12.6 Esercizi

12.6.1 Esercizi dei singoli paragrafi

12.1 - Quadrato di un binomio

12.1. Completa:

a) $(3x+y)^2 = (3x)^2 + 2(3x)(y) + (y)^2 = \ldots\ldots\ldots\ldots$;
b) $(-2x+3y)^2 = (-2x)^2 + 2(-2x)(3y) + (3y)^2 = \ldots\ldots\ldots\ldots$;
c) $(-3x-5y)^2 = (-3x)^2 + 2(-3x)(-5x) + (-5x)^2 = \ldots\ldots\ldots\ldots$;
d) $(3x-y)^2 = (3x)^2 + 2(3x)(-y) + (-y)^2 = \ldots\ldots\ldots\ldots$;
e) $(2x+3y)^2 = (2x)^2 + 2\cdot(2x)(3y) + (3y)^2 = \ldots\ldots\ldots\ldots$;
f) $\left(x^2 - \frac{1}{2}y\right)^2 = (x^2)^{\cdots} + 2\cdot(\ldots\ldots)(-\ldots\ldots) + \left(-\frac{1}{2}y\right)^{\cdots} = \ldots\ldots\ldots\ldots$.

12.2. Quali dei seguenti polinomi sono quadrati di binomi?

a) $a^2+4ab+4b^2$ Sì No
b) $a^2-2ab-b^2$ Sì No
c) $25a^2-15ab+3b$ Sì No
d) $\frac{49}{4}a^4-21a^2b^2+9b^2$ Sì No
e) $a^6+b^4+2a^3b^2$ Sì No
f) $25a^2+4b^2-20ab^2$ Sì No
g) $-25a^4-\frac{1}{16}b^4+\frac{5}{2}a^2b^2$ Sì No
h) $\frac{1}{4}a^6+\frac{1}{9}b^4+\frac{1}{6}a^3b^2$ Sì No

12.3. Completa in modo da formare un quadrato di binomio.

a) $\frac{9}{16}x^2+\ldots+y^2$;
b) $x^2+2x+\ldots$;
c) $4x^2y^2-2xyz\ldots$;
d) $\frac{a^4}{4}-\ldots+4b^4$;
e) $9+6x+\ldots$;
f) $1-x+\ldots$;
g) $x^2+4y^2-\ldots$;
h) $4x^2-4xy+\ldots$;
i) $4x^2-20x+\ldots$

12.4. Sviluppa i seguenti quadrati di binomi.

a) $(x+1)^2$;
b) $(x+2)^2$;
c) $(x-3)^2$;
d) $(2x-1)^2$;
e) $(x+y)^2$;
f) $(x-y)^2$;
g) $(2x+y)^2$;
h) $(x+2y)^2$.

12.5. Sviluppa i seguenti quadrati di binomi.

a) $(-a+b)^2$;
b) $(-a-1)^2$;
c) $(-a+3)^2$;
d) $(-a+2b)^2$;
e) $(2a+3b)^2$;
f) $(2a-3b)^2$;
g) $(3a+2b)^2$;
h) $(-2+3b)^2$.

12.6. Sviluppa i seguenti quadrati di binomi.

a) $\left(\frac{1}{2}a+\frac{3}{4}b\right)^2$;
b) $\left(-2x^2-\frac{7}{4}y\right)^2$;
c) $\left(5x^3-\frac{4}{3}y^2\right)^2$;
d) $\left(-1+\frac{3}{2}a^2x\right)^2$;
e) $\left(3a-\frac{1}{3}a^2\right)^2$;
f) $\left(-2-\frac{1}{2}x\right)^2$;
g) $(x+1)^2$;
h) $(a^2+a)^2$.

12.7. Sviluppa i seguenti quadrati di binomi.

a) $\left(\frac{3}{2}x^2-2x\right)^2$;
b) $\left(x^2-\frac{1}{2}x\right)^2$;
c) $\left(\frac{1}{2}a^2-b^2\right)^2$;
d) $\left(-\frac{2}{3}x-\frac{3}{5}x^2\right)^2$;
e) $\left(x^{2n}-\frac{1}{2}x^n\right)^2$;
f) $\left(-2^2-\frac{1}{2}x^n\right)^2$;
g) $\left(-2x^{2n}-\frac{1}{4}y^m\right)^2$;
h) $\left(x^{n+1}+x^n\right)^2$.

12.8 (*). Semplifica le seguenti espressioni contenenti quadrati di binomi.

a) $(x-2y)^2-(2x-y)^2$;
b) $3(x-y)^2-2(x+2y)^2$;
c) $3(2x+5)^2-4(2x+5)(2x-5)+10(2x-5)^2$;
d) $\left(x^2+1\right)^2-6\left(x^2+1\right)+8$.

12.9 (*). Semplifica le seguenti espressioni contenenti quadrati di binomi.

a) $\frac{1}{2}\left(x-\frac{1}{2}\right)^2-2\left(x-\frac{1}{2}\right)$;
b) $\frac{1}{2}x(y-1)^2-\frac{3}{2}y(x+1)^2+\frac{1}{2}xy(3x-y+8)$;
c) $\left(3x-\frac{1}{2}y\right)^2-\left(\frac{1}{2}x+y\right)^2+3x(2-y)^2-3y^2\left(x-\frac{1}{4}\right)+4x(4y-3)$;
d) $(x-1)^2-(2x+3)^2$.

12.10 (*). Semplifica le seguenti espressioni contenenti quadrati di binomi.

a) $\frac{1}{2}\left(2x+\frac{1}{2}\right)^2-2\left(2x-\frac{1}{2}\right)^2$;
b) $(2a+b)^2(a-b)^2-2(3-b)^2(3+b)^2-(6b+2a^2)^2+a^2b[4a+3(b+8)]$;
c) $\left(\frac{3}{2}x^2-2x\right)^2+\left(x^2-\frac{1}{2}x\right)^2-\left(\frac{3}{2}x^2-2x\right)\left(x^2-\frac{1}{2}x\right)$;
d) $(x+1)^2+(x-2)^2+\left(x-\frac{1}{3}\right)^2-2x\left(x-\frac{1}{2}\right)^2$.

12.2 - Quadrato di un polinomio

12.11. Completa i seguenti quadrati.

a) $(x+3y-1)^2=x^2+\ldots\ldots+1+6xy-2x-6y$;
b) $\left(x^2-\frac{1}{2}y+1\right)^2=x^4+\frac{1}{4}y^2+\ldots\ldots-x^2y+\ldots\ldots-y$;
c) $\left(2x^2-\frac{x}{2}+\frac{1}{2}\right)^2=\ldots\ldots+\frac{x^2}{4}+\frac{1}{4}-2x^{\cdots}+2x^{\cdots}-\frac{\cdots}{\cdots}\ldots$

12.12. Sviluppa i seguenti quadrati di polinomi.

a) $(a+b-c)^2$;
b) $(a-b+c)^2$;
c) $(x^2+x+1)^2$;
d) $(x-x^2+1)^2$;
e) $(3x^2+2z-y^2)^2$;
f) $(-a+b-c)^2$;
g) $(6a-3y^3-2z^2)^2$;
h) $(1-x-x^2)^2$.

12.13. Sviluppa i seguenti quadrati di polinomi.

a) $\left(\frac{1}{3}x^3-\frac{4}{5}x^2-\frac{1}{4}x\right)^2$;
b) $\left(3x^3+\frac{1}{2}y^2-\frac{3}{4}\right)^2$;
c) $\left(5a^3-\frac{1}{2}ab-1-a\right)^2$;
d) $\left(\frac{1}{2}x+2y^2-3\right)^2$;
e) $\left(\frac{2}{3}y^2-3x^4+\frac{7}{4}z\right)^2$;
f) $\left(2a+\frac{1}{2}ab^2-3b\right)^2$;
g) $\left(2x^3y^2-y^2x+5x^2+\frac{1}{2}\right)^2$;
h) $\left(\frac{1}{2}x^2+\frac{3}{4}x^2x-2xy+\frac{3}{8}y\right)^2$;
i) $\left(\frac{2}{3}y^2-3x^2+\frac{3}{4}xy\right)^2$;
j) $\left(a-b+\frac{1}{2}\right)^2$.

12.14 (*). Semplifica le seguenti espressioni che contengono quadrati di polinomi.

a) $(x+y-1)^2-(x-y+1)^2$;
b) $(2a+b-x)^2+(2x-b-a)^2-5(x+a+b)^2+b(4a+3b)$;
c) $(x^2+x+1)^2-(x+1)^2$;
d) $(a+b+1)^2-(a-b-1)^2$.

12.15. Semplifica le seguenti espressioni che contengono quadrati di polinomi.

a) $(a-3b+1)^2-(a-3b)^2-(3b-1)^2+(a-3b)(a+3b-1)$;
b) $\left(\frac{1}{2}a^2-b^2\right)^2+\left(a-b+\frac{1}{2}\right)^2-\left(a+b-\frac{1}{2}\right)^2$;
c) $(a+b-1)^2-(a+b)^2-(a-1)^2-(b-1)^2$.

12.3 - Prodotto della somma di monomi per la loro differenza

12.16. Calcola a mente i seguenti prodotti applicando la regola $(A+B)(A-B)=A^2-B^2$.

a) $18\cdot 22$; b) $15\cdot 25$; c) $43\cdot 37$; d) $195\cdot 205$.

12.17. Esegui i seguenti prodotti applicando la regola $(A+B)(A-B)=A^2-B^2$.

a) $(x-1)(x+1)$;
b) $(a+1)(a-1)$;
c) $(b-2)(b+2)$;
d) $(2a+b)(2a-b)$;
e) $(a+2b)(a-2b)$;
f) $(2a+3b)(2a-3b)$.

12.18. Esegui i seguenti prodotti applicando la regola $(A+B)(A-B) = A^2 - B^2$.

a) $\left(l+\frac{1}{2}m\right)\left(l-\frac{1}{2}m\right)$;
b) $\left(\frac{1}{2}u+v\right)\left(\frac{1}{2}u-v\right)$;
c) $\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)$;
d) $(3a-5y)(-3a-5y)$;
e) $\left(\frac{2}{3}x+\frac{3}{2}y\right)\left(\frac{2}{3}x-\frac{3}{2}y\right)$;
f) $\left(-\frac{2}{5}x-\frac{3}{7}y\right)\left(-\frac{2}{5}x+\frac{3}{7}y\right)$.

12.19. Esegui i seguenti prodotti applicando la regola $(A+B)(A-B) = A^2 - B^2$.

a) $\left(x^2+\frac{1}{2}z\right)\left(x^2-\frac{1}{2}z\right)$;
b) $\left(\frac{2}{3}x^2+3y^2\right)\left(-\frac{2}{3}x^2+3y^2\right)$;
c) $\left(\frac{2}{3}a^3+\frac{1}{2}y^3\right)\left(-\frac{2}{3}a^3+\frac{1}{2}y^3\right)$;
d) $\left(-2a^3-\frac{7}{3}y\right)\left(-2a^3+\frac{7}{3}y\right)$;
e) $\left(5x^2-\frac{6}{5}y^3\right)\left(5x^2+\frac{6}{5}y^3\right)$;
f) $\left(a^5+\frac{1}{2}y^4\right)\left(a^5-\frac{1}{2}y^4\right)$.

12.20. Esegui i seguenti prodotti applicando la regola $(A+B)(A-B) = A^2 - B^2$.

a) $\left(-\frac{8}{3}x^4-\frac{1}{2}x^3\right)\left(\frac{8}{3}x^4-\frac{1}{2}x^3\right)$;
b) $\left(2x^5+\frac{3}{2}y^5\right)\left(2x^5-\frac{3}{2}y^5\right)$;
c) $\left(-x-\frac{1}{2}\right)\left(-x+\frac{1}{2}\right)$;
d) $\left(-x-\frac{1}{2}\right)\left(-\frac{1}{2}+x\right)$;
e) $\left(-\frac{2}{3}x-\frac{3}{5}x^2\right)\left(\frac{2}{3}x-\frac{3}{5}x^2\right)$;
f) $\left(-\frac{2}{3}x-\frac{3}{5}x^2\right)\left(\frac{3}{5}x^2-\frac{2}{3}x\right)$;
g) $\left(\frac{2}{3}x-\frac{3}{5}x^2\right)\left(-\frac{2}{3}x-\frac{3}{5}x^2\right)$;
h) $\left(\frac{2}{3}x+\frac{3}{5}x^2\right)\left(\frac{2}{3}x-\frac{3}{5}x^2\right)$.

12.21 (*). Applica la regola della somma per differenza ai seguenti casi.

a) $(2a+b+1)(2a+b-1)$;
b) $(3x-b+c)(3x+b-c)$;
c) $[(2x+y)+(3y-1)]\,[(2x+y)-(3y-1)]$;
d) $(ab-2b-a)(-ab+2b-a)$;
e) $\left(\frac{1}{2}a+1+b+ab\right)\left(\frac{1}{2}a+1-b-ab\right)$;
f) $(3x-y-1)(3x+y-1)$.

12.22 (*). Semplifica le seguenti espressioni con prodotti notevoli.

a) $(a+b)(a-b)-(a+b)^2$;
b) $[(x-1)(1+x)]^2$;
c) $\left(\frac{2}{3}a-b\right)\left(\frac{2}{3}a+b\right)-\frac{2}{3}(a-b)^2+2\left(\frac{1}{3}a\right)^2$;
d) $-\frac{1}{4}y^2+4x^2+\left(5x-\frac{1}{5}\right)\left(5x+\frac{1}{5}\right)+\left(\frac{1}{5}-5x\right)\left(5x+\frac{1}{5}\right)-\left(2x+\frac{1}{2}y\right)\left(\frac{1}{2}y-2x\right)$.

12.23 (*). Semplifica le seguenti espressioni con prodotti notevoli.

a) $\left(\frac{2}{3}a-b\right)\left(\frac{2}{3}a+b\right)\left(b^2+\frac{4}{9}a^2\right)$;

b) $\left(-\frac{2}{3}x-\frac{2}{3}y\right)\left(\frac{2}{3}x-\frac{2}{3}y\right)+\left(x-\frac{1}{2}\right)\left(-x-\frac{1}{2}\right)+2x\left(x-\frac{1}{4}\right)^2$;

c) $(a+b-1)^2+(a-b)^2+\left(a-\frac{1}{2}b\right)\left(a+\frac{1}{2}b\right)+2a\left(a-\frac{1}{2}\right)-a(5a+3)-(2b-1)$;

d) $(x^2+2x)\left(\frac{1}{2}x+1\right)+\left(\frac{1}{2}x-1\right)^2-\left(\frac{1}{2}x+1\right)\left(-\frac{1}{2}x+1\right)-\frac{1}{2}x^2(x+5)$.

12.4 - Cubo di un binomio

12.24. Riconosci quali dei seguenti polinomi sono cubi di binomi.

a) $-a^3-3a^2b+3ab^2+b^3$;
b) $a^9-6a^4b-12a^2b^2-8b^3$;
c) $8a^9-b^3-6b^2a^3+12a^6b$;
d) $\frac{1}{27}a^6-8b^3+4a^2b^2-\frac{2}{3}a^4b$.

12.25. Sviluppa i seguenti cubi di binomio.

a) $\left(2a+b^2\right)^3=(2a)^3+3\cdot(2a)^2\cdot b^2+3\,(2a)\cdot\left(b^2\right)^2+\left(b^2\right)^3=\ldots\ldots\ldots\ldots\ldots\ldots$
b) $(x-2y)^3=x^{\cdots}-6x^{\cdots}y+12xy^{\cdots}-\ldots y^{\cdots}$
c) $(a+b)^2+(a+b)(a-b)+(a+b)^3-a^3-b^3-a^2-b^2-ab$.

12.26. Sviluppa i seguenti cubi di binomio.

a) $(x+y)^3$;
b) $(x-y)^3$;
c) $(-x+y)^3$;
d) $(a+2)^3$;
e) $\left(\frac{1}{2}a+b\right)^3$;
f) $(a+1)^3$;
g) $(a-1)^3$;
h) $(x+2y)^3$;
i) $(y-2x)^3$;
j) $\left(a-\frac{2}{3}b\right)^3$;
k) $(2x+y)^3$;
l) $\left(x^2y-3\right)^3$;
m) $(xy-1)^3$;
n) $(x^2-2y)^3$;
o) $\left(\frac{1}{2}a-\frac{2}{3}b\right)^3$.

12.27 (*). Sviluppa i seguenti cubi di binomio.

a) $(a-3)^3$;
b) $\left(\frac{1}{2}a^2-\frac{3}{2}a\right)^3$;
c) $\left(\frac{2}{3}x-1\right)^3$;
d) $\left(x-\frac{1}{3}\right)^3$;
e) $\left(\frac{1}{2}xy-2x\right)^3$;
f) $(x+3)^3$;
g) $\left(\frac{2}{5}x^2y-5yx^2a\right)^3$;
h) $\left(\frac{1}{2}x^2+1\right)^3$;
i) $\left(\frac{3}{4}a^2b^3c^2-\frac{1}{3}a^2bc^2\right)^3$;
j) $\left(-\frac{1}{2}+\frac{1}{4}xy^2z^3\right)^3$;
k) $\left(x^2-y^2\right)^3$;
l) $\left(-3xy^2+\frac{3}{2}zx^2\right)^3$;
m) $\left(2x^2z+\frac{2}{3}y^2z^3x\right)^3$;
n) $-\left(\frac{1}{2}x^2-1\right)^3$;
o) $\left(\frac{1}{4}ab^2c-4a^2b\right)^3$.

12.5 - Potenza n-esima di un binomio

12.28. Sviluppa la seguente potenza del binomio.

$$\left(2a-b^2\right)^4 = (2a)^4 + 4\cdot(2a)^3\cdot\left(-b^2\right) + 6\,(2a)^2\cdot\left(-b^2\right)^2 + \ldots(2a)\cdot\left(-b^2\right)^{\cdots} + \left(-b^2\right)^{\cdots}$$

12.29. Sviluppa le seguenti potenze di binomio.

a) $(a+1)^5$;
b) $(x-1)^6$;
c) $\left(a-\frac{1}{2}\right)^4$;
d) $(1-y)^7$;
e) $(a+2)^5$;
f) $\left(\frac{1}{2}a-1\right)^4$;
g) $(a-2)^6$;
h) $(2a-1)^2$;
i) $\left(2-\frac{1}{2}a\right)^5$;
j) $\left(3x^2a-a^2\right)^5$;
k) $\left(2x^2-1\right)^6$;
l) $\left(\frac{1}{3}-2x\right)^5$.

12.30. Trova la regola generale per calcolare il cubo del trinomio $(A+B+C)^3$.

12.6.2 Esercizi riepilogativi

12.31 (*). Risolvi utilizzando i prodotti notevoli.

a) $[a+2\,(b-c)]\,[a-2\,(b-c)]+4b(b-2c)$;
b) $\left[(a-2b)^2-a^3\right]\left[-a^3-(a-2b)^2\right]+a^2(a^2-8ab+24b^2-a^4)$;
c) $x(x-1)^2+(x+1)(x-1)-x(x+1)(x-3)-(x+2)^2$;
d) $(x+1)^2-(x-1)^2$;
e) $(x+1)^3-(x-1)^3-6x^2$.

12.32 (*). Risolvi utilizzando i prodotti notevoli.

a) $(x+1)^2+(x-2)^2-(x-1)^2-(x+1)(x-1)$;
b) $(x+2)(x-2)+(x+2)^2$;
c) $(x+1)^3-(x-1)\left(x^2+x+1\right)+3x(x-1)$;
d) $(x+1)(x-1)+(x+1)^2+(x-1)^2$;
e) $(x+y+1)(x+y-1)+(x+y)^2-2(x+y)(x-y)-(2y-1)(2y+1)$.

12.33 (*). Risolvi utilizzando i prodotti notevoli.

a) $\left(\frac{1}{2}a+\frac{2}{3}-3b+\frac{1}{3}ab\right)\left(\frac{1}{2}a-\frac{2}{3}-3b-\frac{1}{3}ab\right)+\frac{1}{9}ab(31+ab)-\left(\frac{1}{2}a-\frac{2}{3}\right)\left(\frac{1}{2}a+\frac{2}{3}\right)$;
b) $(x-y)^2+(x+y)(y-x)$;
c) $(x+y-z)^2+(x-y+z)^2-2(x-y-z)^2$;
d) $(a-3b)^2+(2a+3b)(2a-3b)-(a+2b)(b-2a)$;
e) $\left[3x^2-(x+2y)(x-2y)\right]^2-2x\left(\frac{1}{2}x-\frac{3}{2}y\right)^2-3xy\left(x+\frac{3}{2}y\right)-\left(2x^2+4y^2\right)^2$.

12.34. Risolvi utilizzando i prodotti notevoli.

a) $\left[(x+2y)^2-\left(x^2-2y\right)^2\right]\left[(x+2y)^2+\left(x^2-2y\right)^2\right]$;
b) $(a+2b-3c)(a+2b+3c)\left(a^2-b\right)\left(-a^2-b\right)+(2a-b)^3$;

c) $\left(x^2+yx+\frac{2}{3}\right)^2-\left(3b^2+\frac{1}{2}a^4+2a^3+\frac{1}{3}a^2\right)^2$;

d) $\left(3x^2-4xy+\frac{2}{5}-y^2x+\frac{1}{2}y^3\right)^2+\left(2x^2y^2+\frac{3}{2}y^2\right)\left(2x^2y^2-\frac{3}{2}y^2\right)$.

12.35 (*). Risolvi utilizzando i prodotti notevoli.

a) $-2x(x-1)^2+2x\left(x-\frac{1}{3}\right)^2-\frac{4}{3}x\left(2x-\frac{4}{3}\right)$;

b) $(a-2b)^4-b(2a-b)^3-a^2(a+6b)^2$;

c) $[(x-1)^2-2]^2-\left(x^2+x-1\right)^2+6x(x-1)(x+1)$;

d) $(x+1)^4-(x+1)^2(x-1)^2-4x(x+1)^2$;

e) $\frac{(x-2)(x+2)}{4}+\frac{(x-2)^2}{(-2)^2}+x$.

12.36 (*). Risolvi utilizzando i prodotti notevoli.

a) $\left(2x-\frac{1}{3}\right)^3+4\left(x+\frac{1}{2}\right)^2$;

b) $(x+1)^3-3(x-1)(-1-x)+(x-4)(x+1)$;

c) $\left(x-\frac{1}{3}\right)^2+\left(x+\frac{1}{3}\right)^2-(x+1)^2-\left(x-\frac{4}{3}\right)\left(x+\frac{4}{3}\right)$;

d) $(x-3)^3-x^2(x-9)-9(x-3)-9$;

e) $x(x-1)^2(x+1)+(x-1)^2-x(x-1)^3$.

12.37 (*). Risolvi utilizzando i prodotti notevoli.

a) $-\frac{1}{2}x\left(x+\frac{3}{4}\right)(2x+1)+\left[x+1\left(x-\frac{1}{2}\right)\left(3x+\frac{1}{2}\right)^2\right]+\frac{1}{8}(5x+1)$;

b) $\frac{1}{9}(x-4)(x+4)+\frac{1}{3}(x-1)^2-\frac{1}{9}x(x-2)+\left(x-\frac{5}{2}\right)\left(x+\frac{1}{3}\right)+\frac{41}{18}$;

c) $\left(\frac{1}{2}x^2+1\right)^3+\frac{1}{6}x^2-\left(\frac{1}{2}x^2-1\right)^3-\frac{1}{6}(x+1)^3-\frac{3}{2}x^4+\frac{1}{6}\left(x^3-11\right)$;

d) $-x^2\left(x^2-1\right)+\left(x^2-4x+2\right)^2+4(x-1)^2+8(x-1)^3$;

e) $x\left(2x^2+3x\right)^2-2x^3\left(2x-\frac{1}{2}\right)^2+x^3(x-2)^3-x^2\left(x^3+2x^2\right)(x-12)$.

12.38 (*). Risolvi utilizzando i prodotti notevoli.

a) $\left\{\left[\left(a-\frac{1}{3}a^2\right)-a^2\right]:\left(-\frac{1}{3}a^2\right)\right\}\left(2a+\frac{1}{3}a^2\right)-\frac{2}{3}\left(\frac{9}{4}a^2-\frac{1}{6}a^4\right)$;

b) $\left[\frac{1}{3}bx-\left(\frac{1}{2}a+\frac{2}{3}b-x\right)^2+\left(\frac{1}{2}a+\frac{2}{3}b+x\right)^2\right]^2-3bx^2(4a+3b)$;

c) $8\left(4x-\frac{1}{2}y\right)\left(\frac{1}{2}y+4x\right)+3(y+z)(y-z)+6\left(4x+\frac{1}{2}z\right)\left(-4x+\frac{1}{2}z\right)$;

d) $\left[\left(x-y-\frac{1}{3}\right)\left(y-x-\frac{1}{3}\right)-\frac{1}{9}\right](x+y)^2+(x-y)^2(x+y)^2$.

12.39 (*). Risolvi utilizzando i prodotti notevoli.

a)
$$\left\{\left[\left(\frac{3}{2}a+\frac{2}{9}\right)^2-\left(\frac{2}{9}-\frac{3}{2}a\right)^2\right]^2-\left(2a^2+\frac{1}{3}\right)^2+\left(2a^2-\frac{1}{3}\right)^2\right\}+ \\ -\left[\left(2a-\frac{1}{3}\right)^2+\left(\frac{1}{3}-2a\right)^2\right]^2;$$

b)
$$\left(b+\frac{1}{2}a\right)^3-b^2\left(\frac{3}{2}a+b\right)+a\left[\frac{1}{2}ab-\left(\frac{1}{4}a+b\right)^2+\left(b+\frac{1}{4}a\right)\left(-\frac{1}{4}a+b\right)\right]+ \\ +\left\{\left[\left(2a^2b^3-\frac{1}{4}a^5\right)(-4a)+(2ab+1)\left(4a^2b^2-2ab+1\right)\right]^2-1\right\}:a^6;$$

c) $\left[(x-4)^3-2(x-1)^2\left(\frac{1}{3}x-8\right)-\frac{1}{3}x^2(x+16)\right]-2\left(\frac{23}{3}x-24\right).$

12.40 (*). Risolvi utilizzando i prodotti notevoli.

a)
$$(x-y)^3-(y-x)^3+2xy(x+y)(x-y)-7(x-y)\left(x^2+xy+y^2\right)+ \\ +5\left(x^3-y^3\right)-2xy(x+y)(x-y+3);$$

b)
$$\left(3ab-\frac{1}{2}a\right)^2+\frac{1}{2}a+2b\left(\frac{1}{2}a-b\right)\left(\frac{1}{2}a+b\right)-\left(1-\frac{3}{2}a\right)^3+ \\ -9a^2\left(\frac{3}{8}a+b^2-\frac{13}{18}\right)+5a\left(\frac{1}{2}ab-1\right);$$

c)
$$\frac{1}{3}x\left\{x^2-1-\left[3x\left(x-\frac{1}{3}\right)^2-\frac{2}{3}x\left(x-\frac{2}{3}\right)^3\right]\right\}-\frac{2}{9}x\left(x-3x^2\right)\left(x+3x^2\right)+ \\ -\frac{1}{9}x^2\left(20x^3-13x^2+\frac{29}{3}x-\frac{43}{27}\right);$$

d)
$$\left(x-\frac{1}{2}y\right)^2-\left(2x+\frac{1}{2}y^2\right)^2+\left(x+\frac{1}{2}y\right)\left(-x+\frac{1}{2}y\right)+(x-y)^3+x^2(3y+4)+ \\ +xy(1-y)+\frac{1}{2}y^2(y-1)(y+1).$$

12.41. Risolvi utilizzando i prodotti notevoli.

a) $\left(\frac{2}{5}zx^3-3x^2y\right)\left(\frac{2}{5}zx^3+3x^2y\right)+\left(2x^2y^2z^3+\frac{1}{2}z^2x^2y\right)^3$;

b) $-2t(t-x)-3t^2+x(x+t)(t-x)+(x-t)^2-\frac{1}{2}\left(x-\frac{1}{2}t\right)^3$;

c) $\frac{1}{9}(x-4)(x+4)+\frac{1}{3}(x-1)^2-\frac{1}{9}x(x-2)^2-x\left(x-\frac{5}{2}\right)\left(\frac{5}{2}-x\right)+\frac{5}{2}\left(\frac{1}{2}x-\frac{1}{3}\right)^2$.

12.42 (*). Risolvi utilizzando i prodotti notevoli.

a) $\left[\left(\frac{1}{3}x+\frac{2}{3}y\right)^2-\left(\frac{1}{3}x\right)^2\right]:\left(\frac{1}{3}y\right)+\left(\frac{1}{3}y-1\right)^3+\frac{1}{3}(y-8)(y-7)+\frac{1}{3}(1+8y)$;

b) $-\left(\frac{1}{4}x+1\right)^2-\frac{1}{16}(2x-1)^2-\frac{1}{2}(3-x)^2-\frac{3}{16}x^2+5+\left(x+\frac{3}{4}\right)^2$;

c) $\left(x-\frac{1}{2}\right)\left(x^2+\frac{1}{4}+\frac{1}{2}x\right)-\left(x+\frac{1}{2}\right)\left(x-\frac{1}{2}\right)-\left(x+\frac{1}{2}\right)^3-\frac{1}{2}\left(7x^2-\frac{3}{4}\right)+\frac{3}{8}(2x-1)$;

d) $(1-x^n)^2-(2x^n-1)^2-\left(2x^{n+1}\right)^2+\left(x^{2n}-1\right)\left(x^{2n}+1\right)$.

12.43 (*). Risolvi utilizzando i prodotti notevoli.

a)
$$\left(\frac{1}{3}ab-\frac{2}{5}xy\right)\left(-\frac{1}{3}ab-\frac{2}{5}xy\right)-4x^2\left(\frac{1}{5}y-\frac{3}{2}\right)^2-\left(x-\frac{1}{3}ab\right)\left(x+\frac{1}{3}ab\right)+ \\ +10x^2\left(1-\frac{6}{25}y\right);$$

b)
$$\left(x+\frac{1}{2}\right)^2+2\left(x-\frac{1}{2}\right)^3-2\left(x+\frac{1}{2}\right)\left(x-\frac{1}{2}\right)-x\left[(x+1)(x+2)+(x+1)^2+\frac{1}{2}x\right]+ \\ +\frac{1}{2}\left(x^2+x-1\right);$$

c)
$$\left(\frac{3}{2}x-2y\right)\left(\frac{3}{2}x+2y\right)\left(\frac{9}{4}x^2+4y^2\right)+x\left(\frac{1}{2}x-2y\right)^2+\left(\frac{3}{2}x+2y\right)^3+ \\ -\frac{3}{4}x\left(x-\frac{2}{3}y\right)\left(x+\frac{2}{3}y\right)+\left(4y^2-\frac{9}{4}x^2\right)\left(4y^2+\frac{9}{4}x^2\right)+ \\ +\frac{1}{2}xy\left(y-\frac{1}{6}x\right)-\left(\frac{5}{2}x+2y\right)^3+\frac{51}{4}x^3;$$

d)
$$\left(x+\frac{1}{3}y\right)\left(x-\frac{1}{3}y\right):\frac{1}{3}-\left(x+\frac{1}{2}xy\right)^2:\left(-\frac{1}{2}x^2\right)+\frac{1}{3}(-3x+y)(3x+y)+ \\ -\frac{1}{2}\left(y^2+4y+4\right).$$

12.44 (*). Risolvi utilizzando i prodotti notevoli.

a)
$$\frac{1}{4}(x+1)^4+\frac{1}{2}(x+1)^2+\frac{1}{8}\left(x^2+1\right)(x+1)(x-1)-\left(2x^2-2x+1\right)^2+ \\ +9x^3\left(\frac{3}{8}x-1\right)+\frac{1}{4}x^2\left(x^2+16\right)+6x-\frac{3}{8};$$

b)
$$\left[2\left(a-\frac{1}{2}b\right)\left(a+\frac{1}{2}b\right)\right]^2-\left(2a^2-b\right)\left(2a^2+b\right)-6a^2(a-2b)(2b-a)+ \\ -b^2\left(22a^2+\frac{1}{4}b^2+1\right)-6a^3(a-4b);$$

c) $\left[(a-b)^2-(a+b)\right](a-b)^2-(a-b)\left[(a-b)^3-(a+b)(a-b)\right]$;

d) $(2x+y)^2(2x-y)^2-\left(4x^2+y^2\right)^2+4\left(xy+\frac{1}{2}\right)\left(xy-\frac{1}{2}\right)$.

12.45 (*). Risolvi utilizzando i prodotti notevoli.

a) $\left(\frac{1}{2}x-a\right)^2\left[\left(x-\frac{1}{2}a+1\right)\left(x+\frac{1}{2}a+1\right)-\left(x+\frac{1}{2}a\right)\left(x-\frac{1}{2}a\right)\right]^2$;

b) $-(x-y)^2(x+y)^2\left(x^2+y^2\right)+\left(x^2+y^2\right)\left(x^4+y^4-x^2y^2\right)$;

c) $3\left(2a-\frac{1}{3}a^2\right)^2-\frac{1}{2}\left(2a^2+\frac{3}{2}a\right)^2+3\left(a^2+\frac{1}{2}a\right)\left(a^2-\frac{1}{2}a\right)$;

d) $\left(4a^2-4x^2\right)^2+\left(\frac{1}{2}a^2-4x^2\right)^2+\left(4a^2-4x^2+2ax\right)\left(-4a^2+4x^2+2ax\right)$;

e) $\left(\frac{1}{2}a+b+\frac{2}{3}\right)\left(\frac{1}{2}a+b-\frac{2}{3}\right)\left(\frac{1}{2}a-b+\frac{2}{3}\right)\left(-\frac{1}{2}a+b+\frac{2}{3}\right)-\left(\frac{1}{4}a^2+b^2+\frac{4}{9}\right)^2$.

12.6.3 Risposte

12.8. a) $3y^2-3x^2$, b) $x^2-14xy-5y^2$.

12.9. a) $\frac{1}{2}x^2-\frac{5}{2}x+\frac{9}{8}$, b) $\frac{1}{2}x-\frac{3}{2}y$, c) $\frac{35}{4}x^2$, d) $-3x^2-14x-8$.

12.10. a) $-6x^2+5x-\frac{3}{8}$, b) $2ab^3-b^4-162$, c) $\frac{7}{4}x^4-\frac{17}{4}x^3+\frac{13}{4}x^2$.

12.14. a) $4xy-4x$, b) $-18ax-16bx$, c) $x^4+2x^3+2x^2$, d) $4ab+4a$.

12.21. d) $a^2-a^2b^2+4ab^2-4b^2$, e) $-a^2b^2+\frac{1}{4}a^2-2ab^2+a-b^2+1$, g) $9x^2-6x-y^2+1$.

12.22. a) $-2ab-2b^2$, b) x^4-2x^2+1, c) $\frac{4}{3}ab-\frac{5}{3}b^2$, d) $8x^2-\frac{1}{2}y^2$.

12.23. a) $\frac{16}{81}a^4-b^4$, c) $\frac{7}{4}b^2-4b-6a+2$, d) x.

12.27. b) $\frac{1}{8}a^6 - \frac{27}{8}a^3 + \frac{9}{8}a^5 + \frac{27}{8}a^4$, g) $\frac{8}{125}x^6y^3 - 125y^3x^6a^3 - \frac{12}{5}x^6y^3a + 30x^6y^3a^2$, l) $-27x^3y^6 + \frac{81}{2}zx^4y^4 - \frac{81}{4}z^2x^5y^2 + \frac{27}{8}z^3x^6$, o) $\frac{1}{64}a^3b^6c^3 - \frac{3}{4}a^4b^5c^2 + 12a^5b^4c - 64a^6b^3$.

12.31. a) $a^2 - 4c^2$, b) $+32ab^3 - 16b^4$, c) -5, d) $4x$, e) 2.

12.32. a) 5, b) $2x^2 - 4x$, c) $6x2 + 2$, d) $3x^2 + 1$, e) $4xy$.

12.33. a) $9b^2$, b) $2y^2 - 2xy$, c) $4xy + 4xz - 8yz$, d) $7a^2 - 3ab - 2b^2$, e) $-\frac{1}{2}x^3 - 9xy^2$.

12.35. a) 0, b) $17b^4 - 38ab^3 - 28a^3b$, c) $3x^2$, d) 0, e) $\frac{1}{2}x^2$.

12.36. a) $8x^3 + \frac{14}{3}x + \frac{26}{27}$, b) $x^3 + 7x^2 - 6$, c) $1 - 2x$, d) $18x - 9$, e) $2x^3 - 3x^2 + 1$.

12.37. a) $8x^3 - \frac{11}{4}x^2$, b) $\frac{4}{3}x^2 - \frac{47}{18}x$, c) $-\frac{1}{2}x - \frac{1}{3}x^2$, d) x^2, e) $52x^4 + \frac{1}{2}x^3$.

12.38. a) $\frac{5}{2}a^2$, b) $4a^2x^2$, c) $32x^2 + y^2 - \frac{3}{2}z^2$, d) 0.

12.39. a) $-\frac{8}{9}a^2$, b) $\frac{3}{4}a^2b + a^6 + 2$, c) 0.

12.40. a) $-12x^2y$, b) $2b^3 - 3$, c) $-\frac{1}{3}x$, d) $x^3 - y^3 + \frac{1}{4}y^4$.

12.42. a) $\frac{4}{3}x + \frac{y^3}{27} + 18$, b) $\frac{17}{4}x$, c) $-6x^2$, d) $-1 + 2x^n - 3x^{2n} - 4x^{2n+2} + x^{4n}$.

12.43. a) 0, b) $-9x^2$, c) $-\frac{43}{6}xy^2 - \frac{313}{12}x^2y$, d) 0.

12.44. a) $-2x^2 + 12x - \frac{3}{4}$, b) 0, c) 0, d) $-12x^2y^2 - 1$.

12.45. a) $x^4 - 4x^3a + 4x^2a^2$, b) $x^4y^2 + x^2y^4$, c) $\frac{4}{3}a^4 - 7a^3 + \frac{81}{8}a^2$, d) $\frac{1}{4}a^4 + 16x^4$, e) $-\frac{1}{8}a^4 - 2b^4 - \frac{32}{81}$.