

Key To

ESSENTIALS OF
TRIGONOMETRY

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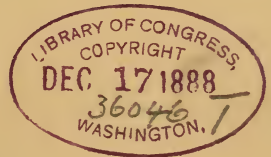
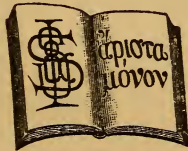
TO

WELLS' ESSENTIALS OF TRIGONOMETRY.

BY

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TO

WELLS' ESSENTIALS OF TRIGONOMETRY.

CHAPTER I.

Art. 9. — Page 3.

$$1. 135^\circ = \frac{3\pi}{4} \qquad 5. 29^\circ 15' = \frac{13\pi}{80}$$

$$2. 198^\circ = \frac{11\pi}{10} \qquad 6. 174^\circ 22' 30'' = \frac{31\pi}{32}$$

$$3. 11^\circ 15' = \frac{\pi}{16} \qquad 7. 128^\circ 34\frac{2}{7}' = \frac{5\pi}{7}$$

$$4. 37^\circ 30' = \frac{5\pi}{24} \qquad 8. 92^\circ 48' 45'' = \frac{33\pi}{64}$$

$$9. \frac{1}{2} = \frac{1}{2} \times 57.2957795^\circ \dots = 28.6478897^\circ \dots$$

$$= 28^\circ 38.873382' \dots = 28^\circ 38' 52.40292'' \dots$$

$$10. \frac{3\pi}{5} = \frac{3}{5} \text{ of } 180^\circ = 108^\circ.$$

$$11. \frac{37\pi}{30} = \frac{37}{30} \text{ of } 180^\circ = 222^\circ.$$

$$12. \frac{5\pi}{4} = \frac{5}{4} \text{ of } 180^\circ = 225^\circ.$$

$$13. \frac{3}{4} = \frac{3}{4} \times 57.2957795^\circ \dots = 42.9718346^\circ \dots$$

$$= 42^\circ 58.310076' \dots = 42^\circ 58' 18.60456'' \dots$$

$$14. 2 = 2 \times 57.2957795^\circ \dots = 114.591559^\circ \dots$$

$$= 114^\circ 35.49354' \dots = 114^\circ 35' 29.6124'' \dots$$

$$15. \quad \frac{2\pi - 1}{3} = \frac{2\pi}{3} - \frac{1}{3}, \quad \frac{2\pi}{3} = \frac{2}{3} \text{ of } 180^\circ = 120^\circ.$$

$$\frac{1}{3} = \frac{1}{3} \times 57.2957795^\circ \dots = 19.0985932^\circ \dots$$

$$= 19^\circ 5.915592' \dots = 19^\circ 5' 54.93552'' \dots$$

$$\therefore \frac{2\pi - 1}{3} = 120^\circ - 19^\circ 5' 54.93552'' \dots = 100^\circ 54' 5.06448'' \dots$$

$$16. \quad \frac{\pi - 1}{4} = \frac{\pi}{4} - \frac{1}{4}, \quad \frac{\pi}{4} = \frac{1}{4} \text{ of } 180^\circ = 45^\circ.$$

$$\frac{1}{4} = \frac{1}{4} \times 57.2957795^\circ \dots = 14.3239449^\circ \dots$$

$$= 14^\circ 19.436694' \dots = 14^\circ 19' 26.20164'' \dots$$

$$\therefore \frac{\pi - 1}{4} = 45^\circ - 14^\circ 19' 26.20164'' \dots = 30^\circ 40' 33.79836'' \dots$$

CHAPTER II.

Art. 15. — Page 8.

3. Here the opposite side = 2, and the adjacent side = 3. Therefore the hypotenuse = $\sqrt{2^2 + 3^2} = \sqrt{13}$. Then,

$$\begin{aligned} \sin A &= \frac{2}{\sqrt{13}}, & \cot A &= \frac{3}{2}, & \text{vers } A &= 1 - \frac{3}{\sqrt{13}}, \\ \cos A &= \frac{3}{\sqrt{13}}, & \sec A &= \frac{\sqrt{13}}{3}, & \text{covers } A &= 1 - \frac{2}{\sqrt{13}}, \\ & & \csc A &= \frac{\sqrt{13}}{2}, & & \end{aligned}$$

4. $\sin A = 1 - \text{covers } A = \frac{2}{5}$.

Here the opposite side = 2, and the hypotenuse = 5. Therefore the adjacent side = $\sqrt{5^2 - 2^2} = \sqrt{21}$. Then,

$$\begin{aligned} \cos A &= \frac{\sqrt{21}}{5}, & \cot A &= \frac{\sqrt{21}}{2}, & \csc A &= \frac{5}{2}, \\ \tan A &= \frac{2}{\sqrt{21}}, & \sec A &= \frac{5}{\sqrt{21}}, & \text{vers } A &= 1 - \frac{\sqrt{21}}{5}. \end{aligned}$$

5. Here the hypotenuse = 4, and the opposite side = 1. Therefore the adjacent side = $\sqrt{4^2 - 1^2} = \sqrt{15}$. Then,

$$\begin{aligned} \sin A &= \frac{1}{4}, & \tan A &= \frac{1}{\sqrt{15}}, & \text{vers } A &= 1 - \frac{\sqrt{15}}{4}, \\ \cos A &= \frac{\sqrt{15}}{4}, & \cot A &= \sqrt{15}, & \text{covers } A &= \frac{3}{4}, \\ & & \sec A &= \frac{4}{\sqrt{15}}, & & \end{aligned}$$

6. $\cos A = 1 - \text{vers } A = \frac{3}{4}$.

Here the adjacent side = 3, and the hypotenuse = 4. Therefore the opposite side = $\sqrt{4^2 - 3^2} = \sqrt{7}$. Then,

$$\begin{aligned} \sin A &= \frac{\sqrt{7}}{4}, & \cot A &= \frac{3}{\sqrt{7}}, & \csc A &= \frac{4}{\sqrt{7}}, \\ \tan A &= \frac{\sqrt{7}}{3}, & \sec A &= \frac{4}{3}, & \text{covers } A &= 1 - \frac{\sqrt{7}}{4}. \end{aligned}$$

7. Here the opposite side = x , and the hypotenuse = y . Therefore the adjacent side = $\sqrt{y^2 - x^2}$. Then,

$$\begin{aligned} \cos A &= \frac{\sqrt{y^2 - x^2}}{y}, & \cot A &= \frac{\sqrt{y^2 - x^2}}{x}, & \text{vers } A &= 1 - \frac{\sqrt{y^2 - x^2}}{y}, \\ \tan A &= \frac{x}{\sqrt{y^2 - x^2}}, & \sec A &= \frac{y}{\sqrt{y^2 - x^2}}, & \text{covers } &= 1 - \frac{x}{y}, \\ & & \csc A &= \frac{y}{x}, & & \end{aligned}$$

8. Here the hypotenuse = 13, and the adjacent side = 5. Therefore the opposite side = $\sqrt{13^2 - 5^2} = 12$. Then,

$$\begin{aligned} \sin A &= \frac{12}{13}, & \tan A &= \frac{12}{5}, & \text{vers } A &= \frac{8}{13}, \\ \cos A &= \frac{5}{13}, & \cot A &= \frac{5}{12}, & \text{covers } A &= \frac{1}{13}, \\ & & \csc A &= \frac{13}{12}, & & \end{aligned}$$

9. Here the adjacent side = x , and the opposite side = 1. Therefore the hypotenuse = $\sqrt{x^2 + 1}$. Then,

$$\begin{aligned} \sin A &= \frac{1}{\sqrt{x^2 + 1}}, & \tan A &= \frac{1}{x}, & \text{vers } A &= 1 - \frac{x}{\sqrt{x^2 + 1}}, \\ \cos A &= \frac{x}{\sqrt{x^2 + 1}}, & \sec A &= \frac{\sqrt{x^2 + 1}}{x}, & \text{covers } A &= 1 - \frac{1}{\sqrt{x^2 + 1}}, \\ & & \csc A &= \sqrt{x^2 + 1}, & & \end{aligned}$$

10. Here the adjacent side = 8, and the hypotenuse = 17. Therefore the opposite side = $\sqrt{17^2 - 8^2} = 15$. Then,

$$\begin{aligned} \sin A &= \frac{15}{17}, & \cot A &= \frac{8}{15}, & \text{vers } A &= \frac{9}{17}, \\ \tan A &= \frac{15}{8}, & \sec A &= \frac{17}{8}, & \text{covers } A &= \frac{2}{17}, \\ & & \csc A &= \frac{17}{15}, & & \end{aligned}$$

11. Here the hypotenuse = $\sqrt{a^2 + b^2}$, and the adjacent side = b . Therefore the opposite side = $\sqrt{a^2 + b^2 - b^2} = a$. Then,

$$\begin{aligned} \sin A &= \frac{a}{\sqrt{a^2 + b^2}}, & \tan A &= \frac{a}{b}, & \text{vers } A &= 1 - \frac{b}{\sqrt{a^2 + b^2}}, \\ \cos A &= \frac{b}{\sqrt{a^2 + b^2}}, & \cot A &= \frac{b}{a}, & \text{covers } A &= 1 - \frac{a}{\sqrt{a^2 + b^2}}, \\ & & \csc A &= \frac{\sqrt{a^2 + b^2}}{a}, & & \end{aligned}$$

CHAPTER III.

Art. 53. — Page 30.

3. If A is acute, $450^\circ - A$ is in the first quadrant. Then,

$$\begin{aligned} \sin(450^\circ - A) &= \cos A, & \cos(450^\circ - A) &= \sin A, \\ \tan(450^\circ - A) &= \cot A, & \cot(450^\circ - A) &= \tan A, \\ \sec(450^\circ - A) &= \csc A, & \csc(450^\circ - A) &= \sec A. \end{aligned}$$

4. If A is acute, $450^\circ + A$ is in the second quadrant. Then,

$$\begin{aligned} \sin(450^\circ + A) &= \cos A, & \cos(450^\circ + A) &= -\sin A, \\ \tan(450^\circ + A) &= -\cot A, & \cot(450^\circ + A) &= -\tan A, \\ \sec(450^\circ + A) &= -\csc A, & \csc(450^\circ + A) &= \sec A. \end{aligned}$$

5. If A is acute, $540^\circ - A$ is in the second quadrant. Then,

$$\begin{aligned} \sin(540^\circ - A) &= \sin A, & \cos(540^\circ - A) &= -\cos A, \\ \tan(540^\circ - A) &= -\tan A, & \cot(540^\circ - A) &= -\cot A, \\ \sec(540^\circ - A) &= -\sec A, & \csc(540^\circ - A) &= \csc A. \end{aligned}$$

6. If A is acute, $540^\circ + A$ is in the third quadrant. Then,

$$\begin{aligned} \sin(540^\circ + A) &= -\sin A, & \cos(540^\circ + A) &= -\cos A, \\ \tan(540^\circ + A) &= \tan A, & \cot(540^\circ + A) &= \cot A, \\ \sec(540^\circ + A) &= -\sec A, & \csc(540^\circ + A) &= -\csc A. \end{aligned}$$

7. If A is acute, $630^\circ - A$ is in the third quadrant. Then,

$$\begin{aligned} \sin(630^\circ - A) &= -\cos A, & \cos(630^\circ - A) &= -\sin A, \\ \tan(630^\circ - A) &= \cot A, & \cot(630^\circ - A) &= \tan A, \\ \sec(630^\circ - A) &= -\csc A, & \csc(630^\circ - A) &= -\sec A. \end{aligned}$$

8. If A is acute, $900^\circ - A$ is in the second quadrant. Then,

$$\begin{aligned} \sin(900^\circ - A) &= \sin A, & \cos(900^\circ - A) &= -\cos A, \\ \tan(900^\circ - A) &= -\tan A, & \cot(900^\circ - A) &= -\cot A, \\ \sec(900^\circ - A) &= -\sec A, & \csc(900^\circ - A) &= \csc A. \end{aligned}$$

9. If A is acute, $-90^\circ + A$ is in the fourth quadrant. Then,

$$\begin{aligned} \sin(-90^\circ + A) &= -\cos A, & \cos(-90^\circ + A) &= \sin A, \\ \tan(-90^\circ + A) &= -\cot A, & \cot(-90^\circ + A) &= -\tan A, \\ \sec(-90^\circ + A) &= \csc A, & \csc(-90^\circ + A) &= -\sec A. \end{aligned}$$

10. If A is acute, $-90^\circ - A$ is in the third quadrant. Then,

$$\begin{aligned} \sin(-90^\circ - A) &= -\cos A, & \cos(-90^\circ - A) &= -\sin A, \\ \tan(-90^\circ - A) &= \cot A, & \cot(-90^\circ - A) &= \tan A, \\ \sec(-90^\circ - A) &= -\csc A, & \csc(-90^\circ - A) &= -\sec A. \end{aligned}$$

11. If A is acute, $-180^\circ + A$ is in the third quadrant. Then,

$$\begin{aligned} \sin(-180^\circ + A) &= -\sin A, & \cos(-180^\circ + A) &= -\cos A, \\ \tan(-180^\circ + A) &= \tan A, & \cot(-180^\circ + A) &= \cot A, \\ \sec(-180^\circ + A) &= -\sec A, & \csc(-180^\circ + A) &= -\csc A. \end{aligned}$$

12. If A is acute, $-180^\circ - A$ is in the second quadrant. Then,

$$\begin{aligned} \sin(-180^\circ - A) &= \sin A, & \cos(-180^\circ - A) &= -\cos A, \\ \tan(-180^\circ - A) &= -\tan A, & \cot(-180^\circ - A) &= -\cot A, \\ \sec(-180^\circ - A) &= -\sec A, & \csc(-180^\circ - A) &= \csc A. \end{aligned}$$

13. If A is acute, $-270^\circ + A$ is in the second quadrant. Then,

$$\begin{aligned} \sin(-270^\circ + A) &= \cos A, & \cos(-270^\circ + A) &= -\sin A, \\ \tan(-270^\circ + A) &= -\cot A, & \cot(-270^\circ + A) &= -\tan A, \\ \sec(-270^\circ + A) &= -\csc A, & \csc(-270^\circ + A) &= \sec A. \end{aligned}$$

14. If A is acute, $-720^\circ + A$ is in the first quadrant. Then,

$$\begin{aligned} \sin(-720^\circ + A) &= \sin A, & \cos(-720^\circ + A) &= \cos A, \\ \tan(-720^\circ + A) &= \tan A, & \cot(-720^\circ + A) &= \cot A, \\ \sec(-720^\circ + A) &= \sec A, & \csc(-720^\circ + A) &= \csc A. \end{aligned}$$

Art. 54.—Page 30.

2. $\cos 152^\circ = \cos(180^\circ - 28^\circ) = -\cos 28^\circ;$
 or, $\quad \quad \quad = \cos(90^\circ + 62^\circ) = -\sin 62^\circ.$

3. $\tan 522^\circ = \tan(540^\circ - 18^\circ) = -\tan 18^\circ;$
 or, $\quad \quad \quad = \tan(450^\circ + 72^\circ) = -\cot 72^\circ.$

4. $\sec(-77^\circ) = \sec(0^\circ - 77^\circ) = \sec 77^\circ;$
 or, $\quad \quad \quad = \sec(-90^\circ + 13^\circ) = \csc 13^\circ.$

5. $\csc 230^\circ = \csc(180^\circ + 50^\circ) = -\csc 50^\circ;$
 or, $\quad \quad \quad = \csc(270^\circ - 40^\circ) = -\sec 40^\circ.$

6. $\cot(-129^\circ) = \cot(-180^\circ + 51^\circ) = \cot 51^\circ;$
 or, $\quad \quad \quad = \cot(-90^\circ - 39^\circ) = \tan 39^\circ.$

7. $\sin 865^\circ = \sin (900^\circ - 35^\circ) = \sin 35^\circ;$
 or, $= \sin (810^\circ + 55^\circ) = \cos 55^\circ.$
8. $\cot 83^\circ = \cot (90^\circ - 7^\circ) = \tan 7^\circ.$
9. $\sin (-50^\circ) = \sin (-90^\circ + 40^\circ) = -\cos 40^\circ.$
10. $\sec 165^\circ = \sec (180^\circ - 15^\circ) = -\sec 15^\circ.$
11. $\cos (-303^\circ) = \cos (-270^\circ - 33^\circ) = \sin 33^\circ.$
12. $\tan 520^\circ = \tan (540^\circ - 20^\circ) = -\tan 20^\circ.$
13. $\csc 768^\circ = \csc (810^\circ - 42^\circ) = \sec 42^\circ.$

Table.—Page 31.

Since $120^\circ = 180^\circ - 60^\circ$, we have

$$\sin 120^\circ = \sin 60^\circ = \frac{1}{2}\sqrt{3}. \quad \cot 120^\circ = -\cot 60^\circ = -\frac{1}{3}\sqrt{3}.$$

$$\cos 120^\circ = -\cos 60^\circ = -\frac{1}{2}. \quad \sec 120^\circ = -\sec 60^\circ = -2.$$

$$\tan 120^\circ = -\tan 60^\circ = -\sqrt{3}. \quad \csc 120^\circ = \csc 60^\circ = \frac{2}{3}\sqrt{3}.$$

Since $135^\circ = 180^\circ - 45^\circ$, we have

$$\sin 135^\circ = \sin 45^\circ = \frac{1}{2}\sqrt{2}. \quad \cot 135^\circ = -\cot 45^\circ = -1.$$

$$\cos 135^\circ = -\cos 45^\circ = -\frac{1}{2}\sqrt{2}. \quad \sec 135^\circ = -\sec 45^\circ = -\sqrt{2}.$$

$$\tan 135^\circ = -\tan 45^\circ = -1. \quad \csc 135^\circ = \csc 45^\circ = \sqrt{2}.$$

Since $150^\circ = 180^\circ - 30^\circ$, we have

$$\sin 150^\circ = \sin 30^\circ = \frac{1}{2}. \quad \cot 150^\circ = -\cot 30^\circ = -\sqrt{3}.$$

$$\cos 150^\circ = -\cos 30^\circ = -\frac{1}{2}\sqrt{3}. \quad \sec 150^\circ = -\sec 30^\circ = -\frac{2}{3}\sqrt{3}.$$

$$\tan 150^\circ = -\tan 30^\circ = -\frac{1}{3}\sqrt{3}. \quad \csc 150^\circ = \csc 30^\circ = 2.$$

Since $210^\circ = 180^\circ + 30^\circ$, we have

$$\sin 210^\circ = -\sin 30^\circ = -\frac{1}{2}. \quad \cot 210^\circ = \cot 30^\circ = \sqrt{3}.$$

$$\cos 210^\circ = -\cos 30^\circ = -\frac{1}{2}\sqrt{3}. \quad \sec 210^\circ = -\sec 30^\circ = -\frac{2}{3}\sqrt{3}.$$

$$\tan 210^\circ = \tan 30^\circ = \frac{1}{3}\sqrt{3}. \quad \csc 210^\circ = -\csc 30^\circ = -2.$$

Since $225^\circ = 180^\circ + 45^\circ$, we have

$$\begin{aligned} \sin 225^\circ &= -\sin 45^\circ = -\frac{1}{2}\sqrt{2}. & \cot 225^\circ &= \cot 45^\circ = 1. \\ \cos 225^\circ &= -\cos 45^\circ = -\frac{1}{2}\sqrt{2}. & \sec 225^\circ &= -\sec 45^\circ = -\sqrt{2}. \\ \tan 225^\circ &= \tan 45^\circ = 1. & \csc 225^\circ &= -\csc 45^\circ = -\sqrt{2}. \end{aligned}$$

Since $240^\circ = 180^\circ + 60^\circ$, we have

$$\begin{aligned} \sin 240^\circ &= -\sin 60^\circ = -\frac{1}{2}\sqrt{3}. & \cot 240^\circ &= \cot 60^\circ = \frac{1}{3}\sqrt{3}. \\ \cos 240^\circ &= -\cos 60^\circ = -\frac{1}{2}. & \sec 240^\circ &= -\sec 60^\circ = -2. \\ \tan 240^\circ &= \tan 60^\circ = \sqrt{3}. & \csc 240^\circ &= -\csc 60^\circ = -\frac{2}{3}\sqrt{3}. \end{aligned}$$

Since $300^\circ = 360^\circ - 60^\circ$, we have

$$\begin{aligned} \sin 300^\circ &= -\sin 60^\circ = -\frac{1}{2}\sqrt{3}. & \cot 300^\circ &= -\cot 60^\circ = -\frac{1}{3}\sqrt{3}. \\ \cos 300^\circ &= \cos 60^\circ = \frac{1}{2}. & \sec 300^\circ &= \sec 60^\circ = 2. \\ \tan 300^\circ &= -\tan 60^\circ = -\sqrt{3}. & \csc 300^\circ &= -\csc 60^\circ = -\frac{2}{3}\sqrt{3}. \end{aligned}$$

Since $315^\circ = 360^\circ - 45^\circ$, we have

$$\begin{aligned} \sin 315^\circ &= -\sin 45^\circ = -\frac{1}{2}\sqrt{2}. & \cot 315^\circ &= -\cot 45^\circ = -1. \\ \cos 315^\circ &= \cos 45^\circ = \frac{1}{2}\sqrt{2}. & \sec 315^\circ &= \sec 45^\circ = \sqrt{2}. \\ \tan 315^\circ &= -\tan 45^\circ = -1. & \csc 315^\circ &= -\csc 45^\circ = -\sqrt{2}. \end{aligned}$$

Since $330^\circ = 360^\circ - 30^\circ$, we have

$$\begin{aligned} \sin 330^\circ &= -\sin 30^\circ = -\frac{1}{2}. & \cot 330^\circ &= -\cot 30^\circ = -\sqrt{3}. \\ \cos 330^\circ &= \cos 30^\circ = \frac{1}{2}\sqrt{3}. & \sec 330^\circ &= \sec 30^\circ = \frac{2}{3}\sqrt{3}. \\ \tan 330^\circ &= -\tan 30^\circ = -\frac{1}{3}\sqrt{3}. & \csc 330^\circ &= -\csc 30^\circ = -2. \end{aligned}$$

Art. 56. — Page 34.

3. Here the ordinate = 1, and the distance = 4. Therefore the abscissa = $\pm\sqrt{4^2 - 1^2} = \pm\sqrt{15}$. Then,

$$\cos A = \pm \frac{\sqrt{15}}{4}, \quad \cot A = \pm \sqrt{15}, \quad \csc A = 4.$$

$$\tan A = \pm \frac{1}{\sqrt{15}}, \quad \sec A = \pm \frac{4}{\sqrt{15}},$$

4. Here the abscissa = 2 and the ordinate = 1, or the abscissa = -2 and the ordinate = -1. Therefore the distance = $\sqrt{2^2 + 1^2} = \sqrt{5}$. Then,

$$\sin A = \pm \frac{1}{\sqrt{5}}, \quad \tan A = \frac{1}{2}, \quad \csc A = \pm \sqrt{5}.$$

$$\cos A = \pm \frac{2}{\sqrt{5}}, \quad \sec A = \pm \frac{\sqrt{5}}{2},$$

5. Here the distance = 3, and the ordinate = -2. Therefore the abscissa = $\pm\sqrt{3^2 - 2^2} = \pm\sqrt{5}$. Then,

$$\sin A = -\frac{2}{3}, \quad \tan A = \mp \frac{2}{\sqrt{5}}, \quad \csc A = \pm \frac{3}{\sqrt{5}}.$$

$$\cos A = \pm \frac{\sqrt{5}}{3}, \quad \cot A = \mp \frac{\sqrt{5}}{2},$$

6. Here the ordinate = 8 and the abscissa = -15, or the ordinate = -8 and the abscissa = 15. Therefore the distance = $\sqrt{8^2 + 15^2} = 17$. Then,

$$\sin A = \pm \frac{8}{17}, \quad \cot A = -\frac{15}{8}, \quad \csc A = \pm \frac{17}{8}.$$

$$\cos A = \mp \frac{15}{17}, \quad \sec A = \mp \frac{17}{15},$$

7. Here the distance = 4, and the abscissa = 3. Therefore the ordinate = $\pm\sqrt{4^2 - 3^2} = \pm\sqrt{7}$. Then,

$$\sin A = \pm \frac{\sqrt{7}}{4}, \quad \tan A = \pm \frac{\sqrt{7}}{3}, \quad \csc A = \pm \frac{4}{\sqrt{7}}.$$

$$\cos A = \frac{3}{4}, \quad \cot A = \pm \frac{3}{\sqrt{7}},$$

8. Here the abscissa = -1, and the distance = 2. Therefore the ordinate = $\pm\sqrt{2^2 - 1^2} = \pm\sqrt{3}$. Then,

$$\sin A = \pm \frac{\sqrt{3}}{2}, \quad \cot A = \mp \frac{1}{\sqrt{3}}, \quad \csc A = \pm \frac{2}{\sqrt{3}}.$$

$$\tan A = \mp \sqrt{3}, \quad \sec A = -2,$$

9. Here the distance = $\sqrt{2}$, and the ordinate = 1. Therefore the abscissa = $\pm \sqrt{2-1} = \pm 1$. Then,

$$\sin A = \frac{1}{\sqrt{2}}, \quad \tan A = \pm 1, \quad \sec A = \pm \sqrt{2}.$$

$$\cos A = \pm \frac{1}{\sqrt{2}}, \quad \cot A = \pm 1,$$

10. Here the ordinate = $2\sqrt{2}$ and the abscissa = 1, or the ordinate = $-2\sqrt{2}$ and the abscissa = -1. Therefore the distance = $\sqrt{8+1} = 3$. Then,

$$\sin A = \pm \frac{2\sqrt{2}}{3}, \quad \cot A = \frac{1}{2\sqrt{2}}, \quad \csc A = \pm \frac{3}{2\sqrt{2}}.$$

$$\cos A = \pm \frac{1}{3}, \quad \sec A = \pm 3,$$

11. Here the abscissa = $-a$, and the distance = b . Therefore the ordinate = $\pm \sqrt{b^2 - a^2}$. Then,

$$\sin A = \pm \frac{\sqrt{b^2 - a^2}}{b}, \quad \cot A = \mp \frac{a}{\sqrt{b^2 - a^2}}, \quad \csc A = \pm \frac{b}{\sqrt{b^2 - a^2}}.$$

$$\tan A = \mp \frac{\sqrt{b^2 - a^2}}{a}, \quad \sec A = -\frac{b}{a},$$

12. Here the ordinate = x , and the distance = 1. Therefore the abscissa = $\pm \sqrt{1 - x^2}$. Then,

$$\cos A = \pm \sqrt{1 - x^2}, \quad \cot A = \pm \frac{\sqrt{1 - x^2}}{x}, \quad \csc A = \frac{1}{x}.$$

$$\tan A = \pm \frac{x}{\sqrt{1 - x^2}}, \quad \sec A = \pm \frac{1}{\sqrt{1 - x^2}},$$

13. Here the abscissa = 1 and the ordinate = x , or the abscissa = -1 and the ordinate = $-x$. Therefore the distance = $\sqrt{1 + x^2}$. Then,

$$\sin A = \pm \frac{x}{\sqrt{1 + x^2}}, \quad \tan A = x, \quad \csc A = \pm \frac{\sqrt{1 + x^2}}{x}.$$

$$\cos A = \pm \frac{1}{\sqrt{1 + x^2}}, \quad \sec A = \pm \sqrt{1 + x^2},$$

14. Here the distance = $\sqrt{a^2 + b^2}$, and the abscissa = a . Therefore the ordinate = $\pm \sqrt{a^2 + b^2 - a^2} = \pm b$. Then,

$$\sin A = \pm \frac{b}{\sqrt{a^2 + b^2}}, \quad \tan A = \pm \frac{b}{a}, \quad \csc A = \pm \frac{\sqrt{a^2 + b^2}}{b}.$$

$$\cos A = \frac{a}{\sqrt{a^2 + b^2}}, \quad \cot A = \pm \frac{a}{b},$$

CHAPTER IV.

Art. 60. — Page 36.

$$\sin A = \tan A \cos A \text{ (Art. 59)} = \frac{\tan A}{\sec A} = \frac{\tan A}{\sqrt{1 + \tan^2 A}} \text{ (Art. 58).}$$

$$\sin A = \frac{1}{\csc A} = \frac{1}{\sqrt{1 + \cot^2 A}} \text{ (Art. 58).}$$

$$\sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - \frac{1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}.$$

$$\cos A = \frac{1}{\sec A} = \frac{1}{\sqrt{1 + \tan^2 A}} \text{ (Art. 58).}$$

$$\cos A = \cot A \sin A \text{ (Art. 59)} = \frac{\cot A}{\csc A} = \frac{\cot A}{\sqrt{1 + \cot^2 A}} \text{ (Art. 58).}$$

$$\cos A = \sqrt{1 - \sin^2 A} = \sqrt{1 - \frac{1}{\csc^2 A}} = \frac{\sqrt{\csc^2 A - 1}}{\csc A}.$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1 - \sin^2 A}} = \frac{\sqrt{1 - \cos^2 A}}{\cos A}.$$

$$\tan A = \frac{1}{\cot A} = \frac{1}{\sqrt{\csc^2 A - 1}} \text{ (Art. 58).}$$

Since the cotangent, secant, and cosecant are the reciprocals of the tangent, cosine, and sine, respectively, we have :

$$\cot A = \frac{\sqrt{1 - \sin^2 A}}{\sin A} = \frac{\cos A}{\sqrt{1 - \cos^2 A}} = \frac{1}{\sqrt{\sec^2 A - 1}}.$$

$$\sec A = \frac{1}{\sqrt{1 - \sin^2 A}} = \frac{\sqrt{1 + \cot^2 A}}{\cot A} = \frac{\csc A}{\sqrt{\csc^2 A - 1}}.$$

$$\csc A = \frac{1}{\sqrt{1 - \cos^2 A}} = \frac{\sqrt{1 + \tan^2 A}}{\tan A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}.$$

CHAPTER V.

Art. 78. — Pages 51 to 53.

3.
$$\frac{\sin(x+y)}{\sin(x-y)} = \frac{\sin x \cos y + \cos x \sin y}{\sin x \cos y - \cos x \sin y}$$

$$= \frac{\frac{\sin x \cos y}{\cos x \cos y} + \frac{\cos x \sin y}{\cos x \cos y}}{\frac{\sin x \cos y}{\cos x \cos y} - \frac{\cos x \sin y}{\cos x \cos y}} = \frac{\tan x + \tan y}{\tan x - \tan y}$$
4.
$$\frac{\cos(x+y)}{\cos(x-y)} = \frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y + \sin x \sin y}$$

$$= \frac{\frac{\cos x \cos y}{\sin x \sin y} - \frac{\sin x \sin y}{\sin x \sin y}}{\frac{\cos x \cos y}{\sin x \sin y} + \frac{\sin x \sin y}{\sin x \sin y}} = \frac{\cot x \cot y - 1}{\cot x \cot y + 1}$$
5.
$$\frac{\sin(x+y)}{\cos(x-y)} = \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y + \sin x \sin y}$$

$$= \frac{\frac{\sin x \cos y}{\sin x \cos y} + \frac{\cos x \sin y}{\sin x \cos y}}{\frac{\cos x \cos y}{\sin x \cos y} + \frac{\sin x \sin y}{\sin x \cos y}} = \frac{1 + \cot x \tan y}{\cot x + \tan y}$$
6.
$$\sin(45^\circ + y) = \sin 45^\circ \cos y + \cos 45^\circ \sin y$$

$$= \frac{1}{\sqrt{2}} \cdot \cos y + \frac{1}{\sqrt{2}} \cdot \sin y \text{ (Art. 16)} = \frac{\sin y + \cos y}{\sqrt{2}}$$
7.
$$\tan(60^\circ - y) = \frac{\tan 60^\circ - \tan y}{1 + \tan 60^\circ \tan y} = \frac{\sqrt{3} - \tan y}{1 + \sqrt{3} \tan y} \text{ (Art. 17)}$$
8.
$$\frac{\sin x + \sin y}{\cos x + \cos y} = \frac{2 \sin \frac{1}{2}(x+y) \cos \frac{1}{2}(x-y)}{2 \cos \frac{1}{2}(x+y) \cos \frac{1}{2}(x-y)} = \tan \frac{1}{2}(x+y)$$
9.
$$\frac{\sin x + \sin y}{\cos x - \cos y} = \frac{2 \sin \frac{1}{2}(x+y) \cos \frac{1}{2}(x-y)}{-2 \sin \frac{1}{2}(x+y) \sin \frac{1}{2}(x-y)} = -\cot \frac{1}{2}(x-y)$$

$$10. \text{ By Arts. 74 and 58, } \sin 2x = \frac{2 \sin x \cos x}{\sin^2 x + \cos^2 x}.$$

Dividing each term of the fraction by $\cos^2 x$,

$$\sin 2x = \frac{\frac{2 \sin x}{\cos x}}{1 + \frac{\sin^2 x}{\cos^2 x}} = \frac{2 \tan x}{1 + \tan^2 x}.$$

$$11. \csc 2x = \frac{1}{\sin 2x} = \frac{1}{2 \sin x \cos x} = \frac{1}{2} \sec x \csc x \text{ (Art. 57).}$$

$$\begin{aligned} 12. \tan x + \cot x &= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \\ &= \frac{1}{\sin x \cos x} = \frac{2}{2 \sin x \cos x} = \frac{2}{\sin 2x}. \end{aligned}$$

$$\begin{aligned} 13. \cot x - \tan x &= \cot x - \frac{1}{\cot x} = \frac{\cot^2 x - 1}{\cot x} \\ &= 2 \left(\frac{\cot^2 x - 1}{2 \cot x} \right) = 2 \cot 2x \text{ (Art. 74).} \end{aligned}$$

$$\begin{aligned} 14. \frac{(1 + \tan x)^2 - (1 - \tan x)^2}{(1 + \tan x)^2 + (1 - \tan x)^2} \\ &= \frac{1 + 2 \tan x + \tan^2 x - 1 + 2 \tan x - \tan^2 x}{1 + 2 \tan x + \tan^2 x + 1 - 2 \tan x + \tan^2 x} \\ &= \frac{4 \tan x}{2 + 2 \tan^2 x} = \frac{2 \tan x}{1 + \tan^2 x} = \sin 2x, \text{ by Ex. 10} \end{aligned}$$

$$\begin{aligned} 15. \sin(x + y) \sin(x - y) \\ &= (\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y) \\ &= \sin^2 x \cos^2 y - \cos^2 x \sin^2 y \\ &= \sin^2 x (1 - \sin^2 y) - (1 - \sin^2 x) \sin^2 y \\ &= \sin^2 x - \sin^2 y. \end{aligned}$$

$$\begin{aligned} 16. \cos(x + y) \cos(x - y) \\ &= (\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y) \\ &= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y \\ &= \cos^2 x \cos^2 y - (1 - \cos^2 x)(1 - \cos^2 y) \\ &= \cos^2 x \cos^2 y - 1 + \cos^2 x + \cos^2 y - \cos^2 x \cos^2 y \\ &= \cos^2 x - (1 - \cos^2 y) = \cos^2 x - \sin^2 y. \end{aligned}$$

$$\begin{aligned}
 17. \sec^2 x \csc^2 x &= \frac{1}{\cos^2 x \sin^2 x} = \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} \\
 &= \frac{\sin^2 x}{\cos^2 x \sin^2 x} + \frac{\cos^2 x}{\cos^2 x \sin^2 x} = \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \\
 &= \sec^2 x + \csc^2 x.
 \end{aligned}$$

$$\begin{aligned}
 18. \cos y + \cos (120^\circ + y) + \cos (120^\circ - y) \\
 &= \cos y + \cos 120^\circ \cos y - \sin 120^\circ \sin y + \cos 120^\circ \cos y + \sin 120^\circ \sin y \\
 &= \cos y + 2 \cos 120^\circ \cos y = \cos y - \cos y \text{ (Art. 55)} = 0.
 \end{aligned}$$

$$\begin{aligned}
 19. \sin A \sin (B - C) + \sin B \sin (C - A) + \sin C \sin (A - B) \\
 &= \sin A (\sin B \cos C - \cos B \sin C) + \sin B (\sin C \cos A - \cos C \sin A) \\
 &\quad + \sin C (\sin A \cos B - \cos A \sin B) \\
 &= \sin A \sin B \cos C - \sin A \cos B \sin C + \sin B \sin C \cos A \\
 &\quad - \sin B \cos C \sin A + \sin C \sin A \cos B - \sin C \cos A \sin B = 0.
 \end{aligned}$$

$$\begin{aligned}
 20. \cos (A + B) \cos (A - B) + \cos (B + C) \cos (B - C) \\
 \quad + \cos (C + A) \cos (C - A) \\
 &= \cos^2 A - \sin^2 B + \cos^2 B - \sin^2 C + \cos^2 C - \sin^2 A, \text{ by Ex. 16,} \\
 &= \cos 2A + \cos 2B + \cos 2C \text{ (Art. 74).}
 \end{aligned}$$

$$\begin{aligned}
 21. \frac{\cos x - \cos 3x}{\sin 3x - \sin x} &= \frac{-(\cos 3x - \cos x)}{\sin 3x - \sin x} \\
 &= \frac{2 \sin \frac{1}{2}(3x + x) \sin \frac{1}{2}(3x - x)}{2 \cos \frac{1}{2}(3x + x) \sin \frac{1}{2}(3x - x)} = \tan 2x.
 \end{aligned}$$

$$\begin{aligned}
 22. \frac{\cos 80^\circ + \cos 20^\circ}{\sin 80^\circ - \sin 20^\circ} &= \frac{2 \cos \frac{1}{2}(80^\circ + 20^\circ) \cos \frac{1}{2}(80^\circ - 20^\circ)}{2 \cos \frac{1}{2}(80^\circ + 20^\circ) \sin \frac{1}{2}(80^\circ - 20^\circ)} \\
 &= \cot 30^\circ = \sqrt{3} \text{ (Art. 17).}
 \end{aligned}$$

$$\begin{aligned}
 23. \sin (x + y + z) &= \sin [(x + y) + z] \\
 &= \sin (x + y) \cos z + \cos (x + y) \sin z \\
 &= (\sin x \cos y + \cos x \sin y) \cos z + (\cos x \cos y - \sin x \sin y) \sin z \\
 &= \sin x \cos y \cos z + \cos x \sin y \cos z + \cos x \cos y \sin z - \sin x \sin y \sin z.
 \end{aligned}$$

$$\begin{aligned}
 24. \cos (x + y + z) &= \cos [(x + y) + z] \\
 &= \cos (x + y) \cos z - \sin (x + y) \sin z \\
 &= (\cos x \cos y - \sin x \sin y) \cos z - (\sin x \cos y + \cos x \sin y) \sin z \\
 &= \cos x \cos y \cos z - \sin x \sin y \cos z - \sin x \cos y \sin z - \cos x \sin y \sin z.
 \end{aligned}$$

$$\begin{aligned}
 25. \quad \sin 3x &= \sin(2x + x) = \sin 2x \cos x + \cos 2x \sin x \\
 &= 2 \sin x \cos^2 x + (1 - 2 \sin^2 x) \sin x \quad (\text{Art. 74}) \\
 &= 2 \sin x (1 - \sin^2 x) + \sin x - 2 \sin^3 x \\
 &= 3 \sin x - 4 \sin^3 x.
 \end{aligned}$$

$$\begin{aligned}
 26. \quad \cos 3x &= \cos(2x + x) = \cos 2x \cos x - \sin 2x \sin x \\
 &= (2 \cos^2 x - 1) \cos x - 2 \sin^2 x \cos x \quad (\text{Art. 74}) \\
 &= 2 \cos^3 x - \cos x - 2(1 - \cos^2 x) \cos x \\
 &= 4 \cos^3 x - 3 \cos x.
 \end{aligned}$$

$$\begin{aligned}
 27. \quad \tan 3x &= \tan(2x + x) = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x} \\
 &= \frac{\frac{2 \tan x}{1 - \tan^2 x} + \tan x}{1 - \frac{2 \tan^2 x}{1 - \tan^2 x}} \quad (\text{Art. 74}) \\
 &= \frac{2 \tan x + \tan x - \tan^3 x}{1 - \tan^2 x - 2 \tan^2 x} = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}.
 \end{aligned}$$

$$\begin{aligned}
 28. \quad \sin(2x + y) - 2 \sin x \cos(x + y) \\
 &= \sin[(x + y) + x] - 2 \sin x \cos(x + y) \\
 &= \sin(x + y) \cos x + \cos(x + y) \sin x - 2 \sin x \cos(x + y) \\
 &= \sin(x + y) \cos x - \cos(x + y) \sin x \\
 &= \sin[(x + y) - x] = \sin y.
 \end{aligned}$$

$$\begin{aligned}
 29. \quad \frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x} &= \frac{\sin 3x \cos x - \cos 3x \sin x}{\sin x \cos x} \\
 &= \frac{\sin(3x - x)}{\frac{1}{2} \sin 2x} \quad (\text{Art. 74}) = 2.
 \end{aligned}$$

$$\begin{aligned}
 30. \quad 1 + \cos 2x \cos 2y &= 1 + (2 \cos^2 x - 1)(1 - 2 \sin^2 y) \\
 &= 2 \cos^2 x + 2 \sin^2 y - 4 \cos^2 x \sin^2 y \\
 &= 2 \cos^2 x (\sin^2 y + \cos^2 y) + 2 \sin^2 y (\sin^2 x + \cos^2 x) - 4 \cos^2 x \sin^2 y \\
 &= 2 (\sin^2 x \sin^2 y + \cos^2 x \cos^2 y).
 \end{aligned}$$

$$\begin{aligned}
 31. \quad 1 + \tan x \tan 2x &= 1 + \frac{\sin x \sin 2x}{\cos x \cos 2x} \\
 &= \frac{\cos 2x \cos x + \sin 2x \sin x}{\cos 2x \cos x} = \frac{\cos(2x - x)}{\cos 2x \cos x} = \frac{1}{\cos 2x} = \sec 2x.
 \end{aligned}$$

$$32. \sin 4x = 2 \sin 2x \cos 2x = 4 \sin x \cos x (1 - 2 \sin^2 x) \quad (\text{Art. 74}) \\ = 4 \sin x \cos x - 8 \sin^3 x \cos x.$$

$$33. \cos 4x = 2 \cos^2 2x - 1 = 2(2 \cos^2 x - 1)^2 - 1 \quad (\text{Art. 74}) \\ = 8 \cos^4 x - 8 \cos^2 x + 2 - 1 = 8 \cos^4 x - 8 \cos^2 x + 1.$$

$$34. \sin 5x = \sin (4x + x) = \sin 4x \cos x + \cos 4x \sin x \\ = 4 \sin x \cos^2 x - 8 \sin^3 x \cos^2 x + 8 \cos^4 x \sin x - 8 \cos^2 x \sin x \\ + \sin x \quad (\text{Exs. 32, 33}) \\ = 4 \sin x (1 - \sin^2 x) - 8 \sin^3 x (1 - \sin^2 x) + 8 (1 - \sin^2 x)^2 \sin x \\ - 8 (1 - \sin^2 x) \sin x + \sin x \\ = 4 \sin x - 4 \sin^3 x - 8 \sin^3 x + 8 \sin^5 x + 8 \sin x - 16 \sin^3 x \\ + 8 \sin^5 x - 8 \sin x + 8 \sin^3 x + \sin x \\ = 5 \sin x - 20 \sin^3 x + 16 \sin^5 x.$$

$$35. \sin 15^\circ = \sin (45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ \\ = \frac{1}{2} \sqrt{2} \cdot \frac{1}{2} \sqrt{3} - \frac{1}{2} \sqrt{2} \cdot \frac{1}{2} = \frac{1}{4} (\sqrt{6} - \sqrt{2}) = \cos 75^\circ.$$

$$\cos 15^\circ = \cos (45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \\ = \frac{1}{2} \sqrt{2} \cdot \frac{1}{2} \sqrt{3} + \frac{1}{2} \sqrt{2} \cdot \frac{1}{2} = \frac{1}{4} (\sqrt{6} + \sqrt{2}) = \sin 75^\circ.$$

$$36. \tan 15^\circ = \frac{1 - \cos 30^\circ}{\sin 30^\circ} = \csc 30^\circ - \cot 30^\circ = 2 - \sqrt{3} = \cot 75^\circ.$$

$$\cot 15^\circ = \frac{1 + \cos 30^\circ}{\sin 30^\circ} = \csc 30^\circ + \cot 30^\circ = 2 + \sqrt{3} = \tan 75^\circ.$$

$$37. \sin 22^\circ 30' = \sqrt{\frac{1 - \cos 45^\circ}{2}} = \sqrt{\frac{1 - \frac{1}{2} \sqrt{2}}{2}} = \sqrt{\frac{2 - \sqrt{2}}{4}} \\ = \frac{1}{2} \sqrt{2 - \sqrt{2}}.$$

$$\cos 22^\circ 30' = \sqrt{\frac{1 + \cos 45^\circ}{2}} = \sqrt{\frac{1 + \frac{1}{2} \sqrt{2}}{2}} = \sqrt{\frac{2 + \sqrt{2}}{4}} \\ = \frac{1}{2} \sqrt{2 + \sqrt{2}}.$$

$$38. \tan 22^\circ 30' = \frac{1 - \cos 45^\circ}{\sin 45^\circ} = \csc 45^\circ - \cot 45^\circ = \sqrt{2} - 1.$$

$$\cot 22^\circ 30' = \frac{1 + \cos 45^\circ}{\sin 45^\circ} = \csc 45^\circ + \cot 45^\circ = \sqrt{2} + 1.$$

CHAPTER VI.

Art. 91.—Page 57.

2. $\log 6 = \log (2 \times 3) = \log 2 + \log 3 = .3010 + .4771 = .7781.$
3. $\log 14 = \log (2 \times 7) = \log 2 + \log 7 = .3010 + .8451 = 1.1461.$
4. $\log 8 = \log (2 \times 2 \times 2) = \log 2 + \log 2 + \log 2 = 3 \log 2$
 $= 3 \times .3010 = .9030.$
5. $\log 12 = \log (2 \times 2 \times 3) = \log 2 + \log 2 + \log 3$
 $= 2 \log 2 + \log 3 = .6020 + .4771 = 1.0791.$
6. $\log 15 = \log (3 \times 5) = \log 3 + \log 5 = .4771 + .6990 = 1.1761.$
7. $\log 21 = \log (3 \times 7) = \log 3 + \log 7 = .4771 + .8451 = 1.3222.$
8. $\log 63 = \log (3 \times 3 \times 7) = \log 3 + \log 3 + \log 7 = 2 \log 3 + \log 7$
 $= .9542 + .8451 = 1.7993.$
9. $\log 56 = \log (2 \times 2 \times 2 \times 7) = 3 \log 2 + \log 7$
 $= .9030 + .8451 = 1.7481.$
10. $\log 84 = \log (2 \times 2 \times 3 \times 7) = 2 \log 2 + \log 3 + \log 7$
 $= .6020 + .4771 + .8451 = 1.9242.$
11. $\log 45 = \log (3 \times 3 \times 5) = 2 \log 3 + \log 5 = .9542 + .6990 = 1.6532.$
12. $\log 98 = \log (2 \times 7 \times 7) = \log 2 + 2 \log 7 = .3010 + 1.6902 = 1.9912.$
13. $\log 105 = \log (3 \times 5 \times 7) = \log 3 + \log 5 + \log 7$
 $= .4771 + .6990 + .8451 = 2.0212.$
14. $\log 112 = \log (2 \times 2 \times 2 \times 2 \times 7) = 4 \log 2 + \log 7$
 $= 1.2040 + .8451 = 2.0491.$
15. $\log 144 = \log (2 \times 2 \times 2 \times 2 \times 3 \times 3) = 4 \log 2 + 2 \log 3$
 $= 1.2040 + .9542 = 2.1582.$
16. $\log 216 = \log (2 \times 2 \times 2 \times 3 \times 3 \times 3) = 3 \log 2 + 3 \log 3$
 $= .9030 + 1.4313 = 2.3343.$

17. $\log 135 = \log (3 \times 3 \times 3 \times 5) = 3 \log 3 + \log 5$
 $= 1.4313 + .6990 = 2.1303.$
18. $\log 168 = \log (2 \times 2 \times 2 \times 3 \times 7) = 3 \log 2 + \log 3 + \log 7$
 $= .9030 + .4771 + .8451 = 2.2252.$
19. $\log 147 = \log (3 \times 7 \times 7) = \log 3 + 2 \log 7 = .4771 + 1.6902 = 2.1673.$
20. $\log 375 = \log (3 \times 5 \times 5 \times 5) = \log 3 + 3 \log 5$
 $= .4771 + 2.0970 = 2.5741.$
21. $\log 343 = \log (7 \times 7 \times 7) = 3 \log 7 = 2.5353.$

Art. 93. — Page 58.

2. $\log \frac{7}{3} = \log 7 - \log 3 = .8451 - .4771 = .3680.$
3. $\log \frac{10}{7} = \log 10 - \log 7 = 1 - .8451 = .1549.$
4. $\log 3\frac{1}{3} = \log \frac{10}{3} = \log 10 - \log 3 = 1 - .4771 = .5229.$
5. $\log 35 = \log \frac{70}{2} = \log (10 \times 7) - \log 2 = \log 10 + \log 7 - \log 2$
 $= 1 + .8451 - .3010 = 1.5441.$
6. $\log \frac{21}{16} = \log 21 - \log 16 = \log (3 \times 7) - \log (2 \times 2 \times 2 \times 2)$
 $= \log 3 + \log 7 - 4 \log 2 = .4771 + .8451 - 1.2040 = .1182.$
7. $\log 125 = \log (5 \times 5 \times 5) = 3 \log 5.$
 $\log 5 = \log \frac{10}{2} = 1 - \log 2 = 1 - .3010 = .6990.$
 $\therefore \log 125 = 3 \times .6990 = 2.0970.$
8. $\log \frac{42}{25} = \log (2 \times 3 \times 7) - \log (5 \times 5) = \log 2 + \log 3 + \log 7 - 2 \log 5.$
 $\log 5 = \log \frac{10}{2} = 1 - \log 2 = .6990.$
 $\therefore \log \frac{42}{25} = .3010 + .4771 + .8451 - 1.3980 = .2252.$
9. $\log 175 = \log (5 \times 5 \times 7) = 2 \log 5 + \log 7.$
 $\log 5 = \log \frac{10}{2} = 1 - \log 2 = .6990.$
 $\therefore \log 175 = 1.3980 + .8451 = 2.2431.$

10. $\log 11\frac{1}{3} = \log \frac{100}{9} = \log 100 - \log (3 \times 3)$
 $= 2 - 2 \log 3 = 2 - .9542 = 1.0458.$
11. $\log 7\frac{1}{7} = \log \frac{50}{7} = \log \frac{100}{14} = \log 100 - \log (2 \times 7)$
 $= 2 - \log 2 - \log 7 = 2 - .3010 - .8451 = .8539.$
12. $\log \frac{35}{6} = \log (5 \times 7) - \log (2 \times 3) = \log 5 + \log 7 - \log 2 - \log 3.$
 $\log 5 = \log \frac{10}{2} = 1 - \log 2 = .6990.$
 $\therefore \log \frac{35}{6} = .6990 + .8451 - .3010 - .4771 = .7660.$
13. $\log 5\frac{4}{9} = \log \frac{49}{9} = \log (7 \times 7) - \log (3 \times 3) = 2 \log 7 - 2 \log 3$
 $= 1.6902 - .9542 = .7360.$

Art. 96. — Page 59.

3. $\log 3^{\frac{3}{5}} = \frac{3}{5} \log 3 = \frac{3}{5} \times .4771 = .2863.$
4. $\log 2^9 = 9 \log 2 = 9 \times .3010 = 2.7090.$
5. $\log 7^5 = 5 \log 7 = 5 \times .8451 = 4.2255.$
6. $\log 5^{\frac{1}{5}} = \frac{1}{5} \log 5.$
 $\log 5 = \log \frac{10}{2} = 1 - \log 2 = .6990.$
 $\therefore \log 5^{\frac{1}{5}} = \frac{.6990}{5} = .1398.$
7. $\log 12^{\frac{2}{3}} = \frac{2}{3} \log 12.$
 $\log 12 = \log (2 \times 2 \times 3) = 2 \log 2 + \log 3 = .6020 + .4771 = 1.0791.$
 $\therefore \log 12^{\frac{2}{3}} = \frac{2}{3} \times 1.0791 = .7194.$
8. $\log 21^{\frac{1}{2}} = \frac{1}{2} \log 21.$
 $\log 21 = \log (3 \times 7) = \log 3 + \log 7 = .4771 + .8451 = 1.3222.$
 $\therefore \log 21^{\frac{1}{2}} = \frac{1.3222}{2} = .6611.$

$$9. \log 14^4 = 4 \log 14.$$

$$\log 14 = \log (2 \times 7) = \log 2 + \log 7 = .3010 + .8451 = 1.1461.$$

$$\therefore \log 14^4 = 4 \times 1.1461 = 4.5844.$$

$$10. \log 25^{\frac{7}{3}} = \frac{7}{3} \log 25.$$

$$\begin{aligned} \log 25 &= \log \frac{100}{4} = \log 100 - \log (2 \times 2) \\ &= 2 - 2 \log 2 = 2 - .6020 = 1.3980. \end{aligned}$$

$$\therefore \log 25^{\frac{7}{3}} = \frac{7}{3} \times 1.3980 = 3.2620.$$

$$11. \log 15^{\frac{5}{6}} = \frac{5}{6} \log 15.$$

$$\log 15 = \log (3 \times 5) = \log 3 + \log 5.$$

$$\log 5 = \log \frac{10}{2} = 1 - \log 2 = .6990.$$

$$\therefore \log 15 = .4771 + .6990 = 1.1761.$$

$$\therefore \log 15^{\frac{5}{6}} = \frac{5}{6} \times 1.1761 = .9801.$$

$$12. \log \sqrt{7} = \frac{\log 7}{2} = \frac{.8451}{2} = .4225.$$

$$13. \log \sqrt[3]{3} = \frac{\log 3}{3} = \frac{.4771}{3} = .1590.$$

$$14. \log \sqrt[7]{2} = \frac{\log 2}{7} = \frac{.3010}{7} = .0430.$$

$$15. \log \sqrt[6]{5} = \frac{\log 5}{6}.$$

$$\log 5 = \log \frac{10}{2} = 1 - \log 2 = .6990.$$

$$\therefore \log \sqrt[6]{5} = \frac{.6990}{6} = .1165.$$

$$16. \log \sqrt[4]{35} = \frac{\log 35}{4}.$$

$$\log 35 = \log (5 \times 7) = \log 5 + \log 7.$$

$$\log 5 = \log \frac{10}{2} = 1 - \log 2 = .6990.$$

$$\therefore \log 35 = .6990 + .8451 = 1.5441.$$

$$\therefore \log \sqrt[4]{35} = \frac{1.5441}{4} = .3860.$$

$$17. \log \sqrt[9]{98} = \frac{\log 98}{9}.$$

$$\log 98 = \log (2 \times 7^2) = \log 2 + 2 \log 7 = .3010 + 1.6902 = 1.9912.$$

$$\therefore \log \sqrt[9]{98} = \frac{1.9912}{9} = .2212.$$

$$18. \log \sqrt[12]{126} = \frac{\log 126}{12}.$$

$$\log 126 = \log (2 \times 3^2 \times 7) = \log 2 + 2 \log 3 + \log 7 \\ = .3010 + .9542 + .8451 = 2.1003.$$

$$\therefore \log \sqrt[12]{126} = \frac{2.1003}{12} = .1750.$$

$$20. \log \left(\frac{10}{3}\right)^5 = 5 \log \frac{10}{3} = 5 (\log 10 - \log 3) = 5 (1 - .4771) \\ = 5 \times .5229 = 2.6145.$$

$$21. \log \frac{7^{\frac{3}{4}}}{5^{\frac{2}{3}}} = \log 7^{\frac{3}{4}} - \log 5^{\frac{2}{3}} = \frac{3}{4} \log 7 - \frac{2}{3} \log 5 \\ = .6338 - .4660 = .1678.$$

$$22. \log (3^{\frac{1}{6}} \times 2^{\frac{3}{5}}) = \log 3^{\frac{1}{6}} + \log 2^{\frac{3}{5}} = \frac{1}{6} \log 3 + \frac{3}{5} \log 2 \\ = .0795 + .1806 = .2601.$$

$$23. \log 3 \sqrt[4]{7} = \log 3 + \log \sqrt[4]{7} = \log 3 + \frac{\log 7}{4} \\ = .4771 + .2113 = .6884.$$

$$24. \log \sqrt{\frac{7}{3}} = \frac{1}{2} \log \frac{7}{3} = \frac{\log 7 - \log 3}{2} = \frac{.8451 - .4771}{2} = .1840.$$

$$25. \log \frac{\sqrt[3]{7}}{\sqrt[5]{2}} = \log \sqrt[3]{7} - \log \sqrt[5]{2} = \frac{\log 7}{3} - \frac{\log 2}{5} \\ = .2817 - .0602 = .2215.$$

$$26. \log \sqrt[3]{\frac{28}{5}} = \frac{1}{3} \log \frac{28}{5} = \frac{\log 28 - \log 5}{3}.$$

$$\log 28 = \log (2^2 \times 7) = 2 \log 2 + \log 7 = .6020 + .8451 = 1.4471.$$

$$\therefore \log \sqrt[3]{\frac{28}{5}} = \frac{1.4471 - .6990}{3} = .2494.$$

$$27. \log \frac{\sqrt{42}}{10^{\frac{2}{3}}} = \log \sqrt{42} - \log 10^{\frac{2}{3}} = \frac{\log 42}{2} - \frac{2}{3} \log 10.$$

$$\log 42 = \log (2 \times 3 \times 7) = \log 2 + \log 3 + \log 7 \\ = .3010 + .4771 + .8451 = 1.6232.$$

$$\therefore \log \frac{\sqrt{42}}{10^{\frac{2}{3}}} = .8116 - .6667 = .1449.$$

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2. $\log 18 = \log (2 \times 3^2) = \log 2 + 2 \log 3 = .3010 + .9542 = 1.2552$.
 $\therefore \log 1.8 = 0.2552$.
3. $\log 225 = \log (3^2 \times 5^2) = 2 \log 3 + 2 \log 5$
 $= .9542 + 1.3980 = 2.3522$.
 $\therefore \log 2.25 = 0.3522$.
4. $\log 196 = \log (2^2 \times 7^2) = 2 \log 2 + 2 \log 7$
 $= .6020 + 1.6902 = 2.2922$.
 $\therefore \log .196 = 2.2922 - 10$.
5. $\log 48 = \log (2^4 \times 3) = 4 \log 2 + \log 3 = 1.2040 + .4771 = 1.6811$.
 $\therefore \log .048 = 1.6811 - 10$.
6. $\log 384 = \log (2^7 \times 3) = 7 \log 2 + \log 3 = 2.1070 + .4771 = 2.5841$.
 $\therefore \log 38.4 = 1.5841$.
7. $\log 54 = \log (2 \times 3^3) = \log 2 + 3 \log 3 = .3010 + 1.4313 = 1.7323$.
 $\therefore \log .0054 = 1.7323 - 10$.
8. $\log 315 = \log (3^2 \times 5 \times 7) = 2 \log 3 + \log 5 + \log 7$
 $= .9542 + .6990 + .8451 = 2.4983$.
 $\therefore \log .000315 = 2.4983 - 10$.
9. $\log 735 = \log (3 \times 5 \times 7^2) = \log 3 + \log 5 + 2 \log 7$
 $= .4771 + .6990 + 1.6902 = 2.8663$.
 $\therefore \log 7350 = 2.8663$.
10. $\log 405 = \log (3^4 \times 5) = 4 \log 3 + \log 5$
 $= 1.9084 + .6990 = 2.6074$.
 $\therefore \log 4.05 = 0.6074$.
11. $\log 448 = \log (2^6 \times 7) = 6 \log 2 + \log 7$
 $= 1.8060 + .8451 = 2.6511$.
 $\therefore \log .448 = 2.6511 - 10$.
12. $\log 3024 = \log (2^4 \times 3^3 \times 7) = 4 \log 2 + 3 \log 3 + \log 7$
 $= 1.2040 + 1.4313 + .8451 = 3.4804$.
 $\therefore \log 302.4 = 2.4804$.
13. $\log 6174 = \log (2 \times 3^2 \times 7^3) = \log 2 + 2 \log 3 + 3 \log 7$
 $= .3010 + .9542 + 2.5353 = 3.7905$.
 $\therefore \log .06174 = 3.7905 - 10$.

$$14. \log (8.1)^7 = 7 \log 8.1.$$

$$\log 81 = \log 3^4 = 4 \log 3 = 1.9084.$$

$$\therefore \log 8.1 = 0.9084.$$

$$\therefore \log (8.1)^7 = 7 \times .9084 = 6.3588.$$

$$15. \log \sqrt[5]{9.6} = \frac{\log 9.6}{5}.$$

$$\log 96 = \log (2^5 \times 3) = 5 \log 2 + \log 3 \\ = 1.5050 + .4771 = 1.9821.$$

$$\therefore \log 9.6 = 0.9821.$$

$$\therefore \log \sqrt[5]{9.6} = \frac{0.9821}{5} = .1964.$$

$$16. \log (22.4)^{\frac{1}{8}} = \frac{1}{8} \log 22.4.$$

$$\log 224 = \log (2^5 \times 7) = 5 \log 2 + \log 7 = 1.5050 + .8451 = 2.3501.$$

$$\therefore \log 22.4 = 1.3501.$$

$$\therefore \log (22.4)^{\frac{1}{8}} = \frac{1.3501}{8} = .1688.$$

Art. 105.—Pages 65 and 66.

$$1. \log (9.238 \times .9152) = \log 9.238 + \log .9152.$$

$$\log 9.238 = 0.9656$$

$$\log .9152 = 9.9615 - 10$$

$$\frac{0.9271}{} = \log 8.454.$$

$$2. \log (130.36 \times .08237) = \log 130.36 + \log .08237.$$

$$\log 130.36 = 2.1151$$

$$\log .08237 = 8.9157 - 10$$

$$\frac{1.0308}{} = \log 10.73.$$

$$3. \log (721.3 \times 3.0528) = \log 721.3 + \log 3.0528.$$

$$\log 721.3 = 2.8581$$

$$\log 3.0528 = 0.4847$$

$$\frac{3.3428}{} = \log 2202.$$

Result, — 2202.

$$4. \log (4.3264 \times .050377) = \log 4.3264 + \log .050377.$$

$$\log 4.3264 = 0.6361$$

$$\log .050377 = 8.7022 - 10$$

$$\frac{9.3383 - 10}{} = \log .2179.$$

5. $\log (.27031 \times .042809) = \log .27031 + \log .042809.$
 $\log .27031 = 9.4319 - 10$
 $\log .042809 = 8.6315 - 10$
 $\hline 8.0634 - 10 = \log .01157.$
6. $\log (.063165 \times 11.134) = \log .063165 + \log 11.134.$
 $\log .063165 = 8.8005 - 10$
 $\log 11.134 = 1.0466$
 $\hline 9.8471 - 10 = \log .7032.$
 Result, $- .7032.$
7. $\log \frac{401.8}{52.37} = \log 401.8 - \log 52.37.$
 $\log 401.8 = 2.6040$
 $\log 52.37 = 1.7191$
 $\hline 0.8849 = \log 7.672.$
8. $\log \frac{7.2321}{10.813} = \log 7.2321 - \log 10.813.$
 $\log 7.2321 = 0.8592$
 $\log 10.813 = 1.0339$
 $\hline 9.8253 - 10 = \log .6688.$
9. $\log \frac{.3384}{.08659} = \log .3384 - \log .08659.$
 $\log .3384 = 9.5294 - 10$
 $\log .08659 = 8.9374 - 10$
 $\hline 0.5920 = \log 3.908.$
 Result, $- 3.908.$
10. $\log \frac{5.163}{.0051422} = \log 5.163 - \log .0051422.$
 $\log 5.163 = 0.9620$
 $\log .0051422 = 7.7112 - 10$
 $\hline 3.2508 = \log 1782.$
11. $\log \frac{22518}{64327} = \log 22518 - \log 64327.$
 $\log 22518 = 4.3525$
 $\log 64327 = 4.8084$
 $\hline 9.5441 - 10 = \log .3500.$

$$12. \log \frac{.007514}{.015822} = \log .007514 - \log .015822.$$

$$\log .007514 = 7.8758 - 10$$

$$\log .015822 = 8.1993 - 10$$

$$\underline{9.6765 - 10} = \log .4748.$$

Result, — .4748.

$$13. \log \frac{3.3681}{12.853 \times .6349}$$

$$= \log 3.3681 + \text{colog } 12.853 + \text{colog } .6349.$$

$$\log 3.3681 = 0.5274$$

$$\text{colog } 12.853 = 8.8910 - 10$$

$$\text{colog } .6349 = 0.1973$$

$$\underline{9.6157 - 10} = \log .4127.$$

$$14. \log \frac{15.008 \times .0843}{.06376 \times 4.248}$$

$$= \log 15.008 + \log .0843 + \text{colog } .06376 + \text{colog } 4.248.$$

$$\log 15.008 = 1.1763$$

$$\log .0843 = 8.9258 - 10$$

$$\text{colog } .06376 = 1.1955$$

$$\text{colog } 4.248 = 9.3718 - 10$$

$$\underline{0.6694} = \log 4.671.$$

Result, — 4.671.

$$15. \log \frac{2563 \times .03442}{714.8 \times .511}$$

$$= \log 2563 + \log .03442 + \text{colog } 714.8 + \text{colog } .511.$$

$$\log 2563 = 3.4087$$

$$\log .03442 = 8.5368 - 10$$

$$\text{colog } 714.8 = 7.1458 - 10$$

$$\text{colog } .511 = 0.2916$$

$$\underline{9.3829 - 10} = \log .2415.$$

$$16. \log \frac{121.6 \times 9.025}{48.3 \times 3662 \times .0856} = \log 121.6$$

$$+ \log 9.025 + \text{colog } 48.3 + \text{colog } 3662 + \text{colog } .0856.$$

$$\log 121.6 = 2.0850$$

$$\log 9.025 = 0.9554$$

$$\text{colog } 48.3 = 8.3161 - 10$$

$$\text{colog } 3662 = 6.4363 - 10$$

$$\text{colog } .0856 = 1.0675$$

$$\underline{8.8603 - 10} = \log .0725.$$

Result, — .0725.

17. $\log (23.86)^3 = 3 \times \log 23.86.$
 $\log 23.86 = 1.3777$
 $\frac{3}{4.1331} = \log 13587.$
18. $\log (.532)^8 = 8 \times \log .532.$
 $\log .532 = 9.7259 - 10$
 $\frac{8}{7.8072 - 10} = \log .006415.$
19. $\log (1.0246)^7 = 7 \times \log 1.0246.$
 $\log 1.0246 = 0.0105$
 $\frac{7}{0.0735} = \log 1.184.$
 Result, $-1.184.$
20. $\log (.09323)^5 = 5 \times \log .09323.$
 $\log .09323 = 8.9695 - 10$
 $\frac{5}{4.8475 - 10} = \log .000007038.$
21. $\log 5^{\frac{2}{3}} = \frac{2}{3} \log 5.$
 $\log 5 = 0.6990; \times \frac{2}{3} = 0.4660$
 $= \log 2.924.$
22. $\log (.8)^{\frac{2}{5}} = \frac{2}{5} \log .8.$
 $\log .8 = 9.9031 - 10$
 $\frac{2}{5)49.8062 - 50}$
 $9.9612 - 10 = \log .9146.$
23. $\log (3.16)^{\frac{4}{3}} = \frac{4}{3} \log 3.16.$
 $\log 3.16 = 0.4997; \times \frac{4}{3} = 0.6663$
 $= \log 4.638.$
24. $\log (.021)^{\frac{5}{2}} = \frac{5}{2} \log .021.$
 $\log .021 = 8.3222 - 10$
 $\frac{5}{2)11.6110 - 20}$
 $5.8055 - 10 = \log .0000639.$

$$25. \log \sqrt{2} = \frac{1}{2} \log 2.$$

$$\begin{aligned} \log 2 &= 0.3010; \div 2 = 0.1505 \\ &= \log 1.414. \end{aligned}$$

$$26. \log \sqrt[4]{5} = \frac{1}{4} \log 5.$$

$$\begin{aligned} \log 5 &= 0.6990; \div 4 = 0.1747 \\ &= \log 1.495. \end{aligned}$$

$$27. \log \sqrt[5]{3} = \frac{1}{5} \log 3.$$

$$\begin{aligned} \log 3 &= 0.4771; \div 5 = 0.0954 \\ &= \log 1.246. \\ \text{Result, } &- 1.246. \end{aligned}$$

$$28. \log \sqrt{.4294} = \frac{1}{2} \log .4294.$$

$$\begin{aligned} \log .4294 &= 19.6329 - 20; \div 2 = 9.8164 - 10 \\ &= \log .6553. \end{aligned}$$

$$29. \log \sqrt[3]{.02305} = \frac{1}{3} \log .02305.$$

$$\begin{aligned} \log .02305 &= 28.3626 - 30; \div 3 = 9.4542 - 10 \\ &= \log .2846. \end{aligned}$$

$$30. \log \sqrt[8]{1000} = \frac{1}{8} \log 1000.$$

$$\log 1000 = 3; \div 8 = 0.3750 = \log 2.372.$$

$$31. \log \sqrt[7]{.00951} = \frac{1}{7} \log .00951.$$

$$\begin{aligned} \log .00951 &= 67.9782 - 70; \div 7 = 9.7112 - 10 \\ &= \log .5142. \\ \text{Result, } &- .5142. \end{aligned}$$

$$32. \log \sqrt[5]{.0001011} = \frac{1}{5} \log .0001011.$$

$$\begin{aligned} \log .0001011 &= 46.0047 - 50; \div 5 = 9.2009 - 10 \\ &= \log .1588. \end{aligned}$$

$$35. \log (2^{\frac{3}{2}} \times 3^{\frac{2}{3}}) = \frac{3}{2} \log 2 + \frac{2}{3} \log 3.$$

$$\log 2 = .3010; \times \frac{3}{2} = .4515$$

$$\log 3 = .4771; \times \frac{2}{3} = .3181$$

$$\underline{\quad\quad\quad} \\ .7696 = \log 5.883.$$

36. $\log \frac{3^{\frac{5}{8}}}{4^{\frac{2}{3}}} = \frac{5}{8} \log 3 - \frac{2}{3} \log 4.$
 $\log 3 = .4771; \times \frac{5}{8} = .2982$
 $\log 4 = .6021; \times \frac{2}{3} = .4014$
 $\underline{9.8968 - 10 = \log .7885.}$
37. $\log \frac{5^{\frac{3}{7}}}{10^{\frac{2}{9}}} = \frac{3}{7} \log 5 - \frac{2}{9} \log 10.$
 $\log 5 = .6990; \times \frac{3}{7} = .2996$
 $\log 10 = 1; \times \frac{2}{9} = .2222$
 $\underline{.0774 = \log 1.195.}$
38. $\log \left(\frac{6}{7}\right)^{\frac{5}{2}} = \frac{5}{2} (\log 6 - \log 7).$
 $\log 6 = .7782$
 $\log 7 = .8451$
 $\underline{9.9331 - 10}$
 $\quad \quad \quad \underline{5}$
 $\quad \quad \quad 2)19.6655 - 20$
 $\quad \quad \quad \underline{9.8327 - 10 = \log .6803.}$
39. $\log \left(\frac{35}{113}\right)^{\frac{3}{8}} = \frac{3}{8} (\log 35 - \log 113).$
 $\log 35 = 1.5441$
 $\log 113 = 2.0531$
 $\underline{9.4910 - 10}$
 $\quad \quad \quad \underline{3}$
 $\quad \quad \quad 8)78.4730 - 80$
 $\quad \quad \quad \underline{9.8091 - 10 = \log .6443.}$
40. $\log \left(\frac{.08726}{.1321}\right)^{\frac{5}{3}} = \frac{5}{3} (\log .08726 - \log .1321).$
 $\log .08726 = 8.9408 - 10$
 $\log .1321 = 9.1209 - 10$
 $\underline{9.8199 - 10}$
 $\quad \quad \quad \underline{5}$
 $\quad \quad \quad 3)29.0995 - 30$
 $\quad \quad \quad \underline{9.6998 - 10 = \log .5010.}$

$$41. \log \sqrt[8]{\frac{21}{13}} = \frac{1}{8} (\log 21 - \log 13).$$

$$\log 21 = 1.3222$$

$$\log 13 = 1.1139$$

$$\begin{array}{r} 8 \overline{) .2083} \\ \underline{.0260} \end{array} = \log 1.062.$$

$$42. \log \sqrt[9]{\frac{3}{7}} = \frac{1}{9} (\log 3 - \log 7).$$

$$\log 3 = .4771$$

$$\log 7 = .8451$$

$$\begin{array}{r} 9 \overline{) 89.6320 - 90} \\ \underline{9.9591 - 10} \end{array} = \log .9102.$$

Result, — .9102.

$$43. \log \left(\sqrt[5]{\frac{2}{3}} \div \sqrt[3]{\frac{3}{5}} \right)$$

$$= \frac{1}{5} (\log 2 - \log 3) - \frac{1}{3} (\log 3 - \log 5).$$

$$\log 2 = .3010$$

$$\log 3 = .4771$$

$$\log 3 = .4771$$

$$\log 5 = .6990$$

$$\begin{array}{r} 5 \overline{) 49.8239 - 50} \\ \underline{9.9648 - 10} \\ 9.9260 - 10 \\ \underline{\hspace{1.5cm}} \\ .0388 \end{array} = \log 1.093.$$

$$\begin{array}{r} 3 \overline{) 29.7781 - 30} \\ \underline{9.9260 - 10} \end{array}$$

$$44. \log (\sqrt[8]{2} \times \sqrt[5]{3} \times \sqrt[7]{.01})$$

$$= \frac{1}{8} \log 2 + \frac{1}{5} \log 3 + \frac{1}{7} \log .01.$$

$$\log 2 = .3010; \quad \div 8 = .0376$$

$$\log 3 = .4771; \quad \div 5 = .0954$$

$$\log .01 = 68 - 70; \quad \div 7 = 9.7143 - 10$$

$$\underline{9.8473 - 10} = \log .7035.$$

$$45. \log \sqrt[5]{\frac{3258}{49309}} = \frac{1}{5} (\log 3258 - \log 49309).$$

$$\log 3258 = 3.5129$$

$$\log 49309 = 4.6929$$

$$\begin{array}{r} 5 \overline{) 48.8200 - 50} \\ \underline{9.7640 - 10} \end{array} = \log .5807.$$

$$46. \log \left(\frac{31.63}{429} \right)^{\frac{3}{17}} = \frac{3}{17} (\log 31.63 - \log 429).$$

$$\log 31.63 = 1.5001$$

$$\log 429 = 2.6325$$

$$\underline{8.8676 - 10}$$

3

$$17 \overline{) 166.6028 - 170}$$

$$9.8002 - 10 = \log .6313.$$

Result, $-.6313$.

$$47. \log \frac{100^{\frac{2}{3}}}{(.7325)^{\frac{3}{7}}} = \frac{2}{3} \log 100 - \frac{3}{7} \log .7325.$$

$$\log 100 = 2; \quad \times \frac{2}{3} = 1.3333$$

$$\log .7325 = 9.8648 - 10$$

3

$$\underline{69.5944 - 70} \div 7 = \frac{9.9421 - 10}{1.3912} = \log 24.62.$$

$$1.3912 = \log 24.62.$$

$$48. \log \frac{\sqrt[3]{.0001289}}{\sqrt[4]{.0008276}} = \frac{1}{3} \log .0001289 - \frac{1}{4} \log .0008276.$$

$$\log .0001289 = 26.1103 - 30; \div 3 = 8.7034 - 10$$

$$\log .0008276 = 36.9178 - 40; \div 4 = \frac{9.2294 - 10}{9.4740 - 10}$$

$$= \log .2979.$$

$$49. \log \frac{(.7469)^{\frac{5}{3}}}{(.2345)^{\frac{7}{2}}} = \frac{5}{3} \log .7469 - \frac{7}{2} \log .2345.$$

$$\log .7469 = 9.8732 - 10 \quad \log .2345 = 9.3701 - 10$$

5

$$3 \overline{) 29.3660 - 30}$$

$$9.7887 - 10$$

$$\underline{7.7953 - 10}$$

$$1.9934$$

$$= \log 98.50.$$

7

$$2 \overline{) 15.5907 - 20}$$

$$7.7953 - 10$$

$$50. \log \frac{\sqrt[11]{.0073}}{(.68291)^{\frac{5}{2}}} = \frac{1}{11} \log .0073 - \frac{5}{2} \log .68291.$$

$$\log .0073 = 107.8633 - 110 \quad \log .68291 = 9.8343 - 10$$

$$\text{Dividing by 11,} = 9.8058 - 10$$

$$\underline{9.5857 - 10}$$

$$.2201$$

$$= \log 1.660.$$

5

$$2 \overline{) 19.1715 - 20}$$

$$9.5857 - 10$$

$$\begin{aligned}
 51. \log \frac{\sqrt{5.955} \times \sqrt[3]{61.2}}{\sqrt[5]{298.54}} \\
 = \frac{1}{2} \log 5.955 + \frac{1}{3} \log 61.2 + \frac{1}{5} \operatorname{colog} 298.54. \\
 \log 5.955 = 0.7748 \quad ; \div 2 = 0.3874 \\
 \log 61.2 = 1.7868 \quad ; \div 3 = 0.5956 \\
 \operatorname{colog} 298.54 = 47.5250 - 50; \div 5 = \frac{9.5050 - 10}{0.4880} = \log 3.076.
 \end{aligned}$$

$$\begin{aligned}
 52. \log (538.2 \times .0005969)^{\frac{1}{8}} = \frac{1}{8} (\log 538.2 + \log .0005969). \\
 \log 538.2 = 2.7310 \\
 \log .0005969 = 6.7759 - 10 \\
 \hline
 8) 79.5069 - 80 \\
 9.9384 - 10 = \log .8678.
 \end{aligned}$$

$$\begin{aligned}
 53. \log [(18.9503)^{11} \times (.1)^{14}] \\
 = 11 \times \log 18.9503 + 14 \times \log .1. \\
 \log 18.9503 = 1.2777; \times 11 = 14.0547 \\
 \log .1 = 9 - 10; \times 14 = 6. \quad - 20 \\
 \hline
 .0547 = \log 1.134.
 \end{aligned}$$

$$\begin{aligned}
 54. \log \sqrt[6]{3734.9 \times .00001108} = \frac{1}{6} (\log 3734.9 + \log .00001108). \\
 \log 3734.9 = 3.5723 \\
 \log .00001108 = 5.0445 - 10 \\
 \hline
 6) 58.6168 - 60 \\
 9.7695 - 10 = \log .5881.
 \end{aligned}$$

$$\begin{aligned}
 55. \log [(2.6317)^{\frac{3}{4}} \times (.71272)^{\frac{2}{5}}] \\
 = \frac{3}{4} \log 2.6317 + \frac{2}{5} \log .71272. \\
 \log 2.6317 = .4203 \quad \log .71272 = 9.8529 - 10 \\
 \hline
 \begin{array}{r}
 3 \\
 4) 1.2609 \\
 .3152 \\
 \hline
 9.9412 - 10 \\
 .2564
 \end{array}
 \qquad
 \begin{array}{r}
 2 \\
 5) 49.7058 - 50 \\
 9.9412 - 10 \\
 \hline
 9.9412 - 10 \\
 .2564
 \end{array}
 \qquad
 = \log 1.805.
 \end{aligned}$$

$$56. \log \frac{\sqrt[3]{.008193} \times (.06285)^{\frac{3}{2}}}{.98342}$$

$$= \frac{1}{3} \log .008193 + \frac{3}{2} \log .06285 + \text{colog} .98342.$$

$$\log .008193 = 27.9134 - 30; \div 3 = 9.3045 - 10$$

$$\log .06285 = 8.7983 - 10$$

$$\frac{3}{2}$$

$$\frac{16.3949 - 20; \div 2 = 8.1974 - 10}{\text{colog} .98342 = 0.0072}$$

$$\text{colog} .98342 = 0.0072$$

$$\frac{7.5091 - 10}{= \log .003229.}$$

$$= \log .003229.$$

$$57. \log (\sqrt{.035} \times \sqrt[6]{.62667} \times \sqrt[3]{.0072103})$$

$$= \frac{1}{2} \log .035 + \frac{1}{6} \log .62667 + \frac{1}{3} \log .0072103.$$

$$\log .035 = 18.5441 - 20; \div 2 = 9.2720 - 10$$

$$\log .62667 = 59.7971 - 60; \div 6 = 9.9662 - 10$$

$$\log .0072103 = 27.8579 - 30; \div 3 = 9.2860 - 10$$

$$\frac{8.5242 - 10}{= \log .03344.}$$

$$= \log .03344.$$

CHAPTER VII.

Art. 110.—Pages 69 to 72.

1. $a = c \sin A.$ $b = c \cos A.$
 $\log c = 1.0492$ $\log c = 1.0492$
 $\log \sin A = \underline{9.8378}$ $\log \cos A = \underline{9.8606}$
 $\log a = \underline{0.8870}$ $\log b = \underline{0.9098}$
 $\therefore a = 7.708.$ $\therefore b = 8.124.$
2. $b = a \tan B.$ $c = \frac{a}{\cos B}.$
 $\log a = 2.8629$ $\log a = 2.8629$
 $\log \tan B = \underline{0.4121}$ $\log \cos B = \underline{9.5576}$
 $\log b = \underline{3.2750}$ $\log c = \underline{3.3053}$
 $\therefore b = 1883.$ $\therefore c = 2019.5.$
3. $a = \frac{b}{\tan B}.$ $c = \frac{b}{\sin B}.$
 $\log b = 1.6785$ $\log b = 1.6785$
 $\log \tan B = \underline{0.2916}$ $\log \sin B = \underline{9.9496}$
 $\log a = \underline{1.3869}$ $\log c = \underline{1.7289}$
 $\therefore a = 24.37.$ $\therefore c = 53.56.$
4. $\sin A = \frac{a}{c}.$ $b = \frac{a}{\tan A}.$
 $\log a = 9.7952$ $\log a = 9.7952$
 $\log c = \underline{9.9590}$ $\log \tan A = \underline{9.9742}$
 $\log \sin A = \underline{9.8362}$ $\log b = \underline{9.8210}$
 $\therefore A = 43^\circ 17.9'.$ $\therefore b = .6622.$
5. $\tan A = \frac{a}{b}.$ $c = \frac{a}{\sin A}.$
 $\log a = 0.6990$ $\log a = 0.6990$
 $\log b = \underline{0.3010}$ $\log \sin A = \underline{9.9678}$
 $\log \tan A = \underline{0.3980}$ $\log c = \underline{0.7312}$
 $\therefore A = 68^\circ 12.2'.$ $\therefore c = 5.385.$

6. $b = \frac{a}{\tan A}$ $c = \frac{a}{\sin A}$
 $\log a = 1.9212$ $\log a = 1.9212$
 $\log \tan A = 0.4912$ $\log \sin A = 9.9785$

 $\log b = 1.4300$ $\log c = 1.9427$
 $\therefore b = 26.91.$ $\therefore c = 87.64.$
7. $a = c \cos B$ $b = c \sin B.$
 $\log c = 8.4359$ $\log c = 8.4359$
 $\log \cos B = 9.9276$ $\log \sin B = 9.7262$

 $\log a = 8.3635$ $\log b = 8.1621$
 $\therefore a = .02309.$ $\therefore b = .01452.$
8. $\cos A = \frac{b}{c}$ $a = b \tan A.$
 $\log b = 0.4604$ $\log b = 0.4604$
 $\log c = 0.7084$ $\log \tan A = 0.1645$

 $\log \cos A = 9.7520$ $\log a = 0.6249$
 $\therefore A = 55^\circ 36.1'.$ $\therefore a = 4.216.$
9. $a = b \tan A.$ $c = \frac{b}{\cos A}$
 $\log b = 3.6281$ $\log b = 3.6281$
 $\log \tan A = 0.1179$ $\log \cos A = 9.7826$

 $\log a = 3.7460$ $\log c = 3.8455$
 $\therefore a = 5571.$ $\therefore c = 7007.$
10. $\tan A = \frac{a}{b}$ $c = \frac{a}{\sin A}$
 $\log a = 2.0043$ $\log a = 2.0043$
 $\log b = 2.0645$ $\log \sin A = 9.8173$

 $\log \tan A = 9.9398$ $\log c = 2.1870$
 $\therefore A = 41^\circ 2.4'.$ $\therefore c = 153.8.$
11. $b = \frac{a}{\tan A}$ $c = \frac{a}{\sin A}$
 $\log a = 2.1995$ $\log a = 2.1995$
 $\log \tan A = 9.9752$ $\log \sin A = 9.8368$

 $\log b = 2.2243$ $\log c = 2.3627$
 $\therefore b = 167.6.$ $\therefore c = 230.5.$

12. $a = c \sin A.$ $b = c \cos A.$
 $\log c = 1.5531$ $\log c = 1.5531$
 $\log \sin A = \underline{9.9314}$ $\log \cos A = \underline{9.7162}$
 $\log a = 1.4845$ $\log b = 1.2693$
 $\therefore a = 30.51.$ $\therefore b = 18.59.$
13. $\sin A = \frac{a}{c}.$ $b = \frac{a}{\tan A}.$
 $\log a = 2.3100$ $\log a = 2.3100$
 $\log c = \underline{2.4398}$ $\log \tan A = \underline{0.0436}$
 $\log \sin A = 9.8702$ $\log b = 2.2664$
 $\therefore A = 47^\circ 52.5'.$ $\therefore b = 184.7.$
14. $a = \frac{b}{\tan B}.$ $c = \frac{b}{\sin B}.$
 $\log b = 0.2158$ $\log b = 0.2158$
 $\log \tan B = \underline{9.7614}$ $\log \sin B = \underline{9.6990}$
 $\log a = 0.4544$ $\log c = 0.5168$
 $\therefore a = 2.847.$ $\therefore c = 3.287.$
15. $a = b \tan A.$ $c = \frac{b}{\cos A}.$
 $\log b = 1.1220$ $\log b = 1.1220$
 $\log \tan A = \underline{9.6114}$ $\log \cos A = \underline{9.9665}$
 $\log a = 0.7334$ $\log c = 1.1555$
 $\therefore a = 5.4125.$ $\therefore c = 14.306.$
16. $a = c \cos B.$ $b = c \sin B.$
 $\log c = 9.8611$ $\log c = 9.8611$
 $\log \cos B = \underline{9.9922}$ $\log \sin B = \underline{9.2747}$
 $\log a = \underline{9.8533}$ $\log b = 9.1358$
 $\therefore a = .7133.$ $\therefore b = .1367.$
17. $\tan A = \frac{a}{b}.$ $c = \frac{a}{\sin A}.$
 $\log a = 2.8051$ $\log a = 2.8051$
 $\log b = \underline{2.7000}$ $\log \sin A = \underline{9.8957}$
 $\log \tan A = 0.1051$ $\log c = 2.9094$
 $\therefore A = 51^\circ 51.9'.$ $\therefore c = 811.7.$

18. $b = \frac{a}{\tan A}$ $c = \frac{a}{\sin A}$
 $\log a = 2.3092$ $\log a = 2.3092$
 $\log \tan A = 0.6832$ $\log \sin A = 9.9908$
 $\log b = 1.6260$ $\log c = 2.3184$
 $\therefore b = 42.27.$ $\therefore c = 208.15.$
19. $\cos A = \frac{b}{c}$ $a = b \tan A.$
 $\log b = 8.3974$ $\log b = 8.3974$
 $\log c = 8.6805$ $\log \tan A = 0.2143$
 $\log \cos A = 9.7169$ $\log a = 8.6117$
 $\therefore A = 58^\circ 35.7'.$ $\therefore a = .0409$
20. $b = a \tan B.$ $c = \frac{a}{\cos B}$
 $\log a = 3.2731$ $\log a = 3.2731$
 $\log \tan B = 8.6085$ $\log \cos B = 9.9996$
 $\log b = 1.8816$ $\log c = 3.2735$
 $\therefore b = 76.13.$ $\therefore c = 1877.$
21. $\tan A = \frac{a}{b}$ $c = \frac{a}{\sin A}$
 $\log a = 1.3922$ $\log a = 1.3922$
 $\log b = 1.5188$ $\log \sin A = 9.7771$
 $\log \tan A = 9.8734$ $\log c = 1.6151$
 $\therefore A = 36^\circ 45.9'.$ $\therefore c = 41.22.$
22. $\cos A = \frac{b}{c}$ $a = b \tan A.$
 $\log b = 0.1574$ $\log b = 0.1574$
 $\log c = 0.5397$ $\log \tan A = 0.3413$
 $\log \cos A = 9.6177$ $\log a = 0.4987$
 $\therefore A = 65^\circ 30'.$ $\therefore a = 3.153.$
23. $a = c \cos B.$ $b = c \sin B.$
 $\log c = 4.5706$ $\log c = 4.5706$
 $\log \cos B = 9.9975$ $\log \sin B = 9.0337$
 $\log a = 4.5681$ $\log b = 3.6043$
 $\therefore a = 36992.$ $\therefore b = 4021.$

24. $a = b \tan A.$ $c = \frac{b}{\cos A}.$
 $\log b = 2.3011$ $\log b = 2.3011$
 $\log \tan A = 0.3122$ $\log \cos A = 9.6415$
 $\log a = 2.6133$ $\log c = 2.6596$
 $\therefore a = 410.5.$ $\therefore c = 456.7.$
25. $\tan A = \frac{a}{b}.$ $c = \frac{a}{\sin A}.$
 $\log a = 2.5316$ $\log a = 2.5316$
 $\log b = 2.3649$ $\log \sin A = 9.9172$
 $\log \tan A = 0.1667$ $\log c = 2.6144$
 $\therefore A = 55^\circ 44.1'.$ $\therefore c = 411.5.$
26. $\sin A = \frac{a}{c}.$ $b = \frac{a}{\tan A}.$
 $\log a = 0.2327$ $\log a = 0.2327$
 $\log c = 0.3012$ $\log \tan A = 0.2155$
 $\log \sin A = 9.9315$ $\log b = 0.0172$
 $\therefore A = 58^\circ 40'.$ $\therefore b = 1.0405.$
27. $b = a \tan B.$ $c = \frac{a}{\cos B}.$
 $\log a = 9.9144$ $\log a = 9.9144$
 $\log \tan B = 9.5968$ $\log \cos B = 9.9685$
 $\log b = 9.5112$ $\log c = 9.9459$
 $\therefore b = .3245.$ $\therefore c = .8828.$
28. $a = c \sin A.$ $b = c \cos A$
 $\log c = 2.4403$ $\log c = 2.4403$
 $\log \sin A = 9.9828$ $\log \cos A = 9.4402$
 $\log a = 2.4231$ $\log b = 1.8805$
 $\therefore a = 264.9.$ $\therefore b = 75.95.$
29. $a = \frac{b}{\tan B}.$ $c = \frac{b}{\sin B}.$
 $\log b = 2.0800$ $\log b = 2.0800$
 $\log \tan B = 9.8329$ $\log \sin B = 9.7503$
 $\log a = 2.2471$ $\log c = 2.3297$
 $\therefore a = 176.64.$ $\therefore c = 213.65.$

$$\begin{array}{ll}
 30. \quad \tan A = \frac{a}{b} & c = \frac{a}{\sin A} \\
 \log a = 1.0046 & \log a = 1.0046 \\
 \log b = 1.2381 & \log \sin A = 9.7027 \\
 \log \tan A = 9.7665 & \log c = 1.3019 \\
 \therefore A = 30^\circ 17.2' & \therefore c = 20.04.
 \end{array}$$

$$\begin{array}{ll}
 31. \quad c = 2b \cos A. & 34. \quad \cos A = \frac{\frac{1}{2}c}{a} \\
 \log 2b = 1.8462 & \log \frac{1}{2}c = 1.7268 \\
 \log \cos A = 9.5553 & \log a = 1.8989 \\
 \log c = 1.4015 & \log \cos A = 9.8279 \\
 \therefore c = 25.206. & \therefore A = 47^\circ 42.9'.
 \end{array}$$

$$\begin{array}{ll}
 32. \quad a = \frac{\frac{1}{2}c}{\cos B} & 35. \quad a = \frac{\frac{1}{2}c}{\cos A} \\
 \log \frac{1}{2}c = 0.1886 & \log \frac{1}{2}c = 9.4489 \\
 \log \cos B = 9.9493 & \log \cos A = 9.5274 \\
 \log a = 0.2393 & \log a = 9.9215 \\
 \therefore a = 1.735. & \therefore a = .8346.
 \end{array}$$

$$\begin{array}{ll}
 33. \quad c = 2b \sin \frac{1}{2}C. & 36. \quad a = \frac{\frac{1}{2}c}{\sin \frac{1}{2}C} \\
 \log 2b = 3.6239 & \log \frac{1}{2}c = 1.6788 \\
 \log \sin \frac{1}{2}C = 9.8116 & \log \sin \frac{1}{2}C = 9.9864 \\
 \log c = 3.4355 & \log a = 1.6924 \\
 \therefore c = 2725.6. & \therefore a = 49.25.
 \end{array}$$

37. Let O be the centre, and AB any side of the pentagon. Join OA , and draw OC perpendicular to AB .

Then $AB = 2 OA \sin AOC = 24 \sin 36^\circ$.

$$\begin{array}{l}
 \log 24 = 1.3802 \\
 \log \sin 36^\circ = 9.7692 \\
 \log AB = 1.1494 \\
 \therefore AB = 14.106.
 \end{array}$$

38. Let A be the point of observation, B the top of the tower, and C its base.

Then $BC = AC \tan A = 100 \tan 38^\circ$.

$$\begin{array}{l}
 \log 100 = 2.0000 \\
 \log \tan 38^\circ = 9.8928 \\
 \log BC = 1.8928 \\
 \therefore BC = 78.12.
 \end{array}$$

39. Let B be the top and C the base of the tower, and A the extremity of its shadow.

$$\begin{aligned} \text{Then} \quad \tan A &= \frac{BC}{AC} = \frac{103.7}{167.3} \\ \log 103.7 &= 2.0157 \\ \log 167.3 &= \underline{2.2235} \\ \log \tan A &= 9.7922 \\ \therefore A &= 31^\circ 47.1'. \end{aligned}$$

40. Let AB be the chord, and O the centre of the circle. Join OA and OB , and draw OC perpendicular to AB .

$$\begin{aligned} \text{Then} \quad \sin AOC &= \frac{AC}{OA} = \frac{513.5}{1634} \\ \log 513.5 &= 2.7105 \\ \log 1634 &= \underline{3.2132} \\ \log \sin AOC &= 9.4973 \\ \therefore AOC &= 18^\circ 18.95'. \\ \therefore AOB &= 36^\circ 37.9'. \end{aligned}$$

41. Let A be the top of the mountain, B the remotest point visible, and O the centre of the earth.

Then in the right triangle OAB , $OA = 3956 + 1\frac{1}{4} = 3957.25$, and $OB = 3956$.

$$\begin{aligned} \text{Hence} \quad AB &= \sqrt{OA^2 - OB^2} \\ &= \sqrt{(OA + OB)(OA - OB)} \\ &= \sqrt{7913.25 \times 1.25}. \\ \log 7913.25 &= 3.8984 \\ \log 1.25 &= \underline{0.0969} \\ &2) \underline{3.9953} \\ \log AB &= 1.9976 \\ \therefore AB &= 99.45. \end{aligned}$$

42. Let AB and BC be consecutive sides of the pentagon. Join AC , and draw BD perpendicular to AC .

$$\begin{aligned} \text{Then} \quad AC &= 2 AB \cos BAC = 14.056 \cos 36^\circ. \\ \log 14.056 &= 1.1478 \\ \log \cos 36^\circ &= \underline{9.9080} \\ \log AC &= 1.0558 \\ \therefore AC &= 11.371. \end{aligned}$$

43. Let A denote the angle of elevation.

$$\begin{aligned} \text{Then} \quad \tan A &= \frac{238}{660} \\ \log 238 &= 2.3766 \\ \log 660 &= 2.8195 \\ \hline \log \tan A &= 9.5571 \\ \therefore A &= 19^\circ 50'. \end{aligned}$$

44. Let A be the position of the buoy, B the top of the light-house, and C its base.

$$\begin{aligned} \text{Then} \quad AC &= BC \cot BAC = 133 \cot 18^\circ 25'. \\ \log 133 &= 2.1239 \\ \log \cot 18^\circ 25' &= 0.4776 \\ \hline \log AC &= 2.6015 \\ \therefore AC &= 399.5. \end{aligned}$$

45. Let H be the position of the headland, S the first point of observation, and S' the second.

Then in the right triangle HSS' , $SS' = 16.38$, and $\angle SHS' = 33^\circ$.

$$\text{Hence} \quad HS = 16.38 \cot 33^\circ,$$

$$\text{and} \quad HS' = \frac{16.38}{\sin 33^\circ}.$$

$$\begin{aligned} \log 16.38 &= 1.2143 \\ \log \cot 33^\circ &= 0.1875 \\ \hline \log HS &= 1.4018 \\ \therefore HS &= 25.22. \end{aligned}$$

$$\begin{aligned} \log 16.38 &= 1.2143 \\ \log \sin 33^\circ &= 9.7361 \\ \hline \log HS' &= 1.4782 \\ \therefore HS' &= 30.07. \end{aligned}$$

46. Let AB be the chord, and O the centre of the circle. Join OA , and draw OC perpendicular to AB .

$$\begin{aligned} \text{Then} \quad OA &= \frac{AC}{\sin AOC} = \frac{20.68}{\sin 72^\circ 48.5'}. \\ \log 20.68 &= 1.3156 \\ \log \sin 72^\circ 48.5' &= 9.9801 \\ \hline \log OA &= 1.3355 \\ \therefore OA &= 21.65. \end{aligned}$$

47. Let O be the centre, and AB any side of the octagon. Join OA , and draw OC perpendicular to AB .

Then $OC = AC \cot AOC = 6 \cot 22^\circ 30'$,

and $OA = \frac{AC}{\sin AOC} = \frac{6}{\sin 22^\circ 30'}$.

$$\log 6 = 0.7782$$

$$\log \cot 22^\circ 30' = 0.3828$$

$$\log OC = 1.1610$$

$$\therefore OC = 14.487.$$

$$\log 6 = 0.7782$$

$$\log \sin 22^\circ 30' = 9.5828$$

$$\log OA = 1.1954$$

$$\therefore OA = 15.682.$$

48. Let A be the position of the observer, B the top of the pole, and C its foot.

Then $AC = BC \cot BAC = 80 \cot 10^\circ$.

$$\log 80 = 1.9031$$

$$\log \cot 10^\circ = 0.7537$$

$$\log AC = 2.6568$$

$$\therefore AC = 453.7.$$

49. Let O be the centre, and AB any diagonal of the pentagon. Join OA , and draw OC perpendicular to AB .

Then $OA = \frac{AC}{\sin AOC} = \frac{16.415}{\sin 72^\circ}$.

$$\log 16.415 = 1.2152$$

$$\log \sin 72^\circ = 9.9782$$

$$\log OA = 1.2370$$

$$\therefore OA = 17.26.$$

50. Let B be the top, and C the foot of the tower, and A the extremity of the base line.

Then $BC = AC \tan BAC = 1000 \tan 21^\circ 16' 37''$.

$$\log 1000 = 3.0000$$

$$\log \tan 21^\circ 16' 37'' = 9.5904$$

$$\log BC = 2.5904$$

$$\therefore BC = 389.4.$$

51. Let AB be the chord, and O the centre of the circle. Join OA , and draw OC perpendicular to AB .

Then $AB = 2 OA \sin AOC = 1446.58 \sin 17^\circ 36.5'$.

$$\log 1446.58 = 3.1604$$

$$\log \sin 17^\circ 36.5' = 9.4807$$

$$\log AB = \underline{2.6411}$$

$$\therefore AB = 437.6.$$

52. Let O be the centre, and AB any side of the hexagon. Join OA , and draw OC perpendicular to AB .

Then $AB = 2 OC \tan AOC = 10 \tan 30^\circ$.

$$\log 10 = 1.0000$$

$$\log \tan 30^\circ = 9.7614$$

$$\log AB = \underline{0.7614}$$

$$\therefore AB = 5.773.$$

53. Let A be the position of the first boat, and B of the second; let C be the top of the light-house, and D its foot.

Then $AD = CD \cot CAD = 200 \cot 14^\circ$,

and $BD = CD \cot CBD = 200 \cot 32^\circ$.

$$\log 200 = 2.3010$$

$$\log \cot 14^\circ = 0.6032$$

$$\log AD = 2.9042$$

$$\therefore AD = 802.$$

$$\log 200 = 2.3010$$

$$\log \cot 32^\circ = 0.2042$$

$$\log BD = 2.5052$$

$$\therefore BD = 320.1.$$

$$\therefore AB = AD - BD = 481.9.$$

54. Let A be the position of the light-house, and B , C , and D , the positions of the ship at 7 A.M., 7.30 A.M., and 10 A.M., respectively.

Then $BC = AB \tan BAC = 10.32 \tan 18^\circ 13'$.

$$\log 10.32 = 1.0136$$

$$\log \tan 18^\circ 13' = 9.5174$$

$$\log BC = 0.5310$$

$$\therefore BC = 3.396.$$

Therefore the rate of the ship is 2×3.396 , or 6.792 miles an hour.

Again, $\tan BAD = \frac{BD}{AB} = \frac{20.376}{10.32}$.

$$\log 20.376 = 1.3091$$

$$\log 10.32 = \frac{1.0136}{}$$

$$\log \tan BAD = 0.2955$$

$$\therefore BAD = 63^\circ 8.4'$$

Therefore the bearing of the light-house at 10 A.M. is $63^\circ 8.4'$ west of north.

Art. 112. — Page 74.

2. $2K = a^2 \cot A$.

$$2 \log a = 2.6916$$

$$\log \cot A = 0.4485$$

$$\log 2K = 3.1401$$

$$2K = 1380.6$$

$$\therefore K = 690.3.$$

6. $4K = c^2 \sin 2A$.

$$2 \log c = 4.5708$$

$$\log \sin 2A = 9.9455$$

$$\log 4K = 4.5163$$

$$4K = 32831$$

$$\therefore K = 8208.$$

3. $2K = a^2 \tan B$.

$$2 \log a = 9.8290$$

$$\log \tan B = 9.6510$$

$$\log 2K = 9.4800$$

$$2K = .302$$

$$\therefore K = .151.$$

7. $2K = b^2 \tan A$.

$$2 \log b = 7.4332$$

$$\log \tan A = 0.2190$$

$$\log 2K = 7.6522$$

$$2K = .00449$$

$$\therefore K = .002245.$$

4. $2K = ab$.

$$\log a = 2.1741$$

$$\log b = 1.8824$$

$$\log 2K = 4.0565$$

$$2K = 11389$$

$$\therefore K = 5695.$$

8. $2K = a\sqrt{(c+a)(c-a)}$.

$$\log a = 9.9694$$

$$\frac{1}{2} \log (c+a) = 0.2851$$

$$\frac{1}{2} \log (c-a) = 0.1341$$

$$\log 2K = 0.3886$$

$$2K = 2.447$$

$$\therefore K = 1.223.$$

5. $2K = b\sqrt{(c+b)(c-b)}$.

$$\log b = 9.4851$$

$$\frac{1}{2} \log (c+b) = 9.9924$$

$$\frac{1}{2} \log (c-b) = 9.7748$$

$$\log 2K = 9.2523$$

$$2K = .17876$$

$$\therefore K = .08938.$$

9. $4K = c^2 \sin 2B$.

$$2 \log c = 2.8718$$

$$\log \sin 2B = 9.7604$$

$$\log 4K = 2.6322$$

$$4K = 428.7$$

$$\therefore K = 107.2.$$

10. $2K = b^2 \cot B$.

$$2 \log b = 9.0574$$

$$\log \cot B = 0.2508$$

$$\log 2K = 9.3082$$

$$2K = .2033$$

$$\therefore K = .1017.$$

CHAPTER IX.

Art. 121.—Page 84.

2. $C = 180^\circ - 115^\circ 10' = 64^\circ 50'$.

$$\begin{aligned}
 b &= a \sin B \csc A. \\
 \log a &= 1.0000 \\
 \log \sin B &= 9.9890 \\
 \log \csc A &= 0.2107 \\
 \hline
 \log b &= 1.1997 \\
 \therefore b &= 15.837.
 \end{aligned}$$

$$\begin{aligned}
 c &= a \sin C \csc A. \\
 \log a &= 1.0000 \\
 \log \sin C &= 9.9567 \\
 \log \csc A &= 0.2107 \\
 \hline
 \log c &= 1.1674 \\
 \therefore c &= 14.703.
 \end{aligned}$$

3. $A = 180^\circ - 154^\circ = 26^\circ$.

$$\begin{aligned}
 a &= b \sin A \csc B. \\
 \log b &= 9.9051 \\
 \log \sin A &= 9.6418 \\
 \log \csc B &= 0.1015 \\
 \hline
 \log a &= 9.6484 \\
 \therefore a &= .445.
 \end{aligned}$$

$$\begin{aligned}
 c &= b \sin C \csc B. \\
 \log b &= 9.9051 \\
 \log \sin C &= 9.9909 \\
 \log \csc B &= 0.1015 \\
 \hline
 \log c &= 9.9975 \\
 \therefore c &= .9942.
 \end{aligned}$$

4. $C = 180^\circ - 80^\circ 35' = 99^\circ 25'$.

$$\begin{aligned}
 a &= c \sin A \csc C. \\
 \log c &= 8.5051 \\
 \log \sin A &= 9.7706 \\
 \log \csc C &= 0.0059 \\
 \hline
 \log a &= 8.2816 \\
 \therefore a &= .01913.
 \end{aligned}$$

$$\begin{aligned}
 b &= c \sin B \csc C. \\
 \log c &= 8.5051 \\
 \log \sin B &= 9.8453 \\
 \log \csc C &= 0.0059 \\
 \hline
 \log b &= 8.3563 \\
 \therefore b &= .02272.
 \end{aligned}$$

5. $B = 180^\circ - 120^\circ 55' = 59^\circ 5'$.

$$\begin{aligned}
 a &= b \sin A \csc B. \\
 \log b &= 1.4625 \\
 \log \sin A &= 9.9996 \\
 \log \csc B &= 0.0666 \\
 \hline
 \log a &= 1.5287 \\
 \therefore a &= 33.78.
 \end{aligned}$$

$$\begin{aligned}
 c &= b \sin C \csc B. \\
 \log b &= 1.4625 \\
 \log \sin C &= 9.7390 \\
 \log \csc B &= 0.0666 \\
 \hline
 \log c &= 1.2681 \\
 \therefore c &= 18.54.
 \end{aligned}$$

6. $A = 180^\circ - 139^\circ 23' = 40^\circ 37'.$

$$\begin{aligned}
 b &= a \sin B \csc A. \\
 \log a &= 0.7340 \\
 \log \sin B &= 9.9954 \\
 \log \csc A &= \underline{0.1865} \\
 \log b &= 0.9159 \\
 \therefore b &= 8.24.
 \end{aligned}$$

$$\begin{aligned}
 c &= a \sin C \csc A. \\
 \log a &= 0.7340 \\
 \log \sin C &= 9.8170 \\
 \log \csc A &= \underline{0.1865} \\
 \log c &= 0.7375 \\
 \therefore c &= 5.464.
 \end{aligned}$$

7. $B = 180^\circ - 158^\circ 54' = 21^\circ 6'.$

$$\begin{aligned}
 a &= c \sin A \csc C. \\
 \log c &= 8.2068 \\
 \log \sin A &= 9.7613 \\
 \log \csc C &= \underline{0.0796} \\
 \log a &= 8.0477 \\
 \therefore a &= .011162.
 \end{aligned}$$

$$\begin{aligned}
 b &= c \sin B \csc C. \\
 \log c &= 8.2068 \\
 \log \sin B &= 9.5563 \\
 \log \csc C &= \underline{0.0796} \\
 \log b &= 7.8427 \\
 \therefore b &= .006962.
 \end{aligned}$$

8. $B = 180^\circ - 114^\circ 28' = 65^\circ 32'.$

$$\begin{aligned}
 b &= a \sin B \csc A. \\
 \log a &= 2.6021 \\
 \log \sin B &= 9.9591 \\
 \log \csc A &= \underline{0.0895} \\
 \log b &= 2.6507 \\
 \therefore b &= 447.4.
 \end{aligned}$$

$$\begin{aligned}
 c &= a \sin C \csc A. \\
 \log a &= 2.6021 \\
 \log \sin C &= 9.9375 \\
 \log \csc A &= \underline{0.0895} \\
 \log c &= 2.6291 \\
 \therefore c &= 425.7.
 \end{aligned}$$

9. $C = 180^\circ - 125^\circ 13' = 54^\circ 47'.$

$$\begin{aligned}
 a &= b \sin A \csc B. \\
 \log b &= 2.4973 \\
 \log \sin A &= 9.9652 \\
 \log \csc B &= \underline{0.0723} \\
 \log a &= 2.5348 \\
 \therefore a &= 342.6.
 \end{aligned}$$

$$\begin{aligned}
 c &= b \sin C \csc B. \\
 \log b &= 2.4973 \\
 \log \sin C &= 9.9122 \\
 \log \csc B &= \underline{0.0723} \\
 \log c &= 2.4818 \\
 \therefore c &= 303.3.
 \end{aligned}$$

10. $A = 180^\circ - 75^\circ 28' 18'' = 104^\circ 31' 42''.$

$$\begin{aligned}
 a &= c \sin A \csc C. \\
 \log c &= 0.8954 \\
 \log \sin A &= 9.9858 \\
 \log \csc C &= \underline{0.1628} \\
 \log a &= 1.0440 \\
 \therefore a &= 11.067.
 \end{aligned}$$

$$\begin{aligned}
 b &= c \sin B \csc C. \\
 \log c &= 0.8954 \\
 \log \sin B &= 9.7248 \\
 \log \csc C &= \underline{0.1628} \\
 \log b &= 0.7830 \\
 \therefore b &= 6.067.
 \end{aligned}$$

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$$2. \tan \frac{1}{2}(A - C) = \frac{a - c}{a + c} \tan \frac{1}{2}(A + C). \quad b = a \sin B \csc A.$$

$$\begin{array}{lll} a - c = 12 & \log = 1.0792 & \log a = 1.4314 \\ a + c = 42 & \text{colog} = 8.3768 & \log \sin B = 9.8569 \\ \frac{1}{2}(A + C) = 67^\circ & \log \tan = 0.3721 & \log \csc A = 0.0080 \end{array}$$

$$\log \tan \frac{1}{2}(A - C) = 9.8281 \quad \log b = 1.2963$$

$$\therefore \frac{1}{2}(A - C) = 33^\circ 56.7'. \quad \therefore b = 19.78.$$

$$\therefore A = \frac{1}{2}(A + C) + \frac{1}{2}(A - C) = 100^\circ 56.7',$$

$$\text{and } C = \frac{1}{2}(A + C) - \frac{1}{2}(A - C) = 33^\circ 3.3'.$$

$$3. \tan \frac{1}{2}(A - B) = \frac{a - b}{a + b} \tan \frac{1}{2}(A + B). \quad c = a \sin C \csc A.$$

$$\begin{array}{lll} a - b = 139 & \log = 2.1430 & \log a = 2.6866 \\ a + b = 833 & \text{colog} = 7.0794 & \log \sin C = 9.8941 \\ \frac{1}{2}(A + B) = 64^\circ 12' & \log \tan = 0.3157 & \log \csc A = 0.0030 \end{array}$$

$$\log \tan \frac{1}{2}(A - B) = 9.5381 \quad \log c = 2.5837$$

$$\therefore \frac{1}{2}(A - B) = 19^\circ 2.7'. \quad \therefore c = 383.5.$$

$$\therefore A = \frac{1}{2}(A + B) + \frac{1}{2}(A - B) = 83^\circ 14.7',$$

$$\text{and } B = \frac{1}{2}(A + B) - \frac{1}{2}(A - B) = 45^\circ 9.3'.$$

$$4. \tan \frac{1}{2}(C - B) = \frac{c - b}{c + b} \tan \frac{1}{2}(C + B). \quad a = b \sin A \csc B.$$

$$\begin{array}{lll} c - b = 1.265 & \log = 0.1021 & \log b = 0.3621 \\ c + b = 5.869 & \text{colog} = 9.2315 & \log \sin A = 9.9459 \\ \frac{1}{2}(C + B) = 59^\circ & \log \tan = 0.2212 & \log \csc B = 0.1987 \end{array}$$

$$\log \tan \frac{1}{2}(C - B) = 9.5548 \quad \log a = 0.5067$$

$$\therefore \frac{1}{2}(C - B) = 19^\circ 44.2'. \quad \therefore a = 3.211.$$

$$\therefore C = \frac{1}{2}(C + B) + \frac{1}{2}(C - B) = 78^\circ 44.2',$$

$$\text{and } B = \frac{1}{2}(C + B) - \frac{1}{2}(C - B) = 39^\circ 15.8'.$$

$$5. \tan \frac{1}{2}(B - A) = \frac{b - a}{b + a} \tan \frac{1}{2}(B + A). \quad c = a \sin C \csc A.$$

$$\begin{array}{lll} b - a = .063 & \log = 8.7993 & \log a = 9.4771 \\ b + a = .663 & \text{colog} = 0.1785 & \log \sin C = 9.9137 \\ \frac{1}{2}(B + A) = 27^\circ 32' & \log \tan = 9.7171 & \log \csc A = 0.3790 \end{array}$$

$$\log \tan \frac{1}{2}(B - A) = 8.6949 \quad \log c = 9.7698$$

$$\therefore \frac{1}{2}(B - A) = 2^\circ 50.2'. \quad \therefore c = .5886.$$

$$\therefore B = \frac{1}{2}(B + A) + \frac{1}{2}(B - A) = 30^\circ 22.2',$$

$$\text{and } A = \frac{1}{2}(B + A) - \frac{1}{2}(B - A) = 24^\circ 41.8'.$$

$$6. \tan \frac{1}{2}(B-C) = \frac{b-c}{b+c} \tan \frac{1}{2}(B+C). \quad a = b \sin A \csc B.$$

$b-c = 835.8$	$\log = 2.9221$	$\log b = 3.0763$
$b+c = 1548.4$	$\text{colog} = 6.8101$	$\log \sin A = 9.6460$
$\frac{1}{2}(B+C) = 76^\circ 52'$	$\log \tan = 0.6320$	$\log \csc B = 0.2254$
$\log \tan \frac{1}{2}(B-C) = 0.3642$		$\log a = 2.9477$
$\therefore \frac{1}{2}(B-C) = 66^\circ 37.1'$		$\therefore a = 886.6.$

$\therefore B = \frac{1}{2}(B+C) + \frac{1}{2}(B-C) = 143^\circ 29.1'$,
and $C = \frac{1}{2}(B+C) - \frac{1}{2}(B-C) = 10^\circ 14.9'$.

$$7. \tan \frac{1}{2}(C-A) = \frac{c-a}{c+a} \tan \frac{1}{2}(C+A). \quad b = a \sin B \csc A.$$

$c-a = 4.039$	$\log = 0.6063$	$\log a = 0.8692$
$c+a = 18.839$	$\text{colog} = 8.7249$	$\log \sin B = 9.9962$
$\frac{1}{2}(C+A) = 48^\circ 47'$	$\log \tan = 0.0575$	$\log \csc A = 0.2411$
$\log \tan \frac{1}{2}(C-A) = 9.3887$		$\log b = 1.1065$
$\therefore \frac{1}{2}(C-A) = 13^\circ 45.1'$		$\therefore b = 12.78.$

$\therefore C = \frac{1}{2}(C+A) + \frac{1}{2}(C-A) = 62^\circ 32.1'$,
and $A = \frac{1}{2}(C+A) - \frac{1}{2}(C-A) = 35^\circ 1.9'$.

$$8. \tan \frac{1}{2}(A-B) = \frac{a-b}{a+b} \tan \frac{1}{2}(A+B). \quad c = a \sin C \csc A.$$

$a-b = 11.66$	$\log = 1.0667$	$\log a = 1.7265$
$a+b = 94.88$	$\text{colog} = 8.0228$	$\log \sin C = 9.9913$
$\frac{1}{2}(A+B) = 50^\circ 43.5'$	$\log \tan = 0.0874$	$\log \csc A = 0.0657$
$\log \tan \frac{1}{2}(A-B) = 9.1769$		$\log c = 1.7835$
$\therefore \frac{1}{2}(A-B) = 8^\circ 32.8'$		$\therefore c = 60.74.$

$\therefore A = \frac{1}{2}(A+B) + \frac{1}{2}(A-B) = 59^\circ 16.3'$,
and $B = \frac{1}{2}(A+B) - \frac{1}{2}(A-B) = 42^\circ 10.7'$.

$$9. \tan \frac{1}{2}(C-B) = \frac{c-b}{c+b} \tan \frac{1}{2}(C+B). \quad a = b \sin A \csc B.$$

$c-b = .02424$	$\log = 8.3845$	$\log b = 8.4262$
$c+b = .0776$	$\text{colog} = 1.1101$	$\log \sin A = 9.9545$
$\frac{1}{2}(C+B) = 32^\circ 6.5'$	$\log \tan = 9.7976$	$\log \csc B = 0.4453$
$\log \tan \frac{1}{2}(C-B) = 9.2922$		$\log a = 8.8260$
$\therefore \frac{1}{2}(C-B) = 11^\circ 5.3'$		$\therefore a = .06699.$

$\therefore C = \frac{1}{2}(C+B) + \frac{1}{2}(C-B) = 43^\circ 11.8'$,
and $B = \frac{1}{2}(C+B) - \frac{1}{2}(C-B) = 21^\circ 1.2'$.

$$10. \tan \frac{1}{2}(C-A) = \frac{c-a}{c+a} \tan \frac{1}{2}(C+A). \quad b = a \sin B \csc A.$$

$c - a = 16.56$	$\log = 1.2191$	$\log a = 1.7108$
$c + a = 119.32$	$\text{colog} = 7.9233$	$\log \sin B = 9.9923$
$\frac{1}{2}(C+A) = 50^\circ 23' 43''$	$\log \tan = 0.0823$	$\log \csc A = 0.1842$
	$\log \tan \frac{1}{2}(C-A) = 9.2247$	$\log b = 1.8873$
	$\therefore \frac{1}{2}(C-A) = 9^\circ 31.4' = 9^\circ 31' 24''.$	$\therefore b = 77.14.$
	$\therefore C = \frac{1}{2}(C+A) + \frac{1}{2}(C-A) = 59^\circ 55' 7''.$	
	and $A = \frac{1}{2}(C+A) - \frac{1}{2}(C-A) = 40^\circ 52' 19''.$	

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3. Here $s = 4.5$, $s - a = 2.5$, $s - b = 1.5$, $s - c = .5$.

$\log(s-a) = 0.3979$	$\log r = 9.8099$
$\log(s-b) = 0.1761$	$\log(s-b) = 0.1761$
$\log(s-c) = 9.6990$	$\log \tan \frac{1}{2} B = 9.6338$
$\text{colog } s = 9.3468$	$\frac{1}{2} B = 23^\circ 17.1'.$
$\quad \quad \quad 2) 9.6198$	$\therefore B = 46^\circ 34.2'.$
$\log r = 9.8099$	$\log r = 9.8099$
$\log r = 9.8099$	$\log(s-c) = 9.6990$
$\log(s-a) = 0.3979$	$\log \tan \frac{1}{2} C = 0.1109$
$\log \tan \frac{1}{2} A = 9.4120$	$\frac{1}{2} C = 52^\circ 14.2'.$
$\frac{1}{2} A = 14^\circ 28.7'.$	$\therefore C = 104^\circ 28.4'.$
$\therefore A = 28^\circ 57.4'.$	

Check, $A + B + C = 180^\circ.$

4. Here $s = 8.5$, $s - a = 4.5$, $s - b = 1.5$, $s - c = 2.5$.

$\log(s-a) = 0.6532$	$\log r = 0.1489$
$\log(s-b) = 0.1761$	$\log(s-b) = 0.1761$
$\log(s-c) = 0.3979$	$\log \tan \frac{1}{2} B = 9.9728$
$\text{colog } s = 9.0706$	$\frac{1}{2} B = 43^\circ 12.4'.$
$\quad \quad \quad 2) 0.2978$	$\therefore B = 86^\circ 24.8'.$
$\log r = 0.1489$	$\log r = 0.1489$
$\log(s-a) = 0.6532$	$\log(s-c) = 0.3979$
$\log \tan \frac{1}{2} A = 9.4957$	$\log \tan \frac{1}{2} C = 9.7510$
$\frac{1}{2} A = 17^\circ 23.2'.$	$\frac{1}{2} C = 29^\circ 24.5'.$
$\therefore A = 34^\circ 46.4'.$	$\therefore C = 58^\circ 49'.$

Check, $A + B + C = 180^\circ 0.2.$

5. Here $s = 7.4$, $s - a = 1.8$, $s - b = 3.1$, $s - c = 2.5$.

$\log(s - a) = 0.2553$	$\log r = 0.1377$
$\log(s - b) = 0.4914$	$\log(s - b) = 0.4914$
$\log(s - c) = 0.3979$	$\log \tan \frac{1}{2} B = 9.6463$
$\text{colog } s = 9.1308$	$\frac{1}{2} B = 23^\circ 53.2'$
$2)0.2754$	$\therefore B = 47^\circ 46.4'$
$\log r = 0.1377$	$\log r = 0.1377$
$\log(s - a) = 0.2553$	$\log(s - c) = 0.3979$
$\log \tan \frac{1}{2} A = 9.8824$	$\log \tan \frac{1}{2} C = 9.7398$
$\frac{1}{2} A = 37^\circ 20'$	$\frac{1}{2} C = 28^\circ 46.7'$
$\therefore A = 74^\circ 40'$	$\therefore C = 57^\circ 33.4'$

Check, $A + B + C = 179^\circ 59.8'$.

6. Here $s = .344$, $s - a = .114$, $s - b = .084$, $s - c = .146$.

$\log(s - a) = 9.0569$	$\log r = 8.8045$
$\log(s - b) = 8.9243$	$\log(s - b) = 8.9243$
$\log(s - c) = 9.1644$	$\log \tan \frac{1}{2} B = 9.8802$
$\text{colog } s = 0.4634$	$\frac{1}{2} B = 37^\circ 11.9'$
$2)7.6090$	$\therefore B = 74^\circ 23.8'$
$\log r = 8.8045$	$\log r = 8.8045$
$\log(s - a) = 9.0569$	$\log(s - c) = 9.1644$
$\log \tan \frac{1}{2} A = 9.7476$	$\log \tan \frac{1}{2} C = 9.6401$
$\frac{1}{2} A = 29^\circ 13'$	$\frac{1}{2} C = 23^\circ 35.3'$
$\therefore A = 58^\circ 26'$	$\therefore C = 47^\circ 10.6'$

Check, $A + B + C = 180^\circ 0.4'$.

7. Here $s = 120.2$, $s - a = 40.9$, $s - b = 26$, $s - c = 53.3$.

$\log(s - a) = 1.6117$	$\log r = 1.3367$
$\log(s - b) = 1.4150$	$\log(s - b) = 1.4150$
$\log(s - c) = 1.7267$	$\log \tan \frac{1}{2} B = 9.9217$
$\text{colog } s = 7.9201$	$\frac{1}{2} B = 39^\circ 51.9'$
$2)2.6735$	$\therefore B = 79^\circ 43.8'$
$\log r = 1.3367$	$\log r = 1.3367$
$\log(s - a) = 1.6117$	$\log(s - c) = 1.7267$
$\log \tan \frac{1}{2} A = 9.7250$	$\log \tan \frac{1}{2} C = 9.6100$
$\frac{1}{2} A = 27^\circ 57.7'$	$\frac{1}{2} C = 22^\circ 10'$
$\therefore A = 55^\circ 55.4'$	$\therefore C = 44^\circ 20'$

Check, $A + B + C = 179^\circ 59.2'$.

8. Here
- $s = 542$
- ,
- $s - a = 221$
- ,
- $s - b = 181$
- ,
- $s - c = 140$
- .

$\log(s - a) = 2.3444$	$\log r = 2.0071$
$\log(s - b) = 2.2577$	$\log(s - b) = 2.2577$
$\log(s - c) = 2.1461$	$\log \tan \frac{1}{2} B = 9.7494$
$\text{colog } s = 7.2660$	$\frac{1}{2} B = 29^\circ 19'$
$2) 4.0142$	$\therefore B = 58^\circ 38'$
$\log r = 2.0071$	$\log r = 2.0071$
$\log(s - a) = 2.3444$	$\log(s - c) = 2.1461$
$\log \tan \frac{1}{2} A = 9.6627$	$\log \tan \frac{1}{2} C = 9.8610$
$\frac{1}{2} A = 24^\circ 42.1'$	$\frac{1}{2} C = 35^\circ 58.9'$
$\therefore A = 49^\circ 24.2'$	$\therefore C = 71^\circ 57.8'$

Check, $A + B + C = 180^\circ$.

9. Here
- $s = .936$
- ,
- $s - a = .295$
- ,
- $s - b = .407$
- ,
- $s - c = .234$
- .

$\log(s - a) = 9.4698$	$\log r = 9.2386$
$\log(s - b) = 9.6096$	$\log(s - b) = 9.6096$
$\log(s - c) = 9.3692$	$\log \tan \frac{1}{2} B = 9.6290$
$\text{colog } s = 0.0287$	$\frac{1}{2} B = 23^\circ 3.1'$
$2) 8.4773$	$\therefore B = 46^\circ 6.2'$
$\log r = 9.2386$	$\log r = 9.2386$
$\log(s - a) = 9.4698$	$\log(s - c) = 9.3692$
$\log \tan \frac{1}{2} A = 9.7688$	$\log \tan \frac{1}{2} C = 9.8694$
$\frac{1}{2} A = 30^\circ 25.4'$	$\frac{1}{2} C = 36^\circ 30.8'$
$\therefore A = 60^\circ 50.8'$	$\therefore C = 73^\circ 1.6'$

Check, $A + B + C = 179^\circ 58.6'$.

10. Here
- $s = 6.989$
- ,
- $s - a = 3.97$
- ,
- $s - b = .258$
- ,
- $s - c = 2.761$
- .

$\log(s - a) = 0.5988$	$\log r = 9.8035$
$\log(s - b) = 9.4116$	$\log(s - b) = 9.4116$
$\log(s - c) = 0.4411$	$\log \tan \frac{1}{2} B = 0.3919$
$\text{colog } s = 9.1556$	$\frac{1}{2} B = 67^\circ 55.3'$
$2) 9.6071$	$\therefore B = 135^\circ 50.6'$
$\log r = 9.8035$	$\log r = 9.8035$
$\log(s - a) = 0.5988$	$\log(s - c) = 0.4411$
$\log \tan \frac{1}{2} A = 9.2047$	$\log \tan \frac{1}{2} C = 9.3624$
$\frac{1}{2} A = 9^\circ 6.2'$	$\frac{1}{2} C = 12^\circ 58.3'$
$\therefore A = 18^\circ 12.4'$	$\therefore C = 25^\circ 56.6'$

Check, $A + B + C = 179^\circ 59.6'$.

Art. 127. — Pages 91 and 92.

6. Since b is $< a$, there is but one solution, corresponding to the acute value of B .

$$\sin B = \frac{b \sin A}{a} \qquad c = a \sin C \csc A.$$

$$\log b = 0.5551$$

$$\log a = 0.7059$$

$$\operatorname{colog} a = 9.2941$$

$$\log \sin C = 9.9884$$

$$\log \sin A = 9.9530$$

$$\log \csc A = 0.0470$$

$$\log \sin B = 9.8022$$

$$\log c = 0.7413$$

$$\therefore B = 39^\circ 21.3',$$

$$\therefore c = 5.511.$$

and

$$C = 180^\circ - 103^\circ 11.3' = 76^\circ 48.7'.$$

7. Since b is $> c$, and C is acute, there will be two solutions, one solution, or no solution, according as $\log \sin B$ is negative, zero, or positive.

$$\sin B = \frac{b \sin C}{c} \qquad a_1 = b \sin A_1 \csc B. \qquad a_2 = b \sin A_2 \csc B.$$

$$\log b = 1.8739$$

$$\log b = 1.8739$$

$$\log b = 1.8739$$

$$\operatorname{colog} c = 8.2062$$

$$\log \sin A_1 = 9.9408$$

$$\log \sin A_2 = 9.0316$$

$$\log \sin C = 9.6615$$

$$\log \csc B = 0.2584$$

$$\log \csc B = 0.2584$$

$$\log \sin B = 9.7416$$

$$\log a_1 = 2.0731$$

$$\log a_2 = 1.1639$$

$$\therefore B_1 = 33^\circ 28.4',$$

$$\therefore a_1 = 118.33.$$

$$\therefore a_2 = 14.58.$$

and $B_2 = 146^\circ 31.6'$.

$$\therefore A_1 = 180^\circ - 60^\circ 46.4' = 119^\circ 13.6',$$

and $A_2 = 180^\circ - 173^\circ 49.6' = 6^\circ 10.4'$.

8. Since c is $< b$, there is but one solution, corresponding to the acute value of C .

$$\sin C = \frac{c \sin B}{b} \qquad a = b \sin A \csc B.$$

$$\log c = 9.2971$$

$$\log b = 9.3687$$

$$\operatorname{colog} b = 0.6313$$

$$\log \sin A = 9.4825$$

$$\log \sin B = 9.9757$$

$$\log \csc B = 0.0243$$

$$\log \sin C = 9.9041$$

$$\log a = 8.8755$$

$$\therefore C = 53^\circ 18.9',$$

$$\therefore a = 0.7508.$$

and

$$A = 180^\circ - 162^\circ 18.9' = 17^\circ 41.1'.$$

9. Since a is $< c$, there is but one solution, corresponding to the acute value of A .

$$\sin A = \frac{a \sin C}{c} \qquad b = a \sin B \csc A.$$

$$\log a = 0.0294 \qquad \log a = 0.0294$$

$$\operatorname{colog} c = 9.7670 \qquad \log \sin B = 9.8916$$

$$\log \sin C = \underline{9.7228} \qquad \log \csc A = \underline{0.4808}$$

$$\log \sin A = 9.5192 \qquad \log b = 0.4018$$

$$\therefore A = 19^\circ 18.1', \qquad \therefore b = 2.522.$$

and $B = 180^\circ - 51^\circ 11.1' = 128^\circ 48.9'.$

$$10. \sin A = \frac{a \sin B}{b} \qquad c_1 = a \sin C_1 \csc A. \qquad c_2 = a \sin C_2 \csc A.$$

$$\log a = 9.2704 \qquad \log a = 9.2704 \qquad \log a = 9.2704$$

$$\operatorname{colog} b = 0.7696 \qquad \log \sin C_1 = 9.7795 \qquad \log \sin C_2 = 9.4314$$

$$\log \sin B = \underline{9.9524} \qquad \log \csc A = \underline{0.0076} \qquad \log \csc A = \underline{0.0076}$$

$$\log \sin A = 9.9924 \qquad \log c_1 = 9.0575 \qquad \log c_2 = 8.7094$$

$$\therefore A_1 = 79^\circ 20', \qquad \therefore c_1 = .11416. \qquad \therefore c_2 = .05121$$

and $A_2 = 100^\circ 40'.$

$$\therefore C_1 = 180^\circ - 143^\circ = 37^\circ,$$

and $C_2 = 180^\circ - 164^\circ 20' = 15^\circ 40'.$

11. Since c is $> a$, and A is obtuse, the triangle is impossible.

12. Since b is $< c$, there is but one solution, corresponding to the acute value of B .

$$\sin B = \frac{b \sin C}{c} \qquad a = b \sin A \csc B.$$

$$\log b = 1.7016 \qquad \log b = 1.7016$$

$$\operatorname{colog} c = 8.1752 \qquad \log \sin A = 9.9232$$

$$\log \sin C = \underline{9.7340} \qquad \log \csc B = \underline{0.3892}$$

$$\log \sin B = 9.6108 \qquad \log a = 2.0140$$

$$\therefore B = 24^\circ 5.4', \qquad \therefore a = 103.3.$$

and $A = 180^\circ - 56^\circ 54.4' = 123^\circ 5.6'.$

$$13. \sin C = \frac{c \sin A}{a} \qquad b = a \tan B.$$

$$\log c = 1.0000 \qquad \log a = 0.9373$$

$$\operatorname{colog} a = 9.0627 \qquad \log \tan B = \underline{9.7623}$$

$$\log \sin A = \underline{9.9373}$$

$$\log b = 0.6996$$

$$0.0000 \qquad \therefore b = 5.007.$$

$$\therefore C = 90^\circ,$$

and $B = 90^\circ - 59^\circ 57' = 30^\circ 3'.$

$$14. \quad \sin C = \frac{c \sin B}{b}. \quad a_1 = b \sin A_1 \csc B. \quad a_2 = b \sin A_2 \csc B.$$

$$\log c = 0.8351 \quad \log b = 0.7127 \quad \log b = 0.7127$$

$$\operatorname{colog} b = 9.2873 \quad \log \sin A_1 = 9.9695 \quad \log \sin A_2 = 9.5939$$

$$\log \sin B = 9.8422 \quad \log \csc B = 0.1578 \quad \log \csc B = 0.1578$$

$$\log \sin C = 9.9646 \quad \log a_1 = 0.8400 \quad \log a_2 = 0.4644$$

$$\therefore C_1 = 67^\circ 10', \quad \therefore a_1 = 6.918. \quad \therefore a_2 = 2.913.$$

and $C_2 = 112^\circ 50'.$

$$\therefore A_1 = 180^\circ - 111^\circ 13' = 68^\circ 47',$$

and $A_2 = 180^\circ - 156^\circ 53' = 23^\circ 7'.$

15. Since a is $< b$, there is only one solution, corresponding to the acute value of A .

$$\sin A = \frac{a \sin B}{b}. \quad c = a \sin C \csc A.$$

$$\log a = 2.3315 \quad \log a = 2.3315$$

$$\operatorname{colog} b = 7.5455 \quad \log \sin C = 9.6825$$

$$\log \sin B = 9.9863 \quad \log \csc A = 0.1367$$

$$\log \sin A = 9.8633 \quad \log c = 2.1507$$

$$\therefore A = 46^\circ 53.3', \quad \therefore c = 141.48.$$

and $C = 180^\circ - 151^\circ 13.3' = 28^\circ 46.7'.$

$$16. \quad \sin B = \frac{b \sin C}{c}.$$

$$\log b = 3.4870$$

$$\operatorname{colog} c = 6.9126$$

$$\log \sin C = 9.9179$$

$$\log \sin B = 0.3175$$

Since $\log \sin B$ is positive, the triangle is impossible.

17. Since c is $< a$, there is only one solution, corresponding to the acute value of C .

$$\sin C = \frac{c \sin A}{a}. \quad b = a \sin B \csc A.$$

$$\log c = 9.7086 \quad \log a = 9.8511$$

$$\operatorname{colog} a = 0.1489 \quad \log \sin B = 9.9363$$

$$\log \sin A = 9.7606 \quad \log \csc A = 0.2394$$

$$\log \sin C = 9.6181 \quad \log b = 0.0268$$

$$\therefore C = 24^\circ 31.4', \quad \therefore b = 1.0637.$$

and $B = 180^\circ - 59^\circ 42.4' = 120^\circ 17.6'.$

$$18. \quad \sin B = \frac{b \sin A}{a} \qquad c = b \sin C.$$

$$\log b = 2.2206 \qquad \log b = 2.2206$$

$$\text{colog } a = 7.9712 \qquad \log \sin C = 9.8843$$

$$\log \sin A = 9.8082 \qquad \log c = 2.1049$$

$$\log \sin B = 0.0000 \qquad \therefore c = 127.32.$$

$$\therefore B = 90^\circ,$$

$$\text{and } C = 90^\circ - 40^\circ 0' 21'' = 49^\circ 59' 39''.$$

$$19. \quad \sin A = \frac{a \sin C}{c} \quad b_1 = a \sin B_1 \csc A. \quad b_2 = a \sin B_2 \csc A.$$

$$\log a = 9.5073 \qquad \log a = 9.5073 \qquad \log a = 9.5073$$

$$\text{colog } c = 0.5673 \quad \log \sin B_1 = 9.9255 \quad \log \sin B_2 = 9.4853$$

$$\log \sin C = 9.8989 \quad \log \csc A = 0.0265 \quad \log \csc A = 0.0265$$

$$\log \sin A = 9.9735 \quad \log b_1 = 9.4593 \quad \log b_2 = 9.0191$$

$$\therefore A_1 = 70^\circ 12', \quad \therefore b_1 = .2879. \quad \therefore b_2 = .1045.$$

$$\text{and } A_2 = 109^\circ 48'.$$

$$\therefore B_1 = 180^\circ - 122^\circ 36' = 57^\circ 24',$$

$$\text{and } B_2 = 180^\circ - 162^\circ 12' = 17^\circ 48'.$$

20. Since c is $< b$, there is only one solution, corresponding to the acute value of C .

$$\sin C = \frac{c \sin B}{b} \qquad a = b \sin A \csc B.$$

$$\log c = 2.7828$$

$$\log b = 2.9092$$

$$\text{colog } b = 7.0908$$

$$\log \sin A = 9.4596$$

$$\log \sin B = 9.9075$$

$$\log \csc B = 0.0925$$

$$\log \sin C = 9.7811$$

$$\log a = 2.4613$$

$$\therefore C = 37^\circ 10',$$

$$\therefore a = 289.3.$$

$$\text{and } A = 180^\circ - 163^\circ 15' 20'' = 16^\circ 44' 40''.$$

Art. 128.—Page 93.

$$2. \quad 2K = ac \sin B.$$

$$3. \quad \text{Here } s = 9,$$

$$\log a = 1.5798$$

$$s - a = 4,$$

$$\log c = 1.7868$$

$$s - b = 2,$$

$$\log \sin B = 9.9670$$

$$\text{and } s - c = 3.$$

$$\log 2K = 3.3336$$

$$K = \sqrt{s(s-a)(s-b)(s-c)}.$$

$$2K = 2155.7$$

$$\therefore K = 1077.9.$$

$$\log s = 0.9542$$

$$\log (s - a) = 0.6021$$

$$\log (s - b) = 0.3010$$

$$\log (s - c) = 0.4771$$

$$\begin{array}{r} 2)2.3344 \\ \hline \end{array}$$

$$\log K = 1.1672$$

$$\therefore K = 14.697.$$

4. $2K = b^2 \sin C \sin A \csc B.$

$$C = 180^\circ - 106^\circ 23' = 73^\circ 37'.$$

$$2 \log b = 0.6320$$

$$\log \sin C = 9.9820$$

$$\log \sin A = 9.9730$$

$$\log \csc B = 0.2268$$

$$\log 2K = 0.8138$$

$$2K = 6.513$$

$$\therefore K = 3.257.$$

5. $2K = bc \sin A.$

$$\log b = 2.0649$$

$$\log c = 2.0000$$

$$\log \sin A = 9.9449$$

$$\log 2K = 4.0098$$

$$2K = 10229$$

$$\therefore K = 5114.$$

6. Here $s = 120,$

$$s - a = 41,$$

$$s - b = 26,$$

and $s - c = 53.$

$$K = \sqrt{s(s-a)(s-b)(s-c)}.$$

$$\log s = 2.0792$$

$$\log (s - a) = 1.6128$$

$$\log (s - b) = 1.4150$$

$$\log (s - c) = 1.7243$$

$$\begin{array}{r} 2)6.8313 \\ \hline \end{array}$$

$$\log K = 3.4156$$

$$\therefore K = 2604.$$

7. $2K = a^2 \sin B \sin C \csc A.$

$$C = 180^\circ - 67^\circ 8' = 112^\circ 52'.$$

$$2 \log a = 0.9892$$

$$\log \sin B = 9.3822$$

$$\log \sin C = 9.9645$$

$$\log \csc A = 0.0966$$

$$\log 2K = 0.4325$$

$$2K = 2.707$$

$$\therefore K = 1.353.$$

8. $2K = b^2 \sin C \sin A \csc B.$

$$B = 180^\circ - 117^\circ 13' = 62^\circ 47'.$$

$$2 \log b = 9.2850$$

$$\log \sin C = 9.8132$$

$$\log \sin A = 9.9880$$

$$\log \csc B = 0.0510$$

$$\log 2K = 9.1372$$

$$2K = .13716$$

$$\therefore K = .06858.$$

9. Here $s = 34,$

$$s - a = 10.9,$$

$$s - b = 14.3,$$

and $s - c = 8.8.$

$$K = \sqrt{s(s-a)(s-b)(s-c)}.$$

$$\log s = 1.5315$$

$$\log (s - a) = 1.0374$$

$$\log (s - b) = 1.1553$$

$$\log (s - c) = 0.9445$$

$$\begin{array}{r} 2)4.6687 \\ \hline \end{array}$$

$$\log K = 2.3343$$

$$\therefore K = 215.9.$$

10. $2K = ac \sin B.$

$$\log a = 9.5089$$

$$\log c = 9.9582$$

$$\log \sin B = 9.9387$$

$$\log 2K = 9.4058$$

$$2K = .2546$$

$$\therefore K = .1273.$$

$$\begin{aligned}
 11. \quad 2K &= c^2 \sin A \sin B \csc C. \\
 A &= 180^\circ - 131^\circ 49' = 48^\circ 11'. \\
 2 \log c &= 3.8088 \\
 \log \sin A &= 9.8723 \\
 \log \sin B &= 9.9932 \\
 \log \csc C &= 0.2791 \\
 \hline
 \log 2K &= 3.9534 \\
 2K &= 8982 \\
 \therefore K &= 4491.
 \end{aligned}$$

$$\begin{aligned}
 13. \quad \text{Here} \quad s &= 8.04, \\
 s - a &= 2.22, \\
 s - b &= 2.04, \\
 \text{and} \quad s - c &= 3.78. \\
 K &= \sqrt{s(s-a)(s-b)(s-c)}. \\
 \log s &= 0.9053 \\
 \log (s-a) &= 0.3464 \\
 \log (s-b) &= 0.3096 \\
 \log (s-c) &= 0.5775
 \end{aligned}$$

$$\begin{aligned}
 12. \quad 2K &= ab \sin C. \\
 \log a &= 8.0072 \\
 \log b &= 8.2607 \\
 \log \sin C &= 9.2924 \\
 \hline
 \log 2K &= 5.5603 \\
 2K &= .00003633 \\
 \therefore K &= .00001817.
 \end{aligned}$$

$$\begin{aligned}
 &2) 2.1388 \\
 \log K &= 1.0694 \\
 \therefore K &= 11.732.
 \end{aligned}$$

Art. 129.—Pages 93 and 94.

1. Let A be the first point of observation, and B the second; and let C be the top of the tower, and D its base.

$$\text{Then} \quad \frac{AC}{AB} = \frac{\sin ABC}{\sin ACB},$$

$$\text{or,} \quad \begin{aligned} AC &= AB \sin ABC \csc ACB \\ &= 100 \sin 35^\circ 16' \csc 17^\circ 23'. \end{aligned}$$

$$\log 100 = 2.0000$$

$$\log \sin 35^\circ 16' = 9.7615$$

$$\log \csc 17^\circ 23' = 0.5247$$

$$\therefore \log AC = 2.2862$$

$$\text{Then} \quad CD = AC \sin CAD = AC \sin 52^\circ 39',$$

$$\text{and} \quad AD = AC \cos CAD = AC \cos 52^\circ 39'.$$

$$\log AC = 2.2862$$

$$\log \sin 52^\circ 39' = 9.9003$$

$$\log CD = 2.1865$$

$$\therefore CD = 153.64.$$

$$\log AC = 2.2862$$

$$\log \cos 52^\circ 39' = 9.7830$$

$$\log AD = 2.0692$$

$$\therefore AD = 117.27.$$

$$\therefore BD = AB + AD = 217.27.$$

2. Denoting the sides opposite the angles A , B , and C of the triangle ABC by a , b , and c , respectively, we have

$$K = \sqrt{s(s-a)(s-b)(s-c)}.$$

Here $s = 351$, $s - a = 115$, $s - b = 40$, and $s - c = 196$.

$$\begin{aligned} \log s &= 2.5453 \\ \log (s - a) &= 2.0607 \\ \log (s - b) &= 1.6021 \\ \log (s - c) &= 2.2923 \\ \hline &2)8.5004 \\ \log K &= 4.2502 \\ \therefore K &= 17792. \end{aligned}$$

Denoting the sides opposite the angles A , C , and D of the triangle ACD by a , c , and d , respectively, we have

$$K = \sqrt{s(s-a)(s-c)(s-d)}.$$

Here $s = 334$, $s - a = 82$, $s - c = 229$, and $s - d = 23$.

$$\begin{aligned} \log s &= 2.5237 \\ \log (s - a) &= 1.9138 \\ \log (s - c) &= 2.3598 \\ \log (s - d) &= 1.3617 \\ \hline &2)8.1590 \\ \log K &= 4.0795 \\ \therefore K &= 12008. \end{aligned}$$

Therefore $\text{area } ABCD = 17792 + 12008 = 29800$.

3. Let A be the position of the first post, and B of the second, and let C be the top of the bluff, and D its foot.

$$\text{Then} \quad \frac{AC}{AB} = \frac{\sin ABC}{\sin ACB}$$

$$\begin{aligned} \text{Whence,} \quad CD &= AC \sin CAD \\ &= \frac{AB \sin ABC \sin CAD}{\sin ACB} \\ &= \frac{1000 \sin 9^\circ 33' \sin 27^\circ 40'}{\sin 18^\circ 7'}. \end{aligned}$$

$$\begin{aligned} \log 1000 &= 3.0000 \\ \log \sin 9^\circ 33' &= 9.2198 \\ \log \sin 27^\circ 40' &= 9.6668 \\ \log \csc 18^\circ 7' &= 0.5073 \\ \hline \log CD &= 2.3939 \\ \therefore CD &= 247.7. \end{aligned}$$

4. Let A be the starting-point, and B and C the positions of the first and second yachts, respectively, at the end of 40 minutes.

Then in the triangle ABC , we have

$$AB = 6.96, \quad AC = 5.14, \quad \text{and } \angle A = 45^\circ.$$

$$\tan \frac{1}{2}(C - B) = \frac{AB - AC}{AB + AC} \tan \frac{1}{2}(C + B).$$

$$AB - AC = 1.82 \qquad \log = 0.2601$$

$$AB + AC = 12.1 \qquad \text{colog} = 8.9172$$

$$\frac{1}{2}(C + B) = 67^\circ 30' \qquad \log \tan = 0.3828$$

$$\log \tan \frac{1}{2}(C - B) = 9.5601$$

$$\therefore \frac{1}{2}(C - B) = 19^\circ 57.5',$$

and $B = \frac{1}{2}(C + B) - \frac{1}{2}(C - B) = 47^\circ 32.5'.$

$$BC = AC \sin A \csc B.$$

$$\log AC = 0.7110$$

$$\log \sin A = 9.8495$$

$$\log \csc B = 0.1321$$

$$\log BC = 0.6926$$

$$\therefore BC = 4.927.$$

5. Let A be the position of the lighthouse, and B and C the first and second positions of the ship.

Then in the triangle ABC , we have

$$BC = 14, \quad \angle B = 105^\circ, \quad \text{and } \angle C = 30^\circ.$$

Whence, $\angle A = 180^\circ - 135^\circ = 45^\circ.$

$$AB = BC \sin C \csc A.$$

$$\log BC = 1.1461$$

$$\log \sin C = 9.6990$$

$$\log \csc A = 0.1505$$

$$\log AB = 0.9956$$

$$\therefore AB = 9.9.$$

$$AC = BC \sin B \csc A.$$

$$\log BC = 1.1461$$

$$\log \sin B = 9.9849$$

$$\log \csc A = 0.1505$$

$$\log AC = 1.2815$$

$$\therefore AC = 19.122.$$

$$\begin{aligned}
 6. \quad AB &= BC \sin C \csc A. \\
 A &= 180^\circ - 158^\circ 23' = 21^\circ 37'. \\
 \log BC &= 2.3187 \\
 \log \sin C &= 9.7218 \\
 \log \csc A &= \underline{0.4337} \\
 \log AB &= 2.4742 \\
 \therefore AB &= 298.
 \end{aligned}$$

7. Let A be the point of observation, B the top of the flag-pole, C its foot, and D the base of the tower.

$$\text{Then,} \quad \frac{AC}{BC} = \frac{\sin ABC}{\sin BAC}$$

$$\text{or,} \quad \begin{aligned} AC &= BC \sin ABC \csc BAC \\ &= 40 \sin 51^\circ 7' \csc 18^\circ 35'. \end{aligned}$$

$$\begin{aligned}
 \log 40 &= 1.6021 \\
 \log \sin 51^\circ 7' &= 9.8912 \\
 \log \csc 18^\circ 35' &= \underline{0.4967} \\
 \therefore \log AC &= 1.9900
 \end{aligned}$$

$$\begin{aligned}
 AD &= AC \cos CAD = AC \cos 20^\circ 18'. \\
 \log AC &= 1.9900 \\
 \log \cos 20^\circ 18' &= \underline{9.9722} \\
 \log AD &= 1.9622 \\
 \therefore AD &= 91.66.
 \end{aligned}$$

$$\begin{aligned}
 CD &= AC \sin CAD = AC \sin 20^\circ 18'. \\
 \log AC &= 1.9900 \\
 \log \sin 20^\circ 18' &= \underline{9.5402} \\
 \log CD &= 1.5302 \\
 \therefore CD &= 33.9.
 \end{aligned}$$

$$8. \quad \begin{aligned} BC &= BD \sin BDC \csc BCD \\ &= BD \sin 60^\circ \csc 20^\circ. \end{aligned}$$

$$\text{But,} \quad \begin{aligned} BD &= AD \sin BAD \csc ABD \\ &= 500 \sin 60^\circ \csc 80^\circ. \end{aligned}$$

$$\text{Whence,} \quad BC = 500 \sin^2 60^\circ \csc 20^\circ \csc 80^\circ.$$

$$\begin{aligned}
 \log 500 &= 2.6990 \\
 2 \log \sin 60^\circ &= 9.8750 \\
 \log \csc 20^\circ &= 0.4659 \\
 \log \csc 80^\circ &= \underline{0.0066} \\
 \log BC &= 3.0465 \\
 \therefore BC &= 1113.1.
 \end{aligned}$$

9.

$$AC = CD \sin ADC \csc CAD.$$

$$\log CD = 2.1761$$

$$\log \sin ADC = 9.6990$$

$$\log \csc CAD = 0.0866$$

$$\log AC = 1.9617$$

$$\therefore AC = 91.56.$$

$$BC = CD \sin BDC \csc CBD.$$

$$\log CD = 2.1761$$

$$\log \sin BDC = 9.9968$$

$$\log \csc CBD = 0.3430$$

$$\log BC = 2.5159$$

$$\therefore BC = 328.$$

$$\tan \frac{1}{2}(BAC - ABC) = \frac{BC - AC}{BC + AC} \tan \frac{1}{2}(BAC + ABC)$$

$$BC - AC = 236.44$$

$$\log = 2.3737$$

$$BC + AC = 419.56$$

$$\text{colog} = 7.3772$$

$$\frac{1}{2}(BAC + ABC) = 77^\circ 30'$$

$$\log \tan = 0.6542$$

$$\log \tan \frac{1}{2}(BAC - ABC) = 0.4051$$

$$\therefore \frac{1}{2}(BAC - ABC) = 68^\circ 31.4',$$

$$\text{and } BAC = \frac{1}{2}(BAC + ABC) + \frac{1}{2}(BAC - ABC) = 146^\circ 1.4'.$$

$$AB = BC \sin ACB \csc BAC.$$

$$\log BC = 2.5159$$

$$\log \sin ACB = 9.6259$$

$$\log \csc BAC = 0.2527$$

$$\log AB = 2.3945$$

$$\therefore AB = 248.$$

10. Denoting the sides opposite the angles A , B , and C of the triangle ABC by a , b , and c , respectively, we have

$$K = \sqrt{s(s-a)(s-b)(s-c)}.$$

Here,

$$s = 87.5,$$

$$s - a = 24.5,$$

$$s - b = 12.5,$$

and

$$s - c = 50.5.$$

$$\log s = 1.9420$$

$$\log (s - a) = 1.3892$$

$$\log (s - b) = 1.0969$$

$$\log (s - c) = 1.7033$$

$$2)6.1314$$

$$\log K = 3.0657$$

$$\therefore K = 1163.2.$$

$$\cos \frac{1}{2} BAC = \sqrt{\frac{s(s-a)}{bc}}.$$

$$\log s = 1.9420$$

$$\log (s-a) = 1.3892$$

$$\text{colog } b = 8.1249$$

$$\text{colog } c = 8.4318$$

$$\hline 2)9.8879$$

$$\log \cos \frac{1}{2} BAC = 9.9439$$

$$\frac{1}{2} BAC = 28^\circ 30'$$

$$\therefore BAC = 57^\circ.$$

Denoting the sides opposite the angles A , B , and D of the triangle ABD by a , b , and d , respectively, we have

$$\cos \frac{1}{2} BAD = \sqrt{\frac{s(s-a)}{bd}}.$$

Here,
and

$$s = 49.5,$$

$$s-a = 7.5.$$

$$\log s = 1.6946$$

$$\log (s-a) = 0.8751$$

$$\text{colog } b = 8.6990$$

$$\text{colog } d = 8.4318$$

$$\hline 2)9.7005$$

$$\log \cos \frac{1}{2} BAD = 9.8502$$

$$\frac{1}{2} BAD = 44^\circ 54.2'$$

$$\therefore BAD = 89^\circ 48.4'.$$

$$\therefore CAD = BAD - BAC = 32^\circ 48.4'.$$

$$2 \text{ area } ACD = AC \cdot AD \cdot \sin CAD.$$

$$\log AC = 1.8751$$

$$\log AD = 1.3010$$

$$\log \sin CAD = 9.7339$$

$$\log (2 \text{ area } ACD) = 2.9100$$

$$2 \text{ area } ACD = 812.8$$

$$\therefore \text{area } ACD = 406.4.$$

$$\therefore \text{area } ABCD = 1163.2 + 406.4 = 1569.6.$$

CHAPTER XI.

Art. 153.—Page 112.

$$\begin{aligned}
 5. \quad \sin A &= \frac{\sin a}{\sin c} \\
 \log \sin a &= 9.5543 \\
 \log \sin c &= \underline{9.8311} \\
 \log \sin A &= 9.7232 \\
 180^\circ - A &= 31^\circ 55'. \\
 \therefore A &= 148^\circ 5'.
 \end{aligned}$$

$$\begin{aligned}
 \cos B &= \frac{\tan a}{\tan c} \\
 \log \tan a &= 9.5842 \\
 \log \tan c &= \underline{9.9646} \\
 \log \cos B &= 9.6196 \\
 \therefore B &= 65^\circ 23.2'.
 \end{aligned}$$

$$\begin{aligned}
 \cos b &= \frac{\cos c}{\cos a} \\
 \log \cos c &= 9.8665 \\
 \log \cos a &= \underline{9.9702} \\
 \log \cos b &= 9.8963 \\
 \therefore b &= 38^\circ 2'.
 \end{aligned}$$

Check.

$$\begin{aligned}
 \sin A &= \frac{\cos B}{\cos b} \\
 \log \cos B &= 9.6196 \\
 \log \cos b &= \underline{9.8963} \\
 \log \sin A &= 9.7233
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \cos a &= \frac{\cos A}{\sin B} \\
 \log \cos A &= 9.8050 \\
 \log \sin B &= \underline{9.9252} \\
 \log \cos a &= 9.8798 \\
 \therefore a &= 40^\circ 41.8'.
 \end{aligned}$$

$$\begin{aligned}
 \cos b &= \frac{\cos B}{\sin A} \\
 \log \cos B &= 9.7322 \\
 \log \sin A &= \underline{9.8864} \\
 \log \cos b &= 9.8458 \\
 180^\circ - b &= 45^\circ 29.2'. \\
 \therefore b &= 134^\circ 30.8'.
 \end{aligned}$$

$$\begin{aligned}
 \cos c &= \cot A \cot B. \\
 \log \cot A &= 9.9187 \\
 \log \cot B &= \underline{9.8070} \\
 \log \cos c &= 9.7257 \\
 180^\circ - c &= 57^\circ 52.5'. \\
 \therefore c &= 122^\circ 7.5'.
 \end{aligned}$$

Check.

$$\begin{aligned}
 \cos c &= \cos a \cos b. \\
 \log \cos a &= 9.8798 \\
 \log \cos b &= \underline{9.8458} \\
 \log \cos c &= 9.7256
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \tan A &= \frac{\tan a}{\sin b} \\
 &+ \\
 \log \tan a &= 9.5611 \\
 \log \sin b &= \underline{9.7941} \\
 \log \tan A &= 9.7670 \\
 180^\circ - A &= 30^\circ 19'. \\
 \therefore A &= 149^\circ 41'.
 \end{aligned}$$

$$\begin{aligned}
 \tan B &= \frac{\tan b}{\sin a} \\
 \log \tan b &= 9.9006 \\
 \log \sin a &= \underline{9.5341} \\
 \log \tan B &= 0.3665 \\
 \therefore B &= 66^\circ 43.8'.
 \end{aligned}$$

$$\begin{aligned}
 8. \quad \sin A &= \frac{\cos B}{\cos b} \\
 \log \cos B &= 9.2397 \\
 \log \cos b &= \underline{9.5798} \\
 \log \sin A &= 9.6599 \\
 \therefore A &= 27^\circ 11.6', \\
 &\text{or } 152^\circ 48.4'.
 \end{aligned}$$

$$\begin{aligned}
 \sin a &= \frac{\tan b}{\tan B} \\
 \log \tan b &= 0.3864 \\
 \log \tan B &= \underline{0.7537} \\
 \log \sin a &= 9.6327 \\
 \therefore a &= 25^\circ 25.2', \\
 &\text{or } 154^\circ 34.8'.
 \end{aligned}$$

$$\begin{aligned}
 \text{Ans. } 1. A &= 27^\circ 11.6', a = 25^\circ 25.2', c = 69^\circ 54'. \\
 2. A &= 152^\circ 48.4', a = 154^\circ 34.8', c = 110^\circ 6'.
 \end{aligned}$$

$$\begin{aligned}
 9. \quad \tan A &= \frac{\cot B}{\cos c} \\
 &+ \\
 \log \cot B &= 9.6064 \\
 \log \cos c &= \underline{9.1525} \\
 \log \tan A &= 0.4539 \\
 180^\circ - A &= 70^\circ 37.5'. \\
 \therefore A &= 109^\circ 22.5'.
 \end{aligned}$$

$$\begin{aligned}
 \cos c &= \cos a \cos b. \\
 \log \cos a &= 9.9730 \\
 \log \cos b &= \underline{9.8935} \\
 \log \cos c &= 9.8665 \\
 180^\circ - c &= 42^\circ 40'. \\
 \therefore c &= 137^\circ 20'.
 \end{aligned}$$

Check.

$$\begin{aligned}
 \cos c \tan A \tan B &= 1. \\
 \log \cos c &= 9.8665 \\
 \log \tan A &= 9.7670 \\
 \log \tan B &= \underline{0.3665} \\
 \log 1 &= 0.0000
 \end{aligned}$$

$$\begin{aligned}
 \sin c &= \frac{\sin b}{\sin B} \\
 \log \sin b &= 9.9661 \\
 \log \sin B &= \underline{9.9934} \\
 \log \sin c &= 9.9727 \\
 \therefore c &= 69^\circ 54', \\
 &\text{or } 110^\circ 6'.
 \end{aligned}$$

Check.

$$\begin{aligned}
 \sin A &= \frac{\sin a}{\sin c} \\
 \log \sin a &= 9.6327 \\
 \log \sin c &= \underline{9.9727} \\
 \log \sin A &= 9.6600
 \end{aligned}$$

$$\tan a = \cos B \tan c.$$

$$\begin{aligned}
 \log \cos B &= 9.5736 \\
 \log \tan c &= \underline{0.8431} \\
 \log \tan a &= 0.4167 \\
 180^\circ - a &= 69^\circ 2.4'. \\
 \therefore a &= 110^\circ 57.6'.
 \end{aligned}$$

$$\sin b = \sin B \sin c.$$

$$\log \sin B = 9.9672$$

$$\log \sin c = 9.9956$$

$$\log \sin b = 9.9628$$

$$180^\circ - b = 66^\circ 38'.$$

$$\therefore b = 113^\circ 22'.$$

Check.

$$\tan A = \frac{\tan a}{\sin b}.$$

$$\log \tan a = 0.4167$$

$$\log \sin b = 9.9628$$

$$\log \tan A = 0.4539$$

10. $\cos A = \cos a \sin B.$

$$\log \cos a = 9.6856$$

$$\log \sin B = 9.9203$$

$$\log \cos A = 9.6059$$

$$\therefore A = 66^\circ 12.1'.$$

$$\tan b = \sin a \tan B.$$

$$\log \sin a = 9.9418$$

$$\log \tan B = 0.1765$$

$$\log \tan b = 0.1183$$

$$180^\circ - b = 52^\circ 42.6'.$$

$$\therefore b = 127^\circ 17.4'.$$

$$\tan c = \frac{\tan a}{\cos B}.$$

$$\log \tan a = 0.2562$$

$$\log \cos B = 9.7438$$

$$\log \tan c = 0.5124$$

$$180^\circ - c = 72^\circ 54.9'.$$

$$\therefore c = 107^\circ 5.1'.$$

Check.

$$\cos A = \frac{\tan b}{\tan c}.$$

$$\log \tan b = 0.1183$$

$$\log \tan c = 0.5124$$

$$\log \cos A = 9.6059$$

11. $\tan A = \frac{\tan a}{\sin b}.$

$$\log \tan a = 0.2683$$

$$\log \sin b = 9.7675$$

$$\log \tan A = 0.5008$$

$$\therefore A = 72^\circ 28.9'.$$

$$\tan B = \frac{\tan b}{\sin a}.$$

$$\log \tan b = 9.8586$$

$$\log \sin a = 9.9446$$

$$\log \tan B = 9.9140$$

$$180^\circ - B = 39^\circ 21.9'.$$

$$\therefore B = 140^\circ 38.1'.$$

$$\cos c = \cos a \cos b.$$

$$\log \cos a = 9.6763$$

$$\log \cos b = 9.9089$$

$$\log \cos c = 9.5852$$

$$180^\circ - c = 67^\circ 22.3'.$$

$$\therefore c = 112^\circ 37.7'.$$

Check.

$$\cos c \tan A \tan B = 1.$$

$$\log \cos c = 9.5852$$

$$\log \tan A = 0.5008$$

$$\log \tan B = 9.9140$$

$$\log 1 = 0.0000$$

$$12. \quad \begin{array}{r} + \\ \sin B = \frac{\cos A}{\cos a} \\ - \\ \log \cos A = 9.2324 \\ \log \cos a = 9.5736 \\ \log \sin B = 9.6588 \\ \therefore B = 27^\circ 7.2', \\ \text{or } 152^\circ 52.8'. \end{array}$$

$$\begin{array}{r} \sin c = \frac{\sin a}{\sin A} \\ \log \sin a = 9.9672 \\ \log \sin A = 9.9936 \\ \log \sin c = 9.9736 \\ \therefore c = 70^\circ 14', \\ \text{or } 109^\circ 46'. \end{array}$$

$$\begin{array}{r} + \\ \sin b = \frac{\tan a}{\tan A} \\ - \\ \log \tan a = 0.3936 \\ \log \tan A = 0.7611 \\ \log \sin b = 9.6325 \\ \therefore b = 25^\circ 24.4', \\ \text{or } 154^\circ 35.6'. \end{array}$$

Check.

$$\begin{array}{r} \sin B = \frac{\sin b}{\sin c} \\ \log \sin b = 9.6325 \\ \log \sin c = 9.9736 \\ \log \sin B = 9.6589 \end{array}$$

Ans. 1. $B = 27^\circ 7.2'$, $b = 25^\circ 24.4'$, $c = 109^\circ 46'$.
2. $B = 152^\circ 52.8'$, $b = 154^\circ 35.6'$, $c = 70^\circ 14'$.

$$13. \quad \begin{array}{r} - \\ \cos A = \frac{\tan b}{\tan c} \\ - \\ \log \tan b = 9.4281 \\ \log \tan c = 9.7196 \\ \log \cos A = 9.7085 \\ 180^\circ - A = 59^\circ 15.7'. \\ \therefore A = 120^\circ 44.3'. \end{array}$$

$$\begin{array}{r} \sin B = \frac{\sin b}{\sin c} \\ \log \sin b = 9.4130 \\ \log \sin c = 9.6668 \\ \log \sin B = 9.7462 \\ \therefore B = 33^\circ 52.6'. \end{array}$$

Check.

$$\begin{array}{r} - \\ \cos a = \frac{\cos c}{\cos b} \\ + \\ \log \cos c = 9.9473 \\ \log \cos b = 9.9849 \\ \log \cos a = 9.9624 \\ 180^\circ - a = 23^\circ 30'. \\ \therefore a = 156^\circ 30'. \end{array}$$

$$\begin{array}{r} \sin B = \frac{\cos A}{\cos a} \\ \log \cos A = 9.7085 \\ \log \cos a = 9.9624 \\ \log \sin B = 9.7461 \end{array}$$

14.

$$\begin{aligned}\cos a &= \frac{\cos A}{\sin B} \\ \log \cos A &= 9.6573 \\ \log \sin B &= \underline{9.7801} \\ \log \cos a &= \underline{9.8772} \\ \therefore a &= 41^\circ 5.5'.\end{aligned}$$

$$\begin{aligned}\cos b &= \frac{\cos B}{\sin A} \\ \log \cos B &= 9.9019 \\ \log \sin A &= \underline{9.9498} \\ \log \cos b &= \underline{9.9521} \\ \therefore b &= 26^\circ 25'.\end{aligned}$$

15.

$$\begin{aligned}\sin a &= \sin A \sin c. \\ \log \sin A &= 9.9809 \\ \log \sin c &= \underline{9.9589} \\ \log \sin a &= \underline{9.9398} \\ \therefore a &= 60^\circ 31.4'.\end{aligned}$$

$$\begin{aligned}\tan B &= \frac{\cot A}{\cos c} \\ \log \cot A &= 9.4822 \\ \log \cos c &= \underline{9.6183} \\ \log \tan B &= \underline{9.8639} \\ 180^\circ - B &= 36^\circ 10'. \\ \therefore B &= 143^\circ 50'.\end{aligned}$$

16.

$$\begin{aligned}\sin A &= \frac{\cos B}{\cos b} \\ \log \cos B &= 9.9129 \\ \log \cos b &= \underline{9.9213} \\ \log \sin A &= \underline{9.9916} \\ \therefore A &= 78^\circ 46.7', \\ &\text{or } 101^\circ 13.3'.\end{aligned}$$

$$\cos c = \cot A \cot B.$$

$$\begin{aligned}\log \cot A &= 9.7075 \\ \log \cot B &= \underline{0.1219} \\ \log \cos c &= \underline{9.8294} \\ \therefore c &= 47^\circ 32.1'.\end{aligned}$$

Check.

$$\begin{aligned}\cos c &= \cos a \cos b. \\ \log \cos a &= 9.8772 \\ \log \cos b &= \underline{9.9521} \\ \log \cos c &= \underline{9.8293}\end{aligned}$$

$$\tan b = \cos A \tan c.$$

$$\begin{aligned}\log \cos A &= 9.4630 \\ \log \tan c &= \underline{0.3406} \\ \log \tan b &= \underline{9.8036} \\ 180^\circ - b &= 32^\circ 27.9'. \\ \therefore b &= 147^\circ 32.1'.\end{aligned}$$

Check.

$$\begin{aligned}\tan B &= \frac{\tan b}{\sin a} \\ \log \tan b &= 9.8036 \\ \log \sin a &= \underline{9.9398} \\ \log \tan B &= \underline{9.8638}\end{aligned}$$

$$\sin a = \frac{\tan b}{\tan B}$$

$$\begin{aligned}\log \tan b &= 9.8202 \\ \log \tan B &= \underline{9.8468} \\ \log \sin a &= \underline{9.9734} \\ \therefore a &= 70^\circ 10', \\ &\text{or } 109^\circ 50'.\end{aligned}$$

$$\sin c = \frac{\sin b}{\sin B}$$

$$\log \sin b = 9.7415$$

$$\log \sin B = \underline{9.7597}$$

$$\log \sin c = 9.9818$$

$$\therefore c = 73^\circ 32.5',$$

$$\text{or } 106^\circ 27.5'.$$

Ans. 1. $A = 78^\circ 46.7'$, $a = 70^\circ 10'$, $c = 106^\circ 27.5'$.

2. $A = 101^\circ 13.3'$, $a = 109^\circ 50'$, $c = 73^\circ 32.5'$.

Check.

$$\sin A = \frac{\sin a}{\sin c}$$

$$\log \sin a = 9.9734$$

$$\log \sin c = \underline{9.9818}$$

$$\log \sin A = 9.9916$$

17.

$$\tan A = \frac{\cot B}{\cos c}$$

$$\log \cot B = 9.5998$$

$$\log \cos c = \underline{9.8291}$$

$$\log \tan A = 9.7707$$

$$\therefore A = 30^\circ 32.1'.$$

$$\tan a = \cos B \tan c.$$

$$\log \cos B = 9.5679$$

$$\log \tan c = \underline{0.0389}$$

$$\log \tan a = 9.6068$$

$$\therefore a = 22^\circ 1.1'.$$

$$\sin b = \sin B \sin c.$$

$$\log \sin B = 9.9681$$

$$\log \sin c = \underline{9.8681}$$

$$\log \sin b = 9.8362$$

$$\therefore b = 43^\circ 17.9'.$$

Check.

$$\tan A = \frac{\tan a}{\sin b}$$

$$\log \tan a = 9.6068$$

$$\log \sin b = \underline{9.8362}$$

$$\log \tan A = 9.7706$$

18.

$$\tan a = \tan A \sin b.$$

$$\log \tan A = 9.5152$$

$$\log \sin b = \underline{9.8769}$$

$$\log \tan a = 9.3921$$

$$180^\circ - a = 13^\circ 51.3'.$$

$$\therefore a = 166^\circ 8.7'.$$

$$\tan c = \frac{\tan b}{\cos A}$$

$$\log \tan b = 0.0588$$

$$\log \cos A = \underline{9.9779}$$

$$\log \tan c = 0.0809$$

$$\therefore c = 50^\circ 18.4'.$$

$$\cos B = \sin A \cos b.$$

$$\log \sin A = 9.4931$$

$$\log \cos b = \underline{9.8181}$$

$$\log \cos B = 9.3112$$

$$180^\circ - B = 78^\circ 11.1'.$$

$$\therefore B = 101^\circ 48.9'.$$

Check.

$$\cos B = \frac{\tan a}{\tan c}$$

$$\log \tan a = 9.3921$$

$$\log \tan c = \underline{0.0809}$$

$$\log \cos B = 9.3112$$

$$19. \quad \begin{array}{r} \overline{\tan A} = \frac{\overline{\tan a}}{\overline{\sin b}} \\ + \\ \log \tan a = 0.3634 \\ \log \sin b = 9.9707 \\ \hline \log \tan A = 0.3927 \\ 180^\circ - A = 67^\circ 57.5'. \\ \therefore A = 112^\circ 2.5'. \end{array}$$

$$+ \quad \overline{\cos c} = \overline{\cos a} \overline{\cos b}.$$

$$\begin{array}{r} \log \cos a = 9.5993 \\ \log \cos b = 9.5500 \\ \hline \log \cos c = 9.1493 \\ \therefore c = 81^\circ 53.6'. \end{array}$$

$$\begin{array}{r} \overline{\tan B} = \frac{\overline{\tan b}}{\overline{\sin a}} \\ + \\ \log \tan b = 0.4207 \\ \log \sin a = 9.9626 \\ \hline \log \tan B = 0.4581 \\ 180^\circ - B = 70^\circ 48'. \\ \therefore B = 109^\circ 12'. \end{array}$$

Check.

$$\begin{array}{r} \cos c \tan A \tan B = 1. \\ \log \cos c = 9.1493 \\ \log \tan A = 0.3927 \\ \log \tan B = 0.4581 \\ \hline \log 1 = 0.0001 \end{array}$$

$$20. \quad \begin{array}{r} \overline{\cos A} = \overline{\cos a} \overline{\sin B} \\ \log \cos a = 9.8652 \\ \log \sin B = 9.9846 \\ \hline \log \cos A = 9.8498 \\ 180^\circ - A = 44^\circ 57.5'. \\ \therefore A = 135^\circ 2.5'. \end{array}$$

$$\tan b = \sin a \tan B.$$

$$\begin{array}{r} \log \sin a = 9.8325 \\ \log \tan B = 0.5674 \\ \hline \log \tan b = 0.3999 \\ \therefore b = 68^\circ 17.3'. \end{array}$$

$$\begin{array}{r} \overline{\tan c} = \frac{\overline{\tan a}}{\overline{\cos B}} \\ + \\ \log \tan a = 9.9674 \\ \log \cos B = 9.4172 \\ \hline \log \tan c = 0.5502 \\ 180^\circ - c = 74^\circ 16'. \\ \therefore c = 105^\circ 44'. \end{array}$$

Check.

$$\begin{array}{r} \overline{\cos A} = \frac{\overline{\tan b}}{\overline{\tan c}} \\ \log \tan b = 0.3999 \\ \log \tan c = 0.5502 \\ \hline \log \cos A = 9.8497 \end{array}$$

$$21. \quad \begin{array}{r} \overline{\cos a} = \frac{\overline{\cos A}}{\overline{\sin B}} \\ + \\ \log \cos A = 9.9129 \\ \log \sin B = 9.9916 \\ \hline \log \cos a = 9.9213 \\ 180^\circ - a = 33^\circ 27.5'. \\ \therefore a = 146^\circ 32.5'. \end{array}$$

$$\begin{array}{r} \overline{\cos b} = \frac{\overline{\cos B}}{\overline{\sin A}} \\ + \\ \log \cos B = 9.2896 \\ \log \sin A = 9.7597 \\ \hline \log \cos b = 9.5299 \\ 180^\circ - b = 70^\circ 11.9'. \\ \therefore b = 109^\circ 48.1'. \end{array}$$

$$\begin{array}{c} + \quad - \quad - \\ \cos c = \cot A \cot B. \end{array}$$

$$\begin{aligned} \log \cot A &= 0.1532 \\ \log \cot B &= \underline{9.2980} \\ \log \cos c &= 9.4512 \\ \therefore c &= 73^\circ 35'. \end{aligned}$$

Check.

$$\begin{aligned} \cos c &= \cos a \cos b. \\ \log \cos a &= 9.9213 \\ \log \cos b &= \underline{9.5299} \\ \log \cos c &= 9.4512 \end{aligned}$$

22.

$$\begin{aligned} \sin A &= \frac{\sin a}{\sin c} \\ \log \sin a &= 9.9710 \\ \log \sin c &= \underline{9.9980} \\ \log \sin A &= 9.9730 \\ \therefore A &= 70^\circ. \end{aligned}$$

$$\begin{aligned} \cos B &= \frac{\tan a}{\tan c} \\ \log \tan a &= 0.4226 \\ \log \tan c &= \underline{1.0125} \\ \log \cos B &= 9.4101 \\ \therefore B &= 75^\circ 6.2'. \end{aligned}$$

$$\begin{aligned} \cos b &= \frac{\cos c}{\cos a} \\ \log \cos c &= 8.9855 \\ \log \cos a &= \underline{9.5484} \\ \log \cos b &= 9.4371 \\ \therefore b &= 74^\circ 7.3'. \end{aligned}$$

Check.

$$\begin{aligned} \sin A &= \frac{\cos B}{\cos b} \\ \log \cos B &= 9.4101 \\ \log \cos b &= \underline{9.4371} \\ \log \sin A &= 9.9730 \end{aligned}$$

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2. In the polar triangle, $a' = 41^\circ$ and $B' = 37^\circ$.

$$\begin{aligned} \cos A' &= \cos a' \sin B'. \\ \log \cos a' &= 9.8778 \\ \log \sin B' &= \underline{9.7795} \\ \log \cos A' &= 9.6573 \\ \therefore A' &= 62^\circ 58.8'. \end{aligned}$$

$$\begin{aligned} \tan b' &= \sin a' \tan B'. \\ \log \sin a' &= 9.8169 \\ \log \tan B' &= \underline{9.8771} \\ \log \tan b' &= 9.6940 \\ \therefore b' &= 26^\circ 18.1'. \end{aligned}$$

$$\begin{aligned} \tan c' &= \frac{\tan a'}{\cos B'}. \\ \log \tan a' &= 9.9392 \\ \log \cos B' &= \underline{9.9023} \\ \log \tan c' &= 0.0369 \\ \therefore c' &= 47^\circ 26'. \end{aligned}$$

Check.

$$\begin{aligned} \cos A' &= \frac{\tan b'}{\tan c'}. \\ \log \tan b' &= 9.6940 \\ \log \tan c' &= \underline{0.0369} \\ \log \cos A' &= 9.6571 \end{aligned}$$

 \therefore in the quadrantal triangle,

$$a = 117^\circ 1.2', B = 153^\circ 41.9', \text{ and } C = 132^\circ 34'.$$

3. In the polar triangle, $a' = 134^\circ 30'$ and $b' = 40^\circ 40'$.

$$\begin{array}{r} \text{---} \\ \tan A' = \frac{\tan a'}{\sin b'} \\ \text{+} \\ \log \tan a' = 0.0076 \\ \log \sin b' = \underline{9.8140} \\ \log \tan A' = 0.1936 \\ \therefore 180^\circ - A' = 57^\circ 22.1'. \end{array}$$

$$\begin{array}{r} \text{---} \quad \text{---} \quad \text{+} \\ \cos c' = \cos a' \cos b'. \\ \log \cos a' = 9.8457 \\ \log \cos b' = \underline{9.8800} \\ \log \cos c' = \underline{9.7257} \\ \therefore 180^\circ - c' = 57^\circ 52.5'. \end{array}$$

Check.

$$\begin{array}{r} \tan B' = \frac{\tan b'}{\sin a'} \\ \log \tan b' = 9.9341 \\ \log \sin a' = \underline{9.8532} \\ \log \tan B' = 0.0809 \\ \therefore B' = 50^\circ 18.4'. \end{array}$$

$$\begin{array}{r} \cos c' \tan A' \tan B' = 1. \\ \log \cos c' = 9.7257 \\ \log \tan A' = 0.1936 \\ \log \tan B' = \underline{0.0809} \\ \log 1 = 0.0002 \end{array}$$

\therefore in the quadrantal triangle,

$$a = 57^\circ 22.1', \quad b = 129^\circ 41.6', \quad \text{and} \quad C = 57^\circ 52.5'.$$

4. In the polar triangle, $A' = 149^\circ 40'$ and $c' = 137^\circ 20'$.

$$\begin{array}{r} \sin a' = \sin A' \sin c'. \\ \log \sin A' = 9.7033 \\ \log \sin c' = \underline{9.8311} \\ \log \sin a' = \underline{9.5344} \\ \therefore 180^\circ - a' = 20^\circ 0.9'. \end{array}$$

$$\begin{array}{r} \text{+} \quad \text{---} \quad \text{---} \\ \tan b' = \cos A' \tan c'. \\ \log \cos A' = 9.9361 \\ \log \tan c' = \underline{9.9646} \\ \log \tan b' = 9.9007 \\ \therefore b' = 38^\circ 30.4'. \end{array}$$

Check.

$$\begin{array}{r} \text{+} \\ \tan B' = \frac{\cot A'}{\cos c'}. \\ \log \cot A' = 0.2327 \\ \log \cos c' = \underline{9.8665} \\ \log \tan B' = 0.3662 \\ \therefore B' = 66^\circ 42.9'. \end{array}$$

$$\begin{array}{r} \tan B' = \frac{\tan b'}{\sin a'}. \\ \log \tan b' = 9.9007 \\ \log \sin a' = \underline{9.5344} \\ \log \tan B' = 0.3663 \end{array}$$

\therefore in the quadrantal triangle,

$$A = 20^\circ 0.9', \quad b = 113^\circ 17.1', \quad \text{and} \quad B = 141^\circ 29.6'.$$

5. In the polar triangle, $b' = 109^\circ 48'$ and $c' = 73^\circ 35'$.

$$\begin{array}{c} - \\ \cos a' = \frac{\cos c'}{\cos b'} \\ - \end{array}$$

$$\log \cos c' = 9.4512$$

$$\log \cos b' = \underline{9.5299}$$

$$\log \cos a' = 9.9213$$

$$\therefore 180^\circ - a' = 33^\circ 27.5'.$$

$$\sin B' = \frac{\sin b'}{\sin c'}$$

$$\log \sin b' = 9.9735$$

$$\log \sin c' = \underline{9.9819}$$

$$\log \sin B' = 9.9916$$

$$\therefore 180^\circ - B' = 78^\circ 46.7'.$$

$$\begin{array}{c} - \\ \cos A' = \frac{\tan b'}{\tan c'} \\ + \end{array}$$

$$\log \tan b' = 0.4437$$

$$\log \tan c' = \underline{0.5307}$$

$$\log \cos A' = 9.9130$$

$$\therefore 180^\circ - A' = 35^\circ 4.4'.$$

Check.

$$\sin B' = \frac{\cos A'}{\cos a'}$$

$$\log \cos A' = 9.9130$$

$$\log \cos a' = \underline{9.9213}$$

$$\log \sin B' = 9.9917$$

\therefore in the quadrantal triangle,

$$a = 35^\circ 4.4', \quad A = 33^\circ 27.5', \quad \text{and} \quad b = 78^\circ 46.7'.$$

6. In the polar triangle, $a' = 74^\circ 7'$ and $A' = 75^\circ 6'$.

$$\sin b' = \frac{\tan a'}{\tan A'}$$

$$\log \tan a' = 0.5459$$

$$\log \tan A' = \underline{0.5750}$$

$$\log \sin b' = 9.9709$$

$$\therefore b' = 69^\circ 16',$$

$$\text{or } 110^\circ 44'.$$

$$\sin B' = \frac{\cos A'}{\cos a'}$$

$$\log \cos A' = 9.4102$$

$$\log \cos a' = \underline{9.4372}$$

$$\log \sin B' = 9.9730$$

$$\therefore B' = 70^\circ,$$

$$\text{or } 110^\circ.$$

$$\sin c' = \frac{\sin a'}{\sin A'}$$

$$\log \sin a' = 9.9831$$

$$\log \sin A' = \underline{9.9851}$$

$$\log \sin c' = 9.9980$$

$$\therefore c' = 84^\circ 30',$$

$$\text{or } 95^\circ 30'.$$

Check.

$$\sin B' = \frac{\sin b'}{\sin c'}$$

$$\log \sin b' = 9.9709$$

$$\log \sin c' = \underline{9.9980}$$

$$\log \sin B' = 9.9729$$

\therefore in the quadrantal triangle,

$$1. \quad B = 69^\circ 16', \quad b = 70^\circ, \quad \text{and} \quad C = 84^\circ 30'.$$

$$2. \quad B = 110^\circ 44', \quad b = 110^\circ, \quad \text{and} \quad C = 95^\circ 30'.$$

CHAPTER XII.

Art. 167. — Page 125.

2. Here $\frac{1}{2}(A - B) = 18^\circ 30'$, $\frac{1}{2}(A + B) = 59^\circ 30'$, $\frac{1}{2}c = 54^\circ$.

$$\tan \frac{1}{2}(a - b) = \frac{\sin \frac{1}{2}(A - B)}{\sin \frac{1}{2}(A + B)} \tan \frac{1}{2}c. \quad \tan \frac{1}{2}(a + b) = \frac{\cos \frac{1}{2}(A - B)}{\cos \frac{1}{2}(A + B)} \tan \frac{1}{2}c.$$

$$\log \sin \frac{1}{2}(A - B) = 9.5015$$

$$\log \cos \frac{1}{2}(A - B) = 9.9770$$

$$\log \csc \frac{1}{2}(A + B) = 0.0647$$

$$\log \sec \frac{1}{2}(A + B) = 0.2945$$

$$\log \tan \frac{1}{2}c = 0.1387$$

$$\log \tan \frac{1}{2}c = 0.1387$$

$$\log \tan \frac{1}{2}(a - b) = 9.7049$$

$$\log \tan \frac{1}{2}(a + b) = 0.4102$$

$$\therefore \frac{1}{2}(a - b) = 26^\circ 52.8'$$

$$\therefore \frac{1}{2}(a + b) = 68^\circ 45'$$

$$\therefore a = \frac{1}{2}(a + b) + \frac{1}{2}(a - b) = 95^\circ 37.8',$$

$$\text{and } b = \frac{1}{2}(a + b) - \frac{1}{2}(a - b) = 41^\circ 52.2'.$$

$$\cot \frac{1}{2}C = \frac{\sin \frac{1}{2}(a + b)}{\sin \frac{1}{2}(a - b)} \tan \frac{1}{2}(A - B).$$

$$\log \sin \frac{1}{2}(a + b) = 9.9694$$

$$\log \csc \frac{1}{2}(a - b) = 0.3447$$

$$\log \tan \frac{1}{2}(A - B) = 9.5245$$

$$\log \cot \frac{1}{2}C = 9.8386$$

$$\frac{1}{2}C = 55^\circ 24.4'$$

$$\therefore C = 110^\circ 48.8'.$$

3. Here $\frac{1}{2}(B - C) = 42^\circ 30'$, $\frac{1}{2}(B + C) = 92^\circ 30'$, $\frac{1}{2}a = 35^\circ 10'$.

$$\tan \frac{1}{2}(b - c) = \frac{\sin \frac{1}{2}(B - C)}{\sin \frac{1}{2}(B + C)} \tan \frac{1}{2}a. \quad \tan \frac{1}{2}(b + c) = \frac{\cos \frac{1}{2}(B - C)}{\cos \frac{1}{2}(B + C)} \tan \frac{1}{2}a.$$

$$\log \sin \frac{1}{2}(B - C) = 9.8297$$

$$\log \cos \frac{1}{2}(B - C) = 9.8676$$

$$\log \csc \frac{1}{2}(B + C) = 0.0004$$

$$\log \sec \frac{1}{2}(B + C) = 1.3603$$

$$\log \tan \frac{1}{2}a = 9.8479$$

$$\log \tan \frac{1}{2}a = 9.8479$$

$$\log \tan \frac{1}{2}(b - c) = 9.6780$$

$$\log \tan \frac{1}{2}(b + c) = 1.0758$$

$$\therefore \frac{1}{2}(b - c) = 25^\circ 28.5'$$

$$180^\circ - \frac{1}{2}(b + c) = 85^\circ 11.9'$$

$$\therefore \frac{1}{2}(b + c) = 94^\circ 48.1'.$$

$$\therefore b = \frac{1}{2}(b + c) + \frac{1}{2}(b - c) = 120^\circ 16.6',$$

$$\text{and } c = \frac{1}{2}(b + c) - \frac{1}{2}(b - c) = 69^\circ 19.6'.$$

$$\cot \frac{1}{2} A = \frac{\sin \frac{1}{2} (b+c)}{\sin \frac{1}{2} (b-c)} \tan \frac{1}{2} (B-C).$$

$$\log \sin \frac{1}{2} (b+c) = 9.9985$$

$$\log \csc \frac{1}{2} (b-c) = 0.3664$$

$$\log \tan \frac{1}{2} (B-C) = \underline{9.9621}$$

$$\log \cot \frac{1}{2} A = 0.3270$$

$$\frac{1}{2} A = 25^\circ 13.1'$$

$$\therefore A = 50^\circ 26.2'.$$

4. Here $\frac{1}{2} (C-A) = 45^\circ 20'$, $\frac{1}{2} (C+A) = 77^\circ$, $\frac{1}{2} b = 20^\circ 20'$.

$$\tan \frac{1}{2} (c-a) = \frac{\sin \frac{1}{2} (C-A)}{\sin \frac{1}{2} (C+A)} \tan \frac{1}{2} b. \quad \tan \frac{1}{2} (c+a) = \frac{\cos \frac{1}{2} (C-A)}{\cos \frac{1}{2} (C+A)} \tan \frac{1}{2} b.$$

$$\log \sin \frac{1}{2} (C-A) = 9.8520$$

$$\log \cos \frac{1}{2} (C-A) = 9.8469$$

$$\log \csc \frac{1}{2} (C+A) = 0.0113$$

$$\log \sec \frac{1}{2} (C+A) = 0.6479$$

$$\log \tan \frac{1}{2} b = 9.5689$$

$$\log \tan \frac{1}{2} b = 9.5689$$

$$\log \tan \frac{1}{2} (c-a) = 9.4322$$

$$\log \tan \frac{1}{2} (c+a) = 0.0637$$

$$\therefore \frac{1}{2} (c-a) = 15^\circ 8.2'.$$

$$\therefore \frac{1}{2} (c+a) = 49^\circ 11.2'.$$

$$\therefore a = \frac{1}{2} (c+a) - \frac{1}{2} (c-a) = 34^\circ 3',$$

and

$$c = \frac{1}{2} (c+a) + \frac{1}{2} (c-a) = 64^\circ 19.4'.$$

$$\cot \frac{1}{2} B = \frac{\sin \frac{1}{2} (c+a)}{\sin \frac{1}{2} (c-a)} \tan \frac{1}{2} (C-A).$$

$$\log \sin \frac{1}{2} (c+a) = 9.8790$$

$$\log \csc \frac{1}{2} (c-a) = 0.5831$$

$$\log \tan \frac{1}{2} (C-A) = \underline{0.0051}$$

$$\log \cot \frac{1}{2} B = 0.4672$$

$$\frac{1}{2} B = 18^\circ 49.8'$$

$$\therefore B = 37^\circ 39.6'.$$

5. Here $\frac{1}{2} (B-A) = 18^\circ 47'$, $\frac{1}{2} (B+A) = 126^\circ 59'$, $\frac{1}{2} c = 63^\circ 16'$.

$$\tan \frac{1}{2} (b-a) = \frac{\sin \frac{1}{2} (B-A)}{\sin \frac{1}{2} (B+A)} \tan \frac{1}{2} c. \quad \tan \frac{1}{2} (b+a) = \frac{\cos \frac{1}{2} (B-A)}{\cos \frac{1}{2} (B+A)} \tan \frac{1}{2} c.$$

$$\log \sin \frac{1}{2} (B-A) = 9.5079$$

$$\log \cos \frac{1}{2} (B-A) = 9.9762$$

$$\log \csc \frac{1}{2} (B+A) = 0.0976$$

$$\log \sec \frac{1}{2} (B+A) = 0.2207$$

$$\log \tan \frac{1}{2} c = 0.2979$$

$$\log \tan \frac{1}{2} c = 0.2979$$

$$\log \tan \frac{1}{2} (b-a) = 9.9034$$

$$\log \tan \frac{1}{2} (b+a) = 0.4948$$

$$\therefore \frac{1}{2} (b-a) = 38^\circ 40.8'.$$

$$180^\circ - \frac{1}{2} (b+a) = 72^\circ 15.2'$$

$$\therefore \frac{1}{2} (b+a) = 107^\circ 44.8'.$$

$$\therefore a = \frac{1}{2} (b+a) - \frac{1}{2} (b-a) = 69^\circ 4',$$

and

$$b = \frac{1}{2} (b+a) + \frac{1}{2} (b-a) = 146^\circ 25.6'.$$

$$\cot \frac{1}{2} C = \frac{\sin \frac{1}{2} (b+a)}{\sin \frac{1}{2} (b-a)} \tan \frac{1}{2} (B-A).$$

$$\log \sin \frac{1}{2} (b+a) = 9.9788$$

$$\log \csc \frac{1}{2} (b-a) = 0.2042$$

$$\log \tan \frac{1}{2} (B-A) = 9.5316$$

$$\log \cot \frac{1}{2} C = 9.7146$$

$$\frac{1}{2} C = 62^\circ 36.1'$$

$$\therefore C = 125^\circ 12.2'.$$

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2. Here $\frac{1}{2}(a-b) = 12^\circ 30'$, $\frac{1}{2}(a+b) = 59^\circ 30'$, $\frac{1}{2}C = 16^\circ 30'$.

$$\tan \frac{1}{2}(A-B) = \frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)} \cot \frac{1}{2} C. \quad \tan \frac{1}{2}(A+B) = \frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \cot \frac{1}{2} C.$$

$$\log \sin \frac{1}{2}(a-b) = 9.3353$$

$$\log \cos \frac{1}{2}(a-b) = 9.9896$$

$$\log \csc \frac{1}{2}(a+b) = 0.0647$$

$$\log \sec \frac{1}{2}(a+b) = 0.2945$$

$$\log \cot \frac{1}{2} C = 0.5284$$

$$\log \cot \frac{1}{2} C = 0.5284$$

$$\log \tan \frac{1}{2}(A-B) = 9.9284$$

$$\log \tan \frac{1}{2}(A+B) = 0.8125$$

$$\therefore \frac{1}{2}(A-B) = 40^\circ 18'.$$

$$\therefore \frac{1}{2}(A+B) = 81^\circ 14.8'.$$

$$\therefore A = \frac{1}{2}(A+B) + \frac{1}{2}(A-B) = 121^\circ 32.8',$$

and

$$B = \frac{1}{2}(A+B) - \frac{1}{2}(A-B) = 40^\circ 56.8'.$$

$$\tan \frac{1}{2} c = \frac{\sin \frac{1}{2}(A+B)}{\sin \frac{1}{2}(A-B)} \tan \frac{1}{2}(a-b).$$

$$\log \sin \frac{1}{2}(A+B) = 9.9949$$

$$\log \csc \frac{1}{2}(A-B) = 0.1892$$

$$\log \tan \frac{1}{2}(a-b) = 9.3458$$

$$\log \tan \frac{1}{2} c = 9.5299$$

$$\frac{1}{2} c = 18^\circ 42.9'$$

$$\therefore c = 37^\circ 25.8'.$$

3. Here $\frac{1}{2}(a-c) = 19^\circ$, $\frac{1}{2}(a+c) = 79^\circ$, $\frac{1}{2}B = 55^\circ$.

$$\tan \frac{1}{2}(A-C) = \frac{\sin \frac{1}{2}(a-c)}{\sin \frac{1}{2}(a+c)} \cot \frac{1}{2} B. \quad \tan \frac{1}{2}(A+C) = \frac{\cos \frac{1}{2}(a-c)}{\cos \frac{1}{2}(a+c)} \cot \frac{1}{2} B.$$

$$\log \sin \frac{1}{2}(a-c) = 9.5126$$

$$\log \cos \frac{1}{2}(a-c) = 9.9757$$

$$\log \csc \frac{1}{2}(a+c) = 0.0081$$

$$\log \sec \frac{1}{2}(a+c) = 0.7194$$

$$\log \cot \frac{1}{2} B = 9.8452$$

$$\log \cot \frac{1}{2} B = 9.8452$$

$$\log \tan \frac{1}{2}(A-C) = 9.3659$$

$$\log \tan \frac{1}{2}(A+C) = 0.5403$$

$$\therefore \frac{1}{2}(A-C) = 13^\circ 4.4'.$$

$$\therefore \frac{1}{2}(A+C) = 73^\circ 55.3'.$$

$$\therefore A = \frac{1}{2}(A+C) + \frac{1}{2}(A-C) = 86^\circ 59.7',$$

and

$$C = \frac{1}{2}(A+C) - \frac{1}{2}(A-C) = 60^\circ 50.9'.$$

$$\tan \frac{1}{2} b = \frac{\sin \frac{1}{2} (A + C)}{\sin \frac{1}{2} (A - C)} \tan \frac{1}{2} (a - c).$$

$$\log \sin \frac{1}{2} (A + C) = 9.9827$$

$$\log \csc \frac{1}{2} (A - C) = 0.6455$$

$$\log \tan \frac{1}{2} (a - c) = \underline{9.5370}$$

$$\log \tan \frac{1}{2} b = \underline{0.1652}$$

$$\frac{1}{2} b = 55^\circ 38.5'$$

$$\therefore b = 111^\circ 17'.$$

4. Here $\frac{1}{2} (b - c) = 24^\circ 50'$, $\frac{1}{2} (b + c) = 95^\circ 30'$, $\frac{1}{2} A = 25^\circ$.

$$\tan \frac{1}{2} (B - C) = \frac{\sin \frac{1}{2} (b - c)}{\sin \frac{1}{2} (b + c)} \cot \frac{1}{2} A.$$

$$\log \sin \frac{1}{2} (b - c) = 9.6232$$

$$\log \csc \frac{1}{2} (b + c) = 0.0020$$

$$\log \cot \frac{1}{2} A = \underline{0.3313}$$

$$\log \tan \frac{1}{2} (B - C) = \underline{9.9565}$$

$$\therefore \frac{1}{2} (B - C) = 42^\circ 8.1'.$$

$$\tan \frac{1}{2} (B + C) = \frac{\cos \frac{1}{2} (b - c)}{\cos \frac{1}{2} (b + c)} \cot \frac{1}{2} A.$$

$$\log \cos \frac{1}{2} (b - c) = 9.9579$$

$$\log \sec \frac{1}{2} (b + c) = 1.0184$$

$$\log \cot \frac{1}{2} A = \underline{0.3313}$$

$$\log \tan \frac{1}{2} (B + C) = \underline{1.3076}$$

$$180^\circ - \frac{1}{2} (B + C) = 87^\circ 10.8'$$

$$\therefore \frac{1}{2} (B + C) = 92^\circ 49.2'.$$

$$\therefore B = \frac{1}{2} (B + C) + \frac{1}{2} (B - C) = 134^\circ 57.3',$$

$$C = \frac{1}{2} (B + C) - \frac{1}{2} (B - C) = 50^\circ 41.1'.$$

and

$$\tan \frac{1}{2} a = \frac{\sin \frac{1}{2} (B + C)}{\sin \frac{1}{2} (B - C)} \tan \frac{1}{2} (b - c).$$

$$\log \sin \frac{1}{2} (B + C) = 9.9995$$

$$\log \csc \frac{1}{2} (B - C) = 0.1734$$

$$\log \tan \frac{1}{2} (b - c) = \underline{9.6664}$$

$$\log \tan \frac{1}{2} a = \underline{9.8393}$$

$$\frac{1}{2} a = 34^\circ 34.4'$$

$$\therefore a = 69^\circ 8.8'.$$

5. Here $\frac{1}{2} (b - a) = 14^\circ 20'$, $\frac{1}{2} (b + a) = 139^\circ 30'$, $\frac{1}{2} C = 70^\circ 10'$.

$$\tan \frac{1}{2} (B - A) = \frac{\sin \frac{1}{2} (b - a)}{\sin \frac{1}{2} (b + a)} \cot \frac{1}{2} C.$$

$$\log \sin \frac{1}{2} (b - a) = 9.3937$$

$$\log \csc \frac{1}{2} (b + a) = 0.1875$$

$$\log \cot \frac{1}{2} C = \underline{9.5571}$$

$$\log \tan \frac{1}{2} (B - A) = \underline{9.1383}$$

$$\therefore \frac{1}{2} (B - A) = 7^\circ 49.8'.$$

$$\tan \frac{1}{2} (B + A) = \frac{\cos \frac{1}{2} (b - a)}{\cos \frac{1}{2} (b + a)} \cot \frac{1}{2} C.$$

$$\log \cos \frac{1}{2} (b - a) = 9.9863$$

$$\log \sec \frac{1}{2} (b + a) = 0.1190$$

$$\log \cot \frac{1}{2} C = \underline{9.5571}$$

$$\log \tan \frac{1}{2} (B + A) = \underline{9.6624}$$

$$180^\circ - \frac{1}{2} (B + A) = 24^\circ 41.2'$$

$$\therefore \frac{1}{2} (B + A) = 155^\circ 18.8'.$$

$$\therefore A = \frac{1}{2} (B + A) - \frac{1}{2} (B - A) = 147^\circ 29',$$

$$B = \frac{1}{2} (B + A) + \frac{1}{2} (B - A) = 163^\circ 8.6'.$$

and

$$\tan \frac{1}{2}c = \frac{\sin \frac{1}{2}(B+A)}{\sin \frac{1}{2}(B-A)} \tan \frac{1}{2}(b-a).$$

$$\log \sin \frac{1}{2}(B+A) = 9.6208$$

$$\log \csc \frac{1}{2}(B-A) = 0.8657$$

$$\log \tan \frac{1}{2}(b-a) = 9.4074$$

$$\log \tan \frac{1}{2}c = 9.8939$$

$$\frac{1}{2}c = 38^\circ 4.2'$$

$$\therefore c = 76^\circ 8.4'.$$

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2. Here $s = 65^\circ 30'$, $s-a = 27^\circ 30'$, $s-b = 14^\circ 30'$, $s-c = 23^\circ 30'$.

$$\log \sin (s-b) = 9.3986$$

$$\log \sin (s-c) = 9.6007$$

$$\log \sin (s-c) = 9.6007$$

$$\log \sin (s-a) = 9.6644$$

$$\log \csc s = 0.0410$$

$$\log \csc s = 0.0410$$

$$\log \csc (s-a) = 0.3356$$

$$\log \csc (s-b) = 0.6014$$

$$2) \underline{9.3759}$$

$$2) \underline{9.9075}$$

$$\log \tan \frac{1}{2}A = 9.6879$$

$$\log \tan \frac{1}{2}B = 9.9537$$

$$\frac{1}{2}A = 25^\circ 59.1'$$

$$\frac{1}{2}B = 41^\circ 57.2'$$

$$\therefore A = 51^\circ 58.2'.$$

$$\therefore B = 83^\circ 54.4'.$$

$$\log \sin (s-a) = 9.6644$$

$$\log \sin (s-b) = 9.3986$$

$$\log \csc s = 0.0410$$

$$\log \csc (s-c) = 0.3993$$

$$2) \underline{9.5033}$$

$$\log \tan \frac{1}{2}C = 9.7516$$

$$\frac{1}{2}C = 29^\circ 26.6'$$

$$\therefore C = 58^\circ 53.2'.$$

3. Here $s = 105^\circ$, $s-a = 4^\circ$, $s-b = 56^\circ$, $s-c = 45^\circ$.

$$\log \sin (s-b) = 9.9186$$

$$\log \sin (s-c) = 9.8495$$

$$\log \sin (s-c) = 9.8495$$

$$\log \sin (s-a) = 8.8436$$

$$\log \csc s = 0.0151$$

$$\log \csc s = 0.0151$$

$$\log \csc (s-a) = 1.1564$$

$$\log \csc (s-b) = 0.0814$$

$$2) \underline{0.9396}$$

$$2) \underline{8.7896}$$

$$\log \tan \frac{1}{2}A = 0.4698$$

$$\log \tan \frac{1}{2}B = 9.3948$$

$$\frac{1}{2}A = 71^\circ 16.4'$$

$$\frac{1}{2}B = 13^\circ 56.3'$$

$$\therefore A = 142^\circ 32.8'.$$

$$\therefore B = 27^\circ 52.6'.$$

$$\begin{aligned}
 \log \sin (s-a) &= 8.8436 \\
 \log \sin (s-b) &= 9.9186 \\
 \log \csc s &= 0.0151 \\
 \log \csc (s-c) &= 0.1505 \\
 &\quad \underline{2)8.9278} \\
 \log \tan \frac{1}{2} C &= 9.4639 \\
 \frac{1}{2} C &= 16^\circ 13.6' \\
 \therefore C &= 32^\circ 27.2'.
 \end{aligned}$$

4. Here $s = 96^\circ$, $s - a = 35^\circ$, $s - b = 57^\circ$, $s - c = 4^\circ$.

$$\begin{array}{ll}
 \log \sin (s-b) = 9.9236 & \log \sin (s-c) = 8.8436 \\
 \log \sin (s-c) = 8.8436 & \log \sin (s-a) = 9.7586 \\
 \log \csc s = 0.0024 & \log \csc s = 0.0024 \\
 \log \csc (s-a) = \underline{0.2414} & \log \csc (s-b) = \underline{0.0764} \\
 & \underline{2)9.0110} & \underline{2)8.6810} \\
 \log \tan \frac{1}{2} A = 9.5055 & \log \tan \frac{1}{2} B = 9.3405 \\
 \frac{1}{2} A = 17^\circ 45.6' & \frac{1}{2} B = 12^\circ 21.3' \\
 \therefore A = 35^\circ 31.2' & \therefore B = 24^\circ 42.6'.
 \end{array}$$

$$\begin{aligned}
 \log \sin (s-a) &= 9.7586 \\
 \log \sin (s-b) &= 9.9236 \\
 \log \csc s &= 0.0024 \\
 \log \csc (s-c) &= 1.1564 \\
 &\quad \underline{2)0.8410} \\
 \log \tan \frac{1}{2} C &= 0.4205 \\
 \frac{1}{2} C &= 69^\circ 12.4' \\
 \therefore C &= 138^\circ 24.8'.
 \end{aligned}$$

5. Here $s = 107^\circ 10'$, $s - a = 44^\circ 50'$, $s - b = 53^\circ$, $s - c = 9^\circ 20'$.

$$\begin{array}{ll}
 \log \sin (s-b) = 9.9023 & \log \sin (s-c) = 9.2100 \\
 \log \sin (s-c) = 9.2100 & \log \sin (s-a) = 9.8482 \\
 \log \csc s = 0.0198 & \log \csc s = 0.0198 \\
 \log \csc (s-a) = \underline{0.1518} & \log \csc (s-b) = \underline{0.0977} \\
 & \underline{2)9.2839} & \underline{2)9.1757} \\
 \log \tan \frac{1}{2} A = 9.6419 & \log \tan \frac{1}{2} B = 9.5878 \\
 \frac{1}{2} A = 23^\circ 40.6' & \frac{1}{2} B = 21^\circ 9.7' \\
 \therefore A = 47^\circ 21.2' & \therefore B = 42^\circ 19.4'.
 \end{array}$$

$$\log \sin (s-a) = 9.8482$$

$$\log \sin (s-b) = 9.9023$$

$$\log \csc s = 0.0198$$

$$\log \csc (s-c) = 0.7900$$

$$\begin{array}{r} 2)0.5603 \\ \hline \end{array}$$

$$\log \tan \frac{1}{2}C = 0.2801$$

$$\frac{1}{2}C = 62^\circ 19'$$

$$\therefore C = 124^\circ 38'.$$

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2. Here $S = 109^\circ$, $S - A = 34^\circ$, $S - B = 27^\circ$, $S - C = 48^\circ$.

$$\log \cos S = 9.5126$$

$$\log \cos (S - A) = 9.9186$$

$$\log \sec (S - B) = 0.0501$$

$$\log \sec (S - C) = 0.1745$$

$$\begin{array}{r} 2)9.6558 \\ \hline \end{array}$$

$$\log \tan \frac{1}{2}a = 9.8279$$

$$\frac{1}{2}a = 33^\circ 55.9'$$

$$\therefore a = 67^\circ 51.8'.$$

$$\log \cos S = 9.5126$$

$$\log \cos (S - B) = 9.9499$$

$$\log \sec (S - C) = 0.1745$$

$$\log \sec (S - A) = 0.0814$$

$$\begin{array}{r} 2)9.7184 \\ \hline \end{array}$$

$$\log \tan \frac{1}{2}b = 9.8592$$

$$\frac{1}{2}b = 35^\circ 52.2'$$

$$\therefore b = 71^\circ 44.4'.$$

$$\log \cos S = 9.5126$$

$$\log \cos (S - C) = 9.8255$$

$$\log \sec (S - A) = 0.0814$$

$$\log \sec (S - B) = 0.0501$$

$$\begin{array}{r} 2)9.4696 \\ \hline \end{array}$$

$$\log \tan \frac{1}{2}c = 9.7348$$

$$\frac{1}{2}c = 28^\circ 30'$$

$$\therefore c = 57^\circ.$$

3. Here $S = 165^\circ$, $S - A = 45^\circ$, $S - B = 35^\circ$, $S - C = 85^\circ$.

$$\log \cos S = 9.9849$$

$$\log \cos (S - A) = 9.8495$$

$$\log \sec (S - B) = 0.0866$$

$$\log \sec (S - C) = 1.0597$$

$$\begin{array}{r} 2)0.9807 \\ \hline \end{array}$$

$$\log \tan \frac{1}{2}a = 0.4903$$

$$\frac{1}{2}a = 72^\circ 4.9'$$

$$\therefore a = 144^\circ 9.8'.$$

$$\log \cos S = 9.9849$$

$$\log \cos (S - B) = 9.9134$$

$$\log \sec (S - C) = 1.0597$$

$$\log \sec (S - A) = 0.1505$$

$$\begin{array}{r} 2)1.1085 \\ \hline \end{array}$$

$$\log \tan \frac{1}{2}b = 0.5542$$

$$\frac{1}{2}b = 74^\circ 24.3'$$

$$\therefore b = 148^\circ 48.6'.$$

$$\begin{aligned}
 \log \cos S &= 9.9849 \\
 \log \cos (S - C) &= 8.9403 \\
 \log \sec (S - A) &= 0.1505 \\
 \log \sec (S - B) &= 0.0866 \\
 &\quad \underline{2)9.1623} \\
 \log \tan \frac{1}{2} c &= 9.5811 \\
 \frac{1}{2} c &= 20^\circ 51.8' \\
 \therefore c &= 41^\circ 43.6'.
 \end{aligned}$$

4. Here $S = 124^\circ 40'$, $S - A = 33^\circ 30'$, $S - B = 39^\circ$, $S - C = 52^\circ 10'$.

$$\begin{array}{ll}
 \log \cos S &= 9.7550 & \log \cos S &= 9.7550 \\
 \log \cos (S - A) &= 9.9211 & \log \cos (S - B) &= 9.8905 \\
 \log \sec (S - B) &= 0.1095 & \log \sec (S - C) &= 0.2123 \\
 \log \sec (S - C) &= 0.2123 & \log \sec (S - A) &= 0.0789 \\
 & & & \underline{2)9.9367} \\
 \log \tan \frac{1}{2} a &= 9.9989 & \log \tan \frac{1}{2} b &= 9.9683 \\
 \frac{1}{2} a &= 44^\circ 55.6' & \frac{1}{2} b &= 42^\circ 54.6' \\
 \therefore a &= 89^\circ 51.2' & \therefore b &= 85^\circ 49.2'.
 \end{array}$$

$$\begin{aligned}
 \log \cos S &= 9.7550 \\
 \log \cos (S - C) &= 9.7877 \\
 \log \sec (S - A) &= 0.0789 \\
 \log \sec (S - B) &= 0.1095 \\
 &\quad \underline{2)9.7311} \\
 \log \tan \frac{1}{2} c &= 9.8655 \\
 \frac{1}{2} c &= 36^\circ 15.9' \\
 \therefore c &= 72^\circ 31.8'.
 \end{aligned}$$

5. Here

$S = 102^\circ 40'$, $S - A = -35^\circ 36'$, $S - B = 71^\circ 29'$, $S - C = 66^\circ 47'$.

$$\begin{array}{ll}
 \log \cos S &= 9.3410 & \log \cos S &= 9.3410 \\
 \log \cos (S - A) &= 9.9102 & \log \cos (S - B) &= 9.5019 \\
 \log \sec (S - B) &= 0.4981 & \log \sec (S - C) &= 0.4043 \\
 \log \sec (S - C) &= 0.4043 & \log \sec (S - A) &= 0.0898 \\
 & & & \underline{2)9.3370} \\
 \log \tan \frac{1}{2} a &= 0.0768 & \log \tan \frac{1}{2} b &= 9.6685 \\
 \frac{1}{2} a &= 50^\circ 2.3' & \frac{1}{2} b &= 24^\circ 59.4' \\
 \therefore a &= 100^\circ 4.6' & \therefore b &= 49^\circ 58.8'.
 \end{array}$$

$$\log \cos S = 9.3410$$

$$\log \cos (S - C) = 9.5957$$

$$\log \sec (S - A) = 0.0898$$

$$\log \sec (S - B) = 0.4981$$

$$\underline{2)9.5246}$$

$$\log \tan \frac{1}{2}c = 9.7623$$

$$\frac{1}{2}c = 30^\circ 3'$$

$$\therefore c = 60^\circ 6'.$$

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$$4. \quad \sin C = \frac{\sin c \sin B}{\sin b}.$$

$$\log \sin c = 9.9549$$

$$\log \csc b = 0.0062$$

$$\log \sin B = 9.9979$$

$$\log \sin C = 9.9590$$

$$\therefore C = 65^\circ 30'.$$

$$\frac{1}{2}(b + c) = 82^\circ,$$

$$\frac{1}{2}(b - c) = 17^\circ 40'.$$

$$\frac{1}{2}(B + C) = 80^\circ 35',$$

$$\frac{1}{2}(B - C) = 15^\circ 5'.$$

$$\cot \frac{1}{2}A = \frac{\sin \frac{1}{2}(b + c)}{\sin \frac{1}{2}(b - c)} \tan \frac{1}{2}(B - C).$$

$$\log \sin \frac{1}{2}(b + c) = 9.9958$$

$$\log \csc \frac{1}{2}(b - c) = 0.5179$$

$$\log \tan \frac{1}{2}(B - C) = 9.4306$$

$$\log \cot \frac{1}{2}A = 9.9443$$

$$\frac{1}{2}A = 48^\circ 40'$$

$$\therefore A = 97^\circ 20'.$$

$$\tan \frac{1}{2}a = \frac{\sin \frac{1}{2}(B + C)}{\sin \frac{1}{2}(B - C)} \tan \frac{1}{2}(b - c).$$

$$\log \sin \frac{1}{2}(B + C) = 9.9941$$

$$\log \csc \frac{1}{2}(B - C) = 0.5847$$

$$\log \tan \frac{1}{2}(b - c) = 9.5031$$

$$\log \tan \frac{1}{2}a = 0.0819$$

$$\frac{1}{2}a = 50^\circ 22.3'$$

$$\therefore a = 100^\circ 44.6'.$$

$$5. \quad \sin B = \frac{\sin b \sin A}{\sin a}.$$

$$\log \sin b = 9.9446$$

$$\log \csc a = 0.1919$$

$$\log \sin A = 9.6946$$

$$\log \sin B = 9.8311$$

$$\therefore B = 42^\circ 40', \text{ or } 137^\circ 20'.$$

$$\frac{1}{2}(b + a) = 79^\circ 10',$$

$$\frac{1}{2}(b - a) = 39^\circ 10'.$$

$$\frac{1}{2}(B + A) = 36^\circ 10', \text{ or } 83^\circ 30'.$$

$$\frac{1}{2}(B - A) = 6^\circ 30', \text{ or } 53^\circ 50'.$$

$$\cot \frac{1}{2}C = \frac{\sin \frac{1}{2}(b + a)}{\sin \frac{1}{2}(b - a)} \tan \frac{1}{2}(B - A).$$

$$\tan \frac{1}{2}c = \frac{\sin \frac{1}{2}(B + A)}{\sin \frac{1}{2}(B - A)} \tan \frac{1}{2}(b - a).$$

Using the first value of B , we have :

$$\log \sin \frac{1}{2}(b + a) = 9.9922$$

$$\log \csc \frac{1}{2}(b - a) = 0.1996$$

$$\log \tan \frac{1}{2}(B - A) = 9.0567$$

$$\log \cot \frac{1}{2}C = 9.2485$$

$$\frac{1}{2}C = 79^\circ 57'$$

$$\therefore C = 159^\circ 54'.$$

$$\log \sin \frac{1}{2}(B + A) = 9.7710$$

$$\log \csc \frac{1}{2}(B - A) = 0.9461$$

$$\log \tan \frac{1}{2}(b - a) = 9.9110$$

$$\log \tan \frac{1}{2}c = 0.6281$$

$$\frac{1}{2}c = 76^\circ 45.1'$$

$$\therefore c = 153^\circ 30.2'.$$

Using the second value of B , we have:

$$\begin{array}{ll}
 \log \sin \frac{1}{2}(b+a) = 9.9922 & \log \sin \frac{1}{2}(B+A) = 9.9972 \\
 \log \csc \frac{1}{2}(b-a) = 0.1996 & \log \csc \frac{1}{2}(B-A) = 0.0930 \\
 \log \tan \frac{1}{2}(B-A) = \underline{0.1361} & \log \tan \frac{1}{2}(b-a) = 9.9110 \\
 \log \cot \frac{1}{2}C = 0.3279 & \log \tan \frac{1}{2}c = 0.0012 \\
 \frac{1}{2}C = 25^\circ 10.3' & \frac{1}{2}c = 45^\circ 4.8' \\
 \therefore C = 50^\circ 20.6' & \therefore c = 90^\circ 9.6'.
 \end{array}$$

$$\begin{array}{l}
 6. \qquad \sin A = \frac{\sin a \sin C}{\sin c} \\
 \log \sin a = 9.9561 \\
 \log \csc c = 0.2562 \\
 \log \sin C = \underline{9.7973} \\
 \log \sin A = 0.0096
 \end{array}$$

Since $\log \sin A$ is *positive*, the triangle is impossible.

$$\begin{array}{ll}
 7. \qquad \sin C = \frac{\sin c \sin A}{\sin a} & \cos B = \frac{\tan a}{\tan c} \\
 \log \sin c = 9.9958 & \log \tan a = 0.4549 \\
 \log \csc a = 0.0252 & \log \tan c = \underline{0.8522} \\
 \log \sin A = \underline{9.9790} & \log \cos B = \underline{9.6027} \\
 \log \sin C = \underline{0.0000} & 180^\circ - B = 66^\circ 23.1' \\
 \therefore C = 90^\circ. & \therefore B = 113^\circ 36.9'.
 \end{array}$$

$$\begin{array}{l}
 \cos b = \frac{\cos c}{\cos a} \\
 \log \cos c = 9.1436 \\
 \log \cos a = \underline{9.5199} \\
 \log \cos b = \underline{9.6237} \\
 180^\circ - b = 65^\circ 8.1' \\
 \therefore b = 114^\circ 51.9'.
 \end{array}$$

$$\begin{array}{ll}
 8. \qquad \sin B = \frac{\sin b \sin C}{\sin c} & \frac{1}{2}(b+c) = 74^\circ 40', \\
 \log \sin b = 9.9770 & \frac{1}{2}(b-c) = 33^\circ 50'. \\
 \log \csc c = 0.1845 & \frac{1}{2}(B+C) = 54^\circ 4', \text{ or } 75^\circ 46'. \\
 \log \sin C = \underline{9.8066} & \frac{1}{2}(B-C) = 14^\circ 14', \text{ or } 35^\circ 56'. \\
 \log \sin B = 9.9681 & \\
 \therefore B = 68^\circ 18', \text{ or } 111^\circ 42'. &
 \end{array}$$

$$\cot \frac{1}{2}A = \frac{\sin \frac{1}{2}(b+c)}{\sin \frac{1}{2}(b-c)} \tan \frac{1}{2}(B-C). \quad \tan \frac{1}{2}a = \frac{\sin \frac{1}{2}(B+C)}{\sin \frac{1}{2}(B-C)} \tan \frac{1}{2}(b-c).$$

Using the first value of B , we have :

$$\begin{array}{ll} \log \sin \frac{1}{2}(b+c) = 9.9843 & \log \sin \frac{1}{2}(B+C) = 9.9084 \\ \log \csc \frac{1}{2}(b-c) = 0.2543 & \log \csc \frac{1}{2}(B-C) = 0.6093 \\ \log \tan \frac{1}{2}(B-C) = \underline{9.4042} & \log \tan \frac{1}{2}(b-c) = \underline{9.8263} \\ \log \cot \frac{1}{2}A = 9.6428 & \log \tan \frac{1}{2}a = 0.3440 \\ \frac{1}{2}A = 66^{\circ} 16.9' & \frac{1}{2}a = 65^{\circ} 37.9' \\ \therefore A = 132^{\circ} 33.8'. & \therefore a = 131^{\circ} 15.8'. \end{array}$$

Using the second value of B , we have :

$$\begin{array}{ll} \log \sin \frac{1}{2}(b+c) = 9.9843 & \log \sin \frac{1}{2}(B+C) = 9.9865 \\ \log \csc \frac{1}{2}(b-c) = 0.2543 & \log \csc \frac{1}{2}(B-C) = 0.2315 \\ \log \tan \frac{1}{2}(B-C) = \underline{9.8602} & \log \tan \frac{1}{2}(b-c) = \underline{9.8263} \\ \log \cot \frac{1}{2}A = 0.0988 & \log \tan \frac{1}{2}a = 0.0443 \\ \frac{1}{2}A = 38^{\circ} 32.3' & \frac{1}{2}a = 47^{\circ} 55' \\ \therefore A = 77^{\circ} 4.6'. & \therefore a = 95^{\circ} 50'. \end{array}$$

$$9. \quad \sin A = \frac{\sin a \sin B}{\sin b}$$

$$\log \sin a = 9.4821$$

$$\log \csc b = 0.5686$$

$$\log \sin B = \underline{9.9134}$$

$$\log \sin A = 9.9641$$

$$\therefore A = 67^{\circ} 1.7', \text{ or } 112^{\circ} 58.3'.$$

Since both values of A are $< B$, while a is given $> b$, the triangle is impossible.

$$10. \quad \sin C = \frac{\sin c \sin A}{\sin a}$$

$$\log \sin c = 9.8241$$

$$\log \csc a = 0.0866$$

$$\log \sin A = \underline{9.8297}$$

$$\log \sin C = 9.7404$$

$$\therefore C = 146^{\circ} 37.9'.$$

$$\frac{1}{2}(c+a) = 96^{\circ} 35',$$

$$\frac{1}{2}(c-a) = 41^{\circ} 35'.$$

$$\frac{1}{2}(C+A) = 94^{\circ} 33.95',$$

$$\frac{1}{2}(C-A) = 52^{\circ} 3.95'.$$

$$\cot \frac{1}{2}B = \frac{\sin \frac{1}{2}(c+a)}{\sin \frac{1}{2}(c-a)} \tan \frac{1}{2}(C-A). \quad \tan \frac{1}{2}b = \frac{\sin \frac{1}{2}(C+A)}{\sin \frac{1}{2}(C-A)} \tan \frac{1}{2}(c-a).$$

$$\log \sin \frac{1}{2}(c+a) = 9.9972$$

$$\log \csc \frac{1}{2}(c-a) = 0.1780$$

$$\log \tan \frac{1}{2}(C-A) = \underline{0.1082}$$

$$\log \cot \frac{1}{2}B = 0.2834$$

$$\frac{1}{2}B = 27^{\circ} 30.3'$$

$$\therefore B = 55^{\circ} 0.6'.$$

$$\log \sin \frac{1}{2}(C+A) = 9.9987$$

$$\log \csc \frac{1}{2}(C-A) = 0.1031$$

$$\log \tan \frac{1}{2}(c-a) = \underline{9.9481}$$

$$\log \tan \frac{1}{2}b = 0.0499$$

$$\frac{1}{2}b = 48^{\circ} 17.2'$$

$$\therefore b = 96^{\circ} 34.4'.$$

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$$\begin{array}{ll}
 2. & \sin b = \frac{\sin B \sin c}{\sin C} & \frac{1}{2}(B + C) = 98^\circ, \\
 & \log \sin B = 9.9537 & \frac{1}{2}(B - C) = 18^\circ. \\
 & \log \csc C = 0.0066 & \frac{1}{2}(b + c) = 99^\circ 25', \\
 & \log \sin c = 9.9976 & \frac{1}{2}(b - c) = 15^\circ 25'. \\
 & \log \sin b = 9.9579 \\
 & \therefore b = 114^\circ 50'.
 \end{array}$$

$$\begin{array}{ll}
 \cot \frac{1}{2} A = \frac{\sin \frac{1}{2}(b + c)}{\sin \frac{1}{2}(b - c)} \tan \frac{1}{2}(B - C). & \tan \frac{1}{2} a = \frac{\sin \frac{1}{2}(B + C)}{\sin \frac{1}{2}(B - C)} \tan \frac{1}{2}(b - c). \\
 \log \sin \frac{1}{2}(b + c) = 9.9941 & \log \sin \frac{1}{2}(B + C) = 9.9958 \\
 \log \csc \frac{1}{2}(b - c) = 0.5754 & \log \csc \frac{1}{2}(B - C) = 0.5100 \\
 \log \tan \frac{1}{2}(B - C) = 9.5118 & \log \tan \frac{1}{2}(b - c) = 9.4405 \\
 \log \cot \frac{1}{2} A = 0.0813 & \log \tan \frac{1}{2} a = 9.9463 \\
 \frac{1}{2} A = 39^\circ 40' & \frac{1}{2} a = 41^\circ 28' \\
 \therefore A = 79^\circ 20' & \therefore a = 82^\circ 56'.
 \end{array}$$

$$\begin{array}{ll}
 3. & \sin a = \frac{\sin A \sin b}{\sin B} & \frac{1}{2}(B + A) = 136^\circ, \\
 & \log \sin A = 9.8711 & \frac{1}{2}(B - A) = 4^\circ. \\
 & \log \csc B = 0.1919 & \frac{1}{2}(b + a) = 97^\circ 12', \text{ or } 119^\circ 48'. \\
 & \log \sin b = 9.9023 & \frac{1}{2}(b - a) = 29^\circ 48', \text{ or } 7^\circ 12'. \\
 & \log \sin a = 9.9653 \\
 & \therefore a = 67^\circ 24', \text{ or } 112^\circ 36'.
 \end{array}$$

$$\cot \frac{1}{2} C = \frac{\sin \frac{1}{2}(b + a)}{\sin \frac{1}{2}(b - a)} \tan \frac{1}{2}(B - A). \quad \tan \frac{1}{2} c = \frac{\sin \frac{1}{2}(B + A)}{\sin \frac{1}{2}(B - A)} \tan \frac{1}{2}(b - a).$$

Using the first value of a , we have :

$$\begin{array}{ll}
 \log \sin \frac{1}{2}(b + a) = 9.9966 & \log \sin \frac{1}{2}(B + A) = 9.8418 \\
 \log \csc \frac{1}{2}(b - a) = 0.3036 & \log \csc \frac{1}{2}(B - A) = 1.1564 \\
 \log \tan \frac{1}{2}(B - A) = 8.8446 & \log \tan \frac{1}{2}(b - a) = 9.7579 \\
 \log \cot \frac{1}{2} C = 9.1448 & \log \tan \frac{1}{2} c = 0.7561 \\
 \frac{1}{2} C = 82^\circ 3.2' & \frac{1}{2} c = 80^\circ 3.2' \\
 \therefore C = 164^\circ 6.4' & \therefore c = 160^\circ 6.4'.
 \end{array}$$

Using the second value of a , we have :

$$\begin{array}{ll}
 \log \sin \frac{1}{2}(b + a) = 9.9384 & \log \sin \frac{1}{2}(B + A) = 9.8418 \\
 \log \csc \frac{1}{2}(b - a) = 0.9019 & \log \csc \frac{1}{2}(B - A) = 1.1564 \\
 \log \tan \frac{1}{2}(B - A) = 8.8446 & \log \tan \frac{1}{2}(b - a) = 9.1015 \\
 \log \cot \frac{1}{2} C = 9.6849 & \log \tan \frac{1}{2} c = 0.0997 \\
 \frac{1}{2} C = 64^\circ 10.3' & \frac{1}{2} c = 51^\circ 31.2' \\
 \therefore C = 128^\circ 20.6' & \therefore c = 103^\circ 2.4'.
 \end{array}$$

$$4. \quad \sin c = \frac{\sin C \sin a}{\sin A}$$

$$\log \sin C = 9.9904$$

$$\log \csc A = 0.0541$$

$$\log \sin a = \underline{9.9555}$$

$$\log \sin c = \underline{0.0000}$$

$$\therefore c = 90^\circ.$$

The triangle is a quadrantal triangle, and the sides a' and c' of its polar triangle are 118° and 78° .

$$\overset{-}{\cos b'} = \frac{\overset{+}{\cos c'}}{\overset{-}{\cos a'}}$$

$$\overset{-}{\cos B'} = \frac{\overset{-}{\tan a'}}{\overset{+}{\tan c'}}$$

$$\log \cos c' = 9.3179$$

$$\log \tan a' = 0.2743$$

$$\log \cos a' = \underline{9.6716}$$

$$\log \tan c' = \underline{0.6725}$$

$$\log \cos b' = \underline{9.6463}$$

$$\log \cos B' = \underline{9.6018}$$

$$\therefore 180^\circ - b' = 63^\circ 42.7'.$$

$$\therefore 180^\circ - B' = 66^\circ 26.2'.$$

Therefore in the given triangle, $B = 63^\circ 42.7'$, and $b = 66^\circ 26.2'$.

$$5. \quad \sin b = \frac{\sin B \sin a}{\sin A}$$

$$\log \sin B = 9.9624$$

$$\log \csc A = 0.1418$$

$$\log \sin a = \underline{9.9948}$$

$$\log \sin b = \underline{0.0990}$$

Since $\log \sin b$ is positive, the triangle is impossible.

$$6. \quad \sin b = \frac{\sin B \sin c}{\sin C}$$

$$\frac{1}{2}(C+B) = 84^\circ 30',$$

$$\frac{1}{2}(C-B) = 62^\circ 10'.$$

$$\log \sin B = 9.5798$$

$$\frac{1}{2}(c+b) = 82^\circ 51.05',$$

$$\log \csc C = 0.2600$$

$$\frac{1}{2}(c-b) = 55^\circ 28.95'.$$

$$\log \sin c = \underline{9.8227}$$

$$\log \sin b = \underline{9.6625}$$

$$\therefore b = 27^\circ 22.1'.$$

$$\cot \frac{1}{2}A = \frac{\sin \frac{1}{2}(c+b)}{\sin \frac{1}{2}(c-b)} \tan \frac{1}{2}(C-B), \quad \tan \frac{1}{2}a = \frac{\sin \frac{1}{2}(C+B)}{\sin \frac{1}{2}(C-B)} \tan \frac{1}{2}(c-b).$$

$$\log \sin \frac{1}{2}(c+b) = 9.9966$$

$$\log \sin \frac{1}{2}(C+B) = 9.9980$$

$$\log \csc \frac{1}{2}(c-b) = 0.0841$$

$$\log \csc \frac{1}{2}(C-B) = 0.0534$$

$$\log \tan \frac{1}{2}(C-B) = \underline{0.2774}$$

$$\log \tan \frac{1}{2}(c-b) = \underline{0.1626}$$

$$\log \cot \frac{1}{2}A = \underline{0.3581}$$

$$\log \tan \frac{1}{2}a = \underline{0.2140}$$

$$\frac{1}{2}A = 23^\circ 40.6'$$

$$\frac{1}{2}a = 58^\circ 34.6'$$

$$\therefore A = 47^\circ 21.2'.$$

$$\therefore a = 117^\circ 9.2'.$$

$$7. \quad \sin a = \frac{\sin A \sin c}{\sin C} \qquad \begin{aligned} \frac{1}{2}(C+A) &= 101^\circ, \\ \frac{1}{2}(C-A) &= 39^\circ 20'. \end{aligned}$$

$$\begin{aligned} \log \sin A &= 9.9446 & \frac{1}{2}(c+a) &= 96^\circ 41.55', \\ \log \csc C &= 0.1950 & & \text{or } 143^\circ 38.45'. \\ \log \sin c &= \underline{9.6946} & \frac{1}{2}(c-a) &= 53^\circ 38.45', \\ \log \sin a &= \underline{9.8342} & & \text{or } 6^\circ 41.55'. \end{aligned}$$

$$\therefore a = 43^\circ 3.1', \text{ or } 136^\circ 56.9'.$$

$$\cot \frac{1}{2}B = \frac{\sin \frac{1}{2}(c+a)}{\sin \frac{1}{2}(c-a)} \tan \frac{1}{2}(C-A). \quad \tan \frac{1}{2}b = \frac{\sin \frac{1}{2}(C+A)}{\sin \frac{1}{2}(C-A)} \tan \frac{1}{2}(c-a).$$

Using the first value of a , we have :

$$\begin{aligned} \log \sin \frac{1}{2}(c+a) &= 9.9971 & \log \sin \frac{1}{2}(C+A) &= 9.9919 \\ \log \csc \frac{1}{2}(c-a) &= 0.0940 & \log \csc \frac{1}{2}(C-A) &= 0.1980 \\ \log \tan \frac{1}{2}(C-A) &= \underline{9.9135} & \log \tan \frac{1}{2}(c-a) &= \underline{0.1330} \\ \log \cot \frac{1}{2}B &= 0.0046 & \log \tan \frac{1}{2}b &= 0.3229 \\ \frac{1}{2}B &= 44^\circ 41.9' & \frac{1}{2}b &= 64^\circ 34.2' \\ \therefore B &= 89^\circ 23.8'. & \therefore b &= 129^\circ 8.4'. \end{aligned}$$

Using the second value of a , we have :

$$\begin{aligned} \log \sin \frac{1}{2}(c+a) &= 9.7730 & \log \sin \frac{1}{2}(C+A) &= 9.9919 \\ \log \csc \frac{1}{2}(c-a) &= 0.9335 & \log \csc \frac{1}{2}(C-A) &= 0.1980 \\ \log \tan \frac{1}{2}(C-A) &= \underline{9.9135} & \log \tan \frac{1}{2}(c-a) &= \underline{9.0695} \\ \log \cot \frac{1}{2}B &= 0.6200 & \log \tan \frac{1}{2}b &= 9.2594 \\ \frac{1}{2}B &= 13^\circ 29.3' & \frac{1}{2}b &= 10^\circ 17.9' \\ \therefore B &= 26^\circ 58.6'. & \therefore b &= 20^\circ 35.8'. \end{aligned}$$

$$8. \quad \sin c = \frac{\sin C \sin b}{\sin B}$$

$$\begin{aligned} \log \sin C &= 9.9950 \\ \log \csc B &= 0.0194 \\ \log \sin b &= \underline{9.9252} \\ \log \sin c &= \underline{9.9396} \end{aligned}$$

$$\therefore c = 60^\circ 28.6', \text{ or } 119^\circ 31.4'.$$

Since both values of c are $< b$, while C is given $> B$, the triangle is impossible.

Art. 173.—Pages 134 and 136.

1. Let B and G denote the positions of Boston and Greenwich, respectively, and P the north-pole.

Denote the arcs PG , PB , and BG by b , g , and p .

Then in the triangle PBG ,

$$\angle P = 71^\circ 4', \quad b = 90^\circ - 51^\circ 29' = 38^\circ 31', \quad g = 90^\circ - 42^\circ 21' = 47^\circ 39'.$$

By Art. 162, we have :

$$\tan \frac{1}{2}(G - B) = \frac{\sin \frac{1}{2}(g - b)}{\sin \frac{1}{2}(g + b)} \cot \frac{1}{2}P,$$

$$\tan \frac{1}{2}(G + B) = \frac{\cos \frac{1}{2}(g - b)}{\cos \frac{1}{2}(g + b)} \cot \frac{1}{2}P.$$

From the data,

$$\frac{1}{2}(g - b) = 4^\circ 34', \quad \frac{1}{2}(g + b) = 43^\circ 5', \quad \frac{1}{2}P = 35^\circ 32'.$$

$$\log \sin \frac{1}{2}(g - b) = 8.9009$$

$$\log \cos \frac{1}{2}(g - b) = 9.9987$$

$$\log \csc \frac{1}{2}(g + b) = 0.1656$$

$$\log \sec \frac{1}{2}(g + b) = 0.1365$$

$$\log \cot \frac{1}{2}P = 0.1462$$

$$\log \cot \frac{1}{2}P = 0.1462$$

$$\log \tan \frac{1}{2}(G - B) = 9.2127$$

$$\log \tan \frac{1}{2}(G + B) = 0.2814$$

$$\therefore \frac{1}{2}(G - B) = 9^\circ 16.1'.$$

$$\therefore \frac{1}{2}(G + B) = 62^\circ 23.2'.$$

$$\therefore B = \frac{1}{2}(G + B) - \frac{1}{2}(G - B) = 53^\circ 7.1',$$

and

$$G = \frac{1}{2}(G + B) + \frac{1}{2}(G - B) = 71^\circ 39.3'.$$

$$\tan \frac{1}{2}p = \frac{\sin \frac{1}{2}(G + B)}{\sin \frac{1}{2}(G - B)} \tan \frac{1}{2}(g - b).$$

$$\log \sin \frac{1}{2}(G + B) = 9.9475$$

$$\log \csc \frac{1}{2}(G - B) = 0.7930$$

$$\log \tan \frac{1}{2}(g - b) = 8.9023$$

$$\log \tan \frac{1}{2}p = 9.6428$$

$$\frac{1}{2}p = 23^\circ 43.1'$$

$$\therefore p = 47^\circ 26.2'.$$

$$47^\circ 26.2' = 2846.2'.$$

$$360^\circ = 21600'.$$

$$\text{Circumference of earth} = 7912 \times 3.1416.$$

$$\therefore p \text{ (in miles)} = \frac{2846.2}{21600} \times 7912 \times 3.1416.$$

$$\log 2846.2 = 3.4542$$

$$\text{colog } 21600 = 5.6655$$

$$\log 7912 = 3.8983$$

$$\log 3.1416 = 0.4971$$

$$\log p = 3.5151$$

$$\therefore p = 3274.3.$$

Therefore the shortest distance between the places is 3274.3 miles; the bearing of Greenwich from Boston is N. $53^\circ 7.1'$ E., and of Boston from Greenwich, N. $71^\circ 39.3'$ W.

2. Let C and V denote the positions of Calcutta and Valparaiso, respectively, and P the north-pole.

Denote the arcs PV , PC , and CV by c , v , and p .

Then in the triangle PCV ,

$$\begin{aligned}\angle P &= 88^\circ 19' + 71^\circ 42' = 160^\circ 1', & c &= 90^\circ + 33^\circ 2' = 123^\circ 2', \\ v &= 90^\circ - 22^\circ 33' = 67^\circ 27'.\end{aligned}$$

By Art. 162, we have :

$$\tan \frac{1}{2}(C - V) = \frac{\sin \frac{1}{2}(c - v)}{\sin \frac{1}{2}(c + v)} \cot \frac{1}{2}P,$$

$$\tan \frac{1}{2}(C + V) = \frac{\cos \frac{1}{2}(c - v)}{\cos \frac{1}{2}(c + v)} \cot \frac{1}{2}P.$$

From the data, $\frac{1}{2}(c - v) = 27^\circ 47.5'$, $\frac{1}{2}(c + v) = 95^\circ 14.5'$, $\frac{1}{2}P = 80^\circ 0.5'$.

$\log \sin \frac{1}{2}(c - v) = 9.6686$	$\log \cos \frac{1}{2}(c - v) = 9.9468$
$\log \csc \frac{1}{2}(c + v) = 0.0018$	$\log \sec \frac{1}{2}(c + v) = 1.0393$
$\log \cot \frac{1}{2}P = 9.2459$	$\log \cot \frac{1}{2}P = 9.2459$

$\log \tan \frac{1}{2}(C - V) = 8.9163$	$\log \tan \frac{1}{2}(C + V) = 0.2320$
---	---

$$\therefore \frac{1}{2}(C - V) = 4^\circ 42.9'.$$

$$180^\circ - \frac{1}{2}(C + V) = 59^\circ 37.5'$$

$$\therefore \frac{1}{2}(C + V) = 120^\circ 22.5'.$$

$$\therefore C = \frac{1}{2}(C + V) + \frac{1}{2}(C - V) = 125^\circ 5.4',$$

$$V = \frac{1}{2}(C + V) - \frac{1}{2}(C - V) = 115^\circ 39.6'.$$

and

$$\tan \frac{1}{2}p = \frac{\sin \frac{1}{2}(C + V)}{\sin \frac{1}{2}(C - V)} \tan \frac{1}{2}(c - v).$$

$$\log \sin \frac{1}{2}(C + V) = 9.9359$$

$$\log \csc \frac{1}{2}(C - V) = 1.0852$$

$$\log \tan \frac{1}{2}(c - v) = 9.7218$$

$$\log \tan \frac{1}{2}p = 0.7429$$

$$\frac{1}{2}p = 79^\circ 45.2'$$

$$\therefore p = 159^\circ 30.4'.$$

$$159^\circ 30.4' = 9570.4'.$$

$$360^\circ = 21600'.$$

Circumference of earth = 7912×3.1416 .

$$\therefore p \text{ (in miles)} = \frac{9570.4}{21600} \times 7912 \times 3.1416.$$

$$\log 9570.4 = 3.9809$$

$$\text{colog } 21600 = 5.6655$$

$$\log 7912 = 3.8983$$

$$\log 3.1416 = 0.4971$$

$$\log p = 4.0418$$

$$\therefore p = 11010.$$

Therefore the shortest distance between the places is 11010 miles; the bearing of Calcutta from Valparaiso is N. $115^{\circ} 39.6' E.$, or S. $64^{\circ} 20.4' E.$, and of Valparaiso from Calcutta is N. $125^{\circ} 5.4' W.$, or S. $54^{\circ} 54.6' W.$

3. Let S and Q denote the positions of Sandy Hook and Queens-town, respectively, P the north pole, and X the intersection of the arc QS with the meridian of $50^{\circ} W.$

Denote the arcs PQ and PS by s and q .

Then in the triangle PQS ,

$$\begin{aligned}\angle P &= 74^{\circ} 1' - 8^{\circ} 19' = 65^{\circ} 42', & q &= 90^{\circ} - 40^{\circ} 28' = 49^{\circ} 32', \\ s &= 90^{\circ} - 51^{\circ} 50' = 38^{\circ} 10'.\end{aligned}$$

By Art. 162, we have :

$$\tan \frac{1}{2}(Q - S) = \frac{\sin \frac{1}{2}(q - s)}{\sin \frac{1}{2}(q + s)} \cot \frac{1}{2}P,$$

$$\tan \frac{1}{2}(Q + S) = \frac{\cos \frac{1}{2}(q - s)}{\cos \frac{1}{2}(q + s)} \cot \frac{1}{2}P.$$

From the data,

$$\frac{1}{2}(q - s) = 5^{\circ} 41', \quad \frac{1}{2}(q + s) = 43^{\circ} 51', \quad \frac{1}{2}P = 32^{\circ} 51'.$$

$$\log \sin \frac{1}{2}(q - s) = 8.9957 \qquad \log \cos \frac{1}{2}(q - s) = 9.9979$$

$$\log \csc \frac{1}{2}(q + s) = 0.1594 \qquad \log \sec \frac{1}{2}(q + s) = 0.1419$$

$$\log \cot \frac{1}{2}P = 0.1900 \qquad \log \cot \frac{1}{2}P = 0.1900$$

$$\log \tan \frac{1}{2}(Q - S) = 9.3451 \qquad \log \tan \frac{1}{2}(Q + S) = 0.3298$$

$$\therefore \frac{1}{2}(Q - S) = 12^{\circ} 28.9'. \qquad \therefore \frac{1}{2}(Q + S) = 64^{\circ} 55.5'.$$

Then in the triangle PQX ,

$$\angle P = 50^{\circ} - 8^{\circ} 19' = 41^{\circ} 41',$$

$$\angle Q = \frac{1}{2}(Q + S) + \frac{1}{2}(Q - S) = 77^{\circ} 24.4', \quad PQ = 38^{\circ} 10'.$$

By Arts. 160 and 161, we have :

$$\tan \frac{1}{2}(PX - QX) = \frac{\sin \frac{1}{2}(Q - QPX)}{\sin \frac{1}{2}(Q + QPX)} \tan \frac{1}{2}PQ.$$

$$\tan \frac{1}{2}(PX + QX) = \frac{\cos \frac{1}{2}(Q - QPX)}{\cos \frac{1}{2}(Q + QPX)} \tan \frac{1}{2}PQ.$$

From the data,

$$\frac{1}{2}(Q - QPX) = 17^{\circ} 51.7', \quad \frac{1}{2}(Q + QPX) = 59^{\circ} 32.7', \quad \frac{1}{2}PQ = 19^{\circ} 5'.$$

$$\log \sin \frac{1}{2}(Q - QPX) = 9.4868 \qquad \log \cos \frac{1}{2}(Q - QPX) = 9.9785$$

$$\log \csc \frac{1}{2}(Q + QPX) = 0.0645 \qquad \log \sec \frac{1}{2}(Q + QPX) = 0.2951$$

$$\log \tan \frac{1}{2}PQ = 9.5390 \qquad \log \tan \frac{1}{2}PQ = 9.5390$$

$$\log \tan \frac{1}{2}(PX - QX) = 9.0903 \qquad \log \tan \frac{1}{2}(PX + QX) = 9.8126$$

$$\therefore \frac{1}{2}(PX - QX) = 7^{\circ} 1.2'. \qquad \therefore \frac{1}{2}(PX + QX) = 33^{\circ} 0.4'.$$

$$\therefore PX = \frac{1}{2}(PX + QX) + \frac{1}{2}(PX - QX) = 40^{\circ} 1.6'.$$

Therefore the latitude of $X = 90^{\circ} - 40^{\circ} 1.6' = 49^{\circ} 58.4' N.$

4. Denote the arcs SP , SZ , and PZ (see Fig., p. 135) by z , p , and s , and their sum by $2s'$.

Then in the triangle SPZ ,

$$z = 90^\circ - 18^\circ 36' = 71^\circ 24', \quad p = 90^\circ - 14^\circ 18' = 75^\circ 42',$$

$$s = 90^\circ - 50^\circ 13' = 39^\circ 47'.$$

By Art. 158,

$$\cos \frac{1}{2} P = \sqrt{\frac{\sin s' \sin (s' - p)}{\sin s \sin z}}.$$

From the data,

$$s' = 93^\circ 26.5', \quad s' - p = 17^\circ 44.5'.$$

$$\log \sin s' = 9.9992$$

$$\log \sin (s' - p) = 9.4839$$

$$\log \csc s = 0.1939$$

$$\log \csc z = 0.0233$$

$$\underline{2)9.7003}$$

$$\log \cos \frac{1}{2} P = 9.8501$$

$$\therefore \frac{1}{2} P = 44^\circ 55', \quad \text{and } P = 89^\circ 50'.$$

To $89^\circ 50'$ corresponds 5 h. 59 m. 20 s. of time.

Therefore the hour of the day is 6 h. 0 m. 40 s. A.M.

The difference between the local and Greenwich time is 2 h. 59 m. 20 s., which corresponds to $44^\circ 50'$ of longitude.

Hence the longitude of the vessel is $44^\circ 50'$ W.

5. Denote the arcs SP , SZ , and PZ (see Fig., p. 135) by z , p , and s . Then in the triangle SPZ ,

$$z = 90^\circ + 12^\circ = 102^\circ, \quad s = 90^\circ - 37^\circ 48' = 52^\circ 12', \quad \angle P = 60^\circ.$$

By Art 162, we have :

$$\tan \frac{1}{2} (Z - S) = \frac{\sin \frac{1}{2} (z - s)}{\sin \frac{1}{2} (z + s)} \cot \frac{1}{2} P,$$

$$\tan \frac{1}{2} (Z + S) = \frac{\cos \frac{1}{2} (z - s)}{\cos \frac{1}{2} (z + s)} \cot \frac{1}{2} P.$$

From the data, $\frac{1}{2} (z - s) = 24^\circ 54'$, $\frac{1}{2} (z + s) = 77^\circ 6'$, $\frac{1}{2} P = 30^\circ$.

$$\log \sin \frac{1}{2} (z - s) = 9.6243$$

$$\log \cos \frac{1}{2} (z - s) = 9.9577$$

$$\log \csc \frac{1}{2} (z + s) = 0.0111$$

$$\log \sec \frac{1}{2} (z + s) = 0.6512$$

$$\log \cot \frac{1}{2} P = 0.2386$$

$$\log \cot \frac{1}{2} P = 0.2386$$

$$\log \tan \frac{1}{2} (Z - S) = 9.8740$$

$$\log \tan \frac{1}{2} (Z + S) = 0.8475$$

$$\therefore \frac{1}{2} (Z - S) = 36^\circ 48.1'.$$

$$\therefore \frac{1}{2} (Z + S) = 81^\circ 54.8'.$$

$$\tan \frac{1}{2} p = \frac{\sin \frac{1}{2} (Z + S)}{\sin \frac{1}{2} (Z - S)} \tan \frac{1}{2} (z - s).$$

$$\begin{aligned} \log \sin \frac{1}{2} (Z + S) &= 9.9957 \\ \log \csc \frac{1}{2} (Z - S) &= 0.2225 \\ \log \tan \frac{1}{2} (z - s) &= \underline{9.6667} \\ \log \tan \frac{1}{2} p &= 9.8849 \end{aligned}$$

$$\therefore \frac{1}{2} p = 37^\circ 29.6', \text{ and } p = 74^\circ 59.2'.$$

Therefore the altitude of the sun is $90^\circ - 74^\circ 59.2'$, or $15^\circ 0.8'$.

6. Denote the arcs SP , SZ , and PZ (see Fig., p. 135) by z , p , and s , and their sum by $2s'$.

Then in the triangle SPZ ,

$$z = 90^\circ + 3^\circ = 93^\circ, p = 90^\circ - 25^\circ 46' = 64^\circ 14', s = 90^\circ + 37^\circ 49' = 127^\circ 49'.$$

By Art. 158,
$$\cos \frac{1}{2} P = \sqrt{\frac{\sin s' \sin (s' - p)}{\sin s \sin z}}.$$

From the data, $s' = 142^\circ 31.5'$, $s' - p = 78^\circ 17.5'$.

$$\begin{aligned} \log \sin s' &= 9.7842 \\ \log \sin (s' - p) &= 9.9908 \\ \log \csc s &= 0.1024 \\ \log \csc z &= \underline{0.0006} \\ &2)9.8780 \\ \log \cos \frac{1}{2} P &= 9.9390 \end{aligned}$$

$$\therefore \frac{1}{2} P = 29^\circ 40', \text{ and } P = 59^\circ 20'.$$

To $59^\circ 20'$ corresponds 3 h. 57 m. 20 s. of time.

Therefore the hour of the day is 8 h. 2 m. 40 s. A.M.

7. Denote the arcs SP , SZ , and PZ (see Fig., p. 135) by z , p , and s .

Then in the triangle SPZ ,

$$z = 90^\circ - 15^\circ = 75^\circ, p = 90^\circ, s = 90^\circ - 42^\circ 21' = 47^\circ 39'.$$

Therefore in the polar triangle of SPZ , we have

$$\begin{aligned} Z &= 180^\circ - 75^\circ = 105^\circ, P' = 180^\circ - 90^\circ = 90^\circ, \\ S' &= 180^\circ - 47^\circ 39' = 132^\circ 21'. \end{aligned}$$

By Art. 144,
$$\begin{array}{c} + \quad - \quad - \\ \cos p' = \cot S' \cot Z'. \end{array}$$

$$\begin{aligned} \log \cot S' &= 9.9598 \\ \log \cot Z' &= \underline{9.4281} \\ \log \cos p' &= 9.3879 \end{aligned}$$

$$\therefore p' = 75^\circ 51.6'.$$

Then in the triangle SPZ , we have

$$P = 180^\circ - p' = 104^\circ 8.4'.$$

To $104^\circ 8.4'$ corresponds 6 h. 56 m. 33.6 s. of time.

Therefore the hour of the day is 5 h. 3 m. 26.4 s. A.M.

USE OF THE TABLES.

Page 2.

- | | |
|--|--|
| 3. $\log 80 = 1.9031.$ | 12. Mantissa of 924 = .9657
$.61 \times 4 = \underline{\quad 2}$
$\therefore \log 92461 = 4.9659$ |
| 4. $\log 6.3 = 0.7993.$ | 13. Mantissa of 403 = .6053
$.22 \times 11 = \underline{\quad 2}$
$\therefore \log .40322 = 9.6055 - 10$ |
| 5. $\log 298 = 2.4742.$ | 14. Mantissa of 717 = .8555
$.8 \times 6 = \underline{\quad 5}$
$\therefore \log .007178 = 7.8560 - 10$ |
| 6. $\log .902 = 9.9552 - 10.$ | 15. Mantissa of 518 = .7143
$.09 \times 9 = \underline{\quad 1}$
$\therefore \log 5.1809 = 0.7144$ |
| 7. Mantissa of 772 = .8876
$.3 \times 6 = \underline{\quad 2}$
$\therefore \log .7723 = 9.8878 - 10$ | 16. Mantissa of 103 = .0128
$.65 \times 42 = \underline{\quad 27}$
$\therefore \log 1036.5 = 3.0155$ |
| 8. Mantissa of 105 = .0212
$.6 \times 41 = \underline{\quad 25}$
$\therefore \log 1056 = 3.0237$ | 17. Mantissa of 866 = .9375
$.76 \times 5 = \underline{\quad 4}$
$\therefore \log .086676 = 8.9379 - 10$ |
| 9. Mantissa of 329 = .5172
$.4 \times 13 = \underline{\quad 5}$
$\therefore \log 3.294 = 0.5177$ | 18. Mantissa of 115 = .0607
$.07 \times 38 = \underline{\quad 3}$
$\therefore \log .11507 = 9.0610 - 10$ |
| 10. Mantissa of 520 = .7160
$.5 \times 8 = \underline{\quad 4}$
$\therefore \log .05205 = 8.7164 - 10$ | |
| 11. Mantissa of 200 = .3010
$.8 \times 22 = \underline{\quad 18}$
$\therefore \log 20.08 = 1.3028$ | |

Page 4.

- | | |
|--|---|
| 4. Number corresponding to
1.8055 = 63.9. | 5. Number corresponding to
9.4487 - 10 = .281. |
|--|---|

$$\begin{array}{r}
 6. \quad 0.2165 \\
 \quad .2148 = \text{mantissa of } 164 \\
 \hline
 \quad \frac{17}{27} = \quad \quad \quad 6
 \end{array}$$

\therefore Number corresponding = 1.646

$$\begin{array}{r}
 7. \quad 3.9487 \\
 \quad .9484 = \text{mantissa of } 888 \\
 \hline
 \quad \frac{3}{5} = \quad \quad \quad 6
 \end{array}$$

\therefore Number corresponding = 8886

8. Number corresponding to
2.7364 = 545.

$$\begin{array}{r}
 9. \quad 8.1648 - 10 \\
 \quad .1644 = \text{mantissa of } 146 \\
 \hline
 \quad \frac{4}{29} = \quad \quad \quad 1
 \end{array}$$

\therefore Number corresponding = .01461

$$\begin{array}{r}
 10. \quad 7.5209 - 10 \\
 \quad .5198 = \text{mantissa of } 331 \\
 \hline
 \quad \frac{11}{13} = \quad \quad \quad 8
 \end{array}$$

\therefore Number corresponding = .003318

$$\begin{array}{r}
 11. \quad 4.0095 \\
 \quad .0086 = \text{mantissa of } 102 \\
 \hline
 \quad \frac{9}{42} = \quad \quad \quad 21
 \end{array}$$

\therefore Number corresponding = 10221

$$\begin{array}{r}
 12. \quad 0.9774 \\
 \quad .9773 = \text{mantissa of } 949 \\
 \hline
 \quad \frac{1}{4} = \quad \quad \quad 2
 \end{array}$$

\therefore Number corresponding = 9.492

$$\begin{array}{r}
 13. \quad 9.3178 - 10 \\
 \quad .3160 = \text{mantissa of } 207 \\
 \hline
 \quad \frac{18}{21} = \quad \quad \quad 9
 \end{array}$$

\therefore Number corresponding = .2079

$$\begin{array}{r}
 14. \quad 1.6482 \\
 \quad .6474 = \text{mantissa of } 444 \\
 \hline
 \quad \frac{8}{10} = \quad \quad \quad 8
 \end{array}$$

\therefore Number corresponding = 44.48

$$\begin{array}{r}
 15. \quad 7.0450 - 10 \\
 \quad .0414 = \text{mantissa of } 110 \\
 \hline
 \quad \frac{36}{39} = \quad \quad \quad 9
 \end{array}$$

\therefore Number corresponding = .001109

$$\begin{array}{r}
 16. \quad 4.8016 \\
 \quad .8014 = \text{mantissa of } 633 \\
 \hline
 \quad \frac{2}{7} = \quad \quad \quad 29
 \end{array}$$

\therefore Number corresponding = 63329

$$\begin{array}{r}
 17. \quad 8.1144 - 10 \\
 \quad .1139 = \text{mantissa of } 130 \\
 \hline
 \quad \frac{5}{34} = \quad \quad \quad 1
 \end{array}$$

\therefore Number corresponding = 0.1301

$$\begin{array}{r}
 18. \quad 2.7015 \\
 \quad .7007 = \text{mantissa of } 502 \\
 \hline
 \quad \frac{8}{9} = \quad \quad \quad 9
 \end{array}$$

\therefore Number corresponding = 502.9

Page 6.

3. $\log \tan 35^\circ 10' = 9.8479$
 $2.7 \times 9 = \underline{24}$
 $\therefore \log \tan 35^\circ 19' = 9.8503$
4. $\log \sin 61^\circ 50' = 9.9453$
 $.6 \times 8 = \underline{5}$
 $\therefore \log \sin 61^\circ 58' = 9.9458$
5. $\log \cot 12^\circ 30' = 0.6542$
 $5.9 \times 4 = \underline{24}$
 $\therefore \log \cot 12^\circ 34' = 0.6518$
6. $\log \cos 26^\circ 50' = 9.9505$
 $.6 \times 6 = \underline{4}$
 $\therefore \log \cos 26^\circ 56' = 9.9501$

7. $\log \tan 82^\circ 0' = 0.8522$
 $9.3 \times 3\frac{1}{3} = \underline{31}$
 $\therefore \log \tan 82^\circ 3' 20'' = 0.8553$
8. $\log \sin 55^\circ 10' = 9.9142$
 $.9 \times 1.8 = \underline{2}$
 $\therefore \log \sin 55^\circ 11.8' = 9.9144$
9. $\log \cos 30^\circ 40' = 9.9346$
 $.8 \times 2.5 = \underline{2}$
 $\therefore \log \cos 30^\circ 42.5' = 9.9344$
10. $\log \cot 48^\circ 0' = 9.9544$
 $2.5 \times 3\frac{4}{10} = \underline{9}$
 $\therefore \log \cot 48^\circ 3' 43'' = 9.9535$

Page 7.

3. 0.9164
 $0.9109 = \log \tan 83^\circ 0'$
 $\frac{55}{10.5} = \underline{5.2}$
 \therefore Angle corresponding = $83^\circ 5.2'$
4. 9.9238
 $9.9236 = \log \cos 33^\circ 0'$
 $\frac{2}{.8} = \underline{2.5}$
 \therefore Angle corresponding = $32^\circ 57.5'$
5. 9.8630
 $9.8629 = \log \sin 46^\circ 50'$
 $\frac{1}{1.2} = \underline{0.8}$
 \therefore Angle corresponding = $46^\circ 50.8'$
6. 0.2154
 $0.2127 = \log \cot 31^\circ 30'$
 $\frac{27}{2.8} = \underline{9.6}$
 \therefore Angle corresponding = $31^\circ 20.4'$

7. 9.2279
 $9.2251 = \log \cos 80^\circ 20'$
 $\frac{28}{7.3} = \underline{3.8}$
 \therefore Angle corresponding = $80^\circ 16.2'$
8. 9.4700
 $9.4669 = \log \cot 73^\circ 40'$
 $\frac{31}{4.7} = \underline{6.6}$
 \therefore Angle corresponding = $73^\circ 33.4'$
9. 9.1891
 $9.1863 = \log \sin 8^\circ 50'$
 $\frac{28}{8} = \underline{3.5}$
 \therefore Angle corresponding = $8^\circ 53.5'$
10. 0.0502
 $0.0481 = \log \tan 48^\circ 10'$
 $\frac{21}{2.5} = \underline{8.4}$
 \therefore Angle corresponding = $48^\circ 18.4'$

Page 8.

$$\begin{aligned}
 2. \log \sin 65^\circ 10' &= 9.9579 \\
 .5 \times 2 &= \underline{1} \\
 \log \sin 65^\circ 12' &= 9.9580 \\
 \therefore \log \csc 65^\circ 12' &= 0.0420
 \end{aligned}$$

$$\begin{aligned}
 3. \log \cos 80^\circ 0' &= 9.2397 \\
 7.3 \times 7.3 &= \underline{53} \\
 \log \cos 80^\circ 7.3' &= 9.2344 \\
 \therefore \log \sec 80^\circ 7.3' &= 0.7656
 \end{aligned}$$

$$\begin{aligned}
 4. \quad 9.5997 &= \log \cos. \\
 9.5978 &= \log \cos 66^\circ 40' \\
 \frac{19}{2.9} &= \underline{6.6} \\
 \therefore \text{Angle corresponding} &= 66^\circ 33.4'
 \end{aligned}$$

$$\begin{aligned}
 5. \quad 9.8112 &= \log \sin. \\
 9.8111 &= \log \sin 40^\circ 20' \\
 \frac{1}{1.4} &= \underline{0.7} \\
 \therefore \text{Angle corresponding} &= 40^\circ 20.7'
 \end{aligned}$$

Page 10.

$$\begin{aligned}
 3. \text{ nat. sin } 3^\circ 30' &= .06105 \\
 .00029 \times 2 &= \underline{.00058} \\
 \text{ nat. sin } 3^\circ 32' &= .06163 \\
 \text{ mantissa of 616} &= .7896 \\
 7 \times .3 &= \underline{2} \\
 \therefore \log \sin 3^\circ 32' &= 8.7898
 \end{aligned}$$

$$\begin{aligned}
 4. \text{ nat. cos } 88^\circ 10' &= .03199 \\
 .000291 \times 7 &= \underline{.00204} \\
 \text{ nat. cos } 88^\circ 17' &= .02995 \\
 \text{ mantissa of 299} &= .4757 \\
 14 \times .5 &= \underline{7} \\
 \therefore \log \cos 88^\circ 17' &= 8.4764
 \end{aligned}$$

$$\begin{aligned}
 5. \text{ nat. tan } 2^\circ 20' &= .04075 \\
 .000291 \times 8.2 &= \underline{.00239} \\
 \text{ nat. tan } 2^\circ 28.2' &= .04314 \\
 \text{ mantissa of 431} &= .6345 \\
 10 \times .4 &= \underline{4} \\
 \therefore \log \tan 2^\circ 28.2' &= 8.6349
 \end{aligned}$$

$$\begin{aligned}
 6. \text{ nat. tan } 50' &= .014545 \\
 .000291 \times 5\frac{14}{10} &= \underline{.001523} \\
 \text{ nat. tan } 55' 14'' &= .016068
 \end{aligned}$$

$$\begin{aligned}
 \text{ mantissa of 160} &= .2041 \\
 27 \times .68 &= \underline{18} \\
 \log \tan 55' 14'' &= 8.2059 \\
 \therefore \log \cot 55' 14'' &= 1.7941
 \end{aligned}$$

$$\begin{aligned}
 7. \quad 7.8702 \\
 .8698 &= \text{ mantissa of } \quad 741 \\
 \frac{4}{6} &= \underline{7} \\
 \text{ Number corresponding} &= .007417 \\
 .007417 &= \text{ nat. sin.} \\
 .005818 &= \text{ nat. sin } 20' \\
 \frac{.001599}{.0002909} &= 5.497' = \underline{5' 29.8''} \\
 \therefore \text{ Angle corresponding} &= 25' 29.8''
 \end{aligned}$$

$$\begin{aligned}
 8. \quad 8.6150 \\
 .6149 &= \text{ mantissa of } \quad 412 \\
 \frac{1}{11} &= \underline{1} \\
 \text{ Number corresponding} &= .04121 \\
 .04121 &= \text{ nat. cos.} \\
 .04071 &= \text{ nat. cos } 87^\circ 40' \\
 \frac{.00050}{.000291} &= 1.718' = \underline{1' 43.1''} \\
 \therefore \text{ Angle corresp.} &= 87^\circ 38' 16.9''
 \end{aligned}$$

$$\begin{array}{r}
 9. \quad 8.2892 \\
 \quad .2878 = \text{mantissa of } 194 \\
 \hline
 \quad \frac{14}{22} = \quad \quad \quad 64
 \end{array}$$

Number corresponding = .019464

$$\begin{array}{r}
 .019464 = \text{nat. cot.} \\
 .017455 = \text{nat. cot } 89^\circ 0' \\
 \hline
 \frac{.002009}{.0002905} = 6.916' = \quad 6' 55'' \\
 \therefore \text{Angle corresp.} = 89^\circ 53' 5''
 \end{array}$$

$$\begin{array}{r}
 10. \quad 8.2131 = \text{cologarithm.} \\
 \quad .2122 = \text{mantissa of } 163 \\
 \hline
 \quad \frac{9}{26} = \quad \quad \quad 35
 \end{array}$$

Number corresponding = .016335

$$\begin{array}{r}
 .016335 \\
 .014545 = \text{nat. tan } 50' \\
 \hline
 \frac{.001790}{.000291} = 6.151' = \quad 6' 9.1'' \\
 \hline
 \quad \quad \quad 56' 9.1'' \\
 \therefore \text{Angle corresp.} = 89^\circ 3' 50.9''
 \end{array}$$

Page 11.

$$\begin{array}{r}
 3. \quad \text{nat. sin } 17^\circ 30' = .3007 \\
 \quad .00028 \times 3 = \underline{.0008} \\
 \therefore \text{nat. sin } 17^\circ 33' = .3015
 \end{array}$$

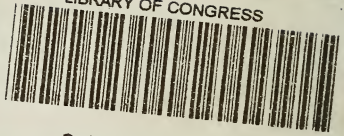
$$\begin{array}{r}
 4. \quad \text{nat. cos } 75^\circ 40' = .2476 \\
 \quad .00029 \times 8.3 = \underline{.0024} \\
 \therefore \text{nat. cos } 75^\circ 48.3' = .2452
 \end{array}$$

$$\begin{array}{r}
 5. \quad .7385 \\
 \quad \frac{.7373}{.0012} = \text{nat. sin } 47^\circ 30' \\
 \quad \quad \quad \underline{.0019} \quad \quad \quad 6.3 \\
 \therefore \text{Angle corresponding} = 47^\circ 36.3'
 \end{array}$$

$$\begin{array}{r}
 6. \quad .9280 \\
 \quad \frac{.9272}{.0008} = \text{nat. cos } 22^\circ 0' \\
 \quad \quad \quad \underline{.0011} \quad \quad \quad 7.3 \\
 \therefore \text{Angle corresponding} = 21^\circ 52.7'
 \end{array}$$



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