AN ADAPTATION OF A MARKOV
CHAIN MODEL FOR ANTISUBMARINE WARFARE
CARRIER AIRCRAFT

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MARKOV CHAIN MODEL FOR

ANTISUBMARINE WARFARE CARRIER AIRCRAFT

by

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ABSTRACT

It is the purpose of this paper to develop a useful mathematical model of ASW aircraft availability. The increasing emphasis of systems studies dictates the use of accurate and representative models of the ASW systems. At present, many studies are using essentially the same models developed during World War II. This paper is an attempt to make use of advanced theory in a more powerful and flexible model and to make the use of the model practical and verifiable.

The writer adapted the time homogeneous bivariate model as developed by F. C. Collins. This is a discrete time Markov process with a stochastic matrix of transition probabilities wherein the maintenance process is modeled as a pulsed input multiple server queue.

The model was programmed in FORTRAN 63 on the CDC 1604 and then modified to allow for variability in the input parameters. Other modifications include an increase in the size of the model to accommodate a 16-aircraft squadron, the largest ASW squadron at present, and an explicit form solution to the maintenance queueing equations.

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TABLE OF SYMBOLS

Symbol	Definition or Meaning
a/c	aircraft
λ	mean repair rate of aircraft
λ_A	mean accident rate
S	the set of all possible outcomes; the probability description space
E	possible outcome(s) or event(s)
$p_{ij}(n, n+1)$	a conditional probability that at time $n+1$ the outcome or state is j given that at time n the state is i
$X_1(t)$	the number of a/c flying at time t, which did not fly the previous cycle
$X_2(t)$	the number of a/c in the maintenance queue at time t
$A(t)$	the number of a/c desired on station at time t
$N(t)$	the total number of a/c of type considered at time t
T	the time interval from the launch to recovery at the start of the cycle
$Q(t_o)$	the probability distribution over all possible states at initial time t_o
$P(t)$	the matrix of transition probabilities at time t

Symbol	Definition or Meaning
$P_{(\alpha, i)(\beta, j)}$	the elements of the P matrix; the probability that $X_1 = \beta$ and $X_2 = j$ at the end of a cycle, given that $X_1 = \alpha$ and $X_2 = i$ at the start of the cycle
γ_{fgh}	the probability given f ready a/c, g are launched, and h enter maintenance
P_γ	probability of entering maintenance just before, during, or immediately after launch
p	the probability of equipment failure during flight requiring maintenance when recovered by the carrier
$\Pi_\alpha(m)$	the probability that of α a/c flying m will enter maintenance upon recovery
D	the number of independent identical maintenance repair stations or "spots"
$p_{ij}(t)$	the probability that $i - j$ a/c are repaired in time interval t

1. INTRODUCTION

The threat to freedom of the seas posed by the vast Soviet submarine fleet is perhaps the most thorny problem facing the U. S. Navy today. Two world wars have produced Pyrrhic victories over limited submarine fleets. During the Second World War operations analysis was born into the Navy to aid in the defeat of the German submarine. The classic antisubmarine warfare (ASW) analyses and models developed by Morse [2] and Koopmans [3] are still being used today, over two decades later, in most of the ASW study efforts for the Navy.

These early ASW analyses assumed a given level of search effort available and directly evaluated the probability that an ASW subsystem could detect and/or kill a submarine. This assumption is not only logical to make the problem tractable, but also practical since no immediate changes in ASW force levels could be expected. Moreover, the studies were conducted during the war, not before it started. It is the purpose of this paper to present a probabilistic model to describe the available effort. Such a model can be used to sharpen the estimates of the effectiveness of an ASW subsystem and to study the characteristics of the associated support system.

Naturally, the current study plays an important but limited role in the overall problem of designing an entire ASW system. The difficulties involved in such a specification are legion. First and foremost

is the quantification of the ASW mission in denying the enemy the effective use of his submarines. Currently, the probability of detecting and/or killing submarines is used as the measure of effectiveness of the mission, and it appears that a more encompassing one has not been developed. Second, the specification of an ASW force level to counter a given threat has many inherent subjective elements. These are due to the existing historical bias in predicting the conduct of a future ASW war with an enemy, particularly one who has never before used a large submarine force in its military operations. The reader can imagine why merely defining terms such as "threat" and "effective counter" becomes quite difficult.

Thus, there is a need to investigate the levels of search effort specified. This may require acceptable models to measure the availability of effort, its effectiveness, and determine the logistic support required for any level of available effort. Specifically, the ASW subsystem to be modeled is the carrier-based aircraft, although the model is adaptable to other systems.

The method of investigating the demand for ASW carrier a/c will assume that the desired number of a/c on station is known as an input parameter. The support required to achieve this measure of available effort depends upon maintenance space, manpower, and supply. Generally, we shall consider how an ASW carrier supports this number of a/c on station with the present or proposed number of a/c embarked

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on the carrier. The parametric input can be subjected to sensitivity analyses.

The operational commander of the ASW force launches the desired number of a/c on station to screen, search, or actively prosecute a submarine contact. Each a/c is relieved on station. Each such relief requires the launching of another a/c prior to the recovery of the initial a/c. The returning a/c must receive varying degrees of maintenance and requires refueling and rearming. This cycle continues until the mission is completed. Loss of a/c due to accidents, insufficient supply, and lack of repair capability cause deviations in this procedure. Naval operations involve the interaction of many quantities which are random in nature. Not all can be considered in a tractable mathematical model. Some quantities which are important are omitted. One example is the length of each cycle time, which is assumed to be a constant value. Including variables of this nature incurs unnecessary mathematical complication. It is hoped that adequacy of the model can be measured by using fleet data available from the Fleet ASW Data Analysis Program (FADAP).

Collins [5] describes a bivariate Markov model for airborne early warning (AEW) and combat air patrol (CAP) jet a/c operating in an attack carrier force. This model is used to evaluate the probability of maintaining a fixed requirement of a/c on station as a measure of effectiveness of the system. It has subsequently been used in a larger

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attack force study for the Navy. The model computes the probabilities
of the number of a/c on station and in or awaiting maintenance at any
given launch period. The comparable ASW problem differs in the following
aspects:

1. Type, range, and speed of a/c;

2. The variable number of a/c required for mission;

3. Attrition due to accidents and supply failures;

4. The greater number of ASW a/c.

It was decided to use the Collins' model with appropriate modification.
For immediate reference, the mathematical content of the model will
be repeated herein.

In order to incorporate these modifications, it was necessary to
spend some time reprogramming on the CDC 1604 digital computer in
FORTRAN 63, the CDC version of the IBM FORTRAN IV. The original
program was not readily available and was written in an early assembler
language. Moreover, the numerical analysis was not sufficiently sharp
to handle the larger input values. Also, double precision (two computer
words instead of one) arithmetic was required in one subroutine for an
accurate explicit solution to the maintenance queueing equations (see
Appendix I). This effected a 50% decrease in the computer time
required for developing a matrix of transition probabilities.

Following this introduction, section 2 contains a brief description of
the operational problems involved and the assumptions made. A brief

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description of Markov chains and the mathematical model are presented in section 3. The details for computing the matrix of transition probabilities are given in section 4. General employment of the model follows. The appendices include the solution mentioned on the preceding page, a logical flow diagram of the program, a copy of the program, and some sample results.

2. ASSUMPTIONS

The real-world employment of carrier a/c is cyclic in nature, and the present state of any given a/c (i.e., flying, in or awaiting maintenance) depends largely on what the previous state was. This fact suggests that a Markovian assumption can logically be made for the a/c transition probabilities. In the search phase, a/c may or may not relieve on station; but, in any part of the contact investigation phase, relief on station will be made. To insure full screening and mission coverage, a/c will relieve on station.

The question of resupply during an operation depends primarily on the availability of carrier on-board delivery (COD). This depends on the geographical location and the mission (convoy protection, strike-force protection, hunter-killer operation, etc.). In practice, resupply is not anticipated within a week's period, and around-the-clock operations have continued for two weeks without resupply.

Standard maintenance procedures aboard carriers preclude major maintenance on the flight deck. It will be assumed that sufficient notice is given so that all major 120-hour checks will be completed prior to the operation. This assumption can be modified with an appropriate adjustment in the mean repair rate. The concept of maintenance crews assigned to hangar deck areas ("spots"), as developed by Collins [3], will be used. Each crew will be capable of all types of maintenance

and will operate independently at the identical mean repair rate λ. The number of spots is determined by the average number of such crews available to work continuously around the clock on a watch basis.

The state of each a/c is assumed to be statistically independent of that of others, and the launching and landing transition probabilities will be developed on the basis of independent Bernoulli trials. The parameters can be determined using the maximum likelihood estimators. The range of the number of a/c desired on station at any given cycle will be set by the user. The number to be launched at any time is assumed equally likely within this range. This input parameter is a function of the estimated submarine density (i.e., expected contact rate). The lower limit will be set at the number of a/c desired on station in the search (screening) phase, and the upper limit is set at the maximum practicable number of a/c to be launched during a multiple-contact phase.

Briefly, the assumptions are:

1. a/c will be relieved on station.

2. Any desired length of operation can be set as an input.

3. Major 120-hour checks will be completed prior to the operation.

4. No resupply to the carrier is available.

5. The launch-to-launch cycle for all ASW a/c is four hours.

6. Minor maintenance, refueling, and rearming only can be performed on the flight deck.

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7. Each maintenance spot is characterized with an independent exponential repair time with mean repair rate of λ for around-the-clock operations.

8. The number of a/c lost due to attrition is a Poisson random variable for each cycle period with parameter λ_A (a/c accident/flying hours for a/c type).

9. Any a/c lost by accident will not be returned to service due to either (a) physical loss at sea, or (b) insufficient maintenance capability aboard ship and lack of major parts.

10. The number of a/c launched for each cycle is uniformly distributed between the upper and lower limits determined by the user.

3. MODEL DESCRIPTION

3.1 The Theory

A stochastic or random process is a collection of random variables indexed on some set T, $(X(t), t \in T)$. In this case, time is the indexing set, and the Markovian assumption states that the future state of the process depends only on the state at the present time and not on its past history. Due to the cyclic nature of our problem, it is possible to increment time $(T = (0, 1, \ldots))$ using the cycle time from launch to launch as the steps of unit time in a discrete Markov chain. It is assumed that the reader is familiar with the notion of a random variable as a function defined on a sample description space (S) on which the family of events or outcomes (E) of a probability function can be defined [4].

A discrete time Markov chain is described by a sequence of discrete valued random variables and is determined when the one-step transition probabilities of the state variables are specified, i.e., a conditional transition probability of a transition at time n for each pair of i, j = 0, 1, . . . , m (m being the number of states in the process) must be given.

$$p_{ij}(n, n+1) = P[X(n+1) = j \mid X(n) = i]$$

If the transition probability functions depend only on the time difference, we have time homogeneity

$$p_{ij}(n + 1, 1) = p_{ij}(0, 1) = p_{ij}.$$

The initial state of the system must be given either as a specific state or randomly as a probability distribution function over the possible states.

The p_{ij} (transition probabilities) are arranged in matrix form and satisfy:

1. $p_{ij} \geq 0$ for $i, j = 0, 1, \ldots, m$;

2. $\sum_{j=0}^{m} p_{ij} = 1$, i.e., the rows of the transition matrix sum to 1 for all i for the states within the description space [4].

3.2 The Model

In order to establish the finite set of states (E) for the model, we shall consider two random variables defined as follows:

$X_1(t)$ = The number of a/c flying at time t not having flown in the previous launch-to-launch interval.

$X_2(t)$ = The number of a/c in or awaiting maintenance at time t.

Now, we will consider the vector $X(t) = [X_1(t), X_2(t)]$ as a pair of random variables and thereby have a bivariate stochastic process with the possible states ranging from (0, 0) to (A, N).

$0 \leq X_1(t) \leq A$ = No. of a/c desired on station, and

$0 \leq X_2(t) \leq N$ = No. of a/c of given type aboard carrier.

We will define an operating cycle as an interval unit of time. Process observations of $X(t)$ will be made at successive unit interval launch times. To develop the p_{ij} elements, consider a given time t for launching until A aircraft are flying or until the supply of ready a/c is depleted. Those a/c failing the launch enter the maintenance state at this idealized point in time t (the total launching time required is much less than the total cycle time). At some time T, less than the launch-to-launch unit time interval, the a/c which were relieved on station return and land at the idealized point in time t + T. Some of these a/c will require maintenance and enter the maintenance queue. Those requiring only refueling and preflight inspection will enter a ready status to be tested for the next launch.

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During the unit time interval, maintenance will be performed on those a/c in the not-ready status, and a certain number of aircraft will be repaired according to assumption 7.

In summary, we start the system in some initial state (such as (0, 0) with no a/c flying or in maintenance) or start with a probability distribution $Q(t_o)$ over the states, E, at time t_o. We launch, recover, and repair a/c in the unit interval and repeat the process over each succeeding unit time interval until the end of the operating period. Knowing the transition probabilities within the unit time interval, we can develop the elements of the transition matrix, P, or $\{p_{(\alpha, i), (\beta, j)}\}$. These are the probabilities of going from the state of α a/c flying and i a/c in maintenance to β a/c flying and j a/c in maintenance over the unit time interval.

It was assumed in section 2 that A, the number of a/c to be launched, and N, the total number of a/c on board, are random variables, whereas they have been treated as constants so far in the development. To be analytically correct in including this feature, one should develop the appropriate quadrivariate process. Such a development leads to too large a state space and the author chose to include these effects by using a Monte Carlo simulation technique. That is, at the beginning of each cycle, a random mechanism is used to determine the values on A and N.

The probability of losing an a/c or changing the desired number to be launched is determined from the specified distributions at the beginning

of each unit interval, and the resulting P matrix containing the

$P_{(\alpha, i), (\beta, j)}$ is then recomputed. The probability distribution $Q(t)$ over the states at any time t may be determined by the appropriate number of successive iterations of the Q vector times the P matrix, i. e.,

$$Q(t) = P[X_1(t) = \beta, X_2(t) = j] = Q(t - 1) \times P .$$

The probability of maintaining α a/c on station over any given period of operation may be obtained at any unit time t (i. e., the beginning of the next cycle) by summing out the appropriate maintenance state probabilities. Thus, $P(\alpha$ a/c are flying at time $t) =$

$$Pr(X_1(t) = \alpha) = \sum_{i=o}^{N} Pr(X_1(t) = \alpha, X_2(t) = i) .$$

A mathematical comment appears to be in order. In the case of fixed A and N, the states of the Markov chain are positive recurrent; and steady-state probabilities can be found for the entire state space. In the case of decreasing N due to a/c attrition, this is not true; and (0, 0) becomes an absorbing state as time (t) goes to infinity. This latter consideration is not a realistic one for the operational period envisioned. Therefore, it is mathematically more feasible to use the former chain in conjunction with the Monte Carlo technique.

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4. DEVELOPMENT OF THE TRANSITION MATRIX

Perhaps the simplest way to view this development is to note the various transition probabilities incorporated in one-unit time cycle defined as follows:

(1) γ_{fgh} = the launching transition probabilities at time t. This is the probability of taking f ready a/c, launching g successfully, and sending h into maintenance. Each a/c to be launched is considered a Bernoulli trial with probability of failure of p_γ, which is estimable and subject to sensitivity analysis. The values of γ_{fgh} are:

    a.   0 if $g > A$, since only A a/c are desired;

    b.   0 if $g + h > f$; it is impossible to launch and send into maintenance more a/c than are available;

    c.   0 if $g < A$, $g + h < f$; launching continues until A a/c are flying or until all f are used up;

    d. $\binom{f}{g} (1 - p_\gamma)^g (p_\gamma)^{f-g}$ if $g < A$, $g + h = f$, standard binomial when all a/c in the ready state are used up but the A a/c are not launched;

    e. $\binom{g+h-1}{h} (1 - p)^g (p)^h$ if $g = A$, $g + h > f$, standard negative binomial for g successes in $g + h - 1$ trials.

(2) $\Pi_\alpha(m)$ = the landing transition probabilities which occur at time $t + T$. We must consider the probability that if there are a/c flying at time t then m a/c will enter maintenance at recovery time $t + T$.

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Π_α (m) will equal a standard binomial where p = the probability of equipment failure in flight:

$$\Pi_\alpha (m) = \binom{\alpha}{m} (1 - p)^{\alpha - m} (p)^m , \quad m = 0, 1, \ldots , \alpha .$$

(3) $p_{ij}(\tau)$ = the maintenance transition probabilities, i.e., the probability of repairing (i - j) a/c in time τ. Two maintenance periods occur: the first starting at time t and ending at time t + T, the second starting at time t + T and ending at the end of the cycle, (t + 1). Under assumption 7, the pulsed input, multiple exponential server queue is developed with D maintenance "spots" or servers each with identical, independent service rates, λ. For each server, then, the probability of remaining occupied (given the server is busy) in time $\tau = e^{-\lambda \tau}$. The probability of becoming free (i.e., repairing an a/c) = $1 - e^{-\lambda \tau}$. The resulting queueing equations are:

A. $dP_{i, n}(t) / dt = -n \lambda P_{i, n}(t) + (n + 1) \lambda P_{i, n+1}(t)$ for $0 \le n < D$;

B. $dP_{i, n}(t) / dt = -D \lambda P_{i, n}(t) + D \lambda P_{i, n+1}(t)$ for $n \ge D$.

Three ranges of i (initial queue state), j (final queue state), and D become significant:

a. When $j \le i \le D$, then not all spots are busy since there are fewer a/c in maintenance than spots. Each spot works independently; therefore, the solution to A is the binomial:

$$P_{ij}(t) = \binom{i}{j} (1 - e^{-\lambda t})^{(i - j)} e^{-\lambda t j} .$$

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b.   When $D \le j \le i$, then all spots are occupied throughout the total

service time, and the closed form solution to B is the Poisson:

$$P_{ij}(t) = \frac{(D \lambda t)^{(i-j)} e^{-D \lambda t}}{(i-j)!} \quad .$$

c.   When $j < D < i$, then all spots are busy at the beginning of the

service period, and some spots become idle during the service

period.   The explicit form solution of equation A is found using

moment generating function transformation:

$$P_{ij}(t) = \sum_{n=j}^{D-1} \binom{n}{j} \binom{D}{n} \left\{ \left(\frac{D}{D-n} \right)^{(i-D)} e^{-\lambda t n} \right.$$

$$\left. - e^{-\lambda t} \sum_{k=o} \left(\frac{\lambda D t}{k!} \right)^k \left(\frac{D}{D-n} \right)^{i-D-k} \right\} \quad .$$

(The derivation of this solution is discussed in Appendix I.)

The figure on the following page will show the relationships of these

transition probabilities within the unit time interval.
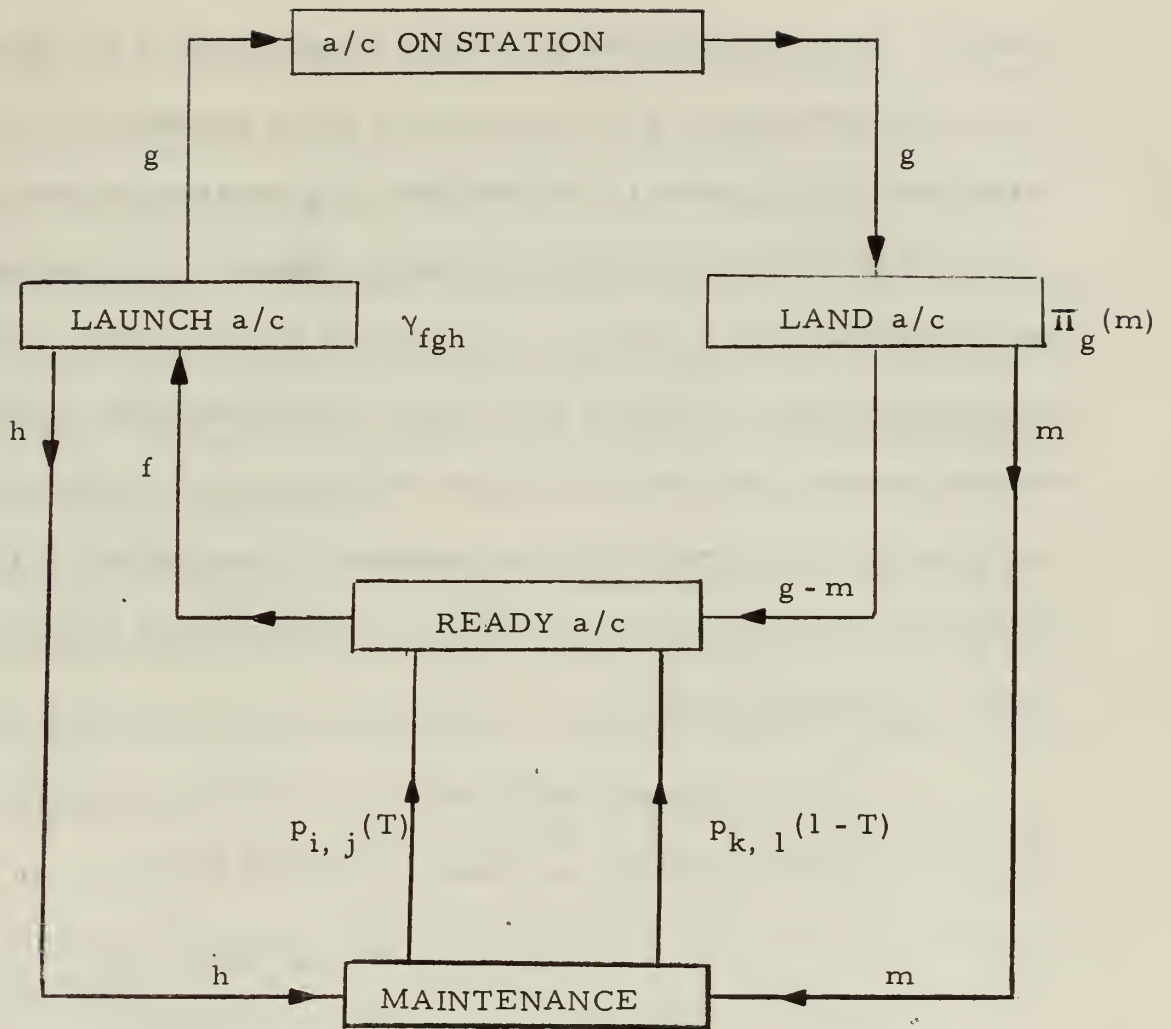
TRANSITION PROBABILITIES WITHIN THE UNIT CYCLE

FIGURE 1

In order to develop each transition probability over the total unit time interval, we must consider all events taking place within the interval. Thus, to obtain the probability of going from α a/c flying and i a/c in maintenance to β a/c flying and j a/c in maintenance, we start at the state (α, i) at time t. At this time, a/c are launched and some l a/c failing the launch enter maintenance. These $i + l$ in maintenance are then serviced until time $t + T$ when some k a/c are still in the maintenance state. At time $t + T$, of the α a/c previously flying, some m enter maintenance and $(\alpha - m)$ enter the ready pool. Maintenance is continued on the $(k + m)$ a/c for the remainder of the cycle $(1 - T)$, until the end of the unit time interval when j a/c remain in the maintenance state. In functional form:

$$P_{(\alpha, i), (\beta, j)} = \sum_{l=0}^{N-\alpha-i} \sum_{k=0}^{i+l} \sum_{m=0}^{\alpha} \gamma_{N-\alpha-i, \beta, l}$$

$$\cdot P_{i+l, k}(T) \cdot \Pi_{\alpha}(m) \cdot P_{k+m, j}(1 - T) \, .$$

5. SUMMARY

Representative values for the mean repair rate and the landing and launching failure rates produced results in agreement with the sensitivity analysis by Collins on these parameters in [5]. For failure probabilities less than .5, and mean repair rate less than 12 hours, the effect of reducing the available maintenance time to 80% of the cycle time was negligible. Optimal loading and cycling policies can be determined for known values of these rates.

The model affords the following checks: (1) the rows of each P matrix are summed as they are computed by the program; and (2) the probability distribution vector (QJ) is summed over the states. Each summation was within 10^{-8} of one in the computer model.

The user may substitute any available distribution over the interval of a/c desired on station. In order to keep A fixed, enter the desired value as both upper and lower limit (A = ALOLIM = LUPLIM). For fixed N, use a very small value for ALAM (such as 10^{-8}). Subroutine KRAN is a uniform generator, using the half open interval (lower limit + 1, upper limit + 2) and a starting number as inputs. KRAN outputs an integer in this interval. Subroutine DRAW was used to provide some intuitive grasp of the results. DRAW was used in binary card form and is not essential to the main program. (The indicated associated statements must be removed, however.)

The results of reasonable arbitrary parameter values, based on the author's experience, have shown that most of the probabilities concentrate over a few states. Moreover, computation time increases rapidly as a function of N (no. of a/c), see Figure 2. This would indicate that a simple approximation to the model could be developed. One method presently being investigated to reduce computation time is to shrink the probability state space to include only those significant states and, thus, reduce the size of the transition matrix. Alternatively, the eigenvector, eigenvalue representation of the P matrix, might be used.

Originally, it was hoped to utilize the data from the Fleet ASW Data Analysis Program (FADAP) to attempt a verification of the model with its real-world counterpart. The only method available at present for obtaining the necessary data is by direct observation or a program of data collection, as suggested by Collins [5].

Many fruitful areas of investigation exist:

(1) Attrition has been simply modeled by the Poisson method. The two components of attrition, accidents and supply shortage, can be more accurately modeled and used to develop logistic schedules for maintenance and supply. One simple technique is to assume each component is independent and Poisson, and estimate a supply failure rate for AOCP attrition from past data. With these assumptions, the total attrition is Poisson, with the parameter equal to the sum of the accident and supply failure rates.

(2) The model can be modified to make the number of maintenance spots available for any cycle a variable function of time, $D(t)$.

(3) An investigation of the Markovian assumption validity as the cycle times become smaller and smaller.

(4) Development of a continuous time model.

(5) Modification of the model to simulate resupply by COD.

(6) A study of the distribution of submarine contacts to determine the validity of the uniform a/c demand assumption.

Time (min)
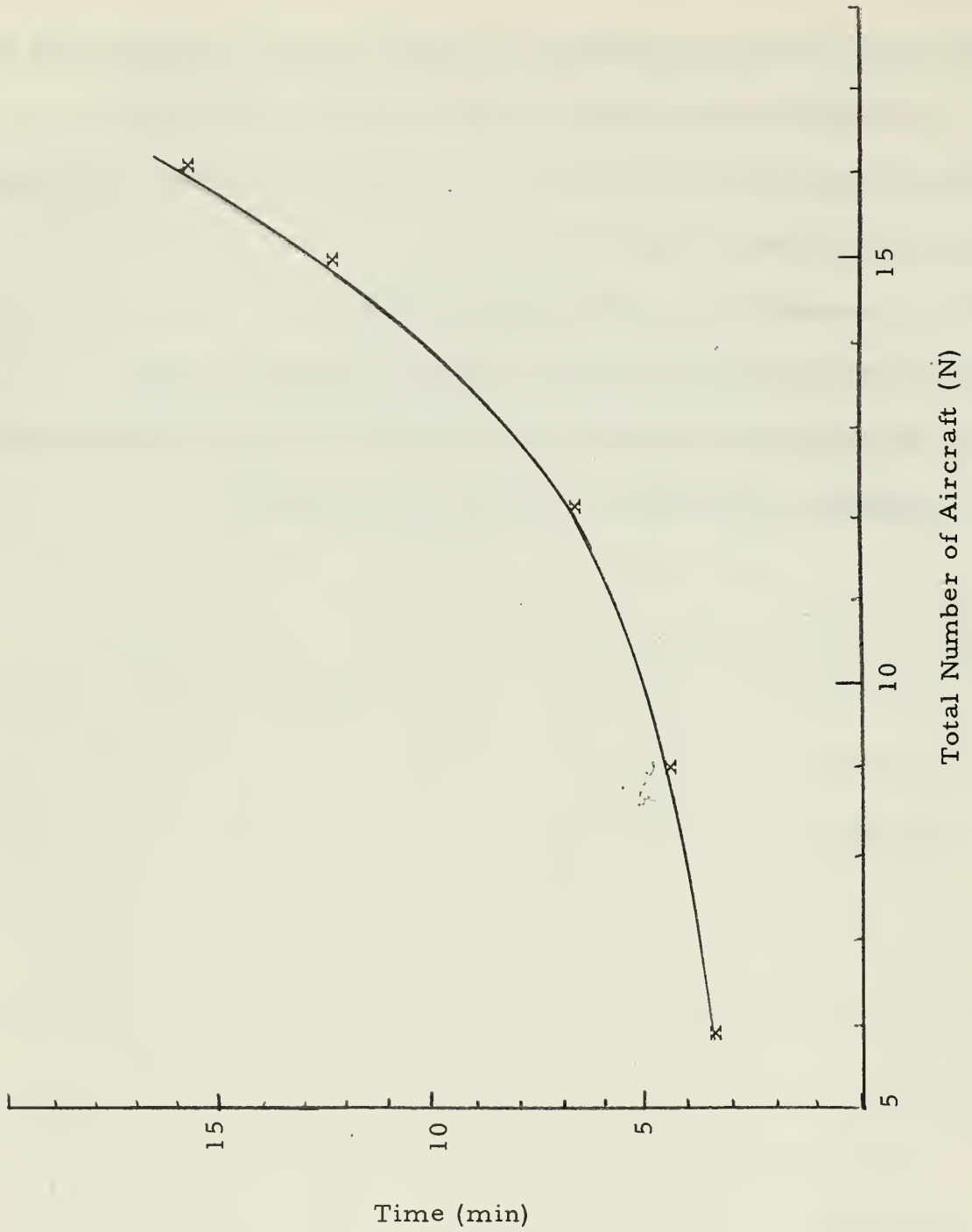
PROGRAM ASSEMBLY AND COMPUTATION TIME
FOR ONE TRANSITION MATRIX (P) AS A FUNCTION
OF THE TOTAL NUMBER OF AIRCRAFT (N)

FIGURE 2

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BIBLIOGRAPHY

1. Feller, W. An Introduction to Probability Theory and Its Applications, Vol. I (2nd ed.), John Wiley & Sons, Inc., 1960.

2. Morse, P. M. and G. E. Kimball. Methods of Operations Research, John Wiley & Sons, Inc., and The Technology Press of Massachusetts Institute of Technology, 1951.

3. Koopmans, B. O. U. S. National Defense Research Committee, A Theoretical Basis for Methods of Search and Screening, 1946.

4. Parzen, E. Stochastic Processes, Holden-Day, Inc., 1962.

5. Collins, F. C. "A Mathematical Model of Carrier Operations in Anti-Air Warfare", Master's Thesis, United States Naval Postgraduate School, Monterey, California, May, 1961.

APPENDIX I

EXPLICIT SOLUTIONS OF THE

MAINTENANCE QUEUEING EQUATIONS

The queueing equations for the pulsed input queue are essentially

the pure death process given in [1] and [4] as problems and developed

by Collins in [5]. The equations are:

A. $\dfrac{dP_{i,\,n}(t)}{dt} = - n\lambda P_{i,\,n}(t) + (n+1)\lambda P_{i,\,n+1}(t)$ for $0 < n < D$

B. $\dfrac{dP_{i,\,n}(t)}{dt} = - n\lambda P_{i,\,n}(t) + D\lambda P_{i,\,n+1}(t)$ for $n \geq D$

where $P_{ij}(0) = \Delta_{ij}$ and $P_{ij}(t) = 0$ for $i < j$, since no input (arrivals)

occur during the service time.

Equation B is solved directly in closed form:

$$P_{i,\,n}(t) = \frac{(D\lambda t)^{(n-i)} e^{-\lambda Dt}}{(n-i)!} \, .$$

Now transforming the first equation (A) using the moment gener-

ating function (MGF),

$$G(s,\,t) = \sum_{n=0}^{D-1} s^{n} P_{n}(t) \,,$$

as outlined in [4] (Chapter 7), and its partial derivatives:

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(1)
$$\frac{dG}{dt} = \sum_{n=0}^{D-1} s^n P_n'(t)$$

(2)
$$\frac{dG}{ds} = \sum_{n=0}^{D-1} n s^{n-1} P_n(t)$$

Where $P_n(t)$ denotes the conditional probability $P_{i,\,n}(t)$, by substituting (A) into (1), properly identifying the first summation with (2), and changing the second summation index to $r = n + 1$, we get:

$$\frac{dG}{dt} = -\lambda s \frac{dG}{ds} + \lambda \sum_{r=0}^{D} r s^{r-1} P_r(t), \qquad \text{or}$$

(3)
$$\frac{dG}{dt} = -\lambda(s-1)\frac{dG}{ds} + \lambda D s^{D-1} P_D(t),$$

since

$$\sum_{r=0}^{D} r s^{r-1} P_r(t) = \frac{dG}{ds} + D s^{D-1} P_D(t).$$

Next, replace the partial differential equation (3) with a system of ordinary differential equations using the Lagrangian auxiliary equations:

$$\frac{dt}{1} = \frac{ds}{\lambda(s-1)} = -\frac{dz}{\lambda D s^{D-1} P_D(t)}.$$

The solution to the first equation (using the first two differentials) is:

$$\lambda t = \ln(s-1) + C'$$

and hence

$$s = C_1 e^{\lambda t} + 1$$

or

$$G_1 = e^{-\lambda t} (s - 1) .$$

The second equation is: (using first and third differentials)

$$dz = -\lambda D (C_1 e^{\lambda t} + 1)^{D-1} P_D(t) dt .$$

Using the solution to (B) where $m = i - D$ to replace $P_D(t)$ and

integrating, term wise, the binomial expansion of $(C_1 e^{\lambda t} + 1)^{D-1}$:

$$z = \frac{(\lambda D)^{m+1}}{m!} \sum_{j=0}^{D-1} \binom{D-1}{j} C_1^j \int t^m e^{-\lambda (D-j)t} dt$$

where the integral is evaluated as:

$$- \sum_{k=0}^{m} \frac{t^k e^{-\lambda (D-j)t}}{(\lambda (D-j))^{m-k+1}} \frac{m!}{k!} + C_2 .$$

Thus,

$$C_2 = z + e^{-\lambda Dt} \sum_{j=0}^{D-1} \binom{D}{j} (s-1)^j \sum_{k=0}^{m} \frac{(\lambda Dt)^k}{k!} \left(\frac{D}{D-j}\right)^{m-k}$$

and the general solution is $\phi (C_1, C_2)$, where ϕ is an arbitrary function

and

$$C_1 = u (s, t, z)$$

and

$$C_2 = v (s, t, z) .$$

36

To get our particular solution, use the boundary conditions for $G(s, t)$:

(1) for $s = 1$,

$$G(1, t) = \sum_{n=0}^{D-1} P_n(t)$$

$$= \Pr[\text{no. in maintenance at } t \text{ is} < D \mid i \text{ at } t = 0]$$

$$G(1, t) = 1 - \sum_{n=0}^{i-D} \frac{e^{-\lambda Dt}(\lambda Dt)^n}{n!} = 1 - \psi_1(t)$$

where

$$u(1, t, z) = C_1 = 0$$

$$v(1, t, z) = C_2 = z + e^{-\lambda Dt} \sum_{k=0}^{m} \frac{(\lambda Dt)^k}{k!}$$

so

$$C_2 = z + \psi_1(t)$$

(2) for $t = 0$,

$$G(s, 0) = \sum_{n=0}^{D-1} s^n P_n(0) = 0, \quad \text{since } i \geq n > D$$

where

$$u(s, 0, z) = (s - 1)$$

$$v(s, 0, z) = C_2 = z + \sum_{j=0}^{D-1} \binom{D}{j} (s - 1)^j \left(\frac{D}{D - j}\right)^m .$$

Thus,

$$G(s, 0) = z + \sum_{j=0}^{D-1} \binom{D}{j} C^j \left(\frac{D}{D - j}\right)^m - C_2 .$$

Substituting the general value for C_2 above:

$$G(s, t) = \phi(u, v) = \sum_{j=0}^{D-1} \binom{D}{j} (s-1)^j e^{-\lambda t j} \left(\frac{D}{D-j}\right)^m$$

$$- \sum_{j=0}^{D-1} \binom{D}{j} (s-1)^j \sum_{k=0}^{m} \frac{(\lambda D t)^k}{k!} e^{-\lambda D t} \left(\frac{D}{D-j}\right)^{m-k} .$$

Rearranging terms,

$$G(s, t) = \sum_{n=0}^{D-1} s^n \sum_{j=n}^{D-1} \binom{j}{n} \binom{D}{j} (-1)^j \left[\left(\frac{D}{D-j}\right)^m e^{-\lambda t j} \right.$$

$$\left. - e^{-\lambda D t} \sum_{k=0}^{m} \frac{(\lambda D t)^k}{k!} \left(\frac{D}{D-j}\right)^{m-k} {}^* \right]$$

where $P_n(t) = $ the coefficient of s^n .

APPENDIX II

THE LOGICAL FLOW DIAGRAM OF THE COMPUTER PROGRAM

SET INPUTS (A, N, p , PGAM TIME, ICYCLE, JCYCLE FLAM, ALAM, AUPLIM, ALOLIM, ETC.)

(805) → DRAW IA (lower limit ≤ A ≤ upper limit)

IS THIS 1st CYCLE? — yes → (11) / no

was 1 a/c lost at t_{-1}? — yes / no

were 2 a/c lost at t_{-1}? — no → IA=IA-1 / yes

were 3 a/c lost at t_{-1}? — no → IA=IA-2 / yes → IA=IA-3

(11) → SET GAMMA(I,J,K)=0

COMPUTE $\pi_\lambda(m)$=PRFM(IAA,MI)

COMPUTE $p_{ij}(t)$ = PTRM(I,IJ,IT)

HAS IA CHANGED? — no / yes

HAS N CHANGED? — yes / no

COMPUTE f_{gh}=GAMMA(IF,IG,IH)

IALAST=IALPH

SET THE TRANSITION MATRIX =0 → (801)

39

(801) → COMPUTE $p_{(\ ,x)(\ ,y)}(t) = PR(I,IBETA,IY)$

$(I = IX+(IALPH-1) \times TOTAL\ A/C)$

PRINT THE p_{ij}'s OF THE (IALPH,IX)TH ROW AND ROW SUM

SET THE QJ VECTOR =0

MULTIPLY QI x P matrix

COMPUTE THE (IBETA, IY)TH ELEMENTS OF QJ(J)

$(J=IY+(IBETA-1) \times N)$

SUM THE ELEMENTS OF THE QJ VECTOR

COMPUTE THE MEAN OF A/C LAUNCHED

PLOT QJ(J) VS J

IS THIS LAST CYCLE? — yes → STOP

no

SET QI = QJ

REINDEX MATRIX
IALAST =IA
NLAST = NEWN
IALASTM1 = IALAST

(805)

40

APPENDIX III

THE COMPUTER PROGRAM

```
-COOP,,LANMAN,O/49/S/1S/2S/E/45=54,15 ,30000,5.
-BINARY,56.                                                              000
(RELOCOM.                                                                000
-FTN,E.                                                                  000
       PROGRAM MARKOV                                                    000
C  THIS PROGRAM IS A NONSTATIONARY BIVARIATE MARKOV CHAIN MODEL OF ASW A/C 000
C  OPERATIONS.  THE RANDOM VARIABLES ARE THE NUMBER OF A/C FLYING AT THE  000
C  BEGINNING OF ANY GIVEN LAUNCH CYCLE.  THE MAXIMUM NO. OF A/C ALLOWED IN 000
C  THE MODEL IS 16(NA).  THE RANGE OF A/C TO BE LAUNCHED AT ANY GIVEN     000
C  INTERVAL IS 0 TO 6 A/C.  THE FOLLOWING INPUTS ARE REQUIRED.           000
C      ID= THE NO. OF INDEPENDENT MAINTENANCE SPOTS                      000
C      NA= TOTAL NO. OF A/C TYPE ON BOARD                               001
C      TIME=TIME FROM LAUNCH TO RECOVERY/LAUNCH TO LAUNCH CYCLE TIME(HRS)001
C      FLAM=MEAN REPAIR TIME PER SPOT/LAUNCH TO LAUNCH CYCLE TIME  (HRS)001
C      PGAM= PROBABILITY OF A/C FAILING LAUNCH(M.L.EST. FROM PAST DATA) 001
C      P= PROBABILITY OF A/C FAILURE DURING FLIGHT REQUIRING MAINTENANCE 001
C          AT LANDING (M.L. ESTIMATOR FROM PAST DATA)                   001
C      QI= THE PROBABILITY DISTRIBUTION VECTOR OVER ALL POSSIBLE STATES  001
C         (7 X 17 = 119) SUCH THAT THE SUM OF ALL QI(I) = 1.    THIS    001
C         IS ESTIMATED BY THE USER AND INPUTTED BY USING  A DATA STATEMENT001
C      ICYCLE = NO. CYCLES DESIRED FOR OPERATION                        002
C      JCYCLE = LAUNCH TO LAUNCH TIME(HRS)(TOT. TIME=ICYCLE X JCYCLE)    002
C      ALAM = ACCIDENT RATE FOR TYPE A/C (ACCIDENT/HOURS)               002
C      ALOLIM = DESIRED LOWER LIMIT ON A                                002
C      AUPLIM = DESIRED UPPER LIMIT ON A                                002
       COMMON FLAM,TIME                                                 002
       TYPE DOUBLE FLAM                                                 002
       COMMON PTRM,GAMMA,PR,PRFMA,ID                                    002
       DIMENSION BC(17),A(17),FBC(17)                                   002
       DIMENSION PTRM(17,17,2),GAMMA(17,7,17),PRFMA(7,7)                002
       DIMENSION PR(119,7,17),QI(119),QJ(119)                          002
       DIMENSION          FJPLOT(119),JT(12)                           002
C      ENTER DATA CARDS HERE                                            002
       DATA((QI(I),I=1,119)=.2,16(.05),102(.0))                        003
       NA=16
       ALAM=.01
       ID = 8
        FLAM=3.0
       PGAM=P=.4
       IYY = 13421773
       TIME = .125
       ICYCLE=20
       JCYCLE=4
       ALOLIM=4.
       AUPLIM=6.
C       END OF DATA CARDS
       AL=ALOLIM+1.  $ AU=AUPLIM +2.                                   0031
       UNITT=1.                                                        0032
       N=NA+1                                                          0033
       IAMAX=7                                                         0034
       IALAST=0                                                        0035
       D=FLOATF(ID)                                                    0036
       NLAST=NEWN=N                                                    0037
       KT=1                                                            0038
   809 IA=KRAN(AL,AU,IYY)                                              0039
       IF(KT.EQ.1) 113,115                                             0040
                                                                       0041
```

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```
115 T1=-LOGF(.000000001 + RANF(-1))*2.30258/ALAM                        0042
    IF(T1-TFLC)130,131,132                                               0043
130 T2=-LOGF(.000000001 + RANF(-1))*2.30258/ALAM                        0044
    IF(T1+T2-TFLC) 230,231,131                                           0045
230 T3=-LOGF(.000000001 + RANF(-1))*2.30258/ALAM                        0046
    IF(T1+T2+T3-TFLC) 331,331,231                                        0047
331 NEWN=NLAST-3  $ GO TO113                                             0048
231 NEWN=NLAST-2  $ GO TO113                                             0049
132 NEWN=NLAST $ GO TO113                                                0050
131 NEWN=NLAST-1                                                         0051
113 PRINT 8882,IA,NEWN                                                   0052
    IF(NEWN-IA) 15,13,13                                                 0053
 15 IA=NEWN                                                              0054
 13 IF(IALAST) 11,12,11                                                  0055
 12 CONTINUE                                                             0056
 FROM THIS NEXT STATEMENT TO NO. 483 IS CONCERNED ONLY WITH THE GRAPH    0057
    DO 482 I=1,12                                                        0058
482 JT(I)=8H                                                             0059
    JT(1)=8HE(A/C) =                                                     0060
    JT(3)=8HSPOTS =                                                      0061
    JT(5)=8H     T =                                                     0062
    JT(7)=8HJ VS QJ                                                      0063
    JT(8)=8HVECTOR                                                       0064
    JT(9)=8H     N =                                                     0065
    JT(11)=8H     A =                                                    0066
    DO 483 I=1,119                                                       0067
    FI=I                                                                 0068
483 FJPLOT(I)=FI                                                         0069
    IALAST=IA                                                            0070
    DO 1235 I=1,17                                                       0071
    DO 1235 J=1,IAMAX                                                    0072
    DO 1235 K=1,17                                                       0073
1235 GAMMA(I,J,K)=0.0                                                    0074
 AT THIS PT THE LANDING TRANSITION PROBABILITIES ARE COMPUTED            0075
    DO 300 IAA=1,IAMAX                                                   0076
    DO 301 MI=1,IAMAX                                                    0077
    IF(IAA-MI)31,32,33                                                   0078
 31 PRFMA(IAA,MI)=0.                                                     0079
    GO TO 301                                                            0080
 32 MM1=MI-1                                                             0081
    PRFMA(IAA,MI)=P**MM1                                                 0082
    GO TO 301                                                            0083
 33 IAM1=IAA-1                                                           0084
    MM1=MI-1                                                             0085
    BC(1)=1.0                                                            0086
    PROD=FLOATF(IAA-MI)                                                  0087
    DO 50 IP=2,MI                                                        0088
    AIP=FLOATF(IP-1)                                                     0089
    PROD = PROD +1.0                                                     0090
 50 BC(IP)=PROD*BC(IP-1)/AIP                                             0091
    IGO=IAA-MI                                                           0092
    PRFMA(IAA,MI)= (BC(MI)*(1.0-P)**(IGO))*P**MM1                        0093
301 CONTINUE                                                             0094
300 CONTINUE                                                             0095
 AT THIS PT THE MAINTENANCE TRANSITION PROBABILITIES ARE COMPUTED        0096
    DO 100 IT=1,2                                                        0097
```

43

```
      IF(IT-1)25,25,26
   25 TAU = TIME                                                           00
      GO TO 28                                                             00
   26 TAU = UNITT-TIME                                                     01
   28 DO101 I=1,N                                                          01
      DO 102 IJ=1,N                                                        01
      IF (I-IJ) 14,199,17                                                  01
  199 IF(I-ID) 19,19,1999                                                  01
 1999 PTRM(I,IJ,IT)=EXPF(-FLAM*TAU*D)                                      01
      GO TO 102                                                            01
   14 PTRM(I,IJ,IT)=0.                                                     01
      GO TO 102                                                            01
   19 FJM1=FLOATF(IJ-1)                                                    01
      PTRM(I,IJ,IT)=EXPF(-FLAM*TAU*FJM1)                                   01
      GO TO 102                                                            01
   17 IF(I-ID-1) 1,1,2                                                     01
    1 BC(1)=1.0                                                            01
      PROD=FLOATF(I-IJ)                                                    01
      DO 10 IP =2,IJ                                                       01
      AIP=FLOATF(IP-1)                                                     01
      PROD = PROD + 1.0                                                    01
   10 BC(IP) =PROD*BC(IP-1)/AIP                                            01
      ELT=EXPF(-FLAM*TAU)                                                  01
      PTRM (I,IJ,IT)=BC(IJ)*(1.-ELT)**(I-IJ)*ELT**(IJ-1)                   012
      GO TO 102                                                            012
    2 IF(IJ-1-ID) 22,24,24                                                 012
   22 CONTINUE                                                             012
      CALL PID(I,IJ,IT)                                                    012
      GO TO 102                                                            012
   24 D=FLOATF(ID)                                                         012
      ELDT=EXPF(-D*FLAM*TAU)                                               012
      FACT = 1.0                                                           012
      A(1)=1.0                                                             012
      MM = I-IJ                                                            013
      DO 20 M=2,MM                                                         013
      FACT=FACT+1.0                                                        013
   20 A(M)=A(M-1)*FACT                                                     013
  201 PTRM(I,IJ,IT)=(D*FLAM*TAU)**(I-IJ)*ELDT/A(I-IJ)                      013
  102 CONTINUE                                                             013
  101 CONTINUE                                                             013
  100 CONTINUE                                                             013
      GO TO 120                                                            013
   11 CONTINUE                                                             013
      IF(IA-IALAST) 120,121,120                                           014
  121 IF(NEWN-NLAST)111,117,111                                           014
  117 IALPH=IALASTM1 $ GO TO 801                                          014
  120 CONTINUE                                                             014
   AT THIS POINT THE LAUNCHING TRANSITION PROBABILITIES ARE COMPUTED      014
      DO 204 IFF=1,N                                                       014
       IGM = XMINOF (IA,IFF)                                               014
      DO 203 IG=1,IGM                                                      014
      IGM1=IG-1                                                            014
      DO 202 IH=1,N                                                        014
      IHM1=IH-1                                                            015
      BPROD=((1.-PGAM)**IGM1)*(PGAM**IHM1)                                015
   86 IF(IG-IA) 91,87,84                                                   015
                                                                          015
```

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```
   91 IF(IG+IHM1-IFF) 84,82,84                                                0154
   87 IF(IG+IHM1-IFF)85,85,84                                                 0155
   84 GAMMA(IFF,IG,IH)=0.                                                     0156
      GO TO 202                                                              0157
   82 BC(1)=1.0                                                              0158
      PROD=FLOATF(IFF-IG)                                                    0159
      DO 30 IP=2,IG                                                          0160
      AIP=FLOATF(IP-1)                                                       0161
      PROD = PROD + 1.0                                                      0162
   30 BC(IP)=PROD * BC(IP-1)/AIP                                             0163
      IHM1=IH-1                                                              0164
      TEMP= PGAM**IHM1                                                       0165
      TEMP1=(1.-PGAM)**IGM1                                                  0166
       BPROD = TEMP*TEMP1                                                    0167
      GAMMA(IFF,IG,IH)=BC(IG)*BPROD                                          0168
      GO TO 202                                                              0169
   85 FBC(1)=1.0                                                             0170
      PROD=FLOATF(IGM1-1)                                                    0171
      DO 40 IP=2,IH                                                          0172
      AIP = FLOATF(IP-1)                                                     0173
      PROD = PROD +1.0                                                       0174
   40 FBC(IP)=PROD*FBC(IP-1)/AIP                                             0175
      GAMMA(IFF,IG,IH)=FBC(IH)*BPROD                                         0176
  202 CONTINUE                                                               0177
  203 CONTINUE                                                               0178
  204 CONTINUE                                                               0179
      REMOVE CARDS FROM HERE TO NO 999 IF PRINT OUT NOT DESIRED             0180
      PRINT 9,(((I,IJ,IT,PTRM(I,IJ,IT),IT=1,2),IJ=1,N),I=1,N)              0181
    9 FORMAT (1H1/(2(6H PTRM(I2,1H,I2,1H,I2,3H) = E14.5)))                  0182
      PRINT 99,(((IFF,IG,IH,GAMMA(IFF,IG,IH),IFF=1,N),IG=1,IA),IH=1,N)     0183
   99 FORMAT(1H1/(2(7H GAMMA(I2,1H,I2,1H,I2,3H) = E14.5)))                  0184
      PRINT 999,((IAA,MI,PRFMA(IAA,MI),IAA=1,IAMAX),MI=1,IAMAX)            0185
  999 FORMAT(1H1/(2(7H PRFMA(I2,1H,I2,3H) = E14.5)))                        0186
  NOW THE TRANSITION MATRIX MUST BE ZEROED                                   0187
  111 CONTINUE                                                               0188
      DO 899 J=1,119                                                        0189
      DO  899 K=1,7                                                         0190
      DO 899 L=1,17                                                         0191
  899 PR(J,K,L)=0.0                                                         0192
  START COMPUTING THE ELEMENTS OF EACH ROW, I=IX+ (ALPHA - 1) X TOTAL A/C    0193
      DO 1000 IALPH=1, IALAST                                               0194
  801 CONTINUE                                                               0195
      DO 1100 IX=1,NLAST                                                     0196
COMPUTE THE P ELEMENTS OF THE IAPH,IX ROW AND SUM THE ROW                    0197
      TSUM=0.                                                               0198
      I=IX+(IALPH-1)*N                                                       0199
      DO 800 IBETA=1,IA                                                      0200
      RSUM=0.0                                                              0201
      DO 900 IY=1,NEWN                                                       0202
      PR(I,IBETA,IY)=0.                                                      0203
      ILIM=NEWN-IALPH-IX+2                                                   0204
      PSUM=0.0                                                              0205
      SUM=0.0                                                               0206
      SUML=0.0                                                             0207
      DO 500 IL=1,ILIM                                                       0208
      KLIM=IX+IL-1                                                          0209
```

45

```
          IXPIL = IX +IL - 1                                                                    021
          SUMM=0.                                                                                021
          DO 600 MI=1,IALPH                                                                      021
          SUMK=0.                                                                                021
     ·.   DO 700 IK=1,KLIM                                                                       021
          IKPMI = IK +MI-1                                                                       021
          IF(IXPIL-NEWN) 701,701,700                                                             021
      701 IF(IKPMI-NEWN) 702,702,700                                                             021
      702 GAMH=GAMMA(ILIM,IBETA,IL)                                                              021
     -    PTRMH1 = PTRM(IXPIL,IK,1)                                                               021
          PRFMAH = PRFMA(IALPH,MI)                                                               021
          PTRMH2 = PTRM(IKPMI,IY,2)                                                              022
          SUM = GAMH * PTRMH1 * PRFMAH * PTRMH2                                                  022
          SUMK=SUMK+SUM                                                                          022
          PSUM=PSUM+SUM                                                                          022
      700 CONTINUE                                                                               022
          SUMM = SUMM + SUMK                                                                     022
      600 CONTINUE                                                                               022
          SUML = SUML +SUMM                                                                      022
          PSUM2 =SUML                                                                            022
      500 CONTINUE                                                                               022
          RSUM=RSUM+PSUM                                                                         023
          PR(I,IBETA,IY)=PSUM                                                                    023
      900 CONTINUE                                                                               023
          TSUM=TSUM+RSUM                                                                         023
      800 CONTINUE                                                                               023
          PRINT 888 ,        TSUM,IALPH,IX                                                       023
      888 FORMAT (                      7H TSUM =   E15.8,2I5)                                    023
     1100 CONTINUE                                                                               023
     1000 CONTINUE                                                                               023
    C     REMOVE   CARD FROM HERE TO 889 IF P MATRIX PRINT OUT NOT DESIRED                       024
          DO 889 J=1,17                                                                          024
          DO 889 K=1,7                                                                           024
          DO889 L=1,17                                                                           024
          I=J+(K-1)*N                                                                            024
      889 PRINT 890,(PR(I,LP,L),LP=1,IAMAX),K,J,L                                                024
      890 FORMAT(7E14.5,2HJ=I2,5HK=1,A,2HL=I2)                                                   024
          DO 898 I=1,119                                                                         024
      898 QJ(I)=0.0                                                                              024
    C NOW   MULTIPLY   QI  AND  P  TO GET QJ                                                     024
      805 PRINT 807,KT,IALAST,IA                                                                 025
      807 FORMAT(1H1,13HQ VECTOR CASE I3/// I5,I5)                                               025
          DO 802 IBETA=1,7                                                                       025
          DO 902 IY=1,17                                                                         025
    CAT THIS POINT CALCULATE THE (IBETA,IY)TH ELEMENT OF THE QJ VECTOR                           025
          J=IY+(IBETA-1)*N                                                                       025
          QP1=0.                                                                                 025
          QP=0.                                                                                  025
          DO 2001 IALPH=1,7                                                                      025
          DO 2201 IX=1,17                                                                        025
          I=IX+(IALPH-1)*N                                                                       025
          QP1=QI(I)*PR(I,IBETA,IY)                                                               026
          QP=QP+QP1                                                                              026
     2201 CONTINUE                                                                               026
     2001 CONTINUE                                                                               026
          QJ(J)=QP                                                                               026
                                                                                                0265
```

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```
      PRINT 8882,IBETA,IY,J,QP                                          0266
 8882 FORMAT(2I4,4H QJ(I3,3H )= E14.8)                                   0267
  902 CONTINUE                                                           0268
  802 CONTINUE                                                           0269
C CHECK THE SUM OF THE Q VECTOR                                          0270
      QSUM=0.                                                            0271
      DO 808 J=1,119                                                     0272
  808 QSUM=QJ(J)+QSUM                                                    0273
      PRINT 8883,QSUM                                                    0274
 8883 FORMAT(6H QSUM=   E15.9)                                           0275
      DO 333 I = 18,119                                                  0276
      K = (I-1)/17                                                       0277
        FK=FLOATF(K)                                                     0278
      FMEAN= FK*QJ(I)+FMEAN                                              0279
  333 CONTINUE                                                           0280
      TFLC=FMEAN*FLOATF(JCYCLE)                                          0281
      PRINT 335,FMEAN                                                    0282
  335 FORMAT(   17HMEAN A/C FLYING = F10.4)                              0283
C   STATEMENTS FROM THIS POINT TO THE CALL DRAW STATEMENT REFER TO  GRAPH 0284
      JT(2)=ICODE(FMEAN)                                                 0285
      JT(4)=ICODE(D)                                                     0286
      FKT=FLOATF(KT)                                                     0287
      JT(6)=ICODE(FKT)                                                   0288
      FN=FLOATF(NEWN-1)                                                  0289
      JT(10)=ICODE(FN)                                                   0290
      FIAA=FLOATF(IA-1)                                                  0291
      JT(12)=ICODE(FIAA)                                                 0292
      CALL DRAW(119,FJPLOT,QJ,0,0,4H        ,JT,0,0,0,0,0,0,8,8,0,LAST  ) 0293
      FMEAN = 0.                                                         0294
C NEXT WE MUST MULTIPLY QJ AND P TO GET QK AND SO ON...(QK+...N)         0295
      KT=KT+1                                                            0296
      IF(KT-ICYCLE) 803,803,806                                         0297
  803 DO 804 I=1,119                                                     0298
  804 QI(I)=QJ(I)                                                        0299
      IALASTM1=IALAST                                                    0300
      IALAST=IA                                                          0301
      NLAST=NEWN                                                         0302
      GO TO   809                                                        0303
  806 STOP 06                                                            0304
      END                                                                0305
      SUBROUTINE PID(I,J,IT)                                             0306
      COMMON FLAM,TIME                                                   0307
      COMMON PTRM,GAMMA,PR,PRFMA,ID                                      0308
      TYPE DOUBLE BC,BDC,PROD ,DID3,DID4,DID5,DEXP                       0309
      TYPE DOUBLE DAN,DID1,DID2,SUM,DN,ANM1,FAC,COF,PSUM,PTR,FLAM,TAU,D  0310
      DIMENSION PTRM(17,17,2),BC(11),BDC(11)                             0311
      DIMENSION GAMMA(17,7,17), PRFMA(7,7), PR(119,7,17)                 0312
      D=FLOATF(ID)                                                       0313
      IDP1=ID+1                                                          0314
      IF(IT-1)25,25,26                                                  0315
   25 TAU = TIME                                                         0316
      GO TO 28                                                           0317
   26 TAU= 1.-TIME                                                       0318
   28 CONTINUE                                                           0319
      IMDP1=I-ID                                                         0320
      PTR=0.0                                                            0321
```

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```
      PSUM=0.                                                          032
      DO 200 NJ = J, ID                                                032
C  DEVELOP N TAKEN J AT A TIME AND D TAKEN N AT A TIME                 032
      BC(1)=1.0                                                        032
      PROD=FLOATF(NJ-J)                                                032
      DO 10 IP=2,J                                                     032
      AIP=FLOATF(IP-1)                                                 032
      PROD = PROD+1.0                                                  032
   10 BC(IP)=PROD* BC(IP-1)/AIP                                        033
      BDC(1)=1.0                                                       033
      PROD=FLOATF(IDP1-NJ)                                             033
      DO 20 IQ=2,NJ                                                    033
      AIQ=FLOATF(IQ-1)                                                 033
      PROD=PROD+1.0                                                    033
   20 BDC(IQ)=PROD*BDC(IQ-1)/AIQ                                       033
      COF=BC(J)*BDC(NJ)*(-1)**(NJ-J)                                   033
      ANM1=FLOATF(NJ-1)                                                033
       DAN=D/(D-ANM1)                                                  033
      DID4=DEXP(-FLAM*TAU*ANM1)                                        034
      DID1=(DAN**(I-IDP1))*DID4                                        034
      SUM=0.                                                           034
      DN=0.                                                            034
      DO 201 K=1,IMDP1                                                 034
      FAC=1.                                                           034
      KM1=K-1                                                          034
      PROD=0.                                                          034
      DO 11 IK=1,KM1                                                   034
      PROD=PROD+1.                                                     034
   11 FAC=FAC*PROD                                                     035
      IMIDK=I-ID-K                                                     035
      SUM=((FLAM*D*TAU)**KM1)*DAN**IMIDK /   FAC                       035
  201 DN=DN+SUM                                                        035
      DID3=DEXP(-FLAM*D*TAU)                                           035
      DID2=DN*DID3                                                     035
      DID5=DID1-DID2                                                   035
      PSUM=COF*DID5                                                    035
  200 PTR =PTR +PSUM                                                   035
  103 CONTINUE                                                         035
      PTRM(I,J,IT)=PTR                                                 035
  102 CONTINUE                                                         03
  101 CONTINUE                                                         03
      END                                                              03
      FUNCTION KRAN(A,B,IY)                                            03
C                                                                      03
C      THIS ROUTINE RETURNS AN UNIFORMLY DISTRIBUTED RANDOM INTEGER    03
C                                                                      03
C      THIS ROUTINE RETURNS A INTEGER RANDOM NUMBER .GE. TO A          03
C      .LT. B                                                          03
C      A = BOTTOM LIMIT (INCLUDED) FOR THE RANDOM NUMBER               03
C      B = TOP LIMIT (NOT INCLUDED) FOR THE RANDOM NUMBER              03
C      SET IY ONLY ONCE IN MAIN PROGRAM FOR EACH SET OF RANDOM NUMBERS 03
C      SOME GOOD STARTING VALUES FOR IY FOLLOW                         03
C      13421773                                                        03
C      33554433                                                        03
C      8426219                                                         03
C      42758321                                                        03
```

48

```
      56237485                                                          0378
      62104023                                                          0379
      ANY  OF  THESE  MAY  BE  USED                                     0380
                                                                        0381
      THIS  ROUTINE  MAY  BE  USED  IN  FORTRAN  60  OR  63             0382
                                                                        0383
      IY = 3125 * IY                                                    0384
      IY = IY - (IY/67108864) * 67108864                               0385
      FY = IY                                                           0386
      KRAN = FY/67108864. * (B-A) + A                                   0387
      RETURN                                                            0388
      END                                                               0389
          FINIS                                                         0390
XECUTER.                                                                0391
```

49

Q VECTOR CASE 1

```
        5       5
    1    1  QJ(   1  )=2.64762775E-02      4    1  QJ(  52  )=3.79764295E-02
    1    2  QJ(   2  )=3.02020831E-02      4    2  QJ(  53  )=3.03224061E-02
    1    3  QJ(   3  )=1.72436823E-02      4    3  QJ(  54  )=1.16230913E-02
    1    4  QJ(   4  )=6.68152510E-03      4    4  QJ(  55  )=2.87808733E-03
    1    5  QJ(   5  )=2.02525670E-03      4    5  QJ(  56  )=5.26888829E-04
    1    6  QJ(   6  )=5.29756823E-04      4    6  QJ(  57  )=7.84720454E-05
    1    7  QJ(   7  )=1.30002033E-04      4    7  QJ(  58  )=1.04516559E-05
    1    8  QJ(   8  )=3.22530060E-05      4    8  QJ(  59  )=1.39145026E-06
    1    9  QJ(   9  )=8.58857000E-06      4    9  QJ(  60  )=2.08960624E-07
    1   10  QJ(  10  )=2.86285667E-06      4   10  QJ(  61  )=4.35334634E-08
    1   11  QJ(  11  )=8.34999861E-07      4   11  QJ(  62  )=7.25557723E-09
    1   12  QJ(  12  )=2.08749965E-07      4   12  QJ(  63  )=9.06947154E-10
    1   13  QJ(  13  )=4.34895761E-08      4   13  QJ(  64  )=7.55789295E-11
    1   14  QJ(  14  )=7.24826268E-09      4   14  QJ(  65  )=3.14912206E-12
    1   15  QJ(  15  )=9.06032836E-10      4   15  QJ(  66  )=               0
    1   16  QJ(  16  )=7.55027363E-11      4   16  QJ(  67  )=               0
    1   17  QJ(  17  )=3.14594735E-12      4   17  QJ(  68  )=               0
    2    1  QJ(  18  )=3.02519433E-02      5    1  QJ(  69  )=4.91642833E-01
    2    2  QJ(  19  )=3.07381552E-02      5    2  QJ(  70  )=1.43718127E-01
    2    3  QJ(  20  )=1.54385405E-02      5    3  QJ(  71  )=2.69531853E-02
    2    4  QJ(  21  )=5.18865107E-03      5    4  QJ(  72  )=3.78709962E-03
    2    5  QJ(  22  )=1.34374437E-03      5    5  QJ(  73  )=4.23732653E-04
    2    6  QJ(  23  )=2.96164302E-04      5    6  QJ(  74  )=3.99381550E-05
    2    7  QJ(  24  )=6.07227021E-05      5    7  QJ(  75  )=3.41914062E-06
    2    8  QJ(  25  )=1.26203649E-05      5    8  QJ(  76  )=2.98561135E-07
    2    9  QJ(  26  )=2.86289503E-06      5    9  QJ(  77  )=3.11088104E-08
    2   10  QJ(  27  )=8.35011051E-07      5   10  QJ(  78  )=4.95114755E-09
    2   11  QJ(  28  )=2.08752763E-07      5   11  QJ(  79  )=5.92108046E-10
    2   12  QJ(  29  )=4.34901589E-08      5   12  QJ(  80  )=4.72886874E-11
    2   13  QJ(  30  )=7.24835981E-09      5   13  QJ(  81  )=1.89139810E-12
    2   14  QJ(  31  )=9.06044977E-10      5   14  QJ(  82  )=               0
    2   15  QJ(  32  )=7.55037481E-11      5   15  QJ(  83  )=               0
    2   16  QJ(  33  )=3.14598950E-12      5   16  QJ(  84  )=               0
    2   17  QJ(  34  )=               0      5   17  QJ(  85  )=               0
    3    1  QJ(  35  )=3.40989043E-02      6    1  QJ(  86  )=               0
    3    2  QJ(  36  )=3.07597531F-02      6    2  QJ(  87  )=               0
    3    3  QJ(  37  )=1.35265107E-02      6    3  QJ(  88  )=               0
    3    4  QJ(  38  )=3.91531781E-03      6    4  QJ(  89  )=               0
    3    5  QJ(  39  )=8.57092801E-04      6    5  QJ(  90  )=               0
    3    6  QJ(  40  )=1.56625190E-04      6    6  QJ(  91  )=               0
    3    7  QJ(  41  )=2.62271034E-05      6    7  QJ(  92  )=               0
    3    8  QJ(  42  )=4.44105168E-06      6    8  QJ(  93  )=               0
    3    9  QJ(  43  )=8.35125955E-07      6    9  QJ(  94  )=               0
    3   10  QJ(  44  )=2.08781489E-07      6   10  QJ(  95  )=               0
    3   11  QJ(  45  )=4.34961435E-08      6   11  QJ(  96  )=               0
    3   12  QJ(  46  )=7.24935725E-09      6   12  QJ(  97  )=               0
    3   13  QJ(  47  )=9.06169656E-10      6   13  QJ(  98  )=               0
    3   14  QJ(  48  )=7.55141380E-11      6   14  QJ(  99  )=               0
    3   15  QJ(  49  )=3.14642242E-12      6   15  QJ( 100  )=               0
    3   16  QJ(  50  )=               0      6   16  QJ( 101  )=               0
    3   17  QJ(  51  )=               0      6   17  QJ( 102  )=               0
```

```
   7    1  QJ(103  ) =                    0
   7    2  QJ(104  ) =                    0
   7    3  QJ(105  ) =                    0
   7    4  QJ(106  ) =                    0
   7    5  QJ(107  ) =                    0
   7    6  QJ(108  ) =                    0
   7    7  QJ(109  ) =                    0
   7    8  QJ(110  ) =                    0
   7    9  QJ(111  ) =                    0
   7   10  QJ(112  ) =                    0
   7   11  QJ(113  ) =                    0
   7   12  QJ(114  ) =                    0
   7   13  QJ(115  ) =                    0
   7   14  QJ(116  ) =                    0
   7   15  QJ(117  ) =                    0
   7   16  QJ(118  ) =                    0
   7   17  QJ(119  ) =                    0
QSUM=1.000000000E 00
EAN A/C FLYING =3.1666E 00

GRAPH TITLED
    E(A/C) =3.17E+00SPOTS = 8.00E+00     T = 1.00E+00
    J VS QJ VECTOR         N = 1.60E+01     A = 4.00E+00
HAS BEEN PLOTTED.
```

APPENDIX IV

SAMPLE RESULTS

The following pages present the values of the elements of the probability distribution vector (Q J) and its graphical plot for five consecutive iterations, i. e., $Q \times P^n$ for n = 1, 2, ... , 5. The inputs are those shown on the first page of Appendix III between statement No. 30 and No. 31. The printouts of the transition matrices and their computational elements are omitted. The plot was made using the DRAW subroutine in the U. S. Naval Postgraduate School computer facility library. Each vector printout contains the values of all 119 states possible (7 x 17) and is headed by the past value of A + 1 and the next value of A + 1. The two indices preceding each element represent $\beta + 1$ and $j + 1$, in the notation of section 3. For example, in the first row on the next page, the "1 1" indicates that the probability of being in state (0, 0) after one iteration is $\doteq$ .026, where the value of A is 4 over the first iteration. Each graph is labeled with the expected value of a/c flying, the number of maintenance spots available, the vector number (T), total number of a/c available (N), and the desired number of a/c on station (A). The "E" notation indicates the power of 10 to multiply by. This sample run demonstrates the loss in total a/c and variable a/c on station.
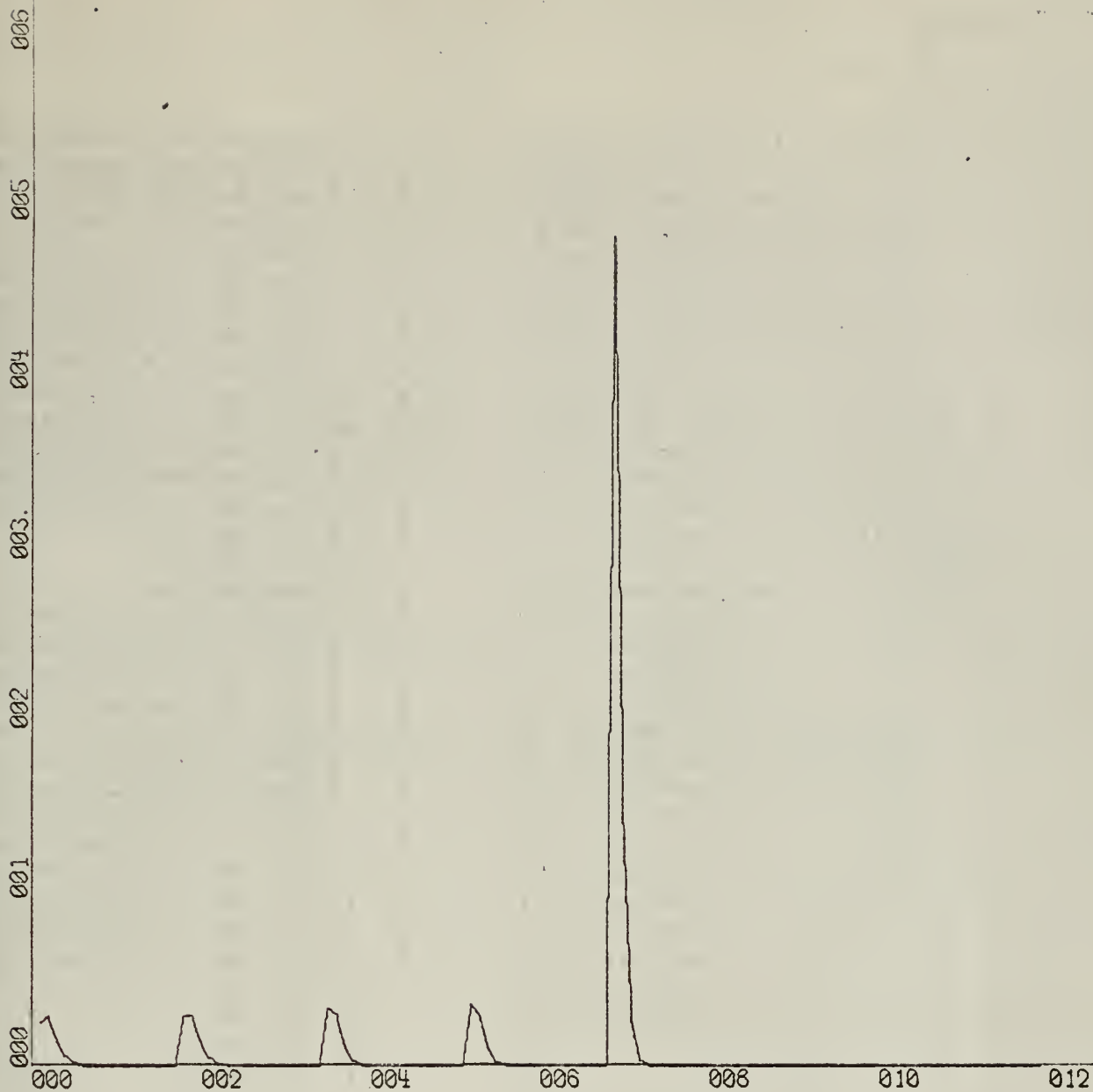
X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 1.00E-01 UNITS/INCH.

E(A/C) =3.17E+00SPOTS = 8.00E+00   T = 1.00E

J VS QJ VECTOR   N = 1.60E+01   A = 4.0

```
   5      5
```

1	1	QJ(1)=9.12184624E-06		4	1	QJ(52)=7.23035242E-03
1	2	QJ(2)=7.94491028E-06		4	2	QJ(53)=4.45693238E-03
1	3	QJ(3)=3.41380533E-06		4	3	QJ(54)=1.28968676E-03
1	4	QJ(4)=9.83913729E-07		4	4	QJ(55)=2.35500532E-04
1	5	QJ(5)=2.20530359E-07		4	5	QJ(56)=3.11156232E-05
1	6	QJ(6)=4.28567319E-08		4	6	QJ(57)=3.30021596E-06
1	7	QJ(7)=7.97994342E-09		4	7	QJ(58)=3.15123146E-07
1	8	QJ(8)=1.56348438E-09		4	8	QJ(59)=3.13674353E-08
1	9	QJ(9)=3.46529220E-10		4	9	QJ(60)=3.81044171E-09
1	10	QJ(10)=1.01221242E-10		4	10	QJ(61)=6.98663591E-10
1	11	QJ(11)=2.58173003E-11		4	11	QJ(62)=1.02568738E-10
1	12	QJ(12)=5.63385201E-12		4	12	QJ(63)=1.13092255E-11
1	13	QJ(13)=1.02295616E-12		4	13	QJ(64)=8.32962386E-13
1	14	QJ(14)=1.48426307E-13		4	14	QJ(65)=3.07550593E-14
1	15	QJ(15)=1.61419113E-14		4	15	QJ(66)=0
1	16	QJ(16)=1.17030587E-15		4	16	QJ(67)=0
1	17	QJ(17)=4.24536876E-17		4	17	QJ(68)=0
2	1	QJ(18)=1.69105396E-04		5	1	QJ(69)=7.70076632E-01
2	2	QJ(19)=1.31058642E-04		5	2	QJ(70)=1.88159529E-01
2	3	QJ(20)=4.93425071E-05		5	3	QJ(71)=2.32480412E-02
2	4	QJ(21)=1.22416819E-05		5	4	QJ(72)=1.93788734E-03
2	5	QJ(22)=2.31695297E-06		5	5	QJ(73)=1.22762068E-04
2	6	QJ(23)=3.73462197E-07		5	6	QJ(74)=6.39420794E-06
2	7	QJ(24)=5.70661929E-08		5	7	QJ(75)=3.02738808E-07
2	8	QJ(25)=9.22154646E-09		5	8	QJ(76)=1.56312395E-08
2	9	QJ(26)=1.72702939E-09		5	9	QJ(77)=1.13519205E-09
2	10	QJ(27)=4.41481881E-10		5	10	QJ(78)=1.50627061E-10
2	11	QJ(28)=9.65869676E-11		5	11	QJ(79)=1.51200491E-11
2	12	QJ(29)=1.75883018E-11		5	12	QJ(80)=1.02093434E-12
2	13	QJ(30)=2.56023495E-12		5	13	QJ(81)=3.47880865E-14
2	14	QJ(31)=2.79434001E-13		5	14	QJ(82)=0
2	15	QJ(32)=2.03393035E-14		5	15	QJ(83)=0
2	16	QJ(33)=7.41007441E-16		5	16	QJ(84)=0
2	17	QJ(34)=0		5	17	QJ(85)=0
3	1	QJ(35)=1.42525319E-03		6	1	QJ(86)=0
3	2	QJ(36)=9.83757697E-04		6	2	QJ(87)=0
3	3	QJ(37)=3.24331350E-04		6	3	QJ(88)=0
3	4	QJ(38)=6.90234010E-05		6	4	QJ(89)=0
3	5	QJ(39)=1.09403657E-05		6	5	QJ(90)=0
3	6	QJ(40)=1.44032446E-06		6	6	QJ(91)=0
3	7	QJ(41)=1.76353392E-07		6	7	QJ(92)=0
3	8	QJ(42)=2.28131763E-08		6	8	QJ(93)=0
3	9	QJ(43)=3.51078290E-09		6	9	QJ(94)=0
3	10	QJ(44)=7.70559333E-10		6	10	QJ(95)=0
3	11	QJ(45)=1.40851171E-10		6	11	QJ(96)=0
3	12	QJ(46)=2.05928494E-11		6	12	QJ(97)=0
3	13	QJ(47)=2.25869291E-12		6	13	QJ(98)=0
3	14	QJ(48)=1.65302266E-13		6	14	QJ(99)=0
3	15	QJ(49)=6.05799194E-15		6	15	QJ(100)=0
3	16	QJ(50)=0		6	16	QJ(101)=0
3	17	QJ(51)=0		6	17	QJ(102)=0

```
7     1  QJ(103  ) =                    0
7     2  QJ(104  ) =                    0
7     3  QJ(105  ) =                    0
7     4  QJ(106  ) =                    0
7     5  QJ(107  ) =                    0
7     6  QJ(108  ) =                    0
7     7  QJ(109  ) =                    0
7     8  QJ(110  ) =                    0
7     9  QJ(111  ) =                    0
7    10  QJ(112  ) =                    0
7    11  QJ(113  ) =                    0
7    12  QJ(114  ) =                    0
7    13  QJ(115  ) =                    0
7    14  QJ(116  ) =                    0
7    15  QJ(117  ) =                    0
7    16  QJ(118  ) =                    0
7    17  QJ(119  ) =                    0
QSUM=1.000000000E 00
EAN A/C FLYING =3.9799E 00

GRAPH TITLED
    E(A/C) =3.98E+00SPOTS = 8.00E+00    T = 2.00E+00
    J VS QJ VECTOR          N = 1.60E+01    A = 4.00E+00
HAS BEEN PLOTTED.
```
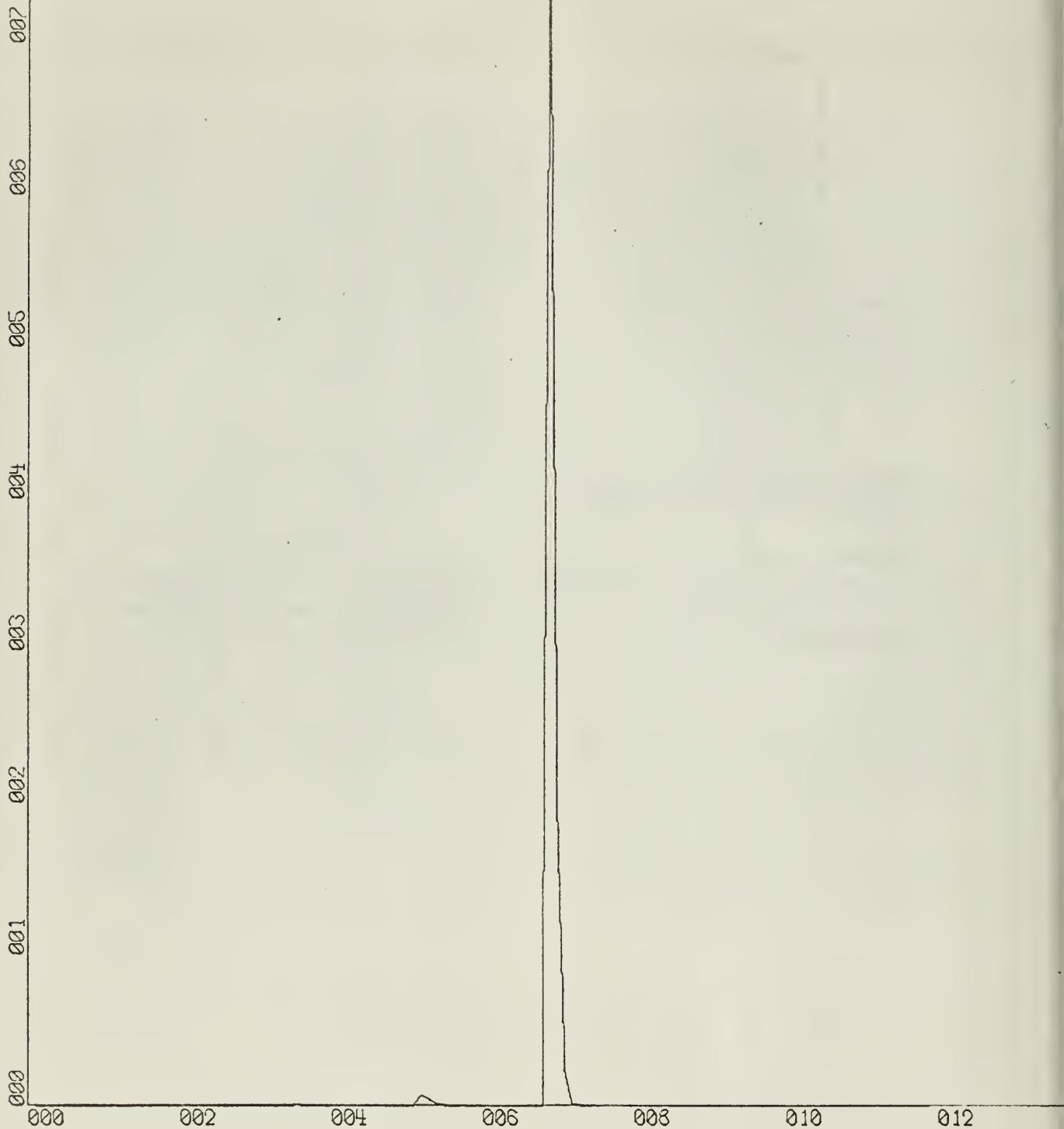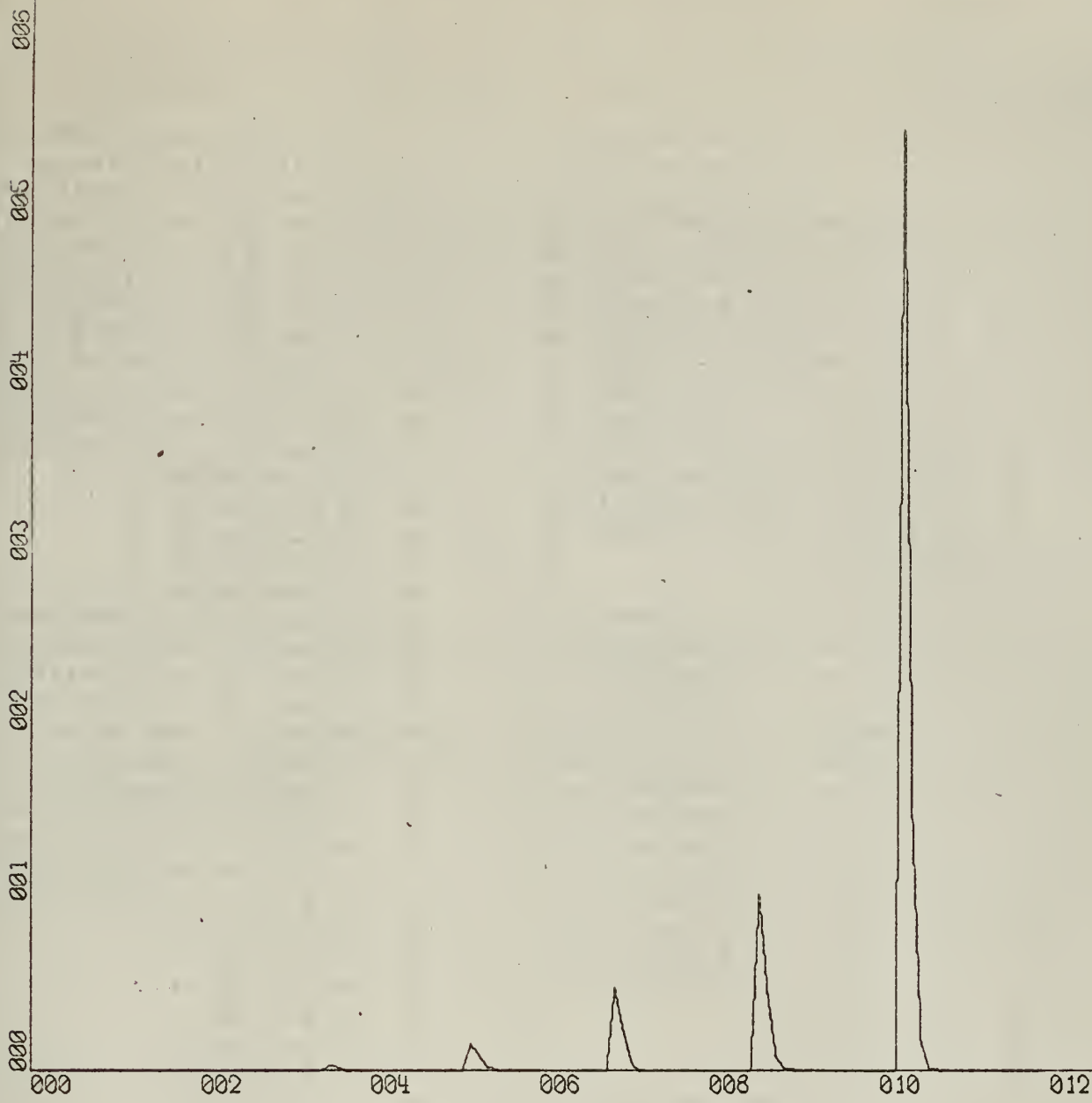
X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 1.00E-01 UNITS/INCH.

E(A/C) = 3.98E+00SPOTS = 8.00E+00   T = 2.00E

J VS QJ VECTOR   N = 1.60E+01   A = 4.0

Q VECTOR CASE 3

     5   7

```
1   1 QJ(  1 )=2.83568314E-05     4   1 QJ( 52 )=1.66499369E-02
1   2 QJ(  2 )=2.15851383E-05     4   2 QJ( 53 )=9.01023249E-03
1   3 QJ(  3 )=7.94343376E-06     4   3 QJ( 54 )=2.23842146E-03
1   4 QJ(  4 )=1.91420746E-06     4   4 QJ( 55 )=3.40006373E-04
1   5 QJ(  5 )=3.49125464E-07     4   5 QJ( 56 )=3.57140995E-05
1   6 QJ(  6 )=5.37405166E-08     4   6 QJ( 57 )=2.82593562E-06
1   7 QJ(  7 )=7.77939446E-09     4   7 QJ( 58 )=1.85894467E-07
1   8 QJ(  8 )=1.18756605E-09     4   8 QJ( 59 )=1.19355508E-08
1   9 QJ(  9 )=2.11283841E-10     4   9 QJ( 60 )=9.51529557E-10
1  10 QJ( 10 )=5.18992660E-11     4  10 QJ( 61 )=1.28538146E-10
1  11 QJ( 11 )=1.08528620E-11     4  11 QJ( 62 )=1.28940846E-11
1  12 QJ( 12 )=1.87739309E-12     4  12 QJ( 63 )=8.53320924E-13
1  13 QJ( 13 )=2.57762883E-13     4  13 QJ( 64 )=2.79262670E-14
1  14 QJ( 14 )=2.63176291E-14     4  14 QJ( 65 )=              0
1  15 QJ( 15 )=1.77499822E-15     4  15 QJ( 66 )=              0
1  16 QJ( 16 )=5.92695364E-17     4  16 QJ( 67 )=              0
1  17 QJ( 17 )=            0       4  17 QJ( 68 )=              0
2   1 QJ( 18 )=4.81711691E-04     5   1 QJ( 69 )=4.98253597E-02
2   2 QJ( 19 )=3.26173533E-04     5   2 QJ( 70 )=2.42738832E-02
2   3 QJ( 20 )=1.04905695E-04     5   3 QJ( 71 )=5.35895703E-03
2   4 QJ( 21 )=2.16142330E-05     5   4 QJ( 72 )=7.10282973E-04
2   5 QJ( 22 )=3.28306882E-06     5   5 QJ( 73 )=6.33588530E-05
2   6 QJ( 23 )=4.09033643E-07     5   6 QJ( 74 )=4.08232868E-06
2   7 QJ( 24 )=4.68083562E-08     5   7 QJ( 75 )=2.05024689E-07
2   8 QJ( 25 )=5.62586373E-09     5   8 QJ( 76 )=9.26759069E-09
2   9 QJ( 26 )=8.10258169E-10     5   9 QJ( 77 )=5.04461339E-10
2  10 QJ( 27 )=1.69317736E-10     5  10 QJ( 78 )=5.13281492E-11
2  11 QJ( 28 )=2.92764116E-11     5  11 QJ( 79 )=3.44456222E-12
2  12 QJ( 29 )=4.01871703E-12     5  12 QJ( 80 )=1.14275058E-13
2  13 QJ( 30 )=4.10297009E-13     5  13 QJ( 81 )=              0
2  14 QJ( 31 )=2.76746676E-14     5  14 QJ( 82 )=              0
2  15 QJ( 32 )=9.24174352E-16     5  15 QJ( 83 )=              0
2  16 QJ( 33 )=            0       5  16 QJ( 84 )=              0
2  17 QJ( 34 )=            0       5  17 QJ( 85 )=              0
3   1 QJ( 35 )=3.67589958E-03     6   1 QJ( 86 )=1.04300023E-01
3   2 QJ( 36 )=2.22029313E-03     6   2 QJ( 87 )=4.52895010E-02
3   3 QJ( 37 )=6.25702594E-04     6   3 QJ( 88 )=8.79453771E-03
3   4 QJ( 38 )=1.10292585E-04     6   4 QJ( 89 )=1.00690538E-03
3   5 QJ( 39 )=1.38900387E-05     6   5 QJ( 90 )=7.55512291E-05
3   6 QJ( 40 )=1.38024736E-06     6   6 QJ( 91 )=3.92418576E-06
3   7 QJ( 41 )=1.21057358E-07     6   7 QJ( 92 )=1.47699319E-07
3   8 QJ( 42 )=1.09262020E-08     6   8 QJ( 93 )=4.43630448E-09
3   9 QJ( 43 )=1.21362937E-09     6   9 QJ( 94 )=1.43899935E-10
3  10 QJ( 44 )=2.09057619E-10     6  10 QJ( 95 )=9.80396467E-12
3  11 QJ( 45 )=2.86055733E-11     6  11 QJ( 96 )=3.30144045E-13
3  12 QJ( 46 )=2.91366712E-12     6  12 QJ( 97 )=              0
3  13 QJ( 47 )=1.96282619E-13     6  13 QJ( 98 )=              0
3  14 QJ( 48 )=6.55549374E-15     6  14 QJ( 99 )=              0
3  15 QJ( 49 )=            0       6  15 QJ(100 )=              0
3  16 QJ( 50 )=            0       6  16 QJ(101 )=              0
3  17 QJ( 51 )=            0       6  17 QJ(102 )=              0
```

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```
7    1  QJ(103  )=5.55639840E-01
7    2  QJ(104  )=1.48858751E-01
7    3  QJ(105  )=1.84227232E-02
7    4  QJ(106  )=1.37794578E-03
7    5  QJ(107  )=6.83380035E-05
7    6  QJ(108  )=2.32262953E-06
7    7  QJ(109  )=5.43332378E-08
7    8  QJ(110  )=8.80653967E-10
7    9  QJ(111  )=1.18536160E-11
7   10  QJ(112  )=3.82356527E-13
7   11  QJ(113  )=              0
7   12  QJ(114  )=              0
7   13  QJ(115  )=              0
7   14  QJ(116  )=              0
7   15  QJ(117  )=              0
7   16  QJ(118  )=              0
7   17  QJ(119  )=              0
QSUM=1.000000000E 00
EAN A/C FLYING =5.5636E 00

GRAPH TITLED
    E(A/C) =5.56E+00SPOTS = 8.00E+00      T = 3.00E+00
    J VS QJ VECTOR        N = 1.50E+01      A = 6.00E+00
HAS BEEN PLOTTED.
```
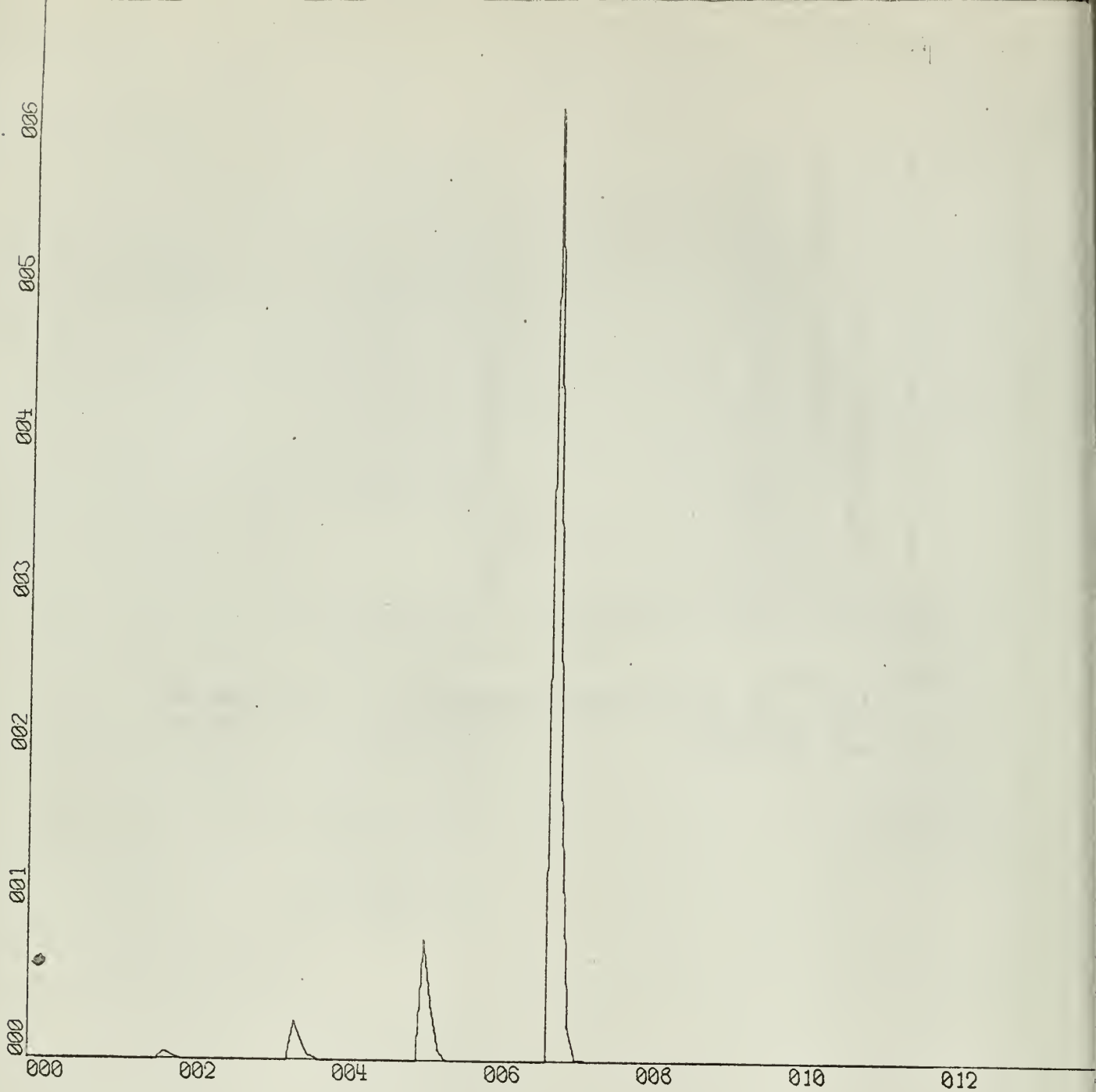
X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 1.00E-01 UNITS/INCH.

E(A/C) =5.56E+00SPOTS = 8.00E+00   T = 3.00E

J VS QJ VECTOR   N = 1.50E+01   A = 6.00

Q VECTOR CASE 4

```
   7      5
```

1	1	QJ(1)=$4.37066460E-04$	4	1	QJ(52)=$8.03893389E-02$
1	2	QJ(2)=$2.66337542E-04$	4	2	QJ(53)=$3.58842827E-02$
1	3	QJ(3)=$7.61707863E-05$	4	3	QJ(54)=$7.24458728E-03$
1	4	QJ(4)=$1.37453763E-05$	4	4	QJ(55)=$8.75756513E-04$
1	5	QJ(5)=$1.79617995E-06$	4	5	QJ(56)=$7.09643835E-05$
1	6	QJ(6)=$1.89112432E-07$	4	6	QJ(57)=$4.12757545E-06$
1	7	QJ(7)=$1.81098891E-08$	4	7	QJ(58)=$1.85478755E-07$
1	8	QJ(8)=$1.84326720E-09$	4	8	QJ(59)=$7.45697706E-09$
1	9	QJ(9)=$2.33815659E-10$	4	9	QJ(60)=$3.67973791E-10$
1	10	QJ(10)=$4.51123736E-11$	4	10	QJ(61)=$3.56621692E-11$
1	11	QJ(11)=$7.18762832E-12$	4	11	QJ(62)=$2.27683580E-12$
1	12	QJ(12)=$9.07642678E-13$	4	12	QJ(63)=$7.17866372E-14$
1	13	QJ(13)=$8.51451858E-14$	4	13	QJ(64)= 0
1	14	QJ(14)=$5.27326607E-15$	4	14	QJ(65)= 0
1	15	QJ(15)=$1.61678538E-16$	4	15	QJ(66)= 0
1	16	QJ(16)= 0	4	16	QJ(67)= 0
1	17	QJ(17)= 0	4	17	QJ(68)= 0
2	1	QJ(18)=$5.18126447E-03$	5	1	QJ(69)=$6.25520845E-01$
2	2	QJ(19)=$2.86433504E-03$	5	2	QJ(70)=$1.72496943E-01$
2	3	QJ(20)=$7.33497609E-04$	5	3	QJ(71)=$2.20943200E-02$
2	4	QJ(21)=$1.16340651E-04$	5	4	QJ(72)=$1.73104557E-03$
2	5	QJ(22)=$1.30125431E-05$	5	5	QJ(73)=$9.18785067E-05$
2	6	QJ(23)=$1.13029797E-06$	5	6	QJ(74)=$3.46552930E-06$
2	7	QJ(24)=$8.54466567E-08$	5	7	QJ(75)=$9.61462191E-08$
2	8	QJ(25)=$6.65517506E-09$	5	8	QJ(76)=$2.13537750E-09$
2	9	QJ(26)=$6.57969504E-10$	5	9	QJ(77)=$5.31168734E-11$
2	10	QJ(27)=$1.05656138E-10$	5	10	QJ(78)=$3.22108252E-12$
2	11	QJ(28)=$1.34342964E-11$	5	11	QJ(79)=$9.66869152E-14$
2	12	QJ(29)=$1.26766175E-12$	5	12	QJ(80)= 0
2	13	QJ(30)=$7.88829950E-14$	5	13	QJ(81)= 0
2	14	QJ(31)=$2.42720088E-15$	5	14	QJ(82)= 0
2	15	QJ(32)= 0	5	15	QJ(83)= 0
2	16	QJ(33)= 0	5	16	QJ(84)= 0
2	17	QJ(34)= 0	5	17	QJ(85)= 0
3	1	QJ(35)=$2.69210521E-02$	6	1	QJ(86)= 0
3	2	QJ(36)=$1.34302284E-02$	6	2	QJ(87)= 0
3	3	QJ(37)=$3.06604116E-03$	6	3	QJ(88)= 0
3	4	QJ(38)=$4.26067113E-04$	6	4	QJ(89)= 0
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3	7	QJ(41)=$1.70443704E-07$	6	7	QJ(92)= 0
3	8	QJ(42)=$9.76651595E-09$	6	8	QJ(93)= 0
3	9	QJ(43)=$7.11954817E-10$	6	9	QJ(94)= 0
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3	14	QJ(48)= 0	6	14	QJ(99)= 0
3	15	QJ(49)= 0	6	15	QJ(100)= 0
3	16	QJ(50)= 0	6	16	QJ(101)= 0
3	17	QJ(51)= 0	6	17	QJ(102)= 0

```
7    1  QJ(103 )=                    0
7    2  QJ(104 )=                    0
7    3  QJ(105 )=                    0
7    4  QJ(106 )=                    0
7    5  QJ(107 )=                    0
7    6  QJ(108 )=                    0
7    7  QJ(109 )=                    0
7    8  QJ(110 )=                    0
7    9  QJ(111 )=                    0.
7   10  QJ(112 )=                    0
7   11  QJ(113 )=                    0
7   12  QJ(114 )=                    0
7   13  QJ(115 )=                    0
7   14  QJ(116 )=                    0
7   15  QJ(117 )=                    0
7   16  QJ(118 )=                    0
7   17  QJ(119 )=                    0
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EAN A/C FLYING =3.7578E 00

GRAPH TITLED
    E(A/C) =3.76E+00SPOTS = 8.00E+00.   T = 4.00E+00
    J VS QJ VECTOR   N = 1.40E+01   A = 4.00E+00
HAS BEEN PLOTTED.

X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 1.00E-01 UNITS/INCH.

E(A/C) =3.76E+00SPOTS = 8.00E+00   T = 4.00E-
J VS QJ VECTOR   N = 1.40E+01   A = 4.00

Q VECTOR CASE 5

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   1    1  QJ(   1 )=7.15520995E-05        4    1  QJ(  52 )=2.93940831E-02
   1    2  QJ(   2 )=4.87189674E-05        4    2  QJ(  53 )=1.43984862E-02
   1    3  QJ(   3 )=1.57753676E-05        4    3  QJ(  54 )=3.19845745E-03
   1    4  QJ(   4 )=3.27727099E-06        4    4  QJ(  55 )=4.26970762E-04
   1    5  QJ(   5 )=5.02868635E-07        4    5  QJ(  56 )=3.84133994E-05
   1    6  QJ(   6 )=6.34199800E-08        4    6  QJ(  57 )=2.50141306E-06
   1    7  QJ(   7 )=7.35880233E-09        4    7  QJ(  58 )=1.27347959E-07
   1    8  QJ(   8 )=8.96925196E-10        4    8  QJ(  59 )=5.85531451E-09
   1    9  QJ(   9 )=1.30710966E-10        4    9  QJ(  60 )=3.24180085E-10
   1   10  QJ(  10 )=2.75401174E-11        4   10  QJ(  61 )=3.33199523E-11
   1   11  QJ(  11 )=4.80498022E-12        4   11  QJ(  62 )=2.26209155E-12
   1   12  QJ(  12 )=6.66122816E-13        4   12  QJ(  63 )=7.60530690E-14
   1   13  QJ(  13 )=6.87539584E-14        4   13  QJ(  64 )=             0
   1   14  QJ(  14 )=4.69377133E-15        4   14  QJ(  65 )=             0
   1   15  QJ(  15 )=1.58863224E-16        4   15  QJ(  66 )=             0
   1   16  QJ(  16 )=             0        4   16  QJ(  67 )=             0
   1   17  QJ(  17 )=             0        4   17  QJ(  68 )=             0
   2    1  QJ(  18 )=1.06771615E-03        5    1  QJ(  69 )=7.70471329E-02
   2    2  QJ(  19 )=6.60751603E-04        5    2  QJ(  70 )=3.36780394E-02
   2    3  QJ(  20 )=1.87506066E-04        5    3  QJ(  71 )=6.58809654E-03
   2    4  QJ(  21 )=3.33939733E-05        5    4  QJ(  72 )=7.60547395E-04
   2    5  QJ(  22 )=4.24508484E-06        5    5  QJ(  73 )=5.76075719E-05
   2    6  QJ(  23 )=4.27752034E-07        5    6  QJ(  74 )=3.02551600E-06
   2    7  QJ(  24 )=3.81603116E-08        5    7  QJ(  75 )=1.15424789E-07
   2    8  QJ(  25 )=3.50828898E-09        5    8  QJ(  76 )=3.52590714E-09
   2    9  QJ(  26 )=3.95927530E-10        5    9  QJ(  77 )=1.16438952E-10
   2   10  QJ(  27 )=6.89194802E-11        5   10  QJ(  78 )=8.00922104E-12
   2   11  QJ(  28 )=9.53886847E-12        5   11  QJ(  79 )=2.72622038E-13
   2   12  QJ(  29 )=9.83824310E-13        5   12  QJ(  80 )=             0
   2   13  QJ(  30 )=6.71883385E-14        5   13  QJ(  81 )=             0
   2   14  QJ(  31 )=2.27773386E-15        5   14  QJ(  82 )=             0
   2   15  QJ(  32 )=             0        5   15  QJ(  83 )=             0
   2   16  QJ(  33 )=             0        5   16  QJ(  84 )=             0
   2   17  QJ(  34 )=             0        5   17  QJ(  85 )=             0
   3    1  QJ(  35 )=7.37121582E-03        6    1  QJ(  86 )=1.39125275E-01
   3    2  QJ(  36 )=4.01163961E-03        6    2  QJ(  87 )=5.35756492E-02
   3    3  QJ(  37 )=1.00338550E-03        6    3  QJ(  88 )=9.10073278E-03
   3    4  QJ(  38 )=1.53687580E-04        6    4  QJ(  89 )=8.95105006E-04
   3    5  QJ(  39 )=1.63163199E-05        6    5  QJ(  90 )=5.62254445E-05
   3    6  QJ(  40 )=1.30932633E-06        6    6  QJ(  91 )=2.34682483E-06
   3    7  QJ(  41 )=8.77478713E-08        6    7  QJ(  92 )=6.58960130E-08
   3    8  QJ(  42 )=5.76387628E-09        6    8  QJ(  93 )=1.26580622E-09
   3    9  QJ(  43 )=4.69636023E-10        6    9  QJ(  94 )=1.99736156E-11
   3   10  QJ(  44 )=6.43465918E-11        6   10  QJ(  95 )=6.91079850E-13
   3   11  QJ(  45 )=6.56029037E-12        6   11  QJ(  96 )=             0
   3   12  QJ(  46 )=4.42292681E-13        6   12  QJ(  97 )=             0
   3   13  QJ(  47 )=1.47863511E-14        6   13  QJ(  98 )=             0
   3   14  QJ(  48 )=             0        6   14  QJ(  99 )=             0
   3   15  QJ(  49 )=             0        6   15  QJ( 100 )=             0
   3   16  QJ(  50 )=             0        6   16  QJ( 101 )=             0
   3   17  QJ(  51 )=             0        6   17  QJ( 102 )=             0
```

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7     1  QJ(103  )=4.84130317E-01
7     2  QJ(104  )=1.18774563E-01
7     3  QJ(105  )=1.31776649E-02
7     4  QJ(106  )=8.59940961E-04
7     5  QJ(107  )=3.59275288E-05
7     6  QJ(108  )=9.79278393E-07
7     7  QJ(109  )=1.69313196E-08
7     8  QJ(110  )=1.69139063E-10
7     9  QJ(111  )=7.45214538E-13
7    10  QJ(112  )=                    0
7    11  QJ(113  )=.                   0
7    12  QJ(114  )=                    0
7    13  QJ(115  )=                    0
7    14  QJ(116  )=                    0
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7    16  QJ(118  )=                    0
7    17  QJ(119  )=                    0
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EAN A/C FLYING =5.3577E 00

GRAPH TITLED
    E(A/C) =5.36E+00SPOTS = 8.00E+00      T = 5.00E+00
    J VS QJ VECTOR           N = 1.40E+01      A = 6.00E+00
HAS BEEN PLOTTED.
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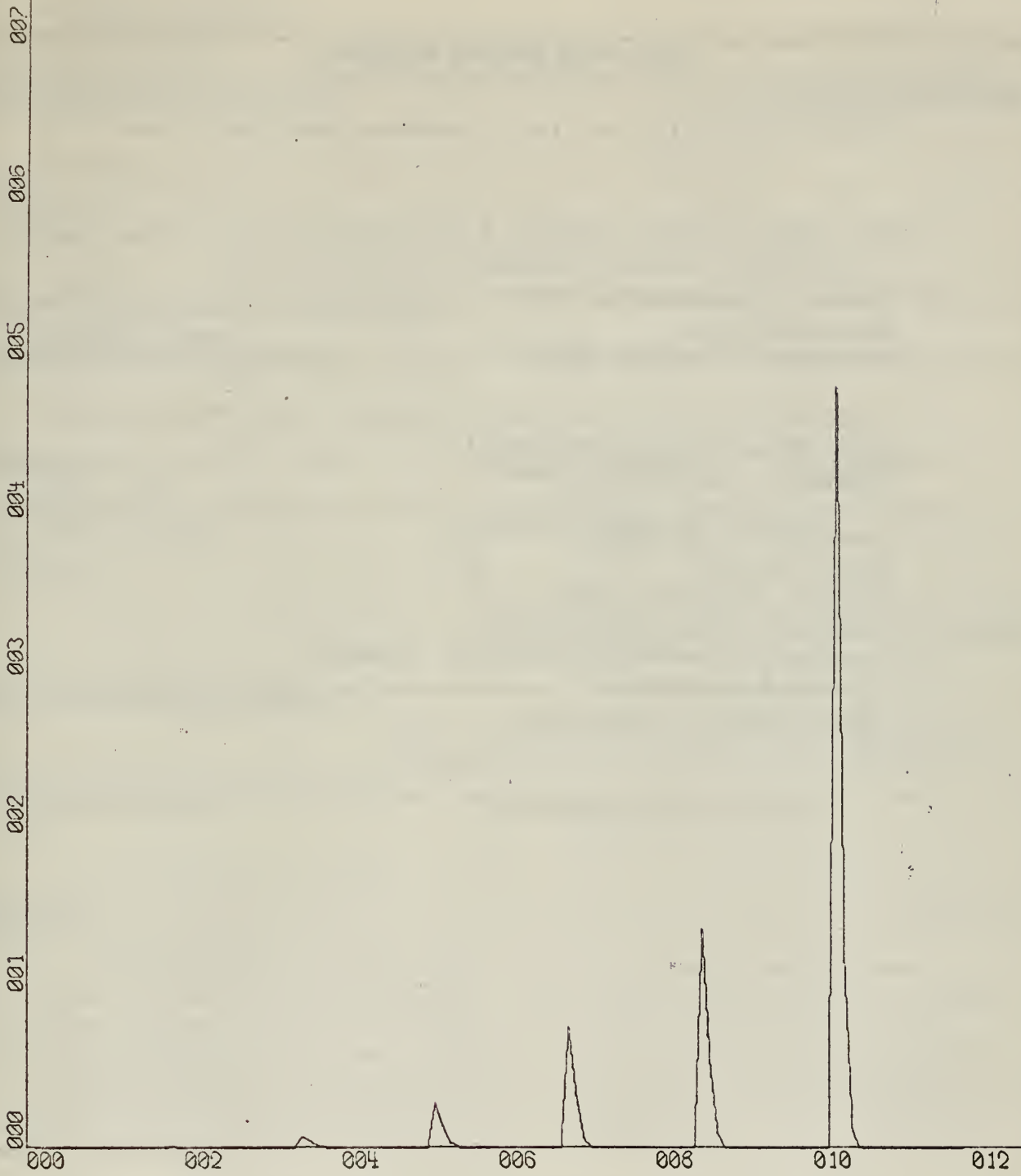
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Master's Thesis

5. AUTHOR(S) (Last name, first name, initial)

Lanman, George M. , Lieutenant Commander, U. S. Navy

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It is the purpose of this paper to develop a useful mathematical model of ASW aircraft availability. The increasing emphasis of systems studies dictates the use of accurate and representative models of the ASW systems. At present, many studies are using essentially the same models developed during World War II. This paper is an attempt to make use of advanced theory in a more powerful and flexible model and to make the use of the model practical and verifiable.

The writer adapted the time homogeneous bivariate model as developed by F. C. Collins. This is a discrete time Markov process with a stochastic matrix of transition probabilities wherein the maintenance process is modeled as a pulsed input multiple server queue.

The model was programmed in FORTRAN 63 on the CDC 1604 and then modified to allow for variability in the input parameters. Other modifications include an increase in the size of the model to accommodate a 16-aircraft squadron, the largest ASW squadron at present, and an explicit form solution to the maintenance queueing equations.

DD FORM 1473 1 JAN 64

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
ASW Aircraft						
Computer Markov Model						
Availability						

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