THE EFFECT OF BEAM DRAFT RATIO ON ROLL DAMPING

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Thesis submitted in partial fulfillment of the requirements for the degree of Naval Engineer.

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by

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NAVAL ENGINEER

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ABSTRACT.

Title of Thesis: "The Effect of Beam-Draft Ratio on Roll Damping."

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This report describes a series of experiments made in an attempt to determine the effect of beam-draft ratio on the roll-damping of ships. The investigation was instagated by a theoretical development of Dr. F. Ursell* which showed that at a beam-draft ratio of 2.52, the wave-making resistance to roll is at a minimum.

The tests were conducted in still water with a series of seven parallel-sided models seven feet long. Beam-draft ratio varied from 1.50 to 3.50. Initial heel angle in the tests averaged about 5°.

Angle of roll versus time records were obtained with a gyro measuring system. Measurements of waves generated by the rolling were also measure by an electrical method.

The resulting data were analyzed on the assumption of linear damping. This assumption was borne out in that with very few exceptions semi-logarithmic plots of roll angle versus time show a linear relationship. The slope of these lines was used to determine k_1, the models' form linear damping constant. This quantity is used as a basis for comparison.

The results obtained are inconclusive. Insofar as the theory is concerned, no correlation is found. This is attributed primarily to changes in damping caused by variations in the position of the rolling axis, and secondarily to the effects of height of the metacentre above the water-line, and of metacentric height, varying from model to model.

Records of waves generated by the rolling models indicate that wave-making accounts for about 80% of the total energy dissipated.

For future work, it is recommended that tests be conducted with the models forced to roll about a point in the water-line. Further tests are also needed to determine the effects of such variables as the height of the centre of gravity on the position of the rolling axis.

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* Ursell, F., "On the Rolling Motion of Cylinders in the Surface of a Fluid" Quarterly Journal of Mechanics and Applied Mathematics, Vol II, 1949, page 335.

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ACKNOWLEDGEMENTS.

The author is indebted to Professor Martin A. Abkowitz, of the Department of Naval Architecture and Marine Engineering, without whose wide experience and guidance this project would not have been attempted.

Thanks are also due to Mr. Selwyn Blake, of the M.I.T. Model Shop for his advice and assistance in building the models, and to Mr. C. Wachnik, for his guidance in operation of much of the equipment used at the Towing Tank.

INTRODUCTION

Any ship in a seaway will roll. The amount which it rolls depends on its damping properties. The latter, in turn, depend primarily on the form of the ship's hull and appendages, on its period and metacentric height. In addition, other things being equal, independent variations in the height of the centre of gravity or height of the metacentre may also affect the rolling characteristics.

Of all the above variables, the least is known about the effect on rolling of changes in the hull form. This work is concerned with the effect of changing a single variable, namely beam-draft ratio.

A ship's resistance to rolling is made up of three components: (1) frictional resistance, (2) eddy resistance, of the type created by bilge keels, (3) wave-making resistance. In the absence of bilge keels, it is generally conceded that more than half of the total energy dissipated by the average rolling ship is in the form of waves.

The effect of beam-draft ratio on roll-damping due to wave-making was first investigated by Dr. F. Ursell*, in a theoretical treatment of the problem. His calculations were based on the following assumptions:

(a) Zero viscosity.

(b) Length of the waves produced is large compared with the beam.

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* Ursell, F., "On the Rolling Motion of Cylinders in the Surface of a Fluid". Quarterly Journal of Mechanics and Applied Mathematics; Volume II, 1949, page 335.

(c) The ship rolls about a point in the water-line.

(d) The rolling is isochronous.

(e) Angles of roll are small.

(f) Two-dimensional (fluid) motion.

(g) The position of the roll axis is independent of the rolling frequency.

(h) The cross-section of the ship is defined by the parametric equations:

$$y = \frac{9}{16}(a+b)\cos\eta - \frac{25}{48}(a-b)\cos\eta - \frac{1}{16}(a+b)\cos 3\eta + \frac{1}{48}(a-b)\cos 5\eta \qquad (1)$$

$$x = \frac{9}{16}(a+b)\sin\eta + \frac{25}{48}(a-b)\sin\eta + \frac{1}{16}(a+b)\sin 3\eta - \frac{1}{48}(a-b)\sin 5\eta \qquad (2)$$

where $a = \frac{B}{2}$, $b = H$.

Based on the above, Dr. Ursell's calculations show the wave amplitude produced by the rolling is given by:

$$a_w = 0.63\, K^2\, \theta_o\, (a+b)(a+1.05b)\left|a-1.26b\right| \qquad (3)$$

where: $K = \omega^2/g$, θ_o = roll amplitude, $\theta = \theta_o \cos \omega t$.

An expression is also given for the same form rolling about an axis a distance d above the water-line:

$$a_w = 0.63\, K^2\, \theta_o\, (a+b)\left[(1.26b-a)(1.05b+a) + 0.015\, d(a+26b)\right] \qquad (4)$$

It is indicated in the paper that this theory may be expected to apply with good precision to the case of the conventional merchant ship form with parallel sides and a flat bottom, a moderate bilge radius, and a fairly long parallel middle body. A criterion in the latter respect is stated to be that the length of the parallel mid-

dle body should be equal to or greater than the length of the waves produced by the rolling.

Considering the wave amplitude equations (3) and (4), several interesting implications may be drawn. First, since the energy content of a wave is proportional to the square of its amplitude, the expressions are measures of energy dissipated due to wave-making by the rolling ship. Secondly, the equations state that the wave amplitude produced by the rolling ship is directly proportional to the roll amplitude, and inversely proportional to the fourth power of the period of roll. Finally, at a = 1.26b, i.e. at a beam-draft ratio of 2.52, both equations pass through a minimum, which implies that roll-damping due to wave-making is at a minimum at this beam-draft ratio. In the case of the first expression, equation (3), in which the ship is rolling about the water-line, the wave-amplitude is zero at the critical beam-draft ratio of 2.52.

The primary concern of this work is, then, an experimental investigation of Dr. Ursell's theory. To this end, eight models were built with beam-draft ratio varying between 1.50 and 3.50. Appropriate instrumentation was arranged at the M.I.T. Towing Tank to measure roll angle versus time, and amplitude versus time of the waves produced by these models rolling in still water. The rest of this report is concerned with a description of the experiments and results, and a discussion of the results in the light of Dr. Ursell's development, and modern rolling theory.

PROCEDURE.

(1) General.

A series of eight parallel sided models was built. These followed conditions required by Dr. Ursell's theory insofar as was practical. Seven of the series were 7 ft. long, with beam varying from 9 to 14½ inches, and beam-draft ratio varying from 1.50 to 3.50. The eighth model was a two-thirds size duplicate of the model with beam-draft ratio 2.52. The models are fully described in Appendix I; the characteristics are given in Table II, page 46.

A gyro roll recording system was used to obtain records of roll angle versus time for the models. This system is fully described in Appendix II. The wave-height recorder recently installed in the M.I.T. Towing Tank was used to measure the waves generated by the rolling models. This system is also described in Appendix II.

It was intended originally that the models would be tested at constant period and metacentric height. In practice, however, it was found impossible to maintain both constant throughout the series. Subsequently, it was decided that only period would be held constant. Considering the assumption in the theory that the length of the waves produced is large compared with the beam of the model, it was desirable that the period be large. Thus for a period of one second, and assuming a trochoidal wave shape, the wave length is:

$$\lambda = \frac{gT^2}{2\pi} = 5.12 \text{ ft.}, \qquad (5)$$

which for the model of 12 ins. beam, gives a wave-length to beam

ratio of 5.12. Although probably a larger ratio is desirable, as the period increases, the waves generated by the rolling become smaller and more difficult to measure. It will be seen that this is a practical and serious difficulty. Eventually, it was decided that the period would be made as small as possible, without going below one second. In practice, in the model with the smallest beam (No. 1), it was found that the smallest period that could be obtained was about 1.1 second.

(2) Experiments.

The main features of the experimental set-up may be seen in Figure III, page 6. The set-up is described in detail in Appendix II, Part (3). For any model, the procedure was as follows below.

The gyro unit was mounted in the model, amidships, with the sensetive axis running fore and aft. The model was then placed in the water, and the gyro temperature control unit was set in operation to allow the gyro to come to operating temperature. The model was then ballasted to give the required draft and period, and an inclining experiment was run to determine the model's metacentric height. The model was then placed in the test position in the towing tank, near the mid-length of the tank, and across its breadth. The wave-height pickup was placed near the model at the mid-width of the tank, and oriented parallel to the side of the model. A constant distance of 39 inches from pickup to the side of the model was maintained throughout the series of tests.

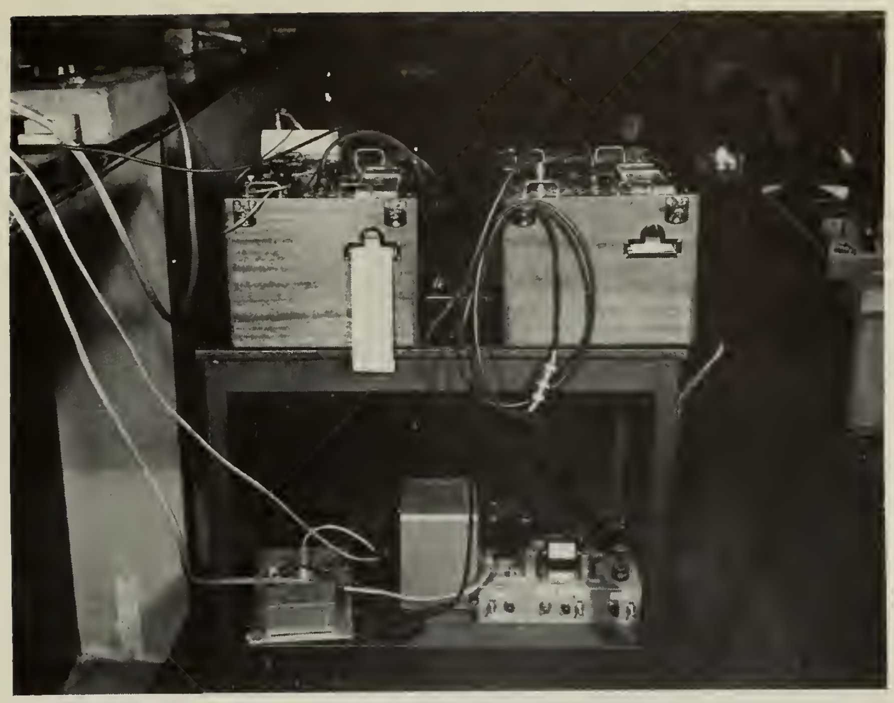


FIG II

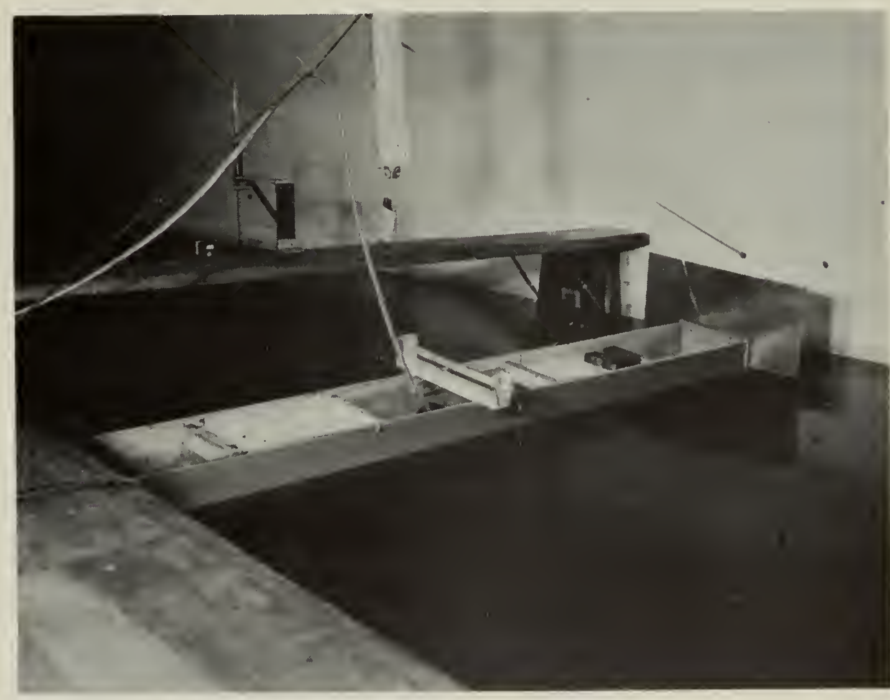


FIG III

ADDENDUM TO FIGURES II AND III

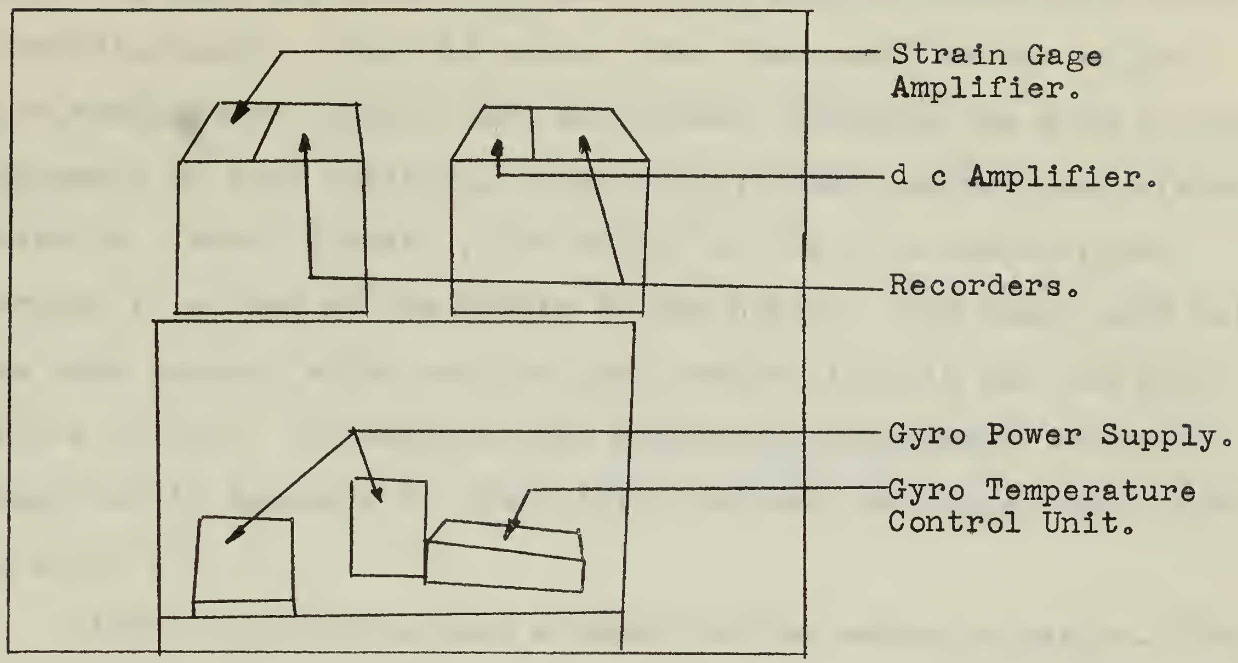


FIG II
MEASURING INSTRUMENTS.

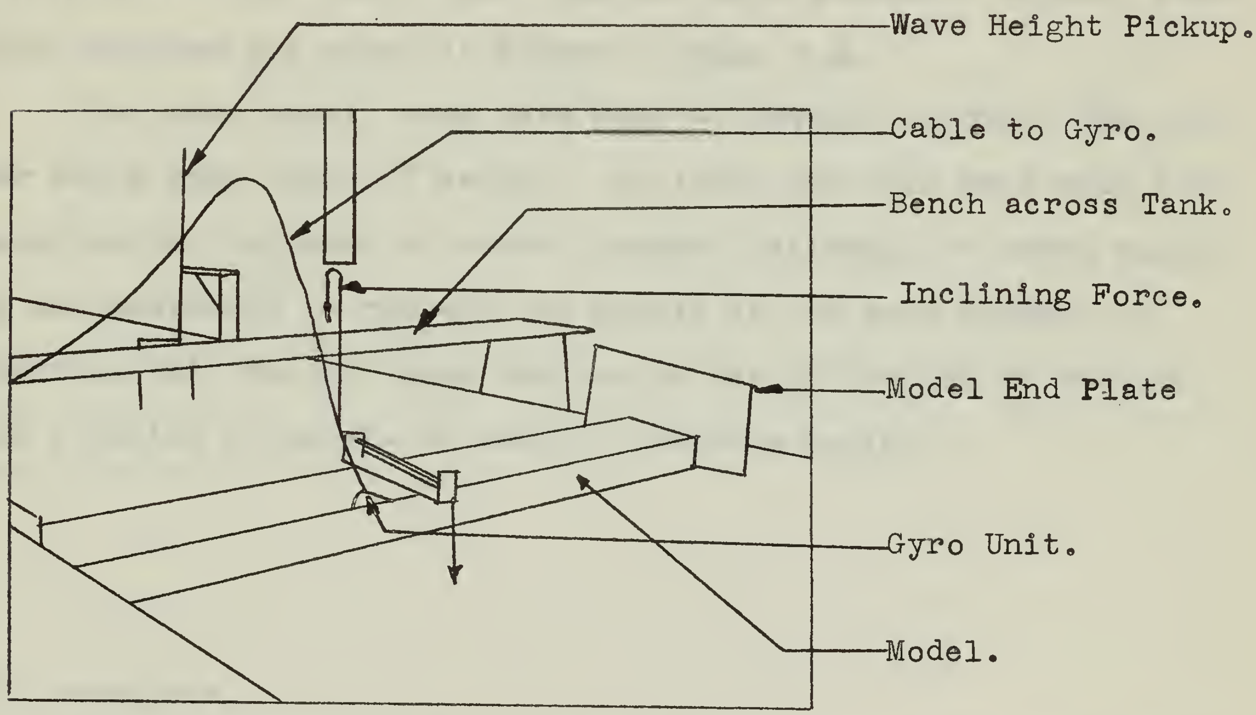


FIG III
EXPERIMENTAL SET-UP.
Model in Inclined Position.

After insuring that the model was upright and had come to rest, the gyro was placed in operation. When the gyro had reached operating speed, (about 15 secs.), the "zero set" switch on the gyro temperature control unit was closed, bringing the gyro to the reference or zero position. Once the gyro had reached the reference position, (about 5 secs.), the stylus on the roll recorder was zeroed; (i.e. set at the middle of the paper). The "zero set" switch was then opened, which set the gyro free so that it was now sensetive to roll. By means of the inclining arrangement, which is described in Appendix II, Part (3), the model was then heeled over to about 5°.

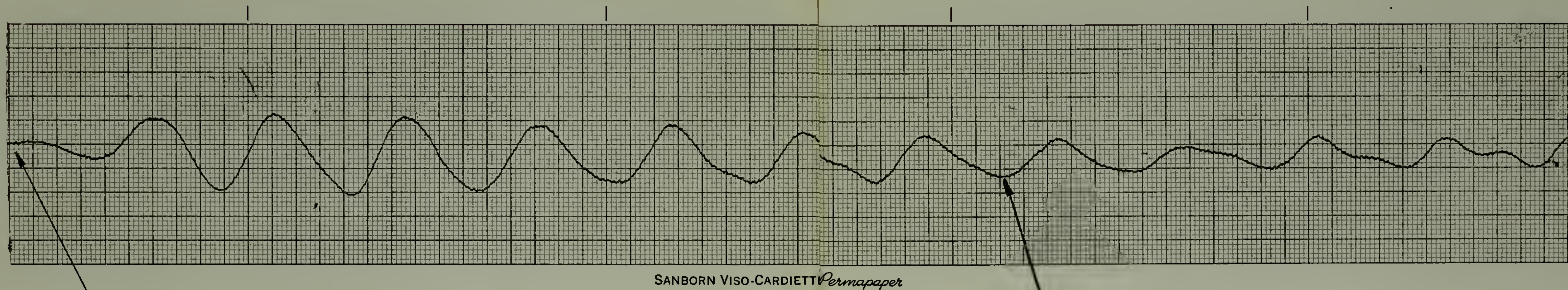
About ten minutes were allowed for the water to settle. The roll and wave-height recorders were then set in operation, and the model was released from the heeled position. When the model rolling was damped out, the recorders were stopped. Typical records obtained are shown in Figure I, page 9 a.

For each model, runs were made at several periods, the latter being kept near 1.1 second. At least two runs were made for each period, in order to obtain checks. Although as noted above it was desirable to run all the models at the same period, in practice this was not done because of the difficulty of ballasting a series of models to exactly the same period.

(3) Analysis.

The differential equation of a ship rolling in still water may be written:

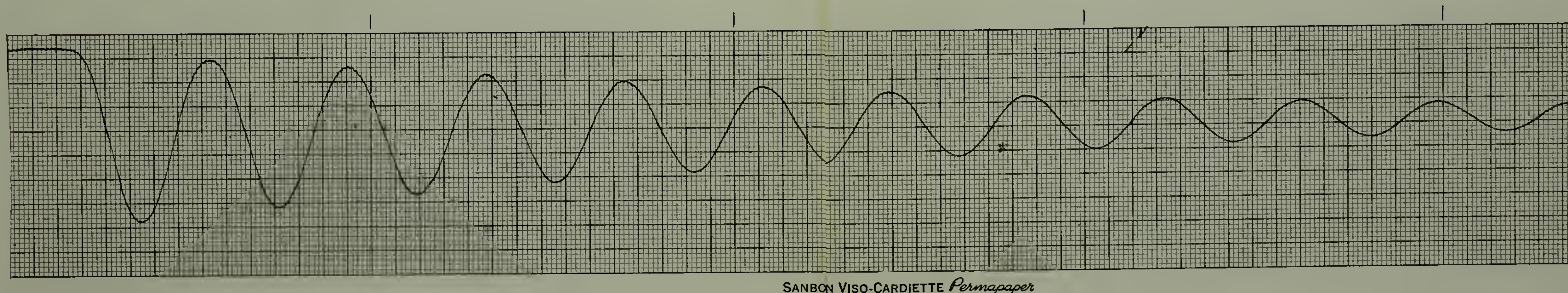
$$\frac{J}{g}\frac{d^2\theta}{dt^2} + A\frac{d\theta}{dt} + B\left(\frac{d\theta}{dt}\right)^2 + \Delta . GZ = 0 \qquad (5).$$



Still Water Level

(a) Typical Wave Record.

Wave Crest.



(b) Typical Roll Record.

FIGURE I

TYPICAL ROLL AND WAVE RECORDS

D.R.M. April 1955.

which states that the envelope of the rolling curve will appear as a straight line on a semi-logarithmic plot of θ versus time, the slope of which line is $-2k_1/T$.

Hence, if the rolling curve of a model is taken, and the envelope of the curve plotted versus time on semi-logarithmic coordinates, the slope of the line so determined will give the damping coefficient k_1.

For any given model run, the envelope was obtained by laying the roll record (see Figure I) on a drawing table, and drawing a smooth curve through the maximum and minimum points. Points on the envelope were then plotted on semi-logarithmic paper against time, at one second intervals, by applying the gyro calibration factor to the ordinates read from the record. (The resulting plot is then one of twice the roll angle versus time; this, of course does not affect the slope.) A mean line was then drawn through the points so obtained.

The period for any run was also obtained directly from the roll record. This was done by counting off a number of cycles on the record and measuring the distance, which determines the time elapsed for that number*. Dividing the total time by the number of cycles then gave the period.

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* The manufacturer's specified paper speed on the recorder is 2.5 cm/sec., which should also then be the horizontal scale of the record. It was found actually that the speed tended to exceed this value, so that for each series of runs it was necessary to check the paper speed and obtain a scale correction factor. The error averaged about 3%, so that 1 second was actually about 2.58 cm.

By writing equation (10):

$$\log_e \Theta_o - \log_e \Theta = \frac{2k_1}{T} t \qquad (11)$$

there results the following expression for k_1:

$$k_1 = \frac{T}{2} \log_e \left(\frac{\Theta_o}{\Theta}\right) . \frac{1}{\Delta t} \qquad (12)$$

where Δt is the time interval between Θ_o and Θ.

Knowing k_1, the damping factor A may than be obtained by writing equation (8b):

$$A = \frac{\Delta GM\ T\ k_1}{\pi^2} \qquad (13)$$

Sample calculations are given in Appendix III.

RESULTS

The principal results are presented in Table I which begins on page 14. k_1 was obtained as indicated in the Analysis section of this report from a semi-logarithmic plot of θ versus time for each run. These plots are presented beginning on page 17. It should be noted that the time scale of these plots has no absolute significance. It is numbered purely for convenience in obtaining time increments between angular decrements.

As will be noted, the majority of the lines presented in the graphs have two sets of points. These represent points obtained for the model in different runs, but at the same period and GM.

The significance of a run number is as follows: a single number is simply the consecutive number of a model run on the first day it was tested; the last figure in a double number has the same meaning, the first figure is the day in which the model was run. Thus run number 23 means that this is the third run of a model, the second day it was tested. This system was used because it is not uncommon in tests of this nature for results to vary from day to day, due, for instance, to changes in disposition of ballast.

No tests were made with Model No. 8, because the wiring arrangement on the gyro unit was such that the gyro could not be mounted in this model.

For reasons which will be noted in the Discussion section of this report, no calculations on the wave data taken are given. In general, however, it can be stated that the magnitude of the

waves generated by the rolling models was such that considerably more than 50% of the energy lost by the models was due to wave-making.

A plot of k_1 versus B/H at various values of period is given on page 16 .

TABLE I

RESULTS FROM DECLINING ANGLE CURVES.

Model No., B/H	T sec.	GM ins.	A_x $ins.^2$	KG ins.	KM ins.	H ins.	K_1	A in-lb-sec.
1	1.103	1.50	53.00	2.85	4.35	6.07	0.0790	2.13
1.50	1.072	1.29		3.06			0.0916	2.07
	1.112	1.32		3.03			0.0786	1.89
	1.249	1.12		3.23			0.0667	1.52
	1.080	1.61		2.74			0.0939	2.66
	1.198	1.30		3.05			0.0769	1.96
2	1.048	1.69	53.99	3.02	4.71	5.31	0.1158	3.40
2.00	1.118	1.52		3.19			0.0755	2.12
	1.146	1.44		3.27			0.0781	2.14
	1.120	1.46		3.25			0.0812	2.20
	1.052	1.52		3.19			0.0944	2.52
	1.155	1.42		3.29			0.0707	1.92
3	1.103	1.48	54.38	3.35	4.83	5.07	0.0742	2.02
2.25	1.072	1.65		3.18			0.0815	2.40
	1.181	1.33		3.50			0.0692	1.80
	1.117	1.39		3.44			0.0774	2.00

Continued, next page.

TABLE I (Cont'd)

Model No., B/H	T ins.	GM ins.	A_x ins.^{2}	KG ins.	KM ins.	H ins.	K_1	A in-lb-sec.
4	1.139	1.33	54.76	3.79	5.12	4.76	0.0793	2.02
2.52	1.081	1.49		3.63			0.0902	2.45
	1.066	1.81		3.31			0.1045	3.38
	1.148	1.91		3.21			0.0900	3.32
	1.082						0.1021	
	1.090	1.89		3.23			0.1015	3.52
	1.152	1.52		3.60			0.0770	2.27
5	1.057	2.11	55.04	3.27	5.38	4.51	0.0904	3.41
2.75	1.117	1.70		3.62			0.0786	2.30
	1.072	2.09		3.29			0.0884	3.34
	1.128	1.79		3.59			0.0758	2.59
6	1.072	2.35	55.39	3.35	5.70	4.38	0.0870	3.73
3.00	1.144	3.03		3.68			0.0680	2.68
	1.079	2.51		3.19			0.0783	3.60
	1.020	2.91		2.79			0.0881	4.45
	1.093	2.11		3.59			0.0722	2.84
	1.143	1.82		3.88			0.0714	2.52
7	1.119	2.69	58.15	3.85	6.54	4.15	0.0683	3.68
3.50	1.055	3.49		3.05			0.0835	5.50
	1.103	3.20		3.34			0.0711	4.49
	1.112	2.78		3.76			0.0662	3.66
	1.178	2.38		4.16			0.0600	2.84

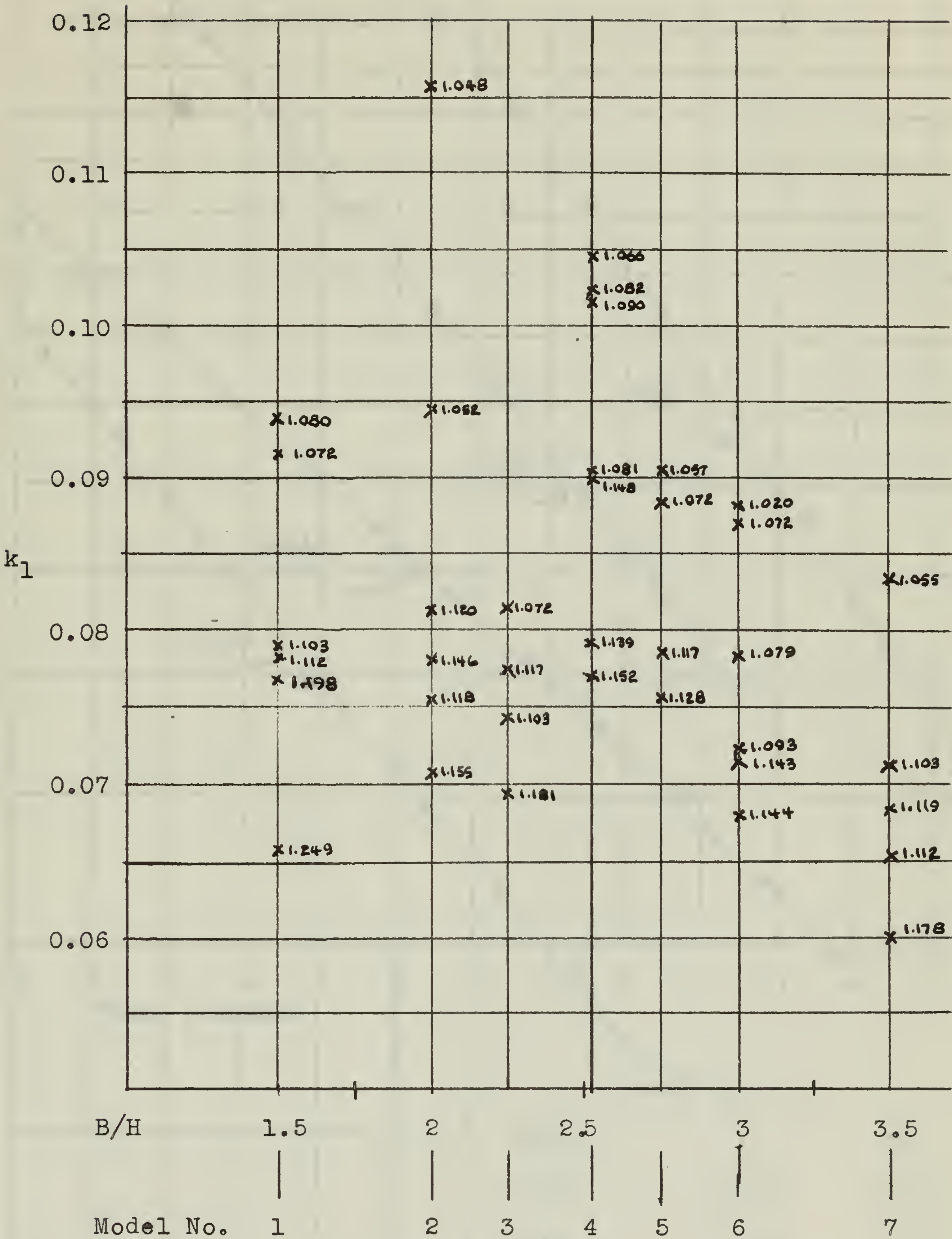
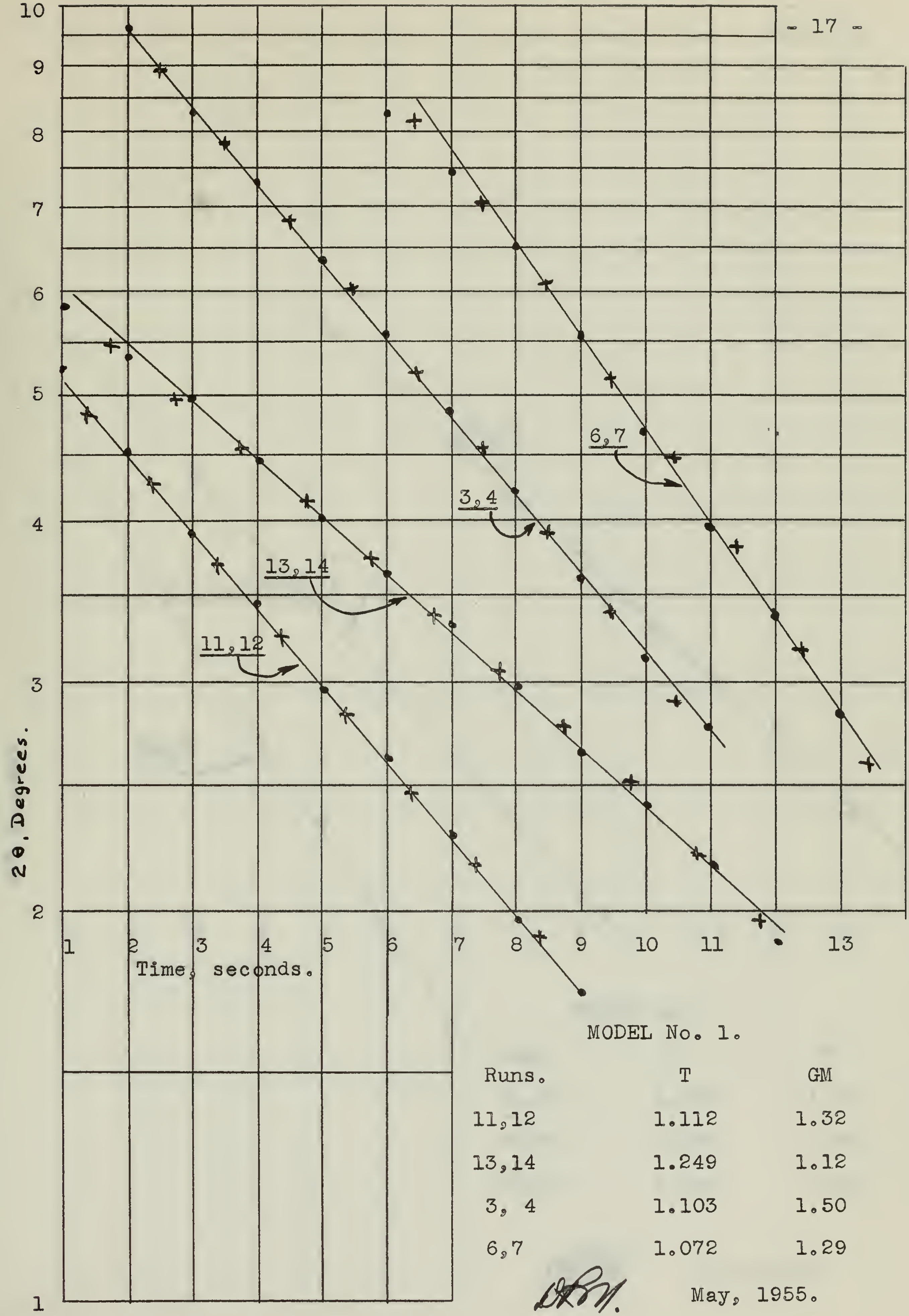


FIGURE IV

k_1 VERSUS B/H.

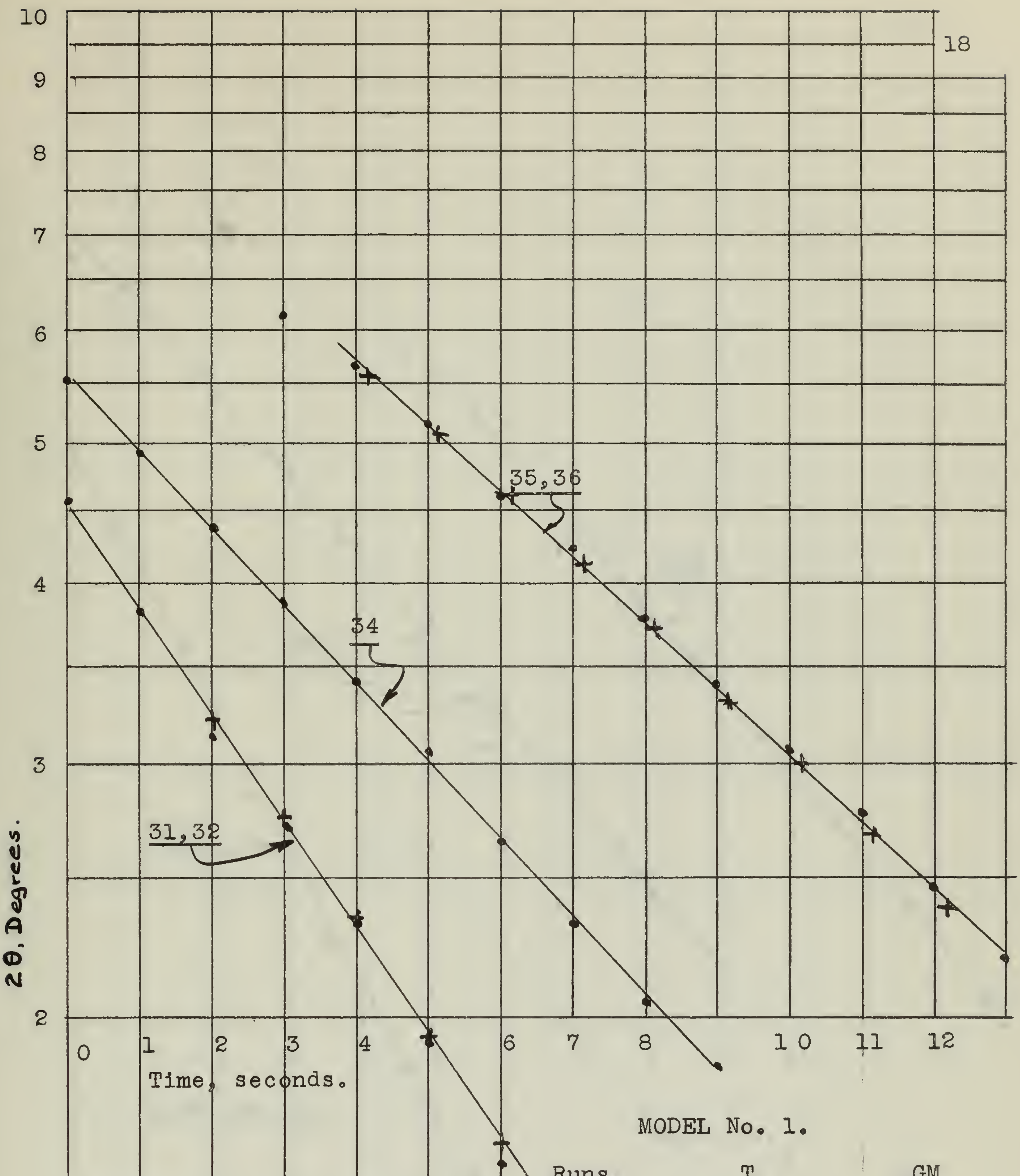
At particular values of
period as shown.



MODEL No. 1.

Runs.	T	GM
11,12	1.112	1.32
13,14	1.249	1.12
3, 4	1.103	1.50
6,7	1.072	1.29

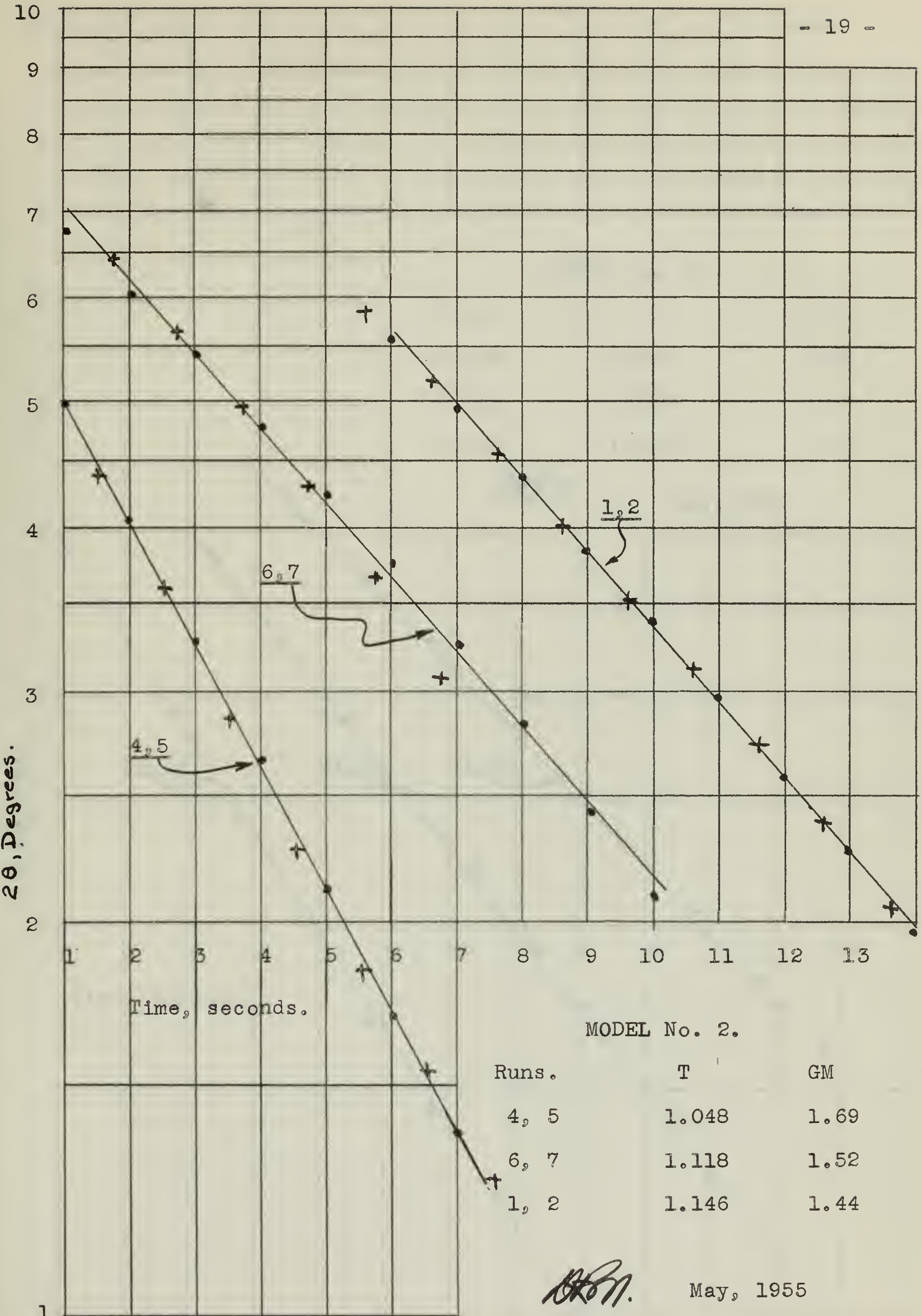
May, 1955.



MODEL No. 1.

Runs.	T	GM
31,32	1.080	1.61
+ 33,34	1.198	1.305
35,36	1.280	1.055

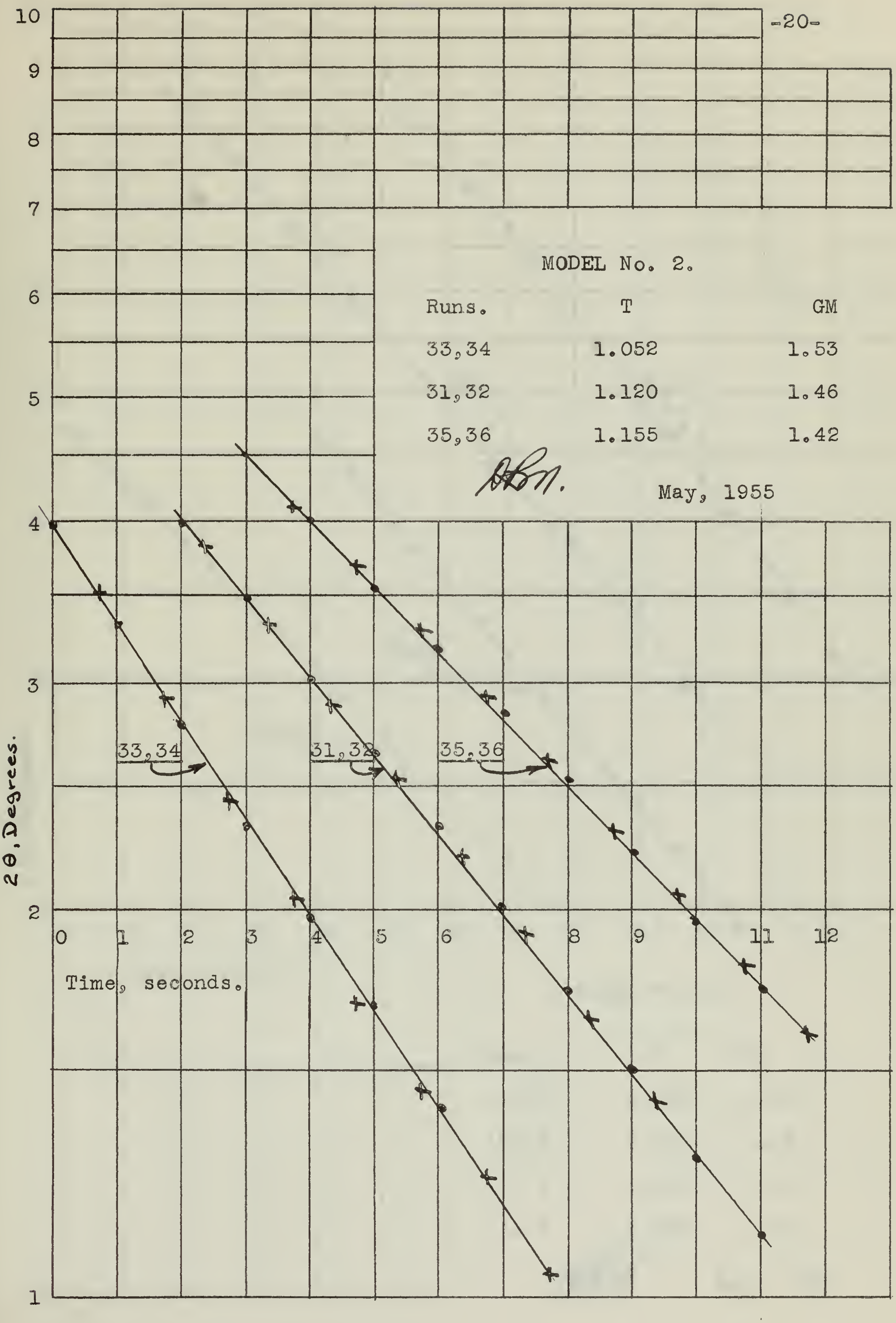
[illegible] May, 1955

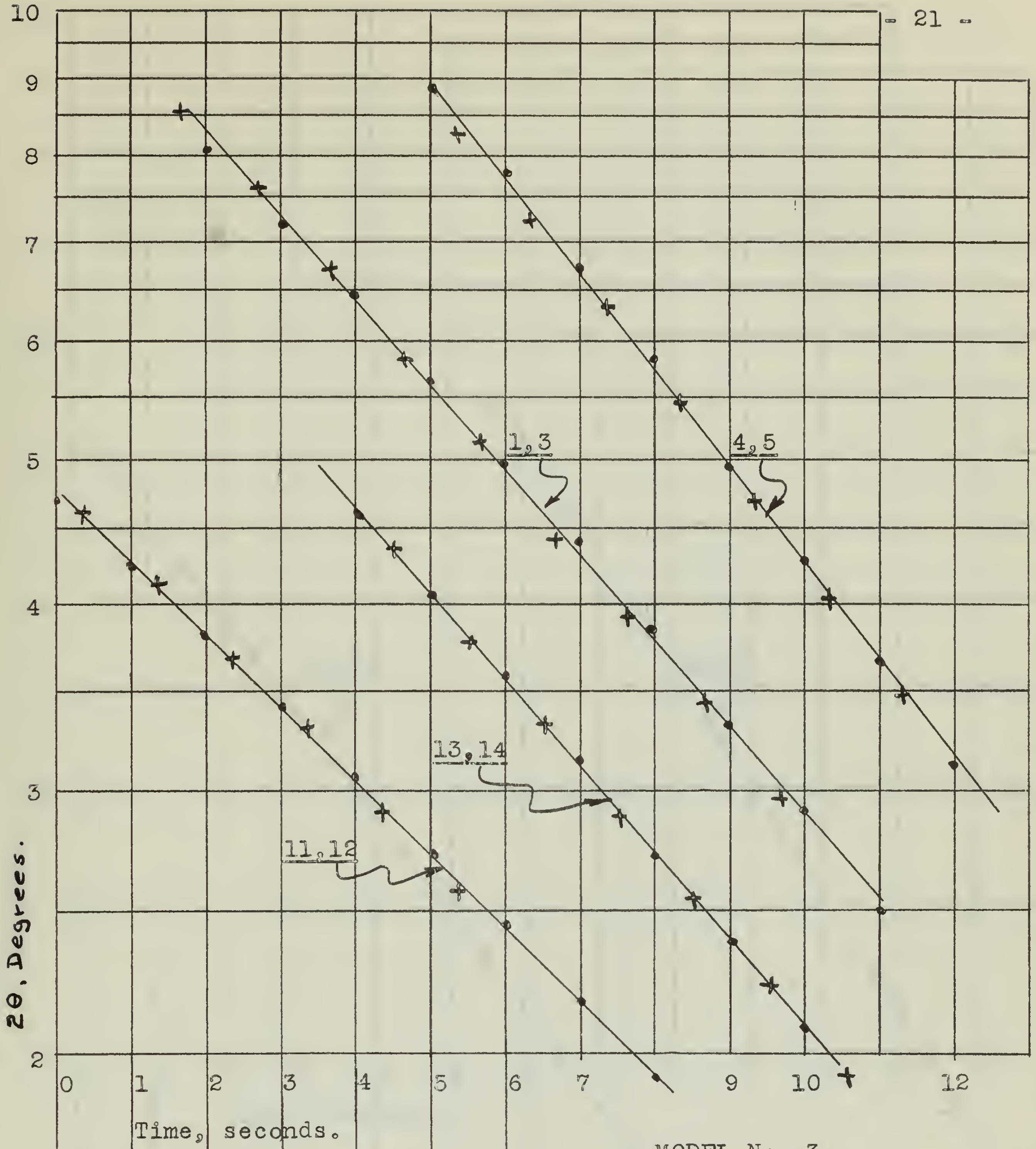


MODEL No. 2.

Runs.	T	GM
4, 5	1.048	1.69
6, 7	1.118	1.52
1, 2	1.146	1.44

May, 1955

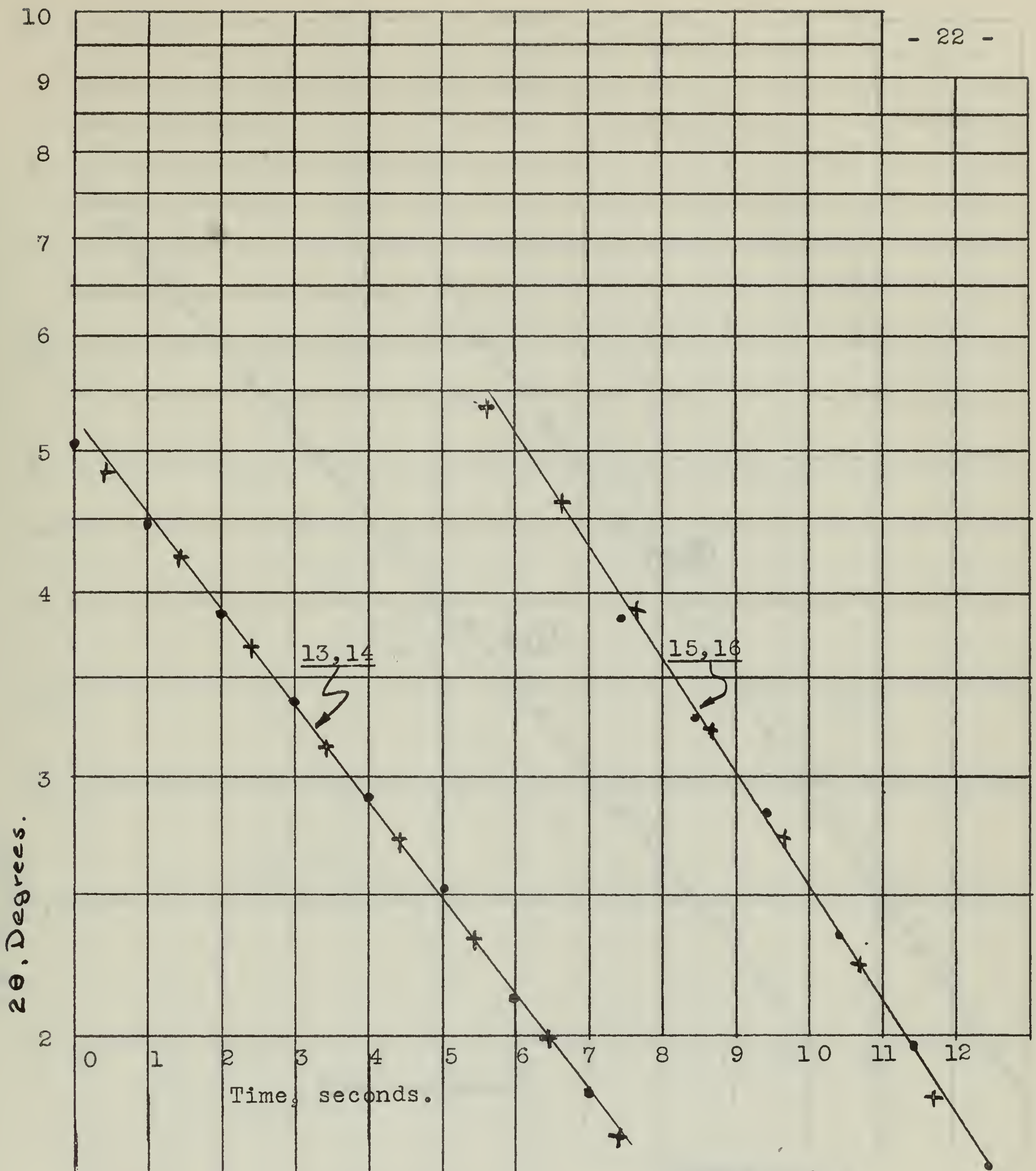

MODEL No. 2.
Runs. T GM
33,34 1.052 1.53
31,32 1.120 1.46
35,36 1.155 1.42
May, 1955
33,34
31,32
35,36
2θ, Degrees.
Time, seconds.
0 1 2 3 4 5 6 8 9 11 12
1 2 3 4 5 6 7 8 9 10



MODEL No. 3.

Runs.	T	GM
11, 12	1.181	1.33
13, 14	1.117	1.39
1, 3	1.103	1.48
4, 5	1.072	1.65

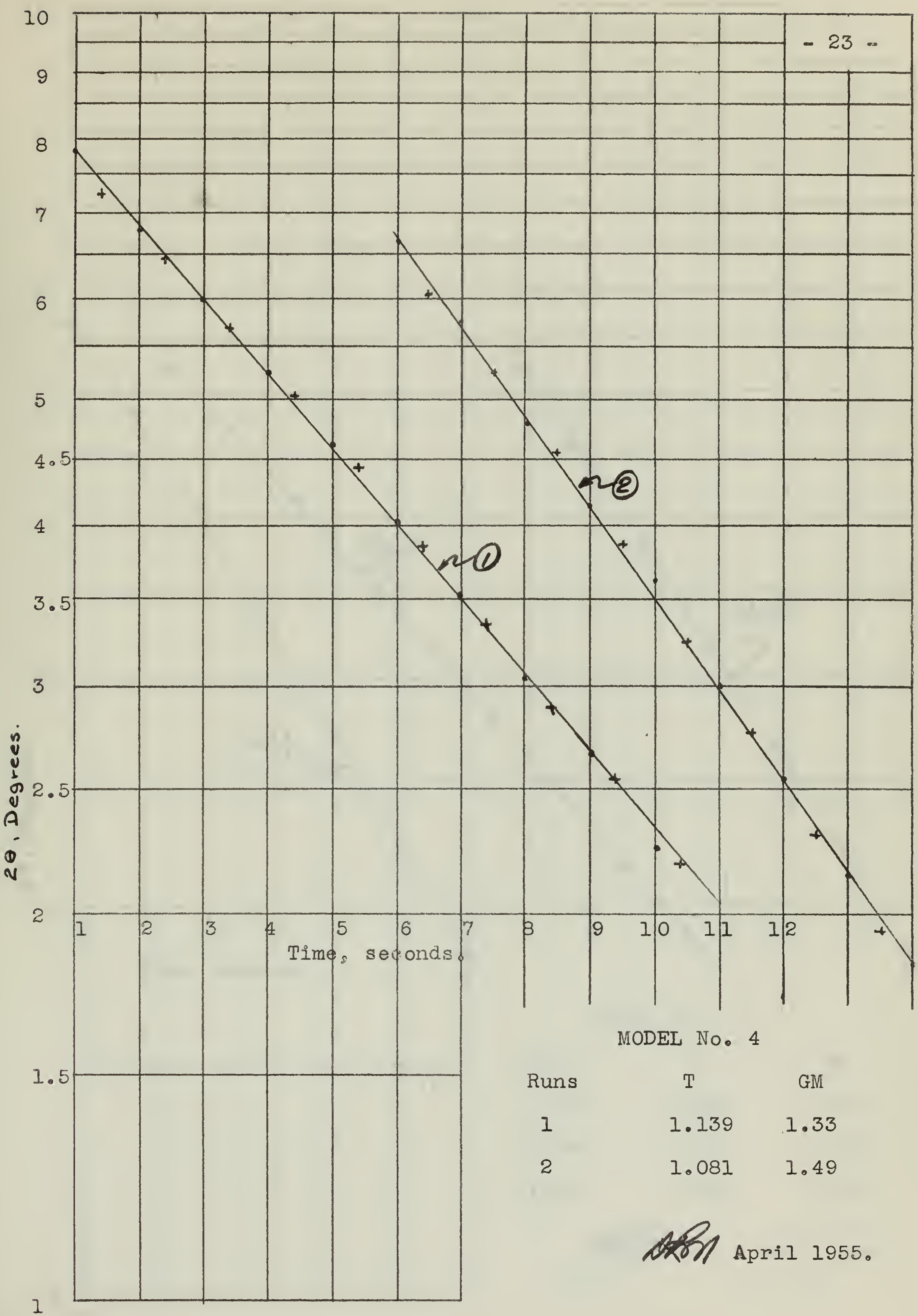
DKM. May, 1955



MODEL No. 4.

Runs.	T	GM
13,14	1.148	1.91
15,16	1.082	Not taken.

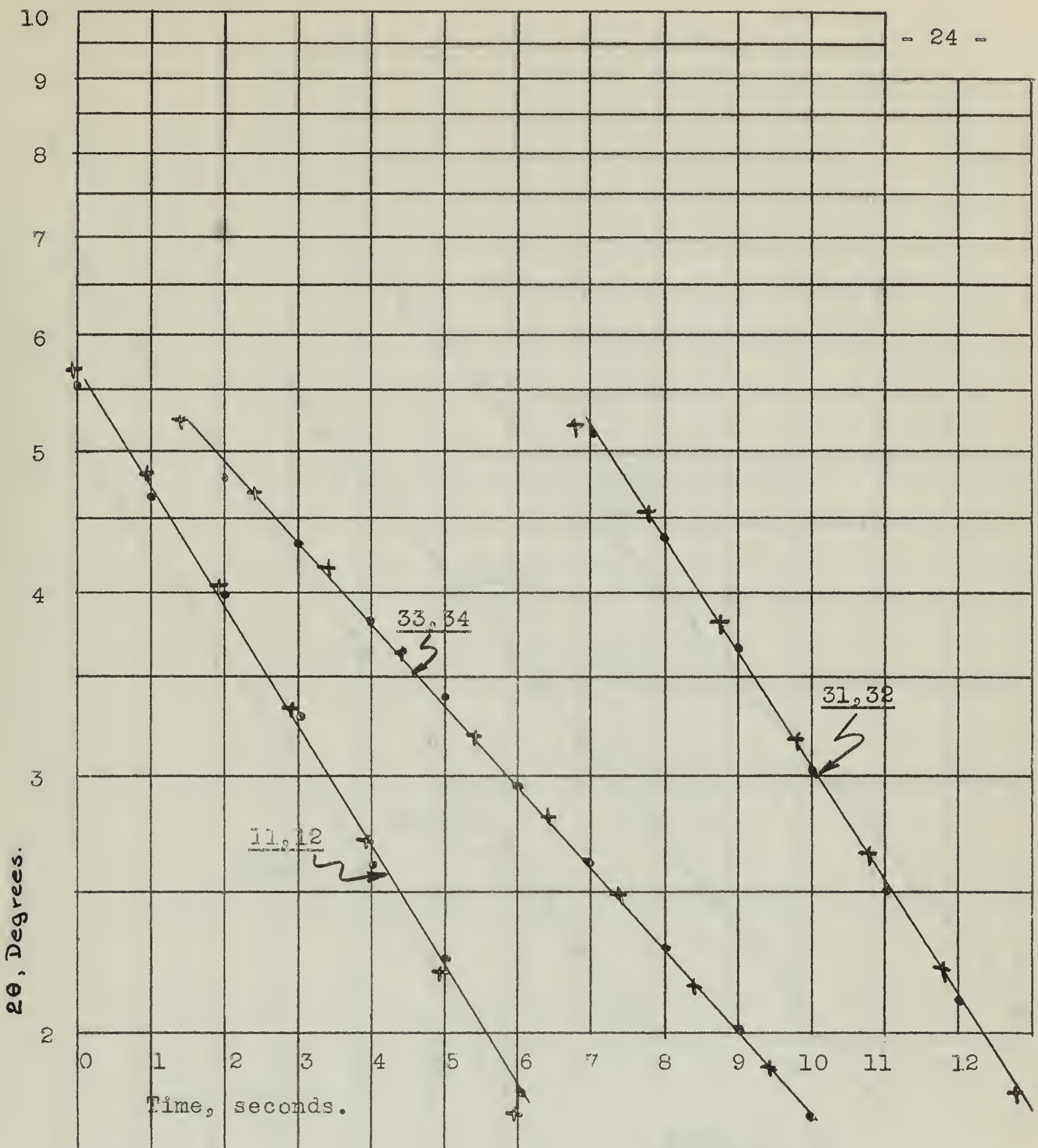
May, 1955



MODEL No. 4

Runs	T	GM
1	1.139	1.33
2	1.081	1.49

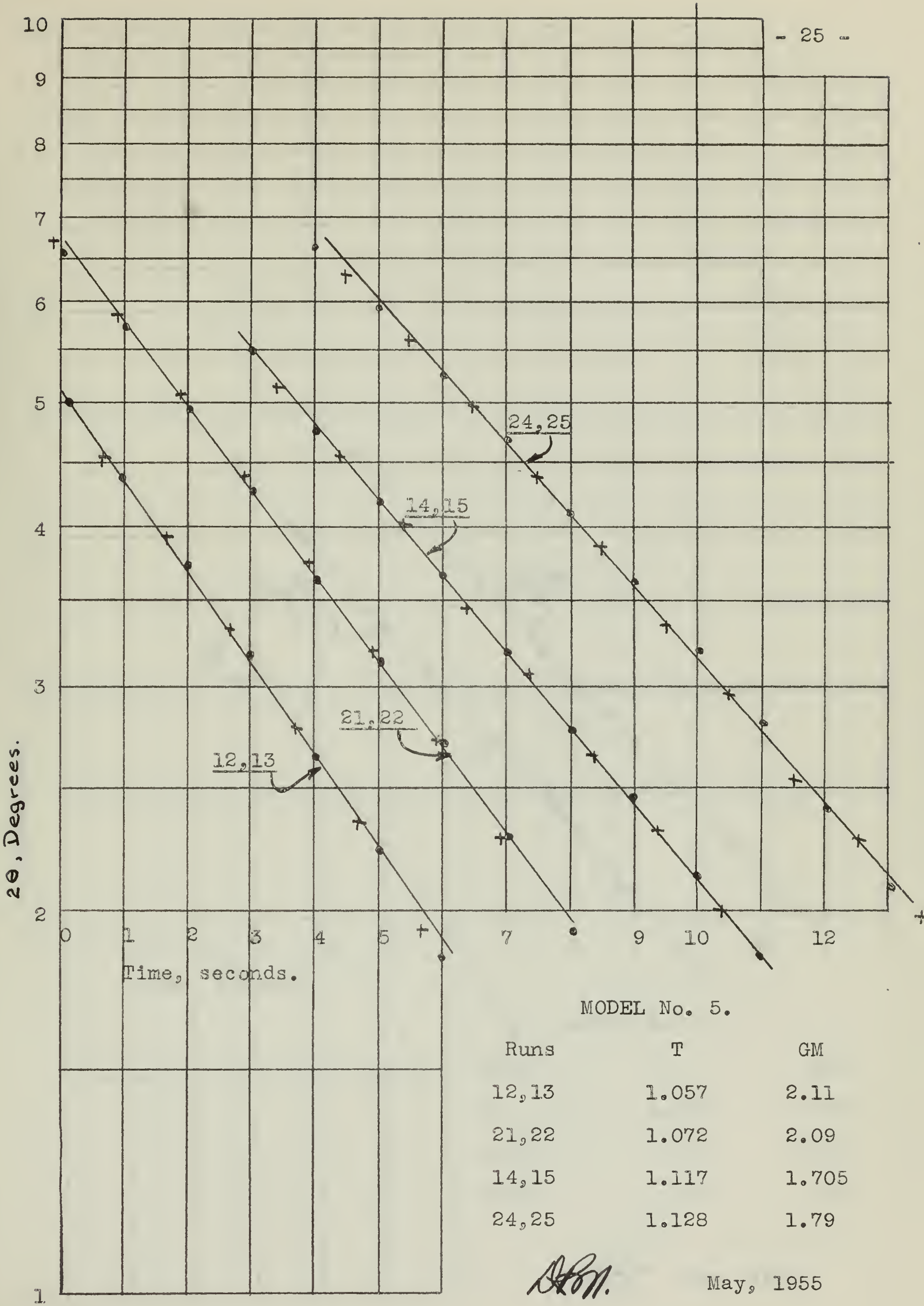
April 1955.



MODEL No. 4.

Runs.	T	GM
11,12	1.066	1.81
33,34	1.152	1.52
31,32	1.090	1.89

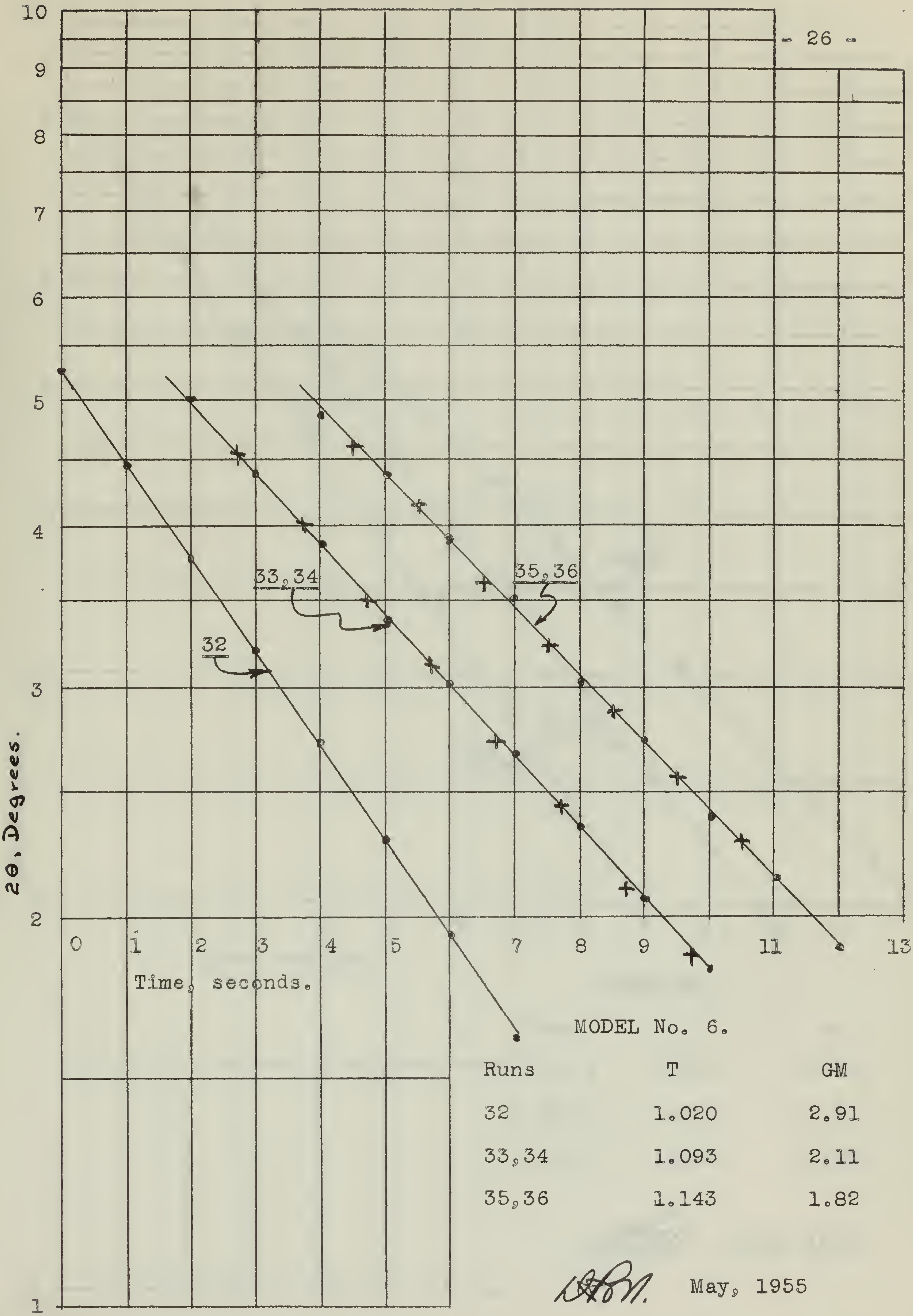
May, 1955



MODEL No. 5.

Runs	T	GM
12,13	1.057	2.11
21,22	1.072	2.09
14,15	1.117	1.705
24,25	1.128	1.79

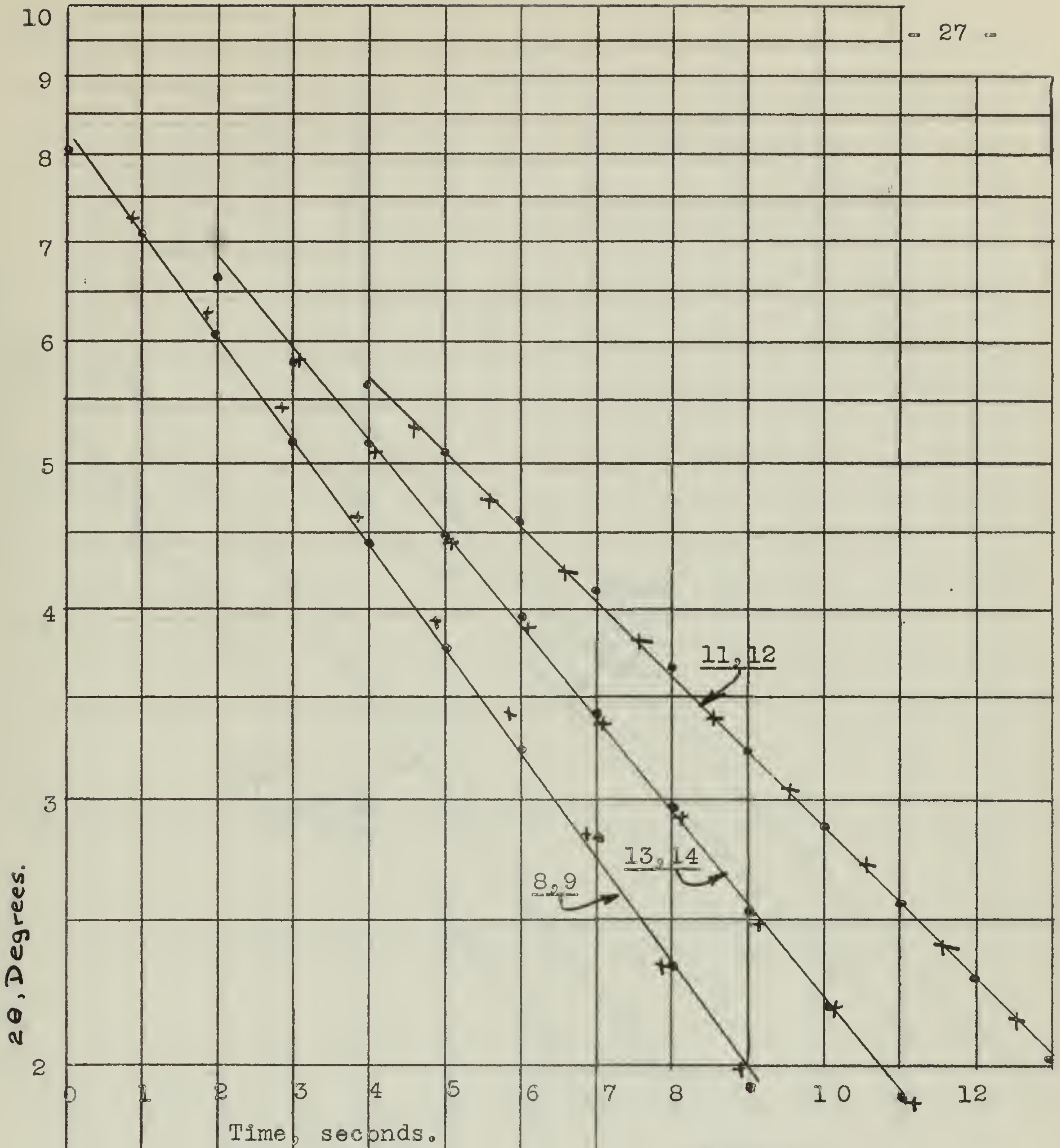
[signature] May, 1955



MODEL No. 6.

Runs	T	GM
32	1.020	2.91
33,34	1.093	2.11
35,36	1.143	1.82

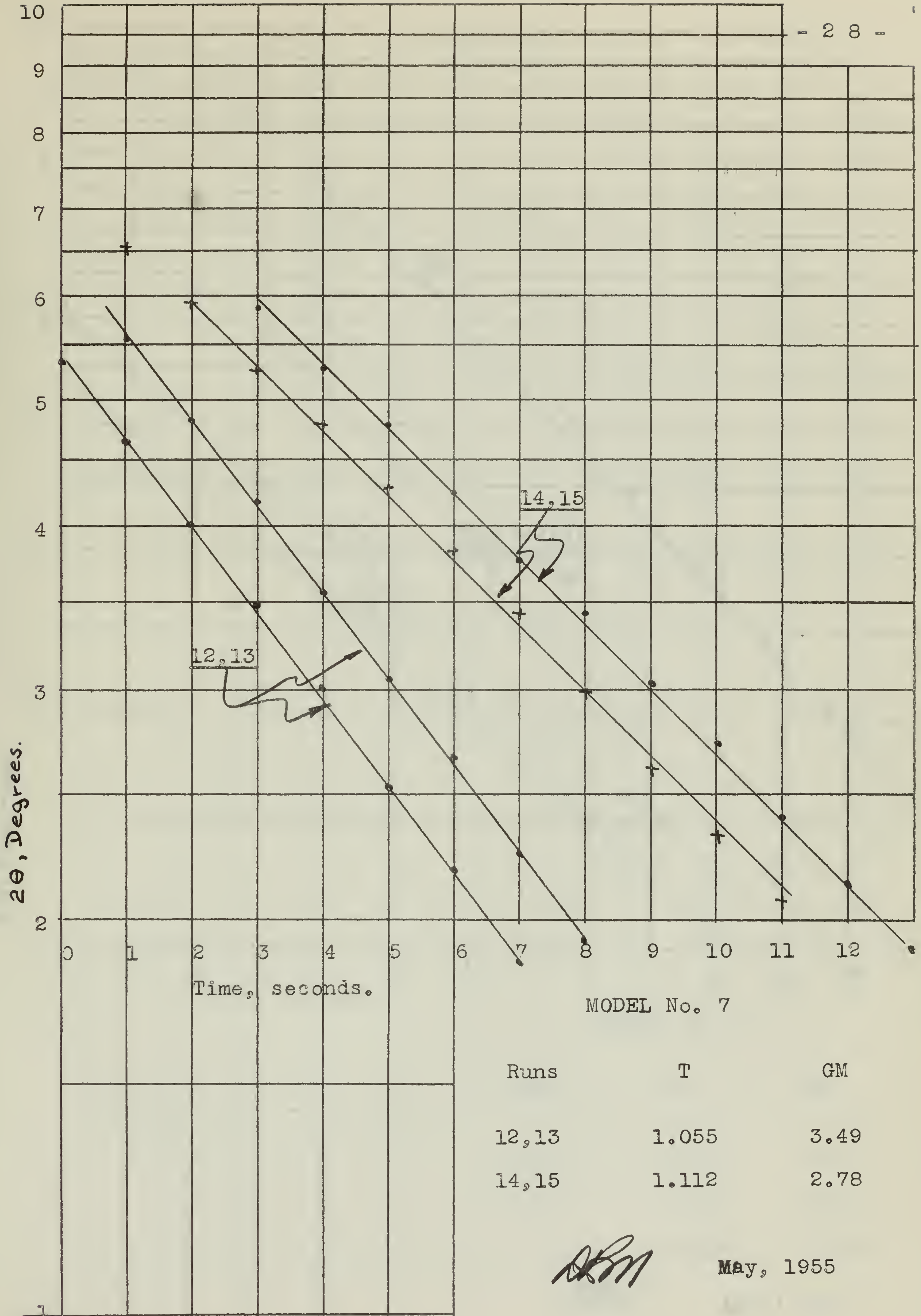
May, 1955



MODEL No. 6

Runs	T	GM
8, 9	1.072	2.35
13,14	1.079	2.51
11,12	1.144	2.02

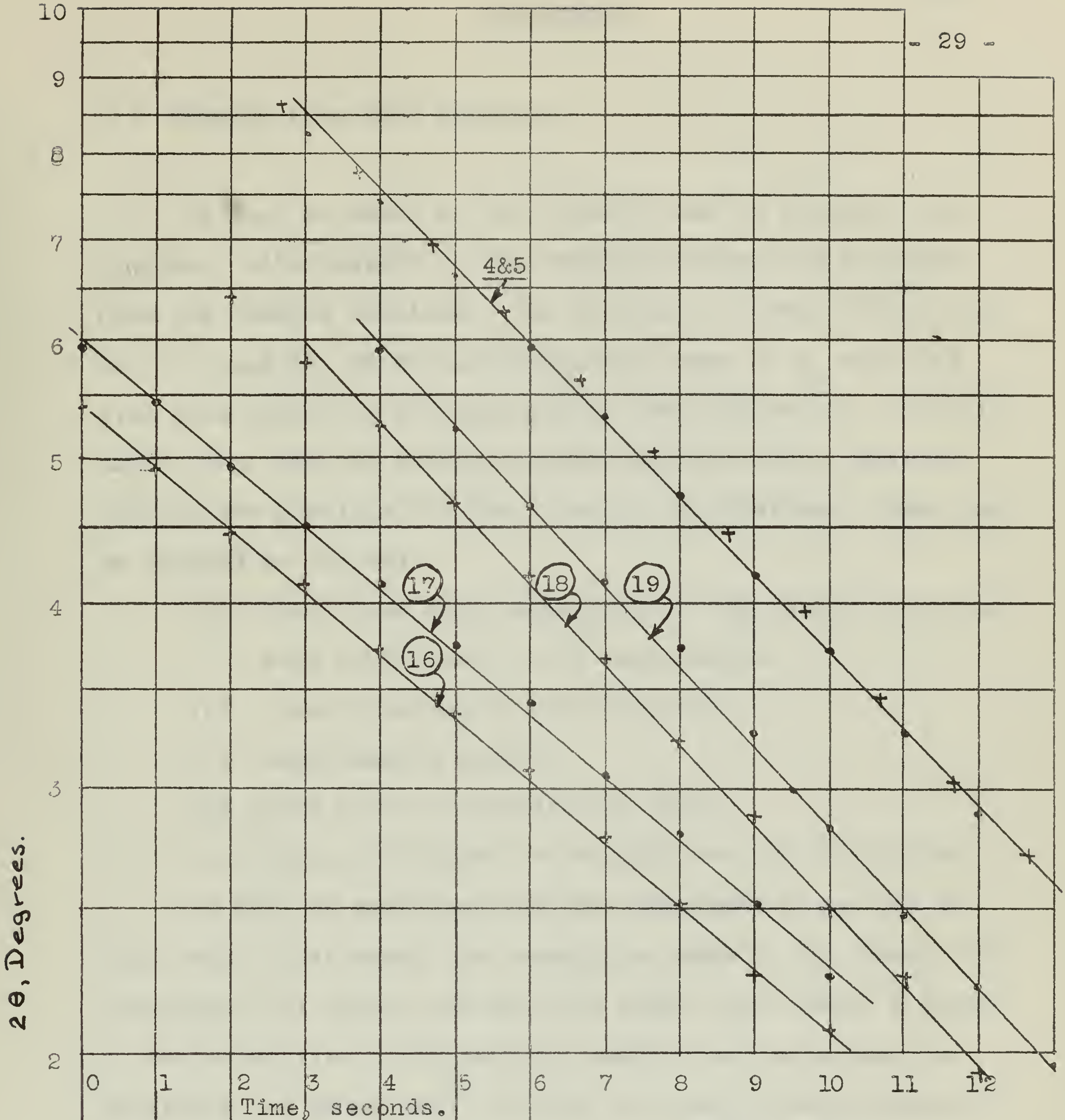
May, 1955



MODEL No. 7

Runs	T	GM
12,13	1.055	3.49
14,15	1.112	2.78

ABM May, 1955



MODEL No. 7

Runs.	T	GM
16,17	1.178	2.38
18,19	1.103	3.20
4, 5	1.119	2.69

DKM.

April 1955

DISCUSSION

(1) Results from Roll Records.

It must be noted at the outset, that no specific conclusions with respect to Dr. Ursell's theory can be drawn from the results obtained. The plot of k_1 versus B/H in Figure IV, page 16, shows no discernible trend of k_1 with B/H even when points at the same period are singled out. This is not to say that the theory has been proved wrong. Several reasons are possible for the lack of correlation. These may be classed as follows:

(a) Conditions and assumptions of the theory not properly duplicated in the experiments.

(b) Invalid method of data analysis.

(c) Experimental errors.

The above will be discussed in turn.

(a) Duplication of Theoretical Assumptions and Conditions.

It will be recalled from the Introduction section of this report that among the assumptions made by Dr. Ursell in developing his theory was that the model rolls about a point in the water-line. Although no observations were made to determine the actual roll axis in the tests, there appears to be little doubt that the rolling axis of the models varied from model to model, that in most cases the axis was not at the water-line, and further, that the axis changed from run to run for the same model as ballasting was changed.

The actual point about which a ship (or model) rolls is still a point of controversy among naval architects. Indeed,

many authorities believe that a true rolling axis does not exist because, it is contended, the instantaneous axis of rotation changes with roll angle. It is clear, however, that a ship must tend to roll about the centre of gravity, as it would if it were surrounded by a homogeneous fluid, and as does a submerged submarine, for instance. Most experiments which have been performed confirm this*.

On the other hand, consider a wall-sided ship of a form similar to that of the models used in these tests. As the ship rolls, the displacement will tend to remain constant. This tendency will want to make the ship roll about a point in the water-line. If the centre of gravity is not in the water-line, (as is usually the case), the ship will probably then tend to roll about a point between the water-line and the centre of gravity. In turn, this implies that the centre of gravity moves during the rolling. Suyehiro's experiments,* in fact, found a slight amount of heaving associated with the rolling. Professor Suyehiro assumed that this was caused by removal of the weight used initially to incline his model, and hence concluded that the model rolled about the centre of gravity. It seems entirely possible, however, that actual movement of the centre of gravity during the rolling caused the heaving motion. It appears logical to conclude, then, that a pure rolling motion can only be expected (for a wall-sided form) when the centre of gravity coincides with the water-line. For forms with sloping sides, it is clear that in the last

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* See for instance: Suyehiro, K., "On the Drift of Ships Caused by Rolling Among Waves," Transactions Institution of Naval Architects, Vol. LXVI, 1934, pages 60 to 67.

analysis, the centre of gravity must tend to move.

Considering the results as given in Table I, page 14, it may be noted that for most of the runs, the centre of gravity was below the water-line. Indeed, in some cases (as in Model No. I), the centre of gravity was as much as 50% of the draft below the water-line. The average distance, it will be noted, decreased with increasing beam-draft ratio.

Under these conditions, as noted above, it may be expected that the model will roll about a point somewhere below the water-line and above the centre of gravity. Hence, a heaving motion will probably be superposed on the pure rolling motion. Furthermore, there must be some lateral movement of the sides of the model in the vicinity of the water-line. Both of these motions will tend to produce waves in addition to those caused by the rolling motion, and hence to augment the energy dissipation.

These considerations explain to a large extent why k_1 in general decreased with increasing beam-draft ratio. In particular, the relatively low values of k_1 for Model No. 7 should be noted, for in this model's runs the centre of gravity was in most cases very near the water-line.

For a single model, however, the variation in k_1 cannot be explained solely on this basis, Although theoretically k_1 should be a function of shape alone, other experiments* have shown that holding other factors constant, (except position of roll axis), independent variations in period and in height

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* Serat, M., & Thews, J., "An Investigation of Some of the Factors Affecting the Rolling of Ships". U.S. Experimental Model Basin Report No. 348. February, 1933.

of the metacentre above the water-line also affect damping.

Although it would be possible to get a value of k_1 at the same period throughout the series, the above considerations indicate beyond doubt that the position of the roll axis would change from model to model, and in addition, throughout the series the height of the metacentre varies from about 2 ins. below to 2 ins. above the water-line.

Finally, it is possible that frictional effects were sufficiently large to affect the results. Further, the frictional effects probably changed from model to model in view of the changing draft.

It appears, then, that the lack of correlation is due to the effects of a large number of variables. Although it cannot absolutely be stated what the relative order of magnitude of the variables is, it seems likely that the damping is more sensetive to variations in the position of the roll axis than anything else.

Several different methods of plotting the results were tried in attempts to separate out the effects of the variables. All of these failed to produce a sensible picture, much less a correlation with the theory.

(b) Method of Data Analysis.

The validity of the method used in analyzing the data, i.e., the validity of the use of k_1 as a basis for comparison, depends on the relative order of magnitude of the frictional and eddy effects which vary with the square of the rolling velocity. There can be little doubt that the assumption of small

angles was met, since the maximum roll angle roll angle in the data does not exceed about 4°.

Two considerations tend to confirm the assumption that frictional and eddy effects were small. First a consideration of the data as presented in the semi-logarithmic plots, shows that with very few exceptions the damping was essentially linear. Secondly, it can be shown that a resistance to rolling varying with the square of the rolling velocity temds to make the motion non-isochronous*. In no case was a rolling record found in which the motion was discernibly non-isochronous.

It is concluded, then, that for the small range of roll angle used in the tests, the damping is for all practical purposes linear, and that hence the value of k_1 alone gives a valid basis for comparison.

(c) Experimental Errors.

It was observed during the experiments that the calibration of the gyro changed from time to time for no apparent reason. It is felt that this was probably due to small changes in the operating temperature of the gyro, since the gyro output is quite sensetive to temperature.

It is, however, completely unrealistic to assume that the gyro calibration changed during the short time required for a single run. Furthermore, the calibration affects only the absolute magnitude of the reported angles, so that k_1, which depends on slope, is independent of calibration. While the

* Robb, A. M., "Theory of Naval Architecture." page 262, Charles Griffin Co., London, 1952.

absolute value of the angles shown in the plots of θ versus time may then be subject to some unknown error, it is felt that the slope of the lines is accurate to the three figures given in the table of results. Furthermore, as will be noted, in nearly all cases two runs are plotted at the same model period and GM, which gives an added check on the slope. The four figures given for period may be optimistic; it is estimated that period is accurate within 0.003 sec.

All the above is not to say there are no "bad points" in the results. In particular, the variation of GM with period is inconsistent in one or two cases. With respect to GM, it is estimated that the values given (where not obviously in error as is the case for the first run listed for Model No. 1), are accurate only within 0.02 in., because of the fairly crude method used to read heel angle during the inclining experiments.

In view of these considerations, it must be concluded that the part played by experimental error in the lack of correlation is insignificant.

(2) Wave Records.

It has been noted in the introduction to the Results section of this report, that very little could be done with the wave data. Theoretically, a k_1 value is associated with the wave record which is the same as that of the rolling record insofar as frictional and eddy effects are small, and as the motion is a pure roll. It was found, however, that the wave records obtained were so irregular, that when plotted to semi-

logarithmic coordinates in an attempt to get k_1, the scatter was such that no mean line could be drawn through the points with any confidence. Some idea of the irregularity of the wave records may be had from Figure I, page 9, which shows one of the better records obtained.

Comparison on a basis of energy was also attempted by calculating the energy dissipated in waves from the amplitudes as read from the wave records. It was again found that the irregularity of the records made the results meaningless. In one case, in fact, the calculated energy dissipated in waves exceeded that lost by the model. This was probably as much due to an inaccurate gyro calibration constant as to anything else.

Several reasons may be advanced for the irregularity of the wave records. First, it is possible that the pickup was too close to the models, and that had it been further away, irregularities in the waves would have had a chance to smooth themselves out. During the initial stages of the project, the distance from the pickup to the models was varied, but little effect on the records resulted. Second, it is felt that the method used of measuring the waves was one which is disadvantageous for measurements of the small waves generated. The amplitude measured varied from less than 1 to a maximum of about 4 millimetres, while the instrument is intended normally for wave-height measurements of the order of inches. Hence, the transducer sensetivity* had to be made as high as possible with a resultant increase in noise to signal ratio. Further, the

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* See Appendix II, Part (2).

characteristics of the pickup wire are such that it is not readily suited to measurements of such small waves. Although a special coating on the wire is supposed to insure uniform wetting, it is probable that at the gain settings used, small residual surface tension effects were distorting the true wave picture. Finally, it is possible that waves other than those caused by the rolling interfered with the wave patterns. Such waves, although small, may have been generated in some cases when the model was released, because the trigger mechanism was such that a small force was required external to the model. This external force may have produced some pitching or yawing which could cause wave patterns out of phase with the rolling waves.

The wave records did, however, serve the purpose of confirming the relatively large amount of energy dissipated by a rolling ship in the form of waves. Indeed the orders of magnitude are such* probably well over 75% of the energy dissipation of the models can be attributed to wave-making.

(3) Instrumentation, Models and Experimental Set-up.

With respect to instrumentation, the drawbacks of the wave-height measuring system, for waves of the order of magnitude encountered, have already been pointed out. A system such such as the one used has probably reached the limit of its reliability and usefulness for amplitudes of about half a centimetre. The solution lies either in using larger models, which is hardly practical, or in a more sensetive measuring system.

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* See Appendix IV.

Probably, an optical, rather than electrical, method will ultimately be the answer.

Disadvantages of the roll recording system are its difficulty and questionable reliability of calibration, its limited linear response range of about 4°, and its drift due to the earth's rotation. Adams* has described a roll measuring system which appears to have considerable merit . This is essentially a transformer in which one winding is mounted in the model, and the other on a fixed frame encircling and external to the model in a horizontal plane. The fixed external winding is excited with alternating current, and as the model rolls its winding develops a voltage proportional to roll angle. Such a system could be designed easily and built quite cheaply. It would havethe advantages of no drift, a much higher measuring range, and simplicity. This system could also easily be adapted for use with a recorder such as the Sanborn Model 127. With respect to the latter, a recorder with controllable paper speed would present several advantages. Among these are the use of slow paper speed for calibration and for inclining experiments.

With respect to the model experimental set-up, as is noted in Appendix II, the method used to incline and release the models was somewhat awkward to use, and doubtless could be improved on quite easily. The method used to read angles during inclining experiments was crude, and should be improved in future work.

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* Adams, H. C., "Some Notes on the Use of Models in the Study of the Rolling of Ships." Society of Naval Architects and Marine Engineers, 1938.

Future experimentation on this subject should concentrate at first on reducing the number of variables. In particular, it would probably be wise at first to force the models to roll about a fixed point, say the water-line, and determine the effects of shape under this condition. Using such a method, the effects of independent variation in period, metacentric height, and height of the metacentre above the water-line, could also more easily be determined and separated from the effects of shape.

Having done this, the models could then be allowed to roll freely. The location of the rolling axis could be determined by a photographic method such as that used by Professor Suyehiro* . The separate effect of location of the rolling axis could then be evaluated.

* Suyehiro, op. cit.

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* Suyehiro, op. cit.

CONCLUSIONS.

(1) On the basis of the results obtained no broad conclusions with respect to Dr. Ursell's theory may be drawn. The lack of correlation is due to unknown interaction of variations in: position of centre of gravity, metacentric height, height of metacentre above water-line, and beam-draft ratio.

(2) The major part of the variation in damping is attributed to variations in the vertical position of the roll axis. The order of magnitude of the effect of this variable appears to be as important as that of shape (B/H).

(3) For small angles of heel, frictional and eddy effects are small, and linear damping may be assumed. Over 80% of energy dissipation is due to wave-making.

RECOMMENDATIONS.

Further Experiments:

(1) Conduct tests with fixed rolling axis to determine effects of shape, and variation of GM and period.

(2) Conduct free rolling tests tp determine the location of the rolling axis. The effects on the position of the roll axis of height of the centre of gravity, metacentre, centre of buoyancy, and of draft should be determined.

(3) Conduct free rolling tests to determine the effect of the location of the roll axis on roll damping.

Improvements in Instrumentation:

(1) A method of measuring roll such as that used by Adams* would appear to present several advantages over the gyro system.

(2) The wave-height recorder at the M.I.T. Towing Tank is not well suited to the measurement of small waves. The possibility of using an optical method should be explored.

(3) The model inclining and releasing arrangement used was somewhat crude and awkward to use. It is felt that the mechanism is basically good, but that refinements can be made.

* Adams, op. cit.

APPENDICES.

APPENDIX I.

MODELS.

Requirements of the models were determined from considerations involved in Dr. Ursell's theoretical development and from practical matters. The assumptions on which the theory was based were outlined in the introduction, but are repeated here for convenience:

(a) Zero viscosity.

(b) Length of waves produced by the rolling is large compared with the beam.

(c) The ship (model) rolls about a point in the waterline.

(d) The rolling is isochronous.

(e) Angles of roll are small.

(f) Two-dimensional (fluid) motion.

(g) The position of the roll axis is independent of the rolling frequency.

(h) The cross-section of the ship (model) is defined by equations (1) and (2).

The length of the models was limited by the width of the tank (8 ft.), but assumption (f) dictates as large a length as possible. This assumption also indicates a fairly liberal length-beam ratio; a value of 7 was felt desirable. In addition, it is indicated in Dr. Ursell's paper that a ship may be expected to follow the two-dimensional theory if the length of its parallel middle body is longer than the length of the waves produced.

In order to get a well defined minimum in a curve of damping versus beam-draft ratio, the models were required to encom-

pass a range of B/H including the critical 2.52, and values substantially greater and less than this. The final requirement was that the models were to be of the same cross-section throughout the length, the section to be defined by the parametric equations (1) and (2).

Based on the above outlined considerations, a length of 7 ft. and a beam of 12 inches were adopted for the control model of beam-draft ratio 2.52, It was decided to use six other models of comparable size, and another model geometrically similar to the first, but two-thirds its size to give an indication of scale effects. The characteristics of the models are given in Table II page 46 . The sections, plotted from offsets calculated from equations (1) and (2), are shown in Figure V page 47 Integration of the offset equations gave the following formula for cross-sectional area below the water-line:

$$A_x = \frac{\pi}{256} \left[39(a+b)^2 - 35(a-b)^2 \right] \qquad (14)$$

The method of construction is indicated in the sketch in Figure V . The models were made from well dried Eastern Pine. A freeboard of about 4 inches was assigned, and platforms were provided for vertical movement of weights during ballasting.

It is possibly of interest to mention, finally, that in the early stages of the project considerable difficulty was experienced with leaks in the models. Additional time spent in the construction stages to insure good joint fits, and the use of caulking compound in all seams exposed to water would have prevented most of these troubles.

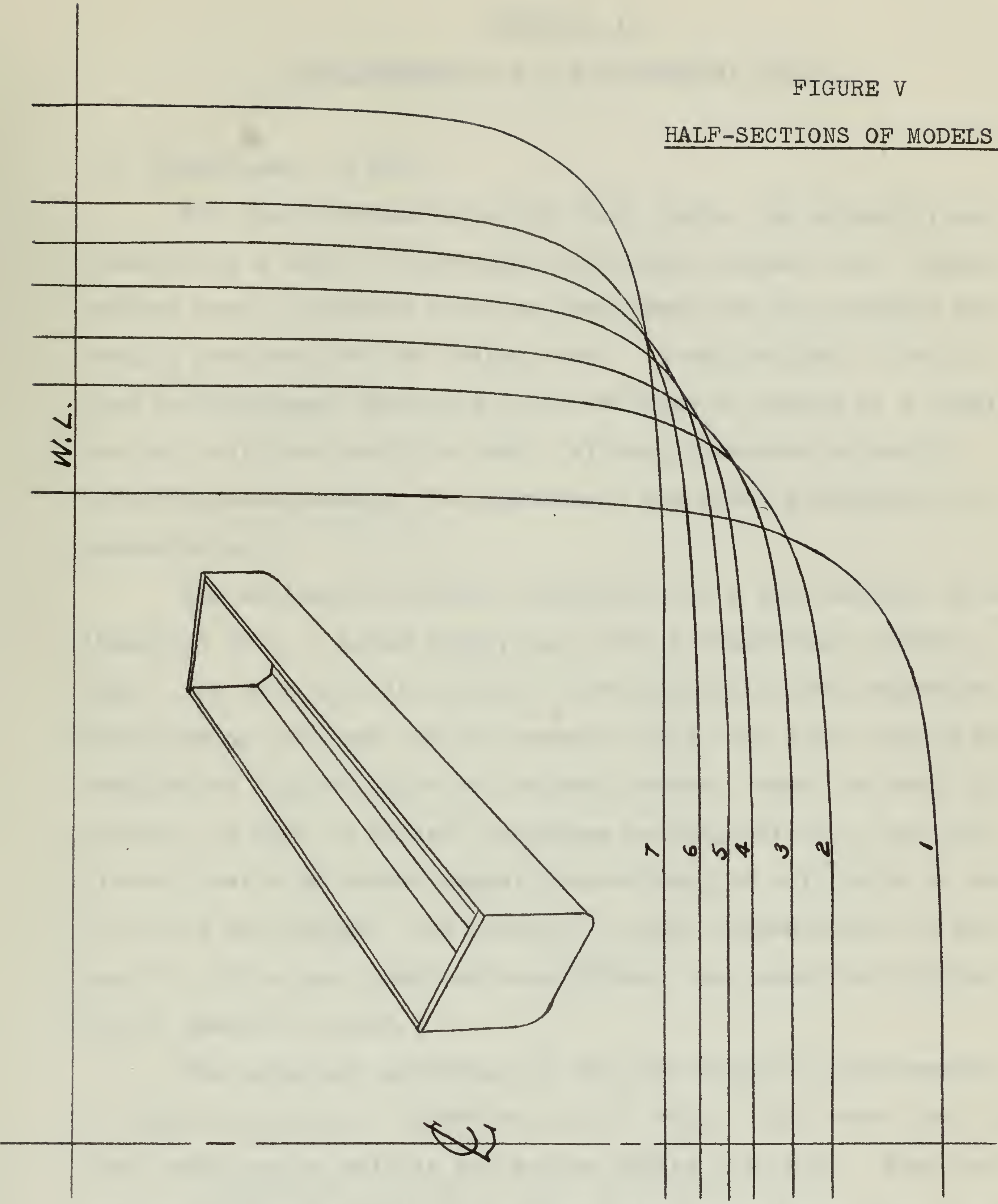
TABLE II

MODEL CHARACTERISTICS.

Model No.	1	2	3	4	5	6	7	8
B, ins.	9.10	10.62	11.30	12.00	12.56	13.15	14.52	8.00
H, ins.	6.07	5.31	5.02	4.76	4.57	4.38	4.15	3.17
B/H	1.50	2.00	2.25	2.52	2.75	3.00	3.50	2.52
L, ins.	84	84	84	84	84	84	84	56
L/B	9.24	7.92	7.43	7.00	6.69	6.39	5.78	7.00
A_x, ins^2.	53.00	53.99	54.38	54.76	55.04	55.39	58.15	24.34
Δ, lbs.	160.8	163.8	164.9	166.1	166.9	168.0	176.4	49.2
KB, ins.	3.16	2.78	2.62	2.49	2.38	2.28	2.16	1.66
KM, ins.	4.35	4.71	4.83	5.12	5.38	5.70	6.54	3.42

FIGURE V

HALF-SECTIONS OF MODELS.



APPENDIX II.

INSTRUMENTATION AND EXPERIMENTAL SET-UP.

(1) Measurement of Roll.

For the instrumentation of these tests, the primary requirement was a means of measuring roll angle versus time. Several methods were considered since no instrument for this purpose was readily available at the Towing Tank. It was decided, finally, that the equipment developed by Lt. William R. Porter as a thesis project this year would be used. Although intended primarily for pitch measurements, the instrument was readily adaptable to measuring roll.

The equipment consists essentially of a gyro mounted in an insulated case, a power supply unit, and a temperature control unit. The gyro unit is mounted in the model with the sensetive axis running fore and aft (to measure roll); the power supply and temperature control units are mounted ashore. When the model is heeled, the gyro in effect integrates rolling velocity, and the system gives a dc output signal proportional to roll angle of about 0.7 volts per degree. The circuit is shown schematically in Figure VI ; for a more detailed description, the reader is referred to Lt. Porter's thesis.*

The principal advantage of the gyro method of measurement is that the system is sensetive only to roll. This means that the model need not be held in any manner during the tests. Many methods used to measure roll in the past have necessitated that the

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* Porter, W., LT., USN, Nav. Eng. Thesis, 1955.

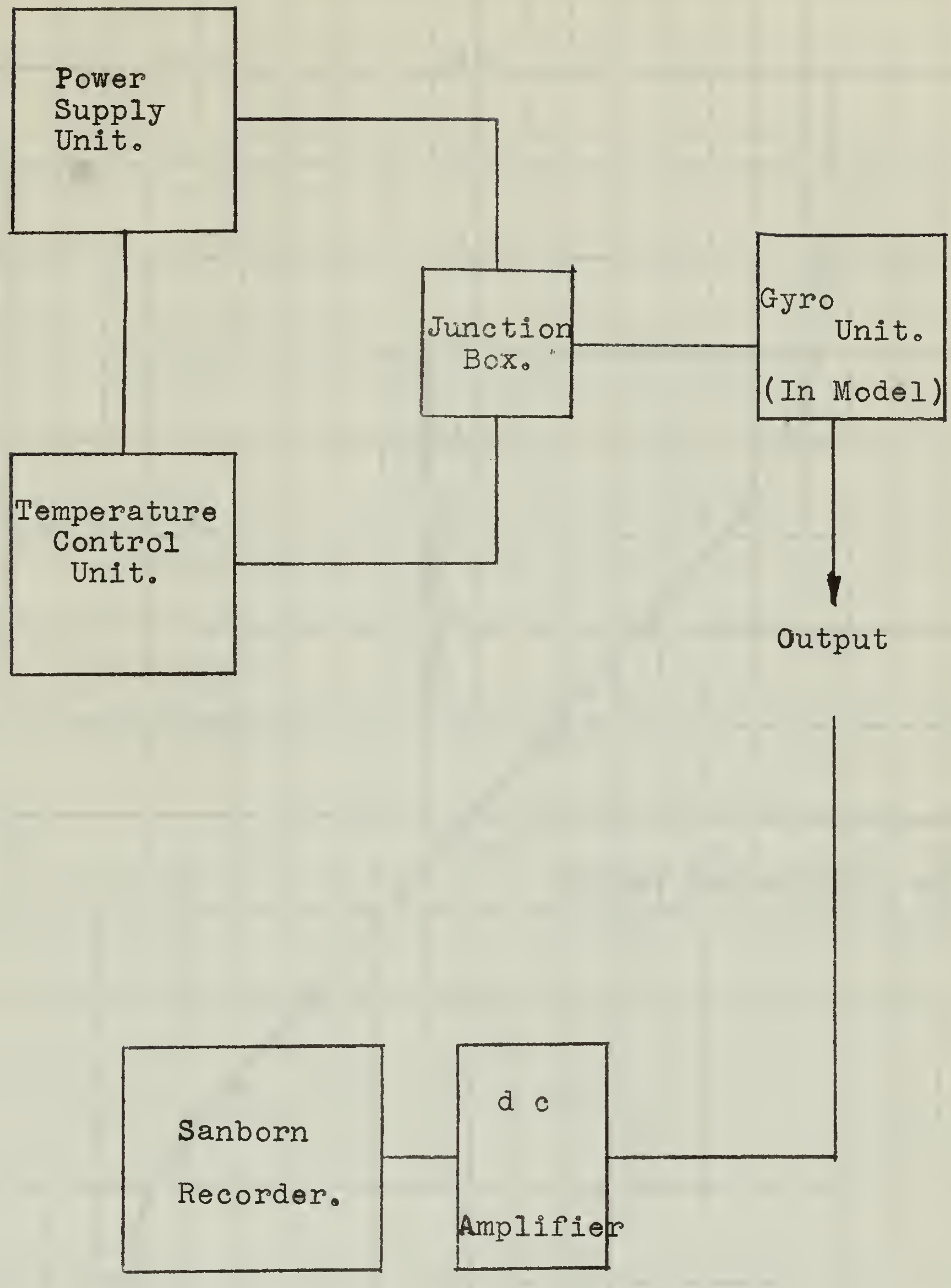


FIGURE VI

SCHEMATIC DIAGRAM OF ROLL RECORDING SYSTEM

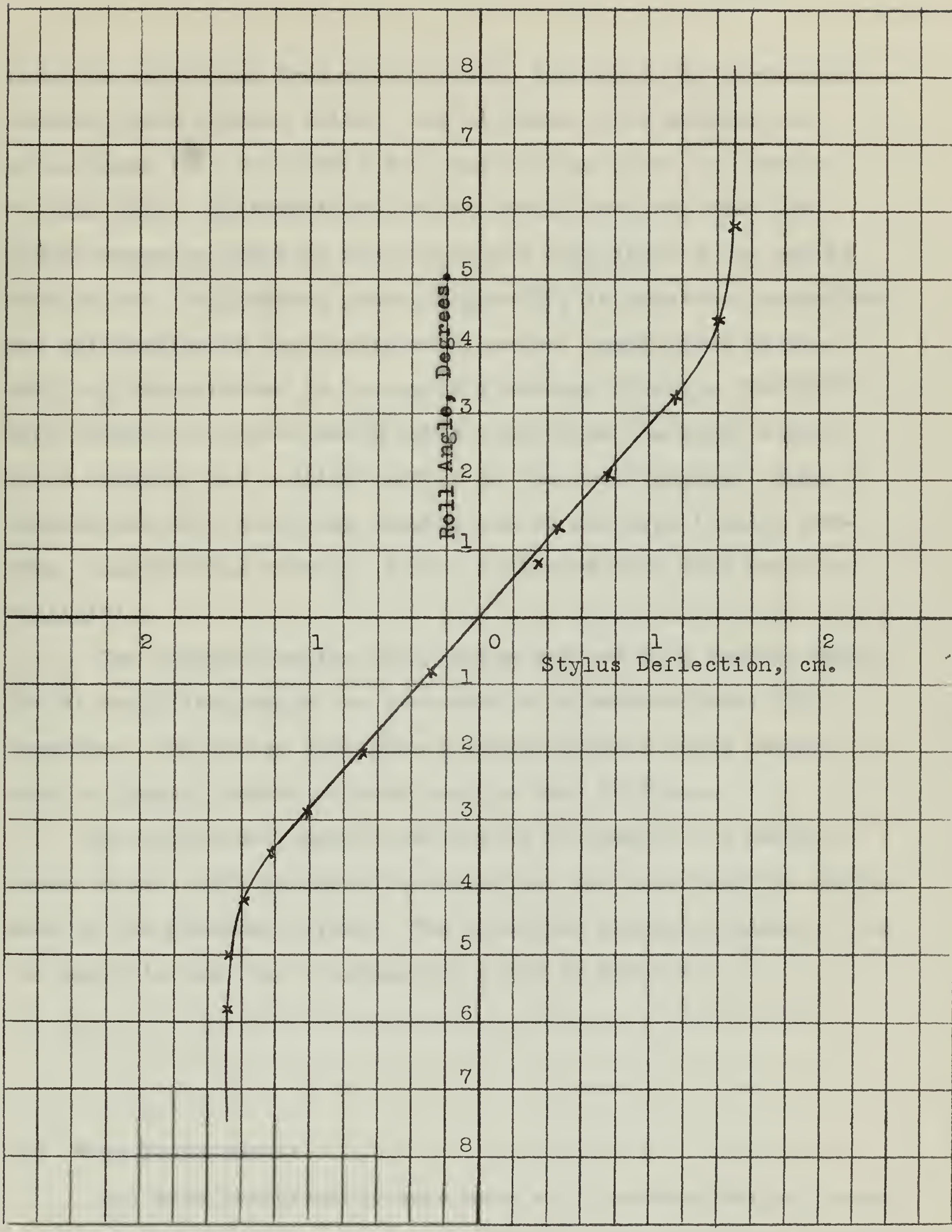


Figure VII GYRO CALIBRATION CURVE. April 1955

model be restrained from motions other than roll, in particular, usually, from lateral drift. Use of these other methods has often meant that the models were not rolling about the natural rolling axis. Disadvantages of the method used are that the linear response limit of the system was only about 4° as may be seen in the calibration curve, Figure VII; in practice, operation and calibration of the system are somewhat complicated by the drift of the gyro due to the earth's motion; finally, the gyro unit requires a rather heavy cable going from the model ashore which probably has a slight effect on the roll damping. Some concern was felt about the damping due to the gyro itself; however, consultation with Lt. Porter indicated that this would be negligible.

The output from the gyro system was fed to a Sanborn Model 126 dc amplifier, which was connected to a Sanborn Model 127 Recorder. The latter then gave a record of roll angle versus time, a typical sample of which may be seen in Figure I .

The system was readily calibrated by heeling the model a known amount, with the gyro in operation, and observing the deflection of the recorder stylus. The principal characteristics of the dc amplifier and the recorder are given in Table III.

(2) Wave Measurements.

All wave measurements were made with the Wave Height Recorder recently installed in the Towing Tank. This instrument consists

essentially of a pickup connected to a Sanborn Model 140 Strain Gage Amplifier, which in turn is connected to a Sanborn Model 127 Recorder.

The pickup, which is illustrated in Figure VIII consists of an aluminum frame on which is mounted an insulated wire. As the water level at the frame changes, the capacitance between the wire and the frame changes in proportion to the change in water level. The output leads from the pickup are connected to the Strain Gage Amplifier as shown in the schematic diagram, Figure IX , page 54 .

Figure IX also illustrates the main features of the Strain Gage Amplifier. The instrument consists essentially of a transducer bridge circuit which is excited by a 2500 cycle oscillator. Output from the bridge is connected to a carrier amplifier stage through a matching transformer and step attenuator. The output from the carrier amplifier stage is fed through mixer and demodulator stages to a final differential output amplifier stage, the output of which is a dc signal proportional to unbalance in the transducer. Referring to Figure IX , the elements R_1 and R_2 of the bridge are fixed resistors built into the instrument, while provision is made for filling the arms R_3 and R_4 externally.

As used in the wave-height recording system, the arms R_3 and R_4 of the transducer are fixed capacitors of equal value, and the wave-height pickup is connected across one of these, as shown in Figure IX .

Controls are provided on the instrument for balancing the bridge, varying the gain of the system, for fixing zero (position of the stylus on the recorder connected to the Strain Gage Amplifier), and for calibration. The latter control is a switch which

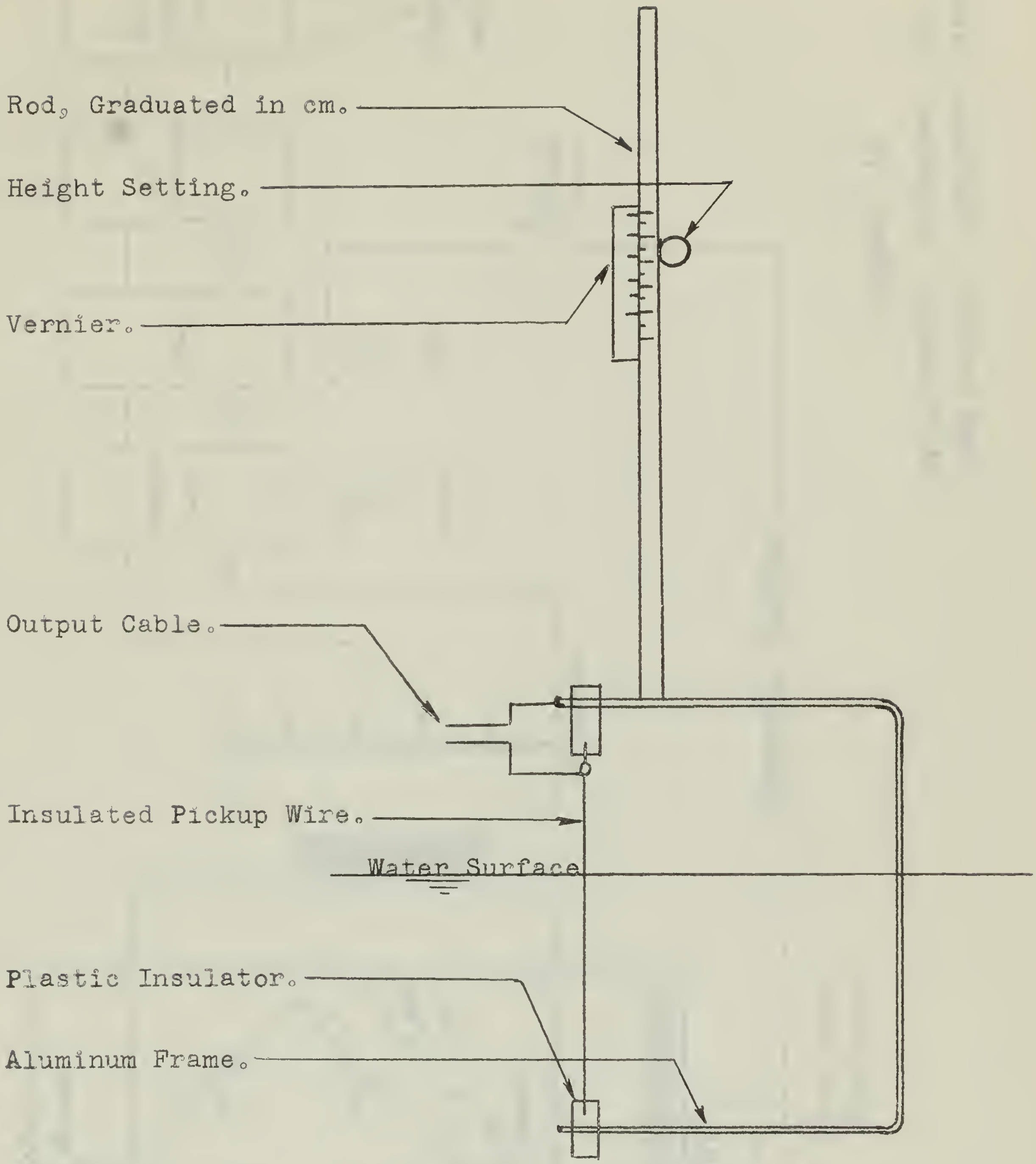


FIGURE VIII

SCHEMATIC DIAGRAM OF
WAVE-HEIGHT PICKUP

Not to Scale. April, 1955

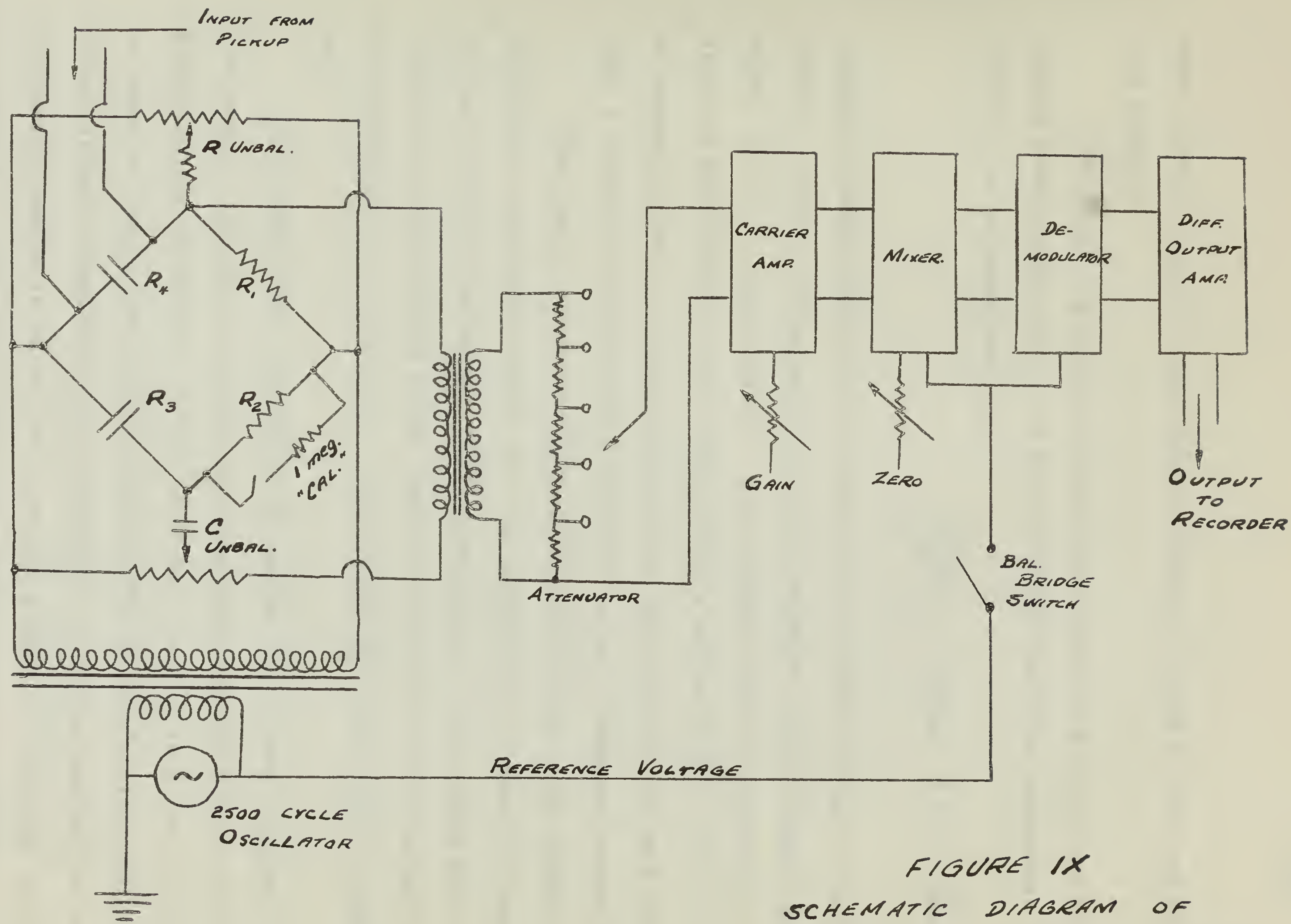


FIGURE IX

SCHEMATIC DIAGRAM OF WAVE-HEIGHT MASURING SYSTEM.

when closed creates a resistance change of 0.01% in one arm of the bridge.

Denoting the value of the fixed capacitors by C_f and the pickup capacitance by C_p, it may be shown that the transducer sensetivity is proportional to:

$$\frac{\Delta C_p}{C_p + C_f}$$, where ΔC_p is the

change in capacitance of the pickup (due to some change in water level). Hence, the sensetivity will increase, other things being equal, as C_f is made smaller. For ordinary work in connection with the wave-maker at the Towing Tank a value of 0.1 mfd for C_f is commonly used. For the purposes of this project, it was found that a value of 0.02 mfd was necessary to give adequate sensetivity.

The operation of the system was as follows. The pickup was adjusted to operating depth, and the bridge was balanced by means of the controls provided*. The sensetivity of the system was then adjusted to the desired value, and the stylus on the recorder set at the desired zero position. The system was then ready to operate.

The system was calibrated by moving the pickup a known vertical amount through the water and observing the deflection of the stylus on the recorder. Rather than do this for each run, a plot was made of stylus deflection when the "Calibrate" switch is closed versus system sensetivity in centimetres of stylus deflection per centimetre change in water level at the pickup. This plot is shown in Figure X , page 56. Just before each run was started, the

* The details of balancing are given in the instrument's instruction book, and need not be mentioned here.

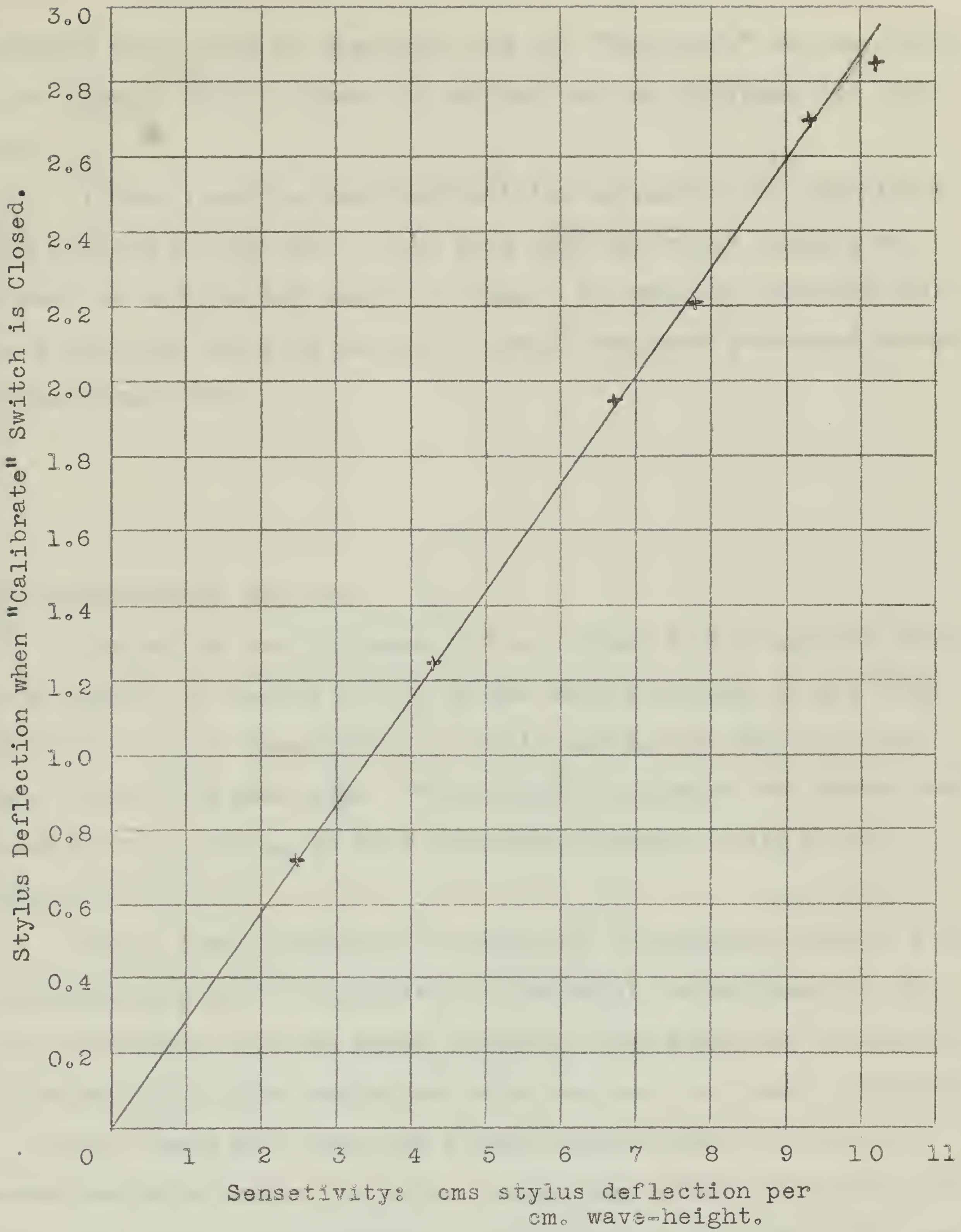


FIGURE X WAVE-HEIGHT RECORDER CALIBRATION. DBM. April '55

recorder was placed in operation and the "Calibrate" switch closed a few times. Thus, a check on calibration was obtained for every run.

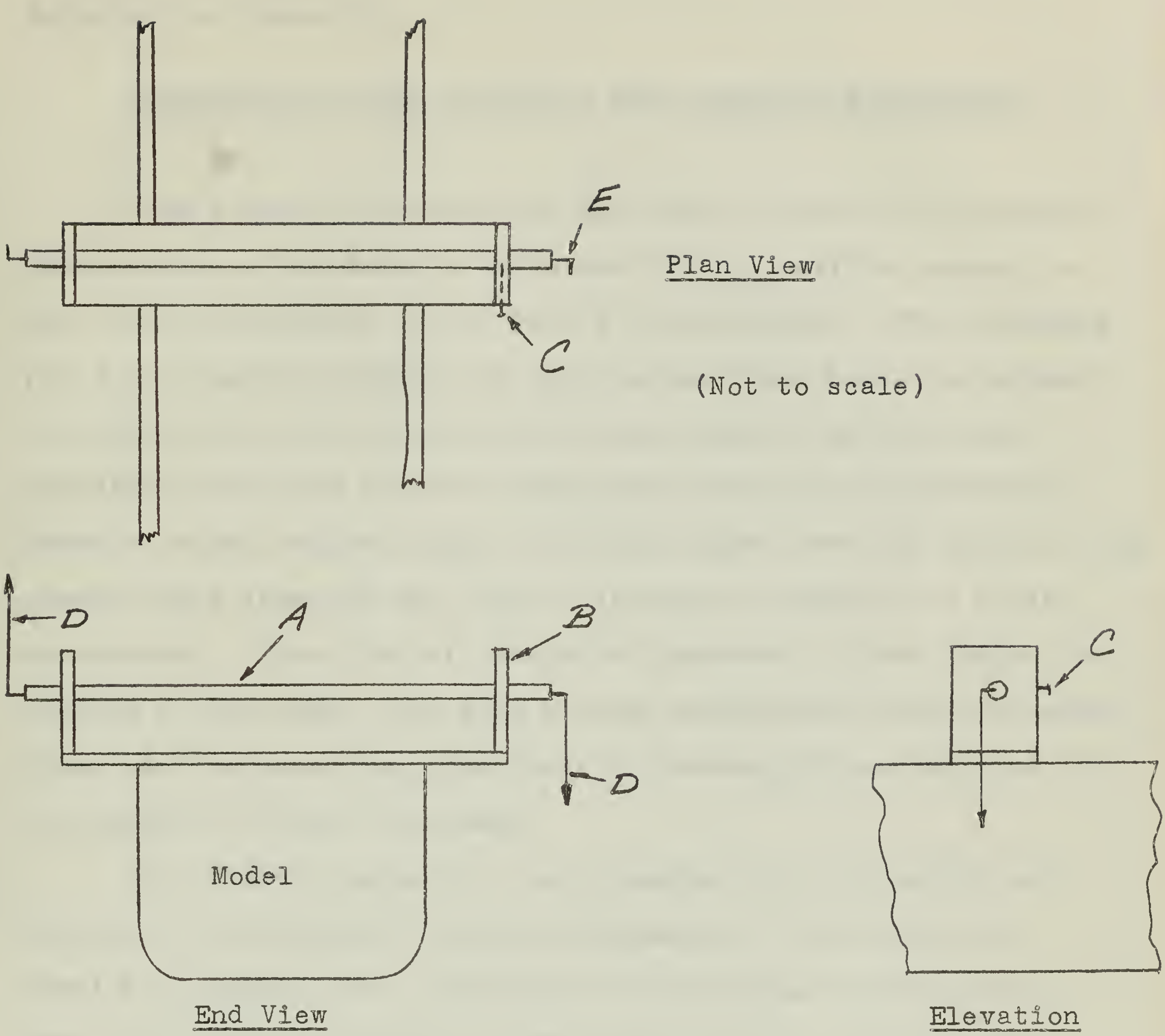
It was found in practice that the properties of the pickup wire changed if the part of the wire near the water surface was allowed to dry for any length of time. In order to overcome this the portion of the wire near the surface was kept submerged except during actual runs.

(3) Experimental Set-up.

During the early stages of the project it was noticed that in attempting to impart a roll to the models by hand it was very difficult, if not impossible, to avoid setting up motions other than roll at the same time. This tended to distort the waves produced by the rolling, so that consistent results could not be obtained.

Thus it was necessary to devise an arrangement whereby a pure heeling moment could be applied to the model, maintained for the period necessary for the water to settle, and released, virtually instantaneously. The mechanism which was used is shown in Figure XI.

It was found that this was rather inconvenient to use, but worked satisfactorily. In order to get approximately the same initial roll angle on each model it was necessary to change the heeling weights from model to model. Another disadvantage was in the trigger mechanism, the operation of which required a small external



A Dowel, with pin E, shaped as shown, in each end.

B Frame supporting bearings for dowel A.

C Releasing pin.

D Vertical inclining forces.

For explanation and method of operation see next page.

FIGURE XI

MODEL INCLINING AND RELEASING

ARRANGEMENT

4/55.

Addendum to Figure XI.

OPERATION OF MODEL INCLINING AND RELEASING ARRANGEMENT.

The frame B is mounted on the model as shown, at the midship section. The dowel A is rotated to the position shown, so that the bent portion of the pins E is horizontal. The releasing pin C is inserted through the hole in the frame B and corresponding hole in the dowel, thus preventing rotation of the dowel. Oppositely directed vertical inclining forces D are exerted by means of equal weights hung on strings looped over the pins E. The upward force shown on one side is attained by means of a pulley arrangement. Thus, for all practical purposes, a pure torque is exerted on the model, the pins E being equidistant from the centre-line. At the same time, the force of the weights on the pins E is tending to rotate the dowel.

To release the model, the releasing pin C is pulled out by means of a triggered spring arrangement. This allows the dowel A to rotate, thus releasing the inclining forces at the same time. An arrangement is provided to catch the inclining weights to prevent disturbance of the water surface.

force which may have imparted some motion other than roll to the model.

An inclining experiment was used to determine GM. For this purpose, a narrow platform, on which a scale in inches was marked, was placed across the middle of the models. The inclining weight was moved on this platform.

Aluminum end plates about 1/8 inch thick and 2 feet square were fitted on the models as shown in Figure III in order to suppress end effects as much as possible. A scale of degrees was marked on one of the plates, and a clinometer arranged by means of a weighted string. This was used in inclining the models and calibrating the gyro.

TABLE III

LIST OF COMMERCIAL INSTRUMENTS USED.

Recorders. Sanborn Model 127. Roll Recorder: No. 396.

Wave-ht. " No. 935.

Galvanometer: Conventional D'Arsonval moving coil type.

Paper Drive: Constant speed, 2.5 cm/sec.

Sensetivity: 10 ma/cm. stylus deflection.

Maximum Undistorted Deflection: 2.5 cm. from centre.

Frequency Response: Linear to 20 cps. with Model 126 dc Amplifier or Model 140 Strain Gage Amplifier.

Controls: Paper Drive On-Off.

Stylus Temperature.

dc Amplifier. Sanborn Model 126, No. 262.

3 dc push-pull stages.

Overall gain: 600.

Input Impedance: 5 megohms.

Internal Output Impedance: 500 ohms.

Controls: Sensetivity,
Stylus centering.
Attenuator.

Strain Gage Amplifier. Sanborn Model 140, No. 342.

See figure IX , page 54, for schematic diagram.

Sensetivity: 10 ma (= 1 cm. on recorder) for ΔR of 100 ppm. in one arm of bridge.

Output Impedance: 500 ohms.

Controls: Calibrate,
Attenuator,
Gain,
Zero Control.
Resistive Balance,
Capacitive Balance,
Coarse-Fine Switch (for balancing)

APPENDIX III

SAMPLE CALCULATION.

Model No. 3. Runs 11 and 12.

T = 1.181 from roll record.

GM = 1.33" from inclining experiment.

Δ = 164.9 lbs.

Referring to the plot of θ versus time for these runs, on page 21 :

at $\theta = 4°$, $t = 1.60$ sec.

at $\theta = 2°$ $t = 7.75$ "

$\Delta t = 6.15$ secs.

From equation (12):

$$k_1 = \frac{1}{\Delta t} \cdot \frac{T}{2} \log_e \frac{\theta_0}{\theta}$$

$$= \frac{1}{6.15} \times \frac{1.181}{2} \times \log_e 2$$

$$= \underline{0.0692}$$

From equation (13):

$$A = \frac{\Delta \, GM \; T \; k_1}{\pi^2}$$

$$= \frac{164.9 \times 1.33 \times 1.181 \times 0.0692}{\pi^2}$$

$$= \underline{1.80 \text{ in-lb-sec.}}$$

APPENDIX IV

Calculation of the wave amplitude required to dissipate all the energy lost in a single roll for a typical run.

Take Model No. 7, Runs 12 & 13. (see page 28).

$T = 1.055$ sec.
$GM = 3.49''$

at $t = 1$, $\theta = 5.42°$
" $t = 1.055$ $\theta = 4.72°$

$\Delta\theta = 0.70°$

$\theta_m = 5.07°$

Energy lost by Model:

$$E = \frac{\Delta}{L} GM\, \theta_m \cdot \Delta\theta$$

$$= \frac{176.4}{7 \times 12} \times 3.49 \times \frac{5.07}{57.3} \times \frac{0.70}{57.3}$$

$$= 0.0795 \quad \text{in.-lbs/in.}$$

Energy in Wave:

$$E = \frac{\rho a_w^2 \lambda}{8} \qquad \lambda = 5.14\,T^2$$

$$= \frac{62.4}{8 \times 12} \times 5.14\,(1.055)^2\, a_w^2 = 0.0795$$

$\therefore \quad a_w^2 = 0.0213\ ; \quad a_w = 0.146$ in. $=$ **3.71 mm.**

APPENDIX V.

NOTATION.

a	half-beam.
a_w	wave amplitude.
A	linear damping constant.
A_x	cross-sectional area.
b	draft.
B	beam; damping constant: coefficient of $(velocity)^2$ term.
GM	metacentric height.
H	draft.
J	Moment of inertia of ship or model about roll axis.
k_1	Ship's linear damping constant.
KG	height of centre of gravity.
KM	height of metacentre.
L	length.
T	period.
Δ	displacement.
η	angular parameter.
θ	roll angle.
θ_0	maximum roll amplitude.
λ	wave length.

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The effect of beam draft ratio on roll damping.