In conclusion it may be observed, that the particular results, (4), (6), (7), (8), are nothing more than immediate consequences of Mr. Jacobi's factorial developments of the trigonometrical functions of the amplitude of an elliptic function, in terms of the function itself.—*Traité des Fonctions Elliptiques,* tom. iii. page 97. It may be seen that they follow at once from these expansions, if we remember that

$$\int_0^\pi \log(1 \pm 2a\cos x + a^2)\, dx = 0$$

when a is less than unity; a theorem proved by Poisson in the seventeenth *cahier* of the *Journal de l'Ecole Polytechnique.*

Sir William R. Hamilton stated the following theorems of central forces, which he had proved by his calculus of quaternions, but which, as he remarked, might be also deduced from principles more elementary.

If a body be attracted to a fixed point, with a force which varies directly as the distance from that point, and inversely as the cube of the distance from a fixed plane, the body will describe a conic section, of which the plane intersects the fixed plane in a straight line, which is the polar of the fixed point with respect to the conic section.

And in like manner, if a material point be obliged to remain upon the surface of a given sphere, and be acted on by a force, of which the tangential component is constantly directed (along the surface) towards a fixed point or pole upon that surface, and varies directly as the sine of the arcual distance from that pole, and inversely as the cube of the sine of the arcual distance from a fixed great circle; then the material point will describe a spherical conic, with respect to which the fixed great circle will be the polar of the fixed point.

Thus, a spherical conic would be described by a heavy point upon a sphere, if the vertical accelerating force were to vary inversely as the cube of the perpendicular and linear distance from a fixed plane passing through the centre.

The first theorem had been suggested to Sir W. Hamilton by a recently resumed study of a part of Sir Isaac Newton's Principia; and he had been encouraged to seek for the second theorem, by a recollection of a result respecting motion in a spherical conic, which was stated some years ago to the Academy by the Rev. C. Graves. In that result of Mr. Graves, the fixed pole was a focus of the conic, and the polar was therefore the director arc; consequently, the sine of the distance from the polar was proportional to the sine of the distance from the pole, and, instead of the law now mentioned to the Academy, there was the simpler law of proportionality to the inverse square of the sine of the distance from the fixed pole or focus.

Professor Graves observed, that he had that morning, in conversation with the President, stated the theorem just announced, respecting the motion of a material point on the surface of a sphere. Sir William Hamilton having, at the last meeting of the Academy, kindly communicated to him his theorem of plane central forces, it occurred to Professor Graves to inquire whether two theorems, which he had stated in January, 1842,* relating to the motion of a point in a spherical conic, might not be included in a more general one, analogous to that first mentioned by Sir William Hamilton. This inquiry led him to perceive the truth of Sir William Hamilton's second theorem.

The mode of proof employed by Professor Graves rests, so far as regards the dynamical part of the question, on the two following elementary propositions:

If a material point, P, constrained to move on the surface of a sphere, be urged by a force acting along a great circle passing through a fixed point, S;

* See Proceedings of the Academy, vol. ii. p. 209.