

# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



APPLICATION OF HOLOGRAPHIC INTERFEROMETRY TO  
DENSITY FIELD DETERMINATION IN  
TRANSONIC CORNER FLOW

by

D. J. Collins

and

R. A. Kosakoski

December 1972

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#### ABSTRACT

The successful application of holographic interferometry to the study of density fields around opaque bodies in wind tunnel experiments has been reported in the literature. The present report extends this technique to the study of the asymmetric flow fields encountered near the wing-fuselage junction of an aerodynamic model in the transonic flow regime. Finite fringe interferometry has been used to investigate the three-dimensional density field about a partially transparent wing-body structure. The resulting asymmetric density field and shock wave structure are shown to be an accurate representation of the phenomena encountered in aerodynamic corner flow.

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## INTRODUCTION

This is a report on holographic investigations directed towards the determination of three-dimensional density fields in transonic corner flows. The report consists essentially of Appendix I, which includes the thesis of LT R. A. Kosakoski on holographic investigations of transonic corner flow. This investigation is the first to use transparent models in holographic interferometry.

A paper based on this thesis, "Application of Holographic Interferometry to Density Field Determination in Transonic Corner Flow," by R. A. Kosakoski and D. J. Collins (No. 73-156) has been presented at the AIAA 11th Aerospace Sciences Meeting in Washington, D. C. The paper is now under consideration for publication in the AIAA Journal.

The integral inversion method used in the thesis and paper has now been essentially superseded by a more efficient method based on Fourier Transforms. This latter method is the subject of another report.

Further work on investigation into the application of fiber optics in holographic interferometry is continuing. This study also involves the consideration of flow with limited fields of view.

APPENDIX I

# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

APPLICATION OF  
HOLOGRAPHIC INTERFEROMETRY  
TO  
DENSITY FIELD DETERMINATION  
IN TRANSONIC CORNER FLOW

by

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Thesis Advisor

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Application of Holographic Interferometry

to Density Field Determination

in Transonic Corner Flow

by

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requirements for the degree of

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## ABSTRACT

The successful application of holographic interferometry to the study of density fields around opaque bodies in wind tunnel experiments has been reported in the literature. The present report extends this technique to the study of the three-dimensional asymmetric flow fields encountered near the wing-fuselage junction of an aerodynamic model in the transonic flow regime. Finite fringe interferometry has been used to obtain fringe information about a partially transparent wing-body structure. A FORTRAN computer program was utilized to invert the fringe information and produce a plot of the density field around the model. The resulting asymmetric density field and shock wave structure are shown to be an accurate representation of the phenomena encountered in aerodynamic corner flow.

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## I. INTRODUCTION

The field of flow measurement has been revolutionized in recent years with the perfection of holography and holographic interferometry techniques. High power Q-switched and dye-switched lasers and sophisticated double-pulsing trigger mechanisms provide exposure times on the order of twenty nanoseconds, thereby "freezing" the flow during the hologram production process. The precision optical quality components and measurement techniques of Mach-Zehnder interferometry have given way to the much less restrictive requirements of holographic interferometry which provide high quality interferograms in three dimensions.

Techniques for the application of holography to interferometry have been reported by Heflinger, et al. [1], and by Brooks [2]. In the determination of the density field around a free jet in the supersonic regime, Matulka [3, 4] expressed the fringe data in a series of orthogonal polynomials and transformed them to polynomials representing density using an inversion technique reported in [5, 6]. The method was extended by Jagota [7, 8] to the determination of the three-dimensional density field around a ten-degree half angle cone in a supersonic wind tunnel. The ability to produce readable holograms in wind tunnel studies using transparent phase objects was verified by Heyer [9]. In the present report the aforementioned techniques

have been combined to study the three-dimensional density field near the wing-fuselage junction of an aerodynamic model in transonic flow. The experiment was conducted at the Naval Ship Research and Development Center, Carderock, Maryland in an eighteen inch transonic blow-down wind tunnel at a Mach number of 0.937, using a semi-transparent model of original design.

Since reasonably small variations in density were anticipated, a finite fringe technique was used in obtaining the interferograms. The horizontal finite fringe field was produced by a vertical translation of a diffusing glass in the scene beam a distance of 0.003 inches between the two exposures of the holographic plate. Fringe data obtained from the interferograms were reduced to density information using a modified form of the inversion computer program used in [7]. A self-testing procedure incorporated in the program verified the resulting density data as an accurate representation of the actual flow around the model.

## II. EXPERIMENTAL APPARATUS

### A. THE WIND TUNNEL

The investigation was conducted in the Naval Ship Research and Development Center blowdown supersonic wind tunnel. Transonic flow conditions were produced through incorporation of slotted upper and lower tunnel walls. The tunnel is a nominal eighteen inch blow-down-to-vacuum facility with a test section fourteen inches by eighteen inches in cross-section and twenty-nine inches in length with the slotted surfaces installed. Optical quality windows twenty-two inches in diameter in the side walls provided complete viewing of the flow in the test section as the model was rolled through 180 degrees for hologram production. A functional schematic of the wind tunnel is shown in Figure 1.

### B. THE MACH NUMBER AND PRESSURE MEASUREMENT PROCEDURE

The Mach number at the test section is determined as a function of total and static pressure measurements and is maintained by carefully controlled butterfly valve settings. Total pressure is determined by recording atmospheric pressure prior to tunnel operation and accounting for the pressure loss between the plenum and the test section during operation. Static pressure is measured directly at a central wall port in the test section. Data recordings

were made on a Beckman Instruments Company 210 Digital Recorder, shown in Figure 2, and were read out on line via a Franklin Strip Tape Printer.

### C. THE HOLOGRAPHIC ARRANGEMENT

The holographic arrangement is illustrated in Figure 3 and shown in photographs included as Figures 4, 5, 6, and 7. Two large wooden tables were constructed and linked together with two-by-four beams under the tunnel to form the experimental platform. Thick rubber pads were attached to the table legs to dampen possible floor vibrations. The bulk of the platform provided sufficient stability and vibration damping for the experiment. The monochromatic light source used was a KORAD K-1 pulsed ruby laser operating at a wavelength of 6943 Angstroms, together with a Pockels cell Q-switching device. The resultant effective exposure time was approximately twenty nanoseconds, eliminating the problems due to possible model vibration during hologram exposure. To maintain the laser head and output etalon at a constant temperature of 27.0 degrees Centigrade, a LAUDA constant temperature circulator Model K2R was used.

The reference beam was directed under the wind tunnel by four front surface mirrors, and the beam size was manipulated by means of lenses (Figure 3). The scene beam was routed through the test section to intersect the reference beam on the holographic plate at an angle of approximately fifty degrees. A diffuse glass, mounted on

a precision X-Y translation table in the scene beam, was used to produce light field holograms. Alignment of the Q-switched laser and system optics was accomplished using a continuous wave, low-power helium-neon laser. Reference grids were mounted accurately on the outer surfaces of the tunnel windows using a survey's transit. Details of the model mounting and reference grids are shown in Figure 7. To enable hologram production during daylight hours, the entire tunnel room was blacked out using drop curtains and light shields.

#### D. THE WIND TUNNEL MODEL

The aerodynamic model used is shown in Figures 8, 9, and 10. The metal portions of the model were stainless steel. The greater part of the modified double wedge platform wing was constructed of optical lucite, as was the portion of the fuselage at the wing root. Detailed model dimensions are shown in Figure 11. The choice of aerodynamic design provided good flow characteristics and a relatively stable lambda-type shock wave on the wing; the largely transparent construction facilitated hologram production through 180 degrees of view.

The model was rotated about its sting mount in the wind tunnel from the zero degree position, wings level, to the 180 degree position, wings level inverted. Alignment for the desired rotation angle was accomplished by manually aligning prescribed lines on the sting mount collar with a scribed mark on the sting support.

### III. ANALYTICAL EVALUATION OF THE DENSITY FIELD

#### A. THE BASIC EQUATION OF INTERFEROMETRY

Interferograms are created when two originally coherent light beams are superimposed and projected on a viewing screen. The two rays will reinforce or annul each other, depending on their relative phase difference at the screen. This phase difference is directly a function of the optical pathlengths traversed by the two waves.

Consider a coherent beam which is split and then recombined on a viewing screen. A difference in optical pathlengths of the two component beams may be achieved by causing the beams to traverse through different media prior to recombination, with their physical pathlengths maintained equal. Each component beam will travel at a speed  $c_0/n$  where  $c_0$  is the speed of light in a vacuum and  $n$  is the index of refraction of the medium traversed. The difference in optical pathlength is then given by

$$L = L(n_2 - n_1) = c_0 \Delta t \quad (1)$$

where  $\Delta t$  is the time difference of travel in the two media. If the optical pathlength is changed by an amount  $N\lambda$ , where  $\lambda$  is the wavelength of the light source and  $N$  is an integer, then the order of interference changes by an amount  $N$ . In other words, a shift of  $N$  fringes occurs in the interference pattern. The fringe shift may be expressed as

$$g = L/\lambda \quad (2)$$

where  $g$  = fringe shift

$\lambda$  = light source wavelength

$L$  = change in optical pathlength

Substituting equation (1) into equation (2) yields

$$g = \frac{L}{\lambda} (n_2 - n_1) \quad (3)$$

The index of refraction for a given medium is a function of density. In the case of gases, since the speed of light is very nearly the same as in a vacuum, the index of refraction is well represented by the first two terms of a Taylor series expansion [10]:

$$n = 1 + \beta \frac{\rho}{\rho_s} \quad (4)$$

where  $\beta$  = dimensionless constant related to the Gladstone-Dale constant by  $K = \beta / \rho_s$

$\rho_s$  = reference density at  $0^\circ$  C, 760 mm. Hg.

The value of  $\beta$  for air at  $\lambda = 5893$  Angstroms (deep red light) is 0.000292; variation with wavelength is very small.

For a fixed difference in the index of refraction between the two component beams the fringe shift relation becomes:

$$g = \beta \frac{L}{\lambda} \left( \frac{\rho_2 - \rho_\infty}{\rho_s} \right) \quad (5)$$

For variable density in the test section, the net change in optical pathlength is the integrated effect along the beam path, or

$$g = \frac{\beta}{\lambda \rho_s} \int_0^L (\rho - \rho_\infty) ds = Q \int_0^L f(x, y, z_c) ds \quad (6)$$

where:

$$Q = \frac{\beta \rho_\infty}{\lambda \rho_s} \quad (7)$$

$$f(x, y, z_c) = \frac{\rho(x, y, z_c)}{\rho_\infty} - 1 \quad (8)$$

$z_c$  = plane of constant  $z$

$ds$  = incremental distance along beam path

Equation (6) is the basic integral equation for the unknown density.

With known fringe shift values from an interferogram, the equation is inverted to obtain the density along a beam path.

## B. THE INTEGRAL INVERSION

The integral inversion procedure utilized in this investigation was first reported by C. D. Maldonado, et al [5, 6]. It was used subsequently by R. D. Matulka [3] and R. C. Jagota [7] to determine the density variation in an asymmetric free jet and about a cone at angle of attack in supersonic flow, respectively. The procedure involves the representation of the function  $f(x, y, z_c)$  of Equation (6) in a complete set of orthogonal functions, with the expansion coefficients evaluated by use of the orthogonality condition between the functions  $f$  and  $g$  of Equation (6). The set of functions used is orthogonal over the entire  $(x, y)$  plane for every  $z_c$  and remains an orthogonal set under a rotation of the coordinate system. The coordinate system used for the inversion is shown in Figure 12. It consists of (a) a set of fixed coordinates  $x, y$  and (b) a set of moving coordinates

$x'$ ,  $y'$  in which the fringe number function is defined and which rotates with respect to  $x$ ,  $y$  as the viewing angle through the test section is varied.

In operator form, Equation (6) can be represented as

$$g(\xi, y', z_c) = \mathcal{T} f(x, y, z_c) \quad (9)$$

and  $f$  is evaluated by inversely transforming the equation to obtain

$$f(x, y, z_c) = \mathcal{T}^{-1} g(\xi, y', z_c) \quad (10)$$

This inversion is achieved by utilizing a pair of orthogonal polynomials  $U_{m+2k}^{+m}(\alpha x, \alpha y)$  and  $H_{m+2k}(\alpha y')$  which are related by the transform relationship

$$\mathcal{T} [U_{2k}(\alpha x, \alpha y) e^{-\alpha^2 x'^2}] = \frac{e^{\pm im\xi}}{[k!(m+k)!]^{1/2}} \cdot \frac{1}{2^{m+2k}} \cdot H_{m+2k}(\alpha y') \quad (11)$$

where  $H_{m+2k}(\alpha y')$  are Hermite polynomials of order  $m+2k$ . The unknown function  $f(x, y, z_c)$  is expanded in a set of functions  $U_{m+2k}^{-m}$  as

$$f(x, y, z_c) = \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \varepsilon_m \left\{ C_{m+2k}^{+m}(\alpha) U_{m+2k}^{+m}(\alpha x, \alpha y) + C_{m+2k}^{-m}(\alpha) U_{m+2k}^{-m}(\alpha x, \alpha y) \right\} e^{-(\alpha^2 x^2 + \alpha^2 y^2)} \quad (12)$$

where  $\varepsilon_m = \frac{1}{2}$  for  $m = 0$ ,  $\varepsilon_m = 1$  for  $m = 1, 2, 3, \dots$ , and  $C_{m+2k}^{\pm m}$  are the unknown coefficients of the expansion.  $\alpha$  is an arbitrary scale factor which may be considered the reciprocal of a non-dimensionalizing coefficient.

Utilizing the transform relation of Equation (11), Equation (6)

becomes

$$g(\xi, y', z_c) = \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \varepsilon_m [k!(m+k)! 2^{2(m+2k)}]^{1/2} \times \left[ C_{m+2k}^{+m}(\alpha) e^{im\xi} + C_{m+2k}^{-m}(\alpha) e^{-im\xi} \right] H_{m+2k}(\alpha) e^{-\alpha^2 y'^2} \quad (13)$$

Equation (13) is subject to the orthogonality condition

$$\int_{-\pi}^{\pi} e^{\pm im\xi} e^{\mp in\xi} d\xi \int_{-\infty}^{+\infty} H_{m+2k}(ay') H_{n+2l}(ay') e^{-\alpha^2 y'^2} dy' = \frac{2\pi^{3/2}}{\alpha} [(m+2k)!(n+2l)! 2^{m+2k} 2^{n+2l} \delta_{mn} \delta_{(m+2k)(n+2l)}] \quad (14)$$

where  $\delta$  is the Kronecker delta function. The solution of Equation (14) applied to Equation (13) yields the series coefficients

$$C_{m+2k}^{\pm m}(\alpha) = \frac{\alpha}{2\pi^{3/2}} \left[ \frac{k!(m+k)!}{(m+2k)!} \right] \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} g(\xi, y', z_c) H_{m+2k}(ay') e^{\mp im\xi} dy' d\xi \quad (15)$$

With the substitution of the coefficients of Equation (15), Equation (7) becomes

$$f(x, y, z_c) = \frac{\alpha}{\pi^{3/2}} \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \varepsilon_m \frac{[k!(m+k)!]^{1/2}}{(m+2k)!} e^{-(\alpha^2 x^2 + \alpha^2 y^2)} \times \text{Re} \left[ \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} g(\xi, y', z_c) e^{-im\xi} H_{m+2k}(ay) dy' d\xi \times U_{m+2k}^{+m}(\alpha x, ay) \right] \quad (16)$$

The functions  $U_{m+2k}^{\pm m}$  are defined as

$$U_{m+2k}^{\pm m}(\alpha x, ay) = (-1)^k \alpha^k \left[ \frac{k!(\alpha^2 x^2 + \alpha^2 y^2)^m}{\pi(m+k)!} \right]^{1/2} e^{\pm im\phi} L_k^m(\alpha^2 x^2 + \alpha^2 y^2) \quad (17)$$

where  $\phi = \tan^{-1}(y/x) - (\pi/2)$  and  $L_k^m$  are the associated Laguerre polynomials

$$\left[ \int_{-k}^m (\alpha^2 x^2 + \alpha^2 y^2) \right] = \sum_{s=0}^{\infty} \frac{(m+k)!}{(k-s)!(m+s)! s!} \left[ (-1)(\alpha^2 x^2 + \alpha^2 y^2) \right]^s \quad (18)$$

Insertion of Equation (17) into Equation (16) yields

$$f(x, y, z_c) = \left( \frac{\alpha}{\pi} \right)^2 \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \varepsilon_m \frac{(-1)^k k!}{(m+2k)!} \left( \alpha^2 x^2 + \alpha^2 y^2 \right)^{m/2} \left[ \int_{-k}^m (\alpha^2 x^2 + \alpha^2 y^2) \right] \\ \times \left[ B_{m+2k}^m(\alpha) \cos(m\phi) + D_{m+2k}^m(\alpha) \sin(m\phi) \right] e^{-(\alpha^2 x^2 + \alpha^2 y^2)} \quad (19)$$

where

$$B_{m+2k}^m(\alpha) = \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} g(\xi, y', z_c) \cos(m\xi) H_{m+2k}(\alpha y') dy' d\xi \quad (20)$$

$$D_{m+2k}^m(\alpha) = \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} g(\xi, y', z_c) \sin(m\xi) H_{m+2k}(\alpha y') dy' d\xi \quad (21)$$

Equations (19), (20), and (21) are the basic equations used to obtain the density distribution from the experimentally determined fringe variations in a completely asymmetric flow field.

### C. THE NUMERICAL PROCEDURE

Because the function  $g(\xi, y', z_c)$  is an experimentally determined quantity the unknown coefficients  $B_{m+2k}^m(\alpha)$  and  $D_{m+2k}^m(\alpha)$  in the series representation of  $f(x, y, z_c)$  in Equation (19) cannot be calculated analytically. It is therefore necessary to evaluate the double integrals of Equations (20) and (21) numerically. This is accomplished by noting in Figure 12 and Equation (8) that there is an area outside which the density is invariant, namely outside the test

section where the known density is  $\rho_\infty$ . Since the function  $f(x, y, z_c)$  = 0 outside this circular domain, the limits of integration of  $+\infty$  and  $-\infty$  in Equations (20) and (21) can be replaced by finite values. The fringe distribution is then approximated by small increments over the test domain, resulting in the representation of the B and D coefficients as double series:

$$B_{m+2k}^m(\alpha) = \sum_{i=1}^{I-1} \sum_{j=0}^{J-1} g(\xi_j + \Delta \xi_j, x_i + \Delta x_i) \int_{\xi_j}^{\xi_{j+1}} \cos(m\xi) d\xi \int_{x_i}^{x_{i+1}} H_{m+2k}(ax) dx \quad (22)$$

and

$$D_{m+2k}^m(\alpha) = \sum_{i=1}^{I-1} \sum_{j=0}^{J-1} g(\xi_j + \Delta \xi_j, x_i + \Delta x_i) \int_{\xi_j}^{\xi_{j+1}} \sin(m\xi) d\xi \int_{x_i}^{x_{i+1}} H_{m+2k}(ax) dx \quad (23)$$

Using the derivative formula for Hermite polynomials, Equations (22) and (23) can be manipulated to yield workable series expressions:

$$B_{m+2k}^m(\alpha) = \left[ \frac{1}{2\alpha m} \cdot \frac{1}{(m+2k+1)} \right] \sum_{i=0}^{I-1} \sum_{j=0}^{J-1} g(\xi_j + \Delta \xi_j, x_i + \Delta x_i) \\ \times \left[ \sin(m\xi_{j+1}) - \sin(m\xi_j) \right] \left[ H_{m+2k+1}(ax_{i+1}) - H_{m+2k+1}(ax_i) \right] \quad (24)$$

$$D_{m+2k}^m(\alpha) = - \left[ \frac{1}{2\alpha m} \cdot \frac{1}{(m+2k+1)} \right] \sum_{i=0}^{I-1} \sum_{j=0}^{J-1} g(\xi_j + \Delta \xi_j, x_i + \Delta x_i) \\ \times \left[ \cos(m\xi_{j+1}) - \cos(m\xi_j) \right] \left[ H_{m+2k+1}(ax_{i+1}) - H_{m+2k+1}(ax_i) \right] \quad (25)$$

Since it is impossible to sum over an infinite number of terms,

Equation (19) is necessarily expressed as the sum of a finite series:

$$f(x, y, z_c) = \left(\frac{\alpha}{\pi}\right)^2 \sum_{k=0}^K \sum_{m=0}^M \epsilon_m (-1)^k \left[ \frac{k!}{(m+2k)!} \right] (\alpha^2 x^2 + \alpha^2 y^2) \quad (26)$$

$$\times \left[ \sum_k^m (\alpha^2 x^2 + \alpha^2 y^2) \left[ B_{m+2k}^m(\alpha) \cos(m\phi) + D_{m+2k}^m(\alpha) \sin(m\phi) \right] e^{-(\alpha^2 x^2 + \alpha^2 y^2)} \right]$$

It has been demonstrated that judicious selection of the parameters  $\Delta \xi$ ,  $\Delta x$ ,  $K$ ,  $M$ , and  $\alpha$  yields density distributions with very good accuracy [3, 6].

#### IV. EXPERIMENTAL PROCEDURE

##### A. LABORATORY TECHNIQUES

In order to visualize the general flow patterns and localize shock or expansion waves about the model a series of standard Schlieren photographs were taken at varying flow Mach numbers. Pictures were produced for roll angles of  $0^\circ$ ,  $45^\circ$ , and  $90^\circ$  at Mach numbers from 0.925 to 1.10. A representative series of Schlieren photographs is shown in Figures 13, 14, and 15. Analysis of the Schlieren photographs dictated a flow Mach number of 0.937 for the experimental study; this Mach number yielded uniform upstream flow conditions and located the lambda-type shock wave ideally near the center of the lucite section of the model wing.

The coherence length of the pulsed ruby laser was approximately ten centimeters for the output power utilized. This reduced the normally critical requirement for pathlength equality in the scene and reference beams that must be fulfilled in the classical Mach-Zehnder interferometric approach. Consequently, a length of string proved to be a sufficiently accurate measuring device to maintain the two beam pathlengths within the coherence length of the laser, a requirement for interferogram production. To compensate for the fact that the scene beam traversed approximately five inches of glass tunnel walls and lucite grids which the reference beam did not, the scene beam

was adjusted to be some 2.5 inches shorter than the reference beam. Reference beam pathlength was maintained at approximately 138 inches throughout the experiment.

Holograms produced using the basic holographic setup shown in Figure 3 exhibited clear, well-defined fringe patterns in nearly every instance. In deciding on the final arrangement, several techniques were tested to improve upon fringe pattern definition. Horizontal, vertical and diagonal translations of the diffuser plate in the scene beam were considered, varying from 0.001 inches to 0.005 inches. A vertical translation of 0.003 inches yielded clear horizontal fringes that were quite easily analyzed. The transverse mode selector aperture was varied from 1.0 mm. to 3.0 mm. in increments of 0.5 mm; best lighting of the model resulted with use of a 2.5 mm. aperture. The temperature of the cooling water circulated through the laser head and etalon was varied from 26.0°C. to 28.0°C. in increments of 0.2°C., with 27.0°C. providing the best fringe definition. Finally, a variety of beam splitters and lenses were tested prior to final selection of the best available optics arrangement for the experiment. A 2:1 reference to scene beam strength ratio was found to yield very good holograms.

Two double exposure holograms were taken for each model viewing angle. The first, labeled a double-static exposure, consisted of two exposures in a no-flow condition with a 0.003 inch vertical translation of the diffuser plate between exposures. The fringe

patterns in this hologram provided a measure of the effect of tunnel wall glass, grid plexiglass and model lucite on the subsequent double exposure. The second, or static-dynamic, exposure consisted of a no-flow exposure, a 0.003 inch diffuser translation, and finally an exposure at flow Mach number 0.937. The fringe deviations recorded in the region behind the lucite portion of the model by the double-static hologram were measured and subtracted from the fringe shifts measured in the corresponding static-dynamic hologram.

Holograms were produced on Agfa-Gaevert 8E75 holographic plates, 4 inches by 5 inches in size. As recommended by Collier, et al. [11] the development process included:

1. Five minutes in Kodak D-19 developer
2. Thirty seconds in a flowing water bath
3. Five minutes in standard rapid fixer
4. Thirty seconds in a flowing water bath
5. One and one-half minutes in Kodak Hypo Clearing agent
6. Five minutes in a flowing water bath
7. Five minutes in methanol bath
8. One minute in a flowing water bath
9. Drying

## B. PHOTOGRAPHIC TECHNIQUES

Normal reconstructions of the original scene were made by illuminating the holograms with a seven milliwatt continuous wave

helium-neon laser beam at a wavelength of 6328 Angstroms. There was some slight distortion in the reconstructed scene because of the difference in wavelengths of the original scene beam and the reconstruction beam; however, the effect was almost totally negated by shrinkage of the holographic plate emulsion during the development process.

A common technique of image reconstruction was employed, utilizing a conjugate reference beam to reilluminate the exposed hologram, as shown in Figure 16. The resulting scene was recorded on photographic film. Individual points on the photograph are produced by a series of non-parallel rays originating from various source points on the diffuser plate in the scene beam. Using a reilluminating beam of small diameter has the effect of a small aperture at the focal point of the imaging lens, filtering out all but a set of nearly parallel rays, as shown in Figure 17. The real images produced in this manner have a large depth of field, permitting simultaneous projection on the film of front and rear grids, the model and the fringe patterns. The imaging lens was focused as near to the plane of the fringes as possible, producing photographs at various planes of constant  $z_c$ .

### C. DATA REDUCTION

Photographic interferograms were obtained using the arrangement shown in Figure 16, with the camera viewing screen in the position of the real image. The line of sight in the plane desired was achieved by translating and elevating the hologram until common points on the

front and rear grids were aligned. The Graphic View camera, with the wide angle lens aperture set fully open at f7.8, was adjusted to yield the best focus on the fringe plane. Exposure times of from 1/5 to 3/4 seconds were used to produce workable interferogram photographs on Polaroid Type 55 P/N film.

Fringe shift analysis was accomplished on 8 inch by 10 inch enlargements of the 4 inch by 5 inch film used to record the images. The enlargements were placed face down on a light table, and the fringes, model contours and shock wave were traced on the back surface at the desired cross-sectional plane. Fringe shift values were recorded by measuring the distance between (1) the intersection of the hypothetically undeviated fringe and the cross-sectional plane and (2) the intersection of the deviated fringe with the same plane. Fringe shifts so obtained were corroborated by placing the negative in a photographic enlarger and tracing the lines of interest directly onto graph paper.

The known model fuselage diameter of 1.1 inches was compared with that measured in each individual photograph to yield magnification factors relating projected dimensions to actual dimensions. These factors were then used as corrections to the fringe shift measurements. A grid reference point located on the cross-sectional plane of interest served as the datum for all fringe shift measurements. A base point was located at the intersection of the cross-sectional plane of interest and the body longitudinal axis. Fringe shift measurements were

corrected using this base point as the new datum so that the inversion circle was properly centered on the body axis. The radius of the inversion circle was selected to be nearly equal to the semi-span of the wing. Fringe shift measurements were converted to fringe numbers using the average free stream spacing, while fringe locations were nondimensionalized using the inversion circle diameter. From the data so obtained, the radial variation of fringe number was plotted for each viewing angle. Fringe numbers at 201 equidistant points, as required for input into Mode 3 of the computer program, HOLOFER, were recorded from the resulting curves. Further details concerning this inversion computer program are outlined in Appendix B. A typical reduction of an interferogram to obtain the radial variation of fringe number at a particular cross-sectional plane is detailed in Appendix A.

## V. EXPERIMENTAL RESULTS AND DISCUSSION

A pair of double exposure holograms was taken of the model at  $11\frac{1}{4}$  degree intervals through a 180 degree field of view. Experimental data from the wind tunnel runs are recorded in Table I. Initial resulting density patterns indicated relatively smooth contours across adjacent intervals; the interval was therefore doubled to  $22\frac{1}{2}$  degrees to simplify and speed the analysis. Fringe data were first inserted into the inversion computer program along nine lines of sight in the 180 degree field of view. A numerical comparison of views from 0 degrees to 90 degrees and from 90 degrees to 180 degrees verified to within 0.20 percent the assumption of a single plane of symmetry in the experiment. The fringe data input was then reduced to five lines of sight in a 90 degree field of view, as shown in Figure 18. The resulting output was an inverted density field along nine radial lines spanning a 180 degree field of view, with a mirror image on the opposite side of the plane of symmetry, as shown in Figure 19.

The static-dynamic photographic interferogram for the 0 degree view, along with its corresponding double-static interferogram, is shown in Figure 20. The diffraction effects caused by the presence of the lucite portions of the model are clearly visible in the double-static exposure, where the free stream fringe lines are bent and displaced toward the model axis. This displacement was measured

and subtracted from subsequent measurements made on the static-dynamic exposure, as outlined in Section IV.A. Photographic interferograms of the remainder of the static-dynamic exposures are shown in Figures 21 through 24. Clearly visible and reduceable in nearly all views were (1) the region of uniform subsonic flow, commonly called the free stream condition, (2) the transition from local subsonic to local supersonic flow, and (3) the lambda-type shock wave on the model wing. These characteristics are shown in schematic representation in Figure 25.

Contour plots of the density function, as expressed in Equation (8) of Section III.A., for successive z-planes of analysis are shown in Figures 26 and 27. The cross-sectional plane of analysis for the plot of Figure 26 was located at 186.75 mm. from the model nose along the longitudinal axis. For Figure 27, the plane of interest was 195.25 mm. from the model nose. It is apparent from both contour plots that the model went to a very small angle of attack under the loading forces produced during tunnel operation; this is evidenced by the compression of the contour lines above the model and the corresponding expansion of the contours below the model. Measurements made from photographic interferograms confirm this angle of attack to be, at most, 0.05 degrees. The closed contours above and adjacent to the wing surface in both figures may very well be the result of a vortex originating at the intersection of the wing leading edge and the

fuselage on either side of the model and traveling aft and outward over the wing surface.

A comprehensive quantitative analysis of the shock wave structure was not undertaken, with the exception of estimating the strength of the shock by comparison of fringe line separation immediately ahead of and aft of the shock wave. Fringe line separation measurements on either side of the shock wave were converted first to density information and thence to pressure information, disregarding compressibility effects. An approximate strength value of 0.207 was computed using the accepted definition of  $(p_2 - p_1)/p_1$ . This corresponds to a local Mach number of 1.08 in the supersonic region just ahead of the shock wave. Qualitative shock wave analysis resulted in the construction of a three-dimensional structural representation as shown in Figure 28, using input information from several interferogram viewing angles. While location of the leading and trailing edges of the lambda-type shock wave was very accurate, interior structure was largely indefinable due to "smearing" and blurring of the fringes transiting the shock wave itself.

As a preliminary step to possible future studies in this field, photographic interferograms were made from holograms produced with the aerodynamic model set at small angles of attack. Orientations included angles of attack of five and ten degrees, with roll angles varying from zero to ninety degrees. Although the holograms themselves were of very good quality, the photographic reproductions

were relatively poor due to the fact that an inferior photographic arrangement had to be used. They were therefore omitted from this report. It was inferred from the holograms, however, that a complete study at angle of attack using the basic procedures followed in the present study would be both totally feasible and rewardingly fruitful.

The original character of the experimental data prevented comparison with published results. Qualitative studies of transonic phenomena are widely available, and the general characteristics of the resulting density field and shock wave structure serve to bear out the schematics based on theoretical and mathematical models. Moreover, the self-testing mode of the inversion computer program, HOLOFER, verified the consistency and reproducibility of the resulting density distributions to within 2.0 percent through proper choice of the input parameters, primarily the slope-matching parameter  $\alpha$ . The errors encountered in the final results are due primarily to errors in the fringe data input to the inversion program. The intrinsic presence of laser speckle, the extended pathlengths of the scene and reference beams and the unavoidable beam scattering and diffraction within the lucite model sections, created difficulty in obtaining precisely the slope of the fringe lines behind the lucite sections. Fringe spacing measurements in the free stream flow were conservatively judged accurate to within 0.5 mm. This assumption was quite reasonable since all measurements were effected with a scale graduated at half millimeter intervals. This figure of 0.5 mm.,

combined with the mean free stream fringe spacing of 5.197 mm. for all interferograms, indicated measurement accuracy to within one tenth (0.1) of a fringe. The mean systematic error of the free stream spacing in each view was computed to be a maximum of 3.9 percent. Associated with this systematic error was a random error of 2.1 percent in the measurement of fringe shifts in each view to a conservative accuracy of 0.5 mm. The resulting error for each viewing angle was therefore a maximum of 6.0 percent, found by merely adding the two types of error for each view. The minimum error limit was found by considering the error resulting from the reproduction of the same interferogram view five separate times. Statistically, with 6.0 percent error in each view, the composite error for the repeated view is 2.6 percent. As five different views, or lines of sight, were used for the data between zero and ninety degrees, the final total error in the analysis was therefore in the interval between 2.6 percent and 6.0 percent. To insure contour clarity and guard against overlapping, the maximum error figure of 6.0 percent was used in construction of the plots shown in Figures 26 and 27. In general, the rather large fringe shifts led to a very low mean fringe sensitivity value of 0.1259. This coefficient indicated a resulting density function (Equation (8)) inaccuracy of less than 1.5 percent for a fringe shift measurement misreading of 0.5 mm.

Physical limitations of beam diameter and hologram plate area dictated the choice of an inversion circle diameter somewhat smaller

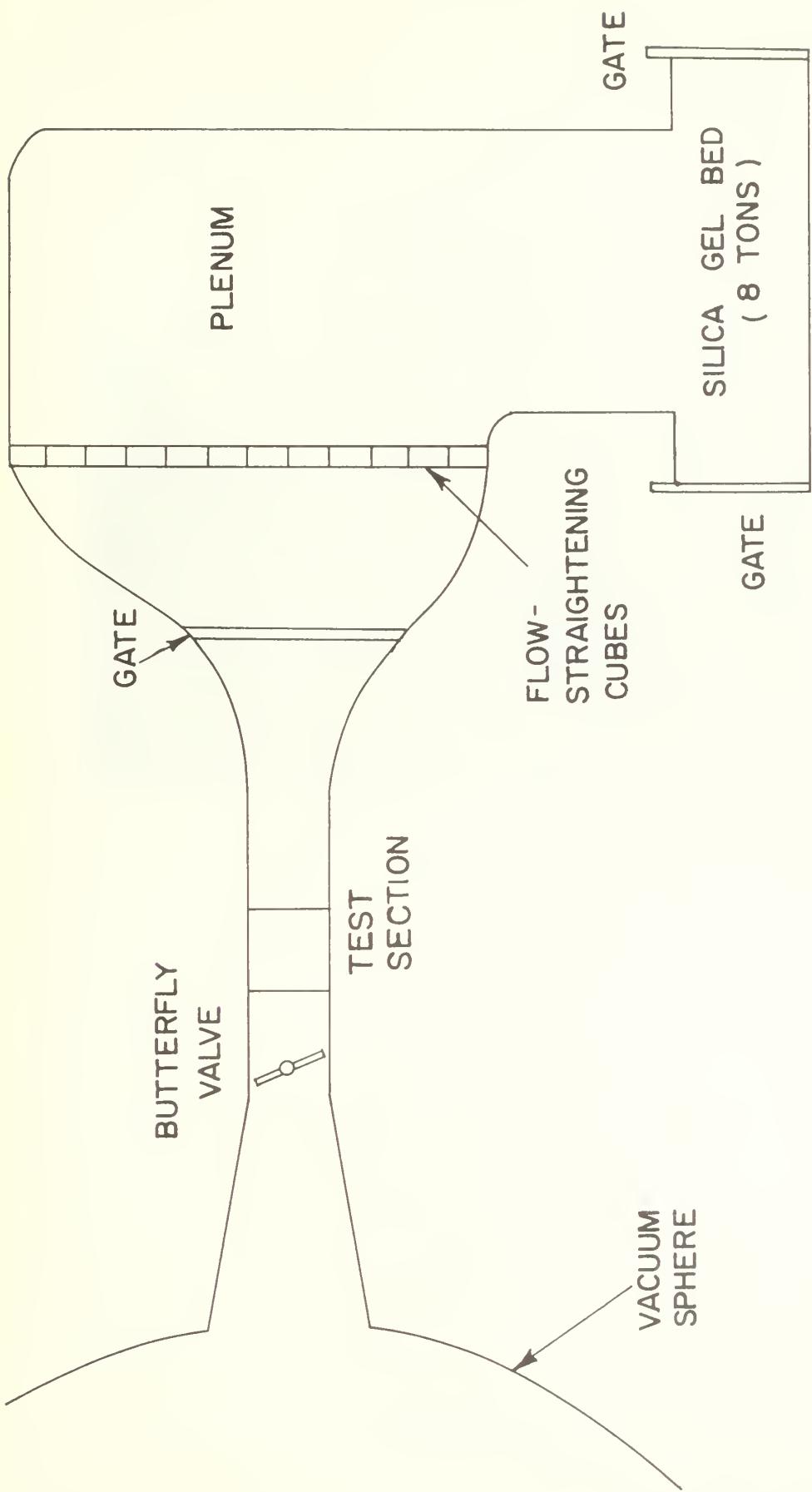
than the full data circle normally used in the finite fringe procedure. This, in effect, introduced an inconsistency in reference density into the analysis since the density on the selected circle and immediately outside it was not the calculated  $\rho_\infty$ ; the density function was therefore not zero outside the actual inversion region. To alleviate this inconsistency, a new, updated reference density was computed for each cross-sectional plane of analysis by averaging the density values on the selected circle from the first inversion process. The actual reference densities used were  $\rho = 1.777 \text{ mg/cc}$  for the 186.75 mm. plane and  $\rho = 1.642 \text{ mg/cc}$  for the 195.25 mm. plane. This procedure was justified since all density values between the selected circle and the full data circle were constant to within approximately fifteen percent. The updated reference densities were then used to produce the final output density field. The net effect was a scaled, uniform shift toward density function values slightly lower than those computed on the basis of the original reference density.

## VI. CONCLUSIONS

The finite fringe procedure for the production of holographic interferograms has been applied successfully to the determination of the three-dimensional density distribution of the flow near the wing-fuselage junction of a partially transparent aerodynamic model in the transonic regime. Density contours accurate to within six percent enabled a thorough analysis of the flow field to be conducted, highlighting flow characteristics and the presence of the shock wave. Subsequent studies of similar models at angle of attack have been shown to be entirely feasible. Procedures used in the experiment also exhibit promise for the direct analysis of duct and inlet flows as well as comprehensive study of shock wave structure.

The inversion computer program, HOLOFER, was found to be adequate in handling a general asymmetric flow field analysis. However, it was considered quite cumbersome and difficult to modify for various experimental situations. Subsequent analysts will find the procedures advocated by Sweeney and Vest [12] for the recording and analyzing of interferograms of considerable interest. In addition, the efforts of Van Houton [13], who utilized the method proposed by Junginger and van Haeringen [14], may prove valuable in reducing computer time significantly.

FIGURE I. FUNCTIONAL SCHEMATIC OF NSRDC WIND TUNNEL



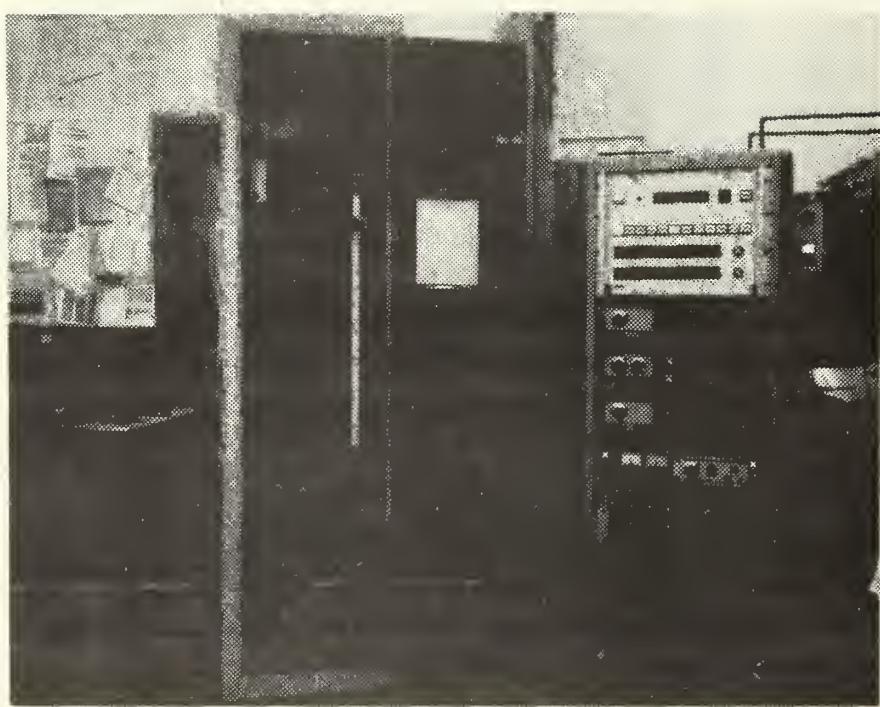


FIGURE 2. BECKMAN 210 DATA RECORDING SYSTEM

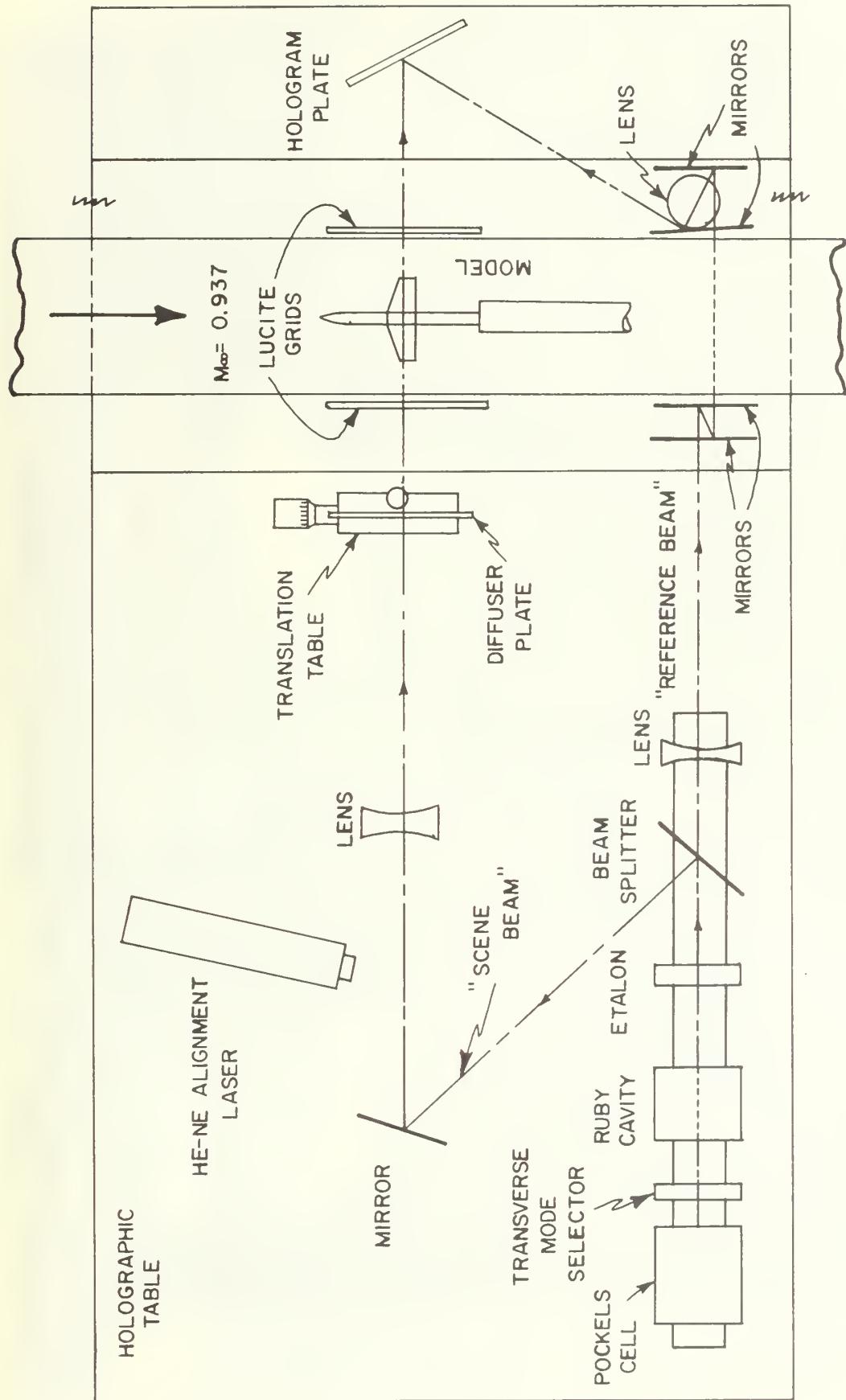
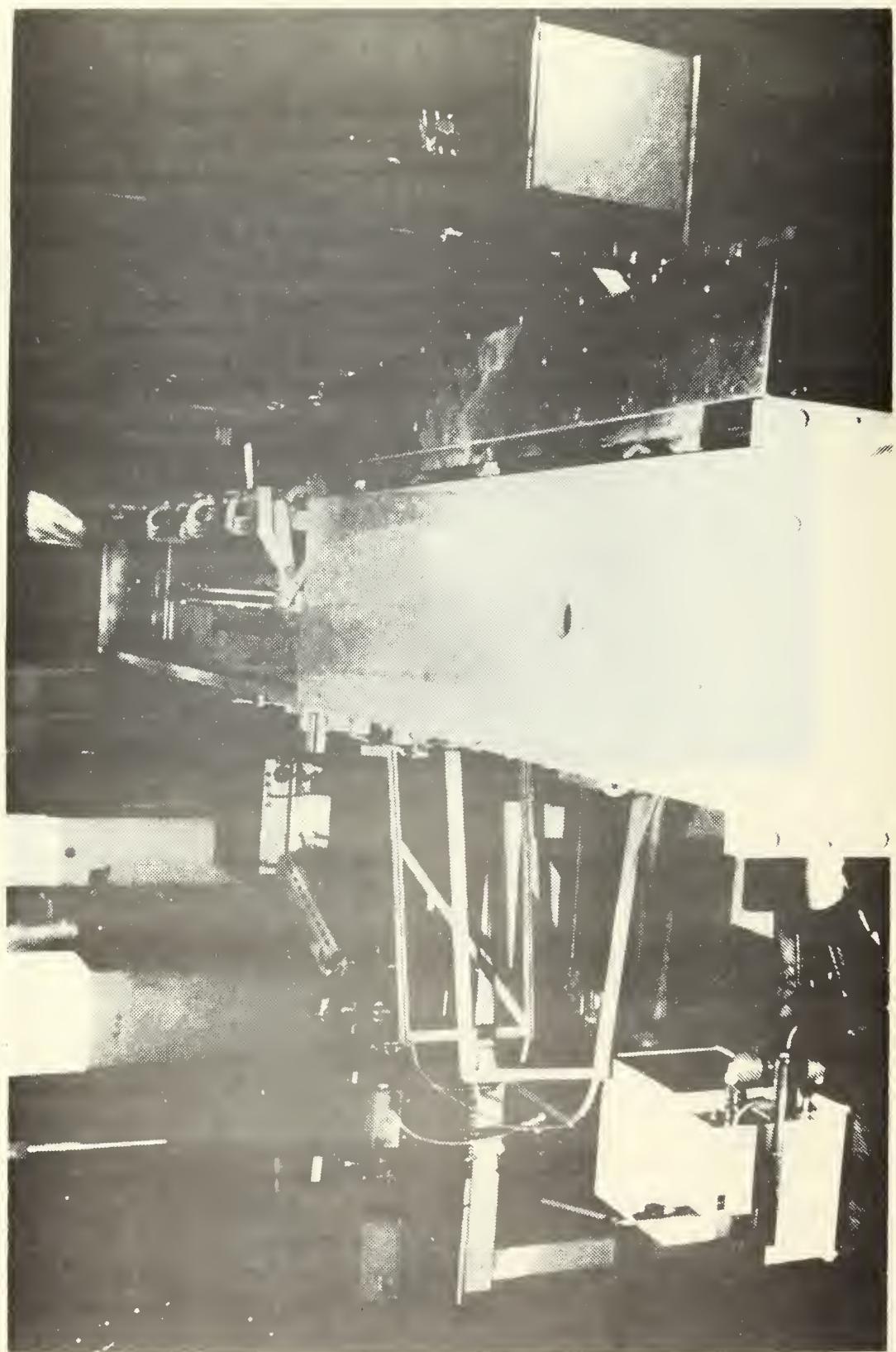


FIGURE 3. SCHEMATIC DRAWING OF THE HOLOGRAPHIC ARRANGEMENT

FIGURE 4. OVERHEAD VIEW OF TUNNEL AND ENTIRE HOLOGRAPHIC SYSTEM



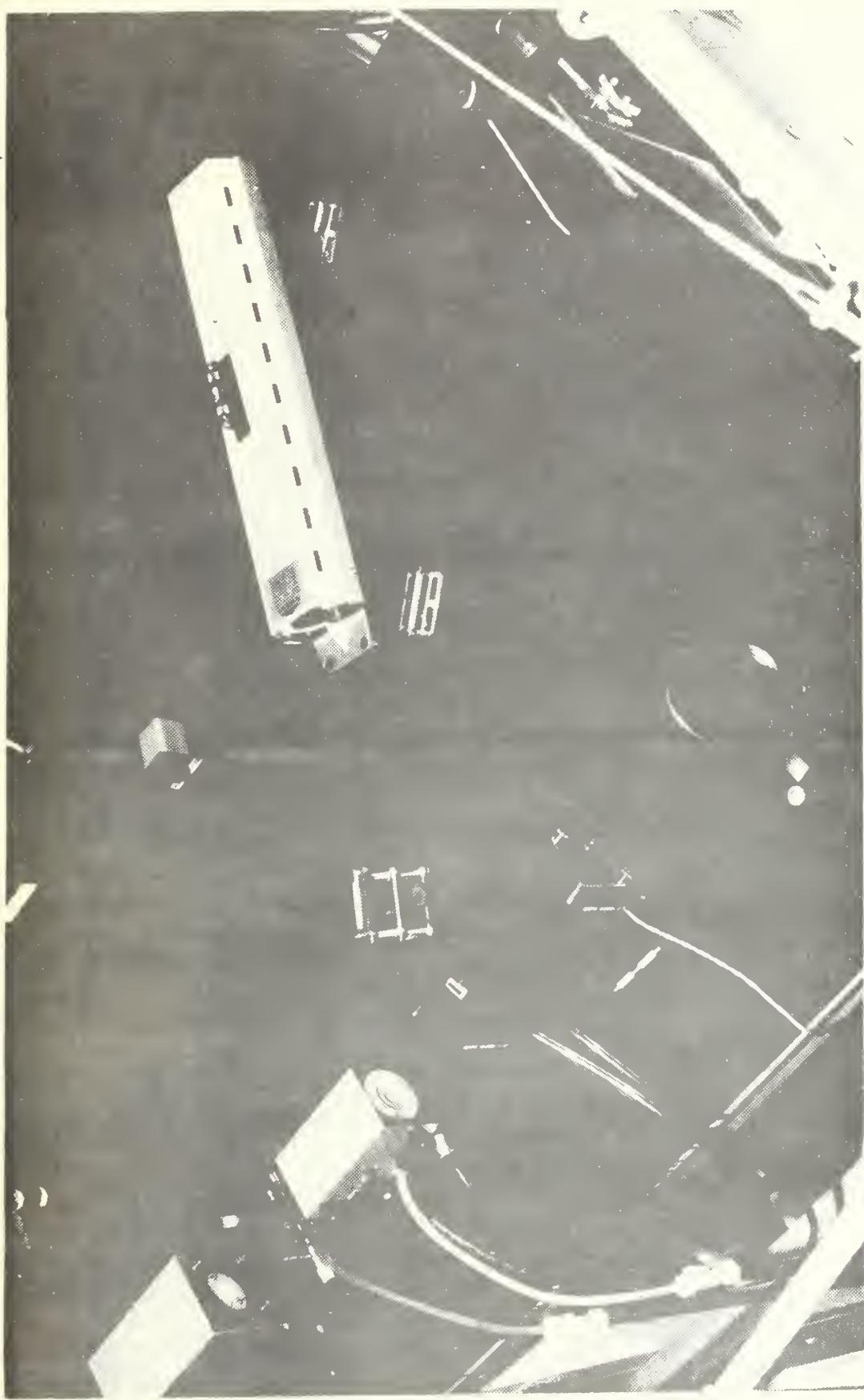


FIGURE 5. OBLIQUE VIEW OF HOLOGRAPHIC TABLE

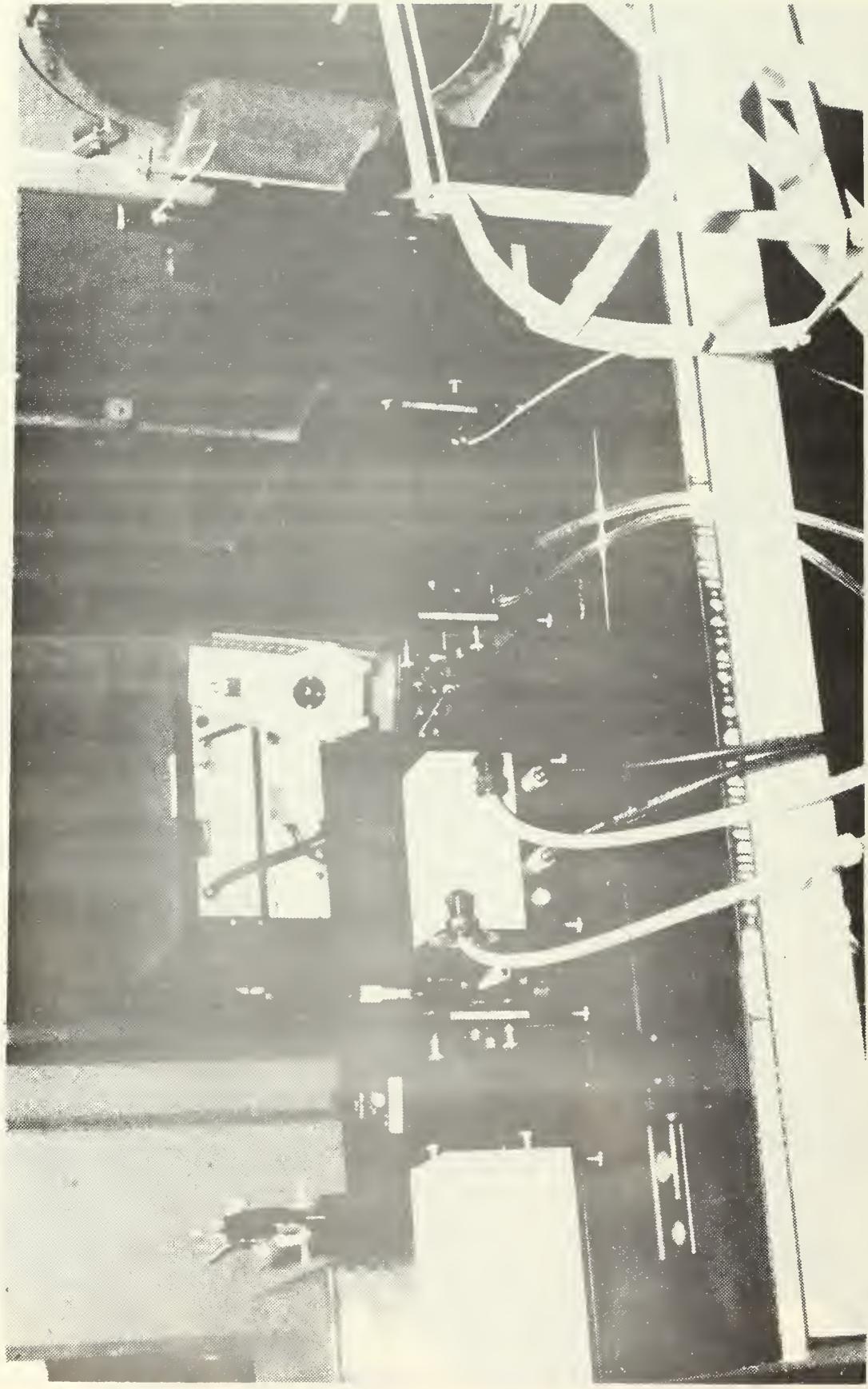
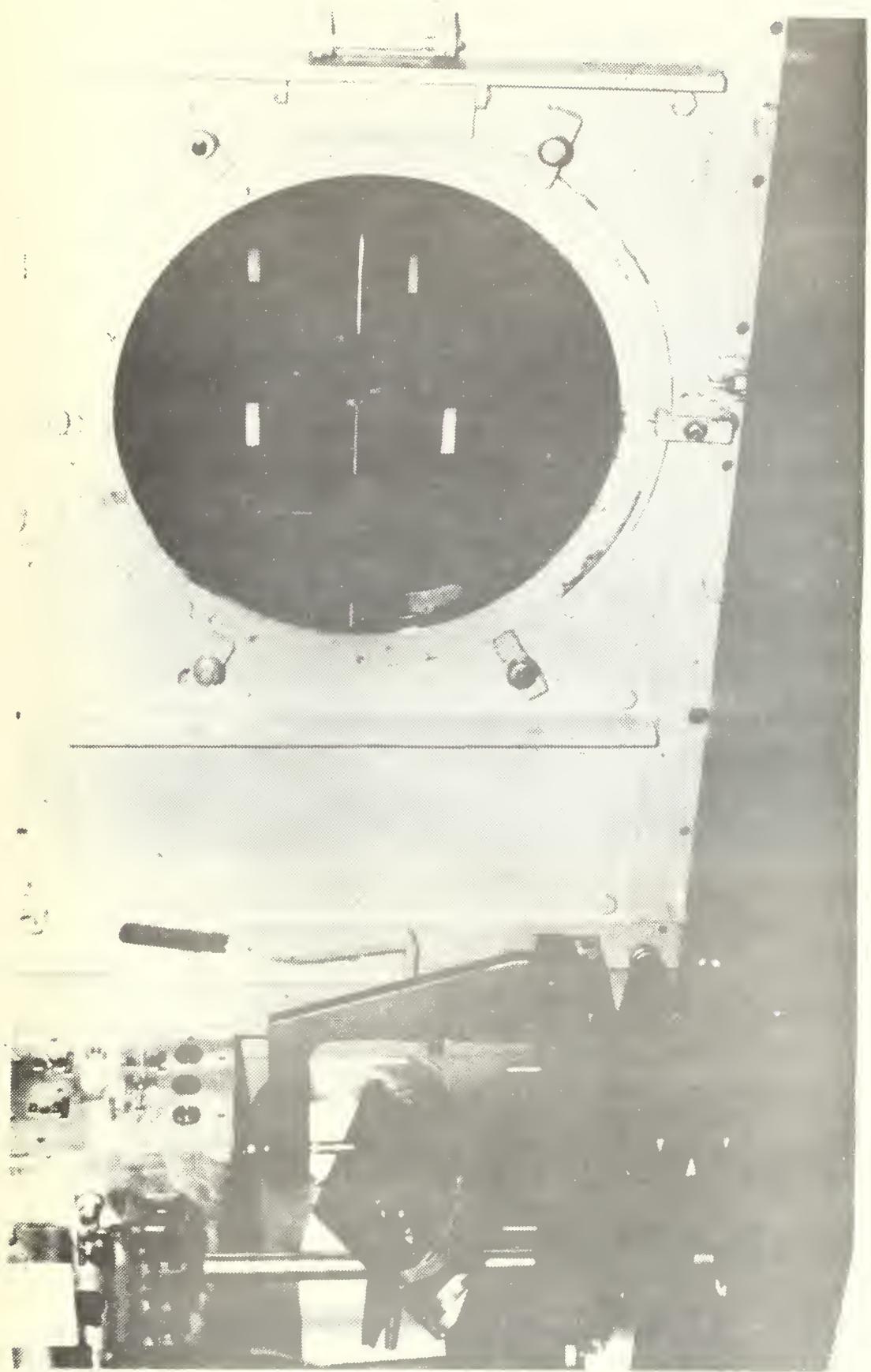


FIGURE 6. OBLIQUE VIEW OF HOLOGRAPHIC TABLE

FIGURE 7. DETAIL OF MODEL MOUNTING AND REFERENCE GRIDS



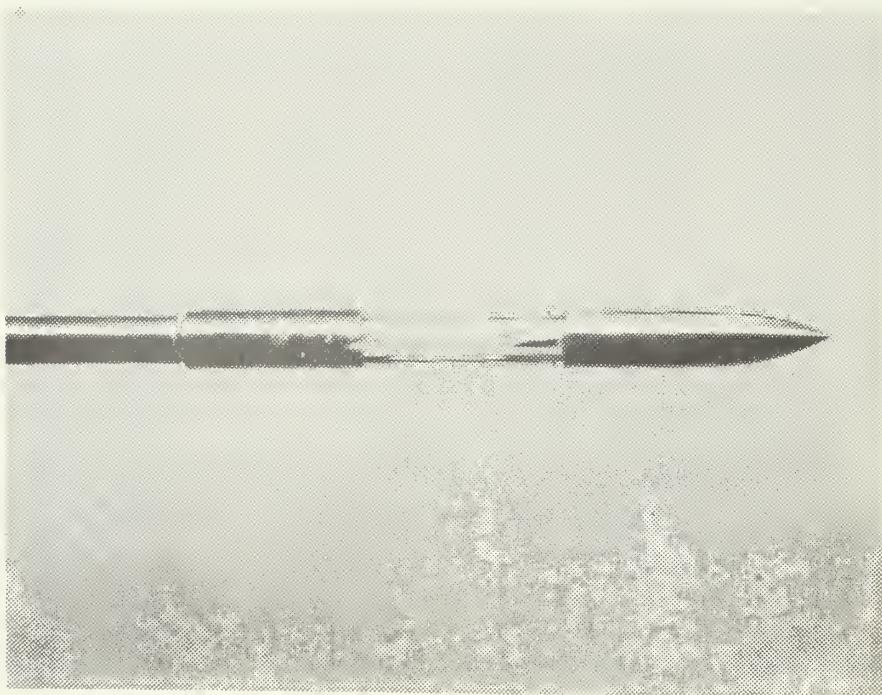


FIGURE 8. AERODYNAMIC TEST MODEL; 0 DEG. ROLL ANGLE

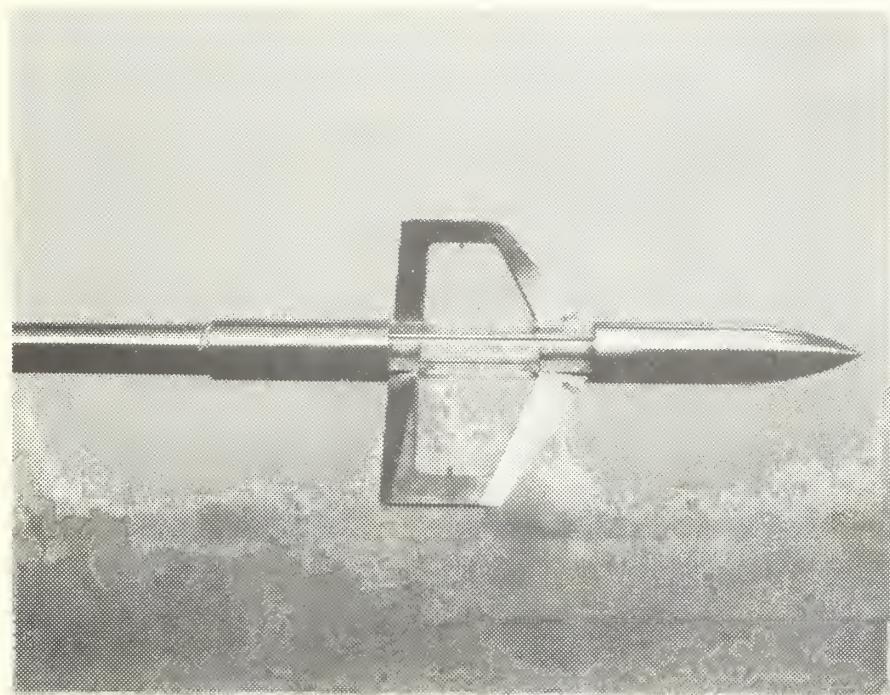


FIGURE 9. AERODYNAMIC TEST MODEL; 45 DEG. ROLL ANGLE

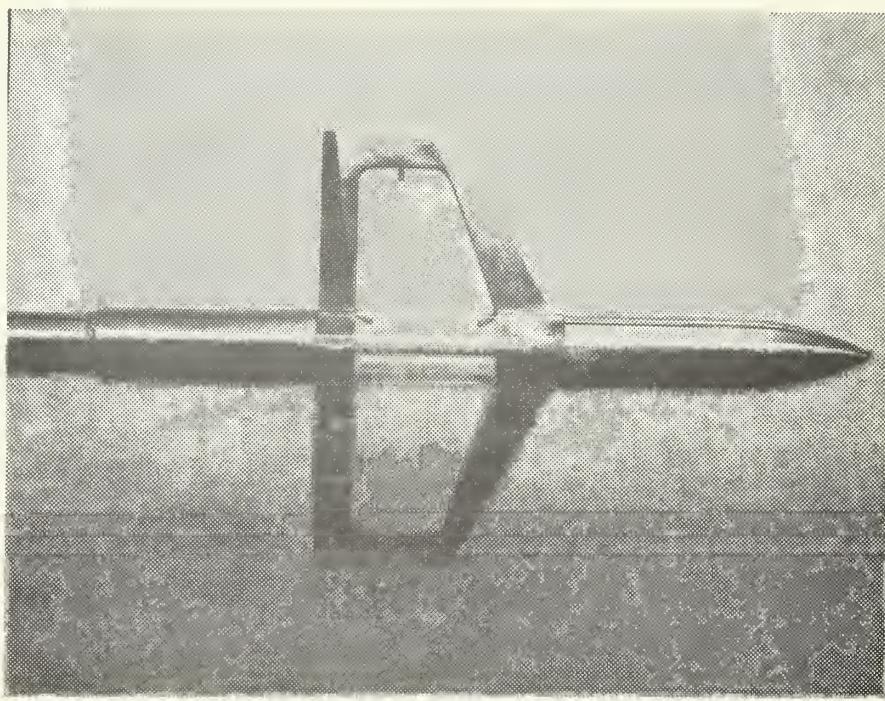


FIGURE 10. AERODYNAMIC TEST MODEL; 90 DEG. ROLL ANGLE

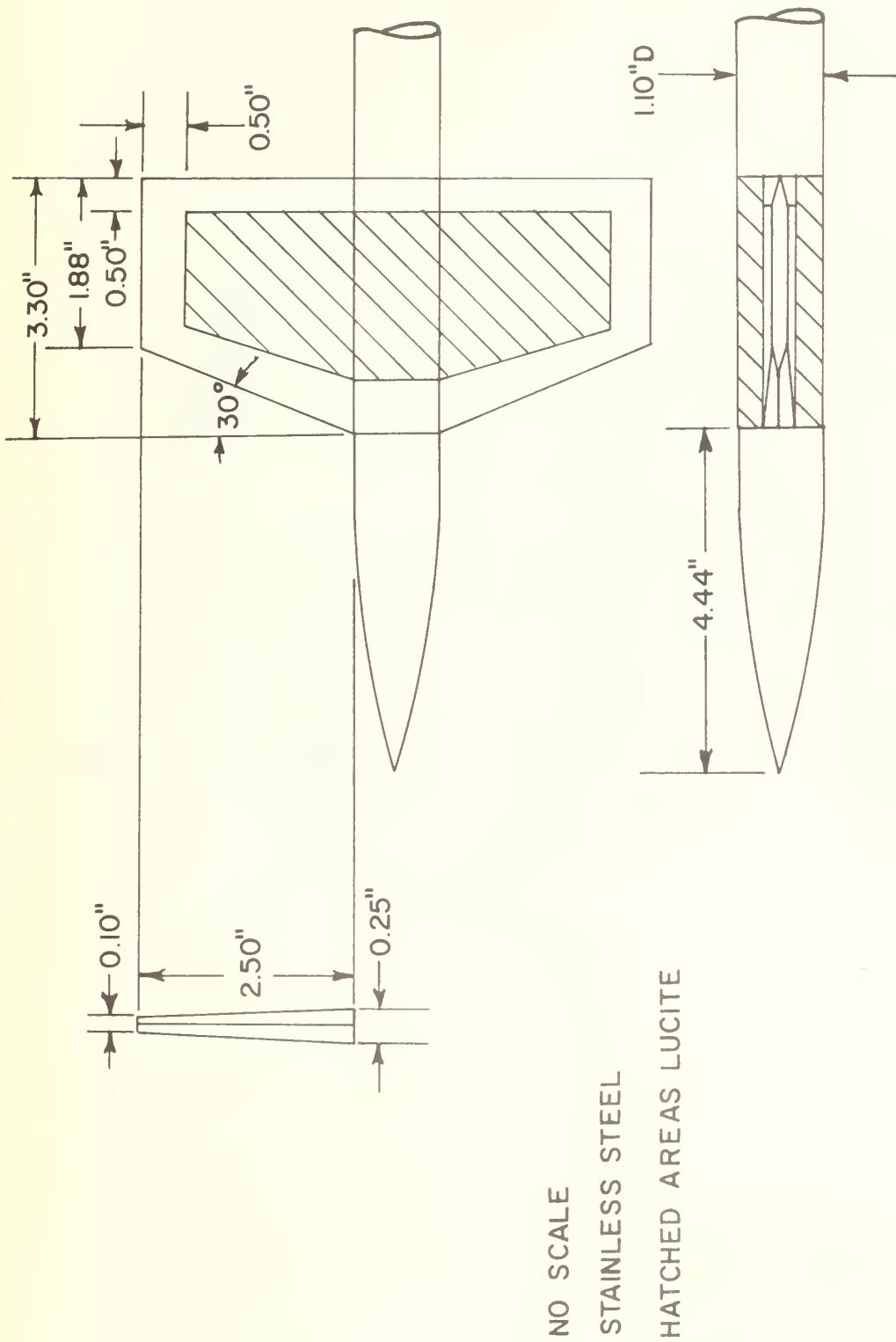


FIGURE 11. DETAILS OF THE AERODYNAMIC TEST MODEL

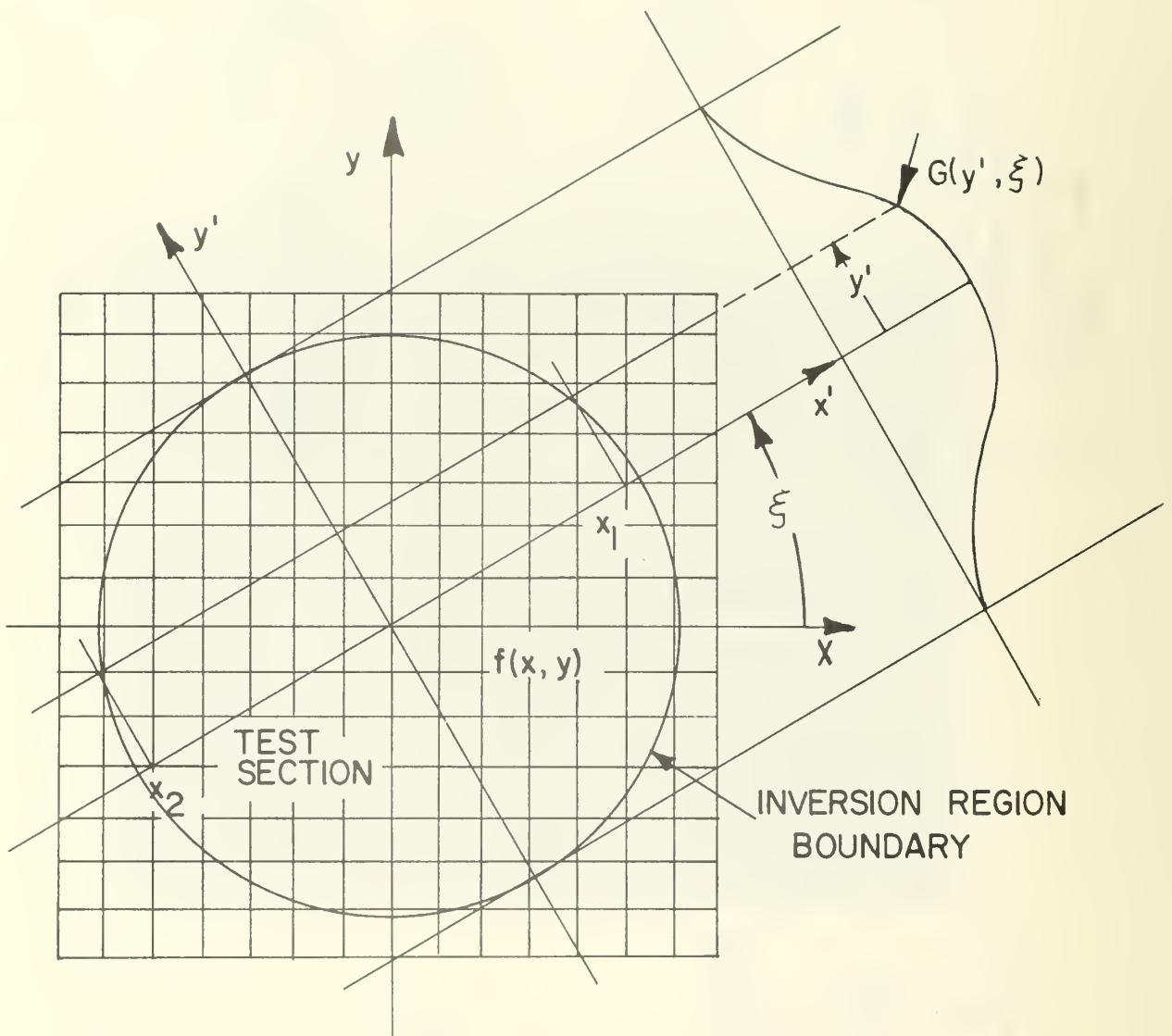


FIGURE 12. CO-ORDINATE SYSTEM USED FOR INVERSION OF FRINGE NUMBER TO DENSITY

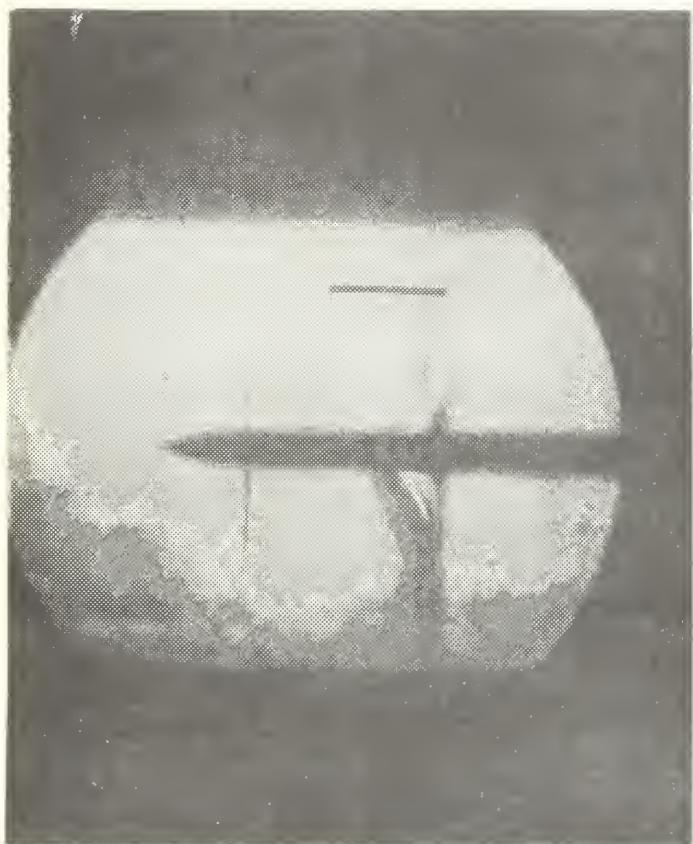


FIGURE 13. SCHLIEREN PHOTOGRAPH; 0 DEG. ROLL ANGLE,  
0.967 MACH NUMBER

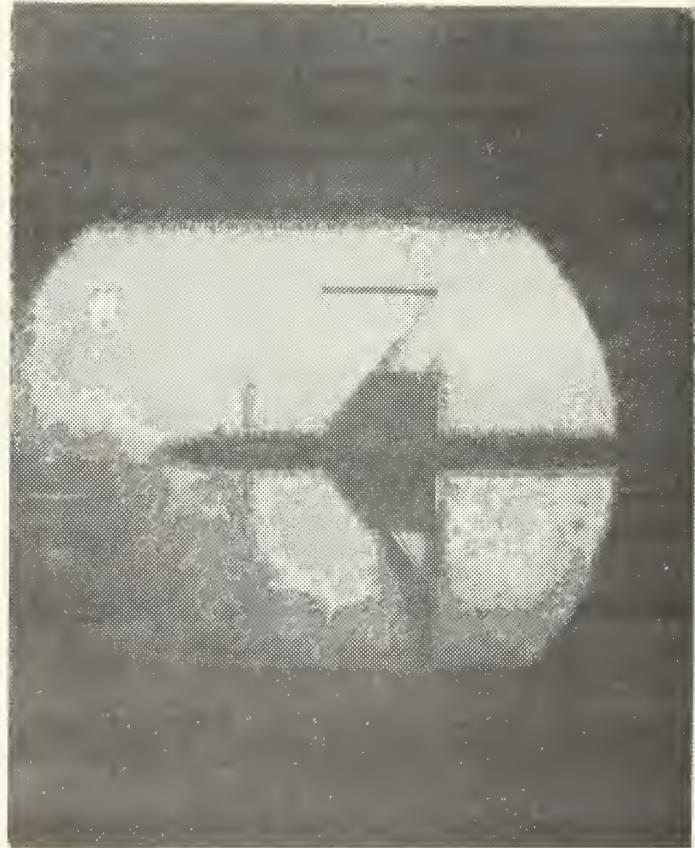


FIGURE 14. SCHLIEREN PHOTOGRAPH; 45 DEG. ROLL ANGLE,  
0.967 MACH NUMBER



FIGURE 15. SCHLIEREN PHOTOGRAPH; 90 DEG. ROLL ANGLE,  
0.967 MACH NUMBER

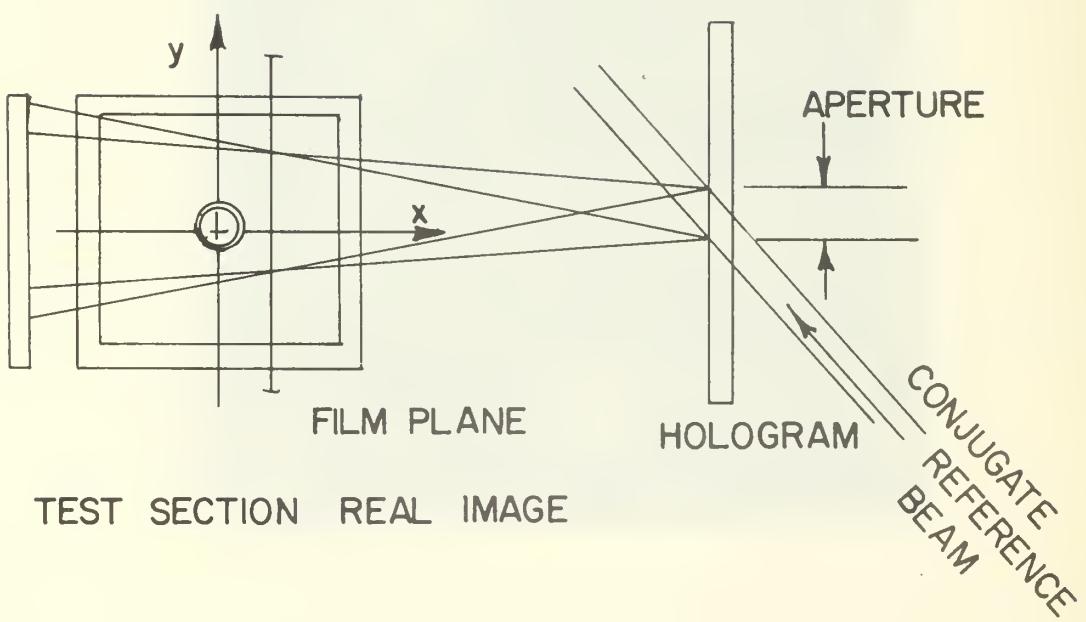


FIGURE 16. LENSLESS PHOTOGRAPHIC TECHNIQUE USING A CONJUGATE REFERENCE BEAM OF SMALL DIAMETER



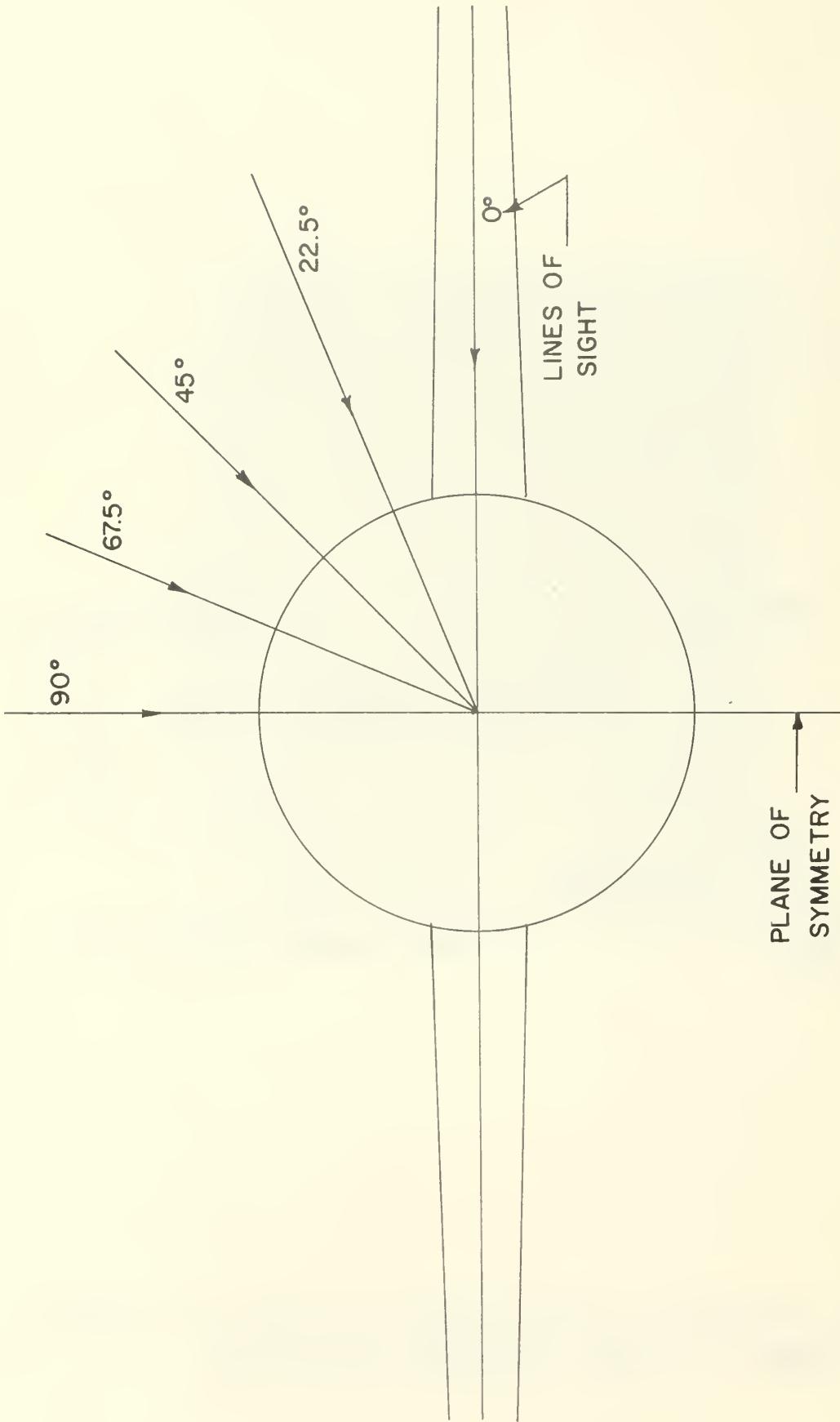


FIGURE 18. FRINGE DATA INPUT INFORMATION

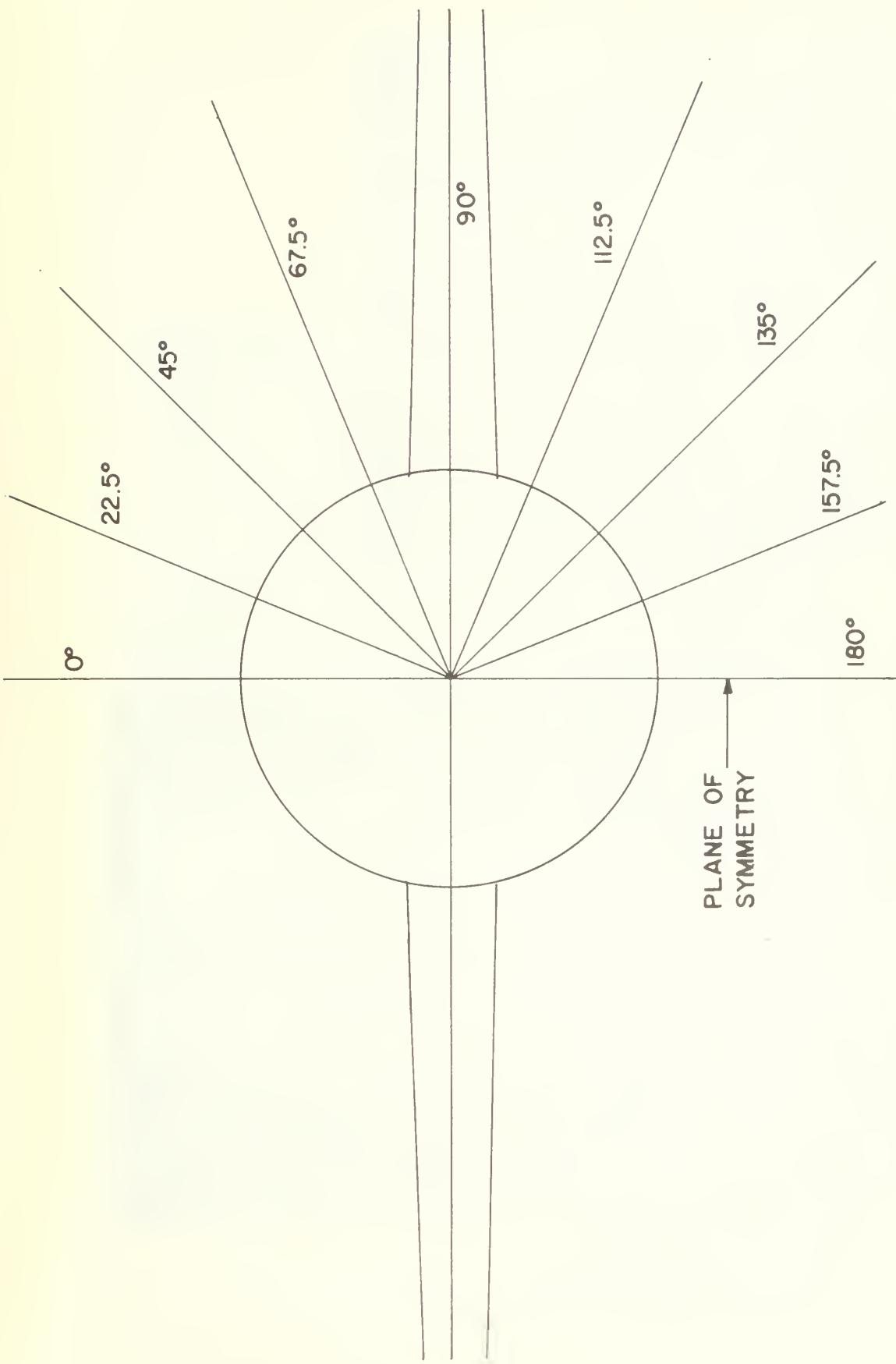
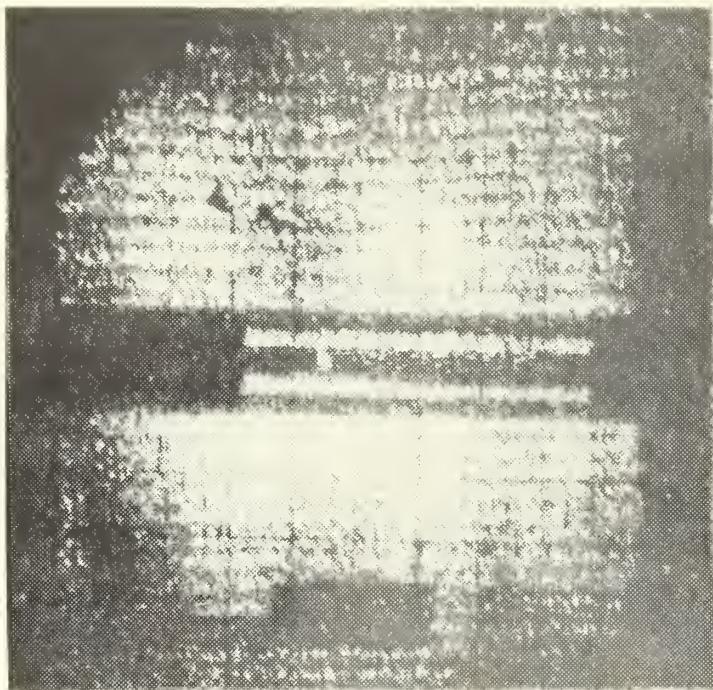
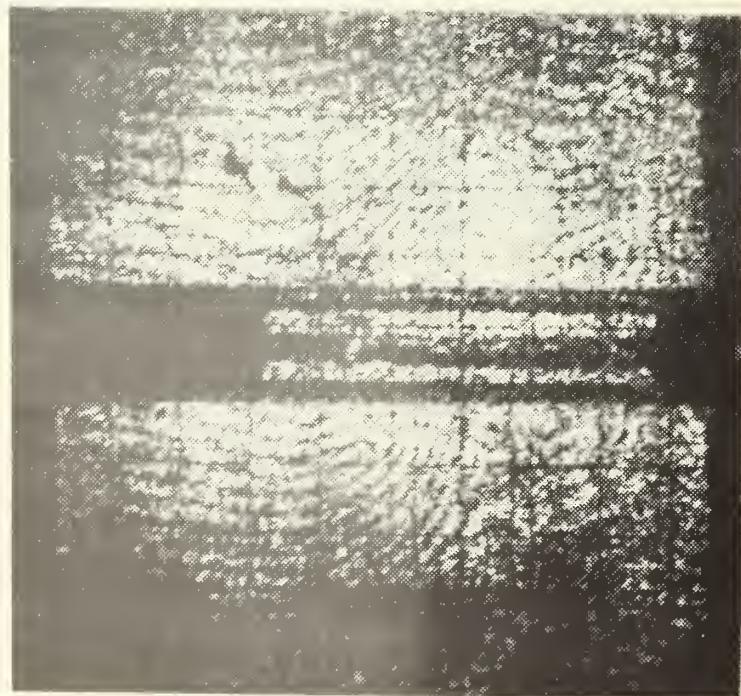


FIGURE 19. DENSITY DATA OUTPUT INFORMATION



DOUBLE-STATIC  
INTERFEROGRAM



STATIC-DYNAMIC  
INTERFEROGRAM

FIGURE 20. PHOTOGRAPHIC INTERFEROGRAMS  
FOR 0 DEG. VIEWING ANGLE

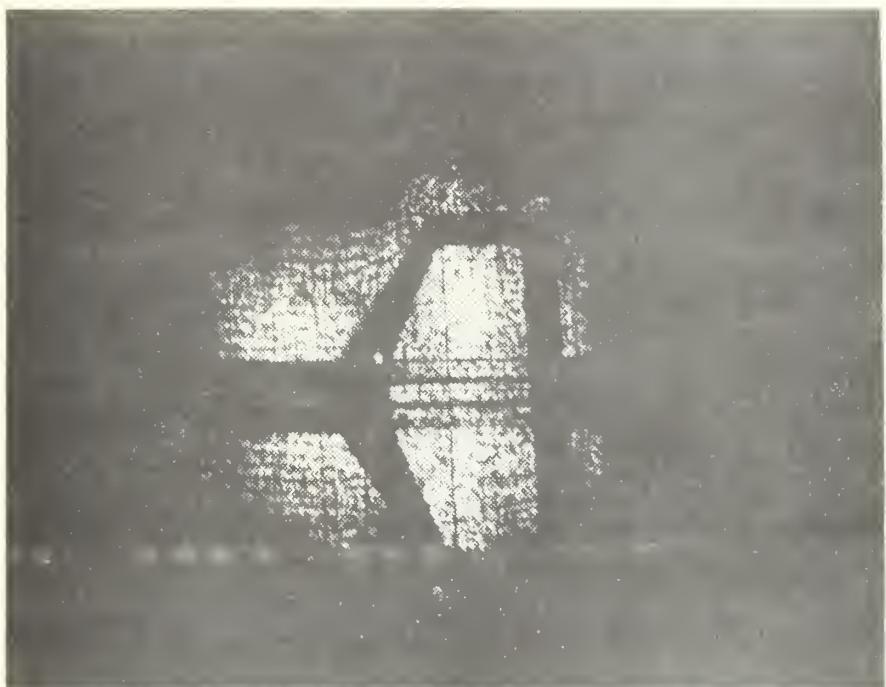


FIGURE 21. STATIC-DYNAMIC INTERFEROGRAM  
FOR  $22\frac{1}{2}$  DEG. VIEWING ANGLE



5

FIGURE 22. STATIC-DYNAMIC INTERFEROGRAM  
FOR 45 DEG. VIEWING ANGLE



33

FIGURE 23. STATIC-DYNAMIC INTERFEROGRAM  
FOR  $67\frac{1}{2}$  DEG. VIEWING ANGLE



C

FIGURE 24. STATIC-DYNAMIC INTERFEROGRAM  
FOR 90 DEG. VIEWING ANGLE

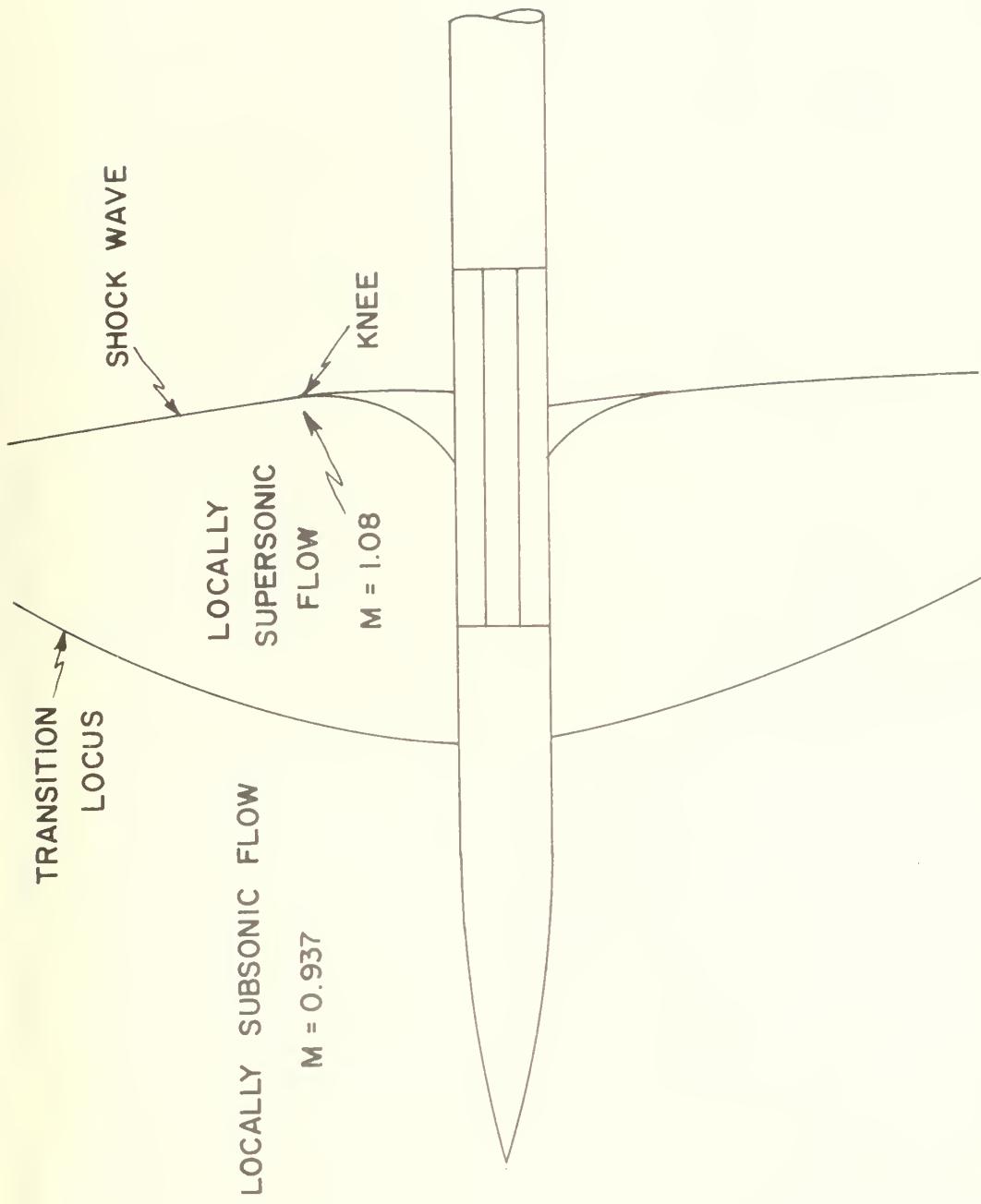


FIGURE 25.  
CHARACTERISTIC TRANSONIC FLOW REGIONS; FROM SEVERAL INTERFEROGRAMS

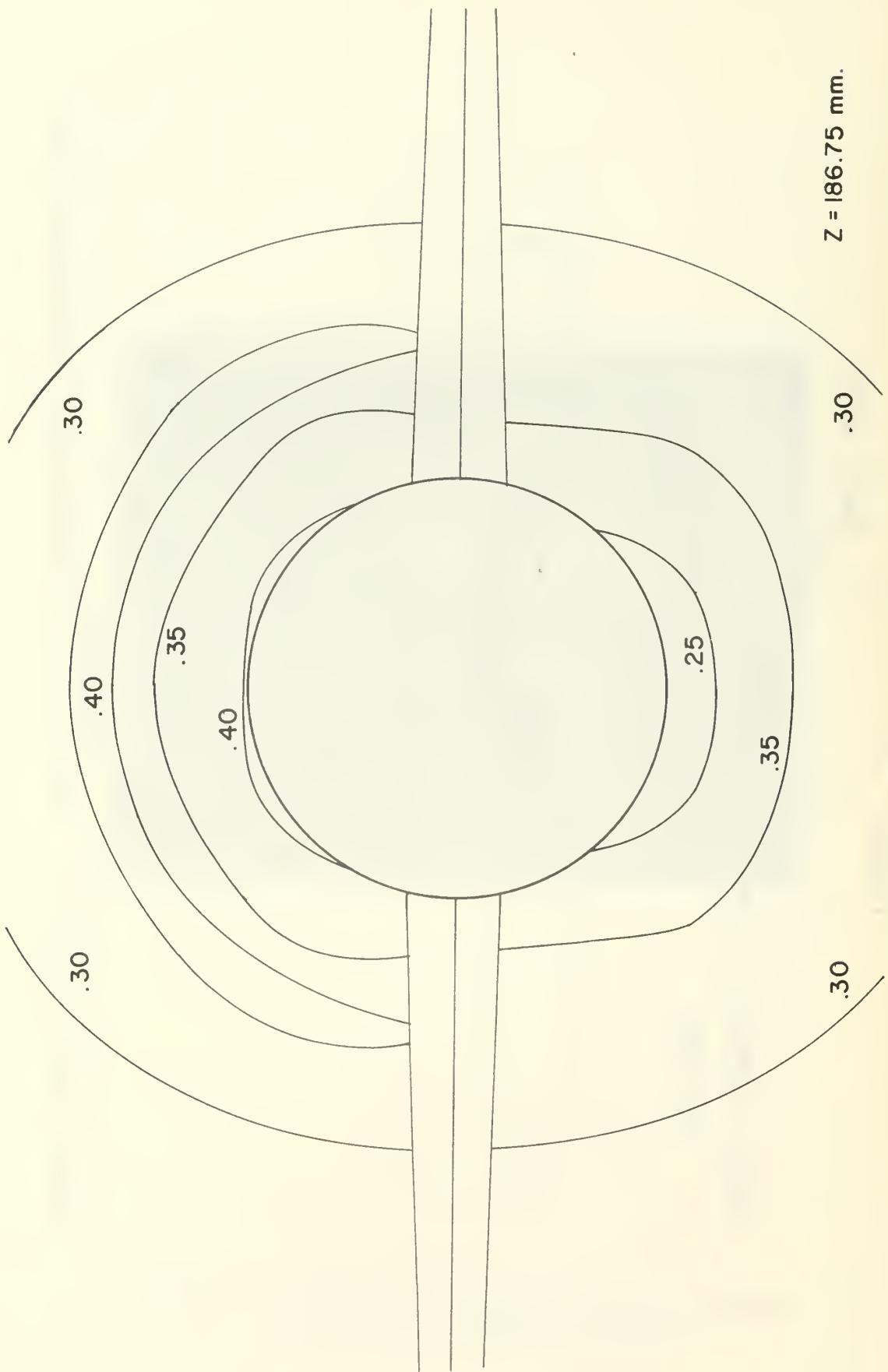


FIGURE 26. CONTOUR PLOT OF DENSITY FUNCTION,  $(\rho / \rho_\infty) - 1$ , FOR GIVEN CROSS-SECTIONAL PLANE

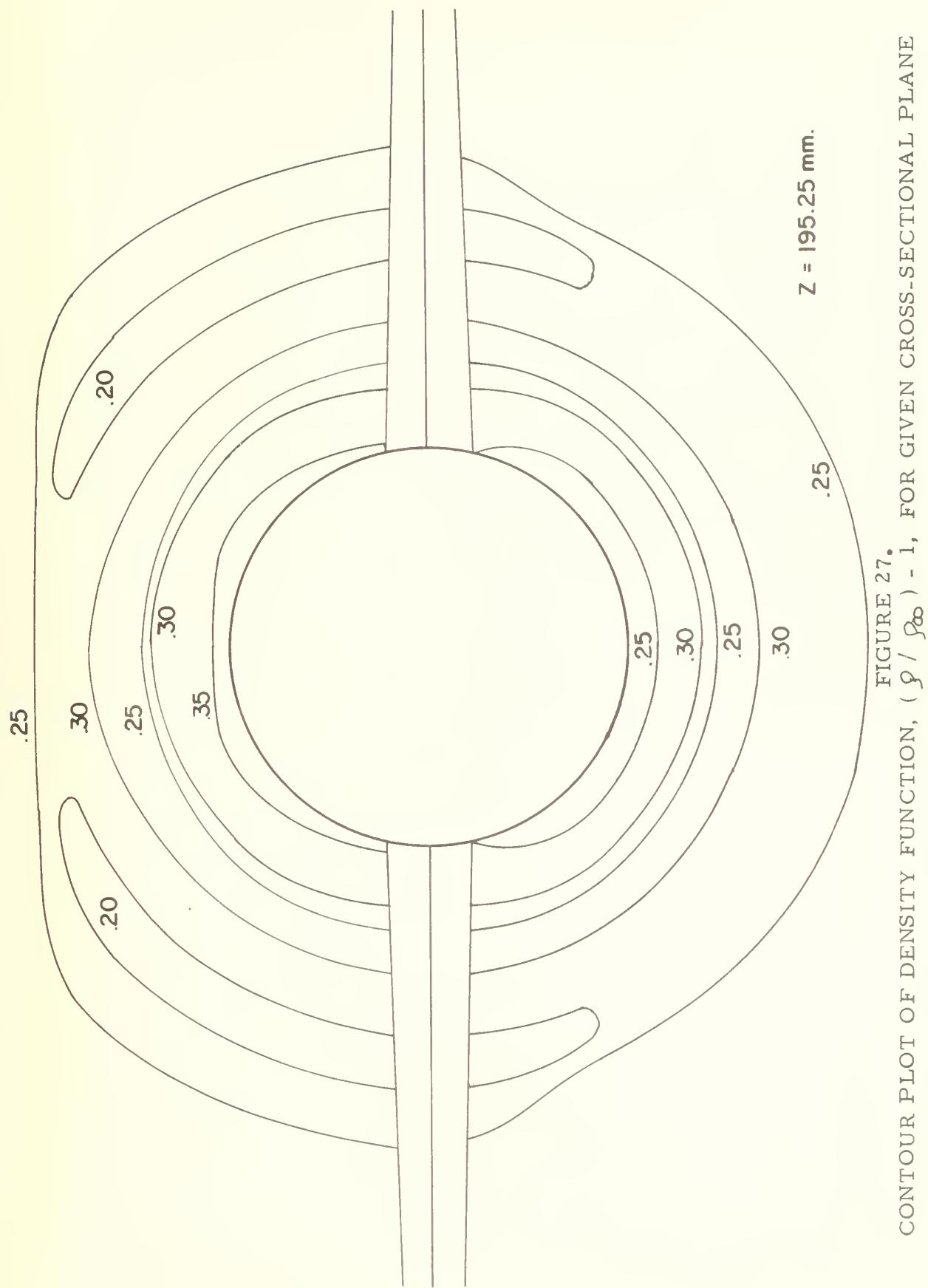


FIGURE 27. CONTOUR PLOT OF DENSITY FUNCTION,  $(\rho / \rho_\infty) - 1$ , FOR GIVEN CROSS-SECTIONAL PLANE

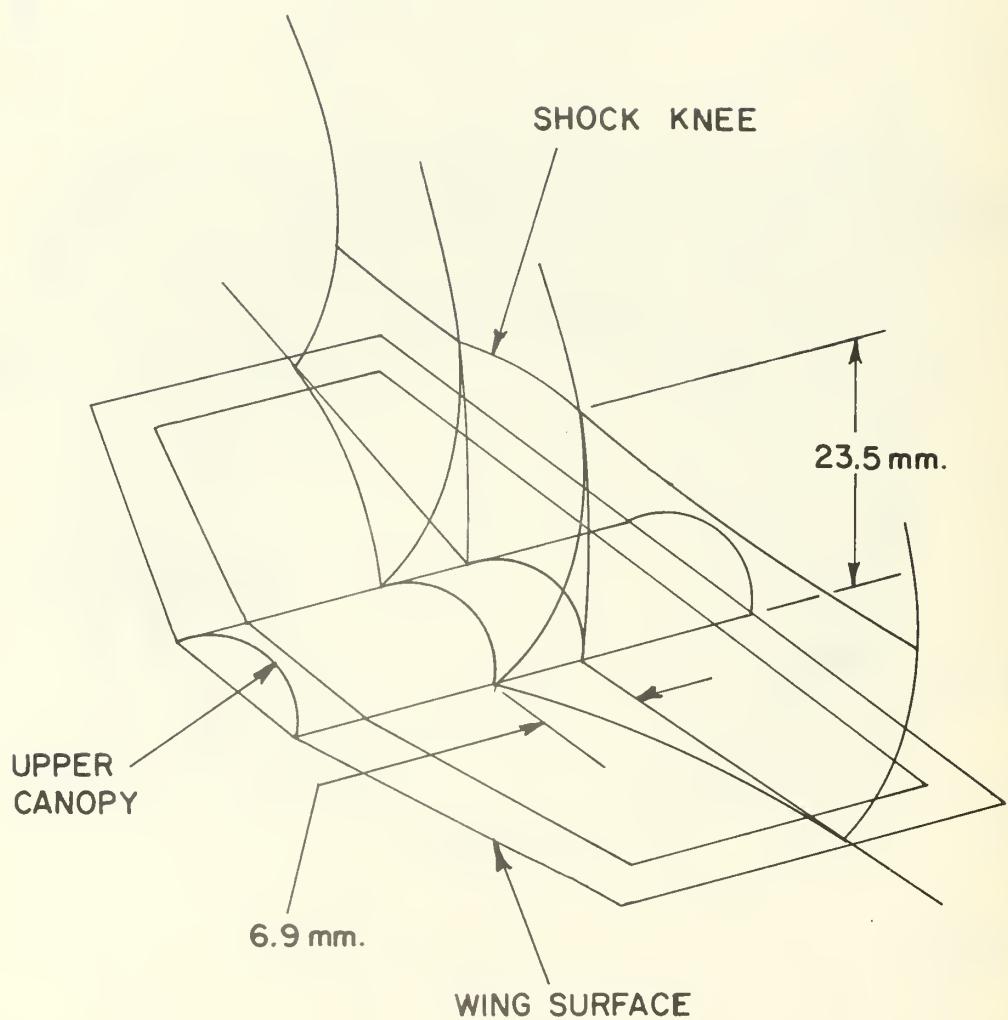


FIGURE 28. THREE DIMENSIONAL SCHEMATIC OF SHOCK WAVE STRUCTURE; CONSTRUCTED USING SEVERAL INTERFEROGRAM VIEWS

$$\text{MEAN} = \frac{x}{7}$$

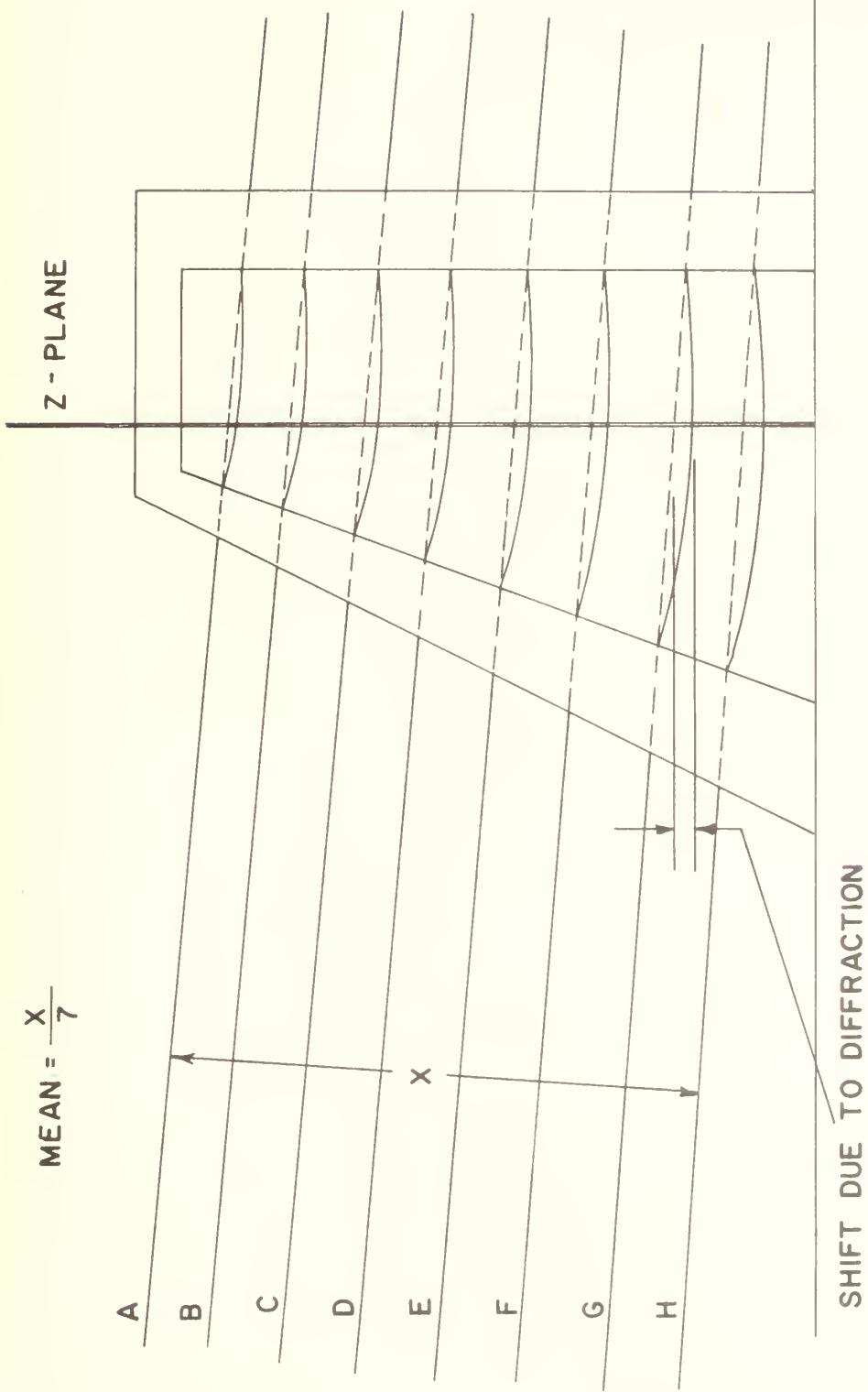


FIGURE 29. DOUBLE-STATIC HOLOGRAM REDUCTION PROCESS;  $Z = 186.75$  mm.

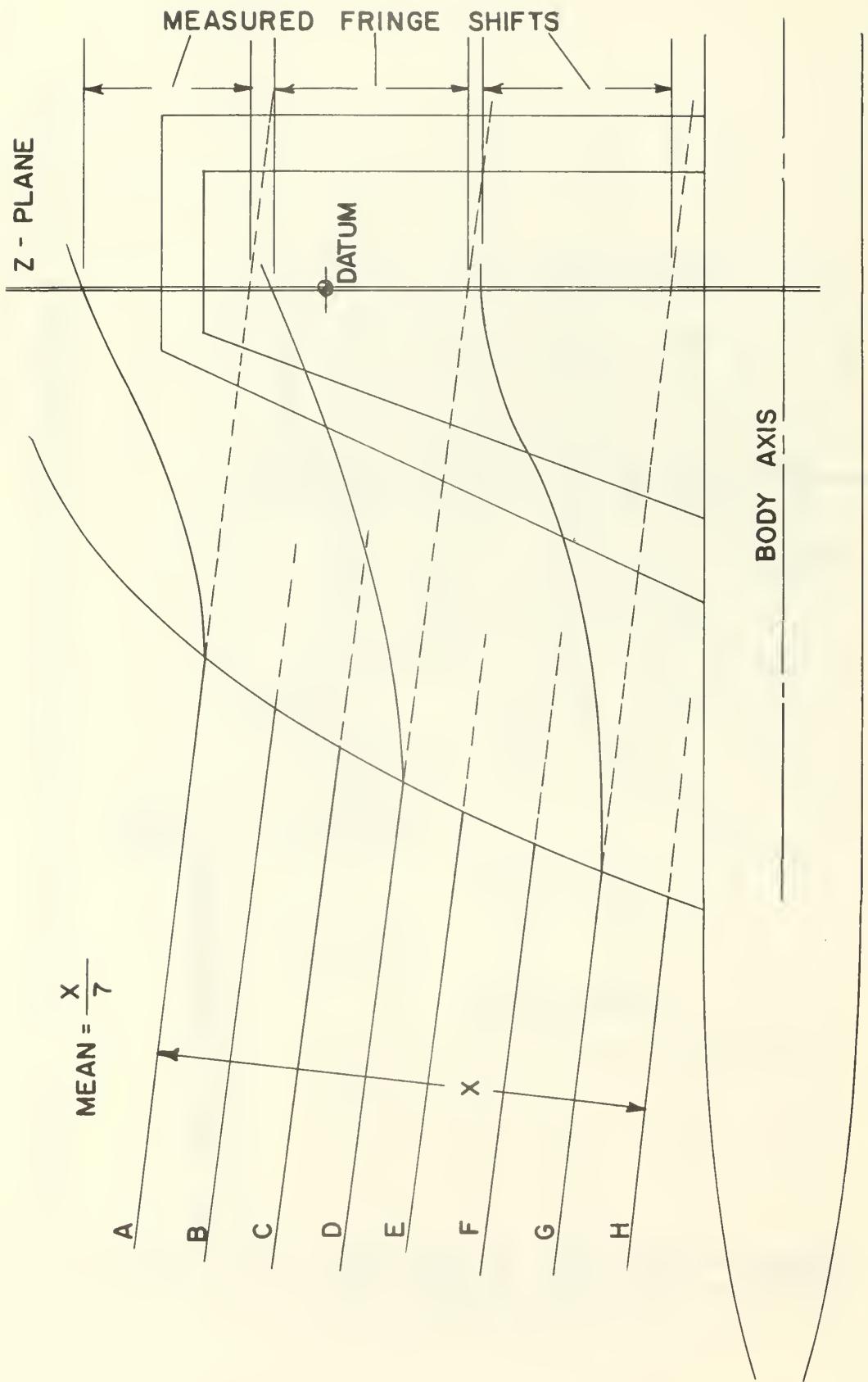


FIGURE 30. STATIC-DYNAMIC HOLOGRAM REDUCTION PROCESS;  $Z = 186.75$  mm.

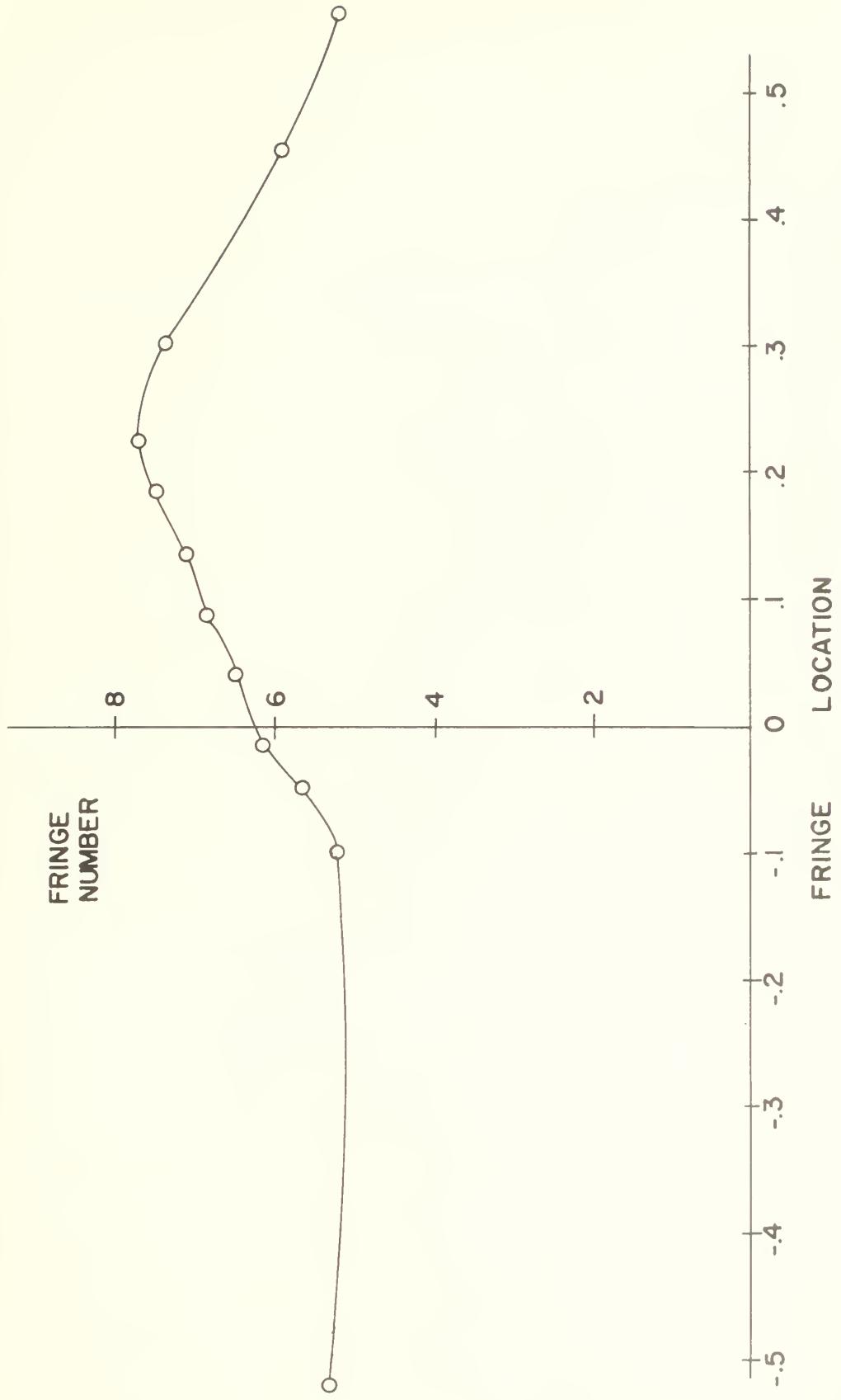


FIGURE 31. RADIAL VARIATION OF FRINGE NUMBER;  
ZERO DEGREE VIEW,  $Z = 186.75$  mm.

RUN NUMBER	HOLOGRAM NUMBER	DOUBLE EXPOSURE	ROLL ANGLE	P <sub>A</sub> (psi)	P <sub>T</sub> (psi)	T <sub>T</sub> (deg. F)	MACH NUMBER
1	9	S/S	0.00	14.781	14.181	64.6	.9371
2	10	S/D	0.00	14.781	14.228	64.2	.9398
3	23	S/S	11.25	14.820	14.187	64.6	.9361
4	24	S/D	11.25	14.820	14.206	64.2	.9392
5	19	S/S	22.50	14.784	14.237	64.3	.9391
6	20	S/D	22.50	14.800	14.195	64.6	.9365
7	15	S/S	33.75	14.785	14.185	64.3	.9342
8	16	S/D	33.75	14.830	14.276	61.8	.9372
9	11	S/S	45.00	14.750	14.195	64.6	.9369
10	12	S/D	45.00	14.787	14.284	62.0	.9352
11	25	S/S	56.25	14.830	14.276	61.8	.9350
12	26	S/D	56.25	14.875	14.284	62.0	.9371
13	21	S/S	67.50	14.787	14.195	64.6	.9365
14	22	S/D	67.50	14.787	14.195	64.3	.9398
15	17	S/S	78.75	14.870	14.285	64.8	.9361
16	18	S/D	78.75	14.870	14.285	64.8	.9342
17	13	S/S	90.00	14.787	14.195	64.6	.9402
18	14	S/D	90.00	14.787	14.195	64.6	.9392
19	35	S/S	101.25	14.781	14.274	61.5	.9371
20	36	S/D	101.25	14.875	14.284	62.5	.9372
21	27	S/S	112.50	14.875	14.284	61.0	.9369
22	28	S/D	112.50	14.875	14.278	62.0	.9352
23	37	S/S	123.75	14.878	14.284	61.0	.9350
24	38	S/D	123.75	14.878	14.284	61.0	.9357
25	29	S/S	135.00	14.875	14.278	62.0	.9371
26	30	S/D	135.00	14.875	14.278	62.0	.9372
27	39	S/S	146.25	14.881	14.286	60.3	.9352
28	40	S/D	146.25	14.881	14.286	60.3	.9350
29	31	S/S	157.50	14.875	14.276	61.8	.9371
30	32	S/D	157.50	14.875	14.276	61.8	.9357
31	41	S/S	168.75	14.881	14.287	61.0	.9371
32	42	S/D	168.75	14.881	14.287	61.0	.9357
33	33	S/S	180.00	14.875	14.282	62.0	.9357
34	34	S/D	180.00	14.875	14.282	62.0	.9357

TABLE I. EXPERIMENTAL DATA FOR WIND TUNNEL RUNS AT NSRDC

	FRINGE I.D.	MEASURED SHIFT	CORRECTED SHIFT	FRINGE NUMBER	DISTANCE FROM DATUM	DEMAG. DISTANCE FROM AXIS	DISTANCE FROM AXIS	NONDIMEN. LOCATION
A	25.0	21.475	3.53	-27.0	-25.1	-63.8	-798	-
B	29.6	26.075	4.29	-23.2	-21.6	-59.8	-748	-
C	30.4	26.875	4.42	-18.5	-17.2	-55.4	-693	-
D	31.6	28.075	4.62	-13.0	-12.1	-50.3	-623	-
E	35.5	31.975	5.23	-9.5	-8.8	-47.0	-588	-
F	36.0	32.475	5.34	-3.6	-3.4	-41.6	-520	-
G	34.0	30.475	5.23	+32.8	+30.5	-7.7	096	-
H	36.4	32.875	5.64	+37.0	+34.4	-3.8	048	-
I	40.0	36.475	6.26	+40.0	+37.2	-1.0	013	-
J	41.5	37.975	6.51	+44.4	+41.3	+3.1	039	+
K	43.5	39.975	6.86	+48.5	+45.2	+7.0	088	+
L	45.3	41.775	7.17	+52.7	+49.1	+10.9	136	+
M	47.4	43.875	7.53	+57.0	+53.1	+14.9	186	+
N	48.6	45.075	7.73	+60.5	+56.3	+18.1	226	+
O	46.8	43.275	7.42	+67.0	+62.4	+24.2	303	+
P	38.5	34.975	5.99	+80.3	+74.8	+36.6	458	+
Q	34.0	30.475	5.23	+89.6	+83.4	+45.2	.565	+

Mean free stream spacing = 5.96 mm.

Diffraction Correction = 3.525 mm.

Magnification factor = 1.074

Body axis location = +38.2 mm

All measurements in millimeters

- = above, + = below

TABLE II.  
RECORDED DATA FOR ZERO DEGREE VIEW AT Z = 186.75 mm. FROM BODY NOSE

## APPENDIX A

### REDUCTION OF AN INTERFEROGRAM TO OBTAIN FRINGE SHIFT DATA

The reduction of data for a cross-sectional plane of interest involved a complete analysis of both doubly exposed holograms, and their corresponding interferograms, for each viewing angle.

For each view, or line of sight, the double-static exposure was first placed face down on a light table. The average fringe spacing was recorded on the back by measuring the perpendicular distance between two sufficiently separated fringe lines in the free stream and dividing by the number of spacings in between. The method is portrayed in Figure 29. Of primary importance was the measurement of the uniform shift of fringe lines due to the beam diffractive quality of the lucite sections of the model; this was taken to be the average distance between hypothetically unaltered fringe lines and the corresponding shifted fringes at their intersection with the z-plane.

The static-dynamic exposure was then placed face down on the light table and the fringe and model contours traced on the back, as shown in Figure 30. Again, the average free stream fringe line spacing was measured and checked against the value from the double-static exposure. A reference point, or datum, was selected at the intersection of a well-defined horizontal grid line and the z-plane of interest. Fringe shifts were computed in the following manner:

1. The distance from datum to a hypothetically undeviated fringe line at its intersection with the z-plane was measured in millimeters.
2. The distance from datum to the actual deviated, or shifted, fringe at its intersection with the z-plane was measured in millimeters. Measured shifts should be made perpendicular to the fringe direction. The present procedure is convenient and only introduces an error of about 1% in the overall level of the density field.
3. The raw fringe shift distance was then the distance in 1. above minus the distance in 2.
4. To correct for diffraction error, the uniform shift measured in the double-static exposure was then subtracted from the distance in 3.

Fringe numbers were calculated by dividing the shift for each fringe by the average free stream spacing for the exposure. Magnification factors were computed for each exposure by comparing photographically recorded model diameter with actual model diameter.

Since the datum location varied slightly from exposure to exposure, it was necessary to reference all measurements to a central point for each plane of analysis. This point was taken to be the intersection of the longitudinal axis of the model fuselage and the cross-sectional plane of interest. The intersections of shifted fringe lines with the z-plane were referenced to this fixed point and demagnified. The resulting distances were then nondimensionalized using the selected inversion circle radius. Table II contains typical data recorded for the zero degree view at the 186.75 mm. plane of interest.

A plot of fringe numbers versus nondimensionalized fringe location was produced for each viewing angle using data obtained from the interferograms. Fringe numbers at 201 equidistant points, as required for input into Mode 3 of the inversion computer program, were recorded from the resulting curves. The curve plotted using the data from Table II is shown in Figure 31.

## APPENDIX B

### APPLICATION OF COMPUTER PROGRAM "HOLOFER"

The computer program used in this study is designed to invert an array of fringe numbers across a field to obtain the associated density field using a special adaptation of the inversion technique first proposed by C. D. Maldonado [5, 6]. Three different modes of operation are available to the operator, as described below:

#### (a) Mode 1

This mode provides the basic self-testing capability of the computer program. It can either generate its own input density field using Subroutine FUNCT or read in a density field through Subroutine FREAD. The fringe number array corresponding to the input density data is first generated; this information is then used as the input for the inversion to obtain the original density distribution once again. This self-testing procedure was utilized in the present investigation to determine the best value of the scale factor,  $\propto$ , required to assure accurate density in the region of inversion.

#### (b) Mode 2

This mode generates the fringe array at regular intervals across the test field from irregularly spaced fringe values read in through Subroutine SHEET. Simulated fringe arrays may be generated

by one of the functions specified in Subroutine FUNCT if NCODE = 1  
is specified.. Mode 2 has not been utilized in the present investigation.

(c) Mode 3

Mode 3 operation reads in a regularly spaced array of input density values directly utilizing Subroutine READ, which is first called by Subroutine GARRAY. The parameters in the first two cards preceding the input fringe data serve to identify the size, location and symmetry of the fringe field. The following parameters were used in calculating the asymmetric density field in the present experiment:

<u>PARAMETER</u>	<u>INPUT</u>
NOF	Run Number
IMAX	201
JMAX	20
ISYM	1
JSYM	1
IMS	201
JMS	5
Z	0.0, 1.0
XO	0.0
YO	0.0
PHISYM	0.0

Amplifying details and applications of the inversion computer program are found in References [3, 7, 9]. A listing of the program is included in this appendix for reference.

```

*****HOLOVERT***** FIELD AT AN ARRAY OF POINTS IN A VARIABLE
***** COORDINATE SYSTEM IT SURVEYS NPTS POINTS EACH FOR A SET OF NLINS
***** LINES ACROSS THE FIELD.

COMMON IMAX,JMAX,IMX,JMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z
COMMON TAB/INDEX,KEXTRA,MEXTRA,KLIMIT,MLIMIT,KOUT,MOUT
COMMON TAB2/IPT,KPT,LPT,BND,NPTS,NLINS,RHOINF,RLAMDA,BETA
COMMON OUT/XP,THEO,CALC,ERR,RHO,CA,FA
COMMON EQPAR/A,B,C,D,E,P,Q,S,T,U,V,W,RO,RA,NO,NA,NOF,NAF
COMMON SYM/ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM
COMMON IO/CMS,INI1,IN2,IN4
DATA BL,PL,ST,EX,OH,SC,DH,BR/1H,1H+,1H*,1HX,1HO,1H-,1H1/
DIMENSION RB(7),TL(62),RO(101),RA(101)
----- DIMENSION G(5151),GA(5151) H(202,5),SCF(73,6),BDA(4000)
DIMENSION THEO(51,11),CALC(51,11),ERR(51),RHO(51)
DIMENSION CA(51,11),FA(51,11),AR(42),XP(51),YP(51)
CMS=0.
----- REWIND 3
IN1=5
IN2=5
IN4=5
IF (CMS.NE.1.) GO TO 20
IN1=1
IN2=2
IN4=4
IF (CMS.EQ.1.) REWIND 4
READ (IN4,89) (AR(I),I=1,42)
INMAX=AR(1)
JMAX=AR(2)**2
IF (JMAX.LE.3) JMAX=1
KLIMIT=AR(3)
MLIMIT=AR(4)
KEXTRA=AR(5)
MEXTRA=AR(6)
JSYM=AR(14)
ALPHA=AR(7)
ALSIZE=AR(8)
EPS=AR(9)
MODE=AR(13)
DGN=AR(17)
RHOINF=AR(10)
RLAMDA=AR(11)*1. E-8
BETA=AR(12)
----- 20
CAL000100
CAL000200
CAL000300
CAL000400
CAL000500
CAL000600
CAL000700
CAL000800
CAL000900
CAL001000
CAL001100
CAL001200
CAL001300
CAL001400
CAL001500
CAL001600
CAL001700
CAL001800
CAL001900
CAL002000
CAL002200
CAL002300
CAL002400
CAL002500
CAL002600
CAL002700
CAL002800
CAL002900
CAL003000
CAL003100
CAL003200
CAL003300
CAL003400
CAL003500
CAL003600
CAL003700
CAL003800
CAL003900
CAL004000
CAL004100
CAL004200
CAL004300
CAL004400
CAL004500
CAL004600
CAL004700

```



```

96 FORMAT (/5X;!* A * B * C * D * E * , SET00420
1 P (* /4X, 6F10.3) *
1 FORMAT (/5X;!* S * T * U * V * W * , SET00440
1 Q (* /4X, 6F10.3)
1 FORMAT (/3X,75AI)
1 IF (DGN .GE .4) WRITE (6,89) (AR(I),I=1,42)
NNN=2
1 IF ( MODE .LT .0 ) NNN=1
1 IF ( MODE .GT .5 ) NNN=3
1 IF ( MODE.GT .5) MODE=MODE-10
NGP=0
1 IF (KL LIMIT .LT .KEXTRA) KEXTRA=KL LIMIT
1 IF (ML LIMIT .LT .MEXTRA) MEXTRA=ML LIMIT
1 IF (IPt .LT .0) NGP=IPt
1 IF (IPt .LT .0) IPt=-IPt
1 ISYM=2*I-(FLOAT(JSYM)/2.-FLOAT(JSYM/2))*2
1 IF (JSYM-EQ.0) ISYM=1
1 IF (JSYM-GT .0) ISYM=2
1 IF (ISYM-EQ.0.1) JMAX=(JMAX+1)/2)*2
RJMX=JMAX
MSYM=JSYM
1 IF ((MSYM-EQ.0).OR.(MSYM.GT .JMAX)) MSYM=1
FCU=ISYM*JSYM*JMAX
1 IF ((JSYM*GT .JMAX).OR.(JSYM.EQ.0)) FCU=JMAX
QSYM=FCU/RJMX
1IMS=(IMAX+ISYM-1)/ISYM
1 JMS=JMAX
1 IF (JSYM-EQ.0.1) JMS=(JMAX/2+1)/2
1 IF (JSYM-EQ.0.1) JMS=JMAX/2
1 MODE=ABS(AR(13))
XO=0.
YO=0.
ZD=0.
PHISYM=0.
HS=SIZE/2.
RHOS=1*286
BOX=RHOINF*BETA/RHOS/RLAMDA
RPTS=NPTS
XPR=0.
IF (NPTS.GT .1) XPR=XPRNG/(RPTS-1.)/2.
PIE=3*141592653589793
MONE=1
WRITE (6,58) IMAX,JMAX,IMS,JMS,ISYM,JSYM,MSYM,QSYM,FCU,
FORMAT (3X,1MAX,JMAX,IMS,JMS,ISYM,JSYM,MSYM,QSYM,FCU,,/
1 NTWO=2
IX=IMAX+JMAX
CAL00750
CAL00760
CAL00770
CAL00780
CAL00790
CAL00800
CAL00810
CAL00820
CAL00830
CAL00840
CAL00850
CAL00860
CAL00870
CAL00880
CAL00890
CAL00890
CAL00910
CAL00920
CAL00930
CAL00940
CAL00950
CAL00960
CAL00970
CAL00980
CAL00990
CAL01000
CAL01010
CAL01020
CAL01030
CAL01040
CAL01050
CAL01060
CAL01070
CAL01080
CAL01090
CAL01100
CAL01120
CAL01130
CAL01140
CAL01150
CAL01160
CAL01170

```

```

NF=INI
IF ((MODE.EQ.1).AND.(NOF.EQ.8).AND.(DGN.GE.1.)) WRITE (6,69)
IF ((MODE.EQ.1).AND.(NOF.EQ.8)) CALL FREAD (NO,RO,NF,ZD)
Z=ZD
IF ((DGN.GE.1.)) WRITE (6,68)
CALL GARRY (G,GA,NOF,DGN,MONE,XO,YO,PHISYM)
LM=1
IF ((LPT.EQ.0).AND.(BND.EQ.0)) LM=0
IIMX=IMAX+1
JJMX=IMAX*JMAX
NBD=1
IF (JSYM.EQ.0) NBD=2
KBD=KLIMIT*NBD
DO 15 IJ=1,1JJMX
GA(IJ)=0
IF (NAF.EQ.0) GO TO 16
NF=IN2
IF ((NAF.EQ.8).AND.(DGN.GE.1.)) WRITE (6,69)
IF ((NAF.EQ.8)) CALL FREAD (NA,RA,NF,ZD)
MST=MODE
MODE=1
IF (DGN.GE.1.) WRITE (6,68)
IF (NAF.NE.3.) CALL GARRY (GA,G,NAF,DGN,NTWO,XO,YO,PHISYM)
DO 16 IJ=1,IJM
GC(IJ)=G(IJ)+GA(IJ)
RLINS=NLINS
IF (NAF.EQ.8) WRITE (6,88) NA,(RA(L),L=1,NA)
IF (NOF.EQ.8) WRITE (6,87) NO,(RO(I),I=1,NO)
IF (LM.EQ.0) GO TO 14
RB(1)=-1.
DO 17 I=2,7
RB(I)=RB(I-1)+.5
TPIE=2.*PIE
MPIE=-PIE
DYP=0.
DXP=0.
16
15

```

16  
15

```

CALJ1180
CAL01190
CAL01200
CALJ1210
CAL01220
CAL01230
CAL01240
CAL01250
CAL01260
CAL01270
CAL01280
CAL01290
CAL01300
CAL01310
CAL01320
CAL01330
CAL01340
CAL01350
CAL01360
CAL01370
CAL01380
CAL01390
CAL01400
CAL01410
CAL01420
CAL01430
CAL01440
CAL01450
CAL01460
CAL01470
CAL01480
CAL01490
CAL01500
CAL01510
CAL01520
CAL01530
CAL01540
CAL01550
CAL01560
CAL01570
CAL01580
CAL01590
CAL01600
CAL01610
CALJ1620
CAL01630
CAL01640
CALJ1650

```

```

IF ((NLINS.GT.1) DYP=YPRNG/(RLINS-1.))
IF ((NPTS.GT.1) DXP=XPRNG/(RPTS-1.))
IF ((DGN.GE.1.) AND((NNN.EQ.2)) WRITE (6,64)
IF ((NNN.EQ.2)) CALL BDGEN (G,H,SCF,DGN,NBD,BDA,KBD)
DO 5 J=1,NLINS
IF (DGN.GE.1.) WRITE (6,67) J
RJM=J-1
PHI=PHIZ+DELPHI*RJM
YPI(J)=YPZERO+DYP*RJM
PSI=(PHI+90.)*PIE/180.

```

```

TAU=PSI-PHI*SYM  GO TO 9
IF (LPT.EQ.0) WRITE (6,78) (ST,I=1,124)
IF (LPT.LE.1) WRITE (6,74) (ST,I=1,95)
IF (LPT.GT.1).AND.(LPT.GT.1) READ (5,79) ZZ
IF (CMS.EQ.1.) WRITE (6,86) Z,PHI,YP(J)
WRITE (6,85) Z,PHI,YP(J)
IF (MODE.EQ.-1) WRITE (6,83) (RB(I),I=1,7)
IF (MODE.GT.1) WRITE (6,80) (RB(I),I=1,7)
WRITE (6,81) (DH,I=1,54),(PL,II=1,13)
IC=0
DO 3 I=1,NPTS
RIM=I-1
THEO(I,J)=0.
CA(I)=0.
ERR(I,J)=0.
CALC(I,J)=0.
RHO(I)=0.
XP(I)=XP*ZERO+XP*RIM
XP(I)=ABS(XP(I))
IF (XP(I).LT.1.*E-1.) XP(I)=0
RS=SQRT(XP(I)*2+YP(J)**2)/HS
IF (RS.GT.1.) GO TO 13
HT=ATANM(YP(J)*XP(I))
IF (XP(I).EQ.0.) HT=0.
SIG=TAU-PIE/2.+HT
IF ((SIG.GT.PIE/2.) SIG=SIG-PIE
IF ((SIG.LT.-MPIE) SIG=SIG+MPIE
SIG=SIG
XS=R*S*COS(SIG)
IF (DGN.GE.1.) WRITE (6,44) SIG
FORMAT(1:SIG=.,E10.3)
44
YS=R*S*SIN(SIG)
IF (DGN.GE.5) WRITE (6,57) PHI,PSI,TAU,THT,SIG,SIGI,XS,YS
FORMAT(1:ANGLES=.,10E10.3)
57
RI=I
F=0.
IF (DGN.GE.2.) WRITE (6,66) I
CALL FUNCT(XS,Y,F,A(I,J),NAF,DGN,NTWO)
IF (MODE.EQ.1) CALL FUNCT(XS,Y,F,NOF,DGN,MONE)
THEO(I,J)=F
IF (NNN.GE.2) REWIND 3
IF (NNN.GE.2) CALL FIELD (RS,SIGI,SOLN,NBD,BDA,DGN,KBD)
IF (NNN.EQ.1) CALL FIELD2 (RS,SIGI,SOLN,G,H,SCF,DGN)
CA(I)=SOLN/BOX/H
CALC(I,J)=CA(I)-FA(I,J)

```

13

```

RHO(I)=RHO INF*(CALC(I,J)+1.)
ERR(I)=CA(I)
IF (MODE.EQ.1) ERR(I)=(CALC(I,J)-THEO(I,J))
IF (MODE.GT.1) THEO(I,J)=FA(I,J)
IF (LPT.EQ.0) GO TO 3
LC=0
TL(1)=BL
TTL=0
IF ((XP(I).GT.XPM).AND.(XP(I).LT.XPR)) TTL=1.
IF ((IC.EQ.5) IC=0
IF ((IC.EQ.0) TL(1)=PL
DO 2 L=2,62
TL(L)=BL
IF ((I.EQ.1).OR.(TL.EQ.1).OR.(I.EQ.NPTS)) TL(L)=PL
IF ((IC.EQ.1) LC=0
IF ((IC.EQ.0)).AND.(LC.EQ.0)) TL(L)=PL
LC=LC+1
TL(2)=PL
TL(62)=PL
IC=IC+1
RLW=(CA(I)+1.)*20.+2.5
LW=RLW
IF (LW.GT.62) LW=62
IF (LW.LT.2) LW=2
TL(LW)=SC
RLY=(FA(I,J)+1.)*20.+2.5
LY=RLY
IF (LY.GT.62) LY=62
IF (LY.LT.2) LY=2
IF (NAF.NE.J) TL(LY)=ST
RLX=(THEO(I,J)+1.)*20.+2.5
LX=RLX
IF (LX.GT.62) LX=62
IF (LX.LT.2) LX=2
IF (MODE.EQ.1) TL(LX)=OH
RLZ=(CALC(I,J)+1.)*20.+2.5
LZ=RLZ
IF (LZ.GT.62) LZ=62
IF (LZ.LT.2) LZ=2
TL(LZ)=EX
WRITE(6,82) MOUT, KOUT, INDEX, THEO(I,J), ERR(I), CALC(I,J), RHO(I),
1 XP(I), TL(L), L=1,62
IF ((NPTS.LE.20).AND.(I.NE.NPTS)) WRITE(6,79)
CONTINUE
IF (LPT.NE.0) WRITE(6,81) (DH,I=1,54), (PL,I=1,13)
TMAX=0.
TMIN=0.

```

2

```

IE=0.
BE=0.
DO 4 I=1,NPTS
TH=THEO(I,J) TMAX=TH
IF (TH.LT.TMAX) TMIN=TH
IF (ABS(CALC(I,J)-TH)>ER) GO TO 4
BE=I
CONTINUE
TMM=TMAX-TMIN
EB=RHOINF*(CALC(I,J)-THEO(I,J))
BE=(CALC(I,J)-THEO(I,J))*100/TMM
IF ((MUD-EQ.1).AND.(LPT.NE.0)) WRITE(6,75) EB,XP(IE),BE
IF (DELPHI.NE.0.) YP(J)=PHI
CONTINUE
IF (BND.EQ.0.) GO TO 14
IF (LPT.EQ.1) WRITE(6,78) (ST,I=1,124)
IF (LPT.GT.1) WRITE(6,74) (ST,I=1,95)
IF ((CMS.EQ.1).AND.(LPT.GT.1)) READ(5,79) ZZ
IF (DGN.EQ.1) WRITE(6,63)
CALL MAP(NPTS,NLINS,Z,BND)
IF (NAF.EQ.0) GO TO 10
NAO=10*NQ+NAF
IF ((DGN.GE.1).AND.(NGP.EQ.-3)) CALL GPUNCH(IZ,XO,YO,PHISYM,NAO,IMAX,JMAX,G)
IF (NGP.EQ.-3) CALL GPUNCH(IZ,XO,YO,PHISYM,NAO,IMAX,JMAX,G)
DO 7 IJ=1,1 JMX
G(IJ)=G(IJ)-GA(IJ)
IF ((IPT.LE.0) GO TO 11
IF (((IPT.EQ.1).OR.((IPT.EQ.3))) WRITE(6,78) (ST,I=1,124)
IF (((IPT.EQ.2).OR.((IPT.EQ.4))) WRITE(6,74) (ST,I=1,95)
IF ((CMS.EQ.1).AND.((IPT.EQ.2).OR.((IPT.GE.4)))) READ(5,79) ZZ
CALL GPRINT(6,MONE)
IF (NGP.EQ.-1) CALL GPUNCH(Z,XO,YO,PHISYM,NOF,IMAX,JMAX,G)
IF (IPT.EQ.3) WRITE(6,78) (ST,I=1,124)
IF ((IPT.GE.4)) WRITE(6,74) (ST,I=1,95)
IF ((CMS.EQ.1).AND.((IPT.GE.4))) READ(5,79) ZZ
IF ((KPT.LE.0) GO TO 12
IF (((KPT.EQ.1).OR.((KPT.EQ.3))) WRITE(6,78) (ST,I=1,124)
IF (((KPT.EQ.2).OR.((KPT.GE.4))) WRITE(6,74) (ST,I=1,95)
IF ((CMS.EQ.1).AND.((KPT.EQ.2).OR.((KPT.GE.4)))) READ(5,79) ZZ
IF (DGN.GE.1) WRITE(6,61)
CALL GPLOT(G,JMS)
WRITE(6,73) (EX,I=1,124)
AGAIN=ST
4
5
14
7
10
11
12

```

```

IF (CMS*NE.1.) READ(5,60) AGAIN
IF (AGAIN.EQ.BL) GO TO 20
WRITE(6,77)
FORMAT(6,77)
FORMAT(6,F12.7) THE INPUT DATA FOR ADD-ON FUNCTION NO.8 (' ,13,
1 POINTS) WAS: '7(1F10.3/)
1 POINTS) WAS: '7(1F10.3/
1 POINTS) WAS: '7(1F10.3/
FORMAT(1H1// THE INVERTED CROSS SECTION FOR:)
FORMAT(10X,Z = 'F8.3, CM.'/10X, PHI='F8.3; ' DEGREES' /10X,
14HY,* =,F8.3, CM.'44X,0 = ORIGINAL FUNCTION/
270X,* = ADD-ON FUNCTION)
FORMAT(,ADJUST PAGE, HIT SPACE AND RETURN.)
FORMAT(,LIMIT MAX ABS-COMPUTED (MG/CC)/* X.,F4.1,
1 TERM FUNCTION ERROR FUNCTION DENSITY .,6H X.,F4.1,
26F10.1) FORMAT(2X,12,1X,13,1X,14,1X,F9.4,2X,F7.3,1X,F7.3,
1F7.3,1X,62A1) FORMAT(3X,54A1,2X,A1,12(4X,A1))
FORMAT(,LIMIT MAX,THE' 4X'DENSITY (MG/CC)/* /CAL03290
1 M K TERM 2X,FUNCTION,5X,SUM,3X,FUNCTION DENSITY ,
2 3H X. FORMAT(1X,47A1,6F10.1)
79 FORMAT(1X,F12.3)
78 FORMAT(1X,124A1)
77 FORMAT(1X,/)

IF (NE.1.) FUNCTION = (RHO/RHO-INFINITY)-1.0',33X,' : = THE'
76 1 INVERTED SUM,70X; X = COMPUTED FUNCTION;) CAL03350
75 1 FORMAT(,LARGEST ERROR: 'F8.6, GMS/CC; AT ' ,3HX' =,F6.3 CAL03370
1 :: 4X, F10.2, PERCENT //) CAL03380
74 1 FORMAT(1X,47A1, SET PAGE, HIT SPACE, RETURN ',48A1) CAL03390
69 FORMAT(,CALL READ)
68 FORMAT(,CALL GARRAY)
67 FORMAT(,LINE',I3, DO LOOP)
66 FORMAT(,POINT',I3, CALL FUNCT)
65 FORMAT(,CALL FIELD)
64 FORMAT(,CALL BDGEN)
63 FORMAT(,CALL MAP)
62 FORMAT(,CALL GPUNCH)
61 FORMAT(,CALL GPOINT)
60 STOP
END

CAL03100
CAL03110
CAL03120
CAL03130
CAL03140
CAL03150
CAL03160
CAL03170
CAL03180
CAL03190
CAL03200
CAL03210
CAL03220
MCAL03230
CAL03240
CAL03250
CAL03260
CAL03270
CAL03280
CAL03290
CAL03300
CAL03310
CAL03320
CAL03330
CAL03340
CAL03350
CAL03360
3CAL03370
CAL03380
CAL03390
CAL03400
CAL03410
CAL03420
CAL03430
CAL03440
CAL03450
CAL03460
CAL03470
CAL03480
CAL03490
CAL03500
CAL03510
CAL03520
SUB00400
SUB00420

```

C000001  
C

```

SUBROUTINE BDGEN ( G,H,SCF,DGN,NBD,BDA,KBD)
C BDGEN EVALUATES THE B AND D COEFFICIENTS FOR ALL M AND K, AND WRITES
C THE ARRAY LINEARLY ON DISK.
C
COMMON /TAB/ IIMX,JJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z
COMMON /INDEX,KEX,TRA,MEXTRA,KLIMIT,KOUT,MOUF
COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM
DIMENSION G(IJMx),H(IIMx,5),SCF(JJMX,6),BDA(KBD)
C INITIALIZE THE VALUES:
INDEX=0
KL2=NBD*KLIMIT
REWIND 3
JJMX6=JJMX*6
IIMX2=(IIMX+1)/2
PIE=3.*141592653589793
RIMAX=IMAX
KLMP=KLIMIT+1
DX=2./RIMAX
RJMAX=JMAX
DXI=2.*PIE/FCU
INITIALIZE THE MODIFIED HERMITE POLYNOMIAL ARRAY; VECTORS:
(1)=H1, (2)=H2, (3)=ALPHA*X(1), (4)=HM+2 STORED, (5)=HM+1 STORED
DO 1 I=1,IIMX2
RII=I
IIM=IIMX-I+1
H(I,I,3)=ALPHA*(RII*DX-DX-1.)
H(I,I,3)=-H(I,I,3)
H(I,I,1)=2.*H(I,I,3)
H(I,I,2)=(H(I,I,3)*H(I,I,1)-1.)/3.
H(I,I,1)=-H(I,I,1)
H(I,I,2)=H(I,I,2)
H(I,I,5)=H(I,I,2)
H(I,I,5)=H(I,I,2)
H(I,I,4)=H(I,I,1)
H(I,I,4)=H(I,I,1)
SIGN=1.
INITIATE THE SIN/COS ARRAY:
DO 2 J=1,JJMX
RJM=J-1
SCF(J,1)=0.
SCF(J,2)=1.
SCF(J,3)=SIN((RJM*DXI-PIE/2.))
SCF(J,4)=COS((RJM*DXI-PIE/2.))
SCF(J,5)=0.
SCF(J,6)=0.
MS=0
1 COMMENCE THE M LOOP:
2

```

```

DO 7 MP=1,MLIMIT
M=MP-1
RM=M
SIGN=-SIGN
IF (DGN.LE.-4) WRITE(6,88) SCF(1,1),SCF(2,1),SCF(2,2)
C TEST FOR SYMMETRY SKIPS:
IF (MS.EQ.MSYM) MS=0
TOTAL=0.
MS=MS+1
IF (MS.NE.1) GO TO 6
C COMMENT THE K LOOP:
DO 5 KP=1,KLIMIT
K=KP-1
PK=KP
RK=K
INDEX=INDEX+1
CALL THE B & D COEFFICIENTS AND WRITE THEM ON DISK:
CALL BD (M,K,G'H'SCF,B'D'J'MX6)
IF (DGN.EQ.3.) WRITE(6,89) M,K,B,D
IF (DGN.LE.-2) WRITE(6,89) M'K'B'D
IF (DGN.LE.-4) WRITE(6,88) H(1,1),H(1,2),H(1,4),H(1,5)
KK=K*NBD+1
K2=KP*NBD
BDA(K2)=D
BDA(KK)=B
GENERATE THE NEXT ORDER OF THE SET OF HERMITE POLYNOMIALS FOR NEW K:
ORDER=M+2*KP+1
HA=SQRT(PK*(PK+RM))/ORDER
HB=2.*SQRT((PK+1.)*(RM+PK+1.))/(ORDER+1.)/(ORDER+2.)
DO 5 II=1,IMX2
IM=IMX-II+1
H(II,1)=2.*((H(II,3)*H(II,2)-HA*H(II,1))
H(II,2)=SIGN#(H(II,1))
H(II,2)=HB*((H(II,3)*H(II,1))-ORDER*H(II,2))
5 ADVANCE THE SIN/COS ARRAY FOR THE NEXT M:
DO 3 J=1,JMX
IF (DGN.LE.-5) WRITE(6,87) (SCF(J,NT),NT=1,6)
3 SCF(J,1)=SCF(J,1)*SCF(J,4)+SCF(J,2)*SCF(J,3)
SCF(J,2)=SCF(J,2)*SCF(J,4)-STEMP*SCF(J,3)
DO 4 J=1,JMAX
SCF(J,5)=SCF(J+1,1)-SCF(J,1)
SCF(J,6)=SCF(J+1,2)-SCF(J,2)
5 WRITE(3)(BDA(I),I=1,KBD)
IF (DGN.LE.-3) WRITE(6,88) (BDA(I),I=1,1)
IF (JSYM.GT.JMAX) RETURN
RM=RM+1

```

```

C      REGENERATE THE HERMITE ARRAY FOR NEW M, K=0:
DO 7   I=1, IMX2
      IM=IMX-I
      H(I,I+1)=H(I,I+4)*SQRT(RM)/(RM+1.)
      H(I,I+2)=H(I,I+5)*(RM+2.)
      H(I,I+1)=H(I,I+1)-SIGN*H(I,I)
      H(I,I+2)=2.*SQR((RM+1)*(H(I,I+3)*H(I,I+1)-(RM+1.)*H(I,I+2)))
      H(I,I+2)=H(I,I+2)/(RM+3.)
      H(I,I+4)=H(I,I+1)
      H(I,I+5)=H(I,I+2)
      M=I4
      FORMAT(2X,10E10.3)
      RETURN
END
C      0000002
C

```

```

SUB00980
SUB00990
SUB01000
SUB01010
SUB01020
SUB01030
SUB01040
SUB01050
SUB01060
SUB01070
SUB01080
SUB01090
SUB01100
SUB01110
SUB01120
SUB01130
SUB01140
SUB01150
SUB01160
SUB01170
SUB01180
SUB01190
SUB01200
SUB01210
SUB01220
SUB01230
SUB01240
SUB01250
SUB01260
SUB01270
SUB01280
SUB01290
SUB01300
SUB01310
SUB01320
SUB01330
SUB01340
SUB01350
SUB01360
SUB01370
SUB01380
SUB01390
SUB01400
SUB01410
SUB01420
SUB01430

SUBROUTINE FIELD (RS,SIG,SOLN,NBD,BDA,DGN,KBD)
C      FIELD EVALUATES THE VALUE OF THE FIELD FUNCTION AT A PARTICULAR
C      POINT DESIGNATED IN CYLINDRICAL COORDINATES BY USING THE INVERSION
C      EQUATION OF MALDONADO, ET.AL. FIELD USES THE ARRAY OF B & D
C      COEFFICIENTS GENERATED IN SUBROUTINE BDGEN.
C
COMMON IMAX,JMAX,IMX,JMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z
COMMON /TAB/ INDEX,KEXTRA,MEXTRA,KLIMIT,KOUT,MOUT
COMMON /SYM/ ISYM,JSYM,MSYM,FCK,IMS,JMS,QLSYM
DIMENSION BDA(KBD),STK(52),STM(52)
C      INITIALIZE THE VALUES:
INDEX=0
MTIMER=0
KOUT=0
MOUT=0
MMAX=0
KMAX=0
TOTAL=0
JJMX6=J*JM*6
REWIND 3
AR=ALPHA*RS
ARG=AR**2
EXPON=EXP(-ARG)
PIE=3.141592653589793
APP=ALPHA/PIE/PIE
M=0
RM=M
RIMAX=IMAX
DX=2./RIMAX

```

```

RJMAX=JMAX
SIGN=1.
STK(1)=0.
STM(1)=0.
SMS=0.
CMS=1.*SIN(SIG)
CM=1.*COS(SIG)
MEP=MEXTRA+1
DO 16 MB=1,MEP
STM(MB)=0.
FM=1.
MS=0
COMMENCE THE M LOOP:
K=0
RK=K
RM=M
ARM=1.*NE*0) ARM=ARM*SIG
KTIMER=0
KEP=KEXTRA+1
DO 15 KB=1,KEP
STK(KB)=0.
SIGNK=-1.
COMPUTE THE K=0 & K=1 ORDERS OF LAGUERRE POLYNOMIAL FOR GIVEN M:
PM=0
P=SQRT(1./FM)
PP=(RM+1.-ARG)*SQRT(1./FM/(RM+1.))
TEST FOR SYMMETRY SKIPS:
IF (MS.EQ.MS) MS=0
MS=MS+1
IF (MS.NE.1) GO TO 7 COEFFICIENTS FOR GIVEN M:
READ A LINE OF B & D COEFFICIENTS
READ (3) (BDA(I),I=1,KBD)
IF (DGN.LE.-6) WRITE (6,88) (BDA(I),I=1,10)
COMMENCE THE K LOOP:
INDEX=INDEX+1
SIGNK=-SIGNK
COMPUTE THE M*K SUMMATION TERM:
KK=K*NBD+1
B=BDA(KK)
D=0
IF (NBD.EQ.2) D=BDA(KK+1)
BRAKE T=B
IF (RM.EQ.0.) GO TO 4
BRAKE T=B*CMS+D*SMS
ADD=SIGNK*BRAKE T*P*ARM
SUB01440
SUB01450
SUB01460
SUB01470
SUB01480
SUB01490
SUB01500
SUB01510
SUB01520
SUB01530
SUB01540
SUB01550
SUB01560
SUB01570
SUB01580
SUB01590
SUB01600
SUB01610
SUB01620
SUB01630
SUB01640
SUB01650
SUB01660
SUB01670
SUB01680
SUB01690
SUB01700
SUB01710
SUB01720
SUB01730
SUB01740
SUB01750
SUB01760
SUB01770
SUB01780
SUB01790
SUB01800
SUB01810
SUB01820
SUB01830
SUB01840
SUB01850
SUB01860
SUB01870
SUB01880
SUB01890
SUB01900
SUB01910

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SUB01920
SUB01930
SUB01940
SUB01950
SUB01960
SUB01970
SUB01980
SUB01990
SUB02000
SUB02010
SUB02020
SUB02030
SUB02040
SUB02050
SUB02060
SUB02070
SUB02080
SUB02090
SUB02100
SUB02110
SUB02120
SUB02130
SUB02140
SUB02150
SUB02160
SUB02170
SUB02180
SUB02190
SUB02200
SUB02210
SUB02220
SUB02230
SUB02240
SUB02250
SUB02260
SUB02270
SUB02280
SUB02290
SUB02300
SUB02310
SUB02320
SUB02330
SUB02340
SUB02350
SUB02360
SUB02370
SUB02380
SUB02390

TOTAL=TOTAL+ADD
IF (DGN*GT .-5) GO TO 5
STOT=TOTAL*EXPON*APP/BOX/SIZE
WRITE (6,89) 'K',STOT,ADD,BRAKET,P,ARM,B,CMS,D,SMS
ESTABLISH CHECK AS THE RELATIVE SIZE OF THE M,K TERM OF THE SERIES:
CHECK=ABS(ADD)
IF (TOTAL.GT.EPS) CHECK=ABS(ADD/TOTAL)
C ADVANCE THE K INDEX:
C K=K+1
RK=K
DO 10 KA=1,KEXTRA
KB=KEXTRA-KA+1
STK(KB+1)=STK(KB)
10 ORDER=TOTAL
ORDER=M+2*K+1
GENERATE THE NEXT ORDER OF LAGUERRE POLYNOMIAL FOR NEW K:
PM=P
P=P*
P=P*(ORDER-ARG)-PM*SQRT(RK*(RM+RK))
P=P/SQRT((RK+1.)*(RM+RK+1.))
SET K TIMER=TIMER+1
KTIMER=KTIMER+1
IF (K.GE.KLIMIT) GO TO 6
IF (CHECK.GE.EPS) KTIMER=0
IF (KTIMER.LE.KEXTRA) GO TO 3
GO TO 7
KOUT=KOUT+1
IF (KEXTRA.EQ.0) GO TO 7
TOTAL=0
DO 11 KA=1,KEXTRA
TOTAL=TOTAL+STK(KA+1)
RKK=KEXTRA
TOTAL=TOTAL/RKK
11 END OF K LOOP; ADVANCE M:
M=M+1
RM=M
STP=SMS*CMI+CMS*SMI
CMS=CMS*CMI-STP*SMI
IF (K.GT.KMAX) KMAX=K
FM=FM*RM
DO 12 MA=1,MEXTRA
MB=MEXTRA-MA+1
STM(MB+1)=STM(MB)
12 STM(2)=TOTAL
SET M TIMER FOR EXTRA M TERMS:
IF (MSS.EQ.1) MTIMER=MTIMER+1
JMAX=JMAX
C GO TO 9

```

```

IF (K•GT•KEXTRA) MTIMER=0
IF (M•GE•MLIMIT) GO TO 13
IF (MTIMER•LE•MEXTRA) GO TO 2
IF (MEXTRA•EQ•0) GO TO 9
TOTAL=0
DO 14 MA=1•MEXTRA
TOTAL=TOTAL+STM(MA+1)
RMX=MEXTRA
TOTAL=TOTAL/RMX
C      END OF M LOOP; COMPUTE OUTPUT SOLN.
9      MOUT=N-1
      IF (KOUT•EQ•0) KOUT=KMAX-1
      SOLN=TOTAL*EXPON*APP/2
      FORMAT(14,14,14,14)
      FORMAT(2X,10E10.3)
      RETURN
END
C000000

```

```

SUBROUTINE BD (M,K,G,H,SCF,B,D,JJMX6)

C BD EVALUATES THE FIRST (B) AND SECOND (D) COEFFICIENTS IN THE
C INVERSION EQUATION FOR A PARTICULAR SET OF INDEXES M & K.
C BD MAKES USE OF THE HERMITIAN POLYNOMIAL ARRAY GENERATED BY
C SUBROUTINE FIELD AS M & K ADVANCE.

COMMON JMAX,JMAXX,IIMX,JJMX,IIJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z
COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IIMS,JMS,QSYM
DIMENSION G(IIMX),SCF(JJMX6),H(IIMX)
PIE=3.141592653589793
B=0.
D=J*M
RM=M
RK=K
RJMAX=X=JMAX
JJMX4=4*IJMXX
DXI=2.*PIE/FCU
FORMAT(1X,110,/)
IF (JSYM.LE.0) GO TO 4
IF (M.NE.J) GO TO 2
S=DXI
DO 1 J=1,JMAX
DO 1 I=1,IMAX
II=I+1
IJ=IMAX*(J-1)+I
200

```

```

1      DH=H(I,I)-H(I,J)
2      B=B+G(I,J)*S*DH
3      B=B*QSYM/2.
      RETURN
4      DO 3 J=1,JMAX
      JS=J+J*MX4
      S=SCF(JS)/RM
      DO 3 I=1,I MAX
      IJ=I+1
      DH=H(I,I)-H(I,J)
      B=B+G(I,J)*S*DH
      B=B*QSYM
      RETURN
      IF (M.NE.0) GO TO 6
      S=DXI
      DO 5 J=1,JMAX
      DO 5 I=1,I MAX
      I=I+1
      IJ=I MAX*(J-1)+I
      DH=H(I,I)-H(I,J)
      B=B+G(I,J)*S*DH
      B=B/2.
      RETURN
      DO 7 J=1,JMAX
      JS=J+J*MX4
      J2=JS+J*MX4
      S=SCF(J2)/RM
      C=SCF(J2)/RM
      DO 7 I=1,I MAX
      IJ=I+1
      IJ=I MAX*(J-1)+I
      DH=H(I,I)-H(I,J)
      B=B+G(I,J)*S*DH
      FORMAT(' ',BD:' ,415,10F6.2)
      D=D-G(I,J)*C*DH
      RETURN
      END
      C0000004
      C
      SUBROUTINE FIELD2 (RS,SIG,SOLN,G,H,SCF,DGN)
      C
      C FIELD2 COMPUTES THE SAME INVERSION AS SUBROUTINE FIELD, EXCEPT THAT
      C THE COEFFICIENTS B AND D ARE COMPUTED INDIVIDUALLY AS USED BY
      C CALLING BD. DISK STORAGE IS NOT REQUIRED, BUT COMPUTING TIME IS
      C MUCH GREATER. FIELD2 IS UTILIZED BY SPECIFYING A NEGATIVE MODE ON
      C
      SUB02850
      SUB02860
      SUB02870
      SUB02880
      SUB02890
      SUB02900
      SUB02910
      SUB02920
      SUB02930
      SUB02940
      SUB02950
      SUB02960
      SUB02970
      SUB02980
      SUB02990
      SUB03000
      SUB03010
      SUB03020
      SUB03030
      SUB03040
      SUB03050
      SUB03060
      SUB03070
      SUB03080
      SUB03090
      SUB03100
      SUB03110
      SUB03120
      SUB03130
      SUB03140
      SUB03150
      SUB03160
      SUB03170
      SUB03180
      SUB03190
      SUB03200
      SUB03210
      SUB03220
      SUB03230
      SUB03240
      SUB03250
      SUB03260
      SUB03270
      SUB03280
      SUB03290
      SUB03300

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```

C THE INPUT PARAMETER. VALUE OF THE FIELD FUNCTION AT A PARTICULAR
C FIELD EVALUATES THE VALUE OF THE FIELD FUNCTION AT A PARTICULAR
C POINT DESIGNATED IN CYLINDRICAL COORDINATES BY USING THE INVERSION
C EQUATION OF MALDONADO, ET AL. FIELD CALLS SUBROUTINES BD & GARRAY.
C
COMMON IMAX, JMAX, IIMX, JMAX, ALPHA, SIZE, EPS, MUDE, BOX, SD, IX, Z
COMMON /TAB/ INDEX, KEXTRA, KLIMIT, FCU, IMSS, JMS, QSYM
COMMON /SYM/ ISYM, JSYM, MSYM
DIMENSION G(IIMX), H(IIMX,5), SCF(IIMX,6)
INITIALIZE THE VALUES:
INDEX=0
MTIMER=0
KOUT=0
MOUT=0
KMAX=0
KMAX=0
TOTAL=0
JIMX6=IIMX*6
JIMX2=(IIMX+1)/2
AR=ALPHA*RS
ARG=AR**2
EXPON=EXP(-ARG)
PIE=3.141592653589793
APP=ALPHA/PIE/PIE
M=0
RM=M
RIMAX=IMAX
DX=2.*RIMAX
RJMAX=JMAX
DXI=2.*PIE/FCU
INITIALIZE THE MODIFIED HERMITE POLYNOMIAL ARRAY; VECTORS:
(1)=H1, (2)=H2, (3)=ALPHA*X(1), (4)=HM+2 STORED, (5)=HM+1 STORED
DO 1 I=1,IIMX
RI=II
IIM=IIMX-I+1
H(II,3)=ALPHA*(RI*DX-DX-1.)
H(II,3)=-H(II,3)
H(II,1)=2.*H(II,3)
H(II,2)=(H(II,3)*H(II,1)-1.)/3.
H(II,1)=-H(II,1)
H(II,2)=H(II,2)
H(II,5)=H(II,2)
H(II,5)=H(II,2)
H(II,4)=H(II,1)
H(II,4)=H(II,1)
SIGN=1.
FM=1.
INITIATE THE SIN/COS ARRAY:

```

```

DO 11 J=1, JJMX
RJM=J-1
SCF(J,1)=0.
SCF(J,2)=1.
SCF(J,3)=SIN(RJM*DXI-PIE)
SCF(J,4)=COS(RJM*DXI-PIE)
SCF(J,5)=0.
SCF(J,6)=0.

11 MS=0
      COMMENCE THE M LOOP:
      SIGN=-SIGN
      K=0
      RM=1.
      IF(M.NE.0) ARM=AR**M
      KIMER=0
      SIGNK=-1
      COMPUTE THE K=0 & K=1 ORDERS OF LAGUERRE POLYNOMIAL FOR GIVEN M:
      PM=0.
      P=SORT(1./FM)
      PP=(RM+1.-ARG)*SQRT(1./FM*(RM+1.))
      ADVANCE THE SIN/COS ARRAY FOR NEW M:
      DO 12 J=1, JJMX
      SCF(J,1)=SCF(J,1)*SCF(J,2)+SCF(J,2)*SCF(J,3)
      SCF(J,2)=SCF(J,2)*SCF(J,4)-SCF(J,1)*SCF(J,3)
      DO 13 J=1, JMAX
      SCF(J,5)=SCF(J+1,1)-SCF(J,1)
      SCF(J,6)=SCF(J+1,2)-SCF(J,2)
      TEST FOR SYMMETRY SKIPS:
      IF(MS.EQ.MSYM) MS=0
      TOTAL=0.
      MS=MS+1
      IF(MS.NE.1) GO TO 7
      RMS=RMS*SIG
      CMS=COS(RMS)
      SMS=SIN(RMS)
      COMMENCE THE K LOOP:
      INDEX=INDEX+1
      SIGNK=-SIGNK
      CALL THE B & D COEFFICIENTS AND COMPUTE THE M,K SUMMATION TERM:
      CALL BD(M,K,G,H,SCFB,D,JJMX6)
      IF(DGN.LE.-2.) WRITE(6,89) M,K,B,D
      BRAKE=B
      IF(FRM.EQ.0.) GO TO 4
      BRAKE=B*CMS+D*SMS
      ADD=SIGNK*BRAKE*P*ARM
      TOTAL=TOTAL+ADD
      ESTABLISH CHECK AS THE RELATIVE SIZE OF THE M,K TERM OF THE SERIES:
      SUB03790
      SUB03800
      SUB03810
      SUB03820
      SUB03830
      SUB03840
      SUB03850
      SUB03860
      SUB03870
      SUB03880
      SUB03890
      SUB03900
      SUB03910
      SUB03920
      SUB03930
      SUB03940
      SUB03950
      SUB03960
      SUB03970
      SUB03980
      SUB03990
      SUB04000
      SUB04010
      SUB04020
      SUB04030
      SUB04040
      SUB04050
      SUB04060
      SUB04070
      SUB04080
      SUB04090
      SUB04100
      SUB04110
      SUB04120
      SUB04130
      SUB04140
      SUB04150
      SUB04160
      SUB04170
      SUB04180
      SUB04190
      SUB04200
      SUB04210
      SUB04220
      SUB04230
      SUB04240
      SUB04250
      SUB04260

```

```

C CHECK=ABS(ADD)
C IF (TOTAL.GT.EPS) CHECK=ABS(ADD/TOTAL)
C ADVANCE THE K INDEX:
C K=K+1
C RK=RK
C ORDER=M+2*K+1
C GENERATE THE NEXT ORDER OF LAGUERRE POLYNOMIAL FOR NEW K:
C PM=P
C P=PP
C PP=P*(ORDER-ARG)-PM*SQRT(RK*(RM+RK))
C PP=PP/SQRT((RK+1.)*(RM+RK+1.))
C GENERATE THE NEXT ORDER OF THE SET OF HERMITE POLYNOMIALS FOR NEW K:
C HA=SQRT(RK*(RK+RM))/ORDER
C HB=2.*SQRT((RK+1.)*(RM+RK+1.))/(ORDER+1.)/(ORDER+2.)
C DO 5 I=1,IMX2
C   M=IMX-I-1
C   H(I,I)=2.*((H(I,I,3)*H(I,I,2))-HA*H(I,I,1))
C   H(I,I,2)=HB*((H(I,I,3)*H(I,I,1))-ORDER*H(I,I,2))
C   SET K TIMER TO PROVIDE EXTRA K TERMS AFTER CHECK < EPS:
C   K TIMER=KTIMER+1
C   IF ((K*GE*KLIMIT) GO TO 6
C   IF ((CHECK*GE*EPS) GO TO 3
C   GO TO 7
C END OF K LOOP: ADVANCE M AND COMPUTE NEW TOTAL:
C   M=M+1
C   IF ((K.GT.KMAX) KMAX=K
C   RM=M
C   FM=FM*RM
C   REGENERATE THE HERMITE ARRAY FOR NEW M, K=0:
C   DO 8 I=1,IMX2
C     M=IMX-I-1
C     H(I,I,2)=H(I,I,4)*SQRT(RM)/(RM+1.)
C     H(I,I,1)=H(I,I,5)*(RM+2.)
C     H(I,I,1)=-SIGN*H(I,I,1)
C     H(I,I,2)=2.*SQR((RM+1.)*(H(I,I,3)*H(I,I,1)-(RM+1.)*H(I,I,2)))
C     H(I,I,2)=(RM+2.)/(RM+3.)
C     H(I,I,4)=H(I,I,1)
C     H(I,I,5)=H(I,I,2)
C   SET M TIMER FOR EXTRA M TERMS:
C   IF (TMS.EQ.1) MTIMER=M TIMER+1
C   IF ((JSYM.GT.JMAX) GOTO 9
C   IF ((K.GT.KEXTRA) MTIMER=0
C   IF ((M.GE.KLIMIT) GOTO 9
C   IF ((MTIMER.LE.MEXTRA) GO TO 2
C   END OF M LOOP: COMPUTE OUTPUT SOLN.

```

```

9      MOUT=M-1
     IF (KOUT.EQ.3) KOUT=KMAX-1
     SOLN=TOTAL*EXP(ON*APP/2)
     FORMAT ('M= ',I4,' , I4, ' ,   K= ',I4,' ,   B= ',E10.4,' ,   D= ',E10.4)
     RETURN
END
C000005

```

SUBROUTINE GARRAY (G,GA,NOF,DGN,NUMB,XO,YO,PHISYM)
 C
 C GARRAY FILLS THE DATA ARRAY OVER AN ORTHOGONAL AREA WITH
 C THE REGULAR DATA OBTAINED BY THE METHOD CORRESPONDING TO THE
 C PARTICULAR MODE:
 C
 MODE 1 - DATA OBTAINED BY SAMPLING A KNOWN FUNCTION SUPPLIED
 C IN SUBROUTINE FUNCT AND SAMPLED IN SUBROUTINE GOLF.
 MODE 2 - DATA OBTAINED BY GENERATING A REGULAR ARRAY FROM
 C IRREGULAR EXPERIMENT INPUT DATA READ IN. CALLS
 C SUBROUTINE SHEET. (EXPERIMENTAL DATA MAY
 C BE SIMULATED, SEE !SHEET.)
 MODE 3 - UTILIZES RAW DATA TAKEN AT THE PROPER INTERVAL INTO THE
 C OR PREVIOUSLY GENERATED, AND READ DIRECTLY INTO THE
 C GARRAY.
 C
 COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM
 COMMON /IO/ CMS,INI1,INI2,INI4
 DIMENSION G(IIMAX,JMAX),GA(IIMAX,JMAX)
 PIE=3.141592653589793
 HS=SIZE/2.0
 IF (IMODE.GT.3) MODE=1
 RIMX=IMAX
 RJMX=JMAX
 DELR=SIZE/RIMX
 DELXI=2.\*PIE/FCU
 IF (MODE.GT.1) GO TO 2
 DO 1 J=1,JMS
 RJ=J
 XI=(RJ-.5)\*DELEXI-PIE
 J2=J+2\*(JMS-J)
 J3=J+JMAX/2
 J4=J2+JMAX/2
 SUB04750
 SUB04760
 SUB04770
 SUB04780
 SUB04790
 SUB04800
 SUB04810
 SUB04820
 SUB00010
 SUB00020
 SUB00030
 SUB00040
 SUB00050
 SUB00060
 SUB00070
 SUB00080
 SUB00090
 SUB00100
 SUB00110
 SUB00120
 SUB00130
 SUB00140
 SUB00150
 SUB00160
 SUB00170
 SUB00180
 SUB00190
 SUB00200
 SUB00210
 SUB00220
 SUB00230
 SUB00240
 SUB00250
 SUB00260
 SUB00270
 SUB00280
 SUB00290
 SUB00300
 SUB00310
 SUB00320
 SUB00330
 SUB00340
 SUB00350
 SUB00360
 SUB00370
 SUB00380

```

DO 1 I=1,IMS
RI=I
I=IMAX+1-I
R=(RI-5)*DELR-HS
CALL GOLF (R,XI,GIJ,NOF,DGN,NUMB)
C(I,J)=GIJ
IF (ISYM.EQ.2) G(I,J)=GIJ
IF (ISYM.EQ.2) GO TO 1
G(I,J3)=GIJ
IF (ISYM.EQ.0) GO TO 1
G(I,J2)=GIJ
G(I,J4)=GIJ
CONTINUE
GO TO 4
IF (MODE.GT.2) GO TO 3
CALL SHEET (G,GA,XO,YO,PHISYM,NOF)
GO TO 4
CALL READ (Z,XO,YO,PHISYM,NOF,IMAX,JMAX,G)
IF (DGN.GE.2) WRITE (6,39)
RETURN
FORMAT (1 GARRY RETURNS)
END
C000006

SUBROUTINE GOLF (R,XI,GIJ,NOF,DGN,NUMB)
C GOLF COMPUTES THE FUNCTION G(R,XI) FOR A PARTICULAR LINE OF SIGHT
C FROM A KNOWN FUNCTION CONTAINED IN SUBROUTINE FUNCT.
C
COMMON IMAX,JMAX,IIMX,JJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z
ZERO=0.
LMAX=IMAX*3
RLMAX=LMAX
DELXP=SIZE/RLMAX
SX1=SIN(XI)
CX1=COS(XI)
DELXS=DELXP*CXI
DELYS=DELXP*SXI
XP=DELXP*.5-SIZE/2.
XS=XP*CXI-R*SXI
YS=XP*SXI+R*CXI
GIJ=0.
DO 1 L=1,LMAX
RL=_
CALL FUNCT (XS,F,NOF,DGN,NUMB)
GIJ=GIJ+F

```

```

1      XS=XS+DELXS
1      YS=Y$+DELY$          GIJ=GIJ*DELXP*BOX
1      IF (GIJ.EQ.0.)        OR*(NUMB.EQ.1.) GO TO 2
1      IF ((SD.EQ.0.)        OR*(NUMB.EQ.1.) GO TO 2
1      IF ((DGN.GE.3)        WRITE(6,28) IX
1      IF ((DGN.GE.3)        CALL GAUSS(IX,SD,ZERO,RV)
1      GIJ=GIJ+RV
1      IF ((DGN.GE.3)        WRITE (6,29) R,XI,GIJ
1      RETURN
1      FORMAT (',R=!,F8.3,',',X1=!,F8.3,',',GIJ=!,F8.3,
2      FORMAT (',GAUSS,IX=!,18)
2      END
C     C000007

```

```

C     SUBROUTINE FUNCT (XS,YS,F,NOF,DGN,NUMB)
C
C     CP67USERID 1395BOXJ
C     FUNCT EVALUATES AS INPUT FUNCTION AT POSITION (X,Y) IN THE TEST
C     SECTION COORDINATE SYSTEM. NOF IDENTIFIES THE EQUATION USED.
C
C     COMMON JMAX,IMAX,JI,MX,JJ,MX,A,PHA,SIZE,EPS,MODE,BOX,SD,IX,Z
C     COMMON /EQPARA/A,B,C,D,E,P,Q,S,T,U,V,W,RO,RA,NO,NA,N1,N2
C     DIMENSION RO(101),RA(101)
C
C     AA=A
C     BB=B
C     CC=C
C     DD=D
C     EE=E
C     PP=P
C     IF (NUMB.LE.1) GO TO 50
C     AA=S
C     BB=T
C     CC=U
C     DD=V
C     EE=W
C     PP=Q
C     PI=3.141592653589793
C     HS=SIZE/2
C     R=SQR((XS**2+YS**2)/HS)
C     F=0
C     IF (R.GT.1.) GO TO 11
C     IF (NOF.LE.0) GO TO 11
C
C     1. AXISYMMETRIC GAUSSIAN:

```

```

1      IF (NOF.GT.1) GO TO 2
2      F=AA*EXP(-1.*(R*HS/BB)**2)
      GO TO 11
C 2.  IF (NOF.GT.2) GO TO 3
F=PP
IF ((ABS(XS-DD).LE.BB).AND.(ABS(YS-EE).LE.CC)) F=AA
GO TO 11
SUB01290
SUB01300
SUB01310
SUB01320
SUB01330
SUB01340
SUB01350
SUB01360
SUB01370
SUB01380
SUB01390
SUB01400
SUB01410
SUB01420
SUB01430
SUB01440
SUB01450
SUB01460
SUB01470
SUB01480
SUB01490
SUB01500
SUB01510
SUB01520
SUB01530
SUB01540
SUB01550
SUB01560
SUB01570
SUB01580
SUB01590
SUB01600
SUB01610
SUB01620
SUB01630
SUB01640
SUB01650
SUB01660
SUB01670
SUB01680
SUB01690
SUB01700
SUB01710
SUB01720
SUB01730
SUB01740
SUB01750
SUB01760

C 3.  DISPLACABLE ELLIPTICAL GAUSSIAN:
C 3.  IF (NOF.GT.3) GO TO 4
F=AA*EXP(-1.*((XS-DD)/BB)**2+((YS-EE)/CC)**2))
GO TO 11
SUB01290
SUB01300
SUB01310
SUB01320
SUB01330
SUB01340
SUB01350
SUB01360
SUB01370
SUB01380
SUB01390
SUB01400
SUB01410
SUB01420
SUB01430
SUB01440
SUB01450
SUB01460
SUB01470
SUB01480
SUB01490
SUB01500
SUB01510
SUB01520
SUB01530
SUB01540
SUB01550
SUB01560
SUB01570
SUB01580
SUB01590
SUB01600
SUB01610
SUB01620
SUB01630
SUB01640
SUB01650
SUB01660
SUB01670
SUB01680
SUB01690
SUB01700
SUB01710
SUB01720
SUB01730
SUB01740
SUB01750
SUB01760

C 4.  CONSTANT:
C 4.  IF (NOF.GT.4) GO TO 5
F=AA
GO TO 11
SUB01290
SUB01300
SUB01310
SUB01320
SUB01330
SUB01340
SUB01350
SUB01360
SUB01370
SUB01380
SUB01390
SUB01400
SUB01410
SUB01420
SUB01430
SUB01440
SUB01450
SUB01460
SUB01470
SUB01480
SUB01490
SUB01500
SUB01510
SUB01520
SUB01530
SUB01540
SUB01550
SUB01560
SUB01570
SUB01580
SUB01590
SUB01600
SUB01610
SUB01620
SUB01630
SUB01640
SUB01650
SUB01660
SUB01670
SUB01680
SUB01690
SUB01700
SUB01710
SUB01720
SUB01730
SUB01740
SUB01750
SUB01760

C 5.  ADJUSTABLE AND DISPLACABLE ELLIPTIC RAMP FUNCTION:
C 5.  IF (NOF.GT.5) GO TO 6
RBC=SQRT(((XS-DD)/BB)**2+((YS-EE)/CC)**2)
F=0.
IF (RBC.LT.1.) F=AA*((1.-RBC)**PP)
GO TO 11
SUB01290
SUB01300
SUB01310
SUB01320
SUB01330
SUB01340
SUB01350
SUB01360
SUB01370
SUB01380
SUB01390
SUB01400
SUB01410
SUB01420
SUB01430
SUB01440
SUB01450
SUB01460
SUB01470
SUB01480
SUB01490
SUB01500
SUB01510
SUB01520
SUB01530
SUB01540
SUB01550
SUB01560
SUB01570
SUB01580
SUB01590
SUB01600
SUB01610
SUB01620
SUB01630
SUB01640
SUB01650
SUB01660
SUB01670
SUB01680
SUB01690
SUB01700
SUB01710
SUB01720
SUB01730
SUB01740
SUB01750
SUB01760

C 6.  DISPLACABLE ELLIPTIC STEP FUNCTION:
C 6.  IF (NOF.GT.6) GO TO 7
RBC=SQRT(((XS-DD)/BB)**2+((YS-EE)/CC)**2)
F=0.
IF (RBC.LT.1.) F=AA
GO TO 11
SUB01290
SUB01300
SUB01310
SUB01320
SUB01330
SUB01340
SUB01350
SUB01360
SUB01370
SUB01380
SUB01390
SUB01400
SUB01410
SUB01420
SUB01430
SUB01440
SUB01450
SUB01460
SUB01470
SUB01480
SUB01490
SUB01500
SUB01510
SUB01520
SUB01530
SUB01540
SUB01550
SUB01560
SUB01570
SUB01580
SUB01590
SUB01600
SUB01610
SUB01620
SUB01630
SUB01640
SUB01650
SUB01660
SUB01670
SUB01680
SUB01690
SUB01700
SUB01710
SUB01720
SUB01730
SUB01740
SUB01750
SUB01760

C 7.  CIRCULAR COSINE-SQUARED FUNCTION OF BB MAXIMA:
C 7.  IF (NOF.GT.7) GO TO 8
F=AA*COS((2.*BB-1.)*PIE*R/2.)***2
GO TO 11
SUB01290
SUB01300
SUB01310
SUB01320
SUB01330
SUB01340
SUB01350
SUB01360
SUB01370
SUB01380
SUB01390
SUB01400
SUB01410
SUB01420
SUB01430
SUB01440
SUB01450
SUB01460
SUB01470
SUB01480
SUB01490
SUB01500
SUB01510
SUB01520
SUB01530
SUB01540
SUB01550
SUB01560
SUB01570
SUB01580
SUB01590
SUB01600
SUB01610
SUB01620
SUB01630
SUB01640
SUB01650
SUB01660
SUB01670
SUB01680
SUB01690
SUB01700
SUB01710
SUB01720
SUB01730
SUB01740
SUB01750
SUB01760

C 8.  NUMERICAL FUNCTION:  REQUIRES AN INPUT ARRAY READ IN BY
C 8.  SUBROUTINE FREAD; N FOLLOWED BY N POINT VALUES. (101 MAX)
A CONSTANT VALUE AA IS ADDED TO THE FUNCTION.
IF (NOF.GT.8) GO TO 9
IF (NUMB.LE.1) N=NO
IF (NUMB.GT.1) N=NA
NM=N-1
NM=N-2
RN=N

```

```

RI=R*(RN-1)+1.
IR=INT(RI)
RIR=FLOAT(IR)
DI=R-I-IR
IF ((NUMB.LE.1) F=RO(IR)
    IF ((NUMB.GT.1) F=RA(IR)
    IF ((IR.NE.N).AND.(NUMB.LE.1)) F=F+DI*(RO(IR+1)-RO(IR))
    IF ((IR.NE.N).AND.(NUMB.GT.1)) F=F+DI*(RA(IR+1)-RA(IR))
F=F*AA+BB
GO TO 11

C 9. SPECIAL FUNCTION: MAY BE WRITTEN FOR THE OCCASION AND
C INSERTED IN SUBROUTINE SPFUN
9   IF (NOF.GT.9) GO TO 10
CALL SPFUN (XS,YS,F)
GO TO 11

C EQUATIONS NO. 10 AND BEYOND ARE SET TO ZERO.
10  F=0.

C 11 IF (DGN.GE.4) WRITE (6,99) XS,YS,F
      XS=*,F8.3,*; YS=*,F8.3,*; F=*,F8.3)
      FORMAT (1X,99)
      RETURN
      END
      C000008

C SUBROUTINE SPFUN (XS,YS,F)
C SPFUN IS A SPECIAL ROUTINE FOR EQN NO. 9. ANY FUNCTION MAY BE
C ENTERED.
C COMMON /EQPARA/A,B,C,D,E,P,Q,S,T,U,V,W,RO,RA,NO,NA,N1,N2
C DIMENSION RO(101),RA(101)
C F=0
C IF ((ABS(XS).LE.B).AND.(ABS(YS).LE.C)) F=A
C RETURN
C END
      C000009

```

SUBROUTINE SHEET ( G,D,XO,YO,PHISYM,NOF )

```

SUB02170
SUB02180
SUB02190
SUB02200
SUB02210
SUB02220
SUB02230
SUB02240
SUB02250
SUB02260
SUB02270
SUB02280
SUB02290
SUB02300
SUB02310
SUB02320
SUB02330
SUB02340
SUB02350
SUB02360
SUB02370
SUB02380
SUB02390
SUB02400
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SUB02490
SUB02500
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SUB02550
SUB02560
SUB02570
SUB02580
SUB02590
SUB02600
SUB02610
SUB02620
SUB02630
SUB02640

SHEET READS IRREGULARLY SPACED VALUES OF THE LINE INTEGRAL, AS
OBTAINED FROM HOLOGRAPHIC INTERFEROGRAMS. THE INTEGRAL LINES MAY BE
DEFINED EITHER BY GRID INTERCEPT POSITIONS, OR BY ANGLE AND RADIIES
ABOUT THE CENTER OF THE LABORATORY COORDINATE SYSTEM. CENTER• LINES
MUST BE ENTERED IN CONSECUTIVE ORDER FROM LOWEST (NEG.) TO HIGHEST
(POS.) RADII. DATA MAY BE SIMULATED BY SPECIFYING NCODE=1.
FOLLOWED BY APERATURE POSITIONS FOR A FUNCTION NUMBER IN •SUBFUNCT•.

COMMON /SYM,JSYM,MSYM,FCU,IMS,JMS,QUA
COMMON /IO,CMS,INI,IN4
DIMENSION G(IMAX,JMAX),D(IMAX,JMAX),XI(303),RR(303)
DIMENSION XG(303),XD(303),YG(303),YD(303),XY(303)
NAR=303
PIE=3.141592653589793
MPIE=-PIE
TPIE=2.*PIE
MPIET=PIE/2.
PIET=PIE/2.

C      ZERO THE ARRAYS:
DO 1 J=1,IMAX
DO 1 I=1,JMAX
G(I,J)=0.
D(I,J)=0.
DO 2 I=1,NAR
XG(I)=0.
XD(I)=0.
YG(I)=0.
YD(I)=0.
XY(I)=0.
RR(I)=0.
1      RIMX=IMAX
DR=SIZE/RIMX
R=(-DR-SIZE)/2.*Y0**2
RZO=SQR((X0**2+Y0**2))
GAM=ATANM(Y0,X0)
TP=3-1SYM
BT=JSYM
DAN=PIE*TP/BT
HS=SIZE/2.

C      READ THE BASIC DATA:
IF(CMS.EQ.1)REWIND 1
READ(INI,59)NOFNCCDE
READ(INI,58)ZMX,YO,PHISYM,XMX,XMN,YMX,YMN
READ(INI,59)JM
RIMX=IMAX
DR=SIZE/RIMX
R=(-DR-SIZE)/2.*Y0**2
RZO=SQR((X0**2+Y0**2))
GAM=ATANM(Y0,X0)
TP=3-1SYM
BT=JSYM
DAN=PIE*TP/BT
HS=SIZE/2.

C      1
2

```

```

MXY=1
IF ((XMX•NE•J.)•OR•(XMN•NE•0..).OR•(YMX•NE•J.).OR•(YMN•NE•0..)) MXY=0
IF ((MXY•EQ•1;) GO TO 3
XMX=XO+HS
YMX=YD+HS
XMN=XO-HS
YMN=YD-HS
C COMMENCE THE READ AND FILL LOOP:
DO 12 J=1, JM
READ (INI,59) IM
MN=0
XH=0.
YH=0.
READ THE LINES, DETERMINE CODE, CALCULATE RADIUS & ANGLE FOR CODE 1:
DO 5 I=1, IM
IF (NCODE•LE•0) READ (INI,58) XD(I), YD(I), XG(I), YG(I), RR(I),
1 XY(I), GE(I)
IF (NCODE•GE•1) CALL SIM(XD(I), YD(I), XG(I), YG(I), RR(I), XI(I),
1 XY(I), XO, YO, PHI, SYM, XMX, XMN, YMX, YMN, NOF, I, IM)
IF (XY(I)•EQ•3) GO TO 5
IF ((XD(I)•NE•J.)•OR•(YD(I)•NE•J.)) XY(I)=1.
IF ((XG(I)•NE•0.)•OR•(YG(I)•NE•0.)) XY(I)=1.
IF ((RR(I)•NE•0.)•OR•(XI(I)•NE•0.)) AND (XY(I)•EQ•0..) XY(I)=2.
IF ((XY(I)•EQ•0.)•AND•(D(I,J)•NE•J.)) XY(I)=2.
IF ((XY(I)•NE•1.) GO TO 4
DEN=SQRT((XG(I)•NE•1.)-XD(I)*2+(YG(I)-YD(I))*2)
IF (DEN•EQ•0.) XY(I)=4.
IF ((XY(I)•EQ•4.) GO TO 4
RR(I)=((XO-XD(I))*(YD(I)-YD(I))-(XG(I)-XD(I)))/DEN
XI(I)=ATANM((YG(I)-YD(I)),(XG(I)-XD(I)))
XIM=XI(I)
IF ((XY(I)•EQ•2.) GO TO 4
XIN=XIM
IF (XY(I)•EQ•2..) RR(I)=RR(I)+RZ0*SIN(GAM-XI(I))
CONTINUE
5 COMPUTE MAX AND MIN ANGLE INDEXES FOR APERTURE POSITION LOCATION:
DO 6 I=1, IM
IF ((XY(I)•NE•1.)•OR•(XY(I)•NE•2..)) GO TO 6
IF ((XI(I)•GT•IM) XIM=XI(I)
IF ((XI(I)•LT•XIN) XIN=XI(I)
IF ((XI(I)•LE•XIN) INT=1
CONTINUE
6 DETERMINE APERTURE LOCATION:
LPR=0
XID=XI(IMT)-XI(INT)
IF (ABS(XID) $\cdot$ LTI $\cdot$ XI(INT))/2.
XIH=(XI(IMT)+XI(INT))/2.

```

```

RRH=10000.0*S(XI-1)
XH=RRH*SIN(XIH)
IF (LPR.EQ.1) GO TO 7
YTX=-RR(INT)*SIN(XI(INT))-YO
XTN=RR(INT)*COS(XI(INT))-XO
XTN=RR(INT)*COS(XI(INT))-XO
UA=TAN(XI(INT))
UC=YT(X-XA*XTX)
UB=YTN-UA*XTN
UD=(UD-UB)/(UA-UC)
XH=XH*UA+UB
RRH=SQRT((XH-XO)**2+(YH-YO)**2)
XIH=ATAN((YH-YO),(XH-XO))
CONTINUE
FILL THE ANGLE AND RADIUS FOR ANY CODE 3 OR 4 LINES:
DO 9 I=1,IM
IF (XY(I).NE.3.) GO TO 8
BAS=SQR((RRH**2-RR(I)**2)
XI(I)=XIH-ATAN(RR(I),BAS)
GO TO 9
XI(I)=ATAN((YH-YD(I)),(XH-XD(I)))
RR(I)=RRH*SIN(XI(I)-XIH)
CONTINUE
ANGLES AND RADII ARE NOW FILLED FOR ALL POINTS IN THIS LINE.
VACATE THE SET OF VECTORS TO BE USED AS TEMPORARY STORAGE:
DO 10 I=1,IM
XD(I)=0.
YD(I)=0.
XG(I)=0.
XY(I)=DR(I)
RR(I)=0.
DI(I,J)=0.
XI(I,J)=0.
CONVERT THE LINE TO REGULAR RADII USING INTERPOLATION:
RR(1)=R+DR
CALL SPLINE(YG,XY,IM,RR(1),D(1,J))
DO 11 I=2,IMAX
RI=I
RR(I)=R+DR*RI
CALL SPLINE(YG,XY,IM,RR(I),D(I,J))
GENERATE THE VECTOR OF ANGLES FOR THIS COLUMN AND STORE IN G ARRAY:
DO 12 I=1,IMAX
BAS=SQR((RRH**2-RR(I)**2)
G(I,J)=XIH-ATAN(RR(I),BAS)

```

```

12 C YG(I)=0. XY(I)
      D(I,J)=0
      XY(I)=0
      COLUMNS ARE NOW ALL REGULARLY FILLED OVER THE ANGLES.
      NEXT, INTERPOLATE EACH ROW REGULARLY OVER THE ANGLES.
      DO 23 I=1,IMAX
      EXPAND THE DATA TO 2 SETS TO ESTABLISH SMOOTH INTERPOLATION.
      JMJ3=3*JMJ
      I1=I MAX+1-I
      IF (I1,J=1,JMS) NEQ 0 GO TO 14
      DO 13 J=1,JMS
      J2=J+JMS
      J3=J2+JMS
      XD(J2)=D(I,J)
      XD(J3)=D(I,J)
      XD(J2)=G(I,J)-TPIE-PHISYM
      XD(J3)=D(I,J)
      XD(J2)=G(I,J)-TPIE-PHISYM
      XD(J3)=G(I,J)-TPIE-PHISYM
      XD(J2)=G(I,J)-TPIE-PHISYM
      XD(J3)=G(I,J)-TPIE-PHISYM
      DO 15 J=1,JMS
      J1=JMS+1-J
      J2=JMS+J-J
      J3=JM3+1-J
      XD(J1)=D(I,J)
      XD(J2)=D(I,J)
      XD(J3)=D(I,J)
      XD(J1)=G(I,J)-2.**(G(I,J)-PHISYM)-PIE-PHISYM
      XD(J2)=G(I,J)-PIE-PHISYM
      XD(J3)=G(I,J)+2.**(DAN+PHISYM-G(I,J))-PIE-PHISYM
      CONTINUE
      JM2=2.*JMS
      JP=JMS/2
      DO 17 J=1,JM2
      XD(J)=XD(J+JP)
      XG(J)=XG(J+JP)
      JJS=JM2+1
      DO 18 J=JJS,JM3
      XD(J)=0.
      XG(J)=0.
      FIND THE SMALLEST ANGLE
      XY(1)=1.
      SA=XG(1)
      DO 19 J=1,JM2
      IF (XG(J).GE.SA) GO TO 19
      SA=XG(J)
      XY(1)=J
      CONTINUE

```

```

C FIND THE MAX ANGLE IN THE ROW:
XY(JM2)=JM2
SB=XG(JM2)
DO 20 J=1,JM2
IF (XG(J).LE.SB) GO TO 20
SB=XG(J)
XY(JM2)=J
CONTINUE
20 DETERMINE THE ORDER OF INCREASING ANGLE IN THE ROW
SB=XG(JM2)
JJ=2
JSA=XY(JJ-1)
SA=XG(JJ-1)
JTS=0
DO 22 J=1,JM2
IF (XG(J)*LE.SA) GO TO 22
IF (XG(J)*GT.SB) GO TO 22
SB=XG(J)
XY(JJ)=J
JTS=1
CONTINUE
22 IF (JTS.EQ.0) JM2=JJ
JJ=JJ+1
IF (JJ.LE.JM2) GO TO 21
DO 23 J=1,JM2
JX=XY(J)
YD(J)=XD(JX)
INTERPOLATE
DXI=2.*PIE/FCU
XI(J)=DXI/2.-PIE-PHISYM
CALL SPLINE (XG,YD,JM2,XI(J),G(I,J))
DO 24 J=2,JMS
XI(I)=XI(J-1)+DXI
CALL SPLINN (XG,YD,JM2,XI(J),G(I,J))
DO 25 J=1,JMS
XI(J)=XI(J)
XY=XMX
IF ((XI.J.GE.0.) .AND. (XI.J.LT.PIE)) XU=XMN
YU=YMN
IF (((XI.J.GE.0.) .AND. (XI.J.LT.PIT)) YU=YMX
XL=XMN
IF (((XI.J.GE.0.) .AND. (XI.J.LT.PIE)) XL=XMX
YL=YMX
IF (((XI.J.GE.MPIT) .AND. (XI.J.LT.PIT)) YL=YMN
SXI=SIGN(XIJ)
CXIJ=COS(XIJ)
RMN=(XC-XL)*SXI-(YO-YL)*(Y0-YU)*CXIJ
RMX=(XO-XU)*SXI-(Y0-YU)*CXIJ
23
24

```

```

DO 25 I=1,IMAX
IF (RR(I).LT.RMN) G(I,J)=0.
IF (RR(I).GT.RMX) G(I,J)=0.
CONTINUE
C EXPAND SYMMETRY SECTOR INTO AN ORTHOGONAL INTERVAL.
IF (ISYM.EQ.2) GO TO 27
DO 26 J=1,JMS
J2 = JMAX/2+1-J
J3 = JMAX/2+J
J4 = JMAX+1-J
DO 26 I=1,IMAX
I1 = IMAX+1-I
G(I1,J2)=G(I,J)
G(I1,J3)=G(I,J)
G(I1,J4)=G(I,J)
RETURN
C FOR EVEN SYMMETRY, AVERAGE THE GARRAY COLUMNS.
27  IMS=(2*IMAX+1)/2
    DO 28 J=1,JMAX
    DO 28 I=1,IMS
I1 = IMAX+1-I
GST=(G(I,J)+G(I1,J))/2.
G(I1,J)=GST
G(I,J)=GST
RETURN
FORMAT (515)
FORMAT (10F7.3)
END
C 003010
C
C FUNCTION ATANM(Y,X)
C COMPUTES THE ARCTAN OF Y/X BETWEEN -P1 AND +P1.
C
PIE=3.141592653589793
P12=PIE/2.
ATANM=SIGN(P12,Y)
IF (X.NE.0.) ATANM=ATAN(Y/X)
IF (X.GE.0.) RETURN
IF (Y.GE.0.) ATANM=PIE+ATANM
IF (Y.LT.0.) ATANM=-PIE+ATANM
RETURN
END
C 000001
C
SUB04570
SUB04580
SUB04590
SUB04600
SUB04610
SUB04620
SUB04630
SUB04640
SUB04650
SUB04660
SUB04670
SUB04680
SUB04690
SUB04700
SUB04710
SUB04720
SUB04730
SUB04740
SUB04750
SUB04760
SUB04770
SUB04780
SUB04790
SUB04800
SUB04810
SUB04820
SUB04830
SUB04840
SUB04850
SUB04860
SUB04870
SUB04880
SUB04890
SUB04900
SUB04910
SUB04920
SUB04930
SUB04940
SUB04950
SUB04960
SUB04970
SUB04980
SUB04990
SUB05000
SUB05010

```

```

SUBROUTINE SIM (XD,YD,XG,YG,D,R,XI,XY,XO,YO,PS,XM,XN,YM,YN,I,IM,
1DG,NF)
C SIMULATES THE FRINGE NUMBER DATA ONE WOULD OBTAIN FROM THE
C HOLOGRAPHIC INTERFEROMETER PROCESS FOR A KNOWN FUNCTION AS
C CONTAINED IN SUBROUTINE FUNCT. THE GRID BOX DIMENSIONS MUST
C EXCEED THE INVERSION CIRCLE SIZE, AND APERATURE POINTS SPECIFIED
C MUST FALL BETWEEN XI=-40 DEGREES, AND XI=+130 DEGREES.
C
COMMON IMAX,JMAX,II,MX,JJMX,I,JM,ALF,SIZ,EPS,MOD,BOX,SD,IX,Z
COMMON /10/,CMSS,IN1,IN2,IN4
READ (1,N1,29) XH,YH
ZERO=0
RIM=IM
RI=1
DX=(XM-XN)/(RIM-1.)
DY=(YM-YN)/(RIM-1.)
XI=XM-(RIM-1.)*DX
YI=YM-(RIM-1.)*DY
XI1=ATANM((YH-Y1)*(XH-X1))-PS
RRH=SQRT(((XH-XO)**2+(YH-YO)**2))
XIO=ATANM((YH-YO)*(XH-XO))-PS
RI=RRH*SIN(XI1-XIO)
IF ((I.GT.1).AND.(I.LT.I)) GO TO 1
IF (ABS(RI).LT.SIZ/2.) RI=SIGN(SIZ/2.,RI)
XY=3.
GO TO 2
CALL GOLF (R,XI,D,NR,ZER,ZER)
XD=XN
YD=YN
IF (XH.NE.X1) YD=Y1-(X1-XN)*(YH-Y1)/(XH-X1)
IF (YS.GE.YN) GO TO 2
YD=YN
IF (YH.NE.Y1) XD=X1-(Y1-YN)*(XH-X1)/(YH-Y1)
RETURN (10F7.3)
END
29 FORMAT (10F7.3)
C000012
C
SUBROUTINE FREAD (NO,RO,NF,ZZ)
C READ READS THE NUMERIC ARRAY WHICH IS USED FOR EQUATION 8 OF
C SUBROUTINE FUNCT. FIRST CARD IS NUMBER OF POINTS (N.GE.1),
C FOLLOWED BY ONE POINT PER CARD.
C
DIMENSION RO(101)
SUB05020
SUB05030
SUB05040
SUB05050
SUB05060
SUB05070
SUB05080
SUB05090
SUB05100
SUB05110
SUB05120
SUB05130
SUB05140
SUB05150
SUB05160
SUB05170
SUB05180
SUB05190
SUB05200
SUB05210
SUB05220
SUB05230
SUB05240
SUB05250
SUB05260
SUB05270
SUB05280
SUB05290
SUB05300
SUB05310
SUB05320
SUB05330
SUB05340
SUB05350
SUB05360
SUB05370
SUB05380
SUB00010
SUB00020
SUB00030
SUB00040
SUB00050
SUB00060
SUB00070
SUB00080
SUB00090

```

```

READ (NF,89) NO,ZZ
WRITE(6,90) NO,ZZ
DO 10 I=1,NO
READ(NF,88) RO(I)
WRITE(6,88) RO(I)
CONTINUE
FORMAT(15,F9.3)
88 FORMAT(F8.5)
90 FORMAT(1X,15,F9.3)
RETURN
END
C000013

```

```

SUB001100
SUB001180
SUB001190
SUB00200
SUB00210
SUB00220
SUB00230
SUB00240
SUB00250
SUB00260
SUB00270
SUB00280
SUB00290
SUB00300
SUB00310
SUB00320
SUB00330
SUB00340
SUB00350
SUB00360
SUB00370
SUB00380
SUB00390
SUB00400
SUB00410
SUB00420
SUB00430
SUB00440
SUB00450
SUB00460
SUB00470
SUB00480
SUB00490
SUB00500

SUB00120
SUB00140
SUB00150
SUB00160
SUB00170

SUBROUTINE GPRINT (G,NUMB)
C   GPRINT PRINTS THE DATA ARRAY "G" WHICH WAS INPUT TO
C   THE PROGRAM IN SUBROUTINE GARRAY.
C
COMMON JMAX,JMAXX,IIMX,JJMX,IJMx,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z
DIMENSION G(IJMx)
DIMENSION X(15)
DATA HYP,VERT/1H-1H/
IF (NUMB.EQ.1) WRITE(6,99) MODE,Z
IF (NUMB.EQ.2) WRITE(6,92) Z
JMAX2=JMAX/2
RIMAX=X=JMAX
RJMAX=JMAX
DX=SIZE/RIMAX
DXI=360/RJMAX
INTRVL SETS THE NUMBER OF TERMS PRINTED PER LINE. IF IT IS ALTERED,
ONE MUST ALSO REDIMENSION X AND ALTER FORMATS 98, 97, AND 95.
INTRVL=15
IB=1
IT=IB+INTRVL-1
IF (IT.GT.IMAX) IT=IMAX
IBT=IT-IB+1
WRITE(6,98) (II,II=IB,IT)
DO 2 I=1,IBT
RI=IB-1+I
X(I)=-SIZE/2.+ (RI-.5)*DX
LMI=7*IBT+1
WRITE(6,97) (X(I),I=1,IBT)
WRITE(6,96) (HYP,L=1,LM),VERT
JMH=JM2+1
DO 3 J=JMH,JMAX
RJ=J

```

```

X1=-180.0+DX1*(RJ-.5)
IGB=(J-1)*IMAX+IB
IGT=IGB-IB+IT
WRITE(6,94) J'X,I,(G(L),L=1,LM),VERT,IGT
IB=IB+INTRVL
ITOLD=IT
IT=IT+INTRVL
IF (ITOLD.LT.IMAX) GO TO 1
WRITE(6,93) THE ARRAY OF INPUT DATA (G), OBTAINED BY GARRAY,
FORMAT(IH1//,1 MODE,1 FOR Z=F7.3,CM: )*
FORMAT(1//11X,1=1517)
FORMAT(11X,X=1,15F7.3)
FORMAT(J=2X,13,F9.2,115A1)
FORMAT(14X,1,F9.2,115A1)
FORMAT(//5X,THE ADD-ON FUNCTION GARRAY FOR Z=1,F7.3,1 CM:1)
RETURN
END
C000014

 99
 1 FORMAT(IH1//,1 MODE,1 FOR Z=F7.3,CM: )*
 98
 97
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 95
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SUBROUTINE GPUNCH (Z,XO,YO,PHS,NOF,IMX,JMX,G)
C GPUNCH PUNCHES OUT THE FIRST NON-SYMMETRIC PORTION OF GARRAY
C (OR WRITES IT ON FILE 7 IN CMS VERSION)
COMMON /SYM/ ISM,JSM,MSM,FCU,IMS,JMS,QSM
COMMON /DIMNS/ IMX,JMX
DIMENS(1,9)=NOF,IMX,JMX,ISM,IMS,JMS,JMS
WRITE(7,39) ((G(I,J),I=1,IMS),J=1,JMS)
39 FORMAT(10I5)
FORMAT(10F7.3)
RETURN
END
C000015

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 16
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 14
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 12
 11
 10
 9
 8
 7
 6
 5
 4
 3
 2
 1
 0

SUBROUTINE READ (Z,XO,YO,PHISYM,NOF,IMAX,JMAX,G)
C READS THE NON-SYMMETRIC PORTION OF THE GARRAY AND EXPANDS IT TO AN
C ORTHOGONAL SET. NOTE, INSURE SUFFICIENT DIMENSIONS IN MAIN PROGRAM.
COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM

```

```

COMMON /IO/ CMS,IN1,IN2,IN4
DIMENSION G(1,39),NOF,IMAX,JMAX,ISYM,JSYM,IMS,JMS
READ (IN1,38) Z,XO,YO,PHISYM
READ (IN1,38) (G(I,J),I=1,IMS),J=1,JMAX
READ (IN1,37) NOF,Z,XO,YO,PHISYM,IMAX,JMAX,JSYM
WRITE(6,37)
RJMX=X=JMAX
MSYM=JSYM
IF ((MSYM.EQ.0).OR. (MSYM.GT.JMAX)) MSYM=1
FCU=ISYM*JMAX
IF (JSYM.GT.JMAX) FCU=JMAX
QSYM=FCU/RJMX
DO 4 J=1, JMS
  IF (ISYM.EQ.1) GO TO 2
  DO 1 I=1,IMS
    I=IMAX+1-I
    G(I,J)=G(I,J)
    GO TO 4
  1   2
    J2=JMAX/2+1-J
    J3=JMAX/2+J
    J4=JMAX+1-J
    DO 3 I=1,IMAX
      I=IMAX+1-I
      G(I,J2)=G(I,J)
      G(I,J3)=G(I,J)
      G(I,J4)=G(I,J)
      CONTINUE
      FORMAT(10I5)
      FORMAT(10F7.3)
      FORMAT(1/, MODE, 3 READS GARRAY DIRECTLY: NOF=14,*
     1      X0=, F7.3;, Y0=, F7.3;, PHISYM=, F7.3;,
     2      JMAX=, I4//, JSYM=, I4//,)
      RETURN
    END
C000016
C
C SUBROUTINE MAP ( IM, JM, A, N, Z, BAND )
C MAP CALLS SUBROUTINE MIMPII AND PLOTS A CONTOUR MAP OF THE ARRAY
C
C DIMENSION A(IM,JM),T(24)
C DATA BL/1H/
C DO 1 I=1,24
C   T(I)=BL
C 1   ICON=1
C IF(BAND.LT.0.) ICON=0
SUB03950
SUB00960
SUB00970
SUB00980
SUB00990
SUB01033
SUB01040
SUB01050
SUB01060
SUB01070
SUB01090
SUB01100
SUB01110
SUB01120
SUB01130
SUB01140
SUB01150
SUB01160
SUB01170
SUB01180
SUB01190
SUB01200
SUB01210
SUB01220
SUB01230
SUB01240
SUB01250
SUB01260
SUB01270
SUB01280
SUB01290
SUB01300
SUB01310
SUB01320
SUB01330
SUB01340
SUB01350
SUB01360
SUB01370
SUB01380
SUB01390
SUB01400

```

```

IF(BAND.LT.0.) BAND=-BAND
IMIN=0.
IJT=0
AZ=1.
BZ=0.
WRITE(6,49) N,Z
CALL MTMPI(A,IM,JM,T,BAND,AZ,BZ,AMIN,IJT,ICON)
FORMAT (1H1,/,*THE FUNCTION SURFACE, TEST NO.*I3,* Z='F5.3//')
RETURN
END
C00017
C

```

```

SUBROUTINE GPOINT (G,GA,JMS)
C
C PLOTS A ROUGH PLOT OF THE LINE INTEGRAL FUNCTIONS IN GARRAY.
COMMON IMAX, JMAX, IJMX, JJMX, ALPHA, SIZE, EPS, MODE, BOX, SD, IX, Z
COMMON /TAB/ INDEX(7), JSYM, ISYM
COMMON /TAB2/ IPT, KPT, LPT, MPT, REST(5)
DIMENSION G(IMAX, JMAX), GA(IMAX, JMAX), ROW(101)
DIMENSION A(201), B(101), C(201), D(101)
JW=101
DATA BL,PL,ST,DH,EX/1H ,1H*,1H-,1H-/,
JMS2=JMAX/2+1
IF(ISYM.EQ.2) JMS2=1
JMS3=JMS2+JMS-1
DO 8 J=JMS2, JMS3
WRITE(6,67) (ST,I=1,120)
DO 1 I=1,IMAX
A(I)=G(I,J)
C(I)=GA(I,J)
AS=.5
BS=0
CALL INTERP (A,IMAX,AS,B,JM,BS)
CALL INTERP (C,IMAX,AS,D,JM,BS)
WRITE(6,69) J
BIG=0.
SMALL=0.
DO 2 I=1,IMAX
IF(A(I).GT.BIG) BIG=A(I)
IF(C(I).GT.BIG) BIG=C(I)
IF(A(I).LT.SMALL) SMALL=A(I)
IF(C(I).LT.SMALL) SMALL=C(I)
RANGE=BIG-SMALL
RINK=RANGE/80.
TOP=BIG+RINK
SUB01410
SUB01420
SUB01430
SUB01440
SUB01450
SUB01460
SUB01470
SUB01480
SUB01490
SUB01500
SUB01510
SUB01520
SUB01530
SUB01540
SUB01550
SUB01560
SUB01570
SUB01580
SUB01590
SUB01600
SUB01610
SUB01620
SUB01630
SUB01640
SUB01650
SUB01660
SUB01670
SUB01680
SUB01690
SUB01700
SUB01710
SUB01720
SUB01730
SUB01740
SUB01750
SUB01760
SUB01770
SUB01780
SUB01790
SUB01800
SUB01810
SUB01820
SUB01830
SUB01840
SUB01850
SUB01860
1
2

```



```

RAT=(RIM-1.*JM)/(RJM-1.*BS)
DO 2 I=1,JM
BI=I
AI=RAT*(BI+BS)-AS
IA=AI
F=AI-FLOAT(IA)
IF((IA*EQ.0).OR.(IA-EQ.JM)) GO TO 1
B(I)=A(IA)+F*(A(IA+i)-A(IA))
GO TO 2
IF((IA*EQ.0) B(I)=A(1)*(F-AS)/(1.-AS)
IF((IA*EQ.JM) B(I)=A(JM)*F/(1.-AS)
CONTINUE
RETURN
END

```

C000019

SUB02330

SUB02340

SUB02350

SUB02360

SUB02370

SUB02380

SUB02390

SUB02430

SUB02410

SUB02420

SUB02430

SUB02440

SUB02450

SUB02460

SUB02470

SPL00010

SPL00020

SPL00030

SPL00040

SPL00050

SPL00060

SPL00070

SPL00080

SPL00090

SPL00100

SPL00110

SPL00120

SPL00130

SPL00140

SPL00150

SPL00160

SPL00170

SPL00180

SPL00190

SPL00200

SPL00210

SPL00220

SPL00230

SPL00240  
SPL00250  
SPL00260  
SPL00270  
SPL00280  
SPL00290  
SPL00300  
SPL00310  
SPL00320  
SPL00330

SUBROUTINE SPLINE  
PURPOSE PROVIDES INTERPOLATED VALUE USING "CUBIC SPLINE FITTING"  
USAGE FIRST CALL TO SUBROUTINE:  
CALL SPLINE(X,Y,M,XINT,YINT)

SUBSEQUENT CALLS:  
CALL SPLINN(X,Y,M,XINT,YINT)

DESCRIPTION OF PARAMETERS  
X: MONOTONICALLY INCREASING ABSCISSA ARRAY  
Y: ONE-FOR-ONE CORRESPONDING ORDINATE ARRAY  
M: NUMBER OF X AND Y VALUES SUPPLIED < OR = 300  
XINT: VALUE OF ABSCISSA FOR WHICH CORRESPONDING ORDINATE  
IS TO BE INTERPOLATED (OR EXTRAPOLATED)  
YINT: INTERPOLATED (OR EXTRAPOLATED) ORDINATE VALUE

REMARKS  
IF SPECIFIED X FALLS OUTSIDE OF RANGE, AN EXTRAPOLATED  
VALUE WILL BE SUPPLIED  
SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED  
SUBROUTINE SPLIN IS INCLUDED IN SPLIN PACKAGE  
MATHEMATICAL METHOD  
UPON FIRST ENTRY TO SPLIN, A CALL TO SPLIN IS MADE TO

DETERMINE THE COEFFICIENTS TO BE USED IN PERFORMING THE  
INTERPOLATIONS. SEARCH FOR BRACKETING ABSCESSA VALUES IS  
ALWAYS MADE FROM THE REFERENCE LAST USED IN INTERPOLATING.

REFERENCE  
PENNINGTON, RALPH H., "INTRODUCTORY COMPUTER METHODS AND  
NUMERICAL ANALYSIS", THE MACMILLAN COMPANY, NEW YORK, 1965

SPL 00430

SUBROUTINE SPLINE(X,Y,M,XINT,YINT)  
DIMENSION X(M),Y(M),C(4,300)  
CALL SPLICO(X,Y,M,C)

K=1  
IF(XINT-X(1))>0,1,2  
K=1

GO TO 7  
1 YINT=Y(1)  
RETURN  
2 IF(XINT-X(K+1))>0,4,5  
YINT=Y(K+1)  
RETURN

4 K=K+1  
IF(M-K) 71,71,3  
K=M-1  
GO TO 7  
6 IF(XINT-X(K))13,12,11  
YINT=Y(K)  
RETURN

12 K=K-1  
GO TO 6  
13 PRINT 101,XINT  
FORMAT(8H0XINT=E18.9,32H,OUT OF RANGE FOR INTERPOLATION)  
11 YINT=(X(K+1)-X(K))\*(C(1,K)\*(X(K+1)-X(K))\*C(2,K)\*C(3,K))\*\*2+C(4,K))  
RETURN  
END

SPL 00440  
SPL 00460  
SPL 00470  
SPL 00480  
SPL 00490  
SPL 00500  
SPL 00510  
SPL 00520  
SPL 00530  
SPL 00540  
SPL 00550  
SPL 00560  
SPL 00570  
SPL 00580  
SPL 00590  
SPL 00600  
SPL 00610  
SPL 00620  
SPL 00630  
SPL 00640  
SPL 00650  
SPL 00660  
SPL 00670  
SPL 00680  
SPL 00690  
SPL 00700  
SPL 00710  
SPL 00720

SPL 00730  
SPL 00750  
SPL 00760  
SPL 00770  
SPL 00780  
SPL 00790

SUBROUTINE SPLICO(X,Y,M,C)  
DIMENSION X(M),Y(M),C(4,300),D(300),P(300),E(300),A(300,3),B(300),  
1Z(300)  
MM=M-1  
DO 2 K=1,MM  
D(K)=X(K+1)-X(K)

```

SPL 00800
SPL 00810
SPL 00820
SPL 00830
SPL 00840
SPL 00850
SPL 00860
SPL 00870
SPL 00880
SPL 00890
SPL 00900
SPL 00910
SPL 00920
SPL 00930
SPL 00940
SPL 00950
SPL 00960
SPL 00970
SPL 00980
SPL 00990
SPL 01000
SPL 01010
SPL 01020
SPL 01030
SPL 01040
SPL 01050
SPL 01060
SPL 01070
SPL 01080
SPL 01090
SPL 01100
SPL 01110
SPL 01120

P(K)=D(K)/6
E(K)=(Y(K+1)-Y(K))/D(K)
DO 3 K=2,MN
B(K)=E(K)-E(K-1)
A(1,2)=-1.*D(1)/D(2)
A(1,3)=D(1)/D(2)
A(2,3)=P(1)*A(1,3)
A(2,2)=2.**(P(1)+P(2))-P(1)*A(1,2)
A(2,3)=A(2,3)/A(2,2)
B(2)=B(2)/A(2,2)
DO 4 K=3,MN
A(K,2)=2.**(P(K-1)+P(K))-P(K-1)*A(K-1,3)
B(K)=B(K)-P(K-1)*B(K-1)
A(K,3)=P(K)/A(K,2)
B(K)=B(K)/A(K,2)
Q=D(M-2)/D(M-1)
A(M,1)=1.*Q+A(M-2,3)
A(M,2)=-Q-A(M,1)*A(M-1,3)
B(M)=B(M-2)-A(M,1)*B(M-1)
Z(M)=B(M)/A(M,2)
MN=M-
DO 6 I=1,MN
K=M-I
Z(K)=B(K)-A(K,3)*Z(K+1)
Z(1)=-A(1,2)*Z(2)-A(1,3)*Z(3)
DO 7 K=1,MN
Q=1./((6.*D(K))
C(1,K)=Z(K)*Q
C(2,K)=Z(K+1)*Q
C(3,K)=Y(K)/D(K)-Z(K)*P(K)
C(4,K)=Y(K+1)/D(K)-Z(K+1)*P(K)
RETURN
END

```

## DESCRIPTION OF PARAMETERS

**REMARKS**

MTMPII REQUIRES A PRINTER WITH 132 PRINT POSITIONS IF NECESSARY. THE MAP WILL BE SEGMENTED COLUMNWISE. S THE ROWS AND COLUMNS ARE NUMBERED ALONG THE EDGES. S THAT A SEGMENTED MAP MAYBE EASILY JOINED TOGETHER. S ONLY THREE SIGNIFICANT FIGURES WILL BE PRINTED AT EACH POINT. THE POSITION OF THE FIRST SIGNIFICANT FIGURE IS DETERMINED BY MAX(1MAX(Y), MIN(Y)). S THE PLOT WILL BE PRODUCED ON A 1 INCH BY 1 INCH GRID. IT WILL BE ASSUMED THAT THE SPACING BETWEEN POINTS IS

SUBROUTINES REQUIRED  
NONE

#### METHOD

THE CONTOUR LEVELS ARE DETERMINED BY SIMPLE LINEAR INTERPOLATION FROM THE FOUR SURROUNDING POINTS.

SUBROUTINE FOR ONE-INCH GRID SPACING

C

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      MET00610
      MET00620
      MET00630
      MET00640
      MET00650
      MET00660
      MET00670
      MET00680
      MET00690
      MET00700
      MET00710
      MET00720
      MET00730
      MET00740
      MET00750
      MET00760
      MET00770
      MET00780
      MET00790
      MET00800
      MET00810
      MET00820
      MET00830
      MET00840
      MET00850
      MET00860
      MET00870
      MET00880
      MET00890
      MET00900
      MET00910
      MET00920
      MET00930
      MET00940
      MET00950
      MET00960
      MET00970
      MET00980
      MET00990
      MET01000
      MET01010
      MET01020
      MET01030
      MET01040
      MET01050
      MET01060

      SUBROUTINE MTMPII(Y,N,M,T,BND,AZ,BZ,AMIN,IJT,ICON)
REAL*4 I1H,KG,ITJZ
DIMENSION A(140)*B(140),C(140),D(140),I1H(20),Y(N,M),TP(10),TPX(10)
Z TPM(10),XMT(10),BTM(10),BTX(10),KG(10),T(24)
DIMENSION E(140),F(140),H(140)

      DATA DUE/4H /,EPL/4H+ /,EMI/4H- /,IH/1H0,1H /,1H1,1H /,1H2,1H /,1H3,1H4,1H5,1H
      1H6,1H7,1H8,1H /,1H9,1H /,KG/
      YMIN=Y(1,1)
      YMAX=Y(1,1)
      DO 20 I=1,M
      DO 12 J=1,N
      YMIN=AMIN1(YMIN,Y(J,1))
      YMAX=AMAX1(YMAX,Y(J,1))
      10 CONTINUE
      DELY=YMAX-YMIN
      IF(BND)25,25,30
      25 BND=DELY/YMIN
      30 IF((AMIN-YMIN)>31,31,32
      31 IF((IJT)33,32,33
      32 PD=YMIN/BND
      PF=ABS(PD-INT(PD))
      1 IF((YMIN)2,1,1
      AMIN=YMIN-PF*BND
      2 AMIN=YMIN-(1.0-PF)*BND
      33 AHLD=AZ
      34 IF(AZ)55,55
      35 SM=AMAX1(ABS(YMIN),ABS(YMAX))
      40 NS=0
      41 NS=NS+1
      42 SM=10.0*SM
      43 IF((SM-1.0)40,50,45
      44 NS=NS-1
      45 SM=SM/10.0
      46 IF((SM-1.0)50,50,45
      47 AHLD=10.0*NS
      48 HBNDF=BND/2.0
      49 PRINT 73
      50 PRINT 6, T

```

```

6 FORMAT(5X,24A4,BZ)
57 PRINT(1H0,65HTHE FOLLOWING TRANSFORMATION WAS PERFORMED ON THE
1 PUT MATRIX /5X,1H(5,1H*E12.5,8H*Y(1,J)+,E12.5,1H) //2X,73HAND THREE
2 DIGITS TO THE RIGHT OF THE DECIMAL POINT ARE PRINTED IN THE MAP )METO1113
C      PRINT 54,YMAX,YMIN
54 FORMAT(/4X,5H,YMAX=,E15.7,5X,5HYMIN=,E15.7)
1 IF (ICON)5,58,5
5 PRINT 11,BND
11 FORMAT(2X,17HTHE BAND WIDTH IS,E12.5,6H UNITS //4X,14HC CONTOUR LEVEL
1 LSS
1 I=0
1 YTOP=A MIN
1 IF(ABS(YMIN-YMAX)-100.0*BND)53,53,58
53
1 YB=YTOP
1 YTOP=YTOP+BND
1 I=I+1
1 J=MOD(I,20)
1 ITJZ=IH(J)
1 IF(YB-YMAX)59,58,58
59 PRINT 61,YB,YTOP,ITJZ
61 FORMAT(/4X,E10.3,4H $\frac{J}{10}$ ,E10.3,2H =,1X,A1)
61 GO TO 53
58 NCCP=0
63 PRINT 70
63 FORMAT(1H1)
70 PRINT 6,T
NLINE=0
NCCP=NCP+1
NCP=NCP+13
73 IF(NCP-M)80,80,75
75 NCP=M
CONTINUE
83
J=-2
NLINEN=NLINEN+1
NLINEN=N-NLINEN+1
85 J=-1
90 DO 100,1 = 1,135 .
100
C SET
100 CONTINUE
110 DO 160,L=NCCP,NCP
J = J+8

```

```

KI=L
1F(KI-100) 130,120,120
LL=KI/100
A(J)=KG(LL+1)
GO TO 135
A(J)=KG(1)
135 J=J+1
1F(KI-10) 150,140,140
140 LL=KI/10
A(J)=KG(LL+1)
KI=KI-10*LL
GO TO 155
150 A(J)=KG(1)
155 J=J+1
A(J)=KG(KI+1)
160 CONTINUE
C SET UP FIRST ROW OF ARRAY
GO TO 260
170 NLINE=NLINE+1
1F(NLINE-N) NLINE+1
DO 190 I=1,135
180 A(I)=BLK
B(I)=BLK
C(I)=BLK
D(I)=BLK
E(I)=BLK
F(I)=BLK
G(I)=BLK
H(I)=BLK
190 CONTINUE
1F(LICON) 195,260,195
NCY=NCCP-1
J=4
1F(NCY) 200,200,210
200 J=5
NCY=NCY+1
1F(NCY-NCP) 220,220,260
220 IF(NCY-M) 230,260,260
NLINEx=NLINEx-1
YD1=Y(NLINE,NCY)-Y(NLINE+1,NCY)
YD2=Y(NLINE,NCY+1)-Y(NLINE+1,NCY)
TP(1)=Y(NLINE,NCY)-0.125*YD1
TPX(1)=Y(NLINE,NCY)-0.250*YD1
TPM(1)=Y(NLINE,NCY)-0.375*YD1
XMT(1)=Y(NLINE,NCY)-0.500*YD1
BTM(1)=Y(NLINE,NCY)-0.625*YD1

```

```

BTX(1)=Y(NLINE•NCY)-0.750*YD1
BT(1)=Y(NLINE•NCY)-0.875*YD1
TP(10)=Y(NLINE•NCY+1)-0.125*YD2
TPX(10)=Y(NLINE•NCY+1)-0.250*YD2
TPM(10)=Y(NLINE•NCY+1)-0.375*YD2
TXMT(10)=Y(NLINE•NCY+1)-0.500*YD2
BTM(10)=Y(NLINE•NCY+1)-0.625*YD2
BTX(10)=Y(NLINE•NCY+1)-0.750*YD2
BT(10)=Y(NLINE•NCY+1)-0.875*YD2

NLINEx=NLINEx+1
D1=0.1*(TP(10)-TP(1))
D2=0.1*(TPX(10)-TPX(1))
D3=0.1*(TPM(10)-TPM(1))
D4=0.1*(XMT(10)-XMT(1))
D5=0.1*(BTM(10)-BTM(1))
D6=0.1*(BTX(10)-BTX(1))
D7=0.1*(BT(10)-BT(1))

DO 240 I=2*9
TP(I)=TP(I-1)+D1
TPX(I)=TPX(I-1)+D2
TPM(I)=TPM(I-1)+D3
XMT(I)=XMT(I-1)+D4
BTM(I)=BTM(I-1)+D5
BTX(I)=BTX(I-1)+D6
BT(I)=BT(I-1)+D7
CONTINUE
DO 250 I=1,10
J=J+1
I1=MOD(IFIX(((TP(I)-AMIN)/BND)*20)+1
I2=MOD(IFIX(((TPX(I)-AMIN)/BND)*20)+1
I3=MOD(IFIX(((TPM(I)-AMIN)/BND)*20)+1
I4=MOD(IFIX(((XMT(I)-AMIN)/BND)*20)+1
I5=MOD(IFIX(((BTM(I)-AMIN)/BND)*20)+1
I6=MOD(IFIX(((BTX(I)-AMIN)/BND)*20)+1
I7=MOD(IFIX(((BT(I)-AMIN)/BND)*20)+1
A(J)=IH(I1)
B(J)=IH(I2)
C(J)=IH(I3)
D(J)=IH(I4)
E(J)=IH(I5)
F(J)=IH(I6)
G(J)=IH(I7)

240
250 CONTINUE
260 GO TO 210
261 NCY=NCCP-1
262 IF(NCY) -2 265,266,270
263 J=-1

```

```

GO TO 330
 270  NCY=NCY+1
      IF(NCY-NCP) 280,280,310
 280  J=J+7
      THLD=AHLD*Y(NLINE,NCY)+BZ
      IF(THLD) 285,290,290
      H(J)=EM1
      GO TO 295
 295  H(J)=EPL
      NUM=INT(ABS(THLD-INT(THLD))*1000.0+0.5)
      NDS=100
      DO 300 KK=1,3
      J=J+1
      KI=NUM/NDS
      H(J)=KG(KI+1)
      NUM=NUM-KI*NDS
      NDS=NDS/10
      CONTINUE
 300  GO TO 270
      IF(NCP-M) 360,320,320
      310  IF(J-127) 330,330,360
      320  IF(J+3
      KI=NLINE
      IF(KI-100) 340,335,335
      330  LL=K1/100
      H(J)=KG(LL+1)
      KI=KI-100*LL
      GO TO 343
      340  H(J)=KG(1)
      343  J=J+1
      IF(KI-10) 350,345,345
      345  LL=K1/10
      H(J)=KG(LL+1)
      KI=KI-10*LL
      GO TO 355
      350  H(J)=KG(1)
      355  J=J+1
      H(J)=KG(KI+1)
      J=J-5
      IF(NCY-1) 360,270,360
      360  IF(NLINE-1) 362,368
      362  PRINT 370,(A(I),I=1,132),(B(IP1),IP1=1,132),(H(IP2),IP2=1,132)
      368  PRINT 370,(A(I),I=1,132),(B(IP1),IP1=1,132),(C(IP2),IP2=1,132),
      1(D(IP3),IP3=1,132),(E(IP4),IP4=1,132),(F(IP5),IP5=1,132),
      2(G(IP6),IP6=1,132),(H(IP7),IP7=1,132)
      370  FORMAT(132A1)
      GO TO 170

```

```

      DO 390 I=1,135
      A(I)=BLK
      B(I)=BLK
      C(I)=BLK
      D(I)=BLK
      CONTINUE
390   J=-2
      IF(NCCP-1) 395,395,400
      J=-1
      DO 430 L=NCCP,NCP
      J=J+8
      KI=L
      IF(KI-100) 410,405,405
      C(J)=KG(LL+1)
      KI=KI-100*LL
      GO TO 412
      C(J)=KG(1)
      J=J+1
      IF(KI-10) 420,415,415
      LL=KI/10
      C(J)=KG(LL+1)
      KI=KI-10*LL
      GO TO 422
      C(J)=KG(1)
      J=J+1
      C(J)=KG(KI+1)
      CONTINUE
430   PRINT 370, (B(IP1),IP1=1,132), (C(IP2),IP2=1,132)
      IF(NCP-M)60,500,500
      RETURN
500

```

USAGE      CALL GAUSS(I,X,S,AM,V)  
DESCRIPTION OF PARAMETERS

IX -IX MUST CONTAIN AN ODD INTEGER NUMBER WITH NINE OR  
 LESS DIGITS ON THE FIRST ENTRY TO GAUSS. THEREAFTER  
 IT WILL CONTAIN A UNIFORMLY DISTRIBUTED INTEGER RANDOM  
 NUMBER GENERATED BY THE SUBROUTINE FOR USE ON THE NEXT  
 ENTRY TO THE SUBROUTINE.  
 S -THE DESIRED STANDARD DEVIATION OF THE NORMAL  
 DISTRIBUTION.  
 AM -THE DESIRED MEAN OF THE NORMAL DISTRIBUTION  
 V -THE VALUE OF THE COMPUTED NORMAL RANDOM VARIABLE

REMARKS  
 THIS SUBROUTINE USES RANDU WHICH IS MACHINE SPECIFIC  
 SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED  
 RANDU

METHOD  
 USES 12 UNIFORM RANDOM NUMBERS TO COMPUTE NORMAL RANDOM  
 NUMBERS BY CENTRAL LIMIT THEOREM. THE RESULT IS THEN  
 ADJUSTED TO MATCH THE GIVEN MEAN AND STANDARD DEVIATION.  
 THE UNIFORM RANDOM NUMBERS COMPUTED WITHIN THE SUBROUTINE  
 ARE FOUND BY THE POWER RESIDUE METHOD.

```

SUBROUTINE GAUSS (IX,S,AM,V)
A=0.0
DO 50 I=1,12
  CALL RANDU(IX,IY,Y)
  IX=IY
  A=A+Y
  V=(A-6.0)*S+AM
  RETURN
END

```

```

C000022
C.....SUBROUTINE RANDU
C.....PURPOSE COMPUTES UNIFORMLY DISTRIBUTED RANDOM REAL NUMBERS BETWEEN
C.....0 AND 1.0 AND RANDOM INTEGERS BETWEEN ZERO AND
C.....2**31. EACH ENTRY USES AS INPUT AN INTEGER RANDOM NUMBER
C.....RAND 10
C.....RAND 120
C.....RAND 300
C.....RAND 400
C.....RAND 500
C.....RAND 600
C.....RAND 700
C.....RAND 800
C.....RAND 900

```

AND PRODUCES A NEW INTEGER AND REAL RANDOM NUMBER.

USAGE CALL RANDU(IX,IY,YEL)

DESCRIPTION OF PARAMETERS  
 IX - FOR THE FIRST ENTRY THIS MUST CONTAIN ANY ODD INTEGER NUMBER WITH NINE OR LESS DIGITS. AFTER THE FIRST ENTRY IX SHOULD BE THE PREVIOUS VALUE OF IX COMPUTED BY THIS SUBROUTINE.  
 IY - A RESULTANT INTEGER RANDOM NUMBER REQUIRED FOR THE NEXT ENTRY TO THIS SUBROUTINE. THE RANGE OF THIS NUMBER IS BETWEEN ZERO AND  $2^{**31}$ .  
 YFL - THE RESULTANT UNIFORMLY DISTRIBUTED, FLOATING POINT, RANDOM NUMBER IN THE RANGE OF 0.0 TO 1.0.

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED  
NONE

## METHOD POWER RESIDUE METHOD DISCUSSED IN IBM MANUAL C20-8011, RANDOM NUMBER GENERATION AND TESTING

```
SUBROUTINE RANDU(IX,IY,YFL)
IY=IX*65539
IF(IY)566
5 IY=IY+2147483647+1
YFL=IY
6 YFL=YFL*.4656613E-9
      RETURN
END
```

```
RAND 540
RAND 550
RAND 560
RAND 570
RAND 580
RAND 590
RAND 600
```

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