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# Design a SSV

## Small solar vehicle

## Case SSV part 1

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# 1. The characteristics of the solar panel

For making a good application out of a solar panel it's evident to know the specific characteristics of the solar panel. In the SSV the solar panel is the main source of the vehicle and will produce the current needed for the given DC motor.

An ideal solar cell model is been illustrated in figure 1.

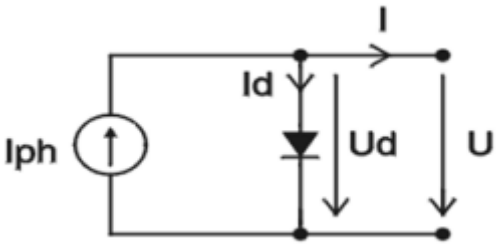



Figure 1: ideal solar panel

The following formula relates to an ideal solar panel that is an abbreviation of reality:

$$I = I_{Ph} - I_s\left(e^{\frac{U}{m*U_r}} - 1\right)$$

$I_{Ph}$ – photocurrent

$I_s$ – reverse saturation current (A)

U – diode voltage (V)

$U_T$ – thermal voltage (V)

m – diode factor

When taking the serial and parallel resistance $R_s$ and $R_P$ into account, the circuit that represents the real solar cell model shown in figure 2 is received.

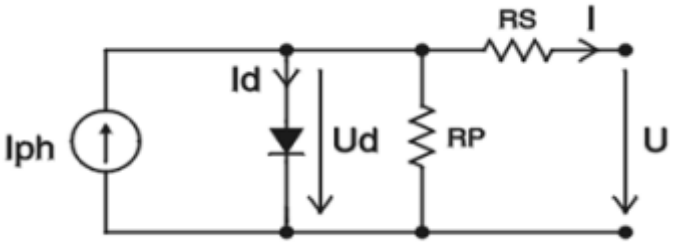



Figure 2: real solar panel circuit

The next formula represents the real solar panel circuit:

$$I = I_{sc} - I_s\left(e^{\frac{U}{m*N*U_r}} - 1\right)$$

$I_{sc}$ – short circuit current (A)

$I_s$ – saturation current (A): $1\times10^{-8}$ A/m²

U – output voltage (V)

$U_r$ – thermal voltage (V): 25,7mV at 25°C

$$U_r = k * \frac{T}{e}$$

k – Boltzmann constant: $1.38\times10^{-23}$ J/K

T – temperature (K)

e – charge of electron (V): $1.6\times10^{-19}$ As

m – diode factor: range of 1 – 5

N – number of solar cells in series: 16

Almost all these values are already defined because they're the solar panel's properties. The other parameters need to be calculated, these parameters are: the short circuit current $I_{sc}$ and the diode factor m.

The first parameter that is calculated is the short circuit current $I_{sc}$. Measuring the current with a multi-meter when the solar panel is short-circuited gives the proper value.

$$I_{sc} = 0{,}64\ A$$

To measure the second parameter an external variable resistor was put into the circuit, a potentiometer like in figure 3.

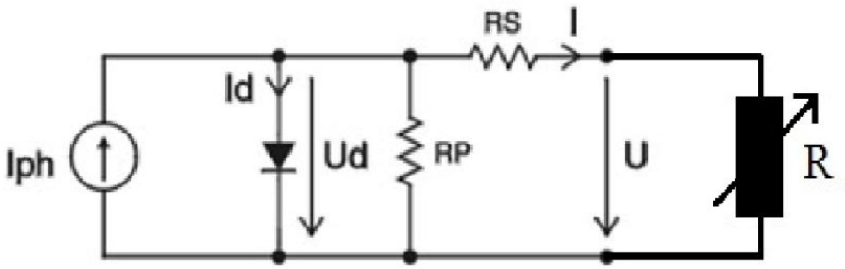



Figure 3: extensive circuit

With this circuit it is possible to measure the values of I and U for different values of the resistor. These different I and U values are plotted in a graph. Table 1 gives the values obtained with the experiment.

| U (V) | I (A) | $I_{sc}$ (A) | $I_s$ (A/m²) | $U_r$ (V) | N | diode factor m | P (W) |
|---|---|---|---|---|---|---|---|
| 0,00 | 0,64 | 0,64 | 0,00000001 | 0,0257 | 15 | 0 | 0,00 |
| 0,17 | 0,64 | 0,64 | 0,00000001 | 0,0257 | 15 | 0 | 0,11 |
| 0,94 | 0,63 | 0,64 | 0,00000001 | 0,0257 | 15 | 0,176 | 0,59 |
| 6,94 | 0,48 | 0,64 | 0,00000001 | 0,0257 | 15 | 1,085 | 3,33 |
| 7,30 | 0,30 | 0,64 | 0,00000001 | 0,0257 | 15 | 1,092 | 2,19 |
| 7,41 | 0,22 | 0,64 | 0,00000001 | 0,0257 | 15 | 1,095 | 1,63 |
| 7,47 | 0,17 | 0,64 | 0,00000001 | 0,0257 | 15 | 1,097 | 1,27 |
| 7,52 | 0,09 | 0,64 | 0,00000001 | 0,0257 | 15 | 1,095 | 0,68 |
| 7,54 | 0,07 | 0,64 | 0,00000001 | 0,0257 | 15 | 1,095 | 0,53 |
| 7,55 | 0,05 | 0,64 | 0,00000001 | 0,0257 | 15 | 1,095 | 0,38 |
| 7,66 | 0,00 | 0,64 | 0,00000001 | 0,0257 | 15 | 1,105 | 0,00 |

| | |
|---|---|
| average diode factor m | 1,095 |

Table 1: measurements + calculated values

For each value of U and I the corresponding diode factor m is calculated with the formula of the real solar panel's circuit.

$$I = I_{sc} - I_s\left(e^{\frac{U}{m*N*U_r}} - 1\right)$$

By taking the average of the calculated values the overall value of the diode factor m is 1,095. Figure 4 shows an example of an ideal voltage-current curve and power curve of the solar panel.

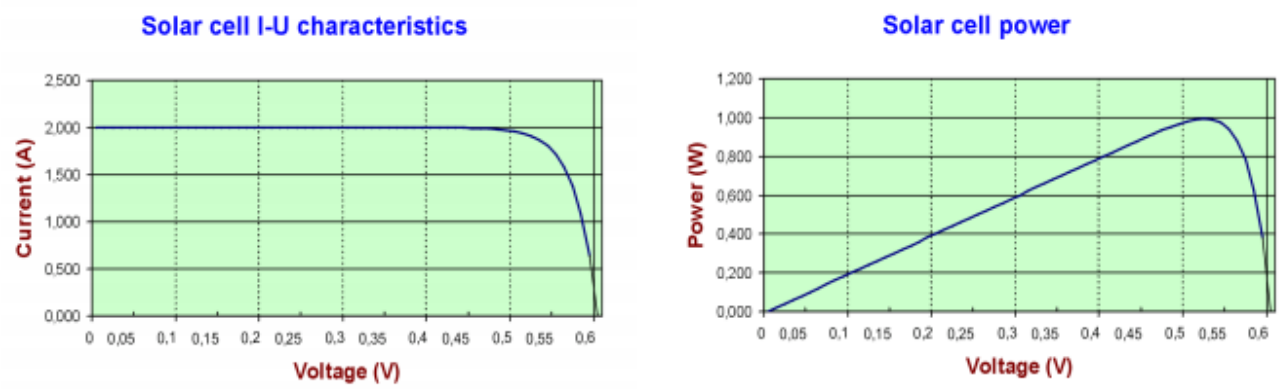



Figure 4: ideal graph for solar panel characteristics

Figure 5 and 6 show the graphs of the experimental measurements. The values aren't exactly the same as the ideal values due to systematic errors. The light source that provides power for the solar panel wasn't turned off between two measurements. This resulted in heating the solar panel which caused the value of $U_r$ to increase and lower the current I and output voltage U.

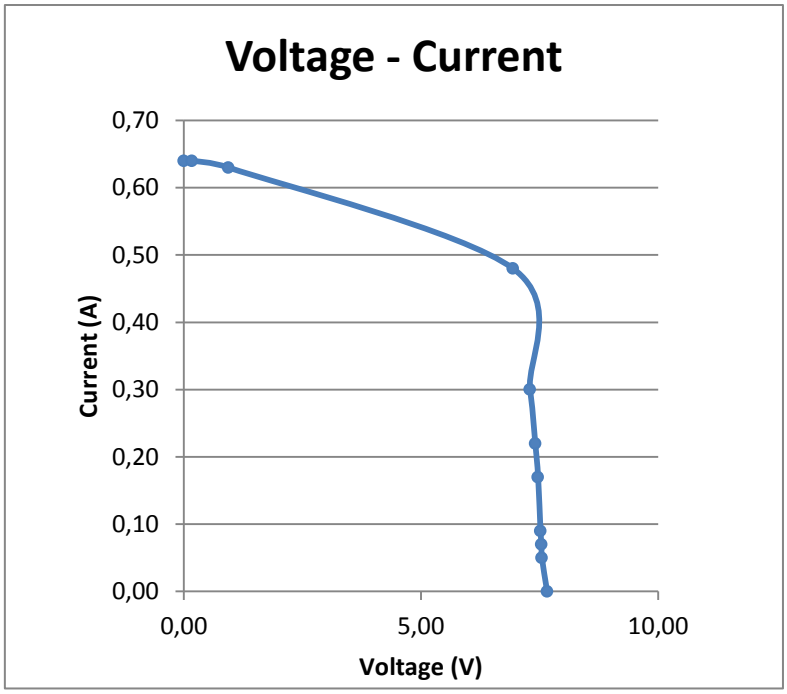



Figure 5: measured values ($U_{out}$ - $I_{in}$)

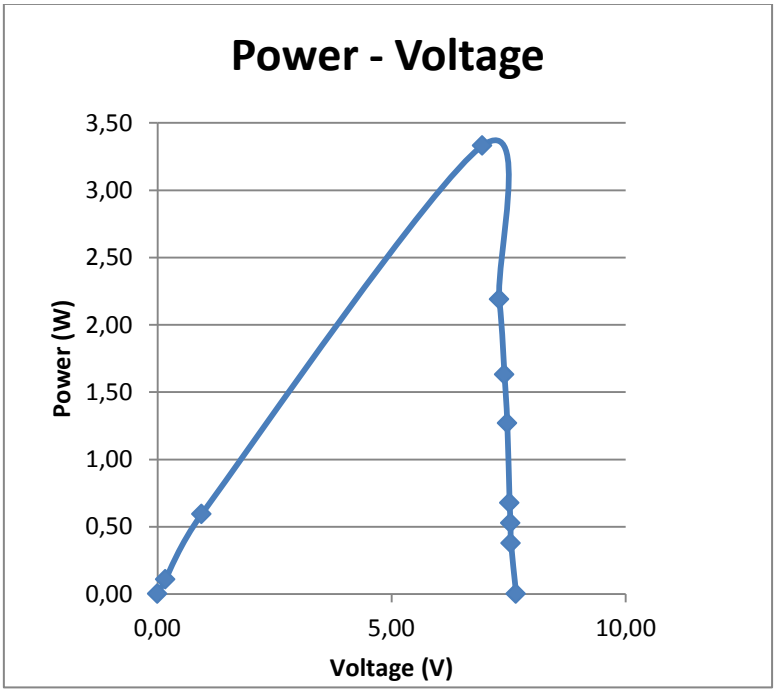



Figure 6: measured values ($U_{out}$ - $I_{in}$)

The more ideal values of $U_{max}$ and $I_{in}$ are 9 V and 0,88 A. Taking this into account, the experimental measured values are rescaled and the more accurate values shown in table 2 are obtained.

| U (V) | I (A) | Isc (A) | Is (A/m²) | Ur (V) | N | Diode factor m | P (W) |
|---|---|---|---|---|---|---|---|
| 0,00 | 0,88 | 0,88 | 0,00000001 | 0,0257 | 16 | 0 | 0,00 |
| 1,48 | 0,88 | 0,88 | 0,00000001 | 0,0257 | 16 | 0 | 1,30 |
| 2,28 | 0,87 | 0,88 | 0,00000001 | 0,0257 | 16 | 0,401 | 1,98 |
| 8,28 | 0,72 | 0,88 | 0,00000001 | 0,0257 | 16 | 1,214 | 5,96 |
| 8,64 | 0,54 | 0,88 | 0,00000001 | 0,0257 | 16 | 1,212 | 4,67 |
| 8,75 | 0,46 | 0,88 | 0,00000001 | 0,0257 | 16 | 1,212 | 4,03 |
| 8,81 | 0,41 | 0,88 | 0,00000001 | 0,0257 | 16 | 1,213 | 3,61 |
| 8,86 | 0,33 | 0,88 | 0,00000001 | 0,0257 | 16 | 1,209 | 2,92 |
| 8,88 | 0,31 | 0,88 | 0,00000001 | 0,0257 | 16 | 1,209 | 2,75 |
| 8,89 | 0,29 | 0,88 | 0,00000001 | 0,0257 | 16 | 1,208 | 2,58 |
| 9,00 | 0,00 | 0,88 | 0,00000001 | 0,0257 | 16 | 1,196 | 0,00 |

| | |
|---|---|
| average diode factor m | 1,209 |

Table 2: more correct values

These values result in new graphs shown on figure 7 and 8.

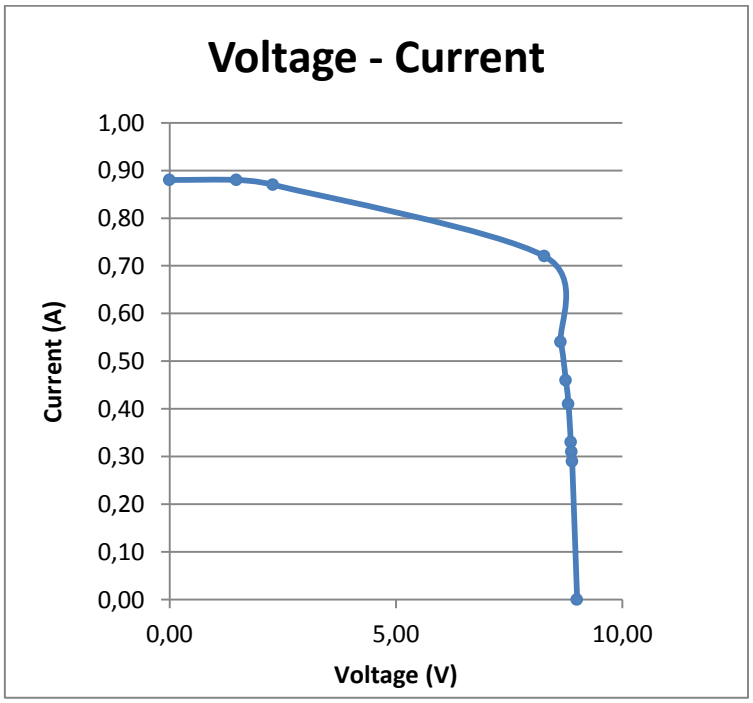



Figure 7: concrete values ($U_{out}$ - $I_{in}$)

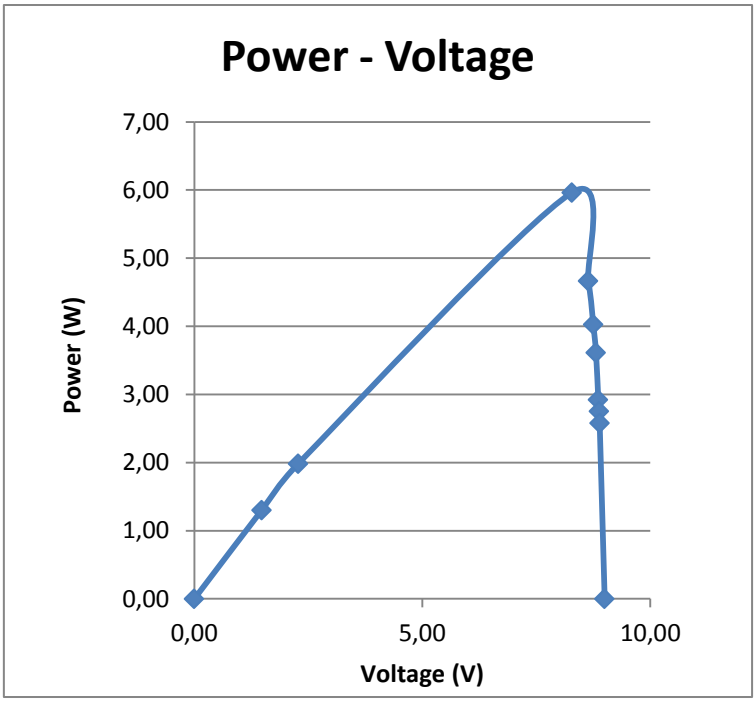



Figure 8: concrete values ($U_{out}$ - $I_{in}$)

## 2. Optimal gear ratio

Gears are used to connect the SSV's wheels with its DC motor in order to move the SSV. The ratio of these gears determines how fast the SSV will complete the racetrack, so this ratio is one of the most important values in the building process. The ratio depends on many variables and in order to calculate it MATLAB was used. With the help of this program, a simulation of the displacement and speed curves of our vehicle on a certain track could be made. To specify the properties of our vehicle and the track in MATLAB following variables were needed:

| **Solar Panel** | |
|---|---|
| Short circuit current (A) | 0,88 |
| Saturation current (A) | $1\times10^{-8}$ |
| Thermal voltage (V) | 0,0257 |
| Diode factor | 1,209 |
| Number of solar cells in series | 15 |
| **DC-motor** | |
| Terminal resistance (Ω) | 3,32 |
| $C_e$ | $8.9285\times10^{-4}$ |
| **Slope** | |
| Gravitational force (N/kg) | 9,81 |
| Gradient (% radians) | 0.12532 |
| **Air resistance** | |
| Drag coefficient | 0,5 |
| Frontal surface area ($m^2$) | 0,02 |
| Density of air ($kg/m^3$) | 1,293 |
| **Rolling resistance** | |
| Rolling resistance coefficient | 0,012 |
| **SSV** | |
| Mass (kg) | 0,850 |
| Wheel radius (m) | 0,04 |
| Gear ratio | Variable (from 1-15) |

Table 3: measurements of solar panel

These blue marked values were found by measuring the solar panel characteristics and using some realistic values for our car's weight, drag coefficient, frontal surface, etc. Of course these values can be slightly different with our real car. The other ones are given properties for the track, the motor, the surroundings and the solar panel.

When using a very small gear ratio, the car will accelerate fast but will have a low top speed. A very high gear ratio means a slow acceleration but a high top speed. It was necessary to find a good value between these two extremes to have a decent acceleration and a top speed high enough so the car would need minimal time to finish the 14-meter track.

The gear ratios were evaluated between 1 and 15 with steps of one in MATLAB. To find the optimal gear ratio, the different race times were put together in a table and in a graph.

| Gear ratio | Race time (s) |
|---|---|
| 3 | 7,7 |
| 4 | 6,35 |
| 5 | 5,56 |
| 6 | 5,08 |
| 7 | 4,83 |
| 7,5 | 4,75 |
| 8 | 4,74 |
| 8,5 | 4,74 |
| 9 | 4,76 |
| 10 | 4,89 |
| 11 | 5,05 |
| 12 | 5,25 |
| 13 | 5,5 |
| 14 | 5,79 |
| 15 | 6,05 |

Table 4: gear ratio versus race time

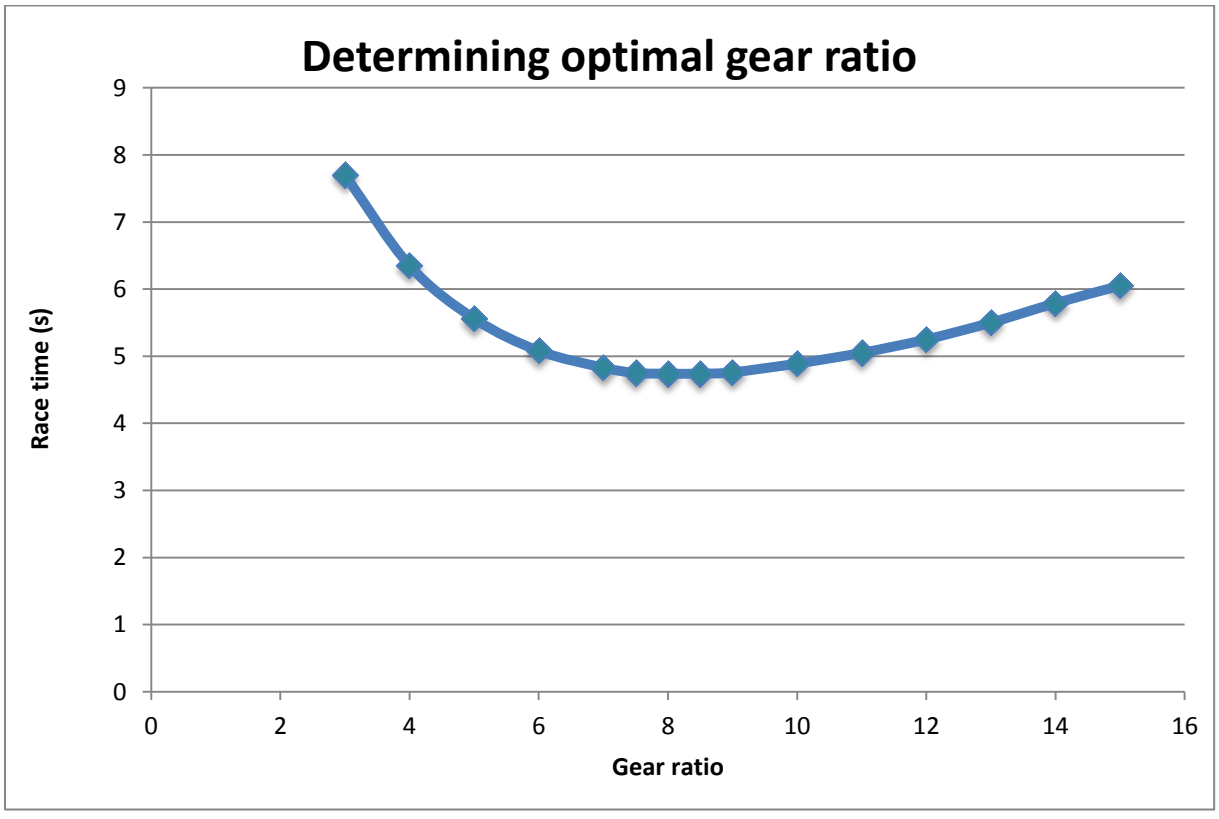
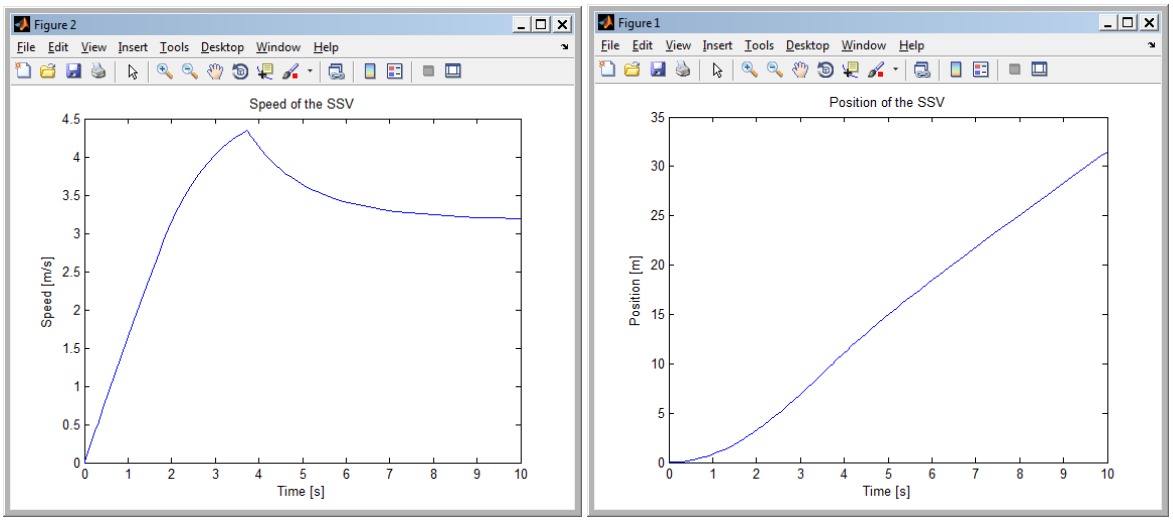



Figure 9: optimal gear ratio

It is clearly visible that the car reaches a minimum of race time with a gear ratio around 8. A couple of gear ratios were evaluated afterwards around the optimum ratio of 8, but no other gear ratio gave the car a real advantage. Considering the practical aspects of the gear ratio -it's way easier to build a car with a gear ratio of 8 than 8.3545- the value of 8 was chosen as the final ratio that will be used on the real car with a solar panel consisting of 15 solar cells in series. For a panel with 16 solar cells, a gear ratio of 9 gives a negligible advantage of 0.01 seconds. So a gear ratio of 8 can be used no matter which solar panel is used (15 or 16 cells).

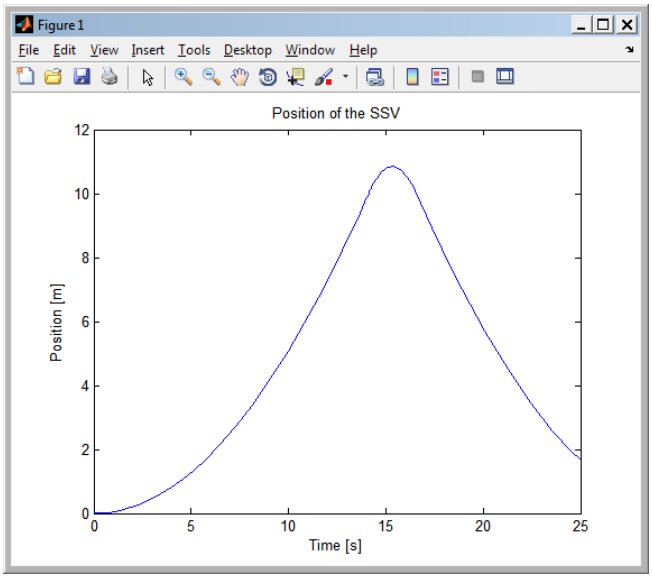
Underneath are some speed curves to illustrate the time the car will need to finish when using different gear ratios. These are all simulated curves for a solar panel with 15 cells except the last one. The last one shows the displacement curve of an SSV with 16 solar cells in series. There is also a curve with the race times for the different gear ratios given for a solar panel of 15 cells.

First the optimal displacement and speed curves are given. The gear ratio has a value of 8. The speed curve shows that the SSV will reach a maximum speed of 4,58 m/s when it reaches the slope; this point is the peak of the graph.

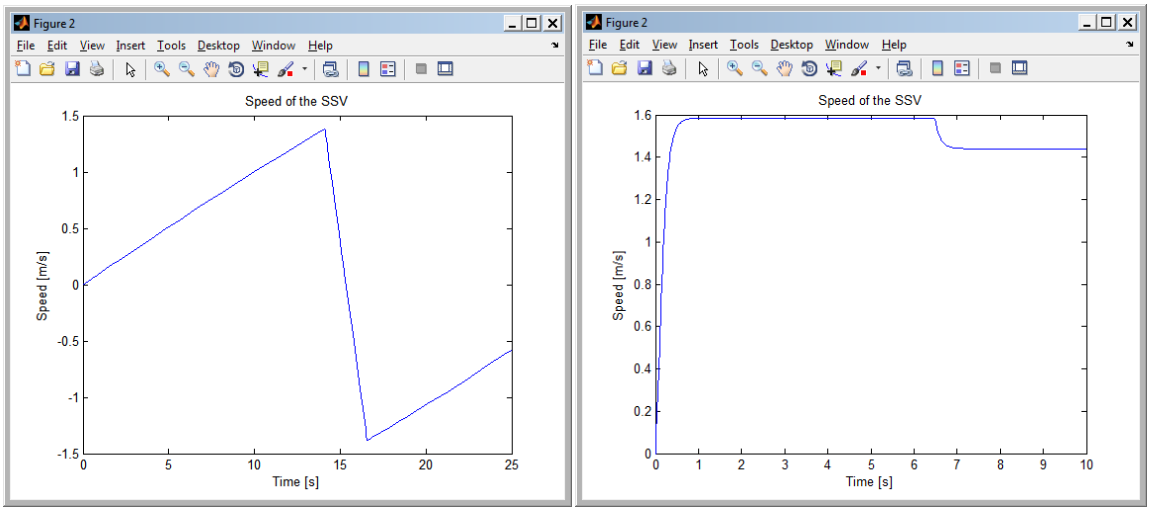



The displacement curve shows the race time. After 4,74 seconds the SSV will reach 14 meters.

Secondly the speed and displacement curves for a gear ratio of 1 are given. These show that when a gear ratio of 1 is used the SSV won't be able to reach the finish. This is because the motor won't be able to put enough force on the wheels to push the vehicle over the slope. After 11 meters (15 seconds) the SSV rolls back.

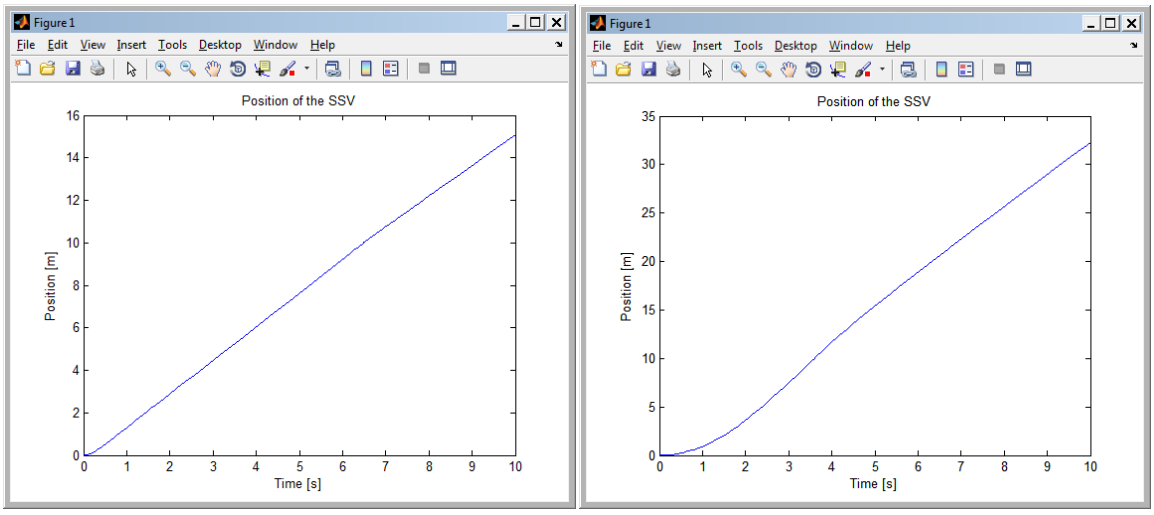



This speed curve of the vehicle shows that the SSV will get a negative speed. This means that it will roll back in the opposite direction.




A speed and displacement curve of the SSV with a gear ratio of 25 is shown below. These show the other extreme possibility. The SSV will accelerate very fast, but immediately reaches a relative low maximum speed.

The vehicle has a constant displacement curve, but the maximum speed is too low to reach the finish in a short time.




With 16 solar cells in series on the panel, the optimal gear ratio is 9. The race time would be 4,63 seconds. Because of the very small difference between 15 and 16 cells, a value of 8 for the gear ratio is chosen.

## 3. Bisection method

In the first paragraphs is shown how to use the bisection method; after it will be used to calculate the acceleration, speed and current during the first second of the race.

Here's an example of how to use the bisection method for following function:

$$f(x) = \frac{1}{2} + \sin\left(\frac{1}{2}x\right) e^{\sin\left(\frac{1}{3}x\right)}$$

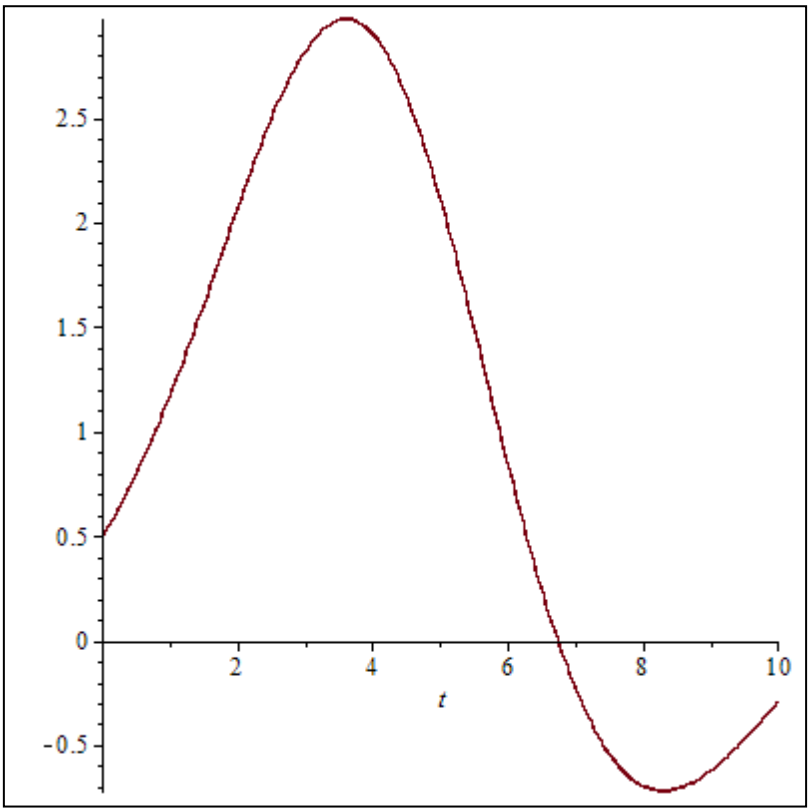



Figure 10: function f(x)

We have to find the root of this function for x-values between 0 and 10. To do this, we will evaluate the function at 5. For this function, y(5) equals 2.119362673. The function is descending so the value we search will be between 5 and 10. The next value we will calculate is 7.5 because it is the average of 5 and 10. When we look at the function value at 7.5 we can see that it is negative (-0.5398626500), so the zero we search will be between 7.5 and 5. We continued this method until we got about 4 zero digits. This should be reasonably accurate for most calculations.

| x | f(x) |
|---|---|
| 5 | 2,119362673 |
| 7,5 | -0,5398626500 |
| 6,25 | 0,5396629439 |
| 6,875 | -0,1180946389 |
| 6,5625 | 0,1852658234 |
| 6,71875 | 0,0264994375 |
| 6,796875 | -0,0476390705 |
| 6,7578125 | -0,0110222724 |
| 6,73828125 | 0,0076265302 |
| 6,748046875 | -0,0017260214 |

Table 5: bisection method

Now this method is used to approximate the current of the solar panel while the acceleration is calculated with these equations:

$$a(t) = -gC_{rr} + \frac{I(t) \times E(t)}{M \times v(t)} - C_w A\rho \times \frac{v^2(t)}{2M}$$

$$E(t) = K_e \times \omega = C_e \times \varphi \times 60 \times v(t) \times \frac{gear\ ratio}{2\pi r}$$

$$I(t) = I_{sc} - I_s\left(e^{\frac{E(t)+I(t)R}{m*N*U_r}} - 1\right)$$

This last equation is the one where the bisection method is needed. It isn't possible to simplify the equation where I(t) equals a constant. Instead, a function f (I(t)) is created and equalized with zero.

| Parameters | |
|---|---|
| g (N/kg) | 9,81 |
| ρ (kg/m$^3$) | 1,293 |
| Ce×φ (V/rpm) | 8,93×10$^{-4}$ |
| $I_{sc}$ (A) | 0,88 |
| m | 1,209 |
| $U_r$ (V) | 0,0257 |
| $I_{sc}$ (A/m$^2$) | 1×10$^{-8}$ |
| N | 16 |
| $R_a$ (Ω) | 3,32 |
| $C_{rr}$ | 0,012 |
| r (m) | 0,04 |
| M (kg) | 0,85 |
| $C_w$ | 0,5 |
| A (m$^2$) | 0,02 |
| $K_e$ (V/(rad/s)) | 8,52×10$^{-3}$ |
| gear ratio | 8 |

Table 6: parameters

Table 6 shows all the parameters that are already defined and stay constant throughout the calculations. To calculate a(0) the normal formula has to be converted to eliminate v(0) since it is not possible to divide by zero. Therefore following formula is used:

$$a(0) = -gC_{rr} + \frac{I(0) \times K_e \times gear\ ratio}{r \times M}$$

Table 7 shows the results of the calculations. The first second of the race is divided in small time intervals of 0,1 s. Therefore there are ten values. In table 8 are the result of the calculation when the first second is divided in time intervals of 0,2 s.

| Time-interval | Acceleration a(t) | Speed v(t) | Distance s(t) | Current I(t) |
|---|---|---|---|---|
| 0,0 | 1,646 | 0,000 | 0,000 | 0,880000 |
| 0,1 | 1,645 | 0,165 | 0,008 | 0,879995 |
| 0,2 | 1,645 | 0,330 | 0,033 | 0,879992 |
| 0,3 | 1,644 | 0,495 | 0,074 | 0,879985 |
| 0,4 | 1,643 | 0,659 | 0,132 | 0,879973 |
| 0,5 | 1,641 | 0,823 | 0,206 | 0,879952 |
| 0,6 | 1,639 | 0,987 | 0,297 | 0,879915 |
| 0,7 | 1,636 | 1,151 | 0,404 | 0,879852 |
| 0,8 | 1,633 | 1,315 | 0,527 | 0,879738 |
| 0,9 | 1,629 | 1,478 | 0,667 | 0,879544 |
| 1,0 | 1,624 | 1,641 | 0,823 | 0,879205 |

Table 7: acceleration with time interval of 0,1s

| Time-interval | Acceleration a(t) | Speed v(t) | Distance s(t) | Current I(t) |
|---|---|---|---|---|
| 0,0 | 1,646 | 0,000 | 0,000 | 0,880000 |
| 0,2 | 1,647 | 0,329 | 0,033 | 0,879991 |
| 0,4 | 1,643 | 0,658 | 0,132 | 0,879973 |
| 0,6 | 1,639 | 0,987 | 0,296 | 0,879915 |
| 0,8 | 1,633 | 1,315 | 0,526 | 0,879738 |
| 1,0 | 1,624 | 1,642 | 0,822 | 0,879202 |

Table 8: acceleration with time interval of 0,2s

Unity list:

Acceleration a(t) – $m/s^2$

Speed v(t) – m/s

Distance – m

Current I(t) – A

There is not much difference between the calculations with the time interval of 0,1s and 0,2s. The results calculated by hand don't differ much from the ones that were simulated with MATLAB.

The complete calculations can be found in the attachments.

# 4. Sankey diagrams

A Sankey diagram is a diagram that puts a visual emphasis on the major transfers or flows within a system. We applied this visual method on the power input and losses of our SSV

Two Sankey diagrams are illustrated in figure 13 and figure 15. The first one illustrates the power consumption of the SSV at half the maximum speed, the other one at maximum speed.

## A) Sankey diagram at maximum speed

The energy consumption of the SSV is visualized in a Sankey-diagram. The input of the diagram consists of the energy that the solar panel supplies. The output contains the energy losses that occur when the SSV is going over the racetrack. These losses are rolling resistance, air resistance, losses due to gear transmission and internal losses of the DC engine.

The energy losses are calculated with the following formulas:

Air resistance:

$$P = 0{,}5 * A * C_w * \rho * v^3$$

With:

P = power
v = linear velocity
$C_w$ = friction coefficient
A = frontal surface
ρ = density of air

Rolling resistance:

$$P = m * g * C_{rr} * v$$

With:

P = power
m = mass
g = gravity constant (9,81 $m/s^2$)
v = linear velocity
$C_{rr}$ = rolling resistance coefficient

The values used for the calculations for drafting the Sankey diagram are listed in table 9.

| | |
|---|---|
| Maximum linear velocity (v) | 4,57 (m/s) |
| Radius SSV wheels | 0,04 (m) |
| Gear ratio | 8 |
| Rolling resistance coefficient (Crr) | 0,012 |
| Friction coefficient (Cw) | 0,5 |
| Density of air (ρ) | 1,293 (kg/m$^3$) |
| Torque constant (Ke) | 8,55 (mNm/A) |
| Frontal surface (A) | 0,02 (m) |
| Mass (m) | 0,850 (kg) |

Table 9: Values used for calculations Sankey diagram

The total power delivered by the DC engine is calculated with this formula:

$$P = U_a * I_a$$

The purpose of the following calculations is to find the right values for $U_a$ and $I_a$. According to the MATLAB calculations the maximum speed of the car is: $V_{max} = 4{,}57 \frac{m}{s}$.

The wheels of the SSV have a radius of: $r = 0{,}04\ m$.

Based on these two values the rotation speed of the wheels can be calculated with:

$$\omega_{wheels} = \frac{V_{max}}{r} = \frac{4{,}57\ m/s}{0{,}04\ m} = 114{,}25\ rad/s$$

With MATLAB an optimal gear ratio of R = 8 has been calculated. This value is necessary to calculate the rotational speed of the DC motor: $\omega_{engine} = 8 * \omega_{wheels} = 914 \frac{rad}{s} = 8728\ rpm$.

According to the datasheet of the DC engine this value of the rotational speed is acceptable because it doesn't exceed the given maximum value of 10600 rpm. For further calculations, a closer look at the DC engine could be useful. The electric circuit of the DC engine is illustrated in figure 11.

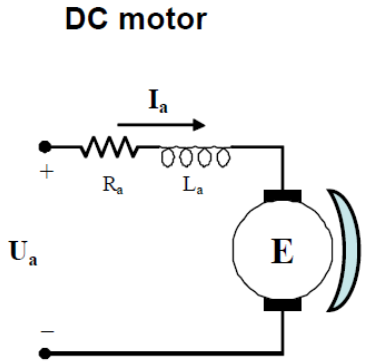



Figure 11: Internal circuit DC engine

The total voltage along the DC engine is given by the following formula:

$$U_a = E + R_a * I_a$$

Where:

$$E = K_e * \omega_{engine} = 8{,}55 \frac{mV}{\frac{rad}{s}} * 914 \frac{rad}{s} = 7{,}815 \text{ V}$$

For calculating the total power given by the DC engine, we need to find $I_a$ in the formula of the solar panel. To find its value we need to apply the bisectional method on the formula:

$$I = I_{sc} - I_s \left(e^{\frac{E+RI_a}{m*N*U_r}} - 1\right)$$

When applying the bisectional method with MATLAB the current is:

$$I_a = 0{,}31799\ A$$

The total power when riding at maximum speed is than while taking the motor efficiency of 84 % into account:

$$P = U_a * I_a = \big((7{,}815 + 3{,}32 * 0{,}31799\ )V * 0{,}31799\ A\big) * 0{,}84 = 2{,}36947\ W$$

The losses when the SSV is riding at maximum speed are:

$$P_{Air\ friction} = 0{,}5 * A * C_w * \rho * v^3 = 0{,}5 * 0{,}02\ m^2 * 0{,}5 * 1{,}293 \frac{kg}{m^3} * 4{,}57^3 \frac{m}{s} = 0{,}666\ W$$

$$P_{rolling} = m * g * C_{rr} * v = 0{,}850\ kg * 9{,}81 \frac{m}{s^2} * 0{,}012 * 4{,}57 \frac{m}{s} = 0{,}4573\ W$$

When taking these losses and the gear efficiency of 95% into account, the resulting power is:

$$P_{resulting} = P_{initial} - P_{losses} = (2{,}36947 - 0{,}666 - 0{,}4573) * 0{,}95 = 1{,}18387\ W$$

Normally the resulting power should be zero otherwise the SSV still has some power to accelerate. But the SSV is already at maximum speed so this acceleration should be impossible. There are several declarations for the fact that there is still some power left. First of all the quality factor m of the solar panel plays an important role on the characteristics of the panel. The m value influences the slope of the solar panel characteristic graph. Its influence is at its maximum in the working point at maximum power: a small increase in voltage could give a major decrease of current. Because of this fact the power values calculated above could be very different. This phenomenon is illustrated in figure 12.

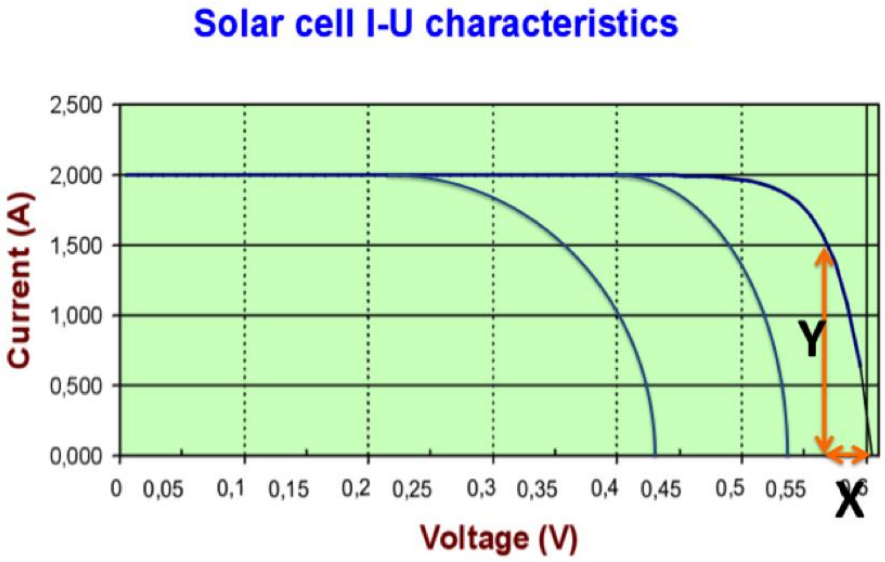



Figure 12: Impact diode factor m

On the other hand there are several losses in the DC-motor. These losses are caused by heat, induction and internal friction. Taking these losses and the inaccurate value of m into account, the resulting power should be zero.

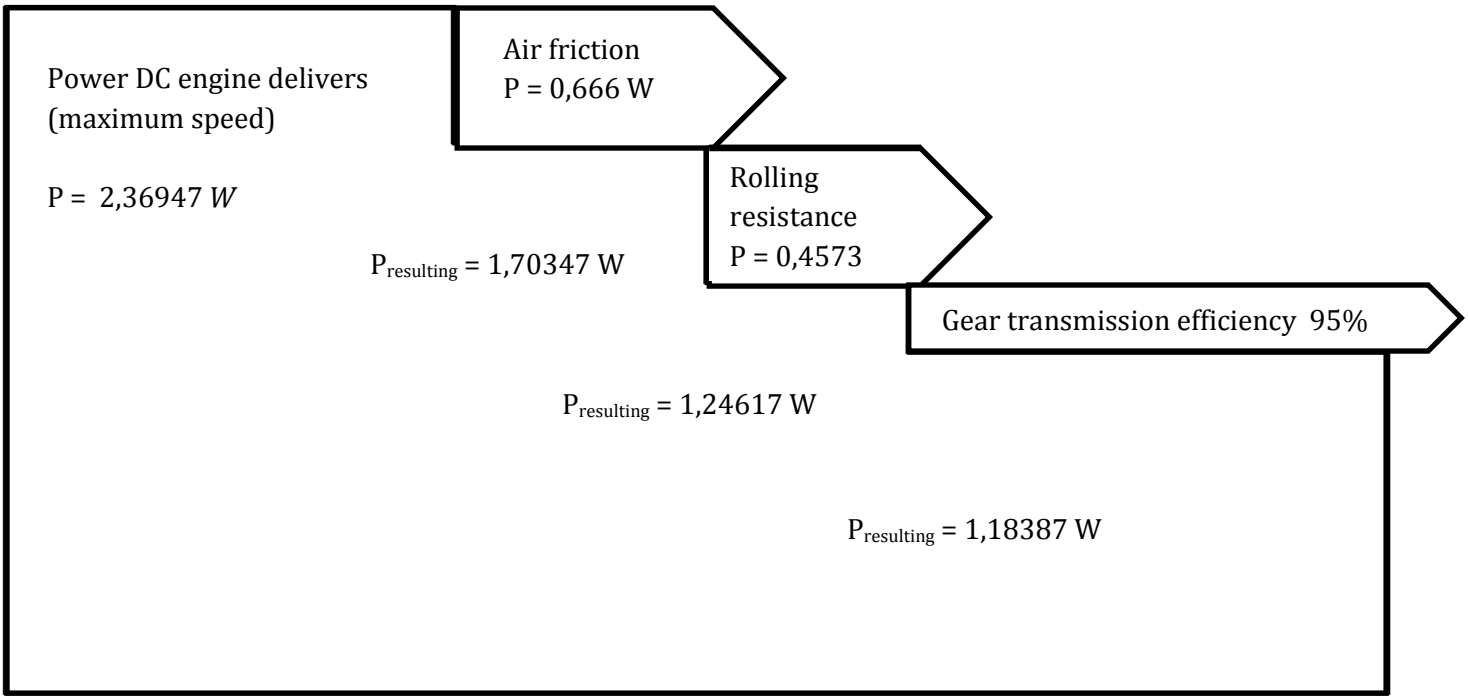



Figure 13: Sankey diagram maximum speed

## B) Sankey diagram at half the maximum speed

The second situation is when the SSV moves at half the maximum speed on the slope. This gives the situation of figure 14.

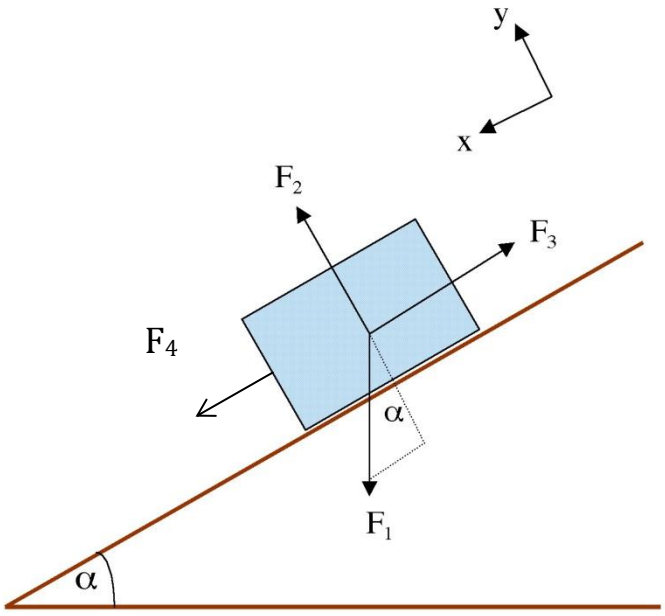



Figure 14: Situation SSV on slope

The speed of the SSV on the slope is half his maximum speed: $V_{slope} = 2{,}285 \frac{m}{s}$.

The radius of the wheels has the same value as the earlier example: $r = 0{,}04\ m$

The same formulas of the previous examples can be used:

$$\omega_{wheels} = \frac{V_{slope}}{r} = \frac{2{,}285\ m/s}{0{,}04\ m} = 57{,}125\ rad/s$$

The rotational speed of the DC motor: $\omega_{engine} = 8 * \omega_{wheels} = 457 \frac{rad}{s} = 4364.02\ rpm.$

The EMF of the DC engine:

$$E = K_e * \omega_{engine} = 8{,}55 \frac{mV}{\frac{rad}{s}} * 457 \frac{rad}{s} = 3{,}907\ \text{V}$$

When applying the bisectional method with MATLAB on the equation of the solar panel the current is:

$$I_a = 0{,}8711\ A$$

The total power when riding at half the maximum speed while taken the motor efficiency into account is:

$$P = U_a * I_a = \left((3{,}907 + 3{,}32 * 0{,}8711)V * 0{,}8711A\right) * 0.84 = 4{,}975\ W$$

The losses when the SSV is riding at half the maximum speed are:

$$P_{Air\ friction} = 0{,}5 * A * C_w * \rho * v^3 = 0{,}5 * 0{,}02\ m^2 * 0{,}5 * 1{,}293 \frac{kg}{m^3} * 2{,}285^3 \frac{m}{s} = 0{,}1543\ W$$

When examining the power losses due to rolling resistance the change of the normal force needs to be taken into account according to figure 14.

$$N = m * g * cos\alpha = 0{,}850\ kg * 9{,}81\ \frac{m}{s^2} * \cos 7° = 8{,}2763\ N$$

$$P_{rolling} = N * C_{rr} * v = 8{,}2763\ N * 0{,}012 * 2{,}285 \frac{m}{s} = 0{,}2269\ W$$

On the other hand there are power losses due to force in the x direction from the gravitation force:

$$F_{x,grav} = m * g * sin\alpha = 0{,}850\ kg * 9{,}81\ \frac{m}{s^2} * sin7° = 1{,}0162\ N$$

The power loss due to this force is: $P_{loss} = F_{x,grav} * v = 2{,}322$ W

When taking these losses and the gear efficiency of 95% into account, the resulting power is:

$$P_{resulting} = P_{initial} - P_{losses} = (4{,}975 - 0{,}1543 - 0{,}2269 - 2{,}322) * 0{,}95 = 2{,}15821\ W$$

The resulting power isn't zero for the same reasons explained in the first section.

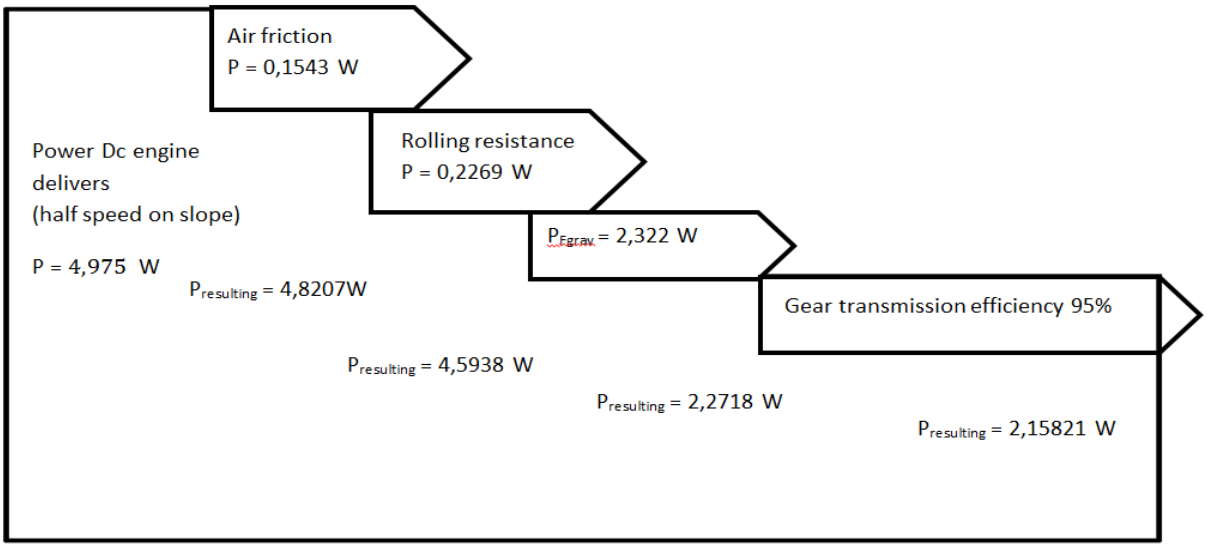



**Figure 15: Sankey diagram at half the maximum speed**

## 5. Attachements

# Bisection method

Acceleration equation:

$$a(t) = -g C_{rr} + \frac{I(t) \times E(t)}{M \times v(t)} - C_w A \rho \times \frac{v^2(t)}{2M}$$

Back EMF equation:

$$E(t) = K_e \times \omega = C_E \cdot \varphi \times 60 \times v(t) \times \frac{\text{gearratio}}{2\pi r}$$

Solar panel equation:

$$I(t) = I_{sc} - I_s \left( e^{\frac{u(t)}{m N U_T}} - 1 \right)$$

$$= I_{sc} - I_s \left( e^{\frac{E(t) + I(t) R}{m N U_T}} - 1 \right)$$

## Parameters

$g = 9,81\ N/kg$
$\rho = 1,293\ kg/m^3$
$C_E \cdot \varphi = 8,93 \cdot 10^{-4}\ V/rpm$
$I_{sc} = 0,88A$
$m = 1,209$
$U_T = 0,0257V$
$I_s = 1 \cdot 10^{-8}\ A/m^2$
$N = 16$
$R_a = 3,32\ \Omega$
$C_{rr} = 0,012$
$r = 0,04\ m$
$M = 0,85\ kg$
$C_w = 0,5$
$A = 0,02\ m^2$

$K_E = 8,52 \cdot 10^{-3}$ V/rad/s
Gear ratio = 8

Initial conditions

$S(0) = 0$ , $v(0) = 0$ , $I(0) = 0,88$ A

Calculations : interval = 0.1

To calculate a(0) the formula is transformed:

$$a(0) = -g C_{rr} + \underbrace{\frac{I(0) \times E(0)}{M \times v(0)}}_{\text{converted to: } \frac{I(0) \times K_E \times \cancel{v(0)} \times \text{gear ratio}}{r \times M \times \cancel{v(0)}}} - \underbrace{\frac{\rho C_w A v^2(0)}{2M}}_{\text{equals to 0}}$$

$a(0) = 1,646$ m/s²

$v(0.1) = a(0) \times T = 0,165$ m/s

$S(0.1) = v(0) \times T + a(0) \times \frac{T^2}{2} = 8,23 \cdot 10^{-3}$ m

$E(0.1) = K_E \times v(0.1) \times \frac{\text{gear ratio}}{r}$

$= 0,281$ V

To calculate I(0.1) we use the actual bisection method where we create: $f(I(t)) = I_{sc} - I_s\left(e^{\frac{E(t) + I(t)R}{m v_t}} - 1\right) - I(t)$

We search for the value of $I(t)$ where $f(I(t)) = 0$.

$f(I(0.1) = 0.88) = -5 \cdot 10^{-6}$
$f(I(0.1) = 0.879995) = -6.55 \cdot 10^{-8}$

$\Rightarrow I(0.1) = 0.879995$ A

$a(0.1) = 1.645$ m/s²

$V(0.2) = V(0.1) + a(0.1) \times T = 0.330$ m/s
$S(0.2) = S(0.1) + V(0.1) \times T + a(0.1) \times \frac{T^2}{2}$
$= 0.033$ m

$E(0.2) =$ Same equation as previous
$= 0.562$ V

$I(0.2)$: $f(I(0.2) = 0.879995) = -4 \cdot 10^{-6}$
$f(I(0.2) = 0.879992) = -9 \cdot 10^{-7}$

$\Rightarrow I(0.2) = 0.879992$ A

$a(0.2) = 1.645$ m/s²

$V(0.3) = 0.495$ m/s
$S(0.3) = 0.074$ m
$E(0.3) = 0.843$ V

$I(0.3)$: $f(I(0.3) = 0.879992) = -7.7 \cdot 10^{-6}$
$f(I(0.3) = 0.879985) = -7 \cdot 10^{-7}$
$\Rightarrow I(0.3) = 0.879985$

$a(0.3) = 1,644\ m/s^2$

$v(0.4) = 0,659\ m/s$
$s(0.4) = 0,132\ m$
$E(0.4) = 1,123\ V$

$I(0.4)$: $f(I(0.4) = 0,879985) = -1,3 \cdot 10^{-6}$
$f(I(0.4) = 0,879973) = -6 \cdot 10^{-7}$

$\Rightarrow I(0.4) = 0,879973\ A$

$a(0.4) = 1,643\ m/s^2$

$v(0.5) = 0,823\ m/s$
$s(0.5) = 0,206\ m$
$E(0.5) = 1,402\ V$

$I(0.5)$: $f(I(0.5) = 0,879973) = -2 \cdot 10^{-6}$
$f(I(0.5) = 0,879952) = -4 \cdot 10^{-7}$
$\Rightarrow I(0.5) = 0,879952\ A$

$a(0.5) = 1,641\ m/s^2$

$v(0.6) = 0,987\ m/s$
$s(0.6) = 0,297\ m$
$E(0.6) = 1,682\ V$

$I(0.6) = f(I(0.6) = 0,879952) = -3,7 \cdot 10^{-6}$
$f(I(0.6) = 0,879915) = 6,35 \cdot 10^{-8}$

$\Rightarrow I(0.6) = 0,879915\ A$

$a(0.6) = 1.639\ m/s^2$

$v(0.7) = 1.151\ m/s$
$s(0.7) = 0.404\ m$
$\mathcal{E}(0.7) = 1.961\ V$

$I(0.7)$ : $f(I(0.7) = 0.879915) = -6.4 \cdot 10^{-5}$
$f(I(0.7) = 0.879852) = -8.2 \cdot 10^{-7}$

$\Rightarrow I(0.7) = 0.879852\ A$

$a(0.7) = 1.636\ m/s^2$

$v(0.8) = 1.315\ m/s$
$s(0.8) = 0.527\ m$
$\mathcal{E}(0.8) = 2.241\ V$

$I(0.8)$ : $f(I(0.8) = 0.879852) = -1.1 \cdot 10^{-4}$
$f(I(0.8) = 0.879738) = 8 \cdot 10^{-7}$

$\Rightarrow I(0.8) = 0.879738\ A$

$a(0.8) = 1.633\ m/s^2$

$v(0.9) = 1.478\ m/s$
$s(0.9) = 0.667\ m$
$\mathcal{E}(0.9) = 2.519\ V$

$I(0.9)$ : $f(I(0.9) = 0.879738) = -2 \cdot 10^{-4}$
$f(I(0.9) = 0.879544) = -3.4 \cdot 10^{-7}$

$\Rightarrow I(0.9) = 0.879544\ A$

$a(0.9) = 1,629 \ m/s^2$

$v(1) = 1,641 \ m/s$
$s(1) = 0,823 \ m$
$E(1) = 2,796 \ V$

$I(1)$: $f(I(1)=0,879544) = -3,4 \cdot 10^{-4}$
$f(I(1)=0,879205) = 7,13 \cdot 10^{-8}$

$\Rightarrow I(1) = 0,879205 \ A$

$a(1) = 1,624 \ m/s^2$

## Calculations : interval = 0.2

$a(0) = 1,646 \ m/s^2$

$v(0.2) = 0,329 \ m/s$
$s(0.2) = 0,033 \ m$
$E(0.2) = 0,561 \ V$

$I(0.2)$: $f(I(0.2)=0,88) = -9 \cdot 10^{-6}$
$f(I(0.2)=0,879991) = 9,6 \cdot 10^{-6}$

$\Rightarrow I(0.2) = 0,879991 \ A$

$a(0.2) = 1,647$

$v(0.4) = 0,658 \ m/s$
$s(0.4) = 0,132 \ m$
$E(0.4) = 1,121 \ V$

$I(0.4)$: $f(I(0.4)=0.879991) = -2\cdot10^{-5}$
$f(I(0.4)=0.879973) = -5\cdot10^{-7}$

$\Rightarrow I(0.4) = 0.879973$ A

$a(0.4) = 1.643$ m/s²

$v(0.6) = 0.987$ m/s
$s(0.6) = 0.296$ m
$E(0.6) = 1.682$ V

$I(0.6)$: $f(I(0.6) = 0.879915) = 6\cdot10^{-8}$

$\Rightarrow I(0.6) = 0.879915$

$a(0.6) = 1.639$ m/s²

$v(0.8) = 1.315$ m/s
$s(0.8) = 0.526$
$E(0.8) = 2.241$ V

$I(0.8)$: $f(I(0.8) = 0.879738) = 8\cdot10^{-7}$

$\Rightarrow I(0.8) = 0.879738$ )

$a(0.8) = 1.633$ m/s²

$v(1) = 1.642$ m/s
$s(1) = 0.822$ m
$E(1) = 2.798$ V

$I(1)$: $f(I(1) = 0,879202) = -1 \cdot 10^{-7}$

$\Rightarrow I(1) = 0,879202$ A

$a(1) = 1,624$ m/s²