intellus LEARNING OPEN COURSES

Fluid Mechanics

Module Overview

Acknowledgments

This presentation is based on and includes content derived from the following OER resource:

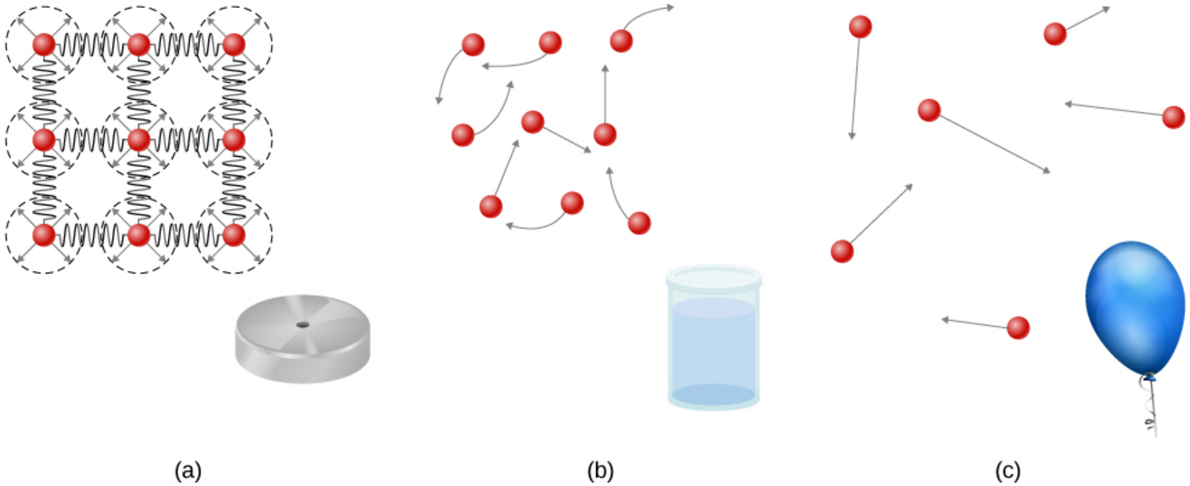
University Physics Volume 1

An OpenStax book used for this course may be downloaded for free at: https://openstax.org/details/books/university-physics-volume-1

Characteristics of Solids vs. Fluids

Solids are rigid and have definite shape and volume. Their atoms are closely packed and they exert strong forces on one another. Large forces are needed to change their shape and they resist shearing forces.

Fluids yield to shearing forces, are more loosely interatomically bonded, and have indefinite shapes. Both gases and liquids behave as fluids.



(University Physics Volume 1. OpenStax. Fig. 14.2.)

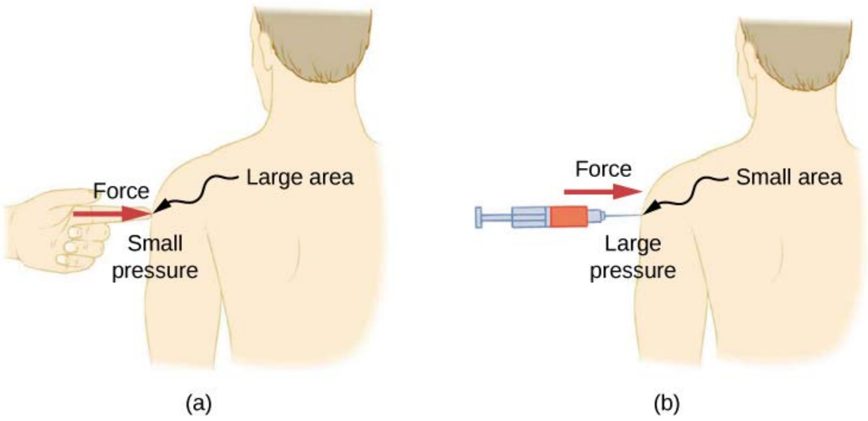
Density

Density, ρ, is the mass per unit volume of a substance or object, $\rho = \frac{m}{V}$. Density can be used to identify a substance and determines whether an object sinks or floats on top of a fluid. If the density of a material is not constant throughout its volume, the local density can be found in the limit where the volume approaches zero, $\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V}$.

The **specific gravity** of a material, often used to compare densities, is the ratio of the material's density to that of water, $\text{Specific gravity} = \frac{\text{Density of material}}{\text{Density of water}}$.

Pressure

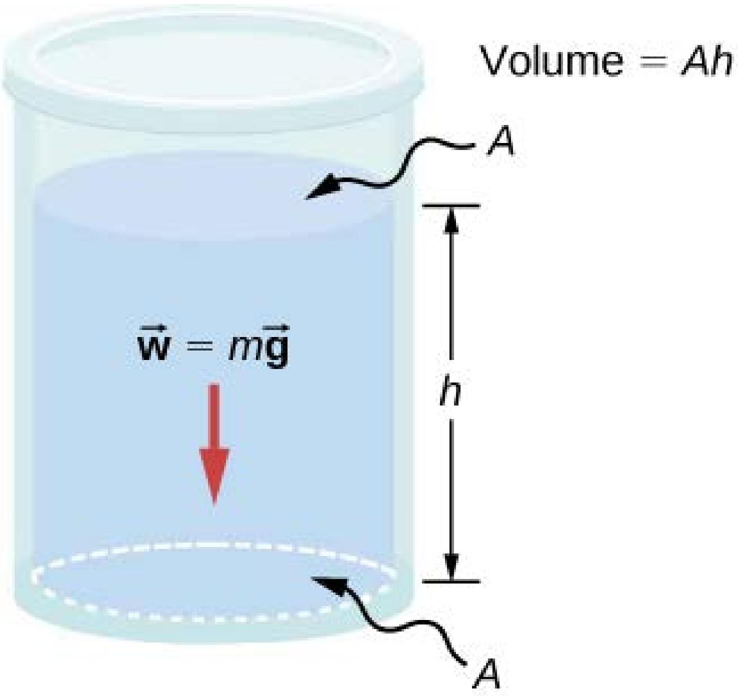
Pressure, p, is the normal force per unit area A over which the force is applied, $p = \frac{F}{A}$. Pressure over a specific point is the force dF over an infinitesimal area dA, $p = \frac{dF}{dA}$. The SI unit of pressure is the pascal, defined as $1 \text{ Pa} = 1 \text{ N/m}^2$.



(University Physics Volume 1. OpenStax. Fig. 14.5.)

Variation of Pressure with Depth

The pressure at the bottom of the fluid is due to both atmospheric pressure p_0 and the weight of the fluid in the layers above. For a fluid with constant density ρ, the weight of the fluid is given by $w = mg = \rho V g = \rho A h g$, resulting in a pressure of $\rho h g$. The total pressure at the bottom of the container is the sum of the atmospheric pressure and the pressure due to the weight of the fluid above, $p = p_0 + \rho h g$.



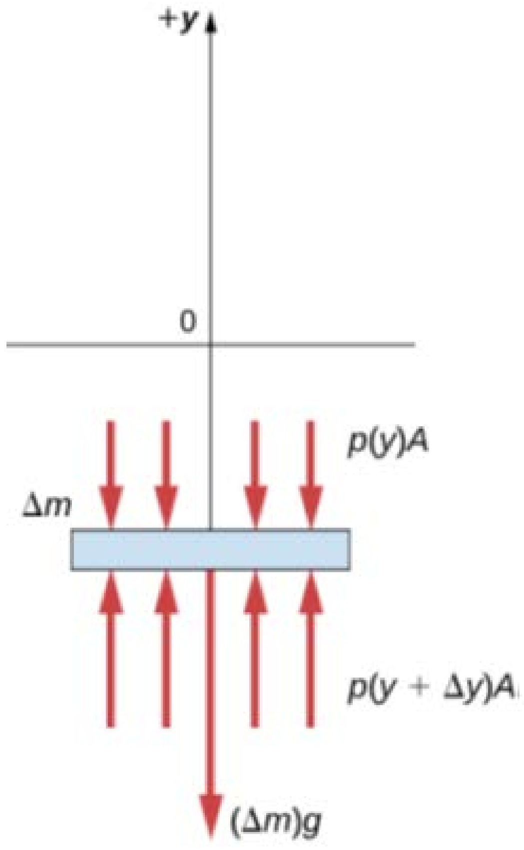
(University Physics Volume 1. OpenStax. Fig. 14.6.)

Pressure in a Static Fluid Due to Uniform Gravity

A static fluid is a fluid that is not in motion. The variation of pressure with depth can be derived by considering the forces on a thin layer of fluid. Since the fluid is static, Newton's second law says that

$$p(y + \Delta y)A - p(y)A - g\Delta m = 0.$$

Writing the mass as $\Delta m = -\rho A \Delta y$, and solving for the change in density yields the variation of pressure with depth in the fluid, $\frac{dp}{dy} = -\rho g$.



(University Physics Volume 1. OpenStax. Fig. 14.8.)

Pressure in Fluid with Constant Density

The variation of pressure with depth can be integrated to find the pressure at a given height in a fluid. For a fluid with constant density, the integral is simple, $\int_{p_0}^{p} dp = -\int_{0}^{-h} \rho g dy$, and evaluating it reproduces the result found previously, $p = p_0 + \rho h g$.

(University Physics Volume 1. OpenStax. Fig. 14.9.)

Variation of Atmospheric Pressure with Height

For a simple model of the atmosphere, we assume the pressure, density, and pressure are related by the ideal gas law, $p = \rho \frac{k_B T}{m}$. Solving this equation for density and substituting into the equation for variation of pressure with height, we find $\frac{dp}{dy} = -p(\frac{mg}{k_B T})$. Integrating this expression yields the variation of pressure with height in the atmosphere, $p(y) = p_0 \exp(-\frac{mg}{k_B T} y)$. The term $\frac{k_B T}{mg}$ is a length scale over which the pressure in the atmosphere drops by a factor of $\frac{1}{e}$. It has a value of about 8800 m.

Pressure is a scalar quantity so it has no direction, but the resulting forces are vectors that point normal to any surface in contact with the fluid.

Gauge Pressure vs. Absolute Pressure

Most pressure gauges are calibrated to read zero at atmospheric pressure, p_{atm}. This reading is called **gauge pressure**, p_{g}. When the pressure increases above atmospheric pressure, the pressure on the gauge reads positive, and when the pressure drops below atmospheric pressure, the gauge pressure reads negative.

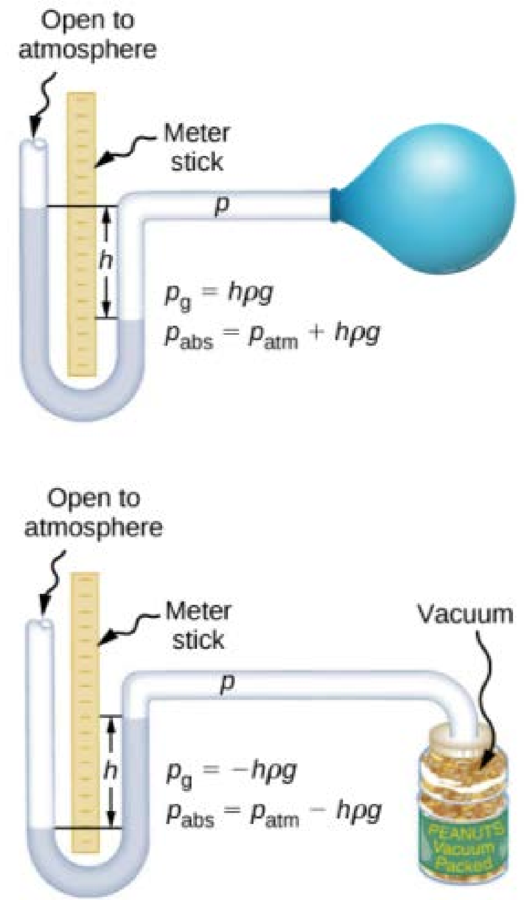
Absolute pressure, p_{abs}, accounts for atmospheric pressure. The two quantities are related by the equation $p_{\mathrm{abs}} = p_{\mathrm{g}} + p_{\mathrm{atm}}$. Under normal circumstances, the absolute fluid cannot be negative, so the smallest possible absolute pressure is zero, and the smallest possible gauge pressure is $p_{\mathrm{g}} = -p_{\mathrm{atm}}$.

Measuring Pressure

Pressure is measured with many devices.

A **manometer** uses a tube with only one end open to atmospheric pressure and measures the height of the fluid exposed to atmospheric pressure.

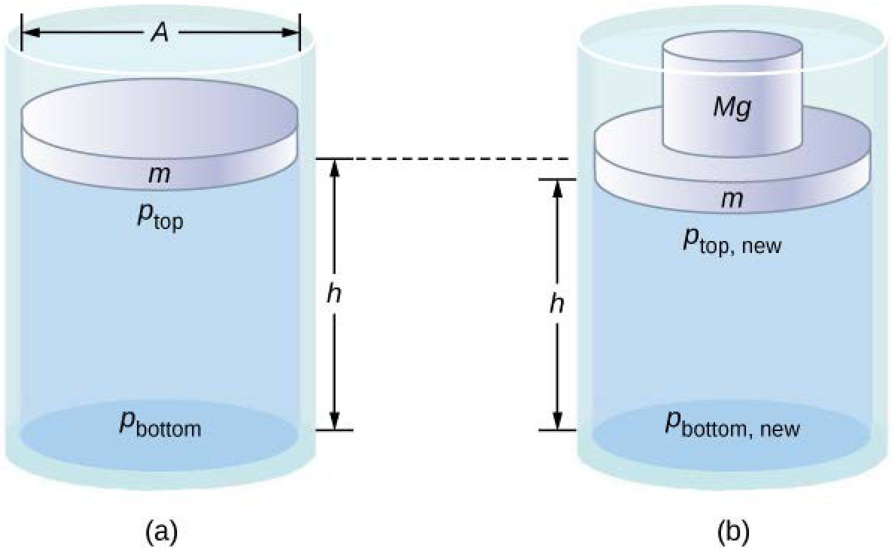
A **barometer** uses a single column of fluid which is pushed up a tube when the fluid at the base of the column is exposed to increased pressure.



(University Physics Volume 1. OpenStax. Fig. 14.12.)

Pascal's Principle

Pascal's principle states that when a change in pressure is applied to an enclosed fluid, it is transmitted undiminished to all portions of the fluid and the walls of its container. Adding a weight Mg to a piston of area A atop a fluid increases the pressure throughout the fluid by $\Delta p = \frac{Mg}{A}$.

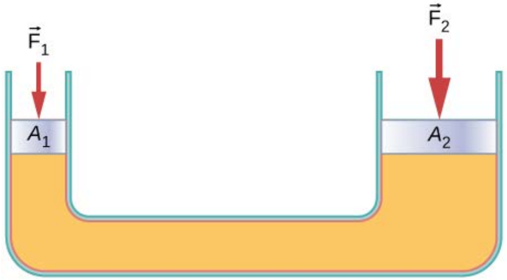


(University Physics Volume 1. OpenStax. Fig. 14.15.)

Hydraulic Systems

Pascal's principle can be applied to lift very heavy weights using a small force. Applying a force F_1 over an area A_1 results in a pressure $p_1 = \frac{F_1}{A_1}$. The same pressure spread over a larger area A_2 results in a larger force F_2 as well, related to the initial force and area by $\frac{F_1}{A_1} = \frac{F_2}{A_2}$.

The **hydraulic jack** uses this relationship to lift cars with a small applied external force.



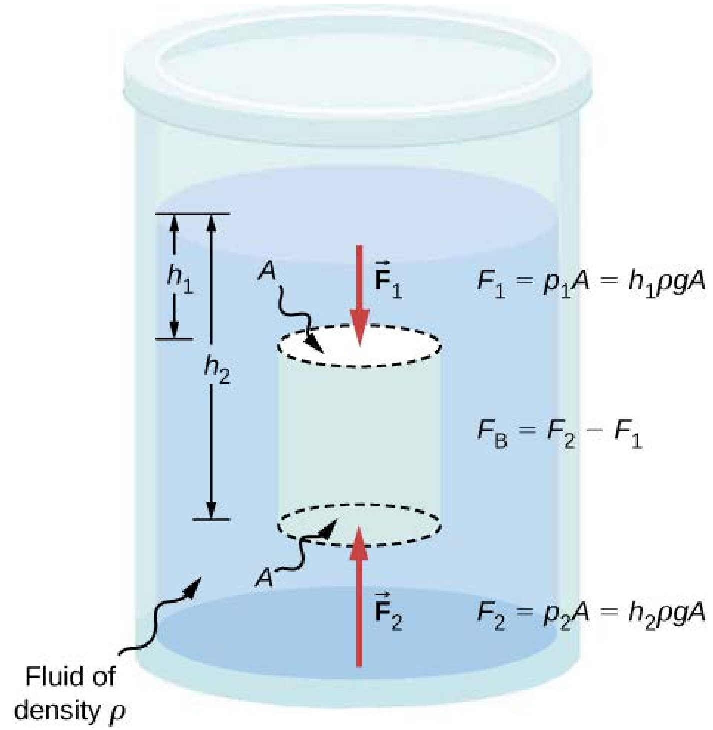
(a)


(b)

(University Physics Volume 1. OpenStax. Fig. 14.15.)

Buoyancy

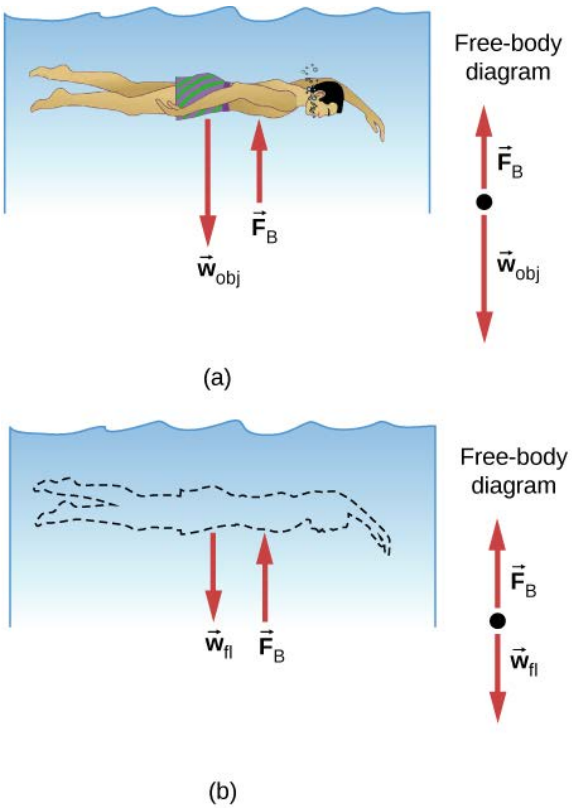
Because pressure increases with depth in a fluid, an upward force called the **buoyant force** is exerted on any object in a fluid. For a submerged cylindrical object, the force on each of the bottom and top faces is $F = h\rho gA$, where ρ is the density of the fluid. The buoyant force is given by the difference in the forces, $F_\mathrm{B} = F_2 - F_1 = \Delta h\rho gA$.



(University Physics Volume 1. OpenStax. Fig. 14.20.)

Archimedes' Principle

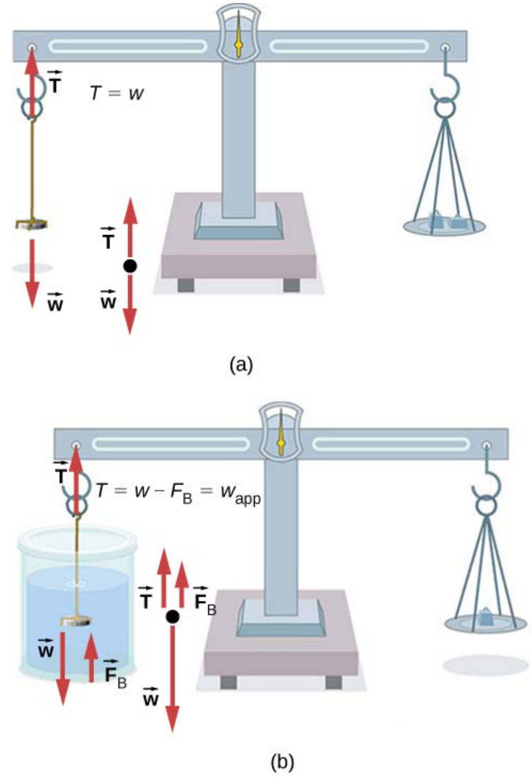
Archimedes' principle states that the buoyant force on an object is equal to the weight of the fluid displaced by the object, $F_{\mathrm{B}} = w$. An object floats atop a fluid if its average density is less than that of the fluid. The fraction submerged is $\frac{V_{\mathrm{sub}}}{V_{\mathrm{obj}}} = \frac{V}{V_{\mathrm{obj}}} = \frac{m/\rho}{m_{\mathrm{obj}}/\rho_{\mathrm{obj}}}$. Since the object floats, $m = m_{\mathrm{obj}}$, and the fraction submerged is $\frac{\rho_{\mathrm{obj}}}{\rho}$.



(University Physics Volume 1. OpenStax. Fig. 14.21.)

Measuring Density

The density of an object can be determined by measuring the weights of the object both in air and in a fluid of known density. The difference in the weight gives the buoyant force, which can be used to calculate the object's density. Conversely, the density of the fluid can be determined if the density of the object is known.



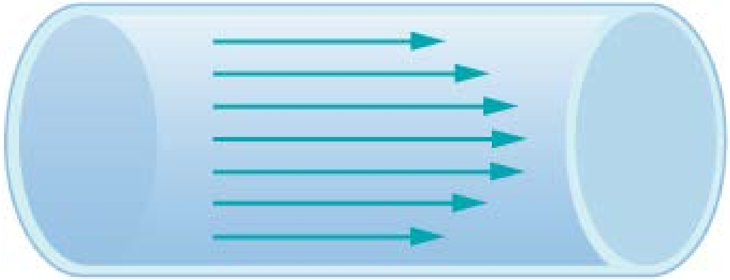
(University Physics Volume 1. OpenStax. Fig. 14.23.)

Characteristics of Flow

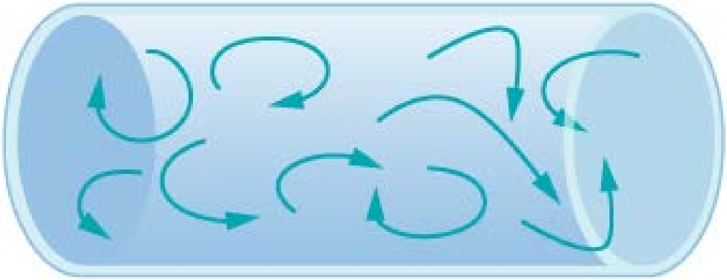
A fluid with no internal resistance to flow, or **viscosity**, is called an **ideal fluid**. Flow can be illustrated as vector field or using streamlines, which follow the direction of flow.

Laminar flow is smooth, steady, and parallel flow.

Turbulent flow is irregular flow that changes over time.



(a) Laminar Flow

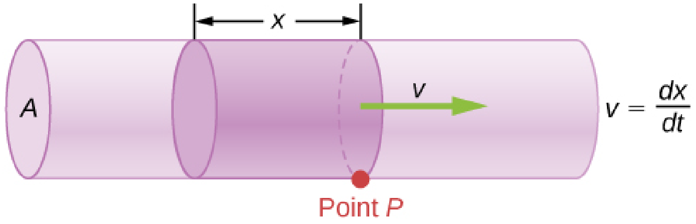


(b) Turbulent Flow

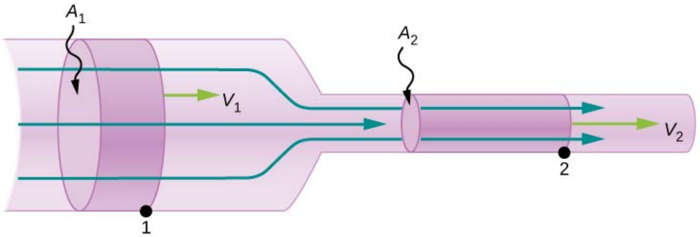
(University Physics Volume 1. OpenStax. Fig. 14.25.)

Flow Rate and Velocity

The **flow rate** Q of a fluid is the volume of fluid passing through an area during a period of time, $Q = \frac{dV}{dt}$. The SI units for flow rate is m^3/s. Flow rate is related to velocity v by the area A of the pipe through which it flows, $Q = Av$. For an incompressible fluid, the flow rate at different points in a closed pipe must be constant, $Q_1 = Q_2$, or in terms of area and velocity, $A_1 v_1 = A_2 v_2$.



$$Q = \frac{dv}{dt} = \frac{d}{dt}(Ax) = A\frac{dx}{dt} = Av$$



(University Physics Volume 1. OpenStax. Figs. 14.26, 14.27.)

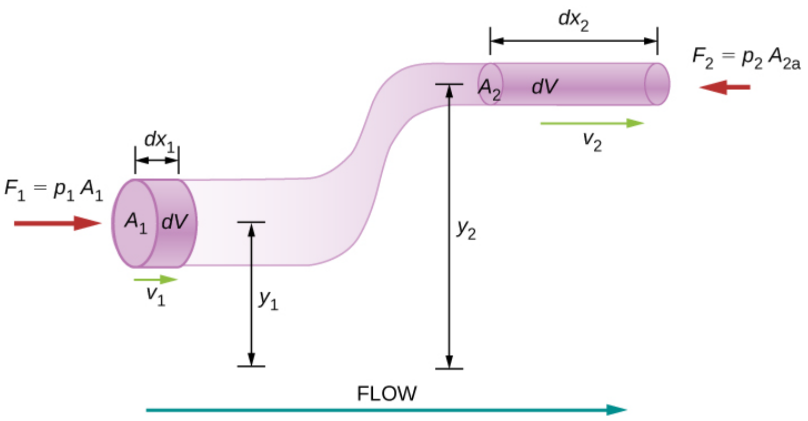
Mass Conservation

Flow is also described in terms of mass flow rate, $\frac{dm}{dt} = \rho A v$. Because the mass entering the pipe must equal the mass exiting the pipe, the continuity equation for mass is written as $(\frac{dm}{dt})_1 = (\frac{dm}{dt})_2$, or $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$.

This more general form of the continuity equation applies to both compressible and incompressible fluids. For an incompressible fluid, the density terms are identical, and the equation reduces to the previous expression, $A_1 v_1 = A_2 v_2$.

Bernoulli's Equation

Consider a slice of an incompressible ideal fluid flowing through a pipe with variable area and height. The work done on the fluid is $dW = (p_1 - p_2)dV$. The work done is due to the gravitational potential, $dU = \rho dV g(y_2 - y_1)$, and the change in the fluid's kinetic energy, $dK = \frac{1}{2}\rho dV(v_2^2 - v_1^2)$. This results in **Bernoulli's equation** for conservation of energy, $p + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$.



(University Physics Volume 1. OpenStax. Fig. 14.30.)

Analyzing Bernoulli’s Equation

For a static fluid, Bernoulli’s equation reduces to the previously derived expression for pressure in a static fluid, $p_2 = p_1 + \rho g h_1$.

For a flowing fluid at constant height, Bernoulli’s equation reduces to **Bernoulli’s principle**, where the height term vanishes, $p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$.

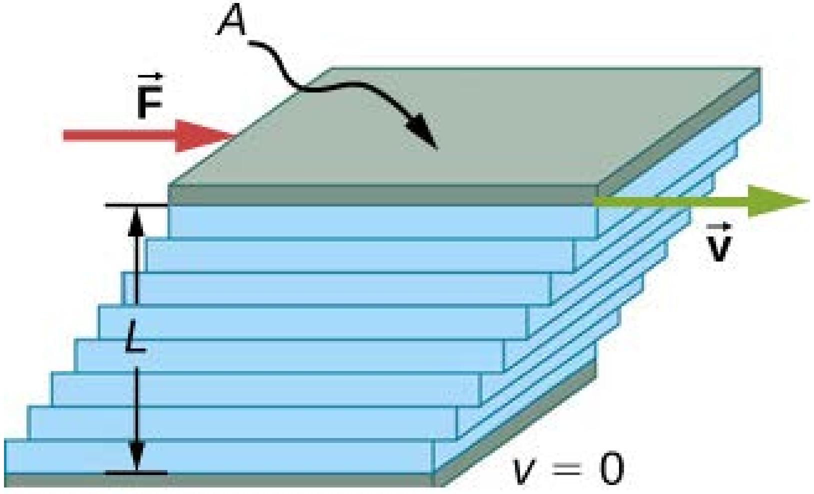
Applications of Bernoulli's Principle

Entrainment is the use of high-velocity fluids to pull other fluids into a stream of flow. As an example, this concept is used to pump perfume out of a bottle.

Bernoulli's principle can be used to measure fluid velocity. If the velocity of the fluid over one side of a manometer is zero, the equation simplifies to $p_1 = p_2 + \frac{1}{2}\rho v_2^2$. The height of the fluid in the manometer rises by a height $h \propto \frac{1}{2}\rho v^2$. Solving for v^2, we have $v^2 \propto \sqrt{h}$.

Viscosity and Laminar Flow

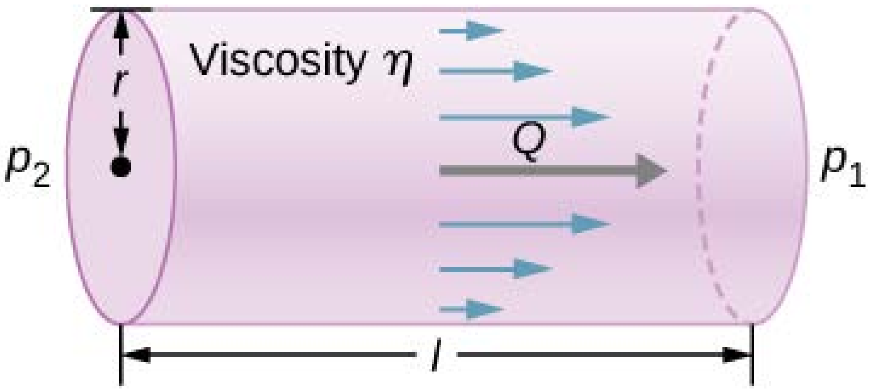
Viscosity, η, is fluid friction within the fluid and between the fluid and its surroundings. It is measured by placing a layer of fluid with thickness L between two plates of area A. The bottom plate is held fixed while a force F is applied to the top plate to move it laterally at a constant velocity v. This measurement method gives us a working definition of viscosity, $\eta = \frac{FL}{vA}$. The SI unit of viscosity is $\mathrm{Pa} \cdot \mathrm{s}$.



(University Physics Volume 1. OpenStax. Fig. 14.36.)

Poiseuille's Law

The flow rate Q of a fluid is caused by a pressure difference, $Q = \frac{p_2 - p_1}{R}$, where R is resistance to flow. Resistance to flow is described in terms of fluid viscosity η, pipe length l, and pipe radius r by **Poiseuille's law for resistance**, $R = \frac{8\eta l}{\pi r^4}$. Flow rate in terms of these variables and pressure difference is called **Poiseuille's law**, $Q = \frac{(p_2 - p_1)\pi r^4}{8\eta l}$.



(University Physics Volume 1. OpenStax. Fig. 14.38.)

Flow, Resistance, Pressure Drops, and Turbulence

Rearranging the expression for flow rate gives the pressure difference on two ends of a pipe, $p_2 - p_1 = RQ$. Resistance is highest in narrow areas, resulting in a larger pressure drop in narrow areas for a given flow rate. Turbulence also creates a high resistance to flow.

A **Reynolds number** N_{R} is a unitless number that describes whether flow is laminar or turbulent. The Reynolds number of flow in a tube of uniform diameter is $N_{\mathrm{R}} = \frac{2\rho v r}{\eta}$. Experimental evidence shows that systems with Reynolds numbers below 2000 are usually laminar and systems with Reynolds numbers about 3000 are typically turbulent. Between these two values, flow is unstable and may oscillate between laminar and turbulent behavior.

How to Study this Module

- Read the syllabus or schedule of assignments regularly.
- Understand key terms; look up and define all unfamiliar words and terms.
- Take notes on your readings, assigned media, and lectures.
- As appropriate, work all questions and/or problems assigned and as many additional questions and/or problems as possible.
- Discuss topics with classmates.
- Frequently review your notes. Make flow charts and outlines from your notes to help you study for assessments.
- Complete all course assessments.

intellus LEARNING OPEN COURSES