*May* 5. 1703.

At a MEETING
OF THE

*ROYAL SOCIETY*,

*Sir* JOHN HOSKYNS, V. P. *in the Chair*,

*Mr.* LOWTHORP *Presented a* Proposal *for Printing an* Abridgment *of the* Philosophical Transactions. *This* Design *was Approv'd by the* Society, *and He was Desir'd to* Proceed *therein.*

---

*May* 12. 1705.

**Imprimatur,**

*Is. Newton*, R. S. Pr.

# THE PHILOSOPHICAL TRANSACTIONS AND COLLECTIONS,

To the End of the Year 1700.

# ABRIDG'D

AND

Dispos'd under GENERAL HEADS.

---

## In Three Volumes.

---

By *JOHN LOWTHORP*, M. A.
and F. R. S.

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*LONDON:*

Printed for *Thomas Bennet* at the *Half-Moon*, *Robert Knaplock* at the *Angel* and *Crown*, and *Richard Wilkin* at the *King's-Head*, in St. *Paul's* Church-yard, MDCCV.


I MEIN WELL
Sr. John Schaw of Greenoak
Baronet

TO HIS

# Royal Highneſs

THE

# PRINCE,

## Lord High ADMIRAL

OF

# ENGLAND, &c.

SIR,

YOUR Royal Highneſs's *Great Condeſcenſion to Subſcribe the Statutes of the* Royal Society, *as one of their* Fellows, *Commands ſome Tribute from every*

*every Member. This,* SIR, *I humbly hope, will in ſome meaſure excuſe the Preſumption of laying theſe* Papers *at Your* Royal Highneſs's *Feet. They have unavoidably, in this Abridgment, loſt much of their Original Beauty: But the* Order *wherein the Remaining Subſtance is Diſpoſed, will give Your* Royal Highneſs *a Nearer Proſpect of the Courſe of thoſe Studies you have the Goodneſs to* Protect.

*It was a Noble Deſign, Worthy of their* Royal Founder, *by* Incorporating *this* Society *to Perpetuate a* Succeſſion *of* Uſeful Inventions: *But the Diſcouraging Neglect of the Great, the Impetuous Contradictions of the Ignorant, and the Reproaches of the Unreaſonable, have unhappily Retarded them in their Purſuit of thoſe Great Ends. To* Reſtore *them therefore to their firſt Vigour, is a Glory reſerv'd for Your*

Royal

Royal Highneſs; *And already*, SIR, *we feel the Chearful Influence of a Returning Spring.*

*The Commands Your* Royal Highneſs *has given, for Publiſhing, at Your own Expence, a moſt Magnificent* Uranography (*far exceeding that of all the* Arabian Princes, *the Noble* Tycho Brahe, *and the Induſtrious* Hevelius) *cannot fail of ſurprizing Effects. All* Art *and* Nature *will exert their Powers upon this Occaſion, to keep pace with* Aſtronomy; *particularly* Navigation, (*being under Your* Royal Highneſs's *Immediate* Care) *will Induſtriouſly apply thoſe* Accurate Obſervations *to all* Nautical *Purpoſes, and by ſome Familiar Method, Deliver the Anxious* Seamen *from the* Fatal Accidents *that frequently attend their* Miſtaken Longitude.

*Thus,*

*Thus*, Sir, *the* Munificence *of the* Prince, *and the* Vigilance *of the* Lord High Admiral, *will be equally a Blessing to the Present, and to all Future Ages. May Your* Royal Highness *ever meet with Returns of Gratitude, Suitable to the Universal* Beneficence. *So Wishes, with great Sincerity,*

*May it please Your* Royal Highness,

*Your* Royal Highness's

*Most* Faithful, *and*

*Most* Obedient Servant.

*John Lowthorp.*

# THE PREFACE.

THE *Philosophical Transactions* having met with General Applause and Encouragement for many Years, it would be a Needless Trouble to give any *History* of them: 'Tis enough to say, That Many of the Discourses were Compos'd, and All of them Collected and Publish'd, by *Particular Members* of the *Royal Society*. I shall therefore employ these very few Pages only to acquaint the Reader with My Own Conduct in this *Abridgment* of them.

When I first Resolv'd upon this Undertaking, I had Two Sorts of Readers in View, whom I was desirous to serve; those who make use of Books for their Private Instruction or Entertainment, and those who Consult them in order to Publish something of Their Own. To a Reader of the Former Class, I thought it sufficient to give him the Substance of so many Curious Papers, in such Order as would best Suit with the Course of those Studies that might Denominate him a *General Scholar*: But for the sake of the Latter, I have, in the Margin, given the Title and Author of each Paper,

 and

and Directed to the *Number* and *Page* of the *Transactions* or *Collections*, where he may meet with the Original it ſelf. To the former I deſign'd this *Abridgment* to be as Uſeful as the Volumes at large, and to ſerve the Latter inſtead of a not inconvenient *Repertorium* : And in the Proſecution of this Deſign, I have Generally confined my ſelf to theſe *Rules*.

I. I have not only *Retain'd* the Eſſential Parts of the Diſcourſes, but I have kept in many Places to the very *Words* of their own Authors, (except where I was forc'd to Vary them a little, to preſerve the Connection:) For I thought it very Unwarrantable to Obtrude any thing of mine, under the Name of another Perſon.

II. But to *Shorten* the Whole Work, wherever I found any *Perſonal Addreſſes*, Long and Unneceſſary *Excurſions*, or Pompous *Citations* of Books, I have taken the Liberty to *Suppreſs* them ; yet, I hope, without Injuring the Force of the Author's Reaſoning.

III. I have *Omitted* all *Accounts* and *Extracts* of *Books*, which now, after ſo many Years Publication, ſeem almoſt Uſeleſs : Yet to put the Readers in mind of them, eſpecially ſuch as are about to Furniſh or Enlarge their Libraries, I have added a *Catalogue* at the End of each Chapter, to which they chiefly belong ; And I have alſo Directed them to ſuch *Additions*, *Emendations*, or *Refutations*, as ought to be conſulted, when thoſe Books fall under their Examination.

IV. I

IV. I have also *Omitted* all *Heads* of *Enquiries*, and Experiments simply Propos'd, without further Prosecution; Believing that the Answers already given to many of them, and other Discourses upon the same, or like Subjects, will sufficiently Direct the Notice of an Inquisitive Reader.

V. The Previous Calculations of *Eclipses*, *Lunar Appulses*, and *Satellite Eclipses* and *Occultations*; also *Tide-Tables*, and many other Curious Papers of that kind; have long ago Out-liv'd the Reason of their Publication,

VI. All Simple *Catalogues* of *Natural Curiosities*, (as of *Shells*, *Minerals*, *Plants*, *Animals*, &c.) without particular Descriptions of them, are little Instructive: And Chiefly serve to enlarge the History of the *Musæum* where they are Deposited; which is no part of the Design of these Volumes.

VII. I have commonly *Omitted* such Papers as have been *Collected* into Just Volumes by their own Authors. For this Reason I have *Omitted* some of those Surprising *Microscopical Discoveries* by the Famous M. *Leeuwenhoeck*: But I further confess, I was also less inclin'd to Insert them here, because most of them Treat of Subjects not at all Convenient (in my Opinion) for Common Readers.

VIII. But to do all the Right I could to the Ingenious Authors of those Papers, which the Limits of this

*Abridgment*

*Abridgment* oblig'd me to *Omit*, I have at the End of each Chapter Annexed their *Titles*, and sometimes a short *Account* of them.

These are the *Rules* I have carefully Observed through the whole Conduct of this Work; Wherein I have Faithfully Aim'd at the *General Good* of all Sorts of Readers. If I have fail'd in the Performance, 'tis for want of Judgment to do it better: But I am bold to say, That if a Kind Reception of this shall encourage a like *Abridgment* of the *Foreign Philosophical Journals*, in the Same Order, it will much Facilitate the many *Discoveries* still Ready to Reward the *Labours* and *Expences* of all Industrious *Promoters* of *Natural Knowledge*.

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A N

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# THE PHILOSOPHICAL TRANSACTIONS AND COLLECTIONS,

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## VOL. I.

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Containing all the

*Mathematical* Papers.

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By *JOHN LOWTHORP*, M. A.
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*LONDON:*
Printed for *Thomas Bennet* at the *Half-Moon*, *Robert Knaplock* at the *Angel* and *Crown*, and *Richard Wilkin* at the *King's-Head*, in St. *Paul's* Church-yard, MDCCV.

n

To the HONOURABLE

Sir *ISAAC NEWTON* Kt.

PRESIDENT,

And to the

COUNCIL and FELLOWS

OF THE

# Royal Society

OF

# LONDON,

FOR THE

Advancement of *Natural Knowledge*,

THESE

*Mathematical* Papers,

ABRIDG'D

AND

Diſpos'd under GENERAL HEADS,

Are Moſt Humbly Dedicated,

By *John Lowthorp*.

THE

# Mathematical Papers,

PUBLISH'D and DISPERS'D

IN THE

PHILOSOPHICAL

## Tranſactions and Collections,

ABRIDG'D;

AND

Diſpos'd under GENERAL HEADS.

## CHAP. I.

### *Geometry, Algebra, Arithmetick, Logarithmotechny.*

I. 1. QUÆ antehac de Augmentis Artium Mathematicarum executus ſum, eorum Summa huc ferè redit; Quamdiu homines Voluntate, Ingenio, Adminiculis, & Otio ad hæc Studia requiſitis deſtituantur, mirum non eſſe, ſi non majores in iis progreſſus faciant: Mihi itaq; videri, ſequentium mediorum ope, Remedium huic Malo ſatis commodè afferri poſſe. Nimirum ſi, *An Idea of Mathematicks, by Dr.* J. Pell, Ann. 1638. *Ph. Col.* n. 5. p. 127.

§. 1. Conſcribatur Conſiliarius Mathematicus (ita appellare libet) qui ad tres haſce Quæſtiones reſpondeat 1. Qui, qualeſq; Fructus ex ſtudiis Mathematicis ſperandi? 2. Quænam aſſequendæ tam fructuoſæ ſcientiæ Auxilia jam extent? 3. Qui Ordo in utendis iſtis Adminiculis obſervandus? Itaque continebit

1. Facilem & perſpicuum diſcurſum, de limitibus ſeu ambitu Artium Mathematicarum, deque ingentibus utilitatibus, primò ad ipſarum ſtudioſos, tum ad gentem complurium in iis peritorum feracem inde redundaturis.

2. Catalogum Mathematicorum & ab iis editorum Operum, qui exhibeat, 1. Synopſin omnis generis Librorum Mathematicorum vel jam Publicatorum, vel Manuſcriptorum in publicis Bibliothecis deliteſcentium, peculiaribus, cuivis Generi, numeris appoſitis. 2. Catalogum Chronologicum omnium Mathematicorum celebrium ſecundum ordinem Annorum quibus ſinguli vixêre, additis ubique annis, quibus Opera ipſorum primò excuſa fuêre. 3. Catalogum ipſorummet Operum ſecundum ordinem Annorum, quibus ullâ Linguâ impreſſa fuêre. In quo concinnando ita procederem, ut notato Anno Domini ſubjungerem, more uſitatorum Catalogorum, Nomina omnia Mathematicorum Librorum in ulla Gente aut Lingua iſto Anno editorum, 1. Indicando tamen in ſingulis, rationem Voluminis, hoc eſt, non tantùm magnitudinem paginarum (*e.g.* 4*to.* 8*vo.* &*c.*) ſed & earundem ſummam, ut moles totius Operis ſic facilè cuivis pateat. 2. Ante ipſummet Titulum exprimendo Annum, ad quem retroſpicere oporteat cogniturum quando opus conſcriptum, quando hâc illâve Linguâ ultimò editum fuerit. 3. Notando in Margine poſt Titulum, 1. Annum quo opus aliquod proximè Typis exſcriptum ſit. 2. Numerum, qui remittat Lectorem ad Synopſin primâ ſtatim Catalogi paginâ deliniatam. Quorum Numerorum beneficio quilibet è Veſtigio omnes unius argumenti libros Mathematicos percurrere queat.

3. Conſilium, inſtruens ſtudioſum, de optimis in quolibet genere libris: Quo ordine & quomodo legendi, quid obſervandum, quid evitandum in lectione quorundam Mathematicaſtrorum, quomodo procedendum, omniaque memoriâ tenenda.

4. Paræneſin, Primò ad omnes, opibus, otio, & ingenio horum Studiorum capaci inſtructos, ut, 1. cum ingentium commodorum ad ipſummet eorum ſtudioſum & ad univerſum humanum genus inde redeuntium. 2. Tum etiam defæcatæ illius voluptatis, quæ ex ſcrutinio abſtruſarum Veritatum, luctâ cum difficilibus Problematibus, & eorundem victoria emergit, reſpectu, ſeriò ad ea excolenda incumbant, tantò quidem magis, quo, 3. Expeditiora adminicula reperta ſunt, quæ laborem, tempus, ſumptus, majoribus noſtris impendenda, nobis compendifaciant. Deinde ad omnes tum judicio ad æſtimandum horum ſtudiorum pretium, tum & opibus (æternæ noſtri memoriæ, ſi in prudentes ſui diſpenſatores inciderint, promicondis) pollentes ut majori curæ ſibi eſſe patiantur hoc ſtudioſorum genus, ampliſque propoſitis præmiis lectiſſimos eorum ſeponant perficiendis inventis, ad quæ ipſo genio ſuo ducuntur. Denique ad omnes Principes & Reſpublicas, qui majus ditionibus ſuis ornamentum vix conſiliaverint,

ſiliaverint, quam operam dando, 1. Ut harum Artium peritis abundent. 2. Via ad eas minus laborioſa ſumptuoſaq; reddatur. 3. Ipſaq; adeo Ingenia Mathematica magis innoteſcant, ſeq; dignis auxiliis gaudeant.

In quem finem utiliſſimum fuerit, ut,

§. 2. Conſtituatur Publica Bibliotheca, quæ & omnes ſupradictos Libros, & unum Inſtrumentorum unquam inventorum contineat, & præterea reditibus ſufficientibus inſtructa ſit. 1. Ad coemenda ſingulorum librorum Mathematicorum, quæ quotannis alibi locorum foras dantur, exemplaria. 2. Ad ſuſtentandum Bibliothecarium, cui incumbat

1. Perlegere omnes hujus Claſſis libros in Regione iſthac publico committendos, 1. Suppreſſis quæ ab artis regulis exorbitant, ne Scriptorum errores Lectoribus fraudi ſint. 2. Monereq; Scriptores, ſi vetera & jam dicta tantùm reponant.

2. Sub periculo exiſtimationis ſuæ approbare inventa inſignia, & ſincerè commendare Inventores Mecænatibus.

3. Recipere, in Catalogum referre, & per Repoſitoria diſponere unum ita perlectorum librorum exemplar, donandum Bibliothecæ bene compactum, Authoris aut Bibliopolæ ſumptibus.

4. Promptum Reſponſum præbere, cuivis ſtudioſo, circa Problema aliquod inquirenti, an ſolutum fuerit nec ne, ni illi neceſſe ſit actum agere, aut contra inventa ſua ſupprimere, metu, ne antiqua jam ſint, & forſan in aliquo Librorum Bibliothecæ jam elaborata.

5. Recipere, &c. omnia Manuſcripta, vel donatione, aut legato Bibliothecæ obvenientia.

6. Diligens cum ſui generis hominibus, extra patriam ſuam agentibus, Literarum commercium exercere, ne ipſum fugiat, quidnam ibi locorum prelo paratum ſit.

7. Obſervare in Conterraneis ſuis, qui docendis hiſce artibus apti ſint.

8. In numerato ſemper habere Artifices omnes conficiendorum Inſtrumentorum Mathematicorum & Repreſentationum, adeoque Operationum in Ligno, Magnete, Metallis, Vitro, &c. peritos.

9. Poſt legitimum Examen, omnis generis practicis, *e.g.* Gubernatoribus Navium, Gæodetis, Logiſtis, &c. Speculativæ Peritiæ & Practicæ Dexteritatis teſtimonium præbere, ne poſthac, quibus hoc hominum genere opus, imperiti magno ipſorum damno imponant.

Catalogus docuerit, in tanta multitudine librorum, mundum tantùm non obruentium, quinam ad hanc tantùm ſtudiorum partem pertineant. Bibliotheca, & ipſorummet Librorum copiam faciet, & ſimul cognoſcendi ubi locorum exemplaria plura venalia ſint: Prætereaq; Promptuarii inſtar erit exteris & indigenis, unde extemplò diſcant, quænam ad hæc ſtudia adminicula Regio iſta ſit ſuppeditatura.

Atque hæc meo judicio expeditiſſima Methodus eſt, utendi illis auxiliis, quibus jam potimur. Si ampliora deſiderentur, conveniens fuerit, ut peritorum Artificum operâ,

§. 3. Conficiantur & publicentur tres hi novi Tractatus,

1. Pandectæ Mathematicæ, complectentes, quam fieri poteſt, perſpicuè, methodicè, compendioſè, & ingenuè quæcunque ex Mathematicis libris & inventis, quæ ante nos fuêre, colligi aut veluti conſectaria inde deduci potuêre, citatis ſub finem cujuſque periodi aut propoſitionis, authoribus, in quibus ea reperitur antiquiſſimis, ſequentibuſque adeo omnibus notâ inuſtâ, ubi vel in furto deprehenſi fuerint, vel mutuati eſſe ſuppreſſo unde ſumpſerint, aut (quod peſſimum omnium) aliorum inventa audacter ſibi arrogâſſe. Hoc pacto magna illa Bibliotheca in multo auguſtius ſpatium contraheretur, majori omnibus poſtfuturis laboris, temporis & ſumptuum compendio, quàm in præſentiarum quis imaginetur. Sed cum & hoc opus portatu minus amplum futurum ſit: Elaboretur præterea,

2. Comes Mathematicus, libro manuali (adeoque quam licet breviſſimè) continens utiliſſimas Tabulas unà cum præceptis de uſu ipſarum in Problematibus ſolvendis, ſeu purè Mathematicis, ſeu ad diverſas materias, prout res requiſierit, applicatas.

Et denique, ne in hoc quoque Eruditionis genere libris amplius aſtricti ſimus: Excogitetur,

3. Mathematicus αὐτάρκης, ſeu Inſtructio quomodo quilibet Mathematicus, laborem non exhorrens, eam peritiam adipiſci poſſit, ut ſine ulla Librorum aut Inſtrumentorum ope, cujuſvis tamen Problematis Mathematici reſolutio æquè facilis ipſi ſit, ac alteri ſolâ librorum Volutatione nixo.

Atque hæc eſt Idea illa Matheſeos, quam, more meo, diu abhinc mihi effinxi, firmiter ſemper perſuaſus, tum demum feliciter magnis rebus vires intendi, ubi exactam prius earundem Ideam, ejuſque aſſequendæ media quàm maximè appoſita animus conceperit: Quam ſi exprimere re ipſa non detur, multum tamen eſſe ab ea quam proximè abfuiſſe. Hancce meam equidem adeo non ſupra vires humanas arbitror, ut vel viri unius, rei domeſticæ aliorumve negotiorum curis non diſtracti, diligentiam parem ei putem. Bibliothecam enim & Catalogum nummis præſentibus facilè parari poſſe, quis non videt? Et ſi mihi Pandectarum contexendarum (quales ſupra deſcripſi) Provincia demandaretur, longè duriores Leges, quam quas ibidem enumeravi, mihi impoſiturus forem. Delinearem, primò omnium, infallibilem Rationis humanæ in indagando, quodcunque objicitur, proceſſum, oſtenſo quomodo à primis principiis ſeu elementis nunquam interruptâ Serie, ad ſublimiſſimas juxta & abjectiſſimas eorum applicationes conſcendendum ſit. Quâ arte forſan haud diu carerent homines, ſi in poſterum ſollicitè veſtigarent, quânam ratione illis quos mirantur tales obortæ eſſent cogitationes, quomodo tali fini, tam appoſita media reperta fuiſſent. Pandectas in Enchiridion, quotidianis uſibus opportunum, conſtringendi ratio forſan haud plures fugerit: At ita mentibus ſuis eas inſcribendi, ut ne Libris quidem amplius opus habeant (quod Mathematicus noſter αὐτάρκης flagitat) vires ingenii humani tranſcendere pleriſque haud dubiè videbitur, cum nemo hucuſque, quod ſciam, tale quid vel animo concipere auſus fuerit. Nihilominus tamen remittent aliquid, credo, de incredulitate ſua homines, poſtquam quæ roborandæ imaginationi, juvandæ

memoriæ

memoriæ, dirigendæ rationi artes excogitari, quam miri effectus conjunctione & jugi exercitio earum produci queant, paulò attentiùs ſecum conſideraverint.

2. Conſultius omnino mihi videtur ac preſtantius, ſi loco tanti apparatus quem Author Ideæ proponit de colligendis variis Mathematicorum ſcriptis, optimi ſolummodo & digniores ſeligerentur; Adductis primùm Authoribus illis Antiquis quorum libri adhuc ſuperſunt, veluti Euclide, Apollonio, Archimede, Theodoſio, Pappo, Ptolemæo, cum reliquis ipſorum Fragmentis & Manuſcriptis quæ nondum viderunt lucem, ac quorum aliqua penes Golium Lugduni Batavorum, quædam verò Romæ aſſervantur. Ipſis deinde recentiores poſſent adjungi, veluti Vieta, Clavius, & noſter Herigonius. Similiter inter Opticos, quis ſeligeret Vitellionem, Keplerum, Aquilonium, & Dom. de Villes. Inter Arithmeticos poſt Diophantum, eminent Cardanus, Tartaglea & Veſter Nepperus. Pro Triangulis Sphæricis & Supputatione per Logarithmos extant Briggius, Gordanus, Pitiſcus, Snellius, & noſter Morinus. Pro Aſtronomicis, poſt Ptolemæum & aliquot Arabes, conducerent omnes illi qui Tabulas compoſuerunt, ſicut Alphonſus, Johannes Regiomontanus, Keplerus, & noſter Duretus. Et ut paucis dicam, pro Fortificatoriis & Muſicis octo vel decem Authores excerpendi eſſent, illi ſcilicet qui meliori inclaruere Praxi. Similiter pro Rebus Mechanicis, Viribus Motuum, Machinis, & Ductibus Aquarum, ita ut enucleatis decem vel duodecim Authoribus, deſiderium ſuum explere poſſet harum rerum cupidus. Et ſi duodecim Viri Intelligentes & Amici cordatè in ſe ſuſciperent hoc negotium, ut quilibet unico Volumine convenienti & perſpicuo compingeret aliquam Scientiam, additis illis quæ deſunt in reliquis Authoribus, & ſublatis non neceſſariis, abſque dubio haberemus breviter & nervosè, in duodecim Voluminibus totum id quod hac in re à quoquam poſſet expeti. Et mea quidem opinione, quicquid ſpectat ad Matheſin, tam Puram quam Mixtam, poterit comprehendi duodecim illis Voluminibus. Sic omnem elegantiorem Philoſophiam tribus, & omnes Artes Liberales & Mechanicas etiam Tribus Libris exponere liceret: Ideoque levi pretio ad Eruditionem quis poſſet pertingere. Quod autem Mathematica illa Inſtrumenta attinet, parum certè prodeſſet apparatus omnium illorum quæ hactenus inventa fuêre. Præſtaret habere quatuor vel quinque, quæ nimirum optima ſunt in eo genere, & magìs commoda præ reliquis judicantur.

*Conſider'd, Oct. 1639. by Merſennus, ibid. p. 135.*

3. Tibi (Vir Clariſſime) ſi mentem tuam rectè aſſequor, Omnia probantur, niſi quòd majorem Apparatum, quam opus ſit, à me deſideratum cenſes: Selectiora tantum, non ut nos, Omnia Volumina atque Inſtrumenta Mathematica, colligi ſuades. Quod conſilium tuum neque ego improbarem, ſi & omnium calculo definitum eſſet quinam in tanta Scriptorum Turba reliquis omnibus Palmam præriperent, & id tantum à me ſpectaretur ut aliquid Laboris, Sumptuumque Compendium harum Artium ſtudioſis ſuppeditaretur. Nunc cum totius Matheſeos Perfectiſſimam Inſtaurationem optem, non aliud Conſilium à me proficiſci debet, quàm quod tali ſcopo per omnia quadret. Hujus Conſilii fundamentum in univerſali ejuſmodi Bibliothecâ haud immeritò collocare

*Anſwer'd by Dr. Pell, ib. p. 137.*

care mihi videor. Non audeo quenquam ſpernere, ſymbolum ſuum quocunq; modo huc conferentem, multo minus condemnare inauditum. Et ſi meo judicio locus ſit, vel Ineptiſſimum ſive Scriptum ſive Inſtrumentum Mathematicum dignum eſſe puto, cujus unum Exemplar, vel propter Errores, in amplæ cujuſcunque regionis loco certo & patente, ſervetur. Videmus enim non pauca ingeniosè inventa in ſuperiorum ſeculorum rudioribus Inſtrumentis, etiam nunc non obſervatu tantùm digna, ſed & Imitatione, quemadmodum etiam Imperiti Scriptores non paucas optimas egregiorum inventorum ἀφορμὰς ſagacioribus ingeniis præbuere; digitum enim intendere poſſumus, quò pervenire non poſſumus. Videmus quamplurima Lemmata rectiſſimè demonſtrata ab hujuſmodi Scriptoribus, & tamen propter unum Pſeudographema inter fundamenta collocatum, quicquid ſuperſtruxerant corruiſſe. Quod ſi non ſolùm falſitate, ſed etiam verboſitate, ſtribligine, &c. offenſus multos rejiciendos putas, cogita quam varium & multiplex ſit hominum Palatum, neque omnes ex tui ingenii perſpicacitate judica. Sunt enim nonnulli qui nihil intelligunt niſi idem (& ferè iiſdem verbis) dictum ſit centies; his credamus ταυτολογίας iſtas accommodatas eſſe. Et quoniam à Notioribus omninò incipiendum eſt, eadem verò non ſunt omnibus notiora, à diverſiſſimis initiis progrediuntur, ut vix reperias Diſcentis Ingenium, cujus Tyrocinio non ſit jamdudum accommodatum Rudius aliquod Scriptum vel Inſtrumentum quod neſcire nullo modo debet, qui ſe Conſiliarium Mathematicum profitetur. Itaque Librorum iſta Perfectior Collectio ſanè neceſſaria mihi videtur.

Quo minus vero nobis placent iſti Mathematicaſtri, eò magis fuerit, cur Bibliothecam hujuſmodi optemus, quippe quæ ſola infinitam iſtam hoc in genere ſcriptitandi pruriginem facilè coërcuerit. Balbutientes enim iſti Magiſtelli qui Tyrocinia & Juventutis captum crepantes, nuſquam non pueriliter ineptiunt, videbunt ſat ſuperque eſſe hujus farinæ Elementarium jam pridem Compilatorum. Qui vero infinitis inventis Matheſin augere ſed fruſtrà cupiunt incautiores, tot κενοδόξων παράδοξα publicè damnata, atque ludibrio omnium expoſita videntes, malo cavebunt ſi ſapiunt. Plagiarii verò, publico odio digniſſimi, non audebunt poſthac veteres aliquos Libellos ſemel forſan typis impreſſos, aut aliquas tantum earum particulas, pro ſuis extrudere. Quin etiam illi, quibus alioqui nec Candor nec Ingenium deeſſet, ad cogitata ſua ſatis commodè proferenda, quia tamen tam multos in ſingulis materiis ſe præceſſiſſe videbunt, ſatis monebuntur, nè alia in publicum edant, quam prioribus indicta. Qualia quæ ſint, ex inſpectâ amplâ ejuſmodi Bibliothecâ, vel ſi hunc laborem gravantur, ex ipſomet, quem ei aſſigno Bibliothecario, facilè cognoſcent. Atque hæ ferè rationes ſunt, quæ conſilii iſtius de tam univerſali Bibliotheca pœnitere me adhuc non ſinunt.

*To the Satisfaction of* Merſennus, *Dec.* 10. 1639. *ib. p.* 143.

4. Statim atque tuas Literas perlegi (Vir Eruditiſſime) non ſolum totus fui tuus, tuæque ſubſcripſi ſententiæ, quam mihi egregiè probatam teſtor; ſed & inſolitus Ardor Animi me propemodum abſtulit, nempe ut tuum illud opus quantumvis arduum Orbis Magnatibus proponendum curarem, ſi liber ad eos foret Additus. Sed quis Regum incipiet? Hoc enim Opus Regium auſim appellare.

5. Ideam

*The Judgment and Approbation of des Cartes. Feb. 1640. ib. p. 144.*

5. Ideam Mathematicam non niſi obiter inſpexi, jamq; tantum memini nihil me in illa reperiſſe à quo multum diſſentirem, & valde probaſſe quod primo loco omnis ſupellex Mathematica ibi enumeretur, & poſtea ipſe Mathematicus tanquam αὐτάρκης ex ſeipſo contentus deſcribatur. In eundem enim fere ſenſum duo ſoleo in Matheſi diſtinguere, Hiſtoriam ſcilicet & Scientiam: Per Hiſtoriam intelligo illud omne quod jam inventum eſt, atque in libris continetur: Per Scientiam verò, Peritiam Quæſtiones omnes reſolvendi, atque adeo inveniendi propriâ Induſtriâ illud omne quod ab humano ingenio in ea Scientia poteſt inveniri, quam qui habet, non ſane multum aliena deſiderat, atque adeo valde propriè αὐτάρκης appellatur. Valde autem optandum foret, ut illa Hiſtoria Mathematica quæ in multis Voluminibus ſparſa, nondum integra & perfecta eſt, in unum Librum tota colligeretur: Neque ad hoc ulli ſumptus in perquirendis aut coëmendis libris eſſent faciendi: Cum enim Authores alii ex aliis multa exſcripſerint, nihil ullibi extat, quod non in quavis mediocriter inſtructâ Bibliothecâ alicubi reperiatur; nec tam diligentia opus eſſet ad omnia colligenda, quam judicio ad ſuperflua rejicienda, & Scientia ad ea quæ nondum inventa ſunt ſupplenda. Atqui ſi talis Liber extaret, facile ex eo unuſquiſque omnem Hiſtoriam Mathematicam, atque etiam aliquam partem Scientiæ addiſceret. Si quis autem omnia quæ ad ejus Praxin pertinent, habere vellet, ut Inſtrumenta, Machinas, Automata, &c. næ ille ſi Rex eſſet, orbis Terrarum impenſis omnibus ad hoc neceſſariis ſufficere nunquam poſſet. Neque verè etiam illis opus habet, ſed ſatis eſt ſi omnium nôrit deſcriptionem, adeo ut ea, cum uſus exiget, vel ipſe facere, vel per Artifices fieri curare poſſit.

*Some of Euclid's Propoſitions, demonſtrated independently from the reſt, by Mr. Aſh, n. 162. p. 672.*

II. THE Propoſitions which I ſhall endeavour to demonſtrate independently from all others, ſhall be theſe; the 32d, and 47th of the *Firſt Book*; moſt of the *Second* and *Fifth Books*; the 1ſt and 16th of the *Sixth*; with their Corollaries. In order to demonſtrate the 32d; I ſuppoſe it known what is meant by an Angle, Triangle, Circle, External Angle, Parallels, and that the Meaſure of an Angle is the Arch of a Circle intercepted between its Sides; That a Right Angle is meaſur'd by a Quadrant, and two Right Angles by a Semicircle. 32. 1. E. I ſay then, that in the Triangle A B C, the External Angle B C E, is equal to the two oppoſite Internal ones A B C, B A C; For let a Circle be drawn, C being the Center, and B C the Radius; and let C D be drawn parallel to A B, thoſe two Lines being always equidiſtant, will both have the ſame Inclination to any third Line falling upon them; that is, (by the Definition of Angle) they will make Equal Angles with it; For if any Part of C D (for Inſtance) did incline more to B C, than did A B, upon that very account they would not be Parallel; it follows therefore that the Angles A B C, B C D, are Equal: Alſo B A C = D C E, becauſe A E falls upon two Parallels; but the External Angle B C E = B C D + D C E, which were before prov'd to be Equal to A B C, B A C. *Q. E. D.* Hence may be inferr'd as a Corollary, That the three Angles of every Triangle are Equal to two Right ones; for the Angles A C B + B C E, are meaſur'd by a

Fig. I.

Semicircle,

Semicircle, and therefore Equal to two Right Angles; Corollaries alſo from
20, 22, 31. 3. E. hence are the 20th, 22d, and 31ſt of the *Third* Book, which contain the Properties of Circles, whoſe Deduction from hence is moſt natural and obvious.

47. 1. E. In order to demonſtrate the 47th; I ſuppoſe the Meaning of the Terms made uſe of, to be known; and that two Angles or Superficies are Equal, when one being put on the other, it neither exceeds, nor is exceeded. This being allow'd, I ſay, the Sides about the Right Angle are either Equal or Unequal;
Fig. II. if Equal, let all the Squares be deſcrib'd; the whole Figure exceeds the Square of the Hypothenuſe B C, by the two Triangles M, V, and exceeds alſo the Squares of the other two Sides, A B, A C, by the two Triangles, A B C, and S; which Exceſſes are equal, for M is equal to A B C, the two Sides about the Right Angle, being two Sides of a Square upon A B, by Suppoſition equal to A C, and the third Side equal to B C; therefore the whole Triangles are Equal. After the ſame manner S and V, are proved to be Equal; therefore the Square of C B, is Equal to the Squares of the two other Sides. *Q. E. D.*

Fig. III. But if the Sides be Unequal, let the Squares be deſcribed, and the Parallelogram L Q compleated, the whole Figure exceeds the Square upon B C, by three Triangles, X, R, Z, and exceeds alſo the Square L A, A D, by the Triangle A B C, and the Parallelogram P Q: which Exceſſes, I ſay, are Equal; for Z is equal to A B C, the Side O C = B C, C D = A C, the Angle D = A, and O C D = B C A; which is manifeſt, by taking the common Angle A C O, out of the two Right Angles B C O, A C D; therefore by Superimpoſition the whole Triangles are Equal. In like manner X is proved equal to A B C, alſo R; and the Parallelogram P Q to be double of the Triangle A B C: Thus the Exceſſes being proved Equal, the Remainders alſo will be Equal; *viz.* the Square of B C, to the Square of A B,
35, 36. 3. E. A C. *Q. E. D.* Manifeſt Corollaries from hence are, the 35th, and 36th,
12, 13. 2. E. of the *Third* Book; alſo the 12th, and 13th, of the *Second.*

1, 2, &c. 2. E. The firſt ten Propoſitions of the *Second* Book are evidently demonſtrated, only by ſubſtituting Species or Letters, inſtead of Lines, and multiplying them according to the Tenor of the Propoſition; thus, to inſtance in one or
Fig. IV. two; Call the whole Line A, and its Parts B and C, therefore $A = B + C$, and conſequently $AA = BB + CC + 2BC$, which is the very Senſe of the
4. 2. E. Fig. V. *Fourth* of the *Second* Book. Thus alſo; Let a Line be cut into equal Parts F, F, and let another Line S, be added thereto; 'tis manifeſt, that $4FF +$
10. 2. E. $4SF + 2SS = 2FF + 2FF + 2SS + 4SF$, which is the *Tenth* Propoſition of the ſame Book.

Almoſt the whole Doctrine of Proportionals, *viz:* Permutation, Inverſion, Converſion, Compoſition, Diviſion of Ratio's, and Proportion *ex æquo*; and conſequently the moſt uſeful Propoſitions of the *Fifth* Book are clearly demonſtrated by one Definition, and that is of Similar or Like Parts, which are ſaid to be ſuch as are after the ſame manner, or equally contain'd in their
Fig. VI. Wholes: Thus the Antecedents A and C, are either Equal to their Conſequents, or Greater, or Leſs; if Equal, the thing is manifeſt; if Leſs, then

(by

(by the Definition of Proportionals) A, and C are like Parts of B and E; therefore what Proportions the whole B and E have to one another, the ſame will A and C have, which is Permutation; likewiſe E : C :: B : A, which is Inverſion; ſo alſo if from Proportionals you take Like Parts, the Remainders will be Proportional; whence Converſion and Diviſion are Demonſtrated: And if to Proportionals you add Like Parts, the Wholes will ſtill be Proportional, which is Compoſition, *&c.* If the Antecedents be greater than the Conſequents, the Conſequents will be like Parts of them, and the Demonſtration exactly the ſame with the former.

The firſt of the *Sixth* Book is prov'd, by conſidering the Generations of the Parallelograms, which are produced by drawing or multiplying the Perpendicular upon the Baſis; that is, taking it ſo often as there be Parts and Diviſions in the Baſe: Therefore the ſame Proportion that R X ſingle, hath to N X ſingle, the ſame hath R X multiplied by X Z, that is, repeated a certain number of times, to N X multiplied by X Z, that is, repeated the ſame number of times; which is as much as to ſay, R X : N X :: Paral. R Z : Paral. N Z. Now that this Propoſition alſo is true in Oblique-angled Parallelograms, is proved, becauſe they are equal to the Rectangled ones upon the ſame Baſis, and between the ſame Parallels, as does thus independently appear; The Triangles R Q X, and M P Z, are equal, for R X = M Z, Q X = P Z, R M = Q P; therefore adding to both M Q, R Q = M P; if therefore from theſe equal Triangles you take what is Common, *viz.* M L Q, the Remainders will be equal, R X L M = Q L Z P; to both which add X L Z, and the whole Parallelograms will be Equal, R Z = Q Z. *Q. E. D.* That Triangles alſo having a Common Baſis, are in the Proportion of their Altitudes, does hence follow, becauſe they are the halves of Parallelograms upon the ſame Baſis. This alſo is true, and the Demonſtration exactly the ſame in Priſms, Pyramids, Cylinders, and Cones, having the ſame Baſis.

1. 5. E.

Fig. 7.

Fig. 7.

To prove the 16th of the *Sixth*, I ſuppoſe the 4 Lines, A, B, C, E, to be Proportional, that is, granting A and C to be the leſſer Terms; the ſame way that A is contained in B, ſo is C in E, and that D is the Denominator of the Ratio; 'twill follow then, that B is made up of A, multiplied by D, and E of C, multiplied by D; ſo that A D = B, and C D = E; draw therefore the Extremes upon one another, that is, A upon C D, and the Means, that is, C upon A D, the Factors being the ſame; I ſay, the Products A C D, and C A D, are the ſame, and conſequently Equal. *Q. E. D.*

16. 6. E.

Fig. 6.

The Problem propos'd by M. *Comier*, (with Oſtentation enough) as if it contain'd ſomething New, tho' in reality it be nothing but the Old Buſineſs of Doubling the Cube a little diſguis'd, is eaſily ſolv'd Algebraically, as follows.

| | | | |
|---|---|---|---|
| Fig. 8. | 8. 6. E. | 1 | $a : 2x :: x : \frac{2x^2}{a} = p$ |
| | 47. 1. E. | 2 | $aa + 2ax - \frac{2x^3}{a} = \frac{4x^4}{aa}$ |
| | $2x$ by $aa$ | 3 | $a^4 + 2a^3x = 4x^4 + 2x^3a$ |
| | $3 \div a + 2x$ | 4 | $a^3 = 2x^3$; that is, The Cube upon $x$, is half the Cube upon $a$. |

*The Squaring of the Hyperbola, by the L. Viscount* Brounker. n. 34. *p.* 645. Fig. 9.

III. Let A B be one Asymptote of the Hyperbola E $d$ C; and let A E, and B C, be parallel to th' other: Let also A E, be to B C, as 2 to 1; and let the Parallelogram A B D E, be equal to 1.

Supposing the Reader knows, that E A, $\alpha\zeta$, K H, $\beta\eta$, $d\theta$, $\gamma\kappa$, $\delta\lambda$, $\varepsilon\mu$, C B, &c. are in an Harmonick Series, or a Series *Reciproca-Primanorum, seu Arithmeticè Proportionalium,* (otherwise he is referr'd for Satisfaction to *Arithm. Infinitor. Wallisii,* Prop. 87, 88, 89, &c.) I say,

$$A B C d E A = \frac{1}{1\times2} + \frac{1}{3\times4} + \frac{1}{5\times6} + \frac{1}{7\times8} + \frac{1}{9\times10}, \&c.$$

$$E d C D E = \frac{1}{2\times3} + \frac{1}{4\times5} + \frac{1}{6\times7} + \frac{1}{8\times9} + \frac{1}{10\times11}, \&c.$$

$$E d C y E = \frac{1}{2\times3\times4} + \frac{1}{4\times5\times6} + \frac{1}{6\times7\times8} + \frac{1}{8\times9\times10}, \&c.$$

in infinitum.

For

Fig. 10, 11, 12.

| For (in *Fig.* 10. and 11.) the *Parallelogram*, | | and (in *Fig.* 12.) the *Triangle*, |
|---|---|---|
| | $CA = \frac{1}{1 \times 2}$ | $EdC = \frac{1}{2 \times 3 \times 4} = \frac{\square dD - \square dF}{2}$ |
| $dD = \frac{1}{2 \times 3}$ | $dF = \frac{1}{3 \times 4}$ | $Ebd = \frac{1}{4 \times 5 \times 6} = \frac{\square br - \square bn}{2}$ |
| $br = \frac{1}{4 \times 5}$ | $bn = \frac{1}{5 \times 6}$ | $dfC = \frac{1}{6 \times 7 \times 8} = \frac{\square fG - \square fk}{2}$ |
| $fG = \frac{1}{6 \times 7}$ | $fk = \frac{1}{7 \times 8}$ | $Eab = \frac{1}{8 \times 9 \times 10} = \frac{\square aq - \square ap}{2}$ |
| $aq = \frac{1}{8 \times 9}$ | $ap = \frac{1}{9 \times 10}$ | $bcd = \frac{1}{10 \times 11 \times 12} = \frac{\square cs - \square cm}{2}$ |
| $cs = \frac{1}{10 \times 11}$ | $cm = \frac{1}{11 \times 12}$ | $def = \frac{1}{12 \times 13 \times 14} = \frac{\square et - \square el}{2}$ |
| $et = \frac{1}{12 \times 13}$ | $el = \frac{1}{13 \times 14}$ | $fgC = \frac{1}{14 \times 15 \times 16} = \frac{\square gu - \square gh}{2}$ |
| $gu = \frac{1}{14 \times 15}$ | $gh = \frac{1}{15 \times 16}$ | &c. |
| &c. | &c. | |

*Note.* $\frac{1}{2} CA = dD + dF$
$\frac{1}{2} dD = br + bn$
$\frac{1}{2} dF = fG + fk$
$\frac{1}{2} br = aq + ap$
$\frac{1}{2} bn = cs + cm$
$\frac{1}{2} fG = et + el$
$\frac{1}{2} fk = gu + gh$
&c.

And that therefore in the first Series half the first Term is greater than the Summ of the two next; and half this Summ of the Second and Third greater than the Summ of the four next; and half the Summ of those Four, greater than the Summ of the next Eight, &c. *in infinitum.* For $\frac{1}{2} dD = br + bn$; but $bn > fG$, therefore $\frac{1}{2} dD > br + fG$, &c. And in the second Series, half the second Term is less than the Summ of the two next, and half this Summ less than the Summ of the four next, &c. *in infinitum.*

That the first Series are the even Terms, *viz.* the 2d, 4th, 6th, 8th, 10th, &c.



and

and the second the odd, *viz.* the 1st, 3d, 5th, 7th, 9th, &c. of the following Series, *viz.*

$\frac{1}{1 \times 2}, \frac{1}{2 \times 3}, \frac{1}{3 \times 4}, \frac{1}{4 \times 5}, \frac{1}{5 \times 6}, \frac{1}{6 \times 7}$, &c. *in infinitum* =:

Whereof $a$, being put for the Number of Terms taken at Pleasure, $\frac{1}{a^2 + a}$ is the last, $\frac{a}{a + 1}$ is the Summ of all those Terms from the Beginning, and $\frac{1}{a + 1}$ the Summ of the rest to the End.

That $\frac{1}{4}$ of the first Term in the third Series, is less than the Summ of the two next, and $\frac{1}{4}$ of this Summ less than the Summ of the four next, and $\frac{1}{4}$ of this last Summ less than the next eight, I thus Demonstrate.

Let $a$, be equal to the Third or Last Number of any Term of the first Column, *viz.* of Divisors;

$$\frac{1}{a \times \overline{a - 1} \times \overline{a - 2}} = \frac{1}{a^3 - 3a^2 + 2a} = \frac{16a^3 - 48a^2 + 56a - 24}{16a^6 - 96a^5 + 232a^4 - 288a^3 + 184a^2 - 48a} = \text{A}.$$

$$\left.\begin{array}{l} \frac{1}{2a \times \overline{2a - 1} \times \overline{2a - 2}} = \frac{1}{8a^3 - 12a^2 + 4a} \\ \frac{1}{\overline{2a - 2} \times \overline{2a - 3} \times \overline{2a - 4}} = \frac{1}{8a^3 - 36a^2 + 52a - 24} \end{array}\right\} =$$

$$\frac{16a^3 - 48a^2 + 56a - 24}{64a^6 - 384a^5 + 880a^4 - 960a^3 + 496a^2 - 96a} = \text{B}.$$

$$\frac{64a^6 - 384a^5 + 928a^4 - 1152a^3 + 736a^2 - 192a}{64a^6 - 384a^5 + 880a^4 - 960a^3 + 496a^2 - 96a} \times \tfrac{1}{4}\text{A} = \text{B}.$$

And $48a^4 - 192a^3 + 240a^2 - 96a$ = Excess of the Numerator above the Denominator.

$$\left.\begin{array}{lrcl} \text{But the Affirmat.} & & > & \text{the Negat.} \\ \text{That is,} & 48a^4 + 240a^2 & > & 192a^3 + 96a \\ \text{Because} & a^4 + 5a^2 & > & 4a^3 + 2a \\ & a^3 + 5a & > & 4a^2 + 2 \end{array}\right\} \text{if } a > 2.$$

$$\text{Therefore} \quad \text{B} > \tfrac{1}{4}\text{A}.$$

Therefore one Fourth of any Number of A, or Terms, is less than their so many respective B; that is, than twice so many of the next Terms. *Q. E. D.*

By

By any one of which three Series it is not hard to Calculate, as near as you please, these and the like Hyperbolic Spaces, whatever be the Rational Proportion of A E, to B C. As for Example; When A E, is to B C, as 5 to 4. (Whereof the Calculation follows, after that where the Proportion is, as 2 to 1; and both by the *Third* Series.)

First then, when (in *Fig.* 9.) A E : B C :: 2 : 1. Fig. 9.

| | | |
|---|---|---|
| 2 × 3 × 4) | 1. (0. 0416666666 | 0. 0416666666 |
| 4 × 5 × 6) | 1. (0. 0083333333 | 0. 0113095237 |
| 6 × 7 × 8) | 1. (0. 0029761904 | |
| 8 × 9 × 10) | 1. (0. 0013888888 | 0. 0029019589 |
| 10 × 11 × 12) | 1. (0. 0007575757 | |
| 12 × 13 × 14) | 1. (0. 0004578754 | |
| 14 × 15 × 16) | 1. (0. 0002976190 | |
| 16 × 17 × 18) | 1. (0. 0002042484 | 0. 0007306482 |
| 18 × 19 × 20) | 1. (0. 0001461988 | |
| 20 × 21 × 22) | 1. (0. 0001082251 | |
| 22 × 23 × 24) | 1. (0. 0000823452 | |
| 24 × 25 × 26) | 1. (0. 0000641026 | |
| 26 × 27 × 28) | 1. (0. 0000508751 | |
| 28 × 29 × 30) | 1. (0. 0000410509 | |
| 30 × 31 × 32) | 1. (0. 0000336021 | |
| 32 × 33 × 34) | 1. (0. 0000278520 | 0. 0001829939 |
| 34 × 35 × 36) | 1. (0. 0000233426 | |
| 36 × 37 × 38) | 1. (0. 0000197566 | |
| 38 × 39 × 40) | 1. (0. 0000168691 | |
| 40 × 41 × 42) | 1. (0. 0000145180 | |
| 42 × 43 × 44) | 1. (0. 0000125843 | |
| 44 × 45 × 46) | 1. (0. 0000109793 | |
| 46 × 47 × 48) | 1. (0. 0000096361 | |
| 48 × 49 × 50) | 1. (0. 0000085034 | |
| 50 × 51 × 52) | 1. (0. 0000075415 | |
| 52 × 53 × 54) | 1. (0. 0000067193 | |
| 54 × 55 × 56) | 1. (0. 0000060125 | |
| 56 × 57 × 58) | 1. (0. 0000054014 | |
| 58 × 59 × 60) | 1. (0. 0000048704 | |
| 60 × 61 × 62) | 1. (0. 0000044068 | |
| 62 × 63 × 64) | 1. (0. 0000040002 | |

0. 0416666666
0. 0113095237
0. 0029019589
0. 0007306482
3) 0. 0001829939 (0. 000060998

0. 05679179
+ 0. 00006100

0. 05685279 $<$ E $d$ C $y$

But 0. 0007306482 }
0. 0001829939 } $\div$
0. 0000458315 }

Therefore 0. 05679179
+ 0. 00004583
+ 0. 00001528

0. 05685290 $>$ E $d$ C $y$

For, it has been Demonstrated, That $\frac{1}{4}$ of any Term in the last Column is less than the Term next after it; and therefore that $\frac{1}{3}$ of the last Term, at which you stop, is less than the remaining Terms; and that the Total of these is less than $\frac{4}{3}$ of a Third Proportional to the two last.

And therefore ABC$y$E being = 0. 75 ——————— 0. 75
and E $d$ C $y$ $>$ 0. 05685279 —— and $<$ 0. 05685290

And A B C $d$ E is $<$ 0. 69314720 —— and $>$ 0. 69314709

Fig. 9. But when A E : B C :: 5 : 4, or as E A, to K H; then will the Space A B C E, or now, the Space A H K E (A H = $\frac{1}{4}$ A B) be found as follows.

8 × 9 × 10) 1. (0. 0013888888 ] 0. 0013888888

16 × 17 × 18) 1. (0. 0002042484 }
18 × 19 × 20) 1. (0. 0001461988 } 0. 0003504472

32 × 33 × 34) 1. (0. 0000278520 }
34 × 35 × 36) 1. (0. 0000233426 }
36 × 37 × 38) 1. (0. 0000197566 } 0. 0000878204
38 × 39 × 40) 1. (0. 0000168691 }

0. 001-

```
               0. 0013888888
               0. 0003504472
            3) 0. 0000878204 (0. 0000292735
               ____________
               0. 0018271564
             + 0. 0000292735
               ____________
               0. 0018564299 < E a b
               ____________

           But 0. 0003504472 )
               0. 0000878204 } ∷
               0. 0000220073 7 )
               ____________
     Therefore 0. 0018271564
             + 0. 0000220074
             + 0. 0000073358
               ____________
               0. 0018564996 > E a b.
               ____________
```

Therefore E M $b$ (*Fig.* 12.) being = 0. 025 ———— 0. 025
E $a$ $b$ > 0. 0018564299 & < 0. 0018564996

E M $b$ $a$ (*Fig.* 12.) or E K M (*Fig.* 9.) > 0. 02685643 < 0. 02685650
A H K E < 0. 22314356 > 0. 22314349

Therefore 3 A B C $d$ E = 2. 07944154
and A H K E = 0. 2231435

A B C $d$ E (when A E : B C :: 10 : 1.) = 2. 3025850

Therefore, The Logar. of 10, is to the Log of 2, As 2. 302585, to 0. 693147.

IV. The Quadrature of the Circle, or the turning it into an equal Square, or any other Right-lin'd Figure, (which depends upon the Ratio of the Circle to the Square of its Diameter, or of the Circumference to its Diameter) may be understood to be Fourfold, to wit, either by Calculation, or by Linear Construction; And each of them again may be either perfectly exact, or else almost, or pretty near. The Quadrature by Accurate Calculation, I call the Analytical; That which is done by Accurate Construction, I call the Geometrical: That which is done by Calculation pretty near, I call the Approach; That which is by Construction pretty near, I call the Mechanical. *The Quadrature of a Circle by* M. Leibnitz, *Phil. Coll.* n. 7. *p.* 204.

The Approaches have been furthest carry'd on by *Ludolph van Ceulen*; *Vieta*, *Hugenius*, and others, have given several Mechanical. The Accurate Geometrical Construction may be had, by which not only an entire Circle may be measur'd, but any Section or Arch of it also, which is by an exact and ordinate Motion, but such notwithstanding as suits with Transcendental Curves,

which

which erroneously are accounted Mechanical, tho' in truth they are as Geometrical as those which are commonly so esteem'd, tho' they are not Algebraical, nor can be reduced to Equations Algebraical or of certain Degrees, they having Degrees proper to themselves, which tho' they be not Algebraical, are yet nevertheless Analytical.

The Analytical Quadrature, or that which is made by Accurate Calculation, may be again subdivided into Three Kinds; namely, into the Analytical Transcendent, the Algebraical, and the Arithmetical. The Analytical Transcendent is to be obtain'd, amongst others, by Equations of Degrees indefinite, hitherto consider'd by none. As if $X^x + X$, be equal to 30, and X, be sought, it will be found to be 3; because $3^3 + 3$, is 27 + 3, or 30.

The Algebraical is done by Vulgar Numbers, tho' irrationally, Vulgar, or by the Roots of common Equations, which for the General Quadrature of the Circle, or its Sectors, is indeed impossible. Now there remains the Arithmetical Quadrature, which is performed by certain Series exhibiting the Quantity of the Circle exact by a Progression of Terms (first) Rational, such as I shall here propound.

I have found therefore, that if the Square of the Diameter be put 1, the Area of the Circle will be $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \frac{1}{15} + \frac{1}{17}$, &c. To wit, the entire Square of the Diameter being diminished (that it may not be too big) by a third part; and again (because hereby too much is taken away) being augmented by one fifth; and again (because by this last too much is added) diminished by one seventh; and so onward continually.

And the first Quantity will be too great, *viz.* 1. but the Error will be less than $\frac{1}{3}$

The next too little, *viz.* $1 - \frac{1}{3}$, but the Error will be less than $\frac{1}{5}$

The 3d too much, *viz.* $1 - \frac{1}{3} + \frac{1}{5}$, but the Error will be $-\frac{1}{7}$

The 4th too little, *viz.* $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}$, but the Error, &c. $-\frac{1}{9}$

&c. &c.

The whole Series contains all the Approaches together, or the Values both greater than they ought to be, and less than they ought to be.

So that by continuing the Series, the Errors may be made less than the Fraction given, and consequently less than any Assignable Quantity. Whence it follows, that the whole Series must give the true Value. And tho' the Summ of the whole Series cannot be express'd by one Number, and that the Series be infinitely continued: yet because it consisteth of one regular Method of Progression, the whole may sufficiently enough be conceived by the Mind. And if *Van Ceulen* could have given a Rule by which his Numbers 314159, &c. could have been continued *in infinitum*, he would have given us the Arith-

metical

metical Quadrature exact in whole Numbers, which we have here done in Fractions.

There are ſeveral things relating to this Quadrature, which might be taken Notice of, eſpecially one, *viz.* That the Terms of this our Series $\frac{1}{1}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{9}$, &c. are of Harmonical Progreſſion, or in a continued Harmonical Progreſſion, as will be evident to any one that ſhall examine them. And a Series made by Skipping, as $\frac{1}{1}$, $\frac{1}{5}$, $\frac{1}{9}$, $\frac{1}{13}$, $\frac{1}{17}$, &c. is alſo of Harmonical Progreſſion. And $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{11}$, $\frac{1}{15}$, $\frac{1}{19}$, &c. is alſo a Series of Harmonical Proportionals. Wherefore ſince the Circle $\frac{1}{1} + \frac{1}{5} + \frac{1}{9} + \frac{1}{13} + \frac{1}{17}$ &c. $- \frac{1}{3} - \frac{1}{7} - \frac{1}{11} - \frac{1}{15} - \frac{1}{19}$, &c. by ſubſtracting the latter partial Series from the former partial Series, the Circle will be the Difference of two Series in Harmonical Progreſſion. And becauſe the Summ of any Number of Terms in Harmonical Progreſſion, how many ſoever, may by ſome Compendium be obtained; hence Compendious Approaches (if after *Van Ceulen* there be any need of them) may be be deduc'd, if one would in this our Series take out the Terms affected with the Sign $-$, by adding the two next into one, $+ \frac{1}{1} - \frac{1}{3}$, and $+ \frac{1}{5} - \frac{1}{7}$, and $+ \frac{1}{9} - \frac{1}{11}$, and $+ \frac{1}{13} - \frac{1}{15}$, and $+ \frac{1}{17} - \frac{1}{19}$, and ſo onward, he will have a new Series for the Magnitude of the Circle, namely $\frac{2}{3}$ (that is $\frac{1}{1} - \frac{1}{3}$) $+ \frac{2}{35}$ (that is $\frac{1}{5} - \frac{1}{7}$) $+ \frac{2}{99}$ (that is $\frac{1}{9} - \frac{1}{11}$) &c. Wherefore

The Square inſcribed being $\frac{1}{4}$

The Area of the Circle ſhall be $\frac{1}{3} + \frac{1}{35} + \frac{1}{99} + \frac{1}{195} + \frac{1}{323}$, &c.

But the Numbers 3, 35, 99, 195, 323, &c. by skipping are taken out of the Series of Square Numbers, 4, 9, 16, 25, &c. diminiſhed by an Unite, and ſo made the Series 3, 8, 15, 24, &c. out of the Members of which Series every fourth after the firſt, is a Number of this our Series. But I have found (which is worth noting) the Summ of this Infinite Series $\frac{1}{3} + \frac{1}{8} + \frac{1}{15} +$ &c. to be $\frac{3}{4}$. Nay, and by culling out by ſingle Skipping, as $\frac{1}{3} + \frac{1}{15} + \frac{1}{35}$, &c. the Summ of this Infinite Series maketh $\frac{2}{4}$ or $\frac{1}{2}$. But if out of this again another Progreſſion be culled out by ſingle Skipping, as $\frac{1}{5} + \frac{1}{35} + \frac{1}{99}$, &c. the Summ of that Infinite Series ſhall be the Semicircle, the Square of the Diameter being 1. Now becauſe by the ſame Means

 the

the Arithmetical Quadrature of the *Hyperbola* is obtain'd, I thought it not amiss to represent to View the whole Harmony.

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | *&c.* | |
| 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 | 121 | 144 | 169 | 196 | 225 | 256 | 289 | 324 | 361 | 400 | *&c.* | |
| 0 | 3 | 8 | 15 | 24 | 35 | 48 | 63 | 80 | 99 | 120 | 143 | 168 | 195 | 224 | 255 | 288 | 323 | 360 | 399 | *&c.* | |
| . | $\frac{1}{3}$ | $\frac{1}{8}$ | $\frac{1}{15}$ | $\frac{1}{24}$ | $\frac{1}{35}$ | $\frac{1}{48}$ | $\frac{1}{63}$ | $\frac{1}{80}$ | $\frac{1}{99}$ | $\frac{1}{120}$ | $\frac{1}{143}$ | $\frac{1}{168}$ | $\frac{1}{195}$ | $\frac{1}{224}$ | $\frac{1}{255}$ | $\frac{1}{288}$ | $\frac{1}{323}$ | $\frac{1}{360}$ | $\frac{1}{399}$ | *&c.* | $= \frac{3}{4}$ |
| . | $\frac{1}{3}$ | . | $\frac{1}{15}$ | . | $\frac{1}{35}$ | . | $\frac{1}{63}$ | . | $\frac{1}{99}$ | . | $\frac{1}{143}$ | . | $\frac{1}{195}$ | . | $\frac{1}{255}$ | . | $\frac{1}{323}$ | . | $\frac{1}{399}$ | *&c.* | $= \frac{2}{4}$ |
| . | . | $\frac{1}{8}$ | . | $\frac{1}{24}$ | . | $\frac{1}{48}$ | . | $\frac{1}{80}$ | . | $\frac{1}{120}$ | . | $\frac{1}{168}$ | . | $\frac{1}{224}$ | . | $\frac{1}{288}$ | . | $\frac{1}{360}$ | . | *&c.* | $= \frac{1}{4}$ |
| . | $\frac{1}{3}$ | . | . | . | $\frac{1}{35}$ | . | . | . | $\frac{1}{99}$ | . | . | . | $\frac{1}{195}$ | . | . | . | $\frac{1}{323}$ | . | . | *&c.* | = to |
| . | . | $\frac{1}{8}$ | . | . | . | $\frac{1}{48}$ | . | . | . | $\frac{1}{120}$ | . | . | . | $\frac{1}{224}$ | . | . | . | $\frac{1}{360}$ | . | *&c.* | = to |

the Circle ABCD, whose Inscribed Square is<br>the Hyperbola CBEHC, whose Power ABCD, is } $\frac{1}{4}$.

Fig. 13. To the Asymptotes AF, AE, at Right Angles to each other, let there be described the Curve Line of an Hyperbola GCH, whose Vertex is C; and ABCD, the Power or Square to which every Rectangle made of the Ordinate, as EH, and the intercepted part AE, is always equal. About this Square let a Circle be drawn, and let the Hyperbola be continued from C to H, so that AE be double to AB. Then putting AE to be 1, AB shall be $\frac{1}{2}$, and its Square ABCD, shall be $\frac{1}{4}$, and the Circle (whose Power ABCD is inscribed) shall be $\frac{1}{3} + \frac{1}{35} + \frac{1}{99}$, *&c.* but the portion of the Hyperbola CBEHC (whose Power inscribed is the same Square $\frac{1}{4}$) which represents the Logarithm of the Ratio of AE to AB, (or of 2 to 1) shall be $\frac{1}{8} + \frac{1}{48} + \frac{1}{120}$, *&c.*

*Tangents to all Geometrical Curves*; by Renatus Fran. Slusius. n. 90. p. 5143.

Fig. 14.

V. 1. Data sit quælibet Curva DQ, cujus puncta omnia referantur ad Rectam quamlibet datam EAB, per Rectam DA; sive EAB sit Diameter seu alia quælibet, sive etiam aliæ simul lineæ datæ sint, quæ, vel quarum potestates, Æquationem ingrediantur; parùm id refert.

In Æquatione Analytica, facilioris explicationis causâ, DA perpetuò dicatur $v$, & BA, $y$; EB verò & aliæ quantitates datæ, Consonantibus exprimantur.

Tum supponatur ducta DC, tangens Curvam in D, & occurrens EB, productæ, si opus sit, in puncto C; & CA perpetuò quoque dicatur $a$. Ad inveniendam AC, vel $a$, hæc erit Regula Generalis;

1. Rejectis ab Æquatione partibus in quibus $y$ vel $v$ non invenitur; statuantur ab uno latere omnes in quibus est $y$, & ab altero illæ in quibus habetur $v$, cum suis signis + vel —. Hoc dextrum, illud sinistrum latus, facilitatis causâ, vocabimus.

2. In latere dextro, præfigatur singulis partibus exponens potestatis quam in illis obtinet $v$; seu, quod idem est, in illum ducantur partes.

3. Fiat idem in latere sinistro, præponendo scil. unicuique illius parti Exponentem potestatis quam in illa habet $y$. Sed & hoc amplius: unum $y$ in singulis partibus vertatur in $a$.

Aio, Æquationem sic reformatam modum ostendere ducendæ Tangentis ad punctum D datum. Cùm enim eo dato, pariter datæ sint $y$ & $v$, & cæteræ quantitates, quæ Consonantibus exprimuntur, $a$ non poterit ignorari.

Si quid fortè sit obscuritatis in Regulâ, aliquot Exemplis illustrabitur: Data sit hæc Æquatio, $by - yy = vv$; in qua EB sit $b$; BA, $y$; DA, $v$; & quæratur $a$, sive AC, talis ut juncta DC, tangat Curvam DQ in D. Ex Regula, nihil rejiciendum est ab hac Æquatione, cùm in singulis ejus partibus reperiatur $y$ vel $v$. Ita quoque disposita est, ut ab uno latere sint omnes illius partes in quibus $y$; ab altero, omnes in quibus $v$. Singulis itaque tantùm præfigendus est Exponens potestatis, quam in illis habet $y$ vel $v$; & in latere sinistro unum $y$ vertendum in $a$, ut fiat $ba - 2ya = 2vv$. Aio nunc, hanc Æquationem ostendere modum ducendæ Tangentis ad punctum D, sive

$$a = \frac{2vv}{b - 2y} = AC.$$

Sic si data fuisset Æquatio, $qq + by - yy = vv$; eadem planè fieret cum priori Æquatio pro Tangente, abjecto scil. $qq$, ut Regula præscribit.

Sic ex $2byy - y^3 = v^3$ fit $4bya - 3yya = 3v^3$, sive $a = \frac{3v^3}{4by - 3yy}$: Ex $bby + zyy + y^3 = qvv$, fit $bba + 2zya + 3yya = 2qvv$, & $a = \frac{2qvv}{bb + 2zy + 3yy}$: Ex $b^4 + by^3 - y^4 = qqvv + zv^3$, fit $3byya - 4y^3a = 2qqvv + 3zv^3$, & $a = \frac{2qqvv + 3zv^3}{3byy - 4y^3}$.

Verùm in similibus Æquationibus nullam arbitror accidere posse difficultatem. Aliqua fortasse in illis occurrit, quarum partes quædam constant ex productis $y$ in $v$: Ut $yv$, $yyv$, $y^3vv$, &c. Sed hæc quoque levis est, ut Exemplis patebit. Detur enim $y^3 = bvv - yvv$. Nihil ab illa rejiciendum erit, cùm in singulis ejus partibus reperiatur $y$ vel $v$.

Sed ut ex Regulæ præscripto disponatur, bis sumendum erit $yvv$, & statuendum tàm in latere dextro, in quo sunt partes quæ habent $v$, quàm in sinistro, cujus partes habent $y$; quandoquidem $yvv$, tàm $y$ quàm $v$ contineat. Faciendum igitur erit $y^3 + vvy = bvv - yvv$. Tum mutatâ, ut priùs, hâc Æquatione in aliam $3yya + vva = 2bvv - 2yvv$, dabitur $a = \frac{2bvv - 2yvv}{3yy + vv}$.

Ita enim intelligenda eſt Regula, ut nempe in latere ſiniſtro non conſideretur poteſtas ipſius $v$, ideoque ipſi $yvv$, Exponens $vv$, præfigi non debeat, ſed tantùm ipſius $y$: Sicut contra ab alio latere, in $yvv$ conſiderari non debet poteſtas ipſius $y$, ſed $v$ tantùm, eique ſuus Exponens præponi. Sic ſi foret $y^5 + by^4 = 2qqv^3 - yyv^3$, faciendum eſſet $y^5 + by^4 + v^3yy = 2qqv^3 - yyv^3$; & haberetur Æquatio pro Tangente $5y^4a + 4by^3a + 2v^3ya = 6qqv^3 - 3yyv^3$ & $a = \frac{6qqy^3 - 3yy.v^3}{5y^4 + 4by^3 + 2v^3y}$.

Atque his Exemplis arbitror, me omnem, quæ dari poſſet, Caſuum varietatem complexum eſſe. Cæterum non erit fortaſſe inutile, ſi ea quæ generatim expoſui, ad lineam aliquem ſingularem applicem. Data ſit igitur Curva
Fig. 15. B D, cujus ea ſit proprietas, ut ſumpto in illa quolibet puncto D, ſi jungatur B D, & erigatur ad illam normalis D E, occurrens rectæ B E in E, recta D E ſit ſemper æqualis datæ rectæ B F. Ut habeatur Æquatio in terminis Analyticis, ſit D A $= v$, B A $= y$, B F vel D E $= q$. Erit itaq; E A $= \frac{vv}{y}$. Et cùm quadratum D E æquale ſit duobus D A, A E; erit Æquatio $qq = \frac{v^4}{yy} + vv$; ſive $qqyy = v^4 + yyvv$; quæ pro Tangente ex Regulæ præſcripto, ſic reformanda erit, $qqyy - vvyy = v^4 + yyvv$, &; $2qqya - 2vvya = 4v^4 + 2yyvv$, & $a = \frac{4v^4 + 2yyvv}{2qqy - 2vvy}$.

Quomodo autem Æquationis hujuſmodi ad faciliores terminos pro conſtructione reduci debeant, id ſanè ſolertem Geometram minimè latebit. Ut ecce in hoc exemplo, quoniam Rectangulum B A E ſupponitur æquale Quadrato A D, ſi E A dicatur $e$, erit $vv = ye$, & $v^4 = yyee$, & $qq = ye + ee$. Itaq; pro illis, poſito in Æquatione eorum valore, fit $a = \frac{4yyee + 2y^3e}{2eey + 2eyy - 2eyy}$, ſive $a = \frac{2ey + yy}{e}$, hoc eſt, $ae = 2ey + yy$, & addito $ee$ utrinque $ae + ee = ee + 2ey + yy$. Erant itaque tres $e$, $e + y$, $e + a$, ſive E A, E B, E C, in continua analogia, & facillima evadet conſtructio.

Cæterùm quoniam hactenus ſuppoſuiſſe videmur, Tangentem verſus partes B ducendum eſſe, cùm tamen ex datis accidere poſſit, ut vel parallela ſit ipſi A B; vel etiam ducenda ad partes contrarias; definiendum nunc ſupereſt, quomodo hæc caſuum diverſitas in Æquationibus diſtinguatur, factâ igitur Fractione pro $a$, ut in Exemplis ſuprà adductis, conſiderandæ ſunt partes tàm Numeratoris quàm Denominatoris, & earum Signa.

1. Nam ſi in utroque, partes vel habeant omnes Signum +, vel ſaltem Affirmatæ prævaleant Negatis, ducenda erit Tangens verſus B.

2. Si Affirmatæ prævaleant Negatis in Numeratore, ſed æquales ſint in Denominatore, recta per D ducta, parallela A B, tanget Curvam in D: hoc enim in caſu, $a$ eſt infinitæ longitudinis.

3. Si

3. Si tàm in Denominatore, quàm Numeratore, partes Affirmatæ minores ſunt Negatis; mutatis omnibus Signis, ducenda erit rurſus Tangens verſus B: hic enim Caſus cum primo in idem recidit.

4. Si in Denominatore prævaleant, in Numeratore minores ſint, vel contra; mutatis Signis illius in quo ſunt minores, ducenda erit Tangens verſus partes contrarias, *h. e.* A C ſumenda erit verſus E.

5. Ac tandem ſi in Numeratore partes Affirmatæ ſint æquales Negatis, quomodocunque ſe habeant in Denominatore, $a$ abibit in nihilum. Itaque vel ipſa A D erit Tangens, vel ipſa E A, aut ei Parallela; quod ex datis facilè dignoſcitur. Horum autem Caſuum Varietas explicari poteſt per Æquationes ad Circulum.

Sit enim Semicirculus, cujus Diameter E B, & in eo punctum D datum, ex quo cadat Normalis D A $= v$. Sit B A $= y$, B E $= b$; erit Æquatio $by - yy = vv$, & ductâ Tangente D C, erit A C, ſive $a \frac{2vv}{b - 2y}$. Nunc ſi $b$ major ſit $2y$, ducenda eſt Tangens verſus B; ſi æqualis, fit Parallela E B; ſin autem minor, ducenda eſt verſus E; ut *n.* 1, 2, & 4. diximus. Fig. 16.

Detur rurſus alius Semicirculus inverſus, cujus puncta referri intelligantur ad rectam Diametro parallelam, & eidem æqualem, ut in Schemate. Denominatis, ut priùs, partibus, & N B $= d$, fit Æquatio $by - yy = dd + vv - 2dv$. Igitur A C, ſive $a = \frac{2vv - 2dv}{b - 2y}$. Cùm verò in Exemplo ſuppoſuerimus, $v$ ſemper eſſe minorem $d$; ſi $b$ ſit major $2y$, ducenda erit Tangens verſus E; ſi æqualis, erit parallela; ſin minor, mutatis omnibus Signis, ducenda erit verſus B; ut *n.* 4, 5, & 3. Nulla autem ducenda eſſet Tangens, ſeu Tangens foret ipſa E B, ſi ſuppoſuiſſemus N B æqualem Semidiametro, ſive $2d = b$; ut *n.* 5. Fig. 17.

Sit tandem alius Semicirculus, cujus Diameter N B normalis ſit ad Rectam B E, ad quam ejus puncta referri intelligantur. N B dicatur $b$, & aliæ partes denominentur ut ſuprà; fiet Æquatio $yy = bv - vv$; & $a = \frac{bv - 2vv}{2y}$. Jam ſi $b$ ſit major $2v$, Tangens ducenda erit verſus B; ſi minor, verſus E; ſi autem æqualis, ipſa D A erit Tangens; ut *n.* 1, 4, & 5. Fig. 18.

Et hæc eſt, ni fallor, Caſuum omnium Varietas, quæ ex Æquationum conſideratione deprehendi poteſt.

*The Lemmata, whereby the preceding Method is demonſtrated, by M. Sluſius. n. 95. p. 6059. n. 97. p. 6126.*

2. (1.) Differentia duarum dignitatum ejuſdem gradûs applicata ad differentiam laterum, dat partes ſingulares gradûs inferioris ex binomio laterum; ut $\frac{y^3 - x^3}{y - x} = yy + yx + xx$. Quod facilè oſtenditur.

(2.) Tot ſunt partes ſingulares ex binomio in gradu quolibet, quot unitates habet Exponens dignitatis immediatè ſuperioris; tres nimirum in Quadrato, quatuor in Cubo, *&c.* Et hoc vulgò notum.

(3.) Si quantitas eadem applicetur ad duas alias, quarum ratio data sit, Quotientes erunt reciprocè in eadem ratione data.

His Lemmatibus Methodus mea facilè demonstratur, cùm eo ordine disposita sint, qui ad illam quasi manu ducit.

*The* Testudo Veliformis Quadrabilis *Ænigmatically propos'd by* V.V. n. 196. *p.* 584.

VI. 1. *ÆNIGMA Geometricum, De miro Opificio Testudinis Quadrabilis Hemisphæricæ, à D.* Pio Lisci Pusillo Geometra *propositum.*

Inter Venerabilia Eruditæ olim Græciæ Monumenta extat adhuc, perpetuò equidem duraturum, Templum Augustissimum Ichnographia Circulari, ALMÆ GEOMETRIÆ dicatum, quod, à Testudine intùs perfectè Hemisphærica, operitur: sed in hac, Fenestrarum quatuor æquales Areæ (circum, & supra Basin Hemisphæræ ipsius dispositarum) tali Configuratione, Amplitudine, tantâque Industriâ, ac Ingenii acumine sunt extructæ, ut, his detractis, superstes Curva Testudinis Superficies, pretioso Opere Musivo ornata, Tetragonismi verè Geometrici est capax.

Quæritur modo, Quæ sit, Quâ Methodo, Quâve Arte, Pars illa Hemisphericæ Superficiei Curvæ Quadrabilis, Tensi ad instar Carbasi, vel Turgidi Veli Nautici, ab Architecto illo Geometra fuerit assecuta? & cui demum Plano Geometricè Quadrabili sit æqualis?

*Solv'd by Dr.* Wallis, *ibid. p.* 587.

2. Accepi, V. C. nudius tertius (noctu decubiturus) Literas tuas quibus heri non vacabat, aliàs occupato, respondere, eisque inclusam Chartulam, Typis impressam, quam ais Florentiâ te accepisse mihi mittendam.

Continet ea Chartula Ænigma Geometricum, quod (verborum involucris exemptum) hoc innuere judico Problema; Ab Hæmisphærii curva Superficie, Segmenta quatuor inter se æqualia sic amputare, ut reliquum sit Tetragonismi capax.

Simulque videtur innuere, in veteris Græciæ Monumentis etiamnum extare quidpiam quo illud fiat.

Hoc esse existimo Hippocratis Chii Quadraturam Lunulæ.

Quippe cùm Archimedes demonstravit, Curvam Hemisphærii Superficiem æqualem duobus Circulis ejusdem Sphæræ maximis, (id est quatuor Semicirculis;) docuitque Hippocrates Chius Lunulam quadrare quandam: Si singulis Hemisphærici hujusce Fornicis Quadrantibus tantundem eximatur, quanto deficit à Semicirculo ea Lunula, Reliquum æquabitur Quadrato, quod Circulo Sphæræ maximo (cui hic insistit Fornix Hemisphæricus) Inscribatur.

Si tamen præter Ænigmaticam Problematis Involutionem, subsit aliquid (de Templo) Historicum; putaverim ego S. Sophiæ (quod est Constantinopoli) Templum hic insinuatum.

*SCHOLIUM.*] Per Hippocratis Chii Quadraturam Lunulæ (primo Physicorum Aristotelis & Simplicii in eum locum Commentariis, indicatam,) Si Semicirculo A B D, in duos Quandrantes A C D, B C D diviso, aptetur A D Subtensa Quadrantalis Arcus, Radio C E bisecta in H: & Centro H scribatur Semicirculus A D F: Erit (propter Quadratum Rectæ A D Subduplum Quadrati Rectæ A B) Semicirculus A D F Subduplus Semicirculi A B D; Fig. 194.

adeoque

adeoque Quadranti A C D æqualis. Et (dempto utrinque communi Segmento A D E) reſidua Lunula A E D F reſiduo Triangulo A D C æqualis. Taleſque quatuor Lunulæ, talibus quatuor Triangulis; hoc eſt, Quadrato toti Circulo inſcripto A D B G.

Porro, per Archimedis demonſtrata, æquatur Sphæræ Superficies, quatuor Circulis in ea Sphæra Maximis; Adeoque Hemiſphærii Superficies Curva, talibus quatuor Semicirculis; taliſque Superficiei Hemiſphæricæ Quadrans, uni Semicirculo.

Circulus A D B G eſto jam Baſis Hemiſphæricæ Superficiei Curvæ; cujus Polus P, Axis C P, plano Baſis perpendicularis, ejuſq; Quadrans unus D P A; qui Plano E P C per Axem tranſeunte, biſecetur. *Fig. 20.*

Ponantur item (ob commodiorem Calculum) Circuli Radius R, Diameter D $=$ 2 R, Peripheria P, Expoſitus Arcûs *a*.

Poſitoque Quadrantali Arcu D E A $= a = \frac{1}{4}$ P; eſt Semicirculus A B D $= a$ R $= \frac{1}{4}$ R P: Triangulum A D C $= \frac{1}{2}$ R$^2$ $= \frac{1}{4}$ R D; reliquumq; Semicirculi (dempto hoc Triangulo) $\frac{1}{4}$ R P — $\frac{1}{4}$ R D; cui æquale auferendum eſt ex D P A (Quadrante Superficiei Hemiſphæricæ Curvæ, æquali Semicirculo A B D) quo Reſiduum æquetur Expoſito Triangulo A D C.

Quod cum variis modis fieri poſſit; per ea quæ nos dudum docuimus *Anno* 1659. (ad Calcem Tractatûs *de Cycloide*, tum editi, *p.* 122. inſerenda ad §. 68.) iterumq; *Anno* 1670. (in Tractatus *de Motu*, Cap. V. Prop. 24.) de Figura Plana, æquali cuivis in Superficie Sphærica Figuræ, Circulis quibuſvis (ſive Maximis, ſive Minoribus) terminatæ. Sic fiat ſimpliciſſimè;

Cum Superficiei Sphæricæ Segmenta, Parallelis Planis Abſciſſa, ſint Axis Segmentis proportionalia (quod de expoſita quadrantalis Cunei Superficie D P A pariter valet:) Si ſumatur, in Axe C P, ut Semicirculus $\frac{1}{4}$ R P, ad Semicirculum dempto Triangulo $\frac{1}{4}$ R P — $\frac{1}{4}$ R D; hoc eſt, ut P ad P — D; ſic C P ad C Y: (ſive, quod tantundem eſt, ut P ad D; ſic C P ad P Y:) Planum per Y Z Baſi parallelum, abſcindet hujus Superficiei Curvæ portionem Polo adjacentem, æqualem Triangulo A D C. Quod cum in reliquis Superficiei Curvæ Quadrantibus, pariter fiat; æquabitur totum Abſciſſum (Polo adjacens) toti Quadrato Baſi inſcripto: Et ſic Tenſum ut oportuit. *Q. E. F.* *Fig. 21.*

Vel ſic brevius. Eſt Hemiſphærii Superficies Curva (utpote duobus Circulis Maximis æqualis) $=$ R P; Quadratum Circulo Maximo inſcriptum $=$ 2 R R $=$ R D; Illudque ad hoc, ut P ad D. Adeoque (propter Segmenta Superficiei Parallelis Planis abſciſſa, Segmentis Axis proportionalia) ſumptis C P ad P Y, ut P ad D, erit tum tota Superficies $=$ R P, tum Portio ad Polum, plano Z Y abſciſſa, $=$ R D, Quadrato Baſi inſcripto. *Q. E. F.*

Si dicatur; Proceſſum hic eſſe ex præſumpti Circuli Quadratura, aut ratione quam habet Circuli Perimeter ad Diametrum: Id omnino verum eſt. Sed non eſt objiciendum. Quia non poſtulat Ænigma propoſitum ut Hemiſphæricæ Superficiei Portiones abſciſſæ, (quas Feneſtras vocat) ſed ut Portio Superſtes, ſit Tetragoniſmi capax. Et quidem ſi utrumq; poſtularet, poſtularet Circuli Quadraturam abſolutè Geometricam; quod haberi non poſſe ſatis conſtat.

Opificium quod ſpectat; ſuper Baſem Planam, extra Baſem Hemiſphærii poſitam, ſed ipſi contiguam, cujus duo Latera in Angulum coëant ad A, intra *Fig. 19.*

protractas

protractas D A, G A Rectas, (quo Fenestrarum quas vocat utrinque adjacentium liber prospectus pateat, non impeditus) extruatur Moles satis firma; ita quidem ut, assurgente Structurâ, promineat ejus acies, angulo suffulta, Circuli Arcum efficiens qualis est D Z, ad altitudinem Y assurgens. Et similiter ad reliquos Angulos D, B, G. Atque his demum Structuris (quasi totidem Columnis) ad eam Altitudinem provectis, imponatur Testudo, sic intus excavata, ut possit Hemisphærica Superficies; Adeoque totum opus imperatum absolvitur.

Fig. 21.

Aliter. Idem fiet si, pro Quadrato Basi inscripto, exponatur Quadratum quodvis Q Q, (quod minus sit quam Hemisphærica Superficies curva). Quippe si sumatur, ut R P (Hemisphærica Superficies curva) ad Q Q (expositum Quadratum, sic) C P (Axis Hemisphærii) ad P Y (Axis Portionem Polo adjacentem:) Planum Z Y (Basi parallelum) abscindet portionem Superficiei Sphæricæ Tetragonismi capacem: Utpote æqualem exposito Quadrato Q Q.

Idem sic aliter absolvi potest: sed majore solicitudine.

Fig. 20.

Cùm sit (ut jam ostensum est) Hemisphæricæ Superficiei curvæ Quadrans D P A æqualis Semicirculo A B D; ejusque Segmenta Planis Basi parallelis abscissa, Segmentis Axis proportionalia: Sumatur in D P Quadrantali arcu, arcus P Q graduum 60; (quod mihi Caswellus suggerit) Polo P descriptus Circulus Q T S bisecabit Axem (propter Sinum Versum grad. 60 = $\frac{1}{2}$ R:) adeoque Quadrantem Hemisphæricæ Superficiei curvæ D P A dirimet in duo Segmenta inter se æqualia. Quorum alterum, D Q T S A Quadrilineum, æquat Quadrantem circularem B C D; reliquumque Trilineum P Q T S æquat Quadrantem A C D. Unde, si porro auferatur Q R S T Bilineum, æquale segmento Circuli A D E: reliquum Trilineum P Q R S, æquabit A D C Triangulum. Taliaq; quatuor, in quatuor Quadrantibus Hemisphærii, æquabunt Quadratum Basi inscriptum. Habebitur autem illud Bilineum per ea quæ nos dudum docuimus locis modò citatis.

Idem universalius sic fiet.

Sumpto Q ubivis in Arcu D Z, (nè major sit D Q quàm D Z;) & quanto deficit Quadrilineum D Q T S A à toto auferendo, tantundem suppleat Bilineum Q R S T: Reliquum æquabit A D C Triangulum.

Et quidem, si sumatur Q in D (quo evanescat Quadrilineum) sumendum erit Bilineum æquale toti auferendo. Sin sumatur Q in Z (ut Bilineo not sit opus) æquabitur Quadrilineum toti auferendo.

Eademque omnia (de Quadrilineo & Bilineo quæ simul compleant totum auferendum) pariter accommodanda erunt (mutatis mutandis) si, pro Quadrato Basi inscripto, substituatur Q Q Quadratum quodvis; quod totâ Superficie curva Hemisphærica non sit majus.

Postquam hæc scripta fuerant, erantque sub prelo, rescivi tandem huic eidem Problemati responsum dedisse Cl. V. D. Leibnitz, illudque in Actis Lipsicis comparere pro Mense Junio, 1692. Quod fecit ut Prelum sufflaminandum curaverim per aliquot septimanas donec illud conspicerem; quod ægrè tandem obtinui. Videoque Cl. Virum juxta mecum sentire, non esse Problema Determinatum, sed mille modis (nedum infinitis) solubile.

3. Ænigmatis

3. Ænigmatis hujus Author Problematis tandem Constructionem ingeniosè admodum & expeditè dedit in Tractatu Italico, *de Formatione & Mensura Testudinum omnium*, ad Serenis. *Etruriæ* Principem, ubi & Nomen suum profiteri dignatus est, nempe à *V. V. Postremo Gallilæi Discipulo*, cum antea dispositis horum verborum ut in Anagrammate elementis, sub ficto nomine *D. Pio Lisci Pusillo Geometra* tectus latuisset. *The Proposer's Solution Demonstrated, by Dr. D. Gregory. n. 207. p. 25.*

Ænigma verùm in sequens Problema ab Authore convertitur; *Super Hemisphærii superficiem, assignare portionem dato quadrato æqualem*: Quod sic construit.

Sphæra cujus Axis æqualis lateri dati Quadrati exponatur per Circulum A C B D in proposita Sphæra verticalem, cujus Diameter Horizontalis est A B, centrum E. Perforetur Sphæra duobus Cylindris rectis quorum communes Sectiones cum plano A C B D sunt Circuli B L E G, A H E I, diametris E B, E A descripti. Dico factum; hoc est à quolibet Hemisphærio *v. g.* superiori A C B ablatas esse per Cylindros perforantes quatuor Figuras bilineares, duas scil. in parte antica & duas in postica æquales similes & similiter positas, ita ut residua superficies Hemisphærica sit æqualis Quadrato rectæ A B. Et quoniam Hemisphærica Superficies, demptis spatiis quatuor Bilinearibus prædictis, refert Velum vento Inflatum & Tensum, Testudinemve Hemisphæricam quatuor Fenestris interruptam, quæ Circulari basi A E B imposita, ipsi ad puncta A, E, E, B, innititur, hanc pro jure suo appellat *Testudinem Veliformem Florentinam Quadrabilem*. *Fig. 22.*

Auctor deinceps in memorato Tractatu plurima ad Praxin attinentia profert, ut ope Torni & Terebræ Cylindricæ tàm hujus quàm reliquarum quinque Testitudinum fiant exemplaria: Atque in hanc rem alia quædam Problemata subtilia construit quorum omnium demonstrationes ab Auctore consulto omissæ facillimè ex nunc proferendis consequentur.

Quod quatuor Fenestræ in Hemisphærio ut dictum est extructæ sint Figuræ æquales similes & similiter positæ satis liquet, reliquum est ut ostendamus reliquam superficiem Hemisphericam Tetragonismi verè Geometrici esse capacem.

Ad planum C A D B, in puncto E, erigi intelligatur Normalis recta æqualis E A, & super Peripheriam A C B D superficies Cylindrica recta ejusdem Altitudinis. Vulgo notum est portionem Superficiei Sphæricæ inter quælibet duo plana Circulo A B C D Parallela comprehensam, æqualem esse portioni Superficiei Cylindricæ inter eadem plana; & horum Annulorum similes portiones resectas à planis in erecta ex E, Normali se mutuo inter secantibus esse etiam æquales. Si jam ducendo innumera plana Basi A C B D parallela dicto modo designari intelligantur in superficie Cylindrica partes respondentibus Sphæricis æquales, quæ è regione Superficiei perforatione ablatæ designatur illi æqualis est. Quare patet residuam à perforatione Superficiem æqualem esse residuæ superficiei Cylindricæ dempta illa quæ è Regione ablatæ per dicta innumera plana designatur. Ducatur Diameter quælibet P M, secans Peripheriam A H E utcunque in H. Jungatur H A, per H ducatur R T normalis ad A B, & parallela ad C D per E ductam, occurrens Peripheria

A C B D in R & T, & Peripheriæ A I E in I. Super R T Diametro fiat Semicirculus cujus Peripheriæ occurrant H S, I Q ad R T normales in S & Q. Hujus Semicirculi Planum intelligatur normaliter erectum ad Circulum A B C D. Unde Peripheria R S Q T erit in Superficie Hemisphærica, rectaque H S nunc ad Planum A C B D normalis, erit Altitudo Superficiei Cylindricæ perforantis supra Baseos punctum H. Idemq; de quolibet puncto superficiei Cylindricæ perforantis verum est, *scil.* ejus altitudinem usque ad Superficiem Sphæræ supra quodvis in Basi punctum H, esse rectam H S, ut dictum est, genitam, sed H S æqualis est H A sinui recto Arcus M A, quoniam tàm hæc quàm illa est Media Geometrica inter P H & H M, altera in Circulo M A P, altera in Circulo Sphæræ etiam Maximo per puncta M, S, & P, transeunte.

Si in erecta in E ad planum A C B D normali, ab E sumatur recta æqualis H S aut H A, & ab extremo ejus puncto ducantur rectæ Parallelæ ad P M & V N, planum per illas extensum erit ad Planum A C B D parallelum, & rectæ hæ per puncta S & Q transibunt, & productæ usque ad Superficiem Cylindricam Hemisphærio circumscriptam abscindent ex lateribus Cylindri rectas ipsis H S vel H A itidem æquales; comprehendentque Arcus æquales & respondentes Arcubus M N & V P. Quod si alterum planum huic ad minimam distantiam parallelum similiter ductum intelligatur, hæc duo per supra ostensa designabunt in Superficie Cylindrica annuli portionem æqualem portioni inter eadem plana à Superficie Hemisphærica perforatione ablatæ. Quod si similis Constructio fieri supponatur ad quodlibet in Peripheria A H E, punctum portiones omnes in Superficie Cylindrica Hemisphæricæ circumscripta dicto modo genitæ & designatæ erunt æquales Superficiei Sphæricæ perforatione ablatæ. Quare residua Superficies Hemisphærica æqualis erit reliquæ superficiei Cylindricæ conflatæ ex rectis omnibus H A ad respectiva puncta M, N, V, & P erectis, seu figuræ sinuum rectorum Semiperipheriarum A C B, A D B, hoc est, per dudum à Geometris cognita, quadruplo Quadrato Radii A E, sive denique quadrato Diametri A B. Cumque duæ integræ Figuræ comprehensæ à communi sectione prædictæ Superficiei Cylindricæ perforantis cum Superficie Sphærica, æquales sint quatuor semissibus earundem, patet residuam superficiem Hemisphæricam A C B, ablatis quatuor spatiis Bilinearibus (ut supra in Constructione) æqualem esse quadrato Diametri A B. *Q. E. D.*

Si Semiperipheria A H E, ita inflectatur ut congruat cum æquali quadrante Peripheriæ A R C; punctum H incident in punctum M ob æquales Arcus A H, A M, & H S altitudo ad H superficiei Cylindricæ super A H E insistentis congruet cum æquali H A altitudine ad M Figuræ Sinuum rectorum super A M C erectæ; idemque in reliquis punctis fiet. Unde curva quæ est communis intersectio Superficiei Sphæricæ cum Superficie Cylindrica super Basi A H E, quamvis non jaceat in eodem Plano inflexa, ut dictum est, congruet, & proinde æqualis est curvæ terminanti figuram Sinuum rectorum; hoc est communi Sectioni Superficiei Cylindricæ supra Quadrantalem Arcum A R C erectæ cum plano secante planum Baseos in recta B A ad Angulos semirectos; sive quadranti curvæ Ellipseos cujus minor Axis est A B, major vero potest hujus duplum. Adeoque Perimeter Veli Quadrabilis Florentini ex hujusmodi quatuor constans æqualis est Perimetro dictæ Ellipseos.

Sed & hoc amplius adnotare non pigebit, Superficies Cylindrorum duorum perforantium intra Sphæram, æquales esse Superficiei Sphæræ post perforationem relictæ, sive duplici Velo Florentino, hoc est duplo quadrato Diametri. Atque hoc exinde patet quod Velum Florentinum æquale sit Figuris quatuor sinuum rectorum Quadrantis & Superficies perforans iisdem etiam sit æqualis, quoniam illis congruit si inflectio fiat ut supra.

Hoc tantum addam, Considerationem Figuræ Sinuum rectorum (cujus etiam partes in Quadrata facilè mutantur) sufficere ad Demonstrationem eorum omnium quæ de aliis solidis Torno elaboratis vel Cylindro perforatis, eorumque Superficiebus ab Acutissimo Geometra V. V. (Vincentio Viviani ni fallor) Dignissimo Galilæi Discipulo proferuntur; dum Fabricam & Mensuram Testudinum docet. Speciatim Superficies Testudinis Scaphoidis Romanæ [*Volta a Schifo alla Romana* ] ex octo Figuris Sinuum rectorum Arcus Quadrantalis constat, ac proinde Testudini Veliformi Florentinæ æqualis est. Unde patet quomodo æqualibus quadratis superimponi possunt duæ Testudines, quarum altera est undique clausa, altera quatuor Fenestris interrupta, utraque Quadrati Baseos dupla.

VII. 1. *Drawing the streight Lines E A, and E B (cutting the Arc A B in G,) and on A G, a Perpendicular E F, (which will therefore pass to the Center C, because Bisecting A G at Right Angles ;) the Right-lined Triangle A F E, is equal to A D E, the proposed Portion of the Lunula.*

*The Quadrature of the Parts of the* Lunula, *by Mr.* J. Perks, *a little vary'd by Dr.* J. Wallis. n. 259. *p.* 411. *Fig.* 23.

The Demonstration is to this purpose; *viz.* A D B being a Quadrantal Arc; the Angle A G B will be three Halves of a Right Angle; (and its conjunct Angle E G A, half a Right Angle) and that Angle (being external to the Triangle A G E, is equal to the two opposite Intervals G E A + E A G. Whereof G E A (because an Angle in the Semicircle A E B) is a Right Angle, and therefore E A G is half a Right Angle, (as are also F E G, and F E A) and the three Triangles A F E, G F E, and G E A, each of them half a Square. And A G to A E, as $\surd$ 2 to 1, (proportional to the Respective Radii of the two Circles). And the like Segments A D G, A E, in their respective Circles (as the Squares of their respective Radii) as 2 to 1. And therefore the Semisegment A F D, equal to the Segment A E. And consequently (one taking from the Triangle, as much as the other adds to it) the portion of the Lunula A D E, equal to the Triangle A F E. *Q. E. D.*

If the Point E chance to be in K (the middle of the Arc A E B) there will be no Intersection at G (the points G, B being then coincident, but without any disturbance to the Demonstration:) If it happen beyond it, toward B; then G will be on the other side; and what is here said of E G B, must be accommodated to E G A.

The Ground of the whole Process is plainly this; The Angle A C E, being an Angle at the Center of the greater Circle, but at the Circumference of the lesser, the Line C D E (as it passeth from C A to C B) doth in the same proportion, divide the Quadrantal Arc A D B, and the Semicircular A E B: whence all the rest doth naturally follow.



2. If

*Improved by Dr.* Gregory, *ibid.* p. 414. Fig. 24.

2. If you compleat the two Circles, whoſe Arcs contain the *Lunula* of *Hippocrates*; the ſame is true as well of the Points in the other Semicircle A C B, as of thoſe in the Semicircle A E B; and for the ſame Reaſons. As appears in the Scheme annex'd, wherein I have mark'd the Points in the Semicircle A C B, (correſpondent to thoſe of Mr. *Perks* in A E B,) with the Correſpondent ſmall Letters of the *Roman* and *Greek* Alphabets.

If Mr. *Perks* had made his Conſtruction Univerſal by making both E A, and E B, meet with the greater Circle, (which he might have done by Protracting theſe Lines and the greater Circle till they meet;) he might have found that the portions of the Spaces A ε C M, B H C N, (ſuppoſing M C N Parallel to A B) are Quadrable as well as thoſe of *Hippocrates*'s *Lunula*: And that E A γ being a ſtreight Line, the Portion A E D of *Hippocrates*'s *Lunula*, is to A ε δ (the Correſpondent of A ε C M) in duplicate proportion of C ε to A ε. For E R ε (at R the Center of the leſſer Circle) is in this Caſe, a Right Angle.

Moreover, If you take any Point ε, in the Semicircle A C B, and proceed according to Mr. *Perks*'s Conſtruction Univerſaliz'd as above ſaid; you will find on the one ſide, the Trilineum A ε δ (contained by the Arcs A ε, A δ, and the ſtreight Line ε δ) equal to the Rectilineal Triangle A ε φ. And on the other ſide, the Trilineum contained by the Arc B ε (the Complement of ε A to the Semicircumference,) and the Arc B *d* (the Complement of A δ to the fourth Part of the Circumference,) and the ſtreight Line ε *d*, (that is the Trilineum B H C *d* diminiſhed by the Segment C ε;) to be equal to the Rectilineal Triangle B ε *f*. And that thoſe two Spaces A ε δ, and the difference of B H C *d*, from the Segment C ε (parts of the *Lunula* A C B *g* γ A) taken together, are equal to the Triangle A C B, as well as the two Spaces A E D, and B E D, Parts of the *Lunula* of *Hippocrates*.

So that upon the whole it appears, that the two Circles (containing the *Lunula* of *Hippocrates*) being compleated, this *Lunula* A E B G A, and the other A C B *g* γ A, make up one Syſtem, and are Conjugate Figures.

For, (drawing a ſtreight Line C D E, or C ε δ, or ε C *d*, at Pleaſure, thro' C the Center of the greater Circle, and cutting thoſe two Circles, the ſpace contained within two Arcs of theſe two Circles, and part of the ſaid ſtreight Line, (as A E D, or A ε δ, or B H ε *d*) is equal to the Rectilineal Triangle A E F, or A ε φ, or B ε *f*, reſpectively.

And it ſo happens, that, if this Line going out from C, be on the ſame ſide of the Diameter M N, with the *Lunula* of *Hippocrates*; the aforeſaid ſpace (which receives a perfect Quadrature) is ſolitary; (ſuch as are the Parts of *Hippocrates*'s *Lunula*, and of the two Spaces A ε C M, B H C N, which therefore are Parts of the *Lunula* more nearly relating to one another).

But if that Line going out from C, be on the other ſide of M N; then the Space which is equal to the Rectilineal Triangle, is the difference of two Mixtilineal Figures (the one a Trilineum, the other a Segment of the leſſer Circle) as is aboveſaid; neither of which can be ſquared ſeverally.

All theſe Particulars are plain from Mr. *Perks*'s Demonſtration; which with a little Variation (ſuch as is uſual in the different Caſes of the ſame Theorem) is applicable to all of them: tho' perhaps he was not aware of it.

The

The like was done (without any Demonſtration) by M. *Tſchirnhauſe* in the *Acta Lipſiæ* 1687. to this purpoſe; If from any Point E, in the Circumference of the leſſer Circle, we let fall on A B, a Perpendicular cutting it in L, and draw the Line C L; the Triangle C A L, is equal to the Portion of the *Lunula* A E D. (And conſequently the Triangle C B L, equal to the Portion B E D.) Which I ſhall Demonſtrate, ſo as the Demonſtration may alſo reach the Portions of the Conjugate Space A C B *g* γ A.

For the Triangles A C B, A E F, are like Triangles, each being the half of a Square: and therefore by 19 *El.* 6, the Triangle A C B is to the Triangle A E F, in the duplicate proportion of B A to A E, that is, by 8 *El.* 6, as B A is to A L. But, by 1 *El.* 6, the Triangle A C B is to the Triangle A C L, as B A is to A L. Therefore by 9 *El.* 5, the Triangles A C L and A E F are equal. But the Triangle A E F is (by Mr. *Perks*) proved equal to the portion A E D. And therefore the ſaid portion A E D, is alſo equal to the Triangle A C L.

3. On the Center B, Mr. *Caſwel* draws by A, a third Circle, which forms another *Lunula*, than that of *Hippocrates*: and he doth (very dexterouſly) ſquare the Portions of this *Lunula*. And doth thereby let us in, to a New Syſtem, which may be purſued in like manner, as Dr. *Gregory* hath done that of *Hippocrates*. *By* Mr. Caſwell *ib. p.* 417.

4. M. *Tſchirnhauſe*, letting fall, from E, (on A B) a Perpendicular E L, determines the Triangle A L C, equal to the Portion A D E. Which being admitted, we may thus divide the *Lunula* in any given Proportion; if we divide A B, at L, in ſuch given Proportion; C L, will, in the ſame Proportion, (becauſe of the common Altitude) divide the Triangle A C B (which is equal to the whole *Lunula*.) And L E (erected at Right Angles on A L B) will determine the point E; from whence if we draw to C, the ſtreight Line E C, this will at D E, divide the *Lunula* in the ſame Proportion. *By Dr.* Wallis *ibid.* Fig. 25.

Mr. *Perks*, on E D C, drawing the Perpendicular A F, determines the Semiquadrate A F E, equal to the propoſed Portion A D E. Which Semiquadrate, is a like Figure, and alike ſituate to A E, as is A C B to A B. Fig. 26.

And therefore (becauſe like Figures are in duplicate Proportion of their Reſpective ſides) if we ſo inſcribe A E, as that the Square of A E be to the Square of A B, in ſuch given Proportion, the *Lunula* will at D E, be ſo divided as is required.

And this will hold (if duly applyed, according as the different Caſes may require) tho' E be taken (in the Continuation of the Semicircle) beyond B. For, ſtill like Figures will be in duplicate Proportion of their reſpective ſides; and C E = C D ± D E. And the ſame is yet improveable much further.

VIII. If upon B C you take any two Points D, E, and draw the Perpendiculars D H, E M, meeting B A in I and L, and cutting a Portion F G M H, of the *Lunula*; the Solid generated by the Converſion of this Portion about the Axis B C, is equal to a Priſm, whoſe Baſe is I L M H, and height the Circumference of a Circle, whoſe Diameter is B C; and the Solid generated by the *The Dimenſion of Solids generated by the Converſion of* Hippocrates *Lunula*, *by M.* Ab. de Moivre, n. 265. *p.* 624.

Fig. 27. the Semicircle B K A, is equal to a Priſm or Semicylinder, whoſe Baſe is the Semicircle B K A, and height the Circumference of a Circle whoſe Diameter is B C.

Having Biſected B A in R, and B C in P, the Surface generated by the Converſion of the Arc H M about the Axis B C, is equal to $\frac{c}{r} \times \overline{BP \times HM + BR \times DE}$ (ſuppoſing the Ratio of the Radius to the Circumference to be as $r$ to $c$) and the Surface generated by the Semicircumference B K A is equal to a Rectangle, whoſe Baſe is the Summ of that Semicircumference and Diameter B A, and height the Circumference of a Circle, whoſe Diameter is B C. As for the Surface generated by the Arc G F, 'tis well known, that it is equal to a Rectangle, whoſe Baſe is the Circumference of a Circle whoſe Radius is B C, and height D E; therefore the Surface generated by the Converſion of the Portion M H F G, is known.

Fig. 28. If upon B A you take any two Points I, L, and draw I N, L V, Perpendicular to it, cutting the Quadrant in O and T, and the Circumference in N and V, the Solid generated by the Converſion of the Portion O N V T about the Axis B A, is equal to a Priſm, whoſe Baſe is I O T L, and height the Circumference of a Circle whoſe Diameter is B A.

Having Biſected B A in R, and drawn C R, meeting the Quadrant in G, the Surface generated by the Converſion of the Arc O T about B A, is equal to $\frac{c}{r} \times \overline{CG \times IL - CR \times OT}$.

Fig. 27. Biſect D E in Y, thro' the Center R draw S Q, Parallel to B C, meeting the Circumference B K A in S, B K Parallel to A C in V, and the Lines D H, E M, in N and O; the Solid generated by the Converſion of the Portion F G M H about the Axis A C, is $\frac{c}{r} \times \overline{\tfrac{1}{3} MO^3 - \tfrac{1}{3} NH^3 + PC \times NOMH + CY \times DNOE - \tfrac{1}{3} EG^3 + \tfrac{1}{3} DF^3}$, and the Solid generated by the Segment K B S is $\frac{c}{r} \times \overline{\tfrac{2}{3} VK^3 + PC \times BVKS}$. Therefore the Solid generated by the Semicircle B K A about A C is $\frac{c}{r} \times \overline{PC \times VQAK + PC \times BCQV - \tfrac{1}{3} AC^3 + \tfrac{2}{3} VK^3 + PC \times BVKS}$, which by due Reduction will be found equal to the Solid generated by the Converſion of the ſame Semicircle about the Axis B C.

Fig. 28. The Solid generated by the Portion O N V T, about the Axis C P, is equal to $\frac{c}{r} \times \overline{\tfrac{1}{3} LV^3 - \tfrac{1}{3} IN^3 - \tfrac{1}{3} QT^3 + \tfrac{1}{3} PO^3 + CS \times PQIL}$.

Fig. 27. From the Points M, H, drop the two Perpendiculars M Z, H W, upon C A prolong'd if need be; the Surface generated by the Converſion

ſion of the Arc H M, about the Axis C A is equal to $\frac{c}{r} \times \overline{PC \times HM}$ $\overline{- RA \times WZ}$, when the point Z is next to C, or $\frac{c}{r} \times \overline{PC \times HM + RA \times WZ}$, when the Point W is next to it.

Thoſe that will think it worth their while to beſtow ſome little Pains to find the Demonſtration of this, may ſolve the following Problem.

*Any two Conic Sections being given, forming a Lunula by their Interſection, and a Right Line being given by Poſition, about which, as an Axis, this Lunula is imagined to turn, to find the Solids generated by the Converſion of any of its Parts, cut off by Lines Perpendicular to that Axis, or Parallel to it, or making any given Angle with it, as alſo the Surfaces made by that Converſion.*

IX. Suppoſe D P V, to be half of an exterior Epicycloid, V B its Axis, V the vertex, V L B half the generant Circle, E its Center; D B the Baſe, C its Center: Biſect the Arc of the Semicircle V B in L, and on the Center C thro' L, draw a Circle cutting the Epicycloid in P: then I ſay the Curvilinear Triangle V L P will be = B E *q* in $\frac{CE}{CB}$; that is, the Square of the Semidiameter of the Generant Circle, will be to the Curvilinear Triangle V L P, as C B the Semidiameter of the Baſe to C E: which C E in the exterior Epicycloid is the Summ of the Semidiameters of the Baſe and Generant, but in the Interior Epicycloid D *p u*, 'tis the difference of the ſaid Semidiameters.

*The Quadrature of a Portion of the Epicycloid, by* Mr. Caſwell. n. 217, *p.* 113. *Fig.* 29.

COROL. I.] In the Interior Epicycloid, if C E is ½ C B, the Epicycloid then degenerating into a Right Line, the Quadrature of the Triangle *l p u*, will be in effect the ſame with the Quadrature of *Hippocrates Chius.*

COROL. II.] If the Semidiameter of the Baſe is ſuppoſed infinite, the Epicycloid then being the Common Cycloid, the Area of the ſaid Triangle will be equal to the Square of the Radius of the Generant; and ſo it falls in with that *Theorem* which *Lalovera* found and calls *Mirabile.*

The general Propoſition from whence I deduced the abovesaid Quadrature, is this; *viz.* the Segments of the Generant Circle are to the Correſpondent Segments of the Epicycloid, as C B, to 2 C E + C B. For Example: Suppoſe F *m* G, the Poſition of part of the Generant, when the point F of the exterior Epicycloid was deſigned, then the Segment F *m* G *n* is to the Segment D F *n* G, as C B to 2 C E + C B. And conſequently the whole Epicycloid to the whole Generant in the ſame Proportion; which is the only Caſe demonſtrated by *M. de la Hire.*

It follows alſo, that in the vulgar Cycloid, its Segments are triple of the Correſpondent Sectors of the Generant, which was firſt ſhewn by Dr. *Wallis.*

X. *Areas*

A general Proposition for measuring all Cycloids and Epicycloids, by Mr. Edm. Halley. n. 218. p. 125.

X. *Area Cycloidis vel Epicycloidis, sive Primariæ sive Contractæ vel Prolatæ, est ad Aream Circuli Generantis; atque etiam Areæ partium genitarum in iisdem Curvis, ad Areas analogorum segmentorum Circuli, ut summa duplæ Velocitatis centri ac Velocitatis motus Circularis, ad Velocitatem motus Circularis.*

Fig. 30.

*Demonstratio.*] Describatur Epicyclois quævis Y P S Q V B, revolutione circuli V L B, super Basi circulari Y M N B; ponatur centrum circuli generantis in *c*, ductâque *c* M K, insistat circulus Basi in puncto M; sitque punctum lineans S. Jam divisis motibus transferatur primum motu circulari punctum S in R, ut augeatur arcus S M particulâ indivisibili R S; deinde progrediatur centrum *c* in C; hoc motu, traducto segmento R S M in situm Q T N, punctum Q tanget Curvam. Patet triangulum R S M esse momentum sive fluxionem Areæ segmenti Circuli: Trapezium vero Q S M N esse Fluctionem Areæ spatii curvilinei simul geniti. Jam cum S M, R M, Q M, non nisi puncto inter se deferre intelligantur, concipe Areolam Q S M N constare ex tribus sectoribus R M S, R M Q, M Q N; adeoque Areolam R M S, esse ad Areolam Q S M N, ut est angulus R M S ad summam trium angulorum R M S + R M Q + M Q N. At anguli R M Q + M Q N, æquantur angulis M C N + M K N, sive angulo *c* M C; propter lineas R M, Q N invicem inclinatis sub angulo ipsi M K N æquali ac propter angulum M Q N ipsius M C N dimidium (per *Eucl.* 3. 20.) Proinde angulus R M S est ad angulos R M S + *c* M C, hoc est (per eandem 3. 20.) arcus $\frac{1}{2}$ R S, ad duos arcus C *c* + $\frac{1}{2}$ R S, sive R S ad 2 C *c* + R S; ut areola R S M ad areolam Q S M N: sive ut momentum segmenti circularis Q T N, ad momentum segmenti in Epicycloide simul geniti Q S Y M N. Cumque hæc momenta semper sint in eadem illa ratione, ubicunque assumpseris punctum Q, constat Areas ipsas Q T N, Q S Y M N his momentis genitas, eandem habere constantem rationem, nempe velocitatis motus circularis R S, ad duplam velocitatem centri addito motu circulari, sive 2 C *c* + R S. Sicut etiam Aream V B Z ad Aream Q V B N, ac proinde semicirculum V L B ad spatium Curvilineum V Q Y N B. Ergo constat propositio. Nulla autem alia est differentia in modo demonstrandi, si Circulus genitor super Arcu Basis Concavæ moveatur, nisi quod angulus *c* M C, hoc in casu, sit differentia angulorum M C N, M K N. Si vero Basis sit linea recta, evanescente M K N, ac ob R M, Q N parallelas, etiam facilior erit probatio. In omnibus autem hujusmodi Curvis portiones analogæ portionibus illis quas in Cycloide primariâ perfectæ Quadraturæ capaces invenit *Cl. Wallisius*, sunt æque quadrabiles, quod quidem facilè consequitur ex præmissis.

Centro K, per punctum Q duc circularem arcum Q Z, ac age Z B abscindens segmentum Z L B, æquale segmento Q T N, dein biseca semicirculum V B in L, ac per punctum L, centro etiam K, describe arcum P L, secantem Epicycloidem in P, circulum Genitorem in T, ac Chordas Q N, Z B in *y* & X. Jam sit Arcus V Z = *a*, ejusque sinus = *s*, Radius Genitoris = *r*, Radius vero Basis = R; sitque arcus C E, sive motus centri = *m*.

Patet

Patet Sectorem C K E eam rationem habere ad ſpatium X $y$ N B, quam habet quadratum ex K E, ad differentiam quadratorum ex K L & K B; ſive ut $RR + 2Rr + rr$ ad $2Rr + 2rr$; hoc eſt ut $R + r$ ad $2r$, vel K E ad B V; ac proinde rectangulum B E in C E ſive $rm$ æquari ſpatio X $y$ N B. Spatium vero V Z B æquale eſt rectang. $\frac{1}{2}ar + \frac{1}{2}sr$; adeoque juxta noſtram propoſitionem, erit ut $a$ ad $2m$, ita $\frac{1}{2}ar + \frac{1}{2}sr$ ad $\frac{mar + msr}{a}$, æquale ſpatio Curvilineo Q V Z L B N Q: Ex hoc ſubduc ſpatium X $y$ N B $= rm$, & remanebit ſpatium Q V Z X $y = \frac{mrs}{a}$: Cumque ſpatia Z X L, Q $y$ T æquenter inter ſe, ſpatium Q V L T Q etiam æquabitur ipſi $\frac{mrs}{a}$: Quoties itaque $a$ ad $m$, ſive motus circularis ad progreſſivum Centri, fuerit in data ratione, dabitur etiam perfecta Quadratura ſpatiorum Curvilineorum Q V L T Q: Totumque ſpatium V P L, ad Quadratum Radii B E, erit in eadem ratione motuum $m$ ad $a$, hoc eſt, in omni Epicycloide primariâ, in ratione radiorum K E ad K B, quæ eſt ipſa *D. Caſwelli* Propoſitio. Spatia autem minora Q V L T Q erunt inter ſe ut Sinus arcuum V Z; ac ſpatia Triangularia Q T P, eodem argumento erunt ut Sinus Verſi arcuum Q T vel Z L: ac proinde etiam quadrantur. Pari modo probabuntur ſpatia P Λ ϒ, $p$ L $u$, $p$ λ ϒ, ſemper eſſe ad Radii B E quadratum (in omnibus his figuris) in prædicta ratione $m$ ad $a$; eorumque portiones $p\ q\ t$, ut Sinus Verſi arcuum interceptorum $q\ t$. Reſidua autem ſegmenta, ut $q\ t$ ϒ Λ, $q\ t$ ϒ λ, &c. erunt ut ſinus recti complimentorum eorundem arcuum $q\ t$.

Componitur autem ratio velocitatum $m$ ad $a$, ex ratione radiorum K E, B E, ac ratione angulorum ſimul æquabiliter deſcriptorum C K E, V E Z: ac proinde datâ etiam illâ angulorum ratione, etiam Quadrabuntur ſpatia omnia Epicycloidalia prædicta.

XI. 1. *Quæritur Curva ejus Proprietatis, ut duo Segmenta* (lineæ rectæ à dato puncto per curvam ductæ) *ad quamcunque potentiam datam elevata & ſimul ſumpta faciant ubique unam eandemque ſummam.* Generalem ſolutionem, Analyſtis eruendam relinquimus. *A Problem propos'd by* M. J. Bernoulli. n. 224. *p.* 387.

2. Problema (ſi rectè intellexi) ſic proponi poteſt. Quæritur Curva K I L ea lege ut ſi recta P K L à dato quodam puncto P, ceu Polo utcunque ducatur, & eidem Curvæ in punctis duobus K & L occurrat, poteſtates duorum ejus ſegmentorum P K & P L, à dato illo puncto P ad occurſus illos ductorum ſi ſint æque altæ (id eſt vel quadrata, vel cubi vel quadrato-quadrata, &c.) datam ſummam P K $q$ + P L $q$, vel P K $^{cub.}$ + P L $^{cub.}$ &c. (in omni rectæ illius poſitione) conficiant. *Solv'd by*: .... *ibid. p.* 389. *Fig.* 31.

*SOLUTIO.*] Per datum quodvis punctum A, ducatur recta quævis infinita poſitione data A D B, recte mobili P K L occurrens in D, & nominentur A D, $x$ & P R, vel P L, $y$, ſinque Q & R quantitates ex quantitatibus quibuſcunque datis & quantitate $x$ quomodocunque conſtantes, & relatio inter $x$ & $y$ definiatur per hanc æquationem $YY + QY + R = 0$. Et ſi R ſit quantitas data, Rectangulum ſub ſegmentis PK & PL dabitur. Si Q ſit quantitas data, ſumma ſegmentorum illorum (ſub ſignis propriis conjunctorum) dabitur. Si $QQ - 2R$ datur, ſumma quadratorum ($PKq + PLq$) dabitur. Si $Q^3 - 3QR$ data ſit quantitas, ſumma cuborum ( $PK^{cub.} + PL^{cub.}$) dabitur. Si $Q^4 - 4QQR + 2RR$ data ſit quantitas, ſumma quadrato-quadratorum ($PKqq + PLqq$) dabitur. Et ſic deinceps in infinitum. Efficiatur itaque ut R, Q, $QQ - 2R$, $Q^3 - 3QR$, &c. datæ ſint quantitates, & Problema ſolvetur. *Q. E. F.*

Ad eundem modum Curvæ inveniri poſſunt quæ tria vel plura abſcindent ſegmenta ſimiles proprietates habentia. Sit æquatio $Y^3 + Qyy + Ry + S = 0$; ubi Q, R & S, quantitates ſignificant ex quantitatibus quibuſcunque datis, & quantitate $x$ utcunque conſtantes; & Curva abſcindet ſegmenta tria. Et ſi S data ſit quantitas, contentum ſolidum illorum trium dabitur. Si Q ſit quantitas data, ſumma trium illorum dabitur. Si $QQ - 2R$ ſit data quantitas, ſumma quadratorum ex tribus illis dabitur.

*The Use of Fluxions in the Solution of Geometrick Problems; by Mr.* Abr. de Moivre. n. 216. p. 52.

XII. Habes hic Methodum de Figurarum Curvilinearum Quadraturis; de Solidorum à Rotatione Plani genitorum eorumque ſuperficierum dimenſione; de Rectificatione Curvarum, deque Centri Gravitatis Calculo. Priuſquam, verùm, ulterius progrediar, hoc te monitum velim me uſurpare illa, quæ demonſtravit Clariſſimus *Newtonus*, in *pag.* 251, 252 & 253. *Princ. Phil.* circa Momentanea Incrementa vel Decrementa Quantitatum quæ Fluxu continuo creſcunt vel decreſcunt, præſertim quod dignitatis cujuſcunque $A^{\frac{n}{m}}$, Momentum ſit $\frac{n}{m} a A^{\frac{n}{m} - 1}$.

Porro data Fluxione $\frac{n}{m} a A^{\frac{n}{m} - 1}$ viciſſim reperiri poteſt Quantitas Fluens $A^{\frac{n}{m}}$, 1° tollendo $a$ de Fluxione, 2° Fluxionis Indicem Unitate augendo, 3° denique Fluxionem dividendo per Indicem ſic Unitate auctum.

Curvæ abſciſſa deſignabitur deinceps per $x$, ejus Fluxio per $\dot{x}$, ordinatim applicata per $y$, ejuſque Fluxio per $\dot{y}$.

His poſitis ut ad Quadraturas deveniamus, 1° aſſumatur valor ordinatim applicatæ ope Æquationis naturam Curvæ exprimentis. 2° Multiplicetur hic valor per Fluxionem abſciſſæ; rectangulum hinc ortum erit Fluxio Areæ. 3°. Data Fluxione Areæ reperiatur quantitas Fluens, habebitur Area quæſita.

Proponatur æquatio $x^m = y^n$ cujuſvis Paraboloidos naturam exprimens, valor ordinatim applicatæ $y$ eſt $x^{\frac{m}{n}}$, qui ſi multiplicetur per $\dot{x}$ Rectangulum

$x^{\frac{m}{n}}\dot{x}$ erit Fluxio Areæ, proindeque Area quæſita erit $\frac{n}{m+n} x^{\frac{m}{n}+1}$; ſeu (poſito $y$ pro $x^{\frac{m}{n}}$) $\frac{n}{m+n} x y$.

Rurſum proponatur Curva cujus Æquatio ſit, $x^4 + aaxx = yy$ (illa ſcilicet quæ inter Exempla *Cl. Craigi* extat prima) aſſumpto $x\sqrt{xx+aa} = y$, Fluxio Areæ erit $x\dot{x}\sqrt{xx+aa}$; cum autem ſub Radicalitate involvatur, ſupponatur $\sqrt{xx+aa} = z$, hinc $xx+aa = z^2$, ideoque $x\dot{x} = z\dot{z}$; poſitiſque $z\dot{z}$ & $z$ pro $x\dot{x}$ & $\sqrt{xx+aa}$, Fluxio à Surdis liberata erit $z^2\dot{z}$, quam ſi ad Originem ſuam $\frac{1}{3}z^3$ revocaverimus, repoſitoque $\sqrt{xx+aa}$ pro $z$, habebitur $\frac{1}{3}\overline{xx+aa}\sqrt{xx+aa}$ pro Area quæſita.

Sed quo magis conſtet quam facili negotio conficiantur hujuſmodi Quadraturæ, unum amplius Exemplum proferre viſum eſt; Æquatio Curvæ talis ſit $\frac{x^2}{x+a} = y^2$, igitur $y = \frac{x}{\sqrt{x+a}}$, ideoque $\frac{x\dot{x}}{\sqrt{x+a}}$ eſt Fluxio Areæ: ſupponatur $\sqrt{x+a} = z$, hinc $x = zz - a$, & $\dot{x} = 2z\dot{z}$, itaque $\frac{x\dot{x}}{\sqrt{x+a}} = 2z^2\dot{z} - 2a\dot{z}$, ac proinde $\frac{2}{3}z^3 - 2az$ ſeu $\frac{2}{3}\overline{x - \frac{4}{3}a}\sqrt{x+a}$ erit Area quæſita.

Verum ſæpe accidit ut quædam Curvæ, quales Circulus, & Hyperbola, ejus naturæ ſint, ut fruſtra tentaveris earum Fluxiones Surdis immunes facere; tunc valore ordinatæ in Seriem infinitam conjecto ſinguliſque hujus Seriei Terminis per Fluxionem abſciſſæ, ut ſupra, multiplicatis, reperiatur ſingulorum Terminorum Quantitas Fluens, orietur nova Series quæ Quadraturam Curvæ exhibebit.

Methodus hæc eadem facilitate ad Dimenſionem Solidorum à Plani circumvolutione genitorum accommodatur, nempe aſſumendo pro eorum Fluxione productum ex Fluxione abſciſſæ per Circulum Baſis; Ratio Quadrati ad Circulum ſibi inſcriptum vocetur $\frac{n}{1}$, Æquatio Circulo competens eſt $yy = dx - xx$; igitur $4\frac{\overline{dx\dot{x} - x^2\dot{x}}}{n}$ eſt Fluxio Portionis Sphæræ, igitur $4\frac{\overline{\frac{1}{2}dxx - \frac{1}{3}x^3}}{n}$ eſt Portio ipſa, huic circumſcriptus Cylindrus eſt $4\frac{\overline{dxx - x^3}}{n}$, ideoq; Ratio Portionis Sphæræ ad circumſcriptum Cylindrum eſt ut $\frac{1}{2}d - \frac{1}{3}x$ ad $d - x$.

Rectificatio Curvarum obtinebitur si Hypothenusa Trianguli Rectanguli cujus latera sunt Fluxiones absciffæ & ordinatæ tanquam Curvæ Fluxio consideretur, sed curandum est ut in expressione istius Hypothenusæ, alterutra Fluxionum solummodo supersit, ac una tantum Indeterminatarum, illa scilicet cujus Fluxio retinetur. Res Exemplis clarior fiet.

Fig. 32. Ex dato sinu recto C B, arcum A C invenire, positis A B $= x$, C B $= y$, O A $= r$; sit C E Fluxio absciffæ, E D Fluxio ordinatim applicatæ, C D Fluxio Arcus C A; ex Circuli proprietate $2rx - xx = yy$, unde $2r\dot{x} - 2x\dot{x} = 2y\dot{y}$, ideoque $\dot{x} = \frac{y\dot{y}}{r-x}$, sed C D $q = \dot{y}\dot{y} + \dot{x}\dot{x} = \dot{y}\dot{y} + \frac{y^2\dot{y}\dot{y}}{rr - 2rx + xx} = \dot{y}\dot{y} + \frac{y^2\dot{y}\dot{y}}{rr-yy} = \frac{rr\dot{y}\dot{y}}{rr-yy}$, igitur C D $= \frac{r\dot{y}}{\sqrt{rr-yy}}$, sed $\frac{r\dot{y}}{\sqrt{rr-yy}}$ factum est ex $\frac{1}{\sqrt{rr-yy}}$ seu $\overline{rr-yy}|^{-\frac{1}{2}}$ in $r\dot{y}$; proindeq; si $\overline{rr-yy}|^{-\frac{1}{2}}$ conjiciatur in Seriem infinitam cujus singula membra per $r\dot{y}$ multiplicentur, & ex unoquoque producto ad Quantitatem Fluentem fiat retrogressus, habebitur Longitudo arcus A C.

Non ab simili modo ex dato Sinu Verso reperietur idem arcus; Resumatur æquatio supra inventa $2r\dot{x} - 2x\dot{x} = 2y\dot{y}$, fit $\dot{y} = \frac{r\dot{x} - x\dot{x}}{2y}$, sed C D $q = \dot{x}\dot{x} + \dot{y}\dot{y} = \dot{x}\dot{x} + \frac{rr\dot{x}\dot{x} - 2rx\dot{x}\dot{x} + x^2\dot{x}\dot{x}}{yy} = \dot{x}\dot{x} + \frac{rr\dot{x}\dot{x} - 2rx\dot{x}\dot{x} + x^2\dot{x}\dot{x}}{2rx - xx}$, seu (omnibus sub eodem denominatore reductis, expunctisque iis quæ sub diversis signis continentur) $= \frac{rr\dot{x}\dot{x}}{2rx - xx}$, unde C D $= \frac{r\dot{x}}{\sqrt{2rx-xx}}$, ideoque Longitudo arcus A C, per ea quæ jam dicta sunt facile obtinebitur.

Fluxio curvæ facilius interdum reperitur per comparationem inter Triangula similia C E D, C B O, institui enim potest hæc proportio C B : C O :: C E : C D, hoc est, pro Circulo $\sqrt{2rx - xx} : r :: \dot{x} : \frac{r\dot{x}}{\sqrt{2rx - xx}}$.

Fig. 33. Curva Cycloidis eadem operâ cognosci poterit. Sit A L K semicyclois, cujus circulus genitor A D L. Assumpto in diametro A L quovis puncto B, ducatur B I parallela Basi L K, peripheriæ circuli in puncto D occurrens; compleatur

pleatur rectangulum A E I B, ducaturque F H rectæ E I parallela, eidemque infinitè vicina, B I productam secans in G, curvamque A K in H; ponatur A L $= d$, A B $=$ E I $= x$, G H $= \dot{x}$; Notum est rectam B G esse ubique aggregatum arcus A D & sinus recti B D, hinc manifestum est Fluxionem I G esse aggregatum Fluxionum arcus A D & sinus recti B D. Porro Fluxio Arcus A D reperta est $= \frac{\frac{1}{2} d \dot{x}}{\sqrt{d x - x x}}$, Fluxio autem Sinus Recti B D reperietur $= \frac{d \dot{x} - 2 x \dot{x}}{2 \sqrt{d x - x x}}$, igitur I G $= \frac{d \dot{x} - x \dot{x}}{\sqrt{d x - x x}}$, ideoque I H $q =$ I G $q +$ G H $q = \frac{d d \dot{x} \dot{x} - d x \dot{x} \dot{x}}{d x - x x}$; Quamobrem I H $=$ $\frac{\dot{x} \sqrt{d d - d x}}{\sqrt{d x - x x}} = \frac{\dot{x} \sqrt{d}}{\sqrt{x}} = d^{\frac{1}{2}} x^{-\frac{1}{2}} \dot{x}$, proindeque A I $= 2 d^{\frac{1}{2}} x^{\frac{1}{2}}$ $= 2 \sqrt{d x} = 2$ A D.

Hæc conclusio minimo cum labore deduci potest ex nota proprietate Tangentis, cum enim illius portiuncula I H semper sit parallela chordæ A D, fit ut Triangula I G H, A B D sint similia, unde A B : A D : : G H : I H, hoc est $x : \sqrt{d x} : : \dot{x} : \frac{\dot{x} \sqrt{d x}}{x}$, igitur I H $= \frac{\dot{x} \sqrt{d x}}{x} = d^{\frac{1}{2}} x^{-\frac{1}{2}} \dot{x}$.

Sed nihil vetat quo minus adhibito Fluxionis I H auxilio, ipsam Cycloidis aream investigemus. Fluxio Areæ A E I est rectangulum E I G $= \frac{d x \dot{x} - x^2 \dot{x}}{\sqrt{d x - x x}} = \dot{x} \sqrt{d x - x x}$, sed Fluxio portionis A B D non alia est ab illa: Itaque Area A E I, correspondensque circuli portio A B D, semper sunt æquales.

Esto A B curva Parabolæ cujus Axis A F, Parameter $a$; Ponatur A E $= x$, E B $= y$, A B $= z$, B D $= \dot{x}$, D C $= \dot{y}$, B C $= \dot{z}$, assumptâ æquatione Parabolæ naturam constituente, videlicet $a x = y y$, fit $a \dot{x} =$ Fig. 34.
$2 y \dot{y}$, unde $\dot{x} = \frac{2 y \dot{y}}{a}$ sed B C $q =$ B D $q +$ C D $q$, hoc est $\dot{z} \dot{z} = \dot{x} \dot{x} + \dot{y} \dot{y} = \frac{4 y^2 \dot{y} \dot{y}}{a a} + \dot{y} \dot{y} = \frac{4 y^2 \dot{y} \dot{y} + a a \dot{y} \dot{y}}{a a}$, ideoque $\dot{z} = \dot{y} \frac{\sqrt{4 y^2 + a a}}{a}$ vel, quod idem est, $\dot{z} = \dot{y} \frac{\sqrt{y^2 + \frac{1}{4} a a}}{\frac{1}{2} a}$: si ergo $\dot{y} \frac{\sqrt{y^2 + \frac{1}{4} a^2}}{\frac{1}{2} a}$, in seriem infinitam transformetur, Curva A B haud difficulter innotescet.

Insuper

Fig. 35. Insuper statim apparet dato Hyperbolico spatio curvam hanc dari, & vicissim. Nam $\frac{1}{2} a \dot{z} = \dot{y} \sqrt{y^2 + \frac{1}{4} a a}$, ac proinde $\frac{1}{2} a z =$ spatio cujus Fluxio est $\dot{y} \sqrt{y^2 + \frac{1}{4} a a}$, sed hujusmodi spatium nihil aliud est quam Hyperbola æquilatera exterior ABEG, cujus semiaxis $AB = \frac{1}{2} a$, abscissa $AE = y$, ordinatim applicata $EG = x$.

Ad dimetiendam superficiem conversione curvæ circa suum Axem descriptam, assumi debet pro ejus Fluxione Cylindrica superficies cujus altitudo est ipsa curvæ Fluxio, cujusque distantia ab Axe est ordinatim applicata huic Fluxioni conveniens.

Fig. 32. Sit *ex. gr.* AC Circuli Arcus qui circa Axem AD revolvendo superficiem Sphæricam generet, quamque dimetiri statuamus; DC arcus Fluxio jam reperta est $\frac{r \dot{x}}{\sqrt{2rx - xx}}$, hanc si multiplicemus per Circumferentiam ad Radium BC pertinentem, hoc est $\frac{c}{r} \sqrt{2rx - xx}$ (posita ratione Circumferentiæ ad radium $= \frac{c}{r}$) habebimus Fluxionem superficiei Shpæricæ $= c \dot{x}$; adeoque superficies ipsa est $c x$.

Ad centra gravitatis quod attinet, repertâ superficiei Solidive Fluxione, hacque ducta in suam à Vertice distantiam, ad quantitatem Fluentem recurrendum est: qua divisa per Superficiem ipsam Solidumve ipsum, prodibit distantia Centri Gravitatis à Vertice.

Inveniendum sit Centrum Gravitatis omnium Paraboloidum horum Fluxio sic generaliter exprimitur $x^{\frac{m}{n}} \dot{x}$, hanc multiplica per $x$, fit $x^{\frac{m}{n}+1} \dot{x}$ cujus quantitatem Fluentem $\frac{n}{m+2n} x^{\frac{m}{n}+2}$, divide per Paraboloidos Aream, puta $\frac{n}{m+n} x^{\frac{m}{n}+1}$; prodibit $\frac{m+n}{m+2n} x$, distantia Centri Gravitatis à Vertice.

Centrum Gravitatis in Portione Sphæræ eodem modo colligitur, namque illius Fluxione $4\frac{\overline{d\dot{x}x - x^2\dot{x}}}{n}$ in $x$ ducta, fit $4\frac{\overline{dx^2\dot{x} - x^3\dot{x}}}{n}$, cujus quantitas Fluens $4\frac{\overline{\frac{1}{3}dx^3 - \frac{1}{4}x^4}}{n}$ per Portionis Soliditatem divisa, puta $4\frac{\overline{\frac{1}{2}dxx - \frac{1}{3}x^3}}{n}$, exhibet $\frac{\frac{1}{3}d - \frac{1}{4}x}{\frac{1}{2}d - \frac{1}{3}x} x$, seu $\frac{4d - 3x}{6d - 4x} x$ Distantiam Centri Gravitatis à Vertice.

XIII. 1. Prop.

XIII. 1. Prop. I. Probl.] *Invenire relationem inter Fluxionem Axios & Fluxionem Ordinatæ in Curva Catenaria.*

*The Catena; by Dr. Dav. Gregory. n. 231. p. 637. Fig. 36.*

Sit Catena F A D ab extremitatibus F & D dependens, cujus punctum imum (seu Curvæ vertex) A, Axis A B ad horizontem erectus, eique applicata B D horizonti parallela. Invenienda est relatio inter B $b$, seu D $\delta$ & $d$ $\delta$; posito $b$ puncto ipsi B proximo, & $b\,d$ ad B D, item D $\delta$ ad B A parallela.

Ex Mechanicis constat potentias tres in æquilibrio positas eandem habere rationem cum rectis tribus ad ipsarum directionis parallelis, vel in dato angulo inclinatis, à mutuo occursu terminatis; adeoque si D $d$ exponat gravitatem absolutam particulæ D $d$ (ut in Catena æqualiter crassa ritè fit) $d\,\delta$ repræsentabit gravitatis partem eam quæ normaliter in D $d$ agit, quaque fit ut $d$ D (ob Catenæ flexilitatem circa $d$ mobilis) in situm verticalem se componere conatur. Adeoque si $\delta\,d$ (sive fluxio ordinatæ B D) constans sit; gravitatis actio in partes correspondentes Catenæ D $d$ normaliter exerta etiam constans erit sive ubique eadem. Exponatur hæc per Rectam $a$. Porro ex supracitato Lemmate Mechanico, D $\delta$ sive fluxio axeos A B, exponet vim secundum directionem ipsius $d$ D exerendam, quæ priori conatui Lineæ gravis $d$ D ad componendam se in situm verticalem æquipolleat, eumque impedire possit. Hæc vero vis oritur à linea gravi D A secundum directionem $d$ D trahente; estque proinde (cæteris manentibus) lineæ D A proportionalis. Est igitur $\delta\,d$ fluxio ordinatæ ad $\delta$ D fluxionem absciss æ, sicut constans recta $a$ ad D A curvam. *Q. E. F.*

*COROL.*] Si recta T D tangat Catenariam, & axi B A producto occurrat in T, erit B D : B T : : ($d\,\delta$ : $\delta$ D : :) $a$ : D A Curvam.

Prop. II. Theorem.] *Si ad perpendiculum A B tanquam axem, vertice A, describatur hyperbola æquilatera A H, cujus semiaxis A C æqualis* a; *& ad eundem axem & verticem, parabola A P cujus parameter æqualis quadruplo axi hyperbolæ, & producatur semper hyperbolæ ordinata H B, donec H F æqualis Curvæ A P: Dico Curvam F A D, in quo puncta F & D versantur (positis B D, B F æqualibus) esse Catenariam.*

Vocetur A B, $x$, erit B $b$ = $\dot{x}$, & B H = $\sqrt{2ax + x^2}$. Unde ex Methodo Fluxionum, Fluxio ipsius B H ( sive $m\,h$ ) = $\frac{a\dot{x} + x\dot{x}}{\sqrt{2ax + x^2}}$, rursus quia parabolæ A P parameter = 8 $a$, erit B P = $\sqrt{8ax}$. Unde $n\,p$ (hoc est Fluxio ipsius B P) æqualis $\frac{2a\dot{x}}{\sqrt{2ax}}$. Quare Fluxio Curvæ A P (= P $p$ = $\sqrt{np\,q + P\,nq}$) = $\sqrt{\frac{4a^2\dot{x}^2}{2ax} + \dot{x}^2}$ = $\frac{\sqrt{2ax\dot{x}^2 + x\dot{x}^2}}{x}$, quæ ducen-

do

do tam numeratorem quam denominatorem in $\sqrt{2a + x}$, $= \frac{2a\dot{x} + x\dot{x}}{\sqrt{2ax + x^2}}$.

Et cum H F fit ubique = A P, erit Fluxio H F rectæ, hoc eft $m h + s f$ $= \frac{2a\dot{x} + x\dot{x}}{\sqrt{2ax + x^2}}$. Sed hactenus inventa eft $m h = \frac{a\dot{x} + x\dot{x}}{\sqrt{2ax + x^2}}$. Unde $s f$, five Fluxio ipfius B F ordinatæ ad axem Catenariæ, eft æqualis $\frac{a\dot{x}}{\sqrt{2ax + x^2}}$. Et igitur Fluxio Curvæ A F (five ipfa $F f = \sqrt{s f q + F s q}$ $= \sqrt{\frac{a^2\dot{x}^2}{2ax + x^2} + \dot{x}^2}) = \frac{a\dot{x} + x\dot{x}}{\sqrt{2ax + x^2}}$, cujus fluens modo oftenfa eft $\sqrt{2ax + x^2}$. Et igitur A F $= \sqrt{2ax + x^2}$. Patetque fluxionen ordinatæ B F five $\frac{a\dot{x}}{\sqrt{2ax + x^2}}$ effe ad $\dot{x}$ fluxionem abfciffæ A B, ficut data $a$ ad Curvam A F, quæ eft fuperius inventa Catenariæ proprietas. Igitur Catenariæ puncta rectè determinantur, per præcedentem conftructionem. *Q. E. D.*

COROL. 1.] Ex Conftructione patet B F ordinatam Catenariæ æquari Curvæ parabolicæ A P, dempta B H correfpondente ordinata hyperbolæ conterminæ A H.

2. Ex demonftratione conftat Catenariam Curvam A F æquari B H correfpondenti ordinatæ conterminæ Hyperbolæ æquilateræ. Cum enim harum linearum Fluxiones æquentur & fimul nafcantur ipfæ lineæ, patet illas ubique effe æquales. Unde datâ Catenâ, dabitur A C five $a$, quippe æqualis femiaxi Hyperbolæ æquilateræ cujus vertex A, & ordinata ad abfciffam A B catenæ A D eft æqualis.

3. Catenariæ omnes funt inter fe fimiles, cum ex fimili fimilium, & fimiliter pofitarum figurarum conftructione generentur. Unde duæ rectæ ad Horizontem fimiliter inclinatæ per Catenarum vertices ductæ abfcindent figuras fimiles & Catenarum portiones abfcindentibus rectis proportionales.

4. Si Catena Q A D fufpendatur à punctis Q & D inæqualiter altis, Curvæ pars F A D eadem manet, ac fi ex punctis æquialtis F & D effet fufpenfa, quoniam nihil refert utrum punctum F affixum fit vel non affixum ad planum verticale.

5. Si Catenæ vis trahens fecundum directionem, $d$ D exponatur per D $d$, dividetur, ut vulgò notum in vim ut $d\,\delta$ fecundum directionem horizontalem, & vim ut $\delta$ D, fecundum directionem verticalem: igitur vis in Catenæ extremo

mo directè accedendi ad axem, eſt ad vim in eodem deſcendendi ſecundum perpendiculum; ſive vis ſuſtinentis pars ſecundum directionem B D agens, eſt ad ejuſdem partem ſecundum directionem D δ agentem, ut ſemi-axis Hyperbolæ conterminæ A H ad D A, longitudinem Catenæ uſque ad verticem Curvæ: Unde datâ Catenâ ratio hæc datur. Et in eadem Catena nunc magìs nunc minùs laxe ſuſpenſa, vis iſta Horizontalis eſt ut Hyperbolæ conterminæ axis, cum D A eadem maneat ſi extrema æquialta ſint.

6. Catenà in Plano verticali, ſed ſitu inverſo, figuram ſervat nec decidit, adeoque arcum ſeu fornicem facit tenuiſſimum: Hoc eſt Sphæræ minimæ rigidæ & lubricæ in inverſa Curva Catenaria diſpoſitæ, arcum conſtituunt cujus nulla pars ab aliis extrorſum vel introrſum propellitur; ſed manentibus infimis punctis immotis, virtute ſuæ figuræ ſuſtinetur. Cum enim punctorum Curvæ Catenariæ ſitus, partiumque inclinatio ad Horizontem eadem ſit, ſive in ſitu F A D, ſive in ſitu inverſo, dummodo Curva ſit in plano ad Horizontem recto, patet illam æque ſervare figuram immutatam in uno ſitu ac in altero. Et è converſo ſolæ Catenariæ ſunt fornices ſive arcus legitimi: Et cujuſcunque alterius figuræ arcus ideo ſuſtinetur, quod in illius craſſitie quædam Catenaria incluſa ſit: Neque, ſi tenuiſſimus eſſet, parteſque haberet lubricas ſuſtineretur. Ex præcedente *Corol.* 5. colligitur, quali vi arcus muros quibus inſiſtit extra propellit; nempe hæc eadem eſt cum parte vis Catenam ſuſtinentis, quæ ſecundum directionem Horizontalem trahit. Quæ enim in Catena introrſum trahit vis, in arcu Catenæ æquali, extrorſum propellit. Alia omnia de murorum quibus fornices imponuntur firmitate requiſita, ex hac theoria Geometricè determinantur, quæ in ædificiorum extructione præcipua ſunt.

7. Si loco gravitatis alia quælibet vis ſimiliter agens in lineam flexilem vires ſuas exerat, eadem producetur linea *v. g.* Si ventus æquabilis ſupponatur, & ſecundum rectas datæ poſitione rectæ parallelas ſpirans, linea vento inflata eadem erit cum Catenaria. Nam cum omnia quæ in gravitate conſideravimus, in altera hac vi obtineant, patet eandem Curvam productum iri.

Prop. 3. Theor.] *Si manente prædicta Hyperbola A H, per A ducatur recta G A L axi A B normalis & deſcribatur Curva K R ejus naturæ, ut B K ſit tertia proportionalis rectis B H & A C, & ad A C applicetur rectangulum A V æquale ſpatio interminato A B K R L A, erit F concurſus rectarum H B, V G ad Catenariam.* Fig. 37.

Nam ex conſtructione eſt $BK = \frac{a^2}{\sqrt{2ax + x^2}}$, quare fluxio ſpatii $ABKRLA = (BKkb = BK \times Bb =) \frac{a^2 \dot{x}}{\sqrt{2ax + x^2}}$. Cumque

 BF =

$BF =$ ſpatio $\frac{ABKRLA}{AC}$, & AC detur, erit fluxio ipſius $BF =$ fluxioni ſpatii $\frac{ABKRLA}{AC} = \frac{a\dot{x}}{\sqrt{2ax + x^2}}$. Sed in præcedentis *Prop.* conſtructione, fluxio ordinatæ $BF = \frac{a\dot{x}}{\sqrt{2ax + x^2}}$. Quare hæc conſtructio eodem redit cum conſtructione *Prop.* præcedentis, & conſequenter punctum F eſt ad Catenariam. *Q. E. D.*

COROL.] Sicut in *Prop.* præced. Catenaria deſcribitur ex data longitudine Curvæ Parabolicæ, ita in hac, illius deſcriptio pendet à quadratura ſpatii in quo $x^2 y^2 = a^4 - 2axy^2$. Nam $y$ (ſive BK) $= \frac{a^2}{\sqrt{2ax + x^2}}$.

Fig. 36. Prop. 4. Theor.] *Spatium AGF ſub Catenaria AF & Rectis FG, AG ad AB, BF parallelis comprehenſum, æquale eſt rectangulo ſub ſemi-axe AC, & DH intervallo applicatarum in Hyperbola & Catenaria.*

Nam $\dot{DH} = (\dot{BH} - \dot{BD} =$, ex *Prop.* 2. hujus, $\frac{a\dot{x} + x\dot{x}}{\sqrt{2ax + x^2}} - \frac{a\dot{x}}{\sqrt{2ax + x^2}} =) \frac{x\dot{x}}{\sqrt{2ax + x^2}}$. Quare fluxio rectanguli ſub data AC & DH $= (\frac{ax\dot{x}}{\sqrt{2ax + x^2}} = x \times \frac{a\dot{x}}{\sqrt{2ax + x^2}} = \dot{fs} \times FG =)$ fluxioni ſpatii AGF. Cumque figuræ hæ ſimul naſcantur, ſequitur rectangulum ſub AC & DH æquari ſpatio AGF. *Q. E. D.*

COROL.] Hinc ſequitur, ſpatium FAD, ſub Catena FAD & recta Horizontali FD comprehenſum, æquari rectangulo ſub FD & BA, dempto rectangulo ſub Hyperbolæ AH axe alterutro, & DH exceſſu rectæ BH, vel Curvæ AD, ſupra ordinatam BD.

Fig. 36. Prop. 5. Theor.] *Si ad rectam AL, applicetur Rectangulum LE æquale ſpatio Hyperbolico ALH, erit E centrum Æquilibrii Curvæ Catenariæ AFD.*

Concipiatur curva gravis FA librari ſuper axe GL. Ex Centro barycis conſtat momentum gravis FA exponi per ſuperficiem Cylindrici recti ſuper FA erecti, & reſecti plano per GL tranſeunte, cum plano Curvæ angulum ſemi-rectum faciente. Et hujus ſuperficiei fluxio, ſive $\dot{FA} \times FG$, æqualis eſt fluxioni ſpatii ALH ſive $\dot{BH} \times HL$; quia $\dot{FA}$, $\dot{BH}$, item FG & HL æquantur.

æquantur. Ac propterea (cum ſimul naſcantur) dicta ſuperficies Cylindrici recti æqualis eſt ſpatio Hyperbolico A L H. Hoc proinde applicatum ad ipſum grave A F, vel illi æqualem rectam A L, facit latitudinem A E æqualem diſtantiæ centri gravitatis ab axe librationis G L. Unde Curvæ F A D, æqualiter ad utramque Axeos A B partem jacentis, centrum æquilibrii eſt E. *Q. E. D.*

*COROL.* 1.] Spatia A B H L, B A H, & A G F ſunt Arithmeticè proportionalia. Nam fluxio ſpatii A L H $= \left(\frac{a\dot{x} + x\dot{x}}{\sqrt{2ax + x^2}} \times x = \frac{ax + x^2 \times \dot{x}}{\sqrt{2ax + x^2}} = \frac{2ax + x^2 - ax \times \dot{x}}{\sqrt{2ax + x^2}} = \dot{x}\sqrt{2ax + x^2} - \frac{ax\dot{x}}{\sqrt{2ax + x^2}} = \right)$ fluxioni ſpatii B A H, multatæ Fluxione ſpatii A G F, per *Prop.* 4. hujus. Cumque hæ tres figuræ ſimul naſcantur, erit B A H — A G F = (A L H =) B L — B A H. Quare 2 B A H = B L + A G F. Unde ſequitur ſpatia B L, B A H & A G F eſſe in proportione Arithmetica.

2. Catenæ centrum gravitatis eſt omnium linearum ejuſdem Longitudinis, eoſdemque terminos habentium, infimum. Nam tantum deſcendet grave quantum poteſt. Cumque tantum deſcendat figura, quantum ejus centrum gravitatis deſcendit, ſe ſic diſponet linea gravis flexilis, ut ejus centrum gravitatis ſit inferius quam ſi aliam quamcunque figuram indueret. Atque ex hoc Symptomate lineæ gravis flexilis, reliqua omnia facilè deduci poſſent.

3. Si ſuper quaſcunque Curvas eandem longitudinem eoſdemque terminos D & F cum Catenaria F A D habentes, erecti Cylindrici recti ſecentur plano per D F tranſeunte; ſuperficierum Cylindricarum ſic reſectarum maxima eſt quæ ſuper Catenariam inſiſtit. Hæ enim ſuperficies (ſi angulus ſub planis fuerit ſemirectus) ad ipſas Curvas (quæ ſunt in caſu præſenti longitudinis ejuſdem) applicatæ, latitudines faciunt æquales diſtantiis centrorum gravitatis Curvarum à D F recta. Cum diſtantia hæc ſit in Catenaria maxima (ob maximum deſcenſum centri gravitatis) erit Cylindrica ſuperficies applicanda etiam maxima. Et quoniam ſuperficierum Cylindricarum reſectarum plano, cum plano baſeos angulum quemvis continente, eadem eſt ratio, atque cum dictus angulus eſt ſemirectus, patet propoſitum univerſaliter.

LEMMA.] *Si in cujuſvis Curvæ A F Q, deſcriptæ evolutione alterius Curvæ K V, ordinatam quamvis F B ad axem A B normalem, à correſpondente in K V puncto V demittatur normalis V R ordinatæ occurrens in R: erunt, manente fluxione axeos A B eadem, fluxio fluxionis ordinatæ B F, fluxio Curvæ A F, & recta F R, continuè proportionales.* Fig. 36.



Producatur

Producatur rectula F $f$, donec proximæ ordinatæ W $\varphi$ occurrat in $o$. Et quoniam ex hypothesi F $s$ — $f$ W, erit $o\,f = $ F $f$, adeoque $o\,\varphi$ erit fluxio ipsius $f\,s$, hoc est fluxio fluxionis ordinatæ. Porro triangula $o\,\varphi\,f$, $f$ F R sunt æquiangula, quia $o\,\varphi\,f =$ alterno $f$ F R, & $f\,o\,\varphi =$ (F $f\,r =$) F $f$ R, quia illorum intervallum R $f\,r$ alterutrius respectu evanescit, cum R $r$ præ $f\,r$ nulla sit. Et igitur $o\,\varphi : \varphi\,f :: f$ F : F R, sed $\varphi\,f$, $f$ F æquales sunt, cum fluxione utriusvis, tantum differant. Quare $o\,\varphi : f$ F $:: f$ F : F R. *Q. E. D.*

Fig. 36. Prop. 6. Probl.] *Invenire Curvam K V cujus evolutione Catenaria A F Q describitur.*

Vocetur ut prius A B, $x$, item B F, $y$. Est, ex *Prop.* 2. hujus, $\dot{y} = \frac{a\,\dot{x}}{\sqrt{2\,a\,x + x^2}}$, sive $2\,a\,x\,\dot{y}^2 + x^2\,\dot{y}^2 = a^2\,\dot{x}^2$. Quare, per satis nunc usurpatam *Newtoni* methodum, $2\,a\,\dot{x}\,\dot{y}^2 + 4\,a\,x\,\dot{y}\,\ddot{y} + 2\,x\,\dot{x}\,\dot{y}^2 + 2\,x^2\,\dot{y}\,\ddot{y}$ ($= 2\,a^2\,\dot{x}\,\ddot{x}$ quæ, propter $\ddot{x} = o$, cum constans $\dot{x}$ non fluat) $= o$. Quare $\ddot{y} = \left(\frac{-\,a\,\dot{x}\,\dot{y} - x\,\dot{x}\,\dot{y}}{2\,a\,x + x^2} =\right) \frac{\overline{a + x} \times a\,\dot{x}^2}{\overline{2\,a\,x + x^2} \times \sqrt{2\,a\,x + x^2}}$, ponendo loco $\dot{y}$ ejus valorem $\frac{a\,\dot{x}}{\sqrt{2\,a\,x + x^2}}$. (Nam signum — quantitati $\ddot{y}$ præfixum, tantum denotat locum puncti R ex F spectati, oppositum esse loco puncti F ex B spectati, cum Curva A F Q est cava versus axem A B) & F $f$, per *Prop.* 2. hujus, $= \frac{\overline{a + x} \times \dot{x}}{\sqrt{2\,a\,x + x^2}}$. Quare per præcedens Lemma, F R $= \left(\frac{\text{F} f\text{q}}{\ddot{y}} = \frac{\overline{a + x}^2 \times \dot{x}^2}{2\,a\,x + x^2} \times \frac{\overline{2\,a\,x + x^2} \times \sqrt{2\,a\,x + x^2}}{\overline{a + x} \times a\,\dot{x}^2} =\right)$ $\frac{\overline{a + x} \times \sqrt{2\,a\,x + x^2}}{a}$. Rursus ob triangula rectangula F $s\,f$, F R V habentia angulos $f$ F $s$, V F R æquales, quia V F $s$ est utriusque complementum ad rectum, est F $s : s\,f ::$ F R : V R, sive $\dot{x} : \frac{a\,\dot{x}}{\sqrt{2\,a\,x + x^2}} :: \frac{\overline{a + x} \times \sqrt{2\,a\,x + x^2}}{a}$ : V R, quæ proinde æqualis $a + x$. Hæc igitur est natura Curvæ K V, ut si A B vocetur $x$, erit F R $= \frac{\overline{a + x} \times \sqrt{2\,a\,x + x^2}}{a}$ & V R $= a + x$. *Q. E. I.*

COROL.

COROL. 1.] AC : CB :: BH : FR. Hæc enim eſt proprietas rectæ FR ſuperius inventa.

2. Recta CB æqualis eſt rectæ BI, ſive VR. Utraque enim eſt æqualis $a + x$.

3. Recta evolvens VF, eſt tertia proportionalis ipſis AC, CB. Nam ob æquiangula triangula $f$ F $s$, VFR, eſt $s$ F : F $f$ :: FR : VF. Sive $x : \frac{ax + xx}{\sqrt{2ax + x^2}} :: \frac{\overline{a + x} \times \sqrt{2ax + x^2}}{a}$ : VF, quæ proinde $= \frac{\overline{a + x}^2}{a}$. Unde $a : a + x :: a + x$ : VF, quæ præterea eſt radius circuli Catenæ in F æquicurvi.

4. Cum punctum F eſt in A, ſive cum vertex evolutione deſcribitur, id eſt cum $x = o$, valor evolventis rectæ VF, quæ in hoc caſu eſt KA, nempe $\frac{\overline{a + x}^2}{a}$ fiet $a$: hoc eſt punctum K ubi Curva VK occurrit axi, tantum extat ſupra Catenæ verticem A, quantum C deprimitur infra eundem. Unde diameter circuli, Catenæ ad verticem æquicurvi, æqualis eſt axi conterminæ Hyperbolæ AH. Adeoque Catenæ AD & Hyperbolæ AH eadem eſt curvatura in vertice A: Nam vulgo notum eſt circulum prædictum, Hyperbolæ æquilateræ AH in vertice A æquicurvum eſſe. Sed & hoc aliunde, ex ipſa Catenæ natura *Prop.* 2. hujus demonſtrata, conſtat. Nam naſcens FH ſive (AP = naſcenti BP =) $\sqrt{8ax}$ dupla eſt naſcentis BH ſive ($\sqrt{2ax + x^2}$, hoc eſt, evaneſcente $x^2$, cum $x$ minima ſit) $\sqrt{2ax}$; Et igitur idem punctum eſt tàm in naſcente Hyperbola quàm naſcente Catenaria; hoc eſt naſcens Hyperbola AH cum naſcente Catenaria AD coincidit, & proinde æquicurvæ ſunt hæ lineæ ad verticem A.

5. Curva KV eſt tertia proportionalis ad rectam AC & curvam AF ſive rectam AL. Ex natura enim evolutionis, KV = (VKA — KA = VF — KA $= \frac{\overline{a + x}^2}{a} - a = \frac{a^2 + 2ax + x^2}{a} - a =$) $\frac{2ax + x^2}{a}$. Et igitur $a : \sqrt{2ax + x^2} :: \sqrt{2ax + x^2}$ : KV. Sed $\sqrt{2ax + x^2}$, ex *Corol.* 2. *Prop.* 2. = AF. Unde AC : AF :: AF : KV.

6. Recta KI dupla eſt ipſius AB. Cum enim BI = (BC =) CA + AB, erit AI = CA + 2AB; At AK = AC, per *Corol.* 4. hujus; igitur KI = 2AB.

7. Rectangulum ſub AC & BR eſt æquale duplo ſpatio hyperbolico BAH. Nam FR × AC = ($\frac{\overline{a + x} \times \sqrt{2ax + x^2}}{a} \times a = \overline{a + x} \times$

$\sqrt{2ax +}$

$\sqrt{2ax+x^2} = x \times \sqrt{2ax+x^2} + a \times \sqrt{2ax+x^2} =$ AB × BH + AC × BH =) AB × BH + AC × BD + AC × DH. Quare FR × AC — BD × AC, hoc eſt BR × AC = AB × BH + AC × DH. Sed per *Prop.* 4. hujus, AC × DH = AGF ſpatio. Et igitur BR × AC = (ABHL + AGF = per *Corol.* 1. *Prop.* 5.) 2 BAH.

Fig. 38. Prop. 7. Theor.] *Si in Curva Logarithmica* LAG *cujus data ſubtangens* HS *æqualis rectæ* a, Corol. 2. Prop. 2. hujus definitæ, *ſumatur punctum A cujus diſtantia ab* HP *aſymptoto, nempe* AC, *æqualis ſit ſubtangenti* HS, & *ex punctis* H & P *utcunque in aſymptoto ſumptis à puncto* C *æqualiter diſtantibus, erigantur* HL, PG *ordinatæ ad Logarithmicam, quarum ſemiſummæ ponatur æqualis* HD *vel* PF, *erunt* D & F *ad Catenariam rectæ* AC *correſpondentem.*

Vocetur AB, $x$, adeoque CB vel DH ſemiſumma ordinatarum HL, PG erit $a+x$; ſemidifferentia earundem vocetur $y$. Unde HL $= a+x+y$, & PG $= a+x-y$. Cumque ex natura Logarithmicæ, CA ſit inter has media proportionalis, erit $a^2+2ax+x^2-y^2=a^2$. Unde $y = \sqrt{2ax+x^2}$. Adeoque HL $= a+x+\sqrt{2ax+x^2}$, & PG $= a+x-\sqrt{2ax+x^2}$. Quare fluxio ipſius HL, ſive ipſa $lm$ eſt $\dfrac{a\dot{x}+x\dot{x}+\dot{x}\sqrt{2ax+x^2}}{\sqrt{2ax+x^2}}$. Et ob æquiangula triangula $lm$L, LHS, eſt LH : HS : : $lm$ : $m$L, unde $m$L ſive $d\delta$ fluxio ipſius BD $= \dfrac{a\dot{x}}{\sqrt{2ax+x^2}}$. Hoc eſt Curva AD ex Logarithmica ſupradicto modo genita, ejus eſt naturæ, ut ſi axis vocetur $x$, ejuſque fluxio $\dot{x}$, fluxio ordinatæ BD ſit $\dfrac{a\dot{x}}{\sqrt{2ax+x^2}}$. Sed hæc ipſa eſt proprietas Catenariæ ad quam $a$ pertinet, *Prop.* 1. hujus demonſtrata. Ergo Curva FAD ſuperius deſcripta eſt hæc ipſa Catenaria. *Q. E. D.*

*COROL.* 1.] Sicut ope Logarithmorum Catenaria deſcribitur, vice verſa ope Catenariæ per ipſam rerum naturam productæ, numeri dati vel potius rationis datæ Logarithmus invenitur. Ut ſi poſita CA unitate, cujus Logarithmus eſt nihilo æqualis, quæratur Logarithmus Numeri CQ ſive rationis inter CA & CQ; Rectis CQ & CA tertia proportionalis ſit CV, ipſarumque CQ, CV ſemiſumma CB; ex B ordinata ad Catenariam, nempe BD eſt Logarithmus quæſitus. Ratio ex propoſitione manifeſta eſt.

2. Viciſſim ſi dato Logarithmo CH vel CP, quæratur correſpondens numerus HL vel PG, ſeu ratio HL ad CA, ſive PG ad CA. Ex H vel P erigatur perpendiculum Catenæ occurrens in D vel F, ipſique HD vel PF hoc eſt CB, fiat æqualis CR ad horizontalem AR terminata; eritque

AR

A R ſemidifferentia quæſitarum L H, G P, ſicut ex ſupra demonſtrata Catenæ natura H D vel C R eſt earundem ſemiſumma: (Nam in tribus quantitatibus Geometricè proportionalibus quales ſunt H L, C A, P G, quadratum ſemiſummæ extremarum multatum quadrato mediæ, æquatur quadrato ſemidifferentiæ extremarum) adeoque C R + A R, & C R — A R ſunt numeri H L vel G P, dato Logarithmo C H vel C P congrui.

3. Ex demonſtratione patet quod ſicut H D ſemiſumma Logarithmicæ ordinatarum H L, P G, ad C H normaliter applicata in H, eſt ordinata Catenariæ, ſic ſemidifferentia earundem H L, P G, ad C A normaliter applicata in B eſt ordinata Hyperbolæ æquilateræ centro C vertice A deſcriptæ: ac proinde, per *Corol.* 2. *Prop.* 2. hujus, æqualis Catenæ A D. Nam $y = \sqrt{2ax + x^2}$. Cumque *Corol. præced.* oſtenſum ſit, A R eſſe etiam ſemidifferentiam rectarum H L, P G, patet A R eſſe æqualem Catenariæ portioni A D. Unde obiter eluceſcit modus, datâ Catenâ A D, inveniendi C centrum Hyperbolæ conterminæ, vel punctum in aſymptoto Logarithmicæ G L. Nam ſi ſumatur A R æqualis Catenæ A D, & ex junctæ rectæ B R puncto medio erigatur ad ipſam B R normalis, hæc occurret B A axi Catenæ in quæſito puncto C, uti patet. Nam ſic erit C R = C B.

4. Hinc etiam ſequitur ſi B D T, angulus fiat æqualis A C R, rectam D T tangere Catenariam in D. Nam ſic fiet in triangulis æquiangulis D B T, C A R; D B : B T :: C A : A R ſive huic æqualem A D curvam. Et igitur per *Corol. Prop.* 1. hujus, D T tangit Catenariam.

5. Sequitur etiam ſpatium A C H D æquari rectangulo ſub C A & A R. Nam quoniam A Y D eſt, per *Prop.* 4. æquale triangulo ſub C A & (A D — B D =, per *Corol.* 3. *hujus Prop.* A R — A Y =) Y R, patet propoſitum. Et quoniam C A datur, conſtat ſpatium A C H D eſſe ſicut A D curva, illiuſque fluxionem H *d* ſicut D *d* fluxio hujus.

6. Si per punctum K, ubi C R ſecat H D, ducatur K Z parallela P H, rectæ A C occurrens in Z, ſumaturque C E æqualis ſemiſummæ ipſarum B C, C Z, erit E centrum Æquilibrii Curvæ F A D.

Intelligatur ſuper F A D erecta ſuperficies Cylindrici recti reſecti plano per P H ad angulos ſemirectos cum plano Curvæ F A D; exponet hæc ſuperficies momentum Curvæ F A D ſuper axe P H libratæ, ejuſque fluxio eſt

$$DH \times Dd + PF \times Ff = 2\,BC \times \dot{AD} = 2 \times \overline{a + x} \times \frac{a\dot{x} + x\dot{x}}{\sqrt{2ax + x^2}}$$

$$= \frac{2a^2\dot{x} + 4ax\dot{x} + 2x^2\dot{x}}{\sqrt{2ax + x^2}} = \frac{a^2\dot{x}}{\sqrt{2ax + x^2}} + \frac{a^2\dot{x} + ax\dot{x}}{\sqrt{2ax + x^2}}$$

$$+ \frac{3ax\dot{x} + 2x^2\dot{x}}{\sqrt{2ax + x^2}},$$ cujus fluens $a \times BD + a\sqrt{2ax + x^2} + x\sqrt{2ax + x^2} = CA \times BD + CB \times AD$. Quare $CA \times BD + CB \times AD =$ (quoniam ſimul naſcitur, dictæ ſuperficiei Cylindricæ =) momento Curvæ F A D ſuper axe P H libratæ. Unde diſtantia centri gravi-

tatis

tatis Curvæ F A D à puncto C est $\frac{CA \times BD + CB \times AD}{2AD}$ sive $\frac{1}{2}\,\frac{CA \times BD}{AD}$ $+ \frac{1}{2}\,CB$. Porro ob Z K parallelam A R, est A D : B D : : (A R : Z K : :) C A : C Z, unde $CZ = \frac{CA \times BD}{AD}$, & igitur C E quæ per constructionem est $= \frac{1}{2}\,BC + \frac{1}{2}\,CZ$, erit $= \frac{1}{2}\,\frac{CA \times BD}{AD} + \frac{1}{2}\,BC$: hoc est Curvæ F A D centrum gravitatis, & E punctum ex constructione definitum æqualiter distant à C; sed & in eadem recta & versus easdem partes sita sunt, ergo coincidunt illa.

Potest & coincidentia puncti E, ut supra determinati, cum centro æquilibrii, *Prop.* 5. hujus definito, Synthetice sic ostendi. Per *Corol.* 1. *Prop.* 5. $2BAX = AYD + BA \times AR$. Unde $AH + 2BAX =$ ($ACHD + BA \times AR =$ per præced. *Corol.*) $AR \times CA + BA \times AR$: hoc est $BD \times AC + 2BAX = AR \times CB$; sive $BD \times AC = AR \times CB - 2BAX$. Unde $BD \times AC + AD \times BC =$ ($AD \times BC + AR \times CB - 2BAX = 2AD \times BC - 2BAX =$) $2AD \times AC + 2AD \times AB - 2BAX$. Et applicando ad 2 A D, erit $\frac{1}{2}\,\frac{BD \times AC}{AD} + \frac{1}{2}\,BC$ $=$ ($AC + \frac{AB \times AD - BAX}{AD} =$) $CA + \frac{ARX}{AR}$. Sed $\frac{ARX}{AR}$ est distantia centri æquilibrii Catenæ à vertice A, per *Prop.* 5. hujus determinata, ac proinde, secundum dictam *Prop.* 5. $CA + \frac{ARX}{AR}$ est distantia puncti E à C, & $\frac{1}{2}\,\frac{BD \times AC}{AD} + \frac{1}{2}\,BC$, est ejusdem E distantia ab eodem C, secundum hoc *Corol.* 6. Unde patet duas istas determinationes puncti E eodem recidere, quoniam $CA + \frac{ARX}{AR} = \frac{1}{2}\,\frac{BD \times AC}{AD} + \frac{1}{2}\,BC$.

7. Spatii P F A D H centrum gravitatis est in I, medio puncto rectæ C E. Cum centrum gravitatis fluxionis ipsius A D, sive D *d* & F *f*, duplo magis distet à P H quam centrum gravitatis fluxionis ipsius A C H D sive D H *h d* & F P *p f*, & $\overline{Dd + Ff} \times AC$ datam, æquale D *d h* H + F *f p* P patet & fluentis F A D centrum gravitatis E duplo magis distare à P H, quam fluentis P F A D H centrum I. Sed libet propositum aliter & ad modum superiorum ostendere.

Intelligatur super figura P F A D H erectus Cylindricus rectus & resectus plano per P H transeunte, cum plano baseos angulum semirectum comprehendente; exponet istud solidum momentum figuræ P F A D H super axe

axe P H libratæ: hujusque solidi sive prædicti momenti fluxio (solida nempe erecta super P F $fp$ & H D $dh$) producitur, si momentum fluxionis, sive fluxio momenti ipsius A D, ducatur in $\frac{1}{2}$ A C datam. Nam per *Corol.* 5. hujus *Prop.* H D $dh$ = D $d$ × A C: Quare ipsum momentum fluens producitur ducendo momentum Curvæ F A D respectu axis P H, superiore *Corol.* determinatum, nempe C A × B D + C B × A D, in $\frac{1}{2}$ A C; eritque proinde $\frac{1}{2}$ A C × A C × B D + $\frac{1}{2}$ A C × C B × A D. Adeoque si hoc applicetur ad figuram libratam P F A D H sive 2 C A × A D, per hujus *Prop. Corol.* 5. fiet distantia centri gravitatis figuræ P F A D H ab axe P H = ($\frac{1}{4}\frac{CA \times BD}{AD} + \frac{1}{4}CB =$) dimidiæ rectæ C E superius determinatæ.

8. Si per N punctum ubi D T tangens Catenariam in D, secat A R, ducatur recta parallela ipsi B C, occurrens rectæ per E ad A R parallelæ in O; erit O centrum gravitatis Curvæ A D. Nam per *Corol.* 6. centrum gravitatis curvæ A D est in recta E O, sed demonstrabitur illud esse in N O recta, & proinde erit ipsum O punctum. Intelligatur D A librari circa H L axem; hujus momentum est curva D A ducta in distantiam centri gravitatis ab H L: & ejus proinde fluxio = D A × H $h$ (H $h$ est fluxio distantiæ axis librationis à gravitatis centro) $= \sqrt{2ax + x^2} \times \frac{a\dot{x}}{\sqrt{2ax + x^2}} = a\dot{x}$. Ac proinde ipsum momentum Curvæ gravis D A circa axem H L libratæ $= ax$. Et igitur distantia centri gravitatis ab eodem axe est $ax$ applicata ad A D, sive $\frac{AC \times DY}{AR}$. Sed quia D T tangit Catenariam, per *Corol.* 4. hujus *Prop.* angulus B D T sive D N Y = A C R, & anguli ad A & Y sunt recti, quare in triangulis æquiangulis R A C, D Y N; R A : A C : : D Y : Y N. Unde $YN = \frac{AC \times DY}{RA}$, hoc est Y N est distantia centri gravitatis Catenæ A D ab axe H L, sive centrum prædictum est in recta N O.

9. Si per I ducatur recta ad A R parallela, rectæ O N productæ occurrens in W, erit W centrum gravitatis spatii A C H D. Nam per *Corol.* 7. centrum gravitatis spatii A C H D est in recta I W, sed ut mox ostendetur, est in N W, & proinde est ipsum W punctum. Eodem enim modo, quo in *Corol.* præced. fluxio momenti spatii A C H D circa H L librati ostenditur esse A C H D × H $h$ = A C × A D × H $h$ $= a \times \sqrt{2ax + x^2} \times \frac{a\dot{x}}{\sqrt{2ax + x^2}} = a^2\dot{x}$. Ac proinde ipsum momentum spatii A C H D circa axem H L librati, æquale est fluenti cujus $a^2\dot{x}$ est Fluxio, hoc est ipsi $a^2 x$. Hoc igitur applicatum ad ipsum spatium A C H D sive $a \times \sqrt{2ax + x^2}$



dat

dat diſtantiam centri gravitatis ſpatii ACHD ab HL $= \frac{ax}{\sqrt{2ax + x^2}}$ $= \frac{AC \times DY}{RA}$. Sed *Corol.* præced. oſtenſa eſt $YN = \frac{AC \times DY}{RA}$. Et igitur centrum gravitatis ſpatii ACHD eſt in NW. Atque ex duobus hiſce ultimis *Corol.* invenitur centrum gravitatis cujuſvis portionis Catenæ etiam ad verticem A non pertingentis; vel cujuſvis ſpatii Catenariæ portione quavis, & aliis rectis præter prædictas comprehenſi.

10. Hinc menſurantur ſuperficies & ſolida genita rotatione Catenæ aut ſpatii ſub illa & rectis comphehenſi, circa axes datos. Nam figura rotatione genita æquatur, uti vulgo notum, figuræ rotatæ ductæ in peripheriam à centro gravitatis inter rotandum percurſam, etiam datam cum detur illius radius ſive diſtantia centri gravitatis ab axe dato. Sic ſi Catena AD rotetur circa axem AB, $\frac{\pi}{\varrho}$ AN eſt peripheria à centro gravitatis O percurſa, ($\frac{\pi}{\varrho}$ denotat rationem peripheriæ circuli ad ſemidiametrum) adeoque ſuperficies rotatione Catenæ AD genita $= (\frac{\pi}{\varrho} \times AN \times AD =) \frac{\pi}{\varrho} \times AN \times AR$. Hoc eſt circulus cujus radius poteſt duplum rectangulum RAN, æquabitur ſuperficiei à Catenæ AD rotatione circa axem AB genitæ. Pari modo ſolidum genitum rotatione ſpatii ACHD circa AC, æquale oſtendetur Cylindro cujus baſis eſt prædictus circulus, altitudo vero æqualis AC. Similiterq; ſuperficies & ſolida, ex rotatione harum figurarum circa alios quoſvis datos axes facta menſurantur. Nam dato centro gravitatis hæc non latebunt.

*The Animadverſions of ..... Anſwered by Dr.* D. Gregory. n. 259. p. 419.

2. Quæ in Animadverſione ad noſtras de Catenaria Demonſtrationes objicit Anonymus *Act. Lipſ. M. Feb. A.* 1699. ſunt hæc; Quod rem ab aliis jam ante ſeptennium inventam & publice expoſitam demonſtrare aggreſſus ſim, modo quodam meo. Ita quidem eſt; Quid vero hic redarguendum ſit non capio. Celeberrimi viri *Hugenius*, *Leibnitius*, & *Bernoullius*, plurimas Catenariæ proprietates detexerunt, & ediderunt, at non demonſtrarunt: Ego quod ſuſcepi, demonſtrationes pertexui.

Sed an res hæc (nempe Catenariæ Natura & proprietates primariæ) ab aliis inventa & publice expoſita fuit? Certè iſta Catenariæ proprietas, *Corol.* 6. *Prop.* 2. aliis indicta eſt penitus ante editas haſce demonſtrationes: Cum tamen ſit ni fallor inter primarias illius proprietates, & omnium longè utiliſſima, & ad vitæ communis uſus facillimè reducenda. Ab omni ævo, in ædificiis publicis Fornices arcuſque tàm ad firmitatem quàm pulchritudinem adhibuerunt Architecti: Qualis tamen ſit Fornicis figura legitima aduſque editas noſtras demonſtrationes ignoratum eſt.

Primum autem quod reprehendat invenit, quod quædam *ex Mechanicis conſtare* dixerim, quæ diſtinctius enuntiare atque etiam applicare operæ pretium fuiſſe ait. Ego qui Geometris demonſtranda Theoremata quædam ſuſceperam, omnia minutim exequenda non credebam.

Verum

Verum ut *Animadversori* gratum faciam, *Lemma* istud (*Prop.* 1.) demonstrabo, cum distinctius enunciare nequeam, quam est hactenus factum in hæc verba.

*Potentiæ tres in æquilibrio positæ eandem habent rationem cum rectis tribus ad ipsarum directiones parallelis, vel in dato Angulo inclinatis, à mutuo occursu terminatis.*

Puta si potentiæ tres trahentes, impellentes vel utcunque agentes, secundum rectas P A, P B, P C sint in Æquilibrio; & inclinentur ad has directiones tres rectæ E F, F D, D E, in angulo quovis dato, hoc est si anguli E A P, F B P, D C P, fuerint æquales, dico potentias A, B, & C, esse inter se ut rectæ F E, F D & D E. Fig. 39.

Producantur rectæ A P, B P, C P in G, H & K.

In Quadrilatero F A B P, cum Angulus externus E A P sit, ex Hypothesi, æqualis interno & opposito P B F, erunt interni duo oppositi F A P, & F B P æquales duobus rectis; cumque omnes quatuor interni quatuor rectis æquentur, erunt reliqui duo F & A P B in eodem quadrilatero oppositi, duobus rectis etiam æquales. Sed A P B & B P G efficiunt duos rectos, & igitur angulus F est æqualis angulo B P G. Similiter ostendentur D & B P K æquales, item E & A P K.

Quoniam tres Potentiæ sunt in Æquilibrio, sunt immotæ, & igitur earum quælibet pro Hypomochlio haberi potest reliquarum duarum respectu quæ in æquilibrio manent. Si B habeatur pro Hypomochlio, per *Mechanicæ notissimum Theorema*, Potentia A est ad Potentiam C, sicut sinus Anguli B P K, ad sinum Anguli B P G, hoc est sinus Anguli D ad sinum Anguli F, hoc est recta F E ad rectam D E. Rursus posito C Hypomochlio, potentia A est ad potentiam B, ut sinus Anguli C P H ad sinum Anguli C P G, sive sinus Anguli B P K ad sinum Anguli A P K, hoc est sinus Anguli D ad sinum Anguli E, hoc est ut recta F E ad rectam F D. Tres igitur Potentiæ A, B, & C, sunt ut rectæ F E, F D, & D E. *Q. E. D.*

De Applicatione hujus *Lemmatis Mechanici* nunc agendum. Si concipiatur (ut supra-dictum *Prop.* 1.) lineolæ *d* D gravitas absoluta per *d* D exposita, in ejus centro gravitatis M collecta, & grave hoc secundum directionem M F ad *d* D normalem vi gravitatis suæ descendere: Potentia secundum M D trahens quæ in æquilibrio est cum prædicto gravi, per præmissum *Lemma*, est ad ejus momentum sive potentiam trahentem secundum M F, sicut δ D ad δ *d*. Nam Angulus δ D *d*, quo D δ inclinatur ad M D, æqualis est angulo *d* ε F, quo *d* δ inclinatur ad M F; *viz.* uterque complementum anguli *d* ad rectum. Atque hoc etiam obtinet, agnoscente *Animadversore*, si ut in vulgari *Mechanica*, prædictum grave, plano M F incumbens, interposita trochlea ad M, trahatur ab alio grave ipsi M D incumbente: Erit hoc ad illud sicut D δ ad *d* δ. Fig. 40.

Quod si, reliquis manentibus, modus applicationis harum potentiarum mutetur, ita ut ad flexilis lineæ *d* D, cujus extremum *d* immotum, punctum medium M applicetur pondus secundum M F vires exerens, quippe arcum, centro *d*, radio *d* M, in descensu descripturum: Erit ponderis hujus vis, ad flexilem lineam rectam ad M incurvandam, infinita respectu vis suæ gravitatis absolutæ; & vis secundum M D trahens ad modo descriptam incurvationem impediendam requisita, etiam infinita respectu ejus quæ prius requirebatur ad pondus M



in

in plano M F sustinendum. Adeo ut Potentiæ quæ in priore applicationis modo exponebantur per $d\,\delta$, $\delta$ D, nunc exponendæ veniant per infinitè majores prioribus proportionales: Nam ut prius pondus M trahit secundum directionem M F, & Potentia illud sustinens secundum M D; & hæc duo esse in æquilibrio ex partium Catenæ quiete constat. Eadem igitur manebit harum ratio quæ prius fuerat. Sed causa quæ lineam flexilem $d$ D (cujus extremum $d$ immotum, cujusque medio puncto M applicatur grave infinitè quidem parvum, sed cujus vires per hunc applicationis modum infinitè majores redduntur, & proinde in *Animadversoris* Phrasi assignabiles fiunt) in rectam extendit, est Catenæ D A gravitas quæ est ipsius longitudini proportionalis. Hæc ergo est ad constantem & assignabilem $a$ (constanti sed inassignabili $d\,\delta$ proportionalem) ut D $\delta$, ad $\delta\,d$. Atque sic *Animadversori* patere credo veram conclusionem absque assumptis erroneis fuisse probatam.

*The Quadrature of Figures Geometrically Irrational, by Mr. J. Craig. n. 232. p. 708.* Fig. 41.

XIV. Sit A C F Semicirculus, cujus Diameter est A F, A D E Curva Geometricè irrationalis, cujus ordinatim applicata B D, secat Semicirculum in C. Quantitates verò sic designentur; Diameter A F $= 2a$, abscissa A B $= y$, Arcus A C $= v$, ordinata B D $= z$: sitque $z = rvy^n$ æquatio generalis exprimens naturas Curvarum Geometricè irrationalium A D E, in qua $r$ denotat quantitatem quamlibet datam & determinatam, & $n$ exponentem indefinitum quantitatis indeterminatæ $y$. Dico Aream

$$\text{ABD} = \frac{rvy^{n+1}}{n+1} - qv + \sqrt{2ay-yy} \times \frac{ra}{\overline{n+1}|^2}\,y^n + \frac{2nra^2+ra^2}{n \times \overline{n+1}|^2}\,y^{n-1} + \frac{aA \times \overline{2n-1}}{n-1}\,y^{n-2} + \frac{aB \times \overline{2n-3}}{n-2}\,y^{n-3}$$

$$+ \frac{aC \times \overline{2n-5}}{n-3}\,y^{n-4} + \frac{aD \times \overline{2n-7}}{n-4}\,y^{n-5} + \frac{aE \times \overline{2n-9}}{n-5}\,y^{n-6}$$

$+$ &c.

De hac Serie Infinita hæc sunt notanda: (1.) Quod Literæ majusculæ A, B, C, D, E, &c. designent coefficientes Terminorum ipsis immediatè præcedentium, scil. $A = \frac{2nra^2+ra^2}{n \times \overline{n+1} \times \overline{n+1}}$, $B = \frac{aA \times \overline{2n-1}}{n-1}$, $C = \frac{aB \times \overline{2n-3}}{n-2}$, & sic porro. (2.) Quod si exponens $n$ sit numerus integer & positivus, aut nihilo æqualis, vel etiam si $2n$ sit numerus impar, tum Quadratura spatii A B D exhibeatur per Quantitatem finitam; serie in his casibus abrumpente. (3.) Quod $q$ designet Terminum ultimo abrumpentem. (4) Quod omnes illæ Figuræ in quibus Series abrumpitur, habeant unam portionem Geometricè Quadrabilem, ex ipsa Serie facillimè assignabilem: Nimirum si capiatur abscissa $y = \overline{r}|^{\frac{1}{n+1}} \times \overline{nq+q}|^{\frac{1}{n+1}}$; erit huic abscissæ competens

competens Area Geometricè Quadrabilis. (5.) Quod solus Terminus Irrationalis $\sqrt{2ay-yy}$ in Terminos ipsum sequentes sit multiplicandus.

*Exemplum* 1.] Sit $z=v$, quia in hoc casu $r=1$, $n=0$, ideo $\frac{ra}{\overline{n+1}|^2}y^n$ est Terminus ultimò abrumpens, quare $q=a$, unde $ABD=vy-av+a\sqrt{2ay-y^2}$: & proinde si (per *Not.* 4.) capiatur Abscissa $y=a$, id est, si ordinata transeat per Circuli Centrum, erit portio huic competens Geometricè Quadrabilis, scil. Area $=aa$, id est, Radii Quadrato.

*Exemplum* 2.] Sit $z=\frac{vy}{a}$, quia in hoc casu $r=\frac{1}{a}$, $n=1$, ideo $\frac{anraa+raa}{n\times\overline{n+1}|^2}y^{n-1}$ est terminus ultimò abrumpens, quare $q=\frac{3a}{4}$; unde $ABD=\frac{vy^2}{2a}-\frac{3av}{4}+\frac{y+3a}{4}\sqrt{2ay-y^2}$, & proinde si (per *Not.* 4.) capiatur $y=\sqrt{\frac{3aa}{2}}$, erit huic abscissæ competens Area Geometricè Quadrabilis, scil. Area $=\sqrt{:\sqrt{6a^4}-\frac{3a^2}{2}}\times\sqrt{\frac{3a^2}{32}}+\frac{3a}{4}$.

*Exemplum* 3.] Sit $z=\frac{vy^2}{aa}$; in hoc casu $r=\frac{1}{aa}$, $n=2$, ideo $\frac{aA\times 2n-1}{n-1}y^{n-2}$ est Terminus ultimò abrumpens, ergo $q=\frac{5a}{6}$; unde per Seriem Infinitam erit $ABD=\frac{6vy^3-15a^3v+\overline{2ay^2+5a^2y+15a^3}\sqrt{2ay-yy}}{18a^2}$, & proinde si (per *Not.* 4.) capiatur $y=\sqrt[3]{\frac{5a^3}{2}}$ erit abscissæ competens Area Geometricè Quadrabilis: scil. Area $=\frac{2ay^2+5a^2y+15a^2}{18a}\times\sqrt{2ay-yy}$.

*Secundo.*] Sit ACF Parabola, cujus Axis AE, vertex A, & latus rectum BA. Sitque ADG Curva Geometricè irrationalis, cujus ordinatim applicata BD secat Parabolam in C. Et vocetur abscissa $AB=y$, Ordinata $BD=z$, Arcus Parabolicus $AC=v$. Sitque æquatio generalis, exprimens Naturas Infinitarum Curvarum irrationalium, hæc, $Z=rvy^n$, in qua $r$ denotat Quantitatem datam & determinatam, & $n$ exponentem indefinitum Quantitatis indeterminatæ $y$. Dico Aream Fig. 42.

ABD

$$ABD = \frac{ry^{n+1} \times v}{n+1} - qv + \sqrt{2ay+yy} - \frac{r}{n+2 \times n+1}y^{n+1}$$

$$- \frac{ra}{\overline{n+2 \times n+1}|^2}y^{n} + \frac{raa \times 2n+1}{n \times \overline{n+2 \times n+1}|^2}y^{n-1} -$$

$$\frac{aA \times 2n-1}{n-1}y^{n-2} + \frac{aB \times 2n-3}{n-2}y^{n-3} - \frac{aC \times 2n-5}{n-3}y^{n-4}$$

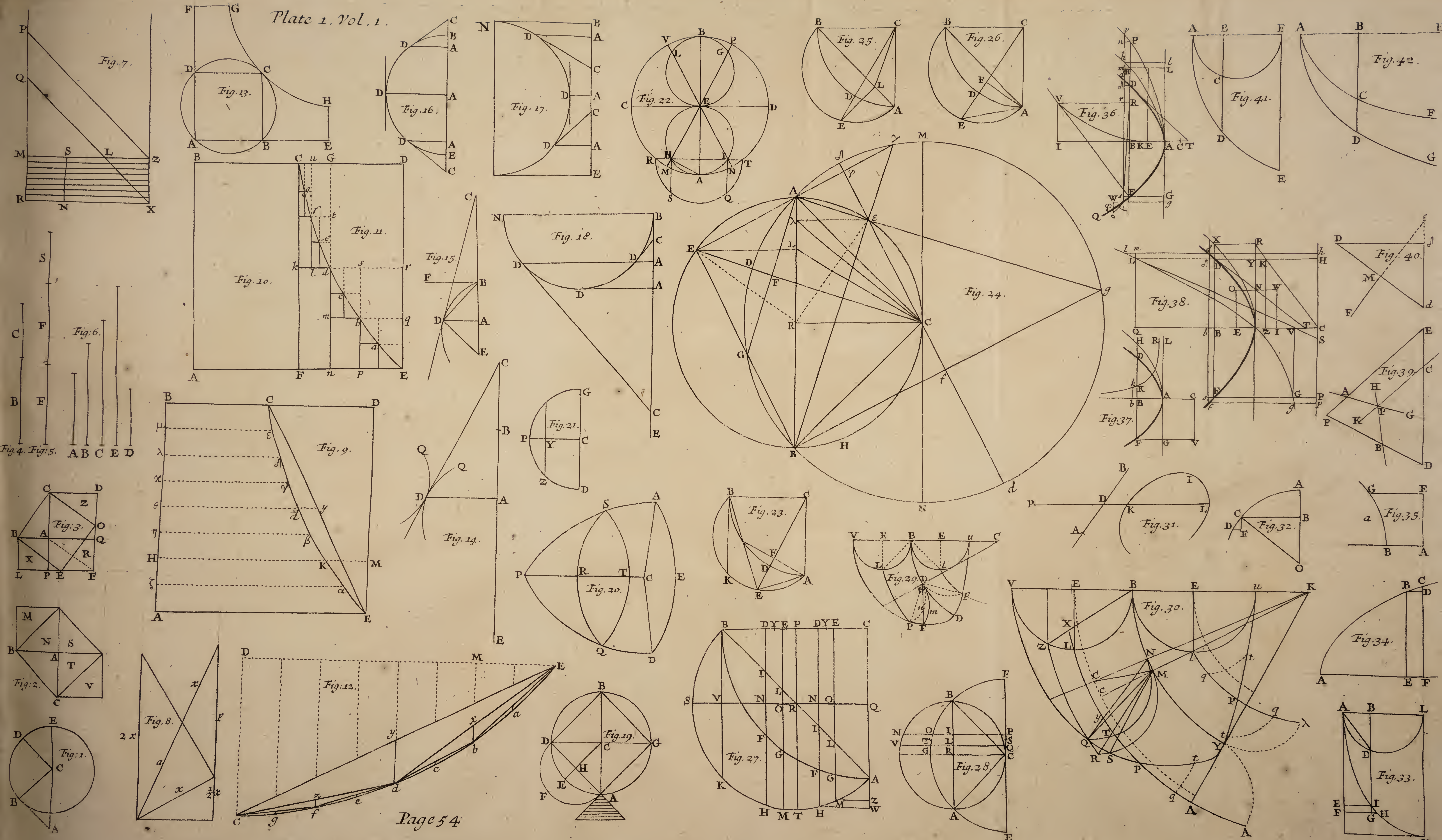
$+$ &c.

De hac Serie hæc ſunt notanda: (1.) Quod Literæ majuſculæ, A, B, C, &c. denotent coefficientes terminorum ipſis præcedentium. (2.) Quòd ſi exponens $n$ ſit integer poſitivus aut nihilo æqualis, aut etiamſi $2n$ ſit numerus impar, tum Quadratura exhibeatur per numerum terminorum finitum; Serie in his caſibus abrumpente. (3.) Quod $+q$ ſit æqualis ultimo termino abrumpenti. (4.) Quod ex terminis Quantitatem $\sqrt{2ay+yy}$ multiplicantibus ultimò abrumpens ſit duplicandus. (5.) Quod omnes illæ Figuræ, in quibus $n$ eſt numerus integer poſitivus & impar, vel generalius, omnes illæ figuræ, in quibus ultimus terminus abrumpens habet ſignum affirmativum ſeu $+$, habeant unam portionem Geometricè Quadrabilem, & ex ipſa ſerie facilè aſſignabilem, ſumendo abſciſſam ut in *Not.* 4. præcedentis ſeriei.

*Exemplum.* 1.] Sit $z = v$, quia in hoc caſu $r = 1$, $n = 0$, ideo terminus ultimò abrumpens eſt $- \frac{ra}{\overline{n+2 \times n+1}|^2}y^n$, unde $+q = -\frac{a}{2}$ (per *Not.* 3.) & quia in hoc caſu $-\frac{a}{2}$ eſt terminus ultimò abrumpens, ideo $-a$ eſt ultimus terminus in $\sqrt{2ay+yy}$ multiplicandus, (per *Not.* 4.) Adeoque $ABD = vy + \frac{av}{2} + \sqrt{2ay+y^2} \times -\frac{1}{2}y - a$.

*Exemplum* 2.] Sit $z = \frac{vy}{a}$ quia in hoc caſu $r = \frac{1}{a}$, $n = 1$, ideo terminus ultimò abrumpens eſt $\frac{raa \times 2n+1}{n \times \overline{n+2 \times n+1}|^2}y^{n-1} = \frac{a}{4}$, unde $q = \frac{a}{4}$, & $\frac{a}{2}$ ultimus terminus in $\sqrt{2ay+yy}$ multiplicandus; adeoque $ABD = \frac{vyy}{2a} - \frac{av}{4} + \sqrt{2ay+y^2} \times -\frac{y^2}{6a} - \frac{y}{12} + \frac{a}{2}$; & ſi capiatur

Plate 1. Vol. 1.

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piatur $y = \sqrt{\frac{aa}{2}}$, erit Area competens hinc Abſciſſæ Geometricè Quadrabilis, ſcil. Area $= \frac{1}{12} \sqrt{: \sqrt{2a^4} + \frac{a^2}{2}} \times 5a - \sqrt{\frac{a^2}{2}}$:

*Tertio.*] Sit A C F ſemicirculus, A D E Curva Geometricè irrationalis, cujus ordinatim applicata B D ſecat ſemicirculum in C. Quantitates verò deſignentur ut prius, ſcil. Diameter A F $= 2a$, Abſciſſa A B $= y$, Arcus A C $= v$, Ordinata B D $= z$; ſitque $z = r v^2 y^n$ Æquatio exprimens Naturas Curvarum A D E, in qua $r$ denotat quantitatem quamlibet datam & determinatam, & $n$ exponentem indefinitum quantitatis indeterminatæ $y$. Dico Aream n. 235. p. 785. Fig. 41.

$$ABD = r v^2 y^{n+1} - q v^2 + v \sqrt{2ay - y^2} \times \frac{2ra}{\overline{n+1}|^2} y^n + \frac{2ra^2 \times \overline{2n+1}}{n \times \overline{n+1}|^2} y^{n-1} + \frac{aA \times \overline{2n-1}}{n-1} y^{n-2} + \frac{aB \times \overline{2n-3}}{n-2} y^{n-3}$$
$$+ \frac{aC \times \overline{2n-5}}{n-3} y^{n-4} + \frac{aD \times \overline{2n-7}}{n-4} y^{n-5} + \frac{aE \times \overline{2n-9}}{n-5} y^{n-6}$$
$$\&c. - \frac{2ra^2}{\overline{n+1}|^3} y^{n+1} - \frac{2ra^3 \times \overline{2n+1}}{n^2 \times \overline{n+1}|^2} y^n - \frac{a^2 A \times \overline{2n-1}}{\overline{n-1}|^2} y^{n-1}$$
$$- \frac{a^2 B \times \overline{2n-3}}{\overline{n-2}|^2} y^{n-2} - \frac{a^2 C \times \overline{2n-5}}{\overline{n-3}|^2} y^{n-3}, \&c.$$

De hoc Theoremate hæc ſunt notanda; (1.) Quod componatur ex duabus ſeriebus infinitis, quarum prior (ſigno + connexa) multiplicatur in $v \sqrt{2ay - y^2}$; termini autem poſterioris (ſigno — affecti) ſunt abſoluti (2.) Quod in priori ſerie literæ majuſculæ, A, B, C, D, E, &c. deſignent coefficientes terminorum ipſis reſpectivè præcedentium; nec non in poſteriori eoſdem obtineant valores, quos in priori. (3.) Quod Quadratura exhibeatur per quantitatem finitam, quando $n$ eſt numerus integer poſitivus, aut nihilo æqualis, vel etiam ſi $2n$ ſit numerus impar: nam in his caſibus utraque Series abrumpitur. (4.) Quod $2q$ ſit æqualis ultimo termino abrumpenti prioris Seriei.

*Exemplum* 1.] Sit $z = \frac{v^2}{a}$. Quia in hoc caſu $n = 0$, $r = \frac{1}{a}$, ideo erit Area A B D $= \frac{yv^2}{a} - v^2 + 2v\sqrt{2ay - y^2} - 2ay$. *Corol.* Integra figura A F E eſt æqualis duplo Quadrato, cujus Latus eſt A C F, dempto diametri Quadrato.

*Exemplum*

*Exemplum* 2.] Sit $z = \frac{y v^2}{a^2}$, quia in hoc casu $n = 1$, $r = \frac{1}{a^2}$, ideo erit Area ABD $= \frac{y^2 v^2}{2 a^2} - \frac{3}{4} v^2 + v \sqrt{2 a y - y^2} \times \overline{\frac{v}{2a} + \frac{3}{2} - \frac{1}{4} y^2 - \frac{3 a y}{2}}$.

*Exempl.* 3.] Sit $z = \frac{y^2 v^2}{a^3}$, quoniam in hoc casu $n = 2$, $r = \frac{1}{a^3}$, ideo erit Area ABD $= \frac{y^3 v^2}{3 a^3} - \frac{5}{6} v^2 + v \sqrt{2 a y - y^2} \times \overline{\frac{2 y^2}{9 a a} + \frac{5 y}{9 a} + \frac{5}{3}} - \frac{2 y^3}{27 a} - \frac{5 y^2}{18} - \frac{5 a y}{3}$.

*The Quadrature of the Logarithmick Curve, by Mr.* J. Craig. n. 245. *p.* 373. *Fig.* 43.

XV. Esto ONF Curva Logarithmica, cujus Asymptotos AR, in qua tale sumatur punctum A, ut ejus prima ordinata AO sit subtangenti seu unitati æqualis: Quæritur spatium Curvilineum AONM a duabus ordinatis AO, MN, Abscissâ AM, & Curvâ Logarithmicâ ON comprehensum.

Ex O ducatur OE ad AM parallela, & secans MN in E; dico quod rectangulum ex segmentis ME, EN sit æquale spatio quæsito.

*Demonstratio.*] Vocetur Ordinata MN, $z$; subtangens AO seu ME, $s$: & ad axem AR construatur alia Curva HGE, cujus æquatio $2 s z = x^2$, ubi ejus ordinata $GM = x$; dico quod sit quadratrix Logarithmicæ juxta Methodi meæ fundamentum; scil. ejus subnormalis est respectivæ hujus Ordinatæ æqualis: ut ex calculo istius Methodi patebit: Ergo (juxta alibi à me exposita) si ad G ducatur GC perpendicularis & æqualis lineæ GM, nec non HD parallela ad GC, & lineis GM, CM occurrens in B & D; erit trapezium GBDC = AONM. Sed GBDC = GMC − BMD = $\frac{1}{2} x^2 - \frac{1}{2}$ BM$q$ = SZ − $\frac{1}{2}$ HA$q$; sed HA = $\sqrt{2}$ AO$q$ ex natura Curvæ HGQ, ergo GBDC = SZ − AO$q$ = AO × $\overline{\text{MN}}$ − AO$q$ = AO × $\overline{\text{MN} - \text{AO}}$ = ME × $\overline{\text{MN} - \text{ME}}$ = ME × EN; Ergo etiam AONM = ME × EN. *Q. E. D.*

*A Quadratrix to the Circle; being the Curve describ'd by its Equable Evolution, by* ....... n. 260. *p.* 445. *Fig.* 44.

XVI. 1. By the Equable Evolution of a Circle, I mean such a gradual approach of its Periphery to Rectitude, as that all its parts do together, and equally, Evolve or unbend; or so that the same Line becomes successively a less and less Arc of a reciprocally greater Circle.

2. Let AHKA be the Periphery of a Circle, AE a Tangent to the point A. Let this Circular Line be suppos'd, cut or divided at A, and then to unbend (like a Spring), its upper end remaining fixt to its Tangent AE, whilst the other

other parts do equally Evolve or extend themſelves thro' all the degrees of leſs Curvature (as in A B D, A M C, &c.) till they become ſtreight in coincidence with the Tangent A E.

3. Let A M C be the Evolving Curve in any middle poſition between its firſt and laſt. Join the fixt end A, and the moving end C, by the Chord-line A C, interſecting the firſt Circle at H. I ſay, that A M C is a like Segment to A *n* H, cut off in the firſt Circle by the Chord A H. For, by the Suppoſition, A M C is the Arc of a Circle, having A E a Tangent common both to it and A *n* H, and both Arcs are terminated in the ſame Right-Line A C.

4. Hence the Curve A D C E (deſcrib'd by the moving end of the Periphery in its Evolution) may be thus conſtructed. Let the Circle A H K A be by Biſections divided into any number of equal Parts. Let H be one of the Points of ſuch diviſion. Then ſay, as the Number of equal Parts in the Arc A *n* H, is to the Number of Parts in the whole Periphery A H K A; ſo is the Chord A H, to a fourth Line, which let be A C in A H produc'd. So is C a point in the Curve A D C E.

5. *Dem.* Upon A C deſcribe A M C, an Arc like to the Arc A *n* H. Whence A H : A C :: A *n* H : A M C. But by conſtruction, A H : A C :: A *n* H : Periph. A H K A, therefore is the Arc A M C equal to the whole Periphery A H K A, and like to the Arc A *n* H. Conſequently A M C repreſents the Evolving Periphery, in a Poſition like to the Arc A *n* H, and C is the deſcribing Point.

6. After the ſame manner may be found other Points, thro' which the Curve may be drawn. But here (as in the old *Quadratrix* of *Dinoſtratus*) the Point E cannot be preciſely determined, but the Curve may be brought ſo near it, that its flexure or tendency will ſo lead to the Point E, that A E ſhall be near enough to the Truth for common Uſes.

7. Suppoſing the Point E found, a Tangent to any point of the Curve may be drawn: and ſuppoſing a Tangent drawn, the point E may be determined; the property of the Tangent being this, that ſuppoſing R T a Tangent to the Point C, and C A, C E, drawn from C to each end of the rectify'd Circle, the Angle A C T (the leſſer Angle that A C makes with the Tangent) is equal to A C E the Angle made by the two Lines drawn from C.

8. Let *c* be a point in the Quadratrix, indefinitely near to C; and draw A *c* interſecting A H K A in *h*, and A M C in *o*. To A *c* as a Chord, draw the Arc A *m c*, like unto the Arc A *n h*. To the point C of the Arc A M C draw the Tangent C L = A E, and joyn L A: ſo is *o* C an indefinitely little particle of the Arc coincident with its Tangent.

9. Becauſe of the like Segments A *n h* A, A M *o* A, A *m c* A, as Chord A *c*, to Chord A *o*; ſo is Arc A *m c* (= A M C), to Arc A M *o*. Or, A *c* : A *o* :: A *mc* (= A M C) : A M *o*. And dividing, A *c* — A *o* (= *c o*) : A *o* :: A *mc* — A M*o* (= C *o*) : A M *o*. That is, *c o* : A *o* :: C *o* : A M *o*, and alternately, *c o* : C *o* :: A *o* : A M *o*. Put A C for A *o*, and A M C for A M *o* (as differing infinitely little) and then 'tis *c o* : C *o* :: A C : A M C. But by *conſtruction* C L = A E = A M C, whence *c o* : C *o* ::

A C : C L, and the Angle L C A = C o c, (o c being infinitely near to A C, is therefore parallel to it) And therefore C o c, A C L, are like Triangles.

10. Becaufe of C L = A E, Angle E A C = L C A, (C L and E A being Tangents to the two ends of the fame Circular Arch A M C, make equal Angles with its Chord A C) and A C common to both, the Triangles E A C, A C L, are like and equal: therefore are all three C o c, A C L, E A C, like Triangles. Whence it follows, That the Angle A C E (in the Triangle E A C) is equal to the Angle o c C (in the Triangle c o C) but o c C = A C T becaufe o c and A C are parallel; therefore the Angle A C E = A C T. *Q. E. D.*

*The Dimenfions of a Sphere and Cylinder compar'd; by. D. Wallis. n. 263. p. 547.*

XVII. In Epiftola quadam mea (*Oper. Mathem.* Vol. III.) inter alias meas Methodos (quibus in Tetragonifmis utor) occurrunt hæ duæ; quarum alteram appello Methodum *Convolutionis* & *Evolutionis*; alteram, Methodum *Complicationis* & *Explicationis*. Quarum ope oftendo (tum aliarum Figurarum, tum fpeciatim) Cycloidis dimetiendæ quis fit modus omnium fimpliciffimus.

Simili Artificio colligetur, tota Sphæræ cum Cylindro collatio: Quod fibi Monumentum fecit Archimedes.

Fig. 45. Quippe fi ad Bafin P (Peripheriæ Circuli æqualem) fumatur altitudo R (æqualis Radio) fiet Parallelogrammum Rectangulum = R P. Quod ex minutis Parallelogrammis æque altis, numero infinitis, (juxta receptam Methodum Indivifibilium) conflatum intelligatur. Quorum fi omnes vertices intel-

Fig. 46. ligantur in unicum punctum contrahi, quo ex illis minutis Parallelogrammis totidem fiant Triangula fuper eifdem Bafibus æque alta; fingula fingulorum, adeoque omnia omnium, dimidia; (curvata Bafi in Circuli Peripheriam) fiet Circulus (Centro C, Radio R,) Parallelogrammi dimidius = $\frac{1}{2}$ R P.

Quæ eft ipfa Archimedis Dimenfio Circuli: æqualis utique Triangulo Rectangulo, cujus laterum (circa Angulum Rectum) æquatur alterum Peripheriæ, alterum Radio expofiti Circuli. Quippe $\frac{1}{2}$ R (femi-altitudo Trianguli) in P (Bafin) ducta, exhibet Magnitudinem iftius Trianguli = $\frac{1}{2}$ R P, circulo æqualem. Idemque accommodabitur Sectori Circulari, fumpto arcu A pro P Peripheria.

Fig. 47. Porro; fi ad illud Parallelogrammum = R P (ut Bafim) fumatur itidem (in ordine ad Hemifphærium) Altitudo R; fiet Parallelepipedum = R R P. Quod pariter, ex minutis Parallelepipedis æque altis, numero infinitis, conflatum intelligatur (minutis areolis iftius Plani infiftentibus; quorum omnium communis altitudo fit R; & Bafium Aggregatum = R P. Quod fi Parallelogrammum hoc (manente magnitudine = R P) intelligatur in Curvam Superficiem Cylindricam curvari (cujus Bafis fit P, jam in Peripheriam circuli convoluta, Altitudo R) quo minuta illa Parallelepipeda in totidem Cuneos, feu Prifmata bafium triangularium, (Parallelepipedorum fingula fingulorum, adeoque omnia omnium, fub-dupla) redigantur; Acies feu Vertices habentia totidem C puncta (feu lineolas minutas) in Axe Cylindri conftituta, eumque complentia, fiet Cylindrus (Parallelepipedi Dimidius) = $\frac{1}{2}$ R R P.

Vel

Vel (in ordine ad Sphæram integram) si ſumatur utrinque Altitudo R, (ut ſit tota Altitudo D = 2 R;) fiet (convolutione pariter facta) Cylindrus (ut prius) ex Cuneis ſeu Priſmatibus numero infinitis (Vertices ſeu Acies habentibus in Axe Cylindri) = R R P = $\frac{1}{2}$ R P × 2 R, æqualis facto ex $\frac{1}{2}$ R P (circulari Baſe) in altitudinem 2 R: ſeu (quod tantundem eſt) = $\frac{1}{2}$ R × 2 R P, æqualis facto ex $\frac{1}{2}$ R (ſemiſſe communis Altitudinis Cuneorum) in (Baſium aggregatum) 2 R P.

Quod quidem Baſium Aggregatum eſt ipſa Cylindrica Superficies curva = P × 2 R (æqualis Facto ex Baſis Circularis Peripheria P in Altitudinem 2 R ducta:) ſeu $\frac{1}{2}$ R P × 4, (æqualis quatuor Circulis in Sphæra maximis:) Quibus ſi accenſeantur, oppoſitæ duæ Baſes circulares; fiet Cylindri (Sphæræ circumſcripti) tota ſuperficies, æqualis ſex Circulis maximis, = $\frac{1}{2}$ R P × 6 = 3 R P. Et Cylindri Magnitudo, = R R P = $\frac{1}{2}$ R P × 2 R, æqualis Facto ex Baſe Circulari $\frac{1}{2}$ R P in Altitudinem 2 R ducta: ut prius.

Quod ſi porro, Cuneorum horum omnium Vertices (Cylindri Axem Complentes) intelligantur in unum punctum contrahi: quo Cunei illi, ſeu Priſmata, jam fiant totidem Pyramides, ſuper iiſdem Baſibus æque altæ; ſingulæ ſingularum, adeoque omnes omnium, ſubſeſqui-tertiæ, ſeu ut $\frac{1}{3}$ ad $\frac{1}{2}$; & Superficies, prius Curva, Cylindrica, jam fiat Sphærica propter ejus omnia puncta æqualiter à Centro remota; manente, quod prius erat, Baſium aggregato = 2 R P, (quatuor Circulis Maximis æquali,) habebitur, tum tota Sphæræ Superficies = 2 R P = $\frac{1}{2}$ R P × 4 (æqualis quatuor Circulis maximis; & quidem toti Curvæ Cylindricæ æqualis & partes partibus reſpective æquales, eaſdem Axis partes reſpicientibus;) tum Sphæræ magnitudo = $\frac{2}{3}$ R R P = $\frac{1}{3}$ R P × 2 R; æquales Facto ex $\frac{1}{3}$ R (triente communis Altitudinis Pyramidum omnium) in 2 R P (Baſium Aggregatum, jam factam ſuperficiem Sphæricam) ducto.

Eſt itaque Cylindri Sphæræ circumſcripti tum Superficies tum Magnitudo, ad Superficiem & Magnitudinem inſcriptæ Sphæræ, ſeſqui-altera, ſeu ut 3 ad 2. (Illic quidem, ut ſex Circuli Maximi = 3 R P, ad quatuor Circulos maximos = 2 R P: Hic vero ut R R P ad $\frac{2}{3}$ R R P.) quod eſt illud ipſum Archimedis Inventum celebre.

Idem paulo brevius haberetur; ſi, in Parallelepipedo illo (ſuper plana Baſe 2 R P cum Altitudine R) ex minutis Parallelepipedis conflato; Horum omnium Vertices immediate cenſeantur in unicum C (punctum) comprimi. Quo, manente ut prius Baſium Aggregato = 2 R P, Parallelepipeda illa in totidem Pyramides redigantur; Vertices habentes ad Sphæræ Centrum coeuntes; cujus Radius R, (communis Pyramidum omnium altitudo;) & Sphærica Superficies, Baſium omnium Aggregatum. Quippe $\frac{1}{3}$ R (triens communis Altitudinis) in 2 R P (Baſium Aggregatum) exhibet Sphæræ magnitudinem (ut prius) $\frac{2}{3}$ R R P; & Sphæræ Superficiem = 2 R P.

Poteſtque hoc itidem Sectori Sphærico accommodari. Ducto $\frac{1}{3}$ R (triente communis Altitudinis Pyramidum inibi omnium) in portionem Sphæricæ Superficiei plano abſciſſam: Quæ eſt ad totam Superficiem Sphæricam, ut eſt Diametri (ſeu Axis) pars abſciſſa ad totum Diametrum; ut ſupra oſtenſum eſt.

 Cujus

Cujus quidem proceſſus totius Ratio his Principiis nititur; nempe, quod figura ex Triangulis eſt dimidia figuræ ex Parallelogrammis, ſuper eiſdem Baſibus, æque-altis: (Illam ego appello Figuram *Convolutam*; hanc *Evolutam*:) Et figura ex Pyramidibus, eſt, triens figuræ ex Parallelepipedis, ſuper eiſdem Baſibus æque-altis: (Illam ego appello Figuram *Complicatam*; hanc *Explicatam*.) Quæ poſſunt mille modis accommodari Figuris Curvilineis (tum Superficialibus tum Solidis) mirum in modum perplexis.

*Improvements in England in the Reſolution of Equations in Numbers, by Mr.* J. Collins. *n. 46. p. 929.*

XVIII. 1. It hath been obſerved by divers of this Nation, that in any Equation, howſoever affected, if you give a Root, and find the abſolute Number or Reſolvend, (which *Vieta* calls *Homogeneum Comparationis*;) and again give Roots and find more Reſolvends; that if theſe Roots, or rather rank of Roots be aſſumed in Arithmetical Progreſſion, the Reſolvends, as to their firſt, ſecond or third differences, &c. imitate the Laws of the pure Powers of an Arithmetical Progreſſion of the ſame degree, that the higheſt Power, or firſt Term of the Equation, is of. *e. g.* In this Equation $aaa - 3aa + 4a = N$.

If $a$ be = 10, 9, 8, 7, 6; Then N or the Abſolutes or Reſolvends will be found to be 740, 522, 352, 224, 132.

| $a$ | N | 1 diff. | 2 diff. | 3 diff. |
|---|---|---|---|---|
| 10 | 740 | | | |
| 9 | 522 | 218 | | |
| 8 | 352 | 170 | 48 | |
| 7 | 224 | 128 | 42 | 6 |
| 6 | 132 | 92 | 36 | 6 |

To wit, the 3d differences of thoſe Abſolutes are equal, as in the Cubes of an Arithmetical Progreſſion.

2. To find what habitude thoſe differences have to the Coefficient, of the Equation, 'tis beſt to begin from an Unite.

3. In any Arithmetical Progreſſion, if you multiply Numbers by Pairs, you ſhall create a rank of Numbers whoſe ſecond differences are equal; and if by Ternaries, then the 3d differences of thoſe Products ſhall be equal. And how to find the greateſt Product of an Arithmetical Progreſſion of any Number of Terms having any common difference aſſign'd, contain'd in any Number propos'd, is ſhewed by *Paſcal* in his Tract *du Triangle Arithmetique*, where he applies it to the Extraction of the Roots of Simple Powers.

4. It appears, how this Rank may be carried eaſily by Addition, till you have a Reſolvend either equal or greater or leſs than that propoſed.

5. When you have a *Majus* and *Minus*, you may interpoſe as many more terms in the Arithmetical Progreſſion as you will, that is to ſay, Subdivide the Common difference in the Arithmetical Progreſſion, and render it leſs; and then renew, and find the Reſolvends, which are eaſily obtained out of the Powers and their Coefficients, which are ſuppoſed known, and may be readily raiſed from a Table of Squares and Cubes, &c. with which kind the Reader may be furniſh'd in *Guldini Centrobaryca*, and *Babington's Fireworks*. By this means you may obtain divers Figures of the Root; and then the General Method of *Vieta* and *Harriot* runs away more eaſily, and is ſo far improved, that after any Figure is placed in the Root, moſt certain Characters

are

are given to know by aid of the Subſequent Dividend and Diviſor, whether the Figure before aſſumed be too great or too ſmall: or laſtly it may be well concluded, that, as in Logarithms, when you propoſe ſuch an one as is not abſolutely given in the Canon, you do by proportional Work, uſing the aid of their firſt differences (when their abſolute Numbers differ by Unite) find the abſolute Number true to 5 or 6 Places farther than the Canon gives it (the reaſon whereof is, that the firſt differences do likewiſe agree to about the ſame number of Places;) that I ſay the like may be done in Equations, after divers of the firſt Figures of the Root are found; provided there be the like agreement in the firſt differences of the Interpoled Reſolvends.

Moreover we ought here to take notice of a more ſubtile kind of Interpolation, common to all Gradual Ranks or Progreſſions of Numbers, wherein Differences happen to be equal: Of which kind the Reader may find Examples in *Briggii Arithmetica Logarithmica & Trigonometria Britannica*, relating to Logarithms, Sines, and the Powers of an Arithmetical Progreſſion: But the Method there delivered may be rendered more eaſy and general, *viz.* by aid of a Table of Figurate Numbers, by deriving Generating differences ſought, from thoſe given; a Doctrine that eaſily flows from *Mercator's Logarithmotechnia*, and of uſe in the Caſe in hand, ſhould we ſuppoſe theſe Powers and their Coefficients unknown, or a Table of Squares and Cubes wanting, and give nothing more than a few Reſolvends belonging to equal Moments or Spaces. And this may likewiſe be of good Uſe in *Gauging*, when having the Contents of a Solid, for every three Inches more or leſs given, without knowing the Dimenſions of the Figure, and even in moſt Caſes, when the differences are Progreſſive of one kind, without knowing the Figure it ſelf, having nothing given but its Contents at ſeveral equal Parallel diſtances, each ſuch diſtance may be ſubdivided, and made as many as you pleaſe, and the reſpective Contents found by this general Method of Interpolation.

After one Root is obtained, the Methods of *Huddenius* and others will depreſs the Equations ſo as to obtain more, and conſequently all of them.

6. It is eaſy by a Table of Figurate Numbers to give the Sum of any ſuch Rank, or any Term in it relating to a known part of the Series of Equals or Roots; but *è converſo*, giving the Reſolvend to find the Root, comes to an Equation as difficult as that propos'd; as in Dr. *Wallis*'s Chapter of *Figurate Numbers.*

7. Some affirm, they can give good Approaches for the obtaining a Root of any pure Power, affected Equation, or for the finding any of the mean Proportionals in any Rank between two Extreams given.

8. Others pretend to have found out a Method (incited thereto by an Example in *Albert Gerard's Invention Nouvelle en Algebre, à Amſterdam.* 1629) ſo much, by comparing of Equations, to increaſe or diminiſh the unknown Root of Equation as to render it a whole Number (or leſs differing therefrom than any Error aſſign'd,) and by *Albert Gerard*'s Method of Aliquot parts to to find the ſame, and thereby the Root ſought, altho' it be a mixt Number, Fraction, or Surd.

Probably

Probably this may Sympathize with what is promised by the Learned *Huddenius* in *Annexis Geometriæ Cartesianæ*, where he saith he intended not then to Publish certain Rules he had ready; whereof one was to find out all the Irrational Roots both of Literal and Numeral Equations. This must be understood, when such Roots are possible; for 'tis certain there are Infinite Equations, whose Roots are no ways explicable, either in whole or mixt Numbers, Fractions or Surds, and can be no otherwise explained, but by a *quam-proxime*.

*Ibid. p. 932.* 9. The *Author of this Narrative* considering, that the Conick Sections may be projected from lesser Circles placed on the Sphere, and thence easily (otherwise than hitherto hath been handled) described by Points, and that by their Intersections, some Spherick Problem is determined; accordingly he found that this following Problem, according to the various Situation of the Eye and of the projecting Plain, would take in all Cases.

*The Distances of an unknown Star are given from two Stars of known Declination and Right Ascension; the Declination and Right Ascension of the unknown Star is required.*

And saith, He hath observed, that, admitting the Mechanism of dividing the Periphery of a Circle into any Number of equal Parts, or (which is equivalent) the use of a Line of Chords, that this Problem, wherever the Eye be placed, may be resolved by plain *Geometry*, and yet the Eye shall be so placed, as to determine it by the Intersections of the Conic Sections; consequently those Points of Intersection (the Species and Position of the Figures being given) may be found without describing any more Points than those sought; and the Lengths of Ordinates falling from thence on the Axes of either Figure Calculated by mixt *Trigonometry*, and hence likewise the Roots of all *Cubick* and *Biquadratick Equations* found by *Trigonometry*.

For giving, from the *Mesolabe* of *Slusius*, the Scheme that finds these Roots, it will then be required to fit those Sections into Cones, which have their Vertex either in the Center, or an assigned Point in the Surface of the Sphere, to which they relate as Projected, and proceed to the Resolution of the Problem proposed: And how to fit in those Sections, see the 7 *Books* of *Apollonius*, *Mydorgius*, the 3d Volume of *des Cartes*'s Letters; *Leotaudi Geometria Practica*, *Andersonii Exercitat. Geometricæ*.

As to the Problem it self, it is determined on the Sphere by the Intersections of the two lesser Circles of Distance, whose Poles are the known Stars. And this Problem hath divers Geometrick ways of Resolution.

1. By plain Geometry (in the sence before mentioned;) supposing a Plain to touch the Sphere at the North Pole: if the Eye be at the South Pole, projecting those Circles into the said Plain, they are still Circles (by reason of the sub-contrary Sections of the visual Cones) whose Centers fall in the sides of the Right-lined Angle, made by the Projected Meridians, that pass thro' the known Stars; and thus the Problem is easily solved in this manner.

2. If it be required to be performed by Conick Geometry; in one Case it may be done, by placing the Eye at the Center of the Sphere, and projecting

as before; to wit, when the longer Axes of the Figures being produced, concur above the Vertex: Here the Problem is determined by the Interſections of two Conick Sections (whereof a Circle cannot be one, unleſs its Center be in the Axis of the other Figure:) And in this ſecond Caſe, theſe points of Interſection fall in the ſame Right-line or projected Meridian, they did before, but at a more remote diſtance from the Pole Point, to wit, in the former Suppoſition, the Polar diſtance was meaſured by a Right Line, that was the double Tangent of half the Arch; here it is the Tangent of the whole Arch. Hence it is evident, how one Projection may beget another, yea, infinite others, altering the Scale; and how the leſſer Circles in the *Stereographick* Projection help to deſcribe the Conick Sections in the *Gnomonick* Projection: But (to reduce the matter to one common Radius) if we ſuppoſe two Sphæres equal, and ſo placed about the ſame Axis, that the Pole point of the one ſhall paſs thro' the Center of the other, and the Touch-Plain to paſs thro' the ſaid Center or Pole-point; and that a leſſer Circle hath the ſame Poſition in the one as in the other; then, if the Eye be at the South Pole of the one, it is at the Center of the other; and any Projected Meridian drawn from the Projected Pole point to paſs thro' both the Projections of theſe leſſer Circles, the diſtances of the Points of Interſection are the Tangents of the half and the whole Arch of the Meridian ſo interſected. But as to the Points of Interſection, which determine the Problem propos'd, they may be found without the aid of the former way, from a *Gnomonick* and *Stereographick* Method of meaſuring and ſetting off the ſides and Angles of Spherical Triangles in thoſe Projections, which is neceſſary in what follows.

3. If the Problem is to be performed by mixt Geometry, as by a Circle, and either a Parabola, Hyperbola, or Ellipſis, the Circle may be conceived to be the ſub-contrary Section of a Cone projected by the Eye at the South Pole, and any of the reſt of the Sections by the Eye at the Center of the Sphere.

4. If by any of the Conick Sections however poſited, the Projecting Plain may remain the ſame; but the Eye muſt be in ſome other Part of the Surface of the Sphære, and not in the Axis.

XIX. 1. Conſtructio quam tradit *Carteſius*, quæque facillime radices Æquationum omnium Cubicarum vel Biquadraticarum, ubi deficit ſecundus terminus, eruit, ut nota ſupponi poteſt; attamen cum cardo ſit à quo ſubſequentia pendent, ex illius Geometria deſumptam placuit Regulam adjungere, pauculis nonnullis in melius (uti reor) tranſpoſitis.

*The Conſtruction of Cubick and Biquadratic Æquations, by a Parabola and a Circle. By M.* Edmund Halley. n. 188. p. 335.

Deficiente ſecundo termino omnes æquationes Cubicæ reducuntur ad hanc formam $z^3 . * . apz . aaq . = 0$, ac Biquadraticæ ad hanc $z^4 . * . apzz . aaqz . a^3 r = 0$, (ubi $a$ deſignat latus rectum Parabolæ cujuſvis datæ, quam in Conſtructione adhibere licet;) vel ſumendo $a$ pro unitate, ad hanc $z^3 . * . pz . q = 0$, vel ad hanc $z^4 . * . pzz . qz . r = 0$.

Jam data Parabola F A G, cujus Axis ſit A C D K L ac latus rectum $a$ vel 1, fiat A C ejus dimidium ac collocetur ſemper a vertice A verſus interiora figuræ; dein ſumatur $CD = \frac{1}{2} p$ in linea illa A C continuata verſus C ſi in æquatione fuerit $-p$, vel verſus alteram partem ſi habeatur $+p$. Porro è Fig. 48.

puncto

puncto D, aut expuncto C ſi non habeatur quantitas $p$, erigenda eſt ad axem perpendicularis DE æqualis $\frac{1}{2}q$, dextrorſum quidem ſi fuerit $-q$, ad alterum vero axis latus ſi fuerit $+q$; ac Circulus, Centro E, Radio AE deſcriptus, ſi æquatio fuerit tantum Cubica, Parabolam tot punctis F & G interſecabit quot veras habet Radices, quarum quidem affirmativæ, ut GK, erunt ad dextram Axis partem, Negativæ, ut FL, ad ſiniſtram.

Aſt ſi Æquatio Biquadratica fuerit, augeri vel minui debet Circuli Radius AE, addendo ſi fuerit $-r$, vel ſubducendo ſi ſit $+r$, ex ejus quadrato rectangulum $ar$, ſeu contentum ſub Latere Recto & quantitate data $r$; id quod nullo fere negotio efficitur Geometricé. Hujus vero Circuli interſectiones cum Parabola omnes veras Biquadraticæ Æquationis Radicis, dimiſſis ad axem perpendiculis, exhibebunt; affirmativas quidem ad dextram Axis, Negativas vero ad ſiniſtram. Totius demonſtrationem *Carteſio*, ejus Inventori, relinquo.

Notandum hic me operam dare ut ſemper habeantur Radices affirmativæ ad dextrum Axis latus, ut evitetur confuſio, à pluribus cautionibus, quarum cauſâ minime evidens eſt, neceſſario oritura.

His præmiſſis, ut aditus pateat ad conſtructionem etiam earum æquationum ubi reperitur terminus ſecundus, conſideranda venit Regula pro tollendo termino ſecundo, ac reducenda æquatione ad aliam quæ methodo præcedente conſtrui poſſit. Omnes vero hujus claſſis æquationes cubicæ ad hanc formam, $z^3.\ bzz.\ apz.\ aaq = 0$, vel ad hanc, $z^3.\ bzz.\ *.\ aaq = 0$; Biquadraticæ vero ad hanc, $z^4.\ bz^3.\ apzz.\ aaqz.\ a^3r = 0$, vel ad hanc, $z^4.\ bz^3.\ *.\ aaqz.\ a^3r = 0$, vel, $z^4.\ bz^3.\ apzz.\ *.\ a^3r = 0$, vel denique ad hanc, $z^4.\ bz^3.\ *.\ *.\ a^3r = 0$, reduci poſſunt: è quibus omnibus, prout ſignis + & — diverſimode connectuntur, ingens oritur varietas; unde Regula generalis omnibus inſerviens obſcura ac maximè difficilis redditur, niſi methodo quam ſubjungimus illuſtrata nodiſque extricata tractetur.

Tollitur in Biquadraticis ſecundus terminus ponendo $x = z + \frac{1}{4}b$, ſi fuerit $+b$ in æquatione, vel $x = z - \frac{1}{4}b$, ſi fuerit $-b$: hinc $x - \frac{1}{4}b$ in primo caſu, & $+\frac{1}{4}b$ in altero æquatur $z$; & in Æquatione quavis propoſita, ſubſtituta loco $z$ quantitate æquali, prodibit nova æquatio termino ſecundo carens, cujus radices omnes $x$ data differentia $\frac{1}{4}b$ vel excedunt vel deficiunt a radice quæſita $z$.

*Exemp.* 1.] $z^4 + bz^3 - apzz - aaqz + aaar = 0$.

ſit $x - \frac{1}{4}b = z$ Et erit

$$xx - \frac{1}{2}bx + \frac{1}{16}bb = zz$$

$$xxx - \frac{3}{4}xxb + \frac{3}{16}xbb - \frac{1}{64}bbb = z^3$$

$$\& \; x^4 - bx^3 + \frac{3}{8}bbxx - \frac{1}{16}b^3x + \frac{1}{256}b^4 = z^4.$$

Hinc

Hinc. $x^4 - bx^3 + \frac{3}{8}bbxx - \frac{1}{16}bbbx + \frac{1}{256}b^4 = +z^4.$

$$+bx^3 - \frac{3}{4}bbxx + \frac{3}{16}bbbx - \frac{1}{64}b^4 = +bz^3$$

$$-apxx + \frac{1}{2}apbx - \frac{1}{16}apbb = -apzz$$

$$-aaqx + \frac{1}{4}aaqb = -aaqz$$

$$+aaar = +aaar$$

Harum omnium ſumma fit æquatio nova ſecundo termino carens, quæque proinde juxta Regulam *Carteſianam* conſtrui poſſit, ſumendo loco $\frac{1}{2}p$ dimidium coefficientis termini tertii per $a$ ſive Latus rectum diviſi, hoc eſt $-\frac{3}{16}\frac{bb}{a} - \frac{1}{2}p$; ac Loco $\frac{1}{2}q$ dimidium coefficientis termini quarti per $aa$ diviſi, ſive $+\frac{1}{16}\frac{bbb}{aa} + \frac{1}{4}\frac{pb}{a} - \frac{1}{2}q$. Cujus partes ſigno + notatæ ſiniſtrorſum ab Axe, ſigno — notatæ dextrorſum collocandæ ſunt, ut habeatur centrum Circuli ad conſtructionem requiſiti, ac cujus interſectiones cum Parabola, dimiſſis in axem perpendiculis, radices omnes veras $x$ deſignet, affirmativas quidem ad dextram axis, negativas vero ad ſiniſtram. Cum vero $x - \frac{1}{4}b = z$, ducendo lineam axi Parallelam, ad dextrum ejus latus & ad diſtantiam $\frac{1}{4}b$, perpendicula illa ad hanc Parallelam terminata deſignabunt omnes radices quæſitas $z$, affirmativas ad dextram, negativas vero ad ſiniſtram. Radium circuli quod attinet, habetur ille addendo partes negativas ac auferendo partes affirmativas termini quinti per $aa$ diviſi, è quadrato lineæ A E, à centro invento E ad verticem Parabolæ A ductæ: id quod maxima ex parte efficitur capiendo loco lineæ A E lineam E O, quæ ad O interſectionem Parabolæ ac parallelæ prædictæ terminatur; ejus enim quadratum omnes termini quinti partes ex ablatione termini ſecundi æquationi novæ ingeſtas complectitur (uti facile probabitur): ac reſtat ſolummodo ut ipſius E O quadratum augeatur, ſi in æquatione habeatur $-r$, vel minuatur, ſi ſit $+r$, additione vel ſubductione rectanguli $ar$, unde conflatur quadratum Radii Circuli quæſiti.

Hæc eſt Methodus inveſtigandi regulam centralem *D. Bakeri* omnibus cautionibus libera, ac ſatis facilis; ac ſola differentia ex eo provenit, quod ego juxta Axem, ille vero juxta Axi parallelam circuli ejuſdem centrum determinat: quodque ego ſemper radices affirmativas ex Axis dextro latere invenio, quas ille nunc dextro nunc ſiniſtro conſtituit.

Æquationes cubicas quod attinet, eæ reduci debent ad Biquadraticas, antequam eadem regula generali conſtrui poſſint; id quod fit ducendo æquationem propoſitam in radicem ſuam $z$, unde provenit æquatio Biquadratica in qua deficit terminus ultimus ſive $r$: quapropter ſublato ſecundo termino & invento centro E, linea E O eſt radius Circuli; cum ſcilicet $ar$ ſit $= 0$, & in nova æquatione totus terminus quintus ex ipſa ablatione termini ſecundi oriatur.



*Exemp.*

*Exemp.* 2. $z^3 - bzz + apz + aaq = 0$: Quæ ducta in $z$ fit

$$z^4 - bz^3 + apzz + aaqz = 0$$

Ad tollendum secundum terminum ponatur $x + \frac{1}{4}b = z$, & fiet

$$x^4 + bx^3 + \frac{3}{8}bbxx + \frac{1}{16}b^3x + \frac{1}{256}b^4 = +z^4$$

$$- bx^3 - \frac{3}{4}bbxx - \frac{3}{16}b^3x - \frac{1}{64}b^4 = -bz^3$$

$$+ apxx + \frac{1}{2}abpx + \frac{1}{16}apbb = +apzz$$

$$+ aaqx + \frac{1}{4}aaqb = +aaqz$$

In hac nova æquatione, tertii termini semi-coefficiens per $a$ divisa, *viz.* $-\frac{3}{16}\frac{bb}{a} + \frac{1}{2}p$, loco $\frac{1}{2}p$ usurpanda est; ac coefficientis termini quarti dimidium, divisum per $aa$ Lateris recti quadratum, *viz.* $-\frac{1}{16}\frac{bbb}{aa} + \frac{1}{4}\frac{pb}{a} + \frac{1}{2}q$, vicem ipsius $\frac{1}{2}q$ in constructione *Cartesii* subit: unde centrum E determinatur. Deinde ducta Axi Parallela ad distantiam $\frac{1}{4}b$ ad sinistrum ejus latus (ob $x + \frac{1}{4}b = z$) cujus intersectio cum Parabola sit O; circulus centro E, Radio E O descriptus Parabolam secet vel tanget in tot punctis quot æquatio veras habet radices: quæ quidem radices seu $z$ sunt perpendicula de punctis illis in Axi parallelam demissa; ad dextram quidem Affirmativæ, Negativæ ad sinistram.

Si in æquatione defuerit terminus tertius vel quartus vel uterque, investiganda regula centrali nulla omnino observanda est methodus differentia, sed deficiente quantitate $p$ vel $q$, deerunt partes illæ linearum C D ac D E ex quantitate illa aliquo modo deductæ, ac producendum est cum reliquis coefficientibus termini tertii & quarti in æquatione nova, sicut in præmissis exemplis præscriptum est.

Hactenus *Cl. Bakeri* methodum generalem pertractavimus, qua quidem nulla alia facilior ac paratior expectanda est, assumpta ad constructionem sive Parabola, sive alia quævis linea curva, cum scilicet æquatio ad Biquadraticam ascendit. Etenim dum hæc scribo mihi occurrit regulæ Centralis Effectio Geometrica præter omnem spem expedita, ac harum rerum Curiosis abunde satisfactura.

Descripta Parabola N A M, cujus vertex A, Axis A B C, ac latus rectum $a$, reducatur æquatio ad hanc formam $z^4.\ bz^3.\ apzz.\ aaqz.\ a^3r. = 0$, vel ad hanc $z^3.\ bzz.\ apz.\ aaq = 0$, si cubica tantum fuerit: dein ad distantiam
Fig. 49. B D $= \frac{1}{4}b$ ducatur linea D H Axi parallela, ad sinistram quidem si fuerit $-b$, ad dextram si $+b$, Parabolæ occurrens in puncto D; de quo demittatur perpendiculum in axem B D. In linea A B continuata versus B fiat B K $= \frac{1}{2}a$, & ducatur linea D K utrinque interminata. Porro sit K C $=$ 2 A B in Axe semper ultra K continuato; ac si habeatur quantitas $p$ signo $-$ affecta, versus easdem partes etiam sumatur C E $= \frac{1}{2}p$, vel in contrarias si habeatur $+p$, ac è puncto E erigatur Axi perpendiculum

diculum EF (vel è puncto C si defuerit quantitas $p$) lineæ DK, si opus est continuatæ, occurrens in puncto F; quod quidem Circuli requisiti centrum est, si defuerit quantitas $q$; ast si habeatur $q$, sumenda est in FE, si opus est continuata, linea $FG = \frac{1}{2}q$, sinistrorsum quidem si fuerit $+q$, dextrorsum si $-q$ collocanda: Et punctum G erit centrum Circuli ad constructionem propositam idonei; ejusque Radius, si defuerit quantitas $r$, hoc est si tantum Cubica fuerit, erit linea GD; cujus quadratum in Biquadraticis augendum est, si fuerit $-r$, vel minuendum, si $+r$, additione vel subductione rectanguli sub $r$ & latere recto. Descripto sic Circulo, ab intersectionibus ejus cum Parabola demissis in lineam DH perpendiculis, quæ ad sinistram sunt, ut NO, radices æquationis negativas semper designant, quæ ad dextram, ut ML, affirmativas.

Aliter ac paulo simplicius Æquationes Cubicæ juxta *Schooteni* Regulam construuntur, quaque etiam radices ad Axem referuntur: quoniam vero ipse inventor nec modum inveniendi, nec demonstrationem inventi exponit, non abs re erit ejusdem fundamentum hic adjicere, simul atque Effectionem Geometricam concinniorem reddere, atque cautionibus quibus implicatur extricare.

Hæc Regula derivatur ex eo quod omnis æquatio Cubica reduci possit ad Biquadraticam, in qua deficiet terminus secundus: Hoc fit ducendo æquationem propositam in $z - b = 0$, si fuerit $+b$ in æquatione, vel in $z + b = 0$, si fuerit $-b$; & æquatio nova producta easdem habebit radices cum Cubica, atque insuper alteram ipsi $-b$ æqualem, si fuerit $-b$ in æquatione; vel contra.

Proponatur construenda $z^3 - z^2 b + apz + aaq = 0$.

Hæc ducta in $z + b$ fit $z^4 - z^3 b + apz^2 + aaqz$
$+ z^3 b - bbzz + abpz + aaqb$.

Hic deficit secundus terminus, ac coefficiens tertii $-bb + ap$ dat $-\frac{1}{2}\frac{bb}{a} + \frac{1}{2}p$ loco $\frac{1}{2}p$ vel CD in Constructione *Cartesii*, & ex dimidio coefficientis termini quarti fit $+\frac{1}{2}q + \frac{1}{2}\frac{bp}{a}$ loco $\frac{1}{2}q$ vel DE usurpanda; adeoque determinatur centrum Circuli quæsiti: atque ob datam unam ex radicibus æquationis novæ, *viz.* $-$ vel $+b$, dabitur etiam punctum in circumferentia, id est Radius ejus. Denique descripto Circulo, ab intersectionibus ejus cum Parabola demissa in axem perpendicula æquationis radices exhibebunt, affirmativas & negativas, eadem lege ac supra.

Investigatur autem centrum Circuli constructione perquam facili, cæterisque omnibus in Cubicis præferenda. Descriptæ Parabolæ AMD sit vertex A, atque Axis AF: ad distantiam ipsi $b$ æqualem ducatur Axi parallela DK, ad dextram si fuerit $+b$ in æquatione, ad sinistram si $-b$, quæ Parabolæ occurrat in puncto D. Centris D & A describantur Radiis æqualibus arcus occulti utrinque sese intersecantes, ac per sectionum puncta ducatur linea interminata BC, quæ medio lineæ suppositæ AD perpendiculariter insi- *Fig.* 50.


stat,

ftat, & Axi occurrat in puncto E. Ab E, inferne quidem si in æquatione habeatur $-p$, vel superne versus A si fuerit $+p$, ponatur $EF = \frac{1}{2}p$; & ex F (vel ex E si defuerit $p$). educatur perpendiculum FG, linea BC occurrens in puncto G; & in GF producta fiat $GH = \frac{1}{2}q$, dextrorsum quidem si in æquatione habeatur $-q$, aliter sinistrorsum, applicanda: ac punctum H erit Centrum quæsitum, HD vero Circuli Radius, qui demissis in Axem Perpendiculis ab intersectionibus suis cum Parabola, ut LM, Radices omnes, ut prius, commonstrabit.

*The Number of Roots in such Æquations, with their Limits and Signs; by Mr.* E. Halley. n. 190. p. 387.

2. Ex *Cartesio* & ex supradictis constat, tam in Cubicis quam in Biquadraticis æquationibus, Radices exponi posse demittendo perpendicula in Axem, datamve diametrum Parabolæ datæ, ab intersectionibus Curvæ illius cum Circulo. Cumque Circulus Parabolam secans, vel in quatuor vel in duobus punctis eam intersecare necesse est, constat in Biquadraticis vel duas vel quatuor Radices veras, Affirmativas vel Negativas, semper haberi; uti etiam, si forte Circulus illam tangat, quo in Casu æqualitas duarum Radicum ejusdem signi concluditur. In Cubicis autem, quoniam una ex intersectionibus ad Constructionem requiritur, non nisi una vel tres reliquæ Radices designant unam vel tres; uti in Casu Contactus, unde constat duas æquales reperiri Radices, Problemaque unde resultat æquatio revera Planum esse.

Cubicæ itaque omnes quomodocunque affectæ una vel triplici Radice explicabiles sunt, utique semper possibiles, nempe si Radices Negativas pro veris admiseris: sic Biquadraticæ, quarum terminus ultimus $r$ signo $-$ affecta est, duabus vel quatuor. Ast si habeatur $+r$ in æquatione, eaque tanta sit, ut $\sqrt{GDq - ar}$, minor sit quam ut Circulus, eo radio ac centro G descriptus, Parabolam contingere in aliquo puncto possit, æquatio data omnino impossibilis est, nec ulla Radice Negativa vel Affirmativa explicabilis: Sed de his plura in sequentibus.

Fig. 49.

Quoniam vero tanta intercedit differentia inter casus Cubicarum & Biquadraticarum, ut simul comprehendi nequeant; primum Cubicas deinde alteras tractabimus. Cubicæ vero infinitis Circulis in data Parabola construuntur, Biquadraticæ autem unico tantum (saltem his methodis): id adeo quia ponendo $z - e$ sive indeterminata aliqua, æqualem nihilo, æquatio Cubica reducitur ad Biquadraticam easdem Radices cum Cubica habentem, atque insuper aliam ipsi $e$ æqualem; unde fit ut tot Circulis diversis construi possit Cubica, quot imaginari velis quantitates $e$, id est infinitis. Inter has vero Constructiones, illa quam superius (§. *ult.*) dedi longe facillima est. Huic tamen non multum cedit alia, quæ ad enucleationem Numeri Radicum, earumque Limitum magis accommodata videtur, quæque ortum trahit ex ablatione secundi termini, ponendo modo vulgari $x = z +$ vel $-$ tertia parte Coefficientis termini secundi. Hæc autem est. Data Parabola ABY ejusque vertice A, Axe AE & Latere recto $a$, reducatur æquatio ad formam consuetam, *viz.* $z^3 . bz^2 . apz . aaq = 0$. Deinde ad distantiam $\frac{1}{3}b$ ducatur Axi parallela BK, dextrorsum quidem si fuerit $+b$, aliter sinistrorsum, Parabolæ occurrens in B; ac lineæ suppositæ AB erigatur perpendicularis utrinque interminata DP, Axi occurrens in puncto G. De B in Axem demitte perpendiculum BC, & ipsi AC fiat GE semper æqualis, ac versus inferiora ponatur. Ab E fiat

Fig. 51.

EH

EH $= \frac{1}{2}p$, ſurſum quidem, ſi in æquatione fuerit $+p$, deorſum vero, ſi $-p$, ac è puncto H (vel ex E ſi defuerit quantitas $p$) educatur perpendiculum H Q interminatæ D P occurrens in puncto O. Denique in linea H Q interminata, fiat O R $= \frac{1}{2}q$, ab O dextrorſum ſi fuerit $-q$, Siniſtrorſum ſi $+q$, collocanda: ac Circulus centro R, radio R A deſcriptus, tot punctis ſecabit Parabolam, quot æquatio propoſita veras habet radices; eæque erunt perpendicula Z Y à punctis interſectionem Y in Axi parallelam B K demiſſa; quarum quæ ad dextram lineæ B K Affirmativæ ſunt, ad Siniſtram Negativæ.

Hujus Conſtructionis commoditas in eo conſiſtit, quod Circulo per verticem tranſeunte paragitur, perinde ac ſi defuiſſet ſecundus terminus; ideoque ad Radicum Numerum determinandum, ſufficit Loci ſive Lineæ Curvæ proprietates perſpectas habere, quæ ſpatia diſcriminat, ubi ſi ponatur Centrum Circuli qui per Parabolæ Verticem tranſeat, circumferentia ejus vel uno vel tribus aliis punctis eam ſecabit; hoc eſt Lineæ curvæ, in quam incidunt centra omnium Circulorum per verticem tranſeuntium ac deinde Parabolam tangentium, naturam definire.

Locus autem ille eſt Parabolois, quam cum *Cl. Walliſio* ſemicubicalem appellare licet, ſive in qua Cubi applicatarum ad Axem ſunt inter ſe ut Quadrata portionum Axis. Cujus Latus rectum eſt, $\frac{27}{8}$ Lateris Recti datæ Parabolæ, Vertex vero punctum V exiſtente A V dimidium lateris recti ejuſdem Parabolæ. Hoc eſt, ſi ponatur Unitas pro Latere Recto datæ Parabolæ, $\frac{8}{27}$ cubi ordinatim applicatæ æquabuntur Quadrato partis diametri, ſive cubus ex $\frac{2}{3}$ V H quadrato ex H R, ſi ſcilicet R ſit Centrum Circuli qui per verticem Parabolæ tranſeat, eamque deinde contingat; Hæc eſt Curva illa quam primus mortalium *Nelius* Noſtras rectæ datæ æqualem demonſtravit, eaque occaſione apud Principes Geometras dudum celebris; ejuſque proprietates *Cl. Walliſius* ſub finem Libri *de Ciſſoide*, & *Hugenius Prop.* 8 & 9, *de Linearum Curvarum Evolutione*, aliique acri ingenio diſquiſivêre, quorum ſcripta conſulat Lector. Hæc Curva utrinque ab Axe Parabolæ deſcripta, *viz.* V N L, V P X, ſpatium complectitur, in quo ſi ponatur centrum Circuli, qui per verticem A tranſeat, interſecabit ille Parabolam in tribus aliis punctis; ſpatia vero ab Axe remotiora Centra præbent Circulis non niſi uno præter verticem puncto Parabolam ſecantibus.

His probe intellectis jam ad determinandum Radicum Numerum accingimur: Ac primum deficiat ſecundus terminus; ſitque Latus Rectum 1, vel A V $= \frac{1}{2}$; in conſtructione V H eſt $\frac{1}{2}p$, H R vero $\frac{1}{2}q$; cumque ſi fuerit $+p$, ab V verſus ſuperiora ponenda ſit $\frac{1}{2}p$, Centrum Circuli extra ſpatium L V X ſemper conſtituitur; ideoque una tantum Radice explicabilis eſt, Affirmativa ſi $-q$, Negativa ſi $+q$: quæ quidem Radices *Cardani* Regulis inveſtigantur. Si vero fuerit $-p$, V H $= \frac{1}{2}p$ inferne ponitur, ac fieri poteſt ut H R cadat inter Axem & Curvam V X vel V L, ſi ſcilicet Cubus ex $\frac{2}{3}$ V H, ſive ex $\frac{1}{3}p$, major ſit quam quadratum ex $\frac{1}{2}q$, ſive $\frac{1}{27}p^3$ major quam $\frac{1}{4}qq$, quo in caſu tres dantur Radices, duæ Negativæ, ſi fuerit $-q$, ac una Affirmativa earum ſummæ æqualis; vel ſi $+q$, duæ Affirmativæ unaque Negativa. Quod ſi $\frac{1}{27}p^3$ minor ſit quam $\frac{1}{4}qq$ una tantum reperitur Radix, Affirmativa ſi $-q$,

Negativa

Negativa si $+q$. Atque hæc passim docentur ab iis qui hanc Geometriæ partem tractarunt.

Jam adsint omnes termini, ac primum proponatur, *e. g.* æquatio hæc $z^3 - z^2 b + zp - q = 0$; cui etiam Figuram (51) adaptavimus. In hujus constructione BC $= \frac{1}{3} b$, VG $= \frac{1}{2}$ AC $= \frac{1}{18} bb$, VE $\frac{1}{6} bb$, VH $\frac{1}{6} bb - \frac{1}{2} p$, GH $\frac{1}{9} bb - \frac{1}{2} p$ vel $\frac{1}{2} p - \frac{1}{9} bb$, hinc HO $= \frac{1}{27} b^3 - \frac{1}{6} bp$, vel $\frac{1}{6} bp - \frac{1}{27} b^3$, atque HR, sive distantia Centri Circuli R ab Axe, est semper differentia inter $\frac{1}{6} bp$ & $\frac{1}{27} b^3 + \frac{1}{2} q$; quæ si æquantur, Centrum cadit in Axe; si $\frac{1}{6} bp$ major sit quam $\frac{1}{27} b^3 + \frac{1}{2} q$ ad Sinistram Axis, sin minor ad Dextram. Si itaque Cubi ex $\frac{2}{3}$ VH, (hoc est ex $\frac{1}{9} bb - \frac{1}{3} p$ quam nominemus $d$) Latus Quadratum sive $\sqrt{ddd}$, majus sit quam HR, sive differentia inter $\frac{1}{27} b^3 + \frac{1}{2} q$ & $\frac{1}{6} bp$; reperitur Centrum R intra spatium NPV, Paraboloidibus VPX, VNL, ac recta interminata DNP, circumscriptum: ac proinde Circulus Parabolam secabit in tribus punctis Y, Y, Y, ad dextram lineæ BK sitis, atque adeo æquatio tres habet Radices Affirmativas. Centro vero extra hoc spatium NVP constituto, non nisi una Radice Affirmativa explicari potest. Hic obiter notandum Rectam DP Paraboloidem VPX tangere in puncto P, existente EP $= \frac{1}{27} b^3$; alteram vero VNL secare in puncto N, ita ut demisso in Axem Perpendiculo NF, VF sit pars quarta ipsius EV, sive $\frac{1}{24} bb$, NF vero $\frac{1}{108} b^3$. VW autem, quæ è puncto V Axi perpendiculariter erecta lineæ DP occurrit in W, æqualis est $\frac{1}{54} b^3$ sive $\frac{1}{2}$ EP.

Hinc tutò concluditur, si in æquatione vel $p$ major sit quam $\frac{1}{3} bb$, vel $q$ major quam $\frac{1}{27} b^3$, non nisi unam eamque Affirmativam Radicem reperiri; Fallit itaque Regula *Cartesii* (Edit. *Amst.* 1659, *pag.* 70.) ubi tot veras dari Radices quot sunt in æquatione mutationes signorum $+$ & $-$ pronunciat, frustra etiam in Commentariis suis Sphalma hoc excusante *Schootenio*; Fingi enim possunt infinitè plures æquationes præcedentis formulæ tres signorum mutationes habentis, quæ unam tantum quam quæ tres habeant Radices. *Propositio* etiam *quinta Sectionis quintæ Artis Analyticæ Harriotti Nostri*, uti *Prob.* 18. *Numerosæ Potest. Resol. Vietæ*, vix satis firma est, cum ex Limitationibus quas ibi posuerunt, toti Parallelogrammo PLVW id conveniat, quod soli spatio NVP jam competere probavimus, hoc est ut centrum præbeat Circulo tribus aliis punctis præter Verticem Parabolam secante.

Quantitas

Quantitas autem $q$, ſive terminus ult., datis, $b$ & $p$ ea lege ut $p$ minor ſit quam $\frac{1}{3}bb$, accurate limitatur ex præcedente æquatione $\sqrt{ddd} = \frac{1}{27}b^3 + \frac{1}{2}q \backsim \frac{1}{6}bp$; cum ſcilicet Circulus Parabolam contingat. Itaque $\frac{1}{2}q$ minor eſſe debet quam $\frac{1}{6}bp - \frac{1}{27}b^3 + \sqrt{ddd}$; at ſi $p$ major fuerit quam $\frac{1}{4}bb$, majorem etiam eſſe oportet $\frac{1}{2}q$ quam $\frac{1}{6}bp - \frac{1}{27}b^3 - \sqrt{d^3}$, ne cadat centrum in ſpatiolo N V W. Atque his conditionibus æquatio ſemper triplici Radice explicabilis erit, aliter non niſi una. Semper vero, ſive tres ſive una, Affirmativæ ſunt, ob poſitionem centri R ad dextram lineæ D P.

Atque hic eſt caſus maxime difficilis, ita ut quicunque præmiſſa bene calleat ſequentia facili negotio intelliget. Detur jam æquatio $z^3 - bz^2 + pz + q = 0$. Hic ut tres habeantur Radices, oportet Centrum Circuli alicubi intra ſpatium P N Δ, rectis P N, P Δ, & curva Paraboloidis N Δ, definitum, reperiri; quapropter cum E F ſit $= \frac{1}{8}bb$, $p$ minor eſſe debet quam $\frac{1}{4}bb$: jam ad determinationem quantitatis $q$, exiſtente $d = \frac{1}{9}bb - \frac{1}{3}p$, ut antea, $\sqrt{ddd} + \frac{1}{7}bbb - \frac{1}{6}bp$ ſemper major eſſe debet quam $\frac{1}{2}q$, ut conſtituatur Centrum Circuli in ſpatio prædicto P N Δ: quod cum ſit æquatio talis duas habet radices Affirmativas ac unam Negativam. Si vero $p$ major eſt quam $\frac{1}{3}bb$, vel $\frac{1}{2}q$ major quam $\sqrt{ddd} + \frac{1}{27}bbb - \frac{1}{6}bp$, non niſi una eaque Negativa Radice explicabilis eſt.

Proponatur jam æquatio $z^3 - bz^2 - pz - q = 0$. Ut hæc æquatio tres habeat Radices, oportet Centrum Circuli alicubi inveniri in ſpatio indefinito, inter rectam D P D & curvam Paraboloidis P X; hic quantitas $p$ non eſt obnoxia limitationibus, $\frac{1}{2}q$ vero ſemper minor eſſe debet quam $\sqrt{ddd} - \frac{1}{27}bbb - \frac{1}{6}bp$, poſito $d = \frac{1}{9}bb + \frac{1}{3}p$: Hoc pacto duæ dantur Radices Negativæ, ac una Affirmativa; aliter vero ſi $\frac{1}{2}q$ major ſit quam $\sqrt{ddd} - \frac{1}{27}bbb - \frac{1}{6}bp$, unica tantum Affirmativa exponi poteſt. Quarto loco ſit æquatio $z^3 - bz^2 - pz + q = 0$, quæ duas Affirmativas habet Radices ac unam Negativam ſi Centrum Circuli reperiatur in ſpatio indefinito inter rectas P Δ, P D, ac curvam Paraboloidis Δ L; hoc eſt, (poſito $d = \frac{1}{9}bb + \frac{1}{3}p$) ſi $\frac{1}{2}q$ minor ſit quam $\sqrt{ddd} + \frac{1}{27}bbb + \frac{1}{6}bp$; ſi vero $\frac{1}{2}q$ major hac quantitate fuerit, una tantum Negativa ineſt Radix.

Quatuor autem æquationes reliquæ, in quibus habetur $+ b$, quoad limitationem Numeri Radicum non differunt à prædictis, si signum termini ultimi mutetur, servato signo termini tertii; quæ vero Affirmativæ erunt Radices in illis hic fiunt Negativæ, & vice versa. Sic in æquatione, $z^3 - bz^2 + pz - q = 0$, una vel tres erant Affirmativæ Radices; in hac vero, $z^3 + bz^2 + pz + q = 0$, vel una vel tres Negativæ sunt, sub iisdem conditionibus; nulla vero omnino Affirmativa. Sic in $z^3 + bz^2 + pz - q = 0$, duæ sunt Negativæ & una Affirmativa, si $p$ minor sit quam $\frac{1}{3} bb$, ac $\frac{1}{2} q$ minor quam $\sqrt{d^3} + \frac{1}{27} b^3 - \frac{1}{6} bp$, quemadmodum in $z^3 - bz^2 + pz + q = 0$, duæ erant Affirmativæ & una Negativa; excedentibus autem leges præscriptas $p$ vel $q$, una tantum hic est Radix Affirmativa, quæ ibi Negativa erat. Pari modo in $z^3 + bz^2 - pz + q = 0$, vel duæ sunt Affirmativæ ac una Negativa, vel una Negativa tantum; denique iisdem de causis in æquatione, $z^3 + bz^2 - pz - q$ duæ sunt Negativæ & una Affirmativa, vel una Affirmativa tantum, quibus in æquatione, $z^3 - bz^2 - pz + q$, duæ erant Affirmativæ & una Negativa, vel una Negativa tantum, nempe prout $\frac{1}{2} q$ major vel minor fuerit quam $\sqrt{d^3} + \frac{1}{27} b^3 + \frac{1}{6} bp$.

Si defuerit terminus tertius, sive $pz$, Centrum R semper cadit in linea I P E Δ, quocirca si fuerit $z^3 - bz^2 . * . - q$, vel $z^3 + bz^2 . * . + q$, una tantum esse potest Radix, si $- b$ Affirmativa, si $+ b$ Negativa. At si fuerit $z^3 - bz^2 . * . + q$, vel $z^3 + bz^2 . * . - q$, duæ possunt esse Affirmativæ ac una Negativa in priore, vel una Affirmativa & duæ Negativæ in posteriore, cadente Centro in linea P Δ inter P ac Δ, hoc est si $\frac{1}{4} q$ minor sit quam $\frac{1}{27} b^3$; sin major fuerit, una tantum Negativa in priore, vel una Affirmativa in posteriore, dari potest.

Hactenus Numerum Radicum in Cubicis æquationibus plenius assecuti sumus, restat ut nonnulla adjiciam de Quantitate Radicum. Hic primum Notandum quod omnis æquatio tres habens Radices ope Tabulæ Sinuum, Trisectione scilicet Anguli, satis expeditè resolvi possit; Ponendo scil. $\sqrt{\frac{4}{9} bb - \frac{4}{3} p}$, vel $\sqrt{4d}$, si fuerit $+ p$ in æquatione, vel $\sqrt{\frac{4}{9} bb + \frac{4}{3} p}$, si $- p$, pro Radio Circuli; Angulum vero trisecandum qui Sinum habeat in Tabula Sinuum $\dfrac{\frac{1}{27} b^3 + \frac{1}{6} bp + \frac{1}{2} q}{\sqrt{d^3}}$: Invento hoc angulo, Sinus tertiæ partis ejus, ut & Sinus tertiæ partis compl. ad Semicirculum, eorumque summa, ex Tabula Sinuum dabuntur. Hi vero Sinus in Radium $\sqrt{\frac{4}{9} bb + \frac{4}{3} p}$ ducendi sunt, & habebuntur Quantitates (y &, y &, y &, in *Fig.* quarum et $\frac{1}{3} b$ vel

Summa

Summa vel differentia, prout caſus poſtulat, veras Radices Æquationis exhibebunt. Hæc omnia ex inventis *Carteſii* derivantur: Ut vero caſus omnes, quantum fieri poſſit, breviter complectar, dico quod Centro R, in prima æquationum formula, cadente in ſpatio V G P, Sectiones duæ Y, Y, cadunt inter A & B, ac proinde utraque ex Minoribus Radicibus Minor eſt quam $\frac{1}{3}b$, tertia autem & Major ſemper ſuperat $\frac{1}{3}b$, ſuperatur verò à $b$. Quod ſi cadat in ſpatio G N V, duæ majores ſunt quam $\frac{1}{3}b$, minores vero quam $\frac{2}{3}b$, tertia vero eſt $b$ — duabus alteris, ac proinde minor quam $\frac{1}{3}b$: Sed adhibita Limitatione Quantitatis $p$, arctioribus terminis Radices includuntur. Maxima enim Radix minor eſt quam $\sqrt{\frac{4}{9}bb - \frac{4}{3}p} + \frac{1}{3}b$, major vero quam $\sqrt{\frac{1}{4}bb - p} + \frac{1}{2}b$; at cum $\frac{1}{4}bb$ minor eſt quam $p$, limes ille fit $\sqrt{\frac{1}{9}bb - \frac{1}{3}p} + \frac{1}{3}b$; Radix media ſemper minor eſt quam $\sqrt{\frac{1}{4}bb - p} + \frac{1}{2}b$; major vero quam $\frac{1}{3}b - \sqrt{\frac{1}{9}bb - \frac{1}{3}p}$; hunc vero limitem nunquam excedit Radix Minima, ſed cum Quantitate $q$ evaneſcit.

In ſecunda formula præſcriptis legibus duæ ſunt Affirmativæ ac una Negativa, ac cadente Centro in ſpatio G P E, altera ex Affirmativis major eſt, altera minor quam $\frac{1}{3}b$, Major vero non excedit $b$, Negativa autem Major non eſſe poteſt quam $\sqrt{\frac{1}{3}bb} - \frac{1}{3}b$, eſt autem differentia ipſius $b$ & ſummæ Affirmativarum. Centro autem in ſpatio E N G Δ poſito, utraque Affirmativa major eſt quam $\frac{1}{3}b$, minor vero quam $\sqrt{\frac{1}{3}bb} + \frac{1}{3}b$, Negativa vero ſemper minor eſt quam $\frac{1}{3}b$. Limites autem propiores ex data $p$ evadunt Radicis quidem maximæ Affirmativæ $\sqrt{\frac{1}{4}bb - p} + \frac{1}{2}b$, qua ſemper minor eſt, ut & major quam $\sqrt{\frac{1}{9}bb - \frac{1}{3}p} + \frac{1}{3}b$; hoc tamen limite minor eſt altera Affirmativa, quæ cum quantitate $q$ minuitur. Negativa vero ſemper minor eſt quam $\sqrt{\frac{4}{9}bb - \frac{4}{3}p} - \frac{1}{3}b$, ac deficiente quantitate $q$ evaneſcit.

In tertia formula duæ Negativæ ſunt ac una Affirmativa: in hac, ut & in quarta, Radices non limitantur à quantitate $b$. Affirmativa vero ſemper minor eſt quam $\sqrt{\frac{4}{9}bb + \frac{4}{3}p} + \frac{1}{3}b$, major tamen quam

$\sqrt{p + \frac{1}{4}bb} + \frac{1}{2}b$: Maxima vero ex Negativis ſemper major eſt quam $\sqrt{\frac{1}{9}bb + \frac{1}{3}p} - \frac{1}{3}b$, minor vero quam $\sqrt{p + \frac{1}{4}bb} - \frac{1}{2}b$. Minor autem ex Negativis ſemper minuitur cum minuta quantitate $q$.

In quarta formula, cadente Centro intra ſpatium L △ P D; ſi duæ ſint Affirmativæ ac una Negativa, Maxima ex Affirmativis major eſſe nequit quam $\sqrt{p + \frac{1}{4}bb} + \frac{1}{2}b$, nec minor quam $\sqrt{\frac{1}{9}bb + \frac{1}{3}p} + \frac{1}{3}b$; Minor vero Radix ab hoc Limite minuitur, minuta quantitate $q$. Negativa autem minor eſt quam $\sqrt{\frac{4}{9}bb + \frac{4}{3}p} - \frac{1}{3}b$; major vero quam $\sqrt{p + \frac{1}{4}bb} - \frac{1}{2}b$.

Notandum vero hic Radices Negativas ubique ſigno Affirmativo notari, quia hæ ſunt Radices Affirmativæ quatuor æquationum illarum, in quibus habetur $+ b$, ac $q$ ſigno contrario notatur; ut ſupra monui. Horum omnium Demonſtratio ex eo conſequitur, quod ubicunque Centrum Circuli R incidit in Lineas curvas V P X, vel V △ L, circumferentia ejus Parabolam tangit in puncto, cujus diſtantia ab axe eſt $\sqrt{\frac{2}{3}}$ V H, eamque ſecat ex altera Axis parte, ad diſtantiam $2\sqrt{\frac{2}{3}}$ VH; cum vero Centrum cadit in Lineam DPD, altera ex Radicibus fit $= 0$, ac proinde Cubica reducitur ad Quadraticam, ſive ad $z^2 - bz + p = 0$, cujus Radices Limites deſignant ubi evaneſcit quantitas $q$: ac quo minor eſt $q$, eo propius ad has limites accedunt Radices. Quadratica eſt etiam cum Centrum cadit in Axe; hoc eſt, cum $\frac{1}{2}q = \frac{1}{6}bp - \frac{1}{27}b^3$, in prima formula; vel $\frac{1}{2}q = \frac{1}{27}b^3 - \frac{1}{6}bp$, in ſecunda; in tertia impoſſibile eſt; at in quarta cum $\frac{1}{2}q = \frac{1}{27}b^3 + \frac{1}{6}bp$; quo in caſu Minor ex Radicibus Affirmativis eſt $\frac{1}{3}b$, Major $\sqrt{\frac{1}{3}bb + p} + \frac{1}{3}b$; Negativa vero $\sqrt{\frac{1}{3}bb + p} - \frac{1}{3}b$. In prima, Radices ſunt $\frac{1}{3}b$, & $\frac{1}{3}b \pm \sqrt{\frac{1}{3}bb - p}$. In ſecunda vero formula, $\frac{1}{3}b$, & $\sqrt{\frac{1}{3}bb - p} + \frac{1}{3}b$, ſunt Affirmativæ: Negativa autem $\sqrt{\frac{1}{3}bb - p} - \frac{1}{3}b$.

Atque hæc in Cubicis ſufficere poſſe videntur; ob eximium vero Uſum Methodi, qua ope Tabulæ Sinuum Radices harum æquationum inveniuntur, placuit unum vel alterum Exemplum adjungere, ut Praxis illius compendium inde innoteſcat. Proponatur Æquatio $z^3 - 39z^2$

$+ 479z$

$+ 479 z - 1881 = 0$; quæruntur Radices $z$. $\sqrt{\frac{1}{9}bb - \frac{1}{3}p} = \sqrt{9\frac{1}{3}}$ $= \sqrt{d}$, cujus duplum $\sqrt{37\frac{1}{3}}$ Radius eſt Circuli; & $\dfrac{\frac{1}{27}b^3 + \frac{1}{2}q - \frac{1}{6}bp}{\sqrt{d^3}} =$ $\dfrac{2197 + 940\frac{1}{2} - 3113\frac{1}{2}}{9\frac{1}{3}\sqrt{9\frac{1}{3}}}$, ſive $\dfrac{24}{9\frac{1}{3}\sqrt{9\frac{1}{3}}}$ eſt Sinus Tabularis Anguli, hoc eſt, facta diviſione ope Logarithmorum, Log. 9. 9251560, cui reſpondet Angulus 57 *gr.* 19 *m.* 11 $\frac{1}{2}$ *s.* Hujus tertia pars 19 *gr.* 6 *m.* 24 *s.* & complementi 40 *gr.* 53 *m.* 36 *s.* Sinus dant Log. 9. 514983, & 9. 816011, qui ducti in Rad. $\sqrt{37\frac{1}{3}}$ producunt Y &, et Y &, Log. 0. 301030. = 2, et Log. 0. 601059 = 4, tertia vero Y &, æqualis eſt eorum ſummæ ſive 6. Ideoque Radices ſunt 13 − 4 = 9, 13 − 2 = 11, & 13 + 6 = 19, ex quibus ſingulis conflatur prædicta æquatio. Ubi Notandum duas Minores Radices non excedere $\frac{1}{3}b$ vel 13, quia centrum R in conſtructione cadit ad Dextram Axis; id eſt $\frac{1}{6}bp$ minor eſt quam $\frac{1}{27}b^3 + \frac{1}{2}q$.

Exemplum alterum ſit $x^3 - 15x^2 - 229x - 525 = 0$, & quærantur Radices $\sqrt{\frac{1}{9}bb + \frac{1}{3}p} = \sqrt{101\frac{1}{3}} = \sqrt{d}$, & Radius Circuli $\sqrt{405\frac{1}{3}}$. $\dfrac{\frac{1}{27}b^3 + \frac{1}{6}bp + \frac{1}{2}q}{\sqrt{d^3}} = \dfrac{125 + 572\frac{1}{2} + 262\frac{1}{2}}{101\frac{1}{3}\sqrt{101\frac{1}{3}}} =$ $\dfrac{960}{101\frac{1}{3}\sqrt{101\frac{1}{3}}}$ = Sinui Tabulari Arcus, cujus Log. 9. 9736426, & Arcus ipſe 70 *gr.* 14 *m.* 22 *s.* hujus pars tertia eſt 23 *gr.* 24 *m.* 47 $\frac{1}{3}$ *s.* & Complementi 36 *gr.* 35 *m.* 12 $\frac{2}{3}$ *s*; quorum Sinus Log. ſunt 9. 599183, & 9. 775275, quibus addito Log. $\sqrt{405\frac{1}{3}}$ fiunt Log. 0. 903089 = 8, & Log. 1. 079181 = 12, & eorum ſumma = 20. Hinc concluditur 20 $+ \frac{1}{3}b$, vel 25, æquari Radici Affirmativæ, & 8 & 12 $- \frac{1}{3}b$, ſive 3 & 7, Negativis. Quod ſi æquatio fuiſſet $x^3 + 15x^2 - 229x + 525 = 0$, 3 & 7 fuiſſent Affirmativæ; 25 vero Negativa. Cæteræ autem Cubicæ unica tantum Radice explicabiles juxta Regulas *Cardani* reſolvendæ ſunt, poſtquam demptus fuerit ſecundus terminus; nec video quo pacto minori calculo hoc negotium peragi poſſit. At ſi deſideretur Radix

 hæc

hæc in Quantitatibus $b$, $p$, $q$, expressa, dico eam esse in prima formula, $\frac{1}{3}b$ + vel — summa vel differentia Radicum Cubicarum ex $\sqrt{\frac{1}{4}qq - \frac{1}{108}p^2b^2 + \frac{1}{27}b^3q - \frac{1}{6}bpq + \frac{1}{27}p^3} \pm \frac{1}{27}b^3 + \frac{1}{2}q - \frac{1}{6}bp$: *viz.* +, si $\frac{1}{27}b^3 + \frac{1}{2}q$ major sit quam $\frac{1}{6}bp$, aliter —; Summa vero quoties $\frac{1}{3}bb$ major est quam $p$; sin minor fuerit $\frac{1}{3}bb$, differentia. Inque cæteris formulis Radix semper conflatur ex iisdem elementis, variatis tamen signis + &—, ut facile percipiet qui velit experiri.

Ope vero Tabulæ Logarithmicæ Sinuum Versorum Radices hæ satis prompte inveniuntur; nempe si coefficientis Numeri sint surdi vel fracti, ac Radices Numeris ineffabiles; ut plerumque fit. Hæc autem est Regula: in prima ac secunda formula, si $\frac{1}{3}bb$ minor sit quam $p$; sit $\frac{1}{3}p - \frac{1}{9}bb = d$, & posita differentia inter $\frac{1}{6}bp$ & $\frac{1}{27}b^3 + \frac{1}{2}q$, hoc est HR, in prima, ac inter $\frac{1}{6}bp + \frac{1}{2}q$ & $\frac{1}{27}b^3$, in secunda, pro Radio; inveniatur Angulus cujus Tangens est $d\sqrt{d}$. Deinde ut Co-sinus hujus Anguli, ad ejusdem Sinum Versum: Ita differentia pro Radio habita, ad quartum; cujus Latus cubicum trisecando Logarithmum habebitur: ac diviso $\frac{1}{3}p - \frac{1}{9}bb$ per hoc Latus Cub. è Quoto subducatur Divisor, Residuum erit quantitas Y &: Hujus Residui ac $\frac{1}{3}b$ summa, si centrum cadit ad dextram Axis, aliter differentia earundem, Radix erit quæsita. Quod si $\frac{1}{3}bb$ major sit quam $p$, posito HR pro Radio, sit $d\sqrt{d}$, sive distantia Paraboloidis ab Axe, Sinus Arcus cujusdem; Hujus Sinus versus ducatur in Radium, sive $\frac{1}{6}bp - \frac{1}{27}b^3 \pm \frac{1}{2}q$, ac trisecto producti Logarithmo, habebitur ejus Latus Cubicum, per quod dividatur $\frac{1}{9}bb - \frac{1}{3}p$. Dico Quoti ac Divisoris summam eadem lege additam vel ablatam ex $\frac{1}{3}b$, Radicem quæsitam exhibere. Ac par est Ratio in tertia ac quarta formulis, nisi quod $\frac{1}{27}b^3 + \frac{1}{6}bp \pm \frac{1}{2}q$ pro Radio assumenda est, ac $\frac{1}{9}bb + \frac{1}{3}p$ in $\sqrt{\frac{1}{9}bb + \frac{1}{3}p}$ sive $d\sqrt{d}$ pro Sinu. Sed hæc præcepta exemplis fortasse melius percipientur.

Sit

Sit æquatio Cubica, $zzz - 17zz + 54z - 350$, ac quæratur Radix $z$: Hic $\frac{1}{3}bb$ major eſt quam $p$, ſed $q$ major eſt quam Cubus ex $\frac{1}{3}b$, ideoque una tantum Affirmativa Radice explicabilis eſt. Jam $\frac{289}{9} - \frac{54}{3}$ eſt $d$, ac $\frac{127}{9}$ $\sqrt{\frac{127}{9}}$ pro Sinu habenda eſt, ad Radium $\frac{4913}{27} + 175 - 153$, hoc eſt $\frac{5507}{27}$: Arcus vero competens fit 15 *gr.* 3 *m.* 49 *s.* Hujus Sinus Verſi Log. 8.5362376, additus Log. Radii 2.3095913, dat 0.8457889, cujus tertia pars 0.2819276 eſt Log. Radicis Cubicæ 1.91394, quo Diviſore diviſo, $\frac{127}{9}$ ſive $d$, fit Quotus 7.37281; Quoti ac Diviſoris ſumma, aucta additione $\frac{1}{3}b$, fit Radix quæſita, nempe 14.9534, &c.

Exactis Cubicis Biquadraticas jam aggrediamur. Hæ ſemper vel nullam, vel duas, vel quatuor Radices veras habent, quarum determinatio, partim à Coefficientibus, partim à ſigno & magnitudine Numeri abſoluti dati, pendet. In Conſtructione æquationis $z^4 - bz^3 + pzz - qz + r = 0$, fit BD $= \frac{1}{4}b$, AB $= \frac{1}{16}bb$, BK $= \frac{1}{2}$, ſive dimidio Lateris recti, KC $= 2$ AB $= \frac{1}{8}bb$, KE $= \frac{1}{8}bb - \frac{1}{2}p$, AE $- \frac{1}{2} = \frac{3}{16}bb - \frac{1}{2}p$, FE $= \frac{1}{16}b^3 - \frac{1}{4}bp$, ac EG $= \frac{1}{16}b^3 - \frac{1}{4}bp + \frac{1}{2}q$; quo facto Circulus, Centro G, Radio $\sqrt{\overline{GD}^2 - r}$, interſecabit Parabolam vel nullo, duobus, aut quatuor punctis; quæ perpendiculis in lineam HD Radices omnes $z$ exhibent. Ut autem quatuor ſint, evidens eſt Centrum Circuli alicubi conſtitui debere intra ſpatium, de cujus puncto quovis tria perpendicula in Curvam Parabolæ demitti poſſint; atque ſimul Radium minorem eſſe maximo ex illis perpendiculis majorem vero medio. Quod ſi Centrum conſtituatur extra hoc ſpatium, ut non niſi una perpendicularis in Parabolam demitti poſſit, qua major ſit Radius; vel ſi minor ſit media ex tribus perpendicularibus, major vero quam minima ex illis, duæ tantum poſſunt eſſe Radices; nulla vero omnino datur, quoties Radius $\sqrt{\overline{GD}^2 - r}$, minor eſt minima ex tribus, vel una illa, quoties una tantum eſt. Jam quale ſpatium hoc ſit, quibuſque limitibus diſcernitur, ac quibus conditionibus Radius Circuli minor vel major ſit prædictis perpendicularibus, nobis reſtat inquirendum; ac primum quo pacto perpendicularis in Parabolam demitti poſſit oſtendendum eſt. *Fig.* 49.

Sit ABC Parabola, AE Axis ejus, AV Semi-Latus Rectum, G punctum de quo demittenda eſt perpendicularis: Ducatur Axi perpendicularis GE, ac biſecetur VE in F, & erecta perpendiculari FH ad idem Axis latus, fiat FH $= \frac{1}{4}$ GE; dico quod Circulus, Centro H, Radio HA deſcriptus, Parabolam *Fig.* 53.

bolam interſecabit in punctis tribus, vel uno, Z; ad quæ ductæ rectæ G Z Curvæ Parabolicæ perpendiculariter inſiſtunt.

Ut autem tres ſint hujuſmodi interſectiones, oportet Centrum Circuli H ita
Fig. 51. collocari, ut ſit intra ſpatium Paraboloidibus incluſum; hoc eſt ut F H minor ſit quam $\surd \frac{8}{27} VF^3$, ſive $FH^2$ minus quam cubus ex $\frac{2}{3} VF$: atque adeo $GE = 4FH$, minor erit quam $4 \surd \frac{8}{27} VF^3$, ſive $4 \surd \frac{1}{27} VE^3$, hoc eſt Quadratum ex G E minus erit quam $\frac{16}{27} VE^3$. Coincidunt itaque hi Limites cum Paraboloidibus duabus ejuſdem generis cum iis quibus in Cubicis uſi ſumus, ſed quarum Latus Rectum duplo minor eſt; *viz.* $\frac{27}{16}$ Lateris Recti Parabolæ, hoc eſt $\frac{27}{8}$ ipſius A V: ideoque ea ipſa eſt linea Curva cujus Evolutione generatur Parabola, ſic demonſtrante *Hugenio*; quamque ſemper con-
Fig. 52. tingit linea D F, quæ Parabolæ perpendiculariter inſiſtit in puncto D. Punctum autem P, ſive in quo contingit recta D F Paraboloidem, Centrum eſt Circuli, qui Radio D P deſcriptus cum Parabola in puncto D coincidit, ſive ejuſdem Curvitatis eſt; ut per ſe ſatis conſtat.

Fig. 52. Deſcriptis itaque hujuſmodi Paraboloidibus V X P, V N Δ, utrinque ab Axe; perſpicuum eſt quod, niſi Centrum Circuli conſtituatur intra hos limites, non poſſit ille pluribus quam duobus in punctis Parabolam interſecare: unde determinare licet quibus ſub conditionibus Coefficientes terminorum intermediorum coercentur, in æquationibus Biquadraticis, ut habeantur quatuor Radices. Ac prima fronte clarum eſt $p$ majorem eſſe non poſſe quam $\frac{3}{8} bb$, (*ſcil.* in formulis ubi habetur $+ p$) nec $q$ quam $\frac{1}{16} b^3$. Generaliter vero $\frac{1}{16} b^3 \mp \frac{1}{4} pb \mp \frac{1}{2} q$, id eſt diſtantia Centri ab Axe E G, minor eſſe debet quam $EH = 4 \surd \frac{1}{27} VE^3$, hoc eſt (ob $VE = \frac{3}{16} bb \mp \frac{1}{2} p$) quam $\frac{1}{4} bb \mp \frac{2}{3} p \sqrt{\frac{1}{16} bb \mp \frac{1}{6} p}$; ſignis + & — in dubio relictis, ut ſecundum æquationis cujuſvis naturam variari poſſint; quemadmodum in Cubicis ſuperius oſtenſum eſt.

Termini autem ultimi $r$ limitatio eadem facilitate inveniri nequit; id adeo, quia Problema ſit Solidum in Curvam Parabolæ demittere perpendicularem, quodque non ſine ſolutione æquationis Cubicæ reſolvi poſſit. Itaque primo loco deficiat ſecundus terminus, vel ſi adfuerit tollatur, ut æquatio habeat formulam, $z^4 . * . pz^2 . qz . r . = 0$. Ac ſi fuerit $- r$, ſemper duabus vel quatuor Radicibus explicari poteſt; ut autem quatuor ſint, oportet Centrum Circuli intra Paraboloides prædictas conſtitui, ſive ut ſit $- p$,

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ac $qq$ minus quam $\frac{8}{27}p^3$, ſive cubo ex $\frac{2}{3}p$. Deinde habeantur Radices æquationis hujus $y^3. * . \frac{1}{2}py. \frac{1}{4}q = 0$, quantitatibus $p$ & $q$ iiſdem ſignis annexis quibus in Biquadratica. Hæ autem Radices auxilio Tabulæ Sinuum ſatis expeditè inveniuntur. Inventis autem tribus illis $y$, (quæ ſunt ordinatim applicatæ ad Axem Parabolæ, de punctis ubi incidunt perpendicula in Curvam ejus *ſcil.* Z Y) $pyy - 3y^4$ ex minore $y$, quantitatem maximam $r$ deſignabit, ſi fuerit $-r$; qua ſi minor fuerit $r$, æquatio quatuor habebit Radices, aliter duas. Aſt ſi fuerit $+r$, oportebit eam minorem eſſe quam $3y^4 - pyy$ ex media $y$, nam ſi major ſit, non niſi duas habere poteſt Radices, ſaltem ſi minor ſit $r$, quam $3y^4 - pyy$ ex maxima $y$. Hac vero ſi major ſit, nulla omnino Radice vera explicabilis eſt æquatio. Hi vero iidem Limites aliter deſignantur ex quantitate $q$, *ſcil.* $\frac{1}{2}qy - y^4$ in primo caſu, $y^4 - \frac{1}{2}qy$ in ſecundo, ac $y^4 + \frac{1}{2}qy$ in tertio. Fig. 53.

Fieri autem poteſt ut duæ minores quantitates $y$ non longe diſtant ab invicem, unde evenit quod utraque ex perpendicularibus major ſit quam recta G A, *ſcil.* cum $qq$ majus ſit quam $\frac{4}{27}p^3$, minus vero quam $\frac{8}{27}p^3$; cadente centro intra ſpatium Paraboloidibus (utriuſque Figuræ 51 & 52) interjectum. Hoc in caſu, ſi fuerit $+r$, non niſi duæ poſſunt eſſe Radices, exiſtente $y^4 + \frac{1}{2}qy$ ex maxima $y$, major quam $r$; aliter nulla. At ſi $\frac{1}{2}qy - y^4$ ex minima $y$, major fuerit quam $r$ ſigno $-$ notata, $r$ vero major quam $\frac{1}{2}qy - y^4$ ex media $y$, tunc habentur quatuor radices; at duæ tantum, ſi vel major priore vel minor poſteriore inventa ſit $r$.

Si vero in æquatione fuerit $+p$, vel ſi ſit $-p$ & $qq$ majus fuerit quam $\frac{8}{27}p^3$, æquatio $y^3. * . \frac{1}{2}py. \frac{1}{4}q$, unica tantum explicatur Radice $y$; hoc eſt, una tantum perpendicularis de Centro Circuli demitti poteſt: unde certo concluditur duas tantum Radices haberi poſſe in æquatione data, quarum Summa, ſi fuerit $-r$, cum quantitate $r$ augetur; at ſi habeatur $+r$, obtenta quantitate $y$, quantitas illa $r$ minor eſſe debet quam $y^4 + \frac{1}{2}qy$; nam ſi ea major ſit, æquatio propoſita abſurda & impoſſibilis eſt.

Longum & ſuperfluum eſſet omnes hujus ſenſus æquationes percurrere, cum ex jam dictis attendenti ſatis evidens ſit, quæ Negativæ quæ Affirmativæ ſint; atque quod Radicum harum Limites ex quantitatibus inventis $y$ petantur. In exemplum vero, quod cuivis in cæteris imitari licet, proponantur indagandi Limites ſive Conditiones ſub quibus in Æquatione Biquadratica 4. Radices Affirmativæ dari poſſint. Hoc autem fit quoties Centrum Circuli G, ponitur Fig. 52.

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in ſpatio VPK, ac ſimul habetur $+r$, ſive Circuli Radius minor quam GD: Unde patet, æquationem de qua agitur hujus eſſe formulæ, $z^4 - bz^3 + pz^2 - qz + r = 0$; $p$ vero majorem eſſe non poſſe quam $\frac{3}{8}bb$, nec $\frac{1}{4}pb$, hoc in caſu, quam $\frac{1}{16}b^3 + \frac{1}{2}q$; deinde opus eſt ut $\frac{1}{4}bb - \frac{2}{3}p$ in $\sqrt{\frac{1}{16}bb - \frac{1}{6}p}$ major ſit quam $\frac{1}{16}b^3 + \frac{1}{2}q - \frac{1}{4}pb$; & ex his Limitibus certo conſtabit Centrum intra ſpatium VPK inveniri. Ut vero definiatur quantitas $r$, ſolvenda primum eſt Cubica, $y^3. * . - \frac{3}{16}b^2 - \frac{1}{2}py = \frac{1}{32}b^3 + \frac{1}{4}q - \frac{1}{8}pb$; & habebuntur puncta, in quæ perpendiculares de Centro in Curvam Parabolæ cadunt.

Inventis autem tribus valoribus hujus $y$, $r$ minor eſſe debet quam $\frac{3}{256}b^4 + \frac{1}{4}bq - \frac{1}{16}bbp + 3y^4 - \frac{3}{8}b^2yy + pyy$ ex media $y$, major vero quam $\frac{3}{256}b^4 + \frac{1}{4}bq - \frac{1}{16}bbp + 3y^4 - \frac{3}{8}b^2yy + pyy$ ex minima $y$. Hos vero Limites ſi excedat $r$, non niſi duæ Radices haberi poſſunt. Deniq; ſi $\frac{3}{256}b^4 + \frac{1}{4}bq - \frac{1}{16}bbp + 3y^4 - \frac{3}{8}bbyy + pyy$ ex maxima $y$, minor fuerit quam $r$, æquatio propoſita impoſſibilis eſt.

Accidit etiam ut quatuor ſint Affirmativæ, cum Centrum G conſtituitur in ſpatiolo VTS, ducta *ſcil.* RTS perpendiculari in medium ſuppoſitæ lineæ AD: hoc autem fit cum $p$ major eſt quam $\frac{5}{16}bb$, ac $\frac{1}{4}bb - \frac{2}{3}p\sqrt{\frac{1}{16}bb - \frac{1}{6}p}$ major quam $\frac{1}{8}pb - \frac{5}{128}bbb - \frac{1}{2}q$. Quo in caſu ſemper duæ, aliquando tres, ex Radicibus fiunt majores quam $\frac{1}{4}b$.

*Fig.* 52. Notandum vero hic limitem illum ex minima $y$ productum, aliquando negativum fieri, ſive minorem nihilo; quoties *ſcil.* maxima ex tribus perpendicularibus major eſt quam GD. Hoc, ſi acciderit quantitas $+r$, à Limite præſcripto ex media $y$, in nihilum minui poteſt. Defectus vero Limitis ex minima $y$ monſtrat quanta poſſit eſſe $-r$ in æquatione, ſi habeantur tres Radices Affirmativæ ac una Negativa; quam ſi excedat, non niſi duæ, altera Affirmativa, altera Negativa, dari poſſunt. Hæc autem omnia demonſtrantur ex eo quod prædicti Limites quantitatis $r$, ſint differentiæ Quadratorum lineæ GD & perpendicularium in Curvam Parabolæ.

Ob perplexas vero cautiones, quas parit in æquationibus hiſce ſignorum diverſitas, præſtat ſemper ſecundum terminum tollere, ac deinde juxta præcepta jam tradita Radicum numerum ac ſigna inquirere; præſertim ſi quantitates illæ $y$ non multum ab invicem. Ex quatuor autem hisce Radicibus Affirmativis, duæ semper sunt minores quam $\frac{1}{4}b$, duæ vero majores; nempe si D G, minor sit quam A G, sive $\frac{1}{4}pb$ quam $\frac{3}{64}b^3+q$. Tres autem minores sunt quam $\frac{1}{4}b$ quoties perpendicularis media, sive ex media $y$ inventa, major est quam A G, sive $\frac{3}{8}bby$ major quam $3y^3-pyy$ ex eadem media $y$; Quarta vero & maxima Radix major est quam maxima $y+\frac{1}{4}b$; æquatur autem differentiæ ipsius $b$ & summæ cæterarum trium Radicum, ideoque minor est $b$. Sed jam manum de Tabula; Fortassis illi qui naturam Parabolæ penitius perspectam habent, majori compendio hæc omnia peragere valebunt; at si quantitates hæ omnes $b.\ p.\ q$ & $r$, absque resolutione Cubicæ æquationis ritè determinari possint, non sine causa ambigitur; quæcunque enim æquationibus planis hac in re fiunt, non veros Limites, sed Approximationes tantum exhibent.

XX. Regulas binas compendiosas admodum pro Approximatione Radicis Cubicæ nuper protulit *D. de Lagny*, alteram rationalem, alteram irrationalem; nempe Cubi $aaa+b$ latus esse inter $a+\frac{ab}{3a^3+b}$ ac $\sqrt{\frac{1}{4}aa+\frac{b}{3a}}+\frac{1}{2}a$. *The Extraction of all Roots without any previous Reduction; by Mr. Ed. Halley. n. 210. p. 136.*

Radicem autem potestatis Quintæ $a^5+b$ sic exprimit $=\frac{1}{2}a+\sqrt{\sqrt{\frac{1}{4}a^4+\frac{b}{5a}}-\frac{1}{4}aa}$ (non $\frac{1}{2}aa$ ut perperam legitur in libro Gallico impresso). Demonstrantur autem Regulæ prædictæ ex Genesi Cubi & Potestatis quintæ. Posito enim Latere Cubi cujusque $a+e$, Cubus inde conflatus fit $aaa+3aae+3aee+eee$, adeoque si supponatur $aaa$ numerus Cubus proxime minor dato quovis non Cubo, $eee$ minor erit Unitate, ac residuum sive $b$ æquabitur reliquis Cubi membris $3aae+3aee+eee$: rejectoque $eee$ ob parvitatem, $b=3aae+3aee$. Cumque $aae$ multo majus sit quam $aee$, $\frac{b}{3aa}$ non multum excedet ipsam $e$; positoque $e=\frac{b}{3aa}$, $\frac{b}{3aa+3ae}$, cui proxime æquatur quantitas $e$, invenietur $=\frac{b}{3aa+\frac{3ab}{3aa}}$ sive $\frac{b}{3aa+\frac{b}{a}}$: hoc est $\frac{ab}{3aaa+b}=e$, adeoque latus Cubi $aaa+b$ habebitur $a+\frac{ab}{3aaa+b}$, quæ est ipsa formula rationalis *D. de Lagny.* Quod si $aaa$ fuerit Numerus Cubus proxime

 major

major dato, Latus Cubi $aaa + b$, pari ratiocinio invenietur $a - \frac{ab}{3aaa + b}$; atque hæc Radicis Cubicæ approximatio satis expedita ac facilis parum admodum fallit in defectu, cum scilicet $e$ residuum Radicis hoc pacto inventum paulo minus justo sit. Irrationalis vero formula etiam ex eodem fonte derivatur, *viz.* $b = 3aae + 3aee$, sive $\frac{b}{3a} = ae + ee$; adeoque $\sqrt{\frac{1}{4}aa + \frac{b}{3a}} = \frac{1}{2}a + e$, atque $\sqrt{\frac{1}{4}aa + \frac{b}{3a}} + \frac{1}{2}a = a + e$, sive Radici quæsitæ. Latus vero Cubi $aaa - b$ eodem modo habebitur $\frac{1}{2}a + \sqrt{\frac{1}{4}aa - \frac{b}{3a}}$. Atque hæc quidem formula aliquanto propius ad scopum collimat, in excessu peccans sicut altera in defectu, ac ad praxim magis commoda videtur, cum restitutio Calculi nihil aliud sit quam continua additio vel subductio ipsius $\frac{eee}{3a}$, secundum ac quantitas $e$ innotescat; ita ut potius scribendum sit $\sqrt{\frac{1}{4}aa + \frac{b - eee}{3a}} + \frac{1}{2}a$ in priori casu, ac in posteriori $\frac{1}{2}a + \sqrt{\frac{1}{4}aa + \frac{eee - b}{3a}}$. Utraque autem formulâ Ciphræ jam cognitæ in Radice extrahendâ ad minimum triplicantur, quod quidem Arithmeticæ studiosis omnibus gratum fore confido, atque ipse Inventori abunde gratulor. Ut autem harum Regularum utilitas melius sentiatur, exemplum unum vel alterum adjungere placuit.

*Exemp.* I. Quæratur Latus Cubi dupli, sive $aaa + b = 2$. Hic $a = 1$, atque $\frac{b}{3a} = \frac{1}{3}$, adeoque $\frac{1}{2} + \sqrt{\frac{7}{12}}$ sive 1, 26 invenietur Latus prope verum. Cubus autem ex 1, 26 est 2, 000376, adeoque 0, 63 + $\sqrt{0,3969 - \frac{0,000376}{3,78}}$ sive 0, 63 + $\sqrt{0,396900529100529}$ = 1, 259921049895,— ; quod quidem tredecem figuris Latus Cubi dupli exhibet, nullo fere negotio, *viz.* unâ Divisione & Lateris Quadrati extractione, ubi vulgari operandi modo quantum desudasset Arithmeticus nôrunt experti. Hunc etiam calculum quousque velis continuari licet, augendo quadratum additione $\frac{eee}{3a}$. Quæ quidem correctio hoc in casu non nisi unitatis in Radicis figurâ decima-quartâ augmentum affert.

*Exemp.*

*Exemplum* II. Quæratur Latus Cubi æqualis menſuræ Anglice *Gallon* dictæ, uncias ſolidas 231 continentis. Cubus proxime minor eſt 216 cujus Latus $6 = a$, ac reſiduum $15 = b$; adeoque pro prima approximatione provenit $3 + \sqrt{9 + \frac{5}{6}} =$ Radici. Cumque $\sqrt{9,8333\ldots}$ ſit $3,1358\ldots$ patet $6,1358 = a + e$. Supponatur jam $6,1358 = a$, & habebimus Cubum ejus $231,000853894712$, ac juxta regulam $3,0679 + \sqrt{9,41201041 - \frac{0,000853894712}{18,4074}}$ æquatur accuratiſſime Lateri Cubi dati, id quod intra horæ ſpatium calculo obtinui $6,135792439661958973$, in octodecimâ figurâ juſtum, at deficiens in decima-nona. Hæc vero formula meritò preferenda eſt rationali, ob ingentem diviſorem, non ſine magno labore tractandum; cum Lateris Quadrati extractio multo facilius procedat, ut experientia multiplex me docuit.

Regula autem pro Radice Surſolidi puri ſive Poteſtatis quintæ paulo altioris indaginis eſt, atque etiam adhuc multo perfectius rem præſtat: datas enim in Radice ciphras ad minimum quintuplicat, neque etiam multi nec operoſi eſt calculi. Author autem nullibi inveniendi methodum ejuſve demonſtrationem concedit, etiamſi maxime deſiderari videatur: præſertim cum in Libro impreſſo non recte ſe habeat; id quod imperitos facile illudere poſſit. Poteſtas autem Quinta Lateris $a + e$ conficitur ex his membris $a^5 + 5a^4e + 10a^3e^2 + 10a^2e^3 + 5ae^4 + e^5 = a^5 + b$, unde $b = 5a^4e + 10a^3e^2 + 10a^2e^3 + 5ae^4$, rejecto $e^5$ ob parvitatem ſuam: quocirca $\frac{b}{5a} = a^3e + 2a^2e^2 + 2ae^3 + e^4$, atque utrinque addendo $\frac{1}{4}a^4$ habebimus $\sqrt{\frac{1}{4}aaaa + \frac{b}{5a}} = \sqrt{\frac{1}{4}a^4 + a^3e + 2a^2e^2 + 2ae^3 + e^4} = \frac{1}{2}aa + ae + ee$. Dein utrinque ſubducendo $\frac{1}{4}aa$, $\frac{1}{2}a + e$ æquabitur $\sqrt{\sqrt{\frac{1}{4}a^4 + \frac{b}{5a}} - \frac{1}{4}aa}$, cui ſi addatur $\frac{1}{2}a$, erit $a + e = \frac{1}{2}a + \sqrt{\sqrt{\frac{1}{4}a^4 + \frac{b}{5a}} - \frac{1}{4}aa} =$ Radici poteſtatis $a^5 + b$. Quod ſi fuiſſet $a^5 - b$, (aſſumptâ $a$ juſto majore) regula ſic ſe haberet, $\frac{1}{2}a + \sqrt{\sqrt{\frac{1}{4}a^4 - \frac{b}{5a}} - \frac{1}{4}aa}$.

Atque hæc regula mirum in modum approximat, ut vix restitutione opus sit; at dum hæc mecum pensitavi, incidi in formularum methodum quandam generalem pro quavis protestate satis concinnam, quamque celare nequeo; cum etiam in superioribus potestatibus datas radices figuras triplicare valeant. Hæ autem formulæ ita se habent tam rationales quam irrationales.

$$\sqrt{aa+b} = \sqrt{aa+b}, \text{ vel } a + \frac{ab}{2aa + \frac{1}{2}b}$$

$$\sqrt[3]{a^3+b} = \tfrac{1}{2}a + \sqrt{\tfrac{1}{4}aa + \frac{b}{3a}}, \text{ vel } a + \frac{ab}{3a^3 + b}$$

$$\sqrt[4]{a^4+b} = \tfrac{2}{3}a + \sqrt{\tfrac{1}{9}aa + \frac{b}{6aa}}, \text{ vel } a + \frac{ab}{4a^4 + \frac{3}{2}b}$$

$$\sqrt[5]{a^5+b} = \tfrac{3}{4}a + \sqrt{\tfrac{1}{16}aa + \frac{b}{10a^3}}, \text{ vel } a + \frac{ab}{5a^5 + 2b}$$

$$\sqrt[6]{a^6+b} = \tfrac{4}{5}a + \sqrt{\tfrac{1}{25}aa + \frac{b}{15a^4}}, \text{ vel } a + \frac{ab}{6a^6 + \frac{5}{2}b}$$

$$\sqrt[7]{a^7+b} = \tfrac{5}{6}a + \sqrt{\tfrac{1}{36}aa + \frac{b}{21a^5}}, \text{ vel } a + \frac{ab}{7a^7 + 3b}$$

Et sic de cæteris etiam adhuc superioribus. Quod si assumeretur *a* radice quæsitâ major, (quod cum fructu fit quoties Potestas resolvenda multo propior sit potestati Numeri integri proxime majoris quam proxime minoris) mutatis mutandis eædem radicum expressiones proveniunt.

$$\sqrt{aa-b} = \sqrt{aa-b}, \text{ vel } a - \frac{ab}{2aa - \frac{1}{2}b}$$

$$\sqrt[3]{a^3-b} = \tfrac{1}{2}a + \sqrt{\tfrac{1}{4}aa - \frac{b}{3a}}, \text{ vel } a - \frac{ab}{3a^3 - b}$$

$$\sqrt[4]{a^4-b} = \tfrac{2}{3}a + \sqrt{\tfrac{1}{9}aa - \frac{b}{6aa}}, \text{ vel } a - \frac{ab}{4a^4 - \frac{3}{2}b}$$

$$\sqrt[5]{a^5-b} = \tfrac{3}{4}a + \sqrt{\tfrac{1}{16}aa - \frac{b}{10a^3}}, \text{ vel } a - \frac{ab}{5a^5 - 2b}$$

$$\sqrt[6]{a^6-b} = \tfrac{4}{5}a + \sqrt{\tfrac{1}{25}aa - \frac{b}{15a^4}}, \text{ vel } a - \frac{ab}{6a^6 - \frac{5}{2}b}$$

$$\sqrt[7]{a^7-b} = \tfrac{5}{6}a + \sqrt{\tfrac{1}{36}aa - \frac{b}{21a^5}}, \text{ vel } a - \frac{ab}{7a^7 - 3b}$$

Atque inter hos duos terminos semper consistit vera radix, aliquanto propior irrationali quam rationali; *e* vero juxta formulam irrationalem inventa, semper peccat in excessu, sicut in defectu à rationali formulâ resultans

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tans Quotus; adeoque si fuerit $+ b$, Irrationalis majorem justo exhibet Radicem, rationalis minorem; è contrario vero si fuerit $- b$. Atque hæc de eliciendis radicibus è Potestatibus puris dicta sunto, quæ quidem ad usus ordinarios sufficientes multo facilius habentur ope Logarithmorum: quoties vero ultra Tabularum Logarithmicarum vires accuratissime definienda est radix, ad hujusmodi methodos necessario recurrendum est. Præterea cum ex harum formularum inventione ac contemplatione, Universalis Regula pro Æquationibus Affectis (quam non sine fructu Geometriæ ac Algebræ studiosis omnibus usurpandam confido) mihi ipsi oblata sit, volui ipsius inventi primordia quâ possim claritate aperire.

Æquationum quidem Affectarum Quadrato-quadratum non excedentium Constructionem generalem concinnam admodum ac facilem, *An. circiter* 1687. jam tum inventam, publici juris feci: ex quo ingens cupido animum incessit, idem Numeris efficiendi. At brevi post D. *Raphson* magna ex parte voto satisfecisse visus est, usque dum *D. de Lagny* etiam adhuc compendiosius rem peragi posse hoc suo libello mihi suggessit. Methodus autem nostra hæc est. Vid. sup. §. XIX. 1.

Supponatur Radix cujusvis æquationis $z$ composita ex partibus $a +$ vel $- e$, quarum $a$ ex hypothesi assumatur ipsi $z$ quantum fieri possit propinqua (quod tamen commodum est, non necessarium) & ex quantitate $a +$ vel $- e$ formentur Potestates omnes ipsius $z$ in Æquatione inventas, iisque affigantur Numeri Coefficientes respectivè: deinde Potestas Resolvenda subducatur è summa partium datarum in prima columna, ubi $e$ non reperitur, quam Homogeneum Comparationis vocant, sitque differentia $\pm b$. Dein habeatur summa omnium Coefficientium ipsius lateris $e$ in secunda Columna, quæ sit $s$; denique in tertia addantur omnes coefficientes quadrati $ee$, quarum summam vocemus $t$, ac Radix quæsita $z$, formulâ rationali habebitur $= a +$ vel $- \frac{sb}{ss + \text{vel} - tb}$; irrationali vero fiet $z = a \frac{\mp \frac{1}{2} s \pm \sqrt{\frac{1}{4} ss \mp bt}}{t}$, id quod exemplis Illustrare fortasse operæ pretium erit. Instrumenti vero loco adsit Tabella, Potestatum omnium ipsius $a +$ vel $- e$ Genesin exhibens, quæ si opus fuerit continuari facile possit. A septima vero incipiam cum pauca Problemata eo usque assurgere deprehendantur. Hanc Tabellam jure optimo *Speculum Analyticum Generale* appellare licet. Potestates autem prædictæ ex continua multiplicatione per $a + e = z$ ortæ, sic proveniunt, cum suis coefficientibus adjunctis.

$$\begin{array}{l} \qquad\qquad\qquad\qquad s \qquad\qquad\qquad t \qquad\qquad\qquad u \\ lz^7 = la^7 + 7la^6e + 21la^5ee + 35la^4e^3 + 35la^3e^4 \\ kz^6 = ka^6 + 6ka^5e + 15ka^4ee + 20ka^3e^3 + 15ka^2e^4 \\ hz^5 = ha^5 + 5ha^4e + 10ha^3ee + 10ha^2e^3 + 5hae^4 \\ gz^4 = ga^4 + 4ga^3e + 6ga^2ee + 4gae^3 + ge^4 \\ fz^3 = fa^3 + 3fa^2e + 3faee + fe^3 \\ dz^2 = da^2 + 2dae + dee \\ cz = ca + ce \end{array}$$

$$\begin{array}{l} \qquad\quad x \qquad\qquad y \\ + 21la^2e^5 + 7lae^6 + le^7 \\ + 6kae^5 + ke^6 \\ + he^5 \end{array}$$

Quod si fuerit $a - e = z$, ex iisdem membris conficitur Tabella, negatis solummodo imparibus Potestatibus ipsius $e$, ut $e$, $e^3$, $e^5$, $e^7$: & affirmatis paribus $e^2$, $e^4$, $e^6$. Sitque summa Coefficientium lateris $e = s$; Summa Coefficientium Quadrati $ee = t$; Cubi $eee = u$; Biquadrati $e^4 = w$; Sursolidi $e^5 = x$; Summa vero Coefficientium Cubo-cubi $e^6 = y$; &c. Cum autem supponatur $e$ exigua tantum pars Radicis inquirendæ, omnes potestates ipsius $e$ multo minores evadunt similibus ipsius $a$ Potestatibus, adeoque pro prima hypothesi rejiciantur superiores (ut in potestatibus puris ostensum est) ac formatâ æquatione novâ, substituendo $a \pm e = z$ habebimus ut diximus $\pm b = \pm se \pm tee$. Cujus rei cape exempla sequentia, quo melius intelligatur.

*Exemp.* I. Proponatur æquatio $z^4 - 3zz + 75z = 10000$. Pro prima hypothesi ponatur $a = 10$, ac consequenter prodibit æquatio.

$$\begin{array}{l} z^4 = + a^4 \pm 4a^3e + 6a^2ee \pm 4ae^3 + e^4 \\ -dz^2 = -da^2 \pm dae - dee \\ + cz = + ca \quad ce \\ = + 10000 \pm 4000e + 600ee \pm 40e^3 + e^4 \\ \quad - 300 \pm 60e - 3ee \\ \quad + 750 \pm 75e \\ \quad - 10000 \\ \hline + 450 \pm 4015e + 597ee \pm 40e^3 + e^4 = 0 \\ \qquad\qquad s \qquad\qquad t \qquad\qquad u \end{array}$$

Signis + ac — (respectu $e$ & $e^3$) in dubio relictis, usque dum sciatur an $e$ sit Negativa vel Affirmativa; quod quidem aliquam paret difficultatem, cum in æquationibus plures radices admittentibus, sæpe augeantur Homogenia Comparationis, ut appellant, à minuta quantitate $a$, ac è contra eâ auctâ minuantur. Determinatur autem signum ipsius $e$ ex signo quantitatis $b$; sublatâ enim Resolvendâ ex Homogenio ab $a$ formato, signum ipsius $se$, ac proinde partium in ejus compositione prævalentium, semper contrarium erit

signo

signo differentiæ $b$. Unde patebit an fuerit $-e$ vel $+e$, sive an $a$ major vel minor radice vera assumpta sit. Ipsa autem $e$ semper æquatur $\frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - bt}}{t}$, quoties $b$ ac $t$ eodem signo notantur; quoties vero diverso signo connectuntur, eadem $e$ fit $\frac{\sqrt{\frac{1}{4}ss + bt} - \frac{1}{2}s}{t}$. Postquam vero compertum sit fore $-e$, in affirmatis æquationis membris negentur $e$, $e^3$, $e^5$, &c. in negatis affirmentur; scribantur *scil.* signo contrario; si vero fuerit $+e$, affirmentur in affirmatis, negentur in negatis. Habemus autem in hoc nostro exemplo 10450 loco Resolvendæ 10000, sive $b = +450$, unde constat $a$ majorem justo assumtam, ac proinde haberi $-e$: Hinc equatio fit $10450 - 4015e + 597ee - 4e^3 + e^4 = 10000$. Hoc est $450 - 4015e + 597ee = 0$. Adeoque $450 = 4015e - 597ee$ sive $b = se - tee$ cujus Radix $e$ fit $\frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - bt}}{t}$, vel si mavis $\frac{s}{2t} - \sqrt{\frac{ss}{4tt} - \frac{b}{t}}$, id est, in præsenti casu, $e = \frac{2007\frac{1}{2} - \sqrt{3761406\frac{1}{4}}}{597}$, unde provenit radix quæsita prope verum 9,886. Hoc vero pro secundâ Hypothesi substituto, emergit $a + e = z$ accuratissime 9, 8862603936495...., in ultima figura vix binario justum superans; nempe cum $\frac{\sqrt{\frac{1}{4}ss + bt} - \frac{1}{2}s}{t} = e$. Atque hoc etiam, si opus fuerit, multo ulterius verificari possit, subducendo $\frac{\frac{1}{2}ue^3 + \frac{1}{2}e^4}{\sqrt{\frac{1}{4}ss + tb}}$ si fuerit $+e$, vel addendo $\frac{\frac{1}{2}ue^3 - \frac{1}{2}e^4}{\sqrt{\frac{1}{4}ss - tb}}$, radici prius inventæ, si sit $-e$.

Cujus compendium eo pluris æstimandum quod quandoque, ex sola prima suppositione, semper vero ex secunda, iisdem conservatis coefficientibus quousque velis calculum continuare possis. Cæterum æquatio prædicta etiam negativam habet radicem, *viz.* $z = 10, 26.....$ quam cuilibet accuratius expiscari licet.

*Exemp.*

*Exemp.* II. Sit $z^3 - 17zz + 54z = 350$; ac ponatur $a = 10$. Ex præſcripto Regulæ,

$$\begin{aligned} zzz &= aaa + 3aae + 3aee + eee \\ -dzz &= daa - 2dae - dee \\ +cz &= ca + ce \end{aligned}$$

$$\begin{array}{lllll} & b & s & t & \\ \text{Id eſt} & +1000 & +300e & +30ee & +eee \\ & -1700 & -340e & -17ee & \\ & +540 & +54e & & \\ & -350 & & & \\ \hline \text{Sive} & -510 & +14e & +13ee & +eee = 0. \end{array}$$

Cum autem habeatur $-510$, conſtat $a$ minorem juſto aſſumi, ac proinde $e$ affirmativam eſſe, ac ex $510 = 14e + 13ee$ fit $\frac{\sqrt{bt + \frac{1}{4}ss} - \frac{1}{2}s}{t} = e = \frac{\sqrt{6679} - 7}{13}$, unde $z$ fit $15, 7 \ldots$ quæ nimia quidem eſt ob late ſumptam $a$; ideo ſupponatur ſecundo $a = 15$, ac pari ratiocinio habebimus $e = \frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - tb}}{t} = \frac{109\frac{1}{2} - \sqrt{11710\frac{1}{4}}}{28}$; ac proinde $z = 14, 954068$. Quod ſi calculum adhuc tertio reſtaurare velis, uſque in vigeſimam quintam figuram vero conformem invenies radicem: Paucioribus vero contentus, ſcribendo $tb \pm teee$ loco $tb$, vel ſubtrahendo aut addendo radici prius inventæ $\frac{\frac{1}{2}eee}{\sqrt{\frac{1}{4}ss \mp tb}}$ ad ſcopum ſtatim perveniet. Æquatio vero propoſita nulla alia radice explicari poteſt, quia Poteſtas Reſolvenda 350 major eſt Cubo ex $\frac{17}{3}$ vel $\frac{1}{3}d$.

*Exemp.* III. Sit Æquatio illa quam in Reſolutione difficillimi Problematis Arithmetici adhibet *Clariſſimus Walliſius*, Cap. LXII. *Algebræ* ſuæ, quo radicem *Vietæ* Methodo accuratiſſime quidem aſſecutus eſt. Eandemque exemplum Methodi ſuæ affert laudatus D. *Raphſon*, pag. 25, 26. nempe $z^4 - 80z^3 + 1998z^2 - 14937z + 5000 = 0$. Hæc autem æquatio ejus formulæ eſt, ut plures habeat radices Affirmativas, ac quod difficultatem ejus augeat, prægrandes ſunt Coefficientes reſpectu Reſolvendæ datæ: Quo melius autem tractetur, dividatur, ac juxta notas punctationum regulas ponatur $-z^4 + 8z^3 - 20z^2 + 15z = 0, 5$ (ubi $z$ eſt $\frac{1}{10}z$ in æquatione

propoſita

proposita) ac pro prima Hypothesi habeamus $a = 1$. Proinde $+ 2 - 5e - 2ee + 4e^3 - e^4 - 0,5 = 0$.

Hoc est $1\frac{1}{2} = 5e + 2ee$; hinc $\frac{\sqrt{\frac{1}{4}ss + bt} - \frac{1}{2}s}{t} = e$ fit $\frac{\sqrt{37} - 5}{4}$ adeoque $z = 1,27$, unde constat $12,7$ radicem esse æquationis propositæ vero vicinum. Secundo loco supponatur $z = 12,7$, ac juxta præscriptum Tabellæ Potestatum oritur.

$$
\begin{array}{lllll}
\quad b & \quad s & \quad t & \quad u & \\
- 26014,4641 & - 8193,532\,e & - 967,74\,ee & - 50,8\,e^3 & - e^4 \\
+ 163870,640. & + 38709,60.\,e & + 3048,..\,ee & + 80,.\,e^3 & \\
- 322257,42.. & - 50749,2..\,e & - 1998,..\,ee & & \\
+ 189699,9... & + 14937,...\,e & & & \\
- 5000,.... & & & & \\
\hline
+ 298,6559 & - 5296,132\,e & + 82,26\,ee & + 29,2\,e^3 & - e^4 = 0
\end{array}
$$

Adeoque $- 298,6559 = - 5296,132\,e + 82,26\,ee$, cujus radix $e$ juxta regulam $= \frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - bt}}{t}$ fit $\frac{2648,066 - \sqrt{6987686,106022}}{82,26} =$ $0,05644080331\ldots = e$ minori vero: ut autem corrigatur, $\frac{\frac{1}{2}ue^3 - \frac{1}{2}e^4}{\sqrt{\frac{1}{4}ss - bt}}$ sive $\frac{0,0026201\ldots}{2643,423\ldots}$ fit $0,00000099117$; ac proinde $e$ correcta $=$ $0,05644179448$; Quod si adhuc plures radicis figuras desideras, formetur ex $e$ correctâ $tue^3 - te^4 = 0,43105602423\ldots$, ac $\frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - bt - tue^3 + te^4}}{t}$ sive $\frac{2648,066 - \sqrt{6987685,67496597577\ldots}}{82,26} =$ $0,05644179448074402 = e$, unde $a + e = z$ radix accuratissima fit $12,75644179448074402\ldots$ qualem invenit *Cl. Wallisius* in loco citato. Ubi observandum redintegrationem calculi semper triplicare notas veras in assumpta $a$, quas prima correctio sive $\frac{\frac{1}{2}ue^3 - \frac{1}{2}e^4}{\sqrt{\frac{1}{4}ss - bt}}$ quintuplices reddit, quæque etiam commode per Logarithmos efficitur. Altera autem correctio post primam, etiam duplum Ciphrarum numerum adjungit, ut omnino assumptas septuplicet; prima tamen plerumque usibus Arithmetices abunde sufficit. Quæ vero dicta sunt de numero Ciphrarum in radice recte assumptarum, ita intelligi velim, ut cum $a$ non nisi decimâ parte distet à vera radice, prima figura recte assumatur; si intra centesimam partem, duæ primæ; si intra millesimam, tres priores rite se habeant; quæ deinde juxta nostram regulam tractatæ statim novem evadunt.

Reſtat jam ut nonnulla adjiciam de noſtra formula rationali, *viz.* $e = \frac{sb}{ss \pm tb}$, quæ quidem ſatis expedita videbitur, nec multum cedet priori, cum etiam datas ciphras triplicare valeat. Formata autem æquatione ex $a \mp e = z$, ut prius, ſtatim patebit an $a$ aſſumpta ſit major vel minor vero, cum *ſcil.* $se$ ſigno ſemper notari debeat contrario ſigno differentiæ Reſolvendæ ac Homogenii ſui ex $a$ producti. Deinde poſito quod $\pm b \mp se +$ vel $- tee = 0$. Diviſor fit $ss - tb$, quoties $b$ ac $t$ iiſdem ſignis notantur; idem vero fit $ss + bt$, ſi ſigna iſta diverſa ſint. Praxi autem magis accommodata videtur ſi ſcriberetur Theorema, $e = \frac{b}{s \pm \frac{tb}{s}}$ nempe cum unâ Multiplicatione ac duabus diviſionibus res peragatur, quæ tres multiplicationes ac unam diviſionem aliàs requireret. Hujus etiam Methodi Exemplum capiamus à prædictæ Æquationis radice 12, 7 . . . : ubi $\underset{+b}{298,6559} - \underset{s}{5296,132\,e} + \underset{+t}{82,26\,ee} + \underset{+u}{29,2\,e^3} - e^4 = 0$, adeoque $\frac{b}{s - \frac{tb}{s}} = e$, hoc eſt, fiat ut $s$ ad $t$ ita $b$ ad $\frac{tb}{s} = 5296,132)\ 298,6559$ in $82,26\ (4,63875 \ldots$; quocirca diviſor fit, $s - \frac{tb}{s} = 5291,49325 \ldots)\ 298,6559\ (0,0564415 \ldots = e$, *viz.* quinque figuris veris adjectis radici aſſumptæ. Corrigi autem nequit hæc formula ſicut præcedens irrationalis; adeoque ſi plures deſiderentur radicis figuræ, præſtat aſſumpta nova Hypotheſi calculum de integro repetere: ac novus Quotus triplicando figuras in radice cognitas ſupputatori etiam maxime ſcrupuloſo abunde ſatisfaciet.

*A Method of Raiſing an Infinite Multinomial, to any given Power; by M. de Moivre. n. 230. p. 619.*

XXI. *Theorem.* $\overline{az + bz^2 + cz^3 + dz^4 + ez^5 + fz^6 + gz^7 + hz^8 + iz^9 \,\&c.}|^m = a^m z^m + \frac{m}{1} a^{m-1} b z^{m+1}$

$+ \frac{m}{1} \times \frac{m-1}{2} a^{m-2} b^2 z^{m+2} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} a^{m-3} b^3 z^{m+3}$

$+ \frac{m}{1} a^{m-1} c \qquad + \frac{m}{1} \times \frac{m-1}{1} a^{m-2} bc$

$+ \frac{m}{1} a^{m-1} d$

$+ \frac{m}{1}$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}a^{m-4}b^{4}z^{m+4}$$
$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}a^{m-3}b^{2}c$$
$$+\frac{m}{1}\times\frac{m-1}{1}a^{m-2}bd$$
$$+\frac{m}{1}\times\frac{m-1}{2}a^{m-2}c^{2}$$
$$+\frac{m}{1}a^{m-1}e$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{5}a^{m-5}b^{5}z^{m+5}$$
$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{1}a^{m-4}b^{3}c$$
$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}a^{m-3}b^{2}d$$
$$+\frac{m}{1}\times\frac{m-1}{1}\times\frac{m-2}{2}a^{m-3}bc^{2}$$
$$+\frac{m}{1}\times\frac{m-1}{1}a^{m-2}be$$
$$+\frac{m}{1}\times\frac{m-1}{1}a^{m-2}cd$$
$$+\frac{m}{1}a^{m-1}f$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{5}\times\frac{m-5}{6}a^{m-6}b^{6}z^{m+6}$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{1}a^{m-5}b^{4}c$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{1}a^{m-4}b^{3}d$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}\times\frac{m-3}{2}a^{m-4}b^{2}c^{2}$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}a^{m-3}b^{2}e$$

$$+\frac{m}{1}\times\frac{m-1}{1}\times\frac{m-2}{1}a^{m-3}bcd$$

$$+\frac{m}{1}\times\frac{m-1}{1}a^{m-2}bf$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}a^{m-3}c^{3}$$

$$+\frac{m}{1}\times\frac{m-1}{1}a^{m-2}ce$$

$$+\frac{m}{1}\times\frac{m-1}{2}a^{m-2}d^{2}$$

$$+\frac{m}{1}a^{m-1}g \text{ \&c.}$$

I ſuppoſe that the Infinite Number Multinomial is $az+bzz+cz^3+dz^4+ez^5$, &c. $m$ is the Index of the Power, to which this Multinomial ought to be rais'd, or if you will, 'tis the Index of the Root, which is to be extracted: I ſay, That this Power or Root of the Multinomial, is ſuch a Series as I have expreſs'd.

For the underſtanding of it, it is only neceſſary to conſider all the Terms by which the ſame Power of $z$ is Multiplied; in Order thereto, I diſtinguiſh two things in each of theſe Terms; 1°. The Product of certain Powers of the Quantities, $a$, $b$, $c$, $d$, &c. 2°. The Unciæ (as *Oughtred* calls 'em) prefixt to theſe Products. To find all the Products belonging to the ſame Power of $z$, to that Product, for inſtance, whoſe Index is $m+r$ (where $r$ may denote any Integer Number) I divide theſe Products into ſeveral Claſſes; thoſe which immediately after ſome certain Power of $a$ (by which all theſe Products begin) have $b$, I call Products of the firſt Claſſis; for Example, $a^{m-4}b^3e$ is a Product of the firſt Claſſis, becauſe $b$ immediately follows $a^{m-4}$; thoſe which immediately after ſome Power of $a$ have $c$, I call Products of the ſecond Claſſis, ſo $a^{m-3}ccd$, is a Product of the ſecond Claſ-

ſis;

ſis; thoſe which immediately after ſome Power of $a$ have $d$, I call Products of the third Claſſis; and ſo of the reſt.

This being done, I Multiply all the Products belonging to $z^{m+r-1}$ (which preceeds immediately $z^{m+r}$) by $b$, and divide 'em all by $a$; 2°. I Multiply by $c$, and Divide by $a$, all the Products belonging to $z^{m+r-2}$, except thoſe of the firſt Claſſis; 3°. I Multiply by $d$ and Divide by $a$, all the Products belonging to $z^{m+r-3}$, except thoſe of the firſt and ſecond Claſſis: 4°. I Multiply by $e$ and Divide by $a$, all the Terms belonging to $z^{m+r-4}$, except thoſe of the firſt, ſecond and third Claſſis; and ſo on, till I meet twice with the ſame Term. Laſtly, I add to all theſe Terms, the Product of $a^{m-1}$ into the letter, whoſe Exponent is $r+1$.

Here I muſt take notice, that by the Exponent of a Letter, I mean the Number which expreſſes what Place the Letter has in the Alphabet, ſo 3 is the Exponent of the Letter $c$, becauſe the Letter $c$ is the 3d in the Alphabet.

It is evident by this Rule, you may eaſily find all the Products belonging to the ſeveral Powers of $z$, if you have but the Product belonging to $z^{m}$, *viz.* $a^{m}$.

To find the Unciæ which ought to be prefix'd to every Product, I conſider the Sum of Units contain'd in the Indices of the Letters which compoſe it (the Index of $a$ excepted) I write as many Terms of the Series $m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3}$, &c, as there are Units in the Sum of theſe Indices, this Series is to be the Numerator of a Fraction, whoſe Denominator is the Product of the ſeveral Series $1 \times 2 \times 3 \times 4 \times 5$, &c. $1 \times 2 \times 3 \times 4 \times 5$, &c. $1 \times 2 \times 3 \times 4 \times 5 \times 6$, &c. the firſt of which contains as many Terms as there are Units in the Index of $b$, the ſecond as many as there are Units in the Index of $c$, the third as many as there are Units in the Index of $d$, the fourth as many as there are Units in the Index of $e$, &c.

*Demonſtration.*] To raiſe the Series $az + bzz + cz^{3} + dz^{4}$, &c. to any Power whatſoever, write ſo many Series equal to it, as there are Units in the Index of the Power demanded. Now it is evident, that when theſe Series are ſo Multiplied, there are ſeveral Products in which there is the ſame Power of $z$; thus if the Series $az + bzz + cz^{3} + dz^{4}$, &c. is raiſed to its Cube, you have the Products $b^{3}z^{6}$, $abcz^{6}$, $aadz^{6}$, in which you find the ſame Power $z^{6}$. Therefore let us conſider what is the Condition that can make ſome Products to contain the ſame Power of $z$; the firſt thing that will appear in relation to it is, that in any Product whatſoever, the Index of $z$ is the Sum of the particular Indices of $z$ in the Multiplying Terms (this follows from the Nature of the Indices); thus $b^{3}z^{6}$ is the Product of $bz^{2}$, $bz^{2}$, $bz^{2}$, and the Sum of the Indices in the Multiplying Terms, is $2 + 2 + 2 = 6$; $abcz^{6}$ is the Product of $az$, $bzz$, $cz^{3}$, and the Sum of the Indices of $z$ in the Multiplying Terms is $1 + 2 + 3 = 6$; $aadz^{6}$ is the Product of $az$, $az$, $dz^{4}$, and the Sum of the Indices of $z$ in the Multiplying Terms is $1 + 1 + 4 = 6$: the next thing that appears, is, that the Index of $z$ in the Multiplying Terms is the ſame with the Exponent of the Letter to which $z$ is joyn'd ; from which two Considerations it follows, that, to have all the Products belonging to a certain Power of $z$, you must find all the Products where the Sum of the Exponents of the Letters which compose them, shall always be the same with the Index of that Power. Now this is the Method I use, to find easily all the Products belonging to the same Power of $z$, let $m + r$ be the Index of that Power, I consider that the Sum of the Exponents of the Letters which compose these Products, must exceed by 1, those which belong to $m^{m+r-1}$, now because the excess of the Exponent of the Letter $b$, above the Exponent of the Letter $a$, is 1, it follows, that if each of the Products belonging to $z^{m+r-1}$ is Multiply'd by $b$, and Divided by $a$, you will have Products, the Sum of whose Exponents will be $m + r$; Likewise the Sum of the Exponents of the Letters which compose the Products belonging to $z^{m+r}$ exceeds by 2 the Sum of the Exponents of the Letters which compose the Products belonging to $z^{m+r-1}$: Now because the Exponent of the Letter $a$ is less by 2 than the Exponent of the Letter $c$, it follows, that if each Product belonging to $z^{m+r-2}$ is Multiplyed by $c$, and Divided by $a$, you will have other Products, the Summ of whose Exponents is still $m + r$; Now if all the Products belonging to $z^{m+r-2}$ were Multiplied by $c$ and Divided by $a$, you would have some Products that would be the same as some of them found before, therefore you must except out of 'em those that I have call'd Products of the first Classis ; what I have said shows why all the Products belonging to $z^{m+r-3}$, except those of the first and second Classis must be Multiplied by $d$, and Divided by $a$: Lastly, you see the Reason why to all these Products is added the Product of $a^{m-1}$ by the Letter whose Exponent is $r + 1$; 'Tis because the sum of the Exponent is still $m + r$.

As for what relates to the Unciæ; observe that when you Multiply $az + bzz + cz^3 + dz^4$, &c. by $az + bzz + cz^3 + dz^4$, &c. each Letter, $a, b, c, d$, &c. of the second Series, is Multiplied by each of the Letters $a, b, c, d$, &c. of the first Series. Thus the Letter $a$ of the second Series is Multiplied by the Letter $b$ of the first Series, and the Letter $b$ of the second Series is Multiplied by the Letter $a$ of the first; therefore you may have the two Planes, $ab$, $ab$, or $2ab$; for the same Reason you have $2ac$, $2ad$, &c. Therefore you must prefix to each Plane of those that compose the Square of the Infinite Series $az + bzz + cz^3$, &c. the Number which expresses how many ways the Letters of each Plane may be changed ; Likewise if you Multiply the Product of the two preceeding Series by $az + bzz + cz^3$, &c. each Letter $a, b, c, d$, of the third Series is Multiplied by each of the Planes form'd by the Product of the first and second Series; Thus the Letter $a$ is Multiplied by the Planes $bc$ and $cb$; the Letter $b$ is Multiplied by $ac$ and $ca$; the Letter $c$ is Multiplied by $ab$ and $ba$; therefore you have the six Solids, $abc$, $acb$, $bac$, $bca$, $cab$, $cba$, or six $abc$; Therefore you must prefix to each Solid whereof the Cube of the Infinite Series is compos'd, the Number which expresses how many ways the Letters of each Solid may be changed. And generally, you must prefix to any Product, whereof any Power of the Infinite Series $az + bzz + cz^3$ &c. is compos'd, the Number which expresses how many ways the Letters of each Product may be changed. Now

Now to find how many ways the Letters of any Product, for instance, $a^{m-n}b^{h}c^{p}d^{r}$ may be changed; this is the Rule which is commonly given: write as many Terms of the Series $1 \times 2 \times 3 \times 4 \times 5$, &c. as there are in the Sum of the Indices, *viz.* $m - n + h + p + r$; let this Series Units be the Numerator of a Fraction, whose Denominator shall be the Product of the Series, $1 \times 2 \times 3 \times 4 \times 5$, &c. $1 \times 2 \times 3 \times 4 \times 5$, &c. $1 \times 2 \times 3 \times 4 \times 5 \times 6$, &c. $1 \times 2 \times 3 \times 4 \times 5$, &c. whereof the first is to contain as many Terms as there are Units in the first Index $m - n$; the second as many as there are Units in the second Index $h$; the third as many as there are Units in the third Index $p$; the fourth as many as there are Units in the fourth Index $r$. But the Numerator and Denominator of this Fraction have a common Divisor, *viz.* the Series $1 \times 2 \times 3 \times 4 \times 5$, &c. continued to so many Terms as there are Units in the first Index $m - n$; therefore let both this Numerator and Denominator be Divided by this common Divisor, then this new Numerator will begin with $m - n + 1$, whereas the other began with 1, and will contain so many Terms as there are Units in $h + p + r$, that is, so many as there are Units in the sum of all the Indices excepting the first; as for the New Denominator, it will be the Product of three Series only; that is, of so many as there are Indices excepting the first. But if it happens withall, that $n$ be equal to $h + p + r$, as it always happens in our *Theorem*; then the Numerator beginning by $m - n + 1$, and being continued to so many Terms as there are Units in $h + p + r$ or $n$, the last Term will be $m$ necessarily, so if you invert the Series, and make that the first Term which was the last, the Numerator will be $m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3}$, &c. continued to so many Terms as there are Units in the Sum of the Indices of each Product excepting the first Index. And whatever is here said of Powers, whose Index is an Integer, may be adapted to Roots or Powers whose Index is a Fraction.

XXII. *Theorem.*] If $az + bzz + cz^3 + dz^4 + cz^5 + fz^6$, &c. $= gy + hyy + iy^3 + ky^4 + ly^5 + my^6$, &c. then will $z$ be $= \frac{g}{a}y + \frac{h - bAA}{a}y^2 + \frac{i - 2bAB - cA^3}{a}y^3 + \frac{k - bBB - 2bAC - 3cAAB - dA^4}{a}y^4 + \frac{l - 2bBC - 2bAD - 3cABB - 3cAAC - 4dA^3B - eA^5}{a}y^5 + \frac{m - 2bBD - bCC - 2bAE - cB^3 - 6cABC - 3cAAD - 6dAABB - 4dA^3C - 5cA^4B - fA^6}{a}y^6$, &c

*The Extraction of a Root of an Infinite Equation; by M. Abr. de Moivre.* n. 240. *p.* 190.

For the understanding of this Series, and in order to continue it as far as we please; it is to be observed, 1. That ev'ry Capital Letter is equal to the Coefficient of each preceding Term; thus the Letter B is equal to the Coeffi-

cient

cient $\frac{h - bAA}{a}$. 2. That the Denominator of each Coefficient is always $a$. 3. That the firſt Member of each Numerator, is always a Coefficient of the Series $gy + hyy + iy^3$, &c. *viz.* the firſt Numerator begins with the firſt Coefficient $g$, the Second Numerator with the Second Coefficient $h$, and ſo on. 4. That in ev'ry Member after the firſt, the Sum of the Exponents of the Capital Letters, is always equal to the Index of the Power to which this Member belongs: Thus conſidering the Coefficient $\frac{k - bBB - 2bAC - 3cAAB - dA^4}{a}$, which belongs to the Power $y^4$, we ſhall ſee that in every Member $bBB$, $2bAC$, $3cAAB$, $dA^4$, the the Sum of the Exponents of the Capital Letters is 4, (where I muſt take notice, that by the Exponent of a Letter; I mean the Number which expreſſes what place it has in the Alphabet; thus 4 is the Exponent of the Letter D) hence I derive this Rule for finding the Capital Letters of all the Members that belong to any Power; Combine the Capital Letters as often as you can make the Sum of their Exponents equal to the Index of the Power to which they belong. 5. That the Exponents of the ſmall Letters, which are written before the Capitals, expreſs how many Capitals there is in each Member. 6. That the Numerical Figures or Unicæ that occur in theſe Members, expreſs the Number of Permutations which the Capital Letters of ev'ry Member are capable of.

For the *Demonſtration* of this; ſuppoſe $z = Ay + Byy + Cy^3 + Dy^4$, &c. Subſtitute this Series in the room of $z$, and the Powers of this Series, in the room of the Powers of $z$, there will ariſe a new Series; then take the Co-efficients which belong to the ſeveral Powers of $y$ in this new Series, and make them equal to the correſponding Co-efficients of the Series $gy + hyy + iy^3$, &c. and the Co-efficients A, B, C, D, &c. will be found ſuch as I have determined them.

But if any one deſires to be ſatisfyed, that the Law by which the Coefficients are inform'd, will always hold, I'll deſire 'em to have recourſe to the

§. XXI. *Theorem* I have given for *Raiſing an Infinite Series to any Power*, or extracting any Root of the ſame; for if they make uſe of it, for taking ſucceſſively the Powers of $Ay + Byy + Cy^3$, &c. they will ſee that it muſt of neceſſity be ſo. I might have made the *Theorem* I give here, much more general than it is; for I might have ſuppos'd, $az^m + bz^{m+1} + cz^{m+2}$ &c. $= gy^m + hy^{m+1} + iy^{m+2}$ &c. then all the Powers of the Series $Ay + Byy + Cy^3$, &c. deſigned by the univerſal Indices, muſt have been taken ſucceſſively; but thoſe who will pleaſe to try this, may eaſily do it, by means of the *Theorem* for raiſing an Infinite Series to any Power, &c.

This *Theorem* may be applyed to what is called the Reverſion of Series, ſuch as finding the Number from its Logarithm given; the Sine from the Arch; the Ordinate of an Ellipſe from an Area given to be cut from any point in the Axis: But to make a particular Application of it, I'll ſuppoſe we have this

this Problem to ſolve; *viz. The Chord of an Arc being given, to find the Chord of an other Arc, that ſhall be to the firſt as* n *to* 1. Let $y$ be the Chord given, $z$ the Chord required; now the Arc belonging to the Chord $y$ is, $y + \frac{y^3}{6dd} + \frac{3y^5}{40d^4} + \frac{5y^7}{112d^6}$ &c. And the Arc belonging to the Chord $z$ is $z + \frac{z^3}{6dd} + \frac{3z^5}{40d^4} + \frac{5z^7}{112d^6}$ &c. The firſt of theſe Arcs is to the ſecond as 1 to $n$; therefore multiplying the Extreams and Means together, we ſhall have this Equation.

$$z + \frac{z^3}{6dd} + \frac{3z^5}{40d^4} + \frac{5z^7}{112d^6} \text{ \&c.} = ny + \frac{ny^3}{6dd} + \frac{3ny^5}{40d^4} + \frac{5ny^7}{112d^6} \text{ \&c.}$$

Compare theſe two Series with the two Series of the Theorem, and you will find $a = 1$, $b = 0$, $c = \frac{1}{6dd}$, $d = 0$, $e = \frac{3}{40d^4}$, $f = 0$, &c. $g = n$, $h = 0$, $i = \frac{n}{6dd}$, $k = 0$, $l = \frac{3n}{40d^4}$, $m = 0$, &c. hence $z$ will be $= ny + \frac{n - n^3}{6dd} y^3$, &c. or $ny + \frac{1 - nn}{2 \times 3dd} yy A$, &c. Suppoſing A to denote the whole preceding Term, which will be the ſame Series as Mr. *Newton* has firſt found.

By the ſame Method, this general *Problem* may be ſolved; *The Abſciſſe correſponding to a certain Area in any Curve being given, to find the Abſciſſe, whoſe correſponding Area ſhall be to the firſt in a given Ratio.*

The Logarithmic Series might alſo be found without borrowing any other Idea, than that Logarithms are the Indices of Powers: Let the Number, whoſe Logarithm we enquire, be $1 + z$, ſuppoſe its Log. to be $az + bzz + cz^3$, &c. Let there be another Number $1 + y$; therefore its Logarithm will be $ay + byy + cy^3$, &c. Now if $1 + z = \overline{1 + y}|^n$, it follows that $az + bzz + cz^3$, &c. : $ay + byy + cy^3$, &c. : : $n$ : 1. that is, $az + bzz + cz^3$, &c. $= nay + nbyy + ncy^3$, &c. therefore we may find a Value of $z$ expreſt by the Powers of $y$; again, ſince $1 + z = \overline{1 + y}|^n$, therefore $z = \overline{1 + y}|^n - 1$, that is $z = ny + \frac{n}{1} \times \frac{n-1}{2} yy + \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} y^3$, &c. Therefore $z$ is doubly expreſt by the Powers of $y$. Compare theſe two Values together, and the Coefficients $a$, $b$, $c$, &c. will be determin'd, except the firſt $a$ which may be taken at Pleaſure, and gives accordingly, all the different Species of Logarithms.

*Doctrine of Exhauſtions; by* Dr. Wallis. n. 255. *p.* 281.

XXIII. De Differentijs Infiniteſſimarum-infiniteſſimis explicandis, non eſt ut ſis porro ſolicitus. Nam, ut tu mihi facilis concedis, quod nihili quodvis multiplum ſit adhuc nihil; eadem ego facilitate tibi permitto, ut Differentias Infiniteſſimas in Infiniteſſimas ductas, tu merito negligas, poteſtque id tuto fieri, modo caute; Quippe, in quovis genere Quantitatum, quæ differunt dato minus, reputanda ſunt Æqualia. Quo nititur *Exhauſtionum* Doctrina tota, Veteribus pariter & Recentioribus neceſſaria.

*The Approximation of the Ancients in the Extracting of Roots, Improv'd*; *by* Dr. Wallis. n. 215. *p.* 2.

XXIV. It is agreed by all, that if a Number propoſed be not a true Square, it is in vain to hope for a juſt Quadratick Root thereof, explicable by rational numbers, Integers or Fracted. And therefore in ſuch caſes, we muſt content our ſelves with Approximations (ſomewhat near the truth) without pretending to Accuracy.

And ſo, for the Cubick Root, of what is not a perfect Cube. And the like for Superior Powers.

Now the Ancients (being aware of this) had their Methods of Approximation, which tho' ſcarce apply'd by them beyond the Quadratick, or perhaps the Cubick Root, yet are equally practicable (by due adjuſtments) to the Superiour Powers alſo.

I ſhall begin with the Square Root: For which rhe Ancient Method is to this purpoſe.

From the propoſed Non-quadrate (ſuppoſe N) ſubſtract (in the uſual manner) the greateſt Square in Integers therein contained (ſuppoſe $Aq$.) The remainder (ſuppoſe $B = 2AE + Eq$) is to the Numerator of a Fraction, for deſigning the near value of E (the remaining part of the Root ſought $A + E = \surd N$,) whoſe Denominator or Diviſor is to be $2A$ (the double Root of the ſubſtracted Square) or $2A + 1$ (that double Root increaſed by 1) the true Value falling between theſe two; ſometime the one, ſometime the other, being neareſt to the true value. But (for avoiding of Negative numbers) the latter is commonly directed.

This Method Mounſieur *De L'agny* affirms to be more than 200 Years old: And it is ſo; for I find it in *Lucas Pacciolus* (otherwiſe called *Lucas de Burgo* or *de Burgo Sancti Sepulchri*) Printed at *Venice* in the Year 1494. (if not even ſooner than ſo, for I find there have been ſeveral Editions of it.) And how much older than ſo, I cannot tell: For he doth not deliver it as a New Invention of his own, but as a received Practice, and derived from the *Moors* or *Arabs*, from whom they had their *Algoriſm*, or Practice of Arithmetick by the 10 Numeral Figures now in uſe.

And it is continued down hitherto in books of Practical Arithmetick in all languages, which teach the Extraction of the Square Root, and (therein) this Method of Approximation, in caſe of a Non-quadrate.

The true ground of the Rule is this: $Aq$ being (by Conſtruction) the greateſt Integer Square contained in N, 'tis evident that E muſt be leſs than 1; (otherwiſe not $Aq$ but the Square of $A + 1$, or ſome greater than ſo, would be the greateſt Integer Square contain'd in N.) Now if the remainder $B = 2AE + Eq$ be divided by $2A$, the reſult will be too great for E, (the Diviſor being too little; for it ſhould be $2A + E$, to make the Quotient E.) But if

(to

(to rectify this) we diminiſh the Quotient, by increaſing the Diviſor, adding 1 to it, it then becomes too little; becauſe the Diviſor is now too big. For (E being leſs than 1) $2A + 1$ is more than $2A + E$; and therefore too big.

As for inſtance; If the Non-quadrate propoſed be $N = 5$, the greateſt Integer Square therein contained is $Aq = 4$. (the Square of $A = 2$:) which being ſubſtracted, leaves $N - Aq = 5 - 4 = 1 = B = 2AE + Eq$. Which divided by $2A = 4$ gives $\frac{1}{4}$: But divided by $2A + 1 = 4 + 1 = 5$, gives $\frac{1}{5}$. That too great, and this too little, for E. And therefore the true Root $(A + E = \surd N)$ is leſs than $2\frac{1}{4} = 2.25$, but greater than $2\frac{1}{5} = 2.2$; And this was Anciently thought an Approach near enough.

If this Approach be not now thought near enough, the ſame Proceſs may be again repeated; and that as oft as is thought neceſſary.

Take now for A, $2\frac{1}{5} = 2.2$, whoſe Square is $4.84 = Aq$, (now conſidered as an Integer in the ſecond place of Decimal parts.) This ſubſtracted from 5. 00, (or, which is the ſame, 0. 84, the exceſs of this Square above the former, from 1, which was then the remainder) leaves a new remainder $B = 0.16$: which divided by $2A = 4.4$, gives $\frac{0.16}{4.40} = \frac{2}{55} = 0.03636 +$, too much. But divided by $2A + 1 = 4.5$, it gives $\frac{0.16}{4.50} = \frac{8}{225} = 0.03555 +$, too little. The true value (between theſe two) being 2. 236 proxime, whoſe Square is 4. 999696.

If this be not thought near enough, Subſtract the Square from 5. 000000: the remainder $B = 0.000304$, divided by $2A = 4.472$, or by $2A + 1 = 4.473$, gives (either way 0. 000068 —; which added to $A = 2.236$, makes 2. 236068 —, ſomewhat to big; but 2. 236067 + would be much more too little.

Which gives us the Square Root of 5, adjuſted to the ſixth place of Decimal parts, at three ſteps. And by the ſame Method, if it be thought needfull, we may proceed further.

For the Cubick Root the Rule is this.

From the Non Cubick propoſed, (ſuppoſe N,) ſubſtract the greateſt Cube in Integers therein contained, (ſuppoſe $Ac$:) the remainder (ſuppoſe $B = 3AqE + 3AEq + Ec$,) is to be the Numerator of a Fraction for deſigning the value of E, (the remaining part of the Root ſought $A + E = \surd c N$.) To this Numerator, if (for the Denominator or Diviſor) we ſubjoyn $3Aq$, the reſult will certainly be too great for E, becauſe the Diviſor is too little: (For it ſhould be $3Aq + 3AE + Eq$, to give the true Value of E.) If for the Diviſor we take $3Aq + 3A + 1$, it will certainly be too little, becauſe the Diviſor is too great. (For E by conſtruction is leſs than 1.) It muſt therefore (between theſe limits) be more than this latter. And therefore this latter reſult being added to A, will give a Root whoſe Cube may be ſubſtracted from the Non-Cubick propos'd, in order to another ſtep.

But if, for the Diviſor, we take $3Aq + 3A$, (or even leſs than ſo) the reſult may be too great; or (in caſe B be ſmall) it may be too little, (and oft is ſo.)

Which comes to pass from hence; because E (by Construction) is less than 1; and therefore 3 A E less than 3 A; and perhaps so much as that the Addition of E $q$ will not redress it. And when it so happens 3 A $q$ + 3 A is a better Divisor than 3 A $q$ + 3 A + 1, (or even somewhat less than either.) But because it doth not always so happen (though for the most part it doth) the Rule doth rather direct the other; as which doth certainly give a Root less than the true value, whose Cube may always be substracted from the Non-Cubick proposed. The Design being to have such a Cube as (being substracted) may leave another B, to be ordered in like manner for a new Approach.

But, for the most part, 3 A $q$ may be safely taken for the Divisor. For tho' the Result will then be somewhat too big, yet the excess may be so small, as to be neglected; or, at least, we may thence easily judge what Number (somewhat less than it) may be safely taken. And if we chance to take it somewhat too big, the inconvenience will be but this, that B for the next step will be a Negative. Of which Case we shall speak anon.

Thus for Instance; if the Non-Cube proposed be 9 = N. The greatest Integer Cube therein contained is 8 = A $c$, (whose Cubick Root is A = 2.) Which Cube substracted, leaves 9 − 8 = 1 = B = 3 A $q$ E + 3 A E $q$ + E $c$. This divided by 3 A $q$ = 12, gives $\frac{1}{12}$ = 0. 08333 +, too big for E. But the same divided by 3 A $q$ + 3 A + 1 = 12 + 6 + 1 = 19, gives $\frac{1}{19}$ = 0. 05263 +, too little. Or if but by 3 A $q$ + 3 A = 12 + 6 = 18, it gives $\frac{1}{18}$ = $\frac{5}{90}$ = 0. 05555 +, yet too little. For the Cube of A + 0. 06, = 2. 06, is but 8. 742 —, which is short of 9. And so much short of it, that we may safely take 2. 07 as not too big: Or perhaps 2. 08, which upon tryal will be found not too big; for the Cube of 2. 08, is but 8. 998912.

If this first step be not near enough, this Cube substracted from 9. 000000, leaves a new B = 0. 001088, which divided by 3 A $q$ = 12. 9796, gives 0. 000084 —; which will be somewhat too big but not much. (For E is now so small, as that 3 A E may be safely neglected, and E $q$ much more.) So that if to 2. 08, we add 0. 000084 —, the Result 2. 080084 will be too big; but 2. 080083 will be more too little. (as will appear if we take the Cube of each.) So that either of them, at the second step, gives the true Root within an Unite in the sixth place of decimal parts. But when I say, taking the Cube of each, (which I do that the thing may be more clearly apprehended) it is not necessary that we trouble our selves with the whole Cube. For, A $c$ being already substracted, for finding B = 3 A $q$ E + 3 A E $q$ + E $c$, we have no more to try, but whither 3 A $q$ E + 3 A E $q$ + E $c$, be greater or less than B, according as we take 0. 000084, or 0. 000083, for E.

Which may conveniently be done in this manner: Take 3 A + E, and multiply this by E, (or E by it) so have we 3 A E + E $q$. To this add 3 A $q$, and multiply the whole by E, (so have we 3 A $q$ E + 3 A E $q$ + E $c$,) to see whether this be greater or less than B.

That is, in the present case, if we take E = 0. 000084, and add to this 3 A = 6. 24, than is 6. 240084 = 3 A + E. This multiplyed by E = 0. 000084, is 3 A E + E $q$ = 0. 000524 +. To which if we add 3 A $q$ = 12. 9792, it is 3 A $q$ + 3 A E + E $q$ = 12. 979724.

Which

Which Multiplied again by $E = 0.000084$, is $0.0010902 + = 3AqE + 3AEq + Ec$, which is more than $B = 0.001088$.

But if we take $E = 0.000083$, and proceed as before, we ſhall have $3AqE + 3AEq + Ec = 0.001077 +$, which is leſs than $B = 0.001088$. And therefore (if we Subſtract that from this) the Remainder, 0.000011, will be another B for the next ſtep, if we pleaſe to proceed further.

Hitherto I have purſued the Method moſt affected by the Ancients, in ſeeking a Square or Cube (and the like of other Powers) always leſs than the juſt value, that it might be Subſtracted from the Number propoſed, leaving B a poſitive remainder; thereby avoiding Negative Numbers.

But ſince the Arithmetick of Negatives is now ſo well underſtood, it may in this (and other Operations of like Nature) be adviſeable, to take the neareſt, whether it be greater or leſs than the juſt value.

According to this Notion, for the Square Root of 5, I would ſay it is $(2 +)$ ſomewhat more than 2; and enquire how much more? But for the Square Root of 8, I would ſay, it is $(3 -)$ ſomewhat leſs than 3; and enquire how much leſs? Taking (in both Caſes) that which is neareſt to the juſt value.

Thus in the Cubick Root before us; I would take for E (in the laſt Enquiry) $0.000084 -$ (where for the next ſtep we have $B = -0.000002$,) rather than $0.000083 +$ (where for the next ſtep we ſhould have $B = +0.000011$.) In the latter Caſe we are to divide $B = +0.000011$, by $3Aq = 12.980236 -$, to find (by the Quotient) how much is to be added to 0.000083. In the other Caſe we are to divide $B = +0.000002$, by $3Aq = 12.980248$, to find (by the Quotient) what is to be abated of 0.000084, ſo have we $\frac{0.000011}{12.980236} = 0.00000085 +$ to be added to 6.240083: Or $\frac{0.000002}{12.980248} = 0.00000015 +$ to be abated of 6.240084. (Or it may ſuffice, in either to divide by $12.98 +$, or even by $13 -$, without being incumbred with a long Diviſor) either of which gives us, for the Root ſought, 2.08008385 proxime. True (at the third ſtep) to the eighth place of Decimal Parts. And if this be not near enough, the Cube of this, compared with the Number propoſed, will give us another B for the next ſtep. And ſo onwards as far as we pleaſe.

Now, what is ſaid of the Cube, is eaſily applicable to the higher Powers.

I ſhall omit that of the Biquadrate; becauſe here perhaps it may be thought moſt adviſable, to Extract the Square Root of the Number propoſed; and then the Square Root of that Root. But if we would do it at once, we are from N (the Number propos'd, being not a Biquadrate) to Subſtract $Aqq$ (the greateſt Biquadrate contained in it) to find the Remainder $B = 4AcE + 6AqEq + 4AEc + Eqq$. Which Remainder, if we divide by $4Ac$, the Quotient will certainly be too big for E, (though perhaps not much:) If by $4Ac + 6Aq + 4A + 1$, it will certainly be too little

little (for reaſons before mentioned.) And we are to uſe our diſcretion in taking ſome intermediate number. And if we chance not to hit on the neareſt, the Inconvenience will be but this, that our Leap will not be ſo great as otherwiſe it might be. Which will be rectify'd by another B at the next ſtep.

For the Surſolid (of five Dimenſions) we are, from N (the Number propoſed, being not a perfect Surſolid) to Subſtract $A\,qc$ (the greateſt Surſolid therein contain'd) to find the Remainder $B = 5\,A\,qq\,E + 10\,A\,c\,E\,q + 10\,A\,q\,E\,c + 5\,A\,E\,qq + E\,qc$. Which (as before) if we divide by $5\,A\,qq$, the Reſult will be ſomewhat too big, (becauſe the Diviſor is too little:) If by $5\,A\,qq + 10\,A\,c + 10\,A\,q + 5\,A + 1$, the Reſult will certainly be leſs than the true E. The juſt value of E being ſomewhat between theſe two, where we are to uſe our diſcretion, what Intermediate Number to take. Which according as it proves too great or too little, is to be rectify'd at the next ſtep.

But for the moſt part it will be ſafe enough (and leaſt trouble) to divide by $5\,A\,qq$, which gives a Quotient ſomewhat too big; which we may either rectify at diſcretion, by taking a Number ſomewhat leſs, or proceed to another B, (Affirmative or Negative, as the Caſe ſhall require) and ſo onward to what exactneſs we pleaſe. Which is for ſubſtance, in a manner co-incident with Mr. *Raphſon*'s Method, even for Affected Equations.

Thus, in the preſent Caſe; If the Number propoſed be $N = 33$, then is $A\,qc = 32$, and $B = 33 - 32 = 1 = 5\,A\,qq\,E + 10\,A\,c\,E\,q + 10\,A\,q\,E\,c + 5\,A\,E\,qq + E\,qc$. Which if we divide by $5\,A\,qq = 5 \times 16 = 80$, the Reſult $\frac{1}{80} = 0.0125$, is ſomewhat too big for E, but not much. And if we examine it, by taking the Surſolid of $2.0125$, or of $2\frac{1}{80}$, we ſhall find a Negative B (for the next ſtep), but not very conſiderable. Or if we think it conſiderable, we may proceed further to another ſtep, or more than ſo.

The like Method may be apply'd (and with more Advantage) in the higher Powers, according as the Compoſition of each Power requires.

And the ſame Method may be of uſe (with good Advantage) in long numbers (if duly applied) even before we come to the place of Units; for the ſame will equally hold there alſo. Which the Reader may eaſily apprehend, without a long Diſcourſe upon it.

*The Proportion of Infinite Quantities; by Mr.* E. Halley. n. 195. p. 556.

XXV. The very Idea of Magnitudes Infinitely great, or ſuch as exceed any aſſignable Quantity, does include a Negation of Limits: yet if we nearly examine this Notion, we ſhall find that ſuch Magnitudes are not equal amongſt themſelves, but that there are really beſides Infinite Length, and Infinite Area, three ſeveral ſorts of Infinite Solidity: all of which are *Quantitates ſui generis*; and that thoſe of each Species are in given Proportions.

Infinite Length, or a Line Infinitely Long, is to be conſidered either as beginning at a Point, and ſo Infinitely extended one way, or elſe both ways from the ſame Point; in which Caſe the one, which is a beginning Infinity, is the one half of the whole, which is the Summ of the beginning and ceaſing Infinity; or as I may ſay, of Infinity *à parte ante* and *à parte poſt*, which is Analogous

analogous to Eternity in Time or Duration, in which there is always as much to follow as is paſt, from any Point or Moment of time: Nor doth the Addition or Subduction of Finite Length or ſpace of Time, alter the Caſe either in Infinity or Eternity, ſince both the one or the other cannot be any part of the whole.

As to Infinite Surface or Area, any Right Line, Infinitely extended both ways on an Infinite Plane, does divide that Infinite Plane into equal parts, the one to the Right, and the other to the Left of the ſaid Line; but if from any Point in ſuch a Plane, two Right Lines be Infinitely extended, ſo as to make an Angle, the Infinite Area, Intercepted between thoſe Infinite Right Lines, is to the whole Infinite Plane, as the Arch of a Circle on the Point of Concourſe of thoſe Lines as a Center, intercepted between the ſaid Lines, is to the Circumference of the Circle; or as the Degrees of the Angle to the 360 Degree of a Circle. For Example, two right Lines meeting at a right Angle do include, on an Infinite Plane, a quarter part of the whole Infinite Area of ſuch a Plane.

But if ſo be two Parallel Infinite Lines be ſuppoſed drawn on ſuch an Infinite Plane, the Area intercepted between them will be likewiſe Infinite; but at the ſame time will be Infinitely leſs than that Space, which is intercepted between two Infinite Lines that are inclined, tho' with never ſo ſmall an Angle, for that in the one Caſe, the given finite diſtance of the Parallel Lines diminiſhes the Infinity in one Degree of Dimenſion; whereas in a Sector, there is Infinity in both Dimenſions: and conſequently the Quantities are the one Infinitely greater than the other, and there is no Proportion between them.

From the ſame Conſideration ariſe the three ſeveral Species of Infinite Space or Solidity; for a Parallelopipid, or a Cylinder Infinitely long, is greater than any Finite Magnitude how great ſoever; and all ſuch Solids ſuppos'd to be formed on given Baſes, are as thoſe Baſes, in proportion to one another. But if two of theſe three Dimenſions are wanting, as in the Space intercepted between two Parallel Planes Infinitely extended, and at a Finite Diſtance; or with Infinite Length and Breadth, with a Finite Thickneſs, all ſuch Solids ſhall be as the given Finite Diſtances one to another; but theſe Quantities, tho' Infinitely greater than the other, are yet Infinitely leſs than any of thoſe wherein all the three Dimenſions are Infinite. Such are the Spaces intercepted between two inclined Planes Infinitely extended; the Space intercepted by the Surface of a Cone, or the ſides of a Pyramid, likewiſe Infinitely continued, &c. of all which notwithſtanding, the Proportions one to another, and to the τὸ πᾶν or vaſt Abyſs of Infinite Space (wherein is the *Locus* of all things that are or can be; or to the Solid of Infinite Length, Breadth and Thickneſs taken all manner of ways) are eaſily aſſignable. For the Space between two Planes is to the whole, as the Angle of thoſe Planes to the 360 Degrees of the Circle. As for Cones and Pyramids, they are as the Spherical Surface intercepted by them, is to the Surface of the Sphere, and therefore Cones are as the verſed Sines of half their Angles, to the Diameter of the Circle: Theſe three ſorts of Infinite Quantity are Analogous to a Line, Surface, and

and Solid, and after the ſame manner cannot be compared, or have no Proportion the one to the other.

*Infinitely-Infinite Fractions; by D.R.Wood. Ph. Col. n. 3. p. 45.*

XXVI. Infinitely-Infinite Fractions, or all the Powers of all the Fractions whoſe Numerator is 1, are all of them together equal to (1) an Unit.

| A. | R. | q. (P.) | c. (P.) | q q. (P.) | |
|---|---|---|---|---|---|
| $1 = \frac{1}{1}$ | $= \frac{1}{2}:$ | $+ \frac{1}{4}$ | $+ \frac{1}{8}$ | $+ \frac{1}{16}$ | &c. |
| $\frac{1}{2}$ | $= \frac{1}{3}:$ | $+ \frac{1}{9}$ | $+ \frac{1}{27}$ | $+ \frac{1}{81}$ | &c. |
| $\frac{1}{3}$ | $= \frac{1}{4}:$ | $+ \frac{1}{16}$ | $+ \frac{1}{64}$ | $+ \frac{1}{256}$ | &c. |
| $\frac{1}{4}$ | $= \frac{1}{5}:$ | $+ \frac{1}{25}$ | $+ \frac{1}{125}$ | $+ \frac{1}{625}$ | &c. |
| $\frac{1}{5}$ | $= \frac{1}{6}:$ | $+ \frac{1}{36}$ | $+ \frac{1}{216}$ | $+ \frac{1}{1296}$ | &c. |
| &c. | &c. | | | | |

Or thus

| | 1 anor + | | 2 anor + | | 3 anor + | | 4 anor + |
|---|---|---|---|---|---|---|---|
| $\frac{1}{1 \times 1} =$ | $\frac{1}{2} +$ | $\frac{1}{1 \times 2} =$ | $\frac{1}{2^2} +$ | $\frac{1}{1 \times 2^2} =$ | $\frac{1}{2^3} +$ | $\frac{1}{1 \times 2^3} =$ | $\frac{1}{2^4}$ + &c. |
| $\frac{1}{2 \times 1} =$ | $\frac{1}{3} +$ | $\frac{1}{2 \times 3} =$ | $\frac{1}{3^2} +$ | $\frac{1}{2 \times 3^2} =$ | $\frac{1}{3^3} +$ | $\frac{1}{2 \times 3^3} =$ | $\frac{1}{3^4}$ + &c. |
| $\frac{1}{3 \times 1} =$ | $\frac{1}{4} +$ | $\frac{1}{3 \times 4} =$ | $\frac{1}{4^2} +$ | $\frac{1}{3 \times 4^2} =$ | $\frac{1}{4^3} +$ | $\frac{1}{3 \times 4^3} =$ | $\frac{1}{4^4}$ + &c. |
| $\frac{1}{4 \times 1} =$ | $\frac{1}{5} +$ | $\frac{1}{4 \times 5} =$ | $\frac{1}{5^2} +$ | $\frac{1}{4 \times 5^2} =$ | $\frac{1}{5^3} +$ | $\frac{1}{4 \times 5^3} =$ | $\frac{1}{5^4}$ + &c. |
| $\frac{1}{5 \times 1} =$ | $\frac{1}{6} +$ | $\frac{1}{5 \times 6} =$ | $\frac{1}{6^2} +$ | $\frac{1}{5 \times 6^2} =$ | $\frac{1}{6^3} +$ | $\frac{1}{5 \times 6 \times} =$ | $\frac{1}{6^4}$ + &c. |

*A.* Is a File or Row of abſolute Numbers, or rather of all the Fractions, whoſe Numerator is 1; which Row is ſuppoſed to be continued *in infinitum* (downwards).

*R.* Is

R. Is another File or Row of all the Roots (whose Numerator is 1) of all the Powers of such Fractions; supposed likewise to be continued *in infinitum* (downwards.)

P. Are all the respective Powers of such Fractions, (*viz.* Squares, Cube, &c.) r so many Ranks of Geometrical Proportionals; supposed to be continued *in infinitum*; both to the Right-hand, and also downwards.

Lemma.] *Each of the said Ranks of Powers, together with their respective Roots, is equal to each of the several Numbers under* A *respectively.*

*Demonstration.*] If from the Line $ab$ you take (for instance) $\frac{1}{4}$ part towards $a$, suppose $ac$; and also from the other end of the same Line $ab$, you take two such Parts (or $\frac{2}{4}$ parts) towards $b$, suppose $bd$, (*viz.* a Number of Parts less by two than the whole Line $ab$, was first supposed to be divided into) there will remain the Line $cd = ac = \frac{1}{4}$ of $ab$. Then again, if from $cd$ you take $\frac{1}{4}$ part thereof towards $a$, suppose $ce$, and from the other end $\frac{2}{4}$ parts of the same $cd$, suppose $df$, there will remain only $ef = ce = \frac{1}{4}$ of $cd$. And if you still go on without ceasing, to take on the side towards $a$, $\frac{1}{4}$ part of what was taken last before, and twice as much on the other side towards $b$, there shall be found between the two Lines last taken always remaining $\frac{1}{4}$ part of the Line from which they were taken. From which $\frac{1}{4}$ part there may still after the same manner be supposed to be taken two other such Lines. But if this be supposed to be done Infinite times actually, then there will nothing more remain (between), and so the continued Division on either side will come exactly to the point $g$, supposing $ag$ to be $\frac{1}{3}$ of $ab$, and $bg = 2ag$. For, because that which was taken away towards $b$, was always twice as much as that, which was taken away towards $a$, the total summ of all the Lines taken away towards $b$, (which all together do make up the Line $bg$) must be twice as much as the Line $ag$, (which is the Total summ of all the Lines taken away towards $a$) *viz.* $bg = 2ag$; and consequently $bg + ag$ (or the whole Line $ab$) is equal to $3ag$: And therefore $ag = \frac{1}{3}$ of $ab$. *Q. E. D.* Fig. 54.

The like *Construction* and *Demonstration* (*mutatis mutandis*) may be made use of in taking away any other part of any quantity, and the like part again of the first mentioned part, and so *in infinitum*. The total sum of all the parts so taken, or supposed to be taken shall be equal to a certain quantity, or part, or Fraction, whose Denominator shall be less by an Unite than the Denominator of the first mentioned part; as $\frac{1}{6} = \frac{1}{7} + \frac{1}{49} + \frac{1}{343}$ &c. $\frac{1}{9} = \frac{1}{10} + \frac{1}{100} + \frac{1}{1000}$ &c. And so, that which the incomparable *Archimedes* (in his Squaring the *Parabola*) has only Demonstrated in one particular Case, *viz.* $\frac{1}{3} = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \frac{1}{1024}$ &c. and that too, not without an huge Apparatus of Preliminary Propositions, amounting to an whole Book, is here Universally De-

 monstrated

monſtrated in all Caſes (which are infinite) and by a very ſimple and eaſie Method (in *Des Cartes*'s way) and on one ſingle page.

Now if each of the ſaid Ranks of Powers, together with their RefpectiveRoots, be equal to the ſeveral Numbers of Fractions under A; (as is Demonſtrated by the *Lemma*) then is A the ſum of them all, or equal to them all: that is to ſay, $R + P = A = R + 1$: For R is the ſame with A wanting $\frac{1}{1}$ or 1, (as appears upon view) or but $(\frac{1}{\infty})$ one infinite part bigger. Wherefore $P = 1$ *Q. E. D. viz. Infinities Infiniti numeri Fracti æquantur Unitati, ſc. minimæ Radici Integræ.*

*Corollaria.*] Hinc patet,

1°. *Dari progreſſum in infinitum.*

2°. *Et non modo in unum Infinitum, ſed etiam in plurima; ſeu potius infinita, Infinita, vel infinities Infinita.*

3°. *Et hoc fieri poſſe, id eſt, hunc Calculum inſtitui, ab ingenio valdè finito aut exiguo.*

4°. *Et totum hunc Progreſſum, vel progreſſus hujuſmodi infinitos, poſſe Numerari, ſive aggregari in unam ſummam.*

5°. *Et in ſummam, non modo non infinitam, ſed adeo tantillam, ut ſit minor omni numero.* Patet ulterius

*Infinitorum alia eſſe æqualia, alia inæqualia.*

*Et unum Infinitum æquari duobus, tribus, pluribus, vel Finitis vel Infinitis.*

For 1. The Infinite Powers of the firſt Rank are $= \frac{1}{2} = \frac{1}{1 \times 2}$, and alſo equal to all the Infinitely-Infinite Powers of all the other Ranks.

The Infinite Powers of the ſecond Rank are $= \frac{1}{6} = \frac{1}{2 \times 3}$

Thoſe of the third Rank are $= \frac{1}{12} = \frac{1}{3 \times 4}$

Thoſe of the fourth Rank are $= \frac{1}{20} = \frac{1}{4 \times 5}$

Thoſe of the fifth Rank are $= \frac{1}{30} = \frac{1}{5 \times 6}$

&c. *in infinitum.*

} *viz.* equal to the reſpective mean proportional number between the SquareNumbers reſpectively. *e. g.*

| | | |
|---|---|---|
| 4, | 6, | 9 |
| 9, | 12, | 16 |
| 16, | 20, | 25 |
| 25, | 30, | 36 |

&c. *in infinitum.*

2. The

2. The Infinite Powers of the two first Ranks are $= \frac{2}{3}$

Those of the three first are $= \frac{3}{4}$

Those of the four first are $= \frac{4}{5}$

Those of the five first are $= \frac{5}{6}$

&c. *in infinitum.*

3. All the Powers of all the Infinite Ranks, except the first, are $= \frac{1}{2}$

all, except the two first, are $= \frac{1}{3}$

all, except the three first, are $= \frac{1}{4}$

all, except the four first, are $= \frac{1}{5}$

&c. *in infinitum.*

These later *Corollaries* may all appear by simple Addition and Subduction; and so may many more.

XXVII. 1. That the Numeral Figures now in use, with the manner of Computation by them (and the names of Algorism, appropriated to that way of Computation) came to us from the Arabs (but somewhat altered, as to the shape of the Figures in succeeding Ages) and to them from the *Indians*, is generally agreed. But it is not so generally agreed, of what Antiquity the use of them, in our *European* parts, hath been. *The Antiquity of the Numeral Figures; by Dr.* J. Wallis. n. 154. p. 399.

*Jo. Gerard Vossius* (*De Scientiis Mathematicis*) thinks they came not into use here, till about the Year of our Lord 1300; or at the farthest, later than the Year 1250. And *P. Mabillion (De Re Diplomatica)* tells us that he hath not found them any where used sooner than the 14 Century. But I think their use in these parts was as old at least as the times of *Hermannus Contractus*, who lived about the Year of our Lord 1050. (that is about the middle of the 11th Century :) If not so frequently in ordinary Affairs; yet at least in Mathematical things, and especially in Astronomical Tables.

But I do not remember, that I have any where seen any Monument of them more Antient than the Mantle-tree of the Parlour Chimney at the dwelling House of Mr. *Will. Richards*, the Rector of *Helmdon* in *Northampton-shire*.

The sides of the Chimney, by which the Mantle-tree is supported, are of Stone: but the Mantle-tree it self is of Wood. It is all over as black as Ink, having by Age and Smoak contracted that Colour. It may yet continue many hundreds of years; for I did not discern in it, any thing either of Worm, or Rottenness, or any tendency to it. The length of it is 5 foot 9 Inches: Its breadth or depth at the ends, (A B) is 11½ inches, but at Fig. 55.



the

the middle, as C D, ſomewhat leſs. It is all carved from end to end: and the lower part of it is Abated, as in the mouldings of other Chimneys. On the Front of the upper part there is (beginning at the middle) on 3 Squares parted from each other by a deep furrow or Channel, the Date (I ſuppoſe, when it was firſt made) deſcribed partly in Numeral Figures A° Do¹ M° 133; on a fourth a Flower, and on a 5th the two Letters W. R. with an Eſcutcheon, repreſenting (I ſuppoſe) the name of him to whom it did then belong. Both the Letters and the Figures are of an Antique form, agreeing well enough with that Age. They are not engraved or cut in, but prominent on their ſeveral Squares (by way of *bas-relief*) the wood being abated round about them. The o over the A, is a round o, but that over the M is a Square ▯.

Hence it appears, that the uſe of ſuch Figures here in *England*, even on ordinary occaſions, is at leaſt as ancient as the Year 1133. And I judge it to have been yet ſomewhat Ancienter, becauſe the ſhape of the Figures, though not come juſt to the ſhape which we now uſe, was even then conſiderably varied from the ſhape of the Arabick Figures; which argues they had then been for ſome time in uſe; ſuch change of ſhape in Figures and Letters coming on gradually with time.

It need not move any ſcruple at all, that part of the Number is expreſſed by the Numeral Letter M (or the word *Milleſſimo*, of which M° is but a Contraction) while the reſt is expreſſed in Numeral Figures. For the like doth oft occur in old *Manuſcripts*; and ſometimes even at this day. And it doth rather favour the ſimplicity of that Age, (not very nice in ſuch things, eſpecially amongſt Mechanicks) than any deſign of Impoſture.

*By Mr.* Tho. Luffkin. n. 255. *p.* 287. n. 266. *p.* 677.

2. Over againſt the *Market-place* in *Colcheſter*, ſtands the Houſe of Mr. *Furley*, a Linnen Draper; ſome of the backermoſt part of which is an Ancient *Roman* building, but the Front is of leſſer ſtanding, and timbred. Upon the bottom Cell (which is almoſt in the form of a Triangular Priſm) of one of the Windows of the Front, between two carved Lions, ſtands an Eſcutcheon, containing only theſe Figures 1090. The Periphery of the Cyphers, and Nine, are rather Fracted than Flected, prominent, large, and very fair; but to make them the more perſpicuous, they are guilded by the Proprietor. The Window looks directly North, the Date being thereby preſerved from the ſcorching heat of the Sun; and by its Inclination (falling from the *Vertex* or *Perpendicular* by an Angle of about 60 degrees) from Rain, Snow &c. If it be objected, that the ſecond and fourth Figures, may repreſent that among the *Arabians* which is with us a 5; I anſwer, that the o is not uſed with all the *Arabs* for 5, but with ſome for a Cypher, and ſo it was uſed by the *Moors* in *Spain*, who firſt brought theſe Figures into our parts; nor is the Square ▯ an *Arabick* Letter, but an *Engliſh* Letter of that Age. And the form of theſe Figures ſoon degenerated from that of the *Arabs*, into ſuch as we now uſe, if not at the firſt reception from the *Arabs*, [or *Moors*] certainly long before 1595, as this *Conſtruction* would make it.

*Fig* 56.

*The Conſtruction of Logarithms; by Mr.* Edm. Halley. n. 216. p. 58.

XXVIII. The Old definition of *Logarithms*, that they are *Numerorum proportionalium æqui-differentes comites*, is too ſcanty to define them fully: For they

they may much more properly be said to be *Numeri Rationum Exponentes*; wherein *Ratio* is consider'd as a *Quantitas sui generis*, beginning from the Ratio of equality, or 1 to 1 = 0; being affirmative when the Ratio is increasing, as of Unity to a greater Number, but Negative when decreasing; and these Rationes we suppose to be measured by the Number of Ratiunculæ contained in each. Now these Ratiunculæ are so to be understood as in a continued Scale of Proportionals, infinite in Number between the two terms of the Ratio, which infinite Number of mean Proportionals is to that infinite Number of the like and equal Ratiunculæ between any other two terms, as the Logarithm of the one Ratio is to the Logarithm of the other. Thus if there be supposed between 1 and 10 an infinite Scale of mean Proportionals, whose Number is 100000, &c. *in infinitum*; between 1 and 2 there shall be 30102, &c. of such Proportionals, and between 1 and 3 there will be 47712, &c. of them, which Numbers therefore are the Logarithms of the Rationes, of 1 to 10, 1 to 2, and 1 to 3; and not so properly to be called the Logarithms of 10, 2, and 3.

This being laid down, it is obvious that if between Unity and any Number proposed, there be taken any infinity of mean Proportionals, the infinitely little Augment or Decrement of the first of those Means from Unity, will be a Ratiuncula, that is, the Momentum or Fluxion of the Ratio of Unity to the said Number: And seeing that in these Continual Proportionals all the Ratiunculæ are equal, their Sum, or the whole Ratio will be as the said Momentum is directly; that is, the Logarithm of each Ratio will be as the Fluxion thereof. Wherefore if the Root of any Infinite Power be extracted out of any Number, the Differentiola of the said Root from Unity, shall be as the Logarithm of that Number. So that Logarithms thus produced, may may be of as many forms as you please, to assume Infinite Indices of the Power whose Root you seek: as if the Index be supposed 100000, &c. infinitely, the Roots shall be the Logarithms invented by the Lord *Napier*; but if the said Index were 2302585, &c. Mr. *Brigg*'s Logarithms would immediately be produced. And if you please to stop at any Number of Figures, and not to continue them on, it will suffice to assume an Index of a Figure or two more than your intended Logarithm is to have, as Mr. *Briggs* did, who to have his Logarithms true to 14 places, by continual extraction of the Square Root, at last came to have the Root of the 140737488355328*th*. Power; but how operose that extraction was, will be easily judged by whoso shall undertake to examine his *Calculus*.

Now, though the Notion of an infinite Power may seem very strange, and to those that know the difficulty of the Extraction of the Roots of High Powers, perhaps impracticable; yet by the help of that admirable Invention of Mr. *Newton*, whereby he determines the Unciæ or numbers prefixt to the members composing Powers, (on which chiefly depends the *Doctrine of Series*) the Infinity of the Index contributes to render the Expression much more easy: For if the Infinite Power to be resolved be put (after Mr. *Newton*'s Method) $\overline{p+pq}$,

$\overline{p+pq}|^{\frac{1}{m}}$

$\overline{p+pq}|^{\frac{1}{m}}$, or $\overline{1+q}|^{\frac{1}{m}}$, instead of $1+\frac{1}{m}q+\frac{1-m}{2mm}qq+$ $\frac{1-3m+2mm}{6m^3}q^3+\frac{1-6m+11mm-6m^3}{24m^4}q^4$, &c. (which is the Root when $m$ is finite,) becomes $1+\frac{1}{m}q-\frac{1}{2m}qq+$ $\frac{1}{3m}q^3+\frac{1}{4m}q^4+\frac{1}{5m}q^5$, &c. $mm$ being *infinitè* infinite, and consequently whatever is divided thereby vanishing. Hence it follows that $\frac{1}{m}$ multiplied into $q-\frac{1}{2}qq+\frac{1}{3}q^3-\frac{1}{4}q^4+\frac{1}{5}q^5$, &c. is the Augment of the first of our mean Proportionals between Unity and $1+q$, and is therefore the Logarithm of the Ratio of 1 to $1+q$; and whereas the Infinite Index $m$ may be taken at pleasure, the several Scales of Logarithms to such Indices will be as $\frac{1}{m}$, or reciprocally as the Indices. And if the Index be taken 10000, &c. as in the case of *Napiers*'s Logarithms, they will be simply $q-\frac{1}{2}qq+\frac{1}{3}q^3-\frac{1}{4}q^4+\frac{1}{5}q^5-\frac{1}{6}q^6$, &c.

Again, if the Logarithm of a decreasing Ratio be sought, the Infinite Root of $1-q$, or $\overline{1-q}|^{\frac{1}{m}}$ is $1-\frac{1}{m}q-\frac{1}{2m}q^2-\frac{1}{3m}q^3-\frac{1}{4m}q^4-$ $\frac{1}{5m}q^5-\frac{1}{6m}q^6$ &c. whence the Decrement of the first of our Infinite Number of Proportionals will be $\frac{1}{m}$ into $q+\frac{1}{2}qq+\frac{1}{3}q^3+\frac{1}{4}q^4+\frac{1}{5}q^5+$ $\frac{1}{6}q^6$ &c. which therefore will be as the Logarithm of the Ratio of Unity to $1-q$. But if $m$ be put 10000, &c. then the said Logarithm will be $q+\frac{1}{2}qq+\frac{1}{3}q^3+\frac{1}{4}q^4+\frac{1}{5}q^5+\frac{1}{6}q^6$, &c.

Hence the terms of any Ratio being $a$ and $b$, $q$ becomes $\frac{b-a}{a}$, or the difference divided by the lesser term, when 'tis an Increasing Ratio; or $\frac{b-a}{b}$ when 'tis Decreasing, or as $b$ to $a$. Whence the Logarithm of the same Ratio may be doubly exprest, for putting $x$ for the difference of the terms $a$ and $b$, it will be either

$\frac{1}{m}$

$$\frac{1}{m} \text{ into } \frac{x}{b} + \frac{x^2}{2bb} + \frac{x^3}{3b^3} + \frac{x^4}{4b^4} + \frac{x^5}{5b^5} + \frac{x^6}{6b^6} \text{ \&c. or}$$

$$\frac{1}{m} \text{ into } \frac{x}{a} - \frac{x^2}{2aa} + \frac{x^3}{3a^3} - \frac{x^4}{4a^4} + \frac{x^5}{5a^5} - \frac{x^6}{6a^6}, \text{ \&c.}$$

But if the Ratio of $a$ to $b$ be ſuppoſed divided into two parts, *viz.* into the Ratio of $a$ to the *Arithmetical Mean* between the terms, and the Ratio of the ſaid *Arithmetical Mean* to the other term $b$, then will the Sum of the Logarithms, of thoſe two Rationes be the Logarithm of the Ratio of $a$ to $b$; and ſubſtituting $\frac{1}{2}z$ inſtead of $\frac{1}{2}a + \frac{1}{2}b$ the ſaid *Arithmetical Mean*, the Logarithms of thoſe Rationes will be by the foregoing Rule.

$$\frac{1}{m} \text{ into } \frac{x}{z} + \frac{xx}{2zz} + \frac{x^3}{3z^3} + \frac{x^4}{4z^4} + \frac{x^5}{5z^5} + \frac{x^6}{6z^6} \text{ \&c. and}$$

$$\frac{1}{m} \text{ into } \frac{x}{z} - \frac{xx}{2zz} + \frac{x^3}{3z^3} - \frac{x^4}{4z^4} + \frac{x^5}{5z^5} - \frac{x^6}{6z^6}, \text{ \&c.}$$

the ſum whereof $\frac{1}{m}$ into $\frac{2x}{z} * + \frac{2x^3}{3z^3} * + \frac{2x^5}{5z^5} * + \frac{2x^7}{7z^7}$ &c. will be the Logarithm of the Ratio of $a$ to $b$, whoſe difference is $x$ and ſum $z$. And this Series converges twice as ſwift as the former, and therefore is more proper for the practice of making of Logarithms: which it performs with that expedition, that where $x$ the difference is but the hundredth part of the ſum, the firſt ſtep $\frac{2x}{z}$ ſuffices to ſeven places of the Logarithm, and the ſecond ſtep to twelve; but if *Brigg's* firſt Twenty *Chiliads* of Logarithms be ſuppoſed made, as he has very carefully computed them, to fourteen places, the firſt ſtep alone is capable to give the Logarithm of any intermediate Number, true to all the places of thoſe Tables.

After the ſame manner, may the difference of the ſaid two Logarithms be very fitly applyed to find the Logarithms of Prime Numbers, having the Logarithms of the two next numbers above and below them: For the difference of the Ratio of $a$ to $\frac{1}{2}z$, and of $\frac{1}{2}z$ to $b$, is the Ratio of $ab$ to $\frac{1}{4}zz$, and the half of that Ratio is that of $\sqrt{ab}$ to $\frac{1}{2}z$, or of the Geometrical Mean to the Arithmetical. And conſequently the Logarithm thereof will be the half difference of the Logarithms of thoſe Rationes, *viz.*

$$\frac{1}{m} \text{ into } \frac{xx}{2zz} + \frac{x^4}{4z^4} + \frac{x^6}{6z^6} + \frac{x^8}{8z^8}, \text{ \&c.}$$

Which is a *Theorem* of good diſpatch to find the Logarithm of $\frac{1}{2}z$. But the ſame is yet much more advantageouſly performed by a Rule derived from the foregoing, and beyond which in my Opinion nothing better can be hoped: For the Ratio of $ab$ to $\frac{1}{4}zz$, or $\frac{1}{4}aa + \frac{1}{2}ab + \frac{1}{4}bb$, has the difference of its terms, $\frac{1}{4}aa - \frac{1}{2}ab + \frac{1}{4}bb$, or the Square of $\frac{1}{2}a - \frac{1}{2}b = \frac{1}{4}xx$, which in the preſent caſe of finding the Logarithms of Prime Numbers is al-

ways

ways Unity, and calling the Sum of the terms $\frac{1}{4} z z + a b = y y$, the Logarithm of the Ratio of $\sqrt{a b}$ to $\frac{1}{2} a + \frac{1}{2} b$, or $\frac{1}{2} z$ will be found

$$\frac{1}{m} \text{ into } \frac{1}{yy} + \frac{1}{3y^6} + \frac{1}{5y^{10}} + \frac{1}{7y^{14}} + \frac{1}{9y^{18}}, \&c.$$

which converges very much faster than any *Theorem* hitherto published for this purpose.

Here note that $\frac{1}{m}$ is all along applyed to adapt these Rules to all sorts of Logarithms. If $m$ be 10000, &c. it may be neglected, and you will have *Napier*'s Logarithms, as was hinted before; but if you desire *Brigg's* Logarithms, which are now generally received, you must divide your Series by 2, 30258509299404568401799145468436420760110148862877297603328, or multiply it by the reciprocal thereof, *viz.* 0,4342944819032518276511289189166050822943970058036665661144 54.

But to save so operose a Multiplication (which is more than all the rest of the work) it is expedient to divide this Multiplicator by the Powers of $z$ or $y$ continually, according to the direction of the *Theorem*, especially where $x$ is small and Integer, reserving the proper Quotes to be added together, when you have produced your Logarithm to as many Figures as you desire; of which Method I will give a Specimen, in the Logarithms of the first Prime Numbers under 20 to sixty places, computed by Mr. *Abraham Sharp*, as they were communicated to me by our common Friend, Mr. *Euclid Speidal*.

| Num. | Logarithms, |
|---|---|
| 2. | 0, 301029995663981195213738894724493026768189881462108541310427 |
| 3. | 0, 477121254719662437295027903255115309200128864190695864829866 |
| 7. | 0, 845098040014256830712216258592636193483572396323965406503835 |
| 11. | 1, 041392685158225040750199971243024241706702190466453094596539 |
| 13. | 1, 113943352306837769206541895026246254561189005053673288598083 |
| 17. | 1, 230448921378273028540169894328337030007567378425046397380368 |
| 19. | 1, 278753600952828961536333475755929317951129337394497598906819 |

The next Prime Number is 23, which I will take for an Example of the foregoing Doctrine; and by the first Rules, the Logarithm of the Ratio of 22 to 23 will be found to be either.

$$\frac{1}{22} - \frac{1}{968} + \frac{1}{31944} - \frac{1}{937024} + \frac{1}{25768160}, \&c. \text{ or}$$

$$\frac{1}{23} + \frac{1}{1058} + \frac{1}{36501} + \frac{1}{1119364} + \frac{1}{32181715}, \&c.$$

As likewiſe that of the Ratio of 23 to 24 by a like Proceſs.

$$\frac{1}{23} - \frac{1}{1058} + \frac{1}{36501} - \frac{1}{1119364} + \frac{1}{32181715}, \text{ \&c. or}$$

$$\frac{1}{24} + \frac{1}{1152} + \frac{1}{41472} + \frac{1}{1327104} + \frac{1}{39813120}, \text{ \&c.}$$

And this is the Reſult of the Doctrine of *Mercator*, as improv'd by the Learned Dr. *Wallis*. But by the ſecond *Theorem*, *viz.* $\frac{2x}{z} + \frac{2x^3}{3z^3} + \frac{2x^5}{5z^5}$, &c. the ſame Logarithms are obtained by fewer ſteps. To wit,

$$\frac{2}{45} + \frac{2}{273375} + \frac{2}{922640625} + \frac{2}{2615686171875}, \text{ \&c. And}$$

$$\frac{2}{47} + \frac{2}{311469} + \frac{2}{1146725035} + \frac{2}{3546361843241}, \text{ \&c.}$$

which was Invented and Demonſtrated in the Hyperbolical Spaces Analogous to the Logarithms, by the Excellent Mr. *James Gregory*, in his *Exercitationes Geometricæ*, and ſince further proſecuted by the aforeſaid Mr. *Speidall*, in a late Treatiſe in *Engliſh* by him Publiſh'd on this Subject. But the Demonſtration, as I conceive, was never till now Perfected without the conſideration of the Hyperbola, which in a matter purely Arithmetical as this is, cannot ſo properly be applied. But what follows, I think, I may more juſtly claim as my own, *viz. That the Logarithm of the Ratio of the Geometrical Mean to the Arithmetical*, between 22 and 24, or of $\sqrt{528}$ to 23, will be found to be either

$$\frac{1}{1058} + \frac{1}{1119364} + \frac{1}{888215334} + \frac{1}{626487882248}, \text{ \&c. or}$$

$$\frac{1}{1057} + \frac{1}{3542796579} + \frac{1}{659676558485285}, \text{ \&c.}$$

All theſe Series being to be Multiplied into 0,4342944819, &c. if you deſign to make the *Logarithm* of *Briggs*. But with great Advantage in reſpect of the Work, the ſaid 0,4342944819, &c. is divided by 1057, and the Quotient thereof again divided by three times the Square of 1057, and that Quotient again by $\frac{3}{5}$ of that Square, and that Quotient by $\frac{5}{7}$ thereof, and ſo forth, till you have as many Figures of your Logarithm as you deſire. As for Example, The Logarithm of the Geometrical Mean between 22 and 24 is found by the Logarithms of 2, 3, and 11, to be

| | | |
|---|---|---|
| | | 1, 36131696126690612945009172669805 |
| 1057) | 43429 &c. ( | - - - - 410874628101468143473158863 68 |
| 3 in 1117249) | 41087 &c. ( | - - - - - - - - 1225852154418182946 0074 |
| $\frac{3}{5}$ in 1117249) | 12258 &c. ( | - - - - - - - - - - - - - 6583235184376175 |
| $\frac{5}{7}$ in 1117249) | 65832 &c. ( | - - - - - - - - - - - - - - - - - 4208829765 |
| $\frac{7}{9}$ in 1117249) | 42088 &c. ( | - - - - - - - - - - - - - - - - - - - - - - 2930 |
| Summa. | | 1, 36172783601759287886777711225117 |

Which is the Logarithm of 23 to Thirty two Places, and obtained by 5 Divisions with very small Divisors, all which is much less Work than simply Multiplying the Series into the said Multiplicator 0, 43429 &c.

From the Logarithm given to find what Ratio it expresses, is a *Problem* that has not been so much considered as the former, but which is solved with the like ease, and demonstrated by a like Process, from the same general *Theorem* of Mr. *Newton*. For as the Logarithm of the Ratio of 1 to 1 + $q$ was proved to be $\overline{1+q}|^{\frac{1}{m}} - 1$, and that of the Ratio of 1 to 1 — $q$ to be $1 - \overline{1-q}|^{\frac{1}{m}}$: so the Logarithm, which we will from henceforth call L, being given, 1 + L will be equal to $\overline{1+q}|^{\frac{1}{m}}$ in the one Case; and 1 — L will be equal to $\overline{1-q}|^{\frac{1}{m}}$ in the other: Consequently $\overline{1+L}|^{m}$ will be equal to $1+q$, and $\overline{1-L}|^{m}$ to $1-q$; that is, according to Mr. *Newton*'s said Rule, $1 + mL + \frac{1}{2}m^2L^2 + \frac{1}{6}m^3L^3 + \frac{1}{24}m^4L^4 + \frac{1}{120}m^5L^5$, &c. will be = to $1+q$, and $1 - mL + \frac{1}{2}m^2L^2 - \frac{1}{6}m^3L^3 + \frac{1}{24}m^4L^4 - \frac{1}{120}m^5L^5$, &c. will be equal to $1-q$, $m$ being any infinite Index whatsoever; which is a full and general *Proposition* from the Logarithm given to find the Number, be the Species of Logarithms what it will. But if *Napier*'s Logarithm be given, the Multiplication by $m$ is saved, (which Multiplication is indeed no other than the reducing the other Species to his) and the Series will be more simple, *viz.* $1 + L + \frac{1}{2}L^2 + \frac{1}{6}L^3 + \frac{1}{24}L^4 + \frac{1}{120}L^5$, &c. or $1 - L + \frac{1}{2}L^2 - \frac{1}{6}L^3 + \frac{1}{24}L^4 - \frac{1}{120}L^5$, &c. This Series, especially in great Numbers, Converges so slowly, that it were to be wished it could be contracted.

If one term of the Ratio, whereof L is the Logarithm, be given, the other term will be had easily by the same Rule: For if L were *Napier*'s Logarithm

rithm of the Ratio of $a$ the leſſer, to $b$ the greater Term, $b$ would be the Product of $a$ into $1 + L + \frac{1}{2} L^2 + \frac{1}{6} L^3$, &c. $= a + aL + \frac{1}{2} a L^2 + \frac{1}{6} a L^3$, &c. But if $b$ were given, $a$ would be $= b - bL + \frac{1}{2} b L^2 - \frac{1}{6} b L^3$, &c. Whence by the help of the *Chiliads*, the Number appertaining to any Logarithm will be exactly had to the utmoſt extent of the Tables. If you ſeek the neareſt next Logarithm, whether greater or leſſer, and call its Number $a$ if leſſer, or $b$ if greater, then the given L, and the difference thereof from the ſaid neareſt Logarithm you call $l$; it will follow that the Number anſwering to the Logarithm L will be either $a$ into $1 + l + \frac{1}{2} l^2 + \frac{1}{6} l^3 + \frac{1}{24} l^4 + \frac{1}{120} l^5$, &c. or elſe $b$ into $1 - l + \frac{1}{2} l^2 - \frac{1}{6} l^3 + \frac{1}{24} l^4 - \frac{1}{120} l^5$, &c. wherein as $l$ is leſs the Series will converge the ſwifter. And if the firſt 20000 Logarithms be given to 14 places, there is rarely occaſion for the three firſt ſteps of this Series to find the Number to as many places. But for *Vlacq*'s great *Canon* of 100000 Logarithms, which is made but to ten places, there is ſcarce ever need for more than the firſt ſtep $a + al$, or $a + mal$ in one Caſe, or elſe $b - bl$, or $b - mbl$ in the other, to have the Number true to as many Figures as thoſe Logarithms conſiſt of.

There is another Series, which is not indeed ſo ſimple and uniform, yet the firſt ſtep thereof is moſt Commodious for Practice, and exact enough for Tables not exceeding 14 Places. It is thus: $a + \frac{al}{1 - \frac{1}{2}l}$, or $b - \frac{bl}{1 + \frac{1}{2}l}$ will be the Number anſwering to the Logarithm given, differing from the truth but by one half of the third ſtep of the former Series. But that which renders it yet more eligible is, that with equal facility it ſerves for *Briggs*'s or any other ſort of Logarithms, with the only variation of Writing $\frac{1}{m}$ inſtead of 1; that is

$$a + \frac{al}{\frac{1}{m} - \frac{1}{2}l}, \text{ and } b - \frac{bl}{\frac{1}{m} + \frac{1}{2}l}; \text{ or } \frac{\frac{1}{m}a + \frac{1}{2}la}{\frac{1}{m} - \frac{1}{2}l} \text{ and } \frac{\frac{1}{m}b - \frac{1}{2}lb}{\frac{1}{m} + \frac{1}{2}l},$$

which are eaſily reſolved into Analogies, *viz.*

As 43429 &c. $- \frac{1}{2} l$ : to 43429 $+ \frac{1}{2} l$ :: ſo is $a$ : } to the Number ſought.
Or As 43429 &c. $+ \frac{1}{2} l$ : to 43429 $- \frac{1}{2} l$ :: ſo is $b$ : }

 If

If more Steps of this Series be desired, it will be found as follows,

$$a + \frac{al}{1 - \frac{1}{2}l} - \frac{\frac{1}{12}al^3}{1 - l} + \frac{\frac{1}{35}al^5}{1 - 2l}$$, &c. as may easily be demonstrated by working out the Divisions in each step, and collecting the Quotes, whose Sum will be found to agree with our former Series, which is no other than an easie *Corollary* to Mr. *Newton*'s general *Theorem* for forming *Roots* and *Powers*.

## XXIX. *Papers of less General Use Omitted.*

*Tangents to Curves.* n. 81. p. 4010.

1. A Breviat of Dr. *Wallis*'s two Methods of drawing Tangents; Extracted by him from his *Con. Sect.* and other parts of his *Mathematical Works*.

*Rectification of Curves.* n. 98. p. 6146, 6149.

2. M. *Huygens* in his *Hor. Oscill.* having given M. *Huraet* the Honour of Inventing a Curve equal to a Straight Line in the Year 1659; Dr. *Wallis* here asserts this Invention to Mr. *William Neile* (Son of Sir *Paul Neile*), who discovered and Demonstrated the Equality of a *Paraboloid* to a Straight Line two Years before. The same was soon after otherwise Demonstrated by my Lord *Brounker*, and Sir *Christopher Wren*, in *June* and *July* 1657; and the Demonstrations inserted by Dr. *Wallis*, in his Tract *de Cycloide* 1659, with a fair Relation of that whole Matter. *Ib.* p. 6150. Besides, Sir *Christopher Wren* found a Straight Line equal to that of a *Cycloide* in the Year 1658: Yet he freely confesses Mr. *Neile*'s Invention of a Curve capable of *Rectification* the Year before.

*Transformation of Curves.* n. 214. p. 233.

3. The Abbot *Galloys* having, in the Year 1693, asserted that Mr. *James Gregory* and Dr. *Barrow* stole their General Propositions concerning the *Transformation of Curves* from Mr. *Robervall*; Dr. *David Gregory* here fully refutes that Assertion. For Mr. *Gregory* Publish'd his Book at *Padua* 1668, and Dr. *Barrow* his *Lectiones Geometricæ* 1674, which Mr. *Robervall* doubtless had a sight of before he dy'd (which was not till *October* 1675), yet he never complain'd of any such Injury done him.

*Cycloidal Spaces perfectly Quadrable.* n. 217. p. 111.

4. Besides that Segment of the *Semicycloidal Figure*, first observed by Sir *Christopher Wren* and after him by Mr. *Huygens*, and a Trilinear part of it, which are capable of being Geometrically Squared, Dr. *Wallis* here produces from his Tracts *de Cycloide*, and *de Motu*, some other Portions thereof equally capable of Quadrature.

*The Cycloid considered long ago.* n. 229. p. 561.

5. Dr. *Wallis* finds among the Mathematical Works of *Bovillus*, Publish'd at several times between the Years 1501 and 1510, that the Curve (which is now call'd the *Cycloid*) was then consider'd. But he also finds that *Bovillus* was not the first who consider'd it: For *Cardinal Cusanus*, as appears by an Ancient M S. of his Works (transcrib'd by *J. Scoblant* in the Year 1451) had consider'd it some time before. The Figure indeed (thro' the unskilfulness of the Transcriber) both in the M S. and the *Basil* Edition, A. 1565, is very ill drawn; but being Corrected according to the true meaning of that *Cardinal*'s own Words, it evidently represents the Modern *Cycloid*. From hence 'tis Manifest, that this Curve was not first taken into Consideration

either

either by *Merſennus* or *Gallilæo*, but ſome Ages before, tho' never well underſtood till this Preſent Age.

6. Some Papers ſent by Mr. *Jo. Collins* to Dr. *Wallis*, giving his thoughts about ſome Defects in Algebra, which he did not live to finiſh. *Defects in Algebra.* n. 159. p. 575.

XXX. *Accounts of Books with Additions, Emendations, &c. Omitted.*

1. EUclidis *Elementa Geometrica, novo ordine ac Methodo demonſtrata.* Lond. 1666. n. 15. p. 261.

2. Archimedis *Opera*; Apollonii Perg: *Conic. Libri* 4; Theodoſii *Sphærica*; *Methodo nova illuſtrata, & ſuccinctè demonſtrata: ab* Iſa. Barrow *R. S. S.* Lond. 1675. *in* 4to. n. 114. p. 314.

3. Ἀρχιμήδους τοῦ Συρακουσίου Ψαμμίτης, καὶ Κύκλου Μέτρησις: Εὐτοκίου Ἀσκαλωνίτου εἰς αὐτὴν ὑπόμνημα, &c. *Cum Verſione & Notis* Jo. Wallis. S S. Th. D. *Oxon.* 1676. n. 123. p. 567.

4. *Theon Smyrnæus*, publiſh'd at *Paris* by *Iſmael Bulialdus* in Greek and Latin. n. 80. p. 3095.

5. Diophanti Alexandrini *Arithmeticorum Libri* 6, *& de Numeris Multangulis Liber unus*; *cum Commentariis* C. G. Bacheti, *& Obſervationibus*, D. P. de Fermat *Senatoris* Tholoſani: *cui acceſſit Doctrinæ Analyticæ Inventum Novum.* Toloſæ 1670. in *Folio.* n. 72. p. 2185.

6. The Works of Monſieur de *Fermat.* n. 1. p. 15.

7. Franciſci du Laurens *Specimina Mathematica, duobus Libris comprehenſa. Horum prior Syntheticus agit de Genuinis Matheſeos Principiis in genere; in ſpecie autem de veris Geometriæ Elementis hucuſque nondum traditis. Poſterior Analyticus de Methodo Compoſitionis atque Reſolutionis fuſè differit, & multa nova complectitur, quæ ſubtiliſſimam Analyſeos Artem mirum in modum promovent.* This Book is here Cenſured, ſome Miſtakes in it Corrected, and the Cenſure vindicated, by Dr. *Wallis.* n. 30. p. 580. n. 34. p. 654. n. 38. p. 744. n. 39. p. 775. n. 41. p. 825.

8. R. P. Andreæ Taquet, *è S. J. Opera Mathematica.* Anwerp. 1669. in *Folio.* n. 43. p. 869.

9. A Mathematical Compendium, Collected out of the Notes and Papers of Sir *Jonas Moore*, by *Nicholas Stevenſon. Lond.* 1674. in *Twelves.* n. 104. p. 83.

10. R. P. Claudii Franc. Milliet de Chales, è S. J. *Curſus ſeu Mundus Mathematicus, univerſam Matheſin tribus Tomis complectens.* Lugd. 1674. in *Folio.* n. 110. p. 229.

11. The Mathematical Works of Dr. *Jo. Wallis, Savilian Profeſſor of Geometry* in the Univerſity of *Oxford*, & F. R. S. in three Volumes in *Folio. Oxon.* n. 216. p. 73. n. 254. p. 259.

12. An Introduction to Algebra, Tranſlated out of *High Dutch* into *Engliſh* by *Tho. Branker* M. A. much altered and augmented by Dr. *J. Pell.* Alſo a Table of ſuch odd Numbers, as are leſs than one Hundred Thouſand, ſhowing thoſe that are *Incompoſit*, and reſolving the reſt into their *Factors*, or *Coefficients. Lond.* in *Quarto.* n. 35. p. 688.

13. *Labyrinthus Algebræ. Auth.* Joh. Jac. Ferguſon. 1667. in *Quarto.* n. 49. p. 996.

n. 90. p. 5152. n. 95. p. 6073. 14. The Elements of that Mathematical Science call'd *Algebra*; by *Jo. Kersey.* Lond. 1673. in *Folio.*

n. 108. p. 192. n. 143. p. 21. n. 173. p. 1095. n. 233. p. 730. 15. A Treatise of *Algebra*, both Historical and Practical; by *Jo. Wallis.* D. D. In the 109th Chapter there are some Numbers mistaken, which are here rectify'd by the Author.

n. 14. p. 253. n. 16. p. 289. 16. *De Principiis & Ratiocinatione Geometrarum; contra Fastum Professorum Geometriæ. Authore* Tho. Hobbes. This Book is here Animadverted on, and Answer'd; by Dr. *Wallis.*

n. 48. p. 971. 17. Thomæ Hobbes *Quadratura Circuli, Cubatio Sphæræ, Duplicatio Cubi, confutata. Auth.* Jo. Wallis. S. T. D. Oxon. 1669. *in Quarto.*

n. 55. p. 1121. 18. Thomæ Hobbes *Quadratura Circuli, Cubatio Sphæræ, Duplicatio Cubi, (secundo edita) denuo refutata. Auth.* Jo. Wallis. S. T. D. Oxon. 1669.

n. 72. p. 2185. 19. *Rosetum Geometricum, cum Censura brevi* Doctrinæ Wallisianæ *de Motu. Auth.* Tho. Hobbes *Malmesburiensi.* Lond. 1671. *in Quarto.* This Book is
n. 73. p. 2202. here Answered by Dr. *Wallis.*

n. 75. p. 2241. 20. Four Papers of Mr. *Hobbs*'s, Publish'd in the Months of *August* and *September*, 1671. which are here Answered by - - - - -

n. 86. p. 5047. 21. *Lux Mathematica, Collisionibus* Johannis Wallisii S. T. D. & Thomæ Hobbesii Malmsburiensis, *excussa, multis & fulgentissimis aucta radiis. Auth.* R. R. *Adjuncta Censura Doctrinæ* Wallisianæ *de Libra, una cum* Roseto Hobbesii.
n. 87. p. 5067. Lond. 1672, *in Quarto.* This Book is here Answered by Dr. *Wallis.*

n. 97. p. 6131. 22. *Principia & Problemata aliquot Geometrica, antè desperata, nunc breviter explicata & demonstrata. Auth.* T. H. Malmsburiensi. Lond. 1673. *in Quarto.*

n. 185. p. 245. 23. *Le Grand & Fameux Probleme de la Quadrature du Cercle resolu Geometriquement par le Cercle & la Ligne droite, par* M. Mallement de Messange. à Paris. 1686. *in Twelves.* This Book is here Refuted by M. D. *Cluverius.* R. S. S.

n. 32. p. 625. 24. *Nouveaux Elemens de Geometrie:* Or a Mathematical Treatise, entituled *New Elements of Geometry.* Paris. 1667. *in Quarto.*

n. 79. p. 3064. 25. *Elemens de Geometrie; par le* P. Ignace Gaston Pardies, *de la Comp. de J.* à Paris 1671. *in Twelves.*

n. 33. p. 640. 26. 1. *Vera Circuli & Hyperbolæ Quadratura, in propria sua Proportionis Specie inventa & Demonstrata,* à Jac. Gregorio Scoto. Patavii. *in Quarto.* This
Ib. p. 641. Subject is here further considered, and the Area of an Hyperbola explain'd; by Mr. *J. Collins.*

n. 37. p. 732. 2. *M. Huygens* having Publish'd Animadversions upon this Book, in the *Journal des Scavans* 1668. Mr. *Gregory* here Answers them. To this *M. Huygens*
n. 44. p. 882. Reply'd in a following Journal of that Year; and Mr. *Gregory*, further to elucidate the Controversy, here returns a second Answer.

n. 216. p. 65. 3. In the 48th page of this Book, Mr. *Halley* has Discovered, and Corrected a small Mistake in the Logarithm of 10.

n. 35. p. 685. 27. *Geometriæ pars Universalis, Quantitatum Curvarum Transmutationi & Mensuræ inserviens. Auth.* Jac. Gregorio *Scoto.* Patavii. 1668. *in Quarto.*

n. 37. p. 738. 28. *De Infinitis Spiralibus inversis, Infinitisque Hyperbolis, aliisque Geometricis. Auth.* F. Stephano de Angelis *Veneto.* Patavii. *in Quarto.*

29. *Michaelis*

29. *Michaelis Angeli Ricci Exercitatio Geometrica.* Romæ. *in Quarto.* Reprinted at *London*, and annext to *Mercator*'s *Logarithmotechnia.* n. 37. p. 738.

30. Renati Franc. Slusii *Mesolabum. Cui accessit pars altera de Analysi, & Miscellanea.* Leodii Eburonum 1668. *in Quarto.* n. 45. p. 903.

31. *Elementa Geometriæ Planæ. Authore* Ægidio Francisco de Gottignies *Bruxellensi. S. J.* Romæ. 1669. *in Twelves.* n. 67. p. 2054.

32. *Synopsis Geometrica; cum tribus Opusculis, de Linea Sinuum & Cycloide; de Maximis & Minimis, Centuria; & Synopsis Geometriæ Planæ. Auth.* Honor. Fabry. *S. J.* Lugduni Galliarum 1669. *in Twelves.* n. 67. p. 2055.

33. *Lectiones* 13. *Geometricæ; in quibus (præsertim) Generalia Linearum Curvarum Symptomata declarantur, ab* Isaaco Barrow. Lond. 1669. *in Quarto.* To these Lectures the Author here adds several *Corollaries* and *Theorems.* n. 75. p. 2260.

34. Erasmi Bartholini *Selecta Geometrica.* Hauniæ. 1674. *in Quarto.* n. 106. p. 137.

35. *Elemens des Mathematiques, ou Principes Generaux de toutes les Sciences qui ont les Grandeurs pour Object. Par* J. P. à Paris 1675. *in Quarto.* n. 126. p. 638

36. *Nouvelle Methode en Geometrie pour les Sections des Superficies Coniques & Cylindriques; qui ont pour Base des Circles, ou des Paraboles, des Ellipses, & des Hyperboliques; par* Ph. de la Hire. a Paris 1673. *in Quarto.* n. 129. p. 745.

37. *De Cycloide & Sectionibus Conicis. Auth.* Ph. de la Hire. *Ibid.* 746.

38. The Geometrical Key, or Construction of all Equations Linear, Quadratick, Cubick, and Biquadratick, by a Circle and one only Parabola; by Mr. *Tho. Baker.* n. 157. p. 549.

39. *Exercitatio Geometrica de Dimensione Figurarum. Auth.* Davide Gregorio. Edinb. 1684. *in Quarto.* n. 163. p. 730.

40. *Methodus Figurarum Lineis Rectis & Curvis comprehensarum Quadraturas determinandi. Auth.* J. Craige. Lond. 1685. *in Quarto*: To this Tract the Author here makes an Addition; and takes Notice of some Remarks made on it in the *Act. Lips.* by M. *Leibnitz*, and M. *J. Bernoulli.* n. 183. p. 185. *Ibid.* p. 186. n. 235. p. 786.

41 *Tractatus Mathematicus de Figurarum Curvilinearum Quadraturis & Locis Geometricis. Auth.* J. Craig. Lond. 1693. *in Quarto.* n. 209. p. 113.

42. *Tractatus de Principiis Calculi Exponentialis. Auth.* D. Bernoullio; wherein a Mistake is here Discovered and Corrected, by Mr. *Craig.* n. 245. p. 374.

43. *Analysis Geometrica, sive nova & vera Methodus Resolvendi, tam Problemata Geometrica, quam Arithmeticas Quæstiones. Pars prima, de Planis. Auth.* D. Antonio Hugone de Omerique *Sanlucarense.* n. 257. p. 351.

44. Stereometrical Propositions, variously applicable, but particularly intended for *Gauging*; by *Rob. Anderson. Lond.* 1668. in *Octavo.* n. 39. p. 785.

45. *Gauging* promoted; being an Appendix to Stereometrical Propositions; by *Rob. Anderson. Lond.* 1669. in *Octavo.* n. 47. p. 960.

46. *Gauging* Epitomized; by *Mich. Dary. Lond.* 1669. upon one Folio Page. n. 52. p. 1054.

47. *Tabula Numerorum Quadratorum decies Millium, una cum ipsorum Lateribus ab Unitate incipientibus, & Ordine Naturali usque ad* 10000 *progredientibus.* Lond. 1672. n. 82. p. 4050.

48. The Description and Use of two Arithmetick Instruments, &c. by Sir *Sam. Moreland. Lond.* 1673. n. 94. p. 6048.

49. Joannis

n. 139 p. 980. 49. Joannis Wallisii S. T. D. *Exercitationes tres.* 1. *De Cometarum Distantiis investigandis.* 2. *De Rationum & Fractionum Reductione.* 3. *De Periodo Juliano.* Lond. 1678.

n. 38. p. 753. 50. *Logarithmotechnia* Nicolai Mercatoris. Lond. 1668. *in Quarto.* This
Ibid. p. 756. Author's Method of squaring the Hyperbola, and of finding the Sum of the
Ibid. p. 759. Logarithms is here improved by Dr. *Wallis*; and further Explicated by the Author himself.

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# CHAP. II.

## *Trigonometry, Surveying.*

*A Chorographical Problem proposed by Mr.* Rich. Townley, *solved by Mr.* John Collins. n. 69. p. 2093.

I. Prob. THE *Distances of three Objects in the same plain being given, as A, B, C; The Angles made at a fourth Place in the same Plain as at S, are observed: the Distances from the Place of Observation to the respective Objects, are required.*

The *Problem* hath six *Cases.*

Fig. 57. CASE I. If the Station be taken without the Triangle made by the Objects, but in one of the sides thereof produced, as at S: find the Angle A C B; then in the Triangle A C S, all the Angles and the side A C are known, whence either or both the Distances S A, or S C, may be found.

Fig. 58. *Case* 2. If the Station be in one of the Sides of the Triangle, as at S: then having the three Sides, A C, C B, B A, given, find the Angle C A B; then again in the Triangle S A B all the Angles, and the Side A B, are known, whence may be found either A S, or S B, Geometrically, if you make the Angle C A D equal to the observed Angle C S B, and draw B S parallel to D A, you determine the Point of Station S.

Fig. 59. *Case* 3. If the three Objects lie in a right line as A C B (suppose it done), and that a Circle passeth through the Station S, and the two Exteriour Objects A, B: then is the Angle A B D, equal to the Observed Angle A S C (by 21. 3. *E*) as Insisting on the same Arch A D: And the Angle B A D in like manner equal to the observed Angle C S B: By this means the point D is determined. Joyn D C, and produce the same, then a Circle passing through the Points A, B, D, intersects D C, produced at S, the place of Station.

*Calculation*] In the Triangle A B D, all the Angles and the Side A B, are known, whence may be found the Side A D.

Then in the Triangle C A D, the two sides C A, and A D, are known, and there contained Angle C A D is known; whence may be found the Angles C D A, and A C D, the Complement whereof to a Semicircle is the Angle S C A: in which Triangle the Angles are now all known, and the side A C: whence may be found either of the Distances S C, or S A.

*Case*

*Case* 4. If the Station be without the Triangle made by the Objects, the Sum of the Angles observed is less than four Right Angles. The *Constructi-on* is the same as in the last *Case*, and the *Calculation* likewise; saving that you must make one Operation more having the three sides A C, C B, B A, there-by find the Angle C A B, which add to the Angle E A D, then you have the two sides, *viz.* A C, being one of the Distances, and A D, (found as in the former *Case*) with their contained Angle C A D given, to find the Angles C D A, and A C D, the Complement whereof to a Semicircle is the Angle S C A: Now in the Triangle S C A, the Angle at C being found, and at S observed, and given by Supposition, the other at A is likewise known, as be-ing the Complement of the two former to a Semicircle, and the side A C given; hence the Distances C S, or A S, may be found. Fig 60.

*Case* 5. If the place of Station be at some Point within the plain of the Triangle, made by the three Objects, the *Construction* and *Calculation* is the same as in the last, saving only that instead of the observed Angle A S C, the Angle A B D is equal to the Complement thereof to a Semicircle, to wit, it is equal to the Angle A S D; both of them insisting on the same Arch A D: And in like manner the Angle B A D is equal to the Angle D S B, which is the Complement of the observed C S B; and in this Case the Sum of the three Angles observed, is equal to four Right Angles. Fig. 61.

In these three latter Cases no use is made of the Angle observed between the two Objects, as A and B, that are made the Base-line of the Construction; yet the same is of ready use for finding the third distance or last side sought; as in the Triangle S A B, there is given the distance A B, its opposite Angle equal to the Sum of the two observed Angles, and the Angle S A B attained, as in the fourth *Case*: Hence the third side, or last distance S B, may be found. Fig. 60.

And here it may be noted, that the three Angles C A S, A S B, S B C, are together equal to the Angle A C B; for, the two Angles C S B and C B S, are equal to E C B, as being the Complement of S C B to two Right Angles; and the like in the Triangle on the other side. *Ergo* &c.

*Case* 6. If the three Objects be A, B, C, and the Station at S, as before, it may happen, according to the former Constructions, that the Points C and D, may fall close together; and so a right Line, joyning them, shall be produced with uncer-tainty; in such Case the Circle may be conceived to pass through the place of Station at S, and any two of the Objects, as through B and C, wherein making the Angle D B C equal to the observed Angle A S C, and B C D equal to the Complement to 180 degrees of both the observed Angles in D S B; thereby the point D is determined, through which, and the Points C, B, the Circle is to be described, and joyning D A, (produced when need requireth) where it intersects the Circle, as at S, is the place of Station sought. Fig. 62.

This *Problem* may be of good use for the due Situation of Sands or Rocks, that are within sight of three Places upon Land, whose distances are well known; or for *Chorographical Uses*, &c. especially now there is a Method of observing Angles nicely Accurate by Aid of the Telescope.

*Three Chorographic Problems solved by a* Member *of the* Philosophical Society *at* Oxford. n. 177. p. 1231.

II. The three following *Problems* may occur at Sea, in finding the Distance and Position of Rocks, Sands, &c. from the Shore; or in Surveying the Sea-Coast; when only two Objects whose distance from each other is known, can be seen at one Station: but especially they may be useful to one, that would make a Map of a Country by a Series of Triangles derived from one or more measured Bases; which is the most exact way of finding the Bearing and Distance of places from each other, and thence their true Longitude and Latitude; and may consequently occur to one that would in that manner measure a degree on the Earth.

Fig. 63. Prob. 1.] *There are two Objects B and C, whose Distance B C is known, and there are two Stations at A and E, where the Objects B, C, being visible, and the Stations one from another, the Angles BAC, BAE, AEB, AEC are known by Observation,* (which may be made with an Ordinary *Surveying Semicircle*, or *Cross-staff*, or if the Objects be beyond the view of the naked Eye, with a *Telescopic Quadrant) to find the Distances or Lines AB, AC, AE, EC.*

Fig. 64. *Construction.*] In each of the Triangles, BAE, CAE, two Angles at A, E, being known, the third is also known; then take any Line $\alpha\epsilon$ at pleasure, on which constitute the Triangles, $\beta\alpha\epsilon$, $\alpha\epsilon\gamma$, respectively equiangular to the Triangles BAE, AEC; joyn $\beta\gamma$. Then upon BC constitute the Triangles BCA, BCE, equiangular to the Correspondent Triangles $\beta\gamma\alpha$, $\beta\gamma\epsilon$; joyn AE, and the thing is manifestly done.

*The Calculation.*] Assuming $\alpha\epsilon$, of any number of parts, in the Triangles $\alpha\beta\epsilon$, $\alpha\gamma\epsilon$, the Angles being given, the Sides $\alpha\beta$, $\alpha\gamma$, $\epsilon\beta$, $\epsilon\gamma$, may be found by Trigonometry: Then in the Triangle $\beta\alpha\gamma$, having the Angle $\beta\alpha\gamma$, and the Legs $\alpha\beta$, $\alpha\gamma$, we may find $\beta\gamma$. Then $\beta\gamma$ : BC :: $\beta\alpha$ : BA :: $\beta\epsilon$ : BE :: $\gamma\alpha$ : CA :: $\gamma\epsilon$ : CE.

Fig. 65. Prob. 2.] *Three Objects B, C, D, are given, or (which is the same) the Sides and consequently Angles, of the Triangle BCD, are given; also there are two points or Stations A, E, such that at A may be seen the three points B, C, E, but not D, and at the Station E, may be seen A, C, D, but not B; that is the Angles BAC, BAE, AEC, AED, (and consequently EAC, AEC) are known by Observation: To find the Lines AB, AC, AE, EC, ED.*

Fig. 66. *Construction.*] Take any Line $\alpha\epsilon$ at pleasure, and at its extremities make the Angles $\epsilon\alpha\gamma$, $\epsilon\alpha\beta$, $\alpha\epsilon\gamma$, $\alpha\epsilon\delta$, equal to the correspondent Observed Angles EAC, EAB, AEC, AED. Produce $\beta\alpha$, $\delta\epsilon$, till they meet in $\phi$; joyn $\phi\gamma$; then upon CB describe (according to 33. 3. E.) a Segment of a Circle, that may contain an Angle $= \gamma\phi\beta$; and upon CD describe a Segment of a Circle capable of an Angle $= \gamma\phi\delta$: Suppose F the common Section of these 2 Circles; joyn FB, FC, FD; then from the point C, draw forth the Lines CA, CE, so that the Angle FCA may be, $= \phi\gamma\alpha$, and FCE $= \phi\gamma\epsilon$; so A, E, the common Sections of CA, CE, with FB, FD, will be the points required, from whence the rest is easily deduced.

*Calculation*

*Calculation.*] Aſſuming $\varphi\epsilon$ of any number, in the Triangles $\alpha\gamma$, $\alpha\epsilon\varphi\epsilon$, all the Angles being given, with the ſide $\alpha\epsilon$ aſſum'd, the ſides $\alpha\gamma, \epsilon\gamma, \alpha\varphi, \epsilon\varphi$, will be known; then in the Triangle $\gamma\alpha\varphi$, the Angle $\gamma\alpha\varphi$, with the Legs $\alpha\gamma$, $\alpha\varphi$, being known, the Angles $\alpha\varphi\gamma, \alpha\gamma\varphi$, with the ſide $\varphi\gamma$, will be known: Then as for the reſt of the work, the Triangle B C D having all its ſides and Angles known, and the Angles B F C, B F D, being equal to the found $\beta\varphi\gamma$, $\beta\epsilon\delta$; how to find F B, F C, F D, by *Calculation* (and alſo *Protraction*) has been already ſhewn above by Mr. *Collins*, as to all its *Caſes*. §. 1.

But here it muſt be noted, that if the Sum of the obſerved Angles, B A E, A E D, is 180 degrees: then A B, and E D, cannot meet, becauſe they are Parallel, and conſequently the given Solution cannot take place; for which reaſon I here ſubjoyn another.

*Another Solution.*] Upon B C deſcribe a Segment, B A C, of a Circle, ſo that the Angle of the Segment may be equal to the obſerved Angle $\beta\alpha\gamma$, (which is ſhewn 33. 3. E.) and upon C D deſcribe a Segment C E D, of a Circle, capable of an Angle equal to the obſerved C E D; from C draw the Diameters of theſe Circles C G, C H; then upon C G deſcribe a Segment of a Circle G F C, capable of an Angle equal to the Obſerved Angle A E C; likewiſe upon C H, deſcribe a Circles Segment C F H, capable of an Angle equal to the Obſerved Angle C A E: ſuppoſe F the Common Section of the two laſt Circles H F C, G F C; joyn F H, cutting the Circle H E C in E; joyn alſo F G, cutting the Circle G A C in A: I ſay, that A, E, are the points required. Fig 67.

*Demonſtration.*] For the Angle B A C is = $\beta\alpha\gamma$, by *Conſtruction* of the Segment, alſo the Angles C E H, C A G, are Right, becauſe each exiſts in a Semicircle: therefore a Circle being deſcribed upon C F, as a Diameter, will paſs thro' E, A, therefore the Angle C A E = C F E = C F H = (by *Conſtruction*) to the obſerved Angle $\gamma\alpha\epsilon$. In like manner the Angle C E A = C F A = C F G = obſerv'd Angle $\gamma\epsilon\alpha$.

If the Stations A, E, fall in a Right Line with the point C; the Lines G A, H E, being Parallel, cannot meet: but in this *Caſe* the Problem is indeterminate, and capable of Infinite Solutions. For, as before, upon C G, deſcribe a Segment of a Circle capable of the obſerved Angle $\gamma\epsilon\alpha$, and upon C H, deſcribe a Segment capable of the obſerved Angle $\gamma\alpha\epsilon$: then through C, draw a Line any way cutting the Circles in A, E, theſe points will anſwer the Queſtion.

Problem. 3.] *Four points, B, C, D, F, or the four ſides of a Quadrilateral, with the Angles comprehended, are given; alſo there are two Stations A and E, ſuch, that at A, only B, C, E, are viſible, and at E, only A, D, F; that is, the Angles B A C, B A E, A E D, D E F, are given: to find the Places of the two Points A, E; and conſequently the Lengths of the Lines A B, A C, A E, E D, E F.* Fig 68.

*Conſtruction.*] Upon B C (by 33. 3. E.) deſcribe a Segment of a Circle, that may contain an Angle equal to the obſerved Angle B A C, then from C, draw the Chord C M, or a Line cutting the Circle in M, ſo that the Angle B C M, may be equal to the Supplement of the obſerved Angle B A E, *i. e.* its Reſidue

 to

to 180 degrees. In like manner on D F deſcribe a Segment of a Circle, capable of an Angle equal to the obſerved D E F, and from D draw the Chord D N, ſo that the Angle F D N, may be equal to the Supplement of the obſerved Angle A E F; joyn M N, cutting the two Circles in A, E: I ſay, A, E, are the two Points required.

*Demonſtration.*] Joyn A B, A C, E D, E F, then is the Angle M A B = B C M, (by 21. 3. E.) = Supplement of the obſerved Angle B A E, by *Conſtruction*; therefore the Conſtructed Angle B A E, is equal to that which was Obſerv'd. Alſo the Angle B A C, of the Segment, is, by Conſtruction of the Segment, equal to the obſerved Angle B A C. In like manner the Conſtructed Angles A E F, and D E F, are equal to the Correſpondent obſerved Angles A E F, D E F; therefore A, E, are the Points required.

*Calculation.*] In the Triangle B C M, the Angle B C M, (= Supplement of B A E) and Angle B M C, (= B A C) are given, with the ſide B C; thence M C may be found; in like manner D N, in the Triangle D N F, may be found. But the Angle M C D (= B C D — B C M) is known, with its Legs M C, C D, therefore its Baſe M D, and Angle M D C, may be known. Therefore the Angle M D N (= C D F — C D M — F D N) is known, with its Legs M D, D N; thence M N, with the Angles D M N, D N M, will be known. Then the Angle C M A (= D M C + D M N) is known, with the Angle M A C, (= M A B + B A C) and M C, before found; therefore M A, and A C, will be known. In like manner in the Triangle E D N, the Angles E, N, with the ſide D N, being known, the ſides E N, E D, will be known; therefore A E (= M N — M A — E N) is known. Alſo in the Triangle A B C, the Angle A, with its ſides B C, C A, being known, the ſide A B will be known, with the Angle B C A; ſo in the Triangle E F D, the Angle E, with the ſides E D, D F, being known, E F will be found, with the Angle E D F. Laſtly, in the Triangle A C D, the Angle A C D, (= B C D — B C A) with its Legs A C, C D, being known, the ſide A D will be known; and in like manner E C, in the Triangle E D C.

*Note*, That in this Problem, as alſo in the firſt and ſecond, if the two Stations fall in a Right Line with either of the given Objects: the *Locus* of A, or E, being a Circle, the particular point of A, or E, cannot be determined from the things given.

As to the other *Caſes* of this third *Problem*, wherein A, and E, may ſhift places, *i. e.* only D, F, E, may be viſible at A, and only A, B, C, at E; or wherein B, D, E, may be viſible at A, and only C, F, A, at E; or wherein A may be on one ſide of the Quadrilateral, and E on the other; or one of the Stations within the Quadrilateral, and the other without it; I preſume that the Surveyor will eaſily direct himſelf, by what has been already ſaid.

The *Solution* of this third *Problem* is General, and ſerves alſo for both the precedent. For ſuppoſe C, D, the ſame point in the laſt figure, and it gives the *Solution* of the ſecond Problem: but if B, C, be ſuppoſed the ſame points with D, F, by proceeding as in the laſt, you may directly ſolve the firſt Problem.

III. The

III. The Variation of the Magnetick Needle is ſo commonly known, that I need not inſiſt much on the Explication thereof; 'tis certain that the true Solar Meridian, and the Meridian ſhewn by a Needle, agree but in very few places of the World; and this too, but for a little time (if a moment) together; the difference between the true Meridian and Magnetick Meridian, perpetually varying and changing in all places, and at all times; ſometimes to the Eaſtward, and ſometimes to the Weſtward.

*An Error of Common Surveyors, in comparing Surveys taken at Long Intervals of time with the Magnetick Nedle, Demonſtrated; by Mr.* William Molyneux. n. 230. *p.* 625.

On which Account 'tis impoſſible to compare two Surveys of the ſame place, taken at diſtant times, by Magnetick Inſtruments (ſuch as the Circumferentor, by which the *Down Survey*, or Sir *William Petty's Survey* of *Ireland* was taken) without due Allowance be made for this Variation. To which purpoſe, we ought to know the difference between the Magnetick Meridian and true Meridian, at that time of the *Down Survey*, and the ſaid Difference at the time, when we make a *New Survey* to compare with the *Down Survey*.

But here I would not be underſtood, as if I propoſed hereby to ſhew, that a Map of the ſame place, taken by Magnetick Inſtruments at never ſo diſtant times, ſhould not at one time give the ſame Figure and Contents as at another time. This certainly it will do moſt exactly, the Variation of the Needle having nothing to do either in the Shape or Contents of the Survey. All that is affected thereby, is the Bearings of the Lines run by the Chain, and the Boundaries between Neighbours. And how this may cauſe a conſiderable Error (unleſs due Allowance be made for it) is what I ſhall prove moſt fully.

In order to which, let us ſuppoſe that about the Year 1657, (at which time the *Down Survey* was taken) the Magnetick Meridian and true Meridian did agree at *Dublin*, or pretty nigh all over *Ireland*; that is to ſay, that there was no Variation. And indeed by Experiment it was at that time found, as I am well aſſured, that at *Dublin* it was hardly half a degree.

Let us ſuppoſe that in the Year 1695, the Variation was 7 Degrees from the North to the Weſtward: that it was really ſo, I believe I am pretty well aſſured, from an experiment thereof made by *my ſelf* with all diligence. But this is not material, let us now only ſuppoſe it.

Let A, B, Repreſent the Survey of two Town Lands, one in the Poſſeſſion of A, and the other in the poſſeſſion of B, taken by the *Down Survey*, *Anno* 1657, when there was no variation. *Fig* 69.

Let the Line N S, running through the Point P, be the true Meridian, and conſequently the Magnetick Meridian alſo at that time, becauſe of the ſuppoſed no Variation, and let this Line N S, be alſo the Boundary between the two Town Lands A, and B.

In the Year 1695, when the Variation is 7 Degrees from the North to the Weſtward, B having a Map of the *Down Survey*, and being ſuſpicious that his Neighbour A, had incroached on him by a Ditch P Q, imploys a Surveyor to enquire into the matter: The Surveyor finds by this Map, that the Boundary between B and his Neighbour A, run from the point P, through a Meadow, directly according to the Magnetick Meridian S P N; but obſerving the Ditch P Q caſt up much to the Eaſtward of the preſent Magnetick Meridian, he concludes that A has incroached upon B, and that the Ditch ought to have

been

been cast up along the Line P *q*, the Angle Q P *q*, being an Angle of 7 Degrees, that is, the present Variation of the Needle, and the Line P *q*, the present Magnetick Meridian: for which Variation not making any Allowance, he positively determines, that B has all the Land in the Triangle Q P *q*, more than he ought to have; and that his Ditch ought to run along the Line P *q*.

'Tis true indeed, if the *Surveyor* go the whole Surround of the Lands A, and B, he will find their figure and Contents exactly agreeable to the Map here expressed. But then the Bearings of the Lines are all 7 Degrees different from the Bearings in the Map, and they will run in and out upon the adjacent Neighbouring Lands, and cause endless differences between their Possessors; as is manifest from the Figure: Wherein the prickt Lines represent the Disagreement in the Bearings of the Lines, protracted from the point P; and we see A incroaching on his Neighbours on the Westward, as he incroaches upon B, and B's Eastward Neighbours incroaching on him, and so forward and clear round. Whereas by a due allowance for the Variation of the Needle, all this Confusion and Disagreement is avoided, and every thing hits right.

Thus, for Instance, in the Case before us, knowing that the Magnetick Variation has caused the present Magnetick Meridian to fall in the Line *n q* P *s*, 7 Degrees from the North to the Westward; to reduce this to the Magnetick Meridian at the time of the *Down Survey*, I must make the Meridian of my Map, to fall 7 Degrees to the Eastward of my Magnetick Meridian; as we see the Line P Q, falls 7 Degrees to the Eastward of the Line P *q*.

What is here said on supposition that the Magnet had no Variation at the time of the first Survey taken, and that it had 7 Degrees Variation Westward at the time of the second Survey, may easily be accommodated to the Supposal of any other Variations at the first and second Surveys, *Mutatis Mutandis*, for knowing the Variations we know their Difference; and if we know their Difference, this gives us the Angle Q P *q*, by which we reduce them to each other. The best way therefore to make Maps invariable, constant, and everlasting, were for the Surveyors, who use Magnetick Instruments, to make always Allowance for the Magnetick Variation, and to protract and lay down their Plats by the true Meridian.

Perhaps it may be objected, That Surveys may be taken without Magnetick Instruments, and that therefore this Error arising from the Magnetick Variation, and Change of the Bearing of Lines, may be avoided. To which I answer, first, That granting a Survey may be taken without Magnetick Instruments, this is nothing against what we have laid down, relating to Surveys that are taken with Magnetick Instruments, as the *Down Survey* actually was, and most *Surveys* at present are actually taken therewith. Secondly, though a Survey may be taken truly without Magnetick Instruments, so as to shew the exact Angles and Lines of the Plat, and consequently the true contents, yet this will not give the true Bearings of the Lines, or shew my Position in relation to my Neighbours, or other parts of the Country. This must be supplyed by the Magnet, or something equivalent thereto, as finding a true Meridian Line on your Land by Celestial Observation. And I doubt not but the Ancient *Ægyptians*, before the Discovery of the Magnet, were forced to some such Expedient in their Surveys, and

and Applotments of Lands, between Neighbour and Neighbour, after the *Inundations* of the *Nile*, which, we are told, gave the first Original to *Geometry* and *Surveying*; Absolute Necessity and Use having introduced these, as Delight and Diversion introduced Astronomy amongst the *Chaldeans*.

And this brings me to another Objection, which may be made against the Instance before laid down: It may be said, That certainly the Surveyor which B imployed was very Ignorant, who would choose to judge of the Line P Q rather by its bearing than by determining the point Q, by measuring from H and G. To this I answer, What if both the points H, and G, were vanished since the *Down Survey* was taken? What if the whole Face of the Country were changed, save only the Point P, and the Line P Q? How shall the Surveyor then judge of the Line P Q, but by its Bearing? That this is no extravagant Supposition, we have an Example in *Ægypt* above-mentioned, where the *Nile* lays all flat before it, and so uniformly covers all with Mud, that there is no distinction. In such a Case your Bearing must certainly help you out, there is no other way.

But I answer, secondly, to say that the Surveyor might have determined the Point Q by Admeasurement from G and H, or any other adjoyning noted Points, as from F, K, I, &c. 'tis very true; but then 'tis against our *Supposition*. I am upon shewing an Error that arises from judging of the Line P Q by *Magnetick Bearing*, and to tell me that this might be avoided by another way, is to say nothing. I my self shew how it may be avoided, by allowing for the Variation; but still it is an Error till it be avoided.

But, thirdly, if B's Surveyor do not allow for the Variation of the Needle, he will never exactly determine even the Points G, F, H, K, &c. or any other Points in the Plat, but instead thereof will fall on the Points *g*, *h*, *f*, *k*.

From what has been laid down, we may see the absolute necessity of allowing for the Variation of the Magnet, in comparing old Surveys with new ones; for want of which great Disputes may arise between Neighbouring Proprietors of Lands: And it were to be wished, that our Honourable and Learned Judges would take this Matter into their Consideration, whenever any business of this kind comes before them.

IV. I have invented a *Levell* with a Tube, with Glasses and a Thread, hanging between four Points, with a Weight in a Box so contrived, that as soon as the Instrument is set down, you have your *Point* of Horizon with a great deal of Exactness. I am making another which playeth on one steel Point, standing on a Diamond. *A new Level; by Mr. Butterfield. n. 141. p. 1026.*

V. *An Account of a Book Omitted; viz.* The Art of Levelling by *M. Mariotte*. *n. 74. p.* 2217.

CHAP. III.

# CHAP. III.

## *OPTICKS.*

*A New Theory about Light and Colours; by Mr. If. Newton.* n. 80. p. 3075.

I. IN the Year 1666 (at which time I applyed my ſelf to the Grinding of Optick Glaſſes of other figures than Spherical,) I procured me a Triangular Glaſs Priſm, to try therewith the Celebrated Phænomena of Colours. And in order thereto, having darkned my Chamber, and made a ſmall Hole in my window-ſhuts, to let in a convenient quantity of the Sun's Light, I placed my Priſm at it's entrance, that it might be thereby Refracted to the oppoſite Wall. It was at firſt a very pleaſing Divertiſement, to view the Vivid and Intenſe Colours produced thereby; but after a while applying my ſelf to conſider them more circumſpectly, I became ſurpriſed, to ſee them in an Oblong Form; which, according to the received Laws of Refractions, I expected ſhould have been Circular. They were terminated at the ſides with ſtreight Lines, but at the Ends, the decay of Light was ſo gradual, that it was difficult to dedetermine juſtly, what was their Figure; yet they ſeemed Semicircular.

Comparing the Length of this Colour'd *Spectrum* with its Breadth, I found it about five times greater, a Diſproportion ſo extravagant, that it excited me to a more than ordinary Curioſity of examining, from whence it might proceed. I could ſcarce think, that the various thickneſs of the Glaſs, or the Termination with Shadow or Darkneſs, could have any influence on Light to produce ſuch an Effect; yet I thought it not amiſs, firſt to examine thoſe Circumſtances, and ſo try'd, what would happen by tranſmitting Light through parts of the Glaſs of divers Thickneſſes, or through Holes in the Window of divers Bigneſſes, or by ſetting the Priſm without, ſo that the Light might paſs through it, and be Refracted, before it was terminated by the Hole: But I found none of thoſe circumſtances material. The faſhion of the Colours was in all theſe Caſes the ſame.

Then I ſuſpected, whether by any Unevenneſs in the Glaſs, or other contingent Irregularity, theſe Colours might be thus dilated. And to try this, I took another Priſm like the former, and ſo placed it, that the Light paſſing thro' them both might be Refracted contrary ways, and ſo by the latter returned into that courſe from which the former had diverted it. For by this means I thought, the Regular Effects of the firſt Priſm would be deſtroyed by the ſecond Priſm, but the Irregular ones more augmented, by the Multiplicity of Refractions. The event was, that the Light, which by the firſt Priſm was diffuſed into an Oblong Form, was by the ſecond reduced into an Orbicular one, with as much Regularity as when it did not at all paſs through them.

n. 83. p. 4061. Fig. 70. That this Experiment may be better apprehended, let E G, deſign the Window; F, the Hole in it, through which the Light arrives at the Priſms; A B C, the firſt Priſm, which Refracts the Light towards P T, painting there the Colour in an Oblong and $\alpha\beta\gamma$, the ſecond Priſm, which Refracts back again the Rays to Q, where the long Image PT is contracted into a Round one. I ſuppoſe the Plane $\alpha\gamma$ Parallel to BC, and $\beta\gamma$ to AC, that the Rays may be equally Refracted contrary ways in both Priſms. The Priſms alſo muſt be placed very near to one another; for if their Diſtance be ſo great, that Colours begin to appear in the Light, before its Incidence on the ſecond Priſm, thoſe Colours will not be deſtroyed by the contrary Refractions of that Priſm. And if a *Lens* be placed in the hole F, or immediately after the Priſms, ſo that its *Focus* be at the Image Q, or PT, the Perimeter of the Image Q, and the ſtraight ſides of the Image PT, will become much better defined than otherwiſe. So that, whatever was the cauſe of that Length, 'twas not any Contingent Irregularity. n. 80. p. 3076.

I then proceeded to examine more critically, what might be effected by the difference of the Incidence of Rays coming from divers parts of the Sun; and to that end, meaſured the ſeveral Lines and Angles, belonging to the Image. Its diſtance from the Hole or Priſm was 22 foot, its utmoſt Length $13\frac{1}{4}$ inches; it's Breadth $2\frac{5}{8}$; the Diameter of the Hole $\frac{1}{4}$ of an inch; the Angle which the Rays, tending towards the middle of the Image, made with thoſe Lines, in which they would have proceeded without Refraction, was 44 deg. 56 min. And the Vertical Angle of the Priſm, 63 deg. 12 min. Alſo the Refractions on both ſides the Priſm, that is of the Incident and Emergent Rays, were, as near as I could make them, Equal, and conſequently about 54 deg. 4 min. And the Rays fell perpendicularly upon the Wall. Now ſubducting the Diameter of the Hole from the Length and Breadth of the Image, there remains 13 inches in the Length, and $2\frac{3}{8}$ the Breadth, comprehended by thoſe Rays, which paſſed through the Center of the ſaid Hole, and conſequently the Angle of the Hole, which that Breadth ſubtended, was about 31 min. anſwerable to the Sun's Diameter; but the Angle which its Length ſubtended, was more than 5 ſuch Diameters, namely 2 deg. 49 min.

Having made theſe Obſervations, I firſt computed from them the Refractive Power of that Glaſs, and found it meaſured by the Ratio of the Sines, 20 to 31; And then by that Ratio, I computed the Refractions of two Rays flowing from oppoſite parts of the Sun's *Diſcus*, ſo as to differ 31 min. in their Obliquity of Incidence, and found, that the Emergent Rays ſhould have comprehended an Angle of about 31 min. as they did before they were Incident.

But becauſe this Computation was founded on the Hypotheſis of the Proportionality of the Sines of Incidence and Refraction, which though by my own Experience I could not imagine to be ſo erroneous, as to make that Angle but 31 min. which in reality was 2 deg. 49 min. yet my Curioſity cauſed me again to take my Priſm. And having placed it at my Window, as before, I obſerved, that by turning it a little about its Axis to and fro, ſo as to vary its obliquity to the Light, more than an Angle of 4 or 5 degrees, the Colours were not thereby ſenſibly Tranſlated from their place on the Wall, and conſequently by that Variation of Incidence, the quantity of Refraction was not ſenſibly varied. By this Experiment therefore, as well as by the former

Computation, it was evident, that the difference of the Incidence of Rays, flowing from divers parts of the Sun, could not make them after Decuſſation Diverge at a ſenſibly Greater Angle, than that at which they before Converged; which being, at moſt, but about 31, or 32 min. there ſtill remained ſome other Cauſe to be found out, from whence it could be 2 deg. 49 min.

Then I began to ſuſpect, whether the Rayes, after their Trajection through the Priſm, did not move in Curve Lines, and according to their more or leſs Curvity tend to divers parts of the Wall. And it increaſed my ſuſpicion, when I remembred that I had often ſeen a Tennis-ball, ſtruck with an Oblique Racket, deſcribe ſuch a Curve Line. For, a Circular as well as a Progreſſive Motion being communicated to it, by that Stroke its parts on that ſide, where the motions conſpire, muſt preſs and beat the contiguous Air more violently than on the other, and there excite a Reluctancy and Reaction of the Air proportionably greater. And for the ſame Reaſon, if the Rays of Light ſhould poſſibly be Globular Bodies, and by their Oblique paſſage, out of one Medium into another, acquire a Circulating Motion, they ought to feel the greater reſiſtance from the ambient Æther, on that ſide, where the motions conſpire, and thence be continually bowed to the other. But notwithſtanding this plauſible ground of ſuſpicion, when I came to examine it, I could obſerve no ſuch Curvity in them. And beſides (which was enough for my purpoſe) I obſerved, that the Difference 'twixt the Length of the Image, and the Diameter of the Hole, through which the Light was tranſmitted, was proportionable to to their Diſtance.

The gradual removal of theſe ſuſpicions at Length led me to the *Experimentum Crucis*, which was this: I took two boards, and placed one of them cloſe *behind the Priſm at the Window*, ſo that the Light might paſs through a ſmall Hole, made in it for the purpoſe, and fall on the other board, which I placed at about 12 feet diſtance, having firſt made a ſmall hole in it alſo, for ſome of that Incident Light to paſs through. Then I placed another Priſm behind this ſecond board, ſo that the Light Trajected through both the boards, might paſs through that alſo, and be again Refracted before it arrived at the Wall. This done, I took the firſt Priſm in my hand, and turned it to and fro ſlowly about its Axis, ſo much as to make the ſeveral parts of the Image, caſt on the ſecond board, ſucceſſively paſs through the Hole in it, that I might obſerve to what places on the Wall the ſecond Priſm would Refract them. And I ſaw by the Variation of thoſe places, that the Light, tending to that end of the Image, towards which the Refraction of the firſt Priſm was made, did in the ſecond Priſm ſuffer a Refraction conſiderably greater than the Light tending to the other end. And ſo the true cauſe of the Length of that Image was detected to be no other, than that *Light* is not ſimilar or Homogenial, but Conſiſts of *Difform Rays, ſome of which are more Refrangible than others*; ſo that without any Difference in their Incidence on the ſame Medium, ſome ſhall be more *Refracted* than others: and therefore that, according to their *particular Degrees of Refrangibility*, they were tranſmitted through the Priſm to divers parts of the oppoſite Wall.

*Ibid. p.* 3081.

I ſhall

I ſhall now proceed to acquaint you with another more Notable *Difformity* in its Rays, wherein the Origin of Colours is unfolded: concerning which I ſhall lay down the Doctrine firſt, and then for its Examination give you an Inſtance or two of the Experiments, as a Specimen of the reſt.

The Doctrine you will find comprehended and illuſtrated in the following Propoſitions.

1. As the Rays of Light differ in Degrees of Refrangibility, ſo they alſo differ in their diſpoſition to exhibit this or that particular Colour. Colours are not Qualifications of Light, deriv'd from Refractions, or Reflections of Natural Bodies (as 'tis generally believed) but Original and Connate Properties, which in divers Rays are divers. Some Rays are diſpoſed to exhibit a Red Colour and no other; ſome a Yellow and no other, ſome a Green and no other, and ſo of the reſt. Nor are there only Rays proper and particular to the more Emiment Colours, but even to all their Intermediate gradations.

2. To the ſame degree of Refrangibility ever belongs the ſame Colour, and to the ſame Colour ever belongs the ſame degree of Refrangibility. The leaſt Refrangible Rays are all diſpoſed to exhibit a Red Colour, and contrarily thoſe Rays which are diſpoſed to exhibit a Red Colour are all the leaſt Refrangible: So the moſt Refrangible Rays are all diſpoſed to exhibit a deep Violet Colour, and contrarily thoſe which are apt to exhibit ſuch a Violet Colour are all the moſt Refrangible. And ſo to all the Intermediate Colours in a continued Series belong Intermediate Degrees of Refrangibility. And this Analogy 'twixt Colours and Refrangibility is very preciſe and ſtrict; the Rays always either exactly agreeing in both, or proportionally diſagreeing in both.

3. The Species of Colour, and Degree of Refrangibility proper to any particular ſort of Rays, is not Mutable by Refraction, nor by Reflection from Natural Bodies, nor by any other cauſe that I could yet obſerve. When any one ſort of Rays hath been well parted from thoſe of other kinds, it hath afterwards obſtinately retained its Colour, notwithſtanding my utmoſt endeavours to change it. I have Refracted it with Priſms, and Reflected it with Bodies, which in day light were of other Colours; I have intercepted it with the Coloured film of Air, interceeding two compreſſed Plates of Glaſs; tranſmitted it through Coloured Mediums, and thro' Mediums Irradiated with other ſorts of Rays, and diverſly terminated it; and yet could never produce any new Colour out of it. It would by contracting or dilating become more brisk, or faint, and by the loſs of many Rays, in ſome Caſes very obſcure and dark; but I could never ſee it chang'd in Specie.

4. Yet ſeeming tranſmutations of Colours may be made, where there is any mixture of divers ſorts of Rays. For in ſuch Mixtures, the component Colours appear not, but, by their mutual allaying each other, conſtitute a midling Colour. And therefore, if by Refraction, or any other of the aforeſaid Cauſes, the Difform Rays, latent in ſuch a mixture, be ſeparated, there ſhall Emerge Colours different from the Colour of the Compoſition. Which Colours are not New generated, but only made apparent by being parted; for

if they be again intirely mixt and blended together, they will again compoſe that Colour, which they did before Separation. And for the ſame reaſon, Tranſmutations made by the Convening of divers Colours are not real; for when the difform Rays are again ſevered, they will exhibit the very ſame Colours which they did before they enter'd the Compoſition; as you ſee Blue and Yellow Powders, when finely mix'd, appear to the naked Eye, Green, and yet the Colours of the component Corpuſcles are not thereby really Tranſmuted, but only blended. For when viewed with a good Microſcope, they ſtill appear Blue and Yellow interſperſedly.

5. There are therefore two ſorts of Colours. The one Original and Simple, the other Compounded of theſe. The Original or Primary Colours are, Red, Yellow, Green, Blue, and a Violet-Purple, together with Orange, Indico, and an indefinite variety of Intermediate gradations.

6. The ſame Colours in Specie with theſe Primary ones, may be alſo produced by Compoſition. For a mixture of Yellow and Blue makes Green; of Red and Yellow makes Orange; of Orange and Yellowiſh Green makes Yellow. And in general, if any two Colours be mixed, which in the Series of thoſe generated by the Priſm are not too far diſtant one from another, they by their mutual Alloy Compound that Colour, which in the ſaid Series appeareth in the midway between them. But thoſe which are ſituated at too great a diſtance, do not ſo. Orange and Indico produce not the intermediate Green, nor Scarlet and Green the Intermediate Yellow.

7. But the moſt ſurpriſing, and wonderful Compoſition was that of Whiteneſs. There is no one ſort of Rays which alone can exhibit this. 'Tis ever Compounded, and to its Compoſition, are requiſite all the aforeſaid Primary Colours, mix'd in a due Proportion. I have often with Admiration beheld, that all the Colours of the Priſm being made to Converge, and thereby to be again mixed, as they were in the Light before it was Incident upon the Priſm, reproduced Light, intirely and perfectly White, and not at all ſenſibly differing from a direct Light of the Sun, unleſs when the Glaſſes, I uſed, were not ſufficiently clear; for then they would a little incline it to their Colour.

8. Hence therefore it comes to paſs, that Whiteneſs is the uſual Colour of Light; for Light is a confuſed aggregate of Rays indued with all ſort of Colours, as they are promiſcuouſly darted from the various parts of Luminous Bodies. And of ſuch a confuſed aggregate, as I ſaid, is generattd Whiteneſs, if there be a due Proportion of the Ingredients; but if any one predominate, the Light muſt incline to that Colour; as it happens in the Blue Flame of Brimſtone; the Yellow Flame of a Candle; and the various Colours of the Fixed Stars.

9. Theſe things conſidered, the manner how Colours are produced by the Priſm is evident. For, of the Rays, conſtituting the incident Light, ſince thoſe which differ in Colour proportionally differ in Refrangibility, they by their unequal Refractions muſt be ſevered and diſperſed into an Oblong Form in an orderly ſucceſſion, from the leaſt Refracted Scarlet, to the moſt Refracted Violet. And for the ſame Reaſon it is, that Objects, when looked upon thro' a Priſm, appear Coloured. For the Difform Rays, by their unequal

Refractions, are made to Diverge towards ſeveral parts of the *Retina*, and there expreſs the Images of things Coloured, as in the former caſe they did the Sun's Image upon a Wall. And by this Inequality of Refractions, they become not only Coloured, but alſo very Confuſed and Indiſtinct.

10. Why the Colours of the Rainbow appear in falling drops of Rain, is alſo from hence evident. For thoſe drops which Refract the Rays, diſpoſed to appear Purple, in greateſt quantity to the Spectator's Eye, Refract the Rays of other ſorts ſo much leſs, as to make them paſs beſide it; and ſuch are the drops on the inſide of the Primary Bow, and on the outſide of the Secundary or Exterior one. So thoſe drops, which Refract in greateſt plenty the Rays, apt to appear Red, toward the Spectator's Eye, Refract thoſe of other ſorts ſo much more, as to make them paſs beſide it; and ſuch are the drops on the Exterior part of the Primary, and Interiour part of the Secondary Bow.

11. The odd Phænomena of an Infuſion of *Lignum Nephriticum*, Leaf-Gold, fragments of Coloured Glaſs, and ſome other tranſparently Coloured Bodies, appearing in one Poſition of one Colour, and of another in another, are on theſe grounds no longer Riddles. For thoſe are Subſtances apt to Reflect one ſort of Light, and tranſmit another; as may be ſeen in a dark Room, by Illuminating them with ſimilar or uncompounded Light. For then they appear of that Colour only, with which they are illuminated, but yet in one Poſition more Vivid and Luminous than in another, accordingly as they are diſpoſed more or leſs to Reflect or Tranſmit the incident Colour.

12. From hence alſo is manifeſt the reaſon of an unexpected Experiment, which Mr. *Hook*, ſomewhere in his *Micrography*, relates to have made with two Wedg-like Tranſparent Veſſels, filled the one with a Red, the other with a Blue Liquor: namely, that though they were ſeverally Tranſparent enough, yet both together became Opake; For if one tranſmitted only Red, and the other only Blue, no Rays could paſs thro' both.

13. I might add more Inſtances of this Nature, but I ſhall conclude with this General one, That the Colours of all Natural Bodies have no other Origin than this, that they are variouſly qualified, to Reflect one ſort of Light in greater plenty than another. And this I have Experimented in a dark Room, by illuminating thoſe Bodies with uncompounded Light of divers Colours. For by that means any Body may be made to appear of any Colour. They have there no appropriate Colour, but ever appear of the Colour of the Light caſt upon them, but yet with this difference, that they are moſt brisk and vivid in the Light of their own Day-light Colour. Minium appeareth there of any Colour indifferently, with which it is illuſtrated, but yet moſt Luminous in Red; and ſo Biſe appeareth indifferently of any Colour, with which it's illuſtrated, but yet moſt Luminous in Blue. And therefore Minium Reflecteth Rays of any Colour, but moſt copiouſly thoſe endowed with Red; and conſequently when illuſtrated with Day-light; that is, with all ſorts of Rays promiſcuouſly blended, thoſe qualified with Red ſhall abound moſt in the Reflected Light, and by their prevalence cauſe it to appear of that Colour. And for the ſame Reaſon Biſe, Reflecting Blue moſt copiouſly, ſhall appear Blue by the exceſs of thoſe Rays in its Reflected Light; and the like of other Bodies. And that this is the in-

tire

tire and Adequate cauſe of their Colours, is manifeſt, becauſe they have no Power to change or alter the Colours of any ſort of Rays incident apart, but put on all Colours indifferently, with which they are enlightened.

Theſe things being ſo, it can be no longer diſputed, whether there be Colours in the dark, nor whether they be the qualities of the Objects we ſee, no nor perhaps, whether Light be a Body. For, ſince Colours are the Qualities of Light, having its Rays for their intire and immediate Subject, how can we think thoſe Rays Qualities alſo, unleſs one Quality may be the Subject of, and ſuſtain another; which in effect is to call it Subſtance. We ſhould not know Bodies for Subſtances, were it not for their ſenſible Qualities, and the Principal of thoſe being now found due to ſomething elſe, we have as good Reaſon to believe that to be a Subſtance alſo.

Beſides, Who ever thought any Quality to be a Heterogeneous Aggregate, ſuch as Light is diſcovered to be? But to determine more abſolutely, what Light is, after what manner Refracted, and by what Modes or Actions it produceth in our Minds the Phantaſms of Colours, is not ſo eaſie: And I ſhall not mingle Conjectures with Certainties.

Reviewing what I have written, I ſee the diſcourſe it ſelf will lead to divers Experiments ſufficient for its Examination: And therefore I ſhall not trouble you further than to deſcribe one of thoſe, which I have already inſinuated.

In a darkened Room make a Hole in the Shut of a Window, whoſe Diameter may conveniently be about a third part of an Inch, to admit a convenient quantity of the Sun's Light: And there place a clear and colourleſs Priſm, to Refract the entring Light towards the further part of the Room, which, as I ſaid, will thereby be diffuſed into an Oblong Coloured Image. Then place a Lens of about 3 foot Radius (ſuppoſe a broad Object Glaſs of a three foot Teleſcope,) at the diſtance of about four or five foot from thence, through which all thoſe Colours may at once be tranſmitted, and made by its Refraction to convene at a further diſtance of about 10 or 12 feet. If at that diſtance you intercept this light with a ſheet of white Paper, you will ſee the Colours converted into Whiteneſs again by being mingled. But it is requiſite, that the Priſm and Lens be placed ſteddy, and that the Paper, on which the Colours are caſt, be moved to and fro; for by ſuch motion you will not only find, at what diſtance the Whiteneſs is moſt perfect, but alſo ſee, how the Colours gradually convene, and vaniſh into Whiteneſs, and afterwards having croſſed one another in that place where they compound Whiteneſs, are again diſſipated, and ſevered, and in an inverted order retain the ſame Colours, which they had before they entred the compoſition. You may alſo ſee, that if any of the Colours at the Lens be intercepted, the Whiteneſs will be changed into the other Colours. And therefore, that the compoſition of Whiteneſs be perfect, care muſt be taken, that none of the Colours fall beſides the Lens.

Fig. 71. Thus, in the deſign of this Experiment, A B C, expreſſeth the Priſm ſet endwiſe to ſight, cloſe by the Hole F, of the Window E G. Its Vertical Angle A C B, may conveniently be about 60 degrees: M N, deſigneth the Lens. Its breadth 2 ½ or three inches. S F, one of the Straight Lines, in which *Difform Rays* may be conceived to flow ſucceſſively from the Sun. F P, and F R, two of

of thoſe Rays unequally Refracted, which the Lens makes to Converge towards Q, and after Decuſſation to Diverge again. And H I, the Paper, at divers diſtances, on which the Colours are projected: Which in Q, conſtitute Whiteneſs, but are Red and Yellow in R, *r*, and $\varrho$, and Blue and Purple in P, *p*, and $\pi$.

If you proceed further to try the impoſſibility of Changing any Uncompounded Colour (which I have aſſerted in the third and thirteenth Propoſitions) 'tis requiſite that the Room be made very dark, leaſt any ſcattering light, mixing with the Colour, diſturb and allay it, and render it Compound, contrary to the deſign of the Experiment. 'Tis alſo requiſite, that there be a perfecter Separation of the Colours, than, after the manner above deſcribed, can be made by the Refraction of one ſingle Priſm; and how to make ſuch further Separations, will ſcarce be difficult to them, that conſider the diſcovered Laws of Refractions. But if tryal ſhall be made with Colours not throughly ſeparated, there muſt be allowed changes proportionable to the mixture. Thus if Compound Yellow Light fall upon Blue Biſe, the Biſe will not appear perfectly Yellow, but rather Green; becauſe there are in the Yellow mixture many Rays indued with Green, and Green being leſs remote from the uſual Blue Colour of Biſe than Yellow, is the more copiouſly reflected by it.

In like manner, if any one of the Priſmatick Colours, ſuppoſe Red, be intercepted, on deſign to try the aſſerted impoſſibility of reproducing that Colour out of the others which are pretermitted, 'tis neceſſary, either that the Colours be very well parted before the Red be intercepted, or that together with the Red the neighbouring Colours, into which any Red is ſecretly diſperſed, (that is, the Yellow, and perhaps Green too) be intercepted; or elſe, that allowance be made for the emerging of ſo much Red out of the Yellow-Green, as may poſſibly have been diffuſed, and ſcatteringly blended in thoſe Colours. And if theſe things be obſerved, the new production of Red, or any intercepted Colour, will be found impoſſible.

*Some Experiments propoſed in relation to this Theory.* n. 83. *p.* 4059.

II. 1. To contract the Beams of the Sun without the Hole of the Window, and to place the Priſm between the Focus of the Lens and the Hole.

2. To cover over both ends of the Priſm with Paper at ſeveral diſtances from the middle; or with moveable Rings, to ſee, how that will vary or divide the Length of the Figure.

3. To move the Priſm ſo, as the End may turn about, the Middle being Steddy.

4. To move the Priſm by ſhoving it, till firſt the one ſide, then the middle, then the other ſide paſs over the Hole, obſerving the ſame Paralleliſm.

*Obſervations on this Propoſal by Mr.* Newton. n. 83. *p.* 4060.

2. I ſuppoſe the deſign of the *Propoſer* of theſe Experiments is, to have their events expreſſed, with ſuch obſervations as may occur concerning them. Touching the *firſt*, I have obſerved, that the Solar Image falling on a Paper placed at the Focus of the Lens, was by the interpoſed Priſm drawn out in length proportional to the Priſm's Refraction or diſtance from that Focus. And the chief obſervable here, which I remember, was, that the Streight Edges of the Oblong Image were diſtincter than they would have been without the Lens.

Con-

Confidering that the Rays coming from the Planet *Venus*, are much lefs inclined one to another, than thofe, which come from the oppofite parts of the Sun's difque; I once tryed an Experiment or two with her Light. And to make it fufficiently ftrong, I found it neceffary to collect it firft by a broad Lens, and then interpofing a Prifm between the Lens and its Focus, at fuch diftance, that all the Light might pafs through the Prifm; I found the Focus, which before appeared like a Lucid point, to be drawn out into a Long fplendid Line by the Prifms Refraction.

Concerning the *fecond* Experiment, I have occafionally obferved, that by covering both ends of the Prifm with Paper at feveral diftances from the middle, the Breadth of the Solar Image will be increafed or diminifhed as much, as is the Aperture of the Prifm, without any Variation of the Length: Or, if the Aperture be augmented on all fides, the Image on all fides, will be fo much and no more augmented.

Of the *Third* Experiment I have occafion to fpeak in my anfwer to another Perfon; where you will find the Effects of two Prifms, in all crofs pofitions of one to another, defcribed. But if one Prifm alone be turned about, the Coloured Image will only be tranflated from place to place, defcribing a Circle, or fome other Conick Section on the Wall, on which it is projected, without fuffering any alteration in its fhape, unlefs fuch as may arife from the obliquity of the Wall, or cafual change of the Prifms obliquity to the Sun's Rays.

The effect of the *Fourth* Experiment I have already infinuated, telling you that Light paffing through parts of the Prifm of divers thickneffes, did ftill exhibit the fame Phænomena.

*The Genuine Method of examining this Theory; by Mr.* Newton. n. 85. p. 5004.

III. I cannot think it effectual for determining truth, to examine the feveral ways by which Phænomena may be explained, unlefs where there can be a perfect Enumeration of all thofe ways. You know, the proper Method for inquiring after the Properties of things, is to deduce them from Experiments. And I told you, that the *Theory* which I propounded, was evinced to me, not by inferring *'tis thus becaufe not otherwife*, that is, not by deducing it only from a confutation of contrary fuppofitions, but by deriving it from Experiments concluding pofitively and directly. The way therefore to examine it is, by confidering, whether the Experiments which I propound do prove thofe parts of the Theory, to which they are applyed; or by profecuting other Experiments which the Theory may fuggeft for its examination. And this I would have done in a due Method; the Laws of Refraction being throughly inquired into and determined, before the Nature of Colours be taken into confideration. It may not be amifs to proceed according to the Series of thefe Queries; which I could wifh were determined by the event of proper Experiments; declared by thofe that may have the Curiofity to examine them.

1. Whether *Rays*, that are alike Incident on the fame Medium, have *Unequal Refractions*; and how great are the *Inequalities* of their Refractions at any Incidence?

2. What

2. What is *the Law* according to which each Ray is *more or less Refracted*; whether it be that the ſame Ray is ever Refracted according to the *ſame Ratio* of the Sines of Incidence and Refraction; and divers Rays, according to divers Ratio's; or that the Refraction of each Ray is greater or leſs without any certain Rule? That is, whether each Ray have a certain degree of Refrangibility, according to which its Refraction is performed; or is Refracted without that Regularity?

3. Whether *Rays*, which are endued with particular degrees of *Refrangibility*, when they are by any means ſeparated, have particular *Colours conſtantly belonging to them*; *viz.* the leaſt Refrangible, Scarlet; the moſt Refrangible, deep Violet; the middle, Sea-Green; and others, other Colours? And on the contrary?

4. Whether *the Colour* of any ſort of *Rays apart* may be changed by *Refraction?*

5. Whether *Colours* by coaleſcing do really *Change* one another to produce a *New Colour*, or produce it by *Mixing* only?

6. Whether *a due Mixture of Rays*, indued with all variety of Colours, produces *Light* perfectly *like that of the Sun*, and which hath all the ſame properties, and exhibits the ſame Phænomena?

7. Whether the *Component Colours* of each mixture be really *Changed*; or be only *Separated*, when from that Mixture various Colours are produced again by Refraction?

8. Whether there be any *other Colours* produced by *Refraction* than ſuch, as ought to reſult from the *Colours* belonging to the *diverſly Refrangible Rays*, by their being ſeparated or mixed by *that Refraction?*

To determine by Experiments theſe and ſuch like Queries, which involve the propounded Theory, ſeems the moſt proper and direct way to a Concluſion. And therefore I could wiſh all Objections were ſuſpended, taken from Hypotheſes or any other heads than theſe two; Of ſhewing the Inſufficiency of Experiments to determine theſe Queries, or prove any other parts of my Theory, by aſſigning the flaws and defects in my Concluſions drawn from them; or of producing other Experiments which directly contradict me, if any ſuch may ſeem to occur. For if the Experiments, which I urge, be Defective, it cannot be difficult to ſhew the Defects; but if Valid, then by proving the Theory they muſt render all Objections Invalid.

IV. 1. Iſtæc tam extraordinaria Hypotheſis, quæ Dioptricæ fundamenta evertit, praxeſque hactenus inſtitutas inutiles reddit, tota nititur illo Experimento Priſmatis Cryſtallini, ubi Radij per foramen feneſtræ intra obſcurum Cubiculum ingreſſi, ac deinde in Parietem impacti, aut in charta recepti, non in rotundum conformati, ut *Cl. Newtono*, ad Regulas Refractionum receptas attendenti, expectandum videbatur, ſed in Oblongam figuram extenſi apparuerunt: Unde concluſit, Oblongam ejuſmodi figuram ex eo eſſe, quod nonnulli Radij minus, nonnulli magis Refringerentur.

*Animadverſions upon this Theory; by R. P. Ign. Gaſton Pardies. n. 84. p. 4087.*

Sed mihi quidem videtur juxta communes & receptas Dioptricæ Leges figuram illam, non Rotundam, ſed Oblongam eſſe oportere. Cùm enim Radii ex oppositis disci Solaris partibus procedentes, variam habeant in ipso transitu Prismatis Inclinationem, variè quoque Refringi debent; ut cùm unorum Inclinatio 30 saltem minutis major sit Inclinatione aliorum, major quoque evadat illorum Refractio.

Igitur Radij oppositi, ex altera superficie Prismatis Emergentes magis Divergunt & Divaricantur, quàm si nullatenus, aut saltem æqualiter, omnes Infracti processissent. Refractio autem ista Radiorum fit solummodo versùs eas partes quæ fingi possunt in planis ad Axem Prismatis rectis; nulla autem Refractionis inæqualitas contingit versùs eas partes, quæ intelliguntur in planis Axi parallelis; ut facile demonstrari potest: Superficies enim duæ Prismatis censeri possunt inter se Parallelæ, ratione habita ad Inclinationem Axis, cùm singulæ ipsi Axi Parallelæ sint. Refractio autem per duas Parallelas planas superficies nulla computatur, quia quantum à prima superficie Radius in unam partem torquetur, tantum ab altera in oppositam partem detorquetur. Igitur cùm Radij Solares è foramine per Prisma transmissi ad latera quidem non frangantur, procedunt ulterius, perinde ac si nulla Prismatis superficies obstitisset, (habitâ, inquam, ratione solùm ad lateralem illam Divaricationem;) at verò cùm ijdem Radij ad superiores seu inferiores partes, alij quidem magis, alij verò minus, utpote inæqualiter Inclinati, Infringantur; necesse est eos magis inter se Divaricari, adeóque & in Longiorem Figuram extendi.

Quin si Calculus ritè obeatur; ut Radij laterales inventi sunt à *Cl. Newtono* in ea latitutudine quæ subtendit Arcum 31 min. qui Arcus respondet Diametro Solis; ita nullus dubito, quin illa inventa quoque Altitudo Imaginis quæ 2 gr. & 49 min. subtendit, sit illa ipsa quæ eidem Diametro Solis post inæquales Refractiones in illo ipso casu respondeat.

Fig. 72. Et revera, posito Prismate ABC, cujus Angulus A sit 60 gr. Radio DE, qui faciat cum perpendiculari EH, Angulum 30 grad. invenio illum, dum Emergit per FG, facere cum perpendiculari FI Angulum 76 gr. 22 min. At verò posito alio Radio *d*E, qui cum perpendiculari EH, faciat Angulum 29 gr. 30 min. invenio illum, dum Emergit per *fg*, facere cum perpendiculari *fi*, Angulum 78 gr. 45 min. Unde isti duo Radij DE, *d*E, qui procedere supponuntur ex oppositis partibus disci Solaris, faciuntque inter se Angulum 30 min. ijdem dum Emergunt per Lineas F*g*, *fg*, ita Divergunt ut constituant Angulum inter se 2 gr. 23 min. Quod si duo alij Radij assumerentur magis accedentes ad perpendicularem EH, (*v. g.* qui cum eadem perpendiculari facerent, unus quidam Angulum, 29 gr. 30 min. alter verò, 29 gr. 0 min.) tunc ijdem Radij emergentes magis adhuc Divergerent, constituerentque Angulum majorem etiam aliquando plus quàm trium Graduum. Et prætereа augetur ulterius ista intercapedo Refractorum Radiorum ex eo, quòd duo Radij DE, *d*E, concurrentes in E, illico incipiunt Divaricari, atque impingunt in duo puncta disjuncta alterius superficiei, nempe in F & in *f*. Quapropter non sufficit ad obeundum ritè Calculum, ex Longitudine Imaginis impactæ in chartam subtrahere magnitudinem Foraminis Fenestræ; quandoquidem etiam posito Foramine Indivisibili E, adhuc fieret aliud veluti Foramen Latum in alia superficie, nempe F*f*.

Quod

Quod etiam vocat *Experimentum Crucis*, mihi quidem videtur quadrare cum vulgaribus & receptis Refractionum Regulis. Nam, ut modo ostendi, Radii Solares qui accedentes & Convergentes faciunt Angulum 30 min. Egredientes deinde etiam post Indivisibile Foramen, Divergunt in Angulum duorum & trium Graduum. Quapropter non mirum, si isti Radii, sigillatim impingentes in alterum Prisma, perexiguo item apertum Foramine, inæqualiter Infringantur, cùm sit inæqualis illorum Inclinatio. Neque refert, quod isti Radii attollantur aut deprimantur per conversionem primi Prismatis, manente immoto secundo Prismate, (quod tamen in omni casu fieri non potest) vel quod manente primo Immobili, secundum moveatur, ut successivè Radios Coloratos totius Imaginis excipiat, & per proprium Foramen transmittat; utrolibet enim modo necesse est Radios illos extremos, hoc est, Rubrum & Violaceum, incidere in secundum Prisma sub inæquali Angulo, adeoque eorundem Refractionem esse inæqualem, ut Violaceorum sit major.

Cùm igitur manifesta causa appareat Oblongæ ejusmodi Figuræ Radiorum, causaque illa ex ipsa natura Refractionis oriatur; non videtur necesse recurrere ad aliam Hypothesin, aut admittere diversam illam Radiorum Frangibilitatem.

Quod deinde excogitavit de *Coloribus*, illud quidem egregiè consequitur ex præcedente Hypothesi; verùntamen nonnullas & ipsum patitur difficultates. Nam quod ait, nullum Colorem, sed potius Candorem apparere, ubi omnes omnium Colorum Radii promiscuè confunduntur, id verò non videtur conforme omnibus Phænomenis. Certè quæ Variationes cernuntur in permistione diversorum corporum, diversis Coloribus imbutorum, eædem omnino observantur in permistione diversorum Radiorum diversis item Coloribus imbutorum: Atque optimè ipse advertit, quòd quemadmodum ex Flavo & Cæruleo corpore exsurgit Viridis Color; ita ex Flavo & Cæruleo Radio Viridis item Color efficitur. Quare si omnes omnium Colorum Radii simul confunderentur, necesse esset in ista Hypothesi, ut ille Color appareret, qui reverà apparet in permistione omnium Pigmentorum. Atqui si ista, hoc est, Rubrum simul & Flavum unà cum Cæruleo & Purpureo aliisque omnibus, si quæ sint, conterantur & confundantur, non jam Candidus, sed Obscurus & Satur Color exsurget. Ergo similis Color appareret in Lumine Ordinario, quod constaret ex aggregatione omnium Colorum.

Præterea nihil primo aspectu magis Ingeniosum magisque aptum videtur, quàm quod ait circa Experimentum Acutissimi *Hookii*, quo duo diversi Liquores, quorum alter Rubeus, alter Cæruleus, uterque sigillatim Pellucidus, simul permixti, Opaci evadunt. Id autem ait Clarissimus *Newtonus* ex eo oriri, quòd unus Liquor solos Rubeos natus sit transmittere, alter verò solos Flavos; unde permisti nullos transmittent. Hoc, inquam, videtur statim valdè appositum; nihilominus tamen ex eo conficeretur, quòd similis opacitas fieret in permistione quorumcunque Liquorum qui essent diversi Coloris; quod tamen verum non est.

2. Refractiones à diversa parte Prismatis quantum potest Inæquales statuit, R. P. *Pardies*, cùm tamen ego tum in Experimentis, tum in Calculo de Experimentis istis inito, Æquales adhibuerim. Sit autem ABC, Prismatis Sec- *Answered by Mr. Newton. n. 84. p. 4091.* *Fig. 73.*

 tio

ctio ad Axem ejus Perpendicularis, FL, & KG, Radii duo in $x$ (medio Foraminis) decussantes, & in Prisma illud Incidentes ad G, & L; sintque eorum Refracti GH, & L$m$, ac denuo HI, & $mn$. Et cùm Refractiones ad Latus AC, æquales esse Refractionibus ad Latus BC, quam proximè suppossuerim; si AC, & BC, statuantur æqualia, similis erit Radiorum GH, & L$m$, ad AB, basin Prismatis Inclinatio; adeoque Ang. CL$m$ = Ang. CHG, & Ang. C$m$L = Ang. CGH. Quare etiam Refractiones in G, & $m$, æquales erunt, ut & in L, & H; atque adeò Ang. KGA = Ang. $nm$B, & Ang. FLA = Ang. BHI; & proinde Refractorum HI, & $mn$, eadem erit ad invicem Inclinatio ac est Incidentium Radiorum FL, & KG. Sit ergo Angulus F$x$K, 30 min. æqualis nempe Solari Diametro, & erit Angulus, quem HI, & $mn$, comprehendunt, etiam 30 min. si modò Radii FL, & KG, æqualiter Refrangibiles statuantur. At mihi Experienti prodiit Angulus ille circiter 2 gr. 49 min. quem Radius HI, extremum Violaceum Colorem, & $mn$, Cæruleum exhibens, constituêre; ac proinde Radios illos diversimodè Refrangibiles esse, sive Refractiones secundùm Disparem sinuum Incidentiæ & Refractionis Rationem peragi necessariò concedendum est.

Addit præterea R. P. quòd non sufficit ad obeundum ritè Calculum, ex Longitudine Imaginis impacta in Chartam subtrahere magnitudinem Foraminis Fenestræ; quandoquidem etiam posito Foramine Indivisibili, adhuc fieret aliud veluti Foramen Latum in posteriori superficie Prismatis. Mihi tamen videtur, his non obstantibus, quòd Refractiones Radiorum, in anteriori æquè ac in posteriori superficie Prismatis decussantium, ex adhibitis Principiis possint ritè computari. Sed si res secùs esset latitudo hiatûs in posteriori superficie, quod ad instar Foraminis est, haud efficeret errorem duorum minutorum secundorum; & in rebus practicis non operæ pretium duco ad minutias istas attendere.

Illi insuper *Experimento*, quod *Crucis* vocaveram, nihil adversatur R. P. dum contendit, Inæquales Radiorum, diversis Coloribus imbutorum, Refractiones ex Inæqualibus Incidentiis effectas fuisse. Nam Radiis per duo admodum perva, ab invicem distantia, & immota Foramina, transeuntibus, Incidentia illæ, prout ergo Experimentum institui, omninò Æquales erant, & tamen Refractiones liquidò Inæquales. Sin ille de Experimentis nostris dubitet, oro, ut Radiorum diversis Coloribus præditorum Refractiones ex Incidentiis paribus mensuret, & sentiet Inæquales esse. Si modus ille, quem ego ad hoc negotium adhibui minùs placeat (quo tamen nullus potest esse Luculentior,) facilè est alios excogitare; sicut & alios ipse haud paucos cum fructu expertus sum.

Contra *Theoriam* de *Coloribus* objicitur, quòd Pulveres diversorum Colorum permisti, non Candidum sed Subobscurum & Fuscum Colorem exhibent. Mihi verò Albus, Niger, & omnes intermedii Fusci, qui ab Albo & Nigro permistis componi possunt, non specie Coloris sed Quantitate Lucis tantùm differre videntur. Et cùm in mistione Pigmentorum, singula corpuscula non nisi proprium Colorem Reflectant, adeoque maxima pars Lucis incidentis supprimatur & retineatur; Lux Reflexa Subobscura evadet, & quasi cum tenebris permista, adeò, ut non intensum Alborem, sed qualem Nigredinis permistio conficit, hoc est Fuscum, exhibere debeat.

Objicitur

Objicitur deinde, quòd à Liquoribus quibuſcunque diverſi Coloris in eodem vaſe commiſtis, æquè ac in diverſis vaſis contentis, opacitas oriri debet; quod tamen, ait, verum non eſſe. Sed non video conſequentiam. Nam plurimi Liquores agunt in ſe invicem & novam ſibi mutuò partium contexturam ſecretò inducunt; unde Opaci, Diaphani; vel variis Coloribus, ex Coloribus permiſtorum nullo modo oriundis, præditi evadere poſſunt. Et hâc de causâ Experimenta hujuſmodi miùs apta ſemper exiſtimavi; à quibus concluſiones deduci poſſint. Subnoto tamen, quòd ad hoc Experimentum requiruntur Liquores ſaturis & intenſis Coloribus præditi; qui per paucos niſi proprii Coloris Radios tranſmittant; quales rarò occurrunt, ut videbitur illuminando Liquores cum diverſis Coloribus Priſmaticis in Obſcurato Cubiculo. Nam pauci reperientur, qui in propriis Coloribus ſatis Diaphani appareant, inque alienis Opaci. Convenit præterea, ut adhibiti Colores ſint inter ſe oppoſiti; quales exiſtimo fore Rubrum & Cæruleum, vel Flavum & Violaceum, vel etiam Viridem & Purpureum illum qui Coccineo affinis eſt. Et ex hujuſmodi Liquoribus nonnulli (quorum partes tingentes non congredientur) fortaſſe permiſti evadent Opaciores. Sed de eventu nihil ſum ſollicitus, tum quod luculentius eſt Experimentum in Liquoribus ſeorſim exiſtentibus, tum quod Experimentum illud (ſicut & Iridis, Tincturæ Nephriticæ, & aliorum Corporum naturalium Phænomena) non ad Probandam ſed ad Illuſtrandam tantùm Doctrinam propoſui.

Quod R. P. *Theoriam* noſtram *Hypotheſin* vocat, amicè habeo, ſiquidem ipſi nondum conſtet. Sed alio tamen conſilio propoſueram, & nihil aliud continere videtur quàm proprietates quaſdam Lucis, quas jam inventas probare haud difficile exiſtimo, & quas ſi non veras eſſe cognoſceram, pro futili & inani ſpeculatione mallem repudiare, quàm pro mea *Hypotheſi* agnoſcere.

3. In ea *Hypotheſi*, quam fuſe explicat noſter *Grimaldus*, in qua ſupponitur Lumen eſſe Subſtantia quædam rapidiſſimè mota, poſſet fieri aliqua diffuſio Luminis poſt tranſitum Foraminis, & decuſſationem Radiorum; Item in ea *Hypotheſi*, qua Lumen ponitur progredi per certas quaſdam Materiæ Subtilis Undulationes, ut explicat Subtiliſſimus *Hookius*, poſſunt explicari Colores per certam quandam *diffuſionem atque Expanſionem Undulationum, quæ fiat ad latera Radiorum ultra Foramen, ipſo Contagio ipſaque materiæ continuatione.* Certè ego talem adhibeo *Hypotheſin* in Diſſertatione de *Motu Undulationis*, quæ eſt *Sexta Pars meorum Mechanicorum*; ut ponam, Colores iſtos apparentes fieri ex ſola illa Communicatione Motionis, quæ ab Undulationibus directè procedentibus ad latera effundatur: Ut, ſi Radii intrantes per Foramen *a*, progrediantur versùs *b*, Undulationes quidem directè terminari deberent (habendo rationem ad motum Rectum & Naturalem) ad lineam rectam *ab*; nihilominus tamen, propter continuitaten materiæ, fit aliqua communicatio commotionis verſus Latera *cc*, ubi tremula quædam & criſpans ſuccuſſio excitatur: Atque ſi in illa laterali criſpatione conſiſtere Colores ſupponatur, exiſtimo omnia Phænomena Colorum explicari poſſe, ut fuſius in ea, quam dixi, Diſſertatione expono. Quibus item poſitis apparet etiam, cur ultra quam ferat Radiorum ipſorum Divaricatio, expandi Colorum Latitudinem neceſſe ſit.

*Some further Objections by* R. P. Pardies. n. 85. p. 5012.

Fig. 7.

Circa *Experimentum Crucis*, nequaquam dubito, quo minus in ſuo Experimento talem ſitum adhibuerit, in quo æqualis Inclinatio fuerit Radiorum inci-

dentium

dentium; quandoquidem id ita à se præstitum expresse affirmat. Verum id non ego poteram conjicere ex iis quæ superius Legeram; ubi ponuntur duo exigua & maximè distantia Foramina, & unum Prisma prope primum Foramen quod est in fenestra; per quod Prisma Radii Colorati erumpentes, Incidunt in alterum distans Foramen. Addebatur autem, quòd ad hoc, ut omnes illi Radii successivè Inciderent in Secundum illud Foramen, convertebatur Primum Prisma supra Axem: Atqui hoc modo necesse est mutari Inclinationem Radiorum, qui Incidunt in secundum Foramen: Atque indicavi ego, quòd perinde sese res haberet, sive manente Primo Prismate Immobili, Secundum Foramen attolleretur aut deprimeretur, ut possit successivè Radios omnes depictæ Imaginis Solaris excipere; sive manente isto Secundo Foramine Immobili, Primum Prisma converteretur, ut ita eadem Imago situm mutaret, atque in Foramen impingere Secundum omnes successivè partes posset. Sed alias sine dubio adhibuit cautiones Solertissimus *Newtonus*.

Quæ circa Colores objeceram, optimè soluta existimo. Quod autem *Theoriam* istam, appellarim *Hypothesin*, id certe ego nullo adhibito consilio feci; atque nomen usurpavi quod primum occurrit: quapropter velim ut ne per contemptum adhibitam vocem ejusmodi existimet.

*Answered by Mr.* Newton. *Ibid. p.* 5014.

4. Ait R.P. quòd absq; varia Diversorum Radiorum Refrangibilitate, possibile sit explicare Longitudinem Colorum; puta ex Hypothesi *P. Grimaldi*, per diffusionem Luminis, quod supponitur esse substantia quædam rapidissimè mota; vel ex Hypothesi *Hookii* nostri, per diffusionem vel expansionem Undulationum, quas statuit in æthere à Lucidis corporibus excitatus quaquaversum propagari. Addo quòd ex Hypothesi *Cartesiana* potest etiam effingi consimilis diffusio conatus vel pressionis Globulorum, perinde ut in explicatione Caudæ Cometæ supponitur. Et eadem diffusio vel Expansio juxta aliam quamvis Hypothesin, in qua Lumen statuitur esse Vis, Actio, Qualitas, vel Substantia quælibet, à Luminosis corporibus undique emissa, effingi potest.

Ut his respondeam, animadvertendum est, quòd Doctrina illa, quam de Refractione & Coloribus explicui, in quibusdam Lucis proprietatibus solummodo constitit, neglectis Hypothesibus per quas proprietates illæ explicari debent. Quamobrem ab Hypothesium contemplatione, tanquam improprio Argumentandi loco, hìc abstinendum esse censui, & Vim Objectionis abstrahendam, ut pleniorem & magis generalem responsionem accipiat.

Itaque per Lumen intelligo quodlibet Ens vel Entis potestatem (sive sit Substantia, sive quævis ejus Vis, Actio, vel Qualitas) quod à corpore lucido rectà pergens aptum sit ad excitandam Visionem; & per Radios Luminis intelligo minimas vel quaslibet indefinitè parvas ejus partes quæ ab invicem non dependent, quales sunt illi omnes Radii, quos Lucentia corpora vel simul vel successivè secundùm Rectas Lineas emittunt. Nam illæ tum collaterales tum successivæ partes Luminis sunt independentes; siquidem unæ absque aliis intercipi possint, & in quaslibet plagas seorsim Reflecti vel Refringi. Et hoc præcognito, Objectionis Vis omnis in eo sita erit; quòd Colores per aliquam Luminis ultra Foramen diffusionem, quæ non oritur ab inæquali diversorum Radiorum (seu Luminis independentium partium) Refrangibilitate, in Longum diduci possint.

Quod

Quod autem non aliunde Oblongentur superius monstravi: & ut omnia summè confirmarem, adjeci *Experimentum* illud quod jam nomine *Crucis* passim insignitur: de cujus conditionibus cùm R. P. dubitaverit, placuit jam designare Schemate. Sit BC Anterior Tabula, cui Prisma A, immediate præfigitur, sitque DE Altera Tabula, quasi 12 pedibus abinde distans, cui suffigitur alterum Prisma F. Tabulæ autem ad *x* & *y*, ita perforentur, ut aliquantulum Lucis ab anteriori Prismate Refractæ trajici possit per utrumque Foramen ad Secundum Prisma, inque eo denuò Refringi. Jam Prisma anterius circa Axem reciproco motu convertatur, & Colores in Tabulam Posteriorem DE procidentes, per vices attollentur ac deprimentur, eoque pacto alius atque alius Color successivè pro arbitrio trajici potest per Foramen ejus *y*, ad Posterius Prisma, dum cæteri Colores in Tabulam impingunt: Et videbis, Radios diversis Coloribus præditos diversam pati Refractionem in illo posteriori Prismate, ex eo quòd ad diversa loca parietis vel cujusvis obstaculi GH, pedibus aliquot ulterius remoti, allabentur; puta Violacei Radii ad H, Rubri ad G, & Intermedii ad loca intermedia: & tamen propter determinatam Positionem foraminum necesse est ut Similis sit Incidentia Radiorum cujusque Coloris per utrumque trajecti. Atque ita ex mensura constat Radios, diversis Coloribus affectos, habere diversas Leges Refractionum. *Fig. 75.*

Sed suspicor unde adductus sit R. P. in dubitationem; nempe videtur collocasse Primum Prisma A, post Tabulam BC, atque ita convertendo circa Axem, verisimile est Inclinationem Radiorum qui interjacent Foramina propter Intermediam Refractionem fuisse mutatam. At, ex descriptione prius expositâ, debuit Tabula illa collocari post Prisma, ut Radii inter Foramina in directum jacerent, quemadmodum ex verbis, *I took two Boards and placed one of them close behind the Prism at the Window*, constare potest. Et usus Experimenti idem innuit.

Ex abundanti placet observare, quod in hoc Experimento Colorata Lux ob Refractionem Secundi Prismatis longe minùs diffunditur ac divaricat, quàm cùm Alba existit, adeò ut Imago ad G, vel H, sit penè Circularis; præsertim si Prismata statuantur parallela, & in contrario situ Angulorum, prout in Schemate designantur. Quinetiam, si præterea diameter foraminis *y*, adæquet Latitudinem Colorum, nulla erit ejusdem Coloratæ lucis in Longum diffusio; sed Imago, quæ à quopiam Colore ad G, vel H, effingitur, (positis Circularibus foraminibus, & Refractione posterioris Prismatis non majori quàm prioris, Radiisque ad obstaculum quàm proximè perpendicularibus) erit planè Circularis. Id quod arguit diffusionem, de qua supra egimus, non ex contagione vel continuitate materiæ undulantis aut celerrimè motæ vel similibus causis ortam esse, sed ex certa Refractionum cujusque Generis Radiorum Lege. Cur autem Imago illa in uno casu sit Circularis, & in aliis nonnihil Oblongata, & quomodo diffusio lucis in Longitudinem in quolibet casu pro arbitrio minui possit, à Geometris determinandum, & cum experientia conferendum, relinquo.

Postquam proprietates Lucis his & similibus Experimentis satis exploratæ fuerint, spectando Radios tanquam ejus sive collaterales sive successivas partes, de quibus experti sumus per independentiam quod sint ab invicem distinctæ;

Hypotheses

Hypotheſes exinde dijudicandæ ſunt, & quæ non poſſunt conciliari rejiciendæ. Sed leviſſimi negotij eſt accommodare Hypotheſes ad hanc Doctrinam. Nam ſi quis Hypotheſin Carteſianam defendere velit, dicendum eſt, Globulos eſſe inæquales; vel preſſiones Globulorum eſſe alias aliis fortiores, & inde diverſimodè Refrangibiles, & aptas ad excitandam ſenſationem diverſorum Colorum. Et ſic juxta Hypotheſin Cl. *Hookii* dicendum eſt, Undulationes Ætheris eſſe alias majores ſive craſſiores aliis. Atque ita in cæteris. Hæc enim videtur eſſe ſummè neceſſaria Lex & Conditio Hypotheſium, in quibus naturalia corpora ponuntur conſtare ex quamplurimis corpuſculis acervatim contextis, ut a diverſis Lucentium corpuſculis, vel ejuſdem corpuſculi diverſis partibus (prout Motu, Figurâ, Mole, aut aliis Qualitatibus differunt) inæquales preſſiones, motiones aut mota corpuſcula per Æthera quaquaverſum trajiciantur, ex quibus confusè miſtis, Lux conſtitui ſupponetur. Et nihil durius eſſe poteſt in iſtis Hypotheſibus quàm contraria ſuppoſitio.

Ex Apertura ſive Dilatatione Lucis in poſteriori facie Priſmatis, quam R. P. dixit eſſe velut Foramen, ſufficit, quod error non emerget ſenſibilis, ſi modo aliquis emergeret. Quòd ſi Calculus juxta obſervationes præciſe ineatur, error erit nullus. Nam diametro foraminis à Longitudine Imaginis ſubductâ, reſtabit Longitudo quam Imago haberet, ſi modò foramen ante Priſma eſſet indiviſibile, idque non obſtante præfata Lucis dilatatione in poſteriori facie Priſmatis; ut facile oſtenditur. Deinde ex data illa Longitudine Imaginis, ac diſtantia à foramine indiviſibili, ut & poſitione & forma Priſmatis, & ad id inclinatione Incidentium Radiorum, ac angulo, quem Refracti Radii ad medium imaginis tendentes, cum à centro Solis incidentibus conſtituunt, cætera omnia determinantur. Et quæ determinant Refractiones & Poſitiones Radiorum, ſufficiunt ad Calculum iſtarum Refractionum ritè ineundum. Sed res non tanti eſſe videtur ut moram inferat.

Quod R. P. Doctrinam noſtram Hypotheſin vocaverit, non aliunde factum eſſe credo, quàm quòd vocabulum uſurpavit quod primum occurrit; ſiquidem mos obtinuit ut quicquid exponitur in Philoſophia dicatur Hypotheſis. Et ego ſanè non alio Conſilio vocabulum iſtud reprehendi, quàm ut ne invaleſceret appellatio quæ rectè Philoſophantibus præjudicio eſſe poſſet.

*To the Satisfaction of* P. Ig. Gaſt. Pardies. n. 85. *p.* 5018.

5. Omnino mihi ſatisfecit noviſſima reſponſio, à D. *Newtono* ad meas Inſtantias data. Noviſſimus Scrupulus, qui mihi hærebat circa *Experimentum Crucis*, penitus fuit exemptus. Atque nunc plane ex Figura ipſius intelligo, quod non intellexeram ante. Experimentum peractum cum fuerit iſto modo, nil habeo quod in eo deſiderem ampliùs.

*Some Conſiderations upon this Theory; by .... Anſwer'd by Mr.* Newton. n. 88. *p.* 5086.

V. The Conſiderations on my *Theories* conſiſt in Aſcribing an Hypotheſis to me, which is not mine; in Aſſerting an Hypotheſis, which, as to the principal parts, is not againſt me; in Granting the greateſt part of my Diſcourſe, if explicated by that Hypotheſis; and in Denying ſome things, the truth of which would have appear'd by an Experimental Examination.

Of theſe particulars I ſhall diſcourſe in order. And *firſt* of the Hypotheſis, which is aſcribed to me in theſe words: *But grant his firſt Suppoſition, that Light is a Body, and that as many Colours or Degrees as there may be, ſo many Bodies there*

*there may be; all which Compounded together would make White, &c.* This, it seems, is taken for my Hypothesis. 'Tis true, that from my *Theory* I argue the Corporiety of Light; but I do it without any absolute positiveness, as the word *perhaps* intimates; and make it at most but a very plausible consequence of the Doctrine, and not a Fundamental Supposition, nor so much as any part of it; which was wholly comprehended in the precedent Propositions. And I somewhat wonder, how the Objector could imagine, that, when I had asserted the *Theory* with the greatest Rigour, I should be so forgetful as afterwards to assert the Fundamental Supposition it self with no more than a *perhaps*. Had I intended any such Hypothesis, I should somewhere have explained it. But I knew, that the Properties, which I declared of Light, were in some measure capable of being explicated, not only by that, but by many other Mechanical Hypotheses. And therefore I chose to decline them all, and to speak of Light in general terms, considering it abstractly, as something or other propagated every way in streight Lines from Luminous Bodies, without determining what that thing is; whether a confused Mixture of Difform Qualities, or Modes of Bodies, or of Bodies themselves, or of any Virtues, Powers, or Beings whatsoever. And for the same reason I chose to speak of Colours according to the Information of our Senses, as if they were Qualities of Light without us. Whereas by that Hypothesis, I must have considered them rather as Modes of Sensation, excited in the Mind by various Motions, Figures, or Sizes of the Corpuscles of Light, making various Mechanical impressions on the Organ of Sense; as I expressed it in that place, where I spake of the Corporiety of Light.

But supposing I had propounded that Hypothesis, I understand not, why the Objector should so much endeavour to oppose it. For certainly it has a much greater Affinity with his own Hypothesis, than he seems to be aware of; the Vibrations of the Æther being as useful and necessary in this, as in his. For, assuming the Rays of Light to be small Bodies, emitted every way from shining Substances, those, when they impinge on any Refracting or Reflecting Superficies, must as necessarily excite Vibrations in the Æther, as Stones do in Water when thrown into it. And supposing these Vibrations to be of several Depths or Thicknesses, accordingly as they are excited by the said corpuscular Rays of various Sizes and Velocities; of what use they will be for explicating the manner of Reflection and Refraction, the Production of Heat by the Sun-beams, the Emission of Light from Burning, Putrifying, or other Substances, whose parts are vehemently agitated, the Phænomena of thin transparent Plates and Bubbles, and of all natural Bodies, the Manner of Vision, and the Difference of Colours, as also their Harmony and Discord; I shall leave to their Consideration, who may think it worth their endeavour, to apply this Hypothesis to the Solution of Phænomena.

In the *second* place, I told you, That the Objector's Hypothesis, as to the Fundamental part of it, is not against me. That Fundamental Supposition is; *That the parts of Bodies, when briskly agitated, do excite Vibrations in the Æther, which are propagated every way from those Bodies in streight Lines, and cause a Sensation of Light by beating and dashing against the bottom of the Eye,* *something after the manner, that Vibrations in the Air cause a Sensation of Sound by beating against the Organs of Hearing.* Now, the most free and natural Application of this Hypothesis, to the Solution of Phænomena, I take to be this: That the agitated parts of Bodies, according to their several Sizes, Figures, and Motions, do excite Vibrations in the Æther of various Depths, or Bignesses, which being promiscuously propagated through that Medium to our Eyes, effect in us a Censation of Light of a White Colour; but if by any means those of unequal Bignesses be separated from one another, the largest beget a Sensation of a Red Colour; the least or shortest, of a deep Violet; and the intermediate ones, of Intermediate Colours; much after the manner that Bodies, according to their several Sizes, Shapes, and Motions, excite Vibrations in the Air of various Bignesses, which, according to those Bignesses, make several Tones in Sound: That the largest Vibrations are best able to overcome the resistance of a Refracting Superficies, and so break through it with least Refraction; whence the Vibrations of several Bignesses, that is, the Rays of several Colours, which are blended together in Light, must be parted from one another by Refraction, and so cause the Phænomena of Prisms, and other Refracting Substances: And that it depends on the Thickness of a thin transparent Plate or Bubble, whether a Vibration shall be Reflected at its further Superficies, or Transmitted; so that, according to the Number of Vibrations interceding the two Superficies, they may be Reflected or Transmitted for many successive Thicknesses. And since the Vibrations which make Blue and Violet, are supposed shorter than those which make Red and Yellow, they must be Reflected at a less Thickness of the Plate: Which is sufficient to explicate all the ordinary Phænomena of those Plates or Bubbles, and also of all natural Bodies, whose parts are like so many Fragments of such Plates.

These seem to be the most plain, genuine, and necessary Conditions of this Hypothesis. And they agree so justly with my Theory, that if the Animadversor think fit to apply them, he need not, on that Account, apprehend a Divorce from it. But yet how he will defend it from other Difficulties, I know not. For, to me the Fundamental supposition it self seems impossible; namely, that the Waves or Vibrations of any Fluid, can, like the Rays of Light, be propagated in streight Lines, without a Continual and very extravagant spreading and bending every way into the Quiescent Medium, where they are terminated by it. I mistake, if there be not both Experiment and Demonstration, to the contrary. And as to the other two or three Hypotheses, which he mentions, I had rather believe them subject to the like difficulties, than suspect the Animadversor should select the worst for his own.

What I have said of this, may be easily applyed to all other Mechanical Hypotheses, in which Light is supposed to be caused by any Pression or Motion whatsoever, excited in the Æther by the agitated parts of Luminous Bodies. For, it seems impossible, that any of those Motions or Pressions can be propagated in Streight Lines, without the like spreading every way into the Shadowed Medium, on which they border. But yet, if any Man can think it possible, he must at least allow, that those Motions, or Endeavours to Motion, caused in the Æther by the several parts of any Lucid Body, that differ

fer in Size, Figure, and Agitation, must necessarily be unequal: Which is enough to denominate Light an Aggregate of *Difform Rays*, according to any of those Hypotheses. And if those Original Inequalities may suffice to difference the Rays in Colour and Refrangibility, I see no reason, why they, that adhere to any of those Hypotheses, should seek for other Causes of these Effects, unless (to use the Objector's Argument) *they will multiply Entities without Necessity*.

The *third* thing to be considered is, the Condition of the Animadversor's Concessions, which is, That I would explicate my Theories by his Hypothesis: And if I could comply with him in that point, there would be little or no difference between us. For he Grants, That, without any respect to a Different Incidence of Rays, there are Different Refractions; but he would have it explicated, not by the different Refrangibility of several Rays, but by the Spliting and Rarefying of Æthereal Pulses. He grants my third, fourth, and sixth Propositions; the sense of which is, That Uncompounded Colours are Unchangeable, and that Compounded ones are Changeable only by resolving them into the Colours, of which they are Compounded; and that all the Changes, which can be wrought in Colours, are effected only by variously mixing or parting them: But he grants them on Condition, that I will explicate Colours by the two sides of a Split Pulse, and so make but two Species of them, accounting all other Colours in the World to be but various Degrees and Dilutings of those two. And he further grants, that Whiteness is produc'd by the Convention of all Colours; but then I must allow it to be not only by Mixture of those Colours, but by a farther Uniting of the Parts of the Ray supposed to be formerly Split.

If I would proceed to examine these his Explications, I think it would be no difficult matter to shew, that they are not only Insufficient, but in some respects to me (at least) Unintelligible. For though it be easie to conceive, how Motion may be dilated, and spread, or how parallel Motions may become Diverging; yet I understood not, by what Artifice any Linear Motion can by a Refracting Superficies be infinitely dilated and rarefied, so as to become Superficial: Or, if that be supposed, yet I understand as little, why it should be split at so small an Angle only, and not rather spread and dispersed through the whole Angle of Refraction. And further, though I can easily imagine, how Unlike Motions may cross one another; yet I cannot well conceive, how they should coalesce into one Uniform Motion, and then part again, and recover the former Unlikeness; notwithstanding that I conjecture the ways, by which the Animadversor may endeavour to explain it. So that the Direct, Uniform, and Undisturb'd Pulses should be Split and Disturbed by Refraction; and yet the Oblique and Disturbed Pulses persist without Splitting or further disturbance by following Refractions, is (to me) as unintelligible; and there is as great a difficulty in the Number of Colours; as you will see hereafter.

But whatever be the Advantages or Disadvantages of this Hypothesis, I hope I may be excused from taking it up, since I do not think it needful to explicate my Doctrine by any Hypothesis at all. For if Light be considered abſtractedly without reſpect to any Hypotheſis, I can as eaſily conceive, that the ſeveral Parts of a ſhining Body may emit Rays of different Colours and other Qualities, of all which Light is conſtituted, as that the ſeveral parts of a falſe or uneven String, or of unevenly agitated Water in a Brook or Cataract, or the ſeveral Pipes of an Organ inſpired all at once, or all the variety of ſounding Bodies in the World together, ſhould produce Sounds of ſeveral Tones, and Propagate them thro' the Air confuſedly intermixt. And if there were any Natural Bodies that could reflect Sounds of one Tone, and ſtifle or tranſmit thoſe of another; then, as the Echo of a confuſed Aggregate of all Tones would be that particular Tone, which the Echoing Body is diſpoſed to Reflect; ſo, ſince (even by the Animadverſor's Conceſſions) there are Bodies apt to Reflect Rays of one Colour, and ſtifle or tranſmit thoſe of another; I can as eaſily conceive, that thoſe Bodies, when illuminated by a Mixture of all Colours, muſt appear of that Colour only which they Reflect.

But when the Objector would inſinuate a difficulty in theſe things, by alluding to Sounds in the String of a Muſical Inſtrument before Percuſſion, or in the Air of an Organ-Bellows before its arrival at the Pipes; I muſt confeſs, I underſtand it as little, as if one had ſpoken of Light in a piece of Wood before it be ſet on Fire, or in the Oyl of a Lamp before it aſcend up the match to feed the Flame.

You ſee therefore how much it is beſides the buſineſs in hand, to diſpute about Hypotheſes. For which reaſon I ſhall now, in the laſt place, proceed to Abſtract the difficulties in the Animadverſor's Diſcourſe, and without having regard to any Hypotheſis, conſider them in general Terms. And they may be reduced to theſe three Queries.

1. *Whether the unequal Refractions made without reſpect to any inequality of Incidence, be cauſed by the different Refrangibility of ſeveral Rays; or by the ſplitting, breaking, or diſſipating the ſame Ray into diverging Parts?*
2. *Whether there be more than two ſorts of Colours?*
3. *Whether Whiteneſs be a Mixture of all Colours?*

The *firſt* of theſe *Queries* you may find determined above by an Experiment, the deſign of which was to ſhew, That the length of the Coloured Image proceeded not from any Unevenneſs in the Glaſs, or any other Contingent Irregularity in the Refractions. Amongſt other Irregularities, I know not, what is more obvious to ſuſpect, than a fortuitous dilating and ſpreading of Light after ſome ſuch manner, as *des Cartes* hath deſcribed in his *Æthereal Refractions* for explicating the Tail of a Comet; or as the Animadverſor now ſuppoſes to be effected by the ſplitting and rarefying of his Æthereal Pulſes. And to prevent the ſuſpicion of any ſuch Irregularities, I told you, that I Refracted the Light contrary ways, with two Priſms ſucceſſively, to deſtroy thereby the Regular Effects of the firſt Priſm by the ſecond, and to diſcover the Irregular Effects by Augmenting them with iterated Refractions. Now, amongſt other Irregularities, if the firſt Priſm had ſpread and diſſipated every Ray into an indefinite Number of Diverging Parts, the ſecond ſhould in like

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manner have ſpread and diſſipated every one of thoſe Parts into a further indefinite Number, whereby the Image would have been ſtill more dilated, contrary to the Event. And this ought to have happened, becauſe thoſe Linear Diverging Parts depend not on one another for the manner of their Refraction, but are every one of them as truly and compleatly Rays, as the whole was before its Incidence; as may appear by intercepting them ſeverally.

The Reaſonableneſs of this proceeding, will perhaps better appear by acquainting you with this further Circumſtance. I ſometimes placed the ſecond Priſm in a Poſition Tranſverſe to the firſt, on deſign to try, if it would make the long Image become four-ſquare by Refractions croſſing thoſe that had drawn the round Image into a long one. For if amongſt other Irregularities the Refraction of the firſt Priſm, did by ſplitting Dilate a Linear Ray into a Superficial, the Croſs Refractions of that ſecond Priſm ought by further ſplitting to Dilate and Draw that Superficial Ray into a Pyramidal Solid. But, upon tryal, I found it otherwiſe; the Image being as regularly Oblong, as before, and inclined to both the Priſms at an Angle of 45 degrees.

I tryed alſo all other Poſitions of the ſecond Priſm, by turning the ends about its middle part; and in no caſe could obſerve any ſuch Irregularity. The Image was ever alike inclined to both Priſms, its Breadth anſwering to the Sun's Diameter, and its Length being greater or leſs, accordingly as the Refractions more or leſs agreed, or contradicted one another.

And by theſe Obſervations, ſince the Breadth of the Image was not augmented by the Croſs Refraction of the ſecond Priſm, that Refraction muſt have been performed without any ſplitting or dilating of the Ray; and therefore at leaſt the Light incident on that Priſm muſt be granted an Aggregate of Rays *Unequally Refrangible* in my ſenſe. And ſince the Image was equally Inclined to both Priſms, and conſequently the Refractions alike in both, it argues, that they were performed according to ſome Conſtant Law, without any Irregularity.

To determine the *ſecond Querie*, The Animadverſor referrs to an Experiment made with two Wedge-like Boxes, recited in the *Micrography* of the Ingenious Mr. *Hock*, *Obſerv.* 10. *Pag.* 73. the deſign of which was to produce all Colours out of a Mixture of two. But there is, I conceive, a double defect in this Inſtance. For it appears not, that by this Experiment all Colours can be produced out of two; and if they could, yet the Inference would not follow.

That all Colours cannot by that Experiment be produced out of two, will appear by conſidering, that the Tincture of Aloes, which afforded one of thoſe Colours, was not all over of one uniform Colour, but appeared Yellow near the edge of the Box, and Red at other Places where it was thicker: affording all variety of Colours, from a Pale Yellow to a deep Red or Scarlet, according to the various thickneſs of the Liquor. And ſo the Solution of Copper, which afforded the other Colour, was of various Blues and Indicoes. So that inſtead of two Colours, here is a great Variety made uſe of for the Production of all others. Thus, for Inſtance, to produce all ſorts of Greens, the ſeveral degrees of Yellow and Pale Blue muſt be mixed; but to compound Purples, the Scarlet and deep Blue are to be the Ingredients. Now,

Now, If the Animadversor contend, that all the Reds and Yellows of the one Liquor, or Blues and Indicoes of the other, are only various degrees and dilutings of the same Colour, and not divers Colours, that is a begging of the Question: And I should as soon grant, that the two thirds or sixths in Musick are but several degrees of the same Sound, and not divers Sounds. Certainly it is much better to believe our Senses, informing us, that Red and Yellow are divers Colours, and to make it a Philosophical Querie, Why the same Liquor doth, according to its various thickness, appear of those divers Colours, than to suppose them to be the same Colour, because exhibited by the same Liquors? For, if that were a sufficient Reason, then Blue and Yellow must also be the same Colour, since they are both exhibited by the same Tincture of Nephritick Wood. But that they are divers Colours, you will more fully understand by the reason, which in my Judgment is this: The Tincture of Aloes is qualified to Transmit most easily the Rays Indued with Red, most difficultly the Rays indued with Violet, and with intermediate degrees of facility, the Rays indued with intermediate Colours. So that where the Liquor is very thin, it may suffice, to intercept most of the Violet, and yet transmit most of the other Colours; all which together must compound a middle Colour, that is, a Faint Yellow. And where it is so much thicker, as also to intercept most of the Blue and Green, the remaining Green, Yellow, and Red, must Compound an Orange. And where the thickness is so great, that scarce any Rays can pass thro' it besides those indued with Red, it must appear of that Colour, and that so much the deeper, and obscurer, by how much the Liquor is thicker. And the same may be understood of the various degrees of Blue, exhibited by the Solution of Copper, by reason of its disposition to intercept Red most easily, and transmit a deep Blue or Indico Colour most freely.

But supposing that all Colours might, according to this Experiment, be produced out of two by Mixture; yet it follows not, that those two are the only Original Colours; and that for a double Reason. First, Because those two are not themselves Original Colours, but compounded of others; there being no Liquor, nor any other Body in Nature, whose Colour in Day-light is wholly Uncompounded. And then, because though those two were Original, and all others might be Compounded of them, yet it follows not that they cannot be otherwise produced. For I said, that they had a double Origin, the same Colours to sence being in some Cases Compounded, and in others Uncompounded; and sufficiently declared in my *third* and *fourth Propositions*, and in the *Conclusion*, by what Properties the one might be known and distinguish'd from the other. But because I suspect, by some Circumstances, that the Distinction might not be rightly apprehended, I shall once more declare it, and further explain it by Examples.

That Colour is Primary or Original, which cannot by any Art be changed, and whose Rays are alike Refrangible: And that Compounded, which is changeable into other Colours, and whose Rays are not alike Refrangible. For instance, To know whether the Colour of any Green Object be Compounded or not, view it thro' a Prism, and if it appear confused, and the Edges

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Tinged with Blue, Yellow, or any variety of other Colours, then is that Green Compounded of ſuch Colours, as at its edges Emerge out of it: But if it appear diſtinct, and well defined, and intirely Green to the very edges, without any other Colours Emerging, it is of an Original and Uncompounded Green. In like manner, if a Refracted Beam of Light being caſt on a White Wall exhibit a Green Colour, to know whether that be Compounded, Refract the Beam with an interpoſed Priſm, and if you find any Difformity in the Refractions, and the Green be transformed into Blue, Yellow, or any variety of other Colours, you may conclude that it was Compounded of thoſe that Emerge: But if the Refractions be Uniform, and the Green perſiſt without any change of Colour, then is it Original and Uncompounded. And the Reaſon why I call it ſo is, becauſe a Green indued with ſuch properties cannot be produced by any mixing of other Colours.

Now, If two Green Objects may to the naked Eye appear of the ſame Colour, and yet one of them thro' a Priſm ſeem confuſed and variegated with other Colours at the edges, and the other diſtinct and entirely Green; or if there may be two Beams of Light, which falling on a White Wall, do to the naked Eye exhibit the ſame Green Colour, and yet one of them, when tranſmitted thro' a Priſm, be uniformly and regularly Refracted, and retain its Colour Unchanged, and the other be irregularly Refracted, and made to divaricate into a multitude of other Colours: I ſuppoſe theſe two Greens will in both Caſes be granted of a different Origin and Conſtitution. And if by mixing Colours, a Green cannot be Compounded with the Properties of the Unchangeable Green, I think I may call that an Uncompounded Colour, eſpecially ſince its Rays are alike Refrangible, and Uniform in all reſpects.

The ſame Rule is to be obſerved in Examining, whether Red, Orange, Yellow, Blue, or any other Colour be Compounded or not. And, by the way, ſince all White Objects thro' the Priſm appear confuſed and terminated with Colours, Whiteneſs muſt, according to this diſtinction, be ever Compounded; and that the moſt of all Colours, becauſe it is the moſt confuſed and changed by Refractions.

There remains now the *third* Querie to be conſidered, which is, Whether Whiteneſs be an Uniform Colour, or a Diſſimilar Mixture of all Colours? The Experiment which I brought to decide it, the Animadverſor thinks may be otherwiſe explained, and ſo concludes nothing. But he might eaſily have ſatisfied himſelf by trying, what would be the reſult of a Mixture of all Colours. And that very Experiment might have ſatisfied him, if he had pleaſed to examine it by the various Circumſtances. One Circumſtance I there declared, of which I ſee no Notice taken; and it is, That if any Colour at the Lens be intercepted, the Whiteneſs will be changed into the other Colours: If all the Colours but Red be intercepted, that Red alone in the Concourſe or croſſing of the Rays, will not conſtitute Whiteneſs, but continues as much Red as before; and ſo of the other Colours: So that the buſineſs is not only to ſhew how Rays, which before the Concourſe exhibit Colours, do in the Concourſe exhibit White; but to ſhew, how, in the ſame place, where the ſeveral ſorts of Rays apart exhibit ſeveral Colours, a Confuſion of altogether

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make White. For inſtance, If Red alone be firſt tranſmitted to the Paper at the place of concourſe, and then the other Colours be let fall on that Red, the Queſtion will be, whether they convert it into White by mixing with it only, as Blue falling on Yellow Light is ſuppoſed to compound Green, or whether there be ſome further change wrought in the Colours by their mutual acting on one another, until, like contrary *Peripatetic Qualities*, they become aſſimilated. And he that ſhall explicate this laſt Caſe Mechanically, muſt conquer a double impoſſibility. He muſt firſt ſhew, that many unlike Motions in a Fluid can by claſhing ſo act on one another, and change each other, as to become one Uniform Motion; and then, that an Uniform Motion can of it ſelf, without any new unequal impreſſions, depart into a great variety of Motions regularly unequal. And after this he muſt further tell me, why all Objects appear not of the ſame Colour; that is, why their Colours in the Air, where the Rays that convey them every way are confuſedly mixt, do not aſſimilate one another, and become Uniform before they arrive at the Spectator's Eye?

But if there be yet any doubting, 'tis better to put the Event on further Circumſtances of the Experiment, than to acquieſce in the poſſibility of any Hypothetical Explications. As, for inſtance, by trying, What will be the Apparition of theſe Colours in a very quick Conſecution of one another. And this may be eaſily performed by the Rapid Gyration of a Wheel with many Spoaks or Coggs in its Perimeter, whoſe Interſtices and Thickneſſes may be equal, and of ſuch a Largeneſs, that, if the Wheel be interpoſed between the Priſm and the white concourſe of the Colours, one half of the Colours may be intercepted by a Spoak or cogg, and the other half paſs through an Interſtice. The Wheel being in this poſture, you may firſt turn it ſlowly about, to ſee all the Colours fall ſucceſſively on the ſame place of the Paper, held at their aforeſaid concourſe; and if you then accelerate its Gyration, until the conſecution of thoſe Colours be ſo quick, that you cannot diſtinguiſh them ſeverally, the reſulting Colour will be a Whiteneſs perfectly like that, which an Unrefracted Beam of light exhibits, when in like manner ſucceſſively interrupted by the Spoaks or Cogs of that circulating Wheel. And that this Whiteneſs is produced by a ſucceſſive intermixture of the Colours, without their being aſſimilated, or reduc'd to any Uniformity, is certainly beyond all doubt, unleſs things that exiſt not at the ſame time may notwithſtanding act on one another.

There are yet other Circumſtances, by which the truth might have been decided; as by viewing the White Concourſe of the Colours through another Priſm placed cloſe to the eye, by whoſe Refraction that Whiteneſs may appear again transformed into Colours: And then, to examine their Origin, if an Aſſiſtant intercept any of the Colours at the Lens before their arrival at the Whiteneſs, the ſame Colours will vaniſh from amongſt thoſe, into which that Whiteneſs is converted by the ſecond Priſm. Now, if the Rays which diſappear be the ſame with thoſe that are intercepted, then it muſt be acknowledged, that the ſecond Priſm makes no new Colours in any Rays, which were not in them before their Concourſe at the Paper. Which is a plain Indication, that the Rays of ſeveral Colours remain diſtinct from one another in the Whiteneſs, and that from their previous diſpoſitions are derived the Colours, of the ſecond Priſm. And, by the way, what is ſaid of their Colours may be applyed to their Refrangibility.

The aforeſaid Wheel may be alſo here made uſe of; and, if its Gyration be neither too quick nor too ſlow, the Succeſſion of the Colours may be diſcern'd thro' the Priſm, whilſt to the naked eye of a By-ſtander they exhibit Whiteneſs.

There is ſomething ſtill remaining to be ſaid of this Experiment. But this, I conceive, is enough to enforce it, and ſo to decide the Controverſie. However I ſhall now proceed to ſhew ſome other ways of producing Whiteneſs by Mixtures, ſince I perſwade my ſelf, that this Aſſertion above the reſt appears Paradoxical, and is with moſt difficulty admitted. And becauſe the Animadverſor deſires an inſtance of it in Bodies of divers Colours, I ſhall begin with that. But in order thereto it muſt be conſider'd, that ſuch Coloured bodies Reflect but ſome part of the Light incident on them; as is evident by *the* 13th *Propoſition*: And therefore the Light Reflected from an Aggregate of them will be much weakned by the loſs of many Rays. Whence a perfect and intenſe Whiteneſs is not to be expected, but rather a Colour between thoſe of Light and Shadow, or ſuch a Gray or Dirty Colour as may be made by mixing White and Black together.

And that ſuch a Colour will reſult, may be collected from the Colour of Duſt found in every corner of an houſe, which hath been obſerved to conſiſt of many Coloured particles. There may be alſo produced the like Dirty Colour, by mixing ſeveral Painters Colours together. And the ſame may be effected by painting a Top (ſuch as Boys play with) of Divers Colours. For when it is made to circulate by whipping it, it will appear of ſuch a Dirty Colour.

Now, the Compounding of theſe Colours is proper to my purpoſe, becauſe they differ not from Whiteneſs in the Species of Colour, but only in a Degree of Luminouſneſs: Which (did not the Animadverſor concede it) I might thus evince. A Beam of the Sun's Light being tranſmitted into a darkned room, if you illuminate a ſheet of White Paper by that Light, Reflected from a Body of any Colour, the Paper will always appear of the Colour of that Body, by whoſe Reflected Light it is illuminated. If it be a Red Body, the Paper will be Red; if a Green Body, it will be Green; and ſo of the other Colours. The Reaſon is, that the Fibres or Threads, of which the Paper conſiſts, are all Tranſparent and Specular; and ſuch Subſtances are known to Reflect Colours without changing them. To know therefore, to what Species of Colour a Grey belongs, place any Grey Body (ſuppoſe a Mixture of Painters Colours) in the ſaid Light, and the Paper being illuminated by its Reflection ſhall appear White. And the ſame thing will happen, if it be illuminated by Reflection from a Black Subſtance.

Theſe therefore are all of one Species; but yet they ſeem diſtinguiſh'd not only by Degrees of Luminouſneſs, but alſo by ſome other Inequalities, whereby they become more harſh or pleaſant. And the diſtinction ſeems to be, that Greys, and perhaps Blacks, are made by an uneven Defect of Light, conſiſting as it were of many little veins or ſtreams, which differ either in Luminouſneſs, or in the unequal diſtribution of diverſly Coloured Rays; ſuch as ought to be caused by Reflection from a Mixture of White and Black, or of diversly Coloured Corpuscles. But when such Imperfectly Mixt Light is by a second Reflection from the Paper more evenly and uniformly blended, it becomes more pleasant, and exhibits a faint or shadowed Whiteness. And that such little Irregularities as these may cause these differences, is not improbable, if we consider, how much Variety may be caused in Sounds of the same Tone, by irregular and uneven Jarrings. And besides, these differences are so little, that I have sometimes doubted, whether they be any at all, when I have considered, that a Black and White Body being placed together, the one in a strong Light, and the other in a very faint Light, so proportioned that they might appear equally Luminous, it has been difficult to distinguish them, when viewed at distance, unless when the Black seemed more Blueish; and the White Body in a Light still fainter, hath, in comparison of the Black Body, it self appeared Black.

This leads me to another way of Compounding Whiteness; which is, that, if four or five Bodies of the more eminent Colours, or a Paper painted all over, in several parts of it with those several Colours in a due Proportion, be placed in the said Beam of Light, the Light Reflected from those Colours to another White Paper, held at a convenient distance, shall make that Paper appear White. If it be held too near the Colours, its Parts will seem of those Colours that are nearest them; but by removing it further, that all its parts may be equally illuminated by all the Colours, they will be more and more diluted, until they become perfectly White. And you may further observe, that if any of the Colours be intercepted, the Paper will no longer appear White, but of the other Colours which are not intercepted. Now that this Whiteness is a Mixture of the severally Coloured Rays, falling confusedly on the Paper, I see no reason to doubt of; because, if the Light became Uniform and Similar before it fell confusedly on the Paper, it must much more be Uniform, when at a greater distance it falls on the Spectator's Eye, and so the Rays, which come from several Colours, would in no Qualities differ from one another, but all of them exhibite the same Colour to the Spectator, contrary to what he sees.

Not much unlike this Instance it is, that, if a Polish'd piece of Metal be so placed, that the Colours appear in it as in a Looking-glass, and then the Metal be made rough, that by a confused Reflection those apparent Colours may be blended together, they shall disappear, and by their Mixture cause the Metal to look White.

But further to enforce this Experiment; If instead of the Paper, any White Froth consisting of small Bubbles, be illuminated by Reflexion from the aforesaid Colours, it shall to the naked eye seem White, and yet through a good Microscope the several Colours will appear distinct on the Bubbles, as if seen by Reflexion from so many Spherical Surfaces. With my naked Eye, being very near, I have also discerned the several Colours on each Bubble; and yet at a greater distance, where I could not distinguish them apart, the Froth hath appeared entirely White. And at the same distance, when I lookt intently, I have seen the Colours distinctly on each Bubble; and yet by straining

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my Eyes as if I would look at ſomething afar off beyond them, thereby to render the Viſion confuſed, the Froth has appeared without any other Colour than Whiteneſs. And what is here ſaid of Froths, may eaſily be underſtood of the Paper or Metal, in the foregoing Experiments. For their parts are Specular Bodies, like theſe Bubbles; And perhaps with an excellent Microſcope, the Colours may be alſo ſeen intermixedly Reflected from them.

In proportioning the ſeverally Coloured Bodies to produce theſe effects, there may be ſome Niceneſs; and it will be more convenient, to make uſe of the Colours of the Priſm, caſt on a Wall, by whoſe Reflexion the Paper, Metal, Froth, and other White Subſtances may be illuminated. And I uſually made my tryals this way, becauſe I could better exclude any ſcattering Light from mixing with the Colours to dilate them.

To this way of Compounding Whiteneſs, may be referred that other, by Mixing Light after it hath been trajected through tranſparently Coloured Subſtances. For Inſtance, if no Light be admitted into a Room but only through Coloured Glaſs; whoſe ſeveral parts are of ſeveral Colours in a pretty equal proportion; all White things in the Room ſhall appear White, if they be not held too near the Glaſs. And yet this Light, with which they are illuminated, cannot poſſibly be Uniform; becauſe, if the Rays, which at their entrance are of divers Colours, do in their progreſs through the Room, ſuffer any alteration to be reduced to an Uniformity, the Glaſs would not in the remoteſt parts of the Room appear of the very ſame Colour, which it doth when the Spectator's Eye is very near it: Nor would the Rays, when tranſmitted into another Dark Room through a little hole in an oppoſite Door or Partition-wall, project on a Paper the Species or Repreſentation of the Glaſs in its proper Colours.

And, by the by, this ſeems a very fit and cogent Inſtance of ſome other parts of my *Theory*, and particularly of the 13th *Propoſition*. For, in this Room all natural Bodies whatever appear in their proper Colours. And all the Phænomena of Colours in Nature, made either by Refraction or without it, are here the ſame as in the open Air. Now, the Light in this Room being ſuch a diſſimilar Mixture, as I have deſcribed in my *Theory*, the Cauſes of all theſe Phænomena muſt be the ſame that I have there aſſigned. And I ſee no reaſon to ſuſpect, that the ſame Phænomena ſhould have other cauſes in the open Air.

The Succeſs of this Experiment, may be eaſily conjectured by the Appearances of things in a Church or Chapel, whoſe Windows are of Coloured Glaſs; or in the open Air, when it is illuſtrated with Clouds of various Colours.

There are yet other ways, by which I have produced Whiteneſs; as by caſting ſeveral Colours from two or more Priſms upon the ſame Place; by Refracting a Beam of Light with two or three Priſms ſucceſſively, to make the Diverging Colours Converge again; by Reflecting one Colour to another; and by Looking through a Priſm on an Object of many Colours; and, (which is equivalent to the above mentioned way of mixing Colours by concave Wedges filled with Coloured Liquors) I have obſerved the Shadows of a Painted Glaſs Window to



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become White, where those of many Colours have at a great Distance interfered. But yet for further satisfaction, the Animadversor may try if he please, the Effects of four or five of such Wedges filled with Liquors of as many several Colours.

Besides all these, the Colours of Water-bubbles and other thin pellucid Substances, afford several instances of Whiteness produc'd by their Mixture; with one of which I shall conclude this particular. Let some Water, in which a convenient quantity of Soap or Washball is dissolved, be agitated into Froth, and, after that Froth has stood a while without further agitation, till you see the Bubbles, of which it consists, begin to break, there will appear a great variety of Colours, all over the top of every Bubble, if you view them near at hand; but if you view them at so great a distance that you cannot distinguish the Colours one from another, the Froth will appear perfectly White.

Thus much concerning the Design and Substance of the Animadversor's Considerations. There are yet some particulars to be taken notice of, before I conclude; as the Denyal of the *Experimentum Crucis*. On this I chose to lay the whole Stress of my Discourse; which therefore was the principal thing to have been objected against. But I cannot be convinced of its Insufficiency by a bare Denyal without assigning a Reason for it. I am apt to believe, it has been misunderstood: for otherwise it would have prevented the Discourses about Rarefying and Splitting of Rays; because the design of it is, to shew, that Rays of divers Colours considered apart, do at Equal Incidences suffer Unequal Refractions, without being Split, Rarefy'd, or any ways Dilated.

*Some Considerations upon this Doctrine of Colours; from* Paris, *by . . . . . . n. 96. p. 6086.*

VI. 1. Methinks that the most Important Objection, which is made against Mr. *Newton* by way of Querie, is that, Whether there be more than two sorts of Colours. For my part, I believe, that an Hypothesis, that should explain Mechanically, and by the Nature of Motion, the Colours Yellow and Blue, would be sufficient for all the rest, in regard that those others, being only more deeply charged, (as appears by the Prisms of Mr. *Hook*) do produce the dark or deep Red and Blue; and that of these four all the other Colours may be Compounded. Neither do I see why Mr. *Newton* doth not content himself with the two Colours, Yellow and Blue; for it will be much more easie to find an Hypothesis by Motion, that may explicate these two Differences, than for so many Diversities as there are of other Colours. And till he hath found this Hypothesis, he hath not taught us, what it is wherein consists the Nature and Difference of Colours, but only this Accident (which certainly is very considerable,) of their *different Refrangibility*.

As for the Composition of White made by all the Colours together, it may possibly be, that Yellow and Blue might also be sufficient for that: Which is worth while to try; and it may be done by the Experiment which Mr. *Newton* proposeth, by receiving against a Wall of a darkned Room the Colours of the Prism, and to cast their Reflected Light upon white Paper. Here you must hinder the Colours of the Extremities, *viz.* the Red and Purple, from striking against the Wall, and leave only the Intermediate Colours, Yellow, Green, and Blue, to see whether the Light of these alone would not make the

Paper appear White, as well as when they all give Light. I even doubt, whether the Lightest place of the Yellow Colour may not all alone produce that effect, and I mean to try it at the first Conveniency; for this thought never came into my Mind but just now. Mean time you may see, that if these Experiments do succeed, it can no more be said, that all the Colours are necessary to compound White, and that 'tis very probable, that all the rest are nothing but degrees of Yellow and Blue, more or less charged.

2. It seems to me, that N. takes an Improper way of Examining the Nature of Colours, whilst he proceeds upon Compounding those that are already Compounded. Perhaps he would sooner satisfie himself by resolving Light into Colours, as far as may be done by Art, and then by examining the Properties of those Colours apart, and afterwards by trying the effects of Re-conjoining two or more, or all of those; and lastly, by separating them again, to examine what changes that Re-conjunction had wrought in them. I have formerly shew'd, That all Colours cannot practically be derived out of the Yellow and Blue, and consequently that those Hypotheses are groundless which imply they may. If you ask what Colours cannot be be derived out of Yellow and Blue? I Answer, None of all those which I defined to be Original; and if he can shew by Experiment, how they may, I will acknowledge my self in an Error. Nor is it easier to frame an Hypothesis by assuming only two Original Colours, rather than an indefinite Variety; unless it be easier to suppose that there are but two Figures, Sizes, and Degrees of Velocity or Force of the Æthereal Corpuscles or Pulses, rather than an indefinite Variety; which certainly would be a harsh Supposition. No man wonders at the indefinite Variety of Waves of the Sea, or of Sands on the Shoar; but, were they all but two Sizes, it would be a very puzling Phænomenon. And I should think it as unaccountable, if the several Parts or Corpuscles, of which a shining Body consists, which must be supposed of various Figures, Sizes, and Motions, should impress but two sorts of Motion on the adjacent Æthereal Medium, or any other way beget but two sorts of Rays. But to examine how Colours may be explained Hypothetically, is besides my purpose. I never intended to shew wherein consists the Nature and Difference of Colours, but only to shew, that *de facto* they are Original and Immutable Qualities of the Rays which exhibit them; and to leave it to others to Explicate by Mechanical Hypotheses, the Nature and Difference of these Qualities; which I take to be no difficult matter. But I would not be understood as if their difference consisted in the *Different Refrangibility* of those Rays; for that *Different Refrangibility* conduces to their Production no otherwise, than by separating the Rays whose Qualities they are. Whence it is, That the same Rays exhibit the same Colours when separated by any other means; as by their *Different Reflexibility*, a Quality not yet discoursed of.

*Answered, by Mr. Newton. n. 97. p. 6108.*

In the next particular, where N. would shew, that it is not necessary to mix all Colours for the Production of White; the Mixture of Yellow, Green, and Blue, without Red and Violet, which he propounds for that end, will not produce White, but Green; and the Brightest part of the Yellow will afford no other Colour but Yellow, if the Experiment be made in a Room well Darkened, as it ought; because the Coloured Light is much weakened by the Reflection, and so apt to be diluted by the mixing of any other scattering Light. But yet there is an Experiment or two formerly mentioned, by which I have produced White out of two Colours alone, and that variously; as out of Orange and a full Blue, and out of Red and Pale Blue, and out of Yellow and Violet, as also out of other pairs of intermediate Colours. The most convenient Experiment for performing this, was that of casting the Colours of one Prism upon those of another, after a due manner. But what N. can deduce from hence, I see not. For the two Colours were Compounded of all others, and so the resulting White, (to speak properly,) was Compounded of them all, and only Decompounded of those two. For Instance, the Orange was compounded of Red, Orange, Yellow, and some Green; and the Blue, of Violet, full Blue, light Blue, and some Green, with all their Intermediate Degrees; and consequently the Orange and Blue together made an Aggregate of all Colours to constitute the White. Thus if one mix Red, Orange, and Yellow Powders to make an Orange; and Green, Blue, and Violet Colours to make a Blue; and lastly, the two Mixtures to make a Grey; that Grey, though Decompounded of no more than two Mixtures, is yet Compounded of all the six Powders, as truly as if the Powders had been all mix'd at once.

This is so plain, that I conceive there can be no further Scruple; especially to them who know how to examine, whether a Colour be Simple or Compounded, and of what Colours it is Compounded; which having explained in another place, I need not now repeat. If therefore N. would conclude any thing, he must shew how White may be produced out of two Uncompounded Colours; which when he hath done, I will further tell him, why he can conclude nothing from that. But I believe there cannot be found an Experiment of that kind; because, as I remember, I once tryed, by gradual Succession, the Mixture of all Pairs of Uncompounded Colours; and though some of them were Paler, and nearer to White, than others, yet none could be truly call'd White. But it being some Years since this tryal was made, I remember not well the Circumstances, and therefore recommend it to others to be tryed again.

*A Reply, by Monsieur N. n. 97. p. 6112.*

3. Seeing that Mr. *Newton* maintains his Opinion with so much concern, I list not to dispute. But what means it, I pray, that he saith, *Though I should shew him, that the White could be produced of only two Uncompounded Colours, yet I could Conclude nothing from that?* And yet he hath affirmed, that to compose the White, all Primitive Colours are Necessary.

*Answered, by Mr.* Newton. n. 96. p. 6087.

4. In my saying, that when *Monsieur N.* hath shewn *how White may be propounded out of two Uncompounded Colours, I will tell him, why he can Conclude nothing from that*; my Meaning was, That such a White, (were there any such) would have different properties from the White which I had respect to, when I described my *Theory*, that is, from the White of the Sun's immediate Light, of the ordinary Objects of our Senses, and of all White Phænomena that have hitherto faln under my Observation. And those different Properties would evince it to be of a different constitution; Insomuch, that such a Production

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of White would be ſo far from contradicting, that it would rather illuſtrate and confirm my Theory; becauſe by the difference of that from other Whites it would appear, that other Whites are not Compounded of only two Colours like that. And therefore if *Monſieur* N. would prove any thing, it is requiſite that he do not only produce out of two Primitive Colours a White, which to the naked Eye ſhall appear like other Whites, but alſo ſhall agree with them in all other Properties.

But to let you underſtand, wherein ſuch a White would differ from other Whites, and why from thence it would follow that other Whites are otherwiſe Compounded, I ſhall lay down this Poſition.

*That a Compounded Colour can be reſolved into no more ſimple Colours than thoſe of which it is Compounded.*

This ſeems to be ſelf evident, and I have alſo tryed it ſeveral ways, and particularly by this which follows. Let α repreſent an Oblong piece of White Paper about ½ or ¼ of an inch broad, and illuminated in a Dark Room, with a Mixture of two Colours caſt upon it from two Priſms, ſuppoſe a deep Blue and Scarlet, which muſt ſeverally be as Uncompounded as they can conveniently be made. Then, at a convenient diſtance, ſuppoſe of ſix or eight Yards, view it through a clear triangular Glaſs or Cryſtal Priſm held parallel to the Paper, and you ſhall ſee the two Colours parted from one another in the faſhion of two Images of the Paper, as they are repreſented at β and γ, where ſuppoſe β the Scarlet, and γ the Blue, without Green or any other Colour between them. Fig. 76.

Now from the aforeſaid *Poſition* I deduce theſe two *Concluſions*. 1. That if there were found out a way to Compound White of two Simple Colours only, that White would be again reſolvable into no more than two. 2. That if other Whites, as that of the Sun's Light, &c. be reſolvable into more than two ſimple Colours (as I find by Experiment that they are) then they muſt be Compounded of more than two.

To make this plainer, ſuppoſe that A repreſents a White Body, illuminated by a direct Beam of the Sun tranſmitted through a ſmall Hole into a dark Room, and α ſuch another Body, illuminated by a Mixture of two ſimple Colours, which, if poſſible may make it alſo appear of a White Colour exactly like A. Then, at a convenient diſtance, view theſe two Whites thro' a Priſm, and A will be changed into a Series of all Colours, Red, Yellow, Green, Blue, Purple, with their intermediate degrees ſucceeding in order, from B to C. But α, according to the aforeſaid Experiment, will only yield thoſe two Colours of which it was Compounded, and thoſe not conterminate like the Colours at B C, but ſeparate from one another, as at β and γ, by means of the different Refrangibility of the Rays to which they belong. And thus by comparing theſe two Whites, they would appear to be of a different Conſtitution, and A to conſiſt of more Colours than α. So that what *Monſieur* N. contends for, would rather advance my Theory by the acceſs of a new kind of White, than conclude againſt it. But I ſee no hopes of Compounding ſuch a White.

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As for *Monsieur N.* his Expression, *That I maintain my Doctrine with some Concern*, I confess it was a little Ungrateful to me, to meet with Objections which had been answered before, without having the least reason given me why those Answers were insufficient. Those Answers were to shew, that there are other simple Colours besides Blue and Yellow; I instanced in a Simple or Homogeneal Green, such as cannot be made by mixing Blue and Yellow or any other Colours. And I also shew'd why, supposing that all Colours might be produced out of two, yet it would not follow that those two are the only Original Colours. The Reasons I desire you would compare with what hath been now said of White. And so the necessity of all Colours to produce White, might have appeared by that Experiment, where I say, That if any Colour at the Lens be intercepted, the Whiteness (which is Compounded of them all) will be changed into (the result of) the other Colours.

Vid. sup. p. 5149. & seq.

However, since there seems to have happen'd some misunderstanding between us, I shall endeavour to explain my self a little further in these things, according to the following Method.

*Definitions.*] 1. I call that Light Homogeneal, Similar, or Uniform, whose Rays are equally Refrangible.

2. And that Heterogeneal, whose Rays are unequally Refrangible.

*Note.* There are but three Affections of Light, in which I have observed its Rays to differ, *viz. Refrangibility*, *Reflexibility*, and *Colour*; and those Rays which agree in Refrangibility agree also in the other two, and therefore may well be defined Homogeneal especially since Men usually call those things Homogeneal, which are so in all Qualities that come under their Knowledge, tho' in other Qualities, that their Knowledge extends not to, there may possibly be some Heterogeneity.

3. Those Colours I call Simple, or Homogeneal, which are exhibited by Homogeneal Light.

4. And those Compound or Heterogeneal, which are exhibited by Heterogeneal Light.

5. Different Colours I call not only the more eminent Species, Red, Yellow, Green, Blue, Purple, but all other the minutest Gradations; much after the same manner that not only the more Eminent Degrees in Musick, but all the least Gradations are esteemed different Sounds.

*Propositions.*] 1. The Sun's Light consists of Rays differing by indefinite degrees of Refrangibility.

2. Rays which differ in Refrangibility when parted from one another, do proportionally differ in the Colours which they exhibit. These two *Propositions* are matter of fact.

3. There are as many Simple or Homogeneal Colours as degrees of Refrangibility. For to every degree of Refrangibility belongs a different Colour, by *Prop.* 2. and that Colour is Simple, by *Def.* 1, and 3.

4. Whiteness

4. Whiteneſs, in all reſpects like that of the Sun's Immediate Light and of all the uſual Objects of our Senſes, cannot be Compounded of two Simple Colours alone. For ſuch a Compoſition muſt be made by Rays that have only two degrees of Refrangibility, by *Def.* 1, and 3; and therefore it cannot be like that of the Sun's Light, by *Prop.* 1; Nor, for the ſame Reaſon, like that of ordinary White Objects.

5. Whiteneſs, in all reſpects like that of the Sun's Immediate Light, cannot be Compounded of Simple Colours, without an indefinite variety of them. For to ſuch a Compoſition there are requiſite Rays endued with all the indefinite degrees of Refrangibility, by *Prop.* 1. And thoſe infer as many ſimple Colours, by *Def.* 1, and 3. and *Prop.* 2, and 3.

To make theſe a little plainer, I have added alſo the *Propoſitions* that follow.

6. The Rays of Light do not act on one another in paſſing thro' the ſame *Medium*. This appears by ſeveral former Paſſages, and is capable of further Proof.

7. The Rays of Light ſuffer not any change of their Qualities from Refraction.

8. Nor afterwards from the adjacent quiet Medium. Theſe two Propoſitions are manifeſt *de facto* in Homogeneal Light, whoſe Colour and Refrangibility is not at all changeable either by Refraction or by the Contermination of a quiet Medium. And as for Heterogeneal Light, it is but an Aggregate of ſeveral ſorts of Homogeneal Light, no one ſort of which ſuffers any more Alteration than if it were alone, becauſe the Rays act not on one another, by *Prop.* 6. And therefore the Aggregate can ſuffer none. Theſe two Propoſitions alſo might be further proved apart by Experiments, too long to be here deſcribed.

9. There can no Homogeneal Colours be educed out of Light by Refraction, which were not commixt in it before; becauſe by *Prop.* 7, and 8, Refraction changeth not the Qualities of the Rays, but only ſeparates thoſe which have diverſe qualities, by means of the different Refrangibility.

10. The Sun's Light is an Aggregate of an Indefinite variety of Homogeneal Colours; by *Prop.* 1, 3, and 9. And hence it is, that I call Homogeneal Colours alſo Primitive or Original.

*Animadverſions on this Theory of Light and Colours; by Mr. Fr.* Linus. n. 110. p. 217.

VII. 1. I doubt not of what Mr. *Newton* affirms; and have my ſelf ſometimes in like Circumſtances obſerved the like Difference between the Length and Breadth of the Coloured *Spectrum*; but never found it ſo when the Sky was Clear and free from Clouds, near the Sun: but then only appeared this Difference of Length and Breadth, when the Sun either ſhined thro' a White Cloud, or enlightned ſome ſuch Clouds near unto it. And then indeed it was no marvel, the ſaid *Spectrum* ſhould be Longer than Broad; ſince the Cloud or Clouds, ſo Enlightned, were in order to thoſe Colours like to a great Sun, making a far greater Angle of Interſection in the Hole, than the true Rays of the Sun do make; and therefore are able to enlighten the whole Length of the Priſm, and not only ſome ſmall part thereof, as we ſee enlightned by the true Sun-beams coming thro' the ſame little Hole. And this we behold alſo in the true Sun-beams, when they enlighten the whole Priſm: for altho' in a Clear Heaven, the Rays of the Sun paſſing thro' the ſaid Hole, never make a Spectrum Longer than Broad, becauſe they then occupy but a ſmall part of the Priſm; yet if the Hole be ſo much bigger as to enlighten the whole Priſm, you ſhall preſently ſee the Length of the Spectrum much exceed its Breadth; which exceſs will be always ſo much the greater, as the Length of the Priſm exceeds its Breadth. From whence I conclude, That the Spectrum, this *Learned Author* ſaw much Longer than Broad, was not affected by the true Sun-beams, but by Rays proceeding from ſome bright Cloud, as is ſaid; and by Conſequence, that the *Theory of Light* grounded upon that *Experiment* cannot ſubſiſt.

What I have here ſaid, needs no other Confirmation than meer Experience, which any one may quickly try; neither have I only tryed the ſame upon this occaſion, but near 30 Years ago ſhewed the ſame, together with divers other Experiments of Light, to that Worthy Promoter of Experimental Philoſophy, Sir *Kenelm Digby*, who coming into theſe Parts to take the *Spaw* Waters, reſorted often-times to my Darkned Chamber *, to ſee theſe various Phænomena of Light, made by divers Refractions and Reflections, and took Notes upon them; which Induſtry if they alſo had uſed, who endeavour to Explicate the aforeſaid Difference between the Length and Breadth of this Coloured *Spectrum*, by the received Laws of Refraction, would never have taken ſo impoſſibly a Task in hand.

* *At* Liege.

*Anſwer'd by* . . . . . . . . n. 110. *p.* 219.

2. Theſe Animadverſions ſeem to need no other Anſwer but this, that you would be pleaſed to conſider the Scheme in Mr. *Newton*'s ſecond Anſwer to *P. Pardies*, and reſt aſſured, That the Experiment, as 'tis repreſented, was try'd in Clear days, and the Priſm placed cloſe to the Hole in the Window, ſo that the Light had no Room to Diverge, and the Coloured Image made not Parallel (as in that conjecture) but Tranſverſe to the Axis of the Priſm.

*A Reply by Mr.* Fr. Linus. n. 121. *p.* 499.

3. *If theſe Aſſertions be admitted*, they do indeed directly cut off what I ſaid of Mr. *Newton*'s being deceived by a *bright Cloud.* But if we compare them with Mr. *Newton*'s firſt Relation of the Experiment, it will evidently appear, they cannot be admitted, as being directly contrary to what is there delivered. For there he tells us, *The Ends of the Coloured Image, he ſaw on the oppoſite Wall, near five times as Long as Broad, ſeemed to be Semicircular.* Now theſe *Semicircular Ends* are never ſeen in a clear day, as Experience ſhews. From whence follows againſt the *firſt* Aſſertion, that the Experiment was not made in a clear day. Neither are thoſe Semicircular Ends ever ſeen when the Priſm is placed cloſe to the Hole; which contradicts the *ſecond* Aſſertion. Neither are they ever ſeen when the Image is Tranſverſe to the Length or Axis of the Priſm, which directly oppoſes the *third* Aſſertion. But if in any of theſe three Caſes, the Image be made ſo much Longer than Broad (as eaſily it may, by turning the Priſm a little about its Axis, near *five times* as Long as Broad) then the one End thereof will run out into a ſharp Cone or Pyramis like the Flame of a Candle, and the other into a Cone ſomewhat more blunt; both which are far from ſeeming Semicircular: Whereas, if the Image be made not in a clear day but with a bright Cloud, and the Priſm not

placed

placed close to the Hole but in a competent distance from the same, then these Semicircular Ends always appear with the Sides thereof streight Lines, just as Mr. *Newton* describes them. Neither is the Length of the Image Transverse, but Parallel to the Length of the Prism. Out of all which evidently follows, that the Experiment was not made in a clear day; nor with the Prism close to the Hole; nor yet with the Image Transverse, but by a bright Cloud and a Parallel Image (as I conjectured;) and I hope you will also now say, I had good Reason so to Conjecture, since it so well agrees with the Relation. And Experience will also shew you, if you please to make Tryal, as it was made in a Dark Chamber, and observe the difference between such an Image made by a bright Cloud, and another made by the Immediate Rays of the Sun: For, the former you shall always find Parallel, with the Ends Semicircular; but the latter you shall find Transverse, with the Ends Pyramidical, as aforesaid, whensoever it appears so much Longer than Broad.

More might be said out of the same Relation, to shew that the Image was not Transverse. For if it had been Transverse, Mr. *Newton*, so well skill'd in *Opticks*, could not have been surprised (as he says he was) to see the Length thereof so much to exceed the Breadth; it being a thing so obvious and easie to be Explicated by the Ordinary Rules of Refraction. That other Place also (where he says, the *Incident* Refractions were made in the Experiment *equal to the Emergent*,) proves again that the said Oblong Image was not Transverse, but Parallel. For it is impossible, the Transverse Image should be so much Longer than Broad, unless those two Refractions be made very Unequal, as both the Computation according to the common Rules of Refraction, and Experience testify.

4. What it is that imposes upon Mr. *Line* I cannot imagine; but I suspect he has not tryed the Experiment since he acquainted himself with my Theory, but depends upon his Old Notions, taken up before he had any hint given to observe the Figure of the Coloured Image. I shall desire him therefore, before he returns any Answer, to try it once more for his Satisfaction, and that according to this manner. *Answer'd by Mr.* Newton. n. 121. *p.* 501. n. 123. *p.* 556.

Let him take any Prism, and hold it so that its Axis may be Perpendicular to the Sun's Rays, and in this Posture let it be placed as close as may be to the Hole through which the Sun shines into a dark Room, which Hole may be about the Bigness of a Pease. Then let him turn the Prism slowly about its Axis, and he shall see the Colours move upon the opposite Wall, first towards that place to which the Sun's Direct Light would pass, if the Prism were taken away, and then back again. When they are in the middle of these two contrary Motions, that is, when they are nearest that place to which the Sun's Direct Ray tends, there let him stop; for then are the Rays equally Refracted on both Sides the Prism. In this Posture of the Prism let him observe the Figure of the Colours, and he shall find it not Round, as he contends, but Oblong, and so much the more Oblong as the Angle of the Prism, comprehended by the Refracting Plains, is Bigger, and the Wall, on which the Colours are cast, more distant from the Prism; the Colours Red, Yellow, Green, Blue, Purple, succeeding in order not from one Side of the Figure to the other, as in Mr. *Line*'s conjecture, but from one End to the

 other;

other; and the Length of the Figure being not Parallel but Transverſe to the Axis of the Priſm. After this manner I uſed to try the Experiment; and it will not ſucceed well if the Day be not Clear, and the Priſm placed Cloſe to the Hole, or ſo near at leaſt, that all the Sun's Light that comes from the Hole may paſs through the Priſm alſo, ſo as to appear in a Round Form, if intercepted by a Paper immediately after it has paſſed the Priſm.

When Mr. *Line* has tryed this, I could wiſh he would proceed a little further, to try that which I called the *Experimentum Crucis*. For when he has tryed them (which by his denying them, I know he has not done yet as they ſhould be tryed) I preſume he will reſt ſatisfied. It may be tryed (though not ſo perfectly) even without darkning a Room, or the Expence of any more Time than half a quarter of an Hour.

*The Experiment of Mr.* Line *Affirmed by Mr.* Gaſcoigne, *n.* 121. *p.* 503. *marg.*

5. Mr. *Linus* (now deceaſed) *tryed the Experiment* again and again, and called divers on purpoſe to ſee it, nor ever made difficulty to ſhew it to any one, who either by chance came to his Chamber as he was doing it, or ſhewed the leaſt deſire to ſee the ſame; ſo that for point of Experience, Mr. *Newton* cannot be more confident on his ſide, than we are here on the other; who are fully perſwaded, that, unleſs the diverſity of placing the Priſm, or the bigneſs of the Hole, or ſome other ſuch circumſtance, be the cauſe of the difference betwixt them, Mr. *Newton*'s Experiment will hardly ſtand.

*Anſwer'd by Mr.* Newton. *Ibid.* n. 123. *p.* 556.

6. By Mr. *Gaſcoigne*'s Letter one might ſuſpect, that Mr. *Linus* tryed the Experiment ſome other way than I did; and therefore I ſhall expect, till his Friends have tryed it according to my late Directions. In which tryal it may poſſibly be a further guidance to them, to acquaint them, that the Priſm caſts from it ſeveral Images. One is, that Oblong one of Colours which I mean; and this is made by two Refractions only. Another there is made by two Refractions and an Intervening Reflection; and this is Round and Colourleſs, if the Angles of the Priſm be exactly equal; but if the Angles at the Reflecting Baſe be not Equal, it will be Colour'd, and that ſo much the more, by how much Unequaller the Angles are; but yet not much Unround, unleſs the Angles be very Unequal. A Third Image there is, made by one ſingle Reflection, and this is always Round and Colourleſs. The only danger is in miſtaking the *Second* for the *Firſt*. But they are diſtinguiſhable not only by the Length and Lively Colours of the *firſt*, but by its different motion too: For, whilſt the Priſm is turned continually the ſame way about its Axis, the *Second* and *Third* move ſwiftly, and go always on the ſame way till they diſappear; but the *Firſt* moves ſlow, and grows continually ſlower till it be Stationary, and then turns back again, and goes back faſter and faſter, till it vaniſh in the Place where it began to appear.

If without Darkning their Room they hold the Priſm at their Window in the Sun's open Light, in a ſuch a poſture that its Axis be perpendicular to the Sun-Beams, and then turn it about its Axis, they cannot miſs of ſeeing the firſt Image; which having found, they may double up a Paper once or twice, and make a round Hole in the middle of it, about $\frac{1}{2}$ or $\frac{1}{4}$ of an inch broad, and hold the Paper immediately before the Priſm, that the Sun may ſhine on the Priſm through that Hole; and the Priſm being ſtayed, and

held

held ſteddy in that Poſture which makes the Image Stationary; if the Image then fall directly on an oppoſite Wall, or on a Sheet of Paper placed at the Wall, ſuppoſe 15 or 20 Foot from the Priſm, or further off; they will ſee the Image in ſuch an Oblong Figure as I have deſcribed, with the Red at one end, the Violet at the other, and a Blueiſh Green in the middle: And if they Obſcure their Room, as much as they can, by drawing Curtains or otherwiſe, it will make the Colours the more conſpicuous.

This Direction I have ſet down, that no Body, into whoſe Hands a Priſm ſhall happen, may find difficulty or trouble in trying it. But when Mr. *Linus*'s Friends have tryed it thus, they may proceed to repeat it in a dark Room with a leſs Hole made in their Window-ſhut. And then I ſhall deſire that they will ſend a full and clear deſcription how they tryed it. I ſhould be glad too, if they will favour me with a deſcription of the Experiment as it hath been hitherto tryed by Mr. *Linus*, that I may have an Opportunity to conſider what there is in that which makes againſt me.

7. Mr. *Gaſcoigne* wanting Convenience to make the Experiment, according to the freſh Directions from Mr. *Newton*, requeſted me to ſupply his Want. *Exceptions by Mr.* Lucas. n. 128. *p.* 692.

The Vertical Angle of my Priſm was 60 Degrees; the Diſtance of the Wall, whereon the Coloured *Spectrum* appeared, from the Window, about 18 Foot; the Diameter of the Hole in the Window-ſhuts about $\frac{1}{4}$ inch, which upon Occaſions I contracted to half the ſaid Diameter; but ſtill with equal Succeſs as to the main of the Experiment. The Refractions on both ſides the Priſm were, as near as I could make them, Equal, and conſequently about 48 deg. 40 min. the Refractive Power of Glaſs being computed according to the Ratio of the Sines 2 to 3. The diſtance of the Priſm from the Hole in the Shuts was about 2 Inches; the Room darkened to that Degree as to Equal the Darkeſt Night, while the Hole in the Shuts was covered.

Now as to the Iſſue of my Tryals; I conſtantly found the Length of the Coloured Image, (Tranſverſe to the Axis of the Priſm) conſiderably greater than its Breadth, as often as the Experiment was made on a Clear Day; but if a Bright Cloud were near the Sun, I found it ſometimes exactly as Mr. *Line* wrote you, namely Broader than Long, eſpecially while the Priſm was placed at a great diſtance from the Hole. Which Experiment will not, I conceive, be queſtion'd by Mr. *Newton*, it being ſo agreeable to the receivd Laws of Refractions. And indeed the Obſervations of theſe two Learne Perſons, as to this particular, are eaſily reconcileable to each other, and bot to Truth; Mr. *Newton* contending only for the Length of the Image (Tranſverſe to the Axis of the Priſm) in a very Clear day; whereas Mr. *Line* only maintained the exceſs of Breadth, Parallel to the ſame Axis, while the Sun is in a Bright Cloud. Though as to what is further delivered by Mr. *Newton*, and oppoſed by Mr. *Line*, namely that the Length of the Coloured Image was *five times* the Diameter of its Breadth; I never yet have found the exceſs above thrice the Diameter, or at moſt $3\frac{1}{2}$, while the Refractions on both Sides the Priſm were equal. So much as to the matter of Fact.

Now as to Mr. *Newton's Theory* of *Light* and *Colours*, I confeſs his neat Sett of very Ingenious and Natural Inferences, was to me upon the firſt Peruſal

usal a strong Conjecture in favour of his New Doctrine; I having formerly observed the like Chain of Inferences upon search into Natural Truths. But since several Experiments of Refractions remain still untouched by him, I conceived a further search into them would be very proper, in order to a further discovery of the Truth of his Assertion. For, accordingly as they are found either agreeing with, or disagreeing from, his New Theory, they must needs much strengthen, or wholly overthrow the same. The Experiments I pitched upon for this purpose are as follow.

1. Having frequently observed, that the Form of Objects viewed in the Microscope (or rather of the Microscope it self) consists almost in an indivisible Point, I concluded, two very small pieces of Silk, the one Scarlet the other Violet Colour, placed near together, should, according to Mr. *Newton's* Theory, appear in the Microscope in a very different degree of Clarity, in regard their Unequal Refrangibility must cause the Scarlet Rays, or Species, to over-reach the Retina, while placed in the due Focus of the Violet ones, and consequently must occasion a sensible Confusion in the Vision of the former, one and the same Point of the Scarlet Object affecting several Nerves in the Retina. Yet upon frequent Tryals I have not been able to perceive any Inequality in this point.

2. The Second Experiment I made in Water. I took a Brass Ruler, and fastning thereunto several pieces of Silk, Red, Yellow, Green, Blue, and Violet, I placed it at the Bottom of a Square Vessel of Water: then I retired from the Vessel so far as not to be able to see the aforesaid Ruler and Coloured Silks, otherwise than by the help of the Refracted Ray. Now, did Mr. *Newton's* Doctrine hold, I conceived I should not see all the mentioned Colours in a Streight Line with the Ruler, in regard the Unequal Refrangibility of Different Rays must needs displace some more than others. Yet in Effect, upon many Tryals, I constantly found them in as Streight a Line, as the bare Ruler had appeared in.

3. To advance this Experiment, I adjoined a Second Refraction to the former of Water, by placing my Prism so as to receive perpendicularly the Refracted Species of the Silk and Ruler; whereby only the Emergent Species suffered a second Refraction. But still with equal Success, as to their appearing in a Streight Line to the Eye placed behind the Prism.

4. To these two Refractions I further added a third, by receiving the Coloured Species obliquely upon the Prism; whereby both Incident and Emergent Species suffered their respective Refractions. But still with the same Success as formerly, as to the Streight Line they appeared in.

For further assurance in this Experiment, lest prepossession, occasion'd from previous Knowledge of the Silks situation in a Streight Line, might possibly prejudice the Judgment of the Eye (as sometimes I have observed to happen to the Judgment the Eye passeth upon the distance of Objects) I called into the Room some unconcerned Persons, wholly Ignorant of what the Experiment aimed at; and demanding whether they saw not the Coloured Silks and Ruler in a Crooked Line? They Answered in the Negative.

5. The

5. The next Experiment I made in uncompounded Colours (as Mr. *Newton* terms them, *Prop.* 5. & 13.) as follows. Having cast two Coloured Images upon the Wall, so as the Scarlet Colour of the one did fall in a Streight Line (Parallel to the Horizon) with the Violet of the other: I then looked upon both through another Prism, and found them still appear in a Streight Line Parallel to the Horizon, as they had formerly done to the Naked Eye. Now According to Mr. *Newton*'s Assertion of Different Refrangibility in Different Rays, I conceive the Violet Rays should suffer a greater Refraction in the Prism at the Eye, than the Scarlet ones; and consequently both Colours should not appear in a Streight Line Parallel to the Horizon.

6. Another Experiment I made, in order to some further Discovery of that surprizing Phænomenon of the Coloured Image, which occasioned Mr. *Newton*'s Ingenious Theory of Light and Colours, as also his excellent Invention of the Reflecting Telescope and Microscope. Having then sometimes suspected that not only the Direct Sun-Beams, but also other Extraneous Light, might possibly influence the Coloured *Spectrum*, I hoped to discover the Truth of this suspicion by means of the Sun-spots, made to appear in the Coloured Image, by placing a Telescope behind the Prism. But my Endeavours proving Ineffectual herein, by Reason of some Intervening difficulties, I thought at Length of a more feasible Method in order to the designed Discovery, as in the following Experiment.

I fastned a very White Paper-Circle (about an Inch in Diameter) upon my Window-shuts; and beholding it thro' my Prism, I found a Coloured Image Painted thereby upon my Retina, answerable in almost all respects to the former of the Sun-beams upon the Wall, especially when the Paper-Circle was indifferently well illuminated. This Image indeed appeared contrary to the former, as to the Situation of Colours, that is, the Scarlet appearing above, the Violet below, tho' but faint. But this I was not surprised at, having observ'd upon dissecting the Eye, that Objects are Painted on the Retina after a contrary Posture to what they appear to sight. Having thus rendered the Coloured Image much more tractable than formerly it was, I conceived good hopes of some further Discovery in the point mentioned. *Fig.* 77.

In pursuance then of my former suspicion, having fixed my Prism in a steady Posture, I caused the Paper C, to be applied close up the Paper-Circle *a b d*: whereupon the former Violet *d*, and the Scarlet Colour of C, vanished into Whiteness. Next I removed the mentioned Circle from the Shuts, and placed it in the open Window, supported only by the edge *d*: whereupon, to my astonishment, all the former Colours exchanged Postures in the Retina, the Scarlet now appearing below, the Violet above; the Intermediate Colours scarce discernable. And here, on the by, 'tis very Remarkable, that during this Observation, I clearly perceived both Blue and Scarlet Light to be Transparent, I being able to discern several Objects thro' both, namely Steeples opposite to my Window. Whence it follows, that these Colours do in great part arise from the Neighbouring Light. Lastly, I placed the Paper-Circle anew, so as the one half *b*, was fastned to the shuts, the other Semicircle *a*, being exposed to the open Air. Whereupon the Semicircle *a*, became

came bordered with Violet above, Scarlet below; but the other Semicircle *b*, quite contrary. Hence I make the following Inferences.

First, That not only the Light Reflected from the Paper-Circle but also from the ambient Air hath great Influence upon the Coloured Image, especially as to the Violet and Scarlet Colours. Whence perchance it will not hereafter seem strange, that the Coloured Spectrum on the Wall is so Long, but only that the Breadth is not greater. Secondly, Were there a more Luminous Body behind the Sun, we should in all likelihood have the Colours of the Spectrum in a contrary Situation to what they appear in at present; Whence (thirdly) it seems to follow, that the present Situation and Order of Colours, ariseth not from any intrinsical Property of *Refrangibility*, (as maintained by Mr. *Newton*) but from Contingent and Extrinsical Circumstances of Neighbouring Objects. For accordingly as the Body behind the Paper-Circle was more or less illuminated than the Circle it self, all the several Colours changed their Situation.

8. The next Experiment was made in Order to Mr. *Newton*'s Doctrine of Primary Colours, as *Prop*. 5. Having covered the Hole in the Window-shuts with a thin slice of Ivory, the Transmitted Light appeared Yellow; but upon adding three, four, or more slices, it became Red. Whence it seems to follow, that Yellowness of Light is not a Primary Colour, but a Compound of Red, &c.

9. The last Experiment was made in reference to Mr. *Newton*'s 12th *Prop.* where from his own Principles he renders a very Plausible Reason of a surprising Phænomenon, related by Mr. *Hooke*; namely of two Liquors, the one Blue, and the other Red, both severally Transparent, yet both, if placed together, became Opake. The Reason whereof, saith Mr. *Newton*, is, because if one Liquor Transmitted only Red, the other only Blue, no Rays could pass thro' both.

In Reference then to this Point; I filled two small Glasses with flat Polished bottoms, the one with *Aqua fortis*, deeply died Blue; the other with Oyl of Turpentine died Red; both to that degree as to represent all Objects thro' them respectively Blue or Red. Then placing the one upon the other, I was able to discern several Bodies thro' both: whereas according to Mr. *Newton*'s Theory, no Object should appear thro' both Liquors; because if one Transmit only Red, the other only Blue, no Rays can pass thro' both.

P. S. *Just upon the close of the adjoyned Letter, I received from Mr.* Gascoigne *yours of* May *the* 4*th; wherein you are pleased to favour us with an exact Account of the Famous Experiment of the Coloured Spectrum, lately exhibited before the* Royal Society. *I was much Rejoyced to see the Tryals of that Illustrious Company agree so exactly with ours here, tho' in somewhat ours disagree from* Mr. Newton.

*Answered, by Mr.* Newton. *ibid.* p. 698.

8. The things opposed by Mr. *Line* being upon Tryals found true and granted me; I begin with the New Question about the Proportion of the Length of the Image to its Breadth. And it is no wonder, that Mr. *Lucas* found the Image

Image ſhorter than I did, ſeeing he tryed the Experiment with a leſs Angle.

The Angle indeed which I uſed was but about 63 degrees, 12 min. and his is ſet down 60 degrees: the difference of which from mine, being but 3 deg. 12 min. is too little to reconcile us, but yet it will bring us conſiderable nearer together. And if his Angle was not exactly meaſured, but the round number of 60 degrees ſet down by gueſs or by a leſs accurate meaſure (as I ſuſpect by the conjectural meaſure of the Refraction of his Priſm, by the Ratio of the Signs 2 to 3, ſet down at the ſame time, inſtead of an Experimental one,) then might it be two or three degrees leſs then 60 deg. if not ſtill leſs: And all this, if it ſhould be ſo, would take away the greateſt part of the difference between us.

But however it be, I am well aſſured, my own obſervation was exact enough. For I have repeated it divers times ſince the receipt of Mr. *Lucas*'s Letter, and that without any conſiderable difference of my Obſervations, either from one another, or from what I wrote before. And that it might appear experimentally, how the increaſe of the Angle increaſes the Length of the Image, and alſo that no body, who has a Mind to try the Experiment exactly, might be troubled to procure a Priſm which has an Angle juſt of the Bigneſs aſſigned by me; I tryed the Experiment with divers Angles, and have ſet down my Tryals in the following Table; where the firſt Column expreſſes the Angles of two Priſms which I uſed, which were meaſured as exactly as I could by applying them to the Angle of a Sector; and the ſecond Column expreſſes in inches the Length of the Image made by each of thoſe Angles; its Breadth being two inches, its Diſtance from the Priſm 18 Feet and four inches, and the Breadth of the Hole in the Window-ſhut $\frac{1}{4}$ of an inch.

| | *Angles* | *Lengths* | | *Angles* | *Lengths* |
|---|---|---|---|---|---|
| The firſt Priſm | 56°. 10′ | $7\frac{3}{4}$ | The ſecond Priſm | 54°. 0′ | $7\frac{1}{3}$ |
| | 60 . 24 | $9\frac{1}{2}$ | | 62 . 12 | $10\frac{1}{8}$ |
| | 63 . 26 | $10\frac{1}{3}$ | | 63 . 48 | $10\frac{3}{4}$ |

You may perceive, that the Length of the Images, in reſpect of the Angles that made them, are ſomething greater in the ſecond Priſm than in the firſt; but that was becauſe the Glaſs, of which the ſecond Priſm was made, had the greater Refractive Power.

The Days in which I made theſe tryals were pretty Clear, but not ſo clear as I deſired; and therefore afterwards meeting with a Day as Clear as I deſired, I repeated the Experiment with the ſecond Priſm, and found the Lengths of the Image made by its ſeveral Angles, to be about $\frac{1}{4}$ of an inch greater than before, the meaſures being thoſe ſet down in this Table.

| | *Angles* | *Lengths* |
|---|---|---|
| The ſecond Priſm | 54°. 0′ | $7\frac{2}{3}$ |
| | 62 . 12 | $10\frac{1}{2}$ |
| | 63 . 48 | 11 |

The reaſon of this difference, I apprehend, was, that in the cleareſt Days in the Light of the White Skies, which dilutes and renders inviſible the fainteſt Colours at the Ends of the Image, is a little diminiſhed in a clear Day, and ſo gives leave to the Colours to appear to a great Length; the Sun's Light at the ſame time becoming Brisker, and ſo ſtrengthning the Colours, and making the faint ones at the two ends more conſpicuous. For I have obſerved, that in Days ſomething Cloudy, whilſt the Priſm has ſtood unmoved at the Window, the Image would grow a little longer or a little ſhorter, accordingly as the Sun was more or leſs obſcur'd by thin Clouds which paſſed over it; the Image being ſhorteſt when the Cloud was brighteſt and the Sun's Light fainteſt. Whence 'tis eaſie to apprehend, that if the Light of the Clouds could be quite taken away, ſo that the Sun might appear ſurrounded with Darkneſs, or if the Sun's Light were much ſtronger than it is, the Colours would ſtill appear to a greater Length.

In all theſe Obſervations the Breadth of the Image was juſt two inches. But obſerving that the Sides of the two Priſms I uſed were not exactly plain, but a little Convex, (the Convexity being about ſo much as that of a double Convex-Glaſs of a ſixteen or eighteen Foot Teleſcope) I took a third Priſm, whoſe ſides were as much Concave as thoſe of the other were Convex; and this made the Breadth of the Image to be two inches and a third part of an inch; the Angles of this Priſm, and the Lengths of the Image made by each of thoſe Angles, being thoſe expreſt in this Table.

| *Angles* | *Lengths* |
|---|---|
| 58°. | $8\frac{1}{2}$ |
| $59\frac{1}{2}$ | 9 |
| $62\frac{1}{2}$ | $10\frac{1}{3}$ |

In this caſe you ſee, the Concave figure of the ſides of the Priſm by making the Rays Diverge a little, cauſes the Breadth of the Image to be greater in proportion to its Length than it would be otherwiſe. And this I thought fit to give you notice of, that Mr. *Lucas* may examine, whether his Priſm have not this fault. If a Priſm may be had with ſides exactly plain, it may do well to try the Experiment with that; but its better, if the ſides be about ſo much Convex as thoſe of mine are, becauſe the Image will thereby become much better defined. For this Convexity of the ſides does the ſame effect, as if you ſhould uſe a Priſm with ſides exactly plain, and between it and the Hole in the Window-ſhut, place an Object-glaſs of an 18 Foot Teleſcope, to make the Round Image of the Sun appear diſtinctly defin'd on the Wall when the Priſm is taken away, and conſequently the Long Image made by the Priſm to be much more diſtinctly defined (eſpecially at its ſtreight ſides) than it would be otherwiſe.

One thing more I ſhall add: That the utmoſt Length of the Image, from the fainteſt Red at one End, to the fainteſt Blue at the other, muſt be meaſured. For in my firſt Letter about Colours, where I ſet down the Length to be five times the Breadth, I called that Length the utmoſt Length of

the

the Image; and I meaſured the utmoſt Length, becauſe I account all that Length to be cauſed by the immediate Light of the Sun, ſeeing the Colours (as I noted above) become Viſible to the greateſt Length in the Cleareſt Days, that is, when the Light of the Sun tranſcends moſt the Light of the Clouds. Sometimes there will happen to ſhoot out from both Ends of the Image a glaring Light a good way beyond theſe Colours, but this is not to be regarded, as not appertaining to the Image. If the Meaſures be taken right, the whole Length will exceed the Length of the ſtreight ſides by about the Breadth of the Image.

By theſe things ſet down thus circumſtantially, I preſume Mr. *Lucas* will be enabled to accord his Tryals of the Experiment with mine; ſo nearly at leaſt that there ſhall not remain any very conſiderable difference between us. For, if ſome little difference ſhould ſtill remain, that need not trouble us any further, ſeeing there may be many various Circumſtances which may conduce to it; ſuch as are not only the different Figures of Priſms, but alſo the different Refractive Power of Glaſſes, the different Diameters of the Sun at divers Times of the Year, and the little Errors that may happen in Meaſuring Lines and Angles, or in placing the Priſm at the Window; though for my part, I took Care to do theſe Things as exactly as I could. However Mr. *Lucas* may make ſure to find the Image as Long or Longer than I have ſet down, if he take a Priſm whoſe Sides are not hollow ground, but plain, or (which is better) a very little Convex, and whoſe Refracting Angle is as much greater than that I uſed, as that he hath hitherto tryed it with is leſs; that is, whoſe Angle is about 66 or 67 degrees, or (if he will) a little greater.

Concerning Mr. *Lucas*'s other Experiments, I am much obliged to him that he would take theſe things ſo far into Conſideration, and be at ſo much Pains for Examining them; and I thank him ſo much the more, becauſe he is the firſt that hath ſent me an Experimental Examination of them. But yet it will conduce to his more ſpeedy and full Satisfaction, if he a little change the Method which he has propounded, and inſtead of a Multitude of things try only the *Experimentum Crucis*. For it is not number of Experiments but weight to be regarded; and where one will do, what need many?

The main thing he goes about to examine is, the *Different Refrangibility of Light*; and this I demonſtrated by the *Experimentum Crucis*. Now if this Demonſtration be good, there needs no further Examination of the thing; if not good, the fault of it is to be ſhewn: For the only way to examine a Demonſtrated Propoſition is, to examine the Demonſtration. Let that Experiment therefore be examined in the firſt Place, and that which it proves be acknowledged, and then if Mr. *Lucas* want my Aſſiſtance, to unfold the difficulties which he fanſies to be in the Experiments he has propounded, he ſhall freely have it. At preſent I ſhall ſay nothing in Anſwer to his Experimental Diſcourſe, but this in general, that it has proceeded partly from ſome miſunderſtanding of what he writes againſt, and partly from want of due caution in trying Experiments; and that amongſt his Experiments there is one, which when duly tryed, is, next to the *Experimentum Crucis*, the most conſpicuous Experiment, I know, for proving the Different Refrangibility of Light, which he brings it to prove againſt.

By the Poſt-ſcript of Mr. *Lucas*'s Letter, one not acquainted with what has paſſed, might think, that he quotes the Obſervation of the *Royal Society* againſt me; whereas the Relation of their Obſervation, which you ſent to *Liege*, contained nothing at all about the juſt proportion of the Length of the Image to its Breadth according to the Angle of the Priſm, nor any thing more (ſo far as I can perceive by your laſt) than what was pertinent to the things then in diſpute, *viz.* that they found them Succeed as I had affirmed. And therefore, ſince Mr. *Lucas* has found the ſame Succeſs, I ſuppoſe, that when he expreſſed, That *he much rejoyced to ſee the Tryals of the* R. Society *agree ſo exactly with his*, he meant only ſo far as his agreed with mine.

P. S. *I had like to have forgotten to adviſe, that the* Experimentum Crucis, *and ſuch others as ſhall be made for knowing the Nature of Colours, be made with Priſms which Refract ſo much, as to make the Length of the Image five times its Breadth, and rather more than leſs; for otherwiſe Experiments will not ſucceed ſo plainly with others as they have done with me.*

*An Optical Experiment; by* Mr. Steph. Gray. n. 221. p. 286.

VIII. I took a ſtiff piece of Brown Paper, and pricking a ſmall Hole therein, I held it at a little Diſtance before me: then applying a Needle to my Eye, I was ſurpriſed to ſee the Point of it Inverted. The nearer the Needle was to the Hole, it was ſo much the more Magnifyed, but leſs Diſtinct; and if it were ſo held, as that its Image was near to the edge of the Hole, its point ſeemed Crooked. So that, it ſeems theſe ſmall Holes, or ſomewhat in them, perform the Effects of a Concave Speculum, and ſo I take leave to call them *Aerial Speculums*.

*A Problem of* Alhazen, *ſolved by* M. Chr. Hugens. n. 97. p. 6119.

IX. 1. Mitto tibi hac occaſione Conſtructionem Problematis *Alhazeni* nuper à me inventam, & à Collegis meis felicem ſatis judicatam. Problema eſt;

*Dato Speculo Cavo aut Convexo, itemque Oculo & Puncto Rei viſæ, invenire Punctum Reflexionis.*

Fig. 78.

Eſto Speculum ex Sphæra quæ Centrum habeat A Punctum, Oculus vero ſit in B, & Punctum Viſibile in C, Planumque ductum per A, B, C, faciat in Sphæra Circulum D *d*, in quo invenienda ſint Reflexionis Puncta. Per tria Puncta A, B, C, deſcribatur Circuli Circumferentia, cujus ſit Centrum Z; occurrat autem ei producta A E, Perpend. B C, in R, & ſit duabus R A, O A, tertia Proportionalis N A, eritque N M, Parallela B C, altera Aſymptoton. Rurſus ſint Proportionales E A, $\frac{1}{2}$ A O, A I, & Summâ I Y æquali I N, ducatur Y M Parallela A Z; eaque erit altera Aſymptotos. Denique ſumptis I X, I S, quæ ſingulæ poſſint dimidium Quadratum A O, una cum Quadrato A I; erunt Puncta *x* & *s* in Hyperbola, aut Sectionibus oppoſitis D *d*, ad inventas Aſymptotos deſcribendis, quarum Interſectiones cum Circumferentia D O, oſtendent Puncta Reflexionis quæſita. Conſtructio hæc, in omni Caſu, quo Problema ſolidum eſt, locum habet, præterquam in uno, ubi non Hyperbola

ſed

ſed Parabola deſcribenda eſt; cum nimirum Circumferentia per Puncta A,B,C, deſcripta, tangit Rectam AE.

2. Cùm Nobiliſſimi *Hugenii* Conſtructionem ad Calculos revocarem, eandem omnino mecum Analyſin ſecutum eſſe deprehendi; ſed cùm ex illa duæ naſcantur Effectiones, utraque per Hyperbolam circa Aſymptotos; Ille unam, Ego alteram, uti faciliorem, ſelegeram. Evidens eſt autem, nihil aliud quæri hoc Problemate (ſi illud ad Terminos merè Geometricos revocemus) niſi in dato Circulo, cujus Centrum A, Radius AP) Punctum aliquod ut P, à quo ductis ad Puncta data E, B, inæqualiter à Centro A, diſtantia, Rectis PE, PB, Recta AP, producta biſecet Angulum EPB. Quod quidem varios Caſus recipit. Vel enim Normalis ex A in Rectum EB, nimirum AO, cadit inter E & B; vel ultra B. Si ultra, vel Rectangulum EOB æquale eſt Quadrato AO, vel majus vel minus. De Caſu æqualitatis videbimus infra; nunc verò tres alios Caſus eadem ferè Conſtructione complectemur. Per tria puncta A, E, B, tranſeat Circulus, ad cujus Circumferentiam producatur AO in D. Ac ſi quidem punctum O cadat inter E & B, Recta AO verſus O producenda erit; ſin autem ultra B, ſitque Rectangulum EOB majus Quadrato AO, producenda erit verſus A; at ſi Rectangulum quadrato minus fuerit, Circulus in ipſo puncto D, rectam AO ſecabit. Tum ductâ AX, parallelâ EB, ſecante Circulum datum in N, fiat ut rectangulum DAO ad quadratum AN, ita $\frac{1}{2}$ AX ad AH, quæ ſumenda erit versùs X, ſi O cadat inter E & B, aut rectangulum EOB minus ſit quadrato OA; at ex parte contraria, ſi ſit majus. Ponatur nunc OQ æqualis AH (in directum EB primo & ſecundo caſu, tertio verò versùs E:) tum fiant proportionales XA, NA, HK, ſumenda omni caſu versùs X: ſectâque AO in V; ut ſit eadem Ratio KA ad AV, quæ AD ad AX; jungatur KV, ac producatur donec occurrat recta EM Parallelæ OA, indefinite productæ, in puncto L; erunt omni caſu KL & QL Aſymptoti Hyperbolæ, quæ per punctum O deſcripta, propoſito ſatisfaciet: hoc tantum diſcrimine, quòd primo & ſecundo caſu Hyperbola per O, Problema ſolvet in ſpeculo Convexo, Sectio verò ei oppoſita in Concavo; at tertio caſu contrà, Hyperpola per O ſerviet Concavo, ejus oppoſita Convexo. Atque id quidem, cùm punctum V cadit inter A & O; nam ſi ultra O caderet, unica Hyperbola inter eaſdem QL, KL, deſcripta, tàm ſpeculo Convexo quàm Concavo ſatisfaceret. Cæterum ſi V caderet in ipſum punctum O, Problema tunc Planum eſſet, & ipſæ Rectæ LQ, LK, illud abſolverent. Unde patet Problematis hujus dari Caſus infinitos, qui per Locum Planum ſolvi poſſunt: quo magis veniâ digni videntur ii, qui illud per eundem Locum universè ſolvi poſſe cenſuerunt, quòd ipſis aliquoties Calculus feliciter cecidiſſet. Nulla enim dari poteſt trium punctorum A, E, B, poſitio, (de caſu æqualitatis Rectanguli EOB, & Quadrati OA, mox videbimus,) quæ non admittat Circulum aliquem ex Centro A deſcribendum, ad cujus Circumferentiam Problema per Locum Planum ſolvi queat. Hujus autem Circuli Radius, ſi tanti eſt, ita invenietur: in primo & ſecundo caſu ſuperioris Conſtructionis fiat ut Quadratum AX unà cum duplo rectangulo OAD, ad duplum Quadratum AD; ita Quadratum AO, ad Quadratum AN, erit AN Radius quæſitus. At in tertio caſu, faciendum eſt, ut Quadratum AX minus Duplo

By M. Sluſius. ibid.

Fig. 79, 80, 81.

Duplo Rectangulo O A D, ad Duplum Quadratum A D; ita Quadratum A O, ad Quadratum A N.

Construendus nunc superest alius casus, æqualitatis nempe Rectanguli E O B, & Quadrati A O, sive in quo Circulus, per puncta A, B, E, descriptus, tangit Rectam A O. Rectè autem monuit Clarissimus *Hugenius* hoc casu describendam esse Parabolam; quod tamen non ita intelligendum est quasi per Hyperbolam solvi non possit, cum & Hyperbolam & Ellipsin, imò Infinitas (si quis Methodo nostrâ uti velit) admittat; sed quod Parabolam quoque recipiat, quam alii casus respuunt. Eadem ratione temperandum est quod ait; Constructionem suam omni casu quo Problema solidum est, locum habere; intelligit enim, levi mutatione semper inveniri Hyperbolam quæ proposito serviat: quod casus à nobis superius Constructos cum ejus Constructione comparanti planum fiet. Ut autem ad casum æqualitatis redeam, & nequid temerè asseruisse videat, Ecce tibi non unam, sed duas Parabolas, ac præterea Hyperbolas Oppositas quæ Propositum absolvunt. Sint, ut priùs, puncta data E, B, Circulus
Fig. 82. ex Centro A, ac alius per tria puncta A, E, B, cujus Tangens sit A O, Centrum D. Ductâ Diametro N A D X, fiant tres Proportionales X A, N A, Z A, cujus dimidium sit A L. Fiant iterum tres Proportionalis 2 O A, N A, I A, cujus dimidium sit K A, & perficiatur Rectangulum L A O V; productaque L V in S, donec V S sit tertia Proportionales ipsarum A I, O V; Axe S L, Latere Recto A I, Vertice S, describatur Parabola; hæc enim Circulum secabit in punctis P, P, quæsitis. Tantundem faciet alia, si perfecto Rectangulo D A K C, & productâ K C in T, ita ut C T sit tertia Proportionalis ipsarum A Z, D C, describatur circa Axem T K, Vertice T, Latere Recto, Z A: occurret enim Circulo in iisdem punctis P, P. Facilior adhuc est Constructio
Fig. 83. per Sectiones Oppositas; factis enim, ut priùs, tribus Proportionalibus X A, N A, Z A, demittatur Z I Normalis, tertia Proportionalis Duplæ A O, & A N. Erit itaque Z I major Z A, cum Dupla A O minor sit X A: tum in puncto I, inclinentur utrinque Angulo Semirecto ad lineam I Z, rectæ I Q, I M, & ab utraque parte indefinitè producantur; demum circa illas tanquam Asymptotos describatur per A Hyperbola, & alia ipsi Opposita; hæc enim satisfaciet Problemati in Speculo Convexo, illa in Concavo. Cùm verò, ut ostendimus, Z I semper major sit Rectâ Z A, Recta I M nunquam transibit per A. Non dabitur itaque Casus, quo ex hac Constructione, velut in præcedentibus, Problema per ipsas Asymptotos solvi possit: et tamen hoc quoque aliquando Locum Planum admittit; cùm scilicet accidit, ut Recta X O ducta ad Centrum D, Tangat Circulum N P P; ipsum enim punctum Contactus quæstionem solvit. Et hæc quidem de Problemate, quod hactenus multorum ingenia exercuit, & cujus solutionem ante aliquot Annos absolvi.

*Otherwise, by* M. Slusius. *ibid. p.* 6123. Fig. 84.

Accipe quæ circa *Alhazeni* Problema, curis secundis meditatus sum.

Datus sit Circulus, cujus Centrum A; puncta data sunt D & *d*. Supponatur factum quod quæritur; sitque Radius Incidens D E, Reflexus E *d*; & ex Puncto Reflexionis E cadat in junctam D A, Normalis E I, & in eandem, ex *d*, Normalis *d* N, occurantque eidem Tangens E C & Radius *d* E, productis in B. Sit nunc $DA = z$. $AI = a$. $NA = n$. $EI = e$. $dN = b$. $BA = y$. $AE = q$. $CA = x$. Igitur, cum Anguli, D E C, C E B, sint æquales, &

Angulus

Angulus CEA Rectus, ex Hypothesi erunt tres, DA, CA, BA, Harmonicè Proportionales (hoc enim facile ostenditur). Erit itaque ut DA, ad BA, ita DC, ad CB; sive in terminis Analyticis, $z : y :: z - x : x - y$; & $2zy - xy = zx$, sive $\frac{2zy}{z+y} = x$. Cum autem Rectangulum CAI, sive $xa$, sit æquale Quadrato AE, sive $qq$, erit $x = \frac{qq}{a}$, & per consequens $\frac{2zy}{z+y} = \frac{qq}{a}$, sive $\frac{zqq}{2za - qq} = y$. Porrò, est ut $d$N, ad EI: ita NB, ad IB; sive $b : e :: y - n : y - a$. Itaque $ye - ne = by - ba$; & $y = \frac{ba - ne}{b - e}$.

Igitur $\frac{zqq}{2za - qq} = \frac{ba - ne}{b - e}$ sive $2zbaa - 2znae - qqba + qqne = bzqq - zqqe$. Quæ æquatio est ad Hyperbolam circa Asymptotos, cujus Constructio cum Circulo dato, Problemati satisfacit. Cùm verò, ob Circulum, sit $qq = aa + ee$, si loco $2bzaa$, ponatur ejus Valor $2bzqq - 2bzee$, habebitur alia pariter ad Hyperbolam circa Asymptotos $bzqq - 2bzee - 2znae - qqba + qqne = -zqqe$. Et hac Methodo, atque illâ, quam in Libello nostro de *Analysi* exposuimus, prodibunt infinitæ Æquationes ad Hyperbolas & Ellipses, quæ cum Circulo dato Problema absolvent; nisi quod Effectiones plerumque intricatiores evadunt quàm ut operæ pretium sit illas aggredi: construi tamen poterunt eo modo, quo ibi usi sumus in Ellipsi.

Retulimus, ut vides, Calculi nostri Summam ad lineam DA; sed satis animadvertis, non majori difficultate referri potuisse ad $d$A (quæ pariter data est) ductis *scil.* lineis, quas in Schemate punctis adumbravimus. Verùm novo Calculi labore non est opus. Si enim Rectæ $d$A, ejusque partibus, eosdem ac prius terminos Analyticos adhibeas, *h. e.* si ipsam $d$A facias æqualem $z$, D$n = b$. $n$A $= n$. AI $= a$. $i$E $= e$. &c. prodibit eadem Æquatio quæ prius; & infinitas alias Hyperbolas & Ellipses obtinebis, quæ cum Circulo dato Problemati satisfacient. Φορτικὸς essem si singulos Casus prosequi vellem, cum illorum Æquationes solâ Signorum + & — variatione discernantur. Unum tamen excipio, nimirum cum Angulus $d$AD est Rectus; ejus enim æquatio habetur, ex punctis à priori æquatione partibus, in quibus $n$ (quæ in nihilum abit) invenitur: nempe hæc, $2zbaa - qqba = bzqq - zqqe$, vel (pro $2zbaa$, posito ejus valore) $zbqq - qqba = 2zbee - zqqe$.

Sed animadvertendum est, quod licet referendo Analysin ad Rectam DA, statim sese offerant in æquatione duæ Hyperbolæ; & aliæ totidem à prioribus diversæ, cùm refertur ad Rectam $d$A; easdem tamen omninò Parabolas haberi, ad utramvis Rectarum $d$A, vel DA, referatur Analysis: cujus rei ratio levi consideratione tibi occurret.

Patere

Patere nunc, *V. Cl.* ut superiorem Analysin omnibus, quæ circa Speculorum Sphæricorum Reflexionem proponi solent, Problematibus applicem, novo facto Schemate. Sit igitur, ut priùs, Circulus, cujus Centrum A, Punctum D datum, & ab eo Radius Incidens D E, cujus Reflexus sit E Q. Junctâ D A, ducatur ad illam Tangens E C, & Normalis E I; & producatur ad eandem, Recta Q E B; Denominentur partes ut priùs. D A $= z$. C A $= x$. A E $= q$. B A $= y$. A I $= a$. I E $= e$. Igitur, propter tres D A, C A, B A, Harmonicè Proportionales, & tres C A, A E, A I, Geometricè, semper habebitur æquatio $y = \frac{zqq}{2za - qq}$, in quodcunque Circuli punctum cadat Radius D E. Itaque si quæratur punctum E, in quod si Radius D E Incidat, Reflectatur παραλλήλως Diametro L A V normali ad D A; Reflexus Q E, productus transibit per I; ut patet; & I ac B coincident. Igitur $a = y = \frac{zqq}{2za - qq}$; sive, $aa - \frac{1}{2}\frac{qqa}{z} = \frac{1}{2}qq$, & Problema per Plana solvetur.

Fig. 85.

Si quæratur punctum, à quo Radius Reflectatur parallelus alteri cuilibet lineæ, ut A K (ductæ ex Centro A;) ducatur ad illam, ex puncto I, Tangens K L $= d$. Evidens est, Triangula A K L, E I B, fore similia, cùm omnia Latera unius parallela sint Lateribus alterius, &c. Itaque A L, ad L K; ut E I, ad I B, sive $q : d :: e : a - y$; & $\frac{qa - de}{q} = y = \frac{zqq}{2za - qq}$; & $zq^3 = 2qzaa - 2zdae - q^3a + qqde$; sive, pro $aa$ posito $qq - ee$, $zq^3 = 2zq^3 - 2zqee - 2zdae - q^3a + qqde$. Utraque autem æquatio est ad Hyperbolam circa Asymptotos, quæ cum Circulo dato Problema absolvit.

Proponatur nunc efficere, ut Radius Reflexus transeat per datum punctum N, (ut in Problemate *Alhazeni*) vel ut productus versus punctum Reflexionis E, occurrat dato Puncto N. Ex N cadat in A L Normalis N O $= n$, sitque A O $= b$. Patet esse, ut A O, ad differentiam ipsarum O N, A B; ita E I, ad I B, h. e. $b : n - y :: e : a - y$; vel $b : y - n :: e : y - a$. Igitur $\frac{ba - ne}{b - e} = y = \frac{zqq}{2za - qq}$. Unde $2zbaa - 2znae - qqba + qqne = bzqq - zqqe$; nim. illa ipsa æquatio Problematis *Alhazeniani* quàm supra innuimus: Vel, secundo casu, $\frac{ba + ne}{b + e} = y = \frac{zqq}{2za - qq}$, sive $2zbaa + 2znae - qqba - qqne = zbqq + zqqe$.

Atque hæc sunt Problemata, quæ circa punctum Reflexionis proponi solent, in quibus tamen Finitam puncti D dati distantiam supposuimus. Sed facilior erit Analysis, si supponamus Infinitam. Secta enim C A bifariam in G, constat ex proprietate trium, D A, C A, B A, Harmonicè Proportionalium, tres D G, C G, B G, fore Geometricè Proportionales, suppositâ quâcunque puncti

puncti D distantiâ. Itaque, si supponatur Infinita, BG abibit in nihilum, & punctum B cum puncto G coincidet. Igitur AB erit perpetuò æqualis BC; erit itaque CA $= 2y$, & Rectangulum CAI, æquale Quadrato AE, dabit, in terminis Analyticis, $2ay = qq$, sive $y = \frac{qq}{2a}$: Cumque distantia puncti D supponatur infinita, erit ED Parallela AC. Itaque, si quæratur Radius Reflexus parallelus AL, quoniam eo casu $a$ & $y$ coincidunt, erit $a = y = \frac{qq}{2a}$, sive $aa = \frac{1}{2}qq$: Si quæratur ut Parallelus sit AK, erit rursus $q : d :: c : a - y$; & $\frac{qa - dc}{q} = y = \frac{qq}{2a}$, sive $2qaa - 2dac = q^3$. Si petatur ut transeat per N, erit, ut supra, $\frac{ba \pm nc}{b \pm c} = y = \frac{qq}{2a}$, & $2baa \pm 2nac = bqq \pm qqc$: Quæ æquationes sunt quoque ad Hyperbolas circa Asymptotos, nisi N punctum esse supponatur in AL; nam cùm tunc $n$ abeat in nihilum, sublatis ab æquatione partibus, in quibus $n$ continetur, residuæ dant æquationem ad Parabolam, ut suprà quoque monuimus.

Non exspectas, *V. Cl.* ut cum Specula Concava hactenus in Exemplum adduxerim, nunc agam de Convexis. Scis enim, eandem esse prorsus Analysin, & Æquationes solâ Signorum + & — variatione distingui. Scis Parabolam vel Ellipsin quæ uni satisfacit, satisfacere alteri; & si Hyperbola in Convexo Problema absolvat, ejus Oppositam paria facere in Concavo. His itaque omissis, addo tantùm, eâdem Analysi haberi in Speculis Concavis Focos & Spatia, quæ Radii occupant in Axe, datâ qualibet Puncti Lucentis distantiâ: Sed mirâ facilitate, cùm Radii supponuntur paralleli; quod tamen nonnullo circuitu à quibusdam demonstrari vidi. Nam in Speculo Concavo EE, cujus Centrum A, si Radius extremus Reflecti intelligatur ad Axem AR in B, ductâ Tangente EC, erit CB = BA. Bisecetur Semi-axis AR in Q; erit itaque Q Focus. Et QB Spatium quæsitum. Est autem QB dimidia CR (ob æquales AQ, QR, AB, BC,) *h. e.* dimidia excessus secantis Arcûs ER supra Sinum totum. Igitur si Arcus ER sit (*e. g.*) grad. 9, erit AC, 101246, & BQ $\frac{623}{100000}$ ipsius AR. Fig. 86.

4. Compendium, quod eodem tempore inveni circa primam Constructionem, ab initio tibi communicatam, tale est: Ductâ lineâ AT, parallelâ CB, eaque bisecta in V, punctum hoc est illud, per quod transire debet una Hyperbolarum Oppositarum, quarum Asymptoti inventæ fuerunt YM, MN. *Otherwise, by M. Hugens. n. 98. p. 6140.* Fig. 87.

Sed en Tibi bonam illam Constructionem, quæ in omnibus Casibus obtinet. Sit Circulus datus ED, cujus Centrum est A; Puncta data, B & C. Fig. 88.

Ductis Lineis AB, AC, fiant Proportionales BA (Radius Circuli) & FA: Eodem modo CA, (Radius Circuli) & GA. Tum jungatur FG, eaque bisecetur in H; & per hoc punctum ducantur Lineæ LHK, MHN, ſe invicem interſecantes ad Angulos Rectos, quarumque L H K ſit Parallela ei quæ biſecat Angulum B A C. Hæ ſunt duæ Aſymptoti Hyperbolarum deſcribendarum per puncta F & G, & quarum una tranſibit etiam per Centrum A, quarum Interſectiones cum Circuli Peripheria notabunt puncta Reflexionis quæſita.

*Further consider'd, by Mr.* Sluſius. *ib. p.* 6141. 5. Videt hic Nob. *Hugenius* qua ratione ad omnes Caſus extendi poſſet Hyperbola Æqualium Laterum, quam in caſu Anguli Recti ſeſe ſtatim offerre præcedentibus meis inſinuaveram. Poſſet quoque ex infinitis Ellipſibus, quæ adhiberi poſſunt, una ſeligi non difficilis Conſtructionis: ſed piget tamdiu in eodem Problemate hærere. Supereſt tamen aliquid, quod contemplationem habet non injucundam ; *nim.* cùm Sectiones, quæ cum Circulo dato ad Problematis ſolutionem adhibentur, illum in quatuor punctis ſecent, quorum duo tantum Reflexioni ſerviunt, quæri poſſet, quodnam Problema ſolvent duo reliqua; & quânam verborum formâ concipienda ſit Propoſitio, ut quatuor illos Caſus complectatur. Deinde annon etiam ijdem quatuor Caſus occurrant cùm Puncta data æqualiter diſtant à Centro?

*Again.* ib. Clar. *Hugenius* non alia utitur Analyſi quam meâ, quæ Parabolam uno tantùm caſu admittit. Quod ut evidentius tibi conſtet, Æquationem quam Conſtruxit hic adſcribam. Repete memoriâ, ſi placet, quæ ſecundis curis ad te ſcripſi, & invenies, me duas æquationes, Problemati per Hyperbolam circa Aſymptos ſolvendo idoneas, aſſignaſſe, has nimirum;

$$2zbaa - 2znac - qqba + qqne = bzqq - zqqe,$$

Et $bzqq - 2znac - qqba + qqne = 2zbee - zqqe$;

ac ſubjeciſſe, levi mutatione, (ſubſtituendo, ex gr. pro $qq$, ejus Valorem $aa + cc$) invenire poſſe infinitas Hyperbolas & Ellipſes, quæ cum Circulo dato Problema ſolverent. Nunc in priori ex his æquationibus pro $bzqq$ ponatur ejus Valor, fiet

$$zbaa - 2znac - qqba + qqne = bzee - zqqe;$$

$$\text{Sive } aa - \frac{qqa}{z} = ee - \frac{qqe}{b} + \frac{2nac}{b} - \frac{qqne}{zb}.$$

Atque hæc eſt Æquatio, quam magno ingenii acumine, ac pari facilitate, conſtruxit Vir Doctiſſimus.

*Again.* ib. Incidi nuper in ſequentem Conſtructionem, qua breviorem cùm dari poſſe vix credam, committere nolui, quin eam judicio ac cenſuræ tuæ ſubmitterem: *Fig.* 89. Sint igitur Puncta data E B, Circulus cujus Centrum A; junctis E A, B A, Secantibus Circulum in F & C; fiant tres Proportionales E A, F A, V A, & tres iterum B A, C A, X A: Tum junctâ V X, ac productâ utcunque, (Vertice X, Latere Tranſverſo V X, ac Recto ipſi æquali) deſcribatur Hyperbola X P, cujus applicatæ ad Diametrum V X G, Parallelæ ſint Rectæ A B: Illa enim ſatisfacit propoſito in caſu Speculi Convexi, ut ejus Oppoſita in caſu Concavi. Si Aſymptotos deſideres, facilè reperiri poſſunt, productâ V X, donec cum E B, pariter productâ, concurrat in L; deinde biſecta V X in I, ac ſumptâ L D aquali L I; junctâ enim D I erit Aſymptoton una, in quam alia normaliter incidit ad punctum I.

Sed

Sed fortasse ingratum tibi non erit intelligere, quâ viâ ad hanc Constructionem pervenerim. Scias itaque, me ex priori mea Analysi deduxisse hoc modo. Datis iisdem quæ priùs, cadat in E B, Normalis A O, sitque punctum quæsitum P, ex quo in A O cadat Normalis P R. Si A O sit $b$; E O, $z$; O B, $d$; A P, $q$; P R, $e$; A R, $a$; facilè colligitur hæc Æquatio, Fig. 90.

$$\frac{2zdae + 2bbae - 2bqqe}{zb - bd} + ee = aa - \frac{qqa}{b},$$ quæ mutari potest in has, $$\frac{zdae + bbae - bqqe}{zb - bd} = aa - \tfrac{1}{2}qq - \frac{\tfrac{1}{2}qqa}{b};$$ Et $$\frac{zdae + bbae - bqqe}{zb - bd} + ee = \tfrac{1}{2}qq - \frac{\tfrac{1}{2}qqa}{b}.$$ Hujus ultimæ Constructionem olim ad te misi; alterius verò, Cl. *Hugenius*. Primam autem, licèt se statim in conspectum dedisset; fermè neglexeram, quòd difficilioris Constructionis esse præsumerem. Sed me vano timore delusum agnovi, cùm in hanc, quam ad te mitto, Constructionem desinere nuper sum expertus. Sit enim, brevioris Calculi causâ, $z - d = k$, $zd + bb = bm$; fiet $ee \frac{-2qqe + 2mae}{k} = aa \frac{-qqa}{b}$. Et additis utrinque $\frac{q^4 + mmaa - 2qqma}{kk}$, erit $ee + \frac{mae - 2qqe}{k} + \frac{q^4 + mmaa - 2qqma}{kk}$, (hoc est quadratum ex $e - \frac{qq + ma}{k}$,) æquale $aa - \frac{qqa}{b} + \frac{q^4 + mmaa - 2qqma}{kk}$.

Fiet igitur, ἀναλογισμὸς $kk : kk + mm :: aa - \frac{kkqqa}{bkk + bmm} - \frac{2qqma + q^4}{kk + mm}$; & quadratum $e - \frac{qq + ma}{k}$: qui ad æquationem faciliorem reduci potest, si, posito $kk + mm = pp$, fiat $\frac{ky}{p} = a$; fit enim tandem, quadratum ex $e \frac{-qq}{k} + \frac{my}{p} = yy \frac{-qqky}{bp} - \frac{2qqmy}{kp} + \frac{q^4}{kk}$; quam Æquationem superiori Constructioni respondere animadvertes, si Calculos applicueris; ac simul observabis, ad quamcunque linearum E A, A B, B E, referatur Analyseos summa, easdem semper habere posse Sectiones, quamvis longiori circuitu & æquationibus valde diversis.

Ex hac Constructione, κατ' ἀναλογίαν deducere licet alterius Problematis effectionem, cùm scil. quæritur Punctum, à quo Radius Reflexus Parallelus sit cuilibet lineæ datæ; ut, si dato Puncto Luminoso B, Circulo ex Centro A, *Fig.* 91. quæreretur Radius Reflexus parallelus rectæ A E. Idem enim est, ac si in alio Problemate, distantia punctorum A & E supponeretur infinita; quo casu Tertia Proportionalis ipsarum E A, F A, abiret in nihilum, & puncta A & V Coinciderent: Itaque V X esset æqualis A X, & A E parallela P E. Applica igitur superiorem Constructionem & Problema absolves. Descripta scil. (Vertice X, Latere Transverso V X, vel A X, & Recto ipsi æquali,) Hyperbolâ X P, cujus Applicatæ ad Diametrum A X, parallelæ sint rectæ A E.

*By* M. Hugens. *Ibid. p.* 6143. 6. Verum est, quin imò mirandum, Constructionem quam antehac ad te misi inveniri quoque per Calculum quem Dn. *Slusius* de ea instituit post mutationem $qq$ in $aa + ee$; at hoc videtur fieri casu, nec ibi apparet Constructionis simplicitas nisi postquam eam peragere sategimus.

*Fig.* 92. *Problema* Alhazeni.] *Dato Circulo, cujus Centrum A, Radius A D, & punctis duobus B, C; invenire punctum H in Circumferentia Circuli dati, unde ductæ H B, H C, faciant ad Circumferentiam Angulos æquales.*

Ponatur Inventum, ductaque A M recta, quæ bifariam secet Angulum B A C, ducatur ei Perpendicularis H F, itemque B M, C L. Jungatur porrò A H, cui Perpendicularis sit H E, Rectisque B H, H C, occurrat A M in punctis K, G.

| | |
|---|---|
| Sit jam $AM = a$ <br> $MB = b$ <br> $AL = c$ <br> $LC = n$ <br> Radius $AD = d$ <br> $AF = x$ <br> $FH = y$ | Quia ergo æquales Anguli K H E & C H Z, sive E H G; estque E H A Angulus Rectus, erit ut K E, ad E G; ita K A, ad A G. Quia verò B M, ad M D; ut H F, ad F K, erit, ut B M + H F ad H E, ita M F ad F K, i. e. $b + y : y :: a - x : \frac{ay - xy}{b + y}$. add. $FA = x$ fit $KA = \frac{ay + bx}{b + y}$. |

Rursùs, quia C L ad L G, ut H F ad F G, erit permutando & dividendo C L — H F ad H F, ut L F ad F G, $n - y : y :: c - x : \frac{cy - xy}{n - y}$, quâ ablatâ ab $AF = x$, fit $GA = \frac{nx - cy}{n - y}$. Est autem $EA = \frac{dd}{x}$, quia Proportionales F A, A H, A E: Ergo E A — G A, hoc est, $EG, = \frac{dd}{x} - \frac{nx + cy}{n - y}$. Et K A — E A, hoc est, $KE = \frac{ay + bx}{b + y} - \frac{dd}{x}$.

Sed

Sed diximus, quod K E ad E G, ut K A ad A G; i. e. $\frac{ay+bx}{b+y} - \frac{dd}{x} : \frac{dd}{x} - \frac{nx+cy}{n-y} :: \frac{ay+bx}{b+y} : \frac{nx-cy}{n-y}$.

Unde invenitur $2anxxy + 2bnx^3 - ddbnx - ddnxy = naddy + nbddx - 2acxyy - 2bcxxy + ddbcy + ddcyy - addyy - bddxy$.

Et quia $n = \frac{bc}{a}$, fit $\frac{2bbc}{a}x^3 - \frac{bbddcx}{a} - \frac{2bbcyyx}{a}$, quia $xx = dd - yy$. Est autem $\frac{2bbc}{a}x^3 = \frac{2bbcddx}{a} - \frac{2bbcyyx}{a}$, quia $xx = dd - yy$. Ergo $-\frac{2bbcxyy}{a} - \frac{ddbcyx}{a} - 2acxyy + ddcyy = -addyy - bddxy$.

Et divisis omnibus per $y$ & ductis in $a$

$$-2bbcxy - ddbcx - 2aacxy + ddcay = -aaddy - bddax$$

$$abddx - cbddx + acddy + aaddy = 2aacxy + 2bbcxy$$

$$\frac{abddx - cbddx + acddy + aaddy}{2aac + 2bbc} = xy;$$ quæ æquatio est ad Hyperbolam.

Vel quia $bc = na$, $\frac{abdd - anddx + acddy + aaddy}{2aac + 2bbc} = xy$.

Sit $\frac{add}{aa+bb} = p$; Ergo $\frac{pbx - pnx + pcy + pay}{2c} = xy$.

Unde porrò non difficulter invenitur sequens Constructio: Jungantur B A, A C, & applicato seorsim ad utramque Quadrato Radii A D, fiant inde A P, A Q; & juncto P Q, dividatur ipsa bifariam in R, & per punctum R ducantur R D, R N, sese ad Rectos Angulos secantes, quorumque R D, sit parallela A D, quæ dividet bifariam Angulum B A C. Erunt jam R D, R N Asymptoti Oppositarum Hyperbolarum, quarum altera per Centrum A transire debet, quæque secabunt Circumferentiam in punctis H quæsitis. Transibunt autem Hyperbolæ per Puncta P, G. Fig. 93.

Ratio Constructionis apparet, ductis P $\gamma$, & Q$\zeta$, perpendicularibus in A M. Fit enim A$\gamma = \frac{add}{aa+bb}$ sive $p$; & A $\zeta = \frac{ap}{c}$. Item P$\gamma = \frac{pn}{c}$, & Q$\zeta = \frac{pb}{c}$. Quare A O $= \frac{pc+pa}{2c}$, & O R $= \frac{pb-pn}{2c}$. Unde Cætera facilia.

7. Mirari

By M. Slusius. ib. p. 6145.

7. Mirari desine, Vir Clarissime, eandem in *Albazeniano Problemate* Constructionem ex diversis Æquationibus deduci, quandoquidem illæ omnes, quibus hactenus usi sumus in una eademque generali Analysi contineantur. Quod ut ostendam, datus sit Circulus cujus Centrum A, puncta H & I; sitque Punctum quæsitum K, ad quod ex punctis I & H ducantur Rectæ H K, I K, & Tangens K D. Tum ex A ducatur quælibet A G, occurrens H K in E, I K in B, Tangenti K D in D (iis nim. productis, quas produci est opus). His positis evidens est, ob Angulos E K D, D K B, æquales, & Angulum A K D Rectum, tres A E, B E, D E fore semper Harmonicè proportionales. Itaque ductis ad A E Normalibus K C, I F, H G, ac denominatis partibus,

Fig. 94.

A K = $q$
A C = $a$
C K = $e$
HG = $b$
A G = $d$
F A = $z$
F I = $n$

habebitur, methodo, quam in secunda hujus Problematis Analysi olim adhibui, hæc generalis Æquatio,

$$ndaa - bzaa - nqqa + bqqa = ndee - zbee + 2bnae + 2zdae - dqqe - zqqe.$$

Finge nunc, A G esse perpendicularem ad H I, nihil varietatis erit in æquatione; nisi quod A F & A G, hoc est, $d$ & $z$, erunt æquales. Posito itaque $d$ pro $z$, fiet

$$ndaa - bdaa - nqqa + bqqa = ndee - dbee + 2bnae + 2ddae - 2dqqe.$$

Sive applicatis omnibus ad $nd - db$,

$$aa - \frac{qqa}{d} = ee + \frac{2bnae + 2ddae - 2dqqe}{nd - bd};$$

Eadem nempe, quam ex prima mea Analysi, licèt aliâ viâ, deduxeram, & quam nuper, modo facili constructam, ad te misi.

Pone deinde, A G coincidere cum A H; abibit igitur H G, sive $b$, in nihilum. Expunctis itaque ab æquatione partibus, in quibus $b$ reperitur, remanebit, $ndaa - nqqa = ndee + 2zdae - dqqe - qqze$. Hanc autem, si meministi, curis secundis inveni, & aliam huic similem, in Casu quo Recta A G transire intelligitur per I.

Supponamus demum, Rectam A G, secare bifariam Angulum H A I; erit ob similitudinem Triangulorum H A G, I A F, ut H G ad G A, ita I F ad F A, sive ut $b$ ad $d$, ita $n$ ad $z$, & $nd = bz$. Ablatis igitur æqualibus, fit $bqqa - nqqa = 2bnae + 2zdae - dqqe - qqze$: Illa ipsa, quam, ut ex literis tuis nuper intellexi, Cl. *Hugenius* construxit.

Intelligatur tandem eadem Recta H G, secare bifariam Rectam H I; erunt igitur æquales H G, I G, hoc est, $b = n$; fietque, ablatis æqualibus, $bdaa - bzaa = bdee - bzee + 2bbae + 2zdae - dqqe - qqze$; quam licèt non admodùm difficilem, nemo nostrûm hactenus construxit. Hæ autem, ut & ipsa Generalis Æquatio, in duas alias dividi possunt, posito, ut nosti, pro $aa$ vel $ee$, ejus valore, $qq - ee$, vel $qq - aa$.

Vides igitur, quicquid hactenus præstitum est, in eandem Analysin resolvi; quæ & infinitas alias Constructiones per Circulum datum & Hyperbolam complectatur. Sed eas investigare non est tanti, cùm in hoc Problemate, ut olim fortassis inopiâ, sic nunc copiâ laboremus. Addam tantùm Constructionem per Parabolam, idque via duplici; quæ licet

cet aliis per Hyperbolam operosior videatur, Lineæ tamen simplicitate, qua Parabola inter reliquas Sectiones commendatur, operam compensat.

Iisdem igitur Datis, Jungatur AI, & producatur in S, donec AS fiat æqualis AH, junctâque HS, & bisecta IS in M, ducatur per M Recta RMQ normalis ad HS, in quam cadat ex A Normalis AQ, & cui Parallelus ducatur Radius AC. Tum factis tribus Proportionalibus IA, AC, AE, fiat ut SA ad AE, ita MQ ad AD, & RS ad AP (in recta AQ versus Q;) & in eadem ab alia parte sumatur DO æqualis DC. Demum bisectâ PD in X, inclinetur per X, Angulo Semi-recto ad AX, Recta VXL, occurrens Normali in D erectæ in puncto V, & in quam ex O cadat Normalis OB. Aio, si fiat ut VX ad XB, ita XB ad BL, punctum L esse Verticem, LV, Axem, XV Latus Rectum Parabolæ, quæ Problemati satisfacit omni casu; Secans nimirum Circulum datum in punctis K, quorum supremum & ultimum ad *Problema Alhazenianum* pertinent, reliqua ad aliud. Fig. 95.

Datur, ut supra indicavi, alia quoque Parabola, quæ cum hac paria facit, & cujus descriptio ex hac adeo facile deducitur, ut nova non sit opus. Sumatur enim A$\delta$, in Directum DA, & ipsi æqualis, & in Directum OA, ipsi quoque æqualis A$\omega$. Tum bisectâ P$\delta$ in $\xi$, ducatur per $\xi$ Recta $\varkappa\xi\beta$, Normalis ad XB, concurrens cum $\delta\varkappa$, Normali ad OA, in $\varkappa$, & in quam cadat Normalis $\omega\beta$; ac fiat ut $\varkappa\xi$ ad $\xi\beta$, ita hæc ad $\beta\lambda$: Erit $\lambda$ Vertex, $\lambda\xi$ Axis, $\varkappa\xi$ Latus Rectum Parabolæ, quæ in iisdem cum priore punctis Circulum datum secabit.

*To find the Principal Focus of Optick Glasses universally; by Mr. Edm. Halley. n. 205. p. 960.*

X. Let BE$\beta$ be a double Convex Lens, C the Center of the Segment EB, and K the Center of the Segment E$\beta$, B$\beta$ the Thickness of the Lens, D a Point in the Axis of the Lens; and it is required to find the Point F, at which the Beams proceeding from the point D, are collected therein, the Ratio of Refraction being as $m$ to $n$. Let the distance of the Object DB $=$ DA $= d$, (the point A being supposed the same with B, but taken at a distance therefrom, to prevent the Coincidence of so many Lines) the Radius of the Segment towards the Object CB, or CA $= r$, and the Radius of the Segment from the Object K$\beta$, or K$\alpha = \varrho$, and let B$\beta$, the thickness of the Lens, be $= t$, and then let the Sine of the Angle of Incidence DAG, be to the Sine of the Refracted Angle HAG, or CA$\varphi$, as $m$ to $n$; and in very small Angles the Angles themselves will be in the same Proportion; whence it will follow, that, As $d$ to $r$, so the Angle at C to the Angle at D, and $d + r$ will be as the Angle of Incidence GAD; and again, as $m$ to $n$, so $d + r$ to $\frac{dn + rn}{m}$, which will be as the Angle GAH $=$ CA$\varphi$. This being taken from ACD, which is as $d$, will leave $\frac{m - nd - nr}{m}$ Analogous to the Angle A$\varphi$D, and the Sides being in this Case proportional to the Angles they subtend, it will Fig. 96.

will follow, that as the Angle A$\varphi$D, is to the Angle AD$\varphi$, so is the Side AD, or BD, to A$\varphi$, or B$\varphi$: That is, B$\varphi$ will be $= \frac{mdr}{m - nd - nr}$, which shews in what Point the Beams proceeding from D would be collected by means of the first Refraction; but if $nr$ cannot be substracted from $m - nd$, it follows, that the Beams after Refraction do still pass on Diverging, and the Point $\varphi$ is on the same side of the Lens beyond D. But if $nr$ be equal to $m - nd$, then they proceed Parallel to the Axis, and the Point $\varphi$ is infinitely distant.

The Point $\varphi$ being found as before, and B$\varphi$ — B$\beta$ being given, which we will call $\delta$, it follows by a Process like the former, that $\beta$F, or the Focal Distance sought, is equal to $\frac{\delta \varrho n}{m - n\delta + m\varrho} = f$. And in the room of $\delta$ substituting $B\varphi - B\beta = \frac{mdr}{m - nd - nr} - t$, putting $p$ for $\frac{n}{m-n}$, after due Reduction this Equation will arise, $\frac{mpdr\varrho - nd\varrho t + npr\varrho t}{mdr + md\varrho - mpr\varrho - m - ndt + nrt} = f$.

Which *Theorem*, however it may seem Operose, is not so, considering the great number of Data that enter the Question, and that one half of the Terms arise from our taking in the thickness of the Lens, which in most Cases can produce no great Effect; however, it was necessary to consider it, to make our Rule perfect. If therefore the Lens consist of Glass, whose Refraction is as 3 to 2, 'twill be $\frac{6dr\varrho - 2d\varrho t + 4r\varrho t}{3dr + 3d\varrho - 6r\varrho - dt + 2rt} = f$. If Of Water, whose Refraction is as 4 to 3, the *Theorem* will stand thus: $\frac{12dr\varrho - 3d\varrho t + 9r\varrho t}{4dr + 4d\varrho - 12r\varrho - dt + 3rt} = f$. If it could be made of Diamant, whose Refraction is as 5 to 2, it would be

$$\frac{\frac{10}{3}dr\varrho - 2d\varrho t + \frac{4}{3}r\varrho t}{5dr + 5d\varrho - \frac{10}{3}r\varrho - 3dt + 2rt} = f.$$

And this is the Universal Rule for the Foci of Double Convex Glasses exposed to Diverging Rays. But if the thickness of the Lens be rejected as not sensible, the Rule will be much shorter, *viz.* $\frac{pdr\varrho}{dr + d\varrho - pr\varrho} = f$; or in Glass, $\frac{2dr\varrho}{dr + d\varrho - 2r\varrho} = f$. All the Terms wherein $t$ is found being omitted, as equal to nothing. In this Case, if $d$ be so small, as that $2r\varrho$ exceed $dr + d\varrho$, then will it be $-f$, or the Focus will be Negative,

gative, which shews that the Beams after both Refractions still proceed Diverging.

To bring this to the other Cases, as of Converging Beams, or of Concave Glasses, the Rule is ever composed of the same Terms, only changing the Signs of + and —; for the Distance of the Point of Concourse of Converging Beams, from the Point B, or the first Surface of the Lens, I call a Negative Distance, or $-d$; and the Radius of a Concave Lens I call a Negative Radius, or $-r$ if it be the first Surface, and $-\varrho$ if it be the second Surface. Let then Converging Beams fall on a double Convex of Glass, and the *Theorem* will stand thus $\frac{-2dr\varrho}{-dr-d\varrho-2r\varrho}=+f$; which shews, that in this Case the Focus is always Affirmative.

If the Lens were a *Meniscus* of Glass, exposed to Diverging Beams, the Rule is, $\frac{-2dr\varrho}{-dr+d\varrho+2r\varrho}=f$. Which is Affirmative, when $2r\varrho$ is less then $dr-d\varrho$, otherwise Negative: But in the Case of Converging Beams falling on the same *Meniscus*, 'twill be $\frac{+2dr\varrho}{+dr-d\varrho+2r\varrho}=f$. And it will be $+f$ whilst $d\varrho-dr$ is less than $2r\varrho$, but if it be greater than $2r\varrho$, it will always be found Negative or $-f$. If the Lens be double Concave, the Focus of Converging Beams is Negative, where it was Affirmative in the Case of Diverging Beams on a double Convex, *viz.* $\frac{-2dr\varrho}{+dr+d\varrho-2r\varrho}=f$; which is Affirmative only when $2r\varrho$ exceeds $dr+d\varrho$: But Diverging Beams passing a double Concave, have always a Negative Focus, *viz.* $\frac{-2dr\varrho}{+dr+d\varrho+2r\varrho}=-f$.

The *Theorems* for Converging Beams are principally of use to determine the Focus resulting from any sort of Lens placed in a Telescope, between the Focus of the Object Glass and the Glass it self; the distance between the said Focus of the Object Glass and the interposed Lens being made $=-d$.

In case the Beams are Parallel, as coming from an infinite distance, (which is supposed in the case of Telescopes) then will $d$ be supposed infinite, and in the Theorem $\frac{pdr\varrho}{dr+d\varrho-pr\varrho}$ the Term $pr\varrho$ vanishes, as being finite, which is no part of the other infinite Terms, and dividing the remainder by the infinite part $d$, the *Theorem* will stand thus $\frac{p\varrho r}{r+\varrho}=f$, or in Glass $\frac{2r\varrho}{r+\varrho}=f$.



In

In caſe the Lens were Plano-Convex expoſed to diverging Beams, inſtead of $\frac{p\,d\,\rho\,r}{d\,r + d\,\rho - p\,r\,\rho}$, $r$ being infinite, it will be $\frac{p\,d\,\rho}{d - p\,\rho} = f$, or $\frac{2\,d\,\rho}{d - 2\,\rho} = f$ if the Lens be Glaſs.

If the Lens be Double-Convex, and $r$ be equal to $\rho$, as being formed of Segments of equal Spheres, then will $\frac{p\,d\,\rho\,r}{d\,r + d\,\rho - p\,r\,\rho}$ be reduced to $\frac{p\,d\,r}{2\,d - p\,r} = f$; and in caſe $d$ be infinite, then it will yet be farther contracted to $\frac{1}{2}\,p\,r$, and $p$ being $= \frac{n}{m - n}$, the Focal diſtance in Glaſs will be $= r$, in Water $1\frac{1}{2}\,r$, but in Diamant $\frac{1}{3}\,r$.

This is not only uſeful to diſcover the Focus from the other propoſed Data, but from the Focus given, we may thereby determine the Diſtance of the Object, or from the Focus and Diſtance given, we may find of what Sphere it is requiſite to take another Segment, to make any given Segment of another Sphere caſt the Beams from the Diſtance $d$ to the Focus $f$. As likewiſe from the Lens, Focus, and Diſtance given, to find the Ratio of Refraction, or of $m$ to $n$, requiſite to anſwer thoſe Data. All which, it is obvious, are fully determined from the Equation we have hitherto uſed, *viz.* $p\,d\,\rho\,r = d\,r\,f + d\,\rho\,f - p\,r\,\rho\,f$. For to find $d$, the *Theorem* is, $\frac{p\,r\,\rho\,f}{r\,f + \rho\,f - p\,\rho\,r} = d$, the Diſtance of the Object; for $\rho$, the Rule is $\frac{d\,r\,f}{p\,d\,r + d\,f + p\,r\,f} = \rho$; but for $p$, it will be $\frac{d\,r\,f + d\,\rho\,f}{d\,\rho\,r + f\,\rho\,r} = p$; which latter determines the Ratio of Refraction, $m$ being to $n$ as r $+ p$ to $p$.

I ſhall not expatiate on theſe Particulars, but leave them for the exerciſe of thoſe that are deſirous to be informed in Optical Matters, which I am bold to ſay are comprehended in theſe Three Rules, as fully as the moſt inquiſitive can deſire them, and in all poſſible Caſes; regard being had to the Signs $+$ and $-$, as in the former Caſes of finding the Focus. I ſhall only ſhew two conſiderable Uſes of them; the one to find the Diſtance whereat an Object being placed ſhall by a given Lens be repreſented in a Species as large as the Object it ſelf, which may be of ſingular uſe, in Drawing Faces and other things in their true Magnitude, by tranſmitting the Species by a Glaſs into a dark Room, which will not only give the true Figure and Shades, but even the Colours themſelves, almoſt as Vivid as the life. In this caſe $d$ is equal to $f$, and ſubſtituting $d$ for $f$ in the Equation, we ſhall have $p\,d\,r\,\rho = d\,d\,r + d\,d\,\rho - d\,p\,\rho\,r$, and dividing all by $d$, $p\,r\,\rho = d\,r + d\,\rho - p\,r\,\rho$, that is,

$2\,p\,r\,\rho$

$\frac{2prg}{r+g} = d$; but if the two Convexities be of the ſame Sphere, ſo as $r = g$, then will the Diſtance be $= pr$, that is, if the Lens be Glaſs $= 2r$, ſo that if an Object be placed at the Diameter of the Sphere diſtant, in this caſe the Focus will be as far within as the Object is without, and the Species repreſented thereby will be as big as the Life; but if it were a Plano-Convex, the ſame diſtance will be $= 2pr$, or in Glaſs, to four times the Radius of the Convexity.

A ſecond Uſe is to find what Convexity or Concavity is required, to make a vaſtly diſtant Object be repreſented at a given Focus, after the one Surface of the Lens is formed; which is but a *Corollary* of our *Theorem* for finding $g$, having $p$, $d$, $r$, and $f$, given; for $d$ being Infinite, that Rule becomes $\frac{rf}{pr-f} = g$, that is in Glaſs $\frac{rf}{2r-f} = g$, whence if $f$ be greater than $2r$, $g$ becomes Negative, and $\frac{rf}{f-2r}$ is the Radius of the Concave ſought.

But to return to our firſt *Theorem*, which accounting for the thickneſs of the Lens, we will here again reſume, *viz.*

$$\frac{mpdrg - ndgt - nprgt}{mdr + mdg - mpgr - m - ndt + nrt} = f.$$

And let it be requir'd to find the Focus where a whole Sphere will collect the Beams proceeding from an Object at the diſtance $d$. Here $t$ is equal to $2r$, and $r = g$; And after due Reduction, the *Theorem* will ſtand thus, $\frac{mpdr - 2ndr + 2nprr}{2nd + 2nr - mpr} = f$: but if $d$ be Infinite, it is contracted to $\frac{mpr}{2n} - r = \frac{2n - m}{2m - 2n} r = f$; wherefore a Sphere of Glaſs collects the Sun's Beams at half the Semidiameter of the Sphere without it, and a Sphere of Water at a whole Semidiameter. But if the Ratio of Refraction $m$ to $n$ be as 2 to 1, the Focus falls on the oppoſite Surface of the Sphere, but if it be of greater Inequality it falls within.

Another Example ſhall be when a Hemiſphere is expoſed to Parallel Rays, that is, $d$ and $g$ being Infinite, and $t = r$, and after due Reduction, the *Theorem* reſults $\frac{nn}{mm - mn} r = f$; that is, in Glaſs it is at $\frac{4}{3} r$, in Water at $\frac{9}{4} r$, but if the Hemiſphere were Diamant, it would collect the Beams at $\frac{4}{15}$ of the Radius beyond the Center.

 Laſtly,

Laſtly, As to the Effect of turning the two ſides of a Lens towards an Object; it is evident, that if the thickneſs of the Lens be very ſmall, ſo as that you neglect it, or account $t = 0$, then in all Caſes the Focus of the ſame Lens, to whatſoever Beams, will be the ſame, without any difference upon the turning the Lens: But if you are ſo curious as to conſider the thickneſs, (which is ſeldom worth accounting for) in the caſe of Parallel Rays falling on a Plano Convex of Glaſs, if the Plain ſide be towards the Object, $t$ does occaſion no difference, but the Focal diſtance $f = 2r$. But when the Convex ſide is towards the Object, it is contracted to $2r - \frac{2}{3}t$, ſo that the Focus is nearer by $\frac{2}{3}t$. If the Lens be Double Convex, the difference is leſs, if a Meniſcus, greater. If the Convexity on both ſides be equal, the Focal Length is about $\frac{1}{6}t$ ſhorter than when $t = 0$. In a Meniſcus, the Concave ſide towards the Object encreaſes the Focal length, but the Convex towards the Object diminiſhes it. A general Rule for the difference ariſing on turning the Lens, where the Focus is Affirmative, is this $\frac{2rt - 2\varrho t}{3r + 3\varrho - t}$, for Double Convexes of differing Spheres. But for *Meniſci*, the ſame difference becomes $\frac{2rt + 2\varrho t}{3r - 3\varrho + t}$; of which I need give no other Demonſtration, but that by a due Reduction it will ſo follow from what is Premiſed, as will the *Theorems* for all ſorts of Problems relating to the Foci of Optick Glaſſes.

*The Generation of an Hyperbolical Cylindroide; by Sir Chriſtopher Wren. n. 48. p. 961.*

*Fig. 97.*

XI. 1. Sint Hyperbolæ Oppoſitæ D B, E C, quarum Axis Tranſverſus eſt B C, Centrum A, & una ex Aſymptotis G P; item per Centrum ſit O M, ducta ad Angulos Rectos ipſi B C. Quare ſi circumducantur Hyperbolæ circa Axin O M, manifeſtum eſt, ex ea Revolutione generari Corpus Cylindroides Hyperbolicum cujus Baſes, ſectioneſque Baſi parallelæ ſunt Circuli. Dico inſuper, ſi idem Corpus ſecetur per Aſymptoton G P, erit ſectio Parallelogrammum.

Secetur per Axin Tranſverſum ſectione Circulari B N C; item per O & M in Circulos æquales & æqualiter à Centro diſtantes; item per Axin in figuram Genetricem cujus ſemiſſis eſt B D E C, in cujus Plano erit Aſymptotos G P, per quam ad Rectos Angulos planum B D E ſecetur in Plano F H P; jungantur denique H O.

Quoniam Triangulum O G H eſt Rectangulum, Ergo quadratum O H, ſive O D, minus quadrato O G, eſt æquale quadrato G H: & quoniam D O parallela eſt ipſi B A, & Aſymptoton ſecat in G, erit (ex proprietatibus Hyperbolæ, quæ in Conicis demonſtrantur) quadratum O G una cum quadrato A B æquale quadrato O D, *h. e.* Quadratum O D minus quadrato O G æquale quadrato A B, ſive quadrato A N. Ergo quadratum G H, æquale eſt quadrato A N. Quare G H & A N æquantur & ſunt ad Angulos Rectos ipſi G A; idemque demonſtratur de omnibus aliis ſectionibus Baſi parallelis. Quare Cylindroides Hyperbolica rite ſecatur per Aſymptoton in Parallelogrammum. *q. e. d.*

*Corol.* 1.

*Corol.*] Hinc patet, in superficie Cylindroidis, quamvis è duplici flexura constet, rectas nihilominus innumeras duci posse: Patet etiam, aliam esse hujus Corporis Generationem, nimir. ex revolutione Parallelogrammi circa Axin manente Angulo ad Axin æquali G A O, vel denique manente Linea Generatrice H R immobili, & massam Volubilem formante aut secante.

Et si acies Dolabri accutissima & rectissima ita disponatur ad Axin, sicut se habet Linea Generatrix, rotante interim Mamphure, manifestum est, Torno tam accuratas posse elaborari Hyperbolas quam Circulos, cum nihil aliud requiratur ad formandam Cylindroidem quam ad Cylindrum, nisi quod in Cylindris acies Dolabri est Axi Parallela, hic vero inclinata.

Itaque notandum est, pro Inclinatione Anguli G A O, variari speciem Hyperbolæ; adeoque facilius accommodatur ad datam Hyperbolam quam ut demonstratione opus habeat: At si manente Angulo Generatrix magis ad Centrum accedat, exsurgit inde minor Hyperbola, sed priori prorsus similis.

2. Sint tria Corpora terendo idonea, P, Q, R; quorum P, & Q, sint æqualia & Columnari forma, R vero Corpus Lenti-forme. P rotetur circa Axin A B; Q, circa C D; & R, circa E G. Sint autem A B, & C D, in diversis Planis, ita tamen ut E G producta, sit ad Rectos Angulos utrique A B & C D: accedant denique ad se invicem Corpora, prout opus fuerit, servata tamen eadem Inclinatione & situ Axium. *The Application thereof to the Grinding of Hyperbolical Opticks Glasses; by Sir Chr. Wren, n. 53. p. 1059.*

Dico ex Revolutione & mutua attritione Corporum prius positorum exsurgere nova Corpora Geometrica, quorum P & Q erunt Cylindroidea Hyperbolica æqualia, R vero Conoides Hyperbolicum, specie & magnitudine datum. *Fig. 98.*

Demonstrationem in promptu habemus, nec non Modulum ipsius Machinæ, terendis Lentibus Hyperbolicis destinatæ; quam operosa Pictura & prolixa Explicatione describere, mihi & Artifici magis fuerit molestum, quam Dædalo cuivis sagaci similem ad-invenire. Postquam enim exposita jam sunt Principia Geometrica, facile erit conjicere, quale sit Instrumentum; nempe, tres sunt Tabulæ Oblongæ, Planæ, Validæ, Labiles, & sibi invicem Impositæ: Infima & Media sustinent inæqualia Capitula (sive Ansas Mamphur sustinentes) alternatim posita; id postulat utriusque Mamphuris Obliquitas & quasi decussatio: Summæ Tabulæ æqualia sunt Capitula in longum Tabulæ disposita; & perforato Citimo Capitulo Mamphur transmittitur. Omitto Rotas, Rotulas, Lora, Pondera, Cochleas, & reliqua ad motum expeditum & Machinæ Firmitudinem necessaria. P pertinet ad infimam Tabulam; Q ad mediam; R ad Summam. R Lens est vitrea; Q Modulus Lentem terens; P Formula Modulum corrigens; quæ dum motu obliquo, & diverso à motu tam Lentis quam Moduli, fertur, delet continuo & deterit, quicquid Vitii imprimitur in Modulum ex Lentis & Materiæ attritione.

Quare, cum adeo simplex & spontanea sit ista Hyperbolici Conoidis genitura, ex solis nempe Motibus Circularibus; cumque Motus sit duplex & varius, credibile est, Lentes Hyperbolicas ex hisce Principiis vel nullis fore explicandas.

XII. This Phænomenon appears very easily Explicable, from the Consideration of placing Glasses in a Tube; which is thus: After the Object Glass, the first Eye Glass is placed so much distant (towards the Eye) from the Focus of the Object Glass, as is the Focus of the Eye Glass; then the middle Eye Glass *Why four Convex Glasses in a Telescope, shew Objects Erect; by Mr. Will. Molineux. n. 183. p. 560.*

Glaſs is placed ſo much diſtant from the Focus of the firſt Eye Glaſs, as is the Focus of this middle Eye Glaſs; laſtly, the neareſt Eye Glaſs is placed ſo much diſtant from the Focus of this middle Eye Glaſs, as is the Focus of this neareſt Eye Glaſs; and the Eye looking through them all is placed in the Focus of this neareſt Eye Glaſs.

I ſay therefore, 1. That one ſingle Convex Glaſs, cannot properly be ſaid by it ſelf to ſhew Objects Erect or Reverſe, but in reſpect of placing of the Eye that looks through it. For if the Eye, that looks through a ſingle Convex Glaſs, be placed nigher thereto than the Glaſs's Focus, the Objects are Erect; if the Eye be placed juſt in the Focus, the Objects are neither Erect nor Reverſed, but all in Confuſion between both; and if the Eye be placed further from the Glaſs than the Focus, the Objects are Reverſed. I mean here diſtant Objects, the Rays flowing from any Point whereof may be counted to come Parallel towards the Object Glaſs.

2. The Object Glaſs of a Teleſcope reverſes the Object, both to the Eye Glaſs and the Eye, that looks through it: For the Eye Glaſs is placed farther from the Object Glaſs than is the Focus of the Object Glaſs. But the Eye Glaſs does nothing towards the Rectification, or Reverſion; the Eye being placed juſt in its Focus.

3. If the ſecond Eye Glaſs (the firſt being that next the Object Glaſs) be placed as it ought in a Teleſcope, Place the Eye nearer to this middle Eye Glaſs than its Focus, and it ſees the Objects Inverted and Confuſed: Place the Eye in the Focus, and it ſees the Objects all in Confuſion, neither Erect nor Reverſed; for here again there is a diſtinct Repreſentation of the Objects to be received on a piece of Paper, as in the Focus of the Object Glaſs, and the Eye being placed at any time at this place (which is uſually call'd the Diſtinct Baſe) ſees all in Confuſion: But then let the Eye be placed farther from this middle Glaſs than its Focus, and it perceives the Objects Erect and Confuſed.

Laſtly, The third, or immediate Eye Glaſs, does nothing towards the Erecting or Reverſing the Species, which it receives Erect from the middle Eye Glaſs; no more than in a Teleſcope of two Convex Glaſſes, the Eye Glaſs does to the Species it receives from the Object Glaſs, as we have ſhewn before. The reaſon that this laſt or immediate Eye Glaſs; has nothing to do in the Erecting or Reverſing the Species, is the ſame, as in a Teleſcope of two Convex Glaſſes, *viz.* The Eye is placed in its Focus, and therefore ſees the Species as 'tis repreſented in the diſtinct Baſe; that is, the Species is Inverted in the diſtinct Baſe of the Object Glaſs, and therefore a ſingle Convex Eye Glaſs brings it to the Eye Inverted; but in the diſtinct Baſe of the middle or ſecond Eye Glaſs the Species is Erect, and therefore the third or immediate Eye Glaſs brings it to the Eye Erect.

Wherefore we are to conſider the Teleſcope conſiſting of an Object Glaſs and three Eye Glaſſes, as two Teleſcopes, each conſiſting of two Convex Glaſſes. The firſt conſiſts of the Object Glaſs and firſt Eye Glaſs, and this Inverts the Species; that is, the Species is Inverted in the Diſtinct Baſe of the Object Glaſs, and ſo brought into the Eye. The ſecond Teleſcope conſiſts of the two immediate Eye Glaſſes, and this Erects what the former Inverted; that is, the Species in the Diſtinct Baſe of the middle Eye Glaſs is

Erect,

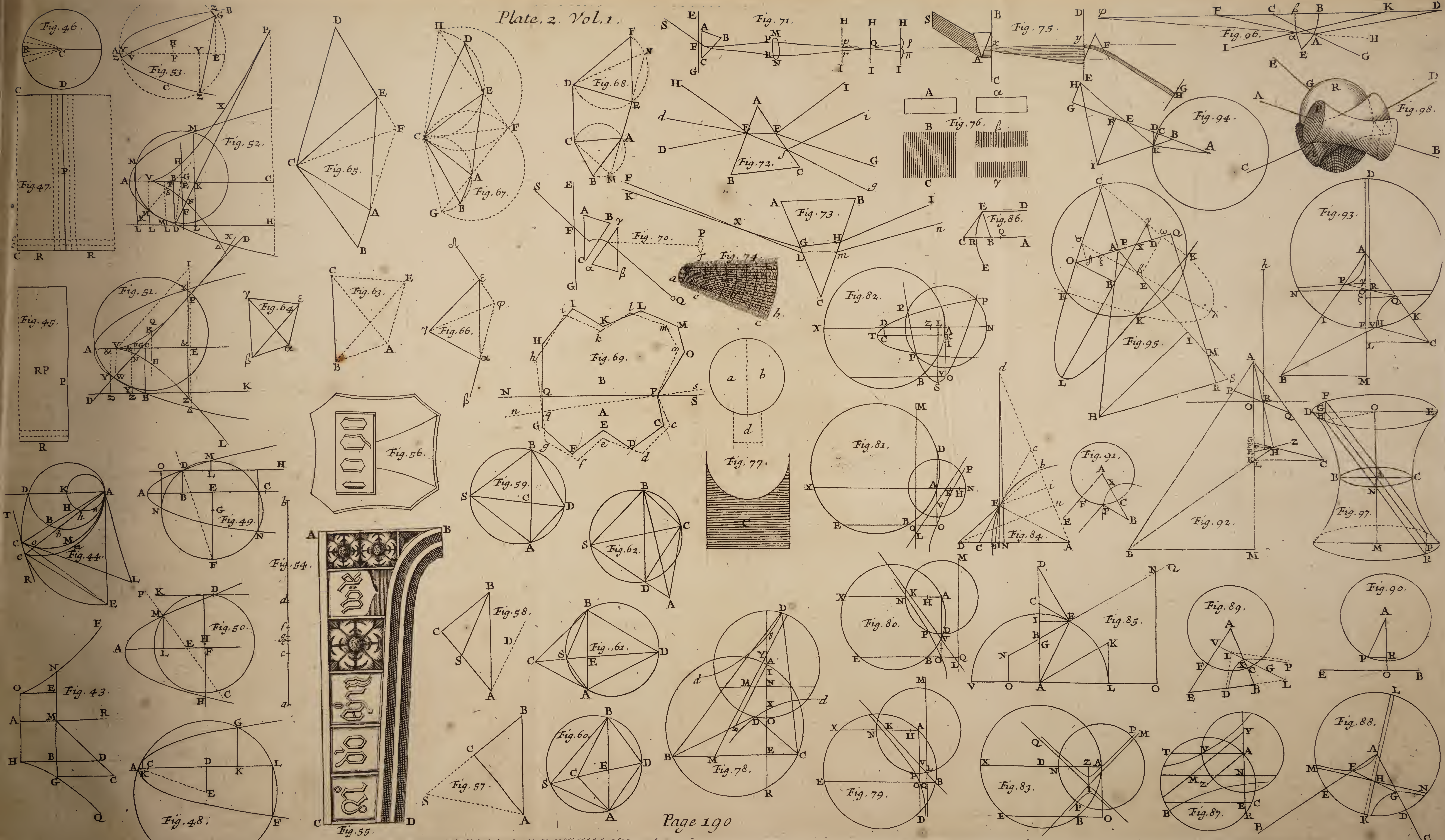
Fig. 43.
Fig. 44.
Fig. 45.
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Fig. 85.
Fig. 86.
Fig. 87.
Fig. 88.
Fig. 89.
Fig. 90.
Fig. 91.
Fig. 92.
Fig. 93.
Fig. 94.
Fig. 95.
Fig. 96.
Fig. 97.
Fig. 98.

Erect, and is ſo brought into the Eye by the Eye Glaſs; the Eye Glaſſes themſelves in neither Caſe having any thing to do with the Erecting or Inverting, but meerly in repreſenting in the ſame poſture the Species immediately before them. So that one Convex Glaſs as poſited in a Teleſcope Inverts; the ſecond (that is the firſt Eye Glaſs) does nothing towards the Erecting or Reverſing, but repreſents the Image as it is in the Diſtinct Baſe of the Object Glaſs before it, that is Inverted; the third Glaſs Erects, or rather Reſtores what was before Inverted; the fourth repreſents the Image as it receives it from the Diſtinct Baſe of the third, that is, Erect.

*The Apertures of Teleſcopes; by M. Auzout. n. 4. p. 55.*

XIII. 1. Mr. *Auzout* has found that the Apertures, which Optick Glaſſes can bear with Diſtinctneſs, are in about a ſubduplicate Proportion to their Lengths: and accordingly he hath made the following Table.

| *Lengths of Glaſſes.* | | *Apertures for* | | | | | | *Lengths of Glaſſes.* | | *Apertures for* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | *Excellent.* | | *Good.* | | *Ordinary.* | | | | *Excellent.* | | *Good.* | | *Ordinary.* | |
| *Feet.* | *In.* | *In.* | *Lin.* | *In.* | *Lin.* | *In.* | *Lin.* | *Feet.* | *In.* | *In.* | *Lin.* | *In.* | *Lin.* | *In.* | *Lin.* |
| | 4 | | 4 | | 4 | | 3 | 25 | | 3 | 4 | 2 | 10 | 2 | 4 |
| | 6 | | 5 | | 5 | | 4 | 30 | | 3 | 8 | 3 | 2 | 2 | 7 |
| | 9 | | 7 | | 6 | | 5 | 35 | | 4 | 0 | 3 | 4 | 2 | 10 |
| 1 | 0 | | 8 | | 7 | | 6 | 40 | | 4 | 3 | 3 | 7 | 3 | 1 |
| 1 | 6 | | 9 | | 8 | | 7 | 45 | | 4 | 6 | 3 | 10 | 3 | 2 |
| 2 | 0 | | 11 | | 10 | | 8 | 50 | | 4 | 9 | 4 | 0 | 3 | 4 |
| 2 | 6 | 1 | 0 | | 11 | | 9 | 55 | | 5 | 0 | 4 | 3 | 3 | 6 |
| 3 | 0 | 1 | 1 | 1 | 0 | | 10 | 60 | | 5 | 2 | 4 | 6 | 3 | 8 |
| 3 | 6 | 1 | 2 | 1 | 1 | | 11 | 65 | | 5 | 4 | 4 | 8 | 3 | 10 |
| 4 | 0 | 1 | 4 | 1 | 2 | 1 | 0 | 70 | | 5 | 7 | 4 | 10 | 4 | 0 |
| 4 | 6 | 1 | 5 | 1 | 3 | 1 | 1 | 75 | | 5 | 9 | 5 | 0 | 4 | 2 |
| 5 | 0 | 1 | 6 | 1 | 4 | 1 | 1 | 80 | | 5 | 11 | 5 | 2 | 4 | 5 |
| 6 | | 1 | 7 | 1 | 5 | 1 | 2 | 90 | | 6 | 4 | 5 | 6 | 4 | 7 |
| 7 | | 1 | 9 | 1 | 6 | 1 | 3 | 100 | | 6 | 8 | 5 | 9 | 4 | 10 |
| 8 | | 1 | 10 | 1 | 8 | 1 | 4 | 120 | | 7 | 5 | 6 | 5 | 5 | 3 |
| 9 | | 1 | 11 | 1 | 9 | 1 | 5 | 150 | | 8 | 0 | 7 | 0 | 5 | 11 |
| 10 | | 2 | 1 | 1 | 10 | 1 | 6 | 200 | | 9 | 6 | 8 | 0 | 6 | 9 |
| 12 | | 2 | 4 | 2 | 0 | 1 | 8 | 250 | | 10 | 6 | 9 | 2 | 7 | 8 |
| 14 | | 2 | 6 | 2 | 2 | 1 | 9 | 300 | | 11 | 6 | 10 | 0 | 8 | 5 |
| 16 | | 2 | 8 | 2 | 4 | 1 | 11 | 350 | | 12 | 6 | 10 | 9 | 9 | 0 |
| 18 | | 2 | 10 | 2 | 6 | 2 | 1 | 400 | | 13 | 4 | 11 | 6 | 9 | 8 |
| 20 | | 3 | 0 | 2 | 7 | 2 | 2 | | | | | | | | |

2. This

*Confider'd, by* Dr. Hook. *ib. p.* 67.

2. This Theory of Apertures, ſeems to me not very clear. For the ſame Glaſs will endure greater or leſſer Apertures, according to the leſſer or greater Light of the Object: If it be for the looking on the Sun or *Venus*, or for ſeeing the Diameters of the fixed Stars, then ſmaller Apertures do better; if for the Moon in the Day light, or on *Saturn*, or *Jupiter*, or *Mars*, then the Largeſt. Thus I have often made uſe of a 12 Foot Glaſs to look on *Saturn* with an Aperture of almoſt 3 inches, and with a ſingle Eye Glaſs of 2 inches double Convex; but, when with the ſame Glaſs I looked on the Sun or *Venus*, I uſed both a ſmaller Aperture, and ſhallower Charge.

*To Meaſure diſtances at one Station*; by M. Auzout. n. 7. p. 123.

XIV. I have found long ſince a way to meaſure, with a great Teleſcope, the diſtance of Objects upon the Earth from one Station. The Practice indeed does not altogether anſwer the Theory, becauſe that the Length of the Teleſcopes admits of ſome Latitude; yet one comes near enough, and perhaps as juſt as by moſt of the ways ordinarily uſed with Inſtruments. That, which I am propoſing, I doubt not but Mr. *Hook* will ſoon underſtand, and ſee the Determination of all Caſes poſſible. I ſhall only ſay, that if we look upon the ſole Theory, we may make uſe of an ordinary Teleſcope, whereof the Eye Glaſs is to be Convex: For by putting the Glaſſes at a little greater diſtance, than they are, porportionably to the diſtance for which it is to ſerve, and by adding to it a new Eye Glaſs, the Object will be ſeen diſtinct, though Obſcure; and if the Eye Glaſs be Convex, the Object will appear Erect. They may be done two manner of ways; either by leaving the Teleſcope in its ordinary Situation, the Object Glaſs before the Eye Glaſs; or by Inverting it, and putting this before that. But if any will make uſe of two Object Glaſſes, whereof the Focuſes are known, the Diſtance of them will be known. If it be ſuppoſed, that the Focus of the firſt be B, and that of the ſecond C, and the diſtance given, B + 2 D, and that D — C, be equal to F; for this diſtance will be equal to B + C + F — r F 2 C 2. And if you have the Focus of the firſt Object Glaſs, equal to B, the diſtance where you will put the ſecond Glaſs equal to B + C + D, the Focus of the ſecond Glaſs will be found equal to $\frac{CD}{C + D}$. And if you will that the Object ſhall be magnified as much with theſe two Glaſſes, as it would be with a ſingle one, whereof the Focus ſhould be of the diſtance given, having the Focus of the Object Glaſs given equal to B, and the diſtance given to B + D; the diſtance between the firſt and ſecond Glaſs will be equal to $\frac{2B^2 + 2BD}{2B + D}$, whence ſubducting B (the Focus of the Object Glaſs given) there remains $\frac{BD}{2B + D}$: and if this Sum be ſuppoſed equal to C, we ſhall eaſily know, by the precedent Rule, the Focus of the ſecond Glaſs.

XV. Prepare

XV. Prepare two Glaſſes, the one exactly flat on both ſides, the other flat on the one ſide, and Convex on the other, of what Sphere you pleaſe. Let the flat Glaſs be a little broader than the other. Then let there be made a Cell or Ring of Braſs, very exactly turned, into which theſe two Glaſſes may be ſo faſtned with Cement, that the Plain Surfaces of them may lie exactly Parallel, and that the Convex ſide of the Plano-convex Glaſs may lie inward ; but ſo, as not to touch the Flat of the other Glaſs. Theſe being Cemented into the Ring very cloſely about the Edges, by a ſmall Hole in the ſide of the Braſs Ring or Cell, fill the interpoſed Space between theſe two with Water, Oyl of Turpentine, Spirit of Wine, Saline Liquors, &c ; then ſtop the Hole with a Skrew : And according to the differing Refraction of the Interpoſed Liquors, ſo ſhall the Focus of this Compound Glaſs be longer or ſhorter. *To make a Plano convex Glaſs of a ſmall Sphere Collect the Rays at a great Diſtance ; by Dr.* Hook. n. 4. *p.* 66. n. 12. *p.* 202.

But this I would have only lookt upon, as one Inſtance of many (for there may be others) of the Poſſibility of making a Glaſs, ground in a ſmaller Sphere, to conſtitute a Teleſcope of a much greater Length : Though (not to raiſe too great Expectation) I muſt add, That, of Spherical Object Glaſſes thoſe are the beſt which are made of the greateſt Sphere, and whoſe Subſtance hath the greateſt Refraction.

XVI. 1. *S. Campani* pretends to have found a Way to Work great *Optick Glaſſes* with a Turn-tool, without any Mould: And that he uſeth three Eye Glaſſes for his great Teleſcopes, without finding any Rainbow Colours. *Teleſcopes and other Optick Glaſſes; by* Campani, *and* Divini. n. 1. *p.* 2. n. 8. *p.* 131.

The *Great Duke* of *Tuſcany*, and Prince *Leopold* his Brother, upon Tryal made of the Glaſſes of *Campani* and *Divini*, have found that thoſe of *Campani* excel the other ; and with them they have been eaſily able to diſtinguiſh People at four Leagues diſtance.

But *Euſtachio Divini* pretends, that in all the Tryals made with them, his great Glaſſes have performed better than thoſe of *Campani* : and that *Campani* was not willing to do what was neceſſary for well comparing the one with the other, *viz.* to put Equal Eye Glaſſes in them, or to exchange the ſame Glaſſes. n. 12. *p.* 209.

2. 'Tis now above 10 Years ſince I invented a peculiar way of Grinding Optick Glaſſes, and reduced it alſo into Practice ; by which 'tis eaſie, without any conſiderable danger of failing, to make and poliſh Optick-glaſſes of any *Conick Section*; and that (which is moſt notable) in any Diſh of any Section of a Sphere. I have already made ſeveral Glaſſes by it, which many Learned Men have ſeen and tryed. *By M.* Hevelius. n. 6. *p.* 98.

M. *Huygens* alſo intends very ſhortly to try ſomething in that kind. *By M.* Huygens. *ib.*

3 *M. du Sons* doth at preſent employ himſelf in *London*, to bring Teleſcopes to perfection, by Grinding Glaſſes of a *Parabolical* Figure. I have ſeen two Eye Glaſſes of that ſhape, about one inch and a half deep, and one inch and a quarter broad, wrought by this *eminent Artiſt* with a rare Steel Inſtrument of his own Contrivance and Workmanſhip, and by himſelf alſo poliſh'd to Admiration. And certainly it will be wondred at by thoſe, who ſhall ſee theſe Glaſſes, how they could be truly wrought to ſuch a Figure, with *By M.* du Sons *ib. p.* 99. n. 7. *p.* 119.

ſuch a Cavity; and yet more, when they ſhall hear the Author undertake to Excavate other ſuch Eye Glaſſes to above two Inches, and Object Glaſſes of five Inches Diameter. He hath likewiſe already begun his Object Glaſſes for the mentioned two Ocular ones, of the ſame Figure of about two Inches Diameter, which are to be left all open, yet without cauſing any Colours.

By M. Burattini. n. 19. p. 348. n. 21. p. 374.

4. The Optick Glaſſes of M. *Burattini* in *Poland*, are perfectly well Wrought and Poliſh'd. He hath ſent two to *Paris*, but they are only the one of 10, the other of 8 Foot. They bear a great Aperture in reſpect of their Length.

By Mr. Francis Smethwick. n. 33. p. 631.

5. Mr. *Fr. Smethwick*, having found a way of grinding Glaſſes *not Spherical*, produced before the Royal Society, *Feb.* 27. 166⅞. certain *Specimina* of that Invention, which were a Teleſcope, a Reading and two Burning-Glaſſes.

The Teleſcope was about 4 Foot long, furniſhed with 4 Glaſſes, whereof the 3 Ocular ones, Plano-convex, were of this newly Invented *Not Spherical* Figure, and the 4th a Spherical Object Glaſs. This being compared with a common, yet very good Teleſcope, longer than it by about 4 Inches, and turned to ſeveral Objects, was found by thoſe of the ſaid Society that look'd through them both, to exceed the other in Goodneſs, by taking in a greater Angle, and repreſenting the Objects more exactly in their reſpective Proportions, and enduring a greater Aperture free from Colours.

The Reading Glaſs of the ſame Figure being compared with a common Spherical Glaſs did far excel it, by magnifying the Letters to which it was applied up to the very edges, and by ſhewing them diſtinctly from one Brim through the Center to the other, which the Spherical Glaſs came far ſhort of.

Laſtly, The two Burning Concaves of this new invented Figure, were the one of 6 Inches Diameter, its Focus 3 Inches diſtant from the Center thereof; the other of the ſame Diameter, but leſs Concave, and its Focus 10 Inches diſtant. Theſe, when approached to a large Candle lighted, did ſomewhat warm the Faces of thoſe that were 4 or 5 Foot diſtant at leaſt, and when held to the Fire, burned Gloves and Garments at the diſtance of about 3 Foot from the Fire.

The Biſhop of *Salisbury*, Dr. *Seth Ward*, was by at another time, when the deeper of his two Concaves turned a Piece of Wood into Flame in the ſpace of 10 Sec. of Time, and the Shallower in 5 Sec. at moſt, in the Seaſon of *Autumn*, about 9 of the Clock in the Morning, the Weather gloomy. The Inventor adds, That the Deeper Concave, when held to a Lucid Body, would caſt a Light ſtrong enough to Read by at a conſiderable diſtance; and that expoſing the ſame to a Northern Window, on which the Sun ſhined not at all, or very little, he had perceived that it would warm one's hand ſenſibly, by Collecting the warmed Air in the Day-time, which it would not do after Sun-ſet.

By an Artiſt at Paris. n. 40. p. 795.

6. We have an Artiſt at *Paris* that Poliſhes Optick Glaſſes on a Turn. I have ſeen a Glaſs of his Workmanſhip which is very good. He turns theſe Glaſſes as he does Wood, that is with the ſame Facility.

7. M. *Bo-*

7. M. *Borelli* hath found out a sure and very easie Method to work all sort of great Glasses. He hath already made one of them very good of 200 Foot, wrought on both sides on the same Rule. His Desire of advancing Astronomical Discoveries hath induced him to make Presents of them to several Persons capable to make use of them. He hath intrusted the Secret to one of the *Royal Academy* of *Sciences*. *By M.* Borelli. n. 128. *p.* 691.

*Campani* and *Divini* have commonly sold their Glasses at a Pistol the Foot. Sometimes they have far exceeded that Price. One of *Divini's* of 12 Foot was sold for 400 Livers; and another of *Campani's*, of 34 Foot, for 2000 Livers. Notwithstanding which *S. Borelli* is willing to part with the best of his own Glasses, of 50, 60, or 65 Foot, for 500 *(French)* Crowns; and the small Glasses, from 6 to 12 Foot, at a *(French)* Crown a Foot; from 12 to 18, at half a Pistol; and from 18 to 26, at a Pistol. n. 140. *p.* 1005.

8. Though it be commonly believed, that *Rock Crystal* is not fit for Optick Glasses, because there are many Veins in it; yet *Eustachio Divini* made one of it, which he saith proved an excellent one, though full of Veins: but perhaps they were only superficial Strictures and slight Scratches, not Veins. *Optick-Lens's of Rock Crystal; by* Eust. Divini. n. 20. *p.* 362.

9. Drops of fair Water being let fall on a piece of plain Glass, form themselves into Plano-convexes, having a Convexity proportionable to the heights from which they descend; from a greater height a less, from a less, a greater degree of Convexity. I applied some of these as Reading Glasses for single Words of small Letters, as on the Globes and Maps, and found no other Inconveniency, than that the Fluidity of the Water obliges one to keep the Glass Horizontal, which I after devised a way to remedy. I took a sufficient quantity of Izing-Glass, and dissolved it in Water over the Fire, and whilst it was warm I dipt a Stick into the Solution, and let some Drops of it fall on the Glass as before; and in a quarter of an Hour they acquire a Consistency, that permits them to be held in any Position, and tho' they are not altogether so transparent, yet this is little or no Impediment to their Use. The Drops of this Solution are more exactly defined than those of common Water, having their edges exactly Circular, and one may make them of a much longer Focus than those. *Of Water; by Mr.* Stephen Gray. n. 228. *p.* 539.

A thin flat Ring of Brass, not exceeding 4 tenths of an Inch Diameter in its interior Circle, being cemented to a plain piece of Glass, and filled with Water, or the Solution now mentioned, then by pressing the Finger into it, till what is superfluous be taken off, there will be formed a Plano-concave, which may serve as an Eye Glass to a Perspective, or to any other Optical use Concave Glasses are applicable.

I have tried what would be the Success of combining Portions of Water by the help of Brass Rings, and plain pieces of Glass, to give them their true Figure and requisite Apertures, and inserted them at the ends of Tubes of several Lengths; and find, that tho' these *Natural Lentes* may serve as Eye Glasses, yet when used as Object ones, either to Telescopes or double Microscopes, their Effects will not compensate the Trouble there is in using them.

*The Advantages of Reflexion to Optick Instruments; by* Mr. Newton. n. 80. *p.* 3079.

XVII. 1. When I had found, That *Light consists of Rays differently Refrangible*, I left off my Glass-works, for I saw that the Perfection of Telescopes was hitherto limited, not so much for want of Glasses truly figured according to the Prescriptions of Optick Authors, (which all Men have hitherto imagined) as because that *Light* it self is a Heterogeneous mixture of *Differently Refrangible Rays*. So that, were a Glass so exactly figured, as to collect any one sort of Rays into one Point, it could not collect those also into the same Point, which having the same Incidence upon the same Medium are apt to suffer a different Refraction. Nay, I wondred, That seeing the Difference of Refrangibility was so great, as I found it, Telescopes should arrive to that Perfection they are now at. For, measuring the Refractions in one of my Prisms, I found, that supposing the common Sine of Incidence upon one of its Plains was 44 Parts, the Sine of Refraction of the utmost Rays on the red end of the Colours, made out of the Glass into the Air, would be 68 parts, and the Sine of Refraction of the utmost Rays on the other end, 69 parts: so that the Difference is about a 24th or 25th part of the whole Refraction. And consequently, the Object Glass of any Telescope cannot Collect all the Rays, which come from one Point of an Object, so as to make them convene at its Focus in less room than in a Circular space, whose Diameter is the 50th part of the Diameter of its Aperture; which is an Irregularity, some hundreds of times greater than a Circularly figured *Lens*, of so small a Section as the Object Glasses of long Telescopes are, would cause by the unfitness of its Figure, were Light Uniform.

This made me take Reflexions into Consideration; and finding them Regular, so that the Angle of Reflexion of all sorts of Rays was equal to their Angle of Incidence, I understood, that by their Mediation Optick Instruments might be brought to any degree of Perfection imaginable, provided a Reflecting Substance could be found, which would Polish as finely as Glass, and Reflect as much Light as Glass Transmits, and the Art of communicating to it a Parabolick Figure be also attained. But there seemed very great Difficulties, and I have almost thought them Insuperable, when I farther considered, that every Irregularity in a Reflecting Superficies makes the Rays stray 5 or 6 times more out of their due Course, than the like Irregularities in a Refracting one: So that a much greater Curiosity would be here requisite, than in figuring Glasses for Refraction.

Amidst these Thoughts, I was forced from *Cambridge*, *Anno* 1666. by the intervening Plague, and it was more than two years before I proceeded further. But then having thought on a tender way of Polishing, proper for Metal, whereby as I imagined the Figure also would be corrected to the last, I began to try what might be effected in this kind, and by degrees so far perfected an Instrument (in the Essential parts of it like that I sent to *London*) by which I could discern *Jupiter*'s 4 Concomitants, and shewed them divers times to two others of my Acquaintance. I could also discern the Moon-like *Phase* of *Venus*, but not very distinctly, nor without some niceness in disposing the Instrument.

From

From that time I was interrupted till this laſt *Autumn*, when I made another. And as that was ſenſibly better than the firſt, (eſpecially for Day Objects) ſo I doubt not, but they will be ſtill brought to a much greater Perfection by their Endeavours, who, as you inform me, are taking care about it at *London*.

2. This new Inſtrument is compoſed of two Metalline Speculums, the one Concave, (inſtead of an Object Glaſs) the other Plain: and alſo of a ſmall Plano-convex Eye Glaſs; as in the Figure, where A B is a Concave Speculum, of which the Radius or Semidiameter is $12\frac{2}{3}$ or 13 Inches. *A new Catadioptrical Teleſcope Invented by Mr.* Newton. n. 81. *p.* 4004. Fig. 99.

C D, another Metalline Speculum, whoſe Surface is Flat, and the Circumference Oval.

G D, an Iron Wire, holding a Ring of Braſs, in which the Speculum C D is fixed.

F, a ſmall Eye Glaſs, Flat above, and Convex below, of the 12th part of an Inch Radius, if not leſs.

G G G, the fore part of the Tube (which is open) faſtned to a Braſs Ring H I, to keep it immoveable.

P Q K L, the hind part of the Tube, faſtned to another Braſs Ring, P Q.

O, an Iron-Hook faſtned to the Ring P Q, and furniſht with a Skrew N, thereby to advance or draw back the hind part of the Tube, and ſo by that means to put the Specula in their due diſtance.

M Q G I, a crooked Iron ſuſtaining the Tube, and faſtned by the Nail R to the Ball and Socket S, whereby the Tube may be turned every way.

The Center of the Flat Speculum C D, muſt be placed in the ſame Point of the Tube's Axe, where falls the Perpendicular to this Axe, drawn to the ſame from the Center of the little Eye Glaſs, which Point is here marked at T.

And to give the Reader ſome Satisfaction to underſtand, in what Degree it repreſents things diſtinct, and free from Colours, and to know the Aperture by which it admits Light, he may compare the Diſtances of the Focus E from the Vertexes of the little Eye Glaſs and the Concave Speculum; that is, E F, $\frac{1}{6}$ of an Inch, and E T V, $6\frac{1}{3}$ Inches, and the Ratio will be found as 1 to 38; whereby it appears, that the Objects will be magnified about 38 times, and be repreſented bigger by $2\frac{1}{2}$ times in Diameter, when ſeen through this, than through an ordinary Teleſcope of about two Foot long.

Thus far as to the Structure of this Teleſcope. Concerning the metalline Matter, fit for theſe Reflecting Speculums, the Inventor hath alſo conſidered the ſame, and gives this Caution, That whilſt Men ſeek for a White, Hard, and Durable Metalline Compoſition, they reſolve not upon ſuch an one, as is full of ſmall Pores, only diſcoverable by a Microſcope. For though ſuch an one may to appearance take a good Poliſh, yet the Edges of thoſe ſmall Pores will wear away faſter in the Poliſhing than the other Parts of the Metal; and ſo, however the Metal ſeem Polite, yet it ſhall not Reflect with ſuch an accurate regularity as it ought to do. Thus Tin-Glaſs mixt with ordinary Bell-Metal makes it more White, and apt to Reflect a greater quan-

tity

tity of Light; but withall its Fumes, raiſed in the Fuſion, like ſo many Aerial Bubbles, fill the Metal full of thoſe Microſcopical Pores. But White Arſenick both Blanches the Metal, and leaves it Solid, without any ſuch Pores, eſpecially if the Fuſion hath not been too violent. What the Stellate Regulus of *Mars*, (which I have ſometimes uſed) or other ſuch like Subſtance will do, deſerves particular Examination.

To this he adds this further Intimation, That Putty, or other ſuch like Powder, with which 'tis Poliſhed, by the ſharp Angles of its Particles, fretteth the Metal, if it be not very fine, and fills it full of ſuch ſmall holes as he ſpeaketh of. Wherefore care muſt be taken of that before Judgment be given, whether the Metal be throughout the Body of it Porous or not.

But not having tried, as he ſaith, many Proportions of the Arſenick and Metal, he does not affirm, which is abſolutely beſt, but thinks there may conveniently be uſed any quantity of Arſenick equalling in Weight between a ſixth and eighth part of the Copper, a greater proportion making the Metal Brittle.

The way which he uſed was this. He firſt melted the Copper alone, then put in the Arſenick, which being melted, he ſtirred them a little together, bewaring, in the mean time, not to draw in Breath near the pernicious Fumes. After this, he put in Tin, and again, ſo ſoon as that was melted, (which was very ſuddenly) he ſtirred them well together, and immediately poured them off.

He ſaith, he knows not, Whether by letting them ſtand longer on the Fire after the Tin was melted, a higher degree of Fuſion would have made the Metal Porous; but he thought that way he proceeded to be ſafeſt.

He adds, That in that Metal, which he ſent to *London*, there was no Arſenick, but a ſmall proportion of Silver; as he remembers, one Shilling in three Ounces of Metal. But he thought withal, that the Silver did as much harm in making the Metal ſoft, and ſo leſs fit to be Poliſht, as good in rendring it White and Luminous.

At another time, he mixed Arſenick one Ounce, Copper ſix Ounces, and Tin two Ounces; And this an Acquaintance of his hath, as he intimates, Poliſht better than he did the other.

As to the Objection, That with this kind of Perſpectives, Objects are difficultly found, he anſwers, That that is the Inconvenience of all Tubes that Magnifie much; and that after a little uſe the Inconvenience will grow leſs, ſeeing that himſelf could readily enough find any Day Objects by knowing which way they were Poſited from other Objects that he accidentally ſaw in it. But in the Night to find Stars, he acknowledges it to be more troubleſome; which yet may, in his Opinion, be eaſily remedied by two Sights affixed to the Iron-Rod, by which the Tube is ſuſtained, or by an ordinary Perſpective Glaſs faſtned to the ſame Frame with the Tube, and directed towards the ſame Object, as *Des Cartes* in his *Diop-tricks* hath deſcribed for Remedying the ſame Inconvenience of his beſt Teleſcopes.

3. I ſee

3. I ſee by the Deſcription you have ſent me of Mr. *Newton*'s admirable Teleſcope, that he hath well conſidered the Advantage, which a Concave Speculum hath above Convex Glaſſes in collecting the Parallel Rays, which certainly according to the Calculation I have made thereof is very great. Hence it is, That he can give a far greater Aperture to that Speculum, than to an Object Glaſs of the ſame diſtance of the Focus, and conſequently that he can much more Magnifie Objects this way, than by an ordinary Teleſcope. Beſides, by it he avoids an Inconvenience, which is inſeparable from Convex Object Glaſſes, which is the Obliquity of both their Surfaces, which vitiateth the Refraction of the Rays that paſs towards the ſides of the Glaſs, and does more hurt than Men are aware of. Again, by the meer Reflexion of the Metalline Speculum there are not ſo many Rays loſt, as in Glaſſes, which Reflect a conſiderable quantity by each of their Surfaces, and beſides intercept many of them by the Obſcurity of their matter. *Approv'd by M. Hugens de Zulichem. Ibid.* p. 4008.

Mean time, the main buſineſs will be, to find a matter for this Speculum that will bear ſo good and even a Poliſh as Glaſſes, and a way of giving this Poliſh without vitiating the Spherical Figure. Hitherto I have found no Specula that had near ſo good a Poliſh as Glaſs: And if Mr. *Newton* hath not already found a way to make it better than ordinarily, I apprehend his Teleſcope will not ſo well diſtinguiſh Objects as thoſe with Glaſſes. But 'tis worth while to ſearch for a Remedy to this Inconviency, and I deſpair not of finding one. I believe that Mr. *Newton* hath not been without conſidering the Advantage, which a Parabolical Speculum would have above a Spherical one in this Conſtruction; but that he deſpairs, as well as I do, of working other Surfaces than Spherical ones with due exactneſs; though elſe it be more eaſie to make a Parabolical than Elliptical or Hyperbolical ones, by reaſon of a certain Propriety of the Parabolick Conoid, which is, that all the Sections Parallel to the Axis make the ſame Parabola.

But though Mr. *Newton* (with M. *Hugens*) deſpairs of performing that work by Geometrical Rules, yet he doubts not but that the thing may in ſome meaſure be accompliſhed by Mechanical Devices. *Ibid. p.* 4009.

4. In my laſt Letter, I gave you occaſion to ſuſpect, that the Inſtrument which I ſent you is in ſome reſpect or other indiſpoſed, or that the Metals are Tarniſhed: And by yours I am fully confirmed in that Opinion. For, whilſt I had it, it repreſented the Moon in ſome Parts of it as diſtinctly as other Teleſcopes uſually do which Magnifie as much as that. Yet I very well know, That that Inſtrument hath its Imperfections both in the Compoſition of the Metal, and in its being badly Caſt, as you may perceive by a Scabrous place near the middle of the Metal of it on the Poliſhed ſide, and alſo in the figure of that Metal near that Scabrous place: And in all thoſe reſpects that Inſtrument is capable of further Improvement. *A further Account of this Inſtrument by Mr.* Newton. *Ibid.*

You ſeem to intimate, That the Proportion of 38 to 1, holds only for its Magnifying Objects at ſmall diſtances. But if for ſuch Diſtances, ſuppoſe 500 Feet, it Magnifie at that rate, by the Rules of Opticks it muſt for the greateſt diſtance imaginable Magnifie more than $37\frac{1}{4}$ to 1, which is ſo inconſiderable a diminiſhing, that it may be even then as 38 to 1.

Here-

Here is made another Inſtrument like the former, which does very well. Yeſterday I compared it with a ſix Foot Teleſcope, and found it not only to Magnifie more, but alſo more Diſtinctly. And to day I found, that I could Read in one of the Philoſophical Tranſactions, placed in the Sun's Light, at an hundred Foot Diſtance, and that at a hundred and twenty Foot Diſtance I could diſcern ſome of the Words. When I made this tryal, its Aperture (defined next the Eye) was equivalent to more than an Inch and a third part of the Object Metal. This may be of ſome uſe to thoſe that ſhall endeavour any thing in Reflexions; for hereby they will in ſome meaſure be enabled to judge of the goodneſs of their Inſtruments.

*The Apertures and Charges of theſe Inſtruments; by* Mr. Newton. n. 82. p. 4032.

5. I know that the Aperture was $1\frac{1}{3}$ of an Inch, by trying that an Obſtacle of that Breadth was requiſite to intercept all the Light, which came from one Point of the Object.

I ſhould tell you alſo that the little plain piece of Metal, next the Eye Glaſs, is not only truly Figured: Whereby it happens, that Objects are not ſo diſtinct at the Middle as at the Edges. And I hope, that by correcting its Figure, (in which I find more difficulty than one would expect,) they will appear all over Diſtinct, and Diſtincter in the Middle than at the Edges. And I doubt not but that the performances will then be greater.

But yet I find, that there is more Light loſt by Reflexion of the Metal which I have hitherto uſed, than by Tranſmiſſion through Glaſſes: For which reaſon a Shallower Charge would probably do better for Obſcure Objects; ſuppoſe ſuch a one, as would make it Magnifie 34 or 32 times. But for Bright Objects at any diſtance, it ſeems capable of Magnifying 38 or 40 times, with ſufficient Diſtinctneſs. And for all Objects, the ſame Charge, I believe, may with Advantage be allowed, if the Steely Matter, imployed at *London*, be more ſtrongly Reflective than this which I have uſed.

The performances of one of theſe Inſtruments of any Length being known, it will appear by this following Table, what may be expected from thoſe of other Lengths by this way, if Art can accompliſh what is promiſed by the Theory. In the firſt Column is expreſſed the Length of the Teleſcope in Feet; which doubled gives the Semidiameter of the Sphere, on which the Concave Metal is to be ground. In the ſecond Column are the proportions of the Apertures for thoſe ſeveral Lengths. And in the third Column are the Proportions of the Charges, or Diameter of the Spheres, on which the Convex Superficies of the Eye Glaſſes are to be ground.

| *Lengths.* | *Apertures.* | *Charges.* | *Lengths.* | *Apertures.* | *Charges.* |
|---|---|---|---|---|---|
| $\frac{1}{2}$ | 100 | 100 | 8 | 800 | 200 |
| 1 | 168 | 119 | 10 | 946 | 211 |
| 2 | 283 | 141 | 12 | 1084 | 221 |
| 3 | 383 | 157 | 16 | 1345 | 238 |
| 4 | 476 | 168 | 20 | 1591 | 251 |
| 5 | 562 | 178 | 24 | 1824 | 263 |
| 6 | 645 | 186 | | | |

The

The Uſe of this Table will beſt appear by Example : Suppoſe therefore a half Foot Teleſcope may diſtinctly Magnifie 30 times with an Inch Aperture, and it being required to know, what ought to be the Analogous Conſtitution and Performance of a four Foot Teleſcope: By the ſecond Column, as 100 to 476 ; ſo are the Apertures, as alſo the number of times which they Magnifie. And conſequently ſince the half Foot Tube hath an Inch Aperture and Magnifieth 30 times, a four Foot Tube proportionally ſhould have $4\frac{76}{100}$ Inches Aperture, and Magnifie 143 times. And by the third Column, as 100 to 168; ſo have their Charges : And therefore if the Diameter of the Convexity of the Eye Glaſs for a half Foot Teleſcope be $\frac{1}{5}$ of an Inch, that for a four Foot ſhould be $\frac{168}{500}$, that is, about $\frac{1}{3}$ of an Inch ; and ſo of other Lengths. But what the Event will really be we muſt wait to ſee determined by Experience. Only this I thought fit to inſinuate, that they which intend to make tryal in other Lengths, may more readily know how to deſign their Inſtruments. Thus for a four Foot Tube, ſince the Aperture ſhould be five or ſix Inches, there will be required a piece of Metal ſeven or eight Inches Broad at leaſt, becauſe the Figure will ſcarcely be true to the Edges. And the thickneſs of the Metal muſt be proportional to the Breadth, leſt it bend in the Grinding. The Metals being Poliſhed, there may be tryals made with ſeveral Eye Glaſſes, to find what Charge may with beſt Advantage be made uſe of.

*Some Objections of M. . . . . . . . Anſwered, by* Mr. Newton. *ib. p.* 4034.

XVIII. 1. I doubt not but *M. A.* will allow the Advantage of Reflexion in the Theory to be very great, when he ſhall have informed himſelf of the *Different Refrangibility* of the ſeveral Rays of Light. And for the Practick part, it is in ſome meaſure manifeſt by the Inſtruments already made, to what Degree of Vivacity and Brightneſs a Mettalline Subſtance may be Poliſhed. Nor is it improbable but that there may be new ways of Poliſhing found out for Metal, which will far excel thoſe that are yet in uſe. And when a Metal is once well Poliſhed, it will be a long while preſerved from Tarniſhing, if diligence be uſed to keep it dry and cloſe, ſhut up from Air: For the principal Cauſe of Tarniſhing ſeems to be, the Condenſing of Moiſture on its Poliſh'd Surface, which by an Acid Spirit, where with the Atmoſphere is impregnated, Corrodes and Ruſts it; or at leaſt, at its Exhaling leaves it covered over with a thin Skin, conſiſting partly of an Earthly Sediment of that moiſture, and partly of the Duſt, which flying to and fro' in the Air had ſetled and adhered to it.

When there is not occaſion to make frequent uſe of the Inſtrument, there may be other ways to preſerve the Metal for a long time ; as perhaps by immerging it in Spirit of Wine or ſome other convenient Liquor. And if they chance to Tarniſh ; yet their Poliſh may be recovered by rubbing them with a ſoft piece of Leather, or other tender ſubſtance, without the Aſſiſtance of any fretting Powders, unleſs they happen to be Ruſty: For then they muſt be new Poliſhed.

I am very ſenſible, that Metal Reflects leſs Light than Glaſs tranſmits; and for that Inconvenience, I gave you a Remedy in my laſt Letter, by aſſigning a ſhallower Charge in proportion to the Aperture, than is uſed in other Teleſcopes. But as I have found ſome Metalline Subſtances to be more ſtrongly Reflective, and to Poliſh better, and be freer from Tarniſhing than others; ſo I hope there may in time be found out ſome Subſtance much freer from theſe Inconveniencies than any yet known.

*The Conſiderations of . . . . . . . Anſwer'd, by Mr.* Newton. n. 88, *p.* 5084.

2. The *Conſiderer* is pleaſed to reprehend me, for laying aſide the Thoughts of Improving Opticks by Refractions. If he had obliged me by a private Letter on this occaſion, I would have acquainted him with my ſucceſs on the Trials I have made of that kind, which I ſhall now ſay have been leſs than I ſometimes expected, and perhaps than he at preſent hopes for. But ſince he is pleaſed to take it for granted, that I have let this Subject paſs without due Examination, I ſhall refer him to my former Letters, by which that Conjecture will appear to be ungrounded. For, what I ſaid there, was in reſpect of Teleſcopes of the ordinary Conſtruction, ſignifying, that their Improvement is not to be expected from the well figuring of Glaſſes, as Opticians have imagined; but I deſpaired not of their Improvement by other Conſtructions, which made me cautious to inſert nothing that might intimate the contrary. For although ſucceſſive Refractions that are all made the ſame way, do neceſſarily more and more augment the Errors of the firſt Refraction; yet it ſeemed not impoſſible for contrary Refractions ſo to correct each others Inequalities, as to make their difference Regular; and if that could be conveniently effected, there would be no further difficulty. Now to this end, I examined, what may be done not only by Glaſſes alone, but more eſpecially by a Complication of divers ſucceſſive Mediums, as by two or more Glaſſes or Cryſtals with Water or ſome other Fluid between them; all which together may perform the Office of one Glaſs, eſpecially of the Object Glaſs, on whoſe Conſtruction the Perfection of the Inſtrument chiefly depends.

To the Aſſertion, That Rays are leſs true Reflected to a Point by a Concave, than Refracted by a Convex, I cannot aſſent; nor do I underſtand, That the Focus of the latter is leſs a Line than that of the former. The Truth of the contrary you will rather perceive by the following Table, computed for ſuch a Reflecting Concave, and Refracting Convex, on ſuppoſition that they have equal Apertures, and Collect Parallel Rays at an equal Diſtance from their Vertex; which Diſtance being divided into 15000 parts, the Diameter of the Concave Sphere will be 60000 of thoſe parts, and of the Convex 10000; ſuppoſing the Sines of Incidence and Refraction to be, in round numbers, as 2 to 3. And this Table following ſhews, how much the exterior Rays, at ſeveral Apertures, fall ſhort of their principal Focus.

*The*

| The Diameter of the Aperture. | The Parts of the Axis intercepted between the Vertex and the Rays | | The Error by | |
|---|---|---|---|---|
| | Reflected. | Refracted. | Reflexion. | Refraction. |
| 2000 | $14991\frac{2}{3}$ | 14865 | $8\frac{1}{3}$ | 135 |
| 4000 | 14966 | 14449 | 33 | 551 |
| 6000 | 14924 | 13699 | 76 | 1301 |
| 8000 | 14865 | 12475 | 135 | 2525 |
| 10000 | 14787 | 9472 | 213 | 5528 |

By this you may perceive, That the Errors of the Refracting Convex are ſo far from being leſs, that they are more than 16 times greater than the like Errors of the Reflecting Concave, eſpecially in great Apertures; and that without reſpect to the Heterogeneous Conſtitution of Light. So that, however the contrary Suppoſition might make the Author of theſe Animadverſions reject Reflexions as uſeleſs for the promoting of Opticks; yet I muſt for this, as well as other Conſiderations, prefer them in the Theory before Refractions.

Whether the *Parabola* be more difficult to deſcribe than the *Hyperbola*, or *Ellipſis*, may be a Quære; But I ſee no abſolute neceſſity of endeavouring after any of their Deſcriptions. For if Metals can be ground truly Spherical, they will bear as great Apertures, as I believe Men will be well able to communicate an exact Poliſh to. And for Dioptrique Teleſcopes, I told you, That the difficulty conſiſted not in the Figure of the Glaſs, but in the Difformity of Refractions; which if it did not, I could tell you a better and more eaſie remedy than the uſe of the *Conic Sections*.

3. We ſee that a Picture made by an Object Glaſs of 12 Foot in a dark Room, is too Diſtinct, and too well Defined, to be produced by Rays, that ſhould ſtray the 50th part of the Aperture. *Objections; by M. . . . . . n. 96. p. 6087.*

To take away this difficulty, I muſt acquaint you, That though I put the greateſt Lateral Error of the Rays from one another to be about $\frac{1}{50}$ of the Glaſs's Diameter; yet their greater Error from the Points on which they ought to fall, will be but $\frac{1}{100}$ of that Diameter: And then, that the Rays, whoſe Error is ſo great, are but very few in compariſon to thoſe, which are Refracted more juſtly; for the Rays which fall upon the middle Parts of the Glaſs, are Refracted with ſufficient Exactneſs, as alſo are thoſe that fall near the Perimeter and have a mean degree of Refrangibility; ſo that there remain only the Rays, which fall near the Perimeter and are moſt or leaſt Refrangible, to cauſe any ſenſible Confuſion in the Picture. And theſe are yet ſo much further weakned by the greater ſpace, through which they are ſcatter'd, that the Light which falls on the due Point, is infinitely more Denſe than that which falls on any other Point round about it. And by this *Anſwer'd, by Mr. Newton. n. 97. p. 6110.*



exceſs

excess of Density, the Light, which falls in or invisibly near the just Point, may I conceive, strike the *Sensorium* so vigorously, that the Impress of the weak Light, which Errs round about it, shall, in comparison, not be strong enough to be animadverted, or to cause any more sensible Confusion in the Picture than is found by Experience. But if this satisfie not, N. may try, if he please, how distinct the Picture will appear, when all the Lens is cover'd excepting a little hole next its Edge on one side only: And if in this case he please to measure the breadth of the Colours thus made at the Edge of the Sun's Picture, he will perhaps find it to approach nearer to my Proportion than he expects.

*A Reply, by M. .... Ibid. p. 6112.*

4. I am satisfied with the manner, whereby Mr. *Newton* reconciles the effect of Convex Glasses with his Theory; but then he is also to acknowledge, that this Aberration of the Rays is not so disadvantageous to Optick-Glasses as he seems to have been willing to make us believe. His Invention is very good; but the defect of the Metal seems to render it as impossible to execute, as the difficulty of the Form obstructs the use of the *Hyperbole* of M. *Des Cartes*.

*Answer'd, by* Mr. Newton. n. 96. p. 6091.

If M. N . . . . . pleases to compute the Errors of a Glass and Speculum that collect Rays at equal Distances, he will find how much he is mistaken; and that I have not been extravagant, as he imagines, in preferring Reflexions. And as for what he says of the difficulty of the *Praxis*, I know it is very difficult, and by those ways which he attempted it I believe it impracticable. But there is a way insinuated above, by which it is not improblable but that as much may be done in large Telescopes, as I have thereby done in short ones, but yet not without more than ordinary Diligence and Curiosity.

*A Cata-Dioptrical Telescope; by M.* Cassegrain. n. 83. p. 4056.

XIX. 1. M. *Cassegrain* has communicated the Figure of a Telescope, almost like that of Mr. *Newton*.

A B C D, is a strong Tube, in the bottom of which there is a great *Concave Speculum* C D, pierced in the middle E.

*Fig.* 100.

F is a Convex Speculum, so disposed, as to its Convexity, that it Reflects the Species, which it receives from the great Speculum, towards the Hole E, where is an Eye Glass, which one looketh through.

The Advantage which I find in this Instrument above that of Mr. *Newton*, is first, That the Mouth or Aperture A B of the Tube may be of what bigness you please; and consequently you may have many more Rays upon the Concave Speculum, than upon that, of which you have given us the Description. 2. The Reflexion of the Rays will be very natural, since it will be made upon the Axis it self, and therefore more Vidid. 3. The Vision of it, will be so much the more pleasing, in that you shall not be incommoded by the great Light, by reason of the bottom C D, which hideth the whole Face. Besides, you'll have less difficulty in discovering the Objects than in that of Mr. *Newton*.

*Consider'd by* Mr. Newton, *Ibid. p.* 4057.

2. When I first applied my self to try the Effects of Reflexions, Mr. *Gregory's Optica Promota*, (Printed in the Year 1663.) being fallen into my hands, where there is an Instrument (described *p.* 94.) like that of Mr. *Cassegrain's*,

with

with a hole in the midſt of the Object Metal, to tranſmit the Light to an Eye Glaſs placed behind it: I had thence an occaſion of conſidering that ſort of Conſtructions, and found theſe Diſadvantages in it; *viz.* 1. There will be more Light loſt in the Metal by Reflexion from the little Convex Speculum, than from the Oval Plane. For it is an obvious Obſervation, That Light is moſt copiouſly Reflected from any Subſtance when Incident moſt obliquely. 2. The Convex Speculum will not Reflect the Rays ſo truly as the Oval Plane, unleſs it be of an Hyperbolick Figure; which is incomparably more difficult to form than a Plane; and if truly formed, yet would only Reflect thoſe Rays truly, which reſpect the Axis. 3. The Errors of the ſaid Convex will be much augmented by the too great diſtance, through which the Rays Reflected from it muſt paſs, before their arrival at the Eye Glaſs. For which reaſon, I find it convenient to make the Tube no wider than is neceſſary, that the Eye Glaſs be placed as near to the Oval Plane, as is poſſible, without obſtructing any uſeful Light in its paſſage to the Object Metal. 4. The Errors of the Object Metal will be more augmented by Reflexion from the Convex than from the Plane, becauſe of the Inclination or Deflexion of the Convex on all ſides, from the Points on which every Ray ought to be Incident. 5. For theſe Reaſons there is requiſite an extraordinary Exactneſs in the Figure of the little Convex, whereas I find by Experience, that it is much more difficult to communicate an exact Figure to ſuch ſmall pieces of Metal, then to thoſe that are greater. 6. Becauſe the Errours at the Perimeter of the Concave Object Metal, cauſed by the Sphericalneſs of its Figure, are much augmented by the Convex, it will not with diſtinctneſs bear ſo large an Aperture as in the other Conſtruction. 7. By reaſon that the little Convex conduces very much to the Magnifying Virtue of the Inſtrument, which the Oval Plane doth not, it will Magnifie much more in proportion to the Sphere, on which the great Concave is ground, than in the other deſign; and ſo Magnifying Objects much more than it ought to do in proportion to its Aperture, it muſt repreſent them very obſcure and dark; and not only ſo, but alſo confuſed, by reaſon of its being over-charged. Nor is there any convenient Remedy for this. For if the little Convex be made of a larger Sphere, that will cauſe a greater inconvenience, by intercepting too many of the beſt Rays; or if the Charge of the Eye Glaſs be made ſo much Shallower as is neceſſary, the Angle of Viſion will thereby become ſo little, that it will be very difficult and troubleſome to find an Object, and of that Object when found there will be but a very ſmall part ſeen at once.

By this you may perceive, that the three Advantages, which Mr. *Caſſegrain* propounds to himſelf, are rather Diſadvantages. For according to his Deſign, the Aperture of the Inſtrument will be but ſmall, the Object dark and confuſed, and alſo difficult to be found. Nor do I ſee, why the Reflexion is more upon the ſame Axis, and ſo more Natural in one caſe than in the other: ſince the Axis it-ſelf is Reflected towards the Eye by the Oval Plane; and the Eye may be defended from external Light, as well at the Sides as at the Bottom of the Tube.

Mr. *Gregory*

Mr. *Gregory* ſpeaking of theſe Inſtruments, in the aforeſaid Book, *Pag.* 95. ſaith; *De Mechanicâ horum Speculorum & Lentium, ab aliis fruſtra tentatâ, ego in Mechanicis minus verſatus nihil dico.* So that there have been Trials made of theſe Teleſcopes, but yet in vain. And I am informed, that about 7 or 8 years ſince, Mr. *Gregory* himſelf, at *London*, cauſed one of 6 foot to be made by Mr. *Reive*, which I take to have been according to the aforeſaid Deſign deſcribed in his Book; but, though made by a skilful Artiſt, yet it was without Succeſs.

*A Cata-Dioptrick Teleſcope; by* S. Salvetti. *n.* 87. *p.* 5065.

XX. *S. Salvetti* hath made a little Proſpective Glaſs, made according to Mr. *Newton*'s new Invention. It was not above half a foot long, it had the ſame Effect of one of two. He is now making another after the Conceit of Mr. *Caſſegrain*, though he agrees not with him in making Convex the little Speculum, which one looks into through the Eye Glaſs, but believes the *French* Author only deviſed that to diſguiſe as much as was poſſible his pretended new Invention, which he endeavours to make Anterior to Mr. *Newton*'s moſt noble one.

*To make the Picture of any thing appear in a Light Room; by Dr.* Hook. *n.* 38. *p.* 741.

XXI. 1. Oppoſite to the Place or Wall, where the Apparition is to be, let a Hole be made of about a foot in Diameter, or bigger: if there be a high Window, that hath a Caſement in it, 'twill be ſo much the better Without this Hole or Caſement opened, at a convenient diſtance, (that it may not be perceived by the Company in the Room) place the Picture or Object, which you will Repreſent, Inverted, and by means of Looking-Glaſſes placed behind, if the Picture be tranſparent, Reflect the Rays of the Sun ſo, as that they may paſs through it towards the place, where it is to be Repreſented; and to the end that no Rays may paſs beſides it, let the Picture be encompaſſed on every ſide with a Board or Cloath. If the Object be a Statue, or ſome Living Creature, then it muſt be very much Enlightned by caſting the Sun-Beams on it by Refraction, Reflexion, or both. Between this Object and the Place where 'tis to be Repreſented, there is to be placed a broad Convex Glaſs, ground of ſuch a Convexity, as that it may Repreſent the Object Diſtinct on the ſaid place; which any one, that hath any inſight in the Opticks, may eaſily direct. The nearer it is placed to the Object, the more is the Object magnified on the Wall, and the further off the leſs; which diverſity is effected by Glaſſes of ſeveral Spheres. If the Object cannot be Inverted (as 'tis pretty difficult to do with Living Animals, Candles, &c.) then there muſt be two large Glaſſes of convenient Spheres, and they placed at their appropriated Diſtances, (which are very eaſily found by Trials) ſo as to make the Repreſentations Erect, as well as the Object.

Theſe Objects, Reflecting and Refracting Glaſſes, and the whole Apparatus; as alſo the Perſons employed to Order, Change, and make uſe of them, muſt be placed without the ſaid high Window or Hole, ſo that they may not be perceived by the Spectators in the Room; and the whole Operation will be eaſily performed.

What-

Whatſoever may be done by means of the Sun-Beams in the Day-time, the ſame may be done with much more eaſe in the Night, by the help of Torches, Lamps, or other bright Lights, placed about the Objects, according to the ſeveral ſorts of them.

2. There are every where made of theſe Lanthorns to repreſent and magnifie Figures upon a Wall, but then 'tis only in the Dark; wherefore to give variety of Colours, take Oil of Spike, and therein mix the ſeveral Colours, wherewith you will have your Glaſs to be ſtained, Paint them finely on, they dry preſently, and penetrate any Glaſs. *The Magick Lanthorn improv'd, by Sir* Rob. Southwell. n. 245. p. 364.

XXII. Having found by many Trials, that ſome Short-ſighted Perſons could find little or no Relief, by the uſe of Concave Glaſſes, for Seeing Objects at any diſtance Diſtinct, and that any one may be made Short-ſighted, and to be able to diſtinguiſh nothing but what is placed very near his Eye, but within certain Limits of Diſtance, by putting on and looking through a very deep pair of Spectacles, ſuch as Ancient Men uſe: I concluded that what Glaſſes ſhould make this Man, whilſt looking thro' theſe Spectacles, to ſee things at a greater Diſtance, would alſo help any other Perſon that ſhould be Short-Sighted by Nature. I then conſidered, That by the help of a Convex Glaſs, placed between the Object and the Eye, the Image of the Object may be made to appear at any Diſtance from the Eye; and conſequently all Objects may thereby be made to appear in any convenient Diſtance from the Eye: ſo that the Short-Sighted Eye ſhall contemplate the Picture of the Object, in the ſame manner as if the Object it ſelf were in that place. But then becauſe the Pictures themſelves are ſo Inverted, and therefore will be uncouth to one, not uſed to ſee them in that poſture, I conſidered of theſe Expedients to help that Defect alſo. *A way to help ſhort-ſightedneſs; by Dr.* Hook. *Ph. Coll.* n. 3. p. 59.

Firſt, If it be only for Reading of a Book, or Writing, there needeth nothing but the Inverſion of the Book, and then holding the Convex at a due Diſtance, for the Picture of the Letters will appear Erected in the due Place, for the Eye to ſee and diſtinguiſh them very plainly.

Secondly, For ſeeing to Write, I thought this would be the beſt expedient, That the Perſon Short-Sighted ſhould firſt learn to Read with his naked Eye, (both Printed Letters and alſo Written Hand) upſide downwards, which is quickly attained to by one that can do both the right way.

Thirdly, For diſtinguiſhing Objects at a Diſtance, I can aſſert by my own Experience, that with a little Uſe of Contemplating Objects Inverted, one ſhall have as good an Idea, and as true a Knowledge of all manner of Objects, as if they were ſeen Erected in their Natural Poſture.

XXIII. 1. *Euſtachio Divini* hath made a Microſcope of a new Invention, wherein inſtead of an Eye Glaſs Convex on both ſides, there are two Plano-Convex Glaſſes, which are ſo placed, as to touch one another in the middle of their Convex Surface: It hath this peculiar, that it ſhews the Objects flat and not crooked; and although it takes in much, yet nevertheleſs magnifieth extraordinarily. *Microſcopes; by* S. Divini. n. 42. p. 842.

It

It is almost $16\frac{1}{2}$ Inches high, and adjusted at four different Lengths. In the first, which is the least, it shews Lines 41 times bigger than they appear to the Naked Eye; In the second, 90 times; In the third, 111 times; And in the fourth, 143 times: Whence one may easily Calculate, how much it augments Surfaces and Solidities.

By S. Piet. Salvetti. n. 87. p. 5065.

2. *S. Salvetti* lately shew'd one of his Microscopes, made in Imitation of those of *Divini* and *Campani*, to the Great Duke of *Tuscany*, which was judged by all much better than any of the best his Highness hath. It was found, for Magnifying, Defining, and Clearness, to be very Excellent.

By M. Leeuwenhoeck. n. 94. p. 6037.

3. M. *Leeuwenhoeck* hath lately contrived Microscopes, excelling those that have been hitherto made by *Eustachio Divini*, and others.

By Mr. Butterfield. n. 141. p. 1026.

4. I have Microscopes of the manner lately brought out of *Holland* by Mr. *Huygens*, of several Fashions ready made. I have tried several ways for the making of Glasses of the bigness of a great Pin's Head and less; as in the Flame of a Tallow Candle, and of one of Wax. But the best way of all I have yet found, to make them Clear and without Specks, is with the Flame of Spirit of Wine well Rectified, and burned in a Lamp. Instead of Cotton I make use of very small Silver Wire, doubled up and down like a Skein of Thread; which being Wet with the Spirit of Wine, and made to burn in the Lamp, giveth through the Veril of the Lamp, a very Ardent Flame. Then take your beaten Glass, being first washed very clean, upon the point of a Silver Needle filed very small, and wet with Spittle. Hold it thus in the Flame till it be quite round, and no longer for fear of Burning it; and if the side of the Glass next the Needle be not melted, you may put it off and take it up with the Needle on the Round side, presenting the Rough side to the Flame, till it be every where Round and Smooth, then wipe and rub one or several of them together with soft Leather, which makes them much the better. Then put them between two pieces of thin Brass, the Apertures very round and without Bur, and that towards the Eye so big almost as the Diameter of the Glass; and so placed in a Frame with the Object conveniently for Observation.

By Mr. Steph. Gray. n. 221. p. 280.

5. I took a small Particle of Glass, about the bigness I designed my Globule, and laying it on the end of a Charcoal, I could by the help of a Blast-Pipe with the Flame of a Candle, soon melt it into a Spherule: and by this means I could make them indifferently Clear, and the smallest very Round, and I could make them much larger, than by the unassisted Heat of the Candle; but these latter were attended with an Inconvenience, they were, on that side that rested on the Coal, flatted, and received a rough Impression from it. To Remedy this Inconvenience, I was wont to Grind them and Polish them on a Brass Plane, and so reduce them to Hemisphærules; but I found the clear small Globules, not to mention that they Magnifie more, shew Objects more Distinctly.

XXIV. 1. A B, I call the Frame of the Microſcope. It may be about $\frac{1}{16}$ of an Inch in thickneſs. At A there is a ſmall Hole, near $\frac{1}{30}$ of an Inch Diameter, this ſerves for the Aperture of the Water, being in the Center of a larger Spherical Cavity, about $\frac{1}{8}$ of an Inch Diameter, and in depth ſomewhat more than half the thickneſs of the Braſs. Oppoſite to this, at the other ſide, there is another Concave but half the breadth of the former; which is ſo deep, as to reduce the Circumference of the ſmall Hole in the Center, to almoſt a ſharp edge. In theſe Cavities the Water is to be placed, being taken upon a Pin, or large Needle, and conveyed into them till there be formed a Double Convex Lens of Water; which, by the Concaves being of different Diameters, will be Equivalent to a Double Convex, of unequal Convexities. By this means, I find the Object is rendred more Diſtinct than by a Plano-convex of Water, or by a Double one, formed on the plain Surface of the Metal. *A Water-Microſcope; by Mr.* Stephen Gray. n. 221. p. 281. n. 223. p. 353. *Fig.* 101.

C D E, is the Supporter, whereon to place the Object; if it be Water, in the Hole G; if a Solid Object, on the Point F. This is fixed to the Frame of the Microſcope, by the Skrew E, where 'tis bent upwards, that its upper part may ſtand at a diſtance from the Frame; 'tis moveable on the Skrew as a Center, to the end that either the Hole C, or the Point F, may be expoſed before the Microſcope; and that the Object may be brought to, and fixed in its Focus. There is another Skrew, about half an Inch in length, which goes through the round Plate into the Frame of the Microſcope A E, the Skrew and Plate taking hold of the Supporter about D, where there is a Slit ſomewhat larger than the Diameter of the Skrew, which is requiſite for the admiſſion of the Hole C, or Point F, according to the Nature of the Object, into the Focus of the Glaſs; for by turning the Skrew G, the Supporter is carried to or from the ſame, which may be ſooner done, if whilſt one turn the Skrew with one hand, the other hold the Microſcope by the end B, and one continue looking through the Water till the Object be ſeen moſt diſtinctly.

The Supporter muſt be made of a thin piece of Braſs well hammered, that by its Spring it may the better follow the Motion of the Skrew. I choſe rather to fix the Supporter by the Skrew E, than by a Rivet; becauſe it may now, by help of a Knife, be unſkrewed, and by the other Skrew G, be brought cloſe to the Frame of the Microſcope without weakening its Spring, and ſo become more conveniently Portable. If the Hole at G be filled with Water, but not ſo as to be Spherical; all Objects that will bear it, are ſeen therein more diſtinctly.

2. Having obſerved ſome Irregular Particles in Globules of Glaſs, and finding them Diſtinct, but prodigiouſly Magnified, when held cloſe to the Eye, I concluded that if I conveyed a ſmall Globule of Water to my Eye, and that there were any opacous or leſs tranſparent Particles than the Water therein, I might ſee them Diſtinctly. I therefore took on a Pin a ſmall Portion of Water, which I knew to have in it ſome minute Animals, and laid it on the end of a ſmall piece of Braſs Wire (there lay then by me) of about $\frac{1}{10}$ *Another.* Ibid. n. 221. p. 280.

of an Inch Diameter, till there was formed ſomewhat more than an Hemiſpherule of Water; then keeping the Wire Erect, I applied it to my Eye, and ſtanding at a proper Diſtance from the Light, I ſaw them and ſome other irregular Particles, as I had predicted, but moſt enormouſly magnified; for whereas they are ſcarce diſcernable by my Glaſs Microſcopes, or firſt Aqueous one, within the Globule they appeared not much different both in their Form, nor leſs in Magnitude than ordinary Peaſe. They cannot well be ſeen by Day-Light, except the Room be darkned, after the manner of the famous Dioptrical Experiment, but moſt diſtinctly by Candle-Light; they may be very well ſeen by the full Moon Light. If the Water be conveyed into the Hole B, (which may be about $\frac{1}{10}$ of an Inch Diameter) till there remain near an Hemiſphere of Water on each ſide of the Hole, the Objects are ſeen more diſtinctly; and the Spherical Form of the Water is this way better ſecured than on the Point of a Pin Wire.

n. 223. p. 355.

The Reaſon of this appearance may be thus Explained. Let the Circle D B B D, repreſent a Sphere of Water, A an Object placed in its Focus, ſending forth a Cone of Rays, two of which are A B, A B, which Opticians know coming into the Water at B and B, will be Refracted from their direct Courſe, and become B D, B D; at D they will, at their paſſing into the Air, be again Refracted into D E, D E, and ſo run Parallel to one another, and to the Axis of the Sphear A F C G. Now 'tis a known and fundamental Principle in Opticks, that the Angle of Reflexion is equal to the Angle of Incidence; wherefore let the Rays B D, B D, be imagined to come from ſome Point of an Object placed within a Sphere of Water, by being Reflected from the Interior Surface of the Sphere at B B, C B D is the Angle of Reflexion, to which making C B F equal, F will be the place where an Object ſending forth a Cone of Rays, two of which are F B, F B, which are Reflected into the Rays B D, B D, and then coming to the other ſide the Sphere at D and D, they are Refracted into D E, D E, as before; and conſequently be as fit for Diſtinct Viſion, whether the Object be placed in F within, or in A without the Sphere, if its Interior Surface be conſidered as a Concave Reflecting Speculum.

n. 221. p. 285. Fig. 102.

*Microſcopes Improved; by* Mr. Newton. n. 88. p. 5096.

XXV. From the Diſtinction I have elſewhere given between *Compounded* and *Uncompounded Colours*, I take Occaſion to Communicate a way for the Improvement of Microſcopes by Refraction; *viz.* By Illuminating the Object in a darkned Room with Light of any convenient Colour not too much *Compounded*: for by that means the Microſcope will with diſtinctneſs bear a deeper Charge and larger Aperture, eſpecially if its Conſtruction be ſuch as I may hereafter deſcribe; for the Advantage in ordinary Microſcopes will not be ſo ſenſible.

*A Reflecting Microſcope; by* Mr. Newton. n. 80. p. 3080. Fig. 103.

XXVI. 1. I have ſometimes thought to make a Microſcope, which ſhould have, inſtead of an Object Glaſs, a Reflecting piece of Metal. For theſe Inſtruments ſeem as capable of Improvement as Teleſcopes, and perhaps more; becauſe but one Reflective piece of Metal is requiſite in them, as you may perceive by the Diagram, where A B repreſenteth the Object Metal, C D the Eye Glaſs, F their common Focus, and O the other Focus of the Metal, in which the Object is placed.

2. A, repreſents a ſmall flat Ring of Braſs, whoſe Interior Circle muſt not much exceed $\frac{4}{10}$ of an Inch Diameter, and about $\frac{1}{30}$ of an Inch thick: This we may call the Frame or Cell of the Glaſs; it muſt be prepared for uſe after the following manner. Take a ſmall Globule of Quick-Silver, and diſſolve it in a few drops of *Aqua Fortis*, to which you may add 10 parts of common Water; dip the end of a Stick in this Liquor, and rub the inward Circle of the Ring with it; ſo it will have acquired a Mercurial Tincture, and being wiped dry, be fit for uſe. Then let it be laid on the Table, and pour a drop of Quick-ſilver within it, which preſs gently with the Ball of the Finger, and it will adhere to the Ring; then cleanſe it with a Hare's Foot, and you will have a Convex Speculum. Take up the Ring and Speculum carrying it Horizontal, and lay it on the Brims of the hollow Cylinder B; ſo will the Mercury become a Concave Reflecting Speculum, which from the ſmallneſs of the Sphere of which it ſeems to be a Section, may be uſed as a Microſcope. The Cylindrick Veſſel B, has a Skrew Hole at the bottom, by which it is skrewed to the top of the Pedeſtal C D; C E F G is the Supporter of the Object Plate, which as you ſee may be raiſed higher, or let lower, as there is occaſion, by the Skrew on the Pedeſtal: The Object Plate muſt be of Glaſs cemented to the Ring G.

*By Mr. Stephen Gray. n. 223. p. 541.*

*Fig. 104.*

This Inſtrument with a little Variation may be made a Microſcope of Water, if inſtead of the Ring G, there be only a ſmall Arm with a Hole in it to receive a Drop of Water, and the Cylindrick Veſſel B, be either taken away or skrewed on with its bottom upwards, ſo as to make an Object Plate. This will be more convenient for viewing the Textures of Opacous Objects, than that above deſcribed, which is more fit for Fluid and Tranſparent ones.

XXVII. 1. The Figure of it is Round, being 30 Inches, and ſomewhat better, in Diameter. On one ſide it hath a Frame of a Circle of Steel, to the end that it may keep its juſt Meaſure: 'Tis eaſie to remove it from place to place, though it be above an hundred weight, and 'tis eaſily put in all ſorts of Poſtures. The Burning Point is diſtant from the Center of the Glaſs about 3 Foot. The Focus is about half a *Louis d'* Or large. One may paſs one's Hand through it, if it be done nimbly; for if it ſtay there the time of a Second Minute, there is danger of receiving much hurt. Green Wood takes Fire in it in an inſtant, as do alſo many other Bodies.

*A Burning Concave at Lyons; made by M. de Vilette. n. 6. p. 95.*

| | *Seconds.* |
|---|---|
| A ſmall Piece of Pot-Iron was melted, and ready to drop down in | 40 |
| A Silver Piece of Fifteen Pence was pierced in | 24 |
| A Groſs Nail (called *le Clau de Paiſan*) was melted in | 30 |
| The end of a Sword Blade of *Olinde*, was burn'd in | 43 |
| A Braſs Counter was pierced in | 06 |
| A piece of Red Copper was melted ready to drop down, in | 42 |

 A

| | Seconds. |
|---|---|
| A piece of a Chamber-Quarry-Stone was Vitrified, and put into a Glaſs drop, in——— | 45 |
| Steel, whereof Watch-makers make their Springs, was found melted, in——— | 09 |
| A Mineral-Stone, ſuch as is uſed in Harquebuſſes *à rovêt*, was Calcin'd and Vitrified, in——— | 01 |
| A piece of Morter was Vitrified, in——— | 52 |

In ſhort, There is hardly any Body which is not deſtroyed by this Fire. If one would Melt by it any great quantity of Metal, that would require much time, the Action of Burning not being performed but within the bigneſs of the Focus, ſo that ordinarily none but ſmall pieces are expoſed to it. One M. *de Alibert* buys it, paying for it 1500 Livres.

*Ibid. p. 97.* You encline to believe, That the Glaſſes of *Maginus* and *Septalius* do approach to that of *Lyons*: But I can aſſure you, they come very far ſhort of it. You may conſult *Maginus* his Book, where he deſcribes his; and there are ſome Perſons here who have ſeen one of his beſt, which had but about 20 Inches Diameter; ſo that this of *Lyons* muſt perform at leaſt twice as much. As to *Septalius*, we expect the Relations of it from Intelligent and Impartial Men. It cannot well be compared to that of *Lyons*, but in bigneſs; and in this caſe, if it have five Palms, (as you ſay) that would be about $3\frac{1}{2}$ Foot *French*, and ſo it were a Foot bigger, which would make it half as much greater in Surface: But as to the Effects, ſeeing it Burns ſo far off, they cannot be very violent. And I have heard one ſay, that had ſeen it, that it did not ſet Wood on Fire but after the time of ſaying *à Miſerere*. You may judge of the difference of the Effects, ſince that of *Lyons* gathers its Beams together within the ſpace of 7 or 8 Lines; and that of *Septalius* muſt ſcatter them in the compaſs of 3 Inches.

*n. 49. p. 986.* It was diſpoſed of to the King of *Denmark*.

*Another by the ſame.* *n. 49. p. 986.* 2. The ſame M. *de Vilette* of *Lyons* hath made another Burning Concave. It is of 34 Inches Diameter, and Melts all ſorts of Metals, and Iron it ſelf of the thickneſs of a Silver Crown, in leſs than a Minute of time, and Vitrifies Brick in the ſame time; and as for Wood, whether Green or Dry, it ſets it on Fire in a moment. The King hath ſeen it, and the Performances of it, with great Satisfaction; and his Majeſty is likely to make it his, and then to beſtow it on his *Royal Academy* of Philoſophers, for making of farther Experiments with it.

*By . . . . . . .* *Ibid. p. 987.* This kind of Concaves Burning the moſt forcibly of any Fire we know of, would be of great uſe, if they could be ſo contrived as to have a Focus of any conſiderable largeneſs, to take in a good quantity of combuſtible matter at once.

*By S. Settalla.* *n. 40. p. 796.* 3. *S. Settalla* at *Milan*, cauſeth to be made a Burning Glaſs of ſeven Foot in Diameter. He pretends to make it Burn at the diſtance of 50 Palms, which is about 33 Foot.

4. The

4. The outer Circle of the Concave Burning Speculum, which I lately caused to be made in *Lusace*, is near 3 *Leipsick* Ells in Diameter, exceeding that great one at *Paris* by ⅔ of such an Ell. It is made of a Copper Plate scarce twice so thick as the back of an ordinary Knife, and may therefore be easily removed from place to place, and ordered for use: and the Workmanship of it may, by the Contrivances I have Invented, be easily and in little time performed by one Man. The Polish thereof is very good, and represents by Distinct Reflexions all those appearances which arise from the Concave Figure thereof.

*A Burning Concave in Germany; by ... n. 188. p. 352.*

The Force of this Speculum is Incredible. For, 1. A piece of Wood put into the Focus (which is two Ells off) flames in a moment, so as a fresh Wind can hardly put it out. 2. Water applied in an Earthen Vessel presently Boils, so as to Boil an Egg; and the Vessel being held there some time, the Water Evaporates all away. 3. A piece of Tin or Lead three Inches thick, as soon as it is put into the Focus, melts away in drops; and held there a little time is in a perfect Fluor, so as in two or 3 Minutes to be quite pierced thorough. 4. A Plate of Iron or Steel placed in the Focus immediately is seen to be red-hot on the backside, and soon after a Hole is burnt through: I have made three such Holes, in a Plate, in 6 Minutes time. 5. Copper, Silver, and the like, applied to the Focus, Melt; which I have tried with several sorts of Coin; among the rest, with a *Rix-Dollar*, and the same happened to it as to the aforesaid Iron Plate in 5 or 6 Minutes. 6. Things not apt to melt, as Stones, Brick, and the like, soon become red-hot like Iron. 7. Slate at first is red-hot, but in a few Minutes turns into a fine sort of black Glass; of which if any part be taken in the Tongs and drawn out, it runs into Glass threads. 8. Tiles, which had suffered the most Intense Heat of Fire, in a little time melt down into a yellow Glass; as do, 9. Pot-Shreads, not only well burnt at first, but much used in the Fire, into a blackish yellow Glass. 10. Pumice-Stone, said to be that of Burning Mountains, in this Solar Fire, melts into a white transparent Glass. 11. A piece of a very strong Crucible put into the Focus, in 8 Minutes, was melted into a Glass. 12. I have seen Bones turned into a kind of Opake Glass, and a Clod of Earth into a yellow or greenish Glass.

These Experiments were made in *August* and *September*, when the Sun has not the same force as when he is about the Summer Solstice. The Beams of the Full Moon, concentred by this Speculum, did not produce any Degree of Heat, tho' the Light was not a little encreased.

5. Some years ago, Dr. *Hook* made a Proposal to the *Royal Society* concerning the same thing. He conceives one may be made of many Foot Diameter, for a small price, being hammer'd out of a Copper Plate, and Tinned over with a mixture of Tin, Lead and Tin-Glass, which is found to bear a very good Polish. Such a Speculum might be of great use, in Perfecting the Art of Pastes, or Factitious Jewels, which require the most Intense Degree of Heat, to bring them to an Exact Mixture.

*By Dr. Hook. Ibid. p. 354.*

XXVIII. A

*Concave Specula nearly of a Parabolick Figure; attempted by* Mr. Stephen Gray. n. 228. p. 542. n. 235. p. 787.

XXVIII. A Linnen Cloth, being first wet in fair Water, and then laid on a Concave Cylinder, as the Verge of a Seive, Keeler, or the like, its Central Parts will descend so as to form a very regular Concave Superficies. And a Thread, being first wet in common Water, and then suspended with its two ends, or any two Points nearer than their utmost extent, so as it might touch the Center of the suspended Cloth, and its two opposite Points on the Ring, was found to have the same Curvature. My Business was then to examine the Figure of the Thread thus suspended, which I did in manner following; On the side of a Wall I described *Parabola's* of several Species, whose Axes were Perpendicular, and Perimeter Horizontal, to which the Line being applied, so as it might touch the Vertex, past very nearly through all the intermediate Points of the *Parabola*, much nearer than the Portion of a *Circle*, which past through the Extremity of the Perimeter and Latus Rectum, would do.

From hence I conclude, That a ponderous and pliable Substance, being suspended on a Ring or hollow Cylinder, so as that its Central Parts may descend, will form it self into a Figure that is more commodious for Burning-Glasses than the Spherical, of which they are now made, being much nearer their most absolute Figure the *Parabola*.

Now if there may be a way found to give to Cloth or Leather a Metalline Surface, or a Varnish that may bear a good Polish; or if this be found Impracticable, perhaps Plates of Metal may be beat out so thin, as being suspended on a large Ring, will by their own Gravity receive their true Figure, one may make Speculums of what largeness he pleaseth.

Upon this Consideration, I devised the following Experiment. There was taken a sufficient quantity of Potter's Clay, of which there was formed a plain circular Plate, by help of an Iron Ring about 13 Inches Diameter. This was laid on a lesser Ring, which was supported by four Feet, and it immediately became of a very regular Concave on its upper, and Convex on its under Superficies: but notwithstanding 'twas set to dry in the Shade, yet before it was dry enough, its Central Parts extended so as to become almost Plain, not without some Defects; if it had continued in its Regularity, I designed to have burned and glazed it in a Potter's Furnace.

*To make the Globe Looking-Glass; by Sir* R. Southwell. n. 245. p. 363.

XXIX. Take Quick-silver, Marchasite of Silver, each three Ounces; Tin and Lead, each half an Ounce; these two first throw on the Marchasite, and last of all the Quick-silver: stir them well together, but they must be taken from the Fire, and be towards Cooling before the Quick-silver be added; let your Glass be well warmed, then pour in the Mixture, and roll it from side to side.

*Note*, This will do also when Cold, but 'tis best when the Glass is heated and very dry.

*Note also*, That if at the Glass-House, your Ball be of yellow Glass, then all will shine like Gold.

XXX. *Papers*

## XXX. *Papers (of less General Use) Omitted.*

1. DR. *Hook* having (in his *Micrographia*) described a new Engine for *Grinding Optick Glasses* of very great lengths, M. *Auzout* (in a small *French Tract*) Objects several Difficulties to this Engine it self: But however, he thinks it impracticable to make any Glasses of above 300 or 400 Foot at most, (and fears that neither Matter nor Art will go even so far) which will be very far from showing us *Plants* or *Animals* in the *Moon*; and then proposes Remedies to some of the Inconveniences of the Turn. To all this, Dr. *Hook* here Replies; He Answers the Objections, and Rejects the proposed Expedients. *Optick Glasses, by a Turn Lathe.* n. 2. p. 31. n. 4. p. 56. *Ibid. p.* 63.

2. *Carlo Ant. Mancini* having, in his *Occhiale all' Occhio*, described a particular way for making Convex-Glasses upon a Plain, his Method is here Translated from the *Italian* into *English*. But 'tis added, That though the contrivance be Ingenious, yet it is conceiv'd by Skilful Artists, that it will be very difficult to put it into Practice. *Upon a Plain.* n. 42. p. 838.

## XXXI. *Accounts of Books, Omitted.*

1. PHysico-Mathesis de Lumine, Coloribus, & Iride, &c.* *Auth.* Franc. Maria Grimaldo, *S. J.* Bononiæ 1665. *in* 4*to.* n. 79. p. 3068.

2. *Cogitationes Physico-Mechanicæ de Natura Visionis. Auth.* Jo. Ott. *Schaphusa Helvetio.* Heidelbergæ 1670. *in* 4*to.* n. 71. p. 2163.

3. *Synopsis Optica. Auth.* Honorato Fabri. *Soc. Jesu.* Lugduni 1667. *in Quarto.* n. 32. p. 626.

4. *L'Occhiale all' Occhio, overo Dioptrica Prattica, del* Carlo Ant. Mancini, *in* Bologna 1660. *in Quarto.* n. 42. p. 837.

5. *Lectiones* 18; *Cantabrigiæ in Scholis publicis habitæ in quibus Opticorum Phænomenωn Genuinæ Rationes investigantur, & exponuntur, ab* Isaaco Barrow. Lond. 1669. *in Quarto.* n. 75. p. 2258.

6. *La Dioptrique Oculaire, par le Pere Cherubin d'* Orleans, *Capucin.* A Paris 1671. *in Folio.* n. 78. p. 3045.

7. Christiani Hugenii *Astroscopia Compendiaria, Tubi Optici molimine Liberata*: Or, The Description of an *Aerial Telescope.* Hague, 1684. *in Quarto.* n. 161. p. 668.

8. A Treatise of Dioptricks. By *Will. Molyneux* Esq; F. R. S. *in Quarto.* n. 205. p. 967.

9. *Catoptricæ & Dioptricæ Elementa. Auctore* Davide Gregorio, *D. M.* Oxon. 1695. *in Octavo.* n. 219. p. 214.

CHAP.

# CHAP. IV.

## ASTRONOMY.

*The Observatory of Tycho Brahe; by Mr. Gourdon. n. 265. p. 692.*

I. THE Island *Ween*, (vulgarly termed the *Scarlet-Island*) famous for the Observations of *Tycho Brahe* that Renowned *Danish Astronomer*, (with all submission to better Judgments) was none of the fittest for Astronomical Observations of all sorts, such as the taking the exact Time of the true Rising and Setting of Celestial Bodies, together with their respective Amplitudes; because the Island lies low, and is Land-lockt on all the Points of the Compass, save three. Besides, The sensible Land Horizon of the *Ween* is extreamly uneven and rugged, the North and Eastern Parts thereof being some rising Hills in the Province of *Schonen*; and the Western part is mostly overspread with Trees on the Island *Zealand*: from the remotest of which Coasts the *Ween* is not distant above three Leagues.

*A new Astronomical Instrument; by M. Weighelius. n. 74. p. 2219.*

II. M. *Weighelius* hath Invented an Instrument, which he calls *Astrodicticum*, by the means whereof very many Persons shall be able at one and the same time to behold one and the same Star. He hath also Invented an exceeding great Globe of the World, capable of 10 Persons to sit in it all at once, and to behold the Motions of the Celestial Bodies, &c.

*A Celestial Globe; by M. Didier Alleman. n. 136. p. 905.*

III. The bigness of this Globe is only of four Inches Diameter. The Body of the Globe of Burnisht Steel, where all the Figures of the Constellations are designed in Silver-colour, but the Stars themselves of all Magnitudes are put on in Emboss'd Gold.

This Globe moves from East to West in 24 hours; and you may there see the Sun exactly Rise and Set as in the great World, together with the Moon, as also the Stars of the Constellations; likewise, how the Sun of this Globe comes to his Meridian, with an admirable Regularity, conform to the *Primum Mobile*. And you may also there perceive the mean Motions of the Sun and Moon from West to East, and all the Lunations; and by the Diurnal Motion of the Moon, it shews the Flux and Reflux of the Sea.

The Meridian serveth for a Needle to shew the Hours, which are marked upon the Zodiack, where the Sun marcheth regularly, which hath two main Rays, one whereof goeth directly Northward, the other Southward. That of the North marks the way or Degree, which the Sun maketh from West to East upon the Signs of the Zodiack, and upon a Circle of Silver, where the 360 Degrees of the Circle are mark'd. The other Ray of the South, marks upon another Circle of Silver the Days of the Month, where the 365 Days are noted. The Circles of the Longitude of the Stars, which separate the Signs,

Signs, and which come from the Poles of the Zodiack, are marked by Gold-Wires; as alſo the Equator, the Tropicks, and the Polar Circles.

There is but one great Spring, the *Primum Mobile*, which puts all the reſt in Motion. It is wound up by the *Antarctique* Pole, and you may wind it up to the Right or Left Hand, without wronging any contrary Motion: And by the *Arctique* Pole, you may advance and retard this Movement, if you ſhould find any Inequality, without altering at all the great Spring.

IV. I applied my ſelf the laſt Summer to the taking of the Diameters of the Sun, Moon, and the other Planets, by a Method which one M. *Picard* and my ſelf have, eſteemed by us the beſt of all thoſe that have been practis'd hitherto; ſince we can take the Diameters to Second Minutes, being able to divide one Foot into 24000 or 30000 Parts, ſcarce failing as much as in one only part, ſo as we can in a manner be aſſured, not to deceive our ſelves in 3 or 4 Seconds. I ſhall not now tell you my Obſervations, but I may very well aſſure you, that the Diameter of the Sun has not been much leſs in his *Apogee*, than 31 min. 37 or 40 ſec. and certainly not leſs than 31 min. 35 ſec. and that at preſent in his *Perigee* it paſſes not 32 min. 45 ſec. and may be leſs by a ſecond or two. That which is at the preſent troubleſome is, that the Vertical Diameter, which is the moſt eaſie to take, is diminiſht, even at Noon, by 8 or 9 ſec. becauſe of the Refractions, which are much greater in Winter than Summer at the ſame height; and that the Horizontal Diameter is difficult, becauſe of the ſwift Motion of the Heavens.

*A Way to Meaſure the Diameters of the Planets, and the Parallax of the Moon; by* M. Auzout. n. 21. *p.* 373.

As for the Moon, I never yet found her Diameter leſs than 29 min. 44 or 45 ſec. and I have not ſeen it paſs 33 min. or if it hath, it was only by a few ſeconds. But I have not yet taken her in all the kinds of Situations of the *Apogees* and *Perigees* which happen, with the Conjunctions and Quadratures. I do not mention all what can be deduced from thence; I ſhall only tell you, that I have found a way to know the *Parallax* of the Moon, by the means of her Diameter: *viz.* If, on a day, when ſhe is to be in her *Apogee* or *Perigee*, and in the moſt Boreal Signs, you take her Diameter towards the Horizon, and then towards the South, with her Altitudes above the Horizon. For if the Obſervation of the Diameters be exact, as in theſe Situations the Moon changes not conſiderably her diſtance from the Earth in 6 or 7 hours, the Difference of the Diameters will ſhew the Proportion there is of her Diſtance with the Semidiameter of the Earth. I do not enlarge, becauſe that as ſoon as one hath this Idea the reſt is eaſie. The ſame would yet be practiſed better in the places where the Moon paſſes through the Zenith, than here; for the greater the difference is of the Heights, the greater is that of the Diameters. I do not Note, (for it eaſily appears) that if one were under the ſame Meridian, or the ſame *Azimuth* in two very diſtant Places, and took at the ſame time the Diameter of the Moon, one would do the ſame thing; though this Method goes not to preciſeneſs.

From what has been ſaid may be Collected the Reaſon of the Obſervation, which M. *Hevelius* made in the laſt Eclipſe of the Sun, (*July* 2. *St. N.* 1666.) touching the Increaſe of the Moon's Diameter about the end. I am exceed-

 ing

ing glad, that a Perſon, who probably knew not the Cauſe of it, has made the Experiment: but it is ſtrange, that until now no Aſtronomer has forſeen, that that ſhould happen, nor given any Precepts for the Change of the Moon's Diameter in the Eclipſes of the Sun, according to the Places where they ſhould happen, and according to the Hour and Height the Moon ſhould have. For, what happened in that Eclipſe of Augmentation, would have fallen out contrarily, if it had been in the Evening; for, the Moon, which in that Eclipſe, that began in the Morning, was higher about the end than at the beginning, was nearer us, and conſequently was to appear bigger: But if the Eclipſe ſhould happen in the Evening, ſhe would be lower at the end, and therefore more diſtant from us, and conſequently appear leſſer. So alſo in two different Places, whereof one ſhould have the Eclipſe in the Morning, and the other at Noon, the Moon ſhould appear bigger to him that hath it at Noon: And ſhe muſt likewiſe appear bigger to thoſe who ſhall have a leſſer Elevation of the Pole under the ſame Meridian, becauſe the Moon will be nearer them.

*An Account of* M. Gaſcoigne's *Micrometer*; *by* Mr. Richard Townley. n. 25. p. 457.

V. 1. I ſhould be look'd upon as a great Wronger of our Nation, ſhould I not let the World know, that I have, out of ſome ſcattered Papers and Letters that formerly came to my Hands of one Mr. *Gaſcoigne*'s, found out, that before our late *Civil Wars* he had not only Deviſed an Inſtrument of as great a Power as M. *Auzout*'s, but had alſo for ſome years made uſe of it; not only for taking the Diameters of the Planets, and Diſtances upon Land; but had farther endeavoured, out of its Preciſeneſs, to gather many Certainties in the Heavens; amongſt which I ſhall only mention one, *viz. The finding the Moon's Diſtance*, from two Obſervations of her Horizontal and Meridional Diameters: which I the rather mention, becauſe the *French Aſtronomer* eſteems himſelf the firſt that took any ſuch Notice, as thereby to ſettle the Moon's Parallax: For our Country-Man fully conſidered it before, and imparted it to an Acquaintance of his, who thereupon propoſed to him the Difficulties that would ariſe in the Calculation; with Conſiderations upon the ſtrange Niceties, neceſſary to give him a certainty of what he deſired. The very Inſtrument he firſt made I have now by me, and two others more perfected by him; which doubtleſs he would have infinitely mended, had he not been Slain unfortunately in His late Majeſty's Service. He had a *Treatiſe of Opticks* ready for the Preſs; but though I have uſed my utmoſt endeavour to retrieve it, yet I have in that point been totally unſucceſsful: But ſome looſe Papers and Letters I have, particularly about this Inſtrument for taking of Angles, which was far from perfect. Nevertheleſs, I find it ſo much to exceed all others, that I have uſed my Endeavours to make it Exact, and eaſily Tractable; which above a year ſince I effected to my own Deſire, by the help of an Ingenious and Exact Watchmaker: Since which time, I have not altogether neglected it, but employed it particularly in taking the *Diſtances* (as occaſion ſerved) of the *Circum-jovialiſts*, towards a perfect ſettling their Motion. I ſhall only ſay of it, That it is ſmall, not exceeding in weight, nor much in bigneſs, an ordinary Pocket-Watch, exactly marking above

above 40000 Diviſions in a Foot, by the help of two Indexes; the one, ſhewing hundreds of Diviſions, the other Diviſions of the hundred; every laſt Diviſion, in my ſmall one, containing $\frac{1}{10}$ of an Inch; and that ſo preciſely, that, as I uſe it, there goes above $2\frac{1}{2}$ Diviſions to a Second. Yet I have taken Land Angles ſeveral times to one Diviſion, though (for the Reaſon mentioned by M. *Auzout*) it be very hard to come to that Exactneſs in the Heavens, *viz.* The Swift Motion of the Planets. Yet, to Remedy that fault, I have Deviſed a *Reſt*, in which I find no ſmall Advantage, and not a little pleaſing thoſe Perſons who have ſeen it, being ſo eaſie to be made, and by the Obſerver manag'd without the help of another; which ſecond Convenience, my yet Nameleſs Inſtrument hath in great Perfection, and is by reaſon of its ſmallneſs and ſhape, eaſily applicable to any Teleſcope.

2. *a a a a*, is a ſmall oblong Braſs Box, ſerving both to contain the Skrews and its Sockets, or Female Skrews, and alſo to make all the ſeveral moveable Parts of the Inſtrument to move very True, Smooth, and in a ſimple Direct Motion. To one end hereof is Skrewed on a round Plate of Braſs *b b b b*, about 3 Inches over; the extream Limb of whoſe out-ſide is divided into 100 Equal Parts, and numbred by 10, 20, and 30, &c. Through the middle of this Plate, and the middle of the Box *a a a*, is placed a very curiouſly wrought Skrew of about the bigneſs of a Gooſe-Quill, and of the length of the Box, the Head of which is by a fixed Ring or Shoulder on the inſide, and a ſmall ſpringing Plate *d d*, on the out ſide, ſo adapted to the Plate that it is not in the leaſt ſubject to ſhake. The other end of this Skrew is by another little Skrew (whoſe ſmall Points fills the Center or Hole made in the end of the longer Skrew for this purpoſe) rendred ſo fixt and ſteady in the Box, that there appears not the leaſt danger of ſhaking. Upon the Head of this Skrew, without the Springing Plate, is put on a ſmall Index *e e*, and above that a Handle *m m*, to turn the Skrew round as often as there ſhall be occaſion, without at all endangering the diſplacing of the Index, it being put on very ſtiff upon a Cylindrical part of the Head, and the Handle upon a Square. The Skrew hath that third of it, which is next the Plate, bigger than the other two thirds of it, by at leaſt as much as the depth of the ſmall Skrew made on it: The Thread of the Skrew of the bigger third is as ſmall again, as that of the Skrew of the other two thirds. To the groſſer Skrew is adapted a Socket *f*, faſtned to a long Bar or Bolt *g g*, upon which is faſtned the moveable Sight *h*, ſo that every turn of the Skrew promotes the Sight *h*, either a Thread nearer, or a Thread farther off from the fixt Sight *i*. The Bar *g g* is made exactly Equal, and fitted into two ſmall Staples *k k*, which will not admit of any ſhaking. There are 60 of theſe Threads; and anſwerable thereto, are made 60 Diviſions on the edge of the Bolt or Ruler *g g*, and a ſmall Index *l*, fixt to the Box *a a a a*, denotes, how many Threads the edges of the two Sights *h* and *i* are diſtant; and the Index *e e*, ſhews on the Circular Plate what part of a Revolution there is more; every Revolution, as was ſaid before, being divided into a 100 Parts. At the ſame time that the moveable Sight *h* is moved forwards or backwards, one or more Threads of the Courſer Skrew, is the Plate *p p*, by the means of the Socket *q*,

*A Deſcription of it; by Dr. Hook.* n. 29. *p.* 542.

*Fig.* 105.

*Fig.* 106.

 to

to which it is Skrewed, moved forward or backward, one or more threads of the finer Skrew: So that this Plate, being fixt to the Teleſcope by the Skrews *r r*, ſo as the middle betwixt the Sights may lie in the Axis of the Glaſs, however the Skrew be turned, the midſt betwixt the Sights will always be in the Axis, and the Sights will equally either Open from it, or Shut towards it.

Fig. 107. It is conceived by ſome Ingenious Men, that it will be more convenient, inſtead of the edges of the two Sights *h* and *i*, to employ two Sights *r* and *s*, fitted with the Hairs *t* and *v*, ſo that they may be conveniently uſed in the place of the ſolid edges of the Sights *h* and *i*.

Fig. 108. The Inſtrument is thus applied to the Teleſcope. The Tube A D is divided into three lengths, of which (as in ordinary ones) B C is to Lengthen or Contract, as the Object requires: But A B is here added, that at A ye may put ſuch Eye Glaſſes, as ſhall be thought moſt convenient, and to ſet them ſtill at the Diſtance moſt proper from the Indexes or Pointers, which here are ſuppoſed to be at B, which length alters alſo in reſpect of divers Perſons Eyes. E is a Skrew, by which the great Tube can be fixt ſo, as by the help of the Figures any ſmaller part of it can immediately be found, meaſuring only, or knowing the Diviſions on B C, the Diſtance of the Object Glaſs from the Pointers. F is the Angular piece of Wood, that lies on the upper Skrew of the Reſt.

Fig. 109.

*Ibid. p.* 556. This Reſt, (by Dr. *Hook*'s Suggeſtion) may be rendred more convenient, if, inſtead of placing the Skrew Horizontal, it be ſo contrived, that it may be laid Parallel to the Equinoctial, or to the Diurnal Motion of the Earth; for, by that means, the ſame thing may be performed by the ſingle Motion of one Skrew, which in the other way cannot be done but by the turning of both Skrews; as will eaſily appear to thoſe that ſhall conſider it.

*More ways to Meaſure ſmall Diſtances, intimated; by Dr.* Hook. n. 25. *p.* 459.

3. I have by me two or three ſeveral ways of Meaſuring the Diameters of the Planets, whether Horizontal, Perpendicular, or Inclined, to the Exactneſs of a Second, by the help of a Teleſcope: as alſo, of taking the Poſition and Diſtance of the ſmall fixt Stars one from another, or from any of the leſs bright Planets, if the Diſtance be not above two or three Degrees.

*Excellence of the Micrometer; by Mr.* Flamſteed. n. 96. *p.* 6099.

4. Micrometro & Tubo Pedum 14, Planetarum frequenter Diametros & à Fixis Diſtantias, ad Secundos fere Scrupulos, quod vix inexpertus credes, Dimenſus ſum.

*Plain Sights rejected; by Mr.* Flamſteed. n. 89. *p.* 5119. n. 96. *p.* 6100.

VI. 1. Pluribus Argumentis evinci poteſt, *Tychonem* ſæpè cum in Locis, tum Latitudinibus, Fixis quibuſdam aſſignatis, duos treſve & interdum quatuor aut quinque totos Scrupulos à vero aberraſſe. Fixarum quidem Reſtitutionem ſuſcepiſſe Celeberrium *Joannem Hevelium* audivimus, attamen quandoquidem Pinacidiis Vitrorum Caſſis fertur ipſum uti, dubium an multum ab ipſo Emendatiores Locos habituri ſimus, quàm reliquit *Tycho*, niſi ubi valde hallucinatus eſt.

2. Pere-

2. Percipio Vestrates non omnes mihi adstipulari in isto Dioptrarum Negotio, de quibus in Machinæ meæ Coelestis *Organographia* tractavi. Verum etiamsi *Cl. Hookius* & *Cl. Flamstedius*, aliique, planè aliter sentiant; experientia tamen quotidiana me edocuit; atque etiamnum docet, rem longè aliter se habere in Magnis illis Organis, Quadrantibus scil. Sextantibus & Octantibus, imprimis Quadrantibus Azimuthalibus, aliisque Quadrantibus Regulis constructis, quæ nempe adeò procliviter commoveri & inverti (dum examinantur Dioptræ Telescopicæ) imò nullo modo possunt, ut quidem Instrumenta illa trium quatuorve Pedum Perpendiculo constructa. Res cumprimis in eo consistit, quòd nullam planè Observationem suscipere possint suis Dioptris Telescopicis, nisi priùs denuò eas Examinent, & Rectificent; in quo tamen Examine variâ viâ idque jugiter, utut studiosissime illud suscipias, hallucinari datur. Ad hæc, in Quadrantibus Azimuthalibus, Octantibus & Sextantibus, quâ ratione Examen illud omni tempore, commodè, & sine magno Temporis Dispendio institui possit, profecto nondum capio.

*Plain Sights preferr'd to Telescopick; by M. Hevelius. n. 102. p. 27.*

Video etiam aliquos (inter quos *Cl. Flamstedius* invenitur) tulisse jam de nostris Observationibus, qualibus qualibus, judicium, priusquam illas viderunt, Examinarunt, vel quicquam de iis Cognoverunt. Nolo quidem vanus esse rerum mearum Jactator, nec unquam imaginatus mihi fui, me in omni isto Negotio, Restitutionis scil. Fixarum rem acu omnino tetigisse, aut tangere pro mea tenuitate posse: Sed hoc mihi penitus imaginor, si quidem totum illud Negotium suscepissem Dioptris Telescopicis, mihi non solum plurimos Annos Examinibus terendos, sed & spe, sine dubio, variâ viâ (qua de re hic non est differendi locus) cadendum fuisse. Exinde gratulor mihi, me ad sententiam illam necdum transiisse, meâque me Methodo omnia perfecisse quicquid præstitum Dei Beneficio fuerit. Quando vero Observationes habebimus 20 & 30 Annorum spatio continuatas utrinque, nimirum tum quæ Dioptris Telescopicis, tum quæ solummodò nostris de Coelo depromptæ fuerint, res omninò clarior futura est. Interea suo quilibet Ingenio fruatur, remque suâ ratione pro Libitu, tentet.

VII. 1. It is well known that the mean Apparent Magnitude of the Moon is 30 min. 30 sec. we will take it *Numero Rotundo* to be 30 min. at a Full Moon in the midst of Winter, and when she's in the Meridian, and at her greatest Northern Latitude, and consequently the utmost that she can be Elevated in our Horizon; 'Tis as well known also, that when she is in this Posture, being looked upon by the Naked Eye, she appears (that we may accommodate all to sensible Measures) to be *Magnitudinis Pedalis*, about a Foot broad. But the same Moon being looked upon just as she Rises, she appears to be three or four Foot broad, and yet if with an Instrument we take her Diameter, both in one Posture and the other, we shall find that still she shall be but 30 min. That this matter of Fact is true, besides the Authority of many Authors, I can Assert that I have accurately tried it my self, and I have so found it: One of the ways I proceeded was thus, I took a very good Telescope of about 6 Foot long, in the inward Focus of whose Eye Glass I apply'd a very fine Lattice made of the single Hairs of a Man's Head; Then

*Why Celestial Objects appear greater when nigh the Horizon than when higher Elevated; Examin'd by Mr. Will. Molineux. n. 187. p. 314.*

looking

looking with this at the Moon, when ſhe was juſt Riſen and looked extraordinary big, I obſerved what number of the Squares of the Lattice were occupied by her Body ; and then obſerving her again, when more Elevated and free from all extravagant Greatneſs, I ſtill found the ſame Squares of the Lattice poſſeſſed by her. This way is Equivalent to that now more uſed, of taking her Diameter by Mr. *Townley*'s Micrometers ; but I have alſo tried and found the ſame thing by an accurate Sextant, taking the Diſtance of the Moon's oppoſite Limbs.

The Celebrated *Des Cartes* attributes this Appearance rather to a Deceived Judgment, than to any Natural Affection of the Organ or Medium of Senſe ; for the Moon (ſays he) being nigh the Horizon, we have a better Opportunity and Advantage of making an Eſtimate of her, by comparing her with the various Objects that incur the Sight, in its way towards her ; ſo that tho' we imagine ſhe looks bigger, yet 'tis a meer deceit : for we only think ſo, becauſe ſhe ſeems nigher the Tops of Trees, or Chimneys, or Houſes, or a ſpace of Ground, to which we can compare her, and Eſtimate her thereby ; but when we bring her to the Teſt of an Inſtrument, that cannot be deluded or impoſed upon by theſe Appearances, then we find our Eſtimate wrong, and our Senſes deceived. Theſe Thoughts, my-thinks, are much below the Accuſtomed Accuracy of the Noble *Des Cartes* ; for certainly if it be ſo, I may at any time Increaſe the apparent Bigneſs of the Moon, tho' in the Meridian ; for it would be only by getting behind a Cluſter of Chimneys, a Ridge of a Hill, or the Tops of Houſes, and comparing her to them in that Poſture, as well as in the Horizon ; beſides, if the Moon be looked at juſt as ſhe is Riſing from an Horizon determined by a ſmooth Sea, and which has no more variety of Objects to compare her to, than the pure Air, yet ſhe will ſeem big, as if lookt at over the rugged top of an uneven Town or rocky Country. Moreover, All variety of adjoyning Objects may be taken off, by looking through an empty Tube, and yet the deluded Imagination is not at all helped thereby.

Fig. 110. The famous *Thomas Hobbs* gives this Solution. Let the point G, be the Center of the Earth, and F the Eye on the Surface of the Earth ; on the ſame Center G, let there be ſtruck the two Arches, E H, determining the Atmoſphere, and A D to repreſent that blue Surface in which we Imagine the Fixed Stars: And let F D be the Horizon. Divide the Arch A D into three equal Parts by the Lines B F, C F ; it is manifeſt that the Angle A F B is greater than the Angle B F C, and this again greater than the Angle C F D. Wherefore, ſays he, to make the Angle C F D equal to the Angle C F B, the Arch C D muſt be greater than the Arch C B ; and conſequently, that the Moon may in the Horizon appear under the ſame Angle as when Elevated ſhe muſt cover a greater Arch, and therefore ſeem greater ; that is, the Moon in the Meridian appearing under the Angle B F C, that ſhe may appear under an equal Angle in the Horizon, as ſuppoſe C F D, 'tis neceſſary that the Arch C D, ſhould be greater than C B ; and conſequently, tho' ſhe appear to Subtend a greater Arch when in the Horizon than when Elevated, yet ſhe appears under the ſame Angle ; And all this without Refraction. The

The Geometry of this Figure is most certainly true and Demonstrable. At this I quarrel not; but it makes no more in our present difficulty than if nothing had been said. For he has made the Circle G F, representing the Earth, very large in Proportion to the Circle A D; and then indeed taking the point F in the Earths Surface, and by Lines from thence dividing the Angle A F D into whatever Equal Parts, the intercepted Arches A B, B C, C D, shall be unequal. But if he had considered, that the Earth is as it were a point in respect of the Sphere of the fix'd Stars, nay the very Annual Orbit of the Earth is almost imperceptible, he would have found that the Lines F B, F C, F D, must be all conceived as drawn from the point G, and then Equal Angles will intercept Equal Arches, and Equal Arches Equal Angles: and so it happens (at least beyond the possibility of the discovery of Sense) to the Eye on the Surface of the Earth; so that his drawing his Lines so far from G as F is, and to another Concentrick Circle so nigh as A D, deceived him in this point.

The famous *Gassendus* has written four large *Epistles* on this Subject, the Substance of all which is, That the Moon being nigh the Horizon, and looked at through a more foggy Air, casts a weaker Light, and consequently forces not the Eye so much as when brighter; and therefore the Pupil does more Inlarge it self, thereby transmitting a larger Projection on the Retina. In this Opinion I find he is not alone, for this Disquisition being lately revived by a *French Abbe*, he therein follows the Sentiment of *Gassendus*, with this addition, That this Contracting and Enlarging of the Pupil causeth a different shape in the Eye; an open Pupil making the Crystalline flatter, and the Eye longer, and the narrower Pupil shortening the Eye, and making the Crystalline more Convex: The first attends our looking at Objects which are remote, or which we think so; the latter accompanies the viewing Objects nigh at hand. Likewise an open Pupil and flat Crystalline attends Objects of a more Sedate Light, whilst Objects of more forcible Rays require a greater Convexity, and narrower Pupil. From these Positions, the *Abbe* endeavoured to give an Account of our *Phænomenon*, as follows. When the Moon is nigh the Horizon, by Comparison with interposed Objects, we are apt to imagine her much farther from us than when more Elevated, and therefore (says he) we order our Eyes as for viewing an Object farther from us; that is, we something Enlarge the Pupil, and thereby make the Crystalline more flat: moreover, the Duskishness of the Moon in that posture does not so much strain the Sight; and consequently the Pupil will be more large, and the Crystalline more flat; hence a larger Image shall be projected on the Fund of the Eye, and therefore the Moon shall appear larger. And this Disposition of the Eye that magnifies her, magnifies also the Divisions of our forementioned Lattice, and consequently she by her Body shall possess no more of the Divisions than when she seems less. These two forementioned Accidents, *viz.* The Moons imaginary Distance and Duskishness, gradually vanishing as she Rises; a different Species is hereby introduced in the Eye, and consequently she seems gradually less and less, till again she approaches nigh the Horizon. These two Opinions of *Gassendus* and the *Abbe* being so nigh a-kin, I shall consider them both

together;

together; and first, I assert, That a wider or narrower Aperture of the Pupil increases not, neither diminishes the Projection on the Retina. I know, *Honoratus Faber* in his *Synopsis Optica* endeavours to prove the clear contrary to this
Fig. 111. my Assertion, and that after this manner. A B is an Object, E F the greater Aperture of the Pupil, admitting the Projection K I on the Retina, whereas the lesser Aperture C D, admits only the Projection G H; but G H is less than K I, wherefore a lesser Aperture diminishes the Projection. I admire that any Man that undertook (as *Honoratus Faber*) to write of *Opticks* more accurately than all that went before him, should be guilty of so very gross an Errour; and I do more admire, that the Celebrated *Gassendus*, and with him the Noble *Hevelius*, should be of the same Opinion: For tho' the aforesaid Demonstration hold most certainly true in direct Projections, as in a dark Room with a plain Hole; yet it will not hold in Projections made by Refraction, as it is in those on the Retina in the Eye, by means of the Cry-
Fig. 112. stalline and other Coats and Humours of the Eye. For let A B be a remote Object, and E F the Crystalline at its large Aperture, projecting the Image I M on the Retina. Let then C D be the lesser Aperture of the Pupil before the Crystalline: I say, the Image I M shall be projected as large as before, for the Cone of Rays E A F consists partly of the Cone of Rays C A D, therefore where the former E A F is projected, the latter C A D, as being a part of the former, shall be projected also. So that no more is effected by this narrow Aperture, but that the sides of the Radiating Cones are intercepted, and consequently the Point I, shall be affected with less Light, but it shall still be in the same place: what is said of that Cone and that Point, may be said of all other Cones and other Points of the Object. From hence appears, First, The Invalidity of the Account given of the Moon's Appearance by *Gassendus* from this Reason. Secondly, The Reason appears why a Telescope's lesser or greater Aperture, makes no difference in the Angle it receives: For imagine E F to be an Object Glass of a Telescope, and 'tis plain. Thirdly, 'Tis Evident why a greater or less Aperture on a Telescope should make the Objects appear lighter or darker, for thereby more or less Rays are admitted to determine on the Projection of each Point. But all this by the by. And this is sufficient for a Confutation of *Gassendus* and *Faber*; But our forementioned *Abbe* superadds to a greater or lesser Aperture of the Pupil, as a necessary consequent, a greater and lesser Convexity of the Crystalline, as also a lengthning and shortning the Tube of the Eye. And this I must confess would do something, if we find it true in our Case; and this let us try. First, (says he) The duskishness of the Moon nigh the Horizon admits the Pupil to enlarge it self, the Crystalline to flatten, and the Eye to lengthen: but what if we change our Object, and instead of the Moon take the Distance between some of the Fixt Stars, (as suppose those of *Orion*'s *Girdle*;) we shall find the same *Phænomenon* in them, and yet I hope neither he nor *Gassendus* will assert, that they at one time strain the Eye more than at another, or that at any time their Fulgur strains the Eye at all; if he do, let him take Stars of the lesser Magnitudes, nay even those that can but just be perceived, and then he will be convinced: Or let him consider whether this

will

will hold in looking at the Sun through very dark Glaſſes, which render the Sight thereof as inoffenſive to the Eye, as that of a Green Field; but perhaps he will then ſay, that this other Reaſon holds, which is, Secondly, That the greater imaginary Diſtance at which we think the Moon near the Horizon, than when more Elevated, makes us Contemplate her as if really ſhe was ſo, *viz.* with Ample Pupils, *&c.* But this I have ſufficiently overthrown in my Remarks againſt *Des Cartes*: Therefore I paſs it over, only ſubjoyning, that if there were any thing in this Surmiſe, my-thinks the Horizontal Moon ſhould be fanſied nigher to us than farther from us; for if we are for trying Natural Thoughts, let us take Children to determine the Matter, who are apt to think, that could they go to the edge of that ſpace that bounds their Sight, they ſhould be able (as they call it) to touch the Sky; and conſequently the Moon ſeems then rather nigher to us than farther from us.

After I had writ thus far, I accidentally caſt my Eye upon *Riccioli*'s Treatiſe of *Refraction*, at the end of his ſecond Volume of the *Almageſt*, *Lib.* 10. *Sect.* 6. *Cap.* 1. *Queſt.* 13. wherein he ſpeaks of our preſent difficulty; but to my wonder I find him aſſert, That he and Father *Grimaldi* had often taken the Horizontal Sun and Moon's Diameters by a Sextant, when to the naked Eye they appeared very large; (*Grimaldus* directing his Sight to the left edge, and *Ricciolus* to the right) and that even by the Inſtrument they always found the Diameters greater than when more Elevated, the Sun often ſubtending an Angle of almoſt a Degree, and frequently 45 Minutes, the Moon alſo 38 or 40 Minutes. This is down-right contrary to the matter of Fact which I have before alledged, and directly repugnant to the matter of Fact aſſerted by the foremention'd French *Abbe*: Whither of us be in the right I leave to accurate Experiment to determine, and ſubmit the whole to the Deciſion of the Illuſtrious *Royal Society*. Only give me leave to add one word againſt *Riccioli*, for had his Experiments been accurately proſecuted, he ſhould have tried them when the Horizontal Moon had look'd 10 times more large in Diameter than ordinary; and then if it be true, that even by an Inſtrument ſhe will be found proportionally broader than really, ſhe ſhould Subtend an Angle of 300 Min. or 5 Deg. for very often I have ſeen the Moon when ſhe appear'd 10 times broader than ordinary, which the ſmall Addition of 8 or 10 Min. to her uſual Diameter will never cauſe.

2. I diſcourſed of this Appearance near 40 years ago with Mr. *Foſter*, then *Profeſſor of Aſtronomy* in *Greſham College*, who did then aſſure me, (from his own Obſervation I ſuppoſe) that the apparent Magnitude taken by Inſtruments, (however the fancy may apprehend it) is not greater at the Horizon than when higher. Mr. *Caſwell* affirms the ſame thing; And I do not doubt but the thing is ſo: For though Refraction near the Horizon alter the Altitude of the thing ſeen; yet it cannot alter the Azimuth at all. For ſince this equally reſpects all Points of the Horizon; let the Refraction be what it will, the whole Horizon can be but a Circle: ſo that there is no room for the breadth of a thing (as to the Angle at the Eye) to be made greater, whatever its tallneſs may (the Refraction not equally affecting all parts in the Circles of Altitude.) Nor is there any Reaſon, why this ſhould

*This Phænomenon conſidered by Dr.* Wallis. *Ibid. p.* 323.

rather thruſt the other, than that the other thruſt this, out of place. Whereas, in the Altitude, it is otherwiſe: For while what is near the Horizon is inlarged, that which is further off is thereby contracted; which as to the Azimuth or Horizontal Poſition cannot be.

Suppoſing then that the Sun's Apparent Horizontal Diameter, taken by Inſtrument, is the ſame near the Horizon, as in a higher Poſition, I take its imaginary greatneſs, which is fanſied near the Horizon, to be only a Deception of the Eye; or rather the Imagination from the Eye.

For ſure it is, that the Imagination doth not Eſtimate the greatneſs of the Object ſeen, only by the Angle which it makes at the Eye; but, by this compared with the ſuppoſed Diſtance. True it is, that *Cæteris paribus*, we judge that to be the greater Object, which makes at the Eye the greater Angle: but not ſo if apprehended at different Diſtances.

For if through a Caſement (or leſſer Aperture) we ſee a Houſe at 100 yards diſtance; this Houſe (though ſeen under a leſs Angle) doth not to us ſeem leſs than the Caſement through which we ſee it, (or this greater than that, becauſe it makes at the Eye the greater Angle:) But the Imagination makes a Comparative Eſtimate from the Angle and Diſtance jointly conſider'd.

So that of two things ſeen under the ſame or equal Angles, if to one of them there be ought which gives the Apprehenſion of a greater Diſtance, that to the Imagination will appear greater. Now, ſure it is, that one great Advantage for Eſtimating the Diſtance of a thing ſeen, is from the variety of intermediate Objects between the Eye and the thing ſeen. For then the Imagination muſt allow room for all theſe things.

Now when the Sun or Moon is near the Horizon, there is a Proſpect of Hills, and Vallies, and Plains, and Woods, and Rivers, and variety of Fields and Incloſures, between it and us; which preſent to our Imagination a great Diſtance capable of receiving all theſe. Or, if it ſo chance, that (in ſome Poſition) theſe Intermediates are not actually ſeen: yet having been accuſtomed to ſee them, the Memory ſuggeſts to us a view as large as is the Viſible Horizon.

But when the Sun or Moon is in a higher Poſition; we ſee nothing between us and them (unleſs perhaps ſome Clouds), and therefore nothing to preſent to our Imagination ſo great a Diſtance as the other is.

And therefore, though both be ſeen under the ſame Angle, they do not appear (to the Imagination) of the ſame bigneſs, becauſe not both fanſied at the ſame Diſtances: But that near the Horizon is judged bigger (becauſe ſuppoſed farther off) than the ſame when at a greater Altitude.

'Tis true, That as to ſmall and midling Diſtances (beſides this Eſtimate from Intermediates) the Eye hath a means within it ſelf to make ſome Eſtimate of the Diſtance. As, when we already know the bigneſs of a thing ſeen, to which we have been accuſtomed; as a Man, a Tree, a Houſe, or the like: If ſuch thing appear to us under a ſmall Angle, and Indiſtinct, and faintly coloured, the Imagination doth allow ſuch Diſtance as to make ſuch a thing ſo to appear. And, if this, through a Proſpective Glaſs, be

repre-

represented to us under a bigger Angle, and more Distinct, it is accordingly apprehended as so much nearer. But the Case is otherwise, when we do not, by the known bigness, judge the Distance; but, by the supposed Distance, judge of the Bigness, as in the Case before us. And accordingly, different Persons, according to different fansied Distances, judge very differently.

Again; In our two Eyes (when the Object is seen by both) there is yet another means of Estimating how far off it is. (And it is this by which we judge of Distances.) Namely, there are, from the same Object, two different Visual Cones, terminated at the two Eyes: whose two Axes contain, at the Object, different Angles, according to different Distances: An Acuter Angle at a great Distance, and more Obtuse when nearer.

Now, that such Object may be seen by both Eyes clearly; it is requisite, that the Eyes be put in such a Position, as that the Sight of each Eye receive the respective Axis at Right Angles; which requires a different Position of the two Eyes, according to the different Distance of the Object; as will manifestly appear, if we look, with Attention, on a Finger (or other small Object) at two or three Inches Distance from the Eye; and then upon another like Object at three or four yards beyond it: (and this alternately several times.) For 'twill be manifest, that while we look intently on the one, we do not see the other, (or but confusedly) though both be just before us. And, as we change our view, from the one to the other, we manifestly feel a Motion of the Eyes (by their Muscles) from one Posture to another.

And according to the different Posture in the Eyes, requisite to a clear Vision by both, we Estimate the Distance of the Object from us.

And hence it is, that they who have lost the Sight of one Eye, are at a great Disadvantage, as to Estimating Distances, from what they could do while they had the use of both.

But now when the Distance grows so great, as that the Position of these Visual Axes become Parallel, or so near to Parallel as not to be distinguishable from it: This Advantage is lost, and we can thenceforth only conclude, that it is far off; but not how far. Hence it is that our view can make no Distinction of the Moon's Distance, from that of the other Planets, or even of the Fixed Stars: But they seem to us as equally remote from us; though we otherwise know their Distances from us to be vastly different. Because the Parallax (as I may so call it) from the different Position of the two Eyes, is quite lost, and undiscernable in Distances much less than the least of these.

So that, though as to small Distances, we may make some Estimate from the known Magnitude of the Object; And as to Middling Distances, from the Parallax (as I may call it) arising from the Interval of the two Eyes: Yet even this latter will hardly reach beyond, if so far as the visible Horizon; and all beyond it is lost. And therefore there being nothing left to assist the fancy in Estimating so great a Distance, but only the intermediate Objects: Where these Intermediates appear to the Eye, (as when the Sun or Moon are near the Horizon) the Distance is fansied greater, than where they appear not, (as when farther from it:) and conſequently, (though both under the ſame or equal Angles) That near the Horizon is fanſied the greater. And this I judge to be the true Reaſon of that Appearance.

*An Experiment of the Refraction of the Air; by* Mr. Lowthorp. n. 257. p. 339.

VIII. We took a Cylinder of Caſt-Braſs, A B C D, and cut one end of it C D, Perpendicular to the Axis *a c x*, the other End A B, Enclined to it at an Angle of about 27 deg. 30 min. and therefore the Perpendicular to this Enclining Plain *p c*, and the Axis of the Cylinder *a c x*, comprehended an Angle *p c a*, of about 62 deg. 30 min. Theſe Ends were ground very true upon a Glaſs-Grinder's Braſs-Tool, and each of them was compaſt about with a narrow Feril of thin Braſs *b b b b*. Into the upper ſide of the Cylinder, at E, was Soldered the Braſs-Pipe E F, and into the under ſide, at G, the other Braſs-Pipe G H; the former of theſe Pipes being about three Inches long, and the later 6 Inches. Upon the Plate *d d d*, were fixt two other Plates, L L, Perpendicular to it and Parallel to each other. Each of theſe two Plates had an Arch of a Circle, (whoſe Diameter was equal to that of the Cylinder) cut out of its upper edge, ſo that when the Pipe, G H, was let through a hole near the middle of the Plate, *d d d*, the Cylinder fell into the Arches; and being faſtned there with Solder, the Axis, *a c x*, laid Parallel to the Plate *d d d*, and about an Inch and half above it. The Perpendicular End of the Cylinder, D C, was cloſed with an Object Glaſs of a $7\frac{1}{2}$ Foot Teleſcope *o o*, and the Enclining End A B, with a well poliſh'd flat Glaſs, *f f*; which was carefully choſen to tranſmit the Object Diſtinct enough, notwithſtanding its Obliquity to the Viſual Rays. The Ferils were filled with Cement round about the edges of the Glaſſes, which laid flat and every where toucht the ſmooth ends of the Cylinder, that they might firmly ſupport the Weight and Preſſure of the Excluded Air. (Fig. 113.)

Fig. 114. Inſtead of a Ciſtern, (as in the *Torricellian* Experiment) we made uſe of the inverted Siphon of Braſs M N O, Solder'd to the Plate *g g g*. One of the Sides M N, ſtood Perpendicular to the Plate *g g g*, and the other ſide, N O, Enclined to it, and was ſupported near the upper end O, with a little piece of Braſs *k k*.

Fig. 115. We then placed the Cylinder upon a Table, which was well faſtned to a firm Floor: The Pipe G H, was let through a Hole in the top of the Table, and the Plate *d d d*, was nailed down to it: The Tube of the Teleſcope *s s s*, with the Eye Glaſs in it, was applied to the Object Glaſs, and a Hair fixt at *x*, the common Focus of both Glaſſes, in the Axis of the Cylinder continued to it. Upon the Floor (under the Cylinder) we Nailed the Plate *g g g*, with the Inverted Syphon upon it, and joyned M to H, by the Inſertion of the Glaſs Tube T. The Joints were very carefully cloſed with Cement, and then covered over with pieces of a Bladder wrapt hard with ſtrong Thread. There was alſo a Bladder tied below each Joint at *m*, and when it was filled with Water it was tied above it at *n*; ſo that no Air could come to the Cement, to inſinuate it ſelf through its Pores or Fiſſures, if any happened to be left uncloſed.

It will not (I hope) be thought more than necessary, that in this Account of the *Apparatus*, I have mention'd so many Minute Circumstances; for, we found it difficult enough to Exclude the Air, and almost impossible to discover the very little Holes through which so Subtile a Fluid would freely Enter, and Possess the Spaces deserted by the *Subsiding Mercury*. But with all this Precaution, the Experiment succeeded at last, as I wisht: after this manner.

We placed the Object *a*, (which was a black Thread fasten'd in a little Frame over a piece of White Paper) in the Axis of the Cylinder *x c a*: We filled the Pipes and Cylinder with *Mercury*; and having stopt the upper end of the Pipe at F, with the little Iron-Stopple K, and closed it, as the upper part of the Tube and other Joints, we let the Mercury run out gently at O, (into the Bladder *u*,) till it remained suspended at the usual Height, (as in the *Barometer*) leaving the upper part of the Tube and the Cavity of the Cylinder between the Glasses *o o*, and *f f*, void of Air. We then saw the Object, which before appeared in the Axis at *x*, raised considerably above it; and we Reduced it to appear again at *x*, by removing it from *a* to *α*. The Axis therefore of the Visual Ray, (which was also the Axis of the Cylinder) *x c a*, falling Perpendicularly on the void Space, past through it without any Refraction: But Emerging Obliquely into the Air, it was Refracted towards the Perpendicular *p c*, and received a new Direction to *α*. And therefore the Distance *a α*, Subtended the Angle of Refraction *a c α*; all which we measur'd, and found as follows; *viz.*

| | *Inches. dec. parts.* |
|---|---|
| The Heighth of the Object above the Axis, or the unrefracted Visual Ray, *a α* ———— | 0, 425 |
| The Distance of the Object from the Refracting Plain *α c*, about 51 Feet, or ———— | 612, 000 |

| | *Deg. Min. Sec.* |
|---|---|
| Therefore the Angle of Refraction *a c α*, was ———— | 00 . 02 . 23 |
| The Angle of Emersion *p c a*, (by the Construction of the Cylinder) was ———— | 62 . 30 . 00 |
| Therefore the Angle of Incidence *p c α* = *p c a* + *a c α*, was ———— | 62 . 27 . 37 |

And therefore *universally*, (according to the known Laws of Refraction)

| | |
|---|---|
| The Sines of the Angles of Incidence being ———— | 100000 |
| The Sines of the Angles of Emersion are ———— | 100036 |
| And the Refractive Power of the Dense Air ———— | 36 |

By the *Refractive Power* of a Pellucid Body, I mean that Property in it whereby the Oblique Rays of Light are Diverted from their Direct Course, and which is measured by the Proportional Differences, (always observed) between the Sines of the Angles of Incidence and Emersion

This

This Property is not always Proportional to the Denſity (at leaſt not to the Gravity) of the Refracting Medium. For the Refractive Power of Glaſs to that of Water is as 55 to 34, whereas its Gravity is as 87 to 34; that is, the *Squares* of their Refractive Powers are (very near) as their reſpective Gravities. And there are ſome Fluids, which tho' Lighter than Water yet have a greater Power of Refraction: Thus the Refractive Power of Spirit of Wine, (according to Dr. *Hook*'s Experiment, *Microgr. Obſ.* lviii. *Pag.* 220.) is to that of Water as 36 to 33, and its Gravity *Reciprocally* as 33 to 36, or $36\frac{1}{2}$. But the Refractive Powers of Air and Water ſeem to obſerve the *Simple* Proportion of their Gravities *directly*, as I have compared them in the following Table. The Numbers there expreſſing the Refraction of Water are taken from the Mean of Nine* Experiments, made at ſo many ſeveral Angles of Incidence *Jan.* 25. $164\frac{1}{1}$. by Mr. *Gaſcoigne*, (the Ingenious firſt Inventor of the Micrometer, and the Ways of Meaſuring Angles by Teleſcopes) and thoſe of Air are produc'd by the preceding Experiment.

| | *Water.* | *Air.* |
|---|---|---|
| The (aſſum'd) Sines of the Angles of Incidence on the Void from | 100000 | 100000 |
| The Sines of the Correſpondent Angles of Emerſion out of | 134400 | 100036 |
| The Refractive Power of | 34400 | 36 |
| The Specifick Gravity (if as 900 to 1 at the time of the Experiment) of | 34400 | 38 |
| Or (if as 850 to 1) of | 34400 | 40 |

From hence it ſeems very probable, That their Reſpective Denſities and Refractive Powers are in a juſt *Simple* Proportion. And if this ſhould be confirmed by ſucceeding Experiments, made at different Angles of Incidence, and with Cylinders continuing Exhauſted through ſeveral Changes of the Air, it would be more than probable that the Refractive Powers of the Atmoſphere are every where, and at all Heights above the Earth, Proportional to its Denſities and Expanſions: And then it would be no difficult matter to trace the Light through it, ſo as to terminate the Shadow of the Earth, and (together with proper Expedients for Meaſuring the quantity of Light Illuminating an Opaque Body) to Examine at what Diſtances the Moon muſt be from the Earth to ſuffer Eclipſes of the obſerved Duration.

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* *I am Indebted for theſe Experiments to the Reverend and very* Accurate *Aſtronomer Mr.* Flamſteed, *who Copied them, together with many other Obſervations and ſeveral Paſſages relating to them, from Mr.* Gaſcoigne'*s Letters to Mr.* Crabtree: *They were happily preſerved, in the time of our Civil War, by the late Sir* Jonas Moor, *and Mr.* Chr. Townley; *and they are now in the Hands of Mr.* Rich. Townley *of* Townley *in* Lancaſhire, *by whom they were Imparted ſome time ago to Mr.* Flamſteed.

IX. Give

IX. Give me leave to ſuggeſt a Speculation, which hath been in my Thoughts theſe fourty years or more; but I have not had the opportunity of reducing it to Practice: It is concerning the Parallax of the Fixed Stars, as to the Earth's Annual Orb.

*To find the Parallax of the Fixt Stars; by Dr.* Wallis, *to Mr.* Will. Molyneux. n. 202. p. 844.

*Galileo* complains of it a great while ſince, (in his *Syſtema Coſmicum*) as a thing not attempted to be obſerved with ſuch diligence as he could wiſh, and I doubt we have the ſame Cauſe of complaining ſtill. I know that Dr. *Hook* and Mr. *Flamſteed* have attempted ſomewhat that way, but have deſiſted before they came to any thing of certainty. What hath been done to that purpoſe abroad I know not.

*Galileo* hath ſuggeſted divers things conſiderable in order to it: as, the Times of Obſervation; the Stars to be Obſerved; and the manner of Obſerving them, which yet I doubt is not Practicable. That which occurred to my Thoughts upon theſe Conſiderations, was to this purpoſe; That ſome Circumpolar Stars (nearer to the Pole of the Equator than is our *Zenith*, and not far from the Pole of the Zodiack) ſhould be made choice of for this purpoſe. And in caſe the Meridional Altitude be diſcernably different at different times, ſo will alſo be their utmoſt Eaſt and Weſt Azimuth, which may be better obſerved than their Riſing or Setting: And this will not be obnoxious to the Refraction, as is the Meridional Altitude; (for though the Refraction do affect the Altitude, yet not the Azimuth at all;) and we may here have choice of Stars for the purpoſe; which, in Obſervations from the bottom of a Well, we cannot have; being there confined to thoſe only which paſs very near our Zenith, though very ſmall Stars.

I would then take it for granted, as a thing at leaſt very probable, that the Fixed Stars are not all (as was wont to be ſuppoſed) at the ſame Diſtance from us, but the Diſtance of ſome vaſtly greater than of others; and conſequently, though as to the more remote, the Parallax may be undiſcernable, it may perhaps be diſcernable in thoſe that are nearer to us.

And thoſe we may reaſonably gueſs (though we are not ſure of it) to be neareſt to us, which to us do appear biggeſt and brighteſt, as are thoſe of the *Firſt* and *Second* Magnitude; and there are at leaſt of the *Second* Magnitude, pretty many not far from the Pole of the Ecliptick; (as that in particular in the *Shoulder* of the *Leſſer Bear*:) And in caſe we fail in one, we may try again and again on ſome other; which may chance to be nearer to us than what we try firſt. And Stars of this bigneſs may be diſcerned by a moderate Teleſcope, even in the Day time; eſpecially when we know juſt where to look for them.

The manner of Obſervation, I conceive, may be thus. Having firſt pitched upon the Star we mean to obſerve, and having then conſidered, (which is not hard to to do) where ſuch Star is to be ſeen in its greateſt Eaſt or Weſt Azimuth; it may be then convenient to fix (very firm and ſteadily on ſome Tower, Steeple, or other high Edifice, (in a convenient Situation) a good Teleſcopick Object Glaſs, in ſuch Poſition as may be proper for viewing that Star. And at a due Diſtance from it near the Ground, build on purpoſe (if already there be not any) ſome little Stone-Wall, or like Place, on which to fix the Eye Glaſs, ſo as to anſwer that Object Glaſs: And having ſo adjuſted it, as through both to ſee that Star in its deſired Station, (which may beſt be done while the Star is to be ſeen by Night in ſuch Situation, near the time of one of the Solſtices) let it be there fixed ſo firmly, as not to be diſturbed, (and the place ſo ſecured, as that none come to diſorder it) and care be taken ſo to defend both the Glaſſes, as not to be endangered by Wind and Weather. In which Contrivance, I am beholden to Mr. *John Caſwell*, M. A. of *Hart-Hall* in *Oxford*, for his Advice and Aſſiſtance, with whom I have many years ſince communicated the whole matter.

This Glaſs being once fixed, (and a Micrometer fitted to it, ſo as to have its Threads Perpendicular to the Horizon, to avoid any inconvenience which might ariſe from Diverſity of Refraction if any be) the Star may then be viewed from time to time (for the following year or longer) to ſee if any Change of Azimuth can be obſerved.

This I thought fit to Recommend to your Conſideration, who do ſo well underſtand Teleſcopes, and the Managery of them: But when I ſuggeſt, (as a convenient Star for this purpoſe) the *Shoulder* of the *Leſſer Bear*, (as being the neareſt to the Pole of the Zodiack of any Star that is of the Firſt or Second Magnitude) I do not confine you to that Star; but (without Retracting that) Suggeſt another; namely, the *Middle Star*, in the *Tail* of the *Great Bear*, which (though ſomewhat further from the Pole of the Zodiack) is a brighter Star than the other, and may be nearer to us.

But I do it principally upon this Conſideration; namely, That there is adhering to it a very ſmall Star, (which the *Arabs* call *Alcor*, of which they have a Proverbial ſaying, when they would Deſcribe a ſharp Sighted-Man, That he can diſcern the *Rider* on the *Middle Horſe* of the *Wayn*; And of one who pretends to ſee ſmall things but overlooks much greater, *Vidit Alcor at non Lunam Plenam:*) Which *Hevelius* in his Obſervations, finds to be Diſtant from it about nine Minutes, and five or ten Seconds: So that beſides the Advantage of diſcovering the Parallax of the greater Star, if diſcernable; the Difference of Parallax of that and of the leſſer Star (being both within the reach of a Micrometer) may do our work as well. For if that of the greater Star be Diſcernable, but that of the leſſer be either not Diſcernable or leſs Diſcernable, their different Diſtances from each other at different times of the Year may perhaps (without farther *Apparatus*) be Diſcerned by a good Teleſcope of a competent length, furniſhed with a Micrometer, if carefully preſerved from

from being diſordered in the Intervals of the Obſervations; and diſcover at once, both that there is a Parallax, and that the Fixed Stars are at different Diſtances from us; wherein, that I be not miſtaken, my meaning is not that the Inſtrument or Micrometer ſhould be removed for the obſerving of the Leſſer Star, but that (when the Azimuth of the greater Star is taken) by a Micrometer (conſiſting of divers fine Threads Parallel and Tranſverſe) may (at the ſame time) be obſerved the Diſtance of the two Stars, each from other, in that Poſition (both being at once within the reach of the Micrometer;) which Diſtance (the Inſtrument remaining unmoved) if it be found (at different times of the year) not to be the ſame, this will prove that there is a different Parallax of theſe two Stars.

This latter part of the Obſervation, (of their different Diſtances at different Times) I ſuggeſt, as more eaſily practicable though not ſo Nice as the former. For it may be done, I think, without any further *Apparatus* there than a good Teleſcope of ordinary Form, furniſhed with a Micrometer, (this being carefully kept unvaried during the Interval of theſe Obſervations.) And if this part only of the Obſervation (without the other) be purſued; it matters not though the two Obſervations (near the two Solſtices) be, one at the Eaſtern, the other at the Weſtern Azimuth (whereby both may be taken in the Night time) for the Diſtance muſt (at both Azimuths) be the ſame. If after obſerving the Azimuth of the greater Star it be neceſſary to move the Micrometer for Meaſuring its Diſtance from *Alcor*, that may be done another Night, (and it is not neceſſary to be done at one Obſervation) for that Diſtance cannot be diſcernably varied in a Night or two.

*Concerning the Diſtance of the Fixt Stars; by Mr. Fr. Roberts. n. 209. p. 101.*

X. Since the *Pythagorean Syſtem* of the World has been revived by *Copernicus*, (and now by all Mathematicians accepted for the True one) there ſeemed ground to imagine, that the Diameter of the Earth's Annual Courſe (which according to our beſt Aſtronomers, is at leaſt 40000 times bigger than the Semidiameter of the Earth) might give a Senſible Parallax to the Fixt Stars, and thereby determine their Diſtance. But there are ſome Conſiderations which make us ſuſpect that even this Baſis is not large enough for that purpoſe.

M. *Hugens* (who is very exact in his Aſtronomical Obſervations) tells us, He could never diſcover any viſible Magnitude in the Fixt Stars, though he uſed Glaſſes which magnified the Apparent Diameter above 100 times.

Now, ſince in all likelihood the Fixt Stars are Suns, (perhaps of a different Magnitude) we may as a reaſonable Medium preſume they are generally about the bigneſs of our Sun.

Let us then ( for Example ) ſuppoſe the *Dog-Star* to be ſo. The Diſtance from us to the Sun being about 100 times the Sun's Diameter, it is evident, That the Angle under which the *Dog-Star* is ſeen in Mr. *Hugens*'s Teleſcope, muſt be near the ſame with the Angle of its Parallax to the Sun's Diſtance, or Semidiameter of the Earth's Annual Courſe; ſo that the Parallax to the whole Diameter, can be but double ſuch a Quantity, as even to M. *Hugens*'s Nice Obſervation is altogether Inſenſible.

The Diſtance therefore of the Fixt Stars ſeems hardly within the reach of any of our Methods to determine : but from what has been laid down, we may draw ſome Concluſions that will much Illuſtrate the prodigious Vaſtneſs of it.

1. That the Diameter of the Earth's Annual Orb (which contains at leaſt 160 Millions of Miles) is but as a Point in Compariſon of it; at leaſt it muſt be above 6000 times the Diſtance of the Sun: For if a Star ſhould Appear through the aforeſaid Teleſcope half a Minute broad, (which is a pretty ſenſible Magnitude) the true Apparent Diameter would not exceed 18''', which is leſs than the 6000th part of the Apparent Diameter of the Sun, and conſequently the Sun's Diſtance not the 6000th part of the Diſtance of the Star.

2. That could we advance towards the Stars 99 Parts of the whole Diſtance, and have only $\frac{1}{100}$ part remaining, the Stars would appear little bigger to us than they do here: For they would ſhow no otherwiſe than they do through a Teleſcope, which Magnifies an hundredfold.

3. That at leaſt 9 parts in 10, of the Space between us and the Fixed Stars can receive no greater Light from the Sun, or any of the Stars, than what we have from the Stars in a clear Night.

4. That Light takes up more time in Travelling from the Stars to us, than we in making a *Weſt-India* Voyage, (which is ordinarily performed in ſix Weeks:) That a Sound would not arrive to us from thence in 50000 years, nor a Cannon Bullet in a much longer time. This is eaſily computed, by allowing (according to Mr. *Newton*) 10 Minutes for the Journey of Light from the Sun hither, and that a Sound moves above 1300 Foot in a Second.

*The Places of the chiefeſt Fixt Stars according to the beſt Antient Obſervers; by Dr.* Edward Bernard, *to Dr.* Robert Huntington. *N.* 158. *p.* 567.

XI. Inter Codices tuos Arabicos in *Muſæo Mertonenſi* (numeras autem plus quadraginta Doctrinæ & Obſervationis Sideralis refertis) in *Tabulis Ilchanicis* ſtudio *Perſæ* præclari *Choagæ Naſirodini Tuſii* factum offendi, ut præcipui Honoris Stellas Fixas, ſecundum variorum Aſtronomorum varia Obſervata, qua Latæ (ut loquuntur) quave illæ Longæ in Cælo deprehendebantur, brevis Pagina repreſentaret. Hoc ergo Canonion, partim ex tuo Peculio, ut vides, partim ab aliis, adauxi. Non equidem quaſi Rem Magnam mihi viderer adeo feciſſe; verum ut noſtri Homines ſenſum

ac

ac opinionem aliquam concipiant Aſtronomiæ Orientis, ubi Ars ea primum nata conſtat. Multa ſane commendant Aſtronomiam Orientalium; Felicitas quidem & Claritas Regionum, ubi Obſervatum; Machinarum Granditas & Accuratio, Quantas plerique noſtri credere nolunt Cœlo ipſos obvertiſſe; Contemplantium inſuper Numerus & Scribentium, decuplo major quam apud Græcos Latinoſque celebratur; Adde decuplo Plures Munificentiores, ac Potentiores Principes, qui viris boni Ingenii ſumtus & Arma Cœleſtia dederunt. Quid vero Aſtronomi Arabum in *Cl. Ptolomæo*, magno Conſtructore Artis Cœleſtis, injuria nulla reprehenderint; quam illi ſolicite Temporis Minutias, per Aquarum Guttulas, Immanibus Sciotheris, imo (mirabere) *Fili Penduli Vibrationibus*, jampridem diſtinxerint & menſurarint; quam etiam peritè & accuratè verſaverint in magno molimine Ingenii Humani, de Ambitu Intervalloque binorum Luminarium & noſtri Orbis, una Epiſtola narrare non debet.

# Canon Præcipuarum e Stellis Fixis ſecundum Obſervata Majorum.

| | | | Magnit. | *Cl. Ptolemæo* in Magna Syntaxi. A. C. 137. Long. S. | ° | ′ | Lat. ° | ′ | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 34 | *Ultima* Fluvii. | 1 | 0 | 10 | 30 al. 50 | 53 | 30 | m. |
| 2 | 12 | *Lucida* Cathedræ Caſſiopeiæ. | 3 | 0 | 7 | 50 | 51 | 40 | b. |
| 3 | 12 | Perſei *Caput Meduſæum.* Algol. | 2 | 0 | 29 | 40 al. 20 | 23 | 0 | b. |
| 4 | 14 | *Oculus* Tauri *Auſtrinus.* | 1 | 1 | 12 | 40 al. 20 | 5 | 10 | m. |
| 5 | 35 | Orionis *Pes Siniſter.* Rigel. | 1 | 1 | 20 | 50 | 31 | 30 | m. |
| 6 | 3 | *Capella* Aurigæ. | 1 | 1 | 25 | 0 | 22 | 30 | b |
| 7 | 2 | Orionis *Humerus Dexter.* | 1 | 2 | 2 | 0 | 17 | 0 | m. |
| 8 | 1 | Sirius. Alhabor. | 1 | 2 | 17 | 40 al. 20 | 39 | 10 | m. |
| 9 | 2 | Procyon. Algomeiſa. | 1 | 2 | 29 | 30 al. 10 | 16 | 10 | m. |
| 10 | 8 | *Cor* Leonis. Regulus. | 1 | 4 | 2 | 30 al. 10 | 0 | 10 | b. |
| 11 | 14 | *Spica* Virginis. | 1 | 5 | 26 | 40 | 2 | 0 | m. |
| 12 | 23 | Arcturus. | 1 | 5 | 27 | 0 | 31 | 30 | b. |
| 13 | 8 | Antares. *Cor* Scorp. | 2 | 7 | 12 | 40 al. 20 | 4 | 0 | m. |
| 14 | 1 | *Caput* Ophiuchi. | 3 | 7 | 24 | 50 | 36 | 0 | b. |
| 15 | 1 | *Lucida* Lyræ. | 1 | 8 | 17 | 20 | 62 | 0 | b. |
| 16 | 3 | Aquilæ *Lucida.* | 2 | 9 | 3 | 50 | 29 | 10 | b. |
| 17 | 5 | *Lucida in Cauda* Cygni. | 2 | 10 | 9 | 10 | 60 | 0 | b. |
| 18 | 3 | *Lucid. in Crure Dext.* Pegaſi. Seat. | 2 | 11 | 2 | 10 | 35 | 0 | b. |
| 19 | 2 | *Alæ* Pegaſi *Ultima.* Algenib. | 2 | 11 | 12 | 10 | 12 | 30 | b. |
| 20 | 7 | Perſei *Lucidum Latus.* | 2 | 1 | 4 | 50 | 30 | 0 | b. |
| 21 | 27 | *Cauda* Leonis. | 1 | 4 | 24 | 30 | 11 | 50 | b. |
| 22 | 1 | *Os* Piſcis Auſt. *in Aqua extrema.* | 1 | 10 | 7 | 0 | 23 | 0 | m. |
| 23 | 44 | Canopus. | 1 | 2 | 17 | 10 | 75 | 0 | m. |

Canon

# Canon Præcipuarum è Stellis Fixis ſecundum Obſervata Majorum.

| | *Ali Abolcaſimo.* A. C. 938. | | | | | *Abdorahmano Sophio.* A. C. 964. * | | | *Ebnolalamo.* A. C. 980. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Long. | | | Lat. | | Long. | | | Long. | | | Lat. | |
| | S. | ° | ' | ° | ' | S. | ° | ' | S. | ° | ' | ° | ' |
| 1 | 0 | 11 | 50 | 53 | 30 | 0 | 12 | 53 | 0 | 16 | 42 | 53 | 30 |
| 2 | 0 | 20 | 20 | 51 | 45 | 0 | 20 | 32 | 0 | 24 | 9 | 51 | 45 |
| 3 | 1 | 12 | 17 | 22 | 45 | 1 | 12 | 22 | 1 | 15 | 1 | 22 | 45 |
| 4 | 1 | 24 | 35 | 5 | 15 | 1 | 25 | 22 | 1 | 28 | 40 | 5 | 15 |
| 5 | 2 | 2 | 1 | 31 | 4 | 2 | 2 | 32 | 2 | 5 | 55 | 31 | 20 |
| 6 | 2 | 6 | 49 | 22 | 50 | 2 | 7 | 42 | 2 | 10 | 50 | 22 | 50 |
| 7 | 2 | 13 | 16 | 16 | 45 | 2 | 14 | 42 | 2 | 17 | 21 | 16 | 45 |
| 8 | 2 | 29 | 30 | 39 | 20 | 3 | 0 | 22 | 3 | 3 | 40 | 39 | 20 |
| 9 | 3 | 10 | 40 | 16 | 0 | 3 | 11 | 12 | 3 | 15 | 45 | 16 | 0 |
| 10 | 4 | 14 | 40 | 0 | 15 | 4 | 15 | 12 | 4 | 18 | 45 | 0 | 15 |
| 11 | 6 | 8 | 23 | 2 | 6 | 6 | 9 | 22 | 6 | 12 | 33 | 2 | 6 |
| 12 | 6 | 8 | 50 | 31 | 12 | 6 | 9 | 42 | 6 | 12 | 55 | 31 | 12 |
| 13 | 7 | 24 | 35 | 4 | 24 | 7 | 25 | 22 | 7 | 28 | 40 | 4 | 24 |
| 14 | 8 | 6 | 53 | 36 | 0 | 8 | 7 | 32 | 8 | 11 | 2 | 36 | 0 |
| 15 | 9 | 0 | 40 | 61 | 45 | 9 | 0 | 2 | 9 | 4 | 45 | 61 | 45 |
| 16 | 9 | 15 | 53 | 29 | 12 | 9 | 16 | 32 | 9 | 20 | 58 | 29 | 14 |
| 17 | 10 | 20 | 4 | 59 | 36 | 10 | 21 | 52 | 10 | 24 | 9 | 59 | 36 |
| 18 | 11 | 14 | 0 | 31 | 10 | 11 | 14 | 52 | 11 | 18 | 54 | 31 | 10 |
| 19 | | | | | | 11 | 24 | 52 | 11 | 24 | 52 | 12 | 30 |
| 20 | 1 | 16 | 15 | 30 | 8 | 1 | 17 | 32 | | | | | |
| 21 | 5 | 6 | 40 | 11 | 50 | 5 | 7 | 12 | | | | | |
| 22 | | | | | | 10 | 18 | 12 | | | | | |
| 23 | | | | | | 2 | 29 | 52 | | | | | |

* Is cum *Ptolemæo* convenit κατὰ πλάτος.

Canon

# Canon præcipuarum e Stellis Fixis secundum Observata Majorum.

| | *Ex Canonibus Hacimicis Johanidæ Ægyptii.* A. C. 996. | | | | | *Choagæ Nasirodino Tusio in Tabulis Ilchanicis.* A. C. 1233. | | | | | *Ex Sultanicis Ologbeci.* A. C. 1437. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Long. | | | Latit. | | Long. | | | Latit. | | Long. | | | Latit. | |
| | S. | ° | ′ | ° | ′ | S. | ° | ′ | ° | ′ | S. | ° | ′ | ° | ′ |
| 1 | 0 | 16 | 20 | 53 | 28 | 0 | 24 | 55 | 51 | 45 | 0 | 15 | 40 | 53 | 45 |
| 2 | 0 | 24 | 34 | 51 | 50 | 0 | 24 | 35 | 51 | 40 | 0 | 28 | 1 | 50 | 48 |
| 3 | 1 | 15 | 24 | 22 | 39 | 1 | 16 | 25 | 23 | 0 | 1 | 18 | 54 | 22 | 0 |
| 4 | 1 | 29 | 7 | 5 | 15 | 1 | 29 | 22 | 5 | 13 | 2 | 2 | 31 | 5 | 15 |
| 5 | 2 | 5 | 30 | 31 | 35 | 2 | 6 | 35 | 31 | 30 | 2 | 9 | 25 | 31 | 18 |
| 6 | 2 | 11 | 32 | 22 | 3 | 2 | 11 | 10 | 22 | 40 | 2 | 14 | 43 | 22 | 42 |
| 7 | 2 | 17 | 46 | 16 | 50 | 2 | 18 | 0 | 16 | 50 | 2 | 21 | 13 | 16 | 43 |
| 8 | 3 | 4 | 2 | 39 | 30 | 3 | 3 | 50 | 39 | 10 | 3 | 6 | 19 | 39 | 30 |
| 9 | 3 | 15 | 52 | 16 | 2 | 3 | 15 | 45 | 16 | 5 | 3 | 18 | 22 | 16 | 0 |
| 10 | 4 | 19 | 10 | 0 | 10 | 4 | 19 | 14 | 0 | 17 | 4 | 22 | 13 | 0 | 9 |
| 11 | 6 | 12 | 58 | 2 | 10 | 6 | 13 | 25 | 1 | 52 | 6 | 16 | 10 | 2 | 9 |
| 12 | 6 | 13 | 49 | 31 | 33 | 6 | 13 | 0 | 31 | 25 | 6 | 16 | 31 | 31 | 18 |
| 13 | 7 | 29 | 9 | 4 | 25 | 7 | 29 | 0 | 4 | 10 | 8 | 2 | 16 | 4 | 30 |
| 14 | 8 | 11 | 34 | 35 | 59 | 8 | 11 | 15 | 35 | 55 | 8 | 15 | 13 | 35 | 51 |
| 15 | 9 | 5 | 0 | 61 | 55 | 9 | 4 | 40 | 61 | 50 | 9 | 8 | 19 | 62 | 0 |
| 16 | 9 | 20 | 34 | 29 | 10 | 9 | 20 | 40 | 29 | 15 | 9 | 24 | 10 | 29 | 15 |
| 17 | 10 | 23 | 31 | 59 | 38 | 10 | 24 | 30 | 59 | 50 | 10 | 28 | 46 | 59 | 42 |
| 18 | 11 | 18 | 44 | 31 | 12 | 11 | 18 | 44 | 31 | 12 | 11 | 21 | 37 | 30 | 51 |
| 19 | | | | | | 11 | 18 | 55 | 31 | 1 | 0 | 1 | 22 | 12 | 34 |
| 20 | | | | | | 1 | 21 | 35 | 30 | 0 | 1 | 25 | 7 | 29 | 21 |
| 21 | | | | | | 5 | 11 | 55 | 11 | 50 | 5 | 13 | 49 | 12 | 0 |
| 22 | | | | | | 10 | 23 | 45 | 23 | 0 | 10 | 20 | 40 | 21 | 30 |
| 23 | | | | | | 3 | 3 | 55 | 75 | 0 | | | * | | |

* Non vidit unquam Nepos *Timuri* Clarum *Canopum*; non cæteri hujus Canonis exceptis *Alexandrinis*.

Canon

# Canon Præcipuarum e Stellis Fixis secundum Observata Majorum.

| | *AbdolgaliloSegazio in Genethliacis.* A. C. 1261. | | | *Ex Persicis Chrysococcæ.* A. C. 1115. | | | *Ex Canonib. Persa. Pembr.* A.C. 1346 *Melixæ* 338. | | | *Ex Cod. Arnoldino apud Gassendum pro.* A.C. 1364. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Long. | | | Long. | | | Long. | | | Long. | | |
| | S. | ° | ′ | S. | ° | ′ | S. | ° | ′ | S. | ° | ′ |
| 1 | 0 | 6 | 45 | 0 | 15 | 10 | | | | 0 | 18 | 20 |
| 2 | 0 | 25 | 2 | 0 | 22 | 50 | | | | | | |
| 3 | 1 | 16 | 52 | 1 | 14 | 40 | | 18 | 10 | 1 | 17 | 45 |
| 4 | 1 | 29 | 45 | 1 | 27 | 0 | 2 | 1 | 30 | 2 | 1 | 0 |
| 5 | 2 | 6 | 52 | 2 | 4 | 50 | | 8 | 20 | 2 | 8 | 0 |
| 6 | 2 | 11 | 43 | 2 | 10 | 50 | | 13 | 3 | 2 | 13 | 10 |
| 7 | 2 | 19 | 12 | 2 | 17 | 0 | | 20 | 30 | 2 | 18 | 10 |
| 8 | 3 | 4 | 28 | 3 | 2 | 40 | | | | 3 | 6 | 0 |
| 9 | 3 | 16 | 11 | 3 | 14 | 10 | | 18 | 0 | 3 | 17 | 20 |
| 10 | 4 | 19 | 42 | 4 | 17 | 30 | | 9 | 31 | 4 | 20 | 40 |
| 11 | 6 | 13 | 26 | 6 | 11 | 40 | | 15 | 10 | 6 | 15 | 10 |
| 12 | 6 | 13 | 47 | 6 | 12 | 0 | | 15 | 30 | 6 | 14 | 10 |
| 13 | 7 | 29 | 45 | 7 | 27 | 40 | 8 | 1 | 10 | 8 | 27 | 40 |
| 14 | 8 | 11 | 49 | | | | | | | | | |
| 15 | 9 | 5 | 38 | 9 | 2 | 20 | | 5 | 0 | 9 | 5 | 30 |
| 16 | 9 | 20 | 56 | 9 | 18 | 50 | | 22 | 20 | 9 | 22 | 0 |
| 17 | 10 | 26 | 15 | 10 | 24 | 10 | | 10 | 20 | 9 | 27 | 18 |
| 18 | 11 | 18 | 58 | 11 | 17 | 10 | | 20 | 40 | 11 | 20 | 20 |
| 19 | | | | 11 | 27 | 10 | | | | | | |
| 20 | 1 | 22 | 2 | 1 | 19 | 50 | | | | | | |
| 21 | 5 | 11 | 38 | | | | 5 | 13 | 0 | 5 | 12 | 40 |
| 22 | 10 | 23 | 53 | 10 | 22 | 00 | | | | | | |
| 23 | 3 | 4 | 3 | 2 | 2 | 10 | | | | | | |

Canon

## Præcipuarum Fixarum Canon juxta Obſervationes Selectas.

| | *Ex Codicibus Savilii & Bodlei. quaſi pro* A. C. 750. | | | *Mohammedi Tizinio.* A. C. 1533. | | | | *Sahebodino Alepenſi ad Ilchanicas, pro* A. C. 1436. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Long. | | | Aſcenſ. Rect. | | Declin. | | Aſcenſ. Rect. | | Declin. | |
| | S. | ° | ′ | ° | ′ | ° | ′ | ° | ′ | ° | ′ |
| 1 | 0 | 12 | 20 | 130 | 17 | 41 | 0 | 132 | 10 | 41 | 30 |
| 2 | | | | 86 | 55 | 56 | 41 | 84 | 2 | 55 | 45 |
| 3 | 1 | 2 | 50 | 130 | 14 | 39 | 43 | 127 | 44 | 39 | 4 |
| 4 | 1 | 19 | 50 | 153 | 0 | 15 | 43 | 150 | 40 | 15 | 20 |
| 5 | | | | 163 | 43 | 9 | 13 | 162 | 20 | 9 | 20 |
| 6 | 2 | 2 | 10 | 161 | 39 | 45 | 0 | 158 | 30 | 44 | 43 |
| 7 | 2 | 9 | 10 | 173 | 15 | 6 | 28 | 171 | 55 | 6 | 15 |
| 8 | 2 | 24 | 50 | 187 | 0 | 15 | 50 | 185 | 45 | 15 | 43 |
| 9 | 3 | 6 | 40 | 199 | 0 | 6 | 5 | 197 | 25 | 6 | 7 |
| 10 | 4 | 9 | 39 | 236 | 0 | 14 | 8 | 234 | 0 | 15 | 10 |
| 11 | 6 | 4 | 50 | 285 | 36 | 8 | 29 | 283 | 40 | 7 | 30 |
| 12 | 6 | 4 | 10 | 299 | 0 | 22 | 14 | 297 | 39 | 33 | 8 |
| 13 | 7 | 19 | 20 | 330 | 57 | 22 | 42 | 331 | 0 | 24 | 12 |
| 14 | | | | 348 | 57 | 13 | 9 | 347 | 5 | 13 | 20 |
| 15 | 8 | 24 | | 38 | 37 | 5 | 29 | 38 | 32 | 4 | 20 |
| 16 | 9 | 15 | 0 | 22 | 5 | 7 | 24 | 20 | 5 | 7 | 3 |
| 17 | | | | 36 | 40 | 43 | 50 | 36 | 20 | 43 | 15 |
| 18 | | | | 71 | 0 | 25 | 16 | 68 | 50 | 24 | 30 |
| 19 | | | | 88 | 10 | 12 | 23 | 85 | 54 | 11 | 30 |
| 20 | | | | 133 | 46 | 47 | 52 | 130 | 20 | 47 | 11 |
| 21 | 5 | 1 | 45 | 261 | 20 | 17 | 55 | 259 | 30 | 17 | 55 |
| 22 | | | | 68 | 45 | 33 | 51 | 67 | 25 | 35 | 2 |
| 23 | | | | 183 | 40 | 51 | 35 | 183 | 30 | 5 | 25 |

Præcipuarum

# Præcipuarum Fixarum Canon juxta Obſervationes ſelectas.

| | *Olaodino Sateridæ in Tabulis Damaſcenis.* A. C. 1480. | | | | | | | *Ex Tabulis Rodolphinis, A. C. 1600. ad Contemplationes Tychonis Brahei.* | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Long. | | | Aſcenſ. | | Declin. | | Long. | | | Lat. | |
| | S. | ° | ′ | ° | ′ | ° | ′ | S. | ° | ′ | ° | ′ |
| 1 | 0 | 20 | 34 | 129 | 30 | 17 | 0 | 0 | 21 | 10 | 53 | 30 |
| 2 | 0 | 18 | 16 | 83 | 50 | 55 | 50 | 0 | 29 | 35½ | 51 | 14½ |
| 3 | 1 | 20 | 4 | 127 | 55 | 38 | 50 | 1 | 20 | 37 | 22 | 22 |
| 4 | 1 | 3 | 4 | 149 | 50 | 15 | 0 | 2 | 4 | 12½ | 5 | 31 |
| 5 | | | | 167 | 50 | 5 | 10 | 2 | 11 | 37 | 31 | 11½ |
| 6 | 2 | 15 | 24 | 157 | 40 | 44 | 40 | 2 | 16 | 16 | 22 | 50½ |
| 7 | 2 | 22 | 24 | 170 | 10 | 6 | 10 | 2 | 15 | 23 | 16 | 63 |
| 8 | 4 | 4 | 4 | 184 | 20 | 15 | 45 | 3 | 8 | 35½ | 39 | 30 |
| 9 | 4 | 19 | 44 | 196 | 20 | 6 | 30 | 3 | 20 | 18½ | 15 | 57 |
| 10 | 4 | 24 | 44 | 233 | 30 | 15 | 5 | 4 | 24 | 17 | 0 | 26½ |
| 11 | 6 | 16 | 30 | 283 | 20 | 7 | 30 | 6 | 18 | 16 | 1 | 59 |
| 12 | 6 | 17 | 24 | 297 | 0 | 23 | 30 | 6 | 18 | 39½ | 31 | 2½ |
| 13 | 8 | 3 | 4 | 322 | 0 | 24 | 15 | 8 | 4 | 13 | 4 | 27 |
| 14 | | | | | | | | 8 | 16 | 50 | 35 | 57 |
| 15 | 9 | 7 | 44 | 4 | 0 | 38 | 30 | 9 | 9 | 43 | 61 | 47½ |
| 16 | 9 | 24 | 14 | 20 | 0 | 7 | 5 | 9 | 26 | 9 | 29 | 21½ |
| 17 | | | | 35 | 10 | 4 | 30 | 10 | 29 | 53½ | 59 | 56½ |
| 18 | 11 | 22 | 34 | 68 | 20 | 24 | 0 | 11 | 23 | 49½ | 31 | 7½ |
| 19 | 0 | 2 | 34 | 85 | 5 | 11 | 30 | 0 | 3 | 28½ | 12 | 35 |
| 20 | | | | 127 | 20 | 1 | 40 | 1 | 26 | 17 | 30 | 5 |
| 21 | 5 | 14 | 54 | 259 | 5 | 18 | 10 | 5 | 16 | 3 | 12 | 18 |
| 22 | 10 | 24 | 24 | | | | | 10 | 28 | 11½ | 21 | 0 |
| 23 | | | | 181 | 15 | 51 | 50 | | | | | |

# Canon Præcipuarum è Stellis Fixis secundum Obſervata Majorum.

*Ricciolo Bononienſi in Aſtronomia Reformata.* A. C. 1660.

| | Long. | | | | Lat. | | | Aſcenſ. | | | Declin. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | S. | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ |
| 1 | 0 | 22 | 0 | 14 | 53 | 30 | 0 | | | | | | |
| 2 | 1 | 0 | 22 | 0 | 51 | 17 | 0 | 357 | 44 | 33 | 57 | 18 | 40 |
| 3 | 1 | 21 | 32 | 10 | 22 | 22 | 40 | 41 | 33 | 54 | 39 | 36 | 30 |
| 4 | 2 | 5 | 1 | 43 | 5 | 30 | 50 | 64 | 6 | 58 | 15 | 46 | 10 |
| 5 | 2 | 12 | 3 | 10 | 31 | 10 | 10 | 74 | 32 | 38 | 8 | 37 | 30 |
| 6 | 2 | 17 | 6 | 15 | 22 | 51 | 45 | 72 | 52 | 20 | 45 | 36 | 0 |
| 7 | 2 | 16 | 13 | 0 | 16 | 52 | 30 | 76 | 45 | 4 | 5 | 59 | 50 |
| 8 | 3 | 9 | 3 | 0 | 39 | 32 | 5 | 97 | 30 | 18 | 16 | 16 | 30 |
| 9 | 3 | 21 | 6 | 30 | 15 | 57 | 10 | 110 | 22 | 32 | 6 | 4 | 0 |
| 10 | 4 | 26 | 4 | 45 | 0 | 26 | 20 | 147 | 31 | 14 | 13 | 36 | 30 |
| 11 | 6 | 19 | 6 | 0 | 1 | 59 | 30 | 196 | 51 | 0 | 9 | 20 | 30 |
| 12 | 6 | 19 | 30 | 40 | 31 | 0 | 40 | 210 | 4 | 38 | 20 | 59 | 50 |
| 13 | 8 | 5 | 1 | 40 | 4 | 26 | 30 | 242 | 10 | 40 | 25 | 33 | 30 |
| 14 | 8 | 17 | 43 | 20 | 35 | 56 | 15 | 259 | 47 | 14 | 12 | 52 | 30 |
| 15 | 9 | 10 | 32 | 40 | 61 | 47 | 0 | 276 | 19 | 32 | 30 | 30 | 40 |
| 16 | 9 | 26 | 58 | 30 | 29 | 20 | 40 | 293 | 31 | 59 | 8 | 1 | 20 |
| 17 | 11 | 0 | 41 | 25 | 59 | 57 | 20 | 307 | 26 | 35 | 44 | 6 | 40 |
| 18 | 11 | 24 | 40 | 20 | 31 | 8 | 20 | 341 | 51 | 48 | 29 | 15 | 50 |
| 19 | 0 | 4 | 26 | 20 | 12 | 37 | 0 | 358 | 57 | 0 | 13 | 19 | 20 |
| 20 | 1 | 27 | 0 | 8 | 30 | 5 | 40 | 44 | 52 | 30 | 48 | 34 | 30 |
| 21 | 5 | 16 | 53 | 0 | 12 | 16 | 20 | 172 | 53 | 58 | 16 | 27 | 40 |
| 22 | 10 | 29 | 2 | 56 | 20 | 59 | 40 | 339 | 37 | 0 | 31 | 20 | 10 |
| 23 | | | | | | | | | | | | | |

Præci-

# Præcipuarum Fixarum Diſtantiæ juxta Obſervationes ſelectas.

| | | | ° ′ | | ° ′ |
|---|---|---|---|---|---|
| 1 | *Ultima* Fluvii. | | | | |
| 2 | *Luc.Cath.*Caſſiop. | à Seat | 33 4 | à *Cauda* Cygni | 33 32½ |
| 3 | Perſei *caput Med.* | à *Latere* Perſei | 9 23⅙ | à Capella | 23 37 |
| 4 | *Ocul.* Taur. *Auſtr.* | à Sirio | 45 58⅓ | ab Algomeiſa | 46 22 |
| 5 | Orion. *Pes Siniſter* | ab *Oculo* Tauri | 26 33 | à Capella | 54 15½ |
| 6 | *Capella* Aurigæ. | ab *Oculo* Tauri | 36 43½ | à Perſei *Latere* | 19 11½ |
| 7 | Orion. *Humer. dex.* | ab *Oculo* Tauri | 15 47⅓ | | |
| 8 | Sirius. | ab Algomeiſa | 25 42 | à Rigel | 23 40 |
| 9 | Procyon. | à Regulo | 37 20 | à Rigel | 38 37½ |
| 10 | *Cor* Leonis. | à Spica | 54 2 | à *Cauda* Cygni | 24 39½ |
| 11 | *Spica* Virginis. | ab Arcturo | 33 2 | Ab Antare | 45 52 |
| 12 | Arcturus. | à Regulo | 59 49 | ab Antare | 56 4½ |
| 13 | Antares. | à *Lucida* Aquilæ | 60 9½ | à *Cauda* Leonis | 24 39½ |
| 14 | *Caput* Ophiuchi. | à Spica | 36 14½ | à Lyræ *Lucida* | 29 33½ |
| 15 | *Lucida* Lyræ. | à Seat | 55 39½ | à Spica | 9 20½ |
| 16 | Aquilæ *Lucida.* | à *Lucida* Lyræ | 34 9 | à *Cauda* Cygni | 38 4 |
| 17 | *Luc.inCauda*Cyg. | à *Lucida* Lyræ | 23 52 | à Seat | 32 57½ |
| 18 | *Luc.inCrure dext.* | ab Algenib. | 20 37 | a*Luc.Cath.*Caſſio. | 22 4 |
| 19 | *Alæ*Peg.*ult.*(Peg. | à *Latere* Perſei | 51 45½ | | |
| 20 | Perſei *Luc. Latus.* | ab *Oculo* Tauri | 36 20½ | | |
| 21 | *Cauda* Leonis. | à Spica | 35 2½ | | |
| 22 | *Os* Piſcis *Auſtralis.* | | | | |
| 23 | Canopus. | | | | |

# Octo Fixarum Nobilium Declinationes ex Curis Antiquis, Consulto etiam Illustri *Braheo.*

| | | *Aristyllo.* A.*a*.C.300. | *Timocharidi* A.*a*.C.295. | *Hipparcho.* A.*a*.C.128. | *Menelao.* A. C. 97. | *Ptolemæo.* A.C. 137. | *Tychoni.* A.C.1600. |
|---|---|---|---|---|---|---|---|
| 4 | *Ocul.*Tauri. | | 8 45 b. | 9 45 b. | | 11 0 b. | 15 38 b. |
| 6 | Capella. — | 40 0 b. | | 40 24 b. | | 41 10 b. | 25 28⅔ b. |
| 8 | Sirius. — | | 15 20 m. | 16 0 m. | | 16 45 m. | 16 11 m. |
| 10 | Regulus. — | | 21 20 b. | 20 40 b. | | 19 50 b. | 13 53½ b. |
| 11 | Spica. — | | 1 24 b. | 0 36 b. | 0 40 m. | 0 36 m. | 9 1 m. |
| 12 | Arcturus. — | | 31 30 b. | 31 0 b. | | 29 50 b. | 21 18½ b. |
| 13 | Antares. — | | 18 20 m. | 19 0 m. | | 20 25 m. | 25 26 m. |
| 16 | Aqu.*Lucida* | | 5 48 b. | 5 48 b. | | 5 50 b. | 7 54 b. |

## Oculi *Tauri* Longitudo.

| | | | | |
|---|---|---|---|---|
| | Hermeti — | ♓ | 25 | 17 |
| | Hipparcho. — | ♈ | 10 | 0 |
| A. C. 140. | Ptolemæo. — | ♈ | 12 | 40 |
| A. C. 851. | Abomasaro. — | ♈ | 19 | 15 |
| A. C. 831. | Thabeto. — | ♈ | 21 | 17 |
| | Arzacheli. — | ♈ | 23 | 20 |
| | Albatanio. — | ♈ | 24 | 30 |
| | R. Grostetio. — | ♈ | 28 | 40 |
| A.C. 1316. | W. Eveshamio. — | ♈ | 29 | 0 |

## *Spicæ* Longitudo.

| | | | | |
|---|---|---|---|---|
| A.*a*.C.295. | Timocharidi | ♍ | 22 | 20 |
| A.*a*.C.283. | Timocharidi | ♍ | 22 | 30 |
| A.*a*.C.128. | Hipparcho. — | ♍ | 24 | 0 |
| A. C. 97. | Menelao. — | ♍ | 25 | 45 |

## *Reguli* Longitudo.

| | | | | |
|---|---|---|---|---|
| A.*a*.C. 128. | Hipparcho. — | ♋ | 29 | 50 |
| A. C. 137. | Ptolemæo. — | ♌ | 2 | 30 |
| A. C. 879. | Albatanio. — | ♌ | 14 | 0 |

Spatio itaque annorum 742. grad. 11. 50′. aut potius gr. 11. 30′. propter *Reguli* Long. *Ptolemaicam* ♌ 2°. 30′. non 2°. 10′. sicut olim legerat Astronomus *Raccensis*.

## Addit Fixarum Longitudinibus Ptolemaicis.

| | | | |
|---|---|---|---|
| A.C. 1303. | Alphonsus. — | 17 | 8 |
| A.C. 1320. | Wymundus. — | 15 | 52 |
| A.C. 1440. | Walterus. *Vigorn.* — | 19 | 5 |

Prima *Arietis* distat ab Æquinoctio verno, 25°. 40′. —

*Hæc omnia* Walterus Vigorniensis, *Cod.* 13. inter Digbianos. —

Progre-

## Progrediuntur autem Stellæ Fixæ Gradum unum Annis Solaribus.

1. *Hipparcho, Ptolemæo, Theoni, Proclo* & *Alfergano.* ——— 100
2. *Timocharidi Alexandrino*, qui observarat *Spicam Cœlestem* Annis *Nabon*, 454. & 466. *Abdorahmano Salehio*, & *D. Petavio*, 72; sive $\frac{60 \times 12}{10}$ : & 50″. quovis Anno.
3. *Johannidæ, Ægyptio*, Canonum *Hacimicorum* Conditori. ——— 70 $\frac{1}{4}$.
4. *Jahiæ* f. *Abomansori*, aliisque Probatæ, quam vocarunt, Astronomiæ Auctoribus: nec non *Nasirodino Tusio, Cotbodino Sirasio, Ologbeco* Mogolorum Domino, *Xacholgio, Abolphetacho, Abenesdræ, Maimonidæ*, & plerisque juniorum. ——— 70. & 51″. 26‴.
5. *Chrysococcæ* in Persicis, & Astron. Anglicis. *Ann. Chr.* 1300. — 68. & 52″. 23‴.
6. Astronomis plerisque Arabum sub *Mamone* Principe ——— 66 $\frac{2}{3}$.
7. *Abdorahmano Sophio, Bahodino Chorcio, Alphonso* Regi, *Albatanio* ex *Racca* (quæ est *Callinicos Mesopotamiæ*), *Abdolgalilo Segazio, Levi* & *Zacuto* Judæis, & Observatorum *Maragensium* nonnullis. ——— 66. & 54″. 33‴.
8. *Copernico, Mæstlino*, aliisque fide illorum ——— fere 71. & 50″. 12‴. 5⁗.
9. Nonnullis apud *Chorcium Arabem.* ——— 54″.
10. *Tychoni Braheo, Keplero, Bullialdo*, ex obliquitate Zod. $23\frac{1}{2}°$. $70\frac{7}{12}$. & 51″.
11. *Longomontano.* ——— $72\frac{1}{3}$. & 49″. 54‴.
12. *Gassendo.* ——— $70\frac{1}{3}$. & 51″. 19‴. 24⁗.
13. *Ricciolo* in Astr. Reform. ex obliq. Zod. — 23°. 30′. 20″. 71. $19\frac{1}{2}$d. 50″. 40‴.
14. *Nobis*, & Ægyptiorum *Hierophantis.* ——— 71. $9\frac{2}{3}$. *mens.* & 50″. $9\frac{1}{4}$‴. fere.

XII. Tubum habeo Pedum $13\frac{2}{3}$, Lentibus Convexis, & Micrometore exactissimo *Towneleiano* Instructum, quocum Noctibus serenioribus Mensium *Octobris* & *Novembris*, nuper elapsorum, Minutas frequenter Pleiadum Stellularum Intercapedines dimensus sum, idque adeo auspicato, ut nunquam 20″, imò perraro 10″, inter se dissiderent repetitæ Observationes; à prægressis etiam Defuncti *D. Gascognii*, & nuperis Generosissimi *Towneleii* (quantorum Hominum!) eadem ratione peractis Observationibus, confirmatæ; quæ quidem Distantiæ limatissimæ sic se habent. *The Pleiades observ'd, in 1671. by Mr. Flamsteed. n. 79. p. 3061, 3062.*

| *Stellæ* | *Distantiæ.* | | | *Stellæ* | *Distantiæ.* | | |
|---|---|---|---|---|---|---|---|
| | Mihi | | Muto | | Mihi | | Muto |
| | ′ | ″ | ′ | | ′ | ″ | ′ |
| a b | 35 | 40 | | a d | 18 | 30 | 22 |
| a e | 27 | 40 | 31 | e d | 26 | 10 | 30 |
| e b | 20 | 00 | 32 | d i | 16 | 25 | |
| c b | 21 | 45 | 22 | a i | 18 | 18 | |
| e c | 10 | 00 | 11 | f i | 29 | 04 | |
| e g | 14 | 40 | | f a | 23 | 00 | 27 |
| c g | 11 | 55 | | h a | 23 | 20 | |
| b d | 22 | 04 | 24 | f h | 04 | 45 | 04 |

Addit *Vinc. Mutus* in Epiſtola ad Doctiſſimum *Ricciolum*, (cujus meminit in Appendice ad *Alm. No. Tom.* 1. *Pag.* 747.) *Occidentalem Lucidiorem tranſiiſſe Meridianum in eadem omnino Altitudine ac Lucidam Pleiadum.* Qua fretus Adverſione & Obſervatis Diſtantiis, Loca Stellis aſſignavi infra ſcripta; *Media* prius *Lucidâ*, eodem omnino Loco & Latitudine donatâ, quæ *Authori Carolino* arrideat; cæteris etiam abinde diſpoſitis; quas tamen omnes, propriam ſi hac in re ſententiam ſequi licuiſſet, tres vel ſaltem duos Scrupulos Primos Promotiores, nec non & Latiores ab Ecliptica proponerem: Ineunte *Ann.* 1672. conſtitutæ,

| *Stellæ.* | | Long. ♉ | | | Lat. Bor. | | | mag. |
|---|---|---|---|---|---|---|---|---|
| *Pleiadum.* | | ° | ′ | ″ | ° | ′ | ″ | |
| Occidentalis Lucidior. | b | 24 | 45 | 15 | 4 | 08 | 51 | 5 |
| Intra hanc & Borealiorem Teleſcopicam | g | 24 | 46 | 47 | 4 | 19 | 21 | 8 |
| Occidentalis Borealior. | c | 24 | 54 | 48 | 4 | 28 | 19 | 6 |
| Suprema in Quadrilatero. | e | 25 | 01 | 24 | 4 | 20 | 39 | 5 |
| Infima Auſtralis oppoſita. | d | 25 | 02 | 18 | 3 | 53 | 59 | 6 |
| Media & Lucida. | a | 25 | 19 | 48 | 4 | 00 | 00 | 3 |
| Quæ in Cuſpide ad Ortum. | f | 25 | 41 | 29 | 3 | 52 | 19 | 5 |
| Orientalium ſuperior Teleſcopica | h | 25 | 42 | 55 | 3 | 56 | 51 | 7 |
| Teleſcopica alia. | i | 25 | 14 | 04 | 3 | 42 | 37 | 9 |

XIII. 1. Inter

XIII. 1. Inter *Canem Majorem* & *Navem* nuper deprehendi *Nebulosam*, visu Pulcherrimam, si magnis Telescopiis inspiciatur, ex Stellis Confertissimis compositam, quæ Coelum mediat cum *Cane Minori*. *A Nebulous Star; by M.* Cassini. n. 123. *p.* 565.

2. Coelos infra Procyonem perlustrans, *Nebulosam* offendi, Latam, & Stellulis Confertissimam. Hanc eandem credo quam *Cl. Cassinus* observavit. *By Mr.* Flamsteed. *Ibid. p.* 567.

XIV. *Ann.* 1664. I discover'd the first Star in the Head of *Aries* to be a Double Star, made of two considerable Stars, so near as not to be discovered two, but by a Glass of 6 or 8 Foot long. *The first of* Aries *a Double Star; by Dr.* Hook. Phil. Coll. n.4.*p.*108.*Marg.*

XV. 1. Desunt in Coelo duæ Stellæ Secundæ Magnitudinis in Puppi *Navis* ejusque Transtris, *Bayero* β & γ, prope *Canem Majorem*, à me & aliis, occasione præsertim *Cometæ. Ann.* 1664. Observatæ & recognitæ. Earum Disparitionem cui Anno debeam, non Novi; hoc indubium, quod à die 10 *April.* 1668. ne Vestigium quidem illarum adesse amplius observo; cæteris circa eas, etiam Quartæ & Quintæ Magnitudinis, immotis. Plura de aliarum Stellarum Mutationibus, plusquam Centenis, at non tanti ponderis, annotavi. *Changes amongst the Fixt Stars; by S.* Montanari. n. 73. *p.* 2202. *Marg.*

2. M. *Cassini* hath discovered many New Stars: *viz.* One of the Fourth Magnitude, and two of the Fifth, in *Cassiopeia.* He hath discovered two others towards the beginning of *Eridanus*, where we were sure they were not yet about the end of the Year 1664. considering that this Place of the Heavens, where passed the then appearing *Comet*, was diligently beheld by many, who perceived divers other small Stars, without observing those two. The same hath also observed, towards the *Arctick* Pole, 4 of the Fifth or Sixth Magnitude. *By M.* Cassini. n. 73. *p.* 2201.

He hath also observed, That the Star which *Bayerus* puts near that which he marketh in the Figure of *Ursa Minor*, appears no more; That that which is marked A, in the Figure of *Andromeda*, is also disappear'd; That in lieu of that which is marked *υ*, at the *Knee* of the same Figure, there are two others more Northward; and that, that which is noted *ξ*, is very much diminisht; the Star, which *Tycho* placeth at the Extremity of *Andromeda*'s *Chain*, and calls it of the Fourth Magnitude, is now so small that one can scarce see it; and that which is in his *Catalogue* the 20th of the Constellation of *Pisces*, is now no more seen.

3. On the 24th of *Sept.* (*St. N.*) 1666. I have observed that New Star in *Pectore Cygni*, (which from the year 1662. until this time, hath been almost altogether hid) not only with my naked Eye, like a Star of the Sixth or Seventh Magnitude, but also with a very great Sextant. It is still in the very same place of the Heavens, where it was from *Ann.* 1661. to almost 1662. For, its Distance from *Scheat Pegasi* hath been by me found 35°. 51′. 20″. and from *Marcab.* 43°. 10′. 50″. which Distances are altogether equal to those which I observed *Ann.* 1658. the first of *November.* For, the Distance from *Scheat* at that time was 35°. 51′. 20″. and from *Marcab*, 43°. 10′. 25″. *The New Star in* Pectore Cygni; *by M.* Hevelius. n. 19. *p.* 349. n. 21. *p.* 372.

where

where that former from *Scheat* exactly anſwers to the Recent; and that from *Marcab* 'tis true, differs in a very few Seconds, but that diſparity is of no moment, ſince it only proceeded from thence, that this New Star is not yet ſo diſtinctly to be ſeen as at that time, when it was of the Third Magnitude. It is therefore certain, that it is the ſelf-ſame Star, which *Kepler* did firſt ſee *Ann.* 1601. and continued till *Ann.* 1662. He that will obſerve this Star, muſt take care leſt he miſtake the Three more Southern ones of the Sixth Magnitude; the Higheſt of which is diſtant from *Scheat Pegaſi*, 36°. 25'. 45''. the Middlemoſt from the ſame 37°. 25'. 20''. and the Loweſt 38°. 4'. 30''.

n. 134. p. 855. Ann. 1665. *Novemb.* 28. Stella illa nova in *Pectore Cygni* quæ aliquamdiu ab *Ann.* 1662. plane diluit, Coelo ſereno quaſi Reviviſcere videbatur.

Ann. 1666. *Septemb.* 21. Nudis Oculis (etiam *Luna* ſplendente) apparuit. *Sept.* 24. Minor erat illis tribus præcedentibus in *Collo* & vix 6 Magn. videbatur.

n. 65. p. 2039. Ann. 1670. *Aug.* 26. Senſim Creſcere videtur quanquam necdum Major eſt Stellis 6 Magn. *Sept.* 3. Adhuc Creſcere videbatur; 8. Paulo adhuc Creſ-
n. 134. p. 855. cere deprehendimus; & *Octob.* 13. Satis clare apparuerit.

Ann. 1671. *April.* 29. Vix major adhuc apparuit quam Anno Præterito; ſi quidem Stellis 6 Magn. æquabatur.

Ann. 1671. *Jun.* 26. Major fere videbatur.

Ann. 1672. *Mart.* 29. Adhuc Creſcere videbatur.

Ann. 1675. *Jul.* 22. Apparuit inſtar 6 Magn.

n. 134. p. 854. Ann. 1677. Nondum ad priorem Magnitudinem (Tertii videlicet Honoris) atque Claritatem & Splendorem (quâ Magnitudine *Ann.* 1657, 1658, & 1659. apparuit) pervenit: ſiquidem non niſi inſtar 6 Magn. adhuc fulget.

Ph. Col. n. 5. p. 162. Ann. 1681. *Aug.* 18. *Nova* in Collo *Cygni*, Nudis Oculis ob ejus tenuitatem parvitatemque haud quidem conſpecta, ſed Teleſcopio tamen deprehenſa eſt.

*The New Star, ſub Capite Cygni;* n. 65. p. 2092. n. 73. p. 2198.

4. 1. Don *Anthelme*, a *Carthuſian* at *Dyon*, on the 20th of *June*, *Ann.* 1670. diſcovered a Star of the Third Magnitude beneath the *Head* of *Cygnus*, ſituated in the Section of the two ſtreight Lines, one of which goeth from *Lyra* to the neareſt of the *Quadrangle* in the *Dolphin*, and the other from the *Eagle* to the Star, which is on the top of the *Upper Wing* of *Cygnus*. He ſent the News of this Diſcovery to M. *L' Abbé Mariotte*, one of the *Royal Academy*, who communicated it to the reſt. They all agree, 'tis a *New* Star; though M. *B . . . . .* oppoſed it at firſt, affirming it to be in *Bayerus*'s Tables: but they prove that Star in *Bayerus* to be another; giving for diſtinguiſhment theſe Meaſures.

| | | ° | ' | '' |
|---|---|---|---|---|
| The Bright Star *ad Roſtrum Cygni*, | It's *Aſcenſio Recta*. | 289 | 22 | 00 |
| | *Declinatio Borealis*. | 27 | 19 | 20 |
| But this *New* Star's *Aſcenſio Recta*, | is———— | 293 | 33 | 00 |
| | *Declinatio Borealis*. | 26 | 33 | 20 |

Longi-

| | | ° | ′ | ″ |
|---|---|---|---|---|
| | *Longitudo.* ♒ | 1 | 55 | |
| | *Latitudo.* | 47 | 28 | 10 |
| It's *Distance* from that *ad Rostrum Cygni* towards *Jaculum.* | | 3 | 47 | 30 |
| From the Tail of *Cygnus* | | 20 | 54 | 30 |
| And from the *Lucida Lyræ.* | | 18 | 39 | 40 |
| It came to the Meridian after the Star *in Rostro Cygni.* | | | 16 | 44 |
| And before the *Lucida Aquilæ.* | | | 0 | 27 |

In the beginning of *July*, this Star was obſerved to Decreaſe: *July* 11. It ſcarce appear'd of the Fourth Magnitude.

*Aug.* 10. It was of the Fifth, and continued to Decreaſe till it wholly diſappear'd.

*Ann.* 1671. *March* 17. D. *Anthelme* ſpied it again of the Fourth Magnitude.

*April* 4. M. *Caſſini* found it greater than the two Stars of the Third Magnitude that are below in the Conſtellation of *Lyra*, and a little ſmaller than that in the *Beak* of *Cygnus*, but more Radiant.

*April* 9. He found it a little Diminiſht, and almoſt equal to the greateſt of the two Stars that are below in *Lyra*.

The 12th, It was equal to the leaſt of theſe two Stars.

The 15th, He perceived that it Increaſed, and found it equal the ſecond time to the greateſt of theſe two Stars.

From the 16th unto the 27th, It appear'd of different Magnitudes, being ſometimes equal to the biggeſt of theſe two Stars, ſometimes equal to the leaſt, and now and then between both.

But the 27th and 28th, It was become as big as the Star in the *Swan*'s *Beak*.

The 30th, It appeared a little Clearer; and the firſt ſix days in *May* it was greater.

The 15th of *May*, It was ſeen ſmaller than the ſame Star.

The 16th, It was in bigneſs between the two Stars that are below in *Lyra*: And ever ſince ſhe hath ſtill Diminiſhed.

Thus this Star hath been twice in her greateſt Splendour; firſt on the 4th of *April*, and the ſecond time in the beginning of *May*.

2. Hiſce te inviſere volui, quo vos de Obſervatione quadam notatu digna certiores facerem, mentemque ſimul meam ea de re vobis exponerem: De *Nova*, *viz.* illa Fixa 3 Mag. ferè, Circa & Infra *Caput Cygni*, inter Informes conſpicua, cujus Longitudo 1°. 52′. 26″. ♒. & Latitudo 47°. 25′. 22″. Bor. modò exiſtit; ut Obſervationes, Die 25 *Julii*, Ann. 1670. à me habitæ, luculenter oſtendunt. *Novam* autem Stellam hanc ipſam omnino eſſe, & in Cœlo ad Ann. 1660. penitus Inconſpicuam fuiſſe, non eſt quod quiſquam dubitet. Accidit enim ut Annis 1659. 1660. & 1661. pleraſque Stellas illas omnes, in Aſteriſmo *Cygni* Apparentes, ſummâ diligentiâ, debitis Organis Dimenſus fuerim, atque ita Omnes illas, etiam circa Collum, &

*By M.* Hevelius. n. 65. *p.* 2037.

& Caput Deprehensibiles notaverim, earumque Distantias à diversis Fixis ceperim; nullam autem Stellam 3 Mag. eo loco ubi jam dicta *Nova* notatur, tum deprehendi; quam tamen optimè, si adfuisset, conspexissem. Sic ut primò hinc Certus sim, Ann. 1660. & 1661. hanc Stellam nondum extitisse visibilem. Deinde clarè etiam patet ex *Bayeri Uranometriâ*, hanc modò dictam *Novam* Stellam neque Ann. 1603. Apparuisse, & per consequens neque *Tychoni*, multo minus *Hipparcho*. Si quidem & *Bayerus* Fixam tantæ Magnitudinis deprehendisset, cùm haud procul ab illa aliam 6 Mag. depinxerit: prout in Asterismo ejus *Cygni* videre est. At, inquies, fortè ea ipsa est, quam tu *Novam* dicis, quippe cùm *Bayerus* Congruis Organis Stellas illas haud Observaverit, fieri facilè potuit, ut à vero ejus loco ad Gradum, vel paulo plus, aberraverit. Sed, crede, id factum non est, quandoquidem Stella illa Parvula adhuc eodem loco, ubi *Bayerus* ferè eam deposuit, commoratur, nec Major est Stella 6 Mag. ut eodem tempore ei videdatur. Distat enim, ut ipse met nuper deprehendi, ab Ore *Pegasi* 32°. 39′. 00″; Dextro Genu *Pegasi* 39°. 32′. 45″; hinc provenit ejus Long. 00°. 06′. 28″. ♒. & Lat. 46°. 11′. 14″. Bor. ad Annum scilicet Currentem 1670. Complet. *Julio*. At *Nova* Elongatur ab Ore *Pegasi* 32°. 31′. 35″. & à Dextro Genu *Pegasi* 38°. 18′. 50″. ex quibus Distantiis Long. 1°. 52′. 26″. ♒. & Lat. 47°. 25′. 22″. Bor. Elicitur. Adeo ut hæc *Nova* planè sit diversa ab illa 6 Mag. à *Bayero* notata; (quanquam hæ duæ nondum ad duos gradus ab invicem removentur.) Atque ex Dictis manifestum fit, hanc *Novam* nec Ann. 1603. nec Ann. 1660. inter cæteras emicuisse Stellas.

Cùm primitus à me observabatur, quoad Magnitudinem & Splendorem, Stellæ in Pectore *Aquilæ* æquabatur, nisi quòd aliquanto Obtusioris fuerit Luminis; quo ad Situm, respectu reliquarum Stellarum, in Linea Recta cum illa in Ancone Alæ Superioris *Cygni*, & illâ in Humero *Aquilæ*, nec non cum *Lucidâ Lyræ*, & illâ in Rhombo *Delphini*, Mediarum Borealiori consistebat; Triangulam verò Æquilaterum cum illa in *Capite* & *Rostro Cygni*) constituebat.

n. 66. p. 2028. Mirum vero in modum M. *Sept.* decrevit, adeò ut 14 *Oct.* nulla ra-
n. 134. p. 855. tione ampliùs Sextante observari à me potuerit; licet omnem ad hibuerim diligentiam.

n. 73. p. 2197. Ann. 1671. *Apr.* 29. Denuo Observavi. Excedit illam in *Rostro Cygni*, nec
n. 134. p. 856. non eam quæ est in Ancone Inferioris Alæ *Cygni*, fereque illi quæ est in Pectore *Cygni* æquatur, nisi quod Lumine paulò Obtusiori & Rubicundiori modò Luceat. Quâ vero Die primùm rursus illuxerit, affirmare adeò certò non possum. Certus interim sum, ad Mens. *Dec. Jan.* imo *Feb.* haud Conspicuum fuisse. Etenim post 14 *Oct.* quo videri desiit, memini me eam sæpius quæsivisse eo in Loco, sed nusquam Apparuisse. Idcirco quantum colligere datur, vix ante initium *Mar.* quin sine dubio adhuc tardius, iterum Prodiit. *Apr.* 30. eam a Reliquis quibusdam Fixis sum dimensus. Distat à Cauda *Cygni* 20°. 55′. 20″. ab Ancone Alæ Superioris *Cygni*, 17°. 47′. 50″; à Capite vero *Serpentarii* 34°. 19′. 40″; sic ut eodem planè Loco adhuc persistat, ubi antea fuerat.

*Maii* 17. Aliquanto Minor videbatur Rostro *Cygni*, & illa in Humero
n. 134. p. 856. *Aquilæ*, tum etiam Lumine Obtusior; Major tamen illâ in Cuspide *Sagittæ*, & æqualis ferè illi Seq. in Jugo *Lyræ*.

*Maii* 25.

*Maii* 25. Minor videbatur quàm Die 29 *Apr.* quâ primùm visa fuit; sic ut decrescere videretur. Minor jam erat Rostro *Cygni*, nec non illâ in Ancone Alæ Austr. etiam Minor illis in Jugo *Lyræ*, & Humero *Aquilæ*; vix Major apparuit Minori duarum in Pede *Cygni*, & illâ in Pectore *Aquilæ*.

*Jun.* 26. Minor apparuit illâ in Collo *Cygni*; sicut notabiliter decreverit.

*Jul.* 3. Minor ferè illâ in Collo *Cygni*; & 18, vix Stellis 5 Mag. æquiparari videbatur.

*Aug.* 2. Vix 6 Magn. apparuit, imò Minor quàm reliquæ omnes circa Caput & Collum *Cygni* existentes; per intervalla tantummodò micabat.

*Sept.* 11. Haud ampliùs Conspecta.

Ann. 1672. *Mar.* 6. I observed it again, but it can hardly be seen with the naked Eye. n. 81. p. 4015.

*Mar.* 29. Vix 6 Magn. apparuit; à quo tempore neutiquam amplius (*viz.* ad Ann. 1677.) in Conspectum venit, utut sæpius illam diligenter Quæsiverim. n. 134. p. 857, 854.

*The* Nebulosa *in the Girdle of* Andromeda; *by M.* Bullialdus.

5. Ann. 1667. in *Jan.* The *Nebulosa* in *Andromeda*'s Girdle (which may well enough be seen by the bare Eye) appeared much obscurer than the year before. In the Months of *February* and *March* I did not see it.

*The* New Star *in* Collo Ceti, *by M.* Bullialdus.

6. 1. Ann. 1667. *Jan.* 20. The *New Star* in the *Neck* of the *Whale*, did approach to the bigness of a Star of the Sixth Magnitude, and grew bigger afterwards. n. 25. p. 459.

*Febr.* 12. I saw it at least of the Fourth Magnitude.

*Febr.* 24. It was equal to the Stars of the Third Magnitude, shining very bright. *Ibid.*

*Feb.* 26. and 27. It appeared yet to Encrease.

*By* M. Hevelius.

2. Ann. 1667. In the beginning of *January* this Star did not appear.

*Jan.* 23. I found a little Star of the Sixth or Seventh Magnitude about the same Place where the said *New Star* uses to appear. But it then seemed to me not the Genuine *New Star*, but another; to wit, preceding the *New*, whose Longitude, in *Ann.* 1660. was defined by me, ♈. 25°. 43′. 3″. and the Latitude 14°. 41′. 32″. n. 25. p. 460. n. 134. p. 855.

*Feb.* 2. It appeared very bright, and that, when the Moon shone, of the bigness of that in the *Mouth* of the *Whale*, or *Nodo Lini*: from which time I always observed it to grow bigger.

*Mar.* 13. I did still find it extreamly bright, but could not by my naked Eye, because of the Vivid Crepuscle, and the low sight of the Star, accurately determine its Magnitude.

Ann. 1668. *Octob.* 26. *Nova* in Collo *Ceti* primum visa; sed instar minutissimæ Stellulæ. n. 134. p. 855.

*Nov.* 7. *Nova* in Collo *Ceti* Mediam ferè in ore æquabat.

Ann. 1669. *Jan.* 28. Minor erat illâ in Ore.

*Sept.* 26. Instar 6 Magn. apparuit.

*Oct.* 16. Illâ in *Ore* Major erat, & Clarior.

*Oct.* 24. Lucidam *Mandib.* æquabat.

*Nov.* 19. Major illâ in *Ore* & Minor *Mandib.*

Ann. 1670. *Aug.* 27. Maximo gaudebat Lumine, æqualis ferè Stellis 2. Mag. & *Mandib. Ceti.*

*Sept.* 3. Admodum Fulgida extitit; Et 8. Æqualis *Mandib. Ceti.*

n. 66. p. 2028. Ad medium usque Mensis *Oct.* *Mandib. Ceti* æqualis fere exstitit Magnitudine, & Claritate eam propemodum superavit; adeo ut hoc Anno secundæ fuerit Magn. ac Major quam præcedentibus Annis, excepto Ann. 1660. quo Major etiam *Mandibula Ceti* à me fuit deprehensa. Aliis temporibus non memini eam tertiæ Magnitudinis Stellas superasse. Certum igitur est, ipsam non eandem semper præ se ferre Magnitudinem nec Claritatem, utut in maximo suo existat Incremento.

n. 134. p. 856. *Dec.* 5. Adeo decreverat ut vix Stellæ 6 Magn. æquaretur.

Ann. 1671. *Aug.* 14. Æquabatur Stellæ ad *Genam*, imò ferè Major paulò videbatur.

*Sept.* 12. Æquabatur illi in *Ore* 4 Magn.

*Oct.* 30. Vix 6 Magn. apparuit.

*Nov.* 3. Non amplius apparuit.

Ann. 1672. *Aug.* 9. Clarissimis fulgebat Radiis, Major erat illâ in *Ore*, & Minor *Mandibulâ.*

*Sept.* 17. Minor illâ ad *Genam*, vix 4. imò 5. Magn. & 25. vix 6 Mag.

Ibid. p. 854. A Mense circiter *Octob.* ad *Decemb.* 23. *Ann.* 1676. ne semel quidem Prodiisse, utut semper omni studio Vigiles Oculos ad eam, quoties Observationibus operam serenis noctibus dedi, direxerim.

Ibid. p. 858. Ann. 1676. *Dec.* 10. Bene memini me *Novam* hanc in *Collo Ceti* haud vidisse, licet eâ in Cœli parte plurimas Stellulas observaverim.

*Dec.* 23. *Novam* hanc in *Collo Ceti* Cœlo admodum sereno clarissime vidimus; & quidem tantâ Claritate & Magnitudine fulgentem, ut *Mandibulam Ceti* non solum æquaret, sed Magnitudine & Claritate vinceret.

*Dec.* 31. Fere Major *Mandib.* *h. e.* 2 Magn.

Ann. 1677. *Jan.* 1. Clarissime rursus Affulgebat, Major ferè *Mandib. Ceti*, Major quoque quàm Extrema *Alæ* & *Marcab. Pegasi*, Colore & Lumine ferè æqualis *Mandib.* Memini tamen me olim observasse, quando Secundæ existebat Magnitud. eam paulo Albicantiorem & Splendidiorem.

Phil. Col. n. 5. p. 162. Ann. 1681. *Aug.* 18. *Nova* Stella in Collo *Ceti*, hac nocte, Luna etsi Plena & Splendente, major erat ea in Ore *Ceti*, sed nondum æquabat Lucidam *Mandibulæ.*

By M. Cassini. n. 123. p. 565. 3. Ann. 1676. *Mar.* Inspecta mihi est Stella *Nova* in *Ore Ceti*, quæ Annos aliquot latuit, Solaribus Radiis tempore Maximæ Fulsionis immersa; nunc vero Stellas 3 Mag. facilè superat.

By Mr. Flamsteed. Ibid. p. 567. 4. *Novam* in *Pectore Ceti* sæpius ante Octo Menses vidi, nec Minorem quam innuit Cl. *Cassinus.*

A New Star in Eridanus; by M. Cassini. n. 35. p. 683. 7. *March* 10. 1668. Not far from that Star in *Eridanus* which is called the 14th by *Bayerus*, there appeared a Star, equal to the Brightest of the Fourth Magnitude, almost in the same Place, where was observ'd the *Comet* of *Ann.* 1664. *Dec.* 31. which Star was not then seen, nor at other times elsewhere, nor is described in any Catalogue, on any Globe or Map, that I can learn; which therefore I deem to be a *New* one, that is of *New Appearance.*

8. The

8. The *Comet*, *Ann.* 1672. had (on the firſt of *April*, *St. N.*) paſs'd 45'. beyond the moſt Northern Star of the *Head* of *Taurus*, and was diſtant 1°. 43'. from the Star that was neareſt to that towards the South. M. *Caſſini* having conſider'd theſe two Stars, obſerv'd, That the Second is not leſs bright than the Firſt, and yet that *Bayerus* hath not marked it; And that at firſt Sight, it ſeems that *Tycho* hath left it out in his Catalogue. For he puts four Stars in the Place he calls *in Quadrilatero Cervicis*, and he ſpeaks not of this which is the fifth, and maketh with the other four an Irregular Pentagone. This Omiſſion of *Bayerus*, and the Denomination which *Tycho* uſeth to denote theſe Stars, which ſutes not with the Number nor the Configuration that now appears, do Adminiſter cauſe to doubt, whether the Star in Queſtion be not one of thoſe that appear from time to time. *A new Star in* Taurus; *by* M. Caſſini. n. 82. p. 4046.

XVI. 1. Supponit Cl. *Caſſinus*, ad Planetam in Ellipſi moventem extendi ab utroque Foco duas Rectas, quarum altera ſit Medii, altera autem Veri Motûs Linea. Conſtructio porro talis eſt: *To find the* Aphelia *of the Planets directly*; *by M.* Caſſini. *Conſidered by Mr.* Nic. Mercator. n. 57. p. 1168.

Fig. 117.

| | |
|---|---|
| L eſt Centrum Concentrici A B C D E. | B I eſt Perpendicularis ad R H G. |
| B L D eſt Diameter. | I eſt Centrum Ellipſeos. |
| B A, B C, B P, ſunt Intervalla Apparentia. | L I eſt Excentricitas. |
| D E, D F, D Q ſunt Intervalla Mediorum Motuum. | I O = L I. |
| B E, B F, B Q; item D A, D C, D P, ſunt Lineæ Rectæ. | O eſt Focus, circa quem ordinatur Medius motus; L, circa quem Verus. |
| B E ſecat D A in H; B F ſecat D C in G; B Q ſecat D P in R. | I M = I N = L B. |
| R H G eſt Linea Recta. | M, eſt Apogeon; N, Perigeon; B L M, Anomalia Vera. |

*Demonſtratio.* 1. Illuſtriſſimus ac Reverendiſſ. *Sethus Wardus*, Epiſcopus *Sariſburienſis*, in Examine Aſtronomiæ Philolaicæ, docuit Methodum, ex data Anomalia Media Planetarum, inveſtigandi Verum; quæ eſt hujuſmodi.

C eſt Centrum Ellipſeos A E P: F, focus circa quem ordinatur Medius Motus. S, focus circa quem ordinatur Verus Motus. A, Apogéon. P, Perigeon. E, Erro ſive Planeta. A F E, Anomalia Media. A S E, Anomalia Vera. F E T, Linea Recta; E T = S E. S T, eſt Linea Recta. Fig. 118.

In Triangulo S F T, dantur, 1. S F, diſtantia focorum: 2. F T = F E + E S = A P. 3. A F T, Angulus Externus, ſive Anomalia Media, æqualis Summæ Angulorum F S T & T. Ergo inveniri poteſt F S E, ſive Anomalia Vera, æqualis Differentiæ Angulorum, F S T & T. Nimirum

*Ut Semi-ſumma Laterum* F T & F S, *ad ſemi-differentiam corundem; Ita Tangens ſemi-ſummæ Angulorum* F S T & T, *ad Tangentem ſemi-differentiæ eorundem.*

Sed Semi-ſumma laterum F T & F S Invenitur, ſubſtituendo pro F T æqualem A P, cujus Semis eſt A C, qui additus C S, ſemiſſi ipſius F S, facit Semi-ſummam A S, Diſtantiam Planetæ Maximam.

Tum

Tum si ex Semi-summa A S, auferatur Latus Minus F S, restat Semi-differentia Laterum F A, æqualis P S, distantiæ Planetæ minimæ; ut sit Regula ex Anomalia Media datâ inveniendi veram.

*Ut* A S, *Distantia Planetæ Maxima, ad* P S, *Distantiam Minimam; Ita Tangens dimidiæ Anomaliæ Mediæ, ad Tangentem dimidiæ Anomaliæ Veræ.*

*Corollar.* 1. Si continuetur S E usque ad V, ita ut E V sit = ipsi F E, & tota S V = Axi A P, erit Trianguli, F S V, Angulus V, Semis Prostaphæreseos F E S, ideoque æqualis Semi-differentiæ Angulorum Anomaliæ Mediæ & Veræ, *h. e.* ipsorum A F E, & A S E; & Externus A F V = Semi-summæ eorundem, A F E & A S E, Angulorum, ablata *scil.* Semi-differentia, V F E, ex Majori A F E; unde oriuntur duæ Analogiæ,

1. *Ut Sinus Semi-summæ Anomaliæ Mediæ & Veræ,* A F V, *ad Sinum Semi-differentiæ eorundem* V; *ita* S V, ( = *Axi transverso* A P ) *ad* S F, *Distantiam Focorum.*

2. *Ut Sinus Semi-summæ Anomaliæ Mediæ & Veræ* A F V, *ad Sinum Anomaliæ Veræ,* F S V, *ita* S V, (*vel Axis* A P) *ad* F V, *Subtensam Anomaliæ Veræ: Ita quoque Semiaxis,* A C, *ad Semi-Subtensam* V X, *vel* F X.

*Corollar.* 2. Si in eodem Triangulo, F S V, ex Subtensæ F V, Puncto medio X, erigatur Perpendicularis X E; secabit illa S V in duas partes, quarum altera V E = est lineæ Medii Motus F E, altera vero S E, est ipsa linea Veri Motus.

Fig. 119. 2. Sit *a* Centrum Concentrici *c h f i*.
*c a d*, Diameter, eademque Linea Apsidum.
*c h*, Arcus Anomaliæ Veræ, cui respondet.
*d i*, Arcus Anomaliæ Mediæ. Itaque
*c d h*, est Angulus dimidiæ Anomaliæ Veræ, &
*d c i*, Angulus dimidiæ Anomaliæ Mediæ.
*c i*, & *d h*, sunt Lineæ Rectæ, secantes se mutuò in *g*.

Ab Intersectionis Puncto *g*, demittatur ad *c d*, Perpendicularis *g b*. Erit igitur,

$db : bg :: \text{Radius} : \text{Tang. } bdg$, vel $cdh$.

Et $cb : bg :: \text{Radius} : \text{Tang. } bcg$, vel $dci$.

Ergo $db \times \text{Tang. } cdh = bg \times \text{Rad.} = cb \times \text{Tang. } dci$.

Quare $db : cb :: \text{Tang. } dci : \text{Tang. } cdh$; hoc est, *d b* erit ad *c b*, ut Tangens dimidiæ Anomaliæ Mediæ ad Tangentem dimidiæ Anomaliæ Veræ; adeoque (per Regulam supra expositam) ut distantia Planetæ Maxima, ad distantiam Minimam. Quamobrem *d b* = erit distantiæ Planetæ Maximæ; & *c b*, Minimæ; & *a b*, Excentricitati.

Cumque idem eodem modo Demonstretur de cæteris omnibus Intersectionum Punctis, nimirum Perpendiculares ab ipsis ad *c d* Lineam incidere in Punctum *b*; oportet, ut Recta, jungens ipsas Intersectiones, congruat Perpendiculari, *b g f*.

3. Ducta

3. Ducta Diametro *h a k*, fiat Arcus *k l* = *i d*, & ducantur *k c*, & *h l*, Secantes ſe mutuo in *p*. Ab *h*, in *b g f*, dimittatur Perpendicularis, *h r*, eademque Parallela Apſidum Lineæ *c d*; erit Angulus *r h s*, Semi-differentia Arcuum Anomaliæ Veræ *c h*, & mediæ *d i*. Tum ab eodem *h* Puncto ducatur Recta *h β*, faciens cum *k h*, Angulum = Angulo *r h s*, & occurrens Lineæ Apſidum in *β*, erit Trianguli *a β h*, Angulus *β a h*, Menſura Arcus *c h*, ſive Anomaliæ Veræ, & *β h a* ſemi-differentia Anomaliæ Veræ & Mediæ (ex Constructione); & Externus *c β h*, (æqualis duobus Internis & Oppoſitis *β a h*, & *β h a*, adeoque compoſitus ex Anomalia Vera & ſemi-differentia ejus à Media) erit ſemi-ſumma Anomaliæ Veræ & Mediæ. Ergo per *Corollarii* primi *Analogiam* priorem; *Ut ſinus c β h, ad ſinum β h a; ita Radius a h, ad Excentricitatem a β.* Sed ſupra Demonſtravimus quoque *a b* æqualem Excentricitati. Ergo Punctum *β* congruit Puncto *b*.

Tum ex *b* excitetur ipſi *h b*, Perpendicularis *b t*; Aio, hanc continuatam Incidere in Punctum Interſectionis *p*. Nam Triangula, *r h s*, & *b h t*, ſunt Similia, ex Conſtructione; quemadmodum & Triangulum *h p k*, ſimile eſt Triangulo *h g i*, cum eidem Peripheriæ *c h*, inſiſtentes Anguli *p k h*, & *g i h*, ſint æquales, nec non æqualibus Peripheriis *k l*, & *i d*, inſiſtentes Anguli *p h k*, & *g h i*, æquales; quare & Tertius *h p k*, æqualis eſt Tertio *h g i*. Et ex æqualibus *p h k*, & *g h i*, ablatis æqualibus *b h t*, & *r h s*, reſtant æquales *p h b*, & *g h r*. Unde ſic Arguo; *s r h* = *t b h*, & *r h s* = *b h t*, Ergo *h s r* = *h t b*; ergò & Complementa horum ad Semicirculum ſunt æqualia, nimirum *r s i* = *b t k*, & *s i g* = *t k p*, Ergo & *i g s* = *k p t*, quibus ablatis ex æqualibus *i g h*, & *k p h*, reſtat *h g s* = *h p t*; & *g h r* = *p h b*, Ergo & *h r g* = *h b p*. Sed *h r g*, eſt Rectus, Ergo & *h b p*, Rectus eſt. Cum verò, & *h b t* Rectus ſit, ex Conſtructione, erit *t b* inde Rectum ipſi *b p*. Cumque idem eodem modo Demonſtretur de quavis alia Interſectione linearum ab *h*, & *k*, ad Congruentia Anomaliæ Veræ & Mediæ Puncta ductarum; patet, non modo Rectam, jungentem Interſectiones, tranſituram per *b* punctum; ſed & *h b*, lineam Perpendicularem fore ad eandem Jungentem. Q. E. D.

*Corollarium.* Si à quovis puncto Anomaliæ Veræ, puta *h*, ad reſpondens punctum Anomaliæ Mediæ *i*, ducatur Recta *h i*; excitata è Centro Excentrici *b*, ipſi *c b d*, Perpendicularis *b f*, ſecabit ipſam *h i* in *s*, eâ ratione quam linea Medii Motus obtinet ad lineam Veri Motus.

Nam per *Corollarii* Primi *Analogiam Poteſteriorem*, *h b*, eſt ſemi-ſubtenſa; ergo per *Coroll.* 2. Perpendicularis erecta ex *b*, nimirum *b t*, ſecat Diametrum *h k*, in *t*, eâ ratione quam linea Medii Motus obtinet ad lineam Veri Motus. Ergo & *r s*, (ſive *b f*,) ſecat *h i*, lineam eadem ratione in *s*; propter Demonſtratam modò Figurarum *t b h k p h b*, &, *s r h i g h r*, Similitudinem.

Cæterum ex laudata ſuperius Reverendiſſ. *Wardi* Methodo inveniendi primam inæqualitatem, non eſt difficile, alium adhuc modum inveſtigandi Apogéa & Excentricitates, non minùs Directum & Geometricum, & Obſervationes quotvis admittentem, producere; quem & paucis exponam. Plures

modos

modos invenient Aſtrophili in Reverendiſſ. Viri *Aſtronomia Geometrica*, ad quam eos remitto. Interim

Fig. 120. Sint *l* & *d*, duo Foci Ellipſeos; *t* & *u*, duo Puncta Veri Motus Planetæ; Arcus Ellipſeos *t u*, ex *l* ſpectatur ſub Angulo *t l u*, & ex *d*, ſub Angulo *t d u*; item Diſtantia Focorum *l d*, ex *t* ſpectatur ſub Angulo *d t l*, & ex *u*, ſub Angulo *d u l*: Aio differentiam Angulorum *t l u*, *t d u*, æqualem eſſe differentiæ Angulorum *d t l*, & *d u l*.

Cum enim Trianguli *l u x*, tres Anguli ſimul ſumpti æquales ſint Trianguli *d t x*, tribus Angulis ſimul ſumptis; ſi auferantur utrinque æquales *l x u*, & *d x t*, reliquorum duorum ſumma, *u l x* + *l u x*, erit = ſummæ reliquorum *t d x* + *d t x*, & ab his æqualibus Summis ſi auferantur inæquales, *v. g.* *u l x*, ex priori, & *t d x*, ex poſteriori; reliquorum *l u x*, & *d t x*, differentia = eſt differentiæ ablatorum *u l x*, & *t d x*; quod erat propoſitum.

Centro *l*, Intervallo Axis Tranſverſi *m n*, deſcribatur Circulus *a b c*, cujus Arcus *a b*, rurſus ex *l* ſpectatur ſub Angulo *a l b*, & ex *d*, ſub Angulo *a d b*; item diſtantia focorum *l d*, ex *a* ſpectatur ſub Angulo *l a d*, & ex *b*, ſub Angulo *l b d*. Ergo rurſus differentia Angulorum *a l b*, & *a d b*, = eſt differentiæ Angulorum *l a d*, & *l b d*. Sed per *Coroll.* 1. Angulus *l a d*, Semis eſt Anguli *l u d*, & Angulus *l b d*, Semis Anguli *l t d*. Ergo horum Angulorum *l a d*, & *l b d*, differentia = eſt Semi-differentiæ Angulorum *l u d*, & *l t d*; Ergo & Angulorum *a l b*, & *a d b*, differentia = eſt Semi-differentiæ Angulorum *u l t*, & *u d t*, quorum prior eſt Intervallum Apparens duarum Obſervationum, poſterior autem, Intervallum Motus Medii. Datâ igitur horum intervallorum differentiâ, datur quoque hujus (differentiæ) Semis, nimirum differentia Angulorum *a l b*, & *a d b*. Sed *a l b*, idem eſt cum *u l t*, dato; Ergo datur quoque *a b d*, Angulus, ſub quo Peripheria *a b*, ſpectatur ex *d*.

Simili modo oſtendetur, differentiam Angulorum *t l y*, & *t d y*, æqualem eſſe Summæ Angulorum *l t d*, & *l y d*; nec non differentiam Angulorum *b l c*, & *b d c*, = eſſe Summæ Angulorum *l b d*, & *l c d*. Cumque *l b d*, ſemis ſit ipſius *l t d*, & *l c d*, ſemis ipſius *l y d*; erit ſanè Summa ipſorum *l b d*, & *l c d*, = ſemi-ſummæ Angulorum *l t d*, & *l y d*; hoc eſt, differentia Angulorum *b l c*, & *b d c*, = erit ſemi-differentiæ Angulorum *t l y*, & *t d y*, quorum prior eſt Intervallum Apparens duarum Obſervationum, poſterior autem, Intervallum Motus Medii. Quare, data horum Intervallorum differentia, datur quoque hujus Semis, nimirum Differentia Angulorum *b l c*, & *b d c*. Sed *b l c*, idem eſt cum *t l y*, dato; Ergo datur quoque *b d c*, Angulus, ſub quo Peripheria *b c*, ſpectatur ex *d*.

Unde liquet, ex datis Intervallis Obſervationum Mediis & Apparentibus dari Angulos, ſub quibus ex *d* ſpectantur Circuli *a b c*, Peripheriæ quotvis, interceptæ à lineis Veri Motus. Ergo per *Herigoni Theor. Plan. l.* 1. *c.* 3. *Prop.* 12. *Schol.* 1. totidem Circuli Segmenta deſcribi poſſunt, capacia Angulorum ſub quibus iſti Arcus conſpiciuntur ex *d*, quæ ſegmenta omnia ſe mutuò interſecabunt in *d*. Poſſunt igitur & hac Methodo inveniri Apogéa

&

& Excentricitates Planetarum, Delineatione Geometricâ, adhibitis Obſervationibus quotvis; nec difficilius eſt, Circulos ducere, quàm Lineas Rectas.

Sed ut demus id, quod verum eſt, Clariſſimi *Caſſini* Delineationem Geometricam non nihil expeditiorem eſſe; verendum eſt interim ne, ſi ἀκρίβειαν Aſtronomis expetitam ſectemur, Diagrammata requirat Enormis Magnitudinis, adeòque operoſior evadat, quàm ipſe Calculus. Ad hunc autem accedentes, utramque Methodum æquipollere deprehendemus. Ne quis vero Apogei & Excentricitatis utraque Methodo inventæ a vero diſcrepantiam cenſeat Errori Calculi imputandam: reſtat ut Hypotheſin Excutiamus.

Et Ellipticæ quidem Orbitæ Inventio, ſine controverſia *Keplero* debetur; ſed quibus Accelerationis & Retardationis gradibus incedant Planetæ, definire, non minus pertinet ad integrandam Hypotheſin, quam ipſius Orbitæ determinatio. Quanquam autem ex Cl. *Caſſini* (vel Interpretis ejus) ſermone id nuſquam apparet; attamen ex Conſtructione Problematis, & ejus Analyſi, manifeſtum eſt, eum ſupponere, Planetam ex Foco ſuperiori videri prorſus æquabili Motu incedere. Fuit ſane, cum idem exiſtimaret *Keplerus*, quod ejus Scripta evolventibus liquere poteſt. Sed cum id Obſervationibus nequaquam Congruere animadverteret, mutavit ſententiam, & Lineam Veri Motus Planetæ æqualibus temporibus æquales Areas Ellipticas verrere profeſſus eſt: Punctum autem, ex quo Planeta exactè æquabili Motu procedere videtur, nullum omnino extare in hoc Univerſo, niſi id libratile ſtatuere libeat. Nulli interim Puncto propiùs æquabilem videri inceſſum Planetæ, quam ipſi Foco ſuperiori Ellipſeos. Neque inventus fuit hactenus, qui Areas *Kepleri* Phænomenis ſatisfacere poſſe negaret; ſed, cum eas Calculo directo exhibere nec ipſe nec poſt eum quiſquam potuerit, cauſati ſunt nonnulli, *Keplerum*, nimis indulgentem cauſis Phyſicis, à Geometria diverſum abiiſſe; quaſi cauſæ Phyſicæ repugnent Geometriæ, aut minus Geometricum ſit Problema; quod, nullâ injectâ Phyſicarum Cauſarum mentione, ſic proponitur: *Data Area Trilinei, inter Lineas Apſidum, & Veri Motus, nec non Peripheriam Ellipticam intercepti, invenire Angulum ad Solem.* Habent igitur à *Keplero* reſponſum, qui illi ἀγεωμετρίαν objiciunt; nim. *Eant ipſi & Schema ſolvant.*

Quamvis autem Religio fuerit *Keplero*, ab Hypotheſi, quam naturalem eſſe, plane perſuaſum habeat, recedere; quidni liberum foret aliis periculum facere, num via quævis alia detur, inæqualitatem Planetarum Primam directo Calculo inveſtigandi? Ideoque Vir Cl. *Iſm. Bullialdus* aggreſſus eſt Ratiocinio Geometrico indagare, quâ ſemitâ, & quibus intenſionis ac remiſſionis gradibus conveniret Planetas ferri, ut ab æquabili inceſſûs Norma, Aſtronomis ante *Keplerum* aſſumptâ, ad eam quam ſpectamus Inæqualitatem perduceremur. Perennant Illuſt. Viri monumenta, unde omnem hujus Inventi rationem haurire licet Aſtrophilis. Amplexus eandem Reverendiſſ. *Seth. Wardus*, primum oſtendit, paria facere cum linea æquabilis Motus circa alterum Ellipſeos Umbilicum gyrata; deinde & Calculi Directi Methodo ornavit, ea, quam paulò antè recitavimus: Ita ut nihil amplius deſiderari poſſet, quam ut Urania felicibus Cœptis annueret. Cujus quidem Nomine ſuſcipere auſus fuit Illuſtriſſ. *Comes Paganus*, edito biennio poſt ejuſdem fere tenoris ſcripto,


adeo

adeo veram esse Hypothesin, ut deprehensam circa Octantes discrepantiam, Astronomorum Inscitiæ tributam mallet. At Cl. *Bullialdus*, audiendum potius ipsam Astronomiam ratus, Observatorum ore loquentem, secundis Curis, adhibita prioribus Inventis Limitatione quadam, discrepantiam illam exterminavit. Unde porrò intelligitur, Hypothesin illam, cui Cl. *Cassinus* Investigationem Apogeorum & Excentricitatum superstruit, tantundem ferè deficere à vero, quantum Cl. *Bullialdi* Limitatio pollet, atque ab illo Defectu pullulare Calculi à Coelo dissensum.

By Mr. Edm. Halley. n. 128. p. 683. 2. Motus Terræ Annuus per Eclipticam, Opticam Inæqualitatem inducit Motibus cæterorum Planetarum; Astronomis *Copernicanis* nomine Parallaxeos Orbis notissimam; quam quidem inæqualitatem, ex Observationibus non multâ operâ datam, Methodi sequentis basin firmissimam constituo; ubi præter Observata nihil aliud supponitur, quàm quòd Orbes Planetarum sint Ellipses, quodque Sol in Foco, omnium Orbibus communi, sit constitutus, & denique, quòd Tempora Periodica singulorum ita innotescant, ut non sentiatur error aliquis, saltem in duabus vel tribus Revolutionibus: His concessis, Motus Terræ, pro cæteris Planetis necessariò requisitus, primò aggrediendus est.

Fig. 121. Sit S, Sol; ABCDE, Orbis Terræ; P, Planeta Mars, (qui in hanc rem plurimus de causis longè præferendus est); & primò observetur Verum Tempus & Locus, quo Mars opponitur Soli; tunc enim Sol & Terra coincidunt in Lineam Rectam cum Marte; vel, (quod ferè semper accidit) si habuerit Latitudinem, cum Puncto, ubi Perpendicularis à Marte demissa in planum Eclipticæ incidit. Sic in Schemate, S, A, & P, sunt in Linea Recta; deinde post 687 dies, Mars revertitur ad idem punctum P, ubi in priori Observatione Soli opponebatur; Terra verò, cum non revertatur ad A, nisi post 730½ dies, in B, Solem respicit in Linea SB, Martem vero in Linea BP, & Observatis Longitudinibus Solis & Martis, omnes Anguli Trianguli, PBS, dantur, & suppositâ PS 100000, in iisdem partibus invenitur Longitudo Lineæ SB; pari ratione post alteram Martis Periodum, Terra existente in C, invenitur Linea SC, nec absimiliter Lineæ SD, SE, SF; differentiæque Observatorum Locorum Solis, sunt Anguli ad Solem ASB, BSC, CSD, DSE: Sic tandem ventum est ad hoc Problema Geometricum; *Datis tribus lineis, in uno Ellipseos Foco coeuntibus, tam Longitudine quàm Positione, invenire Longitudinem Transversæ Diametri, cum distantia Focorum:* Cujus Resolutio extenditur etiam ad reliquos Planetas, si, post Theoriam Motus Terræ cognitam, scrutemur (secundum Methodum propositam à Reverendiss. Episcopo *Sarisburiensi* in *Astronomia* ejus *Geometrica*, *Lib.* 2. *Part.* 2. *Cap.* 5.) tres Distantias Planetæ alicujus à Sole in Positionibus suis. Quoniam verò Rev. *Episcopus* supponit Planetam ita ferri in Orbe suo, ut æqualibus temporibus æquales Angulos ad Focum alterum Ellipseos absolvat, & ei Calculum suum superstruit, non incongruum videtur, ostendere, quomodo id ipsum fieri posset absque ista suppositione, quam Observatio nos rejiciendum monet.

Sit S,

Sit S, Sol; ALBK, Orbis Terræ; P, Planeta, vel Punctum in Plano Eclipticæ, ubi Perpendicularis, à Planeta dimissa, incidit; AB, Linea Apsidum Orbis Terræ: Observentur primò Planetæ, in P, Longitudo & Latitudo, simulque Solis Longitudo à Terra in K; & post Periodum ejusdem Planetæ, Terra existente in L, Observentur denuo Positiones Planetæ Solisque, ut prius: jam ex Observatis Longitudinibus Solis & Aphelii Terræ, Anguli ASK, ASL, dantur, & consequenter Latera SK, SL. Jam in Triangulo KSL, dantur Latera KS, LS, & Angulus KSL, quæruntur Latus KL, & Anguli SKL, SLK: Deinde in Triangulo KLP, dantur KL, KPL differentia Observatarum Longitudinum Planetæ, & PKL Differentia Angulorum SKL, ultimo inventi, & SKP, Elongationis Planetæ à Sole in prima Observatione, quæritur LP: Tum in Triangulo LSP, Latera LS, LP, & Angulus PLS, Elongatio Planetæ à Sole in secunda Observatione, dantur; Latus SP, & Angulus LSP, requiruntur, quibus Inventis, ut SP ad LP, ita Tangens Latitudinis Observatæ ex L, ad Tangentem Inclinationis sive Latitudinis ad Solem; & ut Co-sinus Inclinationis ad Radium, ita SP, Curtata Distantia, ad Veram Distantiam Planetæ à Sole: Sic tandem invenimus Positionem & Longitudinem desideratam. Jam restat ut ostendam, quomodo ex Datis tribus Distantiis à Sole cum Angulis interceptis invenienda sit Media Distantia cum Excentricitate Ellipseos. Fig. 122.

Sit S, Sol, & SA, SB, SC, tres Distantiæ in debita Positione, ductisque AB, BC, sit AB, Distantia Focorum Hyperbolæ, & SA — SB = EH, Transversa Diameter; quibus positis, describatur Linea ista Hyperbolica, cujus Focus Interior est punctum A, extremitas Lineæ Longioris SA: pari modo sint B, C, Foci Alterius Hyperbolæ, cujus Diameter SB — SC = KL; ex quibus describatur Linea Hyperbolica Focum habens Interiorem in puncto B: Dico has duas Hyperbolas sic descriptas sese intersecare in puncto F, qui est alter Ellipseos quæsitæ Focus, ductaque Lineâ FA, FB, vel FC, SA + FA, SB + FB, vel SC + FC, æquabitur Transversæ Diametro, & SF est Distantia Focorum: quibus positis descriptio Ellipseos facillima est. Cum verò hujus Constructionis ratio non omnibus ita facile percipiatur, non abs re erit, illustrationem ejus aliquam afferre; ideo dico, quod ex notissima Ellipseos proprietate SB + FB = SA + FA, & transpositis Æquationis partibus FB — FA = SA — SB, ita ut etiam si FB & FA nos lateant, earum tamen differentia æqualis sit SA — SB, hoc est, EH, cumque sit ex natura Hyperbolæ, ut habeat quasvis duas lineas à suis Focis ad quodvis punctum in sua Curva constanter Differentes quantitate Transversæ Diametri; constat punctum F esse alicubi in Curva Hyperbolæ, cujus Diameter Transversa æquatur SA — SB, & Foci, A, B: Pari modo Demonstrari potest punctum F esse in Hyperbola cujus Diameter est SB — SC, & Foci B, C. Ergo necesse est, ut sit in Intersectione duarum istarum Hyperbolarum, quæ, cum sese intersecent in unico solum puncto, clare ostendunt ubi sit Focus alter Ellipseos quæsitæ. Fig. 123.

Jam ut id ipſum Analyticè expediatur, puta Factum, ſitque FB $= a$, SA — SB $=$ FB — FA $= b$, AB $= c$, SB — SC $=$ FC — FB $= d$, BC $= f$, ſitque Sinus Anguli ABC $=$ S, Co-ſinus ejuſdem $= s$.

Tum ut $c$ ad $b$, ita $2a - b$ ad $\frac{2ab - bb}{c}$; & $\frac{2ab - bb + cc}{2c}$ $=$ BD, per 36. 3. *Eucl.* & ut $f$, ad $d$, ita $2a + d$, ad $\frac{2ad + dd}{f}$ & $\frac{ff - 2ad - dd}{2f} =$ BG, per Eandem; & ut minuatur labor Calculi, ſit $\frac{cc - bb}{2c} = g$, & $\frac{b}{c} = h$, ſimiliter ſit $\frac{ff - dd}{2f} = k$, & $\frac{d}{f} = l$, tunc BD $= g + ha$, & BG $= k - la$; & quoniam in omni Triangulo

{ Obtuſangulo / Acutangulo } Quadratum baſis æquatur { Summæ / Differentiæ }

Quadratorum Laterum, & Dupli Rectanguli Laterum in Co-ſinum Anguli comprehenſi ducti, erit $gg + 2gha + hhaa + kk - 2kla + llaa + 2gks - 2glsa + 2khsa - 2hlsaa$ æqualis quadrato DG: Sed DG æqualis eſt Sinui Anguli DFG, vel DBG, in $a$, id eſt FB, ducto, (eſt enim Quadrilaterum FBDG, Circulo, cujus Diameter eſt FB, Inſcriptum;) ideo SS $aa = gg + 2gha + hhaa + kk - 2kla + llaa + 2gks - 2glsa + 2khsa - 2hlsaa$; quæ æquatio facilè reſolvitur, cùm non excedat Quadraticam Affectam, ſemperque componitur ex iſtis Quadratis & Rectangulis; ſigna tamen + & — ob diverſam trium linearum conſtitutionem multa cautione ſunt Rectangulis adhibenda.

*The Obliquity of the Ecliptick, from the Obſervations of the Ancients; by Dr. Ed. Bernard. n. 163. p. 721.*

XVII. 1. Obliquitatem Zodiaci reperit *Eratoſthenes*, ante Natum *Chriſtum*, Ann. 230. Grad. 23. atque inſuper 51′. 19″. 31‴. 5⁗.

Diſtantia enim Tropicorum ipſi fuit $\frac{11}{83}$ Circuli Meridiani ſive 47°. $\frac{12}{83}$, *Ptol.* Μεγ. Συντ. p. 18. 21. Quare Λόξωσις *Eratoſthenica* minor erat *Ptolemaico* tantum $\frac{3}{83}$ unius Minuti Secundi, re ſane contemnenda.

*Eratoſthenes* apud *Cleomedem*, *Ricciolo* eruente, (ſupra Grad. 23.) 46′. 00″.

*Eratoſthenes* à *Ricciolo* quaſi correctus, 31′. 5″.

*Hipparchus*, (ante *Chriſt.* 140.) *Eratoſthenicam* retinuit. *Ptol.* Συντ. Μεγ. p. 18. & p. 60.

*Theonis*, ὡς ἀκριβῶς εἰλημμένην. 51′. 19″. 31‴. 5⁗.

Tabulæ tamen *Chovareſmicæ*, conditæ poſt *Chriſt.* 830. exhibent Canonicam Λόξωσιν *Alexandrinorum*, juxta MS. Lat. D. *Hattoni*, 51′.

*Pytheas Maſſilienſis*, ante *Chriſt.* 324. *Ricciolo*, 52′. 41″.

*Ariſtarchus*, ante *Chriſt.* 280. Illuſtri *Savilio* ſupputante, 51′. 20″.

*Ariſtarchus*,

*Ariſtarchus*, ex ratiocinio *Riccioli*, 30′. 00″.

*Strabo* Geographus, p. 93. poſt *Chriſt.* 30. $\frac{1}{60}$ Circuli, ſive præter Gradus 23. adhuc unius, ſive 60′.

Nec aliter *Geminus* (tempore *Chriſti*) Cap. IV. Element. Aſtron. Et *Tatius* c. 26. atque *Proclus* de Sphæra. Indique ſive Aſtrologi, apud *Noddamum* Arabem, *Abrahamum Abeneſdram*, &c.

*Noddamus* Aſtronomus, qui floruit circa Ann. Dom. 1200. notat Λόξωσιν neque obſervatam unquam Majorem Gr. 24. neque Minorem 23°. 33′. continuo tamen decreviſſe.

*Cl. Ptolemæus*, poſt *Chriſt.* 140. ſæpius expertus, & Crico ſuo & Plinthide, ſemper reperit proximè eandem cum *Eratoſthenica*, 51′. 20″.

Diſtantia enim Tropicorum verſabatur inter 47 $\frac{2}{3}$. & 47 $\frac{3}{4}$. Sed elegit pro *Selidio* ſuo 47. 42′. 40″. Σωτ. Μεγ. p. 18, 20, 21. & p. 27. capit pene Medium, μοιρ. κγ. να′. κ″. ἔγγιστα. Nec aliter in Hypotheſibus Planetarum. *Theo* vero in Canonibus προχείροις facilitatis cauſâ præteriit Minuta Secunda. Fallitur autem *Ricciolus*, dum ex Climate *Rhodi* colligit Λοξώσεως modum pro *Ptolomæo* 23°. 30′.

*Pappus Alexandrinus* (poſt *Chriſt.* 390.) l. 6. Theor. 35′. *Ricciolo* 30′. *Pappus*, *Fr. Commandino* colligente 50′. 00″. 00‴.

*Theo*, (poſt *Chriſt.* 370.) p. 88. accuratius 51′. 20″. 00‴.

Alibi numero rotundo, ut p. 57. & paſſim in Canonibus ſuis προχείροις nondum vulgatis 51′. 00″. 00‴

*Almamon* Princeps, *Ann. Chriſti* 825. *Hegiræ* 210. 23°. 35′. *Grav.* p. 44. ex *Ebn-Shatir* Damaſceno MS. *Seld.* adſiſtentibus ei plurimis Aſtronomis. Ita enim refert *Abeneſdras* MS. Lat. in Archivis *Digbeanis.* Inſuper *Aſtronomus Incertus* in Arch. *Seld.* affirmat *Jahia Ebn Abimanſur* cum multis aliis Philoſophis, tempore *Almamonis*, τὴν Λόξωσιν Experimento

بالقياس الممتحن deprehendiſſe, 23°. 35′.

Idem tradit de obſervatis *Almamonis* Doctiſſimus *Al Noddam* in Commentariis ſuis ad Aſtronomica *Hoſein Niſaburienſis.* Imo addit ille eodem Ævo ſæpius obſervaſſe *Beni Muſa* modum eundem 23°. 35′. *Bagdadi* in Campis, MS. Arab. *Coll. S. Joan. Oxon.* Hunc etiam placuiſſe pleriſque ſequentium Aſtronomorum. Sanè in eo quieſcit *Alferganus* Aſtron. ſuæ c. 5.

*Mohammed Ebn Gaber Al Batanius*, (*Al Bategnius*,) *Raccæ*; *Ricciolo*, A. D. 880. Ill. *Savilio*, 890. *Gravio*, p. 44. 882. *Hegiræ*, 269. Obiit ille *Hegiræ*, 317. A. D. 929. *Abolfaragi* Hiſt. p. 191. 35′. 00″.

*Al Batanius* hac in re ſuas τηρήσεις præferre non dubitat *Ptolemæi* dictis, c. 4. atque ſe adjutum longiſſima *Albidada*, ſeu Regula Parallactica ad formam, *Ptolemaicarum* cum cura & aſſiduitate reperiſſe apud *Raccam* Tropicorum Diſtantiam, 47°. 10′. (hoc eſt 59°. 36′. minus 12°. 26′.) atque adeo Latitudinem *Raccæ* 35°. quam tamen *Ulugbegus* ſtatuit 36°. 10′. *Schickardus* apud *Curtium*, (p. 33.) & *Ricciolus*, 36°.

*Thales*

*Thabet Ebn Corra*, (*Ricciolo*, A. D. 1210. rectius 901. *Hegiræ* 289.) reperit Λόξωσιν, 33′. 30″.

*Abul Hosein Ebn Suphi*, 35′. 00″.

*Abul Waffi Albuziani*, & *Abn Hamed Saganiensis*, Vir Ingeniosissimus, (A. D. 987. *Heg.* 377.) *Bagdadi* repererunt Λόξωσιν tantum non 35′.

Ita & auctor زيج مفصل Persa in Arch. *Seld.* 35′.

Tabulæ itidem Persicæ *Chrysococcæ*, 35′.

*Al Batrunius Abul Rihan*, (A. D. 995. *Hegiræ* 385. *Abolfaragius* hunc ponit ad *Hegiræ* 463. seu A. D. 1070.) usus Quadrante, cui Radius xv. cubitorum *Grav.* p. 44, ex Cod. Arab. *Birunii.* 35′.

Verum *Abu Jaafer Alchazan*, cum Socio suo *Abufadlo Harwanensi* apud *Edessam*, & istius Ævi alii (A. D. 970.) observarunt τὴν Λόξωσιν ad 23°. 35′. plane non accessisse, sed paulo fuisse Minorem.

*Almæon* F. *Almansoris* (A. D. 1140. *Ricc.*) 33′. 30″. at ille *Clavio* & *Mæstlino* 33′.

*Ismael Abulfeda* Princeps *Hamæ*, (A. D. 1311. *Hegiræ* 711.) in tabulis suis MS. Arab. *Coll. S. Joan.* retinet forte ob *Almamonis* auctoritatem 35′. 00″.

*Prophatius Judæus* (A. D. 1300. *Ricc.* 1303. *Mæstlino* apud *Curtium*, p. 40. 230. annis post *Arzachelem*, inquit *Copernicus*) & *Ricciolo*, & MS. Coll. *Merton.* 32′. 00″.

*Abu Mahmud Al Chogandi* (A. D. 992. *Hegiræ* 382.) tempore *Fecroddaulæ*, Sextante cujus Radius erat Cubitorum XL. limbusque in minuta secunda distinctus, invenerat Λόξωσιν Minorem quam unquam captaverat aliquis Majorum suorum, nimirum 32′. 21″.

Hinc *Noddamus* Astronomus adfirmat ( MS. *Coll. Joan.* ) Solis Declinationem Maximam vix unquam Minorem fuisse repertam 23°. 33″.

*Arzachel Hispanus*, (*Gravio*, p. 44. A. D. 1689. *Hegiræ* 482. *Ricciolo* 1070. *Mæstlino* apud *Curtium*, p. 35. 1075. *Copernico*, l. 3. c. 6. Annis 190. post *Al Batanium*) proposuit Λόξωσιν 23°. 33′. 30″. Ita MS. *Coll. Mert. Oxon.* ubi dicitur differentia 17′. 30″. intercedere inter Λόξωσιν *Ptolemæi* & *Arzachelis.*

Apud *Maragam* Nobilissimus Persa *Chojah Nasiroddinus Tusensis*, A. D. 1269. *Hegiræ* 668. (at *Gravio*, p. 44. 1261. *Hegiræ* 660.) accuratissime observavit τὴν Λόξωσιν. 23°. 30′. 00″.

Hæc est minima ex Maximis Solis Declinationibus, quæ ad hunc usque diem reperta fuit, ait Doctiss. Commentator ad Astronomica *Hosein Nisaburiensis.*

*Ebn Shatir Damascenus*, MS. *Seld.* A. D. 1363. ait se emendasse Λόξωσιν, non neglecta Solis Parallaxi, quæ Horizontalis capta est, 2°. 59′. Huic Solis Max. Declin. 23°. 31′. 00″.

*Olocbegus* Princeps, A. D. 1437. *Hegiræ* 841. cum *Aly Cushgio* aliisque Astronomis, usus summa cura, & maximis Instrumentis (vide *Gravium*, p. 44.) reperit Λόξωσιν 23°. 30′. 17″. Ita MSS. *Coll. D. Joan.* & *Bibliothecæ Savilianæ*, nam MS. *Seld.* exhibet 23°. 30′. 27″.

*Rabbi*

*Rabbi Moyses Ben Maimon Judæorum* Doctissimus ait in *Jad. de Consecratione Calendarum*, c. ult. sect. 4. Maximam Zodiaci Obliquitatem fuisse, A. D. 1174. 23°. 30′. ἔγγιστα.

Scias vix dimidiam partem Astronomorum Orientalium quorum Scripta Academiæ *Oxon.* Bibliothecis servantur à me consultam fuisse. Ex hisce autem Observatis aliisque quæ mecum adhuc cis vulgus servo, unam eandemque suspicor fuisse à primordio Mundi λόξωσιν Zodiacam. Æva enim recentia, quod vides, melioribus Organis errorem excessumque veteris Astronomiæ probè correxerunt.

2. Whether the Poles and Axis of the Earth be really fixt in the Globe, or subject to be transferr'd from place to place, is an old Enquiry, though now lately revived by Mr. *Hook* in his ingenious Essays upon the great Mutations and Catastrophes which in all appearance have happened to the Earth's Surface. A necessary consequence of such a Translation of the Poles would be the Change of the Latitudes of Places, which would Encrease in those Regions towards which the Poles approach, and Decrease in those from which they Recede: and under the Meridian 90 Degrees removed from that in which the Poles shift, the Latitudes continuing the same, the Meridian Line would only alter; but no two Places considerably differing in Longitude can be supposed, wherein if there be any sensible Motion of the Poles, it shall not be perceived by the alteration of the Latitude of one or both of them.

*The Obliquity of the Ecliptick, and Elevation of the Pole continue unaltered; by* n. 190. p. 403.

The accurate M. *Wurtzelbaur*, has lately furnished us with the means of Examining this Hypothesis by Observation, having sent us the Meridian Altitudes of the Sun taken at *Nurenburg* about the two Solstices in the Year 1686. *Jun.* 10. He found the Meridian Altitude of the Sun 64°. 2′. 20″. and the next day, 64°. 2′. 25″. and on *Decemb.* 14. (three days after the Solstice, wherein the Sun was got two Minutes higher) he found the Meridian Altitude 17°. 9′. 10″. wherefore the Solstitial Altitude was 17°. 7′. 10″. These Heights were taken by an Instrument of 6 Foot Radius of Brass; and the Skill and Diligence of the Observer is not to be doubted.

To compare with these, I find among *Bernard Walther*'s Observations made in the same City of *Nurenburg* two hundred years before, *viz.* in the Year 1487. that the Meridian Altitude of the Sun in the Summer Solstice was observed by the Parallactick Instrument of *Ptolemy*, whereby the Chord of the Sun's Distance from the Zenith was observed 44890 Parts of 100000 Radius; the same being confirmed by the Concurrence of the Observations of several Years both before and after. The Arch answering to this Chord gives the Sun's Distance from the Zenith 25°. 56′. 30″. and consequently the Meridian Altitude, its Complement to a Quadrant, 64°. 3′. 30″. Again, The same Year 1487. the Chord of the Meridian Distance of the Sun from the Zenith, on the day of the Winter Solstice, was found 118790, confirmed likewise by many subsequent Observations; the Arch answering to this Chord is 72°. 52′. 40″. and its Complement 17°. 7′. 20″. the Meridian Heighth of the Sun in the Winter Solstice.

Hence

Hence it appears, That the Solsticial Heights were very nearly the same at *Nurenburg* 200 Years ago as now they are, that of the Summer Solstice being but one Minute differing, the other only 10″. both which may possibly arise from the Defects of the Instruments of these Observers, being made with Plain Sights; but what I shall necessarily conclude from hence is, That if there be such a Motion of the Poles, it is either very slow, or else nearly at Right Angles to the Meridian of *Nurenburg*; in which latter case, the Latitudes of Places about *Tunking*, *Siam*, *Malacca*, and *Java*, on the one side; and in our *American* Plantations of *New-England*, *Virginia*, *Jamaica*, *&c.* on the other, ought to change fastest; but I have never yet heard of any such thing observed by any of our Navigators; whence if there be such a Change of the Earth's Poles, it must necessarily require a long time to become sensible.

Besides, from these Observations, it appears, That the Obliquity of the Ecliptick has continued unalterated for these 200 Years last past; that is to say, that the Angle which the Earth's Axis makes with the Plain of the Ecliptick or Orb wherein she moves Annually round the Sun, has been without sensible Change in all that time; which will be very hard to conceive, if we allow a Translation of the Earth's Poles; for the direction of the Axis being perfectly at liberty, it must be purely casual, if it so hit, that after such Change, it make the same Angle with the Ecliptick as before.

A farther Argument of this Slowness of the Change of the Poles, is the Latitude of *Alexandria*, the Habitation of those famous Astronomers of Antiquity, *Eratosthenes*, *Timocharis*, *Hipparchus*, and *Ptolomy*; and for that reason it may be concluded, that this, of all the Latitudes the Ancients have left us, ought to be one of the most Correct. This by *Ptolomy* is said to be 30°. 58′. North, (which he uses in all his Computations in his *Almegist*, and seems derived from the proportion of the Gnomon to its Equinoctial Shadow, as 5 to 3.) but in his *Geography*, 31°. just. In the Year 1638. the Curious and Ingenious Mr. *Greaves*, when he went to visit the *Egyptian Pyramids*, of which he has given so good an account, did with a sufficient Instrument observe the Latitude of *Alexandria*, and found it 31°. 4′. or 6 Minutes more than it is reputed by *Ptolomy*, and before him by *Eratosthenes*; so that in about 2000 Years the Latitude of *Alexandria* has altered only a few Minutes, and so few, that the Accuracy of the Observations of the Ancients may well be questioned: but both being granted, this Motion will amount to no more than a Degree in 20000 Years.

This is said not with intent to Invalidate what Mr. *Hook* hath from so good Grounds advanced, *viz.* That the Ball of the Earth, at least the Fluids thereof, being necessarily of the Figure of a *Sphæroides Prolatus*, or flat Oval, whose shortest Diameter is the Axis, and greatest Circle the Equinoctial; if the Poles be supposed changed, the Equinoctial will be so too; and consequently the Water must rise and cover those Parts from which the Poles recede, and fall off, and leave bare those Places towards which the Poles approach. By this means it may be accounted for, how such strange Marine things are found on the Tops of Hills, and so deep under Ground; and scarce

any

any other way. But from theſe, and the like Obſervations, it will follow, That if theſe Inundations are produced by any regular Motion of the Poles, it would require a prodigious number of Ages to effect thoſe Changes we may be certain have been. Beſides, If the Acceſs and Receſs of the Sea were after ſuch a gradual manner, as when produced by ſuch an eaſie Tranſlation of the Poles, as can by Obſervation be admitted, thoſe Inundations could never be fatal to the Inhabitants, for that they would always give notice of their coming, ſo that the People might provide for their Safety. But the *Holy Scriptures*, and Pagan Tradition, do unanimouſly agree, That the laſt great *Deluge* was brought to paſs in a few days, with no previous notice, ſo that the account we have thereof could not by this *Hypotheſis* be made out, without the ſuppoſition of a great and ſudden alteration in the Poles of the Earth's Diurnal Revolution; for which, whether we ſhould have recourſe to the intelligent Powers that firſt impreſt this Whirling Motion on the Ball, or leave it to be performed Naturally, by the caſual Chock of ſome Tranſient Body, ſuch as a Comet, or the like, whereby the former Axis might be loſt, and a New Revolution produced, differing both in Time and Poſition from the Old, I ſhall not undertake to diſpute; ſuch a Suppoſition would include likewiſe a Change of the Length of the Year and Eccentricity of the Earth's Orb; for which yet we have no ſort of Authority.

3. 1. As I was wondering how an ordinary Mathematician could miſs ſo eaſie a thing as the drawing a true Meridian, ſo far as in the inſtance of the old Meridian in the Church of St. *Petronio* in *Bononia*, which is found by M. *Caſſini* to vary 8 or 9 Degrees from the true Meridian of the Place; and in that of the Meridian of *Uraniburg*, which is found by M. *Picart*, and others, to vary 18′; I hit upon this thought, that Meridians muſt needs Vary. For you know, that (taking it for granted that the Earth moves, *&c.*) beſides the Diurnal and Annual Revolutions, there muſt be alſo a third to account for that ſlow Motion of the Fixed Stars, upon the Poles of the Ecliptick, in about 25000 Years; which is Solved by the Direction of the Earth's Axis from one Point to another of the Polar Circle. And that Direction being nothing but a certain Wabble in the Earth's Motion, muſt needs make the Noon ſhade of a Perpendicular not lie always in the ſame Line.

*A Suppoſed Alteration of the Meridian Line; by* . . . . . . . n. 255. p. 285.

2. This being a New Suggeſtion deſerves to be conſider'd: For it is not probable that ſo careful a man as *Tycho*, and thoſe concerned in the Church of St. *Petronio*, ſhould be ſo much miſtaken in the Meridian Line. But if there be ought of this Nature, it muſt ariſe from a Change of the Terreſtrial Poles (here on Earth) of the Earth's Diurnal Motion; (not of their Pointing to this or that of the Fixed Stars: For if, the Poles of this Diurnal Motion remain fixed to the ſame Place on the Earth; the Meridians (which paſs through theſe Poles) muſt remain the ſame.

*Conſider'd, by* Dr. Wallis. *Ibid.* p. 286.

XVIII. I have had the good hap to meaſure the Diſtances of *Mars* from two Stars the ſame Night; whereby I find, that his Parallax was very ſmall, certainly not 30″: ſo that I believe, the Sun's Parallax is not more than 10″.

*The Parallax of the Sun; by* Mr. Flamſteed. n. 89. p. 5119. n. 96. p. 6100.

*To find the Sun's Ingress into the Tropical Signs; by Mr.* Edm. Halley. n. 215. p. 12.

XIX. It may perhaps pass for a Paradox, if I should assert, That it is an easier matter to be assured of the Moments of the Tropicks, or of the Times of the Sun's entrance into *Cancer* and *Capricorn*, than it is to observe the true Times of the Æquinoctials or Ingress into *Aries* and *Libra*. But I here design to shew a Method to find the Moment of the *Tropicks* capable of all the exactness the most Accurate can desire; and that without any consideration of the Parallax of the Sun, of the Refractions of the Air, of the greatest Obliquity of the Ecliptick, or Latitude of the place: All which are required, to ascertain the Times of the Equinoctials from Observation; and which being faultily assumed, have occasioned an Error of near three Hours in the Times of the Equinoctials deduced from the Tables of the Noble *Tycho Brahe* and *Kepler*, the Vernal being so much later, and the Autumnal so much earlier than by the *Calculus* of these famous Authors.

Now before we proceed, it will be necessary to premise the following *Lemmata*, serving to demonstrate this Method; *viz.*

1. That the Motion of the Sun in the Ecliptick, about the Time of the *Tropicks*, is so nearly Equable, that the Difference from Equality is not sensible, from 5 *gr.* before the Tropick to 5 days after, by reason of the nearness of the *Apogæon* of the Sun to the Tropick of *Cancer*.

2. That for 5 Deg. before and after the Tropicks, the Differences whereby the Sun falls short of the Tropicks, are as the *Versed Sines* of the Sun's Distance in Longitude from the Tropicks, which *Versed Sines* in Arches under 5 Degrees, are beyond the utmost Nicety of Sense, as the Squares of those Arches. From these two follow a third

3. That for 5 days before and after the Tropicks, the Declination of the Sun falls short of the utmost Tropical Declination, by Spaces which are in *Duplicate* Proportion, or as the Squares of the Times by which the Sun is wanting of, or past, the Moment of the Tropick.

Hence it is evident, That if the Shadows of the Sun, either in the Meridian, or any other Azimuth, be carefully observed about the time of the Tropicks, the Spaces whereby the Tropical Shade falls short of, or exceeds those at other times, are always proportionable to the Squares of the Intervals of Time between those Observations and the true Time of the Tropick, and consequently if the Line, on which the Limits of the Shade is taken, be made the Axis, and the correspondent Times from the Tropick *Expounded* by Lines, be erected on their respective Points in the Axis as Ordinates, the extremities of those Lines shall touch the Curve of a *Parabola*. Thus *a*, *b*, *c*, *e*, being supposed Points observed, the Lines *a* B, *b* C, *c* A, *e* F, are respectively Proportional to the Times of each Observation before or after the Tropical Moment in *Cancer*. Fig. 124.

This premised, we shall be able to bring the Problem of finding the true Time of the Tropick by three Observations, to this Geometrical one: *Having three Points in a* Parabola A, B, C, or A, F, C, *given, together with the Direction of the Axis, to find the Distance of those Points from the Axis.*

Of this there are two Cafes; the one, when the time of the fecond Obfervation B is precifely in the middle between A and C: In this Cafe, putting $t$ for the whole time between A and C, we fhall have A $c$, the Interval of the remoteft Obfervation A, from the Tropick, by the following Analogy.

As $2\,ac - bc$ : to $2\,ac - \frac{1}{2}\,bc$ :: So is $\frac{1}{2}\,t$, or A E : to A $c$, the time of the remoteft Obfervation A, from the Tropick.

But the other Cafe, when the middle Obfervation is not exactly in the middle between the other two Times, as at F, is fomething more operofe, and the whole Time from A, to C, being put $= t$, and from A, to F, $= s$, $cc = c$, and $bc = b$, the *Theorem* will ftand thus $\frac{ttc - bss}{2tc - 2bs} =$ A $c$, the Time fought.

To illuftrate this Method of Calculation, it may perhaps be requifite to give you one or two Examples.

*Ann.* 1500. *Bernard Walter* in the month of *June* at *Nuremberg*, obferved the Chord of the Diftance of the Sun from the Zenith, by a large *Parallactick* Inftrument of *Ptolemy*, as follows:

| | | | | | |
|---|---|---|---|---|---|
| *June* 2. | 45467. | | | *June* 8. | 44975. |
| *June* 9. | 44934. | } | and { | *June* 12. | 44883. |
| *June* 16. | 44990. | | | *June* 16. | 44990. |

In both which Cafes, the Middle Time is exactly in the middle between the Extreams, and therefore in the former three, $ac = 533$, $bc = 477$, and $t$, the Time between being 14 days, by the firft Rule, the Time of the Tropick will be found by this Proportion; as 589 : to $827\frac{1}{2}$ :: So $\frac{1}{2}\,t$, or 7 days : to 9 days, 20h. 2′. Whence the Tropick, *Ann.* 1500. is concluded to have fallen *June* 11. 20h. 2′. In the latter three, $ac$ is $= 107$, and $bc = 15$, and the whole Interval of Time is 8 days $=$ to $t$; whence, as 199 : to $206\frac{1}{2}$ :: fo is 4 days to 4d. 3h. 37′; which taken from the 16th day at Noon, leaves 11d. 20h. 23′. for the Time of the Tropick, agreeing with the former to the third part of an Hour.

Again, *Ann.* 1636. *Gaffendus* at *Marfeilles*, obferved the Summer Solftice by a Gnomon of 55 Foot high, in order to determine the Proportion of the Gnomon to the Solftitial Shade, and he hath left us thefe Obfervations, which may ferve as an Example for the Second Rule.

| | | | |
|---|---|---|---|
| *June* 19 | *St. N.* fhadow | 31766 | Parts, whereof the Gnomon was 89428. |
| 20. | | 31753 | |
| 21. | | 31751 | |
| 22. | | 31759 | |



Thefe

These being divided into two Setts of three Obſervations, each; *viz.* The 19th, 20th, and 22d, and the 19th, 21th, and 22d, we ſhall have in the firſt three, $c = 13$, and $b = 7$, $t = 3$ days, $s = 1$; and in the ſecond, $c = 15$, and $b = 7$, $t = 3$, and $s = 2$. Whence according to the Rule, the 19th day at Noon the Sun wanted of the Tropick a Time Proportionate to one day, as $ttc - ssb$ : to $2tc - 2bs$, that is, as 110 : to 64 in the firſt Sett, or 107 : to 62 in the ſecond Sett; that is, $1^d. 17^h. 15'$. in the firſt, or $1^d. 17^h. 25'$. in the ſecond Sett: So that we may conclude the moment of the Tropick to have been, *June* $10^d. 17^h. 20'$. in the Meridian of *Marſeilles*.

Now that theſe two Tropical Times thus obtained, will be found to confirm each others Exactneſs from their near Agreement, appears by the Interval of Time between them, *viz.* $1^d. 2^h. 30'$. leſs than 136 *Julian* Years, whereof $1^d. 1^h. 8'$. ariſes from the Defect of the Length of the Tropical Year from the *Julian*, and the reſt from the Progreſſion of the Sun's *Apogæon* in that time; ſo that no two Obſervations made by the ſame Obſerver in the ſame place, can better anſwer each other, and that without any the leaſt Artifice or Force in the management of them.

What were the Methods uſed by the Ancients to conclude the Hour of the Tropicks, *Ptolemy* has no where delivered; but it were to have been wiſhed, that they had been aware of this, that ſo we might have been more certain of the Moments of the Tropicks we have received from them, which would have been of ſingular uſe to determine the Queſtion, Whether the Sun's *Apogæon* be fixt in the Starry Heaven; or if it move, What is the true Motion thereof? It is certain, That if we take the Account of *Ptolemy*, the Tropick ſaid to be obſerved by *Euctemon* and *Meton*, *Junii* 27. *Manè*, *Ann.* 432. *Ante Chriſtum*, can no ways be reconciled, without ſuppoſing the Obſervation made the next day, or *June* 28. in the Morning. And *Ptolemy*'s own Tropick, Obſerved in the third Year of *Antoninus*, *Ann. Chriſt.* 140. was certainly on the 23d, and not the 24th day of *June*, as will appear to thoſe that ſhall duly conſider and compare them with the Length of the Year deduced from the diligent and concordant Obſervations of thoſe two great Aſtronomical Genii, *Hipparchus* and *Albatani*; eſtabliſhed and confirmed by the Concurrence of all the Modern Accuracy. For theſe Obſervations give the Length of the Tropical Year, ſuch as to Anticipate the *Julian* Account only one day in 300 Years; but we are now ſecure, that the ſaid Period of the Sun's Revolution does Anticipate very nearly 3 days in 400 Years; ſo that the Tables of *Ptolemy* founded on that Suppoſition, do Err about a whole Day in the Sun's Place, for every 240 years. Which Principal Errour, in ſo Fundamental a Point, does Vitiate the whole Superſtructure of the *Almageſt*, and ſerves to *Convict its Author of want of Diligence, or Fidelity, or both.*

But to return to our Method, the great Advantage we have hereby, is, That any very high Building ſerves for an Inſtrument, or the top of any high Tower or Steeple, or even any high Wall whatſoever, that may be ſuf-

ficient

ficient to intercept the Sun, and caſt a true Shade, and make the Spaces large and fair, though the Heighth and Diſtance of the Building, and Poſition of the Plain, upon which you receive the Shade, and of the Line on which you meaſure the Spaces, be not exactly known. But it is convenient that the Plain on which you take the Shade be not far from Perpendicular to the Sun, at leaſt not very Oblique, and that the Wall which caſts the Shade, be ſtraight and ſmooth at the top, and its Direction nearly Eaſt and Weſt. The principal Objection is, That the *Penumbra*, or Partile Shade of the Sun is in its Extreams very difficult to diſtinguiſh from the true Shade, which will render this Obſervation hard to determine nicely. But if the Sun be tranſmitted through a Teleſcope, after the manner uſed to take his *Species* in a *Solar Eclipſe*, and the upper half of the Object Glaſs be cut off by a Paper paſted thereon, and the exact upper Limb of the Sun be ſeen juſt emerging out of, or rather continging the Species of the Wall, (the Poſition of the Teleſcope being regulated by a fine Hair extended in the Focus of the Eye Glaſs) I am aſſured, that the limit of the Shade may be obtained to the utmoſt Exactneſs. I ſhall only further advertiſe, That the Winter Tropick by this Method may be more certainly obtained than the Summer's, by reaſon that the ſame Gnomon does afford a much larger Radius for this manner of Obſervation.

XX. I have found it neceſſary, to make new Solar Numbers, becauſe in my old, I had neglected to apply Refractions in all the Altitudes above 30 Degrees; wherein yet Reaſon and ſome little Experience hath ſhewed me, they are not Inſenſible. I found S. *Caſſini*'s Obſervations, which I took from *Ricciolus* his *Aſtonomia Reformata*, much more Accurate than *Tycho*'s, and therefore ſought out Numbers that might anſwer them. The *Apogæum* I found it neceſſary to promote 44 Minutes; ſo that, *Anno Ineunte* 1655. it might be in ♋. 7°. 30′. 00″. and to make the greateſt Equation only 1°. 54′. 13″. whereby I found, the *Phænomena* would be anſwered much more accurately than I expected, and as near, all things conſidered, as I could deſire.

*The Solar Numbers corrected; by Mr.* J. Flamſteed. n. 110. p. 220.

But ſtill I was uncertain, Whether the Refractions in the ſaid *Caſſini*'s Tables were juſt meaſures or not, and I had no Conveniencies for making Trial. At laſt, I thought on this Expedient, which fully ſatisfied me; *viz.*

I conſidered, That if ſome of thoſe Obſervations of the Diſtances of ♀ from the ☉ by Day, and from the Stars in the Night preceding or following, were skillfully Examined, they might ſhew me the true Quantity of the Equations of the Sun's Orb, or rather the Difference of his Mean and Equal Motion. I turned over his *Progymnaſmata*, and pitched on two. The firſt made, *Ann.* 1585. *March* 5. 4h. 42′. and 7h. 12′. *Poſt Meridiem*: whereby I found, the ☉ at 4h. 42′. was 94°. 47′. in Antecedence of the *Lucida Calcis* ♊. The ſecond made, *Ann.* 1585. *Septemb.* 15. 5h. 15′. and 6h. 55′. *Manè.* Wherefrom, (applying and conſidering the Refractions

fractions in both) I found the Sun at 6h. 55′. to be 74°. 30′. in Consequence of the *Lower Head* of ♊. The Difference of Longitude betwixt these two Stars is 17°. 59′: And therefore now the Sun in Consequence of the *Lucida Calcis* ♊. 92°. 29′. So that the Sun's Apparent Motion betwixt the Year 1582. *Mar.* 5. 4h. 42′. and the Year 1585. *Septemb.* 15. 6h. 55′. *Manè*, (besides the whole Revolutions) was 187°. 16′. but the Mean Motion is 191°. 2′. greater than the Apparent by 3°. 46′. which parted in Proportion to the Equation of the Earth's Motion, collected for those times from my New Tables, gives the greatest Equation of the Orb, 1°. 54′. 15″. consenting to my wonder (without any wresting of the Observations) with that, which I deduced from *Cassini*'s Correct Meridional Altitudes.

The Sun's Motion by the Tables which I now use, grounded on this Equation, is less than *Tycho*'s by no less than 9′. That great Equation made him commit no small Errors, and put him upon strange Shifts to hide and salve them. So that all his Observations of the Planets in their Oppositions to the Sun, are to be Corrected, before we may attempt to represent them by Numbers: For his Errours in the Sun's Place made him Err sometimes 5 or 6 hours in the time of the Opposition; which must be Reformed.

*The Equality of Natural Days; by . . . . . . . Professor of Mathematicks at* Seville. n. 118. p. 426.

XXI. Non parvum adhuc est Dissidium inter Astronomos, quanta sit in æquando Tempore Prosthaphæresis, ita ut *Longomontanus* fateatur, nullam in Astronomico Pulvere majorem Difficultatem se invenisse; quod cum notarem, animadverti in quibusdam Cœli Observationibus à me factis, quid ex illis eveniret; &, cum mihi esset Horologium Rotatile Pendulum, admodum exactum, Lineâ Meridianâ artificialiter Constitutâ, examinabam Solis in Meridianum Ingressum, singulis Diebus; cum quo ad amussim Horologium meum congruebat, & si discrepabat aliquando, rarissimè duobus minutis discriminabat, quod, cum opus erat, emendabam. Quare per triennium continuando, & quotidie Solem in Meridiano observando, cum licebat (quod in hac Regionis Parte sæpè sæpius fit) inveni tandem nullam Diem Naturalem Longiorem Revolutionem, in Uno vel Alio Anni Tempore, aliâ Die habuisse, unde intrepidè dico omnes Dies Naturales æquales esse, & si adhuc aliqua Differentiola intercedit, non esse Sensibilem. Hæc volui notum facere, ut Astronomos hoc Scrupulo liberarem, quod tam multos torsit & indiès torquet, quanquam *Tychonica* Æquatio propter Eclipticæ Obliquitatem non sit rejicienda.

*Refuted; by Mr.* Flamsteed. Ibid. *p.* 430.

2. Dies quomodo Æquales esse possint, & tamen Æquatio *Tychonica* admitti, vix me capere fateor. Ob inæquales etenim æqualium Eclipticæ partium Rectas Ascensiones, Dies unus Æquinoctialis Tropico uno Brevior erit Scrupulis Horæ secundis 40; & Dies 14 Tropici, totidem Æquinoctialibus Longiores sunt sextâ Horæ parte, seu Scrupulis primis 10. Hanc autem Differentiam Majorem credo, quam ut eam in Observationibus suis non perciperet *Professor Hispalensis*, proindeque ipsum in examinandis iis *Tychonicam* Temporis Æquationem adhibuisse Autumem.

Sed

Sed & ponamus, Æquales esse Primi Mobilis Revolutiones (quod nulli, qui *Ptolmaicam Hypothesin* admittant, unquam iverunt inficias) necessario consequitur, nec Æquationem Temporis, ab inæquali Solis in Orbita sua Incessu enatam, rejiciendam esse: Etenim cum Apogeus quotidiè promoveatur tantum 57′. 10″. Perigeus vero 61′. 15″; Apogeus equidem citius 16″, (seu Tempore absumpto dum Primum Mobile revolvit 4′. 5″.) à Meridie in Meridianum Diei sequentis recurret, quam Perigeus: Attamen quandoquidem progenita ex hac causa Æquatio tardius admittit Diurnum Incrementum, Scil. 8″. quotidiè, ad summum cum velocissima, & vix Diebus 15 ad duorum Scrupulorum Quantitatem excrescit, eâ, cujus ille meminit, duorum Scrupulorum Emendatione, in Horologio suo ablatam credo: de qua videat propterea Vir Doctissimus.

Demum vero, si in *Copernicanam Hypothesin* sit promior, quàm in *Ptolemaicam*, in ea etiam, suppositis Terræ Isochronis Revolutionibus, eædem consequuntur Æquationes. Fateor equidem, amoveri posse, & in contrarium trahi, ab Inæquali Incessu Solis in Orbita sua provenientem Temporis Æquationem, si inæquales Terræ vel Primi Mobilis (perinde enim est utrum horum statuerimus) Revolutiones supponamus: Sed si Temporis Naturam benè perpendat, facilè intelliget, impossibile esse, omnem ejus Inæqualitatem removeri.

XXII. Fa-

XXII.
*An Equation Table; by M. Cassini.*
n. 214. p. 243.

## Tabula *Æquationis* Dierum, cum Solis Loco adeunda.

| G. | ♈ ′ Sub. | ″ | ♉ ′ Add. | ″ | ♊ ′ Add. | ″ | ♋ ′ Sub. | ″ | ♌ ′ Sub. | ″ | ♍ ′ Sub. | ″ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 7 | 45 | 1 | 11 | 4 | 3 | 0 | 59 | 5 | 43 | 2 | 8 |
| 1 | 7 | 26 | 1 | 24 | 4 | 0 | 1 | 15 | 5 | 45 | 1 | 53 |
| 2 | 7 | 7 | 1 | 37 | 3 | 56 | 1 | 29 | 5 | 46 | 1 | 37 |
| 3 | 6 | 48 | 1 | 49 | 3 | 51 | 1 | 42 | 5 | 47 | 1 | 21 |
| 4 | 6 | 29 | 2 | 1 | 3 | 45 | 1 | 54 | 5 | 48 | 1 | 5 |
| 5 | 6 | 10 | 2 | 12 | 3 | 39 | 2 | 6 | 5 | 48 | 0 | 48 |
| 6 | 5 | 51 | 2 | 23 | 3 | 32 | 2 | 19 | 5 | 48 | 0 | 30 |
| 7 | 5 | 31 | 2 | 33 | 3 | 25 | 2 | 32 | 5 | 46 | 0 | 12 |
| 8 | 5 | 11 | 2 | 43 | 3 | 17 | 2 | 44 | 5 | 44 | 0 Add | 7 |
| 9 | 4 | 51 | 2 | 53 | 3 | 9 | 2 | 56 | 5 | 40 | 0 | 26 |
| 10 | 4 | 31 | 3 | 3 | 3 | 0 | 3 | 8 | 5 | 36 | 0 | 45 |
| 11 | 4 | 11 | 3 | 13 | 2 | 51 | 3 | 20 | 5 | 31 | 1 | 3 |
| 12 | 3 | 52 | 3 | 22 | 2 | 41 | 3 | 32 | 5 | 25 | 1 | 21 |
| 13 | 3 | 33 | 3 | 30 | 2 | 31 | 3 | 43 | 5 | 19 | 1 | 40 |
| 14 | 3 | 14 | 3 | 37 | 2 | 21 | 3 | 54 | 5 | 13 | 1 | 59 |
| 15 | 2 | 55 | 3 | 43 | 2 | 10 | 4 | 4 | 5 | 6 | 2 | 19 |
| 16 | 2 | 37 | 3 | 48 | 2 | 0 | 4 | 14 | 4 | 58 | 2 | 40 |
| 17 | 2 | 19 | 3 | 53 | 1 | 49 | 4 | 24 | 4 | 49 | 3 | 1 |
| 18 | 2 | 1 | 3 | 57 | 1 | 37 | 4 | 34 | 4 | 39 | 3 | 22 |
| 19 | 1 | 43 | 4 | 1 | 1 | 25 | 4 | 43 | 4 | 30 | 3 | 44 |
| 20 | 1 | 26 | 4 | 5 | 1 | 13 | 4 | 51 | 4 | 20 | 4 | 6 |
| 21 | 1 | 9 | 4 | 8 | 1 | 1 | 4 | 59 | 4 | 9 | 4 | 29 |
| 22 | 0 | 52 | 4 | 10 | 0 | 49 | 5 | 6 | 3 | 57 | 4 | 51 |
| 23 | 0 | 35 | 4 | 12 | 0 | 37 | 5 | 13 | 3 | 45 | 5 | 13 |
| 24 | 0 | 19 | 4 | 13 | 0 | 24 | 5 | 19 | 3 | 32 | 5 | 35 |
| 25 | 0 | 3 | 4 | 11 | 0 | 10 | 5 | 24 | 3 | 19 | 5 | 57 |
| 26 | 0 Add | 12 | 4 | 9 | 0 Sub. | 3 | 5 | 29 | 3 | 5 | 6 | 19 |
| 27 | 0 | 27 | 4 | 8 | 0 | 16 | 5 | 33 | 2 | 51 | 6 | 41 |
| 28 | 0 | 42 | 4 | 6 | 0 | 29 | 5 | 37 | 2 | 37 | 7 | 2 |
| 29 | 0 | 57 | 4 | 5 | 0 | 44 | 5 | 40 | 2 | 23 | 7 | 23 |
| 30 | 1 | 11 | 4 | 3 | 0 | 59 | 5 | 43 | 2 | 8 | 7 | 44 |

Tabula

# Tabula *Æquationis* Dierum, cum Solis Loco adeunda.

| G. | ♎ ′ Add. | ″ | ♏ ′ Add. | ″ | ♐ ′ Add. | ″ | ♑ ′ Add. | ″ | ♒ ′ Sub. | ″ | ♓ ′ Sub. | ″ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 7 | 44 | 15 | 34 | 13 | 25 | 0 | 59 | 11 | 48 | 14 | 36 |
| 1 | 8 | 5 | 15 | 42 | 13 | 7 | 0 | 27 | 12 | 4 | 14 | 29 |
| 2 | 8 | 25 | 15 | 48 | 12 | 48 | 0 Sub. | 5 | 12 | 19 | 14 | 21 |
| 3 | 8 | 45 | 15 | 53 | 12 | 29 | 0 | 35 | 12 | 35 | 14 | 13 |
| 4 | 9 | 5 | 15 | 57 | 12 | 10 | 1 | 4 | 12 | 50 | 14 | 4 |
| 5 | 9 | 25 | 16 | 1 | 11 | 50 | 1 | 33 | 13 | 5 | 13 | 55 |
| 6 | 9 | 44 | 16 | 5 | 11 | 30 | 2 | 3 | 13 | 19 | 13 | 46 |
| 7 | 10 | 3 | 16 | 7 | 11 | 10 | 2 | 32 | 13 | 32 | 13 | 37 |
| 8 | 10 | 22 | 16 | 8 | 10 | 49 | 3 | 1 | 13 | 44 | 13 | 27 |
| 9 | 10 | 41 | 16 | 9 | 10 | 28 | 3 | 29 | 13 | 55 | 13 | 17 |
| 10 | 11 | 00 | 16 | 9 | 10 | 6 | 3 | 57 | 14 | 5 | 13 | 7 |
| 11 | 11 | 19 | 16 | 9 | 9 | 42 | 4 | 25 | 14 | 14 | 12 | 56 |
| 12 | 11 | 38 | 16 | 8 | 9 | 17 | 4 | 53 | 14 | 22 | 12 | 44 |
| 13 | 11 | 57 | 16 | 7 | 8 | 51 | 5 | 20 | 14 | 29 | 12 | 32 |
| 14 | 12 | 15 | 16 | 5 | 8 | 25 | 5 | 48 | 14 | 35 | 12 | 19 |
| 15 | 12 | 33 | 16 | 1 | 7 | 58 | 6 | 15 | 14 | 40 | 12 | 6 |
| 16 | 12 | 50 | 15 | 56 | 7 | 31 | 6 | 42 | 14 | 45 | 11 | 52 |
| 17 | 13 | 7 | 15 | 50 | 7 | 5 | 7 | 9 | 14 | 50 | 11 | 37 |
| 18 | 13 | 22 | 15 | 44 | 6 | 38 | 7 | 34 | 14 | 54 | 11 | 21 |
| 19 | 13 | 36 | 15 | 37 | 6 | 12 | 7 | 58 | 14 | 56 | 11 | 4 |
| 20 | 13 | 49 | 15 | 30 | 5 | 45 | 8 | 21 | 14 | 58 | 10 | 46 |
| 21 | 14 | 2 | 15 | 22 | 5 | 19 | 8 | 45 | 14 | 59 | 10 | 28 |
| 22 | 14 | 14 | 15 | 13 | 4 | 52 | 9 | 8 | 15 | 00 | 10 | 10 |
| 23 | 14 | 26 | 15 | 3 | 4 | 26 | 9 | 31 | 15 | 00 | 9 | 52 |
| 24 | 14 | 37 | 14 | 52 | 3 | 58 | 9 | 53 | 15 | 00 | 9 | 34 |
| 25 | 14 | 47 | 14 | 40 | 3 | 30 | 10 | 13 | 14 | 58 | 9 | 16 |
| 26 | 14 | 57 | 14 | 27 | 3 | 1 | 10 | 32 | 14 | 55 | 8 | 58 |
| 27 | 15 | 7 | 14 | 13 | 2 | 31 | 10 | 51 | 14 | 51 | 8 | 40 |
| 28 | 15 | 16 | 13 | 58 | 2 | 1 | 11 | 10 | 14 | 47 | 8 | 22 |
| 29 | 15 | 25 | 13 | 42 | 1 | 30 | 11 | 29 | 14 | 42 | 8 | 4 |
| 30 | 15 | 34 | 13 | 25 | 0 | 59 | 11 | 48 | 14 | 26 | 7 | 45 |

*Spots observ'd in the Sun; by Mr.* Boyle. *n.* 74. *p.* 2216.

XXIII. *Ann.* 1660. *April* 27. About 8 of the Clock in the Morning there appear'd a Spot in the Lower Limb of the *Sun*, a little towards the South of its Æquator, which was entred about $\frac{1}{40}$ of the Diameter of the *Sun*, it self being about $\frac{1}{160}$ in its shortest Diameter, of that of the *Sun*; its longest, about $\frac{1}{40}$ of the same. It disappeared upon Wednesday Morning, *May* 9. though we saw it the day before about 10 in the Morning to be near about the same Distance from the Westward Limb, a little South of its Æquator, that it first appeared to be from the Eastward Limb, a little South also of its Æquator. It seem'd to move faster in the middle of the *Sun* than towards the Limb. It was a very dark Spot, almost of a Quadrangular Form, and was enclosed round with a kind of duskish Cloud.

We first observed this very same Spot both for Figure, Colour, and Bulk, to be Re-entred the Sun *May* 25. when it appear'd to be in a part of the same Line it had formerly traced; and was Entred about $\frac{4}{33}$ of its Diameter about 7 a Clock in the Afternoon. At the same time there appear'd another Spot, which was just Entered, and appeared to be Entered not above $\frac{1}{152}$ part of the Sun's Diameter. It appeared to be longest towards the North and South, and shortest towards the East and West. There seem'd to be dispersed about it divers small Clouds here and there.

*Spots Observ'd in the Sun; by M.* Picard. n. 74. *p.* 2238. n. 75. *p.* 2253.

XXIV. 1. *Ann.* 1671. M. *Picard*, at Sea near the *Texel*, observ'd a Spot in the Sun from *Aug.* 3. *St. N.* to the 19th. It appear'd at first, like the Tail of a *Scorpion*; but on the 19th day, resembling a Melon-Seed.

*By M.* Cassini. n. 74. *p.* 2238. n. 75. *p.* 2250.

2. *Aug.* 11. (*St. N.*) 1671. About 6 a Clock at Night, M. *Cassini*, with a three Foot Glass, remark'd in the *Sun*'s Disque, two *Spots* very dark, distant from its apparent Center about the third part of his Semi-Diameter. They were in the Southern part of the Sun, and their Elongation from the *Parallel of the Æquator* passing through the Center of the Sun, was about $\frac{1}{60}$ part of his Diameter. The Time which lapsed between the Transite of the *Sun*'s Center and that of the first of these *Spots*, was 22″. or 23″. the Semi-diameter of the *Sun* then passing in 66″. The first of these Spots, being looked upon with a Telescope of 17 Foot long, appear'd of a somewhat Oval Figure; the other was Oblong, and a little Curved, like the *Hebrew* Letter *Jod*; and both together were surrounded by a *Corolla*, or Coronet, made up of little dark Points, which conformed it self to the Figure of the *Spots*, considered as they were joyn'd together.

*Aug.* 12. He perceived that they were nearer his Center. The Time between the passage of the *Sun*'s Center, and that of the Interior edge of the *Coronet* which encompass'd them both, was then of 16″. At 7 a Clock it was but of 15″. and the Southern Limb of the *Coronet* touched the Parallel passing through the *Sun*'s Center. The first *Spot* was composed of two others almost round, and conjoined. The second represented the shape of a *Scorpion*. The third was round. And they were all three environed with a *Coronet*, which was composed, as we said above, of abundance of little obscure Pricks. This *Coronet* appear'd to be clearer than the rest of the *Sun*, when

looked

looked upon with the ſhort Glaſs, and darker when ſeen with the long. Without it there were other Points, but very black ones; *viz.* Five near the round Spot on the South-ſide, and another near the *Scorpion's* Tail on the North-ſide.

At 8 a Clock and 48′. the Figure of the *Scorpion* was ſeen divided into ſeveral pieces, as if his Tail and Arms had been cut off. The Northern Point appear'd no more, there remaining none but thoſe on the South-ſide; and the length of the encloſure of all the *Spots*, comprehended between the Extremities, was of 1′. 15″. and the Breadth of 30″.

The ſame 12th Day, at 6 in the Evening, he found no great Change in the firſt *Spot*. The other two were ſevered into 5 diſtinct ones, compaſs'd about with a *Coronet*, together with 5 black Points, which ſtood in a ſtreight row, and after another manner than they did in the Morning. From 6 at Night unto 7, the time between the paſſage of the *Sun's* Center, and that of the *Coronet's* Limb, was found to be one time, of 8″. and another time of 7½. The Diſtance of the *Spots* unto the Parallel, paſſing through the *Sun's* Center, was near the ſame on the North-ſide with what it had been obſerved to be in the Morning on the South-ſide.

*Aug.* 13. Between the Riſing of the *Sun*, and half an Hour paſt 6 in the Morning, the edge of the *Coronet* was turned to a Point on the South-ſide, and was diſtant from the *Æquator* on the North-ſide, half a Minute; and there was but a Second of Time from the paſſage of the *Sun's* Center unto the paſſage of the ſame anterior edge of the *Coronet*.

At 8 a Clock, 30′. the fore-edge was in the ſame Horary Circle with the Center of the *Sun*: ſo that, in one Day and an half, theſe *Spots* have run through very near the third part of the *Sun's* Apparent Semidiameter, which giveth an Arch of 19 Degrees and an half of the Circumference of the *Sun's* Body; and conſequently their *Diurnal* Motion about the *Sun's* Axe hath been of 13 Degrees; and the time of their *Periodical* Revolution, as far as we could conjecture in ſo little time, muſt be about 27 Days and an half.

*Aug.* 14. At 6 in the Morning, there paſs'd 15″. of Time between the paſſage of the Anterior Limb of the *Crown*, and the paſſage of the *Sun's* n. 78. p. 3020.
Center through the ſame Horary Circle: And then the Southern Limb of the *Crown* was a Minute and an half diſtant, toward the North, from the Parallel of the *Æquator*, paſſing through the ſame Center of the *Sun*. The Figure of the firſt *Spot* was almoſt the ſame with that of the Day before. The ſecond had taken the form of an Heart, the point of which was turned to the North-ſide, and its Baſe between the South and the Eaſt. Three other ſmall *Spots*, diſpoſed Triangle-wiſe, ſtood over the ſaid Baſe, and were accompanied with two others upon a Line turned Southward. And they were all encompaſſed by a *Crown* running out into a point on the South-ſide; and on the North-ſide, Eaſtward, it had an *Appendix*.

*Aug.* 15. At 6 in the Morning, there paſſed 27″. between the paſſage of the Anterior Limb of the *Crown*, and that of the *Sun's* Center through the ſame Horary Circle. The Southern Limb of the ſame *Crown* was two Minutes and an half diſtant from the Parallel of the *Equator*, paſſing through

 the

the Center of the *Sun*, whose Diameter pass'd in 2′. 9″. through the same Horary Circle. The first *Spot* had a little changed its Figure; the second was Quadrangular, longer from East to West, than from North to South: It appeared bigger than ordinary, and had withall on its sides, within the Compass of the *Crown*, three other small *Spots*. There were also seen four more without the said *Crown* on the Southside.

*Aug.* 16. at 6 in the Morning, there was 27″. between the passage of the *Sun*'s Anterior Limb, and the passage of the Anterior Limb of the *Crown* through the same Horary Circle; and 38″. between the passage of the Anterior Limb of the *Crown*, unto the passage of the *Sun*'s Center. The Southern Limb of the *Crown* was 3½′ off from the Parallel of the *Æquator*, passing through the Center of the *Sun*, towards the North. And the Observation having been made yet more exactly at half an hour past 7 of the same Morning, this Distance was found of 3′. 33″. The Figure of the first *Spot* in the beginning of the Observation, differ'd not much from that of the precedent day; but afterwards it was seen divided into two. The second, which likewise seemed to be the same in the beginning, was afterwards divided into three, accompanied with black and dark Points without the *Crown* on the Southside. The same day at 6 a Clock, and 15′. at night, the Figures of these Spots were much changed. There were 5 *Spots* enclosed in the *Crown*; The two foremost were part of that which had been seen in the Morning as one; the two others following those two first, were part of the second in the Morning; and without there were 5 Points on the Southside, and two more a little further to the North, which Points were ranged as in another Area made up of other Points so small, that they could scarce be perceived.

*Aug.* 17. in the Morning, immediately after the Rising of the *Sun*, there appeared three very dark *Spots*, which form'd in a manner these Letters, *F n J*, posited from East to West, and included in their wonted *Crown*, which stretched out, as 'twere, two Arms, or two Handles, one to the South, and the other to the North. There pass'd 18″. between the passage of the foremost Limb of the *Sun*, and that of the foremost Limb of the *Crown*, and 47½″. between the passage of the Anterior Limb of the *Crown* unto the passage of the *Sun*'s Center. The Southern Limb of the same *Crown* was distant 11′. 17″. from the *Parallel* that touched the *Sun* on the Northside, and 4′. 38″. from the *Parallel* that pass'd through his Center.

*Aug.* 18. at 7 in the Morning, the *Spots*, which appear'd through some Clouds, had almost the same shape with those of the day before, only with this difference, that they were a little closer together, drawing from East to West. There lapsed 13″. between the passage of the Anterior Limb of the *Sun*, and that of the Anterior Limb of the *Spot*, through the same Horary Circle, and 52½″. of the foremost Limb of the *Spot* unto the passage of the Center. The Southern Limb of the *Spot* was 9′. 13″. distant from the *Parallel* that touched the Northern Limb of the *Sun*, and 6′. 41″. from the *Parallel* that pass'd through his Center. At 5 a Clock and 55′. at night of the same day, there lapsed 11″. between the passage of the Anterior Limb of the *Sun* through the same Horary Circle, and the passage of the Anterior

Limb

Limb of the *Crown*, and from thence unto the paſſage of the *Sun*'s Center 54$\frac{1}{2}$''. The Limb of the *Crown* next to the *Parallel* paſſing through the Center of the *Sun*, was diſtant from the ſame *Parallel* 7'. 40''.

*Aug.* 19. from 4 to 5 in the Evening, the *Spot* appear'd Oblong near the *Sun*'s Circumference, from which it was diſtant about the breadth of the ſame *Spot.*

*Aug.* 20. in the Morning, which was not the full ſeventh from the day that they were arrived to the middle of the Diſque, they were diſappeared.

The Apparent Velocity nigh the Center was ſuch, that if it had continued the ſame, the *Spots* would have arrived almoſt in 4 days to the Limb of the Diſque; but in this Hypotheſis, that the *Spots* were adherent to the *Sun*'s Surface, or at leaſt, very nigh to it, this Apparent Velocity was to leſſen according as they ſhould remove from the Center, as hath come to paſs in effect. The Diminution of the length of the Miſty *Crown* was in a manner proportionable to the Diminution of the Apparent Velocity; ſince that, when this *Crown* was in the middle, and in a Situation, wherein its true Figure could be beſt ſeen, it appear'd Oblong, and of the Form of an Human Ear, its greateſt Diameter reſpecting Eaſt and Weſt; but being nigh the Limb, this ſame Diameter ſeemed to ſhorten; and having appear'd greateſt in its firſt Situation, it appear'd leaſt in this, becauſe it was almoſt in a Circle that paſs'd through the Center of the *Sun*, whoſe equal Arches are by ſo much the more Oblique, by how much they approach more to the Limb of his Diſque, and conſequently appear leſs, according to the Rules of *Opticks*; mean time, the Diameter, that was turned from South to North, apparently kept the ſame bigneſs it had near the Center, becauſe it was in a Circle almoſt Parallel to the *Horizon* of the *Sun*, which formed the Repreſentation of its Limb, and whoſe equal Arches (by the ſame *Optical* Reaſons) do not appear contracted.

3. Several curious Obſervers at *London* have ſeen one of thoſe *Spots* recurr'd to the Sun's Eaſtern Limb about *Aug.* 25. *St.* N. as M. *Caſſini* Predicted they ſhould return.

*By ſeveral at London.* n. 75. *p.* 5253.

4. *Aug.* 30. 1671. I ſaw a large *Spot* in the Center of the *Sun*'s Face about Noon.

*By Dr.* Hook. n. 77. *p.* 2295.

*Sept.* 1. At 3 a Clock, I ſaw the ſame *Spot* moved about a Quarter of the Diameter of the *Sun* Weſtward: It conſiſted of one greater, and two leſſer black *Spots*, with a *Dusky Cloud* incompaſſing them: The Diameter of the whole Phænomenon was about $\frac{1}{7\frac{1}{2}}$ of the Diameter of the *Sun*, and it was diſtant from the next adjoyning Limb $\frac{18}{72}$ (that is exactly one Quarter) of the Diameter of the *Sun.*

5. Maculæ Solares Obſervatæ fuere nobis *Hamburgi* à 26. *Aug.* *St. V.* (primo ferè die, quo iterum Apparere cœperunt) ad uſque 5. *Sept.* quo ad Limbum quamproximè acceſſere.

*By* M. Hen. Siferus. n. 78. *p.* 3033.

XXV. *Ann.* 1676. *Jun.* 28. St. N. Habemus in ſole ſatis Ingentem *Maculam*, quæ *Solem* ipſum mediavit h. 4. poſt Meridiem, cum Latitudine Auſtrali 4$\frac{1}{4}$; ejus Diſtantiam à Polo Auſtrali *Solis* ex pluribus Obſervationibus ſupputavi *gr.* 78$\frac{1}{4}$. Si ſatis habuerit Conſiſtentiæ ad abſolvendum Circulum expectanda Reſtitutio ejus ad Medium, Diei 25. *Jul.* veſpere, cum majore Latitudine Auſtrali.

*Spots Obſerv'd in the Sun; by M.* Caſſini. n. 127. *p.* 665.

XXVI. 1. Green-

XXVI. 1.

*Spots Obſerv'd in the Sun; by Mr.* Flamſteed, *and Mr.*Halley. n. 128. p. 687.

| | Grenovici, A. D. 1676. | | | Oxonii, A. D. 1676. | | |
|---|---|---|---|---|---|---|
| *Jul. St. V.* | *Tempus Obſervationum.* | *Longit. à* Solis *Centro.* | *Latit. Auſt.* | *Tempus Obſervationum.* | *Long.* | *Lat. Auſt.* |
| | h. ′ | ′ ″ | ′ ″ | h. ′ | ′ ″ | ′ ″ |
| 25 | — | — | — | 6 46 p.m.Con. | 13 40 | 2 8 |
| 27 | 10 3 a. m.Con. | 9 34 | 3 25 | — | — | — |
| 28 | 4 51 — | 5 40 | 2 50 | — | — | — |
| 29 | 10 31 a. m.— | 3 05 | 3 27 | 6 21 a. m.— | 3 55 | 3 22 |
| | 3 54 p. m.— | 2 25 | 3 10 | — | — | — |
| 30 | 9 15 a. m.Ant. | 0 37 | 3 33 | 7 20 a. m.— | 0 00 | 3 30 |
| 31 | — | — | — | 7 40 a. m.— | 3 36 | 3 28 |
| *Aug.*1 | 9 24⅚ a. m— | 6 48 | 4 09 | 7 3 a. m.— | 6 54 | 3 50 |
| | — | — | — | 5 6 p. m.— | 8 7 | 3 53 |
| 2 | 8 8 a. m.— | 9 49 | 3 55 | 7 16 a. m.— | 9 57 | 3 40 |
| 3 | 9 36 a. m.— | 12 28 | 3 27 | 5 9 p. m.— | 13 15 | 3 56 |
| | 4 16⅔ p. m.— | 12 55 | 3 58 | 6 2 p. m.— | 13 25 | 3 26 |
| 4 | 7 38 a. m.— | 14 02 | 4 4 | 7 33 a. m.— | 14 7 | 3 14 |
| | — | — | — | 4 54 p. m.— | 14 43 | 3 23 |

Mr. *Halley* ſaith, That he ſaw the *Spot* again on the fifth day, at 8^h. 30′. *Mane*, very near the Limb of the Sun, ſo that it appeared only as a fine Line; but by reaſon of its fineneſs and the too great Height of the *Sun*, he could not take any meaſures to determine its Place and Latitude by; and that while the *Spot* continued one, as it was *July* 25. he meaſured to the middle of it; as alſo when the pieces were divided, but not far disjoined: Afterwards, when they were ſeparated conſiderably, he obſerved the middle of the bigger *Spot*, which was to the South, apparently, I ſuppoſe; but really, North: for ſo only his Obſervations will agree with thoſe of Mr. *Flamſteed* exactly.

Hence it ſeems very Evident, (ſaith Mr. *Flamſteed*) that the *Spot's* way was not Inclined to the *Ecliptick* ſix or ſeven Degrees, as *Scheiner* and ſome others make it, but much leſs, by the joynt Conſent of the Obſervations of both our Obſervers. Mr. *Halley* adds, That conſidering the Motion of the *Spot* croſs the *Sun's* Diſque, as both their Obſervations give it, it appears that the Latitude was not ſo great at its Entrance into the *Sun* as in the middle of him. And by Mr. *Flamſteed's* Obſervation it was greateſt on the firſt of *Auguſt*, and then again Inclining towards the Ecliptick. If you grant this, it will follow, (infers Mr. *Flamſteed*) that the *Sun's* Axis was Inclined to the Plain of the *Orbis Magnus*; but the quantity of this Inclination muſt not be very great,

great. The *Nodes* of the *Sun*'s Equinox and Ecliptick he guesses to be not far from the beginning of *Cancer* and *Capricorn*; and that from *Cancer* to *Capricorn* the Earth is North of the *Sun*'s *Æquator*; from *Capricorn* to *Cancer*, South of the same: And the *Period* of the *Sun*'s Revolution in respect of the fixed Stars 25 Days, 9½ hours sufficiently exact. Of which things these two Observers say, they might have been more certain, had not the *Spot* in its passage broken into so many parts, and those often varied their Positions to each other.

2. *Solarem Maculam* hic observavimus à die 6 *Aug.* ad 14. *St. N.* Collationeque Observationum didicimus, eam medium Itineris sui in Solis Disco Apparente tenuisse circa Mediam Noctem post 8. diem *Aug.* in distantia Apparenti trium minutorum à Centro Austrum versus. In plures distracta partes est, quæ invicem Boream & Austrum versus indies satis manifesto intervallo disjungebantur, adeo ut, præter Motum Communem circa Solis Axem, singulæ partes Proprium inter se directum habuerint. Hanc porrò *Maculam* diversam esse sentio ab ea quam præcedenti Mense *Junio* observaveramus. Illa quippe cùm medium Itineris sui in Disco Solis Apparente tenuerit die 28 ejusdem Mensis, ad eundem proximè situm reversa esset (si fuisset superstes) die 25 *Julii* Nocte sequente, ut deducitur tum ex ejus Velocitate, Tempore suæ Apparitionis observatâ, tum etiam ex Cursu aliarum *Macularum*, quæ Periodum suam circa Solem à nobis videntur absolvere Spatio dierum 27 cum triente, vel 27 cum semisse. Ejus præterea semita diversa est à præcedenti; prior quippe paulo remotior fuit ab Æquatore *Macularum* quàm posterior. Hæc porrò, si satis habuerit Consistentiæ, ad Medium *Solem* rediditur die 5 *Septembris* manè.

*By M. Cassini. Ibid. p. 689.*

XXVII. *Ann.* 1684. *Apr.* 25. About an hour before Noon I discovered a large *Spot* entred within the *Sun*'s Disk a little distant from his following Limb. These Appearances how ever frequent in the days of *Scheiner* and *Galileo*, have been so rare of late, that this is the only one I have seen in his face since *December* 1676. By the observed Meridional Distances of it, and the *Sun*'s Southern Limb from the *Vertex* at Noon, I found it to have 3′, 40″. more North Declination than the *Sun*'s Center, and at 3h. 35′. after Noon, I measured its Distance from his next Limb 40″.

*Spots Observed in the Sun; by Mr. Flamsteed. n. 157. p. 535.*

Next Morning, *April* 26. I saw it more Remote from his Limb, and by the Observations then made (at 8 h. *manè*,) determined its Longitude from the *Sun*'s Axis 66¼ Deg. and its Declination from the *Solar Equator* 9⅗ Deg. South. Whence supposing the Revolution of any point of the Sun to the same fixed Star to be performed in 25 days 6 hours, the Angle of his *Equator* and our *Ecliptick* 7 Deg. and the Longitude of his Northern Pole ♍ 16 Deg. I designed the Line of its Way or Trace over the Sun, and the Points in it where the *Spot* would appear every Morning after at the same hour, till its Egress on the 8th of *May*, which I found altogether confirmed by such Observations as I made till then; so that I had no reason to doubt of the Theory.

Whea-

When the *Spot* was near the middle of the *Sun*, it appeared very broad; and almost square; the *Nucleus* of the same Figure about 40″. Diameter; but when it was near the Limb much narrower, and almost Oval: It seemed to have Consistence enough to endure a second Return; if it shall, it will enter the Visible Disk of the *Sun* on the 21th of *May* in the Evening, and in its passage over him describe a Line nearly Streight, with greater Latitude from the *Ecliptick*.

*To find in what Proportion the Planets are Enlightned by the Sun; by* M. Auzout. n. 4. p. 63.

XXVIII. One of the means used by M. *Auzout* to Enlighten an Object in what Proportion one pleaseth, is by some great Object Glass, by him called a *Planetary* one, because that by it he shews the difference of Light, which all the *Planets* receive from the *Sun*, by making use of several Apertures, proportionate to their Distances from the Sun, provided that for every 9 Foot Draught, or thereabout, one Inch of Aperture be given for the Earth. Doing this, one sees, (saith he) that the Light which *Mercury* receives, is far enough from being able to Burn Bodies; and yet that the same Light is great enough in *Saturn* to see clear there, seeing that (to him) it appears greater in *Saturn*, than it doth upon our Earth, when it is over-cast with Clouds, which (he adds) would scarce be believed if by means of this Glass, it did not sensibly appear so.

*The Æquinoxes; by* M. Wortzelbaur. n. 265. p. 623.

XXIX. The *Æquinoxes* of this year 1699. according to the Observations of M. *Wortzelbaur* at *Nurenberg*, happened *March* 9. 20h. 35′. 27″. and *Sept.* 12. 10h. 22′. 42″. which by his Tables ought to have been *March* 9. 20h. 40′. 30″. and *Sept.* 12. 10h. 32′. 52″.

*To Observe Solar Eclipses; by* Mr. Flamsteed. n. 55. p. 1104.

XXX. For the well observing Solar *Eclipses*, cast the Species of the *Sun* through a good Telescope of a competent Length, on an extended Paper, placed behind the Eye-Glass so far, as that the said Species may appear at least 6 Inches over; then divide, both his *Periphery* into 360 Degrees for the better Observing the Inclination of the *Cusps* of each Phasis, and his Diameter into Digits and their Parts by Concentrick Circles, for measuring the quantities of the obscured Parts.

*An Eclipse of the Sun*, Ann. 1666. June 22. *at* London; *by* Mr. Willoughby, Dr. Pope, Dr. Hook, *and* Mr. Phillips. n. 15. p. 295.

XXXI. 1. The *Eclipse* began at 5h. 43′.

| | | h. ′ |
|---|---|---|
| It was darkned | $1\frac{1}{2}$ Diam.—at | 6 00 |
| | 4 Digits.—at | 6 7 |
| | 5 Dig.—at | 6 13 |
| | 6 Dig.—at | 6 21 |
| | 7 Dig.—at | 6 $39\frac{1}{2}$ |
| | 6 Dig.—at | 6 57 |
| | 5 Dig.—at | 7 6 |
| | 4 Dig.—at | 7 13 |
| | 3 Dig.—at | 7 20 |
| | 2 Dig.—at | 7 26 |
| | 1 Dig.—at | 7 32 |
| | 0 Dig.—at | 7 37 |

Its *Duration* hence appears to have been one hour and 54′. Its greatest *Obscurity* somewhat more than 7 Digits. About the middle, between the Perpendicular and Westward Horizontal Radius of the Sun, viewing it through

Mr. *Boyle*'s

Mr. *Boyle*'s 60 Foot Teleſcope, there was perceived a little of the Limb of the Moon without the Disk of the Sun: which ſeemed to ſome of the Obſervers to come from ſome ſhining Atmoſphere about the Body either of the Sun or Moon. They affirm to have obſerved the Figure of this Eclipſe, and meaſured the Digits, by caſting the Figure through a 5 Foot Teleſcope, on an extended Paper, fixt at a certain Diſtance from the Eye-Glaſs, and having a round Figure; all whoſe Diameters were divided, by 6 Concentrick Circles, into 12 Digits.

2. The *Eclipſe* Began at $5^{h}$. 44′. 52″. *manè.* It Ended at $7^{h}$. 43′. 6″. So that its whole Duration was $1^{h}$. 58′. 14″. The greateſt Obſcuration was 7 *Dig.* 50′. but it ſeemed to have been greater by 3′. which M. *Payen* imputes to a particular Motion of Libration of the Sun's Globe, which entertained that Luminary in the ſame Phaſis for the ſpace of 8′. and ſome Seconds, as if it had been ſtopped in the midſt of its Courſe; rather than to a Tremulous Motion of the Atmoſphere, as *Scheiner* would have it. The Apparent Diameters were almoſt Equal: for in the Phaſis of 6 *Digits*, the Circumference of the Moon's Disk paſſed through the Center of that of the Sun, ſo as that two Lines drawn through the two Horns of the Sun, made with the common Semidiameter two Equilateral Triangles. *At* Paris; *by* M. Payen. *Ibid. p.* 296.

The Beginning and Middle of the Eclipſe happened to be in the North Eaſtern Hemiſphere, and the End in the South Eaſtern. The firſt Contact (as 'twere) of the two Disks, was obſerved in the Superior Limb of the Sun's Disk in reſpect to the Vertical Line, and in the Inferior in reſpect to the Ecliptick. But the Middle and the End were ſeen in the Superior Limb, in reſpect both to the Vertical and the Ecliptick: And what to M. *Payen* ſeems extraordinary, both the Beginning and the End of this Eclipſe happen'd to be in the Oriental Part of the Sun's Disk.

3. The *Eclipſe* Began about 5 a Clock in the Morning; at $5^{h}$. 15′. The Sun's Altitude was 6°. 55′. *At* Madrid; *by the Earl of* Sandwich. *Ibid. p.* 296.

The Middle of it was at $6^{h}$. 2′. The Sun's Altitude 15°. 5′.

The End was exactly at $7^{h}$. 5′. The Sun's Altitude 25°. 24′.

The Duration $2^{h}$. 4′.

Thirty ſeven Parts of the Sun's Diameter remained Light, and 63 were Darkned.

4. In this *Eclipſe* it is chiefly obſervable, That the Semidiameter of the Moon, from the very beginning to about 5 or 6 Digits of the Increaſing Phaſis, was almoſt equal to the Semidiameter of the Sun: but, after the greateſt Obſcuration, when I again contemplated the Moon's Semidiameter, I found it 8″. or 9″. bigger than that of the Sun; ſo that the Semidiameter of the Moon was not always, during this *Eclipſe*, conſtant to it ſelf. Of this Variation, the Excellent *Iſmael Bullialdus* hath alſo obſerv'd ſomething at *Paris*. For he has written to me, that in the ſame *Eclipſe*, the Semidiameter of the Sun to the Semidiameter of the Moon was, as 16′. 9″. to 16′. 22″. but that in another Phaſis of 6 Digits, the Semidiameters appeared equal. *At* Dantzick; *by* M. Hevelius. n. 19. p. 347.

*Ibid. p. 349. n. 21. p. 370.*

| *Ordo Phasium.* | *Quantitas Phasium.* | *Tem. Æst. Sec. Horol. Ambulat.* | *Temp. ex Sciother.* | *Altitud.* ☉ | *Tempus Correct.* | *Animadvertenda.* |
|---|---|---|---|---|---|---|
| | | h. ′ ″ | h. ′ ″ | ° ′ | h. ′ ″ | |
| | | 5 51 11 | 5 51 00 | 17 45 | 5 53 12 | Quod Sciatericum cum Correcto Tempore non |
| | | 5 57 5 | 5 57 00 | 18 37 | 5 59 28 | omnino convenit, non nisi Lineæ Meridianæ |
| | | 6 00 00 | 6 00 00 | 18 55 | 6 1 28 | imputandum. |
| | Initium. | 6 55 30 | | | 6 57 30 | Initium circa 79 gr. à |
| 1 | $0\frac{1}{8}$ Dig. | 6 57 30 | | | 6 59 30 | puncto Zenith Occa- |
| 2 | $0\frac{3}{4}$ | 7 0 23 | 7 0 0 | | 7 2 23 | sum versus contigit. |
| 3 | $1\frac{1}{8}$ | 7 2 30 | 7 2 0 | | 7 4 30 | |
| 4 | $1\frac{1}{2}$ dig. | 7 4 50 | 7 5 fere. | | 7 6 50 | |
| 5 | $1\frac{3}{8}$ fere. | 7 10 57 | 7 10 | | 7 12 57 | |
| 6 | $3\frac{3}{8}$ | 7 14 59 | 7 15 | | 7 16 59 | |
| 7 | $3\frac{3}{4}$ | 7 17 50 | 7 18 fere. | | 7 19 50 | |
| 8 | $4\frac{3}{8}$ dig. | 7 21 35 | 7 21 | | 7 23 35 | |
| 9 | $4\frac{2}{3}$ | 7 23 43 | 7 23 fere. | | 7 25 43 | |
| 10 | $5\frac{1}{4}$ | 7 27 53 | 7 28 | | 7 29 53 | Hucusq; Semidiameter |
| 11 | 6 | 7 31 50 | 7 32 | | 7 33 50 | Lunæ æqualis extitit |
| | | | | | | Solari. |
| 12 | $6\frac{3}{4}$ | 7 36 55 | 7 37 | | 7 38 55 | |
| 13 | $6\frac{7}{8}$ paul. plus | 7 38 5 | 7 38 | | 7 40 0 | |
| 14 | $7\frac{1}{8}$ | 7 39 45 | 7 39 | | 7 41 45 | |
| 15 | $7\frac{1}{4}$ paul. plus | 7 42 30 | 7 42 | | 7 44 30 | |
| 16 | $7\frac{1}{2}$ | 7 44 6 | 7 44 | | 7 46 6 | |
| 17 | $7\frac{2}{3}$ | 7 46 0 | 7 46 | | 7 48 0 | |
| 18 | 8 fere | 7 48 25 | 7 48 fere. | | 7 50 25 | |
| 19 | $8\frac{1}{3}$ | 7 51 15 | 7 51 | | 7 53 15 | |
| 20 | $8\frac{1}{4}$ paul. plus | 7 53 37 | 7 52 | | 7 55 37 | |
| 21 | $8\frac{3}{4}$ | 7 55 45 | 7 56 fere. | | 7 57 45 | |
| 22 | $8\frac{3}{8}$ pau. min. | 7 59 5 | 7 59 | | 8 1 5 | Max. Obscuratio extitit, |
| 23 | $8\frac{1}{3}$ | 8 6 30 | 8 6 | | 8 8 30 | Dig. 8. 25′. hora 8. 2′. |

| Ordo Phasium. | Quantitas Phasium. | Tem. Æst. Sec. Horol. Ambulat. | Temp. ex Sciother. | Altit. ☉. | Tempus Correct. | Animadvertenda. |
|---|---|---|---|---|---|---|
| | | h. ′ ″ | h. ′ ″ | ° ′ | h. ′ ″ | |
| 24 | $7\frac{3}{4}$ | 8 11 25 | 8 12 | | 8 13 25 | Hic Semidiameter Lunæ |
| 25 | $7\frac{1}{4}$ fere. | 8 17 30 | 8 18 | | 8 19 30 | ad 8″. vel 9″. major |
| 26 | 7 fere. | 8 19 41 | 8 19 | | 8 21 41 | apparuit. |
| 27 | $5\frac{7}{8}$ | 8 28 8 | 8 28 | | 8 30 8 | |
| 28 | $5\frac{1}{2}$ fere. | 8 30 14 | 8 30 | | 8 32 14 | |
| 29 | $4\frac{3}{4}$ | 8 36 25 | 8 36 | | 8 38 25 | |
| 30 | $3\frac{3}{8}$ | 8 43 19 | 8 43 | | 8 45 19 | |
| 31 | $3\frac{1}{4}$ | 8 46 12 | 8 46 fere. | | 8 48 12 | |
| 32 | 3 | 8 47 32 | 8 47 | | 8 49 32 | |
| 33 | $2\frac{1}{4}$ | 8 50 57 | 8 50 | | 8 52 57 | |
| 34 | $2\frac{1}{2}$ fere. | 8 54 15 | 8 54 | | 8 56 15 | |
| 35 | $1\frac{3}{8}$ | 8 58 24 | 8 58 | | 9 0 24 | |
| 36 | $1\frac{1}{8}$ | 8 59 35 | 8 59 | | 9 1 35 | |
| 37 | $0\frac{1}{6}$ | 9 1 38 | 9 1 | | 9 3 38 | |
| 38 | $0\frac{1}{2}$ | 9 3 20 | 9 3 | | 9 5 20 | |
| 39 | Finis. | 9 6 53 | 9 6 | | 9 8 53 | Punctum Finis distitit à |
| | | 9 23 6 | | 47 33 | 9 25 28 | Verticali ad Ortum |
| | | 9 24 16 | | 47 42 | 9 26 45 | 143 gr. |
| | | 9 28 29 | | 48 10 | 9 30 42 | |
| | | 9 30 36 | | 48 28 | 9 33 12 | |



Ordo

XXXII. *An Eclipse of the Sun*, Jun. 23. (*St. N.*) 1675. *at* Dantzick; *by* M. Hevelius. n. 127. *p.* 660.

| Ordo Phasium. | *Animadvertenda.* | *Alt. Solis.* h. | ′ | ″ | *Temp. Cor.* h. | ′ | ″ |
|---|---|---|---|---|---|---|---|
| | Solis Centrum in Horizonte. | | | | 3 | 21 | 30 |
| | Nihil in Sole. | | | | 4 | 42 | 0 |
| | Initium. | | | | 4 | 44 | 0 |
| 1 | $6\frac{1}{2}$ Digiti ferè Obscurati erant. | | | | 5 | 32 | 0 |
| 2 | $6\frac{2}{3}$ Digit. | | | | 5 | 34 | 0 |
| 3 | Maxima ferè obscurat. 6 Dig. 42′. | | | | 5 | 38 | 50 |
| 4 | $6\frac{1}{2}$ Digit. | | | | 5 | 43 | 30 |
| 5 | $6\frac{1}{4}$ Digit. | | | | 5 | 47 | 30 |
| 6 | 6 Digit. | | | | 5 | 49 | 30 |
| 7 | $5\frac{1}{2}$ Digit. Diamet. Lunæ, 14′. 37″. Data nempe | | | | 5 | 55 | 0 |
| 8 | $5\frac{1}{4}$ Digit. — Semidiametro Sol 15′. | | | | 5 | 57 | 0 |
| 9 | 5 Fere Digit. | | | | 5 | 59 | 30 |
| 10 | $4\frac{5}{8}$ Digit. | | | | 6 | 2 | 0 |
| 11 | $4\frac{1}{2}$ Digit. | | | | 6 | 4 | 15 |
| 12 | 4 Digit. | | | | 6 | 6 | 30 |
| 13 | $3\frac{7}{8}$ Digit. | | | | 6 | 9 | 15 |
| 14 | $3\frac{1}{4}$ Digit. | | | | 6 | 14 | 35 |
| 15 | $2\frac{5}{8}$ Digit. | | | | 6 | 18 | 10 |
| 16 | $2\frac{3}{8}$ Digit. | | | | 6 | 20 | 0 |
| 17 | 2 Digit. | | | | 6 | 22 | 0 |
| 18 | $1\frac{1}{2}$ Digit. | | | | 6 | 24 | 0 |
| 19 | $1\frac{1}{4}$ Digit. | | | | 6 | 25 | 25 |
| 20 | 1 Digit. | | | | 6 | 27 | 10 |
| 21 | $\frac{1}{2}$ Digit. | | | | 6 | 30 | 30 |
| | Finis. | | °. | ′ | 6 | 33 | 30 |
| | Altitudo ☉. | | 25 | 20 | 6 | 41 | 22 |
| | Altitudo ☉. | | 28 | 5 | 7 | 0 | 14 |

*An Eclipse of the Sun*, June 1. 1676. *at* Westminster; *by Mr.* Francis Smethwick. n. 126. *p.* 637.

XXXIII. 1. Initium Defectionis. —— 7 h. 50′.
Finis. —— 9 $54\frac{3}{4}$.
Totius Eclipsis Duratio. —— 2 $4\frac{3}{4}$.

Tempus Observatum fuit cum Horologio Oscillatorio, Correcto per Observationes. Tubus adhibitus fuit bonæ notæ, pedum $7\frac{1}{2}$.

2. *At* Wapping; *by Mr.* Colſon. *Ibid.*

| *Temp. juxta Horol. Oſcil.* h. | ′ | ″ | *Phaſes.* | *Solis Alt.* ° | ′ | *Temp. Correct. ex Altit.* h. | ′ | ″ | |
|---|---|---|---|---|---|---|---|---|---|
| 7 | 34 | 50 | | 22 | 46 | 7 | 36 | 0 | |
| 7 | 37 | 14 | | 33 | 10 | 7 | 38 | 40 | |
| 7 | 39 | 10 | | 33 | 30 | 7 | 40 | 48 | |
| 7 | 50 | 40 | $0\frac{1}{4}$ Digit. | | | 7 | 51 | 51 | Tubo Optico æſtim. |
| 8 | 8 | 34 | $1\frac{1}{4}$ Digit. | | | 8 | 9 | 45 | Tub.Optico menſur. |
| 8 | 17 | 25 | $2\frac{1}{10}$ Digit. | | | 8 | 18 | 36 | |
| 8 | 27 | 10 | $3\frac{1}{10}$ Digit. | | | 8 | 28 | 21 | |
| 9 | 39 | | $1\frac{1}{2}$ Digit. | | | 9 | 40 | | Tubo æſtim. |
| 9 | 43 | | $1\frac{1}{4}$ Digit. | | | 9 | 44 | | |
| 9 | 48 | | $0\frac{3}{4}$ Digit. | | | 9 | 49 | | |
| 9 | 54 | 25 | Non Finita. | | | 9 | 55 | 36 | |
| 9 | 55 | 55 | Finita. | | | 9 | 57 | 6 | |
| 4 | 26 | 5 | | 32 | 10 | 4 | 26 | 56 | |
| 4 | 28 | 58 | | 31 | 53 | 4 | 29 | 52 | |
| 4 | 31 | 2 | | 31 | 31 | 4 | 32 | 16 | |

*At* Greenwich; *by Mr.* Flamſteed. n. 127. *p.* 662.

3. Hiſce Obſervationibus peragendis Socium acciveram Amicum meum *Ed. Halleium.* Tubos præparaveram duos, alterum Digitos $196\frac{1}{2}$. longum, quocum & *Micrometo Townleiano* Ego ipſe octo Phaſium priorum cepi Menſuras, alterum Digitorum duntaxat $103\frac{1}{2}$, quocum & *Micrometro* meo, iis adſcriptas Menſuras *Halleius* cepit: in duabus tamen ultimis Animadverſionibus, Ego Minori Tubo & *Micrometro* meo (in hunc uſum altero accommodatiore) Diſtantiam cepi *Azimutharum*, per Solis Limbum Lucidum, & Cuſpidem proximam *Eclipſis* decidentium; *Halleio* interea Partes Lucidas & Cuſpidum Diſtantiam majori Tubo dimetiente. Paulo ante Initium advenerat Nobiliſſimus Præſes Regiæ Societatis Dom. Vice-Comes *Brouncker*, qui Menſuram Diametri Solaris, Tubo Longiori captam, ſuo Judicio probavit. Horâ 7. 45′. Sol primum per Nubes apparuit. Obſervata deinde ſic ſe habuerunt.

*Phaſium*

Fig. 125

| Phasium Ordo. | Hor. Horol. Oscillatorii. h. | ′ | ″ | Correcta. h. | ′ | ″ | Longiori Tubo. | Breviori. |
|---|---|---|---|---|---|---|---|---|
| | 7 | 46 | 00 | 7 | 45 | 00 | Nulla Eclipsis. | |
| 1 | 7 | 54 | 50 | 7 | 53 | 50 | Solis Eluctati è Nubibus Margo Dexter Eclipsatus apparuit. | |
| | | | | | | | ′ ″ | ′ ″ |
| 2 | 7 | 58 | 24 | 7 | 57 | 24 | I C——2640 = 10 10 | |
| 3 | 8 | 04 | 12 | 8 | 03 | 12 | I C——2773 = 13 56 | |
| 4 | 8 | 13 | 40 | 8 | 12 | 40 | I C——3580 = 17 52 | P L–3198 = 26 18 |
| 5 | 8 | 18 | 37 | 8 | 17 | 37 | P L——4975 = 24 50 | I C–2334 = 19 13 |
| 6 | 8 | 21 | 06 | 8 | 20 | 06 | | P L–2989 = 24 35 |
| | | | | | | | Sol. Diamet. 6360 = 31 43 | 3850 = 31 40 |
| 7 | 8 | 28 | 01 | 8 | 27 | 01 | | P L–2888 = 23 57 |
| 8 | 8 | 29 | 01 | 8 | 28 | 01 | P L———4565 = 21 46 | |
| 9 | 8 | 35 | 12 | 8 | 34 | 12 | P L———4478 = 22 18 | A Z–2310 = 19 00 |
| 10 | 8 | 40 | 20 | 8 | 39 | 20 | I C———4417 = 22 00 | A Z–2070 = 17 02 |
| | 10 | 02 | 00 | 10 | 01 | 00 | Sol deinceps sub Nubibus receptus latuit usque ad Emergentis Limbus per Nubes Defectu Liber apparuit; | |
| | 10 | 04 | 00 | 10 | 03 | 00 | Prodibat Clarius, & nihil in ejus Limbo deficere compertum. | |

Pro Correctione Horologii, acceperam pridie *Eclipsis*, *Maii* 31. *Mane.*

| *Hor. Horologii.* h. | ′ | ″ | | *Alt.* ☉. ° | ′ | *Temp. inde sup.* h. | ′ | ″ | *Error.* ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 07 | 12 | Altitudinem Limbi Solis Inferioris. | 27 | 47 | 7 | 06 | 09 | 1 | 03 |
| 7 | 10 | 16 | ejusdem Limbi. | 28 | 16 | 7 | 09 | 19 | 0 | 57 |
| | | | Iterumque *Jun.* 1. *p. m.* | | | | | | | |
| 5 | 32 | 02 | Altitudinem Limbi Solis Inferioris. | 22 | 06 | 5 | 31 | 06 | 0 | 56 |
| 5 | 35 | 23 | Limbi Superioris. | 22 | 06 | 5 | 34 | 34 | 0 | 49 |
| 5 | 45 | 17 | Inferioris. | 20 | 6 | 5 | 44 | 18 | 0 | 59 |
| | | | Denique *Jun.* 2. *m.* | | | | | | | |
| 8 | 9 | 44 | Altitudinem Limbi Solis Inferioris. | 37 | 34½ | 8 | 08 | 45 | 0 | 59 |
| 8 | 13 | 36 | ———— | 38 | 09 | 8 | 12 | 34 | 1 | 02 |
| 8 | 15 | 44 | ———— | 38 | 28 | | | | | |
| 8 | 17 | 51 | ———— | 38 | 47 | | | | | |
| 8 | 20 | 1 | ———— | 39 | 07 | 8 | 18 | 49 | 1 | 11 |

Unde liquet, & Motus constantiam servasse Horologium, & in *Eclipsi* debitè fuisse correctum.

*Hora*

| Hora Horol. Oscillatorii. | Correct. per Lin. Merid. | Mensura | Phasium. | |
|---|---|---|---|---|
| h. ′ ″ | h. ′ ″ | | ′ ″ | ′ ″ |
| 8 06 45 | 8 8 27 | I C 1190 | 16 09 forsan 1109 = | 14 50 |
| 8 11 00 | 8 12 42 | PL 1935 | 26 15 | |
| 8 18 00 | 8 19 42 | I C 1405 | 19 04 | |
| 8 21 00 | 8 22 42 | PL 1805 | 24 30 | |
| 8 26 14 | 8 27 56 | I C 1504 | 20 47 | |
| 8 34 00 | 8 35 42 | PL 1711 | 23 13 | |
| 8 42 15 | 8 43 57 | I C 1551 | 21 03 accuraté. | |
| 8 46 30 | 8 48 12 | PL 1702 | 23 20 vel—-1720 = | 23 15 |
| 8 51 45 | 8 53 27 | I C 1553 | 21 04 accuraté. | |
| 9 00 00 | 9 01 42 | PL 1809 | 24 33 | |
| 9 12 34 | 9 14 16 | I C 1357 | 18 25 | |
| 9 30 55 | 9 32 37 | I C 872 | 11 50 | |
| 9 41 15 | 9 42 57 | Præcise. | | |

4. *At* Townley; *by Mr.* Rich. Townley. *Ibid.* 663.

*Fig.* 125.

Præcisè. Desiit Eclipsis, quantum per Aeris Vibrationem potui discernere. Exitûs Locus adeo Vertici vicinus erat, ut in quam ab ea partem inclinaret, bene non potuerim definire; etiamsi hora 9. 29′. per Horologium Cuspides Horizonti apparerent Parallelæ.

Solis Diameter hora 9. 10′. erat 2334; satis, ut putavi præcise.

Deinde, accedente Sole ad Meridiem per Lineam longam Meridianam, Horologium justo tardius inventum fuit Scrupulis 1′. 42″. Magno autem Æquinoctiali Sciaterico, quo medias minoresve Scrupuli horarii partes possum distinguere, Horologium toto hoc manè tardius duntaxat 45″. Correctioni tamen per Lineam Meridianam quàm Sciaterico fidendum puto.

5. *At* Wingfield, *near* Derby; *by Mr.* Imman. Halton. *Ibid. p.* 664.

7 h. 50′. Nihil sub Sole.
7 50½ Initium accuraté.
7 52 Notabilis defectus.
9 00 Digiti 3½.
9 11 Digiti $3\frac{1}{10}$.
9 21 Digiti $2\frac{1}{100}$.
9 47½ Non Finita; Imminente Fine.

*At* Paris; *by M.* Cassini. *Ibid.*

*Fig.* 126.

6. Cum Sol è nubibus Emergeret, Altitudinem graduum 48 accedens, ad eum direxi Quadrantem, quem ad hanc Altitudinem Immotum tenui. Ex quo, *Solis* Margo superior *c*, tetigit Filum Horizontale *c d*, in Foco Telescopii, ad adventum Centri *b*, fluxêre secundæ horariæ 104 = *a b* vel *b r*; A transitu Centri *b* ad transitum Marginis Lunæ Superioris *o*, Secundæ 11 = *b s*; A transitu Centri *b*, ad Cornu Superioris occidentalis *e*, fluxêre Secundæ 25½ = *e h*; A Transitu Centri *b*, ad transitum Cornu Inferioris & Orientalis *i*, Secundæ 93 = *i k*; Hinc determinatur Linea Cornuum *i e*, (seclusâ variatione) ejusque Inclinatio ad Horizontem *l k*; & Punctum *p* concursus Tangentis Lunam cum Secante *i e p*, & Tangens ipsa *p o*, Media proportionalis inter *p i*, *p c*: & Anguli *n o c*, *t o i*, hinc Angulus *i o c*; & Triangulum *i o c*, Lunari Circumferentiâ inscriptum.

Ex iis, aliisque ex *Astronomia* datis, deduxi

Initium esse debuisse *Parisiis*—7 h. 55′.
Finem vero————————10 12′. vel circiter.

Eclipsis

7. *At* Dantzick; *by* M. Hevelius. *Ib. p.666.*

| *Temp. juxta Sciatericum & Hor. Oscil.* h. ′ ″ | *Altitudines* ☉. ° ′ ″ | *Tempus ex Altitud. Sol. correct.* h. ′ ″ | *Ordo Phasium.* | *Magnitudo Phasium.* | *Animadvertenda.* |
|---|---|---|---|---|---|
| 7 58 10 | 36 17 0 | 7 58 18 | | | |
| 8 1 30 | 36 41 0 | 8 1 6 | | | |
| 8 3 30 | 37 3 0 | 8 3 39 | | | |
| 8 50 30 | | 8 50 0 | | | Sol omnino Purus apparuit. |
| 9 21 30 | | 9 21 0 | | | Nihil adhuc in Sole. |
| 9 22 30 | | 9 22 0 | 1 | Digit. | Initium Eclipſeos. |
| 9 24 10 | | 9 23 40 | 2 | 1/8 feré. | |
| 9 24 55 | | 9 24 25 | 3 | 1/2 Digit. | |
| 9 27 28 | | 9 27 0 | 4 | 3/4 Digit. | |
| 9 29 40 | | 9 29 10 | 5 | 1 Digit. | |
| 9 33 25 | | 9 33 0 | 6 | 1 1/4 Digit. | |
| 9 36 30 | | 9 36 5 | 7 | 1 5/8 feré. | |
| 9 39 35 | | 9 39 10 | 8 | 2 Digit. | |
| 9 45 49 | | 9 45 25 | 9 | 2 1/8 Digit. | |
| 9 54 22 | | 9 54 0 | 10 | 3 1/8 Digit. | |
| 10 3 44 | | 10 3 22 | 11 | 4 1/4 Digit. | |
| 10 8 30 | | 10 8 10 | 12 | 4 3/4 Digit. | |
| 10 18 17 | | 10 18 0 | 13 | 4 3/5 feré. | |
| 10 22 42 | | 10 22 22 | 14 | 4 1/2 pau. plus | |
| 10 26 19 | | 10 26 0 | 15 | 4 1/3 feré. | |
| 10 35 24 | | 10 35 6 | 16 | 4. 22′ | Hora 10. 31′. 0″. ferè, Max. incidit Obſcuratio. |
| 10 38 53 | | 10 38 38 | 17 | 4 1/4 feré. | |
| 10 47 34 | | 10 47 20 | 18 | 4 Dig. feré. | |
| 10 53 49 | | 10 53 30 | 19 | 3 5/8 Digit. | |
| 10 58 17 | | 10 58 8 | 20 | 3 2/8 Digit. | |
| 11 5 27 | | 11 5 20 | 21 | 2 7/8 Digit. | |
| 11 8 50 | | 11 8 44 | 22 | 2 1/4 Digit. | |
| 11 22 13 | | 11 22 8 | 23 | 1 3/4 feré. | |
| 11 29 14 | | 11 29 10 | 24 | 1 1/10 | |
| 11 35 25 | | 11 35 20 | 25 | 1/2 | |
| 11 36 59 | | 11 36 55 | 26 | 1/4 paul. plus | |
| 11 39 15 | | 11 39 15 | | | Nond. Sol omnino Purus Exſtitit. |
| 11 39 40 | | 11 39 40 | | | Finis Eclipſeos. |
| 4 18 10 | 33 11 0 | 11 18 19 | | | |
| 4 20 0 | 32 25 0 | 11 20 36 | | | |

*Semi-*

| | Ex Calculo Rudolph. | Ex Observat. | Diff. | Temp. |
|---|---|---|---|---|
| | ° ′ ″ | ° ′ ″ | ′ ″ | h. ′ ″ |
| Semid. ☉ | 0 15 0 | | | |
| Semid. ☾ | 0 15 3 | 0 13 53 | 1 10 | 10 0 0 |
| | | 0 14 0 | 1 3 | 10 24 0 |
| | | 0 14 50 | 0 13 | 11 0 0 |
| | | 0 15 0 | 0 0 | Ultimo |
| | h. ′ ″ | h. ′ ″ | | |
| Duratio. | 1 50 58 | 2 17 40 | | |

8. Diebus præcedentibus, locum aptissimum elegimus in quo Aëre Puro frueremur, videlicet *Conventum RR. PP. Carmelitarum Discalceatorum*, qui respectu Civitatis *Aven.* ad Ortum vergit & mœnia stringens Aëre, Fumo & Vaporibus urbanis libero gaudet; in medio Horti Cameram Obscuram Tapetibus construximus, & in eâ Instrumenta ad observationem necessaria ritè collocavimus. *At* Avignion *by* M. Gallet. n. 141. *p.* 1028.

Tubospicillum aptavimus Lente Oculari Concavâ, & Objectivâ Convexâ instructum, duplicem habens motum firmo Sustentaculo, Verticalem scil. & Horizontalem, affixam Tabellam immobilem firmatis cochleis secum circumducens, Oculari Vitro semper parallelam, chartâ candidissimâ indutam, in qua Solarem Speciem, distantiâ Tubospicilli determinatam descripsimus, hujus Diametrum Circulis Concentricis in duodecim Digitos divisimus, & quemlibet Digitorum in partes Sexagesimas.

Loco Quadrantis, qui pluribus indiget cautionibus, & nimium obnoxius est vacillationibus, Gnomonem ad captandas umbras Solis in partes 400, optimè divisum disposuimus, ita ut liberè moveretur Situm Verticalem ope perpendiculi conservans. Tandem Horologium Rotatile, minuta prima & secunda indicans, motu Penduli cum Cycloide præparavimus.

Ipsâ die Eclipsis undecimâ *Junii*, horâ unâ circiter post ortum Solis, usque ad Initium & Finem Eclipsis, Speciem ejus Lucidam in Charta, sine intermissione recepimus, & quilibet ex nobis Instrumento sibi destinato semper invigilavit; Dominus de *Beauchamps* Musarum *Avenionensium Mæcenas* Amplissimus, Ego quoque cum illo, Tubospicillo; Dominus de *S. Florent*, Visus perspicacissimi, Gnomoni; Dominus *Moutonier*, Horologio, una cum Domino *Marin* Presbytero, in Mathematicis & præsertim in Horologiis versatissimo.

Statim ac sensibiliter cœpit Umbra Discum inire, quantitatem partium Obscuratarum, Umbram in partibus Gnominis, & Horam horologii notavi è directo primæ Phasis, & ita collegi Phases 39, contentas in sequenti Tabella.

| Num. Phas. | Digiti Ob-scur. | | Umbra Gno-mon. in partibus qualium Gnomen continet. 400 | Altitudo Solis Apparens. | | | Altitudo Solis Vera. | | | Hora Horologii Penduli. | | | Hora correcta per Altitudinem Solis. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | D. | ' | | ° | ' | " | ° | ' | " | h. | ' | " | h. | ' | " | |
| 1 | 0 | 27 | 561 | 35 | 29 | 23 | 35 | 28 | 48 | 7 | 50 | 31 | 7 | 50 | 34 | |
| 2 | 1 | 0 | 536 | 36 | 44 | 0 | 36 | 43 | 28 | 7 | 57 | 25 | 7 | 57 | 28 | |
| 3 | 1 | 30 | 520 | 37 | 34 | 7 | 37 | 33 | 37 | 8 | 2 | 3 | 8 | 2 | 7 | |
| 4 | 3 | 0 | 478 | 39 | 55 | 23 | 39 | 54 | 57 | 8 | 15 | 14 | 8 | 15 | 13 | |
| 5 | 3 | 25 | 466 | 40 | 38 | 30 | 40 | 38 | 6 | 8 | 19 | 0 | 8 | 19 | 14 | |
| 6 | 4 | 30 | 438 | 42 | 24 | 14 | 42 | 23 | 53 | 8 | 29 | 19 | 8 | 29 | 6 | |
| 7 | 4 | 40 | 434 | 42 | 39 | 58 | 42 | 39 | 37 | 8 | 30 | 59 | 8 | 30 | 34 | |
| 8 | 5 | 0 | 424 | 43 | 19 | 53 | 43 | 19 | 32 | 8 | 34 | 34 | 8 | 34 | 18 | |
| 9 | 5 | 30 | 412 | 44 | 9 | 12 | 44 | 8 | 52 | 8 | 39 | 19 | 8 | 38 | 56 | |
| 10 | 6 | 0 | 394 | 45 | 10 | 57 | 45 | 10 | 39 | 8 | 44 | 54 | 8 | 44 | 44 | Cornua Verticalia. |
| 11 | 6 | 40 | 375 | 46 | 35 | 50 | 46 | 35 | 33 | 8 | 53 | 19 | 8 | 52 | 45 | |
| 12 | 6 | 50 | 371 | 46 | 54 | 15 | 46 | 54 | 0 | 8 | 54 | 54 | 8 | 54 | 31 | |
| 13 | 7 | 0 | 366 | 47 | 17 | 30 | 47 | 17 | 14 | 8 | 56 | 44 | 8 | 56 | 44 | |
| 14 | 7 | 20 | 350 | 48 | 30 | 37 | 48 | 30 | 23 | 9 | 3 | 44 | 9 | 3 | 44 | Maxima obscuratio. |
| 15 | 7 | 8 | 339 | 49 | 29 | 10 | 49 | 28 | 58 | 9 | 9 | 14 | 9 | 9 | 15 | |
| 16 | 7 | 0 | | | | | | | | 9 | 11 | 0 | | | | |
| 17 | 6 | 35 | 325 | 50 | 39 | 22 | 50 | 39 | 10 | 9 | 15 | 54 | 9 | 16 | 12 | |
| 18 | 6 | 25 | 321 | 51 | 0 | 12 | 51 | 0 | 0 | 9 | 18 | 14 | 9 | 18 | 11 | |
| 19 | 5 | 25 | 296 | 53 | 10 | 29 | 53 | 10 | 19 | 9 | 31 | 5 | 9 | 31 | 1 | |
| 20 | 5 | 0 | 286 | 53 | 55 | 14 | 53 | 55 | 5 | 9 | 35 | 44 | 9 | 35 | 30 | |
| 21 | 4 | 40 | 283 | 54 | 27 | 6 | 54 | 26 | 57 | 9 | 38 | 39 | 9 | 38 | 43 | |
| 22 | 4 | 35 | | | | | | | | 9 | 42 | 3 | | | | Cornua Parallela Horizonti. |
| 23 | 4 | 0 | | | | | | | | 9 | 47 | 19 | | | | |
| 24 | 3 | 53 | 266 | 56 | 3 | 35 | 56 | 3 | 27 | 9 | 48 | 45 | 9 | 48 | 36 | |
| 25 | 3 | 35 | 262 | 56 | 28 | 29 | 56 | 28 | 22 | 9 | 51 | 29 | 9 | 51 | 11 | |
| 26 | 3 | 30 | 262 | 56 | 37 | 32 | 56 | 37 | 25 | 9 | 52 | 11 | 9 | 51 | 59 | |
| 27 | 3 | 26 | 260 | 56 | 43 | 34 | 56 | 43 | 27 | 9 | 52 | 34 | 9 | 52 | 45 | |
| 28 | 3 | 6 | 254 | 57 | 15 | 59 | 57 | 15 | 53 | 9 | 56 | 5 | 9 | 56 | 10 | |
| 29 | 3 | 0 | | | | | | | | 9 | 57 | 40 | | | | |
| 30 | 2 | 48 | 249 | 57 | 50 | 48 | 57 | 50 | 42 | 9 | 59 | 34 | 9 | 59 | 53 | |
| 31 | 2 | 35 | 246 | 58 | 9 | 32 | 58 | 9 | 26 | 10 | 1 | 34 | 10 | 1 | 53 | |
| 32 | 2 | 25 | 243 | 58 | 26 | 11 | 58 | 26 | 5 | 10 | 3 | 0 | 10 | 3 | 41 | |
| 33 | 2 | 0 | | | | | | | | 10 | 6 | 46 | | | | |
| 34 | 1 | 50 | 236 | 59 | 12 | 33 | 59 | 12 | 28 | 10 | 8 | 56 | 10 | 8 | 47 | |
| 35 | 1 | 0 | 226 | 60 | 16 | 59 | 60 | 16 | 55 | 10 | 15 | 51 | 10 | 16 | 0 | |
| 36 | 0 | 40 | 220 | 60 | 56 | 21 | 60 | 56 | 16 | 10 | 20 | 57 | 10 | 20 | 31 | |
| 37 | 0 | 30 | 217 | 61 | 16 | 11 | 61 | 16 | 6 | 10 | 22 | 54 | 10 | 22 | 50 | |
| 38 | 0 | 20 | 214 | 61 | 36 | 12 | 61 | 36 | 8 | 10 | 25 | 0 | 10 | 25 | 12 | Cornu occidentale verticale cum Centro Solis. |
| 39 | Finis. | | 209 | 62 | 6 | 23 | 62 | 6 | 19 | 10 | 28 | 41 | 10 | 28 | 50 | |

Proportio Diametrorum apparuit æqualis in Eclipsi 6 Digitorum, tunc enim Cornua Solis Verticalia distabant à Verticali Solis hinc inde gradibus circiter 30. Unde patet Centrum Lunæ tunc reperiri in Peripheria Solis, & Lineam Diacentron esse æqualem Semidiametro Solis. Verùm post Medium Eclipsis, mutationem aliquam in Diametro Umbræ deprehendimus; apparuit enim Umbra paululum magis Convexa, & ideo Semidiameter brevior, sed ferè insensibiliter.

XXXIV. *An Eclipse of the Sun, July 2d. 1684. at Greenwich; by Mr. Flamsteed.* n. 162. p. 691.

| *Temp. per Horolog. Oscillator.* h. ′ ″ | *Temp. inde ab observ. correcta.* h. ′ ″ | *Observationes.* | *Partes Obscur. quarum 32 æquales erant Diam.* | *Partes liberæ Micrometro.* ′ ″ |
|---|---|---|---|---|
| 2 12 30 | 2 03 45 | Sol integer; tunc nubes. | ☉ | |
| 2 21 2 | 2 12 17 | ☉ def. perexiguum ima Ora. | | |
| 2 21 25 | 2 12 40 | Partes obscuratæ supra Scenam. | 0½ | |
| 2 32 20 | 2 23 35 | | 4 | |
| 2 46 40 | 2 37 55 | | | 22 19 |
| 2 51 37 | 2 42 52 | | 11 | |
| 3 5 23 | 2 56 38 | | 15 | |
| 3 9 5 | 3 00 20 | Centrum | 16 | |
| 3 10 4 | 3 1 19 | | | 15 07 |
| 3 11 47 | 3 3 2 | | | 14 36 |
| 3 12 15 | 3 3 30 | | 17 | |
| 3 13 23 | 3 4 38 | Cornua Horizontalia | | |
| 3 17 43 | 3 8 58 | | 18 | |
| 3 21 48 | 3 13 3 | | | 12 30 |
| 3 23 0 | 3 14 15 | | 19 | |
| 3 25 40 | 3 16 55 | | | 12 11 |
| 3 29 51 | 3 20 6 | | 19½ | |
| 3 33 54 | 3 25 9 | | 19½ | |
| 3 40 8 | 3 31 23 | | 19 | |
| 3 43 16 | 3 34 31 | | 18½ | |
| 3 45 35 | 3 36 50 | | 18 | |
| 3 49 15 | 3 40 30 | Cuspis inferior ad Nadir ☉s | | |
| 3 50 2 | 3 41 17 | | 17 | |
| 3 54 20 | 3 45 35 | Centrum | 16 | |
| 3 56 45 | 3 48 00 | | 15 | |
| 4 2 58 | 3 54 13 | | 13 | |
| 4 6 0 | 3 57 15 | ferè | 12 | |
| 4 8 45 | 4 00 00 | | 11 | |
| 4 16 32 | 4 7 47 | | 8 | |
| 4 17 44 | 4 9 00 | Cuspides Verticales dist. 84 gr. | | |
| 4 24 8 | 4 15 23 | Inter Cuspides | | 17 32 |
| 4 24 20 | 4 15 35 | | 5 | |
| 4 27 14 | 4 18 29 | Inter Cuspides | | 15 35 |
| 4 29 36 | 4 20 51 | | 2½ | |
| 4 30 32 | 4 21 47 | Inter Cuspides | | 12 22 |
| 4 33 06 | 4 24 21 | | 1 | |
| 4 36 22 | 4 27 37 | Finis accuratè | | |

Deinde ad Errorem *Horologii* investigandum.

| *Hor. Horol.* | *Distantiæ à Vertice.* | | *Hor. inde sup.* | *Error. Hor.* |
|---|---|---|---|---|
| h. ′ ″ | | ° ′ ″ | h. ′ ″ | ′ ″ |
| 4 43 38 | Limbi Solis | 60 11 00 | 4 34 56 | 8 42 |
| 4 45 48 | Inferioris | 60 31 20 | 4 37 7 | 8 41 |
| 4 46 56 | | 60 41 50 | 4 38 15 | 8 43 |
| 4 49 3 | | 61 1 40 | 4 40 22 | 8 41 |

*At* Paris; *by* M. Bullialdus. n. 162. *p.* 693.

2. Alt. ☉. 50°. Initium Elapsum erat, Sole Nubibus tecto, & Digitus circiter deficiebat.

Alt. ☉. 41°. 15′. Paulo amplius quam Digiti 7. Attigit Digitos, 8.

Alt. ☉. 29°. 30′. Finis.

*At the* Observatory; *by* M. Cassini. n. 162. *p.* 693. n. 163. *p.* 715.

3. The Beginning of the Eclipse could not be seen, but was deduced from the following Phases. The Apparent Diameter of the Moon appear'd less than that of the Sun. It was judged that the Dilatation of the Sun's Light, might make the Moon's Diameter seem less.

| *Phases.* | *Time.* | *Phases.* | *Time.* |
|---|---|---|---|
| | h. ′ ″ | | h. ′ ″ |
| Begin. | 2 25 30 | 7⅞ Digits | 3 35 00 |
| or | 2 25 55 | 7 Digits | 3 55 50 |
| 1 Digit | 2 32 50 | 6 Digits | 4 4 10 |
| 2 Digits | 2 40 00 | 5 Digits | 4 12 25 |
| 3 Digits | 2 47 40 | 4 Digits | 4 19 15 |
| 4 Digits | 2 54 10 | 3 Digits | 4 25 50 |
| 5 Digits | 3 2 00 | 2 Digits | 4 32 15 |
| 6 Digits | 3 10 5 | 1 Digits | 4 37 40 |
| 7 Digits | 3 20 10 | End. | 4 43 23 |

4. *By* M. de la Hire *and* Pothenot. *Ib.* *p.* 716.

| *Phases.* | *Time.* | *Phases.* | *Time.* |
|---|---|---|---|
| Begin- | h. ′ ″ | | h. ′ ″ |
| ning. | 2 25 24 | 7 Digits | 3 53 34 |
| 1 Digit | 2 33 2 | 6 Digits | 4 3 53 |
| 2 Digits | 2 40 30 | 5 Digits | 4 11 3 |
| 3 Digits | 2 47 47 | 4 Digits | 4 17 42 |
| 4 Digits | 2 54 41 | 3 Digits | 4 25 14 |
| 5 Digits | 3 2 41 | 2 Digits | 4 31 56 |
| 6 Digits | 3 12 6 | 1 Digit | 4 38 11 |
| 7 Digits | 3 20 54 | End | 4 43 27 |
| 7 Dig. 5′ | 3 36 27 | | |

The

The Beginning was deduced from many Obſervations made ſoon after it. The Moon's Diameter appear'd then not to be more than about 30′; though by the Obſervations of her Diameter made ſome days before and after, it was judged to be 31′. 30″. But the Extremities of the Horns, on which depended the Exactneſs of that Determination, appear'd a little Blunted.

5. *At the College of Lewis the Great; by R. P. Fontenay. Ib. p. 717.*

| Phaſes | Time. | | | Phaſes. | Time. | | |
|---|---|---|---|---|---|---|---|
| Leſs than | h. | ′ | ″ | | h. | ′ | ″ |
| ½ Digit | 2 | 29 | 30 | 7 Digits | 3 | 51 | 20 |
| 1½ Digit | 2 | 37 | 40 | 6 Digits | 4 | 2 | 25 |
| 2 Digits | 2 | 40 | 25 | 5 Digits | 4 | 10 | 50 |
| 3 Digits | 2 | 48 | 34 | 3 Digits | 4 | 24 | 31 |
| 4 Digits | 2 | 54 | 30 | 2 Digits | 4 | 29 | 54 |
| 5 Digits | 3 | 3 | 00 | 0⅓ feré | 4 | 41 | 00 |
| 6 Digits | 3 | 12 | 40 | | | | |
| 7 Digits | 3 | 22 | 18 | | | | |
| 7¾ Digits | 3 | 38 | + | | | | |

6. The Beginning was at 2h. 54′. 30″. The End at 5h. 9′. 9″. The Greatneſs of the Eclipſe 8½ Digits. *At Aix; by M. Gautier. Ib. p. 718.*

7. *At Lyons; by R. P. Paul Hoſte. Ib.*

| Phaſes. | Time. By FixtStars. | | | By theSun. | | |
|---|---|---|---|---|---|---|
| | h. | ′ | ″ | h. | ′ | ″ |
| 1 Digit | 2 | 45 | 3 | 2 | 50 | 3 |
| 8½Digits | 3 | 53 | 52 | 3 | 58 | 52 |
| 1 Digits | 4 | 59 | 20 | 5 | 4 | 20 |

At 3h. 26′. 14″. (by the *Stars*) The Diameter of the *Sun* and *Moon* 30′. 58″. but at 4h. 20′. 34″. The Diameter of the *Sun* 30′. 58″. of the *Moon*, 30′. 5″.

8. *At the Bay de Roſes; by M. Chaſſelles. Ib. p. 719.*

| Phaſes. | Times. h. | ′ | ″ |
|---|---|---|---|
| The Beginning of the Eclipſe | 2 | 40 | 00 |
| The Edge of the Moon at the Sun's Center | 3 | 25 | 00 |
| The Horns Horizontal | 3 | 40 | 00 |
| The Horns Vertical | 4 | 15 | 00 |
| The End of the Eclipſe | 5 | 1 | 30 |

The Greatneſs of the Eclipſe about ¾ of the Sun's Diameter, at which time *Venus* might be ſeen without Pain.

9. The

*At* Honfleur; *by* M. de Glos. *Ib.*

9. The Beginning was at 2h. 15′. 2″. The End, at 4h. 34′. 35″. The Greatneſs, more than 8 Digits, but leſs than 9.

*At* Pau; *by* R. P. Richaud. *Ib.*

10. At $1\frac{3}{4}$h. The Eclipſe was not Begun. At $3\frac{1}{4}$h. at 10 Digits. The End, at $4\frac{3}{4}$h.

11. *At* Avignon; *by* R. P. Bonfa. *Ib.*

| *Phaſes.* | *Time.* | *Phaſes.* | *Time.* |
|---|---|---|---|
| | h. ′ ″ | | h. ′ ″ |
| The Beging. | 2 43 27 | Horns Vert. | 4 24 32 |
| 1 Digit | 2 51 58 | $1\frac{1}{2}$ Dig. | 5 1 16 |
| 9 Digits | 4 2 00 | The End. | 5 4 37 |

The Sun's Diameter 31′. 38″. The Moon's 30′. 6″.

*At* Oxford; *by* Dr. Ed. Bernard. n. 164. p. 747.

12. Phaſes Quadraginta hujus *Defectus* captavit definivitq; bona manus *Wallisii.* Tempus etiam juſtum æquumque ejuſdem Deliquii Altitudinibus aliquot Solaribus comprobarunt Clariſſimi Viri *D. Caſuellus* & *D. Rookius*, ubi Horologiis & Oſcillis noſtris fuerat peccatum.

| *Digiti Obſcuri, cum ſuis decimis.* | *Tempora Oſcillatoria à Cœlo Correcta.* h. ′ ″ | |
|---|---|---|
| | 2 03 00 | *Initium* Eclipſis. |
| 0 6 | 2 07 44 | |
| 1 0 | 2 10 44 | |
| 1 3 | 2 13 19 | |
| 1 6 | 2 15 44 | |
| 2 2 | 2 21 34 | |
| 2 6 | 2 23 44 | |
| 2 9 | 2 25 39 | |
| 3 5 | 2 29 54 | |
| 4 0 | 2 33 04 | |
| 4 4 | 2 36 34 | |
| 4 6 | 2 40 04 | |
| 5 0 | 2 43 14 | |
| 5 3 | 2 48 19 | |
| 5 6 | 2 50 09 | Medium quaſi *Solem* fecit τὸ Ἀσελαγενόν. |
| 6 5 | 2 57 30 | |
| 6 7 | 2 59 14 | |
| 7 4 | 3 08 24 | Obſcuritas Maxima. |
| 7 1 | 3 11 09 | |
| 6 8 | 3 29 19 | |
| 6 7 | 3 31 39 | αὖθις ἐπὶ κέντρον τὸ σκότος. |
| 6 0 | 3 37 24 | |
| 5 7 | 3 39 29 | |

| | | | | | | |
|---|---|---|---|---|---|---|
| 5 | 5 | 3 | 44 | 29 | | |
| 5 | 0 | 3 | 47 | 59 | | |
| 4 | 4 | 3 | 50 | 34 | | |
| 3 | 9 | 3 | 53 | 54 | | |
| 2 | 9 | 4 | 2 | 49 | | |
| 2 | 5 | 4 | 4 | 39 | | |
| 2 | 2 | 4 | 6 | 19 | | |
| 2 | 0 | 4 | 7 | 54 | | |
| 1 | 4 | 4 | 13 | 9 | | |
| 1 | 0 | 4 | 15 | 4 | | |
| 0 | 7 | 4 | 16 | 49 | | |
| 0 | 3 | 4 | 19 | 39 | | |
| | | 4 | 21 | 14 | Finis Eclipsis. | |

13. Mr. *Jacobs* at *Lisbon* Noted. *At* Lisbon. *Ib.* *p.* 749.
The Beginning of this Eclipse at 1h. 30′. exactly.
The Ending at ——————— 4 12.

14. Mr. *Ash* and Mr. *Molyneaux*, toward the Middle of the Eclipse, having a short View of the Sun; they Judged that about 8 Digits were covered: at the Ending also having a faint View thereof; they assigned its End, at 3h. 56′. p. m. *In* Ireland. *Ib.*

The same Eclipse was observed by one Mr. *Osburn*, nigh *Tredagh*. *Initium* 1h. 37.′ 30″. *Finis* 3h. 56′. 20″.

15. Desumebant Solis à Vertice Distantias D. D. *Jo. Ludovicus Donellus*, & *Nicolaus Ignatius Joanettus*; Phasibus determinandis tres aderamus, D. D. *Jo. Galeatius Manzius*, *Hercules Vanottus*, & *Ego*. Observationes scribebat D. *Gregorius Malisardus*; Horas vero in Horologio notabat D. *Bartolomæus Ferrarius*. *At* Bononia; *by* S. Domin. Gulielmini. n. 203. *p.* 858.

| *Horæ Horologii.* | *Horæ erutæ ex Distantiis observatis.* | *Distantiæ Centri Solis à Vertice.* | *Phases.* |
|---|---|---|---|
| h. ′ ″ | h. ′ ″ | ° ′ | ′ |
| 3 34 | | | Dig. 2 30 |
| 3 37 30 | | | Dig. 3 |
| 3 48 14 | 3 47 47 | 51 31 | Dig. 4 20 dubia ubique. |
| 3 52 00 | | | Dig. 5 optima. |
| 3 57 00 | 3 56 2 | 52 58 | Dig. 5 30 |
| 4 2 00 | 4 2 29 | 54 10 | Dig. 6 |
| 4 4 45 | 4 5 59 | 54 44 | Dig. 6 30 optima. |
| 4 9 20 | 4 9 1 | 55 17 | Dig. 7 exacta. |
| 4 14 40 | 4 15 43 | 56 28 | Dig. 7 |
| 4 19 00 | | | Dig. 7 20 dubia. |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 27 | 44 | 4 | 28 | 42 | 58 | 39 | Dig. | 7 | 20 | |
| 4 | 32 | 35 | | | | | | Dig. | 7 | | ſatis exacta. |
| 4 | 34 | 20 | | | | | | Dig. | 6 | 45 | |
| 4 | 44 | 30 | | | | | | Dig. | 6 | 30 | |
| 4 | 47 | 15 | | | | | | Dig. | 6 | 30 | |
| 4 | 51 | 30 | | | | | | Dig. | 6 | | optima. |
| 4 | 54 | 38 | 4 | 53 | 53 | 63 | 16 | Dig. | 5 | 30 | exacta. |
| 4 | 58 | 2 | 4 | 57 | 38 | 63 | 56 | Dig. | 5 | | exacta. |
| 5 | 1 | 55 | 5 | 1 | 2 | 64 | 32 | Dig. | 4 | 30 | ſatis exacta. |
| 5 | 4 | 40 | 5 | 5 | 28 | 65 | 19 | Dig. | 4 | | Diligens ubique. |
| 5 | 7 | 0 | 5 | 6 | 23 | 65 | 28 | Dig. | 3 | 30 | exacta. |
| 5 | 10 | 30 | 5 | 9 | 9 | 65 | 58 | Dig. | 3 | | exacta. |
| 5 | 12 | 35 | 5 | 13 | 28 | 66 | 43 | Dig. | 2 | 30 | ſatis exacta. |
| 5 | 16 | 15 | 5 | 16 | 14 | 67 | 13 | Dig. | 2 | | exacta. |
| 5 | 19 | 50 | 5 | 20 | 24 | 67 | 57 | Dig. | 1 | 30 | exacta. |
| 5 | 22 | 30 | 5 | 23 | 21 | 68 | 28 | Dig. | 1 | | exacta. |
| 5 | 25 | 0 | 5 | 25 | 14 | 68 | 48 | Dig. | 0 | 30 | diligens. |
| 5 | 27 | 40 | 5 | 28 | 7 | 69 | 18 | Dig. | 0 | | accuratiſſima. |

Notabile fuit; cum Quantitas Eclipſis fuerit Dig 7. 20′. quod non modicam Aëris Offuſcationem debebat inducere, (ut aliàs multoties in conſimilibus Defectibus obſervatum eſt,) nihilominus tamen vix ſenſibiliter conſuetum in Sole Libero Aëris ſtatum mutatum fuiſſe; unde plurimis Solem non reſpicientibus orta ſuſpicio, aut Solem non defeciſſe, aut minimum quidem; cujus quidem rei non alia mihi videtur aſſignanda cauſa, quam ingens vis Nubium à Sole maximè illuminatarum, quæ non multum ad eo diſtabant; ab his enim Solis Radius per Reflexionem & Refractionem multiplicatus certè Intenſior redditus deficientem aliundè Splendorem potuit compenſare.

*An Eclipſe of the Sun,* May 1. 1687. *at* Oxford. n. 187. p. 329. *In divers other Places.* n. 189. p. 370.

XXXV. 1. Dr. *Wallis* writes from *Oxford* that this Eclipſe of the Sun was obſerved there about ½ a Digit; between one and two a Clock Afternoon.

2. Hæc *Eclipſis*, etiamſi contemnendæ Quantitatis fuerit, ac nudis oculis non omnino percipi potuerit, tamen ad accuratam determinationem Parallaxis & Latitudinis Lunæ maximè idonea videtur.

*Londini*, ſeorſim obſervantibus *Hookio* & *Halleio*; Initii Momentum, Cœlo licet puriſſimo, ob Obliquam Incidentiam Lunæ, debite Definire non licuit. Sed $1^h. 16'$. jam cœpta erat Eclipſis ſatis notabiliter : circa $1^h. 40'$. prope Medium Eclipſis, Chorda partis Eclipſatæ, ſive inter Cornua, inventa eſt $9'. 30''$. cui reſpondet Arcus $36^{\circ}$. in Diametro vero non niſi $1'. 30''$. Finis, conſenſu utriuſque Obſervatoris, contigit accurate $2^h. 3'. 0''$.

*Grenovici* in *Obſervatorio Regio*, *Flamſteedius* eadem de cauſa, Initium non vidit: Finem vero determinavit $2^h. 4'. 15''$. Medio Eclipſis ſive Maxima Obſcuratione, Chorda Partis Eclipſatæ erat $9'. 54''$.

Apud *Totteridge* prope *Londinum* verſus Corum, Finem vidit D. *Haines*, R. S. S. ad $2^h. 2'$. Quantitatem vero Maximam dimidii Digiti ab Auſtro.

In

In Insula *Barbada*, ad Oppidum *Bridge-Town*, Finem habuit D. *Frank* 1'. 30''. ante quam Solis Altitudo fuit 31°. 47'. ad ortum, hoc est 7^h. 56'. 45''. Quantitatem Maximam æstimatione definivit duorum Digitorum ab Austro.

*Norimbergæ* eandem Eclipsin observavit J. P. *Wurtzelbaur*. Initium quidem accuratè ad 1^h. 58'½; circa Medium, sc. ad 2^h. 36'¼, Quantitatem Maximam duorum Dig. præcise; Finem verò ad 3^h. 18'. 33''.

*Ulmæ Sueviæ*, Observavit *Honoldus* Initium ad 1^h. 48'.; Quantitatem Maximam 2 ⅓ Dig. Finem verò ad 3^h. 16'.

*Lipsiæ*, observatore *Kirchio*, Eclipsis jam satis notabilis ad 2^h. 20'. 10''. ad 2^h. 47'½, Digiti 1⅓ circiter. Finis verò Incidit præcise in 3^h. 15'.

*Uratislaviæ Silesiæ* denique observavit D. G. *Schultzius*. Maximam Obscurationem paulo citius quam 3^h. 12'. fuisse 1½ Dig. Finem verò 3^h. 37'.

XXXVI. 1. I did not see the Beginning of the late Eclipse, but the End happen'd here, precisely 24'. 9''. after 10 a Clock in the Morning, Apparent Time; The Greatest Obscuration, which was 10 Digits and a Quarter, was about 7 Min. after Nine.

*An Eclipse of the Sun*, Sept. 13. 1699. *at* Oxford; *by Dr.* D. Gregory. n. 256. *p.* 330.

2. *At* Nuremberg, *by* M. Worzelbaur. n. 265. *p.* 619.

| *Phases.* | *Quantities Eclipsed.* | | *Times by a Pendulum.* | | | *Phases.* | *Quantities Eclipsed.* | | *Times by a Pendulum.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Dig. | ' | h. | ' | '' | | Dig. | ' | h. | ' | '' |
| Initium | | | 8 | 57 | 14 | 20 | 9 | 49 | 10 | 30 | 10 |
| 1 | 0 | 52 | 9 | 3 | 26 | 21 | 9 | 21 | 10 | 33 | 11 |
| 2 | 1 | 32 | 9 | 8 | 23 | 22 | 8 | 52 | 10 | 35 | 53 |
| 3 | 2 | 28 | 9 | 14 | 14 | 23 | 8 | 30 | 10 | 38 | 46 |
| 4 | 3 | 19 | 9 | 19 | 40 | 24 | 7 | 38 | 10 | 42 | 42 |
| 5 | 4 | 8 | 9 | 24 | 57 | 25 | 7 | 14 | 10 | 46 | 7 |
| 6 | 5 | 15 | 9 | 31 | 57 | 26 | 6 | 33 | 10 | 49 | 42 |
| 7 | 5 | 50 | 9 | 35 | 2 | 27 | 6 | 6 | 10 | 53 | 22 |
| 8 | 6 | 26 | 9 | 38 | 43 | 28 | 5 | 27 | 10 | 56 | 37 |
| 9 | 6 | 53 | 9 | 40 | 36 | 29 | 5 | 9 | 11 | 0 | 00 |
| 10 | 7 | 20 | 9 | 43 | 47 | 30 | 4 | 33 | 11 | 4 | 24 |
| 11 | 7 | 56 | 9 | 50 | 39 | 31 | 3 | 57 | 11 | 8 | 16 |
| 12 | 8 | 39 | 9 | 55 | 9 | 32 | 3 | 13 | 11 | 13 | 3 |
| 13 | 9 | 23 | 10 | 1 | 44 | 33 | 2 | 41 | 11 | 18 | 3 |
| 14 | 9 | 53 | 10 | 5 | 46 | 34 | 2 | 11 | 11 | 21 | 37 |
| 15 | 10 | 24 | 10 | 10 | 34 | 35 | 1 | 32 | 11 | 25 | 38 |
| 16 | 10 | 38 | 10 | 14 | 37 | 36 | 1 | 2 | 11 | 28 | 27 |
| 17 | 10 | 45 | 10 | 17 | 54 | Finis. | 0 | 00 | 11 | 33 | 56 |
| 18 | 10 | 45 | 10 | 22 | 29 | | | | | | |
| 19 | 10 | 12 | 10 | 27 | 31 | | | | | | |

From the 8th to the 12th Phasis, the Opaque Limb of the Moon on the South side, was a little rough, but about the Northern Horn to near a 4th part of the Segment, it was more smooth: But when the Horns of the Eclipse were almost Parallel to the Horizon, before and after the 15th Phasis, the Extremity of the Gibbous Limb of the Moon looking downward, was somewhat Enlightned, and of a kind of a Saffron Colour; but though the Sky was free from Clouds, yet no Stars were visible. Nor was even *Venus* it self visible in the open Air, unless by some more Sharp-sighted than Ordinary.

Amongst many round Plates cut out of thick Paper of divers Magnitudes, differing from one another 5″, about the first Phasis, and after, none agreed to the Limb of the Moon but that which was cut to a Radius or Semidiameter of 15′. 30″. (taking the Radius or Semidiameter of that of the Sun to be 16′. 4″.) and that gradually so swelled or augmented, that larger Plates were necessary to be made use of; and about the 36th Phasis, none less than one described of a Radius of 16′. 5″. would agree with, or equal the Appearance; and consequently that the Diameter of the Moon about the End of the Eclipse did Equalize, if not exceed that of the Sun. Besides, in the 27th Phasis (when the obscure Part was 6. Dig. 6′.) the Body of the Moon did Obscure more than two Thirds of the Sun's Limb; which is an Argument that its Semidiameter at that time was Equal to that of the Sun.

*By Others. ib. p. 623.* 3. This Eclipse by the Observations of M. *Godfred Tuber* at *Ciza*, Began at 9 h. and Ended at 11h. 35′. and increas'd to 11 Dig.; by the Observations of Mr. *Jacob Honold* at *Hervelsing* near *Ulm* of *Suevia*, it Began at 8h. 55′. and Ended at 11h. 31′. and its Greatest Defect was 10 Dig.; And by Observations at *Leipsick*, it Began at 9 h. 11′. and Ended at 12 h. 38′. 30″. The greatest Obscurity was 11 Dig. 20′. which lasted from 10 h. 16′. 45″. for 6′. Ten Digits being obscured, the Sky (being otherwise very clear) began to appear of a more livid or wan Complexion, and more Sad than it usually looks with a clear Sky when the Sun is set, or below the Horizon. The Cocks also, which had hitherto Crowed very frequently, as if silenced, going to Roost left off Crowing, and did not renew it till by the Recovery of the Sun's Light they had recover'd their former Gayety and Mirth: However, we cannot learn that any Star besides that of *Venus* was discover'd by those which were Spectators of it in the Open Air.

*Changes likely to be discover'd in the Moon; by M. Auzout. n. 7. p. 120.* XXXVII. I sometimes think that the Earth must appear to the supposed Inhabitants of the Moon to have a different Face in the several Seasons of the Year; and to have another Appearance in Winter, when there is almost nothing Green in a very great Part of the Earth; when there are Countries all cover'd with Snow, others all cover'd with Water, others all obscur'd with Clouds, and that for many Weeks together; Another in Spring, when the Forests and Fields are Green; Another in Summer, when whole Fields are Yellow, &c. Methinks, I say, that these Changes are considerable enough in the Force of the Reflexions of Light to be observ'd, since we see so many differences of Lights in the Moon. We have Rivers considerable enough to be seen, and they enter far enough into the Land, and have a breadth capable to be observed. There are Fluxes in certain Places, that reach into large Countries, enough to make there some apparent Change, and in some of our Seas there float sometimes

such

ſuch bulky Maſſes of Ice, that are far greater than the Objects, which we are aſſured we can ſee in the Moon. Again, we cut down whole Forreſts, and Drain Marſhes, of an Extent large enough to cauſe a notable Alteration: And Men have made ſuch Works, as have produced Changes great enough to be perceived. In many places alſo are *Vulcans* that ſeem big enough to be diſtinguiſht, eſpecially in the Shadow. And when Fire lights upon Foreſts of great Extent, or upon Towns, it can hardly be doubted, but theſe Luminous Objects would appear either in an Eclipſe of the Earth, or when ſuch parts of the Earth are not Illuminated by the Sun. But yet, I know no Man, who hath yet obſerved ſuch things in the Moon; and one may be rationally aſſur'd, that no *Vulcans* are there, or that none of them Burn at this time. This it is, which all curious Men, that have good Teleſcopes, ought well to attend; and I doubt not, but if we had a very particular Map of the Moon, as I had deſign'd to make one, with a *Topography*, as it were, of all the conſiderable Places therein, that we or our Poſterity would find ſome Change in Her. And if the Maps of the Moon of *Hevelius*, *Divini*, and *Riccioli*, are exact, I can ſay that I have ſeen there ſome places conſiderable enough, where they put Parts that are Clear, whereas I there ſee Dark ones. 'Tis true, that if there be Seas in the Moon, it can hardly fall out otherwiſe, than it doth upon our Earth, where Alluviums are made in ſome Places, and the Sea gains upon the Land in others; I ſay, If thoſe Spots we ſee in the Moon are Seas, as I muſt believe them to be, whereas I have many Reaſons, that make me doubt, whether they be ſo. And I have ſometimes thought, whether it might not be, that all the Seas of the Moon, if there muſt be Seas, were on the ſide of the other Hemiſphere, and that for this Cauſe it might be, that the Moon turns not upon its Axis, as our Earth wherein the Lands and Seas are as it were Ballanced: That thence alſo may proceed the Non-appearance of any Clouds raiſed there, or of any Vapours conſideſiderable enough to be ſeen, as there are raiſed upon this Earth; and that this Abſence of Vapours is perhaps the Cauſe, that no Crepuſcle is there, as it ſeems there is none, my ſelf at leaſt not having been hitherto able to diſcern any Mark thereof: For methinks, it is not to be doubted, but that the reputed Citizens of the Moon, might ſee our Crepuſcle, ſince we ſee, that the ſame is without Compariſon ſtronger, than the Light afforded us by the Moon, even when ſhe is Full; for a little after Sun-ſet, when we receive no more the firſt Light of the Sun, the Sky is far clearer, than it is in the Faireſt Night of the Full Moon. Mean while, ſince we ſee in the Moon, when ſhe is Encreaſing or Decreaſing, the Light ſhe receives from the Earth, we cannot doubt, but that the People of the Moon ſhould likewiſe ſee in the Earth that Light, wherewith the Moon illuminates it, with perhaps the difference there is betwixt their Bigneſs. Much rather therefore ſhould they ſee the Light of the Crepuſcle, being as we have ſaid, incomparably greater. In the mean time, we ſee not any Faint Light beyond the Section of the Light, which is every where almoſt equally ſtrong, and we there diſtinguiſh nothing at all, not ſo much that cleareſt part, which is call'd *Ariſtarchus*, or *Porphyrites*, as I have often try'd; although one may there ſee the Light, which the Earth ſends thither, which is ſometimes ſo ſtrong, that in the Moon's Decreaſe, I have often diſtinctly ſeen all the Parts of the Moon that were not Enlightned by the Sun, together with the Difference of the

 Clear

Clear parts and the Spots, ſo far as to be able to diſcern them all. The Shadows alſo of all the Cavities of the Moon ſeem to be ſtronger, than they would be, if there were a ſecond Light. For although afar off the Shadows of our Bodies, environed with Light, ſeem to us almoſt Dark, yet they do not ſo appear, ſo much as the Shadows of the Moon do; and thoſe that are upon the Edge of the Section, ſhould not appear in the like Manner. But I will determine nothing of any of theſe things.

*To find the Parallax of the Moon; by . . . . . n. 9. p. 151.*

XXXVIII. At certain times agreed on by two Obſervators, making uſe of Teleſcopes Large, Good, and well Fitted for this Purpoſe, by a meaſuring Rod, placed within the Eye-Glaſs at a convenient Diſtance, that it may be diſtinctly ſeen, and ſerve for Meaſuring ſmall Diſtances by Minutes and Seconds, (which is eaſie enough in large Teleſcopes) Let each of ſuch Obſervators, thus furniſht, Obſerve the viſible Way of the Moon among the Fixt Stars, (by taking her exact Diſtance from any Fixt Star, that lies in or very near her Way, together with the exact time of her ſo Appearing) and the then Apparent Diameter of her Disk; continuing theſe Obſervations every time for two or three Hours, that ſo if poſſible, two exact Obſervations of her Apparent Place among the Fixt Stars, being made, at two Places thus diſtant in Latitude, and as near as may be under the ſame Meridian, by theſe Obſervations, concurring at the ſame time, her true and exact Diſtance may be hence collected, not only for that time, but at all other times, by any ſingle Obſervator's Viewing her with a Teleſcope, and meaſuring exactly her Apparent Diameter. It were likewiſe deſirable, that as often as there happens any conſiderable Eclipſe of the Sun, that this alſo might be obſerv'd by them, noting therein the exact Meaſure of the Greateſt Obſcuration compared with the then Apparent Diameter of his Disk. For by this means, after the Diſtance of the Moon hath been exactly found, the Diſtance of the Sun will eaſily be deduced.

As for the Time fitteſt for making Obſervations of the Moon, That will be when ſhe is about a Quarter or ſomewhat leſs Illuminated, becauſe then her Light is not ſo bright, but that with a good Teleſcope, ſhe may be Obſerv'd to paſs cloſe by, and ſometimes over ſeveral Fixt Stars, which is about four or five Days before or after her Change: Or elſe at any other time, when the Moon Paſſes near or over ſome of the Bigger ſort of Fixt Stars, ſuch as of the Firſt and Second Magnitude; which may be eaſily Calculated and foreſeen: Or beſt of all, when there is any Total Eclipſe of the Moon; for then the ſmalleſt Teleſcopical Stars may be ſeen cloſe adjoyning to the very Body of the Moon.

*A Method for Obſerving Lunar Eclipſes; by Mr. Hook. n. 22. p. 388.*

XXXIX. 1. Eclipſes of the Moon are obſerved for two Principal Ends; One Aſtronomical, that by comparing Obſervations with Calculations, the Theory of the Moon's Motion may be Perfected, and the Tables thereof Reformed: the other Geographical, that by comparing among themſelves the Obſervations of the ſame Ecliptick Phaſes, made in divers Places, the Difference of Meridians or Longitudes of thoſe Places may be diſcerned.

The Knowledge of the Eclipſe's Quantity and Duration, the Shadow's Curvity and Inclination, &c. conduce only to the former of theſe Ends. The exact Time of the Beginning, Middle, and End of the Eclipſes, as alſo in Total Ones, the Beginning and End of Total Darkneſs, is uſeful for both of them. But

But becauſe in Obſervations made by the Bare Eye, theſe times conſiderably Differ from thoſe with a Teleſcope; and becauſe the Beginning of Eclipſes, and the End of Total Darkneſs, are ſcarce to be obſerved exactly, even with Glaſſes (none being able clearly to diſtinguiſh between the True-Shadow and Penumbra, unleſs he hath ſeen for ſome time before, the Line ſeparating them, paſs along upon the Surface of the Moon;) and laſtly, Becauſe in ſmall Partial Eclipſes, the Beginning and End, and in Total Ones of ſmall continuance in the Shadow, the Beginning and End of Total Darkneſs are unfit for Nice Obſervations, by reaſon of the ſlow Change of Appearances, which the Oblique Motion of the Shadow then cauſeth. For theſe Reaſons I ſhall propound a Method peculiarly deſigned for the Accompliſhment of the Geographical End in obſerving Lunar Eclipſes, free (as far as is poſſible) from all the mentioned Inconveniencies.

For, Firſt, It ſhall not be Practicable without a Teleſcope. Secondly, The Obſerver ſhall always have Opportunity, before his Principal Obſervation, to note the Diſtinction between the True Shadow and the Penumbra. And, Thirdly, It ſhall be applicable to thoſe Seaſons of the Eclipſe, when there is the ſuddeneſt Alteration in the Appearances. To ſatisfie all which Intents,

Let there be of the Eminenteſt Spots, diſperſed over all Quarters of the Moon's Surface, a ſelect Number generallly agreed on, to be conſtantly made uſe of, to this Purpoſe, in all Parts of the World. As for Example, thoſe, which M. *Hevelius* calleth, *M. Sinai*, *M. Ætna*, *M. Porphyrites*, *M. Serorum*, *Inſ. Besbicus*, *Inſ. Creta*, *Palus Mæotis*, *Palus Mareotis*, *Lacus Niger Major*.

Let in each Eclipſe, not all, but (for Inſtance) three of theſe Spots, which then lie neareſt to the Ecliptick, be exactly obſerv'd, when they are Firſt touch'd by the True Shadow, and again, when they are juſt compleatly Entred into it, and (if you pleaſe) alſo in the Decreaſe of the Eclipſe, when they are firſt fully clear from the True Shadow. For the accurate Determinations of which Moments of time (that being in this Buſineſs of main Importance) let there be taken Altitudes of Remarkable Fixt Stars; on this ſide of the Line, of ſuch as lie between the *Æquator* and *Tropick* of *Cancer*; but beyond the Line, of ſuch as are ſituate towards the Other Tropick; and in all Places, of ſuch as at the Time of Obſervation are about four Hours diſtant from the Meridian.

2. Eclipſis *Lunæ*, Diei 29. *Oct.* An. 1697. Obſervata eſt *Roterodami* per Teleſcopium quatuor ferè pedum Pariſienſium Oculari Convexo, in cujus Foco erant Fila Quatuor, ſeſe in Axe interſecantia ad Angulos Rectos & Semirectos, ad Phaſes dimetiendas, Macularumque Lunarium ſitum determinandum. Hoc Teleſcopium impoſitum erat Fulcro habenti Axem in ſitu parallelo Axi Mundi conſtitutum, ut poſtquam ad Lunam directum eſſet ad unius Phaſis Obſervationem, poſſet ad alias Phaſes obſervandas per Lunæ Semitam ad Occaſum revolvi. Ita autem primo dirigebatur ad Lunam, ut eo immoto permanente Lunæ Limbus Borealis ſuo Motu ad Occaſum raderet unum ex his Filis, quod ideo Parallelum dicimus, licet ob Motum Lunæ in Declinationem Motui Lunæ ad Occaſum multo celeriori commixtum non nihil ab Æquatore declinaret dum Lunæ Diſcus in reliqua tria Fila ſucceſſive incideret. Horum trium Filorum intermedium Angulos Rectos cum Parallelo efficiens, Rectum Perpendiculare & Verticale appellamus. Reliqua duo Obliqua, quorum Primum dicimus in quod prius Luna incidit, Secundum Obliquum in quod Luna incidit poſterius. Initio Eclipſis, quando

*By M. Ja. Caſſini. n. 236. p. 15.*

Vid. infra. §. LIV. 2.

Fig. 127.

quando Lunæ Punctum Borealissimum nondum in Umbra erat Immersum, illud Filo aptavimus Parallelo. Deinde postquam tale Punctum Umbræ Immersum est, eidem Filo aptavimus Australissimum Lunæ Punctum. Unde factum est ut quod Filum initio fuerat Primum in aliarum Phasium determinatione fuerit Postremum, & Primum evaserit quod Postremum fuerat initio. Cum autem Lunæ Limbus Filum Parallelum percurreret, Lunæ Centrum intelligebatur describere Lunarem Semitam huic Filo Parallelam, quæ ab aliis tribus Filis secabatur. Portiones autem hujus Semitæ supponuntur proportionales Temporibus, quibus ipsas Lunæ Centrum percurrit, inæqualitas enim Motus Proprii universali Motui immixti exiguo Tempore imperceptibilis est. Cum igitur Lunæ Limbus Parallelum percurreret, observabatur beneficio Horologii Pendulo instructi, & diebus præcedentibus ad Solem conformati, Tempus adventus Lunæ Macularum aliquot & Lunarium Cornuum ad hæc tria Fila, & deprehensum est dictæ Eclipsis tempore Lunæ Discum transire per Filum Rectum, 2′. 24″. per Fila verò Obliqua 3′. 24″. ideoque Semidiametrum Lunæ transire per Rectum 1′. 12″. per Obliqua verò 1′. 42″. Differentia utriusque transitus existente 30″. Hinc observato uno Appulsu Lunæ ad quodvis horum Filorum, vel uno Egressu, dantur omnes alii ad reliqua Fila. Semidiameter Lunæ A B, jacens in Lunari Semita A B C D E F, pertransit per ejus punctum quodlibet dum Centrum A percurrit spatium sibi æquale A B, ut alia Semidiameter A K, Angulum Rectum efficiens cum alia recta Linea N C K ad punctum K, in quo proinde Lunam continget in K, ab ejus Semita declinans Angulo K C A, transit per ipsum Filum C K, dum Centrum Lunæ percurrit A C, Hypothenusam Trianguli Rectanguli A K C: estque Tempus Transitus Semidiametri A B, per Filum Perpendiculare Lunam contingens in B, ad Transitum Semidiametri A K, per Filum Obliquum N C K, ut A B, vel A K, Sinus Anguli A C K, ad A C, Sinum Auguli Recti, sive Radium. Filo igitur N C K, faciente cum semita Lunæ Angulum K C A Semirectum, & Angulus K A C, in Triangulo Rectangulo, Semirectus erit, ideoque Latera C K, K A, æqualia, erit Transitus Rectus secundum A B, ad Transitum Semidiametri A K, per Filum Obliquum N K, ut Sinus Anguli Semirecti, ad Sinum Anguli Recti, ut 707 ad 1000, sive ut 72″, ad 102″. vel 1′. 42″. ferè, ut observabatur, Lunaris Centri Semita existente A H, Lunæ Semidiametro ipsi perpendiculari A M, ducta M N O, Paralella ipsi A H, ipsa congruet Filo quod Lunæ Limbus motu suo ad Occidentem radet, quod secabitur ab Obliquis N C K, N G I, & à Recto N E P, in puncto N, qua transit Axis Telescopii; faciétque cum his Filis Lunaris Orbita duo Triangula Rectangula N E C, N E G, quæ supponuntur habere Angulos Semirectos ad puncta N, C, G: Sunt ergo Similia & Æqualia, habéntque Latera C E, E G, E N, Æqualia Semidiametro Lunæ A M. Si hinc inde ab Intersectionibus, C, & G, accipiantur in Filis ipsi Semidiametro æquales C K, C S, G I, G R, & in orbita C A, C F, G D, G H, æquales C N, & jungantur A K, F S, D R, H I, erunt ipsæ omnes æquales inter se, efficientque ad Fila Angulos Rectos ad K, S, R, I. Quare Centro Lunæ existente in A, Luna Tanget Primum Obliquum in K, & postquam Centrum Lunæ venit ab A in C, ejus Semidiameter congruet Lineæ C E, ideoque Luna tanget Filum Rectum in E. Postquam autem Centrum Lunæ venerit ab A in D, tanget Secundum Obliquum in R. Est autem A D, æqualis Diametro Lunæ, nam cum G D sit æqualis C A, addendo D C, habebitur A D æqualis G C, qui quidem est Dia-

Fig. 128.

metro

metro Lunæ æqualis. Sed cum G D ſit æqualis C F, ſi ab his auferantur æquales G E, E C, erit F E aqualis E D, & erit D F dupla; tantúmque erit à Contactu primo Secundi Obliqui in R, ad Contactum ultimum Primi Obliqui in S; & poſtquam Centrum Lunæ progreſſum fuerit in G ad diſtantiam Semidiametri unius E G, Luna continget ultimo Filum Rectum in E. Luna Centro progreſſo à G in H, ipſa tanget ultimo Secundum Obliquum in I. Suppoſito igitur tranſitu Recto Lunæ fieri 2′. 24″. ut obſervatum eſt. Et,

| | ′ ″ | Diff. Contact. ′ ″ |
|---|---|---|
| Poſito Centro in A, & contactu Primi Obliqui in K. | 0 0 | |
| Centrum Lunæ erit in C, & continget 1°. Rectum in E | 1 42 | 1 42 |
| Centrum perveniet in D, & continget 1°. 2 Obliquum in R. | 2 24 | 0 42 |
| Lunæ Centrum erit in E, Filo intermedio Perpendiculari | 2 54 | 0 30 |
| Centrum perveniet in F, & continget ultimò 1 Obliquum in S | 3 24 | 0 30 |
| Centrum erit in G, & continget Ultimo Rectum in E | 4 6 | 0 42 |
| Erit tandem in H, & continget ultimò 2 Obliquum in I | 5 48 | 1 42 |

Hinc Calculo correſpondebant ut plurimum Obſervationes in hac Eclipſi intra Secundum unum. Sufficiebat igitur in una Phaſi Obſervare duos ex his Tranſitibus in reliquis Phaſibus unum, ut reliqui omnes innoteſcerent. Quod ad Lunares Maculas attinet; Comparatur Tranſitus Marginis præcedentis Lunæ & Maculæ per Filum Rectum ad habendam Differentiam quam dicimus Longitudinem Maculæ à Margine præcedenti: & Tranſitus Rectus Maculæ comparatur cum Obliquo ad habendam Differentiam quæ æqualis eſt diſtantiæ viæ Maculæ à Semita Puncti Borealiſſimi vel Auſtraliſſimi radentis Filum Parallelum. Cum enim via Maculæ A B C, Parallela ſit viæ Marginis D E F, eoſdem cum eiſdem Filis Angulos facit Semirectos ad A & C, Rectos ad B, unde Angulus ad A, æqualis eſt Angulo ad C, & Latus B A, æquale Lateri B E, Latitudini Maculæ B à Filo F E D. Datâ autem Longitudine & Latitudine Maculæ datur ejus Situs in Luna. Deſcripto quippe circa ipſam Quadrato cujus Latus A B, intelligatur congruere Filo Parallelo, & ſit diviſum in tot æquales partes quot Secundis Luna per Filum Rectum tranſit, Latera verò A C, B D, ipſi Filo perpendicularia ſint in totidem ſimiliter partes æquales diviſa. Sumptâ in Parallelis Longitudine A E, C F, & Ducta F E, & in Perpendicularibus Latitudine A G, B H, quam æqualem dicimus viæ interceptæ inter Rectum & Obliquum, determinatur Situs Maculæ M, in communi harum rectarum Interſectione. Fig. 129. Fig. 130.

Quod ſpectat ad Lunæ Cornua in Eclipſi, ipſa determinari poſſunt ſolâ Longitudine, modo ſciatur quo in Semicirculo Auſtrali vel Boreali ſint. Ut Cornu I, per Longitudinem A E, vel C F, recta quippe F E, Lunæ Marginem ſecat in duobus punctis L, & I, quorum unum eſt in Semicirculo Boreali, alterum in Auſtrali. Poteſt etiam determinari ſolâ Latitudine A K, vel B M, modo ſciatur quo in Semicirculo Orientali vel Occidentali ſit Punctum I. Ex Lineis autem Longitudinis & Latitudinis illa exactius Situm Cornu determinat, quæ propior eſt Centro; ut hic punctum I, exactius determinatur Longitudine quam Latitudine; è contra Punctum O exactius Latitudine quam Longitudine, idque ob minorem Obliquitatem Lineæ rectæ ad Circumferentiam, quâ efficitur ut exigua variatio Diſtantiæ magis ſit in Circumferentia ſenſibilis. Alia ratione per Obliquos Tranſitus

determinatur

Fig. 131. determinatur Situs Macularum & Cornuum Lunæ, si Linea A D, Parallela Semitæ Lunari P Q, ipsius Marginem tangenti, fiat Diameter Quadrati Lunæ Circumscripti quæ dividatur in tot æquales partes quot Secundis Luna per Filum Obliquum pertransit, ut in hâc Eclipsi in partes 204. Hujus Quadrati duo Latera A C, B D, Primum Obliquum representabunt, utpote illi Parallela, reliqua A B, C D, Secundum Obliquum, sumpta autem Differentia inter Transitum Marginis Præcedentis Lunæ & Maculæ M, per Obliquum, in Secundis horariis ab Angulo præcedente ab A in T, & ducta per T, Rectâ E F, Parallela Lateri, A C, & similiter sumptâ ab eodem Augulo A, differentia inter transitum Marginis præcedentis K, & Maculæ M, per Secundum Obliquum A B, ut A V per Punctum V, ducatur Recta G V H, Parallela Lateri A B, representabit Secundum Obliquum secans Priorem in Puncto M, ibique Situm Maculæ determinabit. Eadem ratione determinabitur Situs Cornu E, per Differentiam ipsius Transitus & Marginis per Primum Obliquum sumptam in Diagonali ut A T, Situs Cornu H, per differentiam ipsius Transitus per Secundum Obliquum A B, ut A V, & ducta per V, Recta G H, Parallela Lateri A D, modo sciatur sitne Cornu in Semicirculo Præcedente aut Sequente.

*An Eclipse of the Moon, 1665. Observed at Dantzick; by M. Hevelius. n. 19. p. 348.*

XL. The Tables did indicate an *Eclipse* of the *Moon, July* 27. (*St.n.*) 1665. but though the Sky here was very clear, yet the Moon was not at all Obscured by the True Shadow, but entred only a little into the Penumbra, wherein it continued 50′. The Beginning of its touching the Penumbra did then almost happen, when *Aquila* was elevated 36°. 18′.

*An Eclipse of the Moon, Jun. 6. A. 1666.; by M. Hevelius. ib. p. 348. n. 21. p. 371.*

XLI. In the *Eclipse* of *June* 16. (*St. n.*) 1666. the first *Phasis* of 1 *Digit* 45′. appear'd in the Moon's Altitude of 2°. 30′. when the Greatest Obscuration was already past. The End fell out 9$^{h}$. 27′. about 128°. from the Zenith West-ward.

*An Eclipse of the Moon, Sept. 19. A. 1670; by M. Hevelius. n. 61. p. 2023.*

XLII. Die 29. *Sept.* (*St. n.*) 1670. mane Initium hujus *Eclipsis* incidit 2$^{h}$. 22′. quanquam id ipsum vix omnino accuratè observari potuerit, ob Umbram Terræ Dilutissimam: Siquidem, durante Eclipsi, tota Umbra adeò Tenuis erat atque Diluta, ut omnes præcipuas Maculas per eam, meo viginti pedum Tubo, quin & brevioribus, optimè conspicere potuerim.

Maxima ejus Obscuratio incidit 3$^{h}$. 50′. Finis verò 5$^{h}$. 21′. Tota itaque Duratio fuit 2$^{h}$. 59′; & Quantitas vix amplius 9 Digitorum. Circa Medium hujus Eclipsis 3$^{h}$. 40′. Stellulam quandam Incognitam, ac solo Tubo conspicuam, à Luna circa Lacum Nigrum Majorem tectam, clarissimè conspexi; sed exire eam non deprehendi. Deinde finitâ Eclipsi, jucundissimum quoque observatu erat, bina Luminaria simul supra Horizontem videre. Nam priusquam Luna occideret, Sol oriebatur. Cætera notatu digna ex particulari hoc Typo deprehendetis.

Ordo

| *Ordo Phasium.* | *Phases Lunæ Telescopio observatæ.* | *Altitudines Observatæ.* | *Tempora ex Altitudinibus Correct.* | *Animadvertenda.* |
|---|---|---|---|---|
| | | | h. ′ ″ | |
| | | Caudæ *Cygni*. 50°. 54′ | 12 8 34 | |
| | | Caudæ *Cygni*. 50 25 | 12 11 32 | |
| | | *Pollucis* ♊. 26 39 | 1 28 56 | |
| | | *Pollucis* ♊. 27 16 | 1 33 17 | |
| | Penumb. Init. | | 2 10 0 | |
| | InitiumEclipſ. | | 2 22 0 | Cœpit *Eclipſis* in part. ☽æ ſup. circa Sin. *Hyperb.* |
| 1 | $1\frac{1}{8}$ Dig. | | 2 31 0 | Umbra Sin. *Apollinem*, &. M. *Porphyr.* Stringebat. |
| 2 | $2\frac{1}{4}$ | | 2 36 0 | *Lacus Niger Major* tegi incipieb. |
| 3 | $3\frac{1}{4}$ | | 2 41 40 | Umbra ad *Inſ. Corſicam* pervenerat. |
| 4 | $3\frac{3}{4}$ paul plus. | | 2 45 50 | |
| 5 | $4\frac{3}{8}$ | | 2 49 0 | *Pal. Mæotis* & *M. Ætna* plane tectus. |
| 6 | 5 Dig. fere. | | 2 53 0 | |
| 7 | $5\frac{3}{4}$ | | 2 57 0 | *Inſ. Besbic.* & *Inſ. Macra* tecta. |
| 8 | 6 Dig. | | 3 3 0 | |
| 9 | $6\frac{1}{4}$ | | 3 5 0 | |
| 10 | $6\frac{3}{4}$ | | 3 8 30 | *Inſ. Melos* tegi incipiebat. |
| 11 | $7\frac{1}{8}$ | | 3 11 0 | |
| 12 | $7\frac{5}{8}$ | | 3 16 0 | *Inſ. Rhodus* tecta. |
| 13 | $7\frac{7}{8}$ | | 3 20 0 | |
| 14 | $8\frac{1}{4}$ | | 3 25 30 | |
| 15 | $8\frac{1}{2}$ | | 3 29 0 | *Inſ. Major Caſpii* tegebatur. |
| 16 | $8\frac{3}{4}$ | | 3 33 40 | |
| 17 | $8\frac{7}{8}$ | | 3 37 50 | |
| | | | 3 40 0 | |

| | | | | | | |
|---|---|---|---|---|---|---|
| 18 | 9 Dig. | | 3 | 47 | 0 | *Max. Obscuratio* per *J. Leto.* & *M. Aber.* se extend. |
| 19 | 9 Paulo min. | | 3 | 51 | 10 | *Pal. Maræotis* sub Umbræ Sectione subsisteb. |
| 20 | $8\frac{1}{4}$ | | 3 | 57 | 0 | |
| 21 | $8\frac{1}{2}$ | | 4 | 1 | 40 | |
| 22 | $8\frac{1}{4}$ fere. | | 4 | 6 | 0 | |
| | | *Polluc.* ♊. 49°. 50′. | 4 | 10 | 28 | |
| 23 | 8 paul. plus. | | 4 | 11 | 0 | *Insl. Melos* & *Rhodus* umbra exiverant. |
| 24 | $7\frac{3}{4}$ | | 4 | 17 | 20 | |
| 25 | $7\frac{3}{8}$ | | 4 | 22 | 25 | *M. Porphyrites* illustrari incipiebat. |
| 26 | $6\frac{3}{4}$ | | 4 | 27 | 35 | Umbra *M. Ætnam* jam reliquerat. |
| 27 | $6\frac{1}{4}$ fere. | | 4 | 33 | 40 | |
| 28 | $5\frac{3}{4}$ | | 4 | 37 | 37 | |
| 29 | $4\frac{3}{4}$ | | 4 | 44 | 46 | *M. Argentar.* & *Insl. Besbic.* illustrabantur. |
| 30 | 4 Dig. | | 4 | 50 | 56 | |
| 31 | $3\frac{3}{8}$ | | 4 | 54 | 0 | |
| 32 | $2\frac{3}{4}$ | | 4 | 57 | 0 | |
| 33 | $2\frac{1}{4}$ fere. | | 5 | 1 | 23 | Umbra totam fere *I. Caspiam Maj.* transierat. |
| 34 | $1\frac{5}{8}$ | | 5 | 4 | 45 | |
| 35 | $1\frac{1}{4}$ Dig. | | 5 | 7 | 43 | |
| | | *Cordis* ♌. 25°. 48′. | 5 | 10 | 50 | |
| | Finis *Eclips.* | | 5 | 21 | 25 | Finis circa *Riphæi Montes* contigit. |
| | *Penumb.* Finis. | | 5 | 24 | 0 | |
| | | *Sirii* 18°. 54′. | 5 | 28 | 32 | |

*An Eclipse of the Moon, Sept. 8. 1671. at Ecton, by Mr. Palmer, n. 76. p. 2272.*

XLIII. 1. *Sept.* 8. 1671. Circa Horam sextam vespert. Luna ascendebat totaliter Obscurata *Ectoniæ* in Comitatu *Northamptoniano*. Cœpit Emergere ex Umbra, Centro *Lunæ* Alto 9°. 35′, sive 7h. 18′. Finis, *Arcturo* Alto 16°. 30′. sive 8h. 16′. 20″. Unde computatur Medium Eclipsis fuisse 6h. 28′. 16″.

*At London; by Mr. Street. Ib.*

| | ° | ′ | h. | ′ |
|---|---|---|---|---|
| 2. The Emersion: Alt. of the upper Edge of the ☾ | 10 | 30 | 7 | 21 |
| The End of the Eclipse: Alt. of *Arcturus*. | 16 | 20 | 8 | $16\frac{2}{3}$ |

*by Dr. Hook. n. 77. p. 2296.*

3. 7h. 27′½. I first observed the Moon Eclipsed when it began to be enlightned, the Total Darkness being already past. The Shadow passed through the middle of the Spot called by *Hevelius*, *M. Porphyrius*.

7h. 47′.

7h. 49′. The Shadow passed through the middle of *M. Sinai*, through the middle of the Eastermost of the three Lakes called *Mare Adriaticum*, and just touched the Ridge of the *Appennine* Mountains.

7h. 54′. It passed the middle of the *I. Besbicus* in the *Propontis*.

8h. 0′½ It passed through the *Streights* of the *Pontus Euxinus*, at the Promontories *Acherusia* and *Aristes*.

8h. 6′½. It touched the *Palus Mæotis*, which *Palus Mæotis* was then distant from the Limb of the Moon, next adjacent, one third part of its shorter Diameter or Breadth.

8h. 17′. The Shadow went off the Body of the Moon upon the innermost Limb-line of *Hevelius*'s large Chart of the Moon at the 29 Division, just without the *I. Major* of the *Caspian* Sea. The Duskish *Penumbra* left not the Limb of the Moon quite without some kind of Darkness till 8h. 29′; at which time I found that that side of the Moon which the shadow last left, was full as light and clear as the other.

About four or five Minutes after the Shadow was gone off, I perceived a faint Representation of Colours upon that part of the Body of the Moon, which was most affected with the *Penumbra*, somewhat resembling the Colours of a faint *Halo* about the Moon; this grew fainter and fainter, and after a few Minutes was no more Visible. It did not seem to be caused by any Clouds or Exhalations in the Air, the Sky near the Moon being very clear, and the said Colours not appearing any where, but upon the dusky part of its Phases. Possibly it might be caused by the Refraction of the Light from the Sun through the Atmosphere about the Earth.

| | h ′ ″ | ° ′ ″ |
|---|---|---|
| 4. Observatus est *Finis*, alto *Arcturo* ad occasum | | 13 41 |
| Ablatâ Refractione, alto | | 13 37 45 |
| Hinc datur *Finis* Observatus | 8 29 16 | |
| Finis *Penumbræ* alto *Arcturo* | | 13 0 |
| Correct. | | 12 56 |
| Unde datur | 8 33 40 | |
| Observata est *Mæotis* tota extra Umbram | 8 24 16 | |
| Alto *Arcturo* | | 14 26 0 |
| Correct. | | 14 23 55 |

*At* Paris; *by* M. Bullialdus. n. 76. *p.* 2273.

Exivit *Luna* ex Umbrâ è regione *Petræ Sogdianæ*, *Hevelii*.

5. Hora 8. 30′. Per Nubes Dehiscentes, satis tamen crassas, *Lunam* subobscure animadvertimus, & quidem tanto Lumine jam imbutum, ut dixisses Eclipsin jam esse præteritam. Exinde certum erat, Totalem Obscurationem jam minimum esse eo tempore præteritam, imò aliquanto adhuc citiùs: Siquidem Lunam Rursus adesse nos omnes per Nubes illas satis dilucide deprehendimus; Sic ut Eclipsis in Cœlo ultra dimidiam horam citius ingruerit, quam *Kepleri* Calculus id indicaverit. 8h. 34′. Minimum ad integrum Digitum animadverti jam extra Umbram sese *Lunam* extricasse; & denuo 9h. 41′. ad 1½. Dig. Lumen *Lunæ* jam excrevisse, quantum id dijudicare circiter dabatur.

*At* Dantzick; *by M.* Hevelius. n. 78. *p.* 3028.

*At* Hamburg; *by Dr.* Fogelius. *Ib. p.* 3033.

6. Æque nos hic ac *Hevelius Gedani*, Calculum *Rudolphinum* in nupera Eclipsi aberrasse deprehendimus. Emersisse enim jam *Lunam* ex Umbra Terræ ante horam nonam etiam hic vidimus.

*An Eclipse of the Moon,* Jan. 1. 167$\frac{4}{5}$, *at* London; *by Dr.* Hook. n. 111. *p.* 237.

| | h. | ′ |
|---|---|---|
| XLIV. 1. *Initium* veræ Umbræ | 5 | 22 |
| *Immersio* | 6 | 19 |
| *Emersio* | 7 | 58 |
| *Finis* veræ Umbræ | 8 | 58 |

The Penumbra was seen to continue near half an Hour before it wholly quitted the Body of the Moon.

*At* Derby; *By Mr.* Flamsteed. *Ib.*

2. Mr. *Flamsteed* observed the Beginning of the Entrance of the true Shadow. h. 5. and 19′.

*At* Paris; *by* M. Bullialdus. *Ib. p.* 238.

| | °. | ′ | h. | ′ | ″ |
|---|---|---|---|---|---|
| 3. *Initium* veræ Umbræ Alta *Capella* | 52 | 26 | 5 | 32 | 29 |
| *Immers.* Altâ *Capella* | 62 | 8 | 6 | 33 | 3 |
| *Emers.* Alt. Cap. *Pollucis* | 43 | 46 | 8 | 9 | 30 |
| *Fin.* veræ Umbræ, Alt. *Syrio.* | 20 | 47 | 9 | 10 | 0 |

*At* Paris; *by* M. Cassini, M. Picard, *and* M. Roemer. *Ib. p.* 238. n. 112; *p.* 257.

4. At 5^h. 12′. In the Evening, In the *Royal Observatory*, They began to perceive, that the Oriential part of the Moon, by little and little lost its Light; so that at 5^h. 25′. they saw a manifest *Penumbra*; then at 5^h. 32′. 50″. the Limb over against the Spot called *Hevelius* grew so dark, that they all agreed, that this was the true *Beginning* of the *Eclipse*. At 8^h. 7′. one of the Observers believed the *Emersion*, another at 8^h. 8′. and the third at 8^h. 9′. 30″. but afterwards considering the *Emersion* of the first Spots they all esteem'd it at 8^h. 8′. At 7^h. 21. the Southern Limb of the Moon was come close to a Telescopick Star, at 8^h. 9′. 20″. another Star yet less than the former, came out of the darkest side, almost over against the Spot *Laungrenus*. At 9^h. 9′. 40″. all the three Observers agreed, that the *Moon* then came out of the Shadow. The Diameter of the Moon being measured before the Eclipse, was of 32′. 15″.

The Times were noted by great *Pendulum Watches*, that had been adjusted by the Sun the same Day, and that were afterwards verified the next Day: Besides that, before the Eclipse at 4^h. 45′. 1″. by the Watches, the Star *Capella* was 45 Degrees high towards the East.

| Time. | | | Phases. |
|---|---|---|---|
| h. | ′ | ″ | |
| 5 | 32 | 50 | *Beginning* over againſt the Spot *Hevelius.* |
| 5 | 36 | 00 | The firſt Spot of *Grimaldi. Palus Maræotis.* |
| 5 | 36 | 30 | The ſecond Limb of *Grimaldi.* |
| 5 | 45 | 00 | The middle of *Ariſtarchus. Mons Porphyrites.* |
| 5 | 46 | 00 | *Merſennus.* |
| 5 | 48 | 30 | *Herigone.* |
| 5 | 53 | 00 | *Heraclides.* |
| 5 | 53 | 15 | The firſt Limb of *Copernicus. Ætna.* |
| 5 | 54 | 15 | The middle of *Copernicus.* |
| 5 | 54 | 40 | *Pitheas,* or *Hiera Inſula.* |
| 5 | 55 | 05 | The ſecond Limb of *Copernicus.* |
| 5 | 57 | 40 | The firſt Limb of *Timocharis. Corſica.* |
| 5 | 59 | 35 | The firſt Limb of the *Sinus Medius Æſtuum. Adriatick Sea.* |
| 6 | 01 | 30 | The Middle of the *Sinus Medius.* |
| 6 | 02 | 40 | The firſt Limb of *Tycho,* or *Sinai*: and the firſt Limb of *Plato,* or the *Lacus Niger Major.* |
| 6 | 03 | 50 | The ſecond Limb of *Plato,* and the middle of *Tycho.* |
| 6 | 04 | 30 | The *Center* of the Disk. |
| 6 | 09 | 00 | The middle of *Manilius,* or *Mons Besbicus.* |
| 6 | 12 | 00 | The middle of *Menelaus,* or *Byſantium.* |
| 6 | 13 | 45 | *Dionyſius Areop.* or *Mons Amanus.* |
| 6 | 14 | 30 | *Plinius.* |
| 6 | 15 | 45 | *Vitruvius.* |
| 6 | 20 | 35 | *Endymion,* or *Lac. Hyperbor. ſuperior.* |
| 6 | 21 | 00 | *Promont. Heraclium.* |
| 6 | 24 | 50 | Betwixt *Alcuin* and *Taruntius.* |
| 6 | 26 | 00 | The firſt Limb of the *Caſpian Sea, Mare Criſium. Palus Mæotis.* |
| 6 | 28 | 15 | The middle of the *Caſpian Sea.* |
| 6 | 29 | 40 | The other Limb of the *Caſpian Sea.* |
| 6 | 30 | 5 | The firſt Limb of *Langrenus,* or *Inſula Maj.* |
| 6 | 3 | 5 | The middle of *Langrenus.* |
| 6 | 35 | 46 | *Total Immerſion,* betwixt *Langrenus* and the *Caſpian Sea.* |
| 8 | 8 | 00 | Firſt *Emerſion,* towards *Grimaldus.* |
| 8 | 12 | 35 | The firſt Limb of *Grimaldus.* |
| 8 | 14 | 00 | The ſecond Limb of *Grimaldus.* |
| 8 | 20 | 20 | *Merſennus.* |
| 8 | 24 | 5 | *Herigone.* |
| 8 | 24 | 35 | The middle of *Ariſtarchus* & the middle betwixt *Herigone* & *Morin.* |
| 8 | 26 | 30 | The middle of *Kepler,* or *Loca Paludoſa.* |
| 8 | 28 | 30 | The firſt Limb of *Tycho.* |
| 8 | 29 | 50 | The ſecond Limb of *Tycho.* |

| Time. | | | Phases. |
|---|---|---|---|
| h. | ′ | ″ | |
| 8 | 34 | 5 | The middle of *Copernicus.* |
| 8 | 35 | 35 | The ſecond Limb of *Copernicus.* |
| 8 | 36 | 10 | *Pitheas.* |
| 8 | 36 | 30 | *Heraclides.* |
| 8 | 40 | 0 | The firſt Limb of *Timocharis.* |
| 8 | 42 | 35 | The firſt Limb of *Plato.* |
| 8 | 43 | 45 | The ſecond Limb of *Plato.* |
| 8 | 49 | 30 | The middle of *Manilius.* |
| 8 | 52 | 10 | *Menelaus* and *Dyonyſ. Areopag.* |
| 8 | 55 | 0 | *Poſſidonius.* |
| 8 | 56 | 6 | *Vitruvius.* |
| 8 | 59 | 30 | *Endymion.* |
| 9 | 6 | 20 | The firſt Limb of the *Caſpian Sea.* |
| 9 | 7 | 10 | The midle of the *Caſpian Sea.* |
| 9 | 8 | 40 | The other Limb of the *Caſpian Sea.* |
| 9 | 9 | 40 | The *End*, between the *Caſpian Sea* and *Langrenus.* |

*At* Dantzick; *by* M.Hevelius. *N.* 113. *p.* 289.

5. Vigilantes oculos, per totam Eclipſeos Durationem tubo 20 Pedum, aliiſque præſtantioribus, ad quatuor Fixas (neglectis cæteris minoribus, quas optime etiam conſpiciebam) inter quas Luna eo tempore verſabatur, direxi. Ab *a* Stellula vix quatuor minutis Limbo ſuo Inferiori in ♉, diſtabat; tres verò reliquas, utpote *b*, *c*, & *d*, Luna corpore ſuo omninò texit. Ex omnibus autem his quatuor inſignioribus Stellulis, non niſi unica *c* ab Aſtronomis hactenus obſervata, Globiſque adſcripta eſt; nominatur *Informium inter* ♊ & ♋ *Suprema à tergo Pollucis*; cujus curſus cum ingreſſu, viâ itinerariâ, atque egreſſu imprimis probè notandus. Quippe ex hujus generis obſervationibus, multo procliviùs datur Motum Lunæ redintegrare, ejuſque Nodos Latitudinemque reſtaurare, quàm, meo quali judicio, ex nudis *Solis Eclipſibus.* Stellula *b*, ad montem *Eoum* circiter tecta eſt, & *d* ad ipſum Limbum Lunæ Inferiorem; illa per *Sinum Sirbonis*, *I. Rhodum*, & *S. Athenienſem*; hæc vero per *Deſertum Mingui* tranſiit.

Temp.

| *Temp. sec. Hor. Oscil. ex altitudinibus Cor.* | *Ordo Phasiu.* | *Altitudines Fixarum: Nec non Animadversiones quædam notatu digne.* | *Per quas Maculas transiverint Umbræ Sectiones.* |
|---|---|---|---|
| h. ′ ″ | | | |
| 6 22 18 | | Altitudo *Caudæ Cygni.* 39°. 3′. 0″. | |
| 6 25 4 | | Eadem Altitudo. 38°. 41′. 0″. | |
| 6 35 0 | | Penumbram *Luna* subiit. | |
| 6 41 50 | | *Initium* Eclipseos. | *Incepit* circa 50°. à puncto Nadir Ortum versus. |
| 6 43 45 | 1 | Paludem *Mareotidem* attigit. | Tempore *Initii*, *Sinus Sigaricus*, Ins. *Besbica* & *Ins. Melos* in Linea Recta extiterunt. |
| 6 44 55 | 2 | *Palus Mareotis* omnino obscur. | |
| 6 49 0 | 3 | | Sectio Umbræ per *M. Pentadactylum* & *Eoum* transiit. |
| 6 52 30 | 4 | *Mons Porphyrites* tectus. | Per *Loca Paludosa Montemque Cataract.* |
| 6 57 35 | 5 | | Ad *M. Baronium*, per *M. Petri*, *Athor*, & *M. Troicum.* |
| 7 2 15 | 6 | *M. Ætnam* tegere incepit. | Ad *Sin. Apollinis*, *Ins. Ficariam*, ad radices *Montis Ætnæ*, *Ins. Didymæ*, *Lacumq; Meridionalem.* |
| 7 2 55 | 7 | *M. Ætna* omninò tectus. | Ad *Ins. Sardiniam*, per *Ins. Hieram*, *Insulamque Cretam.* |
| 7 7 40 | 8 | | Per *M. Atlanticos Majores*, *Ins. Vulcaniam*, *Rhodum*, *Montemque Annæ.* |
| 7 10 40 | 9 | M. *Sinai* tectus. | Per *Ins. Ophiusam*, *Ins. Cyprum* & M. *Sinai.* |
| 7 14 0 | 10 | *Lacus Niger Major* tectus. | Ad L. *Nigrum Majorem*, per M. *Sipylum*, *Libanum*; Montemque *Seir.* |
| 7 16 40 | 11 | | Per *L. Nigr. Minorem*, *Ins. Besbicam*, M. *Olympum* & *Didymum.* |

| | | | |
|---|---|---|---|
| 7 20 20 | 12 | Fixa *c*, diſtitit a Limbo ☽ ortum verſus 30′. fere. | Sectio Umbræ tranſiit per M. *Carpathos*, *Byzantium* & *Taurum*. |
| 7 25 0 | 13 | | Per Lacum *Boryſthenem*, *Inſ. Apolloniam*, M. *Moſchum* & *Sogdianos*. |
| 7 28 30 | 14 | | Per Montes *Macrocemnios*, *Promont. Arietis*, *Herculeum*, per Sinum extremum *Ponti*, Montemque *Parapamiſum*. |
| 7 32 0 | 15 | | Per *Paludes Hyperboreos*, *I. Corocondametis*, Montemque *Caucaſum*. |
| 7 34 10 | 16 | | Per *Montes Riphæos*, Palud. *Mæotidem*, Sinum Inferiorem Maris *Caſpii*. |
| 7 42 44 | | *Totalis Immerſio* contigit circa 50°. Limbi à puncto Zenith occaſum verſus. | |
| 7 55 30 | | Stellula *a*, in ipſa ☌ cum ☽; Diſtitit à Limbo ejus inferiori 4 circit. Minut. | |
| 8 0 50 | | Stellula *b*, ad M. *Eoum* Tecta. | |
| 8 35 20 | | Stellula *c*, ad *Lacum Merid.* ſubiit. | |
| 8 51 20 | | Stellula *d*, ad ipſum Limbum Inferiorem occultata. | |
| 9 9 10 | | Stellula *c*, rurſus emerſit, ſub *Monte* ſc. *Nevoſo*; ſic ut ☽ ad 3′. 20″. ſubingreſſa fuerit. Spectaculum ſanè erat Jucundiſſimum, hæcce omnia dilucidiſſima, & quidem ſub ipſa ferè Maxima Obſcuratione, & ☽æ ☌ deprehendere. | |
| 9 12 30 | | *Emerſio* Lunæ ex umbra contigit ad 30°. ferè Limbi ☽ à puncto Nadir Ortum verſus. | |
| 9 20 20 | 17 | | Umbra inceſſit per Pal. *Maræotidem*, *Fontes Amaros* & M. *Eoum*. |
| 9 25 0 | 18 | | Per M. *Audum*, *Ajacem* & *Troicum*. |
| 9 30 20 | 19 | | Per M. *Pentadactylum*, *Carpathum*, S. *Sirbonis*, & M. *Lion*. |

| | | | |
|---|---|---|---|
| 9 32 40 | 20 | Mons *Porphyrites* exiit ex Umbrâ. | Per M. *Porphyrit. Cassium* & *Didymum.* |
| 9 36 5 | 21 | M. *Sinai* illuminari cœpit. | Per M. *Baronium*, ad *I. Siciliam*, per *Cretam* ad radices *Sinai.* |
| 9 42 15 | 22 | M. *Ætna* sub ipsa sectione Umbræ. | Per *Promontorium Appollinis*, M. *Ætnam*, per *Rhodum* Montesque *Seir.* |
| 9 47 10 | 23 | | Per *Inf. Vulcaniam*, M. *Masicytum*, *Cragum* & *Antiliban.* |
| 9 51 0 | 24 | | Ad *L. Nigrum Majorem*, *S. Pastanum*, M. *Didymum*, & *Montes Coibacaronos.* |
| 9 56 50 | 25 | *Insula Besbica* rursus prodiit in Lucem. | Per *S. Nigrum Minorem*, ad *I. Besbicam*, per M. *Uxerios.* |
| 9 59 35 | 26 | | Per *L. Salmidersam*, M. *Horminium*, *Moschum* & *Lacum Thospitis.* |
| 10 5 0 | 27 | | Per *Pontum*, *I. Cyaneam*, *S. Atheniensem*, & M. *Caucasum.* |
| 10 7 0 | 28 | | Per *L. Borysthenem*, ad *I. Apolloniam*, per *Heracleum* ac M. *Tanconem.* |
| 10 19 35 | | Umbra nondum planè exiverat. | |
| 10 20 0 | | *Finis* Eclipseos. | |
| 10 23 0 | | *Penumbra.* | |
| 10 52 58 | | Altitudo ♉. 37°. 12′ | |
| 10 58 35 | | Alt. *Lucidæ* ♈ 28 52 | |
| 11 11 33 | | Alt. *Capellæ.* 70 11 | |

6. Observatio hujus Eclipsis fuit admodum exacta; nam Sole prope Horizontem existente, per hujus Loci Altitudinem Minutorum tempora facilè innotuerunt: Cætera meo Horologio (quod ne unum quidem Minutum ea die discrepabat) notavi, ita ut nulli dubio in observatione detur locus.

*At Seville; by S..... Professor of Mathematiques.* n. 118. p. 428.

| | h. ′ |
|---|---|
| *Initium* veræ Umbræ | 4 56 |
| *Immersio* | 6 1 |
| *Emersio* | 7 33 |
| *Finis* | 8 39 et aliquantulum productior. |

XLV. 1. *An Eclipse of the Moon, Jun.* 27. 1675. *at* London; *by Mr.* Flamsteed *and Mr.* Halley. n. 116. *p.* 371. n. 118. *p.* 432.

| | *Hora Horologii Oscillatorii.* | *Phases.* |
|---|---|---|
| | h. ′ ″ | |
| 1 | 1 1 00 | Lunæ Diameter 3190 = 31′. 40″. } Multoties repetitæ Mensuræ tum à me tum ab adjutore *Edm. Halley.* |
| 3 | 1 11 00 | Rursus Lunæ Di. 3191 = 31 46 |
| 5 | 1 37 50 | Nullum Penumbræ vestigium: jamque Luna nubes subtervolantes subiit, sub quibus latuit usque |
| 6 | 1 46 40 | Dum per earum hiatus Penumbra densa, vel forsan ipsum Initium apparuit; sed certus esse non poteram. |
| 7 | 1 51 40 | E nubibus elevatæ Limbus, notabili satis defectu laborare visus est, obscurato sextante vel minimum octante Peripheriæ. |
| 8 | 1 55 15 | *Pentadactylus* tectus. |
| 9 | 2 2 20 | *Porphyrites* tectus. |
| 10 | 2 5 30 | *Sinæ* Limbus primus. |
| 11 | 2 6 00 | *Ætnæ* Limbus proximus. |
| 12 | 2 8 40 | Partes residuæ illuminatæ 2071=20′. 38″. |
| 13 | 2 12 00 | Fixa exigua Telescopica, minori tubo non visibilis, in majori apparuit dimidium ferè capacitatis ejusdem, vel 15′. à Limbo apparenter inferiori distans. |
| 14 | 2 17 45 | Partes illuminatæ residuæ, 1655=16′. 27″. |
| 15 | 2 23 5 | *Besbici* Limbus prior. |
| 16 | 2 26 3 | *Horminius* tectus. |
| 17 | 2 29 00 | *Æmus* tectus. |
| 18 | 2 30 45 | Partes illustratæ residuæ 1047=10′. 24″. |
| 19 | 2 35 00 | Partes lucidæ residuæ 865 =8 38; jamque tubo longiori fixa alia exigua apparuit, Maculæ *Caspiæ* Longitudinem à limbo Lunæ, Latitudinem ejus à Linea per cuspides ductâ dextram versùs, distans. |
| 20 | 2 37 15 | Umbra tegebat occidentale Littus *Ponti.* |
| 21 | 2 39 30 | Ipsa tetigit Limbum primum *Corocondometis.* |
| 22 | 2 45 00 | Paludem *Mæotidem* tetigit. |
| 23 | 2 50 45 | *Mæotis* tota tecta. |
| 24 | 2 56 10 | Dubium an aliquid veri Luminis supererat. |
| 25 | 2 56 55 | *Immersio*: Certe enim Lux primaria Lunam penitus deseruerat, scil. è regione paulò supra Montes Ripheos & circa gradum Limbi *Heveliani* 330. 2h. 57′. 30″. Limbus Coloris Cineritii per tubum apparuit. |

Aer à septima observatione ad *Immersionem* serenissimus extitit; & Lucula quædam Albicans per totum defectûs tempus Cuspides obscuratæ Lunæ visa est.

eſt inſidere, quæ eam etiam poſt Immerſionem à parte, quæ ultimum in Umbram inciderat, reddebat conſpicuam. Tenuis admodum erat Eclipſis hujus Penumbra, nec major quam *Sinæ* aut *Ætnæ* Latitudo. Palus *Mæotis* Lata apparuit admodum, & quàm maximè ferè potuit à Limbo Lunæ remota. *Mareotis* è contra compreſſa admodum, nec pluſquam dimidium longitudinis ipſius à Limbo Lunæ diſtans.

| | ° | ′ | h. | ′ | |
|---|---|---|---|---|---|
| 2. *Initium* veræ Umbræ altâ *Capellâ* ad ortum | 18 | 45 | 1 | 55 | *At* Paris ; *by M.* Bullialdus. *Ib. p.* 372. |
| Umbra attigit *Paludem Mæot.* altâ *Lyrâ* ad occ. | 50 | 51 | 2 | 55 | |
| *Immerſio* Totalis altâ *Lyrâ* ad occ. | 48 | 50 | 3 | 6 | |

| Temp. | | | Tranſitus Umbræ. | |
|---|---|---|---|---|
| h. | ′ | ″ | | |
| 1 | 56 | 45 | *Initium* Infra *Grimaldum.* | 3. *By M.* Caſſini, *M.* Picart *and M.* Roemer. n. 117. *p.* 388. |
| 1 | 57 | 20 | Per primum Limbum *Grimaldi.* | |
| 1 | 58 | 50 | Secundum Limbum *Grimaldi.* | |
| 2 | 1 | 45 | *Galilæum.* | |
| 2 | 2 | 15 | Primum Limbum *Merſenni.* | |
| 2 | 4 | 15 | Initium *Gaſſendi.* | |
| 2 | 4 | 40 | Medium *Gaſſendi.* | |
| 2 | 5 | 30 | Alterum Limbum *Gaſſendi.* | |
| 2 | 6 | 15 | *Herigonum* & *Seleucum.* | |
| 2 | 7 | 45 | *Morinum.* | |
| 2 | 8 | 18 | Medium *Kepleri.* | |
| 2 | 11 | 35 | *Ariſtarchum* & *Bullialdum.* | |
| 2 | 12 | 40 | *Ariſtarchus* diſparet. | |
| 2 | 16 | 25 | Initium *Tychonis.* | |
| 2 | 16 | 40 | Initium *Copernici.* | |
| 2 | 17 | 25 | Medium *Tychonis* & *Copernici.* | |
| 2 | 18 | 12 | Alterum Limbum *Tychonis.* | |
| 2 | 21 | 45 | *Pytheam* & primum trium *Sinuum Mediorum.* | |
| 2 | 24 | 0 | Medium Secundi *Sinus Medii.* | |
| 2 | 24 | 35 | *Heraclidem* vel *Virginem.* | |
| 2 | 26 | 10 | Primum Limbum *Timocharis.* | |
| 2 | 26 | 40 | Medium *Timocharis.* | |
| 2 | 31 | 40 | *Promontorium* inter *Verginem* & *Platonem.* | |
| 2 | 32 | 20 | *Abulfedam.* | |
| 2 | 34 | 15 | Initium *Manilii.* | |
| 2 | 36 | 15 | *Dionyſium Areopagitam.* | |
| 2 | 38 | 10 | LittusMaris *Tranquillitatis.* | |
| 2 | 38 | 45 | Primum Limbum *Menelai,* & *Platonis.* | |
| 2 | 39 | 20 | *Fracaſtorem.* | |
| 2 | 39 | 50 | Medium *Platonis.* | |

| Temp. | | | Transitus Umbræ. |
|---|---|---|---|
| h. | ′ | ″ | |
| 2 | 41 | 15 | Per Alterum Limbum *Platonis.* |
| 2 | 44 | 15 | Promontor. inter *Consorinum* & *Bedam.* |
| 2 | 50 | 20 | Primum Limbum *Paludis Somnii.* |
| 2 | 50 | 45 | Cornua erant verticalia. |
| 2 | 53 | 55 | Initium *Langreni.* |
| 2 | 55 | 20 | Primum Limbum *Maris Caspii.* |
| 3 | 1 | 10 | Alterum Limbum *Maris Caspii.* |
| 3 | 1 | 45 | *Endimionem.* |
| 3 | 3 | 15 | *Meshalam.* |
| 3 | 7 | 45 | Finis sive *Totalis Immersio*, supra *Mare Caspium.* |

*Mare Caspium* tunc distabat à Limbo Occidentali circiter $\frac{2}{4}$ suæ Latitudinis. Post *Immersionem Totalem* dignoscebatur adhuc totum Corpus Lunæ.

XLVI. 1. *An Eclipse of the Moon* Decemb. 22. 1675. *at* Greenwich; *by Mr.* Flamsteed. n. 121. p. 435.

| Hor. cor. Horol. | | | Phases. |
|---|---|---|---|
| h. | ′ | ″ | |
| 2 | 29 | 30 | Inter Cuspides 2085 = 17′. 16″. |
| 2 | 55 | 45 | *Hæmum* ferè tetigit. |
| 3 | 0 | 30 | *Hæmum* certe tetigerat. |
| 3 | 11 | 30 | Cuspis dexter à *Mariotide* 1235 = 10′. 14″. |
| 3 | 35 | 0 | Partes Lucidæ circiter 2800 = 23. 11. vel paulo forsan amplius; difficile enim erat admodum, Umbræ veræ Terminos, per Aerem Vaporibus fœdatum, definire. |
| 3 | 42 | 30 | Umbra prope *Macram.* |
| 3 | 52 | 45 | Inter Cuspides circiter 2288 = 18′. 57″. |
| 4 | 7 | 15 | *Finis:* Limbus enim apparuit, & nihil videbatur in rotunditate Lunæ desiderari. |
| 4 | 8 | 0 | Limbus admodum dilucide per Tubum conspectus. |
| 4 | 15 | 30 | *Penumbra*, quæ nudis Oculis Eclipsin referebat. |
| 4 | 19 | 30 | ☽æ capta diameter 3757 = 31′. 5″. Sed vix satis certa; quam tamen haud multum à veritate abesse putem. |
| 4 | 23 | 0 | Etiamnum, & postea, Limbus ab *Eclipsi* derelictus Obscurior videbatur ac alter. |

*Corsica* a Limbo ☽æ Remoto distantia 2732 = 22′ 37″.
Limbus ejus proximus à proximo *Lunæ* 1045 = 8. 39.

*Sinæ*

*Sinæ* Limbus Remotior à *Lunæ* Proximo 599 = 4′. 58″. bona.
*Lacus Nigri Majoris* Medium à Limbo proximo 452 = 3 45

Notavi præterea, quòd Umbra semper longè distinctior apparuit ad Cornua, quàm alicubi in Facie Lunæ: In prima Observatione, vel paulo antè, Cornua fuêre Horizonti Paralela.

Tunc etiam *Porphyrites*, & *Lacus Niger Major*, æqualiter ex Umbra extitêre, Longitudinem scil. circiter *Maræotidis*.

Nunquam tamen *Porphyritem* superavit in hac Eclipsi; altè vero illum in Penumbram mersum vidi.

In summa Eclipsi ad *Corsicam* ferè Umbra pertigerit; nunquam tamen eam extinctam vidi, sed altè adeò in Penumbra immersam, ut ægrè eam potuerim discernere.

Nec unquam Umbra vera *Insulam Macram* pervagabatur, sed Penumbra duntaxat densa, per quam difficile erat ipsam percipere.

4^h. 5′½. Limbum videre non potui; nec 4^h. 6′½; sed 4^h. 7′. videre me putabam Limbi Lucem sed languidissimam, & ægrè admodum; 4^h. 7′. 15′. certior factus sum ex Umbris Emersisse, nec aliquid in ejus rotunditate desiderari: Ergo tunc *Finem* observatum statuo.

Exibat Umbra juxta *Lacum Hyperboreum Superiorem* manente Penumbra, quæ Eclipsin nudis Oculis exhibebat usque 4^h. 15′½; sed Limbus ab Eclipsi derelictus Limbi oppositi Claritudinem recuperavit non nisi 4^h. 28′. vel serius.

Tempora Phasium Correcta, ab Altitudinibus *Arcturi* & *Lucidæ Coronæ*, Quadrante Telescopico, pedum trium & amplius Radio, captis; quibus, clarè aliquando in altera Cœli cardine emicantibus, captandis incubui, quoties Lunam subiêre Nubes.

2. *Initium* Eclipsis accidit, antequam ad Instrumenta veni: Quod tamen *Londini* in Vico *Wintoniensi* observavit *Edmundus Halleius*, cùm Lunæ Limbus superior à Vertice distaret — *At London; by Mr. Edm. Halley. Ib. p. 498.*

| | °. | ′. | | h. | ′ |
|---|---|---|---|---|---|
| distaret | 39 | 51 | unde horam supputavit | 2 | 16 |
| alto eodem | 41 | 1 | Cornua Horizonti Parallela | 2 | 25 |
| | 54 | 12 | | 3 | 58 |

3. *Joh. Coelsonus* in Vico *Wappingensi*, ad *Anachoresin*, (Angl. *the Eremitage*) Limbum Lunæ Deficientem aliquantulum vidit 2^h. 17′¼. At 4^h. 9′. 25″. ex Umbra vera ipsum exiisse comperit, densâ duntaxat Penumbrâ remanente. — *By Mr. Colson, Ib.*

4. In hac Eclipsi Duo præcipua à nobis exactè determinata sunt, *Medium* sc. Eclipsis tempus, ejusque Magnitudo. *Medium* deductum est non solùm ex comparatione *Initii*, & *Finis*, sed etiam Duarum æqualium Phasium, determinatu facillimarum, quando *Scil.* distantia Cornuum æqualis erat Lunæ semidiametro, ante Eclipsin captæ, 15′. 28″. Scilicet cum *Initium* Eclipsis existimatum fuerit 2^h. 24′. 35″. — *At Paris; by M. Cassini. n. 123. p. 561.*

| | h. | ′ | ″ |
|---|---|---|---|
| *Finis* verò *Totalis*, relictâ Penumbrâ simili ac fuerit in determinatione *Initii* | 4 | 15 | 25 |
| Duratio totius Eclipsis provenit | 1 | 50 | 50 |
| Dimidia | 0 | 55 | 25 |
| Et Eclipsis *Medium* | 3 | 20 | 0 |
| Sexta verò circumferentiæ pars abscissa est | 2 | 38 | 5 |

Atque

| | h. | ′ | ″ |
|---|---|---|---|
| Atque iterum | 4 | 2 | 25 |
| Intervallum | 1 | 24 | 20 |
| Dimidium | | 42 | 10 |
| Hinc *Medium* Eclipsis. | 3 | 20 | 15 |

Intra quartam minuti partem priori determinationi conveniens.

In Situ Umbræ & Eclipsis Magnitudine D. *Flamstedio* planè convenimus. Ab utrisque quippe Nostrûm annotatum est, Umbram nunquam superasse *Porphyritem*, licet is altè in *Penumbram* fuerit Immersus.

*Porphyriti* proximus est Mons parvus albicans, quem tunc *Aristarchi Comitem* appellavimus, eò quod ab ipso seu *Porphyrite* vix distet sui Diametro. Is Monticulus Immersus est in Umbram $2^h. 51'. 15''$; Emersit autem $3^h. 8'. 25''$. totoque tempore interjecto fuit Umbra *Porphyriti* proxima.

Uterque pariter annotavimus, in summa Eclipsi Umbram ad *Corsicam* ferè pertigisse, nunquam tamen ab ea fuisse tectam, sed relictum exiguum intervallum, cujus termini distantia à Lunari Margine proximo capta est $8'. 17''$, cum *Flamstedius* Insulæ ipsius paulo remotioris distantiam ab eodem Limbo invenerit $8'. 39''$. *Insulam* quoque seu potius *Peninsulam Macram* utrique Umbræ diutissimè adjacentem conspeximus; nos id fieri cœpisse notavimus $3^h. 28'. 15''$. & per horæ quadrantem in eadem distantia perseverasse.

5. *By* M. Bullialdus. n. 125. p. 610.

| Capellæ distant. à Vertice. | | Phases. | Temp. m. | | |
|---|---|---|---|---|---|
| ° | ′ | | h. | ′ | ″ |
| 39 | 36 | *Penumbra* tenuis. | 2 | 6 | 12 |
| 40 | 42 | *Penumbra* crassior. | 2 | 12 | 7 |
| 42 | 30 | *Initium* sensibile è reg. *Sinus Hyperb.* circa gr. 70. | 2 | 23 | 32 |
| 42 | 50 | Digiti fere $\frac{1}{8}$. | 2 | 25 | 48 |
| 44 | 25 | Umbra attigit *Atlantem Minorem.* | 2 | 36 | 11 |
| 47 | 28 | Umbra paulo supra *Baronium*, supra *Ligustinum*, occupaverat *Macr. M.* | 2 | 56 | 27 |
| 48 | 56 | U. attigerat fere *Catenam Mundi.* | 3 | 6 | 20 |
| 49 | 54 | U. attig. *Montunial.* | 3 | 12 | 54 |
| 50 | 30 | Attigit *Sinum Perontic. M.Pyram* & Med.*Paludum Hyperb.* | 3 | 17 | 51 |
| 52 | 7 | Occupavit *Sinum Sagaricum* & *Peronticum*, atque *Promont. Lunæ.* | 3 | 29 | 1 |
| 54 | 52 | *Leucopetra* extra Umbram. | 3 | 48 | 21 |

| | | | | | |
|---|---|---|---|---|---|
| 55 | 40 | *Sinus Peronticus* extra Umbram. | 3 | 54 | 6 |
| 55 | 48 | *Sinus Sagaricus* extra Umbram. | 3 | 55 | 12 |
| 56 | 17 | *Sinus Cercinites* fere Emerserat. | 3 | 58 | 29 |
| 57 | 16 | Pars sub Umb. æqualis fere Latitudini *Paludis Mæotidis.* | 4 | 5 | 38 |
| 58 | 30 | *Finis* verus è reg. *Mont. Macrocem* circa g. 355. | 4 | 13 | 56 |
| 59 | 6 | *Penumbra.* | | | |

*Corsicam* non attigit Umbra, neque *Lacum Thrasymenum*, propterea Eclipsis non excessit 3 Dig. 30′. vel minus etiam. *Initium* uno scrupulo primo, vel 45″, antecessit adnotatum, ita ut statui exactius possit $2^h$. 22′. 32″. Hinc tota *Duratio* satis præcise $1^h$. 51′. 24″. Quare *Maxima Obscuratio* contigit $3^h$. 18′. 14″.

6. *At* Strasburg; *by M.* Richel: *Ib.*

| *Alto Arcturo* | | *Tempor. m.* | | | *Phases.* |
|---|---|---|---|---|---|
| ° | ′ | h. | ′ | ″ | |
| 30 | 30 | 2 | 48 | 48 | *Initium.* |
| 36 | 0 | 3 | 20 | 8 | Umbra per M. *Porphyritem* & *Promont. Lunæ.* |
| 39 | 50 | 3 | 45 | 44 | Umbra strinxit *Lacum Thrasymen. Mont. Baronium,* & *Sinum Cercinitem.* |
| 44 | 15 | 4 | 13 | 20 | Umbra transiit per *Prom. Lunæ* & *M. Cimmerium.* |
| 46 | 25 | 4 | 27 | 36 | Umbra tetigit *Lacum Nigrum Minorem* & *M. Carpathum.* |
| 48 | 30 | 4 | 41 | 44 | Desiit in regione *Hyperborea* media ad *Mare Hyperboreum.* |
| *Tota* itaque *Duratio* $1^h$. 52′. 56″. | | | | | |

7. In hac Eclipsi probè notandum est, quod omnes Sectiones nunquam *Montem Porphyritem* omnino texerint, sed ille per totam Durationem, etiam in ipsa Maxima Obscuratione, in ipso Umbræ Limbo conspicuus perstiterit. *At* Dantzick; *by* M. Hevelius, n. 124. p. 589.

*Temp.*

| Temp. ex Altit. Cor. | | | Altitudines Fixarum. | | | Ordo Phasium. | Per quas Maculas transiverint Umbræ Sectiones. |
|---|---|---|---|---|---|---|---|
| h. | ′ | ″ | ° | ′ | | | |
| 2 | 28 | 38 | 26 | 12 | Dext. Humeri *Orionis.* | | |
| 2 | 33 | 20 | 25 | 35 | Dext. Hum. *Orionis.* | | |
| 2 | 36 | 40 | | | Initium *Penumbr.* | | Ad *Montem Baronium.* |
| 3 | 8 | 10 | | | | | *Penumbra* densior. |
| 3 | 16 | 35 | | | | | Densa *Penumbra.* |
| 3 | 24 | 20 | | | | | Densissima *Penumbra.* |
| 3 | 30 | 0 | | | *Initium* Eclipseos. | 1 | Ad *Sinum Apollinis.* |
| 3 | 36 | 25 | | | | 2 | Per *M. Alabast.* & *Sin. Hyperb.* |
| 3 | 42 | 5 | | | | 3 | Ad *Sinum Apoll.* |
| 3 | 46 | 30 | | | | 4 | Per *M. Baron.* & *L. Nigr. Majorem.* |
| 3 | 52 | 10 | | | | 5 | Per *Insf. Ophiusam.* |
| 3 | 59 | 15 | | | | 6 | Ad *M. Porphyr.* & *L. Nig. Min.* |
| 4 | 7 | 45 | | | Stellula distabat in Limb. 36′. vel. 40′. | 7 | Per *M. Porph.* & *M. Serr.* |
| 4 | 11 | 45 | | | | 8 | Per *Porph. Pr.*☽æ, & *M. Carp.* |
| 4 | 18 | 5 | | | | 9 | Per *Insf. Corf. M. Arg.* & *M. Macroceran.* |
| 4 | 26 | 0 | | | | 10 | Per *M. Porphyrit.* & *Lac. Trasimenum.* |
| 4 | 56 | 20 | | | | 11 | Ad *Sinum Apollinis, M. Christi,* & *Insf. Macr.* |
| 5 | 55 | 28 | 32 | 33 | Lucidæ *Lyræ.* | | |
| 5 | 57 | 33 | 32 | 50 | Lucidæ *Lyræ.* | | |
| 5 | 59 | 0 | | | | | *Penumbra.* |
| 6 | 8 | 0 | | | | | *Penumbra* pene evanuit quoad conjicere potuimus ob nubes. |

XLVII. *An Eclipse of the Moon,* Oct. 29(*St.n.*) 1678. *At* Paris; *by M.* Cassini. *n.* 141. *p.* 1015.

| *Phases Lunæ & Macularum Secundum Denominationem Riccioli.* | In *Observatorio Regio.* *Infra.* | | | In *Observatorio Regio.* *Supra.* | | | In *Collegio Claromontano.* | | |
|---|---|---|---|---|---|---|---|---|---|
| | h. | ′ | ″ | h. | ′ | ″ | h. | ′ | ″ |
| *Incipit* Umbra | 6 | 43 | 30 | 6 | 43 | 40 | 6 | 43 | 54 |
| *Grimaldi* Limbus Sequens | 6 | 45 | 0 | | | | 6 | 45 | 29 |
| *Gallileus* | 6 | 46 | 0 | 6 | 46 | 0 | | | |
| Finis *Galilei* | 6 | 47 | 0 | | | | | | |

*Mersennus*

| | h. ′ ″ | h. ′ ″ | h. ′ ″ |
|---|---|---|---|
| *Merſennus* | 6 48 20 | | |
| Chorda Eclipſis Dig. 6. | | 6 49 00 | |
| *Gaſſendi* Initium | 6 50 50 | 6 50 50 | |
| *Gaſſendi* Medium | | 6 51 30 | 6 51 37 |
| *Schikardi* Initium | 6 51 43 | | |
| II. Digiti Ecliptici | | 6 52 00 | |
| *Ariſtarchi* Initium | 6 52 50 | | |
| *Ariſtarchi* Medium | | 6 53 10 | 6 53 7 |
| *Ariſtarchi* & *Morini* Finis. | 6 54 00 | | |
| *Capuanus* ſive Oculus *Draconis* | 6 56 00 | | |
| Digiti III. | | 6 56 30 | |
| Chorda 9 Digitorum | | 6 57 00 | |
| Initium *Terræ Pruinæ* & *Copernici* | 6 58 54 | | |
| *Copernici* Initium | | 6 59 10 | |
| *Copernici* Medium | 7 00 00 | 7 00 00 | 6 59 30 |
| *Copernici* Finis | 7 00 55 | 7 00 55 | |
| *Pithææ* Initium | 7 1 50 | 7 1 40 | |
| *Pithææ* Finis | 7 2 30 | | |
| *Caput Virginis* | 7 2 45 | 7 2 40 | 7 3 18 |
| *Harpalus* | 7 2 55 | | |
| *Tychonis* Initium | 7 4 20 | 7 4 20 | |
| *Tychonis* Medium | 7 5 00 | | 7 5 48 |
| *Tychonis* Finis | 7 5 55 | | |
| *Eratoſthenes* | 7 6 20 | | |
| Digiti V. | | 7 6 50 | |
| Promontorium inter *Virginem* & *Platonem* | 7 7 00 | | |
| *Inſula* in ultimo *Sinuum Mediorum* | 7 7 30 | | |
| *Clara* ſequens *Tychonem* | 7 8 31 | | |
| Digiti VI. | | 7 11 20 | |
| *Tymocharis* | 7 11 48 | | |
| *Platonis* Initium | | 7 13 20 | 7 13 29 |
| *Platonis* Medium | | 7 13 40 | |
| *Platonis* Finis & Initium *Manilii* | 7 14 00 | | |
| *Platonis* Finis | | 7 14 40 | |
| *Manilius* | | 7 14 50 | 7 15 4 |
| Finis *Manilii* | 7 15 12 | | |
| *Dionyſiii* Initium | 7 17 15 | | |
| *Dionyſius* | | 7 17 25 | |
| *Menelaus* | | 7 18 10 | 7 17 59 |
| *Dionyſii* Finis & *Menelai* Initium | 7 18 28 | | 7 18 11 |
| *Plinius* Incipit | 7 20 56 | | |
| *Plinius* | | 7 21 10 | 7 21 51 |
| *Picolomineus* ſeu *Clara* ſupra *Annulum* | 7 21 30 | | |
| Initium *Fracaſtorii* ſeu *Annuli* | 7 23 00 | 7 23 5 | |
| Initium *Poſſidonii* | 7 23 55 | | 7 24 38 |
| Finis *Fracaſtorii* | 7 24 25 | | |

| | h. ′ ″ | h. ′ ″ | h. ′ ″ |
|---|---|---|---|
| *Clara* ante Angulum *Promontorii Acuti* | | 7 24 45 | |
| Angulus *Promontorii Acuti* | 7 25 10 | | |
| Digiti IX. | | 7 26 10 | |
| *Palus Somni* | 7 28 00 | | |
| Initium *Endimionis* | 7 29 20 | | |
| Initium *Taurentii* | 7 30 30 | | |
| Angulus Cornuum cum Parallelo 77° 15′ | | 7 30 40 | 7 31 16 |
| *Hermetis* Initium | 7 31 5 | | |
| Finis *Taurentii* ſive Capitis *Serpentis* | 7 31 40 | | |
| *Hermetis* Finis | 7 32 00 | | |
| *Proclus* | 7 32 20 | | |
| Limbus *Maris Caſpii* | 7 32 29 | 7 33 3 | 7 33 10 |
| *Macula* Inferior | 7 33 5 | | |
| Initium *Langreni* | 7 34 24 | | |
| *Meſſala* | 7 34 40 | | |
| Finis *Langreni* | 7 35 40 | | |
| *Peninſula* in *Caſpia* | 7 37 50 | | |
| Finis *Caſpiæ* | 7 37 50 | 7 37 50 | 7 38 18 |
| Macula Oblonga *f.* | 7 39 16 | | |
| *Finis* | 7 40 41 | 7 41 00 | 7 41 28 |
| *Initium* Emerſionis | 9 21 30 | 9 21 30 | 9 21 5 |
| *Grimaldus* | 9 22 44 | | 9 22 10 |
| *Grimaldi* Finis | 9 23 40 | 9 23 40 | |
| *Gallileus* | 9 24 35 | | |
| *Mare Humorum* | 9 28 15 | | |
| *Schikardi* Medium | 9 29 00 | | |
| *Ariſtarchi* Initium | 9 29 44 | | 9 29 41 |
| *Ariſtarchi* Medium | | 9 30 20 | |
| *Ariſtarchus* Totus | 9 30 25 | | |
| *Gaſſendus* Totus | 9 31 00 | | |
| *Kepleri* Initium | | 9 31 40 | 9 31 6 |
| *Keplerus* Totus | 9 32 24 | | |
| *Morinus* | 9 30 00 | | |
| Finis Inſulæ *Kepleri* | 9 34 22 | | |
| *Capuanus* Totus | 9 34 40 | | |
| Initium *Terræ Pruinæ* | 9 35 43 | | |
| *Virgo* | 9 37 00 | | |
| *Aurea* inter *Pitheam* & *Keplerum* | | 9 37 20 | |
| *Harpalus* & *Clara* ante *Copernicum* | 9 37 34 | | |
| Digiti III. | | 9 37 40 | |
| *Bullialdus* | 9 38 30 | | |
| Initium *Copernici* | | 9 39 00 | 9 38 36 |
| *Tychonis* Initium | 9 40 00 | | 9 40 22 |
| *Copernicus* Totus | 9 40 18 | | |
| *Pitheas* | | 9 40 30 | |

| | h. ′ ″ | h. ′ ″ | h. ′ ″ |
|---|---|---|---|
| *Tychonis* Medium | 9 41 00 | 9 41 10 | |
| *Tychonis* Finis | | 9 42 10 | |
| Digiti IV. | | 9 42 40 | |
| *Timocharus* Totus | 9 45 30 | | |
| *Plato* Incipit | | 9 45 30 | 9 45 00 |
| Digiti V. | | 9 47 00 | |
| *Plato* Totus | 9 47 00 | 9 47 30 | |
| Inſula ſub *Sinu Medio* | 9 50 00 | | |
| Finis *Sinus Medii* | 9 50 50 | | |
| *Archimedes* | 9 51 37 | | |
| Digiti VI. | | 9 52 00 | |
| *Manilius* | | 9 54 00 | 9 53 52 |
| *Fretum* | 9 54 22 | | |
| *Abifeldeæ* Initium | 9 54 40 | | |
| *Abifeldeæ* Finis | 9 55 30 | | |
| *Ariſtoteles* & *Eudoxus* Toti | 9 56 00 | | |
| Digiti VII. | | 9 56 20 | |
| *Dionyſii* Initium | 9 57 12 | | |
| *Menelaus* | 9 57 50 | 9 57 40 | 9 57 16 |
| *Dionyſii* Medium | | 9 58 30 | |
| *Menelaus* Totus | 9 59 00 | | |
| *Dionyſius* Totus | 10 00 00 | | |
| Digiti VIII. | 10 1 30 | | |
| Oſtium *Lacus Mortis* | | 10 2 00 | |
| *Plinius* | | 10 2 15 | 10 1 28 |
| *Promontorium* ſupra *Dionyſium* | 10 3 00 | | |
| *Plinii* Finis. *Poſſidonius.* | 10 3 40 | | |
| *Promontorium Acutum* | | 10 5 30 | |
| *Hermes* | 10 6 00 | | |
| Digiti IX. | | 10 6 15 | |
| Chorda Umbræ 8 Digitorum | | 10 9 30 | |
| *Meſſala* | 10 11 00 | | |
| Initium *Maris Caſpii* | 10 13 30 | 10 12 10 | 10 11 30 |
| *Langrenus* Totus | 10 16 00 | | |
| Chorda 6 Digitorum | | 10 14 00 | |
| *Caſpii* Prior Finis | | 10 16 50 | 10 16 44 |
| *Caſpii* Alter Finis | 10 17 33 | 10 17 25 | |
| Albedo in *Mari Caſpio* | 10 18 15 | | |
| *Finis* Totalis | 10 20 00 | 10 20 10 | 10 20 42 |

XLVIII. I. *An Eclipse of the Moon*, Aug. 19. *m.* 1681. *at* Greenwich; *by* Mr. Flamsteed, *Ph. Coll.* n. 3. p. 67

| *Temp. per Hor. Oscil* | *Temp. inde per Obser-vat. Corr.* | *Observationes.* |
|---|---|---|
| h. ′ ″ | h. ′ ″ | |
| 1 5 00 | 1 00 00 | Lunæ *Diameter* 6191 = 30′. 53″. |
| 1 46 30 | 1 41 30 | *Penumbra* notabilis. |
| 1 53 40 | 1 48 40 | *Penumbra* densa. |
| 1 55 40 | 1 50 40 | *Initium.* |
| 1 58 00 | 1 53 00 | Umbra ad *Montem Climacem.* |
| 2 3 20 | 1 58 20 | *Sinus Sirbonis* Incipit. |
| 2 5 50 | 2 00 50 | *Marœotis* Incipit. |
| 2 7 30 | 2 2 30 | *Mons Cataractes* Incipit. |
| 2 8 16 | 2 3 16 | *Marœotis* Tota tecta. |
| 2 10 50 | 2 5 50 | *Sina* Incipit. |
| 2 12 56 | 2 7 56 | *Sina* Tota tecta. |
| 2 18 00 | 2 13 00 | *Audus.* |
| 2 27 24 | 2 22 24 | *Circinna.* |
| 2 31 00 | 2 26 00 | ——— *Abulfeda* Ricciolo. |
| 2 32 30 | 2 27 30 | *Pentadactylus* Incipit. |
| 2 33 32 | 2 28 32 | *Ætnæ* Limbus primus. |
| 2 34 26 | 2 29 26 | *Pentadactylus* Totus. |
| 2 35 6 | 2 30 6 | *Ætnæ* Medium. |
| 2 36 00 | 2 31 00 | *Ætna* Tota tecta. |
| 2 40 00 | 2 35 00 | Umbra per Med. *Porphyriten.* |
| 2 44 50 | 2 39 50 | *Hiera Insula* & *Ficaria* simul ad Umbram. |
| 2 46 10 | 2 41 10 | Umbra ad *Horminium.* |
| 2 50 10 | 2 45 10 | *Mons Hercules* Incipit. |
| 2 51 10 | 2 46 10 | *M. Hercules* intra. |
| 2 52 10 | 2 47 10 | Umbra per Med. *Besbici.* |
| 2 54 00 | 2 49 00 | per Med. *Insulæ Majoris.* |
| 2 55 30 | 2 50 30 | per Med. *Byzantium.* |
| 2 57 40 | 2 52 40 | per Med. *Insulæ Cyaneæ.* |
| 3 1 50 | 2 56 50 | ad Limbum primum *Corocondometis.* |
| 3 4 30 | 2 59 30 | ad Limbum primum *Mæotidis.* |
| 3 13 00 | 3 8 00 | *Macra* Incipit obscurari : dub. |
| 3 14 00 | 3 9 00 | *Mæotis* Tota tecta. |
| 3 19 12 | 3 14 12 | Partes Lucidæ residuæ erant 1092 = 5′. 27″. |

| Digits Eclipsed. | Phases. | Time. |
|---|---|---|
| d. ′ ″ | | h. ′ ″ |
| | The *Beginning* at | 1 58 30 |
| | The arrival of the Shadow at *Grimaldus*: | 2 1 30 |
| | At *Tycho*. | 2 8 00 |
| | At the *Center* of the *Moon*. | 2 37 30 |
| | At the Middle of *Copernicus*: | 2 39 30 |
| | At *Ariſtarchus*. | 2 40 00 |
| 8 16 00 | At the Middle of *Manilius*. | 2 54 30 |
| | At *Plinius*. | 3 1 00 |
| 9 2 00 | At the Lower part of the *Caſpian Sea*. | 3 7 30 |
| 9 31 20 | At the Upper ſide of the *Caſpian Sea*. | 3 18 30 |
| 10 1 00 | The *Greateſt Obſcurity*. | 3 35 45 |
| 4 46 00 | The *Return* of the Shadow To *Ariſtarchus*. | 3 47 00 |
| | To the *Center* of the *Moon*. | 4 29 00 |
| | To *Tycho*. | 4 41 30 |
| | The *End* of the *Eclipſe*. | 5 13 00 |
| | The *Whole Duration*. | 3 14 30 |

2. *At* Paris; *by* M. Caſſini. *Ibid. p.* 66.

At 5h. 13′. 56″. the *Apparent Altitude* of the upper Edge of the Sun, was 17′. 10″. and that of the *Moon*, 1°. 11′. 30″.

| Temp. juxta Hor. | Phaſes. | Altitudines. | Temp. Correct. |
|---|---|---|---|
| h. ′ ″ | | ° ′ | h. ′ ″ |
| 1 1 03 | Altitudo *Palilicii*. | 24 14 | 1 4 48 |
| 2 4 30 | Eadem Altitudo. | 32 58 | 2 6 32 |
| 2 9 00 | Rurſus capta. | 33 30 | 2 10 32 |
| 2 45 00 | Veſtigium *Penumbræ*. | | 2 46 30 |
| 2 53 00 | *Penumbra* paulo Denſior. | | 2 54 00 |
| 2 59 00 | Adhuc Denſior. | | 3 00 00 |
| 3 1 00 | Denſiſſima. | | 3 2 00 |
| 3 2 00 | *Initium* Eclipſeos. | | 3 3 00 |
| 3 6 00 | *Prima* Phaſis. | | 3 6 30 |
| 3 12 00 | *Secunda* Phaſis. | | 3 12 30 |
| 3 14 00 | *Tertia* Phaſis. | | 3 14 30 |
| 3 20 00 | *Quarta* Phaſis. | | 3 21 00 |
| 3 22 30 | *Mons Sinai* Totus tectus. | | 3 23 00 |
| 3 29 00 | *Sexta* Phaſis. | | 3 30 00 |
| 3 43 00 | Altitudo *Pollucis*. | 30 00 | 3 43 00 |
| 5 2 00 | Centrum *Solis* Oritur. | | |

3. *At* Dantzick; *by* M. Hevelius. n. 5. *p.* 160.

Colour

Color hujus *Eclipſeos* erat Cinericius ſive Fuligineus. Tempore Obſervationis *Mons Porphyrites* & M. *Ætna* fere in eodem Perpendiculo exiſtebant.

Phaſis I. Per Montem *Eoum* tranſibat.
II. Ad *Paludes Arabiæ* incedebat.
III. Inferiorem Ripam *Paludis Maræotidis* & Extremitatem *Sinus Sirbonis* tangebat.
IV. Per Inſulam *Letoam* incedebat ad ipſum M. *Sinai* uſque, ſed dictus Mons Totus adhuc erat conſpicuus.
V. *Sinum Syrticum* tranſire videbatur, ſic ut totum M. *Sinai* Umbra jam tegeret.
VI. Supra *Sinum Syrticum*, ultra Inſulam *Cretam*, & per *Mare Mortuum* incedebat.

*An Eclipſe of the Moon*, Feb. 11. 1682. *by Mr.* Flamſteed. n. 145. *p.* 89.

XLIX. 1. Ann. 168½. *Feb.* 11. H. 8. 1′. Tubo pedum 16 *Lunæ* cepi Diametrum 6702 = 33′. 25″. deinde diſtantiam Limbi ejus Proximi à Limbo Proximo *Maræotidis* 145 = 00′. 43″. ſed ejuſdem Limbi Maculæ à Limbo Lunæ remotiore 6575 = 32′. 48″. Hujus etiam Tubi ope Tempora, cùm Obſcuratio ad Centrum Lunæ pertigerit, & cùm Radius ejus Arcus in Periphæria Deficientes vel Reſtitutos ſubtenderit, obtinui; è quibus medium derivari poteſt, forſan non minus accuratè, quam ab obſervatis *Initio* & *Fine*, *Immerſione* & *Emerſione*, collatis.

*Phaſes*

| *Phases.* | *Tempora Phasium* per *Horologia Oscillatoria correcta.* | | |
|---|---|---|---|
| | *Greñovici* in *Observatorio* | | *Londini.* |
| | *Mihi.* | D. *Halleio.* | D. *Haynesio.* |
| | h. ′ ″ | h. ′ ″ | h. ′ ″ |
| *Fumus* ad Oram Inferiorem. Nudis Oculis | 8 48 38 | | |
| Umbrago. | 9 4 8 | | |
| Densa *Penumbra.* | 9 11 44 | | |
| *Initium.* | 9 12 32 | 9 13 04 | 9 12 18 |
| Medium Paludis *Maræotidis* tectum. | 9 14 02 | | fortè citiús. |
| Tota Palus tecta. | | 9 14 39 | 9 13 48 |
| Sexta Pars *Peripheriæ Obscurata.* | 9 18 10 | | |
| *Circinna* intra Umbram | 9 20 10 | | |
| *Porphyrites* Medius. | | 9 21 1 | |
| Umbra stringit *Syrbonem.* | | 9 21 9 | |
| Limbus 1us. Montis *Cataractes* vel *Gassendi* | 9 21 32 | | |
| *Oculus Draconis.* | 9 27 58 | | |
| Legebat Limbis primos *Cretæ* & *Ætnæ.* | 9 28 22 | 9 28 44 | 9 28 19 |
| *M. Ætna* Totus tectus. incipit *Hiera.* | 9 29 48 | 9 30 16 | 9 30 12 |
| *Hiera* tot. | | 9 30 44 | |
| Initium *Corsicæ.* | 9 33 46 | | |
| Medium *Corsicæ.* | | 9 34 48 | |
| *Sina* Mons incipit. | 9 36 38 | 9 37 10 | 9 36 20 |
| *Sinæ* Medium. | 9 37 22 | | |
| Totus *Sina* tectus. | 9 38 04 | 9 38 10 | 9 38 20 |
| *Centrum Lunæ* sive *Degiti* VI. | 9 38 48 | | |
| Incipit *Lacus Niger Major.* | 9 39 58 | 9 40 36 | |
| Medium *Lacus Nigri Majoris.* | | | 9 40 48 |
| Totus tegitur Lacus. | | 9 41 40 | |
| Initium *Besbici.* | | 9 43 48 | |
| Medius *Besbicus.* | 9 43 42 | | 9 44 13 |
| Totus *Besbicus* & stringit *Pont. Euxinum.* | | 9 44 36 | |
| Incipit *Byzantium.* | 9 46 58 | | |
| Incipit *Horminius.* | 9 47 22 | | |
| *Carpathes.* | 9 47 50 | | |
| *Mons Serrorum.* | 9 48 30 | | |
| *Apollonia* | 9 50 24 | 9 51 21 | |
| *Macra.* | 9 52 44 | 9 53 33 | |
| *Mons Hercules.* | 9 54 26 | | |
| *Macra* tota tecta. | | 9 54 53 | |
| *Mons Hercules* Totus. | | 9 55 33 | |
| *Corocondometis* Palus. | 9 59 10 | | |
| Per Medium *Lacum Hyperbor. Superiorem.* | | 9 59 53 | |
| *Corocondometes* tota tecta. | 10 00 38 | | |
| Umbra per Montem *Coracem.* | | 10 1 17 | |
| Stringit *Mæotida.* | 10 2 20 | 10 2 38 | |

Per

| | h. | ′ | ″ | h. | ′ | ″ | h. | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| Per *Lacum Hyperboreum Inferiorem.* | | | | 10 | 3 | 53 | | | |
| Per Medium *Insulæ Majoris.* | 10 | 5 | 14 | | | | | | |
| Tota ferè *Mæotis tecta.* | 10 | 6 | 54 | 10 | 8 | 00 | 10 | 6 | 48 |
| *Sexta pars Peripheriæ* Lucida residua. | 10 | 7 | 28 | | | | | | |
| *Immersio.* | 10 | 10 | 14 | 10 | 10 | 11 | 10 | 9 | 48 |
| *Emersio.* | 11 | 47 | 38 | 11 | 47 | 9 | 11 | 47 | 48 |
| *Sexta pars Peripheriæ* Illuminata, cum Limbo 1°. *Maræot.* | 11 | 50 | 02 | 11 | 50 | 7 | | | |
| Medium *Maræotidis.* | 11 | 50 | 42 | | | | 11 | 50 | 48 |
| Tota *Maræotis* Illuminata. | 11 | 51 | 6 | 11 | 50 | 58 | | | |
| Umbra per Medium *Porphyritidis.* | 11 | 52 | 50 | | | | | | |
| Totus *Porphyrites* tectus. | 11 | 53 | 34 | | | | 11 | 52 | 54 |
| *Circinna* tota. | 11 | 58 | 18 | | | | | | |
| Stringit *Sinum Syrbonis.* | | | | 11 | 58 | 59 | | | |
| *Cataractes Syrbonis*, vel *Gassendus* totus. | 12 | 1 | 00 | | | | | | |
| *Hiera* incipit. | | | | 12 | 3 | 14 | | | |
| *Ætna* incipit Emergere. | 12 | 3 | 20 | | | | | | |
| Per Medium *Lacus Nigri Majoris.* | | | | | | | 12 | 3 | 48 |
| Per Medium *Ætnæ.* | | | | 12 | 4 | 19 | | | |
| *Ætna* totus retectus. | 12 | 5 | 18 | 12 | 5 | 4 | 12 | 5 | 45 |
| *Creta* Illuminatur. | 12 | 7 | 34 | | | | | | |
| *Sina* incipit. | 12 | 13 | 38 | | | | 12 | 13 | 43 |
| *Sinæ* Medium. | 12 | 14 | 28 | 12 | 14 | 12 | | | |
| *Sina* totus. | 12 | 15 | 34 | 12 | 14 | 55 | | | |
| Medium *Besbici.* | 12 | 17 | 34 | | | | 12 | 17 | 48 |
| *Centrum Lunæ*, si *Digiti* VI *retecti.* | 12 | 18 | 28 | | | | | | |
| Per Medium *Lacum Hyperbor. Superiorem* | | | | 12 | 19 | 5 | | | |
| Umbra per Medium *Byzantii.* | | | | | | | 12 | 20 | 18 |
| *Byzantium* extra. | 12 | 20 | 50 | 12 | 20 | 25 | | | |
| *Macra* tota. | | | | 12 | 22 | 36 | | | |
| *Insula Apollonia* tota, & *Horminius.* | 12 | 24 | 24 | 12 | 24 | 16 | | | |
| *Lacus Hyperboreus Inferior.* | | | | 12 | 26 | 6 | | | |
| Mons *Hercules.* | | | | 12 | 30 | 56 | 12 | 31 | 18 |
| Medium *Corocondemetidis* ad Umbram. | 12 | 31 | 38 | | | | | | |
| Mons *Corax.* | | | | 12 | 32 | 37 | | | |
| *Mæotis* incipit illustrari. | 12 | 33 | 18 | | | | 12 | 32 | 18 |
| *Mæotis* tota retecta. | | | | 12 | 37 | 57 | 12 | 38 | 3 |
| *Sexta pars Peripheriæ* Obscura residua | 12 | 39 | 6 | | | | | | |
| Media *Insulæ Majoris Caspii.* | | | | 12 | 41 | 22 | 12 | 42 | 18 |
| Tota *Insula* retecta. | 12 | 42 | 18 | | | | | | |
| Dubius *Finis.* | 12 | 45 | 10 | | | | 12 | 44 | 48 |
| *Finis* certè. | 12 | 45 | 38 | 12 | 44 | 35 | 12 | 45 | 18 |

Ingruente *Eclipsi*, Distantias *Lunæ* a Fixis Sextante Ceperant Ds. *Halleius* cum *Tho. Fabro* sequentes;

| *Tempora per Hor. Correct.* | | | *Observationes.* | *Distantiæ.* | | |
|---|---|---|---|---|---|---|
| h. | ′ | ″ | | ° | ′ | ″ |
| 8 | 53 | 07 | Limbi *Lunæ* Remotioris a *Regulo*——— | 8 | 05 | 20 |
| 8 | 55 | 07 | repet.— | 8 | 06 | 20 |
| 9 | 2 | 34 | Limbi *Lunæ* Rem. à *Lucid.* in *Lumbis* Leonis | 14 | 58 | 15 |
| 9 | 4 | 18 | repet.— | 14 | 57 | 55 |
| 9 | 8 | 42 | Limbi *Lunæ* Remot. a *Regulo* iterum —— | 8 | 12 | 50 |
| | | | *Luna* in Umbram penitus mersa, Ds. *Halleius* mecum. | | | |
| 10 | 19 | 21 | Limbi *Lunæ* Proximi a *Regulo*.——— | 8 | 13 | 30 |
| 11 | 38 | 06 | Remoti à *Regulo*.— | 9 | 23 | 50 |
| 11 | 41 | 48 | repet.—– | 9 | 25 | 35 |
| | | | Et finita *Eclipsi*, | | | |
| 12 | 52 | 00 | Limbi Remotioris a *Regulo* iterum.———— | 9 | 58 | 20 |
| 12 | 55 | 10 | denuo.— | 10 | 00 | 00 |

2\.
At *Paris* and *Copenhagen.* n. 146. p. 145.

| *Phases.* | *Parisiis.* In *Observatorio Regio.* A D. *Cassino.* | | | *Parisiis.* In *Observatorio Regio.* A DD. *Picard* & *De la Hire.* | | | *Parisiis.* In *Collegio Claramont.* A R. P. *de Fonteney.* | | | *Hafniæ.* A D. *Roemer*, reductione Meridianorum facta Subtr. 41′. 40″. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | h. | ′ | ″ | h. | ′ | ″ | h. | ′ | ″ | h. | ′ | ″ |
| *Initium.* | 9 | 20 | 55 | 9 | 21 | 58 | 9 | 21 | 25 | 9 | 21 | 50 |
| *Aristarchus* Medius. | 9 | 29 | 57 | 9 | 29 | 33 | 9 | 29 | 28 | 9 | 29 | 50 |
| Medium *Copernici.* | 9 | 38 | 45 | 9 | 38 | 25 | 9 | 38 | 40 | 9 | 38 | 48 |
| Initium *Tychonis.* | 9 | 45 | 52 | 9 | 45 | 48 | 9 | 45 | 48 | 9 | 46 | 20 |
| Finis *Tychonis.* | 9 | 47 | 40 | 9 | 47 | 36 | 9 | 47 | 20 | 9 | 47 | 50 |
| Initium *Platonis.* | 9 | 49 | 47 | 9 | 49 | 08 | 9 | 49 | 25 | 9 | 49 | 50 |
| Centrum *Lunæ.* | | | | 9 | 51 | 10 | | | | 9 | 50 | 20 |
| *Centrum* ex Comparatione *Initii* & *Finis.* | 9 | 50 | 24 | 9 | 50 | 44 | 9 | 50 | 10 | 9 | 50 | 30 |
| *Manilius* Medius. | 9 | 53 | 16 | 9 | 53 | 30 | 9 | 51 | 18 | 9 | 53 | 20 |
| *Menelaus.* | 9 | 56 | 43 | 9 | 56 | 20 | 9 | 56 | 14 | 9 | 56 | 20 |
| *Dionysius.* | 9 | 57 | 09 | 9 | 57 | 40 | | | | 9 | 57 | 20 |
| *Promontorium Acutum.* | 10 | 04 | 31 | 10 | 04 | 15 | 10 | 04 | 12 | 10 | 04 | 20 |
| Initium *Caspiæ.* | 10 | 11 | 40 | 10 | 11 | 25 | 10 | 11 | 20 | 10 | 11 | 50 |
| Finis *Caspiæ.* | 10 | 16 | 27 | 10 | 16 | 30 | 10 | 15 | 50 | 10 | 16 | 20 |
| *Luna* penitus Immersa. | 10 | 19 | 53 | 10 | 19 | 30 | 10 | 18 | 55 | 10 | 19 | 50 |
| Initium *Emers. Grimaldum* inter & *Galileum.* | 11 | 57 | 51 | 11 | 56 | 00 | 11 | 58 | 00 | 11 | 55 | 20 |
| *Aristarchus* Medius. | 12 | 02 | 00 | 12 | 01 | 55 | 12 | 03 | 09 | 12 | 01 | 50 |
| Medium *Platonis.* | 12 | 12 | 06 | 12 | 12 | 10 | 12 | 13 | 26 | 12 | 12 | 20 |
| Medium *Copernici.* | 12 | 14 | 32 | 12 | 14 | 30 | 12 | 14 | 29 | | | |
| Medium *Tychonis.* | 12 | 23 | 35 | 12 | 23 | 55 | 12 | 24 | 21 | 12 | 23 | 20 |
| *Centrum Lunæ.* | | | | 12 | 25 | 50 | | | | 12 | 26 | 20 |
| *Centrum* ex Comparatione *Emers.* Limborum. | 12 | 26 | 09 | 12 | 25 | 15 | 12 | 26 | 08 | 12 | 24 | 50 |
| *Manilius* Medius. | 12 | 26 | 22 | | | | 12 | 26 | 20 | 12 | 26 | 20 |
| *Menelaus* Medius. | 12 | 29 | 08 | 12 | 29 | 20 | 12 | 29 | 22 | 12 | 29 | 05 |
| *Dionysius.* | 12 | 32 | 30 | 12 | 33 | 15 | 12 | 32 | 20 | 12 | 32 | 50 |
| *Promontorium Acutum.* | 12 | 39 | 45 | 12 | 39 | 50 | | | | 12 | 39 | 50 |
| Initium *Caspiæ.* | 12 | 41 | 53 | 12 | 41 | 30 | 12 | 41 | 11 | | | |
| Finis *Caspiæ.* | 12 | 47 | 15 | 12 | 46 | 30 | 12 | 48 | 20 | 12 | 47 | 20 |
| *Finis* Eclipsis. | 12 | 54 | 27 | 12 | 54 | 30 | 12 | 54 | 17 | 12 | 54 | 20 |

*Tempus*

| Tempus secund. Horol. Ambul. | Or. Phas. | Digiti Eclipt. | Altitudines Stellarum. | Per quas Masculas transiverint Umbræ Sectiones, & quæ insuper Notata fuerint. | Tempus ex Alt. Correct. |
|---|---|---|---|---|---|
| h. ′ ″ | | | ° ′ | | h. ′ ″ |
| 8 41 18 | | | *Palilicii* 39 53 | | 8 40 46 |
| 8 44 9 | | | *Pal.* 39 27 | | 8 44 20 |
| 8 48 14 | | | *Pal.* 39 0 | | 8 47 59 |
| 10 8 28 | | | | Initium *Penumbræ* Dilutissima. | 10 9 0 |
| 10 12 50 | | | | *Penumbra* Densior. | 10 13 20 |
| 10 19 5 | | | | *Penumbra* adhuc Crassior. | 10 19 36 |
| 10 23 50 | | | | *Penumbra* Densissima. | 10 24 24 |
| 10 24 30 | | | | *Initium* Eclip. circa 150° Limbi in 95° sc. à Puncto super. Lineæ perpend. Novagesimi Ortum versus contigit. | 10 25 5 |
| 10 29 45 | 1 | $1\frac{1}{4}$ *Digit.* | | Incedebat per M. *Germanician.* ad *Mare Syrtic.* & Mont. *Acabe.* | 10 30 30 |
| 10 32 13 | 2 | 2 *ferè* | | Per M. *Porphyritem, loca Paludosa*, Ins. *Cercinnæ, Synum Syrtic.* per *Mont. Sacr.* & M. *Casium.* | 10 32 50 |
| 10 36 30 | 3 | $2\frac{1}{2}$ *ferè* | | Ad M. *Baronium*, Ins. *Æthusam*, per medium *Sinum Syrbonis* & M. *Pharan.* | 10 37 10 |
| 10 43 25 | 4 | $3\frac{5}{8}$ | | Per *Sinum Apollinis*, M. *Æth.* Ins. Leto. ad Ins. Dydimam. | 10 44 10 |
| 10 47 25 | 5 | $4\frac{5}{8}$ *Dig.* | | Ad Ins. *Majorc. Vulcaniam, Lemnos, Carpath.* & M. *Horeb.* | 10 48 15 |
| 10 52 20 | 6 | $5\frac{3}{4}$ | | Ad *Lacum Nigrum Majorem*, per M. *Argentarium, Sipylum, Masicytum* ; *Tabor, Sinai* & Desertum *Raphidim.* | 10 53 10 |
| 10 55 45 | 7 | $6\frac{1}{2}$ | | Per *Scopulos Hyperboreos*, Ins. *Besbycam*, M. *Olympum, Didymum*, & M. *Antilibanum.* | 10 56 40 |
| 11 0 33 | 8 | $7\frac{1}{4}$ *Dig.* | | Ad Ins. *Cyaneam*, M. *Hormin.* M. *Uxii* & Montes *Coibacaranos.* | 11 1 30 |
| 11 4 30 | 9 | $8\frac{1}{2}$ | | Per M. *Ambonum* ad Ins. *Apolloniam*, & *Sinum Atheniens.* per M. *Mosch. Uxii*, & *Caibacaranos.* | 11 5 30 |
| 11 9 59 | 10 | $9\frac{1}{4}$ | | Per *Lacum Hyperbor. Superior. Pal. Byces*, per Mont. *Hercul.* & Sinum extremum *Ponti.* | 11 10 55 |

3. At *Dantzick*; by *M. Hevel. Ib. p.* 146.



Per

| h. ′ ″ | | | | | h. ′ ″ |
|---|---|---|---|---|---|
| 11 13 30 | 11 | 10 *Dig.* | | Per *Lacum Hyperboreum Inferiorem*, M. *Cimmerium*, & ad Sinum Inferiorem *Maris Caspii.* | 11 14 27 |
| 11 16 30 | 12 | 10½ *ferè* | | Per M. *Riphæos*, Pal. *Mæotidem*, Inf. *Alopeciam*, ad Inf. Majorem *Caspii*, per M. *Nerosum.* | 11 17 17 |
| 11 19 10 | 13 | 11¼ | | Ad Ripam Paludis *Mæotidis* & Mont. *Hippoci.* | 11 20 10 |
| 11 22 0 | 14 | 11½ | | Per M. *Alaunum*, & *Lacum Occident. Minorem.* | 11 23 0 |
| 11 26 30 | | 12 *Dig.* | | *Totalis Obscuratio* circa 297° Limbi in 94° sc. à Puncto Super. Lineæ perpendicularis Nonagesimi Ortum versus deprehensa. | 11 27 30 |
| 11 35 36 | | | *Pollucis* 52° 13′ | | 11 37 4 |
| 11 37 35 | | | *Pollucis* 52 0 | | 11 38 42 |
| 11 38 50 | | | *Pollucis* 51 41 | | 11 39 35 |
| 12 58 20 | | | | *Recuperatio Luminis* circa 118° *Limbi* in 88° sc. à Puncto Superiori Lineæ perpendicularis Nonagesimi Occasum versus extitit. | 1 0 8 |
| 1 0 40 | 15 | ½ *Dig.* | | Ad M. *Alabastrinum*, M. *Pentadactylum*, M. *Audum*, & Pal. *Mareotidem.* | 1 2 31 |
| 1 3 35 | 16 | 1¼ | | Per. M. *Porphyritem*, ad *Mare Syrticum*, & per M. *Eoum.* | 1 5 32 |
| 1 8 10 | 17 | 2 *ferè* | | Per M. *Baronium*, *loca paludosa*, Inf. *Cercinnæ*, inter *Mare Syrticum* & *Ægyptiacum*, ad S. *Syrbonis.* | 1 10 32 |
| 1 11 48 | 18 | 2½ *fere* | | Per *Sinum Apollinis*, Inf. *Taraciniam*, Inf. *Æthusam*, Sinumque *Syrbonis.* | 1 13 54 |
| 1 16 20 | 19 | 3½ | | Ad Inf. *Majorcam*, per M. *Ætnam*, M. *Neptunum*, *Jachyntum*, Inf. *Letoam*, Inf. *Didymam* & M. *Lyon.* | 1 18 33 |
| 1 20 50 | 20 | 4½ *Dig.* | | Ad Inf. *Corsicam*, per Inf. *Vulcaniam*, & *Carpathos*, ad *Mare Mortuum*, per Desertum *Sin.* | 1 23 6 |

Ad

| h. ′ ″ | | | | | h. ′ ″ |
|---|---|---|---|---|---|
| 1 24 39 | 21 | 5¼ *Dig.* | | Ad M. *Argentarium, Mare Pamphilium*, Inſ. *Cyprum*, per M. *Horminium.* | 1 27 0 |
| 1 26 15 | 22 | 6 *ferè.* | | Per Mont. *Apenninum*, ad Lacum *Trasimenum*, ad Mont. *Sipylum* per M. *Infer. Libanum.* | 1 28 40 |
| 1 28 32 | 23 | 6½ | | Ad M. *Carpathos*, per Inſ. *Beſbicam*, M. *Olympum, Didymum* & M. *Dalangueros.* | 1 31 0 |
| 1 31 12 | 24 | 7⅛ *Dig.* | | Per M. *Perce, Byzantium*, ad M. *Horminium*, & per M. *Antitaurum.* | 1 33 44 |
| 1 35 59 | 25 | 8¼ | | Per *Sinum Circinitem*, Inſ. *Macram*, Inſ. *Apolloniam*, Medium Montem *Meſchum*, atque *Montes Sogdianos.* | 1 38 37 |
| 1 39 33 | 26 | 9 *Dig.* | | Per *Lac. Hyperboreum Inferiorem*, Pal. *Byces*, M. *Strobilum*, per Sin. Extremum *Ponti* ac M. *Paropamiſum.* | 1 42 15 |
| 1 41 45 | 27 | 9⅜ *Dig.* | | Inter Pal. *Byces* & *Lacum Corocondametis*, per M. *Herculis* & M. *Caucaſum* Inferiorem. | 1 44 30 |
| 1 44 19 | 28 | 10 | | Per M. *Cimmerium*, M. *Tancon*, & M. *Neroſum.* | 1 47 9 |
| 1 47 20 | 29 | 10½ | | Per Pal. *Mæotidem*, Inſ. *Minorem* Maris *Caſpii, Montemque Neroſum* Superiorem. | 1 50 14 |
| 1 51 44 | 30 | 11 *paulò plus.* | | Per M. *Alaunum*, *M. Sanctum*; *Monteſque Hippoci.* | 1 54 42 |
| 1 56 12 | | 12 *Dig.* | | *Finis*, circa 294° *Limbi*, in 97 *ſc.* gradu à Puncto ſuperiori *Lineæ* perpendicularis Nonageſimi Occaſum verſus contigit. | 1 59 17 |
| 1 57 10 | | | | Denſiſſima *Penumbra.* | 2 0 17 |
| 2 1 15 | | | | Satis adhuc Craſſa. | 2 4 27 |
| 2 2 30 | | | | *Paulo* Dilucior. | 2 5 45 |
| 2 10 20 | | | | *Penumbra* Dilutiſſima. | 2 13 42 |
| 2 12 30 | | | | Finis *Penumbræ.* | 2 15 0 |
| 2 15 39 | | | *Alt. Pol.* 29° 12 | | 2 19 7 |
| 2 17 35 | | | *Ead.Alt.* 28 46 | | 2 22 8 |

Ingruente

Ingruente *Eclipsi* Umbra erat valde diluta, Limbusque ejus quasi Anfractuosus, & minime terminatus, sic ut difficulter admodum ab Initio Phases determinari potuerint, nec accuratè distingui per quas Maculas Umbra transibat, successu tamen temporis crescente *Eclipsi*, distinctiùs omnia deprehendebantur. Color ab *Initio* videbatur satis Tristis, Obscurus, & Fuliginosus, acsi *Eclipsis*, eadem ratione, circa *Maximam Obscurationem*, ut illa Anno 1642. mens. *April.* adeò sese Obumbratam sistere vellet, quo vix conspiceretur; sed res planè aliter cecidit, siquidem Luna cum jam omnino esset Eclipsata, Totus tamen ejus Discus satis clare in oculos incurrebat: Color namque ejus tum omninò Rubidus sive Sanguineus aut Rubiginosus erat, qui eousque perseverabat, donec Luna ad medietatem Lumen suum recuperasset, atque tum rursus satis Obscura & Fuliginosa apparuit.

At *Lisbon*; by Mr. *Jacobs*. Ib. p. 151.

4. The Beginning of this *Eclipse* was observed at *Lisbon* by Mr. *Jacobs* at 8h. 31′. *p. m.*

An Eclipse of the Moon, *June* 17. *m.* 1684. at *Greenwich*; by Mr. *Flamsteed*. n. 162. p. 689.

L. Difficillima fuerit parvæ hujus *Eclipsis* observatio, propter obliquam Lunæ in Umbram Terræ incidentiam, Umbræque ipsius Tenuitatem, per quam Limbum Lunæ, media etiam Eclipsi, satis distinctè cernere potuimus. Partes diametri tunc ab Umbra vera deficientes, propter confusos ejus terminos accuratè definire non licuit. Distantiam ergo cœpi inter Cuspides malè definitos, circa Medium Defectus, è qua dictæ partes facilè deduci possint, & Erroris minore periculo.

| *Tempora per Horolog. Oscillator.* | | | *Tempora Vera ab Obs. Correct.* | | | *Observationes.* | | |
|---|---|---|---|---|---|---|---|---|
| h. | ′ | ″ | h. | ′ | ″ | | ′ | ″ |
| 1 | 40 | 40 | 1 | 31 | 8 | *Eclipsis* Minæ | | |
| 2 | 4 | 30 | 2 | 54 | 58 | *Lunæ* Diameter Tubo ped. 16. erat, 1605= | 31 | 57 |
| 2 | 6 | 40 | 2 | 57 | 8 | ——— rep. 6430= | 32 | 4 |
| 2 | 12 | 0 | 2 | 2 | 28 | *Penumbra* densa, forsan *Initium* ——— | | |
| 2 | 16 | 00 | 2 | 6 | 28 | Umbra Limbum supra *Sinam* temeraverat. | | |
| 2 | 18 | 00 | 2 | 8 | 28 | *Umbra* certè intra discum | | |
| 2 | 21 | 36 | 2 | 12 | 4 | Chorda *Peripheriæ* Obscuratæ 1670= | 8 | 20 |
| 2 | 26 | 00 | 2 | 16 | 28 | ——— rep. 2010= | 10 | 1 |
| 2 | 30 | 00 | 2 | 26 | 28 | ——— iter. 2290= | 11 | 25 |
| 2 | 42 | 00 | 2 | 32 | 28 | Decrevit *Eclipsis* sensibiliter ——— | | |
| 2 | 43 | 00 | 2 | 33 | 28 | Inter Cuspides Obscur. iter. 1895= | 9 | 27 |
| 2 | 50 | 00 | 2 | 40 | 28 | *Finis*; sed Dubius Mihi | | |
| 2 | 54 | 00 | 2 | 44 | 28 | *Finita* certè, Ministro *Fabro* Consentiente. | | |
| 2 | 57 | 00 | 2 | 47 | 28 | *Penumbra* Densa. | | |
| 3 | 11 | 00 | 3 | 1 | 28 | Etiam adhuc. | | |
| 3 | 20 | 00 | 3 | 10 | 28 | *Limbus Austrinus* haud adhuc *Limbi Borei* Claritudinem recuperaverat. Sed lux ejus hebetior quam in *Limbo Boreo*, ut in prima Observatione apparuit. | | |

LI. Artis profecto Laborisque haud exigui erat Tubos, etsi breviores *sc.* 5, 6. & 7, pedum, adeò firmiter continue ad *Lunam* retinere, ut Penumbram rectè discernere potuerimus, Phasesque omnes exactè designare, per quas nimirum Maculas transirent, vel quas omni tempore attingerent. Attamen pro viribus, quousque severa Tempestas atque transeuntes Nebeculæ permittebant, rem peregi. Notandum imprimis habes, quòd Densissima *Penumbra* genuinum *Initium Eclipseos* præcesserit, ita ut vix ac ne vix verum *Initium* discernere quiverim. Colorem quod attinet, hunc in hac *Eclipsi* maxime notandum habes; quippe talem diversitatem raro admodum in ipso Colore deprehendi: modo enim erat Ferrugineus vel Mustelinus; Ingruenti vero *Totali Obscuratione*, Limbus Lunæ circumcirca erat Sublividus, ex parte Sublustris, & Rubicundus; verum in Lunæ Medio, quasi satis Densa, & Obscura Nubecula conspiciebatur, ut vix Maculas rectè in Luna distinguere potuerimus; quæ Nigricantior Umbra paulatim successivè versus Dextram, & Paludem *Mæotidem* promovebatur, sic ut circa Initium *Recuperationis Luminis* tota genuina Umbra admodum Obscura, & Nigricans appareret, atque circa ultimam annotatam Phasin reliqua pars adhuc Obscurata Lunæ, sive ejus Limbus, neutiquam deprehendi vel minimum potuerit.

An Eclipse of the Moon, *Novemb.* 30. (*stn.*) 1685. at *Dantzick*; by M. *Hevelius*. n. 178. *p.* 1256.

| *Ord. Ph.* | *Tempus sec. Horolog. ex Alt. Correct.* | *Notanda.* | *Altitudines* ⊙, ☽, & *Fixarum.* | *Per quas Maculas transiverint Umbræ Sectiones.* |
|---|---|---|---|---|
| | | | ° ′ ″ | |
| | | Altitudo ⊙ Meridiana. | 12 39 0 | |
| | Hor. ′ ″ | | | |
| | 3 19 0 | Altitudo ⊙ | 1 54 0 | |
| | 4 28 0 | Altitudo ☽ | 9 0 0 | |
| | 5 2 30 | Altitudo *Capellæ.* | 31 0 0 | |
| | 5 8 0 | Altitudo *Capellæ.* | 31 2 0 | Dub. |
| | 9 32 25 | *Penumbra.* | | |
| | 9 42 35 | *Penumbra* Densior. | | |
| | 9 48 5 | *Penumbra* Densissima. | | |
| | 9 50 10 | *Init.* quantum collig. dabatur. | | |
| 1 | 9 53 20 | Pal. *Maræotis* jam tota erat tecta. | | Ad M. *Andum*, & Pal. *Maræotidem.* |
| 2 | 9 57 0 | | | Per *Germanicianum*, ad *Mare Syrticum*, & M. *Acabo.* |
| 3 | 10 0 15 | M. *Porphyrit.* nond. planè tect. erat. | | Ad M. *Porphyritem*, per *Mare Syrticum*, Montemque *Ajacem.* |

Per

| | h. | ′ | ″ | | | |
|---|---|---|---|---|---|---|
| 4 | 10 | 3 | 5 | | | Per M. *Petri*, inter Inſ. *Circinnam* & Inſ. *Tarracinniam*, per *Sinum Syrbonis*, & *Paludes Arabiæ*. |
| 5 | 10 | 6 | 25 | | | Ad Inſ. *Ficariam*, *Maltam*, M. *Athos*, per M. *Pharan* & M. *Troicum*. |
| 6 | 10 | 10 | 25 | | | Ad *Sinum Apollinis*, per Inſ. *Erroris*, per M. *Ætnam*, Inſ. *Letoam* & *Didymam*. |
| 7 | 10 | 14 | 55 | Inſula *Melos* jam tota tecta erat. | | Supra *Sin. Apollinis*, ad *Lacum Herculeum*, Inſ. *Siciliæ*, per M. *Parthenium* & *Taigetum*, ad Inſ. *Melos* & Mont. *Lion*. |
| 8 | 10 | 19 | 55 | | | Ad Inſ. *Minorcam*, per Sinum *Paſtanum*, M. *Micalo*, Inſ. *Cyprum*, M. *Hor.* ad M. *Sinai* & Deſertum *Raphidim*. |
| 9 | 10 | 24 | 5 | | | Ad M. *Liguſtinum*, *Panyæum*, inter M. *Sipulum* & *Didymum*, per *M. Libanum* & *Seir*. |
| 10 | 10 | 27 | 10 | | | Supra M. *Nigrum Minorem*, per M. *Apenninum*, Inſ. *Besbicam*, M. *Cimæum*, Montemq; *Calchaſtan*. |
| 11 | 10 | 31 | 27 | | | Ad *Montes Serrorum*, & *Carpathes*, ad *Byzantium*, per M. *Horminium- Taurum*, & M. *Delangueros*. |
| 12 | 10 | 36 | 0 | | | Per *Montes Macrocemnios*, Inſ. *Cyaneam*, M. *Amanum* & *Antitaurum*. |
| 13 | 10 | 41 | 20 | | | Per Inſ. *Macram*, per *Erichtenios Scopulos*, *Sin. Athenienſem*, & Pal. *Arceſam*. |
| 14 | 10 | 45 | 35 | | | Per *Lac. Hyperbor. Superiorem*, Pal. *Byces*, Inſ. *Acam*, *M. Herculis*, Sin. Extremum *Ponti*, Montemque *Parapomiſum*. |
| 15 | 10 | 48 | 40 | | | Ad *Lacum Hyperbor. Inferiorem*, M. *Immerium*, per M. *Coracem*, *Taucan*, Sin. Inferiorem *Maris Caſpii*. |
| 16 | 10 | 52 | 10 | | | Per *Montes Riphæos*, Pal. *Mæotidem*, Inſ. *Majorem Caſpii*, & M. *Neroſum*. |

*Luna*

| | | | | |
|---|---|---|---|---|
| | h. ′ ″ | | | |
| | 10 56 35 | *Luna* nond. plenè *Obſcurata.* | | |
| | 10 58 25 | *Totalis Immerſio.* | | |
| | h. ′ ″ | | ° ′ ″ | |
| | 11 20 45 | Altitud. *Procyonis.* | 31 8 0 | |
| | 11 25 45 | Eadem Altitudo. | 31 38 0 | |
| | 11 36 0 | Obſcura illa Umbra vel Nubec. planè in Med. Diſci Lun. conſiſtebat. | | |
| | 12 43 0 | *Init. Emerſionis* ſeu Recuper. Luminis. | | |
| 17 | 12 46 40 | Palus *Mareotis* jam penitus tecta. | | Ad Mont. *Audum,* haud procul à Palude *Marœotide.* |
| 18 | 12 51 0 | | | Per Mare *Eoum,* Mare *Syrticum,* Montemque *Caſium.* |
| 19 | 12 57 0 | | | Per M. *Atlantem Minorem,* inter Inſ. *Circinnam* & Inſ. *Taraciniam,*per *Mare Ægyptiacum, Sin. Sirbonis,* Montemq; *Pharan.* |
| 20 | 1 5 0 | | | Ad Inſ. *Majorc. Sardin.* & *Hier.* per M. *Ætnam,* Inſ. *Melos,* ad Mont. *Annæ* & *Hajalon.* |

*Initium* Eclipſeos paulo ſupra Paludem *Marœotidem* extitit.
*Totalis Obſcuratio,* ad *Montem Sanctum,* infra Paludem *Maræotidem* accidit.
*Recuperatio Luminis* ferè Ibidem contigit ubi *Initium* cœpit.

2. At *Nuremburg*; by *M. G. C. Eimmart*, n. 182. p. 146.

| h. ′ ″ | |
|---|---|
| 9 19 00 | The *Penumbra* was very Obſcure, and the *Beginning* of the *Eclipſe* was at hand. |
| 9 23 30 | The *Eclipſe* was *Begun*, the Quantity almoſt half a *Digit*, and the Diſtance between the Cuſps was about 42 Degrees of the Moon's Limb, and *Palus Maræotis* was juſt all *Eclipſed*; hence we may conclude the *Beginning* about 9h. 21′ 30″. |
| 10 23 30 | As near as I can collect, was the time of the *Total Immerſion* into the Shadow; to verify which, the Azimuth of the Moon's Center was obſerved to the Eaſt, 41° 18′; 2′ 12″ of time after the ſaid *Immerſion*. |
| 12 13 0 | Or 10′ 13″ before the *Culmination* of the *Right Shoulder* of *Orion*, was the *Emerſion* or firſt Appearance of the Moon out of the Total Darkneſs. |
| 1 14 *ferè* | Was the juſt *End* of the *Eclipſe*, being 2′ 20″ before the *Culmination* of *Syrius*. |

Whence the *Middle* of this *Eclipſe* ſhould have happened at 11h. 18′ *p. m.* at *Nuremburg*: the *Total Duration* 3h. 52′ 30″, and the *Total Darkneſs* 1h. 49′ 30″.

The Meridian Altitude of the Moon's upper Limb was obſerved 63° 23′ 50″, and the Moon's apparent Diameter while totally Eclipſed was found 30′ 07″.

By *M. J. Ph. Wurtzelbaur*, Ib. p. 147.

3. M. *Wurtzelbaur* made uſe of a Pendulum-Clock corrected by Altitudes: According to his Obſervation,

| h. ′ ″ | |
|---|---|
| 9 23 30 | was the *Beginning* of the *Eclipſe*, at about 119 Degrees of the Limb of the Moon in *Hevelius's Selenography*. |
| 9 24 50 | *Palus Maræotis* was all covered. |
| 10 25 20 | The *Total Immerſion*, about the 299th Degree of the Limb of the Moon. |
| 12 11 30 | The Moon began to *Emerge* out of the Shadow, about the 112th Degree of her Limb. |
| 1 14 30 | The *End* of the *Eclipſe* about the 295th Degree of the Limb. |

By theſe Obſervations the *Middle* of the *Eclipſe* ought to have been about 11h. 19′. *p. m.* at *Nuremburg*, differing but one Minute from M. *Eimmart's* Obſervation.

The *Duration* will be 3h. 51′, and the *Total Darkneſs* 1h. 46′.

At *Lisbon*; by Mr. *Jacobs*. n. 184. p. 206.

| 4. | h. | ′ |
|---|---|---|
| *Initium* | 8 | 02 |
| *Immerſio* | 9 | 06 |
| *Emerſio* | 10 | 50 |
| *Finis* | 11 | 57 |

LII. An Eclipse of the Moon, *November*. 19. 1686. at *Dublin*; By Mr. *Will. Molineux*. n. 185. p. 236.

| Phasis. | Temp. h. ′ ″ | |
|---|---|---|
| *Penumbra* Notabilis. | 9 15 00 | |
| *Initium* ob interpositas Nubes præcise determinare haud licuit, ideoque incertius pono *Eclipsin* incepisse ad 9h. 25. vel. | 9 27 00 | * |
| Umbra ad Paludem *Marœotim*. | 9 38 20 | |
| Palus *Marœotis* tecta. | 9 40 20 | * |
| Mons *Sinai* tegitur. | 9 46 30 | * |
| Mons *Thambes* tectus. | 10 01 20 | * |
| Mons *Audus* tegitur. | 10 08 00 | * |
| Mons *Neptunus* tectus. | 10 14 00 | * |
| Umbra ad Montem *Sipylum*. | 10 15 10 | * |
| Insula *Circinna* tegitur. | 10 16 40 | |
| Ad Montem *Didymum*. | 10 18 20 | * |
| Mons *Didymus* tegitur. | 10 20 10 | |
| Emergit M. *Audus*. | 10 23 00 | |
| Umbra ad Paludem *Marœotim*. | 10 48 30 | |
| Emergit *Marœotis*. | 10 52 00 | |
| Ad Montem *Sinai*. | 11 38 00 | |
| Emergit *Sinai*. | 11 39 30 | |
| *Finis* Eclipsis. | 12 04 00 | |

*Notanda*. Tempora sunt Horologii Oscillatorii ad Stellas fixas rectificati. Quæ *Asterismo* notantur Observationes, per hiantes Nubes captæ sunt, adeoque accuratas haberi nolo. Quantitatem hujus Eclipsis sex puto *Digitorum*.

An Eclipse of the Moon, *April* 5. 1688. at *Moscua*; by *M. Timmerman*. n. 192. p. 453.

LIII. Half a Quarter of an Hour after 7 in the Evening, the Moon arose clear, but of a deep Red Colour without any Sign of Eclipse: At $7^{h}\frac{1}{2}$ the Moon went into a thick Cloud, but was again clear at $7^{h}$. 38′. when the Under-side of this Body of the Moon was *Begun* to be *Obscured*, in a clear Sky; she being then in the 25th Degree of *Libra*, and $6^{\circ}\frac{1}{2}$ above the Horizon. (Suppose the Center.) At $9^{h}$. the whole Under-side of the Moon was *Eclipsed*, and about 8′ after $9^{h}$, it was at the height, or rather seemed to decrease. At $9^{h}\frac{1}{2}$ there was still a third Part of the Moon Eclipsed. (Suppose of her Circumference.) About $10^{h}$. it *decreased* apace, and at $10^{h}\frac{1}{2}$ there was but little to be seen: At $10^{h}$. 45′. it was certainly Ended, the Moon being then about 22° High.

LIV. 1. An Eclipse of the Moon, *October* 19. at *Chester*; by Mr. *Ed. Halley*. n. 235. p. 784.

| *Immersions.* | Times | *Emersions.* | Times |
|---|---|---|---|
| | h. ′ | | h. ′ ″ |
| The *Beginning* | 6 $8\frac{1}{2}$ | *Porphyrites* and the Middle of *M. Ætna.* | 8 7 00 |
| *Porphyrites* Immerged | 6 16 | *Horminius* | 8 17 30 |
| North-part of *Marœotis* | 6 $21\frac{1}{2}$ | *Mons Herculis* | 8 18 30 |
| *Lacus Niger Maj.* and South-End of *Mar.* | 6 26 | *Besbicus* | 8 21 00 |
| *Besbicus* | 6 $46\frac{2}{3}$ | *Apollonia* | 8 26 15 |
| *Apollonia* | 6 $49\frac{1}{2}$ | *Byzantium* | 8 29 00 |
| *Byzantium* | 6 53 | *Lacus Niger Major* | 8 32 $\frac{1}{2}$ |
| *Horminius* | 6 59 | South-part of *Mœotis* | 8 35 |
| North-part of *Mœotis* | 7 $2\frac{1}{2}$ | North-part of *Mœotis* | 8 43 |
| *Mons Corax* | 7 $3\frac{1}{2}$ | The *End* | 8 $49\frac{1}{2}$ |
| *Mons Herculis* | 7 10 | | |
| South-part of *Mœotis.* | 7 $12\frac{1}{2}$ | | |

About the *Middle* there remained 9′ 26″. of the Luminous part, and consequently the *Digits* Eclipsed $8\frac{2}{3}$.

2. At *Roterdam*; by M. *Ja. Cassini*. n. 236. p. 20. Vid. sup. §. XXXIX. 2.

| | Temp. Cor. | *Observationes.* |
|---|---|---|
| | h. ′ ″ | |
| A | 6 20 36 | Promontorium Acutum ad Primum Obliquum. |
| B | 6 21 27 | Præcedens *Lunæ* Margo ad Perpendiculare. |
| C | 6 22 3 | Promontorium Acutum ad Perpendiculare. |
| C–B | 0 00 36 | Differentia Transitus per filum Perpendiculare, quæ est Longitudo *Promontorii Acuti* à *Margine Præcedente.* |
| C–A | 0 1 27 | Differentia Transitus *Promontorii Acuti* inter 1. Obliquum & Perpendiculare, quæ ejus est Latitudo à *Margine Boreali.* |
| | 6 32 34 | *Luna* inter Nubes conspecta adhuc apparuit integra. |

*In Prima Phasi.*

| | h. ′ ″ | |
|---|---|---|
| A | 6 41 23 | Initium Maris *Crisii* ad 1. Obl. |
| B | 6 41 50 | Promontorium Acutum ad 1. Obl. |
| C | 6 42 12 | *Plinius* ad 1. Obl. |
| D | 6 42 25 | *Menelaus* ad 1. Obl. |
| E | 6 42 33 | *Manilius* ad 1. Obl. |
| F | 6 42 43 | Primus Margo ad Perpendiculare. |
| G | 6 43 00 | *Proclus* ad Perpendiculare. |
| H | 6 43 20 | Promontorium Acutum ad Perpendiculare. |
| I | 6 43 26 | *Margo Sequens* ad 1. Obliquum. |
| K | 6 43 30 | *Menelaus* ad Perpendiculare. |
| L | 6 44 00 | Cornu præcedens *Lunæ* ad Perpendiculare, ipsa tangit Filum Horiz. |
| M | 6 44 21 | Cornu *Sequens* ad 1. Obl. |
| N | 6 44 35 | *Menelaus* ad 2. Obl. |
| O | 6 44 57 | *Cornu Sequens* ad Verticale. |
| P | 6 45 07 | *Margo Sequens* ad Perpendiculare. |
| Q | 6 45 33 | *Cornu Sequens* ad 2. Obl. |
| R | 6 45 55 | *Grimaldus* ad 2. Obl. |
| S | 6 46 49 | *Sequens Margo* ad 2. Obliquum. |
| P—F | 0 02 24 | Transitus *Lunæ* per Perpendiculare. |
| S—I | 0 03 23 | *Transitus Lunæ* per 2. Obl. |
| H—F | 0 00 37 | *Promontorii Acuti Longitudo à Margine Præcedente.* |
| H—B | 0 01 30 | *Promontorii Acuti Latitudo à Margine Boreali.* |
| K—F | 0 00 47 | *Menelai Longitudo à Margine Præcedente.* |
| K—D } N—K } | 0 01 05 | *Menelai Latitudo à Margine Boreali.* |
| L—F | 0 01 17 | *Cornu Præcedentis Longitudo à Margine Præcedente.* |
| O—F | 0 00 00 | *Latitudo* nulla. |
| O—M | 0 02 14 | *Cornu Sequentis Longitudo à Margine Præced.* |
| Q—O | 0 00 36 | *Latitudo* ejusdem Cornu *à Margine Boreali.* |

*In Secunda Phasi.*

| | h. | ′ | ″ | |
|---|---|---|---|---|
| A | 7 | 6 | 40 | *Margo Præcedens* ad Perpendiculare. |
| B | 7 | 7 | 8 | *Cornu Præcedens* ad Verticale. |
| C | 7 | 7 | 47 | *Cornu Sequens* ad 1. Obl. |
| D | 7 | 8 | 19 | *Margo Sequens* ad 1. Obl. |
| E | 7 | 9 | 4 | *Cornu Sequens* ad Verticale. |
| | 7 | 12 | 22 | Umbra ad *Manilium.* |
| B–A | 0 | 0 | 28 | *Cornu Præced. Longitudo* à Margine præcedente. |
| E–A | 0 | 2 | 24 | *Cornu Sequentis Longitudo* à Margine Orientali. |
| E–C | 0 | 1 | 17 | *Cornu Sequentis Lat.* à Margine Auſtrali. |

*In Tertia Phaſi.*

| | h. | ′ | ″ | |
|---|---|---|---|---|
| A | 7 | 19 | 30 | *Cornu Præced.* ad 1. Obliquum. |
| B | 7 | 21 | 9 | *Margo Præced.* ad Verticale. |
| C | 7 | 21 | 19 | *Cornu Præced.* ad Verticale. |
| D | 7 | 21 | 51 | *Margo Præced.* ad 2. Obliquum. |
| E | 7 | 22 | 24 | *Cornu Sequens* ad 1. Obliquum. |
| F | 7 | 22 | 47 | *Margo Sequens* ad 1. Obl. |
| G | 7 | 23 | 9 | *Cornu Præcedens* ad 2. Obl. |
| H | 7 | 23 | 31 | *Cornu Sequens* ad Verticale. |
| I | 7 | 24 | 40 | *Cornu Sequens* ad 2. Obliquum. |
| | 7 | 26 | 4 | Umbra ad *Dionyſium.* |
| C–B | 0 | 0 | 10 | *Cornu Præcedentis* Longitudo à Margine Orientali. |
| C–A | 0 | 1 | 49 | *Cornu Præced. Latitudo* à Margine Auſtrali. |
| G–C | 0 | 1 | 50 | Eadem *Latitudo.* |
| H–B | 0 | 2 | 22 | *Cornu Sequentis Long.* à Margine præcedente. |
| H–E | 0 | 1 | 7 | *Cornu Sequentis Lat.* à Margine Auſtrali. |
| I–H | 0 | 1 | 9 | Eadem *Latitudo.* |

*In Quarta Phasi.*

| | h. ′ ″ | |
|---|---|---|
| A | 7 40 24 | *Cornu Præcedens* ad 1. Obliquum. |
| B | 7 41 35 | *Cornu Præced.* ad Verticale. |
| C | 7 42 18 | *Margo Præced.* ad 1. Obl. |
| D | 7 42 44 | *Cornu Præced.* ad 2. Obl. |
| E | 7 42 51 | *Cornu Sequens* ad 1. Obl. |
| F | 7 43 14 | *Margo Sequens* ad 1. Obl. |
| G | 7 43 58 | *Margo Sequens* ad Verticale. |
| H | 7 45 04 | *Cornu Sequens* ad 2. Obl. |
| G-2′ 24″=I | 7 41 34 | *Margo Præced.* ad Verticale. |
| B — I | 0 00 01 | *Cornu Præced. Long.* à Margine Præced. |
| B — A | 0 01 11 | } *Cornu Præced. Lat.* à Margine Auſtrali. |
| D — B | 0 01 09 | |
| H — E | 0 2 13 | Differentia Tranſitus *Cornu Sequentis* inter Obl. |
| K | 0 1 6½ | Dimidium, *Lat. Cornu Seq.* à Margine Auſtrali. |
| E + K | 7 43 57½ | *Cornu Sequens* ad Verticale. |
| E + K — I | 0 2 23½ | Longitudo *Cornu Seq.* à Margine Præcedente. |

*In Quinta Phasi.*

| | h. ′ ″ | |
|---|---|---|
| A | 7 50 22 | *Cornu Præced.* ad 1. Obl. |
| B | 7 51 04 | *Promontorium Acutum* in Umbra. |
| C | 7 51 18 | *Cornu Præcedens* ad Verticale. |
| D | 7 51 58 | *Margo Præcedens* ad 2. Obl. |
| E | 7 52 29 | *Cornu Sequens* ad 1. Obl. |
| F | 7 52 56 | *Margo Sequens* ad 1. Obl. |
| D-2′ 24″=I | 7 49 34 | *Margo Præcedens* ad 1. Obl. |
| B — I | 0 01 30 | *Longit. Obl. Promontorii Acuti* à 1. obliq. |
| D-0′ 42″=L | 7 51 16 | *Margo Præcedens* ad Perpendiculare. |
| C — L | 0 00 02 | *Cornu Præced. Long.* à Marg. Præced. |
| C — A | 0 00 56 | *Cornu Præced. Lat.* à Margine Auſtrali. |
| E — I | 0 02 55 | *Long. Obliq.* à 1. obl. ad Max. Long. 3′. 24″. |
| | 0 00 29 | *Complementum.* |
| F — E | 0 00 27 | Idem *Complementum. Medium.* 0′. 28″. |

*In Sexta Phasi.*

| | h. ′ ″ | |
|---|---|---|
| A | 7 54 06 | *Cornu Præcedens* ad 1. Obl. |
| B | 7 54 44 | *Promont. Acutum* ad 1. Obl. |
| C | 7 54 59 | *Cornu Præced.* ad Perpend. |
| D | 7 55 38 | *Margo Præced.* ad 2. Obl. |
| F | 7 55 52 | *Cornu Præced.* ad 2. Obl. |
| G | 7 56 04 | *Cornu Seq.* ad 1 Obl. |
| H | 7 56 36 | *Margo Seq.* ad 1. Obl. |
| I | 7 57 20 | *Cornu Sequens* ad Verticale |
| D—2′ 24″=L | 7 53 14 | *Margo Præced.* ad 1. Obl. |
| D—0 42=K | 7 54 56 | *Margo Præced.* ad Perpend. |
| C — K | 0 00 03 | *Cornu Præced. Long.* à Marg. Præced. |
| C — A }<br>F — C } | 0 00 53 | *Cornu Præced. Lat.* à Marg. Auſtrali. |
| I — K | 0 02 24 | *Longitudo Cornu Sequentis* à Marg. Præced. |
| I — G | 0 01 16 | *Latitudo Cornu Seq.* à Marg. Auſtrali. |
| B — L | 0 01 30 | *Longitudo obliqua Prom. Acuti* à 1. Obl. |

*In Septima Phasi.*

| | h. ′ ″ | |
|---|---|---|
| A | 8 6 58 | *Cornu Præced.* ad 1. Obl. |
| B | 8 7 39 | *Cornu Præced.* ad Perpend. |
| C | 8 8 15 | *Margo Præced.* ad 2. Obl. |
| D | 8 8 21 | *Cornu Præced.* ad 2. Obl. |
| E | 8 8 55 | Medium *Umbræ* ad Perpend. ferè. |
| F | 8 9 14 | *Margo Sequens* ad 1. Obl. |
| G | 8 9 55 | *Cornu Sequens* ad Verticale. |
| H | 8 9 58 | *Margo Sequens* ad Verticale. |
| C—2 24″ =I | 8 5 51 | *Margo Præced.* ad 1. Obl. |
| C—0 42 =K | 8 7 33 | *Margo Præced.* ad Perpend. |
| B — K | 0 0 06 | *Cornu Præced. Long.* à Margine Præced. |
| B — A | 0 0 41 | } *Cornu Præced. Lat.* à Margine Auſtrali. |
| D — B | 0 0 42 | } |
| G — K | 0 2 22 | *Cornu Sequentis Long.* à Marg. Præced. |

*In Octavo Phasi.*

| | h. | ′ | ″ | |
|---|---|---|---|---|
| A | 8 | 23 | 53 | *Cornu Præced.* ad 1. Obl. |
| B | 8 | 24 | 24 | *Cornu Præced* ad Perpend. |
| C | 8 | 24 | 55 | *Margo Præced.* ad 2. Obl. |
| D | 8 | 24 | 57 | *Cornu Præced.* ad 2. Obl. |
| E | 8 | 25 | 55 | *Margo Sequens* ad 1. Obl. |
| F | 8 | 26 | 26 | *Cornu Sequens* ad Perpend. |
| G | 8 | 26 | 38 | *Margo Sequens* ad Vert. |
| H | 8 | 28 | 20 | *Cornu Sequens* ad 2. Obl. |
| C—0′ 42″=K | 8 | 24 | 13 | *Margo Præced.* ad Perpend. |
| B — K | 0 | 00 | 11 | *Longitudo Cornu Præced.* à Marg. Auſtrali. |
| B — A | 0 | 00 | 31 | } *Latitudo Cornu Præced.* à Marg. Auſtrali. |
| D — B | 0 | 00 | 33 | |
| F — K | 0 | 02 | 13 | *Longitudo Cornu Seq.* à Marg. Præcedenti. |
| H — F | 0 | 01 | 54 | *Latitudo Cornu Seq.* à Marg. Auſtrali. |

*In Nona Phasi.*

| | | | | |
|---|---|---|---|---|
| A | 8 | 49 | 25 | *Cornu Præced.* ad 1. Obl. |
| B | 8 | 49 | 42 | *Eclipſis Concavitas* ad 1. Obl. |
| C | 8 | 50 | 08 | *Cornu Præced.* ad Verticale. |
| D | 8 | 50 | 41 | *Margo Præced.* ad 2. Obl. |
| E | 8 | 50 | 51 | *Cornu Præced.* ad 2. Obl. |
| F | 8 | 51 | 34 | *Cornu Seq.* ad Verticale. |
| G | 8 | 51 | 43 | *Margo Seq.* ad 1. Obl. |
| H | 8 | 52 | 26 | *Margo Seq.* ad Vert. |
| H—2′ 24″ =I | 8 | 50 | 02 | *Margo Præced.* ad Perpend. |
| C — I | 0 | 00 | 06 | *Longitudo Cornu Præced.* à Marg. Præced. |
| C — A | 0 | 00 | 43 | } *Latitudo Cornu Præced.* à Marg. Auſtrali. |
| E — C | 0 | 00 | 43 | |
| F — I | 0 | 01 | 32 | *Long. Cornu Seq.* à Marg. Præced. |

*In Decima Phasi.*

| | | | | |
|---|---|---|---|---|
| A | 9 | 4 | 29 | *Cornu Præcedens* ad 1. Obl. |
| B | 9 | 5 | 4 | Umbra recedit à *Plinio.* |
| C | 9 | 5 | 53 | *Cornu Seq.* ad 1. Obl. |
| D | 9 | 6 | 17 | *Cornu Præced.* ad 2. Obl. |
| E | 9 | 7 | 2 | *Margo Seq.* ad 1. Obl. |
| F | 9 | 7 | 47 | *Margo Seq.* ad Vert. |
| E—2′ 42″=H | 9 | 5 | 20 | *Margo Præced.* ad Verticale. |
| D — A | 0 | 1 | 48 | Tranſitus *Cornu Præced.* inter Obliquos. |
| M | 0 | 0 | 54 | Dimidium *Lat. Cornu Præced.* à Marg. Auſtrali. |
| D — M = I | 9 | 5 | 23 | *Cornu Præced.* ad Verticale. |
| I — H | 0 | 0 | 3 | Longitudo *Cornu Præced.* à Marg. Præced. |
| E—3′ 24″ =L | 9 | 3 | 38 | } *Margo Præced.* ad 1. Obl. |
| F—4 06 = L | 9 | 3 | 41 | |
| C — L | 0 | 2 | 25 | *Long. Obliqua Cornu Seq.* à 1. Obliquo. |
| | 9 | 9 | 4 | Umbra recedit à *Langreno.* |
| | 0 | 9 | 19 | Finis *Maris Tranquilitatis.* |
| | 0 | 13 | 40 | *Ariſtoteles.* |
| | 0 | 14 | 39 | Finis *Maris Criſii.* |
| | 0 | 21 | 34 | *Finis.* |

*Juxt.*

LV: A Transit of the Moon above *Venus*. *Oct.* 11. (*st. n.*) 1670. at *Dantzick*; by *M. Hevelius*. n. 66. p. 2026.

| Tempus juxt. Hor. Amb. manè. | | Distantiæ & Altit. | Tempus ex Alt. Corr. |
|---|---|---|---|
| h. ′ ″ | | ° ′ ″ | h. ′ ″ |
| 3 47 20 | Altitudo *Lucidæ* ♈ | 41 45 0 | 3 54 10 |
| 3 50 35 | Eadem Altitudo. | 41 21 0 | 3 57 28 |
| 5 47 44 | Distantia ♀ à ♃ | | 5 54 44 |
| 5 48 35 | Dist. ☽ Limb. Orient. à ♃ | 35 25 30 | 5 55 35 |
| 5 51 59 | Dist. ♀ à ♃ | 35 50 0 | 5 59 0 |
| 5 53 40 | Dist. ☽ Limb. Orient. à ♃ | 35 29 10 | 6 0 40 |
| 6 12 30 | Dist. ♀ à ♃ | 35 50 20 | 6 6 0 |
| 6 14 3 | Dist. ☽ Limb. Orient. à ♃ | 35 37 35 | 6 8 0 |
| 6 37 0 | Dist. ♀ à ☽, quantum nudo oculo dijudicari potuit. | 0 6 0 | 6 51 0 |
| 7 12 0 | Dist. ♀ à ☽ Limbo Infer. | 0 5 0 | 7 7 0 |
| 7 17 0 | Dist. ♀ à ☽ Limb. Infer. | 0 4 0 | 7 12 6 |
| 7 35 0 | *Venus* clare apparuit. | | 7 30 0 |
| 9 11 0 | *Venus* permansit conspicua. | | 9 12 0 |
| 9 15 48 | Altitudo *Solis*. | 19 8 0 | 9 17 20 |
| 9 17 39 | Altitudo *Solis*. | 19 58 0 | 9 19 4 |

LVI. Initium Occultationis accidit 3h. 38′. 27″. mane circa Montem *Germanicianum*. Linea Itineraria, quantum ex solo Ingressu haud obscurè colligere licuit, transiit per M. *Ætnam*, *Centrum* ferè *Lunæ*, per M. *Horminium*, M. *Herculis*, & superiorem partem *Maris Caspii*. Ego quantum memini, bis tantum, si hujus Anni Observationem excipias, intra 41 Annos, *Saturnum* à *Luna* tectum vidi; *Anno* nimirum 1630. Die 29 *Junii*, vesp. h. 11. cùm in *Freto Danico* circa Insulam *Huennam* versarer: Rursus Anno 1661. Die 3. *Augusti* hic *Dantisci* h. 7. 58′. 20″. Vesp.

An Occultation of *Saturn* by the Moon. *June* 1. (*st. n.*) 1671. at *Dantzick*; by *M. Hevelius*. n. 78. p. 3027.

LVII. Sudo admodum Cælo, *Lunam* protinus Orientem, nec non paulò post *Jovem*, Tubo 20. pedum summâ Aviditate excepimus, atque deprehendimus Octante nostro permagno Orichalcico, 9. ferè pedum Radio, *Jovem* à *Lunæ* Limbo Orientali adhuc 1° 23′ 40″. esse remotum; Jovialesque omnes quatuor à Dextra, à qua *Luna* accedebat, adesse. Ipsum quidem Conjunctionis momentum insperatus quidam Casus infelicior deprehendere prohibuit. Cum enim *Jupiter* ad Limbum *Lunæ* Orientalem ad 3′ jam accideret, atque ad 6 duntaxat à Linea Conjunctionis, per utrumque Cornu ducta, distaret, ecce

A Transit of the Moon above ♃ *Sep.* 30. (*st. n.*) 1671. at *Dantzick*; by *M. Hevelius*. Ib. p. 3031.

ſupervenientes Nebeculas, quæ tam *Jovem* quam ipſam *Lunam*, nobis è conſpectu eripuerunt. Tabulæ *Rudolphinæ* Occultationem, eamque multò citiùs promiſere, nulla tamen omninò fuerit, ſed, arctiſſimus ſolummodò ad duos propemodum Digitos, Tranſitus extiterit, *Horâ* ſcil. 7 26′ 0″.

An Occultat. of the *Pleiades* by the *Moon*, Feb. 23. 167½. at *Derby*; by Mr. *Flamſteed*. n. 86. p. 5034.

Fig. 116.

LVIII. Alta *Lunâ* 20° 50′. cepi ipſius Diametrum 32′ 48″; & Altâ ipſâ 19° 23′ rurſus eam cepi, 32′ 47″. Ergo *Lunæ* in Horizonte Semidiameter erat vera 16′ 19″. Plus tamen etiamnum ab Occidentali Stella *Pleiadum* abſuit quàm commode caperet Teleſcopium. At 11h. 19′½, *p. m.* Altâ ✱a *b*, Occidentali *Pleiadum* 9° 50′; ejuſdem Stella diſtantiam cepi à Cornu *Lunæ* Proximo 11′ 58″. divertens deinde ſubitò ad ✱æ altitudinem (oſtenſam Quadrante, 20 digitorum Radio, ad Tubi latus affixo) notandam, & continuo reverſus, Stellam (quippe tunc à *Luna* Tectum) non comperi. Interea *Luna* deſcenderat Minuta 10, ſimulque tantundem *Stella*, quam ſubiiſſe *Lunam* h. 11. 20′½ ex ſequente Phaſi conjicio: Etenim 11h. 30′¼, Alta ✱a *c*. 8° 43′. Stellam *c*. à *Luna* tectam conſpexi. Ejus cum cepiſſem à Cornu Proximo Diſtantiam 16′ 35″. ſpatium Temporis inter hujus & præcedentis Occultationem, editis ſupputationibus, conſtitui 9′ 37″; quæ Tempori hujus Phaſeos ſublatæ, dant utique præcedentis Occultationis Tempus ut conſtitui.

H. 11. 37′½, alta ✱a *c*. 11° 37′½, ipſa *Lunam* ſubiit, me interea diſtantiam ejus dimetiente 22′ 36″. à Cornu *Lunæ* apparenter Inferiori, ſed Superiori verè. Erat, Stellâ evaneſcente, *Lunæ* Semidiameter apparens 16′ 21″. quæ propterea occultata erat 87° 25′. *Peripheriæ Lunaris* à *Cuſpide Superiori*, cujus erat Reclinatio (à Linea per Centrum ejus, *Eclipticæ* ducta perpendiculari) 1° 37′. Sic ſubingreſſus Stellæ fuit 4° 12′. ſupra Lineam per Centrum *Lunæ*, *Eclipticæ* ductam Parallelam, & *Lunæ* Centrum in Antecedentia ✱æ 16′ 18″. cum minori *Latitudine* 1′ 12″.

Fixæ *Locus* Authori *Carolino* ♉ 25° 1′ 24″; *Latitudo* perpetua 4° 20′ 39″, quamobrem *Lunæ* Locus Apparens hora Apparenti *Derbiæ* 11h. 37′½. *p. m.* erat ♉ 24° 45′ 6″, & *Latitudo* viſa 4° 19′ 27″. Bor.

Notatu præterea digniſſimum quod etiamſi omnes ferè omnium Aſtronomorum Hypotheſes, *Lunæ Plenæ Perigeæ* in Quadraturis Majorem tribuant Diametrum, & proinde Minorem à *Terra* Diſtantiam quam in *Syzygiis* aut *Oppoſitionibus Perigeis*: contrarium tamen Cælitus fieri & evenire, *Luna* etenim *Plena Perigea* tranſiens juxta *Pleiadas*, *Nov.* 6. 1671. Majorem habebat Diametrum quam in hoc Tranſitu, quando in eodem ferè loco à *Sole* diſtitit gradus 70, *Lunæ* Semidiameter Horizontalis,

| | *Bullialdo* | *Streetio* | *Obſervata* |
|---|---|---|---|
| *Nov.* 6. 1671. | 71′ 00″ | 16′ 30″ | 17′ 00″. |
| *Feb.* 23. 1672. | 17 50 | 17 13 | 16 19. |
| | + 50 | + 43 | —41. |

Amplius

Amplius non nunc miramur *Lunam* tam diu numerorum recuſaſſe vincula & de Tabulis ſupputata apparentiarum tempora uſque adeò expectationes noſtras fefelliſſe, à falſis quandoquidem Hypotheſibus ipſas plerumque conſtructas fuiſſe liquet.

LIX. *April* 2d. (*ſt. n.*) 6h. 50'. *v.* A Line drawn through the Horns of the *Moon* paſſed through the Star that is at the Point of the *Northern Horn* of *Taurus*, and the Diſtance of this Star to the *Northern Horn* of the *Moon* was by a Minute greater than the Semidiameter of the *Moon*.

The *Moon's* Place, *Mar.* 23. 167½; by *M. Caſſini.* n. 82. p. 4047.

LX. Immerſio Stellæ *Sequentis Duarum in Siniſtro Pede poſteriori Leonis* fuit 10h. 19' 34''. Immerſionis plaga fuit juxta Finem *Schicardi* verſus *Phocilidem* in Selenographia *Riccioli*.

An Occultation of a Fixt Star by the *Moon*, Feb. 29. (*ſt. n.*) 1676. at *Paris*; by *M. Caſſini.* n. 123. p. 564.

Emerſio vero fuit 11h. 16' 40''. in æquali à recta diſtantia à *Vendelino* & *Petavio*.

Per puncta *Immerſionis* & *Emerſionis*, diligenter notata, ducta recta Linea Diametrum illi perpendicularem abſcidit in ratione 6' 45'', ad 26' 5''.

Fuit autem Diameter *Lunæ* ad Meridianum accedentis 32' 50''.

H. 12. 29'. Margo *Lunæ* Superior fuit in eodem Parallelo cum Stella, quæ tunc præcedebat *Lunam* minuto horario 1' 50''.

H. 12. 40' 18''. Stella præcedebat Marginem Occidentalem *Lunæ* minutis horar. 2' 11''. *Lunæ* Diameter per tranſibat 2' 14''.

H. 12. 52' 35''. Stella præcedebat eundem Marginem 2' 25''.

Altitudo Meridiana Limbi Inferioris *Lunæ* capta eſt 39°. 25' 25''.

Hora

LXI.
A Tranſit of the *Moon* above *Jupiter*, *Feb.* 28. *m.* 167$\frac{5}{6}$, at *Greenwich*; by Mr. *Flamſteed*. Ib. p. 566.

| *Hora Horologii correcta.* | | | *Alt. & Diſtantia.* | | |
|---|---|---|---|---|---|
| h. | ′ | ″ | | ′ | ″ |
| 4 | 20 | 15 | ♃ à Limbo *Lunæ* Lucido—— | 26 | 9 |
| 4 | 47 | 0 | ☽ᵉ *Capta Diameter*———— | 31 | 30 |
| 4 | 49 | 30 | ♃ à Cuſpide Proximo———— | 26 | 28 |
| 4 | 52 | 15 | ♃ Rectam per Cuſpides ductam prætererierat decimâ parte Diſtantiæ vel 3′ circiter, oculari per Tubum conjecturâ.———— | | |
| 4 | 56 | 0 | ♃ à Cuſpide———— | 27 | 33 |
| 5 | 1 | 15 | —à Recta per Cuſpides—— | 7 | 53 |
| 5 | 3 | 30 | — àCuſpide ———— | 28 | 22 |
| 5 | 7 | 25 | —à Recta———— | 9 | 58 |
| 5 | 10 | 50 | —ab eadem———— | 11 | 55 |
| 5 | 15 | 50 | —à Cuſpide———— | 30 | 27 |
| 5 | 21 | 20 | —à Limbo Remotiori. dub.— | 62 | 4 |
| 5 | 26 | 0 | —à Cuſpide Proximo—— | 33 | 0 |
| 5 | 31 | 25 | —à Recta per Cuſpides —— | 20 | 9 |
| 5 | 37 | 0 | —à Cuſpide———— | 36 | 15 |
| 5 | 41 | 10 | —☽ᵉ Altæ 10½ *gr.* Diameter circ.———— | 31 | 53 |
| 5 | 48 | 30 | Differentia Altit. Limbi ☽ᵉ Inferioris & ♃ —— | 23 | 1 |
| 5 | 52 | 40 | ♃ à Cuſpide Proximo aberat- | 41 | 40 |
| 6 | 9 | 40 | —à Cuſpide. dub. | 47 | 29 |

An Occultation of *Mars* by the *Moon*, *Aug.* 21. 1676. at *Greenwich*; by Mr. *Flamſteed*. n. 129. p. 723.

LXII. 1. *Aug.* 21. A. 1676. ante meridiem. pro correctione Horologii has Limbi Solaris Altitudines acceperam.

| *Hora Horologii.* | | | *Altitudines.* | | | *Hor. Supp.* | | | *Hor. Err.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| h. | ′ | ″ | | ° | ′ | h. | ′ | ″ | | ′ | ″ |
| 8 | 04 | 31 | Alt. Limbi *Solis* Infer. | 26 | 04 | 8 | 09 | 26 | + | 4 | 55 |
| 8 | 5 | 42 | ———— | 26 | 14 | 8 | 10 | 35 | + | 4 | 53 |
| 8 | 7 | 58 | ———— | 26 | 34 | 8 | 12 | 53 | + | 4 | 55 |
| 8 | 9 | 10 | ———— | 26 | 44½ | 8 | 14 | 03 | + | 4 | 53 |
| 8 | 10 | 15 | ———— | 26 | 54 | 8 | 15 | 12 | + | 4 | 57 |
| 8 | 17 | 15 | ———— | 27 | 54 | 8 | 22 | 09 | + | 4 | 54 |

Deinde poſt Meridiem, cælo ſereniſſimo.

| *Hor. Horol.* | | | *Correcta.* | | | *Distantiæ.* | | | |
|---|---|---|---|---|---|---|---|---|---|
| h. | ' | '' | h. | ' | '' | | | ' | '' |
| 10 | 45 | 03 | 10 | 49 | 58 | *Mars* à Limbo Lucido *Lunæ* — | 5125= | 42 | 08 |
| 11 | 06 | 11 | 11 | 11 | 05 | Eadem Distantia——— | 3829= | 31 | 29 |
| 11 | 20 | 00 | 11 | 24 | 55 | Iterum——— | 3007= | 24 | 44 |
| 11 | 35 | 57 | 11 | 40 | 52 | Denuo——— | 1982= | 16 | 18 |
| 11 | 57 | 31 | 12 | 02 | 26 | ♂ Z. sive Diff.Alt. Limb. Inf. ♂ | 1912= | 7 | 35 |
| | | | | | | Jamq; tubo ped. 16 ♂ à Limbo | 1158= | 5 | 47 |
| 12 | 05 | 00 | 12 | 09 | 55 | Planeta nudis oculis diutius conspici non potuit.——— | | | |
| 12 | 9 | 44 | 12 | 14 | 39 | ♂ Lux cum lumine Lunæ confusa. ♂ Z——— | 1185= | 9 | 44 |
| 12 | 10 | 03 | 12 | 14 | 58 | ♂ penitus Tect. à Cusp. Boreo. | 3475= | 17 | 20 |
| 12 | 18 | 38 | 12 | 23 | 33 | 41ª ♉ in Recta per Cuspides ducta apparuit——— | | | |
| 12 | 20 | 36 | 12 | 25 | 31 | 41ª ♉ à Limbo vel Cusp. Tubo breviori——— | 3912= | 32 | 10 |
| 12 | 24 | 58 | 12 | 29 | 53 | 41ª ♉ à Cuspide iterum eodem Tubo——— | 3935= | 32 | 21 |
| 12 | 46 | 00 | 12 | 50 | 55 | *Lunæ* Diameter longiori tubo— | 5971= | 29 | 47 |
| 1 | 04 | 30 | 1 | 09 | 25 | Iterum eodem tubo——— | 5973= | 29 | 48 |
| 1 | 10 | 56 | 1 | 10 | 51 | ♂ Emers. forsan 4'' vel 5'' citiùs | | | |
| 1 | 13 | 29 | 1 | 18 | 24 | ♂ à Cuspide Boreo——— | 3675= | 18 | 20 |
| 1 | 18 | 15 | 1 | 23 | 10 | Eadem Distantia——— | 4035= | 20 | 08 |
| 1 | 22 | 00 | 1 | 26 | 55 | *Lunæ* Altæ 23° Tubo longiori Diameter——— | 5988= | 29 | 55 |
| 1 | 39 | 00 | 1 | 43 | 55 | *Lunæ* Diameter breviori Tubo. | 3645= | 29 | 58 |

41ª ♉ secundum *Tychonem* Locus nunc est ♉ 17° 58'½. *Latitudo* 1° 20' Australis; unde cum *Lunæ* tum *Martis* Locus accurate deduci potest.

Temp.

2. At Oxford; by Mr. Halley. Ib. p. 724.

| Temp. Corr. | Distantiæ. | | |
|---|---|---|---|
| h. ′ ″ | | | ′ ″ |
| 11 43 30 | The Center of *Mars* from the nearest Limb of the *Moon*. — | 719½= | 12 40 |
| 11 49 2 | Again — | 571= | 10 3 |
| 11 54 58 | Again — | 409= | 7 12 |
| 12 3 25 | The Center of *Mars* from the North Cusp. of ☽ — | 1118= | 19 41 |
| 12 10 28 | The Gibbous part of *Mars* touched the *Moon's* Limb. | | |
| 12 10 42 | *Mars* was wholly covered, being distant from the Cusp — | 963= | 17 14 |
| 1 10 41 | *Mars* did *Emerge*, I suppose, his Center. | | |
| 1 12 45 | *Mars* was distant from the Northern Horn of ☽ | 1018= | 17 55 |
| 1 31 10 | *Mars* passed over a Point noted in the Telescope. | | |
| 1 33 15 | The Southern Limb of *Ætna* passed by the same Point. | | |
| 1 34 00 | The Lucid Limb passed over the same Point. | | |
| 1 52 35 | The *Moon*'s Diam. observed 1698=30′ 1″. *Alt.* ☽ 31° circ. | | |
| 1 57 52 | *Mars* from the Northern Horn of the *Moon* — | 2042= | 36 5 |
| 2 2 53 | *Mars* from the Southern Horn of the *Moon* — | 2266= | 40 3 |

3. At Dantzick; by M. Hevelius. Ib. p. 721.

| Temp. Horol. Sec. Oscil. | Fixarum Nomina. | Altitudines. | Temp. ex alt. corr. | Animadvertenda. |
|---|---|---|---|---|
| h. ′ ″ | | ° ′ | h. ′ ″ | |
| 1 1 25 | *Caudæ Cygni* | 57 10 | 1 0 24 | |
| 1 9 45 | | | 1 8 45 | ♂ Distabat ferè tanto interstitio à Limbo ☽ Lucido, quanto M. *Porphyrites* à M. *Ætna* removetur. |
| 1 36 39 | | | 1 35 42 | *Mars* à *Luna* omninò Tectus. |
| 1 45 25 | *Caudæ Cygni* | 51 17 | 1 44 7 | |
| 2 47 54 | | | 2 46 29 | *Mars* emicuit; *Finis* nempe Occultationis. |
| 3 19 50 | *Scheat Peg.* | 45 3 | 3 18 19 | |

*Mars*

*Mars* obtectus est circa Montem *Audum*, incedens quasi per *Loca Lunæ Paludosa*, per M. *Ætnam*, infra Insulam *Besbicam*, supra Paludem *Acherusiam*, supra M. *Coracem*, per Paludem *Mæotidem*, & paulò supra Insulam *Alopeciam*, & ipsum *Lunæ Centrum*; sicque rursus ad Lacum *Majorem Occidentalem* exiens.

Si quæras, unde viam itinerariam hanc adeò accuratè mihi determinare licuerit, & quidem ad partem *Lunæ* Obscuram, scias, eò evenisse, quod Tubis illis meis præcipuas Maculas majores in parte *Lunæ* Umbrosâ satis distinctè deprehendere potuerim; atque ita dilucidè conspexerim, *Martem* circa medium ferè *Paludis Mæotidis* Emicuisse.

LXIII. Observavit *Bullialdus Initium*, Alto sup. Horiz. ad occasum *Capite Andromedæ* 18° 11′. unde datur à Meridie 7h. 20′ T. A. sed Med. 7h. 29′ 55″. *Finem* vero vidit, Alta ad Occas. *Cinguli Androm. Australiori* Magn. 2. 21° 17′. unde à Meridie colligitur T. A. 8h. 30′ 22″. *An Occultation of* Saturn *by the* Moon, *Feb.* 27. st. n. 1678, *at* Paris; *by M.* Bullialdus. *n.* 139. *p.* 969.

Monere hic necessum est Tabulas *Philolaicas* ♄ Promotiorem in Longitudine ostendere, quam in Cœlo apparet, scrupulis primis ut minimum 19. ita ut ♄ tunc fuerit in Cœlo in ♊ 3° 28′. & Lat. Aust. 1° 38′.

In hac porrò Observatione adhibita Illust. Viri *Joh. Hevelii* Lunaris Disci descriptione, in illa Limbi parte, quæ in recta linea à medio Montis *Berosi* per Montes *Riphæos* ducta, paulò supra *Alanum* Montem, infra Terminos Australes *Paludum Hyperborearum*, sita est, Saturnum Emersisse aspeximus.

LXIV. 1. Etiamsi nunc per quinquaginta Annos ( pro quo D. O. M. immortales & debeo & habeo gratias ) Observationibus Rerum Cœlestium operam dederim, non nisi tamen semel tantummodò *Jovem* à *Luna* vidi obtectum, Anno nempe 1646. die 24 *Decembris*, *St. n.* vesperi, *Sole* scilicet existente sub Horizonte. Gratulor igitur mihi magnoperè, quod hanc observationem, non solùm Cœlo perquam sereno, sed etiam ex voto & quidem cum gratissimo meo Hospite, Claris. & Doctissimo Domino *Edmundo Hallæo* observare potuerim. *An Occultation of* Jupiter, Jun. 5. *(st. n.)* 1679. *at* Dantzick; *by M.* Hevelius. *Ph. Coll.* n. 1. *p.* 29.

*Lunam* intraverit ad M. *Audum*, & quantum conjicere dabatur ex *Jovis* Exitu, viam carpsit per *Loca Paludosa* Insulæ *Cercinnæ*, supra Montem *Ætnam*, per Insf. *Besbicam*, per *Byzantium*, Insf. *Apolloniam* & superiorem partem *Paludis Mæotidis*; sic ut paulò supra *Centrum Lunæ* incesserit, *Lunâ* habenti aliqualem Latitudinem Austrinam. Deinde, quod rarissimum, *Jovis* apparentem Diametrum in hac observatione accuratissimè ( ut mihi videor ) dimensi sumus. Memini quidem me aliquoties *Jovis* Diametrum per Maculas Lunares observasse, eandem videlicet ad 55″. plus minus accedere; sed hac vice diameter *Jovis* longè extitit minor. Cognitâ enim totâ Duratione hujus Occultationis 58′ 10″, atque datâ simul Diametro *Lunari* 32′ 40″, protinus innotescit, ex illa temporis Mora, cum scilicet primum *Jupiter* Limbo suo *Lunæ* Limbum attingeret, & cum rursus Occultaretur ( id quod factum est spatio 55′ ) Diameter *Jovis*, 30″ 53‴.

| Ord. Obser. | Temp. juxta Horol. Oscil. | Altitudines Fixarum & Solis, cum Distantiis Jovis à Lunæ Limbo. | | Temp. ex Alt. corr. |
|---|---|---|---|---|
| | h. ' '' | | ° ' '' | h. ' '' |
| | 1 18 55 | Altit. Capitis *Andromedæ* — | 24 52 0 | 1 20 54 |
| | 1 29 0 | Altit. *Arcturi* — | 31 3 00 | 1 31 24 |
| 1 | 1 52 0 | *Luna* Oriebatur — | | 1 54 25 |
| 2 | 3 26 0 | *Sol* Oritur — | | 3 28 25 |
| 3 | 3 33 0 | Dist. *Jovis* à Limbo *Lunæ* æquabat fere Diametrum *Lunarem* — | | 3 35 0 |
| 4 | 3 41 0 | *Jupiter* à *Lunæ* Limbo in tanta dist. aberat quanta est dist. M. *Porph.* à Mont. *Sinai* — | | 3 43 0 |
| 5 | 3 58 0 | *Jupiter* à Limbo *Lunæ* Orient. ut M. *Ætna* à Palude *Maræotide.* — | | 4 00 0 |
| 6 | 4 13 50 | *Jupiter* distabat à Limbo ☽ duab. Diam. ♃ — | | 4 16 15 |
| 7 | 4 14 40 | *Jupiter* ad unicam Diam. *Jovis* removebatur — | | 4 17 5 |
| 8 | 4 15 40 | *Jupiter* stringebat *Lunæ* Limbum Orient. — | | 4 18 5 |
| 9 | 4 16 9 | *Jupiter* dimidiâ parte Occultabatur — | | 4 18 34 |
| 10 | 4 16 35 | *Jupiter* Totus omninò Tectus — | | 4 19 0 |
| 11 | 5 14 0 | *Jup.* nunc notabili particula prodire videbat — | | 5 16 25 |
| 12 | 5 14 20 | Dimidius *Jupiter* jam exiverat — | | 5 16 45 |
| 13 | 5 14 45 | Totus *Jupiter* omninò prodierat — | | 5 17 10 |
| | 10 22 30 | Altitudo ☉ — | 53 34 44 | 10 25 2 |
| | 10 27 16 | Altitudo ☉ — | 53 59 45 | 10 29 42 |
| | 10 30 8 | Altitudo ☉ — | 54 14 0 | 10 32 26 |
| | 10 38 0 | Altitudo ☉ — | 54 53 40 | 10 40 26 |
| | 10 45 28 | Altitudo ☉ — | 55 27 20 | 10 47 46 |

At Paris; by M. Cassini. ib. p. 33.

2. At $3^h. 0' 11''$. the first *Satellite* was hid by the East Limb of the *Moon*. At $3^h\ 2'\ 0\frac{1}{2}''$. the East-side of the *Moon* touched the West-side of *Jupiter*; then I took the Height of *Jupiter*, which was 8° 01'. at $3^h\ 2'\ 51''$. At $3^h\ 2'\ 57''$. *Jupiter* was intirely hid by the *Moon*. It entered at equal Distance from the two Spots *Grimaldi* and *Aristarchus*; the last of which was in the Section of the *Moon*, which distinguish'd the Light from the Dark-part. At $3^h\ 5'\ 1''$. the Second *Satellite* was hid by the East-side of the *Moon*. At $3^h\ 5'\ 48''$. the third *Satellite* was hid. At $3^h\ 56'$. we perceived by the Eye that *Jupiter* was parted from the obscure Side of the *Moon*.

M. De la Hire took the Height of *Jupiter* two Minutes after parting; and found it 17° 17'.

Temp.

LXV. *An Occultation of the* Bull's Eye, *at* Greenwich, *Sept.* 4. 1680. *by Mr.* Flamsteed. *Ph. Coll.* n. 4. *p.* 99.

| *Temp. Corr. mane.* | *Obſervat.* | | | |
|---|---|---|---|---|
| h. ′ ″ | | | ′ ″ | |
| 2 49 52 | *Lunæ* Cuſpis Auſtrina à *Palilicio* — | 5300 | =26 26 | |
| 2 53 10 | *Palilicium* à Limbo Proximo —— | 1028 | =05 08 | |
| 2 56 54 | à Cuſpide dicta—— | 4830 | =24 06 | |
| 3 01 16 | ——rep.—— | 4370 | =21 48 | |
| 3 05 42 | *Immerſio*, à Cuſpide—— | 4049 | =20 12 | vel |
| 4 08 20 | *Lunæ* Diameter —— | 6595 | =32 54 | paulo |
| | ab Amanuenſi repetita— | 6588 | =32 52 | minus |
| 4 14 02 | *Palilicium* Emerſerat. | | | |
| 4 15 12 | diſtetit à Cuſpide Auſt.—— | 3411 | =17 01 | |
| 4 18 52 | rep.—— | 3725 | =18 35 | |
| 4 22 00 | iter.—— | 3980 | =19 51 | |
| 4 23 52 | denuo—— | 4098 | =20 56 | |

$3^h$ 05′ 40″. Limbo Lunæ Lucido Fixa videbatur adhærere, & poſt duo ſcrupula ſecunda Temporis, nihil in Limbo apparuit. Locus *Immerſionis* erat juxta Auſtraliſſimam revera trium Macularum parvularum medio jacentium inter Paludem *Maræotida* & Montem *Climacem.*

$4^h$ 13′ 45″ Fixa non *Emerſerat*; tunc, vel paulo poſtea, neſcio qua occaſione amovi oculum à Teleſcopio, at cum iterum adhibui $4^h$ 14′ 2″. *Emerſam* vidi & plena Luce effulgentem.

| h. ′ ″ | | ° ′ ″ |
|---|---|---|
| 4 53 20 | *Lunæ* Limb. Prox. à Pede Lucido *Orionis* — | 26 12 25 |
| 4 56 20 | Pes *Orionis* Lucidus à *Palilicio*—— | 26 29 25 |
| 4 58 20 | *Lunæ* Limbus Prox. à Pede *Orionis* iter.—— | 26 12 00 |
| 5 04 02 | *Lunæ* Limbus Prox. à *Polluce* —— | 44 09 10 |
| 5 06 33 | ——rep —— | 44 08 15 |
| 5 11 55 | Lucidus Pes *Orionis* à *Palilicio*—— | 26 29 40 |

LXVI. 1. *An Occultation of the* Bull's Eye *at* Greenwich, *Oct.* 28. 1680; *by Mr.* Flamſteed. *Ph. Coll.* n. 4 *p.* 100.

| *Temp. Corr.* | *Obſervat.* | | |
|---|---|---|---|
| h. ′ ″ | | | ′ ″ |
| 7 11 52 | *Lunæ* Diameter —— | 6745 | =33 39 |
| 7 16 26 | —— rep.— | 6744 | =33 39 |
| 7 19 46 | *Palilicium* à Limbo Lucido —— | 5895 | =29 24 |
| 7 24 44 | —— rep.— | 5095 | =25 25 |
| 7 40 16 | —— | 3203 | =15 59 |
| 7 51 22 | —— | 1817 | =09 04 |

| h. ′ ″ | | | |
|---|---|---|---|
| 7 59 34 | ——— 810 | =4 02 | |
| 8 02 09 | ——— 490 | =2 26 | |
| 8 06 26 | *Attingebat* Limbum | | |
| 8 06 30 | *Evanuit*, Longitudinem Paludis *Miridis* ad Boream ab ejus fine Boreo. | | |

Boreus Limbus *Mæotidis*, idem *Ætnæ*, eandem habuere Declinationem cum Loco subingressus.

Differentia Declinationum Loci *Immersionis* & Limbi *Lunæ* Borei erat 2770=13′ 49″.

9h 02′ 58″. *Emergebat* ab obscuro Limbo, Longitudinem *Insulæ Majoris* ab ejus Boreo Termino.

Itineraria per Locum *Emersionis*, ad Boream à *Creta* ipsius Diametrum, per Limbum Boreum *Sirbonis*, & Montem *Climacem*, transibat.

9h 10′ 26″ *Lunæ* Diameter——— 6791=33′ 52″.

*At* London; *by Mr.* Halley, *and* M. Haines. *Ph. Coll.* n. 5. *p.* 124.

2. At *London*, we noted the *Immersion* at 8h 6′ 00″, and that the Star was newly *Emerged* at, 9h 2′ 52″.

*At* Ballasore *in* India; *by Mr.* Benj. Harry. *ib.*

3. Mr. *Benj. Harry*, Master of the *Berkely Castle*, Riding at Anchor in *Ballasore* Road about 20 Miles E. S. E. from the Town, observed that the *Moon* passed to the Northward of the *Bull's Eye* about 24 or 25 Min. and by his Pendulum Watch, Rectify'd by Altitudes and the Rising and Setting of the Sun, he noted that precisely at 16h 00′ the *Bull's Eye* was in equal Altitude with the *Moon's* Center, and that at 16h 30′ the Star was in equal Altitude with the lower Limb of the *Moon*, and at 17h 12′ the Occidental Limb of the *Moon* was in a Right Line with the *Bull's Eye* and *Capella*.

LXVII. 1. *An Occultation of the* Bull's Eye *at* Dantzick, *Jan.* 1. st. n. 1681. *by M.* Hevelius. *Ph. Coll.* n. 3. *p.* 65.

| *Temp.* | *Observat.* | *Alt.* |
|---|---|---|
| h. ′ ″ | | |
| 6 47 00 | *Palilicium* distabat à confinio Lucis & Umbræ tanto spatio quanto *Mons Christi* removetur à Limbo Lunæ Inferiori.——— | |
| 7 37 00 | Stella occulta est ad *Mare Syrticum* sub Insula *Cercinna*, alto Capite *Andromedæ* ——— | ° ′ 50 32 |
| 7 46 00 | Altitudo Capitis *Andromidæ* ——— | 49 27 |
| 7 49 30 | Eadem Altitudo ——— | 48 55 |
| 8 46 00 | *Palilicium* rursus affulsit ad *Insulam Majorem* Maris *Caspii*. | |
| | Transiit itaque via ejus Itineraria, ratione Macularum *Lunarium*, per Mare *Syrticum*, Montem *Athos*, sub Insula *Lemnos*, & Montem *Didymum*, sub *Sinu Athenienſi*, per *Fretum Ponticum*, atque *Insulam Majorem* Maris *Caspii*. | |
| 8 51 00 | Altitudo Capitis *Andromedæ* ——— | 40 22 |

2. Mr.

2. Mr. *Benj. Harry* in *Ballasore* Road Obſerv'd, that the *Moon* paſt to the Northward of the *Bull's Eye*, and that that Star and the *Moon*'s under Limb were in equal Altitude when they were both 13° 45'. high to the Weſt, which gives the time 14h 49'. and when the *South Horn* of *Taurus* was 23° 30' high, which makes the Time 15h 13'. the Weſtern Limb of the *Moon* was in a Line with *Capella* and the *Bull's Eye*. *At* Ballaſore; *by Mr.* Ben. Harry. *Ph. Col.* n. 5. *p.* 125.

3. The Correct Time of the *Immerſion* was 6h 18' 22''. and the *Emerſion* at 7h. 19' 46''. *At* Avignon. *by* M. *Gallet*. ib.

LXVIII. *Sept.* 27. *ſt. n.* 1682. hora 3. *m. Lunam* tum tres reliquos *Planetas* nudo quidem conſpexi Oculo; ſed *Luna* eo tempore adhuc ad 7. circiter Gradus removebatur, *ſ. ſ. ſ.* Occaſum verſus. *A Tranſit of the* Moon *below the* 3 *Superior* Planets, *and* Regulus. 1682. *at* Dantzick; *by M.* Hevelius. *n.* 143. *p.* 17. n. 151. *p.* 325.

Quantum autem ex *Inclinatione Cornuum Lunæ* quoad Planetarum ductum colligere licuit, protinus proſpiciebam, nullas fore *Occultationes*, ſed tantum *Tranſitus*; ſic ut *Luna* infra illos *Superiores Planetas* incederet. In qua Opinione magis magiſque etiam ſum confirmatus: Cum *Die* ſubſequente, 28 *ſcil. Sept. m.* nec *Regulus* fuerit à *Luna* Tectus, quæ Stella ratione utriuſque Latitudinis potius Occultari debuiſſet. *Regulus* namque in ipſa Conjunctione, *ſcil.* 4h. 6'. Diſtabat à Superiori ☽ Cornu Boream verſus adhuc 31' 17''. id quod optimo *Micrometro* Tuboque egregio, accuratè obſervatum eſt; adeo ut nulla prorſus fuerit *Occultatio Reguli*, ſed tantummodo *Lunæ Tranſitus*. Ita pariter accedit die 25 *Octob.* Nam *Jupiter* & *Saturnus* nec non ♂ Die 26. *Octob. ſt. n.* minime fuerunt à *Luna* Obtecti; ſed *Luna* ſatis longè infra *Planetas* inceſſit. n. 143. *p.* 18. *min.* 31. *ſec.* 17. n. 151. *p.* 326. *min.* 31. *ſec.* 55.

LXIX. *Feb.* 11. ſt. n. 1683. h. 9. cum primum *Luna* in Oculos incurreret, *Regulus* ſatis longè Occaſum verſus removebatur; ita ut ea ipſa Conjunctio (quantum ruditer colligere dabatur) Oriente circiter *Luna*, *ſcil.* 5h vel 6h contigerit: Utrum autem *Regulus* omnino Tectus ſit, an vero tantummodo Tranſitus fuerit, haud adeo accurate deprehendere licuit. *An Occultation of* Regulus *by the* Moon, *Feb.* 11. ſt. n. 1683. *at* Dantzick; *by M.* Hevelius. n. 151. *p.* 331.

Tranſ.

LXX. *An Occultation of Two Fixt Stars by the Moon, and a Transit above a Third,* April. 2. *st. n.* 1683. *at* Dantzick; *by* M. Hevelius. *ib.*

| Temp. Sec. Horol. Ambul. | | Dist. & Altitud. |
|---|---|---|
| h. ′ ″ 9 53 30 | *Initium* Occultationis Stellulæ Majoris A. 5. *Magn.* | |
| 10 08 30 | Conjunctio *Lunæ* & Stellulæ C, distabat à *Lunæ* Cornu Inferiori.———— | ° ′ ″ 0 04 00 |
| 10 29 36 | *Initium* Occultationis Stellulæ B, 6. *Magn.* | |
| 10 52 50 | *Finis* Occultationis Stellulæ A. | |
| 11 45 30 | Altitudo *Lyræ.*———— | 31 44 00 |
| 11 46 30 | Altitudo eadem———— | 31 55 00 |
| 11 47 50 | Denuo ———— | 32 06 00 |

Sectio Luminis & Umbræ hac Die per Montes *Serrorum & Carpatos*, per *Sinum Peronticum*, inter *Byzantium* & Inf. *Cyaneam*, per M. *Amanum*, *Taurum*, *Urijque Montes* incidit.

Prior Stellula A, in Catalogo *Tychonico* non invenitur; sed in meo *Novo* Vocatur *sub Cornu Tauri Austrino Sequens* 5ª *magn.* Versatur hoc tempore in ♊ 19° 11′ 35″. & in Lat. 4° 43′ 44″. Austr. Altera vero B, quantum ex hac observatione colligere potui, existit in ♊ 19° 17′ 00″ & in Latit. 4° 47′ 0″. Austr. At tertia C, quæ forte nudis oculis non conspicitur, degit modo in ♊ 19° 9′ 0″. & in Latit. 5° 2′ 0″ Austr. Cæterum Stella A, *Lunam* subingressa est ad montem *Audum*, transiit per Insulam *Cercinnam*, per M. *Neptunum*, Mare *Adriaticum*, inter M. *Horminium* & M. *Amanum*, per M. *Herculis*; sic ut inter *Paludem Mæotidem* & *Inf. Majorem Caspii* rursus Emerserit: unde liquidum est hanc Stellulam A, fere Centralem cum *Luna* celebrasse Conjunctionem.

Altera vero Stella B, 6ª. *Magn. Lunam* ingressa est ad *Paludem Marœotidem*, Transiit per *Sinum Syrticum*, ad M. *Athos*, per M. *Latmum*, inter Montes *Sipylum* & *Macysitum*, infra *Centrum Lunæ*, per Superiorem M. *Moschum*, per *Fretum Ponticum*, atque sic infra *Insulam Majorem Caspii*.

Maii,

LXXI. *Maii* 2. st. n. 1683. 11h 0′ 0″ vesp. *Luna* supra Stellulam in radice Caudæ *Cancri* transibat, quæ modo versatur in ♋ 27° 53′ 37″. & in Lat. 2° 18′ 41″. Austr. sic ut in ipsa Conjunctione non nisi ad 12′ à *Lunæ Cornu* Infer. abesset. *An Occultation of a Fixt Star, and a Transit above another, May 2. st. n. 1683. at Dantzick; by M. Hevelius. ib.*

12h 0′ 0″. etiam *alia Fixa*, sed minutissima, tecta est, quæ in Catalogo alias non habetur. Quantum conjicere dabatur, hærebat in ♋ 28° 30′ & in Latit. 1° 54′ Austr.

LXXII. Ipsum Momentum *Immersionis* accuratissime notavi, id quod incidit 11h 17′ 20″. vesp. secundum Horolog. Ambulat. Linea itineraria incessit per Mare *Pamphilium*, infra Insulam *Carpathos*, Ins. *Cyprum*, infra Sinum Extremum *Ponti*, & Sinum Inferiorem Maris *Caspii*. 11h 24′ 42″ secundum Horol. Ambul. Altitudo Lucidæ *Lyræ* Observata est 44° 39′ 0″; ex qua *Initium Occultationis* corrigi potest. Sectio Luminis & Umbræ per *Lacum Nigrum Majorem*, ad Ins. *Corsicam*, M. *Nyconium*, per *Lacum Strymonicum*, ad Ins. *Rhodus*, per M. *Sinai* & M. *Techisandam* incidit. *An Occultation of Regulus by the Moon, May 4. st. n. 1683. at Dantzick; by M. Hevelius. ib.*

LXXIII. 1. At 9h 26′ the under Limb of the *Moon* was just Risen, and soon after *Jupiter* appeared near the Eastern Limb of the *Moon*, within a few Minutes of being *Eclipsed*. *An Occultation of Jupiter by the Moon, March 31. 1686. at London; by Dr. Hook, and Mr. Halley. n. 181. p. 85.*

9h 33′ As near as could be guessed, was the Time of the *Central Immersion*, which was very difficult to be observed by reason of the Asperity of the *Moon's* Limb, which Undulated and Sparkled very much, as it appeared through the Vapours near the Horizon: The *Ingress* happened much about the length of the Spot, called by *Hevelius Palus Maræotis*, to the North of the said Spot, or about the 124th Degree of the Outer Limb of his *Selenography*, nearly in the same Latitude with the *Moon's* Center.

10h 30′. The Western Edge of *Jupiter* began to *Emerge* out of the dark Limb of the *Moon*.

10h 31′ 20″. The whole Disk of *Jupiter* was entire, so that he was about a minute and a third in coming out from behind the *Moon*.

The *Emersion* was exactly in a right Line with the *Moon's Center* and the Northern part of *Palus Mæotis*, or about the 324th Degree of the Inner Limb of the *Selenographick Table* of *Hevelius*.

2. *At* Greenwich; *by Mr.* Flamsteed. *n.* 184. *p.* 206.

| Temp. Corr. per hoc Oscil | | | |
|---|---|---|---|
| h. ′ ″ | | | |
| 9 32 30 | ♃s. Limbus tangebat Limbum *Lunæ* Lucidum; *Mareotidis* Diametrum à termino ejus Boreo. | | |
| 9 33 42 | ♃. Totus tectus erat. Quantum per vapores Horizontis, & undulationem Limbi valde turbidam, licuit conjicere. | | |
| 10 11 12 | Differentia Declinationum Limbi *Lunæ* verè Austrini & loci *Ingressus*, Tubo ped. 8. & *Micrometro* | 1546= | ′ ″ 12 42 |
| 10 30 30 | *Jovis* particula *Emerserat* è regione Borealis Limbi *Mæotidis.* | | |
| 10 31 36 | *Jupiter* Totus liber. | | |
| 10 35 50 | Differentia Declinationum Centri *Jovis* & Limbi *Lunæ* Austrini | 3436= | 28 15 |
| 10 41 40 | Repet. | 3585= | 29 28 |
| 10 44 00 | *Lunæ* Diameter. | 3906= | 32 07 |
| 10 45 44 | Repet. | 3915= | 32 11 |

*At* Nuremburg; *by* M. Zimmerman. *n.* 183. p. 177.

3. At 10h 19′ 56″. M. *I. Ja. Zimmerman* observed the first *Contact* of the Limbs of ♃ and the ☽, and at 10h 20′ 47″. ♃ was all *Eclipsed.*

At 11h 22′ 51″. ♃ was wholly *Clear* from the *Eclipse.*

The *Immersion* was about the 117th, the *Emersion* at the 321st. Degree of the Limb, in the *Chart* of *Hevelius.*

At 11h 31′ 06″. The third *Satellite* of ♃ *Emerged.* These times were collected from the Culminations of fixt Stars, and the Vibrations of a Pendulum.

*By* M. Wurtzelbauer. *ib.*

4. At 10h 20′ 50″. ♃ applyed to the Limb of the ☽, over against the *Loca Paludosa Insulæ Cercinnæ.*

At 10h 22′ 00″. he appeared about half *Eclipsed.*

At 10h 22′ 30″. he was wholly *Hid.*

At 11h 19′ 40″. ♃ began to *Emerge.*

At 11h 21′ 20″. he was quite *Free* from the Interposition of the ☽. The point of the *Emersion* was some what to the North of the *Palus Mæotis.* No Spot in the ☽ was so near the apparent *Magnitude* of ♃s Disk at the *Insula Besbicus Hevelii.*

At 11h 40′ 00″. the Altitude of *Procyon* was 8° 37′. whence the pendulum Clock, which had been set by Altitudes of the ☉, the afternoon preceeding, may be examined.

5. Etiam-

5. Etiamsi hucusque per 56 Annos nullam Observationem alicujus Momenti neglexerim, non nisi tres *Jovis* Eclipses rite deprehendere & annotare potuerim: Utpote primam *Anno* 1646, die 24 *Decemb.* vesperi, sed tantummodo ejus *Finem*; Secundam *Anno* 1679, die 5 *Junii* ante meridiem de die, quo tempore res omnis felicius successit; Tertiam hoc *Anno currente* 1686, die 10 *April.* vesperi. *At Dantzick; by M. Hevelius. ib. p. 178.*

Inter alia autem notandum occurrit, quod hæcce *Occultatio* non *Luna* omnino existente *Plena*, sed altera die circiter post ipsum *Plenilunium* vesperi acciderit; & quidem eodem Tempore (quod permirum sanè accidit, & est casus, qualis haud facile unquam continget) eademque facie, ut illa *Occultatio Anni* 1646, die 24 *Decemb.* vesperi visa est, quo Tempore *Luna* jam ad biduum pariter decreverat, & sine dubio eandem *Librationem* etiam exhibuit, quam in hac nostra ultima *Observatione*. Nam Sectio Luminis atque Umbræ plane fuit eadem, & per easdem Maculas transiit (quod satis admirari nequeo) nimirum ad *Lacum Hyperboreum Majorem* & *Minorem*, tum ad *Montes Riphæos*, per Paludem *Mæotidem*, per *Lacum Majorem* Maris *Caspii*, & *Sinum* ejus *Inferiorem*, ad Montem *Nerosum*.

E contrario, *Jovis Occultatio Anno* 1679 à me habita, plane extitit diversa, siquidem illa non circa *Plenilunium*, sed *Novilunium* accidit, tertia circiter die ante *Conjunctionem* ipsam.

| *Horol. ambulat.* | | *Altit. Quadrant captæ.* | *Tempus ex Alt. Corr.* |
|---|---|---|---|
| h. ′ ″ | | Gr. ′ ″ | H ′ ″ |
| 5 10 10 | Altitudo *Solis.* | 13 47 0 | 5 11 43 |
| 5 12 30 | Altitudo *Solis.* | 13 28 0 | 5 13 55 |
| 5 17 40 | Altitudo *Solis.* | 12 41 0 | 5 19 21 |
| 5 23 50 | Altitudo *Solis.* | 11 46 0 | 5 25 43 |
| 8 7 10 | Altitudo *Arcturi.* | 29 55 0 | 8 12 50 |
| 8 11 15 | Altitudo *Arcturi.* | 30 32 0 | 8 17 4 |
| 8 15 10 | Altitudo *Arcturi.* | 30 59 0 | 8 20 51 |
| | *Luna* oritur circit. | | 9 24 0 |
| 9 44 50 | *Jupiter* distabat ab Inf. *Cercinna* 43. circit. minut. | | 9 52 50 |
| 10 21 30 | *Jovis* distantia erat tanta, quanta distantia M. *Sinai* à *Palude Maræotide.* | | 10 31 30 |
| 10 40 35 | *Jovis* distantia erat ferè æqualis distantiæ inter M. *Ætnam* & M. *Porphyritem.* | | 10 51 5 |

| h. ′ ″ | | | h. ′ ″ |
|---|---|---|---|
| 10 51 30 | *Jovis* Limbus à ☽ limbo distabat tanto intersti-tio, quanto *Pal. Maræotis* à *Limbo Lunæ*. | | 11 2 0 |
| 10 56 9 | ♃ Limbo suo Tangere incipiebat *Lunæ* Limbum, atque sic *Initium Occultatiouis* accidit. | | 11 7 9 |
| 10 56 54 | Dimidius *Jupiter* Occultabatur. | | 11 7 54 |
| 10 57 39 | Totus *Jupiter* omnino à ☽ *Tectus*. | | 11 8 39 |
| 11 8 31 | Occultatio *Comitis Jovis ultimi* ad M. *Alabastrinum* accidit. Duo tantummodo *Comites* à parte Orientali conspecti sunt. | | |
| 11 15 54 | Altitudo *Lyræ* | 32° 59′ 0″ | 11 26 0 |
| 11 19 0 | Insula *Besbica* & *Rhodus* reperiebantur sub eodem perpendiculo; id quod ad 35 gr. circ. à Linea ☽ verticali removebatur. | | |
| 11 21 37 | Altitudo *Lyræ*. | 33 50 0 | 11 32 15 |
| 11 24 57 | Altitudo *Lyræ*. | 34 24 0 | 11 36 24 |
| 11 38 15 | *Emersionis* Initium *Jovis*. | | 11 49 15 |
| 11 39 0 | Dimidius *Jupiter* Emergebat. | | 11 50 0 |
| 11 39 45 | Totus *Jupiter* apparebat. | | 11 50 45 |
| | Diameter *Lunæ Micrometro* observata erat 31′ 0″. | | |
| 11 54 10 | Distantia *Jovis* à confinio Lucis & Umbræ erat æqualis distantiæ M. *Ætnæ* à M. *Porphyrite*. | | 12 5 40 |
| 11 57 20 | Distantia *Jovis* à Confinio Lucis & Umbræ elongabatur intervallo inter Insulam *Besbic.* & M. *Ætnam*. Et *Comes* ♃ *Remotissimus* à *Jove* tantum aberat, quantum ipse *Comes* à dicto Confinio Lucis. | | 12 9 20 |
| 12 6 9 | Altitudo *Lyræ*. | 40 19 0 | 12 18 39 |
| 12 9 18 | Eadem Altitudo denuo. | 40 46 0 | 12 21 40 |
| 12 13 20 | Altitudo *Lunæ*. | 16 15 0 | |

Primo liquidum est ex ipsa Observatione, quod *Orbita*, seu Linea *Jovis* Itineraria, per *Montem Alabastrinum*, per M. *Christi*, M. *Carpathes*, infra M. *Macrocemnios*, & per *Lacum Hyperboreum Inferiorem* incesserit. Secundo quod Insula *Besbica* & Insula *Rhodus* sub uno eodemque perpendiculo, tempore *Occultationis*, circiter 11h. 30′ extiterit; sic ut 35 gradus *Lunæ* Limbi culminaverit. Intravit itaque *Jupiter* Limbum *Lunæ* Illuminatum circa 61 gradum,

dum, à Lineâ *scil.* perpendiculari Nonagesimi atque puncto Zenith, Ortum versus; Exivit vero circa 31 gradum à dicta Linea Perpendiculari Nonagesimi occasum versus, ad Limbum *Lunæ* obscuratum. Proinde linea *Jovis* itineraria fuit subtensa 104. ferè graduuum, attenta videlicet parte *Lunæ* Boreali.

Præterea etiam maximè notatu dignum, quod ex hac observatione Diametrum *Jovis* exquisite elicere potuerim; nimirum 51″ 42‴; & tantæ magnitudinis extitit etiam Diameter *Jovis* 50″ circ. quoties illam per *Maculas Lunæ* dimensus sum. Quod autem *Anno* 1679. die 5 *Jun.* cum similem *Jovis* Eclipsim observarem, longè ea extitit minor, nimirum tantum 30″ 53‴. Id ex eo evenisse puto, quod observatio illa, tempore Diurno, Splendente *Sole* fuerit observata; quo Radii Stellarum & Planetarum adventitii magis à Luce *Solis* absterguntur, quam Tempore Nocturno, nocte obscura. Quodsi autem quæras, quamnam Diametrum apparentem veriorem existimem? Scias illam, quam *Anno* 1679. 5 *Jun.* de die, Sole splendente observavi. Non equidem ex eo quod non æquè diligenter hanc quam illam determinaverim; sed quod tempore Nocturno Radii Adventitii magis obstent, sicuti diximus, quam tempore Diurno.

6. At 9h 31′ 6″. *Jupiter* was in a Perpendicular falling on the Limb of the *Moon* over-against the Northern-part of the Spot *Grimaldi*, ( *Maræotis* ) near to *Riccioli* ( *Stag. Miris* ) and distant from the Limb about 4 times as much as the said Spot. *At* Paris; *by M.* Cassini. *n.* 183. p. 175.

9h 40′ 21″. *Jupiter* touched the Circumference of the *Moon*, which undulated by reason of the Vapours near the *Horizon*.

9h 41′ 20″. He quite disappeared in the Inequalities of the *Moon's* Limb, the *Total Immersion* might be some Seconds later. So the *Central Immersion* was at 9h 40′ 51″. *Jupiter* entered over-against that Part of *Grimaldi* next *Riccioli*.

10h 30′ 2″. The *Outermost Satellite* which preceeded *Jupiter* appeared over-against the Middle of the *Caspian Spot* ( *Pal. Mæotis* ) through which the *Section of Light and Darkness* passed, and made nearly an Equilateral Triangle, with the Extremities of that Spot.

10h 40′ 24″. The first Limb of *Jupiter* began to come out of the Dark side of the *Moon*, over-gainst the North-part of the *Caspian* Spot, about *Cleomedes*, ( *ad Montes Riphæos.* )

10h 40′ 56″. The Center of *Jupiter* did *Emerge*. It was difficult to distinguish the Moment when *Jupiter's* Disk was fully clear, but at 10h 41′ 36″. the Eclipse was certainly past.

At the *Emersion* of the Center, the Altitude of *Jupiter* was 11° 31′.

At 10h 42′ 49″. The *Second Satellite* being the nearest of the three that followed the Planet, *Emerged*.

At 10h 45′ 1″. The *Innermost Satellite*, being near its greatest *Elongation*, *Emerged*.

At 10h 50′ 40″. The *Third*, or *Penextimus Satelles*, being likewise near its greatest *Elongation*, began to appear over-against the Northern-Edge of the *Caspian* Spot.

At 11h 45′. The Diameter of the *Moon* was 32′ 27″. and according to the *Calculus* of M. *Cassini*, her *Parallax* was 61 Min.

*At* Avignon; *by* R.P. Bonfa. *ib.* p. 176.

7. The Central *Immersion* was at 9h 42′ 13″. and the Central *Emersion* at 10h 45′ 26″. over-againſt the Southern-part of the *Cuſpian* Spot.

*An Occultation of* Jupiter *by the* Moon, Apr. 28. 1686. *n.* 181. p. 87.

LXXIV. 1. The *Immerſion* was ſeen at *Totteridg* (which Place is about 9 Miles from *London*, and nearly 25″ of Time to the Weſtwards thereof) by Mr. *Ed. Haines*: who between a Gap of the Clouds obſerved the *Contact* of the *Moon*'s Limb and *Jupiters*, at 3h $3\frac{1}{2}$′.

The Clouds cloſing again permitted him to obſerve no more: however from this we may conclude the *Central Immerſion* at *London*, to have been 3h $4\frac{1}{2}$′. *manc.*

The *Emerſion* was obſerved at *London*, by Mr. *Edm. Halley*, to fall out at 3h 49′. for at 3h 50′. *Jupiter* was all out, and the Limbs ſo little ſeparated, that he judged, that a Minute before, the *Center* of *Jupiter* had been upon the *Moon*'s Edge: The Point of the *Emerſion* was over-gainſt the Southern-part of the Spot, call'd by *Hevelius Inſula Macra*, or at the 342d Diviſion of the Inner Limb of his *Map* of the *Moon*.

*At* Avignon; *by* R.P. Bonfa. *n.* 183. p. 177.

2. The *Immerſion* of the *Center* happened at 3h 37′ 23″. on the Eaſt-ſide of the Spot *Xenophanes*. The *Emerſion* was at 4h 28′ 24″. between *Seneca* and *Beroſus*, according to *Riccioli*, or *ad Montes Alanes*, *Hevelii*, a little to the Northward of the *Palus Mæotis*.

3. *At* Dantzick; *by* M. Hevelius. *ib.* p. 184.

| *Secund. Horol. Amb.* | | | *Temp. Corr.* |
|---|---|---|---|
| h. ′ ″ | | ° ′ ″ | h. ′ ″ |
| 3 23 20 | Altitudo *Arcturi.* | 31 16 0 | 3 20 12 |
| 3 24 25 | Eadem Altitudo. | 31 04 0 | 3 21 35 |
| 3 44 30 | *Jupiter* à Limbo *Lunæ* diſtabat Majori adhuc Intervallo quam M. *Sinai* à M. *Ætna*. | | 3 41 30 |
| 3 47 00 | *Jovis* diſtantia erat tanta, quanta M. *Porphyritidis* à *Byzantio*. | | 3 44 00 |
| 3 52 00 | *Jovis* à Limbo *Lunæ* Diſtantia erat æqualis Diſtantiæ Inſulæ *Sardiniæ* & *Paludis Mareotidis*. | | 3 49 00 |
| 3 59 00 | *Jupiter* à Limbo *Lunæ* paulo plus diſtabat quam *Pal. Maræotis* ab *Ætna*. | | 3 56 00 |
| 4 16 40 | Diſtantia *Jovis* à Limbo *Lunæ* æquabatur ferè Diſtantiæ M. *Porphyritidis* ab Inſ. *Cercinna*. | | 4 13 40 |
| | Planetarum *Occaſus* factus eſt. | | 4 17 00 |

LXXV. 1. *An Occultation of* Saturn *by the* Moon, *Mar.* 19. $168\frac{6}{7}$. *at* Totteridge; *by Mr.* Ed. Haines. *n.* 186. p. 268.

| Hora Horolog. ma. | | Temp. Corr. |
|---|---|---|---|
| h. ′ ″ | | ° ′ | h. ′ ″ |
| 12 06 10 | Altitud. *Pollucis.* | 28 35 | 12 12 14 |
| 12 16 47 | ——— | 26 59 | 12 22 47 |
| 1 18 00 | *Lunæ* Limbus tangebat Ansam Occid. ♄ni ——— | | 1 24 00 |
| 1 18 30 | *Immersio* Centri *Saturni* paulo infra *Pal. Mar.cotin.* | | 1 24 30 |
| 1 19 00 | Jam *Saturnus* omnino latuit. | | 1 25 00 |
| 4 01 25 | Altitudo Centri *Solis* ——— | 20 00 | 4 07 17 |
| 4 09 06 | ——— | 18 55 | 4 14 43 |

2. *March* 18. At Night I observed here the *Occultation* of *Saturn* by the *Moon*, which happened at 12h. 13′ 55″. it passed directly under the Midst of the *Moon's Discus.*— *In* Ireland; *by Dr.* Ash, *Bp. of* Cloyn. *n.* 243. p. 293.

LXXVI. 1. *Oct.* 13. 1665. at six of the Clock, with a very good Telescope near 38 Foot long, and a double Eye-glass, *Saturn* appeared to me somewhat otherwise, than I expected, thinking it would have been Decreasing, but I found it as full as ever, and a little hollow above and below. *Phases of* Saturn, *An.* 1665. *at* Mainhead *near* Exeter; *by Mr.* Will. Ball. *n.* 9. *p.* 152. Fig. 132.

2. *Jun.* 29. 1666. between 11 and 12 at Night, I observed the Body of *Saturn* through a 60 Foot Telescope, and found it exactly of the Shape Represented in the Figure. The *Ring* appeared of a somewhat brighter Light than the Body; and the black Lines *a a*, crossing the *Ring*, and *b b*, crossing the Body (whether Shadows or not I dispute not) were plainly visible; whence I could manifestly see, that the Southermost-part of the *Ring*, was on this side of the Body, and the Northern-part, behind or covered by the Body. *An.* 1666. *at* London; *by Dr.* Hook. *n.* 14. p. 246. Fig. 133.

3. *Aug.* 17. 1668. at $11\frac{1}{2}$h these *Parisian* Observers, imploying a Telescope of 21 Foot, saw the *Globe* of *Saturn* in the Middle manifestly appearing above and below, beyond the Ovale of his *Anses*; which was hardly discernable the last Year. They measured divers ways the *Inclination* of the greater Diameter of the *Ovale* to the *Æquator*, and found it of about 9 Degrees. By this Observation and other like ones of this and the preceeding Year, M. *Hugens* finds, that, instead of 23° 30′, the Angle of the Planes of the *Ring* and of the *Ecliptique* must be of 31°, or thereabouts. An. 1668. *at* Paris; *by M.* Hugens *& M.* Piccart. *n.* 45. p. 900.

4. *Aug.* 26. *st. n.* 1670. Telescopium illud 50 Pedes longum, quod non ita pridem mihi transmisistis, Faciem *Saturni* utut *Luna* fuerit præsens nitidissimè ac clarissimè detegebat. Quali autem mihi apparuit, adjecta Delineatio commonstrabit; alia planè Facie, quàm *Cl. Hugenio*, tum vobis *An.* 1666. An. 1670. *at* Dantzick; *by M.* Hevelius. *n.* 65. *p.* 2089. Fig. 134.

tum

tum *Parisiensibus* 1668, videbatur. Siquidem *Annulus*, qui *Saturnum* circundat, multo nunc arctior, compressiorque animadversus quam illo tempore, quasi obliquiori, respectu *Terræ*, nunc via incedit.

*At* Paris; *by* M. Hugens. *ib.* p. 2093. 5. This Summer M. *Hugens* Observed *Saturn* with his Telescope of 22 Feet, and saw his Figure to be very conformable to what it should be according to his *Hypothesis*; viz. the *Ansæ* or Arms to be very narrow, in so much that their opening appeared not but very obscurely.

*At* London; *by Dr.* Hook. *ib.* 6. *Sept.* 16. Dr. *Hook* observed the Phase of Saturn as here represented. *Fig.* 135.

An. 1671. *at* Paris; *by* M. Cassini. *n.* 78. p. 3024. 7. *Saturn* according to the *Hypothesis* of M. *Hugens* was to have retaken his *Round Figure* in the Months of *July* and *August* 1671. But this Appearance hath been perceived ever since the End of *May*, at a Time when he was distant enough from the *Sun* and the *Horizon*, to be well Observed. He hath remained in this Figure unto the 11th of *August*, and M. *Cassini* did then observe him thus; but three Days after he saw him with *Arms*, though very Narrow ones.

*By* M. Hugens. *ib.* p. 3025, 3026. 8. Our *Philosophers* here, know very well, that as soon as M. *Cassini* had told me that the *Arms* of *Saturn* were returned in *August*, I said that assuredly they would disappear before the End of this Year. I still observed them, *Nov.* 6. *st. n.* in the Evening, but they were so faint and obscure, that it was hard to discern them; so that within a few Days they will appear no more at all. This confirms altogether my *Hypothesis* of the *Ring*, which now disappears in Proportion that the Rays of the *Sun* do obliquely illuminate the flat Surface of it, Obververted to our Sight.

*At* Dantzick; *by* M. Hevelius. *ib.* p. 3032. *Fig.* 136. 9. Quali Facie nuper, Die *sc. Sept.* 11. *st. n.* apparuerit, quàm rectissimè & diligentissimè delineavi atque hic transmisi. An vero Mense *Jun. Jul.* & *Aug.* cum planè *Rotundum* conspexeritis, ut volunt *Parisienses*, vix mihi imaginari possum. Etenim utut *Brachia Saturni* ad latera apparuerint arctissima, etiam Tubo 60 vel 70 Pedum; haud tamen credo, ea omnino evanuisse, ita ut ne Vestigium aliquod fuerit reliquum. Fortasse *Parisienses* Telescopiis brevioribus in ipso Crepusculo, Luna præsente, *Saturnum* contemplati sunt.

*At* Darby; *by* Mr. Flamsteed. *ib.* p. 3034. 10. *Oct.* 12. with my less Tube I thought I saw something on each side of *Saturn*, amidst the Colours of my Glass, and the Spurious Rays of his Body. Directing my longer Tube (of 14 Feet) to him, I could see his *Ansæ* somewhat more distinctly, but very slender, and to one, that thought not of them, scarce discernable.

*Nov.* 30. I observed him with my 14 Foot Telescope, the Aperture being $1\frac{1}{2}$ Inch, and its Eye-Glass drawing two Inches. He appeared perfectly *Round*, free from Rays and Colours, and no *Ansæ* to be seen. Mr. *Townley* in his last (*Nov.* 20.) tells me, that he looked at him one Night, and could hardly distinguish his Line of the *Ansulæ*, but plainly saw a dark Line through him near his upper part.

*At* Paris; *by* M. Cassini. *n.* 92. p. 5180. 11. *Dec.* 16. we found that *Saturn* had retaken his *Round Figure*.

An. 1675. *at* London; *by* Mr. Flamsteed. *n.* 116. p. 372. 12. *Jun.* 27. m. 1675. *Saturnum* Patulis auctum *Ansis* vidimus.

*At* Dantzick; *by* M. Hevelius. *n.* 127. p. 661. 13. In *August*, The Figure of *Saturn* appear'd, as *Fig.* 137.

14. *Ex*

14. Ex Schemate *Saturni* à *Cl. Hevelio* ante Annum obſervato video, eum Teleſcopiis, noſtris longè inferioribus, uti. Tunc enim Temporis ( ut & nunc (*Aug.* 1676.) cernebatur nobis in *Saturni* Globo Zona ſubobſcura, paulo Auſtralior Centro, inſtar Zonarum *Jovialium.* Deinde Latitudo *Annuli* dividebatur bifariam, Lineâ obſcurâ apparenter Elliptica, revera Circulari, quaſi in duos *Annulos* Concentricos, quorum Interior Exteriori Lucidior erat. Hanc Phaſim ſtatim poſt Emerſionem *Saturni* è Solaribus Radiis per totum Annum uſque ad ejus Immerſionem conſpexi; primo quidem, Teleſcopio Pedum 35; deinde minori, Pedum 20. *An. 1676. at Paris; by M.* Caſſini. *n.*128. p. 690. *Fig.* 137. *Fig.* 138.

LXXVII. 1. Vidi *Saturnum Aug.* 26. *ſt. n.* 1670. aliâ *Aquila* 24° 32′ 0″. in Diſtantia ab Extrema Alæ *Pegaſi* 33° 48′ 0″. & ab *Ore* ejus 24° 51′ 40″. in 4° 11′. *Piſcium*, in 1° 53′. Latit. Auſtr. in ipſa nempe Oppoſitione *Solis* exiſtentem. *Places of Saturn Obſerved.* An. 1670. *at* Dantzick; *by M.* Hevelius. *n.* 63. p. 2089.

2. *Dec.* 29. *ſt. n.* 1677. 8h 58′. Vidimus in eodem Azimutho inque Nonageſimo Eclipticæ Gradu ab Horizonte *Saturnum* & *Boreum Oculum* ♉, qui infra *Saturnum* erat, unde Planetam & Fixam eandem in *Zodaico* Longitudinem obtinere deprehendimus, *viz.* juxta *Tychonem* ♊ 3° 58′ 53″. An. 1677. *at* Paris; *by M.* Bullialdus. *n.* 139. p. 973.

LXXVIII. 1. About the End of *October* 1671. we diſcovered, by a Teleſcope of 17 Feet, 11 ſmall Stars near *Saturn*, one of which by its particular Motion ſhew'd it ſelf to be a true *Planet*: which we found by comparing it not only to *Saturn* and his ordinary *Satellite*, diſcovered 1655. by *M. Hugens*, but alſo to the Fixt Stars. The Motion of it was very manifeſt in reſpect of the Fixt Stars, but leſs ſenſible in reſpect of Saturn; Yet it appear'd that from *Octob.* 25. unto *November* 1. his Diſtance from *Saturn* increaſed Weſtward, and from that time unto *Novemb.* 6. it diminiſhed: ſo that his greateſt *Digreſſion* from *Saturn* hapned in the Beginning of *November.* *The Outermoſt Satellite of Saturn diſcovered; by M.* Caſſini. *n.* 92. p. 5178.

*Dec.* 16. We found that on the Eaſt of *Saturn*, there was a ſmall Star, far diſtant in a ſtreight Line to *Saturn*, and to his ordinary *Satellite*, which was Oriental alſo but little diſtant from *Saturn*. And *Dec.* 24th we ſaw this *Satellite* in the Weſt, and a Star, Oriental likewiſe, leſs diſtant from *Saturn* than that we had ſeen the 16th.

*Dec.* 13. and 17. 1672. We perceived, with an Excellent Teleſcope, (of 35 Foot made by *Campani*) an Occidental Star, remote from *Saturn*, which in both thoſe Obſervations had a Southern Latitude in reſpect of the Line of his *Wings*; but in the firſt it was further diſtant from *Saturn* than in the Second: ſo that, if this was the ſame Star, as I ſuppos'd it to be, it mov'd towards *Saturn* on the Eaſt, and conſequently (ſuppoſing it to be his *Satellite*) it was in the Superiour part of his Circle.

*Feb.* 6. 1673. We began to ſee him again, and we obſerved him almoſt all the Days following till the 20. This New *Planet* did more and more remove from *Saturn* till the 19th of *Feb.* when we meaſured the Difference between his Paſſage and that of the Center of *Saturn* to be 30″ of an Hour, which gave at leaſt 10 Diameters of *Saturn*, and on the 20. the Diſtance was judged by Eſtimate to be yet greater.

This

Ib. p. 5179, 5184. This Digreſſion being treble to that of the *Ordinary Satellite* enabl'd us at firſt to judge the Time of his Revolution to be Quintuple, applying to the *Satellites* that Proportion, which *Kepler* hath noted in the *Principal Planets*, between the Periodical *Times* and their *Diſtances*. We were afterwards confirmed in this Opinion; For by the Apparent ſwiftneſs of his Motion it was eaſie to ſee that this *Planet* had been in Conjunction with *Saturn*, *Feb*. 3. 1673. and by his Motion on the Weſt it appears, that he was in the Inferior part of his Circle: And becauſe during this time of 17 Days he remov'd more and more from *Saturn*, 'tis certain that he remained in the ſame Quadrant of the Inferior Occidental Circle above 17 Days, and that his whole *Revolution* is more than 68 Days. He was theſe laſt Days at a Diſtonce almoſt equal to that which he had about the end of *Octob*. 1671; ſo that in 480 days or thereabout he made a certain Number of Intire *Revolutions*, which can be no more than 7; ſince each of them is without Queſtion of more than 68 days. If you ſhould count 7 of them, each would be 68½ Days; if you count 6, each would be 80 Days; and if you count but 5, each would be 96 Days. But this laſt Suppoſition can by no means be made to agree with the two Obſervations of *Dec*. 1672, and the firſt doth not agree ſo well with them as the Second.

n. 133. p. 831. M. *Caſſini* has ſince found that this *Outermoſt Satellite* is diſtant from the Center of *Saturn* 10½ Diameters of his *Ring*; that the Period of his *Revolution* in 80 Days is ſo juſt, that he doth not Anticipate 9 Revolutions, which are made in two Years, but by one whole Day; and that in the Conjunctions with *Saturn*, his Latitude encreaſes according as the *Ring* of *Saturn* enlargeth it ſelf; though the Line of his Motion is not Parallel to the Circumference of the *Ring*.

*M. Caſſini* hath alſo diſcovered, after many Revolutions, that this *Satellite* hath a Period of *Apparent Augmentation* and *Diminution*, by which Period he becomes viſible in his greateſt Occidental Digreſſion, and Inviſible in his greateſt Oriental Digreſſion; he begins to *Appear* two or three Days before his Conjunction in the Inferior part, and to *Diſappear* two or three Days after his Conjunction in the Superior part: So that he remains Inviſible in every Revolution of 80 Days for a whole Month together.

This Viciſſitude of *Phaſes* makes it ſeem probable, that one part of his Surface is not ſo capable of Reflecting to us the Light of the Sun, which maketh it viſible, as the other part is. Whence we may conjecture, that the Globe of this *Satellite* hath ſome Diverſity of Parts Analogous to that of the Earth, the one part of whoſe Surface is cover'd by the Sea, which is not ſo fit to Reflect from all parts the Light of the Sun, as the Continent which maketh up the other part: So that this *Planet* by a Converſion about his *Axis*, or by an Expoſition of the ſame Hemiſphere to *Saturn* (much after the manner of the Hemiſphere of the *Moon* to the *Earth*,) ſometimes turns to us the part Analogous to the Continent, ſometimes that part which anſwers to the Sea.

LXXIX.

LXXIX. *Dec.* 23. 1672. We found a ſmall Star Weſtward of *Saturn*, between him and his *Ordinary Satellite*, which was on the Weſt alſo, almoſt at a double Diſtance. *Dec.* 30. We ſaw a little Star, on the Eaſt of him and his *Ordinary Satellite*, which had paſſed alſo to the Eaſt of him. *The Third Satellite of Saturn diſcover'd; by M. Caſſini. n. 92. p. 5181.*

*Jan.* 10. 1673. This little Star appear'd to have returned almoſt to the ſame Poſition in reſpect of *Saturn* and his *Ordinary Satellite*, where it had been *Dec.* 23; *Jan.* 15. the *Ordinary Satellite* was Oriental, and the *New* one Occidental, as it had been in the precedent, but a little nearer to *Saturn*. We had that Evening time enough attentively to Obſerve this *Planet* for a whole Hour together, during which we perceived, it approached to *Saturn* on the Weſt, and conſequently was in the Superiour part of his Circle: which did fully confirm us in the Suppoſition we were inclined to, that it was an *Interior Satellite*.

Comparing the Obſervations together, we began to find the Rule of the Motion of the new *Interior Satellite*. For the two laſt ſhew'd us, that in 5 Days he had made more than a whole Revolution. The firſt Obſervation compared with the 3d made us judge, that in 18 Days he had made a Number of Revolutions almoſt whole ones, which certainly were 4; each of them was of $4\frac{1}{2}$ Days: So that between the 10th and 15th it might be, that there had been one Revolution of $4\frac{1}{2}$ Days, or two Revolutions of $2\frac{1}{4}$ Days each. But the Combination of the firſt with the 2d, made us ſeclude the Period of $2\frac{1}{4}$ Days. We therefore Judged by theſe Obſervations; that this laſt *Planet* finiſhes his Revolution about *Saturn* in $4\frac{1}{2}$ Days; that the Semidiameter of this Circle is $1\frac{1}{8}$ of the Diameter of *Saturn's Ring*; and that he was towards his greateſt Occidental Digreſſion the 23d of *December*, and *Jan.* 1. about 7 a Clock in the Evening. We have ſince found, that his greateſt Digreſſion from the Center of *Saturn* is only $1\frac{2}{3}$ of his *Ring*, *n.* 133. p. 833. and the Period of his Revolution is 4 Days 12h and 27′. His Latitude Augments alſo according as the *Ring* enlargeth, and at the preſent that the largeneſs of the *Ring* is greater than the Diameter of the Globe of *Saturn*, he is to paſs in the Conjunctions without touching either *Saturn* or his *Ring*. Yet notwithſtanding we have not yet been able to diſtinguiſh him in the Conjunctions either in the upper or lower part of his Circle; but only in his Greateſt, as well Oriental, as Occidental Digreſſions.

LXXX. Theſe two *Satellites* were firſt of all ſeen in *Mar. An.* 1684. by two excellent Object Glaſſes of 100 and 136 Feet, and afterwards by two others of 90 and 70 Feet, all made by *S. Campani*, after the Diſcovery of the 3d and 5th *Satellites*, which had been made by others of his Glaſſes of 47 and 34 Feet. We have ſince ſeen all theſe *Satellites* with that of 34 Feet, and continued to obſerve them with Glaſſes of *M. Borelli* of 40 and 70 Feet, and by thoſe which *M. Artouquel* hath lately made, of 80, 155, and 220, Feet. It was eaſie for us to ſee theſe two *Satellites* by theſe different Sorts of Glaſſes, after having found the Rules of their Motion, whereby we might with more particular attention look upon the places where they ought to be. *Two Interior Satellites of Saturn diſcover'd; by M. Caſſini. n. 181. p. 79.*

 The

The *First Satellite* was Observed 45° Distant from its *Perigee*, moving toward the West, *Mar.* 11. 1686. *st. n.* at 10h 40′. at Night, and returned to the same Position on the 14th of *April* at the same Hour.

The *Second* was 36° Distant from the *Perigee* to the West, the 30th of *Mar.* 1686. *st. n.* at 8 a Clock in the Evening.

The *First* or *Innermost Satellite*, is never distant from *Saturn's Ring* above $\frac{2}{3}$ of the apparent length of the same *Ring*; It makes one *Revolution* in 1d. 21h 19′. and the Circle of its Orb is nearly in the same Plain with the *Ring*.

The *Second* or *Penintime Satellite* is $\frac{1}{4}$ of the Length of the *Ring* distant therefrom, and makes his Revolution in 2d 17h 43′.

After a great Number of Choice Observations it was Concluded, that the Proportion of the *Digressions* of the *Second* to that of the *First*, (counting both from the Centre of *Saturn*) is as 22 to 17; and of its *Revolution* as 24¼ to 17. This is that very same Proportion which *Kepler* Observes between the *Distances* and *Periods* of the *Primary Planets*, and which we have found between the other *Satellites* of *Saturn*, and is verified in the *Satellites* of *Jupiter*. There is nothing that better shews the Admirable Harmony of the Particular *Systemes* with the Great *Systeme* of the World.

The Antient Astronomers having translated the Names of their Heroes among the Stars, those Names have continued down to us Unchanged, notwithstanding the Endeavonr of following Ages to alter them; and *Galileo* after their Example, having Honoured the House of the *Medici* with the Discovery of the *Satellites* of *Jupiter*, made by him under the Protection of *Cosmus* II. (which Stars will be always known by the Name of *Sidera Medicea*.) Wherefore the Discoverer Concludes that the *Satellites* of *Saturn*, being much more exalted and more difficult to Discover, are not unworthy to bear the Name of *Louis Le Grand*, under whose Reign, and in whose *Observatory* the same have been detected, which therefore he calls *Sidera Lodoicea*, not doubting but to have Perpetuated the Name of that King, by a Monument much more lasting than those of Brass and Marble, which shall be erected to his Memory.

*M. Hugens's Theory of the Fourth* Satellite *of* Saturn *corrected; by Mr.* Ed. Halley. n. 145. *p.* 82.

LXXXI. The *Fourth* or *Penextime Satellite* of *Saturn*, first Discovered by *M. Hugens* 1655, I have of late frequently Observed with a 24 Foot *Telescope*: and I perceived that *M. Hugens's* Numbers were considerably run out, and about 15° in 20 Years too swift; this made me resolve more nicely to enquire into it's *Period*; and accordingly I waited till I had gotten a Competent Number of Observations, the most Considerable whereof are these.

1682. *Nov.* 13. 13h 00′. *p. m.* the *Satellite* appeared on the North side of *Saturn*, and a Perpendicular let fall from it on the Transverse Diameter of the *Ring*, fell upon the middle of the Dark space of the following *Ansæ*; and the same Night 19h 00′. it had past the Conjunction, and the Perpendicular fell exactly on the Western Edge of the *Globe* of *Saturn*; The Northern Latitude, and Retrograde Motion, made it evident that the *Satellite* was then in *Perigæo*.

That

Again, *Nov.* 21. 16$^h$ 15$'$. this *Satellite* of *Saturn* was on his South-ſide, the Perpendicular on the Line of the *Anſæ* fell on the Middle of the dark Space of the Weſtern *Anſa*, and the ſame Night 19$^h$ 00$'$, the Perpendicular fell preciſely on the Center of *Saturn*, and the Diſtance therefrom was ſomewhat leſs than one Diameter of the *Ring*. By this it was evident that the *Satellite* was in *Apogæo*.

I obſerved it in *Apogæo* again on the 24$^{th}$ of *Jan.* 1683. at 8$^h$ 00$'$ *p. m.* the Perpendicular on the Line of the *Anſæ* fell exactly on the Weſtern Limb of the *Globe* of *Saturn*, and at 9$^h$ 30$'$. *p. m.* the ſaid Perpendicular fell within the *Globe* more than half way to the *Center*, and the Diſtance from the Line of the *Anſæ* towards the South, ſeemed much about one Diameter of the *Ring*.

Laſtly, *Feb.* 9. 1683. 8$^h$ 10$'$. *p. m.* it was again in *Apogæo*, and I could by no means diſcern towards which ſide it inclined moſt, nor whether the Tranſverſe Diameter of the *Ring*, or the Diſtance of the *Satellite* therefrom, were the greater; ſo that at that time it was preciſely *Apogæon*.

To compare with theſe, I choſe two out of thoſe of *Hugens*, which ſeemed the moſt to be confided in; the firſt made 1659. *March* 14. *ſt. n.* 12$^h$ 00$'$ at the *Hague*; when the *Satellite* appeared about one Diameter of the *Ring* under *Saturn*, but it was gone ſo far to the Weſtward, that he concluded, that about 4 Hours before, or 7$^h$ 40$'$ at *London*, it had been in *Perigæo*.

Again, *March* 22. 1659. 10$^h$ 45$'$. the *Satellite* was a whole Diameter above the Line of the *Anſæ*, and the Perpendicular thereon fell nearly upon the Extremity of the Eaſtern *Anſa*.

By the Firſt of my Obſervations it appears that the *Satellite* was in *Perigæo* 1682. *Nov.* 13. 17$^h$ 00$'$. *circiter*, at which time *Saturn* was 30$^s$ 29$^o$ 39$'$ from the firſt Star of *Aries* in the *Ecliptick*, but the *Earth* reduced to *Saturn's Equinoctial*, and the *Satellite* was 9$^s$ 23$^o$ 46$'$. a 1$^a$ ✱ ♈. And *Marh* 4. 1659, 7$^h$ 40$'$. *Saturn*'s Place in the *Ecliptick* was 6s 00 41$'$. but the *Earth* reduced, and conſequently the *Satellite*, in 11$^s$ 28$^o$ 18$'$. *à Prima Stella Arietis*. The Interval of Time is 8655 Days, 9$^h$ 20$'$; in which the *Satellite* had made a certain Number of Revolutions to the Fixt Stars, and beſides 9$^s$ 25$^o$ 28$'$. or 295 Degrees 28$'$, whoſe Complement to a Circle 64$^o$ 32$'$ is 2 Days 20$^h$ 36$'$ Motion of the *Satellite*, according to *Hugens*. So that 8658 Days 5$^h$ 56$'$, or 12467876 Minutes of Time, is the time of ſome Number of intire Revolutions; and dividing that Interval by 15 Days 22$^h$ 39$'$, or 22959$'$ (the Period of *Hugens*) the Quotient 543 ſhews the Number of Revolutions; and again dividing 12467876$'$ by 543, the Quotient 22961$\frac{1}{10}$ *min.* or 15 days, 22$^h$ 41$'$ 6$''$. appears to be the true Time of this *Satellit's Period*. Hence the Diurnal Motion will be 22$^o$ 34$'$ 38$''$ 18$'''$, and the Annual beſides 22 Revolutions 10$^s$ 20$^o$ 43$'$. Having made Tables to this Period, I found that in the *Apogæon* Obſervation of *Hugens*, the *Satellite* was above 3 Degrees faſter than by my *Calculus*, and that in the three other Obſervations of my own, being likewiſe in the Superior part, it was 2$\frac{1}{2}$ *Deg.* ſlower than by the ſame Calculation. Now 'tis evident that theſe Differences muſt ariſe from ſome *Eccentricity* in the *Orbite* of this *Satellite*, and that in *Mar.* 1659, the *Apocronion* ( as I may call it ) was ſomewhere in the Oriental Semicircle, and that in *Nov.* 1682. it was in the Weſtern Semicircle, and ſuppoſing the *Apocronion* fixt, it muſt neceſſarily be between 9ˢ 23° 46′, and 11ˢ 28° 18′, a 1ª ✳ ♈; that being the common Part between thoſe two Semicircles; and becauſe the Difference was greater in *Hugens*'s Obſervation than in Mine, 'twill follow that the *Linea Apſidum*, or *Apocronion*, ſhould be nearer to 9ˢ 23° 46′, than to 11ˢ 28° 18′. I will ſuppoſe 10ˢ 22° 00′ *à Prima Stella Arietis*, ( which happens to be alſo the Place of *Saturn's Equinox* ) and the greateſt Equation about $2\frac{1}{2}$ *Degrees*. Upon the Score of this Inequality the mean Motion of the *Satellite* will be found about 2° 45′. ſlower in $23\frac{1}{2}$ Years, or 7 Minutes in a Year, whence I ſtate the Annual Motion 10ˢ 20° 36′. above 22 Revolutions, and the Correct *Epocha* for the laſt Day of *December* 1682 at Noon in the Meridian of *London* 9ˢ 10° 15′ à 1ª ✳ ♈; from which *Elements* I compoſe the following Table.

*Tabula Motus Medii* Satellitis Saturnii *ab* Hugenio *inventi*, à Prima ✱ ♈.

| *Anno* Christi *Curr.* | Epochæ s. | ° | ' |
|---|---|---|---|
| 1641 | 8 | 29 | 17 |
| 1661 | 10 | 14 | 10 |
| 1681 | 11 | 29 | 3 |
| 1682 | 10 | 19 | 39 |
| 1683 | 9 | 10 | 15 |
| 1684 | 8 | 00 | 51 |
| 1685 | 7 | 14 | 2 |

| *Mens. Ann. Com.* | *Mot. Med.* s. | ° | ' |
|---|---|---|---|
| *Jan.* | 0 | 0 | 0 |
| *Feb.* | 11 | 9 | 53 |
| *Mar.* | 8 | 12 | 2 |
| *April.* | 7 | 21 | 56 |
| *Maii* | 6 | 9 | 14 |
| *Jun.* | 5 | 19 | 7 |
| *Jul.* | 4 | 6 | 26 |
| *Aug.* | 3 | 16 | 19 |
| *Sept.* | 2 | 26 | 12 |
| *Oct.* | 1 | 13 | 31 |
| *Nov.* | 0 | 23 | 24 |
| *Dec.* | 11 | 10 | 43 |

| *Annis.* | *Mot. Med.* s. | ° | ' |
|---|---|---|---|
| 1 | 10 | 20 | 36 |
| 2 | 9 | 11 | 12 |
| 3 | 8 | 1 | 48 |
| 4 | 7 | 14 | 59 |
| 5 | 6 | 5 | 35 |
| 6 | 4 | 26 | 11 |
| 7 | 3 | 16 | 47 |
| 8 | 2 | 26 | 57 |
| 9 | 1 | 20 | 23 |
| 10 | 0 | 11 | 9 |
| 11 | 11 | 1 | 45 |
| 12 | 10 | 14 | 56 |
| 13 | 9 | 5 | 32 |
| 14 | 7 | 26 | 8 |
| 15 | 6 | 16 | 44 |
| 16 | 5 | 29 | 54 |
| 17 | 4 | 20 | 30 |
| 18 | 3 | 11 | 6 |
| 19 | 2 | 1 | 42 |
| 20 | 1 | 14 | 53 |

| *Diebus.* | *Mot. Med.* s. | ° | ' |
|---|---|---|---|
| 1 | 0 | 22 | 35 |
| 2 | 1 | 15 | 9 |
| 3 | 2 | 7 | 44 |
| 4 | 3 | 0 | 18 |
| 5 | 3 | 22 | 53 |
| 6 | 4 | 15 | 28 |
| 7 | 5 | 8 | 2 |
| 8 | 6 | 0 | 37 |
| 9 | 6 | 23 | 12 |
| 10 | 7 | 15 | 46 |
| 11 | 8 | 8 | 21 |
| 12 | 9 | 0 | 55 |
| 13 | 9 | 23 | 30 |
| 14 | 10 | 16 | 5 |
| 15 | 11 | 8 | 39 |
| 16 | 0 | 1 | 14 |
| 17 | 0 | 23 | 48 |
| 18 | 1 | 16 | 23 |
| 19 | 2 | 8 | 58 |
| 20 | 3 | 1 | 32 |
| 21 | 3 | 24 | 7 |
| 22 | 4 | 16 | 42 |
| 23 | 5 | 9 | 16 |
| 24 | 6 | 1 | 51 |
| 25 | 6 | 24 | 25 |
| 26 | 7 | 17 | 00 |
| 27 | 8 | 9 | 35 |
| 28 | 9 | 2 | 9 |
| 29 | 9 | 24 | 44 |
| 30 | 10 | 17 | 18 |
| 31 | 11 | 9 | 53 |
| 32 | 0 | 2 | 28 |

| *Hor.Min* | *Mot.Med* ° / ' | ' / '' |
|---|---|---|
| 1 | 0 | 56 |
| 2 | 1 | 53 |
| 3 | 2 | 49 |
| 4 | 3 | 46 |
| 5 | 4 | 42 |
| 6 | 5 | 39 |
| 7 | 6 | 35 |
| 8 | 7 | 32 |
| 9 | 8 | 28 |
| 10 | 9 | 24 |
| 11 | 10 | 21 |
| 12 | 11 | 17 |
| 13 | 12 | 14 |
| 14 | 13 | 10 |
| 15 | 14 | 7 |
| 16 | 15 | 3 |
| 17 | 16 | 0 |
| 18 | 16 | 56 |
| 19 | 17 | 52 |
| 20 | 18 | 49 |
| 21 | 19 | 45 |
| 22 | 20 | 42 |
| 23 | 21 | 38 |
| 24 | 22 | 35 |
| 25 | 23 | 31 |
| 26 | 24 | 27 |
| 27 | 25 | 24 |
| 28 | 26 | 20 |
| 29 | 27 | 17 |
| 30 | 28 | 13 |

| *Min.* | *Mot.Me.* ' | '' |
|---|---|---|
| 31 | 29 | 10 |
| 32 | 30 | 6 |
| 33 | 31 | 3 |
| 34 | 31 | 59 |
| 35 | 32 | 55 |
| 36 | 33 | 52 |
| 37 | 34 | 48 |
| 38 | 35 | 45 |
| 39 | 36 | 41 |
| 40 | 37 | 38 |
| 41 | 38 | 34 |
| 42 | 39 | 31 |
| 43 | 40 | 27 |
| 44 | 41 | 24 |
| 45 | 42 | 20 |
| 46 | 43 | 17 |
| 47 | 44 | 13 |
| 48 | 45 | 10 |
| 49 | 46 | 6 |
| 50 | 47 | 3 |
| 51 | 47 | 59 |
| 52 | 48 | 56 |
| 53 | 49 | 52 |
| 54 | 50 | 49 |
| 55 | 51 | 45 |
| 56 | 52 | 42 |
| 57 | 53 | 38 |
| 58 | 54 | 35 |
| 59 | 55 | 31 |
| 60 | 56 | 27 |

In *Anno Bissextili* post *Februarium* adde unum diem, motumque ei competentem.

There

I here ſuppoſe the *Linea Apſidum* fixt, as having no Argument from Obſervation to prove the contrary, though it be very probable that as the *Apogæon* of our *Moon* has a Motion about the *Earth* in about 9 Years, ſo that of this *Satellite* ought to have about *Saturn*, but with a much Longer Period, which future Obſervation may diſcover.

The diſtance of this *Satellite* from the Center of *Saturn* ſeems to be much about 4 Diameters of the *Ring*, or 9 of the *Globe*, and the Plane wherein it moves very little or nothing differing from that of the *Ring*, that is to ſay interſecting the Orb of *Saturn* $4^s\ 22^\circ$ and $10^s\ 22^\circ$ *a* $1^a$ ⁎ ♈, with an Angle of $23\frac{1}{2}$ Degrees, ſo as to be nearly Parallel to the *Earth's Equator*, whence the *Latitude* of the *Apogæon* Semicircle from $4^s\ 22^\circ$ to $10^s\ 22^\circ$ of *Saturn's Longitude* from the Firſt Star of ♈, will be Northern, and of the other Semicircle Southern; and the contrary in the other half of *Saturn's Longitude*, to wit, from $10^s\ 22^\circ$ to $4^s\ 22^\circ$ of his diſtance from the Firſt Star of ♈.

It follows now to ſhew how by the help of this Table to compute the place of this *Satellite*, to any time required.

Firſt we muſt have the true *Longitude* of *Saturn* from the *Earth*, and numbred from the Firſt Star of ♈, (or rather the Place of the Earth viewed from *Saturn* together with its Latitude from the Orb of *Saturn*, but that being never fully $\frac{1}{3}$ of a degree we neglect it as a Nicety) and therefrom ſubſtract $10^s\ 22^\circ$ there remains the diſtance of *Saturn* from this *Equinoctial* Point, with which diſtance as with the *Longitude* of the *Sun*, take out the *Right Aſcenſion* and *Declination* thereto ($23\frac{1}{2}$ degrees being the Obliquity common to both) and to the *Right Aſcenſion* adding $10^s\ 22^\circ$ the Summ ſhall be the *Longitude* of the *Satellit's Apogæon*. Then ſay, as *Radius* to *Sine* of the *Declination*, ſo 8 to the greateſt *Latitude* in *Apogæo*, or *Perigæo* in the parts of the Semidiameter of the *Ring*.

Next Collect the *Middle Motion* of the *Satellite*, and from it Subſtract $10^s\ 22^\circ$ the remainder ſhall be the mean Anomaly, with which in the Table of the *Moon's* Primary *Equation*, take out the *Equation* anſwering thereto, and the half thereof added or ſubſtracted to or from the *Middle Motion*, according to the Table, gives the True Motion of the *Satellite*, from which ſubſtract the *Apogæon*, and if the remainder be more than 6 *Signs*, the *Satellite* is Occidental, if leſs Oriental; and as *Radius* to Sine of the remainder, ſo 8 to the Semidiameters of the *Ring*, or 18 to the Semidiameters of the *Globe*, that the *Satellite* is to the Eaſtward or Weſtward of the Center of *Saturn*, according to the aforegoing Precept.

Laſtly, as *Radius* to *Co-ſine* of the ſaid Remainder, ſo is the greateſt *Latitude* from the Line of the *Anſæ*, to the *Latitude* ſought.

Here Note, that I purpoſely neglect the Inequality of the diſtance ariſing from the *Eccentricity*, as being too ſmall to be any way obſervable.

Laſtly to clear all difficulties that may ariſe to them that are but little verſed in this ſort of Calculation, I have added an Example of the Work, that where the precept may ſeem obſcure it may be thereby illuſtrated.

*An.* 1657. *Maii* 19. *st. n.* M. *Hugens* obſerved the *Satellite* very near to *Sa-*
*urn* on the Weſtern ſide, and very little above the Line of the *Anſæ*. I
uppoſe this about 10$^h$ *p. m.* Let us now *Calculate* to that time.

| | | | |
|---|---|---|---|
| 1657 *Maii* | 9$^d$ 9$^h$ 10' *Londini* | | |
| *Saturni Locus* | ♍ | 28° | 57' |
| ♄ *a* 1*a* ✱ ♈. | 5$^s$ | 0 | 32 |
| *Equinoct. Sub.* | 10 | 22 | 00 |
| ♄ *ab Æquinoct.* | 6 | 8 | 32 |
| *Aſcen. Recta* | 6 | 7 | 50 |
| *Apogæon.* | 4 | 29 | 50 |
| *Declin. Auſt.* | | 3 | 23 |

| *Mot. Med. Satel.* | s. | ° | ' |
|---|---|---|---|
| 1641 | 8 | 29 | 17 |
| 16 | 5 | 29 | 54 |
| *Maii* | 6 | 9 | 14 |
| 9$^d$ | 6 | 23 | 12 |
| 9$^h$ 40' | 0 | 9 | 5 |
| *Long. Med. Satel.* | 4 | 10 | 42 |
| *Apocron.* | 10 | 22 | 00 |
| *Anomalia Med.* | 5 | 18 | 42 |
| *Æquatio Sub.* | 0 | 0 | 31 |
| *Long. Vera Satel.* | 4 | 10 | 11 |
| *Apogæon* | 4 | 29 | 50 |
| *Reſiduum* | 11 | 10 | 21 |
| h. e. ante *Apogæum.* | 0 | 19 | 39 |

*Ergo* $2\frac{7}{10}$ *Semid. Annuli ad occaſum* & $\frac{9}{20}$ *ad Boream*; Agreeing exactly ith the Deſcription and Figure of *M. Hugens*.

I here call the Plane of this *Satellit's* Orb, which hitherto I ſuppoſe the ame with that of the *Ring*, *Saturn's Equinoctial*, not that any diſcovery hath een able to prove that the *Axis* of that *Globe* is at right Angles thereto, but ecauſe it hath pleaſed M. *Hugens* to call it ſo, and likewiſe becauſe it is ſo early Parallel to our *Globe's Equinoctial*; Nevertheleſs to ſpeak my Opinion, believe that the *Axis* is Inclined, and that not a little, to the Plane of the *ing*; for as the Reflection of the *Sun's* Light from the *Ring* is a great con- enience to that Hemiſphere of *Saturn*, which beholds its Illuminate ſide; o that the other Hemiſphere is very much Incommoded by the Shaddow of he *Ring*, which for many Months, and in ſome Parallels for ſeveral Years, oc- aſions a continual Night by the Interception of the *Sun's* beams, which is a onſequence that Demonſtratively follows the Poſition of the *Ring* in the lane of *Saturn's Equator*. Now this great inconvenience would be in ſome neaſure relieved by the Oblique Poſition of the *Axis*, for then the Parallels f Latitude interſecting the Plane of the *Ring*, many and in moſt caſes all f them, might for ſome time in every Diurnal Revolution of the Globe, free hemſelves from this *Eclipſe*, which otherwiſe were ſufficient to render his Globe of *Saturn* unfit for any ſetled Habitation; but this is but con- ecture.

The

The other two *Satellites* of *Saturn* discovered by S. *Caſſini* at *Paris An.* 1672. and 1673, I muſt confeſs I could never yet ſee; I have been told that they Diſappear for about $3\frac{2}{3}$ of *Saturn's* Revolution, and were only to be ſeen when the *Anſæ* were very ſmall, it being ſuppoſed that the Light which proceeds from the *Anſæ* when conſiderably opened might hide theſe *Satellites*.

*The Theory of the 5 Satellites of Saturn corrected; by* M. Caſſini. N. 187. p. 299.

LXXXII. 1. La *Diſtance* du *Premier Satellite* du Centre de *Saturne* m' a paru variable, & ſon *Mouvement* ſenſiblement Inegal, plus viſte, en ce temps, dans le Demicercle Occidental, que dans l' Oriental. J' ay diernierement determiné ſa Moyenne Diſtance de $\frac{32}{40}$ du Diametre de l' *Anneau* de *Saturne*, ſon *Mouvement* Journalier de 6s 10° 41′ 31″. Ainſi ſi ſon *Mouvement* eſtoit Egal, la durée de ſa Conjonction avec *Saturne*, c' eſt a dire, tout le temps qu' il met a parcourir ſon *Anneau*, ſeroit de 7h 46′. Elle m' a paru plus grande par les obſervations immediates, mais il eſt a remarquer que je n'ay juſqu'a preſent pû voir ce *Satellite* plus pres de *Saturne*, que d' un quart d' un *Anſe*.

J' ay Calculé l' *Epoque* de ſon *Mouvement*, pour le dernier *Decembre* 1685 a Midi au Meridien de *Paris* en ♑ 24° 50′.

La *Diſtance* du *Second Satellite* du Centre de *Saturne* m' a paru plus Uniforme. Je l' ay determinée d' un Diametre de l' *Anneau* & $\frac{1}{4}$ Son *Mouvement* paroit auſſi plus Egal. J' ay Calculé le Journalier de 4s 11° 31′ 30″. Ainſi la Durée de ſa Conjonction deuroit eſtre de 8h 36′. Je n'ay pas non plus vû juſqu' a preſent ce *Satellite* plus proche de l' *Anneau* de *Saturne* que d' $\frac{1}{4}$ d' un *Anſe*. Comme ce *Satellite* ſe voioit la plus part du temps dedans les Confins de la *Diſtance* du *Premier*, au quel il eſt Egal en Grandeur, & Semblable dans la Coleur, la difficulté de diſtinguer l' un de l' autre a eſté extreme, de ſort que ſans un aſſiduité particuliere aux Obſervations, & ſans une grande multitude de Combinaiſons je n'en ſerois pas venu a bout.

J' ay determiné l' *Epoque* de ce *Satellite* pour le 31 *Decembre* 1685 a Midi en ♍ 9° 10′.

La *Diſtance* du *Troiſieme* du Centre de *Saturne* paroit d' un Diametre de l' *Anneau* & $\frac{1}{4}$. Son *Mouvement Journalier* 2s 18° 41′ 50″. Ainſi ſa Conjonction doit durer 10 heures. L' *Epoque* de ſon *Mouvement* pour le Midi du Dernier del' Année 1685 ♍ 9° 39′.

La *Diſtance* du Quatrieme *Satellite* au Centre de *Saturne* paroit de 4 Diametres de l' *Anneau*. Son *Mouvement Journalier* de 22° 34′ 38″. la Duree de ſa Conjonction 15h 6′. L' *Epoque* de ſon *Mouvement* au meſme temps & lieu que les autres, en ♓ 18° 1′.

La *Diſtance* du *Cinquieſme Satellite* au Centre de *Saturne* de 12 Diametres de l' *Anneau*. Son *Mouvement Journalier* de 4° 32′ 17″. Ses Conjonctions durent 24 heures. L' *Epoque* de ſon *Mouvement* au meſme temps & lieu, en ♓ 16° 19′. Sur ces Principes on peut conſtruire les *Tables*, & les *Ephemerides*.

*By* [illegible] *ib.* p. 300.

2. The Following Tables are Calculated from theſe Elements, and Reduc'd to the *Meridian* of *London*.

Tabula

*Tabula Motus Medii* Intimi Satellitis Saturni, *à* Caſſino *Detecti Anno* 1686.

| Anno Chriſti Curr. | Epochæ | ° | ′ |
|---|---|---|---|
| 1681 | ♒ | 19 | 34 |
| 1685 | ♑ | 10 | 30 |
| 1686 | ♉ | 13 | 4 |
| 1687 | ♍ | 15 | 37 |
| 1688 | ♑ | 18 | 11 |
| 1689 | ♐ | 1 | 26 |
| 1701 | ♌ | 4 | 14 |

| *Menſ. Anni Com.* | Mot. Med. s. | ° | ′ |
|---|---|---|---|
| *Jan.* | 0 | 0 | 0 |
| *Feb.* | 5 | 1 | 27 |
| *Mar.* | 3 | 0 | 49 |
| *April.* | 8 | 2 | 16 |
| *Maii* | 6 | 23 | 2 |
| *Jun.* | 11 | 24 | 29 |
| *Jul.* | 10 | 15 | 15 |
| *Aug.* | 3 | 16 | 42 |
| *Sept.* | 8 | 18 | 9 |
| *Oct.* | 7 | 8 | 54 |
| *Nov.* | 0 | 10 | 21 |
| *Dec.* | 11 | 1 | 7 |

| *Annis.* | Mot. Med. s. | ° | ′ |
|---|---|---|---|
| 1 | 4 | 2 | 34 |
| 2 | 8 | 5 | 7 |
| 3 | 0 | 7 | 41 |
| 4 | 10 | 20 | 56 |
| 5 | 2 | 23 | 30 |
| 6 | 6 | 26 | 4 |
| 7 | 10 | 28 | 38 |
| 8 | 9 | 11 | 52 |
| 9 | 1 | 14 | 26 |
| 10 | 5 | 17 | 00 |
| 11 | 9 | 19 | 34 |
| 12 | 8 | 2 | 48 |
| 13 | 0 | 5 | 22 |
| 14 | 4 | 7 | 56 |
| 15 | 8 | 10 | 29 |
| 16 | 6 | 23 | 43 |
| 17 | 10 | 26 | 17 |
| 18 | 2 | 28 | 51 |
| 19 | 7 | 1 | 25 |
| 20 | 5 | 14 | 39 |

| *Diebus.* | Mot. Med. s. | ° | ′ | *Hor. Mi.* | Mot. Med. ° | ′ | ″ | *Min.* | Mot. Me. ° | ′ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 6 | 10 | 42 | 1 | 0 | 7 | 57 | 31 | 4 | 6 |
| 2 | 0 | 21 | 23 | 2 | 0 | 15 | 53 | 32 | 4 | 14 |
| 3 | 7 | 2 | 5 | 3 | 0 | 23 | 50 | 33 | 4 | 22 |
| 4 | 1 | 12 | 46 | 4 | 0 | 31 | 47 | 34 | 4 | 30 |
| 5 | 7 | 23 | 28 | 5 | 0 | 39 | 44 | 35 | 4 | 38 |
| 6 | 2 | 4 | 9 | 6 | 0 | 47 | 40 | 36 | 4 | 46 |
| 7 | 8 | 14 | 50 | 7 | 0 | 55 | 37 | 37 | 4 | 54 |
| 8 | 2 | 25 | 32 | 8 | 1 | 3 | 34 | 38 | 5 | 2 |
| 9 | 9 | 6 | 14 | 9 | 1 | 11 | 31 | 39 | 5 | 10 |
| 10 | 3 | 16 | 55 | 10 | 1 | 19 | 28 | 40 | 5 | 18 |
| 11 | 9 | 27 | 36 | 11 | 1 | 27 | 24 | 41 | 5 | 26 |
| 12 | 4 | 8 | 18 | 12 | 1 | 35 | 21 | 42 | 5 | 34 |
| 13 | 10 | 19 | 00 | 13 | 1 | 43 | 18 | 43 | 5 | 42 |
| 14 | 4 | 29 | 42 | 14 | 1 | 51 | 15 | 44 | 5 | 50 |
| 15 | 11 | 10 | 23 | 15 | 1 | 59 | 11 | 45 | 5 | 58 |
| 16 | 5 | 21 | 4 | 16 | 2 | 7 | 8 | 46 | 6 | 5 |
| 17 | 0 | 1 | 46 | 17 | 2 | 15 | 5 | 47 | 6 | 13 |
| 18 | 6 | 12 | 28 | 18 | 2 | 23 | 1 | 48 | 6 | 21 |
| 19 | 0 | 23 | 9 | 19 | 2 | 30 | 58 | 49 | 6 | 29 |
| 20 | 7 | 3 | 50 | 20 | 2 | 38 | 55 | 50 | 6 | 37 |
| 21 | 1 | 14 | 32 | 21 | 2 | 46 | 52 | 51 | 6 | 45 |
| 22 | 7 | 25 | 13 | 22 | 2 | 54 | 49 | 52 | 6 | 53 |
| 23 | 2 | 5 | 55 | 23 | 3 | 2 | 45 | 53 | 7 | 1 |
| 24 | 8 | 16 | 36 | 24 | 3 | 10 | 42 | 54 | 7 | 9 |
| 25 | 2 | 27 | 18 | 25 | 3 | 18 | 39 | 55 | 7 | 17 |
| 26 | 9 | 7 | 59 | 26 | 3 | 26 | 35 | 56 | 7 | 25 |
| 27 | 3 | 18 | 41 | 27 | 3 | 34 | 32 | 57 | 7 | 33 |
| 28 | 9 | 29 | 22 | 28 | 3 | 42 | 28 | 58 | 7 | 41 |
| 29 | 4 | 10 | 3 | 29 | 3 | 50 | 25 | 59 | 7 | 49 |
| 30 | 10 | 20 | 45 | 30 | 3 | 58 | 22 | 60 | 7 | 57 |

In *Anno Biſſextili* poſt *Februarium* adde unum diem, motumque ei competentem.

*Tabula Motus Medii* Penintimi Satellitis Saturni, *à* Caſſino *Detecti Anno* 1686.

| *Anno* Chriſti *Curr.* | *Epochæ* | ° | ′ |
|---|---|---|---|
| 1681 | ♓ | 20 | 41 |
| 1685 | ♐ | 28 | 42 |
| 1686 | ♉ | 5 | 20 |
| 1687 | ♍ | 11 | 57 |
| 1688 | ♑ | 18 | 35 |
| 1689 | ♎ | 6 | 44 |
| 1701 | ♒ | 0 | 48 |

| *Menſ. Ann. Com.* | *Mot. Med.* s. | ° | ′ |
|---|---|---|---|
| *Jan.* | 0 | 0 | 0 |
| *Feb.* | 3 | 27 | 16 |
| *Mar.* | 6 | 19 | 58 |
| *April.* | 10 | 17 | 15 |
| *Maii* | 10 | 3 | 0 |
| *Jun.* | 2 | 0 | 16 |
| *Jul.* | 1 | 16 | 1 |
| *Aug.* | 5 | 13 | 18 |
| *Sept.* | 9 | 10 | 34 |
| *Oct.* | 8 | 26 | 19 |
| *Nov.* | 0 | 23 | 36 |
| *Dec.* | 0 | 9 | 21 |

| *Annis.* | *Mot. Med.* s. | ° | ′ |
|---|---|---|---|
| 1 | 4 | 6 | 37 |
| 2 | 8 | 13 | 15 |
| 3 | 0 | 19 | 52 |
| 4 | 9 | 8 | 1 |
| 5 | 1 | 14 | 39 |
| 6 | 5 | 21 | 16 |
| 7 | 9 | 27 | 54 |
| 8 | 6 | 16 | 3 |
| 9 | 10 | 22 | 40 |
| 10 | 2 | 29 | 18 |
| 11 | 7 | 5 | 55 |
| 12 | 3 | 24 | 4 |
| 13 | 8 | 00 | 42 |
| 14 | 0 | 7 | 19 |
| 15 | 4 | 13 | 57 |
| 16 | 1 | 2 | 6 |
| 17 | 5 | 8 | 43 |
| 18 | 9 | 15 | 21 |
| 19 | 1 | 21 | 58 |
| 20 | 10 | 10 | 7 |

| *Diebus.* | *Mot. Med.* s. | ° | ′ |
|---|---|---|---|
| 1 | 4 | 11 | 31 |
| 2 | 8 | 23 | 3 |
| 3 | 1 | 4 | 34 |
| 4 | 5 | 16 | 6 |
| 5 | 9 | 27 | 37 |
| 6 | 2 | 9 | 9 |
| 7 | 6 | 20 | 40 |
| 8 | 11 | 2 | 12 |
| 9 | 3 | 13 | 43 |
| 10 | 7 | 25 | 15 |
| 11 | 0 | 6 | 46 |
| 12 | 4 | 18 | 18 |
| 13 | 8 | 29 | 49 |
| 14 | 1 | 11 | 21 |
| 15 | 5 | 22 | 52 |
| 16 | 10 | 4 | 24 |
| 17 | 2 | 15 | 55 |
| 18 | 6 | 27 | 27 |
| 19 | 11 | 8 | 58 |
| 20 | 3 | 20 | 30 |
| 21 | 8 | 2 | 1 |
| 22 | 0 | 13 | 33 |
| 23 | 4 | 25 | 4 |
| 24 | 9 | 6 | 36 |
| 25 | 1 | 18 | 07 |
| 26 | 5 | 29 | 39 |
| 27 | 10 | 11 | 10 |
| 28 | 2 | 22 | 42 |
| 29 | 7 | 4 | 13 |
| 30 | 11 | 15 | 45 |

| *Hor.Min* | *Mot. Med Sex.* ° | ′ | ″ |
|---|---|---|---|
| 1 | 0 | 5 | 29 |
| 2 | 0 | 10 | 58 |
| 3 | 0 | 16 | 26 |
| 4 | 0 | 21 | 55 |
| 5 | 0 | 27 | 24 |
| 6 | 0 | 31 | 53 |
| 7 | 0 | 38 | 22 |
| 8 | 0 | 43 | 51 |
| 9 | 0 | 49 | 19 |
| 10 | 0 | 54 | 48 |
| 11 | 1 | 00 | 17 |
| 12 | 1 | 5 | 46 |
| 13 | 1 | 11 | 15 |
| 14 | 1 | 16 | 44 |
| 15 | 1 | 22 | 12 |
| 16 | 1 | 27 | 42 |
| 17 | 1 | 33 | 11 |
| 18 | 1 | 38 | 39 |
| 19 | 1 | 44 | 8 |
| 20 | 1 | 49 | 37 |
| 21 | 1 | 55 | 6 |
| 22 | 2 | 00 | 34 |
| 23 | 2 | 6 | 3 |
| 24 | 2 | 11 | 31 |
| 25 | 2 | 17 | 00 |
| 26 | 2 | 22 | 29 |
| 27 | 2 | 27 | 58 |
| 28 | 2 | 33 | 26 |
| 29 | 2 | 38 | 55 |
| 30 | 2 | 44 | 24 |

| *Min.* | *Mot.Me.* ° | ′ |
|---|---|---|
| 31 | 2 | 50 |
| 32 | 2 | 56 |
| 33 | 3 | 1 |
| 34 | 3 | 7 |
| 35 | 3 | 12 |
| 36 | 3 | 17 |
| 37 | 3 | 23 |
| 38 | 3 | 28 |
| 39 | 3 | 34 |
| 40 | 3 | 40 |
| 41 | 3 | 45 |
| 42 | 3 | 50 |
| 43 | 3 | 56 |
| 44 | 4 | 1 |
| 45 | 4 | 7 |
| 46 | 4 | 12 |
| 47 | 4 | 17 |
| 48 | 4 | 23 |
| 49 | 4 | 28 |
| 50 | 4 | 34 |
| 51 | 4 | 39 |
| 52 | 4 | 45 |
| 53 | 4 | 50 |
| 54 | 4 | 56 |
| 55 | 5 | 1 |
| 56 | 5 | 7 |
| 57 | 5 | 12 |
| 58 | 5 | 18 |
| 59 | 5 | 23 |
| 60 | 5 | 29 |

In *Anno Biſſextili* poſt *Februarium* adde unum diem, motumque ei competentem.

*Tabula Motus Medii* Satellitis Saturnii Medii, à Cassino *detecti Anno* 1673.

| Anno Christi Curr. | Epochæ | ° | ′ |
|---|---|---|---|
| 1661 | ♎ | 22 | 50 |
| 1681 | ♍ | 16 | 3 |
| 1685 | ♒ | 2 | 41 |
| 1686 | ♏ | 17 | 10 |
| 1687 | ♍ | 1 | 39 |
| 1688 | ♊ | 16 | 9 |
| 1689 | ♊ | 19 | 20 |
| 1701 | ♌ | 9 | 15 |

| Mens. Anni Com. | Mot. Med. s. | ° | ′ |
|---|---|---|---|
| Jan. | 0 | 0 | 0 |
| Febr. | 9 | 9 | 37 |
| Mar. | 10 | 23 | 8 |
| April. | 8 | 2 | 45 |
| Maii | 2 | 23 | 40 |
| Jun. | 0 | 3 | 17 |
| Jul. | 6 | 24 | 12 |
| Aug. | 4 | 3 | 49 |
| Sept. | 1 | 13 | 25 |
| Oct. | 8 | 4 | 20 |
| Nov. | 5 | 13 | 57 |
| Dec. | 0 | 4 | 52 |

| Annis | Mot. Med. s. | ° | ′ |
|---|---|---|---|
| 1 | 9 | 14 | 29 |
| 2 | 6 | 28 | 58 |
| 3 | 4 | 13 | 27 |
| 4 | 4 | 16 | 38 |
| 5 | 2 | 1 | 8 |
| 6 | 11 | 15 | 37 |
| 7 | 9 | 00 | 6 |
| 8 | 9 | 3 | 17 |
| 9 | 6 | 17 | 46 |
| 10 | 4 | 02 | 15 |
| 11 | 1 | 16 | 45 |
| 12 | 1 | 19 | 55 |
| 13 | 11 | 4 | 24 |
| 14 | 8 | 18 | 54 |
| 15 | 6 | 3 | 23 |
| 16 | 6 | 6 | 34 |
| 17 | 3 | 21 | 3 |
| 18 | 1 | 5 | 32 |
| 19 | 10 | 20 | 1 |
| 20 | 10 | 23 | 12 |

| Diebus | Mot. Med. s. | ° | ′ | Hor. Mi. | Mot. Med. Sex. ° | ′ | ″ | Min. | Motus Med. ° | ′ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 18 | 42 | 1 | 0 | 3 | 17 | 31 | 1 | 41 |
| 2 | 5 | 7 | 24 | 2 | 0 | 6 | 33 | 32 | 1 | 45 |
| 3 | 7 | 26 | 5 | 3 | 0 | 9 | 50 | 33 | 1 | 48 |
| 4 | 10 | 14 | 47 | 4 | 0 | 13 | 7 | 34 | 1 | 52 |
| 5 | 1 | 3 | 29 | 5 | 0 | 16 | 24 | 35 | 1 | 55 |
| 6 | 3 | 22 | 11 | 6 | 0 | 19 | 40 | 36 | 1 | 58 |
| 7 | 6 | 10 | 53 | 7 | 0 | 22 | 57 | 37 | 2 | 1 |
| 8 | 8 | 29 | 35 | 8 | 0 | 26 | 14 | 38 | 2 | 5 |
| 9 | 11 | 18 | 16 | 9 | 0 | 29 | 31 | 39 | 2 | 8 |
| 10 | 2 | 6 | 58 | 10 | 0 | 32 | 47 | 40 | 2 | 11 |
| 11 | 4 | 25 | 40 | 11 | 0 | 36 | 4 | 41 | 2 | 14 |
| 12 | 7 | 14 | 22 | 12 | 0 | 39 | 21 | 42 | 2 | 18 |
| 13 | 10 | 3 | 4 | 13 | 0 | 42 | 38 | 43 | 2 | 21 |
| 14 | 0 | 21 | 46 | 14 | 0 | 45 | 55 | 44 | 2 | 24 |
| 15 | 3 | 10 | 27 | 15 | 0 | 49 | 11 | 45 | 2 | 28 |
| 16 | 5 | 29 | 9 | 16 | 0 | 52 | 28 | 46 | 2 | 31 |
| 17 | 8 | 17 | 51 | 17 | 0 | 55 | 45 | 47 | 2 | 34 |
| 18 | 11 | 6 | 33 | 18 | 0 | 59 | 1 | 48 | 2 | 37 |
| 19 | 1 | 25 | 15 | 19 | 1 | 2 | 18 | 49 | 2 | 40 |
| 20 | 4 | 13 | 57 | 20 | 1 | 5 | 35 | 50 | 2 | 44 |
| 21 | 7 | 2 | 39 | 21 | 1 | 8 | 52 | 51 | 2 | 47 |
| 22 | 9 | 21 | 20 | 22 | 1 | 12 | 8 | 52 | 2 | 50 |
| 23 | 0 | 10 | 2 | 23 | 1 | 15 | 25 | 53 | 2 | 54 |
| 24 | 2 | 28 | 44 | 24 | 1 | 18 | 42 | 54 | 2 | 57 |
| 25 | 5 | 17 | 26 | 25 | 1 | 21 | 59 | 55 | 3 | 00 |
| 26 | 8 | 6 | 8 | 26 | 1 | 25 | 15 | 56 | 3 | 4 |
| 27 | 10 | 24 | 50 | 27 | 1 | 28 | 32 | 57 | 3 | 7 |
| 28 | 1 | 13 | 32 | 28 | 1 | 31 | 49 | 58 | 3 | 10 |
| 29 | 4 | 2 | 13 | 29 | 1 | 35 | 6 | 59 | 3 | 13 |
| 30 | 6 | 20 | 55 | 30 | 1 | 38 | 22 | 60 | 3 | 17 |

In *Anno Bissextili* post *Februarium* adde unum diem, Motumque ei competentem.

*Tabula Motus Medii* Penextimi Satellitis Saturni, *ab* Hugenio *inventi Anno* 1673.

| *Anno* Christi *Curr.* | *Epochæ* | ° | ' |
|---|---|---|---|
| 1641 | ♑ | 24 | 43 |
| 1661 | ♓ | 11 | 19 |
| 1681 | ♈ | 27 | 56 |
| 1685 | ♐ | 13 | 15 |
| 1686 | ♏ | 3 | 56 |
| 1687 | ♍ | 24 | 37 |
| 1688 | ♌ | 15 | 19 |
| 1689 | ♋ | 28 | 34 |
| 1701 | ♊ | 14 | 32 |

| *Menſ. Anni Com.* | *Mot. Med.* s. | ° | ' |
|---|---|---|---|
| *Jan.* | 0 | 0 | 0 |
| *Febr.* | 11 | 9 | 54 |
| *Mar.* | 8 | 12 | 3 |
| *April.* | 7 | 21 | 57 |
| *Maii* | 6 | 9 | 16 |
| *Jun.* | 5 | 19 | 10 |
| *Jul.* | 4 | 6 | 29 |
| *Aug.* | 3 | 16 | 22 |
| *Sept.* | 2 | 26 | 16 |
| *Oct.* | 1 | 13 | 35 |
| *Nov.* | 0 | 23 | 29 |
| *Dec.* | 11 | 10 | 48 |

| *Annis* | *Mot. Med.* s. | ° | ' | *Diebus* | *Mot. Med.* s. | ° | ' | *Hor. Mi.* | *Mot. Me.* ° ' | ' " | *Min.* | *Motus Med.* ' | " |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 10 | 20 | 41 | 1 | 0 | 22 | 35 | 1 | 0 | 56 | 31 | 29 | 10 |
| 2 | 9 | 11 | 22 | 2 | 1 | 15 | 9 | 2 | 1 | 53 | 32 | 30 | 6 |
| 3 | 8 | 2 | 3 | 3 | 2 | 7 | 44 | 3 | 2 | 49 | 33 | 31 | 3 |
| 4 | 7 | 15 | 19 | 4 | 3 | 0 | 18 | 4 | 3 | 46 | 34 | 31 | 59 |
| 5 | 6 | 6 | 00 | 5 | 3 | 22 | 53 | 5 | 4 | 42 | 35 | 32 | 55 |
| 6 | 4 | 26 | 41 | 6 | 4 | 15 | 28 | 6 | 5 | 39 | 36 | 33 | 52 |
| 7 | 3 | 17 | 22 | 7 | 5 | 8 | 2 | 7 | 6 | 35 | 37 | 34 | 48 |
| 8 | 3 | 00 | 39 | 8 | 6 | 0 | 37 | 8 | 7 | 32 | 38 | 35 | 45 |
| 9 | 1 | 21 | 20 | 9 | 6 | 23 | 12 | 9 | 8 | 28 | 39 | 36 | 41 |
| 10 | 0 | 12 | 1 | 10 | 7 | 15 | 46 | 10 | 9 | 24 | 40 | 37 | 38 |
| 11 | 11 | 2 | 42 | 11 | 8 | 8 | 21 | 11 | 10 | 21 | 41 | 38 | 34 |
| 12 | 10 | 15 | 58 | 12 | 9 | 0 | 55 | 12 | 11 | 17 | 42 | 39 | 31 |
| 13 | 9 | 6 | 39 | 13 | 9 | 23 | 30 | 13 | 12 | 14 | 43 | 40 | 27 |
| 14 | 7 | 27 | 20 | 14 | 10 | 16 | 5 | 14 | 13 | 10 | 44 | 41 | 24 |
| 15 | 6 | 18 | 1 | 15 | 21 | 8 | 39 | 15 | 14 | 7 | 45 | 42 | 20 |
| 16 | 6 | 1 | 17 | 16 | 0 | 1 | 14 | 16 | 15 | 3 | 46 | 43 | 17 |
| 17 | 4 | 21 | 58 | 17 | 0 | 23 | 48 | 17 | 16 | 0 | 47 | 44 | 13 |
| 18 | 3 | 12 | 40 | 18 | 1 | 16 | 23 | 18 | 16 | 56 | 48 | 45 | 10 |
| 19 | 2 | 3 | 21 | 19 | 2 | 8 | 58 | 19 | 17 | 52 | 49 | 46 | 6 |
| 20 | 1 | 16 | 36 | 20 | 3 | 1 | 32 | 20 | 18 | 49 | 50 | 47 | 3 |
| | | | | 21 | 3 | 24 | 7 | 21 | 19 | 45 | 51 | 47 | 59 |
| | | | | 22 | 4 | 16 | 42 | 22 | 20 | 42 | 52 | 48 | 56 |
| | | | | 23 | 5 | 9 | 16 | 23 | 21 | 38 | 53 | 49 | 52 |
| | | | | 24 | 6 | 1 | 51 | 24 | 22 | 35 | 54 | 50 | 49 |
| | | | | 25 | 6 | 24 | 25 | 25 | 23 | 31 | 55 | 51 | 45 |
| | | | | 26 | 7 | 17 | 00 | 26 | 24 | 27 | 56 | 52 | 42 |
| | | | | 27 | 8 | 9 | 35 | 27 | 25 | 24 | 57 | 53 | 38 |
| | | | | 28 | 9 | 2 | 9 | 28 | 26 | 20 | 58 | 54 | 35 |
| | | | | 29 | 9 | 24 | 44 | 29 | 27 | 17 | 59 | 55 | 31 |
| | | | | 30 | 10 | 17 | 18 | 30 | 28 | 13 | 60 | 56 | 27 |

In *Anno Biſſextili* poſt *Februarium* adde unum diem, Motumque ei competentem.

Tabula Mediorum Motuum *Extimi Satellitis Saturnii*, à *Cassino* detecti Anno 1671.

| *Anno* Christi *Curr.* | *Epochæ* s. | ° | ' |
|---|---|---|---|
| 1661 | ♓ | 24 | 45 |
| 1681 | ♈ | 25 | 15 |
| 1685 | ♍ | 25 | 21 |
| 1686 | ♉ | 1 | 44 |
| 1687 | ♐ | 8 | 7 |
| 1688 | ♋ | 14 | 31 |
| 1689 | ♒ | 25 | 27 |
| 1701 | ♉ | 25 | 45 |

| *Mens. Anni Com.* | *Mot. Med.* s. | ° | ' |
|---|---|---|---|
| *Jan.* | 0 | 0 | 0 |
| *Feb.* | 4 | 20 | 41 |
| *Mar.* | 8 | 27 | 45 |
| *Apr.* | 1 | 18 | 25 |
| *Maii* | 6 | 4 | 34 |
| *Jun.* | 10 | 25 | 15 |
| *Jul.* | 3 | 11 | 23 |
| *Aug.* | 8 | 2 | 4 |
| *Sept.* | 0 | 22 | 45 |
| *Oct.* | 5 | 8 | 53 |
| *Nov.* | 9 | 29 | 34 |
| *Dec.* | 2 | 15 | 43 |

| *Annis* | *Mot. Med.* s. | ° | ' |
|---|---|---|---|
| 1 | 7 | 6 | 23 |
| 2 | 2 | 12 | 47 |
| 3 | 9 | 19 | 10 |
| 4 | 5 | 00 | 6 |
| 5 | 0 | 6 | 29 |
| 6 | 7 | 12 | 53 |
| 7 | 2 | 19 | 16 |
| 8 | 10 | 0 | 12 |
| 9 | 5 | 6 | 35 |
| 10 | 0 | 12 | 59 |
| 11 | 7 | 19 | 22 |
| 12 | 3 | 0 | 18 |
| 13 | 10 | 6 | 41 |
| 14 | 5 | 13 | 5 |
| 15 | 0 | 19 | 28 |
| 16 | 8 | 0 | 24 |
| 17 | 3 | 6 | 47 |
| 18 | 10 | 13 | 11 |
| 19 | 5 | 19 | 34 |
| 20 | 1 | 0 | 30 |

| *Diebus* | *Mot. Med.* s. | ° | ' | *Hor. Mi.* | *Mot. Me.* ° / ' | ' / '' | *Min.* | *Mot. Me.* ' | '' |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 4 | 32 | 1 | 0 | 11 | 31 | 5 | 51 |
| 2 | 0 | 9 | 5 | 2 | 0 | 23 | 32 | 6 | 3 |
| 3 | 0 | 13 | 37 | 3 | 0 | 34 | 33 | 6 | 14 |
| 4 | 0 | 18 | 9 | 4 | 0 | 45 | 34 | 6 | 25 |
| 5 | 0 | 22 | 41 | 5 | 0 | 57 | 35 | 6 | 37 |
| 6 | 0 | 27 | 14 | 6 | 1 | 8 | 36 | 6 | 48 |
| 7 | 1 | 1 | 46 | 7 | 1 | 19 | 37 | 7 | 00 |
| 8 | 1 | 6 | 18 | 8 | 1 | 31 | 38 | 7 | 11 |
| 9 | 1 | 10 | 50 | 9 | 1 | 42 | 39 | 7 | 22 |
| 10 | 1 | 15 | 23 | 10 | 1 | 53 | 40 | 7 | 34 |
| 11 | 1 | 19 | 55 | 11 | 2 | 5 | 41 | 7 | 45 |
| 12 | 1 | 24 | 27 | 12 | 2 | 16 | 42 | 7 | 56 |
| 13 | 1 | 28 | 59 | 13 | 2 | 27 | 43 | 8 | 8 |
| 14 | 2 | 3 | 32 | 14 | 2 | 39 | 44 | 8 | 19 |
| 15 | 2 | 8 | 4 | 15 | 2 | 50 | 45 | 8 | 30 |
| 16 | 2 | 12 | 36 | 16 | 3 | 1 | 46 | 8 | 42 |
| 17 | 2 | 17 | 8 | 17 | 3 | 13 | 47 | 8 | 53 |
| 18 | 2 | 21 | 41 | 18 | 3 | 24 | 48 | 9 | 4 |
| 19 | 2 | 26 | 13 | 19 | 3 | 35 | 49 | 9 | 16 |
| 20 | 3 | 0 | 46 | 20 | 3 | 47 | 50 | 9 | 27 |
| 21 | 3 | 5 | 18 | 21 | 3 | 58 | 51 | 9 | 38 |
| 22 | 3 | 9 | 50 | 22 | 4 | 9 | 52 | 9 | 50 |
| 23 | 3 | 14 | 22 | 23 | 4 | 21 | 53 | 10 | 1 |
| 24 | 3 | 18 | 54 | 24 | 4 | 32 | 54 | 10 | 12 |
| 25 | 3 | 23 | 27 | 25 | 4 | 43 | 55 | 10 | 24 |
| 26 | 3 | 27 | 59 | 26 | 4 | 55 | 56 | 10 | 35 |
| 27 | 4 | 2 | 31 | 27 | 5 | 6 | 57 | 10 | 46 |
| 28 | 4 | 7 | 4 | 28 | 5 | 17 | 58 | 10 | 58 |
| 29 | 4 | 11 | 36 | 29 | 5 | 29 | 59 | 11 | 9 |
| 30 | 4 | 16 | 8 | 30 | 5 | 40 | 60 | 11 | 21 |

In *Anno Bissextili* post *Februarium* adde unum diem, Motumque ei competentem.

I shall only add that the Proportion of the Squares of the *Times* of the *Periods* to the Cubes of the *Distances*, ( which is proposed as probable by *Kepler*, but now demonstratively found true by Mr. *Newton*, ) gives us nicely the Proportion of the *Distances* of these *Planets* from the Center of *Saturn*; and supposing the *Satellite* of *Hugens* four Diameters of *Saturn's Ring* distant from him, we shall find by the *Periods*, the *Distances* as follows.

| | *Periodus.* | | | *Distantia.* |
|---|---|---|---|---|
| | d. | h. | ′ | *Diam.* |
| *Intimi* | 1 | 21 | 18½ | 0, 964 |
| *Penintimi* | 2 | 17 | 41½ | 1, 235 |
| *Medii* | 4 | 13 | 47¼ | 1, 740 |
| *Penextimi* | 15 | 22 | 41 | 4, 000 |
| *Extimi* | 79 | 7 | 54 | 11, 621 |

These *Distances* may be used, as more accurate than those obtained by Observation, which yet differ but little therefrom.

*The Phasis of* Jupiter; *by Dr.* Hook. *n.* 14. p. 245. Fig. 139.

LXXXIII. *An.* 1666. *Jun.* 26. between 3 and 4 of the Clock in the Morning, I observed the Body of *Jupiter* through a 60 Foot Glass, and found the apparent Diameter of it through the Tube, to be somewhat more than two Degrees, that is about 4 times as big as the Diameter of the *Moon* appears to the naked Eye.

I saw the Limb pretty round, and very well defined without Radiation. The parts of the *Phasis* of it had various Degrees of Light. About *a*, and *f*, the North and South Poles of it, somewhat darker, and by degrees it grew brighter towards *b*, and *e*, two *Belts* or *Zones*; the one of which, *b*, was a small dark *Belt* crossing the Body Southward; Adjoining to which was a small Line of a somewhat lighter Part; and below that again, Southwards was the Great black *Belt c*. Between that, and *e*, the other smaller black *Belt*, was a pretty large and bright *Zone*, but the Middle *d*, was somewhat darker than the Edges.

*The Revolution of* Jupiter *upon his* Axis; *by* S. Campani. *n.* 1, *p.*3.

LXXXIV. 1. *S.Campani* affirms that, by the Goodness of his Glasses, he hath Observed certain *Protuberances* and Inequalities of *Jupiter*: and he is now Observing whether they do not change their Situation.

*By Dr.* Hook. *ib.*

2. *An.* 1664. *May* 9. about 9 a Clock at Night, Mr. *Hook* with an Excellent 12 Foot Telescope Observed a small *Spot* in the biggest of the three *Obscurer Belts* of *Jupiter*, and Observing it from time to time, he found that within two Hours after, the said *Spot* had moved from East to West, about half the Length of the Diameter of *Jupiter*.

By S. Divini; *n.* 12. p. 209.

3. *Eustachio de Divinis* pretends, that the *Permanent Spot* in *Jupiter* hath been first of all discover'd with his Glasses; that *P. Gotignies* is the first that hath

hath thence deduced the *Motion* of *Jupiter* about his *Axis*; and that M. *Caſſini* at firſt oppoſed it: *But that Spot was Obſerved in* England *a good while before.*

4. There are two Sorts of *Spots* at certain times to be ſeen in the Diſque of *Jupiter*. One Sort are nothing but the *Shadows* of his *Satellites*: but the other have ſome reſemblance to thoſe that are ſeen in the *Moon*; and they are perhaps of the ſame Nature with thoſe that are called *Belts*. They Move from the Eaſtern to the Weſtern Limb; their apparent Motion is unequal, and ſwifter near the Center than the Circumference; and they never are ſo well ſeen as when they approach to the Center, they being very narrow and almoſt imperceptible when they approach to the Circumference; which makes us believe, that they are flat and ſuperficial to *Jupiter*.

*By M.* Caſſini. *n.* 8. *p.* 143. *n.* 10. *p.* 172. *n.* 35. *p.* 687. *n.* 82. *p.* 4039.

Among theſe *Spots* there is none ſo ſenſible, as that which is ſituate in the Northern-part of the *Southern Belt*. Its Diameter is about the tenth part of that of *Jupiter*; and at the time that its Center is neareſt to that of *Jupiter*, it is Diſtant from it about the third part of the Semidiameter of that *Planet*.

M. *Caſſini*, after he had made many Obſervations of this *Spot* during the Summer of the Year 1665, found, that the *Period* of its apparent *Revolution* is of $9^h\ 56'$. He continued to Obſerve it till the Beginning of 1666, when *Jupiter* approach'd to the Beams of the *Sun*: But after he was got free of the *Sun-Beams* it was difficult to be diſcern'd. This gave grounds that it might be of the Nature of the *Spots of the Sun* (which after having appeared for a while, diſappear for ever) M. *Caſſini* ceaſed at length to Obſerve them.

But *Jan.* 19. 1672. (*ſt. n.*) when he Obſerv'd *Jupiter*, at $4\frac{1}{4}^h$ in the Morning, he perceived in the ſame Place of his Diſque the Figure of the ſame *Spot*, adhering to the ſame *Southern Belt*. It was already gone beyond the Moiety of this *Belt*, and he ſaw it advance little by little towards the Weſtern Limb, to which it ſeemed to be very near at $6\frac{1}{4}^h$.

By the Celerity of its Motion near the Center, and by the Place where he had begun to ſee it, he judged that it might have been in the midſt of the *Belt* at $4^h\ 35'$ in the Morning. And as he prepared himſelf to make Ephemerides of its Motion for that *Year* 1672, he perceived, that in thoſe, he made for the Year 1666, this *Spot* had been in the midſt of *Jupiter* the ſame Day, namely the 19th of *Jan.* at the ſame Hour. So that in ſix Years, of which one is a *Biſſextile*, it is found to have made, in reſpect of the Earth, at leaſt 5294 Revolutions, each of $9^h\ 55'\ 58''$, compenſating one Revolution by another; and at moſt 5294 Revolutions of $9^h\ 55'\ 51''$; foraſmuch as he was aſſured of the preciſeneſs of one *Mean Revolution* to one eighth of a Minute.

Until then he had never ſeen an immediate Return of this *Spot* after 9 hours and 56 minutes, becauſe it had not happened, that *Jupiter* after the Apparition of the *Spot* had ſtayed, in one and the ſame Night, long enough above the *Horizon*, at leaſt at a ſufficient height, to obſerve him with due Diſtinctneſs. He had only concluded the time of this *Revolution* by Returns obſerved after about 20, 30, and 50 hours; and he had more preciſely limited it by Obſervations more diſtant. But the Night after *Mar.* 1, at $7\frac{1}{2}^h$ in the Evening, he ſaw this *Spot* in the midſt of the *Belt*; and the ſame Night at

$5^h\ 26'$

5ʰ 26′. in the Morning, he ſaw it again returned perciſely to the ſame Place. *Mar.* 3. He together with *M. Buot* and *M. Mariotte*, began to ſee at 8ʰ 4′ the *Spot* already ſomewhat removed from the Oriental Limb, but yet obſcure and ſmall. At 8ʰ 47′. they ſaw it very diſtinctly advancing towards the middle of the *Belt*. From 9ʰ 5′ 40″. until 9ʰ 8′. they ſaw it in the midſt of the *Belt*. At 9ʰ 15′. it was paſt the middle, and was come nearer to the Occidental Limb. And a little after the Heavens being overcaſt, he could obſerve it no further.

*Places of* Jupiter *obſerved*; *by Mr.* Flamſteed *at* Derby. *n.* 82. *p.* 4036.

LXXXV. 1. *An.* 167½. *Feb.* 16. 7ʰ 44½′. Alto *Jove* 18° 10′. ejus Diſtantiam à Fixa Lucis 4ᵗᵃ (cujus Lat. 1° 40′ Bor. Locus *Mihi* ♍ 14° 7′ 16″. at *Streetio*, 14° 3′ 54″) Tubo Longiori dimenſus ſum, 16′ 33″. & Differentiam Altitudinum Centrorum ♃ & ✱ᵃ 1′ 1″.

17 *Febr.* 7ʰ 25′. *p. m.* Alto ♃ 15° 54′. ipſe à *Fixa* diſtitit 21′ 50″; Altitudinum differentia erat 8′ 40″.

18 *Febr.* 7ʰ 0′. *Fixæ* diſtantia à Centro ♃ erat 28′ 15″; Altitudinum differentia circ. 15′ 29″. In utraque Obſervatione *Erro* Altior erat *Fixâ*, à qua ſemper Meridianum verſus Stetit.

Inito dein Calculo ad dies ſingulos & horas obſervationum, inveſtigavi

| *Februar.* | d. h. ′<br>16 7 44½ | d. h. ′<br>17 7 25 | d. ′ ″<br>18 7 0 |
|---|---|---|---|
| | ° ′ ″ | ° ′ ″ | ° ′ ″ |
| *Jovis* à *Fixa* Longitudinem in Antecedentia | 0 9 16 | 0 17 22 | 0 25 12 |
| Latitudinem ad Auſtrum ab ea | 0 13 30 | 0 13 14 | 0 12 45 |
| Ergo ♃is Latitudo Borealis | 1 26 30 | 1 26 46 | 1 27 15 |
| Locus verus ♍ . . . . *Mihi* | 13 58 0 | 13 49 54 | 13 42 4 |
| Locus verus ♍ . . . . *Streetio* | 13 54 38 | 13 46 32 | 13 38 42 |

2. *Mar.* 15. *v.* Obſervare cœpi *Jovis* diſtantias & Poſitiones à *Stella* ♌ 38, cujus Latitudo 1° 20′½ Bor. Locus *Streetio* ♍ 9° 54′ 0″; *mihi* vero ♍ 9° 57′ 20″. 7ʰ 25′. *p. m.* Alto ♃ 32° 52′. Diſtantia Centri Ipſius ab ipſa 33′ 50″. Altitudinum Differentia circit. 20′ 42″.

*Mar.* 16. 7ʰ 48′. Alta *Fixa* 36°. *Jovis* ab ea diſtantia erat 27′ 7″. Altitudo minor 16′ 3″.

*Mart.* 19. Alto ♃ 29° 35′. *i. e.* 6ʰ 45′. *Fixa* Altior erat quam *Planeta* 2′ 24″; à quo 6ʰ 55′. diſtitit. 10′ 21″.

*Hor.* 7. 11. *Satelles* 4ᵗᵘˢ à *Fixa*, 7′ 28″. Etiamnum *Erro* ſemper altior apparuit, ſed verè fuit depreſſior, quam *Fixa*: poſtea humilior viſus eſt, ſed revera fuit Altior.

*Mar.* 20. melius præparato ad Altitudinum Differentias capiendas Micrometro, obſervationes habui (ſic putem) accuratiſſimas, quæ ſequuntur.

*Jov.*

| *Jov. Alt.* | | *Temp.* | | | | |
|---|---|---|---|---|---|---|
| ° | ′ | h. | ′ | | ′ | ″ |
| 30 | 0 | 6 | 44 | *Jovis* Centrum vere altius erat *Stella* ——— | 2 | 13 |
| 30 | 47 | 6 | 51$\frac{1}{3}$ | Altitudinum eadem differentia rursus capta — | 2 | 14 |
| 32 | 0 | 6 | 59 | *Jovis* Centrum à *Fixa* distitit ——— | 7 | 0 |
| 38 | 30 | 7 | 54$\frac{3}{4}$ | Centrum *Jovis* verè altius *Fixa* ——— | 3 | 14 |
| 40 | 50 | 8 | 18$\frac{2}{3}$ | Altitudinum differentia denuo capta ——— | 3 | 42 |
| | | | | Et deinde Centrorum distantia erat ——— | 7 | 5 |

Ad Locum *Jovis* ex his Observationibus acquirendum supputavi, ad

| | h. ′ ″ | h. ′ ″ |
|---|---|---|
| | 6 51 30 | 8 18 40 |
| | ° ′ ″ | ° ′ ″ |
| Angulos *Circuli Verticalis* cum *Ecliptica* ——— | 35 39 — | 46 15 — |
| *Jovis* erat à *Fixa* distantia ——— | 0 7 0 | 0 7 0 |
| Altitudinum differentia ——— | 0 2 14 | 0 3 42 |
| Ergo ♃ erat in consequentia *Fixæ* ——— | 0 2 3 | 0 1 44 |
| Cum Latitudine majori ——— | 0 6 42 | 0 6 47 |
| Quare *Jovis* Latitudo vera ——— | 1 27 12 | 1 27 17 |
| Locus verus ♍ ——— { *Streetio* ——— | 9 56 3 | 9 55 44 |
| { *Mihi* ——— | 9 59 23 | 9 59 4 |

| *Maii.* | *Temp.* | | *Alt. Jov.* | | | | | |
|---|---|---|---|---|---|---|---|---|
| d. | h. | ′ | ° | ′ | | ° | ′ | ″ |
| 24 | 10 | 00 | 24 | 10 | Humilior erat *Jovis* Centrum quam Stella dicta ☊ 38′ ——— | 0 | 7 | 46 |
| | | | | | A qua semel ejus cepi distantiam | 0 | 20 | 00 |
| | | | | | deinde ——— | 0 | 19 | 54 |
| 26 | 8 | 46 | 33 | 30 | Distantia centri ♃ à *Fixa* ——— | 0 | 10 | 4 |
| *Cœlo* | 9 | 00 | 31 | 50 | *Fixa* altior erat quam ♃ ——— | 0 | 6 | 30 |
| *sereno.* | 9 | 20 | 29 | 10 | Altitudinum differentia erat ——— | 0 | 6 | 38 * |
| | 9 | 33 | 27 | 20 | Differ. Azymuthorum ♃ & *Fixæ* — | 0 | 7 | 19 |
| | 9 | 36 | 27 | 00 | Distantia ♃ à *Fixa* denuo capta — | 0 | 10 | 2 |

2. N. 86. p. 5037.

| | h. | ′ | ° | ′ | | ° | ′ | ″ | |
|---|---|---|---|---|---|---|---|---|---|
| 27 *Ventis interdum tubum moventibus.* | 8 | 59 | 31 | 24 | Centri ♃ à *Fixa* distantia | 0 | 6 | 2 | * |
| | 9 | 7 | 30 | 30 | Azymuthorum differentia | 0 | 1 | 50 | |
| | 9 | 16 | 29 | 13 | Eadem rursus differentia | 0 | 1 | 52 | * |
| | 9 | 23 | 28 | 10 | Centri ♃ à *Fixa* distantia | 0 | 6 | 1 | |
| 28 *Ventis valide flantibus & Telescopium concutientibus.* | 8 | 57 | 31 | 8 | A *Fixa* distantia centri ♃ | 0 | 6 | 2 | * |
| | 9 | 19 | 28 | 10 | Differentia Azymuthorum centri, &c. | 0 | 3 | 29 | |
| | 9 | 31 | 26 | 30 | Isthæc Differentia rursus | 0 | 3 | 32 | |
| | 9 | 34 | 26 | 8 | Distantia rursus | 0 | 6 | 7 | |
| 30 *Cælo satis sereno.* | 9 | 36 | 26 | 15 | *Jovis* distantia à *Fixa* | 0 | 15 | 38 | |
| | Hac & præcedente nocte in Consequentia *Fixæ* erat, antea semper in Antecedentia. | | | | | | | | |

Harum observationum certissimas habeo, quas asterisco (*) notavi Azymuthorum differentias; diebus 27 & 28 observatas, nimis strictas acceptas æstimo, ob vacillationem Tubi, quampropter, quando non ut volui eas accuratè dimetiri licebat, ne nimis amplas caperem curabam.

Ad *Jovis* locum ab his observationibus obtinendum supputavi

| | d. | h. | ′ | d. | h. | ′ | d. | h. | ′ |
|---|---|---|---|---|---|---|---|---|---|
| *Maii* | 26 | 9 | 0 | 27 | 9 | 7 | 28 | 9 | 19 |
| | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ |
| *Angulos Parallacticos* sive Circuli *Verticulis* cum *Ecliptica* | 80 | 47 | — | 79 | 49 | — | 78 | 36 | — |
| *Jovis* à *Fixa* distantia observata | 0 | 10 | 4 | 0 | 6 | 2 | 0 | 6 | 3 |
| Differentia obs. Alt. | 0 | 6 | 30 | *Azym.* 1 | | 50 | *Azym.* 3 | | 30 |
| Ergo *Angulus Positionis* | 31 | 1 | | 62 | 38 | 0 | 66 | 3 | 0 |
| Et *Planeta* in antecedentia *Fixa* | 0 | 8 | 38 | 0 | 2 | 51 | *conf.* | 2 | 27 |
| Cum minori Latitud. Boreali | 0 | 5 | 11 | 0 | 5 | 19 | 0 | 5 | 32 |
| *Fixæ* Latitudo *Tychonica* Bor. | 1 | 20 | 30 | 1 | 20 | 30 | 1 | 20 | 30 |
| *Fixæ* Locus ♍ — *Streetio* | 9 | 54 | 9 | 9 | 54 | 9 | 9 | 54 | 9 |
| *Fixæ* Locus ♍ — *Mihi* | 9 | 57 | 30 | 9 | 57 | 30 | 9 | 57 | 30 |
| *Jovis* itaque Locus — *Streetio* | 9 | 45 | 31 | 9 | 51 | 18 | 9 | 56 | 36 |
| *Jovis* itaque Locus — *Mihi* | 9 | 48 | 52 | 9 | 54 | 39 | 9 | 59 | 57 |
| Cum Latitudine Boreali | 1 | 15 | 19 | 1 | 15 | 11 | 1 | 14 | 58 |

3. *An.* 1673. *Mar.* 13. vesp. *Jupiter Aphelius* Pronus ad Phasin *Acronicam*, & *Limitem Orbitæ* Boreum paululum transgressus, *Retrogradus* incessit versus 9am ♍ *Lucis quartæ*, è qua ( Alto eo sex circiter gradus) Limbi ejus Remotissimi distantiam, septem-pedali Tubo & *Micrometro Townleiano* cœpi 4560 52′ 34″. n. 94. p. 6033.

*Mar.* 17 circa horæ dimidiam post exortum *Jovis* ejus eodem Tubo, Limbi Remotissimi à *Fixa* cœpi iterum distantiam 2073 = 23′ 54″.

*Mar.* 20. Sequentes habui observationes. Primam Breviori Tubo, digitorum tantum 85, reliquas Longiori, viz. 164. dig.

| | *Fix. Alt.* | | *Hor. Sup.* | | | | *Limb. ph.* | | *Cent.* | |
|---|---|---|---|---|---|---|---|---|---|---|
| | ° | ′ | h. | ′ | | | ′ | ″ | ′ | ″ |
| 1 | 6 | 0 | 7 | 14 | Limbi ♃ remotior. à *Fixa* dist. | 850 | 9 | 48 | 9 | 24 |
| 2 | ——— | | ——— | | Eadem dist. Tubo Long. capta | 1650 | 9 | 52 | 9 | 28 |
| 3 | 14 | 40 | 8 | 16 | Limb. ♃ Inf. depress. ac *Fixa* | 784 | 4 | 41 | 4 | 17 |
| 4 | 15 | 40 | 8 | 23 | Alt. eadem repetita Differen. | 786 | 4 | 41 | 4 | 17 |
| 5 | ——— | | ——— | | *Jovis* Diameter ——— | 135 | 0 | 48 | | |
| 6 | 16 | 25 | 8 | 29 | Limb. iterum capta distantia— | 1665 | 9 | 57 | 9 | 33 |
| 7 | ——— | | ——— | | Denuo——— | 1658 | 9 | 54 | 9 | 30 |
| 8 | 19 | 00 | 8 | 50 | Differ. Alt. Limbi *Jov.* & *Fixæ* | 838 | 5 | 00 | 4 | 36 |

*Mar.* 26. *vesp.* Alto *Jove* 15° 50′. Limbi sui remotioris à *Fixa* distantiam, eodem minori Tubo, dimensus sum 4205 = 48′ 30″.

Ad *Planetæ* Locum ex his annotationibus eliciendum, structis supputationibus, invenio.

| | d. | h. | ′ | | h. | ′ |
|---|---|---|---|---|---|---|
| *Mar.* | 20 | 8 | 16 | — | 8 | 50 |
| | ° | ′ | ″ | ° | ′ | ″ |
| *Angulum Parallacticum* ——— | 34 | 44 | 0 | 37 | 30 | 0 |
| Centrum *Jovis* à *Fixa* distitit——— | 0 | 9 | 28 | 0 | 9 | 30 |
| Altitudinum differentia erat ——— | 0 | 4 | 17 | 0 | 4 | 36 |
| Ergo, *Angulus* erat *Positionis* ——— | 80 | 6 | 0 | 78 | 21 | 0 |
| Et *Jupiter* in antecedentia *Fixæ*—— | 0 | 1 | 38 | 0 | 1 | 55 |
| Cum Latitudine Minori——— | 0 | 9 | 19½ | 0 | 9 | 18 |

Fixæ *mihi* Locus, accepto motu Annuo 50″, erit ♎ 13° 37′ 11″. quem vult *Author Carolinus* 13° 33′ 47″. Latitudo ejus Borea. 1° 45′. Locus ergo verus *Jovis* erit *Mihi*.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 8h 16′ | ♎ 13° | 35′ | 33″ | Latitudo vera | 1° | 35′ | 40″ ½. |
| 8 50 | ♎ 13 | 35 | 16 | | 1 | 35 | 42 |



At

At *Fixæ* conceſſo Loco *Carolino*, prodibit ♃is Locus ♎ 13° 32′ 09″.

Loco ſic *Planetæ* & Latitudine perceptis Orbitæ *Jovialis* ad *Terreſtris* Orbitæ Planum Inclinationem inde eruere conabimur.

Huic equidem inveniendæ, una cum Loco *Solis*; ejuſdem, *Jovis*, & *Terræ* intermutuæ diſtantiæ poſtulantur: quas à Tabulis quibuſvis probatioribus tutiſſime haurire licet: Ego Tabulis utor plerumque *Carolinis*; quippe quas, accuratiores, & faciliores cæteris omnibus comperi; ex quibus ad 8h 16 p. m. deprompſi;

*Solis* locum verum —————— ♈. 10° 40′ 18″.
diſtantiam a *Terra* —————— 100084.
*Jovis* à *Sole* diſtantiam —————— 544921.
à *Terra* —————— 444952.

Fig. 140. Jam in appoſita Figura ſint, S *Sol*, T *Terra*, ♃ *Planeta*, SE Radius *Eclipticæ*, ad ♃is *Orbitam* protenſæ, & Angulus ♃ TS, viſa *Planetæ* à *Terra* Latitudo 1° 35′ 40″ ½.

Ex datis (in Triangulo ♃ ST) Angulo, ♃ TS, viſæ Latitudinis ad Circulum complemento; ♃ S, & ♃ T, *Planetæ* à *Sole* & *Terra* Diſtantiis, ut ſupra repertis, eruetur Angulus ♃ S E, Latitudo ſive Inclinatio *Planetæ* à *Sole* conſpecta 1o 18′ 7″.

*Jovis* Locus Geo-centricus erat, ♎. 13° 35′ 33″. ab iis ergo datis ♃ S. & *Terræ* à *Sole* diſtantiis, invenietur Locus Helio-centricus *Planetæ* ♎. 13° 03′ 33″; è quo ſubductis ſigilatim iis *Nodi* Locis, quos Autores, quorum Nomina in ſequenti Tabella exaravimus, aſſumpſerunt, annexa produnt Argumenta Latitudinis; è quibus videre eſt, nullis plus *Jovem* à Limite promotum haberi quàm 6° 29′ 56″, nec minus quàm 3° 58′ 59″. quæ quantavis, videtur differentia in maximâ Orbitæ Inclinatione inveſtiganda, Errorem ſcrupulis ſecundis 23″ Majorem inferre nequit.

| *Authores.* | ☊ *Loca.* | | | | *Argumenta Latitudinis.* | | | |
|---|---|---|---|---|---|---|---|---|
| | s. | ° | ′ | ″ | s. | ° | ′ | ″ |
| Keplerus. | 3 | 6 | 33 | 37 | 3 | 6 | 29 | 56 |
| Streetius. | 3 | 6 | 33 | 47 | 3 | 6 | 29 | 46 |
| Wingius. | 3 | 7 | 11 | 39 | 3 | 5 | 21 | 54 |
| Ricciolus. | 3 | 7 | 18 | 00 | 3 | 5 | 45 | 33 |
| Caſſinus. | 3 | 8 | 45 | 00 | 3 | 4 | 18 | 33 |
| Bullialdus. | 3 | 9 | 4 | 34 | 3 | 3 | 58 | 59 |

Fig. 141. Iſte *Nodi* Locus, quem Cl. *Caſſinus* elegerit mihi etiamſi aliquantulùm juſto promotior videtur, Magis tamen cæteris variis de cauſis, placet: ſumptis propterea, Triangulo ♃ A ☊, Argumento Latitudinis, ☊ A, 94° 18′ 33″. & Inclinatione ♃ A, 1° 18′ 07″. eruetur Angulus Inclinationis Plani *Orbitæ*

*Jovialis*

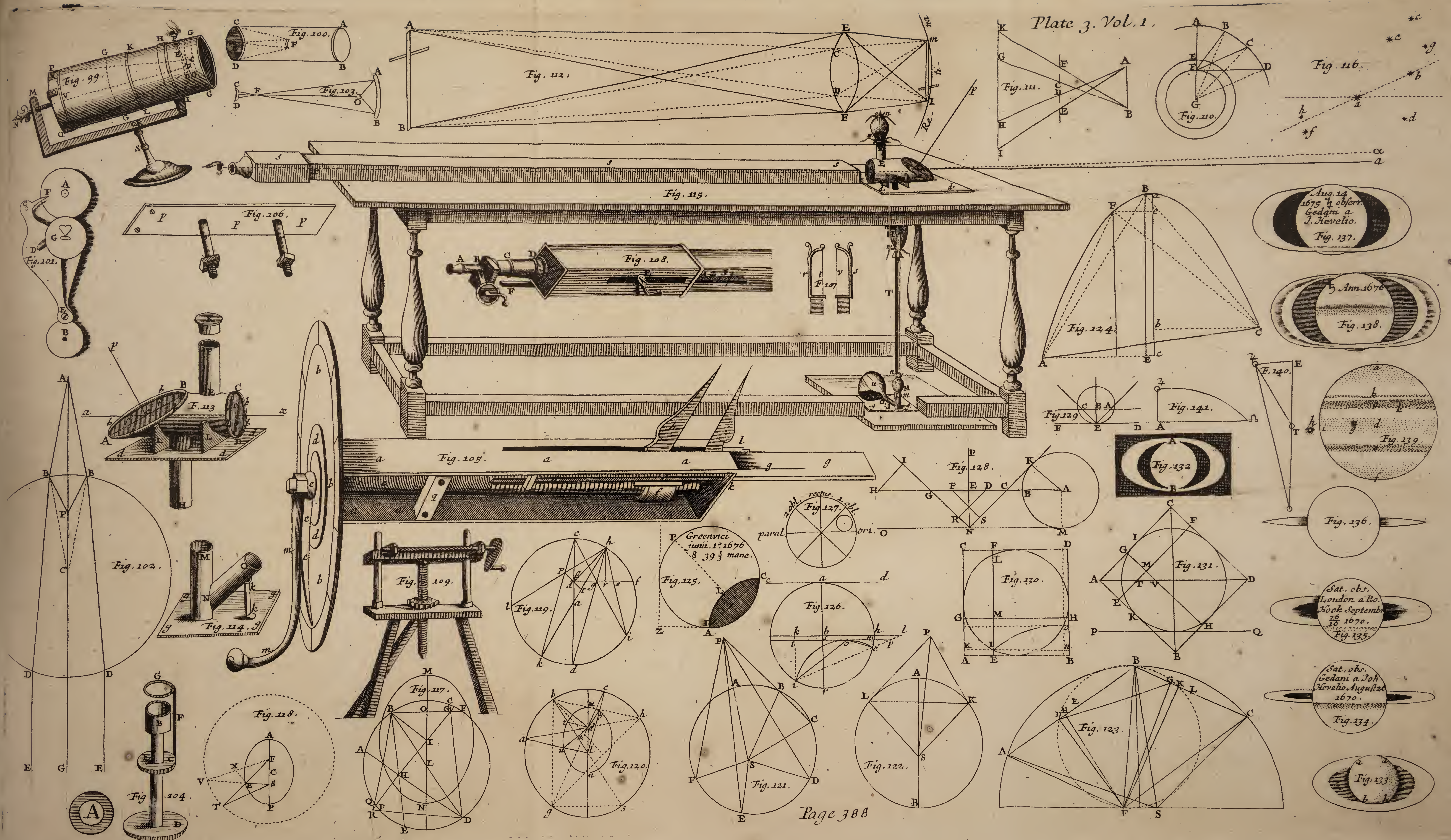
Plate 3. Vol. 1.
Fig. 99.
Fig. 100.
Fig. 101.
Fig. 102.
Fig. 103.
Fig. 104.
Fig. 105.
Fig. 106.
F 107
Fig. 108.
Fig. 109.
Fig. 110.
Fig. 111.
Fig. 112.
F. 113
Fig. 114.
Fig. 115.
Fig. 116.
Fig. 117.
Fig. 118.
Fig. 119.
Fig. 120.
Fig. 121.
Fig. 122.
Fig. 123.
Fig. 124.
Fig. 125.
Greenvici junii 1° 1676 8 39 ¼ mane.
Fig. 126.
Fig. 127.
rectus
obl.
paral.
ori.
Fig. 128.
Fig. 129
Fig. 130.
Fig. 131.
Fig. 132
Fig. 133.
Sat. obs. Gedani a Joh Hevelio August 26 1670.
Fig. 134.
Sat. obs. London a Ro. Hook Septembr 26/16 1670.
Fig. 135.
Fig. 136.
Aug. 14 1675 ♄ observ. Gedani a J. Hevelio.
Fig. 137.
♄ Ann. 1676
Fig. 138.
Fig. 139.
F. 140.
Fig. 141.
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*Jovialis* ad *Eclipticam* 1° 18′ 20″. quem ſtatuunt *Keplerus* 1° 19′ 00″. *Streetius*, 1° 20′ 00″. *Bullialdus* & *Wingius* 1° 21′ 48″. omnes juſto non-nihil Majorem.

Tantamque eſſe Inclinationem, vel ſaltem non Majorem, cum heſternæ noctis, tum Menſium *Februarii Martii* & *Maii Anni Elapſi*, obſervationes ſuadent. Interea verò non diſſimulaudum, poſſe & majorem (ſcilicet 1° 20′ 20″.) a tranſitu ♃s prope 8am. ♍ *Anno* 1649 *Maii* 29. & 30 St. Juliano *Bononiæ* & *Majorcæ* à *Ricciolo* & *Muto*, Viris Doctiſſimis, obſervato Demonſtrari: id quod nobis (ſi quidem Orbitarum Inclinationes ab omnibus invariabiles habentur,) videtur innuere, Errorem vel huic, vel illis *Fixarum* Latitudinibus à *Tychone* aſſignatis, ineſſe aliquem: quæ propterea donec accuratius reſtituantur, à præciſa hujus Inclinationis quantitate determinanda meritò nos arcent: Hoc tantum, quoniam *Fixarum* eæ Latitudines etiam Immutabiles reperiuntur, auſim affirmare; Angulum Maximæ Inclinationis Plani *Orbitæ Jovialis* ad *Eclipticam* Minorem eſſe ſcrupulis 26′ 40″. quam Latitudo *Stellæ* 9æ ♍s *Lucis* 4*tæ* quæ *Tychoni* dicitur, *Ultima quatuor in Siniſtra Ala Virginis*: quæ propterea ſi quando correcta dabitur, eadem certa dabitur Inclinatio.

*Almag. Nov.* Par. 1. *p.* 710.

LXXXVI. Examining our Ancient *Ephimerides* I do not find that three *Conjunctions* of *Saturn* and *Jupiter* have ever hapned in one Years ſpace, ſince they were firſt in uſe to this preſent. Thoſe of *Moletius* Calculated from the *Alphonſine Tables* indeed make three in the ſpace of eight Months betwixt *Auguſt* 1563. and *April* 1564. Incluſive; but the *Ephemerides* of *Stadius* Calculated from the *Prutenick*, make only one, on the 26 of *Auguſt* of which *Junctinus* gives us the following Obſervation in the Preface to his *Aſtronomical Tables*, *An.* 1563. *Aug.* 24 14h 30′. *p. m.* Aurangæ, Jupiter *à parte Septentrionis cooperiebat quaſi* Saturnum, *qui erat à parte Meridionali, utraque autem harum Stellarum, in fine* 28 *gradus* Cancri *deprehendebatur*, *Riccioli* hence concludes, that the Planet ♃ covered ſome part of *Saturn* at this time. But without Reaſon for the words *quaſi cooperiebat* intimate not that the one did corporally cover the other, but rather that there was ſome ſmall Interval betwixt them. The *Caroline Tables* make the viſible Latitude of *Saturn* now 11′ 45″. of *Jupiter* 20′ 10″. both North, the Conjunction being ſome few days paſt: but becauſe their Latitudes alter ſlowly we may hence conclude the Difference 8′ 25″. to have been nearly their Diſtance at that time, theſe Tables being grounded on the *Tychonick* Obſervations made within leſs than 40 Years after, and ſhewing the Latitudes of the *Planets* well at this time near 100 Years later we may Conclude to have anſwered them as well then; and if we conſider how ſmall a ſpace the Diſtance of 8½ minutes appear to the naked Eye in the Heavens, eſpecially betwixt two ſuch bright Planets as *Saturn* and *Jupiter* are, that the *Caroline* Diſtance agrees very well with the Words of *Junctinus* and that *Riccioli* was groſly miſtaken.

*The Conjunctions of* Saturn *and* Jupiter. An. 1682 *and* 1683. *at* Greenwich; *by Mr.* Flamſteed. *n.* 149. *p.* 244.

Their next *Conjunction* according to *Maginus*'s *Ephemerides* founded on the *Prutenick* numbers, was *April* 29. 1583 in 21 *Deg.* of ♓, the *Sun* being then in 17 *Deg.* of ♉. ſo that that Planets Riſing before him in *Signs* of ſhort Aſcenſi-

Aſcenſion and with South Latitude this *Congreſs* could not be obſerved by the Noble *Tycho* who was mindful of it as appears by this note in page 55 of his *Hiſtoria Cœleſtis.* May. 30 A. M. *quo primum poſt* Conjunctionem Saturnum *vidimus, captæ ſunt diſtantiæ inter* Jovem & Saturnum *per Radium.*

| | | | |
|---|---|---|---|
| $1^h$ | 47′ | 3° | 24′. |
| 1. | 50 | 3 | 24. |

The ſame *Ephemerides* ſhew the next *Conjunction* of *Saturn* and *Jupiter* 1603. *December* 14 at noon in 9° 36′ of ♐, but the the Ingenious *Kepler* and our Sir *Chr. Heydon* found it by obſervation 7 days ſooner, or the 7 Day of the ſame Month in the Morning, in near 8 *Degrees* of ♐ the Planets being but then newly Emerged from the Rays of the *Sun.*

The *Ephemerides* of the Learned *Kepler* Calculated from his own *Rudolphine Tables,* make the next *Conjunction* 1623. betwixt the 7th and 8th of *July* in 6° 46′. of ♌ the Planet *Saturn* being then only 4 Minutes to the North of *Jupiter* but this firſt *Conjunction* in the *Fiery Trigon* happening under the *Sun's* beams was not obſervable.

By the ſame *Tables,* and *Ephemerides* of *Eichſtade* Calculated from them; theſe *Planets* met again in the 25° of ♓, betwixt the 15 and 16 of *Febr.* 1643 with a degree Difference, of Latitude.

By the joynt conſent of *Eichſtades* and our *Wing's Ephemerides* the ſame Planets were in *Conjunction* again 1663. on the 10 of *October* at Noon in 13° 30′. of ♐ with one degree Difference of Latitude; this *Conjunction* was obſervable after *Sun-ſet* in our Latitude, but I hear not that any one obſerved it.

In every of theſe Years there hapned only one *Conjunction* of the two *Superiors,* nor is it poſſible that there ſhould be more except the *Heliocentrical Conjunction* fall near the *Oppoſition* of the *Sun*; for then there may be three two *Direct,* and one *Retrograde,* as has been within the Space of 7 Months, betwixt *October* and *May* laſt incluſive, of which the true times are determined from the following Obſervations.

| *An.* 1682. | | | | | | | |
|---|---|---|---|---|---|---|---|
| | d. | | | | ° | ′ | ″ |
| *Oct.* | 5 | 17 | 51 | Betwixt the Centers of *Sat.* and *Jup.* | 0 | 34 | 54 |
| | | 17 | 54 | ———————————rep. | 0 | 34 | 48 |
| | 12 | 13 | 49 | Betwixt their Centers——— | 0 | 16 | 2 |
| | | 13 | 54 | ———————————rep. | 0 | 16 | 4 |
| | | 14 | 3 | Betwixt their Next Limbs——— | 0 | 15 | 22 |

| | d. | h. | ′ | | ° ′ ″ |
|---|---|---|---|---|---|
| *Oct.* | 17 | 14 | 10 | Betwixt their Centers | 0 20 9 |
| | | 14 | 17 | ——— rep. | 0 20 12 |
| | | 14 | 21 | ——— again | 0 20 14 |
| | | 14 | 25 | Betwixt their Next Limbs | 0 19 44 |
| | | 14 | 33 | Betwixt their Remoter Limbs | 0 20 37 |
| | | 15 | 9 | *Saturn* from the *Heel* of *Castor* | 48 32 25 |
| | | 15 | 14 | *Jupiter* from the same *Star* | 48 45 5 |
| | | 15 | 17 | ——— rep. | 48 45 20 |
| | | 15 | 20 | *Saturn* from the same *Star* again | 48 32 20 |
| | | 15 | 50 | Betwixt their Centers again | 0 20 30 |
| | 19 | 15 | 41 | Betwixt their Remote Limbs | 0 26 2 |
| | | 15 | 45 | Their Centers | 0 25 37 |
| | | 15 | 47 | Their Next Limbs | 0 25 11 |
| | 22 | 18 | 25 | Betwixt their Centers | 0 33 19 |
| | | 18 | 29 | ——— rep. | 0 33 26 |

The *Distances* betwixt the *Planets* were measured with the *Micrometer* and 16 Foot Glass, from the *Fixt Stars* with the *Sextant*: those of the 12th by my Assistant, the rest by my self.

On the 22. day, the Planet *Jupiter* was in Consequence of *Saturn* something less Distant from him than he had been observed on the 5th day near the same hour. Hence the Middle Time, betwixt these Observations is pointed out for the time of their true *Conjunction*, but to determine it more accurately I shall Examine the Observations made with the Sextant on the seventeenth day which being nearest the time are most proper for this purpose.

The Correct Longitude of the *Heel of Castor* now ♋ 0° 50′ 42″. its Latitude 51′ 40″. South. The Latitude of *Saturn* by the *Caroline Tables* 56′ 20″. of *Jupiter* 41′ 30″. both North.

By the assumed Latitude of *Saturn* 56′ 20″. and his Distance from the *Heel* of *Castor* observed and corrected 48° 32′ 30″. I find their Difference of Longitude 48° 30′ 37″. therefore *Saturn* in ♌ 19° 21′ 19″.

By the Latitude of *Jupitur* assumed 41′ 30″. and his Distance from the *Star* 48° 45′ 20″. their Difference of Longitude 48° 43′ 56″. and *Jupiter's* Place in ♌, 19° 34′ 39″.

Hence *Jupiter's* Place in Consequence of *Saturn's* 13′ 20″. with which and the Distance of their Centers observed, the same night 20′ 12″. I find the true Difference of their Latitudes 15′ 20″ but half a Minute Different from what I assumed it on the Authority of the *Tables*.

The Apparent Motion of *Jupiter* from the 14 to the 18 day of *October* by an *Ephemeris* exactly calculated and made agreeable to these observations is 29′ 16″. of *Saturn* 15′ 01″. both Direct, hence the Motion of *Jupiter* from *Saturn* in four *days* is 14′ 15″. I say therefore as 4 days Motion or 14′

15″ : 15

15″: is to 4 days, or 96 Hours; so is 13′ 20″. (which *Jupiter* is past the *Conjunction* of *Saturn*;) to 90 Hours or 3 days 18 Hours; the time interlapsed since the *Conjunction*; which taken from the 17 day 15 Hours the time of my Observation, gives the true time of the *Conjunction* of the two Planets on the 13 day one and twenty hours after Noon or according to the Common account, the 14 day at 9 a Clock in the Morning.

At which time *Saturn* is with *Jupiter* in ♌ 19° 07′½ with 15′½. more Northern Latitude.

The *Acta Eruditorum Lipsiensia* p 366 make this *Conjunction* to have happned the same day in the same Longitude with the *Eleventh Star of Leo*; whose place they state in ♌ 19° 04′ Lat. 0′ 16″. N. with 14 Minutes difference of Latitudes betwixt the two *Planets*. But their Observation seems to have been made only by the Judgment of the bare Eye, without an Instrument, which considered, I wonder not that it differs at all, but rather that the Difference is so small from this Determination

On the 19 of *January* following, viewing the Planets then both *Retrograde* with the 16 foot Glass, I found them approached within a measureable Distance of each other.

| *An.* 1683. | | | |
|---|---|---|---|
| d. | h. ′ | | ° ′ ″ |
| *Jan.* 19 | 6 41 | Betwixt their Centers ——— | 0 33 28 |
| | 6 45 | ——— rep. | 0 33 24 |
| | 6 49 | Betwixt their remote Limbs ——— | 0 33 52 |
| 26 | 6 3 | Betwixt their Centers ——— | 0 15 8 |
| *Both being in ☍ to the* Sun. | 6 7 | ——— rep | 0 15 6 |
| | 7 00 | Betwixt their remote Limbs ——— | 0 15 31 |
| | 7 8 | By *T. Smith* ——— rep | 0 15 29 |
| | 7 12 | Betwixt their Centers ——— | 0 15 5 |
| | 7 14 | ——— rep. | 0 15 2 |
| | 7 17 | Betwixt their next Limbs ——— | 0 14 29 |
| | 7 20 | ——— rep | 0 14 31 |
| | 7 21 | ——— again | 0 14 26 |
| | 9 24 | *Jupiter* from the *Heel* of *Castor* ——— | 46 18 10 |
| | 9 26 | ——— rep. | 46 18 5 |
| | 9 28 | *Saturn* from the said *Heel* ——— | 46 8 50 |
| | 9 30 | ——— rep. | 46 8 55 |
| | 9 37 | *Jupiter* from the bright Star of the *Lions Head* E. ——— | 8 42 5 |
| | 9 39 | ——— rep. | 8 42 5 |
| | 9 40½ | *Saturn* from the same Star ——— | 8 29 35 |
| | 9 42½ | ——— rep. | 8 29 40 |

| | | h. ′ | | ° ′ ″ |
|---|---|---|---|---|
| | | 9 48 | *Jupiter* from the *Lions Heart* | 8 18 00 |
| | | 9 50 | rep. | 8 17 55 |
| | | 9 52 | *Saturn* from the same | 8 29 35 |
| | | 9 54 | rep. | 8 29 35 |
| | | 9 59 | The *Lions Heart* from *E* in the *Head* | 12 58 50 |
| | | 10 3 | The *Heel* of *Castor* from the *Lions Heart* | 54 34 20 |
| | | 10 8½ | The *Heel* of *Castor* from E ♌ | 46 24 45 |
| *Jan.* | 30 | 5 28 | Betwixt their Centers | 0 11 36 |
| | | 5 30 | rep. | 0 11 33 |
| | | 5 34 | Betwixt their Remote Limbs | 0 11 58 |
| | | 5 36 | rep. | 0 12 1 |
| | | 5 38 | Betwixt their Next Limbs | 0 11 1 |
| | | 5 41 | rep. | 0 11 00 |
| *Feb.* | 7 | 7 37 | Between their Centers | 0 28 35 |
| | | 7 40 | rep. | 0 28 34 |

From Observations formerly made, I have determined the true Places and Latitudes to this present time of,

| | s | ° | ′ | ″ | | ° | ′ | ″ | |
|---|---|---|---|---|---|---|---|---|---|
| *The Heel of Castor* | ♋ | 0 | 51 | 10 | *Lat.* | 0 | 51 | 40 | *South.* |
| Bright * in the *Lions Head.* E. | ♌ | 16 | 15 | 27. | | 9 | 41 | 07 | *North.* |
| *Lions Heart* | ♌ | 25 | 24 | 45 | | 0 | 26 | 20. | *North.* |

And from the above recited Measures, the true Distances of the *Planets* from these Stars *January* the 26th at $9^h\ 40'$ *p. m.* as follows.

| | ° | ′ | ″ |
|---|---|---|---|
| *Saturn* from the *Heel* of *Castor* | 46 | 09 | 00 |
| *Jupiter* from the same. | 46 | 18 | 10 |
| *Saturn* from the *Lions Heart.* | 8 | 29 | 40 |
| *Jupiter* from the same | 8 | 18 | 00 |
| *Saturn* from the bright * in the *Lions Head* E. | 8 | 29 | 40 |
| *Jupiter* from the same | 8 | 42 | 10 |

Whence I collect the true Places at this time.

| | s | ° | ′ | ″ | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| Of *Saturn.* | ♌ | 16 | 57 | 10 | *Lat.* | 1 | 13 | 10 |
| Of *Jupiter* | ♌ | 17 | 07 | 10 | *Lat.* | 1 | 01 | 30 |
| Difference of *Long.* | | 10 | 00 | | of *Lat.* | | 11 | 40. |



The

The *Retrogade* Motion of *Jupiter* from *Saturn* in 4 days, betwixt the Twenty sixt and thirtieth of this Month, by my correct *Ephemeris* is 12′ 15″. I say therefore as 12′ 15″. is to 4 days or 96 Hours ; so is 10′ 00″. (the Difference of the *Planets* present Longitudes,) to 78 Hours or three days six Hours, which therefore added to the time of that Observation *Jan.* 26d 9h$\frac{2}{3}$ gives the true time of the *Conjunction* Jan. 29*d.* 16*h.* or according to the common account *Jan.* 30 at 4*h.* in the Morning. At which time both the *Planets* are in ☊ 16° 41′$\frac{1}{2}$. with 11′$\frac{1}{2}$. Difference of Latitude or Distance from each other. Which is further confirmed by the Measured Distances of the *Planets* on the 30 at night before recited.

On the 26*th* day at 9*h* 40′ the *Sun's* True Place was by my Tables in our 17° 21′$\frac{1}{2}$ so that he was now about $\frac{1}{4}$ of a Degree past their Opposition.

Towards the Latter, end of the following *April* the Planet *Jupiter* began to approach *Saturn* again both being now Direct ; the 28 at night with the 16 foot Glass and *Micrometer* I measured the Distances

| *An.* 1683. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | d. | h: | ′ | | ° | ′ | ″ |
| *Apr.* | 28 | 10 | 21 | Betwixt their Centers———— | 0 | 32 | 35 |
| | | 10 | 23 | ———— rep. | 0 | 32 | 33 |
| | | 10 | 24 | Betwixt their Next Limbs ———— | 0 | 32 | 4 |
| | | 10 | 26 | ————rep. | 0 | 32 | 2 |
| | | 10 | 29 | Betwixt their Remote Limbs but not accurately Clouds interposing———— | 0 | 33 | 22 |
| *May* | 7 | 8 | 59 | *Jupiter* from the *Lions Heart*———— | 10 | 59 | 00 |
| | | 9 | 1 | ————rep. | 10 | 59 | 00 |
| | | 9 | 3$\frac{1}{2}$ | *Saturn* from the *Lions Heart* ———— | 10 | 58 | 50 |
| | | 9 | 5 | ———— rep. | 10 | 58 | 50 |
| | | 9 | 11 | *Jupiter* from *E* in the *Lions Head*— | 8 | 55 | 35 |
| | | 9 | 15 | ———— rep. | 8 | 55 | 40 |
| | | 9 | 17 | *Saturn* from the same Star———— | 8 | 39 | 40 |
| | | 9 | 18 | ———— rep. | 8 | 39 | 40 |
| | | 9 | 30 | Betwixt their Centers ———— | 0 | 15 | 38 |
| | | 9 | 33 | ———— rep. | 0 | 15 | 37 |
| | | 9 | 35 | Betwixt their Next Limbs———— | 0 | 15 | 3 |
| | | 9 | 36 | ————rep. | 0 | 15 | 00 |
| | | 9 | 40 | Betwixt their Remote Limbs———— | 0 | 16 | 2 |
| | | 9 | 42 | ————rep. | 0 | 15 | 58 |
| | | 10 | 20 | *Jupiter* from *B* in ♍ ———— | 38 | 11 | 45 |
| | | 10 | 23 | ————rep. | 38 | 11 | 45 |
| | | 10 | 26 | *Saturn* from the same Star———— | 38 | 10 | 55 |
| | | 10 | 28 | ————rep | 38 | 10 | 45 |
| *May* | 11 | 9 | 28 | Betwixt their Centers———— | 00 | 20 | 2 |
| | | | 31 | ————rep. | 00 | 20 | 2 |
| | 16 | 9 | 22 | Betwixt their Centers———— | 00 | 24 | 4 |

From

From these Observations I State the Distances of the *Planets* from the Fixed Stars *May* 7th at 9h 5'. *p. m.* as follows,

| | ° | ' | " |
|---|---|---|---|
| *Saturn* from the *Lyons Heart* ———————————— | 10 | 58 | 50 |
| *Jupiter* from the same ———————————— | 10 | 59 | 00 |
| *Saturn* from E in the *Lyons Head* ———————— | 8 | 39 | 40 |
| *Jupiter* from the same ———————————— | 8 | 55 | 35 |

| | ° | ' | " | | ° | ' | " | |
|---|---|---|---|---|---|---|---|---|
| Hence the true Longitude of *Sat.* ♌ | 14 | 27 | 42. | *Lat.* | 1 | 12 | 46. | *North.* |
| of *Jupiter* ♌ | 14 | 26 | 37. | *Lat.* | 0 | 56 | 43. | *North.* |
| Difference of Longitude ———————— | | 1 | 4. | *Lat.* | 0 | 16 | 3. | |

The Difference of Latitudes something exceeds the Distance measured with the *Micrometer*, by reason that the Wind then shaking the *Sextant* permitted us not to be so exact as usually, but the Difference, being less than half a Minute, I esteem inconsiderable.

The Diurnal Motion of *Jupiter* from *Saturn* was now 3' 15". it holds therefore as 3' 15". (one days Motion,) is to one Day or 24 Hours; so 1' 4". (the Distance of *Jupiter* from the ☌ with *Saturn*) to 8 Hours, the Interval betwixt the Observation and following *Conjunction*, which was therefore 17h after Noon, or according to the vulgar reckoning, *May* 8. at 5 a Clock in the Morning. At which time the true Place of the *Planets* is ♌ 14° 28'¾, the Difference of their Latitudes, 15' 40". *Saturn* being so much more Northerly than *Jupiter*.

In all, best esteemed, *Astronomical Tables* extant, the Mean Motions of the *Planet Saturn* are too Swift, of *Jupiter* too Slow considerably: Hence it came to pass that they made the *Direct Conjunctions* some days Later, the *Retrograde* earlier than they were found by Observation.

2. *Oct.* 26. st. n. 1h 40'. mane Situm *Jovis* & *Saturni* Tubo & *Micrometro* ex voto deprehendere mihi obtigit: quo tempore simul Fixa quædam satis conspicua (quod notatu dignum) dictis *Planetis* satis prope adhærebat. *Jupiter* sese cum tribus *Comitibus* tum offerebat, forte & Quartus adfuit, sed ob Nubeculas haud fuit conspectus. *Saturnus* distabat à *Jove* 16' 44"; *Jupiter* à Stella (ni fallor in *Armo Dextro* ♌) 27' 55"; Rursus *Saturnus* à dicta Stella 38' 1". Stella dicta versatur modo juxta nostrum Catalogum in 19° 2' 9". ♌ & Latit. 0° 20' 45". B. Die 30 *Octob.* 5h mane Distantia ♄ & ♃, 25' 5" extitit; Unde certo colligere licuit *Conjunctionem* jam ante complures dies celebratam esse quam Ephemerides Calculusque primum die 3 *Novem.* exhibent.

*At* Dantzick; *by* M.Hevelius. *n.* 143. *p.* 18. & n. 151. *p.* 326.

Id quod subsequentes Observationes adhuc clarius demonstrant. Nam Loco, quod Distantia ♃ & ♄ de die in diem (si ☌ instaret) paulatim Minor fieri debebat, continuo Aucta est. Die 1 *Nov.* hor. 2 mane, ope *Micrometri* nostri dicta Distantia extitit 31' 35". Et Die. 2 *Nov.* eandem Distantiam rursus reperi 35' 21". Die 3. *Nov.* mane hor. 1. jam 39' 9"; Die 4. *Nov.* cælo perquam sereno adhuc paulo Maior dicta Distantia inter ♄ & ♃ deprehensa.

Alteram *Conjunctionem*, quod attinet quæ ex Retrogressionibus horum *Planetarum* incidere debebat secundum Ephemeridum Scriptores die 26. *Januarii* hujus Anni currentis 1683. Observationes nonnullas ac præcipuas habitas hic apponam.

| *An.* 1683. | *Temp. ex alt. corr. vesp.* | | | *Observationes.* | *Distantiæ.* | | |
|---|---|---|---|---|---|---|---|
| | h. | ′ | ″ | | ° | ′ | ″ |
| *Feb.* 1 | 6 | 40 | 0 | Dist. *Satur.* & *Jov.* inventa est—3300 | 0 | 25 | 5 |
| *Feb.* 2 | 9 | 30 | 0 | Dist. *Satur.* & *Jov.*——————2900 | 0 | 22 | 3 |
| *Feb.* 3 | 9 | 00 | 0 | Dist. *Satur.* & *Jov.*——————2500 | 0 | 19 | 0 |
| *Feb.* 4 | 10 | 00 | 0 | Dist. *Satur.* & *Jov.*——————2300 | 0 | 17 | 29 |
| *Feb.* 5 | 8 | 30 | 0 | Dist. *Satur.* & *Jov.*——————2100 | 0 | 15 | 59 |
| *Feb.* 6 | 7 | 51 | 0 | Dist. *Satur.* & *Jov.*——————1850 | 0 | 14 | 6 |
| *Feb.* 7 | 8 | 17 | 19 | Dist. *Satur.* & *Jov.*——————1700 | 0 | 12 | 55 |
| *Feb.* 8 | 6 | 10 | 0 | Dist. *Satur.* & *Jov.*——————1600 | 0 | 12 | 10 |

*Feb.* 9. vesp. $9^h\ 0'\ 8''$. *Planetas* Tubo conspexi per dehiscentes densissimas nubes; Oculoque fugitiv odeprehendebam *Conjunctionem* ipsam jam celebratam esse nocte præcedente inter 8 & 9. *Febr.* Nam dicta Distantia paulo ampliori modo apparebat. Prout etiam 11. *Febr.* vesp. $9^h\ 0'\ 0''$ factum est: nam Distantia inter *Saturnum* & *Jovem* erat 2000, hoc est $0°\ 15'\ 12''$ *Micrometro*, quæ die 8. *Feb.* tantum inventa est $0°\ 12'\ 10''$.

Ad hæc *Conjunctionem* jam esse celebratam, exinde certò constabat, quod uterque *Planeta* cum *Ventre Ursæ Majoris* non amplius, ut quidem die 8. *Feb.* contingebat, in linea subsisteret recta; tum etiam quod *Saturnus* non amplius ad Orbitam *Jovialium* sub Angulo Recto commoraretur.

Quo autem hoc ipsum eo evidentius redderetur, Observationes nonnullas diebus subsequentibus continuatas hic apponam.

| | h. | ″ | | ′ | ″ |
|---|---|---|---|---|---|
| *Feb.* 12 | 9 | 0 | Dist. *Satur.* & *Jov.* capta est—2200 | 17 | 6 |
| 13 | 7 | 15 | ——————————2550 | 19 | 24 |
| 14 | 9 | 0 | ——————————2900 | 22 | 3 |
| 17 | 6 | 0 | ——————————3750 | 28 | 30 |
| 20 | 9 | 0 | ——————————5250 | 30 | 12 |

Ex hisce igitur Observationibus continuatis satis superque liquet, cum de die in diem *Planetæ* ab invicem magis magisque discesserint, quod *Conjunctio* ipsa inter 8 & 9. *Febr.* revera jam fuerit observata.

Denique adjiciendum pariter censeo Astrophilorum gratia, quidnam circa Tertiam eorum *Conjunctionem*, Mense *Maii* observaverim.

*An.* 1683.

| An. 1683. | Temp. Sec. Hor. ambul. vesp. | | Observationes. | Distantiæ. | |
|---|---|---|---|---|---|
| | h. | ′ | Dist. *Saturni* & *Jov.* inventa est *Micrometro* | ′ | ″ |
| *Maii* 8 | 9 | 6 | 4300 | 32 | 41 |
| 10 | 9 | 14 | 3750 | 28 | 30 |
| 11 | 9 | 10 | 3450 | 26 | 13 |
| 12 | 8 | 45 | 3050 | 23 | 10 |
| 13 | 9 | 15 | 2800 | 21 | 17 |
| 14 | 9 | 45 | 2550 | 19 | 23 |
| 15 | 9 | 30 | 2400 | 18 | 15 |
| 16 | 9 | 30 | 2250 | 17 | 6 |
| 17 | 9 | 40 | 2150 | 16 | 10 |
| 18 | 10 | 0 | 2100 | 15 | 58 |
| | | | Hac die extitit Vera *Conjunctio.* | | |
| 20 | 11 | 45 | 2450 | 18 | 37 |
| 21 | 9 | 15 | 2650 | 20 | 9 |
| 22 | 9 | 20 | 2900 | 22 | 3 |
| 23 | 9 | 5 | 3250 | 24 | 43 |
| 24 | 10 | 6 | 3600 | 27 | 22 |
| 25 | 9 | 30 | 4000 | 30 | 25 |
| 26 | 11 | 0 | 4453 | 33 | 50 |
| 27 | 9 | 25 | 4900 | 37 | 15 |
| 28 | 9 | 56 | 5325 | 40 | 29 |

Ex quibus Observationibus cuilibet nunc liquidum est, cum Distantia *Saturni* & *Jovis* de die in diem continuò decreverit ad 18 *Maii*, & ab hac die rursus creverit, *Conjunctionem* horum *Planetarum* eadem ipsa die etiam accidisse, & quidem, ( uti ex observationibus diei 15, & 20. *Maii* patet ) hora antemeridiana 10: quæ secundum Ephemeridum Scriptores die 26. primum ingruere debuit. Sic ut hæc tertia & ultima hujus Anni *Conjunctio Magna* pariter haud mediocriter Tabulas eluserit; ita quidem quod citius ultra 8 integros dies revera contigerit.

De cætero, hanc ultimam *Conjunctionem* jam celebratam die sc. 18 *Maii* vesp. hor. 10. fuisse, ex eo liquet, quod *Saturnus* jam non amplius eo tempore ad Orbitam *Jovialium* sub Angulo versabatur Recto; deinde etiam, ( uti ex subsequentium dierum observationibus videre est, ) quod à die 18 *Maii* ad diem 28 quousque *Micrometro* Distantiam *Saturni* & *Jovis* dirimere potui, continuo aucta fuit.

Ultimo, notandum quoque occurrit, cum die 21 *Maii* vesp. inter reliquas observationes etiam Distantiam utriusque *Planetæ* à Stella *Superiori in Pede*

*Anteriore*

*Anteriore Dextro Ursæ Majoris* ſextante obtinuerim, atque eadem dicta Stella cum utroque *Planeta* in eadem ſimul fere recta tum conſiſterent linea, quod promptum ſit cuilibet Rerum Cæleſtium cultori dijudicare, an obſervationes meæ; *Sextante* noſtro novo ſimul obtentæ, cum obſervationibus *Micrometro* captis omnino etiam conveniant. Sextante Diſtantia *Jovis* à dicta Stella erat 32° 38′ 40″, & *Saturni* 32° 19′ 45″; ſic ut genuina utriuſque *Planetæ* Diſtantia extiterit 18′ 55″; *Micrometro* vero ea ipſa Diſtantia inventa eſt ea ipſa die 21 ſc. *Maii* 20′ 9′; ſic ut *Sextante* obtenta 1′ 14″. Minor extiterit Non eſt autem, mi Amice, quod exiſtimes, hoc vel illo Inſtrumento me aberraſſe, minime profecto; quippe *Saturnus* & *Jupiter* cum dicta Stella non omnino in linea recta ſubſiſterunt; prout cuilibet ex Globo & Calculo patet, hincque neceſſario illa diſtantia, *Sextante* derivata, etiam paulo minor oportuit eſſe.

*The Mean Conjunctions of* Saturn *and* Jupiter; *by Mr.* Flamſteed. *n.* 149. *p.* 254.

LXXXVII. *Riccioli* in the Second Part of the firſt Tome of his *Almageſt*, has given us a Table of all the mean *Conjunctions* of the two Superiors from the Creation till the *Year* of *Chriſt* 2358, but very courſe and incorrect. I have therefore made a new one for 43 *Revolutions* which are completed in 853 *Julian Years*, and 235 Days from their mean Motions which I have corrected by very late Obſervations. This being the *Period* of the *Greateſt Conjunctions*, after which Space of Time they return to the ſame Place of the *Zodiack* within $\frac{4}{5}$ of a Degree.

The *Ordinary Conjunctions* happen once in Twenty Years, or more preciſely in 19 *Julian* Years, and 312 Days, in which time *Saturn's* Mean Motion is 8ˢ 2° 48′$\frac{1}{2}$, *Jupiter's* the ſame above one *Revolution*.

Theſe are commonly termed the *Leſſer* of the *Great Conjunctions*, which continue in *Signs* of the ſame *Triplicity* for 10 *Revolutions* to each other, or 198 Years: each *Conjunction* according to the Mean Motions being 8ˢ 2° 48′$\frac{1}{2}$ removed from the preceeding, ſo that if any *Conjunction* was made upon the firſt Point of ♈, the next following ſhall be in 2° 48′ of ♐, and all the following for 198 Years ſhall fall in ♈, ♌ and ♐, *Signs* of the ſame *Triplicity*.

But the 11 *Conjunction* after, ſhall happen in the firſt Degree of ♍, and the following ten *Conjunctions* in ♉, ♍, and ♑, *Signs* of the ſame *Triplicity*. Of theſe the firſt is called by our Aſtrologers the *Greater Conjunctions*.

But the *Greateſt* is, when after 43 *Conjunctions* compleated in 853 Years, 235 Days, the *Mean Conjunctions* having been made in all the *Signs* return to that Point of the *Ecliptick* from whence they began: tho' I muſt confeſs had I been to name them I ſhould have called thoſe the *Greateſt* which happen in the *Signs* ♋ and ♌, becauſe then the *Planets* Riſe higheſt, and are longeſt viſible in our *Horizon*, as alſo being near their North *Nodes*, they approach neareſt, and if they have any extraordinary Influence (which *Naboyd* thinks either they have not, or if they have, we underſtand not) it muſt according to their Axiomes be ſtrongeſt.

Thoſe which happen in ♑ and ♒ I ſhould call the *Greater* or *Middle*, becauſe the *Planets* being then near their South *Nodes*, may approach each other again

again very nearly though they riſe not high in our *Horizon*, being in Southern *Signs* ; the reſt might be accounted the *Leſſer* or *Ordinary*.

The *Mean Conjunction* of *Saturn* and *Jupiter* this Year 1683. was on the 14th Day of *January* old Stile, at 12 Hours after Noon in the Meridian of *London*, at which time the Mean Motions of both the *Planets* were 4$^{s}$ 11° 45′. This may be the *Radix* of the following *Table*.

A Table of the *Mean Conjunctions* of *Saturn* and *Jupiter*.

| *Intervals.* | | | | | | *Intervals.* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Revolutions Complete.* | *Time.* | | *Motion.* | | | *Revolutions Completed.* | *Time.* | | *Motion.* | | |
| | y. | d. | s. | ° | ′ | | y. | d. | s. | ° | ′ |
| 1 | 19 | 312 | 8 | 2 | 48 | 23 | 456 | 219 | 6 | 4 | 31 |
| 2 | 39 | 258 | 4 | 5 | 37 | 24 | 476 | 165 | 2 | 7 | 19 |
| 3 | 59 | 204 | 0 | 8 | 25 | 25 | 496 | 111 | 10 | 10 | 8 |
| 4 | 79 | 150 | 8 | 11 | 13 | 26 | 516 | 57 | 6 | 12 | 56 |
| 5 | 99 | 96 | 4 | 14 | 1 | 27 | 536 | 3 | 2 | 15 | 44 |
| 6 | 119 | 42 | 0 | 16 | 50 | 28 | 555 | 315 | 10 | 18 | 32 |
| 7 | 138 | 353 | 8 | 19 | 38 | 29 | 575 | 261 | 6 | 21 | 21 |
| 8 | 158 | 299 | 4 | 22 | 26 | 30 | 595 | 207 | 2 | 24 | 9 |
| 9 | 178 | 245 | 0 | 25 | 15 | 31 | 615 | 153 | 10 | 26 | 57 |
| 10 | 198 | 191 | 8 | 28 | 3 | 32 | 635 | 99 | 6 | 29 | 45 |
| 11 | 218 | 137 | 5 | 00 | 51 | 33 | 655 | 45 | 3 | 2 | 34 |
| 12 | 238 | 83 | 1 | 3 | 40 | 34 | 674 | 356 | 11 | 5 | 22 |
| 13 | 258 | 29 | 9 | 6 | 28 | 35 | 694 | 302 | 7 | 8 | 10 |
| 14 | 277 | 340 | 5 | 9 | 16 | 36 | 714 | 248 | 3 | 10 | 59 |
| 15 | 297 | 286 | 1 | 12 | 4 | 37 | 734 | 194 | 11 | 13 | 47 |
| 16 | 317 | 232 | 9 | 14 | 53 | 38 | 754 | 140 | 7 | 16 | 35 |
| 17 | 337 | 178 | 5 | 17 | 41 | 39 | 774 | 86 | 3 | 19 | 24 |
| 18 | 357 | 124 | 1 | 20 | 29 | 40 | 794 | 32 | 11 | 22 | 12 |
| 19 | 377 | 70 | 9 | 23 | 18 | 41 | 813 | 343 | 7 | 25 | 00 |
| 20 | 397 | 16 | 5 | 26 | 6 | 42 | 833 | 289 | 3 | 27 | 49 |
| 21 | 416 | 327 | 1 | 28 | 54 | 43 | 853 | 235 | 0 | 00 | 37 |
| 22 | 436 | 273 | 10 | 1 | 43 | | | | | | |

By this *Table* to find the Time of any *Mean Conjunction*, Past or Future, nearest to any Place of the *Zodiack*; for Times Past, substract the Longitude of the given Place from the Longitude of the *Radix* $4^s$ 11° 45′. the residue seek in the last Column of the *Table*; if you find not the precise Number take the next to it, against this you have in the second Column the Years and Days, and in the first the Number of *Conjunctions* past since any was made in that Place, substract the Years and Days from 1683. *Jan.* 14. and the Motion from $4_s$ $11_o$ 15′. so have you the true Time of the *Mean Conjunction*, and Longitudes of the *Planets* then.

But for Times to come, substract the *Radix* from the given Place, seek the Residue as before in the last Column; if you find it not, take that you find nearest it; against which, as before, you have in the second Column, the Years and Days; in the first, the Revolutions future; for Example.

If it were required to know when the last *Conjunction* happen'd in the first Deg. of ♒, Substracting ♒ or Ten *Signs* from $4_s$ $11_o$ 15′. the Residue is $6_s$ $11_o$ 15′. which seeking I cannot find in the third Column of the *Table*, but I find $6^s$ 12° 56′. which is not two Degrees more, and against them 516 Years, 57 Days, and in the first Column 26. for the Number of *Conjunctions* interlapsed. Substracting 516 Years, 57 Days from 1683. *Jan.* 14. there remains 1166 Years, 322 Days, which shews me that the *Conjunction* was in the *Year* 1166. *Nov.* 18. and substracting the Motion $6_s$ 12° 56′, from $4_s$ 11° 45′. it points me to the Place in $9^s$ 28° 49′.

Or if the Time of the first *Conjunction* in ♎ to come were demanded. I substract the *Radix* $4^s$ 11° 45′. from 6 *Signs*, the Residue $1^s$ 17° 15′. I seek in the *Table* but find it not, I take therefore the next to it, $1^s$ 20° 29′. the next to it, against which stands 357 Years, 124 Days, these added to 1683. *Jan.* 14. give me the Year 2040, and 138 Days, *May* 18. for the Time of this *Conjunction*, and adding the $1^s$ $20_o$ 29′. to the *Radix* $4^s$ $11_o$ 45′. it makes $6^s$ 2° 14′. for the true mean Longitude of this *Conjunction*.

From the *Mean Conjunction* the Apparent may be found by the help of a Planetary Instrument, or the usual Astronomical Tables.

*The Shadows of* Jupiter's Satellites *observed*; *by S.*Campani. *n.* 1. p. 3.

LXXXVIII. 1. *S. Campani* affirms, that he hath remarked in the *Belts* of *Jupiter*, the *Shadows* of his *Satellites*, and followed them, and at length seen them Emerge out of his *Disk*.

*By* M. Cassini *and others*. n. 8. *p.* 143. n. 10. *p.* 171. n. 82. *p.* 4039.

2. *M. Cassini*, after he had discover'd (by the means of those excellent *Glasses* of 35 Foot made by *S. Campani*) the *Shadows* cast by the *Satellites* of *Jupiter* upon his *Disk* when they happen to be between the *Sun* and *Him*; and after He had also distinguisht their Bodies upon the *Disk* of *Jupiter*; made some Predictions when they should appear, to the End that the Curious might be convinc'd of this Matter by their own Observations.

Some of these Predictions have been verified, not only at *Rome*, and in other Places of *Italy*, but also at *Paris* by *M.Auzout*, and in *Holland* by *M. Hugens*; particularly *Sept*, 26. 1665. at half an Hour after seven a Clock, one of these *Shadows* was seen both in *France* and in *Holland*.

These *Spots*, have this Peculiar, which distinguisheth them from all others, that they are found precisely in that Place of *Jupiter*, where some *Satellite* is seen by the *Sun*; that they go from the Oriental Limb to the Occidental of the Disque of *Jupiter*, with a Motion always equal to that of the *Satellite*; that in respect to us they preceed the *Satellite*, before the Opposition of *Jupiter* to the *Sun*, and follow him after the Opposition; that the further *Jupiter* is distant from the Opposition, the greater is the apparent Distance of the same *Satellite*; that at divers Times of the Year, this Distance changeth in Proportion of the Annual *Parallax* of the *Satellite*, according as he is differently seen by the *Sun*, and by the Earth; and that at one and the same time of the Year, when divers *Satellites* happen to be between *Jupiter* and the *Sun*, the *Spots* correspondent to them are distant from them in Proportion of the Semidiameters of the Circles of the same *Satellites*.

3. *An.* 1666. *Jan.* 26. about 3h 15′ in the Morning, I perceived (with a 60 Foot Glass) near the Middle of the Zone *d*, a very round *Spot*, like that represented at *g*, which was not to be percieved about half an Hour before; and I observed it in about 10′. time to be gotten almost to *d*, keeping equal Distance from the *Satellite h*, which moved also Westwardly, and was joined to the Disk at *i*, at 3h 25′, so that it was sufficiently evident that this black *Spot* was nothing else, save the Shadow of the *Satellite h*, Eclipsing a part of the Face of *Jupiter*. The other three *Satellites* in the time of this *Eclipse* were Westwards of the Body of *Jupiter*.

*By Dr.* Hook. *n.* 14. *p.* 246. *Fig.* 130.

LXXXIX. 1. Anni duo & amplius elapsi sunt ex quo Eruditissimus *Richardus Townleius* Armiger, mihi Maximas *Jovialium* Siderum à Centro *Jovis Digressiones*, à seipso observatas, necnon & Motus cujusque Medios, Motuumque illorum Radices, ab Observationibus ejus deductas, *Townleio* suo accommodatas, communicavit. Ab eodem deinceps *Ephemerides* tuas, Clarissime *Cassine*, *Mediceorum Siderum An.* 1668. impetravi; quibus quando cum Motus tum Motuum Radices, necnon & summas Elongationes à Te constitutas, nonnihil à D. *Townleii* inventis dissidere comperui, & Ego, quod ipse impensius hortatus est, nonnullas primâ quoque Occasione Observationes instituere Operæ fore pretium duxi. *Micrometro* itaque & Tubo 14. *Ped. An.* 1672. Mense *Mar. Stil. Jul.* sequentia qua potui Cura Experimenta prima feci, Observationibus, in majorem Certitudinem, identidem quaque Nocte iteratis.

*The Elongations of* Jupiter's Satellites; *by* Mr. Flamsteed. *n.* 82. *p.* 4036, 4037. *n.* 94. *p.* 6033. *n.* 96. *p.* 6094.

*An.* 1672. *Mar.* $19^d\ 7^h\ 11'$. Limb. *Jovis* remotior à 4*to. Satellite* dist. 1601 = 9′ — 34″. 9′ 37″

27. 8. Limb. remotior ab eodem 4*to. Satellite* 1591 = 9 — 30.

*n.* 28. *p.* 4037.

28. 8. Eadem Distantia ——— 1598 = 9 — 33

*Jovis* Diameter pluribus observationibus reperta 128. Ergo Semidiameter ejus 64; quâ divisis Distantiis observatis, apparentes fient *Satellitis* à Limbo *Jovis* Remotiori Distantiæ in Semidiametris ejus,

| | d. | ſd. | ′ | | ſd. | ′ |
|---|---|---|---|---|---|---|
| *Mar.* | 19. | 25 | 1 | Sublatâ ſemidiam. à centro fient | 24 | 1 |
| | 27. | 24 | 51 | | 23 | 51 |
| | 28. | 24 | 58 | | 23 | 58 |

Cujus tunc Motus à *Jove* & diſtantiæ à centro ipſius fuere, ſecundum numeros tuos, ut hic.

| | d. h. ′ | s. o ′ | ſd. ′ |
|---|---|---|---|
| *Mart.* | 19 7 11 | 8 25 33 | 22 56 |
| | 27 8 .. | 2 19 35 | 22 37 |
| | 28 8 .. | 3 11 13 | 22 34 |

Aberat ergo *Satelles* ab extima Elongatione, in prima obſervatione, tantum 4′; in ſecunda 23′; in tertia 26′; Semidiametri ſcrupulos ſexagenarios; quos propterea ſi obſervatis *Elongationibus* modo debito adjiciamus, fient maximæ Digreſſiones, hujus 4*ti Satellitis*, à centro *Jovis* per primam Obſervationem 24ſd 5′; per ſecundam 24ſd 14′; per tertiam, 24ſd 24′; quam *Tu* ſtatuiſti tantum 23ſd. *R. Townleius* 24ſd 72.

Harum Elongationum poſteriores duas, accuratiores exiſtimo, quippe quibus inveſtigandis obſervationes commodas, omni qua cura poteram, peregi; priorem deinde inter plura noctis 19æ. Adverſaria inveni, quam perinde exquiſitè captam haud auſim affirmare. Utcunque tamen Obſervationem adjeci, quippe quæ non adeò à ſequentibus diſſentit, quin eas poſſit confirmare, necnon oſtendere, perparùm (ſi quicquam) minùs, Siniſtram quàm dextram verſus, hunc Satellitem à *Jove* Elongari.

Sed tamen inter obſervandum ſenſi, Aeris & Venti motum, quatiendo vel agitando Tubum, (ad erectam Abietem, ope funis, & trochleæ ſub dio penſilem) obſervationem reddere difficilem; quin & frequenter efficere, ut nimis ſtrictas acciperem Diſtantias. Quamobrem à pluribus hujuſmodi obſervationibus, quæ ſummam curam & præciſionem depoſcunt, eo uſque ſuperſedere conſtitui, donec commodiorem iis inſtituendis locum aptarem, quem tandem paravi. In Feneſtra quadam Ligneam Machinam, brevis adinſtar Scalæ, aptari curavi; cujus ope ei impoſitus Tubus quaquaverſum converti potuit, nec à ventis, niſi admodum turbidis, hinc inde, ut ſubdio fuit, agitari. Huic impoſito Tubo, *An.* 1683. *Apr.* 4. Veſp. Meipſum obſervationibus omni diligentia peragendis accinxi, nec fruſtra quidem: Etenim, cælo tunc admodùm ſereno, omnes quatuor *Satellites*, per Tubum Lentium Convexarum, conſpexi, & eorum infra ſcriptas à Limbo *Jovis*, cuique remotiori, Diſtantias Dimenſus ſum; ſcil.

3 985 iterum—988
2 628 ————636
1 425 ————427
4 272 ————272 Altitudo *Jovis* Quadrante ferè bipedali capta 24°—00′ Ergo hora apparens *Derbiæ* 8h 26′ *p. m.* & tunc 4^tus *Satelles* infra lineam utrinque per extimos *Satellites*, apparuit; ſed vix plenam, ni fallor, Semidiametrum.

*Jovis* Diameter, identidem repetitis obſervationibus, reperta 133; Semidiameter ergo 66½, quæ obſervatis ſublata Diſtantiis, fient interſtitia inter Centrum *Jovis* & *Comitis Primi*, 360; *Secundi*, 569; *Tertii*, 921; *Quarti* 205; quibus per 66½ diviſis, prodibunt viſæ Elongationes à centro *Jovis*, in ipſius Semidiametris.

| | ſd. | ′ | | | s. | ° | ′ | ſd. | ′ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 5 | 25 | Motus *Satellitum* à *Jove* & | 1 | 9 | 4 | 52 | 4 | 59 |
| 2 | 8 | 33 | Remotiones apparentes ſe- | 2 | 2 | 22 | 47 | 7 | 57 |
| 3 | 13 | 51 | dum *Tuas Tabulas*, *Derbiæ* | 3 | 2 | 20 | 26 | 12 | 48 |
| 4 | 3 | 5 | reductas, fuere; | 4 | 4 | 23 | 49 | 2 | 29 |

Defecit ergo *Satelles Primus*, 1′ tantum; *Secundus* 3; *Tertius* 12; Scrupulis Semidiametri ſexagenariis à ſumma Elongatione, quos propterea ſi obſervatis addamus, fient extremæ Digreſſiones.

| | ſd. | ′ | | ſd. | | ſd. | ′ |
|---|---|---|---|---|---|---|---|
| *Primi*, | 5 | 26 | Quas Tu ponis, | 5 | Attamen D. *Townleius*, | 5 | 31 |
| *Secundi*, | 8 | 37 | | 8 | ut in Schedis aliquibus | 8 | 47 |
| *Tertii*, | 14 | 2 | | 13 | reperio, | 13 | 28 |

Commoda rurſus præviſa dari opportunitas *Apr.* 11: veſperi: quamobrem, cum non ab uno aut altero Experimento diſtantias has duxerim definiendas, habitis tunc etiam Obſervationibus, ulterius mecum inquirere inſtitui; quas cùm primum auſpicabar, Cælum circa *Jovem* raris adeo nubibus tectum erat, ut ſubobſcurius non niſi aliquando *Satellites* potuerim conſpicere; quorum tamen à limbo *Jovis* remotiori, ut tulit aer, cæpi diſtantias; nimirum,

*Hor.* 7½ *p. m.* 3 947 iterum 932
2 628————614
1 405. Facto tamen Cælo protinus ad votum ſereno, accuratius notavi.

3 947
2 622
1 405
4 942. Iterum 957, *Alto* ♃^ve. 24° 00′ ergo hora apparens 7h 56′. *Satelles Quartus* paulo ſupra lineam, per *Primum* & *Secundum* ductam, apparuit;

apparuit; *Tertius* infra eam, ſed & aliquando exiſtimavi in ea. *Jovis* capta diameter 132, Semidiameter ergo 66. obſervatis quæ ſubducta Diſtantiis interſtitium dabit inter centrum *Jovis* & *Primi*, 339; *Secundi*, 556; *Tertii*, 881; *Quarti*, 891; quibus ſigillatim per 66 diviſis, prodeunt Elongationes apparentes à centro *Jovis* in Semidiametris ejuſdem, *Primi* quidem 5ſd. 8′; *Secundi*, 8ſd 25′; *Tertii*, 13ſd 21′; *Quarti*, 13ſd 30′.

*Satellitum* Motus Medii à *Pleni-Mediceis*, cum Diſtantiis eorum à centro *Jovis*, ſecundum numeros tuos fuere, ut in hac tabellula exarantur: Unde videre eſt, *Primum* à ſumma Elongatione abeſſe ſcrupulos Semidiametri 10′; *Secundum*, 26′; *Tertium*, 38′; quos propterea ſi obſervatis Elongationibus adjiciamus, fient *Maximæ* hinc deducendæ *Digreſſiones*.

| | s. | ° | ′ | ſd. | ′ |
|---|---|---|---|---|---|
| 1 | 8 | 15 | 35 | 4 | 50 |
| 2 | 2 | 10 | 59 | 7 | 34 |
| 3 | 2 | 12 | 2 | 12 | 22 |
| 4 | 10 | 25 | 8 | 13 | 15 |

| | ſd. | ′ |
|---|---|---|
| *Primi* | 5 | 18 |
| *Secundi* | 8 | 51 |
| *Tertii* | 13 | 59 |

perparum ab iis, quas obſervationibus noctis Quartæ deduximus, diſſentientes.

His tamen utriſque vicibus *Intimus Satelles* ad Lævam, *Secundus* & *Tertius* ad dextram, à *Jove* apparuere; ſed *Apr.* 15. veſp. *Tertium* à ſiniſtra, in maxima Elongatione appariturum prævidi, cui propterea Phænomeno invigilare operæ fore pretium duxi, *nimirum* ut perſpicerem, num eadem eſſet ejuſdem *Satellitis* ad manum utramque à centro *Jovis*, ſumma Remotio. Cælum nocte obſervationi antedicta ſudum erat; ſic pro voto obſervavi circa *hor.* $7\frac{1}{2}$. *Tertii* Diſtantiam 955; & *Jovis* Diam. 131. *Jovis* ergo Semidiameter $56\frac{1}{2}$, obſervatæ quâ ſubductâ Diſtantiæ, fit interſtitium inter centrum *Jovis* & *Satellitem*, 889; quod per eandem Semidiametrum diviſum viſibilem dat Elongationem *Comitis* à Centro *Jovis* in Semidiametris ipſius 13ſd. 35′. Motus *Satellitis* medius erat 3s 14° 9′. Locus *Jovis* verus ♎ 10° 27′. Ergo *Planeta* à *Pleni-Mediceo* 9s 3° 42′ aberat, à ſumma Elongatione tantum ſcrupulos 3; quos ſi obſervatæ digreſſioni 13ſd 35′ adjiciamus, fiet maxima ad *Siniſtram* hac vice 13ſd 38′, parte nimirum tertia Semidiametri minor quam ad *Dextram*, bis conſpirantibus notis, obſervavimus: Quod mihi videtur innuere, eſſe aliquam Centri Orbitæ hujus *Planetæ* à Centro *Jovis Excentricitatem*.

*An Instrument for finding the distances of Jupiters Satellites from his Axis; by Mr.* Flamſteed. *nu.*178. *p.*1262.

2. The Little Circle in the Middle repreſents the Planet *Jupiter*, the four Concentrick Circles, the Proper Orbits of his four *Satellites*, duly proportioned to the Breadth of his Body; the Diſtances betwixt the parallel Lines interſecting them, being each equal to one of his Semidiameters.

Fig. 142.

The 4 divided Circles next without theſe, are diſtinguiſhed into ſo many parts as there are days and hours in each *Sattellite's* Revolution; the innermoſt of them ſerving for the *Firſt* or *Innermoſt Satellite*; that next it, for the 2d; that next without this for the 3d; and the outermoſt for the 4th; above which is a ſmall divided Arch of 15 Degrees.

By this to find the Diſtances of the *Satellites* from ♃. *Axis* to a propoſed Time:

1. Find

1. Find the *Parallax* of *Jupiter*'s Orb to the Time propoſed, and note whether it be to be Added or Subſtracted.

2. Extend the Thread from the Center of the *Inſtrument* over the *Parallax* numbred in the ſmall Arch, it cuts off in the four divided Circles, ſo many hours as each *Satellite* ſpends in paſſing from the *Axis* of the Shadow to the *Axis* of ♃ viewed from our Earth; theſe I call the Simple *Parallactick Intervals*, which if the Parallax was to be added, are alſo Additional, if to be ſubſtracted, Subductive.

3. To theſe *Parallactick Intervals* add the Times of half the Duration of the *Eclipſe* of each *Satellite*, which for the *Firſt* may be aſſumed 1h 10′; for the 2*d*, 1h 30′, (greater exactneſs being needleſs); but for the 3*d*, and 4*th* when *Eclipſed*, (their *Immerſions* into the Shadow and *Emerſion* from it being commonly given in the *Catalogues*) take half the Difference of theſe Times at the next *Eclipſe*, to the Time propoſed, for the half Duration, and Add them to the Simple *Parallactick Intervals*, ſo have you them Augmented. But as often as the 4*th Satellite* is not *Eclipſed*, (which is two Years in every ſix) its Interval needs no Augmentation.

4. Find in the *Tables* the Times of the *Eclipſes* of each *Satellite* next preceeding the Time propoſed, and when the 4*th* is not *Eclipſed*, of its paſſing the *Axis* of the Shadow, to which if the *Parallactick Intervals* Augmented were Additional, Add them to, if Subductive, Subſtract them from, each the Time of its proper *Satellite's Eclipſe*, ſo have you vey near the Apparent Times when each *Satellite* laſt paſt over the *Axis* of ♃ viewed from our Earth.

5. Subſtract each of the Times thus got from the Time propoſed, the Remainders are the Intervals of the Motion of each *Satellite* from ♃'s *Axis*.

6. Extend the Thread, from the Center over each of theſe Intervals of Motion numbred ſeverally in the divided Circles belonging each to its proper *Satellite*, where it cuts the proper *Orbit* of that *Satellite*, whoſe Interval was numbred in its peculiar Circle, it ſhews amongſt the Parallels, how many Semidiameters of ♃, that *Satellite* is diſtant from him, and on which ſide of him 'tis Poſited.

Note further, that the Thread as it lay extended over the *Parallax* of the *Orb* numbred in the ſmall Arch, where it cut the ſeveral proper Orbits of each *Satellite*, ſhewed amongſt the Parallels, how many Semidiameters of ♃, the Center of the Shadow was diſtant from the Center of ♃, viewed from our Earth. And that if the *Parallax* of the *Orb* were Additional, the Shadow lies on the Right-hand from ♃, if Subductive on the Left.

To explain theſe Precepts, I ſhall give two brief Examples. Let it be then propoſed to know how far each *Satellite* appears diſtant from ♃ on the 26th of *Dec.* this preſent Year 1685. at 6h 52′. *p. m.* when the *Third Satellite* falls into the Shadow; alſo on *Jul.* 16. 1686. at 10h 00′. *p. m.* when there is no *Eclipſe*.

Fig. 143. *An.* 1685. *December* 26^d 16^h 52′. *p. m.* the *Parallax* of the *Orb* is 9° 20′ Additional ; Therefore

| | 1 | | | 2 | | | 3 | | | 4 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | d. | h. | ′ | d. | h. | ′ | d. | h. | ′ | d. | h. | ′ |
| The *Simple Parallactick* Intervals *add* | 0 | 1 | 5 | 0 | 2 | 10 | 0 | 4 | 25 | 0 | 10 | 20 |
| The half *Durations* of the *Eclipses* to be added | 0 | 1 | 10 | 0 | 1 | 30 | 0 | 1 | 18 | 0 | 0 | 0 |
| The *Parallactick Intervals* augmented | 0 | 2 | 15 | 0 | 3 | 40 | 0 | 5 | 43 | 0 | 10 | 20 |
| Last *Immersions*, and ♂. *Dec.* | 25 | 9 | 37 | 25 | 5 | 47 | 19 | 12 | 58 | 10 | 00 | 30 |
| Times of last passing *Jupiter's Axis. Dec.* | 25 | 11 | 52 | 25 | 9 | 27 | 19 | 18 | 41 | 10 | 10 | 50 |
| Substracted from the Time proposed. *Dec.* | 26 | 16 | 52 | 26 | 16 | 52 | 26 | 16 | 52 | 26 | 16 | 52 |
| Leaves the *Intervals* of *Motion.* | 1 | 5 | 00 | 1 | 7 | 25 | 6 | 22 | 11 | 16 | 6 | 2 |
| Over which numbred in their peculiar Circles the Thread being severally laid, cuts the proper *Orbit* of each at their visible *Distances* from *Jupiter.* | sd. 5. *dext.* | | | sd. 6½. *sin.* | | | sd. 3. *dext.* | | | sd. 4½. *dext.* | | |

Again

Again, *An.* 1686. *Jul.* 16. 10h *p. m.* the *Parallax* of the *Orb* is 10° 46′. Subductive. Hence *Fig.* 144.

| | 1 | | | 2 | | | 3 | | | 4 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | d. | h. | ′ | d. | h. | ′ | d. | h. | ′ | d. | h. | ′ |
| The *Simple Parallactick Intervals Sub.* | 0 | 1 | 12 | 0 | 2 | 35 | 0 | 5 | 10 | 0 | 12 | 00 |
| Half *Duration* of the *Eclipses* add. | 0 | 1 | 10 | 0 | 1 | 30 | 0 | 1 | 21 | — | — | — |
| The *Parallactick Intervals* augmented. | 0 | 2 | 22 | 0 | 4 | 5 | 0 | 6 | 31 | 0 | 12 | 00 |
| The next last *Emersions*, and passing the *Axis* of the *Shadow*, *Jul.* | 15 | 5 | 55 | 15 | 22 | 2 | 15 | 9 | 19 | 15 | 17 | 52 |
| The Time of last passing the visible *Axis* of *Jupiter* | 15 | 3 | 33 | 15 | 17 | 57 | 15 | 2 | 48 | 15 | 5 | 52 |
| The Time proposed | 16 | 10 | 00 | 16 | 10 | 00 | 16 | 10 | 00 | 16 | 10 | 00 |
| The *Intervals* of *Motion* | 1 | 6 | 27 | 0 | 16 | 3 | 1 | 7 | 12 | 1 | 4 | 8 |
| Therefore *Distan.* from *Jupiter's Axis* | ſd. $5\frac{1}{2}$. | *Dext.* | | ſd. $8\frac{1}{5}$. | *Sin.* | | ſd. $13\frac{1}{2}$. | *Sin.* | | ſd. $10\frac{1}{2}$. | *Sin.* | |

And the *Satellites* ſtand at the two propoſed Times as in the two Figures.

In drawing of which, tho' I have conſidered their *Latitudes* from the Line of their utmoſt *Elongations* paſſing through *Jupiter*'s Center, yet I give no Rules for determining it, the Contrivances and Directions neceſſary on that Account, being too many and troubleſome to be inſerted here: My Deſign is only to ſhew the Ingenious Obſerver, how to find at what Diſtance from ♃, each *Satellite* appears, that ſo he may not miſtake one for another when he is to Obſerve any of their *Eclipſes*.

XC. 1. *An.* 1668. The *French Aſtronomers* have made theſe Obſervations by a 14 Foot Teleſcope.

*Eclipſes and Places of the* Satellites *of* Jupiter *Obſerved, at* Paris; *by* . . . . . . n. 44. p. 892.

*Octob.* 7. 10h 32′ *p. m.* The *Firſt Satellite* (called *Pallas*) Entred upon the Face of *Jupiter*.

*Octob.* 8.

*Octob.* 8. 8h 11′. The *Second Satellite* (called *Juno*) Went out behind *Jupiter.*

*Octob.* 9. 8h 54′. The *Second Satellite* went out from the Face of *Jupiter.*

*Octob.* 16. 10h 4′. The *Second Satellite* entred upon the Face of *Jupiter.*

*Octob.* 22. 10h 41′ 33″. The *First Satellite* entred into the Shadow of *Jupiter.*

*Octob.* 23. 8h 32′. The *First Satellite* entred upon the Face of *Jupiter.*

*Nov.* 12. 10h 40′. The *Second Satellite* entred into the Shadow of *Jupiter.*

*Nov.* 20. 2h 38′ 30″. After Midnight, the *Third Satellite* (call'd *Themis*) entred into the Shadow of *Jupiter.*

*At* Dantzick; *by* M. Hevelius. *n.* 78. *p.* 3029.

2. Cum *An.* 1671. die 25. *Sept.* st. n. manè ex condicto Dn *Cassinus Parisiis*, & Dn *Picard Uraniburgi*, suscepissent, ad Occultationem *Primi Jovialium* attendere, volui haud minus ego huic Phænomeno diligenter invigilare. Itaq; *Hor.*4. 27′. ubi primum *Jupiter* emicuit, deprehendi adhuc *Joviales* omnes adesse, tres sc. à Læva & unum ad dextram. Duo illi propinquiores ad Sinistram haud procul videbantur à Limbo *Jovis*, non minus ille qui ad Dextram apparebat, aliorum Comitum Minimus. Ad quintam usque & 7′ ferè, omnes quatuor (utut Cælum jam livesceret) distincte apparebant. Præter tamen omnem spem, *hor.* 5. 12′ videbatur mihi propinquior ille *Comes* ad Lævam (respectu Tubi mei, qui inverso ordine objecta exhibet) penitus evanescere, remanentibus illis tribus, quanquam ille dexterior magis magisque etiam ad *Jovem* accedebat. An planè momentum ipsum temporis id fuerit *Immersionis* illius *Comitis*, vix ausim adeo certò affirmare; nihilominus tardius non incidit illa *occultatio*; sed anne unico minuto ferè, adhuc citius forte, ingruerit, facilè concesserim.

| *Sec. Horol. amb. mane.* | | | *Observationes.* | *Distantiæ & Altitudines.* | | | *Temp. correct.* | | |
|---|---|---|---|---|---|---|---|---|---|
| h. | ′ | ″ | | ° | ′ | ″ | h | ′ | ″ |
| 4 | 36 | 25 | *Jupiter* primum conspectus——— | — | — | — | 4 | 32 | 0 |
| 5 | 7 | 25 | Altitudo *Procyonis*——— | 34 | 43 | 0 | 5 | 2 | 7 |
| 5 | 16 | 35 | Primus *Jovialium* Evanuit——— | — | — | — | 5 | 12 | 0 |
| 5 | 26 | 5 | Altitudo *Procyonis*——— | 36 | 39 | 0 | 5 | 23 | 27 |

*At* Derby; *by* M. Flamsteed. *n.* 82. *p.* 4036.

3. An. 167½. *Feb.* 17. 7h 25′. *p. m.* Alt. ♃ erat 15° 54′. At 8h 59′. *p. m.* vel forsan 1. min. maturius, *Satelles Primus* ad dextram ♃is, in ipsius Umbram incidit, adeo tamen Evanescentis exigua erat à Limbo Distantia, ut quanta fuerit dijudicare non potuerim.

*Mart.*

*Mart.* 19d. 6h 45′. Alt. *Jov.* 29° 35′. *Satelles Primus* ad Limbum *Jovis* appropinquabat, cui 7 h 51′. jungebatur. *ib.* 4037:

*An.* 1672. *Ap.* 15. $7^h$ 43′. vesp. *Alt.* ♃. 25° 00′. *Satelles Primus* mox *Jovem* à tergo subiturus $\frac{1}{2}$ circiter diametri à Limbo ejus apparuit. *n.* 96. *p.* 6099.

$8^h$ 6′. *Alt.* ♃ 27° 20′. subivit *Jovem*. Alt. *Jove* 27° 26′. *certe* non conspiciebatur.

4. *Jul.* 6. (st. n.) 1675. ante mediam noctem, hora scil. 11. & 16. Secundis præcisè, *Secundus Jovis Satelles* Egredi incipiebat ex Planetæ hujus, qui ipsum obscuraverat, umbra. *At* Paris; *by M.* Cassini. *n.* 117. *p.* 389.

5. An. 1679. *Jun.* 5. st. n. 3h. *m.* I discovered 3 *Satellites* of *Jupiter*: The *First* was distant Westward of the Limb of *Jupiter*, a little less than a Diameter; the *Second* was distant, on the East-side, a little more than a Diameter. The *Third* was more Eastward than the *Second*, by somewhat less than a Diameter of *Jupiter*. *Ph. Col. n.* 1. *p.* 33. *Vid. sup.* §. LXIV. 2.

XCI. 1. Let A be the *Sun*, B *Jupiter*, C the *First Satellite* of *Jupiter*, which enters into the *Shadow* of *Jupiter*, to come out of it at D; and let EFGHLK be the *Earth* placed at divers Distances from *Jupiter*. *The Equation of Light; by M.* Romer. *n.* 136. *p.* 893. Fig. 145.

Now suppose the *Earth*, being in L towards the 2d Quadrature of *Jupiter*, hath seen the *First Satellite* at the Time of its *Emersion*, or Issuing out of the Shadow, in D; and that about $42\frac{1}{2}$ hours after (*vid.* after one Revolution of this *Satellite*) the *Earth* being in K, doth see it returned in D; it is manifest, that if the Light require time to traverse the Interval LK, the *Satellite* will be seen returned later in D, than it would have been if the *Earth* had remained in L, so that the Revolution of this *Satellite* being thus observed by the *Emersions* will be retarded by so much time, as the Light shall have taken in passing from L to K, and that on the contrary, in the other Quadrature FG, where the *Earth* by approaching goes to meet the Light, the Revolutions of the *Immersions* will appear to be shortned by so much, as those of the *Emersions* had appeared to be lengthened.

This new *Equation* of the Motion of Light, which hath been established by the *Royal Academy*, and in the *Observatory* for the Space of 8 Years, was confirmed by the *Emersion* of the *First Satellite* observ'd at *Paris* 1676. *Nov.* 9. $5^h$ 35′ 45″, at Night, 10′ later than it was expected, by deducing it from those that had been Observed in the Month of *August*, when the *Earth* was much nearer to *Jupiter*. *The Theory of* Jupiter's Satellites; *by M.* Cassini. *n.* 128. *p.* 681.

XCII. 1. *M. Cassini*, having formed a new Hypothesis for the *Satellites* of *Jupiter*, different from that of *Galileo*, thinks that the Plain of their *Orbs* is Inclined to the Plain of the *Ecliptick*; He settles their *Nodes* with the *Orbs* of *Jupiter* towards the 13° of *Leo* and *Aquarius*; and finds that the *Obliquity* of their Circles to the Orbite of *Jupiter*, is almost double to the *Obliquity* of this Orbite to the *Ecliptique*. M. Cassini's *Tables, for the Eclipses of the* First Satellite of Jupiter, *Abridged, and Reduced to the Meridian of* London; *by Mr.* Edm. Halley. *n.* 215. *p.* 238.

2. *M. Cassini*, in the last Treatise of a Book, Entitul'd *Recueil d' Observations faites en Plusieurs Voyages*, &c. has employed his Skill, to make easie the

the Calculation of the *Eclipses* of the *First Satellite* of *Jupiter*, which is otherwise Operose even to the Skillful. The Tables have for Principles, that this *Satellite* Revolves to the *Sun* in $1^d\ 18^h\ 28'\ 36''$, so precisely, that in 100 Years the Difference is not sensible; That in the Time of the Revolution of *Jupiter* to his *Aphelion*, which he supposes in $4332^d\ 14^h\ 52'\ 48''$, this *Satellite* makes exactly 2448 Months or Revolutions to the *Sun*; and dividing the Orbite of *Jupiter* into 2448 parts, he has in a Large Table of *Equation* shewn what is the Inequality of the Motion of *Jupiter* in each Revolution reduced to time, assuming, *Thirdly*, the greatest *Equation* of *Jupiter* $5^\circ\ 31'\ 40''$. whence the hourly Motion of the *Satellite* from *Jupiter* being $8^\circ\ 28\frac{1}{2}'$, it follows that the greatest Inequality (*Jupiter* passing the Signs of *Cancer* and *Capricorn*,) amounts to $39'\ 8''$ of Time, to be added in *Cancer*, substracted in *Capricorn*. *Lastly*, As to the *Epocha*, or beginning of this Series of Revolutions, he has determined the *Aphelion* of *Jupiter* about $1\frac{1}{2}$ Degree forwarder than *Astronomia Carolina*, and above two Degrees more than the *Rudolphine* Tables, *viz*. precisely in 9° of *Libra*, in the beginning of this Century, which perhaps he finds the proper Motion of *Jupiter* about the *Sun* at this time to require; and the Number of Revolutions since *Jupiter* was last *in Perihelio*, is here stiled *Num*. I.

A Second Inequality is that which depends on the Distance of the *Sun* from *Jupiter*, which he says Mr. *Romer* did most ingeniously explain by the Hypothesis of the Motion of Light; to which yet *Cassini* by his manner of *Calculus* seems not to assent, though it be hard to imagine how the *Earth*'s Position in respect of *Jupiter* should any way affect the Motion of the *Satellites*. This Inequality he makes to amount to two Degrees in the *Satellite*'s Motion, or $14'\ 10''$ of Time, wherein he supposes the *Eclipses* to happen so much sooner when *Jupiter* Opposes the *Sun*, than when he is in Conjunction with him. The Distribution of this Inequality he makes wholly to depend on the Angle at the *Sun*, between the *Earth* and *Jupiter*, without any Regard to the *Excentricity* of *Jupiter*, (who is sometimes $\frac{1}{2}$ a Semidiameter of the *Earth*'s Orb, farther from the *Sun* than at other times) which would occasion a much greater Difference, than the Inequality of *Jupiter* and the *Earth*'s Motion, both of which are accounted for in these Tables with great Skill and Address. But what is most strange, he Affirms that the same Inequality of two Degrees in the Motion, is likewise found in the other *Satellites*, requiring a much greater Time; as above two Hours in the $4^{th}$ *Satellite*: which if it appeared by Observation would overthrow *M. Romer*'s Hypothesis entirely. Yet I doubt not herein to make it Demonstratively plain, that the Hypothesis of the Progressive Motion of Light is found in all the other *Satellites* of *Jupiter* to be necessary, and that it is the same in all; there being nothing near so great an Annual Inequality as *M. Cassini* supposes in their Motions, by his Table, *p*. 9. and his *Præcepta Calculi*. The Method however used to compute this is very Curious; for having found that whilst the *Sun* Revolves to *Jupiter*, there pass $398^d\ 21^h\ 13'$. wherein are made $225\frac{3}{8}$ Revolutions of the *Satellite* to *Jupiter*, the Number of Revolutions

Vid. Sup. §. XCI.

since

ſince *Jupiter* was laſt in Oppoſition to the *Sun*, is what he calls *Num.* II. in which the Inequality of the *Earth*'s Motion is allowed for in the Months, and that of *Jupiter*'s Orb by a Table of the *Equation* of *Num.* II. amounting in all to $3\frac{1}{2}$ Revolutions of the *Satellite* to *Jupiter*. This in the Tables following I have thought fit to leave out, ſhewing how to find it by the help of the former Equation of *Num.* I. The Numbers are in effect the ſame with *M. Caſſini*'s, only Reduced to our Stile and Meridian, and the Form of them Abridged, and 'tis hoped Amended.

 

*Epochæ Revolutionum* Primi Satellitis *ad* Jovis *Umbram sub Meridiano* Londinensi.

| Anno Jul. Curr. | d. | h. | ′ | ″ | Num. I. | Num. II. |
|---|---|---|---|---|---|---|
| 1660 | 0 | 11 | 5 | 48 | 968 | 200 6 |
| 1661 | 0 | 1 | 17 | 24 | 1174 | 181 2 |
| 1662 | 1 | 9 | 57 | 36 | 1381 | 162 9 |
| 1663 | 1 | 00 | 9 | 12 | 1587 | 143 5 |
| 1664 | 1 | 8 | 49 | 24 | 1794 | 125 1 |
| 1665 | 0 | 23 | 1 | 00 | 2000 | 105 7 |
| 1666 | 0 | 13 | 12 | 36 | 2206 | 86 4 |
| 1667 | 0 | 3 | 24 | 12 | 2412 | 67 0 |
| 1668 | 0 | 12 | 4 | 24 | 171 | 48 6 |
| 1669 | 0 | 2 | 16 | 00 | 377 | 29 2 |
| 1670 | 1 | 10 | 56 | 12 | 584 | 10 9 |
| 1671 | 1 | 1 | 7 | 48 | 790 | 216 9 |
| 1672 | 1 | 9 | 48 | 00 | 997 | 198 5 |
| 1673 | 0 | 23 | 59 | 36 | 1203 | 179 1 |
| 1674 | 0 | 14 | 11 | 12 | 1409 | 159 7 |
| 1675 | 0 | 4 | 22 | 48 | 1615 | 140 3 |
| 1676 | 0 | 13 | 3 | 00 | 1822 | 121 9 |
| 1677 | 0 | 3 | 14 | 36 | 2028 | 102 5 |
| 1678 | 1 | 11 | 54 | 48 | 2235 | 84 1 |
| 1679 | 1 | 2 | 6 | 24 | 2441 | 64 7 |
| 1680 | 1 | 10 | 46 | 36 | 200 | 46 4 |
| 1681 | 1 | 00 | 58 | 12 | 406 | 27 0 |
| 1682 | 0 | 15 | 9 | 48 | 612 | 7 6 |
| 1683 | 0 | 5 | 21 | 24 | 818 | 213 6 |
| 1684 | 0 | 14 | 1 | 36 | 1025 | 195 3 |
| 1685 | 0 | 4 | 13 | 12 | 1231 | 175 9 |
| 1686 | 1 | 12 | 53 | 24 | 1438 | 157 5 |
| 1687 | 1 | 3 | 5 | 00 | 1644 | 138 1 |
| 1688 | 1 | 11 | 45 | 12 | 1851 | 119 7 |
| 1689 | 1 | 1 | 56 | 48 | 2057 | 100 4 |

*Epochæ Revolutionum* Primi Sattellitis *ad* Jovis *Umbram sub Meridiano* Londinensi.

| Anno Jul. Curr. | d. | h. | ′ | ″ | Num. I. | Num. II. | |
|---|---|---|---|---|---|---|---|
| 1690 | 0 | 16 | 8 | 24 | 2263 | 81 | 0 |
| 1691 | 0 | 6 | 20 | 00 | 21 | 61 | 6 |
| 1692 | 0 | 15 | 00 | 12 | 228 | 43 | 3 |
| 1693 | 0 | 5 | 11 | 48 | 434 | 23 | 9 |
| 1694 | 1 | 13 | 52 | 00 | 641 | 5 | 5 |
| 1695 | 1 | 4 | 3 | 36 | 847 | 211 | 5 |
| 1696 | 1 | 12 | 43 | 48 | 1054 | 193 | 1 |
| 1697 | 1 | 2 | 55 | 24 | 1260 | 173 | 7 |
| 1698 | 0 | 17 | 7 | 00 | 1466 | 154 | 4 |
| 1699 | 0 | 7 | 18 | 36 | 1672 | 135 | 0 |
| 1700 | 0 | 15 | 58 | 48 | 1879 | 116 | 6 |
| 1701 | 0 | 6 | 10 | 24 | 2085 | 97 | 3 |
| 1702 | 1 | 14 | 50 | 36 | 2292 | 78 | 9 |
| 1703 | 1 | 5 | 2 | 12 | 50 | 59 | 5 |
| 1704 | 1 | 13 | 42 | 24 | 257 | 41 | 1 |
| 1705 | 1 | 3 | 54 | 00 | 463 | 21 | 8 |
| 1706 | 0 | 18 | 5 | 36 | 669 | 2 | 4 |
| 1707 | 0 | 8 | 17 | 12 | 875 | 208 | 4 |
| 1708 | 0 | 16 | 57 | 24 | 1082 | 190 | 0 |
| 1709 | 0 | 7 | 9 | 00 | 1288 | 170 | 6 |
| 1710 | 1 | 15 | 49 | 12 | 1495 | 152 | 3 |
| 1711 | 1 | 6 | 00 | 48 | 1701 | 132 | 9 |
| 1712 | 1 | 14 | 41 | 00 | 1908 | 114 | 5 |
| 1713 | 1 | 4 | 52 | 36 | 2114 | 95 | 1 |
| 1714 | 0 | 19 | 4 | 12 | 2320 | 75 | 8 |
| 1715 | 0 | 9 | 15 | 48 | 78 | 56 | 4 |
| 1716 | 0 | 17 | 56 | 00 | 285 | 38 | 0 |
| 1717 | 0 | 8 | 7 | 36 | 491 | 18 | 6 |
| 1718 | 1 | 16 | 47 | 48 | 698 | 0 | 3 |
| 1719 | 1 | 6 | 59 | 24 | 904 | 206 | 3 |
| 1720 | 1 | 15 | 39 | 36 | 1111 | 187 | 9 |

Tabula *Revolutionum* Primi Satellitis *Jovis* in *Anno.*

| *Januarius.* | | | | N. I. | N. II. | | *Martius.* | | | | N. I. | N. II. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| d. | h. | ′ | ″ | | | | d. | h. | ′ | ″ | | |
| 0 | 0 | 0 | 0 | 0 | 0 0 | | 1 | 4 | 12 | 24 | 34 | 34 8 |
| 1 | 18 | 28 | 36 | 1 | 1 0 | | 2 | 22 | 41 | 00 | 35 | 35 8 |
| 3 | 12 | 57 | 12 | 2 | 2 1 | | 4 | 17 | 9 | 36 | 36 | 36 8 |
| 5 | 7 | 25 | 48 | 3 | 3 1 | | 6 | 11 | 38 | 12 | 37 | 37 9 |
| 7 | 1 | 54 | 24 | 4 | 4 1 | | 8 | 6 | 6 | 48 | 38 | 38 9 |
| 8 | 20 | 23 | 00 | 5 | 5 2 | | 10 | 00 | 35 | 24 | 39 | 39 9 |
| 10 | 14 | 51 | 36 | 6 | 6 2 | | 11 | 19 | 4 | 00 | 40 | 40 9 |
| 12 | 9 | 20 | 12 | 7 | 7 2 | | 13 | 13 | 32 | 36 | 41 | 41 9 |
| 14 | 3 | 48 | 48 | 8 | 8 2 | | 15 | 8 | 1 | 12 | 42 | 42 9 |
| 15 | 22 | 17 | 24 | 9 | 9 3 | | 17 | 2 | 29 | 48 | 43 | 43 9 |
| 17 | 16 | 46 | 00 | 10 | 10 3 | | 18 | 20 | 58 | 24 | 44 | 44 9 |
| 19 | 11 | 14 | 36 | 11 | 11 3 | | 20 | 15 | 27 | 00 | 45 | 45 9 |
| 21 | 5 | 43 | 12 | 12 | 12 3 | | 22 | 9 | 55 | 36 | 46 | 46 9 |
| 23 | 00 | 11 | 48 | 13 | 13 4 | | 24 | 4 | 24 | 12 | 47 | 47 9 |
| 24 | 18 | 40 | 24 | 14 | 14 4 | | 25 | 22 | 52 | 48 | 48 | 48 9 |
| 26 | 13 | 9 | 00 | 15 | 15 4 | | 27 | 17 | 21 | 24 | 49 | 49 9 |
| 28 | 7 | 37 | 36 | 16 | 16 5 | | 29 | 11 | 50 | 00 | 50 | 50 9 |
| 30 | 2 | 6 | 12 | 17 | 17 5 | | 31 | 6 | 18 | 36 | 51 | 51 9 |
| 31 | 20 | 34 | 48 | 18 | 18 5 | | | | | | | |
| *Februarius.* | | | | | | | *Aprilis.* | | | | | |
| 0 | 20 | 34 | 48 | 18 | 18 5 | | 0 | 6 | 18 | 36 | 51 | 51 9 |
| 2 | 15 | 3 | 24 | 19 | 19 6 | | 2 | 00 | 47 | 12 | 52 | 52 9 |
| 4 | 9 | 32 | 00 | 20 | 20 6 | | 3 | 19 | 15 | 48 | 53 | 53 9 |
| 6 | 4 | 00 | 36 | 21 | 21 6 | | 5 | 13 | 44 | 24 | 54 | 54 9 |
| 7 | 22 | 29 | 12 | 22 | 22 6 | | 7 | 8 | 13 | 00 | 55 | 55 9 |
| 9 | 16 | 57 | 48 | 23 | 23 7 | | 9 | 2 | 41 | 36 | 56 | 56 9 |
| 11 | 11 | 26 | 24 | 24 | 24 7 | | 10 | 21 | 10 | 12 | 57 | 57 9 |
| 13 | 5 | 55 | 00 | 25 | 25 7 | | 12 | 15 | 38 | 48 | 58 | 58 9 |
| 15 | 00 | 23 | 36 | 26 | 26 7 | | 14 | 10 | 7 | 24 | 59 | 59 9 |
| 16 | 18 | 52 | 12 | 27 | 27 7 | | 16 | 4 | 36 | 00 | 60 | 60 8 |
| 18 | 13 | 20 | 48 | 28 | 28 7 | | 17 | 23 | 4 | 36 | 61 | 61 8 |
| 20 | 7 | 49 | 24 | 29 | 29 7 | | 19 | 17 | 33 | 12 | 62 | 62 8 |
| 22 | 2 | 18 | 00 | 30 | 30 8 | | 21 | 12 | 1 | 48 | 63 | 63 8 |
| 23 | 20 | 46 | 36 | 31 | 31 8 | | 23 | 6 | 30 | 24 | 64 | 64 8 |
| 25 | 15 | 15 | 12 | 32 | 32 8 | | 25 | 00 | 59 | 00 | 65 | 65 7 |
| 27 | 9 | 43 | 48 | 33 | 33 8 | | 26 | 19 | 27 | 36 | 66 | 66 7 |
| | | | | | | | 28 | 13 | 56 | 12 | 67 | 67 7 |
| | | | | | | | 30 | 8 | 24 | 48 | 68 | 68 6 |

*Tabula Revolutionum* Primi Satellitis *Jovis* in *Anno.*

| *Maius.* | N. I. | N. II. | | *Julius.* | N. I. | N. II. |
|---|---|---|---|---|---|---|
| d. h. ′ ″ | | | | d. h. ′ ″ | | |
| 0 8 24 48 | 68 | 68 6 | | 1 7 5 48 | 103 | 102 5 |
| 2 2 53 24 | 69 | 69 6 | | 3 1 34 24 | 104 | 103 5 |
| 3 21 22 00 | 70 | 70 6 | | 4 20 3 00 | 105 | 104 4 |
| 5 15 50 36 | 71 | 71 6 | | 6 14 31 36 | 106 | 105 4 |
| 7 10 19 12 | 72 | 72 5 | | 8 9 00 12 | 107 | 106 4 |
| 9 4 47 48 | 73 | 73 5 | | 10 3 28 48 | 108 | 107 3 |
| 10 23 16 24 | 74 | 74 5 | | 11 21 57 24 | 109 | 108 3 |
| 12 17 45 00 | 75 | 75 5 | | 13 16 26 00 | 110 | 109 3 |
| 14 12 13 36 | 76 | 76 4 | | 15 10 54 36 | 111 | 110 2 |
| | | | | 17 5 23 12 | 112 | 111 2 |
| 16 6 42 12 | 77 | 77 4 | | 18 23 51 48 | 113 | 112 2 |
| 18 1 10 48 | 78 | 78 4 | | 20 18 20 24 | 114 | 113 1 |
| 19 19 39 24 | 79 | 79 3 | | 22 12 49 00 | 115 | 114 1 |
| 21 14 8 00 | 80 | 80 3 | | 24 7 17 36 | 116 | 115 1 |
| 23 8 36 36 | 81 | 81 3 | | | | |
| 25 3 5 12 | 82 | 82 3 | | 26 1 46 12 | 117 | 116 0 |
| 26 21 33 48 | 83 | 83 3 | | 27 20 14 48 | 118 | 117 0 |
| 28 16 2 24 | 84 | 84 2 | | 29 14 43 24 | 119 | 118 0 |
| 30 10 31 00 | 85 | 85 2 | | 31 9 12 00 | 120 | 119 0 |
| *Junius.* | N. I. | N. II. | | *Augustus.* | N. I. | N. II. |
| 1 4 59 36 | 86 | 86 1 | | 0 9 12 00 | 120 | 119 0 |
| 2 23 28 12 | 87 | 87 1 | | 2 3 40 36 | 121 | 119 9 |
| 4 17 56 48 | 88 | 88 0 | | 3 22 9 12 | 122 | 120 9 |
| 6 12 25 24 | 89 | 89 0 | | 5 16 37 48 | 123 | 121 9 |
| | | | | 7 11 6 24 | 124 | 122 9 |
| 8 6 54 00 | 90 | 90 0 | | 9 5 35 00 | 125 | 123 8 |
| 10 1 22 36 | 91 | 90 9 | | 11 00 3 36 | 126 | 124 8 |
| 11 19 51 12 | 92 | 91 9 | | 12 18 32 12 | 127 | 125 8 |
| 13 14 19 48 | 93 | 92 9 | | 14 13 00 48 | 128 | 126 8 |
| 15 8 48 24 | 94 | 93 8 | | 16 7 29 24 | 129 | 127 7 |
| 17 3 17 00 | 95 | 94 8 | | 18 1 58 00 | 130 | 128 7 |
| 18 21 45 36 | 96 | 95 7 | | 19 20 26 36 | 131 | 129 7 |
| 20 16 14 12 | 97 | 96 7 | | 21 14 55 12 | 132 | 130 7 |
| 22 10 42 48 | 98 | 97 7 | | 23 9 23 48 | 133 | 131 7 |
| 24 5 11 24 | 99 | 98 6 | | 25 3 52 24 | 134 | 132 7 |
| 25 23 40 00 | 100 | 99 6 | | 26 22 21 00 | 135 | 133 6 |
| 27 18 8 36 | 101 | 100 6 | | 28 16 49 36 | 136 | 134 6 |
| | | | | 30 11 18 12 | 137 | 135 6 |
| 29 12 37 12 | 102 | 101 5 | | | | |

*Tabula Revolutionum* Primi Satellitis *Jovis* in *Anno.*

| *September.* | N. I. | N. II. | | *November.* | N. I. | N. II. |
|---|---|---|---|---|---|---|
| d. h. ′ ″ | | | | d. h. ′ ″ | | |
| 1 5 46 48 | 138 | 136 6 | | 0 9 59 12 | 172 | 170 7 |
| 3 00 15 24 | 139 | 137 6 | | 2 4 27 48 | 173 | 171 8 |
| 4 18 44 00 | 140 | 138 6 | | 3 22 56 24 | 174 | 172 8 |
| 6 13 12 36 | 141 | 139 6 | | 5 17 25 00 | 175 | 173 8 |
| 8 7 41 12 | 142 | 140 6 | | 7 11 53 36 | 176 | 174 8 |
| 10 2 9 48 | 143 | 141 5 | | 9 6 22 12 | 177 | 175 9 |
| 11 20 38 24 | 144 | 142 5 | | 11 00 50 48 | 178 | 176 9 |
| 13 15 7 00 | 145 | 143 5 | | 12 19 19 24 | 179 | 177 9 |
| 15 9 35 36 | 146 | 144 5 | | 14 13 48 00 | 180 | 178 9 |
| | | | | 16 8 16 36 | 181 | 180 0 |
| 17 4 4 12 | 147 | 145 5 | | 18 2 45 12 | 182 | 181 0 |
| 18 22 32 48 | 148 | 146 5 | | 19 21 13 48 | 183 | 182 0 |
| 20 17 1 24 | 149 | 147 5 | | 21 15 42 24 | 184 | 183 0 |
| 22 11 30 00 | 150 | 148 5 | | 23 10 11 00 | 185 | 184 0 |
| 24 5 58 36 | 151 | 149 5 | | 25 4 39 36 | 186 | 185 1 |
| 26 00 27 12 | 152 | 150 5 | | 26 23 8 12 | 187 | 186 1 |
| 27 18 55 48 | 153 | 151 5 | | 28 17 36 48 | 188 | 187 2 |
| 29 12 24 24 | 154 | 152 5 | | 30 12 5 24 | 189 | 188 2 |
| *October.* | | | | *December.* | | |
| 1 7 53 00 | 155 | 153 5 | | 0 12 5 24 | 189 | 188 2 |
| 3 2 21 36 | 156 | 154 5 | | 2 6 34 00 | 190 | 189 2 |
| 4 20 50 12 | 157 | 155 5 | | 4 1 2 36 | 191 | 190 3 |
| 6 15 18 48 | 158 | 156 5 | | 5 19 31 12 | 192 | 191 3 |
| 8 9 47 24 | 159 | 157 5 | | 7 13 59 48 | 193 | 192 3 |
| 10 4 16 00 | 160 | 158 5 | | 9 8 28 24 | 194 | 193 4 |
| 11 22 44 36 | 161 | 159 5 | | 11 2 57 00 | 195 | 194 4 |
| 13 17 13 12 | 162 | 160 5 | | 12 21 25 36 | 196 | 195 5 |
| 15 11 41 48 | 163 | 161 6 | | 14 15 54 12 | 197 | 196 5 |
| 17 6 10 24 | 164 | 162 6 | | 16 10 22 48 | 198 | 197 6 |
| 19 00 39 00 | 165 | 163 6 | | 18 4 51 24 | 199 | 198 6 |
| 20 19 7 36 | 166 | 164 6 | | 19 23 20 00 | 200 | 199 7 |
| 22 13 36 12 | 167 | 165 6 | | 21 17 48 36 | 201 | 200 7 |
| 24 8 4 48 | 168 | 166 6 | | 23 12 17 12 | 202 | 201 8 |
| 26 2 33 24 | 169 | 167 7 | | 25 6 45 48 | 203 | 202 8 |
| 27 21 2 00 | 170 | 168 7 | | 27 1 14 24 | 204 | 203 9 |
| 29 15 30 36 | 171 | 169 7 | | 28 19 43 00 | 205 | 204 9 |
| 31 9 59 12 | 172 | 170 7 | | 30 14 11 36 | 206 | 206 0 |

*Tabula Primæ Æquationis Conjunctionum* Primi Satellitis *cum* Jove.

| N. I. | Æquat. ′ | Æquat. ″ | N. I. | Æquat. ′ | Æquat. ″ | N. I. | Æquat. ′ | Æquat. ″ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 400 | 34 | 20 | 810 | 33 | 21 |
| 10 | 1 | 3 | 410 | 34 | 51 | 820 | 32 | 50 |
| 20 | 2 | 5 | 420 | 35 | 21 | 830 | 32 | 17 |
| 30 | 3 | 8 | 430 | 35 | 47 | 840 | 31 | 44 |
| 40 | 4 | 12 | 440 | 36 | 6 | 850 | 31 | 10 |
| 50 | 5 | 15 | 450 | 36 | 26 | 860 | 30 | 32 |
| 60 | 6 | 16 | 460 | 36 | 47 | 870 | 29 | 56 |
| 70 | 7 | 19 | 470 | 37 | 8 | 880 | 29 | 19 |
| 80 | 8 | 20 | 480 | 37 | 29 | 890 | 28 | 40 |
| 90 | 9 | 23 | 490 | 37 | 44 | 900 | 27 | 59 |
| 100 | 10 | 25 | 500 | 37 | 59 | 910 | 27 | 19 |
| 110 | 11 | 25 | 510 | 38 | 16 | 920 | 26 | 37 |
| 120 | 12 | 25 | 520 | 38 | 29 | 930 | 25 | 53 |
| 130 | 13 | 25 | 530 | 38 | 39 | 940 | 25 | 8 |
| 140 | 14 | 25 | 540 | 38 | 49 | 950 | 24 | 23 |
| 150 | 15 | 22 | 550 | 38 | 55 | 960 | 23 | 37 |
| | | | | | | 970 | 22 | 50 |
| 160 | 16 | 18 | 560 | 38 | 59 | 980 | 22 | 3 |
| 170 | 17 | 17 | 570 | 39 | 3 | 990 | 21 | 15 |
| 180 | 18 | 11 | 580 | 39 | 6 | 1000 | 20 | 26 |
| 190 | 19 | 9 | 590 | 39 | 8 | 1010 | 19 | 37 |
| 200 | 20 | 5 | 600 | 39 | 7 | 1020 | 18 | 47 |
| 210 | 20 | 56 | 610 | 39 | 5 | 1030 | 17 | 56 |
| 220 | 21 | 49 | 620 | 39 | 03 | 1040 | 17 | 5 |
| 230 | 22 | 41 | 630 | 38 | 58 | 1050 | 16 | 13 |
| 240 | 23 | 32 | 640 | 38 | 51 | 1060 | 15 | 19 |
| 250 | 24 | 20 | 650 | 38 | 44 | 1070 | 14 | 25 |
| 260 | 25 | 7 | 660 | 38 | 34 | 1080 | 13 | 32 |
| 270 | 25 | 57 | 670 | 38 | 24 | 1090 | 12 | 37 |
| 280 | 26 | 43 | 680 | 38 | 10 | 1100 | 11 | 42 |
| 290 | 27 | 27 | 690 | 37 | 56 | 1110 | 10 | 47 |
| 300 | 28 | 9 | 700 | 37 | 40 | 1120 | 9 | 52 |
| 310 | 28 | 54 | 710 | 37 | 24 | 1130 | 8 | 57 |
| 320 | 29 | 35 | 720 | 37 | 5 | 1140 | 8 | 00 |
| 330 | 30 | 11 | 730 | 36 | 45 | 1150 | 7 | 3 |
| 340 | 30 | 45 | 740 | 36 | 25 | 1160 | 6 | 7 |
| 350 | 31 | 28 | 750 | 36 | 4 | 1170 | 5 | 1 |
| 360 | 32 | 10 | 760 | 35 | 40 | 1180 | 4 | 13 |
| 370 | 32 | 44 | 770 | 35 | 15 | 1190 | 3 | 15 |
| 380 | 33 | 15 | 780 | 34 | 49 | 1200 | 2 | 19 |
| 390 | 33 | 49 | 790 | 34 | 19 | 1210 | 1 | 21 |
| 400 | 34 | 20 | 800 | 33 | 49 | 1220 | 0 | 2[illegible] |
| | | | 810 | 33 | 21 | 1224. | 0 | 0 |

*Tabula Secundæ Æquationis Conjunctionum* Primi Satellitis *cum* Jove.

| N. II. | Æquat. add. ′ | ″ | N. II. | Æquat. add. ′ | ″ | N. II. | Æquat. add. ′ | ″ | N. II. | Æquat. add. ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 28 | 2 | 4 | 56 | 7 | 0 | 84 | 12 | 0 |
| 1 | 0 | 0 | 29 | 2 | 13 | 57 | 7 | 12 | 85 | 12 | 9 |
| 2 | 0 | 1 | 30 | 2 | 21 | 58 | 7 | 24 | 86 | 12 | 16 |
| 3 | 0 | 2 | 31 | 2 | 30 | 59 | 7 | 36 | 87 | 12 | 24 |
| 4 | 0 | 3 | 32 | 2 | 39 | 60 | 7 | 47 | 88 | 12 | 32 |
| 5 | 0 | 4 | 33 | 2 | 48 | 61 | 7 | 59 | 89 | 12 | 40 |
| 6 | 0 | 6 | 34 | 2 | 58 | 62 | 8 | 11 | 90 | 12 | 47 |
| 7 | 0 | 8 | 35 | 3 | 8 | 63 | 8 | 22 | 91 | 12 | 53 |
| 8 | 0 | 10 | 36 | 3 | 17 | 64 | 8 | 34 | 92 | 13 | 00 |
| 9 | 0 | 14 | 37 | 3 | 27 | 65 | 8 | 46 | 93 | 13 | 6 |
| 10 | 0 | 17 | 38 | 3 | 37 | 66 | 8 | 57 | 94 | 13 | 13 |
| 11 | 0 | 20 | 39 | 3 | 48 | 67 | 9 | 8 | 95 | 13 | 19 |
| 12 | 0 | 23 | 40 | 3 | 59 | 68 | 9 | 20 | 96 | 13 | 24 |
| 13 | 0 | 27 | 41 | 4 | 9 | 69 | 9 | 32 | 97 | 13 | 30 |
| 14 | 0 | 32 | 42 | 4 | 20 | 70 | 9 | 44 | 98 | 13 | 35 |
| 15 | 0 | 37 | 43 | 4 | 31 | 71 | 9 | 54 | 99 | 13 | 39 |
| 16 | 0 | 42 | 44 | 4 | 41 | 72 | 10 | 3 | 100 | 13 | 45 |
| 17 | 0 | 47 | 45 | 4 | 53 | 73 | 10 | 14 | 101 | 13 | 48 |
| 18 | 0 | 53 | 46 | 5 | 4 | 74 | 10 | 25 | 102 | 13 | 51 |
| 19 | 0 | 58 | 47 | 5 | 15 | 75 | 10 | 35 | 103 | 13 | 54 |
| 20 | 1 | 4 | 48 | 5 | 27 | 76 | 10 | 45 | 104 | 13 | 57 |
| 21 | 1 | 11 | 49 | 5 | 39 | 77 | 10 | 55 | 105 | 14 | 00 |
| 22 | 1 | 18 | 50 | 5 | 50 | 78 | 11 | 5 | 106 | 14 | 3 |
| 23 | 1 | 25 | 51 | 6 | 2 | 79 | 11 | 15 | 107 | 14 | 5 |
| 24 | 1 | 32 | 52 | 6 | 14 | 80 | 11 | 25 | 108 | 14 | 7 |
| 25 | 1 | 40 | 53 | 6 | 25 | 81 | 11 | 34 | 109 | 14 | 8 |
| 26 | 1 | 47 | 54 | 6 | 37 | 82 | 11 | 43 | 110 | 14 | 9 |
| 27 | 1 | 56 | 55 | 6 | 49 | 83 | 11 | 52 | 111 | 14 | 10 |
| 28 | 2 | 4 | 56 | 7 | 00 | 84 | 12 | 00 | 112 | 14 | 10 |

*Tabula Dimidiæ Moræ* Primi Satellitis *in Umbra* Jovis.

| N. I. | h. | ′ | ″ | N. I. | h. | ′ | ″ |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 4 | 56 | 1200 | 1 | 5 | 6 |
| 40 | 1 | 4 | 33 | 1240 | 1 | 4 | 48 |
| 80 | 1 | 4 | 12 | 1280 | 1 | 4 | 26 |
| 120 | 1 | 3 | 59 | 1320 | 1 | 4 | 7 |
| 160 | 1 | 3 | 48 | 1360 | 1 | 3 | 54 |
| 200 | 1 | 3 | 39 | 1400 | 1 | 3 | 38 |
| 240 | 1 | 3 | 38 | 1440 | 1 | 3 | 38 |
| 280 | 1 | 3 | 48 | 1480 | 1 | 3 | 44 |
| 320 | 1 | 4 | 1 | 1520 | 1 | 3 | 52 |
| 360 | 1 | 4 | 16 | 1560 | 1 | 4 | 7 |
| 400 | 1 | 4 | 36 | 1600 | 1 | 4 | 24 |
| 440 | 1 | 4 | 56 | 1640 | 1 | 4 | 42 |
| 480 | 1 | 5 | 18 | 1680 | 1 | 5 | 00 |
| 520 | 1 | 5 | 41 | 1720 | 1 | 5 | 22 |
| 560 | 1 | 6 | 1 | 1760 | 1 | 5 | 46 |
| 600 | 1 | 6 | 21 | 1800 | 1 | 6 | 10 |
| 640 | 1 | 6 | 39 | 1840 | 1 | 6 | 28 |
| 680 | 1 | 6 | 53 | 1880 | 1 | 6 | 45 |
| 720 | 1 | 7 | 3 | 1920 | 1 | 6 | 57 |
| 760 | 1 | 7 | 11 | 1960 | 1 | 7 | 7 |
| 800 | 1 | 7 | 15 | 2000 | 1 | 7 | 13 |
| 840 | 1 | 7 | 13 | 2040 | 1 | 7 | 14 |
| 880 | 1 | 7 | 9 | 2080 | 1 | 7 | 15 |
| 920 | 1 | 7 | 2 | 2120 | 1 | 7 | 15 |
| 960 | 1 | 6 | 54 | 2160 | 1 | 7 | 10 |
| 1000 | 1 | 6 | 39 | 2200 | 1 | 6 | 49 |
| 1040 | 1 | 6 | 22 | 2240 | 1 | 6 | 32 |
| 1080 | 1 | 6 | 5 | 2280 | 1 | 6 | 15 |
| 1120 | 1 | 5 | 45 | 2320 | 1 | 5 | 58 |
| 1160 | 1 | 5 | 26 | 2360 | 1 | 5 | 38 |
| 1200 | 1 | 5 | 6 | 2400 | 1 | 5 | 18 |
| | | | | 2440 | 1 | 5 | 2 |

*From these Tables, to any given Year, Month and Day, to find the next Eclipse of the First* Satellite *of* Jupiter *proceed thus.*

1. In the Table of the *Epocha* find the Year of our Lord, and set down the Day, Hours, Minutes and Seconds, with *Num.* I. and *Num.* II. thereto annext; and in the Table of Revolutions, seek the Month, and Day of the Month, with the Hours and Minutes, and *Num.* I. and *Num.* II. affixt, and add them together; and the respective Summs shall shew the Mean Time of the Middle of the *Eclipse* sought, with *Num.* I. and *Num.* II. required. But it must be Observed, that in *Jan.* and *Feb.* in the *Leap-Year*, one Day is to be added to the Day thus found.

2. If *Num.* I. be found less than 1224, with *Num.* I. or if the greater than 2448, substracting 2448 therefrom, with the Residue; enter the Table, and you will have the First Æquation to be Added to the Mean Time before found. But if *Num.* I. be less than 2448, but greater than 1224, substract it from 2448, and entring the same Table with the remainder, you shall have the First Æquation to be Substracted from the Mean Time. Then divide the Minutes of the said first Æquation by 11, or rather $3\frac{4}{3}$, and the *Quote* shall be the Æquation of *Num.* II. (answering to the Eccentrick Motion of *Jupiter*,) to be Added thereto when the first Æquation Substracts, and *è contra* Substracted when that Adds.

3. If *Num.* II. thus Æquated exceed 225,4, Substract 225,4, therefrom; and if the Remainder or *Num.* II. be less than 113, with the said Remainder or Number; or if greater than 113, with the Complement thereof to 225, 4, seek in the Table the second Æquation, which being Added to the Time before found, gives the True Time of the Middle of the *Eclipse*.

4. With *Num.* I. seek the Half Continuance of the Total *Eclipse*, which is to be Added for the *Emersion* when the Æquated *Num.* II. is less than 113, or if more than 225, 4, it be less than 338: But if it exceed 113, or 338, then is the *Semimora* to be Substracted for the *Immersion*.

*Vid. Sup.* §. XXII.

5. Lastly, With the *Sun's* True *Place* take out the Æquation of Natural Days, which Added or Substracted according to the Title, gives the Time of the *Immersion* or *Emersion* sought.

Now how few Figures serve for this Computation, will best appear by an Example.

*An.* 1677. *Sept.* 17 $8^h$ 9′ 40″. at *Greenwich*, Mr. *Flamsteed* observed the *First Satellite* to begin to *Emerge*; that is $8^h$ 9′ 26″. at *London*.

| | d. | h. | ′ | ″ | N. I. | N. II. |
|---|---|---|---|---|---|---|
| 1677. | 0 | 3 | 14 | 36 | 2028 | 102, 5 |
| *Sept.* | 17 | 4 | 4 | 12 | 147 | 145, 5 |
| *Sept.* | 17 | 7 | 18 | 48 | 2175 | 248, 0 |
| *Æquat.* 1. | — | | 26 | 11 | 2448 | 2, 3 |
| *Æquat.* 2. | 17 | 6 | 52 | 37 | 273 | 250, 3 |
| | + | 0 | 1 | 39 | | 225, 4 |
| *Semimora* | + | 1 | 7 | 00 | | 24, 9 |
| *Equal Time* | 17 | 8 | 1 | 16 | 11) 26, 2 (2, 3 | |
| *Æquation* | + | 0 | 9 | 25 | ☉ in ♎ 5°, 00′ | |
| *Apparent T.* | 17 | 8 | 10 | 41 | | |
| *Observ.* | 00 | 8 | 9 | 20 | | |
| *Error* | | | 1 | 21 | | |

An *Immersion* of this *Satellite* being computed after the same manner according to these Tables, ought to have happen'd. *An.* 1683. *Nov.* 30. $16^h$ 52′ 7″. but I observed it at $16^h$ 48′ 40″. so that the Error was — 3′ 27″.

Again, *M. Cassini* observ'd an *Emersion* at *Paris*. *An.* 1693. *Jan.* $14^d$ $10^h$ 40′ 28″. that is, at *London* $10^h$ 30′ 48″. Which these Tables give at $10^h$ 30′ 39″. and therefore the Error was no more than + 9″.

After this manner I have compared these Tables with many good and certain Observations, and scarce ever find them Err above 3 or 4 Minutes of Time; which Errors are exceeding small in comparison of the short Time that the *Satellites* have been discovered.

In the Construction of the Table, which shews the Half-Continuance of these *Eclipses*, the Semidiameter of the Shadow of *Jupiter* is made by *Cassini* just 10 Deg. and that of the *Satellite* 30′; and the *Satellite's Ascending Node* being supposed in 15° of *Aquarius*, at the End of this Century, (that is 55° 20′ before the *Perihelion* of *Jupiter*) it will thence follow, that *Num.* I. being 116, or 2102, *Jupiter* passes the *Nodes* of the *Satellites* Orb, and consequently these *Eclipses* are Central, and of the Greatest Duration. But *Num.* I. being 215, or 1481, the *Satellite* passes the Shadow with the Greatest Obliquity, *viz.* 2° 55′ from the Center, whence the *Semimora* becomes of all the shortest.

3. The Tables of the other 3 *Satellites* not being so perfect or exact as those of the First, are here given in another Form. The Periods of their Revolutions to *Jupiter's* Shade are as follows. *Of the other 3 Satellites. Ib. p. 253.*

Period.

| | d. | h. | ′ | ″ | ‴ | |
|---|---|---|---|---|---|---|
| *Period.* Secundi | 3 | 13 | 17 | 54 | 3 | *five* $2\frac{1}{123}$ *Rev.* Primi. |
| *Period.* Tertii | 7 | 3 | 59 | 39 | 22 | *five* $4\frac{3}{61}$ *Rev.* Primi. |
| *Period.* Quarti | 16 | 18 | 5 | 6 | 50 | *five* $9\frac{7}{15}$ *Rev.* Primi. |

Whence the Table of the First Æquation of the *First Satellite*, or *M. Caſſini*'s larger Table, may by an eaſie Reduction ſerve the other three; the Æquation of the 2*d.* being $2\frac{1}{123}$, or twice the Minutes with half ſo many Seconds as there are Minutes in the Æquation of the *First*, and the greateſt Æquation thereof 1h 18′ 35. Æquation of the 3*d.* is $4\frac{3}{61}$ times greater than that of the *First*, and when greateſt amounteth to 2h 29″. And the Æquation of the 4*th.* being $9\frac{7}{15}$ times that of the *First*, is had by ſubſtracting $\frac{1}{2}$, and $\frac{1}{30}$ from 10 times the Æquation of the *First*; whence the greateſt becomes 6h 10′ 28″. ſo that *Num.* I. and *Num.* II. as here collected for the *First*, may indifferently ſerve all the reſt.

M. Romer's *Æquation of Light defended.* *Ib. p.* 254. *Vid. Sup.*

4. As to the Second Æquation of the other *Satellites*, *M. Caſſini* has, by his *præcepta Calculi* (as is before mentioned) ſuppoſed the Minutes thereof to be increaſed in the ſame Proportion, as inſtead of 14′ 10″, in the *First*, to be 28′ 27″ in the *Second*, 57′ 22″ in the *Third*, and no leſs than 2h 14′ 7″ in the *Fourth*; whereas if this Second Inequality did proceed from the Succeſſive Propagation of Light, this Æquation ought to be the ſame in all of them, which *M. Caſſini* ſays, was wanting to be ſhewn, to perfect *M. Romer*'s Demonſtration; wherefore he has rejected it as ill founded. But there is good Cauſe to believe, that his Motive thereto, is what he has thought not proper to diſcover. And the following Obſervations do ſufficiently ſupply the Defect complained of in the making out of that Hypotheſis.

*An.* 1676, *Oct.* 2. ſt. n. 6h 10′ 37″. *App.* but 5h 59′ 37″. *Eq. Time. M. Caſſini* at *Paris* obſerved the *Emerſion* of the 3*d. Satellite* from *Jupiter*'s Shadow. And again *Nov.* 14. following 6h 20′ 55″. *App.* Time, but 6h 5′ 55″. *Eq. Time*, he obſerved the like Emerſion of the ſame *Satellite*. The obſerved Interval of Time between theſe *Emerſions*, was 43d 0h 6′ 18″, which is 8′ 22″ more than 6 Mean Revolutions of this Satellite, of which 4′ 27″ ariſes from the Difference of the First Æquation, and the greater Continuance of the Latter *Eclipſe*; ſo that the other 4. Minutes is all that is left to anſwer for the Difference of the 2*d* Æquations; and *Num.* II. in that Time increaſing from 48 to 72, gives 4′ 36″ for the Difference of the 2*d.* Æquations of the *First Satellite*. So that here the 2*d* Æquation of the *Third* is found rather leſs than that of the *First*, but the Difference is ſo ſmall, that it may rather be attributed to the Uncertainty of Obſervation. Whereas according to *M. Caſſini*'s Method of Calculating, inſtead of four Minutes it ought to be 18′ 38″. and the Interval of theſe two *Emerſions* 43d 0h 21′. exceeding the Time obſerved by a whole Quarter of an Hour; which that Curious Obſerver could not be deceived in.

The like appears yet more evidently in the *Fourth Satellite.* By the Obſervations of Mr. *Flamſteed* at *Greenwich. An.* 1682. *Sept.* 24. 17h 45′. *T. App.* but 17h 32′. *T. Eq.* the *Fourth Satellite* was ſeen newly come out of the Shadow, ſo that about 17h 30′. *T. Eq.* the firſt beginning of *Emerſion* was conjectured; and after 5 Revolutions, *viz. Decemb.* 17d 11h 16′ or 11h 18′. *T. Eq.* he again obſerved the firſt Appearance of the *Satellite* beginning to *Emerge*, that is, after an Interval of 83d 17h 48′; whereas this *Satellite* makes five mean Revolutions in 83d 18h 25′½. Here we have 37′½ to be accounted for by the ſeveral Inequalities. Of this 21′ is due to the firſt Æquations, which is reduced to 19′ by the Greater Continuance of the latter *Eclipſe*, *Jupiter* then Approaching to his *Deſcending Node.* So that there remains only 18′½ for the Difference of the 2*d.* Æquations, whilſt the Earth approached *Jupiter*, by more than the *Radius* of its own Orb: and the Difference of the 2*d.* Æquations of the *Firſt Satellite*, being, according to *Caſſini* 8′ 30. the ſaid Difference in the *Fourth* ought to be 1h 20′½, inſtead of 18½; whence the Interval of theſe two *Emerſions* would be according to his Precepts, but 83d 16h 46′, inſtead of 83d 17h 48′ obſerved. And whereas 18′½, may ſeem too great a Difference; it muſt be noted, firſt, that *M. Romer* had ſtated the whole 2*d.* Æquation 22′ 00″. which *M: Caſſini* has diminiſhed to 14′ 10″. ſo that inſtead of 8′½. *M. Romer* allows 13′, and ſecondly, that in the firſt of theſe Obſervations, being about half an Hour before Sun riſe, the Brightneſs of the Morning might well hinder the ſeeing of this ſmalleſt and ſloweſt *Satellite*, till ſuch time as a good part thereof was *Emerged.*

XCIII. Having a great Deſire to obſerve the Body of *Mars*, whilſt *Acronycal* and *Retrograde* (having formerly with a Glaſs about 12 Foot long, obſerv'd ſome kind of Spots in the Phaſe of it) though it was not in the *Perihelium* of its Orb, but nearer its *Aphelium*; yet I found that the Face of it, when near its Oppoſition to the Sun (with a Charge, the 36 Foot-Glaſs, I made uſe of, would well bear) appeared very near as big, as that of the Moon to the naked Eye. *The Phaſes and Revolution of* Mars *about his Axis; by Dr.* Hook. *n.* 11. *p.* 138. *n.* 14. *p.* 239.

But ſuch had been the ill Diſpoſition of the Air for ſeveral Nights, that from more than 20 Obſervations of it which I had made ſince its being *Retrograde*, I could find nothing of Satisfaction, though I often Imagined, I ſaw *Spots*, yet the Inflective Veins of the Air (if I may ſo call thoſe parts, which being interſpers'd up and down in it, have a greater or leſs Refractive Power, than the Air next adjoining, with which they are mixt), did make it ſo confuſed and Glaring, that I could not conclude upon any thing.

On the 3*d.* of *Mar.* 0h 30′. in the Morning though the Air was ſtill bad enough, yet I could ſee now and then the Body of *Mars*, which I deſcribed by the Scheme B; as exactly repreſenting what I ſaw through the Glaſs as I could. Fig. 146.

*An.* 166⅚. *Mar.* 10. 0h 20′. in the Morning finding the Air very bad, I made uſe of a very ſhallow Eye-Glaſs, as finding nothing diſtinct with the greater Charge, and ſaw the Appearance of it as in C, which I Imagined might be the Repreſentation of the former *Spots* by a leſſer Charge. About 3 of the Clock Fig. 147.

the

the ſame Morning, the Air being very bad (though to Appearance exceeding clear, and cauſing all the Stars to twinkle, and the Minute Stars to appear
Fig. 148. very thick) the Body ſeemed like D; which I ſtill ſuppoſed to be the Repreſentation of the ſame *Spots* through a more confuſed and Glaring Air.

But obſerving *Mar.* 21. I was ſurpriſed to find the Air (though not ſo clear, as to the Appearance of ſmall Stars) ſo exceeding tranſparent, and the Face of *Mars* ſo very well defined, and round and diſtinct, that I could
Fig. 149. manifeſtly ſee it of the Shape in E, about half an Hour after 9 at Night. The Triangular *Spot* on the Right-ſide (as it was inverted by the Teleſcope) according to the Appearances, through which all the preceding Figures are drawn) appeared very black and diſtinct, and the other towards the left more dim; but both of them ſufficiently plain and defined. About a Quarter before 12 of the Clock the ſame Night, I obſerved it again with the ſame
Fig. 150. Glaſs, and found the Appearance exactly, as in F; which I Imagined to ſhew me a Motion of the former Triangular *Spot.*

*Mar.* 22. about half an Hour after 8 at Night, finding the ſame *Spots* in
Fig. 151. the ſame Poſture (as in G) I concluded that the preceeding Obſervation was only the Appearance of the ſame *Spots* at another Height and Thickneſs of the Air: and thought my ſelf confirmed in this Opinion, by finding them in
Fig. 152. much the ſame Poſture, *Mar.* 23. about half an Hour after 9 (as in H) though the Air was nothing ſo good as before.

*Mar.* 28. about 3 of the Clock, the Air being light (in Weight) though
Fig. 153. moiſt and a little hazy; I plainly ſaw it, to have the Form repreſented in I. which is not reconcileable with the other Appearances, unleſs we allow a Turbinated Motion of *Mars* upon its Center: Which, if ſuch there be, from the Obſervations made *Mar.* 21, 22, and 23. we may gueſs it to be once or twice in about 24 Hours, unleſs it may have ſome kind of Librating Motion; which ſeems not ſo likely.

*The Parallax of* Mars; *by Mr.* Flamſteed. *n.* 89. *p.* 5118. *n.* 96. *p.* 6100.

XCIV. *An.* 1672. *Sept. Micrometro* & Tubo 14 pedum, *Martis* Diſtantias à duabus Fixis eadem Nocte dimenſus ſum; unde didici *Parallaxin* ejus *Acronici* & *Perigei* nunquam majorem eſſe Scrupulis Secundis 25″; unde ſequitur, *Solis* eſſe ſummum 10″, & Diſtantiam 21000 *Terræ* Semidiametros.

*Places of* Mars *Obſerved at* Darby; *by Mr.* Flamſteed. *n.* 86. *p.* 5039.

XCV. 1. *An.* 1672. *Maii* 14 mane, ibat *Martis* Sidus prope Stellam dictam, *Quæ ad Clunes Aquarii*; cujus Latitudo 2° 0′ 0″. Locus tum *mihi*, 24° 12′ 9″. ♒; *Streetio*, 24° 9′ 00″. è qua notabam.

| | ° ′ | | ° ′ ″ |
|---|---|---|---|
| *Altâ Fixâ* verè | 9 40 | 2h 29′ mane, *Martis* in eodem *Azymutho* præciſe *Diſtantiam* | 0 24 17 |
| | 11 12 | eandem denuo *Diſtantiam*—- | 0 24 24 |
| | 12 00 | 2h 51. *Planeta* diſceſſerat ab eodem *Azymutho*, eratque ad Ortum à Linea, *Azym.* Differ. | 0 00 55 |

| | ° | ′ | ″ |
|---|---|---|---|
| 2^h 29′. mane *Angulus Parallacticus* erat | 82 | 26 | 00 |
| Ergo *Mars* in Conſequentia *Fixæ* | 00 | 3 | 11 |
| cum majori Latitudine Auſtrali | 00 | 24 | 4 |
| *Martis* propterea Latitudo | 2 | 24 | 4 |
| Locus verus { *Streetio.* ♒. | 24 | 12 | 17 |
| Locus verus { *Mihi.* ♒. | 24 | 15 | 40 |

2. *An.* 1683. Die 3. *Maii* $9^h$ 12′. *Mars* Conjunctionem celebrabat cum *Penultima* Stella *Alæ Auſtrinæ Virginis*, quæ modo degit in 13° 49′ 14″. ♎. & *Latit.* 1° 48′ 33″. Bor. ſic ut in ipſa Conjunctione vix ultra 40″. *Mars* infra dictam Stellam incederet. *At* Dantzick; *by* M. Hevelius. *n.* 151. *p.* 332.

XCVI. 1. *M. Burattini* hath ſignify'd from *Poland*, that he hath Obſerved Inequalities in *Venus* as in the *Moon*. *Spots in* Venus; *by M.* Burattini. *n.*10. *p.*173.

2. I have at laſt diſcover'd towards the Middle of the Body of *Venus* a part clearer than the reſt, by which one may Judge of the Motion, or the Reſt, of this Planet. *The Rotation of* Venus; *by M.* Caſſini. *n.* 32. *p.* 615. *n.* 35. *p.* 687.

The firſt time I ſaw it, was *Octob.* 14. 1666. $5^h$ 45′. *p.m.* and then this Bright part was very near the Center, on the North-ſide. And at the ſame time I obſerv'd Weſtward two obſcure *Spots*, ſomewhat oblong; but I could not then ſee that Reſplendent part long enough to conclude any thing from thence, nor was I able to ſee any thing well of thoſe parts till *April* 28. 1667. on which Day a quarter of an Hour before Sun-riſing, I ſaw again a Bright part, ſcituated near the Section, and diſtant from the Southern Horn a little more than ¼ of its Diameter. And near the Eaſtern Ring I ſaw a Dark and ſomewhat Oblong *Spot*, which was nearer to the Northern than the Southern Horn. At the Riſing of the *Sun* I perceived, that this Bright part was then no more ſo near the Southern Horn, but diſtant from it ⅓ of its Diameter. This gave me great Satisfaction. But I was ſurpriſed at the ſame time to find, that the ſame Motion, which was made from South to North in the Inferiour part of the Disk was on the Contrary made from North to South in the Superiour part; whence the Determination of the Motion may be better taken: For we have no Example of the like Motion, except it be in that of the Libration of the Moon.

The next Day, at the Riſing of the *Sun*, the ſaid Bright part was not far from the Section, and diſtant from the Southern Horn ¼ of the Diameter. When the *Sun* was 4 Degrees high, the ſame was ſcituated near the Section, remote from the Southern Horn ⅖ of the Diameter. The *Sun* being high 6° 10′. it ſeemed to have paſſed the Center, and that the Section of the Disk did cut the ſame. The *Sun* being 7° high, it appeared yet more advanced Northward, together with two Obſcure *Spots* ſeated between the Section and the Circumference, and equally diſtant from one another, and from each Horn on both Sides. And the Sky being very clear, I obſerved the Motion of the Bright part for $1\frac{1}{8}^h$. which then ſeemed to be exactly made from South to North without any ſenſible Inclination Eaſtward or Weſtward.

 Mean

Mean time I perceived in the Motion of the dark *Spots* ſo great a Variation, that it cannot be aſcribed to any Reaſon in *Opticks*.

*May* 10. and 13. Before *Sun-Riſing*, I ſaw ſtill the Bright part near the Center Northward.

Laſtly, *June* 5. and 6. Before the Riſing of the *Sun*, I ſaw the ſame between the Northern Horn and the Center of this Planet, and I noted the ſame Irregular Variation in the Obſcure *Spots*. But when *Venus* began to be farther removed from the *Earth*, it was more difficult to obſerve theſe *Phænomena*.

It is very difficult to determine any thing of the Motion of theſe *Spots*: Yet this I can ſay (ſuppoſing that this Bright part of *Venus*, which I have obſerv'd, eſpecially this Year 1667, hath always been the ſame) that in leſs than one Day it abſolves its Motion, whether of Revolution or Libration, ſo as in near 23 Hours it returns to the ſame Scituation in this *Planet*, which yet happens not without ſome Irregularity.

*A Place of* Venus, *Obſerv'd at* Dantzick; *by M.* Hevelius. *n.* 154. *p.* 419.

XCVII. *An.* 1683. *Aug.* 4. horâ fere 2. *m. Venus* à Stella Fixa 3. Magnitud. à *Ventre* ſcil. *Pollucis* non niſi 16′. removebatur Auſtrum verſus, id quod ex *Micrometro* accurate comperi.

XCVIII. Mercury *obſerved in the Sun*, 1690. *at* Nuremberg; *by M.* Jo. Phil. Wurtzelbaur. *n.* 192. *p.* 483.

| *Temp. Horol. Oſcil.* | *Obſervationes.* | | | *Tempora ſupputata.* |
|---|---|---|---|---|
| h. ′ ″ | Die 30. *Octob.* p. m. *A.* 1690. | | | h. ′ ″ |
| 6 32 00 | Culminat *Os Pegaſi* ——— | | | 6 28 45 |
| 9 00 00 | Culm. *Cap. Andromedæ* ——— | | | 8 52 17 |
| 9 4 00 | Culm. *Jupiter* ——— | | | |
| | Die 31. *Octob.* m. | | | |
| 8 30 00 | *Sol* è nubibus emerſit, *Mercurius* in diſco ejus ſuperne in Tab. obſervatoria, à Verticali ad dextram (revera ad Lævam) diſtans plus quam Dim. *Dig.* à Limbo exiturus apparuit. | | | |
| 8 36 00 | *Mercurius* poſtquam Undulanti Limbo *Solis* ad Min. Temporis adhæſerat, exiit ad 14° à *Zenith* Septentrionem verſus. | ° | ′ | h. ′ ″ |
| 8 49 00 | Altitudo *Solis* ——— | 10 | 5 | 8 38 38 |
| 8 59 45 | ——— | 11 | 10 | 8 47 48 |
| 9 7 10 | ——— | 12 | 10 | 8 56 24 |
| 9 50 00 | ——— | 16 | 28 | 9 38 7 |
| 11 1 30 | ——— | 21 | 31 | 10 56 32 |

Ratio Diametrorum *Solis* & *Nuclei Mercurii*, dum Lucido *Solis* Diſco immorabatur, quantum per auram haud ſatis defæcatam conjici poterat, erat ut 1000

1000 ad 8½. Poſtea quam ad Limbum *Solis* pervenerat, ejuſque Limbo Undulanti ad Minutum fere adhæſitarat, & genuinam Rotunditatem ſuam (quæ antea ex luce Diſci *Solaris* formam quaſi Ellipticam mentiebatur) recuperarat, erat ut 1000 ad 12⅔.

XCIX. *An.* 1697. *Nov.* 3. ſt.n. 7h 25′ cum *Sol* è nubibus Emerſiſſet directo ad ipſum Teleſcopio Differentia Aſcenſionis Rectæ Centri *Mercurii* Occidentalis & Centri *Solis* obſervata per Horologium fuit Hor. ———————— 0° 11′ 52″

*Mercury obſerved in the Sun. Nov. 3. ſt. n. 1697; by M. Caſſini. n. 245. p. 372.*

Differentia Declinationis *Mercurii* Meridionalis fuit Grad.— 0 6 20

8h 3′. Differentia Aſcenſionis Rectæ Centrorum *Mercurii* Occidentalis & *Solis* fuit Hor. ———————— 0 15 30

Differentia Declinationis Grad.———————— 0 4 42

8h 8′ 38″. *Margo* Præcedens *Mercurii* pervenit ad *Solis* Marginem Præcedentem.

8h 10′ 24″. *Mercurius* totus Emerſit è *Solis* diſco Teleſcopio pedum 18 obſervatus.

Ex his obſervationibus invicem comparatis, quantum ex hoc brevi intervallo inferri potuit, adventum *Mercurii* ad medium ipſius ſemitæ in *Solis* Diſco Trigonometricè deduxi 6h 11′ 18″. poſt Meridiem.

*Nodum* vero *Aſcendentem Mercurii* in ♉ 14° 42′. adhuc promotiorem quam per obſervationes *Anni* 1677.

*Inclinationem* autem *Orbitæ Mercurii* ad *Eclipticam* ex poſtremarum obſervationum comparatione inveni 6° 23′, quam nihilominus ob breve harum obſervationum intervallum præferre non auſim ei quam ex *Sinenſibus* obſervationibus R. P. *Fontenay* longè majori intervallo diſtantibus deduxi. ſc. 6° 40′. propius accedentem ad Tabulas Rudolphinas.

C. *Mercurii Veneriſque* Sidera *Solis* Diſcum ſubintrare, ac inſtar Macularum Nigricantium in Lucido ejus Orbe aliquando conſpici, tam ex Verioris *Aſtronomiæ* Principiis, quam ex indubitata Obſervantium Fide, dudum compertum eſt. Qua vero Lege, quibuſve Conditionibus, quantiſque *Annorum* Intervallis, hæc Phænomena nobis ſpectanda præbentur, neſcio an aliquis ex *Aſtronomis* Hodiernis ritè definiverit: Certe nihil hac de re inter Typis mandata hucuſque mihi viſum eſt. Quapropter non Ingratum fore arbitratus, huic Inquiſitioni ſeriò operam dedi, ac Diſſertatione hac rem maximè perplexam pauciſque intellectam me plenius enucleaturum confido.

*The Viſible Conjunctions of the Inferior Planets with the Sun; by Mr. Halley. n. 193. p. 511.*

Has *Planetarum* Horum Phaſes ſemper in Retrogradientium cum *Sole* Conjunctionibus fieri, cum ſcil. *Sol Nodis* eorum adeo vicinus ſit ut *Planetæ Soli* juncti Latitudo Semidiametrum *Solis* non excedat, per ſe ſatis conſpicuum eſt; quo vero facilius Limites ac Conditiones harum Conjunctionum perveſtigem, cumque Calculi Elementa omninò diverſi ſint, uterque Planeta ſigillatim tractandus eſt: à *Mercurio* itaque exordiamur.

Hujus Planetæ *Nodum Aſcendentem*, juxta nuperas & accuratas Obſervationes, prope 15° *Tauri*, ſeu potius ad 1s 15° 44′. à 1a ✶ ♈. hoc noſtro ſeculo reperiri, pro comperto habemus. *Deſcendentem* vero Oppoſitam ad

 6s 15° 44″.

6s. 15° 44′. *à* 1ª ✱ ♈. Angulum autem quo Planum Orbitæ *Mercurialis* ad *Eclipticam* Inclinatur ſatis bene ſe habet apud *Keplerum*, viz. 6° 54′. Jam ex probatiſſimis Hypotheſibus conſtat *Mercurii* in *Nodo Aſcendente* conſtituti Diſtantiam à *Sole* eſſe partium 31365, quarum media *Solis* Diſtantia à *Terra* ſit 100000; dum vero *Nodum* Alterum occupat, Diſtantia iſta, in iiſdem partibus menſurata, fit 45308. *Sol* vero *Nodo Aſcendenti* Oppoſitus diſtat à *Terra* eidem Juncta partium iſtarum 98955, ad *Nodum* vero *Alterum* eadem Intercapedo fit 101007. Atque idcirco *Mercurius Soli* Conjunctus ad *Nodum Aſcend.* diſtat à *Terra* partibus 67591: ad *Nodum* vero *Deſcend.* partibus 55699. Quæ, cum inter ſe valde diſcrepent, ſeparatim etiam conſiderandæ veniunt Conjunctiones illæ quæ ad diverſos *Nodos* fiunt, Calculi Elementis Compendii gratia Synopticè expoſitis.

*Conjungatur* Mercurius Retrogradus *cum* Sole *Centraliter ad* Nodum Aſcendentem, *Menſe* Octobri; *ac ex prædictis Hypotheſibus habebitur.*

| | | s. | ° | ′ | ″ |
|---|---|---|---|---|---|
| Longitudo *Solis* à *Prima* Stella *Arietis* | | 6 | 15 | 44 | 00 |
| Longitudo *Mercurii* ex *Sole* viſi | | 0 | 15 | 44 | 00 |
| Diſtantia *Mercurii* à *Sole* | 31365 | | | | |
| Diſtantia *Mercurii* à *Terra* | 67591 | | | | |
| Angulus *Inclinationis Orbitæ Mercurii* | | 0 | 6 | 54 | 00 |
| Motus 6 Horar. *Mercurii* ex *Sole* viſi | | 0 | 1 | 30 | 58 |
| Motus *Solis* in iiſdem 6 Horis | | 0 | 0 | 15 | 5 |
| Hinc Motus *Mercurii* à *Sole*, ſex Horis | | 0 | 1 | 15 | 53 |
| Et Motus ejus à *Sole* ex *Terra* viſo, 6 Horis | | 0 | 0 | 35 | 12 |
| Et Angulus Viæ *Mercurii* intra *Solem* viſæ cum *Ecliptica* | | 0 | 8 | 15 | 00 |
| Hinc Motus *Mercurii* in *Orbita* ſua viſibili 6 Horis | | 0 | 0 | 35 | 40 |
| Deinde Motus *Mercurii* in *Anno Siderio* ultra quatuor Revolutiones | | 1 | 24 | 45 | 8 |
| In Annis Tredecim itaque | | 11 | 21 | 46 | 44 |
| Deſunt itaque ad Revolutiones 54 Integras | | 0 | 8 | 13 | 16 |
| | | | d. | h. | ′ |
| Quod ſpatium percurrit *Mercurius* in | | | 2 | 00 | 11 |
| | | | ° | ′ | ″ |
| Quibus promovetur *Solis* Locus; ac ☿ in *Nodo* Situs tantundem diſtat à *Conjunctione Terræ* | | | 2 | 1 | 00 |
| At Arcus iſte ex *Terra* ſpectatus fit | | | 0 | 56 | 10 |
| Unde ex dato Angulo *Viæ* viſæ 8° 15′ provenit *Baſis*, ſive Diſtantia à *Conjunctione Viſibili* | | | 0 | 55 | 34 |
| | | | | h. | ′ |
| Qui Arcus percurritur à *Mercurio* juxta Horariam datam | | | | 9 | 21 |
| Excedunt vero 13 *Anni Siderii* totidem *Julianos* cum *Intercalationibus* tribus | | | | 8 | 0 |

Itaque

| | | | | |
|---|---|---|---|---|
| | | d. | h. | ′ |
| Itaque *Mercurius* revertitur ad *Solem* post *Annos Julianos* 13 atque insuper | | 2 | 17 | 34 |
| Vel cum quatuor *Intercalationibus*, si præcedens Annus sit tertius à *Bissext.* | | 1 | 17 | 34 |
| | | | ′ | ″ |
| Ex arcu vero 56′ 10″. & Angulo dato, fit perpendicularis, sive proxima Distantia ☿ à *Sole* | | 0 | 8 | 3 |
| Itaque ☿ post 13 Annos intra *Solem* conspicuus, 8′ 3″. Borealius Incedit. | | | | |

| | | | | |
|---|---|---|---|---|
| | s. | ° | ′ | ″ |
| Pari Argumento in 46. *Annis Sideriis* movetur ☿ | 11 | 28 | 36 | 8 |
| Desunt itaque ad Revolutiones 191 Integras | 0 | 1 | 23 | 52 |
| | | h. | ′ | |
| Hoc est in Tempore | | 8 | 12 | |
| | | ° | ′ | ″ |
| Quo promovetur *Sol* | | 0 | 20 | 41 |
| Hic arcus è *Terra* visus fit | | 0 | 9 | 36 |
| *Basis* vero ei competens | | 0 | 9 | 30 |
| | | h. | ′ | ″ |
| Tempus vero quo *Mercurius Basin* percurrit fit | | 1 | 30 | 00 |
| Excedunt vero 46 *Anni Siderii* totidem *Julianos* cum 11 Intercalationibus | | 19 | 3 | 00 |
| | | d. | h. | ′ |
| Ac *Mercurius* revertitur ad *Solem* post 46 Annos *Julianos* atque insuper | | 1 | 4 | 51 |
| Vel cum duodecim Intercalationibus, ut fit cum Annus præcedens sit Secundus vel Tertius à *Bissextili* | | 0 | 4 | 51 |
| | | ° | ′ | ″ |
| Perpendicularis vero quo *Mercurius* in Boream provehitur fit | | 0 | 1 | 22 |

| | | | | |
|---|---|---|---|---|
| | | h. | ′ | ″ |
| Periodus vero maxime Accurata *Mercurii* ad *Solem* absolvitur *Annis Sideriis* 263 atque insuper | | 1 | 11 | 30 |
| Hi vero *Siderii* superant totidem *Julianos* cum 66 Intercalationibus | | 10 | 20 | 0 |
| Unde post 263 Annos *Julianos*, *Mercurius* ad *Solem* revolvitur tardius vero | | 11 | 33 | 30 |
| | d. | | | |
| Quod si præcedens Annus *Bissextilis* fuerit addantur | 1 | 11 | 31 | 30 |
| Post hoc demum Intervallum Borealius Incedit | 0 | 0 | 0 | 10 |

Cæteræ vero Periodi Laxiores ex jam Inventis facili negotio eruuntur, suntque vel 6 vel 7 Annorum.

Quæ Septem Annis absolvitur, *Mercurium* deprimit versus Austrum 22′. 47″, ac Septem Dies integros, minus 9 Minutis, citius provenit, si duæ fuerint Intercalationes. At cum una Intercalatione, cum scil. Annus prior *Bissextilis* sit, 6 dies subducendi sunt, additis tantum 9 Minutis, ut prius.

Rarius

Rarius vero post sex Annos in *Solis* disco conspicitur iterum Vagus ille Planeta, qui exacta hoc Periodo 30′ 50″. Borealius transit; idque tardius, 9ᵈ 17ʰ 25′, si Annus præcedens sit Secundus vel Tertius à *Bissextili*, aliter 8ᵈ 17ʰ 25′ addendi sunt.

*Pariter si fiat* Conjunctio *ad* Nodum Descendentem *Mense* Aprili.

| | | s. | ° | ′ | ″ |
|---|---|---|---|---|---|
| Longitudo *Solis* à *Prima* Stella Arietis | | 0 | 15 | 44 | 00 |
| *Mercurii* Longitudo ex *Sole* visi | | 6 | 15 | 44 | 00 |
| Distantia Planetæ à *Sole* ut prius | 45308 | | | | |
| Distantia ejus à *Terra* | 55699 | | | | |
| Motus *Mercurii* à *Sole* visi Sex Horis | | 0 | 0 | 14 | 29 |
| Motus *Solis* in eodem tempore | | 0 | 0 | 43 | 21 |
| Motus *Mercurii* à *Sole* | | 0 | 0 | 28 | 52 |
| Hinc Angulus *Viæ* visæ *Mercurii* intra *Solis* discum cum *Ecliptica* fit | | 0 | 10 | 18 | 00 |
| Motus vero visus à *Terra* in 6 Horis | | 0 | 0 | 23 | 52 |

Unde sequendo methodum calculi præcedentis, evincitur *Mercurium* post 13 Annos atque insuper 3ᵈ 7ʰ 37′ ad *Solis* Conjunctionem revolvi; quod si præcedens Annus fuerit Tertius à *Bissextili*, tunc addendi sunt 2ᵈ 7ʰ 37′. tantum: ac tum *Mercurius* 16′ 55″. Australius incedere reperietur.

Post 46 vero Annos, cum 12 Intercalationibus addantur 0ᵈ 7ʰ 14′. & habebitur Mercurius *Soli* Conjunctus in Tramite Australiore 2′ 23″. si vero Annus Prior *Bissextilis* fuerit, vel ab eo Primus, addendus est 1ᵈ 7ʰ 14′. ut habeatur accurate Synodus. Similiter post 263 Annos, quibus *Mercurius* in Austrum deflectitur 0′ 22″. Addendus est vel 1ᵈ 11ʰ 49′. vel 11ʰ 49′, juxta Legem in priori Casu præscriptam.

At Annis sex vel septem ob viciniam *Terræ* ac *Planetæ*, atque idcirco ob ampliatos Arcus, ad hunc *Nodum* non revertitur ad *Solem*, ut intra Discum appareat. Post Annos autem 33 *Solem* Transit Viâ magis Boreali 14′ 2″. ac habetur momentum Conjunctionis subducendo à Prioris tempore 3ᵈ 0ʰ 23′. si fuerit in Anno Tertio à *Bissextili*; aliter subduc 2ᵈ 0ʰ 23′. tantum.

His Inventis facile erit continuare Calculum pro omnibus hisce Conjunctionibus *Mercurii* cum *Sole*, idque cum summa certitudine, ac sine ulla hæsitatione, an omnes possibiles habeantur necne: Sola Additione obtinentur momenta Conjunctionum ac Distantiæ Planetæ à Centro *Solis*; unde etiam ope Tabellæ depromuntur Durationes harum, ut ita dicam, *Eclipsium*, ut nihil sit quod in hac re desiderari videatur.

*Epochas* vero quod spectat, ex tutius Observatorum industria comparantur, quam calculi cujusvis Subtilitate: adeoque elegimus in primo Casu, notabilem illum Transitum *Mercurii* quem ipse in Insula *Sanctæ Helenæ* perfectissime observavi, *Oct.* 28. *An.* 1677. St. Vet. & cujus Medium ex Initio & Fine determinavi in prædicta Insula quidem 0ʰ 4′. p. m. *Londini* vero 0ʰ 28′. p. m.

Scinita

Semita vero qua incedere visus est Planeta 4′ 40″. Borealior erat *Solis* centro. In altero Casu, viz. cum *Mercurius Soli* conjungitur Mense *Aprili*, ex Cl. *Hevelii Mercurio in Sole viso* p. 72. 75. *Epocham* desumere placuit; nempe quod *Apr.* 23. *An.* 1662. St. Vet. 6h 8′. p. m. *Gedani*, hoc est 4h 52′. *Londini*, *Mercurius Solis* centro proximus apparuit, utpote in medio Transitu, simulque distabat ab eodem Centro 4′ 27″ ad Boream. Hinc juxta præcepta præmissa, omnes ordine visibiles Conjunctiones *Mercurii* cum *Sole* simul exhibere, exigui Laboris opus erat: ac in Exemplum quod cuivis in posterum imitari licet, accipe hujus Seculi, ab invento Telescopio, quotquot usquam apparuere hujusmodi Phænomena, vel quæ etiam in sequentis Seculi posteris apparitura sunt.

*Series Momentorum quibus* Mercurius Soli *Conjunctus intra* Discum *ejus conspicitur, per præsens & futurum Seculum, cum* Distantiis *Planetæ* à Solis *centro.*

*APRILI.*

| *Ann.* | *Temp. Conj.* | | | | *Dist. à Cent* ☉ | | |
|---|---|---|---|---|---|---|---|
| | d. | h. | ″ | | ′ | ″ | |
| 1615 | 22 | 21 | 38 | * * | 7 | 20 | B |
| 1628 | 25 | 5 | 15 | * | 9 | 35 | A |
| 1661 | 23 | 4 | 52 | * | 4 | 27 | B |
| 1674 | 26 | 12 | 29 | | 12 | 28 | A |
| 1707 | 24 | 12 | 6 | | 1 | 34 | B |
| 1720 | 26 | 19 | 43 | * | 15 | 21 | A |
| 1740 | 21 | 11 | 43 | | 15 | 36 | B |
| 1753 | 24 | 19 | 20 | * | 1 | 19 | A |
| 1786 | 22 | 18 | 57 | * | 12 | 43 | B |
| 1799 | 26 | 2 | 34 | * * | 4 | 12 | A |

OCTO-

OCTOBRI.

| Ann. | Temp. Conj. | | | | Dist. à Cent. ☉ | | |
|---|---|---|---|---|---|---|---|
| | d. | h. | ′ | | ′ | ″ | |
| 1605 | 22 | 8 | 29 | | 12 | 48 | A |
| 1618 | 25 | 2 | 3 | * | 4 | 45 | A |
| 1631 | 27 | 19 | 37 | * | 3 | 18 | B |
| 1644 | 30 | 13 | 11 | | 11 | 21 | B |
| 1651 | 23 | 13 | 20 | | 11 | 26 | A |
| 1664 | 25 | 6 | 54 | * | 3 | 23 | A |
| 1677 | 28 | 0 | 28 | * * | 4 | 40 | B |
| 1690 | 30 | 18 | 2 | * | 12 | 43 | B |
| 1697 | 23 | 18 | 11 | * | 10 | 4 | A |
| 1710 | 26 | 11 | 45 | | 2 | 1 | A |
| 1723 | 29 | 5 | 19 | * | 6 | 2 | B |
| 1730 | 22 | 5 | 28 | | 16 | 45 | A |
| 1736 | 30 | 22 | 53 | * * | 13 | 5 | B |
| 1743 | 24 | 23 | 2 | * * | 8 | 42 | A |
| 1756 | 26 | 16 | 36 | | 0 | 39 | A |
| 1769 | 29 | 10 | 10 | | 7 | 24 | B |
| 1776 | 22 | 10 | 19 | | 15 | 23 | A |
| | *Nov.* | | | | | | |
| 1782 | 1 | 3 | 44 | * | 15 | 27 | B |
| | *Oct.* | | | | | | |
| 1789 | 25 | 3 | 53 | * | 7 | 20 | A |

Transitus qui signo * notantur, *Londini* ex parte Visibiles sunt, qui vero signo * *, toti conspici possunt.

Notandum vero est *Solis* Diametrum ad *Nodum* ☿rii *Ascendentem* mense *Octobri* occupare 32′ 34″. atque adeo Maximam Durationem centralis Transitus esse 5h 29′, Mense vero *Aprili* Diameter *Solis* sit 31′ 54″; unde ob tardiorem Planetæ Motum oritur Duratio Maxima 8h 1′: Quod si Obliquè incidat *Mercurius*, Durationes hæ Breviores redduntur pro ratione Distantiæ à centro *Solis*: Quoque perfectior Calculus hic reddatur, sequentes Tabellas adjunxi, quibus exhibentur dimidiatæ Durationes harum *Eclipsium* ad singula minuta Distantiæ Visæ à centro *Solis*; quæ Additæ ac Sublatæ à Conjunctionis momento in priori Tabula invento, *Initium* ac *Finem* totius Phænomeni designant.

*OCTOBRI.*

| Min. Dist. | Semi-durat. |
|---|---|
| ′ | h. ′ |
| 0 | 2 44½ |
| 1 | 2 44 |
| 2 | 2 43 |
| 3 | 2 41½ |
| 4 | 2 39½ |
| 5 | 2 36½ |
| 6 | 2 33 |
| 7 | 2 28½ |
| 8 | 2 23 |
| 9 | 2 17 |
| 10 | 2 10 |
| 11 | 2 1 |
| 12 | 1 51 |
| 13 | 1 39 |
| 14 | 1 24 |
| 15 | 1 4 |
| 15½ | 0 50 |
| 16 | 0 30 |

*APRILI.*

| Min. Dist. | Semi-durat. |
|---|---|
| ′ | h. ′ |
| 0 | 4 0½ |
| 1 | 4 0 |
| 2 | 3 58½ |
| 3 | 3 56 |
| 4 | 3 53 |
| 5 | 3 48½ |
| 6 | 3 43 |
| 7 | 3 36 |
| 8 | 3 28 |
| 9 | 3 18½ |
| 10 | 3 7 |
| 11 | 2 54 |
| 12 | 2 38 |
| 13 | 2 19 |
| 14 | 1 55 |
| 15 | 1 21½ |
| 15½ | 0 56 |

Obſervationes omnes hucuſque Habitas ritè repræſentant hi Numeri, nec eſt quod dubitem de Futuris, cum ex omibus Planetis *Mercurius*, *Soli* proximus, ejus Centro ad eo vicinus ſit, ut aliorum Centrorum interventu minimè cieatur, nec deviationibus illis quæ à cæterorum Syſtemate oriuntur, quibuſque ſuperiores, præſertim *Saturnus*, obnoxii ſunt, quod ſentiri poſſit interturbetur.

Parallaxes conſultò omiſi, ut perexiguas, quæque Locis diverſis diverſæ obvenientes Generaliori Calculo immiſceri non debent; quodque etiam quantæ ſint non ſatis adhuc conſtat, ſed potius ex hujuſmodi Obſervatis tutiſſime derivari poſſint: Diametri etiam *Mercurii* rationem non habui, quia ſupra fidem Parvus perpaucula ſolum Minuta Limbo adhærere videtur. Ex Obſervatione accuratiſſimâ deprehendi vix duo Minuta elapſa dum totus è *Sole* egrederetur *Octob.* 28. 1677. unde concluſi Diametrum ejus 0′ 11″, ac juxta ta rationem Diſtantiarum à *Terra* ad *Nodum Alterum* eſſe 0′ 13″½ fere; adeoque tunc 3½ Temporis Minuta inſumi, dum Totus Planeta *Solis* Limbum Directe pervadit: Oblique vero tranſiens paulo diutius hæret, ſecundum ac Secantes Angulorum Incidentiæ augentur. Æquationes etiam Temporis haud opus eſt ut æſtimemus, quia perplures dies hinc inde in utroque Menſe Conſtantes ac quaſi Invariatæ perſiſtunt.

### *De Viſibili* Veneris *cum* Sole *Conjunctione.*

*Venus*, quamvis Syderum omnium ſpecioſiſſima, more Sexus ſui, ſine mutuato Cultu ac Splendore aſcititio in Conſpectum prodire veretur: Hoc etenim Spectaculum inter *Aſtronomica* longe nobiliſſimum, inſtar Ludorum Secularium, integri Seculi Mortalibus invident Motuum arctæ Leges. Unico vero hoc obſervato, ſumma cum certitudine Diſtantiam *Solis* à *Terra* determinari poſſe, quæ ob Parallaxin alias prorſus Inſenſibilem Vagis Terminis huc uſque definita eſt, poſthac declarabitur. Periodos vero quod attinet, illæ non adeo Accurate ac *Mercuriales* deſcribi poſſunt, cum *Venus* ſemel tantum ab Orbe Condito, idque ab *Horroxio* noſtro, intra *Solis* Diſcum deprehenſa ſit: Correctis autem Motibus, quantum per Rudiores Veterum obſervationes licet, accipe jam ſummam Calculi.

| | | s. | ° | ′ | ″ |
|---|---|---|---|---|---|
| Longitudo *Nodi Aſcendentis Veneris* à *prima* Stella *Arietis* | | 1 | 15 | 16 | 00 |
| *Sol* itaque ei jungitur in puncto oppoſito; hoc eſt, per hæc Secula, circa finem *Novembris* | | 7 | 15 | 16 | 00 |
| Diſtantia *Veneris* à *Sole* partium | 71997 | | | | |
| Diſtantia *Veneris* à *Terra* | 26438 | | | | |
| Inclinatio Orbitæ *Veneris* ad *Eclipticam* | | 0 | 3 | 23 | 0 |
| Motus *Veneris* in 8 *Annis Sydereis*, ſupra 13 Revolutiones | | 0 | 1 | 30 | 28½ |
| Motus *Veneris* in 235 *Annis Sydereis* ſupra 381 Revolutiones | | 11 | 29 | 17 | 39 |
| Motus *Veneris* in 243 *Annis Sydereis* ſupra 395 Revolutiones | | 0 | 00 | 48 | 8 |

Ex his Principiis, inito Calculo juxta Methodum in *Mercurio* expoſitam, proveniunt intervalla Temporum ac Diſtantiarum ut ſequitur.

Poſt 18 Annos *Venus* revolvitur ad *Solem* ſc. ſublatis à prioris Tranſitus momento 2$^d$ 10$^h$ 52′½. Incedit vero Planeta Semitâ 24′ 41″. priori magis Auſtrali.

Poſt Annos 235, Additis 2$^d$ 10$^h$ 9′, *Venus* iterum *Solem* ingredi poteſt, ſed viâ 11′ 33″. Borealiori: Quod ſi præcedens Annus *Biſſextilis* fuerit 3$^d$ 10$^h$ 9′. addendi ſunt.

Poſt Annos 243 *Venus* etiam *Solem* tranſire poteſt, auferendo tantum 0$^h$ 43′. à prioris Tempore; Auſtralius vero incedit 13′ 8″: Quod ſi præcedens Annus *Biſſextilis* fuerit, adde 23$^h$ 17′.

Et

Et in omnibus his Appulsibus *Veneris* ad *Solem*, Mense *Novembri*, Angulus Viæ Visæ *Veneris* cum *Ecliptica* sit 9° 5'. ac Motus ejus Horarius intra *Solem* 4' 7''; cumq; Semidiameter *Solis* sit 16' 21'', provenit Maxima Duratio Transitus centri *Veneris* 7h 56'.

Deinde conjungantur *Sol* & *Venus* ad *Nodum Descendentem* Mense *Maio*; ac juxta Numeros eosdem supputantur intervalla eadem. Post 8 Annos auferendi sunt 2d 6h 55', ac *Venus* Orbitâ 19' 58''. Borealiori pertransibit.

Post Annos 235 adde 2d 8h 18', vel si prior Annus *Bissextilis* fuerit 3d 8h 18'. & habebis *Venerem* Australiorem 9' 21''.

Denique post 243 Annos, adde od 1h 23'. vel si prior Annus *Bissextilis* fuerit 1d 1h 23'. & reperietur *Venus* iterum *Soli* Conjuncta, sed in tramite 10' 37''. magis Boreali.

In omni ad hunc *Nodum* Transitu intra *Solem*, Angulus Viæ visæ *Veneris* cum *Ecliptica* fit 8° 28'; ac Horarius ejus Motus 4' 00''; ac *Solis* Semidiametro subtendente 15' 51''; provenit Duratio Maxima centralis Transitus etiam 7h 56'; præcise eadem ac ad *Nodum Alterum*.

Quoad *Epochas*; Ex Ingressu quem solum vidit *Horroxius* in *Sole* jamjam occasuro concluditur, *Venerem Soli* Junctam fuisse *Londini* 1639. *Nov.* 24d. 6h 37'. sed versus Austrum incessisse 8' 30''. Mense *Maio* vero à nemine Mortalium hucusque intra *Solem* visa est, sed ex Numeris meis quos non multum à Cælo ablusuros confido, constat *Venerem* proximâ vice *Solem* subituram *A.* 1761. *Maii* 25d 17h 55'. mediâ scil. *Eclipsi*, ac tunc Distare à Centro ejus versus Austrum 4' 15''. Hinc & ex præmissis Revolutionibus facili negotio omnia hujus generis Phænomena, per Millenium integrum computavi, ut in sequenti Tabella exhibentur.

*NOVEMBRI.*

| *Ann.* | *Temp. Conj.* | | | *Dist. à Cent.* ☉ | | |
|---|---|---|---|---|---|---|
| | d. | h. | ' | ' | '' | |
| 918 | 20 | 21 | 53 | 6 | 12 | B |
| 1161 | 20 | 21 | 10 | 6 | 55 | A |
| 1396 | 23 | 7 | 20 | 4 | 38 | B |
| 1631 | 26 | 17 | 29 | 16 | 11 | B |
| 1639 | 24 | 6 | 37 | 8 | 30 | A |
| 1874 | 26 | 16 | 46 | 3 | 3 | B |
| 2109 | 29 | 2 | 56 | 14 | 36 | B |
| 2117 | 26 | 16 | 3 | 10 | 5 | A |

*MAIO.*

| *Ann.* | *Temp. Conj.* | | | *Dist. à Cent.* ☉ | | |
|---|---|---|---|---|---|---|
| | d. | h. | ′ | ′ | ″ | |
| 1048 | 24 | 13 | 45 | 3 | 50 | B |
| 1283 | 23 | 8 | 14 | 5 | 31 | A |
| 1291 | 25 | 15 | 9 | 14 | 27 | B |
| 1518 | 25 | 16 | 32 | 14 | 52 | A |
| 1526 | 23 | 9 | 37 | 5 | 6 | B |
| 1761 | 25 | 17 | 55 | 4 | 15 | A |
| 1769 | 23 | 11 | 00 | 15 | 43 | B |
| 1996 | 28 | 2 | 13 | 13 | 36 | A |
| 2004 | 25 | 19 | 18 | 6 | 22 | B |

Durationes harum *Veneriarum Eclipsium* quod attinet, reſpectu Centri eodem modo ſupputari poſſunt ac *Mercuriales*; ſed cum Diameter *Veneris* ſatis ampla ſit, cumque Parallaxes etiam Differentiam valde Notabilem quoad Tempus ingerere poſſunt, Calculus peculiaris pro Locis ſingulis neceſſario ſubeundus eſt.

*Veneris* autem Diameter tanta eſt, ut dum Limbo *Solis* adhæret, ferme 20 Temporis Minuta præterfluunt, cum ſcil. *Solem* Directe aggreditur; Oblique vero Incidens, etiam diutius Limbo immoratur: Occupat autem Diameter iſta, juxta *Horroxii* Obſervationem 1′ 18″; dum ad *Nodum Aſcend. Soli* Jungitur, ac 1′ 12″. ad *Nodum Alterum*. Præcipuus autem harum Conjunctionum uſus eſt, *Solis* à *Terra* Diſtantiam ſive Parallaxin eius accurate determinare, quam quidem fruſtra variis Methodis tentaverunt *Aſtronomi*; dum Inſtrumenta quantumvis Subtilia Angulorum quæſitorum Minutiæ facile eludunt. At in obſervando *Veneris* in *Solem* Ingreſſu & ab eodem Egreſſu, ſpatium Temporis inter Momenta Contactuum Internorum, ad ipſum Temporis Minutum ſecundum, hoc eſt, ad $\frac{1}{15}$ *Minuti Secundi* ſive 4‴. Arcus obſervati, ope mediocris Teleſcopii & Horologii Oſcillatorii, per 6 vel 8ʰ accuratè ſibi conſtantis, obtineri poteſt. Ex duabus autem talibus Obſervationibus in Locis Idoneis debité inſtitutis intra Quingenteſimam partem certo concludi *Solis Diſtantiam* proximâ Occaſione commonſtrabo.

*Fig.* 154, 155. Ne quid Obſcuri Lectoribus *Aſtronomicè* minus Doctis videretur, Schemata pro utriuſque *Planetæ* Tranſitu delineavi, quibus rem oculis ſubjicere conatus ſum.

*The Motion of the Comet. A. 1664. Predicted, by M. Auzout. n. 1. p. 3. n. 2. p. 18.*

CI. 1 *M. Auzout*, after he had ſeen the *Comet* (which was firſt obſerved in *Holland Decemb.* 2. 1664.) 4 or 5 times, made the *Ephemerides* of its Motion upon an *Hypotheſis* that it moveth juſtly enough in the Plain of a great Circle,

Circle, which Inclineth to the *Equinoctial* about 30°, and to the *Ecliptick* 49° or 49°½, cutting the *Æquator* at about 45°½, and the *Ecliptick* at 28° of *Aries*, or a little more.

He takes Notice, that more *Comets* enter into our Systeme by the Sign of *Libra* and about *Spica Virginis* than by all the other parts of the Heavens: For, both the present *Comet* and many others registred in *History* have entred that way, and consequently pass'd out of it by the Sign *Aries*; by which also many have entered. n. 2. p. 19.

2. Till the 6th of *Feb.* this *Comet* always advanced: but after that Day, I found that it returned in Augmenting always its Latitude. I left it *Mar.* 8. at the 18. of the *Horn* of *Aries*, almost in the same Latitude; and I am apt to beleive it will be *Eclipsed* this Evening. *Observed; by M.* Auzout. *ib.*

I shall only add, that on *Feb.* 3. we were surprized, to see the *Comet* again much Brighter than ordinary, and with a considerable *Train*. Some did believe, that it approach'd again to us. But having beheld it with a Telescope, I soon said, that it was joyned with two small Stars, whereof one was pretty Bright, and that this *Conjunction* gave the *Comet* that Brightness. Hence it was, that I assur'd my Friends here, that we should no more see it so Bright.

*M. Auzout* also strongly conceives, that this *Comet* could not be *Feb.* 18. *st. n.* where *M. Hevelius*, in his *Prodromus Cometicus*, hath placed it, viz. *in prima Arietis*; unless it be said that it visited that Star of *Aries* on the 18. and returned thence the 19th, into its ordinary Course: for, according to his, and his several Correspondents Observations, the *Comet* on *Feb.* 17. was distant from that *First Star of Aries* at least 1° 17′; and on *Feb.* 19. (He having missed, as well as his other Friends, the Observation on *Feb* 18.) was Advanced in its way 12′ or 13′, but yet Distant from the said *Star* some Minutes above a whole Degree, and consequently far from having then passed it. After which time *M. Auzout* affirms to have seen it as well as several others, for many Dayes, and that until *Mar.* 17. Observing, that about *Feb.* 26 or 27, when the *Comet* was nearest to the often mentioned *First of Aries*, it approached not nearer than 50′. n. 6. p. 107.

3. Some Eminent *English Astronomers*, who have attentively observ'd the Position of this *Comet*, do joyntly Conclude, that whatever that appearance was, which was seen near the *First Star of Aries* by *M. Hevelius* (the truth of whose Relation concerning the same they do in nowise Question) the said *Comet* did not come near that Star in the *left Ear of Aries*, where the said *M. Hevelius* supposes it to have passed, but took its Course near the *Bright Star* in its *Left Horn*, according to *Bayer*'s Tables. *By some* English Astronomers. *ib.* p. 108. n. 9. p. 150.

4. I have easily found the Principle of *M. Auzout*'s *Ephemerides*: and 'tis this, that this *Comet* moves about a Centre, in a straight Line drawn from the Earth through the *Great Dog*, in so great a Circle, that that Portion which is described, is exceeding small in respect of the whole Circumference thereof, and hardly distinguishable by us from a Straight Line. *The Principles of* M. Auzouts *Hypothesis; and the Motion of that* Comet *observed; by M.* Cassini. n. 2. p. 17.

Concerning the *New Comet* you mention, I observ'd it *Feb.* 11. about the 24° of *Aries*, with a Northern Latitude of 24° 40′.

*The Motion of the* Comet. A. 1665. *Predicted; by M.* Auzont. n. 3. p. 36.

CII. 1. *M. Auzout*, after 3 or 4 Obſervations, hath publiſh'd another *Ephemerides* concerning the Motion of the *Comet* which he firſt began to Obſerve *Apr*. 2. 1665 He affirms that the Line deſcribed by this *Star* reſembles hitherto a Great Circle, as it is found in all other *Comets* in the midſt of their Courſe. He finds the ſaid Circle Inclined to the *Ecliptick* about 26°. 30'. and the *Nodes* where it cuts it, towards the Begining of *Gemini* and *Sagittary* ; that it Declines from the *Æquator* about 26°. and cutts it towards the 11°. and conſequently that its greateſt Latitude hath been towards *Piſces*, and its greateſt Declination towards the 25°. of the *Æquator*. He puts it in its *Perigee* about 15°. of *Piſces* a little more Weſterly then *Marchab*, or the *Wing of Pegaſus*.

*Obſerved; by M.* Auzout. *ib.* p. 37.

2. He obſerves in General, that this *Second Comet* is contrary to the precedent, almoſt in all Particulars : ſeeing that the former moved very ſwift, this pretty ſlow ; that againſt the Order of the *Signs* from Eaſt to Weſt, this, following them, from Weſt to Eaſt ; that, from South to North, this, from North to South, as far as it hath been hitherto, that we hear of, Obſerv'd ; that, on the ſide oppoſite to the *Sun*, this, on the ſame ſide ; that, having been in its *Perigee* at the time of its Oppoſition, this having been there, out of the time of its Conjunction. He taketh alſo notice, that this *Comet* differs in Brightneſs from the other, as well in its *Body*, which is far more Vivid and Diſtinct, as in its *Train*, whoſe Splendor is much greater, ſince it may be ſeen even with great Teleſcopes, which were uſeleſs in the former by reaſon of its Dimneſs.

*A Comet.* An. 1668. *at* Bononia ; *by M.* Caſſini. n. 35. p. 683.

CIII. 1. *An.* 1668. *Mar.* 10d. 1h. of the following Night, ( after the *Italian* way of counting ) I obſerved a Path of Light extended from the *Whale* through *Eridanus* ; which I judged to be the *Train* of a *Comet* both by the Figure and Colour, as alſo becauſe that the Direction of it was to the part oppoſite to the *Sun*, like other *Comets*. By its extream Point it reached to that Star in *Eridanus*, which is call'd the 14. by *Bayerus* : But it iſſued out of the Horizontal Clouds, ſo that I thought the *Head* of the *Comet* was either vailed by them, or hid under the *Horizon*. *Mar.* 11. there was ſeen a Brightneſs in the *Whale*, amongſt the thin Clouds, at leaſt for half an Hour, which was very like the Splendor of *Venus*, likewiſe vail'd with thin Clouds.

*Mar.* 12. when the *Great Dog* was in the *Midheaven*, the ſame *Tail* appear'd again. It paſſed through the Star in *Eridanus*, which *Bayerus* calls the 15. and left to the Southward the 14. where it did terminate *Mar.* 10. Being by the Imagination drawn out to about 3°. and further, it tended to that Southern Star which preceeds the *Ear* of *Lepus*. It was therefore more Northerly than the Day before Yeſterday, and more Eaſterly. We were doubtful whether its head was hid by the Clouds or under the Horizon. But the Line from *Jupiter* to the Extremity of the ſaid *Tail* in the Clouds was Perpendicular to that *Tail* ; ſo that it was in the *Whale*, and the Apparent part of the *Train* reached out in Length about 32°.

2. *Mar.* 5. *st. n.* The *Comet* was first discovered: but for as much as it set few hours after the *Sun*, there could hitherto be taken no Considerable Observations of it. The *Body* thereof is not seen, because it remains hid in the Horizon. Its *Train* is of a Stupendious Length, extended in Appearance over almost the 4th part of the Visible Heaven, from West to East; its apparent Breadth is of a good *Palm*, and its Splendour very great, but it Lasts but a few hours. *At* Lisbon; *by* . . . *ib.* p. 684.

3. At St. *Salvador Mar.* 5. (*st. n.*) at 7 a Clock at Night, *F. Estancel* began to see this *Comet* a little above the Horizon from West to E. S. E. The Beginning of its *Tail*, was a little under the two Lucid Stars, the 15 and 16th of the *Whale*'s *Back*, over which it then passed, its Point being as 'twere at the 8 and 9th which are in the Bottom of the *Whale*'s *Belly*; and thus the whole Length thereof was about 23°. The *Globe* or *Head* of it was so small and thin, that very few could discern it with the Naked Eye. *In* Brasil; *by* P. Valentine Estancel. n. 105. p. 91.

*Mar.* 7. The former Brightness was somewhat less, and become so Thin, that the Eye could easily see the Stars that were behind it, which by Conjecture were the 14th and 20th.

The *Tail* was always directly opposite to the *Sun*; and when it appear'd the first time almost Horizontal, it was seen in the form of a Pillar, the *Head* standing a little under, and on the side, of the Star of the *Whale*, which is in the Lat. of 15°. 46′. and the Long. of 12°. 42′. of *Aries*. And the *Point* did shave the 14th, North of the three that are in the *Belly*, in the Lat. of 20°. 30′. and Long. of 15° 57′. of *Aries*.

This *Comet* was at first very Splendid, and cast it self with that Vividness upon the Sea, that the Rayes thereof were reverberated unto the Shoar, where the Observers stood. But this Brightness lasted only for 3 Days, after which it did considerably Decay. But that which seemed somewhat strange was, that having lost so much of its Light, yet its Bulk was not diminish'd, but continued rather increasing until the *Comet* disappear'd. It pass'd more swiftly than *Venus*, whence he infers that it was under *Venus*; yet the Anticipation was not so great, that it could be believ'd to be under the *Moon*, as he would have it.

4. *P. Pietro Susarte*, Rector of *Macao*, in the *East-Indies*, well versed in Matters *Astronomical*, writes to have seen the same all along the Coast of *Bona Speranza*. *In* Africa; *by* P. Pietro Susarte. *ib.*

CIV. There hath been seen here a New *Comet* from the 2. of *Mar. st. n.* 1671. It is but little, having a *Train* not above a Degree or a Degree and an half long. It is now (*Mar.* 9.) about the Stars in the *Right Arm* of *Andromeda* on her Shoulder Blade. As far as I can collect from one or two Observations, it tends towards the *Lucida* of *Andromeda*'s *Girdle*, and that with a Direct Diurnal Motion of about two degrees in its Course. *A Comet*, A. 167½. *at* Dantzick; *by M.* Hevelius. n. 81. p. 4017.

The 6 of *March* in the Evening 7h. 40′ it was in 7°. ♈. and in 35° of Northern Latitude; as I guess'd by the hasty Inspection of a Globe

*Mar.* 7. in the Morning 3h 30′ its Longitude was about 8° ♈. with a somewhat lesser Latitude than before: in the Evening of the same day it's Longitude was 10° ♈. and Lat. 34°. *fere*.

*Mar.*

*Mar.* 8. in the Morning 4h. the Long. was 12°. ♈. and the Lat. 33°. Which yet I would not have taken precisely, because I cannot yet reduce my Observations to a *Calculus.*

*At* ... *by* Mr Newton. *ib.* p. 4018.

2. Mr. *Isaac Newton* about the 16 of *March st. V.* saw a dull Star South-West of *Perseus*, which he now takes to have been that *Comet*. It was very small, and had not any Visible *Tail*, which made him regard it no further.

*At* Paris; *by* M. Cassini. n. 81. p. 4018. n. 82. p. 4042.

3. The *Mathematicians* of *La Flesche* perceived him from the 16 of *March, st. n.* and gave us here at *Paris* the first notice of it. Those of the *College* of *Clermont* being advertised of it, saw him the 25 of the same Month.

*Mar.* 26. 7h. 30′ in the Evening, M. *Cassini* saw him between the *Head* of *Medusa* and the *Pleiades*; without a Telescope he appeared no otherwise than a Star of the 3d Magnitude; His *Head*, seen with a Telescope of 17 foot, appear'd almost Round; but it was well defined, and distinguisht from the mistiness, which formed a kind of *Cheveluro*, wherewith it was encompassed; and even the middle was a little confused, and seemed to have Inequalities, as are seen in Clouds.

The *Tayl* was almost imperceptible; yet by the Telescope it was seen turned Opposite to the *Sun*, and it appeared of the Length of two Diameters of the *Head* or thereabout: For it was not easie to measure it precisely, because being Thinner according as it was farther from the *Head*, its Extremity was insensibly ost. And so the whole *Comet, Head, Tail*, and *Cheveluro*, taken altogether, took up no more then 3 or 4 Minutes of a Degree. At 7h. 48′. he was in a straight Line with the *Lucida* in the *Head* of *Medusa*, and with the most *Occidental* one of the *Pleiades*; and above the two Clearest Stars of the *Southern Foot* of *Perseus*; so that a straight Line drawn through these two Stars, did almost touch the Southern extremity of his *Cheveluro*. This Place of the *Comet*, transferred upon the Map of the Fixt Stars, fell precisely enough upon 23° 25′. of Taurus, in 14° Northern Latitude.

With a Telescope of 3 foot, we saw near the *Comet* two small Stars, distant one diameter of the *Sun* from one another, which Stars are not in the Catalogues. The *Comet* was in the Straight Line, drawn from one of those two Stars to the other precisely at 9h 15′. but a little nigher to that which was Westward: But 9h 33′. he was equally Distant from them both. It was taken notice of that from 8h 5′. till 10h 26′ *He* made, in respect of these two Stars, an oblique Motion sensible enough, going from North to South in the same time that he advanced from West to East.

*Mar.* 28. 7h 42′. in the Evening the *Comet* was distant from the less bright Star of the *Southern Foot* of *Perseus*, no more than about 24′ Westward. He had almost the same Latitude with this Star; so that he was precisely enough at 26° 8′. ♉, and in the Lat. 12°. 8′. At 8h. 14′. we took, as well as we could, the Distance of the *Comet* to the Star in the Eye of *Taurus*, called *Aldebaran*, 19° 38′. And 8h 29′. the distance of the *Comet* to the Star, called *Capella*, was found to be of 22° 32′.

*Mar.*

*Mar.* 30. 9h 35′. at Night the *Comet*, ſeen without a Teleſcope, appeared no otherwiſe than a Star of the 4th Magnitude: through the Teleſcope he exceeded even thoſe of the Firſt; but he was very Dark, and in what manner ſoever we look'd upon him, we could Obſerve almoſt no *Tail* at all of him. He had paſſed one Degree and an half beneath the *Lucida* of of the *Southern Foot* of *Perſeus*; ſo that this Star was exactly in the midſt of the *Comet* and the little Star of the *Leg* of *Perſeus*, marked *n* by *Bayerus*, which then we ſaw not but by a Teleſcope. A Straight Line drawn from one of theſe Stars to the other, did almoſt touch the *Southern Limb* of the *Comet*, which being transferred upon the Map of the *Fixt Stars*, fell upon 28° 45′ of *Taurus* in the Northern Latit. of 9° 56′. At 9h 45′. the Weſtern Limb of the *Comet* touched a Straight Line, drawn through this leſs bright Star of *Perſeus's Southern Foot* and through the moſt Northern of the *Head* of *Taurus*; but that he was already got ſomewhat nearer to the Latter.

*Mar.* 31 8h. in the Evening, the *Comet* was in a direct Line with the *Lucida* in the *Foot* of *Perſeus*, and with the moſt Northern in the *Head* of *Taurus*; but he was more than twice as much remoter from the firſt than the other, and being transferred upon the Map of the *Fixt Stars*, he was found at 15′. from *Gemini*, in the Latit. of 8° 49′. During the whole time that we could obſerve him this Night, (which was till 10 a Clock) he quitted not this Straight Line, which was almoſt parallel to the Horizon: notwithſtanding that his own particular motion ſhould raiſe him a little above it; as the Parallax, on the contrary, ſhould ſink him beneath it in approaching to the Horizon. It may be, there was a compenſation made of theſe two contrary Motions: poſſibly alſo the effect of both was not ſenſible.

*April* 1. The *Comet* could not be ſeen without a Teleſcope, becauſe the *Moon*, being very near it, hid him from our ſight. But with a Teleſcope only of one foot we diſcerned him eaſily enough, and found that he had paſſed 45′. beyond the moſt Northern Star of the *Head* of *Taurus*, and that he muſt have touch't it by his Southern Limb; as alſo that he was Diſtant 1° 43′, from the Star that was neareſt to that toward the South; which is equally Bright, yet not marked by *Bayerus*. This place being transferred upon the Map of the *Fixt Stars*, we found that he was at 1° 30′ of *Gemini*, in the Northern Latit. of 7° 44′.

*April* 2. 8h. in the even, M. *Caſſini*, having obſerved the *Comet* with a Teleſcope of one foot, which diſcovered 5°. found that he was two Deg. and an half Diſtant from the moſt Northern Star of *Taurus*; and one Deg. from the Star of the *Ear* marked ρ by *Bayerus*, and by *Tycho* called *Sequentis Lateris Borea*.

Two Lines drawn from the moſt Northern Star of *Taurus*, one to the *Comet*, the other to the Star that is wanting in *Bayerus*, made a Right Angle; and the Diſtance of the *Comet* to this Angle, was double to that which is between theſe two Stars. This Place transferred upon the Map of the *Fixt Stars*, fell on 2° 48′. of *Gemini*, in the Northern Latit. of 6° 40′.

*Apr.* 3. 9h. we ſaw him with the one foot Teleſcope. He had paſſed over the Upper Star of the *Ear* of *Taurus*, and he made with this Star the *Baſis* of an *Iſoſceles* Triangle, on the Top whereof was the Inferior Star of the

 *Ear.*

*Ear.* The two Sides of this Triangle were two times and an half bigger than the *Basis* ; so that the *Comet* was 4° of *Gemini*, in the Northern Lat. of 5° 38′.

*Apr.* 5. 8h. at even, the *Comet* had passed the Northern *Ear* of *Taurus*, and was equally Distant from the Upper Star of the Northern *Ear* and from that which was on the *Front* of *Taurus*. He was also as Distant from the Inferior Star of the *Ear* of *Taurus*, as this Star is from the next Westward, by *Tycho* called *Inferior præcedentis Lateris Quadrilateri* ; and a Streight Line, drawn through the *Comet* and the Upper Star of the *Ear*, made an almost Right Angle with another Line, drawn from the *Comet* to the Inferior of the two small Stars, that are above the *Eye* of *Taurus*. This Place being carried over to the Map of the *Fixt Stars*, the *Comet* was found at 6° 18′. of *Gemini*, in the Northern Latit. of 3° 41′. He was so confused this Night, that even with the 17 foot Telescope we could not exactly distinguish the *Head* from the *Chevelure* which environed him. The whole appeared a little bigger than the Disque of *Jupiter*, seen by the same Telescope.

*Apr.* 6. 8h. at even, a Streight Line drawn from the *Comet* to the Star that is in the *Front* of *Taurus*, made a Right Angle with another Streight Line drawn from this same Star to the Inferior of the two that are above the *Eye* : And the Distance of this Latter Star to that of the *Front* of *Taurus* was twice the Distance of the same Star of the *Front* of *Taurus* to the *Comet*. This Place being transferred upon the Map of the *Fixt Stars*, the *Comet* was found at 7° 25′. of *Gemini*, in the Northern Latit. of 2° 45′. At 9h 6′. we saw on the side of the *Comet* a Star sufficiently clear, which was not farther Distant from him than a little more than the Diameter of the *Comet*, and that was at the same height of the Horizon.

*Apr.* 7. 9h. in the Evening, the *Comet* was equally Distant from the Inferior Star of the *Northern Ear* of *Taurus*, and from the Superior of the root of the *Northern Horn*. He was also as far Distant from this latter Star, as this Star is from that of the *Front*. This Place, being carried over to the Map of the *Fixt Stars*, fell on 8° 30′. of *Gemini*, in the Northern Latit. of 1° 56′.

All the Places of the *Comet*, that we have Observed till now, fall into a Line little differing from an Arch of a great Circle, which cuts the *Ecliptique* in 10° 45′. of *Gemini*, and which consequently hath its greatest Latitude in 10° 45′. of *Pisces* ; which Latitude is between 39° and 40° Northward. The same Circle cuts the *Æquator* at 101 degrees of the *Vernal Section* Eastward, and its greatest *Declination* from the *Æquator* Northward is of 38°½.

Having chosen two of our First Observations (because the latter are not so proper for this purpose) and having taken a Mean between the first Observations of the *Mathematicians* of *La Flesche*, we found, by *Our Method* explained in the Theory of the *Comet* of 1665, that this *Comet* had been in his *Perigee* the 12. of *March* at 8 a Clock in the Morning : that in that time, which is that of his greatest apparent Celerity, he made about 2° 32′ a day in the Great Circle of his apparent Motion, and $\frac{444}{1000}$ of his *Perigee* Distance in the Line of his Equal Motion : that he was in his greatest

Declination

Declination the 11th, and 12th of *March*; and that at that time, he passed through the Inferior Meridian at about two a Clock after Midnight.

If we have rightly determined his *Perigee*, and that the *Hypothesis* of the Equality of his Motion be Just for that time, he hath been visible since the Middle of *February*, at which time he was as far Distant from his *Perigee* by Approaching to the *Earth*, as he is at the present by Receding from it. He must then have been at the extremity of the *Southern Wing* of the *Swan*, and arrived at the *Southern Foot* of *Pegasus* on the 23 of *Febr.* of the same bigness that he was have seen to be of *Mar.* 28. He must have arrived at the Stars of the *Northern Arm* of *Andromeda Mar.* 9; at those of her *Girdle*, 12. when he was in his *Perigee*, and in his greatest Declination; to her *Southern Legg*, *Mar.* 15; between her *Southern Legg* and the *Triangle*, *Mar.* 18. very near as he was observed at *La Flesche*; and under the *Head* of *Medusa*, *Mar.* 25. The days ensuing he must have arrived at the Places marked in our First Observations: But in the last he hath been swifter than this *Hypothesis* will bear. To represent these latter Observations, the Line of the Motion ought to have been made Curve, as we did for the end of the Apparent Motion of the *Comet* 1665. with this difference, that instead of that Lines being Convex in regard of the *Earth*, because the Motion was *Retrograde*, this was to be made Concave towards the *Earth*, because that the Motion of this *Comet* is *Direct*.

It's a thing worth Observing, that this *Comet* keeps his Course almost like that of the 2. *Comet* of 1665, and of another of 1577, observed by *Tycho*. For they have passed through almost the same *Constellations*; though this be more Inclined Northward, and cut the *Ecliptick* 5 or 6 degrees more forward than that of 1665. So that it seems that in this Place of the Heavens there is, as 'twere, a *Zodiaque* for *Comets*.

CV. 1. *Cometam* novum D. *Romer* primùm advertit 28. *April.* st. n. 1. 1677. & me statim de rei novitate admonito 4h 6′ 31″. p. m. n. ejus Altitudinem accepimus 12° 22′ 10″. Judicavi eum fuisse in Verticali declinante ab Ortu ad Septentrionem 33°. circiter. Die 29 manè, momento per nubes à D. *Picardo* visus est, 3h 9′ 31″. p. m. n. in Altitudine 4° 39′. *A Comet. A. 1677. at Paris; by M. Cassini. n. 135. p. 868.*

Die 2 *Maii* manè, Ascensione Rectâ Medii Cœli ex Fixis existente 267°, Altitudo Cometæ erat 4° 5′. Distantia Verticalis à Septentrione ad Ortum 42° 8′. circiter. Die 4 manè 3h 30′. p. m. n. Altitudo *Cometæ* fuit 5° 33′. Distantia *Azimuthalis* à Sept. ad Ortum 42° 32′. circiter.

Die 5. 3h 32′. Altitudo *Cometæ* fuit 5° 10′. Distantia *Azimuthalis* à Septent. ad Ortum 44° 10′. circiter.

Observationes quæ habitæ sunt, Initio *Cometam* reponunt in *Triangulo*, postremò prope *Caput Medusæ*, ostenduntque *Cometam* procedere secundum Signorum Seriem per Lineam proximam, & fere Parallelam, Illi quam descripsit *Cometa An.* 1590. Mense *Feb.* Magnitudo *Capitis*, visi Telescopio, videbatur ferè æqualis *Jovis* Disco, aut paulo minùs; nec perfectè Rotundum apparebat, sed Figuræ Ovalis, longiore Diametro Horizonti parallelo; quod Refractioni Horizontali videtur tribuendum.

*Coma* ejus, Telescopio visa, Latior, & ferme Parabolica; Nudo autem Oculo Angusta, & parum Inflexa ad Occasum, videbatur.

 2. Prodiit

*At* Dantzick; by M. Hevelius. *ib.* p. 869.

2. Prodiit hisce diebus *Sidus Crinitum*, quod primâ vice hic *Gedani* die 27. *April.* manè animadversum fuit. Die 29. *Apr.* Oriebatur vel potius in Oculos incurrebat, $1^h$ $52'$, Mesaquilonem versus (*h. e.* N. E. b. N.) *Capite* quidem haud adeò Amplo, sed tamen satis Splendido, ex unico Nucleo clarissimo composito, ad instar illius *An:* 1665. conspecti. *Caudam* Lumine notabilem radiis divaricatis sursùm versus, duorum ferè graduum, exponebat. Linea Directionis continuata *Caudæ* inter *Alamac*, Lucidum sc. Pedem *Andromedæ*, ejusque *Cingulum* incedebat, & quasi Distantiam harum Stellarum in duas æquales partes secabat. Versabatur eo tempore supra *Caput Arietis* in *Triangulo*, inter *Apicem* & *Borialiorem in ejus Basi*, nempe in 5° *Tauri*, & in Latit. 19° Bor. Distabat hoc tempore à *Sole* secundum Longitudinem tantummodò 5°. Suo Circulo verò Maximo 20°. Hincque cum adeò vicinus hic *Cometa* extiterit *Soli*, haud potuit Longiorem *Caudam*, utut meâ opinione reverà longè prolixiorem habuerit, ostendere, imò ut puto proximis diebus aliquanto adhuc Breviorem ostendet.

*Apr.* 30. Deprehensus est in 9°. ♉. & Lat. 18°. Bor. totidemque ferè à *Sole* existente in 12. ♉; *Caudam* rursus duorum grad. & aliquanto Longiorem, ad Borealiorem in *Basi Trianguli* extensam (quæ Stella planè in Cuspide *Caudæ* per Tubos optimè conspecta) exhibebat.

Die 1. *Maii* $2^h$ $32'$. m. in 11°. ♉. repertus est, sub Latitudine Bor. 18°; in ipsa propemodum Conjunctione *Solis*, totidem quoque gradibus à *Sole* distans. *Caudam* adhuc satis Lucidam referebat, sed pauloBreviorem, utut Latiorem, quam ad *Lucidum Pedem Andromedæ* exporrigebat.

A Die 29 *Apr.* quâ primùm à me Observatus, ad hunc usque Diem 1. *Maii*, Motu Proprio propemodum 5° 30′ absolvit.

Quantum ex hisce Observationibus conjicere possum, fertur Motu Directo ad *Sinistrum Pedem Persei*, supra *Taurum*, ad *Pedes Geminorum*, si eo usque perdurabit. *Nodus* Descendens versatur circa 20°. *Geminorum*, (sed ruditer id tantummodo refero) atque sic ibidem *Eclipticam* pertransibit, fietque tum Meridionalis sub inclinatione Orbitæ 27° feré.

Die 2. *Maii* vesp. 8h 45′. etiamsi ea in parte Cœli nullæ adhuc Stellæ emicarent, intensumque Crepusculum existeret, nihilominus *Cometam* Tubo Optico protinus inveni. Paulo post, illum in Altitudine 3° 30′. deprehendi: *Caudam* referrebat, ratione Crepusculi, valde Tenuem, quam inter utrumque *Genu Cassiopeæ*, proprius tamen *Sinistro*, exporrigebat: occidebat eâ Vesp. $10^h$ Circium versus (*h. e.* N. N. W.)

Die 3. *Maii* mane, *Cometa* oriebatur Boream versus (*h. e.* N. N. E.) 1h 23′, quanquam *Cauda* paulò citius à nobis detecta, nempe $1^h$ 18′. Versabatur in 14°. ♉, cum *Sole* feré in ipsa *Conjunctione*, Latitudinem habens 17° & tantam etiam Distantiam fere ab ipso *Sole*. *Caudam* hâc die longé Prolixiorem & Accuratiorem satisque splendidam, 2° vel 3° fere, ostendebat. Hincque à me aliisque Spctatoribus visu pollentibus nudo oculo ad $3^h$ 34′ deprehensus est, & Telescopio ad 3h 40′. in Altitudine 11° 30′. adeo ut *Sol* eo tempore tantummodo 60. infra Hori zontem lateret, imo diutius illum vidissemus, nisi nubeculæ illum nobis eripuissent. Motus Diurnus decrescere videbatur, quantum conjectura absque omni calculo assequi potui: Nam

inter

inter 29 & 30 *April.* 2° 45′ fere extitit; inter 30 *April.* & 1 *Maii* 2° 15′; inter 1 & 2 *Maii* 1° 55′; inter 2 & 3 *Maii* 1° 40′; ſed ipſæ Obſervationes calculuſque id clarius oſtendent. Die 4 *Maii* veſp. Aëre admodum ſudo, 8ʰ 53′, iterum *Cometa* detectus, ſed Obſcurior paulo extitit quam diebus præcedentibus, tum *Cauda* Brevior. Die 5 *Maii* mane 1ʰ 41′. *Caudam Dextrum Genu Caſſiop.* verſus exponens, verſabatur in 17° ♉, in 16° Latit. Bor. pariter in tanta Diſtantia à *Sole.* Motus Proprius à Die 3 ad 5 *Maii* fuit fere 2° 40′, decreſcente Latitudine, ab ipſo Initio ſcil. fere ad 3°. ſic ut à 29 *April.* Motus Proprius *Cometæ* ad 5 *Maii* propemodum fuerit 12°. Die 6 *Maii* mane commorabatur in 18° ♉, & Lat. Bor. 15° 30′. *Sole* exiſtente in 17° ♉; Motus Diurnus erat 50′ circit. Quoad *Caput*, quam *Caudam* multò Tenuior ac Debilior videbatur, ob *Solem* non niſi 16½° à *Cometa* Remotum. Die 6 *Maii* veſp. viſus quidem Tubo optico 8ʰ 35′. *Cauda* adhuc Breviori & Dilutiori; ſed cum in Decliviori Situ, atque in Crepuſculo intenſo exiſteret, nullo modo diſtinctè in Nudos incurrebat oculos.

Die 7 *Maii* deprehenſus primùm 2ʰ 22′. in Alt. 3°, utut valde Tenuis videretur. Occupabat eo tempore 19° ♉, in Lat. 15°. Bor. & Diſtantia à *Sole* 16° feré, *Sole* exiſtente in 18° ♉; Motus ejus Proprius magis magiſque decreſcebat, quantum colligere abſque calculo dabatur. Die 8 *Maii* mane ab Hor. 1. ſedulo nudis quæſitus eſt oculis, ſed nuſquam apparuit: Teleſcopio tamen 12. ped. inventus, *Caudam* quidem adhuc præ ſe ferens, ſed Breviſſimam, paulo à circulo Verticali ſiniſtram verſus extenſam. Quantum conjectura aſſequi potui, verſabatur in 20° ♉, in Diſtantia à *Sole* 15°, qui tum 19° ♉ poſſidebat; ſtabat fere hoc tempore in linea recta cum *Humero Dextro Perſei* & *Algol Meduſæ.* Diameter *Cometæ*, ad *Jovis* Diametrum comparata, vix ad dimidiam partem accedebat. De reliquo, Tubi beneficio ſatis erat adhuc conſpicuus, adeo ut eum ad 3ʰ 45′ diſtinctè conſpicere potuerimus, in Altitudine ſcil. 9° ferè: unde colligere datur, Arcum Viſionis vix 5° tum fuiſſe; *Sol* enim vix 5° ſub Horizonte hærebat, quo tempore omnes jam Stellæ, excepto unico *Jove*, evanuerunt. Die 8 *Maii* veſp. *Cometam* nec nudis oculis, nec ullo Teleſcopio, detegere amplius potuimus.

3. The firſt certain notice I had of this *Comet* was on *Apr.* 21. The 22 of *Apr.* at about 2 a Clock after the Midnight following, I ſaw the *Tail* raiſed almoſt perpendicular to the Horizon; ſoon after the *Head* appeared through a thin Vapour, from which the *Tail* pointed, as near as I could gueſs, upon the * in the *Knee* of *Caſſiopeia*, its Length being about 6 *Deg.* and Breadth at the Top about 7 or 8 *Min.* Viewing the *Head* with a Teleſcope of 16 Foot, I found it was not perfectly Round, but Indented, and not near one *Min.* Diameter. Afterwards I haſtened to Meaſure its Diſtances from ſeveral Fixed Stars, which were as follow. *At* Greenwich, *by Mr.* Flamſteed, *ib. p.* 873.

| h. ′ ″ | | ° ′ ″ |
|---|---|---|
| 2 44 00 | It's *Head* and the *Foot* of *Androm. Alamech* | 11 26 0 |
| 2 47 15 | That Distance repeated | 11 26 50 |
| 2 55 3 | Its *Head* from *Capella* | 31 1 15 |
| 2 59 10 | repeated | 31 1 24 |
| 3 12 2 | Its *Head* from *Algol* in *Medusa's* | 8 16 54 |
| 3 21 22 | from *Mirach* | 19 35 0 |
| 3 27 54 | from *Alamech* again | 11 33 30 |
| 3 36 20 | from *Capella* again | 30 59 45 |

At $3^h\ 21'\frac{1}{2}$ *p. m.* the height of the *Comet* was about $5°\frac{1}{2}$, therefore the Distance of the *Head* of the *Comet* from *Algol* corrected by refraction, 8° 19′.
from *Mirach* ——— 19 37

And admitting with M. *Hevelius* the Place of *Mirach* now in ♈. 21° 40′ 34″. with North Latitude 25° 57′. its Distance from *Algol* will be 23° 42′ 40″. and the Place of the *Head* of the *Comet* in ♉. 14° $48'\frac{1}{6}$, with North Latitude 17° 8′.

At 3h 28′. I State the correct Distance of the *Comet's Head* from *Capella* 31° 00′; from *Alameck* 11° 40′; and therefore its true Place in ♉. 14° $50\frac{1}{2}$, with North Latitude 17° 6′ 25″: agreeing very well with the Place derived from the former Distances from two other and different Stars.

The *Tail* was not, it seems, directly Opposite to the *Sun*: for the *Sun*'s Place was now ♉. 13° 7′; but the *Comet* being in 14° 47′. of the same *Sign*, that is 1° 40′, in the Consequence of the *Sun*, the *Tail* ought, if it had been exactly opposite to the *Sun*, to have lain in Consequence of the *Head*; but the *Knee* of *Cassiopeia* is now in ♉. 13° 24′. in Antecedence of the *Comet*, whose *Tail* lay not therefore in Consequence, but in Antecedence of the Line passing through its *Head* and the *Sun*, at about an Angle of 10°.

Next Night, being that following the 23 of *April*, about $\frac{3}{4}$ of an Hour after two, its *Tail* appeared much shorter than last Morning: At 2h. 51′. its *Head* was from *Mirach* 21° 9′. Hence and from a Course of Observation of it sent me by an ingenious Friend, I found its Motion was Direct, and its Latitude Decreasing.

*A Comet.* An. 1680. *at* Dantzick; *by M.* Hevelius. *Ph. Col. n.* 3. *p.* 65.

CVI. Nuperum *Cometam* Observavi primum m. ante *Solis* Ortum, à Die 2. ad 4 *Dec.* An. 1680. deinde vesp. à 24. *Decemb.* ad *Æquinoctium Vernum.* Mane versabatur in ♎, & ♏. sub Latit. Australi. Vesperi vero in ♑, ♒, ♓, ♈, & ♉. sub Latit. Bor.

*A Comet.* An. 1682. *at* Dantzick; *by M.* Hevelius. *n.* 143. *p.* 16.

CVII. Plurimas distantias à Fixis, tum Altitudines Nuperi *Cometæ* Meridianas, Impetravi: quas autem omnes hic recensere nimis longum foret, nec vacat eas rigidiori Calculo subjicere. Sufficiat hac vice dixisse *Cometam* hunc hic *Gedani* Die 25 *Aug.* st. n. 1682. primum detectum, atque à Die 26. *Aug.* ad 17. *Septemb.*

*Septemb.* debite à me obſervatum eſſe. Qua via autem, qua velocitate, ſub quo Angulo Orbitæ & *Ecliptica* progreſſus fuerit, ex adjecta Tabella patet; quam tamen (quod ſcias velim) non ex accurato Calculo, ſed ex Globo tantummodo Laxiori ratione concinnavi.

| *Menſ. dies.* | | *Long. Cometæ* | *Lat. Cometæ* | *Motus in Pro. Orbit.* | *Motus Diurnus aliquanto Accuratius.* |
|---|---|---|---|---|---|
| | h. ′ | ° ′ s. | ° ′ | ° ′ | ° ′ |
| *Aug.* 26 | 3 0 *Mat.* | 23 30 ♋ | 21 0 Bor. | ——— | ——— |
| *Aug.* 27 | 11 0 *Veſp.* | 5 0 ♌ | 23 30 Bor. | 10 0 fere | 5 28 |
| *Aug.* 28 | ——— *Veſp.* | ——— | ——— | ——— | 5 35 |
| *Aug.* 29 | ——— *Veſp.* | ——— | ——— | ——— | 5 41 |
| *Aug.* 30 | 3 30 *Man.* | 18 0 ♌ | 25 20 Bor. | 13 20 ——— | ——— |
| *Aug.* 30 | 9 0 *Veſp.* | 22 0 ♌ | 25 40 Bor. | 3 30 ——— | 5 46 |
| *Aug.* 31 | 3 30 *Man.* | 24 30 ♌ | 26 0 Bor. | 2 20 ——— | 5 50 |
| *Sept.* 1 | 3 30 *Man.* | 1 0 ♍ | 26 0 fere | 5 45 | |
| *Sept.* 1 | 9 0 *Veſp.* | 6 0 fere ♍ | 25 40 Bor. | 4 45 ——— | 5 46 |
| *Sept.* 2 | ——— | ——— | ——— | ——— | 5 43 |
| *Sept.* 3 | 8 30 *Veſp.* | 20 0 fere ♍ | 24 30 Bor. | 11 30 ——— | 5 40 |
| *Sept.* 4 | ——— *Veſp.* | ——— | ——— | ——— | 5 34 |
| *Sept.* 5 | ——— *Veſp.* | ——— | ——— | ——— | 5 24 |
| *Sept.* 6 | 9 0 *Veſp.* | 5 0 ♎ | 20 30 Bor. | 15 0 ——— | 5 00 |
| *Sept.* 7 | ——— *Veſp.* | ——— | ——— | ——— | 4 30 |
| *Sept.* 8 | 8 0 *Veſp.* | 12 0 ♎ | 18 15 Bor. | 8 0 fere | 4 00 |
| *Sept.* 9 | 8 30 *Veſp.* | 15 30 ♎ | 17 15 Bor. | 3 30 ——— | 3 30 |
| *Sept.* 10 | 8 0 *Veſp.* | 18 30 ♎ | 15 45 Bor. | 3 0 fere | 3 00 |
| *Sept.* 11 | ——— *Veſp.* | ——— | ——— | ——— | 2 40 |
| *Sept.* 12 | 8 0 *Veſp.* | 23 0 ♎ | 14 0 Bor. | 5 0 ——— | 2 20 |
| *Sept.* 13 | 7 30 *Veſp.* | 25 0 ♎ | 13 30 Bor. | 2 0 ——— | 2 00 |

Sic ut *Motu Proprio* in ſuo *Orbita* confecerit à Die 26 *Aug.* ad 13 *Sept.* 83° 27′; & in *Ecliptica* 91° 30′. Latitudo vero Borealis creverit ad 26°. rurſus decreverit ad 12° 30′.

Not. *Nodus Boreus* in 24°. ♉, & *Nodus Auſt.* in 24°. ♏; *Limites* vero in 24°. ♌. & ♒. extiterunt. *Angulus Orbitæ* & *Eclipticæ* fuit 26°. fere. Utrum autem toto Durationis Tempore omnino conſtans cum *Nodis* extiterit? An vero & quouſque ſeſe variaverit? ut ſæpius fieri ſolet, ex Calculo patebit.

Toto Durationis Tempore, Lucidius ac etiam aliquando Majus *Caput*, quam iſte An. 1681. è contrario multo Breviorem *Caudam*, exhibuit. In ipſo *Capite*, beneficio longioris Teleſcopii, non niſi unicum *Nucleum Figuræ Ovalis* & *Gibboſæ* conſtanter notavimus; niſi quod Die præſertim 8 *Sept.* ex dicto

dicto *Nucleo* clarissimus simul Radius, ex parte etiam incurvatus, in *Caudam* exiret; quod notari meretur, cum ejus generis faciem in nullo adhuc *Cometa* (quantum memini) observaverim. Præterea sciendum, quod nonnunquam, ut Die 30 *Aug.* mane, *Caudam* satis præcise in *Oppositum Solis* direxerit; sed sæpius etiam notabilem Deviationem (prout in plurimis *Cometis* sæpius fieri solet) exhibuerit. Longitudinem quoque *Coma* non semper eandem conservavit. Initio *Cauda* ferè 12° videbatur; deinde nonnunquam brevior, interdum etiam longior ad 15° & 16° extitit; circa finem vero quotidie diminuta est.

Fig. 156.

*A Comet. An.* 1683. *at* Dantzick; *by M.* Hevelius. *n.* 154. *p.* 416.

CVIII. Die 30. *Jul.* An. 1683. 11h 30′ in Novo Nostro Sydere, *Tigride* vel *Lynce*, *Sydus Crinitum* hic *Gedani* deprehenderem, *Caudam* haud adeo Longam, inter *Stellam Polarem*, & *Cassiopeiam* sursum cum aliqua Inclinatione exporrigens: constituebat lineam Rectam cum Suprema *Capitis Aurigæ* & *Dextro Humero Persei*; non minus cum *Ventre Ursæ Majoris* & *Dextro Humero Aurigæ*; item cum Media *Caudæ* & *Latere Ursæ Majoris*. Deinde Tubo 10 pedum arrepto, istud Phænomenum contemplatus sum, *Caput* erat quidem satis Amplum, sed Materia non admodum Condensata; sic ut nullus Lucidus *Nucleus* neque distincta Corpuscula, ut quidem alias in plurimis aliis deprehensum est, in eo apparerent. 12h. ferè, Altitudo ejus erat 19° 57′.

Die 31 *Jul.* vesp. scil. 12h 30′. Altus cum esset 21° 28′. Rectam cum *Pede Aurigæ* & *Capella* constituebat. *Cauda* erat dilutissima, ac Rarior quam die hesterna, sed paulo Longior.

Die 4 *Aug.* mane; Removebatur eo tempore tanto spatio à *Dextro Humero Aurigæ*, quanto alias distat dictus *Humerus* à *Capite Hædi*. Cæterum *Sinistra Tibia Persei*, *Capella*, & *Cometa*, Rectam referebant.

Die 16 *Aug.* vesp. hora ferè 11. *Cometa* inter quatuor Stellulas versabatur, quarum una à parte *Cometæ* superiori, in ipsa Conjunctione, non nisi 1′. distabat, adeo arctè limbo adhærebat: quo tempore simul Diametrum *Cometæ* Micrometro meo dimensus sum, nimirum 6′ 5″. existere.

Die 18 *Aug.* vesp. Brevissimam ac Rarissimam *Comam* inter *Capellam* & *Caput Hædi* exporrigebat.

Die 20 *Aug.* vesp. Utroque *Hædo* erat vicinissimus, ita ut cum his Triangulum ferè æquilaterum constitueret, cujus Latera fere Distantiam *Hædorum* (quæ est 47′. circ.) æquabant. Ad hæc *Cometa* cum *Capella* & illa in Planta *Dextri Pedis Persei*, Triangulum Æquilaterum, cujus Basis erat Distantia dictarum Fixarum, exhibebat.

*Aug.* 24. vesp. versabatur inter *Capellam* & *Pleiadas*, sic ut à *Capella* & *Pleiadibus* in eadem fere Remotione videretur. Deinde *Capella*, *Cometa*, & *Pleiades*; item *Alamac*, *Caput Medusæ*, & *Cometa*; nec non *Dexter Humerus Aurigæ*, *Cometa*, & *Sequens Sinistri Pedis Persei*, Lineam fere Rectam constituebant, in hac tamen ultima constitutione, *Cometa* fere infra paulo Rectam jam incedebat.

*Aug.* 25. vesp. cum *Capella* & *Cap. Hædi* Rectam fere constituebat.

*Aug.*

*Aug.* 29. 1h 5′. mane, plurimis minutissimis, & clarissimis Fixis stipatus erat, nempe *Stellis Subiescianis*, atque à Cuspide Occidentali *Pleiadum* sursum versus, non nisi 42′ 35″ removebatur.

Eadem die vesp. longe jam promotior contra S.S. spatio scil. 24 horarum, ad 4°. fere reperiebatur.

*Aug.* 30. vesp. *Cometa* à Stellula quadam bene conspicua non nisi 32′ 41″. aberat; & cum *Musca* & in *Basi Trianguli*, deinde etiam cum præcedente in *Pede* & illa in *Genu Persei*, constituebat Rectam.

*Sept.* 2. mane, *Cometa* inter *Pleiadas* & *Nodum Lini* versebatur, constituens Lineam Rectam cum *Musca* & *Lucida Mandib. Ceti*, & cum infima in *Armo* ♉. & *Mandibula*, Triangulum fere æquicrurum, cujus vertex dicta erat *Mandibula*. Præterea quoque Lineam referebat Rectam cum duabus in *Fronte Ceti*, tum tanto fere spatio ab Occidentaliori distabat, quam alias utraque ab invicem removentur.

Hac die iterum Diametrum *Cometæ Capitis* Micrometro diligenter dimensus sum, 9′ 7″. quæ die 16. *Aug.* eodem Micrometro obtenta tantummodo 6′ 5″. sic ut notabiliter spatio 17. Dierum creverit. Non nemo diceret id factum esse, quod in ultima Observatione vicinior multo fuerit Terræ: Atque ideo Clarius & Lucidius *Caput* exhibere debebat, præsertim si Corpus esset æternum, (ut quidam statuunt) quod rursus certo tempore absoluto suo circulo, nobis in conspectum redit. Sed è contrario *Caput* longe Obtusius, Rariusque ultimo extitit, sic ut distinctissime notari potuerimus materiam *Capitis* sensim se dissolvere; id quod autem multo melius cum nostra convenit Hypothesi.

*Sept.* 4. mane, videbatur *Cometa* in Linea existere Recta cum illa in *Fronte Occid. Ceti* & *Lucida* ♈; item cum illa in *Ore* & *Mandibula Ceti*; ad hæc fere Triangulum æquilaterum cum illa in *Ore* & ad *Genam Ceti* constituebat. De cætero autem ex optato mihi extitit, quod Altitudinem Meridianam in Austro exactissimo Quadrante, Hora, *viz.* Matutina 3h 40′. ferè, nimirum 38° 15′. esse impetraverim.

| *Menſ.* | *Dies* | *Longitudo* | *Latitudo* | *Motus Diurnus* | *Declinatio* | *Aſcenſ. Rect.* |
|---|---|---|---|---|---|---|
| | | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| *Jul.* | 30 | 7 0 ♋ | 29 15 B | | 51 30 B | 100 0 |
| | 31 | 6 25 ♋ | 29 0 B | 0 42 | | |
| *Aug.* | 1 | 5 45 ♋ | 28 45 B | 0 44 | | |
| | 2 | 5 0 ♋ | 28 30 B | 0 46 | | |
| | 3 | 4 10 ♋ | 28 15 B | 0 48 | | |
| | 4 | 3 20 ♋ | 28 0 B | 0 50 | 51 40 B | 96 0 |
| *Aug.* | 5 | 2 20 ♋ | 27 45 B | 0 52 | | |
| | 6 | 1 20 ♋ | 27 30 B | 0 54 | | |
| | 7 | 0 20 ♋ | 27 15 B | 0 56 | | |
| | 8 | 29 20 ♊ | 27 0 B | 0 58 | | |
| *Aug.* | 9 | 28 20 ♊ | 26 40 B | 1 0 | | |
| | 10 | 27 20 ♊ | 26 20 B | 1 2 | | |
| | 11 | 26 20 ♊ | 25 55 B | 1 4 | | |
| | 12 | 25 20 ♊ | 25 30 B | 1 6 | | |
| *Aug.* | 13 | 24 20 ♊ | 25 0 B | 1 8 | | |
| | 14 | 23 20 ♊ | 24 30 B | 1 10 | | |
| | 15 | 22 20 ♊ | 24 0 B | 1 12 | | |
| | 16 | 21 10 ♊ | 23 20 B | 1 14 | 46 0 B | 77 0 |
| *Aug.* | 17 | 19 20 ♊ | 22 30 B | 1 19 | | |
| | 18 | 17 40 ♊ | 21 30 B | 1 25 | 44 0 B | 73 30 |
| | 19 | 16 0 ♊ | 20 30 B | 1 35 | | |
| | 20 | 14 20 ♊ | 19 15 B | 1 45 | 41 0 B | 69 30 |
| *Aug.* | 21 | 12 20 ♊ | 18 0 B | 1 55 | | |
| | 22 | 10 20 ♊ | 16 45 B | 2 5 | | |
| | 23 | 8 20 ♊ | 15 30 B | 2 15 | | |
| | 24 | 6 20 ♊ | 14 15 B | 2 25 | 25 0 | 60 40 |
| *Aug.* | 25 | 3 50 ♊ | 12 45 B | 2 40 | | |
| | 26 | 1 5 ♊ | 11 0 B | 2 55 | | |
| | 27 | 28 15 ♉ | 9 0 B | 3 10 | | |
| | 28 | 25 15 ♉ | 6 30 B | 3 25 | 24 30 B | 51 0 |
| *Aug.* | 29 | 22 15 ♉ | 4 0 B | 3 40 | 21 30 B | 48 30 |
| | 30 | 18 55 ♉ | 1 30 B | 3 55 | 18 00 B | 45 40 |
| | 31 | 16 25 ♉ | 1 0 A | 4 5 | | |
| *Sept.* | 1 | 12 55 ♉ | 3 30 A | 4 20 | | |
| *Sept.* | 2 | 9 55 ♉ | 6 0 A | 4 40 | 10 30 B | 40 0 |
| | 3 | 6 25 ♉ | 9 40 A | 5 5 | | |
| | 4 | 2 35 ♉ | 11 20 A | 5 30 | 1 30 B | 34 0 |

Ex.

Ex quibus nunc luculenter videre eſt, *Cometam* hunc continuo contra S.S. inceſſiſſe; ſic ut in *Ecliptica* 63° 55′. in ſua vero *Orbita* 74° 35′. peragraverit, ſub Angulo, viz. *Orbitæ* & *Eclipticæ* 39° fere, ſub Angulo vero *Orbitæ* & *Æquatoris* 56°. *Latitudo* Initio 29° 15′. *Bor.* & ultimo 11° 20′. *Auſt.* extitit; adeo ut ad 41°. feré eam variaverit.

De *Capite* hæc notandum habeo, quod Initio, quoad Diametrum, longe minus quam ultimo; è contrario Initio longe Lucidius, quam circa Finem extiterit; nullos tamen diſtinctos & fulgentes *Nucleos*, prout in plurimis videre nobis obtingit, exhibuerit, ſed confuſam materiam, & circa Finem multo Tenuiorem. Jure hic *Cometa* (cum plerumque abſque omni *Cauda* viſus) inter *Sidera Comata*, vel *Crinita*, ſive inter *Barbata* & *Hircos* refertur. Nam non niſi ad 18. *Aug.* Breviſſimam & Dilutiſſimam *Comam* ſurſum verſus exporrigebat; quæ poſtmodum vero omnino Evanuit.

CIX. Novus *Cometes* nuper Cælo viſus eſt à Lynceo oculo Abb. *Blanchini*, Diſcipuli Cl. *Geminiani Montanarii*. *Cometes* parvus quidem, ſed in ſua *Orbita* regularis apparuit, Lumine tenui, & tanquam Stella Subobſcura: at Tubo optico exceptus, Luminoſior. *A comet. An. 1684. at Rome; by S. Ciampini. n. 169. p. 920.*

*Jun.* 30. ſt. n. *An.* 1684. *Cometa* primum mihi viſus eſt, in grad. 9. cum aliquibus minutis *Libræ*, Latitudo ejuſdem Borealis fuit graduum 8. & aliquot minutorum.

| *Jul. Die.* | *Long. Cometæ.* | | | *Lat. Bor.* | | *Jul. Die.* | *Long. Cometæ.* | | | *Lat. Bor.* | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | ° | ′ | | ° | ′ | | ° | ′ | | ° | ′ |
| 1 | 11 | 18 | ♎ | 13 | 12 | 10 | 24 | 42 | ♎ | 39 | 40 |
| 2 | 13 | 16 | ♎ | 17 | 57 | 11 | 25 | 44 | ♎ | 40 | 49 |
| 3 | 14 | 58 | ♎ | 22 | 12 | 12 | 26 | 38 | ♎ | 41 | 58 |
| 4 | 16 | 45 | ♎ | 25 | 40 | 13 | 27 | 23 | ♎ | 43 | 5 |
| 5 | 18 | 30 | ♎ | 28 | 50 | 14 | 28 | 32 | ♎ | 44 | 9 |
| 6 | 19 | 50 | ♎ | 31 | 34 | 15 | 29 | 49 | ♎ | 45 | 12 |
| 7 | 21 | 7 | ♎ | 33 | 54 | 16 | 00 | 50 | ♏ | 46 | 20 |
| 8 | 22 | 20 | ♎ | 36 | 6 | 17 | 2 | 8 | ♏ | 47 | 40 |
| 9 | 23 | 32 | ♎ | 38 | 00 | | | | | | |

Quatuor ultimæ meliori indigent Calculo, nam aliquis fortaſſe Minutorum Error irrepſit. Obſervatio Diei Primi *Jul.* accuratiſſima eſt; *Cometa* enim Teleſcopio apparuit una cum Stella *Virginis*, quæ *Bayero* inſcribitur Ω. & accidit ſub Cingulo Septentrionalis partis primæ. Omnium certiſſima eſt Obſervatio Diei Sextæ, qua Die *Arcturus* in Tubo Optico ſimul cum *Cometa* conſpiciebatur. Die pariter 14. *Cometa* & *Stella* χ. in *Colorrobo Bootis*, ſeu *Venabulo*, ſubter *Humerum*, uno Intuitu detegebantur.

*A Comet.* An. 1686. *at* Leipsick; *by M. Kirck. n.* 186. *p.* 256.

CX. *Sept.* 8. *st.v. A.* 1686. 4h. *mane*, about Day-break, *M. Kirk* found this *Comet* in the Conſtellation of *Leo*, to the Right-hand of the *Lucida in Lumbis* ♌. (as is conceived, for the Latin Copy is defective in this place) and reſembling that Star in Colour and Magnitude, with a Thin and Short Tail extended upright. Over the *Comet* in the ſame Vertical was the Star of θ ♌. of *Bayer*, or 21. *Tychoni*, diſtant therefrom, by the Micrometer, exactly a Degree; and a Line drawn from the *Lucida in Lumbis* ♌. to the *Comet* paſſed much about half a Degree to the Right-hand of the ſame θ *Leonis*. The Diſtance of the *Comet* from *Regulus* taken by a Radius was about 17°. The next Morning *Sept.* 9. at 3h 58′. the Diſtance thereof from θ ♌. was found by the Micrometer 2° 23′$\frac{1}{2}$, and at 4h 40′, again 2° 25′$\frac{3}{4}$. To verify the Times, the Altitude of the *Lucida in Lumbis* ♌. was obſerv'd 11° 10′, at 4h 8′. *mane*. A Right-line drawn by the *Comet*, and the ſaid θ *Leonis* towards β *Leonis*, or the *Lucida Colli*, left that Star a little to the Right-hand.

This *Comet* was ſeen by a Countryman, who firſt gave Notice thereof, from the 6th. to the 12th. of *Sept.*

The Reſult of theſe Obſervations is, that the *Comet* was Direct in Motion, that it mov'd about 1$\frac{1}{2}$ Degree *per Diem*, and that it ſeemed rather to Decreaſe in Latitude. On the 7th of *Sept.* it was about 24′. diſtant from θ *Leonis*, but its bearing therefrom is not ſet down.

This Star, θ ♌. was then in 9° 2′. of ♍. with North Lat. 9° 41$\frac{1}{2}$. Whence at the time of the firſt Obſervation it may be Concluded, that the *Comet* was in 9° 55′. of ♍. with North Lat. 9° 15′. And at the 2d Obſervation, the Longitude of the *Comet* will be found about 11° 20′. of ♍. with much the ſame North Latitude as before.

*A omet.* An. 169$\frac{8}{9}$. *at* Paris; *by M.* Caſſini. *n.* 250. *p.* 79.

CXI. *Feb.* 19. ſt. n. *An.* 1699. in *Obſervatorio Regio Pariſienſi*, videri cœpit exiguus *Cometa*, inſtar Stellæ Nebuloſæ tertiæ Magnitudinis; illi perſimilis quæ Menſe *Sept.* 1698. fuit Obſervatus.

Situs erat inter Stellas Informes 6. Magnitud. prope Circulum *Polarem Arcticum* ſupra *Caput Aurigæ*, æquali ferè Intervallo inter *Cubitum Occidentalem Perſei* & *Caput Majoris Urſæ*; illas adſcribit *Tycho* Informibus circa *Urſam Minorem*. Continuatis Obſervationibus, viſus eſt, Proprio Motu, Iter ſuum dirigere *Capellam* verſùs, cum exigua Deviatione ab ejus Circulo Declinationis. Ea erat ejus Velocitas ut unius Diei Spatio, Septem circiter Gradus Magni Circuli perficeret, quo Motu potuit ante Dies 4. ipſi *Polo* fermè adhærere, & *Stellæ Polari* ſociari.

*Hora* 6. p. m. n. Comparavimus *Cometam* cum Stella 6. Mag. quam *Tycho* appellat *Secundam earum quæ ſunt in Linea Recta cum Polo*; *Cometa* in Tranſitu per Circulum Horarium præcedebat hanc Stellam Min. Hor. 15′ 53″. quibus dabitur Differentia *Aſcenſionis Rectæ* 4° 43′. erat autem Septentrionalior eadem Stella 8′. Unde ſuppoſita hujus Stellæ Longitudine & Latitudine *Tychonica* ad hoc Tempus, *Cometa* refertur ad 15° 51′. *Gem.* cum Latitudine Sept. 37° 25′.

Movetur *Cometa* hic ad Cæli partes Oppoſitas illis ad quas tendebat *Cometa An.* præteriti, cum eſſet ferme in eadem Diſtantia à *Polo* in qua noſter hic cum primum viſus eſt, nec valde ab eodem loco remotus.

*Cometa*

*Cometa* autem Menſis *Sept.* eandem proſecutus eſt viam quam inter Sidera tenuerat *Cometa An.* 1652. à Nobis *Bononiæ* Obſervatus, cujus Occaſione editis Literis ad Sereniſſimum *Fran. Eſtenſem Mutinæ* Ducem, eam viam per eadem Sidera quæ noſter tenuit *An.* 1698. diſtinctè deſcripſimus. Ille Menſe *Dec.* ab Auſtralibus Cæli partibus per Aſtra *Leporis, Orionis* & *Tauri*, ubi *Eclipticam* ſecuit cum Inclinatione 76°. & per *Perſeum* & *Caſſiopeiam* pervenit, ubi videri deſiit Menſe *Jan. An.* 1653. Hic videri cœpit Initio Menſis *Sept.* in eadem *Caſſiopeæ* parte ubi ille videri deſierat, indeque pergens per *Humeros* & *Brachia Cephei*, ubi Latitudinem Maximam ab *Ecliptica* habuit 76°. tranſit inter *Draconem* & *Cygnum*, per *Pellem Leonis* in *Hercule*, per *Ophiucum*, uſque ad Conſtellationem *Scorpii*, quam tenebat in ultimis Obſervationibus à Die 24. ad 28. *Sept.* habitis. Ex his autem Obſervationibus collegimus *Cometam* hunc *Perigeum* obtinuiſſe Die 7. *Sept.* Veſperè, cum maxima Velocitate Apparenti fere 10°. unius Diei Spatio.

CXII. Papers *of leſs General Uſe* Omitted.

1. The Conſtellation of *Cygnus*, with the *New* Star *in Pectore* in it, by *Hevelius*; together with the Names of the Stars in that *Conſtellation* by *Tycho*, and of thoſe Added by *himſelf*. Cygnus. *n.* 21. *p.* 372. *n.* 65. *p.* 2088, 2090.

2. Mr. *Flamſteed* having peruſed Mr. *Street*'s Diſcourſe, and conſidered the Contrivance of his *Moon-Wiſer*, aſſures, that for the Motion of Longitude 'tis the very ſame, and for the Motion of Latitude not much better, than Mr. *Horrox*'s. *Mr.* Horrox's *Lunar Syſtem*; *n.* 110. *p.* 219. *n.* 116. *p.* 368.

But Mr. *Flamſteed* hath thought of another Contrivance that will ſhew the *Moon*'s true Place to a Minute.

3. 1. The more Notable *Cœleſtial Appearances* Calculated, by Mr. *Flamſteed*, for the *Year* 1670. *Cæleſtial Phænomena Calculated.* n. 55. *p.* 1099.
2. The ſame for the *Year* 1671. *n.* 66. *p.* 2029.
3. The ſame for the *Year* 1672. *n.* 77. *p.* 2297.
4. The ſame for the *Year* 1673. *n.* 79. *p.* 3061.
5. The ſame for the *Year* 1674. *n.* 86. *p.* 5040. *n.* 89. *p.* 5118. *n.* 99. *p.* 6162.

4. 1. The *Eclipſes* of the *Satellites* of *Jupiter* Viſible at *Uraniburg* the laſt four Months of the *Year* 1671. Calculated by *M. Caſſini*. *Satellite Eclipſes.* *n.* 74. *p.* 2238.
2. The *Eclipſes* of the *Satellites* of *Jupiter* Viſible at *the Obſervatory* at *Greenwich* in the three laſt Months of the *Year* 1683. Calculated by Mr. *Flamſteed*. *n.* 151. *p.* 322.
3. The *Satellite Eclipſes* Calculated by Mr. *Flamſteed* for the *Year* 1684. *n.* 154. *p.* 404.
4. The ſame for the *Year* 1685. *n.* 165. *p.* 760.
5. The ſame, together with the *Parallaxes* of *Jupiter*'s *Orb* and his *Geocentrick Places*, for the *Year* 1686. *n.* 177. *p.* 1215.
6. The ſame for the *Year* 1687. *n.* 184. *p.* 195.
7. The *Satellites Eclipſes* Calculated by Mr. *Halley*, for the *Year* 1688. *n.* 191. *p.* 435.

5. 1. An Account of the *Ephemerides* of the *Comet*, *A.* 1665. Calculated by *M. Auzont*; and the *Principle* of his *Hypotheſis* diſcovered by *M. Caſſini*. *Comets n.* 1. *p.* 3. *n.* 2. *p.* 17, 18.
2. An Account of the *Ephemerides* of the *Comet*, *A.* 1665. Calculated by *M. Auzont*. *n.* 3. *p.* 36.

CXIII.

## CXIII. Accounts *of* Books, *and* Emendations, Omitted.

n. 102. p. 40. 1. A new Size of *Globes* about 15 inches Diameter Rectifyed by *R. Morden* and *Will. Berry*.

Ph.Col. n.1.p.44. 2. A Representation of the Heavens into two large *Hemispheres* of 30 inches Diameter, Stereographically Projected upon the Plain of the *Æquinox*; by Mr. *Fr. Lamb.*

n. 90. p. 5150. 3. Deux Machines propres à fair les *Quadrans*, avec tres grande facilité; par le P. *Ignace Gaston Pardies.* S. J. à *Paris.* 1673. in 12°.

n. 184. p. 213. 4. *Sciotericum Telescopicum*, or a new Contrivance of adapting a Telescope to an *Horizontal Dial*, for Observing the Moment of Time by Day or Night; by *Will. Molineux R. S. S. Dublin* 1686. *in* 4^to^.

n. 241. p. 240. 5. The *Meridian Line* of the *Curch* of St. *Petronio*, Drawn and Fitted for Astronomical Observations, in the *Year* 1655. Revised and Restored in the *Year* 1695; by *Jo. Dom. Cassini.* At *Bononia* 1695. *Fol.*

n. 66. p. 2028. 6. *Joa. Hevelii* Machinæ Cæleſtis Pars prior. *Organographiam Astronomicam*
n. 99. p. 6171. plurimis Iconibus illustratam & exornatam exhibens, *&c. Gedani* 1673. in *Fol.*

n. 109. p. 215. 7. *Animadversions* on the First Part of the *Machina Cælestis* of *Joa. Hevelius*, together with an Explication of some *Instruments* made by *R. Hook. P.*
n. 111. p. 243. of *Geometry* in *Gresh. Coll.* and *R. S. S. London* 1674. in 4^to^. Dr. *Wallis*'s Letter to *M. Hevelius*, concerning *Divisions* by *Diagonals* there inserted, but faultily, is here *Reprinted* more correctly.

n. 175. p. 1162. 8. *Joannis Hevelii* Consulis *Dantiscani* Annus Climactericus. *Gedani* 1685. *Fol.* Wherein (among other things) *M. Hevelius* vindicates the Justness of his *Celestial Observations* against the Exceptions by some made to the *Accuracy*
Ib. p. 1164. of them. The *Controversie* between *Him* and Dr. *Hook*, about the Use of *Te-*
n. 111. p. 244. *lescopick* and *Plain Sights*, and Dr. *Wallis*'s Calculation, for *Dividing* the Limb
n. 175. p. 1176. of Instruments by *Diagonals*, are also here Abridged.

n. 150. p. 308. 9. Excepta ex Literis Ill. & Clariss. Virorum ad Nob. Ampliss. & Consultiss. D. *Jo. Hevelium* Conſ. *Gedanensem* perscriptis, Judicia de Rebus *Astronomicis*, ejusdemque Scriptis, exhibentia; Studio ac Operâ *Joa. Erici Olhoffii* Secretarii *Gedani* 1683. in 4^to^.

n. 118. p. 440. 10. A Description of *Helioscopes*, and some other *Instruments*, made by *R. Hook.* R. S. S. *Lond.* 1675. in 4^to^.

n. 110. p. 233. 11. The Sphere of *M. Manilius* made an *English* Poem, with Annotations, and an *Astronomical Appendix*; by *Ed. Sherburn*, Esq; *Lond.* 1675. in *Fol.*

n. 204. p. 913. 12. *Albatenii* Observationes *Astronomicæ*, Quas ex *Arabico* in *Latinum* Transtulit *Plato Tiburtinus. Noribergæ* 1537; & *Bononiæ* 1645. The *Arabick Copy* of those Observations does not Appear, whereby that *Translation* might be *Examined*: But Mr. *Halley*, by Calculating Tables from the *Principles* there Delivered, hath here *Discovered* and *Corrected* above 30 considerable *Faults* in a few Pages.

n. 43. p. 868. 13. *Historia Cælestis*; ex Libris & Commentariis MS. *Observationum* Vicennalium *Tychonis Brahe*, *Dani. Augustæ Vindelic.* An. 1666. in *Fol.*

n. 102. p. 27, & 29. 14. All the *Manuscripts* of the Famous *Kepler*, (both *Published* and *Unpublished*) which are Purchased, and carefully preserv'd by *M. Hevelius.*

15. *Je-*

15. *Jeremiæ Horroccii Angli* Opera Posthuma: una cum *Guil. Crabtræi* Observationibus Cælestibus; nec non *Jo. Flamstedii* de *Temporis Æquatione* Diatriba, Numerisq; *Lunaribus* ad Novum *Lunæ* Systema *Horroccii*. *Lond.* 1672. in 4to. n. 87. p. 5078.

16. *Astronomia Reformata.* Auctore *Joan. Bapt. Riccioli S. J. Stephano de Angelis*, Conceiving the Arguments of this *Author*, against the *Motion* of the *Earth*, to be none of the strongest, taketh Occasion to let the World see, that they are not more Esteem'd in *Italy*, than in other places: *Manfredi*, in behalf of *Riccioli*, endeavours to Answer the Objections of *Angeli*, and this latter replies to *Manfredi*'s Answer. The Substance of which *Controversie* is here given by Mr. *Ja. Gregory*; with some Remarks and Explications of his own upon it. n. 22. p. 394. n. 36. p. 693.

17. An Attempt to prove the *Motion* of the *Earth* from Observations, made by *R. Hook. F. R. S. Lond.* 1674. *in 4to.* The Method of this Undertaking is Approved and Commended by *M. Chr. Hugens*, and *M. Cassini*. n. 101. p. 12. n. 105. p. 90.

18. *Nicolai Mercatoris* Holsati, è Soc. Regia, *Institutionum Astronomicarum* Libri duo. *Lond.* 1676. in *Octavo*. n. 125. p. 611.

19. Annales Cæli & Temporum perpetui, sive Mysteria *Astronomo-Chronologica* à Seculo Abscondita, nunc per Dei Gratiam Detecta & evidenter Asserta, Libris tribus. *Kiloni.* This Book is Preparing, by Dr. *Wasmuth*. n. 104. p. 74.

20. A *Catalogue* of *Fixed Stars* with their Longitudes, Latitudes, and Magnitudes, according to the *Observations* of *Ulug. Beig. Oxford.* 1666. n. 8. p. 145.

21. *Catalogus Stellarum Australium*, sive Supplementum Catalogi *Tychonici*; exhibens Longitudines & Latitudines *Stellarum Fixarum* quæ prope *Polum Antarcticum* sitæ, in Horizonte *Uraniburgico*, *Tychoni* inconspicuæ fuere. Authore *Edm. Halleio*, è *Col. Reg. Oxon.* in 4to. n. 141. p. 1032.

22. Congietture *Physico Astronomiche* della Natura del *Universo*; da *Pietro M. Cavina.* in *Faenza* 1669. in 4to. n. 65. p. 2012.

23. Prose de *Signori Academici* di *Bologna*; in *Bologna* 1672. in 4to. *S. Montanari's* Discourse concerning the admirable *Changes* and other *Novelties* observed in the *Heavens*. n. 89. p. 5125.

24. *Ismaelis Bullialdi* ad Astronomos Monita duo. Primum de *Stella Nova*, quæ in *Collo* Ceti ante An. aliquot visa est. Alterum de *Nebulosa* in *Andromedæ Cinguli* parte Borea, ante Biennium iterum orta. Approv'd by *M. Hevelius*. n. 21. p. 381. n. 25. p. 460.

25. Three Letters of *Jo. Dominicus Cassinus*, concerning his *Hypothesis* of the *Sun's Motion*, and his Doctrine of *Refractions*. At *Bononia*. in 4to. n. 84. p. 5001.

26. Refractio *Solis Inoccidui*, in Septentrionalibus Oris circa Solstitium Æstivum, *An.* 1695. aliquot Observationibus *Astronomicis* detecta. *Holmiæ.* in 4to. *Translated* into *English*. *Lond.* in 8vo. n. 231. p. 731.

27. *Tabularum Astronomicarum* Pars Prior; de Motibus *Solis* & *Lunæ*, nec non de Positione *Fixarum*, ex ipsis Observationibus deductis: Authore *Ph. de la Hire. Paris* 1687. in 4to. *Some Animadversions on it are here inserted.* n. 191. p. 443.

28. 1. The *Royal Almanack* for the *Year* 1675; by *N. Stevenson.* in 12°. n. 108. p. 192.
2. ————————For the *Year* 1676. n. 120. p. 490.
3. ————————For the *Year* 1677. n. 130. p. 774.

29. Ephemeris, ad *Annum* 1686, exactissime supputata. *Lond.* in 8vo. n. 179. p. 35.

30. The *Celestial World Discovered*, or Conjectures concerning the Inhabitants, Plants, and Productions, of the *Worlds* in the *Planets*. Written in *Latin* by *M. Chr. Huygens.* in 8°. n. 256. p. 337.

31. Rag-

n. 1. p. 2. n. 45. 31. Ragguaglio de Nuove Obſervationi, da *S. Gioſeppe Campani.* This Book
p. 900. n. 4. was Anſwer'd by *M. Auzout*, who gives his Opinion of *Campani*'s Glaſſes, and his
p. 69. new *Obſervations* of *Saturn* and *Jupiter* made with them; To this *S. Campani*
Ib. p. 74. publiſhes a *Reply*, and *M. Auzout* his *Animadverſions* thereon.

n. 35. p. 688. 32. 1. Ephemerides *Mediceorum Siderum*, ex Hypotheſibus & Tabulis *Jo.*
n. 44. p. 892. *Dom. Caſſini. Bononiæ* 1668. in *Fol.* The like *Tables* have been formerly publiſhed by the Learned *J. Bapt. Hodierna* at *Rome*, about 1656.

n. 214. p. 256. 2. The Table of the *Eclipſes* of the *Firſt Satellite* of *Jupiter* by *M. Caſſini*, Publiſht at *Paris* in the *Recueuil d'Obſervations faites en pluſieurs Voyages pour parfectioner l'* Aſtronomie *& la* Geographie, being not printed with the uſual Care of the *Imprimerie Royale*, Mr. *Halley* here amends ſome of the *Errata*.

n. 14. p. 242. 33. *Martis*, circa Axem proprium Revolubilis, Obſervationes *Bononiæ* à
n. 35. p. 687. *Joa. Dominico Caſſino* habitæ 1666. Here *M. Caſſini* Judges it Evident, that the Period of this *Planets Revolution* is not performed in the Space of 12h 20′, but in about 24h 40′; and that thoſe, who affirm the former, muſt have been deceived by not well Diſtinguiſhing the two *Faces*.

n. 134. n. 853. 34. *Mercurius* in *Sole* viſus; à *Jo. Hevelio. Ged.* 1662.

n. 6. p. 104. 35. *Prodromus Cometicus*; by *Hevelius*.

n. 17. p. 301. 36. *Joannis Hevelii* Deſcriptio *Cometæ*, *An. Æræ Chriſtianæ* 1665. exorti: una cum Mantiſſa *Prodromi Cometici*, Obſervationes omnes Prioris *Cometæ* 1654, ex iiſque Genuinum Motum accuratè deductum, cum Notis & Animadverſionibus, exhibens.

n. 40. p. 805. 37. *Jo. Hevelii* Cometagraphia. *Dantzick.* in *Fol.*

n. 35. p. 691. 38. *Staniſlai* de *Lubienietz* Theatrum *Cometicum. Amſtelod.* 1668. in *Fol.*

n. 53. p. 1069. 39. Del Movimento della *Cometa*, apparſa il meſe di *Decembri* 1664. da *Pietro Maria Mutoli.* in *Piſa.* in 4*to.*

n. 53. p. 1071. 40. *Eraſmi Bartholini* de *Cometis*, *An.* 1664, & 1665. Opuſculum; ex Obſervationibus *Hafniæ* habitis adornatum. *Hafniæ.* in 4*to.*

n. 139. p. 980. 41. *Joh. Walliſii*, De *Cometarum* Diſtantiis inveſtigandis. *Lond.* 1678.

Ib. 986. 42. Lectures and Collections made by *R. Hook.* Sec. of the *R. S. Lond.* 1678. in 4*to.*

Ph. Col. n. 4. 43. Obſervat. of the *Comet* of 1680. and 1681. made at the *Col.* of *Cler-*
p. 106. *mont*; by *P. J. de Fontaney* è *S. J. Profeſſ.* of *Mathematicks*. Paris. 1681.

Ib. p. 114. 44. A Treatiſe concerning the late *Comet*, Publiſhed at *Turin* 1681. by *Donato Roſſetti* S. T. D. Canon of *Leghorn*, and *Tutor* in *Mathematicks* to the *Duke of Savoy*.

Ib. p. 116. 45. An Explication of the *Comet* which appeared at the End of 1680. and in the Beginning of 1681. upon the Obſervations of Dr. *Anthelme*, *Carthuſian* of *Dijon*. at *Dijon* 1681. in one ſingle Sheet.

Ph. Coll. n. 7. 46. A ſmall Diſcourſe about *Comets*, publiſhed in the *High Dutch* at *Nu-*
p. 196. *renburg* 1681; by a *Lover of Aſtronomy*.

Ib. p. 199. 47. A new Introduction, ſhewing how the *Motions* of the *Comets* may be reduced to ſome certain and Geometrical Rules, ſo that their *Appearance* may be Predicted. in *High Dutch*; by *Ja. Bernouly.* at *Bazil. An.* 1681.

n. 149. p. 272. 48. *Joannis Jacobi Zimmermanni Cometo-ſcopia.* Or, Three *Aſtronomical Relations* concerning the *Comets* that have been ſeen in the *Years* 1680, 1681, 1682. *Stutgard* 1682. in 4*to.*

# CHAP. V.

## *Mechanicks. Acousticks.*

I. 1. SI Agens ut A Efficit ut E; Agens ut 2 A Efficiet ut 2 E, 3 A ut 3 E, &c. cæteris paribus: Et universaliter, $m$ A ut $m$ E; cujuscunque rationis Exponens sit $m$. *The General Laws of Motion; by Dr.* Wallis. n. 43. *p.* 864.

2. Ergo si Vis ut V moveat Pondus P; Vis $m$ V movebit $m$ P, cæteris paribus, puta per eandem Longitudinem eodem Tempore, *h. e.* eadem Celeritate.

3. Item si Tempore T moveat illud per Longitudinem L; Tempore $n$ T movebit per Longitudinem $n$ L.

4. Adeoque si Vis V, Tempore T, moveat Pondus P, per Longitudinem L; Vis $m$ V, Tempore $n$ T, movebit $m$ P, per Longitud. $n$ L. Et Propterea, ut V T (Factum ex Viribus & Tempore) ad P L (Factum ex Pondere & Longitudine) sic $m\,n$ V T, ad $m\,n$ P L.

5. Quoniam Celeritatis gradus sunt Longitudinibus eodem Tempore transactis Proportionales, seu (quod eodem recidit) Reciproce Proportionales Temporibus eidem Longitudini transigendæ impensis: erit $\frac{L}{T} : C :: \frac{m\,L}{n\,T} : \frac{m}{n} C.$ *h. e.* Gradus Celeritatum, in ratione composita ex directa Longitudinum & Reciproca Temporum.

6. Ergo propter $VT : PL :: mn\,VT : mn\,PL$ : erit $V : \frac{PL}{T} :: mV : \frac{mn\,PL}{nT}$ : *h. e.* $V : PC :: mV : mPC = mP \times C = P \times mC.$

7. Hoc est, si Vis V movere potis sit Pondus P, Celeritate C: Vis $m$ V movebit vel idem Pondus P, Celeritate $m$ C; vel eadem Celeritate, Pondus $m$ P; vel denique quodvis Pondus ea Celeritate, ut Factum ex Pondere & Celeritate sit $m$ P C.

8. Atque hinc dependet omnium Machinarum (pro facilitandis Motibus) construendarum ratio: nempe ut qua ratione augetur Pondus, eadem minuatur Celeritas, quo fiat, ut Factum ex Celeritate & Pondere, eadem Vi movendo, idem sit: puta $V : PC :: V : mP \times \frac{1}{m} C = PC.$

9. Si Pondus P, Vi V, Celeritate C, latum in Pondus Quiescens (non impeditum) $m$P directe impingat; ferentur utraque Celeritate $\frac{1}{1+m}$C. Nam propter eandem Vim, majori Ponderi movendo adhibitam, eadem ratione minuetur aucti Celeritas: nempe $V : PC :: V : \frac{1+m}{1} P \times \frac{1}{1+m} C = PC$. Adeoque alterius Impetus (intellige Factum ex Pondere & Celeritate) fiet $\frac{1}{1+m}$ PC; reliqui $\frac{1}{1+m}$ $m$PC.

10. Si in Pondus P (Vi V) Celeritate C latum, directe impingat aliud, eadem via, majori Celeritate insequens; puta Pondus $m$P, Celeritate $n$C, (adeoque Vi $mn$V latum; ferentur ambo Celeritate $\frac{1+mn}{1+m}$ C. Nam $V : PC :: mnV : mnPC :: V + mnV = \frac{1+mn}{1} V : \frac{1+mn}{1} PC = \frac{1+m}{1} P \times \frac{1+mn}{1+m} C$. Adeoque Præcedentis impetus fiet $\frac{1+mn}{1+m}$ PC; Subsequentis, $\frac{1+mn}{1+m}$ $m$PC.

11. Si Pondera contrariis viis lata, sibi directe occurrant sive impingant mutuo, puta, Pondus P, (Vi V) Celeritate C, dextrorsum; & Pondus $m$P, Celeritate $n$C (adeoque Vi $mn$V) sinistrorsum: utriusque Celeritas, impetus, & directio, sic colliguntur, Pondus dextrorsum latum reliquo si quiesceret, inferret Celeritatem $\frac{1}{1+m}$ C, adeoque Impetum $\frac{1}{1+m}$ $m$PC, dextrorsum, sibique retineret hanc eandem Celeritatem, adeoque Impetum $\frac{1}{1+m}$ PC dextrorsum (per Sect. 9.) Pondusque sinistrorsum latum (simili ratione) reliquo si quiesceret, inferret Celeritatem $\frac{mn}{1+m}$ C, adeoque Impetum $\frac{mn}{1+m}$ $m$PC sinistrorsum; sibique retineret hanc eandem Celeritatem, adeoque Impetum $\frac{mn}{1+m}$ PC sinistrorsum. Cum itaque Motus utrinque fiat; Impetus dextrorsum prius lati, jam aggregatus erit ex $\frac{1}{1+m}$ PC dextrorsum,

trorſum, & $\frac{\bar{m}\,\bar{n}}{1+m}$ P C ſiniſtrorſum ; adeoque reapſe vel dextrorſum vel ſiniſtrorſum, prout hic vel ille major fuerit, eo Impetu quæ eſt duorum differentia : *h. e.* (poſito + ſigno dextrorſum, & — ſiniſtrorſum ſignificante) Impetus erit $+\frac{1}{1+m}$ P C $-\frac{m\,n}{1+m}$ P C $=\frac{1-m\,n}{1+m}$ P C; Celeritas $\frac{1-m\,n}{1+m}$ C ; adeoque dextrorſum vel ſiniſtrorſum, prout 1 vel $m\,n$, major fuerit). Et ſimiliter Impetus prius lati, erit $+\frac{1}{1+m}\,m$ P C $-\frac{m\,n}{1+m}\,m$ P C $=\frac{1-m\,n}{1+m}\,m$ P C ; Celeritas $\frac{1-m\,n}{1+m}$ C : adeoque dextrorſum vel ſiniſtrorſum, prout 1 vel $m\,n$, major fuerit.

12. Si vero Pondera nec eadem directe via procedant, nec directe contraria, ſed oblique ſibi mutuo impingant ; moderandus erit præcedens Calculus pro Obliquitatis menſura. Impetus autem Oblique impingentis, ad ejuſdem Impetum qui eſſet ſi Directe impingeret (cæteris paribus) eſt in ea ratione qua Radius ad Secantem Anguli Obliquitatis ; (quod etiam intelligendum eſt, ubi perpendiculariter, ſed Oblique cadit in percuſſi Superficiem, non minus quam ubi viæ Motuum ſe mutuo Oblique decuſſant : ) quæ quidem conſideratio, cum calculo priori debite adhibita, determinabit, quænam futura ſint ſic Oblique Impingentium Celeritas, Impetus & Directio, *h. e.* quo Impetu, qua Celeritate, & in quas Partes ab invicem reſilient, quæ ſic Impingunt. Eademque eſt ratio Gravitationis gravium Oblique deſcendentium, ad eorundem Perpendiculariter deſcendentium Gravitationem.

13. Si quæ ſic Impingunt Corpora, intelligantur non abſolute dura (prout hactenus ſuppoſuimus) ſed ita Ictui cedentia, ut Elaſtica tamen Vi ſe valeant reſtituere, hinc fieri potuit ut à ſe mutuo reſiliant ea corpora, quæ ſecus eſſent ſimul proceſſura ; (& quidem plus minuſve, prout hæc Vis Reſtitutiva major minorve fuerit,) nempe ſi Impetus ex Vi Reſtitutiva ſit Progreſſivo major.

In Motibus Acceleratis & Retardatis, Impetus pro ſingulis Momentis is reputandus eſt, qui gradui Celeritatis tum acquiſito convenit. Ubi autem per Curvam fit Motus, ea reputanda eſt in ſingulis punctis Motus Directio, quæ eſt rectæ ibidem Tangentis. Et ſi quando Motus tum Acceleratus vel Retardatus ſit, tum & per Curvam fiat (ut in Vibrationibus Penduli) Impetus æſtimandus erit, pro ſingulis punctis, ſecundum tum gradum Accelerationis, tum Obliquitatem ibidem Tangentis.

2. *Lex Naturæ de Colliſione Corporum.* *By Sir* Chriſt. Wren.

Velocitates Corporum propriæ & maximè naturales ſunt ad Corpora reciprocè proportionales. *Ibid. p.* 867.

Itaque Corpora R, S, habentia proprias Velocitates, etiam poſt Impulſum retinent proprias.

 Et

Et Corpora R, S, improprias Velocitates habentia ex Impulſu reſtituuntur ad Æquilibrium ; hoc eſt, Quantum R ſuperat, & S deficit à propria Velocitate ante Impulſum, tantum ex Impulſu abſtrahitur ab R, & additur ipſi S, & è contra.

Quare Colliſio Corporum propriàs Velocitates habentium æquipollet Libræ Oſcillanti ſuper bina Centra æqualiter hinc inde à Centro Gravitatis diſtantia : Libræ verò Jugum, ubi opus eſt producitur.

Itaque Corporum æqualium improprie Moventium tres ſunt Caſus. Corporum verò inæqualium improprie Moventium (ſive ad contrarias ſive ad eaſdem partes) decem ſunt omnino Caſus, quorum quinque oriuntur ex converſione.

Fig. 157. R, S, Corpora æqualia ; vel R, Corpus majus, S, Corpus minus.

*a* Centrum Gravitatis ſive Anſa Libræ. Z, Summa Velocitatum utriuſque Corporis.

{R *e*, S *e*} Veloc. Corp. {R, S} ante Impulſum data. {*o* R, *o* S} Veloc. Corp. {R, S} poſt Impul. quæſita.
vel
{S *o*, R *o*} Veloc. Corp. {S, R} ante Impulſum data. {*e* S, *e* R} Veloc. Corp. {S, R} poſt Impul. quæſita.

Regula. R *e*, S *e*, faciunt *o* R, *o* S : R *o*, S *o*, faciunt *e* S, *e* R.

[Lege Syllabas (quamvis disjunctas) R *e*, S *e*, *o* R, *o* S, vel R *o*, S *o*, *e* S, *e* R, in Linea cujuſlibet Caſus, & harum quæ ſcribitur in Schemate more *Hebraico*, ea indicat Motum contrarium Motui quem notat cujuſvis Syllabæ ſcriptio *Latina*. Syllaba conjuncta quietem Corporis denotat.]

Calcul. $R + S : S :: Z : Ra \mid Re - 2Ra = oR \mid So - 2Sa = eS.$
$R + S : R :: Z : Sa \mid 2Sa \pm Se = oS \mid 2Ra + Ro = eR.$

Natura obſervat regulas Additionis & Subductionis *Specioſæ*.

By M. Hugens, n. 46, p. 927. 3. *Regulæ de Motu Corporum ex mutuo Impulſu.*

1. Si Corpori Quieſcenti duro aliud æquale Corpus durum occurrat, poſt contactum hoc quidem Quieſcet, Quieſcenti vero acquiretur eadem quæ fuit in Impellente Celeritas.

2. At ſi alterum illud Corpus æquale etiam moveatur, feraturque in eadem Linea recta, poſt contactum permutatis invicem Celeritatibus ferentur.

3. Corpus quamlibet magnum à Corpore quamlibet exiguo & qualicunque Celeritate impacto movetur.

4. *Regula Generalis* determinandi Motum, quem corpora dura per occurſum ſuum directum acquirunt, hæc eſt :

Fig. 158. *Sint corpora* A *&* B, *quorum* A *moveatur Celeritate* A D, B *vero ipſi occurrat, vel in eandem partem moveatur Celeritate* B D, *vel denique Quieſcat, hoc eſt, cadit in hoc caſu punctum* D *in* B. *Diviſa Linea* A B *in* C, *(Centro Gravitatis corporum* A, B, *) ſumatur* C E *æqualis* C D. *Dico* E A *habebit Celeritatem corporis* A *poſt Occurſum ;* E B *verò, corporis* B, *& utrumque in eam partem, quam demonſtrat Ordo punctorum* E A, E B. *Quod ſi* E *incidat in punctum* A *vel* B, *ad Quietem redigentur corpora* A *vel* B.

5. Quantitas

5. Quantitas Motus duorum Corporum augeri minuive poteſt per eorum occurſum; at ſemper ibi remanet eadem quantitas verſus eandem partem, ablatâ inde quantitate Motus contrarii.

6. Summa Productorum factorum à Mole cujuſlibet corporis duri ducta in quadratum ſuæ Celeritatis, eadem ſemper eſt ante & poſt occurſum eorum.

7. Corpus durum Quieſcens, accipiet plus Motus ab alio corpore duro, ſe majori minorive, per alicujus Tertii, quod mediæ fuerit quantitatis, interpoſitionem, quam ſi percuſſum ab eo fuiſſet immediaté. Et ſi corpus illud interpoſitum, fuerit medium proportionale inter duo reliqua, fortiſſime omnium aget in Quieſcens.

Conſiderat *Author* in his omnibus (ut ipſe ait) Corpora ejuſdem materiæ, ſive id vult, ut eorum moles æſtimetur ex pondere.

Cæterum ſubjungit, notaſſe ſe miram quandam *Naturæ Legem*, quam Demonſtrare ſe poſſe affirmat in corporibus Sphæricis, quæque Generalis ipſi videtur in reliquis omnibus ſive Duris ſive Mollibus, ſive Directe ſive Oblique ſibi occurrentibus, *viz. Centrum Commune Gravitatis* duorum, trium, vel quotlibet Corporum, æqualiter ſemper promoveri verſus eandem partem in linea recta, ante & poſt occurſum.

*Some Hiſtorical Paſſages relating to theſe Papers; by Mr.* Oldenburg. *Ibid. p.* 925.

4. Cum noviſſimis Menſibus nonnulli è *Societate Regia* in publico ejuſdem conceſſu enixius urgerent, ut graviſſimum illud de *Regulis Motus* Argumentum, non ſemel inter Ipſos ante hac agitatum, ſed pluribus aliis intercurrentibus rebus, nunquam, uti par erat, diſcuſſum expenſumve, tandem aliquando Examini Rigido ſubjectum conficeretur; viſum equidem fuit Illuſtriſſimo iſti *Cœtui* decernere, ut quotquot è *Sociis* ſuis indagandæ Motus Indoli præ cæteris incubuiſſent, rogarentur, ut ſua in rem illam Meditata & Inventa depromere, ſimul & ea, quæ ab aliis Viris Præcellentibus, *Gallilæo* puta, *Carteſio*, *Honorato Fabri*, *Joachimo Jungio*, *Petro Borrelli*, aliiſque, de Argumento iſto fuerant excogitata, congerere & procurare vellent; eo ſcil. fine, ut conſultis hoc pacto collatiſque omnium ſententiis, illa dehinc Theoria, quæ cum Obſervationibus & Experimentis, debitâ cura & fide crebro peractis, quammaximè congrueret, Civitate Philoſophica ſuo jure donaretur.

Edito hoc Celeuſmate, incitati protinus è dicta *Societate* fuerunt, imprimis *Chriſtianus Hugenius*, *Johannes Walliſius*, *Chriſtopherus Wrennus*, ut ſuas de *Motu Hypotheſes* & *Regulas*, quibus condendis aliquamdiu inſudaſſent, maturare atque expedire ſatagerent. Factum hinc, ut ſelectus ille Virorum præſtantiſſimorum Trias, poſt paucarum ſeptimanarum ſpatium, *Theorias* ſuas, eleganter compendifactas, tantum non certatim tranſmitterent, *Regiæque Societatis* ſuper iis ſententiam exquirirent. Primus omnium D. *Walliſius*, ſua de *Motibus* æſtimandis *Principia*, Literis, d. 15 *Novemb.* 1668. datis, ejuſdemque Menſis die 26 traditis & prælectis, communicavit. Mox eum excepit D. *Chriſtopherus Wren*, qui *Naturæ Legem* de *Colliſione Corporum*, proximo Menſe *Decembri*, ejuſque die 17. eidem *Societati* publice exhiberi curavit: quæ in mandatis mox dedit (præ-habito tamen utriuſque hujus Authoris conſenſu) ut ad commodiorem horum ſcriptorum communicationem, diſcuſſionemque diffuſiorem, res tota Typis mandaretur.

Hæc dum apud Nos geruntur, Ecce adfert Nobis Tabellarius d. 4. *Januarii* insequentis (*St. Ang.*) D. *Hugenii* literas ejusdem Mensis d. 5. (at *St. N.*) exaratas, ejusque scripti, de *Motu Corporum* ex mutuo *Impulsu*, priores Regulas quatuor, una cum Demonstrationibus, continentes. Habebam ego in promptu *Theoriæ Wrennianæ* Apographum, idque actutum eodem planè die, sic favente Tabellione Publico, D. *Hugenio*, redhostimenti vice, remittebam, dilata interim Literarum *Hugeniarum* (quibus tale quid includi, ob Molem, & antegressum Authoris promissum suspicabar) resignatione, donec ferret occasio Nobilissimum & Sapientissimum *Regiæ Societatis Præsidem*, D. *Vice-Comitem Brouncker*, compellandi. Quo facto, amborumque Regulis in modo dicta *Societate* collatis, mirus confestim in utroque consensus effulsit; id quod insignem in nobis lubentiam pariebat, utrumque hoc scriptum prælo nostro committendi. Nihil hic Nobis deerat à parte *Hugenii*, quam ejus consensus; absque quo fas nequaquam judicabamus, ipsius Inventum, maximè cum illud haud integrum eo tempore nobis dedisset, in lucem emittere. Curæ interim nobis erat, scriptum Ipsius publicis *Regiæ Societatis* Monumentis inserendi; simul & Authori d. 11. *Januarii* solennes pro Cordata illa Communicatione gratias reponendi, additâ dehinc ( die *Scil.* 4 *Febr.* ) sollicitâ commonefactione, ut suam hanc *Theoriam* vel *Parisiis* (quod proclive erat factu in *Eruditorum*, ut vocant, *Diario*) vel hic *Londini* in *Adversariis Philosophicis*, imprimendam curaret, vel saltem permitteret. Quibus expeditis Literis paulo post secundas accepimus ab *Hugenio*, scripti *Wrenniani* de hoc argumento recte traditi mentionem facientes, nihil tamen quicquam de suimet scripti Editione, vel *Parisiis* vel *Londini* paranda, commemorantes.

Unde liquere omnino autumem, ipsum sibi defuisse *Hugenium* in illa publicatione maturanda; quin imo occasionem dedisse procrastinando, ut Laudatus Dn. *Wren*, pro ingenii sui sagacitate Geminam omnino *Theoriam* eruens in Gloriæ, huic speculationi debitæ, partem jure veniret; cum extra omne sit dubium neutrum horum *Theoriæ* illius quicquam, priusquam scripta eorum simul compararent, resciviſse ab altero, sed utrumque propriâ Ingenii fæcunditate, pulchellam hanc sobolem enixum fuisse.

Solvit equidem *Hugenius*, ante aliquot jam Annos, *Londini* cum ageret, illos de Motu Casus qui ipsi tunc proponebantur; luculento sane Argumento, eum jam tum exploratas habuisse *Regulas*, quarum id evidentia præstaret. At non affirmabit ipse, cuiquam se *Anglorum* suæ *Theoriæ* quicquam aperuisse; quin fateri tenetur, se ab eorum nonnullis ad communicationem ejus solicitatum, nec tamen unquam, nisi nuperrime, ad id faciendum pertractum fuisse.

*The Synchronism of the Vibrations made in a* Cycloide; *Demonstrated by a Person of* Quality. n. 94. p. 6032. Fig. 159.

II. Sint *a b*, *b c*, *c d*, *d e*, *e f*, &c. omnes invicem æquales; & *b* 1, *c* 2, *d* 3, *e* 4, *f* 5, &c. æqualiter crescant ut, 1, 3, 5, 7, 9. &c.

Dico in hac Linea, Grave quodlibet, Cadens ex quovis ejus Puncto, attingere fundum in eodem Temporis spatio, quo eum attingeret si caderet ex quovis ejusdem Puncto alio.

Nam

Nam si ponas, $a = ab = bc = cd$, &c. & $b = b\,1$, & $x$ pro quolibet Numero alterutrorum; tunc si $xa$ ponatur pro $af$, $xxb$ repræsentet oportet $f\delta$, proindeque Tempus Descensus necessario erit $\frac{xxb}{xxaa}$ seu $\frac{b}{aa}$; atque idem in omnibus obtinet Casibus. Ergo, &c.

Dico insuper, Curvam hanc esse *Cycloidem*; quod Demonstratu est facile ex Constructione, atque ex eo quod jam innuo; nempe, Curvam hanc $abcdef\zeta$ æquare duplum Ultimæ Rectarum, *h. e.* $2\zeta\omega$, & $a\omega$ æqualem esse Semicircumferentiæ Circuli cujus $\zeta\omega$ est Diameter; ac universim Triangulum ♈ ♉ ♊ repræsentare Rectam $\zeta\omega$; & Quadratum ♈ ♉ ♊ ♋, Curvam $abcdef\zeta$, & Quadrantem ♈ ♉ ♋ repræsentare Rectam $a\omega$: ac partes unius, partes alterius respectivê. Uti si ♈ ♎ ♍ repræsentat $f\delta$, tunc ♈ ♎ ♌ ♋ repræsentat $a\delta$, & ♈ ♎ ♏ ♋ representat $af$. At non vacat fusiùs hæc prosequi.

Dico denique; Globulum suspensum è Funiculo (Justæ Longitudinis) & intra duas *Cycloides* vibrantem, moveri in *Cycloide*. Quare Vibrationes ejusmodi sunt *Synchronæ*. *Q. E. D.*

III. 1. Probl.] *Determinare Lineam Curvam data duo Puncta, in diversis ab Horizonte Distantiis & non in eadem Rectâ Verticali posita, connectentem, super qua Mobile, Propriâ Gravitate decurrens, & à Superiori Puncto Moveri incipiens, citissimè Descendat ad Punctum Inferius.* *A Problem concerning the Line of Quickest Descent between two Points given; proposed by* M.Jo.Bernoulli. n. 224. *p.* 384.

Sensus *Problematis* hic est, ex Infinitis Lineis quæ duo illa data Puncta conjungunt, vel ab uno ad alterum duci possunt, eligatur illa, juxta quam si incurvetur Lamina Tubi Canalisve Formam habens, ut ipsi impositus Globulus & liberè dimissus iter suum ab uno Puncto ad alterum emetiatur Tempore Brevissimo.

2. Accepi hesterno die duo Problematum à *Joanne Bernoullo* Mathematicorum acutissimo propositorum Exemplaria, *Groningæ* edita, *Cal. Jan.* 1697. Quorum prioris Solutio sit hujusmodi. *Solv'd; by* ..... *Ibid.*

A Dato Puncto A, ducatur Recta Infinita A P C Z Horizonti parallela, & super eadem Recta describatur tum *Cyclois* quæcunque A Q P, Rectæ per alterum Datum Punctum B ductæ (& si opus est productæ) Occurrens in Puncto Q, tum *Cyclois* alia A B C cujus Basis & Altitudo sit ad prioris Basem & Altitudinem respective ut A B ad A Q; Et hæc *Cyclois* Novissima transibit per Punctum B, & erit Curva illa, linea in qua Grave à Puncto A ad Punctum B, Vi Gravitatis suæ, Citissimè perveniet. Q. E. I. *Fig.* 160.

3. Sit A P, Linea Horizontalis; P, Punctum à quo corpus Grave descendit, per Curvam Lineam quæsitam A D E, C & D Puncta duo infinitè propinqua, per quæ Corpus decisurum sit, C D Recta, duo Puncta connectens, D C & $s$ C, D F & S G, F S & G C vel $s$ H, Momenta Curvæ Abscissæ, & Ordinatim applicatæ respective. Capiatur, D $r$ = D $s$, & $t$ C = B C. *The Demonstration; by* Mr. R. Sault. n. 246. *p.* 425. *Fig.* 161.

Quoniam in Lineolis Naſcentibus, Tempus eſt ut Via percurſa directè & Velocitas (*i. e.* in hoc caſu, ut Radix Quadrata Altitudinis corporis deſcenſi) inverſè, per Hypoth. $\frac{Ds}{\sqrt{QD}} + \frac{SC}{\sqrt{QF}} =$ Tempori Minimo. Et quia Velocitas in Punctis æquialtis S & B per Curvam D *s* C & Rectam D B C eadem eſt, Tempus per D C, quod evidenter Minimum eſt, erit ut $\frac{BD}{\sqrt{QD}} + \frac{BC}{\sqrt{QF}}$; æquentur ergo hæc Tempora, & $\frac{Ds}{\sqrt{QD}} + \frac{sC}{\sqrt{QF}} = \frac{DB}{\sqrt{QD}} + \frac{BC}{\sqrt{QF}}$, hoc eſt, $\frac{DB - Ds}{\sqrt{QD}} = \frac{sC - BC}{\sqrt{QF}}$, vel $\frac{Br}{\sqrt{QD}} = \frac{ts}{\sqrt{QF}}$.

Sed Triangula Evaneſcentia B *r s*, B *t s*, æquiangula ſunt Triangulis D *s* F, H *s* C; Ergo $\frac{Bs}{Ds} = \frac{Br}{sF}$, & $\frac{ts}{Hs} = \frac{Bs}{st}$. Componantur hæ duæ rationes æqualitatis, & $\frac{Br}{Ds \times Hs} = \frac{ts}{sF \times st}$. Ex æquo $\frac{\sqrt{QD}}{sF \times st} = \frac{\sqrt{QF}}{Ds \times Hs}$. Quandoquidem autem quidvis ex Elementis æquabiliter Fluere ſupponatur, ponamus D S = S C, & evadet ſimpliciſſima Curvæ expreſſio $\frac{\sqrt{QD}}{sF} = \frac{\sqrt{QF}}{Ds}$ ubique, *i. e.* in Puncto Flexuræ, Curva ſemper erit in Ratione compoſita Velocitatis directè, & Momenti applicatim Ordinatæ inverſé. Sint $\dot{x}$, $\dot{y}$ & $\dot{z}$ Fluxiones Abſciſſæ, Ordinatim applicatæ, & Curvæ reſpectivè, $\frac{x^{\frac{1}{2}}}{\dot{y}}$ conſtans eſt, ut ſupra. Ergo $\frac{x^{\frac{1}{2}}}{\dot{y}} = 1$ ſed poſuimus $\dot{z}$ ( $= \sqrt{\dot{x}\dot{x} + \dot{y}\dot{y}}$ ) conſtans. Ergo ut hæc unitas conſtans ſit & dimenſiones debitas retineat $\frac{x^{\frac{1}{2}}}{\dot{y}} = \frac{a^{\frac{1}{2}}}{\sqrt{\dot{x}\dot{x} + \dot{y}\dot{y}}}$, & poſt Reductionem $\dot{y}\ \frac{x^{\frac{1}{2}}\dot{x}}{\sqrt{a - x}}$. Expreſſio Notiſſima *Cycloidis* P E L. Q. E. I.

IV. Theorema.

IV. Theorema.] *Si in* Cycloide A V D, *cujus Basis* A D *est Horizonti Parallela, Vertice* V *deorsum spectante, ex* A *ducatur utcunque Recta* A B Cycloidi *occurrens in* B, *ex quo ducatur Recta* B C *Curvæ* Cycloidis B D *in* B *Normalis, ad quam ex* A *demittatur Perpendicularis Recta* A C, *Dico Tempus quo Grave è Quiete cadens ex* A, *Vi suæ Gravitatis decurrit Rectam* A B, *esse ad Tempus quo percurrit Curvam* A V B, *sicut Recta* A B *ad Rectam* A C. *How much the Descent is Quicker in the* Cycloid *than in a straight Line, by* ..... n. 225. *p.* 424.

Per B ducatur B L, Parallela *Cycloidis* Axi V E; & B K, Basi A D Parallela, occurrens Axi in G, & Circulo super Diametrum E V descripto in F & H, *Cycloidi* denique in K. Ducatur Recta E F, quæ ex *Cycloidis* natura parallela est Rectæ B C. Unde B M est æqualis E F, & E M æqualis B F; quæ, propter *Cycloidem*, æquatur Arcui V F; & proinde A M est æqualis Arcui E H V F. Fig. 162.

Per *Prop.* 25. *Part.* II. *Horologii Oscillatorii Hugenii*, Tempus quo grave è Quiete cadens percurrit A V, est ad Tempus Casus per E V, ut Semicircumferentia ad Diametrum; & per dictæ Partis *Prop. Ultimam*, Tempus quo Grave percurrit V B, post decursam A V (nempe æquale Tempori quo Grave percurrit K V, post decursam A K) est ad Tempus Lapsus per A V, sicut Arcus V F, ad Semicircumferentiam; adeoque ad Tempus Casus per E V, sicut Arcus F V ad Diametrum. Quare Tempus quo Grave percurrit Curvam A V B, est ad Tempus Casus per E V, sicut Arcus E H V F, ad Diametrum E V. Sed Tempus Casus per E V, est ad Tempus Casus per L B, sive E G, sicut E V ad E F: Igitur ex æquo, Tempus quo Grave percurrit A V B, est ad Tempus Casus per L B, sicut Arcus E H V F ad subtensum E F; hoc est, ut Recta A M, ad Rectam M B. Rursus, Tempus Casus per L B, est ad Tempus Lapsus per A B, ut L B ad A B: Ergo Ratio Temporis quo Grave percurrit A V B, ad Tempus quo percurrit A B, componitur ex Ratione A M ad M B, & Ratione L B ad B A; adeoque æqualis est Rationi $AM \times LB$ ad $MB \times BA$. Sed $AM \times LB$, est æquale $MB \times AC$, quia utrumque æquatur duplo Trianguli A B M: Et igitur Tempus quo Grave è Quiete cadens percurrit Curvam *Cycloidis* A V B, est ad Tempus quo percurrit Rectam A B, sicut $MB \times AC$ ad $MB \times BA$; id est sicut A C ad A B. *Q. E. D.* Similiterque procedet Demonstratio, si Punctum B, sit inter A & V.

V. 1. The upper Plate of the *Watch* is A B: The Circular *Ballance-Wheel* C D, of which the *Arbre* is E F: The *Spring* turned *Spirally*, G H M, fastned to the *Arbre* of the *Ballance-Wheel* in M, and to the piece that is fast to the Watch-Plate, in G, all the *Spires* or Windings of the *Spring* being free without touching any thing. N O P Q, is the *Cock*, in which one of the *Pivots* of the *Ballance-Wheel* turns; R S, is one of the *Indented Wheels* of the *Watch* having a *Ballancing* Motion, which the *Ballance-Wheel* gives to it. And this *Wheel* R S, catches in the *Pinion* T, which holds on the *Arbre* of the *Ballance*, of which by this means the Motion is entertained as much as is necessary. These *Watches* are exact for the *Exact Portable Watches; by M.* Hugens. n. 112. *p.* 272. Fig. 163.

Pocket, and when made greater, will be uſeful to find the *Longitudes* both by Sea and Land.

By Dr. Goth. Guil. Leibnitz. n. 113. p. 285. 2. The Principle I thought upon ſome Years ago for making Exact *Portable Watches*, is altogether different from that of M. *Hugens*; His depending upon a Phyſical Obſervation, but mine upon a meer Mechanical Reflection; which hath not been taken notice of for want of the *Art* of *Combination*, the uſe of which is far more general than that of *Algebra*. For, having conſidered with my ſelf, that a *Spring* being Bent to the ſame Degree, will always Unbend it ſelf in the ſame Time, provided it find the ſame freedom of Unbending it ſelf ſuddenly; I inferred from thence, that there might be imployed two ſuch, one of which ſhould play, whilſt the *Firſt Mover* of the *Watch* did Bend the other again.

Fig. 164. Theſe Thoughts I have executed in the following manner: Let A B, be one of the *Watch-Plates*, C and M, two Indented Barrels, wherein the ſmall *Springs* are incloſed. The Teeth of the Barrels catch thoſe of the Pinions *d d*, which carry the Ballances *e e*, and other Teeth of the ſaid Barrels are catched by thoſe of the Interrupted Wheel F G. Now let us imagine, that this Wheel F G, being moved towards H F, by the force of the *Firſt Mover* of the *Watch*, and turning the Barrel C, Bends the *Spring* incloſed in it, and ſtops with the Barrel as ſoon as it hath Bent this *Spring*. This piece which ſerves to ſtop, is eaſie, and hath not been thought neceſſary to be marked here, to avoid embaraſſing the Figure. But whilſt one Indented part of the Interrupted Wheel F G, *viz.* F, turns the Barrel C, the empty part, oppoſed thereunto, which is G, anſwers to the other Barrel M, and gives Liberty to the *Spring*, it incloſeth, to Unbend it ſelf. Thus whilſt the *Movement* of the *Watch* Bends the ſmall *Spring* of the Barrel C, in the ſame Time the ſmall *Spring* of the other Barrel M, Unbends of it ſelf. I ſay, in the ſame Time, except the *Spring* C, ſhall have done Bending a little ſooner, than the *Spring* M, ſhall have Unbent it ſelf: So that the *Spring* C, being Bent, and the Wheel F G ſtopped; both of them ſtay in this Poſture, till the *Spring* M, when it ſhall be quite Unbent, do, at the end of its Motion, touch a piece which delivers it. And then the *Spring* C Unbends of it ſelf in its turn; the Teeth of the Interrupted Wheel, which continues its Motion the ſame way as before, ſince 'tis delivered, not being any more able to hinder it therefrom, becauſe the Barrel C, doth now meet with the Empty part H, of the ſaid Wheel. But before it hath done with Unbending it ſelf, the Indented part L, being oppoſite to the Empty part H, that turns the Barrel M, Bends its *Spring* again, and having done ſo, ſtops with it; whilſt the *Spring* C, making an end of Unbending it ſelf, delivers them by a Reciprocal good Office, and renders to the *Spring* M, the ſame Services which it had received from it, with an Expectation of receiving the like again.

Which being well conſidered, 'tis manifeſt, That the ſame *Alternative Motions* will continue always: That the Periods, taken from the very Moment that one *Spring* begins to Unbend, until the Moment it once Unbends it ſelf again, will always be of *Equal Duration*, though the two ſmall

*Springs*

*Springs* be not equally ſtrong: That the Ballance of ſuch a *Watch* will be double, and may be Charged more or leſs, and receive Delay, by Advancing or Recoiling along the two Arms two equal Weights, Counter-ballancing one another, that ſo the Change of the Situation may not at all prejudice the *Equality* of the *Watch*. For the reſt, we may in this kind of *Watches* ſpare the *Fuſee*, and conſequently the String or Chain. 'Tis alſo eaſie to judge, that ſuch *Watches* as theſe may be of a Size ſufficiently ſmall; that they will make no more Noiſe than ordinary *Watches*; that they will be as exact as *Pendulums*, and ceaſe not to go whilſt they are Winding up. And though the Motion of the Watch Wheels may be altered by many Accidents, yet the *Periods* of the *ſmall Springs* will not be concerned in all or any of them, provided the Motion of the Watch Wheels have always more Strength than it needs to Bend them again; which is in our Power.

The Objections that have been made againſt this Contrivance, if employed for *Finding Longitudes*, are theſe; That Toſſing of Ships would ſhake the *Springs* as well as other pieces; That Ruſt would ſpoil them, ſince the Saltiſh Humidity of the Sea in remote Voyages, ſpares not the very Needles of Compaſſes though incloſed in Boxes; That the Changes of Seaſons and Climates will ſenſibly alter the *Springs*, eſpecially the great Heats or Rains within the Tropicks, which at length will ſomewhat Untemper the Steel; as is confirmed by the Experiments of the *Illuſtrious Academy of Florence*, ſhewing how eaſily that Heat and Cold do change *Slender Springs*: beſides that, the Air more or leſs condenſed will alſo more or leſs reſiſt the Motion of the Ballance. To which may be added, That *Springs* by working are weakened; And Laſtly, That there will be always ſome little *Friction*, that will make the ſeveral pieces go more or leſs eaſily, and that even in length of time they will wear out.

But I Anſwer, That all theſe Defects, that proceed from the Imperfection of the Matter, may be ſurmounted by a General Remedy, without Examining them here in Particular: And that is, That for executing it in great, we may make uſe of maſſy Springs, as are thoſe of Croſs-Bows, we being Maſters of them, not wanting Force or Place in a Ship, to govern a great Weight that may ſerve to Bend them continually again. Now theſe *Maſſy Springs* may be ſo great, and their Reſtitution ſo ſpeedy, by Augmenting their number, that all the above-named Defects will have no conſiderable Proportion to this Strength, and the Aggregate of their Repetitions will not be ſenſible till after a very long time. And 'tis eaſie to *Demonſtrate*, That by Augmenting the Bigneſs of the Engine, and the Force of the *Maſſy Springs*, we may make the Errour as ſmall as we will, provided we paſs not the Bounds of conveniency, and content our ſelves with Exactneſs ſufficient for their Chief End, *viz.* For finding the *Longitudes*.

*A Clock Aſcendent upon an Inclined Plain*; *by* M. de Gennes. n. 140. p. 1006. Fig. 165.

VI. The Circle, FGH, being placed upon a *Plain Inclined* AB, is divided into two Unequal Parts by the Line GI. To reſtore to the Leſs Secture its *Æquilibrium*, there is faſtned to the Extremity of the *Radius* DF, a Weight F, which is ſufficiently heavy to Recover, what the leſſer Secture

 loſes

loſes by its Situation. That a Wheel or Clock may thus ſtand not only in *Æquilibrium*, but alſo Aſcend upward, there is placed in the middle of the *Clock* a *Drum*, which encloſes the *Spring* of the *Pendulum*; upon which *Drum* is faſtned the *Radius* D F. For thus the *Spring* being Mounted, enforces the *Drum* to turn, and ſo to Raiſe the Weight, which it cannot Raiſe without its becoming more Heavy, in regard that coming to the Point E, it is farther from the Center, than when it was in F, and thus all the *Wheel* turns on that ſide, as the *Spring* gives way.

*A Clock Deſcendent on a Plain Inclined; by* Mr. Maur. Wheeler. n. 161. *p.* 647.

VII. Although the *Marquis of Worceſter* is ſaid to have contrived a *Watch* that ſhould *Move* upon a *Declivity*, and *M. de Gennes* has given ſome Account of a *Clock Aſcendent on a Plain Inclined*; yet neither of them, nor any like them, was ever ſeen by me, and for ought I could ever learn, the *Reaſon* of their *Motions* remains to this Hour as great a Secret, as if they had never been. I ſhall therefore give an Account of a *Movement*, which I have deſign'd to meaſure Time after a Peculiar Manner.

Fig. 166. 1. The Exteriour Structure of it is a Circular Body of $3\frac{1}{2}$ Inches Diameter, conſiſting of two Plates meaſured by the ſame *Radius*, and fixt in a Parallel Poſition to each other by the Hoop *b*, the Breadth of which is about an Inch. This Hoop and the two Plates Form the Caſe of the *Movement*; of which, that which appears in the Front, is towards the Verge thereof Inſcribed with a *Horary Circle*, the Diviſions whereof anſwer the Hours of a Natural Day. The Deep Shades within this Circle are intended to repreſent a Concave, of near half an Inch deep; and the Prominence *g*, in the Middle of this Concave, is a Hemiſphere of Braſs or Silver, riding looſly on a Pin, which lies hid, and is the *Axis* of the *Movement*. The Upper half of this Hemiſphere is hollow, but the Nether filled with Lead; and the ſmall Gentleman that ſits thereon, does with an Erected Finger perform the Office of an Index. But this being only for Ornament, you may Subſtitute in the

Fig. 169. room thereof any other Index, provided the *Axis*, whereon it is ſupported, move freely in the Hole H, and the lower part thereof H L, ſo far preponderate to H P, as always to keep it Pendulous, with its Point to the Vertical Hour.

Fig. 166. 2. For the manner of its Motion, as far forth as it appears outwardly; it is thus: S E repreſents a Board or Shelf, of a Straight and Even Surface, about 6 Foot long, and ſo Thick as not to be apt to Caſt with change of Weather; nor to grow Camber under a ſmall Weight; on this is the *Movement* placed, and here to perform its Courſe; and therefore I call it the *Stage* of the *Movement*. This *Stage* is Raiſed at the end S, about 10 *Deg.* above the Horizon or Line of Level H E; but this Angle of its Declivity D E H, is Variable. The two Plates which Form the *Caſe* of the *Movement*, are to be extant all round without the Hoop *b*, $\frac{1}{8}$ of an Inch, and the Edges of 'em lightly Indented; that while the *Movement* deſcends upon the *Stage*, it may Turn only and not Slide. The *Movement* being placed as high as it may, near the point S, ſhall Move downward towards E, with that ſlowneſs, as to Finiſh one entire Revolution in 24 hours; and while it does ſo, the Diviſions on the

the *Horary Circle* (or Dial-Plate) successively Culminating over the Point of the Index (which is always to keep the same Position) will shew the Hours of the Day and Night. And when by several repeated Revolutions, it has measured out the length of its *Stage*, it is to be replaced at S, as before, which may be done in less than half the time you are Winding up a Watch; and if the *Stage* be 6 Foot long, no oftner than once in a whole Week.

3. The way of Adjusting the Motion to the exact Measure of an Hour, and Rectifying its Errours, is thus: *viz*. By the turning of a Skrew inserted at S, the *Stage* may be Elevated or Deprest, and accordingly the *Movement* will go Faster or Slower: Faster if Raised up, and Slower, if let Down; and by making the *Horary Circle* Moveable, and Inserting several small Bosses or Buttons, here and there upon the Verge thereof, it may with an easie touch of the Finger be moved to the right and left, as there shall be occasion, till the just Time be brought to the Point of the *Suspended Index*.

The Reason of this *Movement* may be thus Explain'd: 1. Let the Circle L O D N, represent any Circular Body, whose Centers both of Gravity and Magnitude are Coincident at M. Let this Circular Body be placed upon some Level Plain G G, and then 'tis Evident that the Angle of its Contact with that Plain at *a*, will also be the Point of its Libration, and consequently it must Rest there: *Quia Momentum & Impedimentum sunt æqualia.* Fig. 167.

2. Let D E, represent a Descending Plain, making an Angle of Contact with this Circular Body at *b*; and here 'tis manifest it cannot Rest; because the Line of Direction *r a*, which (while it Insisted upon a Level) divided the Circular Body by the Centers of Magnitude and Gravity into Parts Æquiponderate, is now removed to L D; which Line L D, falling without, or beside, the Center M, evidently destroys the Æquipoise of its Parts, and therefore must leave it to tumble down towards E. For here *Momentum Impedimento majus*. The Reason therefore of its Descent now, being the Over-ballance of the Parts L N D, to the remaining Section L D O, it must necessarily follow:

3. That if some Weight equal to the Excess of L N D, above L O D, were affixt to the Limb of the Quadrant O *a*, as at P; then the Circular Body would Rest as Quietly at *b*, as it did before at *a*. The Supposition cannot be denied, and the Consequence is unavoidable, because L D O + P = L N D. *i. e. Impedimentum æquatur Momento.*

Let then the Numbers, 1, 2, 3, 4, represent a Train of Wheel-work, wherein there is no material difference from what is found in a common Watch; only the numbers of the Teeth on the Wheels and Pinions are to be so Calculated, that the Motion of the whole Train may correspond to the assigned Revolution of the Body of the *Movement*, which is to be once in 24 Hours: It would be Expedient also, That a *Spiral Spring* were applied to its *Ballance*, as in later *Movements*, is usual; but of a *Fusee* here's no need; for the Turns of the Body of the *Movement*, as it descends upon the *Stage*, answer all the Intentions of a String or Chain; and the *Contranitence* of the Weight P, to the excess of LED, above LQD, serves instead of a Perpetual Spring; and the *Movement* wants only a *Perpetual Descent*, to make its *Mo-* Fig. 168.

*tion:*

*tion* ſo. And whereas the great Wheel in ordinary *Movements*, is placed as near the edge of the Framing Plate *f f*, as it may be; here it muſt (with its *Axis* or Arbre M) poſſeſs the Center of the *Movement*: becauſe this Wheel is to carry the Weight or *Power* P, by the *Vectis* M P, and that Weight P, muſt always keep an Equidiſtance from the Center of the *Movement*; that while the Body thereof (*i. e.* of the *Movement*) performs its Revolutions; the ſaid Weight P, and the great Wheel, (to which it is affixt,) may, without any conſiderable Variation, continue in, or near the ſame Poſition, wherein they now are. Now ſuppoſe this Weight P, with its *Vectis* M P, to be taken quite out of the *Movement*, and the Body of the *Movement* to be placed on a Horizontal Plain H H, its point of Contact in that Plain is T; where it ſhould, but cannot, Reſt; becauſe the Weight of that part of the Train, marked with the Numbers, 2, 3, 4, Removes the Center of Gravity from M, and therefore on the oppoſite part of the *Movement*, as about C Q, the Inſide of the Hoop, which forms the Caſe, is to be loaded with a thin Lining of Lead, which may be a Counterpoiſe to that part of the Train; that ſo, the whole Body of the *Movement*, together with all its Furniture, within and without, (excepting only P, with its *Vectis*) may on that Horizontal Plain, or while it Rides upon its own *Axis*, Reſt indifferently in any Point. This reducing of the *Movement* to an Equilibration of all its Parts in the Center M, muſt be perform'd *Tentando*, *i. e.* by Raſping the Lead at C Q, as much and in ſuch Places as is needful; which to an Artificer, of Ordinary Sagacity, will not be at all difficult.

The Center of Gravity being thus reduced to M, replace the Weight P,
Fig. 170. by the Hole H, on the Arbor of the Central-Wheel M. Then let the Body of the *Movement* be placed on the Declivity D E, and ſuppoſing P + L Q D = L D E, then the Body muſt needs Reſt there: but becauſe the
Fig. 168. Weight P, is not now fixt to any part of the Quadrant Q D, but hangs upon the Train of Wheel-work 1, 2, 3, 4, it evidently follows, That if the Power thereof be Superiour to the Reſiſtance of the Train, then the whole Body of the *Movement* muſt needs Deſcend, towards E. By this you ſee there are two Offices aſſigned to the Weight or Power P. The Firſt is, to be a Counterpoiſe to the exceſs of the Weight of L E D, above L Q D. The Second is, that it be of Force ſufficient to put the Train into a Motion ſo Adjuſted, as may exactly comport with the Time aſſigned for the Revolution of the whole Body. So that if there be any difficulty remaining, it conſiſts in ſuch an exact Stating of the Weight and Power of P, that it may Adequately ſerve both theſe Intentions. Now how very eaſie this is, will be manifeſt from theſe Propoſitions following.

1. That whatever the Intrinſick Weight of P ſhall be, (as ſuppoſe it 4 Ounces *Troy*;) yet the Power of that Weight will be Augmented or Diminiſhed according to the different Degrees of its Elevation in the Quadrant T Q. Thus conſidering P M, as a *Vectis*, its *Hypomochlium* is M, the Point where it exerts its Power on the Train, is at V; I ſay then, whatever Power it has upon the Point V, in its preſent Elevation of 45 *Deg.* it will acquire a greater by being raiſed to 50. 55. *&c.* and the greateſt of all in 90 *Deg.*

at

at Q: and on the contrary, let it Sink to 40°. 35°. *&c.* its Power upon the Point V, will ſtill be Diminiſhed, inſomuch that in T, it will be utterly extinguiſh'd. And therefore if P, be of a competent Weight (*i. e* not utterly too light) to *Move* the Train at all, it will certainly Move it in ſome Degeee of Elevation or other in the Quadrant QT.

2. If the Weight P, be conſidered as to its Office of being a Counterpoiſe to the Body of the *Movement* ; as I need not to prove, that it will perform this no leſs, while it hangs by upon the *Vectis* MP, than if it were faſt Rivetted in the ſame place to the Caſe of the *Movement :* ſo, in what Point of the Quadrant ſoever it will move the Train, it may be alſo a Counterpoiſe to the Body of the *Movement.* For,

1. At what Point ſoever of the Circle LETQ, the Line of Declivity DE, makes an Angle of Contact ; on the ſame Point will the Diameter SD, fall at Right Angles with DE.

2. The Line of Direction LD, will ever fall upon the Point of Contact D, making an Angle with the Diameter, as SDL.

3. The Angle SDL, will be always equal to DEH, *i. e.* As great as is the Elevation of the Line of Declivity DE, above the Horizontal EH, ſo great will the Angle of Diſtance be between the Diameter SD, and the Line of Direction LD. *Fig. 166.*

4. The Greater the Angle of Declivity is, the Leſs will be the Section EQD ; and ſo on the contrary, the Leſs that Angle is, the Greater the Section. And therefore,

5. The Exceſs of the Weight of LED, above LQD, muſt be alſo Greater, by Raiſing up the *Stage* with the Skrew at S: and that Exceſs Leſs by Skrewing it down.

6. The Lighter that part of the Body is, which is repreſented by the Section LQD, the more Heavy ought the Counterpoiſe P, to be ; and that either in its own Intrinſick Weight, (in Ounces and Parts of Ounces) or elſe in its Potential Weight, by being Raiſed Higher in the Quadrant QT.

7. The Skrewing up the *Stage* of the *Movement* at S, will Raiſe the Counterpoiſe Higher in the Quadrant QT, by *Prop.* 3. and therefore Potentially Heavier. And from hence appears, (I take it moſt clearly) both the reaſon of the due Adjuſtment of the Motion of the Train to the exact Meaſure of an Hour, and what Weight is to be aſſign'd to P, that Moves it ; and that we are not confined to Scruples and Grains, but are allowed ſuch a conſiderable latitude, as it is not eaſie to err therein.

Having therefore ſet the *Stage* (by the help of the Arched Skrew) at the Elevation of about 10 *Deg.* place the *Movement* thereon, and try what Weight, hanging at the end of the *Vectis* MP, will ſtir the Train, mean while holding the *Movement* with the Hand in ſuch a Poſition, as the *Vectis* may make an Angle of about 30 *Deg.* with the Perpendicular MT : then let the *Movement* looſe, to Undulate upon the *Stage* ; and when the Vibration ceaſes, obſerve to what Degree of the Quadrant the *Vectis* Points, and at the ſame time mind the Pulſes of the Ballance. If at this Obſervation, the Weight lies low, (as for inſtance, between 25 and 35 *Deg.* of the Quadrant)

and

and the Beats of the Ballance are gueſſed to be not much different from their due time, the Weight P, is well enough proportioned. But if it chance to be much Heavier than is abſolutely needful, that Exceſs will be moderated by Skrewing down the *Stage*; and if it be not abſolutely too Light, its Defect will be compenſated, by Skrewing the *Stage* Higher. Therefore of theſe two Extreams, chooſe the former; for the fewer Degrees that P ariſes in the Quadrant, beyond what is abſolutely neceſſary, it will (for Reaſons very obvious) be ſo much the better.

*The Effects of Gravity in the Deſcent of Heavy Bodies, and the Motion of Projects; by Mr.* Halley. *n.* 179. *p.* 3.

VIII. *Des Cartes* his Notion, I muſt needs confeſs to be to me Incomprehenſible, while he will have the Particles of his *Celeſtial Matter*, by being reflected on the Surface of the Earth, and ſo aſcending therefrom, to drive Down into their Places thoſe Terreſtrial Bodies they find above them: This is as near as I can gather, the Scope of the 20, 21, 22, and 23 *Sections* of the laſt Book of his *Principia Philoſophiæ*; yet neither he nor any of his Followers can ſhew, how a Body ſuſpended *in Libero Æthere*, ſhall be carried downwards by a continual Impulſe tending upwards, and acting upon all its Parts equally: And beſides, the Obſcurity wherewith he expreſſes himſelf, particularly, *Sect.* 23. does ſufficiently argue, according to his own Rules, the *Confuſed Idea* he had of the thing he wrote.

Others, and among them, Dr. *Voſſius*, aſſerts the Cauſe of the Deſcent of Heavy Bodies, to be the *Diurnal Rotation* of the *Earth* upon its *Axis*, without conſidering, that according to the Doctrine of Motion fortified with Demonſtration, all Bodies moved *in Circulo*, would recede from the Center of their Motion; whereby the contrary to *Gravity* would follow, and all looſe Bodies would be caſt into the Air in a Tangent to the Parallel of Latitude, without the Intervention of ſome other Principle to keep them faſt, ſuch as is that of *Gravity*. Beſides the Effect of this Principle is throughout the whole Surface of the Globe found nearly equal, and certain Experiment ſeems to argue it rather leſs near the Equinoctial, than towards the Poles, which could not be by any means, if the *Diurnal Rotation* of the *Earth* upon its *Axis* were the Cauſe of *Gravity*; for where the Motion was ſwifteſt, the Effect would be moſt conſiderable.

Others aſſign the *Preſſure* of the *Atmoſphere* to be the Cauſe of this *Tendency* towards the *Center* of the *Earth*; but unhappily they have miſtaken the Cauſe for the Effect, it being from undoubted Principles plain, that the Atmoſphere has no other Preſſure, but what it derives from its *Gravity*; and that the Weight of the upper parts of the Air, preſſing on the lower parts thereof, do ſo far bend the Springs of that Elaſtick Body, as to give it a Force equal to the Weight that compreſſed it, having of it ſelf no Force at all: And ſuppoſing it had, it will be very hard to Explain the *Modus*, how that Preſſure ſhould occaſion the *Deſcent* of a Body circumſcribed by it, and preſſed equally above and below, without ſome other Force to *Draw* or *Thruſt* it Downwards. But to Demonſtrate the contrary of this Opinion, an Experiment was long ſince ſhewn before the *Royal Society*; whereby it appeared, that the Atmoſphere was ſo far from being the Cauſe of *Gravity*, that the

Effects

Effects thereof were much more vigorous where the Pressure of the Atmosphere was taken off; for a long Glass-Receiver having a light Down-Feather included, being *Evacuated* of Air, the Feather which in the Air would hardly sink, did *in Vacuo* descend with nearly the same Velocity as if it had been a Stone.

Some think to Illustrate this Descent of *Heavy* Bodies, by comparing it with the Virtue of the *Loadstone*; but setting aside the Difference there is in the manner of their Attractions, the *Loadstone* drawing only in and about its Poles, and the Earth near equally in all Parts of its Surface, this Comparison avails no more than to explain *Ignotum per æquè Ignotum.*

Others assign a certain *Sympathetical Attraction* between the Earth and its Parts, whereby they have, as it were, a desire to be united, to be the Cause we enquire after: But this is so far from explaining the *Modus*, that it is little more, than to tell us in other terms, that *Heavy* Bodies Descend, because they Descend.

But though the *Efficient Cause* of *Gravity* be so obscure, yet the *Final Cause* thereof is clear enough; for it is by this Single Principle that the Earth and all the Celestial Bodies are kept from Dissolution: the least of their Particles not being suffered to recede far from their Surfaces, without being immediately brought down again by Virtue of this *Natural Tendency*, which for their Preservation, the Infinite Wisdom of their *Creator* has ordained to be towards each of their Centers; nor can the Globes of the *Sun* and *Planets* otherwise be destroyed, but by taking from them this Power of keeping their parts united.

The Affections or *Properties* of *Gravity*, and its manner of Acting upon Bodies Falling, have been in a great measure discovered, and most of them made out by Mathematical Demonstration in this our Century, by the accurate Diligence of *Galilæus*, *Torricellius*, *Hugenius*, and others; and now lately, by our worthy Countryman Mr. *Is. Newton*. Which *Properties* I shall here enumerate. *The Properties of Gravity. Ibid. p. 6.*

1. The First *Property* is, That by this *Principle* of *Gravitation*, all Bodies do *Descend* towards a *Point*, which either is, or else is very near to, the *Center* of *Magnitude* of the *Earth* and *Sea*, about which the Sea forms it self exactly into a Spherical Surface, and the Prominences of the Land, considering the Bulk of the whole, differ but insensibly therefrom.

2. That this *Point*, or *Center* of *Gravitation*, is *Fixt* within the Earth, or at least has been so, ever since we have any Authentick History: For a Consequence of its Change, tho' never so little, would be the over-flowing of the Low-lands on that side of the Globe towards which it approached, and the leaving new Islands bare on the opposite side, from which it receded; but for this Two Thousand years it appears, that the Low Islands of the *Mediterranean* Sea (near to which the Ancientest Writers lived) have continued much at the same height above the Water, as they now are found; and no Inundations or Recesses of the Sea arguing any such Change, are Recorded in History, excepting the *Universal Deluge*, which can no better way be accounted for, than by supposing this *Center* of *Gravitation* Removed for a time, towards the Middle of the then Inhabited Parts of the World; and a Change

of its place, but the two thousandth part of the Radius of this Globe, were sufficient to bury the Tops of the Highest Hills under Water.

3. That in all Parts of the Surface of the Earth, or rather in all Points equidistant from its Center the *Force* of *Gravity* is nearly *Equal*; so that the length of the Pendulum Vibrating Seconds of Time, is found in all Parts of the World to be very near the same. 'Tis true, at St. *Helena*, in the Latitude of 16 *Deg.* South, I found that the Pendulum of my Clock, which Vibrated Seconds, needed to be made shorter than it had been in *England*, by a very Sensible Space (but which at that time I neglected to observe accurately) before it would keep Time; and since the like Observations have been made by the *French Observers* near the Equinoctial: Yet I dare not affirm, that in mine it proceeded from any other Cause, than the great Height of my Place of Observation above the Surface of the Sea, whereby the *Gravity* being diminished, the Length of the Pendulum, Vibrating Seconds, is proportionably shortned.

4. That *Gravity* does *Equally Affect* all *Bodies*, without regard either to their Matter, Bulk, or Figure; so that the Impediment of the Medium being removed, the most Compact and most Loose, the Greatest and Smallest, Bodies would Descend the same Spaces in Equal Times; the truth whereof will appear from the Experiment I before cited. In these two last Particulars, is shewn, the great Difference between *Gravity* and *Magnetism*, the one affecting only Iron, and that towards its Poles, the other all Bodies alike in every part. As a *Corollary*, from hence it will follow, That there is no such thing as *Positive Levity*, those things that appear *Light*, being only comparatively so; and whereas several things Rise and Swim in Fluids, 'tis because Bulk for Bulk, they are not so Heavy as those Fluids; nor is there any reason, why *Cork*, for instance, should be said to be *Light* because it *Swims* on *Water*, any more than *Iron*, because it *Swims* on *Mercury*.

5. That this *Power Increases* as you *Descend*, and *Decreases* as you *Ascend* from the *Center*, and that in the proportion of the *Squares* of the *Distances* therefrom *Reciprocally*, so as at a double Distance to have but a quarter of the Force: This Property is the Principle on which Mr. *Newton* has made out all the Phænomena of the Celestial Motions, so easily and naturally, that its Truth is past dispute. Besides that, it is highly Rational, that the *Attractive* or *Gravitating Power* should exert it self more vigorously in a Small Sphere, and weaker in a Greater, in proportion as it is Contracted or Expanded; and if so, seeing that the Surfaces of Spheres are as the Squares of their *Radii*, this Power at several Distances will be as the Squares of those Distances *Reciprocally*, and then its whole Action upon each Spherical Surface, be it great or small, will be always Equal. And this is evidently the Rule of *Gravitation* towards the Centers of the *Sun*, *Jupiter*, *Saturn*, and the *Earth*, and thence is reasonably inferred, to be the General Principle observed by Nature in all the rest of the *Celestial Bodies*.

These are the Principal Affections of *Gravity*, from which the Rules of the *Fall* of *Bodies*, and the *Motion* of *Projects*, are Mathematically deducible. Mr. *Is. Newton* hath shewed how to define the Spaces of the Descent of a

Body, let fall from any given Height, down to the Center, ſuppoſing the *Gravitation* to Increaſe, as in the Fifth Property; but conſidering the ſmallneſs of Height, to which any *Project* can be made Aſcend, and over how little an Arch of the Globe it can be Caſt, by any of our Engines, we may well enough ſuppoſe the *Gravity Equal* throughout, and the *Deſcents* of *Projects* in *Parallel Lines*, which, in Truth, are towards the *Center*, the Difference being ſo ſmall as by no means to be diſcovered in Practice.

Prop. I.] *The Velocities of Falling Bodies, are Proportionate to the Times from the beginning of their Falls.* *Propoſitions concerning the Deſcent of Heavy Bodies, and the Motion of Projects. Ibid. p. 9.*

This follows, for that the *Action* of *Gravity* being continual, in every ſpace of Time, the Falling Body receives a new Impulſe, Equal to what it had before, in the ſame Space of Time, received from the ſame *Power*: For inſtance, in the Firſt Second of Time, the Falling Body has acquired a *Velocity*, which in that Time would carry it to a certain Diſtance, ſuppoſe 32 Foot, and were there no new Force, would Deſcend at that rate with an Equable Motion; but in the next Second of Time, the ſame *Power* of *Gravity* continually Acting thereon, ſuperadds a New *Velocity* equal to the former; ſo that at the end of two Seconds, the *Velocity* is double to what it was at the end of the Firſt; and after the ſame manner may it be proved to be Triple, at the end of the Third Second, and ſo on. Wherefore the *Velocities of Falling Bodies, are Proportionate to the Times of their Falls.* Q. E. D.

Prop. II.] *The Spaces deſcribed by the Fall of a Body, are as the Squares of the Times from the beginning of the Fall.*

*Demonſtration.*] Let A B repreſent the Time of the Fall of a Body, B C, Perpendicular to A B, the Velocity acquired at the End of the Fall, and draw the Line A C; then Divide the Line A B, repreſenting the Time, into as many equal Parts as you pleaſe, as *b*, *b*, *b*, *b*, *&c.* and through theſe Points draw the Lines, *b c*, *b c*, *b c*, *b c*, *&c.* Parallel to B C; 'tis manifeſt that the ſeveral Lines, *b c*, repreſent the ſeveral *Velocities* of the Falling Body, in ſuch parts of the Time, as A *b*, is of A B, by the *Former Propoſition.* It is evident likewiſe, that the Area, A B C, is the Sum of all the Lines *b c*. being taken, according to the *Method* of *Indiviſibles* infinitely many; ſo that the Area A B C, repreſents the Sum of all the *Velocities* between none and B C, ſuppoſed infinitely many; which Sum is the Space Deſcended in the Time repreſented by A B. And by the ſame reaſon, the Areas A *b c*, will repreſent the Spaces Deſcended in the Times A *b*; ſo then the Spaces Deſcended in the Times A B, A *b*, are as the Areas of the Triangles, A B C, A *b c*, which by the 20th of the 6th of *Euclid* are as the *Squares* of their *Homologous Sides* A B, A *b*, that is to ſay, of the *Times*: wherefore the *Deſcents* of *Falling Bodies*, are as the *Squares* of the *Times* of their *Fall.* Q. E. D. Fig. 171.

Prop. III.] *The Velocity which a Falling Body acquires in any Space of Time, is double to that, wherewith it would have moved the Space Deſcended by an Equable Motion, in the ſame Time.*

 *Demonſtration.*

*Demonstration.*] Draw the Line E C, Parallel to A B, and A E, Parallel to B C, and compleat the *Parallelogram* A B C E, it is evident that the *Area* thereof may represent the Space, a Body Moved *Equably* with the *Velocity* B C, would Describe in the Time A B, and the *Triangle* A B C, represents the Space Described by the Fall of a Body, in the same Time A B, by the *Second Proposition.* Now the *Triangle* A B C, is Half of the *Parellelogram* A B C E, and consequently the Space described by the *Fall*, is Half what would have been described by an *Equable Motion* with the *Velocity* B C, in the same Time; wherefore the *Velocity* B C, at the End of the Fall, is *Double to that Velocity*, which in the Time A B, would have described the Space Fallen, represented by the *Triangle* A B C, with an *Equable Motion.* Q. E. D.

Prop. IV.] *All Bodies on or near the Surface of the Earth, in their Fall, Descend so, as at the end of the First Second of Time, they have described* 16 *Feet one Inch,* London *Measure, and acquired the Velocity of* 32 *Feet two Inches in a Second.*

This is made out from the 25th *Prop. Par.* 2. *Horol. Oscill. Hugen.* wherein he Demonstrates the Time of the least Vibrations of a Pendulum, to be to the Time of the Fall of a Body, from the Height of Half the Length of the Pendulum, as the Circumference of a Circle to its Diameter: whence, as a *Corallary*, it follows, That as the Square of the Diameter to the Square of the Circumference, so half the length of the Pendulum Vibrating Seconds, to the Space described by the Fall of a Body in a Second of Time: and the Length of the Pendulum Vibrating Seconds, being found 39, 125, or $\frac{1}{8}$ Inches, the Descent in a Second will be found, by the aforesaid Analogy, 16 Foot and one Inch: and by the *Third Proposition*, the Velocity will be double thereto; and near to this it hath been found by several Experiments, which by reason of the swiftness of the Fall, cannot so exactly determine its Quantity.

From these Four Propositions, all Questions concerning the *Perpendicular Fall* of Bodies are easily solved, and either *Time, Height,* or *Velocity* being assigned, one may readily find the other two. From them likewise is the *Doctrine of Projects* deducible, assuming the two following *Axioms*; *viz.*

1. *That a Body set a Moving, will Move on continually in a Right Line with an Equable Motion, unless some other Force or Impediment intervene, whereby it is Accelerated, or Retarded, or Deflected.*

2. *That a Body being agitated by Two Motions at a Time, does by their Compounded Forces, pass through the same Points, as it would do, were the Two Motions divided and Acted successively.* As for instance;

Fig. 172. Suppose a Body moved in the Line G F, from G to R, and there stopping, by another Impulse suppose it Moved in a Space of Time equal to the former, from R towards K to V; I say the Body shall pass through the point V, though these Two several Forces Acted both in the same Time.

Prop. V.] *The Motion of all Projects is in the Curve of a Parabola.*

*Demonstration.*] Let the Line G R F be the Line in which the *Project* is directed, and in which by the *First Axiom* it would Move Equal Spaces in Equal Times, were it not Deflected downwards by the Force of *Gravity.* Let G B be the Horizontal Line, and G C a Perpendicular thereto. Then the Line G R F, being divided into Equal Parts, answering to Equal Spaces of Time, let the Descents of the Project be laid down in Lines Parallel to G C, Proportioned as the *Squares* of the *Lines*, G S, G R, G L, G F, or as the *Squares* of the *Times*, from S to T, from R to V, from L to X, and from F to B, and draw the Lines T H, V D, X Y, B C, Parallel to G F: I say, the Points T, V, X, B, are Points in the *Curve* described by the *Project*, and that that *Curve* is a *Parabola.* By the *Second Axiom* they are Points in the *Curve*; and the Parts of the Descent G H, G D, G Y, G C, = to S T, R V, L X, F B, being as the Squares of the Times, (by the *Second Prop.*) that is, as the *Squares* of the *Ordinates* H T, D V, Y X, C B, Equal to G S, G R, G L, G F, the Spaces measured in those Times; and there being no other *Curve* but the *Parabola*, whose Parts of the *Diameter* are as the *Squares* of the *Ordinates*, it follows that the *Curve* described by a *Project* can be no other than a *Parabola*: And saying, as R V, the Descent in Time, to G R, or V D, the direct Motion in the same Time, so is V D, to a Third Proportional; that Third will be the Line called by all Writers of *Conicks*, the *Parameter* of the *Parabola* to the *Diameter* G C; which is always the same in *Projects* Cast with the same *Velocity*: And the *Velocity* being defined by the number of Feet, moved in a Second of Time, the *Parameter* will be found by dividing the Square of the *Velocity*, by 16 Feet 1 Inch, the Fall of a Body in the same Time.

Lemma.] *The Sine of the Double of any Arch, is equal to twice the Sine of that Arch into its Co-Sine, divided by Radius; and the Versed Sine of the Double of any Arch, is equal to the Square of the Sine thereof divided by Radius.*

Let the Arch B C be Double the Arch B F, and A the Center; Draw the Radii A B, A F, A C, and the Chord B D C, and let Fall B E, Perpendicular to A C, and the Angle E B C, will be equal to the Angle B A D, and the Triangle B C E, will be like to the Triangle A B D; wherefore it will be as A B to A D; so B C, or twice B D, to B E; that is, as *Radius* to *Co-Sine*, so *twice Sine* to *Sine of the Double Arch*; and as A B to B D, so twice B D or B C, to E C; that is as *Radius* to *Sine*, so *twice that Sine* to the *Versed Sine of the Double Arch*; which two *Analogies* resolved into Equations, are the *Propositions* contained in the *Lemma* to be proved. Fig. 173.

Prop. VI.] *The Horizontal Distances of Projections made with the same Velocity, at several Elevations of the Line of Direction, are as the Sines of the doubled Angles of Elevation.*

Let

Fig. 172. Let GB, the Horizontal Diſtance be $= z$, the Sine of the Angle of Elevation, FGB, be $= s$, its Co-Sine $= c$, Radius $= r$, and the Parameter $= p$. It will be as $c$ to $s$, ſo $z$ to $\frac{sz}{c} =$ FB $=$ GC, and by reaſon of the *Parabola* $\frac{psz}{c} =$ to the Square of CB, or GF. Now as $c$ to $r$, ſo is $z$ to $\frac{zr}{c} =$ GF, and its Square $\frac{zzrr}{cc}$ will be therefore $=$ to $\frac{psz}{c}$: which *Equation* Reduced will be $\frac{psc}{rr} = z$. But by the former *Lemma* $\frac{2sc}{r}$ is equal to the Sine of the Double Angle, whereof $s$ is the Sine: wherefore 'twill be as *Radius* to *Sine* of Double the Angle FGB, ſo is Half the *Parameter*, to the *Horizontal Range* or *Diſtance* ſought; and at the ſeveral *Elevations*, the *Ranges are as the Sines of the Double Angles of Elevation*. Q. E. D.

*Coroll.*] Hence it follows, That *Half the Parameter is the greateſt Random, and that that happens at the Elevation of* 45°. *The Sine of whoſe Double is Radius.* Likewiſe, That *the Ranges equally Diſtant above and below* 45°. *are equal, as are the Sines of all doubled Arches, to the Sines of their doubled Complements.*

Prop. VII.] *The Altitudes of Projections made with the ſame Velocity, at ſeveral Elevations, are as the Verſed Sines of the doubled Angles of Elevation.*

As $c$ is to $s$, ſo is $\frac{psc}{rr} =$ GB, to $\frac{pss}{rr} =$ BF, and VK $=$ RV $= \frac{1}{4}$ BF, the *Altitude* of the *Projection* $= \frac{pss}{4rr}$. Now by the foregoing *Lemma* $\frac{2ss}{r} =$ to the *Verſed Sine* of the *Double Angle*, and therefore it will be as *Radius* to *Verſed Sine* of Double the Angle FGB, ſo $\frac{1}{8}$ of *Parameter* to the *Height* of the *Projection* VK; and ſo thoſe *Heights* at ſeveral *Elevations* are as the ſaid *Verſed Sines*. Q. E. D.

*Coroll.*] From hence it is plain, That *the greateſt Altitude of the Perpendicular Projection is a* 4*th of Parameter, or half the greateſt Horizontal Range*: The *Verſed Sine* of 180 Degrees being $= 2r$.

Prop. VIII.] *The Lines* GF, *or times of the Flight of a Project caſt with the ſame Degree of Velocity at Different Elevations, are as the Sines of the Elevations.*

As

As $c$ is to $r$, ſo is $\frac{psc}{rr}$ = GB (by the 6. *Prop.*) to $\frac{ps}{r}$ = GF, that is as *Radius* to the *Sine* of *Elevation*, ſo the *Parameter* to the *Line* GF; ſo the *Lines* GF are as the *Sines* of *Elevation*, and the *Times* are proportional to thoſe *Lines*; wherefore the *Times* are as the *Sines of Elevation*: *Ergo conſtat Propoſitio.*

Prop. IX. Prob. 1.] *A Projection being made, as you pleaſe, having the Diſtance and Altitude, or Deſcent of an Object, through which the Project paſſes, together with the Angle of Elevation of the Line of Direction, to find the Parameter and Velocity*; that is, (having the Angle FGB,) GM, and MX.

*Solution.*] As Radius to Secant of FGB, ſo GM the Diſtance given, to GL; and as Radius to Tangent of FGB, ſo GM to LM. Then LM — MX in *Heights*, or + MX in *Deſcents*; or elſe MX — ML, if the *Direction* be *Below* the *Horizontal-Line*, is the *Fall* in the *Time* that the Direct Impulſe given in G, would have carried the Project from G to L = LX = GY; then by reaſon of the *Parabola*; as LX, or GY, is to GL or YX; ſo is GL, to the *Parameter* ſought. To find the *Velocity* of the *Impulſe*, by *Prop.* 2. and 4. find the *Time* in Seconds that a Body would *Fall* the Space LX, and by that dividing the Line GL, the Quote will be the *Velocity*, or *Space Moved* in a Second Sought, which is always a *Mean Proportional* between the *Parameter* and 16 Feet, 1 Inch. Fig. 172.

Prop. X. Prob. 2.] *Having the Parameter, Horizontal Diſtance, and Height or Deſcent of an Object, to find the Elevations of the Lines of Direction neceſſary to Hit the given Object*; that is, having GM, MX, and the greateſt Random equal to half the Parameter; to find the Angles FGB.

Let the Tangent of the Angle ſought be = $t$, the Horizontal Diſtance GM = $b$, the Altitude of the Object MX = $h$, the Parameter = $p$, and Radius = $r$, and it will be, as $r$ to $t$, ſo $b$ to $\frac{tb}{r}$ = ML; and $\frac{tb}{r} \mp h$ {in Aſcents / in Deſcents} = LX, and $\frac{ptb}{r} \mp ph$ = GL $q$. = XY $q$. *ratione Parabolæ*; but $bb + \frac{ttbb}{rr}$ = GL $q$. (47. 1. *Euclid.*) Wherefore $\frac{ptb}{r} \mp ph = bb + \frac{ttbb}{rr}$; which Equation tranſpoſed, is $\frac{ttbb}{rr} = \frac{ptb}{r} \mp ph - bb$; divided by $bb$, is $\frac{tt}{rr} = \frac{pt}{br} \mp \frac{ph}{bb} - 1$. This Equation ſhews the Queſtion to have two Anſwers, and the Roots thereof are $\frac{t}{r}$

$= \frac{p}{2b} \mp \sqrt{\frac{pp \mp 4ph}{4bb} - 1}$; from which I derive the following Rule. Divide half the *Parameter* by the *Horizontal Distance*, and keep the Quote, *viz.* $\frac{p}{2b}$; then say, as Square of the *Distance* given to the half *Parameter*, so half *Parameter* $\left\{ \begin{matrix} - \\ + \end{matrix} \right\}$ double $\left\{ \begin{matrix} \text{Height} \\ \text{Descent} \end{matrix} \right\}$ to the Square of a *Secant* $= \frac{pp \mp 4ph}{4bb}$, the *Tangent* answering to that *Secant* will be $\sqrt{\frac{pp \mp 4ph}{4bb} - 1}$, or $rr$: so then the Sum and Difference of the afore-found *Quote* and this *Tangent*, will be the Roots of the *Equation*, and the *Tangents* of the *Elevations* sought.

*Note* here, That in *Descents*, if the *Tangent* exceed the *Quote*, as it does when $ph$ is more than $bb$, the *Direction* of the Lower *Elevation* will be below the *Horizon*, and if $ph = bb$, it must be *Directed Horizontal*, and the *Tangent* of the upper *Elevation* will be $\frac{pr}{b}$: *Note* likewise, That if $4bb + 4ph$ in *Ascents*, or $4bb - 4ph$ in *Descents*, be equal to $pp$, there is but one *Elevation* that can *Hit* the *Object*, and its *Tangent* is $\frac{pr}{2b}$; and if $4bb + 4ph$ in *Ascents*, or $4bb - 4ph$ in *Descents*, do exceed $pp$, the *Object* is without the *Reach* of a *Project* cast with that *Velocity*, and so the thing impossible.

From this *Equation* $4bb \mp 4ph = pp$, are determined the utmost *Limits* of the *Reach* of any *Project*, and the Figure assigned, wherein are all the Heights upon each Horizontal Distance beyond which it cannot pass; for by Reduction of that *Equation*, $h$ will be found $= \frac{1}{4}p - \frac{bb}{p}$ in *Heights*, and $\frac{bb}{p} - \frac{1}{4}p$ in *Descents*; from whence it follows, that all the Points $h$ are in the *Curve* of the *Parabola*, whose *Focus* is the Point from whence the *Project* is cast, and whose *Latus Rectum*, or *Parameter ad Axem* is $= p$. Likewise from the same *Equation* may the least *Parameter* or *Velocity* be found capable to Reach the Object proposed; for $bb = \frac{1}{4}pp \mp ph$ being reduced, $\frac{1}{2}p$ will be $= \sqrt{bb + hh} \left\{ \begin{matrix} + h \text{ in } Ascents \\ - h \text{ in } Descents \end{matrix} \right\}$ which is the *Horizontal* Range at 45°. that would just Reach the *Object*, and the *Elevation* requi-

requisite will be easily had; for dividing the so found *Semi-Parameter* by the *Horizontal Distance* given $b$, the Quote into *Radius* will be the *Tangent* of the *Elevation* sought.

But if a *Geometrical Construction* of this *Problem* be required; I think I have one, that is as easie as any can be expected, which I Deduce from the foregoing *Analytical Solution*, *viz.* $\frac{t}{r} = \frac{p}{2b} \pm \sqrt{\frac{\frac{1}{4}pp \pm ph - bb}{bb}}$, and 'tis this; Having made the Right Angle L D A, make D A, DF = $p$, or greatest *Range*, D G = $b$, the *Horizontal Distance*, and D B, D C = $h$, the Perpendicular *Height* of the Object; and draw G B, and make D E = thereto. Then with the Radius A C, and Center E, sweep an Arch, which if the thing be possible, will Intersect the Line A D, in H; and the Line D H, being laid both ways from F, will give the Points K, and L, to which draw the Lines G L, G K; I say, the Angles L G D, K G D, are the *Elevations* required for *Hitting* the *Object*, B. But *Note*, That if B, be below the Horizon, its *Descent* D C = D B, must be laid upon A, so as to have A C = to A D + D C. *Note* likewise, That if in *Descents*, D H, be greater than F D, and so K, fall below D, the Angle K G D, shall be the *Depression* below the Horizon. Fig. 174.

When I gave the preceding Solution of this Problem, *viz. To Hit an Object above or below the Horizontal Line, with the greatest Certainty and least Force*, I was not aware, that the *Elevation* there sought did constantly *Bisect* the Angle between the Perpendicular and the Object, as is Demonstrated from the Difference and Sum of the *Tangent* and *Secant* of any Arch, being always equal to the *Tangent* and *Co-Tangent* of the half Complement thereof to a Quadrant. But having discovered this, I think nothing can be more compendious, or bid fairer to compleat the *Art* of *Gunnery*, it being as easie to Shoot with a *Mortar* at any Object on demand, as if it were on the Level; neither is there need of any Computation, but only simply laying the Gun to pass, in the middle Line between the Zenith and the Object, and giving it its due *Charge*. Nor is there any great need of Instruments for this purpose: For, If the *Muzzle* of the *Mortar* be turn'd truly Square to the Bore of the Piece, as it usually is, or ought to be, a piece of Looking-Glass Plate applied Parallel to the *Muzzle*, will, by its Reflection, give the true Position of the Piece; the *Bombardier* having no more to do, but to look Perpendicularly down on the Looking-Glass, alongst a small Thread with a Plumbet, and to Raise or Depress the *Elevation* of the *Piece*, till the Object appear Reflected on the same Point of the *Speculum* on which the Plumbet falls; for the Angle of *Incidence* and *Reflection* being *Equal*, in this case a Line at Right Angles to the *Speculum*, as is the *Axis* of the *Chase* of the Piece, will *Bisect* the Angle between the Perpendicular and the Object, according as our *Proposition* requires. n. 216. p. 69.

Prop. XI. Prob. 3.] *A Shot being made on an Inclined Plain, having the Horizontal Distance of the Object it strikes, with the Elevation of the Piece, and the Angle at the Gun between the Object and the Perpendicular, to find the greatest Horizontal Range of that Piece, laden with the same Charge; that is, half the* Latus Rectum *of all the* Parabolæ *made with the same* Impetus.

Take Half the Distance of the Object from the *Nadir*, and take the Difference of the given Elevation from that Half; the *Versed Sine* of twice that Difference Substract from the *Versed Sine* of the Distance of the Object from the *Zenith*: Then shall the Difference of those *Versed Sines* be to the Sine of the Distance of the Object from the *Zenith*, as the Horizontal Distance of the Object struck to the greatest Horizontal Range at 45°.

Prop. XII. Prob. 4.] *Having the greatest Horizontal Range of a Gun, the Horizontal Distance and Angle of Inclination of an Object to the Perpendicular, to find the two Elevations necessary to strike that Object.*

Halve the Distance of the Object from the *Nadir*, this Half is always equal to the Half Sum of the two *Elevations* we seek. Then say, As the Greatest Horizontal Range, is to the Horizontal Distance of the Object; so is the Sine of the Angle of Inclination, or Distance of the Object from the Perpendicular, to a 4th Proportional; which 4th being Substracted from the *Versed Sine* of the Distance of the Object from the *Zenith*, leaves the *Versed Sine* of the Difference of the *Elevations* sought; which *Elevations* are therefore had, by Adding and Substracting that Half Difference to, and from, the aforesaid Half Sum.

n. 179. p. 18.] Prop. XIII.] *To determine the Force or Velocity of a Project, in every Point of the Curve it describes.*

To do this, we need no other *Præcognita*, but only the *Third Proposition*, viz. *That the Velocity of Falling Bodies, is double to that which in the same Time would have described the Space fallen by an Equable Motion*: For the *Velocity* of a *Project* is compounded of the constant equal *Velocity* of the impressed Motion, and the *Velocity* of the Fall, under a given Angle, *viz.* The Com-
Fig. 172. plement of the *Elevation*: For instance, In the Time wherein a *Project* would Move from G to L, it Descends from L to X, and by the *Third Proposition* has Acquired a *Velocity*, which in that Time would have carried it by an Equable Motion from L to Z, or twice the Descent L X; and drawing the Line G Z, I say the *Velocity* in the Point X, compounded of the *Velocities* G L, and L Z, under the Angle G L Z, is to the *Velocity* imprest in the Point G, as G Z, is to G L; this follows from our *Second Axiom*; and by the 20th and 21th *Prop. Lib.* 1. *Conic. Midorgii*, X O, Parallel and Equal to G Z, shall touch the *Parabola* in the Point X. So that the *Velocities* in the several Points, are as the Lengths of the *Tangents* to the *Parabola* in those Points, intercepted between any two Diameters: And these again are as the *Secants* of the Angles, which those *Tangents* continued make with the Horizontal Line G B. From what is here laid down, may the comparative Force of

of a Shot in any two Points of the Curve, be either Geometrically or Arithmetically discovered.

*Coroll.*] From hence it follows, That the Force of a Shot is always least at V, or the *Vertex* of the *Parabola*, and that at Equal Distances therefrom, as at T and X, G and B, its Force is always Equal, and that the least Force in V, is to that in G and B, as *Radius* to the *Secant* of the Angle of *Elevation*, F G B.

The *Tenth Proposition* contains a *Problem*, untouch'd by *Torricellius*, which is of the Greatest Use in Gunnery, and for the sake of which this Discourse was principally intended. It was first Solved by Mr. *Anderson*, in his Book of the *Genuine Use and Effects of the Gun*, Printed in the Year 1674. but his *Solution* required so much Calculation, that it put me upon Search, whether it might not be done more easily; and thereupon in the Year 1678. I found out the *Rule* I now Publish, and from it the Geometrical Construction: Since which time, there has a large Treatise of this Subject, Intituled, *L'Art de Jetter les Bombes*, been Publish'd in *France* by M. *Blondel*, wherein he gives the *Solutions* of this *Problem*, by Messieurs *Bout*, *Romer*, and *de la Hire*; But none of them are the same with mine, or in my Opinion more easie.

It was formerly the Opinion of those concerned in *Artillery*, That there was a certain requisite of *Powder* for each *Gun*, and that in *Mortars*, where there Distance was to be varied, it must be done by giving a greater or lesser *Elevation* to the *Piece*. But now our Later Experience has taught us, That the same thing may be more Certainly and Readily performed, by *Increasing and Diminishing the Quantity of Powder*, whether regard be had to the Execution to be done, or to the Charge of doing it. For when *Bombs* are Discharged with great *Elevations* of the *Mortar*, they fall too Perpendicular, and Bury themselves too deep in the Ground, to do all that Damage they might, if they came more Oblique, and Broke upon or near the Surface of the Earth; which is a thing acknowledged by the *Besieged* in all *Towns*, who Unpave their Streets to let the *Bombs* Bury themselves, and thereby stifle the force of their Splinters. A *Second Convenience* is, That at the Extream *Elevation*, the *Gunner* is not obliged to be so curious in the Direction of his *Piece*, but it will suffice to be within a Degree or two of the Truth; whereas in the other Method of *Shooting*, he ought to be very curious. But a *Third* and no less considerable *Advantage* is, in the saving of the *King's Powder*, which in so great and so numerous *Discharges*, as we have lately seen, must needs amount to a considerable Value. And for *Sea Mortars* it is scarce Practicable otherwise to use them, where the *Agitation* of the Sea continually Changes the *Direction* of the *Mortar*, and would render the Shot very uncertain, were it not that they are placed about 45°. *Elevation*, where several Degrees above or under makes very little difference in the *Effect*. n. 216. p. 68.

It only remains by Good and Valid *Experiments* to be assured of the *Force* of *Gun-Powder*; How to make and conserve it equal; And to know the Effect thereof in each *Peice*; that is, How far differing *Charges* will cast the same Shot out of it; which may most conveniently be Engraven on the outside thereof, as a standing Direction to all *Gunners*, who shall from thence *Ibid.* p. 70.

 forward

forward have occasion to use that *Piece*: And were this Matter well ascertained, it might be worth the while to make all *Mortars* of the like *Diameter*, as near as may be alike in *Length* of *Chase*, *Weight*, *Chamber*, and all other *Circumstances*.

n. 179. p. 19. Now the foregoing Rules would be Rigidly true, were it not for the *Opposition* of the *Medium*, whereby not only the Direct Imprest Motion is continually Retarded, but likewise the Increase of the Velocity of the Fall, so that the Spaces described thereby, are not Exactly as the *Squares* of the *Times*: but what this *Opposition* of the *Air* is, against several *Velocities*, *Bulks*, and *Weights*, is not so easie to determine. 'Tis certain, That the *Weight* of the *Air* to that of *Water*, is nearly as 1 to 800. whence the *Weight* thereof, to that of any *Project* is given; 'tis very likely, that to the same *Velocity* and *Magnitude*, but of Different Matter, the *Opposition* should be *reciprocally* as the *Weights* of the *Shot*; as likewise that to *Shot* of the same *Velocity* and *Matter*, but of Different *Sizes*, it should be as the *Diameters reciprocally*: whence generally the *Opposition* to *Shot* with the same *Velocity*, but of differing Diameters, and Materials, should be as their *Specifick Gravities* into their *Diameters reciprocally*; but whether the Opposition, to differing *Velocities* of the same *Shot*, be as the *Squares* of those *Velocities*, or as the *Velocities* themselves, or otherwise, is yet a harder Question. However it be, 'tis certain, That in Large Shot of Metal, whose Weight many thousand times surpasses that of the *Air*, and whose Force is very great in Proportion to the Surface wherewith they press thereon, this *Opposition* is scarce *Discernable*: For by several *Experiments*, made with all Care and Circumspection with a *Mortar-Piece*, extraordinary well fixt to the Earth on purpose, which carried a Solid Brass Shot of $4\frac{1}{2}$ Inches Diameter, and of about 14 Pound weight, the Ranges above and below 45°. were found nearly equal; if there were any Difference, the under Ranges went rather the farthest, but those Differences were usually less than the Errors committed in Ordinary Practice, by the unequal goodness and dryness of the same sort of *Powder*, by the Unfitness of the Shot to the *Bore*, and by the Looseness of the *Carriage*. In a smaller Brass-Shot of about an Inch and half Diameter, cast by a *Cross Bow*, which Ranged it at most about 400 Foot, the Force being much more equal than in the *Mortar-Piece*, this Difference was found more curiously, and constantly, and most evidently, the under *Ranges* outwent the upper. From which Tryals I conclude, That altho' in Small and Light Shot, the *Opposition* of the *Air*, ought and must be accounted for; yet in Shooting of Great and Weighty *Bombs*, there need be very little or no Allowance made; and so these Rules may be put in Practice to all Intents and Purposes, as if this *Impediment* were absolutely removed.

*The Measure of the Airs Resistance to Bodies, moved in it; by* Dr. Wallis. *n. 186. p. 269.*

IX. 1. In order to Compute the *Resistance* of the *Air* to all *Projects*, I first Premise this *Lemma*, (as the most Rational that doth occur, for my first footing,) *That* (supposing other things equal) *the Resistance is proportional to the Celerity*. For in a double *Celerity*, there is to be removed (in the same time) twice as much *Air*, (which is a double *Impediment*;) in a treble, *thrice* as much; and so in other Proportions.

2. Sup-

2. Suppose we then the Force Impressed (and consequently the *Celerity*, if there were no *Resistance*) as 1; the *Resistance* as $r$, (which must be less than the Force, or else the Force would not prevail over the Impediment, to create a *Motion*.) And therefore the Effective Force at a first *Moment*, is to be reputed as $1 - r$: That is, so much as the Force Impressed, is more than the *Impediment* or *Resistance*.

3. Be it, as $1 - r$ to 1, so 1 to $m$, (which $m$ is therefore greater than 1.)

4. And therefore the effective Force, (and consequently the *Celerity*) as to a first *Moment*, is to be $\frac{1}{m}$ of what it would be, had there been no *Resistance*.

5. This $\frac{1}{m}$ is also the remaining Force after such first Moment; and this remaining Force is (for the same reason) to be proportionably abated as to a Second Moment: That is, we are to take $\frac{1}{m}$ thereof, that is, $\frac{1}{mm}$ of the Impressed Force. And for a third Moment (at equal Distance of Time) $\frac{1}{m\,m\,m}$; for a fourth $\frac{1}{m^4}$; and so onward infinitely.

6. Because the Length dispatched (in Equal Times) is proportional to the *Celerities*; the Lines of Motion (answering to those Equal Times) are to be as $\frac{1}{m}$, $\frac{1}{m^2}$, $\frac{1}{m^3}$, $\frac{1}{m^4}$, &c. of what they would have been in the same Times, had there been no *Resistance*.

7. This therefore is a *Geometrical Progression*; and (because of $m$ greater than 1) continually Decreasing.

8. This Decreasing Progression Infinitely continued, (determining in the same Point of Rest, where the Motion is supposed to expire) is yet of a finite Magnitude, and equal to $\frac{1}{m-1}$ of what it would have been in so much time, if there had been no *Resistance*: As is demonstrated in my *Algebra*, *Chap.* 95. *Prop.* 8. For (as I have elsewhere Demonstrated) the *Sum* or Aggregate of a *Geometrical Progression* is $\frac{VR - A}{R - 1}$, (supposing V the greatest Term, A the least, and R the Common Multiplier.) That is, $\frac{VR}{R-1} - \frac{A}{R-1}$. Now in the present Case, (supposing the Progression

Infinitely

Infinitely continued) the leaſt Term A, becomes Infinitely Small, or $= 0$. And conſequently $\frac{A}{R-1}$ doth alſo Vaniſh, and thereby the Aggregate becomes $= \frac{VR}{R-1}$. That is, (as will appear by Dividing V R by $R-1$;) $V + \frac{V}{R} + \frac{V}{RR} + \frac{V}{R^3} +$, &c. $= \frac{VR}{R-1}$; (ſuppoſing the Progreſſion to begin at $V = 1$.) That is, (dividing all by R, that ſo the Progreſſion may begin at $\frac{V}{R} = \frac{1}{m}$:) $\frac{V}{R-1} = \frac{V}{R} + \frac{V}{RR} + \frac{V}{R^3}$ + &c. That is in our preſent Caſe, (becauſe of $V = 1$, and $R = m$) $\frac{1}{m} + \frac{1}{m\,m} + \frac{1}{m^3}$, &c. $= \frac{1}{m-1}$. That is, (putting $n = m - 1$;) $\frac{1}{n}$, of what it would have been, if there had been no *Reſiſtance*.

9. This Infinite Progreſſion is fitly expreſſed by an *Ordinate* in the *Exterior Hyperbola*, Parallel to one of the *Aſymptotes*; and the ſeveral Members of that, by the ſeveral Members of this, cut in *Continual Proportion*. As is there Demonſtrated at *Prop.* 15. For let S H, be an *Hyperbola* between the *Aſymptotes*, A B, A F: And let the *Ordinate* D H, (in the *Exterior Hyperbola*, Parallel to A F) repreſent the Impreſſed Force undiminiſhed; or the Line to be deſcribed in ſuch Time, by a *Celerity* anſwerable to ſuch Undiminiſhed Force. And let B S (a like *Ordinate*) be $\frac{1}{m}$ thereof, which therefore, being leſs than D H, (as being Equal to a part of it) will be further than it from A F. In A B, (which I put $= 1$,) let B $d$, be ſuch a part thereof, as is B S of D H. Now becauſe (as is well known) all the Inſcribed Parallelograms, in the *Exterior Hyperbola*, A S, A H, &c. are Equal; and therefore their Sides Reciprocal: Therefore as A $d = 1 - \frac{1}{m}$, (ſuppoſing B $d$, to be taken from B toward A,) to A B $= 1$, (or as $m - 1$ to $m$:) ſo is B S $= \frac{1}{m}$ D H to $d\,h$, which is therefore Equal to $\frac{1}{m-1}$ of D H; that is, (as will appear by Dividing 1, by $m - 1$;) to $\frac{1}{m} + \frac{1}{m\,m} + \frac{1}{m^3}$ &c. of D H.

Fig. 175.

Or

Or if B $d$ be taken beyond B; then as A $d = 1 + \frac{1}{m}$, to A B $= 1$, or as $m + 1$ to $m$, so is $\frac{1}{m}$ D H, to $d\,h$, which is therefore Equal to $\frac{1}{m+1}$ D H; that is, (as will appear by like Dividing of 1 by $m + 1$;) $=$ to $\frac{1}{m} - \frac{1}{mm} + \frac{1}{m^3} -$, &c. of D H.

10. Let such *Ordinate* $d\,h$, or (Equal to it in the *Asimptote*) A F, be so Divided in L, M, N, &c. (by Perpendiculars cutting the *Hyperbola* in $l, m, n$, &c.) as that F L, L M, M N, be as $\frac{1}{m}$, $\frac{1}{m\,m}$, $\frac{1}{m^3}$, &c. That is, so continually Decreasing, as that each Antecedent be to its Consequent, as 1 to $\frac{1}{m}$, or as $m$ to 1. *Fig.* 176.

11. This is done by taking A F, A L, A N, &c. in such Proportion. For, of Continual Proportionals the Differences are also Continually Proportional, and in the same Proportion. For let A, B, C, D, &c. be such Proportionals; and their Differences, $a$, $b$, $c$, &c. That is, A $-$ B $= a$, B $-$ C $= b$, C $-$ D $= c$, &c.

Then because, A, B, C, D, &c. are in Continual Proportion;
That is, A : B : : B : C : : C : D : :, &c.
And Dividing A $-$ B : B : : B $-$ C : C : : C $-$ D : D : :, &c.
That is, $a$ : B : : $b$ : C : : $d$ : D : :, &c.
And Alternly, $a$. $b$. $c$. &c. : : B. C. D. &c. : : A. B. C. &c.
That is, In Continual Proportion, as A to B, or as $m$ to 1.

12. This being done; the *Hyperbolick Spaces* F $l$, L $m$, M $n$, &c. are Equal, as is Demonstrated by *Gregory San-Vincent*; and as such is commonly admitted.

13. So that F $l$, L $m$, M $n$, &c. may fitly represent Equal Times, in which are dispatched Unequal Lengths, represented by F L, L M, M N, &c.

14. And because they are in Number Infinite, (though Equal to a Finite Magnitude) the Duration is Infinite: and consequently the Impressed Force, and Motion thence Arising, never to be wholly Extinguished (without some further Impediment) but perpetually Approaching to A, in the Nature of *Asymptotes*.

15. The Spaces F $l$, F $m$, F $n$, &c. are therefore as *Logarithms* (in *Arithmetical Progression* Increasing) answering to the Lines, A F, A L, A M, &c. or to F L, L M, M N, &c. in *Geometrical Progression* Decreasing.

16. Be-

16. Becauſe F L, L M, M N, &c. are as $\frac{1}{m}$, $\frac{1}{mm}$, $\frac{1}{m^3}$, &c. (Infinitely) Terminated at A; therefore (by *Prop.* 8.) their Aggregate F A, or *d h*, is to D H, (ſo much Length as would have been diſpatched in the ſame Time, by ſuch Impreſſed Force Undiminiſhed) as 1 to $m - 1 = n$.

17. If therefore we take, as 1 to *n*, ſo A F to D H; this will repreſent the Length to be Diſpatched, in the ſame Time, by ſuch Undiminiſhed Force.

18. And if ſuch D H, be ſuppoſed to be Divided into Equal Parts Innumerable (and therefore Infinitely ſmall;) theſe anſwer to thoſe (as many) Parts Unequal in F A, or *h d*.

19. But, what is the Proportion of *r* to 1, or (which depends on it) of $1 - r$ to 1, or 1 to *m*; remains to be enquired by *Experiment.*

20. If the Progreſſion be not Infinitely Continued; but End (ſuppoſe) at N, and its leaſt Term be A = M N: then out of $\frac{V}{R-1} = \frac{1}{m} + \frac{1}{mm} + \frac{1}{m^3} +$ &c. is to be Subducted $\frac{A}{R-1}$, (as at *Prop.* 8.) that is, (as by Diviſion will appear) $\frac{A}{R} + \frac{A}{R^2} + \frac{A}{R^3} +$ &c. That is, (in our preſent Caſe) $\frac{a}{m} + \frac{a}{mm} + \frac{a}{m^3} +$ &c. And ſo the Aggregate will be $\frac{1-a}{m} + \frac{1-a}{mm} + \frac{1-a}{m^3} + \&c. = \frac{1-a}{n}$.

And thus as to the Line of *Projection*, in which (Secluding the *Reſiſtance*) the Motion is reputed Uniform; diſpatching Equal Lengths at Equal Times. Conſider we next the Line of Deſcent.

21. In the Deſcent of Heavy Bodies, it is ſuppoſed, that to each Moment of Time, there is ſuperadded a new Impulſe of *Gravity* to what was before: And each of theſe, Secluding the Conſideration of the *Airs Reſiſtance*, to proceed equally (from their ſeveral beginnings) through the ſucceeding Moments. As, (in the Erect Lines) I I I I &c. I I I &c. I I &c. I &c. and ſo continually, as in the Line of *Projection*.

I
I I
I I I
I I I I
&c.&c.&c.&c.

22. Hence ariſeth (in the Tranſverſe Lines) for the firſt Moment 1, for the ſecond 1 + 1, for the third 1 + 1 + 1, and ſo forth in *Arithmetical Progreſſion.* As are the *Ordinates* in a Triangle at Equal Diſtance.

23. And ſuch are the continual Increments of the Diameter, or of the *Ordinates* in the *Exterior Parabola*, anſwering to the interior *Ordinates*, or Segments of the *Tangent*, equally Increaſing; As is known, and commonly admitted.

24. If we take in the Confideration of the *Air's Refiftance*; we are then for each of thefe Equal Progreffions, to Subftitute a Decreafing *Progreffion Geometrical*; in like manner (and for the fame Reafons) as in the Line of Projection.

25. Hence arifeth for the firft Moment $\frac{1}{m}$; for the fecond $\frac{1}{m} + \frac{1}{m^2}$; for the third $\frac{1}{m} + \frac{1}{m^2} + \frac{1}{m^3}$; *&c.* And fuch is therefore the Defcent of a Heavy Body falling by its own Weight. The feveral Impulfes of Gravity being fuppofed Equal.

26. That is, as F L, F M, F N, *&c.* in the Line of Defcent, anfwering to F L, L M, M N, *&c.* in the Line of Projection.

| | | | |
|---|---|---|---|
| $\frac{1}{m}$ | | | |
| $\frac{1}{m^2}$ | $\frac{1}{m}$ | | |
| $\frac{1}{m^3}$ | $\frac{1}{m^2}$ | $\frac{1}{m}$ | |
| $\frac{1}{m^4}$ | $\frac{1}{m^3}$ | $\frac{1}{m^2}$ | $\frac{1}{m}$ |
| *&c.* | *&c.* | *&c.* | *&c.* |

Fig. 176.

27. But though the Progreffions for the Line of Projection, are like to each of thofe many in the Line of Defcent: it is not to be thence inferred, that therefore $\frac{1}{m}$ in the one, is equal to $\frac{1}{m}$ in the other: But in the Line of Projection (*fuppofe*) $\frac{1}{m} f$, (fuch a part of the Force Impreffed, and a Celerity anfwerable:) in the Line of Defcent $\frac{1}{m} g$, (fuch a part of the Impulfe of Gravity.)

28. Thofe for the Line of Defcent (of the fame Body) are all equal each to other: Becaufe $g$, (the new Impulfe of Gravity) in each Moment is fuppofed to be the fame.

29. But what is the Proportion of $f$ to $g$, (that is, of the Force Impreffed, to the Impulfe of Gravity, in each Body) remains to be enquired by *Experiment.*

30. This Proportion being found as to one Known Force; the fame is thence Known as to any other Force (whofe Proportion to this is given) in the fame *Uniform Medium.*

31. And this being Known as to one *Medium*, the fame is thence Known as to any other *Medium*, the Proportion of whofe Refiftance to that of this is Known.

32. If a Heavy Body be Projected Downward in a Pendicular Line, it Defcends therefore at the rate $\frac{1}{m}$, $\frac{1}{m^2}$, $\frac{1}{m^3}$, *&c.* of $f$ (the Impreffed Force) increafed by $\frac{1}{m}$, $\frac{1}{m} + \frac{1}{m^2}$, $\frac{1}{m} + \frac{1}{m^2} + \frac{1}{m^3}$, *&c.* of $g$, the Impulfe of Gravity: (by *Prop.* 5. and *P.* 25.) becaufe both Forces are here United.

33. If in a Perpendicular Projection Upwards; it Ascends in the Rate of the former, Abated by that of the latter. Because here the Impulse of Gravity is contrary to the Force Impressed.

34. When therefore this latter (continually Increasing) becomes Equal to that former, (continually Decreasing) it then ceaseth to Ascend; and doth thenceforth Descend at the Rate wherein the latter continually Exceeds the former.

35. In an Horizontal or Oblique Projection: If to a Tangent whose Increments are as F L, L M, M N, *&c.* that is, as $\frac{1}{m} f$, *&c.* be fitted Ordinates (at a given Angle) whose Increments are as F L, F M, F N, *&c.* that is, as $\frac{1}{m} g$, *&c.* The Curve answering to the Compound of these Motions, is that wherein the Project is to move.

36. This Curve (being hitherto without a Name) may be called *Linea Projectorum*, the Line of Projects, or things Projected; which resembles a *Parabola Deformed.*

37. The *Celerity* and *Tendency*, as to each Point of this Line, is determined by a Tangent at that Point.

38. And that against which it makes the greatest Stroke or Percussion, is that which (at that point) is at Right Angles to that Tangent.

39. If the Projection (at *P.* 25.) be not Infinitely Continued, but Terminate (suppose) at N, so that the last Term in the first Column or Series Erect, be $a$; and consequently in the second, $m a$; in the third $m m a$, *&c.* (each Series having one Term fewer than that before it:) then (for the same Reasons as at *P.* 20.) the *Aggregates* of the several Columns (or Erect Series) will be $\frac{1 - a}{n}$, $\frac{1 - ma}{n}$, $\frac{1 - mma}{n}$; and so forth, till (the Multiple of $a$ becoming $= 1$,) the Progression expire.

40. Now all the Abatements here, $a$, $m a$, $m m a$, *&c.* are the same with the Terms of the first Column taken backward. For $a$ is the last, $m a$ the next before it; and so of the rest.

41. And the Aggregate of all the Numerators is so many times 1, as is the number of Terms (suppose $t$,) wanting the first Column; that is, $t - \frac{1 - a}{n}$, or $\frac{nt - 1 + a}{n}$; and this again divided by the common Denominator $n$, becomes $\frac{nt - 1 + a}{nn}$. And therefore, $\frac{nt - 1 + a}{nn} g$, is the Line of Descent by its own Gravity.

42. If

42. If therefore this be Added to a Projecting Force downward in a Perpendicular, or Subducted from such Projecting Force upward; that is, to or from $\frac{1-a}{n}f$: The Descent in the first Case, will be $\frac{1-a}{n}f + \frac{nt-1+a}{nn}g$; and the Ascent in the other case $\frac{1-a}{n}f - \frac{nt-1+a}{nn}g$. And in this latter Case, when the Ablative part becomes equal to the Positive part, the Ascent is at the highest: and thenceforth (the Ablative part exceeding the Positive) it will Descend.

43. In an Horizontal or Oblique Projection having taken $\frac{1-a}{n}f$, in the Line of Projection, and thence (at the Angle given) $\frac{nt-1+a}{nn}g$, in the Line of Descent; the Point in the Curve answering to these, is the place of the Project answering to that Moment.

44. I am aware of some Objections to be made, whether to some Points of the Process, or to some of the Suppositions. But I saw not well how to to wave it, without making the Computation much more perplexed. And in a matter so Nice, and which must depend upon Physical Observations, 'twill be hard to attain such Accuracy, as not to stand in need of some Allowances.

45. Somewhat might have been further Added, to direct the *Experiments* suggested at *P.* 19. and 29. But that may be done at Leisure, after Deliberation had, which way to attempt the *Experiment*.

46. The like is to be said of the different Resistance which different Bodies may meet with in the Same *Medium*, according to their different Gravities, (extensively or intensively considered) and their different Figures and Positions in Motion. Whereof hitherto we have taken no account; but supposed them, as to all these, to be Alike and Equal.

47. The Computation (in *P.* 39, 40, 41.) may, if that be also desired, be *Fig.* 176, 177.
thus represented by Lines and Spaces. The Ablatives *a*, *m a*, *m m a*, *&c.* (being the same with the first Column taken backward) are fitly represented by the Segments of N F, (beginning at N.) and therefore by Parallelograms on these Bases, assuming the common Height of *F h*, or N Q: the Aggregate of which is N *h*, or F Q, and so many times 1, by so many equal Spaces, on the same Bases, between the same Parallels terminated at the *Hyperbola*; The Aggregate of which is *h* F N Q *n*. From whence if we Subduct the Aggregate of Ablatives F Q; the remaining Trilinear *h* Q *n*, represents the Descent.

48. If to this of Gravity, be joined a Projecting Force; which is to the Impulse of Gravity, as *h*K to *h*F, (be it greater, less, or equal) taken in the same Line; the same Parallels determine Proportional Parallelograms, whose Aggregate is K Q.

49. And therefore, if this be a Perpendicular Projection downwards; then *b* K *k n* (the Sum of this with the former) repreſents the Deſcent.

50. If it be a Perpendicular Upwards; then the difference of theſe two repreſents the Motion; which ſo long as K Q is the greater, is Aſcendent: but Deſcendent, when *b* Q *n* becomes greater; and it is then at the Higheſt when they be equal.

51. If the Projection be not in the ſame Perpendicular, (but Horizontal or Oblique) then K Q repreſents the Tangent of the Curve; and *b* Q *n*, the Ordinates to that Tangent, at the given Angle.

52. But the Computation before given, I take to be of better uſe than this Repreſentation in Figure. Becauſe in ſuch Mathematical Enquiries, I chooſe to ſeparate (as much as may be) what purely concerns Proportions; and conſider it abſtractly from Lines or other matter wherewith it is incumbred.

As to the Queſtion propoſed: Whether the Reſiſtance of the Medium do not always take off ſuch a Proportional part of the Force, Moving through it, as is the Specifick Gravity of the Medium to that of the Body Moved in it: (for if ſo, it will ſave us the trouble of Obſervation.) I think this can by no means be admitted; For there be many other things of Conſideration herein, beſide the Intenſive Gravity (or as ſome call it, the Specifick Gravity) of the Medium.

A Viſcous *Medium* ſhall more reſiſt, than one more Fluid, though of like Intenſive Gravity.

And a ſharp Arrow ſhall bore his way more eaſily through the Medium, than a blunt headed Bolt, though of equal Weight and like Intenſive Gravity.

And the ſame Pyramid with the Point, than with the Baſe forward.

And many other like Varieties, intended in my *P.* 46.

But this I think may be admitted, namely, That Different Mediums, equally Liquid, (and other Circumſtances alike) do in ſuch Proportion Reſiſt, as is their Intenſive Gravity. Becauſe there is, in ſuch Proportion, a Heavier Object to be Removed, by the ſame Force which is one of the things to which *P.* 31. refers.

And again: The Heavier Project once in Motion, (being equally ſwift, and all other Circumſtances alike) Moves through the ſame Medium in ſuch Proportion more ſtrongly, as is its Intenſive Gravity: For now the Force is in ſuch Proportion greater, for the Removal of the ſame Reſiſtance. And this, part of what my *P.* 30. Inſinuates.

But where there is a Complication of theſe Conſiderations one with another, and with many other Circumſtances, whereof each is ſeverally to be conſidered; there muſt be reſpect had to all of them.

*Experiments, to Determine the Point Blank Diſtence; the Charge of Powder, and beſt Size of Guns. Propoſed by* Sir Rob. Moray. *n.* 26. *p.* 473.

X. 1. *To know, how far a Gun ſhoots Point Blank, (as they call it) that is, ſo near the Level of the Cylinder of the Peece, that the Difference is either not diſcernable, or not conſiderable.*

Or:

On a fit *Platform* Place and Point the *Gun* at a Mark as large as the Bullet, ſome 50, 60, or more Yards diſtant, ſo as the underſide of the Mark may be in the ſame Level or Line with the underſide of the *Cylinder* of the *Peece*. Then breween the *Gun* and the Mark at convenient Diſtances, place pieces of Canvas, Sheets of Paper paſted together, or the like, upon ſtakes fixt in the ground, ſo as the underſide, being Level with the Horizon, may juſt touch the Viſual Line, that paſſeth from the Eye to the upperſide of the Mark, when the Eye is placed in the Line, that paſſeth from it to the upper ſide of the *Cylinder* of the *Gun*; the Canvas being ſo broad and long, that if the *Bullet* paſs through it 2 or 3 Foot higher than the Level of the Mark, or of either hand, the Hole it makes may make it known, how much it Flieth higher than the Level of that Place. If the *Bullet* falls Lower than the Mark, and touch not the Canvas, the *Gun* may be next time Raiſed a little, and ſo on till the *Bullet* Hit the Mark, or as High as it. If it fall as High as the Mark and Cut the Canvas, the Mark and Canvas may be brought nearer the *Gun*: But if it fall as High as the Mark and do not Cut the Canvas, the Mark may be removed to greater and greater Diſtances.

If this way of *Experiment* be made for further Diſtances and Raiſings of the *Piece*, as High as conveniently may be above the Level, and the Diſtances meaſured, and then all *Randoms* above theſe likewiſe tried and meaſured, the Diſtance of an Object, to be Shot at, being known, and other neceſſary Cautions, beneath to be mentioned, carefully obſerved, good *Gunners* may with great Confidence undertake to Hit the Mark be the Diſtance what it will, ſo it exceed not the Reach of the *Gun*.

2. *To know what Quantity of Powder is the juſt Charge of any Peece, ſo as it maketh the fartheſt Shot, and Fires totally.*

1. Raiſe the *Gun* to a mean *Random*, as of 20°, or 25°, and Shoot with the Ordinary *Charge* of *Powder*, in ſome convenient Ground where the Fall of the Bullet may be eaſily ſeen, and having made a Shot, meaſure the Diſtances with a Chain, between the Hole made by the Bullet, and the *Muzzle* of the *Gun*.

2. Then, inſtead of a Full *Charge* of *Powder* uſed in the firſt Shot, take $\frac{1}{16}$ part leſs, or ſome ſuch Proportion, for the next Trial, doing all things elſe as before.

3. For a third, fourth, or more Trials, diminiſh ſtill the *Quantity* of *Powder* by $\frac{1}{16}$ at a time, till the Shot be conſiderably ſhorter than at firſt.

4. Then take $\frac{1}{16}$ more than the firſt *Charge*, and do all things elſe as before, and ſo continue more Trials, increaſing ſtill the *Quantity* of *Powder* in the ſame Proportion every new Trial, till you find the Increaſe of the *Charge* does not make the *Peece* Shoot further: Only Over-charge not ſo far as to endanger the *Gun*.

5. Three or more Shot are to be made with every different *Charge*, and at every ſeveral Trial that the Certainty may the better appear.

6. The Firſt Shot being Meaſured and Marked, the reſt may all be Meaſured from it, or from one another to ſave Labour.

7. The

7. The *Gun* is to be Pointed, Placed and Ordered, every time in one and the ſame Place and Poſition, aiming ſtill at the ſame Mark, or Pointing ſtill in the very ſame Line or Azimuth; that ſo all the Shot may fall in the ſame Line, as near as is Poſſible.

8. The *Powder* (which ought to be all of the ſame *Goodneſs*) muſt be exactly Weighed, every time the *Peece* is *Charged*, leſt it having been Weighed long before, the Weight may be altered; though *Experiment* may be made with *Cartridges* and without.

9. The *Powder* and *Bullet* is to be *Rammed* home equally at every Shot; though the Looſer the *Powder* lie, it *Fire* the better.

10. When the Right *Charge* of a *Peece* is found, that makes the *fartheſt Shot* in the Ordinary and Plain way of *Charging*, M. *de Sons*'s contrivance of a *Wedge* may be Tried, to make it Shoot farther; which is a piece of Board, ſo long, as being thruſt home to the *Breech* of the *Peece* at one End, the other may reach farther out than the outſide of the Bullet, being *Rammed* up to its place; broad about an Inch, and thin ſo far as the *Wadd* before the *Bullet* reaches on the outſide; there it is to have a Shoulder, from which forward to the end, it is to be cut a-ſlope like a *Wedge*, being of ſuch thickneſs, as that at the place, where the Center of the *Bullet* is to be, it may make it ſtick ſo faſt, that the *Powder* finding more Reſiſtance, may at length Drive it out with the greater Violence.

11. Another of this Nature is a *Wooden Tampion*, like a piece of a Cylinder, big enough to fill the hollow *Cylinder* of the *Gun*, the length ſomewhat more than the Diameter of it, and Hollow'd towards the *Bullet*, ſo as to fit it; and either Flat, or (which is better) Hollow likewiſe towards the *Powder*, and ſerving inſtead of a *Wadd*. Theſe, and ſuch others, will probably render the effect of the *Powder* greater, than otherwiſe it would be: but Care muſt be had that they do not Endanger the *Peece*.

12. The *Strength* of the *Powder* muſt be examin'd by a *Powder Trier*, that raiſeth a Weight, ſuch an one as has been contrived by Mr. *Hook*.

13. The ſame *Bullet* is to be made uſe of, if it can be had, till the Figure of it be Marred; otherwiſe another as near of the ſame Size, Shape and Weight, as is poſſible.

14. Obſerve the *Strength* and *Poſition* of the *Wind*, and at what *Azimuth* the Mark ſtands from the *Gun* at every time of Shooting: and take preciſe notice what effect it hath upon the *Bullet* in carrying it further, in hindring or turning it aſide.

15. Note the Figure, Dimenſions, and Weight of the *Gun*, *Carriage*, and *Wheels*; and Record every thing exactly in a Book, as alſo every Accident and Obſervation.

16. After all other *Experiments* are made, every *Peece* may be Tried with the Right *Charge* of *Powder*, laying every time more and more Weight upon the *Carriage*; and at laſt Fixing the *Gun* ſo, as it may not *Recoil* at all, obſerving every time how far the *Bullet* goes, and how much leſs *Powder* than the full *Charge* will ſerve to Shoot the *Bullet*, when the *Peece* is Fixt, as far as the whole *Charge* does, when it *Recoils* freely.

17. The

17. The *Right Charge* found, the beſt *Random* is to be ſought, by Trying all *Randoms*, by Degrees at a time.

3. *To know what Gun Shoots fartheſt.*

1. A *Gun* to be prepared of *Culverine-Bore*, (as being held the beſt for Shooting Far) but much Longer, (double the Ordinary Length may do well) and without any *Ring* about the *Muzzle*, is to be Placed as in the former *Experiments*, and Charged with the Ordinary Charge of a *Culverine*, or rather with that Quantity, which by the former *Experiments* ſhall be found the Beſt; and being Shot, the Fall of the *Bullet* is to be Mark'd, and Diſtance Meaſured.

2. Then Try Leſs, and More *Powder* in her, as before.

3. Then Cut off two Inches of the *Muzzle* with a Saw, and Place the Pieces ſo cut off in the *Carriage*, or their *Weight* of Lead in a convenient Figure, that the *Recoil* may ſtill be the ſame; and Try as before, doing every thing in the ſame manner: and ſo Cut off ſtill for new Trials, till the Shot begin to fall ſhorter than before.

4. The ſame may be done with *Guns* of Different *Bores*.

2. *Mar.* 18. 1651. At 200 Yards Diſtance from the Platform for great Ordinance at *Woolwich*, there were raiſed three *Butts*, one behind another: the Space between the firſt and ſecond *Butt* was 14 Yards, between the ſecond and the third, eight. The Thickneſs of each *Butt* was 19 Inches, whereof 13 was of Beams of Maſſy Oak faſtned into the Ground, and ſet ſo cloſe that they touched each other: of each ſide were Planks of Oak, 3 Inches a piece in Thickneſs, and theſe were joyned cloſe, and faſtned on both Sides with Iron Bolts, and ſtrong Pins of Wood; and on the Back, at the ends and on the middle, there were 3 Braces of Elm, a Foot in Breadth, and 5 Inches in Thickneſs. *Experiments for Trying the Force of great Guns; by Mr.* Greaves. *n.* 173. *p.*1090.

The firſt *Experiment* was with an *Iron Demy-Cannon*, having a *Cylinder Bore*, of 3500 *lib. Weight*, the *Bullet* 32 *lib.* of Iron, the *Powder* 10 *lib.* which Pierced through the two Firſt *Butts*, and ſtuck in the third, ſo as the Ball was almoſt quite within, but the Timber not ſhivered, (ſmall) nor ſcarce Split. The *Butts* being touched by me, felt not warm; the like *Execution* was done, when it was Charged with 9 *lib.* as alſo when with 8 *lib.* of *Powder*.

The ſecond *Experiment* was with an *Iron Demy-Cannon*, having a *Taper Bore*, and being 3600 *lib.* in *Weight*, and 4 Inches longer than the former, the *Iron Bullet* 32 *lib.* and the *Powder* 7 *lib.* which in three Trials ſeemed to have the ſame Force with the Firſt: One of the *Shots* piercing through the ſecond *Butt*, and Lighting near the Edge of the middle * *Butt* of Elm, Tore it, but by the yielding of it, the *Bullet* glanced aſide off the third *Butt*, and entred into the Earth. * *Brace.*

The third *Experiment* was with a whole *Culverine* in *Braſs*, of 5300 *lib.* in *Weight*, 11 Foot one Inch in *Length*, with a *Taper Bore*; the *Iron Bullet* was 18 *lib.* in *Weight*; the *Powder* in the firſt Trial 10 *lib.* in the ſecond 9 *lib.* in the third 8 *lib.* which laſt Proportion did the beſt *Execution*, and paſſed through the two Firſt *Butts*, Entring gently into the Third, which the former two did touch, but not Enter.

The fourth *Experiment* was with a whole *Culverine* in *Brass*, made at *Amsterdam* for the *French*, with this Mark, 3580, being 10 Foot Long, and not very Thick in the *Breech*; the first Shot with 9 *lib.* of *Powder*, 18 *lib.* of *Bullet Iron*, past through the three *Butts* and entred one Foot into the Ground; it passed by the Joints of the Timber, two Planks having been beat down before. The second Shot with 8 *lib. Powder*, passed through two *Butts* and *Grazed* between them. The third, with 8 *lib.* past Two *Butts*, and 7 Inches into the Third, but the First *Butt* was much Battered before, where it Entred. The Fourth Shot passed, with 8 *lib.* of *Powder*, two *Butts*, and in both *Butts* through the midst of a Massy strong Beam (below) that had not been Battered.

The fifth *Experiment* was with an *Iron Demy-Culverine*, having 9 *lib. Bullet* in *Iron*, and 4 *lib. Powder*; this past one *Butt* (which was Torn before) and Entred the second.

This ½ *Culverine* was Shot 8 Times, as fast as they could *Charge* it with *Powder* and the *Iron Bullet*, and yet was but scarce Lukewarm at the *Breech*, a little more in the Midst, most at the *Muzzle*, and this last scarce so Hot as my Hand, and yet the *Gunners* in Charging her, wet not at all the *Scoop*, or *Spunge*.

The sixth *Experiment* was with a *Brass Demy Culverine*, the *Breech* of her was 13 Inches ⅞, the *Mouth* 9⅛. The First *Shot*, with 4 *lib.* of *Powder*, 9 *lib. Iron Bullet*, past two *Butts*: The Second *Shot* with 3 *lib.* of *Powder*, past almost two *Butts*: This proved to be the best Shot, because the Timbers were the Strongest.

*Shooting by the Rarefaction of the Air; by Dr.* Papin. n. 179. *p.* 21.

XI. Whereas ordinary *Wind Guns* do their Effect by the *Compression* of the *Air*: *Ottho Ghericke* hath found a new sort that Shoots by *Rarefaction*; and he hath published that Device at large in his Book about *Pneumatick Experiments*. I have Contrived another which I take to be Better:

*Fig.* 178.

A A, is a *Pipe*, very Equal from one end to the other.

B B, a small *Pipe* Soder'd to a Hole near the end of the *Pipe* A A, and apply'd to the *Plate* of the *Pneumatick Engine*.

C C C C, Some kind of *Stool*, to bear up the Hinder part of the *Pipe* A A.

D, a *Piece* of *Lead* fitted to the Bore of the *Pipe* A A.

The *Pipe* A A, is to be Shut at both Ends by *Valves* outwardly applied, and so the said *Pipe* A A, though never so Big, may be *Exhausted* of *Air* by means of the *Pneumatick Engine*: Which done, the *Valve* towards D must be suddenly opened, so that the whole Pressure of the Atmosphere Acting upon the *Lead* D, may drive it along the *Pipe* A A, with such a Swiftness, that it will be able to carry it to a great Distance: And because such a *Valve* Shutting a great Hole, would prove very difficult to be Opened, when the *Pipe* A A, is of a great Bore, the Aperture towards D, may be left much smaller than the *Pipe*; the Swiftness of the Air being so great, that even through a pretty small Aperture, it pressed the *Lead* D, as freely almost as if the whole Bore was quite Open.

Having

Having prepared a *Barrel* carrying a *Lead* of two Ounces, the *Experiment* was ſhewn before the *Royal Society*, and the Effect was found very Conſiderable, the Force being little leſs than that of the *Wind-Gun* by *Compreſſion*; the ſame *Experiment* being afterwards repeated with a Longer *Barrel*, 'twas found that the Length in this way of *Shooting* was very little, if any Advantage.

XII. My Way of Computing the *Velocity* of the *Air*, (which I think is better than the Trial made by the *Royal Academy* at *Paris*) is Grounded upon this *Hydroſtatical Principle*, That *Liquors have a Strength to Aſcend as High as their Source is*; and although the *Reſiſtance* of the *Medium* does always hinder *Jeťs d'Eau* in the Open Air from reaching quite ſo High, nevertheleſs the *Liquor* at its firſt Spouting out, hath the neceſſary *Swiftneſs* to come to that *Height*. *The Velocity wherewith the Air Ruſhes into an exhauſted Receiver; by Dr.* Papin. n. 184. *p.* 193.

Prop. I.] From this Principle may be eaſily deduced this Propoſition, That, *of two Differing Liquors driven by the ſame Preſſure, that which is in Specie Lighter muſt Aſcend Higher than that which is Heavier, and their Heights will be Reciprocally in the ſame Reaſon as their Specifick Gravities are.*

Prop. II.] From the foregoing *Propoſition* another may eaſily be Deduced, *viz.* That, *of Differing Liquors bearing the ſame Preſſure, thoſe that are Lighter in Specie muſt acquire a greater Swiftneſs, and their Differing Velocities are to one another* * *as the Roots of the Specifick Gravities of the ſaid Liquors.* * *Reciprocally.*

For we have ſeen, *Prop.* I. That the *Heights* to be Attained are * in the ſame Reaſon as the *Specifick Gravities*; Now *Gallilæus*, *Hugenius*, and others, have Demonſtrated, That the *Velocities* of Bodies are to one another, as the *Square Roots* of the *Heights* to which they may Aſcend: and ſo in this occaſion they are alſo * as the *Roots* of the *Specifick Gravities*. * *Reciprocally.* * *Reciprocally.*

If therefore we would know what is the *Velocity* of the *Air* being driven by any degree of *Preſſure* whatſoever, we ought but to find what would be the *Velocity* of *Water* under the ſame *Preſſure*: and then take the *Square Roots* of the *Specifick Gravities* of theſe two Liquors; becauſe as much as the *Square Root* of the *Specifick Gravity* of *Water*, doth exceed the *Square Root* of the *Specifick Gravity* of the *Air*; ſo much in Proportion will the *Velocity* of *Air* exceed the *Velocity* of *Water*. For Example; When I would Compute what ſhould be the *Swiftneſs* of a *Bullet* ſhot by my *Pneumatick Engine*, I ſhould firſt Compute what was the *Velocity* of the Air it ſelf that drove the Bullet: I did therefore take notice, that in this Occaſion the *Air* bears a Preſſure much about the ſame as that of *Water* when its *Spring* is 32 Foot High. Now ſuch *Water* would ſpout out with a ſufficient *Velocity* to aſcend 32 Foot Perpendicular, and therefore according to the Rules and Obſervations of *Gallilæus*, *Halley*, and others, ſuch Water hath the *Velocity* of 45 Foot in a Second. It remains therefore but to know the Proportion of the *Gravity* of the *Air* to that of *Water*: and we have found it not to be always the ſame; becauſe the Height, the Heat, and the Moiſture, of the *Atmoſphere*, are Variable: nevertheleſs, we may ſay in general, that the Reaſon between the *Specifick Gravities* of *Water* and *Air* is much about 840 to 1. Taking then



their

their *Square Roots* as I have ſaid above, which *Roots* are 29 and 1, we may Conclude that the *Velocity* of *Air* muſt Exceed that of *Water* by 29 Times: and ſo Multiplying 45, the *Velocity* of *Water*, by 29, we ſhall find, that the *Velocity* of the *Air* driven by the whole Preſſure of the *Atmoſphere*, is about 1305 Foot in a Second.

*Wind produced by the Fall of Water; by* Dr. Wal. Pope. n. 2. p. 25. Fig. 179.

XIII. In the *Braſs-Works* at *Tivoli*, the *Water Blows* the *Fire*, not by Moving the *Bellows*, but by affording the *Wind*. Thus: A, is the *River*. B, the *Fall* of it. C, the *Tube* into which it Falls. L G, a *Pipe*. G, the *Orifice* of the Pipe, or *Noſe* of the *Bellows*. G K, the *Hearth*. E, a Hole in the Pipe. F, a Stopper to that Hole. D, a place under-ground, by which the Water runs away. Stopping the Hole E, there is a perpetual ſtrong *Wind*, iſſuing forth at G: and G being ſtopt, the *Wind* comes out ſo Vehemently at E, that it will, I believe, make a *Ball* play, like that at *Freſcati*.

*The best Form of Horizontal Sails for a Mill; by* Dr. Rob Hook. *Phil. Coll.* n. 3. p. 61.

XIV. Whatever Men may imagine concerning *Horizontal Sails*, I doubt there will never be found a better, and more Advantageous way, for receiving the Strength of the *Wind*, or *Motion* of the *Air*, than *Perpendicular Vanes* made of a True Form, ſo as every part thereof may *Draw* alike. But becauſe I find divers have of late attempted *Horizontal Vanes* for *Mills*, I ſhall explain a way of making *Horizontal Vanes* capable of Performing the *Moſt* that is *Poſſible* with *Vanes* of equal *Extenſion*.

The *Invention* is Founded upon the ſame Principle with that of the *Sailing* of *Ships*, and other *Veſſels*, upon the Sea; namely, Upon Diſpoſing and Ordering of the Vane or Sail ſo, as *to ſtand in the beſt Poſture 'tis poſſible to Move the Arms of the Mill, or the Body of the Ship, in that Way it is to be Moved, by the Force of the Wind Blowing thus or thus againſt them.*

The *Firſt Principle* then common to both, is, That the *Vane* or *Sail* be as near as 'tis poſſible, a perfect *Plain* and *Smooth Superficies*, without any Belly-ing, Bunting, or Curvity in the Superficies thereof, upon which the Motion or Force of the *Wind* is Impreſſed.

*Secondly*, That the *Air* may have as Many *Paſſages* between the parts of the *Vane* or *Sail* as may be, that the Moved Air may come to it as *freely* as may be, without being Intercepted by a *Stagnant Air* before it, to Impede or Divert its Force.

*Thirdly*, That the *Plain* of the *Vane* or *Sail* be put in the *Middle Inclination*, between the *Way* of the *Wind* and the *Way* of the *Arm*, or that of the Body of the *Ship*.

The *Contrivance* it ſelf is *This*.

Fig. 180.

Let A B, ſignifie the *Stream* or Current of the Air or *Wind*, Moving from A to B, and let C repreſent the Center of the *Axis* or *Spindle*, ſtanding Perpendicular to the *Horizon*, upon which, at the Top, is fixed at Right Angles, the piece D H, making the two *Arms* C D, and C H, upon the Ends of which the *Vanes* M N, are Moved on *Spindles*; ſo as that the Plain of the *Vane* doth always paſs through the Point D: I ſay, theſe *Vanes* ſo ordered, ſhall be always placed in the moſt *Advantageous Poſture* for Moving

the

the *Arms* round upon the ſaid *Spindle*, whoſe Center is C, in the Order of DEFGHIKLD.

*Firſt*, For the *Vanes* placed at D and H, I ſay, They are ſet in the moſt *Advantageous Poſture* poſſible, in thoſe two Points: For *Firſt*, The *Vane* MN at D being to move Directly againſt the *Wind*, the moſt *Advantageous Poſture* is to turn its edge directly againſt the *Wind*, and thereby to give the leaſt *Reſiſtance* poſſible, that being the only Point in which the *Vane*, ſuppoſed only a Superficies, Draws not. And *Secondly*, For the *Vane* MN placed at H, it ſtandeth the moſt *Advantageouſly*, becauſe its Motion being directly from or before the *Wind*, it ſtandeth full *Croſs*, or Oppoſed to the Motion thereof.

*Secondly*, The *Vanes* at E, F, G, and I, K, L, ſtand the moſt *Advantageouſly*, becauſe they *Divide* the *Angle* between the *Way* of the *Wind* and that of the *Arms* in thoſe Points into two Equal Parts, and conſequently the Wind Impreſſeth the Greateſt Force in the moſt Direct Way: For it is eaſie to be Demonſtrated, That the Force Impreſſed on the *Vane* by the *Wind*, is Perpendicular to the Surface, and conſequently that the Obliquity of the Force to the way of the *Arms*, Increaſed by the *Vanes* ſtanding more full againſt the *Wind*, will have a leſs Proportion of Power to promote the Motion thereof, than in the Poſture here Set. And ſuppoſing the *Vanes* ſet Sharper to the *Wind*, the Diminution of the Force Impreſſed by the *Wind* on its Surface; will be greater than the Augmentation of its Power, by being Moved more Directly to the Way of the *Arms*. This is eaſie enough to be *Geometrically Demonſtrated*.

The *Vane* may be ſo ordered, as always to Stand in this *Poſture* by a great many ways: I ſhall only inſtance in One, not the beſt for *Practice*, but the moſt Eaſie to be underſtood and *Demonſtrated*.

Let the *Vane* be equally Expanded on each ſide of its *Axis*, by which the Preſſure on the extreams of it are always Counterpoiſed, then Faſten upon the Lower end thereof a Wheel, which may be in Diameter about $\frac{1}{2}$ of the Length of the *Arms* from Hole to Hole; then Fix a Wheel upon the Frame in which the *Spindle* of the *Arms* do Move, that ſhall be of Half the Diameter with the former, and to contain Half the number of Teeth. Then by a third ſmall Wheel, Fixed under the *Arms*, of a convenient Bigneſs, Communicate the Motion of the One to the Other; for by this means each *Vane* being ſo provided, they will, being once ſet Right, always continue to be Moved and Diſpoſed in the true Poſture deſired. *Fig.* 181.

This *Contrivance* will not only be Uſeful for all manner of common *Wind-Mills*, but alſo for *Water-Mills* in *Rivers*, where there can no *Dam* be made, as may alſo the *Perpendicular Vanes* of other *Mills*, neither of which has been ſo much as *Hinted* by any Perſon whatſoever that I have hitherto heard of.

XV. 1. The Art of *Flying* hath been in all Ages *Attempted* by many, particularly in the Times of our famous Fryar *Roger Bacon*, who lived about 500 Years ſince. He was Believed a *Magician* or *Conjurer*, and to have performed what was related of him, by the help of *Diabolical Magick*, but from the peruſal of ſeveral of his excellent *Works* yet Extant, I eſteem him no ſuch Perſon, I

*An Account of Flying; by Dr. Hook. Phil. Coll. n. 1. p. 14.*

I rather find him to have been a good *Mathematician*, a knowing *Mechanick*, a rare *Chymist*, and a most Accomplished *Experimental Philosopher*, which was a Miracle for that Dark Age. This Man affirms the Art of *Flying* Possible, and that He himself knew how to make an *Engine*, in which a Man Sitting, might be able to carry himself through the *Air* like a *Bird*: And Affirms, that there was then another Person who had actually Tryed it with good Success. We have not wanted Later Instances in *England*, of several Ingenious Men, who have employed their Wits and Time about this *Design*. Particularly, I have been credibly Informed, That one Mr. *Gascoigne* did about 40 Years since Try it with good Effect; though he since Dying, the thing also Dyed with him. And even now there are not Wanting some in *England*, who Affirm themselves able to do it, and that they have proved as much by *Experiment*. We have little or no account of the Ways they have taken to Effect their Designs; But we may conclude them Defective in somewhat or other, since we do not find them brought into *common Use*.

*The Art of Flying; by S. Besnier. Ibid. p.* 15. *Fig.* 182.

2. The Sieur *Besnier*, a Smith of *Sable* in the County of *Maine*, hath Invented an *Engine* for *Flying*. It consists of two *Poles* or *Rods*, which have at Each End of them an oblong *Chassie* of Taffety, which *Chassies* Fold from above Downwards, as the Frame of a folding *Window Chassie*. He fits these *Poles* upon his Shoulders, so that two of the *Chassies* may be before him, and the other two behind him. The *Order* of *Moving* them, is thus: When the *Right Hand* Strikes down the *Right Wing* before, A, the *Left Leg* by means of the String E, Puls downward the *Left Wing* behind, B; then immediately after, the *Left Hand* Moves or Strikes downwards the *Left Wing* before, C; and at the same time the *Right Foot*, by the String F, Moves or Pulls down the *Right Wing* behind D; and so Successively, or Alternately, the Diagonally Opposite *Wings* always Moving downwards, or Striking the Air together.

*A Flying Chariot, by Fr. Lana. Ibid. p.* 18.

3. 1. *P. Francesco Lana* in his *Prodromo*, finding by an *Experiment*, That the *Weight* of the *Air* is $\frac{1}{640}$ part of the *Weight* of a like quantity of *Water*, he concludes certainly, That if we could make a Vessel of Glass or other Matter that might *Weigh* less than the *Air* that is in it, and should draw out all its *Air*, this Vessel would be *Lighter in Specie* than *Air* it self, and therefore would Swim in it and Ascend on High. This He *supposes* may be done, by making a Round Vessel of Thin *Plate Brass*, (*Weighing* 3 Ounces in a *Square Foot*) of the *Diameter* of 14 Foot. For the *Surface* of the Vessel will be 616 *Square Feet*, and the *Brass* will *Weigh* no more than 1848 Ounces; whereas the *Content* will be $1437\frac{1}{3}$ *Cub. Feet*, and that Quantity of Air will Weigh $2155\frac{2}{3}$ Ounces: so that that Air being evacuated, the Vessel will be $307\frac{2}{3}$ Ounces *Lighter* than *Air*, and therefore will not only Ascend into the Air, but also carry up with it a Weight of $307\frac{2}{3}$ Ounces. And thus by Encreasing the Bulk of the Vessel, without Encreasing the Thickness of the Plates of Brass, he supposes a kind of Ship may be made, to Swim in the Air, and to carry two or three Men it.

2. The Fallacy of the *Author*'s Reaſoning lies in this; He ſuppoſes *Copper* of 3 Ounces in a Foot Square to be of ſufficient Thickneſs to Reſiſt the Preſſure of the *Air* in a *Globe* of 14 Foot *Diameter*, nay of any *Dimenſions*. But in this we can nowiſe Aſſent to him: For the Preſſure from without Inwards, though it be always the ſame upon Equal Surfaces, yet upon Unequal Surfaces the Caſe is quite otherways, for there the Preſſure will be found, not the ſame, but to Encreaſe always in the ſame Proportion with the Surface, and thence conſequently the *Thickneſs* of his *Copper*, or any Metal or Material, which he ſhall make uſe of, muſt Increaſe in the ſame Proportion, with the *Diameter* of the *Sphere*, and conſequently the *Weight* of his *Copper* muſt always *Increaſe* in the ſame Proportion at leaſt to the *Solidity* of his *Sphere*; ſo that by his Augmenting the *Quantity* of his *Sphere*, he has no manner of Advantage of making it proportionably *Lighter* than the *Air*, and proportionably *Strong*, but the contrary: For it is manifeſt, That a Bigger *Sphere* ſo made of any Matter we yet know, has leſs Power of Reſiſting the ſame *Preſſure* of the *Air* than a Leſs, becauſe of the Finite Reſiſtance of Matter to Preſſure, there being ſome degree of *Preſſure* that will *Cruſh* every Body. *Shown Impracticable; by Dr. Hook. Ibid. p. 27.*

XVI. This *Engine* is compoſed of four Principal Parts; the *Serpent* A A, two Foot-Steps or *Treddles* B B, one *Clapper* C, and two *Arms* D D, D D. *An Engine to make Linnen Cloth; by M. de Gennes. n. 140. p. 1007. Fig. 183.*

The *Serpent* or Iron Bar A A, has two Elbows E E, where to the Ends of the Ropes are fix'd that Raiſe and put Down the *Footſteps* B B, F F are two Fourths of a Circle, that ſucceſſively Reſt upon two Arches or Bows of Iron G G, which are above the *Clapper* C, to Raiſe it. H H are two Teeth of Iron, added to the *Serpent* making an Angle of 25 *Deg.* with F F, and K K, which ſerve to put Down a *Baſcule*, or *Sweep*, which is in the Arm that carries the *Shuttle*. The Footſteps or *Treddles* differ in nothing from thoſe which are uſually made uſe of; only the Cords that hold them Pendent from the Ground are fixt in the Elbows of the *Serpent*, which in turning Raiſes and puts them Down by the help of two little Pullies, upon which the Ropes turn.

The *Clapper* is ſupported between two Pillars with a Rope double twiſted, which makes it to make a kind of Spring, and cauſes it naturally to give forwards to Beat the Cloth.

L M, is one of the *Arms* which paſs freely into the Canal or Pipe N N, ſupported by four Pillars of Wood O O O O. The Motion of it proceeds from the following Parts. P Q, is a *Baſcule*, which, though unequally divided by its Supporter R, is yet *in Æquilibrio*, the End P R being made to weigh exactly as much as R Q.

At the Extremity of this *Baſcule* is ty'd a Cord which paſſes through the Pully S, and terminates at the Extremity of the *Arm*, where it is faſtned to a little Bowl M. At the other Extremity of the ſame *Arm*, that is to ſay towards L, is alſo faſtned underneath, a Cord which paſſes through the Pulley T, and which carries the Weight V.

At the ſame end of the *Arm* is added a little *Niche* Z, about the bigneſs of half the *Shuttle*: then over a little Bar X Y, which paſſes athwart the *Arm*,

there

there are two other little pieces of Wood, having at the end of them two Teeth, which enter into the *Niche* Z, through two Holes which are there, of the one side and t'other.

To the Ends of these little pieces of Wood, there is a little *Bow* of Whale-bone or Steel, which keeps the two Ends asunder, and forces the Teeth, which are at the other end, to enter into the *Niche*, before the said pieces can themselves. At the Points 1 1, are two Ropes that pass through the Pullies 2 2, fastned to the Pillars *o* 3, *o* 4, and have each of them a little Weight at the end big enough to keep it from passing through a little Bowl which is under each Pulley.

This *Arm* thus dispos'd, goes and comes in the Hole NN, in the following manner. One Tooth of the *Serpent*, already described, strikes upon the Extremity of the *Bascule* PQ, and so causes the End Q to Rise up, which drawing the Cord fastned to the Point Q, makes the *Arm* LM, to Advance forward. But when afterwards the Tooth of the *Serpent* is come forth again, then the Weight V, tied to the other End of the same *Arm* by a Cord that passes through the Pulley T, forces the said *Arm* by its own Weight to Return again.

When the *Arm* LM, is in its ordinary place, the two little pieces of Wood, into which enters the Bar XY, enclose the *Shuttle* by means of the Whale-bone *Spring*. But when the said *Arm* approaches the other opposite Arm, then the Cords tied to the Points 1 1, being a little too short, and the Weight which is at the end of them not being able to pass through, the *Spring* gives way a little, and so the *Shuttle* is no longer enclosed by the *Arm* which carries it, but is wholly received and grasp'd by the other; which likewise in its Turn, delivers it back again in the same manner.

The Motion of the whole *Machine* is made at the rate as you Move the Handle of the *Serpent*, for then the *Arms* cause the Threads to open, and immediately one of the *Arms* begins to slide in towards the opposite *Arm*, to which it carries the *Shuttle* and Retires, immediately: At the same time, one of the Quarters of a Circle, which held the *Clapper* Elevated, forsakes it, and leaves it for to flap, and then the opposite quarter of a Circle Elevating it self, the other Elbow changes the Threads, and the other *Arm* Retires, and so successively.

The *Advantages* of this *Engine* are these. 1. One *Mill* will set 10 or 12 of these *Looms* at Work. 2. You may make the *Cloth* of what *Breadth* you please. 3. There will be fewer Knots in the *Cloth*, since the Threads will not break so fast as in other *Looms*, because the *Shuttle* that breaks the greatest part, can never touch them. In short, The *Work* will be carried on *Quicker*, and at less *Charge*, in regard that instead of several Work-Folks which are required in making very large *Cloths*, One Boy will serve to tie the Threads of the several *Looms* as fast as they break, and to order the *Quills* about the *Shuttle*.

XVII. I Order'd a *Model* of a part of a *Waggon* to be made conſiſting of four *Wheels*, two *Axes*, and a *Board* nailed upon the *Axes*. The *Leſſer Wheels* were $4\frac{1}{3}$ Inches high, and the *Bigger Wheels* $5\frac{2}{3}$ Inches high, *viz.* $\frac{1}{12}$ of the Ordinary height of the *Wheels* of a *Waggon*: The Weight of the *Model* was almoſt $1\frac{1}{2}$ *lib.* I had alſo two other *Wheels* made $5\frac{2}{3}$ Inches high to be put on inſtead of the Leſſer. The Middle of the two *Axes* were $6\frac{1}{4}$ Inches aſunder. All the *Wheels* Turn'd very eaſily upon the *Axes*.

*Advantages of High Wheels Experimented; by a Member of the* Oxford Society. n. 167. p. 856.

A piece of Lead $50\frac{3}{4}$ *lib. Averdupoiſe*, was laid upon the *Model*, ſo forward, that the *Leſſer Wheels* ſeemed to bear above $\frac{2}{3}$ parts of the *Weight*. Then the *Model* was drawn with a String laid over a Pulley; the top whereof was $\frac{1}{4}$ of an Inch higher then the top of the *Hinder Axes*, and the Middle of this Pulley was $7\frac{1}{2}$ Inches from the Middle of the *Fore Axis*.

The *Leſſer Wheels* being put on, and the String being tied to the top of their *Axis*.

1. *Three* Pound drew the *Model* on the ſmooth level Table.

2. *Twenty* Pound drew the *Leſſer Wheels* over a Squared Rod $\frac{1}{4}$ of an Inch thick.

3. *Thirty* Pound drew *them* over a Round Rod a little more than $\frac{1}{2}$ an Inch thick.

4. *Thirty One* Pound drew *them* over a Square Rod half an Inch thick.

5. *Twelve* Pound drew the *Hinder Wheels* over the bigger Square Rod.

The String being laid under the *Axis*, *viz.* $\frac{5}{8}$ of an Inch lower than before.

6. *Twenty nine* Pound drew the *Leſſer Wheels* over the Bigger Square Rod.

Then the two *Bigger Wheels* being put on inſtead of the *Leſſer*, and the String lying Over the *Axis*.

7. *Three* Pound drew the *Model* on the Table.

8. *Twenty five* Pound drew the *Fore Wheels* over the Round Rod.

9. *Twenty five* Pound drew them over the Bigger Square Rod.

10. The String lying under the *Axis*, 16 Pound drew them over the leaſt Rod.

11. *Twenty three* Pound drew them over the Round Rod.

12. *Twenty three* Pound drew them over the Bigger Square Rod.

13. *Thirteen* Pound drew the *Hinder Wheels* over the Bigger Square Rod.

In all theſe *Experiments*, the *Lead* was laid exactly upon the ſame part of the *Board*, but yet when the *Leſſer Wheels* were taken off, the *Lead* did not lean ſo much forward, ſo that the *Hinder Wheels* were ſomewhat more preſſed than they were before.

By Comparing the ſecond, third, and fourth *Experiments*, with the tenth, eleventh, and twelfth, it appears how much more eaſily a *Waggon*, &c. might be drawn in Rough Ways, if the *Fore Wheels* were as High as the *Hinder Wheels*, and if the *Thills* were fixt under the *Axis*. Such a *Waggon* as this, would likewiſe be Drawn more eaſily, where the *Wheels* cut in *Clay*, or *Sand*, or any Soft Ground. And moreover *High Wheels* would not cut ſo deep as *Low Wheels*.

Low

*Low Wheels* indeed are better for Turning in a narrow Compaſs than High ones: But it ſeems probable that *Waggons* with four *High Wheels*, might be ſo contrived, that there ſhould be no great Inconvenience in that Reſpect; at leaſt, ſuch *Waggons* as ſeldom have occaſion to turn ſhort, as *Carriers Waggons*, and ſuch like.

The Difference which you may obſerve in the eighth and eleventh *Experiments*, is agreeable to what is ſaid by *S. Stevinus*, and Dr. *Wallis*, *viz.* That if a *Coach*, *&c.* muſt be Drawn over Rough, Uneven Places, it is beſt to fix the *Traces* to the Coach Lower than the Height of the Horſes Shoulders.

14. A Table $2\frac{1}{2}$ Foot long, was ſet with one End $8\frac{1}{2}$ Inches Higher than the other End, and the *Model* being loaded as before, leſs Weight by 6 Ounces drew it up the Table, when the four *Bigger Wheels* were on, than when two *Bigger* and two *Leſs* were on. Becauſe in the firſt Caſe there was almoſt the ſame *Direction* of the Motion of the *Model* and of the String that drew it; but not in the ſecond Caſe, when the *Fore Axis* was ſo much Lower than the top of the Pulley.

*A New ſort of Caleſh; deſcribed by Sir* R. B. n. 172. p. 1028.

XVIII. This *Caleſh* goes on Two Wheels; Carries one Perſon, is Light enough; Tho' it Hangs not on *Braces*, yet it is eaſier than the Common Coach; A Common Coach will Overturn, if one Wheel go on a Superficies a Foot and a half Higher than that of the other, but this will admit of the Difference of $3\frac{1}{3}$ Foot in Height of the Superficies, without danger of Over-turning: We Choſe all the irregular Banks, and ſides of Ditches to run over; and I have this day ſeen it at five ſeveral times, turn over and over, and the Horſe not at all diſorder'd. If the Horſe ſhould be in the leaſt unruly, with the help of one Pin, you Diſingage him from the *Caleſh* without any Inconvenience. I my ſelf have been once Overturn'd, and knew it not till I lookt up, and ſaw the Wheel flat over my Head; and if a Man went with his Eyes ſhut, he would imagine himſelf in the moſt ſmooth way, tho' at the ſame time there be three Foot Difference in the Height of the Ground of each Wheel.

*The Contrivance of a Perpetual Motion; by M. . . . . . . . . . Explained; by Dr.* Papin. n. 177. p. 1240. *Fig.* 184.

XIX. Let DEF, be a pair of *Bellows* 40 Inches long, that may be opened by removing the part F, from E: Let them be exactly ſhut every where, but at the Aperture E; and let a Pipe EG, 20 or 22 Inches long, be Sodred to the ſaid Aperture E, having its other end in a Veſſel G, full of *Mercury*, and placed near the Middle of the *Bellows*.

A, is an *Axis* for the *Bellows* to Turn upon.

B, A Counterpoiſe faſtned to the lower end of the *Bellows*.

C, a Weight with a Claſp to keep the *Bellows* upright.

Now if we ſuppoſe the *Bellows* opened only to $\frac{1}{5}$, or $\frac{1}{4}$, ſtanding Upright, and full of *Mercury*, it is plain that the ſaid *Mercury* being 40 Inches High, muſt Fall, as in the *Torricellian Experiment*, to the Height of about 27 Inches, and conſequently the *Bellows* muſt open towards F, and leave a *Vacuity* there. This *Vacuity* muſt be filled with the *Mercury* Aſcending from G through the

Pipe G E, the said Pipe being but 22 Inches long: by this means the *Bellows* must be opened more and more till the *Mercury* continuing to Ascend, makes the Upper part of the *Bellows* so heavy, that the Lower part must get loose from the Clasp C, and the *Bellows* should turn quite upside down; but the Vessel G, being set in a convenient place, keeps them Horizontal, and the part F, Engageth there in another Clasp C; then the *Mercury* by its Weight runs out from the *Bellows* into the Vessel G, through the Pipe E G, and the *Bellows* must shut closer and closer until the part E F comes to be so Light, that the Counterpoise B is able to make the part F, get loose from the Clasp C; then the *Bellows* comes to be Upright again as before; the *Mercury* left in them falls again to the Height of 27 Inches, and consequently all the other Effects will follow, as we have already seen, and the *Motion* will *Continue for ever*. Fig. 185.

Upon this, it is to be observ'd, That the *Bellows* can never be opened by the Internal Pressure, unless the said Pressure be stronger than the External. Now in the Case before us, it is plain, That altho' the Lowermost part of the *Bellows* be Pressed Outward by 40 Inches of *Mercury*, yet the Upper part having no *Mercury* above it, bears none at all; the parts that lie in the Middle near the Axis of the *Bellows* bear but 20 Inches, and so all the rest must bear more or less, according as they lie Higher or Lower: It is evident therefore, That there are as many parts that bear less than 20 Inches, as there are that bear more, and the Increase of Pressure following an *Arithmetical Progression*, it is undeniable, that all these Pressures added together, will do no more than one Uniform Pressure, that would be equal to 20 Inches every where. It is also plain, That the Weight of the *Atmosphere* cannot come at the Inward part of the *Bellows*, but through the Pipe G E, which containing 22 Perpendicular Inches of *Mercury*, doth Counterpoise so much of the Weight of the *Atmosphere*; so that this being supposed to be 27 Inches of *Mercury*, it cannot press the Inward part of the *Bellows* but with a Weight equivalent to 5 Perpendicular Inches of *Mercury*. So that we find, the Inward Pressure both of the *Mercury* and the *Atmosphere* is equivalent but to 25 Inches of *Mercury* in all. Whereas the Pressure of the *Atmosphere* upon the Outside is every where equal to 27 Inches; from whence it appears, That the Pressure without, is stronger than the Pressure within. From this we may conclude, That the *Bellows* standing Upright will rather Shut than Open. *And shown Insufficient by him. Ib. p. 1241. n. 182. p. 138.*

I shall say nothing to the Alterations this *Author* may make in his *Engine*, resolving to leave it to others to shew him, that upon that Principle all he can do signifies nothing. And I doubt not, but if he pleases to consult *M. Perault*, *de la Hire*, or any other at *Paris*, he will find them of the same Opinion with Mr. *Boyle*, and Mr. *Hock*, and others *Here*. n. 186. p. 267.

XX. This *Reflecting Trumpet*, consists of two Parts. The Utmost B *b*, is a large Concave Pyramid, about a Yard long, (or may be of any manageable Length) Open at the Base B, and Closed not with a Flat, but a Concave Head, at the Cone B. Within this is fastned a Bended Tube A *a*. *The Speaking Trumpet Improved; by Mr. J. Conyers.* n. 141. p. 1027.

Fig. 186. This *Trumpet* did at a Meeting of the *Royal Society* at *Arundel-House*, distinctly deliver some Words, cross the Garden and the River *Thames*, and that against the Wind which was then strong; and the Words were written down by one, that was sent over for that purpose: Whereby it appeared, That a *Reflecting Trumpet*, after this, or some other like manner, of Wood, Tin, Pewter, Stone, or Earth, or which may be best of Bell-Metal, will carry the Voice as far, i. not farther, than the Long one Invented by Sir *Samuel Moreland*. Fig. 187. Besides that, it seems to take off from the Astonishing Noise near at hand, which happens in use of the said Long *Trumpet*: Fig. 188. By Sir *Sam. Moreland*'s *Trumpet* Angularly Arched in the Middle, the delivery of Sound to any Distant Place was much shortned; and by another with three large Angular Arches, reaching almost from one End to the Other, the Sound was almost wholly Obstructed.

*The Swiftness of Sounds and their Reflections or Eccboes; by Mr.* Walker. *N.* 247, *p.* 433.

XXI. I provided a *Pendulum*, of small Virginal Wire, with a Pistol Bullet at the end of it, which had two Vibrations in one Second of Time. I took this *Pendulum*, and standing over forgainst a High Wall, I clapt two pieces of small Boards together, and observed how long it was ere the *Echo* returned; and I Removed my Station till I found the place whither the *Echo* Returned in about half a Second. But that I might Distinguish the Time more nicely, I clapt every Second of Time, 10 or 15 times together; so that by this means, I could the better Discover whether the Distances betwixt the Claps and the *Echoes*, and the following Claps, were Equal. And though it be very difficult to be Exact, yet I could come within some few Yards of the place I sought for, thus: I observed the two Places, where I could but just discover that I was too near, and where I was too far off; and from the midway betwixt them I measured to the Wall, which Measure doubled, was the Space that the *Sound Moved* in half a Second.

Here follow the Numbers of *English* Feet which a *Sound Moved* in one Second of Time at several Trials.

| *Trials* | *Feet.* | *Trials* | *Feet.* | *Trials* | *Feet.* |
|---|---|---|---|---|---|
| 1 | 1256 | 5 | 1292 | 9 | 1278 |
| 2 | 1507 | 6 | 1378 | 10 | 1290 |
| 3 | 1526 | 7 | 1292 | 11 | 1200 |
| 4 | 1150 | 8 | 1185 | | |

*Mersennus* mentions an *Experiment* wherein he found the *Motion* of the *Sound* to be 1474 *Feet* in a *Second*. The *Academy del Cimento* caused 6 Harquebusses, and 6 Chambers to be fired one after another at the Distance of 5739 *English Feet*, and from the Flash to the Arrival of the Report of each was 5″: And repeating the *Experiment* at the Midway, the *Motion* was exactly in half the

the time; and Mr. *Boyle* obſerved, That the *Motion* of *Sound* paſſes above 400 *Yards* in a *Second.*

When the *Firſt Trial* was made, there was ſome Wind ſtirring, tho' not much; the 2d, 3d and 6th, were made in a Calm morning. In the 8th, the *Echo* was returned from a Wall at 395 *Yards* Diſtance in two *Seconds*, and in the 9th and 10th, at 213 and 215 *Yards* Diſtance, in one *Second.* The 4th was made at one end of St. *John*'s *Cloiſter* in *Oxford*, which is 104 *Feet* 7 Inches long, where the *Sound* was *Reflected* 11 *times* in two *Seconds*: And the 5th, on the North ſide of *New College Cloiſter*, (which is 160 *Feet* 8 Inches long) where there were about 7¼ *Echoes* in two *Seconds.*

By ſome of thoſe *Experiments* that I tried, I am Inclined to think, That the *Sound* Moved Quicker when it was Calm, than in a Wind, even when the *Sound* Moved half way with the Wind; and that it Moves ſwifter at firſt, than afterwards.

There is ſeldom any *Echo*, where there is not ſome Wall, Wood, Bank, or ſuch like, directly Oppoſite, that may Reflect the *Sound* to the Perſon that makes it; but in St. *John*'s *Grove*, if you ſtand near the Gate leading from the *College* to the *Grove*, and Clap, the *Echo* will Return to you from the *Ball Court*, though a Line drawn from you to the *Ball Court* be not Perpendicular to the Wall there, but as much Oblique as the Line A B, is to the Line BC; where A repreſents the Gate, B C the *Ball Court*-Wall, and BD another Wall. Or, if you ſtand at E, the Corner of the *Grove* next to *Trinity*, and Clap, the *Echo* will Return to you from the *Ball Court.* Fig. 189.

In the ſame *Grove*, I ſtood about 20 Yards from the ſame Gate, and the Gate being ſhut, I Clapt, and at other times Stamped, and the *Echo* Returned from the Gate as loud, if not louder than the Clap or Stamp.

An *Echo* Reflected from a Gate or Door, has uſually a baſer and duller Sound than that which is returned from a Wall, this being much brisker.

As I have been walking towards a Wall, I have Claped my Hands together ſeveral times, and I could diſtinguiſh the *Echo* from the Clap, till I came within 7 or 8 Yards of the Wall.

In the *Cloiſters*, where, as was ſaid before, the *Echo* was Repeated ſeveral times, the firſt Repetition ſeemed to be ſlower than the ſecond or third; but of all the Repetitions, beſides the firſt, the ſubſequent ſeemed ſlower than the precedent.

I have obſerved the Toſſing of a *Sound* forward and back again, in very many Places where there are Parallel Walls; and where the Diſtance of the Walls is leſs, there the *Echoes* follow one another quicker.

Whereſoever a *Sound* was thus toſſed betwixt two Walls, if I ſtood about the Middle, I could hear the *Sound* twice as quick, that is, twice as often Repeated in one Second, as if I ſtood near one Wall: The *Sound* being Reflected to me from both ends, when I ſtood in the middle.

In *Trinity Ball Court*, when I ſtood and Clapt at B, three or four Yards from the End of the Wall C, or at A, which is oppoſite to B, the *Sound* was toſſed betwixt the Oppoſite Walls; but not half ſo long as when I ſtood betwixt the Walls. In Places where there are Parallel Walls, not above ſix or Fig. 190.

 eight

eight Yards asunder, as in *Trinity Ball Court*, and at the Entrance into St. *John*'s *Grove*, *&c.* I have heard the Echoes of a Clap following one another distinctly enough; but there the *Echoes* of a *Musical Note*, which was longer than a Clap, were so confused, that they seemed one continued *Long Sound*: which makes me think, that the *Echo* in some Vaults, is nothing else but the *Sound* tossed betwixt the Side Walls, and betwixt the Top and Bottom. This also makes me conjecture, That the Reason why *stringed Musical Instruments* give a greater and longer *Sound* to the *Strings*, than if the *Strings* were fixt to a single board, may be this; because the *Sound* is tossed from side to side in the Belly of the *Instrument*.

*The Doctrine of Sounds; by* Narcissus, *Bishop of* Ferns *and* Leighlin. n. 156. *p.* 472.

XXII. I cannot better Explain the *Usefulness* of this *Theory of Sounds*, than by making a Comparison 'twixt the Faculties of *Seeing* and *Hearing* as to their Improvements. In order to which, I Observe, That *Vision* is threefold *Direct*, *Refracted*, and *Reflex'd*, answerable whereunto we have *Opticks*, *Dioptricks*, and *Catoptricks*.

In like manner *Hearing* may be Divided into *Direct*, *Refracted*, and *Reflex'd*; where to answer three Parts of our Doctrine of *Acoustick*; which are yet nameless, unless we call them *Acousticks*, *Diacousticks*, and *Catacousticks*, (or in another Sense, but to as good purpose) *Phonicks*, *Diaphonicks*, and *Cataphonicks*.

*Direct Vision* has been Improved two ways.

1. *Ex parte Objecti*, by the Arts of *Producing*, *Conserving*, and *Imitating*, and duly *Applying*, Light, and Colours.

2. *Ex parte Organi vel Medii*, by making use of *Tubes without Glasses*, or, a Man's Closed Hand to look through. So likewise *Direct Hearing*, partly has, and partly may further receive great and notable Improvements, both *ex parte Objecti*, and *ex parte Organi vel Medii.*

1. As to the Object of *Hearing*, which is *Sound*, Improvement has been, and may be made, both as to the *Begetting*; and as to the *Conveying* and *Propogating* (which is a kind of *Conserving*) of *Sounds*.

1. As to the *Begetting* of *Sounds*. The Art of Imitating any *Sound*, whether by Speaking (that is Pronouncing) any kind of Language, (which really is an *Art*; and the *Art* of Speaking perhaps one of the greatest) or by Whistling, or by Singing, (which are allowed *Arts*) or by Hollowing, or Luring, (which the *Huntsman* and *Faulkner* would have to be an *Art* also) or by Imitating with the Mouth (or otherwise) the voice of any Animal; as of Quails, Cats, and the like, or by Representing any *Sound* begotten by the Collision of Solid Bodies, or after any other manner; these are all *Improvements* of *Direct Hearing*, and may be *Improved*.

Moreover, the Skill to make all sorts of *Musical Instruments*, both Ancient and Modern, whither *Wind Instruments* or *String'd*, or of any other Sort, whereof there are very many (as *Drums*, *Bells*, the *Systrum* of the *Egyptians*, or the like) that *Beget* (and not only *Propagate*) *Sounds*: the Skill of Making these, I say, is an *Art*, that has as much Improv'd *Direct Hearing*, as an *Harmonious Sound* exceeds a Single and Rude one, that is an *Immusical Tone*: which *Art* is yet capable of farther Improvement. And I hope, That by the

Rules,

Rules, which may happily be laid down, concerning the *Nature*, *Propagation*, and *Proportion* or *Adapting* of *Sounds*, a way may be found out, both to Improve *Musical Instruments* already in uſe, and to Invent New Ones, that ſhall be more Sweet and Luſhious than any yet known. Beſides, that by the ſame means *Instruments* may be made, that ſhall Imitate any *Sound* in Nature, that is not *Articulate*; be it of Bird, Beaſt, or what thing elſe ſoever.

2. The *Conveying* and *Propagating* (which is a kind of *Conſerving*) of *Sounds*, is much helped by duly *Placing* the *Sonorous Body*, and alſo by the *Medium*.

For if the *Medium* be *Thin* and *Quieſcent*, and the *Sounding Body Placed* conveniently, the *Sound* will be eaſily and regularly *Propagated*, and mightily *Conſerved*.

1. The *Medium* muſt be *Thin* and *Quieſcent*; Hence in a ſtill Evening, or the Dead of the Night, (when the Wind ceaſes) a *Sound* is better ſent out, and to a greater Diſtance than otherwiſe.

2. The *Sonorous Body* muſt be *Placed* conveniently, *viz.* Near a *Smooth Wall*, either *Plane* or *Arched*, (*Cycloidically* or *Elliptically*, rather than otherwiſe; though a *Circular* or any *Arch* will do; but not ſo well.) Hence in a *Church*, the nearer the *Preacher* ſtands to the *Wall*, (and certainly its much the beſt way to place *Pulpits* near the *Wall*) the better is he heard, eſpecially by thoſe who ſtand near the *Wall*, alſo, though at a greater Diſtance from the *Pulpit*; thoſe at the Remoteſt End of the Church, by laying their Ears ſomewhat cloſe to the Wall, may hear him eaſier than thoſe in the Middle.

Hence alſo do ariſe *Whiſpering Places*. For the Voice being applied to one End of an *Arch*, eaſily Rowls to the other. And indeed were the Motion and *Propogation* of *Sounds* but rightly underſtood, 'twould be no hard matter to contrive *Whiſpering Places* of infinite variety and uſe. And perhaps there could be no better or more pleaſant Hearing a *Conſort* of *Muſick*, than at ſuch a Place as this; where the *Sounds* Rowling long together before they come to the Ear, muſt needs Conſolidate and Imbody into one; which becomes a true *Compoſition* of *Sounds*, and is the very Life and Soul of *Conſort*.

2. If the *Sonorous Body* be placed near *Water*, the *Sound* will eaſily be *Convey'd*, yet mollify'd; as Experience teacheth us from a *Ring* of *Bells* near a River; and a great Gun ſhot off at Sea; which differ much in the Strength, and yet ſoftneſs and continuance, or *Propagation* of their *Sounds* from the ſame at Land; where the *Sound* is more Harſh and more Periſhing, or much ſooner Decays.

3. In a *Plane* a Voice may be heard at a far greater Diſtance, than in Uneven Ground. The Reaſon of all which laſt nam'd *Phænomena* is the ſame, becauſe the *Sonorous Air* meeting with little or no Reſiſtance upon a *Plane* (much leſs upon an *Arch'd*) ſmooth *Superficies*, eaſily Rowls along it, without being let or hindred in its Motion, and conſequently without having its Parts Disfigured, and put into another kind of Revolution, than what they had at the firſt *Begetting* of the *Sound*, which is the true Cauſe of its *Preſervation* or *Progreſſion*; and fails much when the *Air* paſſes over an Uneven Surface, according to the Degrees of its Inequality; and ſomewhat alſo, when it paſſes over the *Plane Superficies* of a Body, that is hard and reſiſting.

Where-

Wherefore the *Smooth* Top of the *Water*, (by reaſon of its yielding to the *Arched Air*, and gently riſing again with a kind of *Reſurge*, like to *Elaſticity* though it be not ſo ; by which *Reſurge* it Quickens and Haſtens the *Motion* of the Air Rowling over it, and by its yielding preſerves it in its *Arched Cycloidical* or *Elliptical* Figure) the *Smooth* top of the *Water*, I ſay, for theſe Reaſons, and by theſe means, *Conveys* a *Sound* more entire, and to a greater Diſtance, than the *Plane Surface* of a piece of Ground, a Wall, or any other Solid Body whatever, can do.

2. The *Organ*, which is the *Ear*, is helpt much by *Placing* it near a Wall, (eſpecially at one end of an *Arch*, the *Sound* being *Begotten* at the other) or near the Surface of Water, or of the Earth ; along which the *Sounds* are moſt eaſily and naturally Conveyed ; as was before declared. And 'tis Incredible, how far a *Sound* made upon the Earth, (by the Trampling of a Troop of Horſes, for Example) may be heard in a ſtill Night, if a Man lays his Ear cloſe to the Ground in a large *Plane*.

*Otacauſticks* here come in for helping the *Ear* ; which may be ſo contrived (by a right underſtanding the *Progreſſion* of *Sounds*, which is the Principal Thing to be known for the due regulating all ſuch kinds of Inſtruments) as that the *Sound* might enter the *Ear* without any *Refraction*.

2. *Refracted Viſion* (which is always made *ex parte Medii*,) ariſes from the different *Denſity*, *Figure*, and *Magnitude* of the *Medium* ; which is ſomewhat altered alſo by the divers *Incidence* of the *Viſible Rays*, and ſo it is in *Refracted Hearing*, all theſe Cauſes concur to its *Production* ; and ſome others to be hereafter conſidered.

Now as any Object (a Man for Example) ſeen through a Thickned Air, by *Refraction* appears greater than really he is : So likewiſe a *Sound*, heard through the ſame Thickned part of the *Atmoſphere*, will be conſiderably vary'd from what it would ſeem to be, if heard through a Thinner *Medium*. And this I call a *Refracted Sound*.

Improvements of *Refracted Viſion* have been made, by Grinding or Blowing Glaſſes into a certain Figure, and Placing them at due Diſtances ; whereby the Object may be (as 'twas) enabled to ſend forth its Rays more Vigorouſly, and the *Viſive Faculty* Impowered the better to receive them. Thus,

1. A fine Glaſs Bubble, fill'd with Clear Water, and Placed before a Burning Candle or Lamp, does help it to dart forth its Rays to a Prodigious Length and Brightneſs.

2. The *Viſive Faculty* is much Helped.

1. By *Spectacles* and other Glaſſes which are made to Help the *Purblind* and *Weak Eyes*, to ſee at any competent Diſtance.

2. By *Perſpective Glaſſes* and *Teleſcopes*, which Help the *Eye* to *See* Objects at a very great *Diſtance*, which otherwiſe would not be diſcernable.

3. By *Microſcopes* or *Magnifying Glaſſes*, which Help the *Eye* to ſee *Near Objects*, that by reaſon of their Smallneſs were *Inviſible* before,

4. By *Polyſcopes* or *Multiplying Glaſſes*, whereby one thing is repreſented to the *Eye* as many, whether in the ſame or different Shapes.

After

After the ſame manner, Inſtruments may be contrived for aſſiſting both the *Sonorous Body* to ſend forth its *Sound* more ſtrongly, and the *Acouſtick Faculty* to receive and diſcern it more eaſily and diſtinctly. And thus;

1. An Inſtrument may be Invented, that applied to the *Mouth*, (or any *Sonorous Body)* ſhall ſend forth the *Voice* Diſtinctly as to a prodigious *Diſtance* and *Loudneſs*. For if the *Stentoro-phonicon* (which is but a Rude and Inartificial Inſtrument,) does ſuch great Feats; what might be done with One compoſed according to the Rules of Art? whoſe *Make* ſhould comply with the Laws of *Sonorous Motion* which that does not.

2. There are ſome *Inſtruments*, and more ſuch may be Invented to help the *Ear*: As,

1. *Otacauſticks* (and better may be made) to help *Weak Ears* to hear at a reaſonable Diſtance alſo. Which would be as great a help to the *Infirmity* of *Old Age*, as the other Invention of *Spectacles* is, and perhaps greater; for as much as the *Hearing* what's ſpoken is of more daily uſe and concern to ſuch Men, than to be able to *Read* Books, or to view Pictures.

2. A ſort of *Otacouſticks* may be ſo contrived, as that they ſhall Receive in *Sounds* made at a very great *Diſtance*, which otherwiſe would have been *Inaudible*. And theſe *Otacouſticks*, in ſome Reſpects, would be of greater uſe than *Perſpectives*.

1. In Time of *War* for diſcovering the *Enemy* at a good *Diſtance*, when he Marches or Lies Incamp'd behind a Mountain or Wood, or any ſuch Place of Shelter, which hinder the Sight from reaching very far.

2. At *Sea*, when in dark *Hazy Weather* the Air is too thick, or in *Stormy* Tempeſtuous *Weather*, the Waves Riſe too High, for the *Perſpective* to be made uſe of.

3. In Dark Nights, when *Perſpectives* become almoſt Inſignificant, and yet at ſuch times, generally Soldiers take their March, when they would ſurprize their Enemies.

4. *Microphones*, or *Micracouſticks*, that is, *Magnifying Ear Inſtruments*, which may be Contrived after that manner, that they ſhall render the moſt *Minute Sound* in Nature diſtinctly Audible, by *Magnifying* it to an unconceiveable Loudneſs. By the help whereof we may hear the different Cries and Tones of the ſmalleſt *Animals*.

5. A *Polyphone*, or *Polyacouſtick*, ſo ordered that One *Sound* may be heard, either of the Same, or a Different *Note*. In ſo much that who uſes this *Inſtrument*, he ſhall at the Sound of a *Single Viol* ſeem to hear a whole *Conſort*, and all True *Harmony*. By which means this *Inſtrument* has much the Advantage of the *Polyſcope*.

I have call'd it *Refracted Hearing*, becauſe made through a *Medium*, *viz.* Thick Air, or an Inſtrument, through which the *Sound* paſſing is broken or *Refracted*.

3. *Reflected Viſion* (which is always made *ex parte Objecti)* hath been Improv'd by the Invention of *Looking Glaſſes* and *Poliſh'd Metals*, whether *Plane*, *Concave*, or *Convex*, of ſeveral Figures, and Placed at Determinate Diſtances.

In-

In like manner *Reflex'd Audition* (which is only made *ex parte Corporis Oppositi*) may be Improved by Contriving ſeveral ſorts of *Artificial Echoes*. For (ſpeaking in general) any *Sound* falling *Directly* or *Obliquely* upon any Denſe Body, of a ſmooth (whether Plane or Arch'd) Superficies, is beat back again and Reflected, or does *Echo* more or leſs.

I ſay, (1.) *Falling Directly* or *Obliquely*; becauſe, if the *Sound* be ſent out and Propagated Parallel to the Surface of the Denſe Body, there will be no *Reflexion* of *Sound*, no *Echo*.

I ſay, (2.) Upon a Body of a *Smooth Superficies*; becauſe if the Surface of the *Corpus Obſtans* be Uneven, the Air by *Reverberation* will be put out of its Regular Motion, and the *Sound* thereby broken and extinguiſht: So that, tho' in this caſe alſo the Air be beaten back again, yet *Sound* is not Reflected, nor is there any *Eccho*.

I ſay, (3.) It does *Eccho More* or *Leſs*, to ſhew, that when all things are, as is before deſcrib'd, there is ſtill an *Ecchoing*, though it be not always Heard, either becauſe the *Direct Sound* is too Weak to be beaten quite back again to him that made it; or that it does Return home to him, but ſo weak, that without the help of a good *Otacouſtick* it cannot be diſcerned; or that he ſtands in a wrong Place to receive the *Reflected Sound*, which paſſes over his Head, under his Feet, or to one ſide of him; which therefore may be Heard by a Man ſtanding in that place, where the *Reflected Sound* will come, provided no interpos'd Body, does intercept it; but not by him, that firſt made it.

Theſe *Ecchoes* (like *Reflected Viſion*) may be ſeveral ways *Produced*, as;

1. A *Plane Corpus Obſtans Reflects the Sound* back in its due *Tone* and *Loudneſs*; if allowance be made for the proportionable Decreaſe of the *Sound* according to its Diſtance.

2. A *Convex Corpus Obſtans Repels* the *Sound* (inſenſibly) Smaller; but ſomewhat quicker (though weaker) than otherwiſe it would be.

3. A *Concave Corpus Obſtans Ecchoes* back the *Sound* (inſenſibly) Bigger, Slower, (though Stronger) and alſo *Inverted*; but never according to the order of Words. Nor do I think it poſſible for the Art of Man to Contrive a ſingle *Eccho*, that ſhall Invert the *Sound* and Repeat backwards; becauſe then the Words laſt ſpoken, that is, which do laſt occur to the *Corpus Obſtans*, muſt firſt be *Repell'd*; which cannot be. For where in the mean time ſhould the firſt Words hang and be conceal'd or lie dormant? Or how, after ſuch a Pauſe be Reviv'd and Animated again into Motion? Yet in Complicated or Compound *Ecchoes*, where many Receive from one another, I know not whether ſomething that way may not be done.

From the Determinate *Concavity* or *Archedneſs* of theſe *Reflecting Bodies* it comes to paſs, that ſome of them from a certain Diſtance or Poſiture, will *Eccho* back but one Determinate *Note*, and from no other Place will they *Reverberate* any; becauſe of the undue Poſition of the *Sounding Body*. Such an one (as I remember) is the *Vault* in *Merton College* in *Oxford*.

4. The

4. *The Eccho ing Body*, being Removed farther off, *Reflects* more of the *Sound*, than when nearer. And this is the Reafon, why fome *Ecchoes* Repeat but one Syllable, fome one Word, and fome many.

5. *Ecchoing Bodies* may be fo Contriv'd and Placed, as that *Reflecting* the *Sound* from one to the other, either *Directly* and *Mutually*, or *Obliquely* and by Succeffion, out of one *Sound* fhall many *Ecchoes* be begotten ; which in the firft Cafe will be altogether and fome what Involv'd or Swallowed up of each other ; and thereby Confufed (as a Face in Looking-Glaffes obverted ;) in the other they will be Diftinct, Seperate and Succeeding one another, as moft *Multiple Ecchoes* do.

Moreover, a *Multiple Eccho* may be made, by fo Placing the *Ecchoing Bodies*, at Unequal Diftances, that they *Reflect* all one way, and not One on the Other ; by which means a *Manifold Succeffive Sound* will be heard (not without aftonifhment) ; One Clap of the Hands like many ; One *Ha* like a Laughter ; One fingle Word like many of the fame *Tone* and *Accent* ; and fo one *Viol* like many of the fame kind, Imitating each other.

Furthermore, *Ecchoing Bodies* may be fo ordered, that from any one *Sound* given, they fhall produce many *Ecchoes* different both as to their *Tone* and *Intenfion*. By this means a *Mufical Room* may be fo Contrived, that not only One *Inftrument*, played on in it fhall feem many of the fame Sort and Size ; but even a *Confort* of (fomewhat) Different ones ; only by Placing certain *Ecchoing Bodies* fo, as that any *Note* (played) fhall be Returned by them in 3ds, 5ths, and 8ths, which is poffible to be done otherwife than was mentioned before in *Refracted Audition*.

I have been thus large, that I might give you a little Profpect into the Excellency and Ufefulnefs of *Acouflicks*, and that thereby I might excite others to bend their Thoughts, towards the making of *Experiments* for the Compleating this (yet very Imperfect though Noble) *Science* ; a *Specimen* whereof I will give you in thefe three *Problems*.

Prob. I.] *To make the leaft Sound (by the help of Inftruments) as Loud as the Greateft* ; a *Whifper to become as loud as the Shot of a Cannon.*

By the help of this *Problem* the moft minute Sounds in Nature may be Clearly and Diftinctly heard.

Prob. II.] *To Propagate any (the leaft) Sound to the greateft Diftance.*

By the help hereof any *Sound* may be Conveyed to any, and therefore heard at any *Diftance*, (I muft add, within a certain, though very large Sphear.)

Moreover by this means a *Weather-Cock* may be fo contrived, as that with an Ordinary Blaft of Wind it fhall Cry (or Whiftle) Loud enough to be heard many Leagues. Which happily may be found of fome Ufe, not only for Pilots in mighty Tempeftuous Weather, when *Light Houfes* are rendred almoft ufelefs. But alfo for the Meafuring the Strength of Winds, if allowance be made for their Different Moifture. For I conceive, That the more Dry any Wind is, the Louder it will Whiftle *cæteris paribus* ; I fay *cæteris paribus*, becaufe, befides the Strength and Drynefs of *Winds* or Breath, there are a great many other things (hereafter to be confider'd) that Concur to the Increafe of *Magnifying* of *Sounds*, begotten by them in an Inftrument expofed to their Violence, or Blown into.

Prob. III.] *That a Sound may be convey'd from one Extream to the other (or from one Diftant Place to another) fo as not to be heard in the Middle.*

By the help of this *Problem* a Man may talk to his Friend at a very confiderable Diftance, fo that thofe in the Middle Space fhall hear nothing of what paffed betwixt them.

I fhall here Add, a *Semiplane* of an *Acouftick* or *Phonical Sphear*, as an Attempt to explicate the great Principle in this *Science*, which is, the *Progreffion* of *Sounds*.

You are to conceive this (Rude) *Semiplane* as *Parallel* to the *Horizon*; for if it be *Perpendicular* thereunto, I fuppofe the upper *Extremity* will be no longer Circular, but *Hyperbolical*, and the lower part of it fuited to a greater Circle of the Earth. So that the whole *Phonical Sphear* (if I may fo call it) will be a folid *Hyperbola*, ftanding upon a *Concave Spherical Bafe*. I fpeak this concerning *Sounds* made (as ufually they are) nigh the *Earth*, and whofe *Sonorous Medium* has a free paffage every way. For if they are Generated High in the Air, or Directed one way, the Cafe will be different; which is partly Defigned in the Inequality of the Draught.

XXIII. A Paper, *of Lefs General Ufe*, Omitted. *viz.*

*Carriages.* n. 161. p. 666. EXperiments to be made, relating to* Carriages; *propofed by Sir* Will. Petty.

XXIV. Accounts *of Books* and Additions, Omitted.

n. 32. p. 626. 1. DE vi Percuffionis,* Joh. Alphonf. Borelli: Bononiæ. 1667. *in 4to.*

n. 73. p. 2210. 2. *De Motionibus à* Gravitate *dependentibus Liber*, Jo. Alphonfi Borelli. *in Academia* Pifana *Mathefeos* Profefforis, Regio Julio. 1670. *in 4to.*

n. 67. p. 2057. 3. *Dialogi Phyfici, quorum Primus de Lumine; Secundus & Tertius de* Vi Percuffionis *&* Motu; *Quartus de Humoris Elevatione per Canaliculum; Quintus & Sextus de Variis Selectis. Auth.* Honor. Fabry. *S. Jefu.* Lugduni Galliarum. 1669. *in 8vo.*

n. 54. p. 1086. n. 61. p. 2005. n. 76. p. 2286. n. 87. p. 5074. 4. Mechanica, *five de* Motu, *Tractatus Geometricus*; *Auth.* Joa. Wallis, *S. S. Th. D.* Londini. 1670. 1671. *in 4to. The* Author *here makes fome* Additions *to* Prop. I. Cap. XV. p. 753. *concerning the* Center *of* Gravity *of the* Hyperbola.

n. 61. p. 2008. 5. *Exercitationes Mechanicæ*, Alexandri Marchetti. Pifis. 1669. *in 4to.*

n. 82. p. 4050. 6. *De Refiftentia Solidorum*, Alexandri Marchetti *in* Pifana *Academia Phil. Prof.* Florentiæ. 1665. *in 4to.*

n. 73. p. 2213. 7. *Hypothefis Phyfica nova, five Theoria* Motus Concreti, *una cum Theoria* Motus Abftracti. *Auth.* Gothfredo Gulielmo Leibnitio. *J. V. D.* Lond. 1671.

n. 74. p. 2227. *in* 12°. *Of this Book Dr.* Wallis *here gives his Opinion.*

8. *La*

8. *La Statique, ou la Science des* Forces Mouvantes *par le* P. Ignace Gaston Pardies. *S. J. à* Paris. 1673. *in* 12°. *The First part being of* Local Motion. *Printed at* Paris 1670. *was* Englished *and Printed at* London *the same Year.* *in* 12°. n. 94. p. 6042. n. 65. p. 2010.

9. Christiani Hugenii Zulichemii *Horologium Oscillatorium.* Parisiis. 1673. *in* Fol. n. 95. p. 6068.

10. *A Discourse made before the* Royal Society *concerning the Use of* Duplicate Proportion *in sundry Important Particulars; together with a* New Hypothesis *of* Elastique *or* Springy Bodies: *By Sir* William Petty. n. 109. p. 209.

11. *Traite de la* Percussion *ou* Choq *des* Corps, *&c. par M.* Mariotte, *de* l'Academie Royal *des* Sciences. *A* Paris. 1673. *in* 12°. n. 134. p. 859.

12. *Philosophiæ Naturalis Principia Mathematica. Authore* If. Newton. Lond. *in* 4*to.* n. 186. p. 291. n. 226. p. 445.

13. *Traite de* Mouvement *des* Eaux *&* *des autres* Corps Fluids, *par feu* *M.* Mariotte. *a* Paris. 1686. *in* 8*vo.* n. 181. p. 119.

14. *Mechanick Exercises; or,* The Doctrine of *Handy Works. By Mr.* Jos. Moxon. Lond. 1677. *in* 4*to.* n. 138. p. 967. n. 139. p. 987.

15. *The* Speaking-Trumpet, *as it hath been Contrived, and Published, by Sir* Samuel Moreland; *together with its* Uses *both at* Sea *and* Land. Lond. 1671. n. 79. p. 3056.

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# CHAP. VI.

## *Hydrostaticks. Hydraulicks.*

*To Weigh Water, or other Fluids; by* n. 24. p. 446.

1. 1. TAke a *Viol* with a very *Narrow Body*, and when it is almost full, the Water is to be Dropt into it, drop by drop, till it can hold no more. Then *Weigh* it exactly, and deduct the *Weight* of the *Empty Viol.*

*A New Aræometer; by M.* Homberg: n. 262. p. 530. *Fig.* 192.

2. A, Is a Glass Bottle like a little *Matracium*, of which the *Neck* BC, is so small that a Drop of Water therein takes up the Space of 5 or 6 Lines, near that *Neck* is a little Capillar Tube D, about 6 Lines long, and Parallel to the *Neck* BC; The Opening B is a little dilated, in the Fashion of a Tunnel, for pouring more easily the Liquors into the Bottle, and the little Tube D, is for giving a way to the Air contained in that Vessel to go out, when the Liquor is poured in at B; the Point C, is a little Mark at the same height, as the end of the little Tube D.

When we fill the Vessel, we pour the Liquors into it, by the Opening B, until it goes out by the little Tube D, and if the Height of the Liquor is even to the Mark C, 'tis well; if it is Lower, we must fill more to that Point; if it is Higher, we must strike softly upon the Opening B, till the Overplus of the Liquor be even to the Point C in the *Neck* of the Bottle. By that means we have always exactly the same *Volume* of *Liquor*, and we can know how the same *Volume* of the several Liquors Weighs more one than another precisely. But we must consider the Variation of the Weather when we compare the Weight of a Liquor which we Weigh in Summer time, with the Weight of another, which we have Weighed in the Winter, for the same Liquor being more Rarified in the Hot time, and condensed in the Cold, the same Volume of it will be more Weighty in Cold Weather than in Warm.

*A New Essay Instrument; by Mr. Boyle. n. 24. p. 447. n. 115. p. 329. Fig. 193.*

II. 1. Many Years ago I made use of a little *Glass Instrument*, consisting of a *Bubble*, and furnished with a *Long* and *Slender Stem* to compare the *Specifick Gravities* of *Different Liquors* by its more or less *Sinking* in *Them*: And I have since employed it to discover the *Specifick Gravities* of *Solid*, *several appended*, by its being more or less depressed by them in the *same Liquor*. For 'tis clearly deducible from the Grounds of *Hydrostaticks*, that any Solid Body Heavier than Water, loses in the Water as much of the Weight it had in Air, as Water of equal Bulk to the Immersed Solid would Weigh in the Air; and consequently since Gold is by far the most Ponderous of Metals, a piece of Gold and one of Equal Weight of Copper, Brass, or any other Metal, being proposed, the Gold must be less in bulk, than the Copper or Brass. And by this means, if both of them be Weighed in the Water, the Gold must lose in that Liquor less of its former Weight than the Brass or Copper; because the baser Metal as well as the Gold, grows Lighter by the weight of a Bulk of Water equal to it; and the baser Metal being the more Voluminous, the correspondent Water must Weigh more than that which is correspondent to the Gold. Whence I concluded, that the *Floating Instrument* abovemention'd would be made to sink deeper by an Ounce, for instance of Gold, hanging at it under Water, than by an Ounce of Brass, or any other Metal, which, by reason of its greater Bulk than Gold, losing more of its weight by the Immersion, must needs retain less, and so have less power to Depress the Instrument 'twas fastned to. Which Conclusion will also hold (though the Disparity be not so Great and Conspicuous) in reference to other Metals, as Lead and Tin, that differ in *Specifick Gravity*.

This *Instrument* may be of *Glass*, *Copper*, *Silver*, or almost any other Solid *Body*, that is, or may be made, fit to *Float* in the Water, with a *Guiny*, &c. *Hanging* at it, and of a Texture close enough to keep out the Water. It consists of three Parts; the *Ball* or Globulous part; the *Stem* or Pipe; and that which *Holds* the *Coin*.

*Fig. 194.*

The *Ball* or Round part BCDE (if of *Metal*) consists of two thin Concave Plates, exactly Sodered together in the middle; and at the distantest parts from the Commissure, there ought to be left two opposite *Holes*, one in each Plate, for the two other parts of the Instrument. This middle part, though for Brevities sake we name it the *Ball*, should not be exactly Round; but, of any Shape that shall be found fit to make the *Instrument* keep its Erect posture steadily in the Water. It must contain as much Air, as may serve to keep the whole *Instrument*, when loaded from Sinking beneath the top of the *Stem*.

The *Stem* AB, is to be Soder'd on to the *Ball* at the uppermost of the two mention'd *Holes*. It may be either Hollow or Solid: but it ought to be

made

made very ſlender, that the different Depreſſions of the Inſtrument in the Water may be the more Notable. And for the ſame reaſon, it ought not to be too Short, eſpecially if it be to be applied to other Uſes than the Examining of *Guinys*.

At the Undermoſt of the two *Holes* in the *Ball*, is Inſerted and Soder'd the undermoſt part of the *Inſtrument*, which I call the *Screw*, or the *Stirrup*. The *Screw* F, is a very ſhort piece of Braſs with a broad Slit in it, capable of receiving the edge of the *Guiny*, which with one turn or two of a ſmall and ſlight lateral Screw may be kept faſt in it, and readily the Operation being ended, taken out again. The *Stirrup* G is made of a piece of Wire, that a little beneath the bottom of the *Ball*, is bent round, ſo as to ſtand Horizontally, that the *Guiny* may be laid on it. Fig. 195.

It would be convenient, that the *undermoſt Stem* and the *Screw* be made by it ſelf, that it may be at pleaſure thruſt upon the *Stem* and taken off again. For, by this means, if the *Ball* of the *Inſtrument* be made large enough, you may have room to put on for Ballaſt, as occaſion ſhall require, one, two, or three flat and round pieces of Copper, Lead, &c, with each of them a hole in the middle fitted to the Size of the Stem, ſo that they may be put on as near the Lower part of the *Ball* as you think fit, and then the *Screw* may be thruſt on after them, not only to take hold of the *Coin* or Metalline mixture to be *Examin'd*, but to ſupport the thin Plates. Fig. 196.

To adjuſt this *Inſtrument* for the uſe of examining *Guinys*, which are by far the moſt uſual *Gold Coins* that paſs in *England*, you muſt by the help of the *Stirrup* or *Screw*, Hang, at the bottom of it, a piece of that *Coin* which you know to be Genuine, (and having carefully ſtopt the Orifice of the *Stem* (if it be a Pipe) that no Water may get in at it) Immerſe the *Inſtrument* leiſurely and perpendicularly into a Veſſel full of clean Water, till it be Depreſt almoſt to the top of the *Stem*, and then letting it alone, if, being Setled, it continue in the ſame Station and Poſture, your work is done. If it Emerge, you muſt add a little weight to it, either by putting into the *Stem*, if it be Hollow, ſome Duſt Shot, Filings of Lead, or ſome other Minute and Heavy Body, or elſe by putting on the ſhort *Stem* abovementioned, that comes out beneath the *Ball*, a flat, round and perforated piece of Lead, of Weight ſufficient to enable the *Guiny* to Depreſs the weight as Low as its deſired: But if it Sink quite under Water, you muſt lighten it either with a File, or by ſcraping or grating off a little of the Ballaſt Plate abovementioned; or, if you have put any Weight into the Cavity to poiſe it, by taking out ſome of that, till you have made it Light enough: This being done, a Mark H is to be made juſt at the place where the Surface of the Water touches the *Stem*, and then taking out your *Inſtrument*, ſubſtitute in the place of your *Guiny* a little round Plate of *Braſs*, of the ſame Weight, or a Grain or two heavier, in the Air; and putting the *Inſtrument* into the Water, as before, ſuffer it to Settle, and make another Mark I. at the Interſection of the *Stem* and the Horizontal Surface of the Water. Fig. 193. Fig. 194.

There may (though 'tis like there very ſeldom will) happen a Caſe, wherein, though the Principle, our *Inſtrument* is framed on, will hold good, yet the

the Practical Application may be Unsecure. For if a *Falsifier* of *Money* have the Skill, by *Washing* or otherwise, to take off much of the Quantity or Substance of the *Guiny* without altering or impairing either the *Figure* or *Stamp*, the piece of Coin will not be able to Depress our *Instrument* to the Usual Mark, and may thereby make it to be judged *Counterfeit*, when 'tis indeed but too *Light*. But it presently shews, that the proposed *Guiney*, if it be not *Counterfeit*, is otherwise *Abused*; and though it does not clearly determine, whether that likewise proceed from the want of *Specifick Gravity* in the Metal, or from the Coins having been *Washed* or otherwise fraudulently *Lessned*; yet it probably resolves the doubt, because, if the want of Weight appear by the *Instrument* to be very great, as it usually does, where the pieces has been Robbed of some of its Substance, 'tis a strong Presumption, that 'tis rather *Washed*, *&c.* than *Counterfeited*. However, it will be sure to prompt him that uses it, to employ the Ballance, which will presently assist him to resolve his doubt. For if the Suspected Coin have in the Air its due Weight, 'twill argue that the great Lightness of it in the Water, proceeds from it's not being of the requisite fineness; and, if it want much of its due Weight in the Air, 'tis very probable, that 'tis *Washed*, *&c.* rather than of another Metal than *Gold*.

Any other kind of *Gold Coin*, that is near about the Weight of a *Guiny*, may be Examined by our *Instrument* after the manner above deliver'd. If the *Coin* be Heavier than a *Guiney*, as is a Twenty Shilling piece of *Broad Gold*, the *Ballast*, whether internal or external, of the *Instrument*, must be taken off, that so Heavy a *Coin*, may not quite sink it. But if it be Lighter than a *Guiny*, one may add as much *Gold* (of the same *Alloy*) beaten into thin Plates, as with the Coin proposed, will make up in the Air the Weight of a *Guiny*. For then this Aggregate, being examined as if it were a *Guiny*, will discover in the Water, whether the *Coin* be *Right* or *Counterfeit*.

This *Instrument* may be also made to serve to examine some sorts of *White Money*, less Heavy than Half Crowns. And because it may be useful to know in General, what Coins may, and what may not, be Examined by this or that particular *Instrument* propos'd, I shall here add a general way that is not difficult for finding this out; namely, first by Weighing the piece of *Gold* or *Silver* in the Air, and afterwards in the Water, and Subtracting the latter from the former, to obtain the Difference of the two Weights: And next by Weighing also in the Air and in the Water a piece of Copper or Brass, if this be the likeliest to be employed in *Counterfeiting* the *Coin*, and observing likewise the Difference between those Weights. For the lesser of these Differences being Subtracted from the greater, the Remains will shew, how much the true piece of Coin will out-Weigh the other in the Water, and consequently if so many Grains, as this residue amounts to, being Added to the Weight of the Lighter Metal, do make a sufficiently manifest Depression of it below the Mark it would stay at without that Addition, one may probably Conclude, that the Difference between a True and Counterfeit piece of Coin proposed, will be discoverable by the *Instrument*. But it may be Expedient, for those that have frequent Occasions to Examine Various sorts of Coin,

to

to have a ſeveral *Inſtrument* adjuſted for each of them, to ſave themſelves ſome Pains and Trouble.

With this *Inſtrument*, Pure *Tin* may be certainly Diſtinguiſht from ſuch as is Adulterated. For as *Gold*, being the Heavieſt of Metals, cannot be Allay'd by any other that will not Depreſs our *Inſtrument* leſs than *Gold* can do; ſo *Tin* being the Lighteſt of Metals, cannot be mixed with any other that will not Sink it Lower than unmixt *Tin*, (ſtill ſuppoſing the Weight to be the ſame in the Air.)

After the ſame manner may *Pewter* be Compared and Examin'd. For having once obſerv'd how much the *Inſtrument* is Depreſt by a piece of two, three, or four Drams, or even an Ounce Weight of *Pewter*, which is known to be good, and to contain ſuch a proportion of Lead in reference to the Tin, if you load the *Inſtrument* with an equally Heavy piece of any other Maſs of *Pewter* propounded, if the *Inſtrument* Sink deeper, 'twill be a ſign that the former Proportion of Lead may be very probably argued to exceed in the mixture; I ſay probably, becauſe perhaps 'tis poſſible to Embaſe *Pewter* by Mixing not only Lead but other Mineral Subſtances, whoſe *Specifick Gravity* is not well known: But yet I ſay very probably, becauſe the Addition of too much Lead is the moſt Gainful way of Adulterating *Pewter*.

This *Inſtrument* may alſo aſſiſt us, to make ſuch an Eſtimate as will not much Deceive them of the *Fineneſs* of *Gold* and its differing *Allays* with Silver, or ſome other determinate Metal.

In order to this, the *Inſtrument* may be fitted to ſink to the top of the Pipe with ſome determinate weight of the *Fineſt Gold*, as of 24 *Carats*, as they call that which is moſt *Pure* and *Fine*. But 'twill be convenient, that this Metal, in the Air be juſt an Ounce, or half an Ounce, or ſome ſuch Determinate Weight, that is commodiouſly Diviſible into many aliquot Parts. Then you may make a Mixture that contains a known proportion of the Metal wherewith you *Allay* the *Gold*; as if it hold 19 or 15 parts of *Gold*, and one of *Silver*; and, letting the *Inſtrument* ſettle in the Water, Mark the place where the Surface of the Water cuts the *Stem* or *Pipe*. And then putting in another Mixture, wherein the Silver has a new and greater Proportion to the *Gold*; as if the former be an 18th or a 14th part of the Latter, you may Obſerve, how much leſs than before this Depreſſes the *Inſtrument*, and ſo you may proceed with as many Mixtures or Degrees of *Allays* as you think fit, or can be Diſtinguiſht conveniently on the *Stem*; being always careful, that, whatever be the Proportion of the two Ingredients, the Weight of the Maſs in the Air be juſt the ſame with that of the *Pure Gold*, which we may have lately ſuppoſed to be an Ounce, or half an Ounce.

By the ſame Method may be Examined the differing *Allays* of *Pure Silver*, upon the Admixture of ſuch and ſuch determinate Proportions of Copper, or any other Metal Lighter in Specie than *Silver*; and by the ſame way, with a ſlight Variation, 'twill not be difficult to Eſtimate, how much divers *Coins*, whether of *Silver* or *Gold*, are more or leſs Embas'd by the known Ignobler Metal that is mixt in the piece propoſed. Theſe Eſtimates (which may be made without much Trouble) will come nearer the Truth, not only than

the

the Eſtimates wont to be made by the *Touch-Stone*, but perhaps too, than ſome of thoſe that divers make with Trouble, Inconvenience, and Charge.

It may be alſo Employ'd to Examine other *Mixtures* beſides *Allay'd Coins*, and that if the *Inſtrument* be adjuſted to an Ounce, for inſtance, of Pure Copper, it may help Men to make an Eſtimate of the *Allay* of *Tinn*, or the Quantity of it that is often times added to *Copper*, to make Different Sorts of *Bell Metal*, and of thoſe *Metalline Specula*, whether Plain or Concave, that are call'd Steel Glaſſes, as alſo of Soders conſiſting of certain Proportions of Silver and Braſs, or Copper; in all which, and divers others, the Diſcovery of the Proportion of the Ingredients, may, on ſome Occaſions, be Uſeful to Tradeſmen, as well as deſireable to *Virtuoſi*. And though I have Obſerved, that by Mixture, Tin and Copper acquire a *Specifick Gravity* ſomewhat differing from what their Ingredients promiſe; yet ſince the *Inſtrument* is to be fitted for ſuch Eſtimates, not by Calculation, but by Trials, the Eſtimates may be made near enough to the Truth.

*Further Conſidered; by ... n. 116. p. 553.*

2. Long ſince I took Notice, how Light and Silver-like the Pewter was which deſcended to us; but as ſoon as, to follow the Faſhion, we Changed it, the Weight and the very Colour was altered; and is in every Change more and more Embaſed. And, if our *Silver-Smiths* hold on their Degrading Mixtures, I ſhall Queſtion, whether our Silver-Plate may not ſhortly come down to approach our Fore-Fathers Pewter: I mean in the Country where 'tis never or ſeldom Tried.

*The Weight of Water in Water; by Mr.* Boyle. *n. 50. p. 1001.*

III. A Glaſs Bubble of about the Bigneſs of a Pullets Egg was purpoſely blown at the Flame of a Lamp, with a ſomewhat long Stem turn'd up at the End, that it might the more conveniently be broken off. This Bubble being very well heated to Rarifie the Air, and thereby drive out a good part of it, was nimbly Sealed at the End, and by the help of the Figure of the Stem, was by a convenient Weight of Lead depreſſed under Water, the Lead and Glaſs being tied by a String to one Scale of a good Ballance, in whoſe other there was put ſo much Weight, as ſufficed to Counterpoiſe the Bubble, as it hung freely in the midſt of the Water. Then with a long Iron Forceps I carefully broke off the Seal'd End of the Bubble under Water, ſo as no Bubble of Air appear'd to Emerge or Eſcape through the Water, but the Liquor by the Weight of the Atmoſphere ſprung into the unrepleniſht part of the Glaſs Bubble, and fill'd the whole Cavity about half full; and preſently as I foretold, the Bubble ſubſided, and made the Scale it was faſtened to, Preponderate ſo much, that there needed 4 Drachms and 38 Grains to reduce the Ballance to an *Æquilibrium*. Then taking out the Bubble with the Water in't, we did by the help of the Flame of a Candle, warily applied, drive out the Water (which otherwiſe is not eaſily excluded at a very narrow Stem) into a Glaſs Counterpoiſed before; and we found it, as we expected, to Weigh about 4 Drachms and 30 Grains, beſides ſome little that remained in the Egg, and ſome ſmall matter that may have been Rarify'd into Vapours, which added to the Piece of Glaſs that was broken off under Water and loſt there, might very well amount to 7 or 8 Grains. By which it appears not only, that Water hath

hath some Weight in Water, but that it Weighs very near or altogether as much in Water, as the self same Portion of a Liquor would Weigh in the Air. We Repeated the Experiment with another Seal'd Bubble as big as a great Hen Egg, with like Success.

IV. *Apr.* 7. 1680. Being off of *Pantalara* near *Sicily* in a Calm, I let down a *Bottle* 70 *Fathom*, stopp'd with an excellent good tender Cork, well Fitted, and the Cork came up in the Bottle $\frac{3}{4}$ full of Salt Water. The Bottle was again fitted with an Excellent good Cork, but of a Woodiness or Hardness as some Corks are, with the which, being let down in like manner, the Cork continued in its Place, but as it were Bruised, and the Bottle as before, about $\frac{3}{4}$ full of Salt Water: Whereupon I took a good Ox Bladder, and bound it four fold over the Mouth of the Bottle without any Cork at all, only I put a piece of Leather to keep the Glass from cutting the Bladder; and so ordered, it was let down as before, but taken up without any Water, or the least Moisture in it.

*The Pressure of Water in great Depths; by a Person of Honour.* n. 193. p. 504.

*May* 18. 1680. Being in a Stark Calm some Leagues distant from the Coast of *South Spain*, off the great Hills of *Granada*, we took a Bottle and clapt a Leather on the Mouth of it, tying over that a single part of a Bladder, the which we let down 75 *Fathom*, but it came up again Entire; We then made a Hole in the Leather about the bigness of a large Pea, and let the same down again 75 *Fathom*, but it came up perforated in the Vacant place where the Leather had the Hole in it, and almost full of Water; we then bound over another part of Bladder single, and let it down but 30 *Fathom*, but it came up whole and entire; whereupon immediately we let it down 50 *Fathom*, but it came up broke and full of Water. Then we again fitted the Bottle with the said perforated piece of Leather and a Double Bladder; and let it down 50 *Fathom*, but it again came up Entire: so again, immediately we let it down 75 *Fathom*, but then it came up broken and full of Water.

*June* 24. 1680. Being in 39$\frac{1}{4}$ Degrees of *Latitude*, and by the Ships Account 150 Leagues Westward of *Portugal*, I caused a *Florence Flask* to be well stopped with a Bladder over the Mouth of it, and Lower'd it down 30 *Fathom*, but it was taken up broken. Whereupon imagining that the roughness of the Leads Halling so tender a Body so violently through the Water, might be the breaking thereof, I caused another *Flask* in like manner to be fitted, and close by it I tied likewise another *Flask* so as to be born with the Mouth downwards, as were the other, but which was not Stopp'd; and these I caused to be taken up when they had been but 10 *Fathom* under Water; and found them both Entire, but the Open *Flask* almost full of Water; the which being emptied, were both let down again and taken up at 20 *Fathom*, when the Open Flask was Entire, tho' full of Water, but the Other broken to pieces.

2. *Jun.* 8. 1693. In the *Bay* of *Biscay*, when we had 100 *Fathom* of Water, we took a Quart Glass Bottle stopt with a large Cork: and Fastening it to our Plumbing-Rope with a Lead at the End, we sunk it to the *Bottom* of the Sea,

*By Dr. Oliver.* n. 204. p. 908.

which as ſoon as we perceived, we drew it up again, and found the Cork quite preſſed through the Neck of the Bottle into its Cavity, and the Bottle full of Salt Sea-Water. We Repeated our *Experiment* with another Bottle and Cork in the ſame manner as before, but the Cork being not ſound, the Sea-Water ſoaked through it, and the Bottle was half full of Water, ſo the Cork remained in the Mouth of the Bottle not preſs'd down at all. We Repeated our *Experiment* a third time in 90 *Fathom* of Water, with a very ſound Cork, and much larger than the Mouth of the Bottle. We beat it in with a Hammer as far as it would go, leaving about an Inch of the Cork above the Mouth of the Bottle. The Cork at this *Tryal* was preſſed down only into the Neck, and became Level with the Mouth of the Bottle: But I really believe, had we had 10 or 20 *Fathom* of Water more, it would have ſucceeded as at our firſt *Tryal*.

*The Weight of divers Bodies try'd by the Direction of the* Phil. Society at Oxford. n. 169. p. 926.

V. 1. The following Bodies were poured gently into a Veſſel of well ſeaſoned Oak, whoſe Concave was an exact *Cubic Foot*. Thoſe in the Twelve firſt Experiments were Weighed in *Scales* turning with two Ounces, but the laſt Seven were Weighed in *Scales* turning with one Ounce. The *Pounds* and *Ounces* here mentioned are *Averdupois*.

| | | Lib. | Oun. |
|---|---|---|---|
| 1 | A Foot of *Wheat* (worth 6 *s.* a Buſhel.) | 47 | 8 |
| 2 | *Wheat* of the beſt ſort (worth 6 *s.* 4 *d.* a Buſhel.) | 48 | 4 |
| 3 | The ſame ſort of *Wheat* meaſured a ſecond time. | 48 | 2 |
| | Both ſorts were *Red Lammas Wheat* of the laſt Year. | | |
| 4 | *White Oats* of the laſt Year. | 29 | 8 |
| | The beſt ſort of *Oats* were 2 *d.* in a Buſhel better than theſe. | | |
| 5 | *Blew Peaſe* (of the laſt Year,) and much Worm-eaten. | 49 | 12 |
| 6 | *White Peaſe* of the laſt Year but one. | 50 | 8 |
| 7 | *Barley* of the laſt Year: (the beſt ſort ſells for 1 *s.* 6 *d.* in a Quarter more than this:) | 41 | 2 |
| 8 | *Malt* of the laſt Years *Barley*, made two Months before. | 30 | 4 |
| 9 | Field *Beans* of the laſt Year but one. | 50 | 8 |
| 10 | *Wheaten Meal* (unſifted.) | 31 | 0 |
| 11 | *Rye Meal* (unſifted.) | 28 | 4 |
| 12 | *Pump Water.* | 62 | 8 |
| 13 | *Bay Salt.* | 54 | 1 |
| 14 | White Sea *Salt.* | 43 | 12 |
| 15 | *Sand.* | 85 | 4 |
| 16 | *Newcaſtle Coal.* | 67 | 12 |
| 17 | *Pit-Coal* from *Wednesbury* 63, but this is very uncertain in the filling the Interſtices between the greater pieces. | 63 | 0 |
| 18 | *Gravel.* | 109 | 5 |
| 19 | Wood-*Aſhes.* | 58 | 5 |

| | |
|---|---|
| *Pump-Water.* | 1000 |
| *Fir* Dry | 546 |
| *Elm* Dry. | 600 |
| *Cedar* Dry. | 613 |
| *Walnut-Tree* Dry. | 631 |
| *Crab-Tree* meanly Dry. | 765 |
| *Ash* meanly Dry, and of the Out-side Lax part of the Tree. | 734 |
| *Ash* more Dry, but about the Heart. | 845 |
| *Maple* Dry. | 755 |
| *Yew* of a Knot or Root 16 Years old. | 760 |
| *Beech* meanly Dry. | 854 |
| *Oak* very Dry, almost Worm-eaten. | 753 |
| *Oak* of the Outside sappy part Fell'd a Year since. | 870 |
| *Oak* Dry, but of a very sound close texture. | 929 |
| The *Same* tried another time. | 932 |
| *Logwood.* | 913 |
| *Claret.* | 993 |
| *Meil* Cyder, not Clear. | 1017 |
| *Sea Water*, settled Clear. | 1028 |
| College Plain *Ale* the same. | 1028 |
| *Urine.* | 1030 |
| *Milk* | 1031 |
| *Box* the same. | 1031 |
| *Redwood* the same. | 1031 |
| *Sack.* | 1033 |
| *Beer Vinegar.* | 1034 |
| *Pitch.* | 1150 |
| *Pit-Coal* of *Staffordshire.* | 1240 |
| *Speckled Wood* of *Virginia.* | 1313 |
| *Lignum Vitæ.* | 1327 |
| *Stone* Bottle. | 1777 |
| *Ivory.* | 1826 |
| *Alabaster.* | 1872 |
| *Brick.* | 1979 |
| *Heddington Stone*, the Soft Lax kind. | 2029 |
| *Burford Stone*, an old Dry piece. | 2049 |
| *Paving Stone*, a hard sort from about *Blaidon.* | 2460 |
| *Flint.* | 2542 |
| *Glass* of a Quart Bottle. | 2666 |
| *Black Italian Marble.* | 2704 |
| *White Italian Marble* tried twice. | 2707 |
| *White Italian Marble* of another sort, of a visibly Closer Texture. | 2718 |
| Block *Tin.* | 7321 |
| *Copper.* | 8843 |
| *Lead.* | 11345 |
| *Quick Silver.* | 14019 |
| *Quick Silver.* | 13593 |

2. The Specifick Gravities of *several Bodies, by the Direction of the* Phil. Society *at* Oxford. *Ib. p.* 927.

 The

The laſt *Experiment* was tried with another quantity of *Quick Silver*, which had been uſed in Water in the preceeding *Experiment* : However, I rather truſt the laſt, for that I found a ſmall miſtake (tho' here in the Calculation allowed for) in the Weight of the *Glaſs* containing the *Quick Silver* in the Trial before.

The *Solids* here mentioned, were Examined *Hydroſtatically* by Weighing them in Air and Water ; but the *Fluids*, by Weighing an equal Portion of each in a Glaſs holding about a Quart. The Numbers ſhew the Proportion of *Gravity* of equal Portions of theſe Bodies : but if of theſe Bodies we take Portions equally Heavy, their *Magnitudes* will be *reciprocally* proportional to their correſpondent Numbers : *e. g.* a Cubic Foot of *Water* is to a Cubic Foot of *Alabaſter* in *Gravity* as 1000 to 1872 ; but a Pound Weight of *Water*, is to a Pound Weight of *Alabaſter* in *Magnitude* as, 1872 to 1000. So that, knowing by the former Table, the Weight of a Cubick Foot of *Water*, and by this the proportion in *Gravity* betwixt *Water* and *Alabaſter*; we may by the Rule of Three find the *Weight* of a Cubick Foot of *Alabaſter*, and ſo of any other of theſe Bodies ; or we may know their *Magnitude* by knowing their *Gravity*. So that an Irregular piece or quantity of theſe Bodies being offered, 'tis but *Weighing* them, and we may know their juſt *Magnitude* without farther trouble.

3. By Mr. J. C. n. 199. p. 694.

| | |
|---|---|
| *Pump-Water.* | 1000 |
| *Cork.* | 237 |
| *Saſſafras Wood.* | 482 |
| *Juniper Wood* ( Dry. ) | 556 |
| *Plum-Tree* (Dry.) | 663 |
| *Maſtic.* | 849 |
| *Santalum Citrinum.* | 809 |
| *Santalum Album.* | 1041 |
| *Santalum Rubrum.* | 1128 |
| *Ebony.* | 1177 |
| *Lignum Rhodium.* | 1125 |
| *Lignum Aſphaltum.* | 1179 |
| *Aloes.* | 1177 |
| *Succinum Pellucidum.* | 1065 |
| *Succinum Pingue.* | 1087 |
| *Jet.* | 1238 |
| The Top part of a *Rhinoceros Horn.* | 1242 |
| The Top part of an *Ox Horn* | 1840 |
| The (Blade) *Bone* of an *Ox.* | 1656 |
| An *Humane Calculus.* | 1240 |
| Another *Calculus Humanus.* | 1433 |
| Another *Calculus.* | 1664 |
| *Brimſtone*, ſuch as commonly Sold. | 1811 |
| *Borax.* | 1720 |

| | |
|---|---|
| A Spotted *Factitious Marble.* | 1822 |
| A *Gally-Pot.* | 1928 |
| *Oyster Shell.* | 2092 |
| *Murex Shell.* | 2590 |
| *Lapis Manati.* | 2270 |
| *Selenitis.* | 2322 |
| Wood *Petrified* in *Lough Neagh.* | 2341 |
| *Onyx Stone.* | 2510 |
| *Turcois-Stone.* | 2508 |
| *English Agat.* | 2512 |
| *Grammatias Lapis.* | 2515 |
| A *Cornelian.* | 2568 |
| *Corallachates.* | 2605 |
| *Talc.* | 2657 |
| *Coral.* | 2689 |
| *Hyacinth* (Spurious.) | 2631 |
| *Jasper* (Spurious.) | 2666 |
| A Pellucide *Pibble.* | 2641 |
| *Rock Crystal.* | 2659 |
| *Cryſtallum Diſdiaclaſticum.* | 2704 |
| A Red *Paſte,* | 2842 |
| *Lapis Nephriticus.* | 2894 |
| *Lapis Amiantus* from *Wales.* | 2913 |
| *Lapis Lazuli.* | 3054 |
| An *Hone.* | 3288 |
| *Sardachates.* | 3598 |
| A *Granat.* | 3978 |
| A Golden *Marcaſite.* | 4589 |
| A Blew *Slate* with ſhining Particles. | 3500 |
| A *Mineral Stone*, yielding 1 part in 160 *Metal.* | 2650 |
| The *Metal* thence Extracted. | 8500 |
| The (reputed) *Silver Ore* of *Wales.* | 7464 |
| The *Metal* thence Extracted. | 11087 |
| *Biſmuth.* | 9859 |
| *Spelter.* | 7065 |
| *Spelter Soder.* | 8362 |
| *Iron* of a Key. | 7643 |
| *Steel.* | 7852 |
| Caſt *Braſs.* | 8100 |
| Wrought *Braſs.* | 8280 |
| Hammer'd *Braſs.* | 8349 |
| A Falſe *Guiny.* | 9075 |
| A True *Guiny.* | 18888 |
| *Sterling Silver.* | 10535 |
| A *Braſs* Half Crown. | 9468 |

| | |
|---|---|
| *Electrum* a *British Coin.* | 12071 |
| A *Gold Coin* of *Barbary.* | 17548 |
| A *Gold Medal* from *Morocco.* | 18420 |
| A *Mentz Gold Ducat.* | 18261 |
| A *Gold Coin* of *Alexanders.* | 18893 |
| A *Gold Medal* of *Q. Marys.* | 19100 |
| A *Gold Medal* of *Q. Elizabeths.* | 19125 |
| A *Medal* esteemed to be near *Fine Gold.* | 19636 |

*The Different Weight of several Liquors in* Winter *and* Summer; *by M.* Homberg. n. 262. p. 530. *Vid. Sup.* §. I. 2.

VI. M. *Homberg*, has given us the following Table, of the *Various Weights* of some more Usual Liquors in the *Coldest* Time, and in the *Hottest.*

| | In Summer. | | | In Winter. | | |
|---|---|---|---|---|---|---|
| | ℥ | ʒ | gr. | ℥ | ʒ | gr. |
| The *Areometer* full of *Mercury.* | 11 | 0 | 6 | 11 | 0 | 32 |
| *Oil* of *Tartar.* | 1 | 3 | 8 | 1 | 3 | 31 |
| *Spirit* of *Urine.* | 1 | 0 | 32 | 1 | 0 | 43 |
| *Oil* of *Vitriol.* | 1 | 3 | 58 | 1 | 4 | 4 |
| *Spirit* of *Nitre.* | 1 | 1 | 40 | 1 | 1 | 70 |
| *Spirit* of *Salt.* | 1 | 0 | 39 | 1 | 0 | 47 |
| *Aqua Fortis.* | 1 | 1 | 38 | 1 | 1 | 55 |
| *Vinegar.* | 0 | 7 | 55 | 0 | 7 | 60 |
| *Spirit* of *Wine.* | 0 | 6 | 47 | 0 | 6 | 61 |
| *River Water.* | 0 | 7 | 53 | 0 | 7 | 57 |
| *Distilled Water.* | 0 | 7 | 50 | 0 | 7 | 54 |
| This Empty *Areometer* Weighs. | | | | 0 | 1 | 28 |

*Experiments about the Superficial Figures of* Fluids, *especially Liquors Contiguous to other Liquors, and their Reflective Powers*; *by Mr.* Boyle. n. 131. p. 775.

VII. 1. Having pour'd a strongly *Alcalizat Menstruum* (I used that made of *Fixt Nitre*, dissolved by the moisture of a Cellar) into a Pipe of Glass, sealed at one end, and not full a quarter of an Inch in Bore; that the Cavity, which in a greater breadth would seem less deep, might be the more conspicuous: We gently poured on it some highly Dephlegm'd *Spirit* of *Wine*, which we knew would not Mix with it, but swim above it, and presently as we had guess'd, we found the *Figure* of the *Surface* of the Lower Liquor Changed, and the Cavity quite destroyed; the *Surface* that seemed as it were, common to the two *Contiguous Liquors*, appearing Flat or Horizontal. And such a Level Superficies we had, by putting these two *Liquors* together in a much Wider Glass.

2. We found also, that by Employing *Oyl* of *Turpentine* instead of the *Spirit* of *Wine*, the Liquor did almost totally Loose its Cavity.

3. But

3. But if, instead of *Deliquated Tartar*, we put Common Water into the Pipe, we found this Liquor to retain its Concave Surface, though we put to it some *Oil* of *Turpentine*, and left it to rest upon the Water a good while.

4. Having provided some pure *Oil* of the *Gum* of *Guajacum*, and poured a little of it into a slender Pipe, we found the Upper-Superficies of it to be Concave; almost, if not altogether, like that which Water would have had in the same Pipe. But when I put a little Water upon this *Oil*, it presently changed the Figure of its Surface, which became visibly, though not very much, Protuberant or Convex.

5. Having put some *Oil* of *Tartar* into the slender Pipe, and put some Drops of the *Oil* of *Guajacum* to it, we found, that this Liquor did not manifestly alter the Concave Figure of the Surface of the Liquor *Alcali*, as the *Oil* of *Turpentine* had done: And having for Curiosities sake, warily poured a little Water upon the *Oil* of *Guajacum*, I found as I had reason to suspect, that the upper Superficies of it Changed presently from a Concave Figure to a Convex, so that this *Oil* in the midst of the other Two Liquors appeared like a little red Cylinder; which, instead of having Circular Bases, was Protuberant at both ends, but more at that which touched the *Oil* of *Tartar*.

6. I put some *Essential Oil* (as *Chymists* call it) of *Cloves* into a new slender Pipe, and having observed it to be somewhat Concave at the Top where it was Contiguous to the Air, we caused a little Common *Water* (perhaps a quarter of a Spoonful or less) to be put to it, and found as we expected, the Surface of this *Oil* also to be Tumid. And in regard, this Liquor, as well as the forementioned *Oil* of *Guajacum*, though it were so heavy as to sink into Water, would not do so in *Deliquated Salt* of *Tartar*, we did, into another slender Pipe, put first some of this last named Liquor, then some of the *Aromatic Oil*, and lastly, a little Common Water; by which means we found, that the Little *Cylinder* of *Oil*, did like that of the *Oil* of *Guajacum*, appear Convex at both ends; but was unlike it in one Circumstance, that the *Oil* of *Cloves* appear'd more Convex at the upper end where 'twas Contiguous to the *Water*, than at the Lower, that leaned upon the Surface of the *Oil* of *Tartar*.

7. Having taken a little slender Glass, that was much longer, but of the like Bore, with the former, we put into it a small quantity of *Quick Silver*, and having taken notice how the Upper Superficies swelled in the middle, above the Level of the Parts where it touched the Glass, we poured some Water upon it, and found a Manifest and Considerable Depression of the Surface, though the Protuberance were not quite Suppressed.

8. This *Phænomenon*, having been for greater security several times Repeated, sometimes it seem'd, that when the *Aqueous Cylinder* was much longer, the Depression of the *Mercurial* Surface was somewhat greater. But this did not so constantly happen: But we often observed, that, though a very little Water sufficed by its Contact to make, in the Judgment of the Eye, a manifest Abatement of the Protuberance of the *Quick Silver*, yet it had not the same effect on that Ponderous Fluid, that it had, when being Increased almost as high as the length of the Pipe would permit, a greater Weight of it was

Incumbent

Incumbent on the *Mercury*, for then I manifeftly perceived, and fhewed to others, that the Surface of the *Quick Silver* being Depreffed almoft to a Level in thofe Parts of it that were near the infide of the Glafs, there was about the Middle of the Surface an Elevation of *Mercurial Matter*, that appeared to be rather more than a half Globe, and was to the Height of its full Semidiameter, raifed above the reft of the *Mercurial* Surface, and in that State it continued as long as I thought fit to let it do fo. And left this Tryal fhould Impofe upon me, I caufed it to be more than once Repeated ; and, the better to confirm it, I afterwards caufed the Incumbent *Water* to be Little by Little fuckt up, and found, as I expected, that when the Incumbent *Water* began to be too much fhortn'd, the little Teat or Segment of a Sphere lately mentioned, began to be fomewhat Flattened, and Subfided more and more as the *Water* was further taken off.

9. Having conveyed into one of our *Pneumatical Receivers*, a Couple of fuch flender Pipes as have been already defcribed, one of them Furnifhed with *Common Water*, and the other with *Quick Silver*, we caufed the Common Air to be diligently Pump'd out, without obferving any Senfible Change in the Concave Figure of the *Water* : But as for the *Quick Silver*, I knew not what to Conclude about it. For having Repeated the Tryal twice or thrice, the *Mercury* fometimes feem'd manifeftly to Swell, to be more Protuberant upon the Exhauftation of the *Receiver*, than when it was put in, efpecially when its Figure was attentively Viewed, and the External Air that was Pumpt out but flowly, was fuffered to Re-enter with all convenient Celerity. But that which yet kept me doubtful, was, that I obferved, That upon the diligent withdrawing of the Airs Preffure upon the *Quick Silver*, there difclofed themfelves fome little Bubbles, which, I fear'd, we had not been able to free it altogether from, and which might be fufpected to have fome Intereft in the *Phænomenon*. We alfo convey'd into our *Receiver*, a clear *Chymical Oil* that was heavier than *Water*, and whilft it was Contiguous to it, had not a Concave but a Convex Surface, and having placed the Pipe furnifh'd with both Liquors in the *Pneumatical Receiver*, we Pumpt out the Air without finding that the *Oil* fenfibly altered its Protuberant Surface, as neither did the Water lofe the Concave Figure of it's upper Surface.

10. I took *Fixt Nitre*, (or which is Analogous to it, *Salt* of *Tartar*) refolved *per Deliquium* into a Tranfparent Liquor, and having filled a clear Vial half full with this, I poured on it a convenient quantity of *Vinous Spirit* exactly *Rectified*, that there might be no Phlegm to occafion an Union betwixt the two Liquors, which ought as ours did, to retain Diftinct Surfaces, and fpeedily regain them though the Glafs were well fhaken. Then having found by a Tryal formerly mentioned, that Common *Oil* of *Turpentine*, if employed in a Competent Quantity, will not totally (and much lefs will readily) Diffolve in *Spirit* of *Wine*, and alfo having Obferved (what may feem fomewhat ftrange) that if this *Spirit* of *Wine* be exquifitely Dephlegm'd, the *Oil*, though a *Chymical* One, will not Swim on it, but Sink in it ; I warily let fall fome drops of the *Oil* into the *Spirit*, and had the pleafure to fee, as I expected, that they fell towards the bottom of the Glafs till their Defcent was ftopt by

the Horizontal (for it was not Concave) Surface of the *Alchalizat Liquor* of *Fixt Niter*. And becauſe my deſign was chiefly to obſerve the Superficial Figure of a Fluid Encompaſſed by other Fluids without touching any Solid Body, I ſhall here take notice of the chief *Phænomena* that were produced of that kind, without Staying to Enquire into the Cauſes or the Conſequences of them.

1. If the *Oily Drops* were but ſmall, they ſeem'd to the Eye exactly enough *Spherical*. For the *Oil* differing but very little in the *Specifick Gravity* from the *Spirit* of *Wine*, the Drops did but juſt touch the Surface of the ſubjacent *Alcali*; and the ſame Drops being but ſmall, their own Weight was not great enough viſibly to Depreſs them, and hinder that Roundneſs which the Preſſure of the Ambient *Spirit*, or their own Viſcoſity endeavoured to give them.

2. If an Aggregate of Drops were conſiderably bigger than thoſe newly mentioned, as if it had about a third part of an Inch in Diameter, it would then manifeſtly lean upon the *Alcalizat Liquor* as upon a Floor, and appear ſomewhat *Elliptical*, (for ſome little part of the bottom was a Plain;) the Weight of the upper parts Depreſſing the Drops, and making the Horizontal Diameter ſomewhat longer than the Tranſverſe.

3. If a yet greater Portion of *Oil* were let fall upon the Heavy Liquor, it would for a pretty while appear in the form of a ſomewhat Imperfect *Hemiſphere*, or ſome other large Section of a *Sphere*, the lower part being cut off; (as if a Globe were divided by a Plain) by the Horizontal Surface of the *Deliquated Salt*.

4. But if the Quantity of *Oil* were not too great, 'twas pretty to obſerve, that though at firſt putting in, it did perhaps ſpread it ſelf over the Subjacent Liquor, and lie as it were flat upon it; yet by little and little, (for 'twas but ſlowly) it would by the Action of the Ambient, concurring with its own Tenacity, be raiſed above the Surface of the Fluid *Niter*, and be Reduced to the Figure, either of half a Globe, or of a greater Segment of a Globe, or even of an Imperfect Ellipſis, according to the Bulk or Weight of the *Oil*.

5. Though theſe Globules, or Portions of *Oil*, did oftentimes readily mingle, when they touched one another, yet divers times alſo we obſerved, that having warily approach'd them, we were able to make them touch without Mingling, inſomuch that we have with pleaſure made them ſo far bear againſt one anothers Surfaces, as manifeſtly to preſs them inwards, though being parted they would preſently Reſume their former Figure. But in caſe any of theſe *Oily Portions* came by a more Preſſing Contact to be United, they would then alter the Figures they had whilſt ſeperate, and take another ſuitable to the Bulk of the Aggregate.

6. When a large Portion of *Oil* reſted upon the *Saline Liquors*, if then the Ambient *Spirit* was moderately and warily Agitated, 'twas not unpleaſant to obſerve the various Figurations, which the Convex and Protuberant part of the mutilated Globe would be put into by the Shakes, without any Viſible Solution of Continuity, or conſiderable Motion of the whole Body, which would very quickly recover its former Figure. Though, if the Agitation were too

 ſtrong,

ſtrong, ſome Portions would be quite broken off, and preſently turned into Little Globes.

11. I tried to produce another *Phænomenon*, that would not have been unpleaſant, by putting together in a ſomewhat large Veſſel, with other Liquors, two *Oils*, (whereof one, if I miſtake not, was from *Turpentine*) which firſt by reaſon of the Oleaginous Nature wherein they agreed, might exactly mingle and make a Compounded Liquor; and then by reaſon of there being one Heavier, and the other Lighter in *Specie* than Water, might by this Liquor be again ſeperated, and Include betwixt them the Liquor that had Divided them. But I found that the *Oils* being once United would not be eaſily parted, but according to the Prevalency of the Lighter or Heavier Ingredient, in the mixture, the Compounded *Oil*, would almoſt totally either Emerge to the top of the Water, or Lie beneath at the bottom of it; I ſay almoſt totally, becauſe ſome Parts of the *Oil*, which was not perhaps all Uniformly Mixt, did not keep in a Body with the reſt; but either was Seperated from the Maſs in the form of Globules, or elſe, ſticking to the ſide of the Glaſs, had the other part of its Superficies, which was Contiguous to the Water, very *Variouſly Figured*, according as the Bulk and Degree of Gravity of the adhering *Oil*, and other Circumſtances happen'd to Determine.

Theſe are ſome of the *Phænomena* I obſerved in *Oil* of *Turpentine*, when 'twas invironed only with Fluids; but if it were permitted to be Contiguous to the Inſide of the Glaſs, and ſo to faſten part of its Surface to a Solid, the greater part of the Surface, which remained expoſed to one or both of the Contiguous Liquors, would partly by their Action, and partly by the Gravity of the *Oil* it ſelf, be put into *Figures* ſo *Various*, and ſometimes ſo *Extravagant*, that 'twas much more pleaſant to behold them, than it would be eaſie to Deſcribe them.

12. *Confining Fluids* may have Diſtinct Surfaces, without having, at leaſt in many Poſitions, *Refractions* differing enough, or *Reflexions* ſtrong enough, to make the Plain that Diſterminates them, obvious to the Eye. Thus when the *Oil* of *Tartar*, or *Nitrous Alcali*, that I employed happen'd to be very Clear and Colourleſs, I have more then once made highly *Rectified Spirit* of *Wine* Float upon it ſo, that in moſt Poſitions the Vial ſeem'd to have in it but one uniform Liquor; the Plain that divided the two Fluids being unapt to be diſcerned, but in a Poſition, wherein the Rays of Light paſſing thence to the Eye, fell very Obliquely on it; and indeed, when there was no little Duſt or other Feculency, ſwimming upon the Surface of the *Oil* of *Tartar*; I had ſometimes much ado to convince Ordinary Spectators, that the Vial in two diſtinct Regions of it, contained two *Unſociable Liquors*.

13. We took a *Deliquated Alcali*, made of *Niter* and *Tartar*, and deeply Tinged with *Cochineel*; and, that the Liquors might not only be Heterogeneous, but as differing in *Gravity* and *Denſity* as we could make them, we poured on it a peculiar kind of *Oil* lighter than *Spirit* of *Wine*, and holding the Plain where the two Liquors were Contiguous in a convenient Poſition, in reſpect of the Light and the Eye, I obſerved it to make a ſtrangely vivid *Reflexion* of the Incident Beams of Light: ſo that this Phyſical Surface which

was

was flat, look'd almoſt, for 'twas not ſo *Specular*, like that of *Quick Silver*; and when I kept it till Night, and conſidered it by the Light of a Candle, the bright Figure of the Flame was ſtrongly Reflected almoſt as from a Cloſe Specular Body; which tempted me to ſuſpect, that there might be ſomething elſe than the bare ſmoothneſs of the Surface of the *Alcalizat Liquor* to produce ſo briſk a *Reflection*; and the rather, becauſe I did not obſerve, that the remains of the ſame Tinged *Alcali*, which I kept in another Glaſs, nor a Portion of the ſame *Oil*, which I had alſo by me in a ſeperate Vial, did either of them afford ſo Vivid a *Reflection* from its Surface; though I did the leſs wonder at this, becauſe of the great Diſpoſition to *Reflect Light*; which I had formerly the Curioſity to obſerve in the forementioned *Oil*, when I joyned it with other Liquors. I ſhall add, that looking on this Liquor, as a Body which, though it have all the neceſſary Qualities of an *Oil*, does in regard of its Origin, and ſome Properties I have found in it, differ from common *Chymical Oils*; I was invited the more to obſerve its *Phænomena* in reference to *Reflection*, and I found among other things, (not pertinent to this place,) *First*, That the Confining Plain often mentioned between the Tinged *Alcali* and this *Liquor*, did not appear Red it ſelf, nor communicate that Colour to the Image of the Flame of a Candle Reflected from it. *Secondly*, That when I warily ſhook the Vial, which contained the two Liquors, the uppermoſt would be reduced into a ſeeming Froth, conſiſting of a great number of Imperfectly Globular Bodies, which after a while would make a kind of a Rude Phyſical Plain; which, though neither very Horizontal, nor ſenſibly Smooth, would, at its upper Superficies, ſend back the *Incident Light* with more Briskneſs than one would expect; and when the ſeeming Froth conſiſted of ſmaller Particles, theſe, when they were of a certain Size, and conveniently Placed, in reference to the Flame of a Candle and the Eye, would (as more than one Trial informed me) Reflect the *Incident Light* ſo many ways, and ſo viſibly, that they ſeemed, for multitude and ſplendor, like little Sparkling Corpuſcles of Poliſhed *Silver*; or almoſt like thoſe Gliſtering ones, that appear when a clean Plate of *Copper* is firſt Immerſed into a much allayed Solution of good *Silver*, made in *Aqua Fortis*. *Thirdly*, That though pure *Spirit* of *Wine* be ſo thin a Liquor, and our *Oil* is nevertheleſs ſo Light as to Swim upon it, yet I found the Confining Surface very ſtrongly *Reflexive*.

I have alſo found, that ſome other *Eſſential Oils*, (as *Chymiſts* call thoſe that are Diſtilled with Water in *Limbecks*) and particularly an Unſophiſticated *Oil* of *Limons*, did with our *Tinged Alcali* afford moſt of the ſame Phænomena; but not ſo Brisk a *Reflection*: I ſay moſt, chiefly becauſe with *Spirit* of *Wine* theſe *Subtile Oils*, as I formerly noted, will readily be confounded: though our Anomalous *Oil* be unſociable with it.

14. In Cold Weather we took *Eſſential Oils* of *Anniſeeds*, whoſe Property it is to Coagulate in ſuch Weather, and having in a gentle warmth brought it to be Fluid, we poured it into a Slender Viol more than half filled with *Common Water*, that had been alſo a little warmed, that the *Oil* might not be too haſtily reduced to its former State. This Oil being Lighter than ſo much *Water*, and being poured on in a convenient Quantity, had its Upper Surface n. 132. p. 79..

Kkkk 2 ſome-

ſomewhat Concave, as that of the Water was; but the Lower Surface, Surrounded by the Water, was very Convex, appearing almoſt (for it was not perfectly) of the Figure of a great Portion of a *Sphere*. This being done, the Viol was ſtopt, and ſuffered to reſt for ſome time in a Cold place, by which means the *Water* continuing Fluid as before, the *Oil* of *Anniſeeds* was, as I expected, found *Coagulated* in a Form approaching to that it had whilſt in a Fluid State; I ſay approaching, becauſe it was not eaſie to diſcern the exact Figure in the Viol I was fain to make uſe of: and I ſuſpected, that the *Oil* grown Conſiſtent was become leſs Convex then before. But 'twas worth Obſerving, how great a Difference there was between the dull *Reflection* it made when it was *Coagulated*, and the fine *Reflection* it had made whilſt 'twas a Liquor. The later of which *Reflections* brought into my mind, how Vivid the *Reflective Power* of ſome *Fluids* is in Compariſon of that of the Generality of *Solid Bodies*.

15. Having obſerved, That *Quick-Silver*, and Rectified *Oleum Petræ*, are, the Former of them the Heavieſt, and the Latter the Lighteſt, of all the Viſible Fluids that are yet known to me; I put ſome (Diſtill'd) *Quick Silver* into a ſmall Viol, and held it in ſuch a poſture, that the *Incident Light* was ſtrongly remitted to my Eye, I then ſlowly put to it ſome *Petroleum*, that being well Rectified was very clear, and obſerved, that as this Liquor covered the *Quick Silver*, there was at the Imaginary Plain, where they both confined, a Brisker *Reflection* than the *Quick-Silver* alone had given before. On this occaſion it will not be amiſs to take notice, that either the Surface of the Air it ſelf, as thin and yielding a Fluid as it is, or the Surface of a Solid contiguous to included Air, or ſome interpoſed *Subtile Matter*, may *Reflect* the *Incident* Beams of *Light* more ſtrongly than moſt Men would expect. To this purpoſe, I remember, that a Curious Perſon having one Day brought me a couple of Rarities, which he told me were two pieces of a Solid, but tranſparent Body, that he had caſually found; in one of which there was a *Pearl*, Large, Round, and Orient, and in the other a leſs Perfect One. One of them was opened, and that which had appeared a *Pearl* was found to be but a Cavity, that contained no groſſer Subſtance than Air. And I have by me, a well ſhap'd piece of Glaſs of a good thickneſs, with an Aereal Bubble in the middle, which by ſome Qualities, particularly its Pear-like ſhape and Vivid *Reflection*, does not ill reſemble a fair, though not *Orient Pearl*. But in ſuch-like Obſervations, the Poſition of the Eye, and that wherein the Body receives the Beams of Light, may be very conſiderable. For I have by me a Small Stone (with which I have puzzled a Skillful Jeweller to determine what kind of *Gem* it was) that being laid flat upon ones Hand, or a piece of Paper, and lookt on Directly downwards, looks almoſt like a piece of Common Glaſs, and is tranſparent: But if the Eye be ſo Placed, that the *Incident* Beams of *Light*, by whoſe *Reflection* its ſeen, fall with a convenient degree of Obliquity upon the Stone it, makes an exceeding pretty ſhew, ſometimes appearing like a fine *Opal*, and ſometimes not very unlike an *Orient Pearl*.

16. We

16. We made a competent Quantity of a *Resinous* or *Gummous Substance*, that looked like High Coloured *Amber*, but was easie to melt. This we put into a deep round Glaſs with a wide Mouth, and held it by the Fire-ſide in a moderate Warmth, till it was brought into a Fluid State ; then we Tranſferr'd it into one of our *Pneumatical Receivers*, where we preſumed, that this Temporary Liquor, would, as well as Liquors that are conſtantly ſuch, diſcloſe *Aereal Bubbles*, when the Preſſure of the Air was withdrawn from it ; and accordingly having cauſed the Air to be Pump'd out by degrees, we found, that ſtore of Bubbles appear'd at the Top of the Liquor, and made there a copious Froth, many of them being by reaſon of the Viſcoſity of the Fluid, very large, and divers of them becauſe of the Nature and Texture of it, and the Thinneſs of the Films, being adorned with the Colours of the Rainbow, whoſe Vividneſs, made them pleaſant to behold, and ſuggeſted to us ſome *Optical Conſiderations*. But notwithſtanding this Froth, I cauſed the Pumping to be continued, that thoſe Bubbles that had moſt of Common Air in them, and which therefore are wont to Riſe firſt, might get to the Top, and the Subſequent Bubbles might meet with more reſiſtance from the Liquor ſtill tending to grow Cold, and ſo might be the more Expanded, and yet kept from Emerging by the Concretion of the Reſinous Subſtance ; and anſwerably to this we found, that, when this Subſtance had reſumed its conſiſtent form, there were intercepted, between the Upper and the Lower Surfaces of it, ſome Bubbles that were not ſmall, which yet had a conſiderable *Reflection*, notwithſtanding the ſmall Quantity of the groſſer Particles of the Air, that may be ſuppoſed to be contained in Bubbles ſo very much Expanded.

17. 'Tis taken for granted, That the *Falling Drops* of *Rain* are *Spherical*, yet their Deſcent is ſo ſwift, that I fear 'tis rather *Suppoſed* then *Obſerved* that their Figure is *Spherical* ; which will be the more queſtionable, if it be true, which is vulgarly thought, that *Hail* is but *Rain Frozen* in its paſſage through the *Air*. For 'tis evident, that the Grains of *Hail* are frequently of other Figures than truly Orbicular. But the *Surface* of *Water* may have Differing Figures, according as 'tis totally Encompaſs'd with Heterogeneous Fluids, or as 'tis only in ſome places Contiguous to one or more of them. In the former caſe we found it not ſo eaſie to make an Obſervation, becauſe, we know not of any two Liquors (ſetting *Mercury* aſide) that will not Mingle either with One Another, or with Water. We therefore cautiouſly convey'd into ſome *Chymical Oil* of *Cloves* ſome Portions of *Common Water* of differing Bigneſſes, taking care, as far as we could, that they might not touch one another ; by which means the *Oil* being Tranſparent, and yet ſomewhat colour'd, 'twas eaſie to obſerve, that the Smaller Portions of *Water* were ſo near totally Invironed with the *Oil*, that they were reduced into almoſt Perfect *Globes* ; Thoſe Portions, that were ſomewhat bigger, (as about twice the bigneſs of a Pea,) would be of a Figure ſomewhat approaching to that of an *Ellipſis*, (for 'twas not the ſame) and thoſe Portions that were yet ſomewat Larger, though they ſeemed to be ſunk almoſt totally beneath the *Oil*, yet they held to it by a ſmall Portion of themſelves, whoſe Surface was eaſily enough diſtinguiſhable from that of the *Oil*. Theſe larger Portions of *Immerſed Water* being

almoſt wholly Inviron'd with the other Liquor, were by it reduced into a Round Figure, which was ordinarily ſomewat *Elliptical*, but more Depreſſed in the Middle than that Figure requires.

18. Having into a Slender Pipe, of that ſort that has been deſcrib'd before, put a little *Oil* of *Cloves*, and upon this ſome *Oil* of *Turpentine*, that ſo the *Water* might both above and beneath be touched by Heterogeneous Liquors, I obſerved not the *Oil* of *Cloves* to be very manifeſtly Tumid at the top, nor the Lower Surface of the *Oil* of *Turpentine* (for the Upper was Concave) to be very Convex: for ſomewhat Convex it was Downwards. And from this 'twill be eaſie to conclude, the Figure of the Cylindrical Portion of *Water* intercepted between theſe two Oils.

19. I took *Oil* of *Anniſeeds*, thaw'd by a gentle warmth, and *Common Water*, and having put them together in a conveniently ſhap'd Glaſs, they were ſuffered to ſtand in a cold place till the *Oil* was coagulated; which done, it was Parted from the *Water*, and by the Roughneſs of its Superficies manifeſted, as I expected, that, when its Parts were no longer agitated and kept eaſily Diſplaceable, by the Subtile permeating matter, or whatever other Agent or Cauſe it were to which it owed its *Fluidity*, then the Contiguous *Water* grew unable to Inflect, or otherwiſe place them after the manner requiſite to conſtitute a *Smooth Surface*. And what happen'd to that part of the *Oils* Surface that was touched by the *Water*, happen'd alſo to that which was contiguous to the *Air*; ſave that the *Aſperity* of the laſt named Surface was differing from the other, which whether it were an accidental or conſtant *Phænomenon* farther trial muſt determine. But I have often obſerved, That the Upper Surface of *Oil* of *Anniſeeds*, when this Liquor comes to be Coagulated by the Cold Air, was far enough from being *Smooth*, being variouſly Aſperated by many Flaky Particles, ſome of which lay with their broad, and other with their edged Parts upwards.

20. An Inequality and Ruggedneſs of Superficies I have alſo obſerved in *Water*, when, having covered it with *Chymical Oil* of *Juniper*, and expoſed it in very cold Weather, though the *Oil* continued *Fluid*, yet the *Water* being Frozen had no longer a *Smooth Superficies*, as whilſt in its Liquid ſtate 'twas contiguous to the *Oil*. And the like Inequality, and rather a greater, we obſerved in the Surface of *Water* Frozen, which had *Chymical Oil* of *Turpentine* Swimming over it, yet a no leſs, if not a much greater, Roughneſs may be oftentimes obſerved in the Surfaces of divers Liquors that abound with *Water*, when, thoſe Liquors being Frozen, their Surfaces have an immediate Contact with the Air. I ſhall here add, that having purpoſely cauſed a Strong and Blood-red Decoction of the Soot of Wood to be expoſed in a large Glaſs in a very Cold Night, I was more pleaſed then ſurprized, to find in the Morning a Cake of Ice, that was curiouſly Figur'd, being full of large Flakes ſhap'd almoſt like the broad Blades of Daggers, but neatly Fringed at the Edges. But that which I chiefly mention theſe Figures for, is, that they ſeem to be as it were Imboſt, being both to the Eye and the Touch Raiſed above the Horizontal Plain or Level of the other Ice.

21. I

21. I have ſometimes obſerved the like *Phænomenon* in one and the ſame Liquor, and particularly not long ſince in Froſty Weather, on a Viol where I had long kept *Oil* of *Vitriol*, I perceived that the Cold had reduced far the greateſt part of the *Menſtruum* into a Conſiſtent Maſs, whoſe Upper Surface was very Rugged and odly Figur'd, though it lay covered all over with a pretty deal of high coloured Liquor, that was not Frozen or Coagulated, nor ſeem'd to be diſpoſed to be ſo, at leaſt in that Degree of Cold.

22. This may be alſo Obſerv'd in the beſt ſort of what the *Chymiſts* call *Regulus Martis Stellatus*, where the Figure of a Star, or a Figure ſomewhat like that of the Decoction of the Soot lately mentioned, will frequently appear Imboſt upon the Upper Superficies of the *Regulus*; and ſuch a Raiſed Figure I have ſeen on a Maſs of *Regulus* made of *Antimony* without *Mars*. But if to thoſe two Bodies, *Copper* be alſo Skillfully added, the Superficies will be often times adorned with new Figures according to the Circumſtances; though the moſt uſual I took notice of was that of a Net, that ſeemed to cover the Surface of the compounded *Regulus*. But this is not ſo conſtant, but that I have by me a Maſs of a Conical Figure, conſiſting of two very Contiguous, but eaſily ſuperable Parts, whereof the Lowermoſt, which abounds more in *Metal*, hath its Upper Surface covered with Round Protuberances, in Shape and Bigneſs not unlike to ſmall Peaſe cut in two; and theſe are ſo really Imboſt and Elevated above the reſt of the Superficies, that the other part of the Cone, which is of a more Scorious Nature, has in its Lower Surface, which exactly fits the Upper of the *Regulus*, Cavities, for Number, Shape and Bigneſs, anſwering to the Protuberances lately mentioned; which Argues that the *Regulus* cooled firſt with that Inequality of Surface we have deſcribed, and that the Lighter and more Recrementitious Subſtance, continuing longer Fluid, had thereby Opportunity to accommodate it ſelf to the Superficial Figure of the *Regulus*, on which it firſt leaned, and was afterwards Coagulated.

*Why Bodies Diſſolved Swim in Menſtrua Specifically Lighter than themſelves; by Mr. Will. Molyneux. n. 181. p. 88.*

VIII. 1. My Brother, Mr. *Tho. Molyneux* (in the *Nouvelles de la Republique de Lettres*) has given this Reaſon for the *Phænomenon*, viz. That the *Internal Motion* of the Parts of the Liquor does keep up the Particles of the *Diſſolved Solid*, for they being ſo very minute, are moveable by the leaſt Force Imaginable, and the Action of the *Particles* of the *Menſtruum*, is ſufficient to Drive the Atomes of the *Diſſolved Solid Body* from place to place; and conſequently, notwithſtanding their *Gravity*, they do not *Sink in the Liquor Lighter than themſelves*.

But I conceive another Account may be given of this *Appearance*, and that the Prime Law of *Hydroſtaticks* is a little deficient. 'Tis true indeed, if we conſider only the *Specifick Gravity* of a Liquor, and the *Specifick Gravity* of a Solid Particle Floating therein, the forementioned Rule is exact; but in *Sinking* there is requiſite a *Separation* of the Parts of the Liquor by the *Sinking* Body; and there being a Natural Inclination in the Parts of all Liquors to *Union* ariſing from an Agreement or *Congruity* of their Parts, there is a Reſiſtance therein to any thing that Seperates this Conjunction: Now unleſs a Body have weight enough to overcome this *Congruity* or Union of Parts, ſuch a Body will Float in a Liquor *Specifically Lighter* than it ſelf. But that a Heavy

Body,

Body, as *Mercury* or *Iron*, may have its Parts reduced to that Minuteneſs, that their *Gravity* or Tendency Downwards, is not ſtrong enough to Seperate the *Coheſion* or Union of Parts of a Liquor, will be manifeſt, if we conſider, that the Reſiſtance made by the Medium to a *Falling* Body, is according to the Superficies of the Body: but as the Body Decreaſes in Bulk its Superficies does not proportionably Decreaſe; thus a Sphere of an Inch Diameter, has not *eight* times leſs Superficies, than a Sphere of two Inches Diameter, tho'. it have *eight* times leſs bulk, and conſequently paſſing through a Medium, as ſuppoſe Air or Water, the Sphere of an Inch Diameter, is, proportionably to its Bulk, more *Reſiſted*, than a Sphere of two Inches Diameter in proportion to its Bulk, and hence it will come to paſs, that at laſt a Body may be reduced to that Minuteneſs, that its Gravity *Preſſing* Downwards (which is according to its Bulk) may be leſs than the *Reſiſtance* of the Medium, which operates on the Surface of the Body; ſeeing as I ſaid before, the Surfaces of Bodies do not Decreaſe ſo faſt as their Bulks, theſe Decreaſing in a Triplicate, but thoſe in a Duplicate *Ratio* of the Bodies Diameters.

But becauſe I have ſaid that the forementioned Law of *Hydroſtaticks* is a little Defective, I deſire to explain my ſelf a little further in that point. In Weights *Falling* through the Air, were Gravity only conſidered, the Proportions of their Deſcents would be exactly as *Galileo* has Demonſtrated; but it is allowed by all, that the *Reſiſtance* of the Air, not being conſidered in thoſe Demonſtrations, they are not *Mathematically* True in Practiſe, but that really there is ſomething of that Proportion hindred by the Airs *Reſiſtance*. Now, what is this leſs than to ſay, that the *Reſiſtance* of the Air takes off ſome of the Operation of Gravity, or is able to withſtand or oppoſe part of its Action? And if ſo, what ſhall we ſay, were an *Iron* Sphere let through a Medium of Water? Surely, the Proportions of its Deſcents would be much more Diſturbed herein, as Water is much more ſolid and difficult to be Seperated or paſſed through than Air, and conſequently we muſt needs grant, that more of the Operation of Gravity is taken off or reſiſted by this Oppoſition of the Water, than that of the Air. And if ſo, ſurely there may be a certain Degree of Gravity, that may be quite taken off by the *Reſiſtance* of the Water: Were a Piſtol Bullet let fall through the Air, it would Deſcend imperceptibly nigh the Proportions that *Galileo* has aſſigned, but were a ſingle Grain of Sand ſo let fall, it would be much hindred in its Courſe, and the half of this Grain would be more obſtructed; what ſhall we then ſay of the Ten Thouſandth part, or of a part of the Ten Thouſandth Millionth of this, and again of the Infinite Subdiviſions of that, till at laſt we come to a Part that would be wholly *Reſiſted*, or kept up; ſuch as I conceive the Minute Particles of a Body *Diſſolved* in a *Menſtruum*.

On this Account, 'tis, I ſay, that the forementioned Principle of *Hydroſtaticks* is a little Defective; for it conſiders not the Natural *Congruity* of the Parts of a Liquor, whereby they deſire as 'twere, to Unite and Keep together, juſt as we ſee two drops of Water on a dry Board being brought together, do jump and coaleſce, and therefore Liquors have an Innate Power of *Reſiſting* a certain Degree of Force that would *Seperate* them; ſuch as I ſup-

poſe

pose the Degree of Gravity, in the most minute Particles of a Body *Dissolved* in a *Menstruum*.

The forementioned Rule holds true to the most nice Sense in great Bodies, but in those that are by many Millions of Divisions smaller, it seems to Fail.

I would not however be thought wholly to Reject my *Brother's Solution* of this *Problem*; for certainly that *Motion* (whatsoever it is) in a *Menstruum*, which is able to Dissolve such a Solid Body as Iron, that is, which is able to disturb the Close and Strong Cohesion of the Parts of Iron, may very well be supposed sufficient to disturb or keep up these parts from Resting in the bottom of the Vessel, wherein the *Solution* was made; and certainly no better account can possibly be given of such *Solutions*, than by supposing such an *Internal Motion* in the parts of the *Menstruum* Insinuating themselves into the Solid Body, and loosening its parts. But I leave to others to consider what Kind of *Motion* and Peculiar Conformation of Parts is requisite both in the *Menstruum* and in the Dissolved Body, that a *Solution* may result from their Commixture.

2. Tho' Liquors Consist of Parts United, and tho' this Union be easily Destroyed, yet of Necessity it requires some degree of Force for Effecting it: Yet this *Property* ought not to be rely'd on as the Sole Cause of this *Appearance*; For in this *Solution* of the *Problem*, We first Suppose the Minute Particles of a Heavy Body Rais'd, and then give the Reason of their not Sinking: Whereas, 'tis not to be questioned, but that that Force which Raised them, is the same that Keeps them from Falling to the bottom.

*Considered; by* Mr. Tho. Molyneux. *Ibid. p.* 93.

IX. Sir *Sam. Moreland* undertakes to Demonstrate, (contrary to the Common and Received Opinion through *England* and all *Europe*,)

1. That he will Force Water 60 Foot high with treble the Weight that shall Raise it 20 Foot, and so proportionably, *in infinitum.*

2. That by how much Wider the Barrel is, in which the Forcer Works, than the Pipe through which the Water is forced up, by so much is the *Engine* Pressed with Unnecessary Weight.

*An Undertaking for Raising of Water; by Sir* Sam. Moreland n. 102. *p.* 25.

X. 1. Elapsa nuper Æstate, *Ann.* 1684. in manus incidit Tractatus quidam inscriptus *Sipho Wurtemburgicus*, sive Sipho Inversus Cruribus æquealtis Fluens & Refluens hactenus Inauditus; de hac *Machina* magnifica prædicat Author, sed Lectorem orat, ut ignoscat Patrono ejus Serenissimo quod Mysterium Structuræ ejus sibi servet. Hæc dum Legerem in mentem venit quo modo instrui possit *Sipho*, ut quæ de *Wurtemburgico* illo narrantur præstaret. Habens ergo in manibus *Siphonem* quendam Vitreum, erexi cum supra duo Vascula quantum potui perpendiculariter; dumque in eo situ sisteretur, affudi in unum ex Vasculis Aquam, donec Orificium *Syphonis* paululum superaret, & statim in alterum Vas ut expectabam effluebat Aqua: tunc Evacuato illo Vase in quod primum Aquam Infuderam in alterum Effudi, & immediate Aqua ista Refluebat in prius Vas. Licet non ausim Artificium hujus *Siphonis* mei cum illo *Wurtemburgico* comparare: tamen si Utilitatem spectemus, cum

*A* Siphon *performing the same things with the* Sipho Wurtemburgicus; *by Mr.* J. Davis. n. 167. *p.* 845.

eo certare posse (præsertim si addatur ei Instrumentum quoddam, modo quodam peculiari à me excogitato,) non multum dubito.

By Dr. Papin. Ibid. p. 847. 2. In Tractatu de *Siphone Wurtemburgico*, qui *Stutgardiæ* Authore D°. *Salomone Reiselio* nuper Editus est, magna quædam atque inaudita, si & Utilitatem & Raritatem & Artificium spectes, de novo illo *Siphone* prædicantur; ipsius autem Proprietates Characteristicæ proponuntur his Verbis.

1. *Ut Orificia Crurum Duorum* Siphonis *sui horozontaliter sita Labris inserantur; cum in Veterum inventis Crus longius infra Labrum seu Æquilibrium descendat semper.*

2. *Ut Orificiis vel partim vel ad dimidium Aqua repletis, Effluat tamen Aqua super Montem ducta; cum in reliquis* Siphonibus *totum Orificium Aqua adimpleri seu immergi Aquæ debeat.*

3. *Ut in Siccitate diuturna quiescens* Machina *tamen effectum producat Affluente iterum Aqua.*

4. *Ut Lumine seu Orificio alterutro Aperto, altero vero post Horas demum aut Diem, seu per Epistomium, seu Conum recluso, Effluat tamen Aqua; cum in aliis utraque Simul Lumina Aperiri debeant.*

5. *Ut in Horizontali Linea positis Orificiis, & æqualibus quoad Altitudinem Cruribus exundet Liquor; cum in* Portæ *Aliorumve* Machinis *inæqualia debeant Crura esse, & Perpendiculum majus.*

6. *Ut ab utroque Labro in utrumque Labrum infusa Ascendat & Effluat Aqua; cum in Veteribus non nisi per unicum nempe Longius, Crus Effluat, nunquam Refluat.*

Hæc sunt ipsius verba *Authoris*, qua vero ratione, quibusve auxiliis tanti effectus produci possint, ne verbum quidem: Me itaque jussit *Regia Societas* Machinam construere, quæ eadem illa in *Libro* descripta Phænomena exhiberet. Rem quidem tribus variis modis exequutus sum; ne vero tædiosus nimium videar, sequentem methodum utpote facillimam instar omnium fore arbitror.

Fig. 197. A A, Sunt duo Vasa Metallica quibus duæ *Siphonis* extremitates inseruntur. B C D E D C B, est *Sipho*, cujus *Lumina* B B, In eadem Horizontali Linea disponenda sunt. F, est Tubulus foramini in superiori *Siphonis* parte adferruminatus, diligenterque obturandus, postquam *Sipho* Aqua exacte Repletus fuerit. Jam manifestum est Aquam in partibus C D, contentam, Aeri Externo ingressum prohibere ne ad superiorem *Siphonis* partem E, penetret: *Sipho* igitur Aqua semper plenus (modo debitam Altitudinem non excedat,) effectum suum certissimè producet statim atque Aqua in Vasibus A A, contenta alterutrius Orificii B, partem aliquam Replebit: quumque Ambobus Orificiis Aqua partim Repletis, in utroque Vase A ad eandem Horizontalem Lineam Superficies Aquæ pervenerit, si alterutri Vasi vel tantillum Aquæ infunderis, pars ejus per *Siphonem* statim in alterum Vas deferetur, eademque ratione cætera in *Libro* descripta *Phænomena* exhiberi poterunt.

3. Ne-

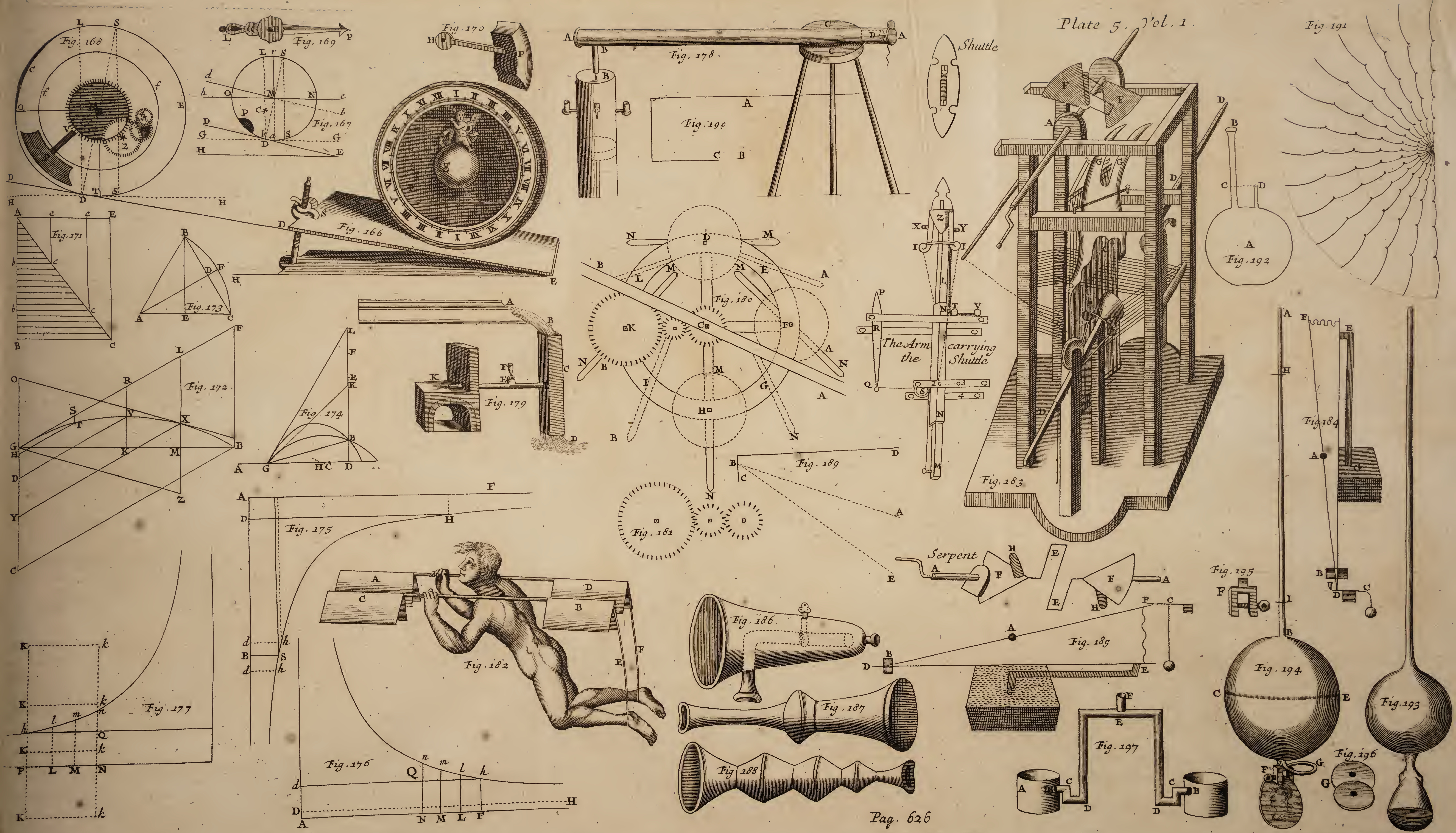
Fig. 166
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Fig 188
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Fig. 190
Fig. 191
Fig. 192
Fig. 193
Fig. 194
Fig. 195
Fig. 196
Fig. 197
Shuttle
The Arm carrying the Shuttle
Serpent

3. Ne diutius *Serenissimæ Societatis* desiderium frustrarer, aut morarer, fateor Excellentissimi Dni. Dris. *Papin Siphonem* ipsissimum esse *Wurtemburgicum*, etiam cum Recurvatione Pedum factum; neque aliud esse Mysterium ut *Inventor* ipse scripsit, in Summitate, quam quod per Infundibulum debeat Impleri, sine qua Impletione non potest transfluere. Quod proximè Typis amplius confirmabitur, quia longum nimis esset & tædiosum hic omnia scribere. *By D.* Salomon Reiselius *himself.* n.178. *p.* 1272

Ast ut in præsenti ostendam me hactenus nonnihil laborasse circa *Siphonis* effectus; ecce inter Experimentandum, hoc quoque mihi occurrit; ut didicerim, quomodo in *Vertice* vel ad *Latera* possit Effluere; quod hactenus multi promisère, vix quisquam efficit.

XI. 1. A A, Is a great Glass made like a Tumbler, but much bigger, and laid upon the Chimney Board, B B. *A New way of Raising Water, Ænigmatically proposed; by Dr.* Papin. n.173. *p.* 1093. Fig. 198.

C C, Is the *Engine* like a small *Rock*, that doth constantly *Spout* out Water by the two Holes D D: This *Rock* is kept at a distance from the bottom of the *Glass* A A; so that it may plainly be seen that it cannot Receive any Water by Subterraneal Tubes.

E E, Is a factitious *Corall*, reaching from the Center of the *Rock* C C, to the Center of the *Crown* F F.

F F, Is a *Crown* bearing upon the Aperture of the *Glass* A A, and holding the *Rock* C C, Suspended at a considerable Distance from the Bottom.

G G, A Glass Open at both ends, apply'd to the *Rock* C C, to keep the Water upon it from Falling down.

The Water in this Engine runs constantly H H, Two *Shells* to receive the Water from the *Jetto*'s.

2. Within the *Rock* C C, there may be a Vessel placed, which shall be made like the Body of a pair of *Bellows*, or those *Puffs* heretofore used by *Barbers*, which being filled with Water, a piece of *Clockwork* put under it may produce the *Jetto*'s; the Water being received into the *Shell* H H, and runing thence into the hollow of the *Coral* E E; may be thereby conveyed into the Follicular Cavity in the same quantity it is Ejected from the two Emerging Tubes; and it will Circulate according to the going of the *Clock-work*. *Solv'd; by Dr.* Nath. Vincent. n.177. *p.* 1238.

3. I conceive that the Air is forced into the *Outer Glass* at the bottom thereof. That it then passes up between the two *Glasses*. *By Mr.* R. A. *Ib. p.* 1239.

That the *Outer Glass* or Case being close Luted at the Head or *Crown* to which the *Inner Glass* is hung by the *Corall*, the Air is forced into the Mouth of the *Inner Glass*.

That the Air so forced pressing on the Surface of the Water that covers the *Rock*, forces the Water to Rise through those two extream Parts that are not at all Clogg'd, or covered with Water.

4. A B D E, signifies a Cylindrical Vessel, Closed on every side, and Divided into two Rooms by the Floor E F. *In a Letter Subscribed,* W. Tenon. n.178. *p.* 1254. Fig. 199.

G L M H, Is another Cylindrical Vessel within that Upper Room, Cemented with its Mouth downwards to the Floor, and full of Water up to the Surface I K; the upper part thereof G I K H, being full of Air.

Q O, R P, Two Pipes, open above and below, and let through the Upper Room into this Veſſel, and reaching almoſt down to the Floor E F.

V W, a Pipe open above and below, and let into the Upper Room. Theſe Pipes muſt be cloſe joyned round about them to the Floors C D, and G H.

X, Y, Two little Hemiſpherical Bladders prepared with Oil or ſome Oily Subſtance, (as Butter and Turpentine) againſt Water, and Cemented with their Mouths upward to the Floor E F, underneath.

$\alpha$ $\beta$, Two Valves Opening out of the Upper Room into the Bladders.

$\gamma$ $\delta$, Two other Valves Opening out of the Bladders into the Inner Veſſel above.

N Z, A *Pendulum* playing upon the Center N, and having two Battle-door Arms *a*, *b*, to ſqueeze alternately the Bladders which reſt upon them.

Let the Upper Room be filled with Water at the Pipe V W, and if the *Pendulum* be made to play by *Clock-work*, the Bladders will perpetually Pump it thence into the Inner Veſſel, and the Compreſt Air G I K H in the Upper part of that Veſſel preſſing upon the Surface of the Water I K, will Force it thence into the Pipes O Q, P R, out of which Spouting with a perpetual even Stream into the Spoons S T, it will run down by the Pipe W V, into the Upper Room again, the *Pendulum* will play moſt eaſily when the Upper Room is filled to the top of the Pipe W V. Inſtead of the Bladders may be other Contrivances, as of Suckers or little Organ Bellows, playing alternately with two Leaves about an Axis in the middle.

By Dr. Papin. Ibid. p. 1274. Fig. 290. 5. A A, Is the *Great Tumbler*, that muſt have ſome little hole in the bottom as I.

I L L, A ſlender Pipe hidden by the Chimney Board B B, whereby the *Tumbler* A A, hath Communication with the *Pump* or *Bellows* M M.

M M, Some kind of *Pump* or *Bellows* well ſhut, and having no other Aperture but through the Pipe I L L. Theſe are put in ſome Secret Place, where a Body may Play the ſame, and not be ſeen.

N N, A ſlender Pipe, that makes a Communication between the *Glaſs* A A, and the *Crown* F F; this Pipe reacheth near to the cover of the *Crown*, that the Water contain'd in it, may not Run Down by that Aperture.

E E, The *Factitious Corall*, Hollow within, Shut at the bottom, and Open at the top.

D D, D D, Two Crooked Pipes, Sodred to the Sides of the *Corall* E E, ſo that the Water running down the *Corall* may Spout out at the Holes D D.

O O, A Pipe hidden in the *Corall* E E, paſſing through the bottom of the ſame where it muſt be well Soder'd, and reaching near to the bottom of the *Rock* C C.

P P, A Pipe to convey the Water from the *Glaſs* G G, into the *Rock* C C; this Pipe is well Soder'd to the *Cover* of the ſaid *Rock*.

Q, A *Valve* working by a Spring at the bottom of the Pipe P P, to keep the Water, that gets in that Way, from returning back.

R, Another *Valve* at the top of the Pipe O O, that the Water getting up that way, may not fall through the ſame.

Now

Now it is plain, that the *Rock* C C, being filled partly with Water, partly with Air; if we open the *Bellows* M M, the Air from the *Crown* F F, muſt run through the Pipe N N, into the *Tumbler* A A: and thence through the Pipe I L L, into M M to fill the Vacuity made therein: the Air in the *Crown* F F, being thus Rarified, gives liberty to the Air in the *Rock* C C, to Rarifie too, by Driving the Water through the Pipe O O. The Water being got up into the *Crown* F F, runs down the *Corall* E E, and through the Crooked Pipes D D, D D, Spouts out at their upper Apertures, and from the *Shells* H H, falls upon the *Rock* C C: if we come afterwards to ſhut the *Bellows* M M, the Air got into their Vacuity, muſt run back into the *Tumbler* A A, and preſs upon the Water at the top of the *Rock* C C: but the Air in the ſaid *Rock* having been Rarified, its Spring is not ſufficient to Reſiſt this Preſſure, and ſo the Water is forced into the ſaid *Rock* through the Pipe P P: and by thus opening and ſhutting the *Bellows* M M, the Water muſt conſtantly circulate by the ways aforeſaid.

*The Uſe of this Contrivance.* As for the Uſes this way for *Raiſing Water* may be applied to, this I do conceive: the Glaſſes, being meerly to conceal the Secret, muſt be left out; and there may be made ſeveral Receptacles above one another to receive the Raiſed Water, ſo as doth the *Crown* F F: and there ſhould be as many *Bellows* to Communicate every one with one *Receptacle*: theſe *Bellows* ſhould be moved by an Axis, ſo that when the *Firſt* is open, the *Second* ſhould be ſhut; the *Third* open, the *Fourth* ſhut; and ſo forth, alternatively; which may be eaſily done: By this means, the Firſt or Loweſt *Receptacle* would give the neceſſary Supply of Water to the *Second*, the *Second* to the *Third*, and the *Third* to the *Fourth*, &c. till the Water would be *Raiſed* to the intended Height; ſuch Receptacles might eaſily be ſet 12 or 15 Foot above one another, and ſo but few of them might *Raiſe Water* to a conſiderable Height, as well as ordinary *Pumps* do; but this New Way would have this advantage, that in the Ordinary Pumps the ſtrength to be applied lieth near the Water to be Raiſed, but by this Contrivance the Stream of a River may be applied to draw Water out of a Mine far Diſtant from it. By the ſame way the Stream of the *Thames* might keep conſtant *Water-Works* in *Windſor-Caſtle*, as eaſie almoſt as in the Loweſt Fields: The River *Seine* might do the ſame at St. *Germain*, and perhaps at *Verſailles* too, notwithſtanding the great Diſtance. For it is to be obſerved, That the Pipes of Communication between the *Bellows* and *Engine*, being meerly for the Conveying of the Air, which moves very ſwiftly, they may be ſlender enough, and ſo contain but a ſmall Quantity of Air to be Rarified; and beſides, they will not be ſubject to Burſt or Leak, ſince the Preſſure they bear, being all external to the Pipe, will rather ſtrengthen than break the ſame. From whence it follows, That the ſaid Pipes need not be ſtrong, but may be made at very ſmall Charges. It is alſo to be obſerved, That thoſe *Bellows* which are open, have the Air in them very much Rarified, ſo that the outward Air lieth heavy upon (to ſhut) them, by which means the Motion of the *Engine* muſt be helpt in Lifting up the oppoſite *Bellows*, that are to be opened: And this Obſervation may

anſwer

answer the greatest difficulty that might be objected against this Contrivance; So that I don't question, but this way for *Raising Water*, may on several Occasions be of a great Advantage.

*Further Explain'd. Fig. 201.* A B, A B, Are several *Receptacles* set above one another, which must be well Shut and Sodered every where.

C D D, C D D, Are two slender Pipes, whereby the First and Third *Receptacles* have a Communication with the *Pump* H H.

E F F, E F F, Two other slender Pipes, whereby the Second and Fourth *Receptacles* have a Communication with the *Pump* I I.

H H, I I, Two *Pumps* whose Plugs are so moved by the Axis L L, that when one goeth down the other goeth up.

M M, A Wheel fastened to the Axis L L, that it may be Moved by the Stream of a River.

N O, P Q, N O, P Q, Are big Pipes for the Water to go up, from a Lower into a Higher *Receptacle*.

O, Q, O, Q, Are *Valves* fitted to the top of the aforesaid Pipes that the Water may not go down thorough the same.

Now it is Plain, that when the *Plug* in the *Pump* H H, is going up, the Air comes in through the Pipes C D D, and so it is Rarified in the First and Third *Receptacles* marked A, A: and by that means the Water may be Driven up into the said *Receptacles* through the Pipes N O, because at the same time the *Plug* in the *Pump* I I, going down, causeth the Air to return to its Ordinary Pressure in the Second and Fourth *Receptacles*, that it may be able to drive up the Water through the said Pipes N O, and the Lowest Pipe draws the Water that Lies open to the Air. By the same reason, when the *Plug* in the *Pump* I I, goeth up, the Air must come in through the Pipes E F F: and so it is Rarified in the Second and Fourth *Receptacles* marked B, B, and by that means the Water may be driven up into the said *Receptacles* through the Pipes P Q, P Q, because at the same time the *Plug* in the *Pump* H H, going down, causes the Air to return to its Ordinary Pressure in the First and Third *Receptacles*, so that it is able to Drive up the Water through the said Pipes P Q.

*Several Objections, made by M. Nuis, Answered; by Dr. Papin. n. 186. p. 263.* 6. 1. To keep the *Receptacles* from being fill'd too much, the Water may be let out by Inserting into each a Crooked Pipe, reaching a pretty way downwards, and having its Lower Aperture shut up with a Valve, whereby the Water may run out when the *Receptacle* shall be Filled to a certain Height: And I may Add, to prevent new Difficulties, that least the *Pumps* should be fill'd to much, a Valve may be made that shall open as soon as the Air in the *Pump* should be more Comprest than the Outward Air: So the Air getting in through any Pores would be constantly let out.

2. I have not positively promised a good Success, but for *Windsor* and St. *Germain*; but when I spoke of *Versailles*, I used the word *perhaps*, thereby shewing, that before any one should go about such a great Undertaking, he should Reflect upon it more than I would then do, not having occasion for such work. But I now make the following Computation;

Let

Let the Diſtance of *Verſailles*, as M. *Nuis* ſuppoſeth, be 12000 Foot, and the Capacity of each *Receptacle* be about one half of a Cubick Foot: I might make the Wheel with the Axis to make their *Revolution* in one Minute of Time, and ſo Order all Things that the Air under the Aſcending *Plugs* might come to be Rarified to ſuch a degree that by its Elaſticity it might not Counterpoiſe more than 7 Foot of Water; but at the ſame time the Air in the *Receptacles* A, A, B, B, would, even in its greateſt Dilatation, be able to Counterpoiſe 17 Foot: ſo it is plain, that the Air will be driven from the *Receptacles* into the *Pumps* by a Strength equivalent to 10 Foot of Water: Now if we Compute the * *Velocity* of Air driven by ſuch a Preſſure: we ſhall find that the ſaid *Velocity* will be about 740 Foot in a Second: ſo that in half a Minute, during which the Plug goeth up, this Air might paſs above 22000 Foot, although it were not Rarified at all; but being Rarified, as we do ſuppoſe it to be, it might go a great deal further.

* *Vid. Sup. Cap.* V. §. XII.

I muſt now take notice, that according to the Honourable Mr. *Boyle*'s Experiments, the *Rarefaction* of the *Air* is much leſſer than Mr. *Nuis* takes it to be: for the Water contained in the Pipe N O, is ſo far from cauſing the Air to fill up a ſpace four times bigger, that it will not Extend it ſelf to a Space once bigger than before; Conſidering therefore the *Velocity* of the Air, and the ſmall Dilatation it doth ſuffer, if any one will take the trouble to Compute, he will find, that if the *Pumps* have in Diameter the Diagonal of a Square Foot, and the ſame Height: and if the ſmall Tubes of Communication be made of $\frac{1}{9}$ part of an Inch in Diameter; ſo that being 12000 Foot long, they may contain about one Cubick Foot of Air, that would be more than Sufficient to make the neceſſary *Rarefaction* in the *Receptacles*.

But for the good Succeſs of the *Engine* it is not enough to make the Air paſs from the *Receptacles* into the *Pumps*, it muſt alſo Return from the *Pumps* into the *Receptacles*: Now for this Intent it would be neceſſary to ſet the *Receptacles* but five Foot above one another; ſo, to Drive the Water up the Pipe N O, it would be enough that the Air in the *Receptacle* B, ſhould Preſs with a ſtrength Equivalent to 23 Foot of Water: For it is plain, that five Foot in the Pipe N O, together with a Preſſure Equivalent to 17 Foot, which I have ſuppoſed to be in the Upper *Receptacle* A, will make but 22 Foot in all; and therefore 23 Foot Preſſing in the *Receptacle* B, muſt prevail, and cauſe the Water to Aſcend: Now the Preſſure in the *Receptacle* being but 23 Foot, and the Air in the *Pump* Returning to its Ordinary Preſſure, which is about 33 Foot, it is plain that the Air going back to the *Receptacle* will be Driven by Strength Equivalent to 10 Foot, as well as it had been in coming from the *Receptacle* towards the *Pump*: and ſo the Bigneſs aſſigned for the *Communication Pipes* will alſo prove more than ſufficient to this Effect.

From what I have been ſaying, it is plain, That in great Diſtances there ſhould be made as many *Pumps* as *Receptacles*, as hath been already propounded: and for to Raiſe Water but 60 Foot High, there ſhould be required 13 or 14 *Receptacles*, and as many *Pumps* of the Bigneſs aforeſaid. Some People may take this for a great Difficulty. But I Anſwer, That in this

this *Engine*, this is not so much as it seems at First; because the Pressure being all from without, there is no need of any great strength to resist it, and so the Metal for the *Pump* will Cost but little: There may also be found Occasions where to make so good use of them, that such an *Engine* as I have Described would in a Years time save Labour enough to pay for many *Pumps*, since it might every hour Raise about 1800 Pounds of Water to the height of 60 Foot: Mean while I don't pretend to have given here the best Proportion for the Bigness of every part of the *Engine*; but it may be, by Altering the Capacity of the *Pumps*, of the *Pipes*, or of the *Receptacles*, a much more considerable Effect might be Produced.

3. The Water doth not at any time Ascend higher than from a Lower *Receptacle* into the next Upper *Receptacle*; which Height is but 12 Foot: so that is is plain enough, that the Pressure of the Air may be sufficient to Drive it up. It is Indifferent, whether it be by Rarefaction or otherwise that the Water comes into the *Receptacle* A; it is enough that the Water is there, and that the Air Presses upon it with such a Strength as will prevail against all that opposeth it.

4. Though the Use of the Pipes be meerly for Conveying of Air: They may nevertheless be easily fill'd with Water when need Requires, and so the Defects in them may as well be found out as in the Pipes that are used for the Conveying of Water.

*An Engine for Raising Water, by the Help of Fire; by Mr.* Tho. Savery. n. 253. *p.* 228. Fig. 202.

XII. A, The *Furnace.*

B, The *Boyler.*

C C, Two *Cocks*, which convey the *Steam* by turns to the *Vessels* D D.

D D, The *Vessels* which receive the Water from the Bottom, in order to Discharge it again at the Top.

E E E E, *Valves.*

F F, *Cocks* which keep up the Water, while the *Valves* on Occasion are Cleans'd.

G, The *Force* Pipe.

H, The *Sucking* Pipe.

I, The *Water.*

*An Hydraulique Engine; by* . . . . . . . n. 128. *p.* 679. Fig. 203.

XIII. This *Engine* is a *Chest* of Copper A, pierced with many Holes above B B, and holds within it the Body of a *Pump* E F M, whose *Sucker* D E, is Raised and Abased by two *Levers* C, O; These *Levers* having each of them two Arms, and each Arm being fitted to be laid hold on by both Hands of a Man. Each *Lever* is pierced in the middle by a Mortaise *a a*, in which an Iron Nail, which passes through the Handle of the *Sucker*, turns when the *Sucker* is raised or lower'd. Near the Body of the *Pump* there is a *Copper Pot* I H K, joined to it by the Tube G, and having another Tube K N L, which in N may be Turned every way.

To make this *Engine* Play, Water is pour'd upon the *Chest* to enter in at the Holes that are in the cover thereof. This Water is drawn into the Body

of

of the *Pump* at the Hole F, at the time when the *Sucker* is Raiſed; and when the ſame is let down, the *Valve* of the ſame Hole F ſhuts, and forces the Water to paſs through the Hole M into the Tube G, of which the *Valve* H being lifted up, the Water enters into the *Pot*, and filling the Bottom, it enters through the Hole K, into the Tube K N L, in ſuch a manner, that when the Water is Higher than the Tube K N L, and the Hole of the Tube G is ſhut by the *Valve* H, the Air Incloſed in the *Pot* hath no iſſue, and it comes to paſs, that, when you Continue to make the Water Enter into the *Pot* by the Tube G, which is much thicker than the Aperture of the end L, at which it muſt iſſue, it muſt needs be, that the Surplus of the Water that enters into the *Pot*, and exceeds that which at the ſame time iſſues through the ſmall end of the *Jet*, compreſſes the Air to find place in the *Pot*: which makes that, whilſt the *Sucker* is Raiſed again, to make new Water to Enter into the Body of the *Pump*, the Air which has been *Compreſs'd* in the *Pot*, Drives the Surplus of the Water by the Force of its *Spring*, mean time that a new *Compreſſion* of the *Sucker* makes new Water to enter, and cauſes alſo a new *Compreſſion* of Air. And thus the Courſe of the Water, which iſſues by the *Jet*, is always entertained in the ſame State.

XIV. A A, The Body of a Square Taper *Pump*, made of Oak, Elm, or Deal Planks; with a *Valve* at bottom *a a*. *A cheap Pump; by Mr.* Conyers. *n.* 136. *p.* 888.

B B, The *Bucket*, in the midſt of which there is a *Valve b*, not viſible in the Figure, being concealed by the Sides of the Leather *b b*.

C C C, The Iron to raiſe the *Bucket*. Fig. 204.

D D, The Wood at the bottom of the *Bucket* containing the *Valve*.

E E, The *Handle* for Raiſing the *Bucket*, to be managed by fewer Hands than Ordinary *Pumps* are; which may be Altered ſo as to employ a Horſe, or Mill, or other ſuch like way, more Advantagious than that of this Handle managed by the Strength of Men.

F F, A Square *Taper-Box* with Holes in the Sides, and open at the Bottom; into the narrower part of which is encloſed the narrower End of the Body of the *Pump*.

G G, An Additional *Bucket* of a larger Dimenſion to be placed on the Iron Work of the *Pump* about H, when it ſhall be needful to Lengthen the Taper of your *Pump*, and thereby to Raiſe the Water more forcibly to a greater Height.

I I, The *Spout* of the *Pump*, to caſt out the Water of the ſame breadth with the ſide of the *Pump*.

K K, The Iron or Wooden Work ſet off, or bent back (if need be) and placed at the back of this *Pump* for the eaſier and more capacious Motion of the *Pump Handle*, in which it moves.

This *Pump* was by me Contrived in 1673. when the *New Canal* of *Fleet-River* in *London* was Enlarged: It was found to Raiſe at leaſt twice as much Water proportionably as thoſe of the ſame, or rather bigger Bore, that were firſt made uſe off and caſt by. It was $8\frac{1}{2}$ Foot long, and 1 Foot 8 Inches broad at the top, and about 8 Inches broad at the Bottom, where it is inserted in the *Box*, and did cast out 8 Gallons at a *Stroke*, and 21 *Strokes* being made in one *Minute*, there was Delivered about 169 Gallons in a *Minutes* time; whence it is easie to Compute, what Quantity is thrown out in an *Hour*. This kind of *Pump* may, by the same Contrivance, be made of a Tree Bored through with a Taper-Bore; and a Basket may be used at the bottom of the *Pump* instead of the *Box-Colender*.

Ph. Col. *n.* 2, *p.* 35. XV. Papers *of less General Use, (Extracted from a Book of* Jo. Alph. Borellius *de Motu Animalium,)* Omitted.

1. A *Way how a Man may* Swim *under* Water, *and* Breath *by the help of a Bag about his Head.*

2. *Another Way of* Breathing *under* Water *by the help of a Leather Pipe kept open by Wreathed Wires, and extended from the* Swimmer's *Head to the top of the Water.*

3. *A way to make a* Submarine Vessel *accommodated with Ways to* Row *it, and to make it* Rise *and* Sink *in the Water.*

XVI. Accounts *of Books* Omitted.

*n.* 8. *p.* 145. *n.* 10. *p.* 173. 1. HYdrostatical Paradoxes, *made out by New* Experiments, *(for the most part Physical and Easie) by the Honourable* R. Boyle *Esq*;

*n.* 226. *p.* 481. 2. Recueil de diverses Pieces touchant quelques *Nouvelles Machines*, &c. Par *D. Papin. M. D. A Cassel*. 1695. in 8*vo*.

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# CHAP. VII.

## *Geography. Navigation.*

*A new Place for the first Meridian propos'd; by a* Professor *of* Math. *at* Seville. *n.* 118. *p.* 425. I. LOngitudo Terrestris *est Arcus Æquinoctialis ab uno ad alium Meridianum interceptus, sive Temporis Spatium, quod per Æquinoctialem numeratur, inter duo Loca; quare consonum fuerit, *Longitudinis Principium* in ipso Æquinoctiali constituere. Insuper cum Circulus ille Æquinoctialis *Globum* in Borealem & Australem partem dividat, si detur hujus *Primarii Meridiani* Fixatio in eo, erit inter Boreales & Australes Æquitas atque Conformitas. Deinceps, oportet, ad Præcisionem, Locum hujus *Primarii Meridiani* esse parvum, ut *Longitudinis* Numeratio exactius exprimatur: non ut aliqui,

aliqui, qui omnes *Insulas Fortunatas* pro hujus *Principio* assumebant, & Distantiam duorum Graduum, inter earum aliquas, non notabant; quod certè absurdum nimis erat. Iterum, Præcautio alia est habenda, ut *Primarius Meridianus* non confundatur cum Terris & Locorum Imaginibus in *Globo* vel *Mappa* exaratis, quod fiet si per Medias Terras Transeat: Et si præcipuas Terræ Partes dividat, ut *Americam*, *Africam*, & *Europam*, per Maria transiens, erit eo aptior & convenientior in *Globi Terraquei* repræsentatione. Quas omnes memoratas causas considerans, Inveni, quod Natura (nihil frustra suppeditans) posuit sub ipso *Æquinoctiali Circulo* Insululam quandam prope *Brasiliam*, olim *Abroxos* nominatam, quæ Insula distat à *Teneriffæ Pico* Grad. 9. Occidentaliores, & ab *Uraniburgo* Grad. 42. Occidentaliores; in qua inveniuntur omnia ad *Primarium Meridianum* conducenda, ut à me æstimatur.

*M. Hugens's Instructions for finding the Longitude with Pendulum Watches; Enlarge by . . . . . . . . . n. 47. p. 937.*

II. 1. Those that intend to make use of *Pendulum Watches* at *Sea*, must have two of them at least; that, if one of them should by mishap or neglect come to stop, or (being by length of time become foul) need to be made clean, there may likely always remain one in Motion.

2. The *Watches* on *Shipboard* are to be Hung in a Close Place, where they may be freest from Moisture or Dust, and out of Danger of being disordered by Knocking or Touching.

3. Before the *Watches* be brought on *Shipboard*, 'tis convenient they be adjusted to a *Middle* or *Mean Day*, the Use of them being then most easie.

*To adjust the Watches.*

4. Here take Notice, That the Sun passeth the 12 Signs, or makes one Entire Revolution in the *Ecliptick* in 365 Days, $5^{h.}$ 48'. or thereabout: and that those Days, reckoned from Noon to Noon, are of different Lengths; as is known to all that are versed in *Astronomy*. Now between the Longest and the Shortest of those Days, a Day may be taken of such a Length, as 365 such Days, $5^{h.}$ 49'. &c. make up, or are Equal to that Revolution: And this is called the *Equal* or *Mean Day*, according to which the *Watches* are to be Set; and therefore the Hour or Minute shewed by the *Watches*, though they be perfectly Just and Equal, must needs differ almost continually from those that are shew'd by the *Sun*, or are Reckoned according to it's Motion. But this Difference is Regular, and is otherwise called the *Æquation*; which is accounted from the first of *February* in the following *Table*.

| | *January.* | | *February.* | | *March.* | | *April.* | | *May.* | | *June.* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ′ | ″ | ′ | ″ | ′ | ″ | ′ | ″ | ′ | ″ | ′ | ″ |
| 1 | 6 | 10 | 0 | 0 | 4 | 46 | 14 | 23 | 19 | 25 | 16 | 24 |
| 2 | 5 | 47 | 0 | 2 | 5 | 03 | 14 | 39 | 19 | 28 | 16 | 13 |
| 3 | 5 | 24 | 0 | 4 | 5 | 21 | 14 | 55 | 19 | 29 | 16 | 01 |
| 4 | 5 | 02 | 0 | 8 | 5 | 39 | 15 | 10 | 19 | 29 | 15 | 49 |
| 5 | 4 | 41 | 0 | 12 | 5 | 57 | 15 | 25 | 19 | 29 | 15 | 37 |
| 6 | 4 | 21 | 0 | 16 | 6 | 15 | 15 | 39 | 19 | 28 | 15 | 24 |
| 7 | 4 | 02 | 0 | 21 | 6 | 33 | 15 | 53 | 19 | 26 | 15 | 11 |
| 8 | 3 | 44 | 0 | 26 | 6 | 51 | 16 | 07 | 19 | 24 | 14 | 58 |
| 9 | 3 | 27 | 0 | 32 | 7 | 09 | 16 | 21 | 19 | 21 | 14 | 45 |
| 10 | 3 | 11 | 0 | 40 | 7 | 27 | 16 | 34 | 19 | 18 | 14 | 32 |
| 11 | 2 | 55 | 0 | 48 | 7 | 45 | 16 | 47 | 19 | 15 | 14 | 19 |
| 12 | 2 | 39 | 0 | 57 | 8 | 03 | 16 | 59 | 19 | 11 | 14 | 06 |
| 13 | 2 | 23 | 1 | 06 | 8 | 22 | 17 | 11 | 19 | 07 | 13 | 53 |
| 14 | 2 | 07 | 1 | 16 | 8 | 41 | 17 | 22 | 19 | 02 | 13 | 40 |
| 15 | 1 | 52 | 1 | 26 | 9 | 01 | 17 | 33 | 18 | 57 | 13 | 27 |
| 16 | 1 | 38 | 1 | 37 | 9 | 21 | 17 | 43 | 18 | 51 | 13 | 15 |
| 17 | 1 | 25 | 1 | 49 | 9 | 41 | 17 | 53 | 18 | 45 | 13 | 03 |
| 18 | 1 | 13 | 2 | 02 | 10 | 01 | 18 | 03 | 18 | 39 | 12 | 52 |
| 19 | 1 | 02 | 2 | 15 | 10 | 21 | 18 | 13 | 18 | 33 | 12 | 41 |
| 20 | 0 | 51 | 2 | 28 | 10 | 40 | 18 | 23 | 18 | 26 | 12 | 30 |
| 21 | 0 | 41 | 2 | 42 | 10 | 59 | 18 | 32 | 18 | 18 | 12 | 19 |
| 22 | 0 | 32 | 2 | 56 | 11 | 18 | 18 | 39 | 18 | 10 | 12 | 08 |
| 23 | 0 | 24 | 3 | 11 | 11 | 37 | 18 | 46 | 18 | 01 | 11 | 58 |
| 24 | 0 | 18 | 3 | 26 | 11 | 56 | 18 | 53 | 17 | 51 | 11 | 48 |
| 25 | 0 | 13 | 3 | 41 | 12 | 15 | 18 | 59 | 17 | 41 | 11 | 38 |
| 26 | 0 | 9 | 3 | 56 | 12 | 34 | 19 | 04 | 17 | 30 | 11 | 28 |
| 27 | 0 | 6 | 4 | 12 | 12 | 53 | 19 | 09 | 17 | 19 | 11 | 18 |
| 28 | 0 | 3 | 4 | 29 | 13 | 12 | 19 | 14 | 17 | 08 | 11 | 09 |
| 29 | 0 | 1 | | | 13 | 31 | 19 | 18 | 16 | 57 | 11 | 00 |
| 30 | 0 | 0 | | | 13 | 49 | 19 | 22 | 16 | 46 | 10 | 52 |
| 31 | 0 | 0 | | | 14 | 06 | | | 16 | 35 | | |

| | *July.* | | *August.* | | *Septemb.* | | *October.* | | *Novemb.* | | *Decemb.* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ′ | ″ | ′ | ″ | ′ | ″ | ′ | ″ | ′ | ″ | ′ | ″ |
| 1 | 10 | 45 | 11 | 07 | 19 | 41 | 29 | 16 | 31 | 13 | 21 | 14 |
| 2 | 10 | 38 | 11 | 16 | 20 | 01 | 29 | 30 | 31 | 03 | 20 | 44 |
| 3 | 10 | 31 | 11 | 25 | 20 | 22 | 29 | 43 | 30 | 53 | 20 | 14 |
| 4 | 10 | 25 | 11 | 36 | 20 | 43 | 29 | 56 | 30 | 43 | 19 | 44 |
| 5 | 10 | 19 | 11 | 48 | 21 | 04 | 30 | 09 | 30 | 32 | 19 | 14 |
| 6 | 10 | 13 | 12 | 01 | 21 | 25 | 30 | 22 | 30 | 20 | 18 | 44 |
| 7 | 10 | 07 | 12 | 14 | 21 | 47 | 30 | 34 | 30 | 08 | 18 | 14 |
| 8 | 10 | 02 | 12 | 28 | 22 | 09 | 30 | 45 | 29 | 55 | 17 | 44 |
| 9 | 9 | 58 | 12 | 42 | 22 | 31 | 30 | 55 | 29 | 40 | 17 | 14 |
| 10 | 9 | 54 | 12 | 57 | 22 | 52 | 31 | 04 | 29 | 23 | 16 | 44 |
| 11 | 9 | 51 | 13 | 12 | 23 | 13 | 31 | 12 | 29 | 06 | 16 | 14 |
| 12 | 9 | 49 | 13 | 27 | 23 | 33 | 31 | 19 | 28 | 48 | 15 | 44 |
| 13 | 9 | 47 | 13 | 43 | 23 | 53 | 31 | 26 | 28 | 30 | 15 | 14 |
| 14 | 9 | 46 | 13 | 59 | 24 | 13 | 31 | 32 | 28 | 11 | 14 | 43 |
| 15 | 9 | 46 | 14 | 16 | 24 | 33 | 31 | 38 | 27 | 51 | 14 | 12 |
| 16 | 9 | 46 | 14 | 33 | 24 | 53 | 31 | 43 | 27 | 30 | 13 | 41 |
| 17 | 9 | 47 | 14 | 50 | 25 | 13 | 31 | 47 | 27 | 08 | 13 | 10 |
| 18 | 9 | 49 | 15 | 08 | 25 | 33 | 31 | 50 | 26 | 45 | 12 | 40 |
| 19 | 9 | 52 | 15 | 26 | 25 | 52 | 31 | 53 | 26 | 22 | 12 | 10 |
| 20 | 9 | 56 | 15 | 45 | 26 | 11 | 31 | 55 | 25 | 58 | 11 | 40 |
| 21 | 10 | 00 | 16 | 04 | 26 | 30 | 31 | 55 | 25 | 34 | 11 | 10 |
| 22 | 10 | 04 | 16 | 23 | 26 | 49 | 31 | 55 | 25 | 10 | 10 | 40 |
| 23 | 10 | 08 | 16 | 42 | 27 | 08 | 31 | 55 | 24 | 45 | 10 | 10 |
| 24 | 10 | 13 | 17 | 01 | 27 | 26 | 31 | 54 | 24 | 20 | 9 | 41 |
| 25 | 10 | 18 | 17 | 21 | 27 | 43 | 31 | 52 | 23 | 55 | 9 | 13 |
| 26 | 10 | 23 | 17 | 41 | 28 | 00 | 31 | 50 | 23 | 30 | 8 | 45 |
| 27 | 10 | 28 | 18 | 01 | 28 | 16 | 31 | 47 | 23 | 04 | 8 | 17 |
| 28 | 10 | 34 | 18 | 21 | 28 | 32 | 31 | 43 | 22 | 38 | 7 | 50 |
| 29 | 10 | 41 | 18 | 41 | 28 | 47 | 31 | 37 | 22 | 11 | 7 | 23 |
| 30 | 10 | 49 | 19 | 01 | 29 | 02 | 31 | 30 | 21 | 43 | 6 | 58 |
| 31 | 10 | 58 | 19 | 21 | | | 31 | 22 | | | 6 | 34 |

By the help of the foregoing *Table* you will always know, what a Clock it is by the *Sun* precifely, and confequently, whether the *Watches* have been fet to the right Meafure of the *Mean Day*, or no; ufing the *Table* as follows.

When you firft Set your *Watch* by the *Sun*, you are to Subduct from the Time obferved by the *Sun*, the *Æquation* adjoyned to that Day of the Month in the *Table*, and to Set the *Watches* to the remaining Hours, Minutes and Seconds; that is, the *Watches* are to be Set fo much Slower than the Time of the *Sun*, as (in the *Table*) is the *Æquation* of that Day; fo that the *Æquation* of the Day Added to the Time of the Clock, is the True Time by the *Sun*. And when after fome Days, you defire to know by the *Watch* the Time by the *Sun*, you are to Add to the Time fhewed by the *Watch*, the *Æquation* of that Day; and the Aggregate fhall be the Time by the *Sun*, if the *Watch* hath been perfectly well Adjufted after the Meafure of the *Mean Days*; for the doing of which, this will be a convenient Way;

Draw a *Meridian Line* upon a Floor, and then hang two Plummets, each by a fmall Thread or Wire, directly over the faid *Meridian*, at the diftance of fome two Foot or more one from the other, as the fmallnefs of the Thread will admit. When the Middle of the *Sun* (the Eye being placed fo, as to bring both the Threads into one Line) appears to be in the fame Line exactly, (for the better and more fecure difcerning whereof, you muft be furnifh'd with a Glafs of a Dark Colour, or fomewhat Blackt with the Smoak of a Candle,) you are then immediately to Set the *Watch*, not precifely to the Hour of 12, but by fo much lefs as is the *Æquation* of that Day; *e. g.* If it were the 12th of *March*, the *Æquation* of that Day being by the Table, 8′. 3″. Thefe are to be Subducted from 12 Hours, and the Remainder will be 11$^h$. 51′. 57″. to which Hours, Minutes, and Seconds, you are to Set the *Index* of the *Watch* refpectively: Then after fome Days you are to obferve again in the fame manner, and likewife to note the Hour, Minute, and Second of the *Watch*; to which you are to add the *Æquation* of thefe Days, taken out of the Table; and if the Aggregate do juft make 12 Hours, the *Watch* is Adjufted to the Right Meafure; but if it differ, you are to Divide the Minutes and Seconds of that Difference by the Number of the Days between both the Obfervations to get the Daily Difference.

Let us fuppofe this Second Obfervation to have been made the 20th of *March*, *viz.* Eight Days after the firft, and finding that the Middle of the *Sun*, being feen in the Meridian in the fame Line with the two Threads, as before,

| | | | |
|---|---|---|---|
| The *Watch* Points,———————————— | 11$^h$. | 51′. | 07″. |
| The *Æquation* of the 20th of *March*, by the *Table*, is——— | 00 | 10 | 40 |
| Which being Added to the Time, fhow'd by the *Watch*, gives— | 12 | 01 | 47 |

If this had been juft 12 Hours, the *Watch* would have been well adjufted, but being 1′. 47″. more than 12, it hath gone fo much too faft in eight Days. And thefe 1′. 47″. that is 107″. being divided by 8, there comes 13⅜ Seconds for the Difference of every 24 Hours; which Difference being known,

known, if you want time, or have no mind to take the Pains, to Adjust the *Watch* to its Right Measure, (this being not necessary, since you may bring it thus on *Ship-Board)* note only the Daily Difference, and regulate your self accordingly. But if you will Adjust it better, you must Remove the less Weight of the *Pendulum* a little downwards, which will make it go Slower; and then you must begin a-new to observe by the *Sun*, as before. If it had gone too Slow, you must have Remov'd the mentioned Weight somewhat upwards. And this is of that Importance in the finding out of *Longitudes*, that if it be not Observed, you may sometimes in the space of Three Months misreckon 7 Degrees, and more yet, (without any fault in the *Watches*;) which under the *Tropicks* will amount to above 400 *English Miles*.

The *Watch* may be also Adjusted *on Board*, when a Ship Rides at Anchor, thus: In the Morning, when the *Sun* is just half above the *Horizon*, Note what Hour, Minute, and Second, the *Watch* points at, if it be going; if not, set it a going, and put the *Indexes* at what Hour, Minute, and Second you please. Let them go till *Sun-Set*, and when the Body of the *Sun* is just half under the *Horizon*, see what Hour, Minute, and Second, the *Indexes* of the *Watch* point at, and note them too; and reckon, how many Hours, *&c.* are passed by the *Watch* between the one and the other. Then take the half of that Number, and Add it to the Hours, *&c.* of the Morning Observation, and you shall have the Hours, *&c.* which the *Watch* did show, when the *Sun* was in the South; whereunto Add the *Æquation* in the Table belonging to that Day, and note the Sum. Then some Days being pass'd, (the more the better,) you are to do just the same: And if the Hour of this last Day be the same that was noted before, your *Watch* is well adjusted; but if it be more or less, the Difference divided by the Number elapsed between the two Observations, will give the Daily Difference. And if you will, you may let it Rest there, or otherwise, Removing the Lesser Weight of the *Pendulum* you may Adjust it better. You may also, instead of the *Sun*'s *Rising* and *Setting*, take two Equal Altitudes of the *Sun*, before and after Noon, and having noted the time given by the *Watches* at the time of both the Observations, proceed with it in the same manner, as was just now directed for Observing the *Sun* in the *Horizon*. In either of which ways there may be some Errour, caused by the *Sun*'s *Refraction*, which is inconsiderable, and therefore needs not to be taken notice of.

*To find by them the* Longitude *at Sea.*

5. Give to each of the *Watches* a Name, or a Mark, as A, B, C; and before you set *Sail*, set them to the Time observed by the *Sun* in the place where you are, and whence you are departing, allowing for the *Æquation* of the Day whereon you make your Observation; which Day you are to note, if the *Watches* be not well adjusted; otherwise it is not necessary.

Then afterwards being at *Sea*, and desiring to know the *Longitude* of the Place where you are; that is, How many Degrees the Meridian of that Place is more Easterly or Westerly, than the Meridian of that Place where you did set the *Watches*; you must observe by the *Sun* or *Stars*, what Time of the Day it is, as precisely as is possible, and Note at the same time, to what Hour,

Minutes,

Minutes and Seconds the *Watches* do point, (which Time, if the *Watches* be not ſet to the Right Meaſure, is by the known Daily Difference to be Adjuſted,) adding thereunto the *Æquation* of the preſent Day, which gives you the time of the Day, ſhewed by the Sun, at the Place where the *Watches* were ſet: And if this time of the Day be the ſame with that obſerved where you are, then you are under the ſame Meridian with the Place where the *Watches* were ſet by the *Sun*; but if the Time of the Day, obſerved where you are, be greater than that ſhewed by the *Watches*, you may be aſſured, that you are come under a more Eaſterly Meridian; and if leſs, you are come under a more Weſterly. And Counting for every Hour of Difference of Time, 15 Degrees of *Longitude*, and for every Minute, 15 Minutes or $\frac{1}{4}$ of a Degree, you ſhall then know, how many Degrees, Minutes, &c. the ſaid *Meridians* do Differ from one another. *E. g.* Suppoſe the *Watches* A, B, C, were Set at the Place, whence you parted, on the 20th of *February*, to the time of Day obſerved by the *Sun*, abating the *Æquation* of the 20th of *February*, (*viz.* 2′. 28″.) and ſuppoſe that the *Watch* A, be ſet to its Right Meaſure, but that B goes every Day 7″. too Slow, and C every Day 12″. too Faſt. Some Days after, ſuppoſe the 5th of *May*, deſiring to know the *Longitude* of the Place where you are at *Sea*;

| | | | |
|---|---|---|---|
| You Obſerve the Time of the Day there to be | 05ʰ | 18′ | 10″. |
| And you find the *Watch* A to point at | 02 | 06 | 00 |
| But the *Watch* B to point at | 1 | 57 | 22 |
| Going too Slow by 7″. every Day, which makes in 74 Days, (*viz.* From the 20th of *Febr.* to the 5th of *May*,) | 00 | 08 | 38 |
| Which being added to its own Time, gives the ſame with that of the *Watch* A, *viz.* | 02 | 06 | 00 |
| You find alſo the *Watch* C to point at | 02 | 20 | 48 |
| Going 12″. too Faſt every Day, which makes in 74 Days | 00 | 14 | 48 |
| Which being Subducted from its own Time, gives again | 02 | 06 | 00 |
| The Time of the Day therefore by the *Watches* being | 02 | 06 | 00 |
| Add thereunto the *Æquation* of the 5th of *May* | 00 | 19 | 29 |
| And ſo you have for the time of Day at the Place where the *Watches* were ſet | 02 | 25 | 29 |
| But the Time Obſerved being | 05 | 18 | 10 |
| Exceeds this by | 02 | 52 | 41 |
| Wherefore the *Meridian* of the Place, where you are *May* 5. is more Eaſterly, than the Place where the *Watches* were Set by. | 02 | 52 | 41 |
| Which being reduced to Degrees, reckoning 15 Degrees for an Hour, comes to | 43°. | 10′ | 15″ |

'Tis true, that from the ſame Reckoning it may be concluded, that you are 180 Degrees more Eaſterly, which happens, becauſe the *Hour Index* goes round in the ſpace of 12 Hours in the *Watches*; but the Difference is ſo great, that one cannot be deceived in it; elſe the *Watch* might be ſo made, that the *Index* ſhall go round about once in 24 Hours.

*To find the Time of the Day at Sea.*

6. Since that for finding the *Longitude*, the Time of the Day, at the Place where you are, muſt be known, (as hath been ſaid above) you muſt have a care

care to Observe that Time as precisely as is possible. For every Minute of Time, that you misreckon, makes $\frac{1}{4}$ Degree in *Longitude*, which amounts, near the *Æquator*, to above 15 *English Miles*, but less elsewhere. Wherefore to find the Time of the Day with certainty, the best way is to Observe the *Sun's Altitude* when it is in the East or West, (the nearer the better:) for being there, its *Altitude* Changes in a short Time more sensibly than before or after; and thus from the Height of the Pole and the Declination of the Sun the Hour may be Calculated.

7. At the *Rising* and *Setting* of the *Sun*, when it is half above the *Horizon*, mark the Time of the Day, which the *Watches* then shew; and though ye have in the mean time sailed on, it is not Considerable. Then Reckon by the *Watches*, what Time is elapsed between them, and Add the half thereof to the Time of the *Rising*, and you shall have the Time by the *Watches*, when the *Sun* was at *South*; to which is to be added the *Æquation* of the present Day by the Table. And if this together makes 12 Hours, then was the Ship at Noon under the same *Meridian*, where the *Watches* were Set with the *Sun*. But if the Summ be more than 12, then was she at Noon under a more *Westerly Meridian*; and if less, then under a more *Easterly*; and that by as many Times 15 *Deg.* as that Summ Exceeds or Comes short Hours of 12; as the Calculation thereof hath been already deliver'd. *An easier Way.*

Suppose, *e. g.* that the *Watches* A and B, as before, were set with the *Sun* at the Place whence you Parted, the 20 of *February*; and the *Indexes* set to the Hour, Min. and Sec. shewed by the *Sun*, abating the *Æquation* of that day, *viz.* 2′ 20″; the *Watch* A being Reduced to the Right Measure, and B going too slow by 7″ a day. Afterwards on the 22[d] of *May*, desiring to know the *Longitude* of the Place to which you are come, you Observe in the *Morning* the *Sun* half above the *Horizon*, when the *Watch* points at—2h 30′ 10″

| | | | |
|---|---|---|---|
| And in the *Evening*, the *Sun* being half under the *Horizon*, when the same *Watch* points at | 3 | 8 | 40 |
| To find the Time Elapsed between them, Subducting the Time of the *Rising* | 2 | 30 | 10 |
| From | 12 | 0 | 0 |
| There Remains | 9 | 29 | 50 |
| Adding thereunto the Time of the *Setting* | 3 | 8 | 40 |
| You have for the time Elapsed between the Observations | 12 | 38 | 30 |
| Whereof the Half | 6 | 19 | 15 |
| Being Added to the Time of *Rising* | 2 | 30 | 10 |
| You have the Time by the *Watch* A, when ☉ was in the *South* | 8 | 49 | 25 |
| And after the same manner you are to seek the Time by the *Watch* B, when the *Sun* was in the *South*; which Let be | 8 | 38 | 48 |
| But this *Watch* going 7″ a day too slow, it is Retarded in 91 days (from the 20[th] of *Febr.* to the 22[d] of *May*) | 0 | 10 | 37 |
| Which therefore Added to the said Time gives | 8 | 49 | 25 |
| That is the same Time given by the *Watch* A. Now Adding to this Time of the *Watches*, the *Æquation* of the 22[d] of *May* | 0 | 18 | 10 |
| You have | 9 | 7 | 35 |

 Which

Which is the same Time of the Day with that of the Place, where the *Watches* were set when the *Sun* was in the same *Meridian* with the Ship, or where the Ship was at Noon. h. ' "

The Difference is ——————————————— 2 52 25

Wherefore this last *Meridian* is by so much more *Easterly*, than the first, which being reduced to Degrees ( as hath ° ' " been formerly directed ) make ——————————— 43 6 15

'Tis manifest, that by this way you find precisely enough the *Longitude* of the Place, where you were at Noon, or the Time of the Sun's being in the South: Which although it differs from the *Longitude* of the Place, where you are when you observe the *Setting* of the *Sun*, yet you may Estimate near enough, how much you have advanc'd, or chang'd the *Longitude* in these few Hours, by the *Log Line*, or other Ordinary Practises of *Reckoning* the Ship's Way; or ( which is the surer way ) by the Degrees passed in 24 Hours by a former day's Observation.

You may also, instead of Observing the *Sun's Rising* and *Setting*, observe the *Setting* first, and then the next Morning the *Rising*; marking at both Times the Time shew'd by the *Watches*; and find thence, after the same manner as before the *Longitude* of the Place where the Ship was at Midnight.

Finally, You may also, instead of the *Rising* and *Setting* of the *Sun*, Obseve before and after Noon two *Equal Altitudes* of the *Sun*, Noting the Time shown by the *Watches*, and Reckoning in the same manner, as hath been said of the *Rising* and *Setting*: Yet it is to be consider'd, that the *Altitudes* of the *Sun* are best taken, when it is about East and West, as hath been already intimated. But note that in Sailing North and South you make not the Observations at the Sun's *Rising* and *Setting*, but at its being due *East* and *West*.

8. But you may put the Rule here prescribed in Practice, by taking two *Equal Altitudes* of some known *Star*, that Riseth high above the Horizon. For you shall thence according to the mention'd Rule, know at what time by the *Watches* the Star hath been in the South, and so the *Right Ascension* of that Star being known, as also the *Right Ascension* of the *Sun*, you may thence easily Calculate, what Time it then was: Which being compared with the Time of the *Watches*, as before, shall give the *Longitude* of the Place where where you were, when you had the Star in the Meridian.

9. If the *Watches* that have gone exactly for a while, should come to differ from one another ( as in length of time it may well happen, that the one or the other fail a Minute, more or less; ) in that case it will be best to Reckon by that which goes fastest; unless you perceive an apparent cause, why it goes too fast (as it may happen when the *Cheeks* retain not their proper Figures) seeing it is not so easie for these *Pendulum Watches* to move faster than at first, as it is to go slower. For the *Wire*, on which the *Pendulum* hangs, may perhaps by the violent Agitation of the Ship, come to stretch a little, but it cannot grow shorter; and the little weight of the *Pendulum* may perhaps slip downward, but cannot get up Higher.

If

If it should be said, that upon any Foulness the *Watch* will go faster by reason of the shorter Vibrations of the *Pendulum*, it is to be considered, that this is only True when the *Watches* have no *Checks*, but when they have them 'tis not so. n. 48. p. 976.

10. When you get Sight of any known Country, Island or Coast, be sure to Note the *Longitude* thereof as exactly as you can by the help of the Rules here prescribed. First, thereby to Correct the *Sea-Maps*, after the *Longitude* of a Place shall have been found at divers times to be the same, so that you doubt no more of it. For all *Maps* are very defective as to the Scituation of Places in respect of East and West, chiefly where Seas are interposed. Secondly, to be able always to know in the Prosecution of your Journey, how far you have sailed from any Place to the East or West. And if by any Notable Mischance or Carelesness all the *Watches* should come to stand still, yet you may at any Place, whereof the *Longitude* is certainly known, set them a Going again, and Adjust them there by the *Sun*, and so reckon the *Longitudes* from that same Meridian. For you are to know, that you are not at all obliged to put one certain Meridian of any known Place as a Beginning of the *Longitude Reckoning*; this happening only in *Maps*, or Tables of *Longitude*: as when you take for that purpose the *Meridian* of the *Pico* in *Teneriffe*, or that of the Islands of *Corvo* and *Flores* (the most Westerly of the *Azores*) or any others. Yet it were very fit, that all *Geographers* agreed and pitched upon one and the same *First Meridian*, that so all Places might be known by the same Degrees as well of *Longitude* as *Latitude*; though in Voyaging, it is sufficient to observe only the Difference of *Longitudes*, beginning to Reckon from the *Meridian* of any Place, you please, as if it were the *First*. n. 47. p. 951.

11. If it happen that being at *Sea* all the *Watches* stop, you must as speedily as is possible, set them a Moving again, that you may know, how much you Advance from that Place towards the *East* or *West*.

12. The *Watches* being distinguish'd by Marks, as A, B, or the like, every Day about Noon, or when most conveniently you can, Observe the Time of the Day by the *Sun*, or by the *Stars* at Night, and subduct thence the Minutes and Seconds, that are adjoined to that Day in the Table, and write the Remainder down in a Paper, wherein 9 Columns or more are mark'd, placing them in the Second Column, having placed the Day of the Month in the First; and at the same Time write down the Hours, Minutes, and Seconds, of each *Watch* in a distinct Column, all Opposite one to another. Then in another Column write down the Difference between the Time taken by Observation, and that given by the *Watches*, or one of them. Then one Column for the *Latitude*: One for the *Longitude* by the Ordinary Way of *Reckoning*: Another for the *Longitude* taken from the Difference between the Time found by Observation, and that given by the *Watches*. And at last a large Column to Note the Accidents, that befall the *Watches*, &c. *The Journal for the Watches.*

2. Major *Holmes* having left the Coast of *Guiny*, and being come to the Isle of St. *Thomas* under the *Line*, he adjusted his *Watches* there, and put to Sea, and Sailed Westward, 7 or 800 Leagues, without Changing his Course; after *The Success of Pendulum Watches; by Maj. Holmes.* n. 1. p. 13.

after which finding the Wind favourable, he steered towards the Coast of *Africk* N. N. E. But having Sailed upon that Line, a matter of two or three hundred Leagues, the Masters of the other Ships under his Conduct, apprehending that they should want Water, before they could reach that Coast, did propose to him to steer their Course to *Barbadoes*, to supply themselves with Water there. Wheeupon the said *Major* having called the Master and Pilots together and caus'd them to produce their Journals and Calculations, it was found that those Pilots did differ in their Reckoning from that of the *Major*, one of them about 80 Leagues, another about an hundred, and the third more; but the *Major* judging by his *Pendulum Watches* that they were only some thirty Leagues distant from the Isle of *Fuego*, which is one of the Isles of *Cape Verde*, and that they might reach it next Day, and having a great Confidence in the said *Watches*, resolved to steer their Course thither, and having given order so to do, they got the very next Day about Noon a Sight of the said Isle of *Fuego*, finding themselves to Sail directly upon it, and so arrived that Afternoon as he had said.

M. *Hugens* being informed of this Success wrote to *Paris* to this effect; I did not imagine that the *Watches* of this First Structure would succeed so well, and I had reserved my main Hopes for the New Ones. But seeing that those have already served so successfully, and that the other are yet more just and exact, I have the more Reason to believe, that the Invention of *Longitudes* will come to its Perfection. In the mean time I shall tell you, that the States did receive my Proposition, when I desired of them a *Patent* for these New *Watches*, and the Recompense set apart for the Invention in Case of Success; and that without any Difficulty they have granted my Request, commanding me to bring one of these *Watches* into their Assembly, to explicate unto them the Invention, and the Application thereof to the *Longitudes*, which I have done to their Contentment.

Ib. p. 14.

*Longitudes from the Moon's Places; by* Math. Professor *at* Seville. n. 118. p. 427.

III. 1. Inveni tandem Modum *Lunæ Locum* sciendi, exili quodam Instrumentiolo tantum adjutus, ad unum vel duo Scrupula; & quod mirum est, nec *Refractiones*, nec *Parallaxes* meis Observationibus obsunt, quia Ingeniosa Methodus his Tricis me liberat. Possum hac Methodo *Terra Marique* uti; ideo Tabulis *Lunæ* Correctis non amplius Modus desideratas *Locorum Longitudines* captandi, in omnibus *Terræ Marisque* Locis, ignorabitur.

*Consider'd by* Mr. Flamsteed. Ib. p. 431.

2. Quod de Instrumentiolo suo scribit *Professor Hispalensis*, Fidem meam (quod bonâ ipsius Veniâ dictum velim) superat. Privari enim *Luna* nec *Refractione*, nec *Parallaxi*, in Horizonte nostro potest, nisi ad *Zenith* aliquando posset pertingere; eousque enim extenduntur, ejusque Locum implicant *Refractiones*: Desinit in Nonagesimo Gradu *Eclipticæ* semel tantum de Die *Longitudinis Parallaxis*, sed *Latitudinis* non perinde. Nec satis capio, quomodo fabricari Instrumentum possit, quod unâ cum *Parallaxi Refractionem*, cujus Incrementum longè diversam habet rationem, consideret.

IV. The

IV. The Obſervation of *Lunar Occultations* is of Singular Uſe to determine the *Longitude* of Places, eſpecially thoſe that are far Remote. Longitudes *from Lunar Occultations by; M.* Halley. n. 181. *p.* 87.

V. The *Revolution* of *Jupiter* upon his *Axis* being the ſwifteſt, and the moſt Regular Motion that is hitherto known in the Heavens, a Traveller alone, even without having any Correſpondence with other Obſervers, may make uſe of it to find the *Longitudes* of the moſt Remote Places of the Earth. Longitudes *by the Revolution of* Jupiter *upon his Axis; by M.* Caſſini. n. 82. *p.* 4042.

VI. 1. Utrum æque bene *Jovialium Eclipſes* ad inveſtigandam *Differentiam Meridianorum* conducant, ut quidem Occultationes Fixarum à *Luna*, habeo fere cur dubitem; præprimis ob minus Tardum *Jovialium* Motum, utut etiam accuratiori Tubo peragantur. Longitudes *by the Satellite Eclipſes; by M.* Hevelius. *n.* 78. *p.* 3030.

2. The *Eclipſes* of the *Satellites* of *Jupiter*, which happen almoſt every Day, afford a fair Way for eſtabliſhing the *Longitudes* over all the Earth. For, beſides that theſe *Eclipſes* are very frequent, the *Emerſion* and *Immerſion* of theſe *Satellites*, eſpecially in the Shadow of *Jupiter*, is ſo Momentany and ſo Senſible, that they may be obſerved with the greateſt Exactneſs, being altogether Exempt from thoſe Eſſential Inconveniencies that accompany the *Eclipſes* of the *Sun* and *Moon*, which alſo are Rare, and whoſe *Beginning* and *End* are always doubtful by reaſon of a certain *Ambiguous Light*. *By M.* Borelli. *n.* 128. *p.* 691.

The *Longitudes* of Places at *Sea*, *Capes*, *Promontories*, and divers *Iſlands*, being once exactly known by this Means, would doubtleſs be of Great Help, and conſiderable Uſefulneſs to *Navigation*.

3. The *Eclipſes* of *Jupiter's Satellits* have been eſteemed, and certainly are, a much better Expedient for the diſcovery of the *Longitude* than any yet known, by reaſon that they happen frequently, and are eaſily obſervable with a Teleſcope of 12 Foot, or for need with one of eight. *By Mr.* Flamſteed. *n.* 151. *p.* 322.

The *Longitude* might be alſo attained by Obſervations of the *Moon*, if we had Tables that would anſwer her Motions exactly; but after 2000 Years experience (for we have ſome Obſervations of *Eclipſes* much Ancienter) we find the beſt Tables extant Erring ſometimes 12 Minutes or more in her apparent Place, which would cauſe a fault of half an Hour, or $7\frac{1}{2}$ *deg.* in the *Longitude* deduced by comparing Her *Place* in the Heavens with that given by the *Tables*. I undervalue not this Method, for I have made it my buſineſs, and have ſucceeded in it, to get a large Stock of good *Lunar Obſervations* in order to the Correction of Her *Theory*, and as a ground Work for better *Tables*; but if we ſhould happily attain what we ſeek, yet the Calculation will be ſo Perplexed and Tedious, that it will be found much more Inconvenient and Difficult than that I propoſe by obſerving the *Eclipſes* of *Jupiter's Satellits*, which however at preſent I muſt prefer. For I am perſuaded, that the *Eclipſes* of the *Firſt* will ſcarcely be found above 4 Minutes of Time different from my Calculations, and I hope it will ſcarce ever be found to Err ſo much. But if the ſame *Eclipſe* may be obſerved in two Diſtant Places at the ſame Time, or compared with an Obſervation of the n. 154. *p.* 404. n. 165. *p.* 760.

ſame

same *Satellit* made within a Week elsewhere, the *Difference* of *Meridians* will be had something better, than by comparing two Observations of the same *Phasis* of a *Lunar Eclipse*, made in Distant Places. For whereas it is somewhat difficult by reason of the *Penumbra* to determine the True Time of the Application of either of the *Moon's* Limbs to the Shadow, the *Satellite Eclipses*, especially those of the *First*, are almost Momentany.

And whereas there can rarely happen 4 *Eclipses* of the *Moon* Visible the same Year, those of the *Satellits* happen so frequently, that there are more of them Visible in one year than we Count Days in it, though the Planet *Jupiter* lie hid under the *Sun's* Rays every Year a whole Month together.

I know our *Navigators* will Object against this Method, that it is difficult to Practice at *Sea*, because Long Telescopes are required which the Motion of the *Ship* will not permit them to manage aboard. But if it be not Practicable at *Sea* they cannot deny but that it is at *Land*; and that the *True Longitude* of Remote Coasts from us are the First thing desired for the Correction of their Charts: Let them attempt these First, and I doubt not but the success will encourage them so much, that they will readily find means to put it in Practice at *Sea*; That the *French* have used this Method successfully both in *Denmark* and in their *Own Country*; That a Telescope of 14 Foot Long at most, or for need one 8 Foot, with broad *Eye Glasses*, will be sufficient for this purpose; That the difficulty cannot be known till it be tryed, and that Use renders many things easie which our first thoughts conceived Unpracticable.

If it be Required to know whether any of those *Eclipses* which are Invisible with us be Visible in any other Given Place, Convert the *Difference* of *Meridians* betwixt it and *London* into Time; and if the Place Lie to the *East* of *London*, Add it to, if to the *West*, Substract it from, the Time of the Appearance at *London*, the Sum or Difference accordingly shall be the True Time of the *Eclipse* under that *Meridian*, at which if *Jupiter* be above the Horizon and the *Sun* beneath it, the *Eclipse* is there Visible, otherways not.

Or, By the help of the *Ephemerides* of the *Planet's* Places and a *Terrestrial Globe*, the space on it in which any of these *Eclipses* will be Visible may be found thus.

First seek the true Places of the *Sun* and *Jupiter* with his *Latitude* in the *Ephemerides*, whereby you may find their Declinations and Right Ascensions, either by the Vulgar Tables, or the Globe it self, exactly enough for this Method.

Bring *London* on the *Globe* to the *Meridian*, and detaining it there, note what *Deg.* of the *Æquator* is cut by it. From this Substract the Time of the *Eclipse* after Noon converted into *deg.* and *min.* the Remainder shews you the *Longitude* of that *Meridian* on the Earth, where it is then Noon when the *Satellit* is *Eclipsed*; which I therefore call the *Meridional Longitude* of the *Eclipse*. Bring this *Meridional Longitude* under the *Meridian*, and Elevate the nearer Pole to the *Sun* as much as is his *Declination*, keep the *Globe* in this Position and if *Jupiter* be in *Consequence* of the *Sun*, Draw a

Line

Line on the *Globe* along the *Eastern Horizon*, it passes over all those Places where the *Sun* is Setting at that Time, but if *Jupiter* be in *Antecedence* of the *Sun*, Draw the said Line on the *Globe* by the *Western* Edge of the *Horizon*, it passes over all those Places where the *Sun* is then Rising. *Jupiter* being in *Consequence* of the *Sun* Add the Difference of *His* and the *Sun's* Right Ascentions to the *Meridional Longitude* afore mentioned, Bring the *Deg.* of the *Æquator* answering their Summ under the Meridian, Raise the *Pole* next *Jupiter* equal to his Declination, and detaining the *Globe* in this position, Draw a Line again by the *Eastern Horizon*, the Space intercepted betwixt This and the Line of the *Sun's* Setting before Described on the *Globe*, comprehends all those places on the Earth where this *Eclipse* is seen from *Sun Setting* till *Jupiter* is *Set*. But if *Jupiter* were in *Antecedence* of the *Sun*, Substract the difference of His and the *Sun's* Right Ascentions from the *Meridional Longitude*, Set the *Degree* of the *Æquator* answering the Remainder under the *Meridian*, and Elevate the *Pole* next *Juqiter* equal to His Declination. Keeping the *Globe* in this position, Draw a Line by the *Western* Edge of the *Horizon*, the Space Included betwixt This and the Line of the *Sun's* Rising contains all those places on the Earth, where this *Eclipse* is Visible betwixt *Jupiter's Rising* and *Sun-rise*.

When any *Eclipse* of these is observed, the Difference betwixt the Noted Time and that given by the *Tables* shall be the *Difference* of *Meridians* betwixt the Place of the Observation and *London*.

As the *Sun* Removes from the Conjunction of *Jupiter*, the *Ingresses* of the *Satellits* into his Shadow become Observable. When he is about 30° from it the *Emersions* of the *Fourth*, and at 60° of the *Third*, begin to be seen betwixt the Shadow and Body, continuing so till the *Sun* be arrived within 60° of the Opposition of *Jupiter*, when the *Emersions* of the *Third* fall behind his Body, but the *Emersions* of the *Fourth* continue Visible till he be less then 30° Distant from the ☍, at which Time they also are hid behind him, all the Appearances being made really to the Right Hand or in Antecedence of *Jupiter*, though with Inverting Telescopes, they appear on the contrary, to the Left.

After the Opposition of the *Sun* and *Jupiter* we begin to see the *Immersions* of all the *Satellits* from the Shadow now on the Left Hand, or in Consequence of *Jupiter*, but through Inverting Glasses on the Right, when the *Sun* is near 30 *deg.* from the Opposition, the *Ingresses* of the *Fourth*, when 60 *deg.* from it, of the *Third*, begin to be Observable betwixt the Body and Shadow, continuing so till the *Sun* arrive at the same or rather within something a wider Distance from the Conjunction of *Jupiter*.

4. Restauratæ *Geographiæ* Fundamenta, Hac Methodo facillimâ ac nullo fere Instrumentorum apparatu præstandâ, sed quæ minime Fallat, Jaceantur. Quæ huc pertinent præcepta *Astronomicæ* Doctos latere non possunt; unicum monere non abs re erit, nempe, Tubo octo vel etiam septem pedum, hoc est, facile portatili, momenta harum *Eclipsium* satis distincte observari posse; præsertim in exterioribus *Satellitibus*, si modo Lentis Objectivæ Apertura 2½ vel 3 pollices pateat. Sic enim Radiorum maxima copia ad oculum Refracta perveniet, unde Minimæ hæ Stellulæ in vicinia *Jovis* conspici possint, quæ *By Mr. Halley. n. 191. p. 435.*

alias

alias Luce ejus nimiâ obfuscarentur; ac quamvis Coloribus tingantur, ac *Jovis* Limbus parum nitidus videatur, tamen cum de momento amissæ vel recuperatæ Lucis unice agatur, sufficit eas Lumine quantum fieri possit auctas in oculos certius incurrere.

n. 214. p. 237. The *Eclipses* of the *First Satellite* of *Jupiter* are found by the *Royal Academy* at *Paris*, in Ascertaining the Geographical Site of the Principal Ports of *France*, almost *Instantaneous*, and with good Telescopes discernable almost to the very Opposition of *Jupiter* to the *Sun*. So that, could the *Satellites* be Observed with Telescopes Manageable on *Shipboard*, a *Ship* at *Sea* might be enabled to find the *Meridian* she was in to very great Exactness, beyond what we can yet Hope to do by the *Moon*, tho' She seem to afford us the only Means Practicable for the *Seamen*. However before *Saylors* can make Use of the Art of *Finding* the *Longitude*, it will be requisite that the *Coast of the whole Ocean* be first Laid down Truly, for which Work this Method by the *Satellites* is most Apposite.

*Long. and Lat. of* Derby; *by Mr.*Flamsteed. *n.* 55. *p.* 1102, 1106. *n.* 111. *p.* 237.

VII. The *Longitude* of *Derby* from *London* W. is 5 or 6 *min.* the *Latitude* 52° 57′ or 58′.

*Lat. of* Ecton. *n.* 76. *p.* 2272.

VIII. *Ectoniæ*, in Com. *Northamptoniano*, *Lat* 52° 15′.

*Long. and Lat. of* Townley. *n.* 127. *p.* 664.

IX. *Townleii* in Com. *Lanc.* Latitudo Observata (ut scribit D. *Townleius*) 53° 44′; *Longitudo* à Meridiano *Londinensi* 9. circiter scr. hor. ad Occasum.

*Lat. of* Tredagh. *n.* 164. *p.* 749.

X. The *Latitude* of *Tredagh* in *Ireland*, is 53° 40′.

*Long. of* Oxford *and* Dantzick; *by Mr.* Halley. *n.* 129. *p.* 724. *vid. sup. Cap.* IV. §. LXII.

XI. Having carefully considered the *Parallaxes* of the *Moon* in the Observations of the Occultation of *Mars*, *Aug.* 21. 1676. at *Dantzick* and *Greenwich*, I find from the *Immersion* the *Difference* of *Meridians* between *Greenwich* and *Oxford* 4′ 57″, between *Greenwich* and *Dantzick* 1h 14′ 50″: By the *Emersion* the *First* of these *Differences* is found 4′ 59″, the latter 1h 14′ 41″; which near Agreement shews the Exactness of the Observations.

*Long. of* Paris; *by M. Cassini.* *n.* 117. *p.* 390. *vid. sup. Cap.* IV. §. XLV.

XII. 1. Observationes D. *Flamsteedii* circa *Lunæ* Eclipsin *Jul.* 7. st. n. 1675. cum nostris in *Regio Observatorio* habitis magna cum voluptate contuli. Ex iis quippe *Differentiam Meridianorum*, quam olim, nostrarum Observationum Collatione, Minutorum 11 definieram, nunc decem novis Comparationibus eandem intra pauca Secunda video confirmari.

*Obser-*

| Observationes D. *Flamstedii.* | h ′ ″ | Observationes *Nostræ.* | h ′ ″ | *Differentia Merid.* ′ ″ |
|---|---|---|---|---|
| *Pentadactil.* Tectus | 1 55 15 | Idem seu *Seleucus.* | 2 06 15 | 11 00 |
| *Porphyrites* Tectus. | 2 02 20 | Idem seu *Aristarchus.* | 2 12 40 | 10 20 |
| *Sinæ* Limbus Pimus. | 2 05 30 | Ejusdem seu *Tychonis* | 2 16 30 | 11 00 |
| | | vel * | 16 25 | 10 55 |
| *Ætnæ* Limb. Primus | 2 06 00 | Ejusd. seu *Copernici.* | 2 16 30 | 10 30 |
| | | vel * | 16 40 | 10 40 |
| *Besbici* Limbus Prior. | 2 23 05 | Ejus. seu *Manilii* Med. | 2 34 15 | Min. quam 11 05 |
| *Horminius.* Tectus. | 2 26 03 | Ad eund. seu *Dionys* | 2 36 15 | Maj. quam 10 12 |
| Tetigit Limbum Primum *Corocondometis* | 2 39 30 | Ejus. seu *Paludis Som.* | 2 50 20 | 10 50 |
| Tetig. *Palud. Mæotid.* | 2 45 00 | Eand. seu *Mare Casp* | 2 55 20 | 10 20 |
| | | vel * | 55 40 | 10 40 |
| *Mæotis* tota Tecta. | 2 50 40 | Eadem. | 3 01 10 | 10 30 |
| *Immersio.* | 2 56 55 | *Immersio.* | 3 07 45 | 10 50 |
| | | vel * | 3 07 40 | 10 45 |

Nota hæc * denotat D *Cassini* peculiarem Existimationem; in reliquis cum DD. *Picardo* & *Romero* consentit.

2. Medium *Eclipseos* Lunaris, *Jan.* 1. st. n. 1675. deductum est ex Comparatione *Initii* & *Finis.* —— 3h 20′ 00″ *n.* 123. *p.* 562.

Duaram *æqualium Phasium* —— 3h 20′ 15″ *vid. s p.Cap.*IV. §. XLVI.

Ex D. *Flamstedii* Observationibus Medium *Eclipsis* pari modo eruatur. Is quippe $2^h\ 29'\ 30''$ Distantiam Cuspidum observavit $17'\ 16''$ & $3^h\ 52'\ 45''$, *Eclipsi* decrescente, Distantiam observavit $18'\ 57''$, uno scil. Minuto $41''$ Majorem. Itaque *Medium Eclipsis* propius est posteriori Observationi quam priori. Medium Tempus inter utramque Observationem fuit $3^h\ 11'\ 7''$. Tardius igitur aliquanto deducitur hinc *Eclipsis Medium*; unde *Differentia Meridianorum* proveniret Minor $9'$; quod minime convenit Observationibus Certioribus *Eclipsis* præcedentis Æstivæ, ex quibus illam deduxi Min. $10\frac{1}{4}$. Prior Observatio nostra cum Priori D. *Flamsteedii*, aliquanto tardiore, comparata, *Differentiam Meridianorum* exhibet Majorem $8'\ 35''$. Posterior nostra, tardior Observatione posteriori D. *Flamsteedii*, *Differentiam Meridianorum* exhiberet Minorem $9'\ 40''$.

| | h. ′ ″ |
|---|---|
| *Finis* à D. *Flamsteedio* Existimatus —— | 4 07 15 |
| Et à *Nobis.* —— | 4 15 25 |
| *Differentiam Meridianorum* inferret —— | 0 08 10 |
| *Initium* à D. *Halleio Londini* Observatum —— | 2 16 00 |
| Cum Observato à *Nobis* —— | 2 24 35 |
| *Differentiam Meridianorum* faceret —— | 0 08 35 |

Ex hac igitur *Eclipsi Differentia Meridianorum* erueretur duobus circiter Minutis minor quam ex *Eclipsi Æstatis* præcedentis, quam tamen huic longè præfero; non solum spectatâ majori Facilitate Determinandi Tempora Appulsuum & Emersionum in ea *Eclipsi Totali*, quam in hac *Par-*

*tiali*; verum etiam ob Aeris ſerenitatem, qua utique æqualiter uſi fuimus in ea *Eclipſi*, cum in hac *Pariſiis* Cœlum ſereniſſimum, *Londini* fuit ſubnubilum. Priori itaque ſtandum cenſeo, donec per Obſervationes *Immerſionum* & *Emerſionum Satellitum Jovis*, quos ad hanc rem exiſtimo maxime Idoneos, rem ſcrupuloſius determinemus.

*Ib. p.* 564.

*By Mr.* Flamſteed. *ib. p.* 565.

3. *Differentiæ Meridianorum*, ab Eclipſi *Lunæ Junii* 27. 1675. *Londini* & *Pariſiis* Obſervata deductæ, vix fidere poſſum, quippe licet Tempora Phaſium à vobis Obſervatarum accuratiſſimè determinata credam; Ego, cum amplior non ſuppeteret, Quadrante uſus fui 20 tantum Digitorum Radio, ad Horologium Corrigendum, quique Nuda duntaxat habuit Pinnacidia; & propterea de Momento Phaſis alicujus certior eſſe vix potui quam ad unum Minutum Horarium. Noviſſimam *Eclipſin Dec.* 22. 1675. Inſtructior obſervavi; cum tamen mihi Aer ſubnubilus extiterit, & propter Obliquam *Lunæ* in Umbram Terræ Incidentiam tardiſſimus fuerit ejus ad Maculas Appulſus, minus apta fuit hæc *Eclipſis* huic Negotio. Incerta igitur inter duo minuta Horaria manet etiamnum *Meridianorum* noſtrorum *Differentia*, quam tamen nullus dubito Nos pro Votis aliquando determinaturos eſſe.

*Long. of* Straſburg *and* Paris; *by M.* Bullialdus. *n.* 125. *p.* 610. *vid. ſup. Cap.* IV. §. XLVI.

XIII. Meridianus *Pariſienſis* ab *Argentoracenſi* diſtat 22′ 48″ ex *Fine Eclipſeos Lunaris Jan.* 1. ſt. n. 1676. Diſtat autem ex hac *Eclipſi Meridianus Pariſienſis* à *Londinenſi* ad Ortum 6′ 38″, qui ex Obſervatione *Eclipſ. Jul.* 7. 1675. apparuit 10′, ut etiam in *Eclipſi Jan.* 11. ejuſdem Anni.

*Long. of* Avignon; *by Mr.* Halley. *Ph. Col. n.* 5. *p.* 126.

XIV. Avignon is 19′ 40″, or 4° 50′ to the Eaſtward of *London*.

*Long. & Lat. of ſeveral Places in* France. *n.* 163. *p.* 718, 719, 720. *vid. ſup. Cap.* IV. §. XXXIV.

XV. *M. Caſſini* having Compared together the Obſervations of the *Solar Eclipſes* of *July* 12. *ſt. n.* 1684. and made ſuch Reductions as the Parallax requires, lays down the *Longitudes* from *Paris* to

*Aix* in *Provence* 14′. E. The *Lat.* by M. *Gautier* is 43° 30′.
*Avignon* 8½. E.
*Lyons* 8′, or 13′. E.
*Roſes* 4′. E. The *Lat.* by M. *Chaſſelles* 42° 10′.
*Honfleur* 7′. W.
*Pau* 11′. W. The *Lat.* by *P. Richaud* 43° 30′.

*Long. of* Lisbon. *n.* 146. *p.* 151. *vid. ſup. Cap.* IV. §. XLIX.

XVI. Mr. *Jacobs* an *Engliſh* Merchant reſiding at *Lisbon*, informed Mr. *Flamſteed* that he Obſerved the *Beginning* of the *Lunar Eclipſe*, Feb. 11. 168½. there at 8h. 31′. *p. m.* which gives the *Difference* of the *Meridians* betwixt the *Obſervatory* at *Greenwich* and *Lisbon*, 41½ *Minutes* of Time, or 10° 22′. conſiderably Different from our *Mapps* and *Sea-Charts*.

*Lat. of* Madrid; *by E. of* Sandwich. *n.* 22. *p.* 390.

XVII. The *Earl* of *Sandwich* Eſteem'd, by the *Sun's* Altitude in the *Solſtice*, and by other Meridian Altitudes, the *Latitude* of *Madrid* to be 40° 10′. which differs conſiderably from that aſſigned by others; The General Chart of *Europe* giving to it 41° 30′, the General Map of *Spain* 40° 27′, and a large Provincial Map of *Caſtile* 40° 38′.

XVIII. 1. Per plures Congruentes Obſervationes *Lunares* & aliorum Planetarum inveni, *Diſtantiam* Civitatis *Hiſpalenſis Longitudinariam* eſſe ab *Vraniburgo* 90′, ſive $1\frac{1}{2}^h$, vel intra 2′ Differentem. *Long. of* Seville *and* Uraniburg; *by* . . Math. Profeſſor *at* Seville. *n.* 118. *p.* 427.

2. Videat D. *Profeſſor* quomodo *Meridiani Hiſpalenſis* ab *Vraniburgico* Interſtitium Scrupulorum 90′ conſtituerit: Deliquii enim *Lunaris* Obſervationes *Jan.* $1\frac{1}{1}$. 1675. *Londini Medium* ponunt $7^h$ $11'\frac{1}{2}$ *p. m.* cui Annotationes *Pariſinæ* conſentiunt; dicti *Profeſſoris* Obſervationes *Medium Hiſpali* ſtatuunt $6^h$ 47′: Noſtrorum ergo *Meridianorum Differentia* $24'\frac{1}{2}$: At Nos inter & *Vraniburgum* non intercedunt niſi Minuta 52′. Eſt igitur *Meridianorum Differentia* nonniſi $1^h$ $16'\frac{1}{2}$ inter *Hiſpalim* & *Vraniburgum*. Vereor tamen, annon Oculis nudis D. *Profeſſoris* factæ fuerint Obſervationes: *Incidentiæ* quippe & *Emerſionis* Tempora faciunt $1^h$ 5′; cum *Noſtræ*, *Pariſinæ*, *Hevelianæque* Obſervationes non faciant ea Tempora plus quam $1^h$ $1'\frac{1}{2}$, forſan aliquanto minus. *By Mr.* Flamſteed. *ib.* *p.* 431. *vid. ſup. Cap.* IV. §. XLIV.

XIX. *Differentia Meridianorum Hafniæ* & *Pariſiorum*, Obſervationibus *Jovialium*, reperta eſt à D. *Picard*, $0^h$ 41′ 40″. *Long. of* Copenhagen; *by* M. Picard. *n.* 146. *p.* 145.

XX. *An.* 1680. *Oct.* 23. *St. v. S. Joſ. Ponthia*, and *Marco Antonio Cellio* with a Teleſcope of 25 Palms, Obſerved the Total *Immerſion* of the *Firſt Satellite* into *Jupiter*'s Shadow at *Rome*, at $10^h$ 7′ 53″. *p. m.* which in our *Obſervatory* here I noted at $9^h$ 15′ 41″, whoſe Difference is the *Difference* of our *Meridians* = 52′ 12″, or 13° 03′. Again, *Jan.* 28. 1685. S. *Francis Blanchini* Obſerved the Total *Immerſion* of the *Firſt* at *Rome*, at 11h $19'\frac{1}{4}$ which I ſaw not here, but my Numbers give at $10^h$ $27'\frac{1}{4}$. Therefore the *Difference* of *Meridians* is $52'\frac{1}{2}$, and *Rome* lies ſo much more *Eaſterly* than the *Obſervatory* of *Greenwich*; agreeing with the former Obſervation. *Long. of* Rome *and* Uraniburg; *by Mr.* Flamſteed. *n.* 177. *p.* 1215.

The Noble *Tycho* judged therefore not much amiſs, when he placed *Vraniburg* and *Rome* under the ſame *Meridian*; for by ſeveral Obſervations of *Satellite Eclipſes* it is Evident, that the *Difference* of *Meridians* betwixt *Vraniburg* and our *Obſervatory* is 51′ 10″ of Time, ſo that *Rome* lies only one *Minute* of Time, or $\frac{1}{4}$ of a *Deg.* to the *Eaſt* of *Vraniburg*.

XXI. 1. *Dantzick* is by many and undoubted Obſervations proved to be $1^h$ 15′ 30″. more *Eaſterly* than *London*. *Long. of* Dantzick; *by Mr.* Halley. *Ph. Col.* n. 5. *p.* 124.

2. An. 1683. Die ipſo *Solſtitii Æſtivi* 21 *Junii* ſt. n. *Gedani*, Altitudo *Solis* in Meridie fuit 59° 7′, Quadrante quidem parvulo Orich. ſed tamen ſatis accurato. Die vero *Æquinoctii Autumnalis* Altitudo *Solis* in Meridie reperta eſt 35° 27′. *Lat. of* Dantzick; *by M.* Hevelius. *n.* 151. *p.* 330. *n.* 154. *p.* 424.

XXII. The *Longitude* of *Nuremburg* has been formerly ſtated 11° from *London*, and ſince found to be ſo by Obſervations of the *Eclipſe* of the *Sun July* $2^d$. 1684, which made it $44'\frac{1}{2}$ of Time. *Long. of* Nuremburg; *by* . . . . . . *n.* 182. *p.* 147.

*Long. of* Moscua, Lipsick *and* Aleppo; *by* . . . . . . *n.* 192. *p.* 453. *vol. 1. p. Cap.* IV. §. LIII.

XXIII. The *Duration* of the *Lunar Eclipse Apr.* 5. 1688. is made by M. *Timmerman* from 7h 38′ to about 10h 45′, which agrees within 8 or 10 *Minutes* with our *Tables*, that never err sensible in the continuance of *Eclipses*; and so much ought to be allowed to an Observer not sufficiently instructed to distinguish the *Penumbra* from the true Shadow, though a small *Telescope* were used in this Observation. Let us conclude then, that the *End* was at 10h 40′ at *Moscua*. We do not find that this *Eclipse* was observed at *London*: However this defect is in good part supplied by an Observation thereof made at *Lipsick*, by M. *Gotfrid Kirck*, and published in his *Ephemerides* for the Year 1689; Where the *End* is determined at 8h 54′ *p. m.* Hence *Moscua* will be 1h 46′ to the *Eastward* of *Lipsick*; and the *Difference* of *Meridians* between *London* and *Lipsick* being already determin'd 49′, it will follow that *Moscua* is 2h 35′ to the *East* of *London*, or 38° 45′ of *Longitude*, which from other Accounts we find to be very near that of the City of *Aleppo* in *Syria*.

*n.* 181. *p.* 86. *it is 52 Min.*

*Lat. of several Places in* Russia. *ib. p.* 454.

By the same hand we have procured the *Latitudes* of the following Places, observ'd, as 'tis said, with a large Quadrant.

| | | |
|---|---|---|
| *Moscua* | 55° | 34′ |
| *Yereslaw* | 57 | 44 |
| *Wologda* | 59 | 19 |
| *Wostak* | 61 | 15 |
| *Arch-Angel* | 64 | 30 |

*Latitudes of some Remarkable* Places; *by Mr.* Francis Vernon. *n.* 124. *p.* 582.

XXIV. I have been as curious as I could in taking the *Latitudes* of some Remarkable Places: as I find them I shall give them you.

| | | | | | |
|---|---|---|---|---|---|
| *Athens* | 38° | 05′ | *Patras* | 38° | 40′ |
| *Corinth* | 38 | 14 | *Delphos* | 38 | 50 |
| *Sparta* | 37 | 10 | *Thebes* | 38 | 22 |
| *Corone* | 37 | 02 | *Negropont* or *Chalcis* | 38 | 31 |

*Latitudes of* Constantinople *and* Rhodes; *directed to* A-B. Usher; *by Mr.* Greaves. *n.* 178. *p.* 1295.

XXV. Upon Intimation of your *Grace's* desires, and upon importunity of some Learned Men, having finished a Table, as a Key to your *Grace's* exquisite disquisition, touching *Asia properly so called*; I thought my self obliged to give both you and them a reason, why in the situation of *Byzantium* and the Island *Rhodus*, (which two Eminent Places I have made the πεϱάσματα and Bounds of the *Chart*,) I dissent from the traditions of the *Ancients*, and from the Tables of our *Late* and best *Geographers*; and consequently dissenting in these, have been necessitated to alter the *Latitudes*, if not *Longitudes*, of most of the remarkable Cities of this Discourse. And first for *Byzantium*, the received *Latitude* of it by *Appianus*, *Mercator*, *Ortelius*, *Maginus*, and some others, is 43° 5′. And this also we find in the *Basil Edition* of *Ptolemy's Geography*, procured by *Erasmus* out of a Greek MS. of *Pettichius*. The same likewise is confirmed by another choice MS. in Greek of the most learned and Judicious *Mr. Selden*, to whom for this favour

and

and ſeveral others I ſtand obliged. And as much is expreſſed in the late *Edition* of *Ptolemy* by *Bertius*, compared and corrected by *Sylburgius*, with a Manuſcript out of the *Palatine Library*. Wherefore it cannot be doubted, having ſuch a Cloud of Witneſſes, but that *Ptolemy* aſſigned to *Byzantium*, as our beſt *Modern Geographers* have done, the *Latitude* of 43° 5′. And this will farther appear, not only out of his *Geography*, where it is often expreſſed, but alſo out of his Μεγάλη Σύνταξις, or *Almageſt* as the *Arabians* terms it, where deſcribing the Parallel paſſing διὰ Βυζαντίου, he aſſigns to it 43° 5′. What was the Opinion concerning *Byzantium* of *Strabo* preceeding *Ptolemy*, or of *Hipparchus* preceeding *Strabo*, or of *Eratoſthenes* Ancienter and it may be Accurater than all of them, ( for *Strabo* (*Lib.* 2.) calls him τελευταῖον πραγματευσάμενον περὶ τῆς γεωγραφίας ) though *Tully* ( *Lib. Ep. ad Att.*) makes *Hipparchus* often reprehend *Eratoſthenes*, as *Ptolemy* after him doth *Marinus*, their Writings not being now extant, ( unleſs thoſe of *Strabo* ) cannot be determined by us. But as for *Strabo*, in our Inquiry we can expect little Satisfaction; for his Deſcription of Places, having more of the *Hiſtorian* and *Philoſopher*, ( both which he hath performed with ſingular Gravity and Judgment ) than the Exactneſs of a *Mathematician*, who ſtrictly reſpects the Poſition of Places, without Inquiſition after their Nature, Qualities, and Inhabitants, ( though the beſt *Geography* would be a Mixture of them all, as *Abulfeda*, an *Arabian Prince* in his *Rectification of Countries* above 300 Year ſince hath done; ) I ſay for theſe Reaſons we can expect little Satisfaction from *Strabo*, and leſs may we hope for from *Dionyſius Afer*, *Arrianus*, *Stephanus Byzantinus*, and others. Wherefore next having recourſe to the *Arabians*, who in *Geography* deſerve the ſecond Place after the *Græcians*, I find in *Naſſir Eddin* the *Latitude* of *Buzantium*, which he terms *Buzantiya*, and *Conſtantiniya*, to be 45°, and in *Uleg Beg's Aſtronomical Tables* the ſame to be expreſſed. *Abulfeda* chiefly follows four Principal Authors as his Guides, in the compiling of his *Geographical Tables*, thoſe are, *Alfaras*, *Albiruny*, *Ibn Saijd Almagraby*, laſtly *Ptolemy*, whoſe *Geography* he terms a *Deſcription of the Quadrant*, ( or the fourth Part of the Earth ) Inhabited; and all theſe, according to his Aſſertion, place *Byzantium* in 45° of *Latitude*. And here it may juſtly be wondred, how this Difference ſhould ariſe between the *Greek Copies* of *Ptolemy*, and thoſe tranſlated into *Arabick* by the Command of *Almamon*, the Learned *Calife* of *Babylon*; for *Abulfeda* expreſly relates, that *Ptolemy* was firſt interpreted in his Time, that is, in the Computation of *Almecinus* in *Erpenius's Edition*, and of *Emir Cond* a *Perſian Hiſtoriographer*, more than 800 Years ſince: Concerning which *Abulfeda* writes thus, *This Book* ( diſcourſing of *Ptolemy's Geography* ) *was tranſlated out of the Grecian Language into the Arabick for Almamon:* And in this I find, ( by three fair MSS of *Abulfeda* ) *Byzantium* to be conſtantly Placed in 45°. and as conſtantly in the *Greek Copies* in 43° 5′. But in the πρόχειροι κανόνες of *Chryſococca*, out of the *Perſian Tables*, made about the Year 1346, in *Scaliger*'s Calculation ) it is Placed in 45°. To reconcile the Difference between the *Greeks* and *Arabians* may ſeem impoſſible; for the common Refuge of flying to the Corruption of Numbers by Tranſcribers, and laying the fault on them which ſometimes is the Author's, will not help us in this particular; ſeeing the *Greek Copies* agree amongſt themſelves, and the *Arabick Copies* amongſt themſelves. The beſt way to end the Diſpute will be, to give credit concerning the *Latitude* of *Byzantium*, neither to the *Greeks* nor *Arabians*. And that I have reaſon for this Aſſertion, appears by ſeveral Obſervations of mine at *Conſtantinople*, with a braſs Sextant of above 4 Foot *Radius*. Where taking, in the *Summer Solſtice*, the Meridian Altitude of the *Sun* without uſing any προσθαφαίρεσις for the Parallax and Refraction, (which at that time was not neceſſary,) I found the *Latitude* to be 41° 6′. And in this *Latitude* in the *Chart* I have placed *Byzantium*, and not in that either of the *Greeks* or *Arabians*. From which Obſervation, being of ſingular Uſe in the *Rectification* of *Geography*, it will follow by way of Corollary, that all *Maps* for the *North-Eaſt* of *Europe*, and of *Aſia*, adjoining upon the *Boſphorus Thracius*, the *Pontus Euxinus*, and much farther, are to be Corrected; and conſequently the Situation of moſt Cities in *Aſia properly ſo called*, are to be brought more Southerly than thoſe of *Ptolemy* by almoſt two intire Degrees, and than thoſe of the *Arabians* by almoſt four.

Concerning *Rhodes*, it may be preſumed, that, having been the Mother and Nurſe of ſo many Eminent Mathematicians, and having long flouriſhed in Navigation, by the Direction of theſe, and by the Vicinity of the *Phœnicians*, they could not be Ignorant of the preciſe *Latitude* of their Country, and that from them *Ptolemy* might receive a true information. Though it cannot be denyed, but that *Ptolemy* in Places remoter from *Alexandria* hath much erred. I ſhall only inſtance in our Own Country, where he ſituates Λονδίνιον, that is *London*, in 54° of *Latitude*; and the τὸ μέσον or the midle of the *Iſle* of *Wight*, (which in the printed Copies is falſely termed Οὐικτησις, but in the MSS. rightly Οὔηκτις,) in 52° and 20′ of *Latitude*. Whereas *London* is certainly known to have for the Altitude of the *Pole*, or *Latitude* of the Place, only 51° and 32′. and the middle of the *Iſle* of *Wight* not to exceed 50°. and ſome minutes.

But in my judgment *Ptolemy* is very excuſable in theſe and the like Errors, of ſeveral other places far diſtant from *Alexandria*; ſeeing he muſt for their Poſition neceſſarily have depended either upon Relations of *Travailers*, or Obſervations of *Mariners*, or upon the *Longitude* of the *Day* meaſured in thoſe times by *Clepſydræ*: all which how uncertain they are, and Subject unto Errour, if ſome Celeſtial obſervations be not joyned with them, and thoſe exactly taken with Large Inſtruments, (in which kind the *Ancients* have not many, and *Our Times*, (excepting *Tycho Brahe*, and ſome of the *Arabians*) but a few,) I ſay no Man, that hath converſed with *Modern Travailers* and *Navigators*, can be ignorant. Wherefore to excuſe theſe Errors of his (or rather of others fathered by him) with a greater Abſurdity, by aſſerting the *Poles* of the *World* ſince his time to have Changed their ſite, and conſequently all Countries their *Latitudes*, as *Mariana* the Maſter of *Copernicus*, and others after him have Imagined: or elſe to Charge *Ptolemy*, being ſo excellent an Artiſt, with Ignorance, and that even of his own Country, as *Cluverius* hath done, (from which my obſervations at *Alexandria*, and

Mem-

*Memphis*, may Vindicate him, ) the former were too great a Stupidity, and the latter too great a Presumption. But to return to *Rhodes*: an Island ( in *Eustathius*'s *Comment* upon *Dionysius*'s περιήγησις ) of 920 furlongs circuit, where according to *Ptolemy* the Parallel passing διὰ Ῥόδου, hath 36° of *Latitude*, and so hath *Lindus*, and Ἰηλυσσός the chief Cities of the Island; the same is confirmed by the MS. but where the Printed Copy and *Eustathius* read Ἰηλυσσός, which *Mercator* renders *Talyssus*, the MS. renders Ἰαλυσός. *Abulfeda* in some Copies situates the Island *Rhodes*, ( for he mentions no Cities there ) in the *Latitude* of 37 *Deg.* and 40 *min.* and the *Geography* of *Said Ibn Aly Algiorgany*, commended by *Gilbertus Gaulmyn*, in 37°, if it be not by a Tranposition in the MS. of the Numerical Letters in *Arabick* 37 for 36, which by reason of their Similitude, are often confounded in *Arabick* MSS. By my Observations under the Walls of the City *Rhodes*, with a fair brass *Astrolabe* of *Gemma Frisius*, containing 14 inches in the Diameter, I found the *Latitude* to be 37° and 50′. A larger Instrument I durst not adventure to carry on Shore in a Place of so much Jealousie. And this *Latitude* in the Chart I have assigned to the City *Rhodes*, ( from the Island so denominated, upon which on the North East side it stands situated) better agreeing with the *Arabians* than with *Ptolemy*, whom I know not how to excuse.

XXVI. In the second Book of the *Voyage de Siam des Peres Jesuites*, are related two observations of the *Satellites* of *Jupiter*, capable, if well made, to ascertain the *Longitude* of the *Cape of good Hope*. The First was there made *June* 2d *st. n.* 1685, when at 11ʰ 29′ 20″. the *First* or Innermost *Satellite* touched the *Western Edge* of *Jupiter*, and at 11ʰ 30′ 50″ it appeared no more: this Observation is said to be made with an excellent Telescope of 12 Foot. The other was on *June* the 4th following *st. n.* when the *Emersion* of the same *Satellite* was observed at 9ʰ 37′ 40′. from which Latter is concluded, that the *Longitude* of the *Cape* is 18° to the *East* of *Paris*, for that the said *Emersion*, according to the *Calculus* of *Cassini*, in the *Meridian* of *Paris*, ought to have happen'd at 8ʰ 26′. This same *Emersion* is computed by *Mr. Flamsteed* at 8ʰ 19′. at *London*, that is 3 *min.* later then by *S. Cassini*; and considering that neither is Verified by observation in *Europe*, the *Longitude* hence deduced is doubtful at least 3 *min.* if this had been the only observation. But the former being considered will yet shew that there is a much greater Doubt still remaining: For from certain Astronomical Principles the *Parallax* of the *Orb*, or difference between the Place of *Jupiter* seen from the *Sun* and *Earth* was, at the time of the first Observation, 9° 9′; Which Arch that *Satellite* moves in 1ʰ 6′. and the utmost Duration of an *Eclipse* thereof in this position of *Jupiter* being scarce 2ʰ 20′. (as appears by the accurate Observations of *M. Cassini* and *Mr. Flamsteed*) it will follow, that from the *Immersion* behind *Jupiter's* Western Edge to the *Emersion* out of the Shadow, there could not be full 3ʰ 26′. Wherefore the *Emersion* out of the Shadow, on *June* 2d, ought according to the time of *Immersion*, to be at 14ʰ 56′. at the latest at the *Cape*; which by

*Long. of the Cape of Good Hope.* n. 185. p. 253.

*Mr. Flamſteed's Calculus* was at *London* $13^h$ 51′. or according to *S. Caſſini* at $13^h$ 58′. at *Paris*. Hence the *Longitude* of the *Cape* will be found but 14 *deg.* and half at moſt to the *Eaſt* of *Paris*; ſo that theſe 2 Obſervations will differ in the reſult about a quarter of an hour, which is a little too much. However there are ſome reaſons that ſeem to argue for this Latter *Longitude* rather than the Former; for it is much eaſier to obſerve what becomes of a Luminous Object that appears, than to wait upon the firſt appearance of a Star Eclipſed: and it is probable that the *Satellite* might, in the Latter time, be ſeveral minutes *Emerged* out of the Shadow, when they might firſt perceive it; but they could not but ſee the Application to the Body of *Jupiter* in the Former, if we may ſuppoſe their Teleſcopes ſo good as they are ſaid to be. And that the *Cape of Good Hope* is not more than an hour to the *Eaſt* of *Paris*, is proved by the conſtant conſent of our *Navigators*, who find by their Reckonings that the Iſland of *St. Helena* is about 22 or 23. *deg.* of *Longitude* to the *Weſtward* of the *Cape*: (and that Sailing both backwards and forwards, 'tis the ſame, which takes away the Objection of Currents) Now by Accurate Obſervations made at *St. Helena*, and compared with others made in *Europe* at the ſame time, the *Longitude* of that Iſle is certainly about $8\frac{1}{2}$ *deg.* to the *Weſt* of *Paris*; it follows therefore that the *Cape* cannot be much more than 14 or 15 *deg.* to the *Eaſt* of *Paris*; and undoubtedly it muſt be leſs than 18°, for 3. *deg.* is much too great an Errour to be committed in ſo ſhort a Diſtance Sailing.

*Long. of* St. Helena. *ib.*

*The Long. of* Madagaſcar; *by Mr.* Flamſteed. *n.* 143. *p.* 15. *vid. ſup. Cap.* IV. §. XLVIII.

XXVII. Mr. *Thomas Heathcot* was Chirurgeon to a Ship, which, *Aug.* 19. 1681. lay at the bottom of a deep Bay on the Weſtern ſhore of *Madagaſcar*, and that part which the *Portugueſe* and our *Mapps* call the *Terra del Gada*; He had with him then on ſhore, a Quadrant of two foot Radius, and a Teleſcope of 9 foot, but no Clock; to ſupply which Defect, he made a *Pendulum* of a String and a Bullet 39 inches long, that each ſingle Vibration might anſwer a Second of Time. Waiting the *Beginning* of the *Eclipſe* with his glaſs, as ſoon as he He ſaw the True Shadow enter on the *Moon's* Limb, he cauſed his Friends who aſſiſted him, to make the Pendulum Vibrate and count its Vibrations; of which they had numbred 140 = 2′ 20″ of Time, when he took the height of *Procyon* (then *Eaſt* of the Meridian) 25° 39′. The next day he obſerved the *Sun's* Meridional Height with the ſame Quadrant, whence he found the *Latitude* of the Place 19° 29′. *South*, hence the time when he took the height of *Procyon* is found $4^h$ 51′ *mane*, and ſubſtracting the 2′ 20″, paſt ſince the obſerved *Beginning* of the *Eclipſe*,

| | h | ′ | ″ |
|---|---|---|---|
| its True *Beginning* was at | 4 | 48 | 40 |
| Which at the *Obſervatory* here, I noted at | 1 | 50 | 40 |
| therefore this part of *Madagaſcar* is more *Eaſterly* | 2 | 58 | 00 |

or 44° 30′, which our *Mapps* make 52°; that is $7\frac{1}{2}$ *Deg.* more Remote from it than it really is.

*The Long. and Lat. of* Ballaſore *in* India; *by Mr.* Edm. Halley. *Ph. Col. n.* 5. *p.* 124. *vid. ſup. Cap.* IV. §. LXVI.

XXVIII. Taking the Obſervations of the *Occultation* of the *Bulls Eye*, *Oct.* 28. 1680. under the Examination of a *Calculus*, I find that at 8h 6′

or

or the *Immersion* at *London*, the true Place of the *Moon* correct by *Parallax* was ♊ 4° 32′ 24″ but at 16$^h$ 00. at *Ballasore Road* ( in the *Lat.* of 21° 20′. N. and about 20 miles E. S. E. from the Town) the True Place of the *Moon* was ♊ 5° 54′. that is 1° 21′ 36″. more than at 8$^h$ 6′. at *London*: Now according to the *Moon's* Velocity at that time, she passed an Arch of 1° 21′ 36″. in 2h 8′ 40″. of time, so then at 10$^h$ 14′ 40″. at *London*, the *Moon* was in the same place as at 16h 00′. at *Ballasore Road*, whence the *Difference* of *Longitude* will be 5$^h$ 45′ 20″. or 86° 20′. *Ballasore* being so much to the *Eastwards* of *London*.

2. By the Calculation of the *Immersion* of the *Bull's Eye Dec.* 22. 1680. I find that at 14h 49′. at *Ballasore* the *Moon's* true Place was ♊ 6° 30′ 30″ and at 7$^h$ 46′ 12″. the Correct Time of the *Immersion* at *Dantzick*, the true Place was ♊ 4° 55′ 11″. that is 1° 35′ 20″. short of the Place deduced from the Observation at *Ballasore Road*, which make in Time 2$^h$ 32′ 40″. whence it follows, that 10$^h$ 18′ 52″. at *Dantzick* makes 14$^h$ 49′. at *Ballasore Road*, and the *Difference* of *Longitude* 4$^h$ 30′ 8″. and *Dantzick* being 1$^h$ 15′ 30″. more *Easterly* than *London*, *Ballasore Road* will be from *London* 5$^h$ 45′ 38″. or 86° 24′. and the same *Difference* of *Meridians* will be found 86° 14′. if you make use of the *Emersion* at *Dantzick*. vid. sup. Cap. IV. §.LXVII.

3. For further confirmation hereof, Mr. *Benj. Harry* being ashore at *Ballasore Town*, he observed with very great Care and Exactness, *Nov.* 18. 1680. that at 9$^h$ 13′. the Star which *Tycho* calls, *in Cotyla dextra* Aquarii *duarum præcedens* (and which was then in *Aquarius* 28° 52′. and *Lat.* 2° 46′. *N.*) was in a Right Line with the *Cusps* of the *Moon*, then near the first Quarter. The Star's Place is confirmed by the agreement of *Hevelius*'s Observations with those of *Tycho*, and the *Theory* of the *Moon* cannot be considerably faulty in that part of the *Orb*, it falling precisely on her greatest *Equation*, wherefore by the Theory and Numbers of *Horrox*, the true Place of the *Moon* at 2$^h$ 53′. at *London* is found ♒ 29° 22′ 10″. but at 9$^h$ 13′. at *Ballasore*, her Place was in ♒ 29° 41′ 17″. that is 19′ 7″ more than at *London*, which in Time gives 36′, so that 3$^h$ 29′ at *London* was 9$^h$ 13′. at *Ballasore*, and the *Difference* of *Long.* 5$^h$ 44′ or 86° 00′. precisely, which the *Dutch Maps* make full out 99°. And the *French Maps* of *Sanson*, pretending to correct them, have made them 5° worse, and the Errour 18° compleatly. What then is to be thought of the *Descriptions* of those Places which have been but seldom Visited.

XXIX. Differentiam *Longitudinum Cantonem* inter & *Parisios* deduxi 7$^h$ 23′. ex Exitu *Mercurii* ex *Solis* Disco *Cantoni* & *Norimbergæ* Observato, & ex *Eclipsibus Lunæ* Observatis *Norimbergæ* & *Parisiis*. *Long. of* Canton; *by M.* Cassini. *n.*245. *p.* 371.

*vid. sup. Cap.* IV.§.XCVIII

XXX. Ex Altitudine Meridiana Maxima *Stellæ Polaris* a *PP. S. I.* Observata Die 31 *Dec.* 1694. Correcto Instrumento 42° 16′ 50″. Supposita Refractione 1′ 17″. & Distantia *Stellæ Polaris* à *Polo* tunc temporis 2° 19′ 57″ eruitur *Altitudo Poli* 39° 54′ 56″. *Lat. & Long. of* Pekin; *by M.* Ja. Cassini. *n.* 237. p. 53.

Ex ejusdem *Stellæ Polaris* Altitudine Meridiana Minima, Observata Diebus 7, 8, 13. *Maii* 1695. Correcto Instrumento 37° 36′ 40″. Supposita Refractione 1′ 28″. & Distantia *Stellæ Polaris* a *Polo* 2° 19′ 50″. Eruitur *Altitudo Poli* 39° 55′ 2″.

| | ° | ′ | ″ |
|---|---|---|---|
| Neglecta Refractione Altitudo maxima *Stellæ Polaris* deducta ex Observatione 31 *Dec.* præcedentis fuisset sub Initium *Maii* | 42 | 16 | 43 |
| Et Altitudo ejusdem Minima tunc fuit | 37 | 36 | 40 |
| Quare Differentia Altitudinum | 04 | 40 | 03 |
| Et Distantia *Stellæ Polaris* a *Polo* | 02 | 20 | $01\frac{1}{2}$ |
| Et *Altitudo Poli* apparens | 39 | 56 | $41\frac{1}{2}$ |
| Ad hanc Altitudinem apparentem Refractio ex mea Tabula est | 00 | 01 | 10 |
| Quare *Altitudo Poli* in *Regia Pekinensi* | 39 | 55 | $31\frac{1}{2}$ |

Pro *Longitudine Pekinensis* Urbis Observata est Immersio *Primi Jovis Satellitis* in *Jovis* Umbram *Die* 18. *Jan.* 1695. $12^h$ 51′ 14″.

Tabulæ nostræ eo Die hanc *Immersionem* repræsentant $5^h$ 18′ 49″ Observationes autem eodem Mense habitæ in *Observatorio Regio Parisiensi* ostendunt Tabulas retardasse tunc Temporis 2′ 30″.

Quare fuit illa *Immersio Parisiis* 5h 16′ 19″. Itaque *Differentia Meridianorum* inter *Pekinum* & Urbem *Parisiensem* erit $7^h$ 34′ 55″.

Cum autem ex aliis Observationibus olim deducta fuerit eadem *Meridianorum Differentia* $7^h$ 36′. sumi poterit $7^h$ $35\frac{1}{2}$′.

*Lat. of St.* Salvadore. *n.* 105. *p.* 91.

XXXI. St. *Salvadore* in *Brasil* is in the *Southern Latitude* of 12° 47′.

*Lat. of* Bridge-Town. *n.* 189. p. 372.

XXXII. *Bridge Town* in *Barbadoes* is in the *Northern Latitude* of 12° 58′.

*A Description of* Nova Zembla; *by M.* Nich. Witsen. *n.* 101. p. 3. Fig. 205.

XXXIII. 1. I herewith send you what I have received out of *Muscovy*, which is a *New Mapp* of *Nova Zembla* and *Weigats*, as it hath been discovered by the Express Order of the *Czar*; and drawn by a Painter, called *Panelapoetski*, who sent it me from *Mosco* for a Present: by which it appears that *Nova Zembla* is not an *Island*, as hitherto it hath been believed to be; and that the *Mare Glaciale* is not a *Sea*, but a *Sinus* or Bay, the Waters whereof are Fresh. Which is the same with what the *Tartars* do also assure us, who have tasted these very Waters in the midst of the *Sinus*. The *Samojeds* as well as the *Tartars* do unanimously affirm, that passing on the back of *Nova Zembla*, at a considerable Distance from the shore, *Navigators* may well pass as far as *Japan*. And 'tis a great Fault in the *English* and *Dutch*, that seeking to get to *Japan* on the South side of *Nova Zembla*, they have almost always pass'd the *Weigats*.

The Letter O in the great River *Oby* marks the Place of a Cataract or Fall of Waters. The Letter K denotes the Conjunction of *Zembla* with the *Continent*. The River marked L, runs from *China*, called *Kitaic*: which is not every where Navigable, by Reason of the *Rocks*, and other Inconveniencies that Obstruct the passing of Vessels. *Weigats* it self is very difficult to pass, because of the great Quantity of *Ice* continually falling into it out of the River *Oby*, whereby that Streight Passage is stopped up. The *Samojeds*

*mojeds* go every Year a Fishing upon the said Sweet Sea and that on *Nova Zembla's* side.

2. I formerly thought *Nova Zembla* had been a *Continent*: But I have since been better Informed, and Retracted that Error. And whereas the late *M. Vossius* would needs perswade himself, as well as he did others to their Ruine, that there was a Passage to *Japan* by the North, and that the *Tartarian* Countries behind *Nova Zembla* did immediately decline towards the South; I did always oppose it, and think I can even Demonstrate the Impossibility thereof. So that what he wrote to encourage Mariners to that attempt, was even directing them to the point of Death, as it afterwards ensued. n. 193. p. 494.

XXXIV. What is noted with a Single Line is exactly Copied from the Map, which *M. Sanson*, one of the most Illustrious *Geographers* of this Age, presented to the *Dauphin An.* 1679. The Names of Cities, whose Situation is also taken from this *Map*, are written in *Italian* Characters; the Correction of the Position of Coasts (which is deduced from the Observations which were made to that End) is marked with a Stroke a little Shadow'd towards the *Sea*, as is Commonly done; and the Names of Cities, whose Situation is Corrected, are set down in *Roman* Characters. *A Map of France; by M.* Picard, *and M.* De la Hire. *n. 226. p.* 443. *Fig.* 205.

The Degrees of *Latitude* are marked on both sides of the Border, and the Degrees of *Longitude* in the same Border above and below; but the Division of them begins at the *Meridian* that passes through the *Observatory* at *Paris*, by going to *East* and *West*, and not at the *Meridian* of the Isle of *Fer*, as hath been Established, because we do not exactly know the Situation of this Island in respect of the *Observatory*.

XXXV. 1. What *Arithmetick* in whole Number and Fractions, as also in Decimals and Logarithms, is necessary for the same? And what Books are best for Teaching so much thereof? 2. What Vulgar Practical Mechanical *Geometry* performable by the Scale and Compass is sufficient? 3. What *Trigonometry*, Right Lined and Spherical, will suffice? 4. How many *Stars* are to be known? 5. What *Instruments* are best for Use at *Sea*, with the Construction of them, and the manner of using them? 6. The whole Skill of the *Magnet* as to the Directive Vertues thereof, and all the Accidents that may befall it? 7. The *Hydrography* of the *Globe* of the Earth, the *Perspective* of the Coasts, and the Description of the under-water-bottom of the *Sea*. 8. The knowledge of Winds and *Meteors*, so far as the same is attainable. 9. The History and Skill of all sorts of *Fishings*. 10. The Art of *Medicine* and *Chyrurgery*, peculiarly for the *Sea*. 11. The *Common Laws* of the *Admiralty*, and *Jurisdiction* of the *Sea*. 12. The several *Victuallings* and *Cloathings* fit for Seamen. 13. The whole Science of *Ebbing* and *Flowing*, as also of Currents and Eddyes at Sea. 14. *Dromometry*, and the Measures of a Ships Motions at Sea. 15. The *Building* of *Ships* of all sorts, with the several Rigging and Sails for each Species, and the use of all the Parts and Motions of a Ship. 16. *Naval Oeconomy* according to several Voyages and Countries. 17. The Art of *Conting*, *Rowing*, and *Sailing*, of all the several sorts of Vessels *What a Compleat Treatise of* Navigation *should contain; by Sir* W. Petty. *n.* 198. *p.* 657.

ſels. 18. The *Gunnery*, Fire-works and other Armatures peculiar to Sea and to Sea-Fights. 19. The Art of *Loading* and *Unloading* the Chief Commodities to the beſt Advantage. 20. The Art of *Weighing Sunken Ships* and Goods, as alſo of *Diving* for Sunken Goods in deep Water. 21. The General *Philoſophy* of the *Motion* and Figures of the *Air*, the *Sea*, and of Seaſons; of *Timber*, *Iron*, *Hemp*, *Brimſtone*, *Tallow*, &c. And of their ſeveral uſes in *Naval* Affairs. 22. An Account of 5 or 6 of the beſt *Navies* of *Europe*, with that of the *Arcenals*, *Magazines*, *Docks*, *Yards*, &c. 23. An Account of all the *Shipping* able to croſs the Seas belonging to each *Kingdom* and *State* of *Europe*. 24. An Account of all the Chief *Commercial* parts of the *World*; with mention of what *Commodities* are originally carried from, and ultimately to, any of them. 25. An Account of the chief *Sea-Fights*, and all other *Naval Expeditions* and Exploits, relating to *War*, *Trade*, or *Diſcovery*, which hath happen'd in this Laſt Century. 26. Of the moſt Advantageous Uſe of *Teleſcopes* for ſeveral purpoſes at *Sea*. 27. Of the ſeveral *Depths* of the *Sea*, and *Heights* of the *Atmoſphere*. 21. The Art of making *Sea-Water Freſh* and *Potable*, and fit for all uſes in Food and Phyſick at Sea.

*The Collection of Secants, and the true Diviſion of the* Meridian *in the Sea-Chart; by Dr.* Wallis. *n.* 176. *p.* 1193. *Fig.* 206.

XXXVI. 1. Though it be well known, that, in the *Terreſtrial Globe*, all the *Meridians* meet at the Pole, (as EP, EP,) whereby the Parallels to the Equator, as they be nearer to the Pole, do continually decreaſe:

2. And hereby a Degree of *Longitude* in ſuch Parallels, is leſs than a degree of *Longitude* in the Equator, or a degree of *Latitude*:

3. And that in ſuch proportion, as is the Co-ſine of *Latitude* (which is the Semidiameter of ſuch Parallel) to the Radius of the *Globe*, or of the Equator.

4. Yet hath it been thought fit (for ſome Reaſons) to repreſent theſe *Meridians*, in the *Sea-Chart*, by Parallel ſtraight Lines; as Ep, Ep.

5. Whereby, each Parallel to the Equator (as LA) was repreſented in the *Sea Chart*, (as la,) as equal to the Equator EE: and a Deg. of *Longitude* therein, as large as in the Equator.

6. By this means, each Degree of *Longitude* in ſuch Parallels, was Increaſed, beyond its juſt proportion, at ſuch rate as the *Equator* (or its Radius) is greater than ſuch Parallel, (or the Radius thereof.)

7. But, in the Old *Sea-Charts*, the Degrees of *Latitude* were yet repreſented (as they are in themſelves) equal to each other; and to thoſe of the Equator.

8. Hereby, amongſt many other Inconveniencies, (as *Mr. Ed. Wright* obſerves, in his *Correction of Errors in Navigation*, firſt publiſhed in the Year 1599,) the repreſentation of the places remote from the Equator, was ſo Diſtorted in thoſe *Charts*, as that (for Inſtance) an *Iſland* in the *Latitude* of 60 degrees, (where the Radius of the Parallel is but half ſo great as that of the Equator) would have its Length (from *Eaſt* to *Weſt*) in compariſon of Breadth (from North to South) repreſented in a double proportion of what indeed it is.

9. For

Plate 7. Vol. 1. Pag. 660.

9. For Rectifying this in some measure (and of some other Inconveniencies) *Mr. Wright* adviseth, that (the *Meridians* remaining Parallel, as before) the Degrees of the *Latitude* remote from the Equator, should at each Parallel, be protracted in like proportion with those of *Longitude*.

10. That is; As the Co-Sine of *Latitude* (which is the Semidiameter of the Parallel) to the Radius of the *Globe*, (which is that of the Equator:) So should be a Degree of *Latitude* (which is every where equal to a Degree of *Longitude* in the Equator,) to such Degree of *Latitude* so protracted (at such Distance from the Equator;) and so to be represented in the *Chart*.

11. That is; every where, in such Proportion as is the respective Secant (for such *Latitude*) to the Radius. For, As the Co-Sine, to the Radius; so is the Radius, to the Secant (of the same Arch or Angle;) or $\Sigma : R :: R : s$. Fig. 207.

12. So that (by this means) the Position of each Parallel in the *Chart*, should be at such Distance from the Equator, compared with so many Equinoctial Degrees or Minutes, (as are those of *Latitude*) as are all the Secants (taken at equal Distances in the Arch) to so many Times the Radius.

13. Which is equivalent (as Mr. *Wright* there notes) to a Projection of the Spherical Surface (supposing the Eye at the Center) on the Concave Surface of a *Cylinder* erected at Right Angles to the Plain of the Equator.

14. And the Division of *Meridians*, represented by the Surface of a *Cylinder* erected (on the Arch of *Latitude*) at Right Angles to the Plain of the *Meridian* (or a Portion thereof.) The Altitude of such Projection (or Portion of such Cylindrick Surface) being, (at each Point of such Circular Base) equal to the Secant (of *Latitude*) answering to such Point. Fig. 208.

15. This Projection (or Portion of the Cylindrick Surface) if expanded into a Plain, will be the same with a Plain Figure, whose Base is equal to a Quadrantal Arch extended (or a Portion thereof) on which (as Ordinates) are erected Perpendiculars equal to the Secants, answering to the respective Points of the Arch so extended: The least of which (answering to the Equinoctial) is equal to the Radius; and the rest continually increasing, till (at the *Pole*) it be Infinite. Fig. 209.

16. So that, as ERSL. (a Figure of Secants erected at Right Angles on EL, the Arch of *Latitude* extended) to ERRL, (a Rectangle on the same Base, whose Altitude ER is equal to the Radius;) so is EL (an Arch of the Equator equal to that of *Latitude*,) to the Distance of such Parallel, (in the *Chart*) from the Equator.

17. For finding this Distance, answering to each Degree and Minute of *Latitude*, Mr. *Wright* (as the most obvious way) Adds all the Secants (as they are found calculated in the *Trigonometrical Canon*) from the beginning, to the Deg. or Min. of *Latitude* proposed.

18. The Summ of all which except the Greatest, (answering to the Figure Inscribed) is too little: The Summ of all except the Least, (answering to the Circumscribed) is too Great; (which is that He follows:) And it would be nearer to the Truth than either, if (Omitting all these) we take the Intermediates; for *Min.* $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, &c. or (the double of these) *Min.* 1, 3, 5, 7, &c. Which yet (because on the Convex-side of the Curve) would be somewhat too Little.

19. But

19. But any of these ways are exact enough for the Use intended, as creating no sensible Difference in the *Chart*.

20. If we would be more exact; Mr. *Oughtred* directs (and so had Mr. *Wright* done before him) to divide the Arch into Parts yet smaller than Minutes, and calculate Secants suiting thereunto.

21. Since the *Arithmetick of Infinites* Introduced, and (in pursuance thereof) the Doctrine of *Infinite Series* (for such Cases as would not, without them, come to a determinate Proportion;) Methods have been found for Squaring some such Figures.

Fig. 207. 22. In order to a *Quadrature* for this Figure of Secants (by an Infinite Series fitted thereunto) Put we, for the Radius of a Circle, R; the Right Sine of an Arch or Angle, S; the Versed Sine, V; the Co-Sine (or Sine of the Complement) $\Sigma = R - V = \sqrt{Rq - Sq}$; the Secant, *s*; the Tangent, T.

23. Then is $\Sigma : R :: R : s$. That is, $\Sigma$) $R^2$ ($s = \frac{R^2}{\Sigma}$; the Secant.

24. And $\Sigma : S :: R : T$. That is, $\Sigma$) SR ($T = \frac{SR}{\Sigma}$; the Tangent.

Fig. 210. 25. Now, if we suppose the Radius CP, divided into equal Parts, and each of them $= \frac{1}{\infty} R$;) and on these, to be erected the Co-Sines of Latitude LA.

26. Then are the Sines of Latitude in Arithmetick Progression.

27. And the Secants answering thereunto, $Ls = \frac{R^2}{\Sigma}$.

28. But these Secants, (answerwing to Right Sines in Arithmetical Progression,) are not those that stand at Equal Distances on the Quadrantal Arch Extended, *Fig.* 209.

29. But, standing at Unequal Distances (on the same extended Arch;) Namely on those Points thereof, whose Right Sines (whilst it was a Curve)

Fig. 211. are in Arithmetical Progression. As *Fig.* 211.

30. To find therefore the Magnitude of REL*s*, *Fig.* 209. which is the same with that of *Fig.* 211. (supposing EL of the same Length in both; however the Number of Secants therein may be unequal) we are to consider the Secants, though at Unequal Distances, *Fig.* 211. to be the same with those of Equal Distances in *Fig.* 210. answering to Sines in Arithmetical Progression.

31. Now these Intervals (or Portions of the Base) in *Fig.* 211. are the same with the Intercepted Arches (or Portions of the Arch) in *Fig.* 210. For this Base is but that Arch Extended.

32. And these Arches (in Parts Infinitely small) are to be reputed equivalent to the Portions of their respective Tangents Intercepted between the same

Fig. 212. Ordinates. As in *Fig.* 210, 212.

33. That is, Equivalent to the Portions of the Tangents of Latitude.

34. And these Portions of Tangents are to the Equal Intervals in the Base, as the Tangent (of *Latitude*) to its Sine.

35. To

35. To find therefore the true Magnitude of the Parallelograms ( or Segments of the Figure; ) we must either Protract the Equal Segments of the Base. *Fig.* 210. (in such Proportion as is the respective Tangents to the Sine) to make them Equal to those of *Fig.* 211.

36. Or else, ( which is equivalent ) retaining the Equal Intervals of *Fig.* 210. Protract the Secants in the same Proportion. (For either way, the Intercepted Rectangles or Parallelograms will be equally Increased) as LM. *Fig.* 212.

37. Namely; as the Sine ( of *Latitude* ) to its Tangent; so is the Secant, to a Fourth; which is to stand ( on the Radius equally divided ) instead of that Secant. Fig. 212.

$$S : \frac{SR}{\Sigma}\ (::\Sigma : R.) :: \frac{R^2}{\Sigma} : \frac{R^3}{\Sigma^2 = R^2 - S^2} = LM.$$

38. Which therefore are as the Ordinates in ( what I call *Arith. Infin. Prop.* 104. ) *Reciproca Secundanorum:* Supposing $\Sigma^2$ to be Squares in the Order of Secundanes.

39. This ( because of $\Sigma^2 = R^2 - S^2$; and the Sines S, in Arithmetical Progression, ) is Reduced ( by Division ) into this *Infinite Series*,

$$R + \frac{S^2}{R} + \frac{S^4}{R^3} + \frac{S^6}{R^5}, \&c.$$

40. That is, ( putting $R = 1.$ ) $1 + S^2 + S^4 + S^6$, &c.

41. Then ( according to the *Arithmetick of Infinites* ) we are to Interpret S, successively, by 1 S, 2 S, 3 S, &c. till we come to S, the greatest. Which therefore Represents the Number of all.

42. And because the first Member doth Represent a Series of Equals, the Second of Secundans; the Third of Quartans, &c. Therefore the First Member is to be Multiplied, by S; the Second by $\frac{1}{3}$ S; the Third by $\frac{1}{5}$ S; the Fourth by $\frac{1}{7}$ S; &c.

43. Which makes the Aggregate, $S + \frac{1}{3}S^3 + \frac{1}{5}S^5 + \frac{1}{7}S^7 + \frac{1}{9}S^9$, &c. = ECLM.

44. This ( because S is always less than $R = 1$ ) may be so far continued, till some *Power* of S become so small as that it ( and all which follow it ) may be safely neglected.

45. Now ( to fit this to the *Sea-Chart*, according to Mr. *Wright's* Design) having the proposed Parallel (of *Latitude*) given; we are to find (by the *Trigonometrical Canon*) the Sine of such *Latitude*; and take Equal to it, CL = S. And, by this find the Magnitude of ECLM, *Fig.* 212. that is of REL*f*. *Fig.* 211. that is, of REL*f*. *Fig.* 209. And then, as RRLE ( or so many times the Radius) to REL*f*, ( the Aggregate of all the Secants; ) so must be a like Arch of the Æquator, ( Equal to the Latitude proposed, ) to the Distance of such Parallel, (representing the *Latitude* in the *Chart* ) from the Æquator: Which is the thing required.

46. The same may be obtained, in like manner, by taking the Versed Sines in Arithmetical Progression. For if the Right Sines ( as here ) beginning at the Æquator, be in Arithmetical Progression, as, 1, 2, 3, &c. Then will the

the Verſed Sines, beginning at the *Pole*, ( as being their Complements to the Radius ) be ſo alſo.

47. The ſame may be applied in like manner, ( though that be not the preſent Buſineſs ) to the Aggregate of *Tangents*, ( anſwering to the Arch Divided into Equal Parts.

48. For, thoſe anſwering to the Radius ſo Divided, are $\frac{SR}{\Sigma}$; ( taking S in Arithmetical Progreſſion. )

49. And then Inlarging the Baſe ( as in *Fig.* 211. ) or the Tangent ( as in *Fig.* 212. ) in the Proportion of the Tangent to the Sine.

$$S : \frac{SR}{\Sigma} \; ( : : \Sigma : R ) : : \frac{SR}{\Sigma} : \frac{SR^2}{\Sigma^2} = \frac{SR^2}{R^2 - S^2}.$$

50. We have by Diviſion this Series, $S + \frac{S^3}{R^2} + \frac{S^5}{R^4} + \frac{S^7}{R^6} + \frac{S^9}{R^8}$ &c.

51. That is, ( putting $R = 1$ ) $S + S^3 + S^5 + S^7 + S^9$, &c.

52. Which ( Multiplying the Reſpective Members by $\frac{1}{2}S$, $\frac{1}{4}S$, $\frac{1}{6}S$, $\frac{1}{8}S$, $\frac{1}{10}S$, &c. ) becomes $\frac{1}{2}S^2 + \frac{1}{4}S^4 + \frac{1}{6}S^6 + \frac{1}{8}S^8 + \frac{1}{10}S^{10}$, &c.

Which is the Aggregate of Tangents to the Arch whoſe Right Sine is S.

53. And this Method may be a Pattern for the like Proceſs in other Caſes of like Nature.

*Two Problems in* Navigation *propos'd; by Mr.* Nich. Mercator. *n.* 13. *p.* 215.

XXXVII. The Line of *Artificial Tangents*, or the *Logarithmical Tangent Line*, beginning at 45°. and taking every Half Deg. for a whole one, is found to agree pretty near with the *Meridian Line* of the *Sea-Chart*, they both growing, as it were, after the ſame Proportion. But the Table of *Meridional* Degrees being Calculated only to every Sexageſimal Minute of a Degree, ſhews ſome ſmall Differenee from the ſaid *Logarithmical Tangent Line*. Hence it may be doubted, whether that Difference do not ariſe from that little Error, which is committed by Calculating the Table of *Meridional* Degrees only to every Minute.

But if a certain Rule could be produced, by which the Agreement or Diſagreement of the ſaid two *Lines* might be ſhewed, the *Helix* or Spiral Line of the *Ship's Courſe* would be reduced to a more preciſe Exactneſs, than ever was pretended by any.

The ſame Rule would alſo diſcover a far eaſier way of making *Logarithms*, than ever was practiſed or known; and therefore might ſerve, whenever there ſhould be Occaſion, to extend the *Logarithms* beyond that Number of Places, that is already extant.

Moreover ſuch a Rule would enable Men to draw the *Meridian Line* Geometrically, that is without Tables or Scales; which indeed might alſo be done by ſetting of the Secants of every whole or half Degree, if there were not this Inconveniency in it ( which is not in my Rule: ) That a Line compoſed of ſo many ſmall Parts would be ſubject to many Errors, eſpecially in a ſmall Compaſs.

The

The ſame Rule alſo will ſerve to find the *Courſe* and *Diſtance* between two Places aſſigned, as far as Practice ſhall require it; and that without any Table of *Meridional Parts*, and yet with as much Eaſe and Exactneſs.

And ſeeng all theſe things do depend on the Solution of this Queſtion, *Whether the Artificial Tangent Line be the True Meridian Line*? It is therefore, that I undertake, by God's Aſſiſtance, to reſolve the ſaid Queſtion. And to let the World know the Readineſs and Confidence I have to make good this Undertaking, I am willing to lay a Wager againſt any one or more Perſons that have a Mind to engage, for ſo much as another *Invention* of mine (which is of no leſs Subtlety, but of a far greater Benefit to the Publick) may be worth to the *Inventor*.

As for the great Advantage, that all *Merchants*, *Mariners*, and conſequently the *Common-Wealth*, may receive from this other Invention, it is, in my Judgment, highly valuable, ſeeing it will often-times make a *Ship Sail*, though according to the common way of *Sailing*, the Wind be quite Contrary, and yet as near to the Place intended, as if the *Wind* had been favourable: Or if you will, it will enable one to gain ſomething in the Intended way, whether the *Wind* be good or no (except only when you go directly South or North) but the Advantage will be moſt where there is moſt need of it, that is, when the *Wind* is contrary: So that one may very often gain a fifth, fourth, third Part, or more of the Intended Voyage; according as it is longer or ſhorter; *viz*. always more in a longer Voyage, where the Gain is more Conſiderable, and more Welcome, not only by ſaving *Time*, but alſo *Victuals*, *Water*, *Fuel*, *Men's Health*, and ſo much *Room* in the *Ship*.

*The Analogy of Logarithmick Tangents to the* Meridian Line, *Demonſtrated; by Mr.* Edm. Halley. *n.* 219. *p.* 202.

XXXVIII. It was firſt Diſcovered by Chance, and as far as I can Learn, firſt Publiſhed by Mr. *Henry Bond*, as an Addition to *Norwood's Epitome of Navigation*, about 50 Years ſince, that the *Meridian Line was Analogous to a Scale of Logarithmick Tangents of half the Complements of the Latitudes.*

For the *Demonſtration* of that *Propoſition* it is Requiſite to premiſe theſe 4 *Lemmata.*

Lemma. 1. *In the Stereographick Projection of the Sphære upon the Plain of the Æquinoctial, the Diſtances from the Center,* which in this Caſe is the Pole, *are laid down by the Tangents of half thoſe Diſtances,* that is, of half the Complements of the Latitudes. This is evident from *Eucl.* 3. 20.

Lemma. 2. *In the Stereographick Projection, the Angles under which the Circles Interſect each other, are in all Caſes equal to the Spherical Angles they repreſent*; which is a very valuable *Property* of this *Projection.*

*Demonſt.* Let EPBL be any Great Circle of the Sphere, E the Eye placed in its Circumference; C its Center, P any Point thereof; and let FCO be ſuppoſed a Plain erected at Right Angles to the Circle EPBL, on which FCO we deſign the Sphere to be Projected. Draw EP croſſing the Plain FCO in *p*, and *p* ſhall be the Point P projected. To the Point P draw the Tangent APG, and on any point thereof, as A, Erect a Perpendicular AD, at right Angles to the Plain EPBL, and draw the Lines PD, AC, DC; and the Angle APD ſhall be equal to the Spherical Angle contained between the Fig. 213.

Plains APC, DPC. Draw alſo AE, DE, interſecting the Plain FCO in the Points *a* and *d*; and join *a d*, *p d*: I ſay, the Triangle *a d p*, is ſimular to the Triangle ADP, and the Angle *a p d* equal to the Angle APD: Draw PL, AK, parallel to FO, and by reaſon of the Parallels, *a p* will be, to *a d*; as AK, to AD: But (by *Eucl.* 3. 32.) in the Triangle AKP, the Angle AKP= LPE, is alſo equal to APK=EPG, wherefore the ſides AK, AP, are Equal, and 'twill be as *a p*, to *a d*; ſo AP, to AD. Whence the Angles DAP, *d a p*, being Right, the Angle APD, will be equal to the Angle *a p d*, that is, the Spherical Angle is equal to that on the *Projection*, and that in all Caſes. Q. E. D.

This *Lemma*, I lately received from Mr. *Ab. de Moivre*, though I ſince underſtand from Dr. *Hook*, that he long ago produced the ſame thing before the *Society*. However the *Demonſtration*, and the reſt of the Diſcourſe is my own.

Lemma. 3. *On the Globe, the Rhumb Lines make equal Angles with every Meridian*, and by the aforegoing *Lemma*, they muſt likeways make *equal Angles with the Meridians in the Stereographick Projection on the Plain of the Equator*: They are therefore in that *Projection*, *Proportional Spirals* about the Pole-point.

Fig. 214. Lemma. 4. *In the Proportional Spiral, it is a known Property that the Angles* BPC, *or the Arches* BD, *are Exponents of the Rationes of* BP *to* PC: For if the Arch BD be divided into innumerable Equal-parts, Right Lines drawn from them to the Center P, ſhall divide the Curve B *c c* C into an Infinity of *Proportionals* between PD and PC, whoſe Number is equal to all the Points *d*, *d*, in the Arch BD: Whence, and by what I have delivered concerning the *Conſtruction of Logarithms*, it follows, that as BD to B *d*, or as the Angle BPC, to the Angle BP *c*, ſo is the *Logarithm* of the *Ratio* of PB to PC, to the *Logarithm* of the *Ratio* of PB to P *c*. vid. ſup. Cap. I. §. XXVIII.

From theſe *Lemmata* our Propoſition is very clearly *Demonſtrated*: For by the *Firſt* PB, P *c*, PC, are the Tangents of half the Complements of the Latitudes in the Stereographick Projection: And by the *Laſt* of them, the Differences of Longitude, or Angles at the Pole between them, are *Logarithms* of the *Rationes* of thoſe Tangents one to the other. But the *Nautical Meridian Line* is no other than a Table of the *Longitudes*, anſwering to each Minute of *Latitude* on the *Rhumb Line* making an Angle of 45 Degrees with the *Meridian*. Wherefore the *Meridian Line* is no other than *a Scale of Logarithmick Tangenss of the half Complements of the Latitudes*. Q. E. D.

*Coroll.* 1. Becauſe that in every Point of any *Rhumb Line*, the Difference of *Latitudes* is to the *Departure*, as the Radius to the Tangent of the Angle that *Rhumb* makes with the Meridian; and thoſe equal *Departures* are every where to the Differences of *Longitude*, as the Radius to the Secant of the *Latitude*; it follows that the Differences of *Longitude* are on any *Rhumb*, *Logarithms* of the ſame Tangents, but of a Differing Species; being proportioned to one another as are the Tangents of the Angles made with the *Meridian*.

*Coroll.* 2.

*Coroll.* 2. Hence any Scale of *Logarithm Tangents* ( as those of the Vulgar Tables made after *Brigg*'s Form; or those made to *Napier*'s, or any other Form whatsoever ) is a Table of the Differences of *Longitude*, to the several *Latitudes*, upon some determinate *Rhumb* or other: And therefore, as the Tangent of the Angle of such Rhumb, to the Tangent of any other Rhumb: So the Difference of the *Logarithms* of any two Tangents, to the Difference of *Longitude*, on the proposed Rhumb, intercepted between the two *Latitudes*, of whose half Complements you took the *Logarithm Tangents*.

Now the *Momentary Augment* or *Fluxion* of the *Tangent Line* at 45 Degrees, is exactly double to the *Fluxion* of the *Arch* of the Circle, ( as may easily be proved ) and the *Tangent* of 45 being *Equal* to the Radius, the *Fluxion* also of the *Logarithm Tangent* will be double to that of the *Arch* if the *Logarithm* be of *Napier*'s Form: But for *Brigg*'s Form, it will be as the same Doubled Arch Multiplied into 0, 43429, &c. or Divided by 2, 30258, &c. yet this must be understood only of the Addition of an Indivisible Arch, for it ceases to be true if the Arch have any Determinate Magnitude.

Hence it appears, that if one Minute be suppose Unity, the Length of the Arch of one Minute being 0, 000290888208665721596154, &c. in parts of the Radius, the Proportion will be as Unity to 2, 908882, &c. so Radius to the Tangent of 71° 1′ 42″, whose *Logarithm* is 10,463726117720718-325204, &c. and under that Angle is the *Meridian Intersected* by that *Rhumb Line*, on which the *Differences* of *Napier's Logarithm Tangents* of the Half Complements of the *Latitudes* are the true Differences of *Longitude*, estimated in Minutes and Parts, taking the first 4 Figures for Integers. But for *Vlacq*'s Table we must say,

As 2302585, &c. to 2908882, &c. so Radius to 1,2633114387424-4569212, &c. which is the *Tangent* of 51° 38′ 9″, and its *Logarithm* 10,10-15104285077209411 62, &c. wherefore in the *Rhumb Line* which makes an Angle of 51° 38′ 9″ with the *Meridian*, *Vlacq's Logarithm Tangents* are the true Differences of *Longitude*. And this compared with our *Second Corollary* may suffice for the Use of the *Tables* already computed.

But if a Table of *Logarithm Tangents* be made by extraction of the Root of the *Infiniteth Power*, whose Index is the Length of the Arch you put for Unity, ( as for Minutes the 0, 000290888 2th. &c. *Power*, ) which we will call $a$; such a Scale of *Tangents* shall be the true *Meridian Line*, or Summ of all the Secants taken infinitely many. Here the Reader is desired to have recourse to my little Treatise of *Logarithms*, that I may not need to repeat it. *vid. sup. Cap.* I. §. XXVIII. By what is there delivered it will follow, that putting $t$ for the Excess or Defect of any Tangent above or under the Radius or Tangent of 45° ; the *Logarithm* of the *Ratio* of Radius to such Tangent will be.

$$\frac{1}{m} \text{ into } t - \tfrac{1}{2} t^2 + \tfrac{1}{3} t^3 - \tfrac{1}{4} t^4 + \tfrac{1}{5} t^5, \text{ \&c.}$$

when the Arch is greater than 45, or

$$\frac{1}{m} \text{ into } t + \tfrac{1}{2} t^2 + \tfrac{1}{3} t^3 + \tfrac{1}{4} t^4 + \tfrac{1}{5} t^5, \text{ \&c.}$$

 when

when it is leſs than 45°. And by the ſame Doctrine putting T for the Tangent of any Arch, and $t$ for the Difference thereof from the Tangent of another Arch, the *Logarithm* of their *Ratio* will be $\frac{1}{m}$ into $\frac{t}{T} + \frac{tt}{2TT} + \frac{t^3}{3T^3} + \frac{t^4}{4T^4} + \frac{t^5}{5T^5}$, *&c.* when T is the greater Term; or, $\frac{1}{m}$ into $\frac{t}{T} - \frac{t^2}{2T^2} + \frac{t^3}{3T^3} - \frac{t^4}{4T^4} + \frac{t^5}{5T^5}$, *&c.* when T is the leſſer Term.

And if $m$ be ſuppoſed, 0,000290888'2, *&c.* $= a$, its reciprocal $\frac{r}{a}$ will be 3437,74677078493925'26, *&c.* which Multiplied into the aforeſaid *Series*, ſhall give preciſely the Difference of the *Meridional Parts*, between the two *Latitudes* to whoſe half Complements the aſſumed Tangents belong. Nor is it material from whether Pole you eſtimate the Complements, whether the Elevated or Depreſſed; the Tangents being to one another in the ſame *Ratio* as their Complements, but Inverted.

In the ſame Diſcourſe I alſo ſhewed that the Series might be made to Converge twice as ſwift, all the Even Powers being omitted; and that putting $\tau$ for the Summ of the two Tangents, the ſame *Logarithm* would be $\frac{2}{m}$ or $\frac{2r}{a}$ into $\frac{t}{\tau} + \frac{t^3}{3\tau^3} + \frac{t^5}{5\tau^5} + \frac{t^7}{7\tau^7} + \frac{t^9}{9\tau^9}$, &c. but the *Ratio* of $\tau$ to $t$, or of the Summ of two Tangents to their Difference, is the ſame as that of the Sine of the Summ of the Arches, to the Sine of their Difference. Wherefore if S be put for the Sine Complement of the *Middle Latitude*, and $s$ for the Sine of half the Difference of *Latitudes*, the ſame Series will be $\frac{2r}{a}$ into $\frac{s}{S} + \frac{s^3}{3S^3} + \frac{s^5}{5S^5} + \frac{s^7}{7S^7} + \frac{s^9}{9S^9}$, *&c.* wherein as the Differences of *Latitude* are ſmaller, fewer Steps will ſuffice. And if the *Equator* be put for the *Middle Latitude*, and conſequently $S = R$, and $s$ to the Sine of the *Latitude*, the *Meridional Parts* reckoned from the *Equator* will be $\frac{s}{a} + \frac{s^3}{3rra} + \frac{s^5}{5r^4a} + \frac{s^7}{7r^6a}$, *&c.* which is Co-incident with Dr. *Wal-*
§. XXXVI. *lis's Solution.* And this ſame Series, being half the *Logarithm* of the *Ratio* of $R + s$ to $R - s$, that is of the *Verſed-ſines* of the Diſtances from both Poles, does agree with what Dr. *Barrow* had ſhewn in his XI *Lecture*.

The

The ſame *Ratio* of $\tau$ to $t$ may be expreſſed alſo by that of the Summ of the Co-ſines of the two *Latitudes*, to the Sine of their Difference: As likewiſe by that of the Sine of the Summ of the two *Latitudes*, to the Difference of their Co-ſines: Or by that of the Verſed-ſine of the Summ of the *Co-Latitudes*, to the Difference of the Sines of the *Latitudes*: Or as the ſame Difference of the Sines of the *Latitudes*, to the Verſed-ſine of the Difference of the *Latitudes*; all which are in the ſame *Ratio* of the Co-ſine of the *Middle-Latitude*, to the Sine of half the Difference of the *Latitudes*. As it were eaſie to Demonſtrate, if the Reader were not ſuppoſed capable to do it himſelf, upon a bare Inſpection of a Scheme duly repreſenting theſe Lines.

This Variety of *Expreſſion* of the ſame *Ratio* I thought not fit to be omitted, becauſe by help of the *Rationality* of the Sines of 30°, in all Caſes where the Summ or Difference of the *Latitudes* is 30°, 60°, 90°, 120°, or 150°; ſome one of them will exhibit a Simple Series, wherein great Part of the Labour will be ſaved. But the former ſeems for all Uſes the moſt convenient, whether we deſign to make the whole *Meridian-Line*, or any Part thereof, *viz.* $\frac{2r}{a}$ into $\frac{s}{S} + \frac{s^3}{3S^3} + \frac{s^5}{5S^5} + \frac{s^7}{7S^7} + \frac{s^9}{9S^9}$, *&c.* wherein $a$ is the Length of any Arch, which you deſign ſhall be the Integer or *Unity* in your *Meridional Parts* (whether it be a Minute, League or Degree, or any other;) S the Co-ſine of the *Middle-Latitude*, and $s$ the Sine of half the Difference of *Latitudes*; but the Secants being the Reciprocals of the Co-ſines $\frac{s}{S}$ will be equal to $\frac{\int s}{rr}$ putting $\int$ for the Secant of the *Middle-Latitude*; and $\frac{2r}{a}$ into $\frac{s}{S}$ will be $= \frac{2\int s}{ra}$. This multiplied by $\frac{\int s}{3SS}$ that is by $\frac{\int\int ss}{3rrrr}$, will give the Second Step; and that again by $\frac{3\int\int ss}{5rrrr}$, the third Step; and ſo forward till you have compleated as many Places as you deſire. But the *Squares* of the *Sines* being in the ſame *Ratio* with the *Verſed-ſines* of the Double Arches, we may inſtead of $\frac{ss}{3SS}$ aſſume for our Multiplicator $\frac{v}{3V}$, or the Verſed-ſine of the Difference of the *Latitudes* divided by thrice the Verſed-ſine of the Summ of the *Co-Latitudes*, &c. which is the utmoſt *Compendium* I can think of for this purpoſe, and the ſame *Series* will become $\frac{2sr}{aS}$ into 1 $+ \frac{v}{3V} + \frac{v^2}{5V^2} + \frac{v^3}{7V^3} + \frac{v^4}{9V^4}$. Hereby we are enabled to eſtimate the Default of the Method of making the *Meridional Line*, by the continual Addition of the Secants of Equidifferent Arches, which as the Differences of thoſe Arches are ſmaller, does ſtill nearer and nearer ap-

proach

proach the Truth. If we assume, as Mr. *Wright* did, the Arch of one Minute to be Unity, and one Minute to be the common Difference of a Rank of Arches: It will be in all Cases, as the Arch of one Minute; to its Chord: So the Secant of the Middle Latitude; to the First Step of our Series. This by Reason of the near Equality between $a$ and $2s$, which are to one another in the *Ratio* of Unity to 1—0, 0000000035256645771 3, &c. will not Differ from the Secant $\int$, but in the 9th Figure; being less than it in that Proportion. The next Step being $+\frac{2\int^3 s^3}{3arr}$ will be Equal to the Cube of the Secant of the *Middle Latitude* Multiplied into $\frac{2sss}{3arr} = 0,0$-000000705132908715; which therefore unless the Secant exceed ten times Radius, can never amount to 1 in the fifth Place. These two Steps suffice to make the *Meridian Line*, or *Logarithm Tangent*, to far more Places than any Tables of Natural Secants yet extant, are computed to; but if the Third Step be required it will be found to be $+\int s$ into $\frac{2s^5}{5ar^4} = 0,00000$-000000000089498; by all which it appears that Mr. *Wright*'s Table does no where exceed the true *Meridian Parts* by fully half a *Minute*; which small Difference arises by his having added continually the Secants of 1′, 2′, 3′, &c. instead of $0\frac{1}{2}'$, $1\frac{1}{2}'$, $2\frac{1}{2}'$, $3\frac{1}{2}'$, &c. But as it is, it is abundantly sufficient for *Nautical* Uses. That in Sir *Jonas Moor's New Systeme of the Mathematicks* is much nearer the Truth, but the Difference from *Wright* is scarce sensible, till you exceed those *Latitudes* where *Navigation* ceases to be Practicable, the one exceeding the Truth about half a Minute, the other being a very small matter Deficient therefrom.

For an Example easie to be Imitated by whoso pleases, I have added the true *Meridional Parts* to the First and Last *Minutes* of the *Quadrant*.

The *First Minute*. 1, 0000000141026586217 8.

The *Second*, ——— 2, 0000000564106380670 7.

The *Last*, or 89° 59′. 30374,9634311414228643, and not 32348, 5279 as Mr. *Wright* has it, by the Addition of the Secants of every whole Minute: Nor 30249, 8 as Mr. *Oughtred*'s Rule makes it, by adding the Secants of every half Minute. Nor 30364, 3 as Sir *Jonas Moor* had concluded it by I know not what Method, though in the rest of his Table he follows *Oughtred*.

The same may be Deduced independently, from the Arch it self. For if the *Latitude* from the Equator be estimated by the length of its Arch A, Radius being Unity, and the Arch put for an Integer be $a$, as before; the *Meridional Parts* answering to that *Latitude* will be $\frac{1}{a}$ into $A + \frac{1}{6}A^3 + \frac{1}{24}A^5 +$

$\frac{1}{84}$

60

$\dfrac{\frac{1}{84}}{60}$ A $^7$ or $\dfrac{61}{5040}$ A $^7$ $+$ $\dfrac{\frac{11}{2880}}{1386}$ A $^9$ or $\dfrac{1385}{362880}$ A $^9$, *&c.* which *Converges* much ſwifter than any of the former *Series*, and beſides has the Advantage of A Increaſing in Arithmetical Progreſſion, which would be of great Eaſe, if any ſhould undertake *De Novo* to make the *Logarithm Tangents*, or the *Meridian Line* to many more Places than now we have them. The *Logarithm Tangent* to the Arch of $45^{\circ} + \frac{1}{2}$ A being no other than the aforeſaid Series A $+ \frac{1}{6}$ A $^3$ $+ \frac{1}{24}$ A $^5$, *&c.* in *Napier*'s Form, or the ſame Multiplied into 0, 43429, *&c.* for *Briggs*'s:

But becauſe all theſe *Series* towards the latter End of the Quadrant do *Converge* exceeding ſlowly, ſo as to render this Method almoſt uſeleſs, or at leaſt very Tedious: It will be convenient to apply ſome other Arts, by aſſuming the Secants of ſome intermediate *Latitudes*; and you may for $s$, or the Sine of $\alpha$, the Arch of half the Difference of *Latitudes*, ſubſtitute $\alpha - \frac{1}{6}\alpha^3 + \frac{1}{120}\alpha^5 - \frac{1}{5040}\alpha^7 + \frac{1}{362880}\alpha^9$, *&c.* according to Mr. *Newton*'s Rule for giving the Sine from the *Arch*; and if $\alpha$ be no more than a Degre, a very few Steps will ſuffice for all the Accuracy that can be deſired.

And if $\alpha$ be commenſurable to $a$, that is, if it be a certain Number of thoſe Arches which you make your Integer, than will $\frac{\alpha}{a}$ be that Number: which if we call $n$, the Parts of the *Meridian Line* will be found to be

$$\frac{sn}{r} \text{ into } \left\{ \begin{array}{l} 1 + \dfrac{ss\,\alpha\alpha}{3r^4} + \dfrac{s^4\,\alpha^4}{5r^8} + \dfrac{s^6\,\alpha^6}{7r^{12}}, \text{ \&c.} \\[2mm] \quad - \dfrac{\alpha\alpha}{6rr} - \dfrac{ss\,\alpha^4}{6r^6} - \dfrac{s^4\,\alpha^6}{6r^{10}}, \text{ \&c.} \\[2mm] \qquad + \dfrac{1}{120}\dfrac{\alpha^4}{r^4} + \dfrac{13}{360}\dfrac{s^2\,\alpha^6}{r^8}, \text{ \&c.} \\[2mm] \qquad\qquad - \dfrac{1}{5040}\dfrac{\alpha^6}{r^6}, \text{ \&c.} \end{array} \right.$$

In this the Firſt two Steps are generally ſufficient for *Nautical Uſes*, eſpecially when neither of the *Latitudes* exceed 60 Degrees, and the *Difference* of *Latitudes* doth not paſs 30 Degrees.

To conclude I ſhall only add, that Unity being Radius, the Co-ſine of the Arch A, according to the ſame Rules of Mr. *Newton*, will be $1 - \frac{1}{2}$ A $^2$ $+ \frac{1}{24}$ A $^4$ $- \frac{1}{720}$ A $^6$ $+ \frac{1}{40320}$ A $^8$ $- \frac{1}{3628800}$ A $^{10}$, *&c.* from which and

the

the former *Series* exhibiting the Sine by the Arch, by Division it is easie to conclude, that the *Natural Tangent* to the Arch A is $A + \frac{1}{3}A^3 + \frac{2}{15}A^5 + \frac{17}{315}A^7 + \frac{62}{2835}A^9$, &c. and the *Natural Secant* to the same Arch $1 + \frac{1}{2}A^2 + \frac{5}{24}A^4 + \frac{61}{720}A^6 + \frac{277}{8064}A^8$, &c. and from the *Arithmetick of Infinites*, the Number of these *Secants* being the Arch A, it follows that the Summ Total of all the Infinite Secants on that Arch is $A + \frac{1}{6}A^3 + \frac{1}{24}A^5 + \frac{61}{5040}A^7 + \frac{277}{72576}A^9$, &c. the which by what foregoes, is the *Logarithm Tangent* of *Napier's* Form, for the Arch of $45° + \frac{1}{2}A$, as before.

And Collecting the Infinite Summ of all the *Natural Tangents* on the said Arch A, there will arise $\frac{1}{2}AA + \frac{1}{12}A^4 + \frac{1}{45}A^6 + \frac{17}{2520}A^8 + \frac{31}{14175}A^{10}$, &c. which will be found to be the *Logarithm* of the Secant of the same Arch A.

*To find the Variation of the Compass at Sea by . . . . . . n. 24. p. 435.*

XXXIX. The Height of the Pole, and the Suns Declination being known, a large Ring-Dial, truly wrought, having a Box with a Compass or needle fixt to its Meridian below, may go as near as any other Instrument, to shew the *Variation* of the *Needle* at Sea. For, when it is set to the just Hour and Minute of the Day, the Meridian of it stands just in its due place; and so shews how far the Needle *Varies* from it, as exactly as the largeness of the Card will permit.

But because these Dials are so rarely Just, &c. though they may be used and taken notice of, yet they are not to be relied on. The thing therefore is to be performed, as followeth:

Find out the Sun's *Azimuthal* Distance from the Meridian some hours before or after noon, and then its *Magnetical Azimuth*, or Distance from the Meridian pointed at by the *Needle*, and the Difference of these two Distances, is the *Variation* of the *Needle*.

To find the Sun's true *Azimuth*, or by how many Degrees, &c. of the Horizon, it is distant from the Meridian: It's Declination, it's Altitude, and the Elevation of the Pole, must all three be known; and thence the true *Azimuth* may be easily *Calculated*. The true *Azimuth* of the Sun being thus found, and the *Magnetical Azimuth* of it accordiug to your *Needle*, being observed, Substract the lesser Number from the greater, and the Remainder is the *Variation* of the *Needle*. If the *Magnetical Azimuth* be less than the other, then the *Variation* is towards the same side of the Meridian, where theSun is; if greater, on the other.

To

To obſerve the Sun's *Azimuth* by the *Needle*, and the *Needle's Variations* to *Degrees*, any *Needle* long enough to afford upon a Card under it a Circle divided into *Degrees*, put in a ſquare Box after the ordinary manner of *Clinatories*, will ſerve turn; by placing the Box ſo, as the Sun may ſhine upon any two oppoſite ſides of it, at the ſame time that the Sun's Height, &c. are taken. For then the *Needle's* diſtance from the Diameter of the Circle on the Card, that is parallel to thoſe ſides, is the *Magnetick Azimuth* required.

The ſame may be done with an ordinary *Sea-Compaſs*, ſo it have a Circle towards the Limb of the Card divided into *Degrees*, by faſtning a ſmall Thread, Lute-ſtring or Wire (not of Iron) ſo upon it, as to paſs juſt over the Center of that Circle; and placing a Strait piece of Wood or Braſs-wire perpendicular on the edge of the Box at the end of the Thread, and turning it to the Sun till the ſhadow of it fall juſt upon the Thread: then obſerve, what *Degree* of the Circle on the Card the Thread cuts, by looking plum upon it; and that is the Sun's *Magnetical Azimuth*.

But to have the *Variation* to *Degrees* and *Minutes* (which is moſt deſirable) then the Obſervation laſt mentioned muſt be made with a *Quadrant*, *Sextant*, or ſome ſuch other *Inſtrument*, ſo large as to admit of the Diviſion of a *Degree* into *Minutes*; which will require the Radius to be about 3 Foot; the larger the better. If a *Quadrant*, then, it being laid Flat and the Square Box with the *Needle* placed upon it, move the *Quadrant* to and again, till that ſide of it, on which the Box is placed, lie parallel to the *Needle* when at quiet: Then the *Sight* of the *Quadrant* being ſlid along the Limb of it, till the Sun ſhine on both its ſides at the ſame time, the Mid-Line, that divides equally the *Sight*, when the Sun Shines upon it through the *Slit*, will mark the *Degree* and *Minute* of the Sun's *Magnetical Azimuth*. All which is eaſie to be put in practice.

To find this *Variation* by the *Stars*, is ſo eaſie, that every Maſter can do it.

XL. It is a received Error, in the Practice of obſerving the *Variation* at *Sea*, to take it by the *Amplitude* of the *Riſing* and *Setting Sun*, when his *Center* appears in the *Viſible Horizon*; whereas he ought to be obſerved when his *Under-Limb* is ſtill above the *Horizon* about $\frac{2}{3}$ of his Diameter, or 20 *Minutes*, upon the Score of the *Refraction*, and the *Height* of the *Eye* of the obſerver above the *Surface* of the *Sea*: Or elſe they are to work the *Amplitude* as they do the *Azimuth*, reckoning the *Sun's* Diſtance from the *Zenith* 90° 36′.

*A Caution for Obſerving the* Variation *at* Sea; *by Mr.* Edm. Halley. n. 195. *p.* 571.

This, though it be of little conſequence near the *Equinoctial*, will make a great Error in *High Latitudes*, where the Sun Riſes and Sets *Obliquely*.

XLI. The *Latitudes* of the *Lizard* and *Scilly* are laid down too far Northerly by near 5 Leagues: For from undoubted Obſervation the *Lizard* lies in 49° 55′. the middle of *Scilly* due Weſt therefrom, and the South part thereof neareſt 49° 50′. whereas in moſt *Charts* and *Books* of

*A Caution to Seamen bound up the* Engliſh Channel; *by* n. 267. *p.* 725.

*Navigation* they are laid down to the Northward of 50°. and in ſome full 50° 10′. Nor was this without a good Effect as long as the *Variation* continued *Eaſterly*, as it was when the *Charts* were made. But ſince it is become conſiderably *Weſterly*, (as it has been ever ſince the Year 1657.) and is at preſent about $7\frac{1}{2}$ *Deg.* all Ships ſtanding in, out of the Ocean, *Eaſt* by the *Compaſs*, go two thirds of a Point to the Northward of their true Courſe, and in every 80 Miles they Sail, alter their *Latitude* about 10′. So that if they miſs an Obſervation for two or three Days, and do not Allow for this *Variation*, they fail not to fall to the Northward of their Expectation, eſpecially if they reckon *Scilly* in above 50°, and to run up the *Briſtol Channel*, not without great *Danger* of all, and the *Loſs* of many, of them. This has been by ſome attributed to the *Indraught* of St. *Georges Channel*: But the *Variation* being allowed, it hath been found that the ſaid *Indraught* is not ſenſible. It is therefore Recommended to all *Maſters* of *Ships* that they ſteer two Watches E. b. S. for one E. which will exactly keep their *Parallel*; as alſo that they come in, out of the *Sea*, on a *Parallel* not more *Northerly* than 49° 40′. which will bring them fair by the *Lizard*.

XLII. Papers *of Leſs General uſe*, Omitted.

*Pendulum Watches. n.* 118. *p.* 440. *vid. ſup. Cap.* V. §. V. Mr. *Oldenburg* having Publiſhed from the *Journal des Scavans*, an Account of M. *Hugens*'s Portable *Watches*, *Dr. Hook*, in the Poſtſcript to his *Deſcription of Helioſcopes*, Complains of it, for not having taken notice, *that this Invention was firſt found out by an Engliſh Man, and long ſince Publiſhed to the World*:
*n.* 128. *p.* 710. To this Mr. *Oldenburg* Anſwers, by relating the *Plain Truth of the Matter*:
*n.* 129. *p.* 749. Whereupon Dr. *Hook* in a Poſtſcript to his *Lampas* further Complains and Reflects on Mr. *Oldenburg's* Integrity and Faithfulneſs in *his Management of the Intelligence of the Royal* Society. This gave Occaſion to the *Council of that* Society to Declare *That Mr.* Oldenburg *had carried himſelf Faithfully and Honeſtly, And had given no juſt Cauſe of ſuch Reflections*; To which Mr. *Oldenburg* Adds part of a Letter from Mr. *Hugens* to him, *Offering ( if Mr.* Oldenburg *believes a Patent in* England *might be worth ſomething) all He might there pretend to*. So that if Mr. *Oldenburg* had a Deſire to take out a *Patent*, it was for no other Contrivance but Mr. *Hugens*'s.

XLIII. Accounts *of Books and* Emendations, Omitted.

*n.* 231. *p.* 670. 1. Volumen Primum *Geographorum Gr.* Minorum. *Oxon.* in 8*vo.*

*n.* 231. *p.* 671. 2. *Dionyſii* Periegeſis, Græce & Latine, cum Scholiis Gr. tam Editis quam Ineditis. Cura *Edv. Thwaites.* M.A. *Oxon.* in 8*vo.*

*n.* 91. *p.* 5172. 3. *Bernhardi Vareni* M. D. *Geographia* Generalis; Aucta & Illuſtrata ab *Iſaaco Newtono.* R. S. S. *Cantab.* 1672. in 8*vo.*

*n.* 231. *p.* 666. 4. *Philippi Cluverii* Introductio in Univerſam *Geographiam*, tam Veterem quam Novam: Tabulis *Geographicis* 46. ac Notis olim Ornata à *Joanne Bunone*; jam vero Locupletata Additamentis & Annotationibus *Jo. Frid. Hekelii* & *Jo. Reiskii.* Amſt. 1697. in 4*to.*

5. *Geo-*

5. *Geography Anatomiz'd*, or a Compleat *Geographical Grammar.* Being a short and Exact *Analysis* of the whole Body of *Modern Geography*, after a New and Curious Method; by *Pat. Gordon.* M. A. F. R. S. The *Second Edition.* n. 255. p. 335.

6. An Account of the Measure of a *Degree* of a *Great Circle* of the *Earth*; by M. Picart. *Paris:* 1671. *Fol.* Translated into *English*; by *Mr. Waller R. S. Sec. Lond.* 1687. *This Book is here Abridg'd; and the Sum of the whole amounts in short to this.* M. Picart *measured on a Plain and straight Ground a Space of* 5663 Toises, *to serve for the First* Basis *to Divers Triangles; by which he hath concluded the* Length *of a* Meridian Line *Equivalent to a* Degree *of* Latitude, *to be* 57060 Toises *or* Fathoms, *that is,* $28\frac{1}{2}$ Leagues *and* 60 Toises. n. 112. p. 261. n. 124. p. 596. n. 189. p. 376.

7. The Seamans Practice; by *Mr. Richard Norwood. Lond.* 1636. in *4to.* *The Measure of a* Degree *is here extracted from that Book.* Mr. Norwood *An.* 1635. *having actually measured, for the most part, the way from* York *to* London, *and having observed the Meridian Altitudes of the* Sun *in both Places, he found the* Difference *of* Latitude *to be* 2° 28'. *and the Distance of their Parallels* 905751 English *Feet; and therefore one* Degree *of a* Great Circle *is* 367196 *Feet, or* Numero Rotundo 367200 *Feet, which is equal to* $69\frac{1}{2}$ English Miles *and* 14 *Poles; Whereas the* French *make it no more than* 365000 *such Feet.* n. 126. p. 636.

8. Longitude found; by *Hen. Bond. Sen.* Lond. 1676. *in 4to. A Mistake in that Book is here Corrected.* n. 95. p. 6065. n. 130. p. 774.

9. A Book published by *Mr. Jo. Moxon*'s, describing a new sort of Terrestrial Globes Invented by the E. of *Castlemain.* n. 139. p. 988.

10. The *English Atlas. Oxford*, for *Moses Pitt.* 1680. *Fol.* Ph. Col. n. 1. p. 43.

11. A new *Map* of *England* full 6 Foot Square, wherein *Computed* and *Measured* Miles are entered in Figures; by *Mr. J. Adams.* Ph. Col. n. 2. p. 39. n. 135. p. 886.

12. A Large and Curious *Map* of *Great Tartary*; by *M. Nich. Witsen.* n. 193. p. 492.

# CHAP. VIII.

## Architecture. Ship-Building.

*Stones fit for Building; by .... n. 93. p. 6010.*

I. THERE is a sort of *Gray Freestone* at *Paris* every where on the South side of the River *Sene*, which is of a reasonable Course Greet, and so Soft when first taken out of the *Quarry*, that 'tis drest and hewen with broad sharp Axes almost as easily as dryed Clay; but grows harder and harder in the Air; 'tis very Durable and Exceeding fit for *Building*. The *Portland Stone* is of a fine Chalky Greet, fit for all Curious hewen and Carved Work, though not so fit for Water or Fire. On the Contrary the *Freestone* in *Kent*, of a Whitish Gray Colour, lasts well in Air and Water; the Greet thereof less fine and Chalky than that of *Portland*. The *Derbyshire Freestone*, though it endure the Fiercest Fire, is yet Brittle, and so unfit for fine and curious Workmanship.

*The Choice and Charges of Slate, for covering Houses; by Mr. Sam. Colepress. n. 50. p. 1009.*

II. 1. Take the thin cleft *Stone*, Slat or Shindle, and so knock it against any hard matter, as to make it yield a sound; if the sound be good, and clear, that sort of *Stone* is not crazy, but firm and good, Or

2. If in hewing it does not break before the Edge of the *Sects* (the Hewing Instrument of the Slatters) you may not much doubt of the Firmness of the *Slate*. But

3. If after it hath been exactly Weighed (and the accompt thereof Laid by) it be put, and for 2, 4, or 8 hours left to remain all under Water in a Vessel; and afterwards taken up and wiped very clean with Cloaths, if then it Weigh more than before 'tis of that kind, which imbibes Water, and therefore not so fit to endure any considerable time without rotting the *Lathes* and *Timber*.

4. These *Stones* may be pretty well guessed at, whether they be of a Close or Loose texture, by their colour: For the over blackish Blew is aptest to take in Water; but the Lighter Blew is always the Firmest and closest. To which may be added the Touch; for, a good *Stone* feels somewhat hard and rough; whereas an open Stone feels very smooth, and as 'twere oily.

5. Place your *Stone* Longways perpendicular in the midst of a Vessel of Water (no matter how Shallow the Water be, so it exceed half a Foot depth;) and be sure, the upper un-immersed part of the *Stone* be not accidentally wetted by the hand, or otherwise; and so let it remain a day, or half a day, or less. If it be a good firm *Stone*, it will not draw (as they

speak)

ſpeak) Water above half an Inch above the Level of the Water, and that perhaps but at the Edges only, the parts of which might be ſomewhat looſened in the Hewing. But a bad *Stone* will draw Water up to the very top, be the *Stone*, as long as it will, all over.

As for the *Charge* of *Covering* Houſes with *Slate*, they may be thus computed.

| | sh. | d. |
|---|---|---|
| 1000 of *Efford Small Blew* at the Ships ſide in *Plymouth* harbor | 5 | 6 |
| 1000 of *Efford Large Blew* | 9 | 9 |
| 1000 of *Can Pelmel* | 7 | 0 |
| 1000 of *Small Blew* of other Quarries | 4 | 0 |
| 1000 of *Large Blew* | 8 | 0 |

3000 of *Small Blew*, accompted 2 Tuns in Carriage by Water.
1000 of *Large Blew*, 1 Tun.
3000 of *Small* will cover 1 *Poole* of Work at the 5 pin plain.

Every *Poole* of Work is either 6 Foot Broad and 14 up, on both ſides, or 168 Foot in *Length*, and one in Breadth.

3000 of *Large*, will cover 2 *Pooles* of Plain Work.

| | sh. | d. |
|---|---|---|
| *Hewing* of all ſorts of *Plain Pelmel per* 1000 | 1 | 6 |
| *Pinning per* 1000, 8 *d.* *Pins per* 1000, 8 *d.* | 1 | 4 |

Three Buſhels (*Winchester* meaſure) of good *Lime* will take 6 buſhels of Freſh Water *Sand*, and ſerves to lay on one *Poole* of Work; though much leſs may ſerve the turn.

300 of *Lathes* to every *Poole* of Work.
1000 of *Lath Nails* to every 3000 of *Laths*.

An able workman may { *Lath* one *Poole* of Work / *Lay on* 2000 or more of *Slate* / *Hew* 1500 plain / *Pin* 4000 } by the day.

*Chequer-Work* conſiſts in Angles, Circles and Semicircles, *&c.* which requires no common Skill and time in Hewing and Laying.

It is worthy obſervation, that if a Side wall happen to take wet by the beating of the Weather, or the like, when nothing elſe will cure it, our *Kerſeing* with *Slate* (which is much uſed in the curious *Fronts* of Houſes, eſpecially in Towns) will quickly Remedy it.

We have ſome ſorts, which by the Conjectures of the moſt experienced *Helliers* (or *Coverors* with *Slate*) have continued on Houſes ſeveral Hundreds of Years, and are yet as firm as when firſt put up.

III. The Cuſtom of *Felling Timber* here in the *South* of *England*, differs from that of *Stafforſhire*, in the time of *Felling*, and manner of *Barking*. It is *Felled* here in the *Spring*, as ſoon as the *Sap* is found to be fully up by the Trees putting out, and then *Barked* after the Trees are proſtrate, the *Sap* yet remaining in the Bodies of them: Whereas *there* it is firſt *Bark'd*, (in the *Spring* as *here*) but before it is *Felled*, the Trees yet Living and Standing all the *Summer*, and not *Felled* till the following *Winter*, when the *Sap* is fully in Repoſe.

*The Beſt Time of Felling of Timber; by Dr.* Rob. Plot. in n. 192. p. 455.

In

In the *Spring* Seaſon and ſome time after, All *Trees* are Pregnant and ſpend themſelves (as Animals do in their Reſpective Off-Springs) in the production of Leaves and Fruits, and ſo become weaker than at other times in the Year; their Cavities and Pores being then Turgid with Juices or *Sap*, which (the Trees being *Felled* at that time) ſtill Remain in the Pores, having no manner of Means of being otherwiſe Spent, and there Putrifie; not only leaving the Tree full of thoſe Cavities which render the *Timber* Weak; but *Secondly* Breeding a Worm, as both *Pliny* and *Mr. Evelyn* Teſtifie, that will ſo exceedingly prejudice it, that it becomes altogether unfit for ſtrong Incumbencies, or other Robuſt Uſes. *Thirdly*, All *Timber Felled* at this time of the Year, whether the Juices Putrifie, or otherwiſe ſweat forth, or Dry away, is not only Subject to *Rift* and *Gape*, but will *Shrink* ſo conſiderably, that a Piece of ſuch *Timber* of a Foot Square will uſually *Shrink* in the Breadth $\frac{1}{4}$ of an Inch; than which, ſays *Vegetius*, nothing is more Pernicious if uſed for the *Building* of *Ships*. To which, *Fourthly*, the Firſt and Greateſt *Roman Emperour Julius Cæſar* adds, that though *Ships* may be made of ſuch moiſt *Timber Felled* in the *Spring*, yet they will certainly be *Sluggs*, not near ſo good *Sailers* as *Ships* made of *Timber Felled* later in the Year.

In all which Circumſtances I find moſt of the *Ancients* ſo very well agree, that none of them adviſe the *Felling* of *Timber* for any ſort of Uſe before *Autumn* at ſooneſt; others, not till the *Trees* have born their Fruit, which ſays *Theophraſtus*, muſt always be proportionably later, as their Fruits are Ripe later in the Year; A third Sort, not till *Mid-winter*: not till *November* ſays *Palladius*, nay not till the *Winter Solſtice* ſays the wiſe *Cato*; and then too in the Decreaſe or *Wane* of the *Moon*, between the 15*th* and 23*d.* day of her Age ſays *Vegetius*, or rather according to *Collumella* between the 20*th* and the *New-Moon*. In general, ſays *Theoprhaſtus*, the *Oak* muſt be *Felled* very late in the *Winter*, not till *December*, as the Emperor *Conſtantinus Pogonatus* poſitively aſſerts, the *Moon* too being then under the *Earth*, as 'tis for the moſt part in the Day time in the firſt part of its *Decreaſe*. And the *Felling* of *Oak* within thoſe Limits, they call *Tempeſtiva Cæſura*, *Felling Timber* in Seaſon, which they all unanimouſly pronounce (if thus *Felled*) will neither *Shrink*, *Warp* nor *Cleave*, nor admit of *Decay*, in many Years; it being tough as *Horn*, and the *Whole* Tree in a manner (as *Theophraſtus* aſſerts) as hard and firm as the *Heart*; with whom alſo agrees our Country-man Mr. *Evelyn*, if you *Fell* not *Oak* (ſays he) till the *Sap* is in Repoſe, as 'tis commonly about *November* and *December*, after the Froſt has well Nipped them, the very *Saplings* thus cut, will continue without *Decay*, as long as the *Heart* of the *Tree*.

And the Reaſon of this is given in ſhort by *Vitruvius*, *quia aeris* Hyberni *vis comprimit & conſolidat Arbores*, becauſe the *Winter* Air doth cloſe the Pores and ſo conſequently conſolidates all Trees, by which means the *Oak* (as he and *Pliny* both expreſs it) will acquire a ſort of *Eternity* in its Duration; and much more will it ſo, if it be *Barkt* in the *Spring*, and left ſtanding all the *Summer*, expoſed to the Sun and Wind, as is uſual in *Staffordſhire*, and the Adjacent Counties, whereby they find by long Experience the *Trunks* of

their

their Trees ſo Dried and Hardened, that the *Sappy* part, in a manner, becomes as firm and durable, as the *Heart* it ſelf.

Which way of *Barking* and *Felling* of *Timber*, tho' it were unknown to the Ancients ( as perhaps it is to all the World beſides theſe few Counties ) yet they ſeem not unacquainted with the Rationality of the Practice. The Great *Vitruvius* prefers the *Timber* on the *South-ſide* the *Apennine* ( where it winds about and incloſes *Tuſcany* and *Campania*, and ſtrongly Reflects the conſtant *Heats* of the *Sun* upon it, as it were from a Concave, ) incomparably before that which grows upon the *North-ſide* of the ſame Hill, in the ſhady moiſt Grounds: of which his Opinion he renders us this Reaſon, for that the *Sun* does not only lick up the ſuperfluous Moiſture of the Earth, whence the *Trees* are ſupplied in ſuch Shady Places with too great a Quantity, but in great meaſure *Exhales* the remaining Juices ( after the Production of Leaves and Fruits ) out of the *Trees* themſelves, Rendering the *Timber* of them the more Cloſe, Subſtantial and Durable; which certainly it would do alſo much more effectually, if the *Bark* were taken off in the *Spring* of the Year, as is accuſtomed in *Staffordſhire*, where the People are content to uſe this Method in their Proviſion of *Timber*, though but for private Uſes.

Much rather then ſhould it be done in ſo *Publick* a Concern as the *Building* of *Ships*, where Tough and Solid *Timber* is much more neceſſary than in Ordinary *Buildings*. There is indeed an *Act* of *Parliament*. 1. *Jac.* 1. *Chap.* 22. which forbids *Felling* of *Timber* for Ordinary Uſes ( in conſideration of the *Tan* ) at any other time but between the Firſt of *April* and the Laſt of *June*, when the *Sap* is up and the *Bark* will Run; made on Suppoſition ( I gueſs ) that ſhould they have admitted *Felling Timber* in any other Seaſon, the *Tanners* would have wanted a Supply of *Bark*. To which I readily anſwer, That I fear the *Legiſlators* that preſſed the making that *Act*, were ignorant that the *Bark* might be taken off in the *Spring*, and that the Tree would notwithſtanding Live and Floriſh till the *Winter* following, as I have ſeen many in *Staffordſhire*: So that though the Tree be not *Fell'd* till the *Winter Solſtice*, or *January* following, yet the *Tanner* is not at all defeated of his *Tan*, but has it here in as due Seaſon, as in any of the *Southern Counties*. The *Legiſlators*, I ſay, were ignorant of this, otherways they would never have made an *Act* ſo Pernicious to the whole Kingdom, as *Felling Timber* at this Seaſon is, for the ſake of a few *Tanners*.

But notwithſtanding this Ignorance, yet then they were ſo Wiſe as to except in that *Act* the *Timber* to be uſed in *Building* of *Ships*, which may be *Fell'd* in *Winter*, or any other time; as I am told all the ancient *Timber* remaining in the *Royal Sovereign* was, it being ſtill ſo hard that 'tis no eaſie matter to drive a Nail into it.

'Tis true indeed that the *Barking* and *Peeling* the Tree Standing is ſomewhat more Troubleſome, and therefore ſomewhat more Chargeable, than when they are Proſtrate; and that 'tis likely, People therefore have uſually *Fell'd* their *Timber*, as well for *Shipping* as other Uſes, in the *Spring* of the Year, for the ſake of the more eaſie and cheap *Barking* it only, rather than any thing elſe. 'Tis true too, that *Timber* is harder to *Fell* in the *Winter*, it being now ſo Compact

pact and Firm, that the Ax will not make so great Impression as it doth in the *Spring*, which will also Increase the Price of the *Felling* some small matter, and it's *Sawing* afterwards; but how inconsiderable these things are in comparison of the great good of this manner of *Felling*, I think is Self-Evident.

The Greatest Objection, that I can foresee will be urged *here* in the *South* against this Practice, is, that if the *Timber* be not *Fell'd* till *Mid-winter* or *January*, where it grows in *Copses* and *Woods*, they cannot perhaps Inclose their young *Springs* so soon as some may imagine needful, and therefore will be backward to *Fell* their *Timber* ( so Growing ) at that Season. To which I Answer, that the *Timber* so *Fell'd* in the *Wood* or *Copses*, may be easily carried off before the Second *Spring*, and so the Prejudice small, and the First it must be there, wherever it is *Fell'd*. But secondly, that which will quite remove this inconsiderable Difficulty is, that perhaps it may be Expedient that no *Timber* whatsoever Growing in *Woods* or *Copses*, be at all bought into the *King's Yards*, for that *Timber* Growing in such Shady Places, ond so fenced from the *Sun* and *Wind*, as *Timber* in *Woods* for the most part is, cannot be so good as that which comes from an *exposed Situation*, such as it usually has in *Forrests*, *Parks*, *Hedge-rows* and Open *Fields* : where too it is indifferent at least, if not better, for the Proprietor, that it be *Fell'd* in *Winter* ( when the Grass and Corn is gone ) than in the *Spring* it self; and the Officers assigned for that Purpose may Buy all their *Timber* under such Conditions as to be *Fell'd* in *Winter*, enjoining the Proprietor to take off the *Bark* in the *Spring* in due time, making him some small Allowance for the Trouble he will have in *Peeling* it *Standing*.

*The Difference of* Timber *in Different* Countries, *and* Fell'd *at different* Seasons; *by M.* Ant. Van. Leuwenhock. *n.* 213. *p.* 224.

IV. It is the Common Opinion, that *Timber* which is *Fell'd* in *Winter* is stronger and more Lasting, as being more Close and Firm, than that which is *Fell'd* in *Summer* : But M. *Leuwenhock's* Sentiment is, that there is no Difference, except in the *Bark* and outermost *Ring* of the Wood, which in the *Summer* are Softer, and so more easily Pierced by the *Worm* ; *Wood* consisting of Hollow Pipes, which in the *Summer* and *Winter* both, are full of Moisture, they do not *Shrink* in the *Winter*, and therefore the *Wood* cannot be Closer at one time than another: For otherwise it would be full of Cracks and Clefts. The Sudden and Unexpected *Rotting* of some *Timber*, he conceives to proceed from some Inward Decay in the *Tree* before it was *Fell'd*: having Observed all *Trees* to begin to decay at first in the Midst or *Heart* of the Tree, though possibly the *Tree* may Stand and Grow for near an Hundred Years afterwards, and Increase in Bigness all along.

2. He says, he was once of Opinion, that *Trees* growing in good Ground, but Increasing slowly, were the best and strongest *Timber*; and that these *Trees* which in few Years grew Large, was the Softest and Britlest; the Contrary to which, upon Enquiry of Experienced Workmen, he found to be true, and Instances in an *Elm* of 80 Years Growth, which was 11 Foot in Circumference, and proved Excellent Tough *Timber*.

3. The *Age* of *Trees* is to be known by the Number of *Rings* to be ſeen when the *Tree* is cut a-thwart, in each of which *Rings* is one Circle of large Open *Pipes*; now the fewer of theſe Large *Pipes*, the Stronger the *Timber* is: wherefore by Conſequence thoſe *Trees* that make the Largeſt Growth in a Year, muſt be the Cloſer and Stronger, and therefore thoſe *Trees* that Grow in *Warm Countries* Grow faſteſt, and are the Beſt and Tougheſt *Timber*; which he confirms by *Riga* and *Dantzick Oak*, which is of Slow Growth, and proves Spongy and Brittle *Timber*, whereas the Contrary is Obſervable in *Engliſh* and *French Oak*, which grows Faſter, and is Excellent *Timber*.

*The Bridge at* St. Eſprit *in* France; *by Dr.* Tankred Robinſon *n.* 160 *p.* 584. *Fig.* 215.

V. 1. This Famous *Roman Bridge* at *Pont St. Eſprit* is very Crooked, Bowing in many places, and making ſeveral unequal Angles, eſpecially in thoſe Places, where the Torrent runs ſtrongeſt, as where the *Turret* ſtands. 4. In which Place the Angle is moſt Unequal, and the greateſt; the *Arches* are very Wide, and have their Feet ſecured by two *Pedeſtals* that encompaſs them. Both theſe *Pedeſtals* have their ſeveral Degrees or Ranks of *Jettings* out, like ſo many Rows of *Stairs* or Steps, the Lowermoſt Order puſhing out moſt, the others being Leſs, and going gradually more in; the Second or Uppermoſt *Pedeſtal* is much leſs than the Firſt or Lowermoſt, being Built a little within its Lines of Circumference; 1, 2. Between the Great *Arches* there are *Windows*, or (as it were) ſmall *Arches*; 3. that come down to the very Plane of the Second, or Uppermoſt *Pedeſtal* dividing the Feet of the great *Arches*. From this my Rude Deſcription it appears to me, that the *Romans* have here contrived all poſſible ways to break gradually the mighty Force of the *Rhoſne*, and to render its paſſage eaſie, and inoffenſive to the Feet of the great *Arches*; for here we ſee ſo many ſeveral *Paliſadoes* and *Sluces*, as may be ſufficient to defend this wonderful Fabrick againſt all Storms of the Torrent; the ſeveral Ranks of *Stairs* jetting from the *Pedeſtals* (for the moſt part Triangularly built, and Faced well with Free-ſtone) oppoſing and breaking the Stream ſeverally, I mean, not altogether or at the ſame time, by reaſon of their various Inequalities in Standing Out: in caſe the Flood ſhould ſwell ſo high (as it frequently does) as to cover both the *Pedeſtals*, then the ſmall *Arches*, Dividing the Feet of the great ones, help to convey the Water through, which otherwiſe might endanger the great *Arches*.

*By Dr.* Liſter *ib. p.* 585.

2. That which ſeems the *Foot* of the *Arch* is an *Horizontal Arch* gradually contracted, every Stone being of vaſt Length and Wedge like, laid Level with the Water. This I ſpeak by *Memory*.

*Compared with ſome other* Bridges; *by Dr.*Tank. Robinſon. *n.* 163. *p.* 712.

3. The Stately *Modern Bridge* at *Avignon* hath yielded in many Places to the extream Rapidity, and violence of the *Rhoſne*. Its fall in my opinion may be Aſcribed to three defects. Firſt, it was not ſo Multangular, as that at St. *Eſprit*: Secondly, it wanted in three or four places, the little *Arches* Dividing the Feet of the Great ones, and in thoſe parts it hath ſuffered moſt; for where thoſe uſeful *Sluces* are, there I obſerved the *Bridge* to ſtand ſtill the moſt intire. Thirdly, the *Pedeſtals* (or as you very properly call them *Horizontal Arches*) were not ſo *Geometrically* and exactly Laid, as thoſe of *Pont St. Eſprit*, their *Jettings* out were few, and they not gradually contracted; ſo that the Force of the Stream muſt be the greater upon the *Fabrick*.

Though the *Tyber* be not so swift as the *Rhosne*, yet it is Subject to greater Inundations, as many Inscriptions assure us. No River ever had so many Bridges Built with that Magnificence and Art, as this; and though they were more Pompous, and Rich in Rare Stones, in Sculpture, &c. than that I formerly sent you a draught of from *Montpelier*; yet they had the like Provision for their Security, and Preservation, and their Design was much the same; which may be seen at *Rome* this very day at the old *Pons Milvius* (now *Ponte Nolle*) near the *Via Flaminia*; in the Marble Remains of the *Pons Æmilius* (repaired with Rich Materials by *Antoninus Pius*) on the side of the *Ripa*, or *Trastavere*, near the Root of the *Aventine Hill*, where first the *Pons Sublicius* stood; as also in the *Pons Fabritius* and the *Cestius*, that leads over to the *Insula Tiberina*; in all which there are still very fair marks of the Old *Roman* Structure, and *Design*; and if that prodigious City had not been knockt so oft to pieces by *Barbarous Sackers*, we might have had still as clear proofs from the other *Bridges*, *Viz.* the *Pons Triumphalis*, the *Senatorius*, &c. But *Gothish* and *Northern Torrents* broke all before them.

A Bridge *without any Pillar under it*; *from the Journal of the* Phil. Society *of* Oxford. *ib.* p. 714. *Fig.* 216.

VI. A *Timber Bridge* may be Built 70 *Foot* Long, or somewhat more, without any *Pillar* under it, which may be useful in some Places where *Pillars* cannot be conveniently Built, after this manner; A C, and B O, are Beams 28 Foot long, and A B, is 32 Foot Long. Under the *Angles* are set two Large *Braces* E L, and S R. At each End is a Wall, on which are laid two Beams BH, and AD, each 20 Foot Long; under these are two *Braces* DE, and RH. There may also be *Braces* at the Ends of the *Arches*, that may lie Obliquely cross the *Bridge*. It may be laid with Planks and Railed. Behind the Walls are *Causeys* F D, and H N. The Length of the *Bridge* C M O, is 70 Foot; the Height K M, is 19 Foot.

*An* Aqueduct *near* Versailles. *n.* 171. *p.* 1016.

VII. 1. The *Aqueduct* which is to be made near *Maintenon*, for the Carrying the River *Eure* to *Versailles*, will have in Length 7000 Fathom; 462 whereof will be 35 Fathom and 4 Foot High, the rest will be lower according to the Difference of the Ground; but no less than 5 Foot and 6 Inches High. There will be to the said *Aqueduct* 861 *Arches*, which, where they are Highest, will have 12 Fathom in Breadth, and 8 Fathom in Thickness, diminishing to 14 foot at the Top. The other *Arches* will be lesser in Breadth, as well as Thickness, according to the Nature of the Ground. The said *Aqueduct* will have 15 Inches *Fall* to every Thousand Fathom in Length; so that for the 7000 Fathom, there will be 8 Foot 8 Inches *Fall*. The *River* is to pass by *Maintenon*, *le Parc Espernon*, *Gajeran*, *Rambouillet*, *les Essars*, *le Perrey*, *Cognieres*, and from thence to *Versailles*. There are 14000 Soldiers that Work there, under the command of the *Marquess d' Uxello*, with three Commissarys of War for their Conduct.

*n.* 176. *p.* 1296. 2. A *Magazine* for the Waters upon the Mountain *Montboron* is already Cut, which will have 2200 Perches of Surface (each Perch being 18 *French* Foot) and 12 Foot in depth. In another place much lower, will be another *Magazine*, to receive the Waters of many *Pools*, the most part of which as yet have no Water in them. In the valley of *Buc* will be an *Aqueduct*, the Middle whereof will be raiſed 22 Fathom High, for conveying the Pools of *Sarle*, which its ſaid contain much Water, though there be nothing but Rain to fill them; this *Aqueduct* is 300 Fathom Long, and paſſes through two Mountains which have been cut through upon that Account. The Valley alſo on both ſides of the *Aqueduct* is Raiſed 11 Fathom High to make Paſſages.

An *Aqueduct* alſo is making near the *Tower* of Stone (where the Mills Raiſe the Water) which will now paſs without Force to the Top of the Mountain; and there be part of it Diſtributed into ſeveral very great *Ciſterns*, which are making above *Marli*, for that Place.

The Elevation of the *Aqueduct* of *Maintenon* is now ſet forth at but 2560 Fathom; whereas it was Deſigned to be carried on more than 8000 Fathom, and the Remainder will be made of Earth, which muſt be brought thither: This Opinion prevails, in regard it gives a Quicker Diſpatch, though it may be doubted, it will not be for the Better.

Theſe 2560 Fathoms contain 242 *Arcades*, whoſe *Aperture* is. 6 Fathom and ½, and the *Face* of each *Pillar* Suſtaining the *Arches*, 4 Fathom; there will be then on the ſide of *Maintenon* 33 *Single Arches*, afterwards 71 *Double* ones; (as having one *Arch* upon another) then 46 *Treble* ones; which will generally be 216 Foot 6 Inches High, (*viz.* up to the Floor of the *Channel*) afterwards 72 *Double* ones; then 20 *Single*, which will reach to the *Mound* of *Earth*, that is to be 50 Foot High.

From the Ground up to the *Second Arcade* are 16 Fathom, from the *Second* to the *Third*, or upper *Arcade*, are 14 Fathom, (which *Arcades* are Double in Number to thoſe they ſtand upon) and 6 Fathom 6 Inches more, to the Floor of the *Channel*, which will at leaſt be 6 Foot High, beſides the *Parapet*.

The *Pillars* by the Ground are 8 Fathom Thick, but what with the *Slopes*, and *Shortnings*, which are made in every *Story*, the Top where the *Channel* goes, will be but 20 Foot Broad: There will likewiſe be at each *Pil-* a *Buttreſs* jetting out one Fathom, and two Fathom wide.

The Intelligent Obſerver, though well Skilled in things of this Nature, as being no Stranger to the Writings of the *Antients*, or the *Famous Ruines* and *Remainders* of their *Fabricks* in *Italy*, and other Places, profeſſes himſelf Surpriz'd with the greateſs of this undertaking at *Verſailles*, and *Maintenon*; for the Magnificence of the Deſign, the Number of Labourers, the Exceſſiveneſs of the Expence, and the admirable Beauty of the *Work*.

VIII. Having been lately at *Edgecot* in *Northamptonſhire*, at the Houſe of *Tobias Chancey* Eſq; he ſhewed me in an Ancient Kitchin (now diſuſed) two *Chimneys*, vaſtly large, of *Stone-Work*: Which I took the more Notice of, becauſe of a Peculiar Way of *Arch-Work* in the Front of them; whereby, without the Advantage of a *Diſcharger* of *Timber* (which is uſual, in ſuch Caſes, to defend the *Arch-Work* from being Overburdened,) an *Arch* of maſſy Stone (in each of them) ſuſtains it ſelf at a great Length, though almoſt upon a Flat,

*A very large Stone Chimney with a Peculiar ſort of* Arch-Work; *by Dr.* J. Wallis. *n.* 166. *p.* 800. Fig. 217.

a Flat, being very little Rais'd in the middle. Over this *Arch* (after ſome walling Interpoſed) there is another *Arch* (to defend the former) more Raiſed from the Flat. The Dimenſions of all, I have thought fit here to Subjoyn.

A B. The *Breadth* between the *Jambs*, from inſide to inſide, 18 Foot.

C D. The *Depth* of the *Stones* in the *Lower Arch*, 22 Inches; Locked one into another, with a Crooked joynt.

D E. The *Diſtance* in Walling, between the *Arches*, 2 Foot and 7 Inches.

E F. The *Depth* of the *Stones* for the *Upper Arch*, 15 Inches: with a *Straight Joynt*.

G H. The Place of two vaſt *Tunnels* of Stone.

K. A *Window* between them.

*A New kind of Stairs*; *by* M. Weighelius. *n.* 74. *p.* 2219.

IX. *M. Weighelius* hath lately Invented an odd *Bridge* or kind of *Stairs*, by which a Man ſhall Deſcend, and yet really be Raiſed upward; and going as 'twere upon a Plain, ſhall from a Lower, by gently ſubſiding, arrive to an upper *Story*.

*Preſerving of Ships from being Worm-eaten*; *by* - - - - - - - - *n.* 11. *p.* 190.

X. In the *Indian Seas* there is a kind of ſmall *Worms*, that faſten themſelves to the *Timber* of the *Ships*, and ſo Pierce them, that they take Water every where; or if they do not altogether Pierce them thorow, they ſo weaken the the Wood, that it is almoſt Impoſſible to repair them. Some have Imployed *Deal*, *Hair* and *Lime*, &c. and therewith Lined their *Ships*; but beſides that this does not altogether affright the *Worms*, it retards much the Ship's Courſe. The *Portugals* ſcorch their Ships, in ſo much that in the *Quick Works* there is made a Coaly Cruſt of about an Inch thick. But as this is dangerous, it happening not ſeldom that the whole Veſſel is burnt; ſo the reaſon why the *Worms* eat not thorow *Portugal Ships*, is conceived to be the exceeding hardneſs of the *Timber*, employed by them. There is in *Holland* a Man that pretends to have found an Admirable Secret to *Remedy* this Evil. And a very worthy Perſon in *London*, ſuggeſts the *Pitch*, drawn out of *Sea Coals*, for a good *Remedy* to ſcare away theſe *Noyſome Inſects*.

*An Account of Lead Sheathing*; *by Mr.* J. Bulteel. *n.* 100. *p.* 6192.

XI. Some few Years ſince, *Sir Phil. Howard* and *Major Watſon*, with great Charge and Induſtry found out a new Way, by a Manufacture of our own, to preſerve the *Hulls* of *Ships* from the *Worm*, &c. which is much ſmoother, and conſequently better for *Sailing* and more Cheap and durable than the Way of *Boards*, *Pitch*, *Tar*, *Roſin*, *Brimſtone*, or any *Sheathing* or *Graving* hitherto uſed. The *King* and *Parliament* being Satisfied, upon Examination of the great Benefit that might redound hereby to his *Majeſty* and Subjects in General, for the Inventors Incouragement to make the ſame Publick, were pleaſed almoſt 4 Years ſince, to grant them an *Act* of *Parliament* for the Sole uſe of this their Invention, with Penalty and Prohibition to all others. In Proſecution whereof, Experiments have been made upon ſeveral of his

*Majeſty's*

*Majesty's Ships*, viz. The *Phœnix*, done three Years agoe, has made two Voyages into the *Streights*, &c. and when ſhe was lately taken into the *Dock* at *Woolwich* to be repaired, upon View of the *Maſter Shipwright* and others, her *Sheathing* was found to be in as good Condition, as at the firſt doing: and the Ship ſo Tight during the whole time, that they were forced to heave in Water to keep her Sweet. The *Dreadnought* a Third Rate, done in *June* 1671; the *Henrietta*, *Lion*, and *Mary*, all Three of the Third Rate, and done a Year and an half ſince, being lately laid on Ground at *Sheerneſs* and *Portsmouth*, are found to be all in as good Condition, and the *Sheathing* to continue as firm and as well as at the Firſt doing; as the *Maſter Builder* and *Aſſiſtant* at *Portsmouth* and others have Certified.

The *Bread-Rooms* alſo of ſome of theſe and many other of his *Majeſty's Ships*, have been Lined within, almoſt in the ſame manner the *Sheathing* is without; which has prov'd a great Preſervation of the Bread, as ſeveral of the *Purſers* and *Officers* of the ſaid Ships have Certifi'd; and by Reaſon of its Duration muſt be much cheaper and better than *Tin*, which is ſo lyable to ruſt, or any Way yet uſed.

Alſo the *Lead* it ſelf (which is the Principal thing uſed herein) they make ſo Cloſe preſſed, Smooth, and Equal, and of what Thickneſs or Thinneſs deſired, that great uſe may be made thereof about ſeveral other things relating to *Shipping*.

XII. *A Paper of leſs General Uſe* Omitted *viz.*

Directions for Inquiries concerning *Stones* and other *Materials* for the uſe of *Building*. *n. 93. p. 6010.*

XIII. Accounts of *Books*, Omitted.

1. *Vitruvius* done into *Engliſh*; by Mr. *Chr. Waſe.*
Les dix Livres d' *Architecture* de *Vitruve*, corrigez, & traduits novellement en Francois, avec des Notes & des Figures; par *Claude Perrault. Paris.* 1673. in Fol. *n. 72. p. 2190. n. 112. p. 279.*

2. Cours d' *Architecture*, enſeigné dans l' *Academie Royale* d' *Architecture*, Premiere Partie; par M. *Francois Blondel.* a *Paris.* 1675. in Fol. *n. 122. p. 549.*

3. *Raphaelis Fabretti Urbinatis* de Aquis & *Aquæductibus* Veteris *Romæ*, Diſſertationes tres. *Romæ.* 1680. *in 4to.* *n. 155. p. 466.*

4. Modern Fortification, *&c.* by *Sir Jonas Moor.* 1673. in *8vo.* *n. 95. p. 6071.*

5. Nouvelle Maniere de *Fortifier* les Places; par M. *Blondel. Hague.* 1684. *n. 158. p. 585.*

6. *Marci Meibomii* de Fabrica *Triremium* Liber. *Amſterlodami.* 1671. in 4to. *n. 79. p. 3071.*

7. *Scheeps-Boow en Beſtier*, that is Naval *Architecture* and *Conduct*; by N. *Witſen. Amſterdam.* 1671. in Fol. *n. 77. p. 3006.*

8. L' *Architecture Navale*, avec le *Routier* des *Indes Orientales* & *Occidentales*; par le Sieur *Daſſié* a *Paris.* 1677. in 4to. *n. 135. p. 879.*

# CHAP. IX.

## *Perſpective. Sculpture. Painting.*

*A Perſpective Inſtrument*; by *Sir* Chriſt. Wren. *n.* 45. *p.* 898.
*Fig.* 218.

I. A is a ſmall *Sight* with a ſhort *Arm* B, which may be turned round about, and moved up and down the ſmall *Cylinder* CD, which is ſcrewed into the piece ED, at D: this piece ED, moving round about the *Center* E; by which means the *Sight* may be removed either towards R, or F.

EF, is a *Ruler* faſtned on the two *Rulers* GG, which *Rulers* ſerve both to keep the ſquare Frame SSSS, perpendicular, and by their ſliding through the ſquare holes TT, they ſerve to ſtay the *Sight*, either farther from or nearer to the ſaid Frame; on which Frame is ſtuck on with a little wax the Paper OOOO, whereon the *Picture* is to be *Drawn* by the *Pen* I. This *Pen* I, is by a ſmall braſs-handle V, ſo fixt to the *Ruler* HH, that the Point I, may be kept very Firm, ſo as always to touch the Paper. H H, is a *Ruler*, that is always, by means of the ſmall Strings *a a a*, *b b b*, moved *Horizontally*, or Parallel to it ſelf; at the End of which is ſtuck a ſmall Pin, whoſe head P, is the *Sight*, which is to be moved up and down on the out-Lines of any *Object*.

The Contrivance of the *Strings* is this. The two *Strings a a a*, *b b b*, are exactly of an equal Length. Two Ends of them are faſtened into a ſmall Leaden Weight QQ, which is moved in a Socket on the back ſide of the Frame, and ſerves exactly to countepoiſe the *Ruler* HH, being of equal Weight with it. The other two Ends of them are faſtned to two ſmall Pins HH, after they have been rolled about the ſmall *Pulleys* N; M,M; L,L; K,K; by means of which *Pulleys* if the *Pen* I, be taken hold of and moved up and down the Paper, the *Strings* moving very eaſily, the *Ruler* will always remain in an *Horizontal* Poſition.

The manner of *Uſing* it is this: Set the *Inſtrument* upon a Table, and fix the *Sight* A, at what Height above the Table, and at what Diſtance from the Frame SSSS, you pleaſe. Then looking through the *Sight* A, and holding the *Pen* I, in your hand, move the Head of the Pin P, up and down the out Lines of the Object, and the Point I, will deſcribe on the Paper OOOO, the ſhape of the Object ſo traced.

*A new way of Delineating by Para'lel Viſual Rays, exactly Obſerving the Symmetry*; by *Mr.* St. Clare. *n.* 96. *p.* 6080.
*Fig.* 219.

II. *Parallelogrammum Proſpographicum* eſt ABCD, Stylus Centralis H F, Calamus Deſignator L C, Index K A, ſive Regula oblonga, Plano *Parallelogrammi*, ope clavis Striati ex ære, ſecundum ſpecimen E, ad Angulos Rectos aptata. Huic Regulæ inſiguntur duæ Dioptræ PR, S V; in medio P R, pertuſum

pertuſum eſt foramen O, in medio SV, erigitur Filum perpendiculare Regulæ RA, in cujus medio eſt Globus quidam parvus, per quem, & Foramen O, Radius ad oculum ( quem inter Delineandum non oportet eſt Fixum, ſed Liberum & Solutum ) ab Objecto protenditur.

1. *Animadvertendum*, Radium per Foramen O, & Globulum, protenſum, ſemper fore perpendicularem Plano *Parallelogrammi*, ſive ejus Diametro, quæ eſt Recta Linea extenſa per Stylum Deſignatorium LC, & Centrum fixum HF, & dictum Globulum parvum, in qua Linea ſemper verſatur iſte Globulus, qualiſcunque ſit *Parallelogrammi* motus.

2. *Not.* Planum Deliniatorium ſenſibile, ſuper quod volutatur apex L, Styli Pictorii LC, ad amuſſim Deſcribentis Imaginem ad motum Indicis KA, & in quod infixus eſt Stylus Centralis HF, eſſe QYXT; Planum verò merè Rationale, ſive *Mathematicum*, priori continuum, eſſe $\epsilon\, \delta\, \beta\, \gamma$.

3. *Not.* Omnes Radios ab Objecto per Globulum & Foramen O, protenſos ad Oculum, ( in tot Medii Diaphani punctis, duce Indice KA, collocatum, quot ſunt puncta in ſuperficie Viſibili Objecti deſcribendi quæ ſunt infinita ) ſemper fore ſibi invicem *Parallelos*.

Objicient forſan quidam, in Objectis longè diſſitis Dioptrarum nullum fore uſum. Quid verò illud noſtrâ intereſt, cum ad tollendas tantùm in *Proſopographia* difficultates, quibus hactenus *Scheineri Parallogrammum* laboravit, hæc noſtra Methodus comparata eſt. Sæpius enim expertus ſum ( licet ob hoc non eſt quod ſequiùs de illius *Inſtrumenti* præſtantia ab Artifice ſtatuatur) nequaquam inter partes *Ectypi* in Plano eam eſſe *Symmetriam*, quæ inter partes *Prototypi* diſſiti.

*A Method of caſting Statues of an Extraordinary Thinneſs; by M. John Weichard Valvaſor. n. 186. p. 259. Fig. 220.*

III. I here ſend you my Method of *Caſting Statues* in *Metal*, in Obedience to the Commands of the *Royal Society*; It is as follows. Firſt, I form out of good Clay, that will endure the Fire, and not crack either in Drying or Burning, ſuch a Figure or *Statue* as I deſire to Caſt; when this is well Dry, I make, all over the Figure, little Holes of no great Depth ( but both Size and Depth proportionate to the Bigneſs of the *Statue* ) into which I let ſmall Pieces of *Metal*, and with ſome of the ſame Clay fix them firmly in the Holes; the Uſe of theſe bits of *Metal*, *a a a a*, is to keep the *Core* and *Mould* from touching one the other, or falling together when the Wax runs out; and that they may remain conſtantly in the ſame fixt Poſture. This done, I Scrape away with ſome proper Inſtrument, as much of the Clay in Thickneſs as I deſign for the Thickneſs of my *Statue*; and then Laying it in a Furnace, I Burn the *Core* till it be Red Hot: (by the *Core* I mean always the *Statue* firſt made in Clay.) When it is cold I rub the *Core* all over with that ſort of Earth or Colour, which our *German Potters* Uſe, to Colour the Joints of the Tiles when they ſet *Stoves* of Tiles or ( *Kachel-Ofens*;). This Colour much reſembles *Black Lead* which is uſed to *Deſign* on Paper, and eaſily wipes out with Bread; but it is not the ſame. This Colour I mix with Water and daub all over the *Core*, becauſe the *Metal* is found to run freely upon it. There are other ſubſtances proper for this Uſe, but I have always made uſe of this eſpecially for Thin *Statues*. This done, I lay on upon the *Core* as much Yellow Wax mix-

ed

ed with Pitch or Rosin, as will make the Thickness of the intended Statue, which I form in the Wax with all the Exactness possible.

Here Note, that the Particles of *Metal* mentioned to be set into the *Core*, to keep it at a distance from the *Mould*, must be so set as to fall in with the Surface of the Wax exactly: and that the reason of mixing Pitch or Rosin with the Wax is, because that when it is Burnt out, it makes a great smoak, and that Smoak adhering to the *Mould*, occasions the *Metal* to run more freely; as I have experienced it. Next I put all over upon the Surface of this *Statue* of Wax, little pieces of Wax which I call the little *Channels c c c c c*, (all which must be contrived so as to enter into the Great *Channels d d d.*) This done I cover the *Core* and Wax all over with the same sort of Clay, that will endure the Fire without Cracking; and so I have my *Concave Statue* or *Mould* made. Upon this I lay the great *Channels* marked *d d d d*, both upright and transverse, Formed likewise in Wax, and placed according to Judgment, so as best to receive the Ends of the little *Channels c c c cc*, for the more easie distribution of the *Metal*. These Great *Channels* must all meet at the top of the *Statue*, so as to come out by one Hole, as at E, where the *Metal* is to be poured in; It is also necessary to have a *Channel* or two to let out the Air as the *Metal* enters, as those marked *f f*, and there must be a Hole or two left at the Foot, as *g g*, where the great *Channels* and *Waxen Statue* joyn; and whereat when the *Mould* is Burnt, the Wax as well of the *Statue* as of the *Chanels* may run out. The great *Channels* being thus placed, the *Mould* must be again laid over with the same sort of Clay; (I use constantly to bind about the *Mould* with Iron Wire and then lay on more Clay) and when this *Mould* is well drie, then I Heat it Red-hot; as I did before the *Core*, so now both together.

I Burn the *Core* first, that there may not need so strong a Fire to Burn the *Mould* as will melt the small bits of *Metal*: But for small manageable *Statues* of not above a Foot or two High, they may be both Burnt together, and there is no need of the Holes *g g*, but the *Mould* may be Inverted, and the Wax run out by the *Channels f f*, and E.

The *Mould* being thus Burnt, I stop with the same Clay the two Holes *g g*, and then I bury it in a Pit, and proceed as is usual in *Casting* of *Bells*, and the like: but care must be taken that the *Metal* be very well in Fusion.

If it be a small *Statue* not above a Foot or two High, whose *Mould* may be Managed in ones Hands; then I make me a *Concave Statue* of Wax, of the thickness I desire, and then place upon it all those great and lesser *Channels*, as afore: Which done I put it all together, into a Liquid Substance made of Plaister and Tile or Brick-dust tempered with Water.

If the *Statue* be intended very Thin, then I take Copper, and when it is well in Fusion I mix with it a good quantity of *Zinc*, without observing any certain Proportion of Weight; the more *Zinc* the better the *Metal* Runs. I have sometimes for small and thin *Statues* put in above a third part of *Zinc*. I have found by experience, that this *Mineral* makes the *Metal* run most freely, and gives it a fair Golden Colour.

The

The *Statue* being *Cast*, I take of the *Mould* and cut of all the little *Chanels*; all which both great and small are filled with *Metal*, which may be kept for farther use. In these there is much more *Metal* than in the whole *Statue*; for if the *Statue* be very Thin, there must be more and bigger *Channels*, and so the cheaper the *Statue* the more weighty the *Channels* and the more *Metal* remaining.

To know the Quantity of *Metal* requisite for my intended Work, I take a Lump of the same Mixture of Wax and Pitch, with which I make the *Mould* of my *Statue*; and having weighed it, I make a *Mould* upon it, and *Cast* in the same a Lump of *Metal* of the same Size; which I Weigh and thereby compute the Proportion of the Weight of the *Metal* and *Wax*; and then observing how many Pounds of Wax I use about the *Figure* and *Channels*, I can calculate to a small matter how much *Metal* I need to Melt.

Hitherto I have *Cast* no *Statue* above 9 foot High, but I doubt not but I could, by the same Methods, *Cast* one of any Bigness desired.

*A Descciption of some Simple Colours; by Mr.* Rich. Waller. *n.* 179. *p.*26,30.

IV. 1. *Spanish White* is made of Chalk and Allum burnt together.

2. I take the *Lapis Armenius* to be the *Blew Bice* sold in the shops, for it is Light and Friable; formerly brought out of *Armenia*, now from the *Silver Mines* of *Germany*, called *Melochites*, in High Dutch *Berghblaw*.

*Simple Blews.*

3. *Ultramarine* is made of the Blewest *Lapis Lazuli*, which is freest from Gold-veins, by Calcination.

4. *Smalt* is made of *Zaffer* and *Pot-ashes* Calcined together in a Glass-furnace.

5. *Litmase* or *Litmose*, I suppose the Juice of a Plant.

6. *Indigo*, said by *Pliny* to be brought from *India*; a kind of Mud adhering to the Froath about Reeds, and that when tryed with a Coal, the true burns with a Purple Flame, and smells of the Sea. *Linschoten* says, it is called *Anil*, that it grows in *Cambaia*, and is a Plant like *Rosemary*, which is gathered and dryed, then wetted with fair Water, and beaten to a Mud.

7. *Indian Ink*, its use known to *Pliny*, though not its Composition; which is yet undiscovered, except it should be burnt Rice, as hath been thought.

*Simple Yellows and Reds.*

1. *Ceruse* is the Rust of *Lead*, made by a vaporous Calcination; *Pliny* Writes thus of it in *cap.*34. *Lib.*18. *Ceruse Psimithium* is made in the Plummers shops, of small Plates of *Lead* laid upon a Vessel of strong Vinegar, what falls into the Vinegar is taken out, and dryed in the Sun: and in *Cap.* 6. *Lib.*35. He says it was made at *Rome* of burnt *Marble Flint* quenched in Vinegar.

2. *Masticot* is a kind of Improper *Calx* of *Tin*.

3. *Gutta Gambæ*, or *Cambodia*, the inspissated Juice of a Plant, not well known: it comes from both the *Indies*. Some think it the Juice of *Euphorbium*; others *Scammony*, or *Tithimal*; others *Ricinus*, others refer it to the greater *Catapulia*, *Esula*, or the Flowers of the *Indian Ricinus*, and will have it Coloured with *Turmerick*: as *Scroder*.

4. *Oker*, a kind of Natural Earth. There are two sorts thereof, the one *Native* formerly brought out of *Attica*, now from *Dacia* and *Hungaria*, and from many places of *England*, especially in the *Forrest* of *Dean*: The other a *Factitious* Substance of *Lead* burnt and quenched in Vinegar. In *Pliny's* time it was made of *Rubrica*, or *Reddle* burnt.

5. *Orpiment*, a fat inflammable Mineral, justly ranked amongst Poysons for its extream Corrosive Quality. *Pliny* says, it was dug up in *Syria* on the Surface of the *Earth*; and that the Emperor *Caligula* had hopes of getting *Gold* out of it; wherefore he caused 14 Pounds of it to be tryed, which afforded him very good *Gold*, but in so small a Proportion that he Lost by the Trial.

6. *Umber* is a Native Earth.

7. *Red-Lead*, a Colour unknown to the Ancients, made of *Litharge* or burnt *Lead* by a *Reverberatory Calcination*, or of *Ceruse* put in a Platter over the Fire, which must be continually stirred till it has acquired a *Red-Lead* Colour. Dr. *Charleton de Fos.*

8. *Burnt Oker* is the Common *Yellow Oker* burnt in the open Fire.

9. *Cinnabar*, or *Vermillian*. There are two sorts; *Native*, or the *Minium* of the *Ancients*, which is the Mineral that yields *Quick-silver*; whereof and of *Sulphur* it chiefly consists; it is found in the Mines of *Istria*. This Colour was amongst the *Ancient Romans* used to Sacred Purposes, and on Festivals *Jupiters* Face was Painted therewith, as likewise the Bodies of those that entred in *Triumph*. The *Factitious Cinnabar* is that which we now use, and is made by a Sublimation of *Mercury* and *Sulphur*.

10. *Carmin*, made of *Cochineel.*

11. *Lake*, thought to be an *Arabick* Word: It is made of Flocks Dyed, or Shavings of Scarlet-cloath, or of the *Cochineel* Insect, or else of *Kermes-berries*, their Tincture being extracted with a Lye of Pot-ashes, and then Precipitated with a Solution of Roch-Alom. After the same manner a *Lake* may be made of any Plant or Flower. There is also another sort of *Lake* made of *Gum-lac*, by extracting its Tincture with Urine.

12. *Sanguis Draconis* is the Gum of a Tree, which looks like dryed Blood; 'tis brought out of several Places in the *East-Indies*.

13. *English Reddle*, or *Ruddle*, is found in many places of *England*; amongst the rest near *Witney* in *Oxfordshire*.

14. *Lamp-black*, by *Pliny* thus described: 'Tis made of the Soot of Rosin or Pitch burnt, Houses being built on purpose for it, that keep in the Smoak.

*To make* China *Varnishes; by Dr.* Will. Sherard. *n.* 262. *p.* 525.

V. This way of making several *China Varnishes* was first sent from the *Jesuits* in *China* to the *Great Duke* of *Tuscany*.

Take of *Crude Varnish* 60 Ounces, Ordinary Water 60 Ounces, mix them well together till the Water disappears, afterwards put this matter into a Wooden Vessel 5 or 6 Palms long, and 2 or 3 broad, Mix them with a Wooden *Spatula*, for a Whole Day in the *Summer's* Sun, and for two in the *Winter*. It is afterwards kept in Earthen Vessels with a Bladder over it; and Cool, this is the *Varnish prepared in the Sun.*

*Boyling the Oyl of Wood.*

Take 20 Ounces of the *Oyl*, called *Oyl of Wood*, of that of the *Fruit* 10 Drams, Give them 5 or 6 Boyls, till it comes to be a little *Yellow*. Let it Cool, and put to it 5 Drams of *Quick-lime* powdered.

*To give the first Grounds call'd* Camiscia.

Take *Swines Blood* and *Quick-lime* powdred, mix them well, lay this Mixture on the *Wood*, and when it is dry, Smooth it with Pumice-Stones.

Take

Take of the *Varnish prepared in the Sun* 60 Ounces, *Stone-black Allum* (supposed to be a sort of *Copperas*) dissolved in a little Water, 3 Drams, 70 Drams of *Lamp-Oyl*, call'd by the *Portuguess Azeite da Candea.* It is prepared in a Wooden Vessel as the *prepared Varnish*, observing to put in the *Lamp-Oyl* at twice. *To make Black Varnish.*

Take of the *Oyl of Wood* Crude (called by the *Portuguese Azeite de Pao*) 40 Drams, of the *Lamp-Oyl*, called *de Candea*, Crude 40 Drams, it is prepared in the Sun in a Wooden Vessel as the *prepared Varnish.* *Pitch-Colour'd Varnish.*

Take 10 Drams of *Cinnabar*, 20 Drams of *Varnish prepared*, a little *Oyl de Candea* or *Lamp-Oyl*, mix them well. *Red Varnish.*

Take of the *Yellow Colour* 10 Drams, 30 Drams of *prepared Varnish*, with some *Lamp-Oyl.* *Yellow Varnish.*

Take of the *Red Varnish* 10 Drams, of the *Black Varnish* 4 Drams, mix them well. *Musk-Colour'd Varnish.*

VI. M. *Colbert*, being pleased some while since to visit the *Academy Royal* for the Improvement of *Painting* and *Sculpture*, expressed himself to this effect, that he thought it proper from time to time that the Works of the most *Excellent Painters* should be *Examined*, and such observations made thereon as would Inform others wherein the *Perfection* of a *Picture* consists. Which hath been ever since practised among them, as the best Means to carry the *Art* of *Painting* to its Highest Perfection; such an *Examen* of the *best Pictures* disclosing many Secrets of that Art, for which there are no Rules, and opening a Door to debate many Important Questions, hitherto not treated of. *An Examen of* Pictures *propos'd; by M.* Colbert. *n. 47. p. 953.*

VII. Here is a Man who makes more lively Counterfeits of Nature in *Wax*, then ever I yet saw in Painting, having an extraordinary Addresſs in *Modeling* the *Figures*, and *Mixing* the *Colours* and *Shadows*; making the Eyes so Lively, that they Kill all things of this Art I ever beheld. *Wax-Work, and a New Kind of Maps in* Low-Relievo, *in* France; *by* . . . *n. 6. p. 99.*

I have also seen a New kind of *Maps* in *Low-Relievo*, or Sculpture; for Example the Isle of *Antibe*, upon a Square of about 8 Foot, made of Boards, with a Frame like a *Picture*: There is Represented the Sea, with Ships and other Vessels Artificially made, with their Canons and other Tackle of Wood fixed upon the Surface after a New and most admirable manner. The Rocks about the Island exactly Form'd, as they are upon the Natural Place; and the Island it self wi h all its Inequalities and Hills and Dales; the Town, the Forts, the little Houses, Plat-form, and Canons mounted; and even the Gardens, and Plat-forms of Trees, with their Green Leaves standing upright, as if they were growing, in their Natural Colours; in fine, Men, Beasts, and whatever you may Imagine to have any Protuberancy above the Level of the Sea. This New Delightful and most Instructive *Form* of a *Map*, or Wooden Country, you are to look upon either Horizontally, or side Long, and it affords equally a very pleasant Object.

VIII. Whether the Way mentioned by *Kircher* in his *Mundus Subterraneus* will succeed or not, is much doubted by some Experienced Men: But 'tis certain *To Colour Marble; by* . . . . . . *n. 7. p. 125.*

 tain

tain that a *Stone-cutter* in *Oxford*, Mr. *Bird*, hath many Years since found out a way of doing the same thing, in effect, that is there mentioned; and hath practised it for many Years. That is, he is able so to apply a *Colour* to the Outside of *Polished Marble*, as that it shall Sink a considerable *Depth* into the *Body* of the *Stone*; and there Represent like *Figures* or *Images* as those are on the Outside; Deeper or Shallower, according as he continues the Application a longer or lesser while.

*An Extraordinary Tincture given to a Stone, by Dr.* Salomon Reisel. *n.* 179. *p.* 22.

IX. *Aurifaber Stutgardianus*, qui & Gemmis & Metallicis Typis Nummorarum Cudendorum Insculpendis Artificiosus est, Nomine *Christophorus Muller*, *An.* 1685. Aurum Aqua-Regis solutum, Oleo Tartari præcipitatum atque edulcoratum, quod *Aurum Fulminans* dicunt, dum in Scutella, quam Maturellam vocant, ex *Lapide Chalcedonico* Coloris unici Pellucidi *Onychini*, seu Cornei, vitro pro Fusione præparato Rubro mixtum & Aqua Fontana Imbutum tereret, ad facienda Encausta seu *Smalta*; de quibus *Ant. Nerius*, vertente *Andrea Frisio*, egit *Lib.* 6. *Artis Vitrariæ*; invenit iterato tertium eodem Labore, quod Color Pulveris istius Puniceus, qui per Dies aliquot siccatus in Vasculo manserat, quousque inter terendum etiam ad Marginem effluxit, relictis tamen puris hinc inde Spatiis, *Onychini* Coloris, durissimam hanc *Gemmam*, quæ Limam spernit, ita profunde penitraverit, non tantum in Scutella, sed & ipso Pistillo, & distrinxerit Maculis atque Circulis sat ordinatè ductis, ut Color hic neque simplici Aqua, neque Lixivia, vel acriori alio Liquore potuerit delere, & quidem sine Politura Elegantioris Detrimento. Talis itaque Tinctura per repetitas Trituras dicti Pulveris tentata denuo aliquoties, in similis Coloris alio Vasculo, neque vero apparuit postea ut ante unquam. Sed hoc imprimis circa Tinctionem hujus Vasculi Observandum est; quod secundum Texturam *Gemmæ*, tam nudo quam armato Oculo, in Tincta Interna, & Sincera Externa parte Vasis, notentur Fibræ seu ductus Circulares; juxta quos Bracteis Succi Lapidei Novi per Intervalla Impositis, in ejusmodi Molem excrevisse credendum est; uti *Bezoar* aliique Lapides Laminis super accrescentibus augentur, & Ligna, in quorum ultimorum Trunco Circuli seu Annuli designant succi Annui Numerum & Incrementa: adeo ut hic Purpureus ille Color Lineis pallidioribus & obscurioribus, prout vel densiores vel rariores Poras molliorem vel duriorem Texturam offendit, Circulares Ambitus circa Verticem aliquam, veluti circa Medullam seu Cor ut appellant, aut Granum aut Paleam in aliis Lapidibus & Lignis, signaverit; intermistis quoque hinc inde Maculis & Spatiis obscurioribus. Veluti Illust. *Boyle*, *Specim. de Orig. & Virt. Gemmarum*, Sect. 1. p. 22, 23. in *Adamante* & *Granatis* acies & commissuras tenuium Bractearum aut Planorum observavit; quod *Granum* Artifices, seu planam Contexturam non dissimilem Fissilitati Ligni, vocant.

Jam vero Tingi posse quoque *Marmora* & *Alabastra* & *Ossa* per Lixiviatos & acres Succos, hinc inde scriptum est: quod fortassis de *Gemmis* sperandum est, quando *Rob. Boyle Cit.* Sect. 2. p. 123. ex iis Tincturam manifestam extractam esse scribit, *alibi* p. 43. & 190. per Vapores Minerales tinctos esse *Crystallos Petrosos*, atque *p.* 45. ipsum *Sapphirum* per Vapores subterraneos.

Cum

Cum denique ex obſervatione noſtra manifeſtum ſit, revera *Tinctam* eſſe Gemmam *Chalcedoniam*, quamvis fortuito acciderit, neque repetito Proceſſu ſimile quid evenerit, merebitur tamen Meditationem, an ex Aſtrorum Fluxu aliave abſcondita potius vi venerit, & Tentamen, an ex Mixtura Salium & Succorum Acrium poſſit imitando produci ejuſmodi Tinctura, & quidem ſine Igne, ut Splendor & Pelluciditas Gemmæ non deſtruatur, Durities autem maneat, adeoque ipſa Gemmæ pretioſitas non tantum ſervetur, ſed & per Tincturam novam creſcat.

X. *Papers* Omitted.

1. A Deſcription of *Scheiner's Stereographick Parallelogram*, and it's *Imperfections* conſidered; by Mr. *J.St.Clare*. vid. ſup. Sect. II. n. 96. p. 6085.

2. A Table of *Simple* and *Mixt Colours* in *Latin*, *Greek*, *French*, and *English*: with a *Specimen* of each *Colour* prefixt to its proper *Name*; by *Mr. Rich. Waller*. n. 179. p. 24, 29.

XI. *Accounts of Books* Omitted. n. 21. p. 383.

1. Entretiens ſur les Vies & ſur les Ouvrages des plus excellens *Peintres*, *Anciens* & *Modernes*; par M. *Felibien*. n. 39. p. 784.

2. An *Idea* of the *Perfection* of *Painting*: Originally written in *French* by *Roland Freart* Sieur de *Cambray*; and rendred *Engliſh* by *J. Evelin*, Eſq; F. R. S. *Lond.* 1668. *in* 8*vo*. n. 47. p. 954.

3. A General *Idea* of the *Art* of *Painting*, and a Relation of Seven *Conferences* held at *Paris* in the *Academy Royal* for the Improvement of the *Arts* of *Painting* and *Sculpture*. n. 86. p. 5048.

4. Optique de *Portraiture* & *Peinture*, contenant la *Perſpective Speculative* & *Pratique* Accomplie, &c. par *Gregoire Huret*, de l' *Academie Royale* de *Peinture* & *Sculpture*. a *Paris*. 1670. *in Fol.*

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CHAP.

# CHAP. X.

## *Musick.*

Of the Trembling of Consonant Strings; by Dr. Wallis. n. 134. p. 839.

I. IT hath been long since observed, that if a *Viol-string*, or *Lute-string*, be touched with the Bow or Hand, another *String* on the same or another Instrument not far from it, (if an *Unison* to it, or an *Octave*, or the like) will at the same time *Tremble* of its own accord. But I can now Add; that not the Whole of that other *String* doth thus *Tremble*, but the several Parts severally, according as they are *Unisons* to the Whole, or the Parts of that *String* which is so struk. For Instance, supposing A C, to be an upper *Octave* to α γ, and therefore an *Unison* to each Half of it, stopped at β. if, while α γ. is *Open*, A C. be struck; the two Halves of this other, that is α β, and β γ, will both *Tremble*; but not the Middle point at β. Which will easily be observed, if a little bit of Paper be lightly wrapt about the *String* α γ, and removed successively from one end of the *String* to the other. (Fig. 221.)

In like manner, if A D, be an Upper *Twelfth* to α δ, and consequently an *Unison* to its three parts equally divided in β, γ; if α δ. being *Open*, A D. be struck, its three parts α β, β γ, γ δ, will severally *Tremble*, but not the Points, β, γ. In the like manner, if A E, be a *double Octave* to α ε; the four Quarters of this will *Tremble*, when that is struck, but not the Points β, γ, δ. So if A G, be a *Fifth* to α η; and consequently each Half of that Stopped in D, an *Unison* to each Third part of this stopped in β, γ; while that is struck, each part of this will *Tremble* severally, but not the Points β, γ; and while this is struck, each of that will *Tremble*, but not the Point D. The like will hold in lesser *Concords*; but the less remarkably, as the Number of Divisions encreases. (Fig. 222. Fig. 223. Fig. 224.)

This was first of all (that I know of) discovered by Mr. *Will. Noble* M. A. of *Merton Colledge*; and by him shewed to some of our *Musicians* about three Years since; and after him by Mr. *Tho. Pigot* A. B. of *Wadham Colledge*, without knowing that Mr. *Noble* had discover'd it before. I add this further (which I took Notice of upon Occasion of making Tryal of the other,) that the same *string*, as α γ, being struck in the midst at β, (each part being *Unison* to the other) will give no *Clear Sound* at all; but very confused. And not only so (which others also have observed, that a *String* doth not sound clear if struck in the midst;) but also if α δ, be struck at β, or γ, where one part is an *Octave* to the other; and in like manner, if α ε, be struck at β, or δ; the one part being a *double Octave* to the other. And so if α ζ, be struck in γ, or δ; the one part being a *Fifth* to the other; and so in other like *Consonant* Divisions; but still the less remarkable as the Number of (Fig. 225.)

Divi-

Divisions encreaseth. This and the former I judge to depend upon one and the same Cause; *viz.* the Contemporary Vibrations of the several *Unison* Parts, which make the one *Tremble* at the Motion of the other: But when Struck at the respective Points of Divisions, the Sound is Incongruous, by Reason that the Point is Disturbed which should be at Rest.

A *Lute-string* or *Viol-string* will also thus answer to a *Consonant Note* in *Wind Instruments*: But not so remarkably to the *Wire-strings* of an *Harpsicord.* And we feel the Wain-scot Seats, on which we sit or lean, to *Tremble* constantly at certain Notes on the *Organ* or other *Wind Instruments*; as well as at the same Notes on a *Base-Viol.* I have heard also (but cannot aver it) of a Thin fine *Venice-Glass* cracked with the strong and lasting sound of a *Trumpet* or *Cornet* (near it) sounding an *Unison* or a *Consonant Note* to that of the *Tone*, or *Ting*, of the *Glass*.

Concerning these *Phœnomena*, an Exquisite Solution is given by Dr. *Narcissus Marsh* in Dr. *Plot's Natural History of Oxfordshire.* n. 135. p. 879.

*The Defects of the* Trumpet, *and* Trumpet-Marine; *by Mr.* Fran. Roberts. n. 195. p. 559.

II. The *Extent* of the *Trumpet* cannot be strictly Determined; it reaches as *High* as the Strength of the breath can force it: But by considering its *Notes* within the *ordinary compass* of the *Scale* of *Musick* (from *Double C-fa-ut* to *C-sol-fa in alt*) the Nature of the Higher Notes will plainly appear. These are all set down in the Table; only take notice that the *Prickt Notes* are imperfect, not exactly in *Tune*, but a little *Flatter* or *Sharper* than the places where they stand, according as *f* or *s* is set over them. Fig. 226.

Here we may make two Inquiries.

1. Whence it comes to pass that the *Trumpet* will perform no other *Notes* (in that *compass*) but only those in the Table, which are usually called by *Musicians Trumpet-Notes*.

2. What is the Reason that the 7th 11th 13th and 14th *Notes* are out of *Tune*, and the others exactly in *Tune*.

In this matter, we may receive some Light from the *Trumpet-Marine*, an Instrument, though as unlike as possible to the *Trumpet* in its frame, one being a *Wind-Instrument*, the other a *Monochord*, yet has a wonderful agreement with it in it's Effect.

The *Sound* is so like as not to be easily distinguisht by the nicest Ear, and as it performs the very same *Notes*, so it has the same Defects as a *Trumpet*, for if the *String* be Stopt in any part but such as produces a *Trumpet-Note*, it yields a harsh and uncouth (not a *Musical*) *Sound*.

Let us therefore proceed to our first Inquiry, and examine what is the Reason that the *Trumpet-Marine* will perform no other but the *Trumpet-Notes*. It is a known *Experiment* of two *Unison Strings*, that Striking one of them *Moves* the other, which probably proceeds from hence, that the Impulses of the Air which are made by one *String*, do more easily set another in Motion which lies in a disposition to have its *Vibrations Synchronous* to them, than a Third, whose Motion would be cross.

We may improve this a little farther, by observing that a *String* will move not only at the Striking of an *Unison*, but an 8th or 12th, though after a different manner.

Fig. 227. If an *Uniſon* is Struck, it makes one intire Vibration in the whole *String*, and the Motion is moſt ſenſibly in the midſt at *m*, for there the Vibrations take the greateſt ſcope.

Fig. 228. If an 8th is Struck, it makes two Vibrations; and then the point *m*, is in a manner Quieſcent, and the moſt ſenſible Motion at *n*, *n*.

Fig. 229. if a 12th be ſtruck, then it makes three Vibrations; and the greateſt Motion at *q*, *m*, *q*, and hardly to be perceived at *p*, *p*. So that in ſhort this *Experiment* holds when any *Note* is Struck which is an *Uniſon* to ſome *Aliquot part* of the *String*, as in the former Example an 8th is *Uniſon* to *Half* the *String*, and a 12th to a *Third* part of it.

In this Caſe, ( the Vibrations of the Equal parts of a String being *Synchronous*) there is no contrariety in their motion to hinder each other, whereas it is otherwiſe if a *Note* is *Uniſon* to S, that does not divide the String into

Fig. 230. equal parts, for then the Vibrations of the remainder *r*, not ſuiting with thoſe of the other parts, immediately make a Confuſion in the whole.

Now in the *Trumpet-Marine*, you do not Stop cloſe as in other *Inſtruments*, but touch the *String* gently with your Thumb, whereby there is a mutual concurrence of the upper and lower part of the String to produce the ſound. This is ſufficiently evident from that, that if any thing Touches the String below the Stop, the Sound will be as effectually ſpoiled as if it were laid upon that part which is immediately Struck with the *Bow*. From hence therefore we may collect, that the *Trumpet-Marine* yields no *Muſical* Sound, but when the Stop makes the upper part of the String an *Aliquot* of the Remainder, and conſequently of the Whole : otherwiſe as we juſt now remarked, the Vibrations of the parts will croſs one another, and make a ſound ſuitable to their Motion, altogether confuſed.

Now that theſe *Aliquot parts* are the very Stops which produce the *Trumpet-Notes* ſhall be plainly ſhown in the treating of the ſecond Inquiry, *viz.* What is the reaſon that the 7th. 11th. 13th. 14th. *Notes* are out of *Tune*; and the reſt exactly in *Tune*.

All Writers of the Mathematical Part of *Muſick* agree

| | | | |
|---|---|---|---|
| That by *Shortning* a String | Half | the Sound is *Raiſed* | an *Eight* |
| | a Third part | | a *Fifth* |
| | a Fourth | | a *Fourth* |
| | a Fifth | | a *Sharp Third* |
| | a Sixth | | a *Flat Third.* |

From this Foundation all the other *Notes* are derived. The *Flat* and *Sharp Sixth* are to be the *Flat* and *Sharp Third* to the *Fourth*, and the *7th* the like to the *Fifth*: the *Second* to be a *Fifth* to the *Fourth* below, &c. By this Rule let us examine what *Notes* a *Monochord* fretted in its *Aliquot parts* will produce.

Fig. 231. Suppoſe the *Monochord* F, to conſiſt of 720 parts, and its Tone *Double* C-*fa-ut*, the firſt *Note* in the *Table*; then Half of it will be 360, and a Third part 240, &c.

Now

Now I ſay, *Fretting*, ( or *Stoping* with the Thumb ) at 360 muſt produce C-*fa-ut*; becauſe 360 being Half 720, the Sound will Riſe an *Eight* from *double* C *fa-ut*. Again 360 being C-*fa-ut*, 240 muſt make G-*ſol-re-ut*, the Third Note in the Table; becauſe 240 being juſt a Third-part leſs than 360, the Sound will Riſe a *Fifth* from that Note. After the ſame manner proceeding Step by Step it will be evident that,

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 180 | which is leſs than | 240 | by juſt | a Fourth | produces | C.-*ſol-fa-ut* the fourth | | Note in the Table. |
| 144 | | 180 | | a Fifth | | E-*la-mi* | fifth | |
| 120 | | 144 | | a Sixth | | G-*ſol-re-ut* | 6th | |
| 90 | | 180 | | Half | | C-*ſol-fa* | 8th. | |
| 80 | | 120 | | a Third | | D-*la-ſol* | 9th | |
| 72 | | 90 | | a Fifth | | E-*la* | 10th | |
| 60 | | 90 | | a Third | | G-*ſol-re-ut* | 12th | |
| 48 | | 60 | | a Fifth | | B-*fa-bi-mi* | 15th | |
| 45 | | 90 | | Half | | C-*ſol-fa* | 16th | |

By the ſame Reaſon,

| | | | | | | |
|---|---|---|---|---|---|---|
| 100 | which is leſs than | 120 | by juſt | a 6th | produces | B-*fa-bi-mi Flat*, |
| $67\frac{1}{2}$ | | 90 | | a 4th | | F-*fa-ut* |
| 54 | | $67\frac{1}{2}$ | | a 5th | | A-*la-mi-re* |
| 50 | | 100 | | Half | | B-*fa-bi-mi Flat*. |

And Conſequently,

| | | | | | |
|---|---|---|---|---|---|
| $102\frac{6}{7}$ the 7th | *Note* in the Table is a little. | *Flatter* | then | B-*fa-bi-miFlat*, |
| $65\frac{5}{11}$ | 11th | | *Sharper* | | F-*fa-ut*, |
| $55\frac{5}{13}$ | 13th | | *Flatter* | | A-*la-mi-re*, |
| $51\frac{3}{7}$ | 14th | | *Flatter* | | B-*fa-bi-miFlat*. |

Which anſwers the Second Inquiry.

Now to apply this ( in a few words ) to the *Trumpet*, where the *Notes* are produced only by the Different Force of the *Breath*; it is reaſonable to imagine that the ſtrongeſt Blaſt *Raiſes* the Sound by breaking the Air within the *Tube* into the ſhorteſt Vibrations, but that no *Muſical* Sound will ariſe unleſs they are ſuted to ſome *Aliquot part*, and ſo by Reduplication exactly meaſure out the whole Length of the *Inſtrument*, as in *Fig.* 229. for otherwiſe a Remainder will cauſe the ſame Inconvenience in this Caſe, as in *Fig.* 230. To which if we Add that a *Pipe*, being ſhortned according to the Proportions we even now diſcourſed of in a *String*, *Raiſes* the Sound in the ſame Degrees, it renders the Caſe of the *Trumpet* juſt the ſame with the *Monochord*.

For a *Corollary* to this Diſcourſe, we may obſerve that the Diſtances of the *Trumpet Notes*, Aſcending continually decreaſed in proportion of $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, *in infinitum*; for, The { Second / Third / Fourth, &c. } Note in the Table, differs from the { First / Second / Third, &c. } by { $\frac{1}{2}$ / $\frac{1}{3}$ / $\frac{1}{4}$ } of the *String*, &c.

*The Division of the* Monochord; *by Dr.* J. Wallis. n. 238. *p.* 80.

III. Any *String* or *Chord* of a *Musical Instrument Open* (or at it's full length) will Sound (what we call) an *Octave* (or *Diapason*) to that of the same *String* stopt in the Middle, or at half it's Length. Hence it is that we commonly assign, to an *Octave*, the *Duple* Proportion (or that of 2 to 1) because such is the Proportion of Lengths (taken in the same *String*) which give those Sounds. And (upon a like Account) we assign to a *Fifth* (or *Diapente*) the *Sesqui-alter* Proportion (or that of 3 to 2.) And to a *Fourth* (or *Dia-tesseron*) the *Sesqui-tertian* (or that of 4 to 3.) And to a *Tone* (which is the Difference of a *Fourth* and *Fifth.*) The *Sesqui-octave* (or that of 9 to 8:) Because Lengths (taken in the same *String*) in these Proportions, do give such Sounds.

And (universally) whatever Proportion of Lengths (taken in the same *String* equally stretched) do give such and such Sounds; such Proportions (of Gravity) we assign to the Sounds so given.

But when an *Eight* (or *Octave*) is said (in common Speech) to consist of 12 *Hemi-tones*, or 6 *Tones*; this is not to be understood according to the Utmost Rigour of *Mathematical* Exactness, (of such 6 *Tones*, as what they call the *Diazentick Tone*, or that of *la*, *mi*, which is the Difference of a *Fourth* and *Fifth*;) but, as exact enough for common Use. For 6 such *Tones* (that is, the Proportion of 9 to 8, 6 times repeated) is somewhat more than that of an *Octave* (or the Proportion of 2 to 1.): And, consequently, such an *Hemi-tone*, is somewhat more than the Twelfth-part of an *Eight*, or *Octave*, or *Diapason*. But the Difference is so little, that the Ear can hardly distinguish it: And therefore (in common Speech) it is usual so to speak.

And, accordingly, when we are directed to take the Lengths (for what are called the 12 *Hemi-tones*) in *Geometrical* Proportion, it is to be understood (not, to be so in the utmost Strictness, but) to be accurate enough for common Use; for placing the *Frets* on the Neck of a *Viol*, or other *Musical Instrument*; wherein a greater Exactness is thought not necessary. And this is very convenient, because (thus) the Change of the *Key* (upon altering the Seat of *mi*) gives no new Trouble, for this doth indifferently serve any *Key*; and the Difference is so small, as not to offend the Ear.

But those who choose to treat of it with more Exactness, go this way to work.

Presupposing the Proportion for an *Octave* (or *Dia-pason*) to be that of 2 to 1; they divide this into two Proportions; not just Equal (for that would fall upon *Surd* Numbers, as $\surd$ 2 to 1;) but near equal (so as to be expressed in small Numbers.) In order to which, instead of taking 2 to 1, they take (the Double of these Numbers) 4 to 2; (which is the same Proportion as before;) and interpose the Middle Number 3. And of these three Numbers, 4, 3, 2, that of 4 to 3, is the Proportion of a *Fourth* (or *Dia-tesseron.*) And that of 3 to 2, the Proportion for a *Fifth* (or *Dia-*

*pente.*)

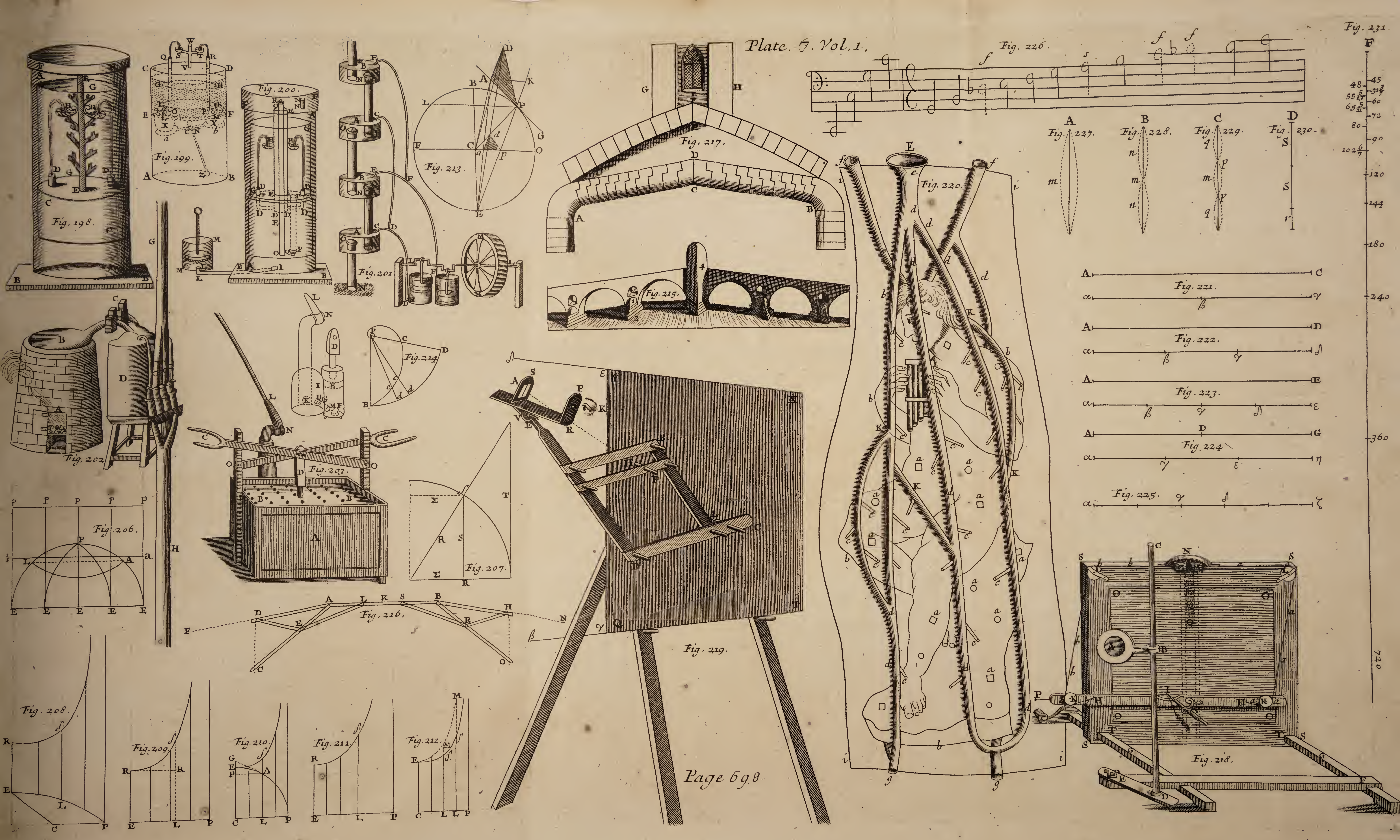
Plate. 7. Vol. 1.
Fig. 198.
Fig. 199.
Fig. 200.
Fig. 201
Fig. 202.
Fig. 203.
Fig. 206.
Fig. 207.
Fig. 208.
Fig. 209.
Fig. 210.
Fig. 211.
Fig. 212.
Fig. 213.
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Fig. 215.
Fig. 216.
Fig. 217.
Fig. 218.
Fig. 219.
Fig. 220.
Fig. 221.
Fig. 222.
Fig. 223.
Fig. 224.
Fig. 225.
Fig. 226.
Fig. 227.
Fig. 228.
Fig. 229.
Fig. 230.
Fig. 231.
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*pente.* ) And theſe two put together, make up that of an *Octave* ( or *Diapaſon,* ) that of 4 to 2, ( or 2 to 1. ) And the Difference of thoſe two, that of a *Tone*, or 9 to 8. As will plainly appear in the ordinary Method of *Multiplying* and *Dividing Fractions.* That is, $\frac{4}{3} + \frac{3}{2} = \frac{4}{2} = \frac{2}{1}$. And $\frac{4}{3}) \frac{3}{2} (\frac{9}{8}$.

Thus in the common *Scale* (or *Gam-ut*) taking an *Octave*, in theſe Notes, *la, fa, ſol, la, mi, fa, ſol, la* ; ſuppoſe, from E to *e* ( placing *mi*, B-*fa-b-mi* ; which is called the *Natural Scale* ; ) the Lengths for the Extremes *la, la,* an *Octave*, are as 2 to 1, or 12 to 6. Thoſe for *la, la,* ( in *la, fa, ſol, la,* ) or *mi, la,* ( in *mi, fa, ſol, la,* ) a *Fourth,* as 4 to 3, or 12 to 9, or 8 to 6. Thoſe for *la, mi,* ( in *la, fa, ſol, la, mi,*) or *la, la,* ( in *la, mi, fa, ſol, la,*) a *Fifth*, as 3 to 2, or 12 to 8, or 9 to 6. Thoſe for *la, mi*, the *Diazeutick-Tone* ( or Difference of a *Fourth* and *Fifth,* ) as 9 to 8. So have we for thoſe four Notes *la, la, mi, la,* their Proportionate Length in the Numbers 12, 9, 8, 6.

Then if we proceed in like manner, to Divide a *Fifth* ( or *Dia-pente* ) *la, fa, ſol, la, mi,* or *la, mi, fa, ſol, la,* or the Proportion of 3 to 2, into near Equals, ( taking double Numbers in the ſame Proportion, 6, 4 ; and interpoſing the middle Number 5 ; ) of theſe three Numbers, 6, 5, 4 ; that of 6 to 5, is the Proportion of a *Leſſer Third,* (called a *Tri-hemitone*, or *Tone* and half, ) as *la, fa,* ( in *la, mi, fa.* ) And that of 5 to 4, is the Proportion of the *Greater Third,* ( commonly call'd a *Ditone*, or two *Tones,* ) as *fa, la,* (in *fa, ſol, la,*) which two put together make a *Fifth,* as 3 to 2 ; that is $\frac{6}{5} \times \frac{5}{4} = \frac{6}{4} = \frac{3}{2}$; And their Difference is, as 25 to 24 : That is $\frac{6}{5}) \frac{5}{4} (\frac{25}{24}$. So have we for theſe 3 Notes *la, fa, la,* their Proportionate Lengths in Numbers, as 6, 5, 4.

In like manner, if we divide a *Ditone,* ( or *Greater Third,* ) as *fa, la,* ( in *fa, ſol, la,* ) whoſe Proportion is as 5 to 4, ( or 10 to 8, ) into two near Equals ( by help of a middle Number 9 ; ) then have we ( in theſe three Numbers 10, 9, 8, ) that of 10 to 9, for ( what they call ) the *Leſſer Tone* : and that of 9 to 8, for ( what they call ) the *Greater Tone*.

But, whether *fa, ſol,* ſhall be made the *Leſſer* ( as 10 to 9, ) and *ſol, la,* the *Greater,* ( as 9 to 8 ; ) or, This the *Leſſer,* (as 10 to 9,) and That the *Greater,* (as 9 to 8,) or ſome time, This, ſome time That, as there is Occaſion, ( to avoid what they call a *Schiſm* ; ) is ſomewhat indifferent. For, either way, the Compound will be, as 5 to 4 ; and the Difference (which they call a *Comma,* ) as 81 to 80. That is $\frac{9}{8} \times \frac{10}{9} = \frac{10}{9} \times \frac{9}{8} = \frac{10}{8} = \frac{5}{4}$. And $\frac{8}{9}) \frac{9}{8} (\frac{81}{80}$.

 Laſtly.

Lastly, if from that of the *Tri-hemitone* ( or *Lesser Third* ) *la, mi, fa*; whose Proportion is as 6 to 5; we take that of the *Tone, la, mi* ( which is the Difference of a *Fourth* and *Fifth* ) as 9 to 8; there remains for the *Hemi-tone, mi, fa,* (or *la, fa,*) that of 16 to 15. That is $\frac{9}{8}$) $\frac{6}{5}$ ( $\frac{48}{45}$ $= \frac{16}{15}$.

Or, the *Tri-hemitone* ( or *Lesser Third*) whose Proportion is as 6 to 5; may be divided into three near Equals, (by taking Triple Numbers, in the same Proportion 18, 15; and interposing the two Intermediates 17, 16; ) which will therefore be as 18 to 17, and as 17 to 16, and as 16 to 15; That is, $\frac{18}{17} \times \frac{17}{16} \times \frac{16}{15} = \frac{18}{15} = \frac{6}{5}$.

Where also the *Greater Tone,* whose Proportion is as 9 to 8, or 18 to 16, is divided into its two near Equals ( commonly called *Hemitones,* ) that of 18 to 17, and that of 17 to 16: That is, $\frac{18}{17} \times \frac{17}{16} = \frac{18}{16} = \frac{9}{8}$.

And the *Lesser Tone,* that of 10 to 9, or 20 to 18, may be in like manner Divided into that of 20 to 19, and that of 19 to 18: That is, $\frac{20}{19} \times \frac{19}{18} = \frac{20}{18} = \frac{10}{9}$.

Which Divisions of the *Greater* and *Lesser Tone,* answer to what is wont to be designed by *Flats* and *Sharps.*

So that (by this Computation,) of these Eight Notes, *la, fa, sol, la, mi, fa, sol, la*; their Proportions stand thus; that of *la, fa,* ( or *mi, fa* ) is as 16 to 15. That of *fa sol* as 10 to 9, and that of *sol la* as 9 to 8: ( or else that of *fa sol,* as 9 to 8, and that of *sol la,* as 10 to 9,) That of *la, mi,* as 9 to 8. And if either of the *Tones* ( *Greater* or *Lesser* chance to be divided ( by *Flats* or *Sharps* ) into ( what they call ) *Hemi-tones,* their Proportions are to be such as is already mentioned.

There may be a like Division of a *Fourth,* (or *Dia-tesseron*) into Two Near equals: And of some others of these, into three Near-equals. Which might be of use for ( what they were wont to call ) the *Chromatick* and *Enarmonick Musick.* But those sorts of *Musick,* having been long since laid aside, there is now no need of these Divisions, as to the *Musick* now in use.

*The Imperfection of an* Organ; *by Dr.* J. Wallis. n. 242. p. 249.

IV. I think 'tis Evident that each *Pipe* in the *Organ* is intended to Express a Distinct Sound at such a *Pitch*; that is, in such a Determinate Degree of *Gravity* or *Acuteness*; or ( as it is now called ) *Flatness* or *Sharpness*: And the Relative or Comparative Consideration of Two ( or more ) such Sounds or Degrees of *Flatness* and *Sharpness,* is the Ground of ( what we call ) *Concord,* and *Discord*; that is, a Soft, or Harsh, Coincidence.

Now,

Now, concerning this, there were amongſt the Ancient *Greeks*, Two (the moſt conſiderable) Sects of *Muſicans*: the *Ariſtoxenians*, and *Pythagorians.*

They Both agreed thus far; that *Dia-teſſeron* and *Dia-pente*, do together make up *Dia-paſon*: that is (as we now ſpeak) a *Fourth* and *Fifth*) do together make an *Eighth* or *Octave*: And, the Difference of thoſe two of a *Fourth* and *Fifth*, they agreed to call a *Tone*, which we now call a *Whole Note.*

Such is that, (in our preſent *Muſick*,) of *la mi*, (or as it was wont to be called *re, mi.*) For *la, fa, ſol, la,* or *mi, fa, ſol, la,* is a perfect *Fourth*: And *la, fa, ſol, la, mi,* or *la, mi, fa, ſol, la,* is a perfect *Fifth*: The Difference of which is *la, mi,* Which is, what the *Greeks* call, the *Dia-zeutick Tone*; which doth Disjoin two *Fourths* (on each ſide of it;) and being added to either of them, doth make a *Fifth*; Which was, in their *Muſick*, that from *Meſe* to *Parameſe*; that is in our *Muſick*, from A to B: ſuppoſing *mi* to ſtand in *B-fa-b-mi*, which is accounted its Natural Poſition.

Now in order to this, *Ariſtoxenus* and his Followers, did take, that of a *Fourth*, as a Known *Interval*, by the Judgment of the Ear; and that of a *Fifth*, likewiſe; And conſequently that of an *Octave*, as the Aggregate of both; and that of a *Tone*, as the Difference of thoſe Two.

And this of a *Tone* (as a Known *Interval*) they took as a common Meaſure, by which they did Eſtimate other *Intervals*. And acccordingly they accounted a *Fourth* to contain two *Tones* and an Half; a *Fifth* to contain Three *Tones* and an Half, and conſequently an Eigth to contain Six *Tones*; or Five *Tones* and two *Half Tones.*

And at this Rate our *Practical Muſicians* talk of *Notes* and *Half Notes* at this Day; ſuppoſing an *Octave* to conſiſt of Twelve *Hemi-tones*, or *Half Notes.*

But *Pythagoras* and thoſe who follow him, not taking the Ear alone to be a Competent Judge in a Caſe ſo nice; choſe to diſtinguiſh theſe, not by *Equal Intervals*, but by *Due Proportions*: And this is followed by *Zarline*, *Kepler*, *Cartes* and others, who treat of *Speculative Muſick* in this and the laſt Age. Accordingly they accounted that of an *Octave*, to be, when the Degree of *Gravity* or *Acuteneſs* of the one Sound to that of the other, is Double, or or as 2 to 1; that of a *Fifth*, when it is *Seſqui-alter*, or as 3 to 2; that of a *Fourth* when *Seſqui-tertian*, or as 4 to 3. Accounting that the Sweeteſt Proportion, which is expreſt in the Smalleſt Numbers, and therefore (next to the *Uniſon*) that of an *Octave*, 2 to 1, then that of a *Fifth*, 3 to 2, and than that of a *Fourth*, 4 to 3.

And thus that of a *Fourth* and *Fifth*, do together make an *Eighth*; For $\frac{4}{3} \times \frac{3}{2} = \frac{4}{2} = \frac{2}{1} = 2$, or the Proportion of 4 to 3, compounded with that of 3 to 2, is the ſame with that of 4 to 2, or 2 to 1. And, conſequently, the Difference of theſe two, which is that of a *Tone*, or Full Note, is that of 9 to 8. For $\frac{4}{3}) \frac{3}{2} (\frac{9}{8}$; or, if out of the Proportion of 3 to 2, we take that of 4 to 3; the Reſult is that of 9 to 8.

Now,

Now, according to this Computation, it is manifeſt, That an *Octave* is ſomewhat leſs than Six *Full Notes*. For (as was firſt demonſtrated by *Euclide*, and ſince by others) the Proportion of 9 to 8, being ſix Times compounded, is ſomewhat more than that of 2 to 1. For $\frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} = \frac{531441}{262144}$, is more than $\frac{524288}{262144} = \frac{2}{1}$.

This being the Caſe; they allowed (indiſputably) to that of the *Diazeutick Tone* (*la mi*,) the full Proportion of 9 to 8, as a thing not to be altered; being the Difference of *Dia-pente* and *Dia-teſſeron*, or the *Fifth* and *Fourth*.

All the Difficulty, was, How the remaining *Fourth* (*mi*, *fa*, *ſol*, *la*,) ſhould be divided into three parts, ſo as to anſwer (pretty near) the *Ariſtoxenians* Two *Tones* and a half: and might, altogether make up the Proportion of 4 to 3, which is that of a *Fourth* or *Dia-teſſeron*.

Many Attempts were made to this purpoſe: And according to thoſe, they gave Names to the Different *Genera* or Kinds of *Muſick*, (the *Diatonick*, *Chromatick*, and *Enarmonick* Kinds,) with the ſeveral *Species*, or leſſer Diſtinctions under thoſe Generals.

The firſt was that of *Euclide* (which did moſt Generally obtain for many Ages:) Which allows to *fa*, *ſol*, and to *ſol*, *la*, the full Proportion of 9 to 8; And therefore to *fa*, *ſol*, *la*, (which we call the *Greater Third*) that of 81 to 64. (For $\frac{9}{8} \times \frac{9}{8} = \frac{81}{64}$. And, conſequently, to that of *Mi*, *fa*, (which is the Remainder to a *Fourth*) that of 256 to 243. For $\frac{81}{64}$) $\frac{4}{3}$ ( $\frac{256}{243}$; that is, if out of the Proportion of 4 to 3, we take that of 81 to 64, the Reſult is that of 256 to 243. To this they gave the Name of *Limma* (λεῖμμα) that is, the Remainder (to wit, over and above two *Tones*.) But, in common Diſcourſe (when we do not pretend to ſpeak nicely, nor intend to be ſo underſtood) it is uſual to call it an *Hemi-tone*, or *Half-Note*, (as being very near it) and the other, Two *Whole Notes*. And this is what *Ptolemy* calls *Diatonum Ditonum*, (of the *Diatonick* kind with *Two Full Tones*.)

Againſt this, it is Objected (as not the moſt convenient Diviſion) that the Numbers of 81 to 64, are too great for that of a *Ditone*, or *Greater Third*; which is not Harſh to the Ear; but is rather Sweeter than that of a Single *Tone*, whoſe Proportion is 9 to 8. And in that of 256 to 243, the Numbers are yet much greater. Whereas there are many Proportions (as $\frac{5}{4}$, $\frac{6}{5}$, $\frac{7}{6}$, $\frac{8}{7}$,) in ſmaller Numbers than that of 9 to 8; of which, in this Diviſion, there is no Notice taken.

To

To Rectifie this, there is another Division thought more convenient; which is *Ptolemey's Diatonum Intensum* (of the *Diatonick* Kind, more *Intense* or *Acute* than that other.) Which instead of two *Full Tones* for *fa, sol, la*, assigns (what we now call) a *Greater* and a *Lesser Tone*; (which by the more Nice *Musicians* of this and the last Age, seems to be more embraced;) Assigning to *fa, sol*, that of 9 to 8, (which they call the *Greater Tone*;) and to *sol, la*, that of 10 to 9, (which they call the *Lesser Tone*:) And therefore to *fa, la*, (the *Ditone* or *Greater Third*) that of 5 to 4. (For $\frac{10}{9} \times \frac{9}{8} = \frac{10}{8} = \frac{5}{4}$.) And consequently to *mi, fa*, (which is remaining of the *Fourth*) that of 16 to 15. For $\frac{5}{4}$ ) $\frac{4}{3}$ ( $\frac{16}{15}$. That is; if out of that of 4 to 3, we take that of 5 to 4, there remains that of 16 to 15.

Many other ways there are (with which I shall not trouble you at present) of dividing the *Fourth* or *Dia-tesseron*, or the Proportion of 4 to 3, into three parts, answering to what (in a looser way of Expression) we call an *Half-Note*, and *two whole Notes*. But this of $\frac{16}{15} \times \frac{9}{8} \times \frac{10}{9} = \frac{4}{3}$, is that which is now received as the most proper.

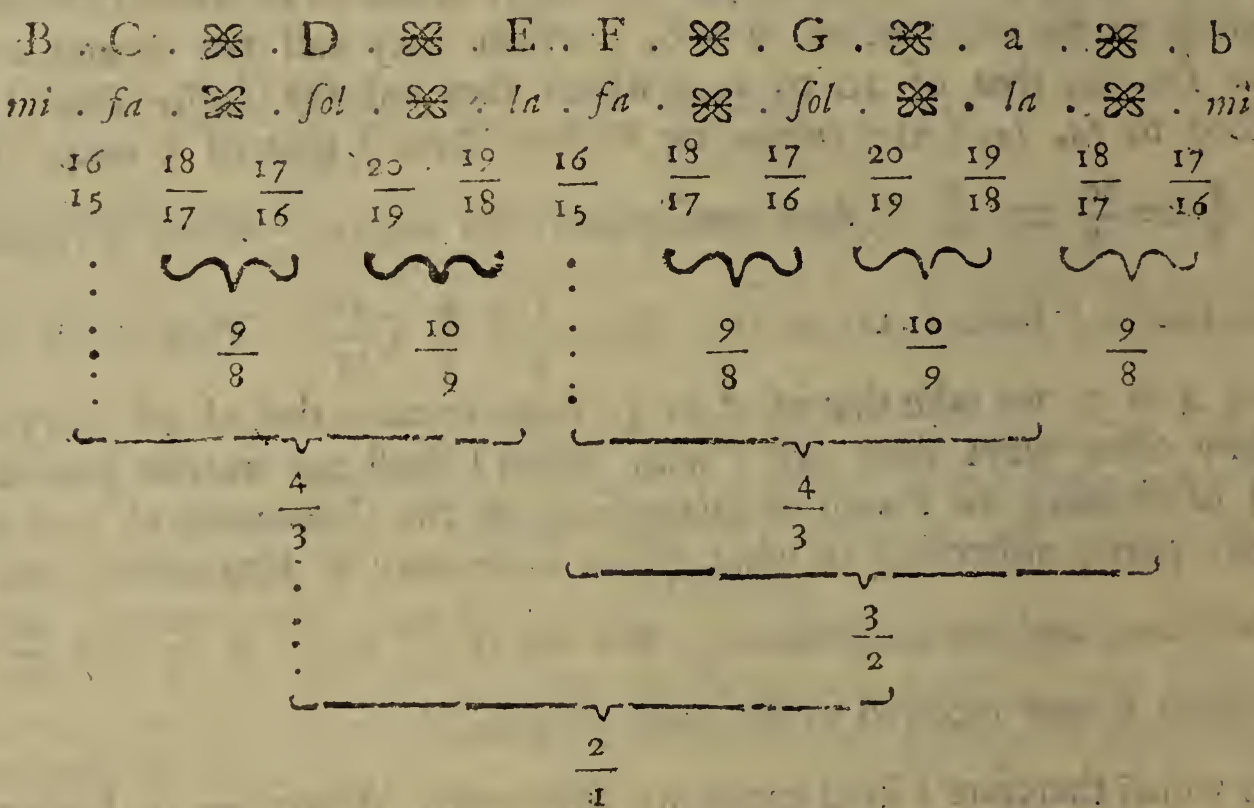
To which therefore I shall apply my Discourse; Where $\frac{16}{15}$ is (what we call) the *Hemitone*, or *Half-Note*, in *mi, fa*; $\frac{9}{8}$ that of the *Greater Tone*, in *fa, sol*, and $\frac{10}{9}$ the *Lesser Tone*, in *sol, la*.

Only with this Addition; That each of those *Tones*, is (upon Occasion by *Flats* and *Sharps* (as we now speak) divided into two *Hemi-tones*, or *Half-Notes*: Which answers to what by the *Greeks* was called *Mutatio quoad Modos* (the Change of *Mood*;) and what is now done by removing *mi* to another *Key*. Namely $\frac{9}{8} = \frac{18}{16} = \frac{18}{17} \times \frac{17}{16}$; and $\frac{10}{9} = \frac{20}{18} = \frac{20}{19} \times \frac{19}{18}$.

Thus by the help of *Flats* and *Sharps* (dividing each *whole Note*, be it the *Greater* or *Lesser*, into two *Half Notes*, or what we call so,) the whole *Octave* is divided into Twelve Parts or *Intervals* (contained between Thirteen *Pipes*) which are commonly called *Hemi-tones* or *Half Notes*, not that each is precisely *Half* a *Note*, but some what near it, and so called. And I say, by *Flats* and *Sharps*; For sometime the one sometime the other, is used. As for Instance, a *Flat* in D, or a *Sharp*; in C, do either of them denote a Midling Sound (though not precisely in the Midst) between C and D; *Sharper* than C, and *Flatter* than D.

Accor-

Accordingly; ſuppoſing *mi* to ſtand in ***B-fa-b-mi*** (which is accounted its *Natural* Seat) the Sounds of each *Pipe* are to bear theſe Proportions to each other, *viz.*




And ſo in each *Octave* ſucceſſively following. And if the *Pipes* in each *Octave* be fitted to Sounds in theſe Proportions of *Gravity* and *Acuteneſs*, it will be ſuppoſed (according to this *Hypotheſis* to be perfectly Proportioned.

But, inſtead of theſe ſucceſſive Proportions for each *Hemi-tone*; it is found neceſſary (if I do not miſtake the Practice) ſo to order the 13 *Pipes* (containing 12 *Intervals* which they call *Hemi-tones*) as that their Sounds (as to *Gravity* and *Acuteneſs*) be in Continual Proportion, (each to its next following, in one and the ſame Proportion;) which, all together, ſhall compleat that of an *Octave* or *Dia-paſon,* as 2 to 1. Whereby it comes to paſs that each *Pipe* doth not expreſs its proper Sound, but very near it, yet ſomewhat varying from it; Which they call *Bearing*; Which is ſomewhat of Imperfection in this *Noble Inſtrument*; the Top of all.

It may be asked, Why may not the *Pipes* be ſo ordered, as to have their Sounds in juſt Proportion, as well as thus *Bearing*?

I anſwer, It might very well be ſo, if all *Muſick* were *Compoſed* to the ſame *Key*, or (as the *Greeks* call it) the ſame *Mode*; As for Inſtance, if, in all Compoſitions, *mi*, were always placed in *B-fa-b-mi.* For then the *Pipes* might be ordered in ſuch Proportions as I have now deſigned.

But *Muſical Compoſitions* are made in great Variety of *Modes*, or with great Diverſity in the *Pitch. Mi*, is not always placed in *B-fa b-mi*; but ſome times in *E-la-mi*, ſometimes in *A-la-mi-re*, &c. And (in Summ) there is none of theſe 12 or 13 *Pipes* but may be made the *Seat* of *mi.* And if they were exactly

exactly fitted to any one of these Cases, they would be quite out of order for all the rest.

As for Instance; If *mi*, be removed from *B-fa-b-mi* (by a *Flat* in B) to *E-la-mi*: Instead of the Proportions but now designed, they must be thus ordered.

B . ⌘ . C . ⌘ . D . ⌘ . E . F . ⌘ . G . ⌘ . a . b

*fa* . ⌘ . *sol* . ⌘ . *la* . ⌘ . *mi* . *fa* . ⌘ . *sol* . ⌘ . *la* . *fa*.

$\frac{18}{17}$ $\frac{17}{16}$ $\frac{20}{19}$ $\frac{19}{18}$ $\frac{18}{17}$ $\frac{17}{16}$ $\frac{16}{15}$ $\frac{18}{17}$ $\frac{17}{16}$ $\frac{20}{19}$ $\frac{19}{18}$ $\frac{16}{15}$

Where 'tis manifest, that the removal of *mi* doth quite disorder the whole *Series* of Proportions. And the same would again happen, if *mi* be removed from E to A (by another *Flat* in E.) And again if remov'd from A to D. And so perpetually. But the *Hemitones* being made all equal; they do indifferently answer all the Positions of *mi* (though not exactly to any:) Yet nearer to some than to others. Whence it is, that the same *Tune* Sounds better at one *Key* than at another.

It is asked, Whether this may not be Remedy'd; by interposing *more Pipes*; and thereby *dividing* a *Note*, not only (as now) into *Half-Notes*, but into *Quarter-Notes* or *Half-Quarter-Notes*, &c.

I answer; It may be thus Remedy'd in part; (that is, the Imperfection might thus be somewhat Less, and the Sounds somewhat nearer to the Just Proportions:) but it can never be exactly true, so long as their Sounds (be they never so many) be in continual Proportion; that is, each to the next Subsequent in the same Proportion.

For it hath been long since Demonstrated, that there is no such thing as a just *Hemi-tone* practicable in *Musick*, (and the like for the Division of a *Tone* into any Number of Equal parts; three, four or more.) For, supposing the Proportion of a *Tone* or *Full Note*, to be $\frac{9}{8}$ (or, as 9 to 8;) that of the *Half-Note* must be as $\sqrt{9}$, to $\sqrt{8}$; that is as 3 to $\sqrt{8}$. or 3 to $2\sqrt{2}$) which are *Incommensurable Quantities*. And that of a *Quarter Note*, as $\sqrt[4]{9}$ to $\sqrt[4]{8}$, which is yet more *Incommensurate*. And the like for any other Number of Equal parts, which will therefore never fall in with the Proportions of Number to Number.

So that this can never be perfectly Adjusted for all *Keys* (without somewhat of *Bearing*) by *Multiplying Pipes*; unless we would for every *Key* or every different Seat of *mi*) have a different *Set* of *Pipes*, of which this or that is to be used, according as (in the *Composition*) *mi* is supposed to stand in this or that *Seat*. Which vast Number of *Pipes* (for every *Octave*) would vastly increase the Charge: And (when all is done make the whole *Impracticable*.

*A New Tuning of the* Lyra Viol; *by* S. Salvetti. *n.* 87. *p.* 5064.

V. *S. Salvetti*, about 4 Years ago Invented a *New Tuning* of the *Ancient Lyra Viol* with the usual 13 *Strings*; by means of which *Tuning* it is rendered wholly Perfect, so that you may express upon it all *Concords, Discords*, and also the *Imperfect Concords*, as *Sevenths, Sixths*, &c. as well as upon any *Virginal* that hath the *Quarters of Notes* upon it. 'Tis true, 'tis only for Melancholly and Passionate matter, and not for *Division*, as is the proper Nature of the *Lyra*. I shall only add, that with the above said *Tuning* the Ascends in *Alt* as high as *G-sol-re-ut*; and Descends as low as *Double C-fa, ut*; and can make every where the same *Concords* as above.

*The strange Effects reported of* Musick *in Former Times, Examined; by Dr.* Wallis. *n.* 243. *p.* 297.

VI. 1. I take it for granted that much of the *Reports* concerning the *great Effects* of *Musick* in *former Times* beyond what is to be found in *Latter Ages*, is highly *Hyperbolical* and next Door to *Fabulous*; And therefore great Abatements must be allowed to the Elogies of their *Musick*.

2. We must consider, That *Musick* (to any tolerable Degree) was then (if not a New, at least) a Rare Thing, which the *Rusticks*, on whom it is reported to have had such *Effects*, had never heard before: and on such a *little Musick* will do great *Feats*; As we find at this Day, a *Fidle* or a *Bag-pipe*, at a Country *Morice Dance*.

3. We are to consider, that their *Musick* (even after it came to some good Degree of Perfection) was much more Plain and *Simple* than ours now-a-days. They had not *Consorts* of two, three, four or more *Parts* or *Voices*: But one Single Voice or single Instrument a-part; which to a *rude* Ear, is much more taking than more *Compounded Musick*. For that is at a Pitch not above their Capacity; whereas this other confounds it, with a great Noise, but nothing Distinguishable to their Capacity.

4. We are to consider, that *Musick* with the *Ancients* was of a larger extent than what we call *Musick* now a-days: For *Poetry* and *Dancing* (or comely Motion) were then accounted parts of *Musick*, when *Musick* arrived to some Perfection. Now we know that *Verse* of it self, if in good Measures and Affectionate Language, and this set to a *Musical Tune*, and Sung by a decent Voice, and accompanied but with *Soft Instrumental Musick* if any, such as not to Drown or obscure the *Emphatick* Expressions (like what we call *Recitative-Musick*) will work strangely upon the Ear, and Move all Affections suitable to the *Tune* and *Ditty*; (whether Brisk and Pleasant, or Soft and Pitiful, or Fierce and Angry, or Moderate and Sedate) especially if attended with a *Gesture* and *Action* suitable. For 'tis well known, that suitable *Acting* on a *Stage* gives great Life to the Words. Now all this together (which were all Ingredients in what *they* called *Musick*) must needs operate strongly on the Fancies and Affections of ordinary People, unacquainted with such kind of Treatments. For, if the deliberate Reading of a *Romance* (when well penn'd) will produce Mirth, Tears, Joy, Grief, Pity, Wrath, or Indignation, suitable to the respective Intents of it, much more would it so do, if accompanied with all those Attendants.

5. You

5. You will ask perhaps, why may not all this be *Now* done, as well as then? I anſwer, no doubt it may, and with like Effect, if an Addreſs be made, in Proper Words with Moving Arguments, in juſt Meaſures (*Poetical* or *Rhetorical*) with the *Emphatick Words*, Words ſet in ſignal Places, pronounced with a good Voice, and a true Accent) and attended with a decent Geſture; and all theſe ſuitably adjuſted to the Paſſion, Affection, or Temper of Mind, particularly deſigned to be Produced, ( be it Joy, Love, Grief, Pity, Courage or Indignation ) will certainly *now*, as well as *then*, produce great Effects upon the Mind, eſpecially upon a Surprize, and where Perſons are not otherwiſe pre-engaged: And if ſo managed, as that you be ( or ſeem to be ) in earneſt; and if not Over-Acted by apparent Affectation.

6. We are to conſider that the uſual Deſign of what we *now* call *Muſick*, is very different from that of the *Ancients*. What we *Now* call *Muſick*, is but what they called *Harmonick*; which was but one part of their *Muſick* ( conſiſting of Words, Verſe, Voice, Tune, Inſtrument, and Acting ) and we are not to expect the ſame Effect of one Piece as of the Whole.

7. When *Muſick* arrived to great Perfection, it was applied to particular Deſigns of Exciting this or that particular Affection, Paſſion or Temper of Mind; the *Tunes* and Meaſures being ſuitably adapted to ſuch Deſigns. But ſuch Deſigns ſeem almoſt quite neglected in our *preſent Muſick*. The chief Deſign now, in our moſt accompliſhed *Muſick*, being to pleaſe the Ear; when by a ſweet Mixture of different *Parts* and *Voices*, with juſt *Cadences* and *Concords* intermixed, a grateful ſound is produced, which only the *Judicious Muſician* can diſcern and diſtinguiſh.

8. 'Tis true, that even this *Compound Muſick* admits of different Characters ſome is more Brisk and Airy; others more Sedate and Grave; others more Languid; as the different Subjects do require. But that which is moſt proper to excite particular *Paſſions* or *Diſpoſitions*, is ſuch as is more *Simple*, and *Uncompounded*: ſuch as a *Nurſes Languid Tune*, Lulling her Babe to Sleep; or a continued Reading in an *Even Tone*; or even the ſoft *Murmur* of a little Rivulet, running upon Gravel or Pibbles; inducing a quiet Repoſe of the Spirits. And contrarywiſe, the Briskneſs of a *Jig*, on a *Kit* or *Violine*, exciting to Dance. Which are more *Operative* to ſuch particular Ends, than an *Elaborate Compoſition* of *Full Muſick*.

9. To Conclude; If we Aim only at pleaſing the *Ear*, by a *Sweet Conſort*, I doubt not but our *Modern Compoſitions* may be equal, if not exceed thoſe of the *Ancients*: Amongſt whom I do not find any Footſteps of what we call *ſeveral Parts* or *Voices*, ( as *Baſe*, *Treble*, *Mean*, &c. Sung in *Conſort* ) anſwering each other to compleat the *Muſick*. But if we would have our *Muſick* ſo adjuſted as to excite *particular Paſſion*, *Affections*, or Temper of Mind ( as that of the *Ancients* is ſuppoſed to have done ) we muſt apply more ſimple *Ingredients*, fitted to the *Temper* we would Produce. And this, I doubt not, but a *Judicious Compoſer* may ſo Effect, ( that with the Help of ſuch *Hyperbole*'s, as with which the *Ancient Muſick* is wont to be ſet off ) our *Muſick* may be ſaid to do as great *Feats* as any of theirs.

VII. *Ac-*

VII. *Accounts of Books*, Omitted.

n. 143. p. 20. 1. *Claudii Ptolemæi Harmonicorum* Libri tres Ex Cod. MSS. undecim, nunc primum *Græce*, Editi. *Jo. Wallis* S.S. Th. D. Recensuit, Edidit, Versione & Notis Illustravit, & Auctuarium adjecit. *Oxon.* 1682. in 4to.

n. 231. p. 668. 2. *Porphyrii* Commentarius in Librum primum *Harmonicorum Claudii Ptolemæi*: atque *Manuelis Bryennii* Commentarius in tres Libros *Harmonicos* ejusdem *Ptolemæi*. ( Qui soli restant ex *Græcis Musicæ* Scriptoribus nondum Editi) *Græce* ac *Latine*, Cura, *Jo: Wallisii* S. Th D. *Fol.*

n. 80. p. 3095. 3. An Essay to the Advancement of *Musick* by *Tho: Salmon* M. A. *Lond.* 1672. *in 8vo.*

n. 90. p. 5153. 4. *Syntagma Musicæ*; Treating of *Musick Philosophically*, *Mathematically*, *and*
n. 100. p. 7000. *Practically*; by *J. Birchensha* Esq; This Book was Preparing for the Press, 1674.

n. 100. p. 6194. 5. *Musica* Speculativa del *Mengoli*. in *Bologna* 1670. *in 4to.*

n. 133. p. 835. 6. A *Philosophical Essay* of *Musick*. *Lond.* 1677. *in 4to.*

n. 208. p. 67. 7. A Treatise of the *Natural Grounds* and *Principles* of *Harmony*; by *Will. Holder*. *D. D. Lond.* 1694. *in 8vo.*

F I N I S.