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THE ACOUSTIC PRESSURE IN A WEDGE-SHAPED
WATER LAYER OVERLYING A FAST FLUID BOTTOM

by

Chil Ki Baek

March 1984

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The Acoustic Pressure in a Wedge-Shaped Water Layer Overlying a Fast Fluid Bottom

by

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A simple equation and computer program for the pressure and phase distribution in a wedge-shaped medium overlying a fast absorbing bottom from a point source at infinite distance from the wedge apex were formulated by using the method of images. The computer program used for calculations was tested for perfectly reflecting boundaries. A sample case using a more realistic bottom is presented and discussed.

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I. INTRODUCTION

Sound radiated from a source in a wedge-shaped medium overlying a fast bottom has been investigated. In 1978, Kawamura and Ioannou [Ref. 1] computed the pressure amplitude and phase distribution along the interface between a tapered fluid layer and an underlying fast fluid bottom. A simple model based on a combination of normal modes and ray theory failed to predict adequately the pressure amplitude and phase along the wedge-bottom interface. In 1980, Bradshaw [Ref. 2] calculated the pressure and phase distribution of sound in a fast fluid medium underlying a tapered fluid medium.

The pressure in a wedge-shaped fluid layer overlying a fast bottom (See Figure 1, 2) can be calculated by using the method of images. If we assume isospeed medium, then it is straightforward to apply the method of images [Refs. 3, 4]. The images lie on a circle (See Figure 3) whose center is the apex of the wedge. The lowest images (closest to the source) correspond to rays of sound which make grazing reflections from the surfaces of the wedge. Higher images correspond to rays with greater angles of elevation and depression; these rays suffer more reflections from the surfaces of the wedge. Finally, images are encountered for which the rays exceed the critical angle at the bottom.

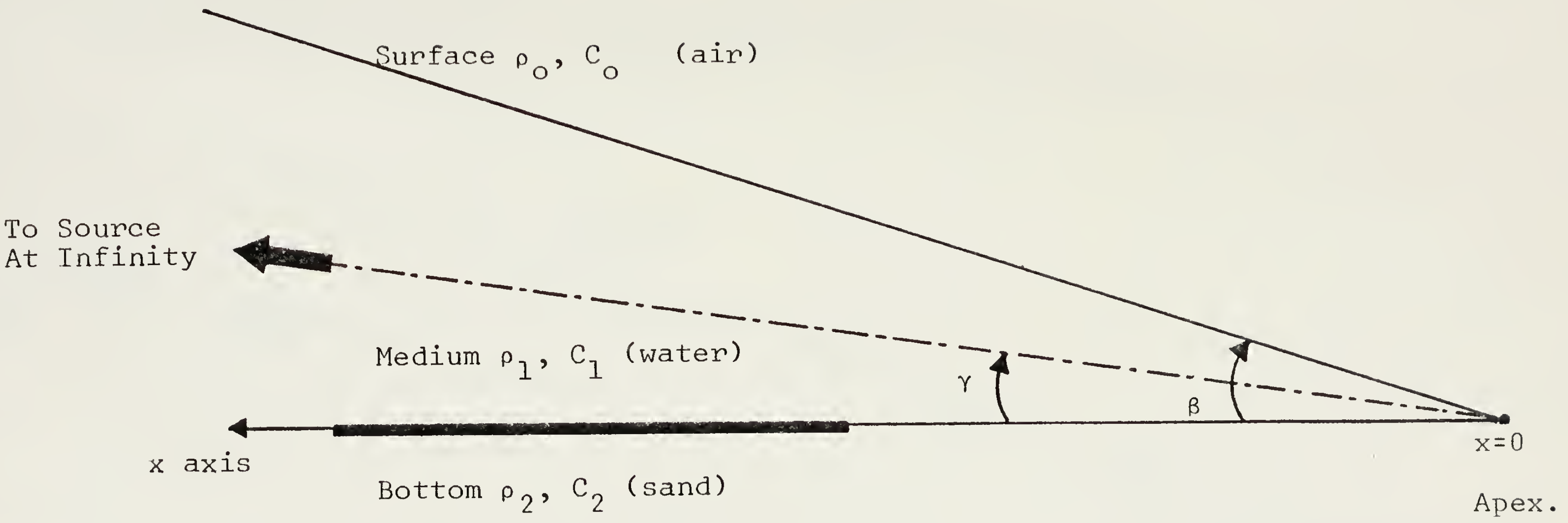

Figure 1 The Cross Section of the Wedge

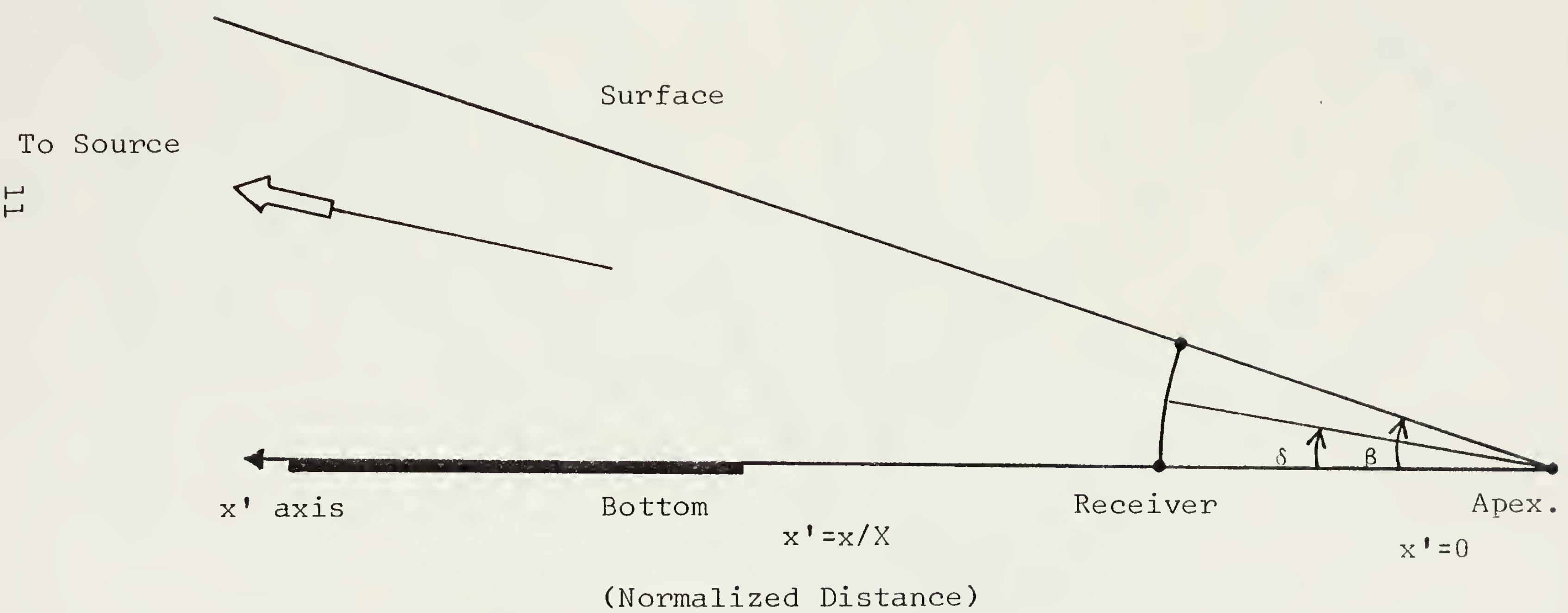

Figure 2 Wedge Problem Geometry

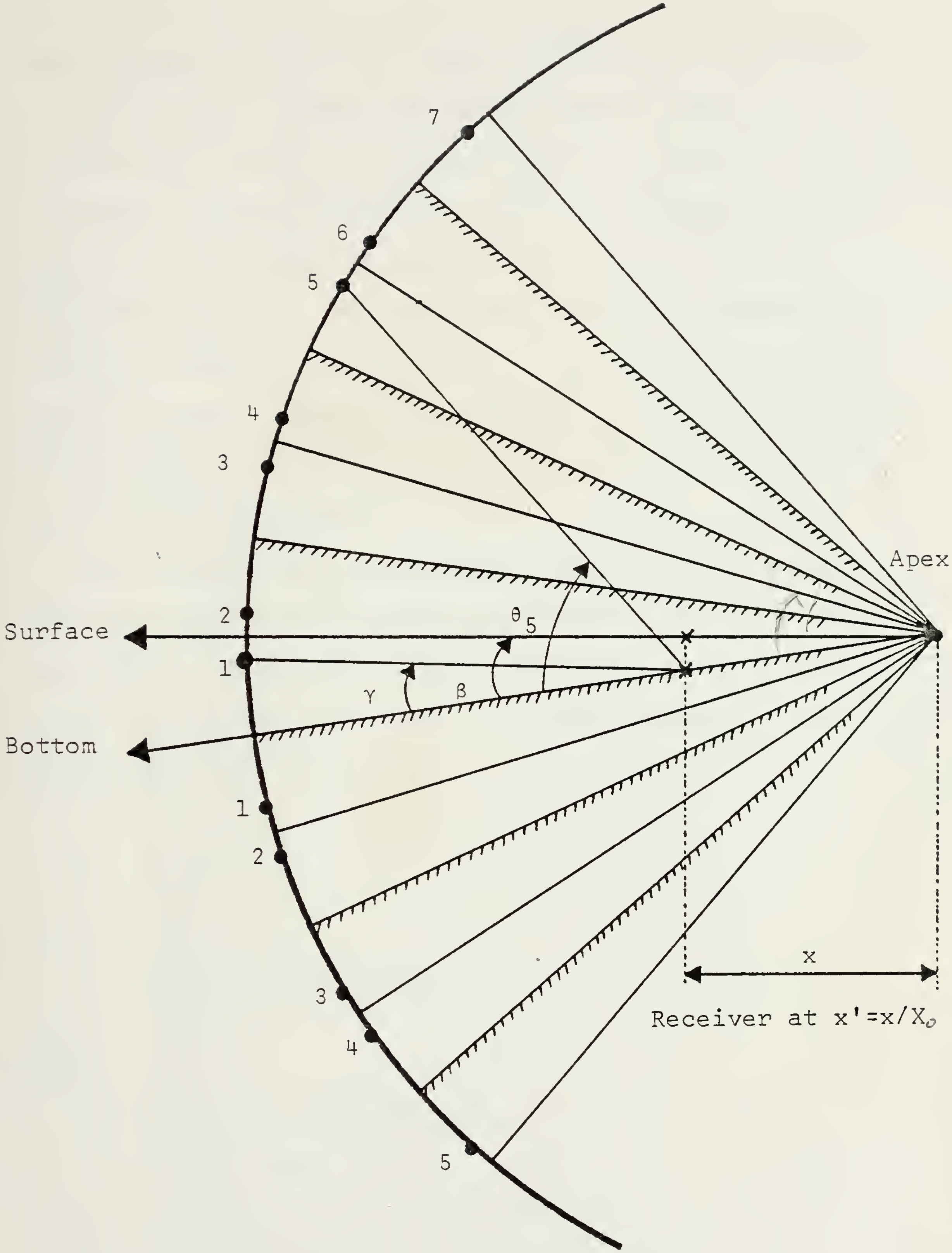

Figure 3 The Geometry of Image Solution

Higher images correspond to more reflections from the bottom so the effective strength of these higher images will be progressively reduced. Summation of the contributions from the source and various images yields the complex acoustic pressure in the wedge if both boundaries are smooth, the propagation loss is only that associated with geometry and absorption losses. A ray will, on each encounter with the sloping bottom, increase its angles of incidence, until the grazing angle will become greater than the critical angle and sound energy enters the bottom.

All distances are normalized to the distance X measured from the apex along the wedge interface at which the critical angle is first exceeded and the lowest mode attains cutoff; this distance X called the dump distance is defined by (See Figure 2), [Ref. 5]

$$X = h / \tan(\beta)$$

$$= \frac{\lambda_1}{4 * \sin(\theta_c) * \tan(\beta)} \qquad (1)$$

where

θ_c = critical grazing angle

β = wedge angle

λ_1 = wavelength in the wedge medium

h = channel depth at X (dump distance)

For a point source of unit pressure at one meter from the source, the complex pressure P(r,t) can be written as

$$\tilde{P}(r,t) = \tilde{P}(r) * \exp(jwt) \qquad (2)$$

where

$$\tilde{P}(r) = (1/r) * \exp(-jkr) \qquad (3)$$

r = distance from a point source to a receiver

w = angular frequency.

For convenience, we assumed that the amplitude of the source is proportional to its distance from the wedge apex. This simplifies calculations considerably, particularly in the limit of large source-apex distance compared to a wavelength.

The purposes of this research are to:

1. Develop a simple expression for the pressure and phase distributions in the wedge.

2. Develop a computer program for calculating the pressure and phase by using the method of images.

II. THEORY

The complex acoustic pressure in the wedge from the point source at the infinite distance can be determined by using the method of images. Assume both the surface and the bottom of the wedge are smooth, and isospeed fluid media. The normalized complex acoustic pressure in the wedge can be expressed as

$$\tilde{P}_N(x) = \exp[jk_1 \, x \cos(\theta_n)] \tag{4}$$

where θ_n is the angle between the line joining Nth image to the apex and the bottom, k_1 is the wave number in the wedge, and x is the distance from the apex. (See Figure 3)

The total pressure of the various images and source along the line of constant x from the apex can be determined by using the method of images (See Figures 4, 5),

$$\tilde{P}_N(x) = \sum_{n=1}^{N} (-1)^{\mathrm{INT}(\frac{n}{2})} [g_{n-2}\exp[jk_1 \, x \cos(\theta_n - \delta)]$$

$$+ g_n \exp[jk_1 \, x \cos(\theta_n + \delta)]] \tag{5}$$

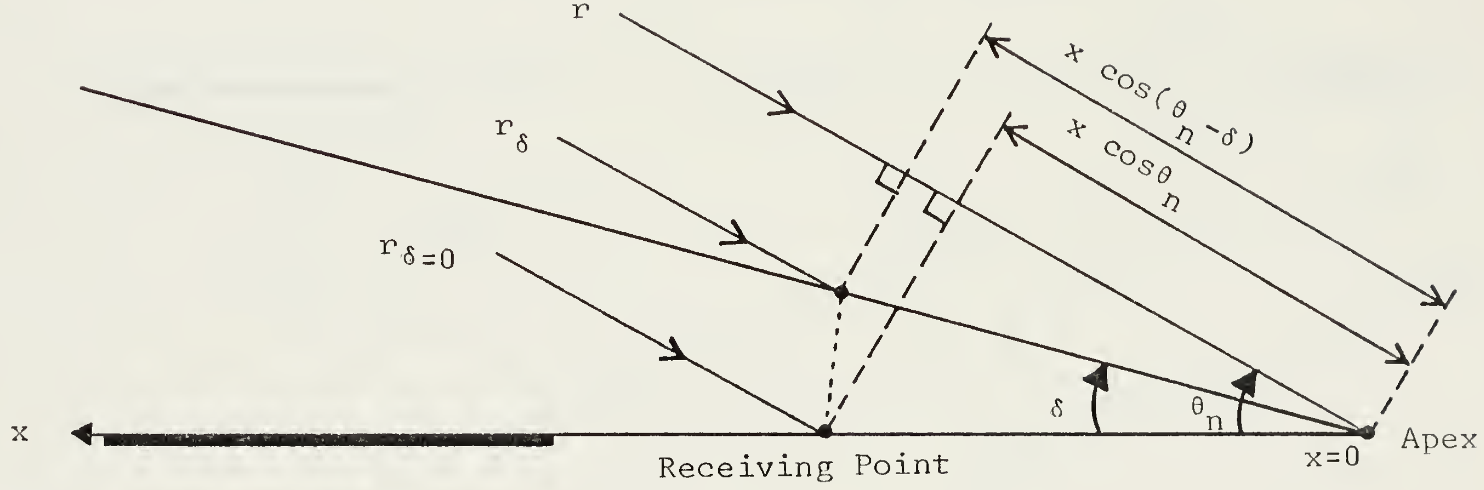

Figure 4 The Sound Field in the Wedge from the Images Above the Wedge-Bottom Interface

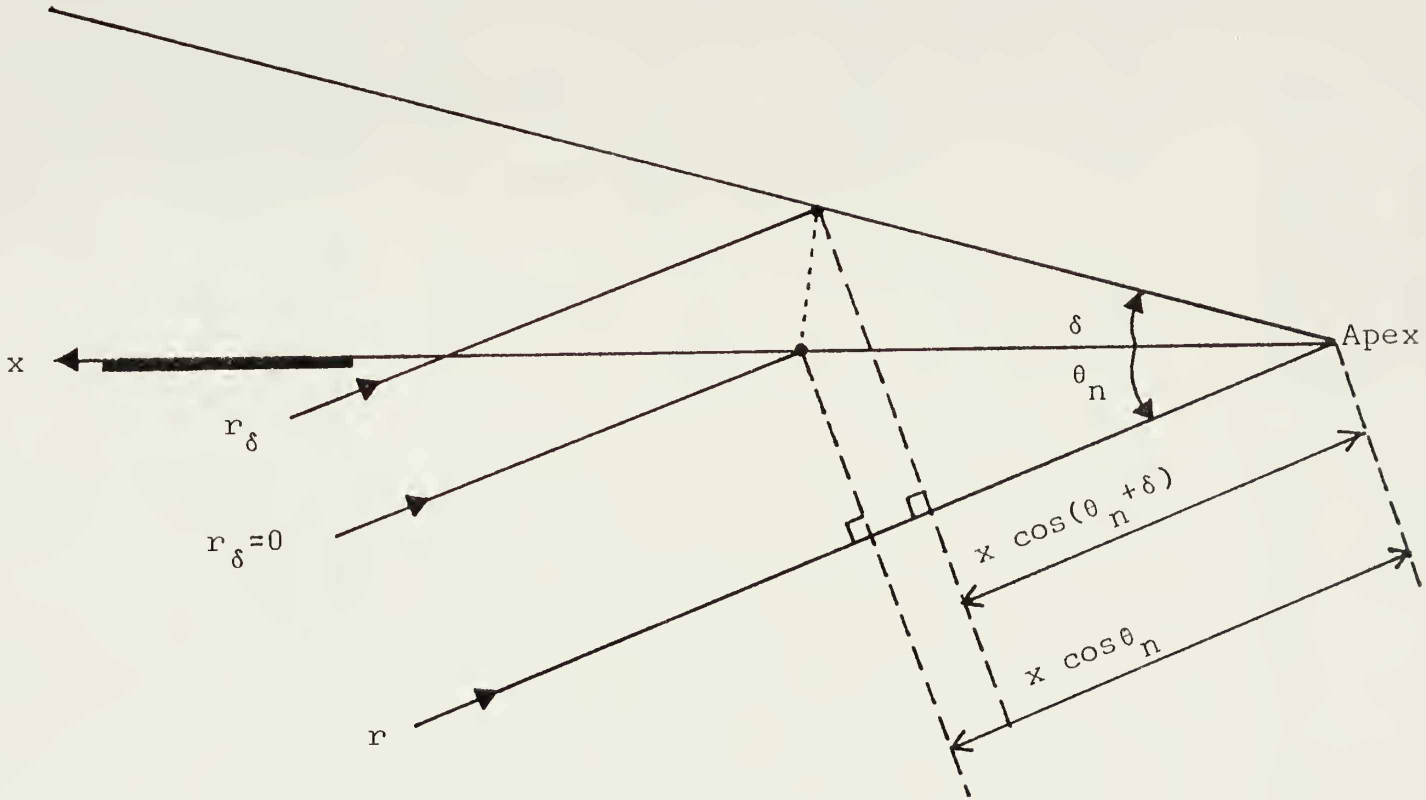

Figure 5 The Sound Field in the Wedge from the Images Below the Wedge-Bottom Interface

where

$N = INT(180/\beta)$,

β = the wedge angle,

δ = the receiver angle,

γ = the source angle. (See Figures 2, 3)

$$\theta_n = (n-1)\beta + \gamma, \quad n = 1,3,5,7...$$

$$\theta_n = n\beta - \gamma, \quad n = 2,4,6,8... \tag{6}$$

$$g_n = R(\theta_n) * g_{n-2}$$

$$= \prod_{\substack{m=1,3,5...\\2,4,6...}}^{n} R(\theta_m) \tag{7}$$

$$(g_{-1} = g_o = + 1.0)$$

g_{n-2} and g_n are path parameters from the nth image and $R(\theta_n)$ is the reflection coefficient of the bottom with grazing angle θ_n, given by Refs. 6 and 7,

$$R(\theta_n) = \frac{D * \sin\theta_n + M_2 + j\, M_1}{D * \sin\theta_n - M_2 - j\, M_1} \tag{8}$$

where

$$D = \rho_2/\rho_1$$

$$M_1 = \sqrt{\sqrt{A^2 + B^2} + A} \;/\; \sqrt{2}$$

$$M_2 = -\sqrt{\sqrt{A^2 + B^2} - A} \;/\; \sqrt{2}$$

$$A = \cos^2\theta_n - n^2$$

$$B = 2\, n\alpha/k_2$$

where α is the absorption coefficient in the bottom (Nepers/m), and k_2 is wave number in the bottom.

$n = C_1/C_2$ (Index of Refraction).

To check the theoretical results, expand Equation (5) into few terms as follows:

$$\tilde{P}_N(x) = \exp[jk_1 x \cos(\theta_1-\delta)] - \exp[jk_1 x \cos(\theta_2-\delta)]$$

$$+ g_1[\exp[jk_1 x \cos(\theta_1+\delta)] - \exp[jk_1 x \cos(\theta_3-\delta)]]$$

$$- g_2[\exp[jk_1 x \cos(\theta_2+\delta)] - \exp[jk_1 x \cos(\theta_4-\delta)]]$$

$$\vdots \qquad\qquad \vdots \qquad (9)$$

a. If a receiver locates at pressure release surface ($\delta=\beta$), then (See Appendix B):

$$\theta_1 - \delta = -\beta + \gamma \qquad \theta_2 - \delta = \beta - \gamma$$

$$\theta_1 + \delta = \beta + \gamma \qquad \theta_3 - \delta = \beta + \gamma$$

$$\theta_2 + \delta = 3\beta - \gamma \qquad \theta_4 - \delta = 3\beta - \gamma$$

$$\vdots \qquad\qquad \vdots$$

All terms in Equation (9) cancel exactly as they should.

b. If source locate at pressure release surface ($\gamma=\beta$), then

$$\theta_1 - \delta = \beta - \delta \qquad \theta_2 - \delta = \beta - \delta$$

$$\theta_1 + \delta = \beta + \delta \qquad \theta_2 + \delta = \beta + \delta$$

$$\theta_3 - \delta = 3\beta - \delta \qquad \theta_4 - \delta = 3\beta - \delta$$

$$\vdots \qquad\qquad \vdots$$

Also,

$$\theta_1 = \theta_2 = \beta \text{ ------------- } g_1 = g_2 = R(\beta)$$

$$\theta_3 = \theta_4 = 3\beta \text{ ------------- } g_3 = g_4 = R(3\beta) * R(\beta)$$

$$\theta_5 = \theta_6 = 5\beta \text{ ------------- } g_5 = g_6 = R(5\beta) * R(3\beta) * R(\beta)$$

$$\vdots \qquad\qquad \vdots$$

Again, all terms in Equation (9) exactly cancel as they should.

III. COMPUTATIONS

A. NORMALIZED DUMP DISTANCE

To use the normalized dump distance in computing Equation (4) the following derivations are given by

$$\cos\theta_c = C_1/C_2 = k_2/k_1 \tag{10}$$

$$X = \frac{\lambda_1}{4 \operatorname{Sin} \theta_c \tan \beta} \quad \text{(from Equation (1))}$$

$$= \frac{\pi}{2 k_1 \operatorname{Sin} \theta_c \tan \beta}$$

$$k_1 X = \frac{\pi}{2 \sin \theta_c \tan \beta} \tag{11}$$

$$k_2 X = k_1 X \cos \theta_c$$

$$= \frac{\pi \cos \theta_c}{2 \sin \theta_c \tan \beta}$$

$$= \frac{\pi}{2 \tan \theta_c \tan \beta} \qquad (12)$$

$$k_1 x = k_2 X \, (k_1/k_2)(x/X)$$

$$= k_2 X \, (1/\cos\theta_c)(x/X)$$

$$= (k_2 C_2 X/C_1)(x/X) \qquad (13)$$

$$\tilde{P}_N(x) = \exp[jk_1 \, x \cos(\theta_n)]$$

$$= \exp[jk_2 \, X \, (C_2/C_1)(x/X)\cos(\theta_n)] \qquad (14)$$

where

$x' = x/X$ = Normalized dump distance

B. EXPANSION OF EQUATION (5)

$$\tilde{P}_N(x) = + \exp[jk_1 x \cos(\theta_1-\delta)] + g_1 \exp[jk_1 x \cos(\theta_1+\delta)]$$

$$- \exp[jk_1 x \cos(\theta_2-\delta)] - g_2 \exp[jk_1 x \cos(\theta_2+\delta)]$$

$$- g_1\exp[jk_1 x \cos(\theta_3-\delta)] - g_3 \exp[jk_1 x \cos(\theta_3+\delta)]$$

$$+ g_2\exp[jk_1 x \cos(\theta_4-\delta)] + g_4 \exp[jk_1 x \cos(\theta_4+\delta)]$$

$$+ g_3\exp[jk_1 x \cos(\theta_5-\delta)] + g_5 \exp[jk_1 x \cos(\theta_5+\delta)]$$

$$- g_4\exp[jk_1 x \cos(\theta_6-\delta)] - g_6 \exp[jk_1 x \cos(\theta_6+\delta)]$$

$$- g_5\exp[jk_1 x \cos(\theta_7-\delta)] - g_7 \exp[jk_1 x \cos(\theta_7+\delta)]$$

$$\vdots \qquad\qquad \vdots \qquad (15)$$

To use Equation (15) in computer program "WEDGE", rearrange to cluster together terms of the same g:

$$\tilde{P}_N(x) = + [\exp(jk_1 x \cos(\theta_1-\delta)] - \exp[jk_1 x \cos(\theta_2-\delta)]]$$

$$+ g_1[\exp[jk_1 x \cos(\theta_1+\delta)] - \exp[jk_1 x \cos(\theta_3-\delta)]]$$

$$- g_2[\exp[jk_1 \, x \cos(\theta_2+\delta)] - \exp[jk_1 \, x \cos(\theta_4-\delta)]]$$

$$- g_3[\exp[jk_1 \, x \cos(\theta_3+\delta)] - \exp[jk_1 \, x \cos(\theta_5-\delta)]]$$

$$+ g_4[\exp[jk_1 \, x \cos(\theta_4+\delta)] - \exp[jk_1 \, x \cos(\theta_6-\delta)]]$$

$$+ g_5[\exp[jk_1 \, x \cos(\theta_5+\delta)] - \exp[jk_1 \, x \cos(\theta_7-\delta)]]$$

$$- g_6[\exp[jk_1 \, x \cos(\theta_6+\delta)] - \exp[jk_1 \, x \cos(\theta_8-\delta)]]$$

$$- g_7[\exp[jk_1 \, x \cos(\theta_7+\delta)] - \exp[jk_1 \, x \cos(\theta_9-\delta)]]$$

$$\vdots \qquad\qquad \vdots$$

IV. CONCLUSIONS AND RECOMMENDATIONS

A. VALIDATION

1. Pressure Release Bottom (R = -1.0)

The acoustic pressure field is separable in x and δ coordinates. It can be seen intuitively and verified mathematically that acceptable eigenfunctions are

$$\tilde{P}_n = \tilde{A}_n \mathrm{Sin}(\delta n\pi/\beta)\ \mathrm{Sin}(\gamma n\pi/\beta)\ \exp[j(wt+k_x x)]$$

Therefore, the total pressure will be of the form

$$\tilde{P} = \sum_{n=1}^{\infty} \tilde{A}_n \mathrm{Sin}(\delta n\pi/\beta)\ \mathrm{Sin}(\gamma n\pi/\beta)\ \exp[j(wt+k_x x)] \quad (16)$$

From this equation any correct solution should show the following: (1) the pressure at the surface ($\delta=\beta$) and at the bottom ($\delta=0$) is zero for all source position; (2) with the receiver at x/X similar to 1.0, all modes except n=1 are evanescent so that only the lowest mode is present for all source positions; (3) with the receiver at x/X similar to 2.0, all modes except n=1 and n=2 are evanescent so only the two lowest modes are present; (4) with only the lowest mode present, changing the source from $\gamma=\beta/2$ to either $\gamma=\beta/4$ or $\gamma=3\beta/4$ will change the pressure amplitude at any depth by

$\sin(\pi/4)$; (5) with only the two lowest modes present, changing the source from $\gamma=\beta/4$ to $\gamma=3\beta/4$ changes the phase relation between the modes by π thereby inverting the pressure distribution. Figures 13 and 14 show that all of these effects are correctly predicted by the image solution.

2. Rigid Bottom (R = +1.0)

a. In this case, analysis (as above) shows that the eigenfunctions should be

$$\tilde{P}_n = \tilde{A}_n \cos(\delta n\pi/2\beta)\cos(\gamma n\pi/2\beta)\exp[j(wt+k_x x)]$$

thus, the total pressure will be of the form

$$\tilde{P} = \sum_{n=1}^{\infty} \tilde{A}_n \cos(\delta n\pi/2\beta)\cos(\gamma n\pi/2\beta)\exp[j(wt+k_x x)] \qquad (17)$$

b. From this equation any correct solution should show the following: (1) the pressure at surface ($\delta=\beta$) is zero and the pressure at the bottom ($\delta=0$) is a maximum; (2) with the receiver at x/X similar to 1.0, all modes except n=1 are evanescent so that only the lowest mode is present for all source positions; (3) with the receiver at x/X similar to 2.0, all modes except n=1 and n=2 are evanescent so only the two lowest modes are present; (4) with only the lowest mode present, changing the source from $\gamma=\beta/2$ to $\gamma=\beta/4$ or $\gamma=3\beta/4$ changes the pressure amplitude by

cos(π/4)/cos(π/8) and cos(π/4)/cos(3π/8). Figures 9 through 11 show that all of these effects are correctly predicted by the image solution.

B. RESULTS FOR A REAL BOTTOM

1. For the case of a real bottom, the coordinates can not be separated as described for the case of pressure release and rigid bottom. However, if the specific acoustic impedance of this bottom is much different than that of the water, and if $C_2>C_1$, then the bottom will look similar to a pressure release bottom for modes high above cutoff and will look similar to a rigid bottom for modes near or below cutoff. For reasonably small β, the modes should be fairly well approximated by a simplistic application of adiabatic normal-mode theory

$$\tilde{P}_n = \tilde{A}_n \, \mathrm{Cos}(\delta n\pi/2\beta)\mathrm{Sin}(\gamma n\pi/\beta) \qquad (18)$$

2. From Figures 6 and 7 it can be seen that at any depth the pressure amplitude with the source at $\gamma=\beta/4$ divided by that with the source at $\gamma=\beta/2$ is within about 10% of the value predicted by Equation 18. Additional computer runs show that for $\beta = 6°$ and the receiver at $x/X=1.0$ the maximum amplitude is obtained when the source is at 2.743° indicating that the eigenfunction at great

distance from the apex is not an exact sine wave. It is also worth noting that the pressure at $x/X=1.0$ is not maximized exactly at the bottom.

C. EXPERIMENTAL DETERMINATION OF k FOR A SAND BOTTOM

As described in Appendix E, the value of k was determined for a specific sand bottom at frequencies suitable for laboratory modeling of the wedge problem. The result was $k = 0.27 \pm 0.06$, which is in reasonable agreement with the empirically obtained value of $k = 0.25$ for the same frequency range. [Ref. 9]

D. RECOMMENDATIONS FOR FURTHER INVESTIGATIONS

1. For the case of a real, fast bottom, identification of the normal modes of the system and investigation of the possibility of normal mode coupling is worth study.

2. For the real bottom, a look at the effects of absorption (α/k_2), sound speed ratio (C_1/C_2), density ratio (ρ_1/ρ_2), and wedge angle β on shapes and phases of the normal modes is worth investigation.

3. Utilize the source angle as a tool to study the amplitude and phase distribution of the normal modes at large distance from the apex.

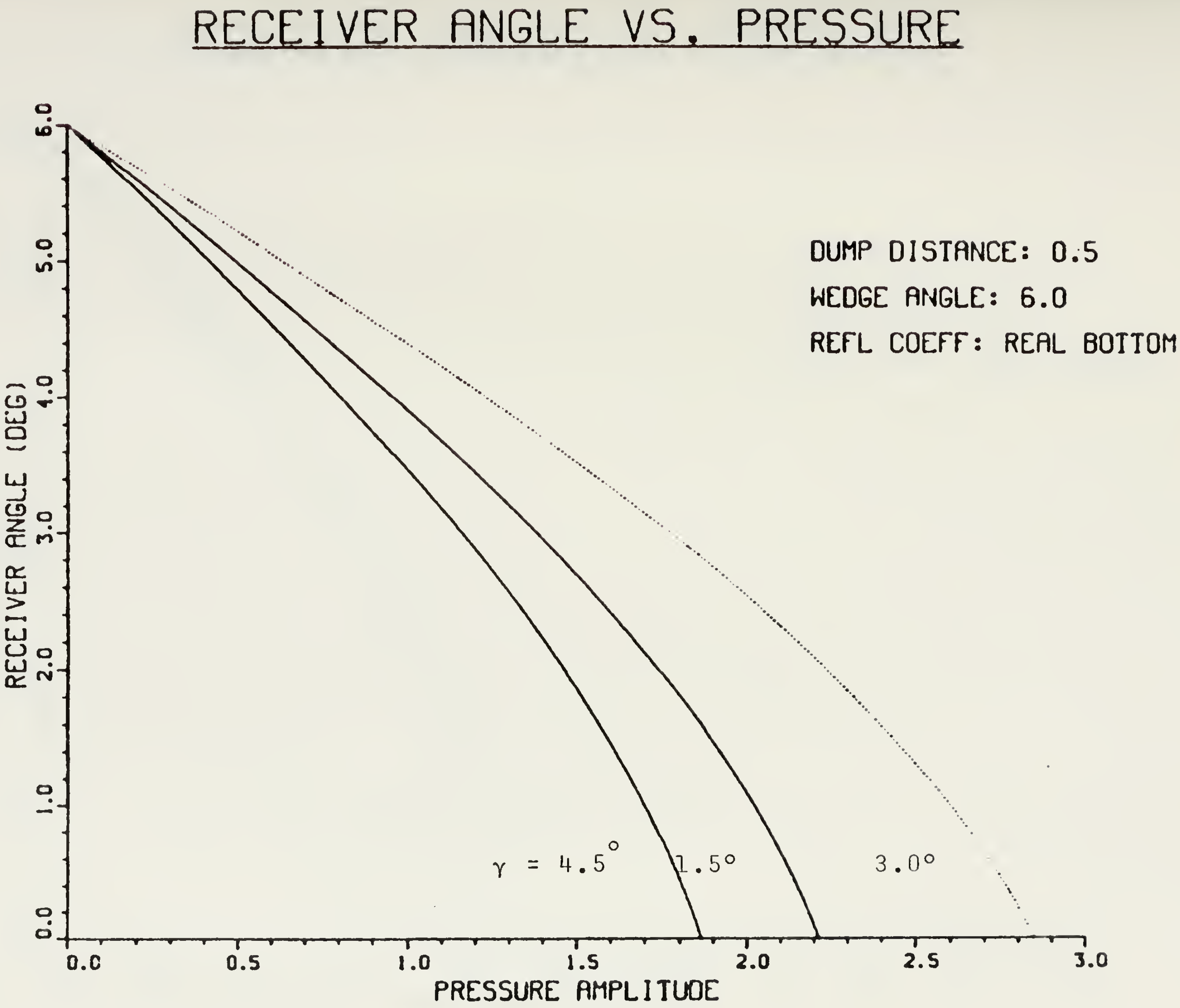

Figure 6 Graph of Data in Table I-A

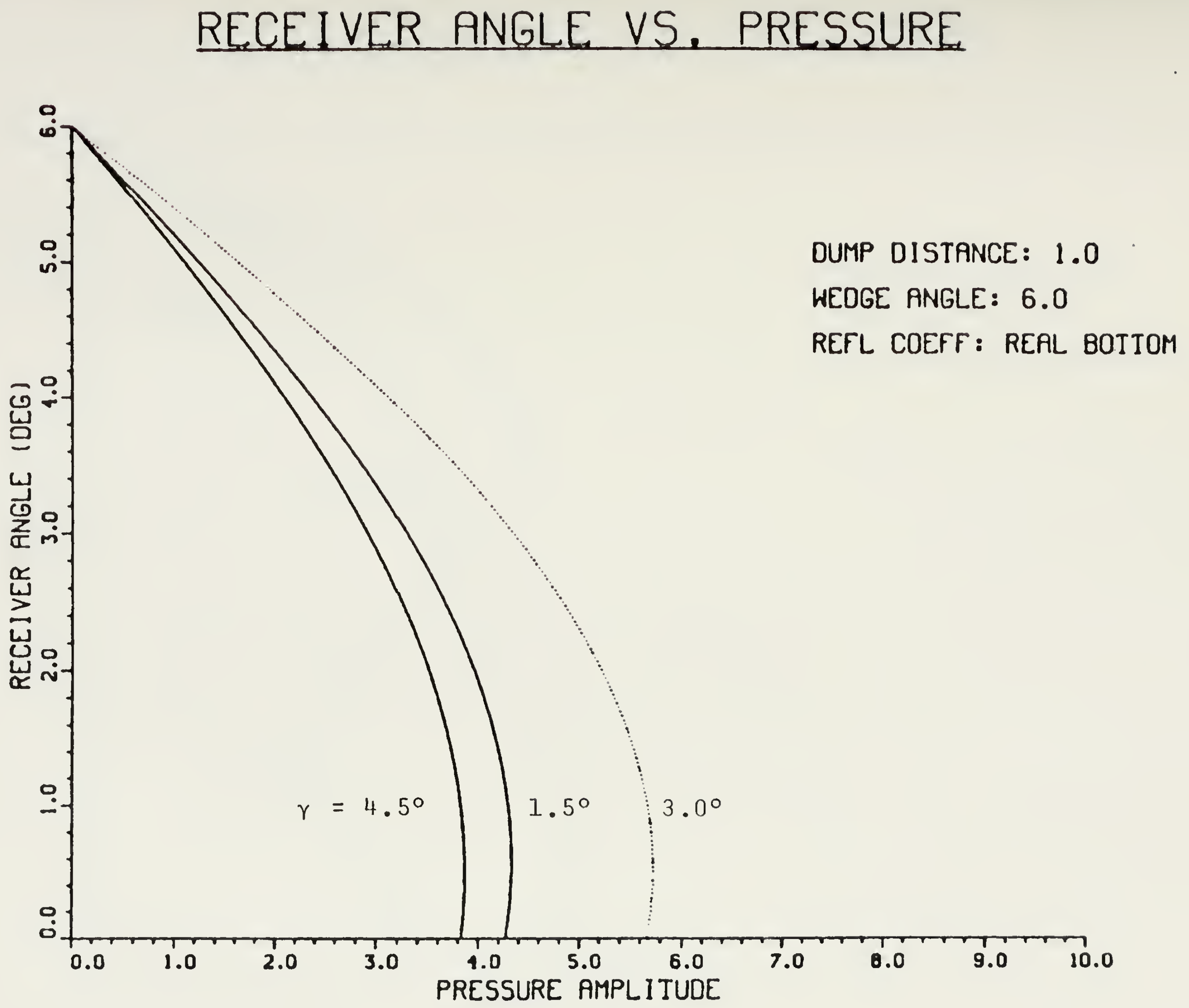

Figure 7 Graph of Data in Table I-B

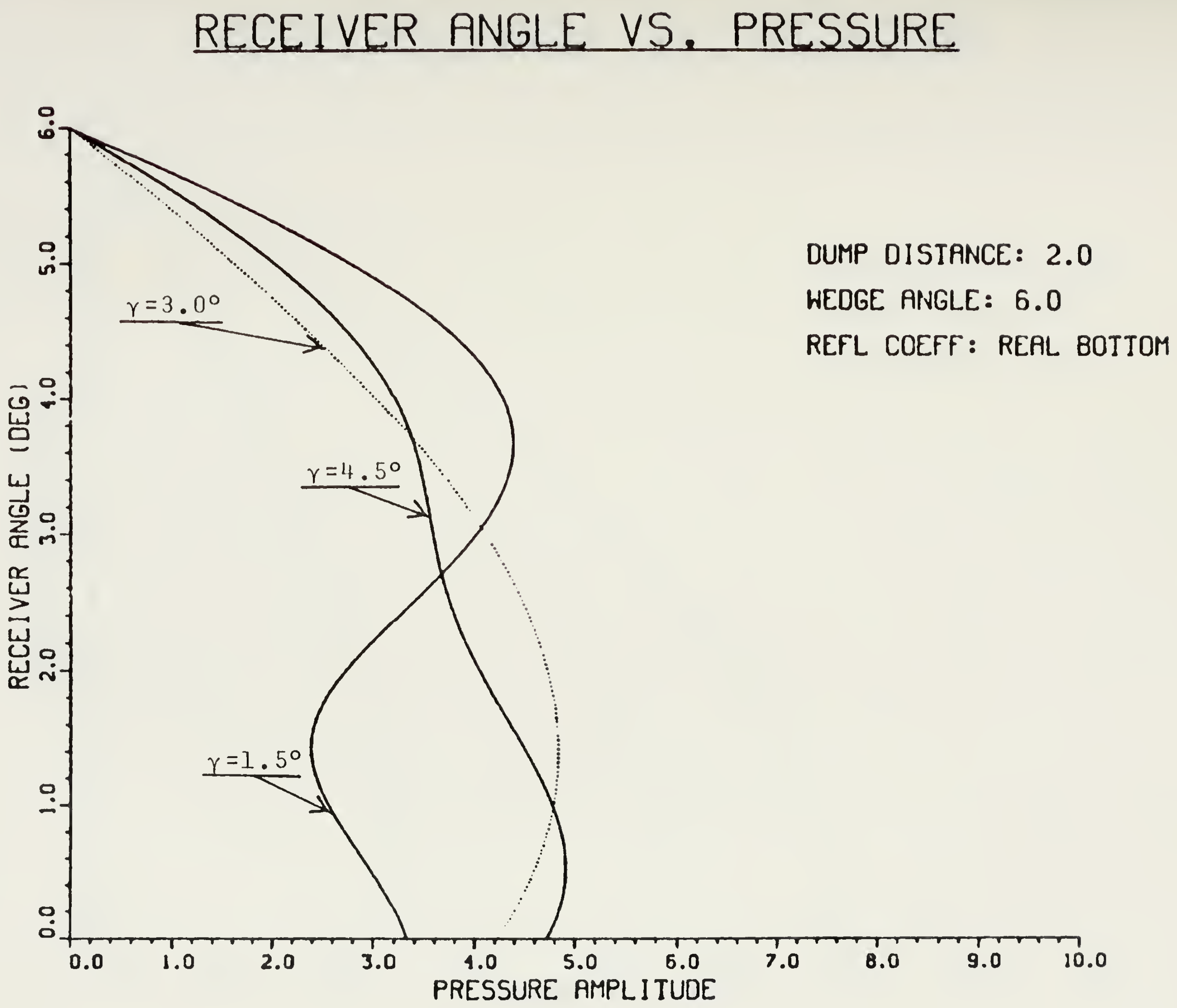

Figure 8 Graph of Data in Table I-C

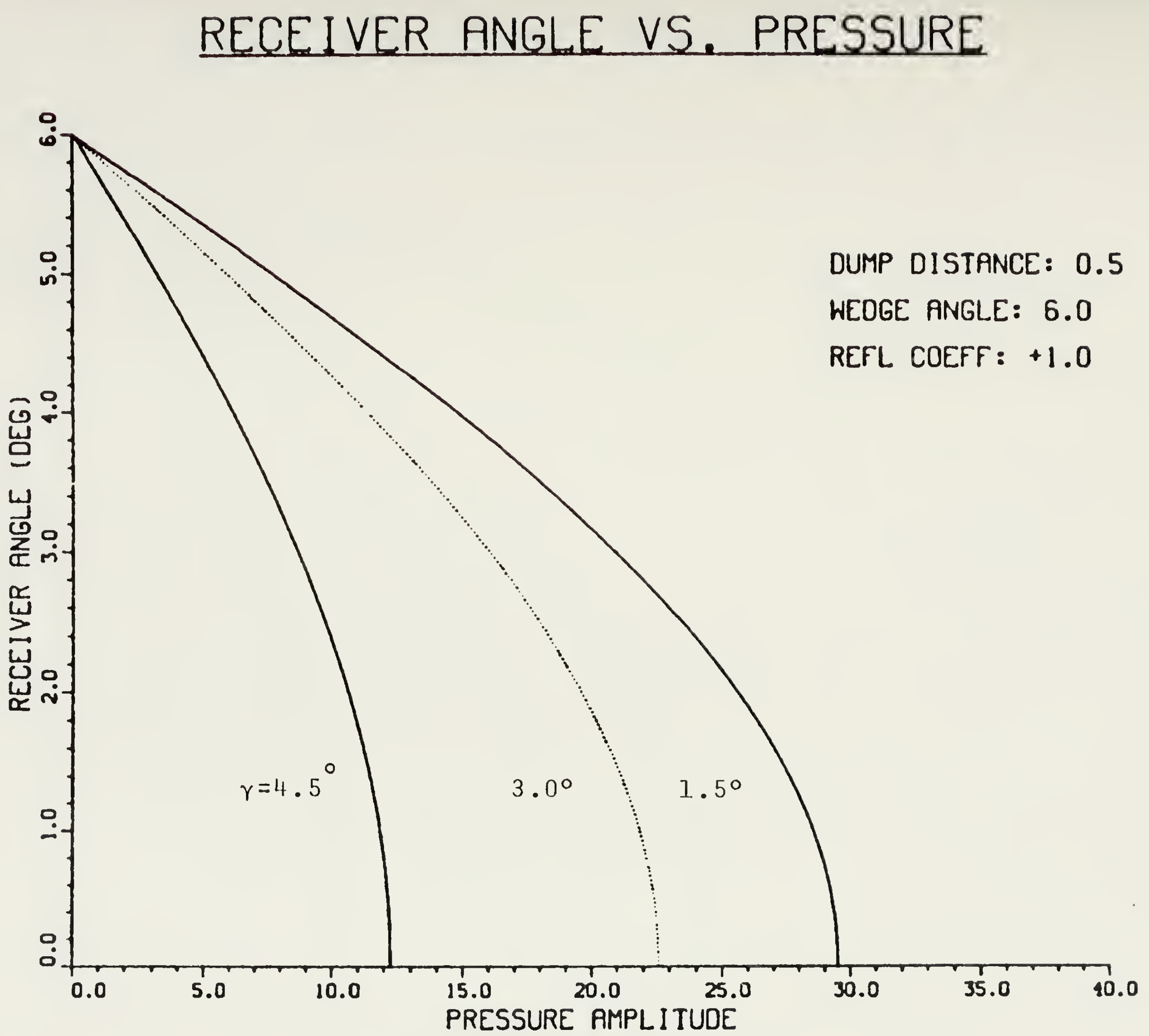

Figure 9 Graph of Data in Table II-A

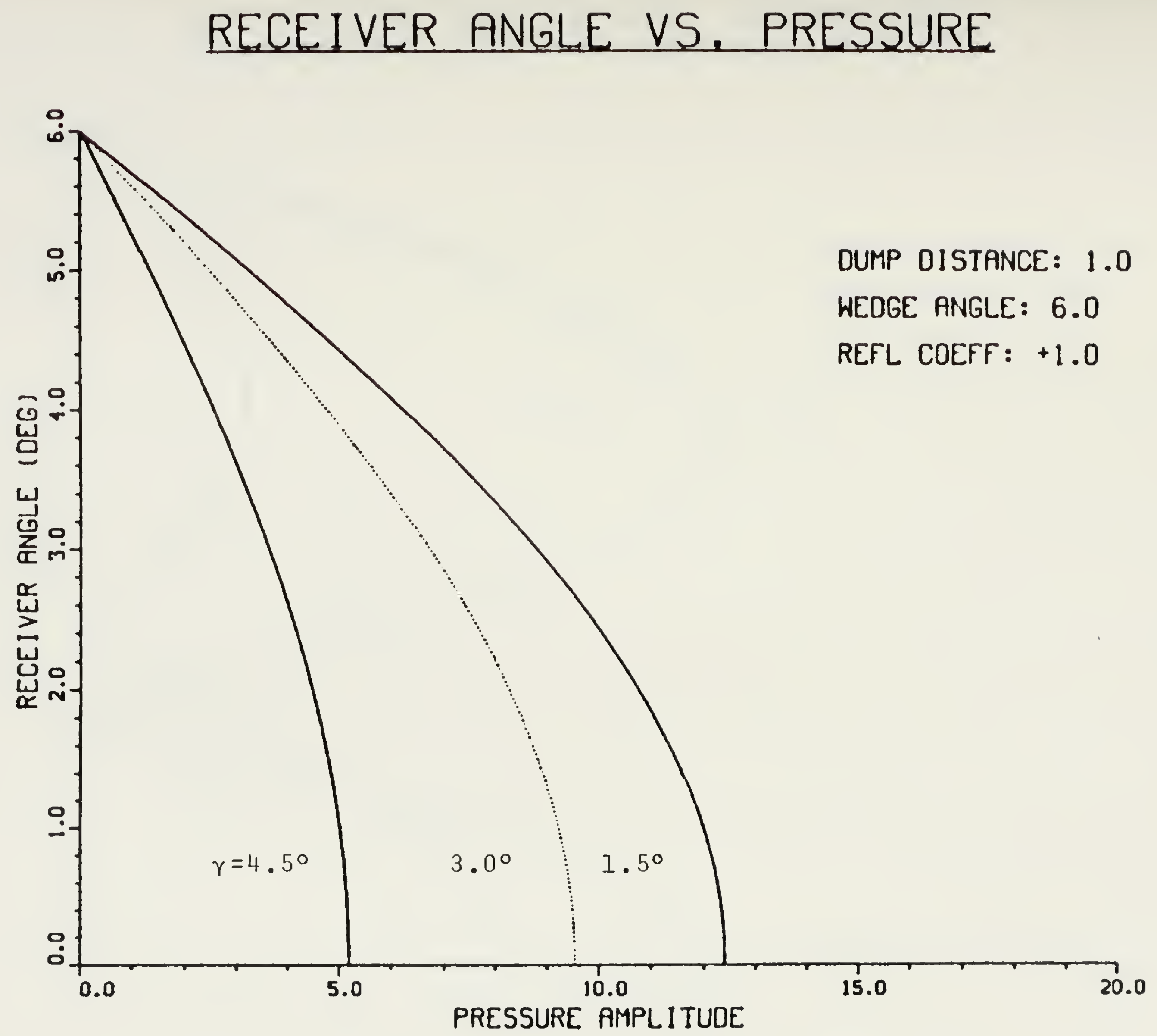

Figure 10 Graph of Data in Table II-B

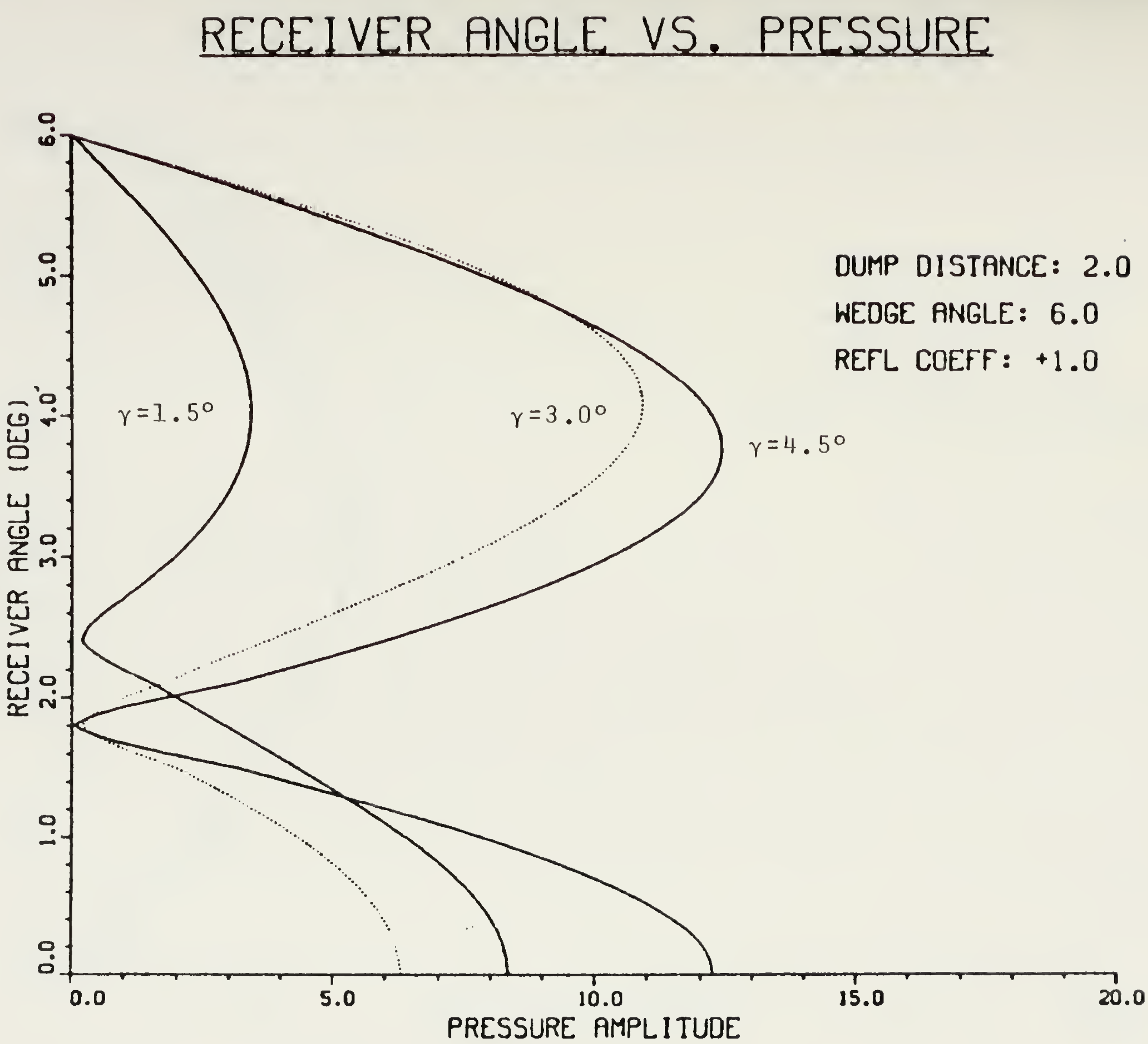

Figure 11 Graph of Data in Table II-C

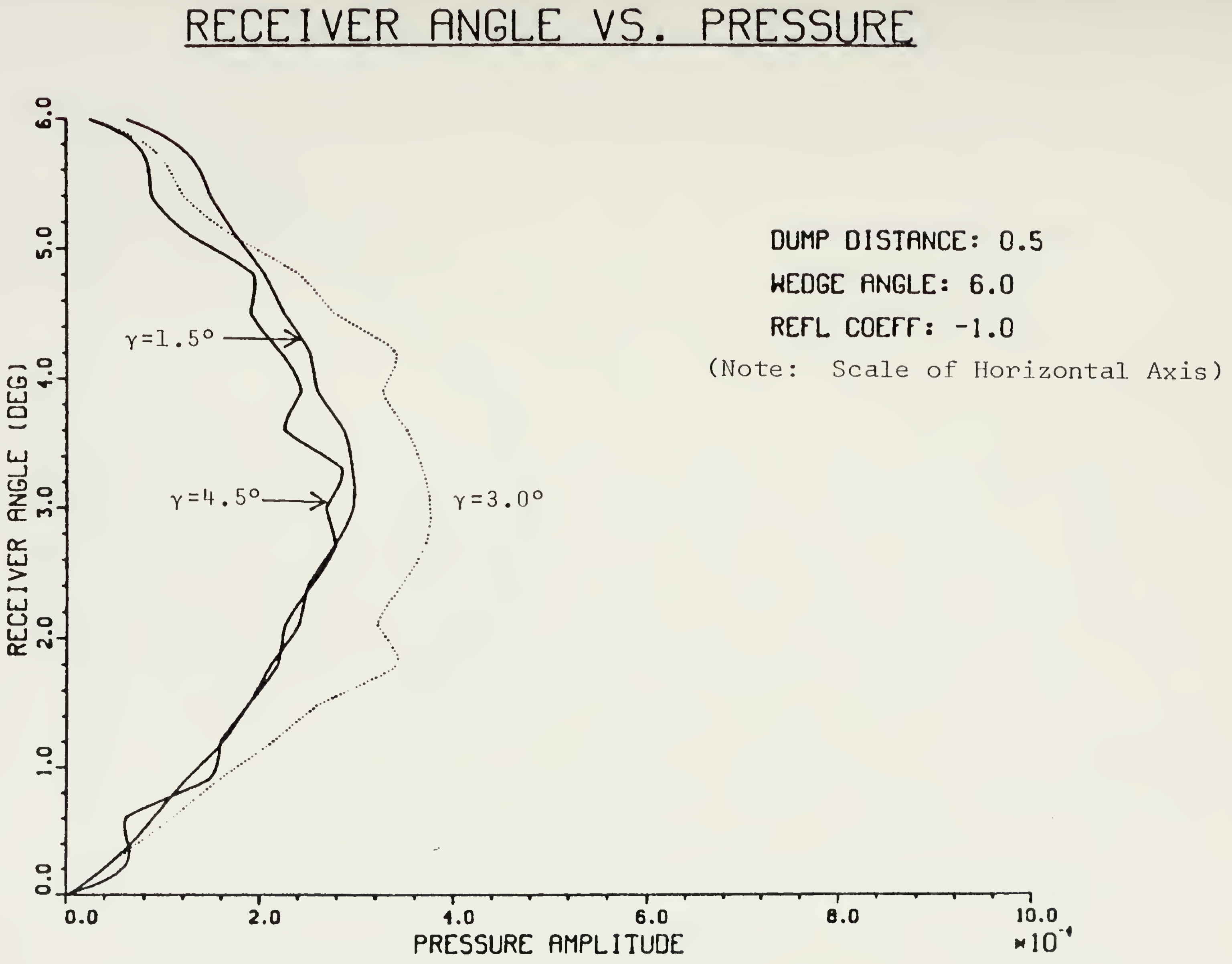

Figure 12 Graph of Data in Table III-A

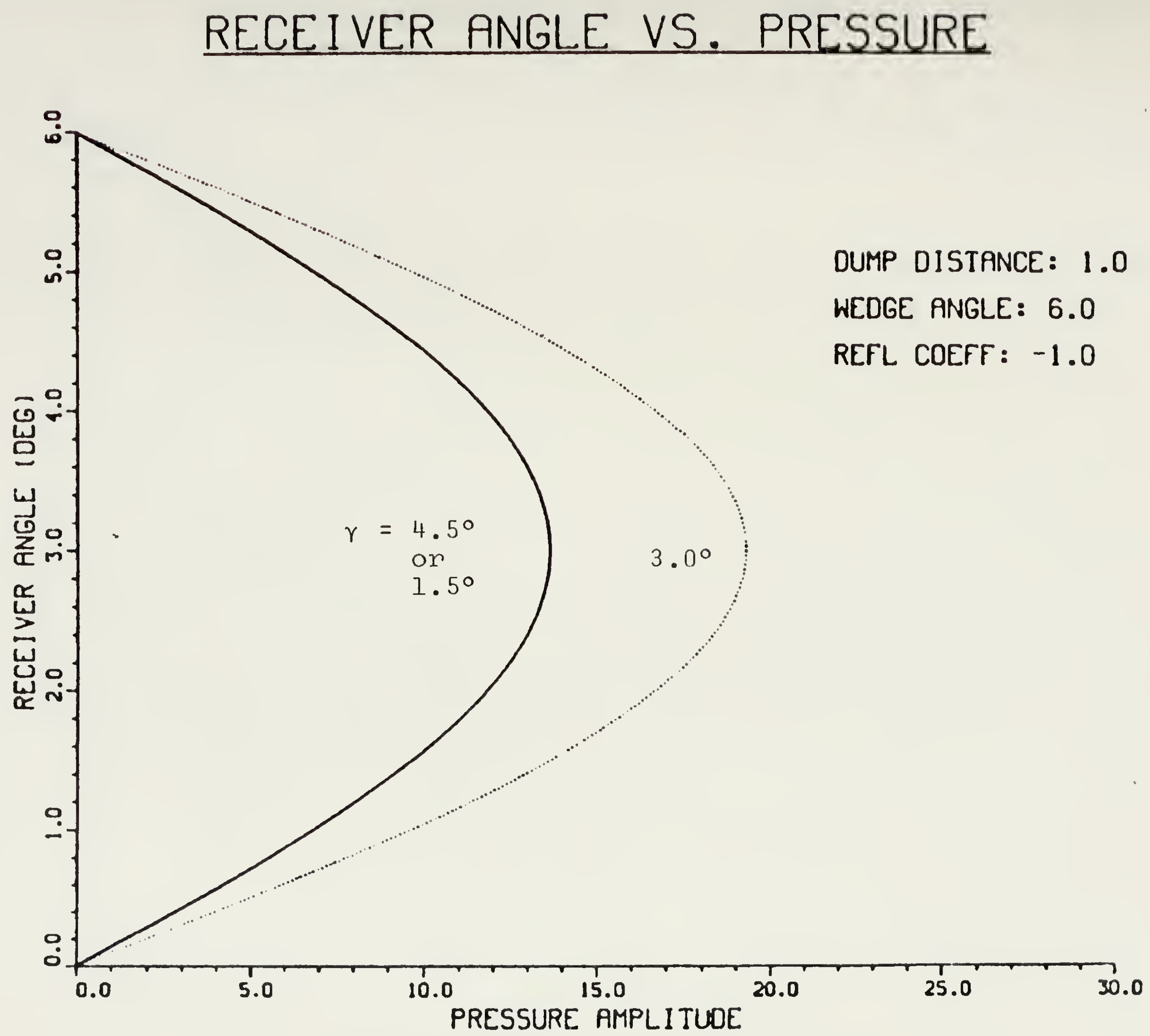

Figure 13 Graph of Data in Table III-B

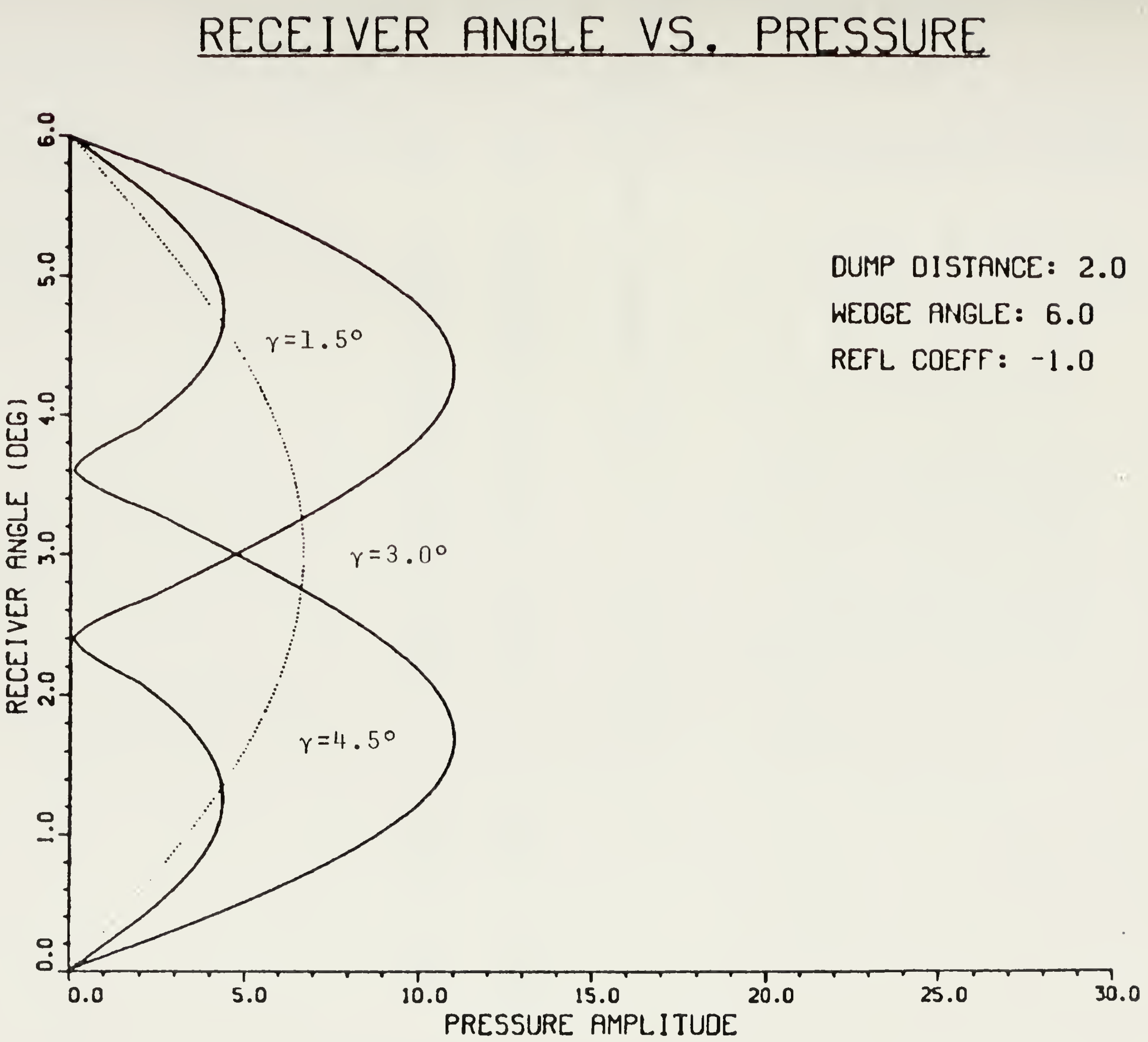

Figure 14 Graph of Data in Table III-C

APPENDIX A

PATH PARAMETER G

From Equation (7)

$$g_1 = R(\theta_1)$$

$$g_2 = R(\theta_2)$$

$$g_3 = R(\theta_3) * g_1 = R(\theta_3) * R(\theta_1)$$

$$g_4 = R(\theta_4) * g_2 = R(\theta_4) * R(\theta_2)$$

$$g_5 = R(\theta_5) * g_3 = R(\theta_5) * R(\theta_3) * R(\theta_1)$$

$$g_6 = R(\theta_6) * g_4 = R(\theta_6) * R(\theta_4) * R(\theta_2)$$

$$g_7 = R(\theta_7) * g_5 = R(\theta_7) * R(\theta_5) * R(\theta_3) * R(\theta_1)$$

$$g_8 = R(\theta_8) * g_6 = R(\theta_8) * R(\theta_6) * R(\theta_4) * R(\theta_2)$$

Therefore another form of g is as follows:

$$g_n = R(\theta_n)R(\theta_{n-2})R(\theta_{n-4})\ldots R(\theta_2) \qquad \text{for } n = \text{even}$$

$$g_n = R(\theta_n)R(\theta_{n-2})R(\theta_{n-4})\ldots R(\theta_1) \qquad \text{for } n = \text{odd}$$

APPENDIX B

GRAZING ANGLE

From Equation (6)

$$\theta_1 = \gamma$$

$$\theta_2 = 2\beta - \gamma$$

$$\theta_3 = 2\beta + \gamma$$

$$\theta_4 = 4\beta - \gamma$$

$$\theta_5 = 4\beta + \gamma$$

$$\theta_6 = 6\beta - \gamma$$

$$\theta_7 = 6\beta + \gamma$$

$$\theta_8 = 8\beta - \gamma$$

$$\theta_9 = 8\beta + \gamma$$

$$\theta_{10} = 10\beta - \gamma$$

$$\theta_{11} = 10\beta + \gamma$$

$$\vdots$$

APPENDIX C

COMPUTER PROGRAM " WEDGE "

A computer program for the calculation of the normalized pressure and phase distribution along the constant distance from the wedge apex by the method of images is following on the next pages.

```
C*****************************************************************
C THIS PROGRAM COMPUTES COMPLEX ACOUSTIC PRESSURE IN THE   *
C WEDGE OVERLYING A FAST FLUID BOTTOM.                      *
C*****************************************************************
C VARIABLE DESIGNATION AND RELATION                         *
C*****************************************************************
C      BL= ALPHA/K2= ABSORPTION COEFFICIENT (NEPERS): INPUT
C      C0=C1/C2= SOUND SPEED RATIO: INPUT
C      D=RHO1/RHO2= DENSITY RATIO: INPUT
C      XV= x/X= DUMP DISTANCE: INPUT
C
C      BETA   (RAD)= BETD (DEG)  = WEDGE ANGLE
C      GAMMA  (RAD)= GAMD (DEG)  = SOURCE ANGLE
C      DELTA  (RAD)= DELD (DEG)  = RECEIVER ANGLE
C      THETAC(RAD)= THETAD(DEG)= CRITICAL ANGLE
C
C      BP= AP*C0= K2*X
C      BX= BL*BP= (ALPHA/K2)*(K2*X)= ALPHA*X
C      AP= K1*X = BP/C0
C      CCN = K1*x
C
C      N   = NUMBER OF DEPTH POINT (RECEIVER POINT)
C      NI  = NUMBER OF IMAGE POINT
C      NP1 = N + 1
C
C      R = REFLECTION COEFFICIENT
C*****************************************************************
      DIMENSION  SUM(200),THETA(200),G(200),Z1(200),Z2(200),
     *           R(200),A(200),Y(200),FM1(200),FM2(200),
     *           ANGLE(200),HH(200),DELD(21),SMAG(21)
      COMPLEX    Z1,Z2,R,G,SUM,SUMI,SUMJ,HI,HR,H1,H2,H3,H4
      INTEGER    I,II,J,K,L,N,NP1,NI
      PHI = ARCOS(-1.0)
C*****************************************************************
C DO LOOP 10 COMPUTES THREE DIFFERENT INPUTS AT EACH TIME.
C*****************************************************************
C        DO 10 M=1,3
C          XV=M
           XV =   2.0
           BETD = 6.0
           GAMD = 4.5
C
           BL  = 0.01
           C0  = 0.89982
           D   = 0.5051
C
           N=20
           AB =-1.0
C************************
C     VARIABLES RELATIONS.
C************************
           TEM  = 180/PHI
           THETAC=ARCCS(C0)
           THETAD=THETAC*TEM
C
           BETA  = BETD/TEM
           BETN  = BETA/N
           GAMMA = GAMD/TEM
C
           BP=PHI/(2*TAN(THETAC)*TAN(BETA))
           BX=BL*BP
           AP=BP/C0
C
           B=2*BL*C0**2
           CON = AP * XV
           S = SQRT(2.0)
C
           NI =INT(180/BETD+0.00001)
           NP1 =N+1
```

```
C
      WRITE (6,100) XV,BETD,GAMD
 100  FORMAT(' 1'/1X,'REFLECTION COEF= - 1.0          '/1X,
     *  'XV= x/X= ',F4.2,4X,'BETD=',F6.2,4X,'GAMD=',F6.2)
C****************************************************************
C DO LOOP 20 COMPUTES 'THETA(N)' AND 'R(N)'
C****************************************************************
       DO 20 I = 1,NI,2
             II=I+1
             THETA(I)= (I-1)*BETA + GAMMA
             THETA(II)= II*BETA - GAMMA
C
             A(I)=CCS(THETA(I))**2 - C0**2
             A(II)=COS(THETA(II))**2 - C0**2
C
             Y(I) = SQRT(A(I)**2 + B**2)
             Y(II) = SQRT(A(II)**2 + B**2)
C
             XX=Y(I) + A(I)
             XY=Y(II) + A(II)
C
             YX=Y(I) - A(I)
             YY=Y(II) - A(II)
C
             IF (XX.LT.0.) XX=0.
             IF (XY.LT.0.) XY=0.
C
             IF (YX.LT.0.) YX=0.
             IF (YY.LT.0.) YY=0.
C
             FM1(I)=SQRT(XX)/S
             FM1(II)=SQRT(XY)/S
C
             FM2(I)= -SQRT(YX)/S
             FM2(II)= -SQRT(YY)/S
C
             Z11R=SIN(THETA(I))/D + FM2(I)
             Z11I= + FM1(I)
             Z12R=SIN(THETA(II))/D + FM2(II)
             Z12I= + FM1(II)
C
             Z1(I)=CMPLX(Z11R,Z11I)
             Z1(II)=CMPLX(Z12R,Z12I)
C
             Z21R= SIN(THETA(I))/D - FM2(I)
             Z21I= - FM1(I)
             Z22R= SIN(THETA(II))/D - FM2(II)
             Z22I= - FM1(II)
C
             Z2(I)=CMPLX(Z21R,Z21I)
             Z2(II)=CMPLX(Z22R,Z22I)
C*****************************************************
C CHOOSE ONE OF REFLECTION COEFFICIENT(DELET"C")
C*****************************************************
              R(I)=Z1(I)/Z2(I)
              R(II)=Z1(II)/Z2(II)
C             R(I) = +1.0
C             R(II) = +1.0
C             R(I) = -1.0
C             R(II) = -1.0
C****************************************************************
 20    CONTINUE
C****************************************************************
C DO LOOP 30 COMPUTES 'G(N)'
C****************************************************************
          G(1) = R(1)
          G(2) = R(2)
       DO 30 I= 3,NI
          G(I)=R(I)*G(I-2)
```

```
C*****************************************************************
 30      CONTINUE
C*****************************************************************
          WRITE(6,210)
 210      FORMAT(/3X,'N',3X,'DELD(DEG)',5X,'PRESSURE'
    *           ,6X,'|PRESSURE|',5X,'PHASE(DEG)'/)
C
C*****************************************************************
C DO LOOP 40 COMPUTES TOTAL SUM (COMPLEX ACOUSTIC PRESSURE)
C*****************************************************************
        DELTA = 0.
        DO 40 J = 1,NP1
          DELD(J) =DELTA*TEM
C
          H11=CON*COS(THETA(1)-DELTA)
          H21=CON*CCS(THETA(2)-DELTA)
          H31=CON*COS(THETA(1)+DELTA)
          H41=CON*COS(THETA(3)-DELTA)
C
          H1=CMPLX(0.00000,H11)
          H2=CMPLX(0.00000,H21)
          H3=CMPLX(C.00000,H31)
          H4=CMPLX(0.00000,H41)
C
C*********************************
C SUMJ TERM IS CONSTANT PART OF SUM.
C*********************************
          SUMJ=CEXP(H1)-CEXP(H2)+G(1)*( CEXP(H3)-CEXP(H4))
C
             AA=AB
             SUMI =(0.00000,0.00000)
             K = 0
C****************************************
C DC LOCP 41 COMPUTES NON-CONSTANT PART OF SUM
C****************************************
            DO 41 I = 2,NI
               IF (K.LT.2) GOTO 411
                  K = 0
                  AA= AA*AB
 411           CCNTINUE
C
             IF (I+2.GT.NI) GOTO 412
               HH(I)=CON*COS(THETA(I)+DELTA)
               HH(I+2)=CON*COS(THETA(I+2)-DELTA )
C
               HI=CMPLX(0.00000,HH(I))
               HR=CMPLX(0.00000,HH(I+2))
C
               SUMI = SUMI + AA*G(I)*(CEXP(HI)-CEXP(HR))
C
               GCTO 413
C
 412              HH(I)=CON*COS(THETA(I)+DELTA)
                  HI=CMPLX(0.00000,HH(I))
                  SUMI=SUMI+AA*G(I)*(CEXP(HI))
 413           CCNTINUE
               K=K+1
C****************************************
 41         CONTINUE
C****************************************
          SUM(J) =SUMJ+SUMI
          SMAG(J) =CABS( SUM(J) )
          RSUM =REAL( SUM(J) )
          RISUM =AIMAG( SUM(J) )
C
          IF (SMAG(J).GT. 0.00000) GOTO 4111
              RISUM=0.
              RSUM=0.
              SPHASE =ATAN2( RISUM,RSUM )
```

```
                     DPHASE =SPHASE*TEM
C
              WRITE(6,220)J,DELD(J),SUM(J),SMAG(J),DPHASE
220           FORMAT(1X,I3,3X,F6.2,4X,F6.2,'+ J',F6.2,4X,F6.2,
     *               9X,F8.2)
C
                 DELTA = DELTA + BETN
                 GO TO 40
C
4111             CONTINUE
                 SPHASE =ATAN2( RISUM,RSUM )
                 DPHASE =SPHASE*TEM
C
            WRITE(6,230)J,DELD(J),SUM(J),SMAG(J),DPHASE
230         FORMAT(1X,I3,3X,F6.2,4X,F6.2,'+ J',F6.2,4X,F6.2,
     *             9X,F8.2)
C
              DELTA = DELTA + BETN
C
C**************************************************************
40            CONTINUE
C**************************************************************
C  NOTICE !
C  Remove C in front of call subroutine if need plotting.
C**************************************************************
C*** FOR PLOTTING BY TEK618 ***
C*******************************
C      CALL  MEDBUF
C      CALL  TEK618
C      CALL  NOBRDR
C      CALL  PAGE(15.,12.)
C      CALL  AREA2D(10.,8.)
C      CALL  XNAME('SMAG (AMPLITUDE OF PRESSURE)$',100)
C      CALL  YNAME('DELD (RECEIVER ANGLE IN DEG)$',100)
C      CALL  YTICKS(5)
C      CALL  XTICKS(5)
C      CALL  GRAF(0.,1.,5.5,0.,1.,8.6)
C      CALL  DOT
C      CALL  GRID(2,2)
C      CALL  HEADIN('DELD VS. PRESSURE$',-100,1.5,1)
C      CALL  MESSAG('X/CAPX=$',100,8.,7.5)
C      CALL  MESSAG('1.0$',100,'ABUT','ABUT')
C      CALL  MESSAG('BETD=$',100,8.,7.)
C      CALL  MESSAG('6.0$',100,'ABUT','ABUT')
C      CALL  MESSAG('GAMD=$',100,8.,6.5)
C      CALL  MESSAG('4.5$',100,'ABUT','ABUT')
C      CALL  MESSAG('RHO1/RHO2=$',100,8.,6.)
C      CALL  MESSAG('0.5051$',100,'ABUT','ABUT')
C      CALL  MESSAG('C1/C2=$',100,8.,5.5)
C      CALL  MESSAG('0.89982$',100,'ABUT','ABUT')
C      CALL  MESSAG('A/K2=$',100,8.,5.)
C      CALL  MESSAG('0.01$',100,'ABUT','ABUT')
C      CALL  MESSAG('R = $',100,8.,4.5)
C      CALL  MESSAG('+1.0$',100,'ABUT','ABUT')
C      CALL  MESSAG('-1.0$',100,'ABUT','ABUT')
C      CALL  MESSAG('REAL BOTTOM$',100,'ABUT','ABUT')
C      CALL  RESET('ALL')
C      CALL  POly3
C      CALL  CURVE(SMAG,DELD,21,0)
C10    CONTINUE
C      CALL  ENDPL(0)
C      CALL  DONEPL
       STOP
       END
```

APPENDIX D

RESULTS OF CALCULATIONS.

COMPUTER OUTPUTS ARE LISTED FROM TABLE I-A1 TO TABLE III-C3, AND PLOTTED FROM FIG. 6 TO FIG. 9. EXPERIMENTAL DATA ARE AS FOLLOWS (See APPENDIX E.)

C_1 / C_2 = 0.89982 (sound speed ratio)

ρ_1 / ρ_2 = 0.5051 (density ratio)

α / k_2 = 0.01 (Nepers)

TABLE I-A1. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= REAL BOTTOM
XV= x/X= 0.50 BETD= 6.00 GAMD= 1.50

N	DELD (DEG)	PRESSURE	|PRESSURE|	PHASE (DEG)
1	0.0	0.52 + J 2.15	2.2153	76.45
2	0.30	0.46 + J 2.12	2.1656	77.67
3	0.60	0.41 + J 2.07	2.1080	78.81
4	0.90	0.36 + J 2.01	2.0425	79.87
5	1.20	0.31 + J 1.94	1.9692	80.85
6	1.50	0.27 + J 1.87	1.8883	81.75
7	1.80	0.23 + J 1.78	1.7999	82.59
8	2.10	0.20 + J 1.69	1.7044	83.36
9	2.40	0.17 + J 1.59	1.6021	84.06
10	2.70	0.14 + J 1.49	1.4932	84.70
11	3.00	0.11 + J 1.37	1.3781	85.27
12	3.30	0.09 + J 1.25	1.2574	85.79
13	3.60	0.07 + J 1.13	1.1314	86.25
14	3.90	0.06 + J 1.00	1.0007	86.65
15	4.20	0.05 + J 0.86	0.8658	87.00
16	4.50	0.03 + J 0.73	0.7272	87.29
17	4.80	0.03 + J 0.58	0.5855	87.52
18	5.10	0.02 + J 0.44	0.4413	87.71
19	5.40	0.01 + J 0.30	0.2953	87.84
20	5.70	0.01 + J 0.15	0.1480	87.92
21	6.00	0.00 + J 0.00	0.0000	28.33

TABLE I-A2. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= REAL BOTTOM
XV= x/X= 0.50   BETD= 6.00   GAMD= 3.00

N	DELD (DEG)	PRESSURE	|PRESSURE|	PHASE (DEG)
1	0.0	1.01 + J 2.66	2.8481	69.12
2	0.30	0.93 + J 2.62	2.7836	70.37
3	0.60	0.86 + J 2.57	2.7090	71.54
4	0.90	0.78 + J 2.50	2.6244	72.62
5	1.20	0.71 + J 2.43	2.5298	73.62
6	1.50	0.65 + J 2.34	2.4255	74.55
7	1.80	0.58 + J 2.24	2.3118	75.40
8	2.10	0.52 + J 2.13	2.1889	76.19
9	2.40	0.47 + J 2.00	2.0573	76.90
10	2.70	0.41 + J 1.87	1.9173	77.56
11	3.00	0.36 + J 1.73	1.7695	78.15
12	3.30	0.32 + J 1.58	1.6144	78.67
13	3.60	0.27 + J 1.43	1.4526	79.14
14	3.90	0.23 + J 1.26	1.2847	79.55
15	4.20	0.19 + J 1.09	1.1115	79.91
16	4.50	0.16 + J 0.92	0.9335	80.20
17	4.80	0.12 + J 0.74	0.7516	80.45
18	5.10	0.09 + J 0.56	0.5665	80.63
19	5.40	0.06 + J 0.37	0.3790	80.77
20	5.70	0.03 + J 0.19	0.1899	80.85
21	6.00	0.00 + J 0.00	0.0000	77.23

TABLE I-A3. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= REAL BOTTOM
XV= x/X= 0.50 BETD= 6.00 GAMD= 4.50

N	DELD(DEG)	PRESSURE	|PRESSURE|	PHASE(DEG)
1	0.0	0.75 + J 1.71	1.8632	66.25
2	0.30	0.70 + J 1.68	1.8211	67.52
3	0.60	0.64 + J 1.65	1.7723	68.70
4	0.90	0.59 + J 1.61	1.7170	69.80
5	1.20	0.54 + J 1.56	1.6552	70.82
6	1.50	0.50 + J 1.51	1.5870	71.76
7	1.80	0.45 + J 1.44	1.5126	72.63
8	2.10	0.41 + J 1.37	1.4323	73.43
9	2.40	0.37 + J 1.30	1.3462	74.15
10	2.70	0.33 + J 1.21	1.2546	74.82
11	3.00	0.29 + J 1.12	1.1580	75.42
12	3.30	0.26 + J 1.02	1.0565	75.95
13	3.60	0.22 + J 0.92	0.9506	76.43
14	3.90	0.19 + J 0.82	0.8408	76.84
15	4.20	0.16 + J 0.71	0.7274	77.20
16	4.50	0.13 + J 0.60	0.6110	77.50
17	4.80	0.10 + J 0.48	0.4919	77.75
18	5.10	0.08 + J 0.36	0.3708	77.94
19	5.40	0.05 + J 0.24	0.2481	78.08
20	5.70	0.03 + J 0.12	0.1243	78.16
21	6.00	0.00 + J 0.00	0.0000	8.92

TABLE I-B1. PRESSURE AND PHASE IN THE WEDGE

REFLECTICN COEF= REAL BOTTOM
XV= x/X= 1.00 BETD= 6.00 GAMD= 1.50

N	DELD (DEG)	PRESSURE	|PRESSURE|	PHASE (DEG)
1	0.0	-3.46 + J 2.51	4.2755	144.05
2	0.30	-3.53 + J 2.49	4.3171	144.74
3	0.60	-3.56 + J 2.46	4.3271	145.37
4	0.90	-3.57 + J 2.41	4.3053	145.95
5	1.20	-3.54 + J 2.35	4.2516	146.49
6	1.50	-3.49 + J 2.27	4.1664	146.98
7	1.80	-3.41 + J 2.18	4.0502	147.43
8	2.10	-3.31 + J 2.08	3.9039	147.84
9	2.40	-3.17 + J 1.96	3.7284	148.22
10	2.70	-3.01 + J 1.84	3.5251	148.56
11	3.00	-2.82 + J 1.70	3.2953	148.87
12	3.30	-2.61 + J 1.56	3.0408	149.15
13	3.60	-2.38 + J 1.41	2.7635	149.40
14	3.90	-2.13 + J 1.25	2.4654	149.61
15	4.20	-1.86 + J 1.08	2.1488	149.80
16	4.50	-1.57 + J 0.91	1.8160	149.95
17	4.80	-1.27 + J 0.73	1.4695	150.08
18	5.10	-0.96 + J 0.55	1.1119	150.18
19	5.40	-0.65 + J 0.37	0.7460	150.25
20	5.70	-0.33 + J 0.19	0.3744	150.29
21	6.00	-0.00 + J 0.00	0.0000	175.89

TABLE I-B2. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= REAL BOTTOM
XV= x/X= 1.00 BETD= 6.00 GAMD= 3.00

N	DELD(DEG)	PRESSURE	|PRESSURE|	PHASE(DEG)
1	0.0	-4.28 + J 3.72	5.6703	138.97
2	0.30	-4.36 + J 3.70	5.7177	139.66
3	0.60	-4.40 + J 3.66	5.7239	140.30
4	0.90	-4.41 + J 3.59	5.6888	140.88
5	1.20	-4.39 + J 3.50	5.6124	141.42
6	1.50	-4.33 + J 3.39	5.4952	141.91
7	1.80	-4.23 + J 3.26	5.3378	142.37
8	2.10	-4.09 + J 3.11	5.1414	142.78
9	2.40	-3.93 + J 2.94	4.9073	143.16
10	2.70	-3.73 + J 2.76	4.6371	143.50
11	3.00	-3.50 + J 2.56	4.3327	143.81
12	3.30	-3.24 + J 2.34	3.9964	144.09
13	3.60	-2.95 + J 2.12	3.6306	144.33
14	3.90	-2.64 + J 1.88	3.2379	144.55
15	4.20	-2.30 + J 1.63	2.8213	144.73
16	4.50	-1.95 + J 1.37	2.3838	144.89
17	4.80	-1.58 + J 1.11	1.9286	145.01
18	5.10	-1.20 + J 0.83	1.4591	145.11
19	5.40	-0.80 + J 0.56	0.9788	145.18
20	5.70	-0.40 + J 0.28	0.4912	145.22
21	6.00	-0.00 + J 0.00	0.0000	97.79

TABLE I-B3. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= REAL BOTTOM
XV= x/X= 1.00 BETD= 6.00 GAMD= 4.50

N	DELD (DEG)	PRESSURE	|PRESSURE|	PHASE (DEG)
1	0.0	-2.80 + J 2.61	3.8250	136.95
2	0.30	-2.85 + J 2.60	3.8538	137.67
3	0.60	-2.88 + J 2.56	3.8552	138.32
4	0.90	-2.89 + J 2.52	3.8290	138.93
5	1.20	-2.87 + J 2.45	3.7753	139.48
6	1.50	-2.83 + J 2.38	3.6945	139.99
7	1.80	-2.77 + J 2.28	3.5871	140.46
8	2.10	-2.68 + J 2.18	3.4536	140.88
9	2.40	-2.57 + J 2.06	3.2951	141.27
10	2.70	-2.44 + J 1.93	3.1126	141.62
11	3.00	-2.29 + J 1.79	2.9075	141.94
12	3.30	-2.12 + J 1.64	2.6811	142.22
13	3.60	-1.93 + J 1.48	2.4352	142.47
14	3.90	-1.73 + J 1.32	2.1714	142.69
15	4.20	-1.51 + J 1.14	1.8917	142.88
16	4.50	-1.28 + J 0.96	1.5981	143.03
17	4.80	-1.03 + J 0.78	1.2928	143.16
18	5.10	-0.78 + J 0.59	0.9780	143.26
19	5.40	-0.53 + J 0.39	0.6560	143.33
20	5.70	-0.26 + J 0.20	0.3292	143.38
21	6.00	-0.00 + J -0.00	0.0000	-178.24

TABLE I-C1. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= REAL BOTTOM
XV= x/X= 2.00 BETD= 6.00 GAMD= 1.50

N	DELD(DEG)	PRESSURE	|PRESSURE|	PHASE(DEG)
1	0.0	-2.47 + J -2.22	3.3254	-138.01
2	0.30	-2.40 + J -2.02	3.1415	-139.91
3	0.60	-2.36 + J -1.68	2.8949	-144.53
4	0.90	-2.34 + J -1.21	2.6350	-152.61
5	1.20	-2.35 + J -0.64	2.4372	-164.68
6	1.50	-2.39 + J -0.01	2.3870	-179.86
7	1.80	-2.44 + J 0.67	2.5321	164.68
8	2.10	-2.51 + J 1.34	2.8441	151.79
9	2.40	-2.57 + J 1.98	3.2452	142.31
10	2.70	-2.61 + J 2.55	3.6546	135.66
11	3.00	-2.63 + J 3.02	4.0077	131.04
12	3.30	-2.61 + J 3.36	4.2573	127.78
13	3.60	-2.54 + J 3.56	4.3700	125.46
14	3.90	-2.41 + J 3.59	4.3251	123.78
15	4.20	-2.21 + J 3.47	4.1129	122.56
16	4.50	-1.96 + J 3.18	3.7345	121.66
17	4.80	-1.65 + J 2.74	3.2004	121.02
18	5.10	-1.29 + J 2.18	2.5301	120.56
19	5.40	-0.88 + J 1.51	1.7506	120.25
20	5.70	-0.45 + J 0.77	0.8949	120.08
21	6.00	-0.00 + J 0.00	0.0000	90.54

TABLE I-C2. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= REAL BOTTOM
XV= x/X= 2.00 BETD= 6.00 GAMD= 3.00

N	DELD(DEG)	PRESSURE	|PRESSURE|	PHASE(DEG)
1	0.0	-3.93 + J 1.58	4.2359	158.06
2	0.30	-4.17 + J 1.60	4.4642	158.99
3	0.60	-4.36 + J 1.59	4.6375	159.91
4	0.90	-4.49 + J 1.56	4.7546	160.81
5	1.20	-4.57 + J 1.51	4.8156	161.69
6	1.50	-4.60 + J 1.44	4.8212	162.56
7	1.80	-4.57 + J 1.36	4.7730	163.40
8	2.10	-4.50 + J 1.27	4.6733	164.22
9	2.40	-4.37 + J 1.17	4.5244	165.01
10	2.70	-4.20 + J 1.06	4.3291	165.77
11	3.00	-3.98 + J 0.96	4.0902	166.49
12	3.30	-3.72 + J 0.85	3.8104	167.16
13	3.60	-3.41 + J 0.74	3.4927	167.78
14	3.90	-3.08 + J 0.63	3.1400	168.35
15	4.20	-2.70 + J 0.53	2.7555	168.86
16	4.50	-2.30 + J 0.44	2.3426	169.29
17	4.80	-1.87 + J 0.34	1.9052	169.66
18	5.10	-1.43 + J 0.25	1.4474	169.95
19	5.40	-0.96 + J 0.17	0.9739	170.16
20	5.70	-0.48 + J 0.08	0.4896	170.28
21	6.00	-0.00 + J 0.00	0.0000	173.56

TABLE I-C3. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= REAL BOTTOM
XV= x/X= 2.00 BETD= 6.00 GAMD= 4.50

N	DELD(DEG)	PRESSURE	|PRESSURE|	PHASE(DEG)
1	0.0	-2.65 + J 3.90	4.7165	124.14
2	0.30	-3.05 + J 3.80	4.8704	128.71
3	0.60	-3.38 + J 3.54	4.8970	133.72
4	0.90	-3.65 + J 3.14	4.8134	139.34
5	1.20	-3.84 + J 2.61	4.6447	145.76
6	1.50	-3.95 + J 2.00	4.4230	153.13
7	1.80	-3.97 + J 1.33	4.1843	161.52
8	2.10	-3.91 + J 0.63	3.9629	170.87
9	2.40	-3.78 + J -0.06	3.7841	-179.16
10	2.70	-3.59 + J -0.69	3.6571	-169.07
11	3.00	-3.34 + J -1.25	3.5710	-159.47
12	3.30	-3.06 + J -1.71	3.4991	-150.83
13	3.60	-2.74 + J -2.03	3.4077	-143.41
14	3.90	-2.40 + J -2.22	3.2643	-137.24
15	4.20	-2.05 + J -2.25	3.0445	-132.25
16	4.50	-1.69 + J -2.15	2.7340	-128.30
17	4.80	-1.34 + J -1.90	2.3289	-125.26
18	5.10	-1.00 + J -1.54	1.8356	-123.01
19	5.40	-0.66 + J -1.08	1.2683	-121.47
20	5.70	-0.33 + J -0.56	0.6480	-120.57
21	6.00	0.00 + J -0.00	0.0000	-77.86

TABLE II-A1. PRESSURE AND PHASE IN THE WEDGE

REFLECTICN COEF= + 1.0
XV= x/X= 0.50   BETD= 6.00   GAMD= 1.50

N	DELD (DEG)	PRESSURE	|PRESSURE|	PHASE (DEG)
1	0.0	0.00 + J-29.52	29.5158	-90.00
2	0.30	0.00 + J-29.42	29.4248	-90.00
3	0.60	0.00 + J-29.15	29.1524	-90.00
4	0.90	0.00 + J-28.70	28.7003	-90.00
5	1.20	0.00 + J-28.07	28.0712	-90.00
6	1.50	0.00 + J-27.27	27.2690	-90.00
7	1.80	0.00 + J-26.30	26.2988	-90.00
8	2.10	0.00 + J-25.17	25.1663	-90.00
9	2.40	0.00 + J-23.88	23.8788	-90.00
10	2.70	0.00 + J-22.44	22.4440	-90.00
11	3.00	0.00 + J-20.87	20.8708	-90.00
12	3.30	0.00 + J-19.17	19.1690	-90.00
13	3.60	0.00 + J-17.35	17.3489	-90.00
14	3.90	0.00 + J-15.42	15.4220	-90.00
15	4.20	0.00 + J-13.40	13.3999	-90.00
16	4.50	0.00 + J-11.30	11.2952	-90.00
17	4.80	0.00 + J -9.12	9.1209	-90.00
18	5.10	0.00 + J -6.89	6.8903	-90.00
19	5.40	0.00 + J -4.62	4.6173	-90.00
20	5.70	0.00 + J -2.32	2.3158	-90.00
21	6.00	-0.00 + J 0.00	0.0000	151.11

TABLE II-A2. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= + 1.0
XV= x/X= 0.50 BETD= 6.00 GAMD= 3.00

N	DELD(DEG)	PRESSURE	|PRESSURE|	PHASE(DEG)
1	0.0	0.00 + J-22.59	22.5904	-90.00
2	0.30	0.00 + J-22.52	22.5208	-90.00
3	0.60	0.00 + J-22.31	22.3123	-90.00
4	0.90	0.00 + J-21.97	21.9662	-90.00
5	1.20	0.00 + J-21.48	21.4848	-90.00
6	1.50	0.00 + J-20.87	20.8708	-90.00
7	1.80	0.00 + J-20.13	20.1282	-90.00
8	2.10	0.00 + J-19.26	19.2615	-90.00
9	2.40	0.00 + J-18.28	18.2760	-90.00
10	2.70	0.00 + J-17.18	17.1779	-90.00
11	3.00	0.00 + J-15.97	15.9738	-90.00
12	3.30	0.00 + J-14.67	14.6713	-90.00
13	3.60	0.00 + J-13.28	13.2783	-90.00
14	3.90	0.00 + J-11.80	11.8035	-90.00
15	4.20	0.00 + J-10.26	10.2558	-90.00
16	4.50	0.00 + J -8.64	8.6450	-90.00
17	4.80	0.00 + J -6.98	6.9808	-90.00
18	5.10	0.00 + J -5.27	5.2736	-90.00
19	5.40	0.00 + J -3.53	3.5339	-90.00
20	5.70	0.00 + J -1.77	1.7724	-90.00
21	6.00	0.00 + J 0.00	0.0000	81.19

TABLE II-A3. PRESSURE AND PHASE IN THE WEDGE

REFLECTICN COEF= + 1.0
XV= x/X= 0.50 BETD= 6.00 GAMD= 4.50

N	DELD (DEG)	PRESSURE	|PRESSURE|	PHASE (DEG)
1	0.0	0.00 + J-12.23	12.2258	-90.00
2	0.30	0.00 + J-12.19	12.1882	-90.00
3	0.60	0.00 + J-12.08	12.0753	-90.00
4	0.90	0.00 + J-11.89	11.8880	-90.00
5	1.20	0.00 + J-11.63	11.6275	-90.00
6	1.50	0.00 + J-11.30	11.2952	-90.00
7	1.80	0.00 + J-10.89	10.8933	-90.00
8	2.10	0.00 + J-10.42	10.4242	-90.00
9	2.40	0.00 + J -9.89	9.8909	-90.00
10	2.70	0.00 + J -9.30	9.2966	-90.00
11	3.00	0.00 + J -8.64	8.6450	-90.00
12	3.30	0.00 + J -7.94	7.9400	-90.00
13	3.60	0.00 + J -7.19	7.1862	-90.00
14	3.90	0.00 + J -6.39	6.3880	-90.00
15	4.20	0.00 + J -5.55	5.5504	-90.00
16	4.50	0.00 + J -4.68	4.6786	-90.00
17	4.80	0.00 + J -3.78	3.7780	-90.00
18	5.10	0.00 + J -2.85	2.8541	-90.00
19	5.40	0.00 + J -1.91	1.9125	-90.00
20	5.70	0.00 + J -0.96	0.9592	-90.00
21	6.00	-0.00 + J 0.00	0.0000	103.86

TABLE II-B1. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= + 1.0
XV= x/X= 1.00   BETD= 6.00   GAMD= 1.50

N	DELD(DEG)	PRESSURE	|PRESSURE|	PHASE(DEG)
1	0.0	0.00 + J-12.40	12.4026	-90.00
2	0.30	0.00 + J-12.36	12.3647	-90.00
3	0.60	0.00 + J-12.25	12.2511	-90.00
4	0.90	0.00 + J-12.06	12.0626	-90.00
5	1.20	0.00 + J-11.80	11.8001	-90.00
6	1.50	0.00 + J-11.47	11.4653	-90.00
7	1.80	0.00 + J-11.06	11.0600	-90.00
8	2.10	0.00 + J-10.59	10.5866	-90.00
9	2.40	0.00 + J-10.05	10.0479	-90.00
10	2.70	0.00 + J -9.45	9.4471	-90.00
11	3.00	0.00 + J -8.79	8.7877	-90.00
12	3.30	0.00 + J -8.07	8.0737	-90.00
13	3.60	0.00 + J -7.31	7.3094	-90.00
14	3.90	0.00 + J -6.50	6.4994	-90.00
15	4.20	0.00 + J -5.65	5.6488	-90.00
16	4.50	0.00 + J -4.76	4.7627	-90.00
17	4.80	0.00 + J -3.85	3.8467	-90.00
18	5.10	-0.00 + J -2.91	2.9064	-90.00
19	5.40	-0.00 + J -1.95	1.9479	-90.00
20	5.70	-0.00 + J -0.98	0.9770	-90.00
21	6.00	-0.00 + J -0.00	0.0000	-166.06

TABLE II-B2. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= + 1.0
XV= x/X= 1.00 BETD= 6.00 GAMD= 3.00

N	DELD (DEG)	PRESSURE	|PRESSURE|	PHASE (DEG)
1	0.0	0.00 + J -9.53	9.5253	-90.00
2	0.30	0.00 + J -9.50	9.4954	-90.00
3	0.60	0.00 + J -9.41	9.4058	-90.00
4	0.90	0.00 + J -9.26	9.2572	-90.00
5	1.20	0.00 + J -9.05	9.0507	-90.00
6	1.50	0.00 + J -8.79	8.7877	-90.00
7	1.80	0.00 + J -8.47	8.4701	-90.00
8	2.10	0.00 + J -8.10	8.1001	-90.00
9	2.40	0.00 + J -7.68	7.6802	-90.00
10	2.70	0.00 + J -7.21	7.2134	-90.00
11	3.00	0.00 + J -6.70	6.7026	-90.00
12	3.30	0.00 + J -6.15	6.1514	-90.00
13	3.60	0.00 + J -5.56	5.5632	-90.00
14	3.90	0.00 + J -4.94	4.9417	-90.00
15	4.20	0.00 + J -4.29	4.2910	-90.00
16	4.50	0.00 + J -3.61	3.6149	-90.00
17	4.80	0.00 + J -2.92	2.9176	-90.00
18	5.10	-0.00 + J -2.20	2.2032	-90.00
19	5.40	0.00 + J -1.48	1.4759	-90.00
20	5.70	0.00 + J -0.74	0.7401	-90.00
21	6.00	0.00 + J 0.00	0.0000	70.94

TABLE II-B3. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= + 1.0
XV= x/X= 1.00   BETD= 6.00   GAMD= 4.50

N	DELD(DEG)	PRESSURE	|PRESSURE|	PHASE(DEG)
1	0.0	-0.00 + J -5.17	5.1728	-90.00
2	0.30	-0.00 + J -5.16	5.1561	-90.00
3	0.60	-0.00 + J -5.11	5.1062	-90.00
4	0.90	-0.00 + J -5.02	5.0235	-90.00
5	1.20	0.00 + J -4.91	4.9086	-90.00
6	1.50	0.00 + J -4.76	4.7627	-90.00
7	1.80	-0.00 + J -4.59	4.5868	-90.00
8	2.10	0.00 + J -4.38	4.3823	-90.00
9	2.40	0.00 + J -4.15	4.1510	-90.00
10	2.70	0.00 + J -3.89	3.8946	-90.00
11	3.00	0.00 + J -3.61	3.6149	-90.00
12	3.30	0.00 + J -3.31	3.3140	-90.00
13	3.60	0.00 + J -2.99	2.9939	-90.00
14	3.90	0.00 + J -2.66	2.6567	-90.00
15	4.20	0.00 + J -2.30	2.3047	-90.00
16	4.50	0.00 + J -1.94	1.9400	-90.00
17	4.80	0.00 + J -1.56	1.5646	-90.00
18	5.10	0.00 + J -1.18	1.1808	-90.00
19	5.40	0.00 + J -0.79	0.7907	-90.00
20	5.70	0.00 + J -0.40	0.3964	-90.00
21	6.00	0.00 + J 0.00	0.0000	20.62

TABLE II-C1. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= + 1.0
XV= x/X= 2.00 BETD= 6.00 GAMD= 1.50

N	DELD (DEG)	PRESSURE	|PRESSURE|	PHASE (DEG)
1	0.0	-0.00 + J 8.34	8.3352	90.00
2	0.30	-0.00 + J 8.15	8.1494	90.00
3	0.60	-0.00 + J 7.61	7.6054	90.00
4	0.90	-0.00 + J 6.74	6.7425	90.00
5	1.20	-0.00 + J 5.62	5.6217	90.00
6	1.50	-0.00 + J 4.32	4.3206	90.00
7	1.80	-0.00 + J 2.93	2.9255	90.00
8	2.10	-0.00 + J 1.52	1.5247	90.00
9	2.40	0.00 + J 0.20	0.2007	90.00
10	2.70	0.00 + J -0.98	0.9756	-90.00
11	3.00	0.00 + J -1.95	1.9496	-90.00
12	3.30	0.00 + J -2.69	2.6851	-90.00
13	3.60	0.00 + J -3.16	3.1649	-90.00
14	3.90	0.00 + J -3.39	3.3894	-90.00
15	4.20	0.00 + J -3.37	3.3733	-90.00
16	4.50	0.00 + J -3.14	3.1426	-90.00
17	4.80	0.00 + J -2.73	2.7302	-90.00
18	5.10	0.00 + J -2.17	2.1726	-90.00
19	5.40	0.00 + J -1.51	1.5072	-90.00
20	5.70	0.00 + J -0.77	0.7711	-90.00
21	6.00	0.00 + J 0.00	0.0000	50.61

TABLE II-C2. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= + 1.0
XV= x/X= 2.00 BETD= 6.00 GAMD= 3.00

N	DELD (DEG)	PRESSURE	|PRESSURE|	PHASE (DEG)
1	0.0	0.00 + J -6.29	6.2853	-90.00
2	0.30	0.00 + J -6.10	6.1035	-90.00
3	0.60	0.00 + J -5.56	5.5619	-90.00
4	0.90	0.00 + J -4.67	4.6721	-90.00
5	1.20	0.00 + J -3.46	3.4562	-90.00
6	1.50	0.00 + J -1.95	1.9496	-90.00
7	1.80	0.00 + J -0.20	0.2045	-90.00
8	2.10	-0.00 + J 1.71	1.7080	90.00
9	2.40	-0.00 + J 3.70	3.6973	90.00
10	2.70	-0.00 + J 5.66	5.6557	90.00
11	3.00	-0.00 + J 7.46	7.4632	90.00
12	3.30	-0.00 + J 9.00	8.9950	90.00
13	3.60	-0.C0 + J 10.13	10.1318	90.00
14	3.90	-0.00 + J 10.77	10.7703	90.00
15	4.20	-0.00 + J 10.83	10.8344	90.00
16	4.50	-0.00 + J 10.28	10.2848	90.00
17	4.80	-0.C0 + J 9.12	9.1250	90.00
18	5.10	-0.00 + J 7.40	7.4047	90.00
19	5.40	-0.00 + J 5.22	5.2178	90.00
20	5.70	0.00 + J 2.70	2.6962	90.00
21	6.00	0.00 + J 0.00	0.0000	8.91

TABLE II-C3. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= + 1.0
XV= x/X= 2.00 BETD= 6.00 GAMD= 4.50

N	DELD (DEG)	PRESSURE	|PRESSURE|	PHASE (DEG)
1	0.0	0.00 + J-12.23	12.2343	-90.00
2	0.30	0.00 + J-11.81	11.8100	-90.00
3	0.60	0.00 + J-10.57	10.5696	-90.00
4	0.90	0.00 + J -8.61	8.6071	-90.00
5	1.20	0.00 + J -6.07	6.0695	-90.00
6	1.50	0.00 + J -3.14	3.1426	-90.00
7	1.80	0.00 + J -0.03	0.0340	-90.00
8	2.10	-0.00 + J 3.05	3.0452	90.00
9	2.40	-0.00 + J 5.90	5.8974	90.00
10	2.70	-0.00 + J 8.35	8.3538	90.00
11	3.00	-0.00 + J 10.28	10.2848	90.00
12	3.30	-0.00 + J 11.61	11.6056	90.00
13	3.60	-0.00 + J 12.28	12.2775	90.00
14	3.90	-0.00 + J 12.30	12.3044	90.00
15	4.20	-0.00 + J 11.73	11.7253	90.00
16	4.50	-0.00 + J 10.61	10.6058	90.00
17	4.80	-0.00 + J 9.03	9.0290	90.00
18	5.10	-0.00 + J 7.09	7.0866	90.00
19	5.40	-0.00 + J 4.87	4.8729	90.00
20	5.70	-0.00 + J 2.48	2.4806	90.00
21	6.00	-0.00 + J 0.00	0.0000	109.30

TABLE III-A1. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= - 1.0
XV= x/X= 0.50   BETD= 6.00   GAMD= 1.50

N	DELD (DEG)	PRESSURE	|PRESSURE|	PHASE (DEG)
1	0.0	-0.00 + J 0.00	0.0000	168.06
2	0.30	-0.00 + J -0.00	0.0000	-180.00
3	0.60	-0.00 + J 0.00	0.0001	180.00
4	0.90	-0.00 + J 0.00	0.0001	180.00
5	1.20	-0.00 + J 0.00	0.0002	180.00
6	1.50	-0.00 + J 0.00	0.0002	180.00
7	1.80	-0.00 + J -0.00	0.0002	-180.00
8	2.10	-0.00 + J -0.00	0.0002	-180.00
9	2.40	-0.00 + J 0.00	0.0002	180.00
10	2.70	-0.00 + J 0.00	0.0003	180.00
11	3.00	-0.00 + J 0.00	0.0003	180.00
12	3.30	-0.00 + J 0.00	0.0003	180.00
13	3.60	-0.00 + J -0.00	0.0002	-180.00
14	3.90	-0.00 + J -0.00	0.0002	-180.00
15	4.20	-0.00 + J 0.00	0.0002	180.00
16	4.50	-0.00 + J 0.00	0.0002	180.00
17	4.80	-0.00 + J 0.00	0.0002	180.00
18	5.10	-0.00 + J -0.00	0.0001	-180.00
19	5.40	-0.00 + J -0.00	0.0001	-180.00
20	5.70	-0.00 + J 0.00	0.0000	180.00
21	6.00	-0.00 + J 0.00	0.0000	91.15

TABLE III-A2. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= - 1.0
XV= x/X= 0.50 BETD= 6.00 GAMD= 3.00

N	DELD (DEG)	PRESSURE	|PRESSURE|	PHASE (DEG)
1	0.0	-0.00 + J 0.00	0.0000	144.21
2	0.30	-0.00 + J -0.00	0.0001	-180.00
3	0.60	-0.00 + J -0.00	0.0001	-180.00
4	0.90	-0.00 + J -0.00	0.0002	-180.00
5	1.20	-0.00 + J 0.00	0.0002	180.00
6	1.50	-0.00 + J 0.00	0.0003	180.00
7	1.80	-0.00 + J -0.00	0.0003	-180.00
8	2.10	-0.00 + J -0.00	0.0003	-180.00
9	2.40	-0.00 + J 0.00	0.0004	180.00
10	2.70	-0.00 + J 0.00	0.0004	180.00
11	3.00	-0.00 + J 0.00	0.0004	180.00
12	3.30	-0.00 + J 0.00	0.0004	180.00
13	3.60	-0.00 + J -0.00	0.0004	-180.00
14	3.90	-0.00 + J 0.00	0.0003	180.00
15	4.20	-0.00 + J 0.00	0.0003	180.00
16	4.50	-0.00 + J 0.00	0.0003	180.00
17	4.80	-0.00 + J 0.00	0.0002	180.00
18	5.10	-0.00 + J -0.00	0.0002	-180.00
19	5.40	-0.00 + J -0.00	0.0001	-180.00
20	5.70	-0.00 + J 0.00	0.0001	180.00
21	6.00	-0.00 + J 0.00	0.0000	137.45

TABLE III-A3. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= - 1.0
XV= x/X= 0.50 BETD= 6.00 GAMD= 4.50

N	DELD (DEG)	PRESSURE	|PRESSURE|	PHASE (DEG)
1	0.0	-0.00 + J 0.00	0.0000	93.18
2	0.30	-0.00 + J 0.00	0.0000	180.00
3	0.60	-0.00 + J -0.00	0.0001	-180.00
4	0.90	-0.00 + J 0.00	0.0001	180.00
5	1.20	-0.00 + J 0.00	0.0002	180.00
6	1.50	-0.00 + J 0.00	0.0002	180.00
7	1.80	-0.00 + J 0.00	0.0002	180.00
8	2.10	-0.00 + J 0.00	0.0002	180.00
9	2.40	-0.00 + J -0.00	0.0002	-180.00
10	2.70	-0.00 + J 0.00	0.0003	180.00
11	3.00	-0.00 + J 0.00	0.0003	180.00
12	3.30	-0.00 + J 0.00	0.0003	180.00
13	3.60	-0.00 + J 0.00	0.0002	180.00
14	3.90	-0.00 + J 0.00	0.0002	180.00
15	4.20	-0.00 + J 0.00	0.0002	180.00
16	4.50	-0.00 + J 0.00	0.0002	180.00
17	4.80	-0.00 + J 0.00	0.0002	180.00
18	5.10	-0.00 + J 0.00	0.0001	180.00
19	5.40	-0.00 + J 0.00	0.0001	180.00
20	5.70	-0.00 + J 0.00	0.0000	180.00
21	6.00	-0.00 + J 0.00	0.0000	115.47

TABLE III-B1. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= - 1.0
XV= x/X= 1.00   BETD= 6.00   GAMD= 1.50

N	DELD (DEG)	PRESSURE	|PRESSURE|	PHASE (DEG)
1	0.0	0.00 + J -0.00	0.0000	-47.78
2	0.30	-2.13 + J -0.00	2.1299	-180.00
3	0.60	-4.21 + J -0.00	4.2074	-180.00
4	0.90	-6.18 + J -0.00	6.1812	-180.00
5	1.20	-8.00 + J -0.00	8.0029	-180.00
6	1.50	-9.63 + J -0.00	9.6275	-180.00
7	1.80	-11.01 + J -0.00	11.0150	-180.00
8	2.10	-12.13 + J -0.00	12.1313	-180.00
9	2.40	-12.95 + J -0.00	12.9489	-180.00
10	2.70	-13.45 + J -0.00	13.4476	-180.00
11	3.00	-13.62 + J -0.00	13.6153	-180.00
12	3.30	-13.45 + J -0.00	13.4476	-180.00
13	3.60	-12.95 + J -0.00	12.9489	-180.00
14	3.90	-12.13 + J -0.00	12.1313	-180.00
15	4.20	-11.01 + J -0.00	11.0150	-180.00
16	4.50	-9.63 + J -0.00	9.6275	-180.00
17	4.80	-8.00 + J -0.00	8.0029	-180.00
18	5.10	-6.18 + J -0.00	6.1812	-180.00
19	5.40	-4.21 + J -0.00	4.2074	-180.00
20	5.70	-2.13 + J -0.00	2.1299	-180.00
21	6.00	0.00 + J -0.00	0.0000	-36.45

TABLE III-B2. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= - 1.0
XV= x/X= 1.00   BETD= 6.00   GAMD= 3.00

N	DELD(DEG)	PRESSURE	|PRESSURE|	PHASE(DEG)
1	0.0	0.00 + J -0.00	0.0000	-22.56
2	0.30	-3.01 + J -0.00	3.0121	-180.00
3	0.60	-5.95 + J -0.00	5.9501	-180.00
4	0.90	-8.74 + J -0.00	8.7415	-180.00
5	1.20	-11.32 + J -0.00	11.3177	-180.00
6	1.50	-13.62 + J -0.00	13.6153	-180.00
7	1.80	-15.58 + J -0.00	15.5775	-180.00
8	2.10	-17.16 + J -0.00	17.1562	-180.00
9	2.40	-18.31 + J -0.00	18.3125	-180.00
10	2.70	-19.02 + J -0.00	19.0178	-180.00
11	3.00	-19.25 + J -0.00	19.2549	-180.00
12	3.30	-19.02 + J -0.00	19.0178	-180.00
13	3.60	-18.31 + J -0.00	18.3125	-180.00
14	3.90	-17.16 + J -0.00	17.1562	-180.00
15	4.20	-15.58 + J -0.00	15.5775	-180.00
16	4.50	-13.62 + J -0.00	13.6153	-180.00
17	4.80	-11.32 + J -0.00	11.3177	-180.00
18	5.10	-8.74 + J -0.00	8.7415	-180.00
19	5.40	-5.95 + J -0.00	5.9501	-180.00
20	5.70	-3.01 + J -0.00	3.0121	-180.00
21	6.00	0.00 + J -0.00	0.0000	-37.20

TABLE III-B3. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= - 1.0
XV= x/X= 1.00 BETD= 6.00 GAMD= 4.50

N	DELD (DEG)	PRESSURE	|PRESSURE|	PHASE (DEG)
1	0.0	0.00 + J 0.00	0.0000	1.53
2	0.30	-2.13 + J -0.00	2.1299	-180.00
3	0.60	-4.21 + J -0.00	4.2074	-180.00
4	0.90	-6.18 + J -0.00	6.1812	-180.00
5	1.20	-8.00 + J -0.00	8.0029	-180.00
6	1.50	-9.63 + J -0.00	9.6275	-180.00
7	1.80	-11.01 + J -0.00	11.0150	-180.00
8	2.10	-12.13 + J -0.00	12.1313	-180.00
9	2.40	-12.95 + J -0.00	12.9489	-180.00
10	2.70	-13.45 + J -0.00	13.4476	-180.00
11	3.00	-13.62 + J -0.00	13.6153	-180.00
12	3.30	-13.45 + J -0.00	13.4476	-180.00
13	3.60	-12.95 + J -0.00	12.9489	-180.00
14	3.90	-12.13 + J -0.00	12.1313	-180.00
15	4.20	-11.01 + J -0.00	11.0150	-180.00
16	4.50	-9.63 + J -0.00	9.6275	-180.00
17	4.80	-8.00 + J -0.00	8.0029	-180.00
18	5.10	-6.18 + J -0.00	6.1812	-180.00
19	5.40	-4.21 + J -0.00	4.2074	-180.00
20	5.70	-2.13 + J -0.00	2.1299	-180.00
21	6.00	0.00 + J -0.00	0.0000	-69.25

TABLE III-C1. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= - 1.0
XV= x/X= 2.00   BETD= 6.00   GAMD= 1.50

N	DELD(DEG)	PRESSURE	|PRESSURE|	PHASE(DEG)
1	0.0	-0.00 + J 0.00	0.0000	122.38
2	0.30	-1.58 + J -0.00	1.5760	-180.00
3	0.60	-2.94 + J -0.00	2.9437	-180.00
4	0.90	-3.92 + J -0.00	3.9165	-180.00
5	1.20	-4.35 + J -0.00	4.3491	-180.00
6	1.50	-4.15 + J -0.00	4.1528	-180.00
7	1.80	-3.31 + J -0.00	3.3058	-180.00
8	2.10	-1.86 + J -0.00	1.8556	-180.00
9	2.40	0.08 + J -0.00	0.0840	-0.00
10	2.70	2.34 + J -0.00	2.3440	-0.00
11	3.00	4.72 + J 0.00	4.7158	0.00
12	3.30	6.97 + J 0.00	6.9714	0.00
13	3.60	8.89 + J 0.00	8.8859	0.00
14	3.90	10.26 + J 0.00	10.2591	0.00
15	4.20	10.94 + J 0.00	10.9359	0.00
16	4.50	10.82 + J 0.00	10.8218	0.00
17	4.80	9.89 + J 0.00	9.8926	0.00
18	5.10	8.20 + J 0.00	8.1982	0.00
19	5.40	5.86 + J 0.00	5.8581	0.00
20	5.70	3.05 + J 0.00	3.0514	0.00
21	6.00	-0.00 + J -0.00	0.0000	-126.23

TABLE III-C2. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= - 1.0
XV= x/X= 2.00 BETD= 6.00 GAMD= 3.00

N	DELD (DEG)	PRESSURE	|PRESSURE|	PHASE (DEG)
1	0.0	-0.00 + J 0.00	0.0000	126.34
2	0.30	1.04 + J 0.00	1.0433	0.00
3	0.60	2.06 + J 0.00	2.0609	0.00
4	0.90	3.03 + J 0.00	3.0278	0.00
5	1.20	3.92 + J 0.00	3.9200	0.00
6	1.50	4.72 + J 0.00	4.7158	0.00
7	1.80	5.40 + J 0.00	5.3954	0.00
8	2.10	5.94 + J 0.00	5.9421	0.00
9	2.40	6.34 + J 0.00	6.3426	0.00
10	2.70	6.59 + J 0.00	6.5868	0.00
11	3.00	6.67 + J 0.00	6.6689	0.00
12	3.30	6.59 + J 0.00	6.5868	0.00
13	3.60	6.34 + J 0.00	6.3426	0.00
14	3.90	5.94 + J 0.00	5.9421	0.00
15	4.20	5.40 + J 0.00	5.3954	0.00
16	4.50	4.72 + J 0.00	4.7158	0.00
17	4.80	3.92 + J 0.00	3.9200	0.00
18	5.10	3.03 + J 0.00	3.0278	0.00
19	5.40	2.06 + J 0.00	2.0609	0.00
20	5.70	1.04 + J 0.00	1.0433	0.00
21	6.00	-0.00 + J -0.00	0.0000	-169.39

TABLE III-C3. PRESSURE AND PHASE IN THE WEDGE

REFLECTION COEF= - 1.0
XV= x/X= 2.00 BETD= 6.00 GAMD= 4.50

N	DELD(DEG)	PRESSURE	|PRESSURE|	PHASE(DEG)
1	0.0	-0.00 + J 0.00	0.0000	112.62
2	0.30	3.05 + J 0.00	3.0514	0.00
3	0.60	5.86 + J 0.00	5.8581	0.00
4	0.90	8.20 + J 0.00	8.1982	0.00
5	1.20	9.89 + J 0.00	9.8926	0.00
6	1.50	10.82 + J 0.00	10.8218	0.00
7	1.80	10.94 + J 0.00	10.9359	0.00
8	2.10	10.26 + J 0.00	10.2591	0.00
9	2.40	8.89 + J 0.00	8.8859	0.00
10	2.70	6.97 + J 0.00	6.9714	0.00
11	3.00	4.72 + J 0.00	4.7158	0.00
12	3.30	2.34 + J -0.00	2.3440	-0.00
13	3.60	0.08 + J -0.00	0.0840	-0.00
14	3.90	-1.86 + J -0.00	1.8556	-180.00
15	4.20	-3.31 + J -0.00	3.3058	-180.00
16	4.50	-4.15 + J -0.00	4.1528	-180.00
17	4.80	-4.35 + J -0.00	4.3491	-180.00
18	5.10	-3.92 + J -0.00	3.9165	-180.00
19	5.40	-2.94 + J -0.00	2.9437	-180.00
20	5.70	-1.58 + J -0.00	1.5760	-180.00
21	6.00	0.00 + J 0.00	0.0000	32.35

APPENDIX E

MEASURED DATA

A. EXPERIMENTAL DESIGN

1. Selected Material

Fresh (tap) water and #30 fine sand were the media used in the experiment. The grain size of #30 fine sand varies from 0.70 mm to 0.15 mm. To remove air bubbles, the water was allowed to settle for a couple of weeks. The bubbles in the sand were removed by using high speed jet of water to agitate the sand-water mixture.

2. Velocity Measurement Techniques

Glass fiber covered wood tank measuring 304 cm long, 117 cm wide, and 90 cm high was available. To avoid particulate scattering, the wavelength must be at least three times the grain size [Ref. 8]. Since the largest grain size in #30 sand is 0.07 cm, a wavelength, equal to 1.7 cm for 100 kHz, readily qualifies.(See Figure 15)

A schematic of the equipment configuration is shown in Figure 16. Output from a General Radio Model 1310 oscillator with a frequency range of 2 Hz to 2 MHz was fed simultaneously into a frequency counter and a tone burst generator. The counter, HP 5233L, would read $\pm$ 10 Hz at 100 kHz. The GR Type 396-A tone burst generator was used to generate either 8 or 16 cycle pulses.

Celesco Industries type LC 10 hydrophones were used as source and receivers. The LC10 is a small (0.97 cm diameter by 2.87 cm long) cylinder, with a receiving range of 0.1 Hz to 120 kHz.

The received signals were amplified 20 dB or 40 dB by a HP-465A Amplifier, then passed through a Spencer-Kennedy Lab. Inc. Model 302 variable electronic filter (set at 20 kHz high pass) to eliminate low frequency mechanical noise present in the laboratory before being passed to the oscilloscope. All measurements were made under far-field conditions.

B. DENSITY

1. Water

According to Lange's "Handbook of Chemistry", the density of distilled water ranges from 0.99913 g/cm^3 at 15° C to 0.99707 g/cm^3 at 25°C. The expected density of room temperature water, to 3 significant figures, was therefore 1.00 g/cm^3.

2. Sand

The density of water saturated sand was measured by partially filling a weighed 100 ml graduated cylinder with saturated sand, observing the volume and the total weight. The density of water saturated sand, from ten separate measurements was 1.98 $\pm$ 0.01 g/cm^3.

C. SOUND SPEED

1. Water

Measurements were made using one LC-10 as receiver, and another as source. The LC-10 source was clamped on the bar above the tank, the second LC-10 was moved along the straight line from the source with same depth. As the receiver was moved, the distance and the time of flight between the receiver and the source were measured. The averaged sound speed was 1446 ± 30 m/sec at 20°C.

2. Sand

The technique was the same as in water except using amplifier 40 dB. The averaged sound speed was 1607 ± 30 m/sec at 20° C.

D. ATTENUATION

Taking the natural logarithm of the well-known equation

$$V = (V_o / r) \exp(-\alpha r)$$

where

V = the measured voltage (volt),

V_o = the source voltage (volt),

r = the distance from the source (meter),

α = the attenuation (nepers/meter),

the linear relation

$$\ln(Vr) = \ln(V_o) - \alpha r$$

is obtained, will be the slope of a graph of ln(Vr) vs r. Graphs of the five data sets of from Table IV-1 to Table IV-5 are shown in Figure 17 through 21.

The well-known Hamilton equation is as follows

$$a = k\ f$$

where

a is absorption in saturated sand (dB/m),

k is proportional constant [(dB/m)/kHz] ,

f is frequency of the source (kHz).

The theoretical value of k is 0.25 [Ref. 9]. In Figure 22, the value of k is shown as 0.27 $\pm$ 0.06.

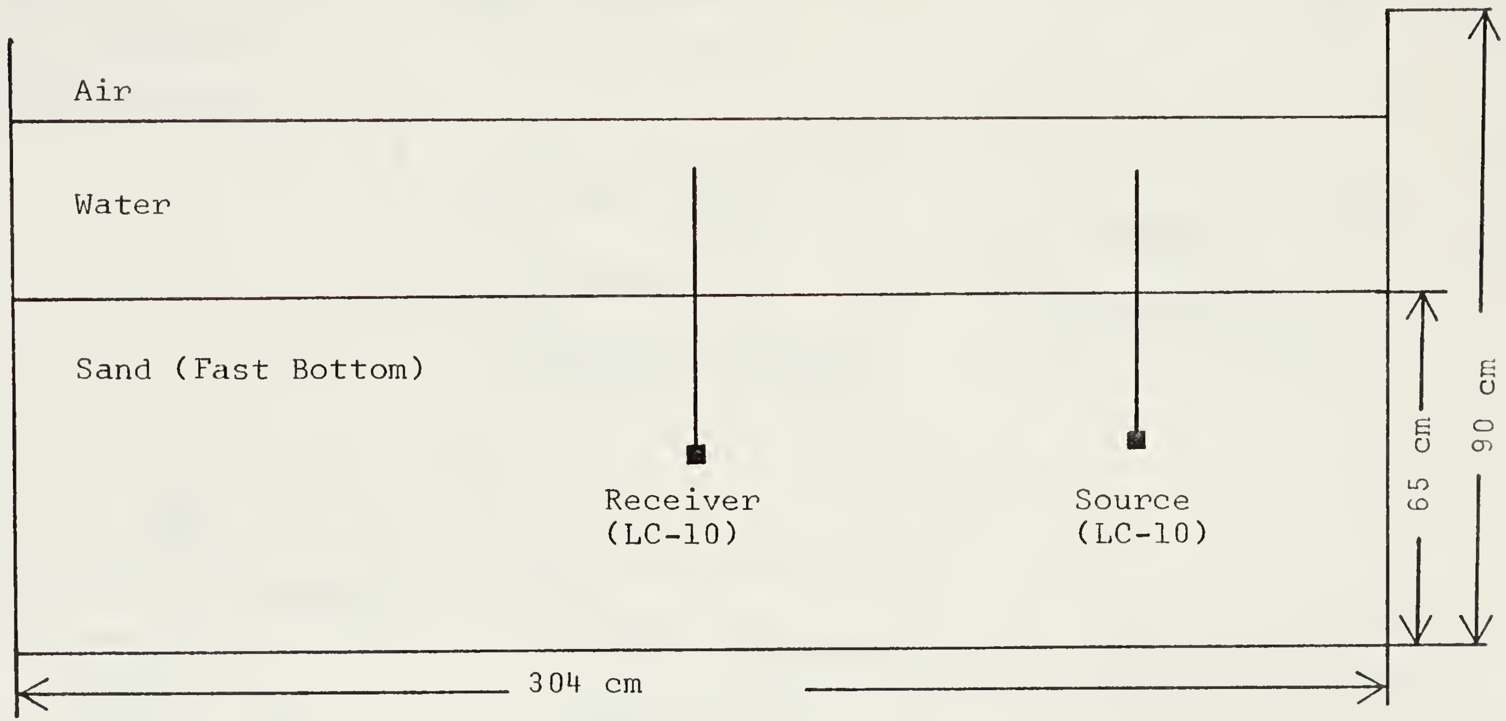

Figure 15 Geometry for the Measurements

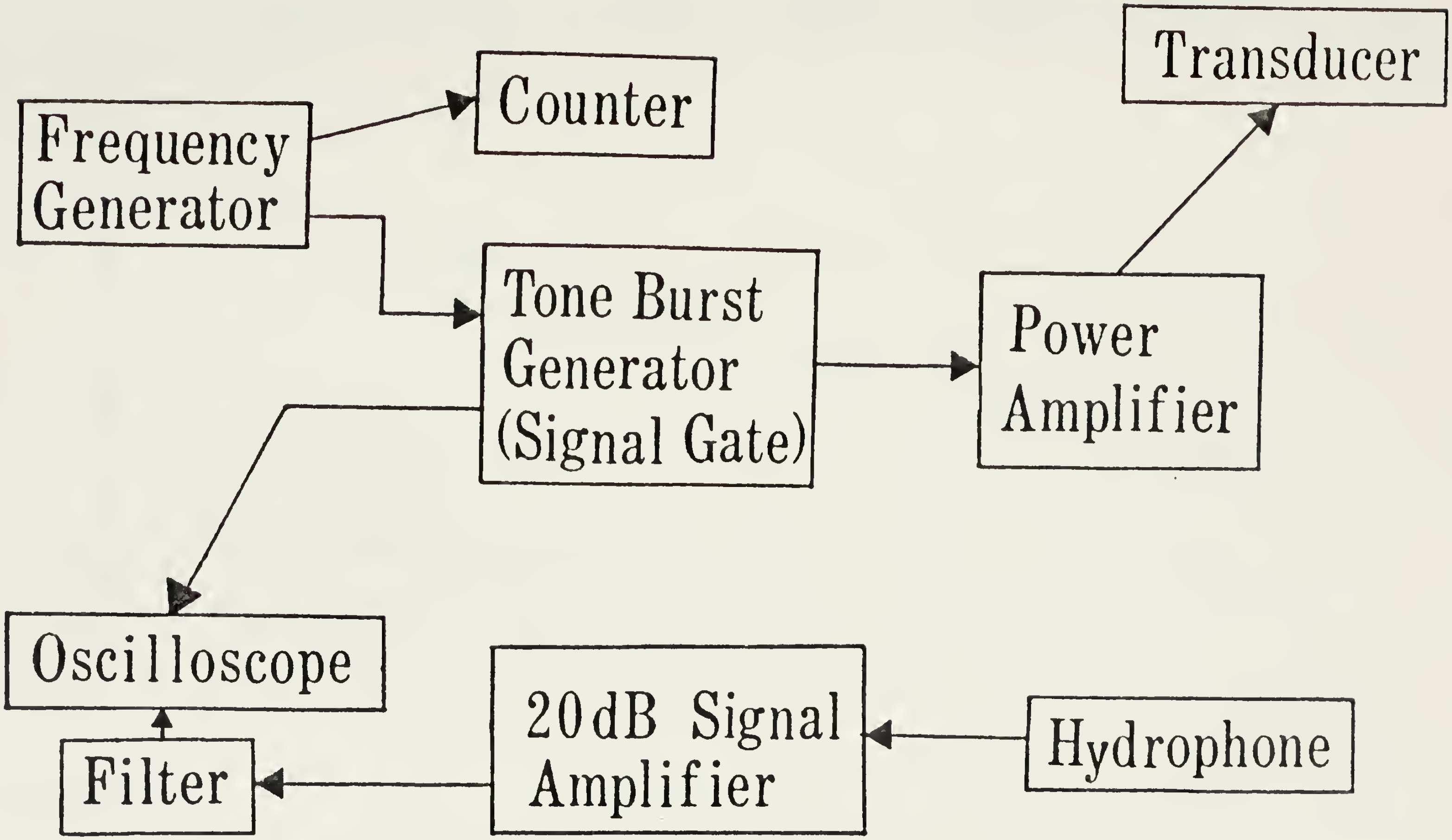

Figure 16 Electronic Equipment Schematic (Adopted from Ref. 8)

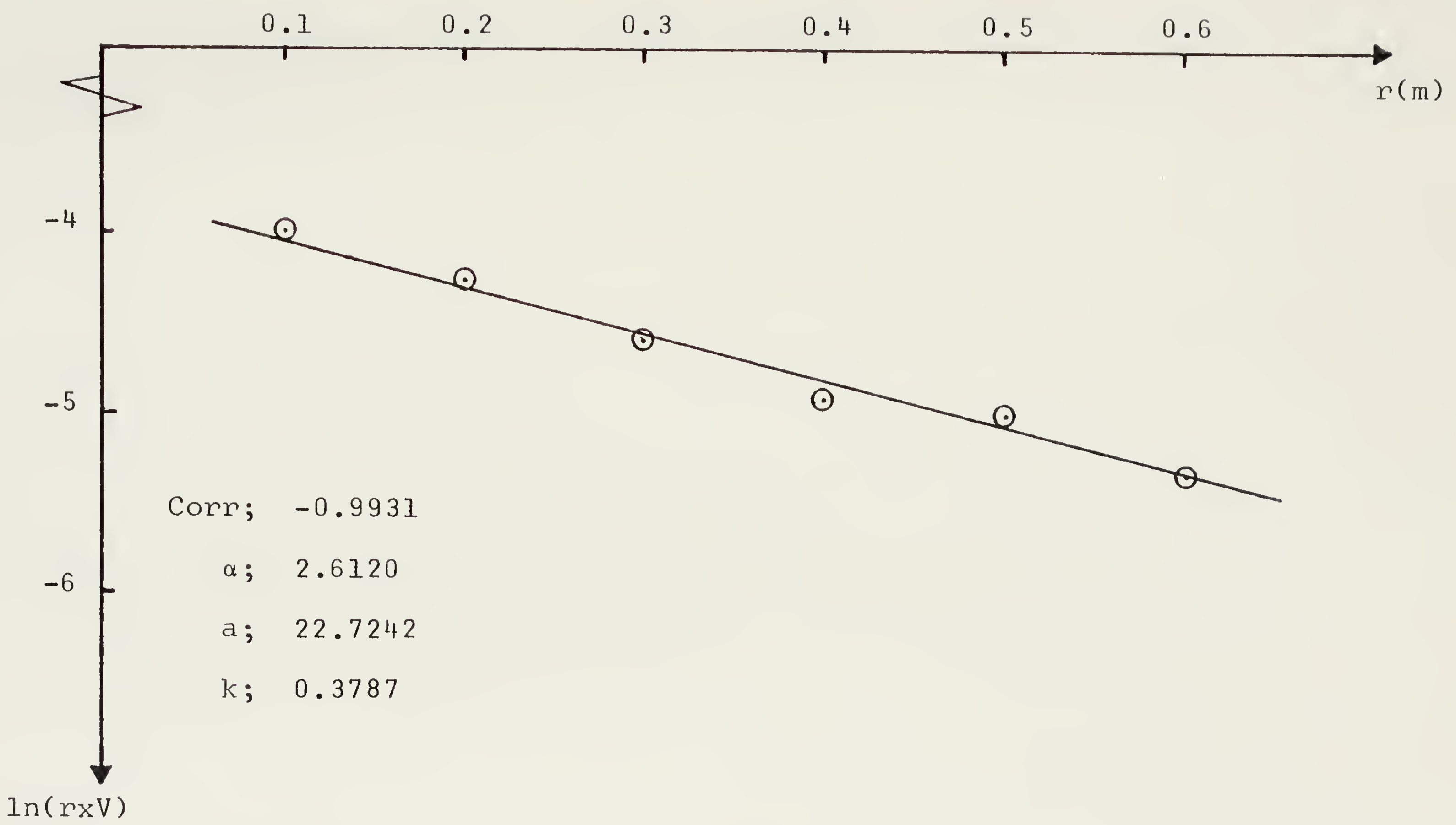

Figure 17 Plot of Data in Table IV-1 (60 kHz)

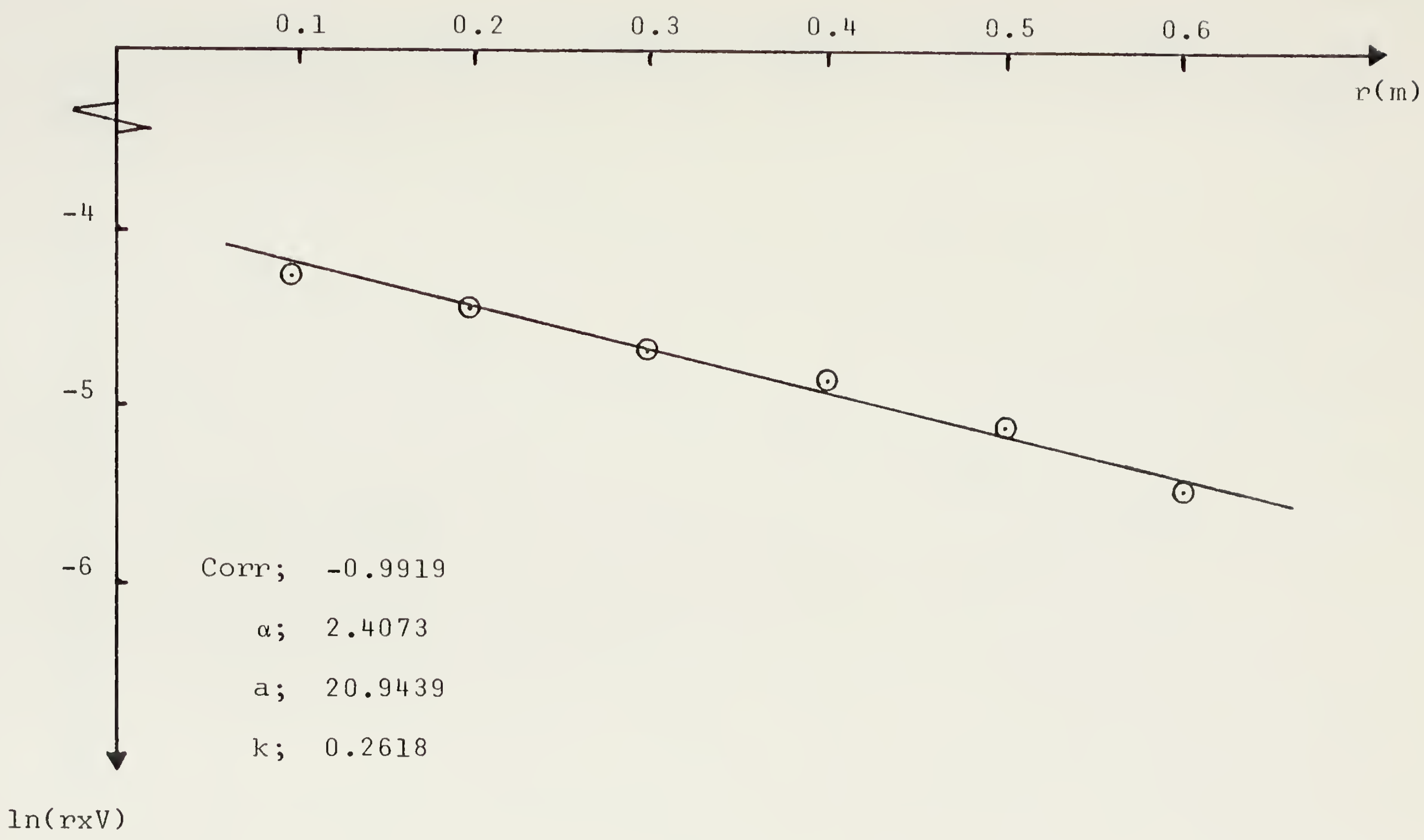

Figure 18 Plot of Data in Table IV-2 (80 kHz)

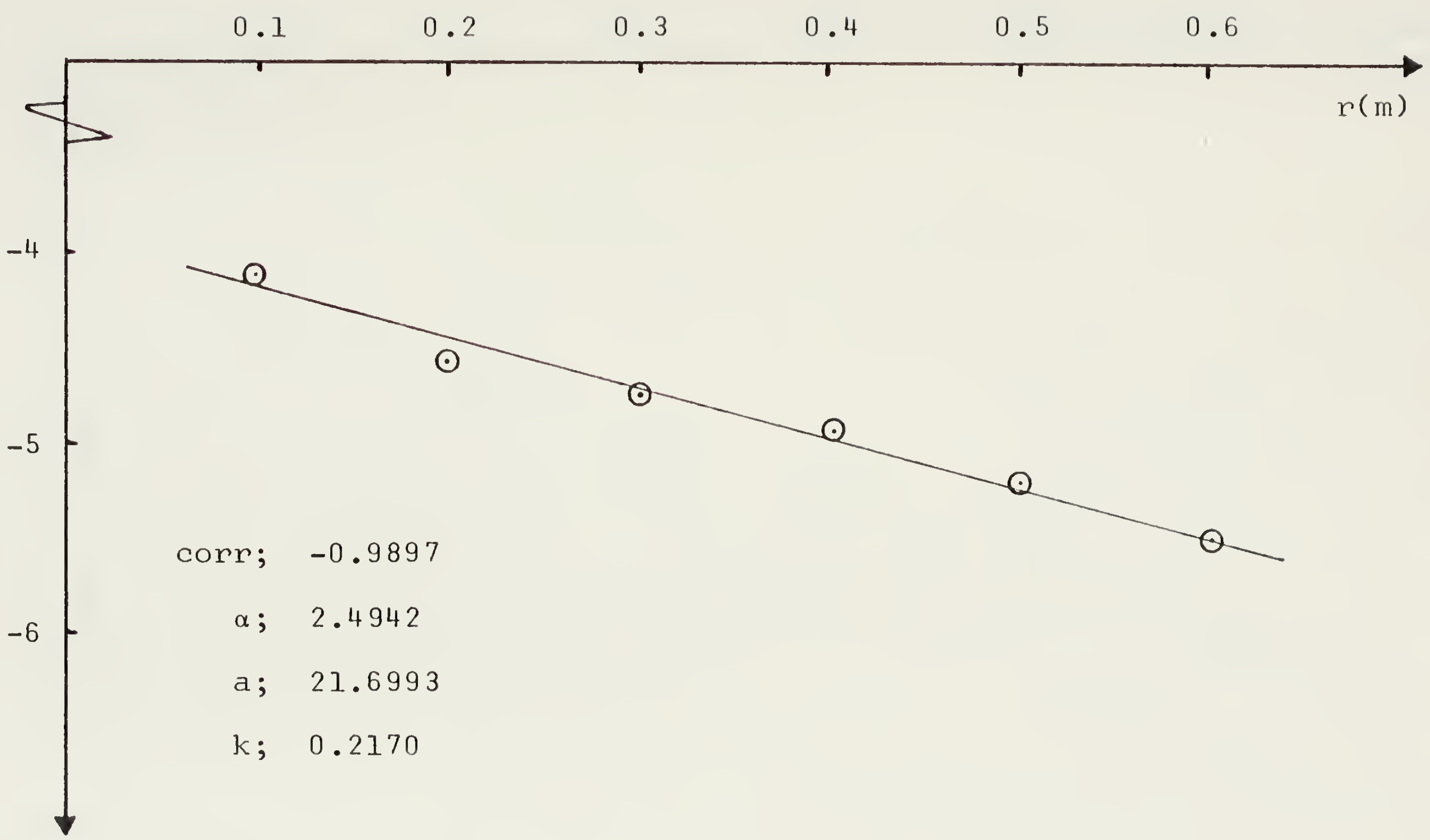

Figure 19 Plot of Data in Table IV-3 (100 kHz)

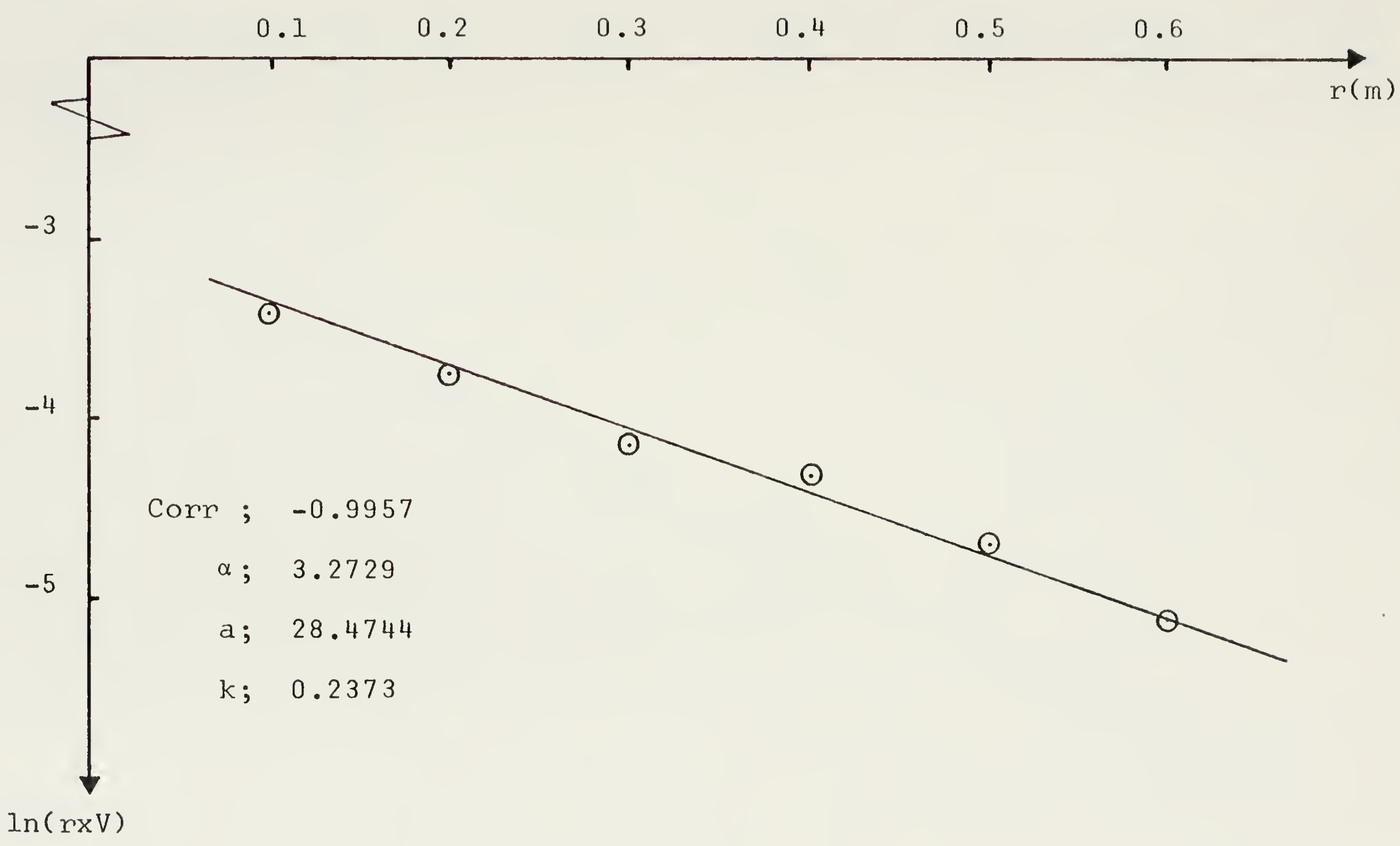

Figure 20 Plot of Data in Table IV-4 (120 kHz)

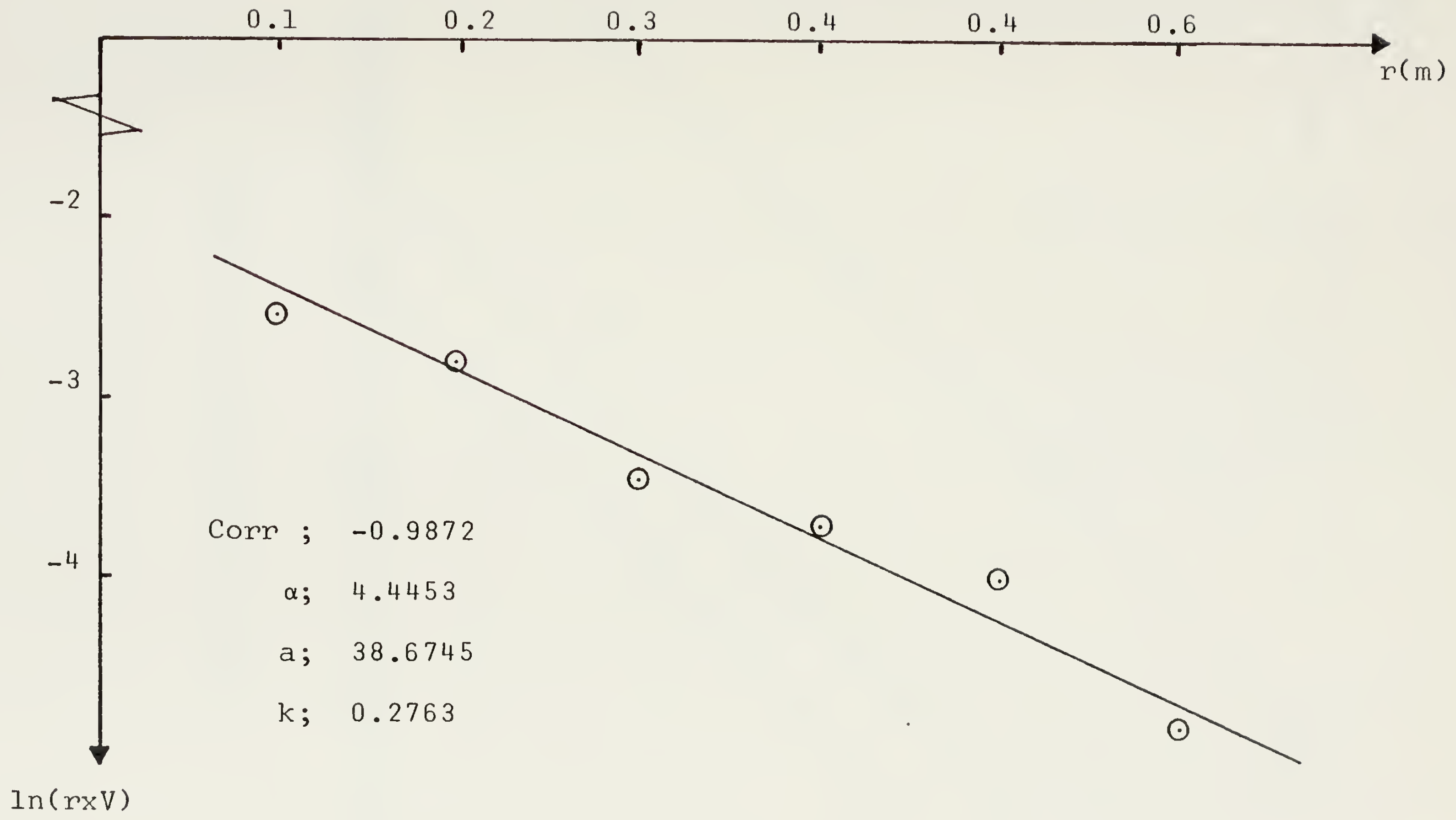

Figure 21 Plot of Data in Table IV-5 (140 kHz)

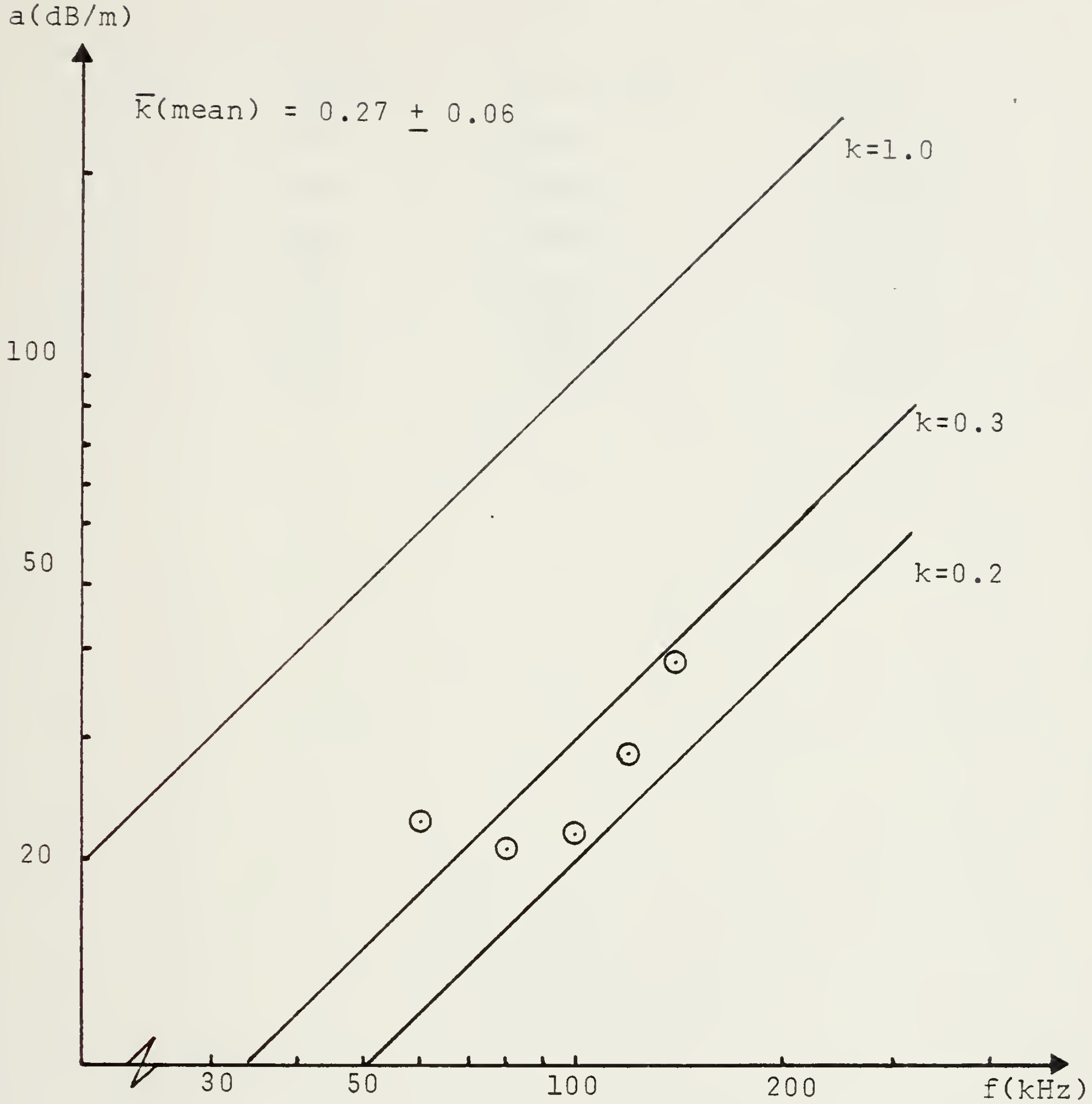

Figure 22 Attenuation in Sand

TABLE IV-1. ATTENUATION IN SAND (60 KHZ)

r (m)	V (volt)	Vr (volt-m)	ln (vr)
0.1	0.18	0.018	- 4.0174
0.2	0.068	0.0136	- 4.2977
0.3	0.033	0.0099	- 4.6152
0.4	0.018	0.0072	- 4.9337
0.5	0.013	0.0065	- 5.0360
0.6	0.008	0.0048	- 5.3391

TABLE IV-2. ATTENUATION IN SAND (80 KHZ)

r(m)	V(volt)	Vr(volt-m)	ln(vr)
0.1	0.145	0.0145	- 4.2336
0.2	0.060	0.0120	- 4.4228
0.3	0.031	0.0093	- 4.6777
0.4	0.020	0.0080	- 4.8283
0.5	0.012	0.0060	- 5.1160
0.6	0.007	0.0042	- 5.4727

TABLE IV-3. ATTENUATION IN SAND (100 KHZ)

r(m)	V(volt)	Vr(volt-m)	ln(vr)
0.1	0.160	0.0160	- 4.1352
0.2	0.051	0.0102	- 4.5854
0.3	0.029	0.0087	- 4.7444
0.4	0.018	0.0072	- 4.9337
0.5	0.011	0.0055	- 5.2030
0.6	0.007	0.0042	- 5.4727

TABLE IV-4. ATTENUATION IN SAND (120 KHZ)

r(m)	V(volt)	Vr(volt-m)	ln(vr)
0.1	0.325	0.0325	- 3.4265
0.2	0.116	0.0232	- 3.7636
0.3	0.052	0.0156	- 4.1605
0.4	0.033	0.0132	- 4.3275
0.5	0.018	0.0090	- 4.7105
0.6	0.010	0.0060	- 5.1160

TABLE IV-5. ATTENUATION IN SAND (140 KHZ)

r (m)	V (volt)	Vr (volt-m)	ln(vr)
0.1	0.780	0.0780	- 2.5510
0.2	0.300	0.0600	- 2.8134
0.3	0.104	0.0312	- 3.4673
0.4	0.060	0.0240	- 3.7297
0.5	0.034	0.0170	- 4.0745
0.6	0.013	0.0078	- 4.8536

LIST OF REFERENCES

1. Kawamura, M., I. Ioannou, Pressure on the Interface Between a Converging Fluid Wedge and a Fast Fluid Bottom, M.S. Thesis, Naval Postgraduate School, Monterey, California, 1978.

2. Bradshaw, R.A., Propagation of Sound in a Fast Bottom Underlying a Wedge-Shaped Medium, M.S. Thesis, Naval Postgraduate School, Monterey, California, 1980.

3. Kinsler, L.E., and others, Fundamentals of Acoustics, p. 427, 3rd ed., Wiley, 1981.

4. Macpherson, J.D., M. J. Daintith, Journal of the Acoustical Society of America, "Practical Model of the Shallow-Water Acoustic Propagation", Vol. 41, pp. 850-854, 1966.

5. Naval Postgraduate School 61-79-002, Two Computer Programs for the Evaluation of the Acoustic Pressure Amplitude and Phase at the Bottom of a Wedge-Shaped, Fluid Layer overlying a Fast Fluid Half Space, by. A. B. Coppens, and others, December 1978.

6. Coppens, A., Notes on Sound Field in a Wedge-Shaped Medium, (Informal).

7. Brekhovskikh, L.M., Waves in Layered Media, p. 18, Academic Press, 1960.

8. Bradshaw, J.A., Laboratory Study of Sound Propagation into a Fast Bottom Medium, M.S. Thesis, Naval Postgraduate School, Monterey, California, 1981.

9. Urick, R.J., "Sound Propagation in the Sea", Defence Advanced Research Projects Agency, pp. 11-7, 1979.

10. Naval Ordnance Laboratory, TR 70-235, "The Propagation of Sound in a Wedge-Shaped Shallow Water Duct", by David Bradley, and A.A. Hudimac, pp. 30-58, November 1970.

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