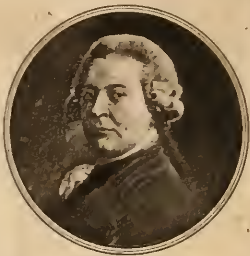

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COCKER's DECIMAL ARITHMETICK,

Wherein is ſhewed the Nature and Uſe of Decimal Fractions in the uſual Rules of Arithmetick, and the Menſuration of Plains and Solids.

Together with Tables of Intereſt and Rebate for the valuation of Leaſes and Annuities, Preſent, or in Reverſion, and Rules for Calculating thoſe Tables.

Whereunto is added

His Artificial Arithmetick, ſhewing the Geneſis or Fabrick of the Logarithms, and their uſe in the Extraction of Roots, the ſolving of Queſtions in Anatociſm, and in other Arithmetical Rules in a Method not uſually practiſed.

ALSO

His Algebraical Arithmetick, containing the Doctrine of Compoſing and Reſolving an Equation; with all other Rules requiſite for the underſtanding of that myſterious Art, according to the Method uſed by Mr. *John Kerſey* in his Incomparable Treatiſe of *ALGEBRA*.

Compoſed by *EDWARD COCKER*, late Practitioner in the Arts of *Writing*, *Arithmetick* and *Engraving*.

Peruſed, Corrected and Publiſhed
By *JOHN HAWKINS*, Writing Maſter at St. *Georges*-Church in *Southwark*.

Cum tua non edas cur hæc mea Zoile Carpis,
Carpere vel noli noſtra, vel eda tua.
Μωμεῖσθαι μὲν ῥαδιόν ἐστι μιμεῖσθαι δὲ χαλεπόν.

The Third Edition.

LONDON,
Printed for *George Sawbridge*, at the *Three Flower de Luces* in *Little-Britain*: And *Richard Wellington*, at the *Dolphin* and *Crown* at the Weſt end of St. *Paul's Church-Yard*. 1703.

To the Right Worſhipful

Sir *PETER DANIEL*, Kt.

AND

PETER RICH, Eſq;

Aldermen of the

CITY

OF

LONDON.

THOMAS LEE, Eſq;

AND

JAMES READING, Eſq;

Juſtices of the Peace for the COUNTY of

SURRY.

JOHN HAWKINS Humbly Dedicateth this Treatiſe of ARITHMETICK.

TO THE READER.

Courteous Reader,

IN the year of our Lord 1677, I publiſhed Mr. *Cocker*'s Vulgar *Arithmetick*; and therein gave an account of the ſpeedy publication of his Decimal, Logarithmical and Algebraical Arithmetick; but other extraordinary occurrences intervening, occaſioned its not ſeeing the light before this time:

By the Vulgar part, the Ingenious Learner may be qualified with ſo much of that moſt neceſſary Art of Arithmetick as is ſufficient for the management of buſineſs in the greateſt concerns of Trade and Commerce; and for thoſe Ingenious Souls, whoſe active fancies lead them to a further ſcrutiny into the ſtudy of the Arts Mathematical was this Treatiſe compoſed, which will fairly lead them by the hand, without any other Guide or Company, into the Contemplation of thoſe moſt ſublime ſpeculations, an inheritance entailed only upon the ingenious, and induſtrious ſons of Art.

The Method throughout the whole is plain, perſpicuous, and clear, and I hope will prove ſatisfactory

tisfactory to those who shall seriously apply themselves to the Rules, Precepts and Examples therein contained.

The use of Decimals (in the solution of questions Arithmetical, and such Geometrical as are necessary in the mensuration of the most usual planes and solids) is as plainly laid down as the Author or my self could possibly contrive it, and particularly in all the varieties of Interest both Simple and Compound, with Tables, and Rules for the Calculation thereof, according to the Method of several Famous Authors (who have bestowed much pains in the management thereof,) and especially of that most Famous, and no less laborious Mathematician of our Age and Nation, Mr. *John Kersey*, whose Memory deserves highly to be honoured by all the Professors of this Science.

The Genesis or Fabrick of the Logarithms, and their use in Arithmetick is laid down after a different but more intelligible manner than hitherto hath been used by other Authors, and I hope the studious Reader will receive that satisfaction therein which our Author earnestly aimed at, or himself can expect.

And as for the Algebraical part I think there is nothing therein expressed that is superfluous, nor any thing omitted that could be thought necessary to render it plain, perspicuous and clear; so that what other Authors treating upon this subject have left intricate, and difficult to be understood is here made obvious (by clear demonstration) to the meanest Capacity; therefore, Courteous Reader, if thou intendest to be a proficient in the Mathematicks, begin chearfully, proceed gradually, and with resolution; and the

the end will crown thy endeavours with ſucceſs; and be not ſo ſloathfully ſtudious as at every difficulty thou meeteſt withal to cry out, *Ne plus ultra*, for pains and diligence will overcome the greateſt difficulty: To conclude, That thou mayeſt ſo read as to underſtand, and ſo underſtand, as to become a proficient, is the hearty deſire of him who wiſheth thy welfare and the progreſs of Arts.

From my School at St. George's *Church in* Southwark, Octob. 27 1684.

JOHN HAWKINS.

Anixo guo Anamſiggino Jorammi Lehkeg *Loſoxroſeholrii Torgiemgig Im Xoniſaſu Diſohemiemgi Puwinaſig ſeho.*

G I H,

IT you Lèpeag ſo zegſod gone ot youh glahe roughèg im lehugims fre toppodims ſeheaſige you dipp frem ze fre zeffeh azpe ſo juws rod I rave glemf nime, amw it ny laimeg freheim nay ze Lhotiſazpe ſo fre luzpixk I rave ny digr, zuf it moſ, if ig moſ a ſoow frims mod imweew I wo gay go.

G I H, I an

Youb ruuz pe gebuamſ
Jorm Radking.

Oxfoz. 30 1684.
Thon Pomwom.

The Advice of a Friend of the Authors to ſuch as are deſirous to attain to the perfection of this moſt uſeful ART, &c.

YOU that peruſe this curious work, obſerve,
That he not meanly does of men deſerve,
Whoſe ſtudious labour brought it to an end,
And as his Maſter-piece did it commend
To thoſe who are deſirous to imploy
Their time the beſt of curious Arts t' enjoy;
An Art by which man's fortunes often rais'd,
An Art by all that Trade or Traffique prais'd:
An Art, or an Aquirement, who ſo wants
His buſineſs, if (important,) quickly faints;
'Tis what's ſo uſeful, that not to be known,
Wou'd ruin each mans occupation:
Therefore let thoſe who fain wou'd riſe, embrace
This, and preferment they have in the chace.
Long ſince it was invented for our good,
Yet till late days, not rightly underſtood;
And not till now to its perfection brought,
Though many ways with tedious trouble ſought.
In theſe choice Pages all is to be found
That does concern the Subject: theſe do bound
The largeſt field of true Arithmetick,
No numbers wanting that mankind wou'd ſeek.
The curious Artiſt with a ſearching eye,
Although turn'd Critick, here no faults can ſpy;
Or if there any be, they are ſo ſmall,
That nearly they reſemble none at all;
For all that have perus'd it, have confeſt
That of this kind, this much exceeds the reſt.

J. A. Teacher of the Mathematicks.

In Commendation of his Friend Mr. JOHN HAWKINS, upon the publication of this Treatise.

THE *Learned* Chymist *can't more truly say*
He can the unseen powers of herbs display;
Or by dissolving their external face
Bring subtil Spirits, Sulphurs, Salts in place;
Exalt their intern Energy; sublime
From Putrefactive Nunc, *Eternal* Time:
Than you by ALGEBRA and Numbers prove
Th' Æquations *true, of all the Orbs above.*
You by subtracting add, and do divide
The self same way by which you multiplyed.
From Numbers small you mighty Powers make,
And from the same the Quintessence you take.
By Infinites, you finite Numbers bind,
By things unknown, you unknown things do find.
Proportions you find out, and as Exact,
As Chymists *you* Æquations *de Extract.*
Thus you the Powers of Numbers do unfold,
And like them, change base Metals into Gold.
The Springs unseen; for no man fully knows
From whence the sacred source of Number flows.
But my poor Mite you need not, nor my praise,
To you my lines can't lasting Trophies raise.
Nor need your Numbers my unlearned defence,
Numerick Truth in its abstracted sense,
Derives its spring from an Eternal Font,
Without beginning endless in Account.
The Universal World it does comprise,
It no beginning had, nor ever dies.
All things i'th' sphear of Sacred Numbers stand,
The most Immense, and the minutest sand. Hea-

Heaven, Earth, the Seas, their furniture ſubmit,
And their num'rous off-ſpring flows with it:
It meaſures place and time; in ſhades of night
It ſees no darkneſs, but Illuſtrious light.
Both Life and Death to it the ſame appear,
And Subjects are within its mighty Sphear.
Thus my affections (friend) make me intrude,
Though with unpoliſh'd lines, and numbers rude.
On ſuch a Theam, Who could forbear to ſing?
To Sacred Fire, who ſhould not Incenſe bring?
Inſpired by thy Great ART, my ſublime Muſe
Th' eternal Truth of Numbers ſhall diffuſe:
Whil'ſt I applaud the object of thy Pen,
The unknown depths of Algebra *and* Men.
Here fix thy Pillars; in this ART aſpire
To light our Tapers with Cœleſtial fire.
In the ſame Zeal proceed: thy numbers fit
With ſpeaking Symbols to the meaneſt Wit.

13*th.* Octob.
1684.

Yours and Truths Servant,

WILLIAM SALMON.

Med. Profeſſ.

To the Ingenious Author of these Decimals, and Algebra, the Famous Arithmetician; and his singular good Friend by choice, EDWARD COCKER.

WIth admiration struck I here shou'd pause,
Not daring trust my Muse in your applause,
Whose fame already has so loud been sung
By the Divinest of the Sacred Throng:
Did not your Rich and Matchless Art inspire
My drowsie soul with a poetick fire;
For who in silence can remain, that views
A Subject worthy such as can infuse
A moving Rapture of the first degree
Into a Breast, before from Phœbus free:
So great a Master-piece as this, mankind
In all their tedious search could never find.
Arithmetick's here to perfection brought,
Here's to be found what never yet was taught:
The curious work so to the Life is drawn,
That all besides are like the Mornings dawn;
Compar'd to day's clear face when *Sol* sits high
In his Meridian Throne in vain some try
To reach your Arts Perfections, but the more
Their Genius flags when to your hights they'd soar;
And at the best their labours do appear
Foils to make your Diamonds shine more clear:
This Book of yours bears record of your fame,
And to all Ages will transfer your Name.
For why, your boundless Wit, and curious Pen
Do still you write the miracle of Men.

R. N. Philo-Math.

In Memory of the deceased Author, Mr. *EDWARD COCKER*: And in praise of this (*Posthumal*) and his former *Works*.

WHO e're (*of old*) *to th'* Common good *apply'd*
Their minds *or* means, *but they were* deifi'd?
And chiefly those, who new Inventions *found*;
Bacchus *for* Wine: Ceres *who* Till'd the Ground:
Whose Fames *and* Memoreis *will ever last*
Till the late Evening *of the* World *be past*.
Now this our Author *by his fluent* Pen
In all Fair-Writing *did exceed most* Men:
And though in Knotting, Gething *did do well*,
Cocker *in* That, *did* Gething *far excell*:
And not with Pen *alone, on* Paper He
Could Write *and* Knot, *but with the* Graver *too*
On Copper plates *He did all* Men *out-do*.
What curious Copy-Books *and* Sculptures *are*
Extant in Print of His, *which may compare*
With any in the World, and no one Hand
Had Pen *and* Graver *both at such* Command?
But leaving now his Writing, *take a view*
Of his Arithmetick, *whose* Books *are Two*:
The one of Plain (*or* Vulgar Numbers) *made*
Fit for Young Scholars, *and for* Men of Trade.
This other's in Three parts, *more* General;
I. *Of* Artificial Numbers DECIMAL:
II. *The second's* Numbers *LOGARITHMICAL*:
III. *The third by Symbols ALGEBRAICAL*,
All fraught with Questions Enigmatical,
Of all Arithmeticks *the GENERAL*.
Consider now what Pains *the* Author *took*,
And Praise Him *as thou benefits by his Book*.
But since the Author's *dead, I'll not defer*
To praise *and thank th'* ingenious Editor.

W. Leybourn.

Ad amicum suum dilectissimum Dominum Joannem Hawkins *de opere hoc mirâ cum eruditione, tum industriâ Correcto & Reviso.*

Ἐγωμιαςικὸν.

Si meruit Laurum, qui Lauro scribere digna
Novit, & ad sophiam pandere callet iter.
Quid meruit qui non tantum novit, sed & ipse;
Præstitit ingenio, vix facienda, suo?
Laura conveniunt non tantum serta capilis,
Aurea sed potiùs, docte, corona tuis.
Aurum vos illi divites concedite, Laurum
Dent alii, nemo se meruisse neget.
Quod si nec Lauri nostro tribuetis honorem
Autori, planè quem meruisse liquet,
Auri nec summam dabitis quam quisq; fatetur
Ingenii meritum non minùs esse sui.
Non Mæcenates eritis, non esse patronos
Posse putat, quorum tam sit avara manus,
Sed potius (veniam petimus, dabimusq; vicissim)
Nominat ingratos vos (scio cur) asinos.

Joannes Robinson.

A Catalogue of the Chapters contained in the *Decimal* Part.

A Catalogue of the Chapters contained in the *Logarithmical* Part.

Division

A Catalogue of the Chapters contained in the *Algebraick* Part.

The

A Catalogue of Chapters.

Advertisements.

THE

NOTATION OF DECIMALS.

CHAP. I.

I. WHAT Arithmetick, and the Subject thereof, (*viz.* Number) is, I have largely defined in the First Chapter of my Vulgar Arithmetick, in which Treatise I have applyed the speices of Numeration to the various Rules of Vulgar Arithmetick, both in Intregers, and Fractions for the solution of various Practical Questions solvable thereby, by such plain and easie Rules as many years experience in the practice thereof had made me capable of, and which I hope might render it Intelligible, and serviceable to the meanest Capacity.

And in this I shall shew you the use of Decimal Fractions in all the Rules of Arithmetick, but Principally in the solving Questions of Interest and Rebate, according to ſeveral Rates of Intereſt, both Simple and Compound, with the true Valuation of Leaſſes and Annuities, either preſent or in Reverſion, and likewiſe their uſe in the calculating of Tables for that purpoſe, *&c.*

II. In Decimal Fractions we ſuppoſe the unite or integer to be divided into ten equal parts, and each of thoſe tenth parts are again divided into ten other equal parts, ſo that then the Unit or Integer will be divided into a 100 equal parts, and then again each of thoſe hundred parts is ſuppoſed to be divided into 10 other equal parts, ſo that then the Unit or Integer, will be divided into 1000 equal parts, *&c.* And ſo by Decimating the firſt and ſubdecimating the ſecond we proceed *ad infinitum.*

III. And hence it is evident that a Decimal Fraction is always either ſo many tenths, or it is ſo many tenths of $\frac{1}{10}$, or 'tis ſo many tenths of $\frac{1}{10}$ of $\frac{1}{10}$, or ſo many tenths of $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$, &c. which compound Decimal Fraction being Reduced, as is taught in the 6 Rule of the 19 Chapter of my Vulgar Arithmetick, will give its equivalent ſimple Decimal Fraction; As for Example, $\frac{8}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$ is .008 that is $\frac{8}{1000}$ and hence it follows that always a Decimal Fraction hath for its Denominator an Unit with a Cypher, or elſe Cyphers annexed to it on the right hand, *viz.* either 10, or 100, or 1000, or 10000, or 100000, &c. *ad infinitum.*

IV. In Decimal Fractions the Denominator is never expreſs'd, but may at firſt ſight be underſtood by the number of places contained in the Numerator; the Denominator being always an unite with as many Cyphers annexed to it, as there are real places in the Numerator; as 8 being a Decimal is $\frac{8}{10}$, *viz.* its Denominator is an

untie

unite with one Cypher annexed to it, and $\frac{85}{100}$ is thus written .85 and $\frac{164}{1000}$ thus written 164; But if the Numerator of a Decimal Fraction consisteth not of so many places as there are Cyphers in the Denominator, then such defect is supplyed by prefixing so many Cyphers before the Numerator, (*viz.*) on the left hand as there are places deficient; as for Example, $\frac{8}{100}$ if it were only set down thus, (.8) then it would be but $\frac{8}{10}$, but by prefixing a Cypher before it thus (.08) it is $\frac{8}{100}$ and $\frac{8}{1000}$ is thus expressed (.008) and $\frac{25}{1000}$ is thus written (.025) and $\frac{25}{10000}$ thus (.0025) *&c.*

V. A Decimal Fraction being written without its Denominator; is known from a whole Number, by having a point or prick prefixed before it thus; .25 is $\frac{25}{100}$ but if it had been expressed without a point thus (25) it would have signified so many unites: The same is to be observed in mixt Numbers, for 29 $\frac{16}{100}$ being written Decimally, will stand thus, (29. 16) and 48$\frac{25}{1000}$ thus, (48. 025) and 48$\frac{28}{10000}$ thus (48.0028) and the like of any others.

But some Authors distinguish Decimals from whole Numbers, by prefixnig a virgula, or perpendicular line before the Decimal, (whether it be alone, or joyned with a whole Number) thus, |8 is $\frac{8}{10}$, and |025 is $\frac{25}{1000}$, and 29|16 is 29$\frac{16}{100}$, *&c.* Others express the same Deciaml Fraction and mixt numbers thus, (*viz.*) |8 |025 29|16, *&c.* Others with a point over the place of Units in the whole number; and then the former Fractions and mixt number will be thus written; *viz.* 08, 0025, 2916 the like of others: And some Authors again put points over all the places or figures in a Decimal Fraction thus: 8, 025, 2916, 48025, *&c.* but being written according to the first direction, I conceive they may be most fit for Calculation.

VI. As whole Numbers do increase their value in a decuple proportion, by annexing a Cypher or Figure to the place of Units, so by prefixing a Cypher or Figure on the left hand of a Decimal, so as actually to take place in the Decimal, its value is decreased in a subdecuple proportion, so the Number 4, by annexing a Figure or Cypher to it; it is increased from 4 to 40, *&c.* But if 4 had been a decimal, *viz.* 4. and if there had been 0 prefixed before it on the left hand, its value had been decreased from $\frac{4}{10}$ to $\frac{4}{100}$ or 04, and by prefixing 5, it is .54; and still by prefixing more Figures or Cyphers, its value will decrease in the same Ratio *ad infinitum.*

VII. And as Cyphers being prefixed before a whole Number, (*viz.*) on the left hand thereof) do neither increase or decrease its value; (for 4, and 04, and 004 being Integers, do still retain one and the same value;) So a Decimal, by having a Cypher, or Cyphers annexed to the Right hand thereof, have not their value either Increased, or Deceased,

Whence it is evident, that all Decimal Fractions may be Reduced to an equal Denomiation at first sight; for suppose .15, and .008, and .73465 were Decimals given to be Reduced to one denomination; In this case I consider, that the denominatior for the given decimal consisting of the most places in 100000, and .15 and .008 whose Denominators are 100 and 1000 may be reduced to decimals of the same value, having like-

likewiſe 100000 for their Denominator, by annexing ſo many Cyphers on the Right hand of the Numerators, as (according to the 4 Definition foregoing) may make each of them to have 100000 for a Denominator, ſo .15 will be .15000, and .068 will be .06800.

VIII. As the order of places in whole Numbers is from the right hand to the left, ſo the order of places in a Decimal Fraction is from the left hand to the right; the firſt place being accounted tenth parts of an Unity, and by ſome it is called primes, the ſecond place is ſo many hundredth parts of unity, or it is called ſeconds, the third place is ſo many thouſandth parts of unity, or it is called thirds, *&c.* which will more fully appear by the following Table.

A Table of Notation of Integers and Decimals.

Integers. — Of Unites.

Of Unites		Integers
C of Mill.	3	Ninth place
X of Mill.	8	Eighth place
Millions	4	Seventh place
C of Thouſ.	3	Sixth place
X of Thouſ.	7	Fifth place
Thouſands	5	Fourth place
Hundreds	8	Third place
Tens	6	Second place
Unites	4	Firſt place

Decimals. — Of Unity

Of Unity		Decimals
Tenth parts	8	Primes
Hundr. parts	2	Seconds
Thouſ. parts	3	Thirds
X Thouſ. parts	0	Fourths
C Thouſ. parts	5	Fifths
Mill. parts	6	Sixths
X Mill. parts	3	Sevenths
C Mill. parts	5	Eights
Tho. Mill. parts	4	Ninths

In the foregoing Table is given a mixt Number of Integers and Decimals; the Integers being ſeparated from the Decimals by a point, or prick, according to the fifth definition beforegoing; ſo that 384375864 ſignify ſo many Integers or Unites, and 823056345 ſignifie ſo many parts of Unity, the Figure 8 in the firſt place being ſo many tenth parts of Unity; and the next Figure, *viz.* the Figure 2 is ſo many hundredth parts of unity, *&c.*

So in the Decimal Fraction .4378, the Figure 4 poſſeſſeth the firſt place, and is. 4 primes, or four tenth parts of an unite, and 3 the ſecond figure is called 3 ſeconds, or 3 hundredth parts of an unite, and ſeven the third Figure is called ſeven thirds, or ſeven thouſandth parts of an unite, and 8 the fourth figure is called eight fourths, or eight ten thouſandth parts of an unite, *&c.*

Whence it appeareth that every place in a Decimal Fraction being conſidered a part by it ſelf, without any reſpect to the reſt, will of it ſelf make a particular Decimal Fraction; ſo in the laſt mentioned Decimal Fraction, *viz.* .4378, each place being conſidered by it ſelf, will make theſe following Decimal Fractions, *viz.* .4 .03 .007 and .0008, or $\frac{4}{10}$, $\frac{3}{100}$, $\frac{7}{1000}$, and $\frac{8}{10000}$; which Fractions being added together, according to the Rules of Addition of Decimals hereafter delivered in the third Chapter, their ſum will be .4378, which is the given Decimal of which they are compoſed.

IX. A Decimal Fraction is expreſſed by ſome Authors, by Primes, Seconds, Thirds, Fourths, *&c:* As if this Decimal .748 were to be expreſſed, they ſay it is ſeven primes, four ſeconds, and eight thirds: Others

Others there are which expreſs it thus, *viz.* ſeven hundred forty eight thirds, but the moſt approved way to expreſs or read a decimal Fraction, is according to the method of reading a vulgar Fraction, and to give it the Denomination of the Figure in the laſt place of the Decimal, and then the Decimal .748 will be thus read *viz*, ſeven hundred forty eight thouſandths, and .036 is thus read, thirty ſix thouſandths, and ſo of any other. This Chapter being well underſtood, all the parts of Numeration, *viz.* Addition, Subſtraction, Multiplication, and Diviſion of Decimals will prove very eaſie.

CHAP. II.

Reduction of Decimals.

To Reduce a given Vulgar Fraction to a Decimal, that ſhall be equivalent thereto.

1. WHen in any Arithmetical Operation your work is ſo mingled with vulgar Fractions, as to render it tedious, or difficult; the beſt remedy you can have, is to reduce your vulgar Fraction or Fractions into a Decimal of

or Decimals, which having done the work, will be as easie in every respect, as if you had to do with nothing but whole Numbers, which you may effect by the following Proportion, *viz.* As the Denominator of the given Vulgar Fraction, is to its Numerator.

So is an Unite, with so many Cyphers as you intend your Decimal shall have places, to the Decimal required.

So if the Fraction to be reduced were $\frac{3}{4}$, and you would reduce it to a decimal consisting of 4 places, I say, the proportion is

As 4 (the Denominator of the given Fraction.)
Is to 3 (its Numerator.)
So is 10000 (the denominator of the decimal required.)
To .7500 (the Decimal required.)

So that I conclude $\frac{3}{4}$ will be reduced to its equivalent Decimal .7500, or .75; for Cyphers on the right hand of a decimal do neither increase nor diminish its value, by the seventh definition of the first Chapter.

Now according to the foresaid proportion, it is evident, that if to the Numerator of any Fraction given to be reduced to a decimal, you annex as many Cyphers as you intend its equivalent decimal shall have places, and then divide it by its denominator, the Quote will be the decimal required.

So let there (again) be given $\frac{3}{4}$ to be reduced (as before) to a decimal of 4 places, in order thereunto I annex 4 Cyphers to the Numerator 3, and it makes 30000, which I divide by the denominator 4, and it Quotes .7500, or .75, for the decimal equivalent to the vulgar Fraction $\frac{3}{4}$.

Note that all vulgar Fractions, cannot be redu-

ced

ced to decimals, having exactly the ſame value, although they may come infinitely near, and the more places that you make your decimal to conſiſt of, ſo much the nearer doth it come to the truth, but 4 or 5 places is exact enough for moſt operations; ſo if it be required to reduce the vulgar Fraction $\frac{9}{11}$ to a decimal of 4 places, it will be found to be .818 which is not exact, but yet it wanteth not $\frac{1}{10000}$ part of an unit of the truth; and if you make it .8182, it will be ſomewhat more than the truth.

Again if you annex 5 Cyphers to the Numerator, and ſo make the Decimal conſiſt of 5 places, it will then be .81818, yet it will want of the truth, but not ſo much as when it had but 4 places, for now it will not want $\frac{1}{100000}$ part of an unite of the exact truth, and if you make it to be .81819, it will then exceed the truth. Thus by increaſing the number of places in the Decimal, you may come infinitely near the truth, but never find a decimal exactly equivalent in many caſes.

Note alſo, that if after you have Reduced your vulgar Fraction to a decimal, according to the foregoing Rule, there be not as many places in the decimal, as you annexed Cyphers to the Numerator of the given vulgar Fraction, then you are to ſupply ſuch defect by prefixing ſo many Cyphers on the left hand of the ſignificant Figures, as there are places wanting, according to the fourth Rule of the Firſt Chapter.

So if $\frac{11}{941}$ were given to be reduced to a decimal of any number of places, as ſuppoſe 6; in order to it, I annex 6 Cyphers to the Numerator 11, and it makes 11000000, for a dividend, which divided by 941, it quotes 11689, which consisteth but of 5 places, but it should have 6 places, wherefore to make it compleat, I prefix a Cypher before it, and it makes .01 1689 for the true decimal Required; and if it had been required to consist of four places, then I annex 4 Cyphers to the Numerator, yet after division is ended, there will be but 3 places in the Quotient, *viz.* 116, therefore to make it consist of 4 places, I prefix a Cypher before it, and it makes .0116 for the decimal sought. Again let there be given $\frac{23}{3842}$ to be reduced to a decimal of (suppose) 5 places it will be found to be .00407; and $\frac{14}{64837}$ will be reduced to .000215.

To Reduce the known parts of Money, Weight, Measure, Time, *&c. to Decimal Fractions.*

II. Hence it is evident that the known parts of Money, Weight, Measure, Time, and Motion, *&c.* may be reduced to decimal Fractions of the same value, or infinitely near it, for if (in Money) a Pound Sterling be an Integer, whatsoever is less than a Pound, is either a part or parts of the same; and when you know what part or parts thereof it is, you may reduce it to a decimal of the same value; by the first Rule of this Chapter; so if you would know what is the decimal of a Pound Sterling equal to 7 Shillings; consider that 7*s.* is $\frac{7}{20}$ of a Pound, and by the said Rule, the decimal answering thereto is .35 *l.* And if I would know the decimal equal to 3 *d.* I consider that 3 *d.* is $\frac{3}{12}$ of $\frac{1}{20}$ of a Pound, or $\frac{3}{240}$ of a Pound, and the decimal equivalent thereto, will

will be found (by the ſaid Rule) to be .0 125; likewiſe if there were given 7 *s.* 3 *d.* to find the decimal equal thereunto: Firſt, I conſider, that 7 *s.* 3 *d.* is 87 pence, which is $\frac{87}{240}$ of a Pound, and the decimal equal thereto will be found to be .3625 *l.*

In like manner if it were required to find the decimal of a pound Troy weight equivalent to 6 *oz.*—12 *pw.* I firſt find that 6 *oz.*—12 *pw.* make 132 *pwts.* which is $\frac{132}{240}$ of a pound Troy weight, and the decimal equivalent thereunto, will be found to be .55 by the ſaid firſt Rule of this Chapter. The like is to be underſtood in the Reducing of any of the known parts of Coyn, Weight, Meaſure, *&c.* into Decimals.

To find the value of a Decimal Fraction, in the known parts of Money, Weight, Meaſure, *&c.*

III. When you would find the value of a decimal Fraction in the known parts of Coyn, Weight, Meaſure, Time, Motion, or the like, obſerve the following

RULE

Multiply the given Decimal by the number of parts in the next Inferiour Denomination that are equal to an Integer in the ſame denomination with the given decimal, and ſee how many places are in the Product, more than were in the ſaid given decimal; and cut ſo many off from the left hand with a daſh of your Pen, and thoſe Fi-

Figures so cut off, are the value of the said decimal in the next inferior Denomination to it, and the Figures (if there be any) Remaining are the decimal of an Integer in the said Denomination, and may be Reduced as low as you please by the same Rule; as in the following Example.

Let it be required to find the value of this decimal of a pound sterling, *viz.* 7635.

First, I Multiply the given decimal by 20, and the Product is 152700 which is of 6 places, and the given decimal is but of 4 places, wherefore I cut off 2 Figures at the left hand, *viz.* 15. which is so many shillings, now when the said 15 is cut off from the rest, there are yet remaining 2700, which I multiply by 12 to find the value thereof in pence, and the product is 32400, which consisting of 5 places, I cut off one Figure, (*viz.* 3) from the left hand, which is so many Pence; so that I conclude the value of the given Decimal to be 15*s.*—03*d.* and the remaining Figures, *viz.* 2400 are the decimal parts of a Peny, which because they do not amount to the value of a Farthing, I do not reduce any lower, see the work in the Margent.

7635
20
———
15|2700
12
———
3|2400

So if .6847*l.* be given, and it be required to find its value, if you work as is before directed, you will find it to be 13*s.*—08*d.*—1.312 *quarters.* And 374*l.* being so reduced, you will find it to make 7*s.*—05*d.*—3.040 *quarters.*

In like manner, if it were required to reduce this decimal of a pound Troy weight, *viz.* .84576*l.* into known parts; First, I multiply it by 12, and it produceth 1014912 from which I cut off the two first Figures to the left hand, (*viz.*

(*viz.* 10) for Ounces, and the remaining Figures, which are 49 12 do I multiply by 20, and the Product is 298240, from which I cut off the first Figure, *viz.* 2. which is two peny weight and 98240 remaineth, which I multiply by 24, and the Product is 2357760, which is 23. 57760 grains; so that I conclude the value of the given Decimal .84576 pound Troy weight to be 10 *oz.*—02 *pw.*—23 *gr.* .57760; the same is to be observed in finding the value of any other decimal whatsoever, whether of Coin, Weight, Measure, Time, or Motion.

```
     .84576
         12
    -------
10| 14912
         20
    -------
     298240
       |  24
    ---|----
     392960
     196480
    ---|----
   23|57760
```

I might here have added Tables of Reduction, shewing the Decimal Fractions of any of the parts of Money, Weight, &c. as divers Authors have already done; but because they are though useful, seldom made use of, and partly by reason of the ease in finding the equivalent decimal of any Fraction whatsoever, according to the Rules herein delivered I shall forbear it.

IV. There is a briefer way of discovering the value of a decimal of a Pound sterling, *viz.* The Figure which standeth in the first place of the decimal, (*viz.* in the place of Primes) being doubled, gives you the number of shillings; then let the Figure possessing the second place of the decimal, *viz.* the place of seconds) be esteemed so many tens, and the Figure in the third place account so many units, which said tens and units being accounted one entire number, and made less by one, will be so many farthings, which said shil-

ſhillings and farthings are the value of the given decimal; but if the Figure in the ſecond place be 5, or elſe exceed 5, then reckon one ſhilling for that, and for the exceſs above 5, eſteem every unite 10, as before.

Example 1.

What is the value of .7365 *l*?

The Figure 7 (ſtanding in the place of primes) being doubled, gives 14, which is ſo many ſhillings, and the Figure in the ſecond place, (which is 3) being accounted ſo many tens is 30, and the Figure in the third place (*viz.* 6.) being eſteemed unites, and annexed to the tens beforeſaid, makes 36, which being leſſened by 1, makes 35 farthings, which is 8 *d.* $\frac{3}{4}$, ſo is 14 *s.*—08 *d.* $\frac{3}{4}$ the value of the given Decimal .7365 *l.*

Example 2.

What is the value of .8896 *l*?

The firſt Figure (8) being doubled, makes 16; and becauſe the next Figure is above 5, I add 1 to 16, which makes 17 ſhillings: Then the exceſs of the ſecond Figure above 5 being 3, I eſteem it ſo many tens, and the Figure (9) in the third place being unites, makes 39; which leſſened by 1, makes 38 Farthings, which is 9 *d.* $\frac{1}{2}$, ſo is 17 *s.*—9 *d.* $\frac{1}{2}$, the value of the given decimal .8896. And after the ſame manner may the value of any decimal of a Pound Sterling, be diſcovered at firſt ſight without loſs of a farthing.

CHAP. III.

Addition of Decimals.

I. THE work of Addition of Decimal Fractions is in every respect the very same with that of whole Numbers of one Denomination in common Arithmetick, respect being had to the right ordering or placing of the Decimals required to be added, which that you may understand, observe this

General Rule.

II. When two or more decimals are given to be added together, you are so to dispose of them one under the other, as that all the Figures on the left hand may stand in order one under the other, that is to say, primes under primes, or tenth under tenths, (whether they be Cyphers or significant Figures) and seconds or hundredths, under seconds or hundredths, *&c.* observing the same order if they consist some of them of never so many places, and others of never so few.

Example.

Let there be given these following Decimals to be added together, *viz.* .00746, and .0832, and .62 and .8: First, I dispose of them in order to the work, as you see in the Margent, where you see the lowermost Figure 8, which is primes, is placed under 6, 0 and 0, which are likewise primes, and the Figure 2 in 62 being in the

.00746
.0832
.62
.8

in the place of ſeconds is placed under 8 and o which are likewiſe ſeconds, or hundredths, and the Figure 3 in the place of thirds, or thouſandths is placed under 7, which is alſo ſo many thirds, &c. The ſame order is to be obſerved in placing of the decimals of mixt numbers to be added, as ſuppoſe there were given theſe following mixt numbers to be added together, *viz.* 168.3572, and 36.864, and 7.42, and .6 : Now in order to their finding out their Sum, I diſpoſe of them in order one under the other as followeth. Where you may obſerve that the whole Numbers themſelves, or integral parts of the given mixt Numbers are placed one under the other, as is directed in Addition of whole Numbers, without any reſpect at all had to the decimals annexed to them, and the decimals are placed under each other, according to the directions given in the laſt Rule, without any reſpect had to the Integers, properly belonging to them.

168.3572
36.864
7.42
.6

III. Having placed your given Decimals in order, according to this Rule, draw a line under them, as in Addition of whole Numbers; under which line you are to place their Sum ; Then proceed in your work in every reſpect, as in Addition of Integers, beginning at the right hand, and ſo proceeding through the Decimals without any regard to them as Decimals, but as if they were all whole Numbers: As for Example, let us take the Decimals given in the firſt example of the laſt Rule

Rule foregoing; And first I put down 6 under the line, because there is no other figure or number to add to it, then I proceed to the next, saying 2 and 4 makes 6, which I also set down in order under the line, then I say 3 and 7 makes 10, so I set down 0, and carry 1 to the next, saying 1 that I carry, and 2, and 8, make 11, for which I set down 1, and carry 1 to the next, saying, 1 that I carry and 8, and 6, make 15, which I put in its place under the line, because it is the last; and because the figure 5 standeth under the place of primes, I put a point before it, that is to say between 1 and 5, and the work is finished; the number 1 being an integer, and the rest a decimal, whereby I find the sum to be 1.51066; that is 1 integer, and .51066 parts of an integer.

```
  .00746
  .0832
  .62
  .8
 -------
 1.51066
```

After the same manner if the mixt numbers in the second example of the foregoing Rule were given to be added, their sum will be found to be 213.2412, that is 213 integers and .2412 decimal parts of an integer, as you may see by the following work.

```
168.3572
 36.864
  7.42
   .6
--------
213.2412
```

Other Examples for the Learners practice may be such as follow.

42.698	4.368	748
16.07	7.573	36.72
26.009	.724	9.564
42.8	.56	.7358
127.487	13.225	795.0198

CHAP. IV.

Subſtraction of Decimal Fractions.

1. WHen two Decimal Fractions are given, and their difference or exceſs is required, you muſt place them (in order to the work) as you were taught in the foregoing Chapter of Addition, and the operation is the very ſame in every reſpect as in Subſtraction of whole Numbers of one Denomination, beginning at the right hand as in the following Example.

Let it be required to ſubſtract the Decimal .634 from the Decimal .728; in order to the work I put them one under the other, *viz.* the biggeſt uppermoſt and take each figure the in lowermoſt out of its Correſpondent Figure in the uppermoſt, putting their reſpective differences in order below the line, and I find, that when I have finiſhed the operation) the Remainder or difference to be .094 as by the work appeareth.

.728
.634
.094

In

In like manner if the mixt Number 42.347 were given to be ſubſtracted from the mixt Number 76.23. I place them in the ſame order as is directed in Addition before-going, only with this Caution be ſure to place the biggeſt uppermoſt, then proceed to take each figure in the lowermoſt out of its correſpondent figure in the uppermoſt, as if they were whole numbers, and having finiſhed the work, the Remainder, or difference will be found to be 33.776 as you ſee it done in the Margent.

76.123
42.347
———
33.776

When the decimal given to be ſubſtracted do not conſiſt of an equal Number of places, ſuch defect muſt be ſupplyed by annexing Cyphers, or ſuppoſing as many Cyphers to be annexed (as are wanting) on the right hand, and then the work will be as in the former Examples.

Example.

Let it be required to ſubſtract .037486 from .84 ; Now becauſe .84 hath in it but 2 places, and the other hath 6, I ſupply that defect by annexing 4 Cyphers thereto as in the Margent, and the work being finiſhed, I find the Remainder or difference to be .802514.

.840000
.037486
———
.802514

The ſame is to be obſerved when a decimal Fraction or mixt Number is given to be Subſtracted from a whole number, as ſuppoſe 15.486 were given to be ſubtracted from 64, becauſe there is no decimal annexed to 64, you are to ſupply the decimal places with Cyphers, and then proceed in the work as before is directed,

64.000
15.486
———
48.514

and having finiſhed the work of Subſtraction, the Remainder will be found to be 48.514 as by the work in the Margent appeareth.

Other Examples for Practice may be theſe following.

From	.3479	84.6	10
Subſtract	.2784	15.0752	0.2358
Remains	.0695	69.5248	9.7642

CHAP. V.

Multiplication of Decimal Fractions.

1. IN Multiplication of Decimals, whether both the Factors are decimal Fractions, or whether they be mixt Numbers, or if the one be a decimal Fraction, and the other a whole or mixt Number the Multiplyer is to be placed under the Multiplicand in the very ſame manner as in multiplication of whole Numbers, and when they are ſo placed, the operation is the ſame in every reſpect, as in Multiplication of whole Numbers, and when you have added the ſeveral particular products together, as is uſual in whole Numbers the value of the product is to be found out by this

General

General Rule.

Look how many Decimal places are in both the Factors, (*viz.* the Multiplicand and Multiplier) so many decimal places must be in the product.

Wherefore cut off so many Figures from the right hand of the Product for decimals, and the figure or figures remaining on the left hand (if there be any) are Integers, as in the following Example.

Let it be required to multiply 34.82 by 7.26 it matters not which you make the Multiplicand, or the Multiplyer, but I take 7.26 for the Multiplyer, because it hath fewest places, and put it in order under 34.82, as if they were both whole numbers, and having finished the work of Multiplication I find the Product to be 252.7932 as you may see by the following work.

```
    34.82
     7.26
   ------
    20892
    6964
   24374
  -------
 252.7932
```

Then to find the value of the Product, I look how many decimal places are in (both) the Multiplicand and Multiplyer, and I find 4, wherefore I mark the 4 first places to the right hand for decimals, by putting a point between them and the other figures on the left hand, and then the Product will appear to be really

252.7932 that is 252 integers, and .7932 decimal parts of an integer.

A ſecond Example may be of a mixt Number given to be multiplyed by a decimal fraction; as thus, let it be required to Multiply 38.5746, by .00463; I prepare the given numbers for operation as is before directed, and having finiſhed the work I find the Product to amount to 178600398. Then to find the true value of the product I conſider the number of decimal places, in both the Factors, which I find to be 9, *viz.* 4 in the Multiplicand and 5 in the Multiplyer, therefore I mark out nine places towards the right hand of the product of a decimal fraction, which indeed is the whole product, and therefore I conclude the true value of the product to be .178600398, as by the following operation appeareth, *viz.*

```
      38.5746
       .00463
   ----------
      1157238
     2314476
    1542984
   ----------
   .178600398
```

A Third Example ſhall be of 2 decimal Fractions, the one being given to be multiplyed by the other, as, let there be given .63478 to be Multiplyed by .8264, having diſpoſed of the given numbers according to order, and finiſhed the work of Multiplication as is before directed, I find the Product to amount to 524582192, which being done, to find the true value thereof, I conſider that there are 9 decimal places in both the Factors,

Factors, *viz.* 5 in the Multiplicand and 4 in the Multiplyer; wherefore I note out 9 places in the product for a decimal Fraction, and so I find the true value of the Product to be .524582192, as by the following operation appeareth.

```
     .63478
      .8264
     ------
     253912
    380868
   126956
  507824
  ---------
  .524582192
```

The like is to be underſtood in any of the like Caſes whatſoever.

II. If it ſo happen (as oftentimes it may) that after your Multiplication is finiſhed, the figures in the product do not conſiſt of ſo many places as there are decimal figures in the Multiplicand and Multiplyer, ſuch defect muſt be ſupplied by prefixing as many Cyphers before it towards the left hand, as it wanteth places, and then mark ſuch product with the ſaid prefixed Cyphers, for a decimal Fraction and the true product required; as in the following Example.

Let it be required to Multiply 0476 by .0642, after the Multiplication is finiſhed, I find the product to be 305692, conſiſting but of 6 places, but the number of decimal places in the Multiplicand and Multiplyer is 8, wherefore to make the product to conſiſt of 8 places, I prefix 2 Cyphers before it, and then the true product will be .00305592; the work followeth.

.0476
.0642
————
952
1904
2856
————
.00305592

In like manner if .376523 were given to be Multiplyed by .1346 you will find the product to be 506799958 consisting of 9 places, but there are 10 Decimal places in both the given Factors; wherefore the Product must be increased to 10 places by prefixing a Cypher which will make it .0506799958, as by the following work.

.376523
.1346
————
2259138
1506092
1129569
376523
————
.0506799958

By this time I doubt not but the diligent Learner is well acquainted with Multiplication of Decimal Fractions, the work being as plain and easie as in whole Nnmbers; The next we come to is Division.

CHAP.

CHAP. VI.

Diviſion of Decimal Fractions.

HAving gone through Addition, Subſtraction, and Multiplication, (The operation being (as you ſee) in every reſpect the very ſame as in whole Numbers) we come now to Diviſion; and although in Decimals, (as well as in whole Numbers) Diviſion may ſeem ſomewhat difficult to the young Practitioner, yet we ſhall endeavour to render it as plain and eaſie as poſſible may be.

I. The operation in diviſion of decimals is in every reſpect the ſame with that of whole numbers, therefore the difficulty in diviſion of Decimals lieth not in the operation, but in finding out the value of the Quotient after the work of Diviſion is ended, a general Rule for finding of which ſhall be given by and by.

II. It is neceſſary many times to annex a Cypher or Cyphers to the dividend, whither it be a whole Number, or a mixt Number, or a Decimal Fraction, for many times the diviſor, conſiſteth of more places than the dividend, and in that caſe there muſt be a competent Number of Cyphers annexed to the Dividend, as, ſuppoſe it were required to divide 73.564 by 46.24897, here you cannot conveniently proceed in the work till you have annexed Cyphers to the dividend, to increaſe the number of places in the decimal part thereof, and you may annex as many as you pleaſe,

for

for by the 7 Rule of the first Chapter, Cyphers annexed to a Decimal Fraction do neither Augment nor diminish its value.

III. When a question to be wrought by Division of decimals is proposed, consider whether there are as many decimal figures in the dividend, as there are in the divisor; if there be any wanting, make them full as many, or rather more by annexing Cyphers thereto, according to the Rule foregoing, but in some cases there must of necessity be more, for when there is an equal number of decimal places in the dividend, and in the divisor, and a division can be made, then the Quotient will infallibly be a whole number without any Fraction, except what is in the Remainder.

IV. In Multiplication of Decimal Fractions, the product containeth as many Decimal figures as there are decimal places in the Multiplicand and Multiplyer, and in Division if you multiply the Quotient by the divisor the product will be equal to the dividend, upon which consideration the true value of the Quotient of any division may infallibly be known by this

General Rule.

After the work of Division is ended, consider how many decimal places are in the dividend more than there are in the divisor, and how many soever the excess is, let so many in the Quotient be separated from the Rest, for a Decimal. But if there are not so many figures in the Quotient, as the said excess is, such defect must be supplyed, by prefixing as many Cyphers one the left hand, putting a point before them, as hath been Taught already; then shall such Decimal

mal as aforesaid, be the true value of the Quotient sought.

I shall explain this Rule by Examples of the several Cases that may happen in the division of Decimals, which are 9, as followeth.

1	a whole Number	*being given to be divided by*	a whole Number
2			a mixt Number
3			a Decimal Fraction
4	a mixt Number		a whole Number
5			a mixt Number
6			a Decimal Fraction
7	a Decimal Fraction		a whole Number
8			a mixt Number
9			a Decimal Fraction

Case 1.

A whole number given to be divided by a whole number.

V. When you are to divide one whole Number by another and they are not commensurable, though there are no decimals in either the dividend or the divisor, yet if you annex a Competent number of Cyphers to the dividend, there will be a decimal in the Quotient consisting of as many places as you annexed Cyphers to the dividend.

Example 1.

Let there be given 5729 to be divided by 438; According to the foregoing Rule, I annex a Number of Cyphers, (suppose 4) to the given dividend which will supply 4 decimal places, and it will be 5729.0000, and after the work of division is finished I find the Quotient to be 130799

438) 5729.0000 (13.0799, *&c*,

Now

Now to find out the value of the Quotient by the General Rule before-going, I consider that there are no decimals in the divisor, but there are 4 in the dividend, and consequently by the said Rule there must be 4 decimal places noted out in the Quotient by setting a point before them, and then the true value of the Quotient will be found to be 13.0799.

Example 2.

Let there be given 48 to be divided by 437.6, you cannot here make any work till you have annexed Cyphers to the dividend, because the divisor is bigger than the dividend, and therefore annex as many as you think convenient, suppose 6, and having finished the work of division you will find the Quotient to be 1095, now to find out its true value, consider that there are no decimal places in the divisor, but there are 6, in the dividend, therefore there must be 6 decimal places in the Quotient, but the Quotient as yet possesseth but 4 places, therefore to make them up 6 according to the said general Rule, I prefix two Cyphers before the other figures, on the left hand of the same, so as they may take place in the decimal by putting a point before them, so will the true Quotient be .001095, *&c.*

43796) 48.000000 (.001095, *&c.*

This first Case may very well serve for a further illustration of the first Rule of the second Chapter of this Book.

Case

Case 2.

Example 3.

A whole number given to be divided by a mixt number.

Let the whole number 586 be given to be divided by the mixt number 36.4865; here you may obſerve that although the dividend be greater than the diviſor, yet there can be no operation untill the dividend is prepared by annexing a competent number of Cyphers to it, and according to the third Rule of this Chapter, I muſt annex at leaſt 4, but here I ſhall take 6 (or more at pleaſure) and then the dividend will be 586.000000, and the work being finiſhed as in Diviſion of whole numbers, the Quotient will be found to be 1606, *&c.*

39.4865) 586.000000 (16.06, *&c.*

Now to diſcover the value of this Quotient, according to the general Rule foregoing, I conſider that there are four decimal figures in the diviſor, and 6 decimal places in the dividend, the exceſs being 2, and conſequently there muſt be two decimal places noted in the Quotient by putting a point before them, and then the true Quotient will be 16.06 as you may prove at your leiſure.

Example 4.

Another Example of the ſecond Caſe may be this, let there be given the number 2, to be divided by the mixt number 28.74, having prepared the dividend, by annexing 6 Cyphers to

it

it, (or more at pleasure) and finished the work of division as in whole numbers, I find the Quotient to be 695, *&c.*

28.74) 2.000000 (0695, *&c.*

Now to find out the true value of this Quotient I consider according to the general Rule, that there are but two decimal places in the divisor, and 6 in the dividend, therefore (the excess being 4) there must be 4 decimal places in the Quotient, but there are but three places, wherefore I make them up 4, by prefixing a Cypher before them, according to the latter part of the said General Rule.

Case 3.

Example 5.

A whole numb. given to be divided by a decimal Fract.

Let there be given the whole Number 48 to be divided by the decimal .0675, after the dividend is prepared by annexing a competent number of Cyphers, as suppose 7, after the work of division is ended, I find the Quotient to amount to 711111 as followeth.

.0675)048.0000000 (711.111, *&c.*

Now to find out the value of the said Quotient, by the foregoing general Rule, I consider that there are 4 decimal places, in the divisor, and 7 in the dividend, the excess being 3; wherefore I conclude that according to the said Rule, there must be 3 decimal figures in the Quote, cut off or separated from the rest by a point, and then the true value of the Quotient will be 711.111 that is 711 integers, and 111 decimal parts of an integer or very near.

Case

Case 4.

Example 6.

A mixt number given to be divided by a whole number.

Let there be given the mixt Number 743.574, to be divided by the whole Number 75.

After the dividend is prepar'd by annexing Cyphers at pleaſure, and the operation (according to diviſion of whole numbers finiſhed) you will find this Quotient, *viz.* 991432.

75) 743.57400 (9.91432

Now to find out the true value of the ſaid Quotient, I conſider that after there are 2 Cyphers annexed to the dividend, that the decimal part thereof will poſſeſs 5 places; and becauſe there are none in the diviſor, therefore the exceſs is 5, and conſequently (according to the ſaid General Rule) I note 5 Places in the Quotient for the decimal part, which being done, I find the true value of it to be 9.91432.

Example 7.

Again, let the dividend in the laſt Example, *viz.* 743.574 be given to be divided by the whole Number 43576, and the Quotient will be found to be 17063, if there be 3 Cyphers annexed to the dividend, and there will be 6 decimal places in it, and not one in the diviſor, wherefore there muſt be 6 decimal places in the Quotient, but there are but 5, therefore to make them 6, according to the ſaid General Rule, I prefix a Cypher, and then the true value of the Quotient will

will be .017063 as upon proof you will easily find.

43567) 743.574000 (.017063, *&c.*

Case 5.

Example 8.

A mixt number given to be divided by a mixt number.

Let the following mixt Number, *viz.* 3.748 be given to be divided by the mixt Number 46.375. Here according to former directions I annex Cyphers (at pleasure) to the dividend, suppose 5, then will the dividend be 3.74800000 and having finished the work of division, as if they were whole numbers, I find the Quote to be 8084, *&c.* but the true value of this Quotient thus found I as yet know not, therefore to make a discovery of its value I consider that in the dividend there are 8 decimal places, and in the divisor there are but three such places, therefore the number of decimal places in the dividend exceeds the number of places in the divisor by 5, so that by the foregoing general Rule I know that there must be 5 decimal places in the Quotient, but there are only 4 figures, *viz.* 8084, but to make them 5 according to the general Rule, I prefix a Cypher before the other figures and it makes .08084, which is the true Quotient sought.

46.375) 3.74800000 (.08084, *&c.*

Case

Case 6.

A mixt number given to be divided by a Dec. Fraction.

Example 9.

Let there be given the mixt number 54. 379 to be divided by the Decimal Fraction .34687, having annexed a competent number of Cyphers, so that there may be 3, or 4, or 5 Decimal places in the dividend more than there are in the divisor, wherefore I annex 6 Cyphers, and then the dividend will be 54.379000000, and when the work of Division is ended the Quotient will be found to be 1567705.

Which being done the next thing in order to the compleating of the work, is to find out the true value of the said Quotient, which is easily done by the said general rule, for I consider that in the divisor there are 5 decimal places but in the dividend there are 9 (*viz.* 3 given significant figures, and 6 Cyphers annexed) so that the excess is 4, therefore I conclude that there must be 4 Decimal places in the Quotient, and the rest are of the Integral part, so that I find the true Quotient is 156.7705, that is 156 Integers and .7705 or $\frac{7705}{10000}$ parts of an Integer, which you may easily prove at your Leisure.

.34687) 54. 379000000 (156.7705, &c.

Example 10.

If there were given the mixt Number 45 3 4 to be divided by .000247; here are not so many decimal places in the dividend as there is in the divisor, therefore do I increase their Number by annexing 5 Cyphers thereto, and then the dividend will be 45.38400000, then do I proceed to the operation, taking no notice at all of the the Cyphers which are before the Divisor, but work as if there were none at all, and when the work of division is finished, I find the Quotient to be 183740.89, *&c.*

.000247) 45.38400000 (183740.89, *&c.*

Now the Quotient being found, I come next to find out its value, which to do I consider that there are 6 decimal places in the divisor, and 8 in the dividend, so that the excess is 2 places, therefore I conclude according to the said General Rule, that there must be two decimal places noted in the Quotient, so that then its true value will be found to be 183740.89, *&c.*

Case 7.

A Decimal Fraction given to be divided by a whole number.

Example 11.

Let it be required to divide the decimal Fraction .07864 by the whole Number 25. here in this Example, there is no need of annexing any Cyphers to the dividend to prepare it for operation, but yet you may at your pleasure, only because there is no necessity I shall forbear it, and proceed to the work according to the Rule of Division in whole Numbers, and the work being finished, I find the Quotient to be ·· then I proceed to find out the value of this Quotient

Quotient by the General Rule foregoing, and because there is no decimal in the divisor, and 5 in the dividend, therefore there must be 5 decimal places in the Quotient, and there are but 3 places as yet, therefore do I prefix two Cyphers before the Quotient thus found, and note them for th true Quotient sought, which is .00314, as by the operation appeareth.

25.) .07864 (.00314

Case 8.

A Decimal Fraction given to be divided by a mixt Number.

Example 12.

Let there be given the Decimal Fraction .846 to be divided by the mixt number 3.476, here I i. prepare the dividend for the work by annexing 4 Cyphers thereto, and having finished the work of Division I find the quotient to be 2433, and to discover its value according to the general Rule, I consider that there are in the dividend (after the 4 Cyphers are thereto annexed) 7 decimal places, and in the divisor there are but 3, so that the excess is 4, therefore I conclude that there must be four decimal places in the Quotient, so that the true Quotient is the Decimal Number .2433, &c. as followeth.

3.476) .8460000 (.2433, &c.

But if to the said dividend .846 there had been annexed 5 Cyphers then the true Quotient would have been .24338, *&c.* and if there had been 6 Cyphers annexed thereto then had the Quotient been .243383, *&c.*

Example 13.

Let it be required to divide this Decimal Fraction, *viz.* .846 by the mixt Number 34. 76, after I have annexed Cyphers to the Dividend to prepare it for the work; and the work of Division being finished, I find (as before) the Quotient to be 2433, but the value of it being found out, by the general Rule, will be different from the former Quote, for having taken the number of decimal places, in the dividend and the divisor, I find the excess to be 5 in the dividend, so that there should be 5 decimal places in the Quotient, but there are now but 4 places, whereof to supply that defect I prefix a Cypher before the said Quote, and put the point before it so as it may take place in the Decimal, and then the true value of the Quotient will be .02433, *&c.* as followeth.

34.76) .8460000 (.02433, *&c.*

And if the Divisor had been 347.6 then the Quote would have been .002433, *&c.*

Case 9.

A Decimal Fraction given to be divided by a Decimal Fraction.

Example 14.

Let there be given the Decimal Fraction .835796 to be divided by .243, here I may annex Cyphers at pleaſure to the Dividend to prepare it for operation, but becauſe there is no neceſſity for it I ſhall forbear, and proceed to the work as in Diviſion of whole numbers, which being finiſhed I find in the Quotient the number 3439, and now I have nothing to do but to find out the true value of this Quotient, and in order thereunto I conſider that in the dividend are 6 decimal places, and in the Diviſor but 3, wherefore the exceſs is 3, which is the number of decimal places in the Quotient, which being ſeparated from the reſt by a point according to former directions, the true value of the Quotient will be found to be 3.439, *&c.*

.243) .835796 (3.439.

But if the dividend had had a Cypher annexed to it, then the Quotient would have been 3.4394 *&c.* and if two Cyphers had been annexed to it, then the Quotient would have been 3.43948, *&c.* But if the dividend had been .0835796, and the diviſor the ſame as before, the operation would have been ſtill the ſame, and the ſame figures would be in the Quotient but not of the ſame value, for they would have been all Decimals,

 viz.

viz. .3439, *&c.* But if the Dividend had been (as before) .835796) and the Divisor had been (.0243) the same as before with a Cypher prefixed before it to depress its value, though the operation be the very same, yet the value of the Quotient would have been 34.39, *&c.* And if the divisor had had two Cyphers prefixed before it thus .00243 then the Quotient would have been 343.9, *&c.* And if the Divisor had been (.000243) the same is before with 3 Cyphers prefixed before it then the Quotient would have been 3439. consisting intirely of Integers, except you have annexed Cyphers to the Dividend. Thus have I largely gone over all the cases that can happen in Division of Decimals, and have given one or more examples in every Case, so that I hope by this time the diligent Reader is made capable of performing any Operation, either in Addition, Substraction, or Multiplication, or Division of Decimals, and if he be so perfected, perhaps he may be desirous to know something of the use and application in the practical parts of Arithmetick, before he comes to the more difficult part of the extraction of Roots, and because I would not dull the edge of his Apetite, I shall give him a taste of their excellent use in the Rule of proportion, and in the Mensuration of some Superficies and Solids, and then come to shew their use in the extracting the Cube and Square Roots, and the calculating of Interest, *&c.*

CHAP. VII.

The Rule of 3 in Decimals.

I Shall not here meddle with the Rule of 3 in its diſtinct kinds, *viz.* Single, Double, Direct, or Inverſe, ſuppoſing the Learner to be acquainted with that already in the practice of vulgar Arithmetick.

I. In the Rule of 3 in decimals, the operation is in every reſpect the ſame as in whole Numbers, ſo is it in all the parts, or Rules of Arithmetick, only when you work in Decimals you muſt have reſpect to the Decimal Rules before taught, for in decimals you muſt Add, Subtract, Multiply, and Divide, when, and after the ſame manner as you do in whole Numbers, a few Examples will make you perfect in the knewledge thereof.

Example 1.

If $1\frac{3}{4}$ *l.* of Tobacco coſt 3 *s.* 6 *d.* how much will $326\frac{1}{4}$ *l.* coſt at that Rate?

When the Fractional parts of the Numbers in this Queſtion are turned into decimals, then it will be read thus, *viz.*

If 1.75 *l.* of Tobacco coſt 3.5 *s.* how much will 326.25 *l.* of the ſame coſt at that Rate?

The numbers being orderly placed as is directed in the 6 Rule of the 10 Chapter of my Vulgar Arithmetick will ſtand as followeth, *viz.*

l.		*s.*		*l.*
1.75	:	3.5	:	326.25

And if you multiply the third number by the ſecond, or the ſecond by the third, which is all one, and divide the Product thereof by the firſt, as is directed in the 10 Rule of the 7 Chap. of my Vulgar Arithmetick; only in Multiplying and Dividing, you muſt have regard to Multiplication and Diviſion of Decimals delivered in the two Chapters foregoing, and when the work is finiſhed, the anſwer will be found to be 652.5 ſhillings, or 32 *l.* 12 *s.* 6 *d.* ſee the following work.

l.		*s.*		*l.*		*s.*
1.75	:	3.5	: :	326.25	:	652.5

```
                 326.25
                    3.5
                 ------
                 163125
                 97875
                 ------          s.
       1.75)  1141.875  (652.5
              1050...
              -------
               918
               875
              -----
               437
               350
              -----
               875
               875
              -----
               (0)
```

Facit 652.5 shillings
or 32 *l.* 12 *s.* 6 *d.*

Example 2.

If 9 *C.* of Tobacco coſt 25 *l.* 7 *s.* what will be the price of 17 *C.* weight of the ſame at that Rate?

The given numbers being Rightly ſtated, together with the whole operation take as followeth.

C.

C.	l.		C.	l.
9	: 25.35	: :	17	: 47.883

```
          25.35
             17
         ------
          17745
          2535
         ------      l.
     9) 430.950 (47.883
        36
        ---
         70
         63
        ---
          79
          72
         ---
          75
          72
         ---
           30
           27
          ---
          (3)
```

Facit 47.883 *l.* or

47 *l.* 17. *s.* 8 *d. fere.*

Here you ſee that the anſwer in Decimals is 47.883 *l.* now the value of this decimal Fraction may be diſcovered at firſt ſight (by that brief way of finding the value of a decimal part of a pound Sterling, delivered in the 4 Rule of the 2 Chapter foregoing) to be 17 *s.* 8 *d. fere* for 8 primes is 16 ſhillings, and 8 ſeconds is 1 ſhilling more, and 3 ſeconds over, which with the 3 in the place of thirds makes 33, from which abating 1, becauſe it is above 25, there remains 32 farthings or 8 pence.

Example

Example 3.

If an ounce of Gold be worth 2 *l.* 19 *s.* 4 *d.* I demand the price of 19 *oz.* 3 *pw.* 5 *gr.* at that Rate?

By the 4 Rule of the 2 Chapter 3 *pw.* 5 *gr.* may be reduced to this decimal of an ounce, *viz.* .160416, so that 19.160416 is a mixt number equal in value to 19 *oz.* 3 *pw.* 5 *gr.* And by the same Rule .9666 is found to be the Decimal part of a pound sterling, equal in value to 19 *s.* 4 *d.* so that the decimals being found out, and the numbers given in the question being stated in order will be as followeth, *viz.*

oz.		*l.*		*oz.*		*l.*
1	:	2.9666	: :	19.160416	:	56.841
				2.9666		

l.
Facit 56.841, *&c.*
l. *s.* *d.*
or 56—16—10 *fere*

```
   19.160416
      2.9666
   ---------
   114962496
  114962496
 114962496
172443744
38320832
------------
56.8412901056
```

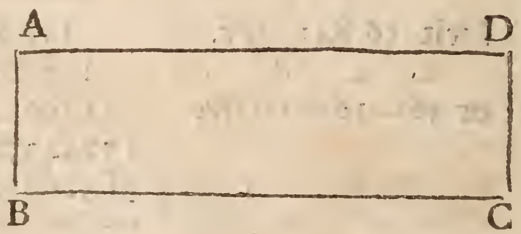
So that I find the answer to the Question to be 56.84129, *&c.* or 56 *l.* 16 *s.* 10 *d.* very near as it may be discovered by the brief way of finding the value of the decimal of a pound sterling delivered before in the 13 page.

CHAP. VIII.

The further uſe of Decimals in the Menſuration of Superficies and Solids.

PROP. I.

To Meaſure a long Square.



THis Figure by Geometricians is called a Rectangular Parallelogram, and it may very fitly be repreſented by a long ſquare Table, or a long Board, or the like, as the figure A B C D, in the Margent, and to find out its content the Rule is

Multiply the length of it in Feet or Inches, by the breadth of it in Feet, or in Inches, and the product will give you the true Superficial Content of it in Feet or Inches.

Example

Example.

There is a Table whose length is 18.75 Feet and its Breadth 3.5 Feet, I demand its content in Feet?

To answer this question, I take 18.75 Feet (the length of it) and multiply it by 3.5 Feet (the breadth of it) and the product is 65.625 feet which is the content of the Table, as was required. See the work.

```
 18.75
   3.5
 -----
  9375
 5625
 -----
```

Facit 65.625 Feet

Here by the way take notice that although amongst Artificers, the two foot Rule is generally divided each foot into 12 inches, &c. Yet for him that is at any time imployed in the practice of Measuring, it would be most necessary for him to have his two foot Rule, each foot divided into 10 equal parts, and each of those parts divided again into 10 other equal parts, so would the whole foot be divided into 100 equal parts and thereby would it be made fit to take the dimensions of any thing whatsoever, in feet and decimal parts of a foot. and thereby the content of any thing may be found as exactly if not more exactly and near, than if the foot were divided into Inches, quarters and half quarters, and

and thereby many times would there be much labour and pains avoided, which the Arteſt is Content to undergo through the want of ſuch Decimal diviſion of this Rule, as we will ſhew in the ſolving of the former propoſition, after the vulgar way. The Queſtion is

There is a Table whoſe length is 18 foot 9 inches, and its breadth 3 foot 6 inches, now I demand its content in feet?

Now before I can find its content, I muſt find its length and breadth in Inches, and then multiply the inches of the length by the inches of the breadth, and then the product will be 9450 which is its content in ſquare inches, and to find its Content in feet I muſt divide the inches by 144 (the number of ſquare inches in a foot) and the Quotient is the content in feet: See the work following

```
 f.  in.              f.  in.
 18—09                3—06
 12                   12
 ______               ______
length 225 inches   breadth 42 inches
         42
       ____
        450
       900
       ____
144) 9450 (65 90/144 feet
     864.
     ____
      810
      720
      ___
       90
```

So

So that you see according to this way the answer is 65 square feet and 90 inches, or $\frac{90}{144}$ of a foot which is the very same with that answer in decimals, and if the Division by 144 had been continued by annexing Cyphers to the Dividend 9450, there would have come out in the Quotient the Decimal .624 as before.

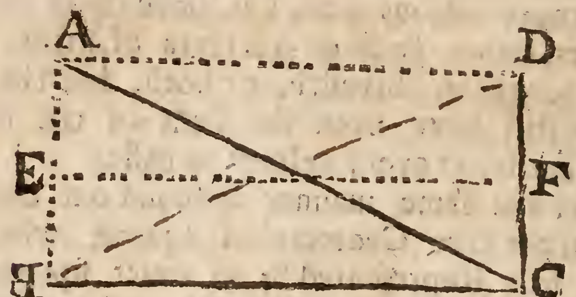
But how tedious a work it is to answer it after the vulgar way, compared with the decimal way I leave the Judicious Reader to judge, and much more tedious would it have been if there had been either halfs or quarters of Inches either in the length or breadth, or both, but the work would still have been the same in the decimal way, that is, in every respect as easie.

After the same manner is found out the Content of the true Geometrical square, which is a figure fitly Represented by an exact square Trencher, that is, having its length and breadth both equal.

PROP. II.

To find the Content of a right angled Triangle.

A Right angled Triangle is a plain figure having 3 sides and 3 Angles as the figure B,C, D, in the Margent, two of which sides *viz.* BC and CD, are perpendicular to each other, now if from the top of the perdendicular at D, there be a line drawn parallel to the base, B, C, B, C, as is the Prickt line A, D, and from the end of the Base at B, there be drawn the prickt line B, A parallel to the perpendicular till it meet the line A, D in A, then will there be made the parallelogram, or long square A, B, C, D, of which the Triangle B, C, D, is half, the Diagonal B, D. dividing the whole parallelogram into 2 equal parts.



Now it is plain from the first proposition, that if you multiply the side B, C, by the side C, D, then the product will be the Content of the whole parallelogram A, B, C, D, and then the half of that Content will be the Content of the given Triangle B, C, D. Or if you take half C, D, which is C, F, and half of B, A, which is B, E, and draw the line E, F, then will E, F, divide the parallelogram A, B, C, D, into two equal parallelograms, and either of them is equal to the given Triangle B, C, D, now if by the first proposition I can find the Content of the parallelogram E, B, C, F, I find also the Content of the Triangle B, C, D, because they are equal, whence it comes to pass that if you multiply the base by half the perpendicular, or the perpendicular by half

half the base of a Rectangular Triangle (which is all one) the product will be the true Content thereof.

Example.

In the former Triangle the base B, C, is 18.28 Feet, and the perpendicular C, D, is 12.26 Feet, I demand its Content in Feet?

Here I first find the Content of the whole Parallelogram, by multiplying the sides together, and the Product is 224.1128 Feet; and the half of that product is the Content of the Triangle B, C, D, which is 112.0564 Feet. See the following work.

```
      18.28  the side B, C,
      12.26  the side C, D.
      ----------
      10968
      3656
     3656
    1828
   -----------
2) 224.1128 (112.0564 Feet
```

The Content would have been the same, if I had multiplyed the one side by half the other, which is indeed the shortest way, and most practical, See the work.

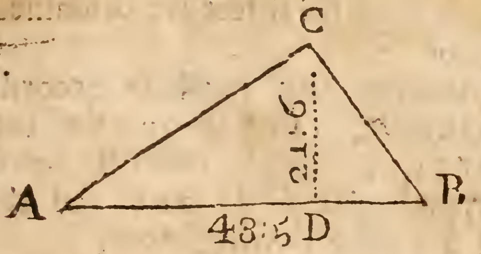
the whole side B C,	18.28
½ the side C D	6.13
	5484
	1828
	10968
Facit	112.0564 Feet

The answer would have been the same, if I had taken the whole side C D, and Multiplyed it by half the side B C.

PROP. III.

To find the Content of any plain Triangle, not Rectangular.

THE best and easiest way is to let fall a perpendicular, upon the longest side from the angle that is opposite to it, which will divide it into two right-angled plain Triangles, as suppose there were given the plain Triangle A B C, as followeth.



Here the ſide A B, being the longeſt ſide, I let fall a perpendicular from its oppoſite angle at C, which falleth upon D, in the line A B, ſo is the line C D the neareſt diſtance between the angular point C, and the line A B, and divideth the given Triangle A B C, into two Right-angled Triangles, (*viz.*) A D C and C D B; and if you find the content of theſe two Right-angled Triangles (according to the directions in the ſecond propoſition) and add them together, their ſum will be the content of the given Triangle A B C. But it may be more artificially found out thus,

Multiply half the line C D, into the whole line A B, the Product will give the Content of the Triangle which was ſought, or if you multiply the whole line C D, into half the line A B, the product will be the Content of the given Triangle, which is very plain from a due conſideration of the method uſed in ſolving the ſecond propoſition.

Example.

Let the baſe or longeſt ſide A B be 48.5 Feet long, and let the length of the perpendicular CD be 21.6 Feet, I deſire to know how many ſquare, or ſuperficial feet are contained in the ſaid Triangle?

To reſolve this queſtion according to the foregoing Rule, I firſt Bi-part the Baſe A B 48.5 which is 24.25 which I multiply by the length of the perpendicular C D 21.6 and the Product is 523.8: See the following work,

```
  24.25     or thus     48.5
   21.6                 14.8
  ------               ------
  14550                 3880
  2425                 4850
 4850                  ------
 ------                523.80
 523.800  facit.
```

So that you ſee the content is the ſame which of the foreſaid ways ſoever you work; obſerve the ſame method in finding the content of any oblique Triangle given.

PROP. IV.

To find the Content of a Trapezium.

A Trapezium is a plane Figure having four unequal ſides, and as many unequal Angles it matters not how unequal they are, and to find out its content obſerve the following directions, *viz.*

Divide

Divide it into two oblique Triangles, by drawing a line from any one of the angles, to the angle that is opposite thereto, which line shall be a common base to both the Triangles.

Then if you find out the content of both these Triangles, according to the method prescribed in the third proposition, the sum of their contents, is the content of the given Trapezium.

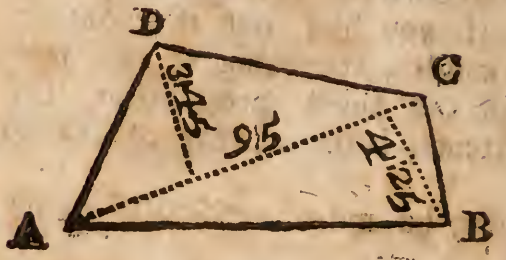
Or it may be more artificially found out thus, *viz.*

Multiply the length of the common base by half the sum of the perpendiculars let fall from the angles opposite to the said common base, and the product will be the content of the whole Trapezium: or else

Multiply the sum of the said perpendiculars by half the said common base, and it will produce the same effect.

Example.

In the following Trapezium A B C D, draw the base A C, which suppose to be 9.5. Feet, then let fall the perpendicular at D, which let be 3.45 Feet, and that at B 4.25 Feet, the sum of the said perpendiculars is 7.7 half of which is 3.85, by which if the common base A C be multiplyed, the product (which is) 36.575 is the content of the Trapezium required. Or if you multiply 7.7 the sum of the perpendiculars by half of the common base 9.5, which is 4.75 the product will be the same. See the following work.



3.45	the perpendicular at D
4.25	the perpendicular at B
7.70	their Sum
3.85	their half Sum
9.5	A C

1925
3465

facit 36.575

4.75 half A C
7.7

3325
3325

facit 36.575

PROP.

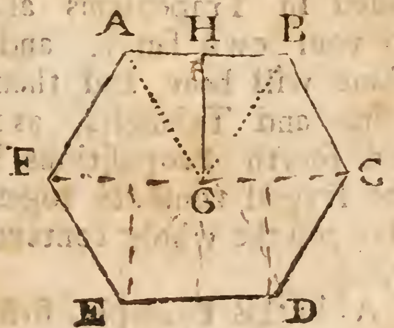
PROP. V.

To find the Content of any regular Polygon.

A Regular Polygon is a plane Figure consisting of equal sides and equal Angles, *viz.* a Pentagon, consisting of 5 equal sides, and 5 equal Angles; an Hexagon, consisting of 6 equal sides, and 6 equal Angles; an Heptagon of seven equal sides, an Octogon of eight, *&c.* and to measure any one of these regular planes, do thus draw a line from the Centre of the Figure to the middle of any one of the sides, and multiply that line into half the sum of the sides, and the Product thence arising is the content of the given plane.

I shall give you an Example of this in the Mensuration of an Hexagon, or plane of 6 equal sides.

Let there be given the Hexagon A B C D E F, having the length of each side 30 then will the length of the perpendicular G H be 26 *fere*, now there being in all 6 sides, and each of them in length 30, the sum of them all is 180 the half of which is 90, which being mul-



tiplyed by 26, the product will be 2340, which is the content of the given Hexagon.

The reason of this manner of working is very plain, if from the Centre you draw the lines G A, and G B, thereby making the Triangle G A B, whose content (by the third proposition) is found by multiplying the perpendicular G H into half the side A B, *viz.* into H A, or H B, but there are 6 such Triangles in the given Polygon; therefore G H, multiplyed into 6 times H B, produceth the content required.

PROP. VI.

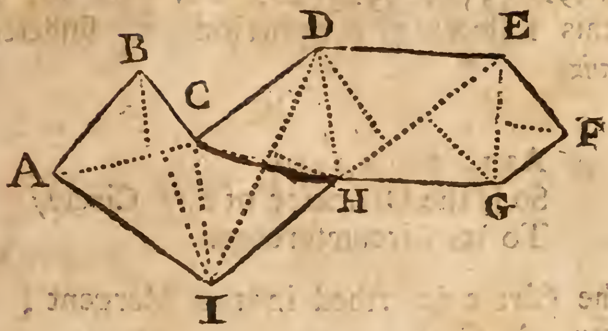
To find the Content of any Irregular Polygon.

LET it be required to measure the following Figure A B C D E F G H I.

First take care that the whole plane be divided in Trapeziums and Triangles according to your own fancy, and as the nature of the plane will bear, and then measure those Trapeziums and Triangles, as is directed in the third and fourth Propositions before going, and add the several contents together, so will the sum give you the whole content of that Irregular Polygon.

As in this Example, first I draw the lines A C, and D H, and E G, so is the whole figure divided

into



into the Trapeziums A B C I, and C I H D, and D H G E, and the Triangle E G F, the contents of which being ſeverally found out by the third and fourth Propoſition, the ſum of them will be the content of the whole Figure.

PROP. VII.

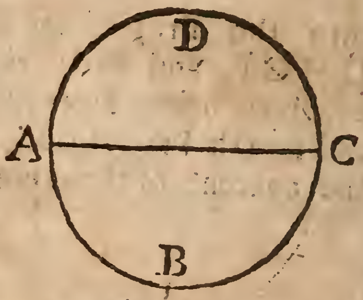
To find the circumference of a Circle having the Diameter given.

A Circle is a Geometrical Figure exactly round, ſo that if from a point in the middle of it called the Centre, there be never ſo many lines drawn to the Circumference, they will all be of equal length. But between the diameter and circumference of a circle there cannot be found a true and exact proportion. *Archimedes* hath demonſtrated the proportion to be near as 7 is to 22; but that of *Van Ceulen* is the

the most exact, who makes it to be as 1, is to 3. 14159265358979323846, *&c.* but for practice this following proportion is sufficiently exact, *viz.*

As 1. is to 3.1416
So is the Diameter of any Circle,
To its Circumference.

In the Circle described in the Margent, the Diameter A C is 28, I demand what is the circumference A B C D?



To answer which I say by the proportion foregoing; As 1 is to 3.1416, so is 28 the diameter to 87.9648, which is the circumference A B C D. The work followeth.

As 1 to 3.1416, so is 28 to 87.9648

```
   28
------
251328
62832
------
87.9648
```

PROP.

PROP. VIII.

To find the content of a Circle having the Diameter given.

FIrst find out the circmference, by the last Proposition, then multiply half the circumference by half the Diameter, and that product is the Content.

Example.

There is a Circle whose Diameter is 14 Inches, I demand how many square Inches are the content of that Circle?

By the foregoing Proposition, I find the circumference to be 43.9824 Inches, the half of which is 21.9912 which being multiplyed by 7, (half the given Diameter) the product is 153.9384 which is the content required. See the work.

21.9912
7

facit 153.9384 square Inches.

PROP. IX.

To find the solid content of a square piece of Timbtr, Stone, &c. Whose bases are equal, that is, whose ends are of the same bigness.

SUch a solid piece by Geometricians is called a Parallepipedon, and its content is thus found out, *viz:*

First find out the superficial content of the base or end, (by the first proposition) then multiply that content by the whole length, and that Product is the solid content of the whole piece.

Example.

There is a square piece of Timber, the two contiguous sides at the end of which are 2.5 Feet, and 1.8 Feet, and its length is 22 Feet, I demand how many solid Feet are in that piece of Timber.

First I multiply 2.5 by 1.8 the sides of the base, and that produceth 4.5 for the content of the base or end, and that Product I multiply by 22 the length, and that produceth 99 Feet, and so many is there contained in that piece of Timber. As you may see in the following work.

2.5 } the 2 sides at the end
1.8 }

200
25

4.50 the superficial content of the end
22

900
900

facit 99.00 Feet for the content Required.

Here note, if the sides of the end, or base, be given in Inches, and its length in Feet, then Reduce the sides of the base into the Decimal parts of a Foot, and proceed as before, or you may find out the content of the base in Inches, and multiply that content by the length in Feet, and that product divided by 144, will give you the content in Feet, or else reduce the length into Inches, and multiply the content of the base thereby, and divide that product by 1728 (for there are so many Cubical Inches in a Foot) and the Quotient will give you the solid content in Feet. But the Decimal way is preferred.

PROP.

PROP. X.

To find the Solid content of a Cylinder, having the Diameter of its Base given.

A Cylinder is a ſolid whoſe baſes are Circular, equal, and parallel, and may fitly be repreſented by a round pillar, or a Roling-ſtone of a Garden, and to find the ſolid content of ſuch a body this is

The Rule.

Firſt find the plane of the baſe, by the 7 and 8 propoſitions foregoing, and then multiply that by the length thereof, which product will give you the ſolid content of the given Cylinder.

Example.

There is a Cylinder, (ſuppoſe a Roling-ſtone) whoſe length is 8.75 Feet, and the Diameter of its baſe 2.8 Feet, I demand the ſolid content thereof?

As 1
Is to 3.1416
So is 2.8 the given Diameter.
To 8.79648 the Circumference of the baſe, half of which (*viz.* 4.39824) being multiplyed by 1.4 the ſemi-diameter will produce 6.157536 for the content of the Baſe, which being

being multiplyed by 8.75 (the length) it produceth 53.87844 for the ſolid content required.

If there had been given the circumference of the Cylinder, then the Diameter of the baſe muſt have been found out by the converſe of the ſeventh Propoſition, as ſuppoſe there had been given 8.75 the length of the Cylinder, and 8.79648 its circumference to find the ſolidity thereof. Firſt I find out the Diameter by the following proportion, *viz.*

As 3.1416
Is to 1
So is 8.79648 the given Circumference
To 2.8 the required Diameter.

And then the reſt of the work is the ſame with that before.

PROP. XI.

To find the Solid Content of a Cone.

A Cone is a Solid Body, having a Circle for its baſe, and its ſuperficies Circular, decreaſing its equidiſtant Diameters from the baſe proportionably, till it remaineth in a point over the Centre of its baſe, and may fitly be repreſented by a Sugar-loaf, or a round Spire Steeple; and to find its ſolid Content this is

The Rule.

By the 7 and 8 Propoſitions foregoing find out the plane of its baſe, and multiply that by ⅓ of its

its height, and that product is the Solid content of the Cone Required.

Example.

There is a Cone the circumference of whose base is 22.5 and its height is 16, I demand the solid content of such a Cone?

As 3.1416
Is to 1.
So is 22.5 the cirumference of the base
To 7.162 the diameter of the base.

Then I multiply half 22.5 which is 11.25 by half 7.162 which is 3.581, and it produceth 40.28625 which is the superficial content of the base; then I take ⅓ of the height of the Cone (16) which is 5.333 very near, by which I multiply 40.28625 (the superficial content of the base) and it produceth 214.84657125. See the work as followeth;

```
   40.28625
      5.333
   --------
   12085875
  12085875
 12085875
20143125
-----------
```

Solid Content is 214.84657125

The same is to be observed in the mensuration of any other Cone.

But here you are to observe that the slanting side of the Cone, (*viz.*) the length from the vertex

vertex to the extremity of its Base) is not to be taken for its true height, but a perpendicular let fall from its vertex or centre of its Base is its true height; and how you may find out that perpendiculars length shall be shown you in the work of the fourth Proposition of the eleventh Chapter.

PROP. XII.

To find the solid Content of a Pyramid.

BEtween the Cone and Pyramid, this is the Difference, as the Cone hath a circular base and superfices, the Pyramid hath a Polygon for its base, so that its base and superficies are Angular, its vertex terminating in a point just over the Centre of its base, and to find out its solidity, here followeth

The Rule.

Find out the superficial content of the base, by the fifth Proposition foregoing, and multiply that by $\frac{1}{3}$ of its height, and it produceth the solid content of the Pyramid.

Example.

There is a Pyramid whose base is an Hexagon the side of which is 30, and its perpendicular height is 54; I demand the ſolid content of ſuch a Pyramid?

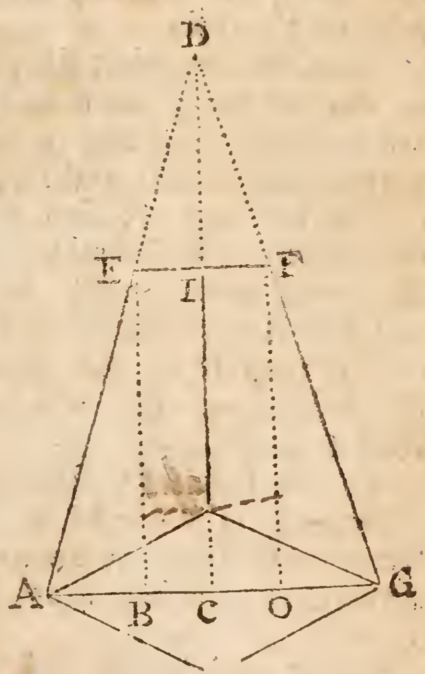
Here by the fifth Propoſition I find the ſuperficial content of the baſe to be 2340; then do I take $\frac{1}{3}$ of the perpendicular height of the Pyramid, which is 18, and thereby do I multiply 2340, (the plain of the baſe) and the product is 42120, which is the ſolid content of the given Pyramid.

Here note by the way, that a line drawn from the point at the top of the Pyramid, to the extremity of any part of the baſis, is not the true height of any Pyramid, but a perpendicular let fall from the Cuſpis (or top) to the centre of the baſe is the true height, and how to find out ſuch perpendicular heights ſhall be ſhewn in the fourth Propoſition of the 11 Chapter.

PROP.

PROP. XIII.

To measure the Frustum of a Pyramid or Cone.



THE Frustum here given to be measured is A G E F, the side of the greater base at A being 24 Inches, and the side of the lesser base at E being 8 Inches, and the length of it I, C, 20 foot equal to EB, or F O.

It is evident that if I find the solidity of the whole Pyramid AGD, & also the solidity of the lesser Pyramid E F D, and then subtract the content of E F D, from the contentof A G D, that there will remain the solidity of the Frustum A G E F; and certainly this way of measuring the Frustum of a Pyramid or Cone, is the most exact of any: And it may be easily measured thus, first of all find out the height of the whole Pyramid CD, which you may do by the following proportion, *viz.*

As the Semi-difference of the ſides of the Baſes,
Is to the height of the Fruſtum,
So is the half ſide of the greater Baſe,
To the height of the whole Pyramid.

And this proportion will hold good if you work by the Semi-difference of the Diameters of the baſes, as well as by the Semi-difference of the ſides of the Baſes.

As in the foregoing figure, let AG be the Diameter of the greater baſe, and EF the Diameter of the leſſer baſe, from E and F let fall the perpendiculars EB and FO, then ſhall BO, be equal to EF, and the ſum of AB and OG are the difference of the Diameters of the baſes, EF and AG; and conſequently AB is the Semi-difference, and BE is the height of the Fruſtum, and AC is half the ſide of the greater baſe, and CD is the height of the whole Pyramid. Then by *Eucl.* 6. 4.

As AB (the Semi-difference of Diameters)
Is to BE (the height of the Fruſtum)
So is AC (half the greater Diameter)
To C D (the height of the whole Pyramid).

So the height of the whole Pyramid AGD, will be found to be 30 foot; for the greater Diameter A G, is 24 Inches, the leſſer 8, the difference 16, the Semi-difference 8, therefore ſhall C D be 30 foot; for

8 : 20 : : 12 : 30

Now

Now having found the height of the whole Pyramid to be 30 foot, I thereby (according to the 12th. Propoſition foregoing) find the content of the whole Pyramid to be 40 foot, then in the leſſer Pyramid EFD there is given the ſide of its baſe EF, 8 Inches, and its height ID 10 Inches for CD 30—CI 20—ID 10, and by the ſaid 12th. propoſition I find the ſolid content of it to be 1.48 Feet, which being Subtracted from 40 (the content of the greater Pyramid) there will remain 38.52 feet for the true ſolid content of the given Fruſtum AGEF.

After the ſame manner is found the ſolidity of the Fruſtum of a Cone, the height of the whole Cone being found out by the difference of the Diameters of its baſes; and by the 11th. propoſition find the ſolidity of the whole Cone, and alſo the ſolidity of the leſſer Cone, that is cut off from the Fruſtum, then Subtract the content of the leſſer from the content of the greater, and the remainder will be the ſolid content of the Fruſtum.

This laſt propoſition is uſeful in the meaſuring of tapering Timber Round or Squared, and f r finding the liquid capacity of Brewers Conical, or Pyramidal Tuns.

Thus have I ſhewed the Uſe of Decimals in the Menſuration of the moſt uſeful Planes and Solids, I might proceed farther to ſhew their Application in the particular Menſuration of Board, Glaſs, Pavement, Plaiſtering, Painting, Wainſcot, Tiling, Flooring, Tapiſtry, Brickwork, Timber and Stone; but it requireth (rather) a particular Treatiſe, than the narrow bounds here allowed for ſuch a work.

CHAP. IX.

The Extraction of the Square Root.

IN the Solution of any Question, or in the working of any ſum whatſoever belonging to any of the Rules of Vulgar or Decimal Arithmetick, there have been (at leaſt) two things or numbers given, whereby the anſwer might be found; but in the extraction of the Square, Cube, and all other Roots, there is but one number given to find out the number ſought; *viz.* there is a ſquare Number given to find its Root, a Cube Number to find its Root, *&c.* And

I. A ſquare Number is that which is produced by multiplying any number by (or into) it ſelf, which Number given to be ſo multiplyed is called the Root; as if the Number 8 were given to be multiplyed by it ſelf, it produceth 64, then is 8 called the Root, and 64 is its ſquare, ſo the Root 12, hath for its ſquare 144.

II. When a ſquare Number is given, and its Root is required, the Operation it ſelf is called the Extraction of the Square Root.

III. Square Numbers are of two kinds, *viz.* either Single or Compound.

IV.

IV. A ſingle ſquare Number is that which is produced by the Multiplication of a Digit, or ſingle Number into it ſelf, and conſequently ſuch a ſquare Number muſt be under 100, which is the ſquare of 10, ſo 25 being given for a ſquare number, it is a ſingle ſquare having for its Root 5. And 81 is a ſingle ſquare Number, having for its Root the Digit 9. All the ſingle ſquare numbers with their Roots, are contained in the following Tablet.

Roots	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	81

V. When the Root of any ſquare Number is required it being leſſer than 100, and yet is not exactly a ſingle quare, expreſſed in the Tablet above, then you are to take the Root of that ſingle ſquare Number expreſſed in the ſaid Tablet, which (being leſs) is neareſt to the given ſquare; as if it were 74 whoſe Root is required I find that 81 (the ſquare of 9) is too much, and 64 (the ſquare of 8) is too little, but yet it is the neareſt ſquare number that is leſſer than 74, and therefore I take 8 to be the ſquare Root of 74, but yet it is plain that 8 is too little for the Root of 74. And to find out the Fractional part of this Root, you ſhall be plainly taught by and by.

VI. A compound ſquare number is that which hath above 9 for its Root.

VII. The Root of a ſingle ſquare Number may be diſcovered at the firſt ſight, but the Extradicti-on of a compound ſqaure Number is more tedious and difficult, its Root conſiſting of two places, at the leaſt, and the ſquare it ſelf, of 100 at the leaſt.

VIII. When a compound ſquare number is giv-en, and it is required to have its ſquare Root ex-tracted, before you can proceed to the Operati-on, your ſquare number muſt be prepared, by pointing it at every ſecond figure, beginning at the place of Units.

As, ſuppoſe you were to extract the ſquare Root of 2304, firſt I put a point over 4 (it ſtanding in the place of Units) and then paſſing over the ſecond place (or place of Tens) which is 0, I put a point over the figure ſtanding in the third place, (or place of Hundreds) which is 3, and the preparative work is done, as you may ſee in the Margent.

$\dot{2}3\dot{0}4$ — 2304.

Now if there had been more places in the given number, then I muſt have put a point over the figure ſtanding in the fifth place, and another over that in the ſeventh &c. And here note, that as many points as you put over the given ſquare number, ſo many Fi-gures there will be in the Root, that is, the Root will conſiſt of ſo many places.

So if there were given the number 33016516, to have its ſquare Root extracted, after I

33016516

have

have pointed it according to the Direction before given, it will ſtand as in the Margent, and becauſe the points that are put over it are in number 4, I conclude the Root it ſelf will conſiſt of 4 places, or figures.

IX. When you have thus prepared your number, then draw a crooked-line on the Right-hand of your number, behind which to place your Root, as you do for a Quotient in Diviſion.

Note that when your number is prepared for Operation, as in the 8 Rule, the numbers containing between point and point, may not unfitly be termed Squares, and in the enſuing work, we ſhall ſo call them, as in the foreſaid number 33016516, being pointed as before, I call 33 the firſt ſquare, 01, the ſecond, 65, the third, and 16 the fourth, and laſt ſquare; every ſquare (except ſometimes the firſt conſiſting of two figures, or places, the laſt of which towards the Right-hand hath always a point over it, and if it ſo happen (as it often doth) that the laſt Figure (in any given ſquare number) toward the left hand hath a point over it then that number alone ſhall be accounted the firſt ſquare.

As if the number 676 were given, when it is pointed for the work according to Direction, as you ſee in the Margent I account 6 for the firſt ſquare and 76 for the ſecond.

$\dot{6}7\dot{6}$

Theſe things being underſtood, we ſhall lay down thoſe general Rules requiſite for the management of the work it ſelf.

X. When your number is prepared, find out the

the ſquare Root of the firſt ſquare, according to the 5 Rule foregoing, and place that Root behind the ſaid crooked line. As

Let it be required to extract the ſquare Root of the ſaid number 2304, here the firſt ſquare number is 23, and (according to the ſaid 5 Rule) its Root is 4, which I place behind the crooked line as you ſee in the Margent.

```
. .
2304 (4
```

XI. Then ſquare the ſaid Root, and place its ſquare which is 16 under the ſaid firſt ſquare 23, and having drawn a line underneath, ſubtract the ſaid ſquare 16, from 23, and place the Remainder, which is 7 underneath the ſaid line as you may percieve by the work in the Margent.

```
. .
2304 (4
16
------
  7
```

XII. Then to the ſaid Remainder bring down the Figures of the next ſquare and annex them thereto on the Right-hand, ſo that they may make one intire number, which (for diſtinctions ſake) we ſhall call the Reſolvend.

As in this Example to the Remainder 7, I bring down the next ſquare 04, and annex it thereto, and it maketh 704 for a Reſolvend, as you may ſee in the Margent.

```
. .
2304 (4
16
------
  704 Reſolv.
```

XIII. Always let the whole Reſolvend (except the laſt Figure on the right hand) be eſteemed a Dividend, on the left hand of which draw a crooked line before which to place a Diviſor, as in Diviſion.

So

So in this example, the Reſolvend 704 is to be made a Dividend, all but the laſt place which is 4, ſo that the Dividend is 70, before which I draw a crooked line, as you ſee in the Margent.

```
 . .
2304(4
16
------
) 704
```

XIV. Let the Quotient expreſſing the Root (or part of the Root ſought) be doubled, or multiplyed by 2, and that double or product ſhall be a Diviſor, and muſt be placed on the left hand of the Reſolvend, before the ſaid crooked line.

So in our Example, the number 4 which was put for part of the Root being doubled makes 8, which I put before the Reſolvend for a Diviſor, as it appears in the Margent.

```
  . .
 2304 (4
 16
 ------
8) 704
```

XV. Then (according to the Rule of Diviſion in whole numbers) ſeek how often the ſaid Diviſor is contained in the ſaid dividend, and put the anſwer down in the Quotient, and alſo on the Right hand of the Diviſor.

As in our Example I ſeek how often the Diviſor 8 is contained in the Dividend 70, which I find to be 8 times, therefore I put 8 in the Quotient for part of the Root, and alſo on the Right hand of the Diviſor. See the work in the Margent.

```
    . .
   2304 (48
   16
   ------
88) 704
```

XXVI. Then

XVI. Then by the Figure laſt put for part of the Root, multiply the ſaid Diviſor together with the Figure that you annexed to it (accounting them both as one intire number) and place the product underneath the ſaid Reſolvend, drawing a line under it, and then ſubtract it out of the ſaid Reſolvend, placing the Remainder beneath the line.

As in our Example, having placed 8 in the Quotient, and alſo on the Right-hand of the Diviſor, then in the place of the Diviſor, their ſtands 88, which I multiply by 8, the number laſt put in the Quotient, and the product is 704, which I place in order under the Reſolvend 704, and having drawn a line underneath, I ſubtract the ſaid product 704, from the Reſolvend 704, and there remaineth 0, ſo is the work finiſhed, and I find the ſquare Root of 2304 to be 48. See the work in the Margent.

```
       2304 (48
       16
       ----
  88)  704
       704
       ----
       (0)
```

Here note that if at any time when you have multiplyed the number ſtanding in the place of the Diviſor, by the Figure laſt placed in the Quotient, or Root (as is directed in the laſt Rule) if the product be greater than the Reſolvend, then conclude the work to be erroneous, to correct which put a leſſer Figure in the Root, and proceed as is before directed.

1. Note.

Note alſo that the work of the 12, 13, 14, 15, and 16 Rules, muſt be repeated as often as there

there are points over the Figures, except for the first square, which is to be wrought according to the Directions given in the 10 and 11 Rules foregoing, and the work of those two Rules is to be observed, but once in the extraction of a square Root, tho' it consist of never so many squares or points. 2. Note.

These things will appear plain and easie in the working of one or two more Examples.

Example 2.

Let it be Required to extract the square Root of 33016516.

Here In order to the work. I first prepare my number by distinguishing it into squares, by pointing it according to the 8 Rule foregiong and thereby I find that 33, is the first square, and (according to the 10 Rule) I take the square Root of 33, which is 5, and place for the first Figure of the Root, then (according to the eleventh Rule) I square the Root (5) and it makes 25, which I place under the said first square number 33, and subtract it therefrom, and the remainder (8) I place below the line, as in the following work.

```
 . . . .
33016516 (5
25
--
 8
```

Then (according to the twelfth Rule) I annex to the said remainder (8) the next square (01) and

and it makes 801 for a Resolvend, then must 80 (according to the thirteenth Rule) be my dividend, and (according to the fourteenth Rule) I double the number (5) in the Root, and it makes 10 for a Divisor, and thereby I divide the said dividend (80) and I find that it Quotes 7, which (according to the fifteenth Rule) I put in the place of the Root after 5, and likewise before the Divisor, (10) so that in the place of the Divisor instead of 10, there is now 107.

Then (according to the sixteenth Rule) I multiply the said 107, by 7, (the figure last placed in the Root) and the product is 749, which I place orderly under the said Resolvend, and subtract it therefrom, and the remainder is 52 which I put below the line, as in the following work.

```
        . . . .
       33016516 (57
       25
       -------
  107) 801 Resolvend.
       749
       ---
        52
```

Then I repeat the same work over again, in finding the next Figure of the Root, as I did in finding the last, *viz.* to the remainder (52) (according to the twelfth Rule) I bring down and thereto annex the next (third) square (65) and it makes 5265 for a new resolvend, then (according to the thirteenth Rule) is 526 a new dividend, and (according to the fourteenth Rule) I take the Root (57) and double it for a new Divisor

visor and it makes 114, which place before the resolvend (5265.)

Then (according to the fifteenth Rule) I seek how often the divisor (114) is contained in the dividend, (526) and I find it will bear 4, which I place in the Root orderly, and also on the right hand of the Divisor, (114) and then there will be in the place of the Divisor, the number 1144, which (according to the sixteenth Rule) I multiply by the Figure (4) last put in the Root, and the product is 4576, which I place orderly under the resolvend (5265) and subtract it therefrom, and the remainder is 689 which I place under the line, as is before directed. See the whole work as followeth.

```
        . . . .
        33016516 (574
        25
        --------
  107)  801 Resolvend
         749 Product
        --------
 1144)  5265 Resolvend
        4576 Product
        --------
         689
```

Then I again repeat the work of the 12, 13, 14, 15, and 16 Rules of this Chapter for finding the next Figure of the Root, *viz.* first I bring down (16) the next square number, and annex it to the remainder 689 (according to the twelfth Rule) and it makes 68916, for a new resolvend, of which (by the thirteenth Rule) 6891 is a new Dividend then (according to the fourteenth Rule)

Rule) I double the Root, and it makes 1148 for a diviſor, which I place on the left ſide the reſolvend, and then ſeek how often it is contained in the ſaid dividend (6891,) and the anſwer is 6, which I place for part of the Root in order, and alſo on the right hand of the ſaid Diviſor, ſo that in the place of the Diviſor 1148, will then ſtand the number 11486, which by the ſixteenth Rule, I multiply by 6, (the Figure laſt placed in the Root) and the product is 11486, which I place in order under the reſolvend, and ſubtract it therefrom, and the remainder is 0, and ſo the work is finiſhed, whereby I find the ſquare Root of 33016516, to be 5746, as by the whole operation appeareth.

```
  .  .  .  .
  33016516 (5746
  25
  ------
107) 801 Reſolvend
     749 Product
     -------
1144) 5265 Reſolvend
      4576 Product
      -------
11486) 68916 Reſolvend
       68916 Product
       -------
        (0)
```

And if the Root had conſiſted of never ſo many places, yet for every Figure put therein (except the firſt, for which you are to obſerve the tenth and eleventh Rule) the work of the 12, 13, 14, 15, and 16 Rules muſt be repeated according

cording to the ſecond note after the ſixteenth Rule foregoing.

Example 3.

A third Example may be this, let it be required to extract the ſquare Root of 8328996.

In the working of this Example you will ſee the uſe of the firſt note upon the ſixteenth Rule, for only the number 8 is the firſt ſquare, as you may ſee by the pointing of the given number, and after the whole work of Extraction is finiſhed, you will find the ſquare Root of the given number, to be 2886, as in the following operation.

```
       8328996 (2886 Root
       4
   48) 432 Reſolvend
       384 Product ſubtract
  568) 4889 Reſolvend
       4544 Product ſubtract
 5766) 34596 Reſolvend
       34596 Product ſubtract.
       (0)
```

XVII. When there is given a number that is not a ſquare number, that is, whoſe root cannot be exactly found, and you are deſirous to find the Fractional part of the root as near as may be,

 you

you are to obſerve the eighth rule in preparing your number for extraction, and then to annex thereto an even number of Cyphers at pleaſure, and note, that as many pairs of Cyphers as you annex thereto, ſo many decimals will there be in the root expreſſed, (which though it come not to be the exact root, yet will it come ſo near the truth, that if the laſt Decimal Figure placed in the root, be increaſed by an unite, it will be too much) and as many points as there are over the given Integral ſquare number, ſo many places will there always be in the Integral part of the Root, as in the following Example, where it is required to extract the ſquare root of 129596.

First I proceed to the work of extraction according to the former rules as if it were an exact ſquare Number, and find the integral root to be 359, as followeth.

```
          129596 (359
          9
          ----------
     65)  395
          325
          --------
    709)  7096
          6381
          --------
           715 Remainder
```

But becauſe (when the work is finiſhed) there is a Remainder of 715, I annex a competent even number of Cyphers, to the given number, as of 4, or 6, or 8, and point them out in the ſame manner

manner as if there were significant figures in an Integer, then bring two of them down to the said remainder (715) and annex them thereto, so have you 71500 for a new resolvend ; Then find out a new divisor by doubling the root, as is before directed, and proceed as if the annexed Cyphers were significant figures, or whole Nnmbers, as far as you please, as in this example, where the work is carried on till there are 3 decimal figures in the Root ; and the work being finished, I find the root to be 359.994, and there is a remainder of 319964. See the work.

```
              129596.000000 (359.694
              9
              ------
      65)     395
              325
              ------
     709)     7096
              6381
              ------
    7189)      71500
               64701
               -------
   71989)      679900
               647901
               -------
  719984)      3199900
               2879936
               -------
               319964 remains.
```

But if you proceed to put another decimal in the root you will find it to be 359.9944, and the remainder will be 3196864. Now you may perceive that the ſaid root, is too little, becauſe there is a remainder, but yet it is ſo near the truth that if the laſt figure thereof were increaſed by an unite, and ſo made 359.9945 it would then be too much, as you may prove at your leiſure.

XVIII. The Square root of a vulgar Fraction that is commenſurable to its root, is thus found, *viz.* extract the ſquare root of the Numerator, for a new Numerator, and likewiſe the ſquare root of the Denominator, for a new Denominator; ſo ſhall that new Fraction be the ſquare root of the given Fraction; as for

Example.

Let it be required to extract the ſquare root of $\frac{25}{36}$ firſt I take the ſquare root of 25, which is 5, and place it for a new Numerator, then I take the ſquare root of the Denominator 36, which is 6, and place it for a new Denominator, ſo is $\frac{5}{6}$ the ſquare root of $\frac{25}{36}$, which was required; in like manner if $\frac{4}{9}$ were given to have its ſquare root extracted, its root would be found to be $\frac{2}{3}$ and $\frac{3}{4}$ is the ſquare root of $\frac{9}{16}$, the like is to be obſerved for any other.

Note. But here note diligently; before you proceed to extract the Square root of any Fraction, that you reduce it to its loweſt Terms, for it may happen that in its given Terms, it may be incommenſurable to its root, but being reduced to its loweſt Terms it may be be commenſurable, and its root exactly found out, ſo $\frac{50}{72}$ is incommenſurable to its root, but being reduced to $\frac{25}{36}$, its ſquare root will be found to be $\frac{5}{6}$ as before.

XIX. The ſquare root of a mixt number that is commenſurable to its root is thus found out, *viz.* reduce the mixt number to an improper Fraction, and then extract the ſquare root of the Numerator, and the ſquare root of the Denominator, for a new Numerator, and a new denominator, as in the laſt Rule.

So if it were required to extract the ſquare root of $1\frac{11}{25}$, firſt I reduce it to an improper Fraction, and ſo it is $\frac{36}{25}$ whoſe Square root is $\frac{6}{5}=1\frac{1}{5}$, ſo if it were required to extract the ſquare root of $3\frac{13}{81}$, firſt I reduce the given mixt number, to the improper Fraction $\frac{256}{81}$, and then extract the ſquare root of the Numerator 256, and it I find to be 16, for a new Numerator, and likewiſe the ſquare root of 81, the denominator, which I find to be 9, for a new denominator, ſo is $\frac{16}{9}=1\frac{7}{9}$ the ſquare root of the given mixt number $3\frac{36}{25}$ which was required.

XX. When you are to extract the ſquare root, of a Fraction that is incommenſurable to its root, prefix before the given Fraction, this Character $\surd$, or $\surd q.$ ſignifying the ſquare root of that before which it is prefixed, ſo the ſquare root of $\frac{24}{29}$ is thus expreſſed, $\surd\frac{24}{29}$ or $\surd q.\ \frac{24}{25}$, the like of any other. But if you would know, as near as may be the ſquare root of any ſuch Fraction, reduce it to a decimal of the ſame value by the firſt Rule of the ſecond Chapter, but let the decimal conſiſt of an even number of places, *viz.* either of two, four, ſix, or eight, *&c.* places;

 and

and the more places it consisteth of, so much the nearer the truth will the root be; Then extract the square root of that decimal (according to the Rules before delivered,) in every respect as if it were a whole number, so shall this root so found be very near the true root; and so near that if it consist of 3 places it shall not want $\frac{1}{1000}$ part of an unit of the true root, and if of 4 places, it shall not want $\frac{1}{10000}$ part of an unite of the truth.

So if I would extract the square root of $\frac{3}{4}$, first I reduce it to a decimal, which I find to be .75 and because I would have the root to consist of 4 places, I annex, 6 Cyphers thereto and it makes .75000000, then extracting the square root thereof as if it were a whole number, I find it to be .8660, and there is a remainder of 4400, but if I would have it consist of 5 places, then I annex 2 more Cyphers to the said remainder, and make it 440000, and proceed, and then I find the root to be .86602, and the remainder to be 93596.

XXI. In like manner if it were required to extract the square root of a mixt number incommensurable to its root, as near as may be, first reduce the Fractional part to a decimal, but let it consist of an even number of places, *viz.* of 2, 4, 6, or 8, *&c.* places, then proceed to extract its square root, according to the Rules formerly delivered in this Chapter, in every respect, as if it were a whole number, so shall the root so found, be very near the truth, and the more places it consisteth of, so much the nearer will it be to the true root. And note that in the root there will be so many decimal places, as you placed

placed points over the decimal part of the ſquare number.

So if it were required to extract the ſquare root of 28 $\frac{6}{13}$ firſt I reduce the fractional part $\frac{6}{13}$ to a decimal and it makes .461538, ſo then the mixt number whoſe ſquare root I am to extract is 28.461538, which being pointed, and the work of extraction finiſhed, according to the former Rules, I find its ſquare root to be very near 5.334 and there is a remainder of 9982, But if I had proceeded yet farther, and made the decimal part to have conſiſted of 7 places, it would have had for its ſquare root 5.3349, which doth not want $\frac{1}{10000}$ part of an unite of the true root.

But if you would not extract the ſquare root of ſuch a mixt number, then prefix before it this character, $\surd$ or $\surd q.$ ſo if the ſaid mixt number 28 $\frac{6}{13}$ were given I would expreſs its ſquare root thus, *viz.* $\surd$ 18 $\frac{6}{13}$ or $\surd q.$ 28 $\frac{6}{13}$ the like is to be underſtood of any other.

XXII. When you are to extract the ſquare root of a decimal Fraction, which hath 2 or 3 Cyphers poſſeſſing the two or three firſt places on the left hand of the given decimal, then cut off 2 of them with a daſh of the Pen, and put a Cypher to poſſeſs the firſt place of the Root, and proceed to extract the ſquare root of the remaining Figures, according to the former Rules as if there had been no ſuch Cyphers before the given decimal; and if the given decimal have 4 Cyphers before it, cut them off with a daſh of the Pen, and put 2 Cyphers in the root, and then proceed as before.

So if it were required to extract the ſquare root of $\frac{5}{846}$, firſt I reduce to it to a decimal Fraction and it makes .005910, then I cut off the two firſt Cyphers, and place one Cypher in the root, then I proceed to extract the ſquare root of the remaining Figures, *viz.* 5910, as if there had been no ſuch Cyphers before them, and I find the root to be very near .077 as you may try at your leiſure.

The proof of the Extraction of the Square Root.

XXIII. The operation in the extraction of the ſquare root is thus proved, *viz.* Multiply the root into it ſelf, and (if there be no remainder after the work of extraction is finiſhed) the product (if the work be truly done) will be equal to the number firſt given. As in the firſt Example, where it is required to extract the ſquare root of 2304, which is there found to be 48. Now if I multiply 48 by it ſelf, it produceth 2304, which is the given number, and therefore I conclude the operation to be true. But if after the work of extraction is finiſhed, there is any remainder, then, when you have multiplied the root by it ſelf, to the product add the ſaid remainder, and if the ſum be equal to the given number, the operation is right, otherwiſe not. As in the Example of the ſeventeenth Rule, where it is required to extract the ſquare root of 129596; and is there found to be 359.994, and the remainder is 319964. Now to prove the work, I multiply the root (359.994) by it ſelf, and it produceth 129595.680036, which ſhould be 129596, therefore to the ſaid product I add the ſaid remainder (319964,) and the ſum is

129596

129596, and therefore I conclude the work to be truly wrought.

CHAP. X.

The Extraction of the Cube Root.

I. A Cube Number is that which is produced by multiplying any Number into it ſelf and again into that product, which ſaid given number is called the Cube Root.

As, ſuppoſe 5 were given to find its Cube, firſt I multiply 5 into its ſelf, and it produceth 25, which is called the Square of 5, then I again multiply 25, (the ſaid Square) by 5, and it produceth 125, which is called the Cube of 5, and here note that as 125, is called the Cube of 5, ſo is 5 called the Cube Root of 125.

II. The extraction of the Cube Root is nothing elſe then when by having a Cube Number given we find out its Cube root; which ſaid Cube number is given always ſuppoſed to be a certain number of little Cubes, comprehended within one intire great Cube, which ſaid Cube may very well be repreſented by a dye, or any other

other ſolid body, having its length, breadth and depth equal; this being ſuppoſed, let there be laid 9 Dyes conſtituting a ſquare, whoſe ſide ſhall be 3, and upon them let there be laid 9 more, Dyes; and upon them let there be laid 9 more, then will there be in all 27 Dyes, which will conſtitute one greater Cube, whoſe length, breadth and depth will be 3 Dyes, and this greater Cube comprehendeth 27 leſſer Cubes. Now the extraction of the Cube root is by having the number of little Cubes (27) comprehended in the greater given Cube to find out how many of the leſſer Cubes make up the ſide of the greater.

III. A Cube number is either Simple or Compound.

IV. A Simple Cube number is that which hath for its root or ſide, one of the 9 Digits, and it is therefore always leſſer than 1000; ſo ſhall you find that 343 is a Simple Cube Number, whoſe ſide or root is 7, for $7 \times 7 \times 7 = 343$; all which ſaid Simple Cubes, and Squares, as alſo their roots are expreſſed in the Tablet following.

Roots	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	81
Cubes	1	8	27	64	125	216	343	512	729

V. A Compound Cube number is that which is produced by the multiplication of a Number conſiſting of two places (at the leaſt 3 times into it it ſelf continually, and is therefore never leſs than 1000, ſo 1728 is a compound Cube number, produced by the multiplication of 12 into its ſelf 3 times; for 12× 12× 12=1728.

VI. When a compound Cube Number is given to have its Cube root extracted, before you can go about it, you muſt prepare it for the work by pointing it; which is thus done, *viz.* put a point over the firſt figure towards the right hand, *viz.* over the place of Units, then (paſſing the two next places) put a point over the fourth figure, or place of Thouſands, and ſo proceed by putting a point over every third figure, as you did over every ſecond figure in the extraction of the Square root, till you have finiſhed your pointing; That being done, on the right hand of the ſaid Cube number draw a crooked line, behind which to place its Cube root, as you do to place the Quotient in Diviſion, as in the following Example.

Example 1.

Let it be required to extract the Cube root of 262144.

In order to prepare this Cube Number, for the Extraction of its Cube root, I firſt put a point over the firſt figure (4) towards the right hand, and then overpaſſing the two next figures (14) I put another point over the forth figure (2) and then is the given number diſtributed into ſeveral parts not unfitly called Cubes, *viz.* 262 (as far as the

$\dot{2}62\dot{1}44$ (

firſt

first point goeth) is the first Cube, and the 144 (from thence to the second point) is the second Cube, and then I draw a crooked line behind it as you see in the Margent.

VII. Having proceeded thus far, find out the Cube Root of the first Cube (262) but because it is not an exact Cube number, take the Cube root of that number in the foregoing Tablet, which being lesser than it is, yet is nearest to it, (which I here find to be 6,) and place it behind the crooked line for the first Figure in the Root, as you see in the following work.

262144 (6

VIII. This being done, Cube the said number which is placed in the root, and subscribe its Cube under the first Cube of the given number. So in this Example 216 being the Cube of 6, I place it under 262 the first Cube of the given number 262144, as followeth

262144 (6
216

IX. Draw a line under the Cube thus subscribed, and subtract it from the first Cube of the given number, placing the remainder orderly underneath the said line. So 216 (the Cube of 6) being subtracted from 262, the remainder is 46, which I place underneath the line as followeth

262144 (6
216

46

X. Bring down the next Cube number and annex it to the ſaid remainder on the right hand therefore. So 144 being the next Cube, I bring it down and annex it to the remainder 46, and it makes 46144, which by Artiſts is uſually called the Reſolveed.

262144 (6
216

46144 Reſolvend

XI. Draw a line underneath the Reſolvend, then Triple the root, that is, multiply it by 3, and place its Triple under the Reſolvend in ſuch order, that the place of units in the ſaid triple may ſtand under the place of tens in the Reſolvend. So the triple of 6, is 18, which I place under the Reſolvend ſo, that 8 (the place of units in the ſaid triple) may ſtand under 4 in the place of tens of the Reſolvend; as you ſee following.

262144

262144 (6
216

46144 Resolvend

18

XII. Square the said root, and then triple the said square of the root, and place the said triple square under the said triple root in such order that the place of unites in the triple square of the root, may stand underneath the place of Tens in the triple root, so in this Example, the square of the root 6, is 36, and the triple thereof is 108, which I place under 18, the triple root so, that 8 the place of unites in the said triple square of the root, may stand under 1, the place of Tens in 18, the said Triple root as followeth

262144 (6
216

46144 Resolvend

18
108

XIII. Draw a line underneath the said triple root, and triple square of the root, as they are placed, and add them together in the same order

as

as they ſtand, ſo ſhall their ſum be a Diviſor. So in our Example, a line being drawn under 18 and 108, and they added together in the ſame order as they ſtand, their ſum is 1098 for a Diviſor, as in the following work.

262144 (6
216
46144 Reſolvend
18
108
1098 Diviſor

XIV. Draw a crooked line on the left hand of the reſolvend, before which to place the ſaid Diviſor, and let the whole reſolvend (except the place of unites therein) be eſteemed a Dividend, then ſeek how often the ſaid Diviſor is contained in the dividend, and put the anſwer for the next Figure in the root. So in our Example ſeek how often 1098 the diviſor is contained in 4614 the dividend, obſerving here the uſual Rules of Diviſion) and the anſwer I find to be 4 which I place for the next figure in the root, as in the Example.

```
262144 (64
216
-----
1098) 46144 Resolvend
-----
   18  the Triple root.
  108  the Triple square of the root.
-----
 1098  Divisor
```

XV. Draw a line underneath the whole work, and then Cube the Figure laſt placed in the root, and place its Cube underneath the Reſolvend in ſuch ſort that the place of units in the one may ſtand under the place of units in the other; ſo in our Example 64 being the Cube of 4 (the Figure laſt placed in the root) I place it under the Reſolvend in ſuch manner that the Figure 4 in the place of unites of the Cube 64, may ſtand under 4, the place of unites in the Reſolvend, and then the work will ſtand as followeth.

```
 .   .
262144 (64
216
      ------
1098) 46144 Resolvend
      ------
        18  the Triple Root
       108  the Triple square of the Root.
      ------
      1098  Divisor
      ------
        64  the Cube of 4
```

XVI. Square the figure last placed in the Root, and multiply its square by the triple Root subscribed underneath the Resolvend, (as is directed in the eleventh Rule of this Chapter) and subscribe the product under the Cube last put down, in such order, that the place of Units in the said product, may stand under the place of Tens, in the said Cube. So in our Example, the figure last placed in the Root is 4, which squared is 16, and 16 multiplyed by 18 (the triple Root before set down) the product is 288, which I place under 64 (the cube of 4) in such sort that 8 (in the place of Units of the said product) may stand under 6 (the place of Tens) in the said cube of 4; view the work.

```
        .  .
        262144 (64
        216  Cube of 6
       ------
1098)   46144 Resolvend
       ------
          18  Triple Root
         108  Triple square of the Root.
       ------
        1098  Divisor
       ------
          64  Cube of 4
         288  the square of 4 in the triple Root.
```

XVII. Multiply the triple ſquare of the Root, (ſubſcribed as is before directed in the twelfth Rule of this Chapter) by the figure laſt placed in the Root, and place the product under the number laſt ſubſcribed, (which is the product of the ſquare of the figure laſt placed in the Root multiplyed by the ſaid Triple Root) in ſuch manner that the place of Units of this, may ſtand under the place of Tens in that; As in this Example, The Triple ſquare of the Root is 108, which multiplyed by 4 (the figure laſt placed in the Root) the product is 432, which I place under 288 (the number laſt ſubſcribed) in ſuch order that the figure 2 (in the place of Units of the ſaid laſt product) may ſtand under 8, which is in the place of Tens, of the ſaid number laſt ſubſcribed; and then the work will ſtand as followeth.

```
   . .
262144 (64
216
------
 46144 Resolvend
------
   18   Triple root.
  108   Triple square of the root.
------
 1098 Divisor
------
   64  Cube of 4
  288  Square of 4 in Triple Root
 432   Triple square of the Root in 4
```

XVIII. Draw another line under the work, and add the 3 numbers together, that were last placed under the Divisor, in the same order as they there stand, and let their sum be called the Subtrahend, which let be subtracted out of the Resolvend, nothing the Remainder; So in this Example I add 64, 288, 432 together in the same order as they stand, and their sum is 46144 the Subtrahend, which I subtract out of 46144 the Resolvend, and there is nothing remaineth, so the whole work is finished, and I find the Cube Root of 262144 to be 64, without any Remainder; See the whole work as followeth,

```
 .  .
262144 (64
216
------
1098)  46144 Resolvend
-------
    18   Triple root.
   108   Triple square of the root.
-------
  1098 Divisor
-------
    64   Cube of 4
   288   Square of 4 in the trip. Root
  432    Triple sq. of the Root in 4.
-------
 46144   Subtrahend.
-------
 (0)
```

Now the Learner is to obſerve three things in general from the Rules before delivered, concerning the extraction of the Cube Root.

Obſerve 1. That the work contained in the 7. 8 and 9 Rules for finding out the firſt Figure of the Root, is not again to be repeated, throughout the whole work of Extraction, although the Root conſiſt of never ſo many places, but the work of all the Rules following is to be repeated as often as a new figure is put in the root.

Obſerve 2. For every particular Cube in the number given, diſtinguiſhed by the points, (except the firſt) there is to be found out a new reſolvend, by annexing the next Cube to the remainder

mainder (according to the 10*th* Rule) and as often as there is a resolvend, so often must there be found a new Divisor (by the 11, 12 and 13 Rules) and as often as there is found a new Divisor so often must there be found a new Subtrahend (according to the 15, 16, 17, and 18 Rule beforegoing.)

Observe 3. When the Subtrahend chanceth to be greater than the resolvend, then you may conclude there is an error in your work, which must be corrected by puting a lesser figure in the Root.

Example 2.

Let it be required to extract the Cube Root of 48627125.

Having prepared the given number for the work of extraction, according to the 6*th*. Rule of this Chapter, I find it to be distributed into 3 Cubes, *viz.* 48, the first, 627, the second, and 125 the third. Then I proceed to the work; and first I find the Cube root of 48, (the first Cube) which is 3, then do I Cube 3, and place its Cube which is 27 under (48) the first cube, and subtract it therefrom

```
48627125 (3
27
------
21
```

and the remainder is 21, according to the 7,8, and 9 Rules of this Chapter, and then will the work stand as you see in the Margent.

Then, to the said remainder 21, do I bring down, and thereto annex the next cube, which

Is 627, and it makes 21627 for a resolvend according to the 10*th*. Rule foregoing. Then do I find out a Divisor according to the 11, 12 and 13 Rules of this Chapter, and first I triple the Root (3) and it makes 9, which I place under (2) the place of Tens in the Resolvend; Then do I square the said Root (3) and that makes 9, then do I triple its square (9) and that makes 27, which I place under the said Triple in such order as is directed in the 12*th* Rule, then drawing a line underneath the work, I add the two said numbers together, (*viz.* the Triple Root, and the Triple Square of the Root) in such order as they are there placed, and their sum is 279 for a Divisor; as *per* Margent.

```
48627125 (3
27
-----
21627 Resolvend
-----
  09
 27
-----
 279 Divisor
```

Then according to the 14*th* Rule I seek how often the said Divisor 279 is contained in 2162 the Dividend, and I find the answer to be 6, which I place for the second figure in the Root, then do I in the next place go about to find out a subtrahend, and in order thereunto first (according to the fifteenth Rule of this Chapter) I cube the Figure (6) last placed in the root, and it maketh 216, which I place under the Resolvend in such order (as is directed in the said fifteenth Rule) that the place of Units of the one may stand under the place of Units of the other; Then (according to the 16*th* Rule of this Chapter) I square the figure (*viz.* 6) last placed in the

the root which makes 36, and then multiply it by (9) the said triple root, and the product is 324 which I place under (216) the said Cube of 6, in such order as is directed in the said 16*th* rule. Then do I multiply the said triple square (*viz.* 27) by the figure (6) last placed in the root, and it produceth 162, which I place under the last product (324) in such manner as is directed in the 1 *th* rule. Then I add these 3 several numbers together in the same order as they stand, and their sum is 9656 for a Subtrahend, which subtracted out of (21627) the Resolvend, the remainder is 1971, as you may see by the following work.

```
    48627125 ( 36
    27
    ------
279) 21627 Resolvend
    ------
      09  Triple root
     27
    ------
     279  Divisor
    ------
     216  Cube of 6
    324   Square of 6 by the triple root
   162    Triple square of the root by 6
   ------
   19656  Subtrahend
   ------
    1971  Remainder
```

Then to the said remainder (1971) do I annex the next cube number (125) according to the 10*th* Rule, and it makes 1971125 for a resolvend;

Then I proceed according to the 11, 12, and 13 Rules to find a Divisor, and therefore I first triple the whole Quotient (36) and it is 108, Then do I square the whole Quote (36) and it makes 1296, which being tripled is 3888, which being orderly placed under the triple Quote, and added thereto in that order, the sum is 38988 for a new Divisor: See the work in the Margent.

108
3888
38988

Then do I seek how often the said Divisor is contained in the Dividend (197112) and I find it to be 5 times contained therein, and accordingly I place 5 in the root, and proceed according to the 15, 16, 17 and 18 Rules to find out a Subtrahend, and therefore first, I cube the number (5) last placed in the root, and it makes 125, then do I square the said 5, and it makes 25, by which I multiply (108) the said triple root, and place the product (2700) under the said cube, as is before directed, then do I by the said 5, multiply (3888) the triple square of the root and the product (19440) do I place under the former product (2700) according to former directions, and add the 3 Numbers together, in the same order as they stand, and their sum is 1971125, (as appears *per* Margent) for a Subtrahend, which taken out of the said Resolvend there remainth (0) and so the work is finished

finiſhed, and I find the cube root of the given number 48627125 to be 365; view the whole work laid down as followeth.

```
              48627125 (365
              27
              ------
     279)     21627 Reſolvend
              ------
                9  the Triple root
               27  the triple ſquare of the root
              ----
              279  Diviſor
              ----
              216  the Cube of 6
             324   the ſq. of 6 in the tr. root
            162    the triple ſquare of the root in 6
            -----
            19656  the Subtrahend
            -----
  38988)    1971125 Reſolvend
            -------
                  108  the triple root
                 3888  the triple ſq. of the root
                -----
                38988  the Diviſor
                -----
                   125  the Cube of 5
                 2700   the ſq. of 5 in the tr. root
                19440   the tr. ſq. of the root in 5
                -------
                1971125  the Subtrahend
                -------
                   (0)
```

XIX. When it is required to extract the cube Root of a Number that is incommenſurable to its

its Root and you are desirous to know the Fractional part of the root as near as may be, you are to annex to the given number a competent number of Cyphers, which number of Cyphers must be always a multiple of 3, *viz.* either 3, 6, 9, 12, *&c.* Cyphers, that is 000, 000000, or 000000000, *&c.* And having observed the 6 Rule for the punctation of the given number, likewise point the annexed Cyphers, in the same manner as if they were significant figures, or integers; and observe that as many points as you put over the integral part, so many places will the integral part of the root consist of, and so many points as are put over the Cyphers, or Decimals, so many decimal places will there be in the root, this being observed the work it self in the extracting the Cube root of a decimal Fraction or of a mixt number of integers and decimals, is the same in every respect as if the number given were an integral Cube number, according to the rules before delivered in this Chapter. As in the following

Example.

Let it be required to extract the Cube root of 13798 which is a number incommensurable to its Cube root, and to find out its root as near as may be I annex to it 9 Cyphers (so by that means I shall have 3 Decimals in the root) and prepare it for Extraction by pointing it as is before directed, and as you see following.

13798.000000000 (

And

And having performed the work of extraction according to the former Rules, I find its Cube root to be 23.984, which (as by the remainder you may perceive) is somewhat too little, but yet so near the truth, that, if the decimal part were increased by an unite and so made 23.985 it would then be too much, and so consequently it cannot want (as it is) $\frac{1}{1000}$ part of an Unite of the truth, and if the root had had another figure placed in it, it would then have come so near the truth that it would not have wanted $\frac{1}{10000}$ part of an unite: for your farther satisfaction see the whole work performed as followeth.

126)

	13	798	000	000	000	(23.984 Root
	8					
126)	5	798				Resolvend
		6				
	1	2				
	1	26				
		27				
		54				
	3	6				
	4	167				Subtrahend
15939)	1	631	000			Resolvend
			69			
		158	7			
		159	39			Divisor
			729			
		55	89			
	1	428	3			
	1	484	919			Subtrahend
1714347)		146	081	000		Resolvend
			7	17		
		17	136	3		
		17	143	47		Divisor
				512		
			458	88		
		137	090	4		
		137	549	792		Subtrahend
172519314)		8	531	208	000	Resolvend
				71	94	
		1	725	121	2	
		1	725	193	14	Divisor
					64	
			1	151	04	
		6	000	484	8	
		6	001	635	004	Subtrahend
			629	572	096	Remains

XX. If at any time it is required to extract the cube root of a vulgar Fraction, let such Fraction be first reduced to its lowest Terms; because it may not be commensurable to its root in the given Terms, but being reduced to its lowest Terms it may, and having so done to perform the work, this is

To extract the Cube Root of a vulgar Fraction.

The Rule.

Extract the Cube Root of the Numerator, (by the former Rules) and place that for a new Numerator, then extract the Cube root of the Denominator, and place that root for a new Denominator, so shall this new Fraction be the Cube root of the given Fraction.

As for Example, Let it be required to extract the Cube root of $\frac{27}{64}$, first I take the Cube root of 27 (the Numerator) which is 3 and place it for a new Numerator. Then I take the Cube root of 64, (the Denominator) which is 4, and place it for a new Denominator, so shall this new Fraction $\frac{3}{4}$ be the Cube root of the given Fraction $\frac{27}{64}$.

In like manner if there were given $\frac{125}{256}$ to have its Cube root extracted, I can easily discover that there cannot be found any Cube root exactly either for the Numerator or Denominator, in the Terms they are given in, but being reduced to their lowest Terms, they are $\frac{27}{64}$, whose cube root is $\frac{3}{4}$ as before.

In like manner the Cube root of $10\frac{16}{24}$ will be found to be $\frac{1}{7} = \sqrt{q}.\ 10\frac{16}{24}$.

XXI. But

To extract the Cube Root of a vulgar Fraction that is incommensurable to its Root.

XXI. But when there is given a vulgar Fraction to have its cube root extracted, it being incommensurable to its root, you may find its cube root very near if you reduce the given vulgar Fraction to a decimal, and then extract the Cube root of that decimal (by the Rules before delivered) in every respect as if it were a whole Number, and then shall that be a decimal cube root, less than the truth, yet so near the truth that if you add an unite to the last decimal figure it will then be greater than the truth.

Note

Here take notice by the way that your vulgar fraction being reduced to a decimal in order to have its cube root extracted, its equivalent decimal must consist of such a number of places as may be a multiple of 3, that is, it must consist of 3, 6, 9, 12, 15, &c. places, and the more places there is in the decimal, the nearer the truth will the root be.

Example.

Let it be required to extract the cube root of $\frac{5}{8}$: In order whereunto I reduce it to this decimal, *viz.* .625, which because it consisteth but of 3 places, (and so consequently can have but 1 figure in its root) I increase to 9 places by annexing 6 Cyphers thereto thus .625000000 and then the root will consist of 3 places, then do I proceed to extract its cube root, (according to the former Rules) and find it to be .854, &c.

and

and there will be a remainder of 2164136 as you may prove at your leisure.

XXII. When your given vulgar Fraction is reduced to a Decimal of the same value, and the 3, or 4 first places towards the left hand are possessed by Cyphers, then in this case you are to cut off 3 of them with a dash of the Pen, and for them place a Cypher to possess the first place in the root, and then proceed to extract the cube root of the remaining figures, according to the former Rules, as if there had been no such Cyphers at all.

As for *Example.*

Let there be given $\frac{4}{8237}$ to have its cube root extracted; First reduce it to a decimal Fraction by the first Rule of the second Chapter of this Book, and it makes .000485613, *&c.* Now to extract the Cube root of this Fraction, I first prepare it, by pointing it in every respect as if it were a whole number, then with a dash of my Pen, I cut off the three first Cyphers and put a (0) to possess the first place in the root, then I proceed to extract the cube root of the remaining figures (485613) as if there had been no Cyphers at all before them; and having finished the work I find its cube root to be .078 as by the following work.

200

```
000|485613 (078
    343
1491) 142613 Resolvend
       21
      147
     1491 Divisor
       512
      1344
     1176
    131552 Subtrahend
     11061 Remainder
```

In like manner if the decimal which is given to have its cube root extracted, have 6 Cyphers placed before the significant figures on the left hand, then cut off those 6 Cyphers with a dash of the Pen, and for them put two Cyphers to possess the two first places in the root, Then proceed to extract the Cube root of the remaining figures as if there had been no such Cyphers, &c.

To extract the Cube Root of a mixt number.

XXIII. When it is required to extract the Cube Root of a mixt number, reduce it to an improper Fraction, and if it hath a perfect Cube root, then extract the Cube root of the Numerator, and

and place it for a new Numerator, and alſo extract the Cube Root of the Denominator, and place it for a new Denominator, ſo ſhall this new Fraction be the Cube Root of the given mixt Number.

Example.

Let it be required to extract the Cube Root of $5\frac{13}{343}$, having reduced it to an improper Fraction, I find it to be $\frac{1728}{343}$, and having extracted the Cube Root of the Numerator (1728) I find its root to be 12, for a Numerator, and the Cube root of 343 the Denominator is 7 for a Denominator, ſo that I conclude $\frac{12}{7}$ or $1\frac{5}{7}$ to be the Cube root of the given mixt Number $5\frac{13}{343}$, as you may prove at your leiſure.

XXIV. But if the given mixt Number, whoſe Cube root is required, have not a perfect root then you are to reduce the Fractional part into a Decimal of the ſame value, (but let the number of decimal places be always a multiple of 3) and then proceed to extract the Cube Root of that mixt Number, as if it were a whole Number, always reſerving ſo many decimal places in the Root as there are points over the decimal part of the mixt Number.

Example.

Let it be required to extract the Cube root of $28\frac{3}{4}$. Firſt, reduce $\frac{3}{4}$ into its equivalent decimal, which is .75, but to make it conſiſt of ſix places, I annex thereto four Cyphers, and then the ſaid mixt number will be 28.750000, which being done, I proceed to the work as followeth.

```
       .  .  .
     28.750000 (3.06, &c.
     27
     ------
279) 1750 Reſolvend
     ------
        9
       27
     ------
      279 Diviſor
     ------
27090) 1750000 Reſolvend
       -------
          90
        2700
       -------
        27090 Diviſor
       -------
          216
         3240
        16200
       -------
      1652616 Subtrahend
       -------
        97384 Remains
       -------
```

So that I find by the work, the Cube root of 28.750000 to be 3.06, *&c.*

XXV. It is uſual amongſt Artiſts to expreſs the Cube Root of a whole Number, mixt Number, or Fraction, either Vulgar, or Decimal, that

that is incommenſurable to its Root, by prefixing this Character, (*viz.* $\sqrt{c.}$) before the incommenſurable number or quantity, ſo the Cube Root of 328 may be thus expreſſed $\sqrt{c}$. 328, and the Cube Root of $24\frac{3}{4}$, thus $\sqrt{c}$. $24\frac{3}{4}$, or in a decimal mixt number thus $\sqrt{c}$, 24.75 and of the fraction $\frac{3}{4}$ thus $\sqrt{c}$, $\frac{3}{4}$, *&c.*

XXVI. The operation in the extraction of the Cube Root is proved thus, *viz.*

The proof of the extraction of the Cube Root.

Cube the Root found out, that is, Multiply it three times into it ſelf, and if any thing remain after the work is done, add it to the laſt product, and if that ſum be equal to the given number, then the work is truly performed, otherwiſe not.

As in our firſt Example, where it is required to extract the Cube Root of 110592, and which is found to be 48; and to prove the work, multiply 48 by it ſelf, whoſe product is 2304, which being again multiplyed by 48, it produceth 110592, which is equal to the given number, and therefore I conclude the work to be right.

Likewiſe to prove the Example of the Nineteenth Rule, where it is required to extract the Cube Root of 13798 which is found to be the mixt number 23.984. Now to prove the work, I Cube the root, as before directed, and find it to be 13796.370427904 to which add the remainder 1629572096 and their ſum maketh the given number 13798 which proves the work to be right.

CHAP. XI.

The Uſe of the Square and Cube Roots in ſolving ſome Queſtions Arithmetical and Geometrical.

PROP. I.

To find a mean proportional between two given Numbers.

MUltiply the given Numbers the one by the other, and extract the ſquare Root of the product, ſo ſhall that ſquare root be the mean proportional ſought.

Example.

Let the given numbers be 12 and 48, and let it be required to find a mean proportional between them; firſt multiply the given numbers 12 and 48 the one into the other, and their product is 576, the Square Root of which is 24, ſo that

that I conclude 24 to be a mean proportional between 12 and 48, for,

12 : 24 : : 24 : 48.

The ſquare of the mean being equal to the product of the extreams.

This propoſition is uſeful in finding the ſide of a ſquare that ſhall be equal to any given paralelogram; for, (according to the firſt Propoſition of the eighth Chapter of this Book,) if you multiply the contiguous ſides of a Rectangular paralelogram the one by the other, that product will be its content, and if you extract the ſquare root of that content, it will give you the ſide of a ſquare, (in the ſame meaſure your paralelogram was) which will be equal to the given paralelogram.

PROP. II.

To find the Side of a Square that ſhall be equal to the Content of any given ſuperficies.

FInd out the Content of the given ſuperficies by the Rules laid down in the Eighth Chapter, and then extract the ſquare root of the Con-

Content, so will that Root be the side of a square equal to the given superficies.

Example.

There is a Rectangled Triangle whose base and perpendicular are 36 and 8, I demand the side of a square that will be equal to the given Triangle.

According to the second Proposition of the Eighth Chapter, I find the Content of this Triangle to be 144, the square Root of which is 12, and is the side of a square equal to the said Triangle.

In like manner if you extract the square root of the Content of a Circle, Pentagon, Hexagon, &c. or of any other figure regular or irregular, it will give the side of a square equal to that superficies.

PROP. III.

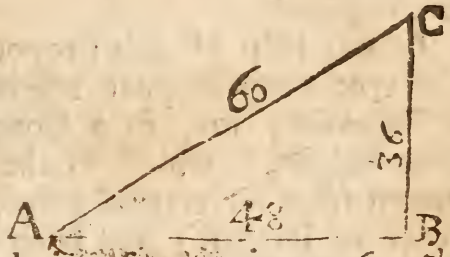
Having any two of the sides of a Right-angled plain Triangle, given to find the third side.

THis most excellent and useful proposition is generally called *Pythagoras* his Theoreme, and in the 47 Pro. of *Euclides* Elements of

Geom. it is demonſtrated, and proved that the Square made of the Hypothenuſe, or ſlant ſide of a right angled plain Triangle is equal to the ſum of the ſquares made of the baſe and perpendicular.

As for Example.

In the Triangle A B C, the Baſe A B is 48, and the perpendicular B C is 36, now I demand the length of the Hypothenuſe A C.



To find out an anſwer to this, firſt I ſquare the baſe A B, (48) which is 2304, then ſquare the Perpendicular (36) and its ſquare is 1296, the ſum of which two ſquares is 3600, which is equal to the Square of the Hypothenuſe A C, therefore the ſquare root of 3600 will give the length of A C, which is 60.

PROP. IV.

THere is a Tower about which there is a Moat that is 48 foot wide, and a ſcaleing Ladder that is 60 Foot long, will reach from the outſide of the Moat, to the top of a Wall, that is within the ſaid Moat, now I demand the height of the ſaid Wall above the Water?

Let the Baſe A B in the foregoing Triangle be the breadth of the Moat, and let the Hypothenuſe A C be the ſcaling Ladder, then is the perpendicular B C the height of the Wall above the Water. Now it is plain that (becauſe the ſquare of A C is equal to the ſum of the ſquares of A B and B C) if from the ſquare of A C which is 3600 you ſubtract the ſquare of A B which is 2304, there will remain 1296, which is the ſquare of C B, therefore I extract the ſquare Root of 1296, and find it to be 36, which is the height of the ſaid Wall above the Water as was required.

By the help of this Propoſition may be found the true perpendicular height of a Cone, or of a Pyramid; for, in a Cone, if you ſquare the ſlant height, (which is the length of a line drawn from its vertical point, to the Circumference of its baſe) and from the ſquare of that, ſubtract the ſquare of the Semidiameter of its baſe, there will remain the ſquare of the perpendicular height of that Cone.

Alſo, In a Pyramid, if from the ſquare of the ſlant height of it, you ſubtract the ſquare of that line which being drawn from the Centre of its baſe, ſhould touch the end of the ſaid ſlant line, (whether they meet at an angle or not) the remainder will be the ſquare of the perpendicular height of that Pyramid, and its ſquare Root will give the height it ſelf.

PROP. V.

By the Content of a Circle to find its Diameter.

The proportion is

AS 22.
Is to 28.
So is the given Content
To the ſquare of the Diameter,

Example.

There is a Circle whoſe ſuperficial Content is 153.9385, I demand its Diameter?

22 : 28 : : 153.9385 : 195.9217.

The ſquare Root of which is 13.99 (very near 14) for the Diameter required.

PROP.

PROP. VI.

By the Content of a Circle to find its Circumference.

The Proportion is

As 7
Is to 88
So is the given Content
To the ſquare of the Circumference.

The ſquare root of which is the Circumference required.

Example.

There is a Circle whoſe ſuperficial content is 153.9385, I demand the Circumference of that Circle!

7 : 88 : : 153.9385 : 1935.2268

The ſquare root of which is 44 *fere* which is the Circumference required.

II. The Cube Root is that by help of which we reſolve all queſtions Mathematical that concern ſolidity, and by which we increaſe ſolid bodies according to any given proportion. By it we diſcover the ſolidity of a body that is capable of length, breadth, and depth, (or thickneſs,) and

and by having the ſolidity given, we diſcover the ſide or diameter of ſuch a body.

Some queſtions pertinent thereto may be ſuch as follow.

PROP. VII.

THere is a Cube whoſe ſide is 4, I demand what ſhall be the ſide of a Cube whoſe ſolidity is double to the ſolidity of that Cube?

To anſwer this propoſition, find out the Cube of 4 (the ſide of the given Cube) which is 64, and double it, which is 128, then extract the Cube root of 128, and it makes 5.0397 *fere*, and that is the ſide of the Cube which is double to the Cube whoſe ſide is 4.

PROP. VIII.

THere is a Cube whoſe ſolidity is 128 foot, I demand the ſide of a Cube whoſe ſolidity is half as much?

Take ½ of 128=64 the Cube root of which (*viz.* 4.) anſwers the queſtion.

PROP. IX.

HAving the ſolid Content of a Globe to find the ſide of a Cube whoſe ſolidity ſhall be equal to the given Globe.

Ex-

Extract the Cube root of the given solid Content of the Globe, and it will give you the side of the Cube required.

Example.

There is a Globe whose solid Content is 1728 Inches, I demand the side of the Cube equal thereto?

Having extracted the Cube root of 1728, I find it to be 12, which is the side of the Cube required.

PROP. X.

HAving the Diameter and Weight of a Bullet, to find the Weight of another Bullet whose Diameter is given.

As the Cube of the given Bullets Diameter,
Is to its Weight or Solidity.
So is the Cube of the Diameter of any other Bullet,
To its Weight or Solidity.

Example.

There is a Bullet whose Diameter is 4 Inches, and its weight is 9 Pound, I demand the weight of another Bullet, whose Diameter is $6\frac{1}{4}$ or 6.25 Inches.

The Cube of 4 is 64.
The Cube of 6.25 is 244.140625
Then I say

64 : 9 : : 244.140625 : 34.33227

So that the weight required is 34.33227 pounds and if you reduce the Decimal to the known parts of Averdupois weight, you will find the anſwer to be 34 ℔—05 *oz.*—05 *dr.*

This kind of Proportion is by Artiſts Termed triplicate proportion.

In like manner, the Diameters of two Bullets, or Globes being given, and the ſolidity of one of them to find out the ſolidity of the other, it may be done by the ſame proportion, only changing the middlemoſt Term.

PROP. XI.

TO find the ſide of a Cube equal to a given paralelepipedon.

Find out the ſolidity of the given paralelepipedon by the Eighth Prop. of the Eighth Chapter, then is the Cube Root thereof, the required ſide.

Example.

There is a paralelepipedon having the ſides of its baſe 10 Foot 4 Inches, and 5 Foot 2 Inches, and its length is 20 Foot 8 Inches, I deſire to know what is the ſide of a Cube whoſe content ſhall be equal to the given paralelepipedon?

The Superficial Content of the baſe is 7688 inches, which drawn into 248 the length in Inches, the product is 1906624 inches for its ſolid Content, the Cube root of which is 124 inches, for the ſide of a Cube equal to the given paralelepipedon.

In like manner if you would find at any time the ſide of the Cube equal to any ſolid Body whether Regular or Irregular: Firſt, Find the ſolid

lid Content of that Body, and then extracting the Cube Root of its solid Content you have your desire.

PROP. XII.

BEtween two given Numbers to find two mean proportionals.

Divide the greater extream by the lesser, and extract the Cube Root of the Quotient, and by the said Cube Root multiply the lesser extream, then will the product give you the lesser mean propotional, then multiply the said lesser mean by the said Cubique Root, and that product will give you the greater mean proportional.

Example.

Let the two given extreams be 6 and 48 between which it is required to find 2 mean proportionals.

First, I divide 48 (the Greater Extream) by 6 (the Lesser Extream) and the Quotient is 8, the Cube Root of which is 2 then by (the Cube Root) 2 I multiply 6 (the lesser extream) and the product is 12 for the lesser mean proportional, and 12 being multiplied by 2 (the Cube Root) the product is 24, for the greater mean proportional sought. Thus have I found 12 and 24 to be two mean proportionals between 6 and 48, for

6 : 12 :: 24 : 48

In

In like manner between 3 and 81 will be found 9 and 27, for two mean proportionals.

PROP. XIII.

THE Concave Diameter of two Guns being known, and the quantity of Gun-powder that will charge one of them, to find out how much will be sufficient to charge the other.

The Capacities are one to another, as are the Cubes of their Diameters, and also the proportion is direct.

Example.

If 25 pound of Gun-powder be sufficient to charge a Gun, whose Concave Diameter is 1 $\frac{1}{2}$ Inches, or 1.5 Inch, how much powder will be sufficient to charge a Gun, whose Concave Diameter is 7 Inches? Answer, 25.47.

The Cube of 1.5 is 3.375 and the Cube of 7 is 343. wherefore the Proportion is as followeth.

3.375 : .25 :: 343 : 25.47

Or thus,

3.375 : 343 :: .25 : 25.47

PROP. XIV.

THE Concave Diameters of two Guns being given, and the quantity of a weaker sort of Gun-powder sufficient to charge one of them, to find out how much Gun-powder of a stronger sort (the proportion of the strength and weakness of the Gun-powder being also given) will be sufficient to charge the other Gun,

This is solved by two operations in the Rule of proportion, first to find out how much of the stronger sort of Gun-powder will be of equivalent strength with the given quantity of the weaker sort, and this proportion is reciprocal; The second is the same with that in the foregoing Proposition.

Example

There is a Gun whose Concave Diameter is $1\frac{1}{2}$ inches, and it requireth 25 pound of powder to Charge it, now there is another sort of Gun-powder which is much stronger than the former, and the proportion between their strength is as 5 to 2, now I demand how much of the strongest powder is sufficient to charge a Gun whose Concave Diameter is 7 inches.

To answer this, First, I find out how much of the strongest powder will charge that Gun, which is $1\frac{1}{2}$ inch in its Concave Diameter, which is done by the following proportion, *viz.*

$$5 : 2 :: .25 : .10$$

Thus have I found that $\frac{1}{10}$ of a pound of the strongest Powder will charge a Gun whose Concave diameter is $1\frac{1}{2}$ inch. And according to the last proportion, I find by a direct Proposition that 10.16 pounds of the same will be sufficient to charge a Gun whose Concave diameter is 7 inches, *viz.*

$$3.375 : 343 :: .10 : 10.16$$

CHAP.

CHAP. XII.

Concerning Simple Intereſt.

I. WHen Money pertaining or belonging to one perſon is in the hands, poſſeſſion, or keeping, or is lent to another, and the Debtor payeth or alloweth to the Creditor, a certain ſum in conſideration of forbearance for a certain time, ſuch conſideration for forbearance is called Intereſt, Loane, or uſe Money; and the money ſo lent, and forborn is called the principal.

II. Intereſt is either Simple or Compound.

III. When for a ſum of Money lent there is loane or intereſt allowed, and the ſame is not paid when it becomes due, and if ſuch intereſt doth not then become a part of the Principal, it is called Simple Intereſt.

IV. In the taking of Intereſt for the continuance or forbearance of Money, reſpect muſt be had to the rate limited by Act of Parliament, which Act now in force, forbiddeth or reſtraineth all perſons whatſoever, from taking more than 6 *l.* for the intereſt of an 100 *l.* for a year, and according to the ſame proportion for a greater or a leſſer ſum, not confining the lender or borrower to the ſpace of one year, no more than it confineth him or them to the limitation of the ſum to be lent, or borrowed, but that the ſum may be either more or leſs than 100 *l.* and may continue in the hands of the Debtor, either a longer, or a ſhorter time than one year, according as the Lender and Borrower do agree, and oblige each other; now for any time greater than one year, the rate or proportion of Intereſt is by Act of Parliament limited, but the Act doth not ſay what part of 6 *l.* ſhall be the intereſt of an 100 *l.* for half a year, a quarter of a year, a month, a day, or for any time leſſer than one year, and in this caſe ſeveral Artiſts do differ in their opinions, ſome would have the true proportional intereſt for any time leſs than a year to be diſcover'd by continual mean proportionals; as ſuppoſe it were required to know the intereſt of 100 *l.* for half a year at 6 *per Cent. per Annum*, they would have the Intereſt to be reckoned after the Rule of Compound intereſt, and ſo 30 *l.* is not the intereſt of a 100 *l.* for half a year, but is too much: But ſay they, to find out the true intereſt thereof: you are to find a mean proportional between 100, and 106, and that made leſs by 100, will give you the intereſt of 100 *l.* for half a year, and ſo by extracting of Roots they find out the intereſt for any time leſs than one year, but this is ſufficiently laborious and painful if it be done without the help of Logarithms; but to perform this work to the 12 power for a Month, or to the 52 for a week, is very tedious, and to the 365 power for one day is ſcarcely poſſible to be effected by natural Numbers, but cuſtom and daily practice tell us that the intereſt of Money for any time leſs than one year ought to to be computed according to the Rules of Simple Intereſt, and ſo 3 *l.* is the undoubted intereſt of 100 *l.* for 6 months, and 30 ſhillings is the intereſt of 100 *l.* for a quarter of a year; but here note by the way that by 6 months is not meant 6 times 4 weeks, or 6 times 28 days, but by ſix months, or half a year is to be underſtood the half of 365 days, and a quarter of a year is $\frac{1}{4}$ of 365 days, and by 1 month is underſtood $\frac{1}{12}$ of 365 days, ſo that a month conſiſteth of 30 $\frac{5}{12}$ days.

Upon the aforeſaid cuſtom of computing the intereſt of Money for time leſs than one year, this following Analogy ſeems to be aſſumed for a ſafe expoſition of the ſtatute (and which is indeed the ground, and reaſon it ſelf of Simple Intereſt) *viz.* That ſuch proportion as 365 days (or one year) hath to the intereſt of any ſum for a year, ſuch proportion hath any part of one year, or any number of days propounded to the intereſt of the ſame ſum, for that time propounded. And this (as was ſaid before) is the whole ground work, and very foundation of the manner of computing of Simple Intereſt.

Vide *Sect. 6 of the 5 chap. of Mr.* Kerſies *Appendix to* Wing. *Arith.*

V. Rebate, or Diſcount, is, when there is an allowance of ſo much *per Cent.* for Money paid before it be due, and as the increaſe of Money at intereſt is found out by continual proportionals Arithmetical or Geometrical increaſing, ſo is the Rebate or diſcount of Money found out by continual proportionals decreaſing Arithmetically or Geometrically, that is according as the al-

Or Rebate, *what it is.*

lowance is, either after Simple or Compound Intereſt; Now the nature of Rebate or Diſcount is thus; when there is a ſum of Money, (ſuppoſe 100 *l.*) to become due at the end of a certain time to come, (*viz.* at the end of 12 Months;) and it is agreed upon by the Debtor and Creditor that there ſhall be made preſent payment of the whole Debt, and it is likewiſe agreed that in conſideration of this preſent payment that the Creditor ſhall allow the Debtor after the rate of 6 *per Cent. per Annum*: Now upon this agreement the Creditor ought to receive ſo much money as being put out to intereſt for the ſame time it was paid before 'twas due, and at the ſame rate of intereſt, that the diſcount was reckoned at, then would it amount or be increaſed to the ſum that was firſt due.

The manner of working Queſtions in Rebate at Simple Intereſt ſhall be ſhewn in the ninth Rule of this Chapter, and of working Queſtions in Rebate at Compound Intereſt ſhall be ſhewn in the Fourth Rule of the next Chapter.

VI. When the intereſt of a 100 *l.* for a year is known, the intereſt of any other ſum, for the ſame time, is alſo found out, by one ſingle rule of direct proportion, *viz.* The intereſt of a 100*l.* for a year by the ſtatute is 6 *l.* I demand what is the intereſt of 75 *l.* for the ſame time, and at the ſame rate of Intereſt? The proportion is as followeth.

l.	*l.*	*l.*	*l.*	*l.*	*s.*
100 :	6 : :	75 :	4.5=	4—	10

Or if you would have the Anſwer to produce both principal and intereſt, then make the ſe-

cond

cond number to be the ſum of the given principal and intereſt, and the fourth proportional will anſwer your deſire. Thus

l.		*l.*		*l.*		*l.*	*l.* *s.*
100	:	106	: :	75	:	79.5	=79—10

VII. When the intereſt of 100 for a year is given, and the intereſt of any other ſum of pounds, ſhillings and pence is required for a year, the anſwer may be eaſily found after the practical method delivered in the following Example.

Let it be required to find the intereſt of 148 *l.*—13 *s.*—04*d.* for one year after the rate of 6 *per Cent. per Annum*, Simple Intereſt?

Firſt, I place the given numbers according to the direction given for the Rule of 3, which will ſtand thus, *viz.*

l.		*l.*		*l.* *s.* *d.*
100	:	6	: :	148—13—04

Now it is evident that if I multiply 148 *l.*—13 *s.*—04 *d.* (which is the third number) by 6 (which is the ſecond number) and divide the product by 100 (which is the firſt number) the Quotient will be the anſwer; Therefore I proceed thus, *viz.* firſt I multiply the pence by 6, which makes 24 pence, or two ſhillings, therefore I ſet down 0 under the pence, and carry 2 to the next, then I go to 13 *s.* ſaying 6 times 13 is 78, and 2 that I carried is 80 *s.* which is 4 *l.* therefore I ſet down 0 under the ſhillings, and carry 4 to the pounds, then I proceed, ſaying 6 times 8 is 48, and 4 that I carry

carry is 52, then I ſet down 2, and carry 5, *&c.* proceeding thus till the work be finiſhed, and then will the product be 890 *l.*—00 *s.*—00 *d.* which product ſhould be divided by 100 (the firſt number) but it being an unite with two Cyphers, I cut off two figures from the right hand of the pounds, with a daſh of the pen, and the figures on the left hand of the ſaid daſh, are ſo many pounds, and thoſe on the right hand of it, are the Decimal parts of a pound, whoſe value may be found out by the 3 Rule of the 2 Chap. But remember, that if there be any ſhillings or pence, in the product you are to add them to their reſpective products in your Reduction.

The work of the foregoing Example is as followeth.

```
 l.       s.        l.   s.   d.
100   :   6   ::   148—13—04
                           6
                  ----------------
                 8|92—00—00
                  |20
                 --
                18|40
                  |12
                 --
                 4|80
                  | 4
                 --
                 3|20
```

So that by the work I find the intereſt of 148 *l.*—13 *s.*—4 *d.* for one year after the rate of 6 *per Cent. per An.* to be 8 *l.*—18 *s.*—04 *d.*—3.2 *qu.*

An-

Another Example may be this, *viz.* I demand the intereſt of 368 *l.*—15 *s.*—3 *d.* for one year, at 6 *per Cent. per An.* Anſwer 22 *l.*—02 *s.*—6 *d.* as by the work following.

```
                  l.    s.    d.
100 : 6 :: 368—15—03
                            6
           ------------------------
        22|12—11—06
          |20
        --|--
         2|51
          |12
        --|--
         6|18
```

VIII. The Intereſt of 100 *l.*: being known for a year, or 365 dayes, the intereſt of any other ſum may be known for any other time, or number of dayes, more or leſs than a year, by two ſingle Rules of 3 Direct, *viz.* Firſt, find out what is the intereſt of the given ſum, for one year, or 365 dayes, according to the laſt Rule, then having found out that, you may (by another ſingle Rule of 3 Direct) find out its intereſt for any other time more or leſs.

Example.

What is the intereſt of 322 *l.* for 6 years after the rate of 6 *per Cent. per Annum*, Simple Intereſt?

First I find what is the Interest of 322 *l.* for a year by the following proportion,

l.		*l.*		*l.*		*l.*
100	:	6	: :	322	:	19.32
				6		

100) 19|32 (19.32

Thus having found the interest of 322 *l.* for a year to be 19.32 *l.* at 6 *per Cent.* by the following proportion I find out its interest for 6 years, to be 115 *l.*—18 *s.*—04 ¾ *d*, and that added to the principal, makes 437 *l.*—18 *s.*—04 ¾ *d.* for the sum due to the Creditor at the end of the said time.

year		*l.*		*year*		*l.*	*l.*	*d.*
1	:	19.32	: :	6	:	115	—18	—04 ¾
		6				322		
		115|92				437	=18	—04 ¾
		|20						
		18|40						
		|2						
		4|80						
		|4						
		3|20						

And here take notice that the second number in this last proportion, must always be only the interest of the sum proposed, and not the sum of of the principal and intereſt, as in the ſecond proportion under the ſixth Rule.

After the ſame manner is the intereſt of 1 *l.* (at the rate of 6 *per Cent. per Annum*, or any other rate of intereſt,) diſcovered for a day, by the help of which the intereſt of any ſum whatſoever may be diſcovered for any number of dayes as ſhall be ſhown by and by.

	l.	*l.*	*l.*	*l.*
Firſt	100 :	6 : :	1 :	.06
	day	*l.*	*day*	
Secondly	365 :	.06 :	1 :	0001643835

So that by the foregoing proportions I have found that the intereſt of 1 *l.* at 6 *per Cent. per Annum* for a day is .0001643835 *l.*

Now if you would know the Intereſt of any other ſum for any number of days more or leſs than 365, you may do it by help of the ſaid number after this manner, *viz.*

Multiply the ſum whoſe intereſt is required by the ſaid number, and that product will give you the intereſt of the ſaid ſum for one day, then multiply that Product by the number of days given, and the laſt Product will give you the intereſt of the ſaid ſum for the number of dayes in the queſtion. Take the following queſtion for an example, *viz.*

What is the intereſt of 568 *l.* for 213 days after the rate of 6 *per Cent. per Annum* ?

.0001643835

```
   .0001643835
           568
   -----------
     13150680
    9863010
   8219175
   -----------
   .0933698280
           213
   -----------
    2801094840
    933698280
  1807390560
  -------------
  19.8877733640
```

	l.	*s.*	*d.*
Facit	19	17	09

Having finished the work as you see, I find the answer to be 19.8877, *&c.* which upon sight I discover to be 19 *l.*—17 *s.*—09 *d.* by the brief way of valuing a decimal Fraction of Coyne laid down in the 4 Rule of the second Chapter beforegoing.

But when the interest of any sum of Money is required for any number of days as aforesaid, at any other rate of interest than at 6 *per Cent. per Annum*, the aforesaid number will not then serve for the work, but you are to find out particular multiplyars for the several rates of interest as is before directed. All which I have expressed from 4 to 10 *per Cent.* in the following Table.

When

When you would find the Intereſt of any ſum for any number of dayes at the rate of	4	*per Cent. per An. the Multiplyar is*	.0001095890
	5		.0001369863
	6		.0001643835
	7		.0001917808
	8		.0002191780
	9		.0002465753
	10		.0002739726

So that when you would find out the intereſt of any ſum of Money for any number of dayes according to the direction before given, at any Rate from 4 to 10 *per Cent. per Annum*, Simple Intereſt, you may perform the work by the multiplyar in the foregoing Table which is placed againſt each reſpective rate of Intereſt.

IX. When the preſent worth of a ſum of Money due at the end of any time to come is required, Rebate being allowed at any rate of Simple Intereſt, it may be found out by the following method; *viz.* Firſt, Find out the intereſt of 100 *l.* for the time that the Rebate is to be allowed for, and at the ſame rate of intereſt propounded, then make the ſum of an 100 pound, and its intereſt for the propoſed time, to be the firſt number in the Rule of 3, and 100 *l.* the ſecond number, and the given ſum whoſe preſent worth is required, let be the third number, and the fourth number in a direct proportion ſhall anſwer the queſtion, as in the following Example, *viz.*

What preſent Money will ſatisfie a debt of 100 *l.* that is due at the end of a year yet to come, Diſcount or Rebate being allowed at the Rate of 6 *per Cent. per Annum.*

According to the foregoing Directions, I ſtate the numbers as followeth, and the fourth proprotional proportional number or anſwer to the queſtion is 94 33962 *l.*=94 *l.*—06 *s.*—09 ½ *d. fere.*

l.		*l.*		*l.*		*l.*
106	:	100	: :	100	:	94 33962

The reaſon of the ſaid Analogy will appear if you conſider, that there ought to be ſo much ready money paid, that if it were put out to intereſt at the ſame rate of Int. that Rebate was allowed for, and for the ſame time, the ſame would then be augmented to the ſum that was at firſt due, as in the laſt queſtion, there is given 100 *l.* which is due at the end of 12 Months, now I ſay, that there ought to be ſo much money paid down to ſatisfie this debt, as being put out to intereſt at 6 *per Cent.* for 12 Months, would then be increaſed to 100 *l.* which is the firſt ſum due, and again it is as evident that if there were 106 *l.* due at the end of 12 Months, or a year, and preſent payment is agreed upon, allowing Rebate at 6 *per Cent, per Annum*, that then there ought to be paid the ſum of 100 *l.* in full diſcharge of the ſaid debt of 106 *l.* for if when I have received the ſaid ſum of 100 *l.* I put it out to intereſt for one year at the rate of 6 *per Cent.* it will then be increaſed to 06 *l.*

Therefore to ſolve the ſaid queſtion, the proportion here uſed is no more than if I ſhould ſay, If 106 *l.* be decreaſed to 100 *l.* what will 100 *l.* be decreaſed to? The anſwer is, to 94 *l.*-6*s.*-9*d.* ½ and for proof, if you will ſeek what that ſum will be increaſed to at the end of 12 Months, at the rate of 6 *per Cent.* you will find it to be 100.

Example 2.

How much preſent Money will ſatisfie a debt of of 82 *l.*—15 *s.* due at the end of 126 dayes, yet to come allowing Rebate after the rate of 6 *per Cent. per Annum*?

First I find the interest of 100 *l.* at the same rate of interest for 126 dayes by the following proportion.

day		*l.*		*day*		*l:*
365	:	6	: :	126	:	2.0712

Then do I add 2.0712 *l.* (the interest of 100 *l.*) to 100 *l.* and the sum is 102.0712 which I make the first number in the Rule of 3, and 100 *l.* the second, and 82.75 *l.* (the sum given to be Rebated) the third number, and the fourth number in a direct proportion is the answer to the question, see the work as followeth.

l.		*l.*		*l.*		*l.*
102.0712	:	100	: :	82.75	:	81.0708
				100		

102.0712) 8275.00 (81.0708

So that by the work it appears that 82 *l.*—15 *s.* due at the end of 126 days yet to come, will be satisfied with the present payment of 81 *l.*—01 *s.*—04 $\frac{3}{4}$ *d.* Rebate be allowed after the rate of 6 *per Cent. per Annum.*

The proof of the Rule.

Find out (by the eighth Rule foregoing) how much the present money that is paid upon Rebate, will amount to being put out to interest for the same time, and at the same rate of interest that Rebate was allowed for, and if the amount be equal to the sum that was due at the end

of

of that time then you may conclude the work to be rightly performed, otherwiſe not.

As for *Example.*

In the foregoing queſtion it was found that 81. 0708 *l.* being paid preſently would ſatisfie a debt of 82. 75 due at the end of a 126 dayes to come, and to prove it, let us ſee whether 81.0708 being put out to intereſt for 126 days at the rate of 6 *per Cent. per Annum*, will be increaſed to 88.75 *l.* (the ſum which was ſaid to be due at the end of 126 days to come) which I do by theſe two proportions following according to the eighth Rule.

	day	*l.*	*day*	*l.*
Firſt,	365 :	6 : :	126 :	2.0712

	l.	*l.*	*l.*	
Secondly,	100 :	2.0712 : :	81.0707 :	1.6791, *&c.*

So you ſee that I have found the intereſt of 81.0708 for 126 days to be 1.6791, *&c.* which added to the principal 81 0708 the ſum is 12.7499 which by the brief way of valuing the Decimal of a pound ſterling is 82 *l.*—15 *s.* and indeed it doth not want $\frac{1}{10}$ part of a farthing of the exact ſum, which is occaſioned by the defective Decimal wherefore I conclude the work to be rightly performed.

Upon the foregoing ninth Rule is grounded the manner of calculating the enſuing Table of Multiplyers, which ſheweth in decimal parts of a pound, the preſent worth of a pound Sterling due at the end of any number of years to come,

not

not exceeding 30, Simple Intereſt being computed at 6 *per Cent. per Annum.*

The firſt number in the Table being found out by this following proportion, *viz.*

As 106 *l.* is to 100 *l.* ſo is 1 *l.* to .943396, and the ſecond number in the Table being the preſent worth of 1 *l.* due at the end of two years to come, is thus found out, *viz.* Firſt I conſider that 12 *l.* is the ſimple intereſt of 100 *l.* for 2 years, which added to 100 *l.* makes 112 *l.* wherefore I ſay as 112 *l.* is to 100 *l.* ſo is 1 *l.* to .892857 *l.* which is the preſent worth of 1 *l.* due at the end of two years to come.

The ſeveral proportions and operations for the whole Calculation being as followeth, *viz.*

1	106	:	100	: :	1	:	.943396
2	112	:	100	: :	1	:	.892857
3	118	:	100	: :	1	:	.847457
4	124	:	100	: :	1	:	.806451
5	130	:	100	: :	1	:	.769230
6	136	:	100	: :	1	:	.734294

And after the ſame manner are all the numbers in the following Table Calculated; which being well underſtood, the way of calculating moſt of the enſuing Tables will eaſily be obtained; and its uſe you will find immediately after the Table it ſelf.

TABLE I.

Which sheweth in Decimal parts of a pound the present worth of 1 *l.* due at the end of any number of years to come under 31, at the rate of *6 per Cent. per Ann.* Simple Interest.

Years		Years	
		11	.602409
		12	.581395
		13	.561797
		14	.543478
		15	.526315
		16	.510204
		17	.495049
		18	.480769
		19	.467289
		20	.454545
1	.943396	21	.442477
2	.892857	22	.431034
3	.847457	23	.420168
4	.806451	24	.409836
5	.769230	25	.400000
6	.735294	26	.390625
7	.704225	27	.381679
8	.675075	28	.373134
9	.649350	29	.364963
10	.625000	30	.357143

After the same method might this Table be continued to any number of years at pleasure, I might also have calculated for other rates of interest, as those are in the next Chapter concerning Compound Interest, but Simple Interest being not so generally in practice, I shall therefore forbear.

The use of the preceding TABLE.

It is evident (by the ninth Rule foregoing) that

that if any ſum be paid with an allowance of Rebate, you are to make 100 *l.* with its intereſt (for the ſame time you Rebate for) both in one ſum, to be the firſt number in the Rule of 3 100 the ſecond, and the ſum to be rebated the third, then will the fourth proportional be the anſwer; and the ſame may be wrought by any other number and its intereſt, as well as by 100 *l.* and its intereſt *mutatis mutandis*: Now in the Table beforegoing there is expreſſed in Decimal parts of a pound, the preſent worth of 1 *l.* due at the end of any number of years to come under 31, *&c.* that is to ſay, if you take the money ſignified by thoſe Decimals, and put it out to intereſt at 6 *per Cent. per Annum*, Simple intereſt for ſo many years as are expreſſed in the Collum of years againſt the ſaid Decimal, then will that ſum at the end of the ſaid Term, be augmented to 1 *l.* wherefore if you have any ſum whatſoever to be rebated for any number of years within the limits of the Table, make 1 *l.* the firſt number in the Rule of 3, and the Decimal in the Table againſt the number of years to be rebated for, make that the ſecond, and the ſum whoſe preſent worth is required the third number, ſo will the fourth proportional be the anſwer. But (becauſe the firſt number (being Unity) neither multiplieth nor divideth if you take the number in the Table, correſpondent to the number of years for which you would reckon Rebate, and thereby multiply the ſum whoſe preſent worth is required, the product will give you the Anſwer.

Examples

There is a ſum of Money, *viz.* 560 *l.* due at the end of 8 years to come, but the Debtor and Creditor agree that preſent payment ſhall be made, and the Debtor to be allowed Rebate after the rate of 6 *per Cent. per Annum*, Simple Intereſt. Now I demand how much preſent money will ſatisfie the ſaid Debt? Anſwer, 378.378 *l.*=378 *l.*—07 *s.*—06 $\frac{3}{4}$ *d.* ſee the following work.

l		*l.*		*l.*		*l.*
1	:	.675675	: :	560	:	378.378

```
   .675675
       560
  --------
  40540500
  3378375
  --------
 378.378000
```

Firſt (the Rebate being to be reckoned for 8 years) I look for 8 in the Collum of years, and juſt againſt it on the right hand, I find .675675 which I multiply by 560 (the ſum whoſe preſent worth is required,) and the product is 378.378, which by the brief way of valuing the fraction of a pound ſterling) I find at firſt ſight to be 378 *l.*=07 *s.*—06 $\frac{3}{4}$ *d.*

This Queſtion if it had been wrought by the foregoing ninth Rule would have produced the ſame anſwer, for, the Int. of 100 *l.* for 8 Mon. is 48 *l.* and 100 + 48=148 wherefore by the Rule of 3 I ſay

l.		*l.*		*l.*		*l.*
148	:	100	: :	560	:	378.378

X. When an Annuity or yearly Income in arrears for any number of years, and you would know the increaſe, or amount of it, allowing Simple Intereſt at a certain rate *per Cent. per Annum*, for each yearly payment from the time it firſt became due, the operation will be ſomewhat more tedious than to find the amount of one ſingle ſum, according to the eighth Rule of this Chap. which will clearly appear by ſolving the following queſtion, *viz.*

There is an Annuity, or an income of 100 *l.* *per Annum.* forborne to the end of 6 years, I demand how much is due at the end of the ſaid Term, allowing intereſt at the rate of 6 *per Cent. per Annum* Simple Intereſt? Anſwer 690 *l.*

In order to the ſolution of this Queſtion, I conſider, Firſt, that

It is evident that for the laſt year, *viz.* the ſixth years payment, there muſt be no intereſt at all Reckoned, becauſe it becomes not due till the end of the ſixth year; Secondly there muſt be reckoned the intereſt of 100 *l.* for one year, *viz.* that which is due at the end of the fifth year; Thirdly, there muſt be reckoned the intereſt of 100 *l.* for two years, *viz.* that which is due at the end of the fourth year. Fourthly, There muſt be reckoned the intereſt of 100 *l.* for three years, *viz.* that which is due at the end of the third year, Fifthly, the intereſt of 100 *l.* for 4 years, *viz.* that which is due at the end of the ſecond year: And Sixthly, The intereſt of 100 *l.* for 5 years, *viz.* that which is due at the end of the firſt year, and is forborne the ſecond, third, fourth, fifth and ſixth years; all which intereſts being added together, and their ſum added to the ſum of each years income, the ſum will exhibit the total ſum, due at the end of the ſaid ſix years, which you may perceive by the following work to be 690 *l.* which is the anſwer to the foregoing Queſtion.

			l.
The Intereſt of 100 *l.* at 6 *per Cent. per An.* Simple Intereſt, for	1	years is	06
	2		12
	3		18
	4		24
	5		30

The ſum of the intereſt is	90
The ſum of the annuities is	600
The Total amount is	690

The Conſtruction of Table II.

UPon the foregoing reaſon is grounded the Calculation of the following Table, which ſheweth the amount of 1 *l.* annuity, being forborn to the end of any number of years under 31, Intereſt being allowed for each yearly payment after the rate of 6 *per Cent. per Annum*, Simple Intereſt.

The firſt number in the Table being 1 *l.* which is that due at the end of the firſt year, no intereſt being due for that; the ſecond number in the

the Table is 2.06, which is the firſt and ſecond years-payment, and the intereſt of 1 *l.* for one year, being that which was due at the end of the firſt year; The third number in the Table is 3.18 *l.* being the increaſe of 1 *l.* for 2 years added to the ſecond number in that Table which is 2.06, for the amount of 1 *l.* at the end of 3 years is 1.12 which added to 2.06 the ſecond number it makes 3.18 for the third number; The fourth number is the amount of 1 *l.* for 3 years which is 1.18 added to the number before it, *viz.* the third number, proceeding in the ſame method, till you have compoſed the Table at your pleaſure, each number in the Table being 1 *l.* and the amount of 1 *l.* (for ſo many years as it ſtandeth againſt in the Table made leſs by one,) added to the number immediately preceeding it.

TABLE II.

Which ſheweth in pounds and Decimal parts of a pound the amount of 1 *l*, annuity being forborne to the end of any number of years under 31, Simple Intereſt being computed after the Rate of 6 *per Cent. per An.*

Years	
1	1.00
2	2.06
3	3.18
4	4.36
5	5.60
6	6.90
7	8.26
8	9.68
9	11.16
10	12.70
11	14.30
12	15.96
13	17.68
14	19.46
15	21.30
16	23.20
17	25.16
18	27.18
19	29.26
20	31.40
21	33.60
22	35.86
23	38.18
24	40.56
25	43.00
26	45.50
27	48.06
28	50.68
29	53.36
30	56.10

The uſe of Table II.

In the preceeding Table in the Collum under the word Years, are ſet down every Year ſucceſſively from 1 to 30, and the number in the Table placed againſt each year, is the amount of 1 *l.* annuity, in pounds and decimal parts of a pound, being forborne ſo many years as it is placed

placed againſt. The uſe of it will plainly appear by the ſolving of one, or two Queſtions, *viz.*

There is an Annuity of 134 *l.*—10 *s.* 6 *d.* all forborn to the end of 4 years; I demand how much is due to the Creditor at the end of the ſaid Term, Simple Intereſt being allowed after the rate of 6 *per Cent. per Annum*?

Facit 586 *l.*—10 *s.*—07 *d.*

To anſwer this Queſtion, firſt, I look for 4 years, in the Collum of years, and the number againſt it is 4.36 which is the amount of 1 *l.* Annuity for 4 years; therefore having turned the 1 *s.*—6 *d.* (in the given annuity) into a Decimal (which is .525) I ſay by the Rule of 3 thus,

l.		*l.*		*l.*		*l.*
1	:	4.36	: :	134.525	:	586.529
				4 36		
				807150		
				403575		
				538.00		
				586.529900		

Thus by the work I find the anſwer to be 58 .529 *l.* the value of which Decimal by the brief way of valuing a Decimal laid down in the 4th. Rule of the 2d. Chapter, I find to be 580 *l.* 10 *s.* 7 *d.*

And it is plain that in ſolving Queſtions by this Table, that (the firſt number in the Rule of 3 being unite) if you multiply the given Annuity by the proper Tabular Number, that then the product will be the anſwer.

Example 2.

What is the amount of an Annuity of 150 *l.* 10 *s.* being forborn to the end of the 7 years, allowing Simple Intereſt after the rate of 6 *per Cent. per Annum?* Anſwer, 1243 *l.*—02 *s.*—07¼*d. fere.*

The given Annuity is	150.5
The Tabular number for 7 years is	8.26
	9030
	3010
	12040
Facit	1243.130

The Rebate of Annuities at Simple Intereſt

XI. When an Annuity or yearly Income, for a certain number of years to come, is to be ſold for ready Money, and the ſeller is to allow the Buyer Rebate at Simple Intereſt for his preſent payment, then in this caſe the buyer ought to pay ſo much preſent Money for each yearly payment, as being put out at Simple Intereſt for ſo many years as it is Rebated for, it would then amount to one yearly payment, and the ſum of all thoſe preſent worths will be the preſent worth of the Annuity required, the Rule will appear very plain by the following Example.

There

There is an Annuity or Leaſe of 100 *l.* *per Annum* to continue 6 years yet to come to be ſold for ready Money, the Seller being to allow the Buyer Rebate at 6 *per Cent. per Annum*, Simple Intereſt: now I deſire to know how much preſent Money will buy out the ſaid Leaſe?

Facit 499 *l.*—09 *s.*—04¼*d. fere.*

It is evident that if we find out the preſent worth of 100 *l.* due at the end of the firſt year, and alſo the preſent worth of 100 *l.* due at the end of the ſecond year, and the preſent worth of 100 *l.* due at the end of the third year, and likewiſe the preſent worth of 100 *l.* due at the end of the fourth, fifth, and ſixth years, and add all theſe preſent worths together, their ſum will be the preſent worth of the given Annuity; which ſeveral preſent worths are found out according to the ninth Rule, by the ſeveral proportions following, *viz.*

years	*l.*		*l.*		*l.*		*l.*
1	106	:	100	: :	100	:	94.339622
2	112	:	100	: :	100	:	89.285714
3	118	:	100	: :	100	:	84.745762
4	124	:	100	: :	100	:	80.645169
5	130	:	100	: :	100	:	76.923076
6	136	:	100	: :	100	:	73.529411

The preſent worth of the ſaid Annuity is } 499.468754

So that you ſee by the foregoing proportions, the preſent worth of 100 *l.* *per Annum* to continue ſix years, allowing Rebate at 6 *per Cent. per An-*

Annum, Simple Intereſt, is 499.468754 *l.*=499 *l.* 9 *s.*—04 $\frac{1}{4}$ *d.*

The conſtruction of the 3 Table.

Upon the foregoing eleventh Rule is grounded the conſtruction and calculation of the following Table which ſheweth the preſent worth of 1 pound annuity to continue any number of years under 31 Simple Intereſt being computed after the rate of 6 *per Cent. per Annum*; the firſt number in the Table is .943396 which is the preſent worth of 1 pound due at the end of a year to come. The ſecond number in the Table is 1.836253, which is the ſum of the preſent worths of 1 *l.* due at the end of two years to come, and if 1 *l.* due at the end of one year to come added together; And the third number in the Table is 2.683710 which is the ſum of the preſent worths of 1 *l.* due at the end of 3, 2, and 1 years to come, after the ſame method is the whole Table calculated.

But the numbers in the ſaid Table may more eaſily be found out thus, *viz.* Look in the firſt Table, and let the firſt number of that be the firſt number of this third Table, and let the ſum of the firſt number in this, and the ſecond number in that be the ſecond number in this Table, and for the third number in this Table take the ſum of the ſecond in this, and the third in that Table, and in this manner you may proceed till you have compoſed the whole Table.

TABLE

TABLE III.

Which ſheweth the preſent worth of 1 *l.* annuity to continue any number of years under 31, Simple Intereſt being computed at *6 per Cent. per An.*

Years	
1	.943396
2	1.836253
3	2.683710
4	3.490161
5	4.259391
6	4.094685
7	5.698900
8	6.374575
9	7.023925
10	7.648925
11	8.251334
12	8.832729
13	9.394526
14	9.938004
15	10.464319
16	10.974523
17	11.469572
18	11.950341
19	12.437630
20	12.892175
21	13.334652
22	13.765686
23	14.175524
24	14 585360
25	14.985360
26	15.375985
27	15.757664
28	16.120798
29	16.485761
30	16.842904

The Uſe of the foregoing Table III.

In the foregoing third Table, in the left hand Colum under the Title of years, are expreſſed all the integral numbers, from one to 30, which ſignifie ſo many years, and the numbers in the Right hand Colum which are placed againſt the number of years are pounds, and decimal parts

parts of a pound ſterling, and every one of them are the preſent worth of 1 pound Annuity to continue ſo many years to come as are placed againſt them in the Collum of years, Rebate being allowed at Simple Intereſt 6 *per Cent. per An.*

As, ſuppoſe there were a Leaſe of 20 ſhillings *per annum* to continue 6 years, to be ſold for preſent Money, allowing the buyer Rebate at 6 *per Cent. per Annum* Simple Intereſt. I deſire to know how much is its preſent worth? To anſwer this, I look in the Collum of years for 6, and in the next Collum on the Right hand juſt againſt 6 you have 4.994685 *l.*=4 *l.*—19 *s.*—10 $\frac{3}{4}$ *d.* which is the anſwer to the Queſtion. And by the help of this Table may the preſent worth of any Annuity to continue any number of years under 31 be found out, allowing Rebate at 6 *per Cent. per Annum*, Simple Intereſt, by one ſingle Rule of 3 Direct, according to the manner of ſolving the following queſtion, *viz.*

Queſt. 1.

There is a Leaſe of 18 years yet to come, of the yearly value of 130 *l.* to be ſold for ready Money, and the purchaſer is to be allowed Rebate after the rate of 6 *per Cent. per Annum*, Simple Intereſt, now I demand how much is the preſent worth of this Leaſe?

Facit 1553 *l.*—10 *s.*—10 $\frac{1}{2}$ *d.*

Firſt, I look in the Table for 18 years, and over againſt it on the right hand I find 11.950341 which is the preſent worth of 1 pound annuity to continue 18 years, *&c.* Therefore by the Rule of 3 Direct, I ſay

1 *l.*

l.		l.		l.		l.
1	:	11.950341	: :	130	:	1553.544330

```
      130
  ─────────
  358510230
 11950341
 ─────────   l.   s.   d.
 1553.544330=1553—10—10¾.
```

So that by the work you find the anſwer to be 1553.544 *l. &c.* or 1553 *l.*—10 *s.*—10¾ *d.* very near, which ſaid anſwer is nothing elſe but the product of the Tabular number, (11.950341 *l.*) multiplied by the given annuity (130 *l.*) For it is evident, that if the preſent worth of 1 pound annuity to continue 18 years be 11.950341 *l.* then the preſent worth of 130 *l. per annum* to continue the ſame number of years (and Rebate being allowed at the ſame rate *per Cent. per An.* for the one as for the other) muſt be 130 times as much. But when rebate is to be allowed after any other rate then 6 *per Cent. per Annum*, then the foregoing Table will not at all be uſeful, but you muſt have recourſe to a Table calculated for the ſame rate of intereſt, which you may eaſily perform at leiſure by the foregoing rules.

Queſt. 2.

What Annuity to continue 18 years will 1553.5443 30 purchaſe, allowing the Buyer Simple intereſt at 6 *per Cent. per Annum* ?

Facit 130 *l.*

This

This Queſtion is but the converſe of the former, and may be thus reſolved, *viz.* Take the Tabular number correſponding to 18 years, which is 11.950341 by which divide the given purchaſe Money, and the Quotient will give you the annuity that it will purchaſe, *viz.*

l.	*l.*	*l.*
11.950341)	1553.54433	(130

So that by the work I find it will purchaſe an Annuity of 130 *l.* to continue 18 years.

The reaſon of the work is plain, for if the Tabular number correſpondent to 18 years be the preſent worth of 1 *l.* Annuity to continue 18 years to come, then it is certain that ſo much Money as is expreſſed by that Tabular number, will purchaſe an Annuity of 1 *l.* to continue 18 years: And conſequently we may find by help of the ſaid Table what annuity any other ſum of Money will purchaſe to continue any number of years not exceeding 30, by a ſingle Rule of 3 Direct, as in the laſt Queſtion, the proportion is as followeth, *viz.*

l.		*l.*		*l.*		*l.*
11.950341	:	1	: :	1553.54433	:	130

And it is no more in effect than a ſum in Diviſion, for the ſecond number (being 1) neither multiplyeth nor divideth, *&c.*

By what hath been ſaid concerning the uſe of the foregoing Table, you may perceive that the preſent worth of an Annuity is found out by multiplication, and to know what annuity any ſum will purchaſe is performed by Diviſion.

I might have made Tables for other Rates of Intereſt, but Simple Intereſt being ſeldom allowed in the purchaſing or valuing of Leaſes and Annuities, they being generally purchaſed at Compound Intereſt, or Intereſt upon Intereſt, makes me forbear, and indeed at Simple Intereſt a Leaſe is over-valued.

CHAP.

CHAP. XIII.

Of Compound Intereſt.

I. WHat hath been ſaid in the laſt Chapter, I judge ſufficient for the underſtanding of the Nature and Uſe of Simple Intereſt, and that being well underſtood, the nature of Compound Intereſt will not ſeem difficult to the ſtudious Learner, and the better he is acquainted with the nature of Simple Intereſt, ſo much the eaſier will he come to the knowledge of the nature, and uſe of Compound Intereſt.

II. Compound Intereſt is, when a ſum of Money is put out to Intereſt, and the Intereſt thereof becoming due is ſtill continued in the hands of the Debtor, ſo as to become part of the principal, intereſt being reckoned for it from the time it becometh due, for which reaſon it is called intereſt upon intereſt: And as Simple Intereſt increaſeth by a ſeries of Arithmetical proportionals continued; ſo doth Compound Intereſt increaſe by a rank or ſeries of continual Geometrical proportionals. For when a ſum of Money is put out to intereſt at any rate *per Cent. per Annum*, (as ſuppoſe a 100 *l.* to be put out to receive at the end of one year 6 *l.* for its intereſt) it is evident that if the intereſt (being 6 *l.*) be continued in the hands of the Debtor, there will be at the end of the ſecond year the increaſe of of 106 *l.* which is 112.36 *l.* and at the third years end there will be the increaſe of 112.36*l.* ſo that every number proceedeth from that going before it, after the ſame rate or reaſon as 100 proceedeth from 100, as you ſee following.

l.		*l.*		*l.*		*l.*
100	:	106	: :	106	:	112.36
100	:	106	: :	112.36	:	119.1016
100	:	106	: :	119.1016	:	126.247696

So that by the Augmentation of 100 *l.* in 4 years you have this rank of Geometrical proportionals continued, *viz.* 100, 106, 112.36, 119.1016 and 126.247696 which is in number 5, *viz.* more by one than is the number of years the laſt of which is the amount of 100 *l.* at 6 *per Cent.* for 4 years reckoning Compound Intereſt, or Intereſt upon Intereſt, and each of theſe proportionals proceedeth from that going before it as 106 proceedeth from 100, that is to ſay, every of the ſaid proportionals, is in ſuch proportion to that which goeth before it as 106 is to 100, or as 100 is to 106, ſo is any one of them, to that which followeth it, or if you take any 3 of them which are placed together, there is this proportion between them, *viz.* As the firſt of thoſe three is to the ſecond, ſo is the ſecond to the third, and the third to the fourth, and the fourth to the fifth, and the fifth to the ſixth, *&c.* whence it is evident that they have amongſt themſelves this following Qualification, *viz.* that the ſquare of any one of them is equal to the Rectangle, or Product, made by that which is placed immediately before it, and that immediately after it, and the same would it be if there were never so many Terms, and is a peculiar property of all numbers that are Geometrical proportionals continued

III. The Interest of 100 *l.* for a year being known, the Compound Interest of any other sum for any number of years may be likewise found out by so many single Rules of 3, as there are given years, for,

As 100 *l.* is to its increase for one year, so is any other sum to its increase for the same time, and so is the first years increase to the second, and the second years increase to the third, and so is the third years increase to the fourth, *&c.*

Example.

Let it be required to find how much 350 *l.* will be increased to, being put out to Interest at 6 *per Cent. per Annum*, Compound Interest for 5 years? Answer, 468 *l.*—7 *s.*—4¾ *d. fere*, See the following work.

l. *l.*	350 : 371
	371 : 393.26
100 : 106 : :	393.26 : 416.8556
	416.8556 : 441.866936
	441.867936 : 468.37895216

Where-

Whereby you ſee that 350 *l.* being put out to intereſt after the rate of 6 *per Cent.* will at the firſt years end be increaſed to 371 *l.* And 371 *l.* being put out for the ſecond year, will be increaſed to 393.26 *l.* and 393.26 *l.* being made a principal, and put out at the ſame rate for the third year, will at the end thereof be increaſed to 416.8556 *l.* and at the end of 5 years it will be increaſed to 468.37895216 *l.*

And upon the aforeſaid Grounds is calculated the following Table 1, whoſe Conſtruction and uſe immediately followeth the ſame.

TABLE I.

Which sheweth what one Pound will amount to, being forborn to the end of any number of years to come, not exceeding 30 Compound Int. being computed at 5, 6, 7, 8, 9, or 10 *per Cent. per Annum.*

Years	5.	6.	7.	8.	9.	10.
1	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000
2	1.10250	1.12360	1.14490	1.16640	1.18810	1.21000
3	1.15762	1.19101	1.22504	1.25971	1.29502	1.33100
4	1.21550	1.26247	1.31079	1.36048	1.41158	1.46410
5	1.27628	1.33822	1.40255	1.46932	1.53862	1.61051
6	1.34009	1.41851	1.50073	1.58687	1.67710	1.77156
7	1.40710	1.50363	1.60578	1.71382	1.82803	1.94871
8	1.47745	1.59384	1.71818	1.85093	1.99256	2.14358
9	1.55532	1.68947	1.83845	1.99900	2.17189	2.35794
10	1.62889	1.79084	1.96715	2.15892	2.36736	2.59374
11	1.71033	1.89829	2.10485	2.33163	2.58042	2.85311
12	1.79585	2.01219	2.25219	2.51817	2.81266	3.13842
13	1.88564	2.13292	2.40984	2.71962	3.06580	3.45217
14	1.97993	2.26090	2.57853	2.93719	3.34172	3.79749
15	2.07892	3.39655	2.75903	3.17216	3.64248	4.17724

A Continuation of the preceding TABLE I.

Years	5.	6.	7.	8.	9.	10.
16	2.18287	2.54035	2.95216	3.42594	3.97030	4.59497
17	2.29201	2.69277	3.15881	3.70001	4.32763	5.05447
18	2.40661	2.85433	3.27993	3.99601	4.71712	5.55991
19	2.52695	3.02559	[illegible]552	4.31570	5.14166	6.11590
20	2.65329	3.20713	3.8[illegible]68	4.66095	5.60441	6.72749
21	2.78596	3.39956	4.14056	5.03383	6.10880	7.40024
22	2.92526	3.60353	4.43040	5.43654	6.65860	8.14027
23	3.07152	3.81975	4.74053	5.87146	7.25787	8.95430
24	3.22509	4.04893	5.07236	6.34118	7.91108	9.84973
25	3.38635	4.29187	5.42743	6.84847	8.62308	10.83470
26	3.55567	4.54938	5.80735	7.30935	9.39915	11.91818
27	3.73345	4.82234	6.21386	7.98806	10.24508	13.10999
28	3.92012	5.11178	6.64883	8.62710	11.16713	14.42099
29	4.11613	5.41838	7.11425	9.31727	12.17218	15.86309
30	4.32194	5.74349	7.61225	10.06265	13.26767	17.44940

The

The Construction of the foregoing TABLE I.

By the third Rule foregoing it is evident that the Interest of 100 *l.* for a year being known, the Compound Interest for any other sum may be found out for any number of years; According to which Rule all the numbers in the said Table are found out, being the amount of 1 *l.* at Compound Interest for any number of years, not exceeding 30, being put out at any of these Rates, *viz.* 5, 6, 7, 8, 9, or 10 *per Cent. per Annum*, which numbers are found out by the Rule of Proportion thus,

$$100 : 105 :: \begin{cases} 1 : 1.05 \\ 1.05 : 1.1025 \\ 1.1025 : 1.157625 \\ 1.157625 : 1.21550625 \end{cases}$$

By which means the four first numbers in the second Colume of the Table (being placed under the number 5) are found, and by a continuation of the same operation are all the rest of the numbers in that Colume found out; which is indeed nothing else but a continual multiplication of the first number, (*viz.* 1.05,) into it self 29 times, and so the last number in that Colume is the thirtieth power of 1.05, and the same Colume may be continued to any other number of years at pleasure above 30; the numbers in this Colume being the interest of 1 *l*, at

5 *per*

5 *per Cent. per Annum* Compound Intereſt for 30 years.

The numbers in the third Colume under the Figure 6, are the increaſe of 1 *l.* at 6 *per Cent. per Annum.* Comp. Int. for 30 years, and are found out by multiplying 1.06 into it ſelf 29 times according to the Rule of Continual Multiplication. The like is to be underſtood of all the reſt.

The uſe of the foregoing TABLE.

In the firſt Colume of the Table under the Title years, are expreſſed the number of years from 1 to 30, and in the ſecond Colume under the figure 5, and againſt every reſpective year are expreſſed the increaſe of 1 *l.* being put out at 5 *per Cent. per Annum*, Compound Intereſt.

In the third Collum under the number 6 is expreſſed the yearly increaſe of 1 *l.* being put out at 6 *per Cent. per Annum*, Compound Int. And ſo in the 4, 5, 6, and 7 Collums, are the yearly amounts of 1 *l.* at 7, 8, 9, and 10 *per Cent. per Annum*, Compound Intereſt.

All which numbers in the ſaid Table are multiplyers, for the producing of the amount or Increaſe of any other ſum being put out at Compound Intereſt, at any rate of Intereſt, and for any number of years therein expreſſed, as will appear by the following Examples.

Example 1.

I demand the full amount of 365 *l.* being put to Interest for 9 years, Interest being Computed after the rate of 6 *per Cent. per Annum*, Compound Interest? Facit 616 *l.*—13 *s.*—01½ *d.*

Here because the sum proposed is put out at 6 *per Cent.* and for 9 years, I look in the Collum of 6 *per Cent.* which is the third Collum of the Table under the figure 6, and just against 9 in the Collum of years, I find 1.68947 which is the increase of 1 *l.* being forborn the same time, and at the same rate of Interest, wherefore by the Rule of 3 I say

l.		*l.*		*l.*		*l.*
1	:	1.68947	::	365	:	61665655

```
    1.68947
        365
    -------
     844735
   1013682
   506841
   --------
   616.65655
```

So that by the work I find that if the sum of 365 *l.* be all forborn to the end of 9 years, and interest be computed for the same at 6 *per Cent. per Annum*, Compound Interest, it will then be increased to 616.65655 which is 616 *l.*—13 *s.* 1½ *d.*

Example 2.

What will 128 *l.*—16 *s.*—08 *d.* be increased to? The utmost improvement thereof being made for 15 years at 7 *per Cent. per Annum*, Compound Interest? Facit 355 *l.*—09 *s.*—01 *d.*

First,

First, turn the 16 *s*—8 *d.* into the Decimal of a pound by the 2d Rule of the 2d Chapter foregoing, and you will find it to be .8333, so that the given sum is 128.8333, *&c.*

Now to answer this question, I look into the foregoing Table, and in the Collum of 7 *per Cent.* and just against 15 years I find 2.75903 which is the uttermost increase of 1 *l.* for 15 years at 7 *per Cent.* Compound Interest, by which if you multiply the given sum, the product will be the answer to the question, as by the following work will plainly appear.

```
1 : 2.75903 :: 128.8333 :
                 2.75903
              ----------
                 3864999
               115949970
               6441665
             9018331
            2576666
            ------------
            355.454939699
```

By the foregoing work the answer is found to be 355.4549, *&c.*=355 *l.*—09 *s.*—01 *d.*

But if any sum be put out at Compound Int. for months, or days over and above the given number of years, then the work will be somewhat different from the former; for first you must find out the amount of the given sum, for the given number of years, and then by the 8th Rule of the foregoing Chapter find out the Interest of that amount for the odd time, being either months or dayes under a year, and that Int. being added to the aforesaid amount, that sum will

will be the answer to the question: this is so obvious that it needeth no Example.

Of Rebated or Discount at Compound Interest.

IV. When a sum of Money due at the end of any number of years to come is to be satisfied with present money, allowing rebate at Compound Interest, there must be found a Rank or Series of continual proportionals, more in number by one than the number of years for which the discount is proposed, of which rank or series of proportionals, the sum to be satisfied by present payment must be the first, and the second must decrease from that after the same rate or proportion as 100 decreaseth from the sum of 100 added to its interest for one year, after the rate of Interest propounded; that is to say, as 100 proceedeth from 106, or 108 if the interest be 6 or 8 *per Cent.* and after the same rate or reason must the third decrease from the second, and the fourth from third, *&c.*

When a question is stated for the rebate of Money at Compound Interest, it is solvable by as many single Rules of 3, as the number of years for which the sum proposed is to be Rebated, and it is nothing else but the inverse of the third Rule of this Chapter, as may be proved by the working of the following question, taken out of the said Rule, where it is proved that 350 *l.* being forborn in the Debtors hands for 5 years at 6 *per Cent.* Compound Interest, it will then be increased to 468.38001216; now let the said Question be inverted thus, *viz.*

There is a sum of Money, *viz.* 468.38001216 due at the end of 5 years to come, now I demand how

how much preſent Money will ſatisfie the ſaid Debt, rebate being allowed after the rate of 6 *per Cent. per Annum*, Compound Intereſt?

First, I ſay, as 106 is to 100, ſo is the ſum due at the end of 5 years, *viz.* 468.38001216, to 441.867936, which is the ſum due at the fourth years end, and ſo is the ſum due at the fourth years end, to the ſum due at the third years end, &c. as by the work appeareth.

		l.		*l.*
		468.38001216	:	441.867936
		441.867936	:	416.8556
106 : 100	: :	416.8556	:	393.26
		393.26	:	371
		371	:	350

So that by the foregoing work you ſee that if 468.38001216 *l.* be due at the end of 5 years to come, and is to be ſatisfied by the payment of preſent money, rebate being allowed at 6 *per Cent. per Annum*, Compound Intereſt, 350 *l.* is the ſum required.

And upon this Rule is grounded the Calculation of the following Table, which ſheweth what 1 *l.* due at the end of any number of years to come, not exceeding 30 is worth in preſent Money, Rebate being reckoned at any of theſe rates, *viz.* 5, 6, 7, 8, 9, or 10 *per Cent. per An.* Compound Intereſt.

TABLE

TABLE II.

Which sheweth the present worth of one pound payable at the end of any number of years to come, not exceeding 30, Rebate being yearly allowed at 5, 6, 7, 8, 9, or 10 *per Cent. per Annum*, Compound Interest.

Years	5.	6.	7.	8.	9.	10
1	.952381	.943396	.934579	.925925	.917431	.909090
2	.907029	.889996	.873438	.857338	.841680	.826446
3	.363837	.839619	.816297	.793832	.772183	.751314
4	.822702	.792093	.762895	.735029	.708425	.683013
5	.783526	.747258	.712986	.680583	.649331	.620921
6	.746215	.704960	.666342	.630169	.596267	.564474
7	.710681	.665057	.622749	.483490	.547034	.513158
8	.676839	.627412	.582009	.540268	.501866	.466507
9	.644608	.591898	.543933	.500248	.460427	.424097
10	.613913	.558391	.508349	.463193	.422410	.385543
11	.584679	.526787	.475092	.428882	.387532	.350494
12	.556837	.496989	.444012	.397113	.355534	.318630
13	.530321	.468839	.414964	.367697	.326178	.289664
14	.505067	.442300	.387817	.340461	.299246	.263331
15	.481017	.417265	.362446	.315241	.274538	.239392

A Continuation of the preceeding TABLE II.

Years	5.	6.	7.	8.	9.	10.
16	.458111	.393646	.338734	.291890	.251869	.217629
17	.436296	.371364	.316574	.270269	.231073	.197844
18	.415520	.350343	.295864	.250249	.211993	.179858
19	.395733	.330512	.276508	.231712	.194489	.163508
20	.376889	.311804	.258419	.214548	.178430	.148643
21	.358942	.294155	.241513	.198655	.163698	.135130
22	.341849	.277505	.225713	.183940	.150181	,122846
23	.325571	.261797	.210947	.170315	.137781	.111678
24	.310067	.246978	.197146	.157699	.126405	.101525
25	.295302	.232998	.184249	.146018	.115967	.092296
26	.281240	.219810	.172195	.135201	.106392	.053905
27	.267848	.207367	.160930	.125186	.097607	.076277
28	.255093	.195630	.150402	.115913	.089548	.069343
29	.242946	.184556	.140562	.107327	.082154	.063039
30	.231377	.174110	.131367	.099377	.075371	.057308

The

The Construction of the foregoing TABLE.

By the Fourth Rule of this Chapter is plainly shewn the manner of finding the present worth of any sum of Money due at the end of any number of years to come, Rebate being computed at Compound Interest, and after the same manner are all the numbers in the foregoing Table found as you may see by the following Example, where the four first numbers in the third Colume are methodically found out by the Rule, that being the Colume of Rebate at 6 *per Cent. per Annum*,

I	106 : 100 : : 1 : .943396226415, *&c.*
II	106 : 100 : : 94339622264 : .88999644, *&c.*
III	106 : 100 : : 88999644 : .83961928, *&c.*
IV	106 : 100 : : 83961928 : .79209460, *&c.*

So that by the foregoing proportions, I say first, if 106 *l.* be decreased to 100 *l.* what will 1 *l.* be decreased to? Answer, to .94339 *l. &c.* =18 *s.* 10 ½. The five first figures thereof being the first number in the third Colume of the foregoing Table, and it sheweth that the present worth of 1 *l.* due at the end of one year to come, Rebate being allowed at 6 *per Cent.* is 94339 *l.*=18 *s.*—10½ *d.*

Secondly, I say, by the Rule of 3, If 106 *l.* be decreased to 100 *l.* what will .94339, *&c.* be decreased to? The answer is .88999 *l. &c.*=17 *s.* 09 ½ *fere.* And this is the second number in the third

third Colume of the ſaid Table, and is placed againſt 2 years in the firſt Colume, and ſheweth the preſent worth of 1 *l.* due at the end of two years to come, Rebate being allowed after the Rate of 6 *per Cent. per Annum*, Compound Intereſt.

And after the ſame manner are all the reſt of the numbers in the ſaid Colume of 6 *per Cent.* found out, and alſo all the other Decimal Fractions in the ſecond, fourth, fifth, ſixth, *&c.* Columes, ſhewing the Rebate of one pound for any number of years not exceeding 30, at 5, 7, 8, 9, and 10 *per Cent.* (*mutatis mutandis.*)

The uſe of the foregoing TABLE II.

The firſt Colume is the number of years for the Rebate of 1 *l.* and the numbers in the reſt of the Columes, are Decimal Fractions ſhewing the preſent worth of 1 *l.* due at the end of ſo many years to come, as they are placed againſt in the Colume of years, Rebate being allowed at the ſame rate of intereſt under which they are placed, the figures 5, 6, 7, 8, 9, and 10 placed at the top, denoting the ſame. An Example or two will make its uſe more plain.

Example 1.

I demand how much preſent Money will ſatisfie a Debt of 684 *l.* due at the end of 6 years to come, allowing rebate after the rate of 8 *per Cent. per Annum*, Compound Intereſt? To anſwer this Queſtion, look in the Colume of 8 *per Cent.* and againſt 6 years I find this number, *viz.* 63017 which ſheweth that if 1 *l.* be due at the end of

6 years

6 years to come, its present worth is .63017 *l.* Rebate being allowed after the rate of 8 *per Cent. per Annum.* Compound Interest.

Therefore I say by the Rule of 3, If 1 *l.* be decreased to .63017 *l.* what will 684 *l.* be decreased to at that rate? *Facit* 431.03628 *l.* = 431 *l.* 0 *s.* 8 ½ *d.* as by the work appeareth.

l.	*l.*	*l.*	*l.*
1	: 63017	: 684	: 431.03628

```
63017
  684
------
252068
504136
378102
------
431.03628
```

l. *s.* *d.*
Facit 431—00—08 ½ *fere*

So that you see the sum proposed being multiplyed by the proper Tabular number, produceth the answer to the question, for the number 1, which is here the first number in the Rule of 3 doth not either multiply or divide, and therefore the answer is found out by the multiplication only. Observe the work of the next Example.

Example.

What is the present worth of 164 *l.*—15 *s.* due at the end of 9 years to come, allowing Rebate after the rate of 6 *per Cent. per Annum,* Compound Interest? *Facit* 97 *l.*—10 *s.*—03 ¼.

Look in the Table aforesaid in the Collum of 6 *per Cent.* and against 9 in the Collum of years you will find this number, *viz.* .59189 which is the present worth of 1 *l.* due at the end of 9

years

years to come, and is the proper multiplyar for finding the anſwer to this Queſtion, as by the work.

l.		*l.*	*l.*
1 :	.59189 :	164.75 :	97.5138775

```
     .59189
      16475
     ------
     295945
    414323
   236756
  355134
  59189
  ---------
  97.5138775
```

The anſwer found by the foregoing operation is 97.5138775=97 *l.*—10 *s.*—03 $\frac{1}{4}$.

But if the given time for the Rebate of any ſum conſiſteth of odd Months or Days, beſides years, then in ſuch caſe, the Rebate (at the given Rate of Intereſt) for the odd time muſt be found by the 9th. Rule of the 12 Chap. foregoing) for the given ſum, and then the preſent worth of the given ſum thus decreaſed, muſt be found for the number of years as in the two laſt Examples.

Example 3.

There is 640 *l.*—10 *s.* due at the end of ſix years, and 3 months to come, what is its preſent worth, Rebate being allowed at the Rate of 7 *per Cent. per Annum*, Compound Intereſt?

Firſt, I find the decreaſe of 640.5 *l.* for Months thus, *viz.*

mon.		*l.*		*mon.*		*l.*
12	:	7.	:	3	:	1.75

So that I find the Int. of 100 *l.* for 3 Months at 7 *per Cent.* to be 1.75 *l.* which added to 100 *l.* makes 101.75, then to find the decreaſe of 640.5 *l.* for 3 Months: I ſay,

l.		*l.*		*l.*		*l.*
101.75	:	100	: :	640.5	:	629.484

So that I find by the laſt proportion, that if at the end of 6 years 3 months: There was due 640.5 *l.* yet at the end of 6 years there will be due but 629.484 *l.* whoſe preſent worth by the foregoing directions will be found to be 419 *l.*—9 *s.* for,

l.		*l.*		*l.*		*l.*	*l.*	*s.*
1	:	.66634	: :	629.484	:	419 450 &c.	=419	—9

Read the 9th. Rule of the Twelfth Chapter foregoing, and you will eaſily underſtand the method here uſed for ſolving Queſtions of this nature.

V. Queſtions in Rebate at Compound Intereſt may be reſolved by the Firſt Table of this Chapter which ſheweth the increaſe of 1 *l.* at Compound Intereſt, &c. But as in the ſecond Table you make the Tabular numbers multiplyars, to find out the preſent worth of a ſum; ſo if if you would find out the preſent worth of a ſum by the firſt Table, you muſt then make thoſe Tabular Numbers Diviſors; the Reaſon whereof is plain, for the firſt Table ſheweth the increaſe of 1 *l.* for 30 years, *&c.* But they may likewiſe ſerve to ſhew what ſum of Money due at the end of any number of years to come under 31 (allowing Rebate according to the rates of Intereſt therein mentioned) 1 *l.* preſent Money will ſatisfie. Now to Reſolve Queſtions in Rebate by this Table, look in the Collum of the propoſed Intereſt or Rebate, and againſt the propoſed number of years is the Tabular number for your work, which muſt be according to the following proportion, *viz.*

As the Tabular Number ſo found,
Is to 1,
So is the ſum propoſed to be Rebated,
To its preſent worth.

To make this a little more plain, I ſhall Anſwer the firſt Queſtion in the Uſe of the ſecond Table, by the help of the firſt Table only, which is as followeth, *viz.*

I demand how much preſent Money will ſatisfie a Debt of 684 *l.* due at the end of 6 years to come, allowing Rebate after the rate of 8 *per Cent. per Annum*, Compound Intereſt?

Look in Table 1. in the Collum of 8 *per Cent.* and againſt 6 years you will find this number, *viz.* 1.58687, therefore the proportion is as followeth.

 1.58687 *l.*

l.	*l.*	*l.*	*l.*
1.58687 :	1 ::	684 :	431.03719

So that by this proportion the answer is 431.03719 *l.* =431 *l.*—00—08 $\frac{3}{4}$ *d.* very near to the answer before found by the second Table of Rebate.

VI. When an annuity is in arrear, and it is required to know its utmost improvement, accounting Interest upon Interest for each particular sum from the time it becomes due, to the end of the given Term of years. The manner how to work such Questions will be apparent by the working of the following Question, *viz.*

The manner of valuing Annuities that are in arrear.

There is an Annuity of 150 *l.* to continue to the end of five years, and the utmost improvement thereof to be made after the rate of 6 *per Cent. per Annum*, Compound Interest; now I demand how much will then be due to the Creditor?

It is evident that there must be found out, first the amount of 150 *l.* for one year, *viz.* that which is due at the end of the fourth year, it lying in the Debtors hands all the fifth year.

Secondly, There must be accounted the improvement of 150 *l.* for 2 years, *viz.* that which is due at the end of the third year, it lying in the Debtors hands the fourth, and fifth years.

Thirdly,

Thirdly, There muſt be accounted the improvement of 150 *l.* for 3 years, *viz.* that which is due at the end of the ſecond year, it lying in the hands of the Debtor the third, fourth, and fifth years.

And in the fourth place there muſt be accounted the utmoſt improvement of 150 *l.* for 4 years, *viz.* that which is due at the end of the firſt year, it lying in the Debtors hands the ſecond, third, fourth, and fifth years.

And beſides there muſt be accounted 150 *l.* due at the end of the fifth year, no Intereſt being reckoned for that, becauſe it becometh not due till the expiration of the laſt year, and then the ſum of all theſe is the utmoſt amount of that annuity.

The ſolving of Queſtions concerning Annuities at Compound Intereſt, will not be any thing different in their operation, from the manner of ſolving a Queſtion concerning a ſingle ſum of money put out for years at Compound Intereſt, by the third Rule before-going. As ſuppoſe that inſtead of an Annuity of 150 *l.* there was a ſingle ſum of 150 *l.* put out for 4 years at Compound Intereſt, at 6 *per Cent.* what would be its utmoſt improvement at the end of the ſaid Term?

Here you will eaſily perceive that in ſolving the one, the other is alſo ſolved.

See the work according to the foregoing third Rule.

l.	l.	
100	: 106 ::	150 : 159
		159 : 168.54
		168.54 : 178.6524
		178.6524 : 189.361544
		: 150
		845.553944

Now if the foregoing proportions be well considered, you will find that

The sum due at the end of the fifth year, being that years Rent is ———— } l. 150

And 150 *l.* due at the end of the fourth year, will at the fifth years end be encreased to ———— } l. 159

And 150 *l.* due at the end of the third year, will at the end of the fifth year be increased to ———— } l. 168.54

And 150 *l.* due at the second years end, will at the fifth years end be increased to ———— } l. 178.6524

And 150 *l.* due at the first years end, will at the fifth years end be increased to ———— } l. 189.371544

The sum of all these being due at the five years end is— } l. 845.553944

So that if an Annuity of 150 *l.* be all forborn to the end of five years, and it be improved to the utmoſt after the rate of 6 *per Cent. per Annum*, Compound Intereſt, it will then be increaſed to the ſum of 845.553944=845 *l.* 11 *s.* 0¾ *d.*

Now if the particular numbers in finding out the augmentation of the ſaid Annuity according to the manner before preſcribed, be well viewed, and the method in finding them out be well conſidered, it will appear, that if an Annuity, payable by yearly payments, be all forborn to the end of any number of years, and the utmoſt improvement thereof be made at Compound Intereſt, the total then due at the end of the ſaid time, or term of years, will be the ſum of a ſeries, or Rank of continual proportionals as many in number as the years of the Annuities forbearance, the firſt being the Annuity, or yearly payment it ſelf, and the ſecond proceeding from the firſt after the ſame Rate or proportion as 100 *l.* and its Intereſt for a year added together, proceedeth from 100 *l.* and after the ſame Rate doth the third proceed from the ſecond, and the fourth from the third, *&c.*

The manner of Calculating the following Third TABLE.

And upon this Rule is grounded the Calculation of the following Table, which ſheweth what 1 *l.* Annuity (being forborn to the end of any number of years to come, not exceeding 30) will be increaſed to Compound Intereſt, being computed after any of the Rates mentioned at the head of the Table.

But conſidering that as an Annuity increaſeth yearly at Compound Intereſt, the ſum due at each years end, is the ſum of a ſeries of continual proportionals equal in number to the yearly payments, and that the firſt number is the annual payment its ſelf, therefore may a Table to ſhew the Annual increaſe of 1 *l.* Annuity with great eaſe be made from the firſt Table, ſhewing the yearly increaſe of 1 *l.* at Compound Intereſt, as will plainly appear by what followeth.

Let us pitch upon making the Collum of 6 *per Cent. per Annum*, in the third Table ? Look in the firſt Table, and you will find the Collum of 6 *per Cent.* to have for its firſt number 1.06, and the ſecond number 1.12360 *&c.* And to make the Collum of 6 *per Cent.* in the third Table proceed thus, for the firſt number in the ſaid third Table put 1, or 1.00000, and for the ſecond number in the third Table, take the ſum of the firſt number in the third Table (which is 1.00000,) and the firſt number in the firſt Table (which is 1.06) and that makes 2.06 for the

the ſaid ſecond number; then add the ſecond number in the third Table, to the ſecond in the firſt, and their ſum is the third number in the third Table; then add the ſaid third number to the third number in the firſt Table, and their ſum is the fourth number in the third Table, *&c.* And after this manner proceed till you have made all the numbers in the ſaid Colum of 6 *per Cent.* And after the ſame method are the reſt of the Colums made, (the firſt number in each being 1. or 1.00000) *mutatis mutandis.*

But here note, that the numbers in the ſaid firſt Table ought to be continued to more places than are there expreſſed, to prevent the errors that elſe may be found in the third Table, by adding of defective Decimals. The uſe of the ſaid Table is ſhewn immediately after the ſame.

CHAP.

TABLE III.

Which sheweth in Pounds and Decimal parts of a Pound the increase or Amount of 1 *l.* Annuity, to continue any number of Years not exceeding 30, Compound Interest being computed at 5, 6, 7, 8, 9, and 10 *per Cent. per Annum.*

Years	5.	6.	7.	8.	9.	10.
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.05000	2.06000	2.07000	2.08000	2.09000	2.10000
3	3.15250	3.18360	3.21490	3.24640	3.27810	3.31000
4	4.31012	4.37461	4.43994	4.50611	4.57312	4.64100
5	5.52563	5.63709	5.75073	5.86660	5.08471	6.10510
6	6.80191	6.97531	7.15329	7.33592	7.52333	7.71561
7	8.14200	8.39383	8.65402	8.92280	9.20043	9.48717
8	9.64910	9.89746	10.25980	10.63662	11.02847	11.43588
9	11.02656	11.49131	11.97798	12.48755	13.02103	13.57947
10	12.57789	13.18079	13.81644	14.48656	15.19292	15.93742
11	14.20678	14.97164	15.78359	16.64548	17.56029	18.53116
12	15.91712	16.86994	17.88845	18.97712	20.14071	21.38428
13	17.71298	18.88213	20.14064	21.49529	22.95338	24.52271
14	19.59863	21.01506	22.55048	24.21492	26.01918	27.97498
15	21.57856	23.27596	25.12902	27.15211	29.36091	31.77248

A Continuation of the preceeding TABLE III.

Years	5.	6.	7.	8.	9.	10.
16	23.65749	25.67252	27.88805	30.32428	33.00339	35.94972
17	25.84036	28.21287	30.84021	33.75022	36.97370	40.54470
18	28.13238	30.90565	33.45024	37.45024	41.30133	45.59917
19	30.53900	33.75999	37.44625	41.44626	46.01845	51.15909
20	33.06595	36.78559	40.76196	45.76196	51.16011	57.27499
21	35.71925	39.99272	44.86517	50.42292	56.76453	64.00249
22	38.50521	43.39228	49.00573	55.45675	62.87333	71.40274
23	41.43047	46.99582	53.43614	60.89329	69.53193	79.54302
24	44.50199	50.81557	58.17667	66.76475	76.78981	88.49732
25	47.72709	54.86451	63.24903	73.10593	84.70089	98.34705
26	51.11345	59.15638	68.67646	79.95441	93.32397	109.18176
27	54.66912	63.70576	74.48382	87.35076	102.72313	121.09994
28	58.40258	68.52810	80.69769	95.33882	112.96821	134.20993
29	62.32271	73.63979	87.34652	103.96593	124.13535	148.63092
30	62.43884	79.05818	94.46078	113.28321	136.30753	164.49402

The

The Use of the third TABLE:

The numbers 5, 6, 7, 8, 9, 10 at the head of the Table are the several Rates of Interest, of 100 *l.* for a year, and the numbers placed in the several Collums under those numbers, shew the yearly increase of 1 pound Annuity, at the same Rate of Interest as it is placed under, and for so many years as it is placed against in the Collum of years on the left hand of the Table; and the use of these numbers will be manifest by the method used in solving the following Question, *viz.*

There is an Annuity of 34 *l.*—8 *s.* payable by yearly payment, forborn unto the end of Twelve years; Now, I demand how much is due at the end of the said Term, Compound Interest being allowed at 6 *per Cent. per Annum*? Facit 580 *l.*—6 *s.*—6 *d.* and somewhat more as will appear by the follwing operation.

The increase of the said Annuity being proposed at 6 *per Cent.* I look in the Collum which hath the number 6 placed at the head of it; and against the number 12 in the Collum of years I find the number 16.86994 which sheweth that if 1 *l.* Annuity be forborn to the end of 12 years, and there be allowed Compound interest at 6 *per Cent.* it will then be increased to 16.86994=16 *l.*—17 *s.*—04 $\frac{1}{2}$ *d.* therefore I say by the Rule of Proportion.

1 *l.*

l.		l.			l.		l.
1	:	16.86994	:	:	344	:	580.324936

```
  16.86994
       344
 ---------
  6747976
 6747976
5060982
 ---------
580325936
```

Whereby it is apparent that those tabular numbers are only Multiplyars for the producing of the amount of any given annuity for any number of years not exceeding 30, any Rate of Compound Interest, being allowed from 5 to 10 *per Cent.* Inclusive, &c.

VII. Questions concerning the increase of Annuities at Compound Interest may be likewise solved by the first Table in this Chapter, according to the following method, *viz.*

When an Annuity is in arrear, and it is required to know to what sum it is augmented, Compound Interest being computed, &c. Find out what principal will in one year gain the Annual Rent proposed, allowing the proposed Rate of Interest. Then (as is taught in the use of the said first Table) find the increase of the said principal for the number of years, and at the rate of interest proposed, and from the amount thereof subtract the said principal, then will that Remainder be the amount of the given Annuity for the given time, as will appear by solving the first Question of the sixth Rule beforegoing

foregoing, which is this, *viz.* there is an Anty of 150 *l.* forborn to the end of 5 years what is its amount at 6 *per Cent. per Annum*, Compound Intereſt?

Now to anſwer this, I find out a principal that at 6 *per Cent.* will gain 150 *l.* in one year, which I do by the following proportion, *viz.*

l.		*l.*		*l.*		*l.*
6	:	100	: :	150	:	2500

So that I find 2500 *l.* to be the anſwer, then ſuppoſing the ſaid principal 2500 *l.* to be put out to intereſt at 6 *per Cent.* Compound Intereſt for 5 years, look in the firſt Table in the Collum of 6 *per Cent.* and againſt 5 years you will find 1.338225, *&c.* which being multiplyed by 2500, produceth 3345.563944 from which if you ſubtract the ſaid principal 2500 *l.* there will remain 845.563944 for the anſwer which is the ſame with that found before.

VIII. When an Annuity to continue any number of years is to be bought with ready money, there ought to be paid ſo much money, as being put out at Compound Intereſt, at any Rate, and for the time of the Leaſes continuance, its total amount may be equal to the utmoſt improvement of the ſaid Annuity, being all forborn to the time of the Leaſes expiration, Compound Intereſt being Computed at the ſame rate. And the manner of finding out ſuch a preſent worth, is as in the following Example, *viz.*

The manner of finding the preſent worth of annuities, Rebate being allowed at Comp. Int.

viz. There is an Annuity of 468.38001216 *l.* to continue 5 years what is its preſent worth, allowing Rebate after the rate of *6 per Cent. per Annum, Compound Intereſt.*

Here it is plain that there muſt firſt be computed the preſent worth of the ſaid Annuity, due at the end of the firſt, ſecond, third, fourth and fifth years, and the ſum of all theſe preſent worths, will be the preſent worth of the ſaid annuity, as will appear by the following work which is wrought by the fourth Rule of this Chapter.

	l.
The preſent worth of 468.38001216 *l.* due at the end of the firſt year is——	441.867936
The ſame ſum due at the end of two years, is in ready money worth. ——	416.8556
The ſame ſum due at the end of three years is worth——	393.26
The ſame ſum due at the end of four years is worth——	371
The ſame due at the end of five years is worth in ready money.——	350
The ſum of the ſaid preſent worths is——	1972.983536

Which is the preſent worth of an Annuity of 468.3001216 to continue 5 years Rebate being al-

allowed at the Rate of *6 per Cent. per Annum*, Compound Intereſt.

The Conſtruction of the following TABLE IV.

And upon the ſame grounds with the ſolution of the laſt Queſtion is calculated, the following fourth Table, which ſheweth the preſent worth of 1 *l.* Annuity to continue any number of years, not exceeding 30, and payable by yearly payments, Rebate being allowed after the rate of 5, 6, 7, 8, 9, and 10 *per Cent. per An.* Compound Intereſt.

But the nature of the following Table being rightly conſidered, you will find the making of it to be eaſily performed by help of the numbers in the ſecond Table of this Chapter.

As for Example.

Let us pitch upon the making of the Collum of 6 *per Cent.* Firſt, I turn to the ſecond Table, and by the numbers in the Collum of 6 *per Cent.* I do the work; The firſt number in the ſecond Table, I make to be the firſt number in the fourth, and to that fourth I add the ſecond number in the ſecond Table, and their ſum is the ſecond number in the fourth Table; then to this ſecond number do I add the third number in the ſecond Table, and their ſum is the third number

ber in the fourth Table and after the ſame manner are all the reſt of the numbers in that Collum made, and alſo thoſe in the reſt of the Collums, *mutatis mutandis.*

But remember when ever you Calculate one Table by the help of another, to continue the Table you make uſe of, to more places than you intend the numbers in your Table to conſiſt of for fear of errors through the Addition of Defective Decimals.

O TABLE

TABLE IV.

Which ſheweth the preſent worth of one pound Annuity payable by yearly payment, and to continue any number of years not exceeding 30, Rebate being allowed at 5, 6, 7, 8, 9, or 10 *per Cent. per Annum*, Compound Intereſt.

Years	5.	6.	7.	8.	9.	10.
1	.95238	.9433	.93457	.92952	.91743	.90909
2	1.85941	1.83339	1.80801	1.80801	1.75911	1.73553
3	2.72324	2.67301	2.62431	2.62431	2.53129	2.48685
4	3.54595	3.46510	3.38721	3.38721	3.23971	3.16986
5	4.32947	4.21236	4.10019	3.10019	3.88965	3.79078
6	5.07569	4.91732	4.76653	4.76653	4.48591	4.35526
7	5.78637	5.58233	5.38928	5.38928	5.03295	4.86841
8	6.46321	6.20979	5.97129	5.97129	5.53481	5.33492
9	7.10782	6.80169	5.51529	6.51523	5.99524	5.75901
10	7.72173	7.36008	7.02358	6.02358	6.41765	6.14456
11	8.30641	7.88687	7.49867	7.49867	6.80519	6.49506
12	8.86325	8.38384	7.94268	7.94268	7.16072	6.81369
13	9.39357	8.85268	8.85765	7.35765	7.48690	7.10335
14	9.89864	9.29498	8.74546	8.74546	7.78614	7.36668
15	10.37965	9.71224	9.10791	9.10791	8.06068	7.60608

A Continuation of the preceeding TABLE IV.

Years	5.	6.	7.	8.	9.	10.
16	10.83776	10.15089	9.44664	8.85136	8.31255	7.82371
17	11.27406	10.47725	9.76322	9.12163	8.54363	8.02155
18	11.68958	10.82760	10.05908	9.37188	8.75562	8.20141
19	12.08531	11.15811	10.33559	9.60359	8.95011	8.36492
20	12.46220	11.46992	10.59401	9.81814	9.12854	8.51356
21	12.82115	11.76407	10.83557	10.01681	9.29224	8.64869
22	13.16300	12.04158	11.06124	10.20074	9.44242	8.77154
23	13.48857	12.30337	11.27218	10.37105	9.58020	8.88329
24	13.79864	12.55035	11.46933	10.52875	9.70661	8.98474
25	14.09394	12.78335	11.65358	10.67477	9.82257	9.07704
26	14.37518	13.00316	11.82577	10.80997	9.92897	9.16094
27	14.64303	13.21053	11.98671	10.93516	10.02657	9.23722
28	14.89812	13.40616	12.13711	11.05107	10.11612	9.30656
29	15.14071	13.59071	12.27767	11.15840	10.19828	9.36960
30	15.39244	13.76482	12.40904	11.25778	10.27365	9.42691

The

The Uſe of the foregoing TABLE.

The firſt Collum is the number of years from 1 to 30, and the number 5, 6, 7, 8, 9 10, at the head of the Table are the Rates of Intereſt of 100 *l.* for a year, and the numbers in each of theſe Collums under the ſaid rates of intereſt are the preſent worths of 1 *l.* Annuity to continue for the number of years which is placed againſt them, allowing Rebate after the rate of Intereſt at the head of each Collum, and are multiplyars ſerving to find the preſent worth of any other Annuity, as will appear by the following

Example.

There is an Annuity of 48 *l.* to continue 12 years, and payable by yearly payments, to be ſold for preſent money, I demand what it is worth, allowing Rebate at 6 *per Cent. per Annum*, Compound Intereſt? Facit 402.424=40 *l.* 8 *s.* 05 $\frac{3}{4}$ *d.* which is thus found out by the foregoing Table, *viz.* look in the ſaid Table, in the Collum of 6 *per Cent.* and againſt 12 in the Collum of years, you have this number, *viz.* 8.38384, which is the preſent worth of 1 *l.* Annuity to continue twelve years, Rebate being allowed, &c. therefore by the Rule of proportion, I ſay

l.	*l.*		*l.*		*l.*
1 :	8.38384	: :	48	:	402.42432
	48				
	6707072				
	3353536				
	402.42432				

So that I find the anſwer to be 402.42432, which is found by multiplying the ſaid Tabular number by 48, as you ſee by the work.

Otherwiſe find a principal which may bear ſuch proportion to the given Annuity that is to be Rebated) as 100 beareth to the Rate of Intereſt allowed in the Rebate. Then find the preſent worth of this principal ſo found, by the Directions given in the uſe of the ſecond Table of this Chapter, then ſubtract the ſaid preſent worth from the principal found as before, and the remainder will be the preſent worth of the given Annuity, Rebate being allowed as propoſed.

Example.

What is the preſent worth of an Annuity of 50 *l.* to continue 3 years, allowing Rebate at 8 *per Cent. per Annum*, Compound Intereſt?

Firſt, I find a principal that ſhall be to the given number 50 as 100 is to 8 which I find to be 625 *l.* by the following proportion, *viz.*

l.		*l.*		*l.*		*l.*
8	:	100	: :	50	:	625

Then by the ſecond Table I find the preſent worth of 625 *l.* which is 496.145 *l.* which I ſubtract from the ſaid principal 625 *l.* and there remaineth 128.855 *l.*=128 *l.*—17 *s.*—1 $\frac{1}{4}$ *d. fere* which is the preſent worth of 50 *l. per Annum*, to continue 3 years, Rebate being allowed at 8 *per Cent. per Annum*, Compound Intereſt.

Moreover by the numbers in the foregoing fourth Table, you may at firſt ſight diſcover how many years purchaſe any Leaſe to continue any number of years, not exceeding 30 is worth in ready money, Compound Int. being Computed on both ſides at any of the rates mentioned at the head of the Table.

Example.

Suppoſe there were a Leaſe iſſuing out of Lands to continue 16 years to be ſold for ready money, allowing Rebate at 8 *per Cent. per Annum*, Compound Intereſt, I demand how many years purchaſe the ſaid Leaſe is worth?

Look in the Table 4, in the Collum of 8 *per Cent.* and againſt 16 years you will find 8.85136 which ſheweth that it is worth 8.85136 years purchaſe which is ſomewhat above 8 years, and 3 quarters; But if the ſaid Leaſe had been of Houſes, and 10 *per Cent.* were thought a convenient allowance for the ſame, then you will find it to be worth 7.82371 years purchaſe which is 7 years, and above 3 quarters purchaſe.

IX. When there is a ſum of money propounded, and it is required to know what annuity to con-

continue any given number of years, it will purchaſe according to any given Rate of Intereſt, you may ſuppoſe any annuity at pleaſure, then by the directions given in the uſe of the fourth Table; or elſe by the eighth Rule of this Chapter, find the preſent worth of the ſuppoſed annuity for the number of years, and at the rate of Intereſt propounded, which being done, you may find what annuity to continue the ſaid number of years, the ſum propounded will purchaſe by the following proportion, *viz.*

Of the purchaſe of Annuities at Comp. Intereſt.

As the preſent worth of the ſuppoſed annuity.
Is to the ſaid annuity.
So is the ſum propounded,
To the annuity required.

As for Example.

Let it be required to find out what annuity to continue 4 years, 800 *l.* preſent money will purchaſe, Compound Intereſt being computed at 6 *per Cent. per Annum*? Facit 230.873 *l.*

Firſt, ſuppoſe an annuity at pleaſure to continue 4 years, as ſuppoſe 150 *l.* then do I find the eighth Rule of this Chapter the preſent worth of the ſaid annuity to be 519.76584 *l.* therefore by the Rule of proportion I ſay

l.	*l.*	*l.*	*l.*
519.76584, *&c.* :	150 ::	800 :	230.873

The Construction of the following TABLE V.

Upon the reason of the foreging Rule is grounded the Calculation of the following Table for the purchasing of Annuities; and it may somewhat more readily be Calculated thus, *viz.*

It is evident by the construction of the first Table of this Chapter, that 1 *l.* present is equivalent to 1.06 due at the end of a year to come; therefore is 1.06 the first number in the Collum of 6 *per Cent.* of the following Table; because 1 *l.* will purchase 1.06 *l.* Then it is also evident by the fourth Table that the present worth of 1 *l.* *Annuity* to continue two years at the same rate is 1.83339, *&c.* that is 1.83339, *&c.* will purchase a Lease of 1 *l. per Annum* to continue two years, Compound Interest, being allowed at 6 *per Cent.* therefore by the Rule of 3 Direct, I say,

l.				*l.*
1.13339, *&c.*	:	1	: : 1 :	.54543, *&c.*

By which I find that 1 *l.* ready money will buy a Lease of .54543 *l. per Annum* to continue 2 years therefore it is the second number in the following Table. Likewise by the fourth Table I find that 2.67301 is the present worth of 1 *l.* Annuity to continue 3 years at the same rate of Interest, wherefore by the Rule of proportion I say,

l.		*l.*		*l.*		*l.*
2.67301, *&c.*	:	1	: :	1	:	.37411, *&c.*

Where-

Whereby I find that 1 *l.* will purchaſe an Annuity of .37411, to continue 3 years. Compound Intereſt being allowed at 6 *per Cent.* wherefore .37411, is the third number in the ſaid Table; whereby it is evident that if you divide 1, or Unite by the ſeveral numbers in the ſaid Collum of 6 *per Cent.* in the fourth Table, ſucceſſively, the ſeveral Quotients will give you the numbers ſucceſſively, for the Collum of 6 *per Cent.* in the fifth Table; And after the ſame manner are all the numbers in the other Collums of the ſaid fifth Table found out (except the firſt number of each Collum, which muſt be the ſame with the firſt numbers in each Collum of the firſt Table) *mutatis mutandis.*

But it is abſolutely neceſſary that the numbers in the ſaid fourth Table, be continued to more places than there are expreſſed, to prevent the errors that otherwiſe will ariſe, by dividing by defective Decimals.

TABLE

TABLE V.

Which ſheweth what Annuity payable by yearly payments to continue any number of years not exceeding 30, one pound will purchaſe, Compound Intereſt, being computed at 5, 6, 7, 8, 9, or 10 *per Cent. per Annum.*

Years	5.	6.	7.	8.	9.	10.
1	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000
2	.53780	.54543	.55309	.56076	.56846	.57619
3	.36720	.37411	.38105	.38803	.39505	.40211
4	.28209	.28859	.29519	.30192	.30866	.31947
5	.23097	.23739	.24389	.25045	.25709	.26379
6	.19701	.20336	.20979	.21631	.22291	.22960
7	.17281	.17913	.18555	.19207	.19869	.20545
8	.15472	.16103	.16746	.17401	.18067	.18744
9	.14069	.14702	.15548	.16007	.16679	.17364
10	.12950	.13586	.14237	.14902	.15582	.16274
11	.12038	.12679	.13335	.14007	.14694	.15396
12	.11284	.11927	.12590	.13269	.13965	.14576
13	.10645	.10296	.11965	.12652	.13356	.14077
14	.10101	.10758	.11434	.12129	.12843	.13574
15	.09634	.10296	.10979	.11682	.12405	.13147

A Continuation of the preceeding TABLE V.

Years	5.	6.	7.	8.	9.	10.
16	.09226	.09895	.10585	.11298	.12029	.12781
17	.08869	.09544	.10242	.10962	.11704	.12466
18	.08554	.09235	.09941	.10670	.11421	.12192
19	.08274	.08962	.09675	.10412	.11173	.11954
20	.08024	.08718	.09439	.10184	.13954	.11745
21	.07799	.08500	.09228	.09983	.10761	.11562
22	.07597	.08304	.09040	.09803	.10590	.11400
23	.07413	.08127	.08871	.09642	.10438	.11257
24	.07247	.07967	.08718	.09497	.10302	.11129
25	.07095	.07822	.08581	.09367	.10180	.11016
26	.06956	.07690	.08456	.09250	.10071	.10915
27	.06829	.07569	.08342	.09144	.09973	.10825
28	.06712	.07459	.08239	.09048	.09885	.10745
29	.06604	.07357	.08144	.08961	.09805	.10672
30	.06496	.07260	.08058	.08882	.09733	.10607

The

The use of the foregoing Table V.

The use of the foregoing Table will appear in the solution of the following Question, *viz.*

A Merchant hath 1500 *l.* by him, which he is willing to lay out upon an Annuity, issuing out of Lands to continue 20 years, beginning presently Compound Interest being Computed on both sides at 6 *per Cent. per Annum.* Now I demand what Annuity the said sum will buy? Facit 130.77 *l.*=130 *l.*—15 *s.*— 05 *d.* very near.

To answer this question. I look in the Collum of 6 *per Cent.* of the foregoing fifth Table, and against 20 in the Collum of years I find .08718, which is the annuity that 1 *l.* present money will purchase to continue 20 years, wherefore by the Rule of three Direct, I say.

l.		*l.*		*l.*		*l.*
1	:	08718	: :	1500	:	130.77

X. Questions concerning the purchasing of Leases and Annuities may be solved very well by the numbers in the fourth Table, if you make them Divisors instead of Multiplyars.

Let the last Question be proposed, and solved by the fourth Table, *viz.*

What Annuity to continue 20 years will 1500 *l.* ready Money purchase, Compound Interest being allowed at 6 *per Cent.*

To answer this I look in the fourth Table, in the Collum of 6 *per Cent.* against 20 years and there I find this number, *viz,* 1146992, which is the present worth of 1 *l.* Annuity to continue 20 years, Compound Interest being allowed at 6 *per Cent.* And if it be the present worth of 1 *l.* Annuity, I conclude it will purchase 1 *l.* An-

Annuity to continue the ſame number of years, wherefore I ſay by the Rule of 3 Direct,

l.		*l.*		*l.*		*l.*
11.46992	:	1	: :	1500	:	130.77

So that the anſwer is the ſame with the former which was found by help of the Fifth Table.

All the foregoing Tables might have been continued to any greater number of years at pleaſure; But although theſe Tables are calculated but for 30 years; yet they may be made ſerviceable for years above 30, as ſhall be ſhewed by and and by.

Arithmetical Queſtions to exerciſe the Learner in the Precedent Tables.

Queſt. 1. There is a Leaſe of 20 years to begin preſently, which in ready Money is worth 1200 *l.* But ſuppoſe the ſaid Leaſe were not to begin till the expiration of 8 years, I demand what would be the preſent worth of the ſaid Leaſe Rebate, being allowed at 8 *per Cent. per Annum*, Compound Intereſt?

The main intent of this Queſtion is to ſhew the uſe of the ſecond Table, for if you find the preſent worth of 1200 *l.* due at the end of 8 years, at 8 *per Cent.* the Queſtion is anſwered, which according to the directions given after the ſaid Table, will be found to be 648.3216 *l.* =648 *l.*—06 *s.*—05 *d.*

Queſt. 2. *A* oweth to *B* 600 *l.* to be paid in 6 years, *viz.* 100 *l.* every year, but being weakned in his Eſtate, is not able to perform; but an Estate being to come into his hands at the end of 10 years; *B* is willing to forbear it all till then, and to be allowed Compound Interest at 8 *per Cent.* for his forbearance; I demand how much will be due to *B* at the 10 years end?

This Question is solved by help of the third and first Tables; for first 100 *l.* is to be paid in the nature of Annuity for 6 years, therefore by the third Table I find the amount of an annuity of 100 *l.* to continue 6 years at 8 *per Cent.* which is 733.592 *l.* and will be due at the expiration of 6 years, and then is that sum to be forborn to the end of 10 years, which is 4 years after the 6 years; which being a single sum, its amount is found by the first Table to be 998.037 *l.* &c. which is the answer to the question.

Quest. 3. There is a Lease to continue 21 years to be sold for 1000 *l.* but the Lessee desireth rather to pay an annual Rent: Now the question is what that annual Rent ought to be Compound Interest being computed at 10 *per Cent. per Annum*?

The intent of this Question is to find what annuity to continue 21 years 1000 will purchase at 10 *per Cent.* which is to be done by the fifth Table thus;

Because the time is for 21 years, look in the Collum of years for 21 and just against it in the Collum of 10 *per Cent.* you will find .11562, by which multiply 1000, and the product is 115.62 *l.* and so much will 1000 *l.* purchase for 21 years at 10 *per Cent.* Compound Interest.

Quest. 4. *A* and *B* have each of them a Lease to continue 20 years; *A* hath 80 *l. per Annum*, and *B* 120 *l. per Annum*, and they agree to make an exchange, upon this condition, that *A* ſhall pay in ready money the exceſs of his Eſtate, allowing him Compound Intereſt at 8 *per Cent.* Now I demand how much ready money *A* ought to give *B* upon this exchange, according to that condition?

Subtract 80 *l.* from 120 *l.* and the Remainder is 40 *l.* and ſo much *per Annum* is the Leaſe of *B* worth more than that of *A*, therefore *A* muſt pay *B* ſo much money as will purchaſe 40 *l.* *per Annum* to continue 20 years at 8 *per Cent.* which by the third Table will be found to be 392.7256 *l.*

Queſt. 5. There is a Houſe to be let by Leaſe for 21 years, for which the Leſſor will have 50 *l.* fine, and 70 *l.* *per Annum*, but the Leſſee is willing to pay the greater fine, that he may have the Rent but 40 *l.* *per Annum*, now I demand what fine he ought to pay upon that condition Compound Intereſt being allowed at 8 *per Cent. per Annum*?

Take the difference between 40 and 70, which is 30 for the abatement in the yearly Rent for 21 years; Then by the fourth Table find the preſent worth of 30 *l.* *per Annum* for 21 years at 8 *per Cent.* which is 300.5043 *l.*=300 *l.*—10 *s.*—01 *d.* which added to the ſaid 50 *l.* fine makes 350 *l.*—10 *s.*—01 *d.* for the fine to be paid upon the ſaid condition.

Queſt. 6. There is a Leaſe to be let of 20 *l.* *per Annum*, and 250 *l.* fine for 24 years, and the Leſſee is willing to pay the greater Rent, that he may pay but 50 *l.* fine, now I demand what Rent he ought to pay upon that condition, Compound Intereſt being computed at 7 *per Cent. per Annum*?

It

It is manifeſt, that if the Leſſor taketh 50 *l.* ne, he abateth 200 *l.* therefore find by the fifth Table what Annuity to continue 24 years, 200 *l.* will purchaſe at 7 *per Cent.* The Tabular number is .08718, which multiplyed by 200 produceth 17.436=17 *l.*—8 *s.*—9 *d.* and ſo much muſt the Leſſee raiſe his rent if he will have 200 *l.* abated of his fine, to which if you add 20 *l.* the propoſed Rent, the ſum is 37 *l.*—8 *s.*—09 *d.* for the yearly Rent to be paid to ſatisfie the ſaid condition.

Queſt. 7. What Annuity to continue 20 years, may I grant preſently, for 900 *l.* to be paid 6 years hence, accompting 6 *per Cent. per Annum*, Compound Intereſt.

Firſt find by the ſecond Table the preſent worth of 900 *l.* due 6 years hence, at 6 *per Cent.* which is 634.464 *l.*=634 *l.*—09 *s.* 03 ½ *d.*

Then by the Fifth Table find what Annuity to continue 20 years 634.464 will purchaſe at 6 *per Cent.* And you will find the anſwer to be 55.31257152 *l.*=55 *l.*—06 *s.*—03 *d.* and ſo much I ought to grant yearly for 20 years for 900 *l.* to be paid me at the end of 6 years.

Queſt. 8. I have 6 years of an old Leaſe, yet to come, and would take a new Leaſe in reverſion for 21 years, after the expiration of the old Leaſe, the annual Rent whereof is 40 *l.* But I would pay ſuch a ſum of Money preſent as a fine, that for my Leaſe in Reverſion for the ſaid 21 years, I may pay but 15 *l. per Annum*, Now I demand how much preſent money I ought to pay the Leſſor, to ſatisfie theſe conditions, Compound Intereſt being computed at 8 *per Cent.*

The difference between 40 and 15 is 25, and ſo much the Leſſee deſireth to have abated in his Rent,

Rent, wherefore by the fourth Table find the preſent worth of 25 *per Annum* for 21 years at 8 *per Cent.* which is 250.42025 *l.*=250 *l.*—08 *s.*—05 *d.* Then by the ſecond Table find the preſent worth of 250.42025 *l.* due at the end of 6 years to come, at 8 *per Cent.* which is 157.807 *l.* &c.= 157 *l.*—16 *s.*—01 ¾ And ſo much ought I to give to ſatisfie the ſaid conditions.

Queſt. 9. There is a Leaſe to be let for 12 years, for 20 *l. per Annum*; and 20 *l.* fine, but the Leſſe deſireth to take a Leaſe of the ſame for 21 years, and to pay the ſame Rent, the Queſtion is, what fine ought to be paid for the Leaſe of 21 years, accounting Compound Intereſt at 6 *per Cent*? Facit 280 *l.*—12 *s.*—05 *d.*

By the fifth Table ſeek what Annuity to continue 12 years, 200 *l.* will purchaſe at 6 *per Cent.* which you will find to be 23.854 *l.* Then by the Fourth Table find the preſent worth of 23.854 *l.* Annuity to continue 21 years at 6 *per Cent.* which is 280.620 *l.* &c.=280 *l.*—12 *s.*—05 *d.* and ſo much ought the Leſſee to pay for a fine, to have his Leaſe for 21 years.

Queſt. 10. A Gentleman hath 1000 *l.* which he would lay out to purchaſe an Annuity of 100 *l.* to be paid by yearly payments; Now the Queſtion is, how many years muſt the ſaid Annuity continue, Compound Intereſt being allowed on both ſides at 8 *per Cent. per Annum*?

Firſt, Divide 1000 by 100, and the Quotient will be 10, which ſheweth that the Buyer giveth 10 years purchaſe for the ſaid Annuity.

Then in the Fourth Table, and in the Collum of 8 *per Cent.* look for the number 10, which cannot be exactly found, but the neareſt to it and leſs than it, is 9.8184 which is placed againſt

20 years, and the nearest to it greater than it is, is 10.01681, therefore I conclude that the Annuity must continue above 20 years, but not 21 years, and to find out how much it must continue more than 20 years, I work thus, *viz.* First, I find the difference between the said Tabular numbers 10.1681 and 9.81814, which is .19867. Then I find the difference between the lesser of the said Tabular Numbers, *viz.* 9.81814 and 10, the Number that I would find in the Table, which is .18186, then by the Rule of proportion, I say.

l.		*year*		*l.*		*year*
.19867	:	1	:	.18186	:	.9153

which is as much as to say, as the greater difference .19867 is to one year, so is the lesser difference to .9153 parts of a year, which is 47 Weeks, and 5 Days, therefore the number of Years sought in the Question is 20 Years, 47 Weeks and 5 Days.

Quest. 11. A Gentleman bought a Lease of 100 *l. per Annum* to continue 18 years, for 960 *l.* now I demand what Rate of Compound Interest was their implyed in such a bargain?

To Answer this, First, I divide 960 by 100, and the Quotient is 9.6 which sheweth how many years purchase it was worth; then because the Lease was to continue 18 years, I look in the fourth Table in the Collum of years for 18, and carry my Eye exactly in the line against it, looking for the said Quotient 9.6 which I cannot find exactly, but the next (lesser) number to it, is, 9.37188 in the Collum of 8 *per Cent.* and the next (bigger) number to it is 10.05908, in the Collum of 7 *per Cent.* wherefore I conclude that the Rate of interest implyed is between 7 and 8 *per*

per Cent. and to know how much it is more than 7, I do thus, take the difference between the two ſaid Tabular numbers which you will find to be 68720 alſo ſubtract (9.6) the ſaid Quotient, from 10.05908 (the greater Tabular number) and the remainder is .45908, then by the Rule of Proportion, I ſay,

l.		*l.*		*l.*		*l.*
.68720	:	.45908	: :	1	:	.6680

That is to ſay, as the difference between the two Tabular numbers is to the leſſer Remainder ſo is 1 *l.* the difference between 7 and 8 *per Cent.* to .668 the proportional part to be added to 7 *l.* which is 13 *s.*—04 ½ *d.* ſo that 7 *l.* 13 *s.* 04 ½ *d.* is very near the Intereſt required.

How to find out Tabular Numbers for years exceeding 30.

It may many times fall out, that the number of years propoſed in a Queſtion, may exceed the number of years limited in the foregoing firſt, ſecond, third, fourth and fifth Tables, and in ſuch caſes that defect may be ſupplyed by the method uſed in the ſolution of the following Queſtions.

Queſt. 12. Suppoſe 80 *l.* were put out to Intereſt at 5 *l. per Cent.* Compound Intereſt for 40 years, I demand how much it will then be amounted to?

This Queſtion is to be ſolved by the firſt Table, thus, *viz.* Take any two Numbers in the Collum of years, which together will make up 40, and then take the Tabular numbers in the

Collum of 5 *per Cent.* which stand against those two numbers, and multiply them together, and then multiply that product by 80 *l.* the given sum, and the last product will be the Answer.

As suppose you take 30 and 10, or 21 and 19, or 31 and 9, or 25 and 15, *&c.*

But we will pitch upon 30 and 10, and the Tabular number against 30 in the Collum of 5 *per Cent.* is 4.32194, and against 10 is 1.62889 which two numbers being multiplyed, produce 7.03996, *&c.* which is the amount of 1 *l.* for 40 years at 5 *per Cent.* then I multiply 7.03996, &c. by 80 *l.* and the product is 563.197 *l.* &c. =563 *l.*—03 *s.*—11 $\frac{1}{4}$*d. fere.*

The Answer would have been the same, if we had pitched upon any other two numbers to have made up 40. And for Tryal hereof, let us pitch upon 25 and 15, the Tabular number against 25 is 3.38635, and the Tabular number against 15 years is 2.07892, and the product of these two Tabular numbers is 7.0399, &c. which multiplyed by 80, produceth 563.197 *l.* as before, and so much will 80 *l.* be increased to in 40 years at 5 *per Cent per An.* Compound Interest. The like is to be understood for any other number of years.

Quest. 13. Suppose 420 *l.* to be payable at the end of 50 years to come, What is its present worth, Rebate being allowed at 5 *per Cent. per Annum*, Compound Interest?

This Question is of the same nature with those belonging to the second Table, and is answered thereby, according to the method used in solving the last Question by the first Table, *viz.* the given time being 50 years; I pitch upon 30 and 20, and the Tabular number against 30 is .23 377

and

and that against 20 is .376889 and the product of these two is .087203, &c. which is the present worth of 1 *l.* due 50 years hence at 5 *per Cent. per An.* wherefore I multiply .087203, &c. by 420, and the product is 36.62544, &c.=36 *l.* 12 *s.* 6 *d.* and so much is the present worth of 420 *l.* due 50 years hence at 5 *per Cent. per Annum.* Compound Interest.

Quest. 14. An Heir being beyond the Sea, did not return till 36 years after an Estate of 30 *l.* *per annum* was fallen to him by the Death of the Proprietor; the Question is, what was then due to him, Compound Interest being computed at 6 *per Cent. per Annum*?

This Question is of the nature of those belonging to the third Table, and the manner of solving it is thus, *viz.*

Find out (by the seventh Rule of this Chapter) what principal will in one year gain 30 *l.* at 6 *per Cent.* by the following proportion.

l.		*l.*		*l.*		*l.*
6	:	100	::	30	:	500

Having found 500 *l.* to be the principal, seek (after the manner of the 12 Question) by the first Table the amount or increase of 500 *l.* for 36 years at 6 *per Cent.* which you will find to be 4073.5998 *l.* &c. from which if you subtract the said principal 500 *l.* the remainder is 3573.5997. &c.=3573 *l.*—12 *s* — 00 *d. fere.* An so much was due to the Heir at his return.

Quest. 15. There is an Annuity of 30 *l.* to continue 37 years, the Question is what it is worth in ready money, Compound Interest being computed at 6 *per Cent. per Annum*?

By the second way of solving Questions under the fourth Table, for a principal which will gain 30 *l.* in one year, at 6 *per Cent.* which is here 500 *l.* then according to the method uſed in ſolving the thirteenth Queſtion foregoing, find the preſent worth of 500 *l.* for 37 years at 6 *per Cent.* which will be found to be 57.896537 which ſubtracted from 500 *l.* leaves 442.1034, *&c.*=442 *l.* 2 *s.* 0 $\frac{3}{4}$ *d.* And ſo much is the preſent worth of the foreſaid Annuity.

Queſt. 16. What Annuity to continue 40 years will 500 *l.* purchaſe Compound Intereſt being computed at 6 *per Cent. per Annum?*

It is evident by the tenth Rule of this Chapter, that if you find out the preſent worth of 1 *l.* Annuity for any number of years, and at any rate of Intereſt, it may eaſily be found what Annuity to continue the ſame number of years any other ſum will purchaſe at the ſame rate of intereſt by one ſingle Rule of 3 Direct: Therefore,

Find out the preſent worth of 1 *l.* Annuity to continue 40 years at 6 *per Cent.* by the method uſed in ſolving the laſt Queſtion, which will be found to be 15.04632 *l.*=15 *l.*—00 *s.*—11 *d.* which ſum of Money will purchaſe an Annuity of 1 *l.* to continue 40 years at 6 *per Cent.* therefore to know what Annuity of 500 *l.* will purchaſe for the ſame time, ſay by the Rule of Proportion.

l.		*l.*		*l.*		*l.*
15.04632	:	1	: :	500	:	33.230, *&c.*

which will be found to be 33. 230 *&c.*= 33 *l.*—04—07 $\frac{1}{4}$ *d. fere*, and ſuch an Annuity to continue 40 years will 500 *l.* purchaſe Intereſt being allowed at 6 *per Cent.*

FINIS.

Cockers ARTIFICIAL ARITHMETICK,

SHEWING

The Genesis or Fabrick of the Logarithmes and their use in the extraction of Roots, solving of Questions in Anatocisme, or Compound Interest, and in the other Rules of Arithmetick, in a Method not usually Practised.

Composed by *EDWARD COCKER*, late Practitioner in the Arts of Writing Arithmetick and Engraving.

Perused, Corrected and Published By *JOHN HAWKINS*, School-Master at St. *Georges*'s Church in *Southwark*.

Nil tam difficile est quod non solertia vincat.

LONDON,
Printed by *James Orme*, in the Year 1703.

The meaning of ſuch Characters as are uſed in the enſuing Treatiſe.

$+$ Is the ſign of Addition, and is as much as to ſay *plus*, ſignifying that the Numbers or Quantities between which it is placed, are to be added together as $4+7$ ſignifieth that 4 and 7 are to be added together.

$-$Is the ſign of Subtraction, and as much as to ſay *minus*, ſignifying that the Number which followeth it is to be ſubtracted out of the Number which produceth it, as $8-5$ ſignifieth that 5 is to be ſubtracted from 8.

$\times$ Is the ſign of Multiplication, and ſignifieth that the Numbers between which it is placed, are to be multiplyed together, as 6×8 ſignifieth that 6 and 8 are to be multiplyed together.

$=$Is a ſign of Equality, and ſignifieth that the Numbers or Magnitudes between which it is placed, are equal as $3+6=7+2$ ſignifyeth that 3 and 6 are equal to 7 and 2 : Likewiſe $18-6=4+8=12$ and $4\times7=28$ *&c.* If this be not a ſufficient Explanation, read the 13, 14, 15, and 19 Sections of the firſt Chapter of my *Algebraical Arethmetick.*

THE

THE SECOND BOOK OF ARTIFICIAL ARITHMETICK,

I. ARtificial Arithmetick is performed by Artificial Numbers, very fitly called Logarithmes.

II. Logarithmes are borrowed Numbers which differ among themselves by Arithmetical proportion, as the numbers which they signified differ by Geometrical proportion.

III. Logarithmetical Arithmetick is an Artificial use of Numbers, invented for ease in Calculation, wherein each natural Number is so fitted with an Artificial, that what is usually produced by Multiplication of natural Numbers, is here effected by the Addition of their Artificial Numbers: And what natural Numbers perform by Division, is here effected by the Subtraction of their artificial Numbers, and what natural Num-

Numbers do perform by long and tedious operations in the extraction of Square, Cube, Biquadrate, *&c.* Roots is here easily effected by Bipartition, Tripartition, Quadrupartion, *&c.* of their artificial Numbers, and so the hardest parts of Calculation is avoided by an easie posthaphæresis, as our Trignometrical Calculators of late have sufficiently experienced, by avoiding very tedious Multiplications and Divisions in the use of the Tables of Natural Sines, Tangents, Secants to the Everlasting Credit of the honourable * Author of this late and incomparable invention.

* *The Lord* Nepair *Baron of* Merchiston *in* Scotland.

IV. The parts of *Artificial Arithmetick* are the same with Natural Arithmetick, but we shall treat them in this order, *viz.* First, of the Nature of Logarithmes; Secondly, of their Genesis, or the Invention of the Table of Logarithmes And Thirdly, of the use of the Logarithmes in Multiplication, Division, the Extraction of Roots, *&c.*

CHAP.

CHAP. II.

Of the nature of Logarithmes.

1. LOgarithmes are Numbers so fitted to proportional numbers, that themselves retain equal differences.

Let there be assigned a series or rank of numbers in Geometrical proportion, as those in the Collum A *viz.* 1, 2, 4, 8, 16. 32, *&c.* And let there be as many other numbers placed over against them in Arithmetical progression, that is having equal differences as those in the Collums B. C. D. E. or any other numbers whatsoever of the like Nature. Then,

A	B	C	D	E
1	0	2	5	0
2	1	4	8	3
4	2	6	11	6
8	3	8	14	9
16	4	10	17	12
32	5	12	20	15
64	6	14	23	18
128	7	16	26	21
256	8	18	29	24
512	9	20	32	27
1024	10	22	35	30
2048	11	24	38	33
4096	12	26	41	36

Forasmuch as these numbers in the Collums B. C. D. E. are of equal difference among themselves, therefore shall they be the Logarithmes of the numbers in the Collum A, each of

the

the respective number against which it is placed. So in the Collum B. the number 4, is the Logarithme of 16 in the Collum A, and in the Collum C the number 10 is the Logarithme of 16 in the Collum A, and in the Collum D, 29 is the Logarithme of 256 in the Collum A, *&c.*

And as the numbers in the said Collums, B, C, D, E, are Logarithmes of the respective numbers in the Collum A, so they may be Logarithmes of any other rank, or series of numbers in Geometrical proportion.

II. If four numbers are Arithmetical proportionals, either Continued, or Discontinued, the sum of the means is equal to the sum of the extreams.

Let us choose 8, 10, 12, 14, in the Collum C. I say that the sum of the Extreams, 8 and 14, are equal to the sum of the two means, 10, and 12. For, $8+14=10+12=22$. Or if they are discontinued as, 10, 12, 22, 24. in the Collum C; for $10+24=12+22=34$. The like of any other, this being a peculiar property of all Numbers that are Arithmetically proportional.

III. If four Numbers are in Geometrical proportion, either continued, or discontinued, the product arising from the Multiplication of the two extreams, is equal to the product of the two means.

So 4, 8, 16, 32, in the Collum of A are Geometrical proportionals continued, and the product of the Extreams 4 and 32, is equal to the product of the means, 8, and 16, for, $4\times32=8\times16=128$.

Also

Alſo, 4, 8, 64, 128 are Geometrical proportionals diſcontinued, and the product of 4 and 128, the extreams, is equal to the product of 8 and 64 the two means, for 4 x 128 = 8 x 64 = 512.

Hence it follows, that what Geometrical proportionals perform by Multiplication, the ſame will the Logarithmes (being Arithmetical proportionals) perform by Addition.

Let there be given four Geometrical proportionals in the Collum A, *viz.* 8, 16, 128, and 256, and let their Logarithmes be 8, 10, 16, and 18 in the Collum C; I ſay that as 8×256 the product of the extreams is equal to 16 x 128 the product of the means, ſo is 8 ÷ 18 the ſum of the Logarithmes of the extreams is equal to 10 ÷ 16 the ſum of the Logarithmes of the means. Therefore.

If 3 Numbers are given to find the fourth proportional, it may be found by Addition and Subtraction of their Logarithmes, (for, as in Natural Numbers if you multiply the ſecond and third together, and divide their product by the firſt, the Quote will be the fourth proportional number ſo if you add the Logarithmes of the ſecond and third together, and from their ſum ſubtract the Logarithmes of the firſt, the remainder will be the Logarithme of the fourth proportional number.

Example.

Let there be given 2, 16, and 64, and let it be required to find a fourth proportional number thereto, which is 512.

The Logarithmes of the given numbers are 3, 12, and 18, Now if you add 12 and 18 together (which are the Logarithmes of the second and third) their sum is 30, (which is the Logarithme of 1024, the product of the second and third) and if from 30 the said sum of the Logarithmes, you subtract 3, (the Logarithme of the first) there will remain 27, which is the Logarithme of the 512 the fourth proportional number sought for.

$$12 + 18 = 30 - 3 = 27$$

And

$$2 : 16 :: 64 : 512$$

IV. By what hath been said, you may perceive that to natural numbers there may be fitted divers kinds of Logarithmes, but we shall pitch only upon that kind which were framed by Mr. *Briggs* at the request of the Baron of *Merchiston*, who hath chosen these Geometrical proportionals, *viz.* 1. 10. 100. 1000. 10000. 100000, &c. To which numbers he hath assumbed the Logarithmes following, *viz.* for the number 1, the Logarithme 0,000000, for 10 the logar. 1.000000, for 100, the logar. 2.000000 for 1000 the logar. 3.000000, for 10000, the logar. 4.000000, &c. as in the following Table.

A	B
1	0.000000
10	1.000000
100	2.000000
1000	3.000000
10000	4.000000
100000	5.000000
1000000	6.000000
10000000	7.000000
100000000	8.000000
1000000000	9.000000
10000000000	10.000000

The numbers in the Collum A are the ſeries of Geometrical proportionals, and the numbers in the Collum B, are the reſpective Logarithmes of each of thoſe Geometrical proportionals, themſelves being Arithmetical proportionols, where-note that the Figures 1, 2, 3, 4, *&c.* which are ſeparated from the reſt by a point or prick, are called the Indices, or Characteriſticks of the logarithme, becauſe they declare how many places the numbers by them ſignified do conſiſt of; the Characteriſtick of any Logarithme being always an unite leſs than the number of places, which the number by it ſignified doth conſiſt of: As in the foregoing Table you may perceive that the logarithme of 1, is 0.000000, and the logarithme of 10 is 1.000000, and the logarthme of 100 is 2.000000, *&c.* ſo that the Index, or Characteriſtick of 1, and of all numbers from 1 to 10 is 0. and the Characteriſtick of 10 and of all numbers from 10 to 100 is 1: And the Characteriſtick of 100 and of all numbers from 100 to 1000 is 2, and the Characteriſtick of each number being an

unite

unite less than the number of places of which the number by it signifyed doth consist, as was said before.

The Logarithmes of this kind ought all to consist of an equal number of places, that is to say they ought not to be, one Log. of 10 places, another of 8, *&c.* but all of them to be of 6, of 7, of 8, *&c.* places.

CHAP. III.

Of the Genesis or Fabrick of the Logarithmes.

I. THE Logarithme of 1 being assumed to be 0.000000, and the Logarithme of 10 to be 1.000000, the Logarithme of 100 to be 2.000000, *&c.* In the next place it will be requisite to shew the way and manner of Calculating the Logarithmes of the intermediate numbers, *viz.* of the numbers between 1 and 10, which are 2, 3, 4, 5, *&c.* and between 10 and 100, which are 11, 12, 13, 14, 15, 16, *&c.* and between 100, and 1000, which are 101, 102, 103, 104, *&c.* which to do, observe the following Rules.

II. Find so many continual means between 1 and 10, till that continual mean which cometh nearest 1, may be a mixt number less than 2, and

and so near 1, that it may have as many Cyphers placed before the significant Figures of the Numerator, as you intend your logarithmes to consist of places; But our Directions here shall be for the making a Table of logarithmes to consist of 7 places; wherefore find so many continual means between 1 and 10, till the last may have 7 Cyphers placed before the significant Figures of its Numerator, in order whereunto, annex to the number 10 a competent number of Cyphers, (*viz.* 28, because the work may be the more exact) and extract the Square Root of that number so enlarged, which being done, you will find its Square Root to be 3. 16227 76601 6837, This being done, annex to the said Root 14 Cyphers more, and extract the Square Root thereof, which you will find to be 1. 77827941003892.

Again annex to the Root last found 14 Cyphers more, and extract the Square Root thereof, which you will find to be 1. 33352143216332, and thus proceeding successively by annexing of Cyphers, and a continual extraction of the Square Root, until you have found a Square Root or Continual mean, having 7 Cyphers placed before the significant Figures of its Numerator, which will be found after 27 several Extractions to be 1. 00000001715559.

So the 3 last continual means between 10 and 1 will be found to be	1.00000006862238 1.00000003431119 1.00000001715559

All which 3 continual means are less than 2, and so near 1, that there are 7 Cyphers placed before the significant Figures of each of their Numerators.

Having found 27 ſeveral means between 10, and 1, place them ſucceſſively one under the other as in the Collum A, of the following Table; Then make another Collum (B) to contain the Reſpective logarithmes of thoſe continual means.

And becauſe biparting the logarithme of any number produceth the logarithme of the Square Root of that number, therefore take the logarithme of 10, which is 1.000000, and place it in the Collum B over againſt 10, then bipart it, (that is, divide it by 2) and you will have 0.500000 which is the logarithme of 3.16227766 0.16837 the Square Root of 10, then take half of that logarithme, *viz.* 0500000 which is 0.50000, and place it for the logarithme of 1.778279410, &c. the ſecond mean proportional, (or Square of Root of 3.162277660, &c.) and ſo by continual bipartition, you will at length find that 0.00000 0007450580, will be the logarithme of the laſt continual mean, *viz.* the logarithme of 1.0000000 171559, as in the following Table.

A Continual means.	B their Logarithmes.
10.000000000000000	1.00000000000000
3.16227766016837	0.50000000000000
1.77827941003892	0.25000000000000
1.33352143216332	0.12500000000000
&c.	&c.
1.00000006862238	0.00000000029802322
1.00000003431119	0.00000000014901161
1.00000001715559	0.00000000007405805

III. Any

III. Any Number whatſoever being given, how to make the Logarithme thereof.

When it is required to make the Logarithme of any number, extract ſo many continual means between the given number and 1, until the mean which cometh neareſt 1, may be a mixt number leſs than 2, and ſo near 1, that it may have 7 Cyphers placed before the ſignificant figures of its Numerator, which being done, you may eaſily find out the Logarithme of that continual mean, by help of the foregoing Table; and then by doubling, and redoubling the Logarithme of the ſaid continual mean, as many times as you found continual means by extraction, ſo ſhall you at laſt have the Logarithme of the given number.

You may make the Logarithme of any number whatſoever by this and the laſt Rule.

As for Example.

Let us pitch upon the number 2, and make its Logarithme.

To do which, annex to the number 2 a competent number of Cyphers, *viz.* 28, and extract the Square Root thereof, which you will find to be 1.4142135623709 for the firſt continual mean, to which ſaid mean annex 14 Cyphers more, and extract the Square Root thereof, and ſo proceed, by annexing of Cyphers and extracting of Roots, till the neareſt mean proportional number to 1, may have ſeven Cyphers placed before the ſignificant Figures of its Numerator, which after 23 ſeveral Extractions you will find to be said to be 1.000000862658.

Then to find out the logarithme of this continual mean, say by the Rule of 3 Direct.

As the significant Figures of the Twenty fifth mean proportional in the foregoing Table, *viz.* 6862238.

Is to its respective Logarithme, 29802322.

So are the significant Figures of the last continual mean found between 1 and 2, *viz.* 8262958.

To its respective Logarithme 35885571.

Now if you prefix before the Logarithme last found 8 Cyphers, it will be 000000003588 5571, which being doubled and redoubled 23 times, (because there were 23 continual means found between 1 and 2) there will at last be produced 0.301029987568, which is the logarithme of the number 2, which was Required, but because we intend the Table of Logarithmes to consist but of 7 places, and because 2 nines follow the sixth place therefore make the Figures 2 to be 3 and so shall the logarithme of 2 be 0.301030 cancelling the following Figures as superfluous.

The Logarithme of 2 being found, you may easily find the logarithmes of 4. 5, 8, 16, 20, 25, 32, 40, 50, 64, &c. by Artificial Multiplication and Division, which is by adding and subtracting of logarithmes; for if you take the logarithme of 2 out of the logarithme of 10, there will remain the logarithme of 5 and the logarithme of 2 Doubled gives you the logarithme of 4, then add the logarithme of 4, to the logarithme of 2, and you have the logarithme of 8, and to the logarithme of 8 add the logarithme of 2, and it gives you the logarithme of 16, and the logarithme of 5 added to the logarithme of 4, gives the logarithme of 20, and the logarithme of 6 doubled

doubled, gives the Logarithme of 25, *&c.*

In the next place you are to get the Logarithmes of 3, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 89, 97, *&c.* by help of which all the reſt may be Calculated.

IV. The firſt figure of every logarithme, which is ſeparated from the reſt by a point or prick is very properly c lled the Index, or Characteriſtick of the logarithme, which ſheweth the Nature of the number by it ſignified, *viz.* whether it be poſitive, or negative, and if poſitive, of what number of places it doth conſiſt, and if negative, what place of the Decimal Fraction the firſt figure of the number by it ſignifyed, ſhall poſſeſs, as in the following Table.

So the logarithme of { … } will be { … }

So the logarithme of	will be
46768	4.670134
4676.8	3.670134
467.68	2.670134
46.768	1.670134
4.6768	0.670134
.46768	— 1.670134
.046768	— 2.670134
.0046768	3 670134
.00046768	— 4.670134

Whereby you may perceive that the logarithmes of abſolute and defective numbers are the ſame, only the Characteriſtick of a defective number is marked with the note of defection, for the logarithme of the abſolute number 46768 is 4.670134, the Characteriſtick 4, ſhewing the number by it ſignified to conſiſt of 5 places, as is already ſaid in the fourth Rule of the ſecond Chapter, and the Logarithme of the mixt number 46.768 is 1.670134 which is the same with the former, only the Characteristick is 1, which sheweth the Integral number by it signified, to consist of two places, the rest being a decimal Fraction. Likewise the Logarithme of the Decimal .46768 is — 1.670134, which is still the same with the former, only its Characteristick being marked with a note of defection sheweth it to be the Logarithme of a Decimal Fraction, and because the Characteristick is — 1, it sheweth that the first figure of the number by it signified doth possess the first place of the Decimal, or place of primes: Again the Logarithme —46703 is still the same, and if you look for it in the Table of Logarithmes, not regarding the Index, you will find it to be the Logarithme of 46768, but because its Index is defective, I conclude it to be the Logarithme of a Decimal, and because the Index, or Characteristick is—4, therefore I conclude that the first Figure of the number signified by it, must possess the fourth place of the Decimal, wherefore place 3 Cyphers before it, and you have .000 6768 for the Decimal signified by the Logarithme—4.670134. This being well understood, the rest will easily be attained by the following Directions.

CHAP.

CHAP. IV.

Of the uſe of the Table of Logarithmes.

THE uſe of the Table of Logarithmes is twofold, *viz.* Firſt, To find therein the logarithme of any given number, or to find the number appropriated to any given logarithme.

Secondly, To reſolve diverſe neceſſary problems in Arithmetick, Geometry, Trigonometry, Aſtronomy, *&c.*

Concerning the firſt of theſe. I ſhall not meddle, becauſe our Limits will not afford ſufficient room to inſert a Table of Logarithmes, and the Tables already publiſhed by others are ſufficiently explained, in that point as Mr. *Briggs*, Mr. *Gunter*, Dr. *Newton*, Mr. *Wingate*, Mr. *Norwood*, Mr. *Phillips*, &c. Every one ſhewing how by their own Tables to find the logarithme, of any number, or the number to any logarithme, therefore I ſhall proceed to ſhew their uſe in Arithmetick, *viz.* how to Multiply, Divide, and Extract Roots, &c. thereby, And Firſt,

To Multiply by the Logarithmes.

In Multiplication by the logarithmes there are 3 Caſes, *viz.* the Characteriſticks of the logarithmes of the Factors are either both affirmative, or both negative: or else they are the one affirmative, and the other negative?

I. When they are both Affirmative.

When the Characteristicks of the logarithmes of the Factors are both Affirmative, then the sum of those logarithmes is the logarithme of the fact or product.

Examples.

Multiply	34	log.	1.531479
by	26	log.	1.414973
Product	884	log.	2.946452
Multiply	28.86	log.	1.460296
by	8.9	log.	0.949390
Product	256.854	log.	2.409686

Note that if you carry 10 to the Characteristick, it is affirmative, as in the last Example.

II. When they are both Negative.

When the logarithmes of the Factors have their Characteristicks both Negative, or defective, then the sum of their logarithmes is the logarithme of their product, the sum of their Characteristicks being also negative as in the following Examples.

Multiply	.004	log.	3.602060
by	.02.	log.	2.301030
Product is	.00008		5.903090

Mul-

Multiply	.025 ———— log.---2.397940	
by	.42 ———— log.---1.623249	
Product is	.01050 ———— log.=2.021189	
Multiply	.093 ———— log.---2.968483	
by	.058 ———— log.---2.763428	
Product	005394 ———— log.---3.731911	

And here note, that when you carry ten to the Characteristicks it is affirmative, and must be abated out of their sum as in the two last examples:

III. When they are Heterogeneal, *viz.* the one Affirmative, and the other Negative.

When the Characteristicks of the Factors are the one Negative, and the other Affirmative, then add the logarithmes together, and when you come to the Characteristicks, take their difference, and place it for the Characteristick of the Product, making it either Affirmative or Negative, according to the affection of that wherein lay the excess; and here note, that if you carry any thing to the Characteristicks, it is Affirmative, and must be added to the affirmative characterist. And in the following Examples.

Multiply	348 ———— log. 2.541579	
by	.64 ———— log.---1.806180	
Product	222.72 ———— log. 2.347759	
Multiply	348 ———— log. 2.541579	
by	.0064 ———— log.---3.806180	
Product	2.2272 ———— log. 0.347759	

Mul-

Multiply	3.48	log.	0.541579
by	.0064	log.	—3.846180
Product	022273	log.	—2.347759

Multiply	3562	log.	3.551693
by	.008	log.	—3.903089
Product	28.496	log.	1.454782

CHAP. V.

Division by the Logarithmes.

I. TO ſubtract the Logarithme of one number out of the logarithme of another is the ſame (and produceth the ſame effect) with Diviſion in Natural Numbers, the Logarithme remaining being the Logarithme of the Quotient.

II. In Diviſion by the Logarithmes there are three Caſes, *viz.* Firſt, when the Characteriſticks of the Dividend, and of the Diviſor are both Affirmative: Secondly, when they are both Negative. And Thirdly, when they are Heterogeneal, *viz.* the one Affirmative, and the other Negative. Of which in their order.

I. When

I. When they are both Affirmative.

III. When the Characteristicks of the Dividend, and of the Divisor, are both Affirmative, then if you subtract the Logarithme of the Divisor out of the logarithme of the Dividend, the remainder will be the logarithme of the Quotient. And if you borrow 10 from the Characteristicks, it is Affirmative.

Examples.

Divide	468	log.	2.670246
by	12	log.	1.079181
Quotient	39	log.	1.591065

Divide	144	log.	2.158362
by	16	log.	1.204120
Quotient	9	log.	0.954242

These Examples are so plain that they need no Explanation.

II. When they are both Negative.

IV. When the Characteristicks of the Dividend, and of the Divisor, are both Negative, subtract the Logarithme of the Divisor from the Logarithme of the Dividend, and the Remainder is the Logarithme of the Quotient, and if you borrow 10, it must be paid to the Index of the Divisor, affirmatively.

Examples.

Examples.

	Divide	.48 ——	log.—1.681241
(1)	by	.12 ——	log.—1 079181
	Quotient	4. ——	log. 0.602060
	Divide	.036 ——	log.—2.556303
(2)	by	.18 ——	log.—1.255272
	Quotient	.2 ——	log.—1.301031
	Divide	.156 ——	log.—1.193125
(3)	by	.39 ——	log.—1.591064
	Quotient	.4 ——	log. 1.602061
	Divide	.0171 ——	log.—2.232996
(4)	by	9. ——	log.—1.954242
	Quote	.019 ——	log.—2.278754.

The first and second of the foregoing Examples are easily understood, and as for the third and fourth, all the difficulty therein is caused by borrowing 10 at the next figure to the Characteristicks, as in the third Example, in subtracting 5 out of 1. Now to make good the 10 borrowed, I pay 1 to the Characteristick of the Divisor, and because the said 1 is affirmative, and the said Characteristick negative, therefore subtract it from the Characteristick of the divisor, and there remains nothing; wherefore I take (0) out of the Characteristick of the Dividend, and there remains—1 for the Characteristick

ſtick of the Quotient. The ſame is to be underſtood in the fourth Example, and in all others of the ſame Nature.

A General Obſervation drawn from the third and fourth Rules foregoing.

V. If when the Characteriſticks of the Dividend and Diviſor be Homogeneal, (that is, both affirmative, or both Negative) the Characteriſtick of the Diviſor is greater than the Characteriſtick of the dividend, then in this caſe ſubtract the Characteriſtick of the Dividend out of that of the Diviſor, placing the remainder for the Characteriſtick of the Quotient, changing its ſign, *viz.* if it be affirmative, make it negative, and if it be negative, make it affirmative. Remembring the Directions under the laſt Rule when you borrow from the Characteriſticks.

Obſerve the following Examples.

	Divide	6.4	log	0.806180
(1)	by	80	log.	1.903090
	Quotient	.08		2.903090

	Divide	6.4	log.	0.806180
(2)	by	800	log.	2.903090
	Quotient.	008	log.	3.903090

	Divide	6.3	log.	0.799340
(3)	by	78.75	log.	1.896251
	Quotient	.08	log.	2.903089

Divide

(4)	Divide 75	log.—1.875061	
	by .0015	log.—3.176091	
	Quotient 500	log.—2.698970	

(5)	Divide .64	log.—1.806180	
	by 008	log.—3.903090	
	Quote 80	log. 1.903090	

(6)	Divide 16.56	log. 1.219060	
	by 460	log. 2.662758	
	Quote .0036	log. 2.556302	

III. When they are Heterogeneal, *viz.* the one Negative, the other Affirmative.

VI. When the Characteristicks of the Dividend and the Divisor are Heterogeneal, proceed as in the two first Cases, till you come to the Characteristicks, and then instead of subtracting the one Characteristick from the other, add them together, so shall their sum be the Characteristick of the Quotient, and it is of the same kind with the Characteristick of the Dividend.

But here note that when you borrow 10 at the next figure to the Characteristick it must be paid to the Characteristick of the Divisor Affirmatively *viz.* If the Characteristick of the Divisor be affirmatively, then add 1 to it to that you borrowed, and if it be negative, subtract 1 from it. As in the following Examples.

(1)	Divide 144	log.—1.158362	
	by 12	log. 1.079181	
	Quote .012	log.—2.069181	

	Divide	64 ——— log. ——	1.806180
(2)	by	.08 ——— log. ——	2.903090
	Quote	800 ——— log.	2.903090

	Divide	.64 ——— log. ——	1.806180
(3)	by	800 ——— log.	2.903090
	Quote.	0008 ——— log.---	4.903090

In the ſecond of the foregoing Examples I borrow 1 (in the place next the Characteriſtick) by ſubtracting 9 out of 8, wherefore to make it good, I ſubtract 1 from —— 2 (the Characteriſtick of the Diviſor,) becauſe it is Negative, and the remainder which is 1 —— I add to 1 (the Characteriſtick of the Dividend) and their ſum is 2 for the Characteriſtick of the Quotient which is Affirmative, becauſe the Characteriſtick of the Dividend is Affirmative.

And in the laſt Example, I likewiſe borrow 1 from the Characteriſtick, wherefore to make it good, I add 1 to the Characteriſtick of the Diviſor, (becauſe it is affirmative) and that makes it 3, which added to —— 1 (the Characteriſtick of the Dividend) makes —4 for the Characteriſtick of the Quotient, which here is negative, becauſe the Characteriſtick of the Dividend is negative.

Other Examples for Exerciſe may be ſuch as follow.

Divide	648 ——— log. ——	2.11575
by	.36 ——— log. ——	1.556303
Quote	1800 ——— log.	3.255272

Divide	6.4	log.	0.806180
by	.08	log	—2.903090
Quote	80	log.	1.903090

Divide	68	log.	1.832509
by	.08	log.	—2.903090
Quote	850	log.	2.929419

CHAP. VI.

To raiſe the Powers of Numbers, *viz.* to find the Square, Cube, Biquadrate, or Squared Square, *&c.* of any number. Alſo to Extract the Square, Cube, Biquadrate, *&c.* Roots of any Number by the Logarithmes.

1. BY the third Section of the Second Chapter of this Book it is evident, that if you add the Logarithmes of two numbers together, the Sum will be the Logarithme of their Product

duct ; And by the first and fourth Sections of the 9th Chapter of my Decimal Arithmetick, it appeareth that any number multiplyed by it self, produceth its Square, wherefore if you double (or multiply by 2) the logarithme of any number, it will produce the logarithme of its Square, which if duly considered you will find that to Square ,Cube, *&c.* any number, is nothing else but to multiply the Logarithme of the given number by the Index of the Power you would rai e it to, *viz.* If you would find the Square of any number, multiply the logarithme thereof by 2, so shall the product thereof be the Logarithme of its Square ; and if you would find the Cube of any number, multiply its logarithme by 3, and the product thereof will be the Logarithme of its Cube ; and if you would find the Biquadrate of any number, multiply its Logarithme by 4, and it will produce the Logarithme of its Biquadrate, *&c.* As in the following Example.

Let it be required to find the Square of 12

The Logarithme of 12 is 1.079181
2
————
2.158362

Which being multiplyed by 2, produceth 2.158362, which is the Logarithme of 144, *viz.* the Square of 12.

Again let it be required to find the Square o
94.

The Logarithme of 94 is —— —— 1.97312 7
2

3.946255

Which being multiplyed by 2, producetl
3.9462556, which is the logarithm of 8836
which is the Square of 94.

II. But if the characteristick of the logarithm
be negative, that is if the given number, whos
Square, Cube, Biquadrate, &c. you would fin
to be a Decimal Fraction, observe, that in multi
plying the next figure to the Characteristick th
ten, or tens to be born in mind are affirmative
and are to be deducted out of the Product of th
negative Characteristicks.

Observe the several Examples following

What is the Square of .7 log.—1.84509
.7 2

Facit .49 log.—1.69019

What is the Square of .09 log—2.95424
.09 2

Facit. 0081 log.—3.90848

What

What is, the Cube of 12? —— log. 1.079181
3

Facit 1728 for 12×12×12=1728 log. 3.237543

What is the Cube of 05 ; —— —— log.—2.698970
3

Facit.000125 for.05×05×05 =00125log.—4.096910

What is the Biquadrate of 9?—log. 0.954242
4

Facit 6561 for 9×9×9×9=6561 log.—3.816968

What is the Biquadrate of .08 ? log.—2.903090
4

Facit .00004096 for.08×.08.×08×08
=00004096 whose logarithme is } —5.612360

What is the fifth Power of 6? — log. 0.778151
5

Facit 7776=6×6×6×6×6 —— — log. 3.890755

The like is to be obſerved of all others.

To Extract the Square, Cube, Biquadrate, &c. Roots of any given numbers by the Logarithmes.

III. From a due conſideration of the firſt Section of this Chapter, it may eaſily be perceived, that

To Extract the Cube Root of any number is to bipart (or divide by 2) its logarithme, ſo ſhall that biparted logarithme be the logarithme of the Square Root desired.

Example.

Let it be required to Extract the Square Root of 75832

q.=75832 ——— ——— log. 4. 879852
2)
√.=275.37——— log.Biparted 2.439926

Let it be required to find the Square Root of 4489.

q.=4489——— ——— log. 3.652149.
2)
√=67——— log. Biparted 1.826074

In the first of these Examples the logarithme of 75832 is 4.879852 which being biparted (or divided by 2 gives 1.826074 for the logarithme of (265.37) the Root required.

And in the second Example 3.652149 (the logarithme of 4489 being biparted gives 1.826074 for the logarithme of (67) its Square Root.

So will the Square Root of 36783 be found to be 191.789 *fere*, and the Square Root of 3866 will be 62.17717 *fere*. And the Square Root of 95 will be found to be 9.7468, *&c.*

To extract the Cube Root of any number is to tripart (or divide by 3) its logarithme, so shall this triparted logarithme be the logarithme of the Cube Root required.

Example

Examples.

Let it be required to Extract the Cube Root of 157464.

The Cube 157464 ———— ——— ——log. 5.197181
3)————
√c. .54————————log. triparted 1.732393

Let it be required to find the Cube Root of 1872376015 80329.

The Cube 187237601580329-log. 14.27239308
3)————
√c. 47209 ——its triparted—log.4.75746436

In the first of these Examples where it is required to extract the Cube Root of 157464, its logar. is 5.197181 which being divided by 3 hath for its third part 1.732393 which is the logarithme of (54) the Cube Root of 157464, which was required. And the same is to be observed in finding the Cube Root of 1872376015 80329 by the Logarithmes; or of any other positive number whatsoever as you may see by the following Examples.

The Cube=8——————————its log. 0.903090
3)————
√c.=2 ———————————— its log. 0.301030

The Cube 125 ———— ——— ——its log. 2.096910
3)————
√c.=5———————— ———— its log. 0.698970

 The

The Cube 15.625 —— —— its log. 1.193820

√c.=2.5 ——— —— ——its log. 0.397940

To Extract the Biquadrate Root of any given number, do thus, *viz.* Take its logarithme, and divide it by 4, so shall the fourth part thereof be the logarithme of the biquad. Root required, as in the following Example.

To extract the Biquadrate Root of any Number.

Let it be required to extract the Biquadrate Root of 256.

Biquadrate number given 256—its log. 2.408239
4)————————
Its √. biquad.=4 ——— ——its log. 0.602059

Here the log. of the given biquadrate number, *viz.* 256 is 2.408239 which being divided by 4, giveth 0.602059 for the *logarithme* of (4) the biquadrate Root required,

In like manner if you would extract the Root of the fifth Power of any number given, Divide its *logarithme* by 5, so shall the Quotient be the *logarithme* of its Root. And if you would find the Root from the sixth power of any Number, divide its *logarithme* by 6. and the Quote is the *logarithme* of the Root desired, &c.

IV. But here you are to observe in Extracting the Square, Cube, Biquadrate, or any other Root of a negative number; or decimal by the *logarithmes*, that if you cannot evenly divide

To extract the Square, Cube, &c. Roots of negative numbers by the Logarithmes.

divide the Index or Characteriſtick of the *logarithme* without the remainder, then add to the ſaid Characteriſtick ſo many units till it may be divided without any remainder, and place the Quotient for a new *Characteriſtick,* (belonging to the Root ;) Then look how many units you lent to the *Characteriſtick,* and eſteem them ſo many tens to be prefixed to the *logarithmetical figure* immediately following the *Characteriſtick,* then proceed to finiſh the work, ſo ſhall this new *logarithme* be the *logarithme* of the Root required, which will alſo be negative.

Examples. follow.

What is the Square Root of .144?

Square given=.144 ——— Its *log.*—1.158362
2)
Its √q=37947 ——its *log.* biparted.--1.579181

What is the Square Root of .00324

Square given = .00324 ——— *log.*—3.510545
2)———
Its √q.=05621 —— *log.* biparted. —2.755272

What is the Cube Root of .000512

Cube given=000512 —— its *log.*—4.709269
3)———
Its √c—.05 ——its *log.* triparted—2.903089

In the firſt of theſe Examples, where it is required to Extract the Square Root of .144, its *logar.* is —1 .158362, which (according to the third Rule) I ſhould bipart, (or divide by 2)

And becauſe its negative *Characteriſtick* (—1) cannot be evenly divided by 2, I increaſe it by an unit, and it makes 2, then will the quote be —1 for the Index of the *log.* of the *Root*, then do I proceed to the next figure, to the *Characteriſtick* which is 1, and becauſe it added 1 to the *Characteriſtick*, therefore I increaſe the next *Figure* by adding 10 to it. (or prefixing 1 before it : and then it is 11, *&c.* So I find the *logarithme* of the *Root* to be—1.5755272, *viz.* .37947.

And in the third Example, where it is required to find the Cube Root of .000512, the Index of its *logarithme* is— 4, which cannot be evenly divided by 3, therefore I add 2 to it to make it 6, and the Quotient is—2 for the Index of the *logarithme* of the *Root*, then becauſe I added 2 to the Index—4, therefore I increaſe the next *Figure* to it with 2 tens, making it 27 *&c.* So is the Cube *Root* required found to be 05.

Obſerve

Observe the like in extracting of any other Roots: Otherwise, you may make use in the following Table.

	A	B
2	—1—2	—1
	—3—4	—2
	—5—6	—3
	—7—8	—4
	—9—10	—5
3	—1—2—3	—1
	—4—5—6	—2
	—7—8—9	—3
4	—1—2—3—4	—1
	—5—6—7—8	—2
5	—1—2—3—4—5	—1
	—6—7—8—9--10	—2
6	—1—2—3—4—5—6	—1
	—7—8—9—10—11-12	—2
	50. 40. 30. 20. 10. 0	

The use of the foregoing Table.

In the foregoing Table the *Figures* 2. 3. 4. 5. 6. placed on the left hand, are in the Indices of Powers, whose *Roots* are required to be extracted, or they are Divisors by which to divide the *logarithme* of any given power, in order to find out its Root: As the number 2 (which is the uppermost) is the Divisor for finding the *logarithme* of the Square Root of any number: And 3 the Devisor for the Cube; and 4 for the Biquadrate, &c. The *Figures* placed between the perpendicular

pendicular line, and the ſeveral lines of connection, and under A are the *Characteriſticks* of the *logarithmes* of Negative or Decimal Numbers, whoſe *Roots* are required to be extracted; And the Figures placed on the right hand of the perpendicular line under B, are the *Characteriſticks* of the *Logarithme* of the ſeveral *Roots*: And the numbers at the bottom of the Table, *viz.* 50, 40, 30, *&c* are the numbers to be added, or rather prefixed to the firſt *Figure* of the *logarithme* next the *Characteriſtick* whole Negative Index is found in the ſame ſeries or Collum even with the Diviſor, *&c.*

Example.

Let it be required to extract the *Cube Root* of .405224.

The *Logarithme* of the given Number is —1.6076951.

And the Diviſor whereby to extract the *Cube Root* is 3, which I find in the foregoing Table on the left hand; then on the right hand of its line of *connection*, I find the *Characteriſtick* of the *Logarithme* —1.6076951 which is —1, and juſt againſt it on the right hand under B I find —1 for the *Characteriſtick* of the *Logarithme* of the *Root*, and in the bottom line under the ſaid —1, in the ſame ſeries, I find 20 which is to be prefixed to the *Figure* in the *Logarithme* next the *Characteriſtick*, &c. and having finiſhed Diviſion, I find the *Logarithme* of its *Root* to be —1.8692317 which is .74.

So if the *Logarithme* —6.418426 were to be divided by 4, Firſt the Diviſor 4 on the left

left hand of the Table, and find the *Characteristick* —6, behind its line of connection just against which on the right hand of the perpendicular line you have —2 for the Index or *Characteristick* of the Quotient; and at the bottom, just underneath—6 you have 20, which being added to (4) the first *figure* of the Dividend next the *Characteristick* makes it 24, in which the Divisor 3 is contained 6 times, &c. See the work.

Biquad. proposed .0000026207 — *log.*—6.418426
4) ————
The √(4)=040235 ———— — *log.* —2.604606

CHAP. VII.

Of the Use of Log. in Comparative Arithmetick.

PROP. I.

Having three Numbers given, to find a fourth proportional.

THis is nothing else but the work of the Rule of 3, and it may be thus performed, *viz.*

Add

Add the *Logarithme* of the ſecond and third Numbers together, and from their ſum ſubtract the *Logarithme* of the firſt, ſo ſhall the remainder be the *Logarithme* of the fourth, as in the following Example.

The 3 given numbers are 3, 24, and 108 unto which it is required to find a fourth proportional.

3 : 24 :: 108 : 864

The Operation by the Logarithmes.

As 3——its *log.* ———————— 0.477121

Is to 24 its *log.* ———————— 1.380211 } add
So 108 its *log.* ———————— 2.033424 }

The ſum of the 2 laſt *log* is 3.413635

From which if you ſubtract the *log.* Of the firſt, the remainder is } 2.936514

Which is the *Logarithme* of 864. the fourth proportional required.

The former work may be ſomewhat ſhortned, if inſtead of the *Log.* of the firſt you take its Complement Arithmetical (which is nothing elſe but to ſet down what every *Figure* wants of 9, till you come to that next the right hand, and then ſet down what it wants of 10) and then add them all 3 together, and cancel the firſt *Figure* of the ſum on the left hand, and then will the ſum be the *Log.* of the Anſwer, as will appear by working the foregoing Examples.

As 3 Comple. Arith. of its *log.*	9.522879		
Is to 24 — its *log.*	1.380211	add	
So is 108 — its *log.*	2.033424		
To 864 — its *log.*	12.936514		

PROP. II.

Between two Numbers given to find a Mean proportional.

When the *Logarithmes* of the numbers propounded are homogeneal, *viz.* both Affirmative, or both Negative, add them together, then bipart that *Logarithmetical* sum, so have you the *Logarithme* of the mean proportional required, which *Logarithme* so found is of the same kind with the *Logarithmes* of the number given

Let it be required to find a mean proportional between 18 and 6.

18	*log.*	1.25527
6	*log.*	0.77815
		2)2.02342
10.392	*log.*	1.01171

In this Example the *Logarithme* of 18, and 6 being added together make 2.02342 which is the *Logarithme* of 108, and that *Logarithme* being divided by 2 (which is the same with extracting the Square Root of its significant number,

ber, as in the third Rule of the ſixth Chapter) the Quotient is 2.02342, the *Logarithme* of 10.392 which is the mean proportional required.

Example 2.

Let it be required to find a mean proportional between .018 and .006.

.018 its *log.* ————	$\bar{2}$.25527
.006 its *log.* ————	$\bar{3}$.77815
	2)—4.03342
.010392 *log.* ————	2.01671

II. But when the *Characteriſticks* of the *Logarithmes* of the given Numbers be Heterogeneal, *viz.* the one Affirmative, and the other Negative; add the *Logarithmes* together as before, till you come to the *Characteriſtick*, then ſubtract the leſſer *Characteriſtick* out of the greater, (according to the third Rule of the fourth Chapter,) which being done, bipart the Logarithmetical production, ſo ſhall the Quote be the *Logarithme* of the mean proportional required, which will always be of the ſame kind with that *Logarithme* of the given Numbers, whoſe Index is greateſt, as in the following Examples.

Example

Example 1.

What is the mean proportional between 36 and .5? Facit 4.2427.

Here the *Log.* of { 36 / .5 } is { 1.556302 / ---1.698970 }

Their sum is 2) 1.255272

which being divided by 2, gives the *Log.* of the me. prop. which is the *Log.* of 4.2427. } 1.627636

Example 2.

What is the mean proportional between 12 and .75? Facit 3.

The *Log.* of { 12 / .75 } is { 1.079181 / ---1.875061 }

Their sum is 2) 0.954242

which being divided by 2 gives the *Log.* of 3 the mean proportional required. } 0.477121

for .75 : 3 :: 312 : 3

PROP.

PROP. III.

Between 2 Numbers given, to find two mean Proportionals.

Whether the numbers given be Homogeneal; or Heterogeneal, ſubtract the *Logarithme* of the leſſer extream from the *Logarithmne* of the greater extream, then take $\frac{1}{3}$ of the difference of the ſaid *Logarithmes*, and add it to the *Logarithme* of the leſſer extream, ſo will the ſum be the *Logarithme* of the leſſer mean; then add the ſame Difference to the ſaid *Logarithme* of the leſſer mean, and the ſum will be the *Logarithme* of the greater mean; ſtill obſerving the Rules delivered in the fourth and fifth Chapters of this book, in adding and ſubtracting of *Logarithmes.*

Examples.

Ex. 1. Let it be required to find 2 mean proportionals between 144 and 12?

The *Log.* of	144	is 2.158362
	12	1.079181
The Difference ——	3)	1.079181
$\frac{1}{3}$ of the Differ.		0.359727
Leſſer mean 27.473 *log.*		1.438908
Greater mean 62.899 *log.*		1.798635

Example

Example. 2. Let it be required to find 2 mean proportionals between .75 and .05 ?

The *Log.* of { 75 / .05 } is { —1.875061 / —2.698970

The Difference ——— 3)1.176091
⅓ part of the Diff. is 0.392030

Lesser mean is .12331 ——— *log.* 1.091000

Greater mean is .30411 —*log.* 1.484030

Example. 3. Let it be required to find 2 mean proportionals, between 125 and .05 ?

The *Log.* of { 125 / .05 } is { 2.096910 / —2.698970

Their Difference ——— 3) 3.397940

⅓ of their Differ. ——— 1.132646

The lesser mean is .67860 *Log.*—1.831616

The greater mean is 9.2101 *Log.* 1.964262

P R O P. IV.

Three Numbers given to find a fourth in a Duplicate Proportion.

Take the Logarithmes of the two Numbers which have one and the same Denomination, and ſubtract the leſſer Logarithme from the greater, and double the remainder, (that is multiply it by 2.) Then if the firſt number be leſs than the ſecond, add the ſaid double difference to the Logarithme of the other Number, ſo will the ſum be the Logarithme of the fourth number, or number required, as in *Example.*

The ſuperficial content of a Circle whoſe Diameter is 14 Inches is 154 Square inches, I demand the Content of another Circle, whoſe Diameter is 25 Inches? Facit 491.07 ſquare Inches. See the operation.

Diam. 14 Inches its *log.*	1.146128
Diam. 25 Inches its *log.*	1.397940
The Difference of the *log.*	0.251812
	2
Their Difference doubled	0.503624
The given content its *log.*	2.187521
The Cont. required 491. 07. *log.*	2.691145

But if the firſt number be greater than the ſecond, then inſtead of adding the doubled difference to the other number, ſubtract it therefrom, ſo ſhall the remainder be the Logarithme of the number required, as in the following Example.

There is a Circle whoſe Diameter is 8 Inches, and its ſuperficial Content is 616 ſquare Inches, I demand what is the Superficial Content of another Circle, whoſe Diameter is 25 Inches? Facit 491.07 Square Inches, as in the former Example

Diameter

Diameter 28 Inches its *log.* ——	1.447158
Diameter 25 Inches its *log.* ——	1.397940
The Difference of the *log.* ——	0.049218
	2
The Difference doubled ——	0.098436
The given Content 616, its *log.*	2.789581
Content required 491.07, its *log.*	2.691145

PROP. V.

Having 3 Numbers given to find a fourth in a Triplicate Proportion.

Triple the Difference of the Logarithmes of the two given Terms, which have the ſame Denomination. Then if the firſt Term be leſs than the ſecond, add the ſaid Triple Difference to the Logarithme of the other Term, ſo ſhall the ſum be the Logarithme of the fourth Term required, as in the following Example.

There is a Bullet whoſe Diameter is 4 Inches, and its weight is 9 *l.* I demand what weight a Bullet of the ſame metal will be, whoſe Diameter is 8 Inches? Facit 72 *l.* view the following work.

Diameter 4 Inches	*log.*	0.602060
Diameter 8 Inches	*log.*	0.903090
The Difference of the *logar.*		0.301030
		3
The Difference Tripled is		0.903090
The given weight 9 *l.*	*log.*	0.954243
The weight required is 72 *l.*	*log.*	1.857333

But if the firſt term be greater than the ſecond then ſubtract the ſaid Tripled Difference from the logarithme of the other term, ſo ſhall the remainder be the logarithme of the fourth number required. As in Example.

There is a Bullet whoſe Diameter is 8 Inches, and its weight is 72 pounds, I demand the weight of another Bullet of the ſame Mettal, whoſe Diameter is 4 Inches? Facit 9 *l.* See the operation, it being the converſe of the former.

Diameter 8 Inches, its *log.*	0.903090
Diameter 4 Inches, its *log.*	0.602060
The Difference of the *log.* is	0.301030
	3
The Difference Tripled is	0.903090
The given weight 72, its *log.*	1.857333
The weight required 9 *l.* its *log.*	0.954243

CHAP.

CHAP. VIII.

Of Anatocisme, or Compound Interest, wherein is shewed how by the Logarithmes to answer all Questions concerning the Increase, or present worth of any Sum of Money or Annuity, for any Term of Years, or at any Rate of Interest. According to the six Fundamental Theorems invented and laid down by Mr. *Oughtred* in his Treatise *De Anatocismo sive Usura Composita*, annexed to his *Clavis Mathematicæ*.

I. WHen any Question in Compound Interest is proposed, it will fall under one of the six Cases following, *viz.*

1. To find the Increaſe or amount of any ſum of money put out at Compound Intereſt for any number of years, and at any Rate of Intereſt propounded.

2. To find the preſent worth of any ſum of money due at the end of any number of years to come, Rebate being allowed at any Rate of Compound Intereſt.

3. To find the increaſe, or amount of an Annuity being forborn for any number of years at any Rate of Compound Intereſt.

4. To find what Annuity any ſum of Money due at the end of any Term of years to come. will purchaſe at any Rate *per Cent.*

5 To find the preſent worth of any Annuity to continue any number of years, allowing Rebate at any Rate *per Cent.*

6. To find what Annuity any Sum of Money will purchaſe for any number of years, and at any Rate of Intereſt propoſed.

II. When any Queſtion in Compound Intereſt is propounded, find out the Intereſt of 1 *l.* and let 1 *l.* with its intereſt be the Rate of Intereſt implyed in the Queſtion, as if any Queſtion were propoſed at 8 *per Cent.* the Int. of 1 *l.* for a year is .08 and the rate of Intereſt is 1.08, if at 6 *per Cent.* the Rate is 1.06, *&c.* of which find out the Logarithme.

III. When any Annuity or Debt the payments be half yearly, Quarterly, or Monthly, *&c.* you are to divide the Logarithme of the ſaid Rate by 2, 4, or 12, *&c.* ſo ſhall that Quotient be the Logarithme of the Rate, as ſuppoſe any Queſtion were propounded at 8 *per Cent,* the

Rate

Rate of Intereſt here implyed is 1.08, for

100 : 108 :: 1 : 10.8

Which ſaid Rate is for yearly payments, the Logarithme whereof is 0.0374204, but if the payments are to be half yearly, then if you divide the ſaid Logarithme by 2, it will give you 0.0187102 for the Logarithme of the Rate, and if the payments be Quarterly, divide the ſaid Logarithme of 1.08 by 4, and it will give you 0.0093551 for the Log. of the Rate, and if the payments be monthly, then if you divide the ſaid Log. of 1.08 by 12, it will give you 0.0031183 for the Log. of the Rate, *&c.* and this is generally the firſt thing to be obſerved in every Queſtion, as you will find by the following Examples.

CASE I.

To find the Increaſe or Amount of any Sum of Money put out at Compound Intereſt for any Term of years, and at any Rate of Intereſt propounded.

Queſt. 1. if 50 *l.*—16 *s.* be but out at 8 *per Cent.* Compound Intereſt. for 7 years, I demand how much will then be due to the Creditor? Facit 87 *l.*—01 *s.*—02$\frac{3}{4}$ *d.*

IV. Multiply the Logarithme of the Rate by the number of years, and the product will give you the Logarithme of the Amount of 1 *l.* for the propoſed time, to which if you add the Log. of the Sum propounded, the ſum will be the Log. of the Anſwer.

 The

The Operation by the Logarithmes.

The log. of 1.08 the Rate prop.	0.0334237	
The numb. of years propounded.	7	
The log. of the Increase of 1 *l* for 7 years.	0.2339659	add
The log. of (50.8 *l.*) the sum proposed.	1.7058637	
The log. of (87 061) the answer.	1.9398296	

which is 87 *l.* —01 *s.*—2 $\frac{3}{4}$ *d. fere*

Quest. 2. What is the amount of 76 *l.*—04 *s.* for 3^{4} years at 8 *per Cent*? Facit 97 *l.*—17 *s.*—01 *d.*

The Operation by the Logarithmes.

The log. of (1.08) the given Rate *per Annum* ?	0.0334237
	4)
which divided by 4, gives the log. of the Quarterly Rate	0.0083559
	13
	0.0250677
	083559
which multiplyed by 13 the Quarters in 3 years give the log. of the increase of 1 *l.*	0.1086267
the log. of (76.2) the given sum	1.8819547
the log. of (97.854) the Answ.	1.9905814

Quest.

Quest. 3. If 50 *l.* be put forth at Intereſt for 20 years at 6 ¼ *per Cent.* I demand how much it will be increaſed to at the end of the ſaid time. Facit 168 *l.*—01 *s.*— 10 *d.*

The rate of Intereſt here propoſed is 6 ¼=6.25 *per Cent.* therefore to find out the Rate of 1 *l.* for a year, ſay by the Rule of proportion.

l.		*l.*		*l.*		*l.*
100.	:	106.25	: :	1	:	1.0625

So that the Rate of Intereſt implyed in the Queſtion fit for Calculation by the Log. is 1.0625 according to the ſecond Rule of this Chapter, behold.

The Operation.

The log. of (1.0625) the given Rate	0.0263289	
The number of years propound.	20	
The log. of the amount 1 *l*, in 20 years	0.5265780	add
The log. of (50) the ſum propoſed,	1.6989700	
The log. of 168.090	2.2255480	

Which is 168 *l.*—01 *s.*—10 *d.* very near, and ſo much will 50 *l.* be increaſed to in 20 years at 6 *l.*—5 *s.* *per Cent.*

CASE

CASE 2.

To find the present worth of any ſum of money due at the end of any number of years to come, Rebate being allowed at any Rate of Compound Intereſt.

V. When it is required to find the preſent worth of any ſum of Money, firſt find the amount of 1 *l.* for the propoſed time, and at the Rate of Intereſt propounded, then find the Logarithme of the ſum propoſed to be Rebated, and from it ſubtract the Logarithme of the amount of 1 *l.* (found as before) and the remainder will be the Logarithme of the preſent worth of the ſum propoſed. As in the following Example.

Queſt. 4. What is 30 *l.* that is due 7 years hence worth to be paid preſently, allowing Rebate at 8 *per Cent*? Facit 17 *l* — 10 *s.* — 01 ½ *d.* as you may perceive by

The Operation by the Logarithmes.

The log. of (1.08) the propoſed Rate	0.033424
The time propoſed	7
The log. of the amount of 1 *l.* for 7 years	0.233968
The log. of 30	1.477121
The log. of (17.506) the Anſwer	1.243153

which is 17 *l.* — 10 *s.* — 01½ *d.* and ſo much is the preſent worth of 30 *l.* due 7 years hence.

Queſt. 5.

Quest. 5. What is the preſent worth of 120 *l.* due 2 years hence, allowing Rebate at 6 *per Cent*? Facit 106 *l.*—15 *s.*—11 *d.*

The Operation of the Logarithmes.

The log. of (1.06) the propoſed Rate	0.025306
The time propoſed	2
The log. of the amount of 1 *l.* for 2 years.	0.050612
The log. of 120	2.079181
The log. of (106.79) the Anſwer.	2.028569

which is 106 *l.*—15 *s.*—11 *d.* and ſo much is the preſent worth of 120 *l.* due 2 years hence.

CASE. 3.

To find the Increaſe, or Amount of an Annuity, being forborn any number of years, at any Rate of Compound Intereſt.

VI. For Reſolving Queſtions concerning the forbearance of Annuities, you are (by the fourth Rule) firſt to find out the Amount of 1 *l.* for the Time, and at the Rate of Intereſt propounded.

Secondly, Find out the Logarithme of the ſaid amount made leſs by 1, and alſo the log. of the Rate made leſs by 1, and ſubtract the latter from the former, ſo ſhall the remainder be the log. of the amount of 1 *l.* Annuity for the term of years propounded, to which if you add the Logarithme of the propoſed Annuity, the ſum will be the Logarithme of the Amount, or Increaſe of the ſaid Annuity. As in the following Example.

Queſt.

Quest. 6. What will be the Amount, or Increaſe of 48 *l.*—16 *s.* *per an.* for 7 years, Compound Intereſt being Computed at 8 *per Cent.* Facit 435 *l.*—08 *s.*—05 ½ *d. fere.* See

The Operation by the Logarithmes.

The log. of 1.08) the given rate is	0.033424	mult.
The time propounded ————	7	
The log. of (1.7138) the amount of 1 *l.* for 7 years. ————	0.233968	
1.7138—1=7138 its log.	—1.853577	ſubt.
1.08—1=08 its log.	—2.903090	
The difference of the log. which is the increaſe of 1 *l.* annuity is ————	0.950487	add
The log. of (48.8) the Annuity propoſed ————	1.688419	
The log. of (435.422) the amount of the propoſed Annuity ———	2.638906	

which is 435 *l.*—8 *s.*—5 ½ *d.* very near, and ſo much will be the Increaſe of an Annuity of 48 *l.* 16 *s.* in 7 years, at 8 *per Cent.* Compound Intereſt.

Quest. 7. There is an Annuity of 50 *l.* forborn to the end of 10 years, I demand how much is then due, Compound Intereſt being computed at 6 ¼ *per Cent.*? Facit 666 *l.*—16 *s.* as you will find by

The

The Operation by the Logarithmes.

The log. of (1.0625) the Rate	0.0263289	mult.
The term of years ———	10	
The log. of (1.8335) the amount of 1 *l.* for 10 years.	0.2632890	
1.8335—1=.8335 its log.	—1.9209056	ſubt.
1.0625—1=.0625 its log.	—2.7958800	
The log. of the amount of 1 *l.* annuity for 10 years ———	1.1250256	add
The log. of (50) the Annuity propoſed. ———	1.6989700	
The log. of (666.80) the amount of the annuity propoſed ———	2.8239956	

which is 666 *l.* — 16 *s.* and ſo much will be due at the end of the ſaid time.

CASE.

To find what Annuity any ſum due at any time to come will purchaſe to continue for any time, and at any Rate of Intereſt propoſed.

VII. The Operation in this Caſe is the ſame in every reſpect with that in the former Caſe, only whereas in the laſt caſe you ſubtracted the log. of the rate leſs 1, from the log. of the increaſe of 1 *l.* leſs 1, ſo in this you muſt ſubtract the log. of the increaſe of 1 *l.* leſs 1, from the log. of the rate leſs 1, as in the following Example.

Queſt.

Queſt. 8. There is 705 *l.* due at the end of 7 years to come, I demand what Annuity to continue 7 years, the ſame will purchaſe, Compound Intereſt being allowed at 8 *per Cent*? Facit 79.015 *l.*=79 *l.*—00 *s.* as you may find if you obſerve

The Operation by the Logarithmes.

The log. of (1.08) the propoſed Rate ————	0.033424	mult.
The propoſed Time——	7	
The log. of the increaſe of 1 *l.* for 7 years 1.738 —— ——	0.233968	
1 08 −1 =.08 its log.	−2.903090	ſubt.
1.7138—1 =.7138 its log.	−1.853577	
The log. of the value of 1 *l.*	−1.050513	
The log. of (705) the purchaſe money. ————	2.848189	

The log. of the purchaſe (79.015) 1.898702 which is 79 *l.*—00 *s.*— 4 *d. fere.*

Queſtions of this Nature may be ſolved at two Operations by the ſecond and ſixth Caſes; Firſt by the Rule in the ſecond Caſe find the preſent worth of the ſum propounded, then by the ſixth, find what Annuity ſuch a ſum will purchaſe.

CASE. 5.

To find the preſent worth of an Annuity to continue any Term of years, howſoever payable, viz. *either yearly, half yearly or Quarterly, Rebate being allowed at any rate* per Cent.

VIII. Find out the Logarithme of the Rate, and multiply it by the number of Years or Quarters,

ters, according as the Annuity is payable, and that will produce the Logarithme of the increaſe of 1 *l.* for the propoſed time, to which add the Log. of the Rate made leſs by 1, and ſubtract that ſum from the Log. of the increaſe of 1 *l.* made leſs by 1, ſo ſhall the remainder be the Log. of the preſent worth of 1 *l.* annuity for the time propoſed to which add the logarithme of the propoſed Annuity, and the ſum will be the *Logarithme* of the preſent worth of the given Annuity. As in Example.

Queſt. 9. What is the preſent worth of an Annuity of 30 *l.* payable by yearly payments, and to continue 30 years, allowing Rebate after the Rate of 8. *per Cent. per Annum?*

Facit 337 *l.*—14 *s.*—09 $\frac{1}{2}$ *d.* as appears in

The Operation by the Logarithmes.

The log. of (1.08) the propoſ. rate	0.033424	mult.
The term of years	30	
The log. of (10.063 *l.*) the increaſe of 1 *l.* for 30 years.	1.002720	add
The log. of (80) the rate leſs 1	2.903090	
	1.905810	ſubt.
The log of 10.063 − 1 = 9.063	0.957272	
The log. of the preſent worth of 1 *l.* Annuity	1.051462	add
The log. of (30) the propoſed Annuity	1.477121	
The log. of (337.74) the preſent worth of the propoſed Annuity.	2.528583	

which is 337 *l.* — 14 *s.* — 09 $\frac{1}{2}$ *d.*

CASE

CASE 6.

To find out what Annuity to continue any term of years any given sum of Money will purchase at any Rate of Compound Interest.

IX. When you would know what Annuity any given ſum will purchaſe, firſt (as in the foregoing Rules) find out the Logarithme of the Rate, which multiply by the propoſed time, ſo will that product be the Logarithme of the encreaſe of 1 *l.* to which add the Log. of the rate made leſs by 1, and from that ſum ſubtract the Log. of the ſaid increaſe of 1 *l.* made leſs by 1, ſo will the remainder be the Log. of what 1 *l.* will purchaſe for the propoſed Time, to which if you add the Log. of the given purchaſe mony, the ſum will be the Log. of the Annuity that the given ſum will purchaſe. As in Example,

Queſt. 10. What Annuity to continue 7 years, and payable by Quarterly payments will 246 *l.* purchaſe. Allowing Rebate at 8. *per Cent* ? Facit 12. 297 *l.*

The Operation by the Logarithmes.

The log. of 1.08 the given Rate *per An.*	0.033424	
	4)	
which divided by 4 gives the log. of (1.0194) the Rate *per Quarter.*	0.008356	mult.
The Quarters in 7 years	28	
	66848	
	16712	
The Log. of the encreaſe of 1 *l.* for 28 Quarters, *viz.* 1.7138	0.233968	add
1.0194 = .0194 its log.	—2.287801	
Sum	—2.521769	
1.7138—1 = .7138 its log.	—1853577	ſubtr.
The log. of the purchaſe of 1 *l.*	—2.668192	add
The log. of the propoſed Sum 264	2.421604	
The log. of (12.297) the Annuity which the ſaid ſum will purchaſe	1.089796	

which is 12*l.*—05 *s*—11 ½ *d.*

More variety of Queſtions might be ſtated, but theſe to the Ingenious are ſufficient.

FINIS.

Cocker's
ALGEBRAICAL
ARITHMETICK,
CONTAINING
The Doctrine of Composing, and Resolving an
EQUATION,

With all other Rules requisite for the understanding of that Mysterious Art, according to the Method used by Mr. *JOHN KERSEY*, in his incomparable Treatise of ALGEBRA.

Composed by EDWARD COCKER, late Practitioner in the Arts of Writing Arithmetick, and Engraving.

Perused, Corrected, and Published By *JOHN HAWKINS*, School-Master at St. *George's* Church in *Southwark*.

Plato foribus Academiæ inscribi jussit; *Nemo Arithmetices Ignarus hic Ingrediatur.*

LONDON,
Printed in the Year 1702.

ALGEBRAICAL DEFINITIONS.

CHAP. I.

Concerning the conſtruction of Coſſick Powers, and the way of expreſſing them by Letters, together with the ſignification of all ſuch Characters or Marks as are uſed in the enſuing Treatiſe.

1. THE Analytical Art generally called Algebra is that by which, when a Prob'em, or hard Queſtion is propounded, we aſſume the Quantity

or Number ſought, as if it were really known; and, with this aſſumed Quantity, and the Quantity and Quantities gven, we proceed by undeniable Conſequences, until the Quantity firſt aſſumed is found to be equal to ſome quantity or, quantities really known, and is therefore it ſelf alſo known

II. Algebra is either Numeral, or Literal.

III. Numeral Algebra is ſo called, becauſe all the given Quantities in any Queſtion are expreſſed by Natural Numbers, and the number or quantity ſought is ſolely repreſented by ſome Letter or Character taken at the pleaſure of the Artiſt.

IV. Literal Algebra is ſo called, becauſe when a queſtion is reſolved after this method, the unknown or given quantities as well as the unknown, are all expreſſed by Letters of the Alphabet, or ſome other Convenient marks or Characters, and this is alſo called, Specious Algebra; and when a queſtion is reſolved after this manner, at the end of the operation, there is diſcovered, a Canon, directing how the queſtion propoſed, or any other of the like nature may be ſolved, and therefore is Literal Algebra, accounted more excellent than Numeral Algebra, for that produceth not a Canon without extraordinary difficulty; becauſe the numbers firſt given are by Arithmetical operations ſo interwoven and confounded, that it may ſeem a task too tedious for the moſt ingenious Artiſt to trace out their footſteps.

V. The

V. The Doctrine of Algebra consists in the knowledg of certain quantities called Cossick Powers, which we shall immediately explain.

VI. In a series, or rank of Geometrical proportionals continued, proceeding from Unity or one, whether they be ascending, or descending, all the numbers or Terms except the first (which is supposed to be unity) are called Cossick Numbers, or Powers, as for Example, in this rank of continual proportionals, *viz.* 1, 2, 4, 8, 16, 32, 64, 128, 156, *&c.* the second Term (2) is called the root or first Power, the third term (4) is called the Square or second power, the fourth Term (8) is called the Cube, or third power; the fifth (16) is called the Biquadrate, or fourth power; (32) is the fifth power, (64) is the sixth power, (128) is the seventh power, *&c.*

In like manner if you take a rank of Geometrical proportionals continued, and descending from unity *viz.* 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, *&c.* or 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$, *&c.* or 1, $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, *&c.* The second Term is called the root or first Power, the third term is called the second Power, the fourth term is called the third Power, *&c.*

VII. Whence it is evident that the Square or Second Power is generated by the Multiplication of the root or first Power into it self, and the Cube or third power is generated by multiplying the second Power by the root, or by multiplying the root 3 times into it self, and the Biquadrate or fourth power is produced by multiplying of the third power by the root, or by multiplying the root 4 times into it self, and the fifth power is produced by multiplying the fourth power by the root, *&c.* As for Example.

If you take 2 for a Root, multiply it by its ſelf, it produceth 4 for the Square or ſecond power of the Root 2: Again, multiply 4 by the Root 2, and it produceth 8 for the third power, or Cube of the Root 2: Again, if you multiply the Cube 8 by the Root 2, it produceth the fourth Power, or Biquadrate of the Root 2, *&c*

In like manner if 3 were propoſed for a Root, it being multiplyed by it ſelf, produceth 9 for the Square, or Second Power of 3, and 9 being multiplyed by the Root 3, produceth 27 for the Cube, or third Power of the Root 3, *&c.*

And alſo if $\frac{1}{2}$ be propoſed for a Root, and it be multiplyed by it ſelf it produceth $\frac{1}{4}$ for the Square or Second power of (the Root) $\frac{1}{4}$, and $\frac{1}{4}$ (the Square) being multiplyed by (the Root $\frac{1}{2}$,) it produceth $\frac{1}{8}$ for the Cube, or third Power of (the Root) $\frac{1}{2}$ *&c.*

Whence it is evident that the 4, 6 or 7 powers of any Roots may be found out, without any reſpect at all had to the intermediate Powers between the Root and the power required; as ſuppoſe there were given the Root 3, and it were required to find the fifth power of it. I take 3, and ſet it down 5 times in order thus, 3, 3, 3, 3, 3, and multiply them all into each other, according to the rule of continual multiplication, and the laſt product (which is 243) is the fifth power of the Root 3, which was required

Again let it be required to find the fourth power of 5, I take 5, and ſet it down 4 times thus, 5, 5, 5, 5, then do I multiply them continually, and find the laſt product to be 625, which

which is the fourth power (of the given Root) as was required. The like may be obſerved in the finding of any other power of any other given Root.

VIII. If there be a ſeries of Geometrical proportionals continued, and againſt each power there be placed numbers orderly repreſenting the number or degree of diſtance of each power from the Root, ſuch numbers are called the Indices or exponents of the powers, becauſe they ſhew how often the Roots is involved into it ſelf for the production of ſuch a power, as in the Rank, or Scale of Algebraical powers placed in the margent, proceeding from the root 2, to the tenth power thereof, which is 1024, under which is written the word Powers, and then againſt each particular power, on the left hand thereof, is expreſſed Index, or Exponent of that Power, ſhewing how often the Root is involved or multiplyed into it ſelf to produce that Power: As for Example, againſt the number 64, is placed the number 6, which ſheweth that 64 is the ſixth power of its Root, or that its Root is multiplyed 6 times into it ſelf to produce the number 64. The like is to be underſtood of any other,

1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024
Indices.	Powers.

Likewiſe if any two or more Indices, or Exponents

ponents be added together, their ſum will be an exponent ſhewing what power will be produced by the multiplication of thoſe Powers belonging to thoſe Exponents or Indices which you add together; as in the foregoing Table let it be required to find out what power of the Root 2 will be produced by multiplying 128 (its ſeventh power) by 8 (its third power,) in order to which I take 3 and 7, the reſpective Indices of the given powers, and add them together, and their ſum is 10, which ſheweth that the third power, and the ſeventh power of any Number, or Root, being multiplyed together, will produce the tenth power of that Root; ſo in our example 128 being multiplyed by 8, produceth, 1024, which is the tenth power of the Root 2.

In like manner, the Indices 3 and 5 being added together, make 8 for a new Exponent, which ſheweth that 32 and 8 (the powers belonging to thoſe Exponents) being multiplyed together, will produce the eighth power, *viz.* 256, as appears by the ſaid Table, the like of any other

So that you ſee that the addition of Indices anſwers to the multiplication of their Correſpondent powers.

And in like manner will the ſubtraction of Indices, or Exponents, anſwer to the Diviſion of their correſpondent powers, obſerving always to make the power correſponding the ſubtrahend (or Index to be ſubtracted) to be the Diviſor.

IX. When

IX. When a Queſtion is propounded, and its ſolution is to be ſearched out by the Algebraick Art, the number or magnitude ſought is generally called a Root, and it muſt be repreſented or ſignified by ſome Character or Symbol, as muſt be alſo all the powers proceeding from the ſaid Root according to the tenure of the Queſtion, in order to which there may be taken ſome letter of the Alphabet at the pleaſure and diſcretion of the Artiſt, as *a*, *b*, *c*, or *d*, *&c.* to expreſs the ſaid Root, but to avoid confuſion in operation, by the commixture of known with unknown Quantities, our Modern Analyſts have been accuſtomed to aſſume vowels to repreſent unknown Quantities, and to put Conſonants to ſignifie known or given quantities.

X. If for the number or quantity ſought there be put or aſſumed the Vowel *a*, then its Square will be *aa*, that is, *a* being multiplyed by it ſelf, produceth *aa*, that is *a* ſquared, or the ſquare of *a*, for *a* time *a* is *aa*, and the Cube or third power raiſed from the Root *a*, is *aaa*; that is, *a* times *aa*, is *aaa*, and the fourth power accordingly is *aaaa*, and after the ſame manner may any higher power of *a* be ſignified:

In like manner if for the quantity or number ſought there be aſſumed, the letter *e*, then ſhall the Square raiſed therefrom be *ee*, and the third power *eee*, and the fourth power *eeee*, and the fifth power *eeeee*, &c.

Alſo if *b*, or any other Conſonant, be put for a given or known Quantity, then its Square will be, *bb*, its Cube *bbb*, and its biquadrate *bbbb*, &c.

but

But by ſome Analyſts the powers of *a*, or any other letter, or Vowel, or Conſonant, are expreſſed by placing the Index or Exponent of the power in a ſmall Character, juſt after the Symbol, even with the head thereof, *viz.* a, a^2, a^3, a , *&c* ſignifie the Root *a*, its Square, its Cube, and its Biquadrate, *&c.* which may be further exemplified by the following Table.

A Table

A Table ſhewing the Powers of Numbers, and how to expreſs the Simple Powers by Alphabetical Characters.

The Root, or firſt Power	1	2	3	4	*a*	a
The Square, or ſecond Power	1	4	9	16	*aa*	a^2
The Cube, or third Power.	1	8	27	64	*aaa*	a^3
The Biquadrate or fourth power	1	16	81	256	*aaaa*	a^4
The fifth Power.	1	32	243	1024	*aaaaa*	a^5
The ſixth Power.	1	64	729	4096	*aaaaaa*	a^6
The ſeventh Power.	1	128	2187	16384	*aaaaaaa*	a^7
The eighth Power.	1	256	6561	65536	*aaaaaaaa*	a^8
The ninth Power.	1	512	19683	262144	*aaaaaaaaa*	a^9
The tenth Power.	1	1024	59049	1048576	*aaaaaaaaaa*	a^{10}

IX. The

XI. The numbers made uſe of in ſolving of Algebraical Queſtions, are either abſolute Numbers, or Numbers prefixt.

Abſolute numbers are thoſe which are disjunct from any kind of Magnitude or Quantity, either known or (unknown) required, but ſtand ſimply of themſelves, without having Relation to any thing elſe, as 5, 10, 20, 100, $\frac{5}{4}$, and $\frac{11}{2}$ are called abſolute numbers.

Numbers prefixt are ſuch as are immediately prefixt to ſome letter or letters, ſignifying an Algebraical quantity, either known, or required ſuch as are $2a$, $4a$, $10a$, $100a$. $\frac{1}{2}a$, $\frac{3}{4}$ a, $3aa$, $5bbb$, $3a$, $5b^3$; which numbers ſo prefixed, ſhew how often the quantity to which they are prefixed, is to be taken, as $4a$, ſignifieth that a is to be taken 4 times, and $5bbb$, or $5b^3$, ſignifieth that the Cube or third power of b is to be taken 5 Times, $\frac{1}{2}a$ is half of a, and $\frac{2}{3}b$ is two thirds of b; The like is to be underſtood of any other. And,

Note, that when you have any Algebraical Quantity or Letter, or Character, not having any number prefixed to it, then 1, or unity muſt be imagined to be prefixed, as a, or $1a$, b, or $1b$, &c.

XII. As in Vulgar and Decimal Arithmetick, in Algebraical Arithmetick, the operations are performed either by Abſolute Numbers, or by Alphabetical characters, in all the fundamental rules, *viz.* Addition, Subtraction, Multiplication, Diviſion, and the Extraction of Roots: and note that

Where it is required to perform the work by abſolute numbers, that the operation is in every reſpect the ſame as in Common Arithmetick.

tick. But where it is performed by Alphabetical Letters, there is an abſolute neceſſity of uſing ſome Characters, to ſignifie the Operation, an explanation of which Characters take as followeth.

XIII. The Character (+) is a ſign of affirmation or Addition, which it is placed between two quantities, ſignifying that the 2 numbers or quantities between which it is placed, are to be added together, and is as much as to ſay *plus*, as 3+6 ſignifieth the ſum of 3 and 6, is as much as to ſay 3 *plus* 6, or 3 more 9, which is 9 and 4+7+9 ſignifieth the ſum of 4, 7, and 9, which is 20; ſo *a*+*b*+*c* ſignifieth the ſum of *a*, *b*, and *c*.

And here note, that when there is no Mark, or Character before any Letter or quantity, then is it Affirmative, and the Mark (+) is ſuppoſed to ſtand before it; as *a*, is +*a* or +1*a*, and *b* is +*b*, or +1*b*, and *bcd* is +*bcd* the like of others.

XIV. This Character (−) is a negative ſign, and always belongeth to the quantity or Number which followeth it denying it to be, and ſignifieth a fictitious Number, or quantity leſs than nothing.

So — 7 is a feigned number, leſs than nothing by 7, *viz*, as the height of the Sun above the Horizon may be affirmed to be 7 *deg.* or +7 *deg.* ſo when it is depreſſed 7 degrees below the Horizon, its height may be ſaid to be—7 *deg.* that is, 7 *deg.* leſs than nothing.

But when the ſaid ſign or Character is placed between two numbers or Quantities, it ſignifies that the number or quantity which followeth

eth it, is to be ſubtracted out of ſome Number or Quantity going before it, as $12-8$ ſignifieth that 8 is to be ſubtracted out of 12, or it ſignifieth the exceſs of 12 above 8, or the Difference between 12 and 8, which is 4, ſo $a-b$ ſignifieth the exceſs of a above b, and it is as much as to ſay (a leſs b,) ſo $a+b-c$ ſignifieth that c is to be from the ſum of a and b.

XV. This ~~Chapter~~ character ($\times$) is the ſign of Multiplication, and ſignifieth that the Numbers or Quantities between which it is placed, are to be multiplyed together, as 4×5 ſignifieth the product of 4 and 5, which is 20; ſo $3\times5\times8$ ſignifieth the product of the continual multiplication of 3, 5, and 8, *viz.* 120.

Likewiſe $b\times c$. ſignifieth the product of the multiplication of b by c, and $b\times c\times d$ ſignifieth the product made by the continual multiplication of b, c, and d, into each other.

But for the moſt part Analyſts ſignifie the multiplication of literal Quantities by ſetting the letters together like letters in a word, as ab is the ſame with $a\times b$ and abc is the ſame with $a\times b\times c$ and this indeed is to be preferred before the other as moſt convenient and fitteſt for operation.

XVI. This Character (∽) ſignifieth the Difference between the two quantities between which it is placed, when it is not known in which of them the exceſs lyeth. So $b \backsim c$ ſignifieth the Difference between b and c, which it is not known whether b be greater or leſſer than c.

XVII. The

XVII. The ſaid 4 Characters defined in the 13, 14, 15, and 16 Sections foregoing, *viz.* $+$, $-$, $\times$ and $\backsim$, may oftentimes have Relation to ſuch a Compound Quantity following the Character, as hath a line drawn over each part of it, as for example, $c + \overline{b \backsim d}$, by which you are to underſtand that the Quantity (*c*) is to be added to the difference between the Quantities (*b* and *d*) in which of them ſoever, the exceſs lyeth.

Likewiſe $a - \overline{b \backsim c}$ which ſignifieth that the difference between *b* and *c* is to be ſubtracted from the Quantity expreſſed by *a*.

Alſo $a \times \overline{b + c}$ ſignifieth that the ſum of *b* and *c* is to be multiplyed by the quantity *a*, where take notice that in regard there is a line drawn over the two quantities *b* and *c* the ſign $\times$ hath reference to the multiplication of *a* into the quantity *c* as well as the quantity *b*, which immediately followeth it, but if the ſaid line were omitted, and the quantities were thus expreſſed, $a \times b + c$, it would ſignifie the quantity *c* to be added to the product of the multiplication of *a* and *b*.

Furthermore $b - \overline{c + d}$ ſignifieth that the quantities *c* and *d* are or muſt be ſubtracted from the Quantity *b*, whereas if there were not a line over the quantities *c* and *d*, it would ſignifie that the quantity *d* is to be added to $b - e$.

And $c \backsim \overline{a + c}$) ſignifieth the difference between the quantity *c*, and the ſum of *d* and *e*, whereas if the line were not over *d* and *e*, it would ſignifie the quantity *e* to be added to the difference between *c* and *d*.

XVIII.

XVIII. This Character ($\surd$) is a radical sign, and signifieth that the Square Root of the quantity or quantities following it, is to be extracted as $\surd\cdot 36$, signifieth the Square Root of 36, *viz.* 6.

So $\surd\ ab$ signifieth the Square Root of the product of the quantities *a* and *b*, and $\surd\ abc$ is the Square Root of the product of the continual multiplication of the quantities of, *a*, *b*, and *c*.

But when you would represent the Root of a Power that is higher than a Square; then immediatly after the said Radical sign, express the index, or exponent of its power in a parenthesis, as followeth, *viz.* $\surd\ (3)\ 64$, signifieth the Cube Root of 64, which is 4, $\surd\ (4)\ 81$ signifieth the Biquadrate Root of 81, *viz.* 3.

Also $\surd\ (3)\ ab$, signifieth the Cube Root of the product of the multiplication of the quantities, *a*, and *b*, and $\surd\ (4)\ cd$ signifieth the Biquadrate Root of the product of the multiplication of the quantities, *c* and *d*.

And the said Radical sign doth oftentimes belong to such a Compound Quantity following it, as hath a line over every part of it. As for Example, $\surd\ \overline{b+c}$ signifieth the Square Root of the sum of the Quantities *b* and *c*. So $\surd\ (3)\ \overline{a+b-c}$ signifieth the Cube Root of the remainder, when the quantity *c* is subtracted from the sum of the quantities, *a* and *b*, and ($\surd\ (4)\ \overline{aa+b-c}$) signifieth the Biquadrate Root of the remainder, when the quantity *c* is subtracted from the sum of the Square of *a* added to *b*.

Likewise $a+\surd\overline{bb+c-d}$ signifieth that to the quantity *a* is to be added the Square Root of the remainder, when the quantity *d* is subtracted from the sum of the Square of the quantity *b*, and

and the quantity c: And these and such like are by Analysts generally called universal Roots.

After the same manner may be expressed the universal Square Root of $b + \overline{aa \div c}$ thus, *viz* $\sqrt{(2)\,b \div \sqrt{aa \div c}}$ which signifieth the Square Root of the sum when b is added to the Square Root of $\overline{aa \div c}$.

XIX. This Character $(=)$ signifieth an Equation, or equality of the magnitudes, or quantities between which it is placed, and imports as much as these words, *viz.* (is equal to) as in the following Example, *viz.* $3+4=7$, which is as much as to say, the sum of 3 and 4, or 3 *plus* 4 is equal to 7; so $7+9=12+4=16$ imports that the sum of 7 and 9 is equal to the sum of 12, and 4 which is equal to 16; and $9=12-3$, signifieth that 9 is equal to the excess of 12 above 3.

Also $4\times5=2\times10=16+4=20$ signifieth that the Rectangle or Product of 4 by 5 is equal to the Rectangle or Product of 10 by 2, which is equal to the sum of 16 and 4, equal to 20.

Likewise $\frac{24}{6}=\frac{8}{2}$ signifieth that the Quotient of 24 divided by 6, is equal to the Quotient of 8 divided by 2.

Again $a+b=c-d$ signifieth that the sum of a and b is equal to the excess c above d and $c+d=f \div g$ signifieth that the sum c and d is equal to the Quotient of f divided by g; and $b\times c=r-s$ signifieth that the Rectangle of b and c is

c is equal to the exceſs or r above s, and $a = \sqrt{cc + \frac{1}{4}cc} - \frac{1c}{2}$ ſignifieth that a is equal to the remainder, when $\frac{1}{2}c$ or $\frac{c}{2}$, is ſubtracted from the univerſal ſquare Root $cc + \frac{1}{4}cc$ this will be made plain and eaſie to the ingenious practitioner by the enſuing Example of this Treatiſe.

XXI. This Character (⊏) ſtands for the word (greater) ſignifying the number, or quantity ſtanding on the left hand of the ſaid Character to be greater than that on the right hand thereof; as 8⊏3 ſignifieth that 8 is greater than 3; alſo $a+b$ ⊏ c ſignifieth that the ſum of a and b is greater than c, &c.

XXII. This Character (⊑) ſtands for the word leſs) and it ſignifieth that the number or quantity ſtanding on the left hand thereof is leſſer than that on the right hand. As 4+3 ⊑ 20—8 ſignifieth that the ſum of 4 and 3 is leſs than the exceſs of20 above 8 Likewiſe $c-d$ ⊑ $b+e$ is thus read, *viz.* the Remainder of d being ſubtracted from c is leſſer than the ſum of b and e.

XXIII. This Character, (: :) is always placed in the middle between 4 Geometrical proportionals, as in the following Examples, *viz.* 2 : 4 : : 9 : 18 is thus to be read, *viz.* as 2 is to 4, ſo 9 is to 18; or after the manner of the Rule of 3, if 2 require 4, 9 will require 18. Alſo $b : c :: d : e$ is thus read, as b is to c, ſo is d to e. And $a+e : b :: c+b : \frac{be+bb}{a+e}$ is as much as to ſay as the Compound Quantity $a+e$ is to the quantity b, ſo is the Compound Quan-

Quantity $c+b$ to the Quotient of the Compound Quantity $bc+bb$ being divided by $a+e$.

XXIV. This Character (∺) placed after any number of quantities exceeding two, declareth the ſaid numbers or quantities after which it is placed to be continual Geometrical proportionals; ſo 2, 4, 8, 16, 32, 64 ∺ ſignifieth the ſaid numbers to be continual proportionals Geometrical, for, as 2 is to 4, ſo is 4 to 8, and ſo is 8 to 16, and ſo is 16 to 32, and ſo is 32 to 64, *&c.* alſo theſe quantities, *viz. a. b. c. d. e,* ∺ are continnal proportionals Geometrical, for, as *a* is to *b*, ſo is *c* to *d*. and ſo is *d* to *e*.

CHAP. II.

Addition of Algebraical Integers.

I AS in Common Arithmetick, ſo in Algebraical, Addition finds out the aggregate, or ſum of two or more given quantities however expreſſed numerally or literally.

II. When the quantities given to be added are alike, and have like ſigns, collect the numbers prefixed to each quantity into one ſum, and thereto annex the letter, or letters of any one of the given quantities, and then prefix the sign of Affirmation or Negation, *viz.* + or — so shall the quantity thus found be the sum desired.

And here note that every quantity which hath no number prefixed to it, is supposed to have the number 1 prefixed, so is $a = 1a$, *and* $b = 1b$.

Example.

What is the sum $3b + b + 2b$? Facit $6b$, for the sum of the numbers prefixed to each quantity, *viz* 3, 1, and 2 is 6, to which if I annex the Character *b*, it will be $6b$, which must have the sign + prefixed to it, or else it must be imagined so to be, then will $+6b$ be the sum of the given quantities. So if $5ab$ the sum of $3ab + 2ab$. And $-4cd$ the sum of $-3cd$.

More Examples of this Rule.

Quantities to be added.	$3aa$	$-4aa$	$+15abc$
	$2aa$	$-aa$	$+11abc$
	$1aa$	$-2aa$	$+25abc$
Sum	$6aa$	$-7aa$	$+51abc$

III. When the quantities given to be added together are like, but have unlike signs, then subtract the lesser number prefixed from the greater, and to the remainder annex the letter or letters by which any one of the given quantities is expressed; and thereto prefix the sign of + or — according to the sign of that prefixed num-

number wherein lay the excess, so shall this new quantity be the sum of the quantities propounded.

Example.

Let it be required to add + 5cd to −2cd, the sum will be found to be + 3cd; for, first, I subtract −2 from + 5 and there remains 3, to which I annex *cd*, so will there be 3*cd*, to which I prefix the sign + because it belongs to the number 5, wherein lay the excess, so have I + 3*cd* for the sum required. See the work.

add { + 5*cd*, −2*cd*

Sum + 3*cd*

Again, if it were required to add + 4*aa* to −7*aa* the sum would be found to be −3*aa*, because the sign − belongs to the number 7 wherein lay the excess, see the work in the Margent.

add { −7*aa*, + 4*aa*

sum 3*aa*

More Examples of the last Rule.

To be added.	5*abcd*	6*aae*	−15*ggg*	−30*bcd*
	−*abcd*	−9*aae*	11*ggg*	14*bcd*
sum	4*abcd*	−3*aae*	−4*ggg*	−16*bcd*

And here note that if the numbers prefixed to the given quantity be equal, and they have different signs, their sum will be 0, so if it were required to add $+8bcd$ to $-8bcd$ their sum will be 0, the negative sign destroying the affirmative.

IV. When the quantities given to be added are more than 2, and have different signs, then according to the second Rule of this Chapter, bring the quantities having like signs into one sum, that is, the affirmative quantities into one sum, and the negative into another, then by the foregoing third Rule add those two quantities together, so shall their sum be the number sought.

Example.

Let it be required to add the sum of $3aa+7aa-2aa-5aa$. First by the said second Rule I find the sum of $3aa+7aa$ to be $10aa$; and the sum of $-2aa-5aa$ to be $-7aa$, then by the said third Rule I find the sum of $10aa-7aa$ to be $3aa$, or $+3aa$, so that I conclude the sum of $3aa+7aa-2aa-5aa$ to be $+3aa$.

More Examples of this Rule follow.

To be added.	$+3cc$	$-2cd$	$-13bcd$
	$-7cc$	$-7cd$	$+17bcd$
	$+8cc$	$+5cd$	$+7bcd$
	$-4cc$	$+9dc$	$-8bcd$
Sum	0	$+5cd$	$3+bcd$

V. When

V. When the ſimple quantities given to be added together, be unlike, then (how many ſoever there be) ſet them one after another in the ſame line without altering their ſigns.

Example.

What is the ſum of $4b$ added to $3cd$? Facit $4b+3cd$ for the ſum.

More Examples of this Rule follow.

To be added.	$2b$ aa	$3bc$ $2cd$	$8ab$ $-3be$
Sum	$2b+aa$	$+3bc+2cd$	$+8ab-3be$

To be added.	$3c$ $2b$ $-ae$	$+2fg$ $-3gh$ $-4rs$
Sum	$3c+2b-ae$	$+2fg-3gh-4rs$

Algebraical Addition of Compound Integers.

VI. The Addition of Compound Algebraical Integers is eaſily performed by the help of the foregoing Rules of this Chapter, whether the Compound quantities to be added are alike, or unlike; as you may eaſily perceive by the work of the following Examples.

Let it be required to find the ſum of $3a+b$ and $5a+3b$. their ſum will be $8a+4b$ for

$3a+5a=8a$, and $3b+b=4b$, whose sum is $8a+4b$ by the second Rule of this Chapter.

Also the sum of $6cd+3bb$ and $2cd-5bb$ will be found to be $8cd-2bb$ for (by the second Rule of this Chapt.) $6cd+2cd=8cd$, and by the third Rule the sum of $3bb-5bb=-2bb$ which two sums added together by the fifth Rule of this Chapter will be $8cd-2bb$.

Moreover if it were required to find the sum of these Compound Quantities, *viz.* $15gg+8a-20$ and $3gg-3a+12$ it will be $18gg+5a-8$ for $15gg+3gg=18gg$ by the second Rule, and the sum of $8a-3a=5a$ by the third Rule, and by the same $12-20=-8$, the sum of which 3 sums is $18gg+5a-8$ by the fifth Rule of this Chapter.

And the sum of $8b-16+2cd$ and $24-5b+3cd$ is $3b+8+5cd$. And here note that in setting down Compound quantities to be added together, it matters not which of them you set first, so that to every quantity there is prefixed its proper sign; as $3a+b-cc$ is the same with $b+3a-cc$ and with $-cc+b+3a$, &c.

More Examples of the Addition of Compound Algebraical Integers.

To be added.	$4ccc-18+3ab$	$10ab+12aa-6d$
	$8ccc+16-8ab$	$-8ab-8aa+9d$
	$-3ccc+12+6ab$	$-2ab-2aa-2d$
Sum	$9ccc+10+ab+$	$+2aa+d$

To

To be added.	$16cde + 4db + 5p$	$9c^3 + 4ef - dg$
	$-8cde - 2db$	$4c^4 + 2dg - ab$
	$-3cde - db$	$2c^2 + ef - dg$
Sum	$5cde + db + 5p$	$c^4 + 5ef - ab$

CHAP. III.

Subtraction of Algebraick Integers.

I Shall not here need to give you a definition of the nature of ſubtraction, but ſhall only give you a general Rule for finding out of the remainder, exceſs, or difference of any two quantities, and that in all caſes whatſoever.

I. When a Quantity Single or Compound, is given to be ſubtracted from another, then change the ſign, or ſigns of the quantity to be ſubtracted, into the contrary ſigns, that is + into —, and — into +; which being done, add the two given quantities together by the Rules of the foregoing ſecond Chapter, ſo ſhall their ſum be the difference, or remainder ſought.

Example.

Example 1.

Let it be required to ſubtract 3 *a* from 8 *a*. The quantity here given to be ſubtracted is 3*a*, which according to the ſecond rule of the ſecond chap. is+3*a*, therefore muſt its ſign+be changed into—, ſo will it be—3*a*, which being added to 8*a* (by the third Rule of the ſecond Chap.) their ſum will be 5*a*, for, 8*a*—3*a*=5*a*, and 8*a* and is the difference between the quantities ſo much 3*a*.

Example 2.

Let it be required to ſubtract—3*bc* from 4*bc*. Here becauſe—3*bc* is the quantity to be ſubtracted, therefore muſt its ſign—be changed into , ſo will it be +3*bc*, which being added to 4*bc*,by the ſecond Rule of the ſecond Chapter, their ſum is 7*bc*, for, 3*bc*+4*bc*=7*bc* , and ſo much is the remainder when—3*bc* is ſubtracted from+4*bc*,

Example 3.

Let it be required to ſubtract —3*bde* from —9*bde*.

Here becauſe—3*bde* is the quantity to be ſubtracted, therefore muſt its ſign—be changed into+, ſo will it be + 3*bde* which being added to —9*bde* according to the third Rule of the ſecond Chapter their ſum will be—6*bde* for + 3*bde*— 9*bde*=6*bde*, and ſo much is the remainder when—3*bde* is ſubtracted from—9*bde*.

Example

Example. 4.

Subtract $3cd$ from $8de$. The sign of $3cd$ being changed, it will be $-3cd$, which being added to $8de$ by the fifth Rule of the second Chapter, their sum will be $8de-3cd$ which is the remainder when $3cd$ is subtracted from $8de$.

Example 5.

What is the remainder when $-3bc$ is subtracted from $2cd$? Facit $3bc+2cd$, or $2cd+3bc$.

In all which Examples you see that the sign of the quantity given to be subtracted is changed into the contrary sign.

More Examples of Subtraction of Simple Algebraick Integers.

	Example 6.	*Example* 7.
From	$3cd$	$-5bc$
Subtract	cd	$-8bc$
Remainder	$3cd-cd$	$5bc-8bc$
Remainder contracted	$\frac{2}{3}cd$	$3bc$

Example

	Example 8.	*Example* 9.
From	$+3da$	$-cde$
Subtract	$-da$	$+2cde$
Remainder	$3da+da$	$cde-2cde$
Remainder contracted	$+4da$	$-3cde$

And when it is required to ſubtract a Compound Integer from a Compound Integer, the operation will not in any wiſe differ from the former, obſerving always to change + into —, and —into +, as will appear by the following Examples.

Examples.

From $3a$ $4b$ let it be required to ſubtract $2a-b$. Here $2a-b$ being the quantity to be ſubtracted from the other, its ſigns muſt be changed into the contrary ſigns. And then inſtead of $2a-b$ you will have $-2a+b$, which being added to $3a$ $4b$ the ſum will be $a+5b=3a$ $4b-2a$ b, and ſo much is the remainder, when ſubtraction is performed according to the tenure of the Queſtion. See the work laid down as followeth.

From

From $3a+4b$
Subtract $2a-b$

Remainder $3a+4b-2a+b$

Remainder contracted $\}$ $a+5b$

Example II.

From $3an-2dc+ab$ let it be required to subtract $aa+3ab-3dc$. The quantity here given to be subtracted is $aa+3ab-3dc$, whose signs being changed, it will then be $-aa-3ab+3dc$, which being added to $3aa-2de+ab$, the sum will be $3aa-2dc+ab-aa-3ab+3dc$, which according to the sixth Rule of the second Chapter is equal to $2aa+dc-2ab$. See the following operation.

From $3aa-2dc+ab$
Subtract $aa+3ab-3dc$

Remain. $3aa-2dc+ab-aa-3ab+3dc$

Remainder contracted $\}$ $2aa+dc-2ab$

But when the given quantities are unlike, then place the quantity to be subtracted immediately after the quantity out of which it is to be subtracted in the same line, changing its signs, which new quantity when the said quantity is so annexed, is the remainder required, which will admit of no Contraction, because the quantities are unlike.

Example.

Example 12.

Let it be required to ſubtract $3ab+2aa$ from $7bc+6cd$, the quantity given to be ſubtracted is $3ab+2aa$ which annexed to the other given quantity, changing its ſigns, will give $7bc+6cd-3ab-2aa$, which is the remainder required. See the following work.

From	$7bc+6cd$
Subtract	$3ab+2aa$
Remainder	$7bc+6cd-3ab-2aa$

More Examples of ſubtraction in Compound Algebraick Integers.

From	$3aa+2bc$	$8rd-3dc$
Subtract	$2aa+4bc$	$-2rd-9cd$
Remain.	$3aa+2bc-2aa-4bc$	$8rd-3dc+2rd+9dc$
Rem. cont.	$aa-2bc$	$10rd+6dc$

From	$6ace+3cd-bc$
Subtract	$4ace-cd-5bc$
Remain.	$6ace+3cd-bc-4ace+cd+5bc$
Rem. cont.	$2ace+4cd+4bc$

From

From $3aa - 2bc + 5ab$
Subtract $2aa - 2bc - ab$

Remain. $3aa - 2bc + 5ab - 2aa + 2bc + ab$

Rem. cont. $aa + 6ab$

From $8a^4 + 3bc - 3d$ | $3cd + 5a$
Subtract $5a^4$ | $2ab - 3c$

Remain. $8a^4 + 3bc - 3d - 5a^4$ | $3cd + 5a - 2ab + 3c$

Rem. cont. $3a^4 + 3bc - 3d$

As in Natural Arithmetick, the remainder and the ſum ſubtracted being added together will be equal to the number from which the ſubtraction is made; ſo is it likewiſe in Algebraical Arithmetick; for if you add the remaining Quantity to the quantity ſubtracted, the ſum will be equal to the quantity out of which the ſubtraction is made. As in the firſt Example of this Chapter, where it is required to ſubtract $3a$ from $8a$, and the remainder is $5a$, now if to $5a$ you add $3a$, the ſum will be $5a + 3a = 8a$; And in the twelfth Example, where it is required to ſubtract $3ab + 2aa$ from $7bc + 6cd$; the remainder is found to be $7bc + 6cd - 3ab - 2aa$, to which if you add the number ſubtracted, *viz.* $3ab + 2aa$, the ſum will be $7bc + 6cd$ equal to the given quantity out of which ſubtraction is made, for $3ab + 2aa$ being added to $-3ab - 2aa$ they de-

ſtroy

ſtroy each other, becauſe their ſigns are unlike, and this may ſerve for a ſufficient (and indeed the only) proof of the work.

CHAP. IV.

Multiplication in Algebraick Integers.

I. IN Multiplication of Algebraical Quantities there are always two quantities given, to find out a third.

II. The two quantities given are called the Factors, and the third quantity invented, or found by the ſaid Factors, is called the Product, Fact, or Rectangle.

III. When the given Factors are ſingle quantities alike, or unlike, if they have not natural numbers prefixed to them, the fact is diſcovered at firſt ſight, and is performed by joyning both the quantities together in one, without any Character between them, like letters in a word.

But ſpecial regard muſt be had to the ſigns of the given quantities, in all kinds of Multiplication,

cation, whether by ſimple or Compound Quantities, and whether with, or without numbers prefixed to them: the nature of the product wholly depending thereupon, *viz.* If the ſigns of the quantities to be multiplied together be alike, that is both + or both —, then the ſign of the product or fact will be +, but if they be of different kinds, *viz.* the one +, and the other —, then the ſign of the product will be —, as you will find by the ſeveral examples following.

Example 1. What is the Product of *a* multiplyed by *b*? Facit *ab*.

Here becauſe both the Factors are ſigned with +, therefore the ſign of the product is +.

In like manner, if the given Factors had been —*a* and —*b* the product would have been (*ab* or *ba*) the ſame as before, becauſe the ſigns of the Factors are both alike, *viz.* both —.

But if the given Factors had been +*a* and —*b*, or —*a* and +*b* then the product or fact would have been —*ba* or —*ab*, becauſe the ſigns of the Factors are unlike, *viz.* the one + and the other —, Obſerve the like in all caſes whatſoever.

Example. 2. What is the product of *abc* multiplyed by *cd*? Facit *abccd* or +*abccd*.

And if you had been to multiply —*abc* by —*cd*, the product would have been the ſame, *viz.* *abccd*, or +*abccd*.

But if the Factors had been —*abc* by +*cd*, or +*abc* by —*cd*, then the Fact would have been —*abccd*, becauſe the ſigns of the Factors are unlike.

More Examples of the like Nature.

Multiplycand	$+ab$	$-ac$	$-aef$
Multiplyar	$+ade$	$-ac$	$+aaf$
The Product	$+aabde$	$+aacc$	$-aaaeff$

IV. When the Quantities given to be ſubſtracted are Single, or Simple Quantities, (whether alike,or unlike)having natural numbers prefixed to them,then in ſuch caſes let the natural numbers be multiplyed together, and to their product annex the product of the given Algebraical Quantities, they being multiplyed together as in the laſt Rule, ſo ſhall this new quantity found be the product required. As in the following Examples.

Example 1. Let it be required to multiply $3a$ by $9a$.

Firſt, I multiply the numbers prefixed to both quantities, the one by the other, *viz.* 9 by 3, and their product is 27, to which I annex the Letters contained in both quantities, *viz.* *aa*, and they make $27aa$, which is the product, or Fact required.

Example 2. Let it be required to multiply $3aa$ by $4b$. Here firſt I multiply the numbers prefixed together, *viz.* 3 and 4, and they make 12; to which product I annex the Letters of both quantities given, *viz.* *aa* and *b*, and they make $12aab$ for the product required.

Example

Example 3. What is the product of $-3abc$ by $-5cd$?

Here first I multiply the given numbers prefixed, *viz.* 3 and 5, and they produce 15, to which I annex the Letter in both the quantities, *viz.* *abc* and *cd*, and they make $15abccd$, to which I prefix the sign +, because the signs of the given quantities were both alike, *viz.* — and then will the product or fact be $+15abccd$.

Example 4. What is the product of $+6ab$ multiply by $-3cd$?

First, multiply the numbers prefixed, *viz.* 6 and 3, and the product is 18, to which I annex the Letters in both the given quantities, *ab* and *cd*, and it makes $18abcd$, to which I prefix the sign — (because the signs of the Factors were unlike, *viz.* the one +, and the other —) and then the product will be $-18abcd$.

More Examples of the like Nature.

Multiplicand	$8fg$	$48bc$	$20dff$	$14ghk$
Multiplyar	$6rs$	$6f$	$3ff$	6
Product	$48fgrs$	$288bcf$	$60df^4$	$84ghk$

V. When Compound quantities are to be multiplyed, the operation (in effect) is the same with multiplication of Simple Quantities delivered in the foregoing Rules, for you are to multiply every particular quantity in the multiplicand by each particular quantity in the multiplyar, (not regarding whether you begin the work at the right hand or the left) and then let the several products be joyned together according to the Rules of the Algebraical Addition, and that ſum will be the product required. The following Examples will make the Rule plain.

Example. 1.

In the firſt place let it be required to multiply a Compound quantity by a ſimple, (*viz.*) $ab+d$ by a. And in order thereto, Firſt, I multiply a into ab, and the product is aab, and then into d, and it produceth ad, ſo is $aab+ad$ the product required, each member of the product being affirmative, becauſe all parts of the Factors were Affirmative.

Example 2. Let it be required to multiply $aa+ab-c$ by b. The Product of aa by b is $+aab$, and the product of ab by b is $+abb$, and the product of $-c$ by b is $-cb$, all which particular products being joyned, and one Compound quantity compoſed thereof, it will give $aab+abb-bc$ for the product required. See the work in the margent.

Multiply	$aa+ab-c$
by	b
Product	$aab+abb-bc$

Example

Example 3. Let it be required to multiply the Compound quantity $ac+dg$ by the Compound quantity $c+d$.

$$\begin{array}{lll} \text{Multiply} & & ac+dg \\ & \text{by} & c\&d \\ \hline & & acc+cdg \\ & & acd+ddg \\ \hline \text{Product} & \multicolumn{2}{l}{acc+cdg+acd+ddg} \\ \hline \end{array}$$

First, Multiply each member of the multiplicand by c, and the product is $acc+cdg$, then multiply each member of the *multiplicand* by d, and the product is $acd+ddg$, which two products being joyned together by the Rule of Algebraical Addition, the sum is $acc+cdg+acd+ddg$, which is the product required, as appears by the operation.

Example 4. What is the product of $da+bc$ multiplyed by $da-ab$?

$$\begin{array}{lll} \text{Multiply} & & da+bc \\ & \text{by} & da-ab \\ \hline & & ddaa+dabc \\ & & \quad -aadb-abbc \\ \hline \text{Product} & \multicolumn{2}{l}{ddaa+dabc-adab-adbbc} \\ \hline \end{array}$$

First, Multiply the *Multiplicand* $da+bc$ by da (the first member of the *multiplyar*) and it produceth $ddaa+dabc$, then multiply the said *Multi-*

plicand by $-ab$ (the ſecond member of the multiplyar) and it produceth $-aadb-abbc$, which two quantities being joyned together give $ddaa+dabc-aadb-abbc$ for the product required. As you may ſee in the Operation.

Example 5. What is the product of $a+b-c$ multiplyed by $a+b-c$?

Multiply	$a+b-c$
by	$a+b-c$
	$aa+ab-ac$
	$+ab+bb-bc$
	$-ac-bc+cc.$
Product	$aa+2ab-2ac+bb-2bc+cc$

Firſt, Multiply each member of the Multiplicand by a (the firſt member of the multiplyar) and it produceth $aa+ab-ac$, then multiply each ſaid member in the multiplicand by b, (the ſecond member of the multiplyar) and it produceth $ab+bb-bc$, then multiply each member of the ſaid multiplicand by $-c$ (the third and laſt member of the multiplyar) and the product is $-ac-bc+cc$; which ſaid three products being joyned together according to the Rules of Algebraical Addition, will give $aa+2ab-2ac+bb-2bc+cc$ which is the Square of $a+b-c$ or product required, as appears by the whole operation.

And if there are natural numbers prefixed to any of the Compound Quantities, the operation will not be different from the foregoing Examples of this Rule, regard being had to the fourth

Rule

Rule of this Chapter, as will appear by the following Example.

Example 6. What is the product of $3b+2c$ multiplyed by $4b-3c$?

Multiplycand	$3b+2c$
Multiplyar	$4b-3c$
	$+12bb+8bc$
	$9bc-6cc$
Product	$12bb-bc-6cc$

First, By the fourth rule multiply each member of $(3b+2c)$ the multiplicand by $4b$, and the product is $12bb+8bc$; then multiply the said multiplicand by $-3c$, and the product is $-9bc-6cc$, which being added to $12bb+8bc$, the sum will be $12bb-bc-6cc$ which is the product required.

Example 7. What is the product of $2a+2e-8$ multiplyed by $2a-5$?

Multiply	$2a+2e-8$
by	$2a-5$
	$4aa+4ae-16a$
	$-10a-10e+40$
Product	$4aa+4ae-26a-10e+40$

First, $2a+2e-8$ being multiplyed by $2a$, produceth $4aa+4ae-16a$, for $2a\times 2a=4aa$ and

$2a \times 2e = 4ae$ and $2a \times -8 = -16a$ which is marked with $-$, because the signs of the Factors are unlike, *viz.* -8 and $+2a$. Secondly $2a+2e-8$ being multiplyed by -5, produceth $+10a-10e+40$; for $-5 \times +2a = -10a$ and $-5 \times +2e = -10e$, and $-5 \times -8 = +40$, all which quantities being joyned together by the Rules of Algebraical Addition will give $+4aa+4ae-26a-10e+40$ which is the product required. See the work.

More Examples in Multiplication of Compound Algebraical Integers.

Multiplicand	$3b+2c$	$3cd+3b-6$
Multipliar	$3b$	8
Product	$9bb+6bc$	$24cd+24b-48$

Multiplicand $2ab+bcd$
Multipliar $3ab-bcd$

$6aabb+3bbacd$
$-2bbacd-bbccdd$

Product $6aabb+bbacd-bbccdd$

Mult. $3a+bb-16$
Mult. $4a-6c-4$

$12aa+4abb-64a$
$-18ac-6bbc+96c$
$-12a-4bd+64$

Product. $12aa+4abb-76a-18ac-6bbc+96c-4bd+64$

Multiplicand

Multiplicand $3ab + 4cc - 2a$
Multiplyar $2ab - 3cc - a$

$$
\begin{array}{l}
6aabb + 8ccab - 4aab \\
\quad - 9abcc - 12c^4 + 6acc \\
\qquad - 3aab - 4acc + 2aa
\end{array}
$$

Product $6aabb + - 7aab - abcc - 12c^4 + - 2acc + 2aa$

CHAP. V.

Division in Algebraick Integers.

I. IN Division Algebraical (as in Division in Common Arithmetick) there are two quantities given to find out a third; which quantity sought is called the Quote, or Quotient ;and of the quantities given that which is to be divided, is called the dividend, and the quantity by which it is to be divided, is called the Divisor.

II. when it is required to divide one number or quantity by another, if you place the Dividend for the Numerator, and the Divisor for the Denominator of a Fraction, that Fraction so composed is equal to the Quotient that would arise

arise by the real Division of the one by the other.

For if it were required to divide 4 by 5, the quotient would be $\frac{4}{5}$, or if it were required to divide 12 by 7, the *Quotient* would be $1\frac{5}{7} = \frac{12}{7}$. The reason of which is the ground of the general part of Division in Algebra; for when one quantity is to be divided by another, set the quantity that is Dividend for the Numerator of a Fraction, and the quantity that is the Divisor set for the Denominator.

So if it were required to divide the quantity b by the quantity a, I would place them thus, *viz.* $\frac{b}{a}$ which signifieth the Quotient of b divided by a.

In like manner if it were required to Divide abe by cd, the Quotient would be $\frac{abe}{cd}$. And if $5ac$ were to be divided by $3cd$, the Quotient would be $\frac{5ac}{3cd}$. And if bcd were to be divided by 7, the Quotient would be $\frac{bcd}{7}$.

The same is to be observed in Division of Compound Algebraick Integers, for if it were required to divide $a+b$ by c, the Quotient would be $\frac{a+b}{c}$, and if $5b$ were to be divided by $3a+bcd$, the Quotient would be $\frac{cb}{3a+bcd}$.

$\frac{5b}{3a+bcd}$

More

More Examples of Division according to the foregoing Rule.

Dividend	cd	$3dce$	$3aa+e-f$
Divisor	fg	$15b$	$a+b+c$

Quotient $\frac{cd}{fg}$ | $\frac{3dce}{15b}$ | $\frac{3aa+e-f}{a+b+c}$

Divid end	a^4b	$15bb$	$6aa$
Divisor	$3b^3$	$7cc$	3

Quotient $\frac{a^4}{3b^3}$ | $\frac{15bb}{7cc}$ $\frac{6aa}{3} = 2aa$

II. When in any quotient that is expressed according to the foregoing rule, there are the same Letter or Letters repeated in every part or member of the Numerator and Denominator, you may cancel such Letter or Letters, but be sure that what you cancel in one part, to cancel the very same in all the rest. So shall this new Quantity be a true quotient, equivalent to what it was before the said Letters were cancelled.

Example. What is the Quotient of bd divided by b? According to the foregoing Rule the Quotient is $\frac{bd}{b}$ but because the Letter b is found both in the Numerator and in the Denominator, therefore cancel b in both of them

them, and then you will find the Quotient to be d, for $\frac{bd}{b}=d$.

Again let it be required to divide $ab+ad$ by acd, the Quotient is $\frac{ab+ad}{acd}$, and because the letter a is found in every member of the Numerator and Denominator, cast it out of every one, and then you will have $\frac{b+d}{cd}$ for the Quotient.

Likewise if you were to divide $ab+abc+abe$ by $abd+abf$ the Quotient you would find to be $\frac{ab+ab+abe}{abd+abf}$ which being contracted by cancelling ab in each member of the Numerator and Denominator, there will be found $\frac{1+c+e}{d+f}$ for the Quotient.

And $bb+b$ being to be divided by b, the Quotient will be $\frac{bb+b}{b}$ and by cancelling b in every part, there will be $b+1$ for the Quotient abbreviated, for $\frac{bb+b}{b}=\frac{1bb+1b}{1b}$ and by cancelling b in every part, there will be $\frac{1b+1}{1}$ and $\frac{1b+1}{1}=b+1$.

The same is to be observed whether the signs be $+$ or $-$; so if it be required to divide $abe+cbe$, by $bae-bce$ the Quotient will be $\frac{abe+be}{bae+bc}$: And because be is found in each quantity, I cancel it, and the Quotient contracted, or abbreviated will be found to be $\frac{a+c}{a+c}$

More

More Examples of Contractions or Abbreviations in Division of Algebraick Integers according to the foregoing Rule.

Dividend	aaa	$ad-ak+a$	$bcd+crd$
Divisor	aa	a	$cgd-cmd$
Quotient	$\frac{aaa}{aa}$	$\frac{ad-ak+a}{a}$	$\frac{bcd+crd}{cgd-cmd}$
Quotient contracted	a	$d-k+1$	$\frac{b+r}{g-m}$

IV. If when it is required to divide a Simple or Compound Algebraick Quantity by a Simple Quantity, there be prefixed to every member a number, or numbers, that may be divided by any other number without any remainder, then instead of the given prefixed numbers prefix the Quotient of each of the said numbers divided by the said Common measurer, not neglecting to cancel any letter that may be found in each part of the Numerator and Denominator, according to the foregoing third Rule. As for Example.

Divide $16bc$ by $4b$. Here according to the foregoing Rule, the Quotient is $\frac{16bc}{4b}$, but because the prefixed numbers 16 and 4 will admit of 4 for a common measure, therefore I divide them both by 4, and the Quotients are 4 and 1, which I prefix to the given Quantities instead of 16 and 4, and then the Quo-

Quotient will be $\frac{4\,bc}{1\,b}$, or $\frac{4bc}{b}$, and becauſe *bc* is contained both in the Numerator, and the Denominator, cancel it, ſo have you $\frac{4c}{1} = 4c$ for the Quotient required.

Moreover $15abc + 12bd$ being given to be divided by $3bg$, the Quotient will be found to be $\frac{5ac+4d.}{g}$

For, I firſt diſcover that the prefixed numbers 15, 12 and 3, have 3 for their common meaſure, by which they being ſeverally divided, give 5, 4, and 1, which being prefixed to the ſaid Quantities after *b*, (which is found in every quantity is cancelled) there will be $\frac{5ac+4d}{g}$ for the true Quotient required.

More Examples of Contraction in Diviſion, according to the two laſt Rules.

Divide	$8bc$	$28cd$	$32abc$
by	$4b$	$16d$	$12acd$
Quotient	$\frac{8bc}{4b}$	$\frac{28cd}{16d}$	$\frac{32abc}{12aod}$
Quotiemt contracted	$2c$	$\frac{7c}{4}$	$\frac{8b}{3d}$

V. When in Compound quantities one or more letter or letters is repeated in every member, then will the remaining letters in each quantity evenly

evenly divide the said Compound quantity without any remainder, and the quotient will be the Letter or letters repeated in each number as aforesaid. As for Example.

1. What is the Quotient of $ba+ca$ divided by $b+c$? Here it is evident that the quotient will be a; for proof whereof take the divisor $b+c$, and multiply by the quotient a, according to the fifth rule of the fourth Chap. and the product will be $ba+ca$ equal to the dividend.

2. Likewise if it were required to divide $bad+cad$ by $b+c$, the quotient will be found to be ad.

3. Also by the same reason if you divide $2ab - 2acd - 2adb$ by $b - cd - db$, the quotient will be $2a$, and if you divide the same dividend by $2b - 2cd - 2db$ the quotient will be a.

More Examples of the like nature.

Dividend $6ab+2adc$	$4bc+3cd-c$
Dividend $6b+2dc$	$4b+3d-1$
a	c

The reason why -1 is the last number of the last example is, because $-c$, or $-1c$ is the last number of the dividend, for according to the thirteenth Rule of the first Chapter, when a quantity hath no number prefixed to it; it is supposed to have the number 1 before it.

And here note, that as in Multiplication of Algebraick Integers $+$ by $+$, and $-$ by $-$ produceth $+$, and $+$ by $-$ produceth $-$, so in Division, if you divide $+$ by $+$, or $-$ by $-$, the

sign

ſign of the Quote will be +, but if you divide + by —, or —, by +, the ſign of the Quote will be — ; ſo if $3ab$ be divided by $3a$, the Quote will be b, or $+b$, and if $-3ab$ be divided by $-3a$, the Quote will be $+b$, for if you multiply $+3a$ by $+b$, the product will be $+3ab$ by the third Rule of the fourth Chapter foregoing. Alſo if you divide $+3ab$ by $-3a$, or $-3ab$ by $+3a$, the Quotient will be $-b$, for $+3a$ being multiplyed by $-b$, produceth $-3ab$; and $-3a$ being mutiplyed by $-b$, produceth $+3ab$.

VI. From a due conſideration of the manner of operating the Examples of the laſt Rule, a way may be diſcovered to divide a compound Quantity by a Simple, or Compound Quantity, and to find out the true Quotient when it likewiſe will be a Compound Quantity, the practice of which will be made plain by the following Examples.

Example 1. Let it be required to divide $ba+ca$, by a. Having placed the Dividend and Diviſor as is uſual in vulgar Arithmetick, and as you ſee in the following operation.

$$a)\ ba+ca\ (b+c$$
$$-ba$$
$$\overline{\quad\quad}$$
$$+ca$$
$$-ca$$
$$\overline{\quad\quad}$$
$$(0)$$

Then do I ſeek how often a is contained in ba (the firſt member of the Dividend) and the anſwer

ſwer is b times, therefore I put b in the Quotient, and thereby I multiply (a) the Diviſor and the product is $+ ba$, which muſt be ſubtracted from ba in the Dividend, and therefore I change its ſign into $-ba$ by the firſt Rule of the third Chapter, and there remaineth o, then do I bring down $+ ca$ the next member of the Dividend, and divide it by a. and the *Quotient is* $+c$, by which I again multiply (a) the Diviſor, and the product is ca, which ſubtracted from ca there remaineth o, and ſo the work of Diviſion is ended, and I find the *Quotient* of $ba+ca$ divided by a to be $b+c$, for proof whereof if you multiply $b+c$ by a (the Diviſor)the product will be $ba+ca$ equal to the given Dividend.

Example 2. Let it be required to divide $ba+ce+be+ce$ by $b+c$

Having diſpoſed of the Dividend and Diviſor in order to the work with a crooked line behind which to place the Quotient, as in Common Arithmetick ; then firſt I ſeek how often b (the firſt member of the Diviſor) is contained in ba, (the firſt member of the Dividend) and there ariſeth a, which I put in the quotient, and thereby mutiply each member of the Diviſor , *viz.* $b+c$, and the product is $ba+ca$, which place under the two firſt quantities of the dividend towards the left hand, *viz.* under $ba+be$, and by the firſt Rule of the ſecond Chapter ſubtract it therefore , ſo will the remainder be o; to which I bring down the remaining part of the dividend, *viz* $be+ce$ and divide be by e, and there ariſeth in the quotient e, by which I multiply the whole Diviſor $b+c$, and the Product is $be+ce$, which ſubtracted from the Divi-

 dend

dend $be+ce$, the Remainder is o. See the whole work as followeth.

$$\begin{array}{llr} & & \textit{Quotient} \\ b+c) & ba+ca+be+ce & (a+e \\ & ba+ca & \\ \hline & 0\quad 0+be+ce & \\ & \qquad be+ce & \\ \hline & \qquad (0\quad 0) & \end{array}$$

So that the Quotient is $a+e$, now to prove the work, multiply the *Divisor* $b+c$ by the *Quotient* $(a+e)$ according to the fifth Rule of the fourth Chapter, and the *Product* you will find to be $ba+ca+be+ce$ which is equal to the given *Dividend*; and therefore I conclude the operation to be truly performed.

Example 3. In like manner, if you divide $ba+bd+ca+cd-ae-de$ by $a+d$, the *Quotient* will be found to be $b+c-e$ according to the following work.

$$\begin{array}{llr} & & \textit{Quotient} \\ a+d) & ba+bd+ca+cd-ae-de & (b+c-e \\ & ba+bd & \\ \hline & 0\quad 0+ca+cd & \\ & \quad +ca+cd & \\ \hline & \quad 0\quad 0-ae-de & \\ & \qquad -ae-de & \\ \hline & \qquad (0\quad 0) & \end{array}$$

The

The work of the last Example explained.

In the foregoing Example, first I divide *ba* (the first member of the Dividend) by *a* the first quantity or member of the Divisor, and there ariseth *b* in the *Quotient*, which is +*b*, (because the signs of the *Dividend* and *Divisor* are +) and thereby I multiply the *Divisor* *a*+*d*. and the *Product* is *ba*+*bd*, which I place under the two first members of the *Dividend* as you see in the work, and subtract it therefrom, and the remainder is 0, to which I bring down the two next quantities, *viz.* +*ca*+*cd*.

Then do I divide +*ca* by *a*, and there ariseth in the *Quotient* +*c*, because the *Dividend* and *Divisor* are both signed +,) by which I multiply the said *Divisor*, and the product is +*ca*+*cd* which I place under the *Dividend*, and subtract it therefrom, and there remaineth 0, to which I annex the two next and last members of the *Dividend*. *viz.* —*ae*—*de*; and divide —*ae* by +*a*, and the *Quotient* is —*e*, (because the signs of the *Dividend* and *Divisor* are different, *viz.* the one +, and the other —) and thereby I multiply the whole *Divisor*, and the *Product* is —*ae*—*de*, which subtracted from (—*ae*—*de*) the *Dividend*, the remander is 0, and so the work is finished, and the *Quotient* arising by this Division is *b*+*c*—*e*, as you may prove at your leisure.

If the quantities or members of the *Dividend* of the foregoing Example are not placed in the same order that is there expressed, the effect of the operation will be the same, as you may see by the following work.

Divisor	*Dividend*	*Quotient*
$a+d)$	$ba+ca-ae+bd+cd-de$	$(b+c-e$
	$ba+bd$	
	$ca-ac$	
	$ca+cd$	
	$-ae-de$	
	$-ac-de$	
	$(0\ 0)$	

First, I divide ba by a, and the Quote is b, by which I multiply the *Divisor* $(a+d)$ and the product is $ba+bd$ which I subtract from $ba+ca$ and the remainder is (by the Rule of the third Chapter) $baca+-ba-bd$ which being contracted+ by the Rules of Addition is ca-bd, (for $+ba$ and $-ba$ expunge each other.) then to this remainder do I bring down the two next quantities of the *Dividend, viz.* $-ae+bd$ which being annexed to the said remainder $ca-bd$ it then (makes for a new dividual) $ca-bd-ce+bd$, but $-bd$ and $+bd$ destroy each other, and therefore the dividual contracted is $ca-ce$, which I divide by $a+d$ as before, and the Quotient is $+c$, by which I multiply the *Divisor*, and the *Product* is $ca+cd$, which being subtracted from the said dividual $ca-ae$, the remainder is ca ae $ca-ed$ which being contracted, is ae cd, to which I joyn the two next quantities in the *Dividend, viz.* $-cd-de$, and it makes ($ae-cd+cd-de$) $=-ae-de$ (for $-cd$ and I cd destroy each other for a new dividual, which I divide by the said *Divisor* $a+d$, and the Quote is $-e$, by which I multiply the *Divisor*

viſor, and the *Product* is $-ae-de$, which ſubtracted from $-ae-de$ (the dividual) the remainder is $-ae-de+ae+de=0$, and ſo the work is finiſhed, and I find the *Quotient* to be $b+c-e$ as before.

But here note by the way that it doth not always fall out that you are to divide the firſt member of the dividual by the firſt of the Diviſor, but by ſome other member which you can diſcover will do the work without making a Fraction. As in the following Example.

Note.

Example 4. Let it be required to divide $aa-ee$ by $a+e$?

Firſt, I divide aa by a, and there ariſeth in the Quotient a, by which I multiply the *Diviſor* $a+e$, and the Product is $aa+ae$, which ſubtracted from the *Dividend* ($aa-ee$) the remainder is $-ee-ae$ for a dividual, and then do I not ſeek how often a is contained in ee, for then the anſwer would be a Fraction, but I divide ee by its correſpondend *Diviſor* $+e$, and there ariſeth $-e$, to be written in the *Quotient* next after a, but not $+e$, becauſe $-$ divided by $+$ quotes $-$. Then I multiply the whole *Diviſor* $a+e$ by $-e$. and the product is $-ae-ee$, which ſubtracted from the ſaid dividual $-ee-ae$, the remainder is 0, ſo is the work ended, and I find the *Quotient* to be $a-e$. See operation.

$$a+e)\ aa-ee\ (a-e$$
$$aa+ae$$
$$\overline{-ee-ae}$$
$$-ae-ee$$
$$\overline{(0\quad 0)}$$

Example 5. If it were required to divide $aaa+abd+baa+bbd$ by $aa+bd$ the Quotient would be found to be $a+b$, as appears by the work.

$$aa+bd)\ aaa+abd+baa+bbd\ (a+b$$
$$aaa+abd$$
$$\overline{0\quad 0+baa+bbd}$$
$$baa+bbd$$
$$\overline{0\quad 0}$$

Example 6. If $aaa-abb+abd+baa-bbb+bbd$ be divided by $a+b$ the Quote will be $aa-bb+bd$, as by the operation.

$$a+b\ )\ aaa-abb+abd+baa-bbb+bbd\ (ad-bb+bd$$
$$aaa+baa$$
$$-abb+abd$$
$$-abb-bbb$$
$$\overline{+abd+bbd}$$
$$+abd+bbd$$
$$\overline{0\quad 0}$$

Example

Example 7. Let it be required to divide $18abbc+9bbcc+24bcc+24abc+16cc$ by $3bc+4c$, the quotient will be found to be $6ab+3bc+4c$. See the following operation.

Divisor Dividend Quotient

$$3bc+4c\ )\ 18abbc+9bbcc+24bcc+24abc+16cc\ (\ 6ab+3bc+4c$$
$$18abbc+24bcc$$
$$\overline{\qquad\qquad}$$
$$+9bbcc+24bcc$$
$$+9bbcc+12bcc$$
$$\overline{\qquad\qquad}$$
$$+12bcc+16cc$$
$$+12bcc+16cc$$
$$\overline{\qquad\qquad}$$
$$0\quad 0$$

When Algebraical Division according to the Rules before delivered, will not exactly perform the work without any remainder, then you may place the Dividend and Divisor Fraction-wise, which is indeed the most general practice amongst Algebrists; or else proceed in Division as far as you can by the preceeding method; and then place the remainder for a Numerator over the Divisor, as in the following Example, where $aa-bb+ac$ is divided by $a+b$, and the quotient is $a-b+\frac{ac}{a+b}$, the remainder being $+ac$.

$$a+b)\ aa-bb+ac\ (a-b\frac{ac}{a+b}$$
$$aa+ab$$
$$\overline{\qquad\qquad}$$
$$-bb-ab+ac$$
$$-ba-bb$$
$$\overline{\qquad\qquad}$$
$$0\quad 0+ac$$

CHAP. VI.

The Doctrine of Algebraical Fractions. And First,

Of Reduction.

I. HE that intends a confiderable proficiency in this myfterious Art, muft be very well acquainted with the Doctrine of Vulgar Fractions, a mean knowledge therein not being fufficient for all operations whatfoever in Algebraick Fractions have their dependence thereupon, being wrought in every refpect as vulgar Fractions, they are by the help of the Rules contained in the feveral Chapters foregoing, and there are very few queftions folved Algebraically, but what have one or more Fractions concerned in its operation.

To reduce Fractions, having unequal Denominators to Fractions of the fame value having a common Dnomin tor.

II. When you would reduce algebraical Fractions to a common Denominator, multiply the Numerator of the firft Fraction into the Denominator or denominators of the reft, fo fhrll the Product be a Numerator equal to the Numerator of of the firſt Fraction, likewiſe multiply the Numerators of the ſecond, third, &c. Fractions into all the Denominators except its own, and the ſeveral products ſhall be ſo many new Numerators; then multiply all the Denominators continually, ſo ſhall the product be a common Denominator to all the Numerators found out as before.

Example 1. Reduce $\frac{a}{b}$ and $\frac{d}{c}$ to a common Denominator. Multiply the Numerator a (of the firſt Fraction) into the Denominator (c) of the ſecond Fraction, and the product is ac, for a Numerator $=a$, then multiply (d) the Numerator of the ſecond Fraction into (b) the Denominator of the firſt, and the product is db, for a Numerator $=d$, then multiply the Denominators together, *viz.* b into c, and the product bc is the Denominator common to both the Numerators, ſo will the 2 new Fractions be $\frac{ac}{bc}$ and $\frac{bd}{bc}$ for $\frac{ac}{bc}=\frac{a}{b}$ and $\frac{db}{bc}=\frac{d}{c}$.

Example 2. What Fractions are $=\frac{a}{c}$, $\frac{b}{a}$ and $\frac{c}{d}$ having an equal or common Denominator?

Facit $\frac{aad,\ bcd}{cad\ \ cad}$ and $\frac{cca}{cad}$, for $a\times a\times d=aad$, $=$ the Numerator a, and $b\times c\times d=bcd=$ the Numerator b, and $c\times c\times a=cca=$ the Numerator, and $c\times a\times d=cad$ which is the common Denominator.

To reduce an Algebraical Fraction to its lowest Terms equivalent.

III. When in the Numerator and Denominator of an Algebraical Fraction, the ſame letter or letters is contained, then cancel the ſame in both and if there be any numbers prefixt, if you can diſcover any number that will divide them both without any remainder, then prefix thoſe quotients inſtead of the numbers prefixed before ; ſo ſhall this new Fraction be of the ſame value with the Fraction propoſed.

So will $\frac{cbd}{cbb}$ be reduced to $\frac{d}{b}$, by cancelling cb in the Numerator and Denominator.

Alſo $\frac{24bde+32bda}{8bdd}$ by being reduced to its loweſt terms will be $\frac{3e+4a}{d}$ by cancelling bd in every part, and dividing the prefixed nnmbers by 8. More Examples follow.

$\frac{cba}{cae} = \frac{b}{e}$	$\frac{16rbsd}{20rsgq} = \frac{4bd}{5gq}$
$e + \frac{ghm}{gh} = e + m$	$\frac{24ade}{12ade+16adb} = \frac{8c}{3e+4b}$

Or if you can (in a Compound Algebraical Fraction) diſcover a quantity that will divide the Numerator and Denominator without any remainder, (according to the ſixth Rule of the fifth Chapter) then ſhall the Quotients be a new

new Numerator and a new Denominator equal to the Fraction in its given Terms. As in the following Examples.

$\frac{ba+da}{b+d}=a$	$\frac{aa+2ae+ee}{a+e}=a+e=e$
$\frac{aa-2ae+ee}{a-e}=ae$	$\frac{aaaa-bbbb}{aa-bb}=aa-bb$

To reduce an Integral quantity to an Algebraical Fraction.

IV. Multiply the given quantity by the intended Denominator, so shall the Product be the Numerator required. As in the following Examples.

Let it be required to reduce the quantity b to a Fraction, having ad for its Denominator. To do which I multiply the given quantity b by ad, and the product is the Numerator, *viz.* bad, so shall $\frac{bad}{ad}$ be the Fraction required, for $\frac{bad}{ad}=b$.

Also if it were required to reduce the quantity e to a Fraction, whose Denominator should be $b+c$, it would be $\frac{be+ce}{b+c}$

V. If it be required to reduce a mixt quantity to a Fraction, multiply the Integral quantity by the Denominator of the Fractional part, and joyn the product to the Numerator of the Fractional part, so shall the sum be the Numerator. As in Example.

Reduce

Reduce $a+b+\frac{c}{d}$ to an improper *Fraction* Mult. the Integral part $a+b$ by the Denominator d, and the *Product* $da+db$ which being added to the numerator c, makes $da+db+c$ for the numerator, to which placing d for a Denominator, it gives $\frac{da+db+c}{d}=a+b+\frac{c}{d}$ for the anſwer.

VI. When you are to expreſs an Algebraick Integer Fraction-wiſe, without an Aſſigned Denominator, then make the given quantity the Numerator, and 1 the Denominator.

So will ab be $\frac{ab}{1}$ and cd will be $\frac{cd}{1}$ and $a+b$ will be $\frac{a+b}{1}$, &c.

Theſe things are ſo plain that they need no further explanation by examples.

CHAP. VII.

Of Addition and Subtraction of Algebraical Fractions.

I. WHen the Fractions given to be added together have an equal or common Denominator, add the Numerators together, and place their ſum for a Numerator over the com-

common Denominator, which new *Fraction* ſhall be the ſum of the given Fractions, but if they have not a common Denominator, reduce them by the ſecond Rule of the ſixth Chapter, and then proceed as before.

Example 1. What is the ſum of $\frac{a}{b}$ and $\frac{ac}{b}$? Facit $\frac{a+ac}{b}$, the ſum of the Numerators, *viz.* a and ac is $a+ac$ which placed over the Denominator b, gives $\frac{a+ac}{b}$ for the ſum required.

Example. 2. So alſo the ſum of $\frac{abc}{f}$ and $\frac{dg}{f}$ will be $\frac{abc+dg}{f}$

Example 3. And the ſum of $\frac{a+b-c}{a}$ $\frac{a+b+c}{a}$ and $\frac{2a-2b}{d}$ will be found to be $\frac{4a}{d}$ for the ſum of the Numerators is $a+b-c+a+b+c+2a-2b = 4a$.

Example 4. What is the ſum of $\frac{a}{b}$ and $\frac{c}{d}$? Facit $\frac{ad+cb}{bd}$ the given Fraction being reduced to a common Denominator by the ſecond Rule of the ſixth Chapter, are $\frac{ad}{bd}$ and $\frac{cb}{bd}$ whoſe ſum is $\frac{ad+cb}{cd}$

Example 5. What is the ſum of $\frac{a}{b}$ $\frac{b}{c}$ and $\frac{3fg}{d}$? Facit $\frac{acd+bbd+3fgbc}{bcd}$

II. When

II. When it is required to gather mixed quantities into one ſum, then add the fractional parts together by the foregoing Rule, and likewiſe bring the Integers into one ſum, and the ſum of theſe two ſums will be the ſum required.

Example.

What is the ſum of $bb + \frac{ab}{c}$ and $cd + \frac{a+b}{d}$?

The Sum of the Fractions being added by the foregoing Rule is $\frac{abd + ca + cb}{cd}$

To which ſum if you add the Integral parts of the propounded mixed Quantities, the ſum required will be $bb + cd + \frac{abd + ca + cb}{cd}$

Subtraction of Algebraical Fractions.

III. If the 2 given Fractions have not a common Denominator, then reduce them to ſuch by the ſecond Rule of the ſixth Chapter, then (by the Rule of the third Chapter) ſubtract the Numerator of the Fraction to be ſubtracted from the Numerator of the other Fraction, and place the remainder for a Numerator over the common Denominator, which new Fraction ſhall be the remainder ſought. As in the following Examples.

If you would ſubtract $\frac{ab}{c}$ from $\frac{bc}{c}$, Take the Numerator ab from the Numerator bc, and the remainder is $bc - ab$ which being placed over the Denomina-

Denominator e, it will give $\frac{bc--ab}{e}$ for the remainder, or difference ſought.

Alſo let it be required to ſubtract. $\frac{ab+c--18}{b+c}$ from $\frac{bc+5c--24}{b+c}$

The Difference of the Numerators of the given Fractions is $\left.\right\}$ $bc+5c-24-ab-c+18 = bc+4c-6-ab$

Which remainder or difference being made a Numerator to the common Denominator will give the difference ſought, which is $\left.\right\}$ $\frac{bc+4c--6--ab}{b+c}$.

And if it were required to ſubtract $a+\frac{cd-bc}{c}$ from $aa+\frac{ab}{c}$ The given mixt quantities will (by the fifth Rule of the ſixth Chapter) be reduced to $\frac{caa+ab}{c}$ and $\frac{ae+cd--be}{e}$

Which will be reduced to theſe Fractions of the ſame value, having a common Denominator, *viz.* $\left.\right\}$ $\frac{caae+abe}{ce}$ and $\frac{cae+ccd...ccb}{ce}$

And if from the Numerator $caae+abe$ you ſubtract the Numerator $cae+ccd-ccb$ there will be given for the remainder ſought, *viz.* $\left.\right\}$ $\frac{caae+abe--cae--ccd-+ccb}{ce}$

In

In like manner if from a it be required to ſubtract $\frac{a+c}{b-c}$ First by the fourth Rule of the ſixth Chapter, reduce the quantity a to the improper Fractional *Quantity* $\frac{ab-ac}{b-c}$ and therefrom ſubtract the given Fraction $\frac{a+c}{b-c}$ ſo will you have the remainder ſought which is $\frac{ab-ac-a-c}{b-c}$

CHAP. VIII.

Multiplication and Diviſion of Algebraical Fractions.

I. WHen it is required to multiply two Algebraical Fractions the one by the other, the work is the ſame as in Vulgar Fractions, for if you multiply the Numerators of the given Fractions together, and likewiſe their Denominators together, and place their Reſpective Products for a new Numerator, and a new Denominator, that new Fraction ſhall be the Product required.

Example.

Example 1. What is the Product of $\frac{9a}{b}$ multiplyed by $\frac{ab}{c}$ Facit $\frac{9aab}{cb}$ for $9a \times ab = 9aab$, which is the Numerator, and $c \times b = cb$ the Denominator.

Example 2. What is the Product of $\frac{3a + 2c}{3cb}$ multiplied by $\frac{4cd}{b + c}$ Facit $\frac{12cda + 8ccd}{2cbb + 2ccb}$; for $4cd \times \overline{3a + 2c} = 12cda + 8ccd$, which is the Numerator.

Example 3. What is the Product of $a + \frac{bc}{d}$ multiplyed by $8c + \frac{ad}{b}$? Facit $\frac{8dacb + 8ccbb + aadd + bdc}{db}$ for $a + \frac{bc}{d} = \frac{da + bc}{d}$ and $8c + \frac{ad}{b} = \frac{8cb + ad}{b}$ by the Rule of the sixth Chapter, and $\frac{da + bc}{d} \times \frac{8cb + d}{b} = \frac{8dacb + 8ccbb + aadd + bdc}{db}$ which is the Product required.

Example 4. What is the product of ab multiplyed by $\frac{a + b}{c}$? Facit $\frac{aab + abb}{c}$; for $ab = \frac{ab}{1}$ and $\frac{ab}{1} \times \frac{a + b}{c} = \frac{aab + abb}{c}$ which is the Product required.

II. If it so chance that you have a Fraction to be multiplyed by an Integer that is equal to the Denominator of the Fraction, then take the Numerator for the Product.

Example. What is the Product of $\frac{aa - 2ae + ee}{a - e}$ being multiplied by $a - e$? Facit $aa - 2ae + ee$.

The reaſon of which is plain, for the Numerator being multiplyed by the Integer, and the ſame Integer being put as a Denominator to the Product, the Quotient ariſing by the Diviſion of the Numerator by the ſaid Denominator, will be equal to the Numerator of the given Fraction; ſo $\frac{a}{b} \times b = \frac{ab}{b} = a.$

III. When it is required to divide one Algebraical Fraction by another, if they have a common Denomitator, cancel the Denominator, and divide the Numerator of the Dividend by that of the Diviſor, ſo ſhall that Quote be the Quotient ſought.

So if it were required to divide $\frac{abc}{d}$ by $\frac{bcc}{d}$ the Quotient will be found to be $\frac{abc}{bcc} = \frac{a}{c}$, for having caſt away the common Denominator d, and divided (abc) the Numerator of the Dividend by (bcc) the numerator of the Diviſor, the Quotient will be $\frac{abc}{bcc}$ which is $\frac{a}{c}$ by cancelling bc in the Numerator and Denominator.

IV. When the given Algebraical Fractions have not a common denominator, then multiply the denominator of the diviſor into the Numerator of the dividend, and the product is a new numerator; alſo multiply the numerator of the diviſor into the denominator of the dividend, and the product is a new denominator; which new Fraction is the quotient ſought, and this is a general Rule in all caſes whatſoever, and is the ſame with diviſion in Vulgar Fractions, only keeping to the Algebraick Rules. *Example*

Example. What is the quotient of $\frac{a}{b}$ being divided by $\frac{a}{b}$; Facit $\frac{ca}{ba}=\frac{c}{b}$, for $c\times a=$, (the new Numerator,) and $a\times b=ba$ (the new Denominator.

Likewise, if it were required to divide $\frac{2a+b}{c}$ by $\frac{2bb}{a+b}$ the Quotient would be $\frac{3aa+4ab+bb}{2bbc.}$ for the numerator of the dividend is $3a+b$, and the denominator of the divisor is $a+b$, and $\overline{a+b}\times\overline{3a+c}=3aa+4ab+bb$ which is the Numerator; and the numerator of the divisor is $2bb$, and the denominator of the dividend is c, and $2bb\times c=2bbc$, which is the Denominator. The like is to be observed in all cases both in Multiplication, and Division of Algebraical Fractions.

CHAP. IX.

The Rule of Three in Algebraick Quantities.

I. THE Rule of Three in Algebraical quantities represented by Letters, (whether it be Direct or Inverse) differs not from the Rule of Three in Vulgar Arithmetick, Respect being had to the Rules of Algebraical Multiplication and Division, before delivered in this Book, for (in a direct proportion) if you multiply the second term by the third, and divide the Product thereof by the first the quotient will be the fourth quantity sought in proportion.

Example 1. If b gives c, what will d give? Facit $\frac{cd}{b}$.

In this Example the second and third quantities, are c and d, which being multiplyed together, produce cd by the Third Rule of the fourth Chapter, which being divided by (b) the first quantity, the quotient is $\frac{cd}{b}$ which is the fourth proportion sought for.

$$b : c :: d \frac{cd}{b}$$

Which may be proved according to the proof of the Rule of Three Direct laid down in the Tenth Chapter of my Vulgar Arithmetick: For,

The *Product* of the second and third Terms is cd. And

The *Product* of the first and fourth Terms is $\frac{bcd}{b}$

And by the 3d Rule of the 6th Chap. $\frac{bcd}{b} = cd$ (the first Term) which was to be proved.

Example

Example 2. If $b+c$ require d, what will $d+e$ require? Facit $\frac{dd+de}{b+c}$. For,

$$b+c \quad : \quad d \quad :: \quad d+e \quad : \quad \frac{dd+de}{b+c}$$

Example 3. If 12 require 36, what will $4ab$ require? Facit $\frac{144ab}{12} = 12ab$. For,

$$12 \quad : \quad 36 \quad :: \quad 4ab \quad : \quad \frac{144ab}{12} = 12ab.$$

II. Nor will the operation be different from the former, if any of the 3 given quantities be a *Fraction*, or if they be all *Fractions*, obſerving the Rules of multiplication and diviſion in Algebraick quantities; or when any of the given Terms is a mixt quantity, let it be reduced to the form of a *Fraction*, by multiplying the Integral part by the denominator, and joyning the *Product* to the *Numerator* of the *Fractional* part, and then multiply and divide as before.

Example 4. If $b+\frac{c}{d}$ require d, what will $\frac{f}{g}$ require? Facit $\frac{ddf}{dbg+gc}$? For if you firſt reduce $b+\frac{c}{d}$ to the form of a *Fraction*, it will be $\frac{db+c.}{d}$ and the ſecond Term d being ſet *Fraction*-wiſe, will be $\frac{d}{1}$ then if you multiply $(\frac{f}{g})$ the third

Term by $(\frac{d}{1})$ the ſecond Term, the *Product* will be $\frac{df}{g}$, which being divided by $(\frac{db+c}{d})$ the firſt Term, the Quotient will be $\frac{ddf}{dbg+gc}$, which is the fourth proportional ſaught. For,

$$\frac{db+c}{d} : \frac{d}{1} :: \frac{f}{g} : \frac{ddf}{dbg+gc}.$$

I ſhall not need here to give any Examples in the Inverſe Rule of proportion in the Algebraick quantities, the manner of the operation being the ſame with the former, only the proportion flows backward, as in the Rule of Three Inverſe in Vulgar Arithmetick.

CHAP.

CHAP. X.

A Collection of ſome eaſy Queſtions wherein the Rules hitherto delivered are Exerciſed, taken out of Mr. *Oughtred's Clavis Mathematica, Chap.* 11. Sir *Jonas More's* Arithmetick in Spices, *Chap.* 10, and Mr. *Kerſey's* Elements of Algebra, *Chap.* 10. of the Firſt Book.

I. THere are two Quantities or numbers, whereof the greater is *a* (=4) and the leſſer is *e* (=2) What is their ſum? What their difference? What the product of their multiplication? What the Quotient of the greater divided by the leſſer? What the Quotient of the leſſer divided by the greater? What the ſum of their Squares? What the difference of their Squares? What is the ſum of their ſum, and difference? What is the difference of their ſum and differences? What is the Product of their ſum and difference? What the Square of their ſum? What

What the Square of their difference? What the Square of their Product?

1. The ſum of the quantities propoſed is	$a+e$
2. Their difference is	$a-e$
3. Their Product by multiplication	ae
4. The quote of the greater divided by the leſſer,	$\frac{a}{e}$
5. The quote of the leſſer by the greater	$\frac{e}{a}$
6. The Sum of their Squares	$aa+ee$
7. The difference of their Squares	$aa-ee$
8. The ſum of their ſum and difference	$2a$
9. The difference of their ſum and difference	$2e$
10. The Product of their ſum and difference	$aa-ee$
11. The Square of their ſum.	$aa+2ae+ee$
12. The Square of the difference	$aa-2ae+ee$
13. The Square of their product	$aaee$

II. There are two quantities whoſe ſum is b ($=12$) and the greater of them is a ($=8$) I demand what is the leſſer? What their difference? What is the product of their multiplication? What is the ſum of their Squares? What the difference of their Squares?

1. The leſſer is	$b-a$
2. Their difference is	$2a-b$
3. The Product is	$ab-aa$

4. The

4. The ſum of their Squares is	$2aa - 2ba + bb$
5. The difference of their Squares is	$2ba - bb$

III. There are two Quantities or Numbers whoſe difference is d, ($=4$) and the greater of them is a ($=8$) I demand what is the leſſer? What is their ſum? What their Rectangle or Product? What the ſum of their Squares? What the difference of their Squares?

1. The difference, or exceſs being ſubtracted from the greater, gives the leſſer.	$a - d$
2. Their ſum is	$2a - d$
3. Their Product or Rectangle is	$aa - da$
4. The ſum of their Squares is	$2aa - 2ad + dd$
5. The difference of their Squares is	$2ad - dd$

IV. There are two Numbers, Magnitudes, or Quantities, whereof the Ratio of the greater to the leſſer is as r to s, (or as 3 to 2) and the greater of them is a ($=12$.) I demand what is the leſſer? What is their Sum? What their difference? What their Rectangle, or Product? What the ſum of their Squares? And what the difference of their Squares?

1. The leſſer is by the Rule of 3.	$\frac{sa}{r}$
2. Their ſum is	$a + \frac{sa}{r}$
3. Their difference is	$a - \frac{sa}{r}$

4. Their

4 Their Rectangle or Product is	$\frac{saa}{r}$
5 The sum of their Squares is	$aa + \frac{ssaa}{rr}$
6 The difference of their Squares is	$aa - \frac{ssaa}{rr}$

But if the Ratio between the lesser and the greater had been given as s to r, (or as 2 to 3) and the lesser had been given e (=8) then,

1 The greater by the Rule of 3 would be	$\frac{re}{s}$
2 Their sum	$\frac{re}{s} + e$
3 Their difference	$\frac{re}{s} - e$
4 Their Rectangle, or Product	$\frac{ree}{s}$
5 The sum of their Squares	$\frac{rree}{ss} + ee$
6 The difference of their Squares,	$\frac{rree}{ss} - ee$

V. There are two numbers or Quantities whereof the Rectangle or Product is b (=96) and the greater quantity is a (=12) What is the lesser? What their sum? What their difference? What the sum of their Squares? And what the difference of their Squares?

1 The Product given being dividend by (a) the lesser quantity is	$\frac{b}{a}$

2 Their

2 Their ſum	$a + \frac{b}{a}$
3 Their difference	$a - \frac{b}{a}$
4 The ſum of their Squares	$aa + \frac{bb}{aa}$
5 The difference of their Squares.	$aa - \frac{bb}{aa}$

But if the Rectangle had been given b, as before, and the leſſer quantity had been given e (=8) Then

1 The greater would have been found by Diviſion to be	$\frac{b}{e}$
2 Their Sum	$\frac{b}{e} + e$
3 Their difference	$\frac{b}{e} - e$
4 The Sum of their Squares.	$\frac{bb}{ee} + ee$
5 The difference of their Squares.	$\frac{bb}{ee} - ee$

CHAP. XI.

Reduction of Equations.

I. AN Equation is an equality between two quantities of different names, whether the

the comparison of Equality be between Simple, or Compound Quantities, or both; between which two Quantities there is always this Character, *viz.* =.

So in this following Equation, *viz.* $a = 3c$, a is said to be the first part, and $3c$ the second part of the Equation, and signifieth that some Number or Quantity represented by a is equal, is three times another Number or Quantity represented by c.

So $a = b + c$ signifieth that some quantity represented by a is equal to the sum of two other Numbers or quantities represented by b and c.

The manner of composing an Equation will be understood by solving of the several questions contained in this and other following Chap. But when known, are mingled with unknown quantities, in an equation they must be so separated or reduced that the unknown quantity or quantities may remain intire on the one side, or part, and the known or given quantities on the other side or part of the Equation, which to perform is the work of Reduction, and which is contained in the several following Rules of this Chapter.

Here note, that the Quantity unknown or sought in every Equation is repesented by the Letter a, or some other Vowel, and the quantity or quantities known or given are represented by Consonants, as b, c, d, f, &c.

Reduction of Addition.

II. If equal numbers or quantities be added to equal numbers or quantities, the sums or totals will be equal, and therefore.

If

If it be granted that	$a - 8 = 20$
Then by adding + 8 to each part of the Equation there ariseth	$a - 8 + 8 = 20 + 8$
Then because in the first part of the Equation there is + 8 and − 8, they destroy each other by the Third Rule of the Second Chap. and it followeth that.	$a = 28$
Again let this Equation be proposed to be reduced, *viz.*	$a - b = d + b$
Then by adding *b* to each part of the Equation, there ariseth	$a - b + b = d + b + b$
And because − *b* and + *b* are in the first part of the equation, they destroy each other, and the Equation is	$a = d + 2b$
Likewise if	$aa - b - c = ff$
Then by adding *b* + *c* to each part of the equation there ariseth	$aa = ff + b + c$

Now from a due consideration of the premises it followeth that if in an Equation there be any Number or Quantity proposed with the sign -- before it, then if it be transferred to the other side of the equation, and cancelled on the side or part, where it now standeth, the effect will be the same as the adding of that Quantity to each part of the Equation, and this

this by Artists is called Transposition. As in the first of the foregoing Examples, where it is granted

That $a-8=20$

And by transposing -8 on the other side of the Equation, making it there $+8$ it giveth $a=28$

And in the second Example where $a-b=d+b$

By transposing $-b$, cancelling it on the first side of the equation, and making it $+b$ on the other, it is $a=d+2b$

And let it be granted that $a-bb-d=cc$

Then by transposing of $-bb$ and $-d$ there ariseth $a=cc+bb+d$

Reduction by Subtraction.

III. If in any Equation there be any number or quantity signed with $+$ (on which side of the equation soever) if it be cancelled on that side, and placed on the other side with the sign - perfixed to it, the work of Reduction is truly performed, and this is also called Transposition, and is only the converse of the foregoing Rule. Examples.

Let it be granted that $a+8=36$

Then if $+8$ be cancelled, and placed on the other part of the equation with the sign $-$ it will give $a=36-8$

Which equation being contracted is $a=28$

Again,

Again let be given $2aa+b=aa+cc$

By the Transposition of $+b$ on the first side the Equation it is $2aa=aa+cc-b$

And by Transposition of aa on the second side of the equation it is $aa=ce-b$

Also if $aa+b+c=ba+dd$

By Transposition of $b+c$ to the second side of the equation it is $aa=ba+dd-b-c$

And by the Transposition of ba to the first side of the equation it is $aa-ba=dd-b-c$

Which method (in reducing of the premised Equation) is deduced from this general Axiom, *viz.*

If from equal Numbers or Quantites, equal Numbers or Quantities are subtracted, the remainder shall be equal.

So in the second Example there is given this equation, *viz.* $2aa+b=aa+cc$

First by subtracting b from each part of the equation, there is $2aa=aa+cc-b$

Then I subtract aa from each part, and there remaineth $aa=cc-b$

Reduction by Multiplication.

IV. When in an Equation one or both parts are Fractions, then let them be reduced to a common denominator by the, 2d, 4th, and 5th Rules of of the ſixth Chapter, and then caſting away the Denominator, uſe only the Numerators, ſo ſhall Equations expreſt by Algebraical Fractions be reduced to other Equations, conſiſting altogether of Integers. As in the following Examples.

If	$\frac{a}{8}=9$
Then by reduciug 9 in the ſecond part of the Equation to a Fraction, having 8 for its Denominator, it is	$\frac{a}{8}=\frac{72}{8}$
And by caſting away the Denominator which is common to both, it is	$a=72$
Again, if	$a=\frac{bcd}{a+b}$
Then by reducing a; on the firſt ſide of the Equation to a Fraction, having $a+b$ for its Denominator, it is	$\frac{aa+ba}{a+b}=\frac{bcd}{a+b}$
And by caſting away the common Denominator $a+b$ the Equation is	$aa+ba=bcd$
Likewiſe, if	$\frac{ab}{c}=\frac{dc}{b}$
The quantities being reduced to a common Denominator, are	$\frac{abb}{cb}=\frac{dcc}{cb}$
And the common Denominator cb being caſt away, the Equation is	$abb=dcc$

V. When

V. When either part of an Equation is Compoſed of a mixed Quantity or Quantities, let the Integral part or parts be reduced to a Fraction or Fractions, and then proceed as in the laſt Example.

It is granted that $b+c+\frac{a}{b}=cd+\frac{bc}{a}$

Firſt, it is reduced to $\frac{bb+bc+a}{b}=\frac{cda+bc}{a}$

Which Fractional Equation being reduced according to the foregoing Rule, is $bba+bca+aa=bcad+bbc$

VI. When ſome power or degree of the number or quantity ſought is multiplyed into each part, and each member of an Equation, then let that degree or power be cancelled in each part and member, ſo will it quite vaniſh, and the Equation will be reduced to more Simple Terms. As for Example.

Let it be granted that $aa+ba=ca$

Foraſmuch as a is a Factor in each part and member of the equation, therefore it being expunged in each, there ariſeth this equation $a+b=c$

VII. When (according to the ſecond, third, forth, and fifth Rules) an Equation is reduced, and that ſome known Number or Quantity is multiplyed into the quantity ſought, then divide each part of the Equation by that known Quantity, to the end that the quantity ſought may have no quantity multiplyed into it but 1 (or unity.) As in Example,

If it be granted that $ba=cd$

Then because the Quantity sought is (a) mulipyed by b, divide each part of the equation by b, and there ariseth $a=\frac{cd}{b}$

VIII. When any one part of an Equation is composed of a surd quantity, (*viz.* such as hath the radical sign, prefixed to it) and the other part is a rational quantity: then let that rational quantity be raised to the power signified by the Radical sign, and then cast away the said radical sign, so shall both parts of the Equation be a rational quantity. As,

If it be proposed that $\sqrt{a}=8$

Square 8 and place its Square in the room of it self, casting away the radical sign from the first part of the Equation, and then it will be $a=64$

Likewise if $\sqrt{a}=cd$

Then by raising the second part of the Equation to its Square, and casting away the radical sign from the first part, there ariseth this Equation, *viz.* $a=ccdd$

Again,

Again, if $\sqrt{a}=b+c$

The ſecond part of the Equation being ſquared, and the radical ſign cancelled in the firſt, there ariſeth $a=bb+2bc+cc$

Reduction by Diviſion.

IX. If equal Quantities be divided by equal Quantities, the Quotients thence ariſing will be equal. For,

If $aa=10a$

Then by dividing each part of the Equation by a, there ariſeth this Equation. $a=10$

And if $aaa=bba+daa$

Then by dividing each part of the Equation by a, there ariſeth $aa=bb+da$

And da in the ſecond part of the Equation being tranſpoſed by the third Rule of this Chapter, there ariſeth this Equation, *viz.* $aa-da=bb$

And if $ba-ca=dd$

Then by dividing each part of the Equation by $b-c$, it is $a=\frac{dd}{b-c}$

CHAP. XII.

To Convert Analogies into Equations, and Equations into Analogies.

I. THIS is deduced from this universal Theorem, *viz.* That if four quantities are Proportionals, the product of the two Means is equal to the product of the two Extreams; and if three numbers are Proportionals, the product of the two Extreams is equal to the Square of the Means.

1. Let there be proposed these four Porportionals. $a : b :: c : d$

Then by the said Theorem this Equation will follow, *viz.* $ad = bc$

2. Let there be proposed these three continual Proportionals, *viz.* $a, b, c \div\!\!\div$

That is to say $a : b :: b : c$

Whence there followeth this Equation, *viz.* $ac = bb$

II. From a due consideration of the Premises it is evident that Equations may oftentimes be resolved into Proportionals, *viz.* when the Product of two quantities is found equal to the product

duct of two other quantities: Then as any one of the Factors in the first side of the Equation is to any one of the Factors in the second part of the Equation, so is the remaining Factor of the second part, to the remaining Factor in the first part: And the Converse,

Suppose that	$bc = ad$
From thence may be drawn this Analogy,	$b : a :: d : c$

The truth of which may be proved by the first Rule of this Chapter, for thereby the said Analogy may be reduced to the given Equation, *viz.* $bc = ad$.

Again if	$3ba = 3dc$
Then from thence may be deduced this Analogy, *viz.*	$3b : c :: 3d : a$
or	$3b : 3c :: d : a$
or	$b : 3c :: d : 3a$

Likewise if	$bc = ca + da$
Then may that Equation be resolved into these Proportionals,	$c + d : b :: c : a$

And if	$dd = 6ba$
Then it will be found that	$6b : d :: d : a$

III. When it happens that there is an Equation between an Algebraical Fraction, and an Integer, if the Numerator of the said Fraction can be resolved into two such quantities, as being multiplyed the one by the other, will produce the said Numerator, then will the said Equation produce this proportion, *viz.*

As the Denominator of the Fraction is to one of the Factors of which the Numerator is produced, so is the other Factor to the Integer, unto which the said Fraction it equal. Examples.

If it be granted, that $\frac{bc}{d} = a$

Then may that Equation be resolved into this Analogy. } $d : c :: b : a$

For, $d : c :: b : \frac{bc}{d}$ (a)

Again, if $\frac{cd}{b+d} = a$

Then may that Equation be resolved into this Analogy, } $b + d : c :: d : a$

And also if $dd = \frac{ab + ad}{cc}$

Then may the said Equation be resolved into this Analogy, *viz.* } $cc : b + d :: a : dd$

The Practice of the two last Rules will be plainly discovered in the next Chapter (in the resolution of Questions producing simple Equations) to be of most excellent use in discovering or laying down of Theorems for the ready solution of the Question proposed, or any other of the same nature, which Theorems are to be kept reserved in store for the finding out of new, and the confirmation of old Truths.

CHAP.

CHAP. XIII.

The Resolution of Arithmetical Questions (Algebraically) which produce Simple Equations

I. AN Equation is two-fold, *viz.* First, Simple, and secondly, Adfected or Compounded.

II. A Simple Equation is when the Quantity sought (solely possessing one part of the Equation) is either expressed by a Single or Simple Root, as *a*, or by a Single or Simple Power as *aa*, or *aaa*, *&c.* as in these Equations, *viz.* $a=32$, and $ac=64$, or $4aaa=256$, and such like.

III. When a Question is propounded, and to be resolved Algebraically, then for the Answer put *a*, and for each of the given Numbers put Consonants, then proceed according to the Tenure of the Question, by Addition, Subtraction, Multiplication, or Division, until an Equation is Composed; and when the Equation is composed, then proceed to reduce it (according to the Rules contained in the Eleventh Chapter) until the Quantity unknown (being *a* or some power of *a*) do solely possess one part of the

the Equation, and the known or given quantities the other part, and then will the quantity sought be also known.

IV. I shall in the Resolution of every *Question* proceed (*gradatim*) step by step, according to the method used by Mr. *Kersey*, each step being numbred orderly in the margent, from the beginning to the end, by 1, 2, 3, 4, *&c.* And I shall only proceed in the operation literally, because otherwise this Treatise would swell to a bigger Volume than is at present intended; but I shall give the Learner a taste of Numeral Algebra, in the solution of two or three of the first *Questions* thereby.

Quest. 1. There are two Numbers whose sum is 48 (or b) and the excess of the greater above the lesser is 14 (or c) I demand what are the Numbers.

The Solution literally.

1. For the greater number put	a
2. From which if you subtract the difference (c) you will have the lesser, which is	$a-c$
3. The greater and lesser being added together, will be equal to (b) the sum whence this Equation	$2a-c=b$
4. And by the Transposition of $-c$ the Equation is	$2a=b+c$
5. Then dividing each part of the Equation by 2, it is	$a=\frac{b+c}{2}$

6. And

6. And if from $\frac{b+c}{2}$ you subtract (c) the excess of the greater above the lesser, the lesser will be	$\frac{b+c}{2}-c$

So that the Numbers sought are 31 and 17, for by the fifth step (a) the greater is found to be $=$ to $\frac{b+c}{2}$ and b is given 48, and c is given 14, the sum of which is 62, which divided by 2, gives 31, for the greater, and by the sixth step, if from the greater you subtract the difference (c) the remainder will give the lesser, which is 17, for $\frac{b+c}{2}-c=17$.

Now if the fifth and sixth steps are duly considered, they will present you with this

Theorem,

The sum of the sum and difference of any two Numbers being divided by 2, will give the greater Number; and the difference of any two Numbers being subtracted from half the sum of the sum and difference, the remainder will give the lesser number.

The Solution Numerally.

1. For the greater number put	a
2. From which if you subtract the difference (14) the lesser is	$a-14$

3. Which

3. Which added together, will be the ſum, whence this Equation. } $2a - 14 = 48$

4. And by tranſpoſition of -14 it will be } $2a = 62$

5. And both parts of the Equation being divided by 2, will give the value of (a) the greater. } $a = 31$

6. From which if you ſubtract (14) the Difference, the remainder will give the leſſer by the ſecond ſtep. } 17

So that the Numbers ſought are 31 and 17, which will ſatisfie the conditions of the Queſtion.

Queſtion 2.

There are two Numbers whoſe Sum is 56 (or b) and the leſſer hath ſuch proportion to the greater, as 2 to 5, (or c to d) I demand what are the Numbers?

1. For the leſſer number put a

2. Then by the Rule of Three find the greater, *viz.* $c : d :: a : \frac{da}{c}$ } $\frac{da}{c}$

3. Wherefore the ſum of the two numbers ſought is } $a + \frac{da}{c}$

4 Which ſum muſt be equal to the given ſum, whence this Equation. } $a + \frac{da}{c} = b$

5. Which

5. Which Equation being reduced by the fourth and fifth Rules of the eleventh Chap. the value of *a* will be found to be	$a = \frac{cb}{c+d}$
6. And by the firſt, ſecond, and fifth ſteps the greater number will be diſcovered to be	$\frac{db}{c+d}$

So that the numbers ſought are 40 and 16, for (*a*) the leſſer is found to be by the fifth ſtep $\frac{cb}{c+d}$ *viz.* the Product of (*cb*) 56 by 2 divided by ($c+d$) the ſum of 2 and 5, *viz.* 7, which is 16, *&c.*

And if (acording to the third Rule of the twelfth Chap.) the two laſt ſteps be turned into proportionals, it will give this

Theorem.

As the ſum of the Terms which repreſent the Ratio of two Numbers, is to the ſum of the numbers themſelves, ſo is the leſſer term to the leſſer number; and ſo is the greater Term to the greater Number.

Therefore if the ſum of two Numbers is given, and alſo their Ratio, the Nnmbers themſelves are alſo given by this *Theorem.*

The

The same Question solved Numerically.

1. For the lesser number put	a
2. Then by the Rule of Three the greater number is found viz. 2 : 5 : : a : $\frac{5a}{2}$	$\frac{5a}{2}$
3. Then will their sum be	$a+\frac{5a}{2}$
4. And according to the tenure of the Question, their sum must be equal to the given sum, whence this equation	$a+\frac{5a}{2}=56$
5. And that Equation being reduced by the fifth and sixth Rules of the eleventh Chap. the value of a will be found to be	$\frac{112}{7}=16$
6. Which being subtracted from the given sum, the greater number is	40

Quest. 3.

A Gentleman asked his Friend (that had four Purses in his hand) what Money he had in each Purse? To whom he answered, that he knew not, but (quoth he) this I know, that in the second Purse there are 8 or (b) Crowns more than in the first or least Purse, and in the third 8 Crowns more than the second, and in the fourth or biggest Purse there are 8 Crowns more than in the third, and twice as many as in the first or least, I demand what number of Crowns he had in each Purse?

1. For

1. For the number of Crowns in the firſt Purſe put	a
2. Then in the ſecond there is	$a+b$
3. And in the third there is	$a+2b$
4. And in the fourth	$a+3b$
5. Which according to the tenure of the Queſtion is double to that in the firſt, whence this Equation	$a+3b=2a$
6. Then by the tranſpoſition of a from the firſt ſide of the Equation, it is	$3b=a$

which diſcovereth the value of a to be $3b$, or 3 times 8, which is 24, *&c.* which is the number of Crowns in the firſt Purſe, and conſequently the number of crowns in each Purſe, is 24, 32, 40, and 48, which will ſatisfie the conditions of the Queſtion.

The ſame Queſtion ſolved Numerically.

1. For the Crowns in the firſt purſe put	a
2. Then in the ſecond there is	$a+8$
3. And in the third	$a+16$
4. And in the fourth	$a+24$
5. Which is double to the number of Crowns in the firſt, whence this equation,	$a+24=2a$
6. Which Equation being reduced by the tranſpoſition of a, diſcovers the value of a, *viz.*	$a=24$

Queſt

Quest. 4.

Three men build a Ship which cost them 2700 *l.* (or b) Pounds, of which *B* must pay double to what *A* must pay, and *C* must pay three times as much as *B*, I demand the share that each must pay.

1. For the sum to be paid by *A*, put	a
2. Then *B* must pay	$2a$
3. And *C* must pay	$6a$
4. The sum of these three quantities are equal to the total charge, whence this Equation	$9a = b$
5. Which being reduced, discovers the value of a, *viz.*	$a = \frac{b}{9}$

which is the sum that *A* must pay, *viz.* 300 *l.* Therefore *B* must pay $\frac{2b}{9} = 600$ *l.* which is twice as much as *A*, and *C* must pay $\frac{6b}{9} = 1800$ *l.* which is three times as much as *B*.

Quest. 5.

There is a Fish whose head is supposed to be 9 (or b) inches, and his Tail is as long as his Head and half his Body, and his Body is as long as his Head and his Tail; I demand the length of such a Fish?

1. For the length of the Body put	a
2. Then will the Tail be	$\frac{a}{2} + b$

3. Then if to the Tail you add the length of the Head, *viz.* b, the ſum is	$\frac{a}{2}+2b$
4. Which according to the tenure of the Queſtion is equal to the length of the Body, whence this Equation	$a=\frac{a}{2}+2b$
5. And the ſecond part of the Equation being clear'd of the unknown quantity a by Reduction, gives the value of a the length of the Body, *viz.*	$a=4b$
6. Then according to the Tenure of the Queſtion, if therefrom you ſubtract (b) the length of the head, the remainder will be the length of the Tail, which is	$3b$

By the fifth Step the length of the Body is found to be $4b=36$, and by the ſixth ſtep the length of the Tail is diſcovered to be $3b=3\times9=27$. So that the length of the head is (given) 9 inches, the length of the Tail 27 inches, and the length of the Body 36 inches, which numbers will ſatisfie the conditions of the Queſtion.

For, $36=27+9$ the Body,
And — $\frac{36}{2}+9=27$ the Tail.

So that the whole length of the Fiſh is $9+27+36=72$ Inches.

QUEST.

QUEST. 6.

A Father lying at the point of death, left to his three Sons *A*, *B*, and *C* all his Estate in Money, and divided it thus, *viz.* to *A* he gave $\frac{1}{2}$, wanting 44 (or b) pounds, and to *B* he gave $\frac{1}{3}$ and 14, (or c) pounds over, and to *C* he gave the rest, which was 82 (or d) pounds less than the share of *B*. Now I demand what was the Father's Estate?

1. For the Father's Estate put — a
2. Then will the share left to *A* be — $\frac{a}{2} - b$
3. And the share of B — $\frac{a}{3} + c$
4. And by the third step the share of C is — $\frac{a}{3} + c - d$
5. The Quantities in the three last steps being added together, give — $\frac{7a}{6} + 2c - b - d$
6. Which must be equal to the Father's Estate, whence this Equation. — $\frac{7a}{6} + 2c - b - d = a$

Which Equation after due reduction and transposition of Quantities, the value of a is discovered to be — $a = 6b + 6d - 12c$

And $6b = 6 \times 44 = 264$, and $6d = 6 \times 82 = 492$, and $12c = 12 \times 14 = 168$, now $264 + 492 - 168 = 588$, so that the Father's Estate was 588 pounds, of which *A* had 250 *l.* *B* 210 *l.* and *C* 128, which Numbers do answer the conditions of the Question.

Quest.

Queſt. 7.

Two perſons thus diſcourſed together concerning their Money, quoth *A* to *B*, give me 3 (or *b*) of your Crowns, and I ſhall have as many as you; nay quoth *B* to *A*, but if you will give me 3 of your Crowns, I ſhall have 5 times as many as you. Now I demand how many Crowns had each perſon?

1. For the number of Crowns which *A* had put	a
2. Then foraſmuch as adding 3 (or *b*) Crowns to *A* will be equal to the Crowns remaining to *B* after he had given 3 Crowns to *A* therefore *B* will then have left.	$a+b$
3. And conſequently if you add thereto the 3 (or *b*) Crowns which he gave to *A* the ſum will be the number of Crowns which *B* had at firſt, which is	$a+2b$
4. Then if from the number of Crowns *A* had at firſt (*a*) you ſubtract 3 (or *b*) crowns, there will remain to *A* $a-b$ crowns, and giving the ſame to *B* he will then have	$a+3b$
5. Which according to the tenure of the Queſtion is five times as much as what *A* had left, whence there ariſeth this Equation.	$5a-5b=a+3b$

 6. Which

6. Which equation being reduced by the second and seventh Rules of the eleventh Chapter, the value of a is discovered to be	$a = 2b$
7. And by the sixth and third steps the number of Crowns which B had at first are found to be	$a + 2b = 4b$

So that it is found that *A* had 6 Crowns, and *B* had 12 Crowns, which numbers will satisfie the conditions of the Question. For,

$$9 + 3 = 12 - 3 = 9$$

And,

$$12 + 3 = 5 \times 6 - 3 = 15$$

Quest. 8.

A Labourer had 576 (or b) pence for threshing 60 (or c) Quarters of Corn, *viz.* Wheat and Barly; for the Wheat he had 12 (or d) pence *per* Quarter, and for the Barly he had 6 (or f) pence *per* Quarter, I demand how many Quarters of each he threshed?

1. For the quarters of Wheat which he threshed put	a
2. Then the quarters of Barley will be	$c - a$
3. The quantity of Wheat in the first step being multiplyed by its price produceth	da

4. The

4. The quantity of Barly in the ſecond ſtep being multiplyed by its price, produceth	$fc-fa$
5. The ſum of the quantities in the two laſt ſteps muſt be equal to the given price of the 60 quarters, whence this equation	$da-fa+fc=b$
6. Which being reduced by the ſecond, third, and fifth Rules of the eleventh Cha. the quantity of Wheat will be diſcovered to be	$a=\frac{b-fc}{d-f}$
7. And by the ſecond and fifth ſteps the quantity of Barly is diſcovered to be	$c-\frac{b-fc}{d-f}$

So that the qnarters of Wheat which he threſhed were 36, and the quarters of Barly 24.

The Proof.

$12\times36=432$
And
$6\times24=144$
And
$432+144=576$, which was to be proved.

Queſt. 9.

A Gentleman bought a Cloak of a Saleſ-man, which coſt him 3 *l.*—10 *s.* or 70 (or *b*) ſhillings, and deſiring the Saleſman to tell him what he gained thereby, he said he gained ¼ (or c) of what it cost him, the question is what the Cloak cost the first penny?

1. Suppose the Cloak cost — a
2. Then he gained — ca
3. The first and second steps being added together, their sum will be equal to the sum which the Gentleman gave for it, whence this equation — $ca+a=b$
4. Which Equation being reduced by the ninth Rule of the eleventh Chap. the value of a will be discovered to be — $a=\frac{b}{b+1}$

So that it cost 56 shillings, ¼ of which is 14 shillings, and $56+14=70$.

And if the quantity in the fourth step be duly considered, you will find that if the gain had been any other part or parts of the first cost, if the price it was sold for had been divided by the Fraction representing part of the gain, increased by 1, the quote would have been the answer.

Question 10.

A Gentleman hired a Labourer to work for him for 40 (or b) days, and made this agreement with him that for every day he wrought he should have 20 (or c) pence, and for every day that he played he should forfeit 8 (or d) pence, and at the end of the said 40 days he received

184 (or f) pence, which was his full due. Now I demand how may days he wrought, and how many days he played?

1. For the number of days he wrought, put	a
2. Then the number of days he played will be	$b-a$
3. And if the time he wrought (in the firſt ſtep) be multiplied by 20 (c) it will produce the total he gained by work, *viz.*	ca
4. And if the time he played (in the 2d ſtep) be drawn into 8 (d) the product will be what he loſt by play	$db-da$
5. And if the total loſs (in the fourth ſtep) be ſubtracted from the gain (in the third ſtep) the remainder will be what he received, whence this Equation	$ca+da-db=f$
6. Which being reduced by the ſecond and ninth Rules of the eleventh Chapter, it will diſcover the value of a to be eighteen which is the days that he wrought.	$a=\frac{f+db}{c+d}$
7. And from the ſixth and ſecond ſteps the number of days he played are diſcovered to be 22 days, *viz.*	$b-\frac{c+d}{f+d}$

So that by the ſixth ſtep it appears he wrought 18 days, and by the ſeventh ſtep it appears that he played 22 dayes.

The proof.

18×20=360 and
22× 8=176 and
360—176=184

Quest. 11.

A person (in the Afternoon) being asked what a Cloak it was, answered that $\frac{3}{5}$ (or b) parts of the time from Noon was equal to $\frac{5}{8}$ (or c) parts of the time remaining to midnight, now, (supposing the time from Noon to Midnight to be divided in 12 (or d) equal parts or hours) I demand what was the present hour of the day?

1. For the hour sought put	a
2. Then the time to midnight will be	$d-a$
3. Then will $\frac{3}{5}$ (or b) parts of the Hour from Noon be	ba
4. And $\frac{5}{8}$ (or c) parts of the time remaining till midnight will be	$cd-ca$
5. Therefore from the third and fourth steps there ariseth this equation.	*viz.* $ba=cd-ca$
6. Which equation being reduced according to the second and ninth Rule of the eleventh Chapter gives the value of a (to be 6 $\frac{48}{552}$ the hour sought) *viz.*	$a=\frac{cd}{b+c}$

So

So that the hour ſought was $6\frac{48}{392}$, and conſequently the time remaining till midnight was $5\frac{281}{392}$ hours, which two numbers will anſwer the conditions of the queſtion, for, $\frac{3}{5}$ parts of $6\frac{381}{392}$, which is $3\frac{33}{19}$ is equal to $\frac{5}{8}$ parts of $5\frac{381}{592}$, as you may prove at your leiſure.

Moreover, If the laſt ſtep be converted into proportionals by the third Rule of the twelfth Chap. it will give this

Theorem.

As the ſum of the parts of any two Numbers (wherein there is an equality) is to the ſum of thoſe Numbers, ſo is the given parts of any one of thoſe Numbers, to the other Number.

As ſuppoſe it were required to find out two Numbers, whoſe ſum is 27, and ſuch, that $\frac{3}{4}$ of the one may be equal to $\frac{3}{5}$ of the other, the ſame may be found out by the ſaid Theorem. For,

$$\frac{3}{4}+\frac{3}{5} : 27 :: \frac{3}{4} : 15$$

which number ſo found is the number ſought, whereof $\frac{3}{5}$ is to be taken; and the other is $27-15=12$, or it may be found by the following proportion, *viz.*

$$\frac{3}{4}+\frac{3}{5} : 27 :: \frac{3}{5} : 12$$

Queſt. 12.

One asked a Shepherd what was the price of his hundred Sheep, quoth he, I have not an hundred, but if I had as many more, and half as many more, and $7\frac{1}{2}$ (or b) sheep, then I should have just 100 (or c) I demand how many sheep he had?

1. For the number of sheep he had; put	a
2. Which being doubled is	$2a$
3. And if to the second step you add half the first, it is	$2a + \frac{a}{2}$
4. And if to the third step there be added $7\frac{1}{2}$ (or b) the sum is	$2a + \frac{a}{b} + b$
5. Which quantity in the fourth step is equal to 100 (or c) whence this equation	$2a + \frac{a}{2} + b = c$
6. Which equation being reduced by the 5th and 7th Rules of the 11th Chap. the value of a will be discovered to be 37, *viz.*	$a = \frac{2c - 2b}{5}$

So that the number of sheep he had were 37 for $37 + 37 + \frac{37}{2} + 7\frac{1}{2} = 100$.

CHAP.

CHAP. XIV.

How to Extract the Root of a Square formed from a Binomial, and how by having any two of the Members of such a Square given to find out the third.

I. A Binomial is a quantity consisting of two names or parts, as $a+b$, or $a-b$, $aa+ec$. $b+d$, *&c.* And when a Square is formed from such a Root, it will consist of three members or parts, *viz.* two Affirmative Squares of the parts of which the Binomial is composed, and the double Rectangle of those parts, which double Rectangle is sometimes affirmative, and sometives negative, *viz.* Affirmative, when the parts of the Binomial are both affirmative, or both negative, that is, when thy are both signed with $+$, or both with $-$; and negative, when one of the parts of the Binomial Root is signed with $+$, and the other with $-$.

So if $a+b$ were given for a Root, its Square would be $aa+2ab+bb$ which is composed of (aa and bb) the Squares of the parts of which the Root is composed, and of ($2ab$) the double

Pro-

Product, or Rectangle made by the multiplication of the ſaid parts (a and b) one by the other. See the work.

$$\begin{array}{l} a+b \text{ the Root} \\ a+b \\ \hline aa+ab \\ \quad ab+bb \\ \hline aa+2ab+bb \text{ the Square.} \end{array}$$

So if it were required to find the Square of the Binomial $a-b$, or $b-a$ it (being multiplyed by it ſelf would be $aa-2ab+bb$, which is compoſed of (aa and bb) the ſum of the Squares of the parts, and their double Rectangle, as before, but ($2ab$) the Double Rectangle of the parts is ſigned with—, ſo that the Squares of the difference of any two numbers or quantities is equal to the ſum of the Squares of the ſaid quantities or numbers made leſs by their double Rectangle. As by the work.

$$\begin{array}{l} a-b \text{ Root} \\ a-b \\ \hline aa-ba \\ -ba+bb \\ \hline aa-2ab+bb \text{ Square} \end{array} \qquad \begin{array}{l} b-a \text{ the Root} \\ b-a \\ \hline bb-ba \\ -ba+aa \\ \hline bb-2ba+aa \text{ the Square} \end{array}$$

So if the Number 10 were divided into 8 and 2, *viz.* $8+2$, its Square would be $64+32+4 = 10\times10 = 100$. And the Square of $8-2$ is $64-32+4 = 6\times6 = 36$ for $8-2 = 6$ and $6\times6 = 36$.

Note,

Note, That a Binomial Root having one of its parts ſigned with —, is by ſome Authors called a Reſidual Root, as $a-b$, and $c-d$, *&c.* are Reſiduals.

II. From what hath been ſaid concerning the Square of a Binomial, may be inferred this

Theorem.

If a Compound quantity conſiſting of 3 members, whereof two are Squares of different names, with the ſign + prefixed to them, and the third is the double Rectangle of the Roots of thoſe Squares, having alſo the ſign + prefixed to it, then ſhall the Sqnare Root of ſuch a compound quantity be the ſum of the Square Roots of the ſaid two ſimple Squares; but if the ſaid double Rectangle hath the ſign — prefixed to it, the Square Root of the ſaid Compound quantity, ſhall be the difference of the ſaid Roots.

So the Square Root of $aa+2ab+bb$ will be found to be $a+b$, for the Square Root of aa is a, and the Square Root of bb is b, which two Roots added together, give $a+b$.

Alſo the Square Root of $aa+8a+16$ will be found to be $a+4$, the 2 Squares in the given quantity are aa and 16, and $8a$ is the double product of (a and 4) the ſaid Roots being multiplyed the one by the other.

Likewiſe the Square Root of $aa-2ab+bb$ is $a-b$, or $b-a$, not $a+b$, becauſe the double Rectangle ($2ab$) is ſigned with —.

Furthermore the Square Root of $9aa+12ba+4bb$ is $3a+2b$: The two Square quantities in the ſaid Compound Square are $9aa$, and $4bb$, whoſe

whose Roots are $3a$ and $2b$, and $12ba$, is the double Product of $3a$ and $2b$ being multiplyed together.

And the Square Root of $aa - 20a + 100$ is $a - 10$, for the two Squares in this Compound Square Quantity are aa and 100, whose Square Roots are a and 10, and $20a$ is the double Rectangle of 10 and a, they being multiplyed together.

The foregoing Theorem being well understood will be of excellent use in the Resolution of Questions, producing Quadratick Equations, as you will find by the Questions contained in the next Chapter.

III. When it is required to extract the square Root of quantity whose Root cannot be exactly extracted, then prefix the radical sign to it, which shall represent its Square Root. So the Square Root of bc is $\sqrt{bc}$, or $\sqrt{2)}\, bc$, and the Square Root of $\overline{aa+cc}$ is thus represented, *viz.* $\sqrt{\overline{aa+bb}}$; or $\sqrt{(2)}\,\overline{aa+bb}$, &c.

IV. From a due consideration of the foregoing Theorem, a way is discovered how by having any two of the members of a Square formed from a binomial Root, the third member may be found out. For,

When two Affirmative Square Quantities are given for two of the members of a Square formed from a binomial Root, then take the Roots of those two Squares and multiply them the one by the other, and double the Product, so shall that Product being doubled be the third member, which being annexed to the two given Squares, either by $+$, or $-$, it will make an exact Compound Square, whose Root shall be a Binomial.

So

So if $aa + bb$ were given for two of the members of a Square, first, I find their Roots to be a and b, which being multiplyed the one by the other, produce ab, and that Product being doubled gives $2ab$, for the middle Term of the Compound Square Quantity to make it a compleat ſquare, the Root whereof is a Binomial, *viz.* $aa + 2ab + bb$, if the ſaid double product be joyned to the ſaid ſum of the Squares by the ſign —, it will give the Compound Square Quantity $aa - 2ab + bb$ whoſe Root is $a - b$.

Alſo if $25aa + 16bb$ were given for two of the members of a Square, whoſe Root is a Binomial. The ſaid Square being compleated, will be $25aa + 40ab + 16bb$, or $25aa - 40ab + 16bb$, whoſe Root is either $5a + 4b$, or $5a - 4b$.

V. When the two given members of a Compound Square Quantity, whoſe Root is a Binomial, are the double product or rectangle, and one of the two affirmative ſquares, divide half the ſaid double product by the Root of the given ſquare, and ſquare the Quotient, ſo ſhall that ſquare be the third member ſought, which being joyned to the two given Quantities with the ſign +, it will give you a compleat ſquare having for its root a Binomial.

As for Example. Let $aa + 2ba$ be propoſed for 2 of the members of a ſquare, whoſe Root is a Binomial: Firſt, I take half of ($2ba$ the ſaid double product and it is ba, which being divided by (a) the Root of aa) the given Square, the Quotient is b, whoſe ſquare is bb for the third member ſought.

Again, Let $25aa + 40a$ be the two propoſed terms of ſuch a ſquare, whoſe Root is a Binomial, and let it be required to find the other ſquare which ſhall make it a compleat ſquare, raiſed from a Binomial Root; in order to which, firſt, I take half ($40a$) the double product, *viz.* $20a$, and divide it by the Root of ($25aa$) the given ſquare, which is $5a$, and the Quotient is 4, which being ſquared, gives 16 for the third member required, which being joyned to the reſt, gives $25aa+40a+16$ for the ſquare compleated.

VI. When the two given members of a ſquare raiſed from a Binomial Root, are ſuch that one of them is a ſquare affirmative without any Number or Quantity prefixed to it, and the other is the Root of the ſaid ſquare multiplyed by ſome other Quantity, then is that other Quantity by Artiſts called the Coefficient, and if you ſquare half the ſaid coefficient, or, (which is all one) take $\frac{1}{4}$ of the ſquare of the coefficient, that ſhall be the third member required, which being joyned to the two given quantities by the ſign $+$, it will give you a compleat ſquare raiſed from a Binomial Root.

Example. Let the two given members of a ſquare be $aa+2ba$, and let it be required to find out the third member. Here the coefficient is $2b$, half of which is b, which being ſquared, gives bb for the third member which was ſought, ſo is the ſquare compleated $aa+2ab+bb$.

In like manner, if the two given members of a ſquare were $aa+ba$, and it were required to find out the third member.

Here the coefficient is b, half of which is $\frac{1}{2}b$, or $\frac{b}{2}$, whoſe ſquare is $\frac{1}{4}bb$, or $\frac{bb}{4}$ for the member

ber ſought. Alſo let the two given members of a ſquare be $aa+8a$, and let it be required to find out the third member. Here the coefficient is 8, half of which is 4, whoſe ſquare is 16, for the third member required, ſo is $aa+8a+16$, a compleat ſquare, whoſe Root is $a+4$.

Again, if the two given members of a ſquare be $aa-ca$, and the third is required; Firſt, I take half the coefficient c, *viz.* $\frac{1}{2}c$, and then ſquare it, and it gives $\frac{1}{4}$ or $\frac{cc}{4}$ for the member ſought, and ſo is the Square compleated $aa-ca+\frac{1}{4}cc$, whoſe Root is $a-\frac{1}{2}c$.

In like manner, if it were required to make $aa+3ba$ a compleat ſquare, take half the coefficient ($3b$) which is $\frac{3}{2}b$, or $\frac{3}{2}b$, whoſe ſquare is $\frac{9}{4}bb$, or $9\frac{bb}{4}$ which being joyned to the two given Terms with the ſign $+$, it gives $aa+3ba+\frac{9}{4}bb$, whoſe Root is $a+\frac{3}{2}b$.

The ſame Rule is to be obſerved for the ſquaring of half the coefficient when it is a Fraction.

As for Example. Let the two members of a ſquare raiſed from a Binomial given be $aa+\frac{bd+3c}{f}a$, and let it be required to find the third member. Here half the coefficient is $\frac{bd+3c}{2f}$ which being ſquared, gives $\frac{bbdd+6bdc+9cc}{4ff}$ for the member ſought, and ſo the ſquare being compleated, is $aa+\frac{bd+3c}{f}a=\frac{bbdd+6bdc+9cc}{4ff}$, whoſe Root is $a+\frac{bd+3c}{f}$

VII. When

VII. When the Root of the given ſquare hath no coefficient, then the number 1 is ſuppoſed to be the co-efficient, half whereof, (*viz.* $\frac{1}{2}$) being ſquared, gives ($\frac{1}{4}$) the third member ſought to make it a compleat ſquare.

So $aa + a$ being given for 2 of the members of a ſquare raiſed from a Binomial, its third member to make the ſquare compleat will be $\frac{1}{4}$, for $aa + a = aa + 1a$, where the Coefficient is 1, whoſe half is $\frac{1}{2}$, which being ſquared, gives $\frac{1}{4}$ for the third member ſought, ſo the Square being compleated, is $aa + a + \frac{1}{4}$, whoſe Root is $a + \frac{1}{2}$.

This Chapter ought to be well underſtood before any further progreſs be made, for the manner how to reſolve Queſtions which produce Quadratick (or ſquare) Equations doth principally depend thereupon.

CHAP. XV.

Concerning the Reſolution of Queſtions producing Quadratick Equations.

I. Quadratick (or ſquare) Equations, are ſuch adfected or Compound Equations as conſiſt of three terms, the higheſt of which is is a ſquare, and is called the higheſt term in the Equation, of which three terms two are always unknown, and the third is always known? the firſt of the three is the ſquare of the Quantity or Number ſought, and the ſecond Term is the Product of the Quantity ſought, being multiplyed by ſome known Number or Quantity, and is called the Middle Term of an Equation, and the third Term is a Number, or Quantity purely known.

So in this Equation, *viz.* $aa+ba=d$, the firſt and higheſt Term or member is aa, which is the ſquare of the Quantity or Number ſought, and ba is the middle term of the Equation which is the Product of the Quantity ſought, it being drawn into b (which is known) and the third term or member of this Equation is b, which is really known, and is uſually called the Abſolute Number or Quantity given.

II. The Equations of this kind are of three Forms, which are laid down by Mr. *Kerſey*, in the fifteenth Chapter of the firſt Book of his Elements of *Algebray*, as followeth, *viz.*

Equations of the firſt Form.

$aa+6a=55$	$aa+ca=b$
$aaaa+8aa=48$	$aaaa+daa=f$
$aaaaaa+4aaa=837$	$aaaaaa+gaaa=h$

Equations of the ſecond Form.

$aa-13a=24$	$aa-ba=k$
$aaaa-6aa=27$	$aaaa-paa=d$
$aaaaaa-2aaa=48$	$aaaaaa-maaa=g$

Equation of the third Form.

$10a - aa = 24$	$ca - aa = n$
$5aa - aaaa = 4$	$raa - aaaa = s$
$9aaa - aaaaaa = 8$	$raaa - aaaaaa = t$

III. The Reſolution of Equations which fall under the firſt Form.

When an Equation is compoſed after any of the three foregoing Forms, and any known Quantities are mixed with unknown, let it be ſo reduced by tranſpoſition (according to the Rules of the Eleventh Chapter) as that the known quantities may poſſeſs one ſide, and the unknown Quantities the other ſide of the Equation.

Example. Let this Equation be given, *viz.* $aa + ba = -ba + bdc$.

By the tranſpoſition of *B* on the firſt part of the Equation, and $-ba$ one the ſecond part, it will be reduced to this Equation, *viz.* $aa + ba = bdc - b$, which is an Equation of the firſt Form.: And when your Equation is ſo reduced, add to each part of the Equation the ſquare of half the coefficient, and ſo will the firſt part of the Equation be an Exact and compleat ſquare, then according to the 2d and 3d Rule of the Fourteenth Chapter extract the ſquare Root of both parts of the Equation, and from the Square Roots of both parts of the Equation ſubtract half the coefficient, and then you will diſcover the value of *a*. As in the following Examples.

Queſt.

Quest. 1.

What number is that which being ſquared, and multiplyed by 8 (or *b*) the ſum of the ſaid Square and Product is equal to 384 (or *c*)?

Reſolution.

1. For the number ſought put	a
2. Whoſe Square is	aa
3. Its Product by 8 (or *b*) is	ba
4. The ſum of the ſecond and third ſteps muſt be equal to 384 (or *c*) whence this Equation.	$aa + ba = c$
5. To each part of the equation add the ſquare of ($\frac{1}{2}b$) half of the coefficient, then will it be	$aa + ba + \frac{1}{4}bb = c + \frac{1}{4}bb$
6. Then by extracting the ſquare root of both parts of the equation by the ſecond and third Rules of the 14 Chap. it will be reduced to	$a + \frac{1}{2}b = \sqrt{:c + \frac{1}{4}bb}$
7. By the tranſpoſition of $\frac{1}{2}b$ to the ſecond part of the Equation the value of *a* is diſcovered to be	$a = \sqrt{:c + \frac{1}{4}bb} - \frac{1}{2}b$

Which Equation is thus expreſſed in words, *viz.* the number ſought is equal to the remainder, when ($\frac{1}{2}b$) 4 is ſubtracted from the ſquare root of the ſum of (*c*) 384 and $\frac{1}{4}$ of the Square of (*b*) 8 (added together) which is 16, ſo that the value of *a* is 16. For $c + \frac{1}{4}bb = 400$ and $\sqrt{(2)}\ 4000 = 2c.$ and $20 - 4 = 16$.

Quest. 2.

What number is that whose Square being multiplyed by 4 (or b) and its Biquadrate (or fourth Power) multiplyed by 6 (or c) and the Products added together, the sum is 3850, (or d)

Resolution.

1. For the number sought put a
2. Its Square multiplyed by b is baa
3. Its Biquadrate multiplyed by c is $caaaa$
4. The sum of the second and third steps must be equal to 3850 (or d) whence this Equation, *viz.* $caaaa + baa = d$
5. And because the highest power of the equation is multiplyed by c, therefore each part being divided by c the equation is $aaaa + \frac{b}{c}aa = \frac{d}{c}$
6. To each part of the equation add half the square of the coefficient $(\frac{b}{c})$ and the equation will be $aaaa + \frac{b}{c}aa + \frac{bb}{4cc} = \frac{d}{c} + \frac{bb}{4cc}$
7. Then the square Root of each part of the equation in the sixth step, being extracted by the second and third Rules of the 4 Chap. the equation then will be $aa + \frac{b}{2c} = \sqrt{:\frac{d}{c} + \frac{bb}{4cc}}$

8. And

8. And by the tranſpoſition of $\frac{b}{2c}$ to the ſecond part of the equation the value of *aa* is found to be } $aa = \sqrt{(2)} \overline{\frac{d}{c} + \frac{bb}{4cc}} \frac{b}{cc}$

9. And becauſe the equation in the 8 ſtep is the value of *aa*, therefore if the ſquare Root of each part of that equation be extracted, the value of *a* it ſelf will be diſcovered to be } $a = \sqrt{(2)}\ \sqrt{:} \overline{\frac{d}{c} + \frac{bb}{4cc}} \frac{b}{2c}$

which in words is as much as to ſay the Number ſought (or *a*) is equal to the ſquare Root of the remainder when $(\frac{b}{2c})\ \frac{1}{3}$ is ſubtracted from the ſquare Root of the ſum of $3\frac{852}{6}$ (or $\frac{d}{c}$) and $1\frac{16}{44}$ (or $\frac{bb}{4cc}$) being added together, ſo that the value of *a*, (or the number ſought is 5. For $\sqrt{:} \frac{d}{c} + \frac{bb}{2cc} = \frac{76}{3}$ and $\frac{76}{3} - \frac{1}{3} (\frac{b}{2c}) = \frac{75}{3} = 25$ and $\sqrt{:}$ $25 = 5$, which is the number ſought.

The Proof.

$$4 \times 5 \times 5 + 6 \times 5 \times 5 \times 5 \times 5 = 3850$$

You muſt remember always to reduce a Fraction to its loweſt Terms before you extract its Root.

III. The Resolution of Equations which fall under the second of the three Forms before mentioned.

Quest. 1.

What unmber is that which having 8 (or *b*) times its self subtracted from its square, the remainder is 48 (or *c*)?

Resolution.

1. For the number sought put	a
2. Then will its square be	aa
3. The first step multiplyed by *b* is	ba
4. If the third step be subtracted from the second, the remainder will be 48 (or *c*) whence this equation	$aa - ba = c$
5. To each part of that equation add the square of ($\frac{1}{2}b$) half the coefficient, and then it will be	$aa - ba + \frac{1}{4}bb = + \frac{1}{4}bb$
6. Extract the square Root of each part of the last equation by the second and third Rules of the 14th Chapter, and it is	$a - \frac{1}{2}b = \sqrt{:c + \frac{1}{4}bb}$
7. And by the transposition of $\frac{1}{2}b$ to the second part of the equation, the value of *a* a discovered to be 12.	$a = \sqrt{:c + \frac{1}{4}bb} + \frac{b}{2}$

which in words is as much as to say, the Number sought (or *a*) is equal to the sum of the universal Square Root of the sum of 48 (or *c*) and a fourth part of the square of *b* (or $\frac{1}{4}bb$) being added

to

to 4 (or $\frac{b}{2}$) which is 12 for 1=48 and $\frac{1}{4}bb$=16, and 48+16=64 and $\sqrt{64}$=8, and 8+4 ($\frac{1}{2}b$)=12.

The Proof.

$$\overline{12\times12}-\overline{8\times12}=48$$

Quest. 2.

What number is that which having 12 (or *b*) times its ſquare ſubtracted from its Biquadrate, or forth power, the remainder is 3328 (or *c*)?

Reſolution.

1. For the number ſought put	a
2. Then its biquadrate is	$aaaa$
3. And its ſquare multiplyed by 12 (or *b*) is	baa
4. The difference of the ſecond and third ſteps muſt be equal to 3328 (or *c*,) whence this equation, *viz.*	$aaaa-baa=c$
5. Square half the coefficient, and add it to each part of the equation, and then it will be	$aaaa-baa+\frac{1}{4}=+\frac{1}{4}bb$
6. Extract the ſquare Root of both parts of the equation by the ſecond and third Rules of the 14 Chap. and then the equation will be	$aa-\frac{1}{2}b=\sqrt{:c+\frac{1}{4}bb}$
7. By the tranſpoſition of $-\frac{1}{2}b$ to the contrary Coaſt the value of (*a*) the number ſought will be diſcovered to be	$aa=\sqrt{:c+\frac{1}{4}bb}+\frac{1}{2}b$

8. By extracting the square root of both parts of the equation in the 17th step the value of a is found to be 8. } $a = (2)\sqrt{:c + \frac{1}{4}bb} + \frac{1}{2}b$

Which is as much as to say, that the Number sought, (or a) is equal to the universal square root of the sum of 6 (or $\frac{1}{2}b$) being added to the universal square Root of the sum of 3328, (or c) and 36 (or $\frac{1}{4}bb$) which upon tryal you will find to be 8.

For, $c = 3328$, and $b = 12$, and $\frac{1}{4}bb = 36$, wherefore $3328 + 36 = 3364$, and $\sqrt{:3364} = 58$, and $58 + (\frac{1}{2}b)\ 6 = 64$, and $\sqrt{:64} = 8$, which is the Number sought.

IV. The manner of resolving Equations which fall under the last of the three forms before mentioned.

Let the equation proposed (if it falls under the third and last form) be reduced to an equation of the second form, by the transposition of its terms, as in the following questions, *viz.*

What Number is that whose square being subtracted from 12 (or b) times it self the remainder is 32 (or c)?

Resolution.

1. For the number sought put — a
2. Its product by 12 (or b) is — ba
3. If from the second step you subtract (aa) the square of the first step, the remainder is } $ba - aa$

4. The remainder in the third ſtep is equal to 32 (or c) whence this equation $\Big\}$ $ba - aa = c$

Now by tranſpoſition I reduce to an equation of the ſecond of the foreſaid forms. And Firſt,

5. By tranſpoſition of aa to the contrary part, the equation is $\Big\}$ $ba = c + aa$

6. Then by tranſpoſition of c in the fifth ſtep, the equation is $\Big\}$ $ba - c = aa$

7. And by tranſpoſition of ba in the ſixth ſtep, the equation will be $\Big\}$ $-c = aa - ba$ or $aa - ba = -c$

So that from a due conſideration of the method uſed in reducing Equations of the third form to equations of the ſecond form you may eaſily perceive that the work of tranſpoſition in the fifth, ſixth and ſeventh ſteps is performed only by changing the ſigns of all the Terms of the Equation in the fourth ſtep, *viz.* by changing $+$ into $-$, and $-$ into $+$.

So the Equation in the fourth ſtep is $ba - aa = c$,

And by changing the ſigns of $ba - aa$ on the firſt part of the Equation, and of c in the ſecond part into $-ba + aa$, and $-c$, the Equation will then be $-ba + aa = -c$, or $aa - ba = -c$, which is the ſame with that in the ſeventh ſtep; and it is now an Equation of the ſecond of the three foregoing forms, ſo that I now proceed to the ſolution of the Equation.

8. The

8. The square of half the coefficient ($\frac{1}{2}b$) in the seventh step to each part of the equation, it will then be	$aa - ba + \frac{1}{4}bb = c + \frac{1}{4}bb$
9. The square root of each part of the last equation being extracted by the second and third Rules of the 14 Chapter the equation will then be	$a - \frac{1}{2}b = \sqrt{: -c + \frac{1}{4}bb}$
10. And by the transposition of $\frac{1}{2}b$ in the ninth step to the contrary part, the value of a will then be found to be (8)	$a = \sqrt{-c + \frac{1}{4}bb} + \frac{1}{2}b$

which is as much as to say that the number sought (or a) is equal to the sum of 6 (or $\frac{1}{2}b$) being added to the Square Root of the remainder, when 32 (or c) is subtracted from 36 (or $\frac{1}{4}bb$) which is 8. For, $\frac{1}{4}bb - c = 4$ whose square Root is 2, and $2 + 6$ (or $\frac{1}{2}b$) $= 8$ which is the number sought.

The Proof.

$$12 \times 8 = 96$$

And $96 - 64$ (aa) $= 32$ (or c) which was propounded.

V. The Resolution of various Questions producing Quadratick Equations.

Quest. 1.

There are two Numbers whose sum is 12 (or b) and the sum of their squares is 80 (or c) I demand what are those numbers?

Reso-

Resolution.

1. For one of the numbers sought put a
2. Then the other will be $b - a$
3. Then the sum of their Squares will be $bb - 2ba + 2aa$
4. Which quantity in the third step is equal to 80 (or c) whence this equation $bb - 2ba + 2aa = c$
5. Which equation being duly reduced by the rules of the eleventh Chapter giveth this equation. $aa - ba = \frac{c}{2} - \frac{bb}{2}$
6. Which Equation being solved according to the third Rule of this Chapter, the value of a is discovered to be $a = \sqrt{\frac{c - bb}{2} + \frac{bb}{4}} + \frac{b}{2}$
7. Wherefore I conclude the numbers sought are 8 and 4, for their sum is 12, and the sum of their squares is 80
8. Moreover the Equation in the sixth step will give this

Canon.

If from half the given sum of the squares you subtract half the square of the given sum, and to the remainder you add half the given sum, the square root thereof being added to the said half sum of the numbers, the sum of this addition will give you the greater number sought, and the greater number being subtracted from the given sum of the numbers, will give the lesser number sought.

Quest.

Question 2.

There are two numbers, the product of whose multiplication is 96 (or b) and the sum of their squares is 208 (or c) I demand what are those numbers?

Resolution.

1. For one of the numbers sought put	a
2. Then by dividing 96 (or b) by a, the Quotient will give other which is	$\frac{b}{a}$
3. The square of the number in the first step is	aa
4. The square of the other number in the second step is	$\frac{bb}{aa}$
5. And the sum of their squares in the third and fourth steps is	$aa + \frac{bb}{aa}$
6. Which sum in the fifth step must be equal to the given sum of the Squares 208 (or c) whence followeth this equation, *viz.*	$aa + \frac{bb}{aa} = c$

7. Which Equation in the last step being duly reduced by the Rules of the eleventh Chapter the value of a will be discovered to be

$$a = \sqrt{(2)}\, \overline{\sqrt{\tfrac{1}{4}cc - bb} + \tfrac{1}{2}c}$$

8. So that I conclude the numbers sought to be 12 and 8, for their product is 96, and the sum of their squares is 208.

9. More-

9. Moreover the Equation in the seventh step giveth this

CANON.

From $\frac{1}{4}$ of the Square of the given sum of the Squares subtract the Square of the given Product of the Multiplication of the numbers sought, and extract the square Root of the remainder, and to the said Square Root add half the given sum of the said squares, and then extract the square root of the sum of that Addition, so shall that square Root be one of the Numbers sought, by which if you divide the given Product, the Quotient will be the other Number sought.

QUESTION 3.

There are two Numbers whose sum is 12 (or b) and the Product of their Multiplication is 20 (or c) what are the Numbers?

RESOLUTION.

1. For one of the Numbers sought put	a
2. Which if you subtract from (12) b the given sum, the remainder will be the other number, *viz.*	$b-a$
3. And if the first and second steps be multiplyed the one by the other, the Product will be	$bb-aa$

4. Which

4. Which Product the Question requires to be equal to 20 (or c) from whence this equation } $ba-aa=c$

5. Which equation is of the third and last form, mentioned in the beginning of this Chapter, which being duly reduced by the Rules of the eleventh Chapter, it will be

$$aa-ba=-c$$

6. Which Equation being solved according to the method used in the fourth Rule of this Chapter, the value of a will be discovered to be

$$a=\sqrt{:\tfrac{1}{4}bb-c}+\tfrac{1}{2}b.$$

7. So that I conclude the Numbers sought to be 10 and 2, whose sum is 12, and their product 20, according to the conditions of the Question. Moreover the Equation in the sixth step, will present you with this

CANON.

From the Square of the half given sum of the Numbers sought, subtracted their given product, and extract the square Root of the remainder, and to its square Root add half the given sum of the numbers sought, so shall the sum of that Addition be the greater number sought, which being subtracted from the said given sum will leave the lesser.

Quest.

QUEST. 4.

There are three Numbers which are Geometrical proportionals continued, the mean whereof is 12 (or b) and the two extreams are ſuch, that their difference is 18 (or c) I demand what are thoſe three Numbers?

RESOLUTION.

1. For the leſſer extream put	a
2. Then the greater will be	$a + c$
3. Then will the product made by the multiplication of the extreams of the firſt and ſecond ſteps be	$aa + ca$
4. Which Product (or Rectangle) in the third ſtep muſt be equal to the ſquare of (12 or b) the mean whence this Equation.	$aa + ca = bb$
5. Which Equation being ſolved by the ſecond Rule of this Chapter, the value of a will be found to be	$a = \sqrt{:bb + \frac{1}{4}cc} - \frac{1}{2}c$

6. I ſay the extream proportionals ſought are 6 and 24, whoſe difference is 18: For,

$$6 : 12 :: 12 : 24 \text{ or } a :: b :: b : \frac{bb}{a}$$

7. The Equation in the fifth ſtep being well conſidered, will preſent you with this

CANON,

CANON.

If to the Square of the given mean you add the ſquare of half the difference of the extreams, or (which is all one) $\frac{1}{4}$ part of the ſquare of the given difference of the extreams, and extract the ſquare root of the ſum of that Addition, and then from that ſquare root ſubtract half the ſaid difference. the remainder will be the leſſer extream, and if thereto you add the given difference, that ſum will be the greater extream.

QUEST. 5.

A Draper ſold a piece of Cloth for 24 *l* (or *b*) and gained as much *per Cent.* (or *c*) as the cloth coſt him, I demand how much it coſt him ?

RESOLUTION.

1. For the price which the cloth coſt, put	a
2. Then will the gain by its ſale be	$b-a$
3. Then by the Rule of three find how much is gained *per Cent.* ſaying, $a : b-a :: c : \frac{cb-ca}{a}$, ſo that his gain *per cent.* was	$\frac{cb-ca}{a}$
4. Which quantity in the 3d ſtep according to the tenure of the queſtion muſt be equal to what the cloth coſt in the firſt ſtep whence this Equation, *viz.*	$a=\frac{cb-ca}{a}$

5. Which

5. Which being reduced by the Rules of the eleventh Chapter, it will be	$aa+ca=cb$
6. Which (being an Equation of the first of the 3 forms delivered in the beginning in the fifteenth Chap.) being solved by the 2d & 3d rule of the 14 Chapter, the value of *a* is discovered to be	$a=\sqrt{:cb+\frac{cc}{4}}-\frac{c}{2}$

I say the cloth cost 20 *l.* which is the value of *a*, for $\sqrt{:cb+\frac{cc}{4}}=70$ and $70-\frac{c}{2}=20$, so that he gained 4 *l.* in laying out 20: For,

l.		*l.*		*l.*		*l.*
20	:	4	: :	100	:	20

and so the conditions of the Question are satisfied.

QUEST. 7.

A Merchant bought a certain number of pieces of cloth, and paid 30 pounds (or *b*) *per* Cloth, and sold them again at such a rate *per* Cloth, that if the pounds he sold a Cloth for be multiplyed by the pounds he gained *per* Cloth, the product will be equal to the Cube of the number of pounds gained *per* Cloth, I demand what he gained *per* Cloth, and what he sold each Cloth for?

RESOLUTION.

1. For the number of pounds gained *per* piece, put	a
2. To which if you add 30 (or b) the ſum will be the number of pounds it was ſold for *per* piece, *viz.*	$a+b$
3. And if (according to the tenure of the Queſtion) the ſecond ſtep be multiplyed by the firſt, the product will be	$aa+ba$
4. Which Product in the third ſtep, muſt (according to the nature of the Queſtion) be equal to the Cube of the pounds gained *per* Cloth in the firſt ſtep, whence this equation, *viz.*	$aa+ba=aaa$
5. Which equation being reduced by the third and ſixth Rules of the eleventh Chap. it will then be	$aa-a=b$
6. Which equation in the 5th ſtep being ſolved by the 7th Rule of the 14th Chap. and the ſixth Rule of this Chap. the value of a will be diſcovered to be	$a=\sqrt{:b+\frac{1}{4}}+\frac{1}{2}$

which is as much as to ſay in words, a (or the gain *per* Cloth) is equal to the ſum when $\frac{1}{2}$ is added to the Square Root of the ſum of 30, and $\frac{1}{4}$ added together, (*viz.* the Square Root of 30 $\frac{1}{4}$) which is $5\frac{1}{2}+\frac{1}{2}=6$.

I ſay,

I say he gained 6 pounds *per* Cloth, and he sold it for 36 pounds *per* Cloth, which two numbers will satisfie the conditions of the question.

The Proof.

$$6 \times 36 = 6 \times 6 \times 6 = 216$$

QUEST. 8.

A Brick-layer, and a Labourer wrought together at the Building of a certain house 42 days, (or b) and the labourer he wrought 4 (or c) days more than the Brick-layer did to gain one pound, and at the end of the 42 days the Brick-layer received for his work $1\frac{3}{4}$ pounds (or d) more than the Labourer, I demand how many days each of them wrought for 1 *l.*

RESOLUTION.

1. For the number of days which the Brick-layer wrought for *l.* put	a
2. Then according to the conditions of the question, the number of days that the labourer wrought for 1 *l.* will be	$a + c$
3. By the Rule of proportion find how many pounds the Brick-layer received for the work of 42 days, as followeth, $a : 1 :: b : \frac{b}{a}$ which is	$\frac{b}{a}$

 4 Then

4. Then find out by the Rule of 3 how many pounds the Labourer received for his work in 42 days thus $ca+c : 1 :: b : \frac{b}{a+c}$ which is — $\frac{b}{a+c}$

5. But the Brick-layer received $1\frac{1}{2}$ (or b) pounds more than the Labourer for 42 days work, wherefore if to the 4th step you add d (or b) it will be equal to what the Brick-layer received in the third step, whence this Equation, *viz.* — $\frac{b \div da \div dc}{a \div c} = \frac{b}{a}$

6. Which Equation being reduced by the fourth, second, and seventh Rules of the eleventh Chapter, it will then be — $aa+ca=\frac{bc}{d}$

7. The Equation in the fifth step being solved by the Rule of this Chapter, the value of a will be discovered, *viz.* — $a=\sqrt{\frac{bc}{d}+\frac{cc}{4}}-\frac{c}{2}$

which is as much as to say a (or the number of days which the Brick-layer wrought) is equal to the difference when $(\frac{c}{2})$ 2 is subtracted from the square root of the sum of $(\frac{bc}{d})\frac{672}{7}$ or 66 and $(\frac{cc}{4})$ 4 which is $100 = 10 - 2 = 8$

I say the Brick-layer wrought 8 days for twenty shillings, and the Labourer wrought $8+4=12$

$8+4=12$ days, which two numbers will satisfie the conditions of the question, as will appear by the

PROOF.

First by the Rule of Three find what the Brick-layer received for the 42 days, saying,

days	*l.*		days		*l.*	*l.*	*s.*
8	: 1	: :	42	:	$\frac{42}{8}$	$=5$	-5

Then find how much the Labourer received for his 42 days work by the Rule of Three, saying,

days	*l.*		days		*l.*	*l.*	*s.*
12	: 1	: :	42	:	$\frac{42}{12}$	$=3$	-10

So that I find the Brick-layer for his 42 days work received 5 *l.*—5 *s.* and the Labourer 3 *l.*—10*s.* which is 1 *l.*—15, or 1 $\frac{3}{4}$ *l.* less than the Brick-layer received.

QUEST. 9.

A Gentleman bought a House, and sold it again for 280 pounds (or *b*) and by its sale he gained so many pounds, that their Square being added to the square of the number of pounds it cost him, the sum will amount to 52000 (or *c*) pounds, now I demand how much the house cost him?

RESOLUTION.

1. For the number of pounds which the houſe coſt, put	a
2. Then will the gain by its ſale be	$b - a$
3. The Square of (a) its firſt coſt is	aa
4. The Square of the gain by Sale is	$aa - 2ba + bb$
5. The ſum of the two quantities in the third and fourth ſteps is	$2a - 2ba + bb$
6. Which quantity in the fifth ſtep is equal to 52000 (or c) whence this Equation	$2aa - 2ba + bb = c$
7. Which equation being reduced by the third and ſeventh Rules of the eleventh Chapter it will be	$aa - ba = \frac{c - bb}{2}$
8. Which equation being ſalved by the third Rule of this Chapter the value of a will be diſcovered, *viz.*	$a = \sqrt{\frac{c - bb}{2} + \frac{bb}{4}} + \frac{1}{2}b$

which is as much as to ſay in words, (a) the price which the houſe coſt is equal to the ſum when half what he ſold it for is added to the Square Root of the ſum of half the given ſum of the Squares added to a fourth part of the Square of what it was ſold for, that ſum being made leſs by half the Square of what it was ſold for; which was 220 *l.* and he gained by the ſale 60 *l.* For,

$\frac{c}{2} = 26000$, and $\frac{bb}{2} = 19600$, and $\frac{bb}{2} = 39200$, now

26000+19600=45600, and 45600 — 39200 =6400, and $\sqrt{(2)}$ 6400=80, and 80+($\frac{1}{2}b$) 140 =220, which is the number of pounds the houſe coſt, and 280—220=60, which is the number of pounds he gained by the Sale of the houſe, as you will find by

The Proof.

220×220=48400, and
60×60= 3600, and
48400+3600=52000

whereby the conditions of the queſtion are anſwered.

QUESTION 10.

A Draper ſold 2 pieces of Cloth (whereof one contained 6 (or *b*) yards more than the other) for two equal numbers of ſhillings, the leſſer piece he ſelleth for 2 (or *c*) ſhillings, *per* yard more than the other, and the number of ſhillings which one piece was ſold for, did exceed the number of yards in both pieces by 186 (or *d*) the queſtion is what was the Number of yards in each piece, and what each piece was ſold for *per* yard?

RESOLUTION.

1. For the number of yards in the leaſt piece put	a
2. Then will the yards in the greater piece be	$a+b$
3. Then will the ſum of the yards in both pieces be	$2a+b$

4. Then if (according to the nature of the Question) to the sum of the yards in the third step, you add 186 (or *d*) the sum will be the number of shillings which each piece was sold for, *viz.* $2a+b+d$

5. And if the quantity in the fourth step, be divided by (*a*) the quantity in the first step the Quotient will give the number of shillings that 1 yard of the least piece was sold for. $\frac{2a+b+d}{a}$

6. And if the said Quantity in the fourth step be divided by the number of yards in the biggest piece, (which is the Quantity in the second step) the Quotient will give the number of shillings that a yard of the biggest piece was sold for, which is $\frac{2a+b+d}{a+b}$

7. If to the Quantity in the sixth step you add 2 (or *c*) shillings, it will then be $\frac{2a+b+d+ca+cb}{a+b}$

8. Which Quantity in the seventh step (as the Question requires) is equal to the Quantity in the fifth step, whence this Equation, *viz.*

$$\frac{2a+b+d+ca+cb}{a+b} = \frac{2a+b+d}{a}$$

9. Which

9. Which equation in the eighth ſtep being reduced by the Rules of the Eleventh Chapter, it will then be

$$aa + \frac{cb - 2b}{c} a = \frac{bb + bd}{c}$$

10. The Equation in the laſt ſtep being ſolved by the third rule of this Chapter, the value of a will be diſcovered to be

$$a = \sqrt{\frac{bb+bd}{c} + \frac{ccbb - 4cbb + 4bb}{4cc}} - \frac{cb-2b}{2c}$$

11. But if you conſider well the Equation in the ninth ſtep, you will find the coefficient to be o, for $\frac{cb-2b}{c} = 0$, and therefore $\frac{cb-2b}{2c} a = 0$, whence the middle Term in that Equation is o, and therefore the middle term being removed, the Equation will be $aa = \frac{bb+bd}{c}$ which is a ſimple Equation, and if the Square root of both parts of that equation be extracted, the value of a will be diſcovered to be $a = \sqrt{\frac{bb+bd}{c}}$ $= 24$, which is the ſame with the value of a in the tenth ſtep, as you may eaſily find upon Tryal, wherefore I ſay,

The number of yards in the leaſt piece is 24.

And the number of yards in the biggeſt piece is $24 + 6 = 30$, which two numbers will ſatisfie the conditions of the queſtions, as will appear by

The

The Proof.

The number of yards in both pieces is 24+30 =44, which if added to 186 (as the Queſtion requires) will give the number of ſhillings which one piece was ſold for, which is 54+186=240, and the leaſt piece was ſold at 10 Shillings *per* yard.

yards		*s.*		*yards*		*l.*	*s.*
24	:	240	:	1	:	$\frac{240}{42}$	=10.

And the price of a yard of the biggeſt piece was 8 *s.* For,

yards		*s.*		*yards*		*l.*	*s.*
30	:	240	: :	1	:	$\frac{240}{40}$	=8

which is two ſhillings *per* yard leſs than the leſſer piece was ſold for *per* yard, and therefore the anſwer is true, and the conditions of the queſtion are ſatisfied.

CHAP. XVI.

The Doctrine of Surd Quantities.

I. ALL quantities or Numbers whatſoever, whether Integral, or Fractional, are called Rational, but when the Root of any power cannot be exactly extracted, ſuch Root is called Irrational or Surd, and is expreſſed by putting the Radical ſign before the number out of which the Root propoſed ought to be extracted; as $\surd$ or $\surd\,(2)$ placed before any number or quantity ſignifieth the Square Root of the quantity or Number, and $\surd\,(3)$ the Cube Root, and $\surd\,(4$ the Biquadrate Root, *&c.* So $\surd\,12$, or $\surd\,(2)\,12$ ſignifieth the Square Root of 12, and $\surd\,(3)\,12$ its Cube Root, *&c.*

II. Surd Numbers are two-fold, *viz.* Simple, and Compound; A Simple Surd quantity is when the Radical ſign is prefixed to a Simple quantity, as $\surd\,(3)\,5$ or $(\surd\,(4)\,ab$.

A Compound Surd quantity conſiſts of ſeveral Simple Surds, which are connected together by + or —, as $\overline{\surd\,4 + \surd\,6}$, and $\surd\,ab + \surd\,ac + \surd\,d$, and $\surd\,(2)\,\overline{ab+dc}$ which laſt Compound Surd is uſually called an univerſal Root.

III. To

III. To Reduce Simple Surd quantities that have different radical ſigns to a common radical ſign.

Let the Indices of the given Powers be reduced to their loweſt Terms by their common meaſurer, and ſet the quotients under their reſpective Dividends, and multiply croſs-wiſe, ſo ſhall the product be the Index required, before which placing $\surd$, it ſhall then be the common radical ſign required: Then raiſe the Powers of the given Roots to the powers ſignified by the ſaid altern quotients, before which ſaid Powers place the common radical ſign found as before, ſo will you have new ſurd quantities equal to the given quantities, and having equal Radical ſigns.

Example. Let it be required to reduce $\surd(6)\,8$, and $\surd(8)\,12$ to two other Roots equivalent to the former, having a common radical ſign.

$$\surd(6)8 \quad \times \quad \surd(8)\,12$$
$$3 \qquad\qquad 4$$
$$\surd(24)4096 \quad \surd(24)1728$$

Firſt, the exponents 6 and 8 are reduced to 3 and 4, which being placed under the given exponents 6 and 8 as you ſee, and having multiplyed Croſs-wiſe, *viz.* 3×8, or 4×6, you have 24 for a new Index, to which prefix $\surd$, and it is $\surd(24)$ for the common radical ſign, and then raiſing 12 to the third power thereof, and 8 to the fourth, you have $\surd(24)\,4096$ and $\surd(24)\,1728$ equal to $\surd(6)8$, and $\surd(8)12$.

So if it were required to reduce $\surd(4)a$, and $\surd(6)b$ to Surd Roots equivalent thereto, having a common Radical ſign, it will be as followeth.

$\surd(4)a$

$$\surd(4)a \qquad \surd(6)b$$
$$2 \qquad\quad 3$$
$$\surd(12)aaa \text{ and } \surd(12)bb$$

IV. Multiplication in Simple Surd Quantities.

1. If the Quantities given to be multiplyed have a common radical ſign, then multiply them together without any regard to the ſign, and to the product prefix the given Radical ſign, which new quantity ſhall be the product ſought.

So if $\surd 6$ be to be multiplyed by $\surd 8$, the Product will be $\surd 48$, and $\surd(3)4$ by $\surd(3)8$ Produceth $\surd(3)32$ and $\surd a$ by $\surd b$, produceth $\surd ab$, and $\surd(3)c$ by $\surd(3)bd$ produceth $\surd(3)cbd$. &c.

2. But if the Quantities given to be multiplyed have not a common Radical ſign, let them be reduced to ſuch by the third Rule foregoing, and then proceed as before.

Example. What is the Product of $\surd(4)a$ by $\surd(6)b$? The ſaid quantities being reduced to a common radical ſign, will be $\surd(12)aaa$ and $\surd(12)bb$, which being multiplyed together, produce $\surd(12)aaabb$ which is the product ſought.

So the $\surd(2)b$ being multiplyed by $\surd(3)c$ they being Reduced to a common radical ſign, are $\surd(6)bbb$, and $\surd(6)cc$ which being multiplyed produce $\surd(6)bbbcc$.

3. When a ſurd quantity is to be multiplyed by a rational quantity, then firſt raiſe the given rational quantity to the power of the given quantity, whoſe Root is irrational or ſurd; and then proceed as before.

So if it were required to multiply $\surd 5$ by 5, the rational number 5 being raiſed to the ſecond power is 25, and then you will have to multiply

$\surd 5$

$\sqrt{5}$ by $\sqrt{25}$, whoſe Product is $\sqrt{125}$.

Likewiſe $\sqrt{(3)}\,b$ being to be multiplyed by a, the Product will be $\sqrt{(3)}\,baaa$, for a being raiſed to the third power is aaa, and $\sqrt{(3)}\,b$ by $\sqrt{(3)}\,aaa$ produceth $\sqrt{(3)}\,baaa$ as before.

V. Diviſion in Simple Surd Quantities.

1. Reduce the Surd Quantities given to be divided to a common Radical ſign by the third Rule of this Chapter, and then divide the Quantity following the Radical ſign of the Dividend by the quantity following the radical ſign of the Diviſor, aud to the quotient prefix the ſaid common Radical ſign, ſo ſhall that Surd quantity be the quotient ſought.

Example. There being given $\sqrt{15}$ to be divided by $\sqrt{3}$, the quotient will be $\sqrt{5}$. And $\sqrt{b}$ being to be divided by $\sqrt{a}$, the quotient will be $\sqrt{\frac{b}{a}}$ and $\sqrt{(2)}\,a$ being given to be divided by $\sqrt{(3)}\,bc$, the quotient will be $\sqrt{(6)}\,\frac{aaa}{bbcc}$, for the given quantities being reduced to a common radical ſign, are $\sqrt{(6)}\,aaa$ and $\sqrt{(6)}\,bbcc$.

VI. Addition and Subtraction of ſimple Surd quantities.

1. When the Surd Roots to be added together, are equal, multiply any one of them by the given number of Surd quantities, ſo ſhall that product be the ſum required, before which prefix the radical ſign given, ſo the ſum of $\sqrt{6}$ and $\sqrt{6}$ is $\sqrt{24}$, for the given number of roots is 2, whoſe ſquare is 4, and $\sqrt{4}\times\sqrt{6}=\sqrt{24}$, ſo $\sqrt{(3)}\,b$ being to be added to $\sqrt{(3)}\,b$, their ſum is $\sqrt{(3)}\,8b$ and $\sqrt{(3)}\,a$ being to be added to $\sqrt{(3)}\,a$, and $\sqrt{(3)}\,a$ their ſum will be

$\sqrt{(3)}\,27a$,

$\sqrt{(3)}27a$; for the given Number of Surds is 3 and $\sqrt{(3)}a$ being multiplyed by 3. *viz.* $\sqrt{(3)}27$ (by the third part of the fourth Rule) the Product is $\sqrt{(3)}27a$ which is the ſum of $\sqrt{(3)}a$, $\sqrt{(3)}a$, and $\sqrt{(3)}a$, which was required.

2. When two unequal Surd Roots which have the ſame Radical ſign prefixed to each of them, be to be added together, or when the leſſer of them is to be ſubtracted from the greater, Then you muſt firſt try whether they be commenſurable, or not; that is, if after they have been divided by their greateſt common meaſurer, the Quotients be rational Quantities, then multiply the ſum of thoſe rational quantities by the ſaid common Diviſor, and the Product ſhall be the ſum of the Surd Quantities propounded; and if the difference of thoſe Rational Quotients be multiplyed by the ſaid common meaſurer, then will the Product be the difference of the Surd Quantities propounded.

Example. Let it be required to find the ſum and difference of $\sqrt{50}$, and $\sqrt{8}$, their greateſt common meaſurer is $\sqrt{2}$, by which they being divided, the Quotients are $\sqrt{25}$ and $\sqrt{4}$, *viz.* 5 and 2; whoſe ſum is 7, which being multiplyed by $\sqrt{2}$, the Product is $7\sqrt{2}$ or $\sqrt{98}$, which is the deſired ſum of the Surd Quantities propounded. And if the difference of the ſaid Rational Quotients, *viz.* $5-2$ (or 3) be multiplyed by the ſaid common Diviſor ($\sqrt{2}$) the Product will be $3\sqrt{2}=\sqrt{18}$, which is the difference of the Surd Quantities given, the leſſer being ſubtracted from the greater.

But if the ſimple Surd quantities given to be added, or ſubtracted, be incommenſurable, neither their ſum nor difference can be expreſt by any ſim-

ſimple Term, or Root, but their ſum and difference muſt be expreſt by + and —? as ſuppoſe you were to add $\sqrt{10}$ and $\sqrt{13}$ together, their ſum would be $\sqrt{13}+\sqrt{10}$, and their difference $\sqrt{13}-\sqrt{10}$. The like of other quantities expreſt by letters.

CHAP. XVII.

The Parts of Numeration in Compound Surd Quantities.

I. *Addition and Subtraction in Compound Surd Quantities.*

THE Addition and Subtraction of Compound Surd quantities is the ſame with the ſimple Surds, having reſpect to the ſigns of Affirmation and Negation, *viz.* + and —.

So if to $6+\sqrt{18}$ $(3\sqrt{2})$ you add $4+\sqrt{8}$ $(2\sqrt{2})$ the ſum will be $10+\sqrt{50}$ $(5\sqrt{2})$ and if from $6+\sqrt{18}$ $(3\sqrt{2})$ you ſubtract $4+\sqrt{8}$ $(2\sqrt{2})$ the difference will be $2-\sqrt{2}$.

Likewiſe if to $\sqrt{320}+\sqrt{108}$ $(85\sqrt{}+6\sqrt{3})$ you are to add $\sqrt{80}-\sqrt{27}$ $(4\sqrt{5}-3\sqrt{3})$ the ſum is $\sqrt{720}+\sqrt{27}$ $(12\sqrt{5}+3\sqrt{5})$ and if you ſubtract the

the latter from the former, the remainder will be $\sqrt{80}+\sqrt{243}(4\sqrt{5}+9\sqrt{3})$.

These two examples are of Compound Surd quantities which are commensurable, and the next is of Compound Surd quantities, partly commensurable, and partly incommensurable. As

Let it be required to add $\sqrt{12}(2\sqrt{3})+\sqrt{5}$, to $\sqrt{27}(3\sqrt{3})+\sqrt{8}$ the sum will be $\sqrt{75}(5\sqrt{3})+\sqrt{8}+\sqrt{5}$, and if the former be subtracted from the latter, the remainder will be $\sqrt{3}+\sqrt{8}-\sqrt{5}$.

The same is to be observed in Addition and Subtraction of Compound Surd quantities altogether incommensurable. As in the following Examples.

To and from	$\sqrt{10}+\sqrt{7}$
Add and Subtract	$\sqrt{3}+\sqrt{2}$
Sum is	$\sqrt{10}+\sqrt{7}+\sqrt{3}+\sqrt{2}$
Or,	$\sqrt{:17+\sqrt{280}:}+\sqrt{:5+\sqrt{24}:}$
Difference is	$\sqrt{10}+\sqrt{7}-\sqrt{3}\ \sqrt{2}$
Or,	$\sqrt{:17+\sqrt{280}:}-\sqrt{:5+\sqrt{24}:}$
To and from	$\sqrt{(3)}10+\sqrt{(3)}7$
Add and Subtract	$\sqrt{(3)}3-\sqrt{(3)}2$
Sum is	$\sqrt{(3)}10+\sqrt{(3)}7+\sqrt{(3)}--\sqrt{(3)}2$
Difference is	$\sqrt{(3)}10+\sqrt{(3)}7--\sqrt{(3)}3+\sqrt{(3)}2$.

II. *Multiplication in Compound Surd Quantities.*

$$\begin{array}{lll} \text{Multiplicand} & \sqrt{180}+\sqrt{48} & (6\sqrt{5}+4\sqrt{3}) \\ \text{Multiplyar} & \sqrt{125}+\sqrt{12} & (5\sqrt{5}+2\sqrt{3}) \\ \hline & & 150+20\sqrt{15} \\ & & \quad +12\sqrt{15}+24 \\ \hline & \text{Product} & 150+32\sqrt{15}+24 \\ & \text{Product contracted} & 174+32\sqrt{15} \end{array}$$

$$\begin{array}{lll} \text{Multiplicand} & \sqrt{abb}+\sqrt{cff} & (b\sqrt{a}+f\sqrt{c}) \\ \text{Multiplyar} & \sqrt{add}+\sqrt{caa} & (d\sqrt{a}+a\sqrt{c}) \\ \hline & & bda+fd\sqrt{ca} \\ & & \quad +ba\sqrt{ca}+fac \\ \hline & \text{Product} & \overline{bda+fd+ba}\times\sqrt{ca}+fac \end{array}$$

$$\begin{array}{ll} \text{Multiplicand} & \sqrt{bc}+a \\ \text{Multiplyar} & \sqrt{bc}-a \\ \hline \text{Product} & ba-aa \end{array}$$

III. *Di-*

III. *Division in Compound Surd Quantities.*

Dividend	$\sqrt{21}+\sqrt{15}$		$\sqrt{(3)}14-\sqrt{(3)}28$
Divisor	$\sqrt{3}$		$\sqrt{(3)}7$
Quotient	$\sqrt{7}+\sqrt{5}$		$\sqrt{(3)}2-\sqrt{(3)}4$

Divisor Dividend

$a+\sqrt{bc})\ ab+b\sqrt{bc}\ (b$ Quotient.

$ab+b\sqrt{bc}$

$0 \quad 0$

$a+\sqrt{bc})aaa+bc\sqrt{bc}\ (aa+bc-a\sqrt{bc}$

$aaa+aa\sqrt{bc}$

$+bc\sqrt{bc}-aa\sqrt{bc}$

$+bc\sqrt{bc}+abc$

$-aa\sqrt{bc}-abc$

$-aa\sqrt{bc}-abc$

$0 \quad 0$

These Examples will not seem difficult to the ingenious, if what is before delivered concerning Surd quantities be duly considered.

CHAP. XVII.

The Parts of Numeration in Universal Surd Roots.

WHEN it is required to extract the Root of any Compound quantity, whether Square, Cube, Biquadrat, &c. if they cannot be exactly extracted without any remainder; then if to such given compound quantity you prefix the Radical sign, such Roots are called Universal Surd Roots, and first, concerning

I. *Multiplication in Universal Surds.*

1. When any Universal Root is to be multiplyed by a Rational quantity, or by any Surd, multiply the Square of the Multiplicand by the Square of the Multiplyar, when the Universal Radical sign is quadratick, or the Cube of the Multiplicand by the Cube of the Multiplyar, when the Universal Radical sign is Cubical, and before that Product prefix the given Universal Radical sign, so shall that new Universal Root be the Product sought.

Example. Let it be required to multiply by 2 this universal Square Root, *viz.* $\surd{:}10+\surd 40{:}$ I take the square of 2, which is 4, and the square of $\overline{\surd 10+\surd 40}$, which is $\surd 10+\surd 40$, and multiply it by 4, and the Product is $\surd 40+4\surd 40$, whose universal square Root is the Product sought, *viz.* $\surd{:}40+4\surd 40{:}$

Also if $\surd(3){:}\surd(3)64+\surd(3)27$ were to be multiplyed by 2, or doubled, take the Cube of the universal Root given, which is $\surd(3)64+\surd(3)27$, and multiply the same by the Cube of 2, which is 8, and the Product is $8\surd(3)64+8\surd(3)27$, the Cube Root of which is the Product sought, *viz.* $\surd{:}(3)8\surd(3)64+8(3)\surd(27$, and it is double to $\surd(3){:}\surd(3)64+\surd(3)27$ the Surd Root given.

In like manner, if it were required to multiply $\surd{:}12+\surd 6{:}+\surd 12-\surd 6{:}$ into its self, or to find its square; the squares of the parts are $12+\surd 6$ and $12-\surd 6$ the sum of which is 24 and the Product made by the Multiplication of the parts one into the other, *viz.* $\surd{:}12+\surd 6{:}$ into $\surd{:}12-\surd 6{:}$ is $\surd 138$, (for the difference of the Squares of 12 and $\surd 6$ is 138, whose square Root is $\surd 138$, and the double of the said Product is $2\surd 138$, which added to 24 (the sum of the squares of the par makes $24+2\surd 138$, which is the square $\surd{:}12+\surd 6+\surd{:}12-\surd 6$.

Likewise if $6+\surd{:}20-\surd 16{:}$ is to be m by $6-\surd{:}20-\surd 16$, the Product will be 20, for if $23-\surd 16$ (which is t ultiplyed $\surd{:}20-\surd 16$) be subtracted from 36 ound to be 6) there will remain $16+\surd 16$ ie square of (the square of hich is 20, the Pro

Product ſought. Alſo $6+\surd:\overline{20-\surd 10}:=10$, and $6-\surd:\overline{20-\surd 16}:=2$ and $2\times10=20$ as before.

Again, if it be required to multiply $\surd:\overline{aa+bb}:$ by a, the ſquares of the given quantities are $aa+bb$ and aa, which being multiplyed the one into the other, the Product will be $aaaa+bbaa$, the univerſal Square Root of which is the Product ſought, *viz.* $\surd:\overline{aaaa+bbaa}:$ which may be more compendiouſly expreſt thus, $a\surd:\overline{aa+bb}$.

II. *Diviſion in Univerſal Surds.*

As in Multiplication you multiplyed the Square of the Multiplicand by the Square of the Multiplyar, the given Radical ſign being Quadratick, *&c.* So in Diviſion of Univerſal Surd Roots you are to divide the ſquare of the Dividend by the ſquare of the Diviſor, when the univerſal Radical ſign is Quadratick, and Divide the Cube of the Dividend by the Cube of the Diviſor, when the univerſal Radical ſign is Cubical, *&c:* ſo ſhall the Quotient, when the univerſal radical ſign given is prefixed thereto, be the Quotient required.

Example. What is the Quotient when $\surd:\overline{40+4\surd 30}$ is divided by 2? Here I divide $40+\surd 40$ (which is the ſquare of $\surd:\overline{40+4\surd 40}:$ the dividend) by 4 (the ſquare of the given Diviſor) and there ariſeth $\surd\,10+\surd 40:$ the univerſal ſquare Root of which, *viz.* $\surd:\overline{10+\surd 40}:$ is the Quotient required.

Alſo if it were required to divide $\surd(3)\overline{8\surd(3)64+8\surd(3)27}$ by 2, the Quotient would be found to be $\surd:(3)\overline{\surd(3)64+\surd(3)27}:$ here the Cube of the given

given Dividend is $8\surd(3)64+8.\ (3)\ 27$ which being divided by 8 (the Cube of 2) there will arise $\surd(3)64+\surd(3)27$, to which if you prefix the universal radical sign of its Cube Root, it will be $\surd:(3)\overline{\surd(3)64+\surd(3)\ 27}$: which is the Quotient sought.

Likewise if it be required to divide $\surd:\overline{aaaa+bbaa}$ by a, the Quotient will be found to be $\surd:\overline{aa+\surd bb}$: for, the square of the Dividend is $aaaa+bbaa$, and the square of the Divisor is aa, and when the Division is ended, there will arise $aa+bb$ the universal square Root of which is $\surd:\overline{aa+bb}$: which is the Quotient sought.

But when the work of Division in universal Surd Quantities happens to be intricate, and its operation cannot be finished without a remainder, you may set the power of the Dividend for a Numerator, and the power of the Divisor for a Denominator, and against the line of Separation, place, or prefix the universal radical sign, which universal Root so signified shall be the Quotient sought.

As if it were required to divide $\surd:\overline{\surd ab+bc}$: by $\surd:\overline{\surd a+c}$: the quotient will be $\surd:\frac{\surd ab+bc.}{\surd a+c.}$

III. *Addition and Subtraction in Universal Surd Quantities.*

1. If two Universal Surd quantities that are commensurable are proposed to be added together, or subtracted, the operation may be performed like simple Surds. As for Example. If the sum and difference of $\surd:\overline{8+4\surd 3}$: and $\surd:\overline{2+\surd 3}$: were required.

Here each of the said quantities being divided by their greatest common measurer, $\overline{\sqrt{:}2+\sqrt{3}:}$ the Quotients are $\sqrt{4}$ and $\sqrt{1}$, *viz.* 2 and 1, which are rational Numbers expressing the proportion of the Surds propounded, therefore if their common Divisor be multiplyed $2+1$ (*viz.* 3.) it giveth $3\overline{\sqrt{:}2+\sqrt{3}:}$ for the sum required, and the said common Divisor being multiplyed by $(2-1)$ the difference of the said 2 and 1, it will produce $\overline{\sqrt{:}2+\sqrt{3}:}$ for the difference of the Roots proposed.

Likewise if it were required to find the sum and difference of $\overline{:aaaa+aabb:}$ and $\overline{\sqrt{:}aabb+bbbb}$.

The said Quantities being reduced, are $a\overline{\sqrt{:}aa+bb}:$ and $b\overline{\sqrt{:}aa+bb}:$

Therefore is their sum $\overline{a+b}\times\overline{\sqrt{:}aa+bb}:$ and their difference is $\overline{a \backsim b}\,\overline{\sqrt{:}aa+bb}$.

2. When the Root of a residual is to be added to, or subtracted from, the Root of its correspondent Binomial, then may those Roots be connected together by the signs $+$ and $-$; and then the whole being multiplyed by it self, the universal Root of the Product shall be the sum or difference of the Roots propounded.

As suppose $\overline{\sqrt{:}12+\sqrt{6}:}$ were propounded to be added to $\overline{\sqrt{:}12-\sqrt{6}}:$ the given Roots being connected together by $+$, make $\overline{\sqrt{:}12+\sqrt{6}:12\ \sqrt{6}:}$ which composed Quantity being multiplyed by it self, produced $24+2\sqrt{138}$, whose universal Square Root $(\sqrt{:}24+2\sqrt{128}:)$ shall be the sum of the Quantities proposed to be added.

But if $\overline{\sqrt{:}12+\sqrt{6}:}-\overline{\sqrt{:}12-\sqrt{6}:}$ be multiplyed into it self, the product will be $24-2\sqrt{138}$, whose

whose universal square Root is the difference of the two given Roots.

3. But if the universal Roots to be added or subtracted are not commensurable, *&c.* then they are to be added by $+$, and subtracted by $-$

So if it were required to add $\sqrt{5+\sqrt{3}}$ to $\sqrt{3-\sqrt{2}}$ their sum would be $\sqrt{5+\sqrt{3}}+\sqrt{3-\sqrt{2}}$: And the latter being subtracted from the former, the remainder would be $\sqrt{5+\sqrt{3}}-\sqrt{3-\sqrt{2}}$: And the sum of $\sqrt{ab+c}$: being added to $\sqrt{d+b}$: will be $\sqrt{ab+c}+\sqrt{d+b}$: and the latter being subtracted from the former, the remainder will be $\sqrt{ab+c}-\sqrt{d+b}$:

IV. *The Extraction of the Square Root out of Binomials, and Residuals.*

Subtract the Square of the lesser part of the given Binomial, from the Square of the greater part, and add the Square Root of the remainder to the greater part, and also subtract it therefrom, and then extract the square Roots of the Sum and remainder, and joyn them together by $+$ if the quantity proposed be a Binominal, but by $-$ if it be a Residual, which Roots so joyned, are the square Root of the given Binomial, or Residual.

Example 1.

Extract the Square Root of this Binomial, *viz.*	$38+\sqrt{1300}$
1. From the square of the greater part 38, *viz.* from	1444

2. Sub-

2. Subtract the ſquare of the leſſer part, *viz.* $\sqrt{1300}$ which is	1300
3. The remainder is	144
4. The ſquare root of the remainder is	12
5. To which root if you add the greater part 38, the ſum is	50
6. The half of which ſum is	25
7. The ſquare root of the ſaid half ſum is the greater part of the root ſought, which is	5
8. From the greater part of the given Binomial, *viz.* 38, ſubtract the ſquare root in the fourth ſtep, *viz.* 12, the remainder is	26
9. The half of which is	13
10. The ſquare root of the ſaid half remainder is the leſſer part of the root ſought.	$\sqrt{13}$
11. To which if you add the quantity in the ſeventh ſtep, the ſum will be the ſquare root ſought, *viz.*	$5+\sqrt{13}$

which is the ſquare root of the given Binomial, but if the given ſurd quantity had been a Reſidual, *viz.* if it had been required to extract the ſquare root of $38-\sqrt{1300}$, then the root would have been $5-\sqrt{13}$.

Example.

Example 2.

Extract the ſquare root of this Binomial, *viz.*	$7 + \sqrt{20}$
1. The ſquare of the greater part 7, is	49
2. From which ſubtract the ſquare of the leſſer part, (*viz.* $\sqrt{20}$,) which is	20
3. The remainder is	29
4. The ſquare root of that remaninder is	$\sqrt{29}$
5. To which ſquare root add the greater part of the given Binomial, *viz.* 7, and the ſum is	$7 + \sqrt{29}$
6. The half of which ſum is	$\frac{7}{2} + \sqrt{\frac{29}{4}}$
7. The ſquare root of the ſaid ſum is the greater part of the root ſought, which is	$\sqrt{:\frac{7}{2} + \sqrt{\frac{29}{4}}:}$
8. From the greater part of the given Binomial, (*viz.* from 7) ſubtract $\sqrt{29}$ in the fourth ſtep, and the remainder is	$7 - \sqrt{29}$
9. The half of which remainder is	$\frac{7}{2} - \sqrt{\frac{29}{4}}$
10. The ſquare root of the ſaid half remainder is the leſſer part of the root ſought, which is	$\sqrt{:\frac{7}{2} - \sqrt{\frac{29}{4}}:}$

11. Which

11. Which being joyned to the greater part of the root ſought in the ſeventh ſtep by the ſign $+$, the ſum will be the ſquare root ſought, which is

$$\sqrt{:\tfrac{7}{2}+\sqrt{^2\tfrac{9}{4}}}:+\sqrt{:\tfrac{7}{2}-\sqrt{^2\tfrac{9}{4}}}$$

But if the leſſer part of the ſaid root found in the tenth ſtep be joyned to the greater part found in the ſeventh ſtep by interpoſing the ſign $-$ inſtead of $+$, it will then be the ſquare root of the reſidual $7-\sqrt{20}$.

Example. 3.

Let it be required to extract the ſquare root of this Binomial, *viz.* $aa+d$ added to $2a\sqrt{d}$, ſuppoſing the greater part of the given Binomial, to be $aa+d$. Then,

1. The ſquare of the greater part is — $aaaa+2bdd+bb$
2. From which ſubtract the ſquare of the leſſer part ($2a\sqrt{d}$) *viz.* $4daa$, and the remainder is — $aaaa-2add+dd$
3. The ſquare root of that remainder is — $aa-d$
4. To which ſquare root add the greater part of the given Binomial, *viz.* $aa+d$, and the ſum is — $2aa$
5. The half of which ſum is — aa
6. The ſquare root of which half ſum is the greater part of the root ſought, which is — a

7. From

7. From ($aa + d$) the greater part of the given Binomial subtract the square root found in the third step ($aa - d$) and the remainder is — $2d$

8. The half of which remainder is — d

9. The square root of which half remainder, is the lesser part of the root sought, *viz.* — $\sqrt{d}$

10. Which said root being joyned to the greater part found in the sixth step by the sign $+$, it will be the root sought, *viz.* — $a + \sqrt{d}$

but if the quantity in the ninth step be joyned to the quantity in the sixth step, by interposing the sign —, it will then be the square root of the residual, $aa + d$ less $2a\sqrt{d}$.

Example. 4.

Let it be required to extract the square root of — $\overline{e + d\sqrt{ed}}$ more $2ed$

supposing the greater part to be — $\overline{e + d\sqrt{ed}}$, then

1. The square of the greater part is — $eeed + 2eedd + eddd$

2. From which subtract the square of the lesser part, which is — $4eedd$

3. And the remainder is — $eeed - 2eedd - eddd$

4. The square root of that remainder is — $\overline{e - d\sqrt{ed}}$

5. To

5. To which if you add the greater part of the given binomial, the ſum is	$2e\surd ed$
6. The half of which ſum is	$e\surd ed$
7. The ſquare root of the ſaid half ſum is the greater part of the root ſought, which is	$\surd\overline{:e\surd ed:}$
8. From the greater part of the given binomial ſubtract the ſquare root found in the fourth ſtep, and the remainder is	$2d\surd ed$
9. The half of which remainder is	$d\surd ed$
10. The ſquare root of the ſaid half remainder is the leſſer part of the root ſought, which is	$\surd\overline{:d\surd ed:}$

11. If to the greater part of the root ſought in the ſeventh ſtep, you join the leſſer part in the eleventh ſtep, by interpoſing the ſign +, it will then be the root ſought, which is

$$\surd\overline{:e\surd ed:}+\surd\overline{:d\surd ed:}$$

But if the two ſaid quantities are joyned together by the interpoſition of the ſign —, it will then be the ſquare root of the reſidual $e+d\surd ed$ leſs $2ed$.

V. *Some Queſtions to exerciſe the Rules of this and the foregoing Chapters.*

QUEST. 1.

Let it be required to divide 100 (or c) into two ſuch unequal parts, that 100 multiplyed by the

the leſſer part may be equal to the ſquare of the greater.

RESOLVTION.

1. For the greater number put	a
2. Then will the leſſer be	$c - a$
3. By which if you multiply 100 (or c) the product will be	$cc - ca$
4. Which quantity in the 3d ſtep muſt be equal to the ſquare of the Quantity in the firſt ſtep, whence this equation.	$aa = cc - ca$
5. Which Equation being reduced by the rules of the 11 Chap. and ſolved, the value of a will be diſcovered to be	$a = \sqrt{:cc + \frac{cc}{4}:} - \frac{1}{2}c$

which Equation in the laſt ſtep being duly conſidered, will preſent you with this

Theorem.

To the Square of the given line or number add a fourth part of its Square, and extract the Square root of that ſum; then from the ſaid ſquare root ſubtract half the given line, ſo ſhall the remainder be the greater ſegment, or number ſought.

QVEST. 2.

What Number is that whoſe ſquare being made leſs by the Rectangle of it ſelf drawn into 12 (or b) the remainder is equal to f?

1. For

1. For the number sought put a
2. The square of which is aa
3. The Rectangle of a in b is ba
4. If the quantity in the third step be subtracted from the quantity in the second step, the remainder is equal to f, whence this Equation. $aa - ba = f$
5. Which Equation being solved by the rules of the 11th Chapter the value of a will be found to be $a = +\frac{1}{2}b + \sqrt{:f + \frac{bb}{4}:}$

The Proof.

6. If $a = \frac{1}{2}b + \sqrt{:f + \frac{1}{4}bb:}$
7. Then by subtracting $\frac{1}{2}b$ from each part of the equation there remaineth. $a - \frac{1}{2}b = \sqrt{:f + \frac{1}{4}bb:}$
8. Then by squaring each part of the equation you have $aa - ba + \frac{1}{4}bb = f + \frac{1}{4}bb:$
9. And by subtracting $\frac{1}{4}bb$ from both sides of the equation there remaineth $aa - ba = f$

which was to be proved

QUEST. 10.

1. Let c and d be put for two such known Quantities that d not $\sqsubset \frac{1}{4}cc$, and let a be put for a quantity unknown, and let it be granted that $ca - aa = d$ what is the value of a?

2. The given equation in the first step is one of the third form mentioned in the beginning of the fifteenth Chapter, and it will be found that the 2 values of a are

$a = \frac{1}{2}c +$

$$a = \frac{1}{2}c + \sqrt{:\frac{1}{4}cc - d:}$$

And

$$a = \frac{1}{2}c - \sqrt{:\frac{1}{4}cc - d:}$$

By either of which values of a the Equation propounded in the first step may be expounded, as will appear by the

DEMONSTRATION.

1. If $a = \frac{1}{2}c + \sqrt{:\frac{1}{4}cc - d:}$
2. Then by the transposition of $\frac{1}{2}c$ to the contrary coast, it is $a - \frac{1}{2}c = \sqrt{:\frac{1}{4}cc = d:}$
3. And by squaring each part of the last Equation, it is $aa - ca + \frac{1}{4}cc = \frac{1}{4}cc - d$
4. And by subtracting $\frac{1}{4}cc$ from each part of the equation, it is $aa - ca = -d$
5. And by changing the signs on the quantities on each side of the equation, it is $ca - aa = d$

Which was to be demonstrated.

6. Again, if $a = \frac{1}{2}c - \sqrt{:\frac{1}{4}cc - d:}$
7. Then by transposition of $\sqrt{:\frac{1}{4}cc - d}$ to the other side it is $a + \sqrt{:\frac{1}{4}cc - d:} = \frac{1}{4}cc$
8. And by transposition of a it is $\sqrt{:\frac{1}{4}cc - d:} = \frac{1}{4}cc - a$
9. And by squaring each part of the equation it will then be $\frac{1}{4}cc - d = \frac{1}{4}cc - ca + aa$

10. And the ſubtracting $\frac{1}{4}cc$ from both parts of the equation, it is $-d = -ca + aa$

11. And the quantities on both ſides of the equation being tranſpoſed to the contrary coaſt, and the ſigns of each thereby changed, the equation will then be $ca - aa = d$

which was likewiſe to be proved.

QUEST. 2.

Let it be required to divide 100 into two ſuch parts that if each part be divided by the other, the ſum of the Quotients may be 3. This is *Queſt.* 1. of the ninth Chapter of the ſecond Book of *Kerſey*'s Elements of Algebra, and it is thus wrought, *viz.*

1. For one of the parts ſought put a
2. Then will the other be $100 - a$
3. Each of which quantities in the firſt and ſecond ſteps being mutually divided by each other (according to the import of the queſtion) this equation ariſeth $\frac{a}{100-a} + \frac{100-a}{a} = 3$
4. Which equation being duly reduced, gives $100a - aa = 2000$
5. Which is an equation of the third form mentioned in Chap. 15. and being ſolved according to the method there given, the two values of a writ be found to be $a = \begin{cases} 50 + 10\sqrt{5} \\ 50 - 10\sqrt{5} \end{cases}$

Which you may eaſily prove at your leiſure.

CHAP.

CHAP. XVIII.

Algebraical Queſtions Reſolved by various Poſitions.

MR. *Kerſey* in the Twelfth Chapter of the ſecond Book of his Elements of Algebra, hath laid down Rules for the ſolution of Queſtions Algebraically by various Poſitions; aſſuming a peculiar letter to repreſent every one of the Quantities ſought, *viz.* *a* for one unknown Quantity, *e* for another, and *y* for a third, *&c.* and for the performance of the work he hath laid down 3 Rules which are as followeth, *viz.*

RULE 1.

When many Quantities are ſought in a Queſtion, let them be repreſented by various letters, and let the tenor of the Queſtion be repreſented by Equations, which done by Tranſpoſition find what any ſingle letter in the firſt equation is equal to; Then whereſoever that Letter is found in the other equations, inſtead thereof, take what it is found equal to, ſo will that letter quite vaniſh out of the following Equations; Then by Tranſpoſition ſet a ſecond letter alone in one of thoſe equations out of which the firſt letter was cancelled and proceed as before, ſo at length one of the letters will be made known, by help of which the rest will be easily discovered.

RULE 2.

When the same Quantity (suppose *a*) is found in two several Equations, and equal Numbers are prefixed to those Quantities, then if their signs be both +, or both —, subtract the lesser Equation from the greater; but if the signs be one +, and the other —, then add those two Equations together, so will the said Quantity *a* quite vanish.

RULE 3.

When the same Quantity (suppose *a*) is found in two several Equations, but the Numbers prefixed to those equal Quantities are unequal, those two Equations may be reduced to two others which shall have equal Numbers prefixed to the said Quantity *a* thus, *viz*, Multiply all the Quantities in the first Equation by the number prefixed to *a* in the second Equation; and also multiply all the Quantities in the second Equation by the number prefixed to the same quantity *a* in the first Equation, so by such alternate multiplication two new Equations will be produced, wherein the Numbers prefixed to the said quantity *a* will be equal to one another, and then proceed according to the second Rule, and expel the same quantity out of the rest of the Equations; proceed in like manner with a second quantity, until at length some one quantity be made known; by which all the rest may be

found

found out. The three foregoing Rules will be exercised in the Resolution of the following *Questions.*

QUEST. 1.

Divide 100 (or c) into two such Numbers, (*viz.* a and e) that $\frac{a}{3}+\frac{e}{5}$ may be equal to 30 (or d) I demand the numbers a and e?

RESOLUTION.

1. If — $a+e=c$
2. And — $\frac{a}{3}+\frac{e}{5}=d$
3. Then by transposition of e in the first step, you will have — $a=c-e$
4. By reducing the Equation in the second step, so as a may solely possess one side thereof, you will have — $a=\frac{15d-3e}{5}$
5. If instead of a in the 4th step, you take what a is equal to in the third step, you will have this equation, *viz.* — $c-e=\frac{15d-3e}{5}$
6. The first part of the equation in the fifth step being multiplyed by 5, will give — $5c-5e=15d-3e$
7. By the transposition of $-5e$ it is — $5c=15d+2e$
8. And by the transposition of $15d$ in the last step, you have — $5c-15d=2e$

9. Each part of the Equation in the last step being divided by 2, will give the value of *e*, *viz.* $\frac{5c - 15d}{2} = e$

I say the value of *e* is 25, and the value of *a* is $100 - 25 = 75$, which will answer the Conditions of the Question. As appears by

The Proof.

$$25 + 75 = 100; \text{ and } \frac{25}{3} + \frac{57}{3} = 30$$

QUEST. 2.

There are two Numbers (*a* the greater, and *e* the lesser) whose difference 4 (or *b*) and the difference of their Squares is 64 (or *c*) what are the Numbers?

RESOLUTION.

1. If $a - e = b$
2. Then by the transposition of *e* you have $a = b + e$
3. And if $aa - ee = c$
4. Then by transposition of *ee* it is $aa = c + ee$
5. If both parts of the equation in the second step be squared, it will be $aa = bb + 2be + ee$
6. And if instead of *aa* in the fifth Equation, you place what it is equal to in the fourth step, the Equation will then be

$$c + ee = bb + 2be + ee$$

7. By

7. By subtracting ce from both parts of the last equation you will have } $c = bb + 2be$

8. By dividing both parts of the last equation by b, you will then have } $\frac{c}{b} = b + 2e$

9. By transposition of b in the last step, the Equation will then be } $\frac{c}{b} - b = 2e$

10. And if both parts of the equation in the last step be divided by 2, the value of e will then be discovered to be } $\frac{c}{2b} \quad \frac{b}{2} = e$

I say the value of e (the lesser number) is 6, and by the second step (a) the greater number is $e + b = 6 + 4 = 10$, which two numbers, (*viz.* 10 and 6) will satisfie the Conditions of the Question, as will appear by

The Proof.

$10 - 6 = 4$, and $\overline{10 \times 10} - \overline{6 \times 6} = 64$

QUEST. 3.

A Maid being at Market sold 10 dozen of Eggs, and twelve Pounds of Butter for thirteen Shillings. And at another time, and at the same rate, she selleth Eight dozen of Eggs, and 18 pounds of Butter for 16 shillings, I demand how she sold her Eggs *per* dozen, and her butter *per* pound?

Let a represent the desired value of a dozen of Eggs, and for the price of a pound of Butter put *e*, and then may the question, being abstracted from words, be stated thus, *viz.*

1. If $10a + 12e = 13$
2. And $8a + 18e = 16$

What are the values of *a* and *e*?

RESOLUTION.

3. By transposition of $12e$ in the first step, that equation will be } $10a = 13 - 12e$
4. And both parts of the last equation being divided by 10, it is } $a = \frac{13 - 12e}{10}$
5. By transposition of $8e$ in the second step, that equation will be } $8a = 16 - 18e$
6. Each part of the last equation being divided by 8, will give } $a = \frac{16 - 18e}{8}$
7. If instead of *a* in the sixth step, you place what it is equal to in the fourth step, the equation will then be } $\frac{13 - 12e}{10} = \frac{16 - 18e}{8}$
8. Both parts of the last Equation being reduced to Integers will give } $104 - 96e = 160 - 180e$
9. By transposition of $180e$ and 104 in the last equation each to the contrary coast, the equation will then be } $84e = 56$

10. If

10. If each part of the last Equation be divided by 84, the value of *e* will be discovered to be $e = \frac{56}{84} = \frac{2}{3}$ which is 8 *d.* for the price of one pound of Butter.

11. By the tenth step the value of *e* is discovered to be $\frac{2}{3}$ *s.* by which means the value of *a* (by the quantities in the fourth and sixth steps) is found to be 6 *d.* for the 4th step, is $a = \frac{13 - 12}{10}$

And it hath been found before, that $e = \frac{2}{3}$ *s*, so that $12e = 12 \times \frac{2}{3} s = 8$ *s.* and $\frac{13 s. - 8 s.}{10} = \frac{5}{10 s.} = 6 d.$ so the Maid sold her Eggs at 6 *d.* per dozen, and her Butter at 8 *d.* per pound, which will answer the conditions of the question.

QUEST. 4.

Three Men, *viz. A, B,* and *C* discourse thus together concerning their Age; quoth *B* to *A*, your age added to mine is 54 (or *b*) years; quoth *C* to *B*, and my age added to yours makes 78 (or *c*) years? and quoth *A* to *C*, my age added to yours is 72 (or *d*) years. I demand the age of each person?

Let the age of each Person be represented by the letters *a e y*, *viz.* for the age of *A* put *a*, for the age of *B* put *e*, and for the age of *C* put *y*; and the *Question* being abstracted from words, will be as followeth, *viz.*

1. If

1. If $a+e=b(=54)$
2. And $e+y=c(=78)$
3. And $y+a=d(=72)$

What are the values of *a e* and *y*?

RESOLUTION.

4. By transpoſition of *e* in the firſt ſtep there will ariſe — $a=b-e$

5. If inſtead of *a* in the third ſtep you put what *a* is equal to in the fourth ſtep, there will ariſe — $y+b-e=d$

6. By the tranſpoſition of *d* and *e* in the laſt ſtep, there ariſeth — $e=y+b-d$

7. And if inſtead of *e* in the ſecond ſtep you take the latter part of the ſixth ſtep, there will then ariſe — $2y+b-d=c$

8. In which laſt equation there is no unknown quantity but *y*, and therefore the equation being duly reduced, will diſcover the value of *y* to be — $y=\frac{c+d-b}{2}$

9. If in the ſixth ſtep inſtead of *y* you take the latter part of the equation in the eighth ſtep, the value of *e* will be found to be — $e=\frac{c+b-d}{2}$

10. And if inſtead of *e* in the fourth ſtep, you take the latter part of the Equation in the ninth ſtep, the value of *a* will be diſcovered, *viz.* — $a=\frac{b+d-c}{2}$

And

And thus the work is finished, and the equations in the eighth, ninth, and tenth steps present you with this

CANON.

From the sum of every two of the three given Numbers subtract the third number remaining, so shall the three remaining numbers being divided by 2 be the Numbers sought. So the Number sought in the Question, *viz.* *a*, *e*, and *y*, are found to be 24, 30, and 48, *viz.* the age of *A* is 24, the age of *B* is 30, and the age of *C* is 48, which three Numbers will satisfie the Conditions of the Question for $24+30=54$, and $30+48=78$, and $48+24=72$:

QUESTION 5.

What two Numbers are those whose sum is 20, (or *b*) and their difference 4 (or *c*)?

Let *a* be put for the greater Number sought, and *e* for the lesser, and then the Question being extracted from words may be stated thus, *viz*

1. If $a+e=b$ (20)
2. And $a-e=c$ (4)

What are *a* and *e*?

RESOLUTION.

3. Forasmuch as $+1a$ is found in each of the equations in the first and second steps, therefore (by the second Rule) they being subtracted, do give this Equation, *viz.* $2e=b-c$

4. And

4. And by dividing both parts of the Equation in the third step by 2, the value of *e* will be discovered, *viz.* $e = \frac{1}{2}b - \frac{1}{2}c$

5. And if instead of *e* in the second step you put what *e* is equal to in the fourth step, you will have this Equation, *viz.* $a - \frac{1}{2}b + \frac{1}{2}c = e$

6. By the transposition of $-\frac{1}{2}b$ and $+\frac{1}{2}c$ to the contrary coast, the value of *a* will be discovered, *viz.* $a = \frac{1}{2}b + \frac{1}{2}c$

From the fourth and sixth steps is raised this

CANON.

If from half the sum of two numbers you subtract half their difference, the remainder will be the lesser number; and if to half their sum you add half their Difference, that sum will be the greater number, whereby the two numbers sought in this *Question* are found to be 12 and 8; for $12 + 8 = 20$ and $12 - 8 = 4$.

QUEST. 6.

What 3 Numbers are those, that if to the first there be added 121 (or *b*) the sum will be equal to the sum of the first and second; and if to the second there be added 121, the sum will be equal to double the sum of the first and third; and if to the third there be added 121, their sum will be trible the sum of the first and second?

If for the number sought you put *a*, *e*, and *y*, *viz.* for the first number *a*, for the second *e*, and

and for the third *y*, then the Question being abstracted from words, may be stated thus, *viz.*

1. If $a+b=e+y$
2. And $e+b=2a+2y$
3. And $y+b=3a+3e$

What are the Numbers $a\ e\ y$?

RESOLUTION.

4. By the transposition of *y* in the first Equation, there ariseth } $a+b-y=e$
5. And if instead of *e* in the second Equation you take what is equal thereto in the fourth equation, there ariseth

$$a+b-y+b=2a+2y.$$

6. The last Equation after due reduction will be } $2b=a+3y$
7. And if instead of $3e$ in the third Equation you take the triple of what *e* is found equal to, in the fourth step, you will find the following Equation to arise, *viz.*

$$y+b=3a+3a+3b-3y.$$

8. Which Equation after due reduction by transposition the quantities will be found to be } $4y=6a+2b$
9. And both parts of the last Equation being divided by 4, there ariseth } $y=\frac{3}{2}a+\frac{1}{2}b$

10. Then

10. Then if instead of $3y$ in the sixth equation there be taken the triple of the latter part of the ninth equation, there ariseth $2b = a + \frac{3}{2}a + \frac{3}{2}b$

11. After due Reduction of the equation in the tenth step, the value of *a* will be discovered, *viz.* $a = \frac{b}{11}$

12. Again, if instead of *a* the sixth Equation you put the lattter part of the Eleventh Equation, there ariseth $2b = \frac{b}{11} + 3y$

13. After due reduction of the equation in the twelfth step, the value of *y* will be discovered to be $y = \frac{7b}{11}$

14. And if for *a* and *y* in the fourth step there be put their equals in the 11th and 13th steps there will arise $\frac{b}{11} + \frac{7b}{11} = e$

15. The Equation in the last step being duly reduced, will discover the value of *e*, *viz.* $e = \frac{5b}{11}$

From the eleventh, thirteenth, and fifteenth steps is gathered this

CANON.

If the number given to be added to the three numbers required be divided by 11, the Quotient will give the first number, and its Quintuple (or Product by 5) being divided by 11, will give the second number, and its septuple (or product

duct by 7) being divided by 11, will give the third number.

By which Canon the numbers required in the Question are 11, 55, and 77, (the second being 5 times as much as the first, and the third is 7 times as much as the first) which said numbers will satisfie the conditions of the Question, as will appear by

PROOF.

$$11+121=55+77=132.$$

$$\text{And } 55+121=2\times\overline{11+77}=176.$$

$$\text{And } 77+121=3\times\overline{11+55}=198.$$

which was to be done.

QUEST. 7.

What two numbers are those that if to 10 times the greater there be added six times the lesser, the sum will be 228 (or b) and if from 4 times the greater you subtract 2 times the lesser, the remainder will be 56 (or c)? For the two numbers put a and e, and then the foregoing question being abstracted from words, may be stated thus. *viz.*

1. If $10a+6e=b$
2. And $4a-2e=c$

What are the numbers a and e?

RESOLUTION.

3. The first equation (according to the third Rule) being multiplyed by 4, which is prefixed to a in the second equation, produceth $\quad 40a+24e=4b$

4. And

4. And the ſecond Equation being multiplyed by 10, which is prefixed to *a* in the firſt, it produceth	$40a - 20c = 10c$
5. And if from the Equation in the third ſtep you ſubtract the equation in the fourth ſtep, becauſe 40*a* is found in both, (according to the ſecond Rule) there ariſeth this equation, *viz.*	$44e = 4b - 10c$
6. Both parts of the equation in the fifth ſtep, being divided by 44 the value of *e* will be diſcovered to be	$e = \frac{4b - 10c}{44}$
7. If inſtead of — 2*e* in the ſecond ſtep you put double the latter part of the equation in the ſixth ſtep, you will have this equation	$4a - \frac{2b - 5c}{11} = c$
8. The ſeventh equation being duly reduc'd, the value of *a* will be diſcover'd to be	$a = \frac{6c + 2b}{44}$

By the ſixth and eighth ſteps the numers ſought are 18 and 8, which will anſwer the conditions of the *Queſtion*, as you may perceive by

The Proof.

$$10 \times 18 + 6 \times 8 = 228$$
And
$$4 \times 18 - 2 \times 8 = 56$$

Soli Deo Gloria.

F I N I S.