THE SPECIALISTS' SERIES

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ELECTRIC TRANSMISSION OF ENERGY.

THE SPECIALISTS' SERIES.

ELECTRIC TRANSMISSION OF ENERGY

AND ITS

TRANSFORMATION, SUBDIVISION, AND DISTRIBUTION.

A PRACTICAL HANDBOOK

BY

GISBERT KAPP,

Member of the Institution of Civil Engineers, Member of the Institution of Electrical Engineers.

WITH 166 ILLUSTRATIONS.

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PREFACE.

During the time which has elapsed since the third edition of this book was published an enormous development has taken place in every branch of electric power transmission. The use of electricity for transmission of power over very long distances has become an accomplished fact, as proved by the works at Lauffen, Rome, Genoa, and the large installation now in course of erection at Niagara Falls ; transmission plants for moderate distances have been established in so many localities that work of this kind has become quite commonplace ; the use of electric mining machinery, and the equipment of ship-yards with electrically driven tools, has made fair progress ; the distribution of power from central stations has become an important and profitable source of revenue to the lighting companies ; the use of electro-motors in lieu of long lines of shafting in factories and mills is slowly but surely gaining favour, and last, but not least, the application of electric traction on tramways and railways has progressed with giant strides. The subject has thus not only become more attractive and practically useful, but has been specialized to an extent which renders its complete treatment in a book of this size impossible. I had, therefore, to face the question, what parts, or how much of each part, should be omitted ? Two ways were open. I might attempt to give a little of each branch of the subject by cutting down the theory to a few pages, and devoting the rest of the book to a necessarily superficial description of examples of all the various applications of electricity to power purposes that are now in use ; or I might omit certain branches of the subject altogether, and treat the others by expounding principles rather than giving examples. For several reasons I have chosen the latter way. In the first place, as the book is intended for students and practical engineers, and as any number of illustrated descriptions of electric power plant can be found in the technical press, I preferred to curtail rather the descriptive than the theoretical part of the previous editions. In the second place, the tendency to specialize in practice has called forth a similar tendency in literature, so that there is now no call for any one book to cover the whole of this vast field ; and lastly, I wished to gain room in this edition for the treatment of multiphase currents in order to draw the attention of English engineers to a branch of electrical work which has hitherto not received sufficient recognition in this country.

These considerations have made it necessary to completely re-write more than half of the book, and omit a corresponding amount of old matter. Thus, the historical account of power transmission, detailed descriptions of plants, comparison of electric with other systems of transmission, underground cables, electric tramways, and telpher lines, have all been omitted. If the amount of capital invested in one particular branch of power transmission be taken as a measure of its importance, then electric propulsion is undoubtedly the most important of our subjects. Nevertheless its omission in this book appeared to me to be justified on the ground that the excellent works of the late Mr. Reckenzaun, and Messrs. Crosby and Bell, give far more information than I could have given in the limited space at my disposal. As regards underground electric lines, the valuable book by Mr. Russell has made my treatment of this subject likewise superfluous. The detailed descriptions of transmission plants given in the third edition have been cut out for want of room, and the same reason has forced me to curtail very much the description of more modern examples. Information of this kind will be found in the technical press, the Frankfort Exhibition Report, Professor Unwin's Howard Lectures, and my Cantor Lectures.

The theory of continuous current machines has been somewhat extended, and that of alternators and multiphasers has been added. In treating the latter I have adopted graphic methods wherever possible, in the hope that practical engineers will find them more handy than complicated analytical expressions. The chapter on the line has been considerably enlarged, especially as regards the law of most economical section. The description of generators and motors given in the last chapter is not intended to form a complete catalogue of the machines built by the best makers, but is merely a collection of some representative types, in order that the reader may gain information as to the general construction of machines used in electric power transmission.

WESTMINSTER, *August*, 1894.

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ERRATA.

Page 118. Line 7 from bottom for (p. 24) *read* (p. 37).

,, 153. Line 11 from bottom for (p. 39) *read* (pp. 51 and 54).

,, 157. Lines 12 and 13 from bottom *read* $\sqrt{E\,R\,\gamma} = \sqrt{80} = 8{\cdot}94$.

,, 171. Line 10 for "service" *read* "surface."

,, 195. Equation 33) *read* $c = -\,g \pm \sqrt{g^2 + \frac{E}{R}(g + \gamma) + \gamma g}$.

,, 305. Equation 62 *read* $S = \frac{\sim_1 - \sim_2}{\sim_1}$

,, 373. *Read* $W = c\sqrt{w\,q\,t\,k\,p}$.

ELECTRIC TRANSMISSION OF ENERGY.

INTRODUCTORY.

THE transmission of energy and its transformation is the fundamental problem of mechanical engineering. No piece of mechanism yet devised is capable of creating energy, but all mechanism has for its object the transmission, transformation, and application of energy already existing in nature in a more or less inconvenient form. A seam of coal some thousand feet below the surface of the ground represents a vast store of energy, but to utilize this energy we require a most elaborate system of human labour aided by mechanical appliances to get the coal, bring it to the surface, and transport it to the place where, by means of a steam-boiler and engine, we transform the energy chemically stored in the coal into mechanical energy, which is then further transmitted and subdivided as may be required. Or take the case of a waterfall in some remote mountain region. There is plenty of energy in the water, but to utilize it by a turbine on the spot would be highly inconvenient. The nature of the ground may be such as to make the erection of a factory in that particular spot impossible, and, even if this were not so, the remoteness of the locality would make the carriage of raw materials to the factory, and of the finished products from the factory, so difficult and costly, that working under these conditions would become commercially impossible. If we wish to utilize the water-power we must place the factory at the nearest convenient locality and convey the water to it in a pipe under pressure. The two cases here cited are typical examples of the transmission of energy, but there is a distinction between them. In the former case we have transmitted not the energy as such, but merely the material from which energy may be obtained by a chemical process, namely, the burning of the coal. In the lump of coal the energy is, so to speak, stored up, and we may therefore consider the carriage of coal from one place to another as the transmission of stored energy. In like manner when we send a load of corn from the field to the farm we transmit stored energy, for the corn, if used as food for horses or other animal machines, is converted into energy to be usefully applied to farm work when and as required. The characteristic feature of this method of transmission is that, as with coal, a chemical process must intervene before the stored energy becomes available, that is, becomes converted into live energy.

With the high-pressure water-main this is not the case. The energy remains all the time in the live form, and can be made available by purely mechanical means, such as a turbine or water-wheel. There is also this further distinction, that the energy must be utilized at the same rate as it is transmitted. When we transmit energy in the stored form we may use it when we like and at any rate we like. Coal can be stored for an indefinite time and burnt on the grate of the steam-boiler at a rate corresponding exactly with the demand for power at the time. Provided we have sufficiently large bunkers we may daily transmit the supply of coal for a week or more and not use our engine at all, or we may get in a week's supply at once and use the engine every day. In this respect the transmission of energy in the live form is not so convenient. Exactly the same number of gallons of water we take in per minute at the top end of the pipe must flow out per minute and into the turbine at the bottom end. We cannot store the water in the pipe, and if from some cause the transmission is interrupted the power is cut off.

The electric transmission of power may be effected both in the stored and live form, though the latter is the method commonly understood by this term, and is also the more important.

If we use a steam-engine and dynamo for charging a secondary battery we store work. Now let us put the battery on a cart and convey it to a place some distance away where power is required. We can there deliver the power by joining the battery by two wires with an electro-motor, and mechanically gearing the motor with the machinery to which power is to be supplied. Here we have an example of electric transmission of power in the stored form. It is, moreover, a method which has already found some application in the working of tramways. At a convenient place on the line is established a charging station, where, by means of steam-power and charging dynamos, current is forced through sets of secondary batteries, and thus work is stored in them. Suitable arrangements facilitate the handling of the batteries, so that each car as it comes in from the road may have its exhausted battery quickly withdrawn and replaced by one fully charged, after which the car takes the road again. We have here really two systems of transmitting energy electrically; first, the transmission from the dynamo into the battery, which takes place in the live form and over a very short distance, and secondly, the transmission of power in the stored form by means of the battery which is carried on the car all over the line.

A similar system is used for the propulsion of small boats. Within the last few years electrically propelled boats and charging stations have been placed upon the Thames and other waters, and in some cases electric boats are carried by large vessels as part of their regular equipment. Where a vessel is already fitted with the electric light the use of such an electric launch is particularly convenient, because the same dynamo which works the lights at night can be used to charge the battery in the launch during the daytime, so that the launch may at any moment be ready to be lowered into the sea, and may contain a sufficient store of power for a run of some hours' duration. When the launch is stowed away on deck its battery may also be used for lighting purposes.

These examples will suffice to show that there is some application for the electric transmission of power in the stored form, but the question to be considered is whether this method is applicable generally or not. If distance of transmission were the only consideration the storage battery sent by railway or canal would certainly have an enormous advantage over the rival system of transmitting live energy over wires, because there is a limit to the distance in the latter case, whilst there is no such limit in the former. But cost is a not less important consideration than distance, and it is impossible to say off hand whether in any given case transmission by wire will be dearer or cheaper than transmission by batteries. Each case must, in fact, be considered on its own merits, and for our present purpose it will suffice if we investigate in a general way the manner in which transmission of power by storage batteries could be carried out. It will be clear, to begin with, that if the original source of energy be coal, and if there be no objection to the establishment of a steam-engine at the place where the power is wanted, it will be better to carry coal there than charged storage batteries, because not only does a sack of coal represent more stored power than a battery of equal weight, but its transmission is easier, cheaper, and requires less precautions than that of the battery. If, however, the original source of energy is falling water, transmission by battery may be feasible. Let us see how such a system could be worked out. At the waterfall we establish a charging station, and to facilitate the transport of the batteries we build a tramline or railway to the place where the power is required, say a factory, the machinery in which is worked by an electromotor, which receives current from the batteries which are delivered there day by day, and when exhausted are returned to the power station for re-charging. The train which carries the batteries to and fro would of course also have to be worked electrically, and thus part of the store of power in the batteries would never reach the factory, but be expended for the purposes of transmission. The remainder would be given up to the motor at the factory, and the ratio of this amount to the total amount of work which could be taken out of the battery if it were discharged at the power station instead of at the factory would represent the efficiency of transmission.

If, for instance, the battery contains a store of 1,000 horse-power hours, and the propulsion of the train requires 50 horse-power hours each way, then only 900 horse-power hours remain available for delivery to the terminals of the electro-motor at the factory, and the efficiency of transmission would be 90 per cent. The farther we can carry the batteries with a given loss of power (in this case 10 per cent.), or the smaller the loss over a given distance, the more perfect will be the system of transmission. Now it is evident that the degree of perfection to which we could bring this system of transmission must depend principally upon two points. First, the batteries themselves, and secondly, the road and arrangements for their transmission. The more power can be stored in a battery of a given weight and bulk, the smaller will be the fraction of this power which must be expended for traction, and the more will remain for delivery. Again, the better and more level the road over which the transmission takes place, and the more perfect the propelling arrangements generally, the less power will be absorbed for the transmission of a given weight of batteries, and the more will remain for working the electro-motor at the factory.

It is interesting to compare such a system of transmission with the well-known systems of transmitting stored power in the shape of corn and coal. In the case of corn the starting-point of our line of transmission is the field where the corn is grown (the ultimate source of energy being of course the sun). At the field we load the corn into waggons and send them to the farm, using horses as our propelling machines. But horses in order to work must eat, and they will eat the more of the corn the harder they work. Consequently a smaller amount of corn will be delivered at the farm than was taken from the field, and the ratio of these two quantities will be the efficiency of transmission, which must depend upon the kind of road available between the field and the farm, the nutritious value of the corn, the condition of the horses, and other considerations. If the efficiency of transmission is, as in the case of batteries, to be again 90 per cent., then for every 100 sacks of corn taken from the field 90 sacks must be delivered at the farm, and the system will be the better the farther we are able to carry the corn with the expenditure of 10 sacks for every 100 taken from the field.

The transmission of power stored in coal is a parallel case. At the pit's mouth we load the coal into waggons and haul them by means of a traction-engine or locomotive to the place where the power is required. Part of the coal is consumed for traction purposes on the outward and homeward journey of the train, leaving the rest for the production of live power at the other end of the line of transmission. The less coal we spend for traction the higher is the efficiency of the system, or in other words, the farther we can carry the coal with the expenditure for traction of 10 tons out of every 100 tons taken out of the mine, the more perfect is our system of transmission.

By tabulating the distances to which we can transmit stored power under different systems we can thus obtain a comparison as to their different values, as is shown in the following table. This table, reproduced from the author's Canton lecture, given at the Society of Arts in 1891, has been calculated for three different lines of transmission, namely, an ordinary carriage road, a tramway, and a railway. Each road is assumed to be level, and the best of its kind. The speed of transmission has been taken at 4, 8, and 20 miles per hour when coal and batteries are the transmitting agents for road, tram, and rail respectively, and at 4 miles per hour for all three kinds of road when the transmission of power is by means of corn transported by horses.

Transmission of Stored Power.

Source of Power.	Distance in miles attainable with 90 per cent. efficiency of transmission over		
	Road.	Tram.	Rail.
Coal and steam-engine . . .	115	270	1,300
Corn and horse	52	170	440
Storage battery and electro-motor .	4	10	26

A glance at this table will show that as far as efficiency of transmission is concerned, the electric system is far behind its two rivals. Even the primitive method of conveying the power stored in corn by means of a horse and cart over a carriage road is twice as efficient as an electric locomotive hauling storage batteries over a railway, and if compared with a steam locomotive hauling coal the difference is still greater. This low efficiency, and not less the great cost of batteries and their want of durability, form a great drawback to the electric transmission of stored power, and tend to restrict the application of such a method to cases where steam-power, animal-power, and other methods of transmission are for special reasons inapplicable. If it should be found possible to greatly reduce the weight and cost of storage batteries, and at the same time increase their durability, the electric transmission of power by their means might yet become commercially feasible, but, taking the batteries as we find them at present, they cannot compete against other methods of transmission.

If we now turn to the electric transmission of live power the case is far more hopeful. Such a system of transmission consists of three essential parts. A generating station where a steam-engine, turbine, or other prime mover, supplies mechanical power to a dynamo machine. This machine converts the mechanical power into electrical power, represented by a current flowing under a certain electric pressure. The current is conveyed along the line of transmission, by means of copper wires, to the motor station, where the electric power of the current is, by means of an electro-motor, again reconverted into mechanical power. We have thus the three parts, generator, line, and motor, which together constitute the transmission plant. The particular arrangements by which power is conveyed from the prime mover to the generator, and the corresponding arrangements by which the power is delivered from the motor, are not, strictly speaking, part of the plant by which the electric transmission of power is effected, although they are necessary adjuncts of the system of transmission taken as a whole.

Going back to our example of the waterfall, and the way in which the power, represented in the falling water, can be utilized for driving a factory or mill, it will be immediately clear how, by the use of dynamos and conducting wires, the distance of transmission can be extended. We have assumed that the waterfall is situated in some remote mountain region which in itself is not adapted as a site for the mill. Hence it was useless to erect the turbine at the fall itself, but we had to take the water in a high-pressure main to the nearest place where a mill and turbine could be conveniently erected. The distance over which water can thus be made to transmit power is limited by the cost of the pipe or other kind of aqueduct, and by the loss of head due to friction. If, then, instead of having to build a costly aqueduct, we need only put up a pair of copper wires supported on insulators and poles, it is obvious that the line of transmission will become cheaper, and, consequently, that we shall be able to afford a longer line, if local conditions should render it desirable to place the mill further away from the waterfall. We see thus that electric transmission offers a means for the utilization of natural sources of power which, by reason of their remoteness, could not be reached by any other method. Of these natural sources, coal, wind, and water, are the most important.

As regards the first-named source of power, electric transmission is as yet rather a matter of the convenient application and subdivision, than the utilization of some natural source of power which would otherwise be wasted. We drive railways and tramways by electrically transmitted power, which power is originally due to the burning of coal. As far as economy of fuel is concerned, the use of separate locomotive steam-cars may not be less advantageous than that of cars propelled by electromotors, but we employ the latter in preference because of their greater convenience and the absence of noise, heat, dirt, and smoke, so objectionable in underground railways and town tramways. Similarly we may replace in machine shops the long lines of main shafting and belt, or other gear, by small electro-motors, each driving its own tool, but we do this simply as a matter of convenience and to prevent the waste of power involved in these mechanical methods of transmission. Here it is not a question of utilizing a source of power which hitherto has been inaccessible. We must bring coal to the railway or factory whether we use a large steam-engine or a number of small ones, but in the former case we burn less of the coal to do a given amount of work, and are able to subdivide the power more conveniently.

Again, if we employ electric transmission of power in coal mining, it is not so much on account of its greater efficiency as compared with transmission by compressed air or water under pressure, but by reason of its greater convenience and the smaller capital outlay for plant. Whether the time will ever come when the power derived from coal will be electrically transmitted over long distances, as is now the case with water-power, it is impossible to predict. It has been suggested to erect large electrical generating stations close to the pit's mouth, and there produce current by steam-power obtained from the small coal which is not worth being carried by rail. The current would be transmitted by wire to industrial towns in the neighbourhood where power is required, and thus the energy contained in the refuse of our coal-fields could be utilized. As yet this suggestion has not been carried out, probably because continuous currents, which have up to the present been almost exclusively considered in connection with transmission work, are not adapted for very long distances. Within the last few years considerable progress has, however, been made in electric transmission of power by means of alternating and multiphase currents, and it is possible that with further improvements in the same direction these various systems may become available for the supply and subdivision of power throughout towns some twenty, thirty, or more miles distant from the coal-field where the power is generated.

This is, however, a problem of the future. For the present we may be satisfied to utilize those water-powers which would otherwise be running to waste. Even with this limitation the field for the use of electric transmission of power is very wide. To take only a few examples. The falls of the Rhine at Schaffhausen represent about 1,750,000 horse-power. Niagara is computed by the Cataract Company to represent some 5,000,000, though Herr Japing estimates it at 16,000,000 horse-power. According to M. Chretien, a French engineer, the total water-power in France is 17,000,000 horse-power. Compared with such figures the amount of horse-power derived from coal is small. The total consumption of coal within the United Kingdom is about 150,000,000 tons annually.

A Royal Commission, sitting in 1870, estimated that 44 per cent. of the total is used in mining and metallurgical work; 26 per cent. is required for domestic purposes, such as heating, gas, and water supply; 5 per cent. for locomotion; and 25 per cent. for manufactures. In this latter item is included the use of coal for the generation of steam-power. If we allow four-fifths of this, or 30,000,000 tons, exclusively for steam-power, it will be a liberal estimate, and reckoning an average of $5\frac{1}{2}$ lbs. of coal per horse-power hour, and 3,000 hours as the average working time of steam-engines throughout the year, we find that the total steam-power in use in the United Kingdom is about 3,500,000 horse-power. This calculation is, of course, very rough, and is only made to show of what order the figure is representing the power derived from coal. If we put the steam-power in use in America and the rest of the world at twice the above figure we find that the steam-power in use throughout the world does not much exceed 10,000,000 horse-power, which is less than the water-power of France, and, of course, only a very small fraction of the total water-power throughout the world.

As far as power alone is concerned the world, taken as a whole, could therefore very well afford to do without coal, although individual countries could not. But even in those countries where water-power is abundant it would be useless if we had not some economical means for transmitting it to considerable distances, and it is in this respect that electric transmission has become of so much importance of late, because it enables us to utilize sources of power which would be otherwise running to waste. It is natural to expect that power transmission generally will become most developed in countries where fuel is dear and waterfalls abound. This is actually the case in Switzerland. This country produces no coal and the supply of wood is not even sufficient for domestic use, so that steam users depend entirely on imported coal, the total value of which is estimated at £800,000 annually. Here we have all the conditions which tend to the better utilization of water-power, and it is therefore natural that Switzerland has, ever since it became an industrial country, paid great attention to the problem of how to get power from waterfalls, and how to transmit this power to the place where it can be used to greatest advantage.

The first system of power transmission to a distance on a large scale was the outcome of experiments made in 1850 by Ferdinand Hirn, in Alsace, who succeeded, by means of flat steel ropes, in transmitting power to a distance of 80 and later of 240 mètres. The success of these installations was so marked that engineers quickly took up the new system, and within ten years there were, in South Germany and Switzerland, some 400 transmissions at work, conveying an aggregate of 4,200 horse-power. The original flat steel ropes had, however, been replaced by round cable running over pulleys with V-shaped grooves. In 1863 a transmission plant of considerable magnitude was proposed by Herr Moser of Schaffhausen, with a view to utilize the power of the Rhine for industrial purposes. By means of a weir placed across the river a fall of from 12 to 16 feet was obtained, and a turbine station established on the left bank. Three turbines, each of 750 horse-power, were erected, and the power was carried across the river and along the right bank by cables to the various mills and factories in Schaffhausen, who took it at a fixed rental of from £5 to £6 per annum per horse-power, according to the greater or lesser amount of power contracted for. This transmission is still at work, though part of it has been replaced by an electric transmission plant erected for the "Kammgarnspinnerei" about three years ago by the Oerlikon works. The annual charge per horse-power is now only £2 16*s.*

The fact that so well designed a work as the Schaffhausen teledynamic transmission is being superseded by electric transmission, shows that the latter must have considerable advantages over the former. Although wire-rope transmission is exceedingly simple and positive in its action, it becomes unwieldy when the power exceeds 600 horse-power, and the distance over which it is economical is limited. The great cost of the cable towers, the influence of temperature on the strain in the cable, and the rapid wear of the latter are also serious drawbacks, and although teledynamic transmissions have been erected and are still working in other places, such as Bellegarde, Freiberg, and Zürich, it is doubtful whether the system will be able to permanently hold the field against its electrical rival. At the present time several thousand horse-power are already being transmitted electrically in Switzerland alone, and the system is being rapidly extended.

If we look to the United States as another country abounding in water-power, we find that its advantages are there also largely appreciated. According to a census in 1880 the total water and steam-power in use in the United States was 3,400,000 horse-power, of which 1,225,000 horse-power were furnished by water. Bearing in mind that this census was taken twelve years ago, when transmission of power as we now understand it was but little developed, it follows that the bulk of this water-power was utilized locally, and that there must be a far larger amount of yet undeveloped water-power capable of being brought into use by means of electric transmission.

In Great Britain there is, of course, not so much scope for electric transmission of water-power, partly because waterfalls of any dynamic magnitude, and which are not already brought into use, are scarce, but principally because coal is cheap. There are, however, also here some very successful examples of electric transmission, amongst which may be mentioned the electric railways of Portrush, and that between Bessbrook and Newry.

With us electric transmission is of importance, not so much on account of its ability to bring into use hitherto inaccessible sources of power, as on account of its convenience for the subdivision and application of power generated at some central point.

In these cases the power is obtained from coal, and although its cost is, generally speaking, higher than that of water power, its electric transmission, and especially its subdivision, constitutes still an economy if compared with the use of many small steam-engines having collectively the same power as the large engine at the central point. This is due partly to the greater efficiency of the large steam-engine, and partly to the fact that the cost of attendance is reduced. Where each small user of power is obliged to have his own boiler and engine, an attendant must be employed to stoke the boiler and drive the engine; but where the power is supplied electrically no special attendant is required. Beyond occasionally oiling the bearings of the motor and turning a switch to start or stop the motor, no attention is required. There is the further advantage that the space occupied by the motor is insignificant as compared with the space required for boiler, engine, feed-tanks, coal bunkers, etc.; that it can be put into any convenient position, and that its efficiency over a wide range of load is very high. The ease with which an electric motor can be stopped and started also tends to economy, because the motor need never be left running idle. With a steam-engine—and still more so with a gas-engine—the reverse is the case. The starting requires a considerable amount of skilled labour, and the result is that stopping and starting is avoided as far as possible, the owner, where the work is intermittent, preferring to run the engine through the idle periods rather than go to the trouble of starting up afresh each time that power is wanted. There are other advantages which the electric motor has over local engines, such as higher efficiency at light loads, freedom from heat, noise, danger of explosion, and greater cleanliness, all of which are so obvious as to need no further comment.

Reference has already been made to the use of motors in factories, and as within the last few years this application of electric power has been greatly developed, a few words of further explanation may here be given. Not only are cranes, lifts, and travellers worked electrically, but in many of the best-equipped works the machine tools are driven by motors which are supplied with power from a central generating station. The advantages of using electric motors for driving machine tools are manifold. In the first place we have greater economy. The large central boiler plant can be provided with mechanical stokers, and thus a cheaper class of coal may be burned, and the number of firemen reduced. A steam-engine of the most economical type may be used, which would not be possible with a number of smaller engines erected in different parts of the works. In the next place there is a saving in power due to the absence of long lines of shafting, and, most important of all, we do away with all or nearly all the belting which obstructs the workshop and involves a considerable expense for upkeep.

In applying electric transmission of power to engineering shops two systems may be used. We may either provide each machine tool with its own motor, or we may have one motor and a short length of counter-shafting to drive two or three tools. The former system is adopted where the tools are large and require each an appreciable amount of power, and the latter for small tools, especially if it be possible to so group them that they will generally have to be worked together. How far such grouping of machines should be carried is a matter which has to be considered carefully in each case. By putting several machines on to one motor we may get a slightly higher efficiency in the motor when all the machines are at work, and we reduce the capital outlay. On the other hand we have more belts and shafting to look after, and the "over all" efficiency of the plant during the time that some of the machines only are being used is lower. The modern tendency is in the direction of using separate motors for all except the smallest machine tools. The over all efficiency of this system as compared with that of long lines of shafting with fast and loose pulleys, belts, bearings and geared wheels is very high. The reason is that only those motors need be supplied with current which at any time are required for working the machine tools, and that the amount of power which must be supplied electrically is approximately proportional to the work done. With mechanical systems of transmission this is not the case. Whether all the tools or only a few of them are required at any time, the whole of the shafting and other gear must be kept in motion, and the power wasted in friction remains almost constant. Hence, at times when only a few of the tools are being worked, the over all efficiency of this method of transmitting power becomes very much lower than the electric transmission. The latter has, further, the advantage of being extremely flexible and easily installed.

No mechanical force can be detected in the conductor carrying the electrical energy such as appears during purely mechanical transmission with shafting, belts, wire-ropes, or in pipes conveying steam, water, or air. The conductor is clean, cold, does not move, and altogether appears inert. It can be bent, moved, or shifted in any manner while transmitting many horse-power. It might be brought round sharp corners, and, having little weight, it can be fixed with greater ease than any mechanical connection. It is thus possible to bring the energy into rooms and places awkwardly situated for mechanical transmission, and there is no noise, smell, dirt, or heat during the transit, nothing to burst or give way. The power is, moreover, under perfect control, and its application exceedingly elastic. The same circuit which is used for supplying light, and may be tapped to give many horse-power, can, at the same time and as conveniently, be used to work a sewing-machine or other small domestic implement, and the power consumed at the generating dynamo is always in proportion to the power obtained from all the motors, so that there is no waste of energy if some of the motors are standing still or are working with less than their full load.

Another and a very important application of electric power concerns the propulsion of vehicles. Allusion has already been made to some electric railways where a water-fall is the original source of power, which latter is transmitted electrically to the train ; but even where the power is originally derived from coal its electric transmission may be of advantage in certain cases. Thus, for working street tram-cars we might employ cable traction or small locomotives, but neither method is so convenient as electric traction. Anyone who has seen the electric cars in the crowded and tortuous streets of Boston (Mass.) must realize that neither the horse, the cable, nor the steam locomotive could cope with such an enormous traffic as easily as the electric motor, to say nothing of the great cost and technical difficulties of establishing a cable system in the centre of the town, and the objection to the use of steam locomotives on account of dirt and noise. The electric cars take sharp turnings with the greatest ease, and are, as regards speed, under perfect control, whilst they occupy less space than horse or steam-cars, which in crowded streets is a great consideration. The only objection—but this is a very serious one as far as England is concerned—is the overhead or "trolley wire" necessary to bring the current to the car. Hence, in this country, electric trolley cars have as yet only been used on suburban or rural lines. For town traffic the trolley is inadmissible, or at least has not yet been sanctioned by the authorities, and electric traction can only be obtained either by the use of an underground conductor and slot in the street, or by the use of storage batteries carried on the car. There is, however, another kind of railway in towns where electric traction is almost indispensable, namely, the underground lines running in cast-iron tunnels. Of this type the City and South London line has been the pioneer, and has now been running successfully for some years. The use of steam locomotives on a line of this type is of course out of the question, and although cable traction was at first proposed, it was abandoned in favour of electric traction, and the technical success of the line has proved the wisdom of this decision. Bills for similar lines crossing London in various directions have been promoted, and have received parliamentary sanction, and there can be no doubt that we shall shortly witness a large development of these underground electric railways.

Thus electric transmission of power has invaded almost every domain of the mechanical engineer. We have the long distance transmission of power in bulk, so to speak, by means of which the energy of falling water at some place far back in the mountains is brought to the town for lighting, for driving mills, metallurgical works, and electric cars; we have the central electric lighting station supplying current not only for lighting, but also for power to the small artizan; we have the power-house and electric railway, or the mine with its generating station, underground cables, and electrically worked mining tools; we have the dock with its electric cranes and hoists, and the workshop with its machine tools worked by electric current.

CHAPTER I.

General Principles—Lines of Force—Relations between Mechanical and Electrical Energy—Absolute Measurements—Ideal Motor and Transmission of Energy—Practical Units.

A PROPER understanding of the principle of the conservation of energy, which exists throughout the whole of nature, must necessarily form the basis of all scientific investigation of mechanical or electrical problems, and of most of the improvements we might attempt to introduce in existing machinery and apparatus. In many cases, the fact that the original amount of energy remains unchanged, whilst the form in which it becomes manifest undergoes many alterations, is easily understood. For instance, if a locomotive engine draws a train behind it on a railway, we are at no loss to explain how the energy of fluid pressure of steam in the boiler is transformed into that of a steady pull overcoming the resistance of the train at a speed of so many miles per hour, and including all the so-called waste caused by deformation, friction, abrasion, and heating of the bodies through which the energy flows. The means by which, in this case, energy is transformed are, for the most part, purely mechanical, and sufficiently familiar to our imagination to allow us to form a mental picture of the different processes taking place. Even the transformation of heat into energy of fluid pressure, although we are not able to represent it by a mechanical model, has, through long familiarity with heat engines in one form or another, become comprehensible to us. With electrical energy, and with that of chemical action, this is not so. We can form no kind of mental picture of the process taking place in a voltaic cell where the energy of chemical action is transformed into that of an electric current, nor can we say what are the connecting links by the aid of which this current, after passing through hundreds of miles of wire, is made to impart mechanical energy to the armature of an electro-magnet, and thereby produce telegraphic signals. There is no mechanical connection between the sending key and the lever of the Morse instrument by which energy could be transmitted in the form of a pull, as is the case in our example of the coupling between a locomotive and its train, and yet energy is unmistakably transmitted. If we neglect waste, that is energy transformed in a way not immediately useful for the purpose in view, we find that the amount of electrical energy received at the distant station is proportional to the amount of chemical energy used up ; and if we take the waste also into account, we shall find that the energy it represents, added to that received in the form of an electric current at the distant station, is again proportional to the amount of chemical energy developed in the voltaic cell. If we know the nature of the chemical process going on in the cell, we can always calculate, by the aid of electro-chemical equivalents, what total amount of electrical energy can be obtained from a given weight of materials.

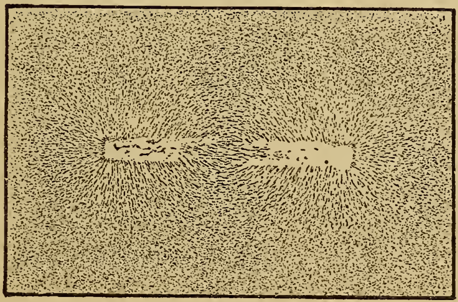
Similarly there exists a definite and constant proportion between electrical and mechanical energy. The relation between the two is somewhat complicated by the development of heat, which, indeed, is inseparable from electric phenomena, but if we make due allowance for the energy wasted in heat, we shall find that a given amount of electrical energy will always produce the same amount of mechanical energy, irrespective of the time required, or the exact manner of transformation. Although we cannot say what are the connecting links between electric current and mechanical force, experiment shows that certain definite relations exist, and we can, on the basis of experimental facts, conceive a mental picture or model by the aid of which these relations are represented in a familiar form. Such a mental picture is the conception of magnetic lines of force, first introduced by Faraday. In adopting this method of rendering electro-mechanical phenomena tangible to our senses, we make no assumption whatever about the reality of the lines of force. Whether they actually exist is a matter of total indifference ; but since all the experiments we can make are compatible with that conception, and since it enables us not only to explain experimental facts, but also to bring them within the region of actual measurement and calculation, it is convenient to make the theory of magnetic lines of force the basis of electro-mechanical investigations.

If a sheet of paper be laid over a straight steel magnet having opposite poles at its ends, and sprinkled with iron filings, it will be found that these arrange themselves in curves, which we take to be the magnetic lines of force,[1] Fig. 1. Each of these lines forms a closed curve

[1] A very convenient way of fixing these curves is by the aid of a sheet of glass, the surface of which has been coated with a thin layer of paraffin. The glass is laid over the magnet, then sprinkled, and carefully lifted off so as not to disturb the filings. It is then gently heated, when the paraffin melts, and upon cooling again the iron filings are fixed to the glass by the coating of paraffin. The glass plate may then be handled as if it were a drawing, and the curves can be reproduced by photography. The drawing in the text has been obtained in this manner.

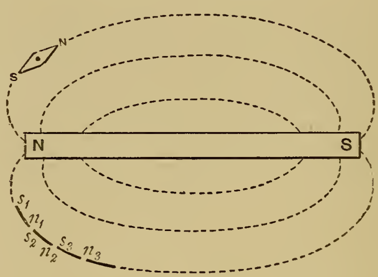
issuing from a point at one end of the magnet, and entering at a corresponding point at the other end. Some of the curves extend far out into space, beyond the surface of the paper, and as far as they are visible, they appear as open lines growing fainter the farther we go from the poles. They must, nevertheless, be considered to be closed lines, only so faint that we cannot trace them throughout their whole length. If the poles of our magnet were two mathematical points, all the curves would pass through those points, but since we

Fig. 1.



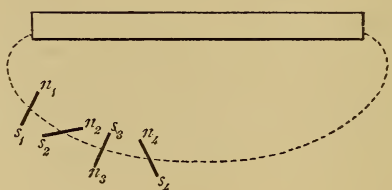
have to deal with a physical magnet, the poles of which are surfaces of some extension, the lines issue from all over these surfaces. To investigate the magnetic properties of these lines we can use a long thin magnetic needle (a magnetized knitting-needle answers very well) suspended vertically by a long thread, so that the lower end of the needle is within a short distance of the paper, and free to move all over it. We shall then find that the lower end of the needle will be repelled by one pole of the magnet and attracted by the other, and in following the combined action of these forces, it will move along that particular line of force upon which it was set on to the paper in the first instance, but it will never move across the lines. We conclude from this experiment that the lines of force are paths along which a free magnet pole is urged under the influence of the magnet. A free magnet pole of opposite sign would travel along the same lines, but in opposite direction, and, if of the same strength, it will be urged along with an equal force. If, instead of a long vertical needle, we take a very short one suspended horizontally in its centre close to the surface of the paper, the two

Fig. 2.



opposite forces will tend to set the needle so as to form a tangent to the line of force passing through its centre, and as then the two forces are equal and opposite, no bodily shifting of the needle can take place. But on whatever point of any of the curves we set the needle, it will always swivel into such a position that its magnetic axis, that is a straight line joining its two poles, becomes a tangent to the curve. (Fig. 2.) It should here be remarked that unless the needle is very short in comparison to the magnet it will, when placed near one of the poles, be drawn right up to it, because in this case there would be a sensible difference in the distance of either of its poles from that magnet pole, and consequently the opposing forces would no longer be in equilibrium. But if the needle is very short, say only the length of a particle of iron filing, this inequality between the attracting and repelling force will at a short distance from the magnet pole become omissible, and then the particle of iron filing will only set itself into the direction of the line of force in that place, but not move bodily along it. We may thus regard each particle of iron filing which has been sprinkled over the paper as a very short magnetic needle, and each line of force as a chain of such needles linked together by their poles of opposite sign—n_1 s_1—n_2 s_2—n_3 s_3—and so on, as shown in Fig. 2. Imagine now that the particles in one such chain, whilst under the influence of the magnet, could by some process be suddenly hardened into steel, or that we had taken steel filings in the first instance, and then remove the magnet. We would then have a succession of little magnets, whose poles of opposite sign touch, and therefore eliminate each other, with exception of the first and last particle of the chain. Here we would have a free N pole at one end, and a free S pole at the other end, these being a finite distance apart, and therefore able to exert magnetic action on other pieces of iron placed into their neighbourhood. But let each particle be turned round its centre (without however, shifting it, bodily) so as to break contact with its neighbour, and we shall have a disjointed line of very small magnets, (Fig. 3), none of which is able to exert any magnetic attraction or repulsion at a distance, because on account of the proximity of the two opposite poles in each particle, their distances from any external point to be acted on would be sensibly equal, and consequently the opposite forces would be in equilibrium. By turning each particle so as to thoroughly break contact with its neighbour, we have completely destroyed the magnetic action of our chain at a distance. If we had turned only a few of the particles, or if we had turned all through a very small angle, so as not to completely interrupt their magnetic continuity, the magnetism of the chain as a whole would have been weakened but not destroyed completely. We can restore our magnetic chain again by turning each particle back into its original position, and if this process should be too laborious to be performed by hand, we can accomplish it in an instant by replacing our magnet under the paper,

Fig. 3.



when its line of force corresponding to the chain of particles, will pass through it again and swivel each into a tangential position, whereby poles of opposite sign are again brought into contact, thus eliminating each other, with the exception of the two free poles at the ends of the chain.

According to the modern theory of magnetism[1] as developed by Weber, Wiedemann, Hughes, and others, what has here been described for a chain of iron filings lying on the sheet of paper, actually takes place within

[1] Proceedings Royal Society, May 10, 1883; also a paper on "The Cause of Evident Magnetism in Iron, Steel, and other Magnetic Metals," read before the Society of Telegraph Engineers and Electricians, and reported in their Journal, vol. xii., No. 49.

the body of any piece of iron or steel whilst being magnetized. According to this theory, each molecule of iron or steel is a complete magnet; it is provided at one end with a definite quantity of magnetic matter of one sign, and at the other end with precisely the same quantity of magnetic matter of the opposite sign, and these magnetic charges are an inseparable attribute of matter like its gravity or chemical or thermal properties, and they can neither be increased nor diminished. In an unmagnetized bar of steel these molecular magnets are supposed to form either chains closed in themselves or disjointed chains, their magnetic axes pointing in all possible directions, and therefore, as was the case in our chain of iron filings after we had rotated them, incapable of magnetic action at a distance. But if, by some means, it were possible to turn all the molecules so as to point one way, without, however, displacing them bodily, we would obtain a number of parallel magnetic chains showing free magnetism at their ends only, and therefore capable of exerting magnetic attraction and repulsion at a distance; in other words, our bar of steel would become a magnet. It will be seen that according to this theory the molecules composing a bar of magnetizable steel must be capable of rotation around their centres, and the more easily and completely they can be rotated, the greater is the degree of magnetization obtained. Since we cannot take hold of each molecule and rotate it mechanically, we must adopt the other method, viz., that of sending lines of force through the bar to perform that work, as we did with the chain of iron filings. This can be done either by the aid of another magnet, or by an electric current. The setting of molecules into continuous chains will be the more complete, the less resistance or internal friction they offer to rotation, and the more powerful are the lines of force which are caused to pass through the bar of steel. In very soft steel, or in soft iron, the molecules rotate freely, and can be set almost completely into continuous chains, but the harder the steel the smaller is the angle through which each molecule can be rotated, and the more magnetizing force is required for this purpose. In such cases the magnetic chains are more or less discontinuous, and the magnetism appearing externally is weaker. On the other hand, the molecules once rotated into position of magnetic continuity are not so easily disturbed again, and thus the harder the steel the more permanent is its magnetization. In soft iron the molecules will lose their magnetic continuity as easily as it was acquired, and the slightest mechanical strain or vibration is sufficient to destroy the greater part of the previous magnetization. To illustrate this we may take a glass tube filled loosely with iron filings, which can be magnetized by drawing the pole of a magnet along it. We shall then see that the particles of filing which previously were lying in all possible directions, have now become more or less parallel to the tube, and the whole appears more like a solid piece of iron of very fibrous texture. The tube has now become a magnet, and if it be carefully handled so as not to disturb the arrangement of the particles, it can be used as if it were a solid steel magnet, and all the usual phenomena of attraction and repulsion at a distance can be obtained. But on tapping or shaking the tube the particles relapse into their former confused position, and all traces of external magnetism of our tube vanish. From this short outline of Professor Hughes' theory it will be seen that the only way in which we can act upon the molecules in the interior of a bar of iron or steel is by sending lines of force through it. The greater the number of lines, or the more powerful the individual lines which we can force through the bar—or, in other words, the greater the magnetizing power—the greater will be the number of molecules which are thereby arranged into more or less complete magnetic chains, and if the metal is hard enough these chains in their turn become the seat and origin of lines of force, and can then be used to magnetize other bars. It will also be clear that after a bar has been magnetized, the space surrounding it becomes filled with lines of force which emanate from it. Strictly speaking, each magnet is surrounded by lines extending into infinite space, but practically they can only be traced throughout the space immediately surrounding the magnet, and this space is called the "*magnetic field.*" Since magnetic lines are not a reality, but only a convenient conception, we can adopt any simple way of expressing their magnitude, or, to speak more correctly, the intensity of the magnetic field at any given point. We can either assume that the lines are of different strength, and that the mechanical force with which a given free magnet pole is urged along any one particular line is dependent on the strength of that line, which may be different from that of any other line belonging to the same field; or we can assume that all the lines are of the same strength, but that the number of lines passing through unit space of the field is different at different points of it. According to this assumption, the intensity of the field in any given spot, and the mechanical force exerted on a free magnet pole, is proportional to the number of *unit lines* passing through unit space at that particular spot. This is the more convenient way of estimating the magnitude of the mechanical forces produced by the magnetic field, but it must not be considered to be a representation geometrically true, and if we try to consider it so, the want of reality in our conception of lines of force becomes at once apparent. This will be seen from the following consideration. If, as we assume, a mechanical force can only be exerted by lines actually passing through the magnet pole, it will be evident that in case the pole be a mathematical point, only one line can pass through it and exert mechanical force on it. This force would therefore be quite independent of the density of lines around the pole. If the pole, although of the same strength, had finite dimensions, more lines would actually pass through it, and more mechanical force would be exerted. Experiment, however, shows that this is not the case, and that within reasonable limits the mechanical force is independent of the extent of the pole, and only depends on its free magnetism. From this we conclude that a strictly geometrical representation of the density of lines in a magnetic field, in the same manner as we might represent the density of trees in a forest, would be incorrect. We cannot pretend to solve the problem of finding a geometrical representation for our conception of the intensity of the magnetic field, and we must be content to use the term in its conventional sense, without having any clear idea of how it could be represented by a mechanical model. Yet this is no reason why we should abandon such an extremely convenient method of representing magnetic action at a distance. Nobody has as yet succeeded in explaining the action of gravitation, or has been able to represent it by a mechanical model. Nevertheless we find no difficulty in using the conventional terms of acceleration, mass, and weight of bodies in our calculations. We know that the weight of a body equals the product of its mass and the acceleration due to gravity. If we put strength of pole for mass and intensity of field for acceleration due to gravity, we find the analogue to weight in the mechanical force with which a free magnet pole is acted on when placed in a magnetic field.

From what has been said above, it will be evident that we must define *magnetic field of unit intensity* as that in which a free magnet pole of unit strength is acted on with unit force. To define a magnet pole of unit strength we must have recourse to the well-known expression for the mechanical attraction or repulsion existing between two poles placed at a certain distance apart. The law has been established experimentally by Coulomb,[1] with the aid of his torsion balance, and verified by Gauss,[2] who used for the purpose a large fixed magnet, and a smaller suspended magnetic needle. It is as follows. If M and m denote the strength of the two poles, and if they are placed at a distance, r, from each other, the mechanical force (attraction or repulsion according to whether the poles are of dissimilar or similar sign) acting between them is $\frac{M m}{r^2}$. If both poles are equal and of the strength m, we have $\frac{m^2}{r^2}$, and if their distance be unity, the force acting between them will equal the square of the free magnetism of one pole. The force will be unity if the free magnetism is unity. We find, therefore, the definition for *unit pole to be a pole of such strength that when placed at unit distance from an equal pole, the two will act upon each other with unit force.* It remains to define unit force

[1] Wüllner, "Experimentalphysik," iv., § 5.
[2] Wiedemann, "Elektricität," iii., p. 116, *ante.*

and unit distance. This might be done on any convenient basis of the measurements of mass, length, and time. In electrical calculations it is customary to use for this purpose

The Gram as the unit of mass.

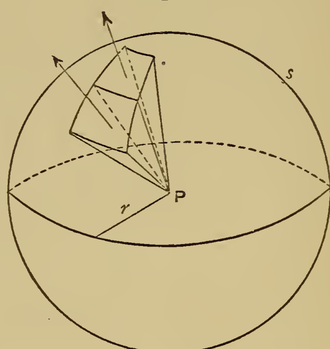
The Centimeter as the unit of length.

The Second as the unit of time.

On these units is based what is known as the *Absolute System of Electro-Magnetic Measurements*. Taking these units as the basis for our calculations, we can find all other units of measurement, because they are all connected in some way with the fundamental units of mass, length, and time. We find thus that the unit of velocity in one centimeter per second, that of acceleration is an increase of velocity of one centimeter per second, and since mechanical forces are measured by the product of mass and acceleration, we define the unit of mechanical force, the *Dyne*, as that force which applied to a mass of one gram, during one second, will give it a velocity of (or accelerate its velocity by) one centimeter per second. The mechanical energy represented by the force of one dyne acting through a distance of one centimeter is the unit of energy, and is called the *Erg*. Having accepted these fundamental and derived units, we can now proceed to establish units for the lines of force, and for the intensity of the magnetic field. We call a *unit line of force* one of such strength that if a unit pole be placed on it, it shall be urged along it with the force of one dyne. A *unit magnetic field* would be one in which a unit pole would be acted on with the force of one dyne. If we find experimentally that an equal force is exerted in all points of a certain portion of the field (as is the case with the magnetic field of the Earth within certain limits), we say that this particular portion of the field is of uniform magnetic intensity, and we consider all the lines of force to be straight, parallel, and equidistant. A *uniform magnetic field of unit intensity* is therefore one in which every square centimeter of transverse section is traversed at right angles by one unit line. We can now determine the number of unit lines which emanate from a free unit pole. Before doing so, a few words of explanation regarding this conception of a free magnet pole are necessary. It has been shown above that magnets are produced by the adjusting of their molecules into continuous chains; and that, therefore, equal quantities of magnetic matter of opposite signs are produced at the poles of the magnet. Experiment shows that it is physically impossible to produce a magnet with one pole only, and that therefore no such thing as a free magnetic pole can be found in nature. But we can get an approximation to the free pole by making the magnet very long in comparison to the strength of its poles. In this way the magnetic influence of each pole will be sensibly felt through a distance considerably smaller than the length of the magnet, and when investigating the magnetic properties of the space immediately surrounding one pole we can neglect the disturbing influence of the other pole. In this case the lines of force emanating from the pole under consideration will be straight radii, shooting out from the pole all around into space. Let, in Fig. 4, P be the pole, and S a sphere described around it as centre, then this sphere will be pierced by the lines of force, in points which are all equidistant from the pole. Let r be that distance, and M the strength of the pole, we find the mechanical attraction exercised upon a unit pole of opposite sign placed at any point on the surface of the sphere, by the expression $\frac{M}{r^2}$.

If, now, a second sphere be described around *P*, with a radius larger than *r* by only an infinitesimal amount, we shall have a spherical shell of infinitely small thickness, within which the intensity of the field is uniform. Into whatever point between the two surfaces of the shell we may place our unit pole, we find that it is attracted with the same force towards *P*, and from this we conclude that the density of lines all over the spherical surface must be uniform. Since in a uniform field the force exerted upon unit pole in the direction of the lines is equal to their

Fig. 4.



density (or number of lines per square centimeter of transverse section), we conclude that through each square centimeter of surface on the sphere, there pass $\frac{M}{r^2}$ unit lines. Now the total surface of a sphere of radius *r*, is $4 \pi r^2$, and consequently the total number of lines emanating from the pole of the strength *M* is

$$4 \pi r^2 \times \frac{M}{r^2} = 4 \pi M.$$

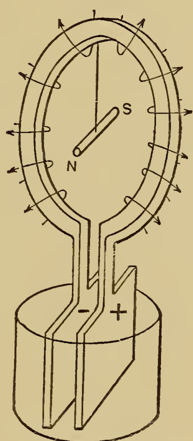
If the pole *P*, instead of having the strength *M*, were a unit pole, the total number of lines would evidently be $4\ \pi$, and thus we find a second definition for *unit pole* as a pole of such strength that $4\ \pi$ unit lines of force emanate from it. This definition is evidently identical with the following: *Unit pole produces unit intensity of field at unit distance.*

Up to the present we have only dealt with magnets and the mechanical forces exerted by them. It will now be necessary to investigate the relations between an electric current and the mechanical force it can exert when

Fig. 5.

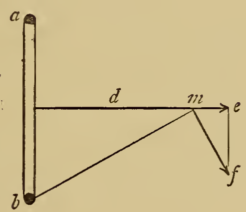
brought into a magnetic field. Experimental facts form now, as before, the basis of our investigation. Let, in Fig. 5, *a* be the cross-section of a wire passing vertically through the surface of the paper, and assume that a current is flowing down the wire. If we sprinkle iron filings on to the paper near the wire, we find that they arrange themselves in concentric circles around it, and if we shift the paper into other places along the wire, we find the same result. From this experiment we conclude that the wire throughout its whole length is surrounded by circular lines of force, or as it is sometimes called, by a magnetic whirl. If we suspend a long thin magnet parallel to the wire, so that its lower end is free to move along the surface of the paper, it will have a tendency to rotate round the wire, but continuous rotation cannot be obtained, because the upper end of the magnet has a tendency to circle round the wire in an opposite direction. If a short magnetic needle, suspended in its centre, is placed horizontally on the paper, it will set itself tangentially to the lines of force, and therefore at right angles to the wire. Each circle of iron filings must be considered as a chain of small magnets closed in itself, and if we were to lay a ring of steel around the wire on the paper, it would become a continuous magnet. Upon removing the ring it would not show any external magnetization, because all along, its molecules are in contact with their opposite poles, but if we interrupt this continuity by cutting the ring open in one place, the ends severed will show opposite polarity when the ring is straightened out. If, instead of a complete ring, we had placed only a segment of a ring or a straight piece of steel at right angles to the wire, it would upon removal at once show magnetic properties. We see from these experiments that it is possible to magnetize a piece of steel by passing an electric current in its neighbourhood at right angles to it. All the experiments detailed above will succeed equally well with a bent wire, and if we employ a coil of wire with a piece of steel inserted at right angles to the plane of the coil, its magnetization will be considerably greater than where only one straight wire is used. The annexed sketch, Fig. 6, will give a clear idea of the lines of force surrounding a circular coil in which a current flows. The zinc and copper plate of a voltaic cell are joined by a stout square wire bent into the form of a circle, as shown, and since all the lines pass around the wire in the same sense it follows that the whole interior space of the circle is filled by a bundle of lines piercing the plane of the coil at a right angle. A free magnet pole would therefore be drawn through the coil in one sense or the other, according to the sign of the pole and the direction of the current. If a small magnetic needle be suspended in the centre, it will set itself at right angles to the plane of the circle and the direction in which its N pole is urged, is given by the following rule due to Ampère: *Imagine a person swimming with the*

Fig. 6.



current and looking towards the needle, then its N pole will be urged towards the left. If, instead of a magnetic needle, we place a non-magnetic piece of steel into the same position it will become magnetized, N at its left and S at its right end. It will be evident that if we approach the N pole of a magnet to the circle from the front, the side turned towards the observer in the figure, it will be repelled, and if we approach a S pole it will be attracted. The opposite takes place on the back. The same would happen if instead of the circular wire traversed by a current, we had a very short magnet of equal diameter. To put the magnet into the same condition as the wire its length would have to be equal to the thickness of the wire, and it would thus become a flat disc, one side of which we assume to be covered with N magnetic matter, and the other side with an equal amount of S magnetic matter. By properly choosing the amount of magnetism distributed over the discs, we can obtain a magnet which in its action at a distance is absolutely equivalent to the circular current, and such a magnet is called the *equivalent magnetic shell.* The action which a physical magnet or a magnetic shell equivalent to a closed current can exert at a distance is most conveniently expressed by the *magnetic moment,* that is the product of strength of poles with their distance. A magnet one centimeter long having unit poles has unit moment. Experiment shows that the magnetic moment of a plane closed circuit is equal to the product of area enclosed by the current and strength of the current, and we can therefore define unit current as *that current which flowing in a plane circuit is equivalent to a magnetic shell the moment of which is numerically equal to the area of the circuit.* Let in Fig. 7, *a b* represent the cross-section through a circular conductor of radius, *r,* traversed by a current, *c,* and *m,* a magnet pole placed at a distance, *d,* from the centre of the coil, then it is found experimentally that each element of the conductor exerts a force on the magnet pole which is numerically equal to the product of strength of current by length of element, by strength of pole divided by the square of the distance ; and the direction of which is at right angles to the plane passing through the element and through the magnet pole. The force due to the element, $d\,l$, situate at b, is therefore $m\,f$, and its amount is $d\,F = \frac{c.\,dl.\,m.}{d^2 + r^2}$. The horizontal component of this force is evidently $d\,H = d\,F \frac{r}{\sqrt{d^2 + r^2}}$ and since the same holds good for any element along the circle, we find the total force by integration between the limits o and $2\,\pi\,r$ . . . . $H = c.\,m.\,\frac{2\,\pi\,r^2}{(d^2 + r^2)^{\frac{3}{2}}}$.

Fig. 7.



If the magnet pole lies in the centre of the coil, $d = o$, and the force is evidently $H = \frac{c.\,m.\,2\,\pi}{r}$.

This equation provides another definition for unit current. It will be seen that if m, r and c are equal to unity, H is equal to $2\,\pi$, and we may define unit current as *that current which, flowing in a wire forming a circle of unit radius, acts on a unit pole placed at the centre with a force of* $2\,\pi$ *dynes.*

If a magnet be inserted into a coil of wire which is connected to a delicate galvanometer, a current will be observed to flow through it for a short time, and if the magnet be withdrawn from the coil, a momentary current in the reverse direction is created. Now since it is impossible that a current should flow without there being an electromotive force in the circuit, we conclude that the act of thrusting a magnet into the coil, or suddenly withdrawing it, sets up an electromotive force in one direction or the other in the wire itself. To explain this phenomenon, we have again recourse to the conception of lines of force. It is evident that in approaching the magnet to the coil we move not only the metal alone, but also *all the lines of force which surround it,* and in so doing we cause these lines, or at any rate some of them, to cut the wire of the coil. The same happens if the magnet remains at rest and we move the coil relatively to it ; the wire cuts through the lines of force and an electromotive force is set up in it in consequence. We cannot explain the why and wherefore of this action, and must rest content to accept it as abundantly proved by experiment. A careful investigation also shows that the strength of the current, and consequently the amount of electromotive force set up, is directly proportional to the speed of movement and to the strength of the magnet. We conclude from this that the electromotive force is proportional to the rate of cutting lines, that is, to the number of lines cut per second by each wire. It is also proportional to the number of wires in the coil. We also find that in thrusting the magnet into the coil we experience a resistance necessitating the expenditure of mechanical energy, the amount of which is proportional to the product of current and electromotive force. This resistance, and the mechanical energy necessary to overcome it, will be the greater the lower the electrical resistance of the coil, provided other things remain equal, and if the coil be open so that no current can pass, there will be no opposing force to the movement of the magnet. In order to investigate this phenomenon of the creation of electromotive force by the movement of a conductor in a magnetic field, we will assume the simplest possible case, viz., that of a uniform field, the lines of which we suppose to be vertical. Let two metallic bars be fixed at equal distance from the ground, and parallel to each other, and let a third bar, which we term a slider, be laid at right angles across these bars, and let it be free to move parallel to itself, but always remaining in contact with them. As soon as the slider is set in motion, a difference of potential will be created between its ends where it rests on the bars, tending to make electricity flow from the bar of higher to the bar of lower potential. Such a flow of electricity will actually take place, and can be made visible if the bars be connected by a galvanometer. Let d be the distance between the bars, v the velocity of the slider, and F the intensity of the field, then the difference of potential between the bars, is $F. d. v$, which product also expresses the number of lines of force cut by the slider per second. If the distance between the bars be one centimeter, and the velocity one centimeter per second, and the intensity of the field be also unity, we obtain the unit of electromotive force. We define, therefore, as the *unit of electromotive force, that which is created in a conductor moving through a magnetic field at such a rate as to cut one unit line per second.* Imagine that the bars and the galvonometer connecting them have absolutely no electrical resistance, but that the slider has a resistance r, then by Ohm's law the current produced through the circuit, whilst the slider is in motion, will be

$$c = \frac{F. d. v}{r}.$$

If the intensity of the field is unity ($F = 1$), and if the bars are one centimeter apart, unit current will be produced at a velocity $v = r$. We find, therefore, that the electrical resistance of the slider, and for the matter of that the electrical resistance of any conductor, can be expressed in the same terms as a velocity. We say that the resistance of a conductor is so many centimeters a second. It is customary to express resistances by reference to a standard resistance, *the ohm.* The relation between this and the unit of resistance in absolute measure will be shown presently. Before doing so, we must, however, say a few words about the energy required to move the slider, and about the relation between current and mechanical force. Let P represent the pull in dynes required to move the slider across the lines of a field of intensity F, with a velocity of v centimeters a second. The energy expended in ergs will evidently be

$$W = P. v.$$

By the principle of the conservation of energy, this must be equal to the electrical energy produced. The question which now presents itself is the determination of the electrical energy of a current, c, flowing under a difference of potential of $F. d. v.$ We have up to the present used the term potential without giving its definition. As the name implies, the potential of a body is its property of allowing energy stored up in it to become potent, that is, to do work. If a weight be raised to a certain height from any given datum level, the mechanical work thereby expended can be recovered by allowing the weight to descend again whilst overcoming the resistance of some piece of mechanism which can be made to do useful work. In its elevated position the weight has, therefore, a certain potential energy, which is equal to the product of the weight multiplied by the distance to which it has been raised. If the weight be unity, this product is numerically equal to the height, and we can say that the mechanical potential of a heavy body raised to a certain height above datum level equals the mechanical energy required to lift unit weight to the same height. By multiplying the potential thus defined with the weight of the body we obtain the total mechanical energy which it is capable of exerting. Similar reasoning applies with regard to the transfer of electricity. It is well known that two bodies charged with electricity of the same sign repel each other, and if one of the bodies is fixed whilst the other is being approached to it mechanical energy must be expended in the act of approaching. This energy can again be recovered (provided there were no losses by dissipation of electricity into the surrounding air) by allowing the movable body to recede from the body at rest whilst doing useful work. To fix ideas, let the stationary body be a very large metallic sphere charged with a certain amount of positive electricity, and let the movable body be a very small gilded pith ball charged with a unit of positive electricity. We assume a great difference in the size of these bodies in order that the charge on the larger body shall not be sensibly altered by the variation of position of the small body. If we remove our pith ball to an infinite distance, so as to be completely beyond the repulsive action of the larger body, we can consider it to be in that position analogous to unit weight placed at datum level. If we now advance the pith ball up to the large sphere, we shall have to perform mechanical work, and, according to Lord Kelvin's definition, the electrical potential of the sphere is measured by the amount of mechanical work performed. If, instead of starting from infinite distance, we had started from another sphere having a different potential from the first, the mechanical work performed in the transfer of the pith ball would be a measure of the *difference of potential* between the two spheres. It will appear self-evident that if, instead of only one pith ball, we transfer two, three, or more, or if the charge of the pith ball, instead of one unit, were two, three, or more units of electricity, the mechanical energy would also be increased in the same proportion. From this it follows that the mechanical energy required to transfer q units of electricity from a sphere or point where the potential is p_1 to a sphere or point where the potential is p_2 will be—

$$-q\,(p_1 - p_2)$$

and this result will not be altered if the transfer, instead of taking place by the aid of our pith ball conveying a definite electrical charge q, were to take place by means of a wire carrying a continuous current, since the latter can be considered as a succession of pith balls. In our experiment with the slider, c is the current or quantity of electricity transferred in one second, and the mechanical energy represented by the current during the interval ot one second is therefore

$$c\,F\,d\,v,$$

which by the principle of the conservation of energy must be equal to the mechanical energy expended during one second in moving the slider. We find, therefore, the relation

$$P = c\,F\,d.$$

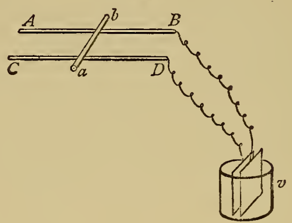
The mechanical force experienced by a straight conductor d centimeters long, carrying a current c, and situate in a uniform field of intensity F, the lines of which are at right angles to the conductor, is equal to the product of length of conductor, current, and intensity of field. This relation is of the utmost importance for the construction of electromotors, inasmuch as the mechanical forces thus determined are the real source of power of these machines. It would, therefore, be desirable to verify the expression obtained above by some other method of reasoning, and this can easily be done if we go back to what has been said about the relation existing between a current and the force exerted by it on a free magnet pole. It was then stated that experiments have shown the force to be equal to the product of length of conductor, current, and strength of pole, divided by the square of the distance. We assume hereby that the conductor stand at right angles to the line joining its centre with the pole, and that it be very small in comparison to the distance from the pole. All the straight lines which can then be drawn from the pole to different points of the conductor intersect it at right angles, and can be considered to be parallel. The conductor lies, therefore, in a uniform magnetic field of the intensity $F = \frac{m}{R^2}$, m being the magnetism of the free pole, and R its distance from the conductor. Let d be the length of the conductor, c the current, and P the mechanical force exerted on the pole, we have

$$P = \frac{m\,c\,d}{R^2},$$

as has already been shown. But since action and reaction must be equal, the conductor acts upon the pole with precisely the same force as that exerted by the pole on the conductor ; and we find that the force tending to lift the conductor out of the plane laid through it and the pole is also equal to P. By substituting F for $\frac{m}{R^2}$ we have therefore $P = c\ F\ d$, the same expression as obtained above.

Returning now to the above example of the two horizontal bars and the slider laid across them, let, in Fig. 8, *A B, C D,* represent the two bars, *a b* the slider, and *V* a voltaic cell connected to the bars by wires, as shown. The lines of force—not shown in the diagram—are supposed to be vertical, and therefore at right angles to the slider and to the bars. From what has been stated above, it will be seen that on establishing connection with the voltaic cell, the current flowing through the slider will

Fig. 8.



generate a force tending to move it along the bars parallel to itself. This force could be utilized by attaching a cord to the slider, which, passing over a pulley, could be made to raise a weight. We have here the most simple case of transforming electrical into mechanical energy. As soon as the slider begins to move, it cuts through lines of force, and, as was explained above, by this action a difference of potential is created at its ends, or, as we can also express it, the slider becomes the seat of an electro-motive force. A moment's reflection will show that this electro-motive force must be directed in opposition to the electro-motive force of the cell, for, were it not so, the original current would be increased by the creation of this second electro-motive force, and we should thus obtain additional electrical energy and mechanical energy at the same time, which is clearly incompatible with the principle of the conservation of energy. If in a circuit there are two electro-motive forces, the current resulting from their combined action is proportional to their sum. Since, in this instance, the electro-motive force of the slider is opposed to that of the cell, we must consider it to be negative, in fact a *counter-electro-motive force,* and the resultant electro-motive force in the circuit will be $E—e$, if by E we denote that of the cell and by e that of the slider. The resultant current is therefore found by dividing $E—e$ by the total resistance of the circuit. As the slider moves along the bars, this resistance is evidently constantly increasing or diminishing, according to the direction in which movement takes place. Not to complicate the problem by the introduction of a variable resistance, we shall therefore assume that the bars are so thick as to have practically no resistance, and in that case the total resistance will consist only of that of the slider, the connecting wires, and the cell. Let that be r as before, and we find the current $c = \frac{E—e}{r}$ by Ohm's law.

The mechanical energy exerted in raising the weight P with a velocity of v is per second : $W = P v$; and that must be equal to the electrical energy which is the product of current and difference of potential between the ends of the slider. Let, as before, F represent the intensity of the field, and d the length of the slider, we have:

$$W = c\ F\ d\ v$$

$$W = \frac{E—e}{r}\ F\ d\ v,$$

and since $e = F\,d\,v$, we have also

$$W = \frac{E - F\,d\,v}{r}\,F\,d\,v.$$

According to our former definition of intensity of field, F represents the number of unit lines of force passing through one square centimeter of surface between the bars, and $d\,v$ is the surface swept over by the slider in one second. The product $F\,d\,v$ represents, therefore, the number of unit lines cut by the slider per second. If we denote this number by z, we have also the following expression for the mechanical energy represented by the raising of the weight:

$$W = \frac{E - z}{r}\,z.$$

This formula will be used later on for the determination of the mechanical energy obtainable with a given electromotor. For the present it will be more convenient to retain the symbol e, and we write,

$$W = \frac{E - e}{r}\,e.$$

Since $e = F\,d\,v$, and $P = c\,F\,d$, we have the relation,

$$P = \frac{c\,e}{v},$$

from which it will be seen that with a constant speed and with a constant current the weight which the slider is capable of hauling up, and therefore its capacity of doing work, is directly proportional to the counter-electromotive force. It will also be seen how mistaken is the notion that counter-electro-motive force in an electromotor is a loss, and that those well-meaning but confused inventors who strive to design motors which shall have as little counter-electro-motive force as possible, so as not to check the flow of current which works the motor, would, if they were successful, obtain machines which could not give out any power at all.

The energy given out by the cell is $E\,c$, that performed by the slider is $e\,c$, and the efficiency of our simple motor is therefore

$$\eta = \frac{e}{E}.$$

In order to find the condition of maximum work performed, we form the differential quotient of W, and equal it to zero, the variable being the counter-electro-motive force e. That gives,

$$o = \frac{d\,W}{d\,e} = E - e + e\,\frac{d}{d\,e}(E - e),$$

$$o = E - 2\,e,$$

$$e = \frac{E}{2}.$$

If the speed of the slider be so regulated, by adjusting the load, that its counter-electro-motive force is equal to half the electro-motive force of the cell, the maximum possible amount of mechanical work will be performed, the efficiency in this case being 50 per cent.

$$W\,max. = \tfrac{1}{4}\,\frac{E^2}{r}.$$

In order to obtain that speed of the slider, we must regulate the weight P attached to the cord, so that,

$$v = \frac{E}{2\,F\,d},\ P = \frac{E\,F\,d}{2\,r},\ \text{and}\ c = \frac{E}{2\,r}.$$

If a heavier weight were attached to the cord, the current would be greater and the speed smaller; if a lighter weight were attached, the current would be less and the speed greater. In both directions there exists a limit which will be reached, on the one hand by reducing the weight to zero, when the speed will be a maximum, and, on the other hand, by using so heavy a weight that the slider cannot move at all, when the current will be a maximum. These limiting values can easily be obtained from the above formulas, and are as follows:

Weight completely removed,

$$P = o, \; c = o, \; e = E, \; v = \frac{E}{F\,d}.$$

Weight just balances force of slider, which remains at rest,

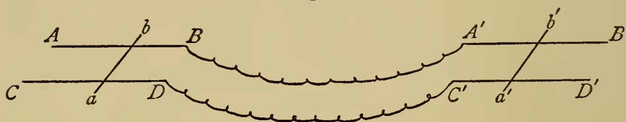
$$P = \frac{E\,F\,d}{r}, \; c = \frac{E}{r}, \; e = o, \; v = o.$$

On comparing these expressions with those found for the condition of maximum work, it will be seen that in the latter case the current is half as great as that obtained with the slider at rest, and the velocity half as great as that of the slider with the weight removed. The statical pull on the slider when doing maximum work is half that obtained with the slider at rest.

These investigations, although at first sight they might seem somewhat abstruse, because no engineer would think of pulling up weights by the arrangement of a slider as described, are nevertheless of great practical importance. Imagine that, instead of having a single slider, we place a number of wires on the surface of the armature of an electro-motor, and that we arrange to have a very intense field by the employment of steel or electro-magnets with suitable devices for commutating the current in the armature wires, which enable us to transform the rectilinear motion of the slider into a continuous rotary motion; and we obtain at once an eminently practical machine. This machine does not differ in principle from our simple slider, and all the general laws we have found above for the latter are therefore applicable to the former. Certain allowances will, of course, have to be made on account of the usual mechanical resistances and losses common to all mechanism, and also on account of certain secondary actions and electrical losses or imperfections inseparable from the adaptation of an abstract or ideal machine to actual work ; but, in general, the laws deduced above hold good in practice. Thus we shall find, that if an electro-motor, having permanent field magnets (either of steel, or electro-magnets excited by a constant current), runs at a speed of 1,000 revolutions a minute when doing no external work, whilst supplied with current at a given electro-motive force, it will do a maximum of work when loaded to such an extent that its speed drops to about 500 revolutions a minute, the electro-motive force remaining the same. If loaded more and more, say by the application of a brake, the speed will be further reduced until the armature of the motor comes to a standstill. In this condition, the statical moment of the armature, or the *torque* as it is also called, will be twice as great as when running at 500 revolutions, and the current passing through it will also be twice its former value. This fact is of importance, as it enables us to calculate the *starting power* of the motor, a point of great interest in the application of motors to tramway or railway carriages. We must at once observe that so large a current should never be allowed to pass through the armature for more than a very few seconds ; and when in regular work, motors are generally so loaded as to run faster than half their idle speed, partly because the current corresponding to half-speed would still be excessive, and heat the wires too much, and partly because we are, as a rule, not content with so low an efficiency as 50 per cent. From the formula for the efficiency given above, it will be seen that the nearer the counter-electro-motive force approaches to the electro-motive force of the source of current (a voltaic cell in our example), the nearer does the coefficient of efficiency approach to unity. But to obtain a high counter-electro-motive force we must allow the motor to run at a high speed.

It has already been shown how a slider, when moved across the lines of a magnetic field over two metallic bars, can be made to produce a current in a wire joining the two bars. It has also been shown how a current sent from an external source into the bars, and through the

Fig. 9.



slider, will cause the latter to move and perform mechanical work. Let, in Fig. 9, *A B, C D,* be the bars receiving the current, and A_1 B_1, C_1 D_1, be the bars in which the current is originated by the movement of the slider a_1 b_1, and it will be clear that by performing mechanical work on the latter slider, we can cause the slider *a b* to give out mechanical work by raising a weight, as explained above. We have here the most simple possible case of the electric transmission of energy. The generating system, A_1 B_1, C_1 D_1, can be at any distance from the receiving system, *A B, C D,* and all that is required are electrical connections (wires to carry the current) between A_1 and *B,* and between C_1 and *D.* Let the intensity of the magnetic field be F_1 at the generator and F at the receiver, and let the pull applied at the generating slider be P_1, whilst that exerted by the receiving slider is P; let also v_1 and v be respectively their velocities, and e_1 and e respectively the electromotive forces, then the following equations evidently obtain:

$$c = \frac{e_1 - e}{r},$$

$$e_1 = F_1\, d_1\, v_1,$$

$$e = F\, d\, v,$$

$$P_1 = \frac{F_1\, d_1\, v_1 - F\, d\, v}{r}\, F_1\, d_1,$$

$$P = \frac{F_1\, d_1\, v_1 - F\, d\, v}{r}\, F\, d,$$

$$\frac{P_1}{P} = \frac{F_1\, d_1}{F\, d}.$$

This equation shows that the pull exerted on the generating slider, and that given out on the receiving slider, bear a fixed proportion to each other which is independent of the speed, but depends on the intensities of the fields and on the dimensions d_1 d of the sliders. The energy expended at the generating system is

$$W_1 = F_1\, d_1\, v_1\, \frac{F_1\, d_1\, v_1 - F\, d\, v}{r},$$

and that given out by the receiving system is

$$W = F\, d\, v\, \frac{F_1\, d_1\, v_1 - F\, d\, v}{r}$$

The ratio between the two, or the efficiency of transmission, is evidently

$$\eta = \frac{F\, d\, v}{F_1\, d_1\, v_1}.$$

If both systems are identical as regards dimensions and strength of field, $\eta = \frac{v}{v_1}$. This would be the case where two identical dynamos are employed, the one as receiver and the other as motor, both machines being series wound, so that the same current circulates around both sets of field magnets. In such cases it has been customary to determine the electrical efficiency of the transmission of energy by simply determining the speeds, and taking their ratio. If no losses and no secondary actions would occur in the connecting wires and in the machines, no objection could be raised to this way of determining the efficiency; but in practice there are some very serious objections to this method. In the first place, the two magnetic fields, although produced by the same magnetizing power, are not of absolutely equal intensity, because the magnetization of the armature produced by the current circulating through its coils has a certain influence in altering the intensity of the magnetic field, and this alteration is different in a motor from what it is in a dynamo. In the second place—and this is a fatal objection—any leak or loss of current taking place at some intermediate point in the wires by which the machines are connected, instead of lowering the efficiency, as determined by the speeds, has actually the effect of making it appear higher than it really is. This will become obvious by reference to the equation for the counter-electro-motive force of the receiving machine. Since $e = F d v$, any reduction in F, consequent upon the loss of some of the magnetizing current through a leak in the line, has naturally the effect of increasing v, the velocity of the receiving machine, and thus it may happen that through the development of a fault in the insulation of the line the ratio of speeds will increase, thus showing apparently an increase of efficiency, whereas in reality the system has become less efficient. The variables in the above equations are v, v_1, P, and P_1; the dimensions of the machines (or sliders) d and d_1, and the intensities of the fields being constant. Since the ratio between the static efforts, P and P_1, is also a constant, the number of variables is reduced to three, and, if two of these are given, the third can be found. As an example, we will take the case that the load P to be put on the receiving machine shall be given (say, for instance, the pull required to haul up a train on a steep gradient, but neglecting for the moment the difference in pull caused by variations of speed) and the speed v_1 of the generating machine shall also be given. We require to know the power necessary to drive the generating machine, and the speed and energy developed by the receiving machine. From the equation for P, we find immediately the speed of the receiving machine,

$$v = v_1 \frac{F_1\, d_1}{F\ d} - \frac{r\, P}{F^2\, d^2}.$$

As will be seen, this speed is not directly proportional to the speed of the generator, and if the latter be increased the speed of the receiver will increase in a somewhat faster ratio. Since the ratio of speeds enters into the formula for the efficiency, it will be evidently advantageous to work the machines at the highest possible speed consistent with mechanical safety. On the other hand, if we lower the speed of the generator beyond a certain point the receiver will not be set in motion at all.

This will happen if $v_1 \frac{F_1\, d_1}{F\ d} = \frac{r\, P}{F^2\, d^2}$,

$$v = \frac{r\, P}{F\, d\, F_1\, d_1}.$$

In this case the efficiency is zero. The minimum speed of the generator is therefore dependent on the dimensions of the two machines and on the strength of the two fields, and is inversely proportional to the product of these four quantities.

The mechanical energy which has to be applied to the generator is $W_1 = P v_1 \frac{F_1 d_1}{F d}$,

and that obtained from the receiver is

$$W = P v_1 \frac{F_1 d_1}{F d} - \frac{r P^2}{F^2 d^2},$$

the difference between the two being lost. This loss, which is represented by the expression $r \left(\frac{P}{F d}\right)^2$, we must regard as energy transformed in a way not suitable for the purpose in view. Since it does not appear in the shape of mechanical energy we must expect to find it appearing in the shape of heat, and this is indeed the case, as can easily be proved. It has been pointed out above that the static pull is the product of current, field-intensity, and the dimension of the machine. The quotient $\frac{P}{F d}$ represents, therefore, nothing else but the current flowing through the circuit, and the above term for the energy lost can also be written in the form

$$r c^2,$$

which, as is well known, represents the heat developed by the passage of the current c through a circuit, the electrical resistance of which is r. Thus the whole of the energy applied at the generator is accounted for, partly by that given out by the receiver, and partly by that used up in heating the circuit. It need hardly be mentioned that the formulas given here for the transmission of energy refer to ideal machines which are free from all other losses, both mechanical and electrical, but that in actual practice these other losses cannot be neglected, and considerably complicate the problems to be solved. The author, nevertheless, has thought it advisable to enter at some detail into the case of transmission of energy by means of ideal machines, not because the formulas obtained are immediately applicable to practical cases, but because they form the basis of other formulas suitably altered for practical purposes, and which will be given in a subsequent chapter. The example cited is also intended to show how easily and simply the system of absolute electro-magnetic measurement can be applied to apparently complicated problems. Before leaving this subject we must refer to the relation between electrical units in absolute measure and those units commonly used in practice. The units as given in the centimeter-gram-second system are of inconvenient magnitude for practical purposes ; some of them are so small that millions and even larger figures are required to express quantities commonly dealt with in practical work, and others are, again, so large as to necessitate the use of fractions. We had already occasion to refer to the three units most often occurring in electro-mechanical problems, viz., current, electro-motive force, and resistance. The unit of quantity of electricity has also incidentally been mentioned as represented by that amount of electrical matter which a given current conveys in one second. For the sake of completing the list we must also mention a property of conductors called their *capacity*, by which term we mean their capacity or power to hold an electrical charge. The capacity is measured by the quantity of electricity with which a body can be charged under an electro-motive force equal to unity. The relation between the so-called practical units and their equivalents in the centimeter-gram-second system is as follows :—

Name of Electrical Quantity.	Practical Unit.	Equivalent c. g. s. Unit.
Current strength	Ampère	10^{1}
Electro-motive force	Volt	10^{8}
Resistance	Ohm	10^{9}
Quantity of electricity	Coulomb	10^{-1}
Capacity	Farad	10^{-9}
	Microfarad	10^{-15}
Rate of doing work	Watt	10^{7}

CHAPTER II.

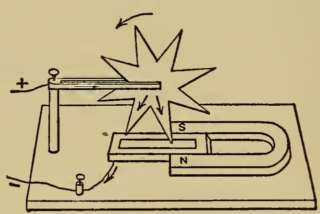
First Electro-motor—Professor Forbes' Dynamo—Ideal Alternating Current Dynamo—Ideal Continuous Current Dynamo—Siemens' Shuttle-Wound Armature—Effect of Self-Induction—Experiments with Electromotors — Hefner-Alteneck Armature — Gramme Armature — Pacinotti Armature—Electro-motive Force created in any armature.

In the preceding chapter it was shown how mechanical energy can be converted into that of an electric current, and how the electric energy represented by a current flowing under a given difference of potential can be reconverted again into mechanical energy and do useful work. The apparatus employed for this double conversion was assumed to be of extremely simple form, in order to limit our investigation to the fundamental laws without obscuring these laws by the introduction of secondary actions and losses. It will now be necessary to confront the subject from a somewhat more practical standpoint, and to show how the conversion between mechanical and electrical energy can be obtained with machinery of a practical form. As a first step towards a practical solution of the problem to produce motive power by an electric current, we must consider Barlow's wheel,[1] invented by Sturgeon and Barlow about seventy years ago. A star-shaped wheel was mounted on a horizontal axis and set over a trough containing mercury in such

[1] Barlow, "On Magnetic Attraction." London, 1823.

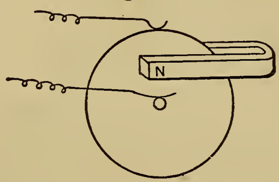
way that during rotation of the wheel one or two spokes were always dipping into the mercury. Fig. 10. A permanent steel magnet *N S* was placed in such position that the lines of force joining its two poles passed transversely across the plane of rotation of the wheel, and upon sending a current through the wheel in the direction indicated by the arrows, rotation was produced in the opposite sense to the hands of a watch as seen from the side on which was placed the *N* pole of the magnet. It will be seen at a glance that this apparatus is nothing else but our arrangement of a slider in rotary form, the lines

Fig. 10.



of the magnetic field being in this case horizontal where they cut through the wheel. Each spoke is a slider coming successively into action as its extremity touches the mercury in the trough and is thus put in electrical connection with the rest of the circuit. It was also found that the experiment succeeded if, instead of a star wheel, a plain metallic disc was employed, the lowest point of the circumference just touching the mercury. In 1831 Faraday reversed the experiment and obtained an electric current from a disc rotating between the poles of a magnet. Fig. 11. The magnet was so placed that the induction between the poles, that is, the lines of force passing from one pole to the other, should pierce the surface of the disc, and the current was taken off by contact springs on the axis and on the circumference; the latter being placed on the radius of greatest induction. Lately Professor George Forbes has constructed dynamos on the same principle, the only difference being that, instead of using a permanent steel magnet, he uses an electro-magnet which becomes excited by the current produced. Professor Forbes' machine [1] is remarkable for the very powerful current it produces as compared to its small size. He has devised several modifications, but

Fig. 11.



for our purpose it will be sufficient to describe one of his arrangements. The armature of this dynamo, which is illustrated in Fig. 12 in longitudinal section, consists of a wrought iron cylinder without any wire on it. The field magnet is a closed iron casing surrounding the armature on all sides, and containing two circular grooves of tapering section into which are laid the exciting coils *E*, formed of insulated copper wire. If a current passes through these coils, it produces lines of force which completely surround each coil, and which pass partly through the iron shell *C D* forming the field magnet, and partly through the armature *A*. The general character of these lines is shown by the dotted curves. It will be seen that as the armature cylinder revolves it becomes the

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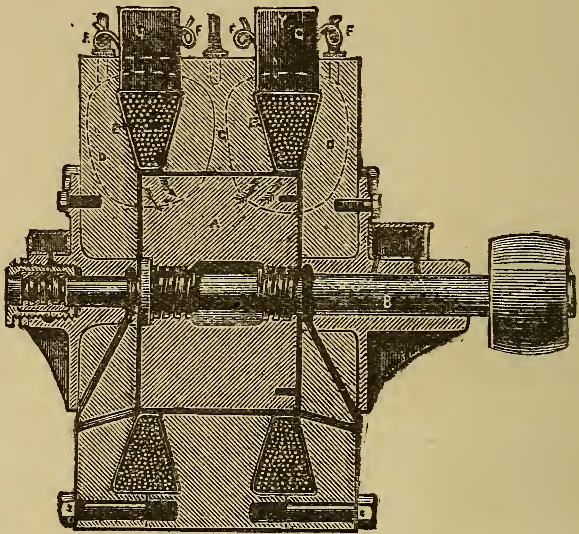
[1] See "The Engineer" of July 17, 1885. The author is indebted to the editor of that paper for the use of the engraving.

seat of electro-motive forces acting at right angles to the lines, as indicated by the arrows, and if we provide rubbing contacts at the ends of the cylinder we can obtain the current due to these electro-motive forces. The contacts are arranged at the inner periphery of the exciting coils, and consist of a series of carbon blocks

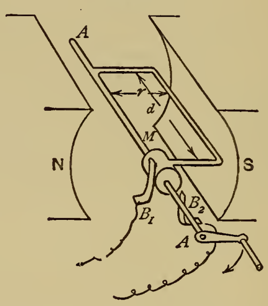
Fig. 12.



FORBES' NON-POLAR DYNAMO.

held in two copper rings, which are connected to the two terminal plates *G G*. The current is thus taken off all around the armature, and the latter contains absolutely no idle portion. This is one of the reasons why the machines are so powerful as compared to their size. The other reason is that the intensity of the magnetic field is very great. As will be shown in a subsequent chapter, when the theory of continuous current motors will be given, the intensity of the magnetic field is the greater the smaller the clearance between the polar surface of the magnet and the core of the armature. In motors or dynamos, which contain copper wire coiled over the armature core, this clearance is necessarily greater than in Professor Forbes' dynamo, where the space between armature and magnet is just sufficient to allow of free rotation. The following figures will serve to give an idea of the relation between the size of these machines and their output of electrical energy. A dynamo having an armature six inches in diameter and nine inches in length, will, when driven at a speed of 2,000 revolutions a minute, give a current of 5,000 Amperes at a difference of potential of two Volts. According to the inventor, an armature four feet in diameter by four feet in length would produce an electro-motive force of sixty Volts when driven at a speed of 1,000 revolutions a minute. If we were to allow the current to increase in the same proportion as the area of the armature cylinder, this machine could produce 320,000 Amperes, and would require about 30,000 h.-p. to drive it. This heavy current would, however, generate more heat in the metal of the armature than could be dissipated at a moderate temperature, and the employment of such an enormous power at the high speed of 1,000 revolutions is of course out of the question, but on purely theoretical grounds it is interesting to notice how easily our simple experiment of the slider when suitably arranged in rotary form will lead to results which on account of their magnitude are quite beyond the capability of modern engineering. Dynamos similar to that just described are generally called *Uni-polar Dynamos.* Professor Forbes prefers the title *Non-polar Dynamos,* and with good reason, for, as was pointed out already in the first chapter, a magnet with only one pole is a physical impossibility. All the dynamos of this class have the disadvantage of requiring to be driven at a very high speed in comparison with the electro-motive force they can produce. The reason lies in this, that the length of conductor cutting through the field is limited by the size of the armature. These machines are practically nothing else but dynamos having only one turn of wire wound on their armature core. An ideal machine of this kind is shown in Fig. 13. The field magnets *N S* are placed

Fig. 13.



IDEAL ALTERNATING CURRENT DYNAMO.

horizontally opposite each other, and their polar surfaces are bored out to form a cylindrical cavity within which one single turn of wire can be revolved by means of a crank. One end of the wire is joined to the axis *A A*, and the other to a metal sleeve *M*, rubbing contact springs B_1 B_2 being arranged in order to take the current off the sleeve and axis respectively. The lines of force pass horizontally across the cylindrical cavity from *N* to *S*, and those which are contained within the space swept by the wire are cut twice during each revolution. The effect is the same as if we had attached our slider to a crank and by turning the latter had caused the slider to assume a reciprocating motion across the lines of the field. In that case, when the crank is vertical, that is, parallel to the lines of the field, the speed of the slider is a maximum, and therefore its electro-motive force is also a maximum. As the crank approaches either of its dead points, where it is horizontal, the speed of the slider and its electro-motive force diminishes and becomes zero at the moment the motion is reversed. From what was said in the preceding chapter, it will also be seen that the direction in which the electro-motive force acts depends on the direction of motion, and the current produced must therefore be alternating. If we plot the angles of the crank on the horizontal, starting from any given position, say, for instance, from its position at the end of the stroke, and the electro-motive forces on the vertical, we obtain a graphic representation of the relation between these two quantities. In a uniform field, where the electro-motive force depends only on the speed of the slider, but not on its position in the field, the electro-motive force is evidently proportional to the sine of the angle of the crank, and is given by the expression

$$E = F\, d\, \omega\, sin\, \alpha,$$

where ω is the circumferential speed of the crank, and α its angular position, the other symbols being the same as before. It will be seen that $E = o$, for $\alpha = o$ and $\alpha = \pi$, whilst for $\alpha = \frac{\pi}{2}$ or $\alpha = -\frac{\pi}{2}$, E attains its greatest numerical value, being positive or negative according to the sign of the angle. The same relations obtain in the ideal alternating current dynamo, Fig. 13. If the crank is in the position shown, the wire is in the middle of the S pole piece and cuts the lines of force at maximum speed; if the crank is vertical, the wire moves parallel to the lines, and its rate of cutting lines is zero. This position corresponds to the end of the stroke with an oscillating slider. When the crank is again horizontal, but pointing to the left, the wire is in the middle of the N pole piece, and again its speed across the lines, or its rate of cutting lines, and the electro-motive force are maxima, but the current will be in an opposite direction to what it was at first. If the crank be turned in the direction indicated by the arrow, the current will leave the machine at the contact spring B_1 during the time the crank is on the right-hand side of the vertical diameter, and it will flow from B_2 through the external circuit, and enter the machine at B_1 during the time the crank is on the left-hand side of the vertical diameter. Let n be the number of revolutions per minute, then $\frac{n}{60} 2 \pi r = \omega$, the circumferential speed of the wire, and the maximum of electro-motive force, irrespective of sign is evidently

$$E = F d \frac{n}{60} 2 \pi r.$$

Now $2 r d$ is the total space swept by the wire, and $F 2 r d$ is the total number of lines passing through that space ; let z be that number, and we find for the maximum of electro-motive force the expression,

$$E = z \pi \frac{n}{60} \quad \ldots\ldots\ldots\ldots \quad 1$$

During one half revolution the electro-motive force increases from zero to this maximum, and then decreases again to zero. As far as practical applications of the dynamo are concerned, it is not the maximum electro-motive force which we require to know, but the *mean electro-motive* force, which acting during the same time as the variable electro-motive force, would cause the same quantity of electricity to flow through the circuit. Let, at any given moment, the wire occupy a position defined by the angle α from the vertical, and let it advance through an angle $\partial \alpha$ during the time ∂t, then the quantity of electricity flowing through the whole circuit of resistance R is evidently

$$\partial q = \frac{F\, d\, \omega \sin \alpha\, \partial t}{R},$$

and since $\omega = r \dfrac{\partial \alpha}{\partial t} \ldots . \partial q = \dfrac{F\, d\, r}{R} \sin \alpha\, \partial \alpha.$

During one half revolution α increases from zero to π, and the integral of the above expression taken between these limits gives

$$q = 2\, \frac{F\, d\, r}{R}.$$

The time occupied in this transfer of q units of electricity is $t = \dfrac{\pi\, r}{\omega}$, and if, during that time, a constant electro-motive force E^1 were acting, the quantity transferred would be $\dfrac{E^1}{R} \cdot \dfrac{\pi\, r}{\omega}$. If this quantity is equal to q, we consider E^1 the average electro-motive force, and its value is given by the equation

$$E^1 = \frac{2}{\pi} F\, d\, \omega.$$

Since $F\, d\, \omega$ is the maximum electro-motive force generated at the moment when the wire is cutting the lines of the field at right angles, we have also

$$E^1 = \frac{2}{\pi} E.$$

It should be noted that the mean value of $E\, M\, F$ as here defined refers to the total quantity of electricity which the apparatus is capable of forcing through a given resistance, but not to the amount of work produced and transformed into heat in the resistance. Inserting the value for E from equation 1, we find the average electro-motive force

$$E^1 = 2\, z \frac{n}{60} \quad \ldots\ldots\ldots\ldots \quad 2$$

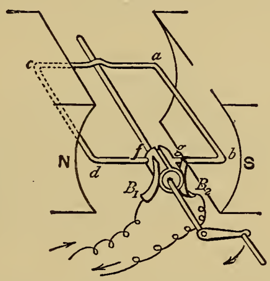
z being, as before, the total number of lines contained in the space swept by the wire, whilst $\frac{n}{60}$ is the number of revolutions per second.

In the ideal alternating current dynamo represented in Fig. 13, the wire in which the currents are generated is arranged to one side of the spindle only. We could easily improve the design by carrying the wire symmetrically to the other side of the spindle, but insulated from it, and attach its end to a second metal sleeve insulated from M. The contact spring or brush B_2 would then have to be set so as to touch this second sleeve, and since the electro-motive forces created in the two wires are at any moment in the same direction as regards the circuit—although opposite as regards a fixed point in space—this improved dynamo with two wires will give double the electro-motive force of the original arrangement. We could still further increase the electro-motive force by coiling the wire several times round the axis, forming a rectangular coil, each convolution being insulated from its neighbours, and if the number of turns counted on both sides of the spindle is $N t$, the average electro-motive force will be

$$E^1 = 2\, z \frac{n}{60} N t.$$

For most practical purposes, and especially for the transmission of energy, alternating currents are, however, not so convenient as continuous currents, and to produce the latter it will be necessary to add to our dynamo a device by which the currents are all directed to flow in the same sense as far as the external circuit is concerned. Such a device is the *commutator*, and its action can be explained by reference to Fig. 14. In the position shown, the electro-motive force created in the wire *a b* will be directed towards the observer, and that created in the wire *c d* will be directed from the observer. The ends

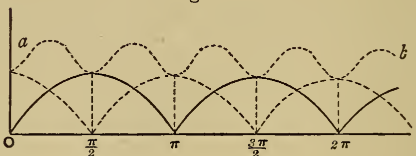
Fig. 14.



IDEAL CONTINUOUS CURRENT DYNAMO.

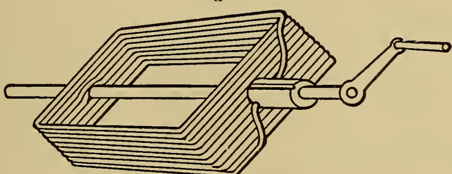
of these wires are joined at the back by a cross connection *a c*, and at the front by two wires *d f* and *b g*, to the two halves of a metal cylinder, which for the purpose of insulation are secured on a wooden hub. The electro-motive forces created in *d c* and *a b* tend to draw a current from the line in the direction of the arrow to the brush B_1, thence through *f d*, *c a*, *b g*, to the brush B_2, and out again into the external circuit. This process will go on until the crank reaches the lower vertical position, the strength of the current meanwhile decreasing to zero. When the crank is vertical, each brush touches simultaneously both halves of the metal cylinder or commutator, as it is technically termed, and a moment later the connections become reversed, the brush B_2 now touching the half cylinder to which the wire f is attached, and the brush B_1 touching the half cylinder to which the wire g is attached. But, at the same time, the direction of electro-motive force in the two wires has been reversed, the wire $c\ d$ entering the right-hand side of the field, and $a\ b$ entering the left-hand side. Consequently the external current flows in the same direction as before, growing from zero to a maximum when the crank stands horizontally on the left, and again diminishing to zero when it is vertical. Graphically represented, the current

Fig. 15.



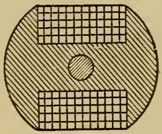
is of the character shown by the curve, Fig. 15, the abscissæ being consecutive angles of the crank, and the ordinates being proportional to the sines of these angles. It should be noted that the reversal of current always takes place when the electro-motive force is zero, and consequently the change in the contact with the brushes from one commutator plate to the other takes place without sparking. To increase the power of the machine, we can replace the single rectangle of wire by a coil of many turns. Fig. 16. Hitherto we have tacitly assumed that the space contained within the wire coils forming the armature contains air or other non-magnetic substance. The lines of force passing between the polar surfaces $S\ N$ have to leap across a considerable air space, and if by some means we could shorten that portion of their path which lies entirely in air, we would facilitate the flow of lines and increase the strength of the magnetic field. Roughly speaking, we may take it that air offers to the lines of force about 800 times the resistance of iron, and if we can contrive to fill part of the space between the polar surfaces with iron, a considerable increase of electro-motive force, and consequently of current, will be the result. The space available for this purpose is that contained within the armature coil ; in other words, to increase the power of the machine we must wind the armature coils over an iron core. An early

Fig. 16.



dynamo constructed on this principle is that of Siemens, invented in 1855, and provided with the so-called shuttle-wound armature. The core consists of an iron cylinder provided with two deep longitudinal grooves placed opposite so that the cross-section resembles a double T with rounded heads. The wire is wound into these grooves, and the two ends of it are joined to the plates of a two-part commutator. Fig. 17 shows a cross-section of this armature. In the first machines the core was in one solid piece, but it was found to heat considerably on account of internal currents. It is well known that if a solid body of metal be rapidly rotated between two powerful magnet poles it becomes hot. The reason for this phenomenon is that the outer portions of the metal in cutting through the lines of force become themselves the seat of electro-motive forces acting at right angles to the direction of motion and to the lines, and powerful currents are started parallel to the axis which run in opposite directions, up on one side and down on the other side of the axis. In a solid armature core there is nothing to check the flow of these currents but the resistance of the metal, which, on account of the large cross-sectional area, is extremely low. These wasteful currents are consequently very strong, and not only absorb much power, but they also weaken the current generated in the copper wire by induction. To avoid

Fig. 17.



SIEMENS SHUTTLE-WOUND ARMATURE.

their creation, it is necessary to subdivide the mass of the core by planes at right angles to the axis, and to insulate as much as possible the subdivided portions from each other. This can be done either by cutting deep narrow circular grooves in the core, or by building it up of thin discs insulated from each other either by paper discs or by being coated with some insulating paint. These armatures are not much used for dynamos at the present day, having been replaced by more perfect forms to be described presently; but they are still extensively employed for small electro-motors. By referring to Fig. 15 it will be seen that the counter-electro-motive force of these motors is a variable quantity depending on the angular position of the armature. If the heads of the double

T core are opposite the field magnet poles, the coil is at right angles to the lines of force and the counter-electromotive force is zero. This happens precisely at the moment when the brushes touch simultaneously both plates of the commutator, and are therefore short circuited. A current sent through the motor while at rest in this position cannot start it, and this condition is expressed by saying that the armature has two dead points. When at work the momentum of the armature is sufficient to carry it over the dead points, and, apart from the inconvenience to have to start the motor occasionally by hand, these dead points present no mechanical imperfection. But it might be thought that they present a serious electrical imperfection for the following reason: The strength of the current which is allowed to pass through the motor at any given moment depends partly on the electrical resistance of the motor, and partly on its counter-electromotive force at that particular moment. But since at the dead points there is no counter-electro-motive force, the strength of the current will be a maximum, whilst at those moments the mechanical energy produced is nil. We assume here that the motor is fed by a current flowing under a constant electro-motive force, which is the case most commonly met with in practice. We have now to distinguish between two cases: the motor may be either series wound or shunt wound. If the former, the current is passing through the motor whilst the armature is at a dead point has only to overcome the resistance of the field magnet coils. If the armature is in the position of greatest counter-electro-motive force the current has to overcome not only that, but also the combined resistance of field magnet and armature coils. In that position the mechanical energy of the armature is at its greatest value, but the strength of the current is a minimum. We find, therefore, on the one hand, that the strength of the field magnets (which depends on the current) is least at the very moments when the armature is in a position to exert most power, and on the other hand, that it is greatest when the armature is at its dead points and cannot exert any power. From the foregoing we should expect that twice during each revolution a great waste of current must take place when momentarily the brushes are short-circuited by the commutator. Although the time during which such short circuits lasts may appear to our senses very brief, it would in comparison with the speed of electric phenomena be still considerable, and have an appreciable effect on the economy of the motor. But there is one circumstance which greatly tends to mitigate the evil effect of the dead points just described, and this is the property of electric currents called *self-induction.* It can best be described as a kind of inertia opposing any sudden change in the strength of the current. If a circuit contains a coil of wire surrounding iron (as in the present case the field magnets) the self-induction is so great that it requires an appreciable time to change the strength of the current. The increase of current at the dead points is, therefore, checked by this property of self-induction, and the current, instead of being subjected to abrupt and violent changes, becomes simply undulatory. The case is different if the motor be shunt-wound and fed from a source of constant electro-motive force. Since the field magnet coils are excited independently from the current which passes through the armature, their self-induction cannot in any way steady that current, and abrupt changes in its strength and great waste of electrical energy must occur at the dead points. This is a matter of considerable practical importance, and shows that motors with shuttle-wound armatures should never be used coupled up otherwise than armature and field magnets in series. If it be absolutely necessary to use a motor of that class, the field magnets of which are either permanent steel magnets or are electro-magnets excited independently, the waste can to a certain extent be prevented by inserting into the armature circuit an electro-magnet which will by its self-induction steady the current. Since this point is of importance, the author has thought it necessary to verify the above theory by experiments. These were undertaken with a twofold object. First, to prove that in a series-wound motor there is no appreciable waste of current at the dead points, and, secondly, to prove that in a motor the field magnets of which are separately excited, such waste occurs. The experiments were carried out as follows. Two small Griscom motors were placed in line behind each other, and their spindles were coupled, so that the armatures stood at right angles to each other, that is to say, when one armature was at its dead point the other was in the position of best action, and its counter-electro-motive force was a maximum. This disposition is represented in Fig. 15 by the dotted curve overlapping that shown in full lines by 90°. The resultant counter-electro-motive force is at any point the sum of the ordinates of the two curves, and is shown by the undulating line *a b*. It will be seen that this curve nowhere touches the horizontal and, therefore, the total counter-electro-motive force of the two motors coupled in series never is zero. An abnormal rush of current at the dead points of any of the armatures can, therefore, not take place. The motors were supplied with a current, the electro-motive force of which was kept as nearly as possible constant during each experiment, whilst the mechanical energy developed was measured on one of the author's absorption dynamometers. The commercial efficiency of the two motors combined was thus ascertained, as shown in Table I. The motors were then coupled parallel, and their efficiency was determined under the same conditions. In this case there were, during each revolution, four dead points, at which the counter-electro-motive force was zero, and when an abnormal rush of current could take place if not checked by the self-induction of the magnet coils. As was to be expected, the current passing through both motors was about double, and its electro-motive force was about half of the former values. But the commercial efficiency was about the same, Table II. One motor alone was then tried, and its commercial efficiency was found to be about the same as that of the two motors combined, Table III. The field magnets of both motors were then excited separately, and the armatures coupled at right angles and connected in series, as per Fig. 15, when the commercial efficiency was found to be rather higher than in the former experiments, Table IV. This is but natural, because the energy necessary to excite the field magnets was not taken into account when calculating the efficiency. The two armatures were then coupled parallel—field magnets still independently excited—and thus during each revolution there were four points where the counter-electro-motive force was zero and waste of current did take place, as is clearly shown by the low efficiency in Table V. One motor alone was then tried under the same conditions and the same result was found, Table VI. These experiments prove conclusively that our above reasoning about the effects of the dead points is correct.

Test of Two Griscom Motors, Numbers 1017 *and* 1027.

Resistance of .	. N° 1017 .	. N° 1027
Armature . .	·328 .	. ·352
Magnets . .	·596 .	. ·522
Total . . .	·924 .	. ·874

Table I. Armatures Coupled at Right Angles, both Field Magnets and Armatures connected in Series.

Revolutions per minute.	Current.	E. M. F.	Foot Pounds on Brake.	Commercial Efficiency °/₀.
2,440	1·31	6·90	0	0
2,368	3·85	18·20	588	19·0
2,440	3·50	16·00	535	21·7

Table II. Armatures Coupled at Right Angles. Each Armature in Series with its Field. Both Motors connected Parallel.

Revolutions per minute.	Current.	E. M. F.	Foot Pounds on Brake.	Commercial Efficiency °/₀.
2,120	2·35	2·94	0	0
2,480	5·25	6·05	206	14·7
2,775	6·60	7·57	432	19·5
2,340	6·80	7·52	366	16·3
2,060	7·50	7·63	450	18·0
2,884	7·90	9·27	748	23·0
2,328	7·60	8·50	578	21·0

Table III. One Motor only. Armature and Field Magnets connected in Series.

Revolutions per minute.	Current.	E. M. F.	Foot Pounds on Brake.	Commercial Efficiency °/o.
1,980	1·02	4·00	0	0
2,024	4·15	8·20	303	28·0
1,772	4·15	8·40	265	17·0
2,334	4·22	9·25	381	22·3
1,954	3·82	8·10	246	18·0
2,241	3·70	8·25	283	20·9
2,118	3·50	7·60	240	20·5
2,070	5·37	12·00	532	18·6

Table IV. Armatures coupled at Right Angles and Connected in Series. Field Magnets excited separately.

Revolutions per minute.	Current.	E. M. F.	Foot Pounds on Brake.	Commercial Efficiency °/o.
1,536	1·42	7·20	0	0
2,030	3·30	11·10	370	22·8
1,632	3·10	9·50	300	23·2
2,190	3·70	12·90	483	22·7
2,264	3·93	13·40	500	21·4

Table V. Armatures Coupled at Right Angles and Connected in Parallel. Field Magnets excited separately.

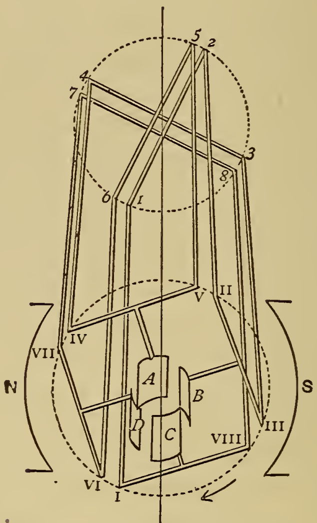
Revolutions per minute.	Current.	E. M. F.	Foot Pounds on Brake.	Commercial Efficiency °/o.
2,000	3·90	4·40	0	0
3,040	4·50	5·20	0	0
1,094	7·50	5·50	242	13·3
1,746	8·50	6·60	385	15·6
1,680	9·10	7·50	396	13·1

Table VI. One Motor only. Field Magnets excited separately.

Revolutions per minute.	Current.	E. M. F.	Foot Pounds on Brake.	Commercial Efficiency °/₀.
1,778	1·65	3·80	0	0
2,330	4·80	5·60	87	7·4
2,422	4·75	5·80	126	10·3

As already mentioned, motors with ordinary shuttle-wound armatures have the disadvantage of requiring to be started by hand if they happen to have stopped on a dead point. They are, consequently, only made of small size, and for larger motors armatures without dead points are used. Such an armature can be evolved out of the simple shuttle-wound pattern by employing two sets of coils placed at right angles to each other. This arrangement is shown in Fig. 18, which represents the Hefner-Alteneck winding invented in 1872. In order to avoid complication the shaft is omitted and the core is indicated by two dotted circles. From what has already been explained it will be seen that in all those wires which at a given moment lie on the right hand side of the vertical centre line, the electro-motive force is directed towards the observer, and in all the wires lying to the left of that line it is directed from the observer. The diameter of commutation joining the points of contact of the brushes with the commutator cylinder will, therefore, be horizontal. In the position shown the negative, or left brush, will touch segment *D*, and the right or positive brush will touch segment *B*. The current enters the armature at the negative brush and splits into two circuits as follows:—One portion goes through VII., 7, 8, VIII., I., 1, 2, II., and out by the positive segment *B*; the other goes through VI., 6, 5, V., IV., 4, 3, III., and out by the same segment *B*. The two currents are, therefore, in parallel connection. When the armature has turned so far as to bring the segment *C* into contact with the negative brush it will touch for a short time both segments *D* and *C*, whilst the positive brush will

Fig. 18.



HEFNER-ALTENECK ARMATURE.

simultaneously touch *A* and *B*. In this position the wires I., VI., V., II. will be in the strongest part of the field, and the wires VII., IV., III., VIII. will stand on the vertical diameter and contribute nothing towards the total electro-motive force. The current now splits into the following two circuits: From *D* to VI., 6, 5, V., to *A*, and from *C* to I., 1, 2, II., to *B*. In this case the total electro-motive force is that due to two wires in the position of best action, whereas in all the other positions it is due to four wires. It has been shown above that the average electro-motive force of a loop such as I., 1, 2, II., consisting of two external wires ($Nt = 2$) is

$$E^1 = 2\, z \frac{n}{60} 2.$$

Since two such loops are placed in series, we find the average electro-motive force of the whole armature

$$Ea = 8\, Z \frac{n}{60}.$$

But 8 is the number of wires counted all round the armature ; and if, instead of a four-part commutator, we had employed a six-part commutator, and had wound the core with three sets of double coils, we would have three coils in series and the expression for Ea would have been

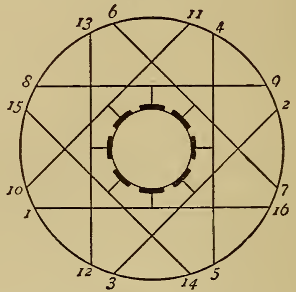
$$Ea = 12\, Z \frac{n}{60},$$

there being twelve external wires on the armature if counted all around. We might thus construct armatures with any even number of external wires. Let Nt be that number, and we have the general expression for the electro-motive force created in the armature of a dynamo, or the counter-electro-motive force created in the armature of a motor :

$$Ea = Nt\, Z \frac{n}{60} \quad \ldots\ldots\ldots\ldots \quad 3$$

For the sake of simplicity we have, in Fig. 18, only shown one wire to each coil. It is, however, obvious that by multiplying the turns or wires in each coil the electro-motive force can be proportionately increased. This case is provided for in formula 3, where N signifies the number of coils, and *t* the number of turns in each coil, the product of the two being equal to the total number of single wires if counted all around the armature. An armature of the Hefner-Alteneck pattern with eight-part commutator, is shown in Fig. 19. Denoting by Roman figures the ends of the wires on the front end of the armature,

Fig. 19.



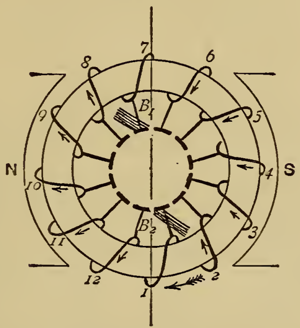
HEFNER-ALTENECK ARMATURE.

and by Arab figures those on the rear end, the winding is as follows:

From the negative brush to	I., 1, 2, II., III., 3, 4, IV., V., 5, 6, VI., VII., 7, 8, VIII. XVI., 16, 15, XV., XIV., 14, 13, XIII., XII., 12, 11, XI., X., 10, 9, IX.	To the positive brush

The greater the number of parts in the commutator the more nearly constant will be the electro-motive force and current. This system of winding armatures has the great advantage of utilizing nearly the whole length of the wire, since, with the exception of the cross connections at the ends, all the wire is active. But it has the practical disadvantage that repairs are troublesome to execute. If a fault of insulation should develop in any of the coils, in order to reach it, a large portion of the wire must be taken off, because the coils—especially at the ends—overlap each other in many layers. In this respect the style of armature known as the Gramme, or Pacinotti type, is preferable. A circular iron ring, Fig. 20, is

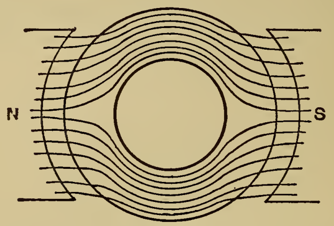
Fig. 20.



GRAMME ARMATURE.

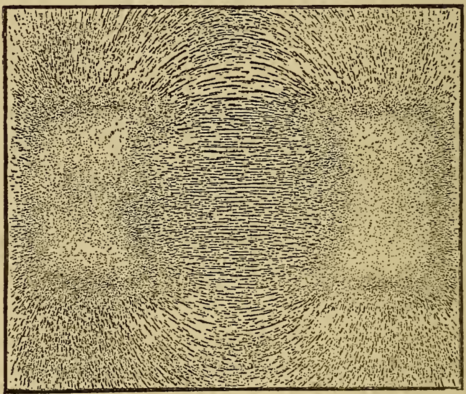
wound with a continuous helix of insulated copper wire, and certain points of the helix are joined by connecting wires, which in our illustration are shown radial, to the commutator plates. Two brushes, B_1 and B_2, serve as connections between the external circuit and the armature wire. The action of the Gramme armature will best be explained by reference to Fig. 21, which shows the lines of force. It has already been pointed out that iron offers very much less resistance to the passage of magnetic lines of force than air. If there be no armature between the field magnet poles, we assume that the majority of the lines will go straight from pole to pole, Fig. 22. If now a circular core is inserted, their course will be so altered that each line takes the path of least resistance—that is, runs as long as possible in iron, and only leaps across the air at the external circumference of the core, because this is the only way in which it can enter the pole piece, Fig. 23. At the internal circumference of the armature

Fig. 21.



there is no necessity for the lines to leave the core, and the central space is therefore almost free of lines. We say almost, because parallel lines exert a repelling action upon

Fig. 22.

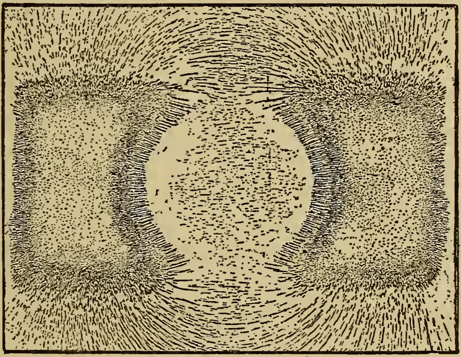


FIELD OF DYNAMO WITH ARMATURE REMOVED.

each other, and it may happen that in case the core is thin, and a large number of lines have to be accommodated, some of them may be elbowed out into the central space.

In well-designed machines the number of lines thus forced across the central space is always so small as to be omissible. The fact of the central space being free from lines; or, as we may also put it, being shielded by the iron of the core from the influence of the magnet poles is of great importance, since in consequence of it the inner wires of the helix are removed from all inductive action. If this were not the case electro-motive

Fig. 23.



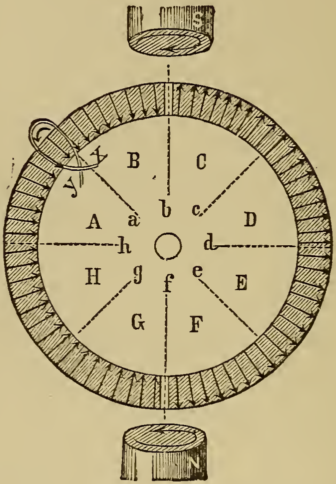
FIELD OF DYNAMO WITH ARMATURE INSERTED.

forces would be created in these wires, which, being opposed to the electro-motive forces developed in the external wires, would weaken the power of the machine. After what has been explained at length with reference to the ideal continuous current dynamo, Fig. 14, it will be easy to trace the direction of electro-motive forces in the external wires of the Gramme armature, Fig. 20. If rotated clock-wise, the electro-motive force will be directed towards the observer in all the wires lying to the right of the vertical centre line, and from the observer in the wires on the opposite side. The two currents resulting from these forces are indicated by the arrows. In the wires 1 and 7, which for the time being move parallel to the direction of the lines of force, there is no electro-motive force generated, whilst in 4 and 10, which move at right angles to the lines, the electro-motive force is a maximum. By virtue of the continuity of the helix the electro-motive forces in the wires 2, 3, 4, 5, 6 are added, and those in 12, 11, 10, 9, 8 are also added, the two circuits being at all times in parallel connection. The current enters the armature at the brush B_2, which is called negative, then splits into the two circuits mentioned, and uniting again at the brush B_1, which is called positive, leaves the armature, and enters the external circuit. It will be seen from the figure that either brush, when touching two consecutive plates of the commutator, establishes a metallic connection between the beginning and end of the corresponding coil, or, in technical language, short circuits that coil. If the brushes are in the position shown—the neutral diameter on the commutator—the short circuit is perfectly harmless, because there is no electro-motive force in the coil; but if we were to shift the brushes into an active part of the field either to the right or left of the neutral line, each coil, as its extremities pass under the brush, would be traversed by an excessive current, causing heavy sparking at the brush, and probably the ultimate destruction of the armature. The best position at which to place the brushes is always found experimentally; it does not accurately coincide with the geometrical neutral line, but is found to be in dynamos slightly in advance of it, and in motors slightly behind it. Opinions are divided as to the reason of this phenomenon. At one time it was ascribed to a certain sluggishness in the iron of the core in taking up and losing magnetism, but this theory has long since been discarded by most practical electricians. Some hold that the shifting of the neutral line is due to the magnetizing influence of the armature current upon the iron core by which the latter is transformed into a double horseshoe magnet with like poles joined, and the magnetic axis of which stands nearly at right angles to that of the field magnets. Others again maintain that the brushes must be set forward in a dynamo and backward in a motor, on account of the influence of self-induction in the armature coils. In reality both the last-mentioned causes have something to do with the position of the brushes, as will be more particularly explained in Chapter IV.

The first electro-motor having an armature wound on the principle above explained, was constructed by Professor Pacinotti, of Pisa, and the design was published in the journal "Il Nuovo Cimento," in 1864. This machine is illustrated in Fig. 24, and the core of the armature differed only in so far from that employed by Gramme seven years later, as it had external projections between the wire coils, which considerably increased the magnetic attraction between the armature and the pole pieces, thus rendering the machine more powerful. Fig. 25 shows part of the core and winding. The core of the Gramme machine consists of iron wire coiled into a ring of oblong cross-section. After being lapped round with tape for the purpose of insulation, it is wound transversely with cotton-covered copper wire. The winding consists of a number of coils which cover the core completely inside and out, and the beginning of each coil is joined with the end of its neighbour to the same commutator plate. When the winding is completed the armature is driven tight over a wooden centre by which it is fastened to the spindle.

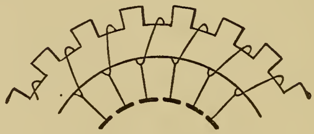
By means of the fundamental formulas established in the previous chapter, we can now determine the electro-

Fig. 24.



motive force of a Gramme armature. Let D be its diameter, b its length, and a the radial depth of the core.

Fig. 25.



PACINOTTI ARMATURE.

Let Nt represent the total number of external wires, counted all around the circumference, t representing the number of wires corresponding to one plate in the commutator, and N the number of plates. If n denotes the speed in revolutions per minute, and z the total number of lines emanating from one pole and entering the halt circumference of the armature, then the average electro-motive force created in each wire is by equation 2,

$$E^1 = 2 z \frac{n}{60}.$$

Since $\frac{Nt}{2}$ wires are for the time being connected in series, the average total electro-motive force in the armature is

$$E_a = z \, Nt \frac{n}{60} \quad . \quad . \quad . \quad . \quad . \quad . \quad 4$$

It might be objected that this expression, which is based on equation 2, will only be correct if the condition under which this equation was obtained is fulfilled in the dynamo. This condition was that the field should be perfectly uniform throughout the space occupied by the armature. In reality this is never the case, and the exact distribution is not accurately known. A doubt might therefore be entertained whether equation 4 be rigorously true in the case where the intensity of the field is not uniform, but varies in different parts of the field. It will consequently be desirable to deduce the formula for the electro-motive force under the supposition that the intensity of the field in any point on the circumference of the armature, is a function of the angle, α, which the radius to that point forms with the neutral line. What that function is we cannot say, nor is it necessary that we should be able to define it. We only make this assumption: that there shall not be any abrupt changes in the strength of the field. We assume that the density of lines varies gradually from place to place. Assume also the number of wires on the armature so large, that their angular distance $\Delta \alpha$ is very small, in fact so small that the intensity of the field can be considered as constant within that angular distance. Since the electromotive force created in the wires is proportional to their speed, we can determine it for any convenient speed, and if it be required for a different speed, we can obtain it by multiplying the result first obtained with the ratio of the two speeds. In the present instance we fix as a convenient speed that which will bring each wire at the end of one second into the position occupied by its immediate neighbour at the beginning of the second, or

$$v = \Delta \alpha \frac{D}{2}.$$

This is a very slow speed, and if we wish to know what will be the electro-motive force at the faster speed of n revolutions a minute, we shall have to multiply the electro-motive force at the low speed with the ratio of $\frac{n}{60} . \pi . D$ and v. Since $\Delta \alpha \, Nt = 2 \pi$ we have also $v = \frac{\pi D}{Nt}$ and the ratio of the two speeds is

$$\frac{\frac{n}{60} \pi D}{\frac{\pi D}{Nt}} = Nt \frac{n}{60}$$

Let $F_1, F_2 \, . \, . \, F_{\frac{Nt}{2}}$ be the intensity of the field at the first, second, . . . $\frac{Nt}{2}$ wire, counted from the neutral line, on one-half of the circumference of the armature, then the electro-motive force in these wires will be given by the expressions,

$$E_1 = F_1\, b\, v$$
$$E_2 = F_2\, b\, v$$
$$\vdots \qquad \vdots \quad \vdots \; \vdots$$
$$\vdots \qquad \vdots \quad \vdots \; \vdots$$
$$E_{\frac{Nt}{2}} = F_{\frac{Nt}{2}}\, b\, v.$$

The sum of all these forces gives the total electro-motive force created within the armature, which we denote in future by E_a.

$$E_a = \Sigma F\, b\, v.$$

But the expression $F_1\, b\, v$ represents the number of lines contained between the first and second wire on the armature, since F_1 is the density, and $b\, v$ the area of the space swept by the first wire in one second. Similarly $F_2\, b\, v$ represents the number of lines between the second and third wire, and so on, the sum of all these expressions representing the total number of lines entering between the first and last wire on one-half circumference of the armature. Let z be that total number, and we find for the electro-motive force at the low speed,

$$E_a = z.$$

At the high speed we have, therefore,

$$E_a = z\, Nt\, \frac{n}{60}, \quad \ldots\ldots\ldots\ldots \quad 4)$$

precisely the same expression as already obtained above. If z be inserted in absolute measure, E_a will also be obtained in absolute measure, and to obtain it in volts the right side of the equation must be multiplied with 10^{-8}. We can also write

$$E_a = \frac{z}{6000}\, N\, t\, n\, 10^{-6},$$

and if we measure the field intensity by means of a unit 6,000 times as great as the absolute unit, we can further simplify the equation to

$$E_a = Z\,N t\,n\,10^{-6}, \;\ldots\ldots\ldots\; 5)$$

Z being the total number of lines in the new system, which is related to the absolute system by the equation

$$Z = \frac{z}{6000}.$$

The cross-sectional area of the armature core is $2\,a\,b$, and if we denote by m the average density of lines per square inch of armature core, we have,

$$Z = 2\,a\,b\,m,$$

and inserting this value in 5), we find for the electro-motive force also the expression,

$$E_a = 2\,a\,b\,m\,N t\,n\,10^{-6}. \;\ldots\ldots\; 6).$$

This expression is sometimes more convenient than the former, because it enables us at once to see how the dimensions of the armature affect the electro-motive force. Experience has shown that the density of lines, m, in the core cannot exceed a certain limit, which is reached when the core is saturated with magnetism. This limit is $m = 30$, but in practical work a lower density is generally adopted, for reasons which will be explained in the following chapter. A fair average value in good modern dynamos and motors is $m = 20$, and the area, $a\,b$, must be taken as that actually filled by iron, and not the gross area of the core. To avoid waste of power and heating, the armature core of dynamos and motors must be subdivided into portions insulated from each other, the planes of division being parallel to the direction of the lines of force, and to the direction of motion. The space wasted by such insulation must be deducted from the gross area of the core, and the remainder—from 70 to 90 per cent. of it—is the portion actually carrying lines of force.

The electrical energy developed in the armature, if a current c be flowing through its coils, is $E_a\ c$, and the horse-power represented by this energy is

$$\text{H-P} = \frac{1}{746}\, c\ 2\ a\ b\ m\ Nt\ n\ 10^{-6}.$$

The power to be applied must naturally be somewhat in excess of this in order to overcome mechanical resistances, as friction in the bearings and air resistance, and also the magnetic resistance due to imperfect subdivision and heating of the core, and reaction of the armature on the magnets. In good dynamos these losses do not exceed about 10 per cent. and may even be less.

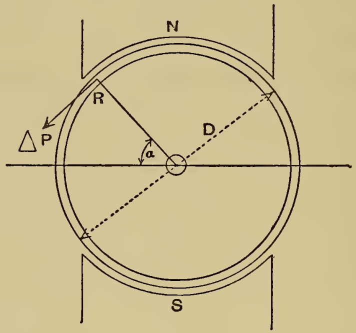
CHAPTER III.

Reversibility of Dynamo Machines—Different conditions in Dynamos and Motors—Theory of Motors—Horse-power of Motors—Losses due to Mechanical and Magnetic Friction—Efficiency of Conversion—Electrical Efficiency—Formulas for Dynamos and Motors.

After what has been explained in the previous chapters it will be evident that dynamo machine and electro-motor are convertible terms. Any dynamo can be used practically as a motor, and in most cases any motor can be used to generate a current. On purely theoretical grounds this should be possible in all cases, but in practice it is found that the speed which is required to make some small motors act as self-exciting dynamos is so high as to render that application mechanically impossible. The reason for this is, that in small motors the polar surfaces are of very limited extent, and consequently the magnetic resistance of the path traversed by the lines of force is excessively high, requiring more electrical energy to excite the field magnets than the armature is capable of developing at a moderate and practical speed. This point will be more fully explained further on. For our present purpose it suffices to note that on purely theoretical grounds the same machine can act as a motor or as a dynamo. A separate investigation as to the theory of motors might, therefore, almost seem superfluous. But, on the other hand, experience has shown that although this reversibility of the dynamo machine exists, it is not always the best dynamo which makes the best motor, and that certain details have to be altered according to the use for which the machine is intended, if we wish to produce the best possible machine for each purpose. The conditions which have to be fulfilled in the case of dynamos are also generally different from those required in motors. The dynamo must have a high efficiency, it must be able to work continuously without undue heating in any of its parts, must not be injured by an occasional excess of current, and must work equally well at extreme variations of electrical output. Its weight is, as a rule, of secondary importance, and in many cases there is no objection to large weights. The motors, on the other hand, are generally required to be of the smallest possible weight, they work intermittently, and high efficiency, although desirable, is not of so much importance, especially not in small motors. In the early days of electric transmission of energy the difference between the conditions in dynamos and motors was overlooked, and the usual arrangement was to employ two identical machines, one acting as generator, the other as receiver, but at the present time this rough-and-ready method does not satisfy all the requirements which can justly be made, and special motors must be provided. It has thus become necessary to study the theory of motors apart from that of dynamos.

Let in Fig. 26, N S be the pole pieces and D the mean diameter of the annular space filled by the external wires on a cylindrical armature of the Gramme or Hefner-Alteneck pattern. Let, as before, b represent the length of the wire and F the intensity of the field at a given point R, the radius to which forms with the neutral line the angle α. All the wires on the upper half of the armature will be traversed by currents flowing in the same direction, say from the observer, and all the wires on the lower half will be traversed by currents flowing towards the observer. Let c be the current in each single wire and let there be Nt external wires counted all around the circumference. If these wires lie close together with only as much space between them as is necessary for

Fig. 26.



mutual insulation, the effect of the current c traversing successively the $\frac{Nt}{2}$ wires on one half of the circumference will evidently be the same as that of a semicircular *sheet of current* of total strength $\frac{Nt}{2}$ c, the width of this sheet measured transversely to the direction of flow being $\frac{\pi \, D}{2}$. The density of current in the sheet, that is, the strength of current per unit of width, is $\frac{Nt}{2} c : \frac{\pi \, D}{2} = \frac{Nt \; c,}{\pi \, D}$ and the current flowing down an elementary section at

R, the angular width of which ($\Delta\ \alpha$) we take to be very small, is,

$$\Delta\ c = \frac{Nt\ c\ D}{\pi\ D\ 2}\ \Delta\ \alpha.$$

The mechanical force tending to rotate the elementary strip of our sheet of current in the direction of the arrow is

$$\Delta\ P = F\ b\ \Delta\ c$$

$$\Delta\ P = F\ b\ \frac{D}{2}\ \Delta\ \alpha\ \frac{Nt\ c}{\pi\ D}.$$

Now F, the intensity of the field, multiplied with $b\ \frac{D}{2}\ \Delta\ \alpha$, the total area of the elementary strip, gives the number of lines of force which enter the core through that area. Let $\Delta\ Z$ represent that number, and we can also write

$$\Delta\ P = \Delta\ Z \frac{Nt\ c}{\pi\ D}.$$

Now consider a second elementary strip of the sheet of current contiguous to the first. The force exerted by this strip will be represented by a similar expression, but in it the value of $\Delta\ Z$ may be different. This will be the case if the field intensity is not uniform, but varies in any way with the angle α. For our purpose it is not necessary to know in what manner the intensity of field F may vary in different points; whatever the law of variation may be, the sum of all the values of $\Delta\ Z$ must always be the same, and equal to the total number of lines passing into the armature core. The mechanical force exerted by the upper semicircular sheet of current, or, which comes to the same thing, by the upper half of the armature winding, $\frac{Nt}{2}$, is therefore

$$Z \frac{Nt\, c}{\pi\, D},$$

Z being the total number of lines. Simultaneously the lower half of the armature exerts the same force, and we have the total force tending to rotate the armature, and acting at a radius equal to that of the winding, $\frac{D}{2}$,

$$P = \frac{2\, Z\, Nt\, c}{\pi\, D}.$$

The turning moment, or torque, is $P \frac{D}{2}$, or

$$T = \frac{Z\, Nt\, c}{\pi}, \quad \ldots\ldots\ldots\ldots 7).$$

If we express the total number of lines by the product of their density within the armature core and the dimensions of the latter, we can also write for the torque

$$T = \frac{2\, a\, b\, m\, Nt\, c}{\pi} \quad \ldots\ldots\ldots\ldots 8).$$

It has already been mentioned that there exists a limit beyond which *m* cannot be increased, however powerful the field magnets may be. Assume that in two motors of different size the field magnets are excited so as to produce equal and maximum density of lines in both armature cores, and assume also that both armatures are wound with wire of the same gauge, then the number of turns will in the larger machine be greater than in the smaller, the proportion being evidently as the squares of their linear dimensions. Since the areas of the cores are also in the same proportion, it follows that the torques or turning moments are in the proportion of the fourth power of the linear dimensions. Thus, if the larger motor be double the linear dimensions of the smaller, its torque will be sixteen times as great. It will be seen from formula 7, that the torque of a motor depends only on the strength of the field and on the current, but does not depend on the speed. This can be shown experimentally in the following manner. Let two series-wound dynamos be connected by a pair of cables, and let one of these act as generator, whilst the other, which is the motor, is provided with a friction brake, on which the energy given out can be measured. Whatever the speed of the motor may be, the brake, if its lever be floating free, indicates the turning moment in the shaft of the motor. This turning moment is equal to the product of the length of the lever and the load suspended. If now the speed of the generator be varied so as to vary the electro-motive force, the speed of the motor will accordingly vary, but the current and the load on the brake will remain unaltered. In dealing with this matter, M. Marcel Deprez, in "La Lumière Electrique" of the 3rd of October, 1885, says:—"If a current traverses a motor having an armature of the Pacinotti type, the turning effort of the latter is independent of its state of movement or rest, and in motion it is independent of the speed, provided the strength of the current is maintained constant. Inversely, if the static moment tending to resist the motion of the armature is maintained constant, the current will thereby automatically be kept constant, whatever means we may employ to vary it. The experiment must be made in the following way. Mount upon the spindle of the motor a self-adjusting dynamometric brake, the load on which is automatically kept constant whatever variation may take place in the friction of the brake or in the speed of the motor, so that the tangential resistance which tends to oppose rotation shall be kept constant. Supply the motor with current from any given source of electricity (a battery or a dynamo machine), and note the strength of the current and its electro-motive force. If the latter be gradually increased from zero we observe that as long as the motor remains at rest the current grows in the same proportion, but as soon as it has reached a certain value and the motor has begun to turn, the current does not further increase, although the rise in the electro-motive force may continue, and with it the rise in the speed of the motor. In an experiment made three years ago the source of electricity was a Gramme dynamo and the motor a Hefner-Alteneck machine, the brake being loaded with $5\frac{1}{2}$ lbs. at a radius of $6\frac{3}{8}$ inches. When the motor began to turn, the needle of the ampere-meter indicated twenty-six divisions. I then augmented the speed of the dynamo until the motor made thirty-two revolutions per second, and yet the ampere-meter only indicated twenty-seven divisions instead of twenty-six."

Since with a constant load on the brake, the energy given out is proportional to the speed, and since the electrical energy supplied to the motor is the product of current and electro-motive force, it follows that if the current is constant the speed must be proportional to the electro-motive force. The following table taken from M. Marcel Deprez's article shows that this is indeed the case. It will be seen that in all the four motors tested the ratio of electro-motive force to speed remained nearly constant throughout a very wide range of speed, and that the current also remained practically constant

Type of motor.	Revolutions per minute.	Current.	$\frac{\text{Electro-motive force.}}{\text{Speed.}}$
Hefner-Alteneck .	425	13·53	·0267
	783	12·68	·0262
	1165	13·65	·0278
	1660	13·00	·0250
A Gramme . . .	270	8·16	·06496
	526	8·16	·06437
	608	8·23	·06768
	742	8·40	·06792
	944	8·23	·06713
	1004	8·23	·06803
	1160	8·23	·06704
	1460	8·23	·06736
Hefner-Alteneck .	356	5·60	·0132
	618	5·78	·0139
	1016	5·42	·0127
	1236	5·60	·0130
	1470	5·95	·0129
	1636	5·60	·0127
	1662	5·42	·0127
High tension machine	200	5·60	1·659
	384	6·30	1·692
	470	6·12	1·775
	606	5·95	1·633
	710	5·95	1·662

Going now back to equation 7), the mechanical energy represented by one revolution of the motor shaft is evidently $2\,\pi\,T$, and if the motor runs at a speed of n revolutions a minute, or $\frac{n}{60}$ revolutions a second, the energy developed during that time is

$$W = Z\,N t\,2 c\,\frac{n}{60} \quad \ldots\ldots\ldots \quad 9).$$

It will be remembered that each half of the armature carries the current c ; $2c$ is consequently the total current passing into the armature at one brush and out at the other. Write C_a (armature current) for $2c$ and we have

$$W = Z\, Nt\, \frac{n}{60}\, C_a \quad \ldots\ldots\ldots \quad 10).$$

But from equation 4) we found that the counter electromotive force of the armature is

$$E_a = Z\, Nt\, \frac{n}{60} \quad \ldots\ldots\ldots\ldots \quad 4),$$

and combining the two equations we find

$$W = E_a\, C_a \quad \ldots\ldots\ldots\ldots \quad 11).$$

The mechanical energy equals the product of current and electro-motive force, that is, equals the electrical energy. This, indeed, is self-evident from the principle of the conservation of energy ; and starting with the equations 4) and 11), we could have deduced the expressions for W and T from these. But on the other hand it is more satisfactory to have determined these values independently, and to find that our conclusions are verified by the principle of the conservation of energy.

All the equations above are based on the absolute system of measurement. For practical purposes, however, the employment of these units is not convenient, and instead of using dynes or ergs we prefer to make our calculation in pounds and horse-powers. It will therefore be necessary to determine the relation between the absolute and practical units.

According to the definition of the dyne given in the first chapter, it is that force which accelerates the mass of one gram by one centimeter in one second. It would not be strictly correct to represent the dyne as equal to a certain fraction of a kilogram or of a pound, because the weight of unit mass (that of one gram) changes according to the position on the surface of the earth where we may happen to measure it. But in all places the following equations hold good:—

$$P = m\,p,$$
$$G = m\,g.$$

P being the force to which corresponds the acceleration p, G being the weight of the body measured by the acceleration of gravity g, and m being the mass of the body

$$P = G\frac{p}{g}.$$

If g be given in meters per second and the weight in kilograms, the force of one dyne is,

$$\text{Dyne} = \frac{10^{-3}\,10^{-2}}{g} \ldots \text{kilograms.}$$
$$\text{Dyne} = \frac{10^{-5}}{g} \ldots\ldots \text{kilograms.}$$

The energy represented by one dyne acting through the distance of one centimeter, the erg, is therefore

$$\frac{10^{-5}}{g} \ldots \text{kilogram-centimeters, or,}$$
$$\text{Erg} = \frac{10^{-7}}{g} \ldots \text{kilogram-meters.}$$

According to equation 11) the number of ergs developed by the armature of the motor is numerically equal to the product of current and electro-motive force in absolute measure. If we wish to insert these values expressed in practical units of amperes and volts we have

$$W = 10^{8} \times 10^{+1} \ldots \text{voltamperes,}$$
$$W = 10^{-7} \ldots\ldots \text{watts.}$$

To obtain the number of watts represented by a certain number of ergs, we have therefore to multiply the latter by 10^{-7}. Similarly to obtain the number of kilogram-meters represented by a certain number of ergs, we have to multiply the latter by $\frac{10^{-7}}{g}$,

$$\text{Watts} = 10^{-7} \times \text{ergs},$$

$$\text{Kilogram-meters} = \frac{10^{-7}}{g} \times \text{ergs}.$$

From these two equations we find that

$$\text{Kilogram-meters} = \frac{\text{Watts}}{g}.$$

The energy required to lift 75 kilograms one meter high in one second is a standard horse-power in the metric system. The acceleration of gravity may be taken as 9·81 meters per second, hence one horse-power is represented by

75 × 9·81 . . . watts, or in round numbers:

736 *watts correspond to one standard horse-power.*

In English measure the standard horse-power is equal to 32,500 foot pounds work done per minute. The usual English horse-power is equal to 33,000 foot pounds. Hence, to obtain the number of watts representing an English horse-power, we must multiply 736 with the ratio of 33,000 to 32,500. This gives the figure 746. Let E_a represent the counter-electro-motive force of the armature in volts, and C_a the current in amperes, then the number of English horse-powers which could be obtained from it, if there were no losses, is

$$H\text{-}P = \frac{E_a \; C_a}{746} \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad 12).$$

Retaining the notation of equations 5) and 6), we have also

$$H\text{-}P = \frac{1}{746} Z\, Nt\, n\, 10^{-6}\, Ca \quad \ldots \quad \ldots 13),$$

$$H\text{-}P = \frac{1}{746} 2\, a\, b\, m\, Nt\, n\, 10^{-6}\, Ca \quad \ldots 14).$$

The power which is actually obtainable is somewhat smaller, as certain losses occur. These might be classified under two headings, mechanical friction and magnetic friction. The former consists of the friction in the journals, of that between the commutator and the brushes, and of the resistance which the air offers to the rapid rotation of the armature, or the "windage," as it is technically termed. The magnetic friction is of a somewhat complicated nature, and may manifest itself in various ways, but more especially in the heating of the armature core and of the pole pieces. If the armature core is not sufficiently subdivided, a fault very common in small motors, currents will be generated in it, which will be the stronger the more intense the field and the quicker the speed. It is as though the motor contained within itself a dynamo working on short circuit, and the power necessary for producing these currents must be supplied by the current flowing through the coils of the armature, and represents therefore so much power withdrawn from external use. Another source of loss is the limited number of the sections in the commutator. In establishing our formulas we have assumed that the aggregate of the currents in the different wires can be represented by a continuous semicircular sheet of current. This assumption is, strictly speaking, only correct if the number of wires and the corresponding number of sections is infinite. But when these numbers are limited, and especially when one section of the commutator corresponds to a wide coil, consisting of a great many turns of wire on the armature, then the change of contact between the brushes and successive commutator strips produces abrupt changes in the magnetizing effect of the current on the core of the armature, and our sheet of current, instead of being fixed in space as first assumed, undergoes violent oscillations, the amplitude of which is equal to the angular distance between two neighbouring coils. It is as though a magnet placed at right angles to the centre line through the pole pieces were kept in rapid oscillation, and since any magnet, if moved in the neighbourhood of metallic masses will heat the latter and absorb power, it follows that the pole pieces will become hot, and part of the energy produced by the motor will be wasted in this way. From what has just been explained, it will be evident that this loss can be reduced by increasing the number of sections in the commutator, and by subdividing the metal of the pole pieces by planes at right angles to the axis of the armature.

Another source of loss in some motors is the discontinuity of the armature core. This loss does not occur in Gramme armatures with smooth cylindrical cores ; but in armatures of the Pacinotti type, the projecting teeth, in sweeping closely by the polar surfaces, react on the latter, and produce eddy currents therein, which in their turn exert a retarding force upon the teeth. That this is really the case is shown in a striking manner in many dynamos having Pacinotti projections, notably in the Brush and Weston machines. Everyone who has examined these machines after some hours' work, must have noticed that the pole pieces, especially where the coils and projections leave them, grow hot. At the entering side the heating is not so great, because there the magnetizing effect of the armature current is to repel and weaken the lines, whereas at the leaving side it is to attract and strengthen them. If the machines be used as motors an opposite effect is produced, the pole pieces becoming hottest at the entering side. Cores with Pacinotti projections are very much in favour with the designers of motors, because it is thought that they increase the magnetic attraction which determines the force of the motor. On purely theoretical grounds this is so. It will be shown presently that the number of lines Z, passing from the pole piece to the armature is the greater, the smaller the distance they have to leap through air, and by allowing the teeth to project so far as to almost touch the polar surfaces, the magnetic resistance of the air space can be very considerably reduced. But in practice such perfection is unattainable on account of the heating and waste of power just explained. It is found necessary to make the clearance between the outer surface of the teeth and the inner surface of the pole pieces much greater than would suffice for free rotation, and it may be doubted whether the Pacinotti core is, after all, so great an improvement over the Gramme core as on purely theoretical grounds it seems to be. There is also another source of loss occurring even in armature cores which are perfectly subdivided and smooth on the outside. This is due to a molecular effect in the iron which has been termed hysteresis by Professor Ewing. In ordinary motors, having two or four field magnet poles, this loss is, however, very small and is generally neglected.

In good motors the sum total of all the losses here enumerated at length amounts to only a small fraction of the total power. The ratio between that and the power actually obtainable on the shaft is called the *efficiency of* *conversion*, and it should never be less than 90 per cent. in medium-sized and large motors.

The *electrical efficiency* of the motor is the ratio of total internal electrical horse-power, as given by our formulas 13) and 14), to the external electrical horse-power applied at the terminals of the motor. Let

E_a represent the electro-motive force created in the armature coils.

E_b represent the electro-motive force appearing at the brushes.

E_t represent the electro-motive force appearing at the terminals.

r_a represent the total resistance of the armature.

r_m represent the total resistance of main coils on field magnets.

r_s represent the total resistance of shunt coils on field magnets.

C, C_a, C_s, C_m represent the external current, the current through the armature, through the shunt coils and main coils on field magnets respectively. Then for a compound-wound dynamo, in which the shunt coils are coupled across the brushes, the following equations evidently obtain:

$$C = C_m, \; C_s = \frac{E_b}{r_s}$$

$$C_a = C_m + C_s \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; 15),$$

$$E_b = E_a - r_a \, C_a \; . \; . \; . \; . \; . \; . \; . \; . \; . \; 16),$$

$$E_t = E_b - r_m \, C_m \; . \; . \; . \; . \; . \; . \; . \; . \; 17).$$

The electrical efficiency is

$$\eta = \frac{E_t \, C}{E_a \, C_a} \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; 18).$$

For an electro-motor, also compound-wound, the equations are

$$C = C_m,\ C_s = \frac{E_b}{r_s},$$

$$C_a = C_m - C_s \ldots\ldots\ldots\ldots 19),$$

$$E_b = E_t - r_m\, C_m \ldots\ldots\ldots 20),$$

$$E_a = E_b - r_a\, C_a \ldots\ldots\ldots\ldots 21),$$

$$\eta = \frac{E_a C_a}{E_t\, C} \ldots\ldots\ldots\ldots 22).$$

If the shunt coils are coupled to the terminals the formulas are for the dynamo,

$$C = C_m - C_s,\ C_s = \frac{E_t}{r_s},\ C_a = C_m,$$

16), 17), and 18) remaining unaltered.

For the motor we have

$$C_m = C - C_s,\ C_s = \frac{E_t}{r_s},\ C_a = C_m,$$

20), 21), and 22) remaining unaltered.

The same formulas are applicable to the case of plain series or shunt machines, whether dynamos or motors, but in the case of series machines we insert $r_s = \infty$, and in the case of shunt machines we insert $r_m = o$.

CHAPTER IV.

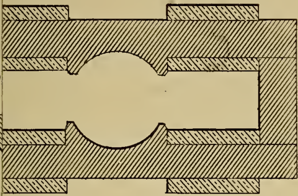
Types of Field Magnets—Types of Armatures—Exciting Power—Magnetic Circuit—Magnetic Resistance—Formulas for strength of Field—Single and Double Magnets—Difficulty in Small Dynamos—Characteristic Curves—Pre-Determination of Characteristics—Armature Reaction—Horse-power Curves—Speed Characteristics—Application to Electric Tramcars.

In the preceding chapter it has been shown how the electro-motive force of an armature can be found if the total number of lines passing through its core be known. It will now be necessary to determine the number of lines, that is the strength of the magnetic field, from the constructive data of the machine. Before entering into a scientific investigation of the subject a cursory glance at the different types of field magnets adopted by the various makers of dynamos and motors, will be of interest. These are shown in Figs. 27 to 51. To make the classification comprehensive the type of armature is written beneath each field and the maker's or designer's name is written above it. We distinguish three types of armature. 1. *The Drum,* wound on the Hefner-Alteneck principle, as explained in Chapter II., and shown in Figs. 18 and 19; 2. *The Cylinder,* wound on the Pacinotti or Gramme principle, also explained in Chapter II., and shown in Figs. 25 and 20; and 3. *The Disc,* wound on the Pacinotti or Gramme principle and only differing from the cylinder by the shape of the core. It is a cylinder of considerable diameter and small length, in fact a flat ring or disc.

All the magnets employed in dynamos or motors are horse-shoes ; straight-bar magnets with poles at the ends being never used. The reason is obvious. We must in all cases bring opposite poles to the same armature, and that necessitates the employment of a bent magnet. It is necessary to distinguish between single, double, and multiple magnets. In the single horse-shoe magnet all the lines passing across the armature go through the magnet in the same direction. As an example we may take the Edison-Hopkinson dynamo, Fig. 27. The lines passing across the armature from *N* to *S* continue all in the same direction, viz., vertically upwards from *S* to *B*, thence across the yoke from *B* to *A*, and finally vertically, downwards from *A* to *N*. A free unit pole would be urged along the closed magnetic circuit *N S B A N*, and there is no other way along which it could travel. Now in a double horse-shoe, as represented for instance by the Weston machine, Fig. 41, there are two ways along which a unit pole might travel. One of these is *N S B A N*, and the other *N S D C N*, or in other words, of the total number of lines passing across the armature, one half will go through the horse-shoe *N A B S*, and the other half will go through the horse-shoe *N C D S*. We may consider the field magnets to consist of these two horse-shoes placed with like poles in contact to the left and right of the vertical center line. The arrangement of the "Manchester" dynamo is similar, but in this case the portions *A B* and *C D*, which in the Weston dynamo constitute the yokes, form the excited or active parts of the magnets and are surrounded by the magnetizing coils. The field magnets of the original Gramme dynamo (or motor) also belong to the double horse-shoe pattern. But in this case a plane laid through the center lines of the cores of the magnets is parallel to and contains the center line of armature shaft, whereas in the Weston type it is at right angles to it. Here, again, the lines are split up to the right and left of the vertical center line into two distinct circuits. Fig. 37 shows a similar arrangement, but with a single magnet. Figs. 39, 40, and 50 show single magnets, the plane of the horse-shoe being at right angles to the armature. Fig. 48 shows a quadruple horse-shoe magnet. Here the lines of force passing across the armature belong to four distinct circuits: *S D A N*, *S D C N*, *S B A N*, and *S B C N*. The field magnets of the Mordey-Victoria machine shown in Figs. 46 and 47 consist of 8 complete horse-shoes, four on each side of the disc, and in some multipolar machines even a larger number of magnetic circuits is sometimes employed. The machines which M. Marcel Deprez employed in his experiments (Fig. 51) had two cylinder armatures mounted on the same spindle *a b*, and around them were placed eight horse-shoes, of which two, *S B A N* and *S C D N*, are shown in the illustration. It is not necessary to enter into a detailed description of all the types shown, as the diagrams are sufficiently clear.

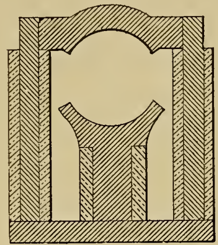
After what has been said above it will be evident that the proper function of the field magnets in a dynamo or motor is to produce lines of force which pass across the armature core. All other lines which miss the armature are useless and may even be detrimental to the working of the machine. The greater the number of useful lines the greater will be the electro-motive force generated at a given speed and with a given armature. Our aim should therefore be to produce a maximum number of

Fig. 33.
PATERSON AND COOPER.



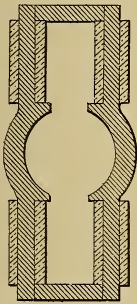
SHORT CYLINDER.

Fig. 34.
GOOLDEN AND TROTTER.



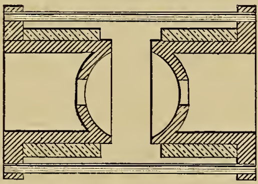
SHORT CYLINDER.

Fig. 42.
MAXIM.



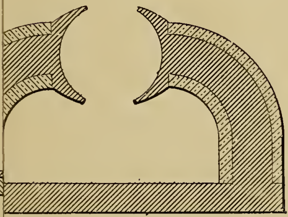
CYLINDER.

Fig. 43.
THOMSON-HOUSTON.



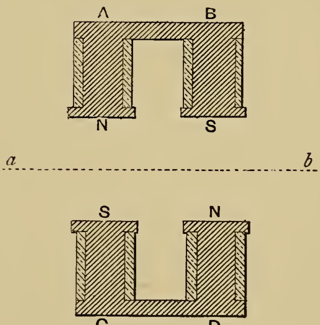
SPHERE.

Fig. 50.
JÜRGENSEN.



CYLINDER.

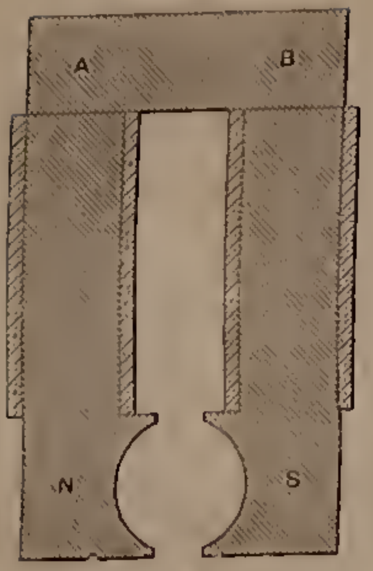
Fig. 51.
MARCEL-DEPREZ.



TWO CYLINDERS.

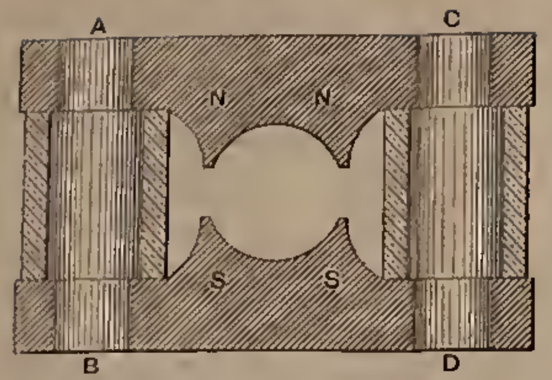
To face page 114.

Fig. 27.
EDISON-HOPKINSON.



DRUM.

Fig. 28.
MANCHESTER.

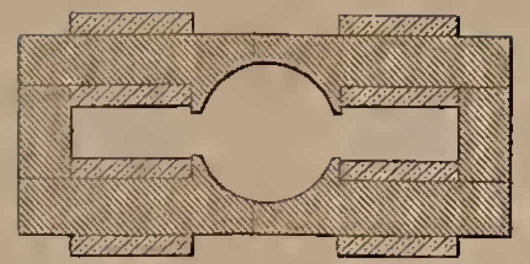


SHORT CYLINDER.

Fig. 29.
SIEMENS.

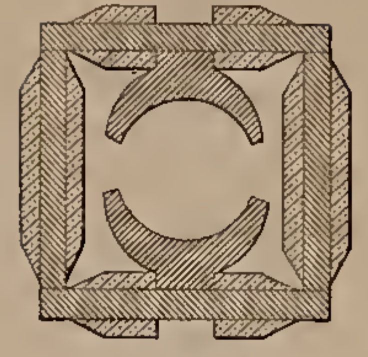
DRUM.

Fig. 30.
CROMPTON.



CYLINDER.

Fig. 31.
ELWELL-PARKER.

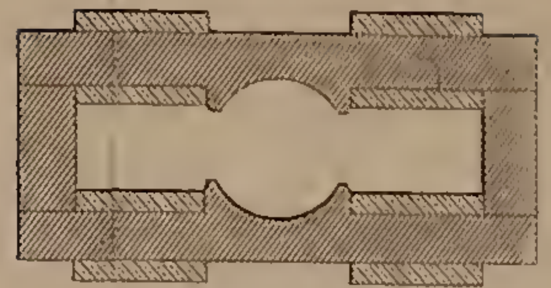


CYLINDER.

Fig. 32.
ELWELL-PARKER.

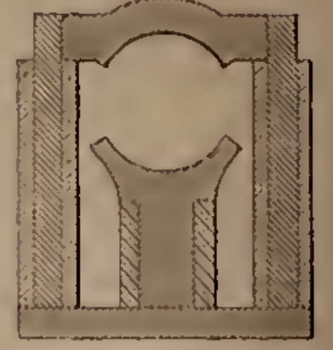
LONG CYLINDER.

Fig. 33.
PATERSON AND COOPER.



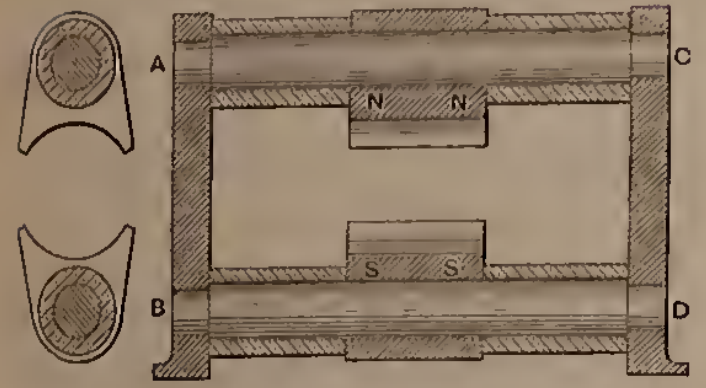
SHORT CYLINDER.

Fig. 34.
GOOLDEN AND TROTTER.



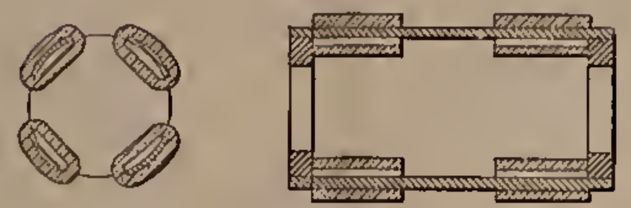
SHORT CYLINDER.

Fig. 35.
GOOLDEN AND TROTTER. GRAMME.



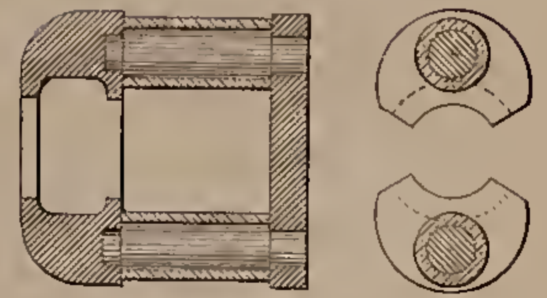
SHORT CYLINDER.

Fig. 36.
ANDREWS.



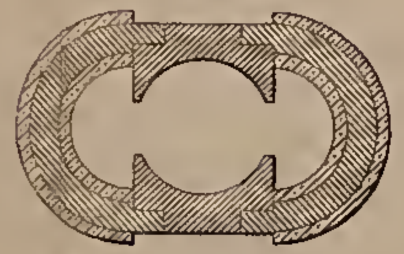
SHORT CYLINDER.

Fig. 37.
JONES.



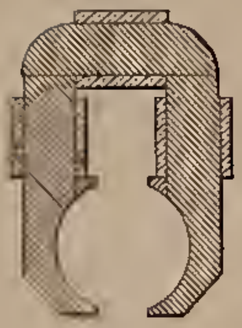
SHORT CYLINDER.

Fig. 38.
KAPP.



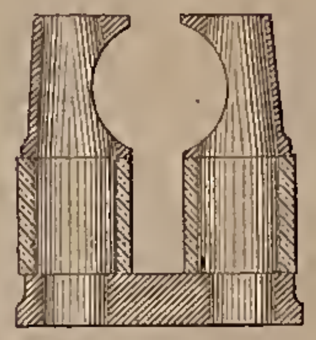
CYLINDER.

Fig. 39.
KAPP.



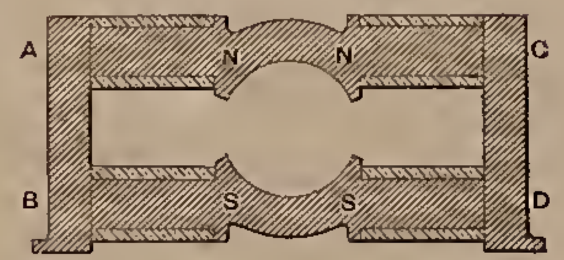
CYLINDER.

Fig. 40.
KAPP.



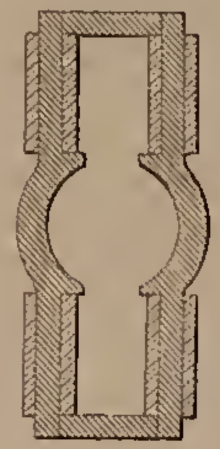
SHORT CYLINDER.

Fig. 41.
WESTON.



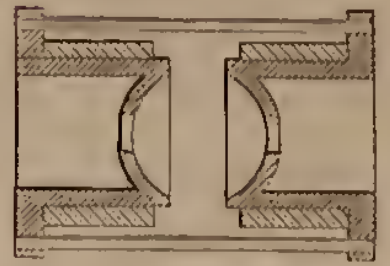
DRUM.

Fig. 42.
MAXIM.



CYLINDER.

Fig. 43.
THOMSON-HOUSTON.



SPHERE.

Fig. 44.
GULCHER.

Fig. 45.

DISC.

Fig. 45.
BRUSH.

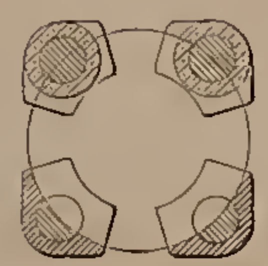
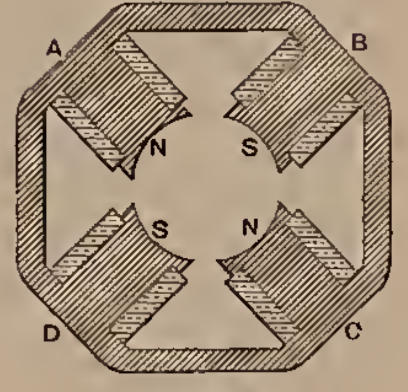


Fig. 47.

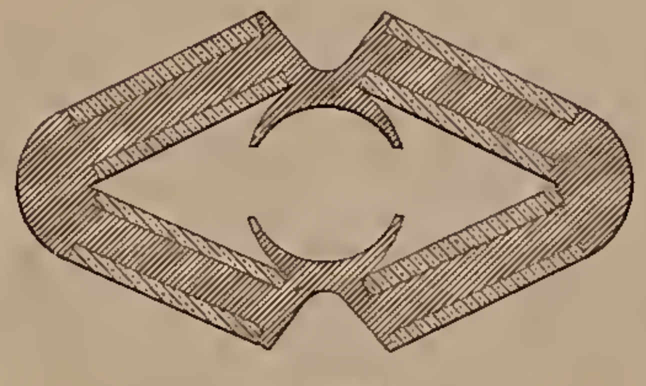
DISC.

Fig. 48.
GRAMME.



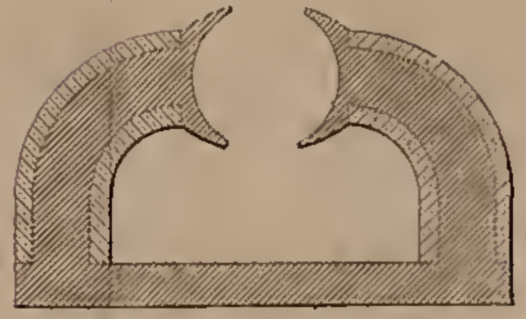
CYLINDER.

Fig. 49.
DE MERITENS.



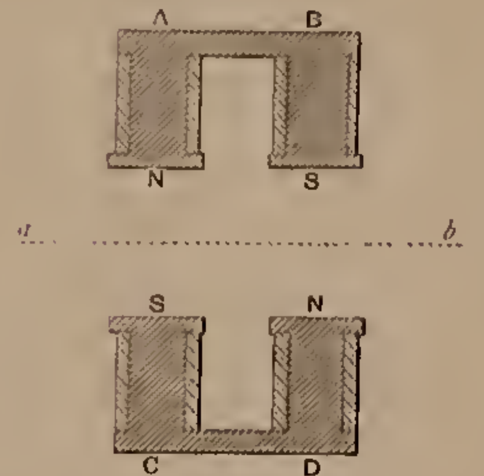
CYLINDER.

Fig. 50.
JÜRGENSEN.



CYLINDER.

Fig. 51.
MARCEL-DEPREZ.



TWO CYLINDERS.

To face page 114.

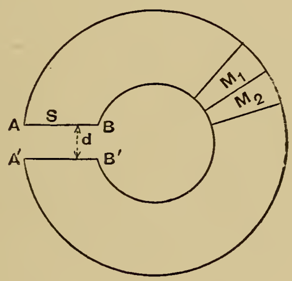
lines, and as a first step towards the realization of this object we must determine the relation between the number of lines and the constructive data of the machine. One of these data is the exciting power, that is the product of the number of turns of wire wound on the magnet, and the magnetizing current sent through the wire. It is customary to reckon the exciting power in *Ampere-Turns*, and it is shown by experiment and theory that the manner in which the product is made up is quite immaterial. We may have a large number of turns of fine wire and a small current, or we may have few turns of stout wire and a large current. The effect will always be the same if the product of amperes and turns be the same. Experiment also shows that for low degrees of magnetization, the electro-motive force produced in the armature is proportional, or nearly so, to the exciting power X applied to the field magnets ; and since electro-motive force and strength of field Z are always proportional, we find that in these cases Z is proportional to X. We can represent this relation mathematically by introducing the conception of *magnetic resistance*. According to this there is in every magnetic circuit a passive force opposing the creation of lines, and the number of lines which are created is the quotient of the magnetizing force and this resistance. Calling the latter R, we have

$$Z = \frac{X}{R} \quad \ldots\ldots\ldots\ldots \quad 23).$$

This formula is rigorously correct, provided we succeed in determining the magnetic resistance for every condition of magnetization. For low degrees of magnetization the resistance is nearly constant, and in these cases there exists simple proportionality between Z and X; for higher degrees of magnetization the resistance increases and the relation between Z and X becomes more complicated. A limit is ultimately approached beyond which we cannot increase the strength of the field although we may increase the exciting power indefinitely. In this case the magnetic resistance has become infinite, and this condition is generally expressed by saying the magnet is *saturated.*

The relations existing between magnetizing power and the magnetic moment have in the case of straight-bar magnets, spheres, and ellipsoids been investigated by Jacobi, Dub, Müller, and others, and a variety of formulas have been proposed to express these relations mathematically. Apart from the fact that these formulas in themselves are only rough approximations but imperfectly fitting the results of experiments, they are for practical purposes almost useless, since the field magnets of dynamos and motors are not straight-bar magnets, but horse-shoes of every possible form and variety. In some cases these formulas are even misleading, and as an example we may cite the original Edison machines. According to the orthodox theory the magnetic moment of a cylindrical bar is proportional to some function of the exciting power, to the square root of the diameter of the bar and to the square root of the cube of its length. Hence to obtain a maximum of magnetic moment with a given weight of iron we must shape it into a long cylinder, and the original Edison machines were constructed on these lines. Experience has since then taught us that this was the worst possible form which could have been adopted, and the Edison machines built subsequently have stout and short magnets. The explanation for this apparent discrepancy between theory and practice is this, that in a dynamo or motor the magnetic moment of each bar composing the field magnet is of no account whatever, the electro-motive force depending only on the total number of lines produced, which is governed by laws totally different from those relating to the magnetic moment. It is very desirable that the relations between strength of field and exciting power should be mathematically established for those forms of magnets which are actually used in the construction of dynamos and motors. As yet no formula rigorously true for all degrees of magnetization has been found, and the difficulty is

Fig. 52.



principally due to the fact that the chemical composition and molecular properties of the iron play an important part which is not easily determinable beforehand. This is especially the case if the magnetization is pushed towards the saturation limit. For lower degrees of magnetization the difficulties are still present, but they are of relatively less importance, and it is possible to establish formulas for the strength of the field which are sufficiently approximate for practical purposes.

Let in Fig. 52 a series of wedge-shaped and very short magnets, M_1 M_2 . . . be placed with polar faces of opposite sign in contact, so as to form a continuous ring interrupted only by the air space, $A\ B$, $A^1\ B^1$. Lines of force will then pass across this air space, and an electro-motive force could be created by moving a conductor or series of conductors, so as to cut these lines. Let the polar surface of each elementary magnet be S, and let the density of magnetic matter, which we imagine to be distributed over the polar surfaces, be σ, then $\sigma\ S$ is the strength of each polar surface. According to Ampere's theory each elementary magnet can be replaced by an equivalent magnetic shell (page 28), consisting of a closed conductor in which a current flows, the product of current and area enclosed being numerically equal to the magnetic moment of the elementary magnet. Imagine now the magnets replaced by a spiral of wire or *solenoid*, then we can without appreciable error consider each turn of wire in the spiral as a current closed in itself, and if there be n such turns, and if the current be C, the total magnetic moment will be in absolute measure $n\ C\ S$. Since with the exception of the two end faces $A\ B$, $A^1\ B^1$, the polar surfaces are in contact and cannot exert any action at a distance, the total magnetic moment of the series of elementary magnets is represented by the product of the magnetism on the end faces, and their distance, d. We have therefore the equation,

$$\sigma\ S\ d = n\ C\ S.$$

It has been shown (page 24), that the total number of lines emanating from unit pole is $4\ \pi$. From a pole of the strength $\sigma\ S$ there must emanate $4\ \pi\ \sigma\ S$ lines. Let Z be the total number of lines, or strength of field within the air space, then we find

$$Z = 4\ \pi\ \sigma\ S,$$

and by inserting the value of $\sigma\ S$ from above equation,

$$Z = \frac{4\, \pi\, n\, C\, S}{d},$$

which can also be written in the form

$$Z = 4\, \pi \frac{n\, C}{\frac{d}{S}}$$

The product $n\, C$ is exciting power in absolute measure, or ampere turns × 10^{-1}. S is the polar surface, and d the distance between the two poles. In deducing the formula for Z we have assumed the polar surfaces to be two parallel planes, but it can be proved that the same law holds good for surfaces of any shape, provided that their distance is very small as compared to their area. We can therefore apply the formula to the case of a cylindrical polar cavity partly filled by a cylindrical armature. Here we have two air spaces, and the polar surface S is the product of length of armature, b and the arc spanned by either pole, λ. Let δ be the distance between the polar surface of the magnets and the external surface of the armature core, and let X represent the exciting power producing Z lines, then the above formula becomes

$$Z = 4\, \pi \frac{X}{\frac{2\, \delta}{\lambda\, b}}$$

$$Z = \frac{X}{\frac{2\, \delta}{4\, \pi\, \lambda\, b}} \quad \ldots\ldots\ldots\ldots \quad 24)$$

The strength of the field is represented by the quotient of exciting power, and an expression which is of the character length divided by area. The analogy with

Ohm's law will be obvious. The electrical resistance of a conductor is found by multiplying its specific electrical resistance with the length, and dividing by the area of the wire. In the same manner the magnetic resistance of the air space is found by multiplying $\frac{1}{4\pi}$ with the length (2δ), and dividing by the area (λb) of the air space. We can therefore regard $\frac{1}{4\pi}$ as the specific magnetic resistance of air. The expression 24) gives the field in absolute lines; to obtain it in such measure as to be directly applicable for the determination of electro-motive force by equation 5) we must divide by 6,000. If for convenience we also use inches instead of centimeters in the dimensions δ, λ and b and Ampere turns, instead of exciting power in absolute measure we find

$$Z = \frac{X}{1880 \frac{2\delta}{\lambda b}} \quad \ldots\ldots\ldots\ldots \quad 24a$$

This formula is only correct under the supposition that there be no other resistance in the magnetic circuit but that of the interpolar air space, and that this space be really filled with air, and not with some other material. Materials differ in regard to the resistance they offer to the passage of lines of force, or as it may also be expressed in regard to the degree to which they are permeable to magnetic lines of force. Iron is more permeable than nickel or cobalt, and these metals are more permeable than copper, whilst copper again is more permeable than air. The *magnetic permeability* of any substance can therefore, be expressed by a coefficient μ, which denotes its relation to the permeability of air, which latter is taken as 1. The equation 24) is valid for a space filled with air or any other substance having permeability 1, but if the space were filled with a substance having permeability μ, the total flux of lines would be given by

$$Z = \frac{\mu X}{\frac{2\delta}{4\pi\lambda b}} \quad \ldots\ldots\ldots\ldots \quad 24b).$$

We have obtained equations 24) and 24b) by considering what flux will be produced across the interpolar space of a dynamo, but it is obvious that these formulas have a much wider application. Nothing prevents us, for instance, to apply the formula to the iron of the armature itself. For this purpose we need only replace the area λb by the cross-sectional area of the iron in the armature A_a, and the length of path 2δ by the length of path L_a, which the lines of force take in flowing across the armature core. Then if we know μ the formula will give us the flux produced by the exciting power X_a, supposing none of the other parts of the magnetic circuit to oppose the flux ; or the formula may be used to determine the exciting power required to drive a given flux Z_a through the armature. In like manner can we calculate the exciting power X_m required to drive a given flux Z_m through the magnets, and by adding these exciting powers we arrive at the total exciting power required for the whole magnetic circuit.

The expression 24b) may thus also be used in the form

$$X = \frac{1}{\mu} Z \frac{L}{4\pi A}$$

if X be desired in absolute measure. If we wish to obtain X in ampère turns we would have

$$X = \frac{1}{\mu} \frac{1}{1{\cdot}256} Z \frac{L}{A}$$

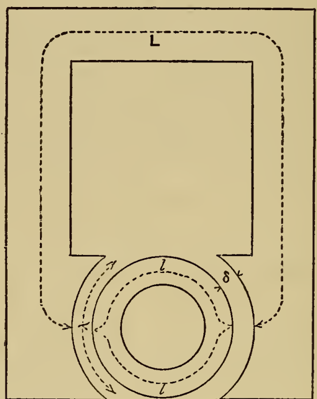
$$X = \frac{{\cdot}8}{\mu} \frac{Z}{A} L \quad \ldots\ldots\ldots\ldots \quad 25),$$

or, if Z be given in English measure and the dimensions in inches:

$$X = \frac{1880}{\mu} \frac{Z}{A} L \quad \ldots\ldots\ldots \quad 25a).$$

The total exciting power required to produce a flux Z_a

Fig. 53.



through the armature and Z_m through the magnets in such a machine as that diagramatically shown in Fig. 53 would therefore be in C.G.S. measure:

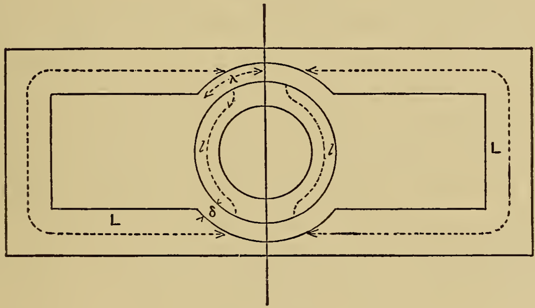
$$X = {\cdot}8 \frac{Z_a}{A_x} 2\,\delta + \frac{{\cdot}8}{\mu} \frac{Z}{A_a} l + \frac{{\cdot}8\, Z_m}{\mu\, A_m} L \quad \ldots \quad 26),$$

$$\text{Or, } X = 1880 \frac{Z_a}{A_\alpha} 2\,\delta + \frac{1880}{\mu} \frac{Z_a}{A_a} l + \frac{1880}{\mu} \frac{Z_m}{A_m} L \quad . \; . \quad 26a),$$

in English measure.

In a double horse-shoe machine such as Fig. 54 the same formulas apply, but only half the total fluxes must be inserted, since we have now two magnetic circuits, each carrying half the lines which produce the E.M.F. It will be noticed that in these equations different symbols are used to denote the total flux through magnets and armature. This is necessary because the two are never equal. The flux is produced within the magnet cores and is *forced* across the armature. In this process of forcing some lines escape laterally and never pass through the armature at all, so that $Z_m > Z_a$. This

Fig. 54.



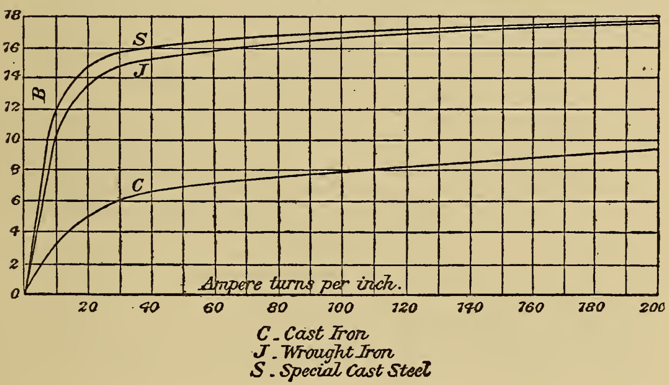
point, which is technically termed magnetic leakage, will be dealt with presently.

To apply either of these formulas it is of course necessary that we know the value of μ in each case. Now this is is not a constant but varies with the induction (number of C.G.S. lines per square centimetre or number of English lines per square inch) passing through the material. The induction is the ratio of total flux to area $\frac{Z}{A}$ and this is generally denoted by the symbol B. Formula 25) may therefore also be written thus:

$$X = \frac{.8}{\mu} B L, \text{ Or, } \frac{X}{L} = \frac{\cdot 8}{\mu} B.$$

The ratio $\frac{X}{L}$ denotes obviously the number of ampère turns required per centimetre of path, in order to produce the induction B in the material. If we know the permeability corresponding to any induction we can then calculate the ampère turns per centimetre or inch of path, and therefore the total ampère turns corresponding to each part of the circuit for various values of the induction. The permeability as a function of the induction must of course be determined experimentally for the particular type of iron used in the construction of armature core or field magnet. It would exceed the scope of this book to describe such experiments and the apparatus required for them in detail. Suffice it to say that instruments for determining the magnetic qualities of samples of iron have been made and used by Ewing, Hopkinson, Thompson, the author, and others. In working with such instruments the permeability is not obtained directly, but as the ratio of two other quantities, namely, magnetizing force and induction ; and since the magnetizing force is given by the expression $\cdot 4\pi\frac{X}{L}$ it is from a practical point of view more convenient to neglect permeability altogether and straightway determine the relation between $\frac{X}{L}$, that is, ampère turns per centimetre or inch of path, and B, the resulting induction. By plotting corresponding values of $\frac{X}{L}$ and B we obtain for each kind of iron a magnetization curve from which we can find at a glance how many ampère turns per centimetre or per inch of path are required to produce a certain induction. Figs. 55 and 56 show such curves obtained with samples of cast iron, ordinary wrought iron, and special cast steel. Fig. 55 gives ampère turns per inch of path when the induction is given in English measure. Fig. 56 gives ampère turns per centimetre of path when the induction is given in C.G.S. measure. To find the exciting power required to produce any given flux of lines through the armature we must therefore proceed in the following manner. We find from the drawing of the machine the

Fig. 55.

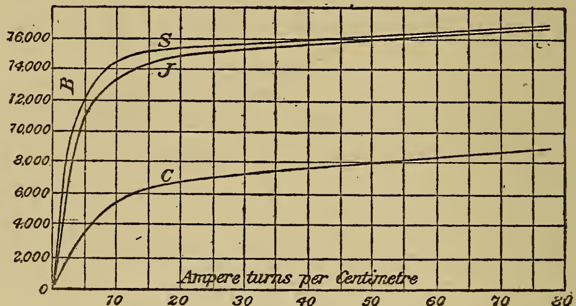


cross-sectional area and the length of the various parts of the circuit, namely, armature core, air space, pole pieces, magnet cores, and yoke. Knowing the total flux and the area in each case we find the induction in each part of the circuit and by reference to such a curve as Fig. 55 or Fig. 56, the exciting power required[1] for each part. By adding

[1] In the third edition of this book an approximate method for determining the exciting power was given which differed from the present and exact method in this, that instead of using an actual magnetization curve,

up we find the total exciting power required to produce the given flux through the armature. In this manner we proceed, assuming various values for the useful flux through the armature, and determining the total exciting power corresponding to each. By plotting these values (Z_a on the vertical and X on the horizontal) we obtain a curve of magnetization for the particular machine under con-

Fig. 56.



C. Cast Iron
J. Wrought Iron
S. Special Cast Steel

sideration. Conversely if it be desired to design a machine having a certain curve of magnetization we can do so by

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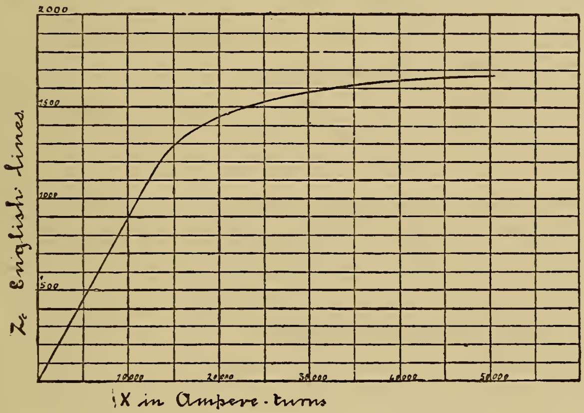
the relation between exciting power and induction was assumed to be proportional to the ratio tan $\left(\frac{\pi}{2}\frac{B}{\beta}\right) \Big/ \frac{\pi}{2}\frac{B}{\beta}$ where β represented the maximum value of B which the iron was capable of attaining under an excessively great magnetizing force; in other words, the number of lines per square inch at saturation. When applied within the working range of dynamos this formula gave fairly accurate results, and was convenient because in the early days of dynamo building very few manufacturers had the necessary appliances for determining the magnetization curve of the iron they used, whereas the point of saturation could be easily found by experimenting with finished machines. Now, however, that instruments for magnetic tests of iron samples of such simple construction as to be fitted for the workshop have become available, the method of design given in the text is the one generally adopted.

starting with a preliminary design, and then see what alterations in dimension and winding are required to produce the desired result. Fig. 57 shows the magnetization curve for a machine of the following dimensions: Armature smooth ring; discs, 18 in. diameter, 14 in. long, $3\frac{1}{2}$ in. deep; length of polar arc $\lambda = 23$ in.; air gap $\delta = \cdot 9$ in.; mean length of path through armature core 16 in.; magnets of the double horse-shoe type, each limb 65 sq. in. in

Fig. 57.



section; length of path through magnet limbs and yoke 66 in.

Before proceeding to show how the magnetization curve, sometimes also called characteristic of magnetization, may be used to determine the actual working condition of the machine, it is necessary to say a few words on the question of magnetic leakage already mentioned a few pages back. It was then pointed out that the flux is forced round the magnetic circuit, and that part of the flux escapes laterally. The pole pieces of the machine send lines through the armature, but at the same time they send lines through the air in the same way as does an ordinary magnet (Fig. 1). There is no means of insulating a surface, so as to prevent the dissipation of lines from it; there must always be a certain amount of flux, not only from one pole piece to the other through the air, but also between pole pieces and magnet cores, bedplate and yoke. In fact, magnetic leakage takes place between any surfaces on a machine where these are under different magnetic pressure, just as in a system of electric conductors immersed in a badly conducting liquid, there must be leakage of electric current between any parts which are under different electric pressure. The magnetic pressure, or in other words, the line integral of magnetic force, is of course proportional to the exciting power, and where the leakage takes place through air (permeability of medium being unity), the amount of flux lost by leakage must obviously be proportional to the exciting power. If we know the position and extent of the surfaces between which leakage takes place, it is possible in certain simple cases to calculate the amount of leakage; but in most dynamos or motors the disposition of the surfaces, and the variation of magnetic pressure between them, are so complex that an exact determination of the leakage is almost impossible. Great exactitude is, however, not required, and we may for practical work be satisfied with an approximate determination. In a completed machine the leakage can, of course, be found experimentally by the use of an exploring coil and ballistic galvanometer. Suppose we have done this with one particular machine, and wish to utilize the result for the predetermination of the leakage to be expected in another machine of the same type, but different dimensions. Since by far the largest part of the total exciting power is required on account of the magnetic resistance of the air gap between polar surfaces and armature surface, we may take $X\alpha$ as the ampère turns to which the total leakage will be proportional. The total flux lost by leakage will therefore be proportional to the quotient $X\alpha$, divided by the average magnetic resistance of the leakage paths. Now assume that the new machine has every way twice the linear dimensions of the machine experimented upon. If we imagine in both machines the whole leakage field mapped out and similarly divided into small sections then the length of path of each section in the large machine will be twice that of the corresponding section in the small machine, and the areas between corresponding points will be as 4 to 1. Since the magnetic resistance of any path in air is proportional to the length divided by the cross sectional area, the average magnetic resistance of the large to that of the small machine will be as $\frac{2}{4}$ to 1, or in other words, the resistance of the leakage field of the large machine will be half that of the small machine, and if the ratio of linear dimensions instead of 2 to 1, had been 3 to 1, or n to 1, the leakage resistance would be one-third, or one-nth that of the small machine. A convenient way of defining the linear dimensions of a machine is to give the diameter and length of the armature. If the different machines of the same type were precisely similar in all respects, either the diameter or the length of the armature alone might be introduced into the formula for the leakage; but as there may between different machines be variations in the ratio of length and diameter of armature, it is convenient to assume that their linear dimensions are determined not by either alone, but by the square root of their product. Thus, if d and l represent diameter and length of the armature, we assume that the magnetic resistance of the leakage field is proportional to $1/\sqrt{d\,l}$. Its absolute value is:

$$\rho = \frac{K}{\sqrt{d\,l}} \quad . \quad . \quad . \quad . \quad (27$$

where K is a co-efficient, depending on the particular type of machine under consideration. For machines of the type shown in Figs. 30 and 39, this co-efficient may be taken at ·21 in cgs., or 460 in English measure. For double horse-shoe fields (Figs. 29, 30, 41), ·25 and 550 respectively ; and for upright machines, like Figs. 40, ·29 and 680. The leakage resistance of multipolar machines of the type shown in Fig. 48, can be found by reducing the design to an equivalent two pole field. Thus, if the machine have 6 poles and an armature 36 inches in diameter, the d to be inserted in the formula for ρ is not 36, but $36/3 = 12$, the diameter being reduced in the ratio of the number of pairs of poles. The co-efficient K may in this case be taken as for overtype machines, namely, ·29 in cgs., and 680 in English measure.

Having thus determined ρ, we find the total flux lost by leakage,

$$\zeta = \frac{X\alpha}{\rho} \quad . \quad . \quad . \quad . \quad (28$$

Strictly speaking, the formula is only valid as long as no current flows through the armature. If the machine is at work, the current flowing through the armature conductors between the pole pieces of opposite sign, assists the field excitation on one side, and opposes it on the other side of the brush, the latter effect being the greater, so that the magnetic pressure between the pole pieces is slightly greater than corresponds to the theoretical value of $X\alpha$. The corrections required on this account is, however, very small, and may in most cases be neglected. The flux through the field magnets can now be found by

$$Zm = Za + \zeta,$$

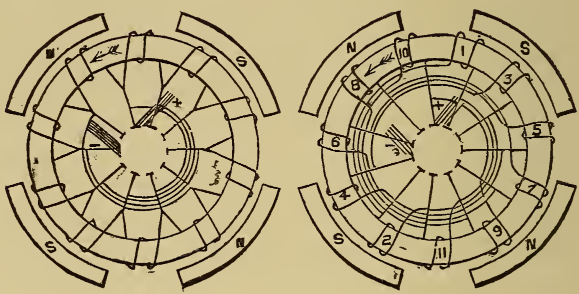
and this is the value to be used in formulas 26) and 26 a).

The investigation above given is sufficient to enable the reader to understand the design and working of two pole dynamos or motors; but it is now necessary to make a slight extension of the theory, in order to include multipolar machines, especially since the latter are generally used in transmission plant, where the power to be dealt with is considerable.

Imagine a ring-wound armature taken out of its own two pole field, and placed in a four-poled field, such as Fig. 48. If the flux emanating from one pole piece be the same in either case, and if the speed of rotation be also the same, it will be obvious that the E.M.F. generated in one quarter of the winding when the armature is rotated in the four-pole field, must be exactly equal to the E.M.F. generated in one-half the winding when the armature is rotated in the two-pole field, for we have in either case the same number of lines cut by the same number of wires at the same speed. The only difference is that, whereas in the two-pole machine the current splits into two circuits when passing through the armature, in the four-pole machine it splits into four circuits, requiring the use of four instead of two brushes. With the same current in each armature wire in both cases the total current obtainable from the four-pole machine will therefore be double that of the two-pole machine. Similarly a six-pole machine will give three times, and an eight-pole machine four times the current, the E.M.F. being in all cases the same.

The number of brushes to be used is, in these cases, equal to the number of poles in the field; but by a slight addition to the winding of the armature, we are able to reduce the number of brushes in all cases to two only. This addition consists in cross connections between

Fig. 58.



diametrically opposite coils, as shown in the left-hand side of Fig. 58 for a four-pole machine.

We have seen that by increasing the number of field poles the output of the machine is proportionately increased, the increase being in the shape of a larger current at the original voltage. It is, however, also possible to obtain the increase of output in the shape of higher voltage with the original current, and for this purpose the winding of the armature must be so altered as to bring a larger number of armature coils into series connection. Several methods of winding have been devised, and as an example may be taken that shown in the right-hand diagram of Fig. 58. For clearness of illustration only eleven armature coils are shown, but it will be understood that this method of winding is applicable to four-pole machines, and ring armatures with any other odd number of coils. One end of each coil is connected to its commutator plate, and the other is brought round to the opposite side, and attached to the wire which connects the opposite coil to the corresponding commutator plate. Thus, the front end of 1 is connected to the back end of 2, and to plate 2 of the commutator ; the front end of 2 is connected to the back end of 3, and to plate 3, and so on, the last connection being the front end of 11 to the back end of 1, and to plate 1. The current entering the armature at the negative brush, where it touches plate 6, splits into two circuits, one going round coil 6, up on the outside of the armature, the other round coil 5, down on the outside of the armature. The former current goes successively up in 7, 8, and 9, leaving the armature at plate 10 by the positive brush, whilst the latter goes successively down in 5, 4, 3, 2, 1, and 11, leaving the armature also at plate 10. If there were 103 coils instead of only 11, the current would go similarly up in 50 coils and down in 53, and by following the direction of the current in the diagram, it will be seen that it coincides with the direction of E.M.F. induced in each coil ; in other words, that the E.M.F. created by one pair of poles is added to that created by the other pair.

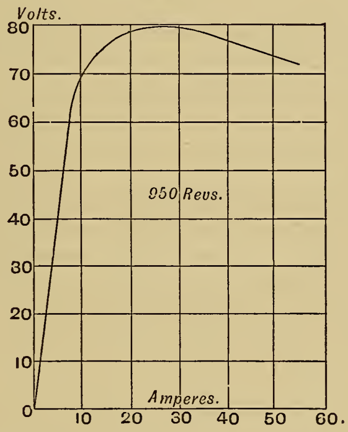
The armature windings up to the present described belong to the ring type, the characteristic feature of which is, that the wire is brought through the internal space of the armature ; this part of the winding acting merely as a conductor, and not assisting in the production of E.M.F. It is, however, also possible to wind multipolar armatures drum fashion when no conductors at all pass through the interior of the armature coil, the connection being made entirely at the ends. Diagrams for this kind of winding and for two pole machines have already been given in Figs. 18 and 19, and it will be easily seen how the system may be extended to multipolar machines. In this case the end connections are arranged to span not half the circumference, but a quarter or a sixth part of it, accordingly as the field has four or six poles; and the winding may be arranged either for parallel or for series connection, accordingly as it is desired to obtain a larger current or a larger E.M.F. In the former case the formulas for E.M.F. previously given, are directly applicable; in the latter case the right-hand side of the equation—4) or 5)—must be multiplied by a number equal to the number of pairs of poles in the field. Thus in a four-pole machine the multiplier would be two, in a six-pole machine it would be three, and so on.

An inspection of formula 26 will show why, as already mentioned in the beginning of Chapter III., small motors sometimes fail to act as dynamos. In very small machines the air-space δ can, for mechanical reasons, not be reduced in exact proportion with the linear dimensions, and the first term on the right-hand side of the equation becomes comparatively large. In other words, the magnetic resistance of the air-space is high, and requires a correspondingly high exciting power. To produce this high exciting power, an amount of electrical energy is required, which may exceed the total output obtainable from the armature, and if this be the case the machine will fail to excite itself. When the machine is used as a motor, this cannot happen, the supply of energy for field excitation being from an outside source, the power of the armature is not affected thereby.

It has already been shown how the relation between exciting power and field can be graphically represented (see Fig. 57) ; and since for constant speed the E.M.F. is proportional to the field strength, it is also possible to represent by a similar curve the relation between field-strength and E.M.F. The exact shape of the curve depends, of course, on the construction of the machine, showing, so to speak, its general character. On this account the name "characteristic curve" has been given to any curve which illustrates the relation between two of the variable quantities concerned in the working of the machine, if all the other quantities are kept constant. For instance, if we keep the speed and the current in the external circuit constant, we can represent the E.M.F. as a function of the exciting power, or in a series machine as a function of the current. At constant excitation and constant speed, the current can be represented as a function of the external resistance. Or the torque in a motor for constant current may be represented as a function of the exciting current, and so on ; the various relations between current, E.M.F. excitation, speed, torque, horse-power, and efficiency, can all be represented graphically by characteristic curves.

The characteristic curves of magnetisation give, as already stated, the total flow of lines of force through the armature, provided no other exciting power except that applied to the field magnets is active. This condition is fulfilled if the field magnets are separately excited, and no current is allowed to flow through the armature, the electro-motive force in which is in this case exactly the same as that which can be measured at the brushes. When a current is allowed to flow through the armature, the electro-motive force which we measure at the brushes is not exactly the same as that generated in the armature, but either smaller or larger, accordingly as the machine is used as a dynamo or motor. We have thus to distinguish between three conditions of working, namely (1) no current, (2) dynamo current, and (3) motor current passing through the armature. The first condition can be obtained by simply opening the external circuit, but we can also imagine that the external circuit remains closed, and that some source of electro-motive force is inserted, acting in opposition to the electro-motive force generated in the armature, and that this is so accurately adjusted as to prevent a flow of current in either direction. We shall then have, so to speak, a static balance between the armature and opposing electro-motive force. And for this reason the author has suggested the name "static characteristic" to any characteristic curve representing this condition of working. If we now imagine the opposing electro-motive force reduced, it will no more be able to balance the electro-motive force of the armature, but will be overpowered by the latter, with the result that a current will flow, doing work upon the opposing electro-motive force. The machine now works as a dynamo, absorbing mechanical and giving out electric energy. Any characteristic curve expressing this condition of working, we call "dynamic characteristic." If, on the other hand, we increase the opposing electro-motive force until the latter overpowers the armature electro-motive force, a current will be forced through the armature doing work in overcoming its electro-motive force, in other words driving the machine as a motor. Any characteristic expressing this condition of working of a machine we call "motor characteristic." For certain reasons which will be given presently, the dynamic characteristic must always be lower than the static characteristic, and the same holds good generally for the motor characteristic, but there are cases when the motor curve lies above the static curve. Confining ourselves for the present to dynamos only, it is easy to see why the dynamic curve must be below the static curve. Allowances must in the first place be made for the drop in electro-motive force due to the electrical resistance of the armature. This drop can of course be calculated by multiplying current and resistance. But beyond this, there is a further reduction of electro-motive force, which can be explained as follows. Let us for the moment assume that we work a two-pole dynamo with the brushes set exactly at right angles to the polar diameter, and let us concentrate our attention on say the positive brush, that is the brush where the current leaves the armature. In all the coils on one side of this brush the current flows in one direction, say towards the commutator on the outside of the armature, whilst in all the coils on the other side of the brush it flows in the opposite direction. There is thus a reversal of current in each coil as it passes under the brush. Whilst under the brush, the coil is short circuited on itself, but as a moment before it was traversed by half the total armature current, there will remain some current flowing during its period of short-circuiting by virtue of the self-induction of the coil. This current becomes gradually weaker, owing to the resistance of the coil, but even if this cause were sufficient to bring the current to zero, it can obviously not reverse it. By the time the coil emerges from under the brush, the previous current in it may or may not yet have died out, but at that instant half the total armature current is forced through it in the opposite direction. This causes a violent spark, which can only be avoided by shifting the brush into a more advanced position. When in that position, the coil whilst under the brush cuts through lines of force which induce in it an electro-motive force opposed to the persisting current, and bring it thus quickly to zero. But these lines of force do more ; they start the current in the opposite direction, so that the coil at the moment of emerging from under the brush carries already the same current as will be forced through it afterwards, whereby the transition from the idle, short-circuited position under the brush to the working position beyond the brush, takes place quite gradually and without sparking. By shifting the brush forward we have killed out the spark, but we have sacrificed some of the lines of force which might otherwise have increased the electro-motive force of the armature, and which, whilst the machine was working on open circuit, have indeed been so utilized. There are therefore more lines of force utilized whilst the machine is working statically, than whilst it is working dynamically, from which it follows that the dynamic electro-motive force must be lower than the static electro-motive force. A further reduction of electro-motive force is due to the fact, thet the armature itself becomes magnetised by the current flowing through its coils, and reacts on the field magnets. If it were possible to work with the brushes on the neutral diameter, the poles developed in the armature would stand exactly midway between the main field poles, and would neither strengthen nor weaken them, but the brushes must for the reason already given be shifted forward, which brings armature and field poles of the same sign nearer together, and the result is, that the main field is somewhat weakened, which again reduces the electro-motive force. In reality, the phenomena here briefly sketched, and which are technically comprised under the term "armature reaction," are not quite as simple as here stated, but it would

Fig. 59.

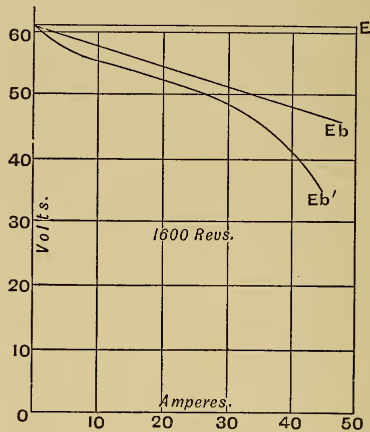


INTERNAL CHARACTERISTIC OF A GRAMME DYNAMO.

exceed the scope of the present work to enter more fully into details, the more so as the total effect of armature reaction is in good dynamos very small, amounting often to less than five per cent. of the total electro-motive force. In badly designed machines it may become considerable, as can be seen from Fig. 59, which represents the internal characteristic of an A gramme dynamo, tested by M. Marcel Deprez.

This behaviour of the dynamo can best be studied with separately excited machines, and Mr. Esson has made very careful trials on the subject, which were published in April, 1884, in "The Electrical Review." The dynamo experimented upon was a "Phœnix" machine with Pacinotti armature. It was separately excited and kept running at a constant speed of 1,600 revolutions a minute, whilst the current which was permitted to flow through the armature was varied by means of a rheostat. The

Fig. 60.



EXPERIMENT WITH PHŒNIX DYNAMO.

line *E*, Fig. 60, represents the internal electro-motive force corresponding to the constant exciting power if there were no reactions. The line *Eb* represents the electro-motive force which would be found at the brushes if there were no reaction, and the line Eb^1 was that actually observed. The difference of the ordinates of *Eb* and Eb^1 represents the loss of electro-motive force due to self-induction, weakening and distorting of the field.

The armature reaction in a motor is very similar to that in a dynamo, except that the electro-motive force lost in resistance must be added to, instead of subtracted from the internal armature electro-motive force. If the machine is properly constructed, there will therefore be very little difference between its dynamic and motor characteristics,[1] both lying below the static characteristic, but if the machine is not so constructed as to give a high efficiency, then it may happen that its motor curve is considerably higher than its dynamic, and even higher than its static curve. The reason is obvious. If a sensible amount of energy is wasted in eddy currents or hysteresis, this energy must be supplied by the motor current in the shape of an increased terminal pressure. Now we find the motor characteristic of electro-motive force by deducting from the electro-motive force at the brushes which can be measured, the calculated electro-motive force necessary to overcome the resistance of the armature. The latter being constant, it follows that the higher the brush electro-motive force, the higher must also be the motor electro-motive force. The author has verified this conclusion by testing a Bürgin machine as a dynamo and as a motor. As was to be expected, the dynamic curve was found to lie below the static curve, but the motor curve, instead of being below the static curve, was found to lie above it. The reason is probably that eddy currents in the corners of the iron wire hexagons which form the armature core, and a certain amount of surging of lines over the surface of the pole pieces produced by these corners, absorb a sensible amount of energy requiring an increased electro-motive force in the

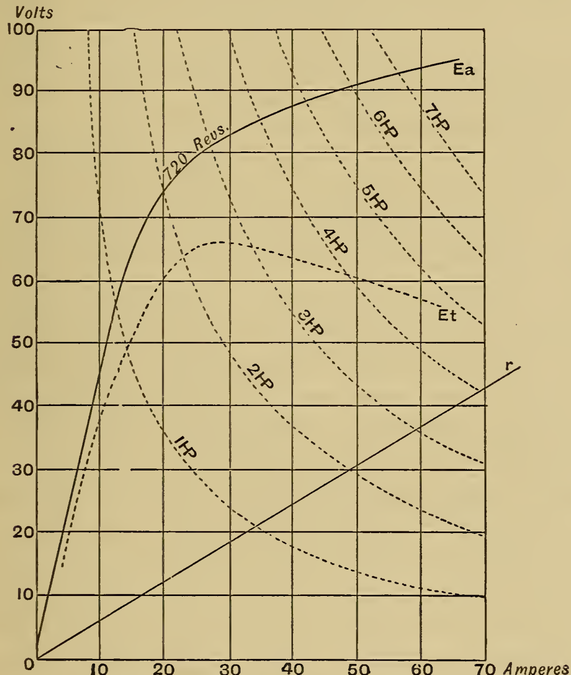
[1] If the loss of E.M.F. due to armature reaction is made equal to the loss of E.M.F. resulting from armature resistance we obtain a motor which will run with practically constant speed under a varying load.

driving current, and thus making it appear as if the counter electro-motive force in the armature were higher than it really is. The fact that some machines show a high counter electro-motive force, has led certain scientists, and notably Professors Ayrton and Perry, to formulate a theory of electro-motors, according to which the armature was credited with some sort of power to increase the strength of the field instead of weakening it, as is actually the case, and it was recommended that motors should have small field-magnets and large armatures. Practical experience has, however, disproved this theory, and the best motors are nowadays designed by the use of the same formulæ as the best dynamos.

We may now proceed to show the use and interpretation of characteristic curves generally.

Fig. 61 shows the internal and external characteristics of a series-wound Siemens dynamo, as given by Dr. Hopkinson in the Proceedings of the Institution of Mechanical Engineers, 1879. The dotted curve $O\,E_t$, represents the electro-motive force at the terminals of the machine, and the curve shown in a full line $O\,E_a$, that in the armature. The latter is obtained from the former by adding to its ordinates the internal losses of electro-motive force due to armature reaction and resistance. This can be represented roughly as the product of current and a certain resistance, which latter was in that particular machine 0·6 ohms. Thus at 50 ampères the loss is 30 volts, and it will be seen from the diagram that the difference between the two ordinates corresponding to 50 on the abscissæ is 30. We can also represent the loss of electro-motive force by a characteristic, and since it is always proportional to the current, the characteristic in this instance becomes a straight line, *O r*. The geometrical tangent of the angle which this line forms with the horizontal is evidently equal to the resistance in question. The ordinates enclosed between $O\ r$ and $O\ E_a$, represent the external electro-motive forces, and therefore the internal

Fig. 61.



CHARACTERISTICS OF SIEMENS DYNAMO.

characteristic, $O\ E_a$, becomes the external characteristic if we take $O\ r$ for the base line instead of the horizontal.

By a very ingenious method due to Professor Silvanus P. Thompson these characteristics can also be used to show at a glance the horse-power which corresponds to any particular current or electro-motive force. As already shown the horse-power represented by a current c flowing under an electro-motive force E_a is H-P $= \frac{c\,E_a}{746}$. One horse-power can be represented by an infinite variety of c and E_a, but these values must all satisfy the equation

$$746 = c\,E_a.$$

A curve representing one-horse power will pass through all such points of which the product of their ordinates is a constant, viz., 746. Similarly a curve representing the value of two horse-power will pass through points of which the product of their ordinates equals 1492, and so on. In other words, all the horse-power curves are rectangular hyperbolas,[1] and by drawing a set of these curves across our diagram—as shown in dotted lines—we can determine at a glance what is the horse-power corresponding to any point on the characteristic. Thus a current of 30 ampères represents about 3·35 H-P of internal electrical energy, and about 2·7 H-P of electrical output or energy delivered into the external circuit. A current of 50 ampères represents a little over 6 H-P internal, and a little over 4 H-P external energy, and so on.

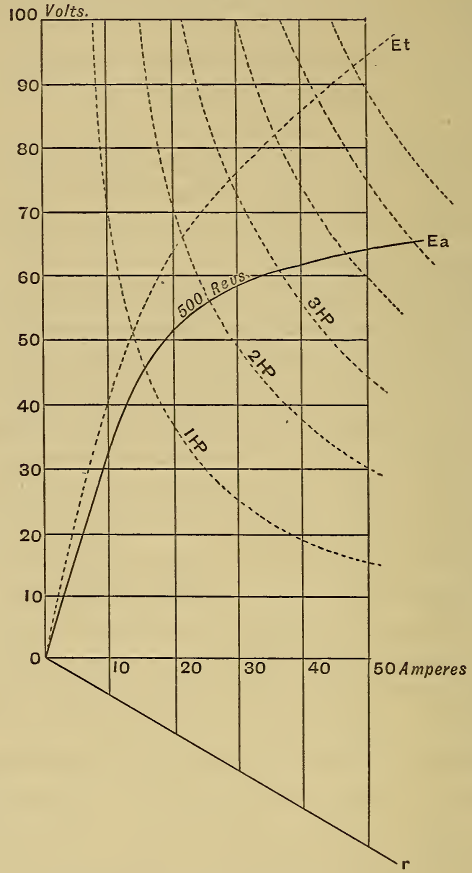
In a dynamo the internal characteristic lies always above the external characteristic. In a motor, however, their position is reversed, since the external electro-motive force must necessarily be greater than the counter-electro-motive force developed in the armature coils. Fig. 62 shows the characteristics of the Siemens dynamo mentioned above if used as motor. Not to get the diagram too long the speed has been reduced to 500 Revolutions. The curve $O\,E_a$ represents the counter electro-motive

[1] The scales for volts and ampères being equal.

force developed in the armature coils, and the curve OE_t, which is shown in a dotted line, represents the terminal electro-motive force. The difference between the ordinates of the two curves represents the electro-motive force necessary to overcome the internal resistance of the machine. By drawing the straight line $O\,r$ under an angle, the tangent of which is numerically equal to the internal resistance, but this time below the horizontal and not above it as in the former example, we can regard it as the new base line, and then the curve $O\,E_a$ becomes the external characteristic.

In diagram (Fig. 62) it is assumed that by some means we keep the speed constantly at 500 revs. a minute. Easy as it is to fulfil such a condition in a dynamo, it presents considerable difficulties if we have to deal with a series-wound motor, because its speed depends on a number of factors which to a certain extent may vary independently of each other. The speed depends on the current and electro-motive force supplied to the motor, and on the amount of mechanical work it has to do. In some cases the work itself, that is the product of turning moment and speed, depends on the latter, and thus it will be seen that the relation between these various quantities is of a rather complex nature. It is however easy to represent these relations graphically by the use of *speed characteristics*, which were first published by the author in "The Electrician," of December 29th, 1883. Assume the case that the external electro-motive force is a fixed and constant quantity. What will be the relation between speed, power, and efficiency of, say, a series-wound motor? Since E_t is constant at all currents, we have practically an unlimited supply of current such as would be obtained from the mains in a system of town supply. The current passing through the motor will depend on its

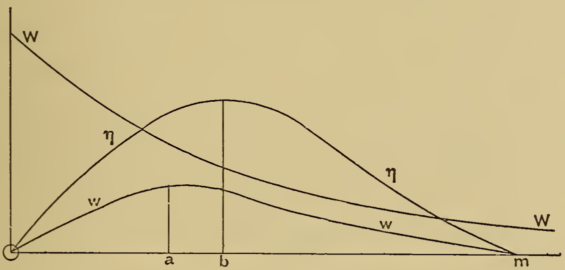
Fig. 62.



CHARACTERISTICS OF SERIES MOTOR.

resistance, and on its counter-electro-motive force. The former is constant, whilst the latter increases with the speed. The faster we allow the motor to run the less current will flow through it, and the less power will be absorbed by it. Let in Fig. 63 the speeds be plotted as abscissæ, and the electrical horse-power absorbed as ordinates, then with a series-wound motor we obtain the curve *W W*. The exact shape of this curve depends, of course, on the construction of the motor, but its general character will be as shown. The easiest way of finding the curve experimentally is by attaching a brake to the motor, and loading it with different weights so as to pro-

Fig. 63.

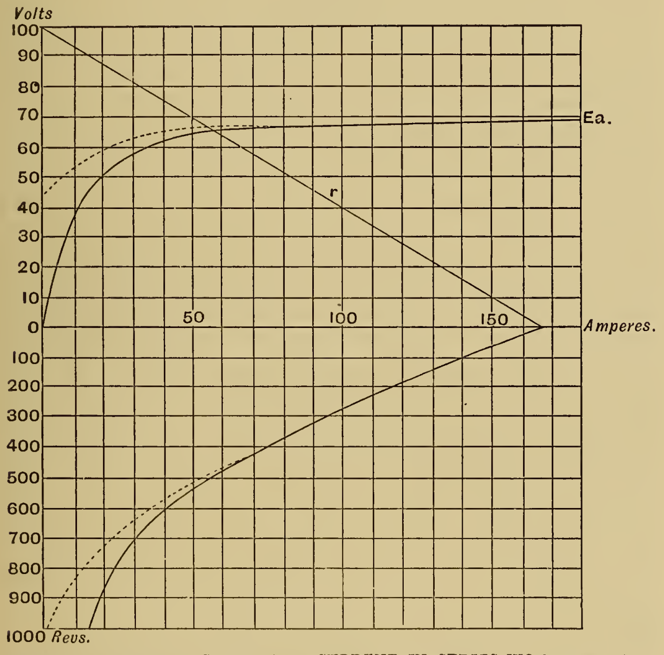


SPEED-CHARACTERISTICS OF SERIES MOTOR.

duce different speeds. The horse-power absorbed by the brake can at the same time be plotted in the curve *w w*. If we begin with an excess of load on the brake, which will hold the motor fast, a maximum of current will flow, and a maximum of electrical energy will be absorbed without producing any external work. On the other hand, if we remove the brake altogether the motor will attain a maximum velocity *o m*, and again no external work will be produced, but in this case very little current will pass, and the electrical energy absorbed will be a minimum. Between these extreme limits of no speed and maximum speed external work will be produced, and there is one particular speed, *o a*, at which this work will be a maximum. The ratio of the ordinates of *W* and *w* can be plotted in a curve, *η η*, drawn to any convenient scale, and this gives the commercial efficiency of the motor as a function of the speed. There is one particular speed, *o b*, at which the efficiency is a maximum, but this is not necessarily the same speed as that for which the work is a maximum. As a rule it is considerably greater, and in actual work the motor should be so geared that it runs at or about the speed of maximum efficiency.

The experimental determination of the most economical speed, as just described, requires the employment of a dynamometer or brake, and if such an apparatus be not at hand, cannot be adopted. In this case a different method can be used, which is fairly reliable, although not quite so accurate as the actual power test. The question to be solved is the relation between speed and current in a given series-wound motor supplied with current at a constant electro-motive force. This question can be solved if we know the internal resistance of the motor and its internal characteristic. Having obtained the relation between speed and current, we can construct the diagram Fig. 63, making a certain allowance for the efficiency of conversion. We assume that the motor derives its supply of current from a pair of mains between which a potential difference of 100 volts is maintained. Let, in Fig. 64, $O\ E_a$, represent internal characteristic for a constant speed of say 500 revs., and let the inclined straight line, *r*, be drawn across the diagram at such an angle with the horizontal that its geometrical tangent is numerically equal to the internal resistance of the motor in ohms, then the ordinates of the line, *r*, represent the counter-electro-motive forces which must be created in the coils of the armature so that any given current may pass. Thus, at 100 ampères, the counter-electro-motive force must be 40 volts. If the armature revolves at a speed of 500 revolutions a minute, we see from the characteristic that its

Fig. 64.



RELATION BETWEEN SPEED AND CURRENT IN SERIES-WOUND MOTOR.

counter-electro-motive force is 68 volts, and to bring the latter down to 40 volts, so that a current of 100 ampères may pass, the speed will have to be reduced in the proportion of 68 to 40. The speed corresponding to a current of 100 ampères is therefore $500 . \frac{40}{68} = 294$ revolutions. Similar calculations can be made for other values of current, and the speeds obtained can be plotted in a curve shown in Fig. 64, below the horizontal. At 166 ampères the speed is zero, because the whole of the constant electro-motive-force available of 100 volts is required to overcome the internal resistance of the motor, leaving nothing to be opposed by counter electro-motive-force. At 16 ampères the speed is 1000 revolutions, and at smaller currents the speed might be still greater. Theoretically, it should be infinite if no current passes, and this would be the case if the motor were free to revolve without doing any work, and if there were no internal mechanical losses. This, of course, is an impossible condition, and a limit is set to the speed by the work which must be done to overcome mechanical and magnetic friction. In good motors this is, however, comparatively small, and consequently the speed of the motor, when running empty, is inconveniently high. This is a great drawback in many cases, especially where motors are required to drive lathes and other machinery offering a variable resistance. The example represented in Fig. 64, applies also to the case where a series-wound motor is worked from a set of secondary cells, having a very low internal resistance, as the electro-motive force is then approximately constant at all currents. To lessen the difference in speed it is usual to insert a rheostat or variable resistance into the circuit between the cells and the motor. A maximum of resistance is inserted when the motor is running empty, and as the load increases resistance is switched out so as to regulate the speed. At best this is a clumsy device, requiring personal attention, and not very efficient, as with it variations in speed can never be altogether avoided. It is also wasteful, the heat developed in the artificial resistance being so much power lost. A better plan is to wind the field magnets of the motor on the compound principle, both main and shunt coils magnetizing in the same direction. This will raise the early part of the characteristic as shown in dotted lines, and will reduce the speed as shown also in a dotted line. This method is not a complete cure for the evil, but it is a palliative for it which in practice proves very successful. To make the motor perfectly self-regulating, it would be necessary to let the main coils on the field magnet excite the latter in an opposite sense to the shunt coils; but then a very valuable quality of the series motor, viz., its great starting power, would be lost. If a motor is employed for railway or tramway work it is very important that there should be an excess of power at starting. This condition is admirably fulfilled by the ordinary series-wound motor, since the current, the strength of the field, and the statical effort or torque are all maxima when the motor is at rest and decrease as it gathers speed. There is thus an automatic adjustment between speed, power, and resistance. Take, as an example, an electric tramcar worked by accumulators. On a heavy gradient or bad part of the road, the speed is low, allowing a large current to pass through the motor, thus providing the extra amount of tractive force necessary ; on a good level road the speed will increase, less current will pass through the motor, and less tractive force will be developed.

CHAPTER V.

Graphic Treatment of Problems—Maximum External Energy—Maximum Theoretical Efficiency—Determination of best Speed for Maximum Commercial Efficiency—Variation of Speed in Shunt Motors—The Compound Machine as Generator—System of Transmission at Constant Speed—Practical Difficulty.

The treatment of problems relating to the electrical transmission of energy is greatly simplified by the use of the curves explained in the preceding chapter, and by other graphic methods, of which we may mention that due to Professor Silvanus Thompson. The problem is as follows. Let a square $A B C D$ be drawn so that the length of one of the sides shall represent the electro-motive force E of the supply to any convenient scale, Fig. 65, and let the counter-electro-motive force e of the motor be represented by the length $A F = A G$. Draw through F and G the lines $F K$ and $G H$ respectively parallel to $A B$ and $A C$. The energy supplied to the motor equals the product of electro-motive force E and current C, whilst the work converted into mechanical energy in the armature of the motor equals the product of counter-electro-motive force e and current C. Let R represent the total resistance in the circuit, then $C = \frac{E - e}{R}$, which in our diagram is represented by the length $F C$ divided by R. The energy delivered to the motor is evidently

$$\frac{E\,(E-e)}{R},$$

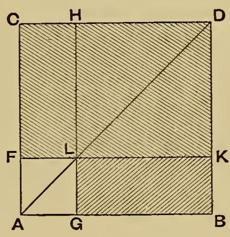
and that converted in the motor is

$$\frac{e\,(E-e)}{R}.$$

Now the area of the rectangle $F\,K\,D\,C = E\,(E-e)$ and the area of the rectangle $G\,B\,K\,L = e\,(E-e)$; and since R is a constant, we find that these areas—shaded in our diagram—are proportional to the work expended and recovered.

Thompson's diagram can immediately be used to solve

Fig. 65.



graphically two of the problems which have already been treated analytically in the first chapter (page 39). These are the following: First, what is the condition of maximum work obtained from the motor? and, secondly, what is the condition of maximum efficiency?

The answer to the first question is easily found by inspecting our diagram, Fig. 65. Since the rectangle $G\,B\,K\,L$, which represents the work of the motor, is inscribed between the diagonal $A\,D$ and the sides $A\,B$, $D\,B$; the question resolves into that of finding which of all possible rectangles inscribed within these lines has a maximum of area. This is evidently a square, the sides of which are half as long as those of the external square. In this case the work expended is represented by a rectangle of half the area of the external square, and the efficiency is therefore 50 per cent.

We have : Work expended $\frac{1}{R}\frac{E^2}{2}$.

„ Work recovered $\frac{1}{R}\frac{E^2}{4}$.

„ Efficiency $\eta = 0{\cdot}50$.

As regards the second question it will readily be seen that the discrepancy in the area of the two rectangles, Fig. 64, is the greater, the nearer the point L is to A, or in other words, the smaller the counter-electro-motive force. In the same measure as the latter increases, point L is pushed further towards D, and the areas of the two rectangles become more and more equal. The efficiency, therefore, tends towards unity as the counter-electro-motive force of the motor tends towards the electro-motive-force of the source of supply of electricity. This statement has already been made in the first chapter, and it is theoretically quite accurate ; but from a practical point of view it requires some qualification. It will be seen that when the counter-electro-motive force of the motor approaches very closely the electro-motive force of the supply, the current becomes very small, and the work expended and converted becomes also very small. Now the work converted in the motor is not all available in the shape of external mechanical energy, and it may well happen that in this case, after the resistance of mechanical and magnetic friction has been overcome, no margin remains for useful external work. The commercial efficiency would therefore be Zero, although the theoretical efficiency is a maximum. To put the matter in another way: a certain minimum of current is required to overcome the friction of the motor, quite apart from any external resistance. It has been shown that with a constant field the torque of the motor depends only on the current which passes through the armature, and is independent of the speed. We may apply this law with sufficient approximation to the present case and assume that at all speeds the current which is required to overcome the internal friction of the motor is constant. Let γ represent this minimum of current, which will just keep the motor alone going, then $\frac{E-e}{R} - \gamma$ is the current doing useful external work, and the commercial efficiency is

$$\eta = \frac{e}{E} \; \frac{\frac{E-e}{R} - \gamma}{\frac{E-e}{R}}$$

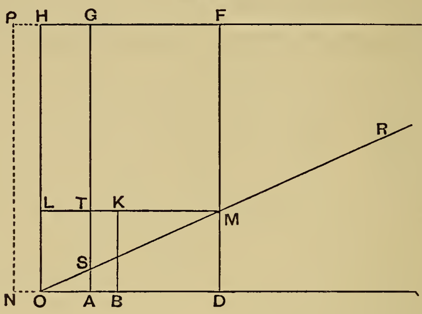
$$\eta = \frac{Ee - e^2 - e\,R\gamma}{E^2 - Ee}.$$

To find the condition under which η becomes a maximum we put $\frac{d\,\eta}{d\,e} = 0$ and obtain

$$(E-e)^2 = E\,R\gamma \; . \; . \; . \; . \; . \; . \; . \; 29).$$

This formula is capable of graphic representation. Let in Fig. 66 *O A* represent the current γ, which is required to keep the motor revolving at or near its normal speed when no external work is being done, and let *O H* represent the electro-motive force *E* of the source, which we suppose to be constant for all conditions. This would be practically the case if the source of current were a self-regulating dynamo, or a set of secondary batteries having a very low internal resistance. The area of the rectangle *O A G H* represents the number of watts required to overcome the friction of the motor at its normal speed when doing no external work, and if the motor be shunt-wound, or compound-wound for constant speed, its strength of field will not greatly vary when external work is being done, and we may with a reasonable degree of approximation regard the area of the rectangle $O\,A\,G\,H$ to represent the internal loss of energy in the motor under all conditions. Draw $O\,R$ at such an angle to the horizontal that its geometrical tangent is numerically equal to the total electrical resistance of the motor and the line, then $S\,A$

Fig. 66.



represents the loss of electro-motive force corresponding to the current $O\,A$, $M\,D$ represents the loss corresponding to the current $O\,D$, and so on. Produce $O\,N = S\,A$ and complete the rectangle $O\,N\,P\,H$ (dotted in the diagram). The area of this rectangle is evidently equal $E\,R\gamma$, and if we produce a square $O\,B\,K\,L$ of equal area, the side $O\,L$ will be equal to the square root of $E\,R\gamma$, and will, according to equation 29, represent $\mathrm{E} - e$. Hence it follows that if we so load the motor that its counter-electro-motive force $e = H\,L$, it will work with maximum commercial efficiency. The energy obtained at the motor spindle is represented by the area of the rectangle *G F M T*, the energy expended at the source of electricity is represented by the area of the rectangle *O D F H*, and the ratio of the two is the commercial efficiency.

In the preceding chapter it was shown how, by the use of an absorption dynamometer, the speed for maximum commercial efficiency can be found experimentally ; it was also shown how, in the case of a series-wound motor, this determination can be made with a fair degree of approximation even without the use of a dynamometer. We can now employ the relations just found to make this determination for shunt or compound-wound motors also, without requiring the use of a brake. This may be explained by an example from actual practice. One of the author's dynamos (shunt-wound and designed to feed sixty glow lamps) was used as a motor. The electro-motive force of the source, which was a compound-wound dynamo, was 100 volts, current through motor when running empty was 4 ampères, speed 1,100 revs., and resistance of line and armature ·2 ohm. We have now $R\gamma = \cdot 8$ and $\sqrt{E R\gamma} = \sqrt{80} = 8\cdot 94.$ To obtain best efficiency the motor must therefore be so speeded that its counter-electro-motive force $e = 100 - 8\cdot 94$

$$e = 91 \text{ volts.}$$

When running empty the counter-electro-motive force is

$$100 - 0\cdot 8 = 99\cdot 2.$$

The best working speed is therefore

$$1100.\frac{91}{99\cdot 2} = 1010 \text{ revolutions.}$$

The current passing at that speed is 45 ampères, of which 4 ampères are required to overcome the internal friction of the motor, leaving 41 ampères to produce useful external work. By gearing the motor to the speed of

1010 revolutions a minute, we shall therefore obtain $\frac{41 \times 91}{736} = 5{\cdot}07$ H-P, actually available on the motor spindle.

But it is not always possible to keep the motor running exactly at the right speed, especially if the load should vary, and in this case it becomes important to know how far on either side of the best speed a variation may take place without seriously reducing the efficiency. For the motor above cited we find the following figures:—

1010 revs.	5·07 H-P.	$e = 91$	$c = 45$	82·8%	Com. effic.
1065 „	2·07 „	$= 96$	$= 20$	76·7	„
944 „	8·20 „	$= 85$	$= 75$	80·0	„

It will be seen from this table that a shunt-wound motor is fairly self-regulating, the range of speed between no load and full load being only about 15% in the present instance. It should be here remarked that the motor described is intended for a working current of 45 ampères, and should not be loaded to more than 5 H-P for continuous work. This reduces the extreme variation in speed to something under 9%. To show the influence of the resistance of the armature on the best speed and efficiency, a table is added, calculated for the same motor and the same electro-motive force, but with an additional resistance of ·3 ohm in the circuit of the armature, making $R = {\cdot}5$

950 revs.	2·82 H-P.	$e = 86$	$c = 28$	73·5%	Com. effic.
860 „	4·30 „	$= 77$	$= 45$	70·5	„
1000 „	1·96 „	$= 90$	$= 20$	72·0	„

In practice, however, the additional resistance would not be placed in the circuit of the armature, but in the line, where, indeed, it is unavoidable if the transmission of energy has to be made over a considerable distance.

By inserting the resistance into the armature circuit only, we have not disturbed the condition under which alone formula 29) gives the best speed, viz., that the strength of the field shall be the same for all currents and speeds. This condition might be fulfilled even in the case of a transmission to a considerable distance if we excite the field of the motor separately or by a pair of separate wires from the distant source, but in practice such an arrangement would be too complicated and, as we shall see presently, it would have no advantage in point of constancy, of speed over the simpler plan of exciting the field of our shunt-motor direct from the line which brings the working current. The effect of an increascd resistance in the line is in the first instance to lower the electro-motive force at the terminals of the motor. With a constant strength of field this would naturally lower the speed of the motor, but if its field magnets are not excited to the saturation point, the reduction of electro-motive force at the terminals of the motor will result in a reduction of the strength of the field, thus allowing more current to pass through the armature by which its torque and speed is increased until its counter-electro-motive force again balances the reduced electro-motive force of the supply. The variation of speed will therefore be smaller than would at first sight appear. But a little consideration will show that the gain in speed due to the increased armature current can never quite compensate for the loss of speed due to the reduced electro-motive force, and thus a pure shunt-wound motor, if fed from a source of constant electro-motive force can never be perfectly self-regulating. It must run faster when the load is thrown off, and it must run slower if more work is put on it. We found the same to be the case with the pure series-wound motor, but in a more marked degree. In this respect the shunt motor is preferable, as will be seen from the above tables (page 158), as its speed when running empty is only slightly higher than when loaded, whereas the speed of the series motor when running empty is excessive. On the other hand, the shunt motor has no starting power, since its armature, when at rest, forms a short circuit of very low resistance. To start a shunt motor it is necessary to arrange the switch in such manner that the field becomes excited before the current is allowed to flow through the armature, and to avoid excessive sparking or heating of the armature, in cases where the motor has to start with the load on, additional resistances must be placed into the armature circuit, which are again cut out as soon as the motor has attained some speed.

We shall now investigate the problem in what manner the electro-motive force of the source of supply must be varied in order to produce constant speed in a shunt-wound motor working under a varying load. Not to complicate the problem too much, we assume that the field magnets of the motor are, with the normal electro-motive force, excited to a very high degree, so that any slight variation in the magnetizing current cannot produce any material difference in the strength of the field. Under this condition the counter-electro-motive force in the armature of the motor will vary directly as the speed; and since the latter is to be constant, the former will also be constant for all loads. Let γ represent the armature current if the motor runs without load, let c be the current when there is a load, and let e be the constant counter-electro-motive force, then $(c-\gamma)\,e$ represents the external mechanical energy; and since e and γ are both constants, a variation of external energy, or, as we call it, a variation in the load of the motor, makes it necessary to vary the current c through its armature. This is done by raising the electro-motive force E of the supply if the load increases, and lowering it if the load decreases. Let R be the resistance of line and armature, then $c = \frac{E - e}{R}$ and $E = e + c\,R$. We neglect as very small the amount of current required for the shunt on the field magnets. The equation shows that to maintain a constant speed of the motor the electro-motive force of the source ought to increase with the load. Its lowest value, when there is no load, will be $E = e + \gamma\,R$, and its highest value will be when load, and consequently current, are both maxima. The difference between the lowest and highest value will be the less, the smaller the resistance R of line and armature, but it can never entirely vanish, for that would require a line and an armature of no resistance. From the above considerations it will be seen that two shunt-wound dynamos can under no circumstances form a system of transmission of energy at constant speed of the receiving machine, because the electro-motive force of the generator —which we suppose to be driven by some prime mover at a constant speed—decreases as the current given out increases, whereas the motor requires exactly the opposite relation between these quantities. A shunt motor might be made to run at a constant speed by using an over-compounded dynamo for the generator. The principle of the compound-wound dynamo, or, as it is also called, of the self-regulating dynamo, is so well known that a few words only of explanation will suffice.

Let the field magnet of a dynamo machine be wound with two coils, one of fine high resistance wire coupled direct to the brushes, and the other of stout low resistance wire, coupled in series with the brushes and the external circuit. If the latter be open, no current passes through the main or series coils, and the magnetism of the machine is entirely due to the exciting power of the shunt coils. If the machine is properly designed, this amount of magnetism should produce an internal electro-motive force exactly equal to that which it is desired to maintain at all currents in the external circuit, provided the dynamo is driven at a constant speed. If a current is permitted to flow through the armature, the electro-motive force measured at the brushes is naturally somewhat less than that created within the armature coils, on account of losses through resistance and self-induction, the loss increasing with the current. To compensate for this loss it is necessary to increase the internal electro-motive force, and this is accomplished by an increase in the strength of the magnetic field. This is brought about automatically by the main current itself, which assists the shunt current in exciting the field magnets. In a correctly compounded machine the increase of magnetization due to the main coils is sufficient, and no more than sufficient, to keep the external electro-motive force constant at all currents which can safely be passed through the machine. We say the machine is accurately compounded for constant terminal pressure.

Now it is easy to see that we can overdo the thing, by putting on somewhat finer shunt wire, which will lower the electro-motive force when the machine works on open circuit or on a circuit of high resistance; and by increasing the number of main coils so as to make the exciting power of the main current preponderate over that of the weaker shunt. In this case the increase of internal electro-motive force will more than counterbalance the loss through self-induction and resistance, and the result will be that up to a certain limit the external electromotive force will rise as the current increases. Such an over-compounded machine could therefore be used as a generator, the receiver being an ordinary shunt machine, and we would thus obtain a system of transmission of energy at constant speed.

Theoretically this is quite correct, but in practice there arises a difficulty due to the fact that the polarity of a compound machine can easily be reversed, especially if the influence of the main coils is considerably greater than that of the shunt coils. The author has attempted to establish such a system of transmission of energy at constant speed, but failed for the above reason. The failure was, however, more instructive than would have been the case had the system worked with theoretical perfection, and an account of it was published at the time in "The Electrician" (April, 1885), of which the following is an abstract:—

"A series-wound dynamo, when used as a motor, runs in the opposite direction to that in which it has to be driven when used as generator. To make the machine run in the same direction (call it forward), the coupling between field and armature must be reversed. With a shunt machine this is not so; the coupling between field and armature remains the same when used as a motor, and it runs always forward. The shunt machine used by the author was driven by a current from an over-compounded dynamo, the shunt of which was weak as compared to the main coils, and when the motor was doing little or no external work it behaved in a most erratic manner, running backward and forward alternately. At every reversal excessive sparking took place at the brushes of both the motor and generator, and it was clear that both machines were overstrained and would speedily come to grief if the circuit were not interrupted. To explain what takes place under these circumstances we will start with the assumption that the generator is kept running at a constant speed, and that the motor is switched on whenever power is required. This is the usual practice where motive power is used at intervals for industrial purposes. Since the leads from the generator remain always charged, the moment we switch the motor on, a large current will pass through its armature and a small current through its magnets. As the motor is at rest there is no counter-electro-motive force to oppose the flow of electricity through the armature, and the result is a momentary excess of current. The immediate effect of this is to start the armature revolving at a high speed before the magnets have had time to become fully excited, for it must be remembered that an armature will revolve in a non-excited field, though with considerable waste of current. The speed required to produce a given counter-electro-motive force is the greater, the weaker the excitation of the field, and hence the motor starts off at a much faster speed than it would have in regular work with its magnets fully excited. On account of self-induction in the shunt field magnet coils, which is considerable, the magnets require some time to become fully excited, and whilst the strength of the field is growing the armature is gathering speed and storing mechanical energy. When at last the field magnets are saturated, the armature of the motor has attained such a speed that its counter-electro-motive force not only equals, but exceeds the difference of potential maintained between the leads by the generating dynamo, and the current is forced back through it. For the time being the motor acts as a generator, the energy stored mechanically in its revolving armature being returned to the circuit in the form of current. This reverses the polarity of the compound dynamo (its shunt coils being weak, as stated above), and now both the generator and the armature of the motor are working in series, the generator assisting instead of opposing the current started by the motor. At this moment we have the following state of things:—The field magnets of the motor have just attained their maximum of magnetization with their original polarity; the polarity of the generator has been reversed, and an excessive current, in an opposite direction to that which produced motion, flows through the armature of the motor. Consequently the latter is quickly brought to rest, and started backward at a high speed. It now opposes a certain counter-electromotive force to the current from the generator, but it is not an increasing force as before. It is a decreasing one, because the original excitation of the motor field magnets is gradually vanishing, by reason of the reversal of polarity in the main leads, from which these shunt coils are fed. Just as it took a certain appreciable time of several seconds for the magnets to become excited, so does it take time for them to lose their magnetism. Eventually there arrives a moment when all the original polarity in these magnets has vanished, and when, therefore, the force impelling the armature to run backward has also ceased, though there is still an excessive current passing through it. A moment later the armature comes to rest, and begins to run forward again at a high rate of acceleration, when the whole cycle of phenomena just described is repeated, but this time with a current in the reverse direction to the first. The third cycle will start with a current in the same direction as the first, the fourth cycle will start with an opposite current, and so on."

A similar phenomenon was observed by M. Gérard-Lescuyer, who used a Gramme series-wound dynamo as a generator, and a magneto machine as a receiver. He called the phenomenon an electro-dynamic paradox, and a description of it will be found in "The Engineer" of Sept. 17, 1880.

CHAPTER VI.

Classification of Systems according to Source of Electricity—Transmission at Constant Pressure—Motors mechanically governed—Self-Regulating Motors—Transmission at Constant Current—Difficulty of Self-Regulation—Motor for Constant Current made Self-Regulating—Application to Transmission over large Areas—Continuous Current Transformator—Transmission between two Distant Points—Loss of Current by Leakage—Theory—Commercial Efficiency—Conditions for Maximum Commercial Efficiency—Self-Regulation for Constant Speed—Practical Example.

It will be necessary to distinguish between different systems of electric transmission of energy, according to the source of electricity. An almost endless variety of cases may present themselves in different applications of electrical transmission, but three systems are of special interest, because most frequently occurring in practice. These are the following:—

1. Transmission of energy from primary or secondary batteries at short distances to one motor only.

2. Transmission of energy from one or several dynamos to a number of motors placed upon the same circuit, but working independently of each other.

3. Transmission of energy between two distant points by means of one generator and one motor.

We may also make another classification according as the motors are intended for a constant or variable load, or a constant or variable speed. Generally speaking, the systems of transmission coming under heading 1) are not required for a constant load, nor is it of any great importance that the speed should remain constant under a variable load. We shall not enter into a minute description of these cases here, as the investigation of electric tramways and railways, worked by accumulators, will afford ample opportunity of entering into details.

System 2) is that presenting most difficulties on account of the condition that all the motors must be independent of each other. The case is further complicated by the requirement that each motor should run with the same speed when empty or loaded. A moment's consideration will show that the last condition is an absolute necessity if we would make the electric transmission of energy of real practical use to small domestic industries. The artisan or small manufacturer would have his motor connected to a common system of service leads, and whenever he required power he would switch the current on to his motor. In doing so he must not disturb any other work which, at the same time, may be done elsewhere from the same service mains, such, for instance, as lighting or working other motors ; and further, his motor should always run at the same safe speed, whether it is giving him little or much mechanical energy. Most operations requiring the use of tools as turning, planing, &c., can only be properly performed at a certain fixed rate of speed, and the machinery must be kept going at that rate at all times.

System 3) presents difficulties of a different nature. Since we have to deal only with one generator and one motor, it is easier to make each fit the other, and as a rule the load is fairly constant, so that regularity of speed is not difficult to obtain. In this case the difficulty lies more in the necessity of proper insulation of line and machinery. Generally speaking, the system is required for long distance transmission, and to obtain an economical arrangement, both as regards first cost and commercial efficiency, the use of a high electro-motive force is necessary. This entails some danger to human life, and some difficulty in maintaining an efficient insulation. Both these points can, however, be satisfactorily dealt with, if proper care is used in the design and execution of the work. As regards the danger to human life involved in the use of electric currents of high pressure, this is generally greatly overrated. It is quite possible for a man who with both hands should touch the positive and negative wires in a non-insulated part, to be killed or severely injured if the pressure is over two or three thousand volts, but the accident can be rendered almost impossible if due precaution is taken. A circular saw if only lightly touched whilst revolving will cut a man's finger off, and what can be more dangerous than a pair of powerful spur wheels? Yet we have found means of protecting life very effectually from destruction by purely mechanical means, and the experience of the past few years has shown that equally efficient protection can be provided from the electrical danger.

System 2) is best described as electric transmission and distribution of energy from one central station to several distant points. This distribution can be made on the parallel or on the series system. In the first case the electro-motive force (or pressure) between the positive and negative mains must be kept constant, and the motors are connected all in parallel from the mains; in the second case the current passing through the mains must be kept constant, and each motor, when at work, is traversed by the same current. The pressure at the station must be the greater the greater the number of motors at work.

In the first case the pressure is kept constant, but the current delivered into the mains must be the greater the greater the number of motors at work. We have thus to distinguish between *distribution at constant pressure* and *distribution at constant current.*

Electric Distribution of Energy at Constant Pressure.

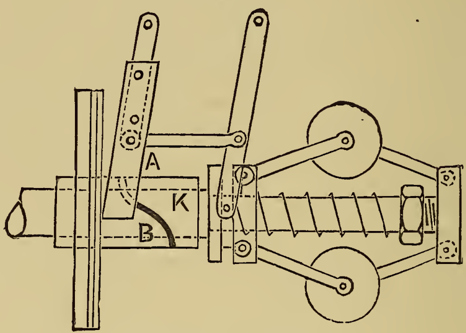
We must now inquire into the theoretical conditions of this case. It will be evident at the outset that for economical reasons any attempt to obtain constancy of speed by the use of artificial resistances can only lead to a partial and not very satisfactory solution of the problem, and had better not be made if other means are at hand. This happily is the case in the present instance. We have two means by which we can without waste regulate the power of the motor to the work and yet keep it running at a constant speed. First we may apply a mechanical device by which the current is periodically cut off in proportion as the work is cut off, and, secondly, we may apply an electric device in the shape of special winding of the field magnets of the motor by which the torque exerted by the armature is automatically regulated so as to correspond to the mechanical load. As regards the first system, Professors Ayrton and Perry have in a paper on Electro-Motors and their Government[1] shown how this can be done. They say: "The method of cutting off the power as hitherto employed has this serious defect, that instead of the power cut off being directly in proportion to the work cut off, the arrangements have been such

[1] "Journal of the Society of Telegraph Engineers and Electricians," No. 49, vol. xii. 1883.

that either all power was cut off or none, so that the motion of the motor was spasmodic, just as in an ordinary gas-engine, which suffers from the same defects, that full charge of gas or no charge are the usual only alternatives. An electro-motor governor of this type, which may be called a 'spasmodic governor,' consists merely of a rotating mercury cup into which dips a wire, which makes in this case contact with the mercury, and so completes the circuit when the speed is slow, but which, on account of the hyperbolic form assumed by the service of the mercury as the speed rises, ceases to dip into the mercury at high speed, and so breaks contact." Later on the inventors say: "The first improvement we made in governing consisted in replacing the 'spasmodic governor' by a 'periodic governor.' With our periodic governor the power is never cut off entirely for any length of time, but in every revolution power is supplied during a portion of the revolution, the proportion of the time in every revolution during which much power is supplied to the time during which less is supplied depending on the amount of work the motor is doing. Our periodic governor, then, differs from the spasmodic governor in the same way that a good loaded steam-engine governor differs from the ordinary governor of a gas-engine. One of the ways of effecting this result is as follows: a brush, *A*, Fig. 67, lies on the rotating piece, *B K*, the cylindric surface of which is formed of two conducting portions connected with one another through any resistance, and the brush, *A*, is moved along the cylinder *B K* under the action of the governor balls. When the brush *A* is touching the contact part *B*, the motor is receiving current directly, but when it rests on the part *K*, the motor receives current through the resistance which is interposed between

B and *K*. If the governor balls fly out, the brush is moved along, *B*, *K*, so that there is contact with *K* during a greater part of the revolution than before ; and if the governor balls come together, the speed of the motor being too small, the brush is moved in the opposite direction so that it makes contact with *B* for a longer time during each revolution. If the motors are in series, we arrange that the periodic governor shunts the current periodically, instead of introducing resistance. In this case the connections are as follows : *B* is made of wood, while *K* is made

Fig. 67.



PERIODIC GOVERNOR.

of metal. *K* is connected to one end of a shunt coil, the other end of the shunt being connected to one of the terminals of the motor and *A* is connected to the other terminal of the motor. If, then, *A* rests on *B*, the shunt is inoperative and all the current passes through the motor ; whereas, if it rests on *K*, the shunt is in operation, and part of the current only passes through the motor." It will be seen that both these governors invented by Professors Ayrton and Perry, have partially, at least, the fault of depending on artificial resistances whether they be used for parallel or series work. The loss of energy thus occasioned can be reduced by making the resistance high for parallel, and low for series work, and on purely theoretical grounds it could even be entirely prevented by making the resistance infinite, that is, breaking the circuit altogether during a portion of each revolution when working in parallel. But this would produce an unequal turning force, and would also entail destructive sparking between the brush, *A*, and the contact pieces *B* and *K*. Even if the resistance between *B* and *K* or that of the shunt coil between *K* and one terminal of the motor is fairly low, there must be some sparking; and the inventors say in their paper that with any such governors it is difficult to entirely prevent sparking, and that on this account motors wound so as to be self-regulating without any mechanical device are preferable.

Broadly speaking, the self-regulating motor is the converse of the self-regulating dynamo wound for constant pressure. In a properly compounded dynamo the pressure at the terminals must remain constant, although the resistance of the external circuit may vary between wide limits, causing an inversely proportional variation in the external current. The power required is approximately proportional to the current. The machine works, therefore, under these conditions: Speed constant—Electro-motive force constant—Current variable—Power required to drive the machine also variable, but proportional to current. Now, in a self-regulating motor the conditions are: Electro-motive force constant—Speed constant—Power variable—Current required to drive the motor also variable, but proportional to power.

It has already been pointed out that in a general way dynamo and electro-motor are convertible terms; and although there are cases when it is impracticable to work a motor as a dynamo, it is always perfectly easy to work a dynamo as a motor. From this general convertibility it is reasonable to expect that a properly compounded dynamo can without any alteration in the connection between its field magnet coils and armature, be used as a self-regulating motor, the only condition being that it shall be supplied with current at a constant electro-motive force. When speaking of a self-regulating motor in the sense that its speed of rotation shall automatically be kept constant whatever variation might occur in the load or mechanical resistance which the armature of the motor has to overcome, it must be understood that this refers only to such cases where the load varies between zero and a maximum not beyond the capability of the motor. If we throw an excess of load on to the motor, it will pull up or slacken speed, and thus cease to be self-regulating, just as the electro-motive force at the terminals of the best compound-wound dynamo will be lowered if we allow an excess of current to flow. But within a reasonable limit of load in the case of the motor, and a reasonable limit of current in the case of the dynamo, both machines can be made self-regulating, and this result is obtained by the same means, that is to say, the same winding which will make the dynamo give a constant electro-motive force, will make the motor run at a constant speed. This result might be expected on the ground of the general convertibility of these machines, but since it is of great practical importance, special proof is desirable. This can be easily obtained from our formulas in Chapter III. According to equation 7) the torque exerted by an armature current, C_a, in a field of Z lines, is in absolute measure:

$$T = \frac{Z\,Nt\,C_a}{\pi}.$$

It is independent of the speed, and since Nt is constant for any given motor, the torque or turning moment exerted by the armature is directly proportional to the product of the strength of field and armature current. By increasing either or both these factors we are able to overcome our increased load. Since the electro-motive force is supposed to be constant, it is evident that a variation in load must be compensated mainly by a variation in current. Assuming that the ends of the shunt coils are coupled to the terminals of the motor—not to the brushes—we have, retaining the notation of Chapter III., the following equations:

$$Cs = \frac{E_t}{r_s} \qquad C_a = C_m.$$

$$E_b = E_t - r_m\,C_m \quad E_a = E_t - (r_m + r_a)\,C_a.$$

The counter-electro-motive force E_a is, according to equation 5) expressed in volts by

$$E_a = Z\,Nt\,n\,10^{6} \;.\;.\;.\;.\;.\;.\;.\;5)$$

$$E_t - (r_m + r_a)\;C_a = Z\,Nt\,n\,10^{-6}.$$

Now the condition under which the motor is to be used is that the electro-motive force at its terminals E_t, shall be kept constant. We have, therefore,

$$\text{Constant } E_t = (r_m + r_a)\;C_a + Z\,Nt\;n\,10^{-6}.$$

Since the speed n must also remain constant if the motor is to be self-regulating at all loads, the only variables are C_a and Z, which have to satisfy the above equation. In other words, we may regard the field Z as a function of the armature current C_a, and the condition that the motor be self-regulating is brought down to this, that the strength of its field shall depend on, and vary in a certain manner with the current passing through the motor.

$$Z = \frac{E_t - (r_m + r_a)\, C_a}{Nt\, n} \times 10^{-6}$$

We see by this equation that Z will be the smaller the greater C_a, and since C_a is almost directly proportional to the mechanical load of the motor, we arrive at this, at first sight, startling result: that the heavier the work we impose upon the motor, the weaker must be its field. It might have been thought that as additional load is thrown on, we ought so to arrange matters that the magnetism of the field magnets becomes strengthened, and able to exert an increased magnetic pull on the armature. But a moment's reflection will show that the effect of such an arrangement would be to reduce the speed. The magnetic pull exerted by the field magnets upon the armature does not depend on the strength of magnetism in the field magnets only, but is the product of that quantity and the current in the armature coils. An increase of pull may therefore be brought about either by making the field stronger, or by increasing the current in the armature, or by both means combined. If we make the field stronger, we not only increase the magnetic pull exerted on the armature, but we also increase the counter-electro-motive force, as will be seen from equation 5), page 82, and thus check, or at least reduce, the flow of current through the armature at the very moment when we want most power. The motor would thus run slower until its reduced counter-electro-motive force again allows a current to pass of sufficient strength for the work imposed on the motor. If, on the other hand, we seek the increase of power by allowing more current to pass through the armature, we do not increase the counter-electro-motive force, but we have a slight increase in the loss of electro-motive force due to the resistance of the armature. To compensate for this slight increase of loss, it is necessary to weaken the field somewhat for heavy currents, and thus bring about the reduction of counter-electro-motive force by an amount corresponding to the increased loss of electro-motive force due to the resistance of the armature. If the motor runs without doing external work C_a is almost zero, and we have the strongest field,

$$Z = \frac{E_t \, 10^6}{Nt \, n},$$

which is entirely due to the shunt coils. Let now a load be thrown on. The immediate effect will be to slightly reduce the speed. The counter-electro-motive force which previously was nearly equal to E_t, will thereby become somewhat reduced, thus allowing a considerable current to pass through the armature and the series coils of the magnets. This again accelerates the armature until the normal speed is reached. The direction of winding of the series coils must be evidently such that the main working current tends to *demagnetize* the field magnets. Now in an ordinary compound-wound dynamo, the series coils are wound and connected in such a way that the main current tends to *increase* the magnetism produced by the shunt coils. If we use such a dynamo as a motor, the current in the shunt coils will remain the same as before, the current in the armature will flow in the reverse direction, and therefore produce motion—instead of resisting it, as is the case when the machine is worked as a dynamo ; and the current through the series coils will also flow in the reverse direction, thus tending to weaken the field magnets. It will be seen that these are precisely the conditions which our theory indicates as necessary, in order to make the motor self-regulating, and we find that it is correct to say that a compound-wound machine can be used either as a self-regulating dynamo or as a self-regulating motor. There may be slight differences in the exact proportion between shunt and series coils in both cases, but the general principle of compounding is the same for either purpose.

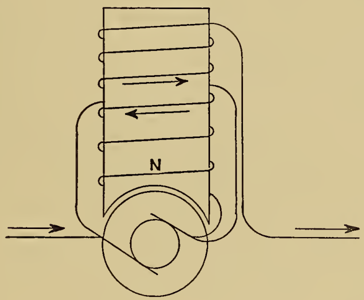
A question of considerable practical importance is that of the relation between the weight of the motor and the maximum of mechanical energy it can give out. Since that maximum must be given out when the field is weakest, whereas in a non-self-regulating motor the arrangements can always be so made that the maximum is given out when the field is strongest, it is evident that, for a given power, the self-regulating motor must be heavier. This is certainly a drawback, and it becomes necessary to know what price, in the shape of increased weight, we have to pay for the advantage of automatic regulation. Our formula for Z enables us to form a rough estimate of this increase in weight. The difference between the initial value of Z and the minimum value is due to the product $(r_m + r_a)\ C_a$. The greater this product, the more must the field be weakened, and the smaller is the maximum of power obtainable with a given weight of motor. It is therefore of importance to keep the product $(r_m + r_a)\ C_a$ as small as possible, and since C_a, which we must consider as the primary source of power, cannot be reduced, it is evident that the resistance of series coils and armature should be as small as possible. Now, in a good modern motor, the loss of electro-motive force occasioned by the resistance of these parts, varies between 5 and 10 per cent. of the electro-motive force applied at the terminals; take 7 per cent. as a fair average, and we find that if the initial field is represented by, say, 1,000 lines, the field at full work will contain 930 lines. Now if the motor were not self-regulating, the field at full power would contain 1,000 lines, and thus be able to develop about $7\frac{1}{2}$ per cent. more mechanical energy. If, on the other hand, we wish the two motors to develop the same maximum of mechanical energy, the field magnets of the self-regulating motor would require to have $7\frac{1}{2}$ per cent. more cross sectional area. Since series and shunt coils act differentially, a larger amount of copper is also required. This excess would probably amount to about $2\frac{1}{2}$ per cent. of the total weight, so that in all the self-regulating motor will weigh 10 per cent. more than an ordinary motor which is not self-regulating. This does not seem too high a price to pay for the safety and general comfort of a self-regulating motor, and the experience gained in American and Continental towns having central electric light stations from which current is supplied to a network of mains on the parallel system has proved that it is perfectly practicable to utilize the same mains for distributing motive power to artisans and small manufacturers by supplying them with such self-regulating motors.

Electric Distribution of Energy at Constant Current.

This problem is not of so easy solution as the distribution of energy at constant pressure, and the difficulty is a fundamental one. It lies in this, that there is no direct connection between the speed of a motor and the current which flows through its armature. There is a direct connection between speed and electro-motive force, and, therefore, self-regulation is possible without the use of any external appliance in the shape of a mechanical governor or other apparatus which controls the power. But where the current is constant, some kind of external governor is necessary. This follows also immediately from M. Marcel-Deprez's experiments cited in Chapter III., page 103. We have seen that the speed was totally independent of the current, the latter remaining throughout the range of each experiment practically constant, whereas the speed was in some cases increased five-fold, by simply increasing the electro-motive force of the source. When a number of motors are coupled in series, as would be the case in a general system of distribution, the difficulties are much increased. To test this matter experimentally the author has placed three precisely similar motors (series-wound) in series into the same circuit. The current was supplied by a dynamo, and the three motors were loaded by brakes to as near as may be the same amount. It was then found quite impossible to keep all three motors going for any length of time at the same speed. The least irregularity in the current, or the least variation in the friction of the brakes, would cause first one and then the other motor to come to rest, whilst the speed of the remaining motor increased to a dangerous extent.

Professors Ayrton and Perry have in the paper above mentioned proposed to make motors self-regulating if worked by a constant current in the following way: The field magnets, Fig. 68, are wound differentially with a fine wire coil, which is a shunt to the armature only, and a thick wire coil which is in series with the armature and main current. The armature and shunt coil constitute a shunt motor, the armature and main coil a brake generator which is intended to absorb any surplus power if the load is thrown off. As far as the author is aware the system has not been tried in actual practice, and there are theoretical reasons for expecting that it would not work. From equation 7) it will be evident that the field must be strongest when the load is greatest. Now suppose that the differential winding could be so proportioned that for a given load the field is exactly of the right strength to produce the normal speed. Now let a very slight additional load be thrown on. The immediate

Fig. 68.

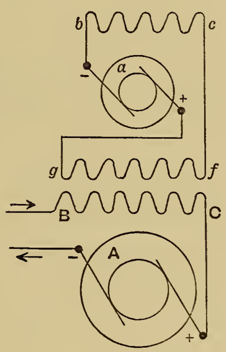


effect will be to slightly reduce the speed, and in consequence of the reduction in speed the magnetizing current in the fine wire coil will also be reduced. The field will thus be slightly weakened. This will further reduce the speed and again weaken the field, and so on, until the armature comes to rest. At that moment the magnetizing influence of the main coils, which is in the opposite direction to that of the shunt coils, will alone exist, and the field magnet instead of presenting a N pole to the armature, as shown in the illustration, will present a S pole to it. The tendency must therefore be to reverse the motion, and thus the slight addition of load has not only brought the armature to rest, but actually caused a tendency to run backwards. Whether it will run backwards depends on the relative magnetizing power of the main and shunt coils.

In a subsequent paper read by the same authors before the Physical Society on May 26, 1888, they suggest to make motors self-regulating for constant current by omitting the series coils on the field magnets altogether, and inserting into the armature circuit a storage battery, the electro-motive force of which helps the current on. As far as the author knows no practical test of this system has been made, and it is easy to see that the objection of instability, which was pointed out above for the first arrangement, also applies in this case. Constant current motors are extensively used in the United States on arc light circuits, but in all cases the regulation of speed is effected by some additional mechanism which either shifts the brushes or alters the exciting power. The attempt to make a constant current motor self-regulating without such additional devices has very little chance of practical success.

An arrangement devised by the author, and which seems to promise somewhat better to fulfil the condition of constant speed, is shown in Fig. 69. *A* is the armature of a series-wound motor mounted upon a spindle, to which is also attached the armature *a*, of a small series-wound dynamo which has no other work to do but to supply current for demagnetizing the field magnets of the motor. The main current magnetizes them in the direction, say, from *B* to *C*, the auxiliary current from the dynamo acts in the direction *f* to *g*, and tends to demagnetize them. *b c* is the field magnet coil of the dynamo. Now for each dynamo working on a closed circuit of constant resistance, as in the present case, there exists a critical speed at which it will begin to give a current of some strength. Below that speed it gives hardly any current, and above that speed it gives almost at once the full current. The motor should be so geared as to run at the critical speed of the little auxiliary dynamo. If now an additional load be thrown on, the immediate result will be to reduce the speed of the motor, thereby causing the armature of the dynamo to run

Fig. 69.



below its critical speed. The dynamo will thus partly or entirely lose its current and the demagnetizing influence which previously has kept the field below its full strength, will to a greater or lesser degree be withdrawn. The strength of the field will thus be increased, and an additional magnetic pull will be brought to act on the armature, by which it can overcome the increased load. In case the load be entirely thrown off, the motor will have a tendency to race, but this tendency will be immediately checked by the auxiliary dynamo, the current from which increases considerably with a very slight increase of speed. Its demagnetizing influence is thus enormously increased, and the field of the motor is weakened to such an extent that there is just power enough left to drive the dynamo but no more. To make this arrangement successful it is necessary that the field magnets of the auxiliary dynamo be made of very soft iron, so as not to retain any considerable amount of permanent magnetism, which would alter the critical point as between an increasing and decreasing speed. The more sensitive and unstable the dynamo can be made, the better. For this reason it is also necessary to place the two armatures a considerable distance apart on the same spindle, so that the field magnets of the motor may not induce magnetism in the field magnets of the dynamo, and thus disturb the critical point. In practice it would probably be found necessary to place a bearing between the two armatures, and that could easily be so shaped as to act as a screen between motor and dynamo.

According to the classification made in the beginning of this chapter we have now to consider

System 3), which comprises the transmission of energy between two distant points by means of one generator and one motor.

Let E_a, E_b, and E_t, represent respectively the electromotive force in the armature, at the brushes and at the terminals of the generator, and let e_a, e_b, and e_t, represent the same for the motor. Let R_a, R_m, represent the resistance of the armature, and magnetizing coils of the generator, and r_a, r_m, represent the same for the motor, then we have, according to the equations 15) to 22), if both machines are series-wound, the following relations :

Generator.	Motor.
$E_a = E_b + CR_a.$	$e_a = e_b - cr_a.$
$E_b = E_t + CR_m.$	$e_b = e_t - cr_m.$
$E_t = E_a - C(R_a + R_m).$	$e_t = e_a + c(r_a + r_m).$

C being the current sent by the generator into the line, and c being the current received by the motor. If the insulation of the line were perfect, these two currents would be equal; but in practice some small leak of current from the positive to the negative circuit, when the line extends over several miles, might take place and then we must assume

$$C > c.$$

The loss of current $C - c$ represents, as far as the generator is concerned, a waste of energy expressed by the product

$$E_t\,(C - c) \text{ watts.}$$

As far as the motor is concerned, this leak not only reduces the current which is available at the receiving station, but it has also the effect of reducing the available electro-motive force e_t beyond the value corresponding to the current c. It will be clear that unless the leak occurs close to the generator, part of the line will have to carry a current larger than c, and thus the loss of electro-motive force due to the resistance of the line must be greater than the product of that resistance and the motor-current c. If the line is throughout its entire length equally well insulated, each unit of its length will have the same insulation resistance, which should be very high in comparison to the conducting resistance itself. In a perfect line it should be infinite, but, as remarked above, this may not be obtainable in an overhead circuit going many miles across country. Let ς represent the conducting resistance of the line, and let i denote the insulation resistance between the positive and negative lead for unit length. If the distance from the generator to the motor be l, the total insulation resistance as measured on a Wheatstone bridge would be $\frac{i}{l}$. Knowing this from actual measurement, it might be thought that by the application of Ohm's law we could easily find the leak, $C - c$, by simply dividing the electro-motive force between the wires by the insulation resistance. This would, however, not be correct, for the simple reason, that the electro-motive force between the wires is not a constant, but diminishes in a certain ratio as we approach the distant end of the line, the actual law by which this diminution takes place depending not only on the resistance of the line and the current, but also on the insulation resistance itself. The question is therefore not so simple as it at first sight appears. An approximate solution sufficiently accurate for practical purposes is the following:

Let ε represent the electro-motive force between the leads at the distance x from the generator; let the distance be increased to $x + dx$ and the leak of current corresponding to length dx be dc, the drop in electro-motive force corresponding to that length being $d\varepsilon$. Then the following equations evidently obtain:

$$- d\,\varepsilon = c \frac{\varsigma}{l} dx.$$

$$- dc = \frac{\varepsilon}{i} dx.$$

From these equations we obtain

$$\varepsilon\, d\,\varepsilon = \frac{\varsigma}{l} i\, c\, dc,$$

and by integration

$$\varepsilon^2 - \frac{\varsigma i}{l} c^2 = Constant.$$

To find the constant we apply the formula to the home end of the line where $\varepsilon = E_t$, $c = C$, and obtain between that and the far end the relation

$$E_t^2 - e_t^2 = \frac{\varsigma}{l} i (C^2 - c^2),$$

from which we find

$$c = \sqrt{C^2 - \frac{l}{\varsigma i} (E_t^2 - e_t^2)}.$$

This gives the current arriving at the motor, but in a somewhat inconvenient form. To simplify the expression we develop the square root into a series, and since the second term is very small in comparison to the first we can neglect the second and subsequent powers.

$$c = C - \frac{1}{2} \frac{l}{\varsigma i} \frac{(E_t^2 - e_t^2)}{C}.$$

Now $E_t^2 - e_t^2 = (E_t + e_t)(E_t - e_t)$ and $\frac{i}{l} = J$, the total insulation resistance of the line. Hence

$$c = C - \left(\frac{E_t + e_t}{2}\right) \frac{1}{J} \cdot \frac{E_t - e}{\varsigma C}.$$

The leak of current is

$$C - c = \left(\frac{E_t + e_t}{2}\right) \frac{1}{J} \cdot \frac{E_t - e_t}{\varsigma C} \quad \ldots\ldots\ldots \quad 30).$$

Now $\frac{E_t + e_t}{2}$ is the average electro-motive force between the out and home lead; and $\left(\frac{E_t + e_t}{2}\right) \frac{1}{J}$ represents the current which under that average electro-motive force would flow through J, the total insulation resistance. This current, multiplied by $\frac{E_t - e_t}{\varsigma\, C}$, gives the actual leak. It will be observed that $\varsigma\, C$, being the product of a resistance and a current, represents a difference of potential, and in this case it represents the electro-motive force which would in a line of perfect insulation be required to drive the full initial current C through the circuit, supposing the far ends were in metallic contact. $\varsigma\, C$ represents, therefore, the loss of electro-motive force if there were no leak. The actual loss, $E_t - e_t$, is naturally somewhat greater, and thus the quotient between the two must always be greater than unity. From this it follows that the loss of current due to leakage along the line is slightly greater than the figure we obtain by dividing the average electro-motive force by the total insulation resistance. Where the insulation resistance is very high, and the conducting resistance very low, the leak will with sufficient accuracy be expressed by

$$C - c = \left(\frac{E_t + e_t}{2}\right)\frac{1}{J},$$

but when the conditions are less favourable formula 30) should be used.

It is necessary in this place to briefly consider the influence of the leak on the total efficiency of a system of electric transmission, especially with reference to the most economical speed of the motor. In text books, and in scientific articles on the subject, the assumption is generally made that the insulation of the line is perfect. This may be so in some favourable cases, but a general theory must include all cases ; it should, therefore, take the leak into account. As far as the writer knows, this has only been done by Professor Oliver Lodge in his treatise on the transmission of power by dynamo-machines, published in "The Engineer," 1883. It is also generally stated that the efficiency is the greater, the nearer the counter-electro-motive force of the motor approaches the electro-motive force of the generator. It has already been pointed out that this is quite wrong (see Chapter V., page 154), even if the motor be worked by a current of constant electro-motive force, such as would be the case if the generator were a self-regulating dynamo placed closed to the motor, and connected with it by leads of practically no resistance and perfect insulation. But when the leads have considerable resistance, and especially if their insulation is not absolutely perfect, the statement above referred to and which is carefully perpetuated by successive writers, becomes still more erroneous. From equation 30) it will be seen that the leak is the greater, the greater e_t. At the same time an increase of e_t has the effect of checking, or at least diminishing, the working current c, thus reducing the amount of energy received. Since the energy lost by leakage increases with the counter-electro-motive force, whilst the energy actually given out by the motor at first increases with the counter-electro-motive force up to a certain point, but beyond it decreases again, it will be clear that high efficiency cannot be obtained by allowing the counter-electro-motive force to approach too near to the electro-motive force of the generator. In the following investigation we shall assume for the sake of simplicity that there is absolutely no leak in the line. The results obtained will, therefore, be to some extent inaccurate, but they can be rectified by using equation 30). Thus with a perfect line we would obtain certain values for $C = c$ and E_t; and the generator would have to give the current and electro-motive force thus determined. Now assume that, after a certain time, the line begins to leak. This will reduce the energy received by the motor, and consequently also that given out by it. It is evident that this loss can be compensated by running the generator at a higher speed; in other words, by increasing E_t and C beyond their original values. A similar plan we follow in the mathematical investigation. We assume at first that the insulation of the line is perfect, and we are thus enabled to use formulas of great simplicity. This gives a certain set of conditions for the generator. If the line is in reality in as perfect a state as assumed, the problem is solved. If, however, the line leaks, we rectify the values for E_t and C by using equation 30). This gives a new set of conditions for the generator, and the mechanical energy necessary for actuating the generator must be calculated for these new conditions. The conditions of the motor are not altered thereby.

The electro-motive force lost in the line is ςc, which must be equal to the difference of electro-motive forces at the terminals of generator and motor

$$E_t = \varsigma c + e_t.$$

The internal electrical energy of the generator is $c\, E_a$, that of the motor is $c\, e_a$, and the proportion between the two is the electrical efficiency of the whole system.

$$\text{Electrical efficiency} = \frac{e_a}{E_a}.$$

By combining this expression with the above equations we find also: Electrical efficiency

$$= \frac{e_a}{e_a + c\,(\varsigma + r_a + r_m + R_a + R_m)} \quad . \; . \; . \; . \; 31).$$

It is evident that whatever may be the resistance of the line ς, or in other words whatever may be the distance to which the energy has to be transmitted, we can always obtain the same electrical efficiency by suitably varying c and e_a. The higher e_a, the counter-electro-motive force of the motor, the greater is the electrical efficiency. Now there are two means by which we can raise the counter-electro-motive force. The one is by increasing the speed, the other by employing machines containing a large number of turns of wire (Nt) on their armatures. The first expedient is limited by the mechanical difficulties generally attendant on the use of excessive speeds, and the latter by the difficulty that the internal resistance of the machines is the greater the more turns of wire they contain. This, in itself, would not effect the result if the electro-motive force would increase in the same proportion as the resistance of the machine. But this is not the case. If a given size armature core be wound with many turns of fine wire, and a precisely similar core with such a number of turns of stouter wire, that both windings fill exactly the same space, the weight of copper contained in the armature wound with stout wire must always be somewhat greater than in the other, because the space wasted by the insulating covering on the wire is less. It is clearly not admissible to reduce the thickness of insulation in the same ratio as the gauge of the wire. A minimum thickness is absolutely necessary for the safe handling during the process of manufacture, and moreover the finer wire is intended for an armature of higher electro-motive force and should for this reason alone have rather better insulation than the thick wire, which is intended for a lower electro-motive force. A good practical rule is to employ a covering of cotton about 8 mils for wires of all sizes up to about 120 mils. The diameter of the covered wire is thus by 16 mils greater than that of the single wire. Now it can be shown that the energy wasted in heating the wire of the armature is inversely proportional to the weight of copper employed, and therefore, with the armature of stouter wire, the same electrical output can be obtained at a smaller cost of energy wasted in heating the wire. The same holds good for the field magnet coils. The dynamo wound with stouter wire, will, therefore, be the most economical of the two, as its internal resistance will be relatively small as compared to its electro-motive force. Inversely, if we wind the machines (generator and motor) with very fine wire in order to obtain a high electro-motive force, we increase their resistances, r_a, r_m, R_a, R_m, in a somewhat quicker ratio, and thus lower their efficiency, taken apart from the line. As regards the line resistance ς, the higher the electro-motive force the better, and it will be evident that taking these two things into consideration there must be one particular value for the electro-motive force for which the electrical efficiency becomes a maximum. This value can be found in each given case by assuming different windings for generator and motor, and calculating their electro-motive forces and resistances. By inserting the data thus obtained successively into equation 31) it can easily be seen which is the best. We suppose that the resistance of the line is given. The electrical data thus obtained can only be regarded as a first approximation to a solution of the problem, because they were obtained on the basis of the highest electrical efficiency, whereas the question of importance is the actual or commercial efficiency. It is sometimes assumed that the commercial efficiency of dynamos and motors bears a fixed proportion to their electrical efficiency, and if that were so we could obtain the actual efficiency of our system of transmission by multiplying equation 31) with that fixed proportion. But this would not be correct. It is evident that the commercial efficiency of a motor cannot be a fixed quantity, but must depend on the power given out, being, generally speaking, the higher the nearer the work done by the motor approaches to the maximum for which it is designed. This relation is best expressed in the manner adopted in Chapter V., by assuming that a certain minimum of current, γ, is necessary to overcome the mechanical and magnetic friction of the motor, and that all the power corresponding to the difference between this minimum and the actual working current is available for external work. Similarly we assume that a certain minimum of current, g, multiplied by the internal electro-motive force of the generator, represents the mechanical energy absorbed by mechanical and magnetic friction. We have, therefore, the following relations :

GENERATOR.

Work absorbed, $W = (c + g)\, E_a$.

MOTOR.

Work given out, $w = (c - \gamma)\, e_a$.

Put R_a + R, = R, and r_a, + r_m = r, then for series-wound generator and motor we have

$$E_a = e_a + c\,(\varsigma + R + r),$$
$$W = (c + g)\,(e_a + c\,(\varsigma + R + r))$$

And the commercial efficiency of the whole system is

$$\eta = \left(\frac{c - \gamma}{c + g}\right)\frac{e_a}{E_a}$$

$$\eta = \left(\frac{c - \gamma}{c + g}\right)\frac{e_a}{e_a + c\,(\varsigma + R + r)} \quad . \; . \; . \; . \; . \quad 32).$$

A question of practical importance is that concerning the working conditions under which, in a given system of transmission, the commercial efficiency becomes a maximum. As already shown, the first condition for attaining this object is to work the generator at as high a speed as mechanically safe. We shall therefore assume that its electro-motive force E_a is a constant and as high as possible. The variables are the current c, and the counter-electro-motive force e_a of the motor. If we allow the motor to run too slowly it will allow a large current to pass, but this will entail a considerable waste of energy in heating the line and the two machines. If we speed the motor too high, this waste will be very small, but the high counter-electro-motive force will only allow very little current to pass, and in this case the work done by the motor will be small, thus again lowering the commercial efficiency. Between these two extreme cases there must evidently exist one current and one counter-electro-motive force for which the commercial efficiency becomes a maximum. To find these values we form the first differential quotient, and equate it to zero. Thus the most favourable current will be found by the equation

$$\frac{d\eta}{dc} = o.$$

and the most favourable counter-electro-motive force will be found by the equation $\frac{d\eta}{d\,e_a} = o.$

Writing for the sake of brevity E for E_a, and e for e_a, and R for the sum of the resistances $\varsigma + R + r$, the first equation gives,

$$(c + g)\,(E - 2\,R\,c + \gamma\,R) - c\,(E + \gamma\,R) +$$
$$R\,c^2 + \gamma\,E = o,$$

c being the only unknown quantity. Resolving the equation we find

$$c = -g + \sqrt{g^2 + \frac{E}{R}(g + \gamma) + \gamma g} \quad . \quad . \quad . \quad . \quad . \quad 33).$$

It will be seen that the quadratic equation has two roots or values for c, the one being positive the other negative. The latter implies that the current travels in the opposite direction, in which case the motor would become the generator and *vice versâ.* This does not concern us here, as it applies to cases where the receiving machine is larger than the generating dynamo, an arrangement which no practical engineer would employ. We have, therefore, only to deal with the positive root, viz.,

$$c = -g + \sqrt{g^2 + \frac{E}{R}(g + \gamma) + \gamma g} \quad . \quad . \quad . \quad . \quad . \quad 34).$$

Having thus determined c, we find the counter-electromotive force of the motor,

$$e = E - Rc \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 35).$$

To obtain a maximum of commercial efficiency the motor must be so speeded that its counter-electro-motive force attains the value $E - Rc$.

By using the equation $\frac{d\eta}{de} = o$, we can also obtain directly the most favourable counter-electro-motive force. The solution gives again two values for e, one smaller than E, the other larger than E. The latter corresponds to the case when motor and generator have changed places and need not be further considered for reasons above stated. The former value for e is alone of importance; it is given by the formula,

$$e = E + Rg - \sqrt{(E + Rg)^2 - (E + Rg)\, E - R\gamma}. \; 36).$$

This equation does not clearly show that e must in all cases be smaller than E, but on developing the expression under the square root on the right, we also obtain,

$$e = E + Rg - \sqrt{R^2 g^2 + R^2 g \gamma + E R (g + \gamma)} \quad . \; 37).$$

The same expression is obtained by inserting,

$$c = \frac{E - e}{R}$$

into formula 34).

It is evident that the square root in 37) must under all circumstances be numerically greater than $R\,g$, and therefore e must under all circumstances be smaller than E. Now, according to the orthodox theory found in text books, maximum efficiency is obtained for $E = e$. This could only be if $g = o$ and $\gamma = o$; that is to say, if the dynamo would absorb no energy whatever when working an open circuit and if the motor could be kept running idle without the expenditure of electrical energy. Both these conditions are evidently absurd.

Since the formulas 32) to 37) have a somewhat complicated appearance, it might be as well to elucidate their application by a practical example. We will assume that in a given system of transmission the generator can be worked at the safe limit of 1,000 volts and 20 ampères, and under these conditions has a commercial efficiency of 80°/₀. Its internal resistance is 5 ohms. Its external electro-motive force at maximum output would therefore be

$$1{,}000 - 20 \times 5 = 900 \text{ volts.}$$

To produce 900 volts and 20 ampères with a machine having 80°/₀ efficiency, requires the expenditure of $18{,}000 \times \frac{100}{80} = 22{,}500$ watts. Of this amount 20,000 watts represents the internal electrical energy developed in the armature, and 2,500 watts represents the energy necessary to overcome the mechanical and magnetic friction of the dynamo. At 1,000 volts this energy is represented by a current of 2·5 ampères. A similar calculation applied to the motor gives, say, 1·5 ampères. We have, therefore,

$$g = 2{\cdot}5 \qquad\qquad \gamma = 1{\cdot}5.$$

Let us assume the distance between generator and motor to be one mile, and the circuit to consist of two miles of copper wire ·134 inch in diameter. At 98 per cent. conductivity the resistance of the line would therefore be 6·2 ohms. Allowing 3 ohms for the resistance of the motor, we have

$$R = 14{\cdot}2.$$

These are all the data necessary for solving the problem as to current and counter-electro-motive force for maximum efficiency. Equation 34) gives immediately

$$c = 14{\cdot}5 \text{ ampères,}$$

and 35) or 36) gives

$$e = 790 \text{ volts.}$$

The maximum commercial efficiency attainable under these conditions is from equation 32)

$$\eta = \frac{14{\cdot}5 - 1{\cdot}5}{14{\cdot}5 + 2{\cdot}5}\ \frac{790}{1{,}000} = 60 \text{ per cent.}$$

Assuming then that the generator be kept running at such a speed, that its electro-motive force is kept at the safe limit of 1,000 volts, we must, in order to obtain the maximum possible return of 60 per cent. of the power expended, so gear and speed the motor that it will oppose an electro-motive force of 790 volts to the current. The strength of the latter will then be 14·5 ampères, and the energy actually given out by the motor will be $\frac{790}{746}(14{\cdot}5 - 1{\cdot}5) = 13{\cdot}8$ horse-power.

To show how a departure from these conditions affects the efficiency and power developed, the following table is added :

Counter-Electro-motive Force.	Current.	Commercial Efficiency, per cent.	Power obtained from motor, H.P.
790	14·5	60	14
876	8	54	7·7
716	20	58·6	18

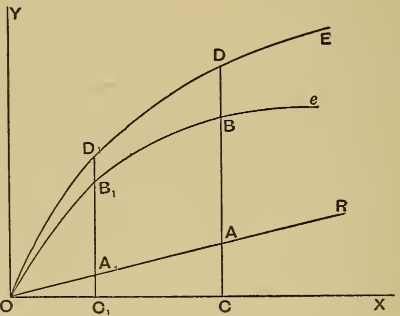
A glance at this table will show that for currents either larger or smaller than 14·5 ampères, the efficiency is less than 60 per cent., but that the falling off is limited to a few per cent., whereas the power transmitted may vary considerably. This is a very valuable property of electric transmission of energy, as it allows a variation of power between wide limits, without serious sacrifice of efficiency, and thus renders the system very elastic. The great importance of this point will become apparent when we compare electric with hydraulic transmission. In the latter the motor consumes always the same quantity of water, whatever work it be doing ; and since the pressure is constant, the efficiency falls very low if the motor is working under its normal power. To remedy this, Mr. Hastie has introduced a water-motor with variable crank-radius, the latter being automatically adjusted to the work done by a spring. A contrivance of this nature, although extremely ingenious, adds considerably to the cost and complication of the machine and represents an additional chance of break-down. On the other hand, electricity can be used without any separate contrivance for regulation, and has thus a great advantage over hydraulic transmission.

The system of transmitting energy by means of two series-wound dynamos has the other advantage of being almost perfectly self-regulating as regards the speed of the motor. It has been shown how a motor intended to be worked by a constant current can be made self-regulating, that is, can be arranged to run always at the same predetermined speed, whatever load may be thrown on it. It has also been shown how motors can be made self-regulating, if supplied with current at constant pressure. In the first case, the electro-motive force must increase as the load increases ; and, in the second place, the current must increase as the load increases, one or the other being kept automatically constant at the generating station. But with a series-wound dynamo, neither the current nor the electro-motive force are constant, but vary in a certain dependence on each other. It might thus, at first sight, seem as if the problem of making the motor self-regulating were thereby rendered very much more difficult. This is not the case. The evil, if we may so regard it, in the dynamo becomes of itself the remedy in the motor.

Let, in Fig. 70, *O E* represent the ordinary characteristic of the series-wound generator, the curve being drawn for a constant speed of, say, 1,000 revolutions a minute. Let *O e* represent the characteristic of the motor also for the speed of, say, 1,000 revolutions. The counter-electro-motive force developed in the armature of the motor at that speed is therefore represented by the ordinates of the curve *O e*. Thus to a current *O C* will be opposed an electro-motive force *C B*, to a current

$O\ C_1$ will be opposed an electro-motive force $C_1\ B_1$, and so on. In the dynamo the electro-motive force corresponding to the current $O\ C$ is $C\ D$, and that corresponding to the current $O\ C_1$ is C_1 D_1. Draw $O\ R$ under such an inclination to the horizontal that the tangent of the angle $R\ O\ X$ represents to the scale of the diagram the numerical value of the sum of the resistances $(R + r + \varsigma)$ of dynamo, motor, and line, then the electro-motive force lost in overcoming these resistances is for the current $O\ C$, evidently $C\ A$, for the current $O\ C_1$, $C_1\ A_1$,

Fig. 70.



and so on. The ordinates between the straight line $O\ R$ and the characteristic curve $O\ E$ represent, therefore, the counter-electro-motive forces which must be developed in the armature of the motor at various currents. If the current is $O\ C$, the counter-electro-motive force is $A\ D$, if the current is $O\ C_1$, the counter-electro-motive force is A_1, D_1, and so on. Now the counter-electro-motive force of the motor, if running at a constant speed of 1,000 revolutions a minute, is given by the curve $O\ e$, and it will be seen that if the ordinates of this curve are for every current equal to the ordinates contained between $O\ R$

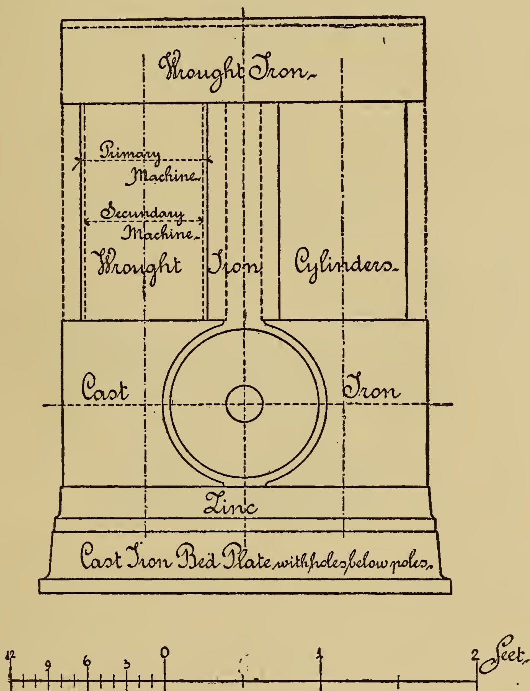
and $O E$, then the motor suits perfectly the requirements of the generator, and it will run at a constant speed. The motor will run at that speed whether the current be $O C_1$ or $O C$, provided that $C_1 A_1 = B_1 D_1$, and $C A = B D$.

The solution of the problem consists, therefore, in the proper choice of motor and dynamo, so that their characteristics fit each other as nearly as possible, as explained. Beyond this, no other precaution or apparatus is necessary to make the system perfectly self-regulating. Even if the characteristics should not fulfil the condition $C A = B D$ over their entire range, it will, as a rule, not be difficult to find two points, C_1 and C, tolerably far apart, for which the condition is fulfilled, and between which the deviation of one curve from the form demanded by the other is very trifling. The system will, therefore, be practically self-regulating between these limits. Several years ago the author had occasion to practically test the soundness of this theory. He had occasion to use electric transmission of energy within the limits of an engineering works, for the purpose of supplying with power the pattern-makers' shop, which on account of its location could not be reached by any mechanical transmission. The power required by the wood-working machines in that shop, including band and circular saws, was, of course, very variable, and it became a matter of the greatest importance to keep the main shaft—from which the different tools were worked by belting—revolving at a constant speed. This object was attained by the method just described. The generator was a Bürgin dynamo, driven at a constant speed from the main engine in another part of the works, and the motor was also a Bürgin dynamo, but wound for a lower electro-motive force. There was a considerable distance between the two characteristics $O\ E$ and $O\ e$, Fig. 70, and to find two points, $O\ C_1$ and $O\ C$, for which the condition $C\ A = B\ D$ should be fulfilled, it was necessary to increase the inclination of the line $O\ R$ by placing a little additional resistance into the circuit. This, of course, entailed some small loss of energy, but was in no way a fault of the system. It was occasioned simply by the necessity of using the two dynamos which happened to be at hand. If the machines could have been designed for this very purpose, no additional resistance would have been required, and the automatic regulation would have been equally good. Within the last few years this method of obtaining constant motor speed in electric transmission of energy has been very extensively applied by Mr. C. E. L. Brown, formerly of the Oerlikon Engineering Works, Switzerland, in the various plants established by that firm on the Continent, and, by careful design of the machines, Mr. Brown has succeeded in reducing the maximum variations in the speed of the motor between running idle and fully loaded to as little as 2 per cent.

Equally good work has recently been done in this direction by Herr von Debrowolsky, and by the courtesy of this gentleman the author is able to give particulars of a transmission plant which he has seen tested in the works of the Allgemeine Elektrizitaets Gesellschaft at Berlin in the spring of 1890. The self-regulation under extreme variations of load was so perfect as to form the best possible proof of the correctness of the above theory, according to which a certain relation between the characteristics of the two machines insures constant speed of the motor at all loads, provided the generator is driven at constant speed. The plant in question consists of a 20 K. W. generator of the

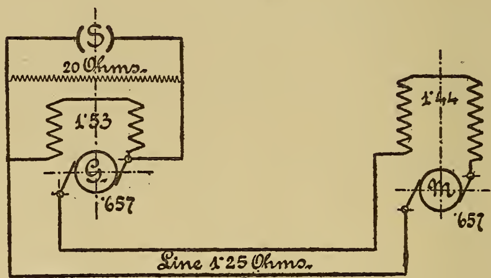
Edison type, working with a normal current of about 25 ampères, a motor of the same type, and a line of 1·25 ohms resistance. The generator is speeded at 1,000 and the motor at 885 revolutions per minute. The general

Fig. 71.



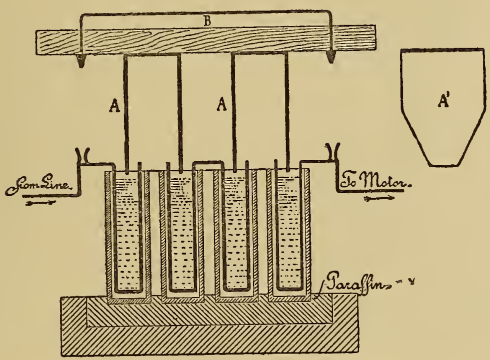
dimensions of the iron parts of the machines can be seen from Fig. 71, which is drawn to scale. The full lines refer to the generator, and the dotted lines to the motor. In the latter, the magnet cores are slightly smaller in diameter, and the yoke is also lighter. The armature core in both machines is $11\frac{5}{8}$ inches in diameter and $12\frac{5}{8}$ inches long, but whereas in the generator only five per cent. of the space is taken up by paper insulation, the space thus taken up in the motor amounts to twenty-five per cent. The winding of the armature is the same in both machines, and consists of 780 external conductors, resistance ·657. The field winding consists of 553 turns on each limb of the magnet, both in the generator and motor. The resistance of the field coils in the generator is 1·53 ohms, and in the motor 1·44 ohms, the reduction in resistance being due to the smaller diameter of magnet

Fig. 72.



cores. In the generator there is also a shunt of 20 ohms, arranged across the terminals of the field coils. This shunt is a resistance coil made of wire doubled back on itself, so as to have no self induction. It will be obvious that by the addition of such a shunt any slight error in the design of the machines can be rectified, that is to say, a more perfect agreement between their characteristics obtained ; but according to Dobrowolsky the shunt does more than this, it increases the rapidity with which one machine corresponds to the other. If the load on the motor be suddenly diminished, the current will be as suddenly checked; but very rapid variations of current through the fields are resisted by the self-induction of the magnets. There must thus always be a tendency to irregularity in the speed owing to a kind of surging of energy to and fro between the two machines, which according to the degree of disturbance will take a longer or shorter time to die out. By the adoption of the shunt, which acts as a kind of electro-magnetic damper or dash-

Fig. 73.



pot, the kick from the magnets is principally taken up by the shunt, and the machines are thereby able to attain more quickly their steady working condition.

Fig. 72 shows diagramatically the arrangement of circuits. As it might break down the insulation if the line were opened by an ordinary switch, the stopping of the current is effected by means of a switch *S*, which is connected with the terminals of the field coil on the generator. By closing this switch the field is caused to vanish, and the current gradually stopped. A liquid rheostat serves for starting the motor. This is shown in

Fig. 74.

Fig. 73. It consists of a series of vessels filled with five

per cent. soda solution, into which dip iron electrodes *A* attached to a cross beam *B*, which can be raised and lowered by means of a crank, rack, and pinion. The electrodes are shaped as shown at A^1. In the vessels there are fixed strips of iron of the form shown, with knife contacts at either end of the apparatus, and when the movable electrodes are placed right down the knife contacts short circuit the electrodes. To prevent creeping the upper parts of the iron plates are painted with some heavy mineral oil. The iron is not attacked by the current. For pressures up to 200 volts, Herr von Dobrowolsky uses in his liquid rheostat two vessels only, and for pressures up to 800 volts he uses four vessels.

The curves, Fig. 74, show the results of tests with the transmission plant above described. The inclined straight line represents the loss of electro-motive force due to the resistance of the circuit, and the two curves represent dynamic and motor electro-motive force of the two machines respectively. The line at the top of the diagram shows the speed of motor actually observed at various currents.

CHAPTER VII.

Importance of alternating currents for long distance transmission—Ideal Alternator—E. M. F. of Alternators—Effective E. M. F. and Effective Current—Clock Diagram—Self-Induction—Power—Different Methods of measuring Power.

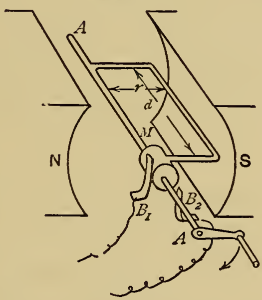
The problems treated in the previous pages had reference to the use of continuous currents only, and in the early days of electric power transmission, dynamos and motors of the continuous current type were the only machines considered practicable for the purpose. As, however, the science of electrical engineering advanced, it began gradually to be perceived that alternating currents might also be used for working electro motors, and this branch of the subject has within the last two or three years been developed with even greater rapidity than the older branch of continuous current work.

There have been principally two causes which have led to this development. In the first place the desire to augment the earning capacity of existing alternating current central stations by the sale of current for power purposes in addition to its use as a lighting agent, and in the second place the natural growth of transmission plant up to and beyond the limits suitable for continuous currents. The first of these causes tended to the invention, design and perfecting of motors of small or moderate power capable of being worked on the existing systems of distributing mains in towns and the other to the development of alternating current generators and motors of large power in combination with extra high pressure transmission plant suitable for long distance work. The question might perhaps be asked, why for such work continuous current plant could not equally well be used. The principal reason is the difficulty of obtaining that degree of insulation in dynamos and motors which is required to withstand the extra high pressure. It has been shown in the previous chapters that the higher the line resistance, that is, the greater the distance to which the power has to be transmitted, the higher must be the working pressure in order to secure a satisfactory degree of efficiency. To reduce the line resistance by increasing the area of the conductor in order to reduce the working pressure is of course always possible, but it is not always financially right, because in so doing we have to increase the capital outlay for the line too much. It is quite conceivable that the annual interest and depreciation on the line would in such a case approach or even exceed the cost of the power if it were produced by an engine at the receiving end of the line, and under these circumstances electric transmission would be commercially a failure. Obviously, a liberal use of copper in the line is no solution of the problem how to carry power to long distances ; the only practicable solution is the employment of high or extra high pressure. As the distance of transmission increases, we thus arrive eventually at a pressure for which continuous current machines cannot easily be insulated. The chief difficulty lies in the commutator. Here we have an organ composed of metallic sections which are necessarily devoid of insulation on the outside, and are therefore subject to surface leakage. The armature conductors themselves can of course be insulated to any desired degree, but as their ends must be connected with the plates in the commutator, there is introduced an element of weakness which lowers the insulation of the machine taken as a whole. With alternators the case is different. Since these machines have no commutator, it is possible to design them in such way that no portion of the armature circuit contains any exposed metallic part, and there is no danger of surface leakage. The whole of the armature circuit may be highly insulated, and thus the working pressure may be made much higher. It is hardly possible to assign an exact limit of pressure at which the continuous current dynamo begins to become impracticable. This depends largely on design, material and workmanship. When no special precautions are taken in the design, but sound material and high-class workmanship are employed, machines for 2,000 volts and even 3,000 volts are fairly reliable. With special designs insuring the insulation of the armature and commutator from the shaft, the magnets from the bedplate, and the latter from the foundation, combined with very careful workmanship, even higher pressures can be obtained ; but whatever may be the limit, it cannot be so high as with alternators, whilst the cost of the machine, owing to these special precautions, becomes high. With alternating currents we have the further advantage that the machine itself need not be subjected to any high pressure at all. We can always employ transformers to increase or reduce the pressure in any desired ratio, and as these transformers contain no moving parts, they can be insulated to any extent we please. Alternating currents are therefore eminently suitable for long distance transmission, and it will now be necessary to place before the reader a short outline of the the theory of alternating current apparatus.

An ideal form of alternator has already been shown in Fig. 13, here reproduced. The E.M.F. induced in a uniform field F is

$$E = F\,d\,\omega\,sin\,\alpha,$$

Fig. 75.



where ω is the circumferential speed of the wire sweeping through the field F, and d its length. At the moment when the plane of the coil is parallel to the lines of the field $\alpha = \frac{\pi}{2}$, and the E.M.F. generated attains its maximum value of $F\,d\,\omega$. It will be convenient to alter this expression by introducing the dimensions of the coil and its rotary speed. Let the symbol $\sim$ represent the number of revolutions made by the coil in one second, thus:

$$\omega = 2\,\pi\,r \sim.$$
$$E = 2\,\pi \sim F\,r\,d\,sin\,\alpha.$$

Now F is the strength of the field expressed in number of lines per square centimeter, and $F\,r\,d$ is therefore the total induction or total number of lines passing through the coil when the plane of the latter is at right angles to the direction of the field. This quantity we have denoted by the symbol z in the formulæ for dynamos. Retaining the same notation we have therefore

$$E = 2 \sim \pi z \sin \alpha$$

as the instantaneous value of the E.M.F. in a coil of one turn containing two active wires. In a coil of two turns there would be four active wires, and the E.M.F. would be double the above amount; whilst in a coil of $\frac{\tau}{2}$ turns containing τ active wires we would obtain

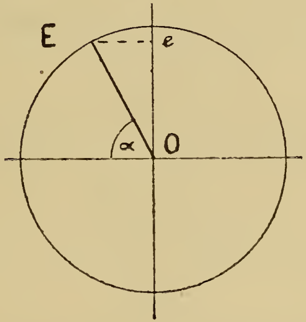
$$E = \tau \pi \sim z \sin \alpha.$$

This is the E.M.F. induced in the coil at the moment when its plane includes the angle α with the position of zero E.M.F. when the coil stands at right angles to the lines of force. As α increases from zero to 2π, the E.M.F. passes through a complete cycle, attaining a positive maximum for $\alpha = \frac{\pi}{2}$, then falling to zero at $\alpha = \pi$; after that attaining a negative maximum for $\alpha = \frac{3}{2} \pi$, and becoming zero again for $\alpha = 2 \pi$. It is obvious that this periodic variation of E.M.F. can be represented graphically by the projection $O\,e$ of a line $O\,E$ (Fig. 76), revolving round O at a speed of $\sim$ complete revolutions per second. If we suppose the radius $O\,E$ to represent the plane of the coil, and the lines of the field to be vertical, then the maximum E.M.F. will be generated when the revolving line $O\,E$ is vertical (plane of coil parallel to lines of force and active wires cutting them at right angles, therefore at maximum rate), and the angle α has to be reckoned from the horizontal. A diagram of the kind shown in Fig. 76 is called a "clock diagram" on account of the resemblance of the motion of the radius to that of the hand of a clock, and it is obvious that we may represent not only an E.M.F., but also a current in the same way, provided it varies according to a sine law, such as

$$i = I \sin \alpha,$$

where i is the instantaneous value of the current, and I its maximum value. Thus imagine the terminals of the coil in our ideal alternator connected to a voltmeter of

Fig. 76.



the hot wire pattern, then the current flowing through the wire must at any instant be proportional to the value of the E.M.F. The total work dissipated in heat during one cycle is in this case $\int_0^T \frac{E^2}{R} \sin^2 \alpha \, d t$; if by E we now denote the maximum value of E.M.F., by R the resistance of the voltmeter, and by T, the time of a complete cycle. Since $\alpha = 2 \pi \sim t$, the integral may also be written thus:

$$\frac{E^2}{2\,\pi \sim R}\int_0^T sin^2\,(2\,\pi \sim t)\,d\,(2\,\pi \sim t).$$

This is the work dissipated in the time $T=\frac{1}{\sim}$. To find the rate per second at which work is dissipated, we must multiply with $\sim$ and have

$$w=\frac{E^2}{2\,\pi\,R}\int_0^T sin^2\,(2\,\pi \sim t)\,d\,(2\,\pi \sim t).$$

The solution of an integral of the form $\int sin^2\,\alpha\,d\,\alpha$ is

$$\frac{\alpha}{2}-\frac{sin\,\alpha\,cos\,\alpha}{2}.$$

The second term cancels out because $sin\,\alpha$ is zero for either limit, and we have

$$w=\frac{E^2}{2\,\pi\,R}\,\frac{2\,\pi}{2},$$

$$w=\frac{E^2}{2\,R}.$$

If the same voltmeter had been connected to the brushes of a continuous current dynamo giving an E.M.F. e, the power dissipated would have been

$$w=\frac{e^2}{R}.$$

Both machines will therefore give the same deflection on the voltmeter, or the same light in a glow lamp if

$$e^2=\frac{E^2}{2}$$

$$e=\frac{E}{\sqrt{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 38)$$

We thus find that the volts measured at the terminals of our ideal alternator compared to its maximum volts are in the ratio of 1: $\sqrt{2}$; or in other words the alternating volts which produce the heating effect in a wire or lighting effect in a glow lamp amount to 70·75 per cent. of the maximum volts induced in the coil. This pressure is called the "effective" or "virtual" pressure.

We can now compare the effective pressure produced by our ideal alternator with the pressure which would be produced by a continuous current dynamo, having the same speed, total field, and number of active conductors on the armature.

The effective E.M.F. produced by the alternator is

$$e_a = \frac{1}{\sqrt{2}} \pi \, z \, \tau \sim$$

and that produced by the continuous current dynamo is

$$e_c = z \, \tau \sim.$$

The two are in the ratio of $\frac{\pi}{\sqrt{2}}$: 1 or very nearly.

$$e_a = 2{\cdot}22 \; e_c.$$

That is to say, to obtain the voltage of the alternator we calculate the E.M.F. as if we had to do with a dynamo and multiply the result with 2·22. This is, however, only correct for alternators in which the wave of induced E.M.F. follows strictly a sine function. The shape of the curve obtained in practice differs to some extent from the true sine curve, and the co-efficient instead of being 2·22 may have some other value. Let this value be K, then the general formula for the E.M.F. of an alternator is

$$e = K \, z \, \tau \sim$$

The exact value of the co-efficient K depends on the shape and configuration of the armature coils and magnet poles, but it does not vary within very wide limits. For most modern and well designed machines K lies between 2·00 and 2·30. In certain cases, notably with toothed armatures producing a very peaky E.M.F. curve K may rise as high as 2·60.

The above formula gives the E.M.F. of an alternator having two poles only. If there are more poles and the armature coils are coupled in series, the E.M.F. is correspondingly increased. Say that we have a machine with p pairs of poles ($2p$ poles) then the E.M.F. would be $p\ K\ z\ \tau\ \sim$, z being again the total induction due to one pole and its fellow. Let n be the speed of the machine in revolutions per minute then $n/60$ is the speed per second. The number of complete cycles through which the E.M.F. passes per second is therefore p times $n/60$ or

$$\sim\ = \frac{p\ n}{60}$$

which inserted in the above formula gives

$$e = p\ K\ z\ \tau\ \frac{n}{60}$$

in absolute measure. To obtain the E.M.F. in volts we multiply by 10^{-8}, and have

$$e = p\ K\ z\ \tau\ \frac{n}{60}\ 10^{-8} \quad . \quad . \quad . \quad . \quad . \quad . \quad 39)$$

It is important to note that z must be inserted in absolute measure, and represents that induction which passes through one coil alternately in one and in the opposite direction. When the induction does not vary from $+z$ to $-z$, but only from $+$ to z zero and back again to $+Z$, as in the Mordey machine, the E.M.F. is obviously only half that given by the above formula. It should also be noted that τ does not mean the number of turns of wire on the armature, but the number of active wires when counted all round the circumference of the armature, precisely as in the case of dynamos.

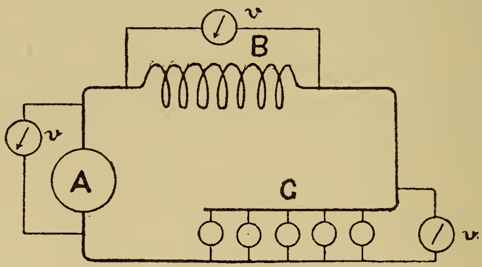
If we give z in English measure, the E.M.F. of an alternator in volts is

$$e = p\,K\,z\,\tau\,n\,10^{6} \quad . \quad . \quad . \quad . \quad . \quad 40).$$

It is now necessary to investigate the power represented by an alternating current. With a continuous current the problem is simple enough ; we have only to multiply ampères and volts to get the watts expended, but with an alternating current this method of computing power is not generally admissible, because the current curve does as a rule not coincide with the E.M.F. curve. To see this clearly let us suppose an alternating current sent through the circuit, shown in Fig. 77, which consists of an alternator, *A*, an electro-magnet, *B*, and a bank of glow lamps, *G*. v, v_1 and v_2 are voltmeters of the electrostatic type, *i.e.* instruments which take no appreciable current. In order to simplify the problem we assume that only the bank of lamps has ohmic resistance, which we call r.

The current passing through the electro-magnet will energize it. Let the core be made of laminated iron so that the magnetization may instantly follow the exciting power, and let the iron be of such good quality that no appreciable amount of power shall be wasted in the process. Let F be the total induction produced when the current i has its maximum value I and τ the number of turns of wire through which the flow of lines passes. If the current passes through $\sim$ complete cycles per second, or, as it may also be expressed, if it has a frequency $\sim$, the coil will in respect to the field created by its own current be in exactly the same condition as the coil of our ideal alternator (Fig. 13), is in respect of the field produced by the system of field magnets. We have seen that when the plane of the coil is at right angles to the lines of force, that is, when the total induction passing through the coil is a maximum, the E.M.F. is zero, and when the coil had turned through 90° so as to bring

Fig. 77.



its plane parallel to the lines of force, making the flux through it zero, the E.M.F. is a maximum, namely $2 \pi \sim F$ for each turn of wire. The electro-magnet B has τ turns of wire and the maximum value of the E.M.F. induced in it, when the flux, and therefore the current producing this flux pass through zero, is

$$E_s = 2 \pi \sim \tau F,$$

This is called the E.M.F. of self-induction, because it is produced by the current itself. Obviously, the total flux F is a function of the current, and since E_s is proportional to F, the E.M.F. of self-induction must be a function of the current. If the core of the electro-magnet be magnetized only to a low degree, say not beyond 5,000 or 6,000 lines per square centimetre, then the permeability may, without serious error, be considered as constant for all points of the magnetic cycle, and in this case the instantaneous value of F would be proportional to i, and the maximum value to be inserted in the above formula would be proportional to I. We have, in fact, $F = \mu \frac{4 \pi I \tau}{l} a$, where l is the length, a the area of the magnetic circuit, and μ its permeability. If we put

$$L = \frac{\mu 4 \pi a \tau^2}{l}$$

we have also

$$E_s = 2 \pi \sim L I.$$

In this equation I and E_s are given in absolute measure. If we wish to have them in ampères and volts we must divide the right hand side by 10 and by 10^8.

Volts of self-induction

$$E_s = 2 \pi \sim L I 10^{-9}.$$

L is called the co-efficient of self-induction, and from the equation above given it is obvious that it must be a length. This equation contains on the right-hand side only numerics (4, π, τ^2, and μ) and the ratio a/l, which is an area divided by a length. This must obviously be a length, and as all our formulæ are based on the *c. g. s.* system, this length must be given in centimetres, and the unit of the co-efficient of self-induction in the *c.g.s.* system must therefore be the centimetre. This, however, is too small a unit for convenient practical use, and a unit a thousand million times as great has been adopted. This is called by the following names: (1.) The "Secohm," from the fact that a length may be considered as the product of a velocity (ohms), and a time (seconds). (2). The "Quadrant," from the fact that 10^9 centimetres represents very nearly the length of an earth quadrant. (3). The "Henry," in honour of the American electrician of this name. The last name for the unit of self-induction seems at present to find most general adoption, having been recommended by the Chicago Electrical Congress.

If then we give L in henrys the factor 10^{-9} must be omitted, and we have the maximum E.M.F. of self-induction in volts.

$$Es = 2\,\pi \sim L\,I \quad . \quad . \quad . \quad . \quad . \quad 41)$$

I being the maximum value of the current (crest of current wave) in ampères. The effective volts of self-induction are of course $\frac{Es}{\sqrt{2}}$, and since effective and maximum current stand to each other in the same relation of $1 : \sqrt{2}$, we find the effective volts of self-induction by simply inserting the effective instead of the maximum value of the current.

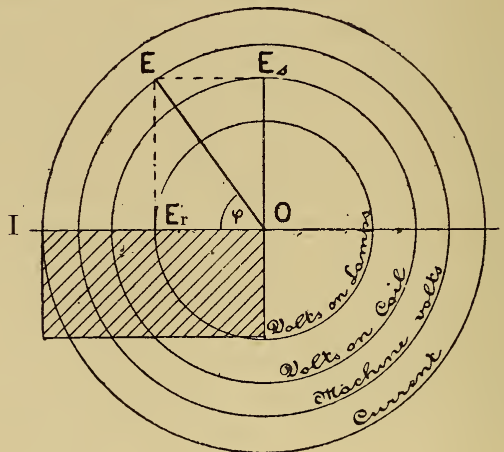
$$e_s = 2\,\pi \sim L\,i \quad . \quad . \quad . \quad . \quad . \quad 42).$$

If we have to do with a continuous current, c, the E.M.F. required to force it through a resistance, r, is $e = r\,c$. Now the E.M.F. required to force the alternating current through a coil having self-induction, is given by a formula of similar construction. We have i for alternating current instead of c for continuous current, and instead of a resistance r given in ohms we have the term $2\,\pi \sim L$. This must be the equivalent of a resistance as we can easily see from the fact that L is a length and $\sim$ is the reciprocal of a time; therefore $\sim L$ is length divided by time, which means a velocity. The expression $2\pi \sim L$ has therefore the same dimensions as a velocity, and as a resistance has also the same dimensions, we find that $2\pi \sim L$ represents a resistance. It is, however, a kind of resistance which does not absorb power, and to distinguish it from a true ohmic resistance it is called "inductance."

Let us now see how the relation between current and E.M.F. of self-induction can be represented in a clock diagram. In the first place it will be obvious that the current line must always be at right angles to the E.M.F. line, for only then is it possible that the projection of one is zero at the moment when the projection of the other is a maximum. There remains only the question how the direction of current and E.M.F. shall be represented in the diagram. Let us agree to call a current positive when it passes through B (Fig. 77) from left to right, and let us call in the clock diagram a current positive when the projection of the current line is above the centre. Assuming that the lines revolve clockwise, and that we start counting the angle of rotation from the horizontal position of the current radius to the left, then currents corresponding to all the angles between $\alpha = o$ and $\alpha = \pi$ would be positive, and currents corresponding to all the angles between $\alpha = \pi$ and $\alpha = 2\pi$ would be negative.

Similarly we call an E.M.F. generated in the coil positive when it tends to produce a positive current, *i.e.*, a current flowing from left to right. Now it is obvious that the E.M.F. of self-induction must, on the whole, tend to prevent both the growth and the decrease of the current, in other words, it must tend to diminish the amplitude of the current wave, for were it to increase the amplitude it would be opposed to Lenz's law. We thus find that at the moment when the current passes through zero and becomes positive, the E.M.F. of self-induction must act in *B* from right to left, that is, it must be negative. The machine *A* must therefore at this moment impress upon the coil an equal and opposite E.M.F.,

Fig. 78.



appearing in the diagram, Fig. 78, as a vertical line OE_s in the upper or positive half of the figure. The current-line at this moment is *OI*. The voltmeter v_1 placed across the terminals of the coil *B* will therefore show the effective voltage which the machine must impress upon this coil in order to balance its E.M.F. of self-induction. But the machine must do more. It must also supply the volts required to drive the current through the lamps. Since the latter have only ohmic resistance, the lamp volts and lamp current must coincide in phase, and their relation is correctly represented by Ohm's law. Let E_r be the volts required to send the current I through the resistance r, then

$$E_r = r\ I \text{ and}$$
$$e_r = r\ i,$$

which voltage is measured by the voltmeter v_2. In the diagram the maximum lamp voltage is represented by the horizontal line $O\ E_r$. The machine has then to give not only the E.M.F. $O\ E_s$, but also the E.M.F. $O\ E_r$. Combining the two in a parallelogram of forces, we thus find that the machine must impress on the whole circuit the maximum voltage of $O\ E$. The diagram was developed on the assumption that the lines revolve clockwise. The volt line $O\ E$ will therefore pass through the vertical position before the current line, that is to say, the maxima of volts and ampères given by the machine do not coincide, but the ampères lag behind the volts by a certain amount represented in the diagram by the angle φ. This angle is called the "lag," or "angle of lag." We have assumed that no power is expended in coil B, so that the whole of the power coming out of the machine is given to the lamps, and amounts to

$$w = i\ e_r = \frac{I\ E_r}{2}.$$

In the diagram the energy expended is therefore represented by half the area of the shaded rectangle, or if we had drawn the diagram so that the different lines represent not the maximum, but the effective values of the different quantities, the energy would be represented by the area of the shaded rectangle. Since $O\ E_r = O\ E$

cos ϕ, it follows that the equation for the power may also be written thus—

$$w = \frac{I E \cos \phi}{2}.$$

or, when effective values are taken,

$$w = i\, e \cos \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 43)$$

To find the power given out by the alternator in watts, we have therefore to multiply the effective ampères by the effective volts and by the cosine of the angle of lag. It is interesting to inquire for what resistance in the lamp circuit this power becomes a maximum. From 42) we have $i = \frac{e_s}{2 \pi \sim L}$, which inserted into 43) gives

$$w = \frac{e_s\, e \cos \phi}{2 \pi \sim L}$$

Since the self-induction of the coil does not vary, $2 \pi \sim L$ is a constant, and the power is proportional to $e_s\, e \cos \phi = e_s\, e_r$. The problem then is to find that position of E on the circle marked "machine volts," for which the product $e_s\, e_r$ becomes a maximum. Obviously this will take place when the line $O E$ includes an angle of 45 deg. with the horizontal, for which position $e_s = e_r$. We obtain maximum power when the volts of self-induction equal the volts applied to the lamps, that is, when the resistance of the lamps has the value $r = 2 \pi \sim L$.

The lag in this case is 45 deg.

It will be seen from the diagram that

$$e^2 = e_r^2 + e_s^2$$

since $e_r = i\, r$, and $e_s = i\, 2 \pi \sim L$, we have also

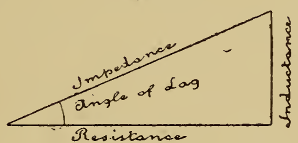
$$e^2 = i^2 \left(r^2 + (2 \pi \sim L)^2\right),$$

from which we find the current

$$i = \frac{e}{\sqrt{r^2 + (2\,\pi \sim L)^2}} \quad . \quad . \quad . \quad . \quad . \quad 44).$$

This formula enables us to predetermine the current when the frequency, E.M.F., resistance, and self-induction of the circuit are given. The analogy with Ohm's law is apparent at a glance. In the formula expressing Ohm's law the denominator is simply a resistance; here it is the square root of the sum of two terms, one being the square of the resistance, and the other the square of the inductance. The denominator in 44) is called the

Fig. 79.



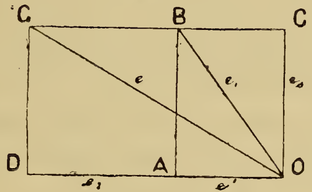
"impedance," and may, according to Dr. Fleming, be represented as the hypothenuse of a right-angular triangle, the sides of which represent resistance and inductance as shown in Fig. 79.

In the above investigation we have arrived at the principle according to which the power given to a current must be computed; and it is now necessary to show how this principle can be applied in practice. In the case represented by Fig. 77 there is no difficulty in measuring the power given by the alternator. We have simply to determine that passing through the lamps in the usual way. But this is only correct under the supposition that the coil *B* does not absorb any power at all, a condition not attainable in practice.

Let us now see how the power given out by the alternator can be correctly measured if the coil *B*, besides having self-induction, also has a certain, but unknown resistance, by virtue of which it absorbs power. We can even go further, and assume that *B* absorbs power not only on account of its ohmic resistance, but because it does useful work in some form or other. It might, for instance, be the primary coil of a transformer or the armature of an alternating current motor. Whatever be the kind of work done in *B* it is clear from what has already been shown that the true watts supplied to *B* are the product of the current and that component of the E.M.F. which coincides in phase with the current. But when an E.M.F. coincides in phase with a current the two are in a relation correctly represented by Ohm's law, and we may therefore represent any kind of useful work by the equivalent of an inductionless resistance which is heated by the current. For the sake of simplicity we will adopt this method of representation, assuming that for the transformer or motor or other apparatus we substitute a coil *B*, which has the same co-efficient of self-induction and a resistance so adjusted that the energy absorbed in it shall be exactly equal to the energy which would be given to the transformer, motor, or other apparatus which it replaced.

Let r_1 represent this unknown resistance of *B*, and r_2 that of the bank of lamps, then $i^2 r_1$ is the energy absorbed by the coil, and $i^2 r_2$ that absorbed in the lamps, the energy supplied by the machine being $i^2 (r_1 + r_2)$. Taking in all cases effective (not maximum) values, we find by the voltmeter v_1 the E.M.F. over the terminals of the coil *B*. Let this be e_1 and let e_2 be the E.M.F. over the lamps. The E.M.F. taken at the terminals of the machine we call e. The E.M.F. e_1 may be considered to be made up of two components; one e' in phase with the current, and therefore equal to ir_1, and the other e_s due to self-induction and in advance over the current by 90°. Let, in Fig. 80, OB represent e_1 then OC is the E.M.F. of self-induction e_s, and $OA = e'$. Similarly let OG represent the terminal E.M.F. of the machine, and OD that component of it which is in phase with the current. There being no self-induction in the lamps, the hight of the point G over the horizontal must be the same as that of point B. Since the E.M.F. over

Fig. 80.



the lamps must be in phase with the current and with e' we have

$$e' + e_2 = OD,$$

and find that the following relations exist:

$$e_s^2 = e_1^2 - e'^2$$

$$e_s^2 = e^2 - (e' + e_2)^2$$

By eliminating e_s we obtain

$$e' = \frac{e^2 - e_1^2 - e_2^2}{2\,e_2}$$

The energy w_1 in coil B is $e'i$, and we find thus

$$w_1 = \frac{i}{2} \frac{(e^2 - e_1^2 - e_2^2)}{e_2} \quad . \; . \; . \; . \; . \quad 45).$$

The determination of the energy requires therefore three measurements of E.M.F. and one current measurement. The latter may be omitted if the resistance r_2 of the lamps or other non-inductive resistance is accurately known. Since $i = \frac{e}{r_2}$ we can also write

$$w_1 = \frac{e^2 - e_1^2 - e_2^2}{2\,r_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad 46).$$

For practical work the equation 45) is preferable to 46), because the exact resistance of *G*, especially if this part of the circuit consists of glow lamps, is not easily determined. This method of determining the power given to an alternate current circuit has been devised by Professor Ayrton and Dr. Sumpner, and is known as the "Three Voltmeter Method." To ensure accuracy it is advisable to make all these volt-measurements with the same instrument, by using suitable connections and switches. It is also advisable to so adjust the non-inductive resistance that e_2 becomes approximately equal to e_1. If there is a large difference between the two, then a small error in the measurements of E.M.F. leads to a large error in computing the power. The power given by the machine is the sum of that given to the coil and that given to the lamps.

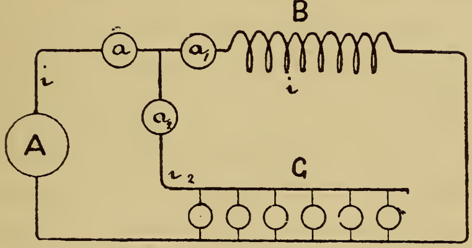
$$w = w_1 + i\,e_2$$

$$w = \frac{i}{2}\,\frac{(e^2 + e_2^2 - e_1^2)}{e_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad 47).$$

$$w = \frac{e^2 + e_2^2 - e_1^2}{2\,r_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 48).$$

It has already been stated that to obtain accurate results e_1 should not differ very much from e_2; this condition implies that e should exceed either by about 40 per cent., and where the object of measurement is to find the total power supplied by the machine, the apparatus may generally be so adjusted as to satisfy this condition. All we need do is to work the transformer or motor B at a lower voltage than that of the machine. But if the problem is to determine the power taken by the transformer or motor B, then we must work it at the voltage for which it has been designed, and the alternator must be worked at a higher voltage. When this is not practicable a modification of the three voltmeter method

Fig. 81.



may be used. This was devised by Dr. Fleming, and is known under the term "Three Ampère Meter Method." Its application will be understood on reference to Fig. 81, where A is the alternator, and a, a_1 and a_2 are ampère-meters indicating respectively the total current and its two components which pass through coil B and lamps G. An investigation analogous to that given with reference to the previous method (but which for want of space is omitted), shows that the power given to the coil is

$$w_1 = \frac{e}{2} \frac{(i^2 - i_1^2 - i_2^2)}{i_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad 49).$$

e being the terminal voltage of the coil. If the resistance r_2 of G is accurately known, the voltage need not be measured, since the power is also given by the formula

$$w_1 = \frac{r_2}{2}(i^2 - i_1{}^2 - i_2{}^2) \quad . \quad . \quad . \quad . \quad 50).$$

Both methods of power-measurement here explained have the disadvantage that an amount of power comparable to that to be measured must be wasted in a non-inductive resistance. There is no difficulty in constructing such a resistance capable of taking up a small amount of power. A bank of glow lamps does very well for this purpose ; and where we wish to measure the power lost in a transformer when working on open circuit, or that lost in a motor when running light, either of the two methods may usefully be employed. But when it becomes necessary to measure the output of a large alternator amounting to some 100 kilowatts or more, an inductionless resistance if it were composed of glow lamps would become cumbersome and costly. We must, therefore, employ some other kind of resistance. If metallic, we cannot be sure of its being inductionless, and thus the three voltmeter and the three ampèremeter methods become alike uncertain. A practical way of measuring consists in employing a metallic resistance formed by wire or strip which is wound, not spirally, but zig-zag fashion on a frame so as to make it as nearly as possible inductionless. The material employed should have as small a temperature coefficient as possible (manganese-steel or platinoid are very suitable alloys), and the total capacity of the resistance should be large enough to take the whole of the power without excessive heating. To find the output of the machine, the current is sent through this resistance and is measured when the latter has reached a steady temperature. The current is then interrupted, and the resistance of the frame is taken at certain intervals of time, the observations being plotted to show by a curve the rate at which the resistance decreases as the frame becomes cooler. This curve, prolonged backwards to the moment when the current was interrupted, gives with a high degree of accuracy the resistance of the frame when the measured current was flowing through it, and the power is then found by multiplying the square of the current with this resistance.

This method of measurement is very suitable for the workshop, before the machine is sent out. It may occasionally be applied to machines after erection, but the necessity of using large resistance frames and delicate instruments for measuring resistance renders it rather cumbersome, whilst it is of course quite unsuitable to all cases where the electric power to be measured is that put into (and not that taken out of) the machine.

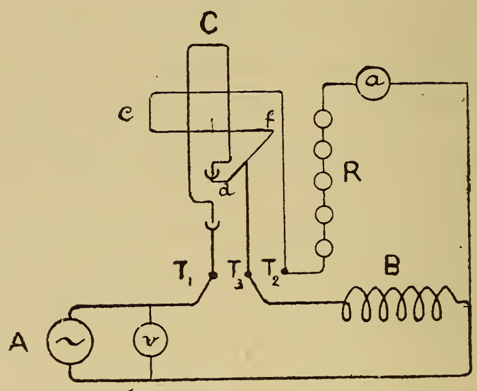
What is required for general use is a method which can be applied equally well in any locality, and which does not itself absorb any appreciable amount of power. An instrument which complies with these requirements is the wattmeter. This is constructed similarly to an ordinary current dynamometer, but instead of passing the same current in series through both coils, the current of which the power is to be measured is passed through one of the coils, and the other coil forms a shunt across the terminals of the apparatus under test. If necessary, an additional resistance may be insertcd in this shunt circuit. Before entering on the theory of the wattmeter, it will be expedient to consider the action of the instrument when used as a dynamometer with an alternating current. Since the direction of the current changes simultaneously in both coils, the force, which tends to deflect the movable coil, acts always in the same direction. It is, however, a variable force changing from zero to a maximum $2 \sim$ times a second, being, of course proportional to the square of the current at any instant. If the spring and movable coil had no mass, the latter would oscillate under the influence of the current, but as it has a considerable mass compared with the force acting on it, and as the frequency with which the force comes and goes is comparatively high, the movable coil can be kept steadily in the zero position by turning the pointer in the same way as if the current were continuous. The acceleration given by a force, f, to a mass, m, is f/m, and the speed attained after T seconds is $\int_0^T \frac{f}{m}\, dt$, the mass being supposed to be at rest at the moment from which the time is counted. In our case the value of the integral is zero, since, owing to the weight of the coil, no movement and, therefore, no speed is imparted to it. We have, therefore, the condition of equilibrium $\frac{1}{T}\int_0^T f\, dt = 0$, where $f = \frac{I^2 \sin^2 \alpha}{K^2}$. K is the co-efficient of the instrument obtained by calibration with a continuous current, when the force is given by the deflection of the pointer. Calling this D, and the continuous current c, we have $c = K\sqrt{D}$ and with an alternating current we have $DK^2 = \frac{1}{T}\int_0^T I^2 \sin^2 \alpha\, dt$. The value of the right-hand term, as already shown, is $\frac{I^2}{2} = i^2$ and we find therefore that

$$i = K\sqrt{D}.$$

This is precisely the same formula as with a continuous current, and we see that a dynamometer calibrated on a continuous current may be used for the measurement of an alternating current, and will give the effective value of the current.

Now let us see how this instrument may be arranged to measure power. As usually made, the dynamometer has an internal connection between the two coils not accessible to the user. There is thus only one external terminal for the movable coil and one for the fixed coil. Generally there are two fixed coils, in order to obtain a greater range, and then three terminals are provided. For the sake of simplicity we shall however assume that we have to do with an instrument having only one fixed coil, and to avoid complication in the diagram we show the movable and fixed coil to consist of one turn each, though in instruments as actually made, any number of turns may, of course, be used. Let, in fig. 82, C represent the movable and c the fixed coil, whilst $d\,f$ is the internal connection, leaving only the terminals T_1 T_2 accessible. If the instrument be inserted in the usual way, the current coming in at T_1 and going out at T_2 will traverse both coils in a clockwise direction, and produce a deflective force on C which can be balanced by moving the pointer in the direction intended by the maker of the instrument. Movement in the opposite direction is generally prevented by a stop. If the currrent comes in at T_2 and goes out at T_1 the direction of current is reversed in both coils and the mechanical effect is the same as before. Since the deflection must always be taken in the same sense, it is of course necessary when altering the connections of the instrument to take care that whatever currents be flowing through the coil they must traverse them in the same sense, that is, either both clockwise or both counter-clockwise. Let us now attach a third terminal, T_3, to the connection $d\,f$ and insert the instrument into an alternating circuit as shown. A is the alternator, B is the apparatus

Fig. 82.



the power given to which is to be measured, R is an inductionless resistance which may conveniently consist of glow lamps arranged in series, and a is an ampèremeter. The fixed coil and resistance R form a shunt to B, and it is obvious that the currents will pass through C and c in such sense as to produce deflection of the movable coil in the desired direction. If we include the resistance of the fixed coil and ampèremeter in the term R, and assume that the movable coil C has only a negligible resistance, and that the self-induction of either coil is also a negligible quantity, we find that the current i passing through c is in phase with the E.M.F. e impressed on B and that it is given by the expression:

$$i = \frac{e}{R}.$$

The current I through the movable coil acted on by the field due to i produces a deflecting force, the instantaneous value of which is proportional to

$$I_o \sin (\alpha - \varphi)\ i_o \sin \alpha,$$

φ being the angle of lag and I_o, i_o the maxima of the two currents. Substituting for i_o its value e_o/R the force producing the deflection D is

$$D K^2 = \frac{1}{T}\int_0^T I_o \sin (\alpha - \varphi)\ \frac{1}{R}\ e_o \sin \alpha\ d t,$$

$$D K^2 = \frac{1}{R}\ \frac{1}{T}\int_0^T I_o \sin (\alpha - \varphi)\ e_o \sin \alpha\ d t,$$

If we were to draw a clock diagram to represent the present case we would have an E.M.F. e_o leading before a current I_o by the angle φ, the power represented being, as already shown, $W = \frac{I_o e_o}{2} \cos \varphi$. Instead of the integral we can therefore also write

$$D K^2 = \frac{1}{R}\ \frac{I_o e_o}{2} \cos \varphi,$$

and since $I_o = I\sqrt{2}$ and $e_o = e\sqrt{2}$ we can also write

$$D K^2 = \frac{1}{R}\ I e \cos \varphi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 51).$$

The power passing through the wattmeter is $I\ e\ cos\ \varphi$, or

$$W = R D K^2 \ldots\ldots\ldots\ldots 52).$$

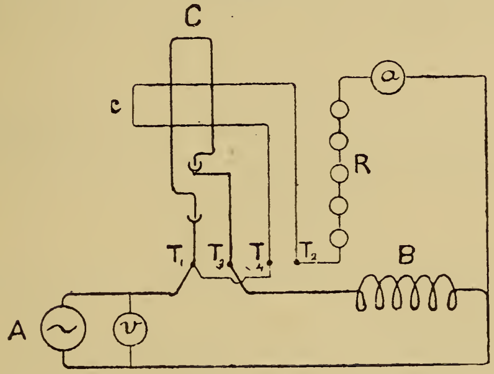
K being, as before, the coefficient of the instrument obtained by calibrating it as a dynamometer on a continuous current. The expression 52) can only be used if the resistance R is accurately known. This may be the case where the resistance consists of platinoid wire or a similar material having a very small temperature coefficient, but when lamps are used the resistance varies so much with the current that for accurate work it is necessary to determine it for each reading. We have then to observe e on a voltmeter v, and i on an ampèremeter a, inserting into 52) for R the ratio e/i. The formula thus becomes

$$W = \frac{e}{i} D K^2 \ldots\ldots\ldots 53).$$

If it be inconvenient to take three readings for each observation (namely, D, e, and i), or if an ampèremeter be not available, we may use the wattmeter itself as a dynamometer and plot beforehand a series of readings connecting e, i, and R so that R is given by a curve as a function of e. We need then only observe D and e and insert the value of R taken from the curve.

If the wattmeter is connected up as shown in Fig. 82 the power measured by it includes that wasted in the resistance, that is to say, it measures the total output coming from the alternator. To obtain the power given to B we must deduct from the measurement taken on the wattmeter the power wasted in the resistance e^2/R. It is, however, possible to connect up the instrument in such way that only the power given to the coil B is measured. This can be done by simply changing the connections between terminals T_1 and T_3 when the current for the fixed coil will be taken off before the movable coil is reached, so that the current in the latter is the same as that passing through *B*. This arrangement has, however, the disadvantage that the direction of current in the two coils is opposed, producing deflection in the contrary sense, for which the instrument is as a rule not suitable. To meet this difficulty it is necessary to remove the internal connection *d f* and to arrange the instrument with four terminals as shown in Fig. 83. By connecting T_1 and T_4 the

Fig. 83.



current for the fixed coil does not pass through the movable coil and the power measured does not include that wasted in *R*. As far as the reading is concerned it is obviously immaterial whether *R* is inserted as shown or between T_1 and T_4, but the latter method of connecting up the instrument is objectionable because it then has to withstand the whole difference of potential between T_1 and T_2, which on high-pressure circuits may lead to a breakdown.

The wattmeter may also be used to determine the phase difference between two currents of the same frequency, or if applied as shown in Fig. 82, the lag of current behind impressed E.M.F. Let I and i be the two currents, then it follows from 51) that

$$D K^2 = I i \cos \varphi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 54).$$

Having determined the deflection D when the instrument is coupled up as a wattmeter, change the connections so as to use it as a dynamometer and measure successively the two currents I and i without changing anything else in the circuit. Let D_1 be the deflection corresponding to the current I and D_2, that corresponding to the current i, then

$$I = K \sqrt{D_1}$$
$$i = K \sqrt{D_2},$$

which inserted with 54 gives—

$$D K^2 = K^2 \sqrt{D_1 D_2} \cos \varphi.$$

$$\cos \varphi = \frac{D}{\sqrt{D_1 D_2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad 55).$$

It will be seen that for the determination of the lag we need not even know the dynamometer constant; all we need do is to take the three readings, using the same coils each time, but coupled up as a wattmeter when we observe D, and as a dynamometer when we observe D_1 and D_2. The above method of measuring lag is due to Mr. T. H. Blakesley, and has been called by him the "Split Dynamometer Method."

When investigating the wattmeter we have assumed that it has no self-induction, that is to say, we have assumed that the current through the fixed coil is in phase with the E.M.F. This condition is of course never absolutely fulfilled. A circuit arranged to produce electro dynamic effects, *i.e.*, mechanical forces, must necessarily have self-induction. It is of course possible to reduce the effect of self-induction by making the resistance R large, but then we also decrease the force tending to deflect the movable coil, and therefore the sensibility of the instrument. Let us then see what the effect of self-induction will be. In the first place it will produce a lag of the current i behind the E.M.F. e. Let this lag be ψ, and let, as before, φ be the lag of I behind e. If l is the coefficient of self-induction of the fixed coil then

$$tan\ \psi = \frac{2\ \pi \sim l}{R}.$$

Similarly, if L is the coefficient of self-induction in the main circuit of which the movable coil forms a part,

$$tan\ \varphi = \frac{2\ \pi \sim L}{r},$$

r being the equivalent resistance of B, as before explained.

Let in Fig. 84 $O\ E$ represent the impressed effective volts, and $O\ I$ the effective current, then projecting E on $O\ I$ we get the point E^1, $O\ E^1$ being the component of E.M.F. in phase with the current. The true power in watts is therefore $O\ I \times O\ E^1$, whilst the apparent power given by the wattmeter is $O\ I \times O\ E_o$. The point E_o is the projection of E_2 on $O\ I$, and E_2 is the projection of E on the line representing i, which makes with e the angle ψ. To get true watts from the reading we must therefore multiply the latter by the ratio

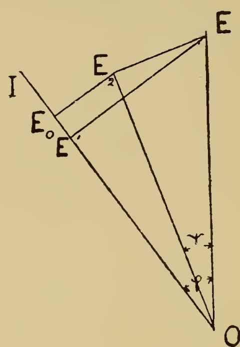
$$\frac{O\ E^1}{O\ E_o} = \frac{O\ E\ cos\ \varphi}{O\ E\ cos\ \psi\ cos\ (\varphi - \psi)}$$

If w be the true and w^1 the apparent watts we have therefore

$$w = w^1 \frac{\cos \varphi}{\cos \psi \cos (\varphi - \psi)} \quad . \; . \; . \; . \; . \; . \; . \; . \quad 56)$$

The angle ψ being a constant appertaining to the instrument can be determined once for all by measuring the self-induction of the coil with a secohm-meter, or by making a power measurement upon an absolutely inductionless resistance which has been inserted into the place

Fig. 84.



of coil B. The lag of I behind i can be determined from 55) (only that in this case we do not get φ but $\varphi - \psi$), so that all the data required for the correction are known.

Formula 56) can also be written as follows:

$$w = w^1 \frac{1 + \tan^2 \psi}{1 + \tan \psi \tan \varphi},$$

from which it will be seen that there are two special cases where no correction is required. The first case occurs if we have a wattmeter so delicately made that a sensible deflection is obtained with a very large R. In such an instrument the effect of self-induction will be so small that ψ approaches sufficiently close to zero to be neglected. In this case the term giving the corrections becomes unity. The second special case occurs if the self-induction of the fixed coil is equal to that of the machine or apparatus under test. In this case $\tan^2\psi = \tan\psi . \tan\varphi$, and the term again becomes unity. If $\psi < \varphi$ the wattmeter gives too large a value for the power, and if $\psi > \varphi$ it gives too small a value. For $\psi > \varphi$ there is no limit to the error, since for $\psi = \frac{\pi}{2}$ the reading becomes zero, and the correction infinite. This case is, however, of no practical importance. Modern wattmeters have very little self-induction, and the angle ψ is therefore always small and generally smaller than φ. It has been shown that no correction is required for $\psi = 0$ and $\psi = \varphi$; between these limits there is a value of ψ, for which the correction becomes a maximum. This occurs obviously when the point E_0^1 in Fig. 82 is farthest from O_1, a position which is reached when the line $E_0\ E_0^1$ forms a tangent to the semicircle that can be drawn over $O\ E$ as diameter. It will be easily seen that in this case $\psi = \frac{\varphi}{2}$. If, then, we have a good instrument in which ψ is under all circumstances small and certainly smaller than φ, then the greatest possible error due to neglecting the correction will be by formula 56) $\cos\varphi/\cos^2\frac{\varphi}{2}$. The error may be less, but it cannot be more. The following table shows the maximum value of the correcting factor for various angles of lag, and also the maximum possible percentage error if the corrections be neglected :

Angle of Lag.	Apparent Watts.	True Watts.	Percentage Error.
5°	1000	998·5	·15
10°	1000	994·6	·54
15°	1000	982·7	1·73
20°	1000	968·8	3·12
25°	1000	950·8	4·92
30°	1000	928·2	7·18

CHAPTER VIII.

Self-induction in Armature—Effect of Armature Reaction on the Field—Best Frequency—Transmission of Power between two Alternators—Margin of Power—Influence of Capacity.

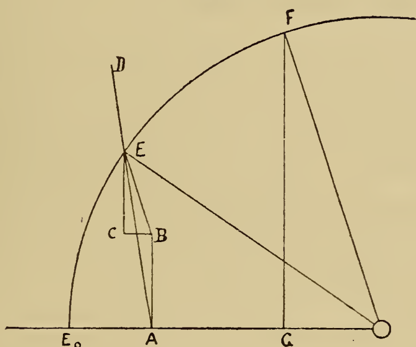
In the foregoing chapter it has been shown how self-induction in the current-receiving circuit produces a lag of current behind the impressed E.M.F. We have simply assumed that a given E.M.F. is applied by the alternator to the terminals of the circuit, but we have not investigated the relations between this E.M.F. and that generated in the armature of the alternator. This we now proceed to do. It has already been shown how the E.M.F. generated in the coils of the armature can be calculated, and it is obvious than on open circuit the terminal E.M.F. must be the same as that generated. It will also be readily understood that by varying the exciting current we can vary the E.M.F., and that the relation between these two quantities may be represented by a curve which (analogous to that of continuous current machines) we may call the static E.M.F. characteristic of the alternator. If the external circuit of the machine be now closed and a current be allowed to flow, we obtain a lower E.M.F. at the terminals, the reduction being due to three causes: (1) Armature resistance, (2) Self-induction, (3) Armature reaction. As regards the loss of E.M.F. by reason of the armature resistance, this is of course easily calculable by Ohm's law and need not be further considered. In phase this E.M.F. coincides of course with the current. The E.M.F. of self-induction is not so easily determined. It is due to the fact that the current passing through the armature coils creates a field, the lines of which encircle the conductors precisely in the same way as we found to be the case with coil *B*, Fig. 77. This E.M.F. lags behind the current by a quarter cycle ; and in order that the current may flow, there must be acting a precisely equal and opposite E.M.F. This must obviously be a component of the total E.M.F. generated in the armature, and must lead before the current by a quarter cycle. If the co-efficient of self-induction were constant and a known quantity, this component could also easily be calculated beforehand.

The total strength of the self-induced armature field must necessarily depend on the relative position of armature coils and field magnet poles, and as this is constantly changing, the co-efficient of self-induction must also change. It must also depend on the excitation of the field magnets, because with a strong main field, when the magnets already carry a large flux, they are less permeable to additional lines of force. The limits within which this change takes place are however not very wide. Professor Ayrton, when testing a Mordey alternator by means of a secohm-meter, found that with a non-excited field the self-induction varied between ·036 and ·038 henry, and that both values decreased by about 14 per cent. when the field was excited.

It would thus seem that for constant excitation we may without serious error assume the self-induction to be the same for all positions of the armature, and calculate the E.M.F. required to balance it by formula 42).

The E.M.F. induced in the armature coils may now be considered the resultant of three different components. First we have the terminal E.M.F. which does useful work and will be in phase with the current if the external circuit contains no self-induction, and will lead before the current if it contains self-induction. Next we have the E.M.F. due to ohmic resistance in the armature which is in phase with the current, and, thirdly, we have the E.M.F. necessary to balance that produced by self-induction which leads in front of the current by a quarter

Fig. 85.

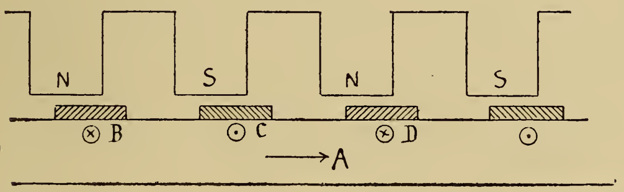


cycle. These relations are shown in the clock diagram, Fig. 85, where OA measured along the current line represents the E.M.F. doing useful work in the external circuit, AB the E.M.F. of self-induction in the external circuit, BC the loss due to ohmic resistance in the armature, and CE the E.M.F. of self-induction in the armature. The length of the lines AB, BC, and CE being proportional to the current, it follows, that if we wish to vary the current and at the same time keep the useful E.M.F. OA constant (the usual condition for lighting and motor work), the point E must remain on the straight line $A D$. For larger currents, E will shift further up, and $O E$ must be made larger by increasing the excitation ; for smaller currents, E will shift downwards, and $O E$ must be made smaller by decreasing the excitation. If provision be not made for altering the excitation in this manner, the useful voltage will vary. The extent of this variation can be seen from the diagram. If $O A$ represents the useful E.M.F. at full load and the load be diminished without altering the excitation until it is zero, the point E will come down on the circle to E_o, and the useful E.M.F. will go up in the ratio $O A$ to $O E_o$.

The diagram also shows what will happen if the machine be short circuited at the terminals. In this case the external self-induction is cut out and the induced E.M.F. is the resultant of only two components, namely, those due to resistance and self-induction in the armature. If we draw a line from O parallel to $B E$, we obtain $O F$ as the new position of induced E.M.F. The triangles $O G F$ and $B C E$ are similar, and since $C E$ is proportional to the current at full load (and may by a suitable choice of scale be made to represent the current), $G F$ measured with the same scale will represent the current on short circuit. In other words, the current on short circuit will exceed ordinary full load current in the ratio of $C E$ to $G F$. This is, however, only an approximation. In reality, the current on short circuit will be smaller, owing to the weakening of the field on account of armature reaction as explained below. For the safety of the machine it is of course desirable that the increase of current on short circuit should not be too large so that the armature may not be overheated. There is when a short circuit occurs also the danger of racing. In a continuous current machine a short circuit will most likely pull up the engine, but in an alternator it takes the load off, and allows the engine to race unless otherwise governed. It will be shown presently that power may be transmitted by two alternators, one acting as generator and the other as motor, the latter running under normal conditions synchronously with the former, but that under excess loads the motor may fall out of step and come to rest. In such an event the circuit would be deprived of the counter E.M.F. in phase with the current, and there would be only the resistance of line and machines and the self-induction of the latter to limit the current. It is thus evident that if we wish to provide for the safety of the plant in the event of the motor being forcibly stopped, the machines must be so designed as to have an appreciable self-induction. On the other hand, large self-induction, as will be shown presently, tends to reduce the total power of the motor, and this may bring about the very evil which it is intended to cure. For the generator too large a self-induction is objectionable for two reasons ; first, because the power given out by the machine is sensibly less than corresponds to the product of ampères and volts (the power being proportional to the cosine of the angle of lag), and, secondly, because the excitation must be regulated within wider limits in order to keep the useful voltage constant. The latter defect is especially objectionable if the current is required not only for power, but also for lighting, because it entails more supervision at the central station. We see thus that safety and convenience in working alternating current plant are to a certain extent contradictory requirements. The best design is necessarily a compromise, and no hard and fast rules applicable to all cases can be given. It may, however, be said that modern alternators by the best makers are designed so as to have an E.M.F. of self-induction (*C E* in Fig. 85) not less than 20 or more than 40 per cent. of the terminal E.M.F.

In the beginning of this chapter it was stated that there are three causes tending to lower the E.M.F. of an alternator. Two of these we have considered above. The third, namely the reaction of the armature current on the field-magnets, must now claim our attention. It will be obvious that if there is no lag or lead of current as compared with the induced E.M.F. the general effect of the armature current on the field-magnets must be zero; because everything being symmetrical, the magnetizing action of the current in a group of wires whilst approaching the centre of the pole piece is precisely balanced by the demagnetizing action when receding from the centre. If, however, there is lag or lead of current, this symmetry is disturbed and one action preponderates over the other, with the result that the field is either weakened or strengthened. The effect is different accordingly as the machine is used as a generator or as a motor, as will be readily seen by reference to the following diagrams. Let, in Fig. 86, *N*, *S* represent the radial magnet poles of an alternator with drum wound armature. For convenience of illustration, the poles are shown in a straight line, so that the part of the armature *A* under consideration would have a rectilinear instead of a rotary movement. Say the movement is from left to right as indicated by the arrow, and let the machine work as a generator. The shaded rectangles *B*, *C*, *D* represent the cross section through groups of wires belonging to the different armature coils. How the wires in each group and the coils are joined up is immaterial to our investigation, provided the connections are so made that the current in all the wires of one group flows in the same way, and the current in all the wires in the next group flows the opposite way, this being obviously the only correct method of winding. The combined effect of the currents in the single wires is then equivalent to that of a broad sheet of current flowing alternately towards and from the observer within the winding spaces *B*, *C*, *D*, etc., as indicated by the little circles with dots and crosses placed under each group of wires. Supposing the current lags, then it will attain its maximum value after the centre of

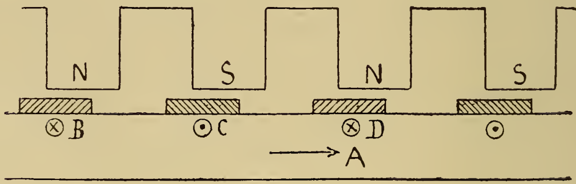
Fig. 86.



each group has passed the centre of the pole piece, and the armature at that instant will occupy the position shown in the diagram. In that position all the wires beyond the edge of the pole piece will obviously exert a demagnetizing influence on the field, and this influence, as far as it is determined by position, will increase as more and more wires emerge; though as far as determined by current strength it will decrease, because the current decreases from that moment. If the current passes through zero before all the wires have passed under the edge of the succeeding pole piece there will be excited a magnetizing action tending to strengthen the field, but owing to want of symmetry brought about by the lag of current behind the E.M.F., this magnetizing action is weaker than the previous demagnetizing action, and the net result is that the field is weakened by the armature current. If, on the other hand, the current leads, and if we again draw the armature in the position it occupies when the current is a maximum, we see from Fig. 87 that the magnetizing action is stronger than the demagnetizing action, and that the armature current will now strengthen the field.

Since in a machine working as motor the current is reversed, these actions will also be reversed, and we find thus the following rules:

Fig. 87.



A lagging current weakens field of generator.
A lagging current strengthens field of motor.
A leading current strengthens field of generator.
A leading current weakens field of motor.

The exciting power in ampère-turns by which the field excitation is altered in consequence of the armature current may be approximately calculated by the formula

$$x = \cdot 0156\, \omega\, i\, \varphi^\circ \quad . \quad . \quad . \quad . \quad . \quad 57),$$

if ω denotes the number of wires in one group, i the effective current through one of these wires, and φ° the angle of lead or lag in degrees.

We have already seen that the self-induction of the armature causes a reduction of terminal E.M.F. at load ;

it also causes lag, and consequently weakening of the field, which again reduces the induced and consequently also the terminal E.M.F. There is thus a kind of cumulative action going on which produces a greater drop in terminal voltage at load than corresponds to self-induction alone, and at short circuit the current will be less than determined from diagram, Fig. 85. Since on short circuit $\varphi°$ becomes nearly 90°, the demagnetizing ampère-turns produced by the armature are $1{\cdot}4\ \omega\ i$, and i need not be so very large to produce a very material reduction in the field strength.

Before discussing the question of the transmission of power by means of alternators, it will be useful to briefly investigate the effect of armature reaction and self-induction on the general design of such a machine, especially as regards the frequency to be adopted. Taking lighting and power purposes together we find that in respect of this point the greatest diversity prevails in practice. In the United States the frequency most customary is 133 complete cycles per second, though a lower frequency is occasionally adopted, especially in more modern designs. In Great Britain the frequency adopted by various makers ranges from 100 to 75, and on the Continent from 65 to 44 or thereabouts. In the case of the Niagara power transmission the frequency adopted by the maker of the first instalment of the plant is only 25, whilst even a lower frequency had been originally suggested by the consulting electrician to the company. When we thus see that in practical work there is so much discrepancy on the question of frequency, it is obvious that no hard and fast rule applicable to all cases can be given, and that each case must be judged on its merits. Thus the method of driving and the speed of the machine, its type, permissible weight, and many other considerations must influence the choice of frequency. As the question is far too complicated to be discussed in its most general aspect, it will be best to leave out of consideration all matters which may be considered as of purely local importance, and to limit the investigation to the weight and cost of the machine and its working, as far as these points are influenced by the frequency.

Let us assume that we have to compare two machines, both designed for the same voltage, speed, and output; but let the frequency of the first machine, A, be twice as great as that of the second machine, B. Thus A may have 20 poles, and run at 540 revs., giving $\sim = 90$, whilst B has only 10 poles, giving at the same speed $\sim = 45$. Since the voltage is proportional to $\tau z p$, it is obvious that τz in B must be twice as great as in A. Let us at first assume that B shall have the same weight as A, then z in B may be twice as great as in A, and τ will be the same in both. Each armature coil in B will have twice as many turns as in A, and since the coil, taken as a whole, is larger (owing to twice the induction having to pass through it), the total length of wire in the armature of B will be larger than in A. On the other hand, the diameter of the armature of B can be made smaller, because less space is lost in the gaps between adjacent poles, there being only 10 gaps in B as against 20 in A. The gaps in A cannot be made too small, as otherwise the magnetic leakage would be too large. Further, as there are in B only half as many field-magnet cores to excite as in A, the excitation in B will, notwithstanding the greater bulk of each of its magnets, require less wire and less exciting energy than that in A. Thus far the balance of advantages lies rather with the machine of lower frequency. Let us now see how the case stands with regard to self-induction and armature reaction. The self-induced field of an armature coil may approximately be considered proportional to the armature current, the number of turns of wire in the coil, and the dimensions of the latter. The current is the same in both machines, but each coil of B has twice as many turns as each coil of A, and has also twice the area. The self-induced field in B will therefore be four times as great as in A. This field cuts the wires of the coil B with a frequency of 45, whilst the corresponding field in A cuts its wires with a frequency of 90. The product $\sim \omega$ (frequency and turns of wire in one armature coil) being necessarily the same in both machines, it follows that the E.M.F. of self-induction in one armature coil of B is four times as great as in one armature coil of A, but since A contains twice as many coils as B, the E.M.F. of self-induction of the whole armature of B is twice as great as that of the whole armature of A. If the lag of current were the same in both armatures, the armature reaction on the field would be twice as great in B as in A, but if the machines both work as generators the lag of B would (owing to greater self-induction) be greater than in A, and consequently the armature reaction would be more than twice as great. The net result of all this will be that the E.M.F. characteristic of A will drop less than that of B.

If we wish to get as small a drop in B as in A we must reduce its E.M.F. of self-induction and its armature reaction. This may be done by increasing z and reducing τ, and also by increasing the resistance of the magnetic circuit, so as to reduce the self-induced field.

The first remedy leads of course to an increase in weight, and the second to an increase in the exciting power required, because greater magnetic resistance means more copper in the field and more exciting energy. Thus, although a low frequency tends to make the machine less costly, because there are fewer parts, and the diameter of the armature may be reduced as compared with a high frequency machine, these advantages may be lost again by reason of the extra weight of field required to provide against the effects of greater self-induction and armature reaction. To sum up:

A machine for high frequency and one having a large number of field poles must be of large linear dimensions, and must be costly on account of multiplicity of parts, though it need not be heavy. Its characteristic has a small drop.

A machine for low frequency, and one having a small number of field poles, is more compact, but also heavier. Its characteristic has a larger drop.

In determining the frequency there are, however, also other considerations which must be taken into account. Thus, if the current is required partly for power and partly for light, the frequency should not be less than about 42 cycles per second, because at a lower frequency alternating current arc lamps work badly. On the other hand, a high frequency may become objectionable on account of the capacity of the line, especially if the latter is long and consists of concentric cable laid underground and the voltage is high. Under these conditions the line absorbs current by virtue of its electrostatic capacity, and this current leads by a quarter period over the E.M.F. This current is, of course, directly proportional to the frequency, and as owing to its lead of 90° it carries no power, whilst wasting power by heating the conductors in circuit, it is desirable to keep it as small as possible. Hence if the line has great capacity the frequency should be low. As regards small motors a low frequency is also desirable. It will be shown later on that all motors, whether single or multiphase, run very nearly at a speed directly proportional to the frequency of the supply current and inversely proportional to the number of poles in the field. To get a moderate speed we must therefore either have a low frequency or build motors with many poles. The latter expedient is easily adopted in large motors, but not so easily in small ones ; hence where the subdivision of power between many small consumers is of importance, the frequency adopted should not be too high. Taking these various requirements together we find that the right frequency for power lines is a compromise, and practical experience goes to show that in the majority of cases a frequency between 40 and 65 is the best.

After this digression we can now return to the problem of power transmission by alternating current. We take, as the simplest case, the transmission of power between two similar single phase alternators by means of a perfectly insulated line having only resistance, but no self-induction and no capacity. The last two conditions are very nearly realized in practice with aërial lines of moderate length. We shall also, for the sake of simplicity, assume that the armature reaction, both in the generating and receiving machine, is negligible, and that the self-inductions in them are constant and known quantities. Assume that both machines are running at the same frequency, and that the excitation of the two fields is so adjusted as to produce E_1 volts in the generator and E_2 volts in the motor. Let the total resistance in circuit (that is, the resistance of the two armatures and line) be r ohms, and let the current be i ampères. If L_1 be the co-efficient of self-induction in the generator, and L_2 that in the motor, the E.M.F. of self-induction is respectively

$$e_{s1} = 2\pi \sim L_1 i$$
$$e_{s2} = 2\pi \sim L_2 i,$$

or, taking the two machines together,

$$e_s = 2\pi \sim (L_1 + L_2)\, i.$$

The loss of pressure by resistance is

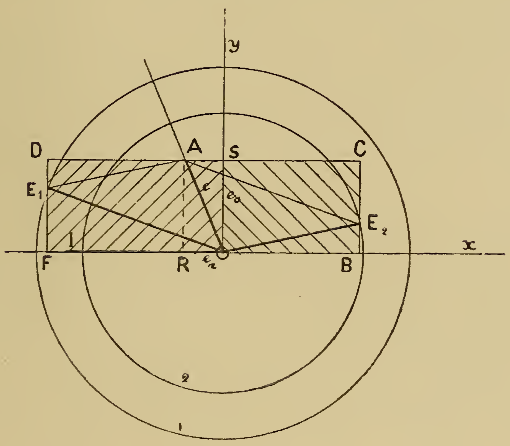
$$e_r = r\, i.$$

We can now construct a clock diagram to show the working of the plant. In drawing such a diagram we may start by fixing the instantaneous position of any one quantity, and build up the whole diagram to suit the first assumption. In the present case it is most simple to start with the current. Thus, let in Fig. 88 $O\,I$ represent the current line taken at the moment when the current passes through zero, and $O\,R$ the voltage lost by resistance. From what has been said in the beginning of this chapter, it will be clear that the E.M.F. required to overcome self-induction must be in advance of the current by a quarter period, that is to to say, the radius representing e_s must at this moment point vertically upwards. Since we know e_s from above formula, we can draw its radius in the diagram. Let this be $O\,S$. We know now that the two machines must combine in such way as to produce in the circuit simultaneously the E.M.F. $e_r = O\,R$ and the E.M.F. $e_s = O\,S$; that is to say, they must produce the resultant $e = O\,A$; because only if this condition is fulfilled can the current OI flow. It is important to note that the direction of this resultant is independent of the strength of the current. It is given by the tangent of the angle $A\ O\ I$, and its expression is

$$\tan A\ O\ I = \frac{2\ \pi \sim (L_1 + L_2)}{r}$$

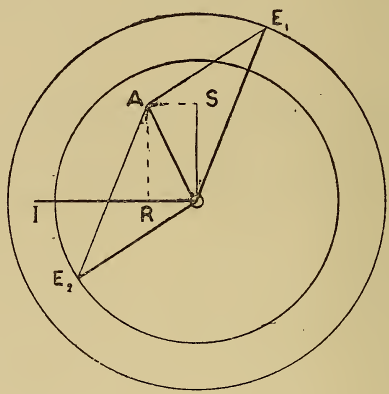
Let the circles 1 and 2 be drawn with radii representing

Fig. 88.



the E.M.F. of generator and motor respectively. The problem is now to find the position of these radii corresponding to the the position of the current line originally fixed. We know that $O\ A$ must be the resultant of the two E.M.F.'s, and we have therefore to find what parallelogram of forces, having E_1 and E_2 for sides, will give $O\ A$ as the resultant. It is clear that only two such parallelograms are possible, one being shown in Fig. 88, and the other in Fig. 89. In the former figure $O\ E_1$ lies on the same side as $O\,I$, and this means that the machine gives power to the current, that is, acts as a generator. The E.M.F. line of the other machine lies on the opposite side to $O\,I$, that is to say, the current passes through the armature in opposition to its E.M.F., and therefore does work on the machine, which acts as motor. Since in the diagram E_1 is shown larger than E_2, we find that the stronger machine is the generator, and the weaker machine is the motor. The position may, however, also be reversed, as

Fig. 89.



shown in Fig. 89. Here the E.M.F. of machine No. 2 lies on the same side as the current, whilst that of machine No. 1 lies on the opposite side. Hence the weaker machine is the generator, and the stronger machine is the motor. In this respect the transmission of power by alternating current differs radically from that by continuous current. In the latter the motor must under all circumstances have a lower E.M.F. than the generator, but with alternating currents the machine of lower voltage may work as generator and drive the machine of higher voltage as a motor.

The power given to the motor in Fig. 88 is the product of *O B* volts (this being the projection of $O E_2$ on the current line) and i ampères. Similarly the power given by the generator to the line is the product of $O F \times i$. For i we may substitute $e_s / 2 \pi \sim (L_1 + L_2)$ and we thus find that the area of the shaded rectangle *O S C B* is proportional to the power given to the motor, and the area of *O S D F* is proportional to the power given out by the generator, whilst the area of *O S A R* is proportional to the power lost in resistance. To get the exact value of any of these powers we measure the rectangles by means of the volt scale which was used in drawing the diagram, and divide the area by $2 \pi \sim (L_1 + L_2)$; the power is then obtained in watts.

The diagram may also be used to show what will happen if the load on the motor is kept constant, whilst its excitation, and therefore its E.M.F., is raised. Since the area of *O S C B* is to remain constant, the point *C* must lie on a rectangular hyperbola, and its position will depend on the E.M.F. of the motor, the E.M.F. of the generator being supposed to remain unaltered. Fig. 90 is drawn to scale for $E_1 = 1000$[1] $r = 1$ $\sim = 50$ $L_1 + L_2 = \cdot 0127$ $2 \pi \sim (L_1 + L_2) = 4$ $e_s = 4\,i$ $e_r = 1\,i$ $W_2 =$ 100,000 watts delivered to motor. The power delivered fixes the position of the hyperbola, and we have now to find the relation between the E.M.F. of the motor and the current for this power. The direction of *O a* is fixed by the condition that the E.M.F. of self-induction is four times that required to overcome resistance. If we now

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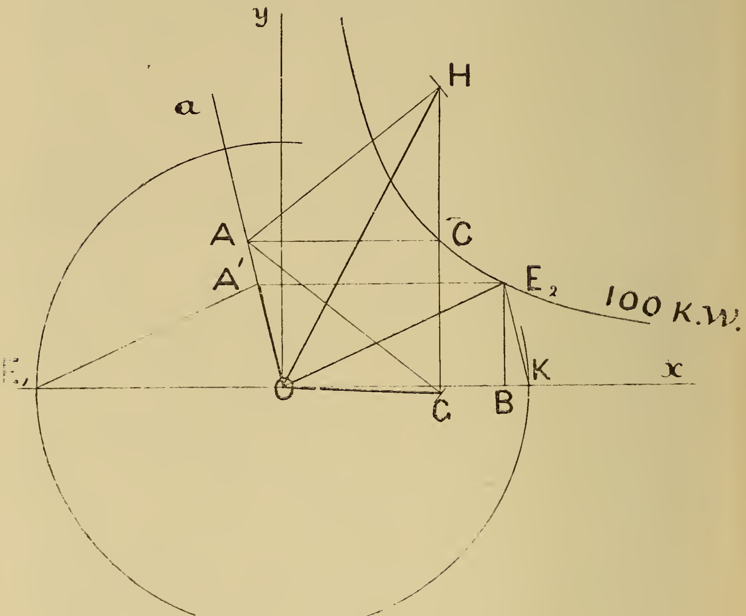
[1] 800 volts to the inch.

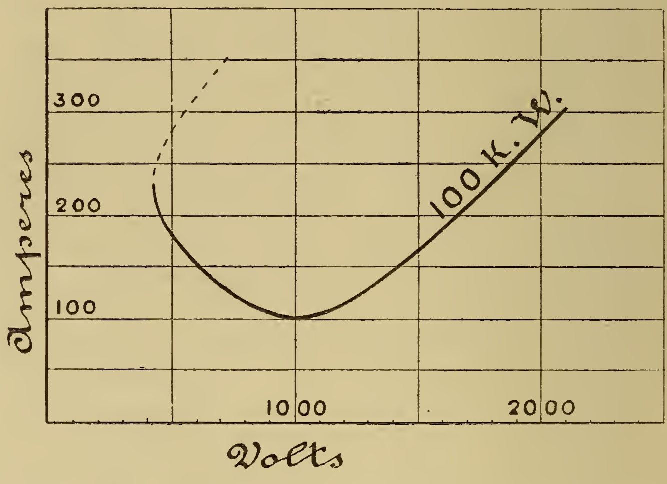
select a point C on the hyperbola, we know that the end of the E.M.F. radius of the motor must lie somewhere on a vertical line through C, and we also know that the end of the radius giving the resultant E.M.F. must lie somewhere on a horizontal line through C. It must also lie on the inclined line Oa, and we find thus the point A by

Fig. 90.



the intersection of the two lines. OA is the resultant E.M.F. If now with a radius corresponding to 1,000 volts, and from A as a centre we intersect the vertical line through C, we find the two points G and H, which are the ends of the radii representing E.M.F. of motor. It will be seen that the same current may be obtained with two E.M.F.'s of motor, which in this case are widely different, namely 680 and 1,370 volts. If we had selected the point C higher up on the hyperbola, there would have been still greater difference between the two voltages at which the motor requires the same current to give 100 Kwt. The current is given by the piece cut off on the vertical $O\,y$ by the horizontal line $C\,A$. In order to waste a minimum of power in heating the resistance of the circuit we must so excite the motor as to make this current a minimum. We must, therefore, endeavour to bring C down to the lowest possible point on the hyperbola. This point is obviously that at which a line drawn from K parallel to Oa intersects the hyperbola at E_2. The corresponding point on the line of resultant E.M.F. is A^1, and the voltage of the motor is $O\,E_2$ (in this case very nearly 1,000 volts). If the motor is excited to give the E.M.F. $O\,E_1$ the line current is a minimum, and therefore the efficiency of transmission a maximum. The E.M.F. of the generator coincides with the current, and the output of the generator is correctly represented by the product of current and E.M.F. It is convenient to represent the relation between motor voltage and working current for constant load also in the way shown in Fig. 91. This diagram is obtained from Fig. 90 by plotting the voltage of the motor, such as $O\,G$, on the horizontal, and the corresponding current on the vertical. The curve resulting shows at a glance how the current varies when the motor voltage is altered. It will be seen that, generally speaking, the same current will result for two different values of the motor E.M.F., the lower value giving a small, and the higher value a large lead of current over the E.M.F. It has already been shown that a leading current weakens the field of the motor, and this effect is the more noticeable the greater the lead. It follows that with a motor too strongly excited armature reaction has rather a beneficial effect as it in part corrects the error of excessive excitation, whilst with a motor too weakly excited the reverse may be the case. On account of safety and efficiency it is therefore better to work the motor at a voltage rather above than below that which, according to Fig. 91, is

Fig. 91.



theoretically the best. It will be seen from the curve that as we reduce the excitation beyond the theoretically best point the current increases until we reach a particular value when the vertical just touches, but does not cut, the curve. This is a critical point. If the excitation be still further reduced the ordinate will miss the curve altogether, and this shows that the motor cannot work in this condition. It must fall out of step and come to rest. The same will happen if, working near the critical point, the load on the motor should be accidentally increased. There is no such critical point to the right of the ordinate corresponding to the best point of working, and in order to have some margin, both as regards an error in adjusting the exciting current and an accidental overload of the motor, it is best to aim at working it at an excitation slightly higher than that which would theoretically be best.

The question of what amount of overload a motor can support without falling out of step is a very important one. In practical work we cannot expect to know to a fraction the exact limit of load which a motor may have at one time or another to support. If, for example, 100 horse-power is under average conditions sufficient to drive a factory, it is conceivable that for short periods, whilst a heavy tool is set in motion, this limit may be considerably exceeded; and as it would be exceedingly inconvenient to have the motor fall out of step and come to rest on some such occasion it is important, when designing the plant, to make provision for a certain amount of spare power. The problem may be stated thus: Assuming that the engine or turbine which drives the generator is so governed as to be able to furnish for a short time any excess of power which may be required, what is the maximum amount of power that the generator can take up and the motor give out without breaking down? We know that with continuous current machines this maximum exceeds by many times the normal power, and we must now investigate how alternating current machines behave in this respect.

For this purpose we may take as an example the machines to which Figs. 90 and 91 refer. Say that the normal power required for driving the factory is 125 horse-power or 93 Kwt. Allowing 7 Kwt. as an outside margin for exciting the motor and for its own frictional eddy current and hysteresis losses (armature resistance being already included in r), then the normal power brought to the motor by the supply current will be 100 Kwt. If the demand for power be reduced to zero, then the supply current will have to bring in merely the 7 Kwt. required to cover the various losses, and if the demand for power be increased beyond 125 horse-power, then the supply current will have to bring in a corresponding excess over the normal 100 Kwt. until a point is reached when the available margin of power in the motor is exhausted and the system breaks down altogether.

We neglect again armature reaction and capacity of line ; and assume that the total resistance in circuit is one ohm whilst the total self-induction in circuit is such as to give at 50 cycles per second $e_s = 4i$. If the transmission were by continuous current, the 100 Kwt. would be delivered at the motor end by a current of 100 α and 1,000v., and if the efficiency of transmission is to be 90 per cent., this would require a pressure of 1,100 volts at the generator end. We shall, therefore, assume that the alternator at the generator end is excited so as to give this effective E.M.F., and then the motor in order to work with a minimum current would have to be excited so as to give a slightly lower pressure (see Fig. 90), but in order to have a margin of safety, as explained a few pages back, we will excite the motor so as to give a somewhat higher E.M.F., say 1,150 volts. It may here be explained that the voltage of the plant has been chosen low merely for the convenience of obtaining a better scale for the diagrams. In practice there is no difficulty whatever in using an E.M.F. up to 5,000 volts, and even higher, without transformers if the distance of transmission is considerable.

We have then the following data on which to construct the working diagram of the plant

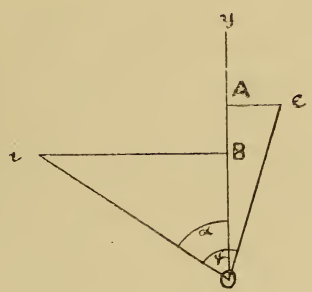
$$E_1 = 1{,}100 \quad E_2 = 1{,}150 \quad e_r = 1.i \quad e_s = 4.i \quad r = 1 \quad \sim = 50$$
$$(L_1 + L_2) = \cdot 0127 \quad e = 4\cdot 123 i \quad i = \cdot 242 e.$$

In the first place we must ascertain whether in the event of the motor coming to rest through being overloaded, the current will remain within safe limits. As with the motor at rest $E_2 = 0$ the whole E.M.F. of the generator (in this case 1,100 volts) will be used up in overcoming the resistance and self-induction of the circuit. The E.M.F. of the generator will, therefore, be the resultant E.M.F. or $e = 1{,}100$ if, as stated above, we neglect armature reaction for the present. We find that under these conditions the current will be $\cdot 242 \times 1{,}100 = 266$ ampères, or about $2\frac{1}{2}$ times the normal working current. Whether this current will injure the armatures by overheating depends on their resistance. The total resistance in circuit is 1 ohm, and the total power absorbed will, therefore, be only 71 Kwt., which is less than the power given out by the motor under normal working conditions. Of that power the greater part is of course absorbed in the line. The remainder is absorbed in heating the two armatures. Now the resistance of armature in modern machines is very low. There is no difficulty in designing alternators so that the loss of pressure due to armature resistance shall not exceed 2 per cent., and even 1 per cent. is a perfectly feasible limit. In the machines with which we are dealing and which are of the author's type, specially designed for power transmission, the loss is about $1\frac{1}{4}$ per cent. at full load. Thus, with 100 ampères flowing, the loss would be about 14 volts in each armature and with 266 ampères, it would be 37 volts, that is under $3\frac{1}{2}$ per cent. of the terminal voltage. The corresponding power is under 10 Kwt. for each armature, which is so small an amount of power that we need not fear any damage to the armatures by overheating. It will be shown later on that even this moderate limit cannot be reached in practice provided the generator is properly designed so as to take advantage of armature reaction, but for the present we may rest satisfied that the plant will be safe against overheating under all conditions. Whether it will be safe against mechanical stresses is another question. Although the power given to the line by the generator in the event of the motor being forcibly stopped is only 71 Kwt., the mechanical stresses between the field poles and armature coils in the generator are much greater than corresponds to the output. This may be clearly seen from Fig. 92 where $o\,e$ is the resultant E.M.F. (in this case coincident with the E.M.F. E_1 of the generator) and $o\,i$ the current. As we are now dealing with instantaneous values, these lines must be drawn to represent maximum and not effective values. Thus, $o\,e$ will not be 1,100 volts, but $\sqrt{2} \times 1{,}100 = 1{,}540$ volts, and $o\,i$ will be $\sqrt{2} \times 666 = 373$ ampères. The mechanical stress on the armature coils is at all times proportional to the product of instantaneous current and instantaneous strength of field, or what comes to the same thing, instantaneous E.M.F. It may, therefore, be represented to a suitable scale by the product of the projections of e and i on the vertical ; thus:

$$\text{Stress} = O\,A \times O\,B.$$

If A and B lie both above O, the stress is in one direction; if one lies above and the other below O, it is in the other direction, whilst it is zero at any time that either e or i changes sign. The question we have to investigate is: at what phase during the cycle is the product $O A \times O B$ a maximum, and what is the relation between this maximum and the normal stress when the machine is working under full load? In the latter case, $i = 140$, and $E = 1{,}540$, and the two may for our present purpose be assumed to coincide. The stress will then

Fig. 92.



never change sign, and its maximum will be proportional to 215,000. When the generator is working on short circuit, the lag ψ between e and i is known from the relation that $e_r : e_s = 1 : 4$, to which corresponds $\psi = 76°$. Let the phase for which the diagram is drawn be represented by the angle α, then we have to find the relation of α and ψ for which the stress becomes a maximum. It is obvious that

$$O A \times O B = ei \cos(\psi - \alpha) \cos \alpha,$$

and by differentiating to α and equating to zero we find

$$tn\,\psi = \frac{2\ tn\ \alpha}{1 - tn^2 \alpha},$$

from which $\alpha = \frac{\psi}{2}$.

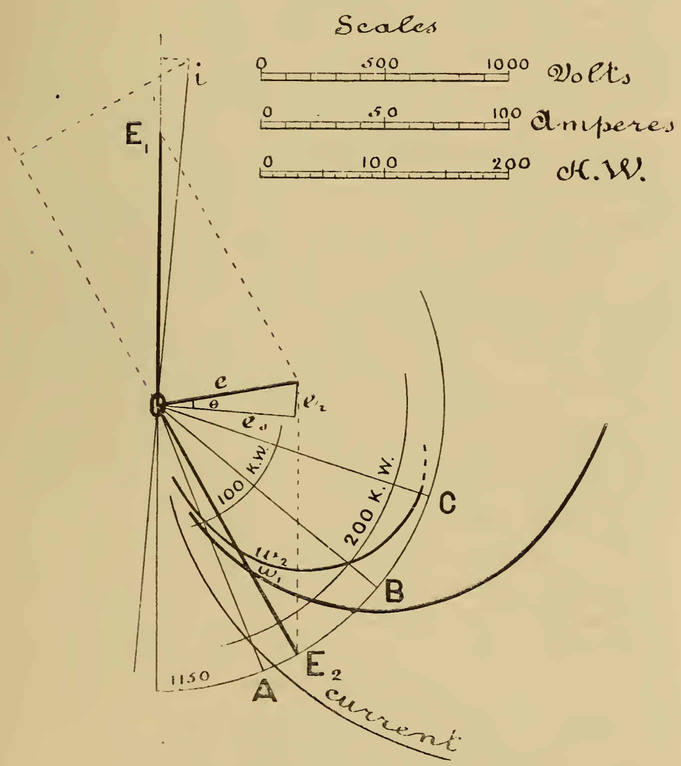
The maximum stress will occur when the vertical is midway between the volt line and ampère line, that is for the phase characterized by $\alpha = 36°$. The cosine of 36° is ·809, and the stress at that moment is

$$OA \times OB = 1{,}540 \times 373 \times 654$$
$$= 377{,}000,$$

or about $1\frac{3}{4}$ times as great as when the machine is working under normal conditions. There should be no difficulty in providing a sufficiently substantial method of construction to render this comparatively small increase in the mechanical stresses perfectly harmless; and having now satisfied ourselves that the plant will be safe against overheating and mechanical failure, we may proceed with our investigation as to its working condition when the load on the motor is varied.

We draw the clock diagram at the moment when the E.M.F. of the generator has attained its positive maximum, OE_1, Fig. 93, and we assume the position of the E.M.F. of the motor at E_2 on the 1,150 volt circle. We then find the resultant e; and since the relation between e_r and e_s is given, the angle θ, by which the E.M.F. opposed to self-induction leads over the resultant E.M.F. is also given (in our case 14°), we can draw the lines representing e_s and e_r. The current line must be at right angles to e_s, and can now be drawn in, its length being determined from the equation $i = ·242\ e$ above given. To find the power corresponding to the selected position of E_2, we prolong $E_2 O$ backwards and project i on it (dotted lines). If we measure this projection with the ampère scale, and multiply with 1,150, we obtain the

Fig. 93.



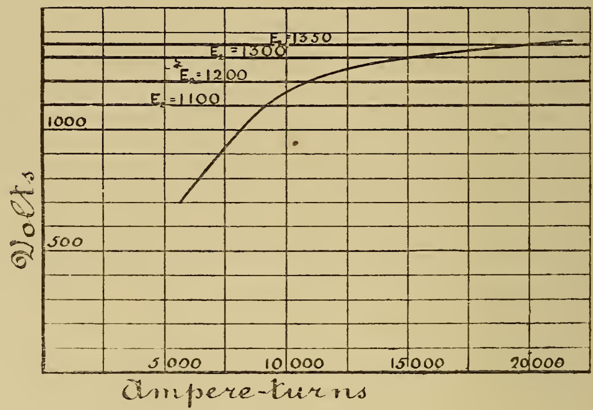
power given to the motor in watts. This may be plotted to any convenient scale on the radius giving the motor E.M.F. We obtain thus the point ω_2. Similarly, if we project the current line on the vertical, and multiply the scaled amount with 1,100, we obtain the power given out by the generator, plotted at ω_1. The difference, $\omega_1 - \omega_2$, is of course the power lost in resistance, ri^2, and the accuracy with which the diagram is drawn may thus be checked. By assuming different positions of E_2 on the 1,150 volt circle we can find a succession of points giving the corresponding power of motor and generator, which, if joined, give the two power curves shown in thick lines. It may here be remarked that if the resistance in circuit may be neglected these curves coincide and become a true circle with its centre in the horizontal axis of the diagram. The power curves show very clearly the working condition of the plant. If, for instance, the load on the motor be such as to require the supply of 100 Kwt., the E.M.F. line of the motor will assume the position *OA*. If the load be reduced this line will advance towards the vertical ; in other words the E.M.F. of the motor will come nearer to the point of exact opposition to the E.M.F. of the generator. Now suppose that the load on the motor is suddenly increased. This will to some extent retard the motion and cause the armature of the motor to lag, thereby increasing the angle between OE_2 and the vertical. As we see from the diagram, this is exactly what is wanted, because it brings the armature into a phase where it is capable of giving out more mechanical power. In this region of the diagram the working is perfectly stable. If the load be increased to 200 Kwt., the E.M.F. radius of the motor will assume the position *OB*. The load may even be increased beyond this point, but not very much, for at 224 Kwt., which corresponds to the position *OC*, we have reached the very utmost limit of power, whilst the current as given by the line marked "ampères" is very great, and the efficiency very low. At this point the motor becomes unstable (dotted part of the power curve). Any slight increase of load brings the E.M.F. radius further back and diminishes the available power. The motor is therefore not able to cope with the increased load, and must fall out of step. We see thus that in the example chosen, and provided we may neglect armature reaction, the load on the motor will have to be more than doubled before the system of transmission breaks down.

But now the question presents itself: are we justified in neglecting armature reaction? This depends on the design of the machines, but as a general rule it will be found that the armature reaction is too important to be neglected. The reason is, that in order to provide for the safety of the plant in the event of the motor stopping, we are obliged to so design the machines as to have a sensible amount of self-induction; the number of armature conductors must therefore not be reduced too much, and this means that the ampère-turns produced by the armature current are considerable. By working the machines with very strongly excited fields, that is, at a point well above the knee of the characteristic curve, we are able to diminish the relative importance of the armature ampère-turns, but even then we cannot render their effect negligible, whilst the use of an abnormally strong excitation is objectionable for obvious reasons.

In order to be able to study the effect of armature reaction we must, of course, know the characteristic of the machine. This can be predetermined in the same manner as already explained in connection with continuous current dynamos. Let Fig. 94 represent the characteristic of the machines previously considered, the ordinates being effective volts induced in the armature at the normal speed of 500 revs. per minute, and the abscissæ being ampère-turns applied to the field. The two machines are alike in every way ; each has 12 poles and 288 armature conductors, giving $\omega = 24$ and by Formula 57) $x = \cdot 373\ i\ \varphi^{\circ}$ as the armature ampère-turns.

Let us first see how armature reaction affects the generator when the motor is forcibly stopped. We have seen that if armature reaction were negligible the current would be 266 ampères, and the angle of lag 76°. These

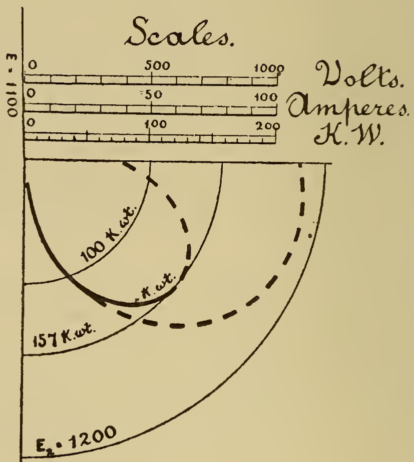
Fig. 94.



values inserted into Formula 57) give $X = 7{,}570$ ampère-turns. From the characteristic Fig. 94 we find that the field excitation, which produces 1,100 volts, is 9,000 ampère-turns. The current lagging behind the E.M.F. must weaken the field, that is to say, if the current of 266 ampères were by any means maintained, the effective field excitation would be $9{,}000 - 7{,}570 = 1{,}430$ ampère-turns and the E.M.F. induced in the armature would only be about 200 volts. Since there is, however, no other means for producing the current than the E.M.F. of the armature, it is clear that the reduction of E.M.F. will not be so great. We can find the exact working condition by a tentative method adopting successively different values for the effective E.M.F. between the two extreme limits of 200 and 1,100 volts, until we arrive at that value of E.M.F. at which the armature current will reduce the field excitation by exactly the proper amount. It is not necessary to give this operation here in detail; the result is that with a current of 150 ampères, to which corresponds an E.M.F. of 600 volts, the armature demagnetizes the field with 4,250 ampère-turns. The field excitation remaining is, therefore, 4,750 ampère-turns, and by referring to the characteristic we find that this excitation will, indeed, produce the 610 volts. When the motor is forcibly stopped the current will, therefore, not be 266 ampères, as found when neglecting armature reaction, but only 150 ampères.

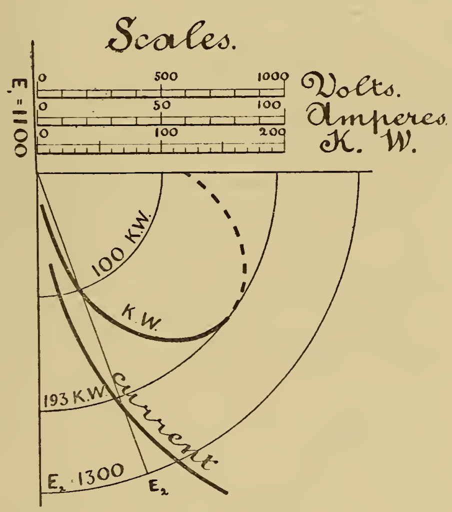
The power given given out by the motor when in ordinary working condition can be found by using the construction shown in Fig. 93, but corrected by reference to the characteristic Fig. 94. It will be found that for a small output the current leads in the generator; for a moderate output it may lag or lead in the generator, but not by any large amount, for a large output approaching the breaking-down point the current in the generator lags. As regards the motor the current leads at all loads. We thus see that as the load increases the field of the generator is at first strengthened and then weakened, whilst the field of the motor is weakened from the beginning; and it is the more weakened the greater the load becomes. For this reason it is advisable to excite the motor to a higher E.M.F. than the generator, as already pointed out. If we work the generator at or slightly below the knee of the characteristic we insure the safety of the plant in the event of a short circuit, whilst to insure a margin of power we must work the motor well above the knee of the characteristic. Let us excite the generator with 9,000 ampère-turns, giving an E.M.F. of 1,100 volts. By assuming successively 10,800, 15,000, and 18,500 ampère-turns as the

Fig. 95.



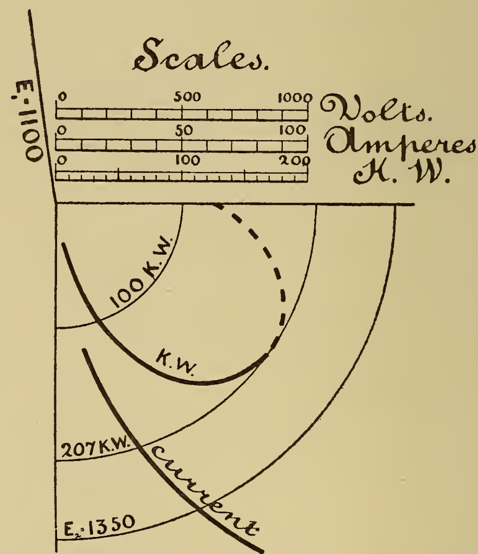
excitation of the motor (corresponding to 1,200, 1,300, and 1,350 volts), we can for each case construct the power curves and see at a glance what margin of power we have. The construction must be made as in Fig. 93, and corrected by reference to Fig. 94, various values for E_1 and E_2 being tentatively adopted, until by trial and error method we arrive at the true values. The construction need not be explained at length; it is quite simple, though somewhat tedious. The result is shown in Figs. 95, 96, and 97. The curve giving the power required by the generator has been omitted in all these diagrams, the power curve marked Kwt. referring to the motor only. In Fig. 95 the power curve resulting from the same construction when armature reaction is neglected, has been shown in a dotted line. It will be

Fig. 96.



noticed that up to about 100 Kwt. (being the normal load) the two power curves coincide, but they diverge for greater loads, and the breaking-down point is reached very much sooner than would be the case if armature reaction were absent. The diagram illustrates in a striking manner the importance of taking armature reaction into account in the design of power transmission plant, especially as regards the determination of the margin of load on the motor. In the present case the breaking-down load corresponds to a supply of 157 Kwt. Since 7 Kwt. is required for excitation and to overcome losses in the motor, 150 Kwt. or about 200 horse-power is the maximum that can be given off. The normal power given off is 125 horse-power, so that the margin of power is only 75 horse-power or 60 per cent. over the normal

Fig. 97.



output. It would scarcely be safe to rely on a transmission plant which is only capable of carrying 60 per cent. excess load. To increase the margin of safety we must excite the motor to a higher degree. Fig. 96 shows the power curve if the motor is excited with 15,000 ampère turns, giving an E.M.F. of 1,300 volts. The current curve has been added, so that by means of the scale of ampères the current corresponding to any load may be read off on the radius representing the phase of E.M.F. of motor at the given load. Thus at a load of 125 horse-power corresponding to a supply of 100 Kwt. the E.M.F. of motor is in the phase represented by the radius $O\ E_2$, and the current is 103 ampères. The loss by resistance is 10·6 Kwt.; losses in both machines, 14 Kwt.; total losses, 24·6 Kwt.; power delivered, 93 Kwt.; power supplied to generator, 117·6 Kwt. Total efficiency, $93/117{\cdot}6 = 79$ per cent. The breaking-down load corresponds to a supply of 193 Kwt. to the motor. Deducting the 7 Kwt. for internal losses we find that 186 Kwt. or very nearly 250 horse-power will be given off by the motor before the system breaks down. This is a margin of 100 per cent.

Should we want a larger margin still we can obtain it by exciting the motor to a still higher degree. Fig. 97 shows the power curve when the motor is excited with 18,500 ampère-turns giving $E_I = 1350$. In this case the breaking down load is 200 Kwt. or 268 horse-power, giving a margin of 134 per cent.

These results are summarized in the following table:

Table showing Working Condition of Transmission Plant. Total Resistance in Circuit 1 *ohm. Total Inductance* 4 *ohms.*

Generator excited to give	1,100	1,100	1,100 volts
Motor excited to give	1,200	1,300	1,350 volts
Normal power given off by motor	125	125	125 HP
Maximum power given off by motor	200	250	268 HP
Margin of excess load causing breakdown of the system	60	100	134 per cent.

A careful study of the diagrams here given for one particular example, and the application of the same methods of investigation to other cases leads to the following general conclusion.

A high resistance, whether this be in the line or in the machines, is objectionable, as it tends to lower the efficiency and the margin of excess load. To insure reliability in working, and an ample margin of power, the total resistance in circuit should be as small as possible. This rule applies not only to power transmission as such, but also to the circuits connecting two or more generators, or two or more motors which work in parallel.

A moderate self-induction and a moderate armature reaction are desirable in the generator, because ensuring the safety of the machine in case of a short circuit.

In the motor there should be as little self-induction and as little armature reaction as possible.

If motor and generator are machines of the same size and type, the motor should be excited to a higher E.M.F. than the generator. It is, however, preferable to give the motor fewer armature conductors than the generator. In this case its self-induction and armature reaction will be lower, and it need not necessarily be worked at a higher E.M.F. than the generator, the essential condition being that the motor should be worked at a point well beyond the knee of its characteristic curve.

The foregoing investigation was based on the assumption that the line has only resistance, but no capacity. In most cases the capacity of the line is so small that it may indeed be neglected. A few miles of overhead line properly erected need not have more than a small fraction of a microfarad of capacity, and the condenser current flowing in and out of the line is so small in comparison with the power current as not to disturb the working of the plant. There are, however, cases where the capacity of the line cannot be neglected, and these occur when the distance of transmission is considerable. We are then compelled to work at extra high pressure, and the line must necessarily have the more capacity the longer it is. Both these circumstances tend to increase the condenser current. Cases may also arise where, owing to climatic or other conditions, an overhead line is not admissible and a concentric cable must be used. Such a cable may have a very sensible capacity. Thus the Ferranti main between Deptford and London has a capacity of ·367 microfarads per mile, and if such a cable were used for a transmission at 10,000 volts over a distance of 20 miles and a frequency of 50 cycles per second the condenser current would be 23 ampères.

If K represents the capacity of a condenser connected to a source of alternating E.M.F. and i the instantaneous current, then the increment of charge flowing into or out of the condenser during the time dt, when e changes by the amount de is $i\,dt = K\,de$. If e is a sine function of the time then $i = K\frac{de}{dt}$ must also be a sine function such as $i = I \sin 2\,\pi \sim t$.

We can, therefore, also write

$$I \sin 2\,\pi \sim t\,dt = K\,de.$$

The total charge of the condenser corresponds obviously to the current flowing in or out during a quarter period from $e = o$ to $e = E$, the maximum value of the E.M.F. applied. This charge is $\int K\,de = KE$ or

$$KE = \int_0^{\frac{T}{4}} I \sin \pi \sim t \, dt$$

$$KE = \left[- \frac{I \cos 2\pi \sim t}{2\pi \sim} \right]_0^{\frac{T}{4}}$$

$$KE = \quad - \frac{I}{2\pi \sim}$$

In this equation both E and I are maximum values, but as the effective value of both bears the same proportion, the formula holds good also for effective values, and we have, therefore, the following formula for the condenser current:

$$i_k = 2\pi \sim e\,K.$$

This is in absolute measure. If we have K in microfarads, i_k in effective ampères and e in effective volts the formula becomes

$$i_k = 2\pi \sim e\,K\,10^{-6} \quad . \quad . \quad . \quad . \quad 58).$$

For a concentric cable in which the inner conductor has a radius of r centimeters, and the outer conductor has an inner radius of R centimetres ($R-r$ being the thickness of the insulation between the two conductors) the capacity may be calculated according to the following formula given in Professor Ayrton's "Practical Electricity" and other text books:

$$K = \frac{2{\cdot}413\ \varepsilon\ l\ 10^{-7}}{\log_{10} R - \log_{10} r} \text{ Microfarads} \quad 59).$$

l being the length of the cable in centimeters and ε the specific inductive capacity of the insulating material. The value of ε varies between 2·5 and 4 according to the nature of this material. Taking 3·3 as an average value for insulating materials consisting mainly of some heavy hydrocarbon compounds, and inserting the length of the cable in miles, we have also

$$K = \frac{\cdot 129\, l}{log_{10} \frac{R}{r}} \text{ Microfarads} \quad . \quad . \quad 60).$$

For overhead lines the capacity may be calculated by the formula given by Steinmetz,[1]

$$K = \frac{1 \cdot 11\, l\, 10^{-6}}{4\, log\, net \left(\frac{d}{r}\right)},$$

in which l is the length of line in centimetres, r the radius of the wire, and d the distance of the two wires. For convenience in use this formula may be transformed into the following expression,

$$K = \frac{\cdot 0192\, l}{log_{10}\left(\frac{d}{r}\right)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad 61),$$

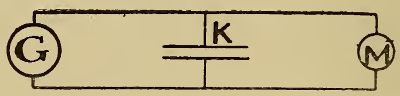
where K is the capacity in microfarads, l the distance of transmission in miles, and the denominator the common logarithm of the ratio between the distance and radius of wires. Thus, if we have wires of 160 mils diameter placed 3 feet apart, this ratio is 450, and the capacity of the line is ·00685 microfarad per mile.

The effect of capacity on the generator and motor may be investigated by means of a clock diagram, in a similar

[1] "Elektrotichnische Zeitschrift," 1893, p. 476.

way to that adopted when dealing with self-induction, except that the capacity current must always be drawn 90 degrees in front of the E.M.F. which produces it. An example may serve to show how the capacity of the line influences the working conditions of a transmission plant. For this purpose we take a rather long underground line working at high pressure, because with short lines at moderate pressure the influence of capacity is too small to show clearly in the clock diagram. Let us then assume that the two machines are connected by a concentric cable 10 miles long, and that the effective pressure in the cable is 10,000 volts. Let the power delivered to the armature of the motor be 500 Kwt. and the total resistance of the line 10 ohms. The conductors would have an area of ·085 sq. in., and the insulation between them a thickness of about ·4 in. The capacity of the whole line would be about 3·2 microfarads. This is distributed over the whole length of the line, and to calculate the exact distribution of the charge would be a very complicated operation, because that depends on the potential difference between the two conductors, and this varies along the cable by reason of its ohmic resistance. The variation is, however, small in comparison with the total potential difference, as will be seen from the following preliminary consideration. The total power transmitted is 500 Kwt. at 10,000 volts. If there were no lag and no capacity the current would be 50 ampères, and the loss of pressure over the whole 10 miles of cable would be 500 volts or 5 per cent. As, however, there will probably be some lag, and as there must be some capacity current, the total current flowing into the line at the generator end must be greater than 50 ampères. How much greater we cannot yet tell with certainty, but we can make a rough approximation. Let the frequency be 50, then we find from formula 58) that a condenser of 3·2 microfarad will at 10,000 volts take 10 ampères. This is a comparatively small current, and as its phase compared with that of the working current must approach more or less a quarter cycle, its effect on the total current cannot be large. As regards the lag produced by self-induction and consequent increase of current, this also cannot be very large for reasons which have been previously explained. We shall probably be within the mark if we assume that the total current passing into the cable will not exceed 60 ampères. In this case the greatest variation of potential difference between any two points of the cable cannot exceed 600 volts out of a total of 10,000, or 6 per cent. If we assume that the whole capacity is concentrated at the generator end of the cable, we would under-estimate the voltage loss due to ohmic resistance by the amount which corresponds to the difference between the true current and that resulting from our assumption. Conversely, if we assume the whole capacity concentrated at the motor end, we would over-estimate the voltage loss. In either case the error can only be a very small fraction of the 600 volts lost in resistance. Under the first assumption the condenser current calculated from formula 58) would be too large, and under the second assumption too small, but the error is in either case exceedingly small. It can be further reduced by assuming the whole capacity concentrated in the middle of the line as shown diagramatically in Fig. 98. G is the generator and M the motor, joined by the outgoing and retiring leads which form the line. We assume that the line itself has no capacity, but its middle points are connected with a condenser, K, the capacity of which is equal to that of the real cable. The object of representing the case in this way is to simplify the investigation, which otherwise would become far too complicated for practical purposes. The error introduced by representing the capacity of the cable as concentrated in one point is, moreover, quite negligible in all cases likely to occur in practice. We have then at K a capacity of 3·2 microfarad ; between K and M a line resistance of 5 ohms, and between K and G also a line resistance of 5 ohms. The resistance of each armature may be taken at 2 ohms so that the total resistance in either half of the circuit is 7 ohms. The inductance of the motor we take at 40 ohms, and that of the generator at 60 ohms.

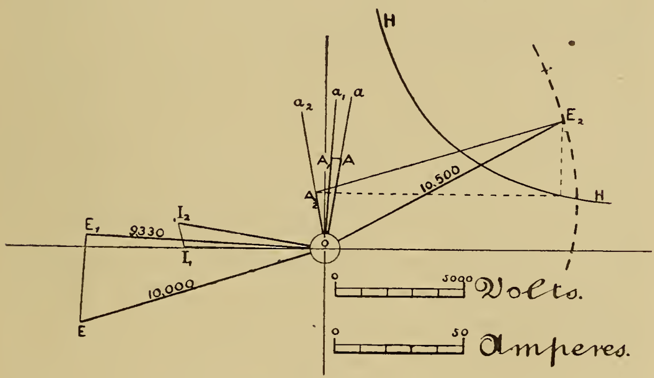
Fig. 98.



The problem may now be represented as follows : At K, we have an E.M.F. of 10,000 volts effective, sending current at a frequency of 50, through a circuit of 7 ohms resistance and 40 ohms inductance, the strength of the current to be such as to deliver 500 Kwt. to M. At G, we have a generator supplying this current and that required for the condenser, K, the generator working at such pressure as to produce 10,000 volts at K, the resistance being 7 ohms, and the inductance 60 ohms. Required to find the working condition of the system. The solution of this problem can best be made graphically by means of a clock diagram. Not to complicate the matter uselessly, we neglect armature reaction. If required, the corrections for armature reaction can be made in accordance with the explanation given a few pages back.

Let, in Fig. 99, HH represent the hyperbola for 500 Kwt., and let O_{a2} be the line of resultant E.M.F. (see also Fig. 90), then, by the construction explained on p. 260, we can find various positions of the E.M.F. radius of the motor, corresponding to an input of 500 Kwt. when the impressed E.M.F. is 10,000 volts. The ends of these radii lie on a curve (dotted line in Fig. 99), and we may select any point on that curve as the working point.

Fig. 99.



Say that in order to get a reasonable margin of power we select the point E_2, corresponding with such an excitation as to produce 10,500 volts in the motor. From E_2 we drop a vertical to the hyperbola, and from its point of intersection we draw a horizontal to the left till it cuts Oa_2 in A_2. The line A_2 E_2 represents the impressed E.M.F. (10,000 volts) in the middle of the cable, and this put in its proper place gives us $O\,E$. The projection of $O\,E_2$ on the horizontal, measures 9,100 volts ; the current to produce 500 Kwt. must, therefore be 500,000/9,100 = 55 ampères, and this is drawn to the left, $O\,I_1$. This is the current flowing through the circuit KM. To find the current flowing through the circuit, GK, we combine with OI_1 the current taken by the condenser. This, at 10,000 volts is 10 ampères, and must, in position, be at right angles to OE. By drawing $I_1 I_2$ perpendicular to OE, and of such length as to represent 10 ampères, we find the point I_2; and OI_2 gives, in direction and magnitude, the current flowing through the generator; this scales 58 ampères. There remains yet the E.M.F. of the generator to be determined. Draw Oa at right angles to OI_2 and Oa_1 at such an angle to Oa that tan aOa_1 is equal to the ratio of resistance and inductance (in our case, 7/60). The E.M.F. required to balance self-induction is $58 \times 60 = 3480$ volts. By measuring this off on Oa, we obtain the point A, and by drawing a line through A, parallel to OI_2, we obtain the point A_1 and the E.M.F. OA_1, which is the resultant between the E.M.F. of the generator and the E.M.F. in the middle of the cable. We thus find OE_1 the E.M.F. to which the generator must be excited; in this case, 9,330 volts.

It is interesting to note that if the cable had no capacity, a similar construction to that just explained shows that the generator would have to be excited to 9,900 volts. The effect of capacity is, therefore, to require a slightly reduced voltage of the generator, and a slightly increased current.

CHAPTER IX.

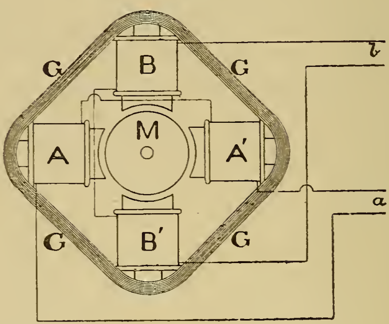
Objections to Single Phase Transmission—Advantages of Poliphase Transmission—Baily's Motor—Arago's Disk—Ferraris' Motor of 1885—Effect of Rotary Field on closed Coil Armature—Theory of Rotary Field Motors—Magnetic Slip—Torque Diagram—Starting Power—Magnetic Leakage—Extension of Theory to Practical Motors—Power Factor—Efficiency—Examples.

The transmission of power by means of single phase alternators is perfectly practicable, and is, moreover, the most simple and reliable system that can be employed for long distance work, especially if no subdivision of power is required. If, however, the power at the delivery end of the line has to be split up between a large number of small motors which must be capable of starting against a load without extraneous help, then a different system of transmission becomes preferable. There are two objecjections against the use of small alternators as motors; one is the necessity of providing some separate source of continuous E.M.F. for excitation, and the other is, their inability to start by themselves. When we have to deal with machines of a moderate or large size, these objections are of very little account. The arrangement usually adopted in such cases is as follows: The alternator is combined with an exciting dynamo, and a storage battery is provided, which, at starting, is connected with the exciter, working it as a motor. Thus the alternator is brought up to speed, and when its frequency is that of the supply current, the latter is switched on. The right moment for connecting is indicated by a synchronizer, and after the power current is switched on, the load may be gradually thrown on. The alternator then supplies power for its own exciter, and the latter may also be utilized to charge up the battery ready for the next start. All this is simple enough, and comparatively inexpensive, when we have to deal with large powers, but for small motors the complications and cost of exciter, battery, synchronizer, and mechanical gear for throwing the load on, become rather objectionable features of this mode of working ; and hence the attention of engineers, especially on the Continent and in the United States has, for the last few years been directed towards a system of power transmission by alternating currents which should be free from these objections. The details of the machinery devised by the various inventors differ considerably, but all motors have this in common, that the armature is acted upon by a magnetic field which progresses round the spindle with a more or less even angular speed. Such motors are therefore called "rotary field motors." Another feature is, that no part of the motor is excited by a continuous current supplied from an external source ; and that all currents circulating in the armature are due to electro-magnetic induction. On this account such motors are also sometimes called "induction motors." Another term sometimes used is "poliphase motors," because two, three, or more distinct alternating currents of the same frequency, but different phase, are used in working them.

Without entering in detail into any question concerning the priority of the many patents which have been taken out in connection with such motors and the various methods of working them, a short historical review may prove of interest. Like so many other remarkable inventions, that of the rotary field motor is much older than commonly supposed. As far as I have been able to ascertain, it dates back to the year 1879, when in a paper read before the Physical Society of London on the 28th June Walter Baily showed how Arago's Rotation could be produced by a number of fixed electro-magnets acting on a copper disk. The paper is published in the "Philosophical Magazine" of October, 1879, where diagrams illustrating the principle of the invention and arrangement of apparatus will also be found. The latter consisted of a copper disk suspended in the centre on a needle point so as to be able to revolve. Below the disk were placed four electro-magnets with their vertical axes equidistant from the centre, and their upper poles in close proximity to the under surface of the disk. It is remarkable that these magnets are shown with laminated cores. The exciting currents were supplied by two batteries, and a commutator was used to alternately reverse the polarity of the field-magnets. Baily evidently recognized that reversals succeeding each other quickly will give an increased effect; for he says: "The experiment with the four electro-magnets may be readily performed by means of a commutator which will reverse the current several times in a second; and a considerable rotation can be given to the disk." He also recognized that by placing other magnets above the disk, that is to say, providing a closed magnetic circuit, the effect on the disk might be much increased. His instrument contained a reversing switch in one of the field-magnet circuits whereby he was able to produce the rotation in either sense. We have here all the important features of the modern two-phase motor embodied in an apparatus invented fifteen years ago, but the invention had been overlooked, because in those days the possibility of using alternating currents for power transmission had not even occurred to engineers.

The next important step in the development of such motors was made by Professor Galileo Ferraris, who, during the summer and autumn of 1885, has constructed several two-phase motors. These machines were on view at the headquarters of the American Institute of Elec-

Fig. 100.

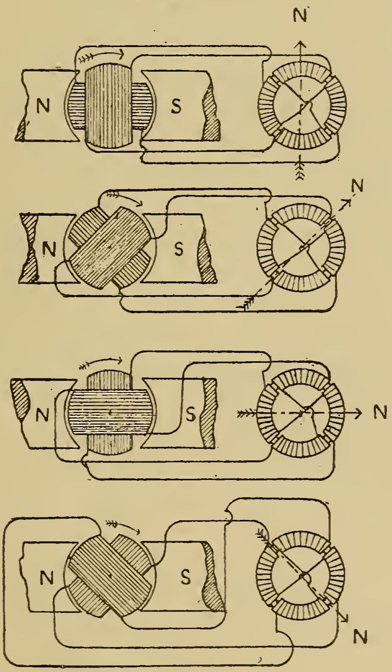


trical Engineers at the Chicago Exhibition. One of these motors, built in 1885, has the form shown in Fig. 100, where $B\ B_1$ and $A\ A_1$ are two pairs of electro-magnets joined by a yoke G of iron wire. Within the polar cavity there is placed an armature M, consisting of a copper cylinder. Armatures consisting of solid iron and others of an iron core with a copper coating were also tried. The description of this motor was only published in March, 1888, but before then two other patents for motors of this kind were taken out; one by the Helios Company of Cologne in May, 1887, and the other by

Borel and Paccaud in February, 1888. In May, 1888, we have Tesla's very complete patent specification for power transmission by two-phase alternating current, and after that come a host of others.

The production of a rotating field in the motor is

Fig. 101.



explained by Tesla by means of diagrams some of which are reproduced in Fig. 101. *N S* are the field-magnets of the generator which is provided with an armature having two coils as shown. The field system of the motor consists of a ring of laminated iron surrounded by four coils which are connected in pairs with the four line wires. The armature of the motor is not shown. When the armature of the generator occupies the position shown in the top figure, only the horizontal coil is active and sends current through the right and left coil of the motor field, producing lines of force in the direction indicated by the arrow *N.* When the armature of the generator has reached the position shown in the second figure, both coils are active and all four coils on the motor magnets are energized, producing by their combined action a field in the direction of the inclined arrow. In the next position of the generator armature, only the top and bottom coil of the motor-magnet are energized, producing a field in a horizontal direction and so on, the general effect being that the field in the motor rotates with the same speed as the armature of the generator. It is obvious that a similar action is produced in the Ferraris motor, Fig. 100. Calling a and b the two currents energising the magnets AA_1 and BB_1 respectively, then with a phase difference of 90° a will be a maximum when b is zero, and *vice versâ.* Thus, at the moment when a is a maximum, the field passing through the armature will be horizontal ; an eighth's period later when the a and b currents are equal (though a is decreasing and b increasing), the resultant field will pass through the armature at an angle of 45° to the former direction. After a further eighth's period only the coils BB_1 are energised, and the field will have a vertical direction, and so on ; the resultant field revolving round the centre with a speed corresponding to the frequency of the supply currents.

It will thus be seen how a rotating field can be produced by the employment of two alternating currents of equal frequency, but having a phase difference of 90°.

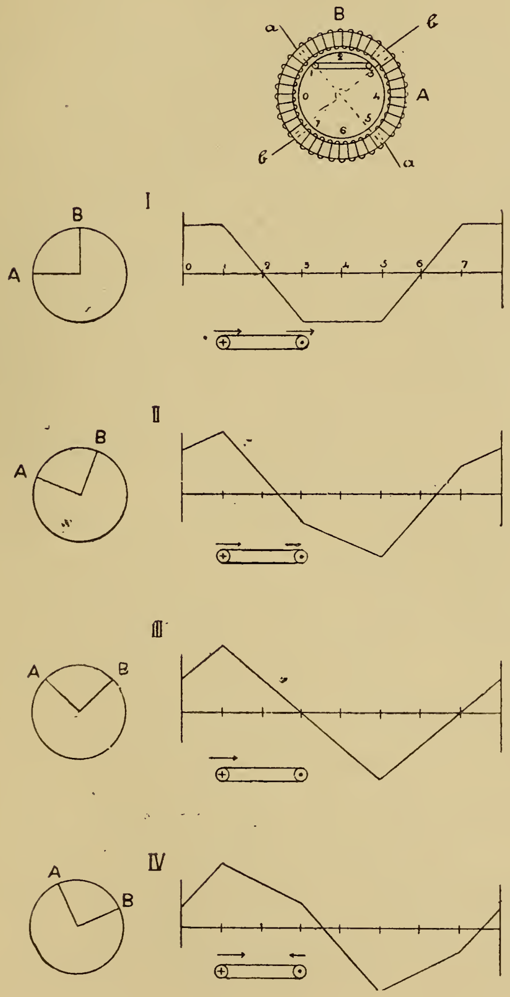
If we build a Ferraris motor with three instead of two pairs of magnets, and energise them by three alternating currents with 120° phase difference between any two, we shall also obtain a rotary field, and so on for any larger number of phases. There is, however, no advantage in employing more than three phases, whilst the multiplication of circuits constitutes an undesirable complication; so that for practical work the choice is limited to either two or three phases. The system of power transmission by polyphase currents consists, then, of a generator or set of generators, producing the currents at the sending end of the line, and a rotary field motor at the receiving end of the line, the line itself consisting of at least three wires, as will be explained later on. For the present we shall confine ourselves to the motor only, and endeavour to establish a working theory in the same way as has already been done in previous pages of this book for continuous current and single phase alternating current machines.

The well known Arago disk may serve as a starting point for our theory. In this instrument we have a copper disk mounted on a vertical axis passing through its centre, and so arranged that it can be set into rapid rotation. Over the disk, and in close proximity to it, is placed a compass-needle, which points north-south when the disk is at rest. If, however, the disk be rotated in either direction, the compass-needle is deflected correspondingly, and if the rotation is made rapid enough the needle itself is finally caused to revolve in the same sense. The explanation of this experiment is as follows: The compass-needle, being a magnet, produces throughout the space surrounding it a magnetic field, part of which passes through the copper disk. The latter when rotating cuts the lines of force and currents are thereby produced in the copper. It is not necessary to inquire into the exact paths of these currents, which, indeed, are extremely complicated; but it will be obvious that in close proximity to the poles these currents must be more or less radial, that is, at right angles to the lines of the field and the direction of movement. These currents flow, therefore, in a direction more or less parallel to the needle, and the latter is acted upon in the same manner as in the well-known Oersted experiment, where a conductor carrying a current is placed north-south either over or under the compass-needle. The result is that the needle is deflected, and if the deflection exceeds 90° it is set in rotation. Obviously the experiment might also be reversed. We might rotate the magnet, and then the disk would be acted upon by a certain mechanical twisting couple tending to set it in rotation. This is the most simple form of rotary field motor. The magnet produces the field, which in this case must be rotated by power, and the disk represents the armature of the motor. Such an arrangement, although useful as a starting-point for a theory of rotary field motors, because it facilitates the understanding of their action, would, of course, be quite useless for practical purposes. Since we must have mechanical power on the spot to revolve the magnet, we can use that power direct, and need not pass it through the motor at all. The motor only becomes of practical value when no mechanical power is available on the spot, and we must therefore so alter the construction of the machine, that the revolving physical magnet is replaced by some equivalent device which receives the power electrically. We must, in fact, produce a rotating field, without actually rotating a physical magnet, and this object is attained by the inventions of Ferraris, Tesla, and others, as already explained.

When we rotate a physical magnet, the strength and configuration of the field acting upon the armature conductors are of course the same for all positions during one revolution, and to get an exact equivalent by means of a field electrically produced, the latter would have to satisfy the following conditions: (1) Strength and configuration must be independent of the direction in space; (2) the speed of rotation must be constant. Now it is easy to see that these conditions are not fulfilled in the Ferraris motor, Fig. 100, and, indeed, cannot be fulfilled in any motor that can be practically constructed. If we build the motor with well defined field-poles, we must have a more or less jerky movement of the resultant field, and its strength and configuration must necessarily be subject to fairly large variations. Even if we avoid the use of pole pieces, the number of exciting coils must be finite, and there must be variations in the strength of the field. It will be shown later on that these variations are automatically corrected or reduced by armature reaction, and are, therefore, not nearly so harmful as might at first sight be supposed; but for our present purpose it is not necessary to enter into these details. What we have to determine now is whether, with a field such as can be produced practically, a twisting couple will be exerted on the armature. It is obvious that the larger the number of phases, the less jerky will become the rotary movement of the field. We shall, therefore, investigate the worst possible case in this respect, namely, a motor-field energised by two currents only differing in phase by a quarter period. Apart from armature reaction, the ratio between minimum and maximum field strength would in this case be as $1 : \sqrt{2} = 1 : 1{\cdot}41$, whilst in a three-phase motor the ratio would be $\frac{\sqrt{3}}{2} : 1 = 1 : 1{\cdot}16$. The shape of field-magnet usually adopted in modern machines differs from the original Ferraris arrangement in this respect, that no sharp distinction exists between magnet cores, poles, and yoke, all these parts being more or less merged in one cylindrical ring of subdivided iron. As a typical form, we may take a Gramme ring, as shown on the top of Fig. 102, the winding consisting of two pairs of coils, *A* and *B*, covering the four quarters of the ring. The supply leads for one current are connected to the wires *a a*, and those for the other current to the wires *b b*. It will be easily understood that the field may also be wound drum fashion ; but as such a winding cannot be so easily followed in a diagram, the Gramme winding has been adopted in Fig. 102. Within the field is mounted the armature, which consists of a cylindrical core of iron plates, provided with external conductors. Two of these conductors, forming a single coil, closed on itself as shown, the width of the coil being such as to embrace one quarter of the circumference of the armature, which disposition must obviously give maximum induction through the coil.

The distribution of the field at various times is shown by the diagrams I. to IX. in this figure and in Fig. 103. In order to simplify the representation, the circumference of the interpolar space is straightened out, and the induction per centimetre of circumference is represented by the ordinates of the broken line. The coil is shown below each diagram in the same position ; and the direction of the current induced in it by the advanc-

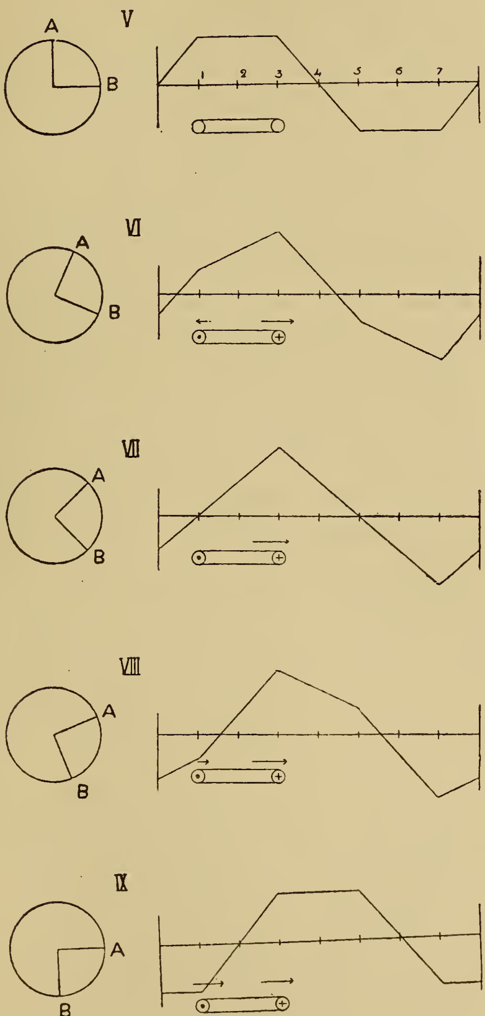
Fig. 102.



ing field is indicated by dots and crosses in the usual way. The circle on the left of each figure represents the phase of both currents. Thus in diagram I. the current in A is zero, and that in B a positive maximum. The field produced by B passes through zero, and changes its sign at the circumferential points, 2 and 6. The coil is traversed by that part of the field which lies between the points 1 and 3. Since within this space half the field is positive and half negative, the total field passing through the coil at this moment is zero. By reference to diagram II. it will be seen that a moment after the total field passing through the coil has a positive value, and since by Lenz's law the coil must oppose this change, we see that in diagram I. the current must circulate as indicated by the dot and cross. The effect of this current, combined with the field in 1 and 3, is to produce in each conductor a mechanical force acting towards the right, as indicated by the two small arrows.

In the position II., which occurs later than I. by one-sixteenth of a complete cycle, the force is still to the right in both wires, though greater in that under point 1, because the strength of the field in that point is greater than in 3. In the next position the current flows as before, but only the wire in point 1 produces a force to the right, the field strength in 3 being now zero. When position IV. is reached, we have still the same direction of current, but the force acting on the wire in point 3 has now been reversed, and opposes the force on the wire in point 1. In V. there is no current, and consequently no mechanical force ; in VI. the forces are opposed, but their difference is towards the right and so on. It will be clear from an examination of these diagrams, that on the whole the coil is acted upon by a mechanical force

Fig. 103.



directed towards the right, and tending to produce clockwise rotation, that is to say, the coil will try to follow the rotation of the field. What has been shown here to take place with one armature coil, must obviously take place with all the armature coils, their collective effect being to impart to the armature a powerful twisting couple in a clockwise direction.

The above explanation is sufficient to show in a general way how the torque in the armature of a rotary field-motor is produced ; but it is not sufficient for an exact determination of the torque, because it does not take account of a very important effect, namely, armature reaction. The lines indicating the configuration of the field in Figs. 102 and 103, are obviously only correct if the only currents producing the field are those passing through the coils *A* and *B*. We have, however, seen that the coils on the armature also carry currents, and these must necessarily affect the distribution and total strength of the field. Although this is, strictly speaking, only a secondary effect, it is by no means negligible, and any theory of rotary field-motors which does not take account of armature reaction must lead to results at variance with practical experience. Our next step must therefore be to investigate the effect of the currents produced in the armature conductors.

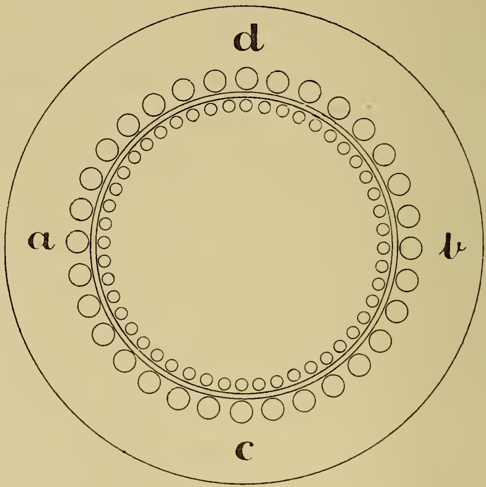
To attempt such an investigation for the most general case, that is, for a field of irregular configuration and irregular speed, would lead to mathematical expressions of such complexity as to be quite useless for practical purposes. We must, therefore, be content with the approximate, but at the same time much more simple solution of the problem, which results from the assumption that the broken line in Figs. 102 and 103, representing the configuration of the field produced by the supply currents, may, without serious error, be replaced by a true sine curve, advancing with a constant speed. In making this assumption, we do not violate physical probabilities to any great extent. In the first place the sharp corners of our broken line are impossible when there is an air space between the iron surfaces of field and armature. Our curve then, to begin with, must have the corners rounded. Next, any great variation in the highest point of the curve is impossible for the following reason. The whole surface of the armature is covered by coils closed on themselves. Any variation of field strength (apart from the variation due to the even progression of the field), must therefore immediately produce in the armature conductors currents which oppose the change, and thus the general effect of these currents must be to equalize the maxima, and still further to round off the corners of the broken line, so that to assume it to be a sine curve is, after all, not such a very great stretch of imagination as may at first sight appear. That the effect of the armature currents is to obliterate more or less the changes in field strength is proved practically by the fact that two-phase motors work as well as three-phase motors, yet in the former the maximum exceeds the minimum by apparently 41, and in the latter by only 16 per cent.

In order to be able to deal by means of simple mathematics with the working condition of a rotary field motor, we assume that the induction within the interpolar space between field and armature varies according to a simple sine law. Whether this induction is due to the current in the field coils alone, or to the combined effect of field and armature currents, we need at present not stop to inquire; all we care to know is that such an induction does actually exist when the motor is at work, and that the sinusoidal field which it represents revolves with a speed corresponding to the frequency of the supply currents. Thus, if there be 4 field coils, as in Fig. 102, and the frequency is 50, we would have a two-pole field revolving 50 times a second, or 3,000 times a minute, round the centre of the armature, and if there were no resistance to the movement of the latter it would be dragged round by the field at a speed of 3,000 revolutions per minute. It is obvious that the actual speed must be smaller. If the speed of the armature coincided exactly with that of the field, then the total induction passing through any armature coil, or between any pair of conductors on the armature would remain absolutely constant, and there would be no E.M.F., and, therefore, no current induced in the armature wires. Where there is no current there can be no mechanical force, and the armature could, therefore, not be kept in rotation. In order that there may be a mechanical force exerted, it is obviously essential that there shall be a variation in the magnetic flux passing through any armature coil, and that necessitates a difference in the speed of rotation between field and armature. This difference is called the "magnetic slip" of the armature. If, for instance, the speed of the field in our two-pole motor, Fig. 102, is 50 revolutions per second, and the speed of the armature 48 revolutions per second, we would have a magnetic slip of 2 revolutions out of 50, or 4 per cent. In modern machines the slip at full load averages about 4 per cent., and rarely reaches as high as 10 per cent., so that good rotary field motors are in point of constancy of speed under varying load about equal to continuous shunt motors.

It was mentioned above that the motor shown in Fig. 102 would have a frequency of 50 revolutions at a speed only by 4 per cent. short of 3,000 revolutions per minute. This is an inconveniently high speed for any but very small sizes. To reduce the speed is, however, quite easy. We need only increase the number, and proportionately reduce the length of the field coils. Thus, if instead of 4 coils, each spanning 90° of the circumference, we use 8 coils, each spanning 45°, and connect them so as to produce two rotary fields, the speed will be reduced to one half of its former value. By using 12 coils we obtain a six-pole motor, in which the speed will be reduced to one-third, or about 1,000 revolutions per minute ; with 16 coils we get down to 750 revolutions, and so on. In order to avoid unnecessary complexity we shall, however, commence the investigation on a two-pole machine, having only one revolving field, and leaving the transition to a multipular machine running at lower speed until the more simple case has been dealt with.

Such a machine is shown in Fig. 104. The field consists of a stationary cylinder, composed of insulated iron plates, and provided close to the inner circumference with holes through which the winding passes. The armature is also a cylinder made up of insulated iron plates provided with holes near its outer circumference for the reception of the conductors. The use of buried conductors, although not absolutely necessary, has two important advantages—first, mechanical strength and protection to the winding ; and, secondly, reduction of the magnetic resistance of the air gap, which, it will be seen later on, is an essential condition for a machine in which the difference between the true watts and apparent watts shall not be too great. The armature conductors may be connected so as to form single loops, each passing across a diameter, or they may all be connected in parallel at each end face by means of circular conductors, somewhat in the fashion of a squirrel cage. Either system of winding does equally well, but as the latter is mechanically more simple, we will assume it to be adopted in Fig. 104. The circular end connections are supposed to be of very large area as compared with

Fig. 104.



the bars, so that their resistance may be neglected. The potential of either connecting ring will then remain permanently at zero, and the current passing through each bar from end to end will be that due to the E.M.F. acting in the bar divided by its resistance. It is important to note that the E.M.F. here meant is not only that due to the bar cutting through the lines of the revolving field, but that which results when armature reaction and self-induction are duly taken into account.

Let us now suppose that the motor is at work. The primary field produced by the supply currents makes $\sim_1$ complete revolutions per second, whilst the armature follows with a speed of $\sim_2$ complete revolutions per second. The magnetic slips is then

$$s = \frac{\sim_1 - \sim_1}{\sim_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 62)$$

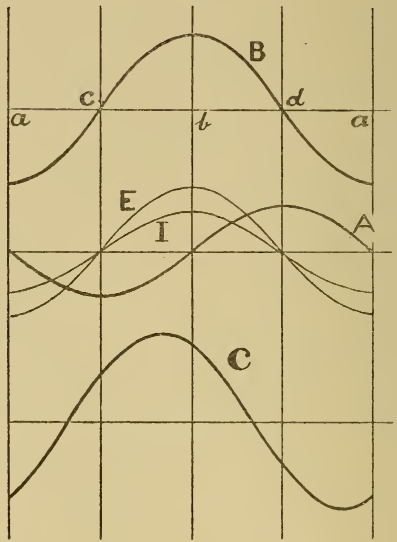
If the field revolves clockwise, the armature must also revolve clockwise, but at a slightly slower rate. Relatively to the field, then, the armature will appear to revolve in a counter clockwise direction, with a speed of

$$\sim = \sim_1 - \sim_2$$

revolutions per second. As far as the electro-magnetic action within the armature is concerned, we may therefore assume that the primary field is stationary in space, and that the armature is revolved by a belt in a backward direction at the rate of $\sim$ revolutions per second. The effective tangential pull transmitted by the belt to the armature will then be exactly equal to the tangential force which in reality is transmitted by the armature to the belt at its proper working speed, and we may thus calculate the torque exerted by the motor as if the latter were worked as a generator backward at a much slower speed, the whole of the power supplied being used up in heating the armature bars. The object of approaching the problem from this point of view is of course to simplify as much as possible the whole investigation. If we once know what torque is required to work the machine slowly backward as a generator, it will be an easy matter to find what power it gives out when working forward as a motor at its proper speed.

Let in Fig. 105 the horizontal *a*, *c*, *b*, *d*, *a* represent the interpolar space straightened out, and the ordinates of the sinusoidal line, B, the induction in this space, through which the armature bars pass with a speed of ~ revolutions per second. We make at present no assumption as to how this induction is produced, except that it is the resultant of all the currents circulating in

Fig. 105.



the machine. We assume, however, for the present that no magnetic flux takes place within the narrow space between armature and field wires, or, in other words, that there is no magnetic leakage, and that all the lines of force of the stationary field are radial. The rotation being counter clockwise, each bar travels in the direction from *a* to *c* to *b*, and so on. The lines of the field are directed radially outwards in the space $d\ a\ c$, and radially inward in the space $c\ b\ d$. The E.M.F. will, therefore, be directed downwards in all the bars on the left, and upwards in all the bars on the right of the vertical diameter in Fig. 104. Let E represent the curve of E.M.F. in Fig. 105, then, since there is no magnetic leakage the current curve will coincide in phase with the E.M.F. curve, and we may represent it by the line I. It is important to note that this curve really represents two things. In the first place it represents the instantaneous value of the current in any one bar during its advance from left to right; and in the second place it represents the permanent effect of the current in all the bars, provided, however, the bars are numerous enough to permit the representation by a curve instead of a line composed of small vertical and horizontal steps. The question we have now to investigate is: what is the magnetising effect of the currents which are collectively represented by the curve I? In other words, if there were no other currents flowing but those represented by the curve I, what would be the disposition of the magnetic field produced by them? Positive ordinates of I represent currents flowing upwards or towards the observer in Fig. 104, negative ordinates represent downward currents. The former tend to produce a magnetic whirl in a counter clockwise direction, and the latter in a clockwise direction. Thus the current in the bar which happens at the moment to occupy the position b, will tend to produce a field, the lines of which flow radially inwards on the right of b, and radially outwards on the left of b. Similarly the current in the bar occupying the position a, tends to produce an inward field, *i.e.*, a field the ordinates of which are positive, in Fig. 105, to the left of a, and an outward field to the right of *a*. It is easy to show that the collective action of all the currents represented by the curve *I* will be to produce a field as shown by the sinusoidal line *A*. This curve must obviously pass through the point *b*, because the magnetizing effects on both sides of this point are equal and opposite. For the same reason the curve must pass through *a*. That the curve must be sinusoidal is easily proved, as follows: Let *i* be the current per centimetre of circumference in *b*, and let *r* be the radius of the armature; then the current through a conductor distant from *b* by the angle α, will be $i \cos \alpha$ per centimetre of circumference. If we take an infinitesimal part of the conductor comprised within the angle $d\,\alpha$, the current will therefore be

$$di = i\, r \cos \alpha\, d\, \alpha,$$

and the magnetizing effect in ampère-turns of all the currents comprised between the conductor at *b*, and the conductor at the point given by the angle α will be

$$\int_0^\alpha di = -\, i\, r \sin \alpha,$$

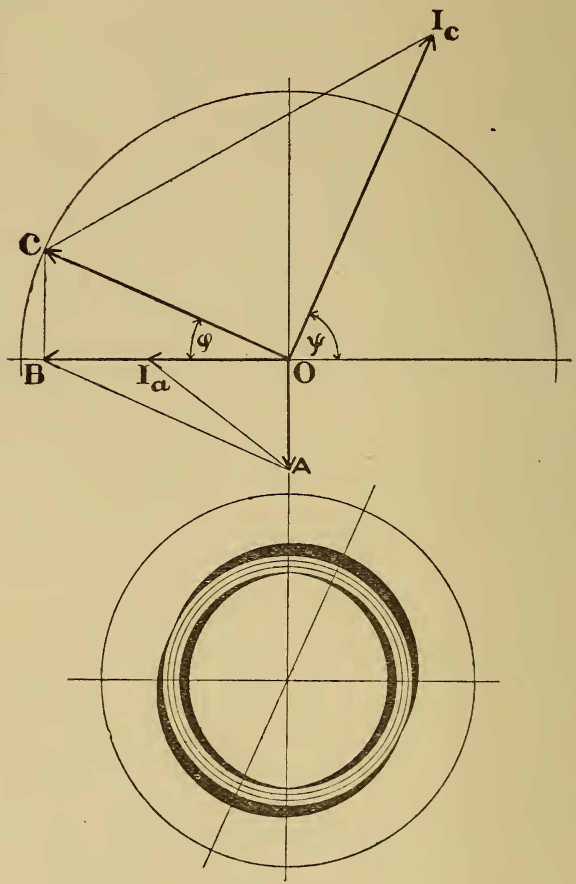
and since the conductors on the other side of *b* act in the same sense, the field in the point under consideration will be produced by $2\, i\, r \sin \alpha$ ampère-turns, *i* being the current per centimetre of circumference at *b*.

Since for low inductions, which alone need here be considered, the permeability of the iron may be taken as constant, it follows that the field strength is proportional to ampère-turns, and that consequently *A* must be a true sine curve.

When starting this investigation, we have assumed that the field represented by the curve *B* is the only field which has a physical existence in the motor ; but now we find that the armature currents induced by *B* would, if acting alone, produce a second field, represented by the curve *A*. Such a field, if it had a physical existence, would, however, be a contradiction of the premiss with which we started, and we see thus that there must be another influence at work which prevents the formation of the field *A*. This influence is exerted by the currents passing through the coils of the field magnets. The primary field must therefore be of such shape and strength, that it may be considered as composed of two components, one exactly equal and opposite to *A*, and the other equal to *B*. In other words, *B* must be the resultant of the primary field and the armature field *A*. The curve *C* in Fig. 105 gives the induction in this primary field, or as it is also called, the "impressed field," being that field which is impressed on the machine by the supply currents circulating through the field coils. It will be noticed that the resultant field lags behind the impressed field by an angle which is less than a quarter period.

The working condition of the motor, which has here been investigated by means of curves, can also be shown by a clock diagram. Let in Fig. 106, the maximum field strength within the interpolar space (*i.e.*, number of lines per square centimetre at *a* and *b* of Fig. 104), be represented by the line $O\,B$, and let $O\,I_a$ represent the total ampère-turns due to armature currents in the bars to the left or the right of the vertical, then $O\,A$ represents to the same scale as $O\,B$ the maximum induction due to these ampère-turns. We need not stop here to inquire into the exact relation between $O\,I_a$ and $O\,A$, this will be explained later on. For the present it is only necessary to note that under our assumption that there is no magnetic leakage in the machine, *O A* must

Fig. 106.



stand at right angles to $O\ I_a$, and therefore also to *O B*, and that the ratio between $O\ I_a$ and *O A* (*i.e.* armature ampère-turns and armature field) is a constant. By drawing a vertical from the end of B and making it equal to $O\,A$, we find $O\,C$ the maximum induction of the impressed field. The total ampère-turns required on the field magnet to produce this impressed field are found by drawing a line from C under the same angle to $C\,O$, as $A\,I_{\rho}$ forms with $A\,O$, and prolonging this line to its intersection with a line drawn through O at right angles to $O\,C$. Thus we obtain $O\,I_c$, the total ampère-turns to be applied to the field. The little diagram below shows a section through the machine, but instead of representing the conductors by little circles as before, the armature and field currents are shown by the tapering lines, the thickness of the lines being supposed to indicate the density of current per centimetre of circumference at each place.

At this stage it will be convenient to indicate in general terms the programme of the investigation. A great deal has already been published in books and periodicals concerning the theory of rotary field motors, and it would have been an easy matter to simply give an abstract of one or the other of these theories, leaving to the reader the task of fitting it to practical requirements.[1] Such a task may be within the power of a professor of mathematics, but as this book is intended for those who are or wish to become practical engineers, an abstract of existing theories, which bristle with high mathematics, would be of little use to them. I have

[1] One of the most admirable investigations is that recently published by Prof. Ferraris, "Un Metodo per la Trattazione dei Vettori Rotanti," Carlo Clausen, Turin. This I have only seen after my own theory was written, but even had it been otherwise I could not have used it, for Ferraris does not deal with the all important questions of current and E. M. F. in the supply circuits.

therefore sacrificed mathematical brevity and elegance in favour of a more roundabout treatment, which, although it occupies more space, is also more easily understood by practical men, because it retains in all its stages the connection between physical quantities and the formulæ intended to represent them. The problem we have finally to solve concerns the working condition of a motor supplied with two or three alternating currents of given voltage. Required to know are the strength of supply currents, their lag, the speed, power, and efficiency of motor. To attempt the solution of this problem at one operation would be too difficult, and we shall therefore approach the solution gradually under the following programme. We assume first that we have to deal with a motor in which there is no other loss but that arising from the ohmic resistance of the armature bars, and in which there is no magnetic leakage. This motor we can work under two different conditions. We may keep the strength of the supply currents constant, which means that the impressed field of the motor is a constant quantity; or we may keep the E.M.F. of the supply currents constant, which means that the resultant field is a constant quantity. We shall then deal with a motor such as can practically be built. In such a motor there must be magnetic leakage and there must be various losses, the effect of which will be that if the motor is working on a constant pressure circuit, the resultant field will not be constant, but must decrease as the load increases, thus bringing the performance of this practically possible motor somewhere in between the performances of the two perfect motors first mentioned.

I. Perfect Motor; Supply Currents Constant.

Going back to our original conception of the armature worked backwards by a belt, whilst the impressed field is kept constant and stationary in space, it will be obvious that the tangential force which must be supplied by the belt is proportional to the product of $O\,B$ and $O\,I_a$, and since the latter is proportional to $O\,A$, the force is also proportional to $O\,A \times O\,B$, or, which comes to the same thing, the torque is proportional to the area of the triangle $O\,C\,B$. The E.M.F. generated in each armature bar, the current in it, and the collective current or armature ampère-turns, $O\,I_a$, are all proportional to the speed of rotation and to the strength of the resultant field. We have therefore

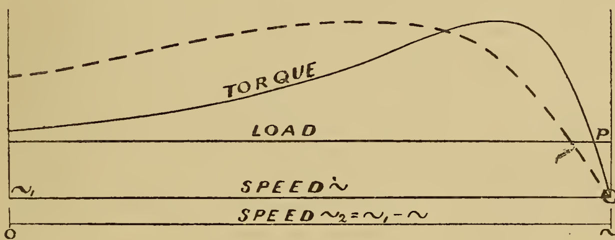
$$O\,I_a = K \sim O\,B,$$

where K is a constant depending on the construction of the machine. Let us now see what will happen if the field excitation $O\,I_c$, remaining the same, we increase or diminish the belt speed, so as to vary $\sim$ between wide limits. If $\sim$ is small, it means that the machine, when working as a motor, will run at a speed only very little below synchronism; if $\sim$ is large, but smaller than $\sim_1$, it means that the machine when working as a motor, will run slowly, and if $\sim = \sim_1$, it means that the machine is on the point of starting as a motor from rest. It is especially this latter case which is of great importance in the design of such machines, for their ability to start against a load is their chief advantage, as compared with ordinary synchronous alternators.

The field excitation being kept constant, it follows that the impressed field will remain of the same magnitude, though relatively to OB it will shift its direction as the speed $\sim$ is raised. In other words, to each position of C on the circle representing the impressed field corresponds a particular speed $\sim$ and a particular torque. Speed and torque are the two quantities of which we require to know the relation.

From the above equation it will be seen that the speed is proportional to the ratio between OI_a and OB. Now OI_a is proportional to OA, and the latter quantity is equal to BC, so that the tangent of angle φ is simply a measure for the speed. It should not be forgotten that the term "speed" refers to the machine driven by belt as a generator on short circuit. A low speed in this sense means a small slip, and consequently a high speed, when working as a motor, and *vice versâ*. The relation between speed and torque can now easily be seen from the diagram. The speed is proportional to tan φ, and the torque to the area of the triangle OCB. For zero speed (motor running synchronously) the area of the triangle is zero, and the motor gives no torque. For a very small speed (motor running with very little slip) the triangle is narrow and the torque small. If we now increase the speed (motor running with more slip) the torque increases to a maximum which is reached when $\varphi = 45°$. A further increase of speed again reduces the area of the triangle, and therefore the torque, and the reduction is the greater the greater the speed. It will be clear from this that for stable working the slip of the motor must be such as corresponds to an angle φ between zero and $45°$, for were the slip greater, then a very slight increase of load would bring the motor into a working condition in which the torque is less than before, and the motor would consequently stop. On the other hand, if the slip is small so that φ is considerably less than 45°, then an increase of load will bring the motor into a working condition where the torque is greater, and there will be no danger of the motor pulling up. It is obvious that if the motor is at all able to carry the load it will automatically adopt the working condition in which $\varphi < 45°$. At starting the motor is necessarily in an unstable working condition because $\varphi > 45°$. The result is that the motor very quickly runs up to a speed not far short of synchronism, whilst the torque first rises to the maximum correspond-

Fig. 107.



ing with $\varphi = 45°$, and then drops to that value which corresponds with the load.

The curve of torque as a function of the speed is roughly represented in Fig. 107. The exact shape depends of course on the constructive details of the machine, but the diagram is sufficient to show the general character. Two speed lines are shown; on the upper we count the speed of the machine when driven backwards by belt as a generator from right to left, and on the lower we count the speed of the machine when running as a motor from left to right. In order that the machine may start ($\sim_2 = 0$, lower line), the load line must lie wholly below the curve, and the working point will then be at P. The resistance of the armature bars has a very important bearing on the shape of the curve of torque. To make this point clear let us assume that we could by some means suddenly double the resistance of the armature bars of the machine to which Fig. 106 refers. With the same resultant field and speed we would then only obtain half the armature ampère-turns there shown, and consequently only half the armature field. To get back to the old conditions we would therefore have to double the speed; that is to say, the tangent of the angle φ will now represent twice as much speed as formerly, and this will alter the torque diagram by making the abscissæ of the curve twice as long as before without altering its ordinates. The curve so altered is shown by a dotted line in Fig. 107, and it will be immediately seen that by increasing the resistance of the armature we have also increased the torque at starting. At the same time the working point has been shifted further to the left, causing the motor to run with rather more slip than before, which means of course a greater variation of speed between full load and no load. The maximum torque is the same in either case, so that for the same load the margin of power is also the same; but the motor with high resistance armature is able to start with a heavier load. On the other hand, it should be remembered that high armature resistance is objectionable on account of loss of power and possible overheating. To avoid these defects and yet retain the advantage of great torque at starting the armatures of large motors are sometimes wound with several distinct circuits (instead of the squirrel cage winding) which are not closed on themselves, but connected to contact rings. The circuits are completed by means of brushes and external resistances, the latter being used for starting only. When the motor has run up to speed these resistances are short circuited. By winding the armature with three circuits 120° apart, the resulting armature field (curve A, Fig. 105) is almost perfectly sinusoidal, and only three contact rings are required.

We have so far only dealt generally with the problem; and to make the investigation of practical use we must now establish the exact numerical relation between ampère-turns and induction, and between induction and torque. We take the latter relation first. Let B represent, as before, the maximum value of the resultant induction in the interpolar space, and let the armature be wound as a squirrel cage with τ conductors, each having a resistance of ρ ohms. The resistance of the end rings we neglect as already mentioned. The strength of the field at any point of the circumference is $B \sin \alpha$, the angle being counted from a radius at right angles to the radius along which the induction has the maximum value B. If r be the radius of the armature, and v its circumferential speed, we have

$$v = 2 \pi r \sim$$

centimetres per second, and the E.M.F. generated at a point distant by the angle α from the point of no E.M.F. is

$$e = B \sin \alpha \, v \, l \, 10^{-8}$$

volts, if l represents the length of the armature in centimetres. Since there are τ bars in a length of $2 \pi r$ centimetres, there are $\frac{\tau}{2 \pi r}$ bars per centimetre and $\frac{r \, d \alpha \, \tau}{2 \pi r}$ bars within an element of the circumference of length $r \, d \alpha$ over which the induction may be regarded as constant. The resistance of the group at bars within the element of the circumference is $\rho / \frac{r\, d\alpha\, \tau}{2\pi r}$ or $\frac{\rho\, 2\pi}{\tau\, d\alpha}$, and the collective current flowing through the group of bars is

$$di = \frac{B \sin\alpha\, v\, l\, 10^{-8}}{\rho\, 2\pi}\, \tau\, d\alpha$$

ampères, or one-tenth of this in absolute measure. Inserting the value for v, we have the elementary current in absolute measure.

$$di = \frac{B \sin\alpha\, r \sim l\, 10^{-9}\, \tau\, d\alpha}{\rho}.$$

The strength of field being $B \sin\alpha$, the tangential force due to the elementary group of armature conductors of length l is in dynes

$$d\,P = \frac{B^2 \sin^2\alpha\, l^2 r \sim 10^{-9}\, \tau\, d\alpha}{\rho}.$$

If we integrate the expression between the limits $\alpha = 0$ and $\alpha = \pi$, we get the tangential force in dynes exerted by one half the armature. The integral of $\sin^2\alpha\, d\alpha$ between these limits is $\frac{\pi}{2}$, and since both halves of the armature produce forces acting in the same sense, we find the total tangential force exerted by the armature to be

$$P = \frac{B^2\, l^2\, r \sim 10^{-9}\, \tau\, \pi}{\rho}$$

dynes. This expression may also be written as follows:

$$P = B\, \frac{2\pi \sim r\, l\, B\, 10^{-8}}{10\,\rho}\, l\, \frac{\tau}{2}$$

Now $\frac{2\pi \sim r\, l\, B\, 10^{-8}}{\rho}$ is the maximum current in ampères passing through that bar which at the time is in either of the two strongest parts of the resultant field. Call this current I, then the above equation becomes

$$P = \frac{B\,I\,l}{10}\,\frac{\tau}{2}\text{ dynes, or } \frac{B\,I\,l}{9{,}810{,}000}\,\frac{\tau}{2}\text{ Kilogrammes} \quad . \;.\;. \quad 63).$$

It is interesting to compare with this expression that which we obtained for a continuous current motor. Using the same notation, but writing c for the continuous current in ampères, we have the tangential force $\frac{B\,c\,l}{10}\,\frac{2\,\tau}{\pi}$. To make the comparison between the two machines fair we must assume the same B in both, as well as the same number and resistance of conductors. We must also adjust the currents so as to get the same heating effects, and this will obviously be the case if the effective alternating current equals the continuous current or $I/\sqrt{2} = c$. We find thus the following relation between the two machines:

Tangential Force of Rotary Field Motor.	Tangential Force of Continuous Current Motor.
$\frac{B\,c\,l\,\tau}{10}\sqrt{\frac{1}{2}}$	$\frac{B\,c\,l\,\tau}{10}\,\frac{2}{\pi}$

If, therefore, the rotary field motor gives a tangential pull of $70\frac{1}{2}$ lbs., the equivalent continuous current motor will only give a pull of $63\frac{1}{2}$ lbs. For equal weight of armature copper the rotary field motor pulls about 11 per cent. more than the continuous current motor. In addition to this advantage there is the greater simplicity and more substantial mechanical construction.

The armature current tends to produce a field A at right angles to the impressed field B. To find the ampère-turns tending to produce the field A, we must integrate the current flowing through half the armature bars. In the point where B has its maximum value the current is I, and the current density per centimetre of circumference is $\frac{\tau I}{2 \pi r}$. At a point distant by the angle α from the point where there is no current the current density is $\frac{\tau I}{2 \pi r} \sin \alpha$, and the current in an elementary group of wires is $\frac{\tau I}{2 \pi} \sin \alpha \, d \alpha$, which integrated over half the armature ($\alpha = o$ to $\alpha = \pi$) gives the ampère-turns producing or rather tending to produce the field A,

$$X_a = \frac{I \tau}{\pi} \quad \ldots\ldots\ldots\ldots \quad 64).$$

If δ is the clearance in centimetres between field iron and armature iron, the field which this existing power if acting alone would produce is

$$A = \frac{X_a}{1 \cdot 6 \, \delta} \quad \ldots\ldots\ldots\ldots \quad 65)$$

lines of force per square centimetre.

The total power developed by the armature of the rotary field motor is found by multiplying P with its circumferential speed $2 \pi r \sim_2$. This gives

$$2 \pi r \sim_2 \frac{B I l}{10} \frac{\tau}{2}$$

dyne centimetres per second, or

$$w = 2 \pi \sim_2 r B I l \frac{\tau}{2} 10^{-8} \text{ watts.}$$

Since I is proportional to the resultant field, it is evident that the power of the motor, other things being equal, increases as the square of the maximum induction within the interpolar space. The above expression for W may be written more simply if for $2\, r\, l\, B$ we substitute the symbol F, signifying the total magnetic flux passing between armature and field. The quantity F is equivalent to what was called the total useful field passing through the armature in continuous current machines. We thus obtain

$$W = \pi \sim_2 F\, 10^{-8}\, \tau\, \frac{I}{2}.$$

The term $\pi \sim_2 F\, 10^{-8}$ is the maximum value of the E.M.F. generated in one bar if the bar passes at a speed of $\sim_2$ revolutions per second through a stationary field F. Call this E.M.F. E, then we have

$$W = \frac{E\, I}{2}\, \tau.$$

Let e be the effective value of E.M.F., and i the effective value of the current in one bar, we find the following simple expression for the power given out by the two pole armature with squirrel-cage winding :

$$W = e\, i\, \tau \quad \ldots\ldots\ldots\ldots \quad 66).$$

In this expression the symbols have the following values :

τ = number of armature bars counted all round the circumference.

i = effective current through one bar produced by the bar cutting through the field F at the rate of $\sim = \sim_1 - \sim_2$ complete revolutions per second, and given by the equation

$$i = \frac{1}{\sqrt{2}}\, \frac{1}{\rho}\, \pi \sim F\, 10^{-8} \quad \ldots\ldots\ldots \quad 67).$$

where ρ is the resistance of one bar in ohms.

e = effective E.M.F. in one bar produced by the bar cutting through the field F at the rate of $\sim_2$ complete revolutions per second, and given by the equation

$$e = \frac{1}{\sqrt{2}} \pi \sim_2 F \, 10^{-8} \quad . \; . \; . \; . \; . \; . \; . \; . \quad 68).$$

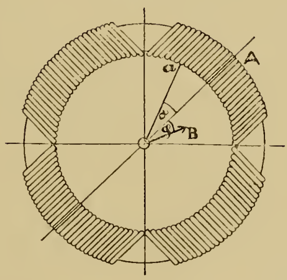
The power wasted in heating the armature bars is $\tau \, i^2 \rho$, or $\tau \, i \, i \, \rho = \frac{W}{e} i \, \rho$. Since $\frac{i \, \rho}{e} = \frac{\sim}{\sim_2}$, it follows that the power wasted is

$$W \frac{\sim}{\sim_2}.$$

In order that this may be small, $\sim$ must be small as compared with $\sim_2$. The slip is $s = \frac{\sim}{\sim_1}$, and since $\sim_1$ is under the usual working condition not very different from $\sim_2$, we may also express the power wasted by the term $W S$, from which it will be seen that the slip given as a percentage is also a direct measure of the power wasted in heating the armature conductors. Thus, if the slip be 3 per cent, then 3 per cent. of the power is wasted in heating the armature conductors. High efficiency and great starting torque are thus to a certain extent contradictory conditions. High efficiency means that the resistance of the armature must be low, making the slip small. The curve of torque therefore rises rapidly, and its tail end runs down near to the axis, so that at starting the torque can be but small, though when once running the motor is able to carry a considerable overload. A large slip, which results from having an armature of high resistance, raises the tail end of the curve of torque, and thus provides ample power for starting ; but the motor is rendered thereby less efficient, whilst the greater speed variations due to a changing load may also become objectionable. The general practice nowadays is to design these motors for a slip of from 2 to 5 per cent.

It remains yet to determine the effect of the revolving resultant field B on the field-magnet coils. For the sake of simplicity we again assume that the field is wound gramme fashion, as shown in Fig. 108, opposite coils being coupled in series and traversed by the same

Fig. 108.



current. We limit the investigation to one of the four coils, say for instance the coil A. Instead of revolving the field with a speed of $\sim_1$ revolutions per second in a clockwise direction, we may imagine the field to stand still in space, and the coil to revolve counter clockwise with the same speed. Then the inner turns of the gramme winding will cut through the lines of the field, and E.M.F.'s will be induced in them.

Let $O\ B$ represent the resultant field, then the field strength at a is $B \cos(\varphi + \alpha)$, and the E.M.F. acting in one turn of wire at a is $B \cos(\varphi + \alpha)\ l\ 2\ \pi\ r \sim 10^{-8}$ volts. The angle φ is the angle which the radius to the centre of the coil forms at this moment with the direction of maximum induction B. To find the instantaneous value of the E.M.F., not only in one turn but in the whole coil, we must integrate between the limits of $\alpha = \frac{\pi}{4}$ and $\alpha = -\frac{\pi}{4}$. If t is the number of turns in the coil, the number of turns within an element $r\, d\, \alpha$ of the circumference is $\frac{2\,t}{\pi}\, d\, \alpha$, and the total E.M.F. is,

$$e = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} B \cos(\varphi + \alpha)\, l\, 4\, r \sim_1 t\, 10^{-8}\, d\, \alpha$$

$$e = B\, l\, 2\, r \sim_1 t\, 2\sqrt{2} \cos \varphi\, 10^{-8}.$$

For the two opposite coils coupled in series the E.M.F. has twice this value. It is a maximum for $\varphi = o$ when the diameter joining the two centres of the coils coincides with the direction of maximum induction, and the E.M.F. is zero for $\varphi = 90°$. The E.M.F. in each set of field-coils varies then according to a sine function, the maximum value of which is

$$E = 4 \sim_1 F\, t \sqrt{2}\, 10^{-8}$$

when $F = B\, l\, 2\, r$ and t is the number of wires in one quarter of the field. It is obvious that F represents the total resultant field passing between armature and field magnet. If the winding were concentrated in one narrow coil instead of being spread over two quarters of the circumference, the expression for E would be $2\, \pi \sim_1 F\, t\, 10^{-8}$. It will be seen that this expression is similar to that obtained above, except that the co-efficient here is

2π, whereas in our previous case it was $4\sqrt{2}$. The difference in the two co-efficients, namely, 6·28 in one case, and 5·65 in the other, is due to the spreading of the coils over two quarters of the field circumference. Since the ratio between effective and maximum volts is $\sqrt{2} : 1$, we find the following expression for the effective E.M.F. induced by the revolving resultant field in each of the two field windings.

$$e = 4 \sim_1 F\,t\,10^{-8} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 69)$$

In this expression t represents the number of field wires contained in one-quarter of the circumference. The voltage e acts more or less in opposition to the supply current, and it is due to this back E.M.F., or, to speak correctly, to that component of it which is in phase with the supply current, that the latter must deliver power to the motor. The total power delivered by both currents is,

$$W = 2\,i \cos \psi\, 4 \sim_1 F\,t\,10^{-8}$$

where i is the effective current in each of the two supply circuits, and ψ the angle of lag between supply current and back E.M.F. The lag is, of course, the same for both circuits, and is represented in Fig. 106 by the angle ψ. It is obvious that since φ must for stable working be less than 45°, ψ must be greater than 45°, so that the true watts given out by the motor must be something less than $1/\sqrt{2}$, or less than 70 per cent. of the apparent watts brought in by the two supply circuits. This result might at first sight appear to be very unfavourable to rotary field motors. If this limit of $\varphi = 45°$ were really the limit in practical machines, it would mean that the whole of the plant, namely, generator, line, and motor, would have to contain at least 50 per cent. more material than corresponds to the maximum power actually transmitted. But it must be remembered that the case we have investigated does not deal with a practical machine, either as regards construction or method of working. As regards construction, the practical machine is worse than the one which has formed the basis of our investigations, because it must have certain losses not yet taken into account, and it must also have magnetic leakage ; as regards the method of working, the practical machine is very much better. We have assumed that the impressed field is constant, and this means that the supply currents are constant. The method of working was to connect the motor in series with supply circuits of constant current strength, and under this condition the performance of the machine is indeed not very satisfactory. The practical method of working is to connect the motor in parallel to supply circuits of constant potential difference, and on investigating this case we shall find that its performance is very much better. Before, however, passing on to this case, we must yet extend our investigation of the constant-current motor to the relations between ampère-turns applied to the field coils and the resultant induction B. Since the latter is the resultant between A and C, and since their relative positions are known from the clock diagram, the problem will also be solved if we determine the relations between field ampère-turns and the impressed field, supposing that no other currents are flowing. Thus, if we insert an armature without conductors, the field excitation corresponding to the length of line $O\ I\ c$ in Fig. 106 would give the induction $O\ C$, that is, C lines per square centimetre through the air gap at the point where the induction is a maximum. Let δ be the length of air gap in centimetres, and neglect the ampère-turns which are required to drive the flux through the iron of field and armature, then

$$X = \cdot 8\ C\ 2\ \delta$$

gives the ampère-turns to be applied to produce the maximum induction C. The induction actually produced is smaller, namely B; but since the armature ampère-turns act partly against the field ampère-turns, the latter must be provided to produce the resultant induction B. Each of the four field coils contains t wires, and the supply current in each circuit can now be determined. At the moment when one circuit passes through zero the other should give $\frac{X}{t}$ ampères. One-eighth period later, when all coils carry equal currents, its strength would be $\frac{X}{2t}$, two neighbouring coils acting together. Now the maximum current corresponding to $\frac{X}{2\,t}$ is $\sqrt{2}\,\frac{X}{2\,t} = \frac{1}{\sqrt{2}}\,\frac{X}{t}$, whereas in the former phase the maximum current was $\frac{X}{t}$. We thus find that there is a discrepancy between the two methods of determining the current, which arises from the fact that with a two-phase transmission the impressed field is not constant. It has already been pointed out that the fluctuation of the field strength is prevented by compensating currents flowing in the closed winding of the armature. These currents have no other effect than to slightly increase the loss in the armature due to ohmic resistance. To find the true field current required to produce the induction B, we may take the mean between the two determinations. This is very nearly

$$i = \cdot 6 \frac{Xc}{t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 70)$$

effective ampères in each circuit. The same value of i is found by equating the power given to the field, namely $2\ i\ e \cos \psi$, to the power given out by the armature, plus loss by resistance, or $W\left(1 + \frac{\sim}{\sim_2}\right)$. The angle ψ is taken from the clock diagram, Fig. 106, and we thus obtain

$$i = \frac{W\left(1 + \frac{\sim}{\sim_2}\right)}{2\ e \cos \psi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 71)$$

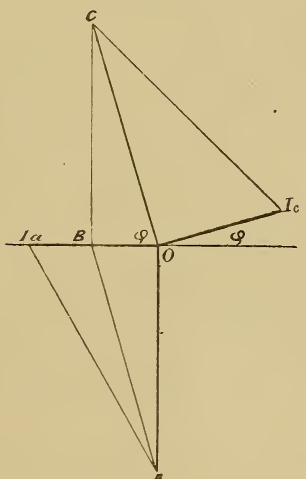
in which expression i and e refer to the field and not to the armature.

II. Perfect Motor; Supply Voltage Constant.

The term "perfect motor" has the same meaning as before; namely, a motor having no magnetic leakage and no losses, except the loss due to the ohmic resistance of the armature bars. The back E.M.F. produced by the revolving resultant field in the coils of the field-magnet must therefore be equal to the E.M.F. in the supply circuits, and as the latter is constant, it follows that the resultant field B must be constant, whatever may be the load on the motor. The clock diagram expressing the working condition of such a motor is very simple.

Let in Fig. 109 $O\,B$ represent, as before, the resultant field, and $O\,A$ the field due to armature reaction produced by the ampère-turns OI_a. Then $O\,C$ must be the field impressed by the current in the magnet coils, and $O\ I_c$ the corresponding ampère-turns, the angles $O\ C\ I_c$ and $O\ A\ I_a$ being, of course, equal. The armature ampère-turns are proportional to the speed $\sim = \sim_1 - \sim_2$, and the length of the line $O\ A$ may therefore be taken to represent to a suitable scale the speed. The torque is proportional to the product of resultant field and arma-

Fig. 109.

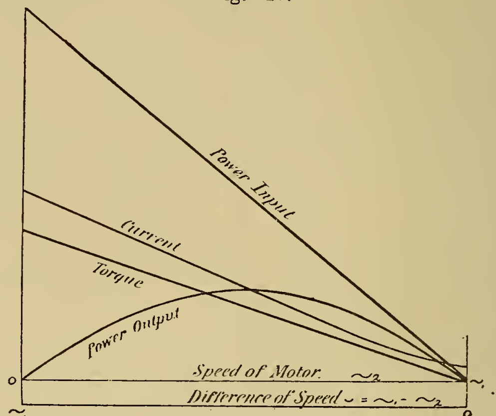


ture current, and since the former is constant, and the latter proportional to $O\ A = B\ C$, we find that the line $B\ C$ may be made to represent directly the torque if the scale be suitably chosen. The length of this line represents, therefore, two things, namely, speed and torque, from which it follows that the torque is directly proportional to the speed $\sim$; and the torque diagram is simply an inclined line passing through zero for $\sim = O$

when $\sim_2 = \sim_1$; and attaining the highest point for $\sim = \sim_1$ when $\sim_2 = O$, as shown in Fig. 110.

In the same diagram is shown the curve representing the power developed by the armature as a function of the speed. The ordinates of this curve being proportional to $(\sim_1 - \sim_2) \sim_2$, it is obvious that the curve must be a parabola passing through zero for $\sim_2 = O$ at starting; and again for $\sim_2 = \sim_1$ when the motor runs synchronously.

Fig. 110.



The power put into the field magnet by the supply currents is represented by a straight line, and the current by a line which is nearly straight over the greater part of the diagram, and slightly curved at the lower end. For synchronous running, when the impressed and the resultant fields are equal, the current is a minimum, and the lag $\psi = 90°$. As load is put on, the lag diminishes and the current increases. The power obtainable from the motor becomes a maximum for a slip of 50 per cent. when the power wasted equals that usefully given out.

The efficiency is then 50 per cent. For a smaller slip the efficiency is higher, and for a larger slip lower.

If a motor, such as we have here assumed, could be practically built, the working would be perfectly stable. To an increase of load causing a reduction of speed would correspond a proportional increase of torque ; and at starting the torque would be a maximum, which is just what is wanted. Such perfection is, however, not attainable in practice. Apart from the fact that we cannot build motors having no leakage, it is obvious that the field winding could not possibly carry the immense current which the motor would take at starting. In order to get good efficiency the motor would have to work with a small slip ; the working point would therefore be near the right hand end of the diagram, and the size of the field wire would be proportioned to carry the corresponding current. As a consequence this winding would not be capable of passing the enormously greater current which the diagram shows to be necessary at starting. The diagram merely gives the working condition of an ideal machine fed with current at constant pressure, in the same way as Fig. 107 showed the working condition of an ideal machine fed with constant current. The case of a practical machine lies between these two extremes, and shall now be considered.

III. Practical Motor ; Supply Voltage Constant.

To investigate the working condition of a motor, such as can practically be built, we have to take into consideration all the losses and imperfections of the machine. The losses are of three kinds, mechanical, magnetic, and electrical, and are occasioned by journal friction, air resistance, hysteresis, eddy currents, and the electrical resistance of the field winding. The imperfections are due to magnetic leakage brought about by the fact that the magnetizing effects of the currents in field and armature are more or less opposed, and thus cause a certain flux of lines to pass along the annular space between the field wires and armature wires. The smaller this annular space can be made, that is, the smaller the clearance δ and the nearer the conductors are placed to the circumference, the more restricted becomes the space through which leakage can take place, and the more perfect will be the machine, but it is obvious that for mechanical reasons this space can never be reduced to zero. The effect of leakage is to set up in each conductor a counter E.M.F. in phase at right angles to the phase of the current, and in magnitude proportional to the current, thereby producing lag and reducing the power of the motor. The diagram, Fig. 109, is thus no longer correct. There we have assumed that the current in any armature bar is a maximum when that bar passes the point where the resultant field is a maximum. Owing, however, to the fact that in a practical motor there must be a certain amount of space available for leakage (which is equivalent to saying that each armature bar has not only resistance, ρ, but also a certain coefficient L of self-induction), the current in each bar becomes a maximum after the position of maximum resultant field has been passed, and the line $O\,I_a$ in Fig. 109 will no longer coincide with $O\,B$, but will include with it a certain angle, ψ. This angle increases with the speed $\sim$, as can easily be seen from the following consideration. Retaining our original conceptions of a stationary field in which the armature is revolved backwards by belt, and calling E the E.M.F. induced in each bar as it passes the point where B is a maximum, we have

$$E = F \, \pi \sim .$$

This is the resultant of two components; one E_s the E.M.F. required to overcome self-induction, and E_r the E.M.F. required to overcome ohmic resistance.

$$E_s = 2 \, \pi \sim L \, I \quad E_r = \rho \, I$$

when I is the maximum value of the current in one bar. Hence

$$\tan \psi = 2 \, \pi \sim \frac{L}{\rho}.$$

Since L and ρ are constants for any given machine, the angle ψ can be determined for any speed, and having ψ we find the current from the relation $E_r = E \cos \psi$

$$I = \frac{F \, \pi \sim}{\rho} \cos \psi \; 10^{-8}$$

in ampères. The effective current is

$$i = \frac{1}{\sqrt{2}} \frac{1}{\rho} \pi \sim F \cos \psi \; 10^{-8} \quad \ldots\ldots \quad 72).$$

Comparing this equation with 67) it will be seen that the current is reduced in the ratio of 1 : cos ψ, which means that the torque will also be reduced. The larger the self-induction, that is, the larger the annular space between the two windings, the larger becomes tan ψ and the smaller cos ψ, so that on this account alone magnetic leakage reduces the power of the motor. The reduction does, however, not stop here. Besides the weakening of the torque due to reduction of current, there is a further weakening due to the fact that the maxima of current and resultant field do no longer coincide, but differ by the angle ψ. To find the tangential pull produced by one half the armature we must integrate the expression $B \cos (\psi + \alpha) \frac{l \tau I}{2 \pi} \cos \alpha \, d \alpha$ between the limits $\alpha = \frac{\pi}{2}$ and $\alpha = -\frac{\pi}{2}$; and to get the total pull exerted by the whole armature we double the result. This gives

$$P = B \frac{l \tau}{2} \frac{I}{10} \cos \psi \text{ dynes, or}$$

$$P = \frac{B l I}{9{,}810{,}000} \frac{\tau}{2} \cos \psi \text{ Kilogrammes} \quad . \; . \; . \quad 73).$$

Comparing this expression with 63) it will be seen that for the same resultant field, and the same armature current the pull has been reduced in the ratio of 1 : cos ψ. Equation 63) may also be written in the form

$$r P = \frac{2 r}{2} \frac{B l I}{10} \frac{\tau}{2}$$

or if by T_o we denote the torque in kilogramme-centimetres

$$T_o = \frac{F I_o \tau}{39{,}240{,}000} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad 74).$$

In this expression $I_o = \frac{F \pi \sim 10^{-8}}{\rho}$ ampères . . . . . . 75).

In the case under consideration when there is magnetic leakage the corresponding formulæ are

$$T = \frac{F I \tau}{39{,}240{,}000} \cos \psi \text{ and } I = \frac{F \pi \sim 10^{-8}}{\rho} \cos \psi$$

or $$T = \frac{F I_o \tau}{39{,}240{,}000} \cos^2 \psi \text{ kilogramme-centimetres.}$$

$$T = T_o \cos^2 \psi \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad 76).$$

T_0 is the torque which would be given by a machine in which there is no magnetic leakage, the armature-current being I_0. The torque actually obtained in the practical machine, in which there is a magnetic leakage causing the lag ψ, is reduced in the ratio of $1 : \cos^2 \psi$, and the armature-current is reduced in the ratio $1 : \cos \psi$.

$$I = I_0 \cos \psi.$$

The power of the motor in watts is found by multiplying P with $2 \pi r \sim_2 10^{-7}$, and we thus find

$$W = e\, i\, \tau \cos^2 \psi \quad \ldots\ldots\ldots\ldots \quad 77),$$

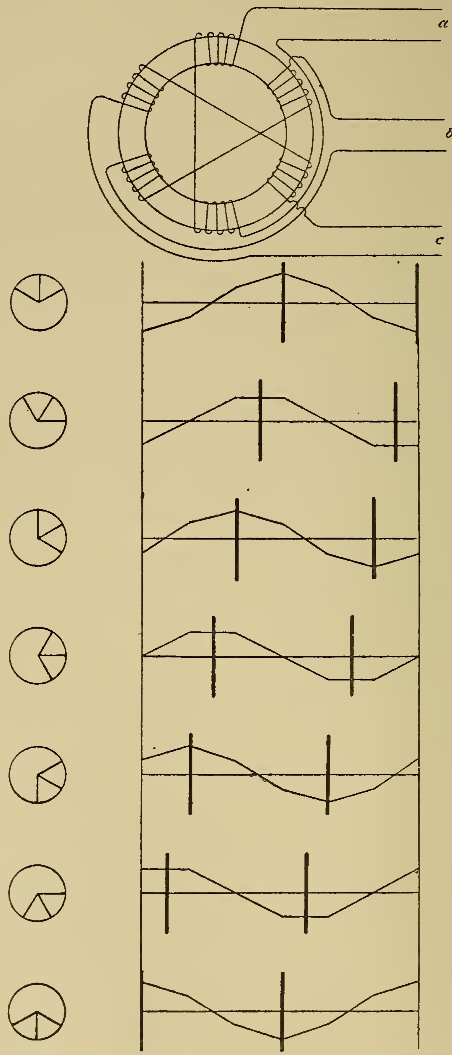
where i and e have the values specified in equations 67) and 68) respectively. Equation 77) shows that the power is by magnetic leakage reduced in the ratio of $1 : \cos^2 \psi$. It is therefore highly important to so design the motor that there shall be as little magnetic leakage as possible. As already pointed out this means that the air space δ shall not be greater than necessary to allow for free running mechanically, and that the holes in armature and field should be placed as near the circumference as possible.

The angle ψ is not a constant, but varies with the speed $\sim$. Under normal running conditions $\sim$ is small and ψ is therefore also small. The reduction of torque owing to magnetic leakage is therefore less important, once the motor has attained its normal speed ; but at starting when $\sim = \sim_1$, the angle ψ is large and $\cos^2 \psi$ consequently very small. It follows that magnetic leakage is particularly detrimental because it reduces the starting power of the motor. How great this reduction is depends on the relative values of ρ and L. As will be seen from 75) ρ should be small in order to have a large armature-current, and therefore a large torque and high efficiency when running at normal speed. On the other hand, if ρ be very small ψ becomes large, and the torque at starting is too much reduced. The two conditions are to a certain extent contradictory, and the difficulty is solved in practice by making ρ large at starting and reducing it gradually as the motor attains its proper speed. For this purpose the squirrel cage winding is in larger machines replaced by a winding consisting of three coils set 120° apart, which are connected amongst each other, and to three insulated contact rings on which there are three brushes. The brushes are connected with three non-inductive rheostats, and at starting the whole of the resistance is used. As the motor gets up speed the resistance is gradually reduced and finally short circuited.

We have hitherto tacitly assumed that the resultant field B remains the same under all working conditions of the machine ; but this is obviously impossible when the E.M.F. of the supply current is kept constant, which is the only case of practical importance. Owing to self-induction and resistance of the field-coils an increase of current is necessarily accompanied by a decrease in that component of the supply voltage which is in phase with B, and consequently the total field F in 74) and 75) is not a constant, but becomes smaller as the supply current increases. This again causes a reduction in torque which we now proceed to investigate. To facilitate the investigation we neglect at first hysteresis, eddy current, and mechanical losses, or rather we substitute for them a certain power expressed in watts which we deduct from the power given out by the machine. Since the speed of the machine under normal working conditions is very nearly constant, the total of these losses (which in any case amounts only to a small percentage of the total power) may be considered constant. The net power of the machine is to be found by deducting this constant loss from the total power. The problem may now be stated as follows.

Given a certain supply voltage and frequency, to find the torque and the power at different speeds of the armature. The first question to be investigated concerns the relation between field current and impressed field and between resultant field and back E.M.F. This has already been done for a two-phase field and we now proceed to apply the same treatment to a three-phase field, assuming for the sake of simplicity that the winding is so arranged as to produce a two-pole field. In Fig. 111 is shown a gramme ring, wound with three pairs of field-coils, the angular distance between two neighbouring coils being 60°. If the winding were drum fashion there would be three single coils (not pairs) set 120° apart. The magnetic effect is the same; but we adopt the gramme winding as more easily shown in the diagram. The diagram is constructed in the same way as Figs. 102 and 103 and needs no further explanation. It will be seen that the broken line representing the strength of the field at different points of the circumference approximates fairly closely to a sine curve. The locality of the poles produced is in each case indicated by thick lines, thus showing at a glance how the poles advance with the changing phases of the three-field currents. It will also be noticed that the number of ampère-turns producing maximum induction at the poles varies between narrow limits, namely, $2\,It$ and $\sqrt{3}\,It$, when I is the maximum current in each circuit and t the number of active wires in one coil, or one-sixth the number of wires counted over the whole circumference. This term applies, of

Fig. 111.



course, equally to a drum-wound field. The average exciting power producing the field is the mean between these two values, or $1{\cdot}865\, I\, t$, and if for the maximum value of the current we insert the effective value i, we have the exciting power in ampère-turns supplied by the field coils

$$X_c = 2{\cdot}635\; i\; t.$$

The current required in each of the three branches to produce a field excitation of X_c ampère-turns is therefore

$$i = {\cdot}38 \frac{X_c}{t} \quad \ldots\ldots\ldots\ldots\ldots \quad 78).$$

It is interesting to compare this expression with 70) which gives the current required in a two-phase field. In that case t is one quarter the total number of field wires T instead of one-sixth as in a three-phaser. Let us now assume that we have two fields of exactly the same size and containing the same total number of wires T, but arranged in one case for two and in the other for three phases. The field currents will then be

$i = 2{\cdot}28 \frac{X_c}{T}$ in the three-phase machine, and

$i = 2{\cdot}40 \frac{X_c}{T}$ in the two-phase machine.

The current in the three-phase machine is thus by five per cent. smaller than in the two-phase machine, and the area of the field wire for equal resistance loss might be reduced by five per cent. As far as weight of copper in the field goes the three-phase arrangement gives thus a slight advantage. It also gives a slight advantage as regards power, inasmuch as the counter E.M.F. produced in a given number of turns is somewhat greater, as can be seen from the following consideration. Let φ be the angle which the radius to the centre of a field coil includes at a given instant with the direction of B, and α the angular distance of an element of winding from the centre of the coil. Since within an angle of $\frac{\pi}{3}$ there are t wires, there must be $\frac{3}{\pi} t\, d\alpha$ wires within an angle of $d\alpha$. The induction in the point occupied by the elementary group of wires in $B \cos(\alpha - \varphi)$ and the E.M.F. produced is

$$d e = B \cos(\alpha - \varphi)\, 2 \pi r \sim_1 l \frac{3}{\pi}\, t\, d\alpha.$$

This equation integrated between the limits of $\frac{\pi}{6}$ and $-\frac{\pi}{6}$ gives

$$e = B\, 2\, r\, l \sim_1 3\, t \cos \varphi$$

in absolute measure. The E.M.F. induced in one field coil is therefore a sine function having the maximum

$$E = F \sim_1 3\, t.$$

In the opposite coil, belonging to the same pair, an equal E.M.F. is, of course, induced, and since the two coils are coupled in series, we have the E.M.F. induced in each circuit given by

$$E = 6\, F_1 \sim t\, 10^{-8} \text{ volts.}$$

This is the maximum value ; to find the effective value we divide by $\sqrt{2}$, and obtain

$$e = 4{\cdot}26 \sim_1 F\, t\, 10^{-8} \text{ volts} \quad . \quad . \quad . \quad . \quad . \quad . \quad 79)$$

The coefficient found in 69) for the E.M.F. induced in one circuit of a two-phase field was 4 ; now the coefficient is 4·26, showing that for the same total field and the same number of turns per circuit, the back E.M.F. in a three-phaser is $6\frac{1}{2}$ per cent. greater than in a two-phaser. To make the comparison between the two machines fair, we must, however, not assume t to be the same in both. We have two equal fields wound with the same total number, T, of wires, but connected up for three and two phases respectively. Let i and e refer to the two-phaser, and i^1 and e^1 to the three-phaser, then if the size of the field wire be the same in both, we have $i^1 = 1{\cdot}05\ i$ as previously shown. In the two-phaser we have $e = \sim_1, F\ T\ 10^{-8}$, and in the three-phaser we have $e^1 = {\cdot}71 \sim_1 F\ T\ 10^{-8}$, so that

$$e^1 = {\cdot}71\ e.$$

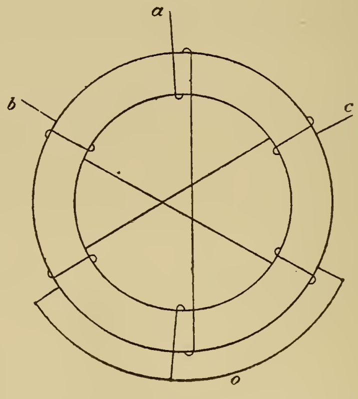
The apparent watts in each circuit of the two phases are $w = e\ i$, and those in each circuit of the three-phases are $w^1 = e^1\ i^1$; and the total watts in all the circuits collectively are:

$$W = 2\ e\ i.$$
$$W^1 = 3\ e^1\ i^1.$$
$$W^1 = 3 \times 1{\cdot}05\ i \times {\cdot}71\ e.$$
$$W^1 = 2{\cdot}23\ e\ i.$$
$$W^1 = 1{\cdot}115\ W.$$

For the same expenditure of material, therefore, the three-phase motor will give about 11 per cent. more power, or to put it another way, a three-phase motor can be built about 10 per cent. lighter than a two-phase motor of the same power and speed.

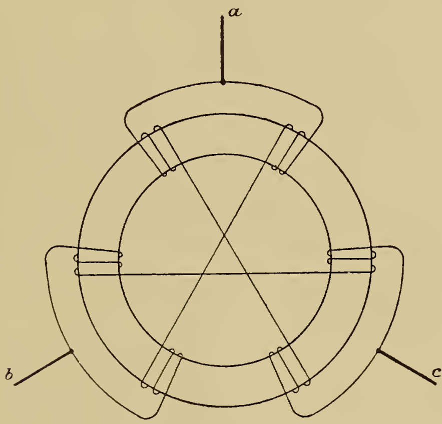
In Fig. 111 the three field circuits are shown entirely distinct from each other, and if arranged in this way six wires are required between generator and motor. The number of wires can, however, be reduced to three if the circuits are connected, as in Fig. 112. The coils forming one pair are cross connected as before, thus leaving six free terminals. Three of these are connected together by the wire *o*, and the three others, marked *a b c*, are connected with the three line wires. It is easily seen that with this arrangement (commonly known as the "star coupling," on account of the three circuits radiating, as it were, from the electrical centre *o* of the star), the exciting power of the coils on the field iron is

Fig. 112.



precisely the same as with three totally independent circuits. The reason is, that the sum of the currents, $a+b+c$, must at all times be zero, and that by bunching the three return wires into one, this latter would have to carry zero current, and may therefore be omitted altogether. The common centre *o* is generally earthed, by which means the potential of each line wire is kept within the limit corresponding to the voltage of the generator. There is another method of coupling up the field coils. This is shown in Fig. 113, and is called "link coupling." In this system there is no electrical centre which can be earthed, but the coils are connected to form a triangle or closed link, tapped by the line wires in three points electrically equidistant. On examining the diagram, and tracing the currents through the coils, it will be found that the excitation differs from that given by the star coupling. At the

Fig. 113.



moment when one current, say *a*, passes through zero, the other two, *b* and *c*, produce an excitation of 1·3 $I\,t$ ampère-turns, whilst a twelfth of a period later, when *b* is a maximum and *a* and *c* have half the maximum value, the excitation is $I\,t$. The fluctuation of exciting power is thus considerably greater with the link coupling than with the star coupling. It is interesting to compare the two windings by means of the following table :

	Field excitation in ampère-turns produced by coupling as a		
	Star.	Link.	Combined.
Any current passes through			
zero value . . .	$1{\cdot}73\ I t_1$	$1{\cdot}3\ I t_o$	$1{\cdot}515\ I t$
maximum value . .	$2\ I t_1$	$I t_o$	$1{\cdot}500\ I t$

If by t_1 we denote the number of wires in one-sixth of the field in circumference when coupled as a star, and by t_o the corresponding number when coupled as a link. To produce the same average excitation, t_o must be greater than t_1, the relation being $t_o = 1{\cdot}61\ t_1$. The variation in the exciting power applied to the field has the effect of calling forth in the armature conductors compensating currents whereby the variation in field strength is nearly completely prevented. These compensating currents must, however, waste some power, and to obviate this defect Mr. Dobrowolsky has devised a combination of the two systems of winding under which the ampère-turns applied to the field remain practically constant.

If $t = t_1 + t_o$ is the number of wires in the star and link winding collectively, and we make $t_1 = t_o = \frac{t}{2}$, the combined effect of the two windings is therefore $1{\cdot}515\ It$, and $1{\cdot}500\ I t$, as shown in the third column. The greatest variation from the mean exciting power is therefore

With star coupling	$7\frac{1}{4}$	per cent.
With link coupling	13	,,
With Dobrowolsky coupling . . .	$\frac{1}{2}$	,,
With two-phase current the variation is .	17	,,

We have now all the data required to determine the working condition of the motor by means of a clock diagram. The relation between self-induction and resistance of armature bars is supposed to be known, so that the lag ψ of armature current behind the resultant field B can be determined for any speed $\sim$, the term speed being here applied to signify the relative speed between field and armature. The total field F and the maximum induction B are chosen as high as considerations of efficiency and heating limits will allow. The clearance δ being known, we can from

$$X_b = B\ 1{\cdot}6\ \delta$$

determine the resultant ampère-turns required to produce the maximum induction B. The two components are armature ampère-turns 64) and field ampère-turns X_c. The latter is found graphically, and the corresponding field current is given by 78), whilst the effective back E.M.F. in one field circuit is found from 79). The position and magnitude of the impressed E.M.F. can also be found graphically if resistance and inductance of the field winding are known. In this construction we find the impressed E.M.F. and field current to produce a certain B at a certain speed $\sim$. This is, however, not the form in which the problem is met with in practice. What we want to know is torque and power as a function of the speed $\sim$ if the impressed E.M.F. has a constant value given beforehand. The transition from one solution to the other is, however, very simple. We assume a certain B and $\sim$, and find the impressed E.M.F. If this differs from the given value we need only enlarge or reduce all the lines in the diagram in whatever proportion is required to make the impressed E.M.F. come out at the correct value.

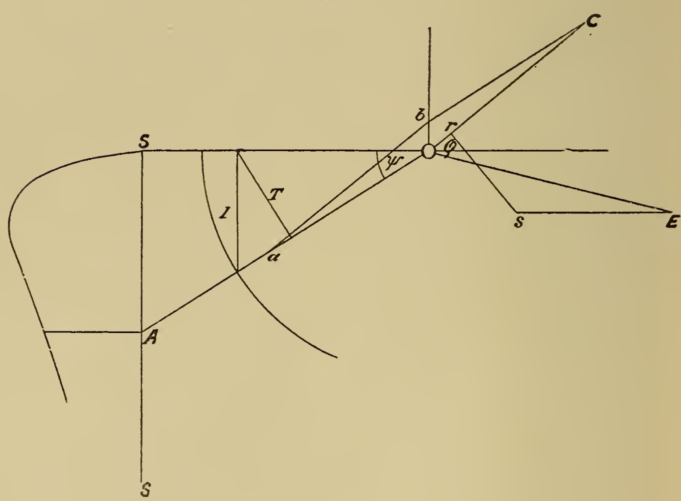
Fig 114 shows a clock diagram so constructed. All the constants of the machine being known, we find the angle ψ for any speed $\sim$ from

$$\tan \psi = 2 \pi \sim \frac{L}{\rho};$$

hence the speed can be plotted on the vertical $S S$, which is placed at a distance from O equal to $\frac{\rho}{2 \pi L}$, the scale for $\sim$ being arbitrarily chosen.

We assume a certain induction B or total field strength

Fig. 114.



F, and find the armature current I in ampères from either of the two following equations :

$$I = \frac{F \pi \sim}{\rho} \cos \psi \; 10^{-8}.$$

$$I = \frac{F}{2 L} \sin \psi \; 10^{-8}.$$

The latter is the more convenient expression, as it permits to read off the value of I by means of a circle of radius $\frac{F\,10^{-8}}{2\,L}$. We can now calculate the armature ampère-turns

$$X_a = \frac{I\,\tau}{\pi}$$

and plot the corresponding value on the radius $O\,A$. Let this be $O\,a$. Let to the same scale $O\,b$ represent the ampère-turns required to produce the resultant induction B, so that

$$O\,b = 1{\cdot}6\,B\,\delta\,;$$

then the length $O\,c$ gives to the same scale the field ampère-turns X_c. To find the current in each field coil we have for a three-phaser

$$i = {\cdot}38\,\frac{X_c}{t}.$$

The loss of pressure by ohmic resistance in each field circuit can now be calculated, and plotted on the line $O\,c$ to a suitable volt-scale. Let this be $O\,r$; and let $e_s = r\,s$ represent to the same scale the E.M.F. required to balance self-induction in the field circuit. The numerical value of this E.M.F. can be found if the coefficient of self-induction is known, the frequency being, of course, $\sim_1$. The back E.M.F. produced by the resultant field, F, is given by

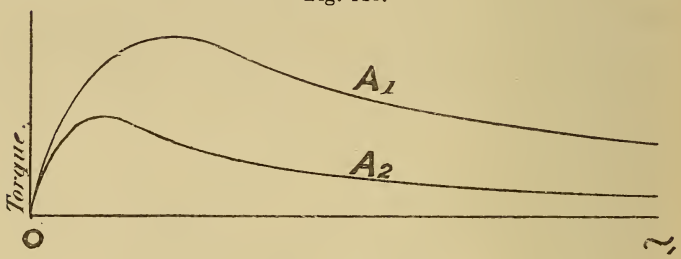
$$e_b = 4{\cdot}26\,\sim_1\,F\,t\,10^{-8},$$

and must be plotted to the right from s, giving the total supply E.M.F. $e = O\,E$, which is in advance over the current by the angle φ. The total power supplied is

$$W = 3\,e\,i\,\cos\varphi.$$

Since for a constant field the torque is proportional to $I \cos \psi$, it is obvious that the length of the line T (which is drawn at right angles to the radius $O A$), gives to a suitably chosen scale the torque corresponding to any speed ; always provided that the field is really constant. This, however, cannot possibly be the case. The only constant quantity in the working of the motor is the supply E.M.F. If, then, the length $O E$, resulting from the above construction, is either greater or smaller than the given value of the supply E.M.F., all the lines of the diagram, with exception of $O S$ and $S S$, must be

Fig. 115.



reduced or enlarged in proportion, the angles remaining unaltered. Call m the ratio between the given supply E.M.F., and that resulting from the diagram, then the true currents and true volts will be m times the corresponding values as plotted in the diagram ; the true induction will be $m B$ and the true field $m F$. The power put into the motor will be $m^2 W$, and the true torque will be $m^2 T$. The true torque and power given out can now be plotted as a function of the speed over the line $S S$. The problem is thereby solved.

It is interesting to note the great effect which magnetic leakage (self-induction in armature and field) has upon the performance of such a motor. The curve A_1 in the torque diagram, Fig. 115, refers to a motor in which $\frac{2\pi L}{\rho} = \cdot 0\ 2$, and the E.M.F. of self-induction in the field at full load is 15 per cent. of the supply E.M.F. The curve A_2 refers to a motor in which both these values are doubled.

Up to the present we have assumed that the self-induction in the armature and field is known. For obvious reasons it is, however, impossible to obtain it merely by calculation, and we must have recourse to experiment, so as to obtain data which can be used in future designs. Suppose a motor is made and ready for testing. Run it at full load, and note the current and supply voltage. This gives us the length of the lines $O\ C$ and $O\ E$ in Fig. 114, but not their relative position. To find this we must measure the power supplied, so as to determine the true watts, and the ratio of true watts to volt-ampères is the cosine of the angle of lag, or as it is also called the "power factor" of the motor at full load. Having found the angle φ, we can complete the diagram from the known data of the motor, and thus find the angle ψ, which determines the ratio $\frac{L}{\rho}$.

The self-induction in the field can be determined approximately by the following experiment. Let the armature spindle be gripped tightly, so that it cannot turn, and reduce the supply voltage to such an extent, that under this condition no more than the full working current passes through the field winding. We have now $\sim\ = \sim_1$, and owing to this high frequency, an extremely feeble resultant field, B, will suffice to produce very large armature currents. That is to say, in Fig. 114 the length of the line $O\ b$ will shrink to almost nothing, and the back E.M.F. (length of line $S\ E$) will also approach zero. Since the loss through ohmic resistance of field coils is under all circumstances very small, the length of the line $O\ E$ will become very nearly equal to the length of $r\ s$, or in other words, the measured volts give approximately the E.M.F. of self-induction at the current then passing, which by the condition of the experiment is the normal working current at full load. The maximum strength of the resultant field may be found by running the motor light at normal voltage. In this case an extremely feeble current in the armature bars will suffice to keep the armature in motion, and the ampère-turns in the armature will be so small as to not seriously react on the field. The line $O\ C$ will then become vertical, and the E.M.F. of self-induction, and the back E.M.F. will come into line, so that the supply E.M.F. is the sum of the two. Since the E.M.F. of self-induction for any current can be determined by the experiment previously described, we are able to determine the back E.M.F. as the difference between the supply E.M.F. and the E.M.F. of self-induction. Having the back E.M.F. and knowing the field winding, we can easily find the total strength of the resultant field from the formulæ previously given.

It is thus possible to determine approximately by a few simple experiments some of the electrical data which cannot be found beforehand by calculation. To investigate the working condition of a multiphase motor thoroughly, more elaborate experiments must, of course, be made, and amongst these the most important is the direct determination of the stray field or leakage when the motor is running free and loaded. The ratio of the flux actually utilized in the armature to that produced by and passing through the field coils is termed the leakage factor, and it will be obvious that the nearer the leakage factor is to unity the smaller is the self-induction in armature and field, and the more perfect is the motor.

The following experiment for the direct determination of the leakage factor has been made by Mr. E. Kolben on a 9 horse-power six polar three-phase motor. The field winding consists of thirty-six coils of seven turns each, and the armature winding of ninety bars in holes arranged in six polar drum connections so as to form three independent circuits each closed in itself. For the leakage test at no load the proper armature winding is replaced by an experimental winding passing through only thirty of the ninety holes, with two wires to each hole, and all connected in series with the two free ends brought to a voltmeter. We now have in each phase of the field $7 \times 12 = 84$ turns, and in the armature 60 turns. The field is supplied with current at a given voltage and frequency, and the E.M.F. induced in the experimental armature winding is observed on the voltmeter. The armature is turned slowly by hand so as to occupy different positions relatively to the field, and readings of the induced E.M.F. are taken at these different positions.

Owing to the three-phase winding, these readings vary very little, their average being in the present case 60·5 volts. We thus know that a magnetic flux of such strength is passing through the armature as to produce an effective E.M.F. of 60·5 volts in the 60 turns of the armature coil. At the same time a somewhat larger flux is passing through the field coil of 84 turns, and produces a back E.M.F. of 98 volts as observed on the voltmeter connected across the terminals of the field coil. If the flux were the same in both coils then the voltages should be in the same ratio as the windings, but as some of the flux is lost by leakage between field and armature, the E.M.F. in the armature coil is correspondingly reduced. We have thus,

$$\text{Leakage factor} = \frac{84}{60} \times \frac{60 \cdot 5}{98} = \cdot 865.$$

For every 1,000 lines generated in the field, 155 lines are lost in leakage between the field and armature winding. This applies to the condition of the experiment when no sensible current passes through the armature; it applies of course also to the motor provided with its proper armature if the latter is running free, because also in this case the armature current is extremely feeble. It does, however, not apply to the case when the armature is running under a load, because the field and armature ampère-turns are then much increased and the magnetic pressure producing leakage is also much increased. To find the leakage in this case the motor was fitted with its proper armature and an exploring coil of 8 turns was placed over one of the field poles, and as close as possible to the armature, and the E.M.F. produced in the exploring coil was observed with the motor running free and loaded. The fall of E.M.F. when the motor was loaded is an indication of the amount by which the leakage factor has been reduced.

The following table gives a summary of the experiments.

Remarks.	Speed.	~	E.M.F. on Field.	Current in Field.	E.M.F. induced in		Watts absorbed.	Leakage Factor.
					Armature Coil.	Exploring Coil.		
Not running .	0	50	98	20·2	60·5	—	365	·865
Running free .	980	49	98	22·3	—	26	500	·865
Over loaded to 12·5 H.-P. .	930	49	98	45·7	—	24·5	10,260	·815

The leakage factor at overload is found by multiplying the leakage factor at no load (·865) with the ratio of the voltages of the exploring coil. The efficiency of this motor at 12·5 horse-power is 90 per cent. and the power factor is the ratio of the watts absorbed to the volt ampères supplied, or

$$10{,}260/3 \times 98 \times 45{\cdot}7 = {\cdot}775.$$

Mr. Kolben has also tested some larger three-phasers on the brake with the object of determining the efficiency, slip and power factor. The results of the tests of two of these machines are given in the annexed table.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	
Metric H.-P. at the brake of armature pulley.	Watts equiv. to eff. H.-P. = 736 × Eff. H.-P. I.	Current in each branch Ampères.	Volts betw. the neutral and each terminal.	Apparent Watts 3 × Volts IV. × Amp. III.	Real Watts intake, measured at motor terminals by Watt meter.	Coefficient $\frac{\text{True Watts VI.}}{\text{Voltamp. V.}}$ ("Power factor.")	% *Slip* at load 50 ~	Efficiency $\frac{\text{Watts II.}}{\text{Watts VI.}}$	Notes.
60 42 20 0	44,160 30,910 14,720 0	318 252 150 125	60 60 60 60	57,240 45,360 27,000 22,500	48,300 36,800 17,700 —	0·844 0·81 0·655 —	2 % 1·3 — —	0·91 0·84 0·83 —	Motor I. Theoretical speed 750. Squirrel cage armature. A. E. G. make.
53·8 46 0	39,600 33,900 0	180 158 40	93 95 98	50,220 45,000 11,760	42,100 37,440 1,710	0·84 0·83 0·145	4 % 3 % 0	0·94 0·905 —	Motor II. Theoretical speed 750. Drum armature with 11 divisions, each div. short-circuited in itself. Oerlikon make.

CHAPTER X.

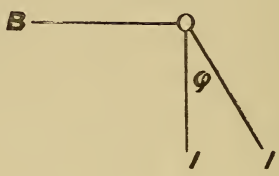
Single Phase Motor—General Explanation of its Working—Theory of Single Phase Motors—Self-induction necessary—Torque Diagram—Practical Examples—Starting Device.

If an armature with short circuited winding is placed into an oscillating field produced by a single phase alternating current, it shows no tendency to rotation as long as it is left to itself. If, however, such an armature be started in either direction, and the speed gradually increased by the application of external power, a point is soon reached when the armature of itself continues to revolve in the same direction, running faster and faster until synchronism is nearly reached. A load may then be thrown on, and the machine worked as a motor. The reason why the armature once set in rotation developes a torque in the same direction, may be explained, in a general way, as follows: Let the lines of the oscillating field be vertical, and consider one single turn armature coil of area A, which at a given moment includes the angle β with the lines of the field. If B is the maximum induction, the total flux passing through the coil at the time t, is $B\ A \sin\alpha \sin\beta$, where $\alpha = 2\pi \sim t$. The E.M.F. generated in the coil is $2\pi \sim B\ A \sin\beta \cos\alpha\ 10^{-8}$, and this will produce a certain current, depending on the resistance and self-induction of the coil. Let us first assume that the coil has no self-induction, but only resistance. The current will then be in phase with the E.M.F., that is to say, in the clock diagram, Fig. 116 the current-line $O\,I$ will be at right angles to the line of induction $O\,B$. The maximum value of the current is

$$I = \frac{2\,\pi \sim B\,A \sin \beta\, 10^{-8}}{\rho}$$

if by ρ we denote the ohmic resistance of the coil. Now the inter-action between this current and the field produces horizontal forces acting alternately to the right and left, and the first question to consider is whether these forces integrated over the time of a complete period will

Fig. 116.



produce a turning moment. The turning moment at the time, $t = \alpha/2\,\pi \sim$, is obviously proportional to $I \sin \alpha\, B \cos \alpha$, and the average effect over a complete period is proportional to $\sim \int_0^{2\pi} \sin \alpha \cos \alpha\, d\,\alpha$. This integral is zero, and it follows that the coil will have no tendency to turn in either direction.

This, however, is opposed to experiment. We know that if we place a coil at an angle to the lines of an oscillating field it has a tendency to set itself parallel to the lines. The reason of the discrepancy between theory and experience is that our assumption, that the coil has no self-induction, was wrong. The effect of self-induction is to retard the current so that the current-line in Fig. 116 will not be at right angles to $O\,B$ as assumed, but will occupy the position $O\,I^1$, lagging behind the E.M.F. line by the angle φ. The turning moment at the time t is therefore proportional, not to $\sin \alpha \cos \alpha$ as previously stated, but to $\sin \alpha \cos (\varphi - \alpha)$, and the average effect over a complete period is proportional to

$$\sim \int_0^{2\pi} \sin \alpha \cos (\varphi - \alpha)\, d\,\alpha = \sim \pi \sin \varphi.$$

There will thus be a torque exerted on the coil, and the latter will, if left to itself, assume a position parallel to the direction of the field. It is easy to see that for a complete armature, the whole surface of which is covered by coils, the torque will be clockwise for half the coils, and counter-clockwise for the other half, so that no resultant torque is produced as long as the armature is at rest. It is also obvious that if the resistance is small, and the self-induction fairly large, the angle φ will approach 90°, that is to say, the armature currents pass through their maximum values very shortly before the field passes through its maximum value, the combined effect of all armature currents being of course to oppose the field which induced them. Let, in Fig. 117, the circle represent a section through the armature bars (shown for simplicity as a continuous sheet of copper), then at the moment that the field passes through its maximum value, and is directed vertically downwards, there will be a downward current through all the bars lying to the right of the vertical diameter, and an upward current through all the bars on the left of that diameter.

The mechanical effect due to the inter-action between the field and the currents is as follows: Quadrant *a b* tends to pull the armature round in a clockwise sense, quadrant *b c* tends to pull it round counter clockwise. In the lower half quadrant *c d* is acting clockwise, and quadrant *d a* counter clockwise, the net result being that the armature remains at rest. Now let us suppose we rotate the armature by mechanical power in a clockwise sense, and see what happens. If there were no self-in-

Fig. 117.

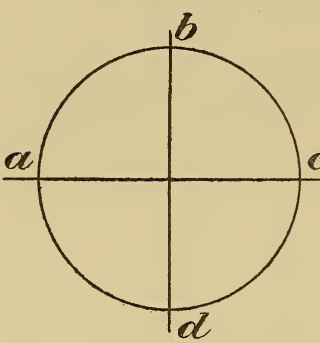
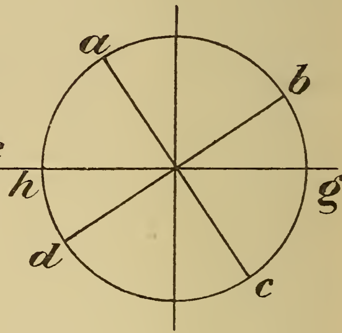


Fig. 117*.



duction, it is obvious that not only would the currents pass through their maxima at the times that the field passes through zero, and therefore each quadrant taken by itself could produce no effect, but the currents would remain symmetrically distributed in space on either side of the vertical diameter *b d*, no matter how fast the armature is turned. Since, however, there is self-induction, this symmetrical distribution will be disturbed. A certain time must elapse from the moment that the applied E.M.F. has reached its maximum to the moment that the current produced thereby has reached its maximum.

At the moment that the field passes through zero, the E.M.F. is a maximum, but the current will only attain its maximum value something short of a quarter period later. If, in Fig. 117, we assume that the field is just passing through zero and about to grow vertically downwards, then the E.M.F. will be directed downwards in quadrants *b c* and *c d*, and vertically upwards in quadrants *d a* and *a b*. The corresponding currents cannot be produced instantly, but require a certain time till they reach their maximum values. During that time the armature has turned through a certain angle and attained the position indicated in Fig. 117*. At this instant we have very nearly maximum field strength, the lines being directed downwards, and we have maximum currents, which are downwards in *b c d* and upwards in *d a b*. The turning force produced by these currents is obviously clockwise in the region *h a b* and *g c d*; and it is counter clockwise in the region *d h* and *b g*. The clockwise force is necessarily the greater, and for low initial speeds must increase with the speed. It is therefore necessary that the initial speed should reach a certain value before the resultant torque has become large enough to overcome the frictional and other resistances opposing rotation. This value passed, the torque increases and the motor runs quickly up to a speed not far short of synchronism.

The above explanation of the working of a single phase motor makes no pretence to completeness or scientific accuracy. It is merely given to show, in a general way, how it is that such a motor can work at all, and to draw attention to the fact that self-induction in the armature is an essential condition of its working.

The same conclusion is reached when treating the subject in the manner first adopted by Professor Ferraris, which is based on the fact that magnetic fields are vector quantities, and may be combined in the same way as are forces in a parallelogram. An oscillating field may, therefore, be considered to be the resultant of two equal constant fields revolving in opposite directions, with a speed equal to the frequency of the oscillating field. Thus, if we have a field oscillating with a frequency of 50 between the values $F = 100{,}000$ and $F = -100{,}000$ *c. g. s.* lines, we may imagine this replaced by two fields, each giving a constant flux of $F = 50{,}000$, and revolving with a frequency of 50 in opposite directions. Now imagine that field 1 revolves clockwise and field 2 counter clockwise ; imagine also that the armature is rotated clockwise by power at a speed of 50 revolutions per second. The armature conductors will then relatively to field 1 be at rest, and since this field is of constant strength, no inductive action between the armature and field 1 can take place. Relatively to field 2 the speed of the armature is 100 revolutions per second, and the inductive action between this field and the armature will be precisely the same as if we had a true revolving field of strength $F = 50{,}000$ and frequency 100 acting on a stationary armature. Similarly, if we revolve the armature with a speed of 48, the relative speed to field 1 will be 2 per second, and that to field 2 98 per second. Now both fields act inductively on the armature, and the current flowing in any armature bar may be considered the resultant of two currents, one induced by field 1 and the other by field 2. As far as each field and the armature currents induced by it are concerned, it is obvious that the machine may be regarded as a combination of two rotary field motors running respectively at the speeds of 2 and 98 revolutions per second, the frequency being 100 cycles per second. What is, however, not obvious at first sight is that we may determine the torque produced by each field, as if it were acting alone. In other words, are we justified in assuming that the armature current produced by field 1 is for the production of torque only acted upon by this field, and not also by field 2? The following consideration will show that this is, indeed, the case. The current induced by field 1 in any armature bar has a frequency of 2, whilst the same armature bar cuts through field 2 with a frequency of 98, that is, at a frequency 49 times as great. Denote this ratio generally by m, then we have the instantaneous value of the force resulting from the interaction of field 2 on the current produced in the armature bar by field 1 given by an expression of the form

$$K \sin(m\,\alpha) \sin \alpha$$

where K is a constant, and the angle α refers to the slower of the two periods. To find the sustained mechanical effect we must integrate over a whole period or over several periods. The integration over one period gives

$$\sim \int_0^{2\pi} K \sin(m\,\alpha) \sin \alpha \, d\,\alpha$$

The value of this integral is

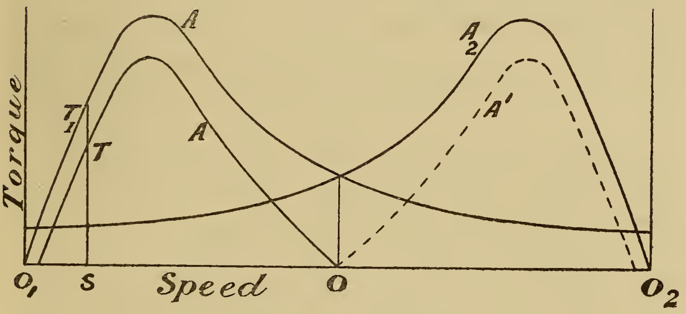
$$\sim K \left[\frac{\sin(m-1)\,\alpha}{2\,(m-1)} - \frac{\sin(m+1)\,\alpha}{2\,(m+1)} \right]_0^{2\pi} = O$$

for all cases where m is a whole number. Thus for the case assumed, where $m = 49$, there is no mechanical effect of one field on the armature current produced by the other field. The same would be the case if we revolved the armature of the motor with a speed of 46 revolutions, making the frequencies 4 and 96, or $m = 24$, whilst at the intermediate speed of 47 revolutions when the frequencies are 3 and 97, m is not a whole number, and the above expression has a certain, though obviously very small, value which may be either positive or negative. This refers, of course, to one period only, since we have extended the integration only over the time of one period. It is obviously permissible to extend the integration over two, three, or more periods, because the work stored in the revolving mass of the armature is immensely great in comparison with the small increment or decrement of work that is produced by this small force acting during several periods first in the direction of motion, and then during an equal number of periods in opposition to the motion. If, then, we extend the integration over a sufficient number of periods, the value of the integral is always zero. To get the actual torque at various speeds, we may, therefore, combine the two torque diagrams which would result from the consideration of each field by itself. Thus, let for instance in Fig. 118 A_1 represent the torque diagram of field 1, revolving with a frequency of 100 in a clockwise direction relatively to the armature. The length O_2 O_1 would then represent 100, and the relative speed of the armature would be measured to the left from O_2; its absolute speed in space to the left from O. If the absolute speed of the armature is $O\ s$ its speed relatively to field 1 is $O_2\ s$, and the torque exerted on it by this field is given by the length of the line $s\ T_1$. This torque is exerted in the sense in which the armature is rotating, and is therefore positive. The relative speed of the armature as regards field 2 is similarly $O\ O_1 - O\ s = s\ O_1$, and the torque produced by field 2 is given by the length of the line $s\ T_2$. This is exerted in a direction opposed to the movement, and is therefore negative. The resultant torque is the difference between these two torques or

$$s\ T = s\ T_1 - s\ T_2.$$

By determining this difference for various speeds $O\ s$ we obtain the resultant torque curve A. It will be seen that

Fig. 118.



this curve passes through zéro for speed zero, that is, when the armature is at rest; and that it again passes through zero at a speed slightly short of synchronism. Between these two values the torque is positive, and has one maximum. The working is stable for armature speeds exceeding that which corresponds to maximum torque and unstable for lower speeds. The direction in which the motor runs is indifferent. We supposed the motor was started in a clockwise direction when its torque is given by the curve A. Had we started it in a counter clockwise direction its torque would be represented by the dotted curve A^1 symmetrically placed to the other side of O.

It is obviously of advantage to so design the motor that the ordinates of A shall be large, and this will be the case if the torque curve A_1 rises high in its left branch and tails off low in its right branch. Now the condition for such a curve is that the motor shall have a sensible amount of self-induction. It is easily seen that a motor without self-induction, such as on page 328 was taken as a type of "perfect motor," could not possibly work on a single phase circuit. In such a motor (if it were possible to build it) the torque curve A_1, instead of being of the shape shown in Fig. 118, would be a straight line sloping from O_1 upwards to the right, and A_2 would be a symmetrical line sloping from O_2 upwards to the left. The resultant torque curves would also be straight lines sloping downwards from O to the right and left, that is to say, having negative ordinates. Such a motor would therefore not only refuse to start by itself, but would also resist the motion impressed by an external source, the more, the faster it is driven. A motor of this kind (that is, one having no self-induction) would therefore be absolutely unworkable on a single phase circuit, though used as a true rotary field or multiphase motor it would be perfection. Such perfection is, of course, not attainable in practice, and some self-induction must always be present; but whereas in a multiphase motor self-induction is one of its objectionable features, it is an absolute necessity in a single phase motor.

It was shown in the former chapter how the self-induction (or what comes to the same thing, the magnetic leakage) in a multiphase motor may be experimentally investigated, and some figures were given from tests made by Mr. Kolben with such a motor. The same motor was also tested for leakage when working on a single phase circuit by having all its thirty-six field coils connected in series. The total number of field turns is then 252, and the following results were obtained :

Frequency, 50 ; E.M.F. on field, 180 ; current in field, 18·8 ; watts absorbed, 432. The maximum E.M.F. induced in the experimental armature coil of 60 turns was 54 volts when the armature was placed in certain positions relatively to the field coils ; in other positions the E.M.F. was smaller, varying down to zero. When plotted the E.M.F. curve was found to be almost exactly of sine shape, and the average E.M.F. may therefore be taken as $54 / \frac{\pi}{2} = 34{\cdot}4$. We find thus the leakage factor $\frac{252}{60} \times \frac{34{\cdot}4}{180}$ $= {\cdot}800$.

Of every 1,000 lines passing through the field coils, 200 are lost by leakage and 800 pass through the armature when running light. When running under load the proportion passing through the armature would be still further reduced, as explained in the previous chapter. The following is a brake test made by Mr. Kolben on a 3 H.P. Oerlikon single phase motor at 50 frequency, and the reader should compare these figures with those given previously for three-phase motors. It will be seen that the power factor is smaller and the slip greater.

B H.P.	B Watts.	Field Current.	Field Voltage.	Apparent Watts.	True Watts Input.	Power Factor.	Slip per cent.	Efficiency per cent.
3·6	2,650	45	110	4,950	3,766	76	4·5	70·5
0	0	21·8	112	2,440	410	16·8	0	0

In larger motors the power factor may reach or even exceed 90 per cent., as will be seen from the following table giving results of four tests made by Riccardo Arnò[1] on a 15 H.P. Brown single phase motor. This motor is remarkable in so far as it has no sliding contacts whatever. The resistance of the armature winding is therefore not artificially increased during the period of starting, and a correspondingly large current is taken at starting, as will be seen from the first line in the table. The means employed for making such motors self-starting are explained later on. The motor in question is built for a frequency of 40 cycles per second, and having a six pole winding on the field, its speed should be slightly under 800 revolutions per minute. During the test the frequency of the supply current was, however, over 40, which accounts for the speed, even under load, exceeding 800 revolutions. The supply pressure is 150 volts. During the test the motor was loaded by a brake and all the electrical readings were taken with carefully calibrated instruments.

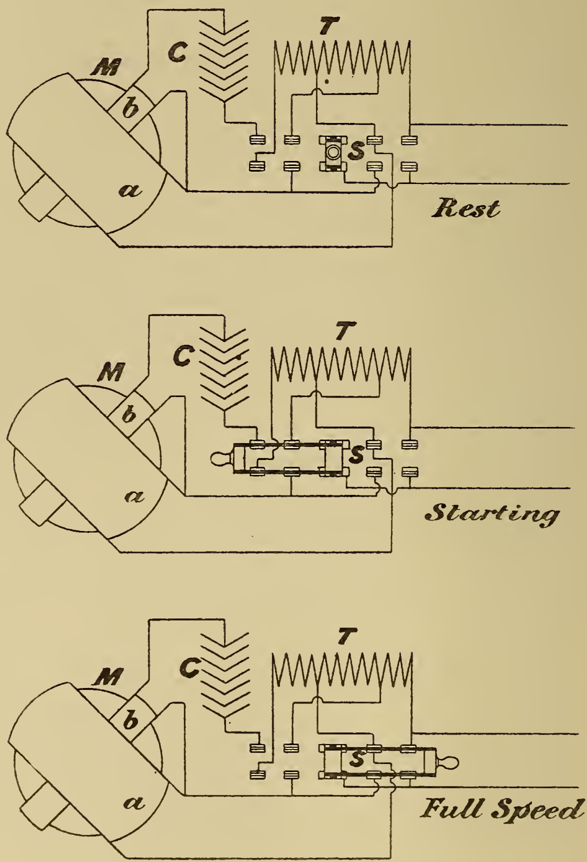
Test of 15 H.P. Single Phase Brown Motor.

Speed.	Watts on brake.	True Watts supplied.	Volts.	Ampères.	Apparent Watts.	Power Factor per cent.	Efficiency per cent.
0	0	17,595	132	150	19,800	89	0
876	0	688	157	27	4,252	16	0
862	574	1,173	156	27	4,303	27	49
863	1,943	2,652	155	31	4,774	56	73
866	2,870	3,774	157	36	5,630	67	76
868	4,136	5,176	156	42	6,520	79	80
863	5,085	6,171	154	47	7,307	84	82
862	5,652	6,732	154	51	7,792	86	84
859	6,940	7,854	152	57	8,694	90	88
858	7,728	8,823	151	64	9,634	92	88
856	9,980	11,398	149	82	12,218	93	88
851	11,385	13,923	146	102	14,943	93	82
812	11,886	15,478	143	118	16,903	92	77
816	12,320	16,626	144	128	18,432	90	74

[1] "L'Elettricista," III. No. 7, p. 149.

The working of single phase motors has been already explained both in a general way, and more particularly by reference to the torque diagram. It, however, remains yet to explain how such motors are started. In small sizes a vigorous pull on the belt by hand is generally sufficient to give the armature a speed sufficiently high to make the corresponding torque exceed the frictional resistance of running light, and a small motor may thus be started by hand, if there is a fast and loose pulley provided on the shaft which receives the power from the motor. Larger motors are, however, too heavy to be started in this way, and some special starting appliance is necessary. A variety of such appliances have been designed, but they all depend upon the expedient of splitting up the single phase current brought by the supply leads into two components of different phase. The field of the motor is wound with two sets of coils, and these are connected with the two branch circuits in such a way as would produce a true rotary field if the phase difference between the branches were 90°. If the phase difference has a smaller value, the resultant field will still be rotary, although it will not be of constant strength, but such as it is, it suffices to start the motor, and after a certain speed has been attained the starting appliance is cut out, and the connections with the field coils are altered, so as to form one circuit only, producing an oscillating field. The motor then runs quickly up to a speed near synchronism, after which the load may be thrown on. The difference in phase between the two components of the supply current may be produced by inserting ohmic resistance into one branch, and inductance into the other ; or inductance and capacity, or only capacity into one branch, the natural inductance of the field coils in the other branch being sufficient to produce a sensible phase difference. This is the arrange-

Fig. 119.



ment adopted by Mr. C. E. L. Brown, and which is shown diagramatically in Fig. 119. The condenser used by Mr. Brown is of the liquid type, and has an enormous capacity ; the pressure between any two neighbouring plates must, however, not be too high, hence a number of dish-shaped plates are put in series, and the first and last plates are connected to the circuit. The current at starting being in any case large, whilst the E.M.F. required is small, Mr. Brown combines with his condenser a transformer, so that whilst the line is only called upon to give a moderate current at full voltage, the motor receives a large current at reduced voltage. The necessary changes in the connections from "rest" to "starting" and "full speed," are performed by one switch, as will be seen from the three diagrams in fig. 119. *M* is the motor, *C* the condenser, and *S* the switch. The leads on the right bring in the supply current ; the other leads serve as connections between the motor and its starting device. The winding of the motor consists of two coils, *a* and *b* ; the latter is merely an auxiliary coil of finer wire than coil *a ;* and it is only in use during the period of starting, when it is put in series with the condenser. The current in it is in advance over the current *a* by in something less than a quarter period, thus producing a rotary field. The switch at starting is thrown over to the left. When the switch is thrown over to the right the main coil *a* is coupled directly in parallel with the supply leads, and the auxiliary coil *b* and the whole starting device is completely cut out of circuit.

CHAPTER XI.

The Line—Relation between Capital Outlay and Waste of Energy—Most Economical Method of Working—Weight of Copper in relation to Power, Distance, Voltage and Efficiency—Phase Rectifier—Weight of Copper required with Different Systems of Transmission—Material of Conductor—Stress in Conductor—Insulators—Joints—Lightning Guards.

Both as regards first cost and economy of working, the line forms a very important item in any extended system of electric transmission of energy. We have to consider two separate cases. The one, where energy from a central station is transmitted to and divided between a number of small working centres all grafted upon a network of conductors forming the main circuit, and the other, where all the energy is conveyed to a single receiving station along a pair of conductors without any ramifications. The first case would occur in a system of town supply where electricity is furnished for lighting and power purposes, and where the lamps and motors are all connected in parallel to the mains. The second is that occurring when energy from an hitherto inaccessible source is conveyed to a convenient point of application, the distance being considerable. Whatever particular form of transmission and distribution the system may have, it will be clear that the first cost of the conductors, and the annual expenditure represented by the energy wasted in heating the conductors, follow opposite laws. To economize energy it is necessary to employ leads of low resistance, and, therefore, of considerable cross-sectional area. To reduce the first cost we would, on the other hand, employ leads of small weight—that is, of small sectional area. We see that first cost and the subsequent working expenses are both governed by the area of conductor chosen, but whilst the former increases with the area, the latter decreases as the area increases, and it is evident that in each system of electric transmission of energy there must exist at least one particular area of conductor for which the sum of interest on its first cost, and annual cost of energy wasted, becomes a minimum. The subject is very complicated as elements enter into the calculation which cannot well be brought into mathematical form. To simplify the problem we at first shall neglect all these bye-issues, and treat the question in a purely academical way ; we shall then extend the theory to practical cases.

The items which more especially affect the most economical size of conductor are : 1. The rate of interest to be charged on capital outlay ; 2. The cost of one horse-power-hour at the terminals of the generator ; 3. The number of hours per annum that the energy is required ; 4. The cost of unit weight of the conducting material ; 5. The cost of insulation ; 6. The cost of supports if an overhead line, or troughs if an underground line ; 7. The cost of labour in laying. If it be permissible to consider the capital outlay as proportional to the total weight of conducting material, then for a given line we have the relation $p\,K = k\,a\,p$, where K is the total cost of the line, k a constant and p the annual rate of interest. The resistance of the line is inversely proportional to the area a, and the energy wasted equals resistance multiplied by the square of the current. Let q represent the cost of one electrical horse-power-hour at the terminals of the dynamo, and let t represent the number of hours per annum during which the current c is flowing—there being always the full amount of energy transmitted—then we have the annual value of energy wasted,

$$W = \frac{w}{a} q t c^2$$

w being a constant. The total expenditure will be a minimum if $\frac{d K p}{d a} + \frac{d W}{d a} = o.$

This gives $p K = \frac{w q t c^2}{a^2}$, and

$$a = c \sqrt{\frac{w q t}{p k}}.$$

By inserting this value into the equations for K and W we find

$$p K = c \sqrt{w q t k p} \text{ and}$$
$$W = c \sqrt{w q t k p}$$

Hence $\quad p K = W,$

or the most economical area of conductor, will be that for which the annual interest on capital outlay equals the annual cost of energy wasted. This law is commonly known as Lord Kelvin's law, and was first published by him in a paper on "The Economy of Metal Conductors of Electricity," read before the British Association in 1881. It should be remembered that this law in the form here given only applies to cases where the capital outlay is strictly proportional to the weight of metal contained in the conductor. In practice this is, however, seldom correct. If we have an underground cable, the cost of digging the trench and filling in again will be the same whether the cross-sectional area of the cable be one tenth of a square inch or one square inch; and other items, such as insulating material, are if not quite independent of the area, at least dependent in a lesser degree than assumed in the formula. In an overhead line we may vary the thickness of the wire within fairly wide limits without having to alter the number of supports, and thus there is here also a certain portion of the capital outlay which does not depend on the area of the conductor. It would, therefore, be more correct to write

$$K = Ko + ka,$$

where Ko represents that part of the capital outlay which is constant and independent of the area of the conductor. This addition on the right-hand side of the formula makes no alteration in the differential equation, for $\frac{d\,Ko}{d\,a} = o.$

We obtain, therefore, again,

$$a = c\sqrt{\frac{w\,q\,t}{p\,k}},$$

but the value of $p\,K$ is altered.

$$p\,K = p\,K\,o + c\sqrt{w\,q\,t\,k\,p}$$
$$W = \sqrt{w\,q\,t\,k\,p}.$$

The interest on capital outlay, and the annual cost of energy wasted are now in the relation

$$p\,K = p\,Ko + W.$$

They are no longer equal, but the interest on capital outlay must be greater than the annual cost of energy wasted. By writing the above equation in the form

$$p\,(K - Ko) = W,$$

we find that the most economical area of conductor is that for which the annual cost of energy wasted is equal to the annual interest on that portion of the capital outlay which can be considered to be proportional to the weight of metal used.

Up to this point we have treated the subject in what was at the beginning of this chapter called the purely academical sense. Let us now see how the result can be practically applied. That the deduction here arrived at is not directly applicable to a transmission plant is obvious, because certain important premisses have been disregarded. To see clearly that the law of maximum economy indiscriminately applied may lead to a wrong conclusion, we need only remember that according to this law the sectional area of the conductor is proportional to the current and does not depend on the voltage of the generator or the distance of transmission. If the law as above enunciated were universally applicable it would also have to fit a case where the voltage is low and the distance great; and under such circumstances it is quite conceivable that the whole of the available voltage is required for overcoming the ohmic resistance of the line and that no power at all is delivered at the motor end. The law would thus seem to indicate that the most economical method of working a line of transmission is one under which we get no return at the motor end for our annual outlay at the generator end, which is obviously absurd. The fallacy does not, however, lie in the law, but in our application of it. The law itself is perfectly true. It says that if a certain current has to be sent through a given circuit, then there is one particular area of cross-section for which the annual outlay of sending the current round becomes a minimum. The fallacy lies in this, that we assume the earning power at the motor end of the circuit to become a maximum when the cost of sending the current through the circuit becomes a minimum. This is not the case. The law also assumes that the power has the same money value all along the line. This can obviously not be the case, for if the annual horse-power could be produced locally at the motor end as cheaply as it can be produced at the generator end, then there would be no need at all for a transmission plant. Further, the law assumes that the electrical horse-power has a certain annual value quite irrespective of the voltage at which the current is delivered. This, again, is not correct. Generators for a very high voltage cost more to build and maintain than generators for a moderate voltage. It is thus obvious that the voltage at which the plant is intended to be worked must be taken into account in designing the line for greatest economy. We must further take into account not only the interest and depreciation of the line, but also the interest and depreciation of the machinery at either end. The whole problem is exceedingly complicated and cannot be solved by reference to the line only; the investigation must extend over the whole of the plant.

In order to present the problem in the simplest possible form we assume that the transmission of power is to be effected by continuous current. The transition to alternating and polyphase currents can then be made by establishing the relations existing between the weight of copper required with continuous currents and other systems of transmission. The conditions under which the problem is generally met with in practice are the following. The maximum voltage at generator terminals is either given or may be selected with due regard to constructive reasons and local conditions. It is generally found that the economy in working is greatly influenced by the voltage, and for this reason alternative designs at various voltages should be prepared so that the best may be finally selected. The annual value of the brake H.P. at the generating station is known, as is also the brake H.P. required at the motor end, distance of transmission and cost of machinery and regulating appliances per H.P. Having decided on the type of line to be employed, we also know the cost of supports for the conductor per mile and the cost of the conductor itself per ton of copper erected. The data required are area of conductor, working current, power required at generating station, total efficiency, total outlay, and total annual cost per brake H.P. delivered. The latter to be a minimum.

We assume the efficiency of the generator and motor to be 90 per cent. for each ; and the resistance of the line to be ·088 ohm for one square inch of conductor for one mile out and home. Assume the following notations :

D	Distance in miles.
a	Section of conductor in square inches.
E	Terminal volts at generator.
e	Terminal volts at motor.
HP_g	Brake horse-power required to drive generator.
HP_m	Brake horse-power obtained from motor.
c	Current in ampères.
	Efficiency of generator 90 per cent. ; efficiency of motor 90 per cent.
g	Cost in £ per electrical horse-power output of generator.
m	Cost in £ per brake horse-power output of motor including regulating gear.
$G = \cdot 9g\ HP_g$ . .	Cost in £ of generator.
$M = m\ HP_m$ . .	Cost in £ of motor and regulating gear.
$t = 18\cdot 2\ Da$ . .	Weight in tons of copper in line.
K	Cost in £ per ton of copper, including labour in erection.
s	Cost in £ of supports of line per mile run.
p	Cost in £ of one annual brake horse-power absorbed by generator.
q	Per-centage for interest and depreciation on the whole plant.

The power delivered to the motor is 830 HP_m watts; and that wasted in the line is $Ec - 830\ HP_m$ watts. This must obviously be equal to resistance of line multiplied by the square of the current.

$$Ec - 830\ HP_m = \cdot 088 \frac{D}{a} c^2.$$

The sectional area of conductor is from this equation.

$$a = \frac{\cdot 088\ D c^2}{Ec - 830\ HP_m}.$$

The capital outlay for the conductor and erection irrespective of supports is

$$tK = 18 \cdot 2\ D a K,$$

and by inserting the value for a we have

$$\text{Cost of conductor} = \frac{1 \cdot 6\ K D^2 c^2}{Ec - 830\ HP_m}.$$

The total capital outlay on which interest and depreciation has to be charged consists of this item plus the cost of supports Ds; plus cost of machinery at the motor end, $m\ HP_m$; plus cost of machinery at the generator end, $g \frac{Ec}{746}$. We thus obtain the capital outlay,

$$A = g \frac{Ec}{746} + m\ HP_m + Ds + \frac{1 \cdot 6\ K D^2 c^2}{Ec - 830\ HP_m},$$ and the

Annual cost per brake horse-power delivered $= q \frac{A}{HP_m} + p \frac{HP_g}{HP_m}$.

Put $B = \frac{Ep}{670} + q \frac{Eg}{746}$;

$\gamma = \frac{830}{E} HP_m$, the current which would be required if the line had no resistance;

and $\beta = \gamma^2 \dfrac{E\,B}{1{\cdot}6\,q\,K\,D^2 + E\,B}$; then the most economical current at the given voltage E is

$$c = \gamma \left(1 + \sqrt{1 - \frac{\beta}{\gamma^2}}\right)$$

$$c = \gamma \left(1 + \sqrt{\frac{1{\cdot}6\,q\,K\,D^2}{1{\cdot}6\,q\,K\,D + B\,E}}\right).$$

Having found the current c, we can from $e^1\,c = 830\,H\,P_m$ calculate the voltage e^1 at the motor end of the line and thus find $E - e^1$, the voltage lost in line resistance. The line resistance itself and therefore the section and weight of line can now also be found as well as all the other data required, including the total annual cost per H.P. delivered. By making the calculation for several values of E, due account being taken of the influence of voltage on the cost of machinery and insulation of the line, it will be found that the cost per H.P. delivered varies with the voltage, and that there is for every case one particular voltage at which the plant will work with the greatest economy. Provided local conditions permit it, this voltage should be chosen. It is interesting to note that the square root in the equation for the current can never be greater than unity. It approaches unity the more nearly the smaller the voltage, and the greater the distance of transmission. For economical working we would thus always find

$$c < 2\,\gamma.$$

If c were equal to $2\,\gamma$, then half the total power would be lost in the line, but since c must be smaller it follows that under no circumstances can it be economical to lose as much as half the power in the line. Apart from economical reasons the loss of power in the line is with alter-

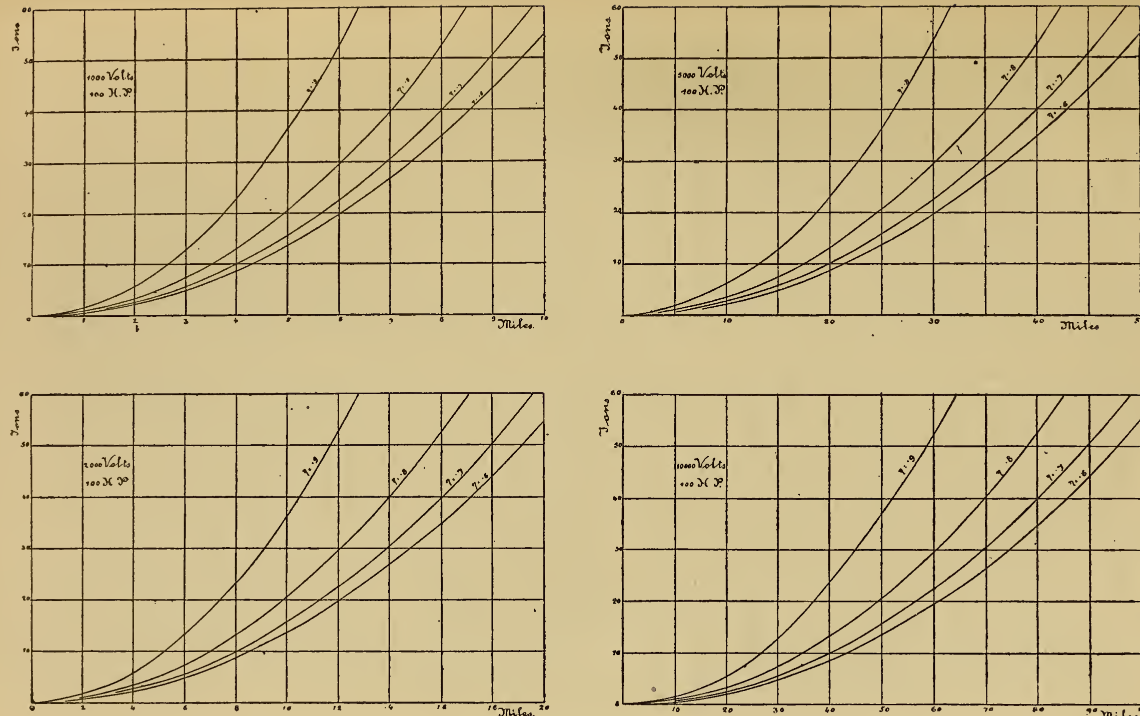

1000 Volts
100 H.P.
5000 Volts
100 H.P.
2000 Volts
100 H.P.
10000 Volts
100 H.P.
Tons
Miles.

Fig. 120.

nating currents further restricted by the necessity of providing a sufficient margin of power as already explained in Chapter VIII.

For preliminary or approximate calculations as to the weight of copper required for any line, it is convenient to use tables or diagrams which can be prepared once for all. Fig. 120 shows a set of such diagrams for continuous current power transmission at 1,000, 2,000, 5,000 and 10,000 volts. Although continuous currents would not be used at a pressure of 10,000 volts, it is convenient to make the continuous current transmission the standard as regards weight of copper to which other systems may be referred, as will be explained later on. In all the four diagrams Fig. 120 distance of transmission is plotted on the horizontal, and weight of copper given in tons on the vertical. The motor is supposed to have an efficiency of 91 per cent., and the power actually delivered at the motor spindle is supposed to be 100 brake horse-power. For a larger or a smaller power the weight of copper scaled off the diagrams would have to be proportionately altered. The voltage written in each diagram is that measured at the terminals of the generator, that is to say, the highest voltage on the line. The length of conductor is taken at 5,400 feet (not 5,280 feet) per mile to allow for the sag of the wire between the posts. Each diagram contains 4 curves, marked respectively $\eta = \cdot 6$; $\eta = \cdot 7$; $\eta = \cdot 8$; $\eta = \cdot 9$. This means that the efficiency of the line itself is respectively 60, 70, 80, and 90 per cent. The use of these diagrams is very simple, and may be explained by an example. Say we have to transmit power over 20 miles, and wish to do so with a line efficiency of 80 per cent., and a pressure of 5,000 volts on the terminals of the generator. We find from the diagram that for every

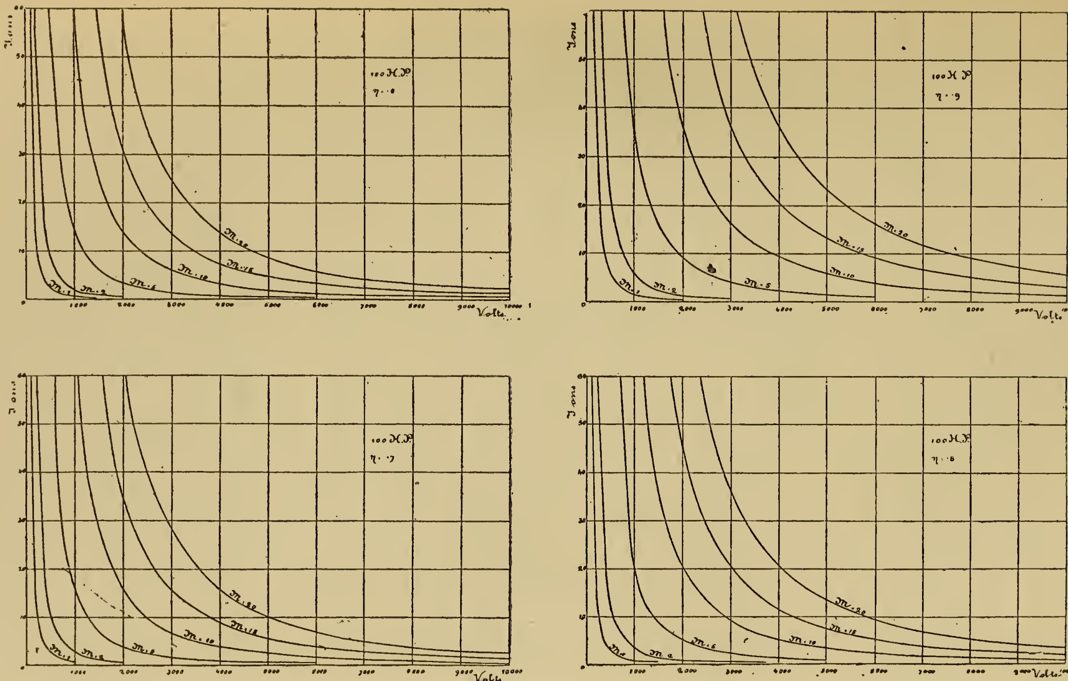
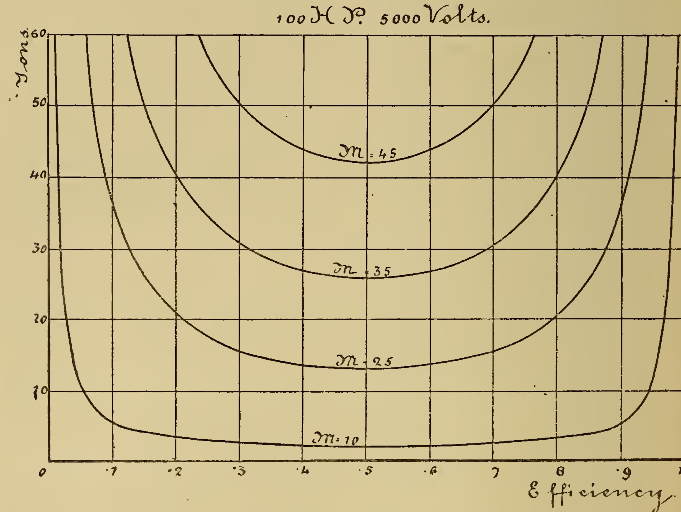


Fig. 121.

100 brake horse-power delivered we would require 13½ tons of copper in the line.

Fig. 121 gives similar diagrams in a different form. The volts are plotted horizontally and tons of copper vertically, whilst the curves refer respectively to 1, 2, 5, 10, 15, and 20 miles of distance over which the power has to be transmitted. Given voltage, distance and

Fig. 122.



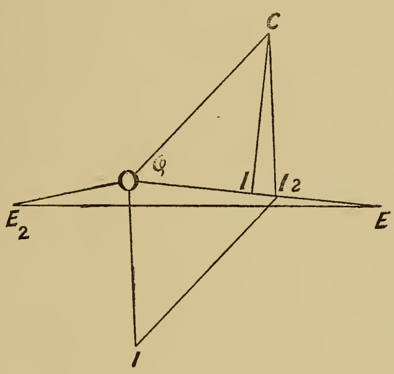
efficiency, the copper weight can be scaled off the diagram. Fig. 122 shows the relation between efficiency and copper weight for various distances, but in all cases for the same pressure, namely, 5,000 volts. For instance, a 90 per cent. efficiency line 25 miles long contains 37 tons of copper for every 100 horse-power delivered. If we are satisfied with 80 per cent. efficiency the weight is only 20 tons, at 70 per cent. only 16 tons, and so on, the lowest weight being reached at 50 per cent. efficiency. For a less efficiency than 50 per cent. the weight increases again, so that on account of prime cost in copper it would obviously be wrong to make the line efficiency less than 50 per cent. This result is in accordance with that found a few pages back when we deduced from $c < 2\gamma$ that under no circumstances can it be economical to lose more, or even as much as half the power, in the line.

When comparing different systems of transmission as regards weight of copper in the line, we must first of all agree on a suitable standard of comparison. That the comparison can only be fair if we take the same power delivered, the same distance and the same efficiency, will be obvious. The only other condition affecting weight is the voltage, and this must, therefore, be our standard of comparison. The question now arises as to the exact meaning of the term voltage in this connection. In a continuous current transmission the meaning is perfectly definite, but with alternating currents there is a doubt whether we ought to take effective volts, or the volts corresponding to the crest of the pressure wave. If the question were merely one of using machines as put on the market by various makers, then effective volts would have to be taken, because the machines are designed and listed to work at pressures given in effective volts, and their insulations may be assumed to be sufficient for their pressures. In this case a 2,000 volt alternating current transmission would require the same weight of copper in the line, as a 2,000 volt continuous current transmission provided, in the former case there is no lag between E.M.F. and current, and that the current and E.M.F. waves are sinusoidal. But this is not the correct way of considering the question. The limiting condition is not the difficulty of designing a winding that will give a high voltage, but the difficulty of effectually insulating that winding so that it will safely stand the high voltage. In other words, the stress on the insulation and not the effective voltage is the factor which must be taken into consideration, and which must form the basis on which different systems of power transmission must be compared. Thus an alternator which gives a very peaky E.M.F. curve when working at 1,000 volts effective pressure, may strain its insulation as much or more than another alternator giving a sine curve, and working at 2,000 volts effective pressure. Obviously the line, if supplied from the first machine, would require to be four times as heavy as if supplied from the second machine, and it will thus be seen that with regard to safety it is advantageous to employ machines which give an E.M.F. curve of sinusoidal shape. Let us then assume that single and multiphase alternators are all so designed as to give true sine curves and let us compare the following systems with a continuous current plant: *a*) Simple alternating; *b*) two-phase alternating with two independent circuits requiring four wires; *c*) two-phase alternating with one wire common to both phases, and requiring therefore only three wires; and *d*) three-phase alternating with three wires. We also assume for the sake of simplicity that in neither case is there any lag between E.M.F. and current. As a rule there is lag, and the effect of this is that the current must be larger than would otherwise be the case, so that the weight of copper will be increased in the ratio of 1 to cos ψ. In a single phase transmission the lag may be avoided by suitably exciting the motor as explained on page 261. In a multiphase transmission the same expedient is not applicable;

but we may use another expedient to reduce the lag in the line to zero. This is the invention of Von Dobrowolsky, and consists in the addition of an idle running alternator to the receiving plant at the motor end of the line.

Such a machine might appropriately be called a "phase-rectifier," and its armature must be wound for two or three circuits according to the number of phases produced by the generator. To explain the principle on

Fig. 123.



which the action of the phase-rectifier depends, we must go back to Fig. 114, where *O E* represents the impressed E.M.F. (that is, in the case of a transmission plant the E.M.F. at which the current enters the motor), and *O C* the current. The diagram refers, of course, only to one of the phases. It will be seen that under no circumstances can the angle of lag ϕ become zero, although in practical work it is generally much smaller than was shown for the sake of clearness in the diagram. The lines *O C* and *O E* are reproduced in Fig. 123. The length of the line $O\ C$ may be taken to represent the current flowing through the cable (or in a three-phase plant through one of the three conductors of which the cable is composed), whilst the current doing useful work is smaller, namely $O\ I$, which is the projection of $O\ C$ on the E.M.F. line. The problem now is to reduce the line-current without altering the working conditions of the motor, and this problem Von Dobrowolsky has solved by coupling in parallel with the motor an alternator excited so as to give a higher E.M.F. than that of the supply current. Such an alternator, if running light, will take a leading instead of a lagging current, and by suitably designing the plant, it is possible to so arrange matters that the lead of the current taken by the phase-rectifier exactly compensates for the lag of the current taken by the motor, so that the resultant current flows through the line without lead or lag. In the diagram the length of $E\ E_2$ is the E.M.F. of the phase-rectifier, which combined with the supply E.M.F. $O\ E$, gives the resultant E.M.F. $O\ E_2$. To this resultant E.M.F. corresponds the current $O\ I_1$, taken by the phase-rectifier. The resultant between this current and the current taken by the motor is $O\ I_2$, and this is the current which actually flows through the line. It is made up of two components ; the one, $O\ I$, which is the true power current of the motor, and the other $I\ I_2$, which is the power current required to keep the phase-rectifier in motion. The latter is of course exceedingly small, amounting to some two or three per cent. of the motor-current only ; in the diagram it has, however, been shown large in order to make the construction clear. In practice there is no need to make any elaborate calculations as to the exact E.M.F. to which the rectifier should be excited. All we need do is to provide a rheostat by which the excitation can be varied, and to turn its handle into that position for which the ampèremeter in the supply circuit indicates minimum current. Either more or less excitation will result in an increase of supply current, and we thus know that for that particular excitation, and at the given load on the motor, we have coincidence in phase of current and E.M.F. in the line. The rectifier may be excited by a little dynamo on its own shaft, and may be started by the motor by means of a friction-clutch or other mechanical device capable of being disconnected after synchronism is reached.

After this digression, intended to show how alternating current lines may be worked without lag, we return to the main problem, which is the comparison of the five systems of transmission above enumerated. Taking the continuous current system as our standard of comparison, and putting the condition that the factor of safety against a breakdown of insulation shall be equal in all the systems, we can express the amount of copper required for each system in terms of the amount required by the continuous current system. The conditions of equal safety requires that at no moment shall the E.M.F. between any two-line wires, or between any line wire and earth, exceed the corresponding values obtaining in the continuous current plant.

Take the single phase alternating current first. The power transmitted is proportional to the effective E.M.F., the stress on the insulation to the maximum E.M.F. The latter must be equal to the E.M.F. of the continuous current. The effective E.M.F. of system *a*) is thus only $1/\sqrt{2}$, of that of the standard system, and as the copper weight varies inversely with the square of the

E.M.F., the ratio of copper weights is 1/2, that is to say, the alternating current requires twice as much copper as the continuous current.

The two-phase system with four-line wires being simply a duplication of the previous case, the same relation holds good.

The two-phase system with three-line wires might, at first sight, appear to result in a saving of copper because the common wire carries not twice, but $\sqrt{2}$ times the current carried by each of the two other wires. This is, however, a fallacy. By joining two of the wires into one, we have forcibly raised the potential difference between the two other wires to $\sqrt{2}$ times the previous amount; and to get back to our original conditions of safety we must correspondingly lower the E.M.F. in each circuit. This results in a considerable increase of copper in the line. It is not necessary to give the calculation in detail; the result is that this system requires 2·9 times the standard weight of copper.

In a three-phase system with star-coupling, the maximum potential difference between any two wires is obviously $2\sqrt{E^2 - \frac{E^2}{4}} = E\sqrt{3}$, when E is the maximum E.M.F. in one phase. The equivalent continuous current system would therefore be worked at a pressure of $E\sqrt{3}$, and if the current be C, the power put into the line is $C\,E\sqrt{3}$. Let i be the alternating current in each phase, then the power put into the line is $3\,i\,E/\sqrt{2}$, from which we find $i = C\sqrt{\frac{2}{3}}$. Let R and r be the resistance of one-line wire in the continuous and alternating system respectively, then for equal loss we have $2\,R\,C^2 = 3\,r\,i^2$, and since from the above equation for i,

$3\ i^2 = 2\ C^2$, it follows that $R = r$, that is to say, a wire of the same section must be used in both systems. Since there are three wires in the alternating and only two in the continuous current system, the weight of copper is as 3 to 2, or 1·5 to 1. For a three-phase plant with link coupling, a similar investigation leads to the same conclusion. Summarizing these results, and assuming as a basis that 10 tons of copper are required with the continuous current system, we require for an equal factor of safety, against breakdown of insulation:

	tons.
a) with single phase alternating current . .	20
b) with two-phase alternating current and four wires	20
c) with two-phase alternating current and three wires	29
d) with three-phase current and three wires .	15

The three-phase system thus has an appreciable advantage, in so far as the weight of copper in the line is concerned. It must be remembered that the comparison with the continuous current system has been made merely with a view to adopt a convenient standard. In practice it is of course not possible to work a continuous current system at so high a pressure as any alternating current system, as already explained in a former chapter, and where the distance is great, the adoption of alternating current in preference to continuous current must always result in a saving of copper for the line.

The question whether the line should be carried overhead or placed underground, depends on a number of local circumstances. In many cases, and especially for distribution of power within populous districts, an underground system is the only admissible one, and then any types of high pressure cable which have stood the test of practical experience in lighting may be used. It may also happen that although no objection exists on the part of public authorities to an overhead line, yet the underground line is preferable on account of climatic conditions. In districts where the winter is very severe, or where during the summer thunderstorms are both frequent and violent, the upkeep of an overhead line is necessarily costly, and in such cases it may be good policy to adopt that system which is not so liable to interruption, although it may be the more expensive at the first start. Apart from such special cases, however, the overhead line, consisting of posts, insulators, and naked wires, is the one more generally used for power transmission, especially for long distances.

The material used for the conductor is generally hard-drawn copper, but silicon bronze and phosphor bronze are also used. The properties required in the metal are great strength and high conductivity, conditions which to a certain extent are contradictory. Thus pure, soft copper has the highest conductivity, namely 100, but its breaking strength is only 16 or 17 tons per square inch. In hard-drawn copper the conductivity is only 97 per cent., but the tenacity is 29 tons, whilst phosphor bronze of 45 tons has a conductivity of only 26 per cent. The following table gives the tenacity and conductivity for different materials:

Material.	Tenacity. Tons per square inch.	Conductivity.
Soft copper, pure	16	100
Hard-drawn copper . . .	29	97
Silicon bronze	28	97
" "	35	80
" "	48	45
Phosphor bronze	45	26
Cast steel	59	10

The line taken by a wire between two supports is a catenary, but as the sag is always small, and as the supports are generally at equal heights, we may without great error substitute a parabola. In this case the tangent of the angle with the horizontal at which the wire leaves the support is given by the ratio of twice the sag divided by half the span, and this is obviously also the ratio between the vertical and horizontal forces of the support. The vertical force is the weight of half the span, and the horizontal force (which without great error we may consider as equal to the stress or tension in the wire) is therefore

$$T = \frac{W\,S}{8\,s}$$

where W is the weight of wire in one span, S is the span, and s the sag. If a be the area of the wire in square inches, the weight per foot is 3·85 a lbs., and $W = 3{\cdot}85\,a\,S$, which substituted in the equation for T gives

$$T = {\cdot}48\,a\left(\frac{S}{s}\right)S.$$

The stress per square inch of section is

$$\sigma = {\cdot}48\left(\frac{S}{s}\right)S$$

for a wire of hard-drawn copper, and as the density of the various bronzes is about the same as that of copper, the formula may be used also for wires of silicon or phosphor bronze. Thus a wire sagging 4 feet in a 200 feet span would be subjected to a tensile stress of $\cdot 48 \times 5 \times 200 = 4,800$ lbs., or 2·15 tons per square inch. If the dead weight were the only force to be considered, such a wire of hard-drawn copper would have a factor of safety of $29/2 \cdot 15 = 13 \cdot 5$. There are, however, other forces to consider, namely, wind pressure, accumulations of snow and ice, and influence of temperature. The latter, if not duly allowed for at the time the line is erected, may cause a considerable addition to the calculated stress. The effect of cold weather is to cause the wire to shorten, thus reducing the sag and increasing the stress σ. Let L be the length of wire in one span, then from the theory of the parabola we have

$$s = \sqrt{\frac{3}{8} S (L - S)},$$

which inserted into the above expression for σ gives

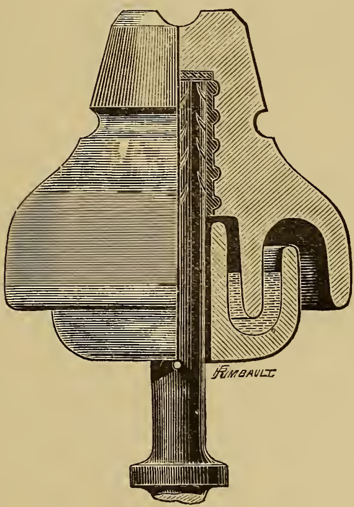
$$\sigma = \cdot 785\ S^{\frac{3}{2}} / \sqrt{L - S}.$$

This expression shows that the stress is increased if L becomes lessened, owing to a reduction of the temperature. The temperature co-efficient of copper for expansion may be taken as ·00172, so that the variation in L with a temperature variation of t centigrade is $\Delta L = \cdot 00172\ t\ L$.

If we know the summer and winter temperatures, we can find the two extreme values for L and the two extreme values for the sag and stress from the condition of the line at the time of its erection; or conversely, we can so settle these conditions when erecting the line that the greatest possible stress in winter shall still leave a sufficient factor of safety.

The posts, insulators, method of erection, use of guards, brackets, etc., are all similar to telegraph practice, though the work is generally of a heavier character and a higher degree of insulation is required. This is generally obtained by means of the Johnson and Phillips's oil

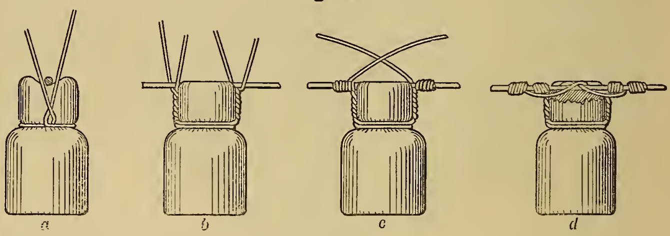
Fig. 124.



insulator, which makes surface leakage impossible. Many forms of these insulators are in use, but the one illustrated in Fig. 124 may serve to show the principle. The insulator has an internal and external bell, and the former dips into an annular trough of china, which can slide on the stalk of the insulator, and is secured in its higher position by a pin as shown. The trough is filled with oil, and this cuts off completely any surface leakage that might otherwise take place. For cleansing and refilling with oil the inner trough is lowered.

All insulators before being put up should be tested to make sure that the glazing is perfect, and that there are no cracks through the material which might allow electrical leakage to take place. To test an insulator it is placed upside down, the inner space is filled with acidulated water, and it is then immersed to near the rim in a bath of acidulated water. If the insulation is perfect, it must be impossible to pass even the most minute current measurable on a delicate galvanometer from the

Fig. 125.

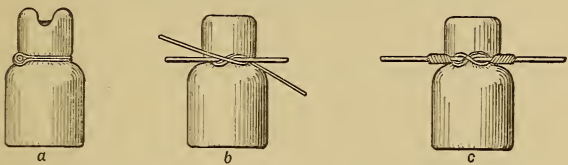


liquid on the inside through the insulator to the liquid on the outside.

The wire may be attached to the insulator either on the groove at the top or at the side, the latter if there should be a bend in the line occasioning a considerable lateral strain. The method of attachment in both cases will be seen from Figs. 125 and 126, where the views *a, b, c, d* and *a b c,* represent respectively the different stages of the process.

Since wire and cables can only be obtained and carried to the place of erection in limited lengths, it is frequently necessary to make joints. A joint should not only be as strong as the wire or cable itself, but it must have an absolutely perfect contact, as otherwise the passage of the current would heat and ultimately destroy it. It is also desirable to avoid the use of other metals than that of the conductor, so as to prevent electrolytic action. The use of solder is, for many joints, a necessity, and must

Fig. 126.

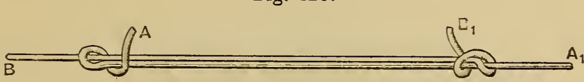


be exempt from this rule ; but it is not advisable to use iron couplings for a line of copper, or any other combination of two different metals. With thin wires a strong

Fig. 127.

joint is made, as shown in Fig. 127, which explains itself. To improve the contact, the middle portion is soldered over. Fig. 128 shows another form of joint suitable for

Fig. 128.



thin wires, which can easily be bent. A_1 A is one wire, B_1 B the other ; the ends A and B_1 are left long enough to allow of being lapped round the middle portion of the joint until they meet, and are then twisted together, as shown in Fig. 129.

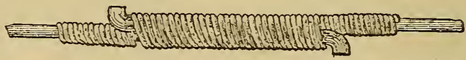
If the wire is too thick to allow of its being easily twisted into a knot, the joint shown in Fig. 130 is sometimes used. The two ends of the wire are bent short at right angles, and placed side by side, so that the ends point outwards. In this position they are held by a

Fig. 129.

clamp whilst being served with a layer of binding wire of the same material as the conductor. When the space between the two ends is completely filled by the binding wire it is soldered over.

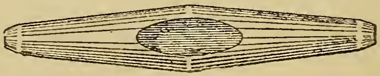
Cables may be joined either by careful splicing or by

Fig. 130.



couplings. A very neat coupling has lately been introduced by Mr. Lazare Weiller; it consists of a double hollow cone (Fig. 131) with an opening in the middle. The end of the cable is inserted at one end, brought out at the central opening, then doubled over and pushed

Fig. 131.



back again through the opening. A pull is applied to the cable as if to draw it out of the coupling, and this has the effect of jamming the end of the cable tightly in the cone. The end of the second cable is treated in the same manner, and to secure perfect contact, and prevent any slipping back, melted solder is poured into the central opening. Fig. 132 shows the coupling in section and the

Fig. 132.

cables in place. As a suitable composition for the solder, Lazare Weiller recommends two parts of block tin to one part of lead. The wire cable and the coupling are both

Fig. 133.

made of silicon bronze, and thus electrolytic action is avoided.

A very ingenious joint requiring no solder has recently

Fig. 134.

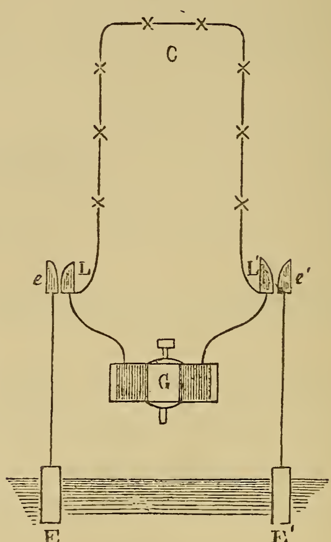
been introduced by Messrs. Schmidmer & Co., of Nürnberg. This consists of a sleeve made of very ductile copper which is placed over the wire ends to be joined,

Fig. 135.

and is then twisted up by special tongs. Fig. 133 shows the sleeve and wire ends before and Fig. 134 after twisting. Even fairly large cables and two heavy cables of different sections can thus be jointed as will be seen from Fig. 135. The strength of the joint is very nearly equal to that of the conductor itself.

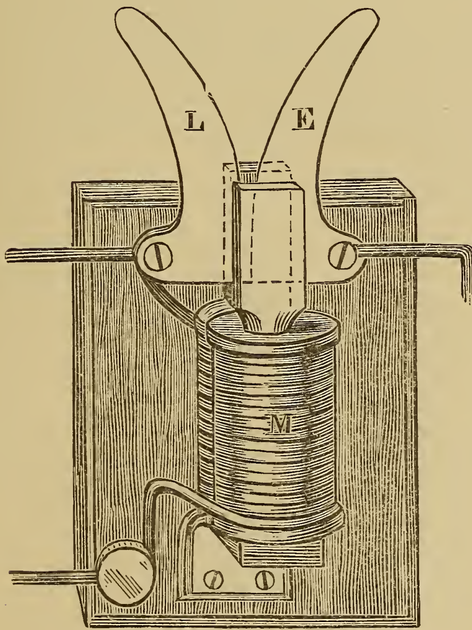
A very important matter in connection with overhead lines is their protection from lightning. The difficulty lies not so much in devising apparatus that will conduct

Fig. 136.



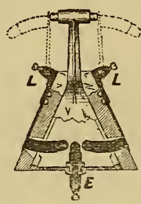
the lightning discharge to earth as in preventing the power current from following the arc established by the lightning flash. The ordinary telegraphic lightning guards are therefore mostly useless for power lines and a large number of special instruments had to be devised for this purpose. The simplest and one of the most reliable is the Wirt guard, which consists of a number of serrated metallic cylinders insulated from each other and set parallel to each other in a row with very little clearance. There are generally seven of these cylinders and the centre cylinder is connected to earth, whilst the two outer cylinders are connected to the two line wires. One such group is used for an alternating current circuit up to

Fig. 137.



1,000 volts, and for higher pressures more groups are put in series and the connections are suitably altered. The principle on which this instrument is based is that the ordinary power current has not a sufficient pressure to leap over from one cylinder to the other, whilst a lightning flash does so easily. The cylinders are made up of an alloy of zinc which is incapable of sustaining an arc. The precise reason why this Wirt alloy should quickly extinguish any arc is not known; it seems, however, probable that on the first passage of the current zinc is burned and the zinc oxide volatilized, thus forming an insulating atmosphere which prevents the further flow of current. For continuous current lines Professor Elihu Thomson has devised the arrangement of lightning guards shown in Figs. 136 and 137. *G* is the generator and *C* the line (in this case an arc circuit), each terminal of which is protected. The lightning stroke passes from

Fig. 138.



L to *E* and thus to earth, and if the power current attempts to follow, the arc thus established is blown upwards by the poles of the electro-magnet *M*. The arc is thus transferred to the upper region of the terminals, *L E*, where the distance is greater and the metal not yet heated, with the result that the arc is immediately broken. A very ingenious lightning guard introduced by the Westinghouse Company and applicable to alternating and continuous current lines is shown in sections in Fig. 138. It consists of a closed box with a fixed carbon electrode, *E*, projecting through the bottom and two movable carbon electrodes, *L*, passing through holes in the sides. The movable electrodes are attached to pivoted arms and connected with the lines, the fixed electrode E is connected to earth. If a lightning flash leaps from one carbon to the other and the power current attempts to follow the heat suddenly generated within the box expands the air with almost explosive force and the two movable electrodes are shot outwards, thus rupturing the arc. The motion is arrested by indiarubber buffers at the top and the arms immediately fall back into their normal position so as to be ready for the next stroke.

CHAPTER XII.

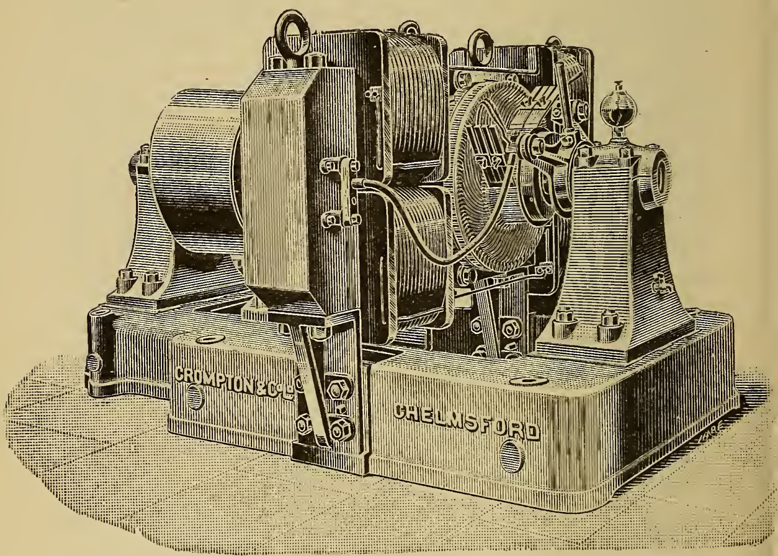
Examples of Dynamos and Motors—Crompton—Edison-Hopkinson—Wolverhampton—Siemens—Brush—Mordey—Kapp—Brown—Wenström—Thury—Oerlikon—A. E. G. Company of Berlin.

The previous chapters dealt with the general principles of electric power transmission, and with the conditions to be fulfilled by the various parts of the plant, without entering into the details of any inventor's so-called "system" or any maker's special apparatus. The question of system—in so far as the term applies to some patented arrangement for which the inventor claims that it is the best under all circumstances—is becoming of less importance year by year. Experience has shown that in power transmission, as in lighting, no inventor's particular system is universally applicable, and that the success of such works is mainly dependent upon the common sense and engineering skill of the men who plan them, and the less they are trammelled by adherence to any cut and dried system the better.

An electric power transmission is a very complex piece of engineering. To begin with, at one end we have the apparatus for producing the power. If this be derived from water, it means impounding reservoirs, dams, sluices, heavy foundations for the machinery, turbines or water-wheels, provision for floods, regulating appliances, and all the gear for getting the power to the generators. Then we have the generators themselves, with their regulating appliances, the line, the motors at the other end, more regulating appliances, and the gear for delivering the power where wanted. The transmission plant as a whole requires, therefore, the services of a civil engineer, a mechanical engineer, and an electrical engineer. With the work of the civil and mechanical engineer this book is not concerned. As regards the electrical part of the subject, this has been treated in the previous chapters, except as regards the more minute details of apparatus. A comprehensive treatment of this part of the subject would, however, require far more space than can be given in this book, for it would be nothing less than a treatise on the design and manufacture of dynamos, alternators, transformers, switchboards, and other accessories Of books treating those subjects more or less in detail, there are already several in the market, and it is therefore not necessary to describe such machinery here at any length. In order, however, to give the student some idea of the practical construction of dynamo-electric apparatus, a few examples are given in the following pages. In some cases the dynamos described were constructed for lighting ; but, after what has been said in the earlier parts of this book, it will be clear to the reader that any lighting machine can also be used for power purposes. The list of machines described is necessarily incomplete ; in fact, some amongst the best machines are not included, the reason being partly want of space and partly the maker's reluctance to publish details.

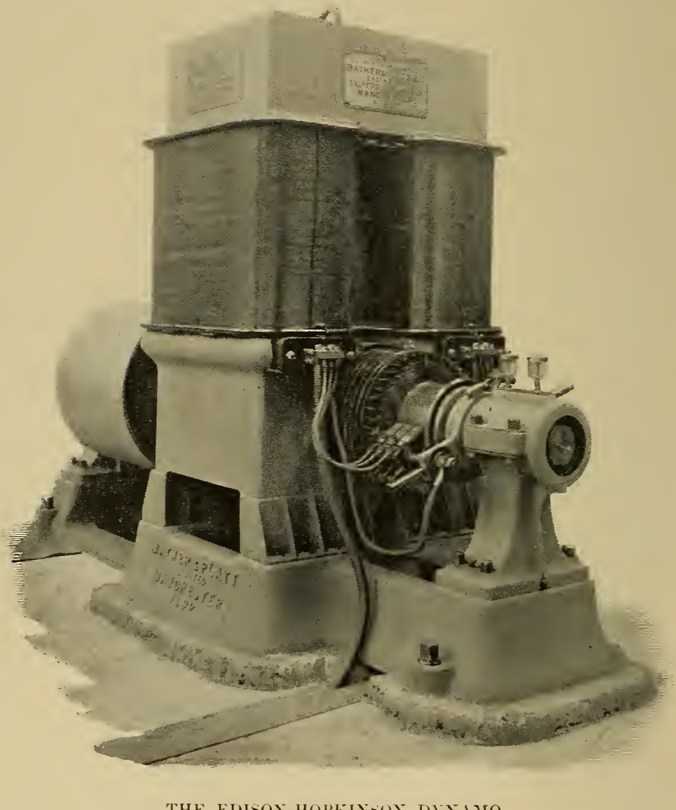
The Crompton Dynamo.—Fig. 139 shows a perspective view, and Fig. 140 an elevation of a 100 horse-power Crompton motor, largely used for mining work. The field consists of two horse-shoe magnets producing four poles, and the armature is a four-pole series-wound drum, two sets of brushes only being required. The armature core is 24 in. diameter, and consists of a number of thin charcoal-iron discs, smooth on the outside, but provided on the inside with notches placed equidistantly. Into the grooves thus formed radial bars are fitted, by which

Fig. 139.



the armature is supported on the central hub, which is keyed to the spindle, as shown. The discs are insulated from each other, and the armature-bars, which consist of stranded and compressed rectangular cable, are held in place by a large number of fibre driving-horns, sunk into the surface of the armature core. The motor is rated at 100 B. horse-power at 600 revolutions per minute when supplied with current at 455 volts. The magnet cores

Fig. 141.



THE EDISON-HOPKINSON DYNAMO.

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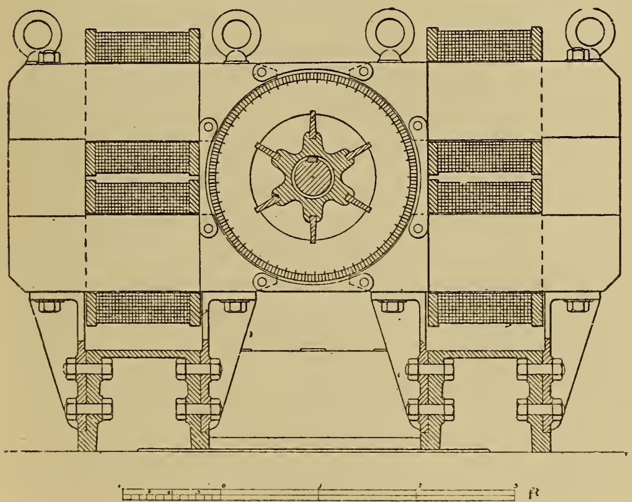
are of wrought-iron, carried on gun-metal brackets. The armature contains 358 conductors, and the commutator 179 bars. The induction in the armature core is 10,000, and that in field magnet cores is 15,000 C.G.S. lines per square centimetre. The efficiency of this machine is given by the makers as 93 per cent.

The Edison-Hopkinson Dynamo.—This machine is the

Fig. 140.



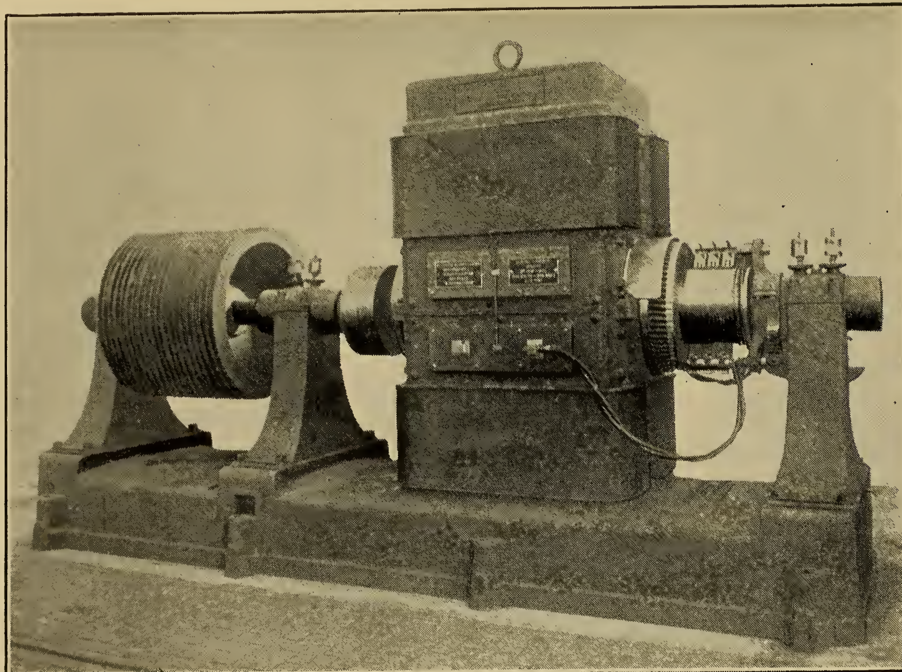
result of important improvements made by Dr. Hopkinson in the original Edison machine. Fig. 141 illustrates one of the latest machines of the improved type as made by Messrs. Mather and Platt for the Central Electric Light Station of the Manchester Corporation. The following particulars are given by the makers :—Speed 400 revolutions, output 590 ampères at 410 volts, armature resistance ·017 ohms, field magnet shunt-wound resistance

52·7 ohms. Two of these machines were tested by working one as a generator, and the other as a motor with spindles rigidly coupled. The power is thus circulated between the two machines, and only the loss of power has to be supplied from outside, which in this case was done by placing a third machine in series with the two machines to be tested. The makers found that the efficiency of the two combined machines when tested in this manner was 90·5 per cent giving $\sqrt{90{\cdot}5} = 95$ per cent. efficiency for each. The total weight of the machine is a little over 24 tons, and that of the armature alone is close upon 3 tons.

The Wolverhampton Dynamo.—The machines made by the Electric Construction Corporation of Wolverhampton are all bi-polar, with either single or double horseshoe magnets, the latter for larger sizes. One of these is illustrated in Fig. 142. Four machines of this size are in use at the power-house of the Liverpool Electric Railway, the output of each is 475 ampères at 500 volts, the speed being 400 revolutions per minute. A good feature in the mechanical design is the coupling between the armature and the rope pulley, whereby it is rendered possible to remove the armature for repairs without having to take the driving ropes off. Machines of this type have been built up to 2,000 volts.

Siemens Dynamo.—English and Continental practice differ sometimes very much. Thus Messrs. Siemens and Halske of Berlin prefer for large dynamos the multipolar type, whereas the London firm of Messrs. Siemens Brothers are in favour of the two-pole type for all sizes. The multipolar type is undoubtedly lighter in large sizes, whilst the two-pole type has the advantage of simplicity in construction and fewness of parts. The armature is

Fig. 142.



THE WOLVERHAMPTON DYNAMO.

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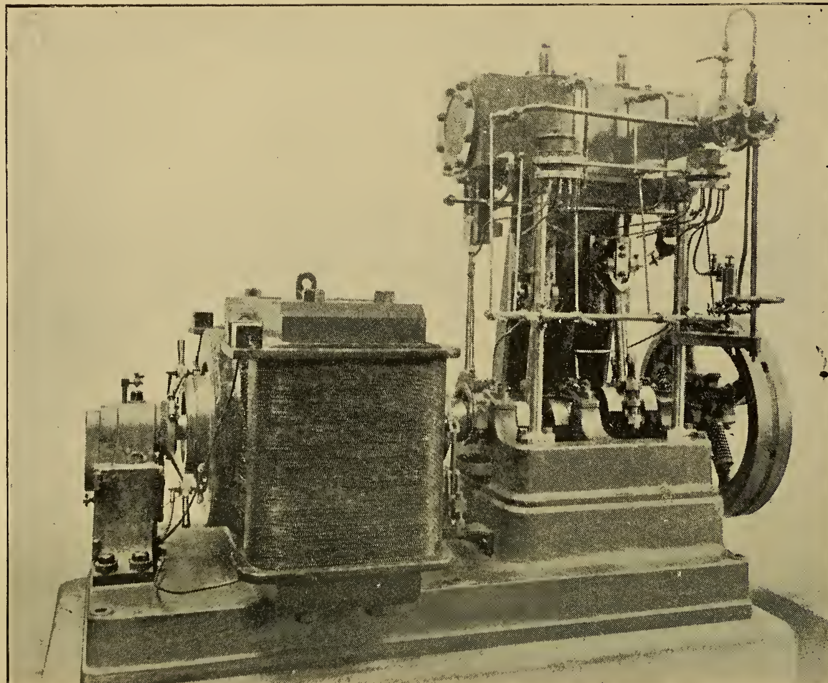
Fig. 143.



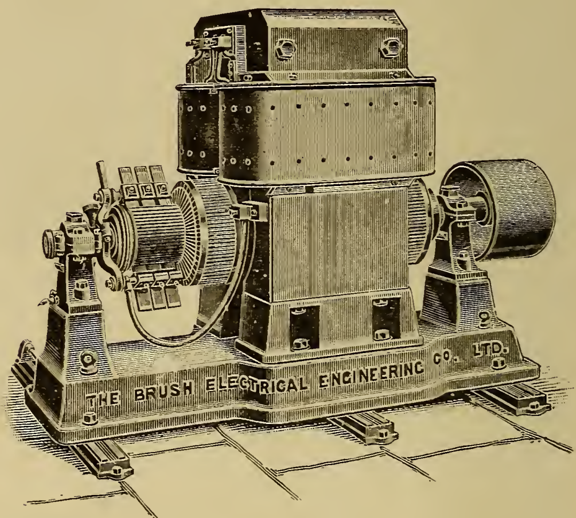
SIEMENS DYNAMO.

To face page 407.

smaller in diameter, and contains a smaller number of active conductors. When wages are high and materials cheap the two-pole type may thus be economically possible even for large sizes. Messrs. Siemens, of London, have introduced a high voltage machine with automatic regulation for constant current. The regulation is effected by shifting the brushes into positions of higher or lower E.M.F. on the commutator. The rocking frame which carries the brushes is geared with the dynamo shaft by ratchet wheels, crank motor pawl. The latter is controlled by a solenoid through which the external current passes, and accordingly as the current is above or below its normal value, the solenoid throws the pawl into gear with either one or other ratchet wheel, thus shifting the brushes into a position of lower or higher E.M.F. The dynamo illustrated in Fig. 143 is direct-driven by a compound steam engine at 300 revolutions per minute, and is intended for an output of 10 ampères at a maximum pressure of 1,800 volts. The field is of the double magnet Manchester type, and is series wound. The armature is of the ring type with a core 30 inches diameter by 17 inches long, and the commutator has 200 parts.

The Brush Dynamos.—Fig. 144 and 145 illustrate the two types of dynamos at present made by the Brush Electrical Engineering Company. Both types are now much in favour with many English makers and need no special description. Fig. 146 shows the Mordey Victoria alternator, with exciting dynamo mounted on the same spindle. The revolving field magnet system consists of two heavy iron castings, with projecting polar extensions facing each other, but with sufficient clearance for the fixed armature between them. The latter is built up of segmental coils, consisting of flat copper strip wound with insulating material over slate cores, and then covered with a special enamel to insure high insulation. Each coil is held by german silver attachments and bolts in a gun-metal frame which is made removable for repairs. Owing to the absence of iron within the armature coils

Fig. 144.



the self-induction, and consequently the drop of pressure between no load and full load, is remarkably small.

Kapp Dynamos.—One of the earliest forms of the author's dynamos, as made by Messrs. Allen and Co. and used chiefly for ship lighting, is shown in Fig. 147. The type is so well known that no special description is required. A multipolar type adopted by Messrs. Johnson and Phillips, for central station and power purposes, is shown in Fig. 148. The machine illustrated is driven by

Fig. 145.

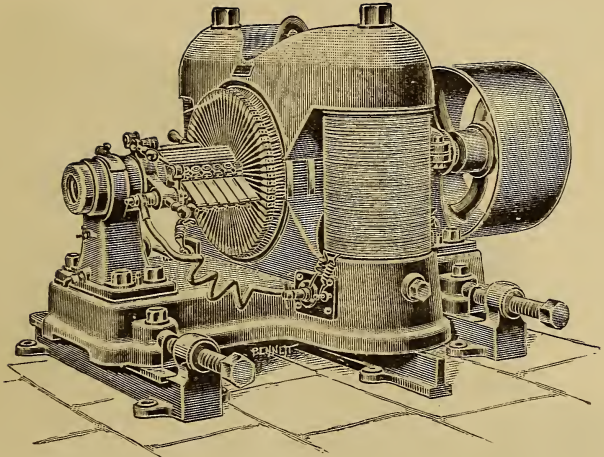
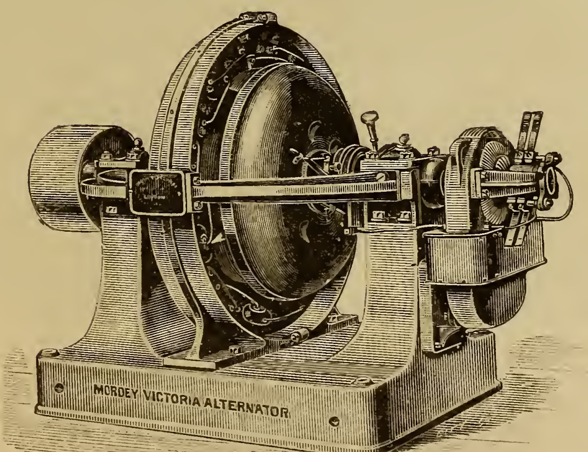


Fig. 146.



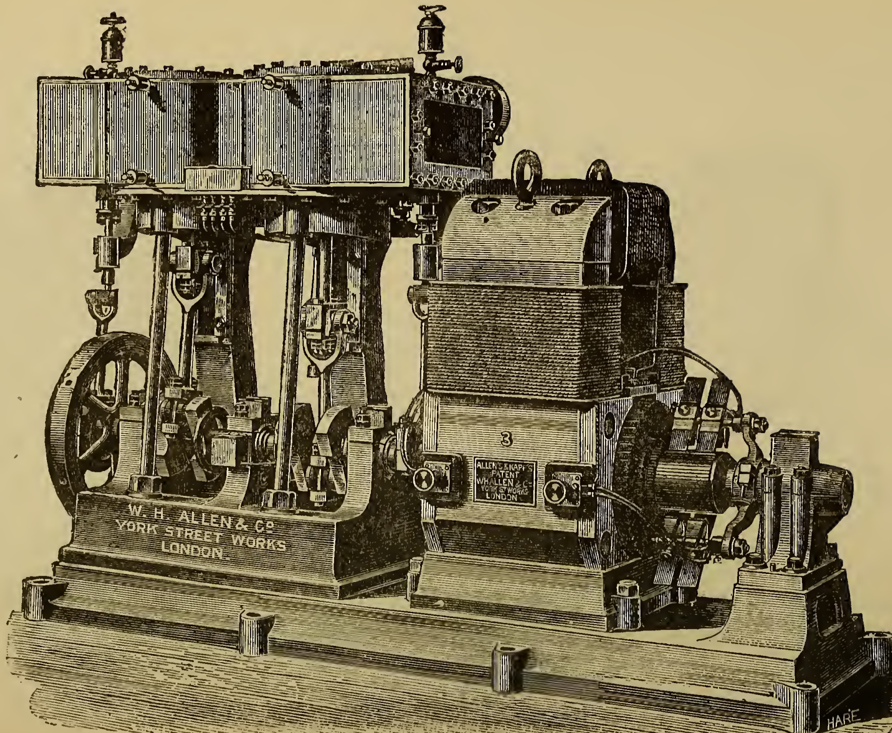
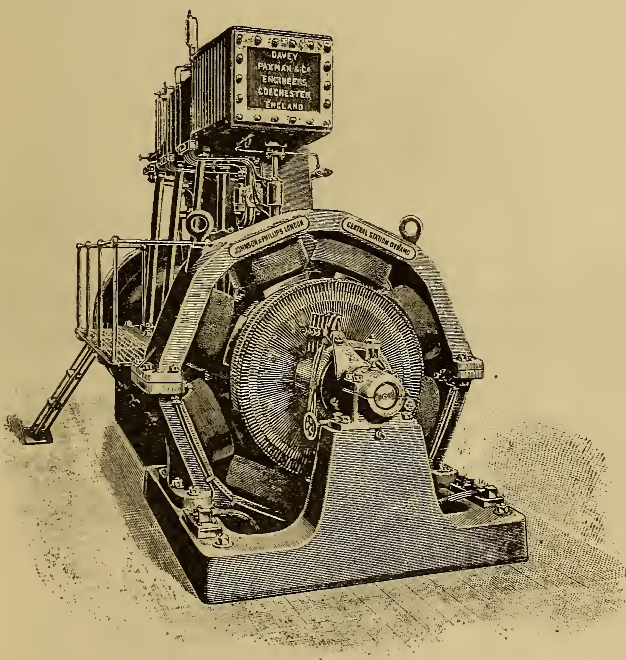


Fig. 147. COMBINED KAPP DYNAMO AND ALLEN ENGINE.

a triple expansion Davy-Paxman engine at 120 revolutions per minute, and has an output of 500 ampères at 260 volts or 130 K.W. The field has 8 poles and the armature is drum-wound with series end connections so as to necessitate only two sets of brushes 135° apart. A

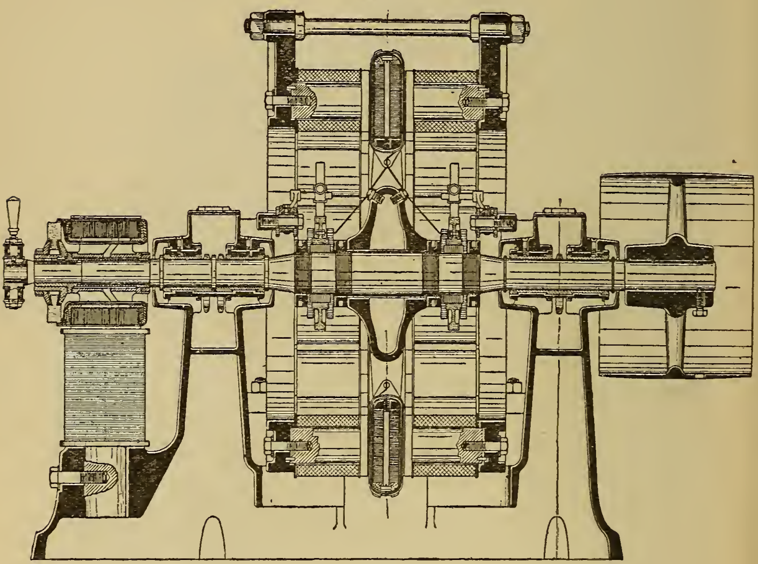
Fig. 148.



machine of the same type, but with a four pole field and supplying current for a military electric railway, has the following constructive data—output 47 K.W. at 450 volts, speed 200 revolutions per minute, armature core 31 inches diameter by 20 inches long, wound with 470 bars with series end connections. Field magnets of cast steel, diameter of radial magnet cores 15 inches. The field magnets are compound wound, the shunt exciting power being 18,000 and the series exciting power 5,000 ampère-turns.

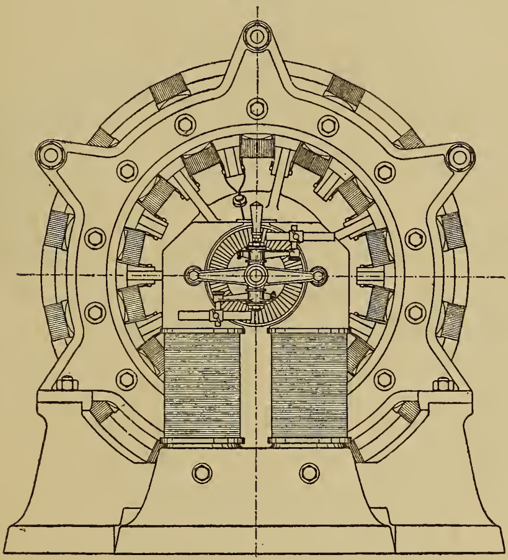
The original type of the author's alternator, as made by the Oerlikon works and largely used in Continental

Fig. 149.



lighting and power stations, is shown in Figs. 149 and 150. The field system consists of two sets of magnets, one on either side of the armature, with rectangular pole faces and circular yokes. The armature core is composed of charcoal iron strip coiled up with paper insulation over a gun-metal supporting wheel, to form a narrow and deep ring to which lateral strength is given by steel bolts inserted radially as shown. The core thus formed is carefully insulated with mica and press-spahn and the armature coils are wound transversely over it, and held in place at the outer and inner circumference by driving pieces of insulating material. A more modern form of the author's alternator is shown in Fig. 151, which represents a type recently introduced by Messrs. Johnson and

Fig. 150.



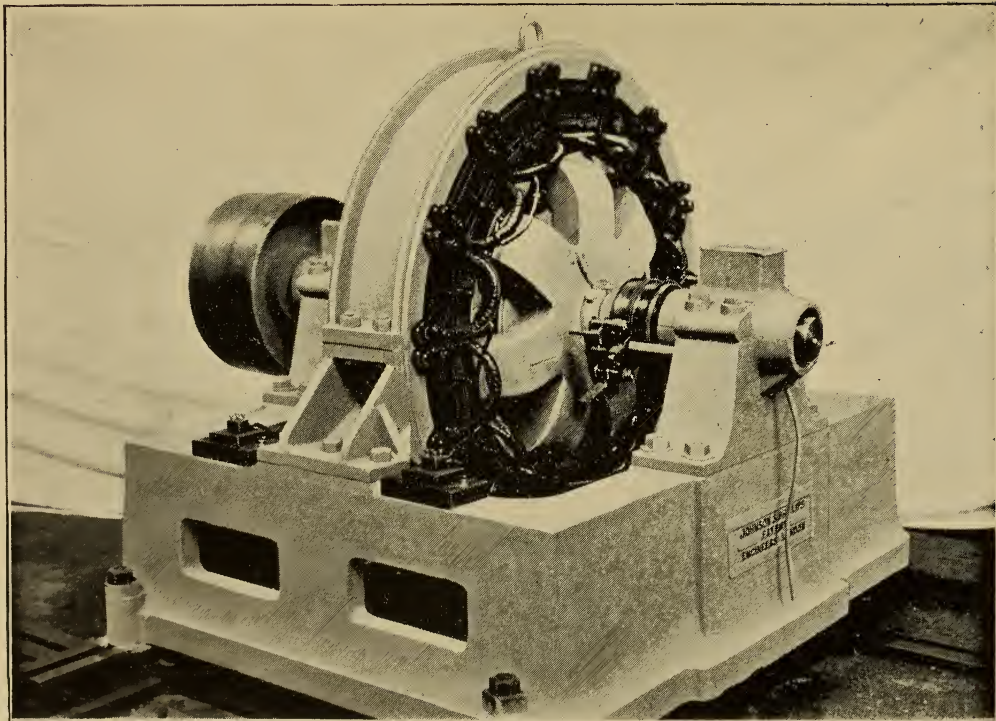
Phillips for lighting and power purposes. In this type the armature is fixed and the field magnet system revolves. The latter consists of two claw-shaped steel castings with exciting coil between them, whereby opposite polarity is produced on alternate polar faces. The armature consists of as many segments as there are poles, each segment separately supported by gun-metal brackets and steel bolts in an external cast-iron shell, and therefore easily removable. Each segment consists of a core of charcoal iron plates of such shape as to form two longitudinal grooves in which is placed the armature coil. The latter may, before the core plates are inserted, be insulated all over its surface to any desired thickness, thus rendering this type of machine specially suitable for long distance transmission at high pressure.

A plant of this kind has recently been made by Messrs. Johnson and Phillips for the Sheba Gold Mines in South Africa, where power is transmitted by two phase current at 3,300 volts over a distance of five miles. At the generating station where water power is available there are two 132 Kwt. two phase alternators of the type illustrated in Fig. 150, and at the motor station the pressure is reduced by step-down transformers to 100 volts, and this low pressure current is supplied to the various motors driving the stamps and other machinery.

The Brown Dynamos.—For continuous current transmission plant Mr. Brown employs the type of machine shown in Fig. 152. The field is of the Manchester type, with cast-iron pole pieces and circular magnet cores, the latter being fitted flat against the pole pieces, and not let in as is the usual practice. The bottom pole piece is cast in one with the bed-plate and bearings. The armature is of the ring type, and very carefully insulated, so that the use of these machines becomes possible up to 3,000 volts, and even more.

For alternating current work Mr. Brown has adopted the type of generator shown in Fig. 153. The field magnet system consists of a massive yoke ring with magnet cores projecting radially inward. The armature is of cylindrical shape, and the core is built up in the

Fig. 151.



NEW TYPE KAPP ALTERNATOR.

To face page 414.

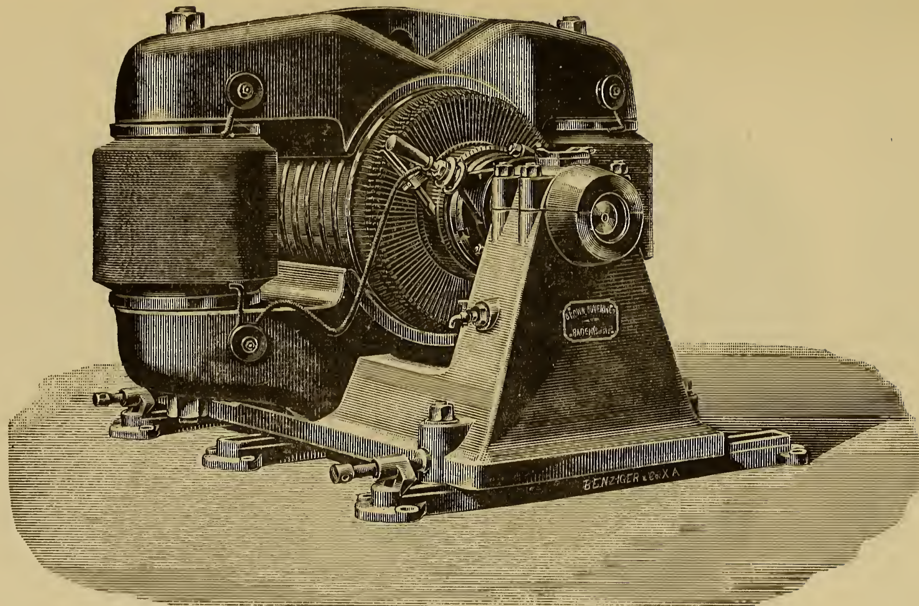


Fig. 152.

usual way of iron disks. The armature is wound drum fashion, and the ends of the coils on either side are protected by special gun-metal rings. In some cases the machine is built with a vertical shaft, so that the armature may be carried directly on the turbine spindle.

Fig. 153.

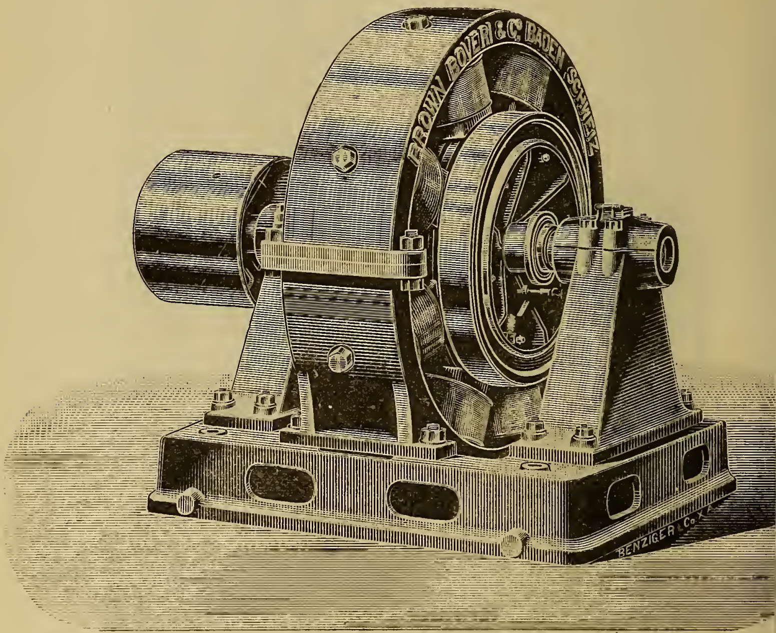
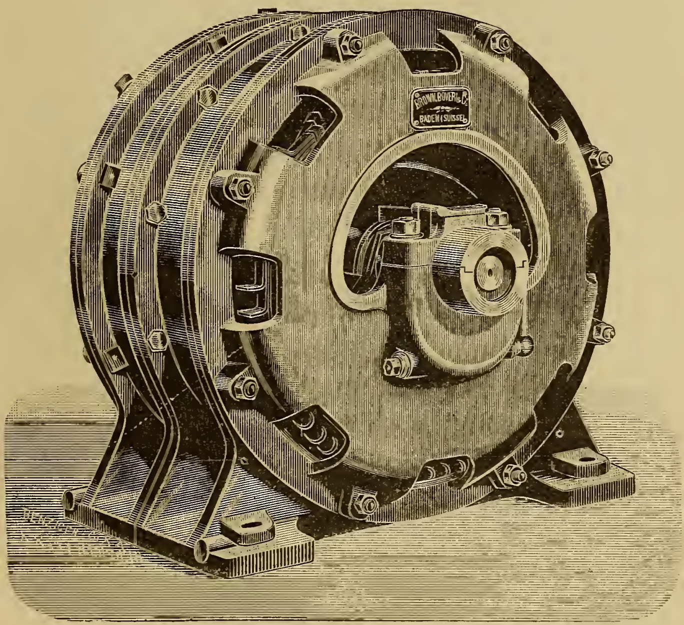


Fig. 154 shows a Brown multiphase motor. Both the field and the armature conductors are laid in holes very close to the circumference so as to reduce magnetic leakage. The armature in larger machines is wound with three distinct circuits, so as to form a star connection with one common centre and three free ends, which are brought out to three insulated contact rings, each provided with a brush, the three brushes are joined to a rheostat with multiple contact switch. At starting the whole of the resistance is in circuit, and as the motor begins to work the resistance is gradually reduced until at full speed the resistance is completely cut out. The

Fig. 154.

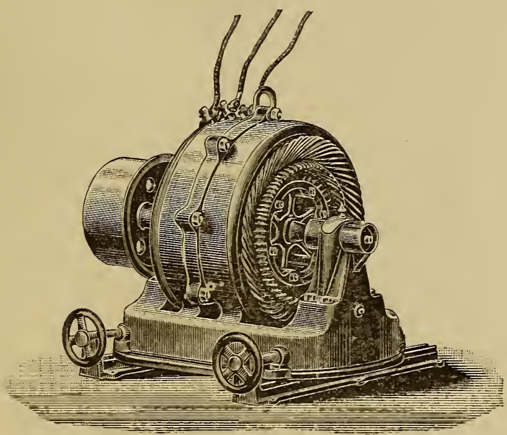


single phase motors are of the same appearance as Fig. 154, and the starting is effected by means of the transformer and condenser arranged as shown in Fig. 119. The test recorded in Chapter X. refers to a motor of this kind.

Wenström's Dynamo.—The late Mr. Wenström has been one of the pioneers in multiphase working, and was also the first to employ buried conductors in the armature. The alternator now made by his firm in Vesteras, Sweden, is of the same type as Brown's, and is made for either single or three-phase currents. The armature is drum wound with star connection, the conductors being heavy bars placed in insulated holes close to the external circumference of the core. The following description refers to a 100 horse-power three-phase generator, of which four are used in the transmission plant of Hellsjön Grängesberg, Sweden. The field has 14 poles, and the speed is 600 revolutions per minute, giving a frequency of 70 cycles per second. The armature contains 84 holes, and there are two conductors in each hole, making in all 168 conductors, or 56 conductors per phase. The current in each phase is 190 ampères, and the effective pressure 150 volts, giving an output of 85·5 apparent Kwt., which, with a power factor of 88 per cent., makes the true output 100 horse-power. The armature weighs 15 cwt., and the complete machine 3 tons. One of the advantages of machines having iron in the armature is their perfect safety in case of a short circuit on the line. In the event of a short circuit the self-induction of the armature prevents the current rising too much. The machine under notice gives when working at full load, and under full exciting current on short circuit, only double its normal current. Of the four machines in the power-house two are working in parallel for power purposes, one is used for lighting, and the fourth is kept in reserve. Each machine is driven by its own turbine. In the lighting machine only two of the phases are used, whereby only two-line wires are required. The power line, which is 8·5 miles long, consists of three 160 mils.

bare copper wires, carried on Johnson and Phillips' oil insulators on wooden posts 28 feet high, underground cable being used for crossing railways and high roads. The pressure is raised at the power-house by step-up transformers to 5,200 volts in each branch, making the difference of potential between any two-line wires 9,000 volts. The power is taken off at various points on the line by means of step-down transformers and three-phase

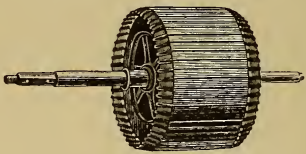
Fig. 155.



motors, of which there are five ot 45 horse-power, three of 30 horse-power, and one of 9 horse-power. The type of motor employed is illustrated in Fig. 155. The small motor is provided with a squirrel cage armature, as shown in Fig. 156, but the larger motors have armatures wound with three closed circuits and provided with contact rings, as shown in Fig. 157.

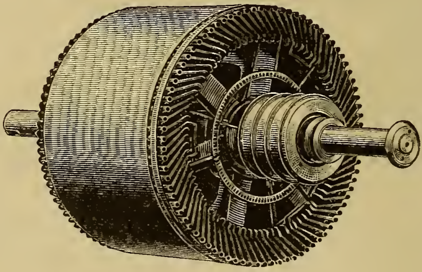
Thury Dynamo.—The makers of this machine (L'Industrie Electrique, of Geneva) have developed a very interesting system of power transmission and distribution by means of constant current and variable voltage, according to the requirements of the customers receiving power from the line. The generators employed in these plants

Fig. 156.



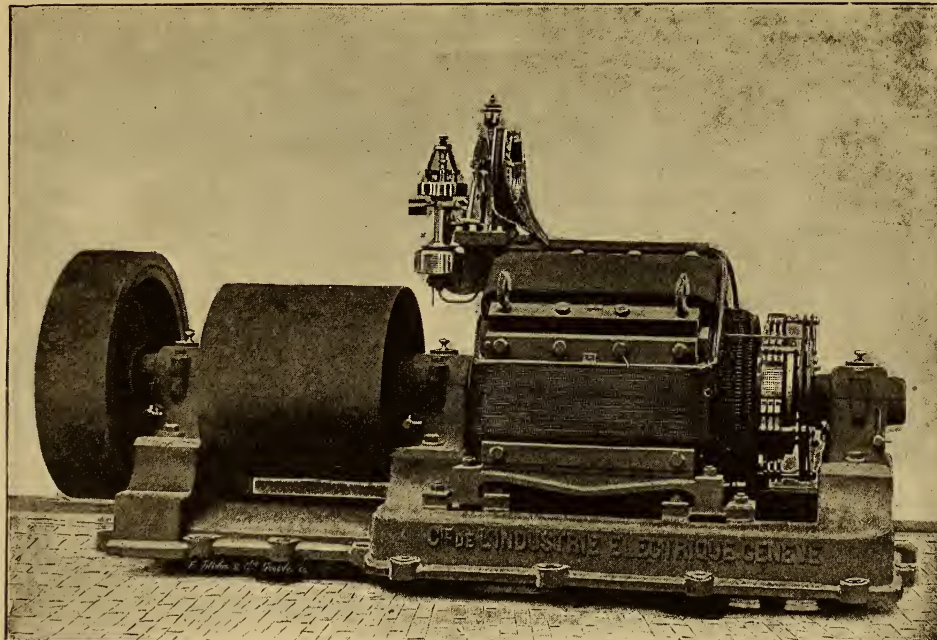
are of the type shown in Fig. 158. They are six-pole continuous-current dynamos, with drum-wound armatures. Great precautions are taken as regards insulation, not only of the winding from the core, but of the

Fig. 157.



machine itself. Thus the armature core is insulated from the shaft, and to prevent sparking from the armature conductors to the pole-pieces, the whole armature, after winding, is wrapped round with layers of paper and shellaced cloth. The field-magnets are insulated from the bed-plate by thick layers of mica, and the bed-plate

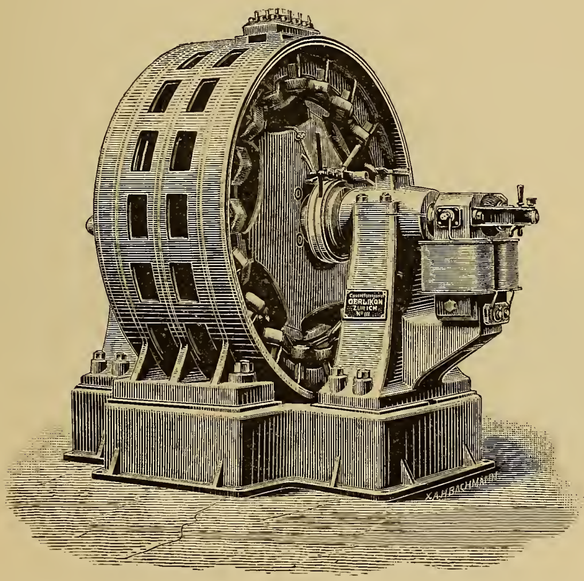
Fig. 158.



itself is insulated from the ground by porcelain insulators, with circular channels filled with resin oil. The machine illustrated has an output of 47 ampères at 1,100 volts when driven at a speed of 475 revolutions. Eighteen of these machines are employed in the power transmission between the river Gorzente and the town of Genoa. The total available fall of water is 1,200 feet, and this is taken up in three power stations, which divide the head of water between them. The two larger stations are each fitted with eight generators, which are coupled in series, and the current is sent to Genoa by an overhead line supported on oil insulators, the wire being 350 mils. diameter. Special automatic apparatus is employed to keep the current constant at 47 ampères, whatever may be the variation in pressure produced by the stopping or starting of the motors on the circuit. The motors are all series-wound Thury machines, and are of the two-pole type up to 15 horse-power, four-pole type between 15 and 30 horse-power, and six-pole type for larger powers. These motors are placed on the premises of the power company's customers, and are all connected in series on the line, which is about twenty-seven miles long. The speed of the motors is regulated by a centrifugal governor acting upon the field-winding in such a way that the exciting power is decreased as the speed increases, and *vice versâ*. To stop a motor the customer has simply to short circuit it. The total power thus distributed in Genoa is about 1,000 horse-power.

Oerlikon Dynamos.—The continuous-current machines for moderate powers, built by the Oerlikon Works, are of the " Manchester " type, Fig. 28, and for larger sizes of the multipolar type, the latter with vertical spindles for direct coupling to the turbine shaft where such an arrangement is expedient. In some cases alternators are also arranged in this manner, and to relieve the thrust bearing or foot step of the turbine from the weight of the revolving masses, these makers have introduced a most ingenious magnetic balancing arrangement, consisting of an annular electro-magnet and ring-shaped armature of

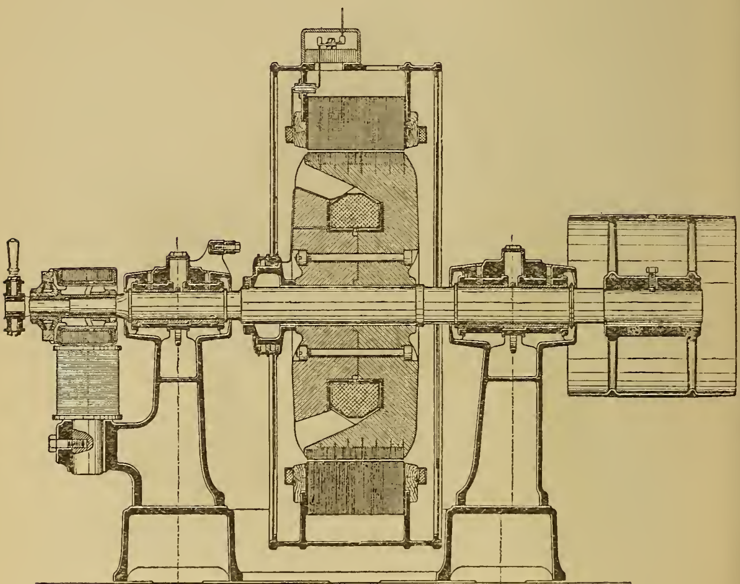
Fig. 159.



coiled iron strip. The latter is firmly attached to the turbine shaft, and revolves in close proximity under the poles of the fixed magnet. The magnet is excited by a continuous current, and by suitably proportioning the surface and number of poles any desired magnetic pull upwards can be obtained, thus relieving the bearings of down thrust. The action of this device is more certain and the loss of power smaller than with any form of hydraulic balancing gear yet introduced.

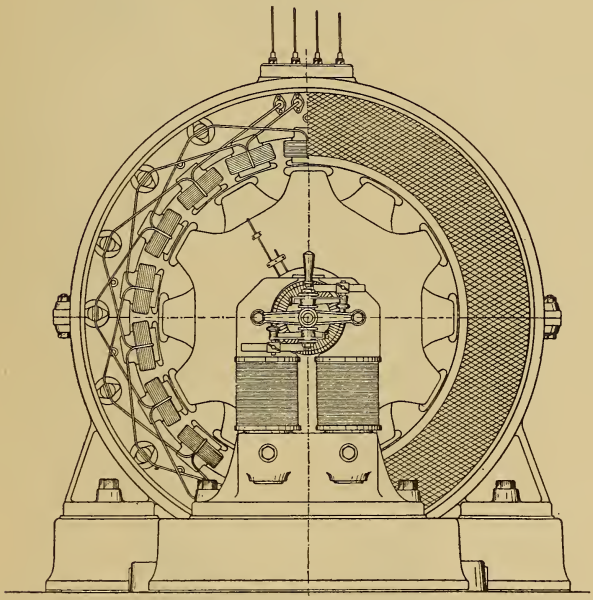
In addition to the author's types of alternators, which have already been illustrated, the Oerlikon Works make a three-phase generator of the type shown in Figs. 159, 160, and 161. The field-magnet system is of the single

Fig. 160.



coil claw type, and the armature consists of a continuous ring of iron plates with teeth projecting inwards. The teeth are widened out at their inner ends, but not sufficiently to meet. The object of leaving spaces between the ends of the teeth is to permit of the armature coils being inserted and afterwards squeezed tight against the narrow part of the tooth by a special clamping tool. The number of poles is to the number of coils as 2 : 3, and the coils are coupled up to form the three circuits of the three-phase system. As the gaps in the inner surface of the armature produce fluctuations in the magnetic flux which would cause waste of power and heat the face of the pole-pieces if these were made solid, subdivision of

Fig. 161.



the metal is obtained by tipping the pole-pieces with thin sheet-iron stampings, as shown in Fig. 160.

Figs. 162 and 163 show details of the construction of the Oerlikon transformer. The coils are wound on separate paper sleeves and are placed over the central core, which is built up of plates of different widths so as to approach a circular shape. The magnetic circuit is

Fig. 162.

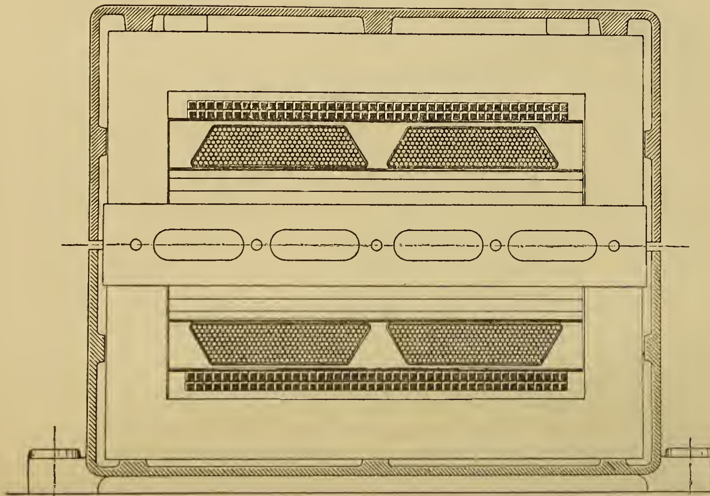
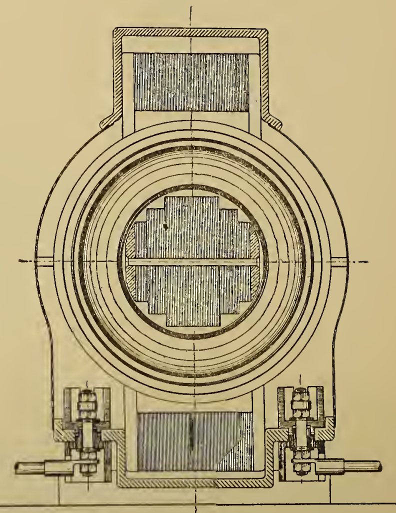
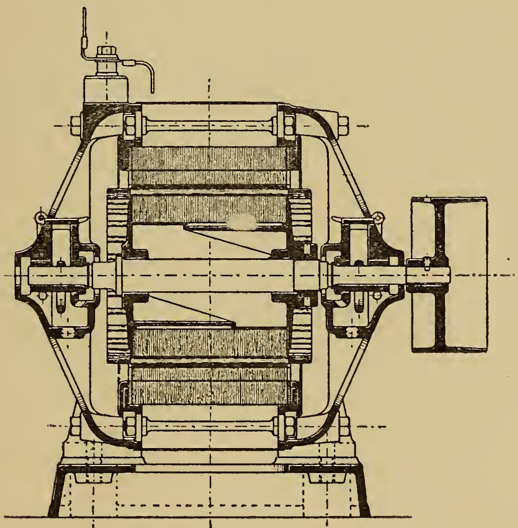


Fig. 163.



closed by a top and bottom yoke clamped tightly in the two halves of the cast-iron case. For three-phase transformers a vertical design is adopted, consisting of three cores joined at both ends by annular yokes. The surface of small and moderate-sized transformers is sufficient to radiate all the heat generated into the air, but as with

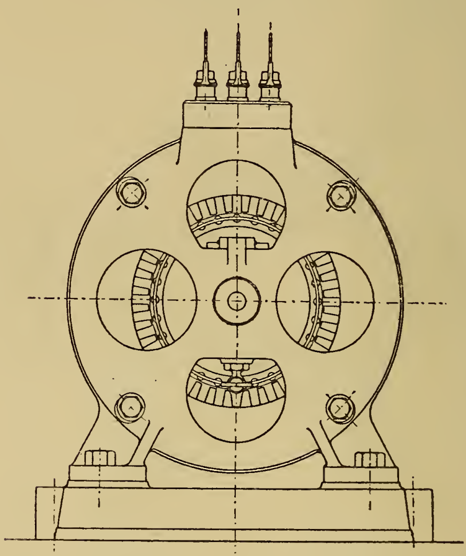
Fig. 164.



increase of size the surface augments only as the square of the linear dimensions, whereas the bulk and power wasted grow as the cube of the linear dimensions, a point is finally reached where even the most efficient type of transformer cannot be kept cool enough by air circulation, and it becomes necessary to fill the transformer case with rosin oil, and in some cases to circulate the oil by means of a pump through cooling tanks.

The general design of single and multiphase motors made by the Oerlikon Works is shown in Figs. 164 and 165. Small motors have an armature with squirrel cage winding, and in larger motors the armature is drum wound with three distinct circuits and the winding is connected to contact rings as already explained.

Fig. 165.



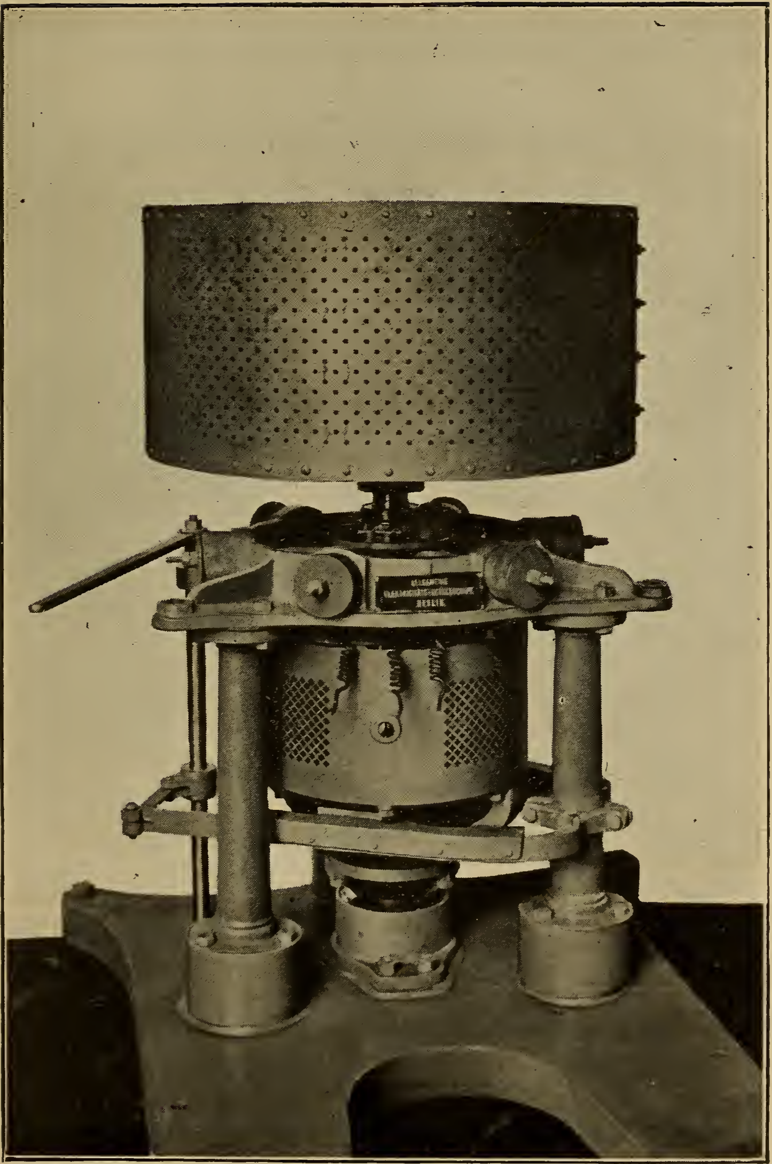
The Oerlikon Works are driven by electric power brought from Bülach, over a distance of fourteen and a half miles. In the generating station there are three 200 horse-power turbines, and to the vertical shaft of each is coupled direct a three-phaser giving at 180 revolutions in each phase 1,500 ampères at 50 volts. The frequency is 48. Each generator is connected with the low pressure winding of a transformer, and the high pressure winding is arranged to raise the pressure in each phase to 7,500 volts or 13,000 volts between any two line wires. The coupling of the phases in the generators and in both windings of the transformer is star fashion, the common centres being in metallic connection and also well earthed. Ordinarily only two generators are in use and the corresponding transformers are coupled parallel on the high pressure side to the three line wires, which consist of 160 mil bare hard drawn copper wire carried on oil insulators. The posts are of wood 33 feet high, and are placed at an average distance of 330 feet. In addition to the three power wires they carry two telephone wires, lead and return. At the motor station in the Oerlikon Works the current is transformed down in two transformers, with primaries in parallel and secondaries in series, and is then led into a three-phase motor which drives the whole of the works. A phase rectifier is used on the secondary circuit at the motor station. At starting one turbine only is at first used, and the armature circuits of the motor are closed by a three pole switch. When the motor is under way the second turbine is started and the corresponding transformer thrown into parallel with the first. No synchronizer or other appliance is used, the machines pulling each other into place without any difficulty when the speeds are approximately equal.

A similar transmission plant, but combined with distribution of current between a number of small consumers of light and power, is that between Killwangen and Zürich, a distance of twelve and a half miles. The generator is a 300 horse-power three-phaser driven by a turbine, and a step-up transformer raises the pressure in each phase from 50 to 3,000 volts for transmission to Zürich on an overhead line. From a central point in

Zürich branch lines are taken to the different customers for the supply of light and motive power by means of step-down transformers. The motors used are 3 horse-power, 10 horse-power, and 20 horse-power. Variations of pressure in the power circuits do, of course, not affect the speed of the motors as this depends only on the frequency, but such variations would affect the lighting service, and to overcome this difficulty a fourth line wire is used connecting the centres of the star coupling at either end. Similar three-phase plants for light and power combined are at work or in course of erection at Wangen (Würtemberg) for 350 horse-power, Percine (Tyrol) for 100 horse-power, St. Etienne (France) for 1,000 horse-power, and Florensac (France) for 100 horse-power.

The A. E. G. Dynamos.—Some account of continuous current plant for power transmission, made by the Allgemeine Elektrizitäts Gesellschaft of Berlin, has already been given in a previous chapter. In addition to continuous current work this firm make a speciality of three-phase power transmission, in which branch their electrician, Herr Dolivo von Dobrowolsky, has been one of the earliest pioneers. The results of tests with one of his motors has already been given in Chapter X, and Fig. 166 shows a novel application of his motors for industrial purposes. The centrifugal machines in sugar refineries have hitherto been worked by belt, but considerable difficulties were experienced with the bearings on account of the side stresses produced by the belt, whilst much time was lost in bringing the machine up to speed after each charge. These drawbacks have now been overcome by the use of a three-phase motor coupled direct on the spindle of the centrifugal machine. Not only are

Fig. 166.



CENTRIFUGAL MACHINE WITH DOBROWOLSKY MOTOR.

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all side stresses thereby completely avoided, reducing wear and tear and economizing power, but the machines may be brought up to full speed much more quickly, and the output of the works is thus very considerably increased.

THE END.

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