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A comparison of ship maneuvering characteristics for rudders and podded propulsors

Betancourt, Michelle K.

Monterey, California. Naval Postgraduate School

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

A COMPARISON OF SHIP MANEUVERING CHARACTERISTICS FOR RUDDERS AND PODDED PROPULSORS

by

Michelle K. Betancourt

June 2003

Thesis Advisor: Fotis Papoulias

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A comparison of a high speed container ship using a rudder versus a podded propulsor is made to study replacing a rudder with a pod. A mathematical model is altered to simulate a ship operating with a rudder and with a pod to maneuver. The model incorporates the nonlinear maneuvering equations and couples the surge and sway forces, yaw and roll moment, and the roll angle induced during a steady turn with varying rudder and pod angles. The model uses the hydrodynamic derivatives and coefficients for a high speed container ship. The equations are numerically integrated in order to predict the roll angle, sway and surge velocities, and the ship's position in the xy-plane. Both transient and steady state results are utilized to quantify the relative efficiency of each system. The results are used as a preliminary study into replacing a rudder on a ship with a podded propulsor. The results indicate that the ship responds faster and has a shorter turning radius with the pod at lower initial speeds and pod angles, while the rudder responds better at high speeds regardless of angle. Further research is necessary to study the effects of changing the pod's position and increasing the number of pods used.

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A COMPARISON OF SHIP MANEUVERING CHARACTERISTICS FOR RUDDERS AND PODDED PROPULSORS

Michelle K. Betancourt
Ensign, United States Navy
B.S., University of Arizona, 2002

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Author: Michelle K. Betancourt

Approved by: Fotis Papoulias
Thesis Advisor

Young S. Kwon
Chairman, Department of Mechanical Engineering

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ABSTRACT

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I. INTRODUCTION

A. GENERAL

Most modern ships use a rudder to provide the necessary forces and moments to allow the ship to turn. However, in recent years the development of podded propulsors has allowed ships with pods to increase their propulsive efficiency and maneuverability [Ref. 1]. The use of pods allows the ship to operate with a diesel-electric power plant. If the pods are aft on the ship, the pod can increase the forces generated while maneuvering at low speeds and eliminate the need for stern thrusters [Ref. 1]. Podded propulsors are currently used in cruise ships, where passenger comfort is the main concern, dredging vessels, and naval vessels such as mine hunters. The pod allows the ship to maneuver more rapidly with less adverse effect on the ship during turns, especially at slow speeds. Thus the ship can turn faster at low speeds and will have smaller roll angles. The ship also has a smaller turning diameter at slow speeds and pod deflections [Ref. 2]. Another advantage of pods is that they can be steered over 360º, allowing the ship to maneuver in any direction fairly easily. Pods also take up less space on a ship because the need for long shaft-lines, conventional rudders, and reduction gears is eliminated [Ref. 2].

Pods operate in a similar fashion as rudders. To turn a ship, one simply positions the rudder at the desired angle to accomplish the turn at a certain rate. A ship with a pod can apply the necessary angle by rotating one or all of the pods, if more than one pod is installed [Ref. 1]. Because of the weight of the pod, turning by using a pod is considered to cause more strain on the equipment than steering with rudders. However, rudders experience stall at about 35 degrees, whereas pods do not experience stall at all due to the fact that the force is in the same direction as the propeller thrust [Ref. 1]. Rudders are also used to help maintain the course of a ship when attempting to travel in a straight line. The ship may veer from its intended course due to ocean waves and currents. According to Toxopeus and Loeff, vessels with podded propulsion perform well at course keeping and yaw checking [Ref. 1].

However, pods do have some disadvantages. Pods can respond very rapidly and therefore may respond faster than a human operator, thus making it difficult for a helmsman to control the ship [Ref. 2]. One solution may be to implement computer controls to allocate the thrusts of the propeller in such a way that the environmental loads do not have a detrimental effect on the ship's performance. Pods also tend to cause large heel angles. According to a study performed by MARIN, most passengers tend to panic at heel angles of 7º, and pods have exhibited heel angles of up to 25º in some ship models [Ref. 2]. Also, in order for the pod to operate in the most effective manner possible, the ship hull has to be altered and usually more than one pod is required to provide adequate power and maneuverability [Ref. 1]. However, because pods have proven so versatile, further study should be done to see if they can be used in more applications.

B. SCOPE OF THIS THESIS

The purpose of this thesis is to compare the performance of a ship using a pod with that of a ship using a rudder with different speeds and rudder and pod deflections. The comparison is made to help determine the viability of replacing rudders on naval vessels with pods. Since naval vessels may need to maneuver rapidly at high speeds, for example to avoid a torpedo, it is important to determine if a pod will perform well under such conditions.

The comparison of the two ships is made using a model that incorporates data for a high speed container ship. The model, described in Chapter II, uses the equations of motion for a ship to develop equations for the derivatives of several parameters. The original model simulates a ship with a rudder. The model is modified to simulate the use of a podded propulsor rather than a rudder. It is important to note that the modifications made on the model do not take into account any change in the hull that may be required for use of a pod, nor the possible need of more than one pod to properly maneuver. The model simply removes the rudder and assumes that the propeller, whose characteristics have not been altered, can now rotate in the same manner as a pod.

The analysis performed will help determine the general operating characteristics of a podded propulsor over a range of speeds and pod deflections. A description of the transient analysis performed is found in Chapter III. The steady state analysis performed is described in Chapter IV. The parameters for each ship that are compared are: the total speed U, the sway velocity v, the roll angle ϕ, and the ship's position in the xy-plane.

These parameters are plotted for each ship configuration and are then used to study the pod's operating characteristics.

The conclusions drawn from the analysis performed are shown in Chapter V. Areas of further research are also recommended.

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II. MATHEMATICAL MODEL

A. EQUATIONS OF MOTION

The nonlinear equations of motion in a body-fixed coordinate system for a ship are shown in equations (2.1). The equations assume symmetry along the longitudinal axis (y_G=0). The orthogonal, right-hand coordinate system is shown in Figure 1 and the parameters are defined in Table I.

$$\text{Surge: } m\left[\dot{u} - x_G\left(q^2 + r^2\right) + z_G\left(\dot{q} + pr\right) - rv + wq\right] = X \tag{2.1.a}$$

$$\text{Sway: } m\left[\dot{v} + x_G\left(pq + \dot{r}\right) + z_G\left(qr - \dot{p}\right) + ur - wp\right] = Y \tag{2.1.b}$$

$$\text{Heave: } m\left[\dot{w} + x_G\left(rp - \dot{q}\right) - z_G\left(p^2 + q^2\right) + pv - qu\right] = Z \tag{2.1.c}$$

$$\text{Roll: } \begin{array}{c} I_x\dot{p} + \left(I_z - I_y\right)qr - I_{xy}\left(pq + \dot{r}\right) - mz_G\left(\dot{v} + ur - wp\right) = \\ K - \Delta GZ\left(\phi\right) \end{array} \tag{2.1.d}$$

$$\text{Pitch: } \begin{array}{c} I_y\dot{q} - \left(I_z - I_x\right)rp - I_{xz}\left(r^2 - p^2\right) + \\ m\left[z_G\left(\dot{u} + qw - rv\right) - x_G\left(\dot{w} + pv - qu\right)\right] = M \end{array} \tag{2.1.e}$$

$$\text{Yaw: } I_z\dot{r} + \left(I_y - I_z\right)pq + I_{xz}\left(qr - \dot{p}\right) + mx_G\left(\dot{v} + ur - wp\right) = N \tag{2.1.f}$$

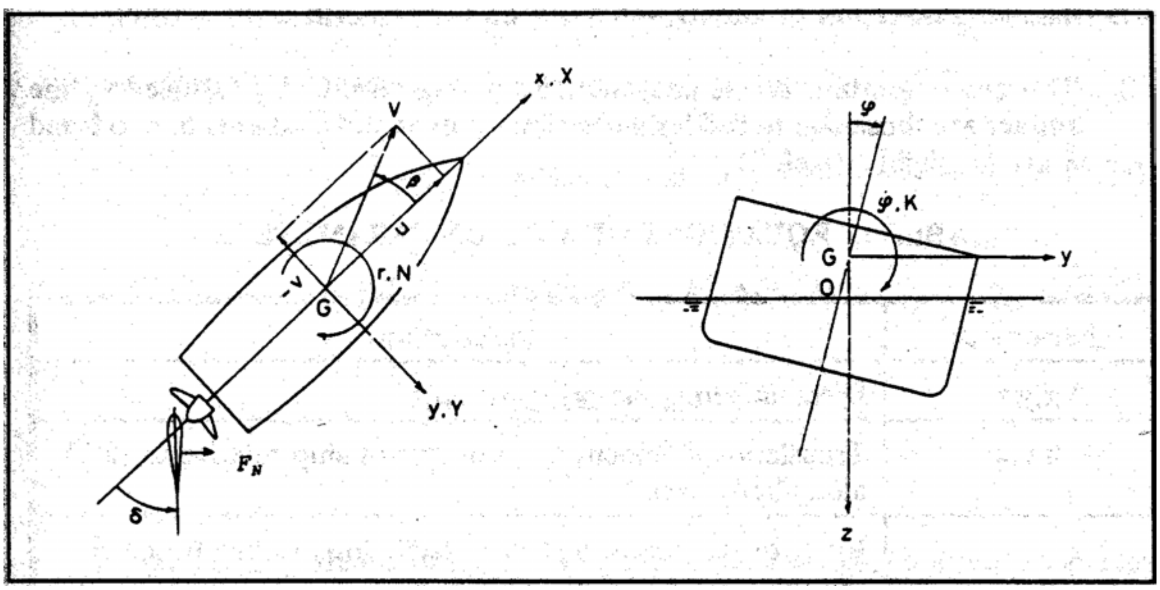

Figure 1. Coordinate System [Ref. 7]

Table 1 Equations of Motion: Parameters

Parameter	Description
x, y, z	Distance along the principal axes
u, v, w	Translational velocity components of ship relative to fluid along body axes
p, q, r	Rotational velocity components of ship relative to inertial reference system along body axes
X, Y, Z	Hydrodynamic force components along body axes
K, M, N	Hydrodynamic moment components along body axes
ψ, θ, ϕ	ψ yaw angle: bow to starboard positive θ pitch angle: bow up positive ϕ roll angle: starboard down positive
m	Mass of ship
x_G, y_G, z_G	Coordinates of the center of gravity in the body axis system
I_X, I_Y, I_Z	Moments of inertia about the body axis system
I_{XZ}, I_{YZ}, I_{XY}	Products of inertia about the body axis system
∇	Displacement volume of ship
Δ	Displacement weight of ship
GZ(ϕ)	Righting moment as a function of roll angle
δ	Rudder angle in radians
V	Initial velocity of ship
ρ	Mass density of sea water
L	Ship length between perpendiculars (LBP)

B. SIMPLIFYING ASSUMPTIONS

To simulate a ship maneuvering in the ocean without changes in depth, the following assumptions were made [Ref. 3]:

1. The rotational velocity and acceleration about the y-axis are zero. ($q = 0$ and $\dot{q} = 0$)
2. The translational velocity and acceleration in the z direction are zero. ($w = 0$ and $\dot{w} = 0$)
3. The vertical heave and pitch motions are decoupled from the horizontal plane motions.
4. The product of inertia I_{XZ} is small and can be neglected.

5. The surge equation is substituted by an algebraic equation which is a function of u, V, and δ.

6. The longitudinal center of gravity, (LCG), and the longitudinal center of buoyancy, (LCB), are at midship.

7. The vertical center of gravity, (VCG), is on the centerline.

8. The only important forces and moments acting on the ship induced by the rudder (or pod) are due to rudder (or pod) deflection. Forces and moments due to $\dot{\delta}$ and $\ddot{\delta}$ are negligible. [Ref. 4]

Applying these assumptions to Equations (2.1), the following simplified equations of motion are obtained [Ref. 5]:

$$\textit{Sway: } m\left[\dot{v}+x_G\dot{r}-z_G\dot{p}+ur\right]=Y \tag{2.2.a}$$

$$\textit{Roll: } I_x\dot{p}-mz_G\left(\dot{v}+ur\right)=K-\Delta GZ\left(\phi\right) \tag{2.2.b}$$

$$\textit{Yaw: } I_z\dot{r}+mx_G\left(\dot{v}+ur\right)=N \tag{2.2.c}$$

To simplify the analysis even further, equations (2.2) are converted to a nondimensional form. The nondimensional relationships are shown in Table 2.

To demonstrate the conversion to nondimensional terms, the appropriate values from Table 2 are substituted in Equation (2.2a) to yield the following:

$$\left(\frac{1}{2}\rho L^3\right)m'\left[\left(\frac{V^2}{L}\right)\dot{v}'+\left(Lx_G{}'\right)\left(\frac{V^2}{L^2}\dot{r}'\right)-\left(Lz_G{}'\right)\left(\frac{V^2}{L^2}\dot{p}'\right)+\left(Vu'\right)\left(\frac{V}{L}r'\right)\right]=\left(\frac{1}{2}\rho L^2V^2\right)Y'$$

Simplifying by factoring out (V^2/L) gives

$$\left(\frac{1}{2}\rho L^2V^2\right)m'\left[\dot{v}'+x_G{}'\dot{r}'-z_G{}'\dot{p}'+u'r'\right]=\left(\frac{1}{2}\rho L^2V^2\right)Y'$$

Equation (2.2a) can then be rewritten as Equation (2.3) as shown.

$$m'\left[\dot{v}'+x_G{}'\dot{r}'-z_G{}'\dot{p}'+u'r'\right]=Y' \tag{2.3}$$

Because Equations (2.2) and (2.3) are of the same form, the prime notation will no longer be included and all equations will be treated as nondimensional unless otherwise specified.

Table 2 Nondimensional Parameter Relationships

$v' = \dfrac{v}{V}$	$\dot{v}' = \dot{v}\left(\dfrac{L}{V^2}\right)$	$Y' = Y\left(\dfrac{1}{1/2\,\rho L^2 V^2}\right)$
$r' = r\left(\dfrac{L}{V}\right)$	$\dot{r}' = \dot{r}\left(\dfrac{L^2}{V^2}\right)$	$N' = N\left(\dfrac{1}{1/2\,\rho L^3 V^2}\right)$
$\phi' = \phi$	$\dot{\phi}' = \dot{\phi}\left(\dfrac{L}{V}\right)$	$K' = K\left(\dfrac{1}{1/2\,\rho L^3 V^2}\right)$
$p' = p\left(\dfrac{L}{V}\right)$	$\dot{p}' = \dot{p}\left(\dfrac{L^2}{V^2}\right)$	$u' = \dfrac{u}{V}$
$m' = m\left(\dfrac{1}{1/2\,\rho L^3}\right)$	$x_G{}' = x_G\left(\dfrac{1}{L}\right)$	$z_G{}' = z_G\left(\dfrac{1}{L}\right)$
$I_x{}' = I_x\left(\dfrac{1}{1/2\,\rho L^5}\right)$	$I_z{}' = I_z\left(\dfrac{1}{1/2\,\rho L^5}\right)$	$\Delta GZ(\phi)' = \Delta GZ(\phi)\left(\dfrac{1}{1/2\,\rho L^3 V^2}\right)$

C. FORCE AND MOMENT REPRESENTATION

Using the method described by Abkowitz and Strom-Tejsen [Ref. 4], the sway force, roll moment, and yaw moment can be expressed as Equations (2.4).

$$\textit{Surge Force:}\ X = f_1\left(u, v, r, \phi, \delta\right) \tag{2.4.a}$$

$$\textit{Sway Force:}\ Y = f_2\left(u, v, r, \dot{u}, \dot{v}, \dot{r}, \phi, \dot{\phi}, \delta\right) \tag{2.4.b}$$

$$\textit{Roll Moment:}\ K = f_3\left(u, v, r, \dot{u}, \dot{v}, \dot{r}, \phi, \dot{\phi}, \ddot{\phi}, \delta\right) \tag{2.4.c}$$

$$\textit{Yaw Moment:}\ N = f_4\left(u, v, r, \dot{u}, \dot{v}, \dot{r}, \phi, \dot{\phi}, \delta\right) \tag{2.4.d}$$

Using a third order Taylor expansion for f_1, f_2, and f_3, equations (2.5) are obtained.

$$f_1 = X_{uu}u^2 + X_{vr}vr + X_{vv}v^2 + X_{rr}r^2 + x_{\phi\phi}\phi^2 + X\left(\delta\right) + X_o \tag{2.5.a}$$

$$\begin{aligned} f_2 = {} & Y_v v + Y_{\dot{v}}\dot{v} + Y_{vvv}v^3 + Y_r r + Y_{rrr}r^3 + Y_{vvr}v^2 r + Y_\phi \phi + \\ & Y_{\dot{\phi}}\dot{\phi} + Y_{vv\phi}v^2\phi + Y_{v\phi\phi}v\phi^2 + Y_{rr\phi}r^2\phi + Y\left(\delta\right) + Y_o \end{aligned} \tag{2.5.b}$$

$$f_3 = K_v v + K_{vvv} v^3 + K_{vvr} v^2 r + K_r r + K_{rrr} r^3 + K_\phi \phi + K_{\dot{\phi}} \dot{\phi} + K_{\ddot{\phi}} \ddot{\phi} + K_{vv\phi} v^2 \phi + K_{v\phi\phi} v \phi^2 + K_{r\phi\phi} r \phi^2 + K(\delta) + K_o - \Delta GZ(\phi) \tag{2.5.c}$$

$$f_4 = N_v v + N_{\dot{v}} \dot{v} + N_{vvv} v^3 + N_{vrr} v r^2 + N_r r + N_{\dot{r}} \dot{r} + N_{rrr} r^2 + N_\phi \phi + N_{\dot{\phi}} \dot{\phi} + N_{vv\phi} v^2 \phi + N_{v\phi\phi} v \phi^2 + N_{rr\phi} r^2 \phi + N_{r\phi\phi} r \phi^2 + N(\delta) + N_o \tag{2.5.d}$$

The coefficients in Equation (2.5) are called the hydrodynamic coefficients and represent the partial derivative of the force or moment with respect to the subscripted variable. For example, Y_v represents $\frac{\partial Y}{\partial v}$ and Y_{vrr} represents $\frac{\partial}{\partial v}\left(\frac{\partial^2 Y}{\partial r^2}\right)$. The terms Y_o, K_o, and N_o are the sway force, roll moment, and yaw moment induced by the propeller, respectively. Similarly, Y(δ), N(δ), and K(δ) represent the force and moments induced by the rudder. The righting moment due to the static stability of the ship is represented by the term $\Delta GZ(\phi)$.

D. RUDDER FORCE AND MOMENT REPRESENTATION

The expressions used to determine the rudder force and moments are shown in Equations (2.6) and (2.7) [Ref. 5]. The parameters are defined in Table 3.

$$X(\delta) = c_{RX} F_N \sin(\delta) \tag{2.6.a}$$

$$Y(\delta) = -(1 + a_H) F_N \cos(\delta) \tag{2.6.b}$$

$$K(\delta) = (1 + a_H) z_R F_N \cos(\delta) \tag{2.6.c}$$

$$N(\delta) = -(x_R + a_H x_H) F_N \cos(\delta) \tag{2.6.d}$$

$$F_N = \left(\frac{6.13\Lambda}{\Lambda + 2.25}\right)\left(\frac{A_R}{L^2}\right) V_R^2 \sin(\alpha_R) \tag{2.7.a}$$

$$V_R = \sqrt{{u_R}^2 + {v_R}^2} \tag{2.7.b}$$

$$u_R = u_P \varepsilon \sqrt{1 + \frac{8kK_T}{\pi J^2}} \tag{2.7.c}$$

$$J = \frac{u_P V}{nD} \tag{2.7.d}$$

$$u_P = u\left[\left(1 - w_P\right) + \tau\left\{\left(v + x_P r\right)^2 + c_{pv} v + c_{pr} r\right\}\right] \tag{2.7.e}$$

$$v_R = \gamma v + c_{\delta r} r + c_{\delta rrr} r^3 + c_{\delta rrv} r^2 v \tag{2.7.f}$$

$$\alpha_R = \delta + \tan^{-1}\left(\frac{v_R}{u_R}\right) \tag{2.7.g}$$

Table 3 Rudder Force and Moment Parameters

Parameter	Description
a_H	Rudder to hull interaction coefficient
F_N	Normal force action on the rudder
z_R	z coordinate of point on which rudder force Y_δ acts
x_R	x coordinate of point on which rudder force Y_δ acts
x_H	x coordinate of point on which normal force F_N acts
Λ	Rudder aspect ratio
A_R	Rudder area
V_R	Effective rudder inflow velocity
α_R	Effective rudder inflow angle
u_R, v_R	Components of rudder effective inflow velocity
ε	Constant in Equation (2.7c)
u_P	Effective propeller inflow velocity
k	Constant in Equation (2.7c)
K_T	Thrust coefficient
J	Advance coefficient
n	Number of propeller revolutions per second
D	Propeller diameter
w_P	Effective propeller wake fraction
τ	Constant in Equation (2.7e)
x_P	x coordinate of propeller position
c_{pv}, c_{pr}	Propeller flow rectification coefficients
γ	Flow rectification coefficient
$c_{\delta rr}$, $c_{\delta rrr}$, $c_{\delta rrv}$	Rudder wake coefficients

E. FORCES AND MOMENTS FOR CONTAINER SHIP WITH RUDDER

The following equations are used to determine the forces and moments acting on the ship, including the forces and moments caused by the rudder. Equations (2.8) are used in the mathematical model and they combine all of the above equations with the rudder [Ref. 6]. The values of the hydrodynamic derivatives and coefficients are given in Table 4.

$$\textit{Surge Force:}\quad \begin{aligned} X &= X_{uu}u^2 + (1-t)T + X_{vr}vr + X_{vv}v^2 + X_{rr}r^2 \\ &+ X_{\phi\phi}\phi^2 + c_{RX}F_N\sin(\delta) + (m+m_y)vr \end{aligned} \tag{2.8.a}$$

$$\textit{Sway Force:}\quad \begin{aligned} Y &= Y_v v + Y_r r + Y_p p + Y_\phi \phi + Y_{vvv}v^3 + Y_{rrr}r^3 + Y_{vvr}v^2 r \\ &+ Y_{vrr}vr^2 + Y_{vv\phi}v^2\phi + Y_{v\phi\phi}v\phi^2 + Y_{rr\phi}r^2\phi + Y_{r\phi\phi}r\phi^2 + \\ &(1+a_H)F_N\cos(\delta) - (m+m_x)ur \end{aligned} \tag{2.8.b}$$

$$\text{Roll Moment:} \quad \begin{aligned} K = {} & K_v v + K_r r + K_p p + K_\phi \phi + K_{vvv} v^3 + K_{rrr} r^3 + \\ & K_{vvr} v^2 r + K_{vrr} v r^2 + K_{vv\phi} v^2 \phi + K_{v\phi\phi} v \phi^2 + K_{rr\phi} r^2 \phi + \\ & K_{r\phi\phi} r \phi^2 - \left(1 + a_H\right) z_R F_N \cos\left(\delta\right) + m_x l_x u r - W\left(GM\right)\phi \end{aligned} \tag{2.8.c}$$

$$\text{Yaw Moment:} \quad \begin{aligned} N = {} & N_v v + N_r r + N_p p + N_\phi \phi + N_{vvv} v^3 + N_{rrr} r^3 + \\ & N_{vvr} v^2 r + N_{vrr} v r^2 + N_{vv\phi} v^2 \phi + N_{v\phi\phi} v \phi^2 + \\ & N_{rr\phi} r^2 \phi + N_{r\phi\phi} r \phi^2 + \left(x_R + a_H x_H\right) F_N \cos\left(\delta\right) \end{aligned} \tag{2.8.d}$$

F. POD FORCE AND MOMENT REPRESENTATION

The expressions used to determine the pod force and moments are shown in Equations (2.9). These equations were derived by comparing the thrust term in the equations of motion to the rudder term. The thrust terms were then modified by the same sign, distance (for moments), and trigonometric relationship as the rudder term, and the rudder term was removed from the equations.

$$X_o = \left(1 - t\right) T \sin\left(\delta_{pod}\right) \tag{2.9.a}$$

$$Y_o = \left(1 - t\right) T \cos\left(\delta_{pod}\right) \tag{2.9.b}$$

$$K_o = -z_R \left(1 - t\right) T \cos\left(\delta_{pod}\right) \tag{2.9.c}$$

$$N_o = \frac{1}{2}\left(1 - t\right) T \cos\left(\delta_{pod}\right) \tag{2.9.d}$$

G. FORCES AND MOMENTS FOR CONTAINER SHIP WITH POD

The following equations are used to determine the forces and moments acting on the ship, including the forces and moments caused by the pod. Equations (2.10) are used in the mathematical model and they combine all of the above equations with the pod. The values of the hydrodynamic derivatives and coefficients are given in Table 4.

$$\text{Surge Force:} \quad \begin{aligned} X = {} & X_{uu} u^2 + \left(1 - t\right) T \sin\left(\delta_{pod}\right) + X_{vr} v r + X_{vv} v^2 + \\ & X_{rr} r^2 + X_{\phi\phi} \phi^2 + \left(m + m_y\right) v r \end{aligned} \tag{2.10.a}$$

$$\text{Sway Force:} \quad \begin{aligned} Y = {} & \left(1 - t\right) T \cos\left(\delta_{pod}\right) + Y_v v + Y_r r + Y_p p + Y_\phi \phi + Y_{vvv} v^3 + \\ & Y_{rrr} r^3 + Y_{vvr} v^2 r + Y_{vrr} v r^2 + Y_{vv\phi} v^2 \phi + Y_{v\phi\phi} v \phi^2 + \\ & Y_{rr\phi} r^2 \phi + Y_{r\phi\phi} r \phi^2 - \left(m + m_x\right) u r \end{aligned} \tag{2.10.b}$$

Roll Moment:

$$K = -z_R\left(1-t\right)T\cos\left(\delta_{pod}\right) + K_v v + K_r r + K_p p + K_\phi \phi + K_{vvv}v^3 + K_{rrr}r^3 + K_{vvr}v^2 r + K_{vrr}vr^2 + K_{vv\phi}v^2\phi + K_{v\phi\phi}v\phi^2 + K_{rr\phi}r^2\phi + K_{r\phi\phi}r\phi^2 + m_x l_x ur - W\left(GM\right)\phi \tag{2.10.c}$$

Yaw Moment:

$$N = \frac{1}{2}\left(1-t\right)T\cos\left(\delta_{pod}\right) + N_v v + N_r r + N_p p + N_\phi \phi + N_{vvv}v^3 + N_{rrr}r^3 + N_{vvr}v^2 r + N_{vrr}vr^2 + N_{vv\phi}v^2\phi + N_{v\phi\phi}v\phi^2 + N_{rr\phi}r^2\phi + N_{r\phi\phi}r\phi^2 \tag{2.10.d}$$

Table 4 Hydrodynamic Derivatives and Coefficients [Ref. 3]

A. Hull only

Y_v	-0.0116000	N_v	-0.0038545	K_v	0.00030260
Y_r	0.0024200	N_r	-0.0022200	K_r	-0.00006300
Y_ϕ	-0.0000630	N_ϕ	-0.0001424	K_ϕ	-0.00002100
Y_{vvv}	-0.1090000	N_{vvv}	0.0014920	K_{vvv}	0.00284300
Y_{vrr}	-0.0405000	N_{vrr}	0.0015600	K_{vrr}	0.00105650
Y_{rrr}	0.0017700	N_{rrr}	-0.0022900	K_{rrr}	-0.00004620
Y_{vvr}	0.0214000	N_{vvr}	-0.424000	K_{vvr}	-0.00055800
$Y_{vv\phi}$	0.0460500	$N_{vv\phi}$	-0.0190580	$K_{vv\phi}$	-0.00120120
$Y_{v\phi\phi}$	0.0030400	$N_{v\phi\phi}$	-0.0053766	$K_{v\phi\phi}$	-0.00007930
$Y_{rr\phi}$	0.0093250	$N_{rr\phi}$	-0.0038592	$K_{rr\phi}$	-0.00024300
$Y_{r\phi\phi}$	-0.0013560	$N_{r\phi\phi}$	0.0024195	$K_{r\phi\phi}$	0.00003569
X_{uu}	-0.0004226	X_{vr}	-0.00311	X_{vv}	-0.00386
X_{rr}	0.00020	$X_{\phi\phi}$	-0.00020	x_G	0.0
m	0.00792	m_x	0.000238	m_y	0.000238
I_x	0.0000176	I_y	0.0	I_z	0.000456
J_x	0.0000034	α_y	0.05	J_z	0.000419
l_x	0.0313	l_y	0.0313		

B. Propeller and rudder

a_H	0.237	τ	1.090	(1-t)	0.825
x_H	-0.480	ε	0.921	x_R	-0.500
c_{RX}	0.710	k	0.631	x_P	-0.526
z_R	0.033	$c_{\delta r}$	-0.156	K_T	0.527-0.455J
c_{Rr}	-0.156	c_{Rrrr}	-0.275	c_{Rrrv}	1.960
c_{pv}	0.000	$c_{\delta rrr}$	-0.275	γ	0.088 v>0 0.193 v≤0
c_{pr}	0.000	$c_{\delta rrv}$	1.960	N_P	79.10 Fn=0.2 118.64 Fn=0.3 158.19 Fn=0.4

H. DIMENSIONAL STATE DERIVATIVES

Equation (2.11), which defines the derivatives of the desired variables, is used to find the variation of the following parameters over time: surge velocity u, sway velocity v, yaw velocity r, pitch rate p, roll angle ϕ, heading angle ψ, position in x direction, position in y direction, rudder deflection angle δ (or δ_{pod}), and n (the actual shaft velocity). Using Euler Integration to find the desired variable, the derivative is multiplied by a specified time step and then added to the previous (or initial) value of the variable. The variable is then equal to the new calculated value. Only u, U, v, r, ϕ, and x and y were used for analysis. The equation is in dimensional form.

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\psi} \\ \dot{x} \\ \dot{y} \\ \dot{\delta} \\ \dot{n} \end{bmatrix} = \begin{bmatrix} X\left(\frac{V^2}{L}\right) \Big/ (m+m_x) \\ -\frac{(I_x+J_x)(I_z+J_z)Y + (-m_y l_y)(I_z+J_z)K + (m_y\alpha_y)(I_x+J_x)N}{(m+m_y)(I_x+J_x)(I_z+J_z) - (-m_y l_y)^2(I_z+J_z) - (m_y\alpha_y)^2(I_x+J_x)}\left(\frac{V^2}{L}\right) \\ \frac{-(-m_y l_y)(I_z+J_z)Y + K(m+m_y)(I_z+J_z) - K(m_y\alpha_y)^2 + (-m_y l_y)(m_y\alpha_y)N}{(m+m_y)(I_x+J_x)(I_z+J_z) - (-m_y l_y)^2(I_z+J_z) - (m_y\alpha_y)^2(I_x+J_x)}\left(\frac{V^2}{L^2}\right) \\ \frac{-(m_y\alpha_y)(I_x+J_x)Y + (-m_y l_y)(m_y\alpha_y)K + N(m+m_y)(I_x+J_x) - N(-m_y l_y)^2}{(m+m_y)(I_x+J_x)(I_z+J_z) - (-m_y l_y)^2(I_z+J_z) - (m_y\alpha_y)^2(I_x+J_x)}\left(\frac{V^2}{L^2}\right) \\ p\left(\frac{V}{L}\right) \\ \cos(\phi) r\left(\frac{V}{L}\right) \\ (\cos(\psi)u - \sin(\psi)\cos(\phi)v)V \\ (\sin(\psi)u + \cos(\psi)\cos(\phi)v)V \\ \delta_{commanded} - \delta \\ \frac{1}{T_m}(n_{commanded} - n)\times 60 \end{bmatrix} \tag{2.11}$$

The commanded rudder/pod angle is the desired deflection angle of the rudder/pod for the turn. The actual angle is initially zero since the ship is traveling in a straight line, and will increase each time step as $\dot{\delta}$ is integrated. The results obtained using Euler Integration are then plotted to observe and compare the ship response to a rudder versus a podded propulsor. The parameters compared are the total speed over time, the sway velocity over time, the roll angle over time, and the ship's position in the xy-plane. Analysis of the ship's position in the xy-plane enables a comparison of the turning path of each ship configuration. The definitions for the turning path of a ship are shown in Figure 2.

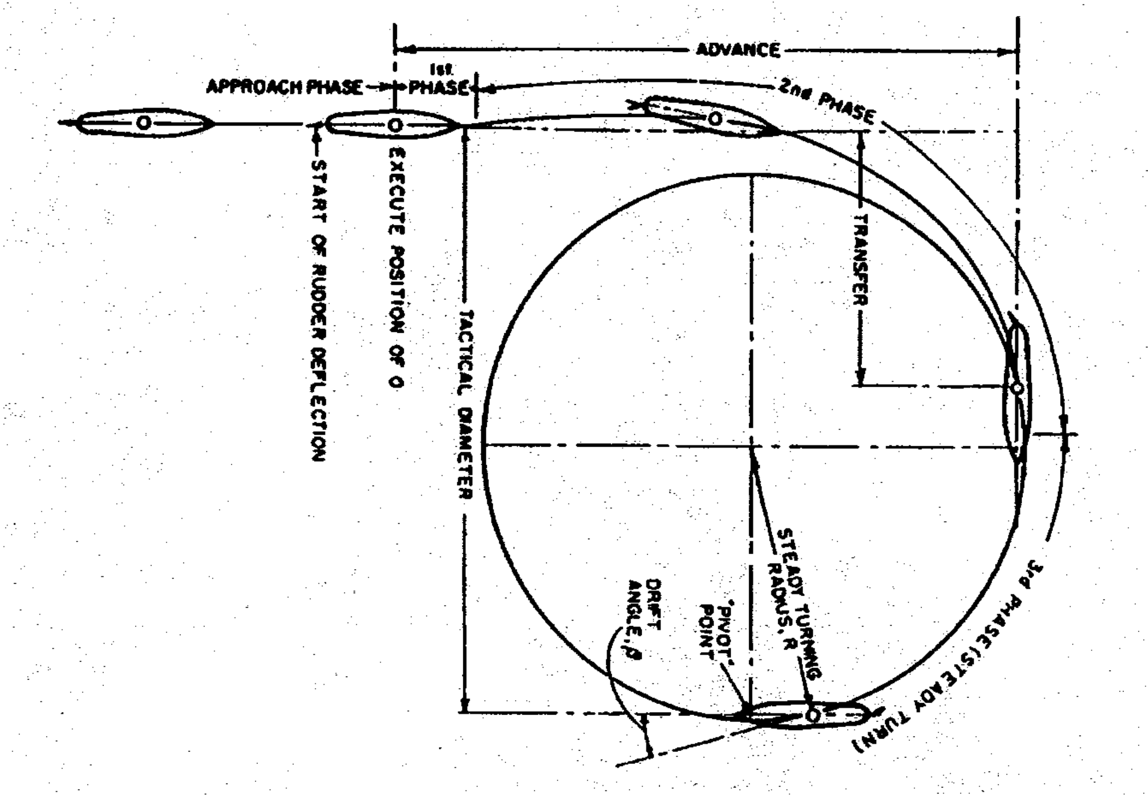

Figure 2. Turning Path of a Ship [Ref. 7]

III. TRANSIENT ANALYSIS

A. INPUT PARAMETERS

The ship's behavior is simulated at various initial speeds U and the rudder (or pod) angles as shown in Table 5.

Table 5 Values of Varied Parameters

Speed U [knots]	Rudder or Pod Angle δ [deg]
5	5
10	10
15	15
20	20
25	
30	

At each speed, the rudder and pod angles are set to the values shown above. The results are then plotted. The following parameters are plotted: total speed U versus time, roll angle ϕ versus time, sway velocity v versus time, and position in the xy-plane. Each parameter is then studied at the varying speeds and rudder/pod angles.

B. TOTAL SPEED U VERSUS TIME

For the ship with a pod, the total speed U increases as pod angle increases (with the initial speed constant). The speed U does not vary as initial speed increases (with constant pod angle). For the ship with a rudder, the total speed U decreases as rudder angle increases (with constant initial speed). The speed U again does not change as initial speed increases (with constant rudder angle). The behavior of the speed for each ship is shown in Table 6 for each speed and deflection angle modeled. For both ships modeled the speed of the ship increases from its initial value for an initial speed of 5 knots and all deflections. At an initial speed of 10 and 15 knots, the ship with the pod slows down at deflections of 5 and 10 degrees, while the ship with the rudder speeds up (except at 15 knots and 10 degrees where it slows down). At all other initial speeds and deflections, both ships will slow down.

The model indicates that for the same initial speed U, the ship with a rudder tends to slow down as it maneuvers along a turn whereas the ship with a pod tends to speed up. This pod behavior is consistent with other studies performed on the high speed performance of ships with pods installed [Ref. 1]. The plots for a total speed of 5 knots and varying rudder/pod angles are shown in Figures 3 through 6.

Table 6 Ship Speed Behavior at Different Initial Speeds and Deflection Angles

Total Speed U [knots]	Rudder or Pod Angle [deg]	Ship Speed Behavior for Rudder	Ship Speed Behavior for Pod
5	5	↑	↑
5	10	↑	↑
5	15	↑	↑
5	20	↑	↑
10	5	↑	↓
10	10	↑	↓
10	15	↑	↑
10	20	↑	↑
15	5	↑	↓
15	10	↓	↓
15	15	↓	↓
15	20	↓	↓
20	5	↓	↓
20	10	↓	↓
20	15	↓	↓
20	20	↓	↓
25	5	↓	↓
25	10	↓	↓
25	15	↓	↓
25	20	↓	↓
30	5	↓	↓
30	10	↓	↓
30	15	↓	↓
30	20	↓	↓

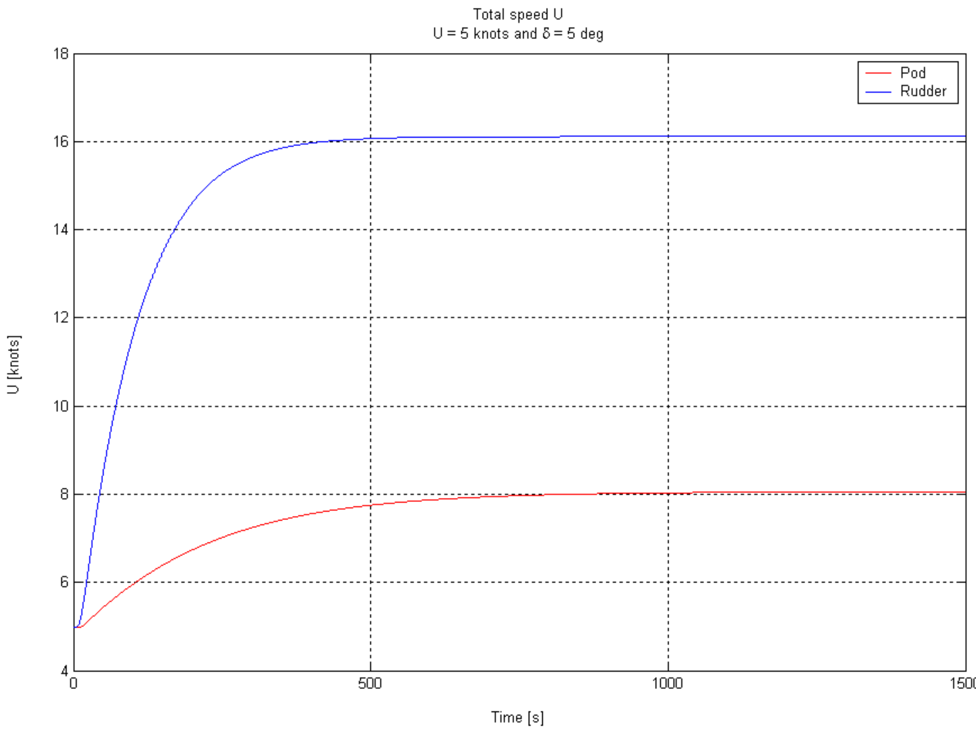

Figure 3. U vs. Time, U = 5 knots and δ = 5º

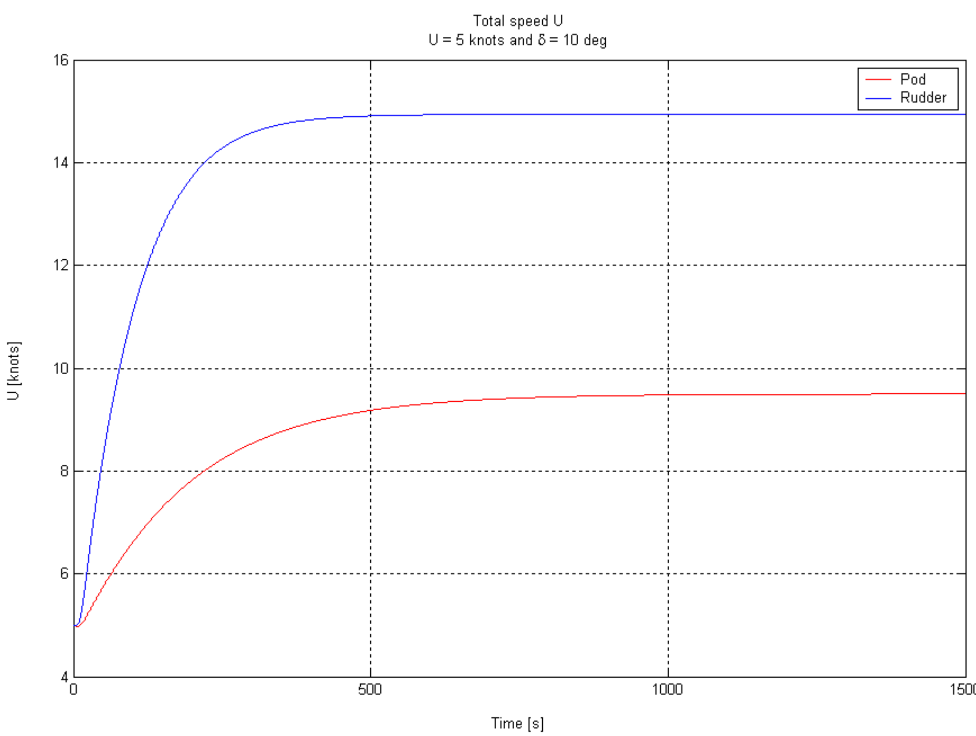

Figure 4. U vs. Time, U = 5 knots and δ = 10º

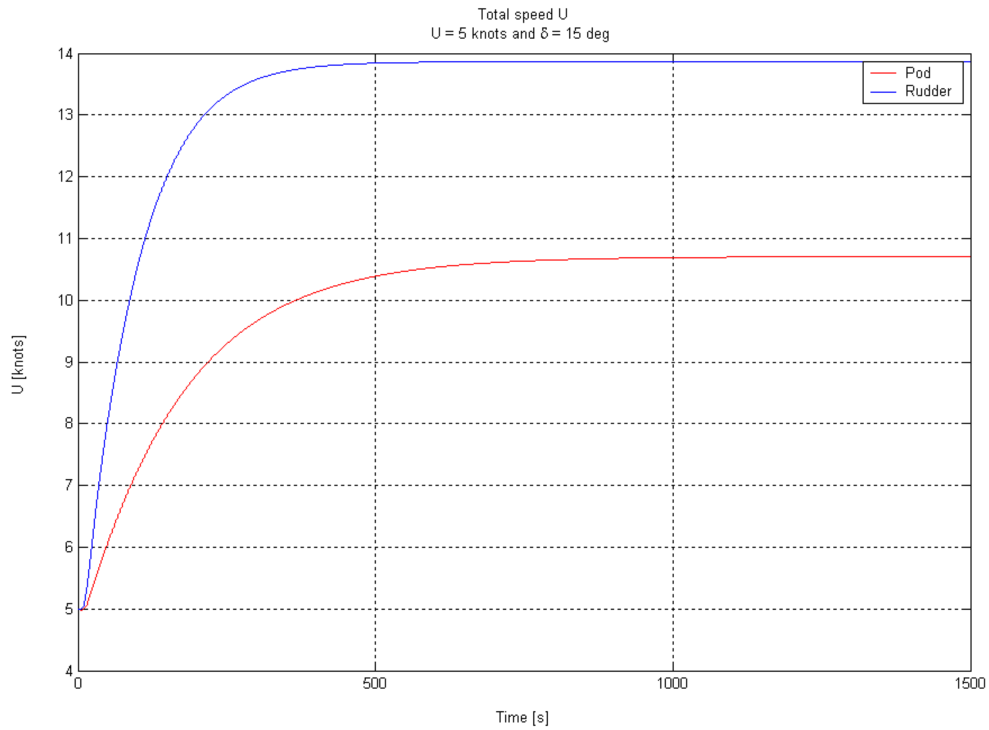

Figure 5. U vs. Time, U = 5 knots and $\delta = 15^{o}$

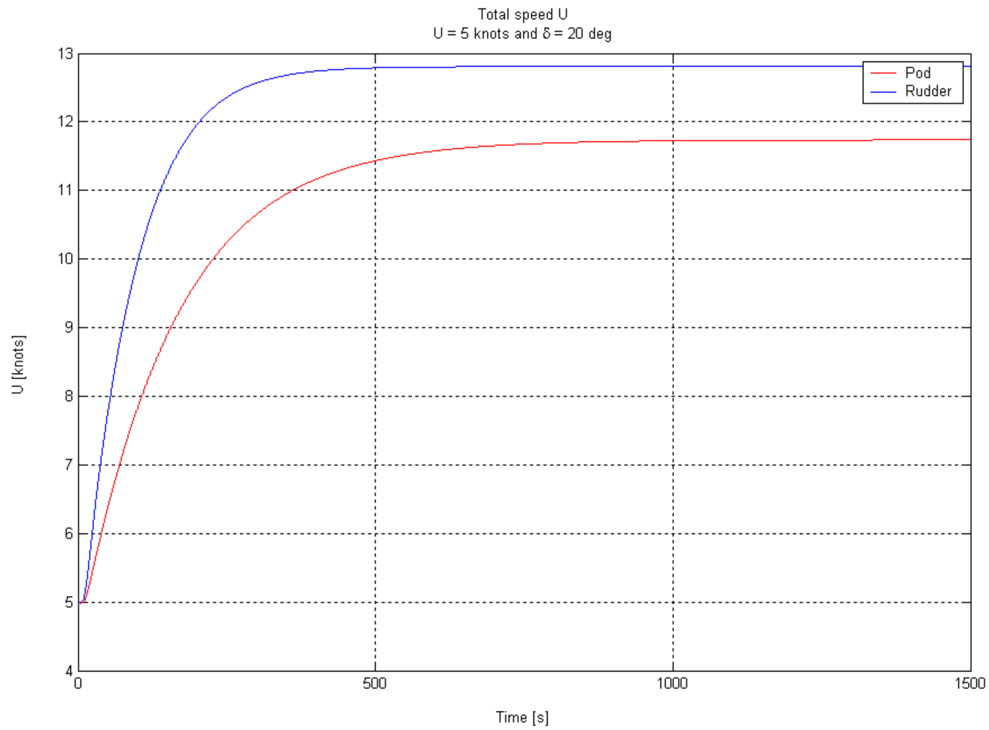

Figure 6. U vs. Time, U = 5 knots and $\delta = 20^{o}$

C. ROLL ANGLE ϕ VERSUS TIME

For the ship with a pod, the roll angle ϕ decreases as the pod angle δ increases (with constant initial speed U). The roll angle ϕ is approximately constant for varying total speed U and same pod angle δ. For the ship with a rudder, the results are the same as for the pod, although the magnitude of the roll angle is less than that of the ship with the pod for the same rudder/pod angle. The model therefore indicates that the ship with the pod will tend to have a larger magnitude roll angle than the ship with the rudder. This means that the ship with the pod will be tilted at a steeper angle during a turn than the ship with the rudder. These results are consistent with other studies performed on ships with rudders [Ref. 1]. The plots for a total speed of 5 knots and varying rudder/pod angles are shown in Figures 7 through 10. The plots for a rudder/pod angle of 5 degrees and varying total speed are shown in Figures 7 through 16.

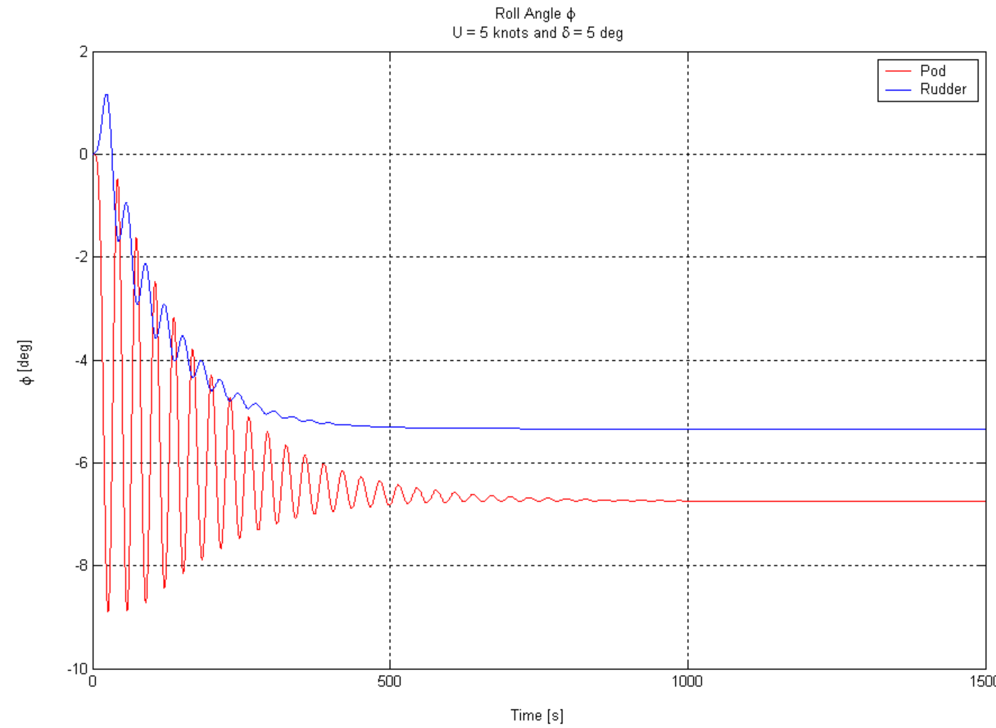

Figure 7. ϕ vs. Time, U = 5 knots and $\delta = 5^{o}$

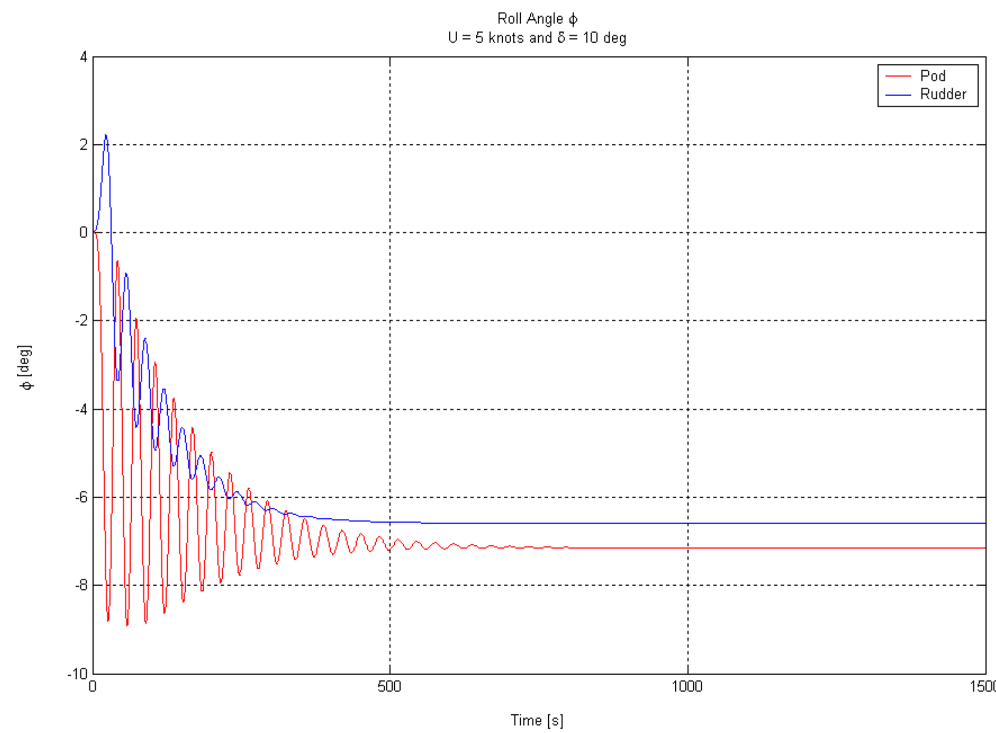

Figure 8. ϕ vs. Time, U = 5 knots and $\delta = 10^{o}$

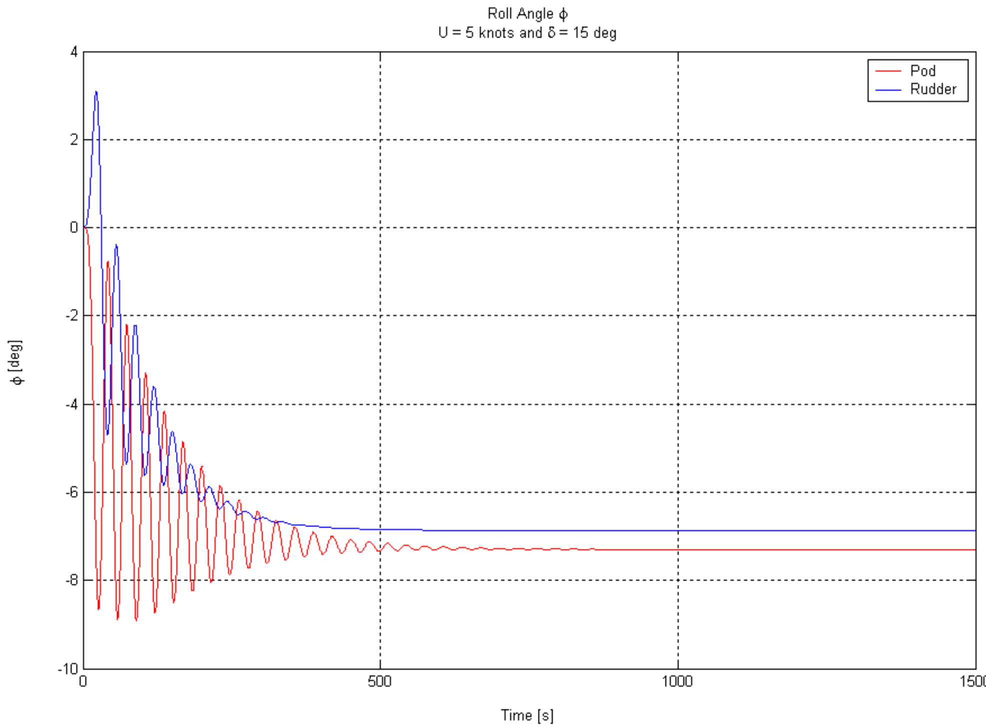

Figure 9. ϕ vs. Time, U = 5 knots and $\delta = 15^{\circ}$

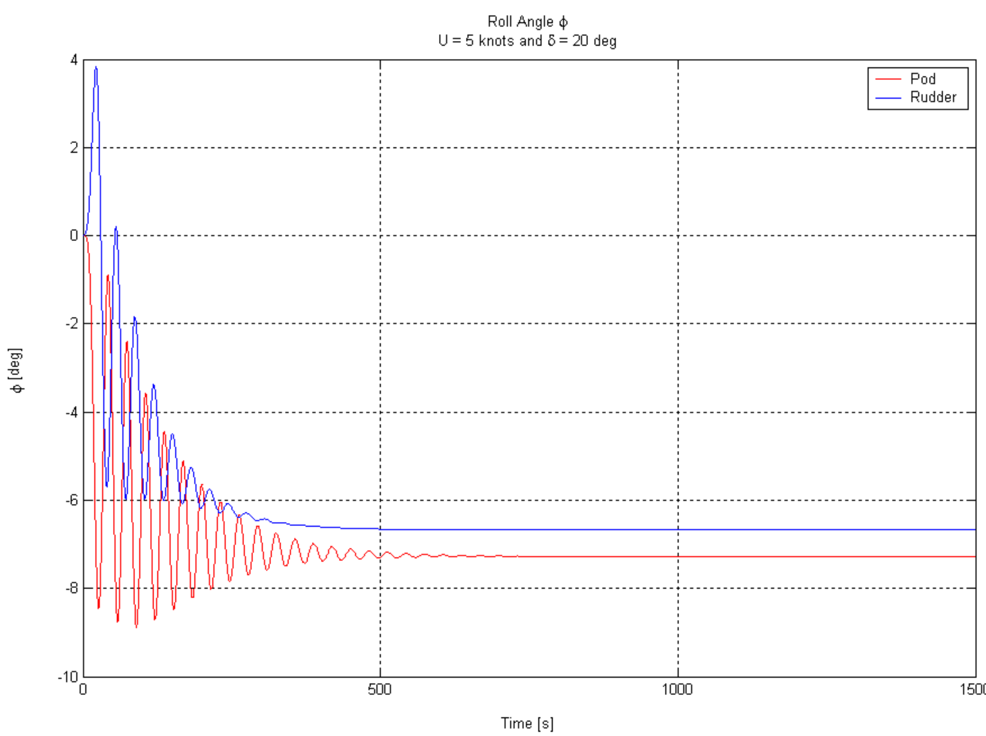

Figure 10. ϕ vs. Time, U = 5 knots and $\delta = 20^{\circ}$

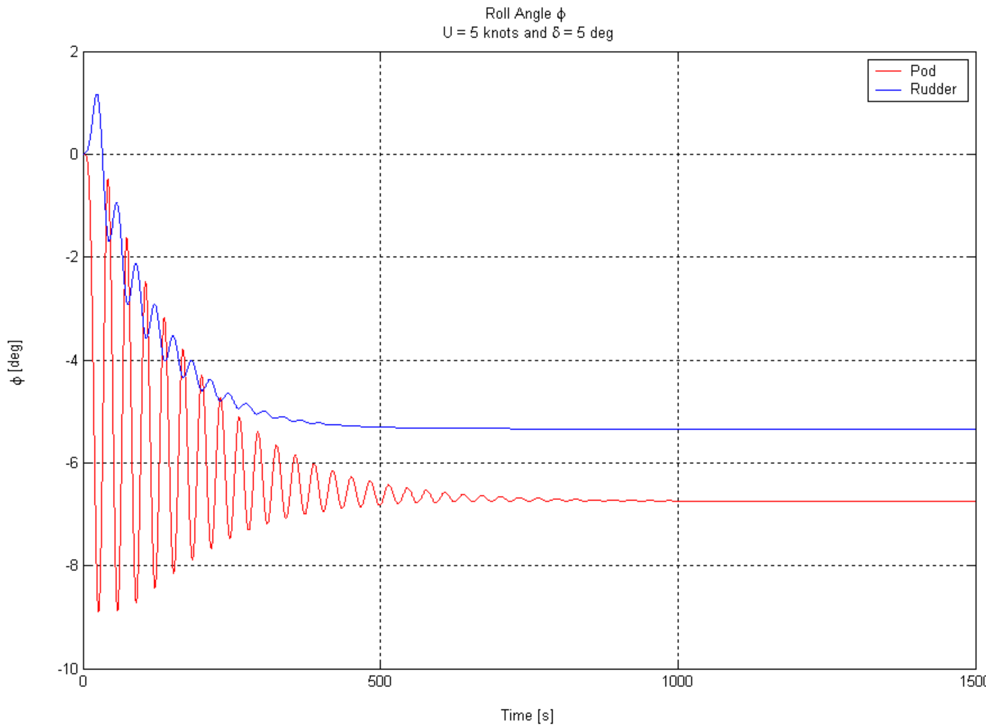

Figure 11. ϕ vs. Time, U = 5 knots and δ = 5º (same as Figure 7)

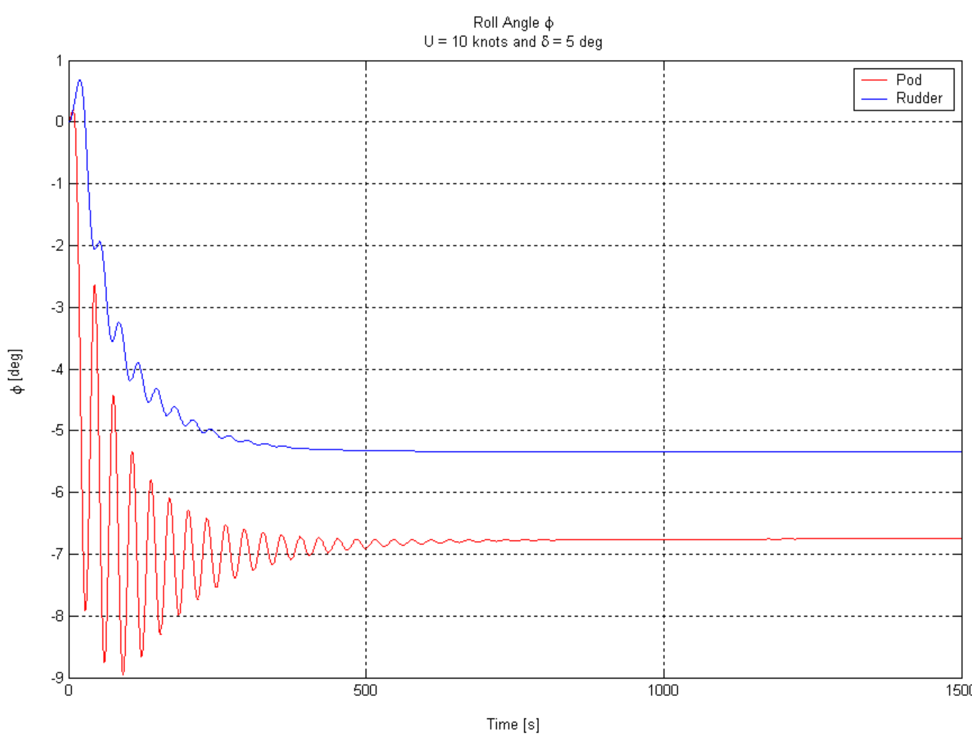

Figure 12. ϕ vs. Time, U = 10 knots and δ = 5º

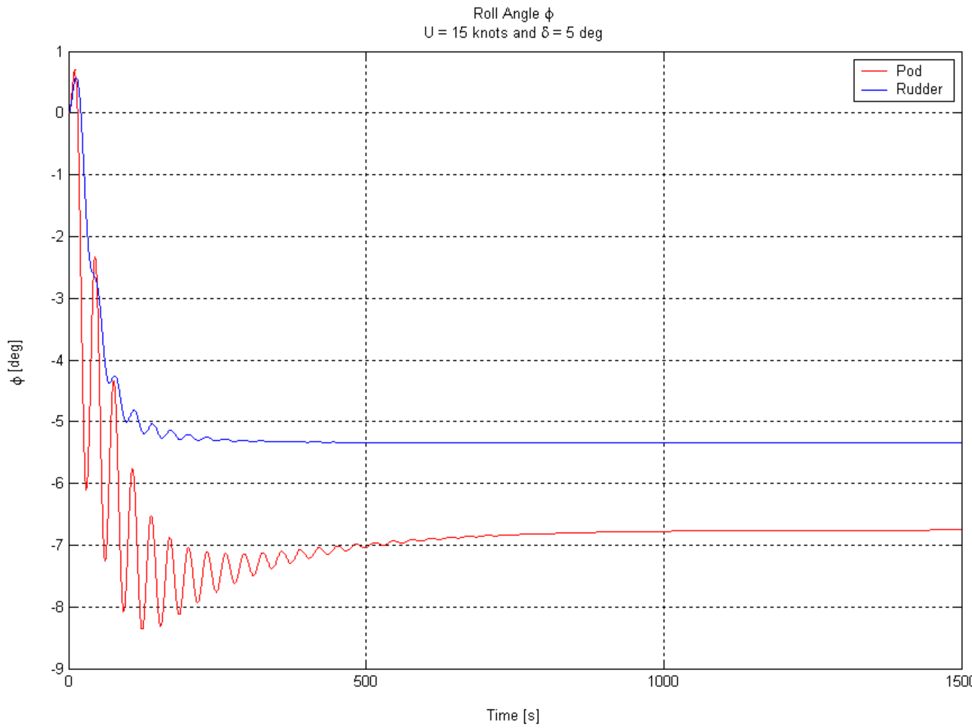

Figure 13. ϕ vs. Time, U = 15 knots and $\delta = 5^{\circ}$

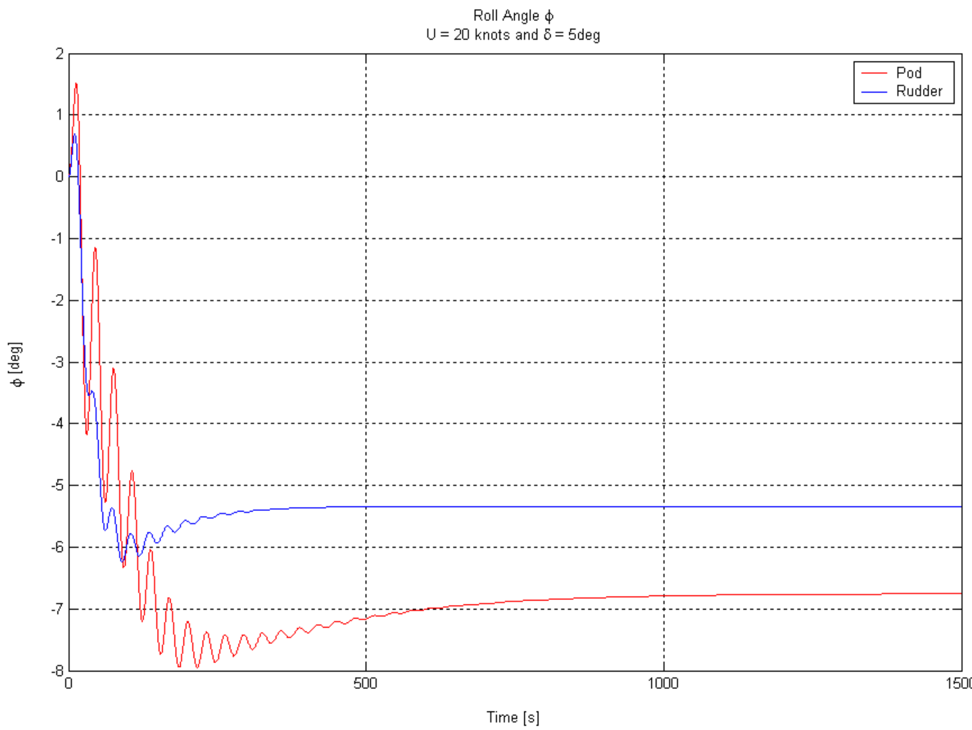

Figure 14. ϕ vs. Time, U = 20 knots and $\delta = 5^{\circ}$

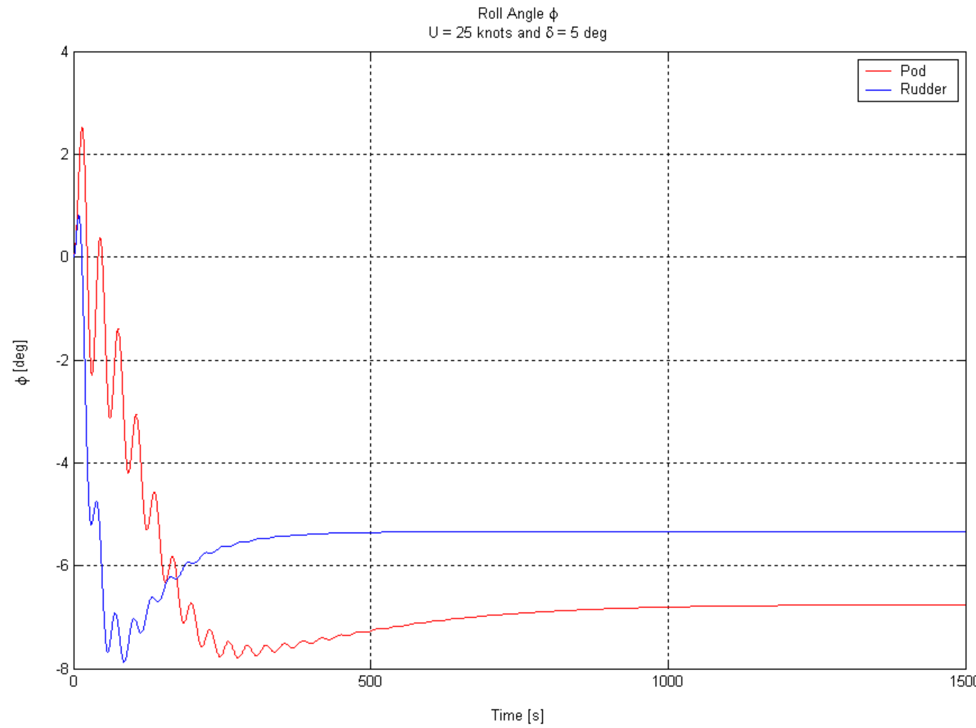

Figure 15. ϕ vs. Time, U = 25 knots and $\delta = 5^{\circ}$

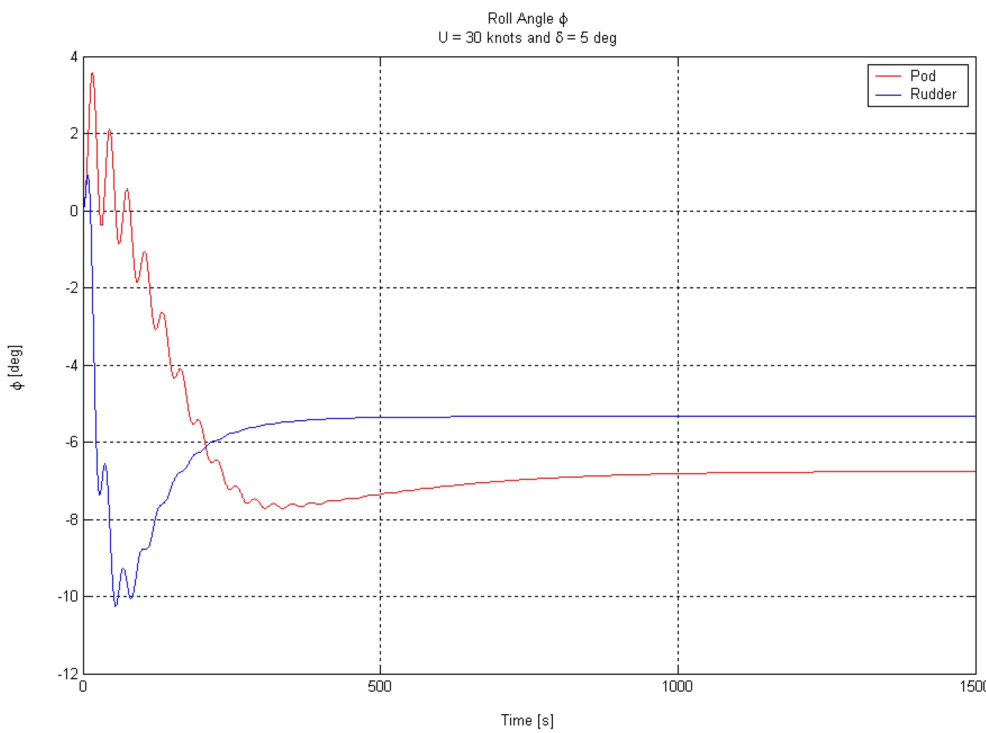

Figure 16. ϕ vs. Time, U = 30 knots and $\delta = 5^{\circ}$

D. SWAY VELOCITY V VERSUS TIME

For the ship with a pod, the sway velocity v is approximately constant as the speed U increases (with constant pod angle δ). The sway velocity v increases in magnitude as the pod angle δ increases (with constant speed U). For the ship with a rudder, the sway velocity v is constant as the initial speed U increases (with constant rudder angle δ). The sway velocity v increases in magnitude as the rudder angle δ increases (with constant speed U). The plots for a speed of 5 knots and varying rudder/pod angle are shown in Figures 17 through 20.

As seen from the plots, the sway velocity is always negative for the rudder and is negative for pod angles greater than 5˚, regardless of the speed. However, the magnitude of the sway velocity is always larger for the rudder than for the pod.

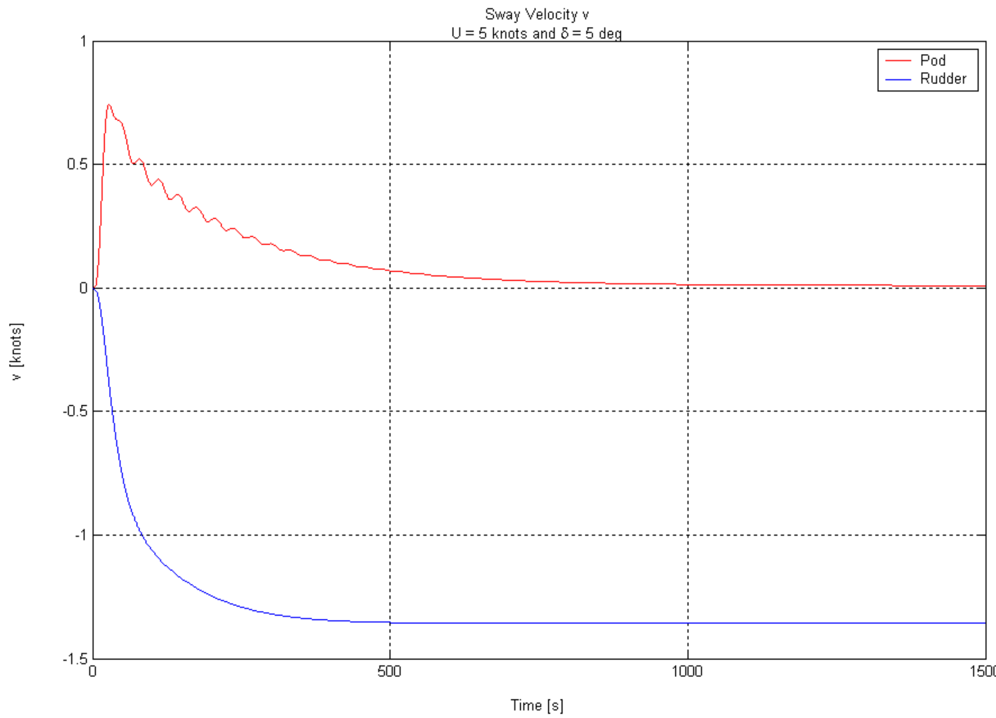

Figure 17. v vs. time, U = 5 knots and δ = 5º

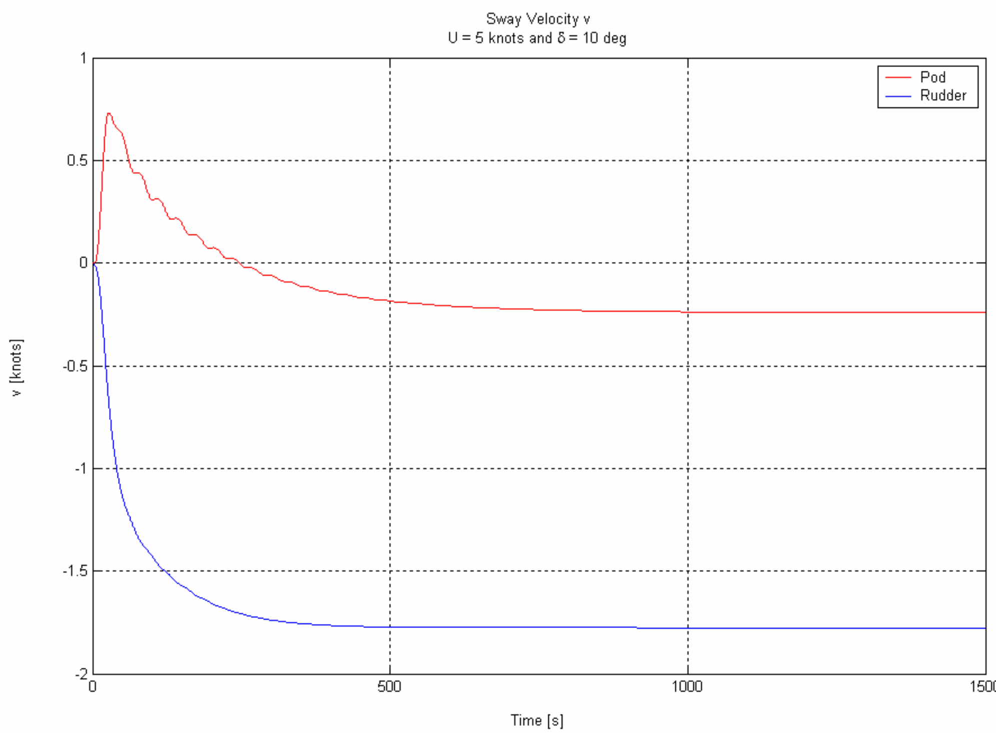

Figure 18. v vs. time, U = 5 knots and δ = 10º

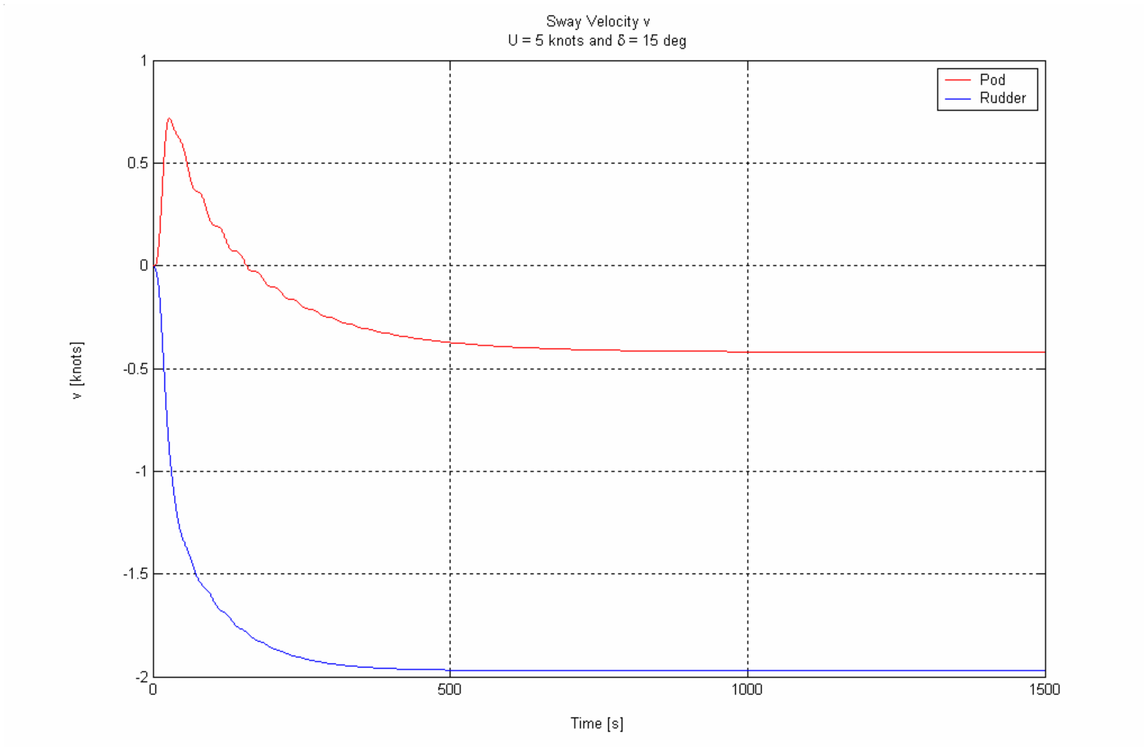

Figure 19. v vs. time, U = 5 knots and $\delta = 15^{o}$

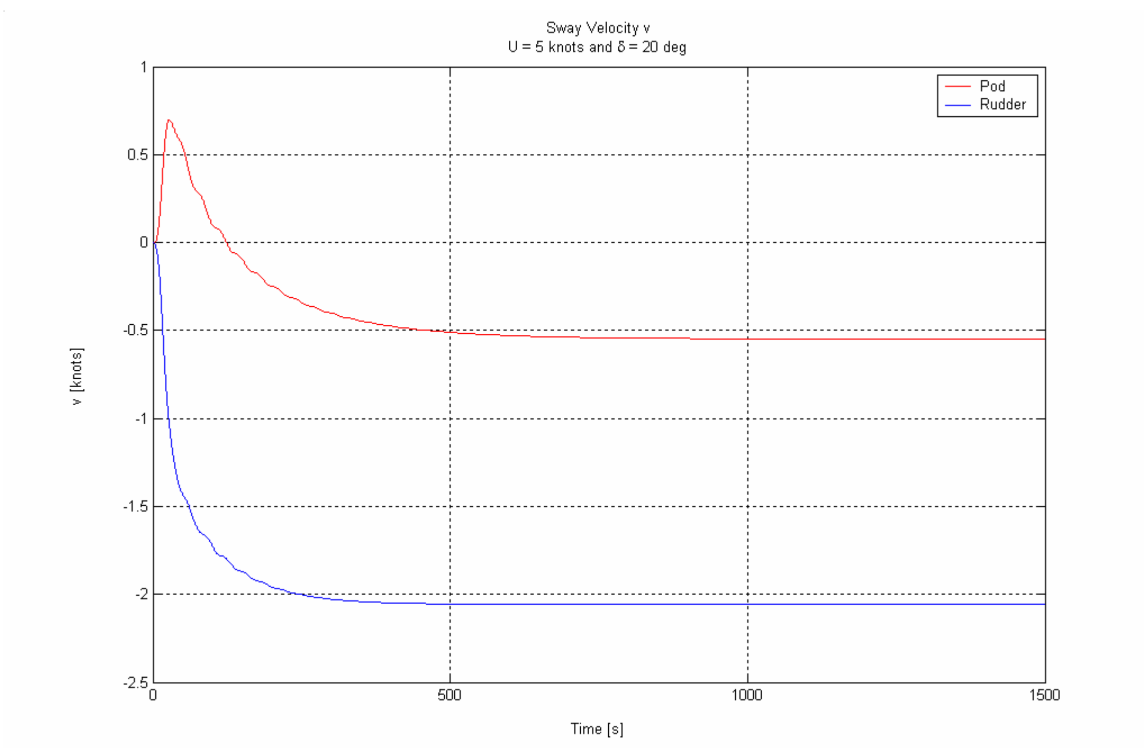

Figure 20. v vs. time, U = 5 knots and $\delta = 20^{o}$

E. POSITION IN XY PLANE VERSUS TIME

The ship with a pod has a smaller turning radius than the ship with a rudder. The ship with the pod also has a smaller advance and transfer at the commanded pod angle for increasing pod angles until the total speed is 15 knots and the rudder/pod angle is 15 degrees. After this point, the ship with the rudder has a smaller advance. Also, as the rudder/pod angle increases, the pod progressively has a larger turning radius than before for the same rudder/pod angle (and different speed U). However, the ship with the pod still has a smaller turning radius than the ship with a rudder. As the total speed U increases, the advance of the ship with the pod is greatly increased. The advance increases to as much as 2000 m for a total speed of 30 knots and pod angle of 20 degrees. The transfer also increases so that at higher speeds U the pod requires more distance to begin the turn. These results are consistent with previous studies [Ref. 1]. The plots for varying rudder/pod angle and varying total speed U are shown in Figures 21 through 32.

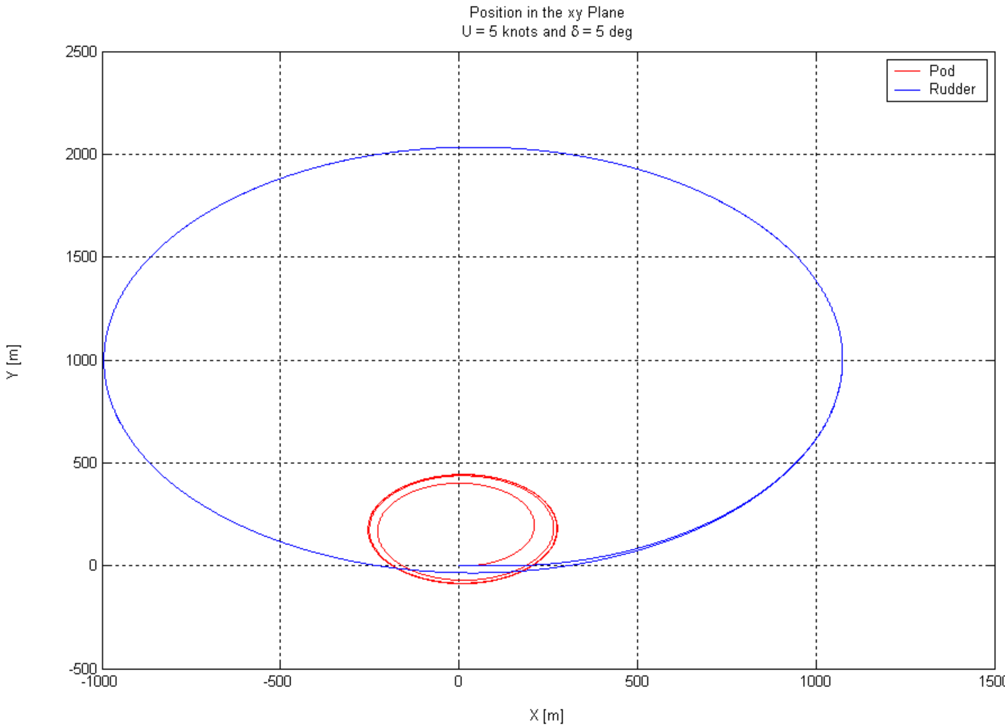

Figure 21. Position in XY Plane, U = 5 knots and δ = 5º

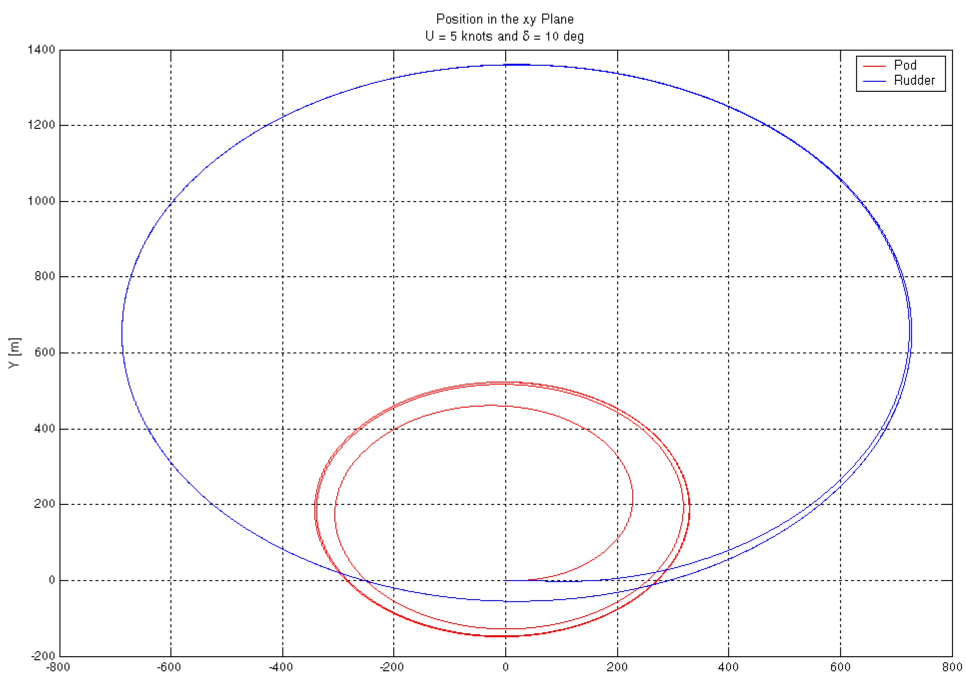

Figure 22. Position in XY Plane, U = 5 knots and δ = 10º

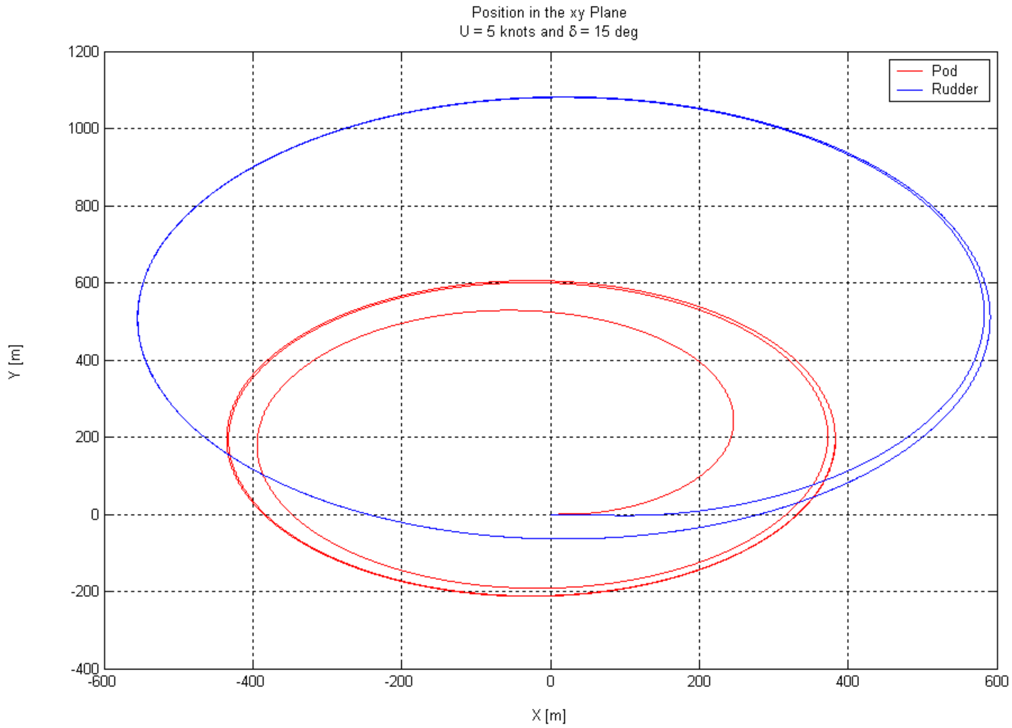

Figure 23. Position in XY Plane, U = 5 knots and δ = 15º

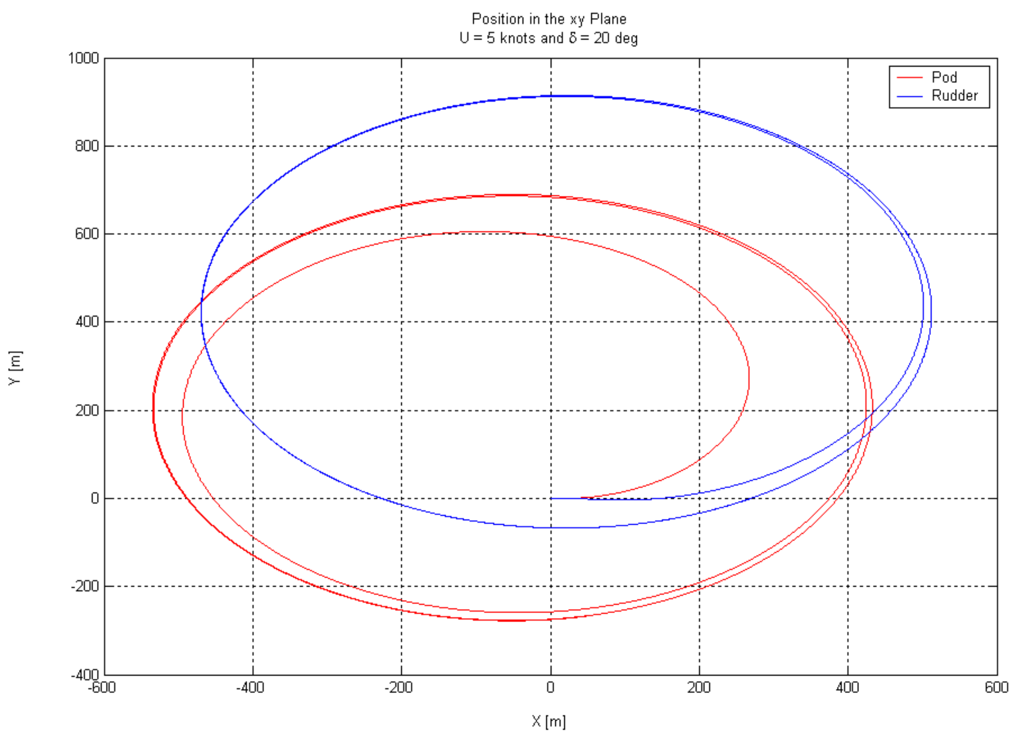

Figure 24. Position in XY Plane, U = 5 knots and δ = 20º

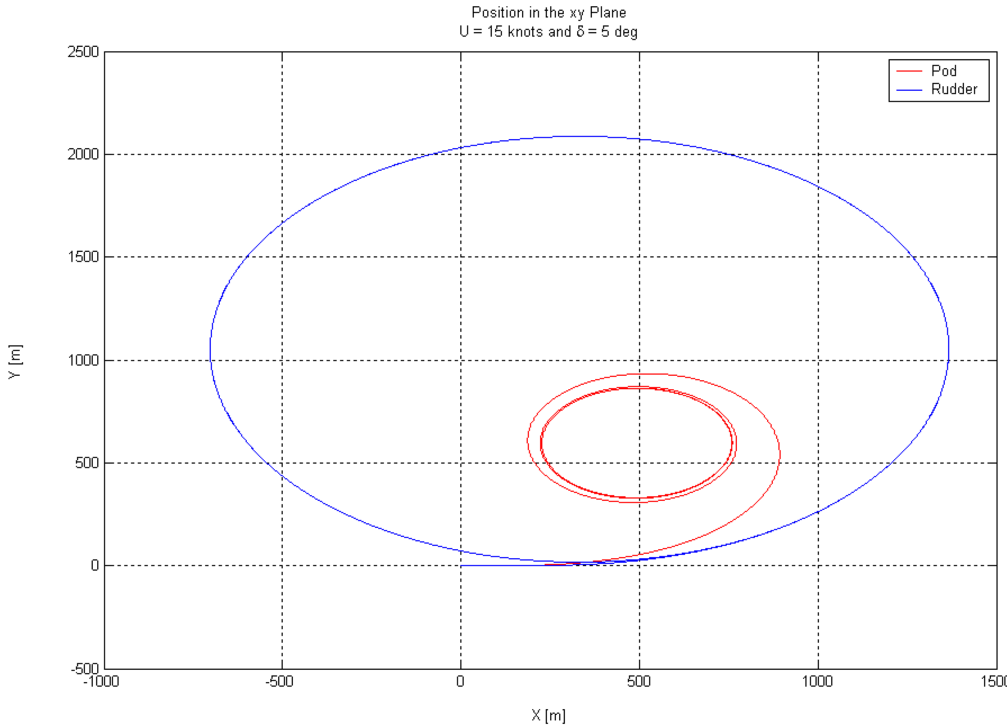

Figure 25. Position in XY Plane, U = 15 knots and $\delta = 5^o$

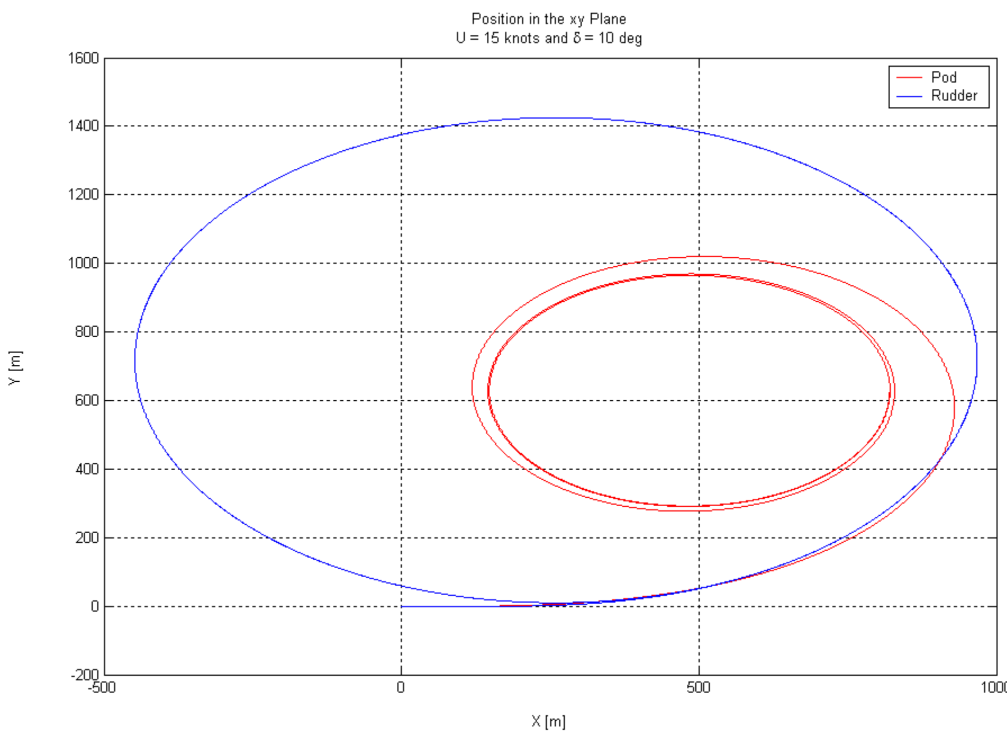

Figure 26. Position in XY Plane, U = 15 knots and $\delta = 10^o$

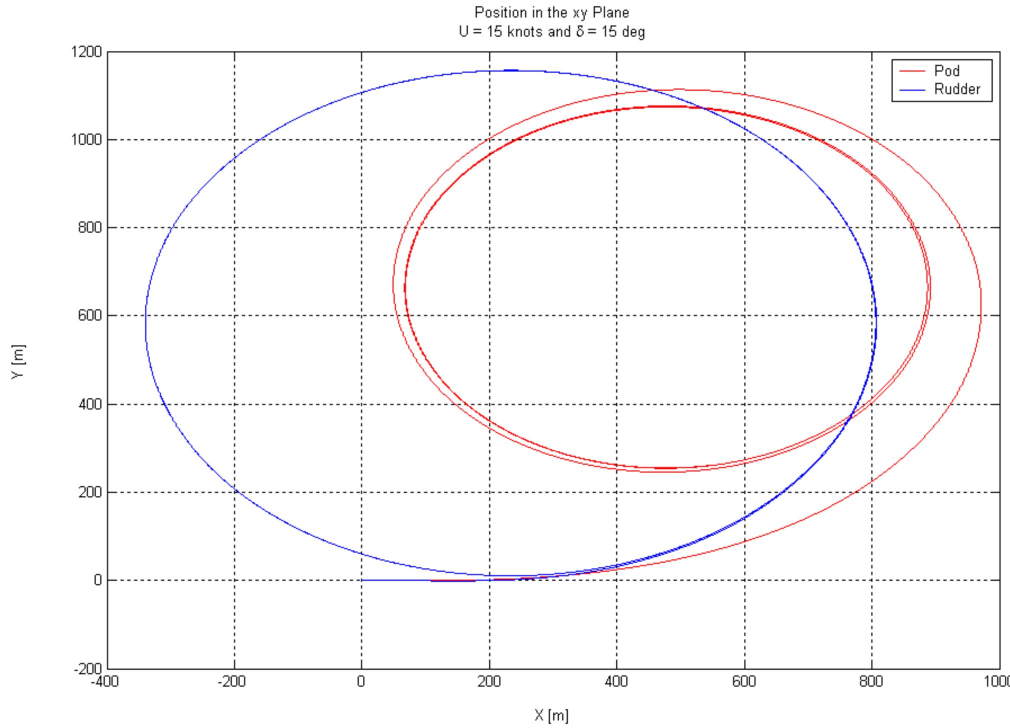

Figure 27. Position in XY Plane, U = 15 knots and δ = 15º

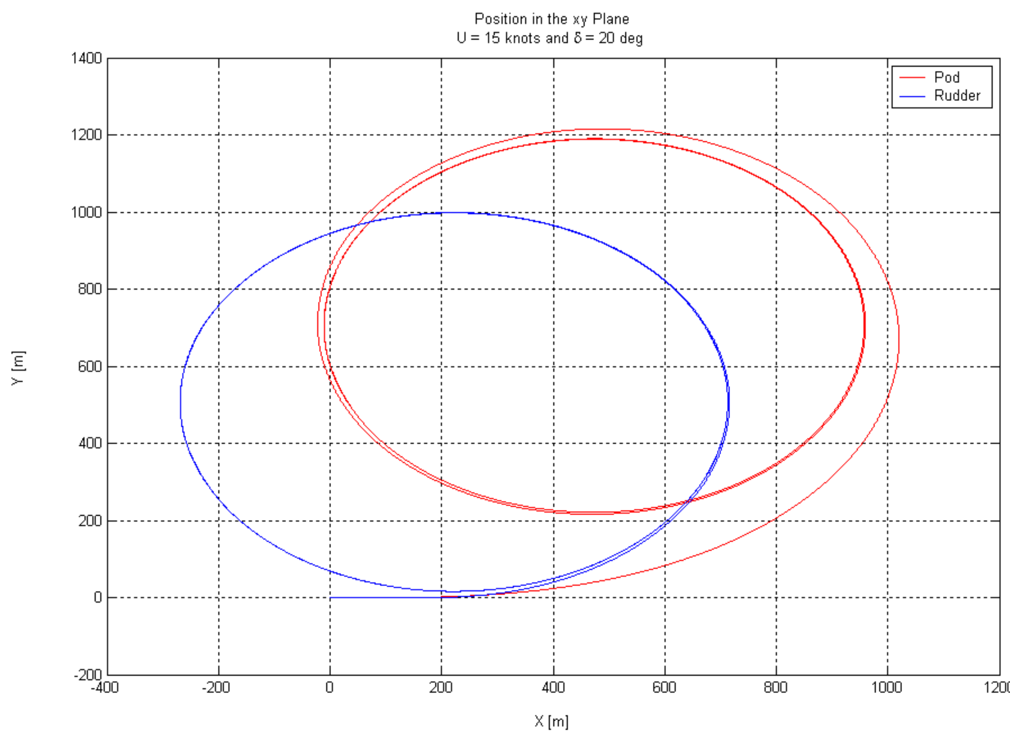

Figure 28. Position in XY Plane, U = 15 knots and δ = 20º

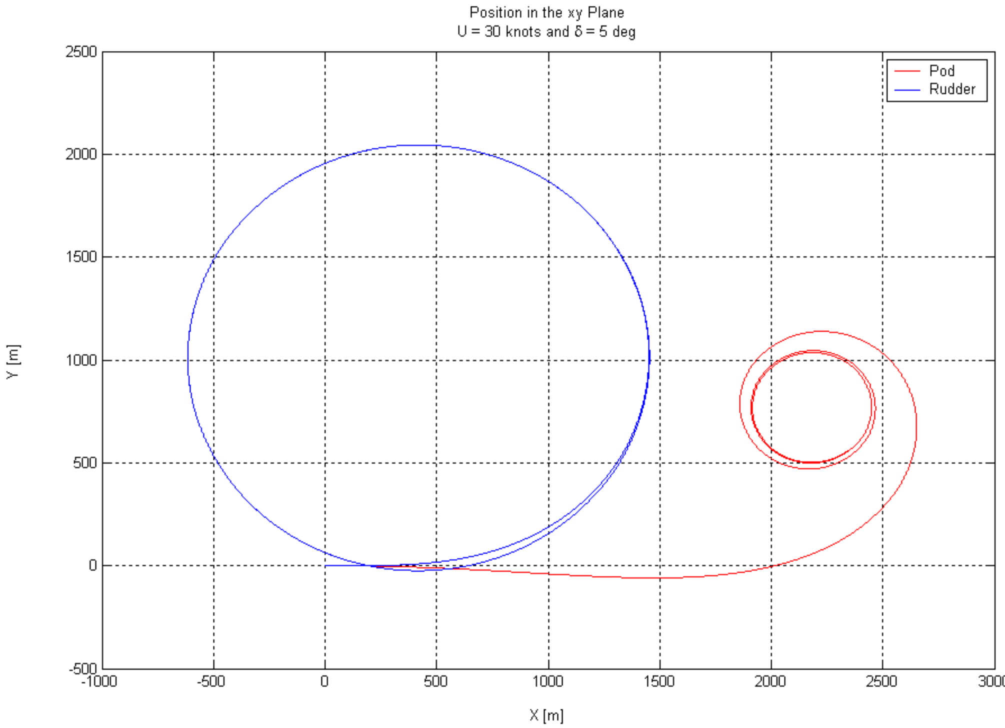

Figure 29. Position in XY Plane, U = 30 knots and δ = 5º

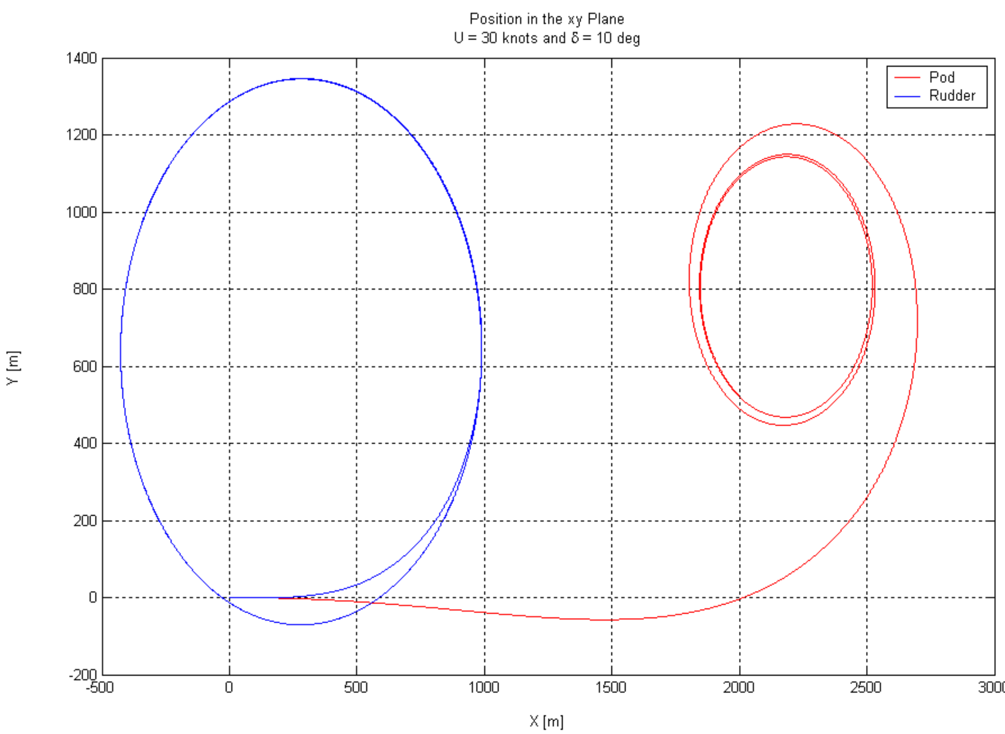

Figure 30. Position in XY Plane, U = 30 knots and δ = 10º

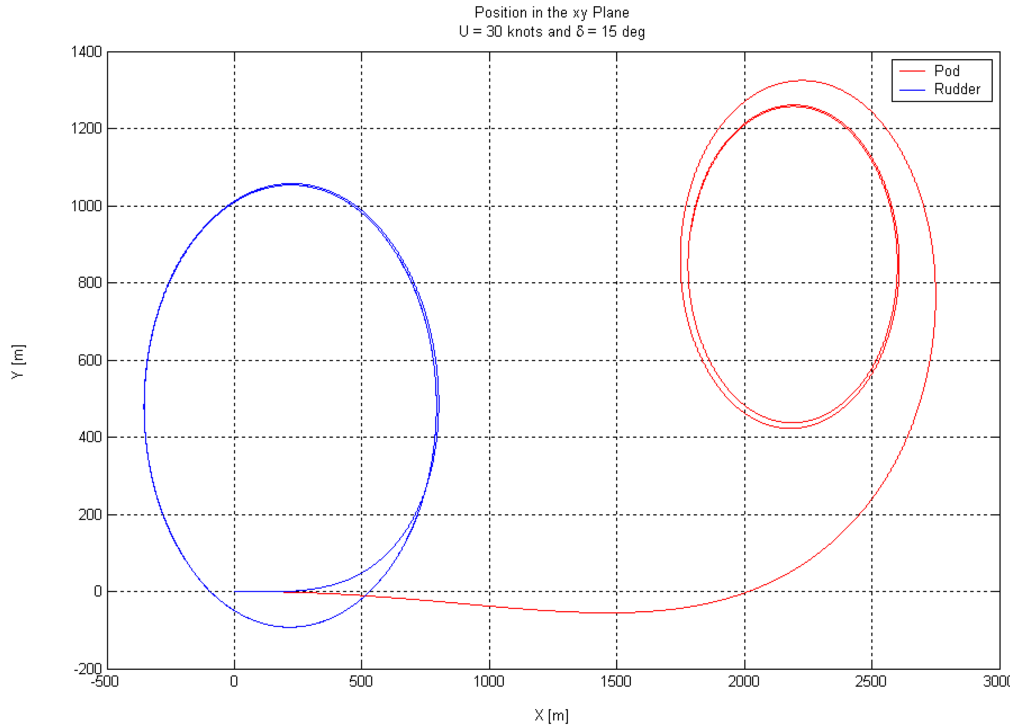

Figure 31. Position in XY Plane, U = 30 knots and $\delta = 15^{\circ}$

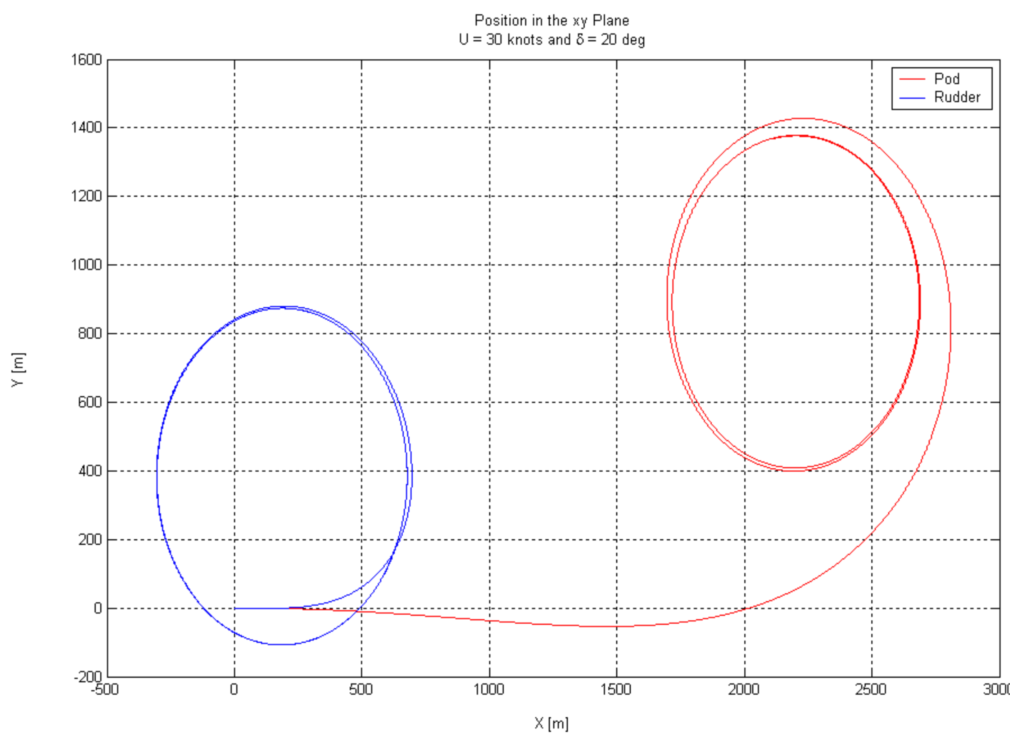

Figure 32. Position in XY Plane, U = 30 knots and $\delta = 20^{\circ}$

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IV. STEADY STATE ANALYSIS

A. PARAMETERS OF INTEREST

The final integrated value of each parameter found from numerical integration of Equation 2.11 is obtained from Matlab; this value is the steady state value of the parameter. For the steady state analysis, the following parameters are studied: sway velocity v, yaw velocity r, and roll angle ϕ.

B. ANALYSIS OF PARAMETERS

The results are plotted for different initial speeds U and different rudder/pod angles δ. As observed from the plots, the steady state values of each parameter do not vary with a change in the initial speed U (with the same rudder/pod angle). This is the same result obtained from the transient analysis for the parameters studied. The parameters vary slightly when plotted using constant initial speed U and varying rudder/pod angle.

Figure 33. Steady State Sway Velocity for Pod with Varying Initial Speed

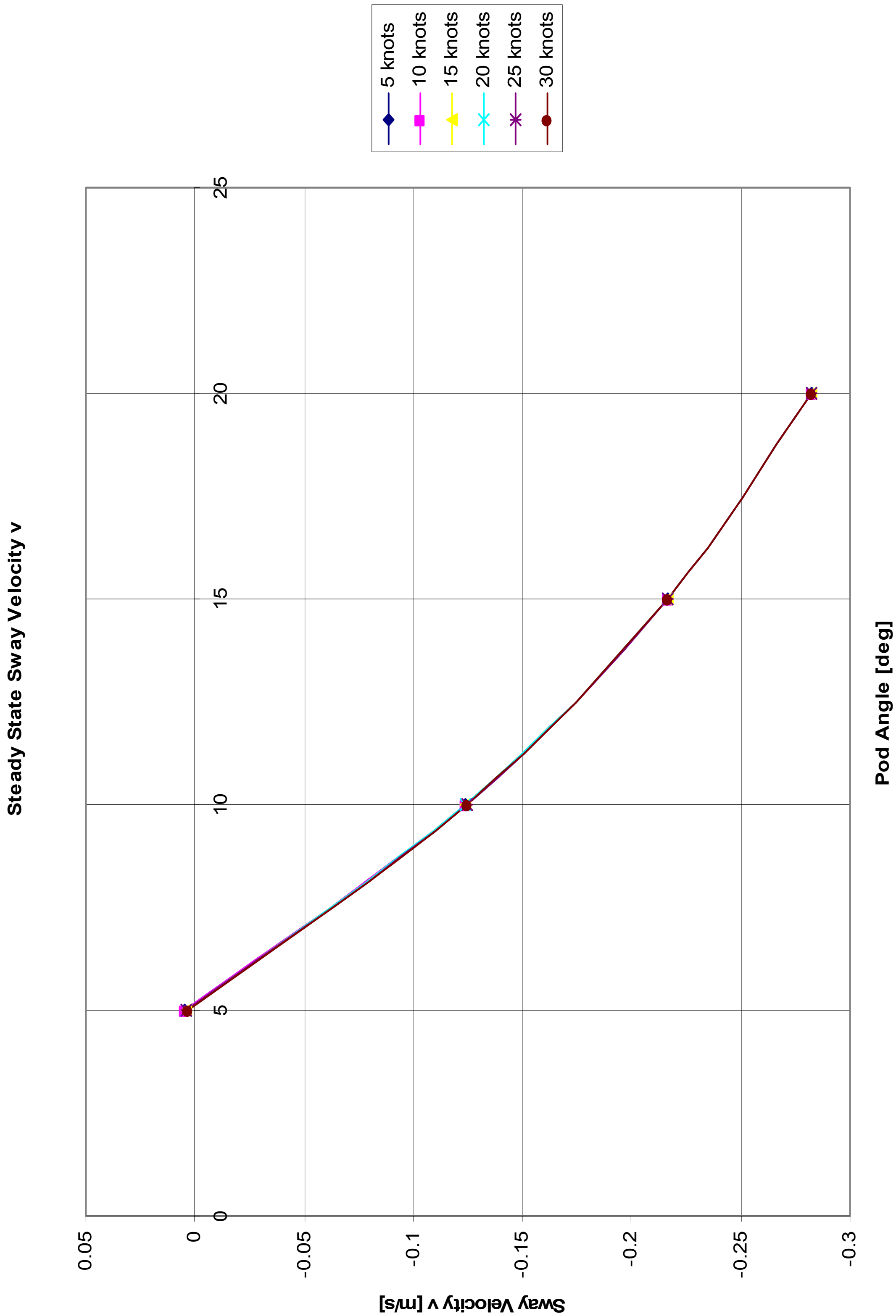


Figure 34. Steady State Sway Velocity for Rudder with Varying Initial Speed

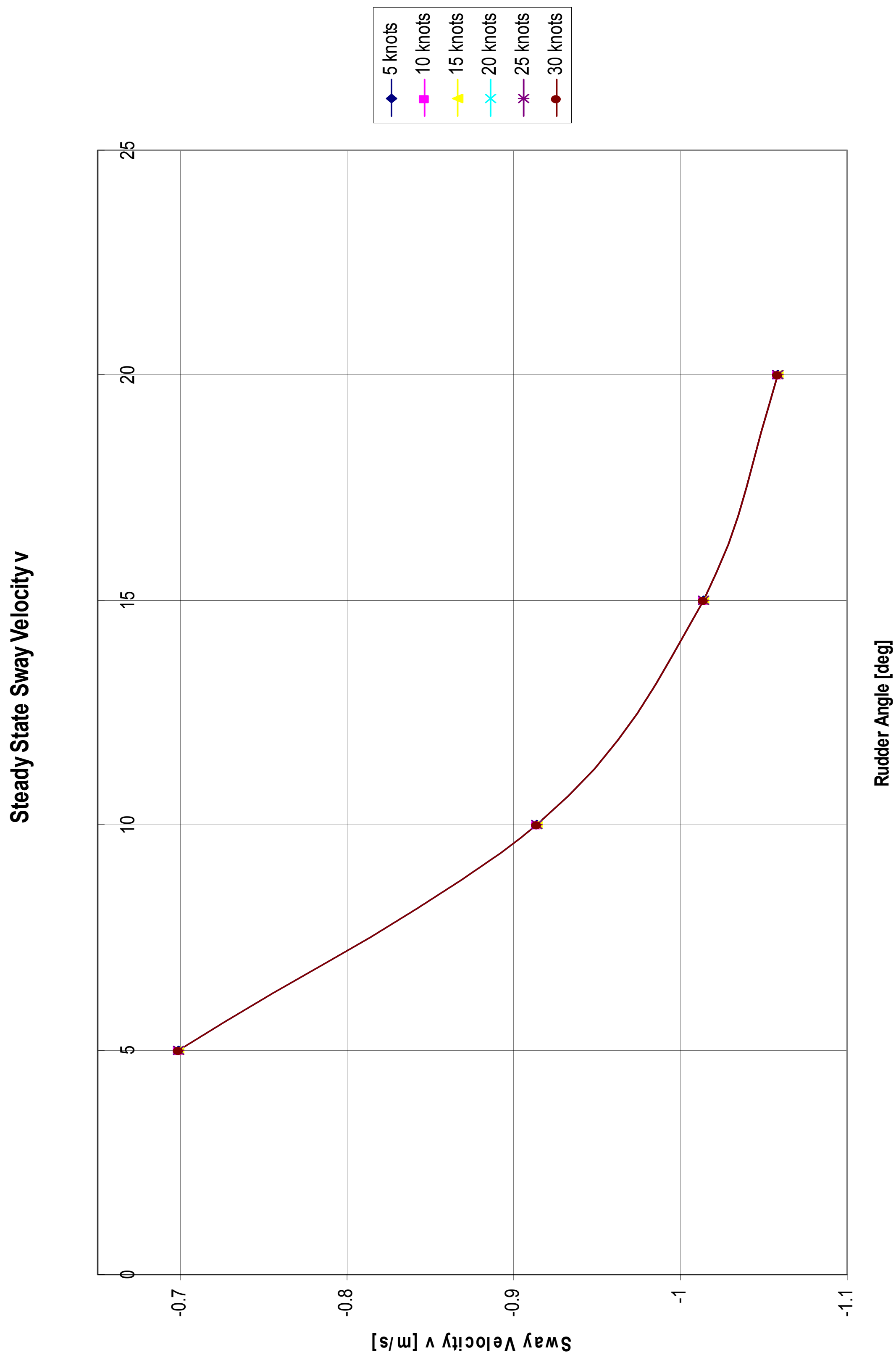


Figure 35. Steady State Yaw Velocity for Pod with Varying Initial Speed

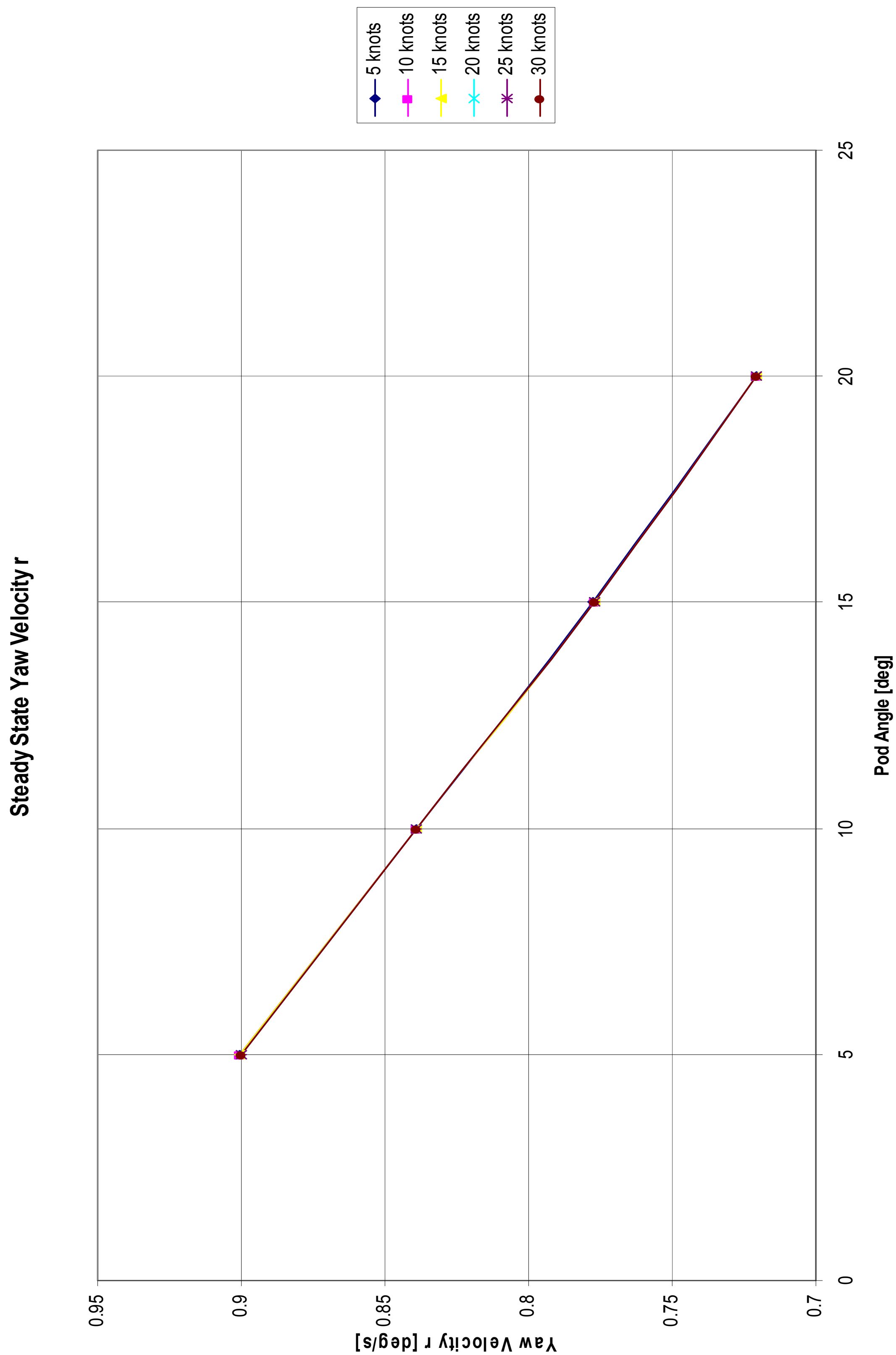
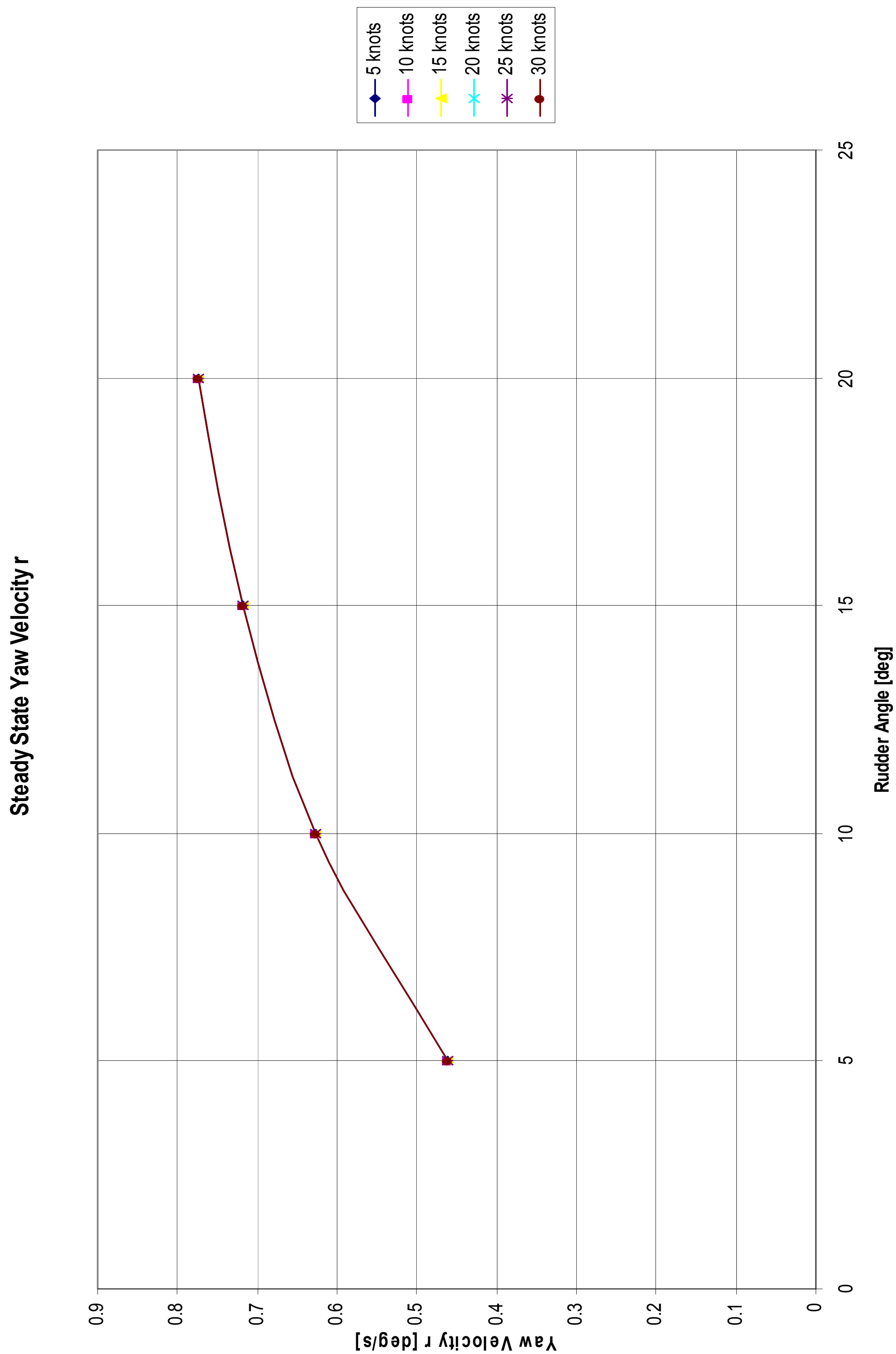


Figure 36. Steady State Yaw Velocity for Rudder with Varying Initial Speed



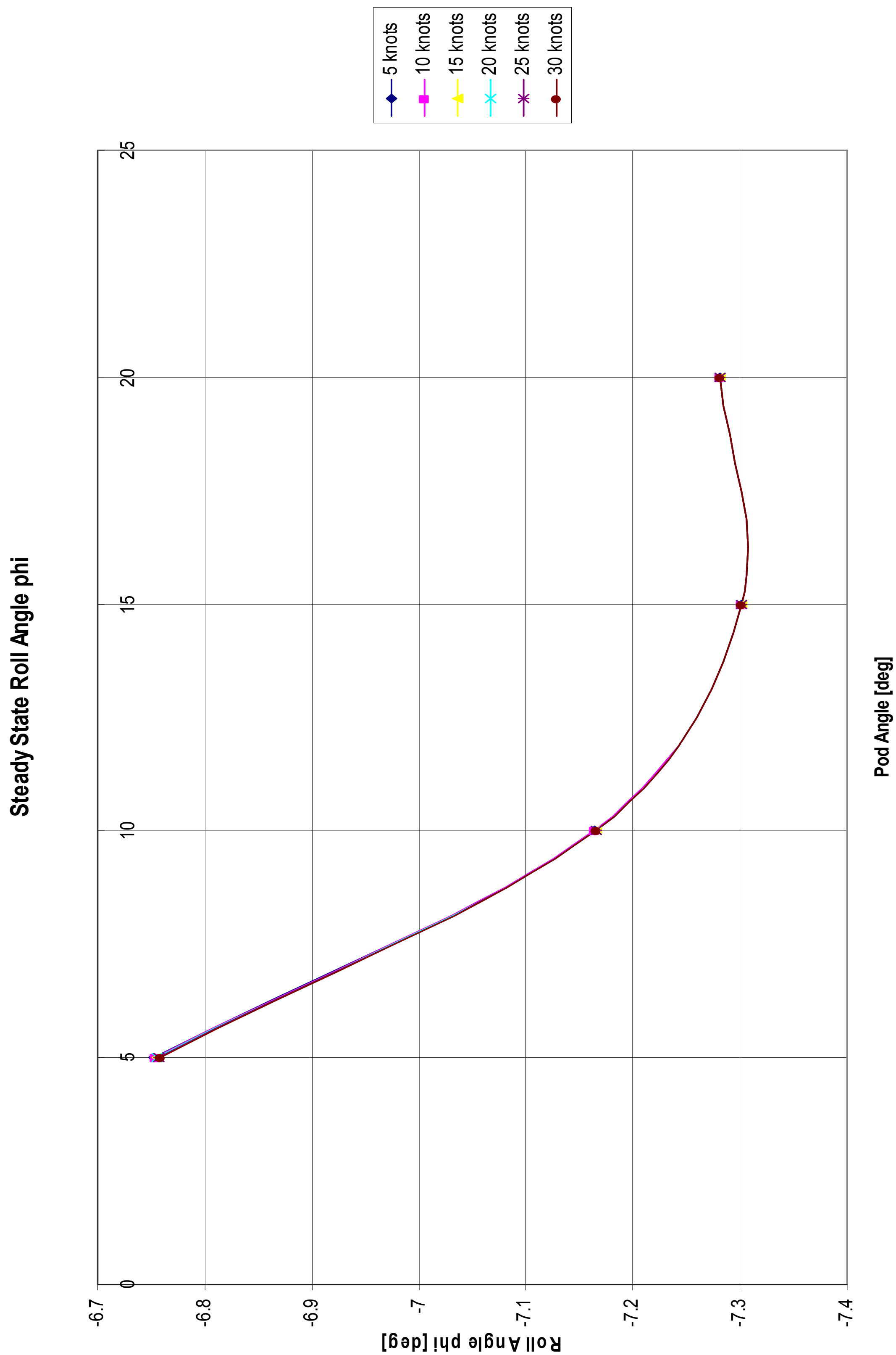
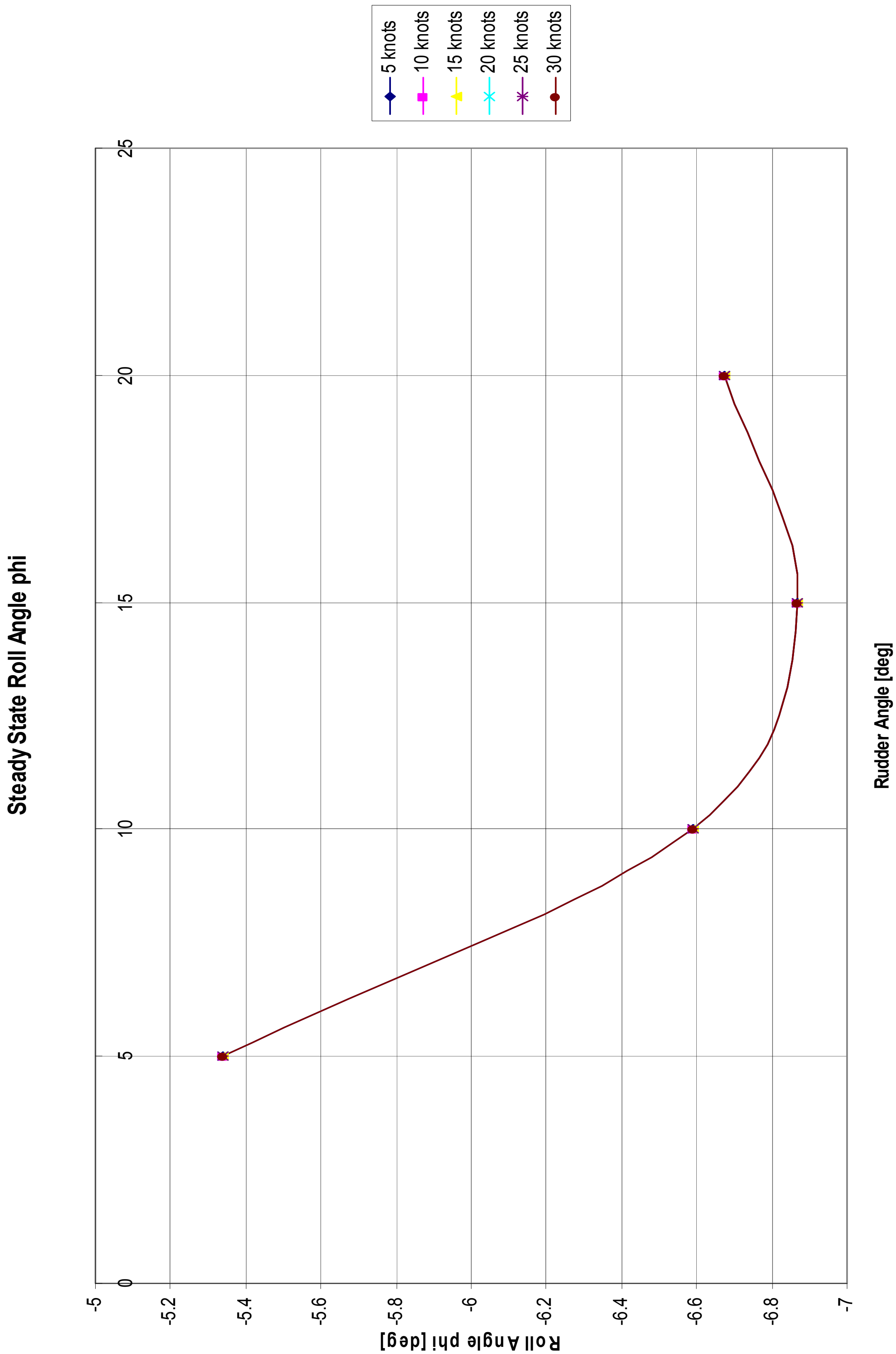


Figure 37. Steady State Roll Angle for Pod with Varying Initial Speed

Figure 38. Steady State Roll Angle for Rudder with Varying Initial Speed



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V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

In this thesis, a mathematical model was modified to simulate the motion of a ship operating with a rudder and operating with a pod. The model was first used to simulate the ship operating with a rudder. The model was then modified to simulate operation with a pod. The two results were then compared to determine how well a ship with a pod operates at various speeds and pod deflections. Based on the comparison made, the following results were observed for the ship with the pod:

1. The results suggest that a ship with a pod will operate very well at speeds less than or equal to 15 knots and pod deflections of less than or equal to 15 degrees. After this point, the pod tends to have a larger advance and transfer as initial speed and deflection angle increase.
2. The results indicate that for a ship with a pod, the ship's total speed will increase during a turn if the initial speed is 5 knots or less. At all other speeds and deflections, the ship with the pod will slow down. This behavior is similar to that of a ship with a rudder.
3. The results show that a ship with a pod will tend to have a larger magnitude roll angle than a ship with a rudder; however, the roll angle does not exceed 8^{o} regardless of the initial speed.
4. The results indicate that the sway velocity v will be approximately constant as the initial speed U increases if the pod deflection is constant. The sway velocity will decrease as the pod deflection increases and the initial speed is held constant.

Based on the results observed using the model as described, the rudder performs better than the pod at high speeds and high deflection angles. Naval vessels must have the capability of responding rapidly in an emergency, regardless of the initial speed of the vessel. The pod does not provide the necessary performance requirements to meet this constraint in the current configuration.

B. RECOMMENDATIONS

The results produced in this thesis were intended as an initial study into the viability of replacing a rudder on a naval ship with a podded propulsor. In order to accomplish this, an existing model was modified to simulate a podded propulsor in place of a propeller and no rudder. The model did not take into account any change in the hull configuration that may be required for use of a pod, nor the possible need of more than one pod to properly maneuver the ship. One area of further study is to further modify the model used to account for this, or to create a new model to simulate a ship that already uses a pod.

Further study is also necessary to determine the best possible pod and hull configuration for a given application. Previous studies have shown that pods do not operate as well as rudders at high speeds. Further study is needed to investigate the benefits of adding control surfaces to counteract this negative performance aspect of a pod.

APPENDIX A. MATLAB CODE FOR SHIP WITH RUDDER

A. V07_CONT

```
function u1 = V07_cont(k,x,u)
u1 = [ 20*pi/180;          % delta (rad)
          85     ];
```

B. V07_DATA

```
% V07_DATA   datafile for v07.m. Contains initial state
%            vector, initial input and main dimensions.
%
% NTH 1994 Trygve Lauvdal
clc
echo on
% Default parameters for High Speed Container Ship:
%
% Max rudder angle   : 0 deg
% Max rudder rate    : 0 deg/s
% Max shaft velocity : 160 rpm
% Length of ship     : 175 m
% Initial states, inputs and sample time (to be edited by user):
u     =    15.433;     % surge velocity       (m/s)
v     =    0;          % sway velocity        (m/s)
p     =    0;          % roll velocity        (rad/s)
r     =    0;          % yaw velocity         (rad/s)
phi   =    0;          % roll angle           (rad)
psi   =    0;          % yaw angle            (rad)
xpos  =    0;          % surge position       (m)
ypos  =    0;          % sway position        (m)
delta =    0;          % actual rudder angle  (rad)
n     =    10;         % actual shaft speed   (rpm)
```

```
d_c      =     20*pi/180;     % commanded rudder angle   (rad)
n_c      =     0;             % commanded shaft speed    (rpm)

h        =     0.1;           % sample time          (s)

x        =     [u  v  p  r  phi psi xpos ypos delta n]';     % state vector
u        =     [d_c  n_c]';                                  % input vector

echo
current = pwd;
if ~exist(['sim.m']),
saveon  = [current '/v07_data x u h'];
else
saveon  = [current '/v07/v07_data x u h'];
end
eval(['save v07_data'])
```

C. V07_RUDDER

```
function xdot = V07_rudder(x,ui)
%
% xdot = V07(x,ui); returns the time derivate of the state vector :
%
% x = [ u v p r phi psi xpos ypos delta n ]'  where
%
% u     =   surge velocity          (m/s)
% v     =   sway velocity           (m/s)
% p     =   roll velocity           (rad/s)
% r     =   yaw velocity            (rad/s)
% phi   =   roll angle              (rad)
% psi   =   yaw angle               (rad)
% xpos  =   position in x-direction (m)
% ypos  =   position in y-direction (m)
% delta =   actual rudder angle     (rad)
% n     =   actual shaft velocity   (rpm)
%
% The input vector is:
%
% ui    =   [ delta_c n_c ]'  where
%
% delta_c   =   commanded rudder angle      (rad)
% n_c       =   commanded shaft velocity    (rpm)
%
% NTH 1994, Trygve Lauvdal
%
% Reference : Son og Nomoto (1982).
%           On the Coupled Motion of Steering and
%           Rolling of a High Speed Container Ship,
%           Naval Architect of Ocean Engineering,
%           20: 73-83. From J.S.N.A. , Japan, Vol. 150, 1981.
```

```
% Check of input and state dimensions
if ~(length(x) == 10),error('x-vector must have dimension 10 !');end
if ~(length(ui) == 2),error('u-vector must have dimension  2 !');end

% Normalization variables
L       =       175;                % length of ship (m)
sqrV    =       (x(1)^2 + x(2)^2);
V       =       sqrt(sqrV);         % service speed  (m/s)

% Check of service speed
if V == 0,error('The ship must have speed greater than zero');end

delta_max       =       20;         % max rudder angle          (deg)
Ddelta_max      =       5;          % max rudder angle rate     (deg/s)
n_max           =       160;        % max shaft velocity        (rpm)

% Non-dimensional states and inputs
delta_c         =       ui(1);
n_c             =       ui(2)/60*L/V;

u    = x(1)/V;    v   = x(2)/V;
p    = x(3)*L/V; r    = x(4)*L/V;
phi  = x(5);      psi = x(6);
delta = x(9);    n   = x(10)/60*L/V;

% Parameters, hydrodynamic derivatives and main dimensions
m = 0.00792;     mx     = 0.000238;        my = 0.000238;
Ix = 0.0000176;  alphay  = 0.05;            lx = 0.0313;
ly = 0.0313;     Ix     = 0.0000176;       Iz = 0.000456;
Jx = 0.0000034;  Jz     = 0.000419;        xG = 0;

B    = 25.40;          dF = 8.00;      g    = 9.81;
```

```
dA    = 9.00;           d  = 8.50;      nabla = 21222;%Displacement Volume
KM    = 10.39;          KB = 4.6154;  AR    = 33.0376;%Rudder Area
KG    = 10.09;          Delta = 1.8219; %Aspect Ratio
D     = 6.533; %Propeller Diameter
GM    = .3/L;
rho   = 1000;           t  = 0.175;     T     = 0.0005;

W     = rho*g*nabla/(rho*L^2*V^2/2);

Xuu     = -0.0004226;  Xvr   = -0.00311;   Xrr     = 0.00020;
Xphiphi  = -0.00020;   Xvv   = -0.00386;

Kv      =  0.0003026;          Kr    = -0.000063;    Kp      = -0.0000075;
Kphi     = -0.000021;          Kvvv   =  0.002843;   Krrr     = -0.0000462;
Kvvr     = -0.000588;          Kvrr   =  0.0010565;  Kvvphi   = -0.0012012;
Kvphiphi = -0.0000793;         Krrphi = -0.000243;   Krphiphi =  0.00003569;

Yv      = -0.0116;      Yr    =  0.00242;      Yp      =  0;
Yphi     = -0.000063;  Yvvv   = -0.109;        Yrrr     =  0.00177;
Yvvr     =  0.0214;     Yvrr   = -0.0405;      Yvvphi   =  0.04605;
Yvphiphi =  0.00304;  Yrrphi =  0.009325;    Yrphiphi = -0.001368;

Nv      = -0.0038545; Nr    = -0.00222;       Np      =  0.000213;
Nphi     = -0.0001424;Nvvv   =  0.001492;     Nrrr     = -0.00229;
Nvvr     = -0.0424;     Nvrr   =  0.00156;      Nvvphi   = -0.019058;
Nvphiphi = -0.0053766;  Nrrphi = -0.0038592;  Nrphiphi =  0.0024195;

kk     =  0.631;          epsilon =  0.921;       xR    = -0.5;
wp     =  0.184;          tau     =  1.09;        xp    = -0.526;
cpv    =  0.0;            cpr     =  0.0;         ga    =  0.088;
cRr    = -0.156;          cRrrr   = -0.275;       cRrrv =  1.96;
cRX    =  0.71;           aH      =  0.237;       zR    =  0.033;
xH     = -0.48;
```

```
% Masses and moments of inertia
m11 = (m+mx);
m22 = (m+my);
m32 = -my*ly;
m42 = my*alphay;
m33 = (Ix+Jx);
m44 = (Iz+Jz);

% Rudder saturation and dynamics
if abs(delta_c) >= delta_max*pi/180,
  delta_c = sign(delta_c)*delta_max*pi/180;
end

delta_dot = delta_c - delta;
if abs(delta_dot) >= Ddelta_max*pi/180,
  delta_dot = sign(delta_dot)*Ddelta_max*pi/180;
end

% Shaft velocity saturation and dynamics
n_c = n_c*V/L;
n    = n*V/L;
if abs(n_c) >= n_max/60,
  n_c = sign(n_c)*n_max/60;
end

if n > 0.3,Tm=5.65/n;else,Tm=18.83;end
n_dot = 1/Tm*(n_c-n)*60;

% Calculation of state derivatives
  vR     = ga*v + cRr*r + cRrrr*r^3 + cRrrv*r^2*v;
  uP     = cos(v)*((1 - wp) + tau*((v + xp*r)^2 + cpv*v + cpr*r));
   J     = uP*V/(n*D);
  KT     = 0.527 - 0.455*J;
```

```
uR     = uP*epsilon*sqrt(1 + 8*kk*KT/(pi*J^2));
alphaR = delta + atan(vR/uR);
FN     = - ((6.13*Delta)/(Delta + 2.25))*(AR/L^2)*(uR^2 + vR^2)*sin(alphaR);
T      = 2*rho*D^4/(V^2*L^2*rho)*KT*n*abs(n);

% Forces and moments
X   = Xuu*u^2 + (1-t)*T + Xvr*v*r + Xvv*v^2 + Xrr*r^2 + Xphiphi*phi^2 + ...
      (m + my)*v*r + cRX*FN*sin(delta);

Y   = Yv*v + Yr*r + Yp*p + Yphi*phi + Yvvv*v^3 + Yrrr*r^3 + ...
      Yvvr*v^2*r + Yvrr*v*r^2 + Yvvphi*v^2*phi + Yvphiphi*v*phi^2 +
          Yrrphi*r^2*phi + ...
      Yrphiphi*r*phi^2 - (m + mx)*u*r + (1 + aH)*FN*cos(delta);

K   = Kv*v + Kr*r + Kp*p + Kphi*phi + Kvvv*v^3 + Krrr*r^3 + ...
      Kvvr*v^2*r + Kvrr*v*r^2 + Kvvphi*v^2*phi + Kvphiphi*v*phi^2 +
          Krrphi*r^2*phi + ...
      Krphiphi*r*phi^2 + mx*lx*u*r - W*GM*phi - (1 + H)*zR*FN*cos(delta);

N   = Nv*v + Nr*r + Np*p + Nphi*phi + Nvvv*v^3 + Nrrr*r^3 + ...
      Nvvr*v^2*r + Nvrr*v*r^2 + Nvvphi*v^2*phi + Nvphiphi*v*phi^2 +
          Nrrphi*r^2*phi + ...
      Nrphiphi*r*phi^2 + (xR + aH*xH)*FN*cos(delta);

% Dimensional state derivatives
xdot =[                           X*(V^2/L)/m11
   -(-m33*m44*Y+m32*m44*K+m42*m33*N)/(m22*m33*m44-m32^2*m44-
                                  m42^2*m33)*(V^2/L)
       (-m32*m44*Y+K*m22*m44-K*m42^2+m32*m42*N)/(m22*m33*m44-
                                  m32^2*m44-m42^2*m33)*(V^2/L^2)
       (-m42*m33*Y+m32*m42*K+N*m22*m33-N*m32^2)/(m22*m33*m44-
                                  m32^2*m44-m42^2*m33)*(V^2/L^2)
                                  p*(V/L)
                              cos(phi)*r*(V/L)
```

```
                     (cos(psi)*u-sin(psi)*cos(phi)*v)*V
                     (sin(psi)*u+cos(psi)*cos(phi)*v)*V
                                delta_dot
                                 n_dot          ];
```

D. V07_SIM

```
% SIM.m
%
% Main program for contship simulation
%
% NTH 1994, Trygve Lauvdal
clear

v07_data                                % set model data
load v07_data                           % load model data

N     = 15000;                          % number of samples
table = zeros(N,length(x)+length(u));% storage vector
disp('Simulating');

for i = 1:N+1,                          % START OF MAIN LOOP
  table(i,:) = [ x' , u'] ;             % data storage
  u  = v07_cont(i*h,x,u);               % input
  xdot = v07_rudder(x,u');              % xdot(i) = f(x(i),u(i))
  x    = x + h*xdot ;                   % euler integration x(i+1)
end                                     % END OF MAIN LOOP
```

E. V07RUDDER_PLOT

```
function plot_data = V07rudder_plot(table,h,N)

figure (1)
x = table(:,7);
y = table(:,8);
plot(x,y)
legend('Pod','Rudder')

figure(2)
time = (0:N)*h;
u = table(:,1)*1.94384449244;
v = table(:,2)*1.94384449244;
U = sqrt(u.*u + v.*v);
plot(time,U),
legend('Pod','Rudder')

figure(3)
plot(time,v),
legend('Pod','Rudder')

figure(4)
phi = table(:,5)*(180/pi);
plot(time,phi)
legend('Pod','Rudder')
```

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APPENDIX B. MATLAB CODE FOR SHIP WITH POD

A. V07_CONT

```
function u1 = V07_cont(k,x,u)
u1 = [ 20*pi/180;          % delta_pod (rad)
       85    ];
```

B. V07_DATA

```
% V07_DATA   datafile for v07.m. Contains initial state
%            vector, initial input and main dimensions.
%
% NTH 1994 Trygve Lauvdal
clc
echo on
% Default parameters for High Speed Container Ship :
%
% Max pod angle       : 0 deg
% Max pod rate        : 0 deg/s
% Max shaft velocity  : 160 rpm
% Length of ship      : 175 m
% Initial states, inputs and sample time (to be edited by user):
u     =     15.433;      % surge velocity     (m/s)
v     =     0;           % sway velocity      (m/s)
p     =     0;           % roll velocity      (rad/s)
r     =     0;           % yaw velocity       (rad/s)
phi   =     0;           % roll angle         (rad)
psi   =     0;           % yaw angle          (rad)
xpos  =     0;           % surge position     (m)
ypos  =     0;           % sway position      (m)
delta_pod=  0;           % actual rudder angle (rad)
n     =     10;          % actual shaft speed  (rpm)

d_c   =     20*pi/180;   % commanded rudder angle   (rad)
```

```
n_c     =     0;                % commanded shaft speed     (rpm)

h       =     0.1;              % sample time          (s)

x       =     [u  v  p  r  phi psi xpos ypos delta n]';          % state vector
u       =     [d_c  n_c]';                                        % input vector

echo
current = pwd;
if ~exist(['sim.m']),
saveon  = [current '/v07_data x u h'];
else
saveon  = [current '/v07/v07_data x u h'];
end
eval(['save v07_data'])
```

C. V07_POD

```
function xdot = V07_pod(x,ui)
%
% xdot = V07(x,ui); returns the time derivate of the state vector :
%
% x = [ u v p r phi psi xpos ypos delta n ]'  where
%
% u       =    surge velocity          (m/s)
% v       =    sway velocity           (m/s)
% p       =    roll velocity           (rad/s)
% r       =    yaw velocity            (rad/s)
% phi     =    roll angle              (rad)
% psi     =    yaw angle               (rad)
% xpos    =    position in x-direction (m)
% ypos    =    position in y-direction (m)
% delta_pod=   actual pod angle        (rad)
% n       =    actual shaft velocity   (rpm)
%
% The input vector is:
%
% ui      =    [ delta_c n_c ]'  where
%
% delta_c      =    commanded pod angle          (rad)
% n_c          =    commanded shaft velocity     (rpm)
%
% NTH 1994, Trygve Lauvdal
%
% Reference : Son og Nomoto (1982).
%           On the Coupled Motion of Steering and
%           Rolling of a High Speed Container Ship,
%           Naval Architect of Ocean Engineering,
%           20: 73-83. From J.S.N.A. , Japan, Vol. 150, 1981.
```

```
% Check of input and state dimensions
if ~(length(x) == 10),error('x-vector must have dimension 10 !');end
if ~(length(ui) == 2),error('u-vector must have dimension  2 !');end

delta_pod = 20;                    % pod angle     (deg)
delta_pod = delta_pod*pi/180;      % pod angle     (rad)

% Normalization variables
L       =       175;               % length of ship (m)
sqrV    =       (x(1)^2 + x(2)^2);
V       =       sqrt(sqrV);        % service speed  (m/s)

% Check of service speed
if V == 0,error('The ship must have speed greater than zero');end

delta_max    =    20;       % max rudder angle          (deg)
Ddelta_max   =    5;        % max rudder angle rate     (deg/s)
n_max        =    160;      % max shaft velocity        (rpm)

% Non-dimensional states and inputs
delta_c      =    ui(1);
n_c          =    ui(2)/60*L/V;

u    = x(1)/V;          v    = x(2)/V;
p    = x(3)*L/V;        r    = x(4)*L/V;
phi  = x(5);            psi = x(6);
delta_pod = x(9);       n   = x(10)/60*L/V;

% Parameters, hydrodynamic derivatives and main dimensions
m = 0.00792;     mx      = 0.000238;        my = 0.000238;
Ix = 0.0000176;  alphay  = 0.05;            lx = 0.0313;
ly = 0.0313;     Ix      = 0.0000176;       Iz = 0.000456;
```

```
Jx = 0.0000034;  Jz       = 0.000419;          xG = 0;

B     = 25.40;            dF = 8.00;       g     = 9.81;
dA    = 9.00;             d  = 8.50;       nabla = 21222;%Displacement Volume
KM    = 10.39;            KB = 4.6154;  AR    = 33.0376;%Rudder Area
KG    = 10.09;            Delta = 1.8219; %Aspect Ratio
D     = 6.533; %Propeller Diameter
GM    = .3/L;
rho   = 1000;             t  = 0.175;      T     = 0.0005;

W     = rho*g*nabla/(rho*L^2*V^2/2);

Xuu      = -0.0004226;  Xvr   = -0.00311;    Xrr     = 0.00020;
Xphiphi  = -0.00020;    Xvv   = -0.00386;

Kv       =  0.0003026;          Kr     = -0.000063;    Kp       = -0.0000075;
Kphi     = -0.000021;           Kvvv   =  0.002843;   Krrr     = -0.0000462;
Kvvr     = -0.000588;           Kvrr   =  0.0010565;  Kvvphi   = -0.0012012;
Kvphiphi = -0.0000793;          Krrphi = -0.000243;   Krphiphi =  0.00003569;

Yv       = -0.0116;       Yr     =  0.00242;       Yp      =  0;
Yphi     = -0.000063;  Yvvv   = -0.109;          Yrrr    =  0.00177;
Yvvr     =  0.0214;       Yvrr   = -0.0405;        Yvvphi  =  0.04605;
Yvphiphi =  0.00304;  Yrrphi =  0.009325;    Yrphiphi = -0.001368;

Nv       = -0.0038545; Nr     = -0.00222;       Np      =  0.000213;
Nphi     = -0.0001424;Nvvv   =  0.001492;     Nrrr    = -0.00229;
Nvvr     = -0.0424;       Nvrr   =  0.00156;       Nvvphi  = -0.019058;
Nvphiphi = -0.0053766;  Nrrphi = -0.0038592;  Nrphiphi =  0.0024195;

kk     =  0.631;          epsilon =  0.921;       xR   = -0.5;
wp     =  0.184;          tau     =  1.09;        xp   = -0.526;
cpv    =  0.0;            cpr     =  0.0;         ga   =  0.088;
```

```
cRr   = -0.156;         cRrrr  = -0.275;        cRrrv =  1.96;
cRX    =  0.71;         aH     =  0.237;        zR    =  0.033;
xH    = -0.48;
% Masses and moments of inertia
m11 = (m+mx);
m22 = (m+my);
m32 = -my*ly;
m42 = my*alphay;
m33 = (Ix+Jx);
m44 = (Iz+Jz);

% Pod saturation and dynamics
if abs(delta_c) >= delta_max*pi/180,
   delta_c = sign(delta_c)*delta_max*pi/180;
end

delta_dot = delta_c – delta_pod;
if abs(delta_dot) >= Ddelta_max*pi/180,
   delta_dot = sign(delta_dot)*Ddelta_max*pi/180;
end

% Shaft velocity saturation and dynamics
n_c = n_c*V/L;
n     = n*V/L;
if abs(n_c) >= n_max/60,
   n_c = sign(n_c)*n_max/60;
end

if n > 0.3,Tm=5.65/n;else,Tm=18.83;end
n_dot = 1/Tm*(n_c-n)*60;

% Calculation of state derivatives
  vR     = ga*v + cRr*r + cRrrr*r^3 + cRrrv*r^2*v;
```

```
uP     = cos(v)*((1 - wp) + tau*((v + xp*r)^2 + cpv*v + cpr*r));
 J     = uP*V/(n*D);
KT     = 0.527 - 0.455*J;
uR     = uP*epsilon*sqrt(1 + 8*kk*KT/(pi*J^2));
T      = 2*rho*D^4/(V^2*L^2*rho)*KT*n*abs(n);

% Forces and moments
X   = Xuu*u^2 + (1-t)*T*sin(delta_pod) + Xvr*v*r + Xvv*v^2 + Xrr*r^2 +
        Xphiphi*phi^2 + (m + my)*v*r;


 Y   = (1-t)*T*cos(delta_pod) + Yv*v + Yr*r + Yp*p + Yphi*phi + Yvvv*v^3 +
        Yrrr*r^3 + ...
      Yvvr*v^2*r + Yvrr*v*r^2 + Yvvphi*v^2*phi + Yvphiphi*v*phi^2 +
        Yrrphi*r^2*phi + ...
      Yrphiphi*r*phi^2 - (m + mx)*u*r;


 K   = - zR*(1-t)*T*cos(delta_pod) + Kv*v + Kr*r + Kp*p + Kphi*phi +
        Kvvv*v^3 + Krrr*r^3 + ...
      Kvvr*v^2*r + Kvrr*v*r^2 + Kvvphi*v^2*phi + Kvphiphi*v*phi^2 +
        Krrphi*r^2*phi + ...
      Krphiphi*r*phi^2 + mx*lx*u*r - W*GM*phi;


 N   = 0.5*(1-t)*T*cos(delta_pod) + Nv*v + Nr*r + Np*p + Nphi*phi +
        Nvvv*v^3 + Nrrr*r^3 + ...
      Nvvr*v^2*r + Nvrr*v*r^2 + Nvvphi*v^2*phi + Nvphiphi*v*phi^2 +
        Nrrphi*r^2*phi + ...
      Nrphiphi*r*phi^2;

% Dimensional state derivatives
xdot =[                    X*(V^2/L)/m11
   -(-m33*m44*Y+m32*m44*K+m42*m33*N)/(m22*m33*m44-m32^2*m44-
                           m42^2*m33)*(V^2/L)
        (-m32*m44*Y+K*m22*m44-K*m42^2+m32*m42*N)/(m22*m33*m44-
                           m32^2*m44-m42^2*m33)*(V^2/L^2)
        (-m42*m33*Y+m32*m42*K+N*m22*m33-N*m32^2)/(m22*m33*m44-
                           m32^2*m44-m42^2*m33)*(V^2/L^2)
```

```
p*(V/L)
cos(phi)*r*(V/L)
(cos(psi)*u-sin(psi)*cos(phi)*v)*V
(sin(psi)*u+cos(psi)*cos(phi)*v)*V
delta_dot
n_dot          ];
```

D. V07_SIM

```
% SIM.m
%
% Main program for contship simulation
%
% NTH 1994, Trygve Lauvdal
clear

v07_data                          % set model data
load v07_data                     % load model data

N     = 15000;                    % number of samples
table = zeros(N,length(x)+length(u));% storage vector
disp('Simulating');

for i = 1:N+1,                    % START OF MAIN LOOP
  table(i,:) = [ x' , u'] ;       % data storage
  u  = v07_cont(i*h,x,u);         % input
  xdot = v07_rudder(x,u');        % xdot(i) = f(x(i),u(i))
  x    = x + h*xdot ;             % euler integration x(i+1)
end                               % END OF MAIN LOOP
```

E. V07POD_PLOT

```
function plot_data = V07pod_plot(table,h,N)
clf

figure (1)
```

```
x = table(:,7);
y = table(:,8);
plot(x,y,'r')
grid on, xlabel('X [m]'), ylabel('Y [m]')
title('Position in the xy Plane')
hold on

figure(2)
time = (0:N)*h;
u = table(:,1)*1.94384449244;
v = table(:,2)*1.94384449244;
U = sqrt(u.*u + v.*v);
plot(time,U,'r'),
grid on, xlabel('Time [s]'), ylabel('U [knots]')
title('Total speed U')
hold on

figure(3)
plot(time,v,'r'),
grid on, xlabel('Time [s]'), ylabel('v [knots]')
title('Sway Velocity v')
hold on

figure(4)
phi = table(:,5)*(180/pi);
plot(time,phi,'r')
grid on, xlabel('Time [s]'), ylabel('\phi [deg]')
title('Roll Angle \phi')
hold on
```

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