through concentrated sulphuric acid, and through U-tubes containing calcium chloride; it then passed over copper sulphate. The absence of a blue tinge in the latter showed the gas to be dry. It must here be observed that although precautions were taken to keep the caustic soda dry, doubtless it contained a proportion of moisture. Commercial caustic soda usually contains water up to 10 per cent., and in exceptional cases up to 25 per cent. Analyses were made of the gaseous mixture entering and leaving the caustic compartment at frequent intervals. Their results enabled an average to be struck for that period of the percentage of CO_2 extracted at each of the temperatures of the tests. When these average extractions were graphed against temperature, curve A (fig. 2) was obtained. It was observed, at any given temperature, that the caustic soda became most effective after a lapse of from ten to twenty minutes, and that after that time the efficiency of absorption rapidly fell away. The examination of the material taken from the tray after an experiment showed the granules to be substantially unaltered in form and to have received only a thin coating of carbonate. The greater part of each granule was unaltered caustic soda.

II. *Gaseous Mixture saturated with Water Vapour at* 12° *C.*

The tests were carried out in the same manner as those just described, and as a result curve B (fig. 2) was obtained. The efficiency of absorption is evidently better in this case than in the last. The maximum efficiency of absorption took place in the circumstances of this test at 70°–90° C., where it amounted to a 72 per cent. extraction of the carbon dioxide.

* J. S. Haldane, *Methods of Air Analysis*, p. 48.

Another feature clearly shown by contrasting curves A and B is that while in the dry condition represented by the former, a temperature over 100° C. brought about the most marked diminution in efficiency of absorption, such a temperature in the second series was not so deleterious. In the latter series, in the circumstances admitting of maximum extraction, the granules

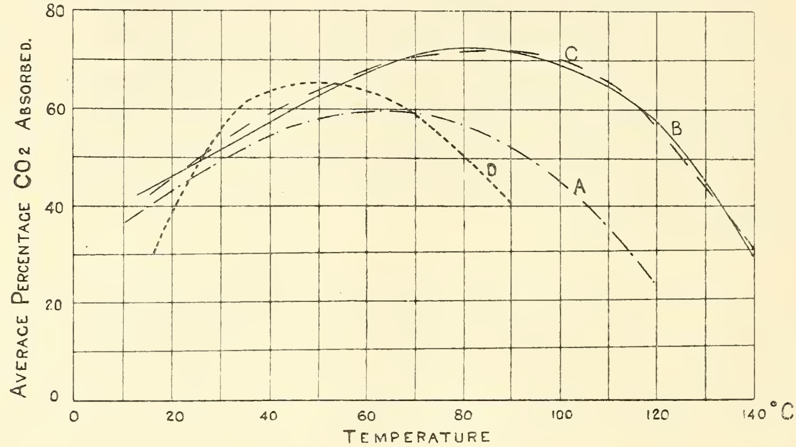

Fig. 2.

on being taken from the test cartridge were found to have considerably increased in size and to contain relatively little unused caustic soda.

III. *Gaseous Mixture before entering the Caustic Compartment was saturated at* 37° *C.*

The temperature and hygrometric state of the stream of air and carbon dioxide were kept under observation. In this series the lower compartment of the case contained no water. On the results being recorded in graphical form (curve C, fig. 2), the curve was found considerably to resemble that of the last series; at the optimum temperature extraction was again between 70° and 90° C. The maximum extraction was 72 per cent. of the carbon dioxide. The remarks as to the condition of the caustic soda in the last series equally apply to this.

IV. *Gaseous Mixture saturated at the Temperature at which the Caustic Soda was maintained.*

In each of these experiments the lower compartment E (fig. 1) was half filled with water, which was then kept at a definite temperature. The ingoing gaseous mixture flowed over the surface of the water before entering the caustic compartment. The results of the series are shown in curve D (fig. 2). The maximum extraction was 65 per cent., and the best temperature was 50° C. The maximal extraction in this case being less than that in the last two cases would indicate that the greater amount of water held by the ingoing stream of gas in series 4 was detrimental to absorption. The curve D shows how much efficiency of absorption depends on the temperature of the containing vessel; for though the extraction was 65 per cent. at 50° C., it was only 30 per cent. at 16° C. and 40 per cent. at 90° C.

Discussion of Results.

A conceivable explanation for some part of the difference in efficiency of absorption is that, owing to variations of temperature and hygrometric state, the actual rate of flow over the caustic was different in different experiments. This effect, however, was found to be of negligible moment by carrying out special tests, in which the same mass of carbon dioxide was passed over the caustic per second, though contained in air currents which flowed at greatly different rates. A more serious criticism on the method of test is that the recorded temperature of the bath surrounding the caustic compartment was not necessarily the temperature of the caustic granules themselves. Undoubtedly there would be very considerable local heating, and the severity of this local heating may be judged when it is stated that in certain other experiments in which caustic granules were held on blotting-paper trays the blotting-paper caught fire from the heat of reaction. The writer has no means of judging of the temperature of the active surfaces at which the absorption was actually taking place. The general conduction of heat away from the granules, however, must have been moderately efficient, owing to the mass of metal with which they were in contact. It is advisable to state again that the whole set of experiments was carried out to give information which would be applicable for rescue apparatus in the cartridges of which the caustic soda is supported in much the same way as in the experiments. A slight error in the percentage of carbon dioxide in the outgoing sample, as given by the results of the analyses, was due to the condensed moisture in the sampling-bottles having dissolved some CO_2; but on calculation it was ascertained that this error was negligible.

Some remarks have already been made as to the appearance of the caustic soda on removal from the tray. There was a marked difference in appearance after tests at high, low, and intermediate temperatures. At the lowest and highest of the temperatures in the series I., II., and III., the granules retained much of their original shape; but at the high temperatures in series IV., where a great deal more moisture was present in the air, the granules coalesced in a sticky state with corresponding reduction in surface, and therefore in the efficiency of absorption.

In the circumstances giving the best absorption the caustic particles were found in all cases to have swollen sometimes to three times their original linear dimensions, and in many cases they were found to be hollow inside. This action is of great importance to the designer of mine rescue apparatus or other appliances in which these granules are used for the abstraction of carbon dioxide from air. The action appears to be much as follows:—

A granule begins its active life by taking up a considerable weight of moisture, and attains a plastic state. The action of carbon dioxide upon such a body is to coat it with a layer of spongy dry carbonate. The next effect is a penetration, by capillary attraction, of the soft caustic through its skin of spongy carbonate and a further attack by the carbon dioxide on the soda thus brought within its reach. The continuance of such an action will evidently be to cause the formation of a carbonate shell with a hollow interior and greatly to increase the apparent bulk of the granule. When absorption is at its best, this action continues until there is no caustic soda left behind; in other cases until a small granule of caustic is left behind, apparently lying detached inside the shell of carbonate.

Conclusions.

The following conclusions, of value especially to the designer of rescue apparatus, follow from these tests:—

1. It is evident that too much and too little moisture is detrimental to the action. The best results are obtained when the proportion of moisture carried is approximately that carried by air saturated at temperatures between 60° C. and 90° C.

2. The best results for absorption are obtained when the container is kept at a temperature of from 60° to 100° C. Provided the moisture conditions are right, the average proportional absorption is then about 70 per cent., when using the relatively small amount of caustic employed in the tests. Below and above those temperatures the absorption efficiency falls off rapidly. Below 10° C. and above 100° C. absorption is extremely poor. It is worthy of remark that in actual use with rescue apparatus in France during the war and in certain cold countries it was found by experience that caustic is inactive at temperatures near freezing-point, and the practical precaution was taken on cold days of warming the caustic prior to use by breathing through it.

3. It is evident that measures must be taken to secure good conduction

of heat, as it is important to keep an equal temperature in all parts. Otherwise at some parts the action may become violent, with consequent rise of temperature and stoppage of the reaction, while at others the temperature may never rise sufficiently high to give the best results.

4. It is important that the particles be of such a size as to give a fair surface of exposure for absorption. Excessively large particles are for this reason to be avoided. Provided the conditions as to moisture and temperature are at their optima, however, there is no need to decrease the size of the particles unduly in order to increase the surface; *e.g.*, given such conditions, granules 6 or 7 mm. in diameter will, by the swelling action described, be entirely or almost entirely used up.

5. The particles must be so spaced as to give sufficient room for swelling. There should indeed be a greater volume of interstitial space in a canister fitted for absorption of carbon dioxide than of actual solid caustic soda itself. A disregard of this requirement leads to the coalescence of the granules due to swelling, and to the blocking of the air passages. This, in turn, involves a greatly increased resistance which is highly detrimental in breathing apparatus, and at the same time decreases the availability of the caustic to the carbon dioxide.

(*Issued separately September* 5, 1921.)

XV.—On the Criterion for Stable Flow of a Fluid in a Uniform Channel. By H. Levy, M.A., D.Sc., Assistant-Professor of Mathematics, Imperial College of Science, South Kensington.

(MS. received May 27, 1921. Read June 20, 1921.)

§ 1. The conditions determining the stable flow of a viscous fluid in a uniform channel and in a uniform circular pipe were investigated experimentally for water by Osborne Reynolds in a searching series of papers.*

For a channel he concluded that so long as the non-dimensional group Ul/ν, where

U = mean velocity in the channel,
l = breadth of the channel,
ν = kinematic viscosity of the fluid,

was maintained below a certain value, any slight disturbance imposed on the steady flow tended to die out, and the steady streaming persisted, but in the neighbourhood of and above this critical value a condition of turbulence and eddying set in immediately the steady stream-line motion was disturbed.

Many theoretical investigations of this problem, having for their object the mathematical formulation of the conditions determining this critical state, have been attempted, but no satisfactory analysis has yet been forthcoming. The mathematical difficulties of a direct attack are so formidable that simplifications are invariably introduced; but these, while they may render the mathematical development amenable to treatment, nevertheless involve physical assumptions whose interpretation is frequently obscure. This is amply borne out by the fact that theoretical estimates of the critical value of Ul/ν vary over exceedingly wide ranges. In the present paper it is proposed to approach this question from a new standpoint.

§ 2. On general grounds of dimensions it is clear that any problem in the flow of a viscous fluid with given boundary will centre round the particular value assigned to Ul/ν, and the author has indicated how this fact may be utilised to approach a solution of any such problem in general.† A full experimental investigation of the question in relation to the critical flow of air, water, and oil in pipes has been conducted by Stanton and

* *Scientific Papers*, vol. ii, p. 51 *et seq.*; *Phil. Trans.*, 1883.

† *Phil. Mag.*, xli, April 1921, "On a Method of Analysis suitable for the Differential Equations of Mathematical Physics."

Pannell,* who have shown that the conclusions arrived at by Osborne Reynolds as regards water are independent of the state of the fluid—that, in fact, the critical value of Ul/ν is a universal constant for given geometry of boundary. From another standpoint, these results have since received verification in all hydro- and aero-dynamical experiments, where it is found that the resisting force R of a body of given shape in a viscous fluid of density ρ and kinematic viscosity v is always expressible in the form

$$\frac{R}{\rho U^2 l^2} = f\left(\frac{Ul}{\nu}\right),$$

where l fixes the scale of the body. In effect these experiments may be regarded as justifying the assumption that the properties of real fluids in motion should require nothing further for their explanation than the assumption that the fluid is viscous and dense. It is important in this connection to note that compressibility plays no part in this question, and that for the present purpose the air may be justifiably regarded as inelastic. Examination of the resistance of projectiles indicates that compression effects do not become apparent until the velocity of sound is approached.†

§ 3. Returning to the flow of a fluid in a channel, let it be supposed that a disturbance is communicated to the fluid, say, by dipping a small obstacle into the fluid and withdrawing it. For a given shape of obstacle inserted in any given manner, for a given speed U of the central stream line of the channel, and for given viscosity and density of the fluid, it is clear that the vortex distribution immediately resulting from this disturbance will be physically quite determinate. Let the strength of the vorticity at a geometrically given position be κ, then κ can only be dependent on U, l the breadth of the channel, ν the kinematic viscosity, and ρ the density, and on nothing else.

Hence

$$\kappa = f(U, l, \nu, \rho).$$

Now κ being an angular velocity distributed over an area has dimensions $\frac{1}{T} \times L^2$.

$$\therefore \quad [\kappa] = L^2T^{-1}; \quad [U] = LT^{-1}; \quad [\nu] = L^2T^{-1}; \quad [\rho] = ML^{-3}.$$

Assuming

$$\kappa = \sum A_r U^r l^s \nu^t \rho^u,$$

* *Phil. Trans.*, A, 214, pp. 199–224, "Similarity of Motion in Relation to the Surface Friction of Fluids."

† Rayleigh's *Scientific Papers*, vol. v, p. 534 ; *Aeronautical Journal*, June 1919, "From Model to Full Scale in Aeronautics," H. Levy.

where A_r is a number, and equating the dimensions of each term on the right to $[\kappa]$, it is easily found that

$$r = s = 1 - t\,;\ u = 0.$$

$$\therefore\quad \kappa = \sum A_r (Ul/\nu)^r v,$$

where r is indeterminable so far. The quantities A_r and r, in fact, must depend on the shape of the boundaries.

$$\therefore\quad \kappa = vf_1(Ul/\nu) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or

$$Ul/\kappa = F(Ul/\nu) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where the form of the functions depends on the geometry of the problem.

From (1) it follows that for a given value of Ul/ν the strength of the vorticity is directly proportional to the kinematic viscosity. Equation (2) indicates that the discussion of the stability of flow in a viscous fluid to a given disturbance, and all the circumstances of the motion generally, may equally well be centred round the non-dimensional group Ul/κ; that in fact we may imagine a given disturbance in vorticity, specified by κ, applied to the fluid, and seek to determine the value of Ul/κ, which is critical in the sense that it separates the region of values of this non-dimensional grouping for which the motion is stable from those for which the motion is unstable. Having determined this critical value of Ul/κ, how to determine the exact relation (2) from which to evaluate the critical quantity Ul/ν is clearly the next step, and if this can be completely effected the problem will definitely be solved. For the present I propose to restrict myself to determining whether such a critical value of Ul/κ exists at all, and if so to evaluating it. It need scarcely be said that the discussion so far is not limited to the question of flow in a channel, although that is the case we have had in view.

§ 4. It will be presumed that a fluid is moving between and parallel to the walls of a uniform channel, with the parabolic distribution in velocity corresponding to the steady motion of a viscous fluid in such a case, so that in the usual notation

$$u = U(1 - y^2/a^2)\,;\quad v = 0,$$

where $2a$ is the breadth of the channel.

Let it be disturbed in such a manner as to give rise to two vortices of strengths $-\kappa$ and $+\kappa$ situated at the points $(x=0,\ y=h)$, $(x=0,\ y=-h)$ respectively. These may be imagined to have been produced by the sudden even insertion of a plate of breadth $2h$ stemming the flow symmetrically about the axis of x. Experiment shows that two vortices would immediately be formed at the two edges, of opposite signs, and from symmetry necessarily of equal strengths. It will be presumed that the whole disturbance is initially concentrated in these two vortices.

§ 5. Two difficulties immediately present themselves. In the first place, if the vorticity thus imposed on the fluid be imagined as concentrated at the two points, the condition that there is no slipping at the boundary, as will shortly become apparent, is immediately violated. Whether or not this is a serious deviation from the real physical conditions is not even yet quite definite. Ample experimental evidence exists to show that for moderately small speeds—that is to say, for values of Ul/ν well below the critical—there does not exist any appreciable relative motion at the surface between the body and the moving fluid, and this condition is satisfied by the solution for steady motion, $u = U(1 - y^2/a^2)$, which has been presumed. For velocities in the neighbourhood of and above the critical, however, where turbulent motion sets in, experimental results are not so conclusive, and do not appear to provide sufficiently definite evidence beyond the fact that as the surface is approached the relative velocity does not drop very rapidly except when a very close approach is made to the surface. It is, in fact, the extremely short distance from the surface within which all the fall is to take place that constitutes the real experimental difficulty. Whether or not the assumption that a small amount of slipping does take place is a serious cause of discrepancy will, I hope, be discussed in a later paper, when some of the results of the present discussion will be developed; but in any case it will become evident that the slipping that results from the assumption of concentrated vortices is comparatively small in general, although in certain circumstances it may be considerable.

§ 6. The second difficulty is of a different nature, and is not vital. A vortex formed in a viscous fluid will not for long maintain its spin unimpaired, because of the viscous action of the fluid; its energy will be gradually dissipated into heat. A consideration of the rate of decay of a single vortex, say, along the axis of a circular cylinder filled with a viscous fluid indicates, in fact, that for a fluid of such small viscosity as water, for example, or even air, the rate of decay is very small, and that as far as any consideration of the effects immediately subsequent to the formation of the vortex is concerned, no serious error will be involved by the assumption that κ remains constant. This may be illustrated in the following manner.

The general equations of motion of a viscous fluid in two dimensions are known to be

$$\frac{\partial \zeta}{\partial t} = \nu \nabla^2 \zeta - u\frac{\partial \zeta}{\partial x} - v\frac{\partial \zeta}{\partial y} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where

$$2\zeta = \nabla^2\Psi = \text{spin} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

using the customary notation. Where the vortex is situated along the axis of the cylinder, the motion at any point must be purely a function of r, in which case (3) takes the form

$$\frac{\partial\zeta}{\partial t} = \nu\left(\frac{1}{r}\frac{\partial\zeta}{\partial r} + \frac{\partial^2\zeta}{\partial r^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

the terms in u and v cancelling out.

Writing $\zeta = Ze^{-a^2\nu t}$ and inserting in (5), assuming that Z is a function of r only,

$$\frac{d^2Z}{dr^2} + \frac{1}{r}\frac{dZ}{dr} + a^2Z = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This is Bessel's equation of order zero, the solution which is finite for $r = 0$ being $J_0(ar)$.

Hence a solution of (5) is

$$\zeta = AJ_0(ar)e^{-a^2\nu t}.$$

If $\zeta = 0$ for $t = 0$ at $r = R$, the radius of the cylinder, then $J_0(aR) = 0$, a transcendental equation determining an infinite series of values of a, viz.

$$\frac{aR}{\pi} = \cdot 7655, \quad 1\cdot 7571, \quad 2\cdot 746, \ . \ . \ .$$

Accordingly we may write for the complete solution of (5)

$$\zeta = \sum A_n J_0(a_n r)e^{-a_n{}^2\nu t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where the coefficients A_n are to be found by expanding the initial distribution of ζ as a series of Bessel functions, as specified above.

If ζ is initially concentrated mostly in the region of $r = 0$, the first few terms only of this series will be of consequence, and these decay on account of the term $e^{-a_n{}^2\nu t}$. Now for water $\nu = \cdot 01$ c.g.s. units, and, taking $R = 10$ cm., say, $a = \frac{\pi}{10} \times \cdot 76 = \cdot 24$.

$$\therefore \quad e^{-a_1{}^2\nu t} = e^{-\cdot 006t},$$

indicating that a relatively considerable time must elapse before ζ decays, as far as this term is concerned. The terms, of course, decay more rapidly as we proceed further up the series, but they themselves become small. From the point of view of the present discussion, where we are primarily concerned with what occurs in an extremely short interval subsequent to a disturbance to the vortex, it clearly suffices to assume that the strength of the vortices $+\kappa$ and $-\kappa$ are sensibly constant.*

* The full justification for this assumption can ultimately only be found by a comparison with experiment. The experimental investigation of the rate of decay of eddies is at present being conducted by the author.

§ 7. In practice the ideal case where the vortices are formed exactly in symmetrical positions is, of course, never realised. The question immediately arises whether, if the exact arrangement is one of equilibrium or steady motion, it is also one of stability; or more precisely, under what conditions is the symmetrical arrangement of two equal and opposite vortices in a uniform channel along which fluid is moving steadily with the parabolic distribution in velocity of a viscous fluid across the channel, one of stability?

§ 8. *Examination of the Stability of the Vortex Pair.*—The general motion in the channel is given by $u=\mathrm{U}(1-y^2/a^2)$. Let the centres of the two vortices P and Q of strengths $-\kappa$ and $+\kappa$ respectively be situated initially at the points $(0, a-\alpha)$ and $(0, -a+\alpha)$. We may dispense with the walls and deal with the infinite fluid provided an infinite row of vortices of equal but alternating strengths $+\kappa$ and $-\kappa$ be placed along the y-axis at the points whose co-ordinates are given by

$$+\kappa;\quad \{0,\ (4n\pm 1)a+\alpha\}\,;\qquad -\kappa;\quad \{0,\ (4n\pm 1)a-\alpha\}.$$

Let P receive small displacements (ξ, η) and Q (ξ', η') parallel to the x and y axes respectively; then, in virtue of the fact that the walls are rigid and that the fluid does not leave them, the displacements of the images are immediately determined.

Regarding the field as a complex plane, the co-ordinates of the system of vortices now become

$$\left.\begin{aligned} &+\kappa;\quad \xi+i(\overline{4n+1}a+\alpha-\eta),\quad \xi'+i(\overline{4n-1}a+\alpha+\eta')\\ &-\kappa;\quad \xi+i(\overline{4n+1}a-\alpha+\eta),\quad \xi'+i(\overline{4n-1}a-\alpha-\eta')\end{aligned}\right\}\quad . \quad . \qquad (8)$$

the vortices P and Q corresponding to

$$\xi+i(a-\alpha+\eta)\quad \text{and}\quad \xi'+i(-a+\alpha+\eta')\ \text{respectively.}$$

If (u, v) and $(u'\ v')$ are the component velocities of the vortices P and Q, they will be composed of two types of terms:

(a) the contribution in velocity due to direct effect of the infinite row of vortices;

(b) the general translational motion in the channel.

Consider first the effect of (a). The contribution to $-u+iv$ is

$$\frac{i\kappa}{2\pi}\sum_{-\infty}^{+\infty}\frac{1}{\{\xi+i(a-\alpha+\eta)\}-\{\xi+i(\overline{4n+1}a+\alpha-\eta)\}}+\text{etc.}\quad . \quad . \qquad (9)$$

four terms in all, in each of which the first member in double brackets in the denominator is the same, while the second member is successively each of the four terms in (8), with the appropriate sign for κ in front of the Σ. Expanding each typical term and neglecting quantities of higher order than the first in the small quantities ξ and η, it is easily found that (9) takes the form

$$\frac{i\kappa}{2\pi}\Big[\sum_{-\infty}^{+\infty}\frac{1}{4nai}+\sum_{-\infty}^{+\infty}\frac{1}{2ai(2n-1)}-\sum_{-\infty}^{+\infty}\frac{(\xi-\xi')+i(\eta+\eta')}{4a^2(2n-1)^2}-\sum_{-\infty}^{+\infty}\frac{1}{2i(2na+\alpha)}$$

$$+\sum_{-\infty}^{+\infty}\frac{i\eta}{2(2na+\alpha)^2}-\sum_{-\infty}^{+\infty}\frac{1}{2i(\overline{2n-1}a+\alpha)}+\sum_{-\infty}^{+\infty}\frac{(\xi-\xi')+i(\eta-\eta')}{4(\overline{2n-1}a+\alpha)^2}\Big]$$

$$=\frac{i\kappa}{2\pi}\Big[-\sum_{-\infty}^{+\infty}\frac{1}{2i(na+\alpha)}-\sum_{-\infty}^{+\infty}\frac{(\xi-\xi')+i(\eta+\eta')}{4a^2(2n-1)^2}+\sum_{-\infty}^{+\infty}\frac{(\xi-\xi')+i(\eta-\eta')}{4(\overline{2n-1}a+\alpha)^2}$$

$$+\sum_{-\infty}^{+\infty}\frac{i\eta}{2(2na+\alpha)^2}\Big] \quad . \quad (10)$$

since the first two terms vanish and the fourth and sixth combine together into one simple series.

In the same way the contribution to $-u'+iv'$ due to (a) is

$$\frac{i\kappa}{2\pi}\sum_{-\infty}^{+\infty}\frac{1}{\{\xi'+i(-a+\alpha+\eta')\}-\{\xi+i(\overline{4n+1}a+\alpha-\eta)\}}+\text{etc.} \quad . \quad . \quad (11)$$

there being four such terms identical with (9), except that the first member in the denominator in each case is the expression for the point Q.

Abandoning once more terms of higher order than the first in $\xi\ \eta\ \xi'\ \eta'$, on expanding (11) becomes

$$\frac{i\kappa}{2\pi}\sum_{-\infty}^{+\infty}\Big[\frac{1}{2i(\overline{2n+1}a-\alpha)}-\frac{(\xi'-\xi)+i(\eta'-\eta)}{4(\overline{2n+1}a-\alpha)^2}+\frac{1}{2i(2na-\alpha)}$$

$$-\frac{i\eta'}{2(2na-\alpha)^2}-\frac{1}{2i(2n+1)a}-\frac{1^*}{4nai}+\frac{(\xi'-\xi)+i(\eta'+\eta)}{4a^2(2n+1)^2}\Big]$$

$$=\frac{i\kappa}{2\pi}\sum_{-\infty}^{+\infty}\Big[\frac{1}{2i(na-\alpha)}+\frac{(\xi'-\xi)+i(\eta'+\eta)}{4a^2(2n+1)^2}-\frac{(\xi'-\xi)+i(\eta'-\eta)}{4(\overline{2n+1}a-\alpha)^2}-\frac{i\eta'}{2(2na-\alpha)^2}\Big] \quad (12)$$

The infinite series in (10) and (12) are all summable as follows:

$$\sum_{-\infty}^{+\infty}\frac{1}{na+\alpha}=\sum_{1}^{\infty}\frac{1}{na+\alpha}-\sum_{1}^{\infty}\frac{1}{na-\alpha}+\frac{1}{\alpha}=-\sum_{-\infty}^{+\infty}\frac{1}{na-\alpha}$$

$$=\frac{1}{\alpha}-2\alpha\sum_{1}^{\infty}\frac{1}{n^2a^2-\alpha^2}=\frac{\pi}{a}\cot\frac{\alpha\pi}{a} \quad (13)$$

by a well-known expansion.

$$\sum_{-\infty}^{+\infty}\frac{1}{(2n-1)^2}=\sum_{-\infty}^{+\infty}\frac{1}{(2n+1)^2}=\frac{\pi^2}{4} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

$$\sum_{-\infty}^{+\infty}\frac{1}{(\overline{2n-1}a+\alpha)^2}=\sum_{-\infty}^{+\infty}\frac{1}{(\overline{2n+1}a-\alpha)^2}=\frac{\pi^2}{4a^2}\sec^2\frac{\pi\alpha}{2a} \quad . \quad . \quad (15)$$

$$\sum_{-\infty}^{+\infty}\frac{1}{(2na+\alpha)^2}=\sum_{-\infty}^{+\infty}\frac{1}{(2na-\alpha)^2}=\frac{\pi^2}{4a^2}\operatorname{cosec}^2\frac{\pi\alpha}{2a} \quad . \quad . \quad . \quad (16)$$

* Omitting the term $n=0$.

Expressions (10) and (12) now reduce to

$$-u+iv=\frac{i\kappa}{2\pi}\left[-\frac{\pi}{2ai}\cot\frac{\alpha\pi}{a}-\frac{(\xi-\xi')+i(\eta+\eta')}{16}\cdot\frac{\pi^2}{a^2}+\frac{(\xi-\xi')+i(\eta-\eta')}{16}\frac{\pi^2}{a^2}\sec^2\frac{\pi\alpha}{2a}\right.$$
$$\left.+\frac{i\eta\pi^2}{8a^2}\operatorname{cosec}^2\frac{\pi\alpha}{2a}\right]$$
$$=\frac{i\kappa}{2\pi}\left[\frac{i\pi}{2a}\cot\frac{\alpha\pi}{a}+(\xi-\xi')\frac{\pi^2}{16a^2}\tan^2\frac{\pi\alpha}{2a}+\frac{i\eta\pi^2}{16a^2}\left(\tan^2\frac{\pi\alpha}{2a}+2\operatorname{cosec}^2\frac{\pi\alpha}{2a}\right)\right.$$
$$\left.-\frac{i\eta'\pi^2}{16a^2}\left(\sec^2\frac{\pi\alpha}{2a}+1\right)\right] \quad . \quad (17)$$

$$-u'+iv'=\frac{i\kappa}{2\pi}\left[-\frac{\pi}{2ai}\cot\frac{\alpha\pi}{a}+\frac{(\xi'-\xi)+i(\eta+\eta')}{16}\cdot\frac{\pi^2}{a^2}-\frac{(\xi'-\xi)+i(\eta'-\eta)}{16}\cdot\frac{\pi^2}{a^2}\sec^2\frac{\pi\alpha}{2a}\right.$$
$$\left.-\frac{i\eta'}{8a^2}\operatorname{cosec}^2\frac{\pi\alpha}{2a}\right]$$
$$=\frac{i\kappa}{2\pi}\left[\frac{i\pi}{2a}\cot\frac{\alpha\pi}{a}+(\xi-\xi')\frac{\pi^2}{16a^2}\tan^2\frac{\pi\alpha}{2a}+\frac{i\eta\pi^2}{16a^2}\left(1+\sec^2\frac{\pi\alpha}{2a}\right)\right.$$
$$\left.-\frac{i\eta'\pi^2}{16a^2}\left(\tan^2\frac{\pi\alpha}{2a}+2\operatorname{cosec}^2\frac{\pi\alpha}{2a}\right)\right] \quad . \quad (18)$$

For the contribution to $-u+iv$ due to (b), it is to be noted that since the vortices move with the fluid, and the total velocity in the undisturbed channel at a point $x+iy$ is $\mathrm{U}\left(1-\frac{y^2}{a^2}\right)$, there must be added to (17) and (18) the terms

$$-\mathrm{U}\left(1-\frac{\overline{a-\alpha+\eta}^{\,2}}{a^2}\right) \quad \text{and} \quad -\mathrm{U}\left(1-\frac{\overline{-a+\alpha+\eta'}^{\,2}}{a^2}\right)$$

respectively; or, retaining terms up to the first order only,

$$-\mathrm{U}\left\{1-\frac{\overline{a-\alpha}^{\,2}}{a^2}-\frac{2\eta}{a^2}(a-\alpha)\right\} \quad \text{and} \quad -\mathrm{U}\left\{1-\frac{\overline{a-\alpha}^{\,2}}{a^2}+\frac{2\eta'}{a^2}(a-\alpha)\right\} \quad . \quad (19)$$

If u_0 v_0, u_0' v_0' are the component velocities of P and Q when in the undisturbed position, these may be derived from (17), (18), and (19) by making $\xi=\eta=\xi'=\eta'=0$, and equating real and imaginaries. Hence

$$u_0=\frac{\kappa}{4a}\cot\frac{\alpha\pi}{a}+\mathrm{U}\left(1-\frac{\overline{a-\alpha}^{\,2}}{a^2}\right)=u_0'\,;\quad v_0=0=v_0' \quad . \quad . \quad (20)$$

indicating that the two vortices in the undisturbed position would move steadily along the channel with the velocity given by (20). For a given value of $\mathrm{U}a/\kappa$ (positive) the vortex will move up channel only if α/a is greater than some definite number itself greater than $\frac{1}{2}$, for only if $\alpha/a>\frac{1}{2}$ can the first term in u_0 become negative, while the second term is always positive.

§ 9. Let the axes of co-ordinates be in steady motion with the speed given by (20), so that in the undisturbed position the vortices would appear at rest relative to these axes. The component speeds may now be written when the vortices are disturbed as

$$\frac{d\xi}{dt}, \frac{d\eta}{dt}; \frac{d\xi'}{dt}, \frac{d\eta'}{dt}.$$

$$\therefore \; -\frac{d\xi}{dt}+i\frac{d\eta}{dt}=\frac{i\kappa\pi}{32a^2}\left[(\xi-\xi')\tan^2\frac{\pi\alpha}{2a}+i\eta\left(\tan^2\frac{\pi\alpha}{2a}+2\operatorname{cosec}^2\frac{\pi\alpha}{2a}\right)-i\eta'\left(1+\sec^2\frac{\pi\alpha}{2a}\right)\right]$$
$$+\frac{2U}{a^2}\eta(a-\alpha) \quad . \quad (21)$$

$$-\frac{d\xi'}{dt}+i\frac{d\eta'}{dt}=\frac{i\kappa\pi}{32a^2}\left[(\xi-\xi')\tan^2\frac{\pi\alpha}{2a}+i\eta\left(1+\sec^2\frac{\pi\alpha}{2a}\right)-i\eta'\left(\tan^2\frac{\pi\alpha}{2a}+2\operatorname{cosec}^2\frac{\pi\alpha}{2a}\right)\right]$$
$$-\frac{2U}{a^2}\eta'(a-\alpha) \quad . \quad (22)$$

From which, by separating real and imaginary terms, it follows that

$$\frac{32a^2}{\pi\kappa}\frac{d\xi}{dt}=\eta\left(\tan^2\frac{\pi\alpha}{2a}+2\operatorname{cosec}^2\frac{\pi\alpha}{2a}\right)-\eta'\left(1+\sec^2\frac{\pi\alpha}{2a}\right)-\frac{64U}{\kappa\pi}(a-\alpha)\eta \quad . \quad (23)$$

$$\frac{32a^2}{\pi\kappa}\frac{d\eta}{dt}=(\xi-\xi')\tan^2\frac{\pi\alpha}{2a} \quad . \quad (24)$$

$$\frac{32a^2}{\pi\kappa}\frac{d\xi'}{dt}=\eta\left(1+\sec^2\frac{\pi\alpha}{2a}\right)-\eta'\left(\tan^2\frac{\pi\alpha}{2a}+2\operatorname{cosec}^2\frac{\pi\alpha}{2a}\right)+\frac{64U}{\kappa\pi}(a-\alpha)\eta' \quad . \quad (25)$$

$$\frac{32a^2}{\pi\kappa}\frac{d\eta'}{dt}=(\xi-\xi')\tan^2\frac{\pi\alpha}{2a} \quad . \quad (26)$$

This is a system of linear equations with constant coefficients.

The solutions are consequently in general of the form

$$(\xi, \eta, \xi', \eta')=\sum_1^4 A_n e^{\frac{32a^2}{\pi\kappa}\lambda_n t},$$

and the equation to determine the λ's is

$$\begin{vmatrix} \lambda, & -X, & 0, & Y \\ -\tan^2\frac{\pi\alpha}{2a}, & \lambda, & \tan^2\frac{\pi\alpha}{2a}, & 0 \\ 0, & -Y, & \lambda, & X \\ -\tan^2\frac{\pi\alpha}{2a}, & 0, & \tan^2\frac{\pi\alpha}{2a}, & \lambda \end{vmatrix}=0 \quad . \quad (27)$$

where

$$X=\tan^2\frac{\pi\alpha}{2a}+2\operatorname{cosec}^2\frac{\pi\alpha}{2a}-\frac{64U}{\kappa\pi}(a-\alpha)$$

$$Y=1+\sec^2\frac{\pi\alpha}{2a}.$$

On expanding (27), it follows that

$$\lambda^2\left[\lambda^2+2\tan^2\frac{\pi\alpha}{2a}.(Y-X)\right]=0$$

or

$$\lambda^2=0, \quad \text{and} \quad \lambda^2=4-\frac{128U}{\kappa\pi}(a-\alpha)\tan^2\frac{\pi\alpha}{2a} \quad . \quad (28)$$

Equations (24) and (26) indicate that if the initial displacement is purely in η and η', then $\frac{d\eta}{dt}$ and $\frac{d\eta'}{dt}$ are both zero. Similarly (23) and (25) show that if the initial disturbance is purely in ξ and ξ', then $\frac{d\xi}{dt}$ and $\frac{d\xi'}{dt}$ are both zero. In each of these cases the system is neutral to the disturbance in question. This corresponds, of course, to the two zero values for λ.

The other expressions for λ^2 being real, it follows that the arrangement is stable or unstable according as

$$4 - \frac{128\mathrm{U}}{\kappa\pi}(a - \alpha)\tan^2\frac{\pi\alpha}{2a} \lesseqgtr 0,$$

i.e. as

$$\frac{\mathrm{U}(a - \alpha)}{\kappa} \gtrless \frac{\pi}{32}\cot^2\frac{\pi\alpha}{2a} \quad . \quad . \quad . \quad . \quad . \quad (29)$$

If $2h$ is the distance apart of the two vortices, $h = a - \alpha$, and the criterion (29) takes the form

$$\frac{\mathrm{U}h}{\kappa} \gtrless \frac{\pi}{32}\tan^2\frac{\pi h}{2a} \quad . \quad . \quad . \quad . \quad . \quad (30)$$

according as there is stability or instability.

§ 10. The inequalities (29) and (30) provide the criterion sought for, and furnish the analogue of the experimentally known critical value for $\mathrm{U}a/\nu$ already referred to. At first sight there appears a serious dissimilarity between the two critical conditions; whereas Reynolds has found that for $\mathrm{U}a/\nu$ *less* than a definite number stability existed, the analysis of the present paper shows that for $\mathrm{U}a/\kappa$ (or what is in effect the same, $\mathrm{U}h/\kappa$) *greater* than a definite quantity a stable state of affairs would exist. The inconsistency is, however, only apparent. For a legitimate comparison of the two conditions, equations (1) or (2) expressing κ in terms of ν and $\mathrm{U}a/\nu$ are required; κ/ν may be a comparatively complicated function of $\mathrm{U}a/\nu$, and the inequalities (29) and (30) would require to be transformed accordingly. If, for example, it were found that under certain circumstances $\kappa \propto \nu(\mathrm{U}l/\nu^2)$, the inequalities in question would immediately revert to the Reynolds form. How to establish the appropriate relation of the form (1) or (2) is, however, a question for a future paper. For the moment it suffices to state that the criterion (29) or (30) determines the maximum intensity permissible for the two eddies, that they may stably maintain their arrangement for a given forward velocity of fluid. It should be noted that where $\mathrm{U}h/\kappa$ is already above the critical, and therefore stability already exists, the gradual decay of the vortex strengths due to the viscosity will tend to force up the value of Uh/κ further into the region of stability, and to maintain the vortices in their equilibrium positions. No such corresponding assertion can, however, be made when Uh/κ is initially below the critical, and the vortices move off along paths determined by their initial displacements.

§ 11. For the evaluation of the velocities in the fluid when the vortices are in their undisturbed position, we may most conveniently specify the positions of the latter thus:—

$$+\kappa ;\ (\overline{2n+1}a+\alpha)i. \qquad -\kappa ;\ (\overline{2n+1}a-\alpha)i.$$

Hence for the effect of the vortices alone

$$w=\frac{i\kappa}{2\pi}\sum_{-\infty}^{+\infty}\{\log[z-(\overline{2n+1}a+\alpha)i]-\log[z-(\overline{2n+1}a-\alpha)i]\}$$

$$=\frac{i\kappa}{2\pi}\log\prod_{-\infty}^{+\infty}\frac{z-(\overline{2n+1}a+\alpha)i}{z-(\overline{2n+1}a-\alpha)i}$$

$$=\frac{i\kappa}{2\pi}\log\prod_{0}^{\infty}\frac{1+\dfrac{(z-\alpha i)^2}{(2n+1)^2a^2}}{1+\dfrac{(z+\alpha i)^2}{(2n+1)^2a^2}}$$

$$=\frac{i\kappa}{2\pi}\log\frac{\cosh\dfrac{\pi}{2a}(z-\alpha i)}{\cosh\dfrac{\pi}{2a}(z+\alpha i)}.$$

$$\therefore \qquad -u+iv=\frac{dw}{dz}=\frac{i\kappa}{4a}\left[\tanh\frac{\pi}{2a}(z-\alpha i)-\tanh\frac{\pi}{2a}(z+\alpha i)\right]$$

$$=\frac{\kappa}{2a}\cdot\frac{\sin\dfrac{\pi\alpha}{a}}{\cosh\dfrac{\pi z}{a}+\cos\dfrac{\pi\alpha}{a}}.$$

To this must be added at every point the motion due to the steady streaming in the channel; thus

$$-u+iv=\frac{\kappa}{2a}\frac{\sin\dfrac{\pi\alpha}{a}}{\cosh\dfrac{\pi z}{a}+\cos\dfrac{\pi\alpha}{a}}-U\left(1-\frac{y^2}{a^2}\right).$$

§ 12. The motion in the channel at great distances from the vortices approaches the steady undisturbed motion of the fluid. Along the walls $y=\pm a$,

$$-u_a+iv_a=\frac{\kappa}{2a}\cdot\frac{\sin\dfrac{\pi\alpha}{a}}{\cosh\left(\dfrac{\pi x}{a}\pm i\pi\right)+\cos\dfrac{\pi\alpha}{a}}.$$

$$\therefore \quad u_a = \frac{\kappa}{2a} \frac{\sin \frac{\pi a}{a}}{\cosh \frac{\pi x}{a} - \cos \frac{\pi a}{a}}, \quad v_a = 0.$$

The slip at the boundary therefore attains a maximum value of $\frac{\kappa}{2a} \cot \frac{\pi a}{2a}$ when $x = 0$ and falls off exponentially as x increases, becoming zero as $|x| \rightarrow \infty$. The maximum velocity of slip may likewise be written, of course, $\frac{\kappa}{2a} \tan \frac{\pi h}{2a}$. Associating this with the conditions for stability and instability (30) in the form $\kappa \lesseqgtr \frac{32}{\pi} \mathrm{U}h \cot^2 \frac{\pi h}{2a}$, it follows that for a stable system the maximum velocity of slip is always less than $\frac{16}{\pi} \frac{h}{a} \mathrm{U} \cot \frac{\pi h}{2a}$, and for an unstable system the slip is always greater than this quantity. The fact that as the passage is made from the stable to the unstable region the tendency to slip increases, should be associated with the remarks in § 5 of this paper relative to the insufficiency of experimental evidence on the question of slip at the boundary when turbulent flow has set in.

(*Issued separately December* 13, 1921.)

XVI.—Note on Conditions for Mirage on the Queensferry Road. By Alex. G. Ramage.

(Read July 4, 1921. MS. received September 21, 1921.)

The surface of the Queensferry Road, from about the bend above Blackhall, past Marchfield, and on towards Cramond Bridge, was remade in the spring 1919.

It was made in the modern fashion for a motoring road, with road metal and liberal supplies of bitumen, and small pieces of quartz scattered on top of the bitumen, the whole being rolled by steam roller.

After this had been done, although I watched carefully throughout the summer, no signs of the mirage, so common on this road during the previous summer, made their appearance until August, and then but faintly. This in my opinion points to a triturated condition of the quartz under road traffic, as being an essential factor in the phenomenon. During the summers of 1920 and 1921, on bright days, mirage was much in evidence on this road at the places described in my paper, and at other parts in the vicinity.

The following observations made during August 1921 may be of interest.

I have had occasion to use the Craigleith Road (which branches off from the Queensferry Road near Craigleith Station and joins the Comely Bank Road) frequently this year, and have carefully looked for mirage, but saw none. The surface is "*old*," and not to be distinguished from that of the Queensferry Road when mirage was observable. It is smooth, with the small stones well embedded in the bitumen. On the south side there are open railings, so that the sun reaches it without obstruction. Compared with the Queensferry Road there is little traffic.

Recently the Queensferry Road between Lord Salvesen's house and Craigleith Station has been resurfaced, and considerable traffic has been diverted round the Craigleith Road. On the 15th August 1921, weather hot and sunny, I walked from the bus terminus at Comely Bank along the Craigleith Road and saw an isolated spot of *water* on this road, but no reflection. Walking on through Blackhall on Queensferry Road, I found the two yellow advertisement boards clearly reflected from the road, and some children's coloured clothes were well reflected just at the bend of the road from Blackhall towards Marchfield. This is the first reflection of coloured garments I have seen on the Queensferry Road since the surface was remade in early summer, although some days earlier I observed the two yellow advertisement boards reflected, but not so distinctly as on the 15th August.

The surface of the Queensferry Road where the garments were seen to be reflected is not yet smooth, the stones are still protruding through the bitumen in places. The traffic has been very great.

I have just returned from an extensive motoring tour in England, taking in both main highways and byways. Only on the highways where there was great traffic did I see mirage on the road. On the byways I looked for it in vain.

(*Issued separately December* 13, 1921.)

XVII.—The Annual Incidence of Intelligence, and its Measurement by the American Army Tests. By M. M'Callum Fairgrieve, M.A.

(MS. received May 4, 1921. Read June 20, 1921.)

(1) The following investigation was undertaken to verify by the American Army Tests a result previously obtained by the use of Mr Cyril Burt's tests of intelligence (*Journal of Experimental Pedagogy*, vol. vi, No. 1), *i.e.* that there appears to be some likelihood that boys born in the spring months are slightly less intelligent on the average than those born in other months. Of these American tests (see *Mental Tests in the American Army*, C. S. Yoakum and R. M. Yerkes, pp. 220–230) the groups of tests marked "form 6" were used; but "test 8" was modified to suit British conditions by using test 8 of group 9 (p. 274, *loc. cit.*) as a basis, and replacing tests 1, 3, 7, 8, 9, 11, 13, 19, 35, 36, 38 by 8, 1; 6, 3; 7, 39; 8, 39; 6, 35; 7, 36; 7, 38; 6, 39, and by three others of local interest.

(2) The tests were applied in the manner directed to 368 boys—nearly the whole of the upper school; whereas the previous Burt test had been applied to 192 boys only. But it should be noted that

(*a*) the tests could not be given to all classes simultaneously;
(*b*) different classes were examined at different times;
(*c*) all forms of test 1 were used to minimise possible coaching;
(*d*) the test took about a month to complete, but
(*e*) a partial repetition of the test, using "form 7," showed that this extension of time was probably immaterial.

(3) The total marks gained by boys of the same age in months was then added to the marks gained by those a month younger and a month older, and the average mark obtained. The method and result are given in Table I. Ages date from October 1.

(4) These average marks are plotted in fig. 1, and show clearly minima in the late spring or early summer months at 12 years 5 months; 14 years 6 months: 15 years 4 months; 16 years 3 months; and maxima in the late autumn months at 11 years 11 months; 12 years 10 months (or 13 years 0 months); 13 years 10 months; and 14 years 10 months, and perhaps 16 years 0 months.

(5) (*a*) Table II. gives the average mark obtained in three different ways of boys born in each month.

(*b*) The result marked B is the most reliable average for the usual reasons, the boys aged eleven being rather a clever sample.

(*c*) The result of the Burt test is added for comparison.

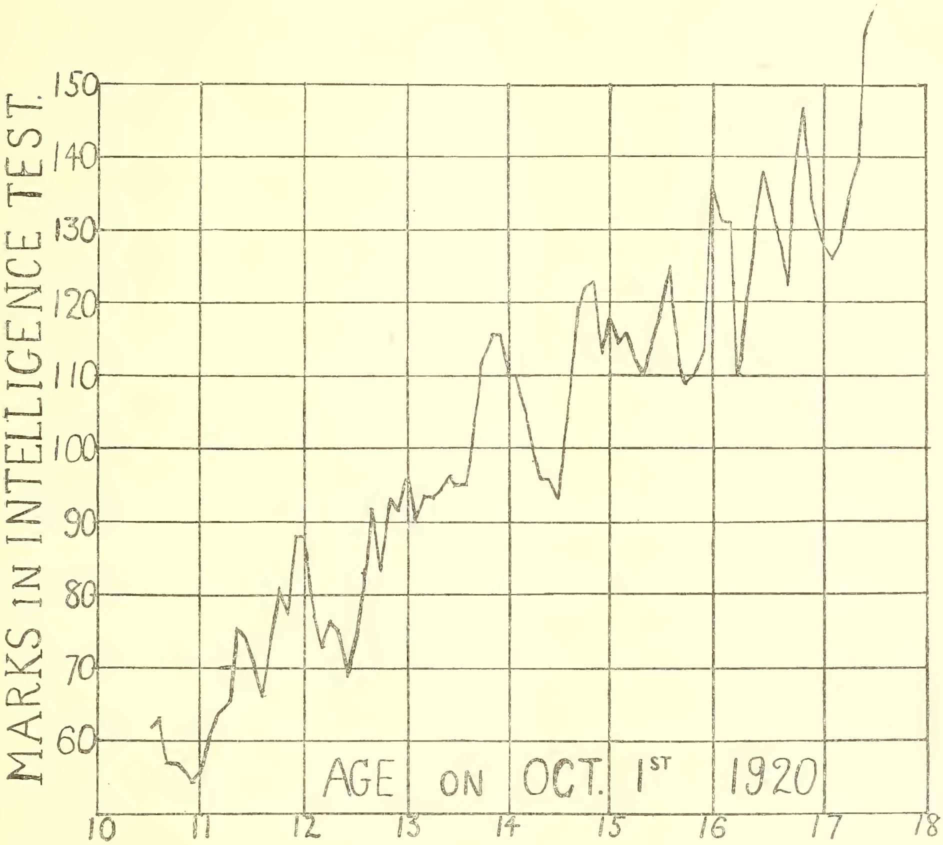

Fig. 1.

(6) Both diagram and table thus give clear indication that, from some cause, boys born in the late spring months—say March to July—are in danger of developing less intelligence than those born about October to December. But

(*a*) many of the cleverest boys in the school have birthdays in the less intelligent period;

(*b*) some of the duller boys are brilliant enough athletically; hence

(*c*) some effect of environment may possibly be indicated;

(*d*) the experiment requires to be repeated in other localities.

Table I.—Average Mark gained by a Boy aged x Years and y Months at October 1, 1920.

The marks have been smoothed in sets of three months. Column 3 gives the number of boys of each age, and column 4 the number in each set of three months.

(1) Age.	(2) Total Mark.	(3) No. of Boys.	(4) No in Group.	(5) Average Mark.	(1) Age.	(2) Total Mark.	(3) No. of Boys.	(4) No. in Group	(5) Average Mark.	(1) Age.	(2) Total Mark.	(3) No. of Boys.	(4) No. in Group.	(5) Average Mark.
x y					x y					x y				
10 5	171	3	...	...	12 10	403	5	12	93	15 3	904	8	16	112
6	275	4	9	62	11	548	5	16	91	4	588	5	20	110
7	115	2	7	63	13 0	500	6	16	96	5	713	7	18	114
8	52	1	3	57	1	487	5	12	90	6	765	6	17	118
9	...	...	4	57	2	93	1	18	93	7	521	4	13	125
10	175	3	8	56	3	1094	12	20	93	8	337	3	9	116
11	285	5	10	54	4	670	7	24	94	9	197	2	11	109
11 0	78	2	9	56	5	499	5	15	96	10	661	6	10	110
1	147	2	8	61	6	275	3	13	95	11	243	2	10	113
2	262	4	8	64	7	457	5	14	95	16 0	228	2	13	136
3	111	2	12	66	8	605	6	19	103	1	1293	9	14	131
4	423	6	10	76	9	899	8	18	112	2	320	3	15	131
5	228	2	15	74	10	519	4	16	116	3	350	3	9	110
6	444	7	13	70	11	440	4	15	116	4	316	3	10	119
7	240	4	14	66	14 0	784	7	19	110	5	522	4	13	130
8	246	3	11	74	1	865	8	19	110	6	853	6	14	138
9	321	4	11	81	2	447	4	16	106	7	551	4	11	133
10	324	4	14	77	3	389	4	12	100	8	81	1	7	128
11	438	6	17	88	4	368	4	10	96	9	266	2	3	122
12 0	625	7	14	88	5	202	2	11	96	10	...	...	4	147
1	78	1	14	77	6	388	5	12	93	11	321	2	5	134
2	380	6	14	73	7	534	5	16	104	17 0	350	3	12	129
3	568	7	21	76	8	722	6	17	118	1	880	7	14	126
4	640	8	19	75	9	743	6	13	122	2	547	4	13	128
5	217	4	20	69	10	118	1	9	123	3	244	2	8	134
6	520	8	18	74	11	251	2	11	113	4	282	2	6	139
7	595	6	18	81	15 0	874	8	16	118	5	311	2	6	157
8	337	4	12	92	1	768	6	17	114	6	348	2	4	162
9	169	2	11	83	2	295	3	17	116					

(7) At the right hand of Table II. is also given the average mark of boys of a definite yearly age and the number of boys in each yearly group.

(*a*) Marks for ages 12, 13, 14, and 15 may be taken as fairly reliable.

(*b*) The differences between these suggest a parabolic form as very suitable for the normal curve.

(*c*) The highest mark (140) on this supposition would correspond well with many of the averages of Yoakum and Yerkes. In particular, it is nearly identical with that given for the 15,385 cases of the "White Officers' Principal Sampling" (139).

(*d*) Hence Table III. is suggested as giving a suitable normal mark for boys of these ages, the figures in brackets being calculated from the parabola of best fit.

TABLE II.—AVERAGE SCORE OF BOYS BORN IN DIFFERENT MONTHS.

(A) is average score of all years available (10·6–17·6).
(B) is score for best years available (11·6–16·6).
(C) is score for best years available (11·6–16·6), unsmoothed figures.

Birth month	Apr.	Mar.	Feb.	Jan.	Dec.	Nov.	Oct.	Sept.	Aug.	July.	June.	May.	Apr.	Mar.	Average Age.	Average Mark.	No. of Boys.
x years + .	6.	7.	8.	9.	10.	11.	0.	1.	2.	3.	4.	5.	6.	yrs.		
10 . .	62	63	57	57	56	54	56	61	64	66	76	74	70	11	63	43
11 . .	70	66	74	81	77	88	88	77	73	76	75	69	74	12	76½	62
12 . .	74	81	92	83	93	91	96	90	93	93	94	96	95	13	91½	61
13 . .	95	95	103	112	116	116	110	110	106	100	96	96	93	14	104½	61
14 . .	93	104	118	122	123	113	118	114	116	112	110	114	118	15	115	63
15 . .	118	125	116	109	110	113	136	131	131	110	119	130	138	16	122	47
16 . .	138	133	128	122	147	134	129	126	128	134	139	157	162	17	137	31
Totals .	650	667	688	686	722	709	733	709	711	691	709	736	750			
Levelling correction	50	42	34	25	17	9	0	−9	−17	−25	−34	−42	−50			
Average (A)	100	101	103	102	105	103	105	100	*99*	*95*	*96*	*99*	100			
" (B)	*97*	100	105	105	106	105	109	104	101	*94*	*95*	*95*	*97*			
" (C)	*94*	104	103	104	106	108	101	109	*93*	*96*	*93*	*92*	*94*			
Burt average mark	*14½*	*13½*	*14½*	17 −	17 −	17 +	17 +	17 −	17 −	17	*16½*	*15½*	*14½*			

TABLE III.—AGE NORMS FOR AMERICAN ARMY TESTS.

Age . . .	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
Differences of average mark			(21)	(19)	(17)	15	13	11	9	(7)	(5)	(3)	(1)
Normal mark .	0	(19)	(40)	(59)	76	91	104	115	124	(131)	(136)	(139)	(140)

(*Issued separately December* 13, 1921.)

XVIII.—Experiments with an Electrified Pith Ball in an Ionised Atmosphere. By Dr Dawson Turner and Mr D. M. R. Crombie.

(Read May 2, 1921. MS. received June 16, 1921.)

A PITH ball suspended from the centre rod of a charged Leyden jar provides us with an interesting and very delicate method of demonstrating the ionised atmosphere surrounding flames and incandescent bodies. In an ionised atmosphere of sufficient intensity the ball, provided that the suspending fibre is of high insulating quality, rapidly loses its charge and falls back toward the centre rod of the jar, from which it receives a fresh charge and is violently repelled, only to repeat the performance; thus an oscillation is set up the rapidity of which is governed chiefly by the length of the suspending fibre.

A screen of any material, even wire gauze, interposed between the ionising source and the ball, and not too near the latter, at once stops the oscillation.

The violence of the oscillations seems from the experiments performed to depend upon the following factors:—

(1) The distance of the source of ionisation. The nearer the source is to the ball, the more violent are the oscillations.

(2) The nature of the source of ionisation. Thus the bulk of experiments showed that a bunsen burner was the most effective in producing oscillations. Less effective sources included a naked Nernst filament, a glowing platinum wire, a candle flame, and a spirit-lamp flame. An electric arc gave comparatively poor results. This was somewhat surprising, but may be due to the powerful electric field between the carbons preventing the escape of ions to the surrounding atmosphere. To test if the effect had any relation to the actinic quality of the light, a piece of burning magnesium ribbon was tried, but was found to be less effective than a candle flame.

Mere bulk of flame does not make much difference to the oscillations, since a jet of gas burning from an open pipe, though of large dimensions, was not so effective as the bunsen.

(3) The third factor influencing the oscillations is the potential of the jar. Effects can be produced at a greater distance when the jar is highly charged than when it has only a small charge. This might seem to show the directive influence of the jar and ball in attracting a stream of the opposite kind of ions towards themselves.

(4) The position of the source of ionisation can be shown to have an important bearing on the oscillations. Thus a candle held directly opposite the ball at a distance too great to be effective may be made so at the same distance if held at a lower level than the ball. From this we may conclude that the ions formed in the flame are carried upwards by the convection currents of air, and entirely disposes of any suspicion of projection of ions from the flame. Also, should the ball happen to be in any position other than directly between the source of ionisation and the centre rod of the jar, the source may be approached very close indeed to the ball without oscillations taking place. This is in confirmation of the previous deduction arrived at, namely, that the highly charged centre rod of the jar exerts a directive influence on the ions.

In addition to the above, the atmospheric condition seems to play an important part. This appears to be a reasonable explanation of the varying maximum distance at which effects could be produced on different days when to all appearance other conditions were identical.

The conclusion drawn from the whole series of experiments was that the nature of the charge given to the jar had no bearing on the results obtained, and though there were occasions when this did not seem to be the case, subsequent experiments showed that the discrepancies were probably due to variation in some of the factors just mentioned.

In performing the experiments we employed the following method :—

The jar was charged from a Wimshurst machine and placed on a bench with the pith ball standing out towards the ionising source, which was placed 30 inches off, with a screen interposed between it and the ball. A reading telescope was mounted in such a position as to have the ball in the field of view of its graduated scale. The screen was then removed and the ball carefully watched for any slight drop. This distance was invariably too great to show any effect, but on most occasions, when the distance was decreased to 25 inches, a slight drop could be noticed. As a rule, at 20 inches the drop of the ball was large enough to be seen easily with the naked eye, and occasionally oscillations commenced at that distance. This was when the ionising source was a bunsen burner; but as a rule oscillations did not commence until the bunsen was about 18 inches from the ball, though sometimes, under unfavourable conditions, much shorter distances were necessary before effective oscillations could be produced. Once oscillations had commenced, it was usually found that the source could be moved further off and the movement maintained at the increased distance.

The introduction of a platinum cup containing a sodium salt into the bunsen flame did not increase the distance at which oscillations could be produced, but seemed to have the opposite effect.

As already mentioned, a Nernst filament, a candle flame, and a spirit flame were all found to be inferior to the bunsen as sources of ionisation; but especial mention is required in the case of the electric arc, which was unable to produce oscillations at a greater distance than 6 inches, while 2 inches of platinum wire raised to a low white heat by an electric current were quite effective at 13 inches.

An interesting effect was noticed while working with a glass screen which was held so near the jar that the pith ball was only a matter of $1\frac{1}{2}$ to 2 inches from the screen. When a source of ionisation (in this case the Nernst) was brought up to the side of the screen remote from the jar, the ball was visibly attracted toward the screen, and when the Nernst was brought nearer still the ball jumped to the screen and stuck there. The effect was more marked if in place of the glass screen a thin insulated metal screen was used; but in this latter case the ball, instead of sticking to the screen, oscillated between it and the centre rod of the jar. As before, this effect was independent of the nature of the charge in the jar. If, however, the charged pith ball were detached from the jar and suspended from a glass rod at a distance of 2 inches, as before, from the screen, the effect was no longer obtainable, and the approach of the Nernst to the other side of the screen produced no visible attraction between the screen and the ball.

A possible explanation of these effects is as follows:—The knob of the jar being in close proximity to the screen induces on the side next to it an opposite and repels to the far side a similar charge. These opposite charges in close proximity to one another can have little influence on the ball, and the thinner the screen the less will this influence of the opposite induced charge be in attracting the ball towards it. If, however, the repelled similar charge be removed by any means, then the opposite charge is left free to attract the ball. This is what happened when the Nernst lamp was brought toward the far side of the screen, for then the ionised atmosphere surrounding it rapidly discharged the repelled similar charge, leaving the induced opposite charge to attract the pith ball. The fact that when the ball alone was present no effect was produced seems to show that it was dependent on the proximity of the highly charged knob of the Leyden jar.

When X-rays were used as the source of ionisation the oscillation effects, though in the main similar to those produced by flames, etc., differed in one or two important respects. Firstly, as was to be expected, the distance at which oscillations could be produced was very much greater; secondly, the effect could be produced as a rule at a greater distance when the potential of the jar was low than when it was high; also, as the distance increased the oscillations took longer to commence, during which period the ball was slowing dropping toward the centre rod. Eventually, when the extreme limit at which oscillations could be produced was being approached, an interval of two or three minutes elapsed before oscillations began. At distances too great for the production of oscillations a dropping of the ball could be observed each time the X-rays were turned on. The interposition of an iron screen 0·35 mm. thick stopped the effect at once.

In all the previous cases the potential of the central rod of the jar was the active means whereby a stream of ions was drawn from the source of ionisation so as to pass through the ball, hence when, instead of a single ball, four balls were attached round the circumference of the rod, one in each quadrant and equidistant from one another, only the ball in the direct alignment was affected, but in the case of the X-rays all the balls were almost equally affected, being all equally immersed in a sea of ions.

Similar results, but at much shorter distances, were obtained by using the rays from a 50-mm. capsule of radium bromide. The interposition of the iron screen did not stop the effect.

Summary of Conclusions.

(1) A pith ball suspended from the centre rod of a charged Leyden jar will oscillate in an ionised atmosphere and can serve as a very delicate indicator of the electrical condition of its surroundings.

(2) By its means the ionised atmosphere around flames can be detected at a considerable distance, and the relative intensities of various sources of ionisation can be compared.

(3) The ions are concentrated along the line joining the centre rod of the charged jar and the source of ionisation, for unless the pith ball be in this line it will be unaffected, except in the case of X-rays and the rays from radio-active bodies.

(4) The ions tend to be carried upwards by convection currents.

(5) The ionisation of the atmosphere does not depend upon the luminous or actinic intensity of a flame.

(6) The effect upon the electrified pith ball appears to be independent of the nature of its charge.

(*Issued separately December* 13, 1921.)

OBITUARY NOTICES.

Robert Munro, M.A., M.D., LL.D. By Dr George Macdonald, C.B.

(MS. received July 4, 1921. Read October 24, 1921.)

Robert Munro was born at Assynt, in the Ross-shire parish of Alness, on 21st July 1835. After spending some years at the Free Church School of Kiltearn, he was sent to finish his education at the Royal Academy, Tain. Though he was alert and observant from the first, his intellectual powers seem to have developed somewhat slowly: it was not until his career at Tain was drawing to a close that his capacity for University work was realised. The financial difficulty was serious. But his own mind was definitely made up, and with characteristic determination he set himself to overcome all obstacles. As a means to the end, he took to teaching, and in 1860 he found himself a graduate in Arts of the University of Edinburgh. His original intention had been to proceed to the New College, with a view to entering the Church. In 1859, however, his whole outlook in life had been changed by the appearance of Darwin's *Origin of Species*, which he read with avidity, and which made an immediate appeal to his scientific instincts. In the Free Church of those days there was no room for a Darwinian, and there was nothing for it but to abandon all thought of the profession at which he had been aiming.

For two years after obtaining his degree he remained doubtful as to how he should shape his future. Ultimately, with great courage and also (as the event proved) with great wisdom, he resolved to face the discipline of the medical curriculum. In 1862, at the age of twenty-seven, he matriculated once more at Edinburgh. Even then his course was not destined to proceed on normal lines. What should have been his third winter of medical study was spent on the Riviera, in charge of a semi-invalid. At his time of life the interruption might well have seemed serious. But he never saw reason to regret it. His receptive mind derived real profit from his sojourn abroad. The fauna, the flora, and the geology of the Mediterranean all had an interest for him. And in various other ways his horizon was appreciably widened. If, however, the interlude was educationally valuable, it had the incidental effect of postponing for a whole year the accomplishment of his immediate purpose. He did not finally "qualify" till 1867. He was then thirty-two, and had no resources behind him save the priceless assets of ability and character.

His first appointment was as assistant to a busy doctor in a colliery district of Ayrshire. He at once became deeply absorbed in his everyday duties, utilising to the full the opportunities for instruction which they offered. His own description may be quoted: "The sudden transition from a scholastic atmosphere and the teaching of medical science in lecture-rooms and well-equipped hospitals to the practice of the healing art among a mining population was to me like going into a new world. Therapeutic theories and book-learning had to be tested by action there and then." The sound knowledge thus acquired of the origin, progress, and correct treatment of disease stood him in excellent stead when he aspired to a position of greater independence. This he did after an apprenticeship of some two years' duration. Looking round for an opening, he decided upon a partnership in Kilmarnock. Before settling down, however, he received an invitation to make an extended tour in the Near East as companion to the son of a well-known Ayrshire proprietor. The offer came at an opportune moment, and he gladly availed himself of it. Doubtless he was ultimately responsible for the comprehensive itinerary which, beginning with the more important cities of France and Italy, led through Sicily and Malta to Egypt and the Nile, the Holy Land, Baalbek, Athens, Constantinople, Rustchuk, Budapest, Vienna, and thence home by Munich and the Rhine.

There followed sixteen years of arduous general practice, diversified by short holidays abroad. No figure in Kilmarnock was better known in those days than Dr Munro's. His regular patients were as numerous as he could wish for, and the reputation he had won during his assistantship brought many miners from Cumnock and its neighbourhood to his consulting-room. At the same time he was in great demand as a popular lecturer on scientific and social subjects, invariably speaking his mind with a singularly refreshing frankness. His influence in the community grew steadily, and to outsiders it must have seemed as if his highest ambition had been satisfied. There was, therefore, general surprise and regret when, in 1885, he announced that he had made up his mind to retire. Friends came to remonstrate. But he was inflexible: "I divide my life into three periods: during the first I struggled hard for my education, during the second I served the public to the best of my ability, and for the rest of my life I mean to please myself." Ten years earlier he had married Miss Anna Taylor, a lady of singular charm, who was to be his devoted companion for thirty-two years in all, and in 1879 the death of his father-in-law had made him a shareholder in two local engineering companies, then on the threshold of a prosperous career. In 1880 he became chairman of one of them, and he was subsequently elected chairman of a combine that included both. The income from these undertakings, added to what he had been able to save from his professional earnings, had put him in possession of a comfortable competency.

His decision to relinquish his practice was more immediately prompted by a threatened breakdown in health. But there was a larger reason behind it. "I began to realise that I was gradually becoming enslaved to a monotonous existence of mere routine work, with the prospect of premature decay. My real object in joining the medical profession had no higher motive than to secure an honourable livelihood, an object which had now, in a small but efficient way, been attained; but yet worldly prosperity did not bring with it the realisation of my earlier ideals of an intelligent human existence. The preliminary studies on which the laws of organic development of the human body, both in health and disease, are supposed to be founded, are most fascinating; but the art of healing, which in practice is largely based on empiricism, soon engenders in the mind of the conscientious physician doubts as to the efficiency of many so-called remedies. In many instances of serious illness it is often as clear as noonday to the skilful physician that palliation of symptoms is all that can be done; but yet, if the doctor expressed a hint of this truth, he would in all probability instantly lose his patient. Here lies a dangerous pitfall which sometimes leads to quackery and hypocrisy." This outspoken confession throws a curious and interesting light on the writer's own temperament. He can hardly have been one of those medical men "whose visits make it a pleasure to be ill," as R. L. Stevenson puts it. Rather, he must have resembled Chaucer's "Doctour of Phisyk"—

> "He was a verrey parfit practisour.
> The cause y-knowe, and of his harm the rote,
> Anon he yaf the seke man his bote.
> . . . .
> His studie was but litel on the bible."

As soon as he was free, he set out for Rome, where he rapidly threw off the painful illness that had attacked him. His physical vigour restored, he devoted all his energies to a line of research which he had resolved to make his own. In 1877 he had been enrolled as an original member of the Ayrshire and Galloway Archæological Association. Hitherto his interest in antiquities had been very detached, although during a holiday in

Switzerland his scientific curiosity had been aroused by the fine series of objects from lake-dwellings displayed in the museum at Zurich. By a fortunate chance, the very first piece of work undertaken by the Association was the excavation of a crannog, or artificial island, whose remains had been accidentally discovered on the farm of Lochlee, familiar from its association with Robert Burns. The late Mr Cochran Patrick, who was the mainspring of the organisation, promptly enlisted Munro as a helper, and a magnificent recruit he proved. Several other crannogs were explored during the next two or three years, Munro taking a prominent part in every case, and ultimately becoming leader. In 1882 the results were brought before the public in his *Scottish Lake-dwellings*, a performance which made it clear that he had laid a firm grasp on the essentials of the subject.

The writing of the book had, however, convinced him of the importance of extending the scope of his inquiries by the study of analogous phenomena on the Continent. The great collection of relics from the lake-dwellings and terramara settlements of the Po Valley, preserved in the Museo Preistorico at Rome, was systematically examined as soon as his health was sufficiently recovered, Mrs Munro lending invaluable assistance through her deftness in sketching. Then followed a series of visits to public and private collections elsewhere in Italy, as well as to every locality in which lake-dwellings or relics of their inhabitants were to be found. On returning to Scotland in the summer of 1886, he received an invitation to deliver the Rhind Lectures for 1888, the subject suggested being "The Lake-dwellings of Europe." These lectures were issued in book form in 1890, and with their appearance his reputation as an archæologist was made. The best testimony to their enduring quality is that they were translated into French eighteen years after they were originally issued. They have definitely taken their place as the standard work on the subject. The mass of material passed in review is so extensive that any serious modification of the conclusions reached is not likely to be called for.

About 1890 Dr and Mrs Munro had settled in Edinburgh, where their house in Manor Place speedily became a centre of hospitality for antiquaries on the one hand, and men of science on the other. Munro had hosts of friends in both camps, and he liked to stimulate young men of promise by introducing them to the notice of those who had already achieved distinction. He had been elected a Fellow of the Society of Antiquaries of Scotland in 1879. In 1888 he was appointed Honorary Secretary, a post which he continued to hold for eleven years. In 1891 he joined the Fellowship of the Royal Society, where he was speedily at home in most congenial company.

Honorary and Corresponding Memberships flowed in on him from various learned bodies in other countries. He delighted to attend archæological and scientific congresses, largely because it gave him a colourable excuse for the travel which he so thoroughly enjoyed. His experiences in Bosnia, Herzegovina, and Dalmatia are recorded in a volume which has gone through more than one edition. But his most comprehensive tour was undertaken in 1897, when he and his wife went to Toronto to attend the British Association meeting, and made the return journey by Japan, China, India, and the Mediterranean.

In 1892 he played a prominent part in rousing public interest in the newly discovered lake-village at Glastonbury. Next year he was President of the Anthropological Section of the British Association. By this time he had pushed his researches back from the lake-dwellers to the makers of palæolithic implements, and he chose for the subject of his Presidential address "The relation between the Erect Posture and the Physical and Intellectual Development of Man," maintaining the view that "man's mental superiority over all other animals was primarily due to his attainment of the erect attitude which, by entirely eliminating the fore-limbs from participating in the function of locomotion, enabled him to utilise these limbs exclusively for prehensile and mechanical purposes." The theory attracted widespread attention, and the address, which was afterwards published, was always regarded by its author as one of his most important contributions to anthropology. Such criticism as it received, he welcomed. Nothing pleased him better than intelligent discussion. Even controversy had a certain attraction for him: witness the zest with which he used to recall the main incidents of the dispute about the great "Clyde Mystery" long after time had justified the attitude he himself had so consistently adopted. So, too, he thoroughly enjoyed being summoned to give evidence before Lord-Justice Farwell in a lawsuit over certain Irish gold ornaments, when the point regarding which he had to testify was the date of the last upheaval of the land that formed the raised beaches along the shores of the North of Ireland and Scotland. This was in 1903.

The same year was marked by an incident that indicated an impending change in his way of life. He purchased a house at Largs. He was now sixty-eight, and he was beginning to feel that the bustle of foreign travel was something of a strain. He hoped to find in the quieter pursuits of a country environment a more restful form of the variety that he loved. At first his new home was a summer residence only. But he gradually became more and more attached to his garden at Elmbank. The death of his wife in 1907 was a very heavy blow. Thereafter Edinburgh saw him only at rare intervals. As the list of his contributions to learned periodicals shows, he continued to work strenuously at his subject, seeking in this way to gain relief, first from the grievous personal loss that had befallen him, and afterwards from a painful neuritic affection which laid hold of him in 1909, and slowly but inexorably tightened its grasp until the end.

The evening of his life was brightened by an interest that sprang directly from his own liberality. In 1910 he handed over to the University Court of the University of Edinburgh a substantial capital sum for the endowment of a permanent lectureship in anthropology and prehistoric archæology. By a happy inspiration the Court invited the donor himself to be the first lecturer under the new foundation, and the vigour and freshness of the inaugural course which he delivered in 1912 are still vividly remembered by many. During the next year or two he watched with all a parent's solicitude the development of the experiment he had initiated. It was a matter of peculiar satisfaction to him that his friend Professor Geikie should have been appointed his immediate successor. Similarly, he journeyed to Edinburgh in the early months of 1914 to welcome and entertain Mr D. G. Hogarth, the third Munro Lecturer. Then came the war, an incidental result of which was to postpone for six years the series which the Abbé Breuil had promised to deliver. The postponement was a great disappointment to Munro, who had been looking forward keenly to the visit of the distinguished French scholar, of whose work he had a high appreciation.

And, when the Abbé did come to Scotland in 1921, the founder of the Lectureship was no longer alive to receive him. As early as 1916 his strength had been so seriously undermined that he took the gloomiest view of the future. But, despite much suffering, his splendid constitution and his determined will enabled him to hold out for four years more, and even to write, to lecture, and to publish in the interval. He died on 18th July 1920. when he was within three days of attaining the age of eighty-five. The last piece of work to which he set his hand was a short sketch of his own life, which was composed for the information of his closest friends, and which has since been printed for private circulation. From it not a little of the material for the foregoing notice has been drawn. It is a plain record of a strenuous and useful career, of real distinction achieved through native ability and steadfast concentration of purpose. Those who knew Munro can readily fill in the outline for themselves and colour it by their recollection of his frank sincerity, his genuine kindliness, his love of all good fellowship.

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1921. *Robert Munro, M.A., M.D., LL.D.* Autobiographic Sketch. Glasgow: MacLehose, Jackson & Co.

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"Notice of the Discovery of Five Bronze Celts and a Bronze Ring at the 'Maidens,' near Culzean Castle," *ibid.*, vol. iv, pp. 1–8.

1885. "The Lake-dwellings of Wigtonshire," *ibid.*, vol. v, pp. 74–124.

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1886. "Notes on Lake-dwellings in Lough Mourne, Co. Antrim, Ireland," *Proc. Soc. Antiq. Scot.*, vol. xx, pp. 321–330. (An ironsocketed Celt with a loop was found here, and is figured on p. 330.)

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John George Bartholomew, LL.D. (Edin.), F.R.G.S., Geographer and Cartographer to the King. By Geo. G. Chisholm, M.A., B.Sc., Reader in Geography, Edinburgh University, Secretary to the Royal Scottish Geographical Society. *Communicated by* The General Secretary.

(MS. received October 19, 1921. Read November 7, 1921.)

It was in the latter part of 1883 or early in 1884 that I became acquainted with the subject of this notice. At that time I was settled in London, and on the occasion of a short visit to Edinburgh I called on the late Professor Geikie, who said to me, "There's a man I want you to know, who has got his head screwed on the right way on the subject of maps." He named Mr Bartholomew, and recommended me to call on him, which I at once did. I found him at his office in Chambers Street, engaged on the actual work of map-drawing, and he straightway proceeded to give me his ideas on this subject and to indicate the methods which he wished to see displaced. Twenty-five years or so passed, during which, owing to the distance between our abodes, our meetings were infrequent; still there was, I believe, scarcely a visit of either of us to either end without our meeting somewhere—mostly at the London end, where the increasing business and reputation of the firm with which Mr Bartholomew was connected frequently brought him. Naturally, our meetings were more frequent when Edinburgh once more became my home in 1908, and still more so after my appointment to the secretaryship of the Royal Scottish Geographical Society.

Meantime the remark which Professor Geikie had made in first speaking of him to me had been amply verified. At that time Mr Bartholomew was a young man, under twenty-four years of age. He was born at Edinburgh on the 22nd of March 1860. Yet he had already for several years taken an active share in the work of the cartographical establishment then belonging to his father. From 1888, when accordingly he was only twenty-eight, he had the entire management of the business. In 1889 he married; and in that year, too, the business was transferred from Chambers Street to Park Road and became known as the Edinburgh Geographical Institute—a name retained at the new premises in Duncan Street, to which the business was removed in 1911.

Dr Bartholomew's management of the business was signalised from an early date by the inception of a number of enterprises of great boldness, and those which were carried out raised the reputation of the firm to a high pitch. First came *The Survey Atlas of Scotland*, in 1895; but this, it should be mentioned, was mainly the uniting in one whole of sectional sheets on the scale of half an inch to the mile, which had been appearing for several years and formed the first topographical maps in which the method of representing the inequalities of the surface by layering, or the distinguishing of areas between successive contour lines by different colours and tints, was applied on a large scale. It had previously been made use of at Mr J. G. Bartholomew's suggestion, at least as early as 1880 in maps prepared for Baddeley's *Guide to the English Lake District*. The method has since been adopted on topographical maps prepared by many other geographical establishments, including the Ordnance Survey Department at Southampton, but by none with greater taste and effectiveness than by the firm which first so used it. *The Survey Atlas of England and Wales* followed in 1903. Both atlases have, besides the large-scale sheets, more comprehensive maps on a smaller scale, showing the geology and climatic and other features of the geography of the countries represented. In both, the maps by Bosse showing the density of population are particularly noteworthy. For Scotland this map was brought up to date in maps prepared by Mr Bartholomew for publication in *The Scottish Geographical Magazine*, in accordance with the censuses of 1901 and 1911, the latter included also in the 1912 edition of the *Atlas of Scotland*. The three together form an interesting conspectus of census results, although of course they cannot but exhibit the inevitable defects of all density of population maps arising from the necessary arbitrariness in the choice of the limits of density distinguished by different colours or shades, and the mode in which town populations are allowed to influence the density tint of the areas to which they belong.

Before the issue of the second of the two atlases mentioned there appeared, in 1899, the first volume to be issued of the grandest enterprise of the Institute—a physical atlas designed on a scale of hitherto unparalleled magnitude. The prospectus of the whole work was given to the public along with the *Atlas of Meteorology*, which was the first published of the seven volumes of which the whole work was designed to consist, and of which this volume was to form the third.

The whole work was then planned in all its essential details. The first volume, besides containing a general introduction dealing with the Extent of Land and Sea Surveys, was to be devoted to Geology; the second to Orography, Hydrography, and Oceanography. The third, as already stated, is an atlas of Meteorology. The fourth was to be devoted to Botany, the fifth to Zoology, the sixth to Ethnography and Demography, and the seventh to General Cosmography and Terrestrial Magnetism. It was to include in all 212 plates, the titles of which are given in the prospectus. The prospectus states that the other sections will follow that on Meteorology in rapid succession, and, if the fact that this anticipation proved too sanguine will surprise no one who has had anything to do with the preparation of comprehensive works even on a much smaller scale than this, it may be taken as a typical illustration of the patient tenacity that characterised Dr Bartholomew in all his work that a second volume of the series, the *Atlas of Zoogeography*, was at last published in 1911, as well as that many other plates belonging to other sections not yet published were prepared under Dr Bartholomew's direction.

It will serve to give some idea of the magnitude of the whole undertaking to compare the two sections of the atlas which have been published with the corresponding sections of the atlas of Physical Geography that had the first place at the time when that of the Edinburgh Geographical Institute began to be published, Berghaus *Physikalischer Atlas*. To begin with, the size of the plates in the Edinburgh atlas is considerably larger than those of Berghaus—measured from the outer limit of the border (exclusive of margin) $19\frac{1}{2}'' \times 15\frac{3}{4}''$, as against $16'' \times 13''$. The section on Meteorology in the Edinburgh atlas has 34 plates (35, including the frontispiece plate showing the distribution of meteorological stations in the world at the time of publication) as against 12 in Berghaus, and an introductory text of 40 pages, besides an appendix of 12 pages (4 giving a list of meteorological stations, 4 a bibliography, 2 a glossary, and 2 tables), as against 10 in Berghaus; that on Zoogeography has 36 plates as against 9 in Berghaus, together with an introductory text of 56 pages, exclusive of a bibliography of 11 pages, as against a text of 8 pages in Berghaus.

All those primarily responsible for the *Atlas of Meteorology* are now dead. It was prepared by Dr Bartholomew himself in association with the late Professor Herbertson, under the editorship of the late Alexander Buchan, LL.D., F.R.S. Among its new features may be mentioned several maps illustrating isanomalies of temperature, maps showing isonephs, or lines marking the limits of equal degrees of cloudiness, and isohels, or similar lines marking the limits of equal extent of sunshine, and maps showing the paths of barometric minima.

It may be mentioned as another characteristic fact that when the *Atlas of Zoogeography* did appear it contained even more than was promised in the prospectus—36 instead of 35 plates. In this case the long interval that elapsed between the drawing up of the prospectus and the appearance of the volume resulted in a great change in the selection and arrangement of the plates. The scheme as originally prepared was that of the late Philip Lutley Sclater, but the zoologists under whose care the volume was actually prepared were W. Eagle Clarke, F.R.S.E., F.L.S., Keeper, and Percy H. Grimshaw, F.R.S.E., F.E.S., Assistant Keeper of the Natural History Department, the Royal Scottish Museum; and the classification adopted naturally answered to the state of zoological science at a later date than that of the prospectus.

Though the other volumes of the atlas have not yet appeared, it may be taken for granted that some of the work done with a view to their publication has been utilised in other works. Thus the volume on Ethnography and Demography was designed to include plates illustrating the Production of Edible and Drinkable Commodities, International Commerce at the End of the Nineteenth Century, and others on the same subjects as some of those in the folio *Atlas of the World's Commerce* (176 plates), published by Newnes early in the present century. Dr Bartholomew was also responsible for the preparation of the atlas accompanying the *Imperial Gazetteer of India* (1908). At the time of his death he had supervised the preparation of nearly all the plates for the important political atlas recently completed and published under the title of *The "Times" Survey Atlas of the World.*

Inevitably Dr Bartholomew's zeal for geography was manifested in many ways apart from the work carried out in the Geographical Institute. Most conspicuously was this the case in connection with the Royal Scottish Geographical Society. He was one of the most active and enthusiastic of those who encountered and vanquished all the difficulties that had to be overcome in getting it founded in 1884. From the beginning till the time of his death he acted as one of its honorary secretaries. He was the contributor both of maps and articles to its magazine—the articles on "The Mapping of the World," in vols. vi and vii. He took a special interest in the preparation of the Edinburgh number issued in 1919, and for it he presented to the Society the interesting "Chronological Map of Edinburgh showing Expansion of the City from the Earliest Times to the Present" (a "present," however, previous to the last extension of the city boundaries).

He bequeathed to the Society the sum of £500.

He took great interest in the establishment of the lectureship in Geography in Edinburgh University, and was a generous benefactor to the department when the lectureship was founded and equipment required.

From 1909 to 1912 he was a member of Council of this Society.

Only those who knew Dr Bartholomew personally could be aware of the extraordinary difficulties under which the above-enumerated series of persevering labours were carried on, and the extraordinary resolution revealed in carrying them through, and only those who knew him in his earlier years could realise the whole nature of the man. For a great part of his life, and, above all, in his later years, he had to contend against constant weak and too frequently ill health. Sometimes he was absolutely laid aside, but, except on those occasions, he went on steadily and calmly with his work to the limit of his strength, and never lost his interest in those things which he had at heart. Again and again, before Council meetings of the Geographical Society, I had interviews with him in bed, and the advice that he had to give on those occasions was always eagerly looked for by the other members of Council.

This constant fight with ill-health naturally gave to him in his later years a somewhat melancholy expression; but it was always a calm, grave, and dignified melancholy untouched by any hint of complaint. It was, however, an expression that made it difficult to realise the buoyant and exuberant energy which characterised him when young, and brought out other sides of his character. I remember particularly one occasion in the early days of our acquaintance when seated on a brake in the island of Jersey I was hailed by him from another brake which was going on the same tour. The two brakes stopped at the same place for lunch, and Mr Bartholomew, as he then was, entered with sympathetic zest into the enjoyments of the youngest and most frivolous. Then it was quite easy to picture to oneself the energy which he had shortly before shown at the foundation of the Geographical Society.

His later years were further saddened for him, as for others, by the War, but in connection with it also his character was revealed. He took the War as a call to national and personal duty, but—though he lost a son in the War and had another maimed—without any admixture of national or personal hatred, but always regarding it as a great human tragedy. It may be mentioned here that he was for many years an elder in the United Free Church of St George's, Edinburgh.

In the later years of his life he frequently had to leave his home in search of improved health. It was on one of those occasions that he met his end. Early in 1920 he went to Esterel in Portugal, accompanied by his wife and daughters. Having been taken up to Cintra in the hope that the hill air would benefit him, he died there on the 13th of April in the same year, and there he is buried. He left a widow, two sons, and two daughters, the elder of the two sons now the managing director of the firm styled Messrs John Bartholomew & Son, Limited.

Both at home and abroad the value of Dr Bartholomew's services to science were recognised in various ways. He was an honorary member of many foreign geographical societies, including those of Paris, Portugal, Budapest, and Chicago. In 1905 the Royal Geographical Society awarded to him the Victoria Medal "for his successful effort to raise the standard of cartography." In 1918 the Geographical Society of Chicago conferred on him the Helen Culver Gold Medal. In 1909 Edinburgh University, his *Alma Mater*, bestowed on him the honorary degree of LL.D.

In spite of the drawback of ill-health the private life of Dr Bartholomew was singularly, though quietly, happy, a natural result of the qualities in him which inspired confidence and affection among all those who came into intimate contact with him. This notice may be concluded by testimony on this head borne by a Russian admirer, General Jules de Schokalsky, President of the Russian Geographical Society, in a communication to this Society, dated Petrograd, October 1920, just after he had heard the news of Dr Bartholomew's death. After speaking in the highest terms of the value of Dr Bartholomew's cartographical work, taking as an illustration the remarkable precision even of his "ordinary" work in the map on Lambert's equivalent area projection accompanying the paper by Dr (afterwards Sir John) Murray "On the Height of the Land and the Depth of the Ocean" in *The Scottish Geographical Magazine*, January 1888—a precision such as to enable General A. Tillo to obtain valuable results working from a much reduced copy of it,—the writer goes on to say:—

"My personal acquaintance with J. G. Bartholomew began by correspondence. Being interested in geographical and cartographical matters, I was introduced to him by Sir J. Murray, and we remained a long time only in correspondence. At the opportunity of the Geographical Congress at Geneva in 1908 I paid a visit to Edinburgh, and was for a fortnight the guest of Mr and Mrs J. G. Bartholomew; and later we met at Geneva, staying in the same hotel and working side by side on the Congress business, and became true friends. In 1912 I came on a second visit to Edinburgh, and stayed about ten days at the J. G. Bartholomew's home.

"These opportunities of meeting and talking with J. G. Bartholomew and observing his system of working, his relation to his aids in the Institute and surrounding scientists and other people, revealed his true character as a man. . . . He was the personified truth itself, and at the same time with such unselfishness and goodness as charmed anyone who approached him.

"Geographical science lost in him one of its best workers, his nearest and his friends true support in their hard moments of life.

"Coming myself not from a cold-blooded origin, I have no shame when in writing this my eyes are full of tears, and his country can remember that there rarely lived a greater *gentleman*."

John Aitken, LL.D., F.R.S. By C. G. Knott, D.Sc., LL.D., F.R.S.

(Read January 10, 1921.)

John Aitken, born at Falkirk on September 18, 1839, was the fourth son of Henry Aitken of Darroch, Falkirk, head of a well-known legal firm in that town. He was educated at the Falkirk Grammar School and the University of Glasgow, where he studied with a view to a career as an engineer. Two years of his apprenticeship he served in Dundee, and three years with Messrs Napier & Sons, shipbuilders, Glasgow. After finishing his apprenticeship as a marine engineer he broke down in health, and was compelled to abandon all thought of carrying out his profession. Thenceforward his interest lay in the line of scientific and especially physical research, for which he received a great inspiration while attending Lord Kelvin's (then Sir William Thomson's) classes in natural philosophy.

His early training as an engineer was of incalculable value all through the long series of physical investigations which made his name famous in the ranks of experimenters. Most of the apparatus used in his researches was not only devised by him but constructed with his own hand. The drawing-room of the house he occupied latterly in Falkirk was transformed into a laboratory and workshop, with a fine turning-lathe placed in front of the window and supplied with all kinds of tools of the most approved pattern. A carpenter's bench and work-tables laden with glass-work, blow-pipes, and many odds and ends of apparatus in the course of construction or of apparatus which had served its purpose, covered the floor space, while cabinets along the walls contained drawers full of thermometers and other delicate meteorological instruments.

The earliest line of work which brought out his experimental skill was a discussion of colour sensations in a paper read before the Royal Scottish Society of Arts in 1872. He devised new methods of experimenting, and elaborated a modified form of Young's three-colour theory of sensation, supporting it by means of many ingenious experiments. Another early line of thought led him to discuss the conditions of boiling of liquids and condensation of vapours, which he showed to depend on the presence of free surfaces separating different states; and it was by following up some of the ideas suggested by this work that he hit upon what will probably be regarded as his greatest contribution to physical science. This was the demonstration that water vapour in the atmosphere will not condense to form clouds unless it has some solid or liquid nucleus to condense upon.* Dr Aitken worked out this whole research with unswerving zeal, clearing away by a magnificent series of control experiments many objections which seemed at first sight difficult to meet and even inconsistent with the broad theory. He brought into prominence the vast importance of the dust in the atmosphere, not only visible dust but the impalpable dust particles which provide nuclei for the condensation of vapour and the formation of visible drops of rain or mist. By an interesting process of evolution he gradually constructed a form of apparatus by which, from the number of raindrops produced in a closed region of saturated air, he was able to calculate the number of dust particles in this region. A slight expansion by means of an air-pump in connection with the closed region produced a cooling in the saturated air, from which the vapour condensed on the dust particles and formed tiny drops of water. These, falling on a silvered surface ruled in small squares, were readily counted. This was the so-called Dust-counter, the final portable form of which was an instrument of considerable precision in the hands of the skilful meteorologist.

The production of a fog cloud in a receiver from which saturated air was being extracted was a phenomenon which had often been seen by experimenters; but it was reserved for John Aitken not only to give a complete explanation of the phenomenon but to open up an entirely new line of research.

Aitken's experiments proved that when the saturated air was free of dust no cloudy condensation took place on slight expansion, for there were no particles to serve as nuclei. He found, however, that once the air was cleared of dust by filtration through cotton-wool, a more rapid expansion sometimes led to cloudy condensation. The explanation of this was subsequently given by C. T. R. Wilson, who showed that ionised air, although dust free, produced cloudy condensation when a considerable expansion with accompanying cooling took place. There has consequently been a tendency in some quarters to explain condensation of vapour in terms of the presence of ions, arguing that Aitken's dust particles were unnecessary as a factor in the process. But such a view shows an absolute lack of appreciation of the whole meaning of the phenomenon. The sudden expansion and cooling required to produce cloudy condensation on ions are much greater than can ever occur in nature. On the other hand, when dust particles are present a very slight expansion with accompanying slight

* See "On Dust, Fogs, and Clouds," *Trans. Roy. Soc. Edin.*, xxx, 1880–1; and various papers on dust particles in the air, *Trans. Roy. Soc. Edin.*, vols. xxxv to xxxix, 1887–1899; and many papers in the *Proceedings*.

cooling suffices. An experiment often made is to hold a bunsen flame for a moment within a receiver, set the receiver immediately on the air-pump plate with a dish of water within it, and then pump some of the air out. A dense fog cloud is formed, and this is not unfrequently referred simply to the ionisation due to the flame. But the argument is faulty, for of course there are numerous dust particles also produced by the flame, and it is impossible in such an experiment to discriminate between the effect of the particles as fog producers and the effect of the ions. Moreover, Aitken himself proved that when dust particles were undoubtedly present electrification of the air did not increase the cloudy condensation.

When we recognise that dust particles are always present in the atmosphere, and that a slight cooling of the saturated air is the cause of the production of raindrops, and when we further bear in mind the beautiful demonstration given by Aitken that no cloudy condensation is produced in saturated dustless air on slight cooling, there is no escape from the conclusion that mist, fog, and cloud require for their formation the presence of dust particles.

Another important direct result of Aitken's experiments on cloudy condensation, and especially of his methods of counting the raindrops formed, is worthy of mention. Sir J. J. Thomson in his classical experiments on the mass and charge of an electron made use of Aitken's method of condensation in obtaining one of the measurements on which the determination of these two small quantities depended.

Meanwhile, Aitken himself pushed his own investigations in many directions, such as the meteorological and industrial conditions governing the production of dust particles in the air, the influence of locality and altitude, the effect of prevalent winds and of cyclonic and anticyclonic distributions.

Closely connected with this whole research is his important paper on the formation of dew.* His views, though now generally accepted, were strongly combated by certain authorities at the time of their first promulgation. What he showed by skilfully arranged experiments was that the vapour which condenses as dew on cold surfaces comes mainly, if not entirely, from the ground below and not from the air above. He also showed that the so-called dewdrop on leaves of plants was not dew at all, but was exuded sap. He has also placed on record some interesting observations on hoar frost; and in a paper published in the *Journal of the Scottish Meteorological Society* he has given a remarkably clear description of the formation of ground ice.

In his presentation of papers before our Society, in whose *Transactions*

* See "On Dew," *Trans. Roy. Soc. Edin.*, xxxiii, 1885.

and *Proceedings* his most important work is published, Dr Aitken spared no pains in bringing before his audience the very experiments he had devised in following out his ideas. Thus he imitated on a large experimental scale the production of cyclones and the manner of their trend over the earth's surface.* Whether, in view of the new information we have in regard to the vertical distribution of temperature in cyclonic and anti-cyclonic distributions, Aitken's own views as to the genesis and maintenance of cyclones will continue to meet with acceptance, it is perhaps too soon to give a judgment. He himself believed that apparent discrepancies could be explained, and his latest paper on this subject, published in the *Proceedings* of the Royal Society of London, discusses many of the physical relations in an interesting and profound way. In this kind of work, however, he was handicapped from lack of mathematical equipment.

With a mind keenly alive to all problems of a meteorological character, John Aitken entered in 1884 upon a long series of experiments on the measurement of air temperatures. In the majority of our meteorological stations the thermometers are placed within what is known as the Stevenson screen. This form of screen was long ago found to be quite unsuitable for hot climates, and in India the thermometers are placed under a broad shed through which the air courses freely. Aitken soon satisfied himself that in this country also the temperature given by thermometers hung within the Stevenson screen read several degrees too high when the day was fine and sunny. After many experiments on various forms of screen, he finally devised a form free from the defects of the Stevenson screen, and incidentally made many other interesting and important observations on temperatures of air and soil and solar radiation. At his death on November 14, 1919, he left in manuscript what might be called his matured views after thirty years of experimenting, wherein he lamented that meteorologists still continued to use a demonstrably inefficient method of screening the thermometer from the effects of radiation, direct and indirect. This paper has been published in the *Proceedings* of the Royal Society of Edinburgh, and it may well be regarded in the light of a scientific legacy from a great natural philosopher.

The bulk of his estate Dr Aitken left in the hands of trustees to use (1) for the benefit of the poor of Falkirk; (2) to establish a temperance public-house in Falkirk. He also left a fund of £1000 to the Council of the Royal Society of Edinburgh to meet the cost of publication of a collected edition of his more important papers. This is now being prepared.

* See "Notes on the Dynamics of Cyclones and Anticyclones," *Trans. Roy. Soc. Edin.*, xl, 1900; also *Proc. Roy. Soc. Edin.*, xxxvi, 1916.

I have touched only on the outstanding problems which Aitken tackled and solved; but his many papers on physical subjects show that he possessed in a singular degree the power of clear thinking and a real intuition for the devising of illustrative experiments. His one aim was to get at the truth. He was severely critical of his own experiments, making sure at every step that all precautions had been taken against possible flaws of adjustment or fallacies of reasoning. He was never satisfied until a clear issue was established. Thus for many years he was unable to find a convincing explanation of the great increase in the number of dust particles at certain times of day at Kingairloch, a small holiday resort on Loch Linnhe, to the south of Fort William. Time and again he returned to the inquiry; and at length he was rewarded by the discovery that the source of the particles was the foreshore under action of the rays of the sun at certain conditions of tide.*

After the publication of his great paper on Fogs and Clouds, Aitken was recognised as one of the original experimenters of his day. In due course he received honours and medals from various scientific societies, including the Keith and Gunning Prizes from the Royal Society of Edinburgh, and a Royal Medal from the Royal Society of London. In 1899 he received the degree of Doctor of Laws from the University of Glasgow.

Much though his friends desired it of him, Aitken would never accept office as president or vice-president in the societies to which he brought credit and renown. To the end he remained the same quiet, modest investigator, keenly interested in all true scientific progress, and never accepting any theory which seemed to him insufficiently supported by physical reasoning. Every problem which presented itself was studied in his own way and by his own methods, and to him in a peculiar sense we might well apply the Horatian line from which the Royal Society of London has taken its motto—

"Nullius addictus jurare in verba magistri."

Handicapped through his long life by ill-health, but blessed with a competency which made him independent, John Aitken was fortunate in being able to cultivate his inborn powers undistracted by official duties. Still more fortunate the country which could claim him as one of its distinguished sons.

* See "The Sun as a Fog Producer," *Proc. Roy. Soc. Edin.*, xxxii, 1912.

Yves Delage. By Professor J. H. Ashworth, F.R.S.

(Read January 9, 1922.)

The loss of Professor Yves Delage is deeply felt by a wide circle of friends and fellow-workers by whom he was held in the highest esteem for his outstanding merits as an investigator and for his unaffected simplicity. He was a pupil of Lacaze-Duthiers, and obtained his doctorate in 1881 for his "Contribution à l'étude de l'appareil circulatoire des crustacés édriophthalmes marins." His next important work was his famous memoir (1884) on Sacculina, in which he described the structure and made known for the first time the complete life-history of this remarkable parasitic crustacean and its extraordinary relations to its host (crabs). Succeeding memoirs dealt with Balænoptera, the histology of Convoluta, the functions of the semicircular canals—Delage was not only a first-rate morphologist, but a keen physiologist,—with the otocysts of invertebrates as organs of orientation, and with the development of siliceous and fibrous sponges.

In 1895 appeared his stimulating book on *L'hérédité et les grands problèmes de Biologie générale*, an impressive example of the great extent of his knowledge and the clearness of his outlook. In this volume, Delage discussed in masterly fashion the structure of protoplasm, development, heredity, the evolution of species, and the principal theories relative to these subjects. In his views on evolution he was definitely Lamarckian, and on many points opposed to Weismannism. By this time he had turned his attention to the factors—external and internal—which determine the development of the egg, and he propounded a theory to account for the activation of the egg which led him to the experiments on artificial parthenogenesis with which his name (and that of Jacques Loeb) will ever be associated. He was able, by treating eggs of sea-urchins in hypertonic sea water alternately with tannin and ammonia, to activate them, and he succeeded in rearing some of the resulting larvæ through their metamorphosis. His researches in this domain extended over some ten years. His last published work (1920) is an important volume on dreams, in which he discussed the principal theories, psychological and physiological. A list of his publications shows not only his wonderful output of first-rate work—for his industry was extraordinary—but also his many-sided intellect.

Delage rendered notable service to biology by founding in 1895

L'Année biologique, some twenty volumes of which have appeared giving critical abstracts of memoirs on general biology. Reference should also be made to the *Traité de Zoologie concrète*, written in collaboration with Professor Hérouard, the five volumes of which are marked by great lucidity of exposition.

No account of his services to biological science would be complete which omitted to record an appreciation of his work for the marine biological station at Roscoff, first as assistant-director under Lacaze-Duthiers, and, since 1900, as director. In 1908–9 he reconstructed, extended, and re-organised the station, and for a generation has been an inspiration to many grateful workers there.

He succeeded Lacaze-Duthiers twenty years ago in the chair of Zoology at the Sorbonne, and was in active work there practically to the end of his life. For several of the last years an affection of the eyes made him almost blind, so that he was unable to carry on himself researches requiring delicate technique; but his intellectual vision remained as keen as ever, and younger workers continued to receive freely of his encyclopædic knowledge and his warm encouragement. He was elected an Honorary Fellow of the Society on 21st June 1920, and died on 7th October 1920, aged sixty-six.

Edward William Prevost, Ph.D., F.I.C. By Dr Henry Barnes, O.B.E., M.D.

(Read January 9, 1922.)

By the death of Dr Prevost, which took place at his residence, The Sandiway, Ross-on-Wye, on 7th October 1920, the Society loses one of its oldest Fellows and one keenly interested in its welfare.

Dr Prevost was born at Carlisle in 1851, being the elder son of Colonel T. W. Prevost, at that time Staff Officer of Pensions in the district. His early education was under private tutors and at Glenalmond, which school he left in 1864 for Rugby, where he remained till 1867. After two years on the science side at Edinburgh University he proceeded to Leipzig and Heidelberg, where he studied chemistry under Bunsen and graduated Ph.D. Returning to England, he worked for two and a half years at the Royal College of Science and Art, South Kensington, undertaking chemical research for Norman Lockyer. For several years he was a tutor in chemistry at Oxford, where he was employed under Odling as teacher of quantitative chemistry. He carried out in Oxford an investigation into the nutritive value of the turnip grown in (*a*) unmanured, (*b*) manured, soil. From May 1879 to the spring of 1881 he was Professor of Chemistry at the Royal Agricultural College at Cirencester. On leaving in 1881 he retired into private life and took up farming, first at Elsmere, near Tamworth, and afterwards at Elton, Newnham, where in 1890 the first part of the Cumberland Dialect Glossary was written. In 1882 he received the Gold Medal of the Royal Highland Agricultural Society for a paper on the cultivation of potatoes. In 1878 he obtained the Fellowship of the Institute of Chemistry of Great Britain and Ireland. While living at Newnham he joined the Gloucester Garrison Volunteers, afterwards attaining the rank of major. He became a Fellow of the Society in 1875, one of the proposers being Professor Piazzi Smyth, then Astronomer Royal for Scotland, and a paper by him on "An Ammonium-Cupric Zinc Chloride" appears in our *Proceedings*, vol. ix, 5th February 1877. It is the analysis of the substance which grows on the brass binding screws and carbon of a Leclanché cell.

It is, however, by his Glossary of the dialect of Cumberland that Dr Prevost will be best remembered. He began his study of this subject nearly thirty years ago, taking for groundwork the Glossary formed by the late Mr William Dickinson of Workington, published by the English Dialect Society (1878–81). The first results of his labours, in which he received assistance from many correspondents throughout the country, was the publication in 1899 of *A Glossary of the Words and Phrases pertaining to the Dialect of Cumberland*, by W. Dickinson, F.L.S., rearranged, illustrated, and augmented by E. W. Prevost, Ph.D., F.R.S.E. The amplifications and fresh matter contained in this volume constituted practically a new work, which would in itself have made Dr Prevost's reputation as a painstaking and persevering compiler, but from then to the time of his death his labours in search of unrecorded words, elucidations of meanings, and illustrations of dialect usage were unceasing. In 1905 he published a supplementary volume which, like the earlier volume, contained a comprehensive Digest of the Phonology and Grammar of the Dialect, by Mr S. Dickson Brown, M.A. Lond., F.R.G.S. Continuing his efforts with undiminished ardour, Dr Prevost has for the last fifteen years been adding to the stock of information already garnered; and when he learned how fragile was his hold upon life, he devoted all his energies to complete his work while he still had power to do so. The application that this has involved has been continuous and close. Many pages of correspondence were devoted to discovering the exact significance of a single word, slight shades in difference of meaning being cleared up, and illustrative quotations sought for. New words kept cropping up constantly, for in the compilation of a glossary there is no finality.

Dr Prevost just lived to draw to a close the task he had set for himself of compiling a second supplement of the Glossary; and it was his last wish that, should he himself be prevented from finishing this supplement and seeing it through the press, the duty should be undertaken by Mr James Walter Brown of Carlisle, who had been in close collaboration with him upon dialect work for more than twenty years.

The cost of publishing this supplement has been assured by the Philological Society of London, and it will be issued at an early date. Dr Prevost was deeply interested in music, and was a skilful organist. He had explored and inspected the mechanism and equipments of many of our cathedral organs, and in a letter to a friend three days before his death were the words, "I am off to Gloucester to see how the reconstruction of the organ is getting on."

In December 1918 Dr Prevost was found to have an ailment which might cause his death at any moment, and from that time he was condemned to a life of physical inactivity. For the last few weeks he had been able to take more exercise, and so recently as last September he was well enough to go to Devonport to see H.M.S. *Hood*, in which his younger son is a midshipman. Soon after his return the threatened seizure occurred, and ended in his death at the age of sixty-nine years.

Sir Thomas R. Fraser. By Harry Rainy, M.A., M.B., C.M., F.R.C.P. Ed.

(Read January 23, 1922.)

By the death of Sir Thomas Richard Fraser on 4th January 1920 the Royal Society and the University of Edinburgh have lost one of their most distinguished ornaments, who, by the extent and carefulness of his research work, gained a European reputation, and left behind him an example that must serve as an inspiration to his successors in the department to which he devoted himself. He was born in Calcutta on the 5th of February 1841, and his education, both at school and at the university, was Scottish. In the Edinburgh University he had the advantage of having such men as Sir Lyon Playfair, Hughes Bennett, Sir James Simpson, and Sir Robert Christison as his teachers, whilst amongst his fellow-students and colleagues in later life he reckoned men like Professor Rutherford, Sir Thomas Grainger Stewart, Sir William Turner, Professor Crum Brown, Professor Sanders, and Lord Lister. Under the stimulus of such associates it is not surprising to find that he made his mark even whilst he was a student, and that his graduation thesis in 1862, of which the subject was "On the Characters, Actions, and Therapeutic Uses of the Ordeal Bean of Calabar" (*Physostigma venenosum*), embodied a research of the highest order, which brought him into prominence both at home and abroad, winning for him, in association with a later paper on the same subject which appeared in the *Transactions of the Royal Society*, the Barbier Prize of the Academy of Sciences of Paris, which he received in 1868. Shortly after graduation he acted as resident physician in the Royal Infirmary, and in 1869 he was appointed assistant physician to the Infirmary. In 1863, largely on the strength of his research work, he was chosen assistant to the Professor of Materia Medica, a post which he held until 1870, when he relinquished it in order to become a lecturer on materia medica and therapeutics in the Extra-Mural School of Medicine. Four years later he was appointed medical officer of health for Mid-Cheshire, a district which at that time included a population of over 123,000 persons. In those days the work which fell to be discharged by a medical officer was less sharply defined than it is at the present time, and Dr Fraser's methodical mind and organising capacity led to the development of his work in a way which can scarcely be appreciated by those who nowadays have their duties much more explicitly laid down. At this time the Public Health Act had just been placed on the Statute-Book, and the amount of organising work that fell to Dr Fraser may be more readily realised when it is observed that not only did the administrative area extend over fully one half of Cheshire, but that it was divided into twelve districts under twelve boards of guardians both rural and urban, and that the medical officer of health had several inspectors and assistant inspectors under him, whose work was entirely guided by him, and who constantly reported directly to him on all matters of importance. His time was thus from the first occupied by the public service, and he never undertook any private practice in the district. Sir Thomas Fraser himself always laid stress on the importance of this time, not only because of its opening up a new field of work to him, but also because the comparatively regular duties enabled him to indulge in a certain amount of open-air recreation, such as working in his garden and even riding to hounds on occasion. To this he attributed the fact that his health, which had been considerably strained by the arduous tasks of original research, was much improved, and the relaxation enabled him on his return to Edinburgh to undertake further research with a better prospect of being physically able to stand the strain of the work. Another factor which greatly assisted him in the discharge of all these duties was the hearty co-operation of his wife, Lady Fraser, a daughter of the Rev. R. Duncan, whom he married in 1874, without whose constant care for over forty-five years he would scarcely have been able to complete the tasks to which he devoted his life.

On many occasions Sir Thomas Fraser was accustomed to say that he thought that in all research work there is a very considerable element of chance in the success of the undertaking. But whether this is so or not, it is a striking fact that nothing that he touched failed to become interesting and valuable, and, without denying the possibility of a certain degree of good fortune in the way in which his topics developed, a great deal more is attributable to the sound judgment with which he selected the various subjects of his investigations, and the extreme conscientiousness and ability which he displayed in even the smallest details of the research.

In 1877 the death of Sir Robert Christison made a vacancy in the chair of Materia Medica, and Dr Fraser at once became an applicant. By that time his contributions in various branches of research in materia medica had become extensive, and his application contained the titles of no less than twenty-one important contributions, as well as references to numerous papers which were written for the *Edinburgh Medical Journal* and other scientific publications. Amongst the most important of these contributions, in addition to researches founded on his thesis on the Calabar bean, were a series of papers in which he was associated with Professor Crum Brown, "On the Connection between Chemical Constitution and Physiological Action," which in many respects formed the basis of much of the work which was done in the production of new drugs in the following fifty years, and which are still of outstanding importance. They were early recognised as epoch-making, and received recognition by the award of the Makdougall Brisbane Prize by the Royal Society of Edinburgh in 1868. During the same time he published papers on the "Antagonism between the Actions of Physostigma and Belladonna," thus showing how the opposing actions of their active principles could be made to fulfil the century-old desideratum of Goethe for drugs which would precisely neutralise each other's action. During the same time he also published a short investigation in the *Journal of Anatomy and Physiology* on the effects of rowing on the circulation, which emphasises the fact that he was always a keen supporter and at one time a member of the Edinburgh University Boat Club. Although latterly his health prevented his taking any practical share in the work, he never lost his interest in the club, and was always its loyal supporter in every difficulty.

It was striking to note, in the case of so young a man as Dr Fraser was when he applied for the chair, how many of the leaders in pharmacology throughout the whole Continent knew him by his work and were supporters of his candidature. No fewer than five eminent French professors, including Dujardin-Beaumetz and Paul Bert, were reckoned amongst their number, whilst eleven German and three American teachers also warmly supported his claims, giving him strongly worded testimonials. It is only necessary as an illustration to quote the last paragraph of the testimonial which he received from Schmiedeberg, who was then the illustrious professor of pharmacology in Strasbourg: "It would be superfluous for me to enter into further details concerning your successful scientific career; not only in your own department, but in still wider circles, it is sufficiently known and it is valued and esteemed by all. I have thus always been of the opinion that no one is so worthy of the honour of being Sir Robert Christison's successor as you, most honoured colleague, for you are already his intellectual successor. In the interest of that science which I also have the honour to represent, it is my sincere wish that you may be successful in obtaining this chair, so that you may be in a position to enrich, as heretofore, the sciences of materia medica and toxicology by new observations and discoveries." To those who know how careful Schmiedeberg always was in giving testimonials of this nature, such an encomium proves better than almost any other how greatly Fraser's work had impressed the minds of these Continental leaders whose opinions were worth consideration.

His accession to the chair brought increased responsibilities in connection with his duties at the Royal Infirmary, where, as a clinical professor, he now took charge of wards with the status of a full physician. But, in spite of the extra labours thus thrown upon him, we find a continuous succession of papers on subjects of the utmost scientific and clinical value rapidly succeeding one another in the following years. The subjects of some of these researches had already occupied his attention before he returned as a professor to Edinburgh. Amongst these a series of papers on Strophanthus and other digitalis bodies was specially noteworthy, and the subject was only laid aside from time to time with a view to returning to it later. Thus his paper, published in early days, on the Kombé arrow poison was followed by a long series of communications on the various varieties of Strophanthus and their selective actions, many of which appeared in the *Transactions of the Royal Society*. This series of papers was recognised by the awarding to him of the Keith Prize by the Royal Society of Edinburgh for the period 1891–93. During this time he also wrote on various other arrow poisons, on snake venom, and the anti-venomous properties of bile.

Whilst carrying on all this research, as well as the ordinary duties of lecturing systematically on materia medica, Professor Fraser developed great powers as a clinical teacher, and many of the students who knew comparatively little of his laboratory research work felt that his teaching in the wards, whilst he was still in full vigour, was an inspiration. The scrupulous care with which he established the clinical facts of each case he examined, the clear logic of his deductions, and the reasoned lines of treatment did much to impress upon his hearers the conviction that clinical medicine could almost be raised to the level of an exact science. Three qualities he possessed in notable degree: accurate observation, clearness of vision which refused to be drawn away by side issues, and an indomitable will which triumphed to the end over ill-health and bodily weakness. His intimate knowledge of materia medica found the fullest scope in combating disease in the wards under his charge, and whilst, to the mere research worker, there may have seemed to be a lack of application of his researches to clinical uses, those who were privileged to be his students were able to contradict that impression by their daily experience in the wards. As a clinical teacher he has left as a legacy to the school that tradition of high ideals and of exacting standards of duty which were characteristic of all his work.

A break occurred in his teaching in the school during 1898, when he went to India to act as president of a commission appointed to inquire into the whole question of bubonic plague, its origin, transmission, and treatment. His capacity and accuracy in detail made itself manifest at once, and a voluminous report was issued in 1901 which threw considerable light on the subject of inquiry.

In addition to his academic work, Professor Fraser filled many important positions in the scientific world, and even before he received his commission to the Edinburgh chair he was engaged, at the request of the Admiralty, as a member of the committee to inquire into and report on the causes of scurvy which broke out in Sir G. Nares' Arctic expedition. On a subsequent occasion he acted as president of the section of materia medica and pharmacology at the London meeting of the International Congress.

It was natural that a man of Sir Thomas Fraser's clear thinking and business capacity should become interested in professional matters outside the duties of his chair, and we find that he occupied many responsible positions where his powers in this direction could find scope. For twenty years, from 1880 onwards, he acted as Dean of the Medical Faculty in Edinburgh University. He was also a member of the University Court from 1904 to 1913, and in 1905 he became University representative on the General Medical Council, of which body he remained a member for ten years; and during that time, amongst other tasks, he took an active part in the 1914 issue of the British Pharmacopœia. For many years he was the valued principal medical adviser of the Standard Life Assurance Company, and in that capacity his sound judgment and keen critical instincts made him a more than usually competent guide in the difficult problems which constantly emerge in such work. He also discharged for nearly twenty-four years the duties of consulting medical adviser to the Prison Commissioners for Scotland, in which capacity his tact and judgment repeatedly proved of great service.

His distinguished career as a research worker also brought him many well-earned honours. At an early age, even before he had obtained a chair in Edinburgh, he had been elected a member of the Royal Societies of Edinburgh and London, and had been laureated by the Institute of France. At subsequent dates he received the recognition of the Turin Academy of Medicine and of the College of Physicians of Philadelphia. He was created an Honorary M.D. of Dublin, an Honorary Sc.D. of Cambridge, and received the degree of LL.D. from the Aberdeen and

Glasgow Universities. On the death of Sir William Gairdner in 1907, Sir Thomas Fraser succeeded him as Honorary Physician to His Majesty the King in Scotland, the honour of knighthood having previously been conferred on him in 1902. During the years 1900 to 1902 he filled the Presidential chair at the Royal College of Physicians of Edinburgh, and discharged its duties with great efficiency. In 1913 he was elected a member of the Athenæum Club under the special rule authorising the committee to elect nine members in each year because of their distinguished eminence in Science, Literature, or the Arts, or for public services. This distinction Sir Thomas Fraser highly appreciated.

On retiring from his Professorship in 1919 he was laureated LL.D. of the Edinburgh University, and his portrait, painted by Mr Robert Home, was presented to him by a large number of his former students, colleagues, and professional friends.

During his earlier years, Professor Fraser, though never very robust, was of a wiry constitution, and took great pleasure in many forms of outdoor exercise. But, as the years passed on, he suffered from repeated attacks of bronchitis which sapped his vitality, and rendered strenuous efforts difficult or impossible, and at the age of seventy he fractured his femur, thus rendering himself still less fit for physical exercise. For many years he spent his summer holidays at Druimbeg, a small country property which he purchased for himself on the shores of Loch Shiel in Argyllshire, where he delighted in showing hospitality to his friends, and where his garden, stocked with numerous plants of therapeutic interest and of beauty of foliage, gave him constant interest and pleasure. Whilst his strength enabled him, he also enjoyed the sports of trout-fishing, shooting, and hill-climbing, though he gradually had to abandon these amusements as his health became less satisfactory. To the end, however, his mind was as clear as ever, and when overtaken by his last illness he was busily engaged in working up the material which he had accumulated in the course of his long life of clinical and laboratory research.

In many respects Sir Thomas Fraser's position in the school was unique. He formed a link between the older ideals represented in Sir Robert Christison and the newer methods of separating the laboratory worker much more completely from the physician in charge of patients, and this change, though probably inevitable, is not without great drawbacks, for it eliminates much of the human element from the life-work of those who now guide the profession in therapeutic matters; and although this permits of a higher degree of specialisation, it certainly debars the worker from that close contact with the problems confronting the practising physician which so often in former days opened up new lines of thought and action, and led to some of the most important advances in the region of therapeutics.

In Sir Thomas Fraser the two outlooks were harmoniously combined, with the result that he was not only respected as a research worker, but loved and trusted as a physician.

T. Lindsay Galloway, M.A., F.G.S., A.M.Inst.C.E., M.Inst.M.E.
By Professor W. P. Ker, F.B.A., M.A.

(Read January 9, 1922.)

THE following lines are written not as part of a biography but in order to bring out what Lindsay Galloway's contemporaries recognised in him, his combination of scientific attainments and ability with practical good sense and very wide interests in literature and philosophy, and to pay honour to a man whose friends found him always unfailing in honour and goodwill.

Lindsay Galloway distinguished himself at Glasgow University in mathematics and natural philosophy. Sir William Thomson (not yet Lord Kelvin) regarded him as one of his most brilliant students, and entrusted him, when he was not much more than twenty, with the task of testing the new piano-wire invention for deep-sea soundings in a voyage to Brazil. Galloway was so successful that the Brazilian Government asked him to do the soundings for them along their coast from Para to Pernambuco. He might have found a career there, but he had determined to work as a mining engineer, and it was with the experience that he had acquired on the Tyne that he settled down in Kintyre at the Drumlemble coal-pit.

Lindsay Galloway had sat under both William Thomson and Edward Caird, and he followed all his life the studies of science and philosophy which he began under these masters. His work in mathematics and physics gave him a great advantage over most moral philosophers, while his philosophy made him sceptical about the more positive and dogmatic theories of Nature which were popular fifty years ago. He was also a good scholar and a lover of poetry. Indeed, very few men of his time have come nearer to the old ideal of the humanities; there was no province of knowledge in which he could not find his way. He shared with Mr Macdonald of Largie in the foundation of the Archæological Society of Kintyre, and was ready to give his time and energy to an adventure which would have been profitable in results for the history of Scotland —which yet may be successful in spite of the sudden loss of two of the leaders.

Within the last few years Galloway gave some attention to the improvement of a method of determining "the relative directions of two reference lines or bases for mining surveys" which he had imagined, and was engaged in making observations with the magnetometer or magnetic reflector he had invented for this purpose up to the very day of his death. He described the magnetic reflector and the method of employing it in conjunction with a theodolite in a paper read before the Institution of Mining Engineers in 1918 (*Trans. Inst. M.E.*, vol. lvi, p. 222), and a method of employing two magnetic reflectors when greater precision is required in a second paper read before the same institution in 1920 (vol. lx, p. 235). His other papers include the following: "On the Present Condition of Mining in some of the Principal Coal-producing Districts of the Continent," *Proc. N. of E. Inst. of Mining and Mech. Engineers*, vol. xxvii, 1878; and one on "James Watt," *Proc. Roy. Phil. Soc. Glasgow*, 1920.

He was elected a Fellow of the Royal Society of Edinburgh in 1918, and died on 22nd September 1921 at Kilchrist, near Campbeltown.

Henry Barnes, O.B.E., M.D., LL.D. Contributed by his daughter, Miss E. Barnes.

(Read January 9, 1922.)

Henry Barnes, O.B.E., M.D., LL.D., was the third son of Mr Joseph Barnes, yeoman, and was born at Aikton, Cumberland, on 20th July 1842. He was educated at St Bees Grammar School, and afterwards at Edinburgh University. He graduated M.D. with honours in 1864, and in the same year he became a member of the Royal College of Surgeons (England). He settled in Carlisle in 1866, and in 1873 he was appointed honorary physician to the Cumberland Infirmary, which position he held till 1903. He was then appointed consulting physician, in which office he remained up to the time of his death. From 1904 to 1910 he was chairman of the committee of management. In 1906 he was elected one of the vice-presidents, and in November 1920 he accepted the office of president. It will thus be seen that his labours for the Cumberland Infirmary extended over nearly half a century. Large extensions were made to the institution in 1908. A new Nurses' Home was built, a new ophthalmic and X-ray department were provided, and additional wards added. An inscription over the door of the Nurses' Home states that it was named "the Barnes Wing in recognition of the services rendered by Henry Barnes, M.D., in the enlargement of the Infirmary." It was also mainly due to his efforts that the Infirmary received a grant of £30,000 from the British Red Cross Society at the end of the war.

Dr Barnes was keenly interested in the work of the British Medical Association. It was through him that the Cumberland and Westmorland Branch was formed. Later on the area was extended and the title was changed to the Border Counties Branch, and in 1876 he was elected president. In 1896 the annual meeting of the British Medical Association was held in Carlisle under Dr Barnes' presidency. Two years later, when the annual meeting was held in Canada, the degree of LL.D. from the M'Gill University was conferred upon him. He was also a Fellow of the Royal Society of Edinburgh (1882), a Fellow of the Royal Society of Medicine, and a Fellow of the Botanical Society of Edinburgh.

In addition to his professional work for the Cumberland Infirmary, Dr Barnes was physician to the Carlisle Dispensary, the Carlisle Fever Hospital, the Border Counties Home for Incurables, and consulting physician to the Silloth Convalescent Institution. In 1907 he helped to found the Cumberland Branch of the British Red Cross Society, and acted as honorary secretary from the time it was founded to 1918. During the war his work was most strenuous, and in recognition of his services he received from the King the O.B.E.

Dr Barnes was made a Justice in 1887, a county magistrate in 1889, and in 1904 he was appointed chairman of the Cumberland Ward Justices. He was also intensely interested in antiquarian and archæological research, and papers by him have appeared in the *British Medical Journal*, the *Lancet*, the *Edinburgh Medical Journal*, and the medical journals of New York, Philadelphia, St Louis, and Toronto. He also contributed papers on the historical side of medicine to the *Transactions* of the Cumberland and Westmorland Antiquarian Society.

Dr Barnes founded and was president of the Carlisle Branch of the Medical Mission Auxiliary of the Church Missionary Society, and was deeply interested in its welfare.

Dr Barnes was able to work, as he would have wished, to the end, passing away on 12th April 1921 after only a week's illness, from heart failure following bronchitis. His kindness of heart, courtesy of manner, social disposition, and well-stored mind endeared him to all classes of the community.

The following Notices have been prepared by the Assistant Secretary, Mr George A. Stewart.

(Read January 9, 1922.)

Alford, Robert Gervase, M.Inst.C.E., son of the late Rt. Rev. Bishop Alford, died at his residence "Three Gables," Tunbridge Wells, on the 10th May 1921, aged 72 years. He was elected a Fellow of this Society in 1895.

Anderson, Sir Rowand, LL.D. (born 1834), the son of Mr James Anderson, an Edinburgh solicitor, was educated at George Watson's Old Hospital School, which occupied part of the site of the Edinburgh Royal Infirmary. He was for a time in the office of Messrs Keegan & Welsh, solicitors. His bent, however, was in another direction. From early years he had shown a fondness for drawing. He studied for a time at the Trustees' Academy, Royal Institution, under the late Mr Christie, a contemporary of Dick Lauder. Afterwards he entered the office of Mr John Lessels, one of the leading Edinburgh architects of the day; and to his young assistant was entrusted in 1857 the work of superintending the restoration of the roof of Greyfriars Church, which had been destroyed by fire. Subsequently he made a tour on the Continent, and for a time worked in the office of Cuypers, Amsterdam, then in great repute. He also served for a period in the London office of Sir Gilbert Scott, the great Gothic architect, who had a high idea of the abilities of the young Scotsman. On returning to Edinburgh he was employed in the architectural branch of the Royal Engineers; then as partner to the distinguished Scottish architect, David Bryce. In the sixties he settled down to business on his own account in an office in Dundonald Street.

From the outset Sir Rowand was looked upon as a rising man, and was kept busily employed in the designing of churches, especially Episcopal churches, of which denomination he was a member. Sir Rowand Anderson's designs were accepted for the following buildings:—Fountainbridge, Stockbridge, and Causewayside Board Schools, the stately New Medical School of the University, and the M'Ewan Hall. It was in connection with the completion of the New Medical School that at the memorable celebration of the Tercentenary of the University of Edinburgh in 1884, he received the honorary degree of LL.D. To him was also entrusted the task of completing Adams' design of the Old University by crowning it with a dome. His name is also identified in the public mind as the designer of the Scottish National Portrait Gallery and Museum of Antiquities, Queen Street. The palace reared by the late Marquis of Bute at Mountstuart may be cited as the most important example of the abilities of the architect in this branch of his art. Other architectural works of note were executed by Sir Rowand during his long professional career.

In connection with church architecture, Sir Rowand Anderson was frequently consulted regarding schemes for the restoration of ancient buildings; and it was he who carried out the restoration of Dunblane Cathedral, the Chapel of King's College, Aberdeen, Borthwick Parish Church, and Culross Abbey. He was employed by the late Marquis of Lothian to devise measures for the preservation of Jedburgh Abbey, and these were renewed previous to this ancient religious house being handed over to the Ancient Monuments Commission.

A scheme with which he was intimately associated, and in connection with which he did lasting work for his profession and for the arts and crafts of the city, was the founding of the College of Applied Art, which had its headquarters in the Royal Institution, Mound, and was under the ægis of the Board of Manufactures. In the formation of the Edinburgh College of Art Sir Rowand also took much interest. He was elected an Associate of the Royal Scottish Academy in 1876, but resigned in 1883 on a question as to the architectural status of Members of the Academy. During the Presidentship of Sir George Reid, Sir Rowand Anderson in 1896 was elected an Honorary Member of the Academy, and the feud was in that way peacefully ended. In 1902 he received the honour of knighthood. It may be mentioned that he rendered good service to this Society during the transference from the Royal Institution to our present building. He was a man of wide sympathies, and despite a certain reserve in temperament, had a large circle of friends. He was elected a Fellow of the Society in 1883, and died at his residence in Colinton on the 1st June 1921, at the age of 87.

Beck, Hon. Sir J. Henricus Meiring, Kt., M.D., J.P., M.R.C.P.E., M.L.A., son of the late C. Beck, J.P. of Worcester (Cape), was born at Worcester (Cape Province) in 1855. He was educated at Worcester Public School, South African College, University of Edinburgh, Berlin, and Vienna; was one of the delegates from the Cape of Good Hope to the South African National Convention, and was a Member of Council of the University of the Cape of Good Hope from 1888–1912. For a period he filled the position of President of the Cape Medical Council. From 1916 he was

Minister of Posts and Telegraphs, Union of South Africa, was a Senator from 1910, and Chairman of Committees of Senate.

He was elected a Fellow of the Society in 1892, and died in 1919.

Bridger, Adolphus Edward, B.Sc. (Paris), B.A., M.D. (Edin.), F.R.C.P.E., held the following posts:—Senior Physician, St Pancras Dispensary; Consulting Physician in Tuberculosis, Borough of St Pancras; Physician Superintendent, London Hospital for Women; Physician, Chest Hospital, Margaret Street; and Anæsthetist, Royal Dental and National Dental Hospitals. He was a member of the Society of Authors, and published the following:—*Digestion, Perfect and Imperfect; Depression; Biliousness; Man and his Maladies*, 1889; *The Treatment of Consumption*, 1891; *Minds in Distress*, 1913; *The Function of the Sympathetic Nervous System in Psychic Phenomena*, 1913.

Dr Bridger was elected a Fellow of the Society in 1912, and died in London on 2nd February 1920.

Brown, David, F.C.S., Manufacturing Chemist, was born in Edinburgh in 1840. He succeeded his father, Mr D. R. Brown, as senior partner of the firm of J. F. Macfarlan & Co. He was educated at the Royal High School, and studied Chemistry at the University of Edinburgh. For some time he assisted Professor George Wilson in Edinburgh, and afterwards Professor Anderson in Glasgow. At a later period he joined the staff of the Apothecaries' Hall in London, thus gaining a knowledge of applied pharmacy. Thereafter he was engaged in the manufacture of chemical substances used in medicine.

Though engrossed in the management of the business, Mr Brown was the author of several valuable papers on chemical subjects published in the *Pharmaceutical Journal* and elsewhere. Notes on Chloroform and on the location of Salicin in Willow Bark are among his published papers. Though he refrained from publishing anything on the opium alkaloids, he recognised oxynarcotine as an undescribed substance, and handed it over to Dr Wright, who made an analysis and gave the results to the Chemical Society.

The preparation of salicin from willow bark was begun in Mr Brown's time. He was a Justice of the Peace and a member of the Merchant Company in Edinburgh.

Mr Brown was elected to the Fellowship of this Society in 1893, and died on 21st June 1921 at his residence, Willowbrae House, Edinburgh. His son, Mr Rainy Brown, died a few months earlier.

Brown, David Rainy, was educated at the Edinburgh Institution, under Dr Ferguson, and at Edinburgh University. He also studied in Germany. Entering the firm of J. F. Macfarlan & Co., Manufacturing Chemists, Edinburgh, as a young man, he later succeeded his father, Mr David Brown, the subject of the immediately preceding notice. Mr Brown accomplished a considerable amount of research work on Opium and Chloroform and published papers in the *Pharmaceutical Journal.* An old member of the Queen's Edinburgh Volunteers, he served during the war in connection with Coast Defence Work.

He was elected a Fellow of the Society in 1911, and died at Edinburgh on 9th January 1921, at the age of 50.

Carter, William Allan, O.B.E., J.P., M.Inst.C.E., was a son of Mr Frederick Hayne Carter, C.A., and was born in Edinburgh in 1847. He received his early education at the Edinburgh Academy, and was afterwards trained as a Civil and Consulting Engineer. For a long period Mr Carter was a member of the Dean of Guild Court of Edinburgh, and from 1909 to 1913 he served as Lord Dean of Guild—an office which gave him a seat as a Member of the Town Council of Edinburgh. He acted also for some time as Engineer to the Convention of Royal Burghs, was Secretary to the Royal Scottish Society of Arts, and Hon. President of the East of Scotland Engineers' Association.

Mr Carter started his professional career in paper engineering, and specialised in the recovery of soda in that connection. He was engaged later in the construction of bridges, water supplies, and drainage schemes, etc., for County Councils and Burghs. Appearing often in the Court of Session as an expert witness, he was greatly respected for his broad and moderate views and courteous manner. Mr Carter frequently acted as Examiner for the degree of D.Sc. in the University of Edinburgh. He was known to a wide circle of acquaintances as a man of outstanding ability and high professional skill, a capable administrator, and also a gentleman of wide culture.

Mr Carter was elected to the Fellowship of the Society in 1898, served on the Council from 1911–1914, and represented the Society on George Heriot's Trust from 1911–1918. He died at his residence at Gullane on 7th September 1921.

Dundas, William John, W.S., LL.D., was the son of the late Lord Manor, and an elder brother of Lord Dundas. Born in 1849, he was educated at the Edinburgh Academy under Mr James Carmichael, and, on leaving school, entered the writing chambers of Messrs Dundas & Wilson, as an apprentice. He was admitted a Writer to the Signet in 1871, and a few years later became a partner. In 1913 he retired from business. On the formation of the present Government he returned for two years to the post of Crown Agent, which he had held from 1895–1905. Dr Dundas was for many years a Director of the Standard Life Assurance Company, and was also on the Board of the Edinburgh Academy. In recent years he rendered admirable service as one of the Carnegie Trustees, and in 1914 received the degree of LL.D., from the University of Edinburgh. He stood in the direct line of the best Scottish legal traditions, and his place will be difficult to fill.

Dr Dundas, who was interested in the study of the higher mathematics, was elected a Fellow of the Society in 1919, and died at Edinburgh on 9th July 1921.

Gatehouse, Tom Ernest, A.M.Inst.C.E., M.I.Mech.E., M.I.E.E., was born in 1854 at Norwich, in Norfolk. He was the son of Mr Tom Gatehouse, a mechanical engineer; naturally, therefore, he had a bent towards engineering as a profession, and about 1870 he became a pupil of Robert Sabine, one of the most eminent and able pioneers in the electrical industry. Later he was associated with Sir Charles Wheatstone and Sir Samuel Canning, foremost exponents of the art of telegraphy on land and by submarine cable. Amongst the various undertakings with which he was closely connected were the first electric lighting of Aldgate Station with Lontin arc lamps, and the development of the Werdermann arc lamp and the Gramme dynamo; in regard to the Lontin lamp he patented an improvement. He was also interested in the development of the telephone and experimented with a view to its improvement. The introduction of the incandescent lamp, however, indirectly proved to be a turning point in the career of Mr Gatehouse, who invented a device (afterwards re-invented in connection with the Nernst lamp) for the patent rights of which he received a substantial sum. About this time his fellow-pupil under Sabine, Mr H. R. Kempe (subsequently Electrician to the Post Office), had become associated with Mr H. Alabaster, proprietor of the *Telegraphic Journal and Electrical Review*, and invited Mr Gatehouse to throw in his lot with them. Mr Gatehouse assumed the office of editor in 1881. In recent years, owing to failing health, he has not taken a very active part in the production of the *Electrical Review.*

Mr Gatehouse was elected to the Fellowship of the Society in 1899, and died in London on Thursday 31st March 1921.

Gilruth, Geo. Ritchie, L.R.C.P., L.R.C.S.Edin. 1865, held the following appointments:—Surg. Lt.-Col. (V.D.) 1st. Edin. (City) V. Artillery; Asst. Demonstrator of Anatomy R.C.S. Edin.; Res. Surgeon Consett Infirmary; and was the author of *Physiological Effects of Injuries of the Spinal Cord in the Lower Animals*, and papers to the *Lancet* and *Edinburgh Medical Journal.*

He was elected a Fellow of the Society in 1880, and died at Allanton, Bridge of Allan, on 15th August 1921.

Helme, Thomas Arthur, M.D. Edin. (Gold Medal) 1889, M.B., C.M. (Hon.) 1885, M.R.C.S. Eng. 1894, M.R.C.P. Lond. 1894, was educated at Edinburgh University, University College, London, and the University of Strasburg. He was Freeland-Barbour Research Scholar R.C.P. Edin. in 1890; Leckie-Mactier Fellow 1886; Buchanan Scholar 1885, University, Edinburgh; Exhib. Univ. Lond. 1884; Hon. Phys. Northern Hosp. Women and Children, Manchester; V.P. N. of England Obstetrical Society; and President Lancs. and Ches. Branch B.M.A. He held the following appointments:—Physician, Women's Dispensary, Edin.; Res. Surg. Roy. Maternity Hosp. Edin.; Res. Obst. St Mary's Hosp. Women and Children, Manchester; and communicated a paper to the *Transactions* of this Society (vol. xxxv) on "Histological Observations on the Muscular Fibre and Connective Tissue of the Uterus during Pregnancy and the Puerperium." He was author of the *Anatomy and Physiology of the Uterus*, 1889, and published papers in the *Medical Chronicle*, 1893, and in the *British Medical Journal*, 1907.

Dr Helme was elected to the Fellowship of this Society in 1890, and died on 5th September 1921.

Hunter, James, F.R.C.S.E., F.R.A.S., was one of the oldest Fellows of the Royal College of Surgeons, Edinburgh, and was well known in medical circles. For a number of years he was Lecturer in Physiology to the School of Medicine of the Royal Colleges, and latterly he acted as Examiner in Biology and Physiology for the Royal College of Surgeons in the triple qualification. He was a keen amateur astronomer, interested in optics, and published several papers in the Scottish Microscopical Society's *Transactions* and *Proceedings*, and a joint paper with Mr E. Sang in the *Proceedings* of this Society, vol. viii, 1873, p. 126: "Observations and Experiments on the Fluid in the Cavities of Calcareous Spar."

Dr Hunter was elected a Fellow of the Society in 1887, and for many years regularly attended its meetings. He died at Edinburgh on 15th February 1921.

Martin, Sir Thomas Carlaw, LL.D., J.P., was born on 10th April 1850 near Linlithgow, of farming stock. He was trained in the country schools, until, in early manhood, he attended the Watt (now the Heriot-Watt) College and Edinburgh University, in which institution he was Gold Medallist in the class of Political Economy. After some years spent in the Post Office, he became associated with journalistic work, and eventually made it his profession. Sir Thomas was the first and only editor of *The Leader*, which continued for some years as a Gladstonian-Liberal paper in Edinburgh. As editor of the *Dundee Advertiser* for eighteen years he became a prominent figure in Scottish journalism. St Andrews University conferred on him the LL.D. degree. While essentially a student and reflective observer of public affairs, he was especially interested in agriculture and economics. In 1904 he was a member of the Scottish Agricultural Commission to Denmark, and the report of that commission contains many suggestive and readable pages from his pen. In 1908 he was Chairman of the Scottish Agricultural Commission in Canada, and in the same year received the honour of knighthood. In 1910–1911 he was Chairman of the Australian Agricultural Commission, and in 1911 was appointed Director of the Royal Scottish Museum. An able administrator, he did much to improve the working conditions there, and developed its educational possibilities, keeping in view not only the needs of the student but of the large number of casual visitors and children from the schools. For these he wrought out schemes, and with the introduction of guides and lectures brought the wealth of the Museum before a growing public. After his retiral from the Directorship of the Museum he acted as Chairman of the Transport Committee, and contributed largely to its report. At the time of his death he was Chairman of the local Employment Committee under the Ministry of Labour.

Sir Thomas was elected a Fellow of the Society in 1912, and died at Edinburgh on 26th October 1920.

Mylne, Rev. Robert Scott, M.A., B.C.L., Oriel College, Oxon., F.S.A. L. and E., Rector of Furthoe, Northants, died at his residence Great Amwell, Ware, Herts., on 23rd November 1920, in his 67th year.

He was elected to the Fellowship of the Society in 1902.

Oliphant, James, M.A., formerly Headmaster of Charlotte Square Institution, Edinburgh, died at his residence, 11 Heathfield Park, London, N.W. 2, on 19th February 1921, in his 67th year.

He was elected a Fellow of the Society in 1888.

Roberts, David Lloyd, M.D. St And. 1859, F.R.C.P. Lond. 1878, M.R.C.S. Eng., and L.S.A. 1857. Held the following appointments:—Cons. Obst. Phys., Manch. Roy. Infirmary: Phys. St Mary's Hospital, Manch.; Lect. on Clin. Midwf. and Dis. of Women, Owens Coll., Manch. He was a Fell. Med. Soc. Lond.; Mem. Lit. and Philos. Soc. Manch.; late Pres. N. of Eng. Obst. and Gynæcol. Soc.; Vice-Pres. Obst. Sect. Brit. Med. Assoc., and Pres. Manch. Med. Soc., and was the author of the following books:—*The Scientific Knowledge of Dante,* 1914; *The Practice of Midwifery*; *Cases of Ovariotomy*; *Clinical Papers*; *The Various Methods of Treating the Pedicle in Ovariotomy*; Edr. Sir Thomas Browne's *Religio Medici,* etc. He also published papers in the *Obstetrical Transactions,* and in the *Transactions* St. And. Med. Grad. Assoc.

Dr Roberts was elected to the Fellowship of this Society in 1880, and died on 20th September 1920.

Sprague, Thomas Bond, M.A. Camb., LL.D. St Andrews, F.I.A., Hon. F.F.A., was born on 29th March 1830, and was the eldest son of Thomas Sprague of London, Wholesale Stationer. He was educated at Tarvin Hall, a private school near Chester, under the Headmastership of Dr John Brindley. While at school he distinguished himself so much in mathematics that Dr Brindley advised his father to send him to Cambridge, and he accordingly entered St John's College as a Sizar. At the end of his first year he became a Proper Sizar, and in consequence of the excellent places he took in the College examinations, he became in due course a Scholar. In 1853 he was Senior Wrangler and First Smith's Prizeman, and was elected a Fellow of St John's College, and appointed one of the College Lecturers. About this time he qualified in law and was called to the Bar, having in view the law as his future profession. Progress, however, being slow, he sought a career which would be more immediately remunerative, and entered the Eagle Insurance Company in 1855 as a pupil of the late Charles Jellicoe.

After holding some minor appointments, Dr Sprague was, for a short time, Actuary of the then Liverpool and London Assurance Company, until in 1861 he was appointed, at the age of 31, Actuary and Secretary of the Equity and Law Life Assurance Society, with which Society he remained for twelve years. In 1873 he became Manager of the Scottish Equitable Life Office. This involved his removal to Edinburgh. In 1900, after being twenty-seven years Manager of the Scottish Equitable, he retired from active business life. In 1893 the University of Aberdeen conferred on him the degree of LL.D.

Dr Sprague was elected Associate of the Institute of Actuaries in 1856. His first paper is to be found in the Institute *Journal*, vol. vi, on a certain method of distributing surplus. He was elected a Fellow of the Institute in 1857, and was elected to the Council in 1863. He remained a member until his retirement from business in 1900, a period of thirty-eight years, which is a record. He edited the *Journal* of the Institute from 1867 to 1883, and the volumes published during his editorship bear witness to his skill and zeal. Elected President of the Institute in 1882, he occupied the Presidential Chair for four years. During his Presidency the Institute received its Charter. In 1874 he was elected a Fellow of the Faculty of Actuaries in Scotland, and was President from 1894–1896. No one else has occupied the Presidential Chairs of both the Institute and the Faculty. He was also President of the Actuarial Society of Edinburgh on three occasions, 1874, 1882, and 1891.

Besides being an Actuary of unusual erudition, Dr Sprague was a first-class business man. The business of life assurance, using the words "life assurance" in their broadest sense, was the business of his life, and in all his investigations he kept in view their immediate practical application, and refrained from acting on any theories he might have formed until he had put them to practical test. His writings all bear witness to this; and one would like to refer to them in some detail, but in this short notice it is impossible. The communications to the *Journal* of the Institute alone occupy nearly five and a half pages of the index. Of other actuarial publications special mention may be made of his volume on *Life Insurance Accounts* (1874); his contributions to the 9th Edition of the *Encyclopædia Britannica*, particularly an exhaustive article on "Annuities," in some parts highly mathematical, which superseded the article by Joshua Milne written many years before; and sundry contributions to the *Transactions* of the Actuarial Society of Edinburgh. The construction and use of his monumental Select Tables were fully explained in two elaborate papers to the *Journal* of the Institute of Actuaries, vol. xxi, p. 229, 1878, and vol. xxii, p. 391, 1881, and in 1896 they were published separately in book form, with extensive monetary tables at four rates of interest. He also took a great part in the preparation of what are now known as the Institute of Actuaries' Life Tables.

Dr Sprague's energies were not confined to matters actuarial. He published various mathematical papers in the *Transactions* and *Proceedings* of the Royal Society of Edinburgh and of the Edinburgh Mathematical Society; and was besides an active member of the Edinburgh Field Naturalists and Microscopical Society, to the publications of which he made several contributions. A list of these and other papers is given below:—

"Note on the Probability that a Marriage entered into by a Man above the Age of 40 will be fruitful," *Proc. Roy. Soc. Edin.*, vol. x, 1880, p. 202.

"On the Nature of the Curves whose Intersections give the Imaginary Roots of an Algebraic Equation," *Trans. Roy. Soc. Edin.*, vol. xxx, 1883, p. 467.

"On a New Algebra by means of which Permutations can be transformed in a Variety of Ways and their Properties investigated," *Trans. Roy Soc. Edin.*, vol. xxxvii, 1893, p. 399.

"On the Adjustment of Numerical Results derived from Observation," *Brit. Assoc. Report*, 1883, p. 446.

"Note on the Evaluation of Functions of the Form 0°," *Proc. Edin. Math. Soc.*, 1884–85.

"On the Different Possible Non-linear Arrangements of Eight Men on a Chess Board," *Proc. Edin. Math. Soc.*, 1889–90.

"On the Transformation and Classification of Permutations," *Proc. Edin. Math. Soc.*, 1890–91.

"On the Geometrical Interpretation of *i*," *Proc. Edin. Math. Soc.*, 1893–94.

"On the Eight Queens' Problem," *Proc. Edin. Math. Soc.*, 1898–99.

"On the Singular Points of Plane Curves," *Proc. Edin. Math. Soc.*, 1902–03.

The following were communicated to the Edinburgh Field Naturalists and Microscopical Society:—

"Bones and Shells taken from a Kitchen Midden on Inchkeith during 1881."

"The 'Green Balls' of Loch Kildonan."

"On the Growth of Leaves."

"Fibre Balls."

"Ripple Marks on Sand."

"On the Occurrence of the Fresh-Water Mussel."

"Notes on the Bournemouth Cliffs."

"Notes on the Entomostraca (Water Fleas) of Midlothian."

Dr Sprague was twice married: first, in 1859, to Miss Margaret Vaughan Steains, younger daughter of Mr James Steains of Liverpool, by whom he had eleven children, eight of whom survive; and second, in 1908, to Miss Jean Elizabeth Stuart of Edinburgh, who also survives him.

He was elected to the Fellowship of the Royal Society of Edinburgh in 1874, and served on the Council from 1885–1887. He died at West Holme, Woldingham, Surrey, on 29th November 1920.

Walker, Robert, M.A. (Aberd. et Cantab.), LL.D., was born in Aberdeen in 1842. He was well known to several generations of students in connection with the part he took in arranging and supervising a number of the more noteworthy functions of the University of Aberdeen, and his services were especially outstanding in connection with the quatercentenary celebrations and the visit of King Edward VII. to open the extension of Marischal College in 1906. The following is a brief *résumé* of his career:—B.A. (fifteenth Wrangler) at Cambridge in 1865, and in 1868 M.A. Fellow of Clare College, Cambridge, from 1866–1878. He taught mathematics for some time in the Edinburgh Academy, and was examiner in that subject in the Universities of Edinburgh and Aberdeen. In 1877 he was appointed University Librarian at Aberdeen, and in 1893 Secretary to the Aberdeen University Court, Registrar of the University and Clerk of the General Council, which position he resigned in 1906. The following year he received the honorary degree of LL.D. from Aberdeen University in recognition of his valuable services.

Dr Walker was elected a Fellow of the Royal Society of Edinburgh in 1873, and died at Aberdeen on 26th October 1920.

Woods, Geo. Arthur, L.R.C.P. Edin., and L.M. 1868, M.R.C.S. Eng. 1868, L.S.A. 1868 (St Bart.), Hunterian Gold Medal, was the author of the following work:—*Anat. Physiol. and Pathol. of the Third, Fourth, and Sixth Nerves, as illustrated by Observation and Experiments in Health and by References to Effects of Injury and Disease* (Coll. Trien. Prize, R.C.S. Eng.), and published papers in the *Liverpool Medical Surgical Reports*, *Lancet*, *International Medical Congress*, and the *Liverpool M.-C. Journal*.

He was elected to the Fellowship of the Society in 1884, and died, after an illness of many years, on 30th June 1921.

APPENDIX.

CONTENTS.

PROCEEDINGS OF THE STATUTORY GENERAL MEETING
Beginning the 138th Session, 1920–1921.

At the Statutory Meeting of the Royal Society of Edinburgh, held in the Society's Lecture Room, 24 George Street, on Monday, October 25, 1920, at 4.30 p.m.,

Professor FREDERICK O. BOWER, President, in the Chair,

the Minutes of Meeting of October 27, 1919, were read, approved, and signed.

The CHAIRMAN nominated as Scrutineers, Dr INGLIS CLARK and Mr GALL INGLIS.

The ballot for the election of Office-Bearers and Members of Council was then taken.

The SECRETARY submitted the following Report:—

The number of papers read at our Meetings during the last Session was 27, as compared with 23 the previous Session. Of these, three were Addresses, of which two were delivered by Professor R. A. SAMPSON, "On Gravitation and Relativity," and one by Principal Sir JAMES ALFRED EWING, "On the Molecular Energy of Gases." Of the others, 4 were in Mathematics, 7 in Physics, 4 in Meteorology, 1 in Chemistry, 1 in Zoology, 1 in Botany, 5 in Geology, and 1 in Medicine. 23 have been, or are being, published in the *Proceedings* or *Transactions*.

Last Session the Society elected 20 new Fellows. Two Fellows have resigned, and we have lost by death 3 Honorary Fellows and 16 Ordinary Fellows.

The Keith Prize was awarded to Professor JOHN STEPHENSON, Lt.-Col. I.M.S., and the Neill Prize to Professor JOHN TAIT.

Owing to the continued increase in the cost of publication the Council has to exercise great care in order to meet the necessary expenses. The income of the Society remains practically the same; but the Special Fund to which many Fellows contributed has enabled us to meet the comparatively small deficit this year. An Appeal by the Council to the Carnegie United Kingdom Trust for aid in binding the loose journals in the Library was very generously met by the Trust, and already £1500 (half the promised grant) has been received from the Trust and paid over to the binders. The Society takes this opportunity of expressing its thanks to the Trust for their generous help.

An interesting event of the year was the election of the Prince of Wales as an Honorary Fellow. Also the names of seven distinguished scientific men were added to the roll of Foreign Honorary Fellows.

All who have had occasion to use the Library will hear with regret that Miss LE HARIVEL has been obliged, for reasons of health, to resign her position as Assistant Librarian. Steps are being taken to fill up the vacancy thus occasioned.

The Society is being called upon to take an increasing share in national developments. Thus Dr E. M. WEDDERBURN has been appointed the Society's representative on the newly constituted Meteorological Committee under the Air Ministry. In connection with the International Association of Scientific Bodies, Professor WHITTAKER and Dr KNOTT have been appointed representatives on the National Union of Mathematics, Dr H. R. MILL on the National Union of Geography, and Dr CRICHTON MITCHELL and Dr ERSKINE MURRAY on the National Union of Radio-Telegraphy.

It will interest the Fellows to know that the Society is associated with the Municipality and the University in preparing for the Meeting of the British Association to be held in Edinburgh in September 1921.

In order to give a distinct scientific flavour to the Statutory Meeting, the Council has adopted a suggestion made last year by the previous President, Dr Horne, and asked the President to deliver his Address at this Meeting instead of a week later at the first Ordinary Meeting of the Session.

The TREASURER, in submitting his Report for the year, compared the income and expenditure with those of the previous year, and called attention to the fact that the deficit of £34, 9s. 5d. on the year's working had been met from the Special Subscription Fund, which now stands at £792, 11s. 5d. The full list of subscribers up to the present is given on pp. 208–209 (vol. xl).

Dr JOHN HORNE moved the adoption of the Reports, and the reappointment of Messrs LINDSAY, JAMIESON & HALDANE, C.A., as auditors of the accounts for the ensuing Session.

This was unanimously agreed to.

The Scrutineers reported that the Ballot Papers were in order, and that the following Office-bearers and Members of Council had been elected :—

Professor Frederick O. Bower, M.A., D.Sc., LL.D., F.R.S., F.L.S., President.

Professor D. Noël Paton, M.D., B.Sc., LL.D., F.R.C.P.E., F.R.S.,
Professor Arthur Robinson, M.D., M.R.C.S.,
Sir George A. Berry, M.B., C.M., LL.D., F.R.C.S.E.,
Professor William Peddie, D.Sc.,
Principal Sir James Alfred Ewing, K.C.B., M.A., D.Sc., LL.D., M.Inst.C.E., F.R.S.,
Professor John Walter Gregory, D.Sc., F.R.S.,
} Vice-Presidents.

Cargill G. Knott, D.Sc., LL.D., F.R.S., General Secretary.

Professor E. T. Whittaker, Sc.D., F.R.S.,
Professor J. H. Ashworth, D.Sc., F.R.S.,
} Secretaries to Ordinary Meetings.

James Currie, M.A., LL.D., Treasurer.

A. Crichton Mitchell, D.Sc., Hon. D.Sc. (Geneva), Curator of Library and Museum.

ORDINARY MEMBERS OF COUNCIL.

Professor R. A. Sampson, M.A., D.Sc., F.R.S.
Professor J. Lorrain Smith, M.A., M.D., F.R.S.
W. A. Tait, D.Sc., M.Inst.C.E.
Maj.-General W. B. Bannerman, C.S.I., I.M.S., M.D., D.Sc.
Henry Moubray Cadell, of Grange, B.Sc.
Professor Arthur Robertson Cushny, M.A., M.D., LL.D., F.R.S.
Professor Francis Gibson Baily, M.A., M.Inst.E.E.
George James Lidstone, F.F.A., F.I.A.
Robert Campbell, D.Sc.
Professor James Colquhoun Irvine, C.B.E., Ph.D., D.Sc., F.R.S.
The Hon. Lord Salvesen.
Professor J. Arthur Thomson, M.A., LL.D.

SOCIETY'S REPRESENTATIVE ON GEORGE HERIOT'S TRUST.

W. A. Tait, D.Sc., M.Inst.C.E.

The President, in the name of the Society, thanked the Scrutineers for their services.

PROCEEDINGS OF THE ORDINARY MEETINGS, Session 1920–1921.

FIRST ORDINARY MEETING.

Monday, November 1, 1920.

Professor Frederick O. Bower, M.A., D.Sc., LL.D., F.R.S., F.L.S., President, in the Chair.

The following Communications were submitted:—

1. "Isle of Wight" Disease in Hive Bees.

 a. The Etiology of the Disease. By ELSIE J. HARVEY, JOHN RENNIE, D.Sc., and P. BRUCE WHITE, B.Sc.
 b. The Organism associated with the Disease. By JOHN RENNIE, D.Sc.
 c. The Pathology of the Disease. By P. BRUCE WHITE, B.Sc.
 d. Experiments in Infection. By ELSIE J. HARVEY.

Trans., vol. lii, pp. 737–779.

2. On the Equations of Motion of a Single Particle. By Professor J. H. MACLAGAN WEDDERBURN, M.A., D.Sc. *Proc.*, vol. xli, pp. 26–33.

SECOND ORDINARY MEETING.

Monday, November 22, 1920.

Principal Sir J. A. Ewing, K.C.B., LL.D., M.Inst.C.E., F.R.S., Vice-President, in the Chair.

The Chairman announced that the Council had resolved to recommend to the Fellows certain changes in the Bye-laws as follows:—

"Each Fellow shall, before he is admitted to the privileges of Fellowship, pay an admission fee of three guineas, and a subscription of three guineas for the year of election. He shall continue to pay a subscription of three guineas at the beginning of each session so long as he remains a Fellow. A Fellow may compound for the annual subscriptions by a single payment of fifty guineas, or on such other terms as the Council may from time to time fix.

"Each Fellow who was elected previous to December 1916 shall pay four guineas at the beginning of each session until he has made ten annual payments, and thereafter three guineas at the beginning of each session until he has completed his twenty-fifth annual payment. Each Fellow who has completed or shall complete his payments shall be invited to contribute one guinea per annum so long as he remains a Fellow, or to pay a single sum of ten guineas."

The following Communications were submitted:—

1. Obituary Notice of Surgeon-General W. C. Gorgas, Honorary Fellow. By Major-General W. B. BANNERMAN. *Proc.*, vol. xl, pp. 187–190.
2. On Fechner's Law and the Self-luminosity of the Eye. By Professor W. PEDDIE, D.Sc. *Proc.*, vol. xli, p. 60. (Abstract.)
3. Aether and the Quantum Theory. By Dr H. STANLEY ALLEN. *Proc.*, vol. xli, pp. 34–43.

THIRD ORDINARY MEETING.

Monday, December 6, 1920.

Professor Frederick O. Bower, M.A., D.Sc., LL.D., F.R.S., F.L.S., President, in the Chair.

The following Communications were submitted:—

1. Observations on the Behaviour of the Endodermis in the Secondarily Thickened Root of *Dracæna fruticosa* (Koch). By Miss ANNETTE G. MANN. Communicated by Professor F. O. BOWER, F.R.S. *Proc.*, vol. xli., pp. 50–59.

2. On Cretaceous Ammonoidea from Angola, collected by Professor J. W. Gregory, F.R.S. (With notes by the late G. C. Crick, F.G.S.) By L. F. Spath, M.Sc., F.G.S. Communicated by Professor J. W. Gregory, F.R.S. (*With Lantern Illustrations.*) *Trans.*, vol. liii, pp. 91–160. Note by Professor Gregory. Vol. liii, pp. 161–163.

The President drew the attention of the Fellows to the subjoined intimation on the Billet:—

It will be proposed and seconded at the Ordinary Meeting on 10th January 1921 that Rule VI read as follows:—

"Each Fellow shall, before he is admitted to the privileges of Fellowship, pay an admission fee of three guineas, and a subscription of three guineas for the year of election. He shall continue to pay a subscription of three guineas at the beginning of each session so long as he remains a Fellow. A Fellow may compound for the annual subscriptions by a single payment of fifty guineas, or on such other terms as the Council may from time to time fix.

"Each Fellow who was elected previous to December 1916 shall pay four guineas at the beginning of each session until he has made ten annual payments as reckoned from the year of election, and thereafter three guineas at the beginning of each session until he has completed his twenty-fifth annual payment. Each Fellow who has completed or shall complete his payments shall be invited to contribute one guinea per annum so long as he remains a Fellow, or to pay a single sum of ten guineas."

FOURTH ORDINARY MEETING.

Monday, January 10, 1921.

Professor Frederick O. Bower, M.A., D.Sc., LL.D., F.R.S., F.L.S., President, in the Chair.

The following Communications were submitted:—

1. Obituary Notice of Dr John Aitken, F.R.S. By Dr C. G. Knott, F.R.S. *Proc.*, vol. xli, pp. 177–181.
2. Thermometer Screens. By the late Dr John Aitken, F.R.S. Communicated by Dr C. G. Knott, F.R.S. *Proc.*, vol. xl, pp. 172–181.
3. The Avoidance of Relativity which is not of the Galileo-Newtonian Type. By Professor Wm. Peddie, D.Sc.
4. On the Transverse Galvanomagnetic and Thermomagnetic Effects in several Metals. By F. Unwin, M.Sc. Communicated by Professor F. G. Baily. *Proc.*, vol. xli, pp. 44–49.
5. The Confluent Hypergeometric Functions of two Variables. By Pierre Humbert. Communicated by Professor E. T. Whittaker, F.R.S. *Proc.*, vol. xli, pp. 73–96.

The proposed Amendment to Law VI was discussed and some slight alteration proposed in the wording and arrangement of certain sentences. The President ruled that, under the circumstances, the final vote on the proposed Amendment should be postponed until the next Ordinary Meeting (February 7).

FIFTH ORDINARY MEETING.

Monday, February 7, 1921.

Professor Frederick O. Bower, M.A., D.Sc., LL.D., F.R.S., F.L.S., President, in the Chair.

The President proposed that Rule VI in its amended form should read as follows, and this was agreed to with the suggestion that an explanatory footnote should be added to obviate any ambiguity.

"Each Fellow shall, before he is admitted to the privileges of Fellowship, pay an admission fee of three guineas, and a subscription of three guineas for the year of election. He shall continue to pay a subscription of three guineas at the beginning of each session so long as he remains a Fellow.

"Each Fellow who was elected subsequent to December 1916 and previous to December 1920 shall also pay a subscription of three guineas at the beginning of each session so long as he remains a Fellow.

"Each Fellow who was elected previous to December 1916 and who has not completed his twenty-five annual payments shall, at the beginning of each session, pay three guineas or four guineas, according as he has or has not made ten annual payments as reckoned from the year of election. Each Fellow who has completed or shall complete his payments shall be invited to contribute one guinea per annum or to pay a single sum of ten guineas.

"A Fellow may compound for the annual subscriptions by a single payment of fifty guineas, or on such other terms as the Council may from time to time fix."

The following Communications were submitted :—

1. The Relation of the Soil Colloids to the Conductivity of the Soil. By T. Bedford Franklin, B.A. *Proc.*, vol. xli, pp. 61–67.

2. The Natural History of Pack Ice as observed in the Weddell Sea, December 1914 to April 1916. By J. M. Wordie, M.A., F.G.S. Communicated by Professor J. W. Gregory, F.R.S. *Trans.*, vol. lii, pp. 795–829.

3. Shackleton Antarctic Expedition, 1914–1917 : Bathymetrical Observations in the Weddell Sea. By J. M. Wordie, M.A., F.G.S. Communicated by Professor J. W. Gregory, F.R.S. *Trans.*, vol. lii, pp. 781–793.

SIXTH ORDINARY MEETING.

Monday, March 7, 1921.

Professor Frederick O. Bower, M.A., D.Sc., LL.D., F.R.S., F.L.S., President, in the Chair.

The Ballot for the election of Ordinary Fellows was taken. The Scrutineers were Mr Gall Inglis and Dr J. M'Lean Thompson. The undernoted Gentlemen were declared duly elected to the Ordinary Fellowship of the Society :—Nelson Annandale, William Arthur, Bevan Braithwaite Baker, Archibald Barr, John Bartholomew, Alexander Bruce, Andrew Campbell, Rasik lal Datta, John Dougall, Charles Vickery Drysdale, George Topham Forrest, Walcot Gibson, John Wm. Heslop Harrison, James Alex. George Lamb, Rev. Albert Ernest Laurie, Neil M'Arthur, Dougald Black M'Quistan, Thomas Murray MacRobert, John M'Whan, John Mathieson, Sir Geo. Herbert Pollard, Edward Burns Ross, Rt. Hon. James Parker Smith, Norman Kemp Smith, Ian Struthers Stewart.

The following Communications were submitted :—

1. On a Graphical Method of Determining Shear Influence Lines, and Diagrams of Maximum Shearing Force, for a Beam subjected to a Series of Concentrated Rolling Loads. By Professor A. R. Horne, B.Sc. *Proc.*, vol. xli, pp. 68–72.

2. Studies in Floral Morphology. No. 2 : The Staminal Zygomorphy of *Couroupita guianensis*, Aubl. By Dr J. M'Lean Thompson. *Trans.*, vol. liii, pp. 1–15.

SEVENTH ORDINARY MEETING.

Monday, March 21, 1921.

Professor Frederick O. Bower, F.R.S., President, and afterwards Professor R. A. Sampson, F.R.S., Vice-President, in the Chair.

The following recently elected Ordinary Fellows were admitted and signed the Roll:—Mr J. A. G. Lamb, the Rev. A. E. Laurie, Mr D. B. M'Quistan, Professor E. B. Ross, and the Rt. Hon. J. Parker Smith.

The following Communications were submitted :—

1. An Experimental Analysis of the Losses by Evaporation of Liquid Air contained in Vacuum Flasks. By Professor Henry Briggs, D.Sc. *Proc.*, vol. xli, pp. 97–110.

2. A Generalisation of Lagrange's Equations of Motion and their Hamiltonian Forms. By Dr John Marshall, M.A.

3. Note on a Continuant of Cayley's of the year 1874. By Sir Thomas Muir, C.M.G., LL.D., F.R.S. *Proc.*, vol. xli, pp. 111–116.

EIGHTH ORDINARY MEETING.

Monday, May 2, 1921.

Professor Frederick O. Bower, M.A., D.Sc., LL.D., F.R.S., F.L.S., President, in the Chair.

The following recently elected Ordinary Fellows were admitted and signed the Roll:—Mr B. B. Baker and Dr Walcot Gibson.

The following Communications were submitted :—

1. Behaviour of an Electrified Pith Ball in an Ionised Atmosphere, illustrated with Experiments. By Dr Dawson Turner and Mr D. M. R. Crombie. *Proc.*, vol. xli, pp. 154–157.

2. On Old Red Sandstone Plants showing Structure, from the Rhynie Chert Bed, Aberdeenshire. Part IV: Restorations of the Vascular Cryptogams, and discussion of their bearing on the General Morphology of the Pteridophyta and the Origin of the Organisation of Land-Plants. Part V: The Thallophyta occurring in the Peat-Bed; the succession of the Plants throughout a Vertical Section of the Bed, and the conditions of Accumulation and Preservation of the Deposit. By Dr R. KIDSTON, F.R.S., and Professor W. H. LANG, D.Sc., F.R.S. *Trans.*, vol. lii, pp. 831–854, 855–902.

NINTH ORDINARY MEETING.

Monday, June 6, 1921.

Professor Frederick O. Bower, M.A., D.Sc., LL.D., F.R.S., F.L.S., President, in the Chair.

By request of the Council, Lt.-Col. WM. GLEN LISTON, M.D., Indian Medical Service, gave an address on "Plague and Rats."

A vote of thanks to Col. LISTON was proposed by the President, seconded by Major-Gen. BANNERMAN, and carried unanimously.

The President announced the award of the Gunning Victoria Jubilee Prize to Mr C. T. R. WILSON, F.R.S., in recognition of his important discoveries in relation to Condensation Nuclei, Ionisation of Gases, and Atmospheric Electricity; and the Makdougall-Brisbane Prize for the Period 1918–1920 to Professor J. H. MACLAGAN WEDDERBURN, M.A., D.Sc., for his many valuable memoirs in universal algebra which he has published in the Society's *Transactions* and *Proceedings* and elsewhere. One of these papers, "On the Equations of Motion of a Single Particle," falls within the period of award.

The Prizes will be presented at the Meeting on July 4.

TENTH ORDINARY MEETING.

Monday, June 20, 1921.

Professor Frederick O. Bower, F.R.S., and afterwards Professor R. A. Sampson, F.R.S., Vice-President, in the Chair.

The following Ordinary Fellows were admitted and signed the Roll:—Professor NORMAN KEMP SMITH, Dr NELSON ANNANDALE, and Mr IAN BARTHOLOMEW.

The President announced that the Council had approved of the following nominations for Honorary Fellowship of the Royal Society of Edinburgh:—

BRITISH.

WILLIAM HENRY PERKIN, F.R.S., Waynflete Professor of Chemistry in the University of Oxford; Sir RONALD ROSS, K.C.B., K.C.M.G., F.R.S., Consultant in Malaria, Ministry of Pensions, London; Sir ERNEST RUTHERFORD, Kt., F.R.S., Cavendish Professor of Experimental Physics in the University of Cambridge; Sir JETHRO J. H. TEALL, Kt., F.R.S., lately Director of the Geological Survey of Great Britain and of the Museum of Practical Geology.

FOREIGN.

REGINALD ALDWORTH DALY, Professor of Geology, Harvard University, Cambridge, Mass.; JOHAN HJORT, Director of Norwegian Fisheries, Bergen; CHARLES LOUIS ALPHONSE LAVERAN, Nobel Laureate, Medicine, 1907, Paris; HEIKE KAMERLINGH ONNES, Nobel Laureate, Physics, 1913, Leiden, Holland; SALVATORE PINCHERLE, Professor of Mathematics in the University of Bologna.

The following Communications were submitted:—

1. The Annual Incidence of Intelligence and its Measurement by the American Army Tests. By MUNGO M'CALLUM FAIRGRIEVE, M.A. *Proc.*, vol. xli, pp. 150–153.

2. Shackleton Antarctic Expedition, 1914–1917: Geological Observations in the Weddell Sea Area. By J. M. WORDIE, M.A., F.G.S. Communicated by Professor J. W. GREGORY, F.R.S. *Trans.*, vol. liii, pp. 17–27.

3. On the Criterion for Stable Flow of a Fluid in a Uniform Channel. By HYMAN LEVY, M.A., D.Sc. *Proc.*, vol. xli, pp. 136–147.

4. The Genus Clisiophyllum. By Professor P. MACNAIR, F.G.S., and COLIN M. LEITCH, O.B.E., B.Sc.

5. Exhibition of Lantern Slides by Dr NELSON ANNANDALE, Director Zoological Survey of India.

ELEVENTH ORDINARY MEETING.

Monday, July 4, 1921.

Professor Frederick O. Bower, M.A., D.Sc., LL.D., F.R.S., F.L.S., President, in the Chair.

A Ballot was taken for election to the Honorary Fellowship of the Society. Dr H. S. Allen and Dr G. A. Carse were nominated as Scrutineers and the undernoted gentlemen were declared duly elected to the Honorary Fellowship:—

British Honorary Fellows.

William Henry Perkin.
Sir Ronald Ross.
Sir Ernest Rutherford.
Sir Jethro J. H. Teall.

Foreign Honorary Fellows.

Reginald Aldworth Daly.
Johan Hjort.
Charles Louis Alphonse Laveran.
Heike Kamerlingh Onnes.
Salvatore Pincherle.

Dr T. M. MacRobert and Mr Wm. Arthur signed the Roll and were admitted as Ordinary Fellows of the Society.

The President announced the awards of the Gunning Victoria Jubilee and Makdougall-Brisbane Prizes, in the following terms, and presented the Prizes.

The Gunning Victoria Jubilee Prize is awarded to Mr C. T. R. Wilson, F.R.S., in recognition of his important discoveries in relation to Condensation Nuclei, Ionisation of Gases, and Atmospheric Electricity.

Mr Wilson's first group of important contributions to science (*Phil. Trans.*, vol. clxxxix, 1897: vol. cxciii, 1899) related to Condensation Nuclei, a subject which had been initiated by our distinguished Fellow—Dr John Aitken. He discovered the existence of two definite limits to the supersaturation in moist dust-free gases, a lower limit beyond which rain-like condensation occurs, and an upper limit beyond which cloud-like condensation occurs. He showed that nuclei identical with those which give rise to the rain-like condensation are produced in great numbers by X-rays and other agents which give conducting power to a gas; and proved that such nuclei are removed by an electric field and are therefore identical with the charged "ions" to which the conducting power of the gas is due. This discovery furnished much more direct evidence than any previously available for the existence of discrete "ions," each individual ion being made visible by the condensation of water upon it. The earliest determinations of the charge carried by an ion were made by J. J. Thomson and H. A. Wilson by applying this method.

Mr Wilson's next group of researches (Camb. *Phil. Soc. Proc.*, 1900–1902; *Proc. Roy. Soc.*, vol. lxviii, 1901; vol. lxix, 1902) related to the natural ionisation of gases. He showed that air and other gases conduct electricity to a minute extent under normal conditions, even when dust-free and contained in closed vessels. This conduction he proved to be due to the continual production of ions, the number produced per sec. being determined. The method introduced for the purpose of this investigation has since been extensively used in radio-active and similar work.

After this Mr Wilson's attention was turned to Atmospheric Electricity (Camb. *Phil. Soc. Proc.*, vol. xiii, 1905–06; *Proc. Roy. Soc.*, vol. lxxx, 1908). He devised a new method for measuring the surface charge on the ground (and hence the vertical electric force) and at the same time determined directly the current from the atmosphere into the ground. These experiments, which were made at Peebles, gave the first direct measures of this current.

In 1911–1912 (*Proc. Roy. Soc.*, vol. lxxxv, 1911; vol. lxxxvii, 1912) he published important work on the Tracks of Ionising Particles. The expansion apparatus of the earlier condensation experiments was modified so that the ions were made visible as minute drops and photographed in the position which they occupied immediately after their liberation. The tracks of the individual alpha and beta rays from radium and its products, and of the electrons emitted in air when exposed to X-rays, were thus made visible. In the case of the fast β-rays the individual ions liberated along the track of the corpuscle are made visible in the photographs. This work made clear the nature of the ionisation of a gas by X-rays, viz., that it consists in the ejection, from the atoms of the gas, of electrons, each of which ionises the gas along the course of its flight.

In more recent years Mr Wilson has published further researches in Atmospheric Electricity and Thunderstorms (*Proc. Roy. Soc.*, vol. xcii. 1916; *Phil. Trans.*, vol. ccxxi, 1920). The method of measuring the surface charge on the ground was modified so as to allow of automatic records being obtained, a special type of capillary electrometer being devised for the purpose of measuring charges. This was applied to study the variations in the electric field during thunderstorms, including the sudden changes produced by the passage of lightning flashes at various distances. The "electric moments" of a large number of flashes were thus measured, and estimates obtained of the quantity of electricity which passes in a flash. The results of these observations have been applied to the Theory of Thunderstorms.

The Makdougall-Brisbane Prize for the period 1918–1920 is awarded to Professor J. H. Maclagan Wedderburn of Princeton University for the many valuable memoirs which he has published in our *Transactions* and *Proceedings* and elsewhere. One of these memoirs falls within the period of award of the prize—"On the Equations of Motion of a Single Particle." Among the other memoirs may be mentioned—

Isoclinal Lines of a Differential Equation of the 1st Order. *Proc.R.S.E.*, vol. xxiv, p. 400, 1903.

General Scalar Functions of a Vector. *Proc.R.S.E.*, vol. xxiv, p. 409, 1903.

Applications of Quaternions in the Theory of Differential Equations. *Trans.R.S.E.*, vol. xl, p. 709, 1903.

And a number of important papers on Hypercomplex Numbers published in the Transactions of the American Mathematical Society.

By request of the Council, Mr C. T. R. Wilson, M.A., B.Sc., F.R.S., gave an address on "Some Recent Work on Lightning and Thunderstorms."

The following Communications were submitted :—

1. The Adsorption of Gas under Pressure. By Professor Henry Briggs, D.Sc., and Wm. Cooper, M.A., B.Sc. *Proc.*, vol. xli, pp. 119–127.

2. The Utilisation of Solid Caustic Soda in the Absorption of Carbon Dioxide. By Miss Elizabeth Gilchrist, M.A., B.Sc. Communicated by Professor Henry Briggs, D.Sc. *Proc.*, vol. xli, pp. 128–135.

3. On the Development of the Feathers of the Duck during the Incubation Period. By Miss Augusta Lamont, B.Sc. Communicated by Professor J. Cossar Ewart, F.R.S.

4. Note on Conditions for Mirage. By Alex. G. Ramage. *Proc.*, vol. xli, pp. 148–149.

PROCEEDINGS OF THE STATUTORY GENERAL MEETING

Ending the 138th Session, 1920–1921.

At the Statutory Meeting of the Royal Society of Edinburgh, held in the Society's Lecture Room, 24 George Street, on Monday, October 24, 1921, at 4.30 p.m.,

Professor F. O. Bower, F.R.S., President, in the Chair,

the Minutes of the last Statutory Meeting of October 25, 1920, were read, approved, and signed.

The Chairman nominated as Scrutineers Mr C. H. Milne and Dr J. B. Ritchie.

The Ballot for the Election of Office-Bearers and Members of Council was then taken.

The Secretary submitted the following Report:—

The number of papers read at our meetings during the last Session was 35, as compared with 27 the previous Session. Of these, 3 were addresses: the first by the President on "Size a Neglected Factor in the Internal Morphology of Plants"; the second by Lt.-Col. Liston on "Plague and Rats"; and the third by Mr C. T. R. Wilson on "Some Recent Work on Lightning and Thunderstorms." Of the others, 5 were in mathematics, 1 in statistics, 9 in physics, 2 in chemistry, 5 in zoology, 2 in botany, 5 in geology, 2 in oceanography, and 1 in engineering. Twenty-eight of these have been, or are being, published—11 in the *Transactions* and 17 in the *Proceedings*.

Last Session the Society elected 25 new Ordinary Fellows, and 9 Honorary Fellows—4 British and 5 Foreign. Three Fellows have resigned, and we have lost by death 4 Honorary Fellows and 22 Ordinary Fellows.

The Makdougall-Brisbane Prize was awarded to Professor J. H. Maclagan Wedderburn of Princeton, U.S.A.; and the Gunning Victoria Jubilee Prize to Mr C. T. R. Wilson of Cambridge.

The vacancy in the post of Assistant Librarian, occasioned by the resignation of Miss Le Harivel, has been filled up by the appointment of Mr Ernst M. Stewart.

The high cost of printing and publication still continues to hamper the work of the Society. To meet to some extent the increased expense, the Council asked the Society to amend Law VI and raise the annual subscription by one guinea. This was agreed to, and the amendment of Law VI was duly carried out at the Meeting of February 7, 1921.

The Grant of £3000 from the Carnegie United Kingdom Trust for binding the journals in the Library is now nearly all expended; and again the Society takes the opportunity of expressing its thanks to the Trust for their generous help.

The Conjoint Board of Scientific Societies has prepared an important report on the question of aid to Scientific Societies for publication of papers, and our representatives have been instructed to look specially to our interests in any further developments which may occur.

One of the great events of the year was the Meeting of the British Association for the Advancement of Science. This our Society helped by a donation of £100 towards the local expenses and by inviting the Members to a free use of the Library and Reading-room. The Secretaries also organised, by request of the Council, a Royal Societies dinner, at which our Society entertained Fellows of the Royal Society of London, Members of the Royal Irish Academy, and distinguished Foreign visitors who were members of their National Academies. The dinner was held in the Freemasons' Hall on Monday, September 12, when our President, with the Maharaj Rana of Jhalawar on his right hand, presided over a gathering of one hundred and eighty-one scientific men from many lands.

During the summer, and especially in view of the British Association Meeting, the Council arranged for cleaning and painting the west staircase and lecture room, and laying new matting on the stairs.

As will be seen from the Treasurer's Report, the financial condition of the Society is fairly satisfactory, notwithstanding these exceptional but necessary outlays during the Session.

The Treasurer in submitting his Report for the year compared the Income and Expenditure with those of the previous year.

Dr R. Kidston, F.R.S., moved the adoption of the Reports and the reappointment of Messrs Lindsay, Jamieson & Haldane, C.A., as auditors of the accounts for the ensuing Session.

This was unanimously agreed to.

The Scrutineers reported that the Ballot Papers were in order, and that the following Office-Bearers and Members of Council had been elected :—

Professor Frederick O. Bower, M.A., D.Sc., LL.D., F.R.S., F.L.S., President.

Sir George A. Berry, M.B., C.M., LL.D., F.R.C.S.E.,
Professor William Peddie, D.Sc.,
Principal Sir James Alfred Ewing, K.C.B., M.A., D.Sc., LL.D., M.Inst.C.E., F.R.S.
Professor John Walter Gregory, D.Sc., F.R.S.
Major-General W. B. Bannerman, C.S.I., I.M.S., M.D., D.Sc.
W. A. Tait, D.Sc., M.Inst.C.E.
} Vice-Presidents.

Cargill G. Knott, D.Sc., LL.D., F.R.S., General Secretary.

Professor E. T. Whittaker, Sc.D., F.R.S.,
Professor J. H. Ashworth, D.Sc., F.R.S.,
} Secretaries to Ordinary Meetings.

James Currie, M.A., LL.D., Treasurer.

A. Crichton Mitchell, D.Sc., Hon. D.Sc. (Geneva), Curator of Library and Museum.

ORDINARY MEMBERS OF COUNCIL.

Henry Moubray Cadell, of Grange, B.Sc.
Professor Arthur Robertson Cushny, M.A., M.D., LL.D., F.R.S.
Professor Francis Gibson Baily, M.A., M.Inst.E.E.
George James Lidstone, F.F.A., F.I.A.
Robert Campbell, M.A., D.Sc., F.G.S.
Principal James Colquhoun Irvine, C.B.E., Ph.D., D.Sc., LL.D., F.R.S.
The Hon. Lord Salvesen.
Professor J. Arthur Thomson, M.A., LL.D.
Herbert Stanley Allen, M.A., D.Sc.
Sir Robert Blyth Greig, M.A., LL.D., F.Z.S.
James Ritchie, D.Sc.
Ernest Maclagan Wedderburn, M.A., LL.B., W.S., D.Sc.

SOCIETY'S REPRESENTATIVE ON GEORGE HERIOT'S TRUST.

W. A. Tait, D.Sc., M.Inst.C.E.

The Chairman, in the name of the Society, thanked the Scrutineers for their services

PUBLIC BUSINESS.

An Obituary Notice of Robert Munro, M.A., M.D., LL.D., by Dr George Macdonald, C.B., was read by Dr C. G. Knott, General Secretary. *Proc.*, vol. xli, pp. 158–169.

The President summarised the position of the Society on the working of the year just closed, as regards the volume of work presented for publication and the showings of the Balance Sheet. He pointed out that the volume of work was at present considerably below the average of pre-war years, and forecasted the strong probability that in the near future, when the immediate effects of the war had been worked off, an even larger volume of work would be offered than the average of the years before the war.

He drew attention to the importance of keeping in close touch with the younger men, who are at the outset of their productive period : and he suggested as being worthy of consideration by the Society, the possible constitution of a junior grade of membership to which the younger men might belong. It might stand to the full Fellowship in somewhat the same relation as in the Royal Academy the A.R.A. stands to the rank of full Academician.

THE KEITH, MAKDOUGALL-BRISBANE, NEILL, GUNNING VICTORIA JUBILEE, AND JAMES SCOTT PRIZES.

The above Prizes will be awarded by the Council in the following manner:—

I. KEITH PRIZE.

The Keith Prize, consisting of a Gold Medal and from £40 to £50 in Money, will be awarded in the Session 1923–1924 for the "best communication on a scientific subject, communicated,* in the first instance, to the Royal Society of Edinburgh during the Sessions 1921–1922 and 1922–1923." Preference will be given to a paper containing a discovery.

II. MAKDOUGALL-BRISBANE PRIZE.

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded before the close of the Session 1922–1923, for an Essay or Paper having reference to any branch of scientific inquiry, whether Material or Mental.

2. Competing Essays to be addressed to the Secretary of the Society, and transmitted not later than 8th July 1922.

3. The Competition is open to all men of science.

4. The Essays may be either anonymous or otherwise. In the former case, they must be distinguished by mottoes, with corresponding sealed billets, superscribed with the same motto, and containing the name of the Author.

5. The Council impose no restriction as to the length of the Essays, which may be, at the discretion of the Council, read at the Ordinary Meetings of the Society. They wish also to leave the property and free disposal of the manuscripts to the Authors; a copy, however, being deposited in the Archives of the Society, unless the paper shall be published in the Transactions.

* For the purposes of this award the word "communicated" shall be understood to mean the date on which the manuscript of a paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

6. In awarding the Prize, the Council will also take into consideration any scientific papers presented * to the Society during the Sessions 1918–19, 1919–20, whether they may have been given in with a view to the prize or not.

III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr Patrick Neill of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate:

1. The Neill Prize, consisting of a Gold Medal and a sum of Money, will be awarded during the Session 1923–1924.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented * to the Society during the two years preceding the fourth Monday in October 1923,—or failing presentation of a paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award.

IV. GUNNING VICTORIA JUBILEE PRIZE.

This Prize, founded in the year 1887 by Dr R. H. Gunning, is to be awarded quadrennially by the Council of the Royal Society of Edinburgh, in recognition of original work in Physics, Chemistry, or Pure or Applied Mathematics.

Evidence of such work may be afforded either by a Paper presented to the Society, or by a Paper on one of the above subjects, or some discovery in them elsewhere communicated or made, which the Council may consider to be deserving of the Prize.

The Prize consists of a sum of money, and is open to men of science resident in or connected with Scotland. The first award was made in the year 1887. The next award will be made in 1924–1925.

In accordance with the wish of the Donor, the Council of the Society may on fit occasions award the Prize for work of a definite kind to be undertaken during the three succeeding years by a scientific man of recognised ability.

V. JAMES SCOTT PRIZE.

This Prize, founded in the year 1918 by the Trustees of the James Scott Bequest, is to be awarded triennially, or at such intervals as the Council of the Royal Society of Edinburgh may decide, "for a lecture or essay on the fundamental concepts of Natural Philosophy."

The first award will be in the year 1922.

* For the purposes of this award the word "presented" shall be understood to mean the date on which the manuscript of a paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

RESOLUTIONS OF COUNCIL IN REGARD TO THE MODE OF AWARDING PRIZES.

(*See Minutes of Meeting of January* 18, 1915.)

I. With regard to the Keith and Makdougall-Brisbane Prizes, which are open to all Sciences, the mode of award will be as follows :—

1. Papers or essays to be considered shall be arranged in two groups, A and B, —Group A to include Astronomy, Chemistry, Mathematics, Metallurgy, Meteorology and Physics; Group B to include Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, and Zoology.
2. These two Prizes shall be awarded to each group in alternate biennial periods, provided papers worthy of recommendation have been communicated to the Society.
3. Prior to the adjudication the Council shall appoint, in the first instance, a Committee composed of representatives of the group of Sciences which did not receive the award in the immediately preceding period. The Committee shall consider the Papers which come within their group of Sciences, and report in due course to the Council.
4. In the event of the aforesaid Committee reporting that within their group of subjects there is, in their opinion, no paper worthy of being recommended for the award, the Council, on accepting this report, shall appoint a Committee representative of the alternate group to consider papers coming within their group and to report accordingly.
5. Papers to be considered by the Committees shall fall within the period dating from the last award in groups A and B respectively.

II. With regard to the Neill Prize, the term "Naturalist" shall be understood to include any student in the Sciences composing group B, namely, Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, Zoology.

AWARDS OF THE KEITH, MAKDOUGALL-BRISBANE, NEILL, AND GUNNING PRIZES.

I. KEITH PRIZE.

1st Biennial Period, 1827–29.—Dr Brewster, for his papers "on his Discovery of Two New Immiscible Fluids in the Cavities of certain Minerals," published in the Transactions of the Society.

2nd Biennial Period, 1829–31.—Dr Brewster, for his paper "on a New Analysis of Solar Light," published in the Transactions of the Society.

3rd Biennial Period, 1831–33.—Thomas Graham, Esq., for his paper "on the Law of the Diffusion of Gases," published in the Transactions of the Society.

4th Biennial Period, 1833–35.—Professor J. D. Forbes, for his paper "on the Refraction and Polarization of Heat," published in the Transactions of the Society.

5th Biennial Period, 1835–37.—John Scott Russell, Esq., for his researches "on Hydrodynamics," published in the Transactions of the Society.

6th Biennial Period, 1837–39.—Mr John Shaw, for his experiments "on the Development and Growth of the Salmon," published in the Transactions of the Society.

7th Biennial Period, 1839–41.—Not awarded.

8th Biennial Period, 1841–43.—Professor James David Forbes, for his papers "on Glaciers," published in the Proceedings of the Society.

9th Biennial Period, 1843–45.—Not awarded.

10th Biennial Period, 1845–47.—General Sir Thomas Brisbane, Bart., for the Makerstoun Observations on Magnetic Phenomena, made at his expense, and published in the Transactions of the Society.

11th Biennial Period, 1847–49.—Not awarded.

12th Biennial Period, 1849–51.—Professor Kelland, for his papers "on General Differentiation, including his more recent Communication on a process of the Differential Calculus, and its application to the solution of certain Differential Equations," published in the Transactions of the Society.

13th Biennial Period, 1851–53.—W. J. Macquorn Rankine, Esq., for his series of papers "on the Mechanical Action of Heat," published in the Transactions of the Society.

14th Biennial Period, 1853–55.—Dr Thomas Anderson, for his papers "on the Crystalline Constituents of Opium, and on the Products of the Destructive Distillation of Animal Substances," published in the Transactions of the Society.

15th Biennial Period, 1855–57.—Professor Boole, for his Memoir "on the Application of the Theory of Probabilities to Questions of the Combination of Testimonies and Judgments," published in the Transactions of the Society.

16th Biennial Period, 1857–59.—Not awarded.

17th Biennial Period, 1859–61.—John Allan Broun, Esq., F.R.S., Director of the Trevandrum Observatory, for his papers "on the Horizontal Force of the Earth's Magnetism, on the Correction of the Bifilar Magnetometer, and on Terrestrial Magnetism generally," published in the Transactions of the Society.

18th Biennial Period, 1861–63.—Professor William Thomson, of the University of Glasgow, for his Communication "on some Kinematical and Dynamical Theorems."

19th Biennial Period, 1863–65.—Principal Forbes, St Andrews, for his "Experimental Inquiry into the Laws of Conduction of Heat in Iron Bars," published in the Transactions of the Society.

20th Biennial Period, 1865–67.—Professor C. Piazzi Smyth, for his paper "on Recent Measures at the Great Pyramid," published in the Transactions of the Society.

21st Biennial Period, 1867–69.—Professor P. G. Tait, for his paper "on the Rotation of a Rigid Body about a Fixed Point," published in the Transactions of the Society.

22nd Biennial Period, 1869–71.—Professor Clerk Maxwell, for his paper "on Figures, Frames, and Diagrams of Forces," published in the Transactions of the Society.

23RD BIENNIAL PERIOD, 1871–73.—Professor P. G. TAIT, for his paper entitled "First Approximation to a Thermo-electric Diagram," published in the Transactions of the Society.

24TH BIENNIAL PERIOD, 1873–1875.—Professor CRUM BROWN, for his Researches "on the Sense of Rotation, and on the Anatomical Relations of the Semicircular Canals of the Internal Ear."

25TH BIENNIAL PERIOD, 1875–77.—Professor M. FORSTER HEDDLE, for his papers "on the Rhombohedral Carbonates," and "on the Felspars of Scotland," published in the Transactions of the Society.

26TH BIENNIAL PERIOD, 1877–79.—Professor H. C. FLEEMING JENKIN, for his paper "on the Application of Graphic Methods to the Determination of the Efficiency of Machinery," published in the Transactions of the Society; Part II having appeared in the volume for 1877–78.

27TH BIENNIAL PERIOD, 1879–81.—Professor GEORGE CHRYSTAL, for his paper "on the Differential Telephone," published in the Transactions of the Society.

28TH BIENNIAL PERIOD, 1881–83.—THOMAS MUIR, Esq., LL.D., for his "Researches into the Theory of Determinants and Continued Fractions," published in the Proceedings of the Society.

29TH BIENNIAL PERIOD, 1883–85.—JOHN AITKEN, Esq., for his paper "on the Formation of Small Clear Spaces in Dusty Air," and for previous papers on Atmospheric Phenomena, published in the Transactions of the Society.

30TH BIENNIAL PERIOD, 1885–87.—JOHN YOUNG BUCHANAN, Esq., for a series of communications, extending over several years, on subjects connected with Ocean Circulation, Compressibility of Glass, etc.; two of which, viz., "On Ice and Brines," and "On the Distribution of Temperature in the Antarctic Ocean," have been published in the Proceedings of the Society.

31ST BIENNIAL PERIOD, 1887–89.—Professor E. A. LETTS, for his papers on the Organic Compounds of Phosphorus, published in the Transactions of the Society.

32ND BIENNIAL PERIOD, 1889–91.—R. T. OMOND, Esq., for his contributions to Meteorological Science, many of which are contained in vol. xxxiv of the Society's Transactions.

33RD BIENNIAL PERIOD, 1891–93.—Professor THOMAS R. FRASER, F.R.S., for his papers on *Strophanthus hispidus*, Strophanthin, and Strophanthidin, read to the Society in February and June 1889 and in December 1891, and printed in vols. xxxv, xxxvi, and xxxvii of the Society's Transactions.

34TH BIENNIAL PERIOD, 1893–95.—Dr CARGILL G. KNOTT, for his papers on the Strains produced by Magnetism in Iron and in Nickel, which have appeared in the Transactions and Proceedings of the Society.

35TH BIENNIAL PERIOD, 1895–97.—Dr THOMAS MUIR, for his continued communications on Determinants and Allied Questions.

36TH BIENNIAL PERIOD, 1897–99.—Dr JAMES BURGESS, for his paper "on the Definite Integral $\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$, with extended Tables of Values," printed in vol. xxxix of the Transactions of the Society.

37TH BIENNIAL PERIOD, 1899–1901.—Dr HUGH MARSHALL, for his discovery of the Persulphates, and for his Communications on the Properties and Reactions of these Salts, published in the Proceedings of the Society.

38TH BIENNIAL PERIOD, 1901–03.—Sir WILLIAM TURNER, K.C.B., LL.D., F.R.S., etc., for his memoirs entitled "A Contribution to the Craniology of the People of Scotland," published in the Transactions of the Society, and for his "Contributions to the Craniology of the People of the Empire of India," Parts I, II, likewise published in the Transactions of the Society.

39TH BIENNIAL PERIOD, 1903–05.—THOMAS H. BRYCE, M.A., M.D., for his two papers on "The Histology of the Blood of the Larva of *Lepidosiren paradoxa*," published in the Transactions of the Society within the period.

40TH BIENNIAL PERIOD, 1905–07.—ALEXANDER BRUCE, M.A., M.D., F.R.C.P.E., for his paper entitled "Distribution of the Cells in the Intermedio-Lateral Tract of the Spinal Cord," published in the Transactions of the Society within the period.

41ST BIENNIAL PERIOD, 1907–09.—WHEELTON HIND, M.D., B.S., F.R.C.S., F.G.S., for a paper published in the Transactions of the Society, "On the Lamellibranch and Gasteropod Fauna found in the Millstone Grit of Scotland."

42ND BIENNIAL PERIOD, 1909–11.—Professor ALEXANDER SMITH, B.Sc., Ph.D., of New York, for his researches upon "Sulphur" and upon "Vapour Pressure," appearing in the Proceedings of the Society.

43RD BIENNIAL PERIOD, 1911–1913.—JAMES RUSSELL, Esq., for his series of investigations relating to magnetic phenomena in metals and the molecular theory of magnetism, the results of which have been published in the Proceedings and Transactions of the Society, the last paper having been issued within the period.

44TH BIENNIAL PERIOD, 1913–15.—JAMES HARTLEY ASHWORTH, D.Sc., for his papers on "Larvæ of Lingula and Pelagodiscus," and on "Sclerocheilus," published in the Transactions of the Society, and for other papers on the Morphology and Histology of Polychæta.

45TH BIENNIAL PERIOD, 1915–17.—ROBERT C. MOSSMAN, for his work on the Meteorology of the Antarctic Regions, which originated with the important series of observations made by him during the voyage of the "Scotia" (1902–1904), and includes his paper "On a Sea-Saw of Barometric Pressure, Temperature, and Wind Velocity between the Weddell Sea and the Ross Sea," published in the Proceedings of the Society.

46TH BIENNIAL PERIOD, 1917–19.—JOHN STEPHENSON, Lt.-Col. I.M.S., for his series of papers on the Oligochæta and other Annelida, several of which have been published in the Transactions of the Society.

II. MAKDOUGALL-BRISBANE PRIZE.

1ST BIENNIAL PERIOD, 1859.—Sir RODERICK IMPEY MURCHISON, on account of his Contributions to the Geology of Scotland.

2ND BIENNIAL PERIOD, 1860–62.—WILLIAM SELLER, M.D., F.R.C.P.E., for his "Memoir of the Life and Writings of Dr Robert Whytt," published in the Transactions of the Society.

3RD BIENNIAL PERIOD, 1862–64.—JOHN DENIS MACDONALD, Esq., R.N., F.R.S., Surgeon of H.M.S. "Icarus," for his paper "on the Representative Relationships of the Fixed and Free Tunicata, regarded as Two Sub-classes of equivalent value; with some General Remarks on their Morphology," published in the Transactions of the Society.

4TH BIENNIAL PERIOD, 1864–66.—Not awarded.

5TH BIENNIAL PERIOD, 1866–68.—Dr ALEXANDER CRUM BROWN and Dr THOMAS RICHARD FRASER, for their conjoint paper "on the Connection between Chemical Constitution and Physiological Action," published in the Transactions of the Society.

6TH BIENNIAL PERIOD, 1868–70.—Not awarded.

7TH BIENNIAL PERIOD, 1870–72.—GEORGE JAMES ALLMAN, M.D., F.R.S., Emeritus Professor of Natural History, for his paper "on the Homological Relations of the Cœlenterata," published in the Transactions, which forms a leading chapter of his Monograph of Gymnoblastic or Tubularian Hydroids—since published.

8TH BIENNIAL PERIOD, 1872–74.—Professor LISTER, for his paper "on the Germ Theory of Putrefaction and the Fermentive Changes," communicated to the Society, 7th April 1873.

9TH BIENNIAL PERIOD, 1874–76.—ALEXANDER BUCHAN, A.M., for his paper "on the Diurnal Oscillation of the Barometer," published in the Transactions of the Society.

10TH BIENNIAL PERIOD, 1876–78.—Professor ARCHIBALD GEIKIE, for his paper "on the Old Red Sandstone of Western Europe," published in the Transactions of the Society.

11TH BIENNIAL PERIOD, 1878–80.—Professor PIAZZI SMYTH, Astronomer-Royal for Scotland, for his paper "on the Solar Spectrum in 1877–78, with some Practical Idea of its probable Temperature of Origination," published in the Transactions of the Society.

12TH BIENNIAL PERIOD, 1880–82.—Professor JAMES GEIKIE, for his "Contributions to the Geology of the North-West of Europe," including his paper "on the Geology of the Faroes," published in the Transactions of the Society.

13TH BIENNIAL PERIOD, 1882–84.—EDWARD SANG, Esq., LL.D., for his paper "on the Need of Decimal Subdivisions in Astronomy and Navigation, and on Tables requisite therefor," and generally for his Recalculations of Logarithms both of Numbers and Trigonometrical Ratios, —the former communication being published in the Proceedings of the Society.

14TH BIENNIAL PERIOD, 1884–86.—JOHN MURRAY, Esq., LL.D., for his papers "On the Drainage Areas of Continents, and Ocean Deposits," "The Rainfall of the Globe, and Discharge of Rivers," "The Height of the Land and Depth of the Ocean," and "The Distribution of Temperature in the Scottish Lochs as affected by the Wind."

15TH BIENNIAL PERIOD, 1886–88.—ARCHIBALD GEIKIE, Esq., LL.D., for numerous Communications, especially that entitled "History of Volcanic Action during the Tertiary Period in the British Isles," published in the Transactions of the Society.

16TH BIENNIAL PERIOD, 1889–90.—Dr LUDWIG BECKER, for his paper on "The Solar Spectrum at Medium and Low Altitudes," printed in vol. xxxvi, Part I, of the Society's Transactions.

17th Biennial Period, 1890–92.—Hugh Robert Mill, Esq., D.Sc., for his papers on "The Physical Conditions of the Clyde Sea Area," Part I being already published in vol. xxxvi of the Society's Transactions.

18th Biennial Period, 1892–94.—Professor James Walker, D.Sc., Ph.D., for his work on Physical Chemistry, part of which has been published in the Proceedings of the Society, vol. xx, pp. 255–263. In making this award, the Council took into consideration the work done by Professor Walker along with Professor Crum Brown on the Electrolytic Synthesis of Dibasic Acids, published in the Transactions of the Society.

19th Biennial Period, 1894–96.—Professor John G. M'Kendrick, for numerous Physiological papers, especially in connection with Sound, many of which have appeared in the Society's publications.

20th Biennial Period, 1896–98.—Dr William Peddie, for his papers on the Torsional Rigidity of Wires.

21st Biennial Period, 1898–1900.—Dr Ramsay H. Traquair, for his paper entitled "Report on Fossil Fishes collected by the Geological Survey in the Upper Silurian Rocks of Scotland," printed in vol. xxxix of the Transactions of the Society.

22nd Biennial Period, 1900–02.—Dr Arthur T. Masterman, for his paper entitled "The Early Development of *Cribrella oculata* (Forbes), with remarks on Echinoderm Development," printed in vol. xl of the Transactions of the Society.

23rd Biennial Period, 1902–04.—Mr John Dougall, M.A., for his paper on "An Analytical Theory of the Equilibrium of an Isotropic Elastic Plate," published in vol. xli of the Transactions of the Society.

24th Biennial Period, 1904–06.—Jacob E. Halm, Ph.D., for his two papers entitled "Spectroscopic Observations of the Rotation of the Sun," and "Some Further Results obtained with the Spectroheliometer," and for other astronomical and mathematical papers published in the Transactions and Proceedings of the Society within the period.

25th Biennial Period, 1906–08.—D. T. Gwynne-Vaughan, M.A., F.L.S., for his papers, 1st, "On the Fossil Osmundaceæ," and 2nd, "On the Origin of the Adaxially-curved Leaf-trace in the Filicales," communicated by him conjointly with Dr R. Kidston.

26th Biennial Period, 1908–10.—Ernest MacLagan Wedderburn, M.A., LL.B., for his series of papers bearing upon "The Temperature Distribution in Fresh-water Lochs," and especially upon "The Temperature Seiche."

27th Biennial Period, 1910–12.—John Brownlee, M.A., M.D., D.Sc., for his contributions to the Theory of Mendelian Distributions and cognate subjects, published in the Proceedings of the Society within and prior to the prescribed period.

28th Biennial Period, 1912–14.—Professor C. R. Marshall, M.D., M.A., for his studies "On the Pharmacological Action of Tetra-alkyl-ammonium Compounds."

29th Biennial Period, 1914–16.—Robert Alexander Houstoun, Ph.D., D.Sc., for his series of papers on "The Absorption of Light by Inorganic Salts," published in the Proceedings of the Society.

30th Biennial Period, 1916–18.—Professor A. Anstruther Lawson for his Memoirs on "The Prothalli of *Tmesipteris Tannensis* and of *Psilotum*," published in the Transactions of the Society, together with previous papers on Cytology and on The Gametophytes of various Gymnosperms.

31st Biennial Period, 1918–20.—Professor J. H. Maclagan Wedderburn of Princeton University for his Memoirs in Universal Algebra, etc., published in the Transactions and Proceedings of the Society, and elsewhere.

III. THE NEILL PRIZE.

1st Triennial Period, 1856–59.—Dr W. Lauder Lindsay, for his paper "on the Spermogones and Pycnides of Filamentous, Fruticulose, and Foliaceous Lichens," published in the Transactions of the Society.

2nd Triennial Period, 1859–61.—Robert Kaye Greville, LL.D., for his contributions to Scottish Natural History, more especially in the department of Cryptogamic Botany, including his recent papers on Diatomaceæ.

3rd Triennial Period, 1862–65.—Andrew Crombie Ramsay, F.R.S., Professor of Geology in the Government School of Mines, and Local Director of the Geological Survey of Great Britain, for his various works and memoirs published during the last five years, in which he has applied the large experience acquired by him in the Direction of the arduous work of the Geological Survey of Great Britain to the elucidation of important questions bearing on Geological Science.

4th Triennial Period, 1865–68.—Dr William Carmichael M'Intosh, for his paper "on the Structure of the British Nemerteans, and on some New British Annelids," published in the Transactions of the Society.

5th Triennial Period, 1868–71.—Professor William Turner, for his papers "on the Great Finner Whale; and on the Gravid Uterus, and the Arrangement of the Fœtal Membranes in the Cetacea," published in the Transactions of the Society.

6th Triennial Period, 1871–74.—Charles William Peach, Esq., for his Contributions to Scottish Zoology and Geology, and for his recent contributions to Fossil Botany.

7th Triennial Period, 1874–77.—Dr Ramsay H. Traquair, for his paper "on the Structure and Affinities of *Tristichopterus alatus* (Egerton)," published in the Transactions of the Society, and also for his contributions to the Knowledge of the Structure of Recent and Fossil Fishes.

8th Triennial Period, 1877–80.—John Murray, Esq., for his paper "on the Structure and Origin of Coral Reefs and Islands," published (in abstract) in the Proceedings of the Society.

9th Triennial Period, 1880–83.—Professor Herdman, for his papers "on the Tunicata," published in the Proceedings and Transactions of the Society.

10th Triennial Period, 1883–86.—B. N. Peach, Esq., for his Contributions to the Geology and Palæontology of Scotland, published in the Transactions of the Society.

11th Triennial Period, 1886–89.—Robert Kidston, Esq., for his Researches in Fossil Botany, published in the Transactions of the Society.

12th Triennial Period, 1889–92.—John Horne, Esq., F.G.S., for his Investigations into the Geological Structure and Petrology of the North-West Highlands.

13th Triennial Period, 1892–95.—Robert Irvine, Esq., for his papers on the Action of Organisms in the Secretion of Carbonate of Lime and Silica, and on the solution of these substances in Organic Juices. These are printed in the Society's Transactions and Proceedings.

14th Triennial Period, 1895–98.—Professor Cossar Ewart, for his recent Investigations connected with Telegony.

15th Triennial Period, 1898–1901.—Dr John S. Flett, for his papers entitled "The Old Red Sandstone of the Orkneys" and "The Trap Dykes of the Orkneys," printed in vol. xxxix of the Transactions of the Society.

16th Triennial Period, 1901–04.—Professor J. Graham Kerr, M.A., for his Researches on *Lepidosiren paradoxa*, published in the Philosophical Transactions of the Royal Society, London.

17th Triennial Period, 1904–07.—Frank J. Cole, B.Sc., for his paper entitled "A Monograph on the General Morphology of the Myxinoid Fishes, based on a Study of Myxine," published in the Transactions of the Society, regard being also paid to Mr Cole's other valuable contributions to the Anatomy and Morphology of Fishes.

1st Biennial Period, 1907–09.—Francis J. Lewis, M.Sc., F.L.S., for his papers in the Society's Transactions "On the Plant Remains of the Scottish Peat Mosses."

2nd Biennial Period, 1909–11.—James Murray, Esq., for his paper on "Scottish Rotifers collected by the Lake Survey (Supplement)," and other papers on the "Rotifera" and "Tardigrada," which appeared in the Transactions of the Society—(this Prize was awarded after consideration of the papers received within the five years prior to the time of award: see Neill Prize Regulations).

3rd Biennial Period, 1911–13.—Dr W. S. Bruce, in recognition of the scientific results of his Arctic and Antarctic explorations.

4th Biennial Period, 1913–15.—Robert Campbell, D.Sc., for his paper on "The Upper Cambrian Rocks at Craigeven Bay, Stonehaven," and "Downtonian and Old Red Sandstone Rocks of Kincardineshire," published in the Transactions of the Society.

5th Biennial Period, 1915–17.—W. H. Lang, F.R.S., M.B., D.Sc., for his paper in conjunction with Dr R. Kidston, F.R.S., on *Rhynia Gwynne-Vaughani*, Kidston and Lang, published in the Transactions of the Society, and for his previous investigations on Pteridophytes and Cycads.

6th Biennial Period, 1917–19.—John Tait, D.Sc., M.D., for his work on Crustacea, published in the Proceedings of the Society, and for his papers on the blood of Crustacea.

IV. GUNNING VICTORIA JUBILEE PRIZE.

1st Triennial Period, 1884–87.—Sir William Thomson, Pres. R.S.E., F.R.S., for a remarkable series of papers "on Hydrokinetics," especially on Waves and Vortices, which have been communicated to the Society.

2nd Triennial Period, 1887–90.—Professor P. G. Tait, Sec. R.S.E., for his work in connection with the "Challenger" Expedition, and his other Researches in Physical Science.

3rd Triennial Period, 1890–93.—Alexander Buchan, Esq., LL.D., for his varied, extensive, and extremely important Contributions to Meteorology, many of which have appeared in the Society's publications.

4th Triennial Period, 1893–96.—John Aitken, Esq., for his brilliant Investigations in Physics, especially in connection with the Formation and Condensation of Aqueous Vapour.

1st Quadrennial Period, 1896–1900.—Dr T. D. Anderson, for his discoveries of New and Variable Stars.

2nd Quadrennial Period, 1900–04.—Sir James Dewar, LL.D., D.C.L., F.R.S., etc., for his researches on the Liquefaction of Gases, extending over the last quarter of a century, and on the Chemical and Physical Properties of Substances at Low Temperatures: his earliest papers being published in the Transactions and Proceedings of the Society.

3rd Quadrennial Period, 1904–08.—Professor George Chrystal, M.A., LL.D., for a series of papers on "Seiches," including "The Hydrodynamical Theory and Experimental Investigations of the Seiche Phenomena of Certain Scottish Lakes."

4th Quadrennial Period, 1908–12.—Professor J. Norman Collie, Ph.D., F.R.S., for his distinguished contributions to Chemistry, Organic and Inorganic, during twenty-seven years, including his work upon Neon and other rare gases. Professor Collie's early papers were contributed to the Transactions of the Society.

5th Quadrennial Period, 1912–16.—Sir Thos. Muir, C.M.G., LL.D., F.R.S., for his series of Memoirs upon "The Theory and History of Determinants and Allied Forms," published in the Transactions and Proceedings of the Society between the years 1872 and 1915.

6th Quadrennial Period, 1916–20.—C. T. R. Wilson, Esq., F.R.S., in recognition of his important discoveries in relation to Condensation Nuclei, Ionisation of Gases, and Atmospheric Electricity.

ABSTRACT

OF

THE ACCOUNTS OF JAMES CURRIE, ESQ., LL.D.

As Treasurer of the Royal Society of Edinburgh.

SESSION 1920–1921.

I. GENERAL FUND.

CHARGE.

1. Arrears of Contributions at 30th September 1920		£92 8 0
2. Contributions for present Session:—		
1. 234 Fellows at £2, 2s. each	£491 8 0	
51 Fellows at £3, 3s. each	160 13 0	
	£652 1 0	
Deduct—		
Commutation of Contributions of one Fellow—proportion thereof included in above	2 2 0	
	£649 19 0	
2. Fees of Admission and Contributions of twenty-five new Fellows at £6, 6s. each	157 10 0	
3. Commutation Fee in lieu of future Contributions of one Fellow	7 17 6	
		815 6 6
3. Extra Contributions for 1920–1921 under Amended Law, No. 6—		
Voluntary Contributions—34 Fellows at £1, 1s. each	£35 14 0	
Additional Contributions—173 Fellows at £1, 1s. each	181 13 0	
Commutations—12 Fellows at £10, 10s. each	126 0 0	
		343 7 0
4. Interest received—		
Interest on £7830 five per cent. War Loan, 1929–47, Untaxed	£391 10 0	
Annuity from Edinburgh and District Water Trust, less Tax, £15, 15s.	36 15 0	
Interest on Deposit Receipts	55 19 10	
		484 4 10
5. Transactions and Proceedings		101 7 5
6. Annual Grant from Government		600 0 0
7. Income Tax repaid for year to 5th April 1921		15 15 0
8. Receipts from Sale of Napier Tercentenary Memorial Volume		4 6 7
9. Anonymous Donation towards Napier Tercentenary Expenses		5 0 0
Amount of the Charge		**£2461 15 4**

DISCHARGE.

1. Taxes, Insurance, Coal and Lighting:—		
Inhabited House Duty	£0 6 3	
Insurance	25 2 4	
Coal, etc., to 18th March 1921	38 8 8	
Gas to 10th May 1921	4 19 0	
Electric Light to 15th September 1921	9 19 4	
Water, 1920–21	4 4 0	
		£82 19 7
Carry forward		£82 19 7

2. Salaries :—	Brought forward . .		£82 19 7
General Secretary, 1920–21		£100 0 0	
Librarian and Assistant Secretary		252 0 0	
Assistant Librarian		49 16 8	
Office Keeper		122 9 0	
Treasurer's Clerk		35 0 0	
			559 5 8
3. Expenses of Transactions :—			
Neill & Co., Ltd., Printers		£269 12 0	
Hislop & Day, Ltd., Engravers		31 12 1	
M'Farlane & Erskine		16 3 6	
The Zinco-Collotype Co.		56 7 6	
Less Receipts—		£373 15 1	
Grants by Carnegie Trustees towards Messrs Thompson's, Kidston's, and Gordon's Papers	£170 0 0		
Grant by Scott Polar Research Fund towards J. M. Wordie's Papers	161 13 3		
		331 13 3	
			42 1 10
4. Expenses of Proceedings :—			
Neill & Co., Ltd., Printers		£560 10 0	
Hislop & Day, Ltd., Engravers		19 5 1	
Macmillan & Co., Ltd.		0 8 0	
Less—		£580 3 1	
Carnegie Trustees—Grant towards Dr J. R. Milne's Paper		3 17 6	
			576 5 7
5. Books, Periodicals, Newspapers, etc. :—			
James Thin, Bookseller		£148 7 1	
R. Grant & Son, Booksellers		6 8 9	
W. Green & Son, Ltd., Booksellers		1 10 0	
Robertson & Scott, News Agents		8 12 7	
Ray Society, Subscription		1 1 0	
Palæontographical Society, Do.		1 1 0	
Board of Scientific Societies, London, Donation		5 0 0	
Williams & Norgate		2 0 0	
Dowell's, Ltd.		0 2 6	
Edinburgh and Leith Post Office Directory, Ltd.		0 16 6	
			174 19 5
6. Orrock & Son, Bookbinders		£2000 0 0	
Less—Received from Carnegie Trust		2000 0 0	
7. Loan to Makerstoun Magnetic Meteorological Observation Fund £100, *less* repaid £48, 12s. 11d.		£51 7 1	
8. Other Payments :—			
Neill & Co., Ltd., Printers		£133 7 11	
J. & T. Scott		37 3 6	
S. Duncan & Sons, Tailors (uniforms)		11 19 0	
S. Heddle—Bonus		25 0 0	
Andrew H. Baird		10 11 0	
Lindsay, Jamieson & Haldane, C.A., Auditors		10 10 0	
A. Muirhead & Son		99 12 6	
A. Cowan & Sons, Ltd.		8 18 4	
Orrock & Son, Bookbinders		8 15 0	
Gillies & Wright, Joiners		8 2 0	
R. Graham, Slater		17 0 0	
Burn Brothers, Plumbers		2 6 8	
A. Black & Co., Brushmakers		5 12 0	
R. G. Hislop		5 5 6	
G. Waterston & Sons, Ltd.		6 0 6	
Jas. Gray & Son		3 10 6	
Travelling Expenses of Delegate to London		11 11 8	
W. Blackwood & Sons		6 5 6	
John Wright		22 12 8	
Petty Expenses, Postages, Carriage, etc.		243 0 3	
			677 4 6
	Carry forward . .		£2112 16 7

Brought forward . . .		£2112 16 7
9. Arrears of Contributions written off		17 17 0
10. Arrears of Contributions outstanding at 30th September 1921:—		
Present Session	£50 8 0	
Previous Sessions	35 14 0	
		86 2 0
Amount of the Discharge . .		£2216 15 7

Amount of the Charge	£2461 15 4
Amount of the Discharge	2216 15 7
Excess of Receipts over Payments for 1920–1921 transferred **to Special Subscription Fund**	£244 19 9

SPECIAL SUBSCRIPTION FUND.

To 30th September 1921.

Charge.

1. Balance at 30th September 1920:—	
Due by Union Bank of Scotland, Ltd., on Deposit Receipt	£700 0 0
Due by Union Bank of Scotland, Ltd., on Account Current	88 11 11
Due by Treasurer	3 19 6
	£792 11 5
2. Amount transferred from General Fund being surplus for year to 30th September 1921	244 19 9
	£1037 11 2

Discharge.

1. Balance at 30th September 1921:—	
Due by Union Bank of Scotland, Ltd., on Deposit Receipt	£350 0 0
Due by Union Bank of Scotland, Ltd., on Account Current	629 18 8
Balance of Loan to Makerstoun Magnetic Meteorological Observation Fund .	51 7 1
Due by Treasurer	6 5 5
	£1037 11 2

Note:—

Total Subscriptions received towards Special Subscription Fund		£1128 17 9
Sums transferred to General Fund to meet Deficiency on Accounts—		
To 30th September 1919	£301 16 11	
To 30th September 1920	34 9 5	
	£336 6 4	
Amount transferred from General Fund, being Surplus for Year to 30th September 1921	244 19 9	
Net Amount transferred to General Fund out of Special Subscription Fund as at 30th September 1921 . .	£91 6 7	
		91 6 7
Amount of Special Subscription Fund at 30th September 1921, as above		. £1037 11 2

II. KEITH FUND

To 30th September 1921.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Account Current at 30th September 1920	£39 14 10
2. Interest Received:—	
On £650 five per cent. War Loan, 1929–47, Untaxed	32 10 0
	£72 4 10

DISCHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1921		£72 4 10

III. NEILL FUND

To 30th September 1921.

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current, at 30th September 1920		£15 10 6
2. INTEREST RECEIVED :—		
On £300 five per cent. War Loan, 1929–47, Untaxed		15 0 0
		£30 10 6

DISCHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1921		£30 10 6

IV. MAKDOUGALL-BRISBANE FUND

To 30th September 1921.

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., at 30th September 1920 :—		
On Deposit Receipt		£35 15 6
On Account Current		20 0 0
		£55 15 6
2. INTEREST RECEIVED :—		
On £400 five per cent. War Loan, 1929–47, Untaxed	£20 0 0	
On Deposit Receipt—Union Bank of Scotland, Ltd.	2 18 5	
		22 18 5
		£78 13 11

DISCHARGE.

1. Prof. J. H. M. Wedderburn—Money Portion of Prize, 1918–20		£15 0 0
2. Alex. Kirkwood & Son, Engravers, Gold Medal		22 10 0
3. BALANCE due by Union Bank of Scotland, Ltd., at 30th September 1921 :—		
On Deposit Receipt	£38 13 11	
On Account Current	2 10 0	
		41 3 11
		£78 13 11

V. MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND

To 30th September 1921.

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 30th September 1920		£36 2 11
2. INTEREST RECEIVED :—		
On £250 five per cent. War Loan, 1929–47, Untaxed		12 10 0
3. BORROWED from General Fund £100, *less* repaid £48, 12s. 11d	£51 7 1	
4. BALANCE due to General Fund at 30th September 1921		51 7 1
		£100 0 0

DISCHARGE.

1. DONATION to British Association for the Advancement of Science		£100 0 0

VI. GUNNING VICTORIA JUBILEE PRIZE FUND

To 30th September 1921.

(Instituted by Dr R. H. Gunning of Edinburgh and Rio de Janeiro.)

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., at 30th September 1920 :—		
On Deposit Receipt		£57 14 2
On Account Current		71 5 0
		£128 19 2
2. Interest Received :—		
On £570 five per cent. War Loan, 1929–47, Untaxed	£28 10 0	
On Deposit Receipt—Union Bank of Scotland, Ltd.	4 14 11	
		33 4 11
		£162 4 1

DISCHARGE.

1. C. T. R. Wilson, Esq.—Money Portion of Prize 1916–20	£105 0 0
2. Balance due by Union Bank of Scotland. Ltd., on Deposit Receipt, at 30th September 1921	57 4 1
	£162 4 1

VII. JAMES SCOTT PRIZE FUND

To 30th September 1921.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1920	£264 4 0
2. Interest Received :—	
On Deposit Receipt—Union Bank of Scotland, Ltd.	22 10 6
	£286 14 6

DISCHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1921	£286 14 6

VIII. DR JOHN AITKEN FUND

(For Publication of his Scientific Work.)

To 30th September 1921.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1920	£1040 1 11
2. Interest Received :—	
On Deposit Receipts, Union Bank of Scotland, Ltd.	43 6 0
	£1083 7 11

DISCHARGE.

1. Accounts Paid :—		
Zinco Collotype Co., Portraits	£20 0 0	
Hislop & Day, Ltd., Line Blocks	21 19 1	
		£41 19 1
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1921		1041 8 10
		£1083 7 11

STATE OF THE FUNDS BELONGING TO THE ROYAL SOCIETY OF EDINBURGH

As at 30th *September* 1921.

1. GENERAL FUND—

1. £7830 five per cent. War Loan, 1929–47, at 88½ per cent.			£6929	11	0
2. £52, 10s. Annuity of the Edinburgh and District Water Trust, equivalent to £875 at 100 per cent.			875	0	0
3. Deposit Receipt Union Bank of Scotland, Ltd., being balance of Legacy received during 1917–18, from the Trustees of the late Mr Robert Mackay Smith, £500 *less* legacy duty £50			450	0	0
4. Arrears of Contributions, as per preceding Abstract of Accounts			86	2	0
5. Balance of Special Subscription Fund			1037	11	2
6. Balance of Loan to Makerstoun Magnetic Meteorological Observation Fund			51	7	1
Amount			£9429	11	3

Exclusive of Library, Museum, Pictures, etc., and Furniture in the Society's Rooms at George Street, Edinburgh.

2. KEITH FUND—

1. £650 five per cent. War Loan, 1929–47, at 88½ per cent.			£575	5	0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt			72	4	10
Amount			£647	9	10

3. NEILL FUND—

1. £300 five per cent. War Loan, 1929–47, at 88½ per cent.			£265	10	0
2. Balance due by Union Bank of Scotland, Ltd. on Deposit Receipt			30	10	6
Amount			£296	0	6

4. MAKDOUGALL-BRISBANE FUND—

1. £400 five per cent. War Loan, 1929–47, at 88½ per cent.				£354	0	0
2. Balance due by Union Bank of Scotland, Ltd. :—						
On Deposit Receipt	£38	13	11			
On Account Current	2	10	0			
				41	3	11
Amount				£395	3	11

5. MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND—

1. £250 five per cent. War Loan, 1929–47, at 88½ per cent.	£221	5	0
Less—Balance of Loan from General Fund	51	7	1
Amount	£169	17	11

6. GUNNING VICTORIA JUBILEE PRIZE FUND—Instituted by Dr Gunning of Edinburgh and Rio de Janeiro—

1. £570 five per cent. War Loan, 1929–47, at 88½ per cent.	£504	9	0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	57	4	1
Amount	£561	13	1

7. JAMES SCOTT PRIZE FUND—

Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	£286	14	6

8. TAIT MEMORIAL FUND—

This Fund consists of War Loan, and is to mature for a period of about ten years from 1918, when it is expected to yield about £75 per annum.

9. DR JOHN AITKEN FUND—

Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt . . .	£1041 8 10

EDINBURGH, 13*th October* 1921.—We have examined the preceding Accounts of the Treasurer of the Royal Society of Edinburgh for the Session 1920–1921, and have found them to be correct. The securities of the various Investments at 30th September 1921, as noted in the foregoing Statement of Funds (with the exception of No. 8), have been exhibited to us.

LINDSAY, JAMIESON & HALDANE, C.A.,
Auditors.

LIST OF VOLUNTARY CONTRIBUTORS OF TEN GUINEAS (Single Payment) under Law VI (end of para. 3) up to 30th September 1921.

Dr John Alison.
Sir James Dewar, F.R.S.
Mr W. F. King.
Prof. Graham Lusk.
Geo. A. Mitchell, M.A.
Sir F. G. Ogilvie, LL.D.
Sir Wm. Peck.
Alex. Philip, M.A., LL.B.
Dr J. Stephenson.
Prof. E. Talmage.
Dr A. F. Tredgold.
James Watt, W.S.

LIST OF VOLUNTARY CONTRIBUTORS OF ONE GUINEA under Law VI (end of para. 3) up to 30th September 1921.

The late R. G. Alford, M.Inst.C.E.
Maj.-Gen. W. B. Bannerman.
G. W. W. Barclay, M.A.
Prof. T. Hudson Beare.
J. P. F. Bell, F.Z.S.
Prof. A. C. Boon.
Mr T. C. Day.
Sir Arch. Denny.
Dr L. Dobbin.
John S. Ford, F.C.S.
Dr John Fraser.
Prof. Andrew Gray, F.R.S.
Sir R. B. Greig.
Dr D. Fraser Harris.
Prof. Sir W. A. Herdman, F.R.S.
Dr John Horne, F.R.S.
Dr W. F. Hume.
Sir J. H. Kemnal.
Dr R. Kidston, F.R.S.
Em.-Prof. Sir P. R. Scott Lang.
Dr D. F. Lowe.
Dr A. M. M'Aldowie.
Dr P. M'Bride.
Dr Geo. M'Gowan.
Dr John Macintyre.
Dr H. R. Mill.
Prof. W. Peddie.
H. A. Reid, O.B.E., F.R.C.V.S.
Prof. Sutherland Simpson.
Mr James Sorley.
Prof. Sir J. Walker, F.R.S.
A. C. Wilson, F.C.S.
Mr J. C. Wright.
Sir R. P. Wright.

THE COUNCIL OF THE SOCIETY.

October 1921.

PRESIDENT.

PROFESSOR FREDERICK O. BOWER, M.A., D.Sc., LL.D., F.R.S., F.L.S.

VICE-PRESIDENTS.

SIR GEORGE A. BERRY, M.B., C.M., LL.D., F.R.C.S.E.

PROFESSOR WILLIAM PEDDIE, D.Sc., Professor of Natural Philosophy in University College, Dundee.

PRINCIPAL SIR JAMES ALFRED EWING, K.C.B., M.A., D.Sc., LL.D., M.Inst.C.E., F.R.S., Principal, University of Edinburgh.

PROFESSOR JOHN WALTER GREGORY, D.Sc., F.R.S., Professor of Geology, University of Glasgow.

MAJOR-GENERAL W. B. BANNERMAN, C.S.I., I.M.S., M.D., D.Sc.

W. A. TAIT, D.Sc., M.Inst.C.E.

GENERAL SECRETARY.

CARGILL G. KNOTT, D.Sc., LL.D., F.R.S.

SECRETARIES TO ORDINARY MEETINGS.

PROFESSOR E. T. WHITTAKER, Sc.D., F.R.S., Professor of Mathematics, University, Edinburgh.

PROFESSOR J. H. ASHWORTH, D.Sc., F.R.S., Professor of Zoology, University, Edinburgh.

TREASURER.

JAMES CURRIE, M.A., LL.D.

CURATOR OF LIBRARY AND MUSEUM.

A. CRICHTON MITCHELL, D.Sc., Hon. D.Sc. (Geneva).

COUNCILLORS.

HENRY MOUBRAY CADELL, of Grange, B.Sc.

PROFESSOR ARTHUR ROBERTSON CUSHNY, M.A., M.D., LL.D., F.R.S.

PROFESSOR FRANCIS GIBSON BAILY, M.A., M.Inst.E.E.

GEORGE JAMES LIDSTONE, F.F.A., F.I.A.

ROBERT CAMPBELL, M.A., D.Sc., F.G.S.

PRINCIPAL JAMES COLQUHOUN IRVINE, C.B.E., Ph.D., D.Sc., F.R.S.

THE HON. LORD SALVESEN.

PROFESSOR J. ARTHUR THOMSON, M.A., LL.D.

HERBERT STANLEY ALLEN, M.A., D.Sc.

SIR ROBERT BLYTH GREIG, M.A., LL.D., F.Z.S.

JAMES RITCHIE, D.Sc.

ERNEST MACLAGAN WEDDERBURN, M.A., LL.B., W.S., D.Sc.

ALPHABETICAL LIST OF THE ORDINARY FELLOWS OF THE SOCIETY,

Corrected to January 31, 1922.

N.B.—*Those marked* * *are Annual Contributors.*
,, ,, † *have commuted Voluntary Contribution* (*see* 3*rd Paragraph, Law VI*).

B. prefixed to a name indicates that the Fellow has received a Makdougall-Brisbane Medal.
K. ,, ,, ,, Keith Medal.
N. ,, ,, ,, Neill Medal.
V. J. ,, ,, ,, the Gunning Victoria Jubilee Prize.
C. ,, ,, ,, contributed one or more Communications to the Society's TRANSACTIONS or PROCEEDINGS.

Date of Election.			Service on Council, etc.
1898	C.	* Abercromby, the Rt. Hon. Lord, LL.D., 62 Palmerston Place, Edinburgh	
1898		Adami, J. G., C.B.E., M.A., M.D. (Camb., M'Gill, and Belfast), LL.D., F.R.S., Vice-Chancellor of the University of Liverpool.	
1896		† Affleck, Sir Jas. Ormiston, M.D., LL.D., F.R.C.P.E., 38 Heriot Row, Edinburgh	
1889		† Alison, John, M.A., LL.D., Head Master, George Watson's College, Edinburgh	
1894		Allan, Francis John, M.D., C.M., Edin., M.O.H. City of Westminster, Westminster City Hall, Charing Cross Road, London 5	
1888	C.	Allardice, R. E., M.A., Professor of Mathematics in Stanford University, Palo Alto, Santa Clara Co., California	
1920	C.	* Allen, Herbert Stanley, M.A. (Cambridge), D.Sc. (London), Lecturer in Natural Philosophy in the University of Edinburgh	1921–
1906		Anderson, Daniel E., M.D., B.A., B.Sc., Green Bank, Merton Lane, Highgate, London, N.	
1920	C.	* Anderson, Ernest Masson, M.A., B.Sc., F.G.S., Geologist, H.M. Geological Survey of Scotland, 5 Riselaw Road, Edinburgh	
1905		* Anderson, William, M.A., Head Science Master, George Watson's College, Edinburgh, 6 Lockharton Crescent, Edinburgh 10	
1903		Anderson-Berry, David, M.D., LL.D., F.R.S.L., M.R.A.S., F.S.A. (Scot.), Versailles, Highgate, London, N.	
1905		* Andrew, George, M.A., B.A., H.M.I.S., Balgillo Cottage, Seafield Road, Broughty Ferry	
1881	C.	Anglin, A. H., M.A., LL.D., M.R.I.A., Professor of Mathematics, Queen's College, Cork	
1921	C.	* Annandale, Nelson, B.A. (Oxon.), D.Sc. (Edin.), Director of the Zoological Survey of India, and Vice-Chairman of the Trustees of the Indian Museum, Calcutta	
1915		Anthony, Charles, M.Inst.C.E., M. Am. Soc. C.E., F.R.San.I., F.R.Met.S., F.R.A.S., F.C.S., General Manager, Water Works Company, Vieytes y Gorriti, Bahia Blanca, Argentina 15	
1906		Appleton, Colonel Arthur Frederick, F.R.C.V.S., 19 Cumberland Road, Bromley, Kent	
1910	C.	Archibald, E. H., B.Sc., Professor of Chemistry, University of British Columbia, Vancouver, Canada	
1907		* Archibald, James, M.A., 31 Leamington Terrace, Edinburgh	
1921		* Arthur, William, M.A., Assistant to the Professor of Mathematics in the University of Glasgow. 149 Stanmore Road, Mount Florida, Glasgow	
1911	C. K.	* Ashworth, James Hartley, D.Sc., F.R.S., Professor of Zoology, University of Edinburgh (SECRETARY), 69 Braid Avenue, Edinburgh 20	1912–14, 1915–18. Sec. 1918–
1907		* Badre, Muhammad, Ph.D., Almuneerah, Cairo, Egypt	

Date of Election.			Service on Council, etc.
1920		* Bagnall, Richard Siddoway, 15 Grey Street, Newcastle-on-Tyne	
1920	C.	* Bailey, Edward Battersby, M.C., B.A., F.G.S., District Geologist, H.M. Geological Survey of Scotland, 23 Pentland Terrace, Edinburgh	
1896	C.	Baily, Francis Gibson, M.A., M.Inst.E.E., Professor of Electrical Engineering, Heriot-Watt College, Edinburgh, Newbury, Colinton, Midlothian	1909–12. 1920–
1921		* Baker, Bevan Braithwaite, M.A., B.Sc. (Lond.), Lecturer in Mathematics in the University of Edinburgh. 30 Murrayfield Gardens, Edinburgh 25	
1877	C.	Balfour, Sir I. Bayley, K.B.E., M.A., Sc.D., M.D., LL.D., F.R.S., F.L.S., King's Botanist in Scotland, Professor of Botany in the University of Edinburgh and Keeper of the Royal Botanic Garden, Inverleith House, Edinburgh	1888–91.
1905	C.	Balfour-Browne, William Alexander Francis, M.A., F.Z.S., F.E.S., Barrister-at-Law, Lecturer in Zoology (Entomology) in the University of Cambridge), Oaklands, Fenstanton, near St Ives, Hunts	
1892	C.	Ballantyne, J. W., M.D., F.R.C.P.E., 19 Rothesay Terrace, Edinburgh	
1918		* Balsillie, David, B.Sc., F.G.S., Department of Mineralogy and Geology, Royal Scottish Museum, Edinburgh	
1902	C.	Bannerman, W. B., C.S.I., I.M.S., M.D., D.Sc., Maj.-General, Indian Medical Service, 11 Strathearn Place, Edinburgh (Vice-President) 30	1919–21. V-P 1921–
1889		Barbour, A. H. F., M.A., M.D., LL.D., F.R.C.P.E., 4 Charlotte Square, Edinburgh	
1886		Barclay, A. J. Gunion, M.A., 3 Chandos Avenue, Oakleigh Park, London, N.	
1883	C.	Barclay, G. W. W., M.A., Raeden House, Aberdeen	
1903		Bardswell, Noël Dean, M.D., M.R.C.P. Ed. and Lond., King Edward VII Sanatorium, Midhurst	
1914	C.	* Barkla, Charles Glover, M.A., D.Sc., F.R.S., Professor of Natural Philosophy in the University of Edinburgh, Nobel Laureate, Physics, 1917, 20 Hermitage Drive, Edinburgh 35	1915–18.
1921		* Barr, Archibald, D.Sc., LL.D. (Glasgow and Birmingham), Em.-Professor of Engineering in the University of Glasgow. Westerton of Mugdock, Milngavie	
1904		Barr, Sir James, C.B.E., M.D., LL.D., F.R.C.P. Lond., 72 Rodney Street, Liverpool	
1874		Barrett, Sir William F., Kt., F.R.S., M.R.I.A., formerly Professor of Physics, Royal College of Science, Dublin, 31 Devonshire Place, London, W. 1	
1921		* Bartholomew, John, M.C., M.A., F.R.G.S., Geographical Institute, Edinburgh	
1895	C.	Barton, Edwin H., D.Sc., F.R.S., F.P.S.L., F.Inst.P., Professor of Physics, University College, Nottingham 40	
1904		* Baxter, William Muirhead, Glenalmond, Sciennes Gardens, Edinburgh	
1913		Beard, Joseph, F.R.C.S. (Edin.), M.R.C.S. (Eng.), L.R.C.P. (Lond.), D.P.H. (Camb.), Medical Officer of Health and School Medical Officer, City of Carlisle, 8 Carlton Gardens, Carlisle	
1888		Beare, Thomas Hudson, B.A., B.Sc., M.Inst.C.E., J.P., D.L., Professor of Engineering in the University of Edinburgh	1907–09. V-P 1909–15.
1897	C.	Beattie, Sir John Carruthers, K.B., D.Sc., Vice-Chancellor and Principal, The University, Cape Town	
1893	C. B.	Becker, Ludwig, Ph.D., Regius Professor of Astronomy in the University of Glasgow, The Observatory, Glasgow, Millbank Terrace, Crieff 45	
1882	C.	Beddard, Frank E., M.A. Oxon., D.Sc., F.R.S., formerly Prosector to the Zoological Society of London, 20 Sherriff Road, Kilburn, London, N.W. 6	
1887		Begg, Ferdinand Faithfull, 46 Saint Aubyns, Hove, Sussex	
1906		Bell, John Patrick Fair, F.Z.S., Springbank, Ayton, Berwickshire	
1916		* Bell, Robert John Tainsh, M.A., D.Sc., Professor of Mathematics in the University of Otago, New Zealand	
1915		Bell, Walter Leonard, M.D.Edin., F.S.A.Scot., 123 London Road, North Lowestoft, Suffolk 50	
1893	C.	Berry, Sir George A., M.B., C.M., LL.D., F.R.C.S.E. (Vice-President), 31 Drumsheugh Gardens, Edinburgh	1916–19. V-P 1919–
1897	C.	Berry, Richard J. A., M.D., F.R.C.S.E., Professor of Anatomy in the University of Melbourne, Victoria, Australia	
1880	C.	Birch, De Burgh, C.B., M.D., Emeritus Professor of Physiology in the University of Leeds	
1907		* Black, Frederick Alexander, Solicitor, 59 Academy Street, Inverness	
1884	C.	Black, John S., M.A., LL.D., 125 St James' Court, London, S.W. 1 55	1891–94, 1916–18. Cur. 1906–16.

Date of Election.			Service on Council, etc.
1897	C.	Blaikie, Walter Biggar, LL.D., The Loan, Colinton	1914–17.
1904	C.	* Bles, Edward J., M.A., D.Sc., Elterholm, Cambridge	
1918		* Blight, Francis James, Chairman and Managing Director of Charles Griffin & Co., Ltd., Publishers, Tregenna, Wembley, Middlesex	
1894		Bolton, Herbert, M.Sc., F.G.S., F.Z.S., Director of the Bristol Museum and Art Gallery, Bristol, 58 Coldharbour Road, Redland, Bristol	
1915		* Boon, Alfred Archibald, D.Sc., F.I.C., B.A., Professor of Chemistry, Heriot-Watt College, Edinburgh 60	
1872	C.	Bottomley, J. Thomson, M.A., D.Sc., LL.D., F.R.S., F.C.S., 13 University Gardens, Glasgow	
1886	C.	Bower, Frederick O., M.A., D.Sc., LL.D., F.R.S., F.L.S. (President), Regius Professor of Botany in the University of Glasgow, 1 St John's Terrace, Hillhead, Glasgow	1887–90, 1893–96, 1907–09, 1917–19. V-P 1910–16. P 1919-
1884	C.	Bowman, Frederick Hungerford, D.Sc., F.C.S. (Lond. and Berl.), F.I.C., A.Inst.C.E., A.Inst.M.E., M.Inst.E.E., etc., 76 Acomb Street, Whitworth Park, Manchester	
1901		Bradbury, J. B., M.D., Downing Professor of Medicine, University of Cambridge	
1916		Bradley, His Honour Judge (Francis Ernest), M.A., M.Com., LL.D., Barrister-at-Law, Examiner to the Council of Legal Education, Bank of England Chambers, Tib Lane, Manchester 65	
1903	C.	* Bradley, O. Charnock, M.D., D.Sc., Principal, Royal Dick Veterinary College, Edinburgh, President of the Royal College of Veterinary Surgeons, London	1907–10, 1915–17.
1886		Bramwell, Byrom, M.D., F.R.C.P.E., LL.D., 23 Drumsheugh Gardens, Edinburgh	1890–93.
1907		* Bramwell, Edwin, M.D., F.R.C.P.E., F.R.C.P. Lond., 23 Drumsheugh Gardens, Edinburgh	
1918		* Bremner, Alexander, M.A., D.Sc., Headmaster, Demonstration School, Training Centre, Aberdeen, 13 Belgrave Terrace, Aberdeen	
1916	C.	* Briggs, Henry, D.Sc., A.R.S.M., Professor of Mining, Heriot-Watt College, Allermuir, Liberton, Midlothian 70	
1895		Bright, Sir Charles, M.Inst.C.E., M.Inst.E.E., F.R.Aë.S., F.Inst.Radio.E., F.R.A.S., F.R.G.S., Leigh Grange, Kent, and Athenæum Club, Pall Mall, London, S.W.	
1893		Brock, G. Sandison, M.D., 6 Corso d'Italia, Rome, Italy	
1901	C.	* Brodie, W. Brodie, M.B., Camden House, Bletchingley, Surrey	
1907		Brown, Alexander, M.A., B.Sc., Professor of Applied Mathematics, The University, Cape Town	
1864	C. K. B.	Brown, Alex. Crum, M.A., M.D., D.Sc., F.R.C.P.E., LL.D., F.R.S., Emeritus Professor of Chemistry in the University of Edinburgh, 8 Belgrave Crescent, Edinburgh 75	1865–68, 1869–72, 1873–75, 1876–78, 1911–13. Sec. 1879–1905. V-P 1905–11.
1883	C.	Brown, J. J. Graham, M.D., F.R.C.P.E., 3 Chester Street, Edinburgh	
1885	C.	Brown, J. Macdonald, M.D., F.R.C.S., 64 Upper Berkeley Street, Portman Square, London, W.	
1909	B. C.	* Brownlee, John, M.A., M.D., D.Sc., the National Institute for Medical Research, Mount Vernon, Hampstead, N.W. 3	
1921		* Bruce, Alexander, B.Sc. (Edin.), Government Agricultural Chemist and City Analyst, Colombo, Ceylon	
1912		* Bruce, Alexander Ninian, D.Sc., M.D., 8 Ainslie Place, Edinburgh 80	
1898	C. K.	* Bryce, T. H., M.A., M.D. (Edin.), Professor of Anatomy in the University of Glasgow, 2 The University, Glasgow	1911–14.
1870	C. K.	Buchanan, John Young, M.A., F.R.S., Athenæum Club, Pall Mall, London, S.W.	1878–81, 1884–86.
1905		Bunting, Thomas Lowe, M.D., 27 Denton Road, Scotswood, Newcastle-on-Tyne	
1902		* Burgess, A. G., M.A., Rector of The Academy, Rothesay, Blythswood, Rothesay	

Date of Election.			Service on Council, etc.
1887		† Burnet, Sir John James, A.R.A., R.S.A., LL.D., Architect, 1 Montague Place, Bedford Square, London, W.C. 1. 85	
1888		Burns, Rev. T., D.D., J.P., F.S.A. Scot., Minister of Lady Glenorchy's Parish Church, Croston Lodge, Chalmers Crescent, Edinburgh	
1917		* Burnside, George Barnhill, M.I.Mech.E., 104 Beechwood Drive, Glasgow, W.	
1915		* Butchart, Raymond Keiler, B.Sc., Ph.D., University College, Dundee, 5 Briarwood Terrace, West Park Road, Dundee	
1896		Butters, J. W., M.A., B.Sc., Rector of Ardrossan Academy	
1887	C.	Cadell, Henry Moubray, of Grange, B.Sc., D.L., Linlithgow 90	1919–
1910		* Calderwood, Rev. Robert Sibbald, Minister of Cambuslang, The Manse, Cambuslang, Lanarkshire	
1893	C.	Calderwood, W. L., Inspector of Salmon Fisheries of Scotland, South Bank, Canaan Lane, Edinburgh	
1894		Cameron, James Angus, M.D., Medical Officer of Health, Firhall, Nairn	
1905	C.	Cameron, John, M.D., D.Sc., M.R.C.S. Eng., Dalhousie University, Halifax, Nova Scotia	
1921		* Campbell, Andrew, Advisory Chemist, Burmah Oil Co., Ltd., and Anglo-Persian Oil Co., Ltd. The Coppice, Beckenham, Kent 95	
1904		* Campbell, Charles Duff, Scottish Liberal Club, Princes Street, Edinburgh	
1918		* Campbell, John Menzies, L.D.S. (Glas.), D.D.S. (Toronto), L.D.S. (Ontario), 14 Buckingham Terrace, Glasgow, W.	
1915	C. N.	* Campbell, Robert, M.A., D.Sc., F.G.S., Lecturer in Petrology, University of Edinburgh, 2 Woodhall Road, Colinton	1920–
1899	C.	* Carlier, Edmund W. W., M.D., M.Sc., F.E.S., Professor of Physiology, University, Birmingham	
1910		Carnegie, Col. David, C.B.E., M.Inst.C.E., M.Inst.Mech.E., M.I.S.Inst., "Woodlands," Beckenham Hill, Kent 100	
1920	C.	* Carruthers, R. G., F.G.S., District Geologist, H. M. Geological Survey, High Barn, Stocksfield-on-Tyne	
1905	C.	* Carse, George Alexander M.A., D.Sc., Lecturer on Natural Philosophy, University of Edinburgh, 3 Middleby Street, Edinburgh	
1901		Carslaw, H. S., M.A., D.Sc., Professor of Mathematics in the University of Sydney, New South Wales	
1905		Carter, Joseph Henry, F.R.C.V.S., Avalon, Western Road, Henley-on-Thames	
1898		Carus-Wilson, Cecil, F.R.G.S., F.G.S., Waldegrave Park, Strawberry Hill, Middlesex, and Sandacres Lodge, Parkstone-on-Sea, Dorset 105	
1908		Cavanagh, Thomas Francis, M.D., The Hospital, Bella Coola, B.C., Canada	
1882		Cay, W. Dyce, M.Inst.C.E., Junior Carlton Club, Pall Mall, London, S.W. 1	
1899		Chatham, James, Actuary, c/o Robert Murrie, Esq., 28 St Andrew Square, Edinburgh	
1912		Chaudhuri, Banawari Lal, B.A. (Cal.), B.Sc. (Edin.), Assistant Superintendent, Natural History Section, Indian Museum, 120 Lower Circular Road, Calcutta, India	
1874		Chiene, John, C.B., M.D., LL.D., F.R.C.S.E., Emeritus Professor of Surgery in the University of Edinburgh, Barnton Avenue, Davidson's Mains 110	1884–86, 1904–06.
1891		Clark, John B., M.A., Head Master of Heriot's Hospital School, Lauriston, Garleffin, 146 Craiglea Drive, Edinburgh	
1911		* Clark, William Inglis, D.Sc., 22 Buckingham Terrace, Edinburgh	
1903		Clarke, William Eagle, I.S.O., LL.D., F.L.S., Honorary Supervisor of the Bird Collection and formerly Keeper of the Natural History Collections in the Royal Scottish Museum, Edinburgh. 35 Braid Road, Edinburgh	
1909		Clayton, Thomas Morrison, M.D., D.Hy., B.Sc., D.P.H., Medical Officer of Health, Gateshead, 13 The Crescent, Gateshead-on-Tyne	
1913		* Cleghorn, Alexander, M.Inst.C.E., Marine Engineer, 14 Hatfield Drive, Kelvinside, Glasgow 115	
1904	C.	Coker, Ernest George, M.A., D.Sc., Hon. D.Sc. (Sydney), F.R.S., M.Inst.C.E., M.Inst.E.E., Professor of Civil and Mechanical Engineering, University of London, University College, Gower Street, London, W.C.	
1904		Coles, Alfred Charles, M.D., D.Sc., York House, Poole Road, Bournemouth, W.	
1888	V. J. C.	Collie, John Norman, Ph.D., D.Sc., LL.D., F.R.S., F.C.S., F.I.C., F.R.G.S., Professor of Organic Chemistry in the University College, Gower Street, London	
1904		* Colquhoun, Walter, M.A., M.B., 18 Walmer Crescent, Ibrox, Glasgow	
1909	C.	* Comrie, Peter, M.A., B.Sc., Head Mathematical Master, Boroughmuir Junior Student Centre, 19 Craighouse Terrace, Edinburgh 120	
1886		Connan, Daniel M., M.A.	
1905		* Corrie, David, F.C.S., 159 Lauderdale Mansions, Maida Vale, London, W. 9.	

Date of Election.			Service on Council, etc.
1914		* Coutts, William Barron, M.A., B.Sc., Senior Lecturer in Range Finding and Optics, Artillery College, Red Barracks, Woolwich, S.E. 18.	
1911		* Cowan, Alexander, Papermaker, Valleyfield, Penicuik, Midlothian	
1920		Craib, William Grant, M.A. (Aberdeen), Regius Professor of Botany in the University of Aberdeen 125	
1916	C.	Craig, E. H. Cunningham, B.A. (Cambridge), Geologist and Mining Engineer, The Dutch House, Beaconsfield	
1908		Craig, James Ireland, M.A., B.A., Woolwich House, The Drive, Sydenham, London, S.E. 26	
1875		Craig, William, M.D., F.R.C.S.E., Lecturer on Materia Medica to the College of Surgeons, 71 Bruntsfield Place, Edinburgh	
1903		Crawford, Lawrence, M.A., D.Sc., Professor of Pure Mathematics, The University, Cape Town	
1870		Crichton-Browne, Sir Jas., Kt., M.D., LL.D., D.Sc., F.R.S., Lord Chancellor's Visitor and Vice-President and Treasurer of the Royal Institution of Great Britain, 45 Hans Place, S. W., and Royal Courts of Justice, Strand, London 130	
1916		* Crombie, James Edward, M.A., LL.D., Millowner, Parkhill House, Dyce, Aberdeenshire	
1886		Croom, Sir John Halliday, Kt., M.D., F.R.C.P.E., formerly Professor of Midwifery in the University of Edinburgh, late President, Royal College of Surgeons, Edinburgh, 25 Charlotte Square, Edinburgh	
1914		* Cumming, Alexander Charles, D.Sc., O.B.E., Roselands, Crescent Road, Blundell Sands, Liverpool	
1917		* Cunningham, Brysson, D.Sc., B.E., M.Inst.C.E., Lecturer on Waterways, Harbours, and Docks, University College, London, 16 Beechwood Road, Sanderstead, Surrey	
1898		* Currie, James, M.A. Cantab., LL.D. (Treasurer), Larkfield, Goldenacre, Edinburgh 135	Treas. 1906–
1919		* Cushny, Arthur Robertson, M.A., M.D., LL.D., F.R.S., Professor of Materia Medica and Pharmacology, University, Edinburgh	1919–
1904		* Cuthbertson, John, Secretary, West of Scotland Agricultural College, 6 Charles Street, Kilmarnock	
1885		Daniell, Alfred, M.A., LL.B., D.Sc., Advocate, The Athenæum Club, Pall Mall, London	
1921		* Datta, Rasik Lal, D.Sc., Assistant Professor of Chemistry, University of Calcutta. 78 Manicktola Street, Calcutta, India	
1884		Davy, R., F.R.C.S. Eng., Consulting Surgeon to Westminster Hospital, Burstone Manor, Bow, North Devon 140	
1917		* Day, T. Cuthbert, Partner of the firm of Hislop & Day, 36 Hillside Cres., Edinburgh	
1894		Denny, Sir Archibald, Bart., LL.D., Cardross Park, Cardross, Dumbartonshire. Somerset Lodge, Somerset Road, Wimbledon Common, S.W. 19 (temporary address)	
1869	C. V. J.	† Dewar, Sir James, Kt., M.A., LL.D., D.C.L., D.Sc., F.R.S., F.C.S., Jacksonian Professor of Natural and Experimental Philosophy in the University of Cambridge, and Fullerian Professor of Chemistry at the Royal Institution of Great Britain, London	1872–74.
1905		* Dewar, James Campbell, C.A., 27 Douglas Crescent, Edinburgh	
1906		* Dewar, Thomas William, M.D., F.R.C.P., Kincairn, Dunblane 145	
1884		Dickson, the Right Hon. Charles Scott, Lord Justice-Clerk, K.C., LL.D., 22 Moray Place, Edinburgh	
1888	C.	Dickson, Henry Newton, C.B.E., M.A., D.Sc., formerly Professor of Geography at University College, Reading. 18 Bedford Square, London, W.C. 1.	
1876	C.	Dickson, J. D. Hamilton, M.A., Senior Fellow and formerly Tutor, St Peter's College, Cambridge	
1885	C.	Dixon, James Main, M.A., Litt. Hum. Doctor, Professor of English, University of Southern California, University Avenue, Los Angeles, California, U.S.A.	
1897		Dobbie, James Bell, F.Z.S., 12 South Inverleith Avenue, Edinburgh 150	
1904		* Dobbie, Sir James Johnston, Kt., M.A., D.Sc., LL.D., F.R.S., formerly Principal of the Government Laboratories, London. Fairlie Cottage, Fairlie, Ayrshire	1905–08.
1881	C.	Dobbin, Leonard, Ph.D., Lecturer in Chemistry in the University of Edinburgh, 6 Wilton Road, Edinburgh	1904–07, 1913–16.
1918		* Dodd, Alexander Scott, B.Sc., F.I.C., F.C.S., City Analyst for Edinburgh, 20 Stafford Street, Edinburgh	
1905		* Donaldson, Rev. Wm. Galloway, J.P., F.R.G.S., F.E.I.S., The Manse, Forfar	
1882	C.	Dott, David B., F.I.C., Memb. Pharm. Soc., Ravenslea, Musselburgh 155	
1921	B. C.	* Dougall, John, M.A., D.Sc., Publisher's Reader, 26 Underwood Street, Langside, Glasgow	

Date of Election.			Service on Council, etc.
1901 & 1918		* Douglas, Carstairs Cumming, M.D., D.Sc., Professor of Medical Jurisprudence and Hygiene, Anderson's College, Glasgow, 2 Royal Crescent, Glasgow	
1910		* Douglas, Loudon MacQueen, Author and Lecturer, 29 W. Saville Terrace, Newington, Edinburgh	
1908	C	Drinkwater, Harry, M.D., M.R.C.S. (Eng.), F.L.S., Lister House, Wrexham, North Wales	
1901		* Drinkwater, Thomas W., L.R.C.P.E., L.R.C.S.E., Chemical Laboratory, Surgeons' Hall, Edinburgh 160	
1917		* Dron, Robert W., A.M.Inst.C.E., 11 W. Regent Street, Glasgow	
1921		* Drysdale, Charles Vickery, D.Sc. (Lond.), M.I.E.E., F.Inst.P., O.B.E., Superintendent of the Admiralty Research Laboratory, Teddington, Middlesex	
1904		* Dunlop, William Brown, M.A., 4A St Andrew Square, Edinburgh	
1903		Dunstan, John, M.R.C.V.S., Inversnaid, Liskeard, Cornwall	
1892	C.	Dunstan, M. J. R., M.A., F.I.C., F.C.S., Principal, South-Eastern Agricultural College, Wye, Kent 165	
1906	C.	Dyson, Sir Frank Watson, Kt., M.A., D.Sc., LL.D., F.R.S., Astronomer Royal, Royal Observatory, Greenwich	1907–10.
1893		Edington, Alexander, M.D., Howick, Natal	
1904		* Edwards, John, LL.D., 4 Great Western Terrace, Kelvinside, Glasgow	
1904		* Elder, William, M.D., F.R.C.P.E., 4 John's Place, Leith	
1875		Elliot, Daniel G., American Museum of Natural History, Central Park West, New York, N.Y., U.S.A. 170	
1913 & 1921		* Elliot, George Francis Scott, M.A. (Cantab.), B.Sc., F.R.G.S., F.L.S., Drumwhill, Mossdale	
1906	C.	* Ellis, David, D.Sc., Ph.D., Lecturer in Botany and Bacteriology, Royal Technical College, Glasgow	
1897	C.	Erskine-Murray, James Robert, D.Sc., 16 Elmfield Road, Bromley, Kent	
1884		Evans, William, F.F.A., 38 Morningside Park, Edinburgh	
1879	C. N.	Ewart, James Cossar, M.D., F.R.C.S.E., F.R.S., F.Z.S., Regius Professor of Natural History, University of Edinburgh, Craigybield, Penicuik, Midlothian 175	1882–85, 1904–07. V-P 1907–12.
1902		* Ewen, John Taylor, B.Sc., M.I.Mech.E., H.M. Inspector of Schools, Clairmont, 54 Albert Drive, Pollokshields, Glasgow	
1878	C.	Ewing, Sir James Alfred, K.C.B., M.A., D.Sc., LL.D., M.Inst.C.E., F.R.S., J.P. (Vice-President), Principal of the University of Edinburgh, formerly Director of Naval Education, Admiralty, 16 Moray Place, Edinburgh	1888–91, 1919–20. V-P 1920–
1900	C.	Eyre, John W. H., M.D., M.S. (Dunelm), D.P.H. (Camb.), Professor of Bacteriology, Guy's Hospital, London	
1910	C.	* Fairgrieve, Mungo M'Callum, M.A. (Glasg.), M.A. (Cambridge), Master at the Edinburgh Academy, 37 Queen's Crescent, Edinburgh	
1907	C.	Falconer, John Downie, M.A., D.Sc., F.G.S., Lecturer on Geography, The University, Glasgow 180	
1888	C.	Fawsitt, Charles A., Coney Park, Bridge of Allan	
1883	C.	Felkin, Robert W., M.D., F.R.G.S., Whare Ra, Havelock North, Hawkes Bay, New Zealand	
1899		* Fergus, Andrew Freeland, M.D., LL.D., c/o Messrs. Mackay & Boyd, 50 Wellington Street, Glasgow	
1907		* Fergus, Edward Oswald, c/o 22 Blythswood Square, Glasgow	
1904		* Ferguson, James Haig, M.D., F.R.C.P.E., F.R.C.S.E., 7 Coates Crescent, Edinburgh 185	
1898		* Findlay, Sir John R., K.B.E., M.A. Oxon., 3 Rothesay Terrace, Edinburgh	
1899		* Finlay, David W., B.A., M.D., LL.D., F.R.C.P., D.P.H., Emeritus Professor of Medicine in the University of Aberdeen, Honorary Physician to His Majesty in Scotland, Balgownie, Helensburgh	
1911		Fleming, John Arnold, F.C.S., etc., Pottery Manufacturer, Locksley, Helensburgh, Dumbartonshire	
1906		* Fleming, Robert Alexander, M.A., M.D., F.R.C.P.E., Physician, Royal Infirmary, 10 Chester Street, Edinburgh	
1900	C. N.	* Flett, John S., M.A., D.Sc., LL.D., F.R.S., O.B.E., Director of the Geological Survey of Great Britain and of the Museum of Practical Geology, London, 28 Jermyn Street, S.W. 1 190	1916–19.
1872		Forbes, George, M.A., M.Inst.C.E., M.Inst.E.E., F.R.S., F.R.A.S., formerly Professor of Natural Philosophy in Anderson's College, Glasgow. 11 Little College Street, Westminster, S.W.	

Date of Election.			Service on Council, etc.
1892		Ford, John Simpson, F.C.S., 7 Corrennie Drive, Edinburgh	
1921		* Forrest, George Topham, Architect to the London County Council, and Superintending Architect of Metropolitan Buildings, New County Hall, Westminster Bridge, London, S.W.	
1920	C.	* Franklin, Thomas Bedford, B.A. (Hons. Mathematics), Cambridge, Stancliffe Hall, near Matlock, Derbyshire	
1910		* Fraser, Alexander, Actuary, 15 S. Learmonth Gardens, Edinburgh 195	
1896		Fraser, John, M.B., F.R.C.P.E., formerly one of H.M. Commissioners in Lunacy for Scotland, 54 Great King Street, Edinburgh	
1915		* Fraser, Rev. Joseph Robert, U.F. Manse, Kinneff, Bervie	
1914		* Fraser, William, Managing Director, Neill & Co., Ltd., Printers, 212 Causewayside, Edinburgh	
1891		Fulton, T. Wemyss, M.D., Scientific Superintendent, Scottish Fishery Board, 41 Queen's Road, Aberdeen	
1907		* Galbraith, Alexander, "Ravenswood," Dalmuir, Dumbartonshire 200	
1888	C.	Galt, Alexander, D.Sc., late Keeper of the Department of Technology, Royal Scottish Museum, Edinburgh, St Margaret's, Craiglockhart, Edinburgh	
1901		Ganguli, Sanjiban, M.A., Principal, Maharaja's College, and Director of Public Instruction, Jaipur State, Jaipur, India	
1909	C.	* Geddes, Rt. Hon. Sir Auckland C., K.C.B., M.D., D.C.L., British Ambassador to the U.S.A., The British Embassy, Washington	
1880	C.	Geddes, Patrick, Professor of Botany in University College, Dundee, and Lecturer on Zoology, Ramsay Garden, University Hall, Edinburgh	
1861	C. B.	Geikie, Sir Archibald, O.M., K.C.B., D.C.L. Oxf., D.Sc., LL.D., Ph.D., Late Pres. R.S., Foreign Member of the Reale Accad. Lincei, Rome, of the National Acad. of the United States, of the Academies of Stockholm, Christiania, Göttingen, Corresponding Member of the Institute of France and of the Academies of Berlin, Vienna, Munich, Turin, Belgium, Philadelphia, New York, etc., Shepherd's Down, Haslemere, Surrey 205	1869–72, 1874–76, 1879–82.
1914		Gemmell, John Edward, M.B., C.M., Hon. Surgeon, Hospital for Women and Maternity Hospital; Hon. Gynæcologist, Victoria Central Hospital, Liscard, 28 Rodney Street, Liverpool	
1909		* Gentle, William, B.Sc., 12 Mayfield Road, Edinburgh	
1920	C.	* Ghosh, Sudhamoy, M.Sc. (Cal.), D.Sc. (Edin.), F.C.S., Government Research Chemist, Medical College, Calcutta, 9/1 Rammohan Dutt Road, Bhowanipur, Calcutta, India	
1914		* Gibb, Sir Alexander, G.B.E., C.B., formerly Director-General of Civil Engineering, Ministry of Transport. 91 Victoria Street, Westminster, London, S.W.	
1916		* Gibb, A. W., D.Sc., Lecturer in Geology, The University, Aberdeen, 1 Belvidere Street, Aberdeen 210	
1910	C.	* Gibb, David, M.A., B.Sc., Lecturer in Mathematics, Edinburgh University, 15 South Lauder Road, Edinburgh	
1917	C.	* Gibson, Alexander, M.B., Ch.B., F.R.C.S. (Eng.), 661 Broadway, Winnipeg, Canada	
1910		* Gibson, Charles Robert, Lynton, Mansewood, by Pollokshaws	
1890		Gibson, George A., M.A., LL.D. Professor of Mathematics in the University of Glasgow, 10 The University, Glasgow	1905–08, 1912–13. V-P – 1917–20.
1921		* Gibson, Walcot, D.Sc., F.G.S., Assistant Director, H.M. Geological Survey (Scotland), 33 George Square, Edinburgh 215	
1911		Gidney, Henry A. J., L.M. and S. Socts. Ap. (Lond.), F.R.C.S. (Edin.), D.P.H. (Camb.), D.O. (Oxford), Army Specialist Public Health, c/o Thomas Cook & Sons, Ludgate Circus, London	
1900		Gilchrist, Douglas A., B.Sc., Professor of Agriculture and Rural Economy, Armstrong College, Newcastle-upon-Tyne	
1907		Gilruth, John Anderson, M.R.C.V.S., D.V.Sc. (Melb.), Administrator, Government House, Darwin Northern Territory, Australia	
1909		* Gladstone, Hugh Steuart, M.A., M.B.O.U., F.Z.S., Capenoch, Thornhill, Dumfriesshire	
1911		Gladstone, Reginald John, M.D., F.R.C.S. (Eng.), Lecturer and Senior Demonstrator of Anatomy, King's College, University of London, 22 Regent's Park Terrace, London, N.W. 220	
1898		* Glaister, John, M.D., F.R.F.P.S. Glasgow, D.P.H. Camb., Regius Professor of Forensic Medicine and Public Health in the University of Glasgow, 3 Newton Place, Glasgow	

Date of Election.			Service on Council, etc.
1910		Goodall, Joseph Strickland, M.B. (Lond.), M.S.A. (Eng.), Lecturer on Physiology, Middlesex Hospital, London, Annandale Lodge, Vanbrugh Park, Blackheath, London, S.E.	
1901		Goodwillie, James, M.A., B.Sc., Liberton, Edinburgh	
1920	C.	* Gordon, William, B.Sc., A.M.I.Mech.E., Lecturer in Engineering in the University of Edinburgh, 3 Wellington Street, Edinburgh	
1913	C.	* Gordon, William Thomas, M.A., D.Sc. (Edin.), M.A. (Cantab.), Professor of Geology, University of London, King's College, Strand, W.C. 225	
1897		Gordon-Munn, John Gordon, M.D., Heigham Hall, Norwich	
1898	C.	* Gray, Albert A., M.D., 4 Clairmont Gardens, Glasgow	
1883	C.	Gray, Andrew, M.A., LL.D., F.R.S., Professor of Natural Philosophy in the University of Glasgow	1903–06. V-P 1906–09.
1910		Gray, Bruce M'Gregor, C.E., A.M.Inst.C.E., Westbourne Grove, Selby, Yorkshire	
1909	C.	* Gray, James Gordon, D.Sc., Professor of Applied Physics in the University of Glasgow, 11 The University, Glasgow 230	1913–15.
1918		* Gray, Wm. Forbes, F.S.A. (Scot.), Editor and Author, 8 Mansionhouse Road, Edinburgh	
1897		Greenlees, Thomas Duncan, M.D. Edin., Viresco, Fordingbridge, Hants	
1905	C.	* Gregory, John Walter, D.Sc., F.R.S. (Vice-President), Professor of Geology in the University of Glasgow, 4 Park Quadrant, Glasgow	1908–11. V-P 1920–
1906		Greig, Edward David Wilson, C.I.E., M.D., D.Sc., Lt.-Col., H.M. Indian Medical Service, Pasteur Institute, Kasauli, India	
1905		Greig, Sir Robert Blyth, M.C., LL.D., F.Z.S., Chairman of the Board of Agriculture for Scotland, 29 St Andrew Square, Edinburgh 235	1921
1910		* Grimshaw, Percy Hall, Assistant Keeper, Natural History Department, The Royal Scottish Museum, 49 Comiston Drive, Edinburgh	
1899		* Guest, Edward Graham, M.A., B.Sc., 5 Newbattle Terrace, Edinburgh	
1907	C.	* Gulliver, Gilbert Henry, D.Sc., A.M.I.Mech.E., 99 Southwark Street, London, S.E.	
1911	C.	* Gunn, James Andrew, M.A., M.D., D.Sc., Professor of Pharmacology in the University of Oxford	
1888	C.	Guppy, Henry Brougham, M.B., Rosario, Salcombe, Devon 240	
1911		* Guy, William, F.R.C.S., L.R.C.P., L.D.S.Ed., Consulting Dental Surgeon, Edinburgh Royal Infirmary; Dean, Edinburgh Dental Hospital and School; Lecturer on Human and Comparative Dental Anatomy and Physiology, 11 Wemyss Place, Edinburgh	
1911		Hall-Edwards, John Francis, L.R.C.P. (Edin.), Hon. F.R.P.S., Senior Medical Officer in charge of X-ray Department, General Hospital, Birmingham, 141A and 141B Great Charles Street (Newhall Street), Birmingham	
1918		* Hardie, P. S., M.A., B.Sc., Lecturer in Physics, Sultania Training College, Cairo, Egypt	
1896	C.	Harris, David Fraser, B.Sc. (Lond.), D.Sc. (Birm.), M.D., F.S.A. Scot., Professor of Physiology in the Dalhousie University, Halifax, Nova Scotia	
1914		Harrison, Edward Philip, Ph.D., Professor of Physics, Presidency College, University of Calcutta, The Observatory, Alipore, Calcutta 245	
1917		* Harrison, John, C.B.E., D.L., J.P., LL.D., Chairman of the Edinburgh Public Library, Rockville, Napier Road, Edinburgh	
1921		* Harrison, John William Heslop, D.Sc. (Durham), Lecturer in Genetics, Armstrong College, Newcastle. The Avenue, Birtley, Co. Durham	
1914	C.	Harvey-Gibson, Robert John, C.B.E., D.L., J.P., M.A., Mem. Roy. Dub. Soc., formerly Professor of Botany, University of Liverpool. "Beckallars" Grasmere, Westmorland.	
1880	C.	Haycraft, J. Berry, M.D., D.Sc., Professor of Physiology in the University College of South Wales and Monmouthshire, Cardiff	
1892	C.	Heath, Thomas, B.A., formerly Assistant Astronomer, Royal Observatory, Edinburgh, 11 Cluny Drive, Edinburgh 250	
1893		Hehir, Sir Patrick, K.C.I.E., C.B., C.M.G., M.D., F.R.C.S.E., M.R.C.S., L.R.C.P.E., Retired Maj.-General I.M.S., 3 Nelson Terrace, Westward Ho! N. Devon	
1900		Henderson, John, D.Sc., A.Inst.E.E., Kinnoul. Gregory's Road, Beaconsfield, Bucks	
1908		* Henderson, William Dawson, M.A., B.Sc., Ph.D., Lecturer, Zoological Laboratories, University, Bristol	
1890	C.	Hepburn, David, C.M.G., M.D., Professor of Anatomy in the University College of South Wales and Monmouthshire, Cardiff	

Date of Election.			Service on Council, etc.
1881	C. N.	Herdman, Sir W. A., Kt., C.B.E., D.Sc., LL.D., F.R.S., Past Pres. L.S., Pres. Brit. Assoc., Emeritus Professor of Natural History in the University of Liverpool. Croxteth Lodge, Ullet Road, Liverpool, and Rowany, Port Erin, I.O.M. 255	
1916		* Herring, Percy Theodore, M.D., F.R.C.P.Ed., Professor of Physiology, University of St Andrews, Hepburn Gardens, St Andrews	1917–20.
1894		Hill, Alfred, M.D., M.R.C.S., F.I.C., Valentine Mount, Freshwater Bay, Isle of Wight	
1902		Hinxman, Lionel W., B.A., formerly of the Geological Survey of Scotland. 4 Morant Gardens, Ringwood, Hants	
1904		Hobday, Major Frederick T. G., C.M.G., F.R.C.V.S., Officier du Merite Agricole, Cavaliere dei S.S. Maurizio e Lazaro, Hon. Veterinary Surgeon to H.M. the King, Editor of the Veterinary Journal, 165 Church Street, Kensington, London, W.	
1885		Hodgkinson, W. R., C.B.E., M.A., Ph.D., F.I.C., F.C.S., Professor of Chemistry and Physics at the Ordnance College, Woolwich, 89 Shooter's Hill Road, Blackheath, Kent 260	
1911		Holland, William Jacob, LL.D. St Andrews, etc., Director Carnegie Institute, Pittsburg, Pa., 5545 Forbes Street, Pittsburg, Pa., U.S.A.	
1920	C.	* Horne, Alexander Robert, O.B.E., B.Sc., M.I.Mech.E., A.M.I.C.E., Professor of Engineering, Robert Gordon's Technical College, Aberdeen, 374 Great Western Road, Aberdeen	
1881	C. N.	Horne, John, LL.D., F.R.S., F.G.S., formerly Director of the Geological Survey of Scotland, 20 Merchiston Gardens, Edinburgh	1902–05, 1906–07, 1914–15. V-P 1907–13. P 1915–19.
1896		Horne, J. Fletcher, M.D., F.R.C.S.E., The Poplars, Barnsley	
1904	C.	* Horsburgh, Ellice Martin, M.A., D.Sc., Reader in Technical Mathematics, University of Edinburgh, 11 Granville Terrace, Edinburgh 265	
1897		Houston, Sir Alex. Cruikshanks, K.B.E., C.V.O., M.B., C.M., D.Sc., 19 Fairhazel Gardens, South Hampstead, London, N.W.	
1912	C. B.	* Houstoun, Robert Alexander, M.A., Ph.D., D.Sc., Lecturer in Physical Optics, University, Glasgow, 45 Kirklee Road, Glasgow	
1893		Howden, Robert, M.A., M.B., C.M., D.Sc., Professor of Anatomy in the University of Durham, 14 Burdon Terrace, Newcastle-upon-Tyne	
1883	C.	Hoyle, William Evans, M.A., D.Sc., M.R.C.S., Director of the Welsh National Museum : Crowland, Llandaff, Wales	
1910		Hume, William Fraser, D.Sc. (Lond.), Director, Geological Survey of Egypt, Helwân, Egypt 270	
1916		* Hunter, Charles Stewart, L.R.C.P.E., L.R.C.S.E., D.P.H., Walden, Anerley Road, London, S.E. 20	
1911		Hunter, Gilbert Macintyre, M.Inst.C.E., M.Inst.E.S., M.Inst.M.E., Resident Engineer, Nitrate Railways, Iquique, Chile, and Maybole, Ayrshire	
1887	C.	Hunter, William, M.D., M.R.C.P.L. and E., M.R.C.S., 103 Harley Street, London	
1908		Hyslop, Theophilus Bulkeley, M.D., M.R.C.P E., 5 Portland Place, London, W.	
1920		* Inglis, James Gall, Publisher and Editor of Educational Works, Edinburgh, 36 Blacket Place, Edinburgh 275	
1912		* Inglis, Robert John Mathieson, M.Inst.C.E., District Engineer, North British Railway. Tantah, Peebles	
1904	C.	Innes, R. T. A., Director, Government Observatory, Johannesburg, Transvaal	
1917		* Irvine, James Colquhoun, C.B.E., Ph.D., D.Sc., LL.D., F.R.S., Principal of the University of St Andrews	1920–
1914		Jack, John Noble	
1875		Jack, William, M.A., LL.D., D.Sc., Emeritus Professor of Mathematics in the University of Glasgow 280	1888–91.
1889		James, Alexander, M.D., F.R.C.P.E., 9 Randolph Crescent, Edinburgh	
1901		* Jardine, Robert, M.D., M.R.C.S., F.R.F.P.S. Glas., 20 Royal Crescent, Glasgow	
1912	C.	* Jeffrey, George Rutherford, M.D. (Glasg.), F.R.C.P. (Edin.), etc., Bootham Park Private Mental Hospital, York	
1906	C.	* Jehu, Thomas John, M.A., M.D., F.G.S., Professor of Geology in the University of Edinburgh : 35 Great King Street, Edinburgh	1917–20.
1900		* Jerdan, David Smiles, M.A., D.Sc., Ph.D., 26 Avenue du Château d' Eau, Saventhem, Belgium 285	

Date of Election.			Service on Council, etc.
1895		Johnston, Col. Henry Halcro, C.B., D.Sc., M.D., F.L.S., late A.M.S., Orphir House, Kirkwall, Orkney	
1903	C.	* Johnston, Thomas Nicol, M.B., C.M., Pogbie, Humbie, East Lothian	
1874		Jones, Francis, M.Sc., Lecturer in Chemistry, 17 Whalley Road, Whalley Range, Manchester	
1888		Jones, John Alfred, M.Inst.C.E., Fellow of the University of Madras, Sanitary Engineer to the Government of Madras, c/o Messrs Parry & Co., 70 Gracechurch Street, London	
1915		Kemnal, Sir James Hermann Rosenthal, Managing Director and Engineer-in-Chief of Babcock & Wilcox, Ltd., Kemnal Manor, Chislehurst, Kent 290	
1912		Kennedy, Robert Foster, M.D. (Queen's Univ., Belfast), M.B., B.Ch. (R.U.I.), Assistant Professor of Neurology, Cornell University, New York, 20 West 50th Street, New York, U.S.A.	
1909		Kenwood, Henry Richard, M.B., Chadwick Professor of Hygiene in the University of London, 126 Queen's Road, Finsbury Park, London, N.	
1908		* Kerr, Andrew William, F.S.A. Scot., 81 Great King Street, Edinburgh	
1891		Kerr, Joshua Law, M.D., 16 High Street, Swindon, Wilts.	
1913		* Kerr, Walter Hume, M.A., B.Sc., Lecturer on Engineering Drawing and Structural Design in the University of Edinburgh 295	
1908		Kidd, Walter Aubrey, M.D., 2 Suffolk Square, Cheltenham	
1886	C. N.	Kidston, Robert, LL.D., D.Sc., F.R.S., F.G.S., 12 Clarendon Place, Stirling	1891–94, 1903–06. Sec. 1909–16. V-P 1917–20.
1907		* King, Archibald, M.A., B.Sc., formerly Rector of the Academy, Castle Douglas; H.M. Inspector of Schools, Inverspey, Fochabers, Morayshire	
1880		† King, W. F., Lonend, Russell Place, Trinity, Leith	
1918		* Kingon, Rev. John Robert Lewis, M.A. (Edin. and Cape of Good Hope), D.Sc. (Ghent), F.L.S., U.F. Church of Scotland, Box 17, Port Elizabeth, C.P., South Africa 300	
1878		Kintore, The Right Hon. the Earl of, P.C., G.C.M.G., M.A. Cantab., LL.D. Cambridge, Aberdeen, and Adelaide, Keith Hall, Inverurie, Aberdeenshire	
1901		* Knight, Rev. G. A. Frank, M.A., 5 Granby Terrace, Hillhead, Glasgow	
1907		* Knight, James, M.A., D.Sc., F.C.S., F.G.S., Head Master, Queen's Park High School. Enterkin, Douglas Gardens, Uddingston, by Glasgow	
1880	C. K.	Knott, C. G., D.Sc., LL.D., F.R.S., Reader in Applied Mathematics in the University of Edinburgh, formerly Professor of Physics, Imperial University, Japan (GEN. SECRETARY), 42 Upper Gray Street, Edinburgh	1894–97, 1898–1901, 1902–05. Sec. 1905–12. Gen. Sec. 1912–
1921		* Lamb, James Alexander George, Banker, 11 Braid Crescent, Edinburgh 305	
1920		* Lamont, John Charles, Lieut.-Col., I.M.S. (retired), C.I.E., M.B., C.M. (Edin.), M.R.C.S. (Eng.), 7 Merchiston Park, Edinburgh	
1878	C.	Lang, Sir P. R. Scott, Kt., M.A., B.Sc., Emeritus Professor of Mathematics, University of St Andrews	
1910	C.	* Lauder, Alexander, D.Sc., Lecturer in Agricultural Chemistry, Edinburgh and East of Scotland College of Agriculture, 13 George Square, Edinburgh	1917–20.
1885	C.	Laurie, A. P., M.A., D.Sc., J.P., Principal of the Heriot-Watt College, Edinburgh	1908–11, 1913–16.
1894	C.	Laurie, Malcolm, B.A., D.Sc., F.L.S., 4 Wordsworth Road, Harpenden, Herts 310	
1921		* Laurie, The Rev. Albert Ernest, M.C., C.F., Rector of Old St Paul's, Edinburgh, and Canon of St Mary's Cathedral, Edinburgh. Lauder House, Jeffrey Street, Edinburgh	
1910	C. B.	* Lawson, A. Anstruther, B.Sc., Ph.D., D.Sc., F.L.S., Professor of Botany, University of Sydney, New South Wales, Australia	
1905		* Lawson, David, M.A., M.D., L.R.C.P. and S.E., Druimdarroch, Banchory, Kincardineshire	
1910	C.	* Lee, Gabriel W., D.Sc., Palæontologist, Geological Survey of Scotland, 33 George Square, Edinburgh	
1903		* Leighton, Gerald Rowley, O.B.E., M.D., Medical Officer, Scottish Board of Health, 125 George Street, Edinburgh 315	
1910		Levie, Alexander, F.R.C.V.S., D.V.S.M., Rannock, Carlton Road, Derby	

Date of Election.			Service on Council, etc.
1916	C.	*Levy, Hyman, M.A., D.Sc., Assistant Professor of Mathematics, Imperial College of Science and Technology, London, S.W. 7, "Eskbank," 105 Cambridge Road, Teddington, Middlesex	
1914	C. N.	Lewis, Francis John, D.Sc., F.L.S., Professor of Biology, University of Alberta, Edmonton South, Alberta, Canada	
1918		*Lidstone, George James, F.F.A., F.I.A., Manager and Actuary of the Scottish Widows' Fund Life Assurance Society, 8 Eglinton Crescent, Edinburgh	1919–
1905		*Lightbody, Forrest Hay, 53 Queen Street, Edinburgh 320	
1889		Lindsay, Rev. James, M.A., D.D., B.Sc., F.R.S.L., F.G.S., M.R.A.S., Corresponding Member of the Royal Academy of Sciences, Letters and Arts, of Padua, Associate of the Philosophical Society of Louvain, Annick Lodge, Irvine	
1912		*Lindsay, John George, M.A., B.Sc. (Edin.), Rector of Dunfermline High School	
1920	C.	*Lindsay, Thomas A., M.A. (Hons.), B.Sc., Head Master, Higher Grade School, Bucksburn, Aberdeenshire	
1912		*Linlithgow, The Most Honourable the Marquis of, Hopetoun House, South Queensferry	
1903		†Liston, William Glen, M.D., Lt.-Col. Indian Medical Service, Director Bombay Bacteriological Laboratory, Paree, Bombay, India 325	
1903		*Littlejohn, Henry Harvey, M.A., M.B., B.Sc., F.R.C.S.E., Professor of Forensic Medicine, and late Dean of the Faculty of Medicine in the University of Edinburgh, 11 Rutland Street, Edinburgh	
1898		*Lothian, Alexander Veitch, M.A., B.Sc., Training College, Jordanhill, Glasgow	
1884		Low, George M., Actuary, 11 Moray Place, Edinburgh	
1888		Lowe, D. F., M.A., LL.D., formerly Headmaster of Heriot's Hospital School, Lauriston, 19 George Square, Edinburgh	
1900		†Lusk, Graham, Ph.D., M.A., Professor of Physiology, Cornell University Medical College, New York, N.Y., U.S.A. 330	
1894		Mabbott, Walter John, M.A., Rector of County High School, Duns, Berwickshire	
1887		M'Aldowie, Alexander M., M.D., 8 Holland Road, Cheltenham	
1917		*Macalister, Sir Donald, K.C.B., M.D., M.A., B.Sc., Principal of the University of Glasgow, The University, Glasgow	
1907		MacAlister, Donald Alexander, A.R.S.M., F.G.S., 10 St Alban's Road, Kensington, London, W. 8	
1921		*M'Arthur, Neil, M.A., B.Sc., Lecturer in Mathematics, Glasgow University. c/o Mrs Croll, 56 West End Park Street, Glasgow 335	
883		M'Bride, P., M.D., F.R.C.P.E., 20 South Drive, Harrogate	
1903		M'Cormick, Sir W.S., M.A., LL.D., Chairman of the Advisory Council, Department of Scientific and Industrial Research, 16–18 Old Queen Street, Westminster, S.W. 1	1910–13.
1918		*M'Culloch, Rev. James David, D.D., 43 Brougham Street, Greenock	
1905		*Macdonald, Hector Munro, M.A., F.R.S., Professor of Mathematics, University of Aberdeen, 52 College Bounds, Aberdeen	1908–11.
1897	C.	Macdonald, James A., M.A., B.Sc., H.M. Inspector of Schools, Stewarton, Kilmacolm 340	
1904		*Macdonald, John A., M.A., B.Sc., King Edward VII School, Johannesburg, Transvaal	
1920		*M'Donald, Stuart, M.A., M.D., F.R.C.P.E., Professor of Pathology, School of Medicine, Newcastle-on-Tyne	
1904		Macdonald, William, M.S.Agr., Sc.D., Ph.D., D.Sc., Editor, *Agricultural Journal* of South Africa, Rand Club, Johannesburg, Transvaal	
1886		Macdonald, William J., M.A., LL.D., 15 Comiston Drive, Edinburgh	
1901	C.	*MacDougall, R. Stewart, M.A., D.Sc., Professor of Biology, Royal Veterinary College, Edinburgh, 9 Dryden Place, Edinburgh 345	1914–17.
1910		Macewen, Hugh Allen, M.B., Ch.B., D.P.H. (Lond. and Camb.), Local Government Board, Whitehall, London, S.W.	
1888	C.	M'Fadyean, Sir John, Kt., M.B., B.Sc., LL.D., Principal, and Professor of Comparative Pathology in the Royal Veterinary College, Camden Town, London	
1885	C.	Macfarlane, J. M., D.Sc., LL.D., Emeritus Professor of Botany, 4320 Osage Avenue, Philadelphia, Pennsylvania, U.S.A.	
1897		MacGillivray, Angus, C.M., M.D., D.Sc., F.S.A. (Scot.), 23 South Tay Street, Dundee	
1878		M'Gowan, George, F.I.C., Ph.D., 21 Montpelier Road, Ealing, London, W. 5 350	
1903		*M'Intosh, Donald C., M.A., D.Sc., Education Offices, Elgin	
1911		M'Intosh, John William, A.R.C.V.S., Dollis Hill Farm, Cricklewood, London, N.W. 2	

Date of Election.			Service on Council, etc.
1869	C. N.	M'Intosh, William Carmichael, M.D., LL.D., F.R.S., F.L.S., Emeritus Professor of Natural History in the University of St Andrews, Pres. Ray Society, 2 Abbotsford Crescent, St Andrews	1885–88.
1895	C.	Macintyre, John, M.D., LL.D., 179 Bath Street, Glasgow	
1914		* M'Kendrick, Archibald, F.R.C.S.E., D.P.H., L.D.S., 12 Rothesay Place, Edinburgh 355	
1873	C. B.	M'Kendrick, John G., M.D., F.R.C.P.E., LL.D., F.R.S., Emeritus Professor of Physiology in the University of Glasgow, Maxieburn, Stonehaven	1875–78, 1885–88, 1893–94, 1900–02. V-P 1894–1900.
1912	C.	M'Kendrick, Anderson Gray, M.B., Major, Indian Medical Service, Superintendent, Research Laboratory, Royal College of Physicians, Edinburgh	
1900	C.	* M'Kendrick, John Souttar, M.D., F.R.F.P.S.G., 2 Buckingham Terrace, Hillhead, Glasgow	
1910	C.	* Mackenzie, Alister, M.A., M.D., D.P.H., Principal, College of Hygiene and Physical Training, Dunfermline	
1916	C.	* Mackenzie, John E., D.Sc., Lecturer in Chemistry, University of Edinburgh, Major-Adjutant, O.T.C., 2A Ramsay Garden, Edinburgh 360	
1894		Mackenzie, Robert, M.D., Napier, Nairn	
1904	C.	* Mackenzie, Sir W. Leslie, M.A., M.D., D.P.H., LL.D., Medical Member of the Scottish Board of Health, 14 Belgrave Place, Edinburgh	
1918		* Mackie, Wm., M.A., M.D., D.P.H., 13 North Street, Elgin	
1910		* MacKinnon, James, M.A., Ph.D., Professor of Ecclesiastical History, Edinburgh University, 12 Lygon Road, Edinburgh	
1904		* Mackintosh, Donald James, C.B., M.V.O., M.B., C.M., LL.D., Supt. Western Infirmary, Glasgow 365	
1899		Maclean, Ewan John, M.D., M.R.C.P. Lond., J.P., Professor of Obstetrics and Gynæcology, Welsh National School of Medicine, 12 Park Place, Cardiff	
1888	C.	Maclean, Magnus, M.A., D.Sc., LL.D., M.Inst.C.E., M.I.E.E., Professor of Electrical Engineering in the Royal Technical College, 51 Kerrsland Terrace, Hillhead, Glasgow	1916–19.
1913		* M'Lellan, Dugald, M.Inst.C.E., District Engineer, Caledonian Railway, 20 Kingsburgh Road, Murrayfield, Edinburgh	
1916	C.	* M'Lintock, W. F. P., D.Sc. (Edin.), Museum of Practical Geology, 28 Jermyn Street, London, S.W. 1	
1907	C.	* Macnair, Professor Peter, Curator of the Natural History Collections in the Glasgow Museums, Kelvingrove Museum, Glasgow 370	
1917		* Macpherson, Rev. Hector Copland, M.A., F.R.A.S., Guthrie Memorial U.F. Church. 30 Pilrig Street, Edinburgh	
1921		* M'Quistan, Dougald Black, M.A., B.Sc., Head Mathematical Master in Allan Glen's School, Glasgow. 29 Viewpark Drive, Rutherglen, near Glasgow	
1921		* MacRobert, Thomas Murray, M.A., D.Sc., Lecturer in Mathematics in the University of Glasgow. 6 Lothian Gardens, Kelvinside, N., Glasgow	
1921	C.	* M'Whan, John, M.A. (Glasgow), Ph.D. (Gött.), Lecturer in Mathematics in the University of Glasgow. 59 Greenlees Road, Cambuslang	
1898	C.	Mahâlanobis, S. C., B.Sc., Professor of Physiology, Presidency College, Calcutta, India 375	
1913		Majumdar, Tarak Nath, D.P.H. (Cal.), L.M.S., F.C.S., Health Officer, Calcutta, IV, 37 Lower Chitpore Road, Calcutta, India	
1917		* Malcolm, Louis William Gunther, M.A. (Melbourne), Capt. R.G.A., Bristol Museum and Art Gallery	
1908		Mallik, Devendranath, Sc.D., B.A., Professor of Mathematics, Astronomical Observatory, Presidential College, Calcutta, India	
1912		Maloney, William Joseph, M.D. (Edin.), Professor of Neurology at Fordham University, New York City, N.Y., U.S.A.	
1913		Marchant, Rev. Sir James, K.B.E., LL.D., F.R.A.S., F.L.S., Director, National Council for Promotion of Race-Regeneration, 20 Bedford Square, London, W.C. 11 380	
1909	C. B.	* Marshall, C. R., M.D., M.A., Professor of Materia Medica, Marischal College, Aberdeen	1915–18.
1882	C.	Marshall, D. H., M.A., Em.-Professor of Physics, Queen's University, Elmtree House, Union Street, W., Kingston, Ontario, Canada	
1901	C.	Marshall, F. H. A., Sc.D., Lecturer on Agricultural Physiology in the University of Cambridge, Christ's College, Cambridge	

Date of Election.			Service on Council, etc.
1920	C.	* Marshall, John, M.A., D.Sc. (St Andrews), B.A. (Cantab.), Senior Lecturer in Mathematics, University College, Swansea	
1913		Masson, George Henry, M.D., D.Sc., M.R.C.P.E., Port of Spain, Trinidad, British West Indies 385	
1885	C.	Masson, Orme, M.A., D.Sc., F.R.S., Professor of Chemistry in the University of Melbourne	
1898	C. B.	* Masterman, Arthur Thomas, M.A., D.Sc., F.R.S., formerly Superintending Inspector, H.M. Board of Agriculture and Fisheries. Mill House, Wooburn Green, Bucks	1902–04.
1911		Mathews, Gregory Macalister, F.L.S., F.Z.S., Foulis Court, Fair Oaks, Hants	
1921		* Mathieson, John, F.R.S.G.S., late Division Superintendent, Ordnance Survey (retired), 42 East Claremont Street, Edinburgh	
1906		* Mathieson, Robert, F.C.S., St Serf's, Innerleithen 390	
1902		Matthews, Ernest Romney, A.M.Inst.C.E., F.G.S., Chadwick Professor of Municipal Engineering in the University of London, University College, Gower Street, London, W.C.	
1917		* Maylard, A. Ernest, M.B., B.Sc. (Lond.), F.R.F.P.S. (Glasgow), 1 Windsor Terrace, W., Great Western Road, Glasgow	
1901	C.	Menzies, Alan W. C., M.A., B.Sc., Ph.D., F.C.S., Professor of Chemistry, Princeton University, Princeton, New Jersey, U.S.A.	
1917		* Merson, George Fowlie, Manufacturing Technical Chemist, 9 Hampton Terrace, Edinburgh	
1888		Methven, Cathcart W., M.Inst.C.E., F.R.I.B.A., Durban, Natal, S. Africa 395	
1902	C.	Metzler, William H., A.B., Ph.D., Corresponding Fellow of the Royal Society of Canada, Professor of Mathematics, Syracuse University, Syracuse, N.Y., U.S.A.	
1885	C. B.	Mill, Hugh Robert, D.Sc., LL.D., Hill Crest, Dorman's Park, E. Grinstead, London	
1908		* Miller, Alexander Cameron, M.D., F.S.A. Scot., Craig Linnhe, Fort-William, Inverness-shire	
1910		* Miller, John, M.A., D.Sc., Professor of Mathematics, Royal Technical College, 2 Northbank Terrace, North Kelvinside, Glasgow	
1909		Mills, Bernard Langley, M.D., F.R.C.S.E., M.R.C.S., D.P.H., Lt.-Col. R.A.M.C., formerly Army Specialist in Hygiene, c/o National Provincial Bank, Fargate, Sheffield 400	
1905		* Milne, Archibald, M.A., D.Sc., Lecturer on Mathematics and Science, Edinburgh Provincial Training College, 108 Comiston Drive, Edinburgh	
1905		* Milne, C. H., M.A., Head Master, Daniel Stewart's College, 19 Merchiston Gardens, Edinburgh	
1904	C.	* Milne, James Robert, D.Sc., Lecturer in Natural Philosophy, University of Edinburgh, 17 Manor Place, Edinburgh	
1886		Milne, William, M.A., B.Sc., 70 Beechgrove Terrace, Aberdeen	
1899		* Milroy, T. H., M.D., B.Sc., Professor of Physiology in Queen's College, Belfast 405	
1889	C.	Mitchell, A. Crichton, D.Sc., Hon. Doc. Sc. (Genève), formerly Director of Public Instruction in Travancore, India (CURATOR OF LIBRARY AND MUSEUM), The Observatory, Eskdalemuir, Langholm, Dumfriesshire	1915–16. Cur. 1916–
1897		† Mitchell, George Arthur, M.A., 9 Lowther Terrace, Kelvinside, Glasgow	
1900		* Mitchell, James, M.A., B.Sc., Monydrain, Lochgilphead	
1911		Modi, Edalji Manekji, D.Sc., LL.D., Litt.D., F.C.S., etc., Proprietor and Director of Arthur Road Chemical Works, Meher Buildings, Tardeo, Bombay, India	
1906	C.	Moffat, Rev. Alexander, M.A., B.Sc., Professor of Physical Science, Christian College, Madras, India 410	
1890	C.	Mond, R. L., M.A. Cantab., F.C.S., Combe Bank, near Sevenoaks, Kent	
1887	C.	Moos, N. A. F., D.Sc., L.C.E., J.P., Director of Bombay and Alibag Observatories (retired), Gowalia, Tank Road, Bombay, India	
1896		Morgan, Alexander, M.A., D.Sc., Principal, Edinburgh Provincial Training College, 1 Midmar Gardens, Edinburgh	
1919		* Morris, Robert Owen, M.A., M.D., C.M. (Edin.), D.P.H. (Liverpool), Tuberculosis Institute, Newtown, N. Wales	
1892	C.	Morrison, J. T., M.A., B.Sc., Professor of Physics and Chemistry, University, Stellenbosch, Cape Colony 415	
1914		Mort, Spencer, M.D., Ch.B., F.R.C.S.E., Lieut.-Col., R.A.M.C., North Middlesex Hospital, Upper Edmonton, London, N. 18.	
1901		Moses, O. St John, I.M.S., M.D., D.Sc., F.R.C.S., Lt.-Col. I.M.S., Professor of Medical Jurisprudence, c/o Messrs King, Hamilton & Co., 4 and 5 Koila Ghat Street, Calcutta, India	
1892	C. K.	Mossman, R. C., Oficina Meteorologica Argentina, Paseo Colon 974, Buenos Aires	
1916		* Muir, Robert, M.A., M.D., Sc.D., F.R.S., Professor of Pathology, University of Glasgow, 16 Victoria Crescent, Dowanhill, Glasgow	

Date of Election.				Service on Council, etc.
1874	C. K. V. J.	Muir, Sir Thomas, C.M.G., M.A., LL.D., F.R.S., lately Superintendent-General of Education for Cape Colony, Education Office, Cape Town, and Elmcote, Sandown Road, Rondebosch, South Africa	420	1885–88. V-P 1888–91.
1888	C.	Muirhead, George, Commissioner to His Grace the Duke of Richmond and Gordon, K.G., Speybank, Fochabers		
1907		Muirhead, James M. P., J.P., F.R.S.L., F.S.S., c/o Royal Societies Club, St James's Street, London, S.W.		
1887		Mukhopâdhyay, Asûtosh, M.A., LL.D., F.R.A.S., M.R.I.A., Professor of Mathematics at the Indian Association for the Cultivation of Science, 77 Russa Road North, Bhowanipore, Calcutta, India		
1894 & 1921		* Munro, J. M. M., M.Inst.E.E., Assoc.M.Inst.C.E., Consulting Electrician and Engineer, 11 Randolph Place, Edinburgh.		
1896		Murray, Alfred A., M.A., LL.B., 20 Warriston Crescent, Edinburgh	425	
1907		Musgrove, James, M.D., F.R.C.S. Edin. and Eng., LL.D., Emeritus-Professor of Anatomy, University of St Andrews, The Swallowgate, St Andrews		
1888		Napier, A. D. Leith, M.D., C.M., M.R.C.P., 4 Kent Street, Hawthorn, Unley, S. Australia		
1897		Nash, Hon. Alfred George, M.L.C., B.Sc., F.R.G.S., C.E., Belretiro, Mandeville, Jamaica, W.I.		
1898		Newman, Sir George, K.C.B., M.D., D.C.L., F.R.C.P., Chief Medical Officer of the Ministry of Health and the Board of Education, Whitehall, S.W. 1		
1884		Nicholson, J. Shield, M.A., D.Sc., Professor of Political Economy in the University of Edinburgh, 3 Belford Park, Edinburgh	430	1885-87, 1892–95, 1897–1900.
1880	C.	Nicol, W. W. J., M.A., D.Sc., 15 Blacket Place, Edinburgh		
1878		Norris, Richard, M.D., M.R.C.S. Eng., 3 Walsall Road, Birchfield, Birmingham		
1888		† Ogilvie, Sir F. Grant, Kt., C.B., M.A., B.Sc., LL.D., Principal Assistant Secretary, Department of Scientific and Industrial Research, 15 Evelyn Gardens, London, S.W.		1901–03.
1886		Oliver, James, M.D., F.L.S., Physician to the London Hospital for Women, 123 Harley Street, London, W.		
1895	C.	Oliver, Sir Thomas, Kt., M.D., LL.D., F.R.C.P., Professor of Physiology in the University of Durham, 7 Ellison Place, Newcastle-upon-Tyne	435	
1915		* Orr, Lewis P., F.F.A., Manager of Scottish Life Assurance Co., 19 St Andrew Square, Edinburgh		
1914		* Oswald, Alfred, Lecturer in German, Glasgow Provincial Training College, 11 Nelson Terrace, Hillhead, Glasgow		
1908		Page, William Davidge, F.C.S., F.G.S., M.Inst.M.E., 10 Clifton Dale, York		
1905		Pallin, Lt.-Col. William Alfred, C.B.E., D.S.O., F.R.C.V.S., Headquarters, Eastern Command, Nainital, India		
1914		Pare, John William, M.D., C.M., L.D.S., Lecturer in Dental Anatomy, National Dental Hospital, 9a Cavendish Square, London, W.	440	
1901		* Paterson, David, F.C.S., Lea Bank, Rosslyn, Midlothian		
1918		* Paterson, Rev. William Paterson, D.D., LL.D., Professor of Divinity, University, Edinburgh, 3 Royal Terrace, Edinburgh		
1886	C.	Paton, D. Noël, M.D., B.Sc., LL.D., F.R.C.P.E., F.R.S., Professor of Physiology in the University of Glasgow, University, Glasgow		1894–97, 1904–06, 1909–12. V-P 1918–21
1919	C.	* Patterson, Thomas Stewart, D.Sc. (London and Glasgow), Ph.D. (Heidelberg), Professor of Organic Chemistry in the University of Glasgow, 10 Oakfield Terrace, Hillhead, Glasgow		
1892		Paulin, Sir David, Actuary, 6 Forres Street, Edinburgh	445	
1881	C. N.	Peach, Benjamin N., LL.D., F.R.S., F.G.S., formerly District Superintendent and Acting Palæontologist of the Geological Survey of Scotland, 72 Grange Loan, Edinburgh		1905–08, 1911–12. V-P 1912–17.
1907		* Pearce, John Thomson, B.A., B.Sc., School House, Tranent		
1914		Pearson, Joseph, D.Sc., F.L.S., Director of the Colombo Museum, and Marine Biologist to the Ceylon Government, Colombo Museum, Ceylon		
1904		* Peck, James Wallace, C.B., M.A., Scottish Board of Health. 10 South Learmonth Gardens, Edinburgh		
1889		† Peck, Sir William, Kt., F.R.A.S., Town's Astronomer, City Observatory, Calton Hill, Edinburgh	450	

Date of Election.			Service on Council, etc.
1887	C. B.	Peddie, Wm., D.Sc. (Vice-President), Professor of Natural Philosophy in University College, Dundee, The Weisha, Ninewells, Dundee	1904–07, 1908–11. V-P 1919–
1893		Perkin, Arthur George, F.R.S., Grosvenor Lodge, Grosvenor Road, Leeds	
1913	C.	† Philip, Alexander, M.A., LL.B., Writer, The Mary Acre, Brechin	
1889		Philip, Sir R. W., Kt., M.A., M.D., LL.D., F.R.C.P.E., Professor of Tuberculosis, University of Edinburgh, 45 Charlotte Square, Edinburgh	
1907	C.	Phillips, Major Charles E. S., O.B.E., Castle House, Shooters Hill, Woolwich, S.E. 18 455	
1914		* Pilkington, Basil Alexander, "Kambla," Davidson's Mains	
1905		* Pinkerton, Peter, M.A., D.Sc., Rector, High School, Glasgow, 7 Park Quadrant, Glasgow, W.	
1908	C.	* Pirie, James Hunter Harvey, B.Sc., M.D., F.R.C.P.E., Superintendent of the Routine Division of The South African Institute for Medical Research, P.O. Box 1038, Johannesburg, South Africa	
1911		* Pirie, James Simpson, M.Inst.C.E., 28 Scotland Street, Edinburgh	
1906		Pitchford, Herbert Watkins, C.M.G., F.R.C.V.S., Lt.-Col., Oaklands Drive, Weybridge, Surrey 460	
1921		* Pollard, Sir George Herbert, K.B., M.D., C.M. (Edin.), Barrister-at-Law, Inner Temple. 79 Albert Road, Southport	
1919		* Porritt, B. D., M.Sc. (Lond.), F.I.C., Research Association of British Rubber and Tyre Manufacturers Chemistry Dept., University College, Gower Street, London, W.C. 1	
1888		Prain, Sir David, Lt.-Col., Indian Medical Service (retired), Kt., C.M.G., C.I.E., M.A., M.B., LL.D., F.L.S., F.R.S., For. Memb. K. Svensk. Vetensk. Akad.; Hon. Memb. Soc. Lett. ed Arti d. Zelanti, Acireale; Pharm. Soc. Gt. Britain; Corr. Memb. Bayer Akad. Wiss., etc.; Director, Royal Botanic Gardens, Kew, Surrey	
1902		* Preller, Charles du Riche, M.A., Ph.D., A.M.Inst.C.E., M.I.E.E., F.G.S., 61 Melville Street, Edinburgh	
1892	C.	Pressland, Arthur J., M.A. Camb., Edinburgh Academy 465	
1915		Price, Frederick William, M.D., M.R.C.P. Edin., Physician to the Great Northern Hospital, London, 133 Harley Street, London, W.	
1903		* Pullar, Laurence, Dunbarney, Bridge of Earn, Perthshire	
1911		Purdy, John Smith, D.S.O., M.D., C.M. (Aberd.), D.P.H. (Camb.), F.R.G.S., Town Hall, Sydney, N.S.W., Australia	
1920		* Purser, George Leslie, M.A. (Cantab.), Assistant in the Natural History Department of the University of Edinburgh, c/o Muir, 21 Buccleuch Place, Edinburgh	
1898		* Purves, John Archibald, D.Sc., 52 Queen Street, Exeter 470	
1897	C.	Rainy, Harry, M.A., M.D., F.R.C.P.Ed., 16 Great Stuart Street, Edinburgh	
1899	C.	* Ramage, Alexander G., Marchfield, Davidson's Mains, Midlothian	
1914		* Ramsay, Peter, M.A., B.Sc., Head Mathematical Master, George Watson's College, 63 Comiston Drive, Edinburgh	
1911		* Rankin, Adam A., British Astronomical Association, West of Scotland Branch, 24 Woodend Drive, Jordanhill, Glasgow	
1891		Rankine, Sir John, K.C., M.A., LL.D., Professor of the Law of Scotland in the University of Edinburgh, 23 Ainslie Place, Edinburgh 475	
1904		Ratcliffe, Joseph Riley, M.B., C.M., c/o The Librarian, The University, Birmingham	
1900		Raw, Nathan, C.M.G., M.D., M.P., 45 Weymouth Street, Harley Street, London, W. 1.	
1883	C.	Readman, J. B., D.Sc., F.C.S., Frankleigh House, Bradford-on-Avon, Wilts	
1902		Rees-Roberts, John Vernon, M.D., D.Sc., D.P.H., 11 Oak Hill Park, Hampstead, London, N.W. 3	
1902		Reid, George Archdall O'Brien, M.B., C.M., 9 Victoria Road South, Southsea, Hants 480	
1913		Reid, Harry Avery, O.B.E., F.R.C.V.S., D.V.H., Bacteriologist and Pathologist, Department of Agriculture, Wellington, New Zealand	
1908	C.	* Rennie, John, D.Sc., Lecturer on Parasitology and Experimental Zoology, University of Aberdeen, 60 Desswood Place, Aberdeen	
1914		Renshaw, Graham, M.D., M.R.C.S., L.R.C.P., L.S.A., Editor of the *Avicultural Magazine*, Sale Bridge House, Sale, Manchester	
1913		* Richardson, Harry, M.Inst.E.E., M.Inst.M.E., General Manager and Chief Engineer, Electricity Supply, Dundee and District, Dudhope Crescent Road, Dundee	

Date of Election.			Service on Council, etc
1908		Richardson, Linsdall, F.G.S., 10 Oxford Parade, Cheltenham, Glos. 485	
1875		Richardson, Ralph, W.S., 29 Eglinton Crescent, Edinburgh	
1916	C.	* Ritchie, James, M.A., D.Sc., Keeper of the Natural History Department in the Royal Scottish Museum, 20 Upper Gray Street, Edinburgh	1921–
1914	C.	* Ritchie, James Bonnyman, D.Sc., Headmaster, Academy, Forres	
1906	C.	* Ritchie, William Thomas, M.D., F.R.C.P.E., 14 Rothesay Place, Edinburgh	
1898	C.	Roberts, Alexander William, D.Sc., F.R.A.S., Lovedale, South Africa 490	
1919		* Roberts, Alfred Henry, O.B.E., M.Inst.C.E., Superintendent and Engineer, Leith Docks, 2 Cargil Terrace, Trinity, Edinburgh	
1900		* Robertson, Joseph M'Gregor, M.B., C.M., 26 Buckingham Terrace, Glasgow	
1902	C.	* Robertson, Robert A., M.A., B.Sc., Lecturer on Botany in the University of St Andrews	
1919		* Robertson, William Alexander, F.F.A., Century Insurance Co., Ltd., 18 Charlotte Square. 12 Lonsdale Terrace, Edinburgh	
1896	C.	Robertson, W. G. Aitchison, D.Sc., M.D., F.R.C.P.E., The Grange, Ashford, Middlesex 495	
1910	C.	* Robinson, Arthur, M.D., M.R.C.S., Professor of Anatomy, University of Edinburgh, 35 Coates Gardens, Edinburgh	1910-12. Sec. 1912–18. V-P 1918–21.
1916		* Ronald, David, M.Inst.C.E., Chief Engineer, Scottish Board of Health, 125 George Street, Edinburgh	
1881		Rosebery, The Right Hon. the Earl of, K.G., K.T., LL.D., D.C.L., F.R.S., Dalmeny Park, Edinburgh	
1909	C.	* Ross, Alex. David, M.A., D.Sc., F.R.A.S., Professor of Mathematics and Physics, University of Western Australia, Perth, Western Australia	
1921		* Ross, Edward Burns, M.A. (Edin. and Camb.), Professor of Mathematics in the Madras Christian College, Madras 500	
1906		* Russell, Alexander Durie, B.Sc., Mathematical Master, Falkirk High School, 14 Heugh Street, Falkirk	
1902	C. K.	* Russell, James, 22 Glenorchy Terrace, Edinburgh	
1906		Saleeby, Caleb William, M.D., 10 Campden Mansions, Kensington, London, W. 8	
1916		* Salvesen, The Hon. Lord, Judge of the Court of Session, Dean Park House, Edinburgh	1920–
1914		* Salvesen, Theodore Emile, 37 Inverleith Place, Edinburgh 505	
1912	C.	* Sampson, Ralph Allen, M.A., D.Sc., F.R.S., Astronomer Royal for Scotland, Professor of Astronomy, University, Edinburgh, Royal Observatory, Edinburgh	1912–15, 1919–21. V-P 1915–18.
1903		* Samuel, Sir John S., K.B.E., D.L., J.P., F S.A. (Scot.), 13 Park Circus, Glasgow, W.	
1903		* Sarolea, Charles, Ph.D., D.Litt., Professor of French, University of Edinburgh, 21 Royal Terrace, Edinburgh	
1900	C.	* Schafer, Sir Edward Albert Sharpey, M.D., LL.D., D.Sc., F.R.S., Professor of Physiology in the University of Edinburgh	1900–03, 1906–09, 1918–19. V-P 1913-17.
1919		* Scott, Alexander, M.A., D.Sc., Carnegie Scholar, 1912–14 ; 1851 Exhibition Scholar, 1914–16 ; lectured (temp.) on Petrology, Oxford, 1914–15, and at Glasgow University, 1917–18 ; Physical Chemist in charge of Radiometric Laboratory, Glasgow University, 1916–18 ; Chief Assistant to Principal, Pottery Laboratory, Stoke-on-Trent 510	
1885	C.	Scott, Alexander, M.A., D.Sc., F.R.S., 34 Upper Hamilton Terrace, London, N.W.	
1919		* Scott, Alexander Ritchie, B.Sc. (Edin.), D.Sc. (Lond.), Principal London County Council, Beaufoy Institute, Prince's Road, Vauxhall Street, London, S.E. 11	
1917		* Scott, Henry Harold, M.D., M.R.C.P., L.R.C.P. (London), M.R.C.S. (Eng.), D.P.H., Bacteriologist and Pathologist to the Government of Hong Kong	
1908		* Simpson, George Freeland Barbour, M.D., F.R.C.P.E., F.R.C.S.E., 43 Manor Place, Edinburgh	
1900	C.	* Simpson, James Young, M.A., D.Sc., Professor of Natural Science in the New College, Edinburgh, 25 Chester Street, Edinburgh 515	
1911	C.	Simpson, Sutherland, M.D., D.Sc. (Edin.), Professor of Physiology, Medical College, Cornell University, Ithaca, N.Y., U.S.A., 118 Eddy Street, Ithaca, N.Y., U.S.A.	

Date of Election.			Service on Council, etc.
1900		Sinhjee, Sir Bhagvat, G.C.I.E., M.D., LL.D. Edin., H.H. the Thakur Sahib of Gondal, Gondal, Kathiawar, Bombay, India	
1903		* Skinner, Robert Taylor, M.A., J.P., Head Master, Donaldson's Hospital, Edinburgh	
1901		* Smart, Edward, B.A., B.Sc., Tillyloss, Tullylumb Terrace, Perth	
1920		* Smellie, William Robert, M.A., B.Sc., D.Sc., Geologist on the Staff of the Anglo-Persian Oil Company, Mayfield, Mossend, near Glasgow 520	
1891	C. K.	Smith, Alexander, B.Sc., Ph.D., LL.D., Department of Chemistry, Columbia University, New York, N.Y., U.S.A.	
1882	C.	Smith, C. Michie, C.I.E., B.Sc., F.R.A.S., formerly Director of the Kodaikânal and Madras Observatories, Winsford, Kodaikânal, South India, c/o Messrs H. S. King & Co., 9 Pall Mall, London, S.W. 1	
1915		* Smith, James Lorrain, M.A., M.D., F.R.S., Professor of Pathology, University of Edinburgh, 9 Carlton Terrace, Edinburgh	1918-21.
1921		* Smith, The Right Hon. James Parker, P.C., M.A. (Camb.), 41 Drumsheugh Gardens, Edinburgh	
1921		* Smith, Norman Kemp, M.A., D.Phil., Professor of Logic and Metaphysics, University of Edinburgh 525	
1911		* Smith, Stephen, B.Sc., Engineer, 31 Grange Loan, Edinburgh	
1907	C.	Smith, William Ramsay, D.Sc., M.D., C.M., Permanent Head of the Health Department, South Australia, Belair, South Australia	
1880		Smith, Sir William (Robert), M.D., D.Sc., LL.D., Principal of The Royal Institute of Public Health, Em.-Professor of Forensic Medicine and Toxicology in King's College, University of London, 36 Russell Square, London, W.C. 1	
1919		* Smith, William Wright, M.A. (Edin.), Assistant Keeper, Royal Botanic Garden, Edinburgh, 6 Lennox Row, Trinity, Edinburgh	
1899		Snell, Ernest Hugh, M.D., B.Sc., D.P.H. Camb., Medical Officer of Health, Coventry 530	
1880		Sollas, W. J., M.A., D.Sc., LL.D., F.R.S., Fellow of University College, Oxford, and Professor of Geology and Palæontology in the University of Oxford	
1910		* Somerville, Robert, B.Sc., Science Master, High School, Dunfermline, 31 Cameron Street, Dunfermline	
1889		Somerville, Wm., M.A., D.Sc., D.Oec., Sibthorpian Professor of Rural Economy and Fellow of St John's College in the University of Oxford, 121 Banbury Road, Oxford	
1911	C.	* Sommerville, Duncan M'Laren Young, M.A., D.Sc., Professor of Pure and Applied Mathematics, Victoria College, Wellington, New Zealand	
1882		Sorley, James, 73 Onslow Square, London, S.W. 7 535	
1896		Spence, Frank, M.A., B.Sc., 25 Craiglea Drive, Edinburgh	
1906		Squance, Major Thomas Coke, M.D., M.S., F.R.M.S., F.S.A.Scot., Physician and Pathologist in the Sunderland Infirmary, President Sunderland Antiquarian Society, Sunderland Naturalists' Association, The Cottage, Newbiggin, Aysgarth, S.O.	
1891		Stanfield, Richard, Professor of Mechanics and Engineering in the Heriot-Watt College, Edinburgh	
1885 & 1915		* Steggall, John Edward Aloysius, M.A., Professor of Mathematics at University College, Dundee (St Andrews University), Woodend, Perth Road, Dundee	
1912	C. K.	† Stephenson, John, M.B., D.Sc. (Lond.), Lt.-Col. I.M.S., Zoological Department, University, Edinburgh 540	
1910		* Stephenson, Thomas, F.C.S., Editor of the *Prescriber*, Examiner to the Pharmaceutical Society, 6 South Charlotte Street, Edinburgh	
1916		* Steuart, D. R., F.I.C., Chemist to the Broxburn Oil Company, 20 Hillview, Blackhall, Midlothian	
1921		* Stewart, Ian Struthers, M.D. (Edin.), Nordrach-on-Dee, Banchory	
1886	C.	Stevenson, Charles A., B.Sc., M.Inst.C.E., 28 Douglas Crescent, Edinburgh	
1884		Stevenson, David Alan, B.Sc., M.Inst.C.E., 84 George Street, Edinburgh 545	
1919		* Stevenson, David Alan, B.Sc., A.M.Inst.C.E., 28 Douglas Crescent, Edinburgh	
1888	C.	Stewart, Charles Hunter, D.Sc., M.B., C.M., Professor of Public Health in the University of Edinburgh, Usher Institute of Public Health, Warrender Park Road, Edinburgh	
1902		* Stockdale, Herbert Fitton, LL.D., Director of the Royal Technical College, Glasgow, Clairinch, Upper Helensburgh, Dumbartonshire	
1889	C.	Stockman, Ralph, M.D., F.R.C.P.E., F.F.P.S.G., Professor of Materia Medica and Therapeutics in the University of Glasgow	1903-05.

Date of Election.			Service on Council, etc.
1906		Story, Fraser, formerly Professor of Forestry, University College, Bangor, North Wales. 4K Artillery Mansions, Victoria Street, London, S.W. 1 550	
1907		* Strong, John, C.B.E., M.A., LL.D., Professor of Education in the University of Leeds	
1903		Sutherland, David W., M.D., M.R.C.P., C.I.E., Lt.-Col. I.M.S., Principal and Professor of Medicine, Medical College, Lahore, India	
1905		Swithinbank, Commander Harold William, Crag Head, Bournemouth, Hants	
1912		* Syme, William Smith, M.D. (Edin.), 11 Lynedoch Crescent, Glasgow	
1885	C.	Symington, Johnson, M.D., LL.D., F.R.C.S., F.R.S., formerly Professor of Anatomy in the Queen's University, Belfast. Towercliffe Private Hotel, West Cliff, Bournemouth 555	1892-95.
1917	C. N.	* Tait, John, D.Sc., M.D., Professor of Physiology, M'Gill University, Montreal, Canada	
1904		* Tait, John W., B.Sc., Rector of Leith Academy, 18 Netherby Road, Leith	
1898	C.	Tait, William Archer, D.Sc., M.Inst.C.E. (Vice-President), 72A George Street, Edinburgh (Society's Representative on George Heriot's Trust)	1914–17, 1918–21. V-P 1921–
1895		† Talmage, James Edward, D.Sc., Ph.D., F.R.M.S., F.G.S., formerly Professor of Geology, University of Utah, 47 East S. Temple Street, Salt Lake City, Utah, U.S.A.	
1890	C.	Tanakadate, Aikitu, Professor of Natural Philosophy in the Imperial University of Japan, Tokyo, Japan 560	
1870		Tatlock, Robert R., F.C.S., City Analyst's Office, 156 Bath Street, Glasgow	
1899		* Taylor, James, M.A., Mathematical Master in the Edinburgh Academy	
1917	C.	* Taylor, William White, M.A., D.Sc., Lecturer on Chemical Physiology, University, Edinburgh, Park Villa, Liberton, Edinburgh	
1892		Thackwell, J. B., M.B., C.M., D.P.H., Carlton House, 1 Prince of Wales Road, Battersea Park, London, S.W.	
1885	C.	Thompson, D'Arcy W., C.B., D.Litt., F.R.S., Professor of Natural History, University, St Andrews, 44 South Street, St Andrews 565	1892–95, 1896–99, 1907–10, 1912–15. V-P 1916–19.
1917	C.	* Thompson, John M'Lean, M.A., D.Sc., F.L.S., Professor of Botany, University of Liverpool	
1905		* Thoms, Alexander, 7 Playfair Terrace, St Andrews	
1887		Thomson, Andrew, M.A., D.Sc., F.I.C., 145 Bruntsfield Place, Edinburgh	
1896		Thomson, George Ritchie, C.M.G., M.B., C.M., Professor of Surgery, University College, Johannesburg, Transvaal	
1903		Thomson, George S., F.C.S., Ferma Albion, Marculesci, Roumania 570	
1906		* Thomson, Gilbert, M.Inst.C.E., 164 Bath Street, Glasgow	
1887	C.	Thomson, J. Arthur, M.A., LL.D., Regius Professor of Natural History in the University of Aberdeen	1906–08, 1920–
1906	C.	Thomson, James Stuart, M.Sc., Ph.D., Zoological Department, University, Manchester	
1880		Thomson, John Millar, LL.D., F.R.S., Em.-Professor of Chemistry in King's College, London, 55 Bedford Gardens, Kensington, London, W. 8	
1899		* Thomson, R. Tatlock, F.C.S., 156 Bath Street, Glasgow 575	
1912	C.	Thomson, Robert Black, M.B. Edin., Professor of Anatomy, The University, Cape Town	
1870		Thomson, Spencer C., Actuary, 10 Eglinton Crescent, Edinburgh	
1882		Thomson, Wm., M.A., B.Sc., LL.D., Registrar, University of South Africa, Somerset House, Vermeulen Street, Pretoria	
1876	C.	Thomson, William, Royal Institution, Manchester	
1917		* Thorneycroft, Wallace, J.P., Coal and Iron Master, Plean House, Plean, Stirling 580	
1920		* Todd, John Barber, B.Sc., A.M.I.Mech.E., Lecturer in Engineering in the University of Edinburgh. 39 Warrender Park Terrace, Edinburgh	
1917		* Tovey, Donald Francis, B.A., Professor of Music, University, Edinburgh, 2 St Margaret's Road, Edinburgh	
1914		† Tredgold, Alfred Frank, M.D. (Durham), M.R.C.P., Lecturer on Mental Deficiency at London University, and Bethlem Royal Hospital, "St Martins," Guildford	
1915		* Trotter, George Clark, M.D., Ch.B. Edin., D.P.H. (Aberdeen), Medical Officer of Health, Metropolitan Borough, Islington. Public Health Department, 20 Compton Terrace, Upper Street, Islington, London, N. 1	

Date of Election.			Service on Council, etc.
1905		* Turner, Arthur Logan, M.D., F.R.C.S.E., 27 Walker Street, Edinburgh 585	
1906	C.	* Turner, Dawson F. D., B.A., M.D., F.R.C.P.E., M.R.C.P., Lecturer on Medical Physics, Surgeons' Hall, Physician in charge of Radium Treatment, Royal Infirmary, Edinburgh, 37 George Square, Edinburgh	
1895		Turton, Albert H., M.I.M.M., 233 George Road, Erdington, Birmingham	
1898	C.	* Tweedie, Charles, M.A., B.Sc., formerly Lecturer on Mathematics in the University of Edinburgh. Marine View, Belhaven, Dunbar	
1918	C.	* Tyrrell, G. W., A.R.C.Sc., F.G.S., Chief Assistant and Lecturer in Petrology, Geological Department, University, Glasgow	
1910		Vincent, Swale, M.D. Lond., D.Sc. Edin., etc., Professor of Physiology in the University of London, Physiological Laboratory, Middlesex Hospital Medical School, Berners Street, London, W. 1 590	
1891	C. B.	Walker, Sir James, Kt., D.Sc., Ph.D., LL.D., F.R.S., Professor of Chemistry in the University of Edinburgh, 5 Wester Coates Road, Edinburgh	1903–05, 1910–13. V-P 1916–19.
1902		* Wallace, Alexander G., M.A., 56 Fonthill Road, Aberdeen	
1886	C.	Wallace, R., M.A., F.L.S., Professor of Agriculture and Rural Economy in the University of Edinburgh	
1898		Wallace, Wm., M.A., Belvedere, Alberta, Canada	
1891		Walmsley, R. Mullineux, D.Sc., Principal of the Northampton Institute, Clerkenwell, London 595	
1920		* Walmsley, Thomas, M.D. (Glasgow), Professor of Anatomy in the University of Belfast, 59 South Street, Greenock	
1901	C.	* Waterston, David, M.A., M.D., F.R.C.S.E., Professor of Anatomy, University, St Andrews	1916–19.
1911		* Watson, James A. S., B.Sc., etc., Lecturer in Agriculture, University of Edinburgh, 30 Mayfield Terrace, Edinburgh	
1900		* Watson, Thomas P., M.A., B.Sc., Principal, Keighley Institute, Keighley	
1907		* Watt, Andrew, M.A., 10 Rothesay Place. 6 Woodburn Terrace, Edinburgh 600	1912–14.
1911		† Watt, James, W.S., F.F.A., Craiglockhart House, Slateford, Edinburgh	
1911		* Watt, Rev. Lauchlan Maclean, D.D., Minister of St Stephen's Parish, 7 Royal Circus, Edinburgh	
1896		Webster, John Clarence, B.A., M.D., F.R.C.P.E., Professor of Obstetrics and Gynæcology, Rush Medical College, Shediac, N.B., Canada	
1907	B. C.	* Wedderburn, Ernest Maclagan, M.A., LL.B., W.S., D.Sc., 6 Succoth Gardens, Edinburgh	1913–16. 1921–
1903	B. C.	* Wedderburn, J. H. Maclagan, M.A., D.Sc., P.O. Box 53, Princeton, N.J., U.S.A. 605	
1904		Wedderspoon, William Gibson, M.A., LL.D., Indian Educational Service, Senior Inspector of Schools, Burma, The Education Office, Rangoon, Burma	
1896		Wenley, Robert Mark, M.A., D.Sc., D.Phil., Litt.D., LL.D., D.C.L., Professor of Philosophy in the University of Michigan, Ann Arbor, U.S.A.	
1916		* White, J. Martin, Esq., of Balruddery, Balruddery, near Dundee	
1896	C.	White, Philip J., M.B., Professor of Zoology in University College, Bangor, North Wales	
1911		* Whittaker, Charles Richard, F.R.C.S. (Edin.), F.S.A. (Scot.), Lynwood, Hatton Place, Edinburgh 610	
1912	C.	* Whittaker, Edmund Taylor, Sc.D., F.R.S., Professor of Mathematics in the University of Edinburgh (SECRETARY), 35 George Square, Edinburgh	1912–15. c. 1916–
1918		* Whyte, Rev. Charles, M.A., LL.D., F.R.A.S., U.F. Church Manse, Kingswells, Aberdeen	
1918		* Wight, John Thomas, General Manager, Hydraulic Gears, Ltd., Beavor Lane, Hammersmith, London, W. 6; Dartbeigh, Ascot, Berks.	
1879		Will, John Charles Ogilvie, of Newton of Pitfodels, M.D., 17 Bon-Accord Square, Aberdeen	
1908		* Williamson, Henry Charles, M.A., D.Sc., Naturalist to the Fishery Board for Scotland, Marine Laboratory, Aberdeen 615	
1910	C.	* Williamson, William, F.L.S., 79 Morningside Drive, Edinburgh	
1900		Wilson, Alfred C., F.C.S., Voewood Croft, Stockton-on-Tees	
1911		* Wilson, Andrew, M.Inst.C.E., 66 Netherby Road, Trinity, Edinburgh	
1902	V.	* Wilson, Charles T. R., M.A., F.R.S., 14 Cranmer Road, Cambridge, Sidney Sussex College, Cambridge	
1895		Wilson-Barker, Sir David, Kt., R.N.R., F.R.G.S., late Captain-Superintendent Thames Nautical Training College, H.M.S. "Worcester," off Greenhithe, Kent, Flimwell Grange, near Hawkhurst, Kent 620	

Date of Election.			Service on Council, etc.
1882		Wilson, George, M.A., M.B., LL.D.	
1920		* Wilson, Malcolm, D.Sc. (London), Lecturer in Mycology and Bacteriology in the University of Edinburgh. Royal Botanic Garden, Edinburgh	
1908		* Wood, Thomas, M.D., Eastwood, 182 Ferry Road, Bonnington, Leith	
1911		* Wrigley, Ruric Whitehead, B.A. (Cantab.), Assistant Astronomer, Royal Observatory, Edinburgh	
1890		Wright, Johnstone Christie, Conservative Club, Edinburgh 625	
1896		Wright, Sir Robert Patrick, LL.D., formerly Chairman of the Board of Agriculture for Scotland. Kingarth, Colinton, Midlothian	
1882		Young, Frank W., F.C.S., Scottish Education Department (Ex-Service Student's Branch), 3 Parliament Square, Edinburgh	
1892		Young, George, Ph.D., "Bradda," Church Crescent, Church End, Finchley, London, N.	
1896	C.	Young, James Buchanan, M.B., D.Sc., Dalveen, Braeside, Liberton	
1904		Young, R. B., M.A., D.Sc., F.G.S., Professor of Geology and Mineralogy in the South African School of Mines and Technology, Johannesburg, Transvaal 630	

LIST OF HONORARY FELLOWS OF THE SOCIETY

At January 31, 1922.

HIS MOST EXCELLENT MAJESTY THE KING.
HIS ROYAL HIGHNESS THE PRINCE OF WALES.

FOREIGNERS (LIMITED TO THIRTY-SIX BY LAW I).

Elected

1916 Charles Barrois, Professor of Geology and Mineralogy, Université, Lille, France: 37 rue Pascal, Lille.

1905 Waldemar Christofer Brögger, Professor of Mincralogy and Geology, K. Frederiks Universitet, Christiania, Norway.

1916 Douglas Houghton Campbell, Professor of Botany, Leland Stanford Junior University, California, U.S.A.

1920 William Wallace Campbell, Director of the Lick Observatory, Mt. Hamilton, California, U.S.A.

1921 Reginald Aldworth Daly, Professor of Geology, Harvard University, Cambridge, Mass.

1910 Hugo de Vries, Professor of Plant Anatomy and Physiology, Lunteren, Holland.

1916 Marcel Eugène Emile Gley, Professor of Biology, Collège de France, Paris, Membre de l'Académie de Médecine, Paris: 14, rue Monsieur le Prince, Paris.

1910 Karl F. von Goebel, Professor of Botany, Universität, München, Germany.

1916 Camillo Golgi, Professor of Pathology, Università, Pavia, Italy.

1916 Gio. Battista Grassi, Professor of Comparative Anatomy, Regia Università, Roma, Italy: Via Agostino Depretis N. 91, Rome.

1905 Paul Heinrich von Groth, Professor of Mineralogy, Universität, München, Germany.

1913 George Ellery Hale, Director of Mount Wilson Solar Observatory (Carnegie Institution of Washington), Pasadena, California, U.S.A.

1921 Johan Hjort, Director of Norwegian Fisheries, Bergen.

1910 James Cornelius Kapteyn, Professor of Astronomy, Universiteit, Gröningen, Holland.

1921 Charles Louis Alphonse Laveran, Nobel Laureate, Medicine, 1907. Rue du Montparnasse 25, Paris.

1920 Hendrik Antoon Lorentz, Nobel Laureate, Physics, 1902, Professor of Physics, Leiden University.

1910 Albert Abraham Michelson, Nobel Laureate, Physics, 1907, Professor of Physics, University, Chicago, U.S.A.

1897 Fridtjof Nansen, Professor of Oceanography, K. Frederiks Universitet, Christiania, Norway.

1921 Heike Kamerlingh Onnes, Nobel Laureate, Physics, 1913, Universiteit, Leiden, Holland.

1908 Henry Fairfield Osborn, Professor of Zoology, Columbia University and American Museum of Natural History, New York, N.Y., U.S.A.

1908 Ivan Petrovitch Pavlov, Emeritus Professor of Physiology, Kais. Inst. Exper. Med., Petrograd: Wedenskaja Strasse 4, Petrograd, Russia.

1920 Ch. Emile Picard, Perpetual Secretary, Academy of Sciences, Paris.

1921 Salvatore Pincherle, Professor of Mathematics in the University of Bologna.

1889 Georg Hermann Quincke, Emeritus Professor of Physics, Bergstrasse 41, Heidelberg, Germany.

1913 Santiago Ramón y Cajal, Nobel Laureate, Medicine, 1906, Professor of Histology and Pathological Anatomy, Universidad, Madrid, Spain.

1920 Charles Richet, Professor of Physiology, Faculty of Medicine, Paris.

1920 Georg Ossian Sars, formerly Professor of Zoology, Christiania, and Director of Norwegian Fisheries.

1913 Vito Volterra, Professor of Mathematical Physics, Regia Università, Rome, Italy.

1916 Eugenius Warming, Emeritus Professor of Botany at the University of Copenhagen and Director of the Botanical Garden: Bjerregaardsvej 5, Copenhagen, Valby.

Total, 29.

BRITISH SUBJECTS (LIMITED TO TWENTY BY LAW I).

1916 Sir Francis Darwin, Kt., D.Sc., M.B., F.R.S., Hon. Fellow, Christ's College, Cambridge, 10 Madingley Road, Cambridge.

1900 Sir David Ferrier, Kt., M.A., M.D., LL.D., F.R.S., Emer.-Professor of Neuro-Pathology, King's College, London, 34 Cavendish Square, London, W.

Elected

1900 Andrew Russell Forsyth, M.A., Sc.D., LL.D., Math.D., F.R.S., Chief Professor of Mathematics in the Imperial College of Science and Technology, London, formerly Sadleirian Professor of Pure Mathematics in the University of Cambridge, Imperial College of Science and Technology, London, S.W.

1910 Sir James George Frazer, D.C.L., LL.D., Litt.D., F.R.S., Fellow of Trinity College, Cambridge, 1 Brick Court, London, E.C. 4.

1916 James Whitbread Lee Glaisher, M.A., Sc.D., F.R.S., Fellow of Trinity College, Cambridge.

1908 Sir Alexander B. W. Kennedy, Kt., LL.D., F.R.S., Past Pres. Inst. C.E., A7, Albany, Piccadilly, London, W.

1913 Horace Lamb, M.A., Sc.D., D.Sc., LL.D., F.R.S., lately Professor of Mathematics in the University of Manchester. 65 Grange Road, Cambridge.

1916 John Newport Langley, Sc.D., LL.D., F.R.S., Fellow of Trinity College, Professor of Physiology in the University of Cambridge, Hedgerley Lodge, Madingley Road, Cambridge.

1908 Sir Edwin Ray Lankester, K.C.B, D.Sc., LL.D., F.R.S., 44 Oakley Street, Chelsea, London, S.W. 3.

1910 Sir Joseph Larmor, Kt., M.A., D.Sc., LL.D., D.C.L., F.R.S., M.P. University of Cambridge since 1911, Lucasian Professor of Mathematics in the University of Cambridge, St John's College, Cambridge.

1900 Archibald Liversidge, M.A., LL.D., F.R.S., Emer.-Professor of Chemistry in the University of Sydney, Fieldhead, Coombe Warren, Kingston, Surrey.

1921 William Henry Perkin, M.A., Ph.D., Sc.D., LL.D., F.R.S., Waynflete Professor of Chemistry in the University of Oxford.

1921 Sir Ronald Ross, K.C.B., K.C.M.G., F.R.S., Consultant in Malaria, Ministry of Pensions, London, 36 Harley House, Regent's Park, N.W. 1.

1921 Sir Ernest Rutherford, Kt., M.A., D.Sc., B.A., LL.D., F.R.S., Nobel Laureate, Chemistry, 1908, Cavendish Professor of Experimental Physics in the University of Cambridge.

1916 Sir Arthur Schuster, Kt., Ph.D., D.Sc., LL.D., D. ès Sc. Geneva, Foreign Secretary of the Royal Society, Honorary Professor of Physics in the University of Manchester, Yeldall, Twyford, Berks.

1908 Sir Charles Scott Sherrington, G.B.E., M.A., M.D., LL.D., P.R.S., Waynflete Professor of Physiology in the University of Oxford, Physiological Laboratory, Oxford.

1921 Sir Jethro J. H. Teall, Kt., M.A., Sc.D., LL.D., F.G.S., F.R.S., lately Director of the Geological Survey of Great Britain and of the Museum of Practical Geology.

1913 Sir William Turner Thiselton-Dyer, K.C.M.G., C.I.E., M.A., LL.D., F.R.S., formerly Director of the Royal Botanic Gardens, Kew: The Ferns, Witcombe, Gloucester.

1905 Sir Joseph John Thomson, Kt., O.M., D.Sc., LL.D., Past Pres.R.S., Nobel Laureate, Physics, 1906, lately Cavendish Professor of Experimental Physics, University of Cambridge, Trinity College, Cambridge.

1900 Sir Thomas Edward Thorpe, Kt., C.B., D.Sc., LL.D., F.R.S., formerly Principal of the Government Laboratories, Emeritus-Professor of Chemistry, Imperial College of Science and Technology, South Kensington, London, S.W. Whinfield, Salcombe, South Devon.

Total, 20.

CHANGES IN FELLOWSHIP DURING SESSION 1920–1921.

ORDINARY FELLOWS OF THE SOCIETY ELECTED.

NELSON ANNANDALE.
WILLIAM ARTHUR.
BEVAN BRAITHWAITE BAKER.
ARCHIBALD BARR.
JOHN BARTHOLOMEW.
ALEXANDER BRUCE.
ANDREW CAMPBELL.
RASIK LAL DATTA.
JOHN DOUGALL.
CHARLES VICKERY DRYSDALE.
GEORGE TOPHAM FORREST.
WALCOT GIBSON.
JOHN WM. HESLOP HARRISON.
JAMES ALEX. GEORGE LAMB.
Rev. ALBERT ERNEST LAURIE.
NEIL M'ARTHUR.
DOUGALD BLACK M'QUISTAN.
THOMAS MURRAY MACROBERT.
JOHN M'WHAN.
JOHN MATHIESON.
Sir GEO. HERBERT POLLARD.
EDWARD BURNS ROSS.
Rt. Hon JAMES PARKER SMITH.
NORMAN KEMP SMITH.
IAN STRUTHERS STEWART.

HONORARY FELLOWS ELECTED.

4th July 1921.

BRITISH.

WILLIAM HENRY PERKIN.
Sir RONALD ROSS.
Sir ERNEST RUTHERFORD.
Sir JETHRO J. H. TEALL.

FOREIGN.

REGINALD ALDWORTH DALY, Cambridge, Mass.
JOHAN HJORT, Bergen.
CHARLES LOUIS ALPHONSE LAVERAN, Paris.
HEIKE KAMERLINGH ONNES, Leiden.
SALVATORE PINCHERLE, Bologna.

ORDINARY FELLOWS DECEASED.

ROBERT GERVASE ALFORD.
Sir R. ROWAND ANDERSON.
HENRY BARNES.
Sir J. H. MEIRING BECK (died 1919).
ADOLPHUS EDWARD BRIDGER.
DAVID BROWN.
DAVID RAINY BROWN.
WM. ALLAN CARTER.
WM. JOHN DUNDAS.
T. LINDSAY GALLOWAY.
T. E. GATEHOUSE.
GEO. RITCHIE GILRUTH.
T. ARTHUR HELME.
JAMES HUNTER.
Sir T. CARLAW MARTIN.
Rev. R. S. MYLNE.
JAMES OLIPHANT.
E. W. PREVOST.
D. LLOYD ROBERTS.
THOMAS BOND SPRAGUE.
ROBERT WALKER.
G. A. WOODS.

HONORARY FELLOWS DECEASED.

JULIUS VON HANN.
GABRIEL LIPPMAN.
CARL MENGER.
ALFRED G. NATHORST.

ORDINARY FELLOWS RESIGNED.

Sir DUNCAN A. JOHNSTON.
G. C. PRINGLE.
Sir GEO. ADAM SMITH.

List of Additions to Library by Gift or Purchase.

Bergen's Museum, 1825–1900. En Historisk Fremmstilling af Dr J. Brunchorst. 8vo. Bergen, 1900. (*Presented by Mr Wm. Williamson.*)

Bhattacharyya, D. Vector Calculus. 8vo. Calcutta, 1920. (*Presented by Calcutta University.*)

Bibliographie Scientifique Française. (*Presented by Ministère de l'Instruction Publique.*)

Bright, Sir Charles. Inter-Imperial Communication through Cable, Wireless, and Air. 8vo. London, 1919. (*Presented by the Author.*)

British (Terra Nova) Antarctic Expedition, 1910–1913: Meteorology. Vols. I and II. 4to. 1919. (*Presented.*)

——— Terrestrial Magnetism. By Charles Chree. 4to. London, 1921.

Calcutta, University of: Journal of the Department of Letters. Vol. I– . 8vo. Calcutta, 1920 (Univ. Studies Series.)

Campbell, Andrew. Petroleum Refining. 8vo. London, 1918. (*Presented by the Author.*)

Catalogue of Scientific Papers. (Royal Society of London.) Fourth Series, 1884–1900. Vol. XVII (Marc–P). 4to. Cambridge, 1921. (*Purchased.*)

Cavendish, Henry, The Scientific Papers of.
Vol. I. The Electrical Researches. Edited by James Clerk Maxwell, Revised by Sir Joseph Larmor. 4to. Cambridge, 1921.
Vol. II. Chemical and Dynamical. Edited by Sir Edward Thorpe. 4to. Cambridge, 1921. (*Purchased.*)

Constantinesco, George. Theory of Wave Transmission. (Theory of Sonics.) A Treatise on Transmission of Power by Vibrations. Vol. I. 8vo. London, 1920. (*Presented by Walter Haddon.*)

Cotton Research Board (Ministry of Agriculture, Egypt). Annual Report, 1920. La. 8vo. Cairo, 1921. (*Presented.*)

Cushman, Allerton S. Chemistry and Civilisation. 8vo. Boston, 1921. (*Presented by Wagner Free Institute.*)

Granton, Edinburgh, Marine Station for Scientific Research. 8vo. Edinburgh, 1884.

——— Scottish Marine Station for Scientific Research: its Work and Prospects. 8vo. Edinburgh, 1885. (*Presented by Mr Wm. Williamson.*)

Gribble, J. D. B. (*see* Hehir, Patrick).

Gude, G. K. The Fauna of British India: Mollusca. Vol. III: Land Operculates. 8vo. London, 1921. (*Presented by the Under Secretary of State for India.*)

Hehir, Patrick. Prophylaxis of Malaria in India. 8vo. Allahabad, 1910.

——— Prevention of Disease and Inefficiency, with Special Reference to Indian Frontier Warfare. 2nd edition. 8vo. Allahabad, 1911.

Hehir, Patrick. The March: Its Mechanism, Effects, and Hygiene. 8vo. Calcutta, 1912.

——— Hygiene and Diseases of India. A Popular Handbook. 3rd edition. 8vo. Madras, 1913. (*Presented by Sir Patrick Hehir.*)

Hehir, Patrick, and J. D. B. Gribble. Outlines of Medical Jurisprudence for India. 5th edition. 8vo. Madras, 1908. (*Presented by Sir Patrick Hehir.*)

Herdman, W. A. Reports of the Grain Pests (War) Committee. No. 10. (R.S.L.) 8vo. London, 1921. (*Presented by the Author.*)

Horsburgh, E. M. Calculating Machines. (Inst. Engineers and Shipbuilders in Scotland.) 8vo. Glasgow, 1920.

Howard, William Trevis, Jr. The Natural History of Typhoid Fever in Baltimore, 1851–1919. (Reprinted from the *Johns Hopkins Hospital Bulletin,* Vol. XXXI, Nos. 354–355, 1920.) 8vo. Baltimore, 1920. (*Presented by Raymond Pearl.*)

Huygens, Christiaan. Œuvres Complètes publiées par la Société Hollandaise des Sciences. Vol. XIII, fasc. 1 and 2, and Vol. XIV. 4to. La Haye, 1916 and 1920. (*Presented by the Société Hollandaise des Sciences de Harlem.*)

Kanthack, R. Tables of Refractive Indices. Vol. I.—Essential Oils. Vol. II.—Oils, Fats, and Waxes. Edited by J. N. Goldsmith. 8vo. London, 1918 and 1921. (*Presented by Adam Hilger, Ltd.*)

Kaploun, Albert. Psychologie Générale tirée de l'Etude du Réve. 8vo. Lausanne, 1919.

Kidd, Walter. Initiative in Evolution. 8vo. London, 1920. (*Presented by the Author.*)

Lancashire and Cheshire Fauna Committee. Sixth Annual Report and Report of the Recorder for 1919. 8vo. Manchester, 1920.

Lister, Lord, Photograph of. (*Presented by Lady Lister and Sir R. J. Godlee, Bt.*)

Lowell, J. Reed. The Mathematics of Biometry. (Reprinted from the *American Mathematical Monthly,* Vol. XXVII, November 1920.) 8vo. Baltimore, 1920. (*Presented by Raymond Pearl.*)

Macgillivray, Angus. A Bacteriological and Clinical Study of the External Diseases of the Eye. 8vo. London, 1916. (*Presented by the Author.*)

Manchester University Publications.

Historical Series—

Tout, T. F., and Tait, James. Historical Essays. 8vo. 1907.

Petit-Dutaillis, Charles. Studies and Notes Supplementary to Stubb's *Constitutional History.* Vols. I and II. 8vo. 1911.

Clemesha, H. W. A History of Preston in Amounderness. 8vo. 1912.

Holden, Joshua. A Short History of Todmorden. 8vo. 1912.

Gill, Conrad. The Naval Mutinies of 1797. 8vo. 1913.

Tait, James. Chronica Johannis de Reading et Anonymi Cantuariensis, 1346–1367. 8vo. 1914.

MANCHESTER UNIVERSITY PUBLICATIONS—*Historical Series*—continued.

Tout, T. F. The Place of the Reign of Edward II in English History. 8vo. 1914.

Deansley, Margaret. The *Incendium Amoris* of Richard Rolle of Hampole. 8vo. 1915.

Pirenne, Henri. Belgian Democracy: its Early History. Translated by J. V. Saunders. 8vo. 1915.

Tout, T. F. Germany in the Nineteenth Century. 8vo. 1915.

Muir, Ramsay. The Making of British India, 1756–1858. 8vo. 1917.

Little, A. G. Studies in English Franciscan History. 8vo. 1917.

Hovell, the late Mark. The Chartist Movement. Edited and completed, with a Memoir, by Prof. T. F. Tout. 8vo. 1918.

Unwin, George. Finance and Trade under Edward III. 8vo. 1918.

M'Lauchlan, H. The Methodist Unitarian Movement. 8vo. 1919.

Tout, T. F. Chapters in the Administrative History of Mediæval England. Two vols. 8vo. 1920.

Daniels, George W. The Early English Cotton Industry. 8vo. 1920.

Lecture Series—

Neville, Ralph. Garden Cities. (Warburton Lecture.) 8vo. 1904.

Thomson, J. J. On the Light Thrown by Recent Investigations on Electricity on the Relation between Matter and Ether. (Adamson Lecture.) 8vo. 1908.

Thorburn, William. The Evolution of Surgery. 8vo. 1910.

Ward, A. W. Leibniz as a Politician. (Adamson Lecture.)

Waterhouse, Paul, and Raymond Unwin. Old Towns and New Needs: also the Town Extension Plan. (Warburton Lectures.) 4to. 1912.

Wilkinson, Henry S. Learners as Leaders. 8vo. 1918.

Hart, Bernard. The Modern Treatment of Mental and Nervous Disorders. 8vo. 1918.

Celtic Series—

Lewis, Timothy. A Glossary of Mediæval Welsh Law. 8vo. 1913.

Classical Series—

Norwood, Gilbert. The Riddle of the Bacchæ. 8vo. 1908.

Economic Series—

Dehn, R. M. R. The German Cotton Industry. (A Report.) 8vo. 1913.

Ethnological Series—

Jackson, J. Wilfrid. Shells as Evidence of the Migrations of Early Culture. 8vo. 1917.

Miscellaneous (*Botanical Section*)—

Weiss, F. E., A. D. Imms, and Wilfrid Robinson. Plants in Health and Disease. 8vo. 1916.

MANCHESTER UNIVERSITY PUBLICATIONS—*Miscellaneous*—continued.

Calendar, 1920–1921. 8vo. 1920.

Journal of the Manchester Egyptian and Oriental Society, 1911; 1912–13; 1913–14; 1914–15; 1915–16; 1916–17; 1917–18; 1918–19; 1920 (No. 9).

Maxwell, J. Clerk. Matter and Motion. Reprinted: with Notes and Appendices by Sir Joseph Larmor. (Society for Promoting Christian Knowledge.) 8vo. London.

Mindeskrift i Anledning af 100-aaret for J. Steenstrups Födsel. Two vols. 4to. Köbenhavn, 1914. (From the Royal Danish Society of Sciences.)

Munro, Robert, Autobiographic Sketch of. 8vo. Glasgow, 1921. (*Presented by Mr Hugh Munro and the Misses Munro.*)

Natal (Province) Descriptive Guide and Official Handbook. La. 8vo. Durban, Natal, 1911. (*Presented by High Commissioner, Union of South Africa.*)

Norman Lockyer Observatory. Handbook to the Norman Lockyer Observatory. Compiled by Major William J. S. Lockyer. Sm. 8vo. London, 1921.

——— Director's Annual Report, 1921. 4to. London, 1921. (*Presented.*)

Pearl, Raymond. Some Landmarks in the History of Vital Statistics. (Reprinted from *Quarterly Publications of the American Statistical Association*, June 1920.) 8vo. Baltimore, 1920. (*Presented by the Author.*)

——— The Relative Influence of the Constitutional Factor in the Etiology of Tuberculosis. (Reprinted from the *American Review of Tuberculosis*, Vol. IV, No. 9, November 1920.) 8vo. Baltimore, 1920. (*Presented by the Author.*)

Quincke, G. Spaltung und Erwärmung von Metalldrähten und isolierenden Stäben durch elektrische Longitudinalschwingungen. (*Sitz. Heidelberger Akad. Wissenschaftlichen, Math.-Nat. Kl.*) 8vo. Heidelberg, 1920.

Radium (a Monthly Magazine). Edited by Charles H. Viol, Ph.D., and William H. Cameron, M.D. Vol. XVII– . 8vo. Pittsburgh, Pa., 1921. (*Presented by Watson & Sons (Electro-Medical) Ltd., Kingsway, W.C. 2.*)

Rayleigh, Baron. Scientific Papers. Vol. VI. 1911–1919. 4to. Cambridge, 1920. (*Purchased.*)

Report of the Third Entomological Meeting (Pusa). In three vols. Edited by T. Bainbrigge Fletcher. La. 8vo. Calcutta, 1920.

Salter, M. de Carle S. The Rainfall of the British Isles. 8vo. London, 1921. (*Presented by the University of London Press.*)

Spiller, G. A New System of Scientific Procedure. 8vo. London, 1921. (*Presented.*)

Sweden: Historical and Statistical Handbook. Edited by J. Guinchard. Two vols. 2nd edition. English issue. 8vo. Stockholm, 1914. (*Presented by the Royal University of Upsala.*)

LAWS OF THE SOCIETY.

Adopted July 3, 1916; amended December 18, 1916.

(Laws VIII, IX, and XIII amended May 3, 1920. Law VI amended February 7, 1921.)

I.

THE ROYAL SOCIETY OF EDINBURGH, which was instituted by Royal Charter in 1783 for the promotion of Science and Literature, shall consist of Ordinary Fellows (hereinafter to be termed Fellows) and Honorary Fellows. The number of Honorary Fellows shall not exceed fifty-six, of whom not more than twenty may be British subjects, and not more than thirty-six subjects of Foreign States.

Fellows only shall be eligible to hold office or to vote at any Meeting of the Society.

ELECTION OF FELLOWS.

II.

Each Candidate for admission as a Fellow shall be proposed by at least four Fellows, two of whom must certify from personal knowledge. The Official Certificate shall specify the name, rank, profession, place of residence, and the qualifications of the Candidate. The Certificate shall be delivered to the General Secretary before the 30th of November, and, subject to the approval of the Council, shall be exhibited in the Society's House during the month of January following. All Certificates so exhibited shall be considered by the Council at its first meeting in February, and a list of the Candidates approved by the Council for election shall be issued to the Fellows not later than the 21st of February.

III.

The election of Fellows shall be by Ballot, and shall take place at the first Ordinary Meeting in March. Only Candidates approved by the Council shall be eligible for election. A Candidate shall be held not elected, unless he is supported by a majority of two-thirds of the Fellows present and voting.

IV.

On the day of election of Fellows two scrutineers, nominated by the President, shall examine the votes and hand their report to the President, who shall declare the result.

V.

Each Fellow, after his election, is expected to attend an Ordinary Meeting, and sign the Roll of Fellows, he having first made the payments required by Law VI. He shall be introduced to the President, who shall address him in these words:

In the name and by the authority of THE ROYAL SOCIETY OF EDINBURGH, I admit you a Fellow thereof.

PAYMENTS BY FELLOWS.

VI.

Each Fellow shall, before he is admitted to the privileges of Fellowship, pay an admission fee of three guineas, and a subscription of three guineas for the year of election. He shall continue to pay a subscription of three guineas at the beginning of each session so long as he remains a Fellow.

Each Fellow who was elected subsequent to December 1916 and previous to December 1920 shall also pay a subscription of three guineas at the beginning of each session so long as he remains a Fellow.

Each Fellow who was elected previous to December 1916, and who has not completed his twenty-five annual payments,* shall, at the beginning of each session, pay three guineas or four guineas according as he has or has not made ten annual payments as reckoned from the year of election. Each Fellow who has completed or shall complete his payments shall be invited to contribute one guinea per annum or to pay a single sum of ten guineas.

A Fellow may compound for the annual subscriptions by a single payment of fifty guineas, or on such other terms as the Council may from time to time fix.

VII.

A Fellow who, after application made by the Treasurer, fails to pay any contribution due by him, shall be reported to the Council, and, if the Council see fit, shall be declared no longer a Fellow. Notwithstanding such declaration all arrears of contributions shall remain exigible.

* The following is an extract from the previous law affecting Annual Subscribers elected prior to December 1916:—"Every Ordinary Fellow, within three months after his election, shall pay Two Guineas as the fee of admission, and Three Guineas as his contribution for the Session in which he has been elected; and annually at the commencement of every Session, Three Guineas into the hands of the Treasurer. This annual contribution shall continue for ten years after his admission, and it shall be limited to Two Guineas for fifteen years thereafter."

ELECTION OF HONORARY FELLOWS.

VIII.

Honorary Fellows shall be persons eminently distinguished in Science or Literature. They shall not be liable to contribute to the Society's Funds. Personages of the Blood Royal may be elected Honorary Fellows at any time on the nomination of the Council, and without regard to the limitation of numbers specified in Law I.

IX.

Honorary Fellows shall be proposed by the Council. The nominations shall be announced from the Chair at the First Ordinary Meeting after their selection. The names shall be printed in the circular for the last Ordinary Meeting of the Session, when the election shall be by Ballot, after the manner prescribed in Laws III and IV for the Election of Fellows.

EXPULSION OF FELLOWS.

X.

If, in the opinion of the Council, the conduct of any Fellow is injurious to the character or interests of the Society, the Council may, by registered letter, request him to resign. If he fail to do so within one month of such request, the Council shall call a Special Meeting of the Society to consider the matter. If a majority consisting of not less than two-thirds of the Fellows present and voting decide for expulsion, he shall be expelled by declaration from the Chair, his name shall be erased from the Roll, and he shall forfeit all right or claim in or to the property of the Society.

XI.

It shall be competent for the Council to remove any person from the Roll of Honorary Fellows if, in their opinion, his remaining on the Roll would be injurious to the character or interests of the Society. Reasonable notice of such proposal shall be given to each member of the Council, and, if possible, to the Honorary Fellow himself. Thereafter the decision on the question shall not be taken until the matter has been discussed at two Meetings of Council, separated by an interval of not less than fourteen days. A majority of two-thirds of the members present and voting shall be required for such removal.

MEETINGS OF THE SOCIETY.

XII.

A Statutory Meeting for the election of Council and Office-Bearers, for the presentation of the Annual Reports, and for such other business as may be arranged by the Council, shall be held on the fourth Monday of October. Each Session of the Society shall begin at the date of the Statutory Meeting.

XIII.

Meetings for reading and discussing communications and for general business, herein termed Ordinary Meetings, shall be held, when convenient, on the first and third Mondays of each month from November to July inclusive, with the exception that in January the meetings shall be held on the second and fourth Mondays.

XIV.

A Special Meeting of the Society may be called at any time by direction of the Council, or on a requisition to the Council signed by not fewer than six Fellows. The date and hour of such Meeting shall be determined by the Council, who shall give not less than seven days' notice of such Meeting. The notice shall state the purpose for which the Special Meeting is summoned ; no other business shall be transacted.

PUBLICATION OF PAPERS.

XV.

The Society shall publish Transactions and Proceedings. The consideration of the acceptance, reading, and publication of papers is vested in the Council, whose decision shall be final. Acceptance for reading shall not necessarily imply acceptance for publication.

DISTRIBUTION OF PUBLICATIONS.

XVI.

Fellows who are not in arrear with their Annual Subscriptions and all Honorary Fellows shall be entitled gratis to copies of the Parts of the Transactions and the Proceedings published subsequently to their admission.

Copies of the Parts of the Proceedings shall be distributed by post or otherwise, as soon as may be convenient after publication ; copies of the Transactions or Parts thereof shall be obtainable upon application, either personally or by an authorised agent, to the Librarian, provided the application is made within five years after the date of publication.

CONSTITUTION OF COUNCIL.

XVII.

The Council shall consist of a President, six Vice-Presidents, a Treasurer, a General Secretary, two Secretaries to the Ordinary Meetings (the one representing the Biological group and the other the Physical group of Sciences),* a Curator of the Library and Museum, and twelve ordinary members of Council.

ELECTION OF COUNCIL.

XVIII.

The election of the Council and Office-Bearers for the ensuing Session shall be held at the Statutory Meeting on the fourth Monday of October. The list of the names recommended by the Council shall be issued to the Fellows not less than one week before the Meeting. The election shall be by Ballot, and shall be determined by a majority of the Fellows present and voting. Scrutineers shall be nominated as in Law IV.

XIX.

The President may hold office for a period not exceeding five consecutive years; the Vice-Presidents, not exceeding three; the Secretaries to the Ordinary Meetings, not exceeding five; the General Secretary, the Treasurer, and the Curator of the Library and Museum, not exceeding ten; and ordinary members of Council, not exceeding three consecutive years.

XX.

In the event of a vacancy arising in the Council or in any of the offices enumerated in Law XVII, the Council shall proceed, as soon as convenient, to elect a Fellow to fill such vacancy for the period up to the next Statutory Meeting.

* The Biological group includes Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, Zoology; the Physical group includes Astronomy, Chemistry, Mathematics, Metallurgy, Meteorology, Physics.

POWERS OF THE COUNCIL.

XXI.

The Council shall have the following powers :—(1) To manage all business concerning the affairs of the Society. (2) To decide what papers shall be accepted for communication to the Society, and what papers shall be printed in whole or in part in the Transactions and Proceedings. (3) To appoint Committees. (4) To appoint employees and determine their remuneration. (5) To award the various prizes vested in the Society, in accordance with the terms of the respective deeds of gift, provided that no member of the existing Council shall be eligible for any such award. (6) To make from time to time Standing Orders for the regulation of the affairs of the Society. (7) To control the investment or expenditure of the Funds of the Society.

At Meetings of the Council the President or Chairman shall have a casting as well as a deliberative vote.

DUTIES OF PRESIDENT AND VICE-PRESIDENTS.

XXII.

The President shall take the Chair at Meetings of Council and of the Fellows. It shall be his duty to see that the business is conducted in accordance with the Charter and Laws of the Society. When unable to be present at any Meetings or attend to current business, he shall give notice to the General Secretary, in order that his place may be supplied. In the absence of the President his duties shall be discharged by one of the Vice-Presidents.

DUTIES OF THE TREASURER.

XXIII.

The Treasurer shall receive the monies due to the Society and shall make payments authorised by the Council. He shall lay before the Council a list of arrears in accordance with Rule VII. He shall keep accounts of all receipts and payments, and at the Statutory Meeting shall present the accounts for the preceding Session, balanced to the 30th of September, and audited by a professional accountant appointed annually by the Society.

DUTIES OF THE GENERAL SECRETARY.

XXIV.

The General Secretary shall be responsible to the Council for the conduct of the Society's correspondence, publications, and all other business except that which relates to finance. He shall keep Minutes of the Statutory and Special

Meetings of the Society and Minutes of the Meetings of Council. He shall superintend, with the aid of the Assistant Secretary, the publication of the Transactions and Proceedings. He shall supervise the employees in the discharge of their duties.

DUTIES OF SECRETARIES TO ORDINARY MEETINGS.

XXV.

The Secretaries to Ordinary Meetings shall keep Minutes of the Ordinary Meetings. They shall assist the General Secretary, when necessary, in superintending the publication of the Transactions and Proceedings. In his absence, one of them shall perform his duties.

DUTIES OF CURATOR OF LIBRARY AND MUSEUM.

XXVI.

The Curator of the Library and Museum shall have charge of the Books, Manuscripts, Maps, and other articles belonging to the Society. He shall keep the Card Catalogue up to date. He shall purchase Books sanctioned by the Council.

ASSISTANT-SECRETARY AND LIBRARIAN.

XXVII.

The Council shall appoint an Assistant-Secretary and Librarian, who shall hold office during the pleasure of the Council. He shall give all his time, during prescribed hours, to the work of the Society, and shall be paid according to the determination of the Council. When necessary he shall act under the Treasurer in receiving subscriptions, giving out receipts, and paying employees.

ALTERATION OF LAWS.

XXVIII.

Any proposed alteration in the Laws shall be considered by the Council, due notice having been given to each member of Council. Such alteration, if approved by the Council, shall be proposed from the Chair at the next Ordinary Meeting of the Society, and, in accordance with the Charter, shall be considered and voted upon at a Meeting held at least one month after that at which the motion for alteration shall have been proposed.

INDEX.

Index of Papers published in the "Transactions" during Session 1920–21.

(Arranged under the Authors' Names.)

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MODEL INDEX.

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Cells, Liver,—Intra-cellular Canaliculi in.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

Liver,—Injection within Cells of.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

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