Connection spaces And their link to Graph theory

This booklet is of 3 *parts.*

The first one is a gentle introduction.
The second one is all about the diverse and beauty of connections and connection spaces
The third part is about logical completeness and pseudo-spaces.

Table of Content

Introduction

Logic can be expressed abstractly like this :-
A is taller than B. C is shorter than B. D is taller than B. A is taller than D.

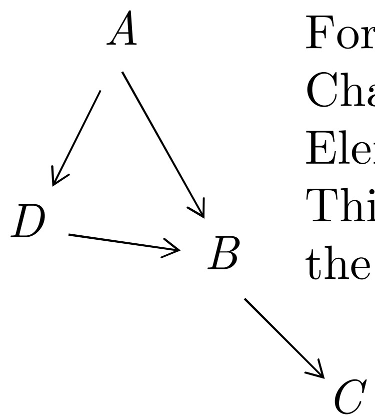
$\longrightarrow$

The arrow means taller than

$A \longrightarrow B$
$B \longrightarrow C$
Since c being shorter than b is *isomorphic* to b being taller than c.
$D \longrightarrow B$
$A \longrightarrow D$

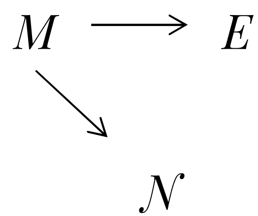
**This will cover direct connections two lay the foundation for Advanced structures.*

Motivation



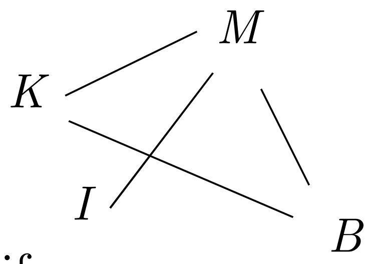
In this visual way we can see that A is the tallest
We can also see, by following the arrows, that
For A the smallest value in the chain (Lowest
Chain Element {LCE}) is C. For C the smallest
Element is C itself. In fact for every element in
This diagram/chain C is the smallest and A is
the largest...

Consider the second diagram, in here LCE for the Element E is *undefined* because there is no connection that joins E to some other element. In fact, since there is no connection the LCE for E is indeterminate.



Motivation cont.2

Consider this configuration :-



In this configuration there is no notion of tall or short it's connection (which I will call W) is *symmetric* so if

$$M \xrightarrow[W]{} B \quad = \quad B \xrightarrow[W]{} M$$

(read as: M *connected* to B by W)

If this property holds for two M;B then W is a symmetric connection for M & B if this holds for any two variables Then it is a Total symmetric – Connection.

Chapters

-Statements as rules/or structures
-*Connection space*
-Elements in a *Connection space*
-Connection in a *Connection space*
-Meaning of Definitions
- Isomorphism Between spaces
-Examples and Counterexamples
-Types of Connection
-Beauty and diversity of connections
-All this as another view to graph theory

-Summary
-List of Jargon
-Start of Part II

Red means under construction

Blue means exists

Statements as rules/Or structures

Statements are a list of symbols satisfying these conditions :-
-A statements contains 2 or more arguments.
-A statement connects a element to itself or some other element or multiple elements.
-Statements can have a connection of any type.
-Statements must not have a empty argument.

Statements are the smallest entities of logic.

Collections of Statements is called a BLOCK.
A block must satisfy these conditions as well :-
-A Block must be logically complete and must not contain contradictory statements.
-A Block must contain more than 1 statement.
-A Block must contain statements and proper statements.

Blocks give a connection space it's structure because blocks specify how connections are made and they give the actual meaning of a *connection space.*

Connection space

A connection space is a *Set* of Elements called the VARIABLES and a *set* of *subsets* of this *set* called the RELATION. The relation is given by a block of associated statements called the Logic. The specific connection given in the statements is abbreviated as W. and a Connection space can have one such connection for all elements (equipped with that connection) or have multiple connections. The Logic of a connection space is called it's structure. The structure is unchanged under isomorphism of connection spaces.

The connection space is denoted as C
it's Variables are denoted as V
Relations are denoted as R
The block of statements is denoted B
And finally the *specific* connection is denoted W
although there could be *multiple connections.*
We can think of W as being the set of all these connections

Elements in a Connection space

Elements are entities related to each other by a connection in a connection space C
Elements can be any mathematical or logical quantity. The set of all elements of a Connection space is called the VARIABLES of the CONNECTION SPACE. An element must be a relatable entity.

Elements in a connection space are abbreviated as variable names
A,B,C,D,E etc.

Elements are the easiest part of a Connection space.
Note : The definition of the elements is vague and informal.

Connection in a Connection space

The connection is the reason why connection spaces are called ‘Connection space’. Connection specifies relationship between elements. The connection relates elements in statements. Connections are of 3 types :-

And (*direct* connections) $A \rightarrow B$ (pronounced as : A to B)

Or (*conditional* connections) $A \overset{\sim}{\longrightarrow} B{:}C$ (pronounced as : A to B or C)

Not (*indirect* connections) $A \not\rightarrow B$ (pronounced as : not to B)

Connections do not contain any logic . When they form statements and assemble logically and form blocks then they are logically complete. Therefore Connections are not logically complete.

There are two types of Connections : Ordered and Unordered Connections

Meaning of Definitions

Ok I admit, I got a bit too rigorous on the definition part. But now we will *explore* them step by step. Before all of that let's have a talk why I gave such *rigorous* definitions. Well, rigorous definitions are Complete, in the sense that they capture all the points, Concise ,in the sense that use the least words, and General , in the sense that apply to all situations . In *Mathematics* most definitions are infuriating and so we need to *crack open what they mean*. And so we also need to unpack what I meant by those *definitions*.

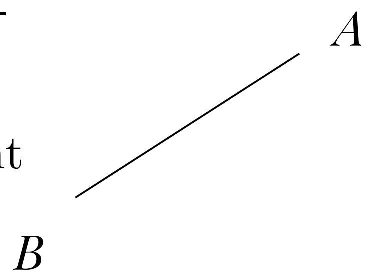
Statements Explanation

A statement looks like this :-

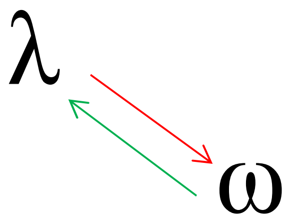
'$A \xrightarrow[W]{\qquad} B$' in this statement ,the arrow is the connection W, and A is being connected by W to B. A and B are the *arguments* of this connection and the arguments of the *statement.*

A *statement* is the smallest entity of logic because it represents the smallest entity which has some logical meaning. Whereas a Connection W is not itself a entity of logic, however when it connects 2 elements or more (forming a statement) it acquires *logical meaning.*
Visually a statement may infer :-

This structure has a statement
$A \underset{W}{\overline{\qquad}} B = B \underset{W}{\overline{\qquad}} A$



Here the connection W is *symmetric* so a to b is the same as b to a.



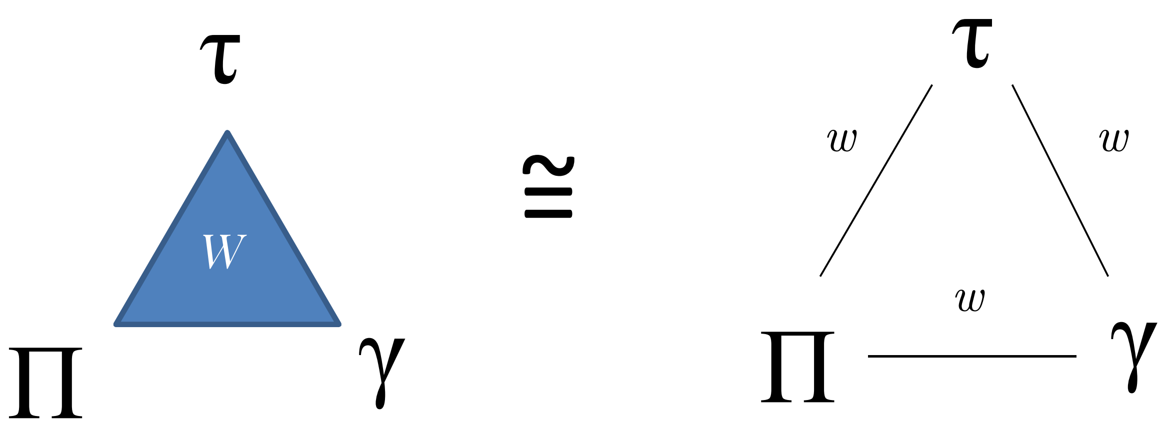
For this structure we have :-

$$\omega_G \longrightarrow \lambda$$

$$\lambda_R \longrightarrow \omega$$

This structure has a *non symmetric connection* between lambda and omega . So, omega is connected to lambda by G (omega to lambda by G) and lambda is connected to omega by R (lambda to omega by R).

$$\tau_W \longrightarrow \Pi : \gamma \longleftarrow$$ Ok now let's find out, What this one means?



Notice that the statement earlier says , Tau to Pi and Gamma by W
But the result is *isomorphic* to a smaller connection w joining all the elements together. The connection W is a Trielement connection whereas w is a bielement Connection.

In fact all multielement connections are isomorphic to multiple bielement connections. We do not need a proof for this since that is what they are defined as
A $\longrightarrow$ B:C:D.... is isomorphic to A$\rightarrow$ B $\wedge$ C$\rightarrow$ D $\wedge$.... *note : This only applies to and statements but that is what we were doing so far.

Types of statements

1.Ordered statements
i)And-Ordered Statements
ii)Or-Ordered Statements
iii)Not-ordered Statements (often confused with unordered statements)

2. Unordered Statements
i)And-Unordered Statements
ii)Or-Ordered Statements
iii)Not-Unordered Statements

We will talk about all of these types of statements in PART II

Blocks

Blocks are a specific collection of Statements

Block are logically complete and do not contain any contradictory statements
For eg. A $\not\rightarrow$ B ; A $\longrightarrow$ B these 2 are *contradictory* statements if they are put in a block together, The block, they constitute, will disqualify as a block since it has *contradictory* statements. In some cases though, we may call them *Pseudoblocks.*

Please go back and visit the rigorous definition of Blocks, because I will build over that definition

i)'A block must contain a statement'. I guess that's clear to you, since a empty block has no logical meaning.
ii)'A block must be logically complete' means that it contains complete information about the structure of the connection space.
iii)A block must not have contradictory statements to be logically complete. This assures that no logical paradox arises.
iv)Since a block captures information about a Connection space, it must have a statement to be a connection space without one it doesn't have any relation.

Types of Blocks

1. Pure Blocks- Blocks that contain one type of statements.
i) And-Blocks – Blocks with and-statements
ii) Or-Blocks – Blocks with or-statements
iii)Not Blocks – Blocks with not-statements

2. Mixed Blocks- Blocks that contain multiple type of statements.

We will talk about diversity of Blocks in Part II

Types of Block actually don't care about ordered or unordered statements.

Connection space

The Connection space is the most important topic in the Theory of Connection spaces (as evident by it's name).

A Connection space is a Set of Elements called the VARIABLES of the Connection space and a set of subsets of this set called the RELATIONS of the Connection space. The subsets are given by a associated Block called the LOGIC of the Connection space....

What does this mean? To explore we need to a little bit of Set theory.

Basic Set theory

There is more to *Set theory* than what we will talk about. If you are interested it is easily available at the internet.

So what is *Set theory*? Well, Set theory is the study of *Sets*, and certain operations on them .Therefore We must list off, What a Set is?

A set is a collection of entities, called it's *elements*, which does not include itself.

This means a *set* is a collection of stuff which does not include the set itself.

For eg. A = {1,2,3,4,5,6,7,8,9,0} since A does not contain *itself*, it is a set.
B = {a,b,c,d,d,e,f,g,h} now, B is also a set but it contains 'd' twice but we do not care about the order of the elements, thus B = {a,b,c,d,e,f,g,h}.

The curly braces denote the *elements* of the set.

$\mathbf{N} = \{x \mid x \text{ is a } \textit{natural number}\}$
This is read as "**N** is the set of all x such that x is a *natural number*"
The '|' means such that.
'∈' means 'a element of' For eg :-
1 is a *natural number*
thus $1 \in \mathbf{N}$
The special set **N** is called the set of natural numbers.

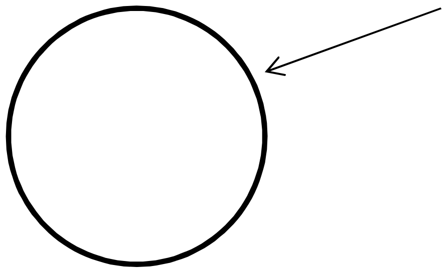
The special set which has no element is called the Empty set
The empty set is represented as ' $\varnothing$ '.

The special set **U** is defined as the set of *everything* excluding **U** itself.
This set is called the UNIVERSAL SET.

The set of the Rational Numbers is called '**Q**'. The set of the Irrational number is called **Q'** (read as Q prime). The set of Real numbers is called '**R**'. The set of Complex Numbers is called '**C**'. The set of whole numbers is called $\mathbf{N}_0$.

Union and Intersection of sets

A set can be *visualised* like this :-

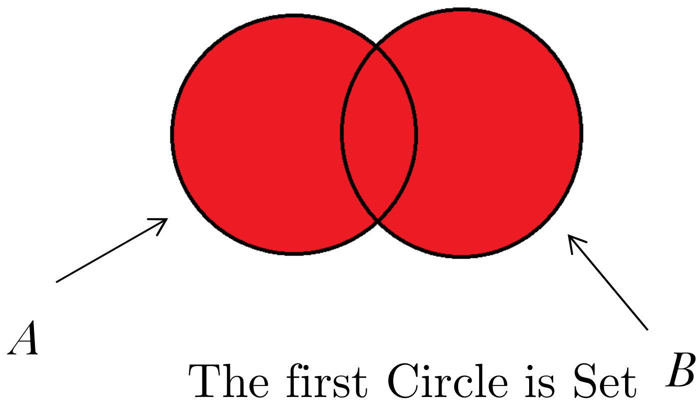


Every point in this Circle represents a Element.
Of course, a set must not contain just points,
But this method of reasoning is quite helpful.

The *union* of two sets is another set which contains both their *elements*.

The *union* is *represented* as '∪'.

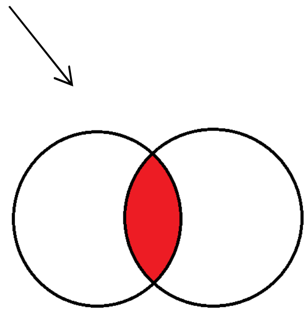
$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$



The first Circle is Set A and second Circle is Set B. The red part is the union of A&B

Intersection

The intersection is the *red* part.



The intersection is represented as '∩'.

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

The *intersection* is everything two sets have in common.

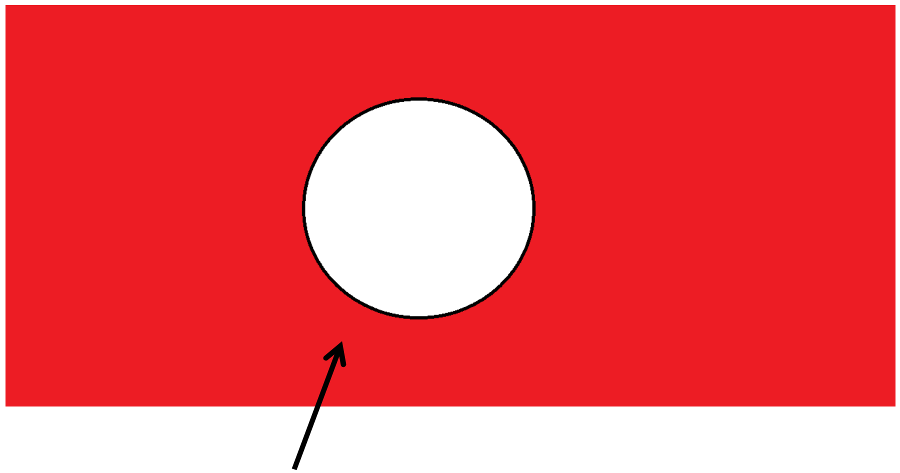
Note : The *intersection* is not the inverse of the union.

Set complement

The *complement* of a set is everything it does not contain.

The complement is denoted A^c for some set A.

$$A^c = \{ x \mid x \notin A \}$$



The red region is the complement.