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特71-885

平面三角法問題集解答

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M43.5

CAE-0001

平面三角法問題集解答

全

265
54

特別
8月

恒等式(鋭角)

(1)

$\cos A \tan A = \sin A$

$左辺 = \cos A \frac{\sin A}{\cos A} = \sin A = 右辺$

(2)

$\cos A = \cot A \sin A$

$右辺 = \frac{\cos A}{\sin A} \sin A = \cos A = 左辺$

(3)

$\tan^2 A - \sin^2 A = \tan^2 A \sin^2 A$

$左辺 = \frac{\sin^2 A}{\cos^2 A} - \sin^2 A = \frac{\sin^2 A - \sin^2 A \cos^2 A}{\cos^2 A}$

$= \frac{\sin^2 A(1-\cos^2 A)}{\cos^2 A} = \frac{\sin^2 A}{\cos^2 A}(1-\cos^2 A)$

$= \tan^2 A \sin^2 A = 右辺$

(4)

$\frac{1}{\tan^2 A+1} + \frac{1}{\cot^2 A+1} = 1$

$左辺 = \frac{1}{\sec^2 A} + \frac{1}{\operatorname{cosec}^2 A} = \cos^2 A + \sin^2 A = 1$

$= 右辺$

(5)

$\cot^2 A - \cos^2 A = \cot^2 A \cos^2 A$

$左辺 = \frac{\cos^2 A}{\sin^2 A} - \cos^2 A = \frac{\cos^2 A(1-\sin^2 A)}{\sin^2 A}$

$= \frac{\cos^2 A}{\sin^2 A}(1-\sin^2 A) = \cot^2 A \cos^2 A = 右辺$

(6)

$\operatorname{cosec} A - \sin A = \cot A \cos A$

$左辺 = \frac{1}{\sin A} - \sin A = \frac{1-\sin^2 A}{\sin A} = \frac{\cos^2 A}{\sin A}$

$= \frac{\cos A}{\sin A}\cos A = \cot A \cos A = 右辺$

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図書

恒等式(鋭角)

(7)

$\cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

左辺 $= \cos^2 A - (1 - \cos^2 A) = 2\cos^2 A - 1$

$= 1 - \sin^2 A - \sin^2 A = 1 - 2\sin^2 A$

即ち 左辺＝中辺＝右辺

(8)

$\sec^2 A + \operatorname{cosec}^2 A = \sec^2 A \operatorname{cosec}^2 A$

左辺 $= \dfrac{1}{\cos^2 A} + \dfrac{1}{\sin^2 A} = \dfrac{\sin^2 A + \cos^2 A}{\cos^2 A \sin^2 A}$

$= \dfrac{1}{\cos^2 A \sin^2 A} = \dfrac{1}{\cos^2 A} \cdot \dfrac{1}{\sin^2 A}$

$= \sec^2 A \operatorname{cosec}^2 A =$ 右辺

(9)

$(\tan A + \cot A)\sin A \cos A = 1$

左辺 $= \left(\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}\right)\sin A \cos A$

$= \dfrac{\sin^2 A + \cos^2 A}{\cos A \sin A} \cdot \sin A \cos A = 1$

$=$ 右辺

(10)

$\tan A \sin A + \cos A = \sec A$

左辺 $= \dfrac{\sin A}{\cos A} \cdot \sin A + \cos A = \dfrac{\sin^2 A + \cos^2 A}{\cos A}$

$= \dfrac{1}{\cos A} = \sec A =$ 右辺

(11)

$\sec A - \cos A = \tan A \sin A$

左辺 $= \dfrac{1}{\cos A} - \cos A = \dfrac{1 - \cos^2 A}{\cos A} = \dfrac{\sin^2 A}{\cos A}$

$= \dfrac{\sin A}{\cos A} \sin A = \tan A \sin A =$ 右辺

恒等式(鋭角)

(12)

$\sin^4 A - \cos^4 A = \sin^2 A - \cos^2 A$

左辺 $= (\sin^2 A + \cos^2 A)(\sin^2 A - \cos^2 A)$

$= 1 \times (\sin^2 A - \cos^2 A) = \sin^2 A - \cos^2 A =$ 右辺

(13)

$\cot A + \tan A = \sec A \operatorname{cosec} A$

左辺 $= \dfrac{\cos A}{\sin A} + \dfrac{\sin A}{\cos A} = \dfrac{\cos^2 A + \sin^2 A}{\sin A \cos A}$

$= \dfrac{1}{\sin A \cos A} = \dfrac{1}{\sin A} \cdot \dfrac{1}{\cos A}$

$= \sec A \operatorname{cosec} A =$ 右辺

(14)

$\sin^3 A + \cos^3 A = (\sin A + \cos A)(1 - \sin A \cos A)$

左辺 $= (\sin A + \cos A)(\sin^2 A + \cos^2 A - \sin A \cos A)$

$= (\sin A + \cos A)(1 - \sin A \cos A) =$ 右辺

(15)

$(\sec B - \tan B)(\sec B + \tan B) = 1$

左辺 $= \sec^2 B - \tan^2 B = \dfrac{1}{\cos^2 B} - \dfrac{\sin^2 B}{\cos^2 B}$

$= \dfrac{\cos^2 B}{\cos^2 B} = 1 =$ 右辺

(16)

$(1 + \sin A)^2 + (1 + \cos A)^2 = 3 + 2(\sin A + \cos A)$

左辺 $= 1 + 2\sin A + \sin^2 A + 1 + 2\cos A + \cos^2 A$

$= 2 + \sin^2 A + \cos^2 A + 2(\sin A + \cos A)$

$= 3 + 2(\sin A + \cos A) =$ 右辺

恒等式（鋭角）

(17)

$$\sin^2 C\tan^2 C+\cos^2 C\cot^2 C=\tan^2 C+\cot^2 C-1$$

$$\text{左辺}=\sin^2 C\tan^2 C+\sin^2 C-\sin^2 C+\cos^2 C\cot^2 C+\cos^2 C-\cos^2 C$$

$$=\sin^2 C(\tan^2 C+1)+\cos^2 C(\cot^2 C+1)-(\sin^2 C+\cos^2 C)$$

$$=\sin^2 C\sec^2 C+\cos^2 C\operatorname{cosec}^2 C-1$$

$$=\frac{\sin^2 C}{\cos^2 C}+\frac{\cos^2 C}{\sin^2 C}-1=\tan^2 C+\cot^2 C-1$$

$$=\text{右辺}$$

(18)

$$\sin^2 A\tan A+\cos^2 A\cot A+2\sin A\cos A=\tan A+\cot A$$

$$\text{左辺}=(1-\cos^2 A)\tan A+(1-\sin^2 A)\cot A+2\sin A\cos A$$

$$=\tan A-\cos^2 A\tan A+\cot A-\sin^2 A\cot A+2\sin A\cos A$$

$$=\tan A-\cancel{\cos A\sin A}+\cot A-\cancel{\cos A\sin A}+\cancel{2\sin A\cos A}$$

$$=\tan A+\cot A=\text{左辺}$$

(19)

$$(\sin A+\cos A)^2=1+2\sin A\cos A$$

$$\text{左辺}=\sin^2 A+\cos^2 A+2\sin A\cos A$$

$$=1+2\sin A\cos A=\text{右辺}$$

(20)

$$\sec A=\tan A\operatorname{cosec} A$$

$$\text{左辺}=\frac{\sin A}{\cos A}\cdot\frac{1}{\sin A}=\frac{1}{\cos A}=\sec A=\text{右辺}$$

恒等式（鋭角）

(21)

$$(1+\sin A+\cos A)^2=2(1+\sin A)(1+\cos A)$$

$$\text{左辺}=1+\sin^2 A+\cos^2 A+2\sin A+2\cos A+2\sin A\cos A$$

$$=2+2\sin A+2\cos A+2\sin A\cos A$$

$$=2(1+\sin A+\cos A+\sin A\cos A)$$

$$=2\{(1+\sin A)+\cos A(1+\sin A)\}$$

$$=2(1+\sin A)(1+\cos A)=\text{右辺}$$

(22)

$$\frac{1-\sin A}{1+\sin A}=(\sec A-\tan A)^2$$

$$\text{左辺}=\frac{(1-\sin A)^2}{(1+\sin A)(1-\sin A)}=\frac{(1-\sin A)^2}{1-\sin^2 A}$$

$$=\frac{(1-\sin A)^2}{\cos^2 A}=\left\{\frac{1-\sin A}{\cos A}\right\}^2$$

$$\text{右辺}=\left\{\frac{1}{\cos A}-\frac{\sin A}{\cos A}\right\}^2=\left\{\frac{1-\sin A}{\cos A}\right\}^2=\text{左辺}$$

(23)

$$(\tan A+\sec A)^2=\frac{1+\sin A}{1-\sin A}$$

$$\text{左辺}=\left(\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)^2=\frac{(1+\sin A)^2}{\cos^2 A}=\frac{(1+\sin A)^2}{1-\sin^2 A}$$

$$=\frac{(1+\sin A)^2}{(1+\sin A)(1-\sin A)}=\frac{1+\sin A}{1-\sin A}=\text{右辺}$$

(24)

$$\frac{\sin x}{1+\cos x}=\frac{1-\cos x}{\sin x}$$ 所以＝ $\sin x(1+\cos x)$ ヲ乘ジテ

$\sin^2 x=1-\cos^2 x$ 即 $\sin^2 x=\sin^2 x$

恒等式(鋭角)

(25)

$$(\sin A+\cos A)(\tan A+\cot A)=\sec A+\operatorname{cosec} A$$

$$\text{左辺}=(\sin A+\cos A)\left(\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}\right)$$

$$=(\sin A+\cos A)\frac{\sin^2 A+\cos^2 A}{\sin A\cos A}$$

$$=\frac{\sin A+\cos A}{\sin A\cos A}=\frac{\sin A}{\sin A\cos A}+\frac{\cos A}{\sin A\cos A}$$

$$=\frac{1}{\cos A}+\frac{1}{\sin A}=\sec A+\operatorname{cosec} A=\text{右辺}$$

(26)

$$\frac{\tan A+\tan B}{\cot A+\cot B}=\tan A\tan B$$

$$\text{左辺}=\frac{\dfrac{\sin A}{\cos A}+\dfrac{\sin B}{\cos B}}{\dfrac{\cos A}{\sin A}+\dfrac{\cos B}{\sin B}}=\frac{\dfrac{\sin A\cos B+\sin B\cos A}{\cos A\cos B}}{\dfrac{\sin B\cos A+\sin A\cos B}{\sin A\sin B}}$$

$$=\frac{\sin A\sin B}{\cos A\cos B}=\tan A\tan B=\text{右辺}$$

(27)

$$2(\sin^6 A+\cos^6 A)-3(\sin^4 A+\cos^4 A)+1=0$$

$$2(\sin^6 A+\cos^6 A)=2\{(\sin^2 A)^3+(\cos^2 A)^3\}$$

$$=2(\sin^2 A+\cos^2 A)(\sin^4 A-\sin^2 A\cos^2 A+\cos^4 A)$$

$$=2\times 1\{(\sin^4 A+\cos^4 A)-\sin^2 A\cos^2 A\}$$

$$=2\{(\sin^2 A+\cos^2 A)^2-2\sin^2 A\cos^2 A-\sin^2 A\cos^2 A\}$$

$$=2\{1-3\sin^2 A\cos^2 A\}$$

$$\text{左辺}=2(1-3\sin^2 A\cos^2 A)-3(1-2\sin^2 A\cos^2 A)+1$$

$$=2-\cancel{6\sin^2 A\cos^2 A}-3+\cancel{6\sin^2 A\cos^2 A}+1$$

$$=0=\text{右辺}$$

恒等式(鋭角)

(28)

$$\cot^4 A+\cot^2 A=\operatorname{cosec}^4 A-\operatorname{cosec}^2 A$$

$$\text{左辺}=\cot^2 A(\cot^2 A+1)=\frac{\cos^2 A}{\sin^2 A}\cdot\frac{1}{\sin^2 A}$$

$$=\frac{1}{\sin^4 A}\cos^2 A=\frac{1}{\sin^4 A}(1-\sin^2 A)$$

$$=\frac{1}{\sin^4 A}-\frac{1}{\sin^2 A}=\operatorname{cosec}^4 A-\operatorname{cosec}^2 A$$

$$=\text{右辺}$$

(29)

$$\sec^4 A+\tan^4 A=1+2\sec^2 A\tan^2 A$$

$$\text{左辺}=(1+\tan^2 A)^2+\tan^4 A$$

$$=1+2\tan^2 A+\tan^4 A+\tan^4 A$$

$$=1+2\tan^2 A(1+\tan^2 A)=1+2\tan^2 A\sec^2 A$$

$$=\text{右辺}$$

(30)

$$\frac{1+\cos\theta}{1-\cos\theta}-\frac{1-\cos\theta}{1+\cos\theta}=4\cot\theta\operatorname{cosec}\theta$$

$$\text{左辺}=\frac{(1+\cos\theta)^2-(1-\cos\theta)^2}{1-\cos^2\theta}=\frac{2\times 2\cos\theta}{\sin^2\theta}$$

$$=4\frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\sin\theta}=4\operatorname{cosec}\theta\cot\theta=\text{右辺}$$

(31)

$$(1+\sin A-\cos A)^2+(1+\cos A-\sin A)^2=4(1-\sin A\cos A)$$

$$\text{左辺}=\{1-(\cos A-\sin A)\}^2+\{1+(\cos A-\sin A)\}^2$$

$$=2+2(\cos A-\sin A)^2$$

$$=2+2\cos^2 A+2\sin^2 A-4\cos A\sin A$$

$$=4-4\cos A\sin A=4(1-\cos A\sin A)$$

$$=\text{右辺}$$

恒等式(鋭角)

(32)

$$\sec A+\tan A=\frac{1}{\sec A-\tan A}$$

$$\text{右辺}=\frac{1}{\frac{1}{\cos A}-\frac{\sin A}{\cos A}}=\frac{\cos A}{1-\sin A}$$

$$=\frac{\cos A(1+\sin A)}{1-\sin^2 A}=\frac{\cos A(1+\sin A)}{\cos^2 A}$$

$$=\frac{1}{\cos A}+\frac{\sin A}{\cos A}=\sec A+\tan A=\text{左辺}$$

(33)

$$(1-\sin A-\cos A)^2(1+\sin A+\cos A)^2=4\sin^2 A\cos^2 A$$

$$\text{左辺}=\{1-(\sin A+\cos A)^2\}^2$$

$$=\{1-\sin^2 A-\cos^2 A-2\sin A\cos A\}^2$$

$$=\{-2\sin A\cos A\}^2=4\sin^2 A\cos^2 A=\text{右辺}$$

(34)

$$1+\operatorname{cosec}^4 A-\cot^4 A=2\operatorname{cosec}^2 A$$

$$\text{左辺}=1+\left(\frac{1}{\sin^4 A}-\frac{\cos^4 A}{\sin^4 A}\right)$$

$$=1+\frac{(1+\cos^2 A)(1-\cos^2 A)}{\sin^4 A}$$

$$=\frac{\sin^4 A+(1+\cos^2 A)\sin^2 A}{\sin^4 A}=\frac{1+1}{\sin^2 A}$$

$$=2\cdot\frac{1}{\sin^2 A}=2\operatorname{cosec}^2 A=\text{右辺}$$

(35)

$$2\sec^2\theta=\frac{1}{1-\sin\theta}+\frac{1}{1+\sin\theta}$$

$$\text{右辺}=\frac{1+\sin\theta+1-\sin\theta}{1-\sin^2\theta}=\frac{2}{\cos^2\theta}$$

$$=2\sec^2\theta=\text{左辺}$$

恒等式(鋭角)

(36)

$$\sin^6 A-\cos^6 A=(2\sin^2 A-1)(1-\sin^2 A+\sin^4 A)$$

$$\text{左辺}=(\sin^3 A+\cos^3 A)(\sin^3 A-\cos^3 A)$$

$$=(\sin A+\cos A)(\sin^2 A-\sin A\cos A+\cos^2 A)$$

$$\times(\sin A-\cos A)(\sin^2 A+\sin A\cos A+\cos^2 A)$$

$$=(1-\sin A\cos A)(1+\sin A\cos A)$$

$$\times(\sin A+\cos A)(\sin A-\cos A)$$

$$=(1-\sin^2 A\cos^2 A)(\sin^2 A-\cos^2 A)$$

$$=\{1-\sin^2 A(1-\sin^2 A)\}\{\sin^2 A-(1-\sin^2 A)\}$$

$$=(1-\sin^2 A+\sin^4 A)(2\sin^2 A-1)=\text{右辺}$$

(37)

$$\operatorname{cosec}^6 A-\cot^6 A=1+3\operatorname{cosec}^2 A\cot^2 A$$

$$\text{左辺}=(1+\cot^2 A)^3-\cot^6 A$$

$$=1+3\cot^2 A+3\cot^4 A+\cancel{\cot^6 A}-\cancel{\cot^6 A}$$

$$=1+3\cot^2 A(1+\cot^2 A)$$

$$=1+3\cot^2 A\operatorname{cosec}^2 A=\text{右辺}$$

(38)

$$(1+\tan A)^2+(1+\cot A)^2=(\sec A+\operatorname{cosec} A)^2$$

$$\text{左辺}=1+2\tan A+\tan^2 A+1+2\cot^2 A+\cot^2 A$$

$$=(1+\tan^2 A)+(1+\cot^2 A)+2(\tan A+\cot A)$$

$$=\sec^2 A+\operatorname{cosec}^2 A+2\left(\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}\right)$$

$$=\sec^2 A+2\left(\frac{1}{\cos A}\cdot\frac{1}{\sin A}\right)+\operatorname{cosec}^2 A$$

$$=(\sec A+\operatorname{cosec} A)^2=\text{右辺}$$

恒等式(鋭角)

(39)

$$\sec^4 B - \sec^2 B = \tan^4 B + \tan^2 B$$

$$左辺 = \sec^2 B(\sec^2 B - 1) = \sec^2 B \tan^2 B$$

$$= \frac{1}{\cos^2 B} \cdot \frac{\sin^2 B}{\cos^2 B} = \frac{\sin^2 B + \cos^2 B}{\cos^2 B} \cdot \frac{\sin^2 B}{\cos^2 B}$$

$$= \frac{\sin^4 B + \cos^2 B \sin^2 B}{\cos^4 B}$$

$$= \frac{\sin^4 B}{\cos^4 B} + \frac{\cos^2 B \sin^2 B}{\cos^4 B}$$

$$= \tan^4 B + \tan^2 B = 右辺$$

(40)

$$(1 - \tan^2 A)\cos^2 A + \tan^2 A = 1$$

$$左辺 = \cos^2 A - \frac{\sin^4 A \cos^2 A}{\cos^2 A} + \frac{\sin^2 A}{\cos^2 A}$$

$$= \cos^2 A - \frac{\sin^2 A}{\cos^2 A}(\sin^2 A - 1)$$

$$= \cos^2 A + \frac{\sin^2 A}{\cos^2 A} \cdot \cos^2 A = 1 = 右辺$$

(41)

$$(\cos^2 A + \cot^2 A)\tan^2 A = \sec^2 A + (\cos^2 A - 1)\tan^2 A$$

$$左辺 = \cos^2 A \tan^2 A + \cot^2 A \tan^2 A$$

$$= \sin^2 A + 1$$

$$右辺 = \frac{1}{\cos^2 A} + \cos^2 A \tan^2 A - \frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{1 - \sin^2 A}{\cos^2 A} + \sin^2 A$$

$$= 1 + \sin^2 A = 左辺$$

恒等式(鋭角)

(42)

$$\cos^4 A - \sin^4 A = (\cos^2 A - \sin^2 A)(1 - 2\sin^2 A \cos^2 A)$$

$$左辺 = (\cos^2 A + \sin^2 A)(\cos^2 A + \sin^2 A)(\cos^2 A - \sin^2 A)$$

$$= \{(\cos^2 A + \sin^2 A)^2 - 2\sin^2 A \cos^2 A\}(\cos^2 A - \sin^2 A)$$

$$= (1 - 2\sin^2 A \cos^2 A)(\cos^2 A - \sin^2 A) = 左辺$$

(43)

$\sin\theta + \sin^2\theta = 1$ ヨリ $\sin\theta = 1 - \sin^2\theta = \cos^2\theta$

$\cos^2\theta + \cos^4\theta = 1$ ／左辺ニ上式ヲ入レテ

$$左辺 = \cos^2\theta + (\sin\theta)^2 = \cos^2\theta + \sin^2\theta = 1$$

(44)

$$\frac{1 + \cos A}{1 - \cos A} = (\operatorname{cosec} A + \cot A)^2$$

$$右辺 = \left\{\frac{1}{\sin A} + \frac{\cos A}{\sin A}\right\}^2 = \left\{\frac{1 + \cos A}{\sin A}\right\}^2$$

$$= \frac{1 + 2\cos A + \cos^2 A}{\sin^2 A}$$

$$= \frac{\sin^2 A + \cos^2 A + 2\cos A + \cos^2 A}{\sin^2 A}$$

$$= \frac{2\cos^2 A + 2\cos A + \sin^2 A}{\sin^2 A}$$

$$= \frac{2\cos A(\cos A + 1) + (1 - \cos^2 A)}{1 - \cos^2 A}$$

$$= \frac{(1 + \cos A)(2\cos A + 1 - \cos A)}{(1 - \cos A)(1 + \cos A)}$$

$$= \frac{\cos A + 1}{1 - \cos A} = 左辺$$

恒等式（鋭角）

(45)

$$(\sin A+\sec A)^2+(\cos A+\operatorname{cosec} A)^2=(1+\sec A\operatorname{cosec} A)^2$$

$$\text{左辺}=\sin^2A+2\cdot\frac{\sin A}{\cos A}+\frac{1}{\cos^2A}$$

$$+\cos^2A+2\cdot\frac{\cos A}{\sin A}+\frac{1}{\sin^2A}$$

$$=\sin^2A+\cos^2A+\frac{1}{\sin^2A}+\frac{1}{\cos^2A}$$

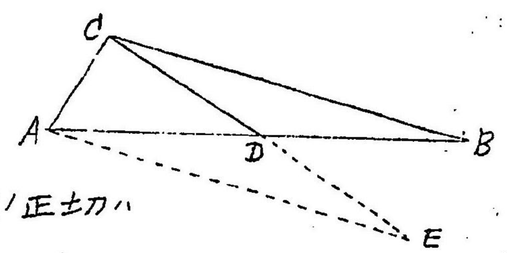
$$+2\left(\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}\right)$$

$$=1+\frac{1}{\sin^2A\cos^2A}+2\cdot\frac{1}{\sin A\cos A}$$

$$=1+2\cdot\frac{1}{\cos A}\cdot\frac{1}{\sin A}+\frac{1}{\cos^2A}\cdot\frac{1}{\sin^2A}$$

$$=(1+\sec^2A\operatorname{cosec}^2A)^2=\text{右辺}$$

(46)



三角形ABCニ於テ
Cヨリ對辺ニ到ル中
線ガACニ垂線ナル
トキハABC角ノ補角ノ正切ハ
Aノ正切ノ二倍ナリ

CDヲABヘノ中線トシ之ヲ延長シテ DE＝CD ナラシメ
AEヲ結付ケレバ △AE ハ $A\hat{C}B$ ノ補角ナルコト明ナリ
而シテ $A\hat{C}D=\hat{R}$ ナルヲ以テ
$\tan CAE=2\cdot\tan CAB$
故ニ $A\hat{C}B$ ノ補角ノ正切ハAノ正切ノ二倍ナリ

恒等式（鋭角）

(47)

$$\sin^2(A+45^\circ)+\sin^2(45^\circ-A)=1$$

$A+45^\circ$ ト $45^\circ-A$ トハ互ニ補角ナリ

$$\text{左辺}=\sin^2(A+45^\circ)+\cos^2(A+45^\circ)=1=\text{右辺}$$

(48)

$$\sec(90^\circ-A)-\cot A\cos(90^\circ-A)\tan(90^\circ-A)=\sin A$$

$$\text{左辺}=\operatorname{cosec}A-\cot A\sin A\cot A$$

$$=\frac{1}{\sin A}-\frac{\cos^2A}{\sin^2A}\cdot\sin A=\frac{1-\cos^2A}{\sin A}$$

$$=\frac{\sin^2A}{\sin A}=\sin A=\text{右辺}$$

(49)

$$\frac{\cot^2A\sin^2(90^\circ-A)}{\cot A+\cos A}=\tan(90^\circ-A)-\cos A$$

$$\text{左辺}=\frac{\cot^2A\cos^2A}{\cot A+(\cot A\sin A)}$$

$$=\frac{\cot^2A(1-\sin^2A)}{\cot A(1+\sin A)}$$

$$=\frac{\cot A(1+\sin A)(1-\sin A)}{1+\sin A}$$

$$=\cot A-\sin A\cot A=\cot A-\cos A$$

$$=\tan(90^\circ-A)-\cos A=\text{右辺}$$

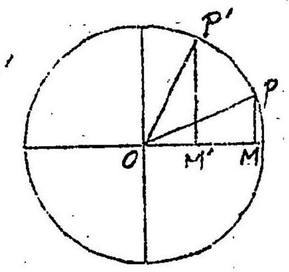
(50)

$\sin 30^\circ = \cos 60^\circ$

$P\hat{O}M = 30^\circ,\ P'\hat{O}M' = 60^\circ$

$\therefore \frac{MP}{OP} = \frac{M'P'}{OP'}$

$\therefore \sin 30^\circ = \cos 60^\circ$



(51)

$\sin 195^\circ = \cos(-105^\circ)$

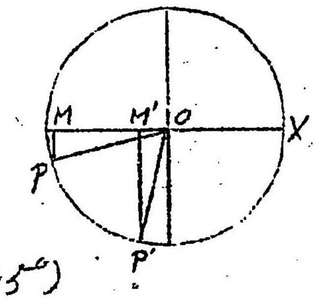
$90^\circ - 195^\circ = -105^\circ$

而シテ $P\hat{O}X = 195^\circ$

$P'\hat{O}X = -105^\circ$

$\therefore \frac{MP}{OP} = \frac{M'O}{OP'}$

$\therefore \sin 195^\circ = \cos(-105^\circ)$



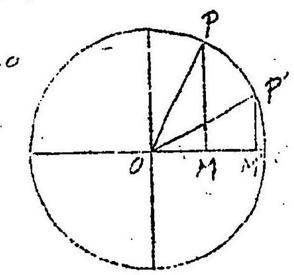
(52)

$\sin 65^\circ = \cos 25^\circ$

$P\hat{O}M = 65^\circ,\ P'\hat{O}M = 25^\circ$

$\therefore \frac{MP}{OP} = \frac{M'O}{OP'}$

$\therefore \sin 65^\circ = \cos 25^\circ$



(53)

$\operatorname{cosec}(90^\circ - A) + \tan(180^\circ + A)\sin(-A) = \sin(90^\circ + A)$

左辺 $= \sec A + \tan A(-\sin A) = \frac{1}{\cos A} - \frac{\sin^2 A}{\cos A}$

$= \frac{1-\sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A$

$= \sin(90^\circ + A) =$ 左辺

(54)

$\cos 275^\circ = \sin(-185^\circ)$

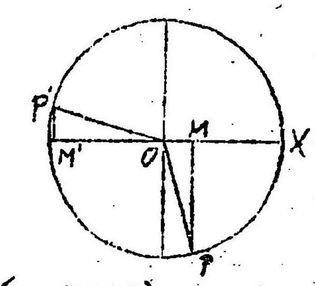
$90^\circ - 275^\circ = -185^\circ$

而シテ $X\hat{O}P = 275^\circ$

$X\hat{O}P' = -185^\circ$

$\therefore \frac{OM}{OP} = \frac{M'P'}{OP'}$

$\therefore \cos 275^\circ = -\sin(-185^\circ)$



(55)

$\cos 310^\circ = \sin 140^\circ$

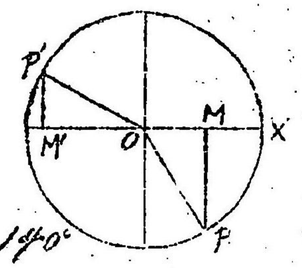
$90^\circ - 310^\circ = -220^\circ$

$360^\circ - 220^\circ = 140^\circ$

而シテ $X\hat{O}P = 310^\circ$

$X\hat{O}P' = -220^\circ = 140^\circ$

$\therefore \frac{OM}{OP} = \frac{M'P'}{OP'}$ $\therefore \cos 310^\circ = \sin 140^\circ$



(56)

$\tan(-27^\circ) = \cot 117^\circ$

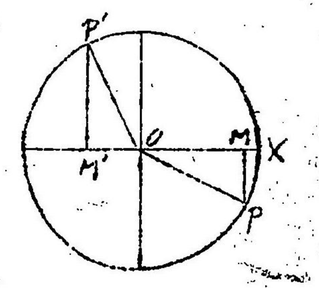
$90^\circ - (-27^\circ) = 117^\circ$

而シテ $X\hat{O}P = -27^\circ$

$X\hat{O}P' = 117^\circ$

$\therefore \frac{PM}{OM} = \frac{OM'}{P'M'}$

$\therefore \tan(-27^\circ) = \cot 117^\circ$



(57)

$\sin\frac{B}{2} = \cos\frac{A+C}{2}$. $A+C$ ハ B ノ補角, $\frac{A+C}{2}$ ハ $\frac{B}{2}$ ノ余角

$\therefore \sin\frac{B}{2} = \cos\frac{A+C}{2}$

(58)
$\cos\frac{A}{2}=\sin\frac{B+C}{2}$

$B+C$ ハ A ノ補角ナルヲ以テ $\frac{B+C}{2}$ ハ $\frac{A}{2}$ ノ余角ナリ

$\therefore\ \cos\frac{A}{2}=\sin\frac{B+C}{2}$

(59)
$\cot\frac{C}{2}=\tan\frac{A+B}{2}$, $\frac{A+B}{2}$ ハ $\frac{C}{2}$ ノ余角ナルヲ以テ

$\therefore\ \cot\frac{C}{2}=\tan\frac{A+B}{2}$

(60)
$\sec\frac{A}{2}=\operatorname{cosec}\frac{B+C}{2}$, $\frac{B+C}{2}$ ハ $\frac{A}{2}$ ノ余角ナリ

$\therefore\ \sec\frac{A}{2}=\operatorname{cosec}\frac{B+C}{2}$

(61)
$\sin 60^\circ=\sin 120^\circ$

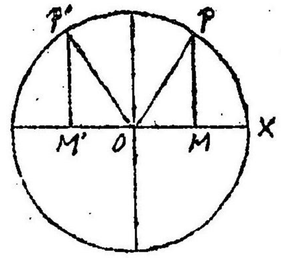
$180^\circ-60^\circ=120^\circ$

而シテ $P\hat{O}X=60^\circ$

$P'\hat{O}X=120^\circ$

$\therefore\ \frac{PM}{OP}=\frac{P'M'}{OP'}$

$\therefore\ \sin 60^\circ=\sin 120^\circ$



(62)
$\sin A=\sin(B+C)$, $B+C$ ハ A ノ補角ナリ

$\therefore\ \sin A=\sin(B+C)$

(63)
$\tan B=-\tan(C+A)$, $C+A$ ハ B ノ補角ナリ

$\therefore\ \tan B=-\tan(C+A)$ ..... 終

(64)
$\sec 320^\circ=-\sec(-140^\circ)$

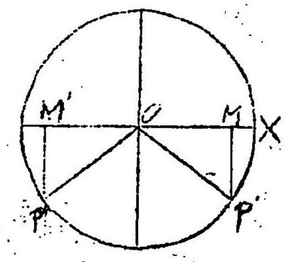
$180^\circ-320^\circ=-140^\circ$

而シテ $P\hat{O}X=320^\circ$

$P'\hat{O}X=-140^\circ$

$\therefore\ \frac{MP}{OM}=-\frac{M'P'}{OM'}$

$\therefore\ \sec 320^\circ=-\sec(-140^\circ)$



(65)
$\cot 190^\circ=-\cot 350^\circ$

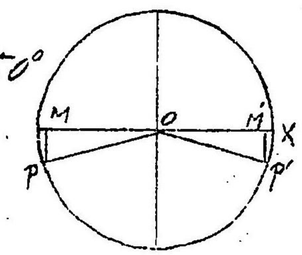
$180^\circ-190^\circ=-10^\circ=350^\circ$

而シテ $P\hat{O}X=190^\circ$

$P'\hat{O}X=350^\circ$

$\therefore\ \frac{MO}{PM}=-\frac{M'O}{P'M'}$

$\therefore\ \cot 190^\circ=-\cot 350^\circ$



(66)
$\tan(-40^\circ)=-\tan 220^\circ$

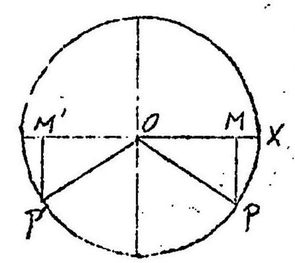
$180^\circ-(-40^\circ)=220^\circ$

而シテ $P\hat{O}X=-40^\circ$

$P'\hat{O}X=220^\circ$

$\therefore\ \frac{MP}{OM}=-\frac{M'P'}{OM'}$

$\therefore\ \tan(-40^\circ)=-\tan 220^\circ$



(67)
$\cos C=-\cos(A+B)$, $A+B$ ハ C ノ補角ナリ

$\therefore\ \cos C=-\cos(A+B)$

等式(一般角)

(68)

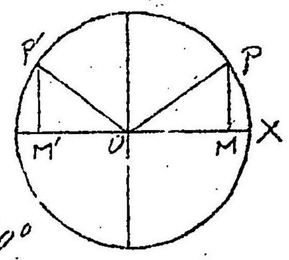
$\sin(-340^\circ) = \sin 160^\circ$

$180^\circ - (-340^\circ) = 520^\circ$

$520^\circ - 360^\circ = 160^\circ$

而シテ $P\hat{O}X = -340^\circ$

$P'\hat{O}X = 520^\circ$ 即 160°



$\therefore \dfrac{PM}{OP} = \dfrac{P'M'}{OP'}$

$\therefore \sin(-340^\circ) = \sin 160^\circ$

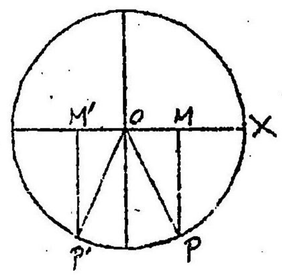
(69)

$\cos 290^\circ = -\cos(-110^\circ)$

$180^\circ - 290^\circ = -110^\circ$

而シテ $X\hat{O}P = 290^\circ$

$X\hat{O}P' = -110^\circ$



$\therefore \dfrac{OM}{OP} = -\dfrac{OM'}{OP'}$

$\therefore \cos 290^\circ = -\cos(-110^\circ)$

(70)

$\operatorname{cosec} A = \operatorname{cosec}(B+C)$, $B+C$ ハ A ノ補角ナリ

$\therefore \operatorname{cosec} A = \operatorname{cosec}(B+C)$ 第7

(71)

$\cos(270^\circ + A) = -\sin A$

左辺 $= \cos\{90^\circ + (180^\circ + A)\} = -\sin(180^\circ + A)$ ... 第8

$= \sin A$ 第6

(72)

$\sin(270^\circ - A) = -\cos A$

左辺 $= \sin\{90^\circ + (180^\circ - A)\} = \cos(180^\circ - A)$ ... 第8

$= \cos A$ 第7

等式(和,差角)

(73)

$\sin(A - 60^\circ) = \frac{1}{2}(\sin A - \sqrt{3}\cos A)$

左辺 $= \sin A \cos 60^\circ - \cos A \sin 60^\circ$

$= \frac{1}{2}\sin A - \frac{\sqrt{3}}{2}\cos A = \frac{1}{2}(\sin A - \sqrt{3}\cos A) =$ 右辺

(74)

$\cos\alpha = \dfrac{a}{\sqrt{a^2+b^2}}$ ナル故ニ: $\sin\alpha = \dfrac{b}{\sqrt{a^2+b^2}}$

$a\cos A \pm b\sin A = \sqrt{a^2+b^2}\cos(A \mp \alpha)$

右辺 $= \sqrt{a^2+b^2}(\cos A\cos\alpha \pm \sin A\sin\alpha)$

$= \sqrt{a^2+b^2}\left(\cos A\dfrac{a}{\sqrt{a^2+b^2}} \pm \sin A\dfrac{b}{\sqrt{a^2+b^2}}\right)$

$= \sqrt{a^2+b^2}\,\dfrac{a\cos A \pm b\sin A}{\sqrt{a^2+b^2}} =$ 左辺

(75)

$\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

左辺 $= (\sin A\cos B + \cos A\sin B)(\sin A\cos B - \cos A\sin B)$

$= \sin^2 A\cos^2 B - \cos^2 A\sin^2 B$

$= (1-\cos^2 A)\cos^2 B - (1-\cos^2 B)\cos^2 A$

$= \cos^2 B - \cancel{\cos^2 A\cos^2 B} - \cos^2 A + \cancel{\cos^2 A\cos^2 B}$

$= \cos^2 B - \cos^2 A =$ 右辺

$= 1 - \sin^2 B - 1 + \sin^2 A$

$= \sin^2 A - \sin^2 B =$ 中辺

(76)

$\sin A + \sin(120^\circ + A) - \sin(120^\circ - A) = 0$

左辺 $= \sin A + 2\cos 120^\circ \sin A$ 公式

$= \sin A + 2\left(-\frac{1}{2}\right)\sin A = 0 =$ 右辺

等式(和,差角)

(77)

$$\sin\alpha+\sin\beta+\sin\gamma-\sin(\alpha+\beta+\gamma)=4\sin\frac{\beta+\gamma}{2}\sin\frac{\gamma+\alpha}{2}\sin\frac{\alpha+\beta}{2}$$

$$\text{左辺}=(\sin\alpha+\sin\beta)-\{\sin(\alpha+\beta+\gamma)-\sin\gamma\}$$

$$=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}-2\cos\frac{\alpha+\beta+2\gamma}{2}\sin\frac{\alpha+\beta}{2}$$

$$=2\sin\frac{\alpha+\beta}{2}\left(\cos\frac{\alpha-\beta}{2}-\cos\frac{\alpha+\beta+2\gamma}{2}\right)$$

$$=2\sin\frac{\alpha+\beta}{2}\left\{-2\sin\frac{\alpha+\gamma}{2}\left(-\sin\frac{\beta+\gamma}{2}\right)\right\}$$

$$=4\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha+\gamma}{2}\sin\frac{\beta+\gamma}{2}=\text{右辺}$$

(78)

$$\cot\alpha\pm\tan\beta=\frac{\cos(\alpha\mp\beta)}{\sin\alpha\cos\beta}$$

$$\text{左辺}=\frac{\cos\alpha}{\sin\alpha}\pm\frac{\sin\beta}{\cos\beta}=\frac{\cos\alpha\cos\beta\pm\sin\beta\sin\alpha}{\sin\alpha\cos\beta}$$

$$=\frac{\cos(\alpha\mp\beta)}{\sin\alpha\cos\beta}=\text{右辺}$$

(79)

$$\tan\alpha\pm\tan\beta=\frac{\sin(\alpha\pm\beta)}{\cos\alpha\cos\beta}$$

$$\text{左辺}=\frac{\sin\alpha}{\cos\alpha}\pm\frac{\sin\beta}{\cos\beta}=\frac{\sin\alpha\cos\beta\pm\cos\alpha\sin\beta}{\cos\alpha\cos\beta}$$

$$=\frac{\sin(\alpha\pm\beta)}{\cos\alpha\cos\beta}=\text{右辺}$$

(80)

$$\sin2A\cos A+\cos4A\sin A=\sin3A\cos2A$$

$$\text{左辺}=\frac{1}{2}(2\cos A\sin2A+2\cos4A\sin A)$$

$$=\frac{1}{2}(\cancel{\sin3A}+\sin A+\sin5A-\cancel{\sin3A})$$

$$=\frac{1}{2}(\sin5A+\sin A)$$

$$=\frac{1}{2}\times2\sin3A\cos2A=\text{右辺}$$

等式(和,差角)

(81)

$$\sin^2\alpha-\sin^2\beta=\frac{\sin(\alpha+\beta)\sin(\alpha-\beta)}{\cos^2\alpha\cos^2\beta}$$

$$\text{左辺}=\frac{\sin^2\alpha}{\cos^2\alpha}-\frac{\sin^2\beta}{\cos^2\beta}=\frac{\sin^2\alpha\cos^2\beta-\sin^2\beta\cos^2\alpha}{\cos^2\alpha\cos^2\beta}$$

$$=\frac{(\sin\alpha\cos\beta+\sin\beta\cos\alpha)(\sin\alpha\cos\beta-\sin\beta\cos\alpha)}{\cos^2\alpha\cos^2\beta}$$

$$=\frac{\sin(\alpha+\beta)\sin(\alpha-\beta)}{\cos^2\alpha\cos^2\beta}=\text{右辺}$$

(82)

$$\cos\alpha+\cos3\alpha+\cos5\alpha+\cos7\alpha=4\cos\alpha\cos2\alpha\cos4\alpha$$

$$\text{左辺}=(\cos3\alpha+\cos\alpha)+(\cos7\alpha+\cos5\alpha)$$

$$=2\cos2\alpha\cos\alpha+2\cos6\alpha\cos\alpha$$

$$=2\cos\alpha(\cos6\alpha+\cos2\alpha)$$

$$=2\cos\alpha(2\cos4\alpha\cos2\alpha)=\text{右辺}$$

(83)

$$\cos A+\cos(120^\circ+A)+\cos(120^\circ-A)=0$$

$$\text{左辺}=\cos A+2\cos120^\circ\cos A$$

$$=\cos A+2\left(-\frac{1}{2}\right)\cos A=\cos A-\cos A$$

$$=0=\text{右辺}$$

(84)

$$\sin60^\circ+\sin30^\circ=2\sin45^\circ\cos15^\circ$$

$$\text{左辺}=2\sin\frac{60^\circ+30^\circ}{2}\cos\frac{60^\circ-30^\circ}{2}=2\sin45^\circ\cos15^\circ$$

$$=\text{右辺}$$

(85)

$$\tan(45^\circ\pm A)=\frac{1\pm\tan A}{1\mp\tan A}$$

$$\text{左辺}=\frac{\tan45^\circ\pm\tan A}{1\mp\tan45^\circ\tan A}=\frac{1\pm\tan A}{1\mp\tan A}=\text{右辺}$$

(86)

$$\cot(A-B)=\frac{\cot A\cot B+1}{\cot B-\cot A}$$

$$左辺=\frac{1+\tan A\tan B}{\tan A-\tan B}=\frac{1+\frac{1}{\cot A}\cdot\frac{1}{\cot B}}{\frac{1}{\cot A}-\frac{1}{\cot B}}$$

$$=\frac{\frac{\cot A\cot B+1}{\cot A\cot B}}{\frac{\cot B-\cot A}{\cot A\cot B}}=\frac{\cot A\cot B+1}{\cot B-\cot A}=右辺$$

(87)

$$\cos A+\sin A=\sqrt{2}\sin(45^\circ+A)$$

$$右辺=\sqrt{2}(\sin 45^\circ\cos A+\sin A\cos 45^\circ)$$

$$=\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos A+\frac{1}{\sqrt{2}}\sin A\right)$$

$$=\cos A+\sin A=左辺$$

(88)

$$\tan(A-45^\circ)+\cot(A+45^\circ)=0$$

$$左辺=\frac{\tan A-\tan 45^\circ}{1+\tan A\tan 45^\circ}+\frac{1-\tan A\tan 45^\circ}{\tan A+\tan 45^\circ}$$

$$=\frac{\tan A-1}{1+\tan A}+\frac{1-\tan A}{\tan A+1}$$

$$=\frac{\tan A-1+1-\tan A}{\tan A+1}=0=右辺$$

(89)

$$\sin(\alpha+\beta)+\cos(\alpha-\beta)=(\sin\alpha+\cos\alpha)(\sin\beta+\cos\beta)$$

$$左辺=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$+\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$=\cos\beta(\sin\alpha+\cos\alpha)+\sin\beta(\cos\alpha+\sin\alpha)$$

$$=(\sin\alpha+\cos\alpha)(\sin\beta+\cos\beta)=右辺$$

(90)

$$\cos 47^\circ-\cos 61^\circ-\cos 11^\circ+\cos 25^\circ=\sin 7^\circ$$

$$左辺=(\cos 25^\circ-\cos 11^\circ)-(\cos 61^\circ-\cos 47^\circ)$$

$$=-2\sin 18^\circ\sin 7^\circ+2\sin 54^\circ\sin 7^\circ$$

$$=2\sin 7^\circ(\sin 54^\circ-\sin 18^\circ)$$

$$=2\sin 7^\circ(2\cos 36^\circ\sin 18^\circ)$$

$$=\cancel{2}\sin 7^\circ\left(\cancel{2}\times\frac{\sqrt{5}+1}{\cancel{4}}\times\frac{\sqrt{5}-1}{\cancel{4}}\right)=\sin 7^\circ=右辺$$

(91)

$$\cos(A+45^\circ)+\sin(A-45^\circ)=0$$

$$左辺=\cos A\cos 45^\circ-\sin A\sin 45^\circ$$
$$+\sin A\cos 45^\circ-\cos A\sin 45^\circ$$

$$=\frac{1}{\sqrt{2}}\cos A-\cancel{\frac{1}{\sqrt{2}}\sin A}+\cancel{\frac{1}{\sqrt{2}}\sin A}-\cancel{\frac{1}{\sqrt{2}}\cos A}$$

$$=0=右辺$$

(92)

$$4\sin A\sin(60^\circ+A)\sin(60^\circ-A)=\sin 3A$$

$$左辺=4\sin A\{\sin(60^\circ+A)\sin(60^\circ-A)\}$$

$$=4\sin A\{\cos^2 A-\cos^2 60^\circ\}$$ 公式22

$$=4\sin A(1-\sin^2 A)-\cancel{4}\sin A\times\frac{1}{\cancel{4}}$$

$$=4\sin A-4\sin^3 A-\sin A=3\sin A-4\sin^3 A$$

$$=\sin 3A=右辺$$

(93)

$$\sin(A+45^\circ)=\frac{1}{\sqrt{2}}(\sin A+\cos A)$$

$$左辺=\sin A\cos 45^\circ+\cos A\sin 45^\circ$$

$$=\frac{1}{\sqrt{2}}\sin A+\frac{1}{\sqrt{2}}\cos A=\frac{1}{\sqrt{2}}(\sin A+\cos A)$$
$$=右辺$$

(94)

$$\sin^6\theta+\cos^6\theta=1-\frac{3}{4}\sin^2 2\theta$$

$$\text{左辺}=(\sin^2\theta+\cos^2\theta)(\sin^4\theta-\sin^2\theta\cos^2\theta+\cos^4\theta)$$

$$=\sin^4\theta+2\sin^2\theta\cos^2\theta-2\sin^2\theta\cos^2\theta$$

$$-\sin^2\theta\cos^2\theta+\cos^4\theta$$

$$=(\sin^4\theta+2\sin^2\theta\cos^2\theta+\cos^4\theta)$$

$$-3\sin^2\theta\cos^2\theta$$

$$=(\sin^2\theta+\cos^2\theta)^2-3(\sin\theta\cos\theta)^2$$

$$=1-\frac{3(2\sin\theta\cos\theta)^2}{4}=1-\frac{3}{4}\sin^2 2\theta=\text{右辺}$$

(95)

$$\cot(A+B)=\frac{\cot A\cot B-1}{\cot A+\cot B}$$

$$\text{右辺}=\frac{1}{\tan(A+B)}=\frac{1-\tan A\tan B}{\tan A+\tan B}$$

$$=\frac{1-\frac{1}{\cot A\cot B}}{\frac{1}{\cot A}+\frac{1}{\cot B}}=\frac{\frac{\cot A\cot B-1}{\cot A\cot B}}{\frac{\cot B+\cot A}{\cot A\cot B}}$$

$$=\frac{\cot A\cot B-1}{\cot B+\cot A}=\text{左辺}$$

(96)

$$\cos\alpha\sin(\beta-\gamma)+\cos\beta\sin(\gamma-\alpha)+\cos\gamma\sin(\alpha-\beta)=0$$

$$\text{左辺}=\frac{1}{2}\{2\cos\alpha\sin(\beta-\gamma)+2\cos\beta\sin(\gamma-\alpha)$$

$$+2\cos\gamma\sin(\alpha-\beta)\}$$

$$=\frac{1}{2}\{\sin(\alpha+\beta-\gamma)-\sin(\alpha-\beta+\gamma)$$

$$+\sin(\beta+\gamma-\alpha)-\sin(\beta-\gamma+\alpha)$$

$$+\sin(\gamma+\alpha-\beta)-\sin(\gamma-\alpha+\beta)\}$$

$$=0=\text{右辺}$$

(97)

$$\frac{\sin 3\theta+\sin 2\theta}{\cos 3\theta-\cos 2\theta}=\cot\frac{\theta}{2}$$

$$\text{左辺}=\frac{2\sin\frac{5}{2}\theta\cos\frac{1}{2}\theta}{2\sin\frac{5}{2}\theta\sin(-\frac{1}{2})\theta}=\frac{\cos\frac{\theta}{2}}{-\sin\frac{\theta}{2}}$$

$$=-\cot\frac{\theta}{2}=\text{右辺}$$

(98)

$$\cot B\pm\cot A=\frac{\sin(A\pm B)}{\sin A\sin B}$$

$$\text{左辺}=\frac{\cos B}{\sin B}\pm\frac{\cos A}{\sin A}=\frac{\sin A\cos B\pm\sin B\cos A}{\sin A\sin B}$$

$$=\frac{\sin(A\pm B)}{\sin A\sin B}=\text{右辺}$$

(99)

$$\frac{\sin A-\sin B}{\cos B-\cos A}=\cot\frac{A+B}{2}$$

$$\text{左辺}=\frac{\cancel{2}\cos\frac{A+B}{2}\cancel{\sin\frac{A-B}{2}}}{\cancel{2}\sin\frac{B+A}{2}\cancel{\sin\frac{B-A}{2}}}=\frac{\cos\frac{A+B}{2}}{\sin\frac{B+A}{2}}$$

$$=\cot\frac{A+B}{2}=\text{右辺}$$

(100)

$$\cos 20^\circ\cos 40^\circ\cos 80^\circ=\frac{1}{8}$$

$$\text{左辺}=\cos 20^\circ\cdot\frac{1}{2}(2\cos 80^\circ\cos 40^\circ)$$

$$=\frac{1}{2}\cos 20^\circ(\cos 120^\circ+\cos 40^\circ)$$

$$=\frac{1}{2}\cos 20^\circ(\cos 40^\circ-\frac{1}{2})$$

$$=\frac{\cos 20^\circ\cos 40^\circ}{2}-\frac{\cos 20^\circ}{4}$$

$$=\frac{\cos 60^\circ+\cancel{\cos 20^\circ}}{4}-\frac{\cancel{\cos 20^\circ}}{4}=\frac{\frac{1}{2}}{4}=\frac{1}{8}=\text{左辺}$$

(101)

$$\frac{1+\sin\theta}{1-\sin\theta}=\left(\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}\right)^2$$

$$\text{右辺}=\left\{\frac{1+\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}}{1-\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}}\right\}^2=\frac{\left(\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cancel{\cos\frac{\theta}{2}}}\right)^2}{\left(\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cancel{\cos\frac{\theta}{2}}}\right)^2}$$

$$=\frac{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$=\frac{1+\sin\theta}{1-\sin\theta}=\text{左辺}$$

(102)

$$\cos\theta\cos(120^\circ+\theta)+\cos\theta\cos(120^\circ-\theta)+\cos(120^\circ+\theta)\cos(120^\circ-\theta)=-\frac{3}{4}$$

$$\text{左辺}=\frac{1}{2}\{2\cos(120^\circ+\theta)\cos\theta+2\cos\theta\cos(120^\circ-\theta)+2\cos(120^\circ+\theta)\cos(120^\circ-\theta)\}$$

$$=\frac{1}{2}\{\cos(120^\circ+2\theta)+\cos 120^\circ+\cos 120^\circ+\cos(120^\circ-2\theta)+\cos 240^\circ+\cos 2\theta\}$$

$$=\frac{1}{2}\{\cos 120^\circ\cos 2\theta-\sin 120^\circ\sin 2\theta+2\cos 120^\circ+\cos 120^\circ\cos 2\theta+\sin 120^\circ\sin 2\theta+\cos 240^\circ+\cos 2\theta\}$$

$$=\frac{1}{2}\{-\frac{1}{2}\cos 2\theta-\frac{\sqrt{3}}{2}\sin 2\theta-2\times\frac{1}{2}-\frac{1}{2}\cos 2\theta+\frac{\sqrt{3}}{2}\sin 2\theta-\frac{1}{2}+\cos 2\theta\}$$

$$=\frac{1}{2}\{2(-\frac{1}{2}\cos 2\theta)+\cos 2\theta-1-\frac{1}{2}\}$$

$$=\frac{1}{2}\times(-\frac{3}{2})=-\frac{3}{4}=\text{左辺}$$

(103)

$$\frac{\cos x}{1-\sin x}=\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}$$

$$\text{右辺}=\frac{1+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}=\frac{\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cancel{\cos\frac{x}{2}}}}{\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cancel{\cos\frac{x}{2}}}}$$

$$=\frac{(\cos\frac{x}{2}+\sin\frac{x}{2})(\cos\frac{x}{2}-\sin\frac{x}{2})}{(\cos\frac{x}{2}-\sin\frac{x}{2})^2}$$

$$=\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\cos\frac{x}{2}\sin\frac{x}{2}}$$

$$=\frac{\cos x}{1-\sin x}\ (\text{公式}23,24)=\text{左辺}$$

(104)

$$\cos\alpha+\cos\beta+\cos\gamma+\cos(\alpha+\beta+\gamma)=4\cos\frac{\beta+\gamma}{2}\cos\frac{\gamma+\alpha}{2}\cos\frac{\alpha+\beta}{2}$$

$$\text{左辺}=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}+2\cos\frac{\alpha+\beta+\gamma}{2}\cos\frac{\alpha+\beta}{2}$$

$$=2\cos\frac{\alpha+\beta}{2}\left(\cos\frac{\alpha-\beta}{2}+\cos\frac{\alpha+\beta+2\gamma}{2}\right)$$

$$=4\cos\frac{\beta+\gamma}{2}\cos\frac{\gamma+\alpha}{2}\cos\frac{\alpha+\beta}{2}=\text{右辺}$$

(105)

$$\cos(A-30^\circ)=\frac{1}{2}(\sqrt{3}\cos A+\sin A)$$

$$\text{左辺}=\cos A\cos 30^\circ+\sin A\sin 30^\circ$$

$$=\frac{\sqrt{3}}{2}\cos A+\frac{1}{2}\sin A$$

$$=\frac{1}{2}(\sqrt{3}\cos A+\sin A)=\text{右辺}$$

(106)

$$\tan(m+n)A-\tan mA-\tan nA$$
$$=\tan(m+n)A\tan mA\tan nA$$

$$\text{左辺}=\frac{\tan mA+\tan nA}{1-\tan mA\tan nA}-\tan mA-\tan nA$$
$$=\frac{\tan mA+\tan nA-\tan mA-\tan nA+(\tan mA\tan nA)\tan mA\tan nA}{1-\tan mA\tan nA}$$
$$=\tan(m+n)A\tan mA\tan nA$$
$$=\text{右辺}$$

(107)

$$\sec(45^\circ+\alpha)\sec(45^\circ-\alpha)=2\sec 2\alpha$$

$$\text{左辺}=\frac{1}{\cos(45^\circ+\alpha)}\cdot\frac{1}{\cos(45^\circ-\alpha)}$$
$$=\frac{2}{2\cos(45^\circ+\alpha)\cos(45^\circ-\alpha)}$$
$$=\frac{2}{\cos 90^\circ+\cos 2\alpha}=2\cdot\frac{1}{\cos 2\alpha}$$
$$=2\sec 2\alpha=\text{右辺}$$

(108)

$$\sin 40^\circ-\sin 10^\circ=2\cos 25^\circ\sin 15^\circ$$
$$\text{左辺}=2\cos\frac{40^\circ+10^\circ}{2}\sin\frac{40^\circ-10^\circ}{2}$$
$$=2\cos 25^\circ\sin 15^\circ=\text{右辺}$$

(109)

$$\sin(60^\circ+A)-\sin(60^\circ-A)=\sin A$$
$$\text{左辺}=2\cos 60^\circ\sin A=2\times\frac{1}{2}\sin A$$
$$=\sin A=\text{右辺}$$

(110)

$$\cot A-\tan A=2\cot 2A$$

$$\text{左辺}=\frac{\cos A}{\sin A}-\frac{\sin A}{\cos A}=\frac{\cos^2 A-\sin^2 A}{\sin A\cos A}$$
$$=\frac{2(\cos^2 A-\sin^2 A)}{2\sin A\cos A}=2\cdot\frac{\cos 2A}{\sin 2A}$$
$$=2\cot 2A=\text{右辺}$$

(111)

$$\frac{\cos 2A+\cos 12A}{\cos 6A+\cos 8A}+\frac{\cos 7A-\cos 3A}{\cos A-\cos 3A}+2\cdot\frac{\sin 4A}{\sin 2A}=0$$

$$\text{左辺}=\frac{2\cos\frac{12A+2A}{2}\cos\frac{12A-2A}{2}}{2\cos\frac{8A+6A}{2}\cos\frac{8A-6A}{2}}$$
$$+\frac{2\sin\frac{3A+7A}{2}\sin\frac{3A-7A}{2}}{2\sin\frac{3A+A}{2}\sin\frac{3A-A}{2}}+2\frac{\sin 4A}{\sin 2A}$$
$$=\frac{\cos 7A\cos 5A}{\cos 7A\cos A}+\frac{-\sin 5A\sin 2A}{\sin 2A\sin A}+2\frac{\sin 4A}{\sin 2A}$$
$$=\frac{\cos 5A}{\cos A}-\frac{\sin 5A}{\sin A}+2\frac{\sin 4A}{\sin 2A}$$
$$=\frac{\sin A\cos 5A-\cos A\sin 5A}{\sin A\cos A}+2\frac{\sin 4A}{\sin 2A}$$
$$=\frac{\sin(A-5A)}{\frac{\sin 2A}{2}}+2\frac{\sin 4A}{\sin 2A}=0=\text{右辺}$$

(112)

$$\cos(45^\circ+A)+\cos(45^\circ-A)=\sqrt{2}\cos A$$
$$\text{左辺}=2\cos 45^\circ\cos A=2\times\frac{1}{\sqrt{2}}\cos A=\sqrt{2}\cos A$$
$$=\text{左辺}$$

等式(和,差角)

(113)

$$\operatorname{cosec}A-\cot A=\tan\frac{A}{2}$$

$$\text{左辺}=\frac{1}{\sin A}-\frac{\cos A}{\sin A}=\frac{1-\cos A}{\sin A}$$

$$=\frac{\cancel{2}-(\cancel{2}-2\sin^2\frac{A}{2})}{2\sin\frac{A}{2}\cos\frac{A}{2}}=\frac{\cancel{2}\sin^2\frac{A}{2}}{\cancel{2}\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$=\frac{\sin\frac{A}{2}}{\cos\frac{A}{2}}=\tan\frac{A}{2}=\text{右辺}$$

(114)

$$\sin(B-C)+\sin(C-A)+\sin(A-B)$$
$$+4\sin\frac{B-C}{2}\sin\frac{C-A}{2}\sin\frac{A-B}{2}=0$$

$$\sin(B-C)+\sin(C-A)+\sin(A-B)$$

$$=2\sin\frac{B-A}{2}\cos\frac{A+B-2C}{2}+2\sin\frac{A-B}{2}\cos\frac{A-B}{2}$$

$$=2\sin\frac{A-B}{2}\left\{\cos\frac{A-B}{2}-\cos\frac{A+B-2C}{2}\right\}$$

$$=2\sin\frac{A-B}{2}\left\{2\sin\frac{A+B-2C+A-B}{4}\sin\frac{A+B-2C-A+B}{4}\right\}$$

$$=2\sin\frac{A-B}{2}\left\{2\sin\frac{A-C}{2}\sin\frac{B-C}{2}\right\}$$

$$=4\sin\frac{A-B}{2}\left(-\sin\frac{C-A}{2}\right)\sin\frac{B-C}{2}$$

$$=-4\sin\frac{B-C}{2}\sin\frac{C-A}{2}\sin\frac{A-B}{2}$$

$\therefore$ 左辺 $=0=$ 右辺

(115)

$$\operatorname{cosec}A+\cot A=\cot\frac{A}{2}$$

$$\text{左辺}=\frac{1}{\sin A}+\frac{\cos A}{\sin A}=\frac{1+\cos A}{\sin A}=\cot\frac{A}{2}$$

公式36

$=$ 右辺

等式(和,差角)

(116)

$$\left(\cos\frac{A}{2}\pm\sin\frac{A}{2}\right)^2=1\pm\sin A$$

$$\text{左辺}=\cos^2\frac{A}{2}\pm2\cos\frac{A}{2}\sin\frac{A}{2}+\sin^2\frac{A}{2}$$

$$=1\pm2\cos\frac{A}{2}\sin\frac{A}{2}=1\pm\sin A=\text{右辺}$$

(117)

$$\frac{1-\cos\theta}{1+\cos\theta}=\tan^2\frac{\theta}{2}$$

$$\text{左辺}=\frac{\cancel{2}\sin^2\frac{\theta}{2}}{\cancel{2}\cos^2\frac{\theta}{2}}\ (\text{範式}24)=\tan^2\frac{\theta}{2}=\text{右辺}$$

(118)

$$1+\tan(A+B)\tan(A-B)=\frac{1-2\sin^2A}{\cos^2A-\sin^2A}$$

$$\text{左辺}=1+\frac{\sin(A+B)}{\cos(A+B)}\cdot\frac{\sin(A-B)}{\cos(A-B)}$$

$$=1+\frac{\sin^2A\cos^2B-\cos^2A\sin^2B}{\cos^2A\cos^2B-\sin^2A\sin^2B}$$

$$=1+\frac{\sin^2A-\sin^2B}{\cos^2B-\sin^2A}\ (\text{公式}22,23)$$

$$=\frac{\cos^2B-\cancel{\sin^2A}+\cancel{\sin^2A}-\sin^2B}{\cos^2A-\sin^2B}\ (\text{公式}23)$$

$$=\frac{1-2\sin^2B}{\cos^2A-\sin^2B}=\text{右辺}$$

(119)

$$\cos^2A(1-\tan^2A)=\cos2A$$

$$\text{左辺}=\cos^2\left(1-\frac{\sin^2A}{\cos^2A}\right)=\cos^2A-\sin^2A$$

$$=\cos2A\ (\text{範式}21)=\text{右辺}$$

等式（和，差角）

(120)

$$\frac{1\pm\sin A}{1\mp\sin A}=\tan^2\left(45^\circ\pm\frac{A}{2}\right)$$

$$\text{左辺}=\frac{\left(\sin^2\frac{A}{2}+\cos^2\frac{A}{2}\right)\pm 2\sin\frac{A}{2}\cos\frac{A}{2}}{\left(\sin^2\frac{A}{2}+\cos^2\frac{A}{2}\right)\mp 2\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$=\frac{\left(\sin\frac{A}{2}\pm\cos\frac{A}{2}\right)^2}{\left(\sin\frac{A}{2}\mp\cos\frac{A}{2}\right)^2}$$

$$\text{右辺}=\left\{\frac{\tan 45^\circ\pm\tan\frac{A}{2}}{1\mp\tan 45^\circ\tan\frac{A}{2}}\right\}^2=\left\{\frac{1\pm\tan\frac{A}{2}}{1\mp\tan\frac{A}{2}}\right\}^2$$

$$=\left\{\frac{\dfrac{\cos\frac{A}{2}\pm\sin\frac{A}{2}}{\cancel{\cos\frac{A}{2}}}}{\dfrac{\cos\frac{A}{2}\mp\sin\frac{A}{2}}{\cancel{\cos\frac{A}{2}}}}\right\}^2=\text{左辺}$$

(121)

$$\cos(A+B+C)=\cos A\cos B\cos C-\cos A\times\sin B\sin C-\cos B\sin C\sin A-\cos C\times\sin A\sin B$$

$$\text{左辺}=\cos\{A+(B+C)\}$$

$$=\cos A\cos(B+C)-\sin A\sin(B+C)$$

$$=\cos A\cos B\cos C-\cos A\sin B\sin C-\sin A\sin B\cos C-\sin A\cos B\sin C=\text{右辺}$$

(122)

$$\cos 65^\circ+\cos 55^\circ+\cos 175^\circ=0$$

$$\text{左辺}=(\cos 65^\circ+\cos 55^\circ)+\cos 175^\circ$$

$$=2\cos 60^\circ\cos 5^\circ-\cos(180^\circ-175^\circ)$$

$$=2\times\tfrac{1}{2}\cos 5^\circ-\cos 5^\circ=0=\text{右辺}$$

等式（和，差角）

(123)

$$\tan\alpha+\tan\beta+\tan\gamma=\tan\alpha\tan\beta\tan\gamma+\frac{\sin(\alpha+\beta+\gamma)}{\cos\alpha\cos\beta\cos\gamma}$$

$$\text{右辺}=\frac{\sin\alpha}{\cos\alpha}\cdot\frac{\sin\beta}{\cos\beta}\cdot\frac{\sin\gamma}{\cos\gamma}+\frac{\sin\alpha\cos\beta\cos\gamma}{\cos\alpha\cos\beta\cos\gamma}+\frac{\cos\alpha\sin\beta\cos\gamma}{\cos\alpha\cos\beta\cos\gamma}+\frac{\cos\alpha\cos\beta\sin\gamma}{\cos\alpha\cos\beta\cos\gamma}-\frac{\sin\alpha\sin\beta\sin\gamma}{\cos\alpha\cos\beta\cos\gamma}$$

$$=\frac{\sin\alpha}{\cos\alpha}+\frac{\sin\beta}{\cos\beta}+\frac{\sin\gamma}{\cos\gamma}$$

$$=\tan\alpha+\tan\beta+\tan\gamma=\text{左辺}$$

(124)

$$\frac{1}{\tan 3A-\tan A}+\frac{1}{\cot A-\cot 3A}=\cot 2A$$

$$\text{左辺}=\frac{1}{\frac{\sin 3A}{\cos 3A}-\frac{\sin A}{\cos A}}+\frac{1}{\frac{\cos A}{\sin A}-\frac{\cos 3A}{\sin 3A}}$$

$$=\frac{\cos 3A\cos A}{\sin(3A-A)}+\frac{\sin 3A\sin A}{\sin(3A-A)}$$

$$=\frac{\cos(3A-A)}{\sin 2A}=\frac{\cos 2A}{\sin 2A}=\cot 2A=\text{右辺}$$

(125)

$$\cos 3A-\cos 5A=2\sin 4A\sin A$$

$$\text{左辺}=2\sin\frac{5A+3A}{2}\sin\frac{5A-3A}{2}$$

$$=2\sin 4A\sin A=\text{右辺}$$

(126)

$17A=180^\circ$ ナルヲシテ $17A-3A=14A$ ハ3Aノ補角, $17A-5A=12A$ ハ5Aノ補角ナリ

$$\frac{\cos A\cos 13A}{\cos 3A+\cos 5A}=-\frac{1}{2}$$

$$\text{左辺}=\frac{2\cos 13A\cos A}{2(\cos 3A+\cos 5A)}=\frac{\cos 14A+\cos 12A}{2(-\cos 14A-\cos 12A)}$$

$$=\frac{\cancel{\cos 14A+\cos 12A}}{-2\cancel{(\cos 14A+\cos 12A)}}=-\frac{1}{2}=\text{右辺}$$

(127)

$$\frac{\sin\theta-\sin 3\theta+\sin 5\theta}{\cos\theta-\cos 3\theta+\cos 5\theta}=\tan 3\theta$$

$$\text{左辺}=\frac{\sin 5\theta+\sin\theta-\sin 3\theta}{\cos 5\theta+\cos\theta-\cos 3\theta}$$

$$=\frac{2\sin 3\theta\cos 2\theta-\sin 3\theta}{2\cos 3\theta\cos 2\theta-\cos 3\theta}$$

$$=\frac{\sin 3\theta(2\cos 2\theta)}{\cos 3\theta(2\cos 2\theta)}=\tan 3\theta=\text{左辺}$$

(128)

$$\frac{\sin\theta}{1+\cos\theta}=\frac{1-\cos\theta}{\sin\theta}=\tan\frac{\theta}{2}$$

$$\text{左辺}=\frac{2\sin\frac{\theta}{2}\cancel{\cos\frac{\theta}{2}}}{2\cos^2\frac{\theta}{2}}=\tan\frac{\theta}{2}=\text{右辺}$$

$$\text{中辺}=\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}=\tan\frac{\theta}{2}=\text{右辺}$$

(129)

$$\cos^3 A\frac{\sin 3A}{3}+\sin^3 A\frac{\cos 3A}{3}=\frac{1}{4}\sin 4A$$

$$\text{左辺}=\cos^3 A\frac{3\sin A-4\sin^3 A}{3}+\sin^3 A\frac{4\cos^3 A-3\cos A}{3}$$

$$=\frac{3\cos^3 A\sin A-4\cos^3 A\sin^3 A+4\sin^3 A\cos^3 A-3\sin^3 A\cos A}{3}$$

$$=\frac{3\sin A\cos A(\cos^2 A-\sin^2 A)}{3}$$

$$=\frac{2\sin A\cos A(1-2\sin^2 A)}{2}$$

$$=\frac{2(\sin 2A\cos 2A)}{4}\quad(\text{公式}30,31)$$

$$=\frac{\sin 4A}{4}=\text{左辺}$$

(130)

$$\tan(45^\circ+A)-\tan(45^\circ-A)=2\tan 2A$$

$$\text{左辺}=\frac{\sin(45^\circ+A)}{\cos(45^\circ+A)}-\frac{\sin(45^\circ-A)}{\cos(45^\circ-A)}$$

$$=\frac{\sin(45^\circ+A)\cos(45^\circ-A)-\sin(45^\circ-A)\cos(45^\circ+A)}{\cos(45^\circ+A)\cos(45^\circ-A)}$$

$$=\frac{\sin 2A}{\frac{\cancel{\cos 90^\circ}+\cos 2A}{2}}\quad\left\{\begin{array}{l}\text{分子ハ公式18,分母ハ公}\\\text{式28ノ3}\end{array}\right\}$$

$$=2\cdot\frac{\sin 2A}{\cos 2A}=2\tan 2A=\text{左辺}$$

(131)

$$2\operatorname{cosec}2A=\sec A\operatorname{cosec}A$$

$$\text{左辺}=\frac{2}{2\cos A\sin A}=\frac{2}{\sin 2A}=2\operatorname{cosec}2A$$

$$=\text{左辺}$$

等式（和差角）

(132)

$$\cos A = \frac{\cos B - K}{1 - K\cos B} \quad \text{ナルトキハ}$$

$$\tan\frac{A}{2} = \sqrt{\frac{1+K}{1-K}}\tan\frac{B}{2}$$

$$\text{左辺} = \sqrt{\frac{1-\cos A}{1+\cos A}} \underset{(\text{公式}35)}{=} \sqrt{\frac{1-\frac{\cos B - K}{1-K\cos B}}{1+\frac{\cos B - K}{1-K\cos B}}}$$

$$= \sqrt{\frac{1-K\cos B - \cos B + K}{1-K\cos B + \cos B - K}} = \sqrt{\frac{(1+K)(1-\cos B)}{(1-K)(1+\cos B)}}$$

$$= \sqrt{\frac{1+K}{1-K}}\cdot\sqrt{\frac{1-\cos B}{1+\cos B}} = \sqrt{\frac{1+K}{1-K}}\tan\frac{B}{2} = \text{右辺}$$

(133)

$$\sin 2A + \sin 4A + \sin 6A = \frac{\cos A - \cos 7A}{2\sin A}$$

$$\text{左辺}\times 2\sin A$$

$$= 2\sin 2A\sin A + 2\sin 4A\sin A + 2\sin 6A\sin A$$

$$= (\cos A - \cos 3A) + (\cos 3A - \cos 5A) + (\cos 5A - \cos 7A)$$

$$\text{左辺} = \frac{\cos A - \cos 7A}{2\sin A} = \text{右辺}$$

(134)

$$1 + \cos 6A - \cos 10A - \cos 4A = 4\sin 2A\cos 3A\sin 5A$$

$$\text{左辺} = (1-\cos 6A) - (\cos 10A + \cos 4A)$$

$$= 2\cos^2 3A \,(\text{公式}31) - 2\cos 7A\cos 3A$$

$$= 2\cos 3A(\cos 3A - \cos 7A)$$

$$= 2\cos 3A\{-2\sin 5A\sin(-2A)\}$$

$$= 4\sin 2A\cos 3A\sin 5A = \text{右辺}$$

(135)

$$\tan\beta\tan\gamma + \tan\gamma\tan\alpha + \tan\alpha\tan\beta = 1 - \frac{\cos(\alpha+\beta+\gamma)}{\cos\alpha\cos\beta\cos\gamma}$$

$$\text{左辺} = \frac{\sin\beta\sin\gamma}{\cos\beta\cos\gamma} + \frac{\sin\gamma\sin\alpha}{\cos\gamma\cos\alpha} + \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}$$

$$= \frac{\cos\alpha\sin\beta\sin\gamma + \sin\alpha\cos\beta\sin\gamma + \sin\alpha\sin\beta\cos\gamma}{\cos\alpha\cos\beta\cos\gamma}$$

$$\text{右辺} = \frac{\cancel{\cos\alpha\cos\beta\cos\gamma} - \cancel{\cos\alpha\cos\beta\cos\gamma} + \cos\alpha\sin\beta\sin\gamma + \sin\alpha\cos\beta\sin\gamma + \sin\alpha\sin\beta\cos\gamma}{\cos\alpha\cos\beta\cos\gamma}$$

$$= \frac{\cos\alpha\sin\beta\sin\gamma}{\cos\alpha\cos\beta\cos\gamma} + \frac{\sin\alpha\cos\beta\sin\gamma}{\cos\alpha\cos\beta\cos\gamma} + \frac{\sin\alpha\sin\beta\cos\gamma}{\cos\alpha\cos\beta\cos\gamma} = \text{左辺}$$

(136)

$$(\cos A + \cos B)^2 + (\sin A + \sin B)^2 = 4\cos^2\frac{A-B}{2}$$

$$\text{左辺} = \cos^2 A\cos^2 B + 2\cos A\cos B + \sin^2 A + \sin^2 B + 2\sin A\sin B$$

$$= (\cos^2 A + \sin^2 A) + (\cos^2 B + \sin^2 B) + 2(\cos A\cos B + \sin A\sin B)$$

$$= 2 + 2\cos(A-B) \,(\text{公式}12)$$

$$= 2\{1 + \cos(A-B)\} = 2\left\{2\cos^2\frac{A-B}{2}\right\} (\text{公式}31)$$

$$= 4\cos^2\frac{A-B}{2} = \text{右辺}$$

(137)

$$\tan(A+B+C)$$

$$=\frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan B\tan C-\tan C\tan A-\tan A\tan B}$$

$$\text{左辺}=\tan\{A+(B+C)\}$$

$$=\frac{\tan A+\tan(B+C)}{1-\tan A\tan(B+C)}$$

$$=\frac{\tan A+\dfrac{\tan B+\tan C}{1-\tan B\tan C}}{1-\tan A\dfrac{\tan B+\tan C}{1-\tan B\tan C}}$$

$$=\frac{\dfrac{\tan A-\tan A\tan B\tan C+\tan B+\tan C}{1-\tan B\tan C}}{\dfrac{1-\tan B\tan C-\tan A\tan B-\tan A\tan C}{1-\tan B\tan C}}=\text{左辺}$$

(138)

$$\tan A+2\tan 2A+4\tan 4A=\cot A-8\cot 8A$$

$$\text{左辺}=\cot A-\tan A+\tan A-8\cot 8A$$

$$=2\cot 2A+\tan A-8\cot 8A \quad (\text{問}110)$$

$$=2(\cot 2A-\tan 2A)+2\tan 2A+\tan A-8\cot 8A$$

$$=4\cot 4A+2\tan 2A+\tan A-8\cot 8A$$

$$=4(\cot 4A-\tan 4A)+4\tan 4A+2\tan 2A+\tan A-8\cot 8A$$

$$=8\cot 8A+4\tan 4A+2\tan 2A+\tan A-8\cot 8A=\text{左辺}$$

(139)

$$\sec A\pm\tan A=\tan\left(45^\circ\pm\frac{A}{2}\right)$$

$$\text{左辺}=\frac{1}{\cos A}\pm\frac{\sin A}{\cos A}=\frac{\sin\frac{A}{2}\pm\cos\frac{A}{2}}{1-2\sin^2\frac{A}{2}}$$

$$=\frac{\left(\sin\frac{A}{2}\pm\cos\frac{A}{2}\right)^2}{\sin^2\frac{A}{2}-\cos^2\frac{A}{2}}$$

$$=\frac{\left(\sin\frac{A}{2}\pm\cos\frac{A}{2}\right)^2}{\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)\left(\sin\frac{A}{2}-\cos\frac{A}{2}\right)}$$

$$=\frac{\sin\frac{A}{2}\pm\cos\frac{A}{2}}{\sin\frac{A}{2}\mp\cos\frac{A}{2}}=\tan\left(45^\circ\pm\frac{A}{2}\right) \quad (\text{問}120)$$

$$=\text{左辺}$$

(140)

$$\tan\theta=\frac{b}{a}\ \text{より}\ \frac{b}{a}=\frac{\sin\theta}{\cos\theta}\ \text{より}$$

$$\therefore a=\frac{b\cos\theta}{\sin\theta}\cdots\cdots\cdots\cdots(1)$$

$$\sqrt{\frac{a-b}{a+b}}+\sqrt{\frac{a+b}{a-b}}=\frac{2\cos\theta}{\sqrt{\cos 2\theta}}$$

$$\text{左辺}=\frac{a-b+a+b}{\sqrt{a^2-b^2}}=\frac{2a}{\sqrt{a^2-b^2}}\quad \text{此に}a=(1)\text{を入れて}$$

$$=\frac{\dfrac{2b\cos\theta}{\sin\theta}}{\sqrt{\dfrac{b^2\cos^2\theta}{\sin^2\theta}-b^2}}=\frac{\dfrac{2b\cos\theta}{\sin\theta}}{b\dfrac{\sqrt{\cos^2\theta-\sin^2\theta}}{\sin\theta}}$$

$$=\frac{2\cos\theta}{\sqrt{\cos 2\theta}}\ (\text{公式}31)=\text{左辺}$$

等式(和,差角)

(141)

$$\tan\frac{A}{2}=\frac{-1\pm\sqrt{1+\tan^2 A}}{\tan A}$$

$\tan 2A=\dfrac{2\tan A}{1-\tan^2 A}$ (公式32) ヨリ

$\tan A=\dfrac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$ ヲ得分母ヲ拂ヒテ

$$\tan A-\tan A\tan^2\frac{A}{2}-2\tan\frac{A}{2}=0$$

$$\tan A\tan^2\frac{A}{2}+2\tan\frac{A}{2}-\tan A=0$$

之レ $\tan\frac{A}{2}$ ニ付キ二次方程式ナリ

$$\therefore\ \tan\frac{A}{2}=\frac{-1\pm\sqrt{1+\tan^2 A}}{\tan A}$$

Aガ何レノ象限ニアルモ $\sqrt{1+\tan^2 A}$ ノ絶對値ハ1ヨリ大ナリ, 故ヘニ $-1+\sqrt{1+\tan^2 A}$ ハ正ニシテ $-1-\sqrt{1+\tan^2 A}$ ハ負ナリ

而シテAガ第一象限内ニアレバ $\frac{A}{2}$ モ同象限内ニアルベク從テ $\tan\frac{A}{2}$ ハ正ニシテ右辺ノ√前ノ符号ハ十ヲ採ルベシ

Aガ第四象限内ニアレバ $\frac{A}{2}$ ハ第二象限内ニアルベク從テ $\tan\frac{A}{2}$ ハ負ニシテ且 $\tan A$ モ亦負ナルベシ故ヘニ右辺ノ√前ノ符号ヲ正トスレバ分子ハ正トナル然ルトキハ右辺ハ負トナルコトナシ故ヘニAガ第四象限内ニアレバ√前ノ符号ハ正ヲ採ルベシ

Aガ第三, 第二象限内ニアルトキモ之ニ準シテ類推スベシ

等式(和,差角)

(142)

$$\cot A+\tan A=2\operatorname{cosec}2A$$

$$左辺=\frac{\cos A}{\sin A}+\frac{\sin A}{\cos A}=\frac{\cos^2 A+\sin^2 A}{\sin A\cos A}$$

$$=\frac{2}{2\sin A\cos A}=2\operatorname{cosec}2A=右辺$$

(143)

$$\sin 2A=\frac{1-\tan^2(45^\circ-A)}{1+\tan^2(45^\circ-A)}$$

右辺 $=\cos 2(45^\circ-A)$ (公式31系1)

$=\cos(90^\circ-2A)=\sin 2A=$ 左辺

(144)

θガ $4m\times180^\circ$ ト $(4n+2)\times180^\circ$ トノ間ニアリテ

nガ正ノ整数ナル故ヘニ $\frac{\theta}{2}$ ハ $\frac{4m\times180^\circ}{2}$ ト $\frac{(4n+2)\times180^\circ}{2}$ トノ間即チ $2m\times180^\circ$ ト $(2n+1)\times180^\circ$ トノ間即チ第一象限ト第二象限トノ間ニアリテ正弦ハ正ナリ

$$\sin\frac{\theta}{2}=+\sqrt{\frac{1-\cos\theta}{2}}$$

(145)

$$\cos\frac{A}{2}=-\sqrt{\frac{1+\cos A}{2}}$$

Aガ 180° ト 540° トノ間ノ角ナルヲ以テ $\frac{A}{2}$ ハ 90° 及ビ 270° ノ間即チ第二象限ト第三象限トノ間ニアルベシ故ヘニ余弦ハ負ナリ

(146)

$$2\sin\frac{A}{2}=+\sqrt{1+\sin A}+\sqrt{1-\sin A}$$

$A=160^\circ$ ナルヲ以テ $\frac{A}{2}=80^\circ$ ナリ故ヘニ

$$\left.\begin{aligned}\sin\frac{A}{2}+\cos\frac{A}{2}&=+\sqrt{1+\sin A}\\ \sin\frac{A}{2}-\cos\frac{A}{2}&=+\sqrt{1-\sin A}\end{aligned}\right\}$$ (107頁例), 故ヘニ本題ハ証セラル

等式（和、差角）

(147)

$$2\cos\frac{A}{2} = -\sqrt{1+\sin A} - \sqrt{1-\sin A}$$

Aが270°ト360°トノ間ニアルヲ以テ $\frac{A}{2}$ ハ135°ト180°トノ間 即第二象限内ノ後半ニアリ故ヘニ

$$\sin\frac{A}{2} + \cos\frac{A}{2} = -\sqrt{1+\sin A}$$

$$\sin\frac{A}{2} - \cos\frac{A}{2} = +\sqrt{1-\sin A}$$

即本題ハ証明セラル

(148)

$$\tan\frac{A}{2} = -\cot A + \operatorname{cosec} A$$

$270° < A < 360°$ ∴ $\tan A$ ハ負ナリ

$135° < \frac{A}{2} < 180°$ ∴ $\tan\frac{A}{2}$ ハ負ナリ

而シテ $\tan\frac{A}{2} = \frac{-1 \pm \sqrt{1+\tan^2 A}}{\tan A}$ （範式37）

此等式ノ左辺ガ負ニシテ分母ガ負ナル故ヘニ分子ハ正ナラザルベカラズ故ヘニ

$$原式左辺 = \frac{-1+\sqrt{1+\tan^2 A}}{\tan A} = \frac{-1+\sec A}{\tan A}$$

$$= -\frac{1}{\tan A} + \frac{\sec A}{\tan A} = -\cot A + \operatorname{cosec} A$$

（注意）$270° < A < 360°$ ナルヲ以テ $\sec A$ ハ正ナル符号ヲ有ス故ヘニ $\sqrt{\sec^2 A}$ ハ根号ヲ去リテ正トス

等式（角辺面積）

(149)

$$\sin\frac{3A+3B}{2} = -\cos\frac{3C}{2}$$

$$左辺 = \sin 3\left(\frac{A+B}{2}\right) = \sin 3\left(90° - \frac{C}{2}\right)$$

$$= \sin\left(3\times 90° - \frac{3C}{2}\right) = -\cos\frac{3C}{2} = 右辺$$

（藤原氏教科書 50頁参ニスル）

(150)

$$\cos A + \cos B + \cos C = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} + 1$$

$$左辺 = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} + 1 - 2\sin^2\frac{C}{2}$$

$$= 2\sin\frac{C}{2}\cos\frac{A-B}{2} - 2\sin^2\frac{C}{2} + 1$$

$$= 2\sin\frac{C}{2}\left(\cos\frac{A-B}{2} - \cos\frac{A+B}{2}\right) + 1$$

$$= 2\sin\frac{C}{2}\left(2\sin\frac{A}{2}\sin\frac{B}{2}\right) + 1$$

$$= 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} + 1 = 右辺$$

(151)

$$\sin A \sin B - \sin C = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$$左辺 = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} - 2\sin\frac{C}{2}\cos\frac{C}{2}$$

$$= 2\cos\frac{C}{2}\left\{\cos\frac{A-B}{2} - \cos\frac{A+B}{2}\right\}$$

$$= 2\cos\frac{C}{2}\left\{2\sin\frac{A}{2}\sin\frac{B}{2}\right\}$$

$$= 4\cos\frac{C}{2}\sin\frac{A}{2}\sin\frac{B}{2}$$

$$= 右辺$$

等式(角辺面積)

(152)

$$\sin\frac{A}{2}+\sin\frac{B}{2}+\sin\frac{C}{2}=4\sin\left(45^\circ-\frac{A}{4}\right)\sin\left(45^\circ-\frac{B}{4}\right)\sin\left(45^\circ-\frac{C}{4}\right)+1$$

$$\text{左辺}=2\sin\frac{A+B}{4}\cos\frac{A-B}{4}+\cos\frac{A+B}{2}$$

$$=2\sin\frac{A+B}{4}\cos\frac{A-B}{4}+1-2\sin^2\frac{A+B}{4}$$

$$=2\sin\frac{A+B}{4}\left(\cos\frac{A-B}{4}-\sin\frac{A+B}{4}\right)+1$$

$$=2\sin\frac{A+B}{4}\left\{\sin\left(90^\circ-\frac{A-B}{4}\right)-\sin\frac{A+B}{4}\right\}+1$$

$$=2\sin\frac{A+B}{4}\left\{2\cos\left(\frac{90^\circ-\frac{A-B}{4}+\frac{A+B}{4}}{2}\right)\sin\left(\frac{90^\circ-\frac{A-B}{2}-\frac{A+B}{2}}{2}\right)\right\}+1$$

$$=2\sin\frac{A+B}{4}\left\{2\cos\left(45^\circ+\frac{B}{4}\right)\sin\left(45^\circ-\frac{A}{4}\right)\right\}+1$$

$$=2\sin\frac{180^\circ-C}{4}\left\{2\sin\left(90^\circ-45^\circ-\frac{B}{4}\right)\sin\left(45^\circ-\frac{A}{4}\right)\right\}+1$$

$$=2\sin\left(45^\circ-\frac{C}{4}\right)2\sin\left(45^\circ-\frac{B}{4}\right)\sin\left(45^\circ-\frac{A}{4}\right)+1$$

$$=\text{右辺}$$

(153)

$$\sin(B+C-A)-\sin(C+A-B)+\sin(A+B-C)=4\cos A\sin B\cos C$$

$$\text{左辺}=2\cos C\sin(B-A)+\sin\{(A+B)-C\}$$

$$=2\cos C\sin(B-A)+\sin(A+B)\cos C-\sin C\cos(A+B)$$

$$=2\cos C\sin(B-A)+\sin(A+B)\cos C+\sin(A+B)\cos C$$

$$=2\cos C\{-\sin(B-A)+\sin(A+B)\}$$

$$=2\cos C\{2\sin B\cos(-A)\}$$

$$=4\cos A\sin B\cos C=\text{右辺}$$

等式(角辺面積)

(154)

$$\cos^2A\cos^2B+\cos^2C=1-2\cos A\cos B\cos C$$

$$\text{左辺}=\frac{1}{2}\{(1+\cos2A)+(1+\cos2B)+2\cos^2C\}$$

$$=\frac{1}{2}\{2+(\cos2A+\cos2B)+2\cos^2C\}$$

$$=\frac{1}{2}\{2\cos(A+B)\cos(A-B)+2\cos^2C+2\}$$

$$=-\cos C\cos(A-B)+\cos^2C+1$$

$$=\cos C\{\cos C-\cos(A-B)\}+1$$

$$=\cos C\{-\cos(A+B)-\cos(A-B)\}+1$$

$$=\cos C\{-2\cos A\cos B\}+1$$

$$=1-2\cos A\cos B\cos C=\text{右辺}$$

(155)

$$\cos(A+B-C)=-\cos2C$$

$$\text{左辺}=\cos\{(A+B)-C\}$$

$$=\cos(A+B)\cos C+\sin(A+B)\sin C$$

$$=-\cos C\cos C+\sin C\sin C$$

$$=-(\cos^2C-\sin^2C)=-\cos2C=\text{右辺}$$

(156)

$$a\sin(B+C)+b\sin(C-A)+c\sin(A-B)=0$$

$$\text{左辺}=m\sin A(\sin B\cos C-\cos B\sin C)$$

$$+m\sin B(\sin C\cos A-\cos C\sin A)$$

$$+m\sin C(\sin A\cos B-\cos A\sin B)$$

$$=m\{\sin A\sin B\cos C-\sin A\cos B\sin C+\cos A\sin B\sin C$$

$$-\sin A\sin B\cos C+\sin A\cos B\sin C-\cos A\sin B\sin C\}$$

$$=0=\text{右辺}$$

(注意) $m=\frac{a}{\sin A}=\cdots\cdots$ トス

(157)

$$\sin(A-B)+\sin(B-C)+\sin(C-A)$$
$$=4\sin\frac{A-B}{2}\sin\frac{A-C}{2}\sin\frac{B-C}{2}$$

$$\text{左辺}=2\sin\frac{A-C}{2}\cos\frac{A+C-2B}{2}-\sin(A-C)$$
$$=2\sin\frac{A-C}{2}\cos\frac{A+C-2B}{2}-2\sin\frac{A-C}{2}\cos\frac{A-C}{2}$$
$$=2\sin\frac{A-C}{2}\left\{\cos\frac{A+C-2B}{2}-\cos\frac{A-C}{2}\right\}$$
$$=2\sin\frac{A-C}{2}\left\{-2\sin\frac{A-B}{2}\sin\frac{C-B}{2}\right\}$$
$$=4\sin\frac{A-B}{2}\sin\frac{A-C}{2}\sin\frac{B-C}{2}=\text{右辺}$$

(158)

$$\frac{a-b}{c}=\frac{\cos B-\cos A}{1+\cos C}$$

$$\text{左辺}=\frac{\sin A-\sin B}{\sin C}=\frac{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{2\sin\frac{A+B}{2}\cos\frac{A+B}{2}}$$

$$=\frac{\sin\frac{A-B}{2}}{\sin\frac{A+B}{2}}$$ 両項ニ $\sin\frac{A+B}{2}$ ヲ乗ジテ

$$=\frac{2\sin\frac{A-B}{2}\sin\frac{A+B}{2}}{2\sin^2\frac{A+B}{2}}=\frac{2\sin\frac{A+B}{2}\sin\frac{A-B}{2}}{2\cos^2\frac{C}{2}}$$

$$=\frac{\cos B-\cos A\ (\text{範式}19)}{1+\cos C\ (\text{範式}27)}=\text{右辺}$$

(159)

$$\sin 2A=\frac{2ab}{c^2}$$

$$\text{左辺}=2\sin A\cos A=2\cdot\frac{a}{c}\times\frac{b}{c}=\frac{2ab}{c^2}=\text{右辺}$$

(160)

$$\frac{\cos A}{\sin B\sin C}+\frac{\cos B}{\sin C\sin A}+\frac{\cos C}{\sin A\sin B}=2$$

$$\text{左辺}=\frac{-\cos(B+C)}{\sin B\sin C}+\frac{-\cos(A+C)}{\sin C\sin A}+\frac{-\cos(A+B)}{\sin A\sin B}$$
$$=\frac{-\cos B\cos C+\sin B\sin C}{\sin B\sin C}+\frac{-\cos A\cos C+\sin A\sin C}{\sin C\sin A}$$
$$+\frac{-\cos A\cos B+\sin A\sin B}{\sin A\sin B}$$
$$=1-\cot B\cot C+1-\cot A\cot C+1-\cot A\cot B$$
$$=3-(\cot B\cot C+\cot A\cot C+\cot A\cot B)$$
$$=3-1=2=\text{右辺}\ (\text{問}193)$$

(161)

$$\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$$

$$\text{左辺}=2\sin(A+B)\cos(A-B)+2\sin C\cos C$$
$$=2\sin C\cos(A-B)+2\cos C\sin C$$
$$=2\sin C[-\{\cos(A+B)-\cos(A-B)\}]$$
$$=2\sin C(2\sin A\sin B)=4\sin A\sin B\sin C=\text{右辺}$$

(162)

$$a+b+c=(b+c)\cos A+(c+a)\cos B+(a+b)\cos C$$

$$\text{左辺}=(b\cos C+c\cos B)+(c\cos A+a\cos C)$$
$$+(a\cos B+b\cos A)$$
$$=b\cos C+a\cos C+c\cos B+a\cos B$$
$$+c\cos A+b\cos A$$
$$=(b+c)\cos A+(c+a)\cos B+(a+b)\cos C$$
$$=\text{右辺}$$

(163)

$$\cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2}=4\cos\left(45^\circ-\frac{A}{4}\right)\times\cos\left(45^\circ-\frac{B}{4}\right)\cos\left(45^\circ-\frac{C}{4}\right)$$

$$\text{左辺}=2\cos\frac{A+B}{4}\cos\frac{A-B}{4}+\sin\frac{A+B}{2}$$

$$=2\cos\frac{A+B}{4}\cos\frac{A-B}{4}+2\sin\frac{A+B}{4}\cos\frac{A+B}{4}$$

$$=2\cos\frac{A+B}{4}\left(\cos\frac{A-B}{4}+\sin\frac{A+B}{4}\right)$$

$$=2\cos\frac{180^\circ-C}{4}\left\{\cos\frac{A-B}{4}+\cos\left(90^\circ-\frac{A+B}{4}\right)\right\}$$

$$=2\cos\left(45^\circ-\frac{C}{4}\right)2\cos\frac{90^\circ-\frac{A+B}{4}+\frac{A-B}{4}}{2}\times\cos\frac{90^\circ-\frac{A+B}{4}-\frac{A-B}{4}}{2}$$

$$=4\cos\left(45^\circ-\frac{C}{4}\right)\cos\left(45^\circ-\frac{B}{4}\right)\cos\left(45^\circ-\frac{A}{4}\right)$$

$=$ 右辺

(164)

$\cos^2\frac{A}{2}=\frac{b+c}{2c}$, $\cos A=2\cos^2\frac{A}{2}-1$ ヨリ

$$\cos^2\frac{A}{2}=\frac{\cos A+1}{2}=\frac{\frac{b}{c}+1}{2}=\frac{b+c}{2c}$$

(165)

$$c(\sin^2A+\sin^2B)=\sin C(a\sin A+b\sin B)$$

$$\text{左辺}=c\sin A\sin A+c\sin B\sin B$$

$$=a\sin C\sin A+b\sin C\sin B \quad (\text{正弦比例式ヨリ})$$

$$=\sin C(a\sin A+b\sin B)=\text{右辺}$$

(166)

$$\frac{1+\cos(A-B)\cos C}{1+\cos(A-B)\cos B}=\frac{a^2+b^2}{a^2+c^2}$$

$$\text{左辺}=\frac{1+\cos(A-B)\{-\cos(A+B)\}}{1+\cos(A-C)\{-\cos(A+C)\}}$$

$$=\frac{1-\cos^2B+\sin^2A}{1-\cos^2C+\sin^2A}=\frac{\sin^2B+\sin^2A}{\sin^2C+\sin^2A}$$

$$=\frac{a^2+b^2}{a^2+c^2}=\text{右辺}$$

(167)

$$\frac{a-b}{a+b}=\tan\frac{A-B}{2}$$

$$\text{左辺}=\frac{\frac{a}{c}-\frac{b}{c}}{\frac{a}{c}+\frac{b}{c}}=\frac{\sin A-\sin B}{\sin A+\sin B}=\frac{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}$$

$$=\frac{\cos 45^\circ\sin\frac{A-B}{2}}{\sin 45^\circ\cos\frac{A-B}{2}}=\tan\frac{A-B}{2}$$

(168)

$$\frac{a+c}{2b}=\sin(30^\circ+C)$$

$$\text{左辺}=\frac{\sin A+\sin C}{2\sin B}=\frac{2\sin\frac{A+C}{2}\cos\frac{A-C}{2}}{2\sin B}$$

$$=\frac{\cos\frac{B}{2}\cos\frac{A-C}{2}}{\sin B}=\frac{\cos 30^\circ\cos\frac{A-C}{2}}{\sin 60^\circ} \quad (\text{仮設})$$

$$=\cos\frac{A-C}{2}=\sin\left\{90^\circ-\frac{A-C}{2}\right\}$$

$$=\sin\left\{\frac{A+B+C}{2}-\frac{A-C}{2}\right\}=\sin\frac{B+2C}{2}$$

$$=\sin\left(\frac{60^\circ}{2}+C\right)=\sin(30^\circ+C)=\text{右辺}$$

等式(角辺,面積)

(169)

$$\log a = \log(b+c) + \log\cos\phi$$

$$\sin\phi = \frac{2\sqrt{bc}}{b+c}\cos\frac{A}{2}$$ ナルヲシルベシ

$$\therefore \sin^2\phi = \frac{4bc}{(b+c)^2}\cos^2\frac{A}{2} = \frac{\cancel{4}^{2}bc}{(b+c)^2}\cdot\frac{1+\cos A}{\cancel{2}}$$

$$1-\sin^2\phi = 1 - \frac{2bc+2bc\cos A}{(b+c)^2}$$

$$\cos^2\phi = \frac{(b+c)^2-2bc-2bc\cos A}{(b+c)^2}$$

$$= \frac{b^2+2bc+c^2-2bc-2bc\cos A}{(b+c)^2}$$

$$= \frac{a^2}{(b+c)^2}$$

$$\therefore \cos\phi = \frac{a}{b+c} \quad \therefore a = \cos\phi(b+c)$$

$$\therefore \log a = \log(b+c) + \log\cos\phi$$

(170)

$$a(\cos B+\cos C) = 2(b+c)\sin^2\frac{A}{2}$$

$$\text{左辺} = a\cos B + a\cos C$$

$$= (c\cos A + a\cos C) + (a\cos B + b\cos A)$$

$$-(c\cos A + b\cos A)$$ (1項ノ順ヲ換ヘテ $c\cos A$, $a\cos A$ ヲカロ減ス)

$$= b+c-\cos A(c+b)$$

$$= (b+c)(1-\cos A)$$

$$= (b+c)\left(2\sin^2\frac{A}{2}\right)$$ (公式31)

$$= 2(b+c)\sin^2\frac{A}{2} = \text{右辺}$$

等式(角辺,面積)

(171)

$$\frac{\sin A-\sin B}{\sin A+\sin B} = \tan\frac{C}{2}\tan\frac{A-B}{2}$$

$$\text{左辺} = \frac{\cancel{2}\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{\cancel{2}\sin\frac{A+B}{2}\cos\frac{A-B}{2}} = \frac{\sin\frac{C}{2}\sin\frac{A-B}{2}}{\cos\frac{C}{2}\cos\frac{A-B}{2}}$$

$$= \frac{\sin\frac{C}{2}}{\cos\frac{C}{2}}\cdot\frac{\sin\frac{A-B}{2}}{\cos\frac{A-B}{2}} = \tan\frac{C}{2}\tan\frac{A-B}{2} = \text{右辺}$$

(172)

$$\sin(2A+2B) = -\sin 2C$$

$$\text{左辺} = \sin(360^\circ-2C) = -\sin 2C = \text{右辺}$$

(173)

$$\frac{\sin A+\sin B-\sin C}{\cos A+\cos B-\cos C+1} = \tan\frac{A}{2}\tan\frac{B}{2}\cot\frac{C}{2}$$

$$\text{左辺} = \frac{2\sin\frac{A+B}{2}\cos\frac{A-B}{2} - 2\sin\frac{C}{2}\cos\frac{C}{2}}{2\cos\frac{A+B}{2}\cos\frac{A-B}{2} \cancel{-1} + \sin^2\frac{C}{2} \cancel{+1}}$$

$$= \frac{\cancel{2}\sin\frac{A+B}{2}\left(\cos\frac{A-B}{2}-\cos\frac{A+B}{2}\right)}{\cancel{2}\cos\frac{A+B}{2}\left(\cos\frac{A-B}{2}+\cos\frac{A+B}{2}\right)}$$

$$= \frac{\sin\frac{A+B}{2}\left\{-2\sin\frac{A}{2}\sin\left(-\frac{B}{2}\right)\right\}}{\cos\frac{A+B}{2}\left\{2\cos\frac{A}{2}\cos\left(-\frac{B}{2}\right)\right\}}$$

$$= \frac{\cancel{2}\cos\frac{C}{2}\left\{\sin\frac{A}{2}\sin\frac{B}{2}\right\}}{\cancel{2}\sin\frac{C}{2}\left\{\cos\frac{A}{2}\cos\frac{B}{2}\right\}}$$

$$= \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}}\cdot\frac{\sin\frac{B}{2}}{\cos\frac{B}{2}}\cdot\frac{\cos\frac{C}{2}}{\sin\frac{C}{2}}$$

$$= \tan\frac{A}{2}\tan\frac{B}{2}\cot\frac{C}{2} = \text{右辺}$$

(174)

$$\sin\frac{A}{2}\cos\frac{A}{2}+\sin\frac{B}{2}\cos\frac{B}{2}+\sin\frac{C}{2}\cos\frac{C}{2}$$
$$=2\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

$$\text{左辺}=\frac{1}{2}\left(2\sin\frac{A}{2}\cos\frac{A}{2}+2\sin\frac{B}{2}\cos\frac{B}{2}+2\sin\frac{C}{2}\cos\frac{C}{2}\right)$$
$$=\frac{1}{2}\left(\sin A+\sin B+2\sin\frac{C}{2}\cos\frac{C}{2}\right)$$
$$=\frac{\cancel{1}}{\cancel{2}}\left(\cancel{2}\sin\frac{A+B}{2}\cos\frac{A-B}{2}+\cancel{2}\sin\frac{C}{2}\cos\frac{C}{2}\right)$$
$$=\cos\frac{C}{2}\cos\frac{A-B}{2}+\sin\frac{C}{2}\cos\frac{C}{2}$$
$$=\cos\frac{C}{2}\left(\cos\frac{A-B}{2}+\cos\frac{A+B}{2}\right)$$
$$=\cos\frac{C}{2}\left(2\cos\frac{A}{2}\cos\frac{B}{2}\right)$$
$$=2\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}=\text{右辺}$$

(175)

$$\sin A\sin(A+2C)+\sin B\sin(B+2A)+\sin C\sin(C+2B)=0$$

$$\text{左辺}=\frac{1}{2}\{2\sin(A+2C)\sin A+2\sin(B+2A)\sin B$$
$$+2\sin(C+2B)\sin C\}$$
$$=\frac{1}{2}\{-\cos(2A+2C)+\cos 2C-\cos(2B+2A)$$
$$+\cos 2A-\cos(2C+2B)+\cos 2B\}$$
$$=\frac{1}{2}\{-\cos(360^\circ-2B)+\cos 2C-\cos(360^\circ-2C)$$
$$+\cos 2A-\cos(360^\circ-2A)+\cos 2B\}$$
$$=\frac{1}{2}\{\cancel{-\cos 2B}+\cancel{\cos 2C}\cancel{-\cos 2C}+\cancel{\cos 2A}$$
$$\cancel{-\cos 2A}+\cancel{\cos 2B}\}$$
$$=0=\text{右辺}$$

(176)

$$\cos^2\frac{A}{2}+\cos^2\frac{B}{2}-\cos^2\frac{C}{2}=2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$$

$$\text{左辺}=\frac{1}{2}\left(2\cos^2\frac{A}{2}+2\cos^2\frac{B}{2}-2\cos^2\frac{C}{2}\right)$$
$$=\frac{1}{2}\left\{(1+\cos A)+(1+\cos B)-2\cos^2\frac{C}{2}\right\}\ (\text{能式24})$$
$$=\frac{1}{2}\left\{2+(\cos A+\cos B)-2\left(1-\sin^2\frac{C}{2}\right)\right\}$$
$$=\frac{\cancel{1}}{\cancel{2}}\left\{\cancel{2}+\cancel{2}\cos\frac{A+B}{2}\cos\frac{A-B}{2}-\cancel{2}\left(1-\sin^2\frac{C}{2}\right)\right\}$$
$$=\cancel{1}+\cos\frac{A+B}{2}\cos\frac{A-B}{2}-\cancel{1}+\cos^2\frac{A+B}{2}$$
$$=\cos\frac{A+B}{2}\left(\cos\frac{A-B}{2}+\cos\frac{A+B}{2}\right)$$
$$=\sin\frac{C}{2}\left(2\cos\frac{A}{2}\cos\frac{B}{2}\right)$$
$$=2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}=\text{右辺}$$

(177)

$$\frac{a}{1-\tan\frac{B}{2}\tan\frac{C}{2}}=\frac{b}{1-\tan\frac{C}{2}\tan\frac{A}{2}}=\frac{c}{1-\tan\frac{A}{2}\tan\frac{B}{2}}$$

$$\text{左辺}=\frac{a}{1-\frac{1}{p-b}\sqrt{\frac{(p-a)(p-b)(p-c)}{p}}\times\frac{1}{p-c}\sqrt{\frac{(p-a)(p-b)(p-c)}{p}}}$$
$$=\frac{a}{1-\frac{1}{\cancel{(p-b)(p-c)}}\times\frac{\cancel{(p-a)(p-b)(p-c)}}{p}}$$
$$=\frac{a}{\frac{p-p+a}{p}}=p$$

同様にして中辺も右辺も皆 p に等しきを知る故に

左辺＝中辺＝右辺

(178)

$$\sin\frac{B+C-A}{2}+\sin\frac{C+A-B}{2}+\sin\frac{A+B-C}{2}-1$$
$$=4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$$\text{左辺}=\left(\sin\frac{B+C-A}{2}+\sin\frac{C+A-B}{2}\right)+\sin\left(\frac{A+B}{2}-\frac{C}{2}\right)-1$$
$$=2\sin\frac{C}{2}\cos\frac{B-A}{2}+\left(\sin\frac{B+A}{2}\cos\frac{C}{2}-\cos\frac{B+A}{2}\sin\frac{C}{2}\right)-1$$
$$=2\cos\frac{B+A}{2}\cos\frac{B-A}{2}+\sin^2\frac{B+A}{2}-\cos^2\frac{B+A}{2}-1$$
$$=2\cos\frac{B+A}{2}\cos\frac{B-A}{2}+\left(-\cos^2\frac{B+A}{2}-\cos^2\frac{B+A}{2}\right)$$
$$=2\cos\frac{B+A}{2}\left\{\cos\frac{B-A}{2}-\cos\frac{B+A}{2}\right\}$$
$$=2\sin\frac{C}{2}\left\{2\sin\frac{B}{2}\sin\frac{A}{2}\right\}$$
$$=4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=\text{右辺}$$

(179)

$$\cot\frac{A}{2}\cot\frac{C}{2}=3,$$

$$\cot\frac{A}{2}+\cot\frac{C}{2}=2\cot\frac{B}{2}\quad\left(\cot\frac{A}{2},\ \cot\frac{B}{2},\ \cot\frac{C}{2}\ \text{ガ A.P ヲナス}\right)$$
$$=2\tan\frac{A+C}{2}=\frac{2}{\cot\left(\frac{A}{2}+\frac{C}{2}\right)}$$
$$=\frac{2}{\dfrac{\cot\frac{A}{2}\cot\frac{C}{2}-1}{\cot\frac{A}{2}+\cot\frac{C}{2}}}=\frac{2\left(\cot\frac{A}{2}+\cot\frac{C}{2}\right)}{\cot\frac{A}{2}\cot\frac{C}{2}-1}$$

$$\therefore\ \cancel{\cot\frac{A}{2}+\cot\frac{C}{2}}=\frac{2\left(\cancel{\cot\frac{A}{2}+\cot\frac{C}{2}}\right)}{\cot\frac{A}{2}\cot\frac{C}{2}-1}$$
$$\therefore\ \cot\frac{A}{2}\cot\frac{C}{2}-1=2$$
$$\therefore\ \cot\frac{A}{2}\cot\frac{C}{2}=3$$

(180)

$$bc=4R^2(\cos A+\cos B\cos C)$$
$$\text{左辺}=(2R\sin B)(2R\sin C)=4R^2\sin B\sin C$$
$$=4R^2(\sin B\sin C-\cos B\cos C+\cos B\cos C)$$
$$=4R^2\{-(\cos B\cos C-\sin B\sin C)+\cos B\cos C\}$$
$$=4R^2\{-\cos(B+C)+\cos B\cos C\}$$
$$=4R^2(\cos A+\cos B\cos C)=\text{右辺}$$

(181)

$$\sin 3A+\sin 3B+\sin 3C=-4\cos\frac{3A}{2}\cos\frac{3B}{2}\cos\frac{3C}{2}$$
$$\text{左辺}=2\sin\frac{3A+3B}{2}\cos\frac{3A-3B}{2}+2\sin\frac{3C}{2}\cos\frac{3C}{2}$$
$$=2\sin\frac{3(180^\circ-C)}{2}\cos\frac{3A-3B}{2}+2\sin\frac{3C}{2}\cos\frac{3C}{2}$$
$$=2\sin\left(3\times 90^\circ-\frac{3C}{2}\right)\cos\frac{3A-3B}{2}+2\sin\frac{3C}{2}\cos\frac{3C}{2}$$
$$=-2\cos\frac{3C}{2}\cos\frac{3A-3B}{2}+2\sin\frac{3C}{2}\cos\frac{3C}{2}$$
$$=2\cos\frac{3C}{2}\left\{\sin\frac{3C}{2}-\cos\frac{3A-3B}{2}\right\}$$
$$=2\cos\frac{3C}{2}\left\{-\cos\left(90^\circ+\frac{3C}{2}\right)-\cos\frac{3A-3B}{2}\right\}$$
$$=-2\cos\frac{3C}{2}\cdot 2\cos\frac{90^\circ+\frac{3C}{2}+\frac{3A-3B}{2}}{2}\cos\frac{90^\circ+\frac{3C}{2}-\frac{3A-3B}{2}}{2}$$
$$=-4\cos\frac{3C}{2}\cos\left(180^\circ-\frac{3B}{2}\right)\cos\left(180^\circ-\frac{3A}{2}\right)$$
$$=-4\cos\frac{3A}{2}\cos\frac{3B}{2}\cos\frac{3C}{2}=\text{右辺}$$

(182)

$$\sin 2A=\sin 2B,\qquad 2A+2B=180^\circ$$

即 2A+2B トハ互ニ補角ナリ故ニ

$$\sin 2A=\sin 2B$$

(183)

$$\frac{1}{r'}+\frac{1}{r''}+\frac{1}{r'''}=\frac{1}{r}$$

$$左辺=\frac{p-a}{S}+\frac{p-b}{S}+\frac{p-c}{S}$$ {遠藤氏教科書100頁系1ヨリ}

$$=\frac{p-a+p-b+p-c}{S}=\frac{3p-(a+b+c)}{S}$$

$$=\frac{3p-2p}{S}=\frac{p}{S}=\frac{1}{r}$$ (公式45)=右辺

(184)

$$\sin(A-B)+\cos 2A=0$$

$$左辺=\sin A\cos B-\sin B\cos A+\cos^2 A-\sin^2 A$$

$$=\sin A\sin A-\cos A\cos A+\cos^2 A-\sin^2 A$$

$$=\sin^2 A-\cos^2 A+\cos^2 A-\sin^2 A$$

$$=0=右辺$$

(185)

$$c^2-b^2=ab$$

$$左辺=4R^2(\sin^2 C-\sin^2 B)$$

$$=4R^2\{\sin(C+B)\sin(C-B)\}$$

$$=4R^2\{\sin A\sin(2B-B)$$

$$=2R\sin A\cdot 2R\sin B=ab=右辺$$

(186)

$$\frac{c-a}{b}=\tan\frac{B}{2},\qquad C=90^\circ$$

$$左辺=\frac{\frac{c}{c}-\frac{a}{c}}{\frac{b}{c}}=\frac{1-\cos B}{\sin B}=\frac{2\sin^2\frac{B}{2}}{2\sin\frac{B}{2}\cos\frac{B}{2}}$$

$$=\frac{\sin\frac{B}{2}}{\cos\frac{B}{2}}=\tan\frac{B}{2}=右辺$$

(187)

$$\tan A+\tan B+\tan C=\tan A\tan B\tan C$$

$$左辺=\frac{\sin A}{\cos A}+\frac{\sin B}{\cos B}+\frac{\sin C}{\cos C}$$

$$=\frac{\sin A\cos B+\cos A\sin B}{\cos A\cos B}+\frac{\sin C}{\cos C}$$

$$=\frac{\sin(A+B)}{\cos A\cos B}+\frac{\sin C}{\cos C}=\frac{\sin C}{\cos A\cos B}+\frac{\sin C}{\cos C}$$

$$=\sin C\left\{\frac{1}{\cos A\cos B}+\frac{1}{\cos C}\right\}$$

$$=\sin C\left\{\frac{\cos C+\cos A\cos B}{\cos A\cos B\cos C}\right\}$$

$$=\sin C\left\{\frac{-\cos(A+B)+\cos A\cos B}{\cos A\cos B\cos C}\right\}$$

$$=\sin C\left\{\frac{\cancel{-\cos A\cos B}+\sin A\sin B\cancel{+\cos A\cos B}}{\cos A\cos B\cos C}\right\}$$

$$=\frac{\sin A\sin B\sin C}{\cos A\cos B\cos C}=\tan A\tan B\tan C$$

$$=右辺$$

(188)

$$b\cos B+c\cos C=a\cos(B-C)$$

正弦比例ヨリ $a=2R\sin A,\ b=2R\sin B,\ c=2R\sin C$

$$左辺=2R\sin B\cos B+2R\sin C\cos C$$

$$=R(2\sin B\cos B+2\sin C\cos C)$$

$$=R(\sin 2B+\sin 2C)$$

$$=R\{2\sin(B+C)\cos(B-C)\}$$ (公式22)

$$=2R\sin A\cos(B-C)=a\cos(B-C)$$

$$=右辺$$

(189)

$$\tan\phi = \frac{b+c}{b-c}\tan\frac{A}{2} + \text{レバ}$$

$$\log a = \log(b-c) + \log\cos\frac{A}{2} - \log\cos\phi$$

$$\tan^2\phi = \left(\frac{b+c}{b-c}\right)^2\tan^2\frac{A}{2}$$

$$\sec^2\phi - 1 = \left(\frac{b+c}{b-c}\right)^2\left(\sec^2\frac{A}{2} - 1\right)$$

$$= \left(\frac{b+c}{b-c}\right)^2\sec^2\frac{A}{2} - \left(\frac{b+c}{b-c}\right)^2$$

$$\frac{1}{\cos^2\phi} - 1 = \frac{(b+c)^2}{(b-c)^2\cos^2\frac{A}{2}} - \frac{(b+c)^2}{(b-c)^2}$$

$$\frac{1}{\cos^2\phi} = \frac{(b+c)^2 - (b+c)^2\cos^2\frac{A}{2} + (b-c)^2\cos^2\frac{A}{2}}{(b-c)^2\cos^2\frac{A}{2}}$$

$$= \frac{(b+c)^2 - 4bc\cos^2\frac{A}{2}}{(b-c)^2\cos^2\frac{A}{2}}$$

$$\cos^2\phi = \frac{(b-c)^2\cos^2\frac{A}{2}}{b^2+c^2+2bc-\cancel{\frac{4}{2}}bc\frac{1+\cos A}{\cancel{2}}}$$

$$= \frac{(b-c)^2\cos^2\frac{A}{2}}{b^2+c^2+\cancel{2bc}-\cancel{2bc}-2bc\cos A}$$

$$= \frac{(b-c)^2\cos^2\frac{A}{2}}{a^2}$$ {菊池氏教科書151頁例題1ニヨル}

$$\cos\phi = \frac{(b-c)\cos\frac{A}{2}}{a}$$

$$\therefore a = \frac{(b-c)\cos\frac{A}{2}}{\cos\phi}$$

$$\therefore \log a = \log(b-c) + \log\cos\frac{A}{2} - \log\cos\phi$$

(190)

$$\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$$

$$\text{左辺} = \left\{\frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} + \frac{\cos\frac{B}{2}}{\sin\frac{B}{2}}\right\} + \cot\frac{C}{2}$$

$$= \frac{\cos\frac{A}{2}\sin\frac{B}{2} + \sin\frac{A}{2}\cos\frac{B}{2}}{\sin\frac{A}{2}\sin\frac{B}{2}} + \cot\frac{C}{2}$$

$$= \frac{\sin\left(\frac{A}{2}+\frac{B}{2}\right)}{\sin A\sin B} + \frac{\cos\frac{C}{2}}{\sin\frac{C}{2}}$$

$$= \frac{\cos\frac{C}{2}}{\sin\frac{A}{2}\sin\frac{B}{2}} + \frac{\cos\frac{C}{2}}{\sin\frac{C}{2}}$$

$$= \cos\frac{C}{2}\left\{\frac{1}{\sin\frac{A}{2}\sin\frac{B}{2}} + \frac{1}{\sin\frac{C}{2}}\right\}$$

$$= \cos\frac{C}{2}\left\{\frac{\sin\frac{C}{2} + \sin\frac{A}{2}\sin\frac{B}{2}}{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}\right\}$$

$$= \cos\frac{C}{2}\left\{\frac{\cos\left(\frac{A}{2}+\frac{B}{2}\right) + \sin\frac{A}{2}\sin\frac{B}{2}}{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}\right\}$$

$$= \cos\frac{C}{2}\left\{\frac{\cos\frac{A}{2}\cos\frac{B}{2} - \cancel{\sin\frac{A}{2}\sin\frac{B}{2}} + \cancel{\sin\frac{A}{2}\sin\frac{B}{2}}}{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}\right\}$$

$$= \frac{\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}} = \cot\frac{A}{2}\cos\frac{B}{2}\cot\frac{C}{2}$$

=右辺

(191)

$$\frac{a - c\cos B}{b - c\cos A} = \frac{b}{a}$$

$$\text{左辺} = \frac{(b\cos C + c\cos B) - c\cos B}{(c\cos A + a\cos C) - c\cos A} = \frac{b\cos C}{a\cos C} = \frac{b}{a}$$

=右辺

(192)

$$R=\frac{abc}{4S}$$

$c=2R\sin C$ （菊池氏教科書範式68ヨリ）

$$S=\frac{ab\sin C}{2}\ (\text{範式}65)=\frac{abc\sin C}{2c}$$

$$=\frac{abc\,\cancel{\sin C}}{2\times 2R\,\cancel{\sin C}}=\frac{abc}{4R}$$

$$\therefore\ R=\frac{abc}{4S}$$

(193)

$$\cot B\cot C+\cot C\cot A+\cot A\cot B=1$$

$$\text{左辺}=\cot C(\cot B+\cot A)+\cot A\cot B$$

$$=\frac{\cos C}{\sin C}\left(\frac{\cos B}{\sin B}+\frac{\cos A}{\sin A}\right)+\cot A\cot B$$

$$=\frac{\cos C}{\sin C}\cdot\frac{\sin A\cos B+\cos A\sin B}{\sin A\sin B}+\cot A\cot B$$

$$=\frac{\cos C}{\cancel{\sin C}}\cdot\frac{\cancel{\sin(A+B)}}{\sin A\sin B}+\frac{\cos A\cos B}{\sin A\sin B}$$

$$=\frac{-\cos(A+B)+\cos A\cos B}{\sin A\sin B}$$

$$=\frac{\cancel{-\cos A\cos B}+\sin A\sin B\cancel{+\cos A\cos B}}{\sin A\sin B}=1=\text{右辺}$$

(194)

$$S=2R^2\sin A\sin B\sin C$$

$a=2R\sin A,\ b=2R\sin B,\ c=2R\sin C$

$$S=\frac{abc}{4R}\ (\text{ヨリ}192)=\frac{8R^3(\sin A\sin B\sin C)}{4R}$$

$$=2R^2\sin A\sin B\sin C$$

(195)

$$a(b^2+c^2)\cos A+b(c^2+a^2)\cos B+c(a^2+b^2)\cos C=3abc$$

遠藤氏三角法公式41ヨリ

$$a\times bc=b^2c\cos C+bc^2\cos B$$

$$b\times ac=ac^2\cos A+a^2c\cos C$$

$$c\times ab=a^2b\cos B+ab^2\cos A$$

以上三式辺々相加ヘテ後両辺ヲ互換スレバ

$$ab^2\cos A+ac^2\cos A+bc^2\cos B+a^2b\cos B+a^2c\cos C+b^2c\cos C=3abc$$

$$\therefore\ a(b^2+c^2)\cos A+b(c^2+a^2)\cos B+c(a^2+b^2)\times\cos C=3abc$$

(196)

$$a\cos A+b\cos B-c\cos C=2c\cos A\cos B$$

$$\text{左辺}=(b\cos C+c\cos B)\cos A+(c\cos A+a\cos C)\cos B-(b\cos A+a\cos B)\cos C$$

$$=\cancel{b\cos A\cos C}+c\cos A\cos B+c\cos A\cos B+\cancel{a\cos B\cos C}-\cancel{b\cos A\cos C}-\cancel{a\cos B\cos C}$$

$$=2c\cos A\cos B=\text{右辺}$$

(197)

$$S=\frac{a^2\sin B\sin C}{2\sin(B+C)}$$

$$S=\frac{1}{2}bc\sin A\ (\text{公式}43)=\frac{1}{2}\frac{a\sin B}{\sin A}\cdot\frac{a\sin C}{\cancel{\sin A}}\times\cancel{\sin A}$$

$$=\frac{a^2\sin B\sin C}{2\sin(B+C)}$$

(198)

$$r=\frac{a\sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{A}{2}}$$

$$r=\frac{S}{s}\ (\text{範式}69)=\sqrt{\frac{s(s-a)(s-b)(s-c)}{s^2}}$$

$$=a\sqrt{\frac{(s-b)(s-c)}{ca}}\times\sqrt{\frac{(s-c)(s-a)}{ab}}\div\sqrt{\frac{s(s-a)}{bc}}$$

$$=a\sin\frac{B}{2}\times\sin\frac{C}{2}\div\cos\frac{A}{2}\ (\text{範式}45,47\text{等})$$

$$=\frac{a\sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{A}{2}}$$

(199)

$$\frac{r}{R}=4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$$\text{左辺}=\sqrt{\frac{(p-a)(p-b)(p-c)}{p}}\div\frac{a}{2\sin A}$$

$$=\frac{\sqrt{(p-a)(p-b)(p-c)}}{\sqrt{p}}\div\left(\frac{2\times2}{bc}\cdot\frac{a}{\sqrt{p(p-a)(p-b)(p-c)}}\right)$$

$$=\frac{\frac{4}{bc}\cdot\sqrt{(p-a)(p-b)(p-c)}\times\sqrt{p(p-a)(p-b)(p-c)}}{a\sqrt{p}}$$

$$=\frac{\frac{4}{bc}\cdot\sqrt{p}\,(p-a)(p-b)(p-c)}{a\sqrt{p}}=\frac{4(p-a)(p-b)(p-c)}{abc}$$

$$\text{右辺}=4\sqrt{\frac{(p-b)(p-c)}{bc}}\times\sqrt{\frac{(p-c)(p-a)}{ca}}\times\sqrt{\frac{(p-a)(p-b)}{ab}}$$

$$=\frac{4(p-a)(p-b)(p-c)}{abc}=\text{左辺}$$

(200)

$$S=(rr_1r_2r_3)^{\frac{1}{2}}$$

$$r=\frac{S}{s},\ r_1=\frac{S}{s-a},\ r_2=\frac{S}{s-b},\ r_3=\frac{S}{s-c}$$

$$\therefore\ rr_1r_2r_3=\frac{S^4}{s(s-a)(s-b)(s-c)}$$

$$=\frac{S^4}{S^2}=S^2$$

$$\therefore\ S=\sqrt{rr_1r_2r_3}=\text{右辺}$$

(201)

$$(b^2-c^2)\cot A+(c^2-a^2)\cot B+(a^2-b^2)\cot C=0$$

$$\text{左辺}=(b^2-c^2)\frac{R(b^2+c^2-a^2)}{abc}+(c^2-a^2)\frac{R(c^2+a^2-b^2)}{abc}$$

$$+(a^2-b^2)\frac{R(a^2+b^2-c^2)}{abc}\ \left\{\begin{array}{l}\text{遠藤氏三角法ノ}\\\text{頁注意ニヨル}\end{array}\right\}$$

$$=\frac{R(b^2-c^2)(b^2+c^2-a^2)}{abc}+\frac{R(c^2-a^2)(c^2+a^2-b^2)}{abc}$$

$$+\frac{R(a^2-b^2)(a^2+b^2-c^2)}{abc}$$

$$=\frac{R\{b^4-c^4-a^2b^2+a^2c^2+c^4-a^4-b^2c^2+a^2b^2+a^4-b^4-a^2c^2+b^2c^2\}}{abc}$$

$$=\frac{R\times0}{abc}=0=\text{右辺}$$

(202)

$$R(\sin A+\sin B+\sin C)=s$$

$$\text{左辺}=R\left\{\frac{a}{2R}+\frac{b}{2R}+\frac{c}{2R}\right\}=R\times\frac{a+b+c}{2R}=\frac{a+b+c}{2}$$

$$=s=\text{右辺}$$

(203)

$\tan\frac{A}{2}=\frac{5}{6}$, $\tan\frac{B}{2}=\frac{20}{37}$ ナレバ $\tan C=\frac{20}{21}$

$$\tan\left(\frac{A}{2}+\frac{B}{2}\right)=\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}$$

$$=\frac{\frac{5}{6}+\frac{20}{37}}{1-\frac{5}{6}\times\frac{20}{37}}=\frac{5}{2}$$

而シテ $\tan\left(\frac{A}{2}+\frac{B}{2}+\frac{C}{2}\right)=\tan\left\{\left(\frac{A}{2}+\frac{B}{2}\right)+\frac{C}{2}\right\}$

$$=\frac{\tan\left(\frac{A}{2}+\frac{B}{2}\right)+\tan\frac{C}{2}}{1-\tan\left(\frac{A}{2}+\frac{B}{2}\right)\tan\frac{C}{2}}\cdots\cdots(1)$$

然ルニ $\tan\left(\frac{A}{2}+\frac{B}{2}+\frac{C}{2}\right)=\tan 90^\circ=\infty$ ナルカラ

以テ (1)ノ分母ハ 0ナラザルベカラズ

故ニ $1-\frac{5}{2}\tan\frac{C}{2}=0$ ∴ $\tan\frac{C}{2}=\frac{1}{\frac{5}{2}}=\frac{2}{5}$

故ニ $\tan C=\frac{2\tan\frac{C}{2}}{1-\tan^2\frac{C}{2}}=\frac{\frac{4}{5}}{1-\frac{4}{25}}=\frac{20}{21}$

(204)

$(b-c)\cos\frac{A}{2}=a\sin\frac{B-C}{2}$

左辺 $=a(\sin B-\sin C)\cos\frac{A}{2}$ (正弦比例ヨリ)

$=a\,2\cos\frac{B+C}{2}\sin\frac{B-C}{2}\cos\frac{A}{2}$

$=a\,2\sin\frac{A}{2}\cos\frac{A}{2}\sin\frac{B-C}{2}$

$=a\sin A\sin\frac{B-C}{2}=a\sin\frac{B-C}{2}=$ 右辺

(205)

$a\cos A+b\cos B+c\cos C=2a\sin B\sin C$

左辺 $=a\cos A+\{a\cos(B-C)\}$ (問188)

$=a\{\cos A+\cos(B-C)\}=a\{\cos(B-C)-\cos(B+C)\}$

$=2a\sin B\sin C=$ 右辺

(206)

$\tan\phi=\frac{2\sqrt{bc}}{b-c}\sin\frac{A}{2}$ ナレバ

$\log a=\log(b-c)-\log\cos\phi$

$$\tan^2\phi=\frac{4bc}{(b-c)^2}\sin^2\frac{A}{2}=\frac{2bc}{(b-c)^2}\cdot\frac{1-\cos A}{2}$$

$$1+\tan^2\phi=1+\frac{2bc-2bc\cos A}{(b-c)^2}$$

$$\sec^2\phi=\frac{(b-c)^2+2bc-2bc\cos A}{(b-c)^2}$$

$$\cos^2\phi=\frac{(b-c)^2}{b^2-2bc+c^2+2bc-2bc\cos A}=\frac{(b-c)^2}{a^2}$$

∴ $\cos\phi=\frac{b-c}{a}$ ∴ $a=\frac{b-c}{\cos\phi}$

∴ $\log a=\log(b-c)-\log\cos\phi$

(207)

$(a+b)\sin\frac{C}{2}=c\cos\frac{A-B}{2}$ ヨリ $\frac{a+b}{c}=\frac{\cos\frac{A-B}{2}}{\sin\frac{C}{2}}$

$$\frac{a+b}{c}=\frac{\sin A+\sin B}{\sin C}=\frac{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}{2\sin\frac{C}{2}\cos\frac{C}{2}}$$

$$=\frac{\cos\frac{C}{2}\cos\frac{A-B}{2}}{\sin\frac{C}{2}\cos\frac{C}{2}}=\frac{\cos\frac{A-B}{2}}{\sin\frac{C}{2}}=$$ 右辺

(208)

$b\cos A=a\cos B$ ナルトキハ $a=b$

即 $b\times\frac{b^2+c^2-a^2}{2bc}=a\times\frac{a^2+c^2-b^2}{2ca}$

即 $b^2+c^2-a^2=a^2+c^2-b^2$, $2a^2=2b^2$, $a=\pm b$

a, b ハ何レモ三角形ノ辺ナルヲ以テ負値ヲ捨テテ正値ノミヲ採リ依リテ $a=b$ トス

等式(角,辺,面積)

(209)

$$S = Rr(\sin A + \sin B + \sin C)$$ 先ヅ次ノ恒等式ヲ作リテ置クコトス

$$S = 2S \times \frac{abc}{4S} \times \frac{a+b+c\,(=2s)}{abc}$$

$$= \frac{2Sabc}{4S}\left(\frac{1}{bc} + \frac{1}{ca} + \frac{1}{ab}\right)$$

$$= \frac{abc}{4S}\left(\frac{2S}{bc} + \frac{2S}{ca} + \frac{2S}{ab}\right)\frac{S}{S}$$

$$= \frac{abc}{4S}\cdot\frac{S}{S}(\sin A + \sin B + \sin C)$$ (範式47)

$$= Rr(\sin A + \sin B + \sin C)$$

(210)

$$\sin\frac{A}{2} = \sqrt{\frac{(p-b)(p-c)}{bc}}$$

$$\sin^2\frac{A}{2} = \frac{1-\cos A}{2} = \frac{1-\frac{b^2+c^2-a^2}{2bc}}{2}$$ (公式42)

$$= \frac{a^2-b^2+2bc-c^2}{4bc} = \frac{a^2-(b-c)^2}{4bc} = \frac{(a+b-c)(a-b+c)}{4bc}$$

$$= \frac{2(p-c)2(p-b)}{4bc} \quad \therefore \sin\frac{A}{2} = \sqrt{\frac{(p-b)(p-c)}{bc}}$$

(注意) $\frac{A}{2}$ ハ鋭角ナルヨリシテ $\sqrt{\ }$ 前ノ符号ハ正ノミヲオモル

(211)

$$S = s^2\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}$$

$$右辺 = s^2\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}\cdot\sqrt{\frac{(s-c)(s-a)}{s(s-b)}}\cdot\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

$$= s^2\sqrt{\frac{(s-b)(s-c)(s-c)(s-a)(s-a)(s-b)}{s^3(s-a)(s-b)(s-c)}}$$

$$= s^2\sqrt{\frac{s(s-a)(s-b)(s-c)}{s^4}}$$

$$= \sqrt{s(s-a)(s-b)(s-c)} = S$$ (範式66)

等式(角,辺,面積)

(212)

$$\cos\frac{A}{2} = \sqrt{\frac{p(p-a)}{bc}}$$

$$\cos^2\frac{A}{2} = \frac{1+\cos A}{2} = \frac{1+\frac{b^2+c^2-a^2}{2bc}}{2} = \frac{b^2+2bc+c^2-a^2}{4bc}$$

$$= \frac{(b+c+a)(b+c-a)}{4bc} = \frac{2p\cdot 2(p-a)}{4bc}$$

$$\therefore \cos\frac{A}{2} = \sqrt{\frac{p(p-a)}{bc}}$$ (注意) $\sqrt{\ }$ 前ノ符号ハ(210)ニ同ジ

(213)

$$S = ra\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

$$右辺 = ra\sqrt{\frac{bc}{(s-b)(s-c)}}\cdot\sqrt{\frac{s(s-b)}{ca}}\cdot\sqrt{\frac{s(s-c)}{ab}}$$

(範式45—48)

$$= ra\sqrt{\frac{s^2}{a^2}} = ra\frac{s}{a} = rs = S$$

(214)

$$a\cos A = b\cos B$$

$$a\cdot\frac{b^2+c^2-a^2}{2bc} = b\cdot\frac{c^2+a^2-b^2}{2ca}$$

$$a^2(b^2+c^2-a^2) = b^2(c^2+a^2-b^2)$$

$$a^2b^2+a^2c^2-a^4 = b^2c^2+a^2b^2-b^4$$

$$(a^2-b^2)c^2 = a^4-b^4$$

$$(a^2-b^2)(c^2-a^2-b^2) = 0$$

$\therefore$ $a^2-b^2=0$ ヨリ $a=b$ 符号ハ(208)ニヨル

$c^2-a^2-b^2=0$ ヨリ $a^2+b^2=c^2$

此最後ノ式ニヨレバ此三角形ハ直角三角形(ピタゴラスノ定理ニヨリテ)ナリ故ニ辺 c ノ対角 C ハ 90° ナリ

等式(角辺,面積)

(215)

$$r_2r_3+r_3r_1+r_1r_2=s^2$$

$$左辺=\frac{s}{s-b}\times\frac{s}{s-c}+\frac{s}{s-c}\times\frac{s}{s-a}+\frac{s}{s-a}\times\frac{s}{s-b}$$

$$=\frac{s^2}{(s-b)(s-c)}+\frac{s^2}{(s-c)(s-a)}+\frac{s^2}{(s-a)(s-b)}$$

$$=\frac{s^2\{(s-a)+(s-b)+(s-c)\}}{(s-a)(s-b)(s-c)}$$

$$=\frac{s^2\{3s-(a+b+c)\}s}{s(s-a)(s-b)(s-c)}\ (\text{等式}66)\text{ヨリ}$$

$$=s(3s-2s)=s\cdot s=s^2$$

(216)

A,B,Cガ A·Pヲナセバ $b=\sqrt{c^2-ca+a^2}$

$A-B=B-C$ 從テ $A+C=2B$

然ルニ $A+B+C=180^\circ$ ナルヲ以テ $B=60^\circ$

而シテ $b^2=c^2+a^2-2ca\cos B$ (公式42)

$$=c^2+a^2-2ca\times\frac{1}{2}$$

$$\therefore\ b=\sqrt{c^2-ca+a^2}$$

(注意) b ハ $B(60^\circ)$ ニ對スル辺ナルヲ以テ符号ヲ正トス

(217)

$$\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}=\frac{a^2+b^2+c^2}{2abc}$$

$$左辺=\frac{\frac{b^2+c^2-a^2}{2bc}}{a}+\frac{\frac{c^2+a^2-b^2}{2ac}}{b}+\frac{\frac{a^2+b^2-c^2}{2ab}}{c}$$

$$=\frac{b^2+c^2-a^2+c^2+a^2-b^2+a^2+b^2-c^2}{2abc}$$

$$=\frac{a^2+b^2+c^2}{2abc}=右辺$$

(218)

$$S=p^2\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}$$

$$右辺=p^2\left\{\frac{1}{p-a}\sqrt{\frac{(p-a)(p-b)(p-c)}{p}}\times\frac{1}{p-b}\sqrt{\frac{(p-a)(p-b)(p-c)}{p}}\right.$$

$$\left.\times\frac{1}{p-c}\sqrt{\frac{(p-a)(p-b)(p-c)}{p}}\right\}\ (\text{公式}46)$$

$$=p^2\left\{\frac{1}{(p-a)(p-b)(p-c)}\left(\sqrt{\frac{(p-a)(p-b)(p-c)}{p}}\right)^3\right.$$

$$=\frac{p^2}{(p-a)(p-b)(p-c)}\cdot\frac{(p-a)(p-b)(p-c)}{p}\cdot\sqrt{\frac{(p-a)(p-b)(p-c)}{p}}$$

$$=\sqrt{p(p-a)(p-b)(p-c)}=S\ (\text{公式}44)$$

(注意) 本解ハ217ノ別解ナリ

(219)

$a\sin\theta+b\cos\theta=c$ ...(1)　$a'\sin\theta+b'\cos\theta=c'$ ...(2)

ヨリ θ ヲ消去スベシ

(1)ニ a' ヲ (2)ニ a ヲ各両辺ニ乗ジテ相減ズレバ

$$aa'\sin\theta+ab'\cos\theta=a'c$$
$$aa'\sin\theta+ab'\cos\theta=ac'$$

即チ　$ab'\cos\theta-ab'\cos\theta=a'c-ac'$

$$(ab'-ab')\cos\theta=a'c-ac'$$

$$\therefore\quad \cos\theta=\frac{a'c-ac'}{ab'-ab'}$$

又 (1)ニ b' ヲ (2)ニ b ヲ各両辺ニ乗ジテ相減ズレバ

$$ab'\sin\theta+bb'\cos\theta=b'c$$
$$ab'\sin\theta+bb'\cos\theta=bc'$$

即チ　$(ab'-ab)\sin\theta=b'c-bc'$

$$\therefore\quad \sin\theta=\frac{b'c-bc'}{ab'-ab}$$

備　$\sin^2\theta+\cos^2\theta=\left(\frac{b'c-bc'}{ab'-ab}\right)^2+\left(\frac{a'c-ac'}{ab'-ab'}\right)^2=1$

$$\frac{(b'c-bc')^2}{(ab'-ab)^2}+\frac{(a'c-ac')^2}{(ab'-ab)^2}=1$$

$$\therefore\quad (b'c-bc')^2+(a'c-ac')^2=(ab'-ab)^2 \text{ 答}$$

(220)

$\sin^4A+(\cos^2A+2\sin^2A)\cos^2A$ ノ値ハ恒ニ一定スルコトヲ示セ.

$$\text{此式}=\sin^4A+\cos^4A+2\sin^2A\cos^2A$$
$$=(\sin^2A+\cos^2A)^2=1$$

即チ $\sin A$, $\cos A$ ノ如何ニ関セズ恒ニ一定ナリ

(221)

$\operatorname{cosec}A$ ヲ用ヒテ $(\sec A-\tan A)^2$ ヲ表ハセ

$\operatorname{cosec}^2A=1+\cot^2A$ ヨリ $\cot A=\sqrt{\operatorname{cosec}^2A-1}$

ヲ得, 従テ　$\tan A=\dfrac{1}{\sqrt{\operatorname{cosec}^2A-1}}$ ヲ得

又　$\cos A=\cot A\sin A=\sqrt{\operatorname{cosec}A-1}\cdot\dfrac{1}{\operatorname{cosec}A}$

$$=\frac{\sqrt{\operatorname{cosec}A-1}}{\operatorname{cosec}A}$$

従テ　$\sec A=\dfrac{\operatorname{cosec}A}{\sqrt{\operatorname{cosec}A-1}}$

$$\therefore\quad \text{原式}=\left\{\frac{\operatorname{cosec}A}{\sqrt{\operatorname{cosec}A-1}}-\frac{1}{\sqrt{\operatorname{cosec}A-1}}\right\}^2$$

$$=\frac{(\operatorname{cosec}A-1)^2}{\operatorname{cosec}^2A-1}=\frac{(\operatorname{cosec}A-1)^{\cancel{2}}}{(\operatorname{cosec}A+1)\cancel{(\operatorname{cosec}A-1)}}$$

$$=\underline{\underline{\frac{\operatorname{cosec}A-1}{\operatorname{cosec}A+1}}}$$

(222)

$\sin B$ ヲ用ヒテ $(\sec B-\tan B)^2$ ヲ表ハセ

$\sec B=\dfrac{1}{\sqrt{1-\sin^2B}}$,　$\tan B=\dfrac{\sin B}{\sqrt{1-\sin^2B}}$

$$\therefore\quad \text{原式}=\left\{\frac{1}{\sqrt{1-\sin^2B}}-\frac{\sin B}{\sqrt{1-\sin^2B}}\right\}^2=\left\{\frac{1-\sin B}{\sqrt{1-\sin^2B}}\right\}^2$$

$$=\frac{(1-\sin B)^2}{1-\sin^2B}=\frac{(1-\sin B)^{\cancel{2}}}{\cancel{(1-\sin B)}(1+\sin B)}$$

$$=\underline{\underline{\frac{1-\sin B}{1+\sin B}}}$$

式ノ変化

(223)

$a^2\tan^2\theta+b^2\cot^2\theta$ ノ最小値ヲ求メヨ

此式 $=(a\tan\theta-b\cot\theta)^2+2ab\tan\theta\cot\theta$

$=(a\tan\theta-b\cot\theta)^2+2ab$

偖 ab ハ常数ナルヲ以テ原式ヲ最小ナラシメンニハ

$a\tan\theta-b\cot\theta=0$ ナルヲ要ス

故ニ原式最小値ハ $2ab$ ニシテ

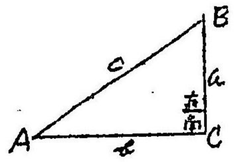
$a\tan\theta=b\cot\theta$ ナルトキナリ

(224)

$\sin A=\frac{a}{c}$ $\therefore a=c\sin A$

$\cos B=\frac{a}{c}$ $\therefore a=c\cos B$

$\tan A=\frac{a}{b}$ $\therefore a=b\tan A$



(225)

$\frac{\sin(90^\circ+A)\cos(90^\circ+A)}{\cos(180^\circ+A)}+\frac{\sin(180^\circ-A)\cos(90^\circ+A)}{\sin(180^\circ+A)}$ ヲ簡単ニスルニ

$=\frac{\cos A(-\sin A)}{-\cos A}+\frac{\sin A(-\sin A)}{-\sin A}$

$=\sin A+\sin A=\underline{\underline{2\sin A}}$

(226)

$\sin 150^\circ$ ヲ鋭角ノ同名函数ニテ表ハセ

$=\sin(1\times180^\circ-30^\circ)=\sin(180^\circ-30^\circ)=\sin 30^\circ$ (範式10)

(227)

$\sin 90^\circ+\tan^2(180^\circ-\alpha)-\operatorname{cosec}^2(90^\circ-\alpha)$ ヲ簡単ニスベシ

$=1+\tan^2\alpha-\sec^2\alpha$

$=\sec^2\alpha-\sec^2\alpha=0$

式ノ変化

(228)

次ノ各函数ヲ鋭角ノ同名函数ニテ表ハセ

$\tan(-80^\circ)=-\underline{\underline{\tan 80^\circ}}$ (範式5)

$\cos 220^\circ=\cos(180^\circ+40^\circ)=-\underline{\underline{\cos 40^\circ}}$ (範式10)

$\sec 300^\circ=\sec(2\times180^\circ-60^\circ)=\sec(-60^\circ)$

$=\underline{\underline{\sec 60^\circ}}$ (範式5)

$\cot(-500^\circ)=\cot(-3\times180^\circ+40^\circ)$

$=\cot(180^\circ+40^\circ)=\underline{\underline{\cot 40^\circ}}$ (範式10)

$\operatorname{cosec}(-36^\circ)=-\underline{\underline{\operatorname{cosec} 36^\circ}}$ (範式5)

(229)

$\frac{\sin(180^\circ-A)}{\tan(180^\circ+A)}\cdot\frac{\cot(90^\circ-A)}{\tan(90^\circ+A)}\cdot\frac{\cos(360^\circ-A)}{-\sin(-A)}$ ヲ簡単ニセヨ

$=\frac{\sin A}{\tan A}\cdot\frac{\tan A}{-\cot A}\cdot\frac{\cos A}{-\sin A}=\frac{\cos A}{\cot A}=\underline{\underline{\sin A}}$

(230)

次ノ函数ヲ鈍角ノ同名函数ニテ表ハセ

$\sin 25^\circ=\sin(180^\circ-25^\circ)=\underline{\underline{\sin 155^\circ}}$

$\cot 16^\circ=-\cot(180^\circ-16^\circ)=\underline{\underline{-\cot 164^\circ}}$

(231)

$\tan(180^\circ+A)\sin(90^\circ+A)+\cos(180^\circ-A)\cot(180^\circ-A)$ ヲ簡ニセヨ

$=\tan A\cos A+(-\cos A)(-\cot A)$

$=\tan A\cos A+\cos A\cot A$

$=\sin A+\frac{\cos^2 A}{\sin A}=\frac{\sin^2 A+\cos^2 A}{\sin A}$

$=\frac{1}{\sin A}=\underline{\underline{\operatorname{cosec} A}}$

(232)

次ノ各函數ヲ鋭角ノ正弦ヲ用ヒテ表ハセ

$\sin 112^\circ = \sin(180^\circ - 112^\circ) = \underline{\underline{\sin 68^\circ}}$

$\cos(-350^\circ) = \cos(2\times 180^\circ - 350^\circ) = \cos 10^\circ$

$= \underline{\underline{\sin 80^\circ}}$

(233)

$\cos(180^\circ + x)\cos(90^\circ + y) - \sin(90^\circ - x)\sin(180^\circ - y)$

ヲ簡單ニセヨ

原式 $= -\cos x(-\sin y) - \cos x \sin y$

$= \cos x \sin y - \cos x \sin y = \underline{\underline{0}}$

(234)

$\sin 3A \sin 2B$ ヲ二函數ノ差ニ変形セヨ

$= \dfrac{2\sin 3A \sin 2B}{2}$

$= \dfrac{1}{2}\{\cos(3A - 2B) - \cos(3A + 2B)\}$

(235)

次ノ函數ヲAノ函數ニテ表ハセ

$\sin(270^\circ \pm A) = \sin(3\times 90^\circ \pm A) = \underline{\underline{-\cos A}}$

$\cos(270^\circ \pm A) = \cos(3\times 90^\circ \pm A) = \underline{\underline{\sin A}}$

(236)

$\cos(A+B)\cos(A-B)$ ヲ二函數ノ和ニ変形セヨ

$= \frac{1}{2}\{2\cos(A+B)\cos(A-B)\}$

$= \frac{1}{2}\{\cos(A+B+A-B) + \cos(A+B-A+B)\}$ (第三法)

$= \frac{1}{2}(\cos 2B + \cos 2A)$

(237)

次ノ二ツヲ最小正角ノ函數ニテ表ハセ

$\sin 1275^\circ = \sin(3\times 360^\circ + 2\times 90^\circ + 15^\circ) = \underline{\underline{-\sin 15^\circ}}$

$\tan(-2232^\circ) = \tan(-6\times 360^\circ - 90^\circ + 18^\circ) = \underline{\underline{-\cot 18^\circ}}$

(注意) $-90^\circ + 18^\circ$ ハ第四象限ノ角ニシテ $\pi/2$ ヲ以テ

此答ヲ得

(238)

$2\cos\frac{3A}{2}\cos\frac{A}{2}$ ヲ二函數ノ和ニ変形セヨ

$= 2\cos\frac{2A+A}{2}\cos\frac{2A-A}{2} = \cos A + \cos 2A$

(239)

$\sin\alpha + \sin\beta = a \ldots(1)$, $\cos\alpha + \cos\beta = b \ldots(2)$, $\cos(\alpha-\beta) = c \ldots(3)$

ヨリ α, β ヲ消去セヨ

(1)ヲ平方シテ $\sin^2\alpha + 2\sin\alpha\sin\beta + \sin^2\beta = a^2$

(2)ヲ平方シテ $\cos^2\alpha + 2\cos\alpha\cos\beta + \cos^2\beta = b^2$

此両式辺々相加フレバ

$\sin^2\alpha + \cos^2\alpha + \sin^2\beta + \cos^2\beta + 2\sin\alpha\sin\beta$

$+ 2\cos\alpha\cos\beta = a^2 + b^2$

$2 + 2\sin\alpha\sin\beta + 2\cos\alpha\cos\beta = a^2 + b^2$

$2(1 + \sin\alpha\sin\beta + \cos\alpha\cos\beta) = a^2 + b^2 \ldots(4)$

(3)ヨリ $\cos\alpha\cos\beta + \sin\alpha\sin\beta = c$

之ヲ(4)ニ代入シテ

$2(1 + c) = a^2 + b^2$ 即 $2 + 2c = a^2 + b^2$

$\therefore\ a^2 + b^2 - 2c = 2$

式ノ変化

(240)

$\cos\frac{5A}{2}\sin\frac{3A}{2}$ ヲ二函數ノ差ニ変形セヨ

$$=\frac{2\cos\frac{4A+A}{2}\sin\frac{4A-A}{2}}{2}=\frac{1}{2}(\sin 4A-\sin A)$$

(241)

次ノ二式ヲ簡單ニセヨ

$$\cos(15^\circ-A)\sec 15^\circ-\sin(15^\circ-A)\operatorname{cosec}15^\circ$$

$$=\frac{\cos(15^\circ-A)}{\cos 15^\circ}-\frac{\sin(15^\circ-A)}{\sin 15^\circ}$$

$$=\frac{\sin 15^\circ\cos(15^\circ-A)-\sin(15^\circ-A)\cos 15^\circ}{\sin 15^\circ\cos 15^\circ}$$

$$=\frac{\sin\{15^\circ-(15^\circ-A)\}}{\sin 15^\circ\cos 15^\circ}=\frac{2\sin A}{2\sin 15^\circ\cos 15^\circ}$$

$$=\frac{2\sin A}{\sin 30^\circ}=\frac{2\sin A}{\frac{1}{2}}=\underline{\underline{4\sin A}}$$

$$1+\tan\alpha\tan\frac{\alpha}{2}=1+\frac{\sin\alpha}{\cos\alpha}\cdot\frac{1-\cos\alpha}{\sin\alpha}\quad(\text{公式36})$$

$$=\frac{\sin\alpha\cos\alpha+\sin\alpha-\sin\alpha\cos\alpha}{\sin\alpha\cos\alpha}=\underline{\underline{\sec\alpha}}$$

(242)

次ノ二式ヲ二函數ノ和又ハ差ニ変形セヨ

$$\cos 10^\circ\sin 50^\circ=\frac{1}{2}\cdot 2\cos 10^\circ\sin 50^\circ$$

$$=\frac{1}{2}\{\sin(10^\circ+50^\circ)-\sin(10^\circ-50^\circ)\}=\frac{1}{2}(\sin 60^\circ+\sin 40^\circ)$$

$$\cos 45^\circ\sin 15^\circ=\frac{2\cos 45^\circ\sin 15}{2}$$

$$=\frac{\sin(45^\circ+15^\circ)-\sin(45^\circ-15^\circ)}{2}$$

$$=\frac{1}{2}(\sin 60^\circ-\sin 30^\circ)$$

式ノ変化

(243)

$\sec 2A-\cos 2A$ ヲ $\tan A$ ニテ表ハセ

$$=\frac{1}{\cos 2A}-\cos 2A=\frac{1-\cos^2 2A}{\cos 2A}$$

$$=\frac{1-(2\cos^2 A-1)^2}{\cos^2 A-\sin^2 A}=\frac{1-4\cos^4 A+4\cos^2 A-1}{\cos^2 A-\sin^2 A}$$

$$=\frac{4\cos^2 A(1-\cos^2 A)}{\cos^2 A-\sin^2 A}$$ 分母子ヲ $\cos^2 A$ ニテ割レバ

$$=\frac{4(1-\cos^2 A)}{1-\tan^2 A}=\frac{4\left(1-\frac{1}{\sec^2 A}\right)}{1-\tan^2 A}$$

$$=\frac{4\left(1-\frac{1}{1+\tan^2 A}\right)}{1-\tan^2 A}=\frac{4\cdot\frac{1+\tan^2 A-1}{1+\tan^2 A}}{1-\tan^2 A}=\underline{\underline{\frac{4\tan^2 A}{1-\tan^4 A}}}$$

(244)

$2\cos 2\theta\cos\theta-2\sin 4\theta\sin\theta$ ヲ簡單ニセヨ

$$=\cos(2\theta-\theta)+\cos(2\theta+\theta)-\cos(4\theta-\theta)+\cos(4\theta+\theta)$$

$$=\cos\theta+\cos 5\theta=2\cos\frac{5\theta+\theta}{2}\cos\frac{5\theta-\theta}{2}$$

$$=2\cos 3\theta\cos 2\theta$$

(245)

$\sin 5A$ ヲ $\sin A$ ニテ表ハセ

$$\sin 5A=\sin 5A+\sin A-\sin A$$

$$=2\sin 3A\cos 2A-\sin A$$

$$=2(3\sin A-4\sin^3 A)(1-2\sin^2 A)-\sin A$$

$$=6\sin A-8\sin^3 A-12\sin^3 A+16\sin^5 A-\sin A$$

$$=5\sin A-20\sin^3 A+16\sin^5 A$$

式ノ変化

(246)

$\dfrac{\cos\alpha-\cos 5\alpha}{\sin\alpha-\sin 5\alpha}$ ヲ簡単ニセヨ

$$=\frac{2\sin\frac{5\alpha+\alpha}{2}\cdot\sin\frac{5\alpha-\alpha}{2}}{2\sin\frac{\alpha+5\alpha}{2}\cdot\cos\frac{\alpha-5\alpha}{2}}$$

$$=\frac{\sin\frac{5\alpha-\alpha}{2}}{\cos\frac{5\alpha-\alpha}{2}}=\frac{\sin 2\alpha}{\cos 2\alpha}=\underline{\underline{\tan 2\alpha}}$$

(247)

$\cos^2 A+\cos^2(A+120^\circ)+\cos^2(A+240^\circ)$ ヲ簡単ニセヨ

$$=\frac{1+\cos 2A}{2}+\frac{1+\cos(2A+240)}{2}$$

$$+\frac{1+\cos(2A+480)}{2}\quad(\text{公式}31)$$

$$=\frac{3}{2}+\frac{\cos 2A+\cos(2A+240^\circ)+\cos(2A+480^\circ)}{2}$$

$$=\frac{3}{2}+\frac{\cos 2A+2\cos(2A+360^\circ)\cos 120^\circ}{2}\quad(\text{公式}29)$$

$$=\frac{3}{2}+\frac{\cos 2A+2\cos 2A\left(-\frac{1}{2}\right)}{2}$$

$$=\frac{3}{2}+\frac{\cos 2A-\cos 2A}{2}=\underline{\underline{\frac{3}{2}}}$$

(248)

Aガ180°ト360°トノ間ニアルトキ$\cos A$ヲ用ヒテ$\sin\frac{A}{2}$ヲ表ハセ

$\frac{A}{2}$ハ第二象限内ニアルヲシテ$\sin\frac{A}{2}$ハ正ナリ

故ニ $\sin\frac{A}{2}=+\sqrt{\frac{1-\cos A}{2}}$

式ノ変化

(249)

二鋭角α, βノ和ガ定角δニ等シキトキ$\cos\alpha\cos\beta$ノ最大値ヲ求メヨ

$$\cos\alpha\cos\beta=\frac{2\cos\alpha\cos\beta}{2}$$

$$=\frac{1}{2}\cos(\alpha+\beta)+\frac{1}{2}\cos(\alpha-\beta)$$

$$=\frac{1}{2}\cos\delta+\frac{1}{2}\cos(\alpha-\beta)$$

今原式ヲ最大ナラシメンニハ上式中ハ$\frac{1}{2}\cos(\alpha-\beta)$ヲ最大ナラシムルヲ要ス而シテ余弦ノ最大値ハ1ナルヲ以テ上式ハ次ノ如クナル

$$\frac{1}{2}\cos\delta+\frac{1}{2}\times 1=\frac{1}{2}(1+\cos\delta)$$

$$=\cos^2\frac{\delta}{2}\quad(\text{公式}31)$$

∴ 原式$=\cos^2\frac{\delta}{2}$ナルトキ最大ナリ

(250)

Aガ$(4n+1)\times 180^\circ$ト$(4n+3)\times 180^\circ$トノ間ニアリテnガ正ノ整数ナルトキ, $\cos A$ヲ用ヒテ$\cos\frac{A}{2}$ヲ表ハセ

$\frac{A}{2}$ハ$\frac{(4n+1)\times 180^\circ}{2}$ト$\frac{(4n+3)\times 180^\circ}{2}$トノ間即チ

$2n\times 180^\circ+90^\circ$ト$2n\times 180^\circ+270^\circ$トノ間即チ

第二象限ト第三象限トノ中ノ角ナリ

故ニ $\cos\frac{A}{2}=-\sqrt{\frac{1+\cos A}{2}}$

(251)

$7\log_a\frac{15}{16}-6\log_a\frac{3}{8}+5\log_a\frac{2}{5}-\log_a\frac{25}{32}$ ヲ簡単ニセヨ

$$=\log_a\left\{\left(\frac{15}{16}\right)^7\div\left(\frac{3}{8}\right)^6\times\left(\frac{2}{5}\right)^5\div\frac{25}{32}\right\}=\underline{\log_a 3}$$

(252)

$\sin 3\theta+\sin 2\theta+2\sin\frac{3\theta}{2}\cos\frac{\theta}{2}$ ヲ簡単ニセヨ

$$=2\sin\frac{3\theta+2\theta}{2}\cos\frac{3\theta-2\theta}{2}+2\sin\frac{3\theta}{2}\cos\frac{\theta}{2}$$

$$=2\sin\frac{5\theta}{2}\cos\frac{\theta}{2}+2\sin\frac{3\theta}{2}\cos\frac{\theta}{2}$$

$$=2\cos\frac{\theta}{2}\left(\sin\frac{5\theta}{2}+\sin\frac{3\theta}{2}\right)$$

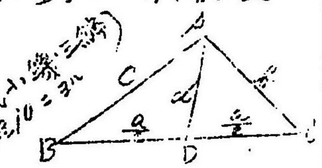
$$=2\cos\frac{\theta}{2}\left\{2\sin\frac{5\theta+3\theta}{4}\cos\frac{5\theta-3\theta}{4}\right\}$$

$$=4\cos\frac{\theta}{2}\sin 2\theta\cos\frac{\theta}{2}=\underline{\underline{4\cos^2\frac{\theta}{2}\sin 2\theta}}$$

(253)

三角形ノ三辺ヲ a, b, c トセバ a ニ對スル中線ノ長サハ

$\frac{1}{2}\sqrt{b^2+c^2+2bc\cos A}$ ナリ



(証明ノ際三角形 ABD ニ於テ)

今中線 AD ノ長サヲ d トセバ

$$b^2+c^2=2\left\{\left(\frac{a}{2}\right)^2+d^2\right\}$$

$$=\frac{a^2}{2}+2d^2=\frac{b^2+c^2-2bc\cos A}{2}+2d^2$$

$$2(b^2+c^2)=b^2+c^2-2bc\cos A+4d^2$$

$$4d^2=2(b^2+c^2)-(b^2+c^2)+2bc\cos A$$

$$d^2=\frac{1}{4}(b^2+c^2+2bc\cos A)$$

$$\therefore\ d=\frac{1}{2}\sqrt{b^2+c^2+2bc\cos A}$$

(254)

a, b, c ガ A·P ヲナストキハ $\cot\frac{A}{2}, \cot\frac{B}{2}, \cot\frac{C}{2}$ モ亦 A·P ヲナス

$b-a=c-b$ (a, b, c ガ A·P ヲナスカラ出発ス)

$\therefore\ \frac{a-b}{r}=\frac{b-c}{r}$ (上ノ等式ノ両辺ヲ r ニテ除シタル)

即 $\frac{p-b}{r}-\frac{p-a}{r}=\frac{p-c}{r}-\frac{p-b}{r}$

即 $\cot\frac{B}{2}-\cot\frac{A}{2}=\cot\frac{C}{2}-\cot\frac{B}{2}$

(公式46ヨリ)

$\therefore\ \cot\frac{A}{2}, \cot\frac{B}{2}, \cot\frac{C}{2}$ ハ A·P ヲナス

(255)

$\sin A+\sin B>\sin C$

$$\sin A+\sin B=\frac{a}{2R}+\frac{b}{2R}\ (\text{正弦比例ヨリ})$$

$$=\frac{a+b}{2R}$$

$$\sin C=\frac{c}{2R}$$

而シテ $a+b>c$ (幾何学定理)

$\therefore\ \sin A+\sin B>\sin C$

(256)

二鋭角 α, β ノ和ガ定角 δ ニ等シキトキ $\cos\alpha+\cos\beta$ ノ最大値ヲ求ム

$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$ 此式ニ於テ $2\cos\frac{\alpha+\beta}{2}$ 即 $2\cos\frac{\delta}{2}$ ハ定値ヲ有スルヲ以テ此式ノ最大値ハ $\cos\frac{\alpha-\beta}{2}$ ノ最大ナルヲ要ス故ニ所要ノ最大値ハ $\underline{2\cos\frac{\delta}{2}}$ ナリ

式ノ変化

(257)

$\sin^4 A - \cos^4 A$ ノ最大値ヲ求メヨ

$= (\sin^2 A + \cos^2 A)(\sin^2 A - \cos^2 A) = 1 \times (1 - 2\cos^2 A)$

$= 1 - 2\cos^2 A$

某式ヲ最大ナラシメンニハ $2\cos^2 A$ ヲ 0 ナラシムル事ヲ

即 $2\cos^2 A = 0$, $\cos A = 0$

故ニ $A = 90^\circ$ or 270° or ……ナルトキ原式ハ

最大ニシテ其値ハ 1 ナリ

(258)

$(a+b)\cos C + c(\cos A + \cos B) > a\cos B + b\cos A$

左辺 $= a\cos C + b\cos C + c\cos A + c\cos B$

$= (a\cos C + c\cos A) + (b\cos C + c\cos B)$

$= a + b$ (公式41)

右辺 $= c$

即チ三角形ノ二辺ノ和ハ他ノ一辺ヨリ大ナルヲ以テ

右式ハ証明セラル

(259)

$\sin^{-1} a = \cos^{-1}\sqrt{1-a^2} = \tan^{-1}\frac{a}{\sqrt{1-a^2}} = \cot^{-1}\frac{\sqrt{1-a^2}}{a}$

$= \sec^{-1}\frac{1}{\sqrt{1-a^2}} = \operatorname{cosec}^{-1}\frac{1}{a}$

$\sin^{-1} a = A$ トセバ $\sin A = a$

$\cos A = \sqrt{1-\sin^2 A} = \sqrt{1-a^2}$ $\therefore \cos^{-1}\sqrt{1-a^2} = A$

$\tan A = \frac{\sin A}{\cos A} = \frac{a}{\sqrt{1-a^2}}$ $\therefore \tan^{-1}\frac{a}{\sqrt{1-a^2}} = A$

同様ニ $\cot^{-1}\frac{\sqrt{1-a^2}}{a} = A$, $\sec^{-1}\frac{1}{\sqrt{1-a^2}} = A$

$\operatorname{cosec}^{-1}\frac{1}{a} = A$

故ニ此ノ六ツノ逆函數ハ相等シ

(260)

$\cos^{-1} a = \sin^{-1}\sqrt{1-a^2} = \tan^{-1}\frac{\sqrt{1-a^2}}{a} = \cot^{-1}\frac{a}{\sqrt{1-a^2}}$

$= \sec^{-1}\frac{1}{a} = \operatorname{cosec}^{-1}\frac{1}{\sqrt{1-a^2}}$

$\cos^{-1} a = A$ トセバ $\cos A = a$

$\sin A = \sqrt{1-\cos^2 A} = \sqrt{1-a^2}$ $\therefore \sin^{-1}\sqrt{1-a^2} = A$

$\tan A = \frac{\sin A}{\cos A} = \frac{\sqrt{1-a^2}}{a}$ $\therefore \tan^{-1}\frac{\sqrt{1-a^2}}{a} = A$

同様ニ $\cot^{-1}\frac{a}{\sqrt{1-a^2}} = A$, $\sec^{-1}\frac{1}{a} = A$

$\operatorname{cosec}^{-1}\frac{1}{\sqrt{1-a^2}} = A$

故ニ此六ツノ逆函數ハ相等シ

(261)

$$\tan^{-1}a=\sin^{-1}\frac{a}{\sqrt{1+a^2}}=\cos^{-1}\frac{1}{\sqrt{1+a^2}}=\cot^{-1}\frac{1}{a}$$
$$=\sec^{-1}\sqrt{1+a^2}=\operatorname{cosec}^{-1}\frac{\sqrt{1+a^2}}{a}$$

$\tan^{-1}a=A$ トセバ $\tan A=a$

$\sec A=\sqrt{1+\tan^2 A}=\sqrt{1+a^2}$ $\therefore \sec^{-1}\sqrt{1+a^2}=A$

$\cos A=\frac{1}{\sec A}=\frac{1}{\sqrt{1+a^2}}$ $\therefore \cos^{-1}\frac{1}{\sqrt{1+a^2}}=A$

$\sin A=\tan A\cos A=\frac{a}{\sqrt{1+a^2}}$ $\therefore \sin^{-1}\frac{a}{\sqrt{1+a^2}}=A$

同様ニ $\cot^{-1}\frac{1}{a}=A$, $\operatorname{cosec}^{-1}\frac{\sqrt{1+a^2}}{a}=A$

故ニ此六ツノ逆函數ハ相等シ

(262)

$$\cot^{-1}a=\sin^{-1}\frac{1}{\sqrt{1+a^2}}=\cos^{-1}\frac{a}{\sqrt{1+a^2}}=\tan^{-1}\frac{1}{a}$$
$$=\sec^{-1}\frac{\sqrt{1+a^2}}{a}=\operatorname{cosec}^{-1}\sqrt{1+a^2}$$

$\cot^{-1}a=A$ トセバ $\cot A=a$

$\operatorname{cosec}A=\sqrt{1+\cot^2 A}=\sqrt{1+a^2}$ $\therefore \operatorname{cosec}^{-1}\sqrt{1+a^2}=A$

$\sin A=\frac{1}{\operatorname{cosec}A}=\frac{1}{\sqrt{1+a^2}}$ $\therefore \sin^{-1}\frac{1}{\sqrt{1+a^2}}=A$

$\cos A=\cot A\sin A=\frac{a}{\sqrt{1+a^2}}$ $\therefore \cos^{-1}\frac{a}{\sqrt{1+a^2}}=A$

同様ニ $\tan^{-1}\frac{1}{a}=A$, $\sec^{-1}\frac{\sqrt{1+a^2}}{a}=A$

故ニ此六ツノ逆函數ハ相等シ

(263)

$$\cos^{-1}a\pm\cos^{-1}b=\cos^{-1}\{ab\mp\sqrt{(1-a^2)(1-b^2)}\}$$

$\cos^{-1}a=A$ トセバ $\cos A=a$

$\cos^{-1}b=B$ トセバ $\cos B=b$

偖 $\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$

$=ab\mp\sqrt{1-\cos^2 A}\sqrt{1-\cos^2 B}$

$=ab\mp\sqrt{1-a^2}\sqrt{1-b^2}$

$\therefore A\pm B=\cos^{-1}\{ab\mp\sqrt{(1-a^2)(1-b^2)}\}$

$\therefore \cos^{-1}a\pm\cos^{-1}b=\cos^{-1}\{ab\mp\sqrt{(1-a^2)(1-b^2)}\}$

(264)

$$2\sin^{-1}a=\sin^{-1}2a\sqrt{1-a^2}$$

$\sin^{-1}a=A$ トセバ $\sin A=a$

又 $\sin 2A=2\sin A\cos A=2\sin A\sqrt{1-\sin^2 A}$

$=2a\sqrt{1-a^2}$

$\therefore 2A=\sin^{-1}2a\sqrt{1-a^2}$

$\therefore 2\sin^{-1}a=\sin^{-1}2a\sqrt{1-a^2}$

(265)

$$2\tan^{-1}a=\tan^{-1}\frac{2a}{1-a^2}$$

$\tan^{-1}a=A$ トセバ $\tan A=a$

又 $\tan 2A=\frac{2\tan A}{1-\tan^2 A}=\frac{2a}{1-a^2}$

$\therefore 2A=\tan^{-1}\frac{2a}{1-a^2}$

$\therefore 2\tan^{-1}a=\tan^{-1}\frac{2a}{1-a^2}$

計算（辺，角）

(266)

正五辺形ノ一角ヲ度數ニテ示セ

幾何學ノ公式ニヨリ，正五辺形ノ一角 $=\frac{2(n-2)}{n}$ 直角

$=\frac{2(5-2)}{5}\times 90$ 度 $=$ 108度

正八辺形，正十辺形，正十五辺形等ハ夫々次ノ直角度ニ直セバヨシ

$\frac{2(8-2)}{8}$，$\frac{2(10-2)}{10}$，$\frac{2(15-2)}{15}$

(267)

0.678直角 $=(.678\times 90)^\circ=61.02^\circ$

$0.02^\circ=(.02\times 60)'=1.2'$

$0.2'=(.2\times 60)''=12''$

$\therefore$ 0.678直角 $=61^\circ 1' 12''$

(268)

半径一尺ノ円ニ於テ一尺ノ弧ノ上ニ立ツ中心角ハ何度ナルカ

半径ヲ單位トシテ円周ノ數ハ 2π ナリ ニテ円周ノ對スル中心角ハ 4直角ナルヲ以テ

所要中心角 $=\frac{\cancel{4}^{2}\times 90}{\cancel{2}\pi}$ 度 $=\frac{180}{\frac{22}{7}}$ 度 $=$ 57.3°強

(269)

$\frac{11}{16}$ 直角 $=(\frac{11}{16}\times 90)$ 度 $=61\frac{9}{8}$ 度

$\frac{7}{8}$ 度 $=(\frac{7}{8}\times 60)$ 分 $=52\frac{1}{2}$ 分

$\therefore$ $\frac{11}{16}$ 直角 $=61^\circ 52' 30''$

計算（辺，角，弧）

(270)

半径五寸ノ円ノ中心角30°ナル弧ノ長ヲ幾何

所要弧 $=5\times\frac{22}{7}\times\frac{30}{180}$ 寸 $=$ 2.62寸

(271)

或ル角ヲ單位トシテ15°及ビ0.2直角ヲ計リテ得タル値ノ和ガ $0.7\dot{3}$ ナルトキ單位ナル角ハ何度ナルカ

15°及ビ0.2直角即18°ヲ別々ニ計リテ得タル數ノ和ハ $15^\circ+18^\circ=33^\circ$ ヲ同單位ヲ以テ計リタル數ニ等シ故ヘニ其單位ハ

$33^\circ\div 0.7\dot{3}=33^\circ\times\frac{90}{66}=45^\circ$

(272)

半径4000哩ノ円ノ中心角1′ニ對スル弧長ハ如何

所要弧 $=4000\times\frac{22}{7}\times\frac{1}{180\times 60}$ 哩 $=$ 1.16哩強

(273)

$49^\circ=\frac{49}{90}$ 直角 $=$ 0.54 直角

$11^\circ 15'=(\frac{11}{90}+\frac{15}{90\times 60})$ 直角 $=$ 0.125 直角

$45'5.4''=(\frac{45}{90\times 60}+\frac{54}{90\times 60\times 10\times 10})$ 直角 $=$ 0.00083 直角

(274)

35′30″ ノ弧度ハ如何

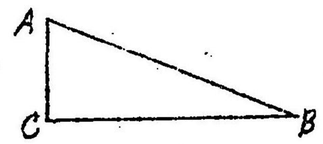
$\frac{\theta}{\pi}=\frac{D}{180}$ ヨリ $\theta=\frac{D\pi}{180}$

故ヘニ所要ノ弧度 $\theta=\frac{3.1416\times\frac{355}{60}}{180}=\frac{92939}{900000}$

$=0.01033$

(275)

直角三角形ノ三辺ガ 3, 4, 5 ナルトキ最小角ノ正弦, 余弦如何



$AC=3,\ CB=4,\ AB=5$

トセバ 最小角ハ B ナリ

故ニ $\sin B=\frac{3}{5},\quad \cos B=\frac{4}{5}$

(276)

前問ノ圖ニ於テ $AC:CB:BA=1:2:\sqrt{5}$ ナルトキ大鋭角ノ正弦, 正切ヲ求メヨ

三角函數ハ比ナルヲ以テ三角形ノ辺ノ比ガ変ラザレバ辺ノ長短ニハ関係ナシ, 故ニ $AC=1,\ CB=2,\ BA=\sqrt{5}$ ト見做スコトヲ得, 又大鋭角ハ A ナルヲ以テ

$\sin A=\frac{2}{\sqrt{5}},\quad \tan A=2$

(277)

前問ノ圖ニ於テ $AC=28,\ CB=45$ ナルトキ大鋭角ノ正弦ヲ求メヨ

此斜辺 $=\sqrt{45^2+28^2}=53$ ナリ, 而シテ大鋭角ハ A ナルガ

故ニ $\sin A=\frac{45}{53}$

(278)

前問ノ圖ニ於テ $BC=1,\ AC=0.7$ ナレバ $\tan A=1.02$ ナルベシ

$CB=\sqrt{1-.49}=\sqrt{.51}$

$\tan A=\frac{\sqrt{.51}}{.7}=\frac{.714}{.7}=1.02\cdots\cdots$

(279)

三角形ノ三ツノ角ガ等差級數ヲナシ最小角ヲ秒ニテ表ハシタル數ガ最大角ヲ分ニテ表ハシタル數ノ四倍ナルトキ三ツノ角ノ度數ニテ表ハセ

最大角ヲ a 分トセバ最小角ハ $4a$ 秒ニテ中角ハ等差級數ノ公式ニヨリテ

$\frac{a分+4a秒}{2}=\frac{64a}{2}秒=32a秒$ ナリ

依リテ次ノ方程式ヲ得之ヲ解キテ

$60a+32a+4a=180\times60\times60$

$96a=648000,\quad a=6750$

即最大角ハ $6750'=112.5^\circ$

最小角ハ $112.5^\circ\times\frac{4}{60}=7.5^\circ$

中角ハ $(112.5^\circ+7.5^\circ)\div2=60^\circ$

(280)

$\cos30^\circ=\frac{\sqrt{3}}{2}$ ナリ今 30° ノ角ヲ有スル直角三角形ノ斜辺ガ 12 寸ナレバ他ノ二ツノ辺ハ各幾寸ナルカ

問題 275 ノ圖ニ於テ $B=30^\circ,\ AB=12$ 寸トセバ

$\cos30^\circ=\frac{BC}{AB}=\frac{BC}{12}=\frac{\sqrt{3}}{2}$

$\therefore\ BC=\frac{12\sqrt{3}}{2}=6\sqrt{3}$ 寸 $AC=\sqrt{12^2-(6\sqrt{3})^2}=6$ 寸

(281)

$\frac{\pi}{13}$ ノ度數ヲ求メヨ. $\frac{\theta}{\pi}=\frac{D}{180}$ ヨリ $D=\frac{180\theta}{\pi}$

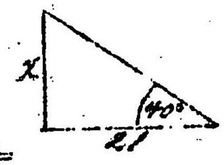
故ニ 所要ノ度數 $=\frac{180\times\pi/13}{\pi}=\frac{180\pi}{13\pi}$

$=13.846153$

計算（辺，角，弧）

(282)

$\tan 40^\circ = 0.8391$ ヨリ今一ツノ鋭角ガ 40° ナル直角三角形ニ於テ直角ト此鋭角ニ接スル辺ノ長サ21米ナルトキハ此鋭角ニ對スル辺ハ幾何ナルカ



$\tan 40^\circ = \frac{x}{21} = 0.8391$

x米 $= 0.8391 \times 21 = \underline{17.62米}$

(283)

n辺ヲ有スル正多角形ノ一内角ノ弧度ハ如何

n辺ノ正多角形ノ一内角ハ $\frac{90 \times 2(n-2)}{n}$ 度ヨリ(問281)

而シテ $\theta = \frac{D\pi}{180}$ (問274) ナルヲ以テ

所要弧度 $= \frac{180(n-2)\pi}{n \times 180} = \underline{\underline{\frac{(n-2)\pi}{n}}}$

(284)

次ノ角ハ夫々何レノ象限ニアルヤ. 120°, 500°, -410° 及ビ $(2n+1) \times 180^\circ - 40^\circ$

$120^\circ = 1 \times 180^\circ - 60^\circ$ 即チ第二象限

$500^\circ = 3 \times 180^\circ - 40^\circ$ 即チ第二象限

$-410^\circ = (-2) \times 180^\circ - 50^\circ$ 即チ第四象限

$(2n+1) \times 180^\circ - 40^\circ$ $2n+1$ ハ奇數ナルヲ以テ此角ハ第二象限ノ角ナリ

(285)

下記諸角ノ餘角, 補角ヲ問フ. 80°, 270°, -25°

$90^\circ - 80^\circ = \underline{10^\circ}$ $180^\circ - 80^\circ = \underline{100^\circ}$

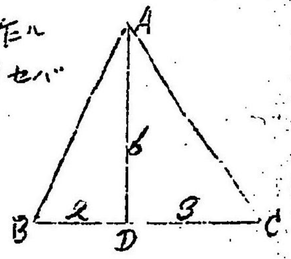
$90^\circ - 270^\circ = \underline{-180^\circ}$ $180^\circ - 270^\circ = \underline{-90^\circ}$

$90^\circ - (-25) = \underline{115^\circ}$ $180^\circ - (-25) = \underline{205^\circ}$

計算（辺，角，弧）

(286)

△ABC ノ頂 A ヨリ BC ニ垂線 AD ヲ作ルトキ BD, CD, AD ガ 2, 3, 6 ニ比例セバ頂角ノ大サハ如何



$\tan B = \frac{6}{2} = 3$

$\tan C = \frac{6}{3} = 2$

$\tan(B+C) = \frac{\tan B + \tan C}{1 - \tan B \tan C} = \frac{3+2}{1-3\times2} = -1$

$\therefore$ $B + C = 135^\circ$ (三角形ノ二内角ノ和ハ決シテ 180° ヨリ大ナルモノハ採ラズ)

$\therefore$ $A = 180^\circ - 135^\circ = \underline{\underline{45^\circ}}$

(287)

半径4尺ノ円ニ於テ10尺ノ弧ノ上ノ中心角ヲ六十分法ニテ表ハセ

半径 r ナル円ニ於テ l ナル弧ノ上ニ立ツ中心角ノ弧度ハ $\frac{l}{r}$ ナルヲ以テ本題ノ弧度 $\theta = \frac{10}{4} = \frac{5}{2}$ ヨリ

故ニ $D = \frac{180\theta}{\pi}$ (問題281) ニヨリ

所要度數 $= \frac{180 \times \frac{5}{2}}{3.1416}$ 即 $\underline{\underline{143^\circ 14' 21''}}$

(288)

底辺12寸頂角 60° ナル三角形ノ外接円ノ半径如何

$2R = \frac{a}{\sin A}$ ヨリ $R = \frac{a}{2\sin A}$

$\therefore$ $R = \frac{12}{2\sin 60^\circ} = \frac{12}{\sqrt{3}} = \underline{\underline{4\sqrt{3}}}$

計算（辺，角）

(289)

$A=67°40'$, $C=60°10'$, $a=20$間、B, bヲ求ム

$$B=180°-(67°40'+60°10')=\underline{52°10'}$$

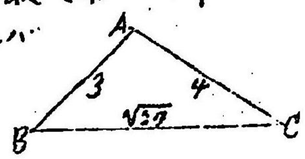
$$b=\frac{a\sin B}{\sin A}=\frac{20\times\sin 52°10'}{\sin 67°40'}$$

$$=\frac{20\times 0.7898}{0.925}=\underline{17.08\text{(間)}}$$

(290)

$3, 4, \sqrt{37}$ ナル三辺ヲ有スル三角形ノ最大角ハ如何

$a=\sqrt{37}$, $b=4$, $c=3$ トセバ



$\hat{A}$ ノ最大ナルコト明ナリ

$$\cos A=\frac{b^2+c^2-a^2}{2bc}\ (\text{公式}42)$$

$$=\frac{16+9-37}{2\times4\times3}=-\frac{1}{2}=\cos 120°$$

$$\therefore\ A=\underline{120°}$$

(291)

$A=109°$, $B=48°$, $c=273$尺, a, bヲ問フ

$$C=180°-(109°+48°)=23°$$

$$a=\frac{c\sin A}{\sin C}=\frac{c\sin 109°}{\sin 23°}=\frac{273\times.9455}{.3907}$$

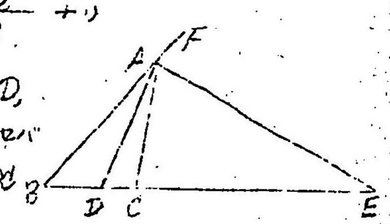
$$=\underline{660.7}$$

$$b=\frac{c\sin B}{\sin C}=\frac{c\sin 48°}{\sin 23°}=\frac{273\times.7431}{.3907}$$

$$=\underline{519.2}$$

(292)

$\triangle ABC$ ニ於テ A ノ内角及ビ外角ノ二等分線ノ夫々ニ $\dfrac{2bc\cos\frac{A}{2}}{b+c}$ 及ビ $\dfrac{2bc\sin\frac{A}{2}}{b\sim c}$ ナリ



A ノ内角ノ二等分線ヲ AD, 外角ノ二等分線ヲ AE トセバ

$\triangle ABC=\triangle ABD+\triangle ADC$ ナルヲ以テ

$$\frac{1}{2}bc\sin A=\frac{1}{2}c\cdot AD\sin\frac{A}{2}+\frac{1}{2}b\cdot AD\sin\frac{A}{2}$$

即チ $\frac{1}{2}bc\times2\sin\frac{A}{2}\cos\frac{A}{2}=\frac{1}{2}c\cdot AD\sin\frac{A}{2}+\frac{1}{2}b\cdot AD\sin\frac{A}{2}$

即チ $bc\sin\frac{A}{2}\cos\frac{A}{2}=\frac{1}{2}AD\sin\frac{A}{2}(b+c)$

$$\therefore\ AD=\frac{bc\sin\frac{A}{2}\cos\frac{A}{2}}{\frac{1}{2}\sin\frac{A}{2}(b+c)}=\underline{\frac{2bc\cos\frac{A}{2}}{b+c}}$$

又 $\triangle ABC=\triangle ABE-\triangle ACE$ ナルヲ以テ

$$\frac{1}{2}bc\sin A=\frac{1}{2}c\cdot AE\sin\left(A+90°-\frac{A}{2}\right)$$
$$-\frac{1}{2}b\cdot AE\sin\left(90°-\frac{A}{2}\right)$$

即チ $2bc\sin\frac{A}{2}\cos\frac{A}{2}=AE\left(c\cos\frac{A}{2}-b\cos\frac{A}{2}\right)$

$$=AE\cdot\cos\frac{A}{2}(c-b)$$

$$\therefore\ AE=\frac{2bc\sin\frac{A}{2}\cos\frac{A}{2}}{\cos\frac{A}{2}(c-b)}=\underline{\frac{2bc\sin\frac{A}{2}}{b\sim c}}$$

(293)

$38°26'47''$ ノ余角及ビ補角ヲ求ム

$$90°-38°26'47''=\underline{51°33'13''}$$

$$180°-38°26'47''=\underline{141°33'13''}$$

計算(辺,角)

(294)

$A=33°34'1''$　$C=53°30'35''$　$b=538.28$間

ナルトキ B, a, c ヲ求メヨ

$$B=180°-(33°34'1''+53°30'35'')$$
$$=92°55'24''$$

$$a=\frac{b\sin A}{\sin B}$$

$$\log a=\log b+\log\sin A-\log\sin B$$
$$=\log 538.28+\log 33°34.02'-\log\sin 92°55.4'$$
$$=2.73078+(9.74189-10)-(9.9994-10)$$
$$\quad +23 \qquad +76 \qquad -4$$
$$=2.47422,\ 此真數ハ\ 298$$

$\therefore\ a=\underline{\underline{294間}}$

$c=\frac{a\sin C}{\sin A}$ ヨリ上ト同様ニシテ $c=\underline{\underline{433.31}}$

(295)

A, B, P ヲ知リテ $A\overset{\frown}{B}C$ ニ External 弦ヲ引ク

$$\frac{a+b+c}{\sin A+\sin B+\sin C}=\frac{a}{\sin A}=\frac{a+b+c}{4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}$$

$$\therefore\ a=\frac{2P\sin A}{4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}=\frac{2P\cdot 2\sin\frac{A}{2}\cos\frac{A}{2}}{4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}$$

$$=\frac{P\cdot\sin\frac{A}{2}}{\cos\frac{B}{2}\cos\frac{C}{2}}$$ — A, B ハ已知ナルヲ以テ C ヲ知ルコトヲ得故ニ此最後ノ式ハ a ノ値ヲ表ハス

同様ニシテ次ノ如ク b, c ヲ知ルコトヲ得

$$b=\frac{P\sin\frac{B}{2}}{\cos\frac{A}{2}\cos\frac{C}{2}},\quad c=\frac{P\sin\frac{C}{2}}{\cos\frac{A}{2}\cos\frac{B}{2}}$$

計算(角,辺,面積)

(296)

$b=295.6$, $c=999.2$, $A=108°29.6'$ ノ c, a ヲ問フ

$$\frac{C+B}{2}=90°-\frac{A}{2}=90°-54°14.8'=35°45.2'$$

$$\tan\frac{C-B}{2}=\frac{c-b}{c+b}\cot\frac{A}{2}\ (遠藤氏三角法104頁)$$

$$=\frac{703.6}{1294.8}\cot 54°14.8'$$

$\log 703.6 = 2.8474$

$-\log 1294.8 = \bar{4}.8878$

$\log\cot 54°14.8' = \bar{1}.8573$

$\log\cot\frac{C-B}{2} = \bar{1}.5925$

$\therefore\ \frac{C-B}{2}=21°22.2'$

而シテ $C=\frac{C+B}{2}+\frac{C-B}{2}=35°45.2'+21°22.2'$
$=\underline{\underline{57°7.4'}}$

$$a=\frac{(c-b)\cos\frac{A}{2}}{\sin\frac{C-B}{2}}=\frac{703.6\times\cos 54°14.8'}{\sin 21°22.2'}$$

$\log 703.6 = 2.8474$

$\log\cos 54°14.8 = \bar{1}.7666$

$-\log\sin 21°22.2 = 0.4384$

$\log a = 3.0524$

$\therefore\ a=\underline{\underline{1128.2}}$

(297)

$B=26°30'$, $a=167.42$, $c=341.52$, A,C ヲ問フ

$$\frac{C+A}{2}=\frac{180°-B}{2}=76°45'$$

$$\tan\frac{C-A}{2}=\frac{c-a}{c+a}\tan\frac{C+A}{2}=\frac{174.1}{508.9}\tan 76°45'$$

$$\log\tan\frac{C-A}{2}$$

$$=\log 174.1+\log\tan 76°45'-\log 508.9$$

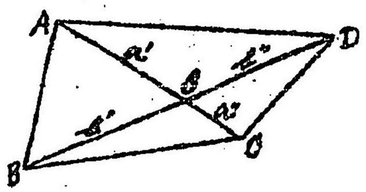
$$=2.24080+0.62807-(2.70586+77)$$

$$=.16224$$

$$\therefore\ \frac{C-A}{2}=55°28.4'\quad \therefore C=\underline{132°13'}$$

(298)

四辺形ノ對角線AC, BDノ長サ a, b トシ θ ノ其交角トセバ其ノ面積ハ $\frac{1}{2}ab\sin\theta$ ナリ



AC, BD ヲ圖ノ如ク a', a'' b', b'' ニ分チバ

$$A\hat{O}C=\frac{1}{2}a'b'\sin(180°-\theta)\ (\text{公式4})=\frac{1}{2}a'b'\sin\theta$$

$$B\hat{O}C=\frac{1}{2}b'a''\sin\theta,\quad A\hat{O}D=\frac{1}{2}a'b''\sin\theta$$

$$C\hat{O}D=\frac{1}{2}a''b''\sin(180°-\theta)=\frac{1}{2}a''b''\sin\theta$$

四辺形ABCD $=A\hat{O}B+B\hat{O}C+C\hat{O}D+A\hat{O}D$

$$=\frac{1}{2}\{a'(b''+b')+a''(b''+b')\}\sin\theta$$

$$=\frac{1}{2}(a'+a'')(b'+b'')\sin\theta$$

$$=\frac{1}{2}ab\sin\theta\quad \because \begin{cases} a'+a''=a \\ b'+b''=b \end{cases}$$

(299)

三角形ノ二辺ノ比ハ 5:3 ナリ又其夾角ハ 40° ナリ他ノ角ハ各如何

二辺ヲ a, b トシ其夾角ヲ C トセバ $a:b=5:3$ ニシテ三角函數ハ比ナルヲシテ辺ノ大小ニ関係ナシ、故ニ $a=5$, $b=3$ トナスヲ得、依リ問題296ニ準シテ解ケバヨシ、但シ數箇ナルヲシテ對數ヲ用ヒザルヲ便トス

(300)

$B=43°$, $b=40$間, $c=28$間, C ヲ問フ

$$\sin C=\frac{c\sin 43°}{b}=\frac{28\times .682}{40}=.4774$$

$$\therefore\ C=\underline{28°31'}$$

(注意) $b>c$ ナルヲシテ $B>C$ ナリ依リテ C ハ鋭角ナラズ即チ唯一ツノ鋭角ハ

(301)

$A=38°40'$, $a=1000$尺, $b=1276$尺, B ヲ問フ

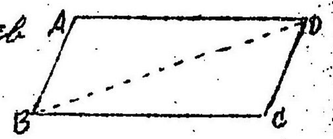
$$\sin B=\frac{b\sin 38°40'}{a}=\frac{1276\times .6248}{1000}=.7972$$

$$\therefore\ B=\underline{52°52'}\ \text{又ハ}\ 180°-52°52'=\underline{127°8'}$$

(注意) $b>a$ ナルヲシテ $B>A$ ナリ依リテ B ハ鋭角鈍角ノ二ツアリ(菊池氏三角法160頁而ハニヨル)

(302)

平行四辺形ABCDノ $AB=a$, $AD=b$ トセバ其面積ハ $ab\sin A$ ナリ



$$\triangle ABD=\frac{1}{2}ab\sin A$$

$$\triangle BCD=\frac{1}{2}ab\sin C,\ \text{而シテ}\ C=A$$

且ツ $\square ABCD=\triangle ABD+\triangle BCD$

$$\therefore\ \square ABCD=ab\sin A$$

(303)

$A=114°13'.5,\ a=4513,\ c=2500,$ C ヲ問フ

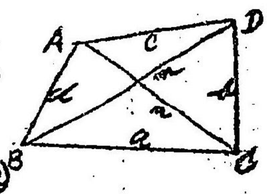
$$\sin C=\frac{c\sin 65°46.5'}{a}=\frac{2500\times .919}{4513}=.50515$$

$$\therefore\ C=\underline{30°20.6'}$$

(注意) A ガ鈍角ナルヲ以テ C ハ鋭角ニ限ル

(304)

四辺形ノ三辺 $d, a, b,$ ニ對角線 m, n ヲ已知ニテ残リノ一辺 c ヲ求ムル方法ヲ示セ



$$\cos ABC=\frac{d^2+a^2-m^2}{2ad}\ (\text{公式42})$$

ヨリ $A\hat{B}C$ ヲ得ベシ

$$\cos DBC=\frac{m^2+a^2-d^2}{2ma}$$ ヨリ $D\hat{B}C$ ヲ得ベシ

$$A\hat{B}C-D\hat{B}C=A\hat{B}D$$

$$\cos ABD=\frac{d^2+m^2-c^2}{2md}$$ ヨリ

$$c^2=d^2+m^2-2md\cos ABD$$

$$\therefore\ c=\sqrt{d^2+m^2-2md\cos ABD}$$

(305)

$a=10$寸, $b=12$寸, $c=14$寸. 最小角ヲ問フ

a ガ最小ナルヲ以テ最小角ハ A ナリ

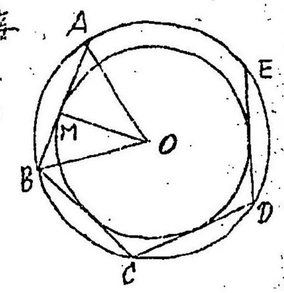
$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{144+196-100}{2\times 12\times 14}$$

$$=\frac{240}{336}=.7143$$

$$\therefore\ A=44°\underset{+5'}{20'}=\underline{\underline{44°25'}}$$

(306)

n 辺ノ正多角形ノ AB 等ヲ a, 外接円ノ半径 OA 等ヲ R, 内接円ノ半径 OM 等ヲ r トセバ此等ノ中何レカ一ツヲ知ルトキハ其ノ面積ハ次式ノ一ツニテ計算セラル



(1) $\frac{1}{4}na^2\cot\frac{180°}{n}$

(2) $\frac{1}{2}nR^2\sin\frac{360°}{n}$, (3) $nr^2\tan\frac{180°}{n}$

$$A\hat{O}B=\frac{360°}{n},\ A\hat{O}M=\frac{180°}{n},\ AM=\frac{a}{2}$$

$$OM=AM\cot AOM=\frac{1}{2}a\cot\frac{180°}{n}$$

而ニテ $S=nA\hat{O}B$ ニテ

$$A\hat{O}B=\frac{1}{2}AB\cdot OM=\frac{1}{2}a\cdot\frac{1}{2}a\cot\frac{180°}{n}$$

$$\therefore\ S=\frac{1}{4}na^2\cot\frac{180°}{n}$$ 即チ (1)

又 $A\hat{O}B=\frac{1}{2}\cdot OA\cdot OB\cdot\sin AOB$ (公式43)

$$\therefore\ S=\frac{1}{2}nR^2\sin\frac{360°}{n}$$ 即チ (2)

又 $AM=OM\tan\frac{180°}{n}=r\tan\frac{180°}{n}$

而ニテ $A\hat{O}B=2A\hat{M}O=AM\cdot MO=r\cdot r\tan\frac{180}{n}$

$$\therefore\ S=nr^2\tan\frac{180°}{n}$$ 即チ (3)

(307)

$A=30°,\ a=100,\ c=250$ ナルトキ解アリヤ否ヤ

$c\sin A=250\cdot\sin 30°=125$, 而ニテ $a=100$

$\therefore\ c\sin A>a$ $\therefore$ 而イニヨリテ解ナシ

(308)

$a=23.46,\ b=35.79,\ A=28°35.4'$ ナルトキCヲ問フ

$\sin B=\frac{b\sin A}{a}=\frac{35.79\times\sin 28°35.4'}{23.46}$

$\left.\begin{array}{ll}\log 35.79 & =1.5538\\ \log\sin 28°35.4' & =\bar{1}.6799\\ -\log 23.46 & =\bar{2}.6297\end{array}\right\}+=\log\sin B=\bar{1}.8634$

$\therefore\ B=46°54.2'$, 或ハ $132°5.8'$

従テ $C=180°-(A+B)=180°-(28°35.4'+46°54.2')$

$=\underline{\underline{104°30.4'}}$

又ハ $C=180°-(28°35.4'+133°5.8')=\underline{\underline{18°18.8'}}$

$c=\frac{b\sin C}{\sin B}=\frac{35.79\times\sin 104°30.4'}{\sin 46°54.2}$

$\left.\begin{array}{ll}\log 35.79 & =1.5538\\ \log\sin 104°30.4' & =\bar{1}.9859\\ -\log\sin 46°54.2' & =0.1366\end{array}\right\}+=\log c=1.6763$

$\therefore\ c=\underline{\underline{47.46}}$

或ハ B, C ノ第二ノ値ヲ採レバ

$c=\frac{35.79\times\sin 18°18.8'}{\sin 132°5.8'}$

$\left.\begin{array}{ll}\log 35.79 & =1.5538\\ \log\sin 18°18.8' & =\bar{1}.4972\\ -\log\sin 132°5.8' & =0.1296\end{array}\right\}+=\log c=1.1806$

$\therefore\ c=\underline{\underline{15.11}}$

(309)

$C=49°38'40''$ $a=167.39$, $c=402.79$. A, B, b ヲ問フ

本問ハ前問ノAガCトナリ aガcトナリ bガaトナリタルニテ

ナレバ方法ハ前問ト異ナルコトナリ唯 $A=18°27'46''$ ガ

一ツヨリ他ニナキコトノ異ナルノミ

偖テ何故ヘ一ツノミナカトスヘバ $c>a$ ナルヲ以テ

$C>A$ ナリ故ヘAハ鈍角ナルコト証ハザレバナリ

(310)

$B=35°51.3'$, $a=583.1$, $b=367.2$. A, C, c ヲ問フ

$\sin A=\frac{a\sin B}{b}=\frac{583.1\times\cdot 58\overset{+2}{5}4}{367.2}=\cdot 93006$

$\therefore\ A=\underline{\underline{68°27'}}$ 又ハ $\underline{\underline{111°33'}}$

従テ $C=180°-(68°27'+35°51.3')=\underline{\underline{75°41.7'}}$

又ハ $C=180°-(111°33'+35°51.3')=\underline{\underline{32°35.7'}}$

$c=\frac{b\sin C}{\sin B}=\frac{367.2\sin 75°41.7'}{\sin 35°51.3'}$

$=\frac{367.2\times\cdot 96842}{\cdot 58\overset{+34}{54}}=\underline{\underline{607.5}}$

又ハ $c=\frac{367.2\sin 32°35.7}{\sin 35°51.3'}=\underline{\underline{337.9}}$

(311)

$A=120°$, $a=125$, $c=250$ ノ解ノ有無如何

$c>a$ ナルヲ以テ $C>A$ ナリ

而シテ $A=120°$ ナルヲ以テ $C>120°$ 即チCハ

鈍角ナリ, 然レバ三角形ノ内角ノ二ツガ何レモ鈍

角ナルコトハルベシ不合理ナリ故ヘ本題ハ解ナシ

(312)

$a=375.9,\ b=298.7,\ c=400.8$ ナルトキ A, B ハ如何

$$p=\frac{375.9+298.7+400.8}{2}=537.7$$

$p-a=161.8,\ p-b=239,\ p-c=136.9$

$$\tan\frac{A}{2}=\frac{1}{p-a}\sqrt{\frac{(p-a)(p-b)(p-c)}{p}}$$

$$=\sqrt{\frac{(p-a)(p-b)(p-c)}{(p-a)^2p}}=\sqrt{\frac{239\times136.9}{161.8\times537.7}}$$

$$\log\tan\frac{A}{2}=\frac{1}{2}(\log239+\log136.9-\log161.8-\log537.7)$$

$$=\frac{1}{2}(2.3784+2.1364-2.2090-2.7306)$$

$$=\frac{1}{2}\times\bar{1}.5752=\bar{1}.7876$$

$\therefore\ \frac{A}{2}=31^\circ31.5'$ 従テ $A=\underline{63^\circ3'}$

$$\tan\frac{B}{2}=\sqrt{\frac{(p-a)(p-c)}{p(p-b)}}=\sqrt{\frac{161.8\times136.9}{537.7\times239}}$$

$$\log\tan\frac{A}{2}$$

$$=\frac{1}{2}(\log161.8+\log136.9-\log537.7-\log239)$$

$$=\frac{1}{2}(2.2090+2.1364-2.7306-2.3784)$$

$$=\bar{1}.2364\times\frac{1}{2}=\bar{1}.6182$$

$\therefore\ \frac{B}{2}=22^\circ32.8'$ 従テ $B=\underline{45^\circ5.8'}$

(313)

$a=261.5,\ b=252.7,\ c=137.1,$ A ハ如何

$$s=\frac{261.5+252.7+137.1}{2}=325.65$$

$s-b=72.95,\quad s-a=64.15$

$$\sin\frac{C}{2}=\sqrt{\frac{(s-a)(s-b)}{ab}}$$

$$\log\sin\frac{C}{2}=\frac{1}{2}(\log64.15+\log72.95-\log261.5-\log252.7)$$

$$=\frac{1}{2}(1.807\overset{+34}{8}6+1.862\overset{+30}{7}3-2.417\overset{+42}{8}4-2.401\overset{+46}{4}6)$$

$$=\bar{1}.42508$$

$\therefore\ \frac{C}{2}=15^\circ25'=15^\circ26'$ 従テ $C=\underline{30^\circ52'}$

(314)

$a=222,\ b=318,\ c=406,$ A ハ如何

$$s=\frac{222+318+406}{2}=473$$

$s-a=251,\ s-b=155,\ s-c=67$

$$\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}=\sqrt{\frac{155\times67}{473\times251}}$$

$$\log\tan\frac{A}{2}=\frac{1}{2}(\log155+\log67-\log473-\log251)$$

$$=\frac{1}{2}(2.19033+1.82607-2.67486-2.39967)$$

$$=\bar{1}.47094$$

$\therefore\ \frac{A}{2}=16^\circ28.5'$ 従テ $A=\underline{32^\circ57'}$

(315)

$a:b:c=4:5:6$, $\log 2=0.3010300$, $\log 3=0.4771213$, $\log\cos 41^\circ 25'=9.8750142-10$ 60″ニ對スル差 0.0001115 ヲ知リテ C ヲ求ムルコト

$$S=\frac{4+5+6}{2}=7.5,\quad S-c=1.5$$

$$\log 4=0.3010300\times 2=0.6020600$$

$$\log 5=1-0.3010300=0.6989700$$

$$\log 7.5=0.6989700\times 2+0.4771213-1=0.8750613$$

$$\log 1.5=0.4771213+0.6989700-1=0.1760913$$

$$\cos\frac{C}{2}=\sqrt{\frac{S(S-c)}{ab}}=\sqrt{\frac{7.5\times 1.5}{4\times 5}}$$

$$\log\cos\frac{C}{2}=\frac{1}{2}(0.8750613+0.1760913-0.6020600-0.6989700)$$

$$=9.8750613-10$$

$$\therefore\ \frac{C}{2}=41^\circ 25'-25''.3=41^\circ 24'\,34''.7$$

$$\therefore\ C=\underline{\underline{82^\circ 49'\,9''.4}}$$

(316)

$A=30^\circ$, $a=200$, $c=125$ ノ解ノ有無ヲ問フ

$c<a$ ナルヲシリテ $C<A$ ナリ

即チ C ハ 30° ヨリ小ナルヲシリテ鋭角ナリ

故ニ此ニヨリテ唯一ツノ解アリ

(317)

$a=2$, $b=\sqrt{6}$, $c=1+\sqrt{3}$ ノ三角形ノ各角如何

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{6+(1+2\sqrt{3}+3)-4}{2\times\sqrt{6}\times(1+\sqrt{3})}$$

$$=\frac{6+2\sqrt{3}}{2\times\sqrt{2}\sqrt{3}+\sqrt{2}\sqrt{3}}=\frac{\cancel{2(\sqrt{3}+3)}}{2\times\sqrt{2}\cancel{(\sqrt{3}+3)}}$$

$$=\frac{1}{\sqrt{2}}=0.7071\times\quad\therefore\ A=\underline{\underline{45^\circ}}$$

$$\cos B=\frac{c^2+a^2-b^2}{2ac}=\frac{(1+2\sqrt{3}+3)+4-6}{2\times 2(1+\sqrt{3})}$$

$$=\frac{2+2\sqrt{3}}{4+4\sqrt{3}}=\frac{1}{2}\qquad\therefore\ B=\underline{\underline{60^\circ}}$$

(318)

$A=30^\circ$, $a=200$, $c=250$ ナレバ二ツノ解アルコトヲ証シ且ツ鈍角ニ付テ之ノ解ヲ

$c>a$ ナルヲシリテ $C>A$ ニシテ

$c\sin A=250\times\frac{1}{2}=125$ ナルヲシリテ

$c\sin A<a$ ナリ. 故(丙)(ハ)ニ依リテ二ツノ解アリ

$$\sin C=\frac{c\sin A}{a}=\frac{250\times .5}{200}=0.625$$

$\therefore\ C=38^\circ 40.93'$ 又ハ $141^\circ 19.1'$

C ノ鈍角ヲ採レバ $B=8^\circ 40.9$ トナル

$$b=\frac{c\sin B}{\sin C}=\frac{250\times\sin 8^\circ 40.9'}{\sin 38^\circ 40.93}$$

$$=\frac{250\times .15089}{.625}=61.3$$

(319)

$A=120^\circ$, $a=250$, $c=125$. 解ノ有無ヲ問フ

$c<a$ ナルヲシリテ $C<A$ ナリ故ニ問316ト同様ニ唯一ツノ解アリ

計算（角辺）

(320)

$C=90^\circ,\ c=934,\ a=257.\ b$ ヲ問フ

$b=\sqrt{c^2-a^2}=\sqrt{(c+a)(c-a)}=\sqrt{1191\times677}$

$\log b=\frac{1}{2}(\log 1191+\log 677)$

$=\frac{1}{2}(3.07591+2.83059)=2.95325$

$\therefore\ b=\underline{\underline{897.9}}$

(321)

$C=90,\ A=34^\circ15',\ a=843.2.\ c$ ヲ問フ

$\sin 34^\circ15'=\frac{843.2}{c}\ \therefore\ c=\frac{843.2}{\sin 34^\circ15'}$

$\log c=\log 843.2-\log\sin 34^\circ15'$

$=2.925^{+1}_{-4}3-(9.74^{+2}_{-4}3-10)=3.17557$

$\therefore\ c=149^{+1}_{-8}2=\underline{\underline{1498.2}}$

(322)

$C=90^\circ,\ B=32^\circ41'54'',\ c=314.08,\ b$ ヲ問フ

$\sin B=\frac{b}{c}\ \therefore\ b=c\sin B=314.08\sin 32^\circ41'54''$

$\log b=\log 314.08+\log\sin 32^\circ41'.9$

$=2.49^{+1}_{-6}693+9.732^{+3}_{-1}79-10=2.2296$

$\therefore\ b=169.^{+1}_{-6}6=\underline{\underline{169.67}}$

(323)

$A=30^\circ,\ a=125,\ c=250$ ノ解アリヤ否ヤ

$c\sin A=250\times\frac{1}{2}=125=a$

而シテ $C>A$ ナリ

故ニ(丙)(ロ)ニヨリテ唯一ツノ解アリ

計算（角辺）

(324)

$C=90^\circ,\ A=18^\circ47',\ b=169.4,\ a$ ヲ問フ

$\tan A=\frac{a}{b},\ a=b\tan A=169.4\tan A$

$\log a=\log 169.4+\log\tan 18^\circ47'$

$=2.22891+9.53^{+5}_{-3}078-10=1.76052$

$\therefore\ a=57.^{+1}_{-6}6=\underline{\underline{57.7}}$

(325)

$C=90^\circ,\ a=132.7,\ b=261.9.\ A,B,c$ ヲ問フ

$\tan B=\frac{b}{a}=\frac{2619}{1327}$

$\log\tan B=\log 2619-\log 1327$

$=3.41^{+1}_{-7}664-3.12287=0.29526$

$\therefore\ B=\underline{\underline{63^\circ7'46''}},\ A=\underline{\underline{26^\circ52'14''}}$

$c=\frac{b}{\cos A}=\frac{261.9}{\cos 26^\circ52'14''}$

$\log c=\log 261.9-\log\cos 26^\circ52'14''$

$=2.41813-(9.95^{-1}_{-4}52-10)$

$=2.46775$

$\therefore\ c=\underline{\underline{293.59}}$

計算（函數）

(326)

$\tan A = \frac{8}{15}$ ナルトキ $\sin A, \cos A$ ヲ問フ

$1+\tan^2 A = \sec^2 A = \frac{1}{\cos^2 A} = \frac{1}{1-\sin^2 A}$

$\therefore\ 1-\sin^2 A = \frac{1}{1+\tan^2 A}$

$\therefore\ \sin^2 A = 1-\frac{1}{1+\tan^2 A} = \frac{\tan^2 A}{1+\tan^2 A}$

$\therefore\ \sin A = \frac{\tan A}{\sqrt{1+\tan^2 A}} = \frac{\frac{8}{15}}{\sqrt{1+\frac{64}{225}}} = \underline{\underline{\frac{8}{17}}}$

$1+\tan^2 A = \frac{1}{\cos^2 A}\quad \therefore\ \cos^2 A = \frac{1}{1+\tan^2 A}$

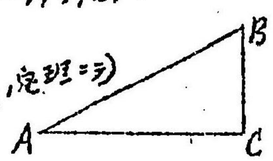
$\therefore\ \cos A = \frac{1}{\sqrt{1+\tan^2 A}} = \frac{1}{\sqrt{1+\frac{64}{225}}} = \underline{\underline{\frac{15}{17}}}$

(327)

$\sin A = \frac{3}{5}$ ナルトキ $\tan A, \operatorname{cosec} A$ ヲ問フ

$AB=5,\ BC=3$ トスルヲ得

從ッテ $AC=4$ トナル（ピタゴラスノ定理ニヲ）



$\therefore\ \tan A = \underline{\underline{\frac{3}{4}}}$

$\operatorname{cosec} A = \underline{\underline{\frac{5}{3}}}$

(328)

$\tan A = \frac{4}{3}$ ナルトキ $\sin A, \sec A$ ヲ問フ

前問ノ圖ニ於テ $AC=3,\ BC=4$ トナスニトヲ得 從テ $AB=5$ トナル

$\therefore\ \sin A = \underline{\underline{\frac{4}{5}}}$

$\sec A = \underline{\underline{\frac{5}{3}}}$

計算（函數）

(329)

$1+\sin^2\theta = 3\cos\theta\sin\theta$ ヨリ $\tan\theta$ ヲ求メヨ

$1+\frac{\tan^2\theta}{1+\tan^2\theta} = 3\cdot\frac{1}{\sqrt{1+\tan^2\theta}}\cdot\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$

（問326ニヨル）

即 $\frac{1+\tan^2\theta+\tan^2\theta}{1+\tan^2\theta} = \frac{3\tan\theta}{1+\tan^2\theta}$

即 $2\tan^2\theta-3\tan\theta+1=0$ ヲ解キテ

$\tan\theta = \frac{3\pm\sqrt{9-8}}{4} = \frac{3\pm1}{4} = \underline{\underline{1}}$ 又ハ $\cancel{\frac{1}{2}}$

問題ニ適セズ

(330)

$\cos A = \frac{1}{3}$ ナルトキ $\sin A, \cot A$ ヲ求メヨ

問題327ノ圖ニ於テ $AB=3,\ AC=1$ トスルヲ得 從ッテ $BC=\sqrt{8}=2\sqrt{2}$ トナル

$\therefore\ \sin A = \underline{\underline{\frac{2\sqrt{2}}{3}}},\quad \cot A = \underline{\underline{\frac{1}{2\sqrt{2}}}}$

(331)

$\tan A+\sec A = 2$ ナルトキ $\sin A$ ヲ求メヨ

$\tan A+\sec A = \frac{\sin A}{\cos A}+\frac{1}{\cos A} = \frac{\sin A+1}{\cos A} = 2$

而シテ $\cos A = \sqrt{1-\sin^2 A}$ ナルヲシメテ

$\sin A+1 = 2\sqrt{1-\sin^2 A}$

$\sin^2 A+2\sin A+1 = 4-4\sin^2 A$

$5\sin^2 A+2\sin A-3=0$ ヲ解キテ

$\sin A = \frac{-1\pm\sqrt{1+15}}{5} = \frac{-1\pm4}{5} = \underline{\underline{\frac{3}{5}}}$ 又ハ $\cancel{-1}$

-1 ハ問題ニ適セズ

(332)

22.5°ノ正弦,余弦ヲ求メヨ

$$\tan 22.5^\circ = \tan\frac{45^\circ}{2} = \frac{\sin 45^\circ}{1+\cos 45^\circ}$$

第二地氏三角法頁説明ニヨル

$$= \frac{\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}+1} = \sqrt{2}-1$$

従ッテ $\cot 22.5^\circ = \frac{1}{\sqrt{2}-1} = \sqrt{2}+1$

又タ $\operatorname{cosec} 22.5^\circ = \sqrt{1+\cot^2 22.5^\circ} = \sqrt{1+(\sqrt{2}+1)^2}$

$$= \sqrt{4+2\sqrt{2}}$$

従ッテ $\sin 22.5^\circ = \frac{1}{\sqrt{4+2\sqrt{2}}} = \underline{\underline{\frac{1}{2}\sqrt{2-\sqrt{2}}}}$

又タ $\cos 22.5^\circ = \cot 22.5^\circ \cdot \sin 22.5$

$$= (\sqrt{2}+1)\left(\frac{1}{2}\sqrt{2-\sqrt{2}}\right)$$

$$= \frac{1}{2}\sqrt{(\sqrt{2}+1)^2(2-\sqrt{2})} = \frac{1}{2}\sqrt{(2-\sqrt{2})(3+2\sqrt{2})}$$

$$= \frac{1}{2}\sqrt{6+\sqrt{2}+4} = \underline{\underline{\frac{1}{2}\sqrt{2+\sqrt{2}}}}$$

(333)

$(2n-1)\times 180^\circ + 30^\circ$ ノ正弦,余弦,正切ヲ求メヨ

$2n-1$ ハ奇數ナルヲ以テ本問ノ角ハ第三象限内ニアリ

$$\sin\{(2n-1)\times 180^\circ + 30^\circ\} = -\sin 30^\circ = \underline{\underline{-\frac{1}{2}}}$$

$$\cos\{(2n-1)\times 180^\circ + 30^\circ\} = -\cos 30^\circ = \underline{\underline{-\frac{\sqrt{3}}{2}}}$$

$$\tan\{(2n-1)\times 180^\circ + 30^\circ\} = \tan 30^\circ = \underline{\underline{\frac{1}{\sqrt{3}}}}$$

(334)

$\tan A = .75$ ナルトキ $\sin^6 A + \cos^6 A$ ノ値ヲ問フ

$$\tan A = .75 = \frac{75}{100} = \frac{3}{4}$$

今垂線ヲ3トセバ底辺ハ4トスルコトヲ得従ッテ斜辺ハ5トナル故ニ

$$\sin^6 A + \cos^6 A = 1 - 3\sin^2 A\cos^2 A \quad (\text{問}27)$$

$$= 1 - 3\times\frac{3^2}{5^2}\times\frac{4^2}{5^2} = 1 - \frac{432}{625} = \underline{\underline{\frac{193}{625}}}$$

(335)

$\cot B = \sqrt{3}$ ナルトキ $\sin B, \cos B$ ヲ求メヨ

$\cot B = \sqrt{3} = \frac{\sqrt{3}}{1}$ ナル故ニ垂線ヲ1,底辺ヲ$\sqrt{3}$トスルコトヲ得,従テ斜辺ハ2トナル

$$\therefore \sin B = \underline{\underline{\frac{1}{2}}}, \quad \cos B = \underline{\underline{\frac{\sqrt{3}}{2}}}$$

(336)

75°ノ正弦,余弦ヲ問フ

$$\sin 75^\circ = \cos(90^\circ - 75^\circ) = \cos 15^\circ$$

$$= \underline{\underline{\frac{\sqrt{6}+\sqrt{2}}{4}}}$$ (第二地氏三角法27頁例題ニヨル)

$$\cos 75^\circ = \sin 15^\circ = \underline{\underline{\frac{\sqrt{6}-\sqrt{2}}{4}}}$$

(337)

$\frac{1}{3}\sin^2 60^\circ - \frac{1}{2}\sec 60^\circ \tan^2 30^\circ + \frac{4}{3}\sin^2 45^\circ \tan^2 60^\circ$ ノ値ヲ求メヨ

$$= \frac{1}{3}\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{1}{2}\cdot 2\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{4}{3}\left(\frac{1}{\sqrt{2}}\right)^2(\sqrt{3})^2$$

$$= \frac{1}{3}\times\frac{3}{4} - \frac{1}{2}\times 2\times\frac{1}{3} + \frac{4}{3}\times\frac{1}{2}\times 3$$

$$= \frac{1}{4} - \frac{1}{3} + 2 = \frac{23}{12} = \underline{\underline{1\frac{11}{12}}}$$

(338)

150°ノ正弦,余弦,正切ヲ問フ

$150° = 180° - 30°$ 即第二象限ノ角ナリ

$\therefore \sin 150° = \sin 30° = \frac{1}{2}$

$\cos 150° = -\cos 30° = -\frac{\sqrt{3}}{2}$

$\tan 150° = -\tan 30° = -\frac{1}{\sqrt{3}}$

(339)

750°ノ正弦ハ如何

$750° = 4 \times 180° + 30°$ 即一象限ノ角

$\therefore \sin 750° = \sin 30° = \frac{1}{2}$

(340)

$\sec A = 1.0\dot{3}$ ナルトキ $\sin A$, $\tan A$ ヲ求メヨ

$\sec A = 1.0\dot{3} = \frac{31}{30}$

而シテ $\sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec^2 A}$ (遠藤氏三角ノ5頁例ニヨル)

$= \frac{\sqrt{\left(\frac{31}{30}\right)^2 - 1}}{\frac{31}{30}} = \frac{\sqrt{61}}{31}$

$\tan A = \sqrt{\sec^2 A - 1}$ (16頁例)

$= \sqrt{\left(\frac{31}{30}\right)^2 - 1} = \frac{\sqrt{61}}{30}$

(341)

330°ノ正弦,余弦ヲ求メヨ

$330° = 2 \times 180° - 30°$ 即第四象限ノ角

$\therefore \sin 330° = -\sin 30° = -\frac{1}{2}$

$\cos 330° = \cos 30° = \frac{\sqrt{3}}{2}$

(342)

$1 - \cos A$ ガ 0°ヨリ 360°マデノ間ノ値ノ変化如何

$\cos A$ ハ第一象限ニ於テハ 1ヨリ 0マデニ減ジ第二象限ニ於テハ 0ヨリ −1ニ減ジ第三象限ニ於テハ −1ヨリ 0マデ増シ第四象限ニ於テハ 0ヨリ 1マデ増ス

故ニ $1 - \cos A$ ハ

第一象限ニ於テハ 0ヨリ 1マデ増ス

第二象限ニ於テハ 1ヨリ 2マデ増ス

第三象限ニ於テハ 2ヨリ 1マデ減ズ

第四象限ニ於テハ 1ヨリ 0マデ減ズ

(343)

$\sin^3 60° \cot 30° - 2\sec^2 45° + 3\cos 60° \tan 45° - \tan^2 60°$ ノ値ヲ計算セヨ

原式 $= \left(\frac{\sqrt{3}}{2}\right)^3 \sqrt{3} - 2(\sqrt{2})^2 + 3 \times \frac{1}{2} \times 1 - (\sqrt{3})^2$

$= \frac{9}{8} - 4 + \frac{3}{2} - 3 = -4\frac{3}{8}$

(344)

$\tan 11°15'$ ノ値ヲ問フ

$\tan 11°15' = \tan \frac{22.5°}{2} = \frac{\sin 22.5°}{1 + \cos 22.5°}$

$= \frac{\frac{1}{2}\sqrt{2 - \sqrt{2}}}{1 + \frac{1}{2}\sqrt{2 + \sqrt{2}}} = \frac{\sqrt{2 - \sqrt{2}}}{2 + \sqrt{2 + \sqrt{2}}}$

(注意) 22.5°ノ函數値ハ皆 $\sqrt{2}$ ニヨル

(345)

$\sec\theta = 4$ ナルトキ $\sin\theta$, $\cot\theta$ ヲ求メヨ

$\sec\theta = 4 = \frac{4}{1}$ ナル故ニ斜辺ハ4,底辺ハ1トスルコトヲ得,従ッテ垂線ハ $\sqrt{16-1} = \sqrt{15}$ トナル

$\therefore \sin\theta = \frac{\sqrt{15}}{4}$ $\cot\theta = \frac{1}{\sqrt{15}}$

(346)

$\tan 25°26'.7$ ヲ求メヨ

$\tan 25°30' = .4770,\ \tan 25°20' = .4734$

此差ハ $.0036$ ニシテ角ノ差ハ $26'.7 - 20' = 6'.7$ ナリ

故ニ次ノ比例式ヲ作ル

$10' : 6.7' = .0036 : x,\ x = .0024$

$\therefore\ \tan 25°26'.7 = .4734 + .0024 = \underline{\underline{.4758}}$

(347)

$67°30'$ ノ三角函數ヲ求メヨ

$90° - 67.5° = 22.5°$

$\therefore\ \sin 67.5° = \cos 22.5° = \underline{\underline{\frac{1}{2}\sqrt{2+\sqrt{2}}}}$

他ノ函數ハ問332ヨリ容易ニ求ムルコトヲ得

(348)

$\sin 240°,\ \tan 240°$ ヲ求メヨ

$240° = 180° + 60°$ 即第三象限ノ角ナリ

$\therefore\ \sin 240° = -\sin 60° = \underline{\underline{-\frac{\sqrt{3}}{2}}}$

$\tan 240° = \tan 60° = \underline{\underline{\sqrt{3}}}$

(349)

$\operatorname{cosec} 41°18.2'$ ヲ求メヨ

$\operatorname{cosec} 41°10' = 1.519$ $\quad 18.2'$

$\operatorname{cosec} 41°20' = 1.514$ $\quad 10'$

$10' \quad .005 \quad 8'.2$

$10' : 8.2' = 0.005 : x,\ x = 0.004$

$\therefore\ \operatorname{cosec} 41°18.2' = 1.519 - 0.004$

$= \underline{\underline{1.515}}$

(350)

$-45°$ ノ正弦,余弦,正切ヲ問フ

$-45° = 0 \times 180° - 45°$ 即第四象限ノ角ナリ

$\therefore\ \sin(-45°) = -\sin 45° = \underline{\underline{-\frac{1}{\sqrt{2}}}}$

$\cos(-45°) = \cos 45° = \underline{\underline{\frac{1}{\sqrt{2}}}}$

$\tan(-45°) = -\tan 45° = \underline{\underline{-1}}$

(351)

$1 - \sin A$ ガ $0°$ ヨリ $360°$ マデノ中ノ値ノ変化如何

$\sin A$ ハ第一象限ニ於テハ 0 ヨリ 1 マデ増シ第二象限ニ於テハ 1 ヨリ 0 マデ減シ第三象限ニ於テハ 0 ヨリ -1 マデ減シ第四象限ニ於テハ -1 ヨリ 0 マデ増スヲ以テ $1 - \sin A$ ハ

第一象限ニ於テハ 1 ヨリ 0 マデ減ズ

第二象限ニ於テハ 0 ヨリ 1 マデ増ス

第三象限ニ於テハ 1 ヨリ 2 マデ増ス

第四象限ニ於テハ 2 ヨリ 1 マデ減ズ

(352)

$\sin 100° \sin(-160°) + \cos 200° \cos(-280°)$ ノ値ヲ問フ

$= \sin 100°(-\sin 160°) + \cos 200° \cos 280°$

$= \cos 280° \cos 200° - \sin 100° \sin 160°$

$= \cos(2\times 90° + 20°)\cos(2\times 90° + 10°) - \sin(90° + 10°)\sin(2\times 90° - 20°)$

$= -\cos 20° \sin 10° - \cos 10° \sin 20°$

$= -(\cos 20° \sin 10° + \cos 10° \sin 20°)$

$= -\sin(10° + 20°) = -\sin 30° = \underline{\underline{-\frac{1}{2}}}$

計算（函數）

(353)

-225° ノ正弦，余弦，正切ヲ問フ

$-225^\circ=(-1)\times180^\circ-45^\circ$ 即第二象限ノ角ナリ

$\therefore\ \sin(-225^\circ)=\sin45^\circ=\frac{1}{\sqrt{2}}$

$\cos(-225^\circ)=-\cos45^\circ=-\frac{1}{\sqrt{2}}$

$\tan(-225^\circ)=-\tan45^\circ=-1$

(354)

$A=18^\circ$ ナレバ $\sin2A=\cos3A$ ナルコトヲ証明シ依リテ以テ $\sin18^\circ=\frac{\sqrt{5}-1}{4}$ ヲ証明セヨ

$\sin(2\times18^\circ)=\cos(3\times18^\circ)$（余角ナル故ニ）

$\sin2(18^\circ)=\cos3(18^\circ)$ ..... (1)

即 $\sin2A=\cos3A$

又ハ(1)ヨリ $2\sin18^\circ\cos18^\circ=4\cos^3 18^\circ-3\cos18^\circ$

即 $2\sin18^\circ=4\cos^2 18^\circ-3$

即 $2\sin18^\circ=4(1-\sin^2 18^\circ)-3$

即 $4\sin^2 18^\circ+2\sin18^\circ-1=0$ 之ヲ解キテ

$\sin18^\circ=\frac{-1\pm\sqrt{1+4}}{4}=\frac{-1+\sqrt{1+4}}{4}$

$=\frac{\sqrt{5}-1}{4}$，$\sin18^\circ>0$ ヲ以テ(−)ハ捨ツ

(355)

-300° ノ正弦ヲ問フ

$-300^\circ=(-2)\times180^\circ+60^\circ$ 即第一象限ノ角ナリ

$\therefore\ \sin(-300^\circ)=\sin60^\circ=\frac{\sqrt{3}}{2}$

計算（函數）

(356)

$\sec38^\circ27'.7$ ノ値ヲ問フ

$\sec38^\circ30'=1.278$		$38^\circ27'.7$
$\sec38^\circ20'=1.275$		$38^\circ20'$
$10'$	0.003	$7'.7$

$10':7'.7=0.003:x$，$x=0.002$

$\therefore\ \sec38^\circ27'.7=1.275+0.002=1.277$

(357)

$\sin^2A$ ガ 0° ヨリ 360° マデ増ノ値ノ変化如何

$\sin A$ ハ問題351ニ説明セル通リナルヲ以テ

$\sin^2A$ ハ

第一象限ニ於テハ 0 ヨリ 1 マデニ増ス

第二象限ニ於テハ 1 ヨリ 0 マデニ減ズ

第三象限ニ於テハ 0 ヨリ 1 マデニ増ス

第四象限ニ於テハ 1 ヨリ 0 マデニ減ズ

(358)

-135° ノ正弦，余弦，正切ヲ求メヨ

$-135^\circ=(-1)\times180^\circ+45^\circ$ 即第三象限ノ角ナリ

$\therefore\ \sin(-135^\circ)=-\sin45^\circ=-\frac{1}{\sqrt{2}}$

$\cos(-135^\circ)=-\cos45^\circ=-\frac{1}{\sqrt{2}}$

$\tan(-135^\circ)=\tan45^\circ=1$

(359)

105° ノ正弦ヲ問フ

$105^\circ=180^\circ-75^\circ$ 即第二象限ノ角ナリ

$\therefore\ \sin105^\circ=\sin75^\circ=\frac{\sqrt{3}+\sqrt{2}}{4}$

計算(函數)

(360)

$(1+\sin 45^\circ+\sin 30^\circ)(1-\cos 45^\circ+\cos 60^\circ)$ ノ値ヲ求メヨ

$$=\left(1+\frac{1}{\sqrt{2}}+\frac{1}{2}\right)\left(1-\frac{1}{\sqrt{2}}+\frac{1}{2}\right)=\underline{\underline{1\frac{3}{4}}}$$

(361)

195° ノ正弦, 正切, 余弦ヲ求メヨ

$195^\circ=180^\circ+15^\circ$ 即第三象限ノ角ナリ

$$\therefore\ \sin 195^\circ=-\sin 15^\circ=\underline{\underline{-\frac{\sqrt{6}-\sqrt{2}}{4}}}$$

$$\tan 195^\circ=\tan 15^\circ=\underline{\underline{2-\sqrt{3}}}$$

$$\cos 195^\circ=\underline{\underline{-\frac{\sqrt{6}+\sqrt{2}}{4}}}$$

(362)

$\cos^2 A$ ガ 0° ヨリ 360° マデノ中ノ値ノ変化如何

$\cos A$ ハ問題342ニ説明セル如クナルヲ以テ

$\cos^2 A$ ハ

第一象限ニ於テハ 1ヨリ0マデニ減ズ

第二象限ニ於テハ 0ヨリ1マデニ増ス

第三象限ニ於テハ 1ヨリ0マデニ減ズ

第四象限ニ於テハ 0ヨリ1マデニ増ス

(363)

$\cos 63^\circ 37.3'$ ノ値ヲ問フ

$\cos 63^\circ 30'=0.4462$		$63^\circ 37.7'$
$\cos 63^\circ 40'=0.4436$		$63^\circ 30'$
$10'$	0.0026	$7.7'$

$10':7.3'=0.0026:x,\quad x=0.0019$

$\therefore\ \cos 63^\circ 37.7'=0.4462-0.0019=\underline{\underline{0.4443}}$

計算(函數)

(364)

$\sin A+\cos A$ ノ値ノ変化ヲ問フ

$\sin A+\cos A$ ハ

$A=0^\circ$ ナレバ	$0+1=1$	ニナリ
$A=45^\circ$ トナレバ	$\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\sqrt{2}$	トナリ
$A=90^\circ$ トナレバ	$1+0=1$	トナリ
$A=135^\circ$ トナレバ	$\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}=0$	トナリ
$A=180^\circ$ トナレバ	$0-1=-1$	トナリ

A ガ 180° 以上トナレバ順次絶対値等シク符号ノ異ナル値ノ変化タルニ過ギズ

(365)

$\sin 22^\circ+\cos 22^\circ$ 及ビ $\sin 22^\circ-\cos 22^\circ$ ノ符号ヲ求メヨ

22° ハ第一象限ノ角ニシテ第一象限ニ於テハ角ノ増スホド正弦ハ値ヲ増シ余弦ハ減ジ 45° ナルトキ共ニ相等シ

故ニ 22° ニ於テハ正弦ハ余弦ヨリ小ナリ

故ニ $\sin 22^\circ+\cos 22^\circ$ ハ正ニシテ

$\sin 22^\circ-\cos 22^\circ$ ハ負ナリ

(366)

$\sin A=0.9479$ ナルトキ A ノ値ヲ求メヨ

$\sin 71^\circ 30'=0.9478$		0.9479
$\sin 71^\circ 20'=0.9474$		0.9474
$10'$	0.0009	0.0005

$9:5=10':x,\quad x=6'$

$\therefore\ A=71^\circ 20'+6'=\underline{\underline{71^\circ 26'}}$

(367)

$\sin A - \cos A$ ノ値ノ変化如何

$\sin A - \cos A$ ハ

$A = 0$ ナレバ $0 - 1 = -1$ ニシテ

$A = 45^\circ$ トナレバ $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$ トナリ

$A = 90^\circ$ トナレバ $1 - 0 = 1$ トナリ

$A = 135^\circ$ トナレバ $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$ トナリ

$A = 180^\circ$ トナレバ $0 + 1 = 1$ トナリ

Aガ180°ヲ越ユレバ順次絶對値等シク符号ノ異ナル値ノ変化スルニ過ギズ

(368)

$\sin 191^\circ - \cos 191^\circ$ ノ符号如何

191°ハ第三象限ノ角ニシテ此象限ニ於テハ正弦、余弦共ニ負ニシテ絶對値ノ増減ハ問題365ト同様ナリ 但シ45°ニ代フルニ225°ヲ以テス

故ニ191°ニ於テハ正弦ハ余弦ヨリ絶對値小ナリ

故ニ此式ノ符号ハ正ナリ

(369)

A, Bガ鋭角ニシテ $\sin A = \frac{3}{5}$, $\cos B = \frac{12}{13}$ ナルトキ A+B ノ正弦 余弦ヲ問フ

$\sin A = \frac{3}{5}$ ナレバ $\cos A = \frac{4}{5}$ } A,Bガ直角三角形ノ二鋭角ナリ

$\cos B = \frac{12}{13}$ ナレバ $\sin B = \frac{5}{13}$ }

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \frac{3}{5} \times \frac{12}{13} + \frac{4}{5} \times \frac{5}{13} = \underline{\underline{\frac{56}{65}}}$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$= \frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13} = \underline{\underline{\frac{33}{65}}}$

(370)

$\sin 290^\circ + \cos 290^\circ$ ノ符号ハ如何

$\sin 290^\circ + \cos 290^\circ = -\cos 20^\circ + \sin 20^\circ$

而シテ絶對値ニ於テ $-\cos 20^\circ$ ガ $\sin 20^\circ$ ヨリ大ナリ

故ニ此式ノ符号ハ負ナリ

(371)

$\sin 345^\circ - \cos 345^\circ$ ノ符号如何

$\sin 345^\circ - \cos 345^\circ = -\cos 75^\circ - \sin 75^\circ$

故ニ此式ノ符号ハ負ナリ

(372)

$\tan A = 0.1723$ ナルトキ A ハ如何

$\tan 9^\circ 50' = 0.1733$ 　　0.1733

$\tan 9^\circ 40' = 0.1703$ 　　0.1723

10' 　0.0030 　　0.0010

$3 : 1 = 10' : x$, 　$x = 3'$

$\therefore A = 9^\circ 40' + 3' = \underline{\underline{9^\circ 43'}}$

(373)

$\tan A + \cot A$ ノ値ノ変化ヲ問フ

$\tan A + \cot A$ ハ

$A = 0$ ナレバ $0 + \infty = \infty$ ニシテ

$A = 45^\circ$ トナレバ $1 + 1 = 2$ トナリ

$A = 90^\circ$ トナレバ $\infty + 0 = \infty$ トナリ

$A = 135^\circ$ トナレバ $-1 - 1 = -2$ トナリ

$A = 180^\circ$ トナレバ $0 - \infty = -\infty$ トナリ

Aガ180°ヲ越ユルニ至レバ順次絶對値等シクシテ符号ノ異ナル値ノ変化ニ過ギズ

計算(函數)

(374)

$\sin(-22°)-\cos(-22°)$ ノ符号ヲ問フ

$-22°$ ハ $338°$ ニシテ第四象限ノ角ナリ故ヘニ此ノ題 371 ト同様ナリ

(375)

$\cot A=1.346$ ナルトキ A ノ値ヲ求メヨ

$\cot 36°30'=1.351$　　1.346

$\cot 36°40'=1.343$　　1.343

$10'$　0.008　　0.003

$8:3=10':x,\quad x=4'$

$\therefore A=36°30'+4'=\underline{\underline{36°34'}}$

(376)

$\cos^2A-\sin^2A$ ノ値ノ変化ヲ問フ

$\cos^2A-\sin^2A=(\cos A+\sin A)(\cos A-\sin A)$

$A=0$ ナレバ $1\times(-1)=-1$ ニシテ

$A=45°$ トナレバ $\sqrt{2}\times 0=0$ トナリ

$A=90°$ トナレバ $1\times 1=1$ トナリ

$A=135°$ トナレバ $0\times\sqrt{2}=0$ トナリ

$A=180°$ トナレバ $(-1)\times 1=-1$ トナリ

A ガ $180°$ ヲ越ニル ニ至レハ唯此値ヲ繰返スノミ

(377)

$\cos 0°\sin 270°+2\cos 180°\tan 45°$ ノ値ヲ求メヨ

此式 $=1\times(-1)+2(-1)\times 1$

$=\underline{\underline{-3}}$

計算(函數)

(378)

$\sin(-270°)-\cos(-270°)$ ノ符号如何

$-270°$ ハ $85°$ ニシテ第一象限ノ角ナリ故ヘニ正弦, 余弦共ニ符号ハ正ニシテ値ハ正弦ガ大ナリ故ヘニ此ノ式ノ符号ハ正ナリ

(379)

$\tan A$ ガ知レタルトキ $\sin\frac{A}{2}$, $\cos\frac{A}{2}$ ニ夫々幾ツノ値アリヤ

$$\cos A=\frac{1}{\sec A}=\frac{\pm 1}{\sqrt{1+\tan^2A}}=2\cos^2\frac{A}{2}-1$$

$$\therefore\ 2\cos^2\frac{A}{2}=1\pm\frac{1}{\sqrt{1+\tan^2A}}$$

即チ $\cos^2\frac{A}{2}=\dfrac{\sqrt{1+\tan^2A}\pm 1}{2\sqrt{1+\tan^2A}}$

$$\therefore\ \cos\frac{A}{2}=\pm\sqrt{\frac{\sqrt{1+\tan^2A}\pm 1}{2\sqrt{1+\tan^2A}}}$$

即チ $\cos\frac{A}{2}$ ニハ四ツノ値アルヲ知ル

$$\sin A=\frac{1}{\cos\sec A}=\frac{\pm 1}{\sqrt{1+\frac{1}{\tan^2A}}}=2\sin\frac{A}{2}\cos\frac{A}{2}$$

$$\therefore\ \sin\frac{A}{2}=\pm\frac{1}{2\sqrt{1+\frac{1}{\tan^2A}}\cos\frac{A}{2}}$$

即チ $\sin\frac{A}{2}$ ニハ二ツノ値アルモ $\cos\frac{A}{2}$ ナル因數ヲ除數ニ有スルガ故ヘニ都合ハ二ツノ値アリ, 然レドモ此中二ツヅツハ同一トナルヲ以テ四ツノ値トナル

即 $\sin\frac{A}{2}$, $\cos\frac{A}{2}$ ノ何レモ四ツノ値アリ

(380)

$\tan A = -\frac{4}{3}$ ナルトキ $\sin A$ 及 $\cos A$ ノ値如何

$\tan A = -\frac{4}{3} = \frac{-4}{3} = \frac{4}{-3}$ トナスヲ得

〃 〃 中 各項ノ符ニシテ

サリ故ヘニAカ第二象限ノ角ナレバ

$\sin A = \frac{4}{5}$, $\cos A = -\frac{3}{5}$

又Aガ第四象限ノ角ナレバ

$\sin A = -\frac{4}{5}$, $\cos A = \frac{3}{5}$

(381)

$\frac{\sin A}{1+\cos A}$ ノ値ヲ変化シ問フ

此式 $= \tan\frac{A}{2}$ (問題128) トナスヲ得ルヲ以テ

$A=0°$ ナレバ $\frac{A}{2}=0°$ ∴ $\tan\frac{A}{2}=0$

$A=90°$ トナレバ $\frac{A}{2}=45°$ ∴ $\tan\frac{A}{2}=1$

$A=180°$ トナレバ $\frac{A}{2}=90°$ ∴ $\tan\frac{A}{2}=\infty$

$A=270°$ トナレバ $\frac{A}{2}=135°$ ∴ $\tan\frac{A}{2}=-1$

$A=360°$ トナレバ $\frac{A}{2}=180°$ ∴ $\tan\frac{A}{2}=0$

Aガ360°ヲ越ユルニ至レバ順次此値ヲ繰返スニ過ギズ

(382)

$3\sin 0° \sin 180° + 2\cos 90° - \cos 360°$ ノ値ヲ問フ

此式 $= 3\times 0\times(-0) + 2\times 0 - 1 = 1$

(383)

$\tan A = \frac{\sqrt{3}}{4-\sqrt{3}}$, $\tan B = \frac{\sqrt{3}}{4+\sqrt{3}}$ ナルトキ $\tan(A-B)$ ノ値ヲ問フ

$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$$

$$= \frac{\frac{\sqrt{3}}{4-\sqrt{3}} - \frac{\sqrt{3}}{4+\sqrt{3}}}{1+\frac{\sqrt{3}}{4-\sqrt{3}}\cdot\frac{\sqrt{3}}{4+\sqrt{3}}} = \frac{3}{8}$$

(384)

$\sin(-470°) - \cos(-470°)$ ノ符号ヲ問フ

$-470°$ ハ $250°$ ニシテ第三象限ノ角ナリ故ヘニ正弦余弦共ニ符号ノ負ニシテ絶對値ノ正弦ガ大ナリ故ヘニ此式ノ符号ノ負ナリ

(385)

$15°$ ノ三角函數ヲ求メヨ

$$\sin 15° = \sin(45°-30°)$$
$$= \sin 45° \cos 30° - \cos 45° \sin 30°$$
$$= \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\times\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\cos 15° = \sqrt{1-\sin^2 15°} = \sqrt{1-\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)^2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\tan 15° = \frac{\sin 15°}{\cos 15°} = \frac{\frac{\sqrt{6}-\sqrt{2}}{4}}{\frac{\sqrt{6}+\sqrt{2}}{4}} = 2-\sqrt{3}$$

(386)

$\tan\frac{A}{2}=\frac{-1\pm\sqrt{1+\tan^2 A}}{\tan A}$ (前141) ヨリ $\tan 7.5^\circ$ ヲ求メヨ

$7.5^\circ=\frac{15^\circ}{2}$ ナリ 而シテ $\tan 15^\circ=2-\sqrt{3}$ (前問)

$$\therefore\ \tan 7.5^\circ=\frac{-1\pm\sqrt{1+\tan^2 15^\circ}}{\tan 15^\circ}$$

$$=\frac{-1\pm\sqrt{1+(2-\sqrt{3})^2}}{2-\sqrt{3}}=\frac{-1+\sqrt{8-4\sqrt{3}}}{2-\sqrt{3}}$$

$$=\underline{\underline{(2\sqrt{2-\sqrt{3}}-1)(2+\sqrt{3})}}$$

(387)

$\sin 1000^\circ+\cos 1000^\circ$ ノ符号如何

1000° ハ第四象限ノ角ニシテ正弦ハ負, 余弦ハ正ナリ

而シテ其絶對値ハ正弦大ニシテ余弦小ナリ

故ニ此式ノ符号ハ負ナリ

(388)

α,β ガ鋭角ニシテ $\cot\alpha=2$, $\operatorname{cosec}\beta=\sqrt{10}$ ナルトキ

$\alpha+\beta$ ハ如何

$\cot\alpha=2$ 從テ $1+\cot^2\alpha=1+4=\operatorname{cosec}^2\alpha$

$\therefore\ \operatorname{cosec}\alpha=\pm\sqrt{5}$ α ガ鋭角ナルヲ以テ(−)ハ捨ツ

$\therefore\ \sin\alpha=\frac{1}{\sqrt{5}},\quad \cos\alpha=\frac{2}{\sqrt{5}}$

又 $\operatorname{cosec}\beta=\sqrt{10}$ ヨリ $\sin\beta=\frac{1}{\sqrt{10}},\ \cos\beta=\frac{3}{\sqrt{10}}$

而シテ $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$$=\frac{1}{\sqrt{5}}\times\frac{3}{\sqrt{10}}+\frac{2}{\sqrt{5}}\times\frac{1}{\sqrt{10}}=\frac{1}{\sqrt{2}}$$

$\therefore\ \alpha+\beta=\underline{\underline{45^\circ}}$

(389)

$\tan x=2,\ \tan y=\frac{1}{3}$ ナルトキ $\tan\{2(x+y)\}$ ノ値ヲ求メヨ

$$\tan\{2(x+y)\}=\tan 2(x+y)$$

$$=\frac{2\tan(x+y)}{1-\tan^2(x+y)}\ (\text{公式}32)=\frac{2\cdot\frac{\tan x+\tan y}{1-\tan x\tan y}}{1-\left(\frac{\tan x+\tan y}{1-\tan x\tan y}\right)^2}$$

$$=\frac{2\times\frac{2+\frac{1}{3}}{1-2\times\frac{1}{3}}}{1-\left(\frac{2+\frac{1}{3}}{1-2\times\frac{1}{3}}\right)^2}=\frac{14}{1-49}=\underline{\underline{-\frac{7}{24}}}$$

(390)

$\cos 315^\circ=\frac{1}{\sqrt{2}}$ ヲ知リテ $\sin 157.5^\circ$ 及 $\cos 157.5^\circ$ ノ値ヲ求メヨ

$157.5^\circ=\frac{315^\circ}{2}$

$$\therefore\ \sin 157.5^\circ=\sqrt{\frac{1}{2}(1-\cos 315^\circ)}\ (\text{公式}33)$$

$$=\sqrt{\frac{1}{2}\left(1-\frac{1}{\sqrt{2}}\right)}=\underline{\underline{\frac{\sqrt{2-\sqrt{2}}}{2}}}$$

又ハ $\cos 157.5^\circ=\sqrt{\frac{1}{2}(1+\cos 315^\circ)}$ (公式34)

$$=\sqrt{\frac{1}{2}\left(1+\frac{1}{\sqrt{2}}\right)}=\underline{\underline{\frac{\sqrt{2+\sqrt{2}}}{2}}}$$

(391)

$\cos 2A=\frac{3}{5}$ ナルトキ $\sin^4 A+\cos^4 A$ ノ値ヲ求メヨ

$\sin^4 A+\cos^4 A$

$$=(\sin^2 A+\cos^2 A)^2-2\sin^2 A\cos^2 A$$

$$=1-\frac{4\sin^2 A\cos^2 A}{2}=1-\frac{1}{2}\sin^2 2A$$

$$=1-\frac{1}{2}(1-\cos^2 2A)=1-\frac{1}{2}+\frac{1}{2}\times\frac{9}{25}=\underline{\underline{\frac{17}{25}}}$$

(392)

種々ノ角度ニ於ケル三角函數ヲ計算スレバ次表ノ如シ

角度	π·90°+A	sin A	cos A	tan A	cot A	sec A	cosec A
120°	90°+30°	$\cos 90° = \frac{\sqrt{3}}{2}$	$-\sin 90° = -\frac{1}{2}$	$-\cot 30° = -\sqrt{3}$	$-\tan 30° = -\frac{1}{\sqrt{3}}$	$-\sec 90° = \frac{2}{\sqrt{3}}$	
135°	90°+45°	$\cos 45° = \frac{1}{\sqrt{2}}$	$-\sin 45° = -\frac{1}{\sqrt{2}}$	$-\cot 45° = -1$	$-\tan 45° = -1$	$-\sec 45° = -\sqrt{2}$	
150°	90°+60°	$\cos 60° = \frac{1}{2}$	$-\sin 60° = -\frac{\sqrt{3}}{2}$	$-\cot 60° = -\frac{1}{\sqrt{3}}$	$-\tan 60° = -\frac{1}{\sqrt{3}}$	$-\sec 60° = -\frac{2}{\sqrt{3}}$	
210°	2×90°+30°	$-\sin 30° = -\frac{1}{2}$	$-\cos 30° = -\frac{\sqrt{3}}{2}$	$\tan 30° = \frac{1}{\sqrt{3}}$	$\cot 30° = \sqrt{3}$	$-\sec 30° = -\frac{2}{\sqrt{3}}$	$-\operatorname{cosec} 30° = -2$
225°	2×90°+45°	$-\sin 45° = -\frac{1}{\sqrt{2}}$	$-\cos 45° = -\frac{1}{\sqrt{2}}$	$\tan 45° = 1$	$\cot 45° = 1$	$-\sec 45° = -\sqrt{2}$	$-\operatorname{cosec} 45° = -\sqrt{2}$
240°	2×90°+60°	$-\sin 60° = -\frac{\sqrt{3}}{2}$	$-\cos 60° = -\frac{1}{2}$	$\tan 60° = \sqrt{3}$	$\cot 60° = \frac{1}{\sqrt{3}}$	$-\sec 60° = -2$	$-\operatorname{cosec} 60° = -\frac{2}{\sqrt{3}}$
300°	3×90°+30°	$-\cos 30° = -\frac{\sqrt{3}}{2}$	$\sin 30° = \frac{1}{2}$	$-\cot 30° = -\sqrt{3}$	$-\tan 30° = -\frac{1}{\sqrt{3}}$	$\sec 30° = \frac{2}{\sqrt{3}}$	$-\operatorname{cosec} 30° = -2$
315°	3×90°+45°	$-\cos 45° = -\frac{1}{\sqrt{2}}$	$\sin 45° = \frac{1}{\sqrt{2}}$	$-\cot 45° = -1$	$-\tan 45° = -1$	$\sec 45° = \sqrt{2}$	$-\operatorname{cosec} 45° = -\sqrt{2}$
330°	3×90°+60°	$-\cos 60° = -\frac{1}{2}$	$\sin 60° = \frac{\sqrt{3}}{2}$	$-\cot 60° = -\frac{1}{\sqrt{3}}$	$-\tan 60° = -\sqrt{3}$	$\sec 60° = 2$	$-\operatorname{cosec} 60° = -\frac{2}{\sqrt{3}}$

(393)

$\sin A + \cos A = \frac{5}{4}$ ナルトキ $-\sin 2A$ 及ビ $\sin^3 A + \cos^3 A$ ノ値ヲ求メヨ

$$\sin 2A = 2\sin A\cos A \quad (\text{公式}26)$$
$$= (\sin A + \cos A)^2 - (\sin^2 A + \cos^2 A)$$
$$= \frac{25}{16} - 1 = \underline{\underline{\frac{9}{16}}}$$

$$\sin^3 A + \cos^3 A = (\sin A + \cos A)(\sin^2 A + \cos^2 A - \sin A\cos A)$$
$$= \frac{5}{4}\left(1 - \frac{2\sin A\cos A}{2}\right) = \frac{5}{4}\left(1 - \frac{\frac{9}{16}}{2}\right)$$
$$= \underline{\underline{\frac{105}{128}}}$$

(394)

$\sin 32°53'$ ノ値ハ次ノ如シ

$\sin 32°50' = 0.5422$, 表差24

$10' : 3' = 0.0024 : x$, $x = 0.00018$

$\therefore \sin 32°53' = \underline{\underline{0.5429}}$

計算（對數）

(395)

$\log_3 81$ ハ如何

$3^4 = 81 \quad \therefore \log_3 81 = \underline{\underline{4}}$

(396)

底ノ如何ニ係ラズ1ノ對數ハ0ナリ

$a^0 = \frac{a}{a} = 1 \quad \therefore \log_a 1 = \log_a a^0$ 即1ノ對數ハ

底ノ對數ハ1ナリ

$\log_a a = \log_a a^1$ 即チaノ對數ハ1ナリ

(397)

$\log_2 1024 = x$ トセバ $2^x = 1024 = 2^{10}$

$\therefore x = \underline{\underline{10}} = \log_2 1024$

(398)

$\log_2 32 = x$ トセバ $2^x = 32 = 2^5$

$\therefore x = \underline{\underline{5}} = \log_2 32$

(399)

$\log_3 \sqrt{27} = x$ トセバ $3^x = \sqrt{27} = 3^{\frac{3}{2}}$

$\therefore x = \underline{\underline{\frac{3}{2}}} = \log_3 \sqrt{27}$

(400)

$\log_5 125 = x$ トセバ $5^x = 125 = 5^3$

$\therefore x = \underline{\underline{3}} = \log_5 125$

$\log_9 243 = y$ トセバ $9^y = 243 = 9^{\frac{5}{2}}$

$\therefore y = \underline{\underline{\frac{5}{2}}} = \log_9 243$

計算（對數）

(401)

$\log_4 0.125 = x$ トセバ $4^x = \frac{125}{1000} = \frac{1}{8} = \frac{1}{4^{\frac{3}{2}}}$

$\therefore x = -\frac{3}{2} = \underline{\underline{-1.5}} = \log_4 .125$

(402)

$\log_{16} 8 = x$ トセバ $16^x = 8 = 16^{\frac{3}{4}}$

$\therefore x = \frac{3}{4} = \underline{\underline{0.75}} = \log_{16} 8$

(403)

$\log_{2.25} 3.375 = x$ トセバ $2.25^x = 3.375 = 2.25^{\frac{3}{2}}$

$\therefore x = \frac{3}{2} = \underline{\underline{1.5}} = \log_{2.25} 3.375$

(404)

$\log_4 64 = \log_4 4^3 = \underline{\underline{3}}$

$\log_4 8 = \log_4 4^{\frac{3}{2}} = \underline{\underline{\frac{3}{2}}}$

$\log_4 \frac{1}{2} = \log_4 \frac{1}{4^{\frac{1}{2}}} = \log_4 4^{-\frac{1}{2}} = \underline{\underline{-\frac{1}{2}}}$

(405)

$\log_{49} \sqrt[3]{7} = x$ トセバ $49^x = \sqrt[3]{7} = 49^{\frac{1}{6}}$

$\therefore x = \underline{\underline{\frac{1}{6}}} = \log_{49} \sqrt[3]{7}$

(406)

$\log_a a^3 = \underline{\underline{3}}. \quad \log_a a^{\frac{10}{3}} = \underline{\underline{\frac{10}{3}}}. \quad \log_a \sqrt[4]{a} = \underline{\underline{\frac{1}{4}}}$

$\log_a \sqrt[3]{a^2} = \underline{\underline{\frac{2}{3}}}. \quad \log_a \frac{1}{a^{\frac{5}{2}}} = \log_a a^{-\frac{5}{2}} = \underline{\underline{-\frac{5}{2}}}$

$\log_4 0.0625 = \log_4 \frac{1}{16} = \log_4 \frac{1}{4^2} = \log_4 4^{-2} = \underline{\underline{-2}}$

$\log_4 \sqrt[3]{16} = \log_4 4^{\frac{4}{3}} = \underline{\underline{\frac{4}{3}}}$

計算(對數)

(407)

$\log_{0.01}10 = x$ とすれば　$0.01^x = 10$,　$\left(\frac{1}{10^2}\right)^x = 10$,

$10^{-2x} = 10^1$　$\therefore -2x = 1$　$\therefore x = -\frac{1}{2}$

$\therefore \log_{0.01}10 = \underline{\underline{-\frac{1}{2}}}$

$\log_5 0.04 = x$ とすれば　$5^x = 0.04 = \frac{1}{5^2} = 5^{-2}$

$\therefore x = \underline{\underline{-2}} = \log_5 0.04$

(408)

$\log_{10}1000 = x$ とすれば　$10^x = 1000 = 10^3$,　$\therefore x = \underline{\underline{3}}$

$\log_{100}0.001 = \log_{100}\frac{1}{\sqrt{100^3}} = \log_{100}100^{-\frac{3}{2}} = \underline{\underline{-\frac{3}{2}}}$

$\log_{27}9 = \log_{27}27^{\frac{2}{3}} = \underline{\underline{\frac{2}{3}}}$

(409)

$\log_{10}42 = \log_{10}(2\times3\times7)$

$= \log_{10}2 + \log_{10}3 + \log_{10}7$

$= 0.30103 + 0.47712 + 0.84510$

$= \underline{\underline{1.62325}}$

(410)

$\log_{10}140 = \log_{10}(2^2\times5\times7)$

$= \log_{10}2^2 + \log_{10}\frac{10}{2} + \log_{10}7$

$= \cancel{2\log_{10}2 + \log_{10}10 - \log_{10}2 + \log_{10}7}$

$= 0.30103 + 1 + 0.84510 = \underline{\underline{2.14613}}$

計算(對數)

(411)

$7\log_2\frac{16}{15} + 5\log_2\frac{25}{24} + 3\log_2\frac{81}{80}$

$= \log_2\left\{\left(\frac{16}{15}\right)^7\times\left(\frac{25}{24}\right)^5\times\left(\frac{81}{80}\right)^3\right\} = \log_2 2 = \underline{\underline{1}}$

(412)

$\log_{10}7.29 = \log_{10}\frac{3^6}{100} = \log_{10}3^6 - \log_{10}10^2$

$= 6\times0.47712 - 2 = \underline{\underline{0.86272}}$

(413)

$\log_{10}3600 = \log_{10}(100\times3^2\times2^2)$

$= \log_{10}100 + 2\log_{10}3 + 2\log_{10}2$

$= 2 + 2\times0.47712 + 2\times0.30103 = \underline{\underline{3.5563}}$

(414)

$\log_{15}\sqrt{54} - \log_{15}\left(\frac{27}{27}\right)^2 + \log_{15}\frac{8}{3}\sqrt{\frac{2}{3}}$

$= \log_{15}\left\{\sqrt{54}\div\left(\frac{36}{5\times27}\right)^2\times\frac{8}{3}\sqrt{\frac{2}{3}}\right\}$

$= \log_{15}15^2 = \underline{\underline{2}}$

(415)

$\log_{10}(6.4\div\sqrt[3]{21}) = \log_{10}\left\{\frac{2^6}{10}\div(3^{\frac{1}{3}}\times7^{\frac{1}{3}})\right\}$

$= 6\log_{10}2 - 1 - \frac{1}{3}\log_{10}3 - \frac{1}{3}\log_{10}7$

$= 6\times0.30103 - 1 - \frac{1}{3}\times0.47712 - \frac{1}{3}\times0.84510$

$= \underline{\underline{0.36577}}$

$\log_{10}\sqrt[5]{10^2} = \log_{10}10^{\frac{2}{5}} = \underline{\underline{\frac{2}{5}}}$

計算（對數）

(416)

$\log_{10} 5^{\cdots} = \log_{10} \frac{10000}{2} = \log_{10} 10000 - \log_{10} 2$

$= 4 - 0.30103 = \underline{\underline{3.69897}}$

(417)

次ノ對數ノ指標如何

8973 ハ整數四桁ナルヲ以テ $\underline{3}$ ナリ

0.1 ハ小數点以下直チニ有効數字ナルヲ以テ $\underline{-1}$ ナリ

3.68 ハ整數一桁ナルヲ以テ $\underline{0}$ ナリ

78.5241 ハ整數二桁ナルヲ以テ $\underline{1}$ ナリ

108000 ハ整數六桁ナルヲ以テ $\underline{5}$ ナリ

0.000000086 ハ小數点以下有効數字マデノ0ノ大ニ付 $\underline{7}$

(418)

次ノ對數ノ指標ニヨリテ真數ノ小數点ノ位置決メヨ

3.571 ハ指標3ナルヲ以テ千位ノ數ナリ

0.268 ハ指標0ナルヲ以テ一位ノ數ナリ

5.4625 ハ指標5ナルヲ以テ十万位ノ數ナリ

2 ハ指標ノミナル故ハ其真數ハ百位ナリ

10 ハ11桁ノ整數ナリ

(419)

$a^m = b^n$ ナレバ $n \log_b a = m \log_b x$ ナリ

$a^m = b^n = x$ トセバ

$\log_b a x = m \cdots (1)$ $\log_b x = n \cdots (2)$

(1)ニnヲ, (2)ニmヲ乘ジテ辺々相減ズレバ

$n \log_b x = m \log_b x$ トナル

計算（對數）

(420)

次ノ對數ノ整ナル真數ノ整數部分ノ桁數ヲ問フ

$\log 2^{18} = 18 \log 2 = 18 \times 0.30103 = 5.41854$

指標5ナルヲ以テ整數部分六桁ナリ

$\log 2^{30} = \log 2^{30} = 30 \log 2 = 9.0309$

指標9ナルヲ以テ整數部分ハ十桁ナリ

(421)

$\log_n m \times \log_m n = 1$

$\log_n m = x \cdots (1)$ $\log_m n = y \cdots (2)$

(1)ヨリ $m^x = n$ (2)ヨリ $n^y = m$

$\therefore (m^x)^y = n^y = m$

$\therefore m^{xy} = m^1$ $\therefore xy = 1$

$\therefore \log_m n \times \log_n m = 1$

(422)

$\log_2 \sin 45^\circ = x$ トセバ $2^x = \sin 45^\circ = 2^{-\frac{1}{2}}$

$\therefore x = -\frac{1}{2} = \underline{\underline{-0.5}} = \log_2 \sin 45^\circ$

(423)

$\log\{37.59 \times (0.004962)^3\}$

$= \log 37.59 + 3 \log 0.004962$

$= 1.57507 + \bar{3}.69566$

$= 1.57507 + (-9 + 2.08698)$

$= -9 + 3.66205 = \underline{\underline{\bar{7}.66205}}$

(424)

$\left(\frac{203}{200}\right)^{2x}=2$ ヲ解ケ　$\log\left(\frac{203}{200}\right)^{2x}=\log 2$

$2x\log 203-2x\log 200=\log 2$

$2(\log 203-\log 200)x=\log 2$

$x=\frac{\log 2}{2(\log 203-\log 200)}=\frac{0.3010}{2(2.3075-2.3010)}$

$=\underline{\underline{23.16}}$

(425)

$\log 2^{\frac{100}{7}}$，整數部ノ桁數ハ何

$=\frac{100}{7}\log 2=\frac{100}{7}\times 0.30103=4.30043$

此指標ニ4ヲ加ヘタル整數ノ桁數ハ五ツ

(426)

$\log(67.26\div 3.759)$

$=\log 67.26-\log 3.759$

$=1.82776-0.57507=\underline{\underline{1.25269}}$

(427)

$\log\sqrt{0.0000 6726}=\frac{1}{2}\log 0.0000 6726$

$=\frac{1}{2}\times\bar{5}.82776=\frac{1}{2}\times(-6+1.82776)$

$=\underline{\underline{\bar{3}.91388}}$

(428)

$\log 16^{50}=\log 2^{200}=200\log 2$

$=200\times 0.30103=60.206$ 此整數部ガ61

(429)

$\log(375.4\times 49.62)=\log 375.4+\log 49.62$

$=2.57507+1.69566=\underline{\underline{4.27073}}$

(430)

$8^{5-3x}=12^{4-2x}$ ヲ解ケ

$\log 8^{5-3x}=\log 12^{4-2x}$

$(5-3x)\log 8=(4-2x)\log 12$

$5\log 8-3x\log 8=4\log 12-2x\log 12$

$2x\log 12-3x\log 8=4\log 12-5\log 8$

$(2\log 12-3\log 8)x=4\log 12-5\log 8$

$x=\frac{4\log 12-5\log 8}{2\log 12-3\log 8}=\frac{4.3168-4.5155}{2.1584-2.7093}$

$=\underline{\underline{0.486}}$

(431)

$\log(4.962^2\div 0.06726)$

$=2\log 4.962-\log 0.06726$

$=2\times 0.69566-\bar{2}.82776$

$=1.39132-\bar{2}.82776=\underline{\underline{2.56356}}$

(432)

$\sqrt[6]{2713}$ ヲ計算セヨ

$\log\sqrt[6]{2713}=\frac{1}{6}\log 2713$

$=\frac{1}{6}\times 3.432$ 等

$=0.57224=\log\underline{\underline{3.7346}}$

(433)

log sin 40°6′ ノ値ヲ問フ

log sin 40° = 9.80807−10, 表差150

10′ : 6′ = 150 : x, x = 90

∴ log sin 40°6′ = 9.80807 + 90 = 9.80897−10

(434)

$$x = \frac{(34.73)^{\frac{4}{5}} \times \sqrt[6]{2.539}}{\sqrt[4]{4.397} \times (3.456)^3}$$ ヲ對數ニテ計算セヨ

$$\log x = \frac{4}{5}\log 34.73 + \frac{1}{6}\log 2.539 - \frac{1}{4}\log 4.397 - 3\log 3.456$$

$$= 1.2326 + 0.0674 - 0.1608 - 1.6158$$

$$= -0.4766 = \bar{1}.5234$$

∴ x = 0.3334

(435)

log tan A = 0.12051 ノ A ヲ求メヨ

log tan 52°50′ = 0.12026, 表差263

0.12051 − 0.12026 = 0.00025

263 : 25 = 10″ : x, x = 0.95

∴ A = 52°50′.95 = 52°50′57″

(436)

log cot 27°13′24″ ノ値ヲ問フ　24″ = 0.4′

log cot 27°10′ = 0.28472, 表差311

10′ : 3.4′ = 311 : x, x = 106

∴ log cot 27°13.4′ = 0.28472 − 106 = 0.28366

(437)

$25^m = 421$ ヲ解ケ

$\log 25^m = \log 421$, $m \log 25 = \log 421$

$1.39794m = 2.62428$

$$m = \frac{2.62428}{1.39794} = 1.88$$

(438)

$$\frac{(20.13)^2 (0.00593)^{\frac{3}{2}}}{(0.009123)^4}$$ ヲ對數ニテ計算セヨ

$$\log \frac{(20.13)^2 (0.00593)^{\frac{3}{2}}}{(0.009123)^4}$$

$$= 2\log 20.13 + \frac{3}{2}\log 0.005 - 4\log 0.009123$$

$$= 2 \times 1.3039 + \frac{3}{2} \times \bar{3}.7731 - 4 \times \bar{3}.9602$$

$$= 2.6078 + \bar{4}.6597 - \bar{9}.8408$$

$= 7.4267$　此真數 = 26720000

(439)

L cos A = 9.31687 ナルトキ A ヲ求メヨ

log cos 78° = 9.31788−10, 表差298

9.31788−10 − (9.31687−10) = 101

298 : 101 = 5′ : x, x = 1.7′ = 1′42″

∴ A = 78°1′42″

(440)

$$\log \sqrt[3]{0.003759} = \frac{1}{3}\log 0.003759$$

$$= \frac{1}{3} \times \bar{3}.57507 = \bar{1}.19169$$

計算（對數）

(441)

$2^m \times 5^n = 1$ ....(1)　$5^{m+1} \times 2^n = 2$ ....(2)

ヨリ m, n ヲ求メヨ

(1)ヨリ $m\log 2 + n\log 5 = \log 1$ ....(3)

(2)ヨリ $m\log 5 + \log 5 + n\log 2 = \log 2$ ....(4)

(3)×$\log 5$　$m\log 2 \times \log 5 + n(\log 5)^2 = 0$ ..(5)

(4)×$\log 2$　$m\log 2 \times \log 5 + n(\log 2)^2$

$= (\log 2)^2 - \log 5 \times \log 2$ ..(6)

(5)ヨリ(6)ヲ減ジテ

$n\{(\log 5)^2 - (\log 2)^2\} = \log 2(\log 2 - \log 5)$

即 $n\{(0.699+0.3010)(0.699-0.3010)\}$

$= 0.301(0.3010 - 0.699)$

即 $n(1 \times 0.398) = 0.3010(-0.398)$

$n = \underline{\underline{-0.301}}$

(3)ヨリ $m\log 2 - 0.301\log 5 = 0$

$0.301m = 0.301 \times 0.699$, $m = \underline{\underline{0.699}}$

驗　$\log(2^{0.699} \times 5^{0.301})$

$= 0.699\log 2 - 0.301\log 5$

$= 0.699 \times 0.301 - 0.301 \times 0.699$

$= 0$

$\therefore\ 2^m \times 5^n = 1$

(2)モ同様ノ方法ヲ驗シ得ベシ

方程式

(442)

$\sin\theta = \frac{\sqrt{3}}{2}$ ヲ解ケ（最小角）　$\theta = \underline{\underline{60^\circ}}$

(443)

$\sin\theta = \sin 20^\circ$ ヲ解ケ（0°ト直角トノ間）

$\theta = n \times 180^\circ + (-1)^n A$ ニヨリ

$n = 0$ トセバ　$\theta = 0 \times 180^\circ + 1 \times 20^\circ = \underline{\underline{20^\circ}}$

$n = 1$ トセバ　$\theta = 1 \times 180^\circ - 1 \times 20^\circ = \underline{\underline{160^\circ}}$

$n = 2$ トセバ　$\theta = 2 \times 180^\circ + 1 \times 20^\circ = \underline{\underline{380^\circ}}$

$n = 3$ トセバ　$\theta = 3 \times 180^\circ - 1 \times 20^\circ = \underline{\underline{520^\circ}}$

$n = 4$ トセバ　$\theta = 4 \times 180^\circ + 1 \times 20^\circ = 740^\circ$

(444)

$4\sin^2 x - 2(\sqrt{3}+1)\sin x + \sqrt{3} = 0$ ヲ解ケ（銳角）

$(2\sin x - 1)(2\sin x - \sqrt{3}) = 0$

$\therefore\ 2\sin x - 1 = 0,\ 2\sin x = 1,\ \sin x = \frac{1}{2},$

$\therefore\ \sin x = \sin 30^\circ\ \therefore\ x = \underline{\underline{30^\circ}}$

又 $2\sin x - \sqrt{3} = 0,\ 2\sin x = \sqrt{3},$

$\therefore\ \sin x = \frac{\sqrt{3}}{2} = \sin 60^\circ\ \therefore\ x = \underline{\underline{60^\circ}}$

(445)

$\sin A = -\frac{1}{2}$ ヲ解ケ　$-\frac{1}{2} = \sin(-30^\circ)$

$A = n \times 180^\circ + (-1)^n(-30^\circ)$

(446)

$\sin\theta = 0$ ヲ解ケ　$0 = \sin 0^\circ$

$\theta = n \times 180^\circ + (-1)^n 0^\circ$

(447)

$\sin^2 x+\sqrt{3}\cos x-\frac{7}{4}=0$ ヲ解ケ (90°以内)

$1-\cos^2 x+\sqrt{3}\cos x-\frac{7}{4}=0$

$\cos^2 x-\sqrt{3}\cos x+\frac{3}{4}=0$

$(\cos x-\frac{\sqrt{3}}{2})^2=0 \quad \therefore \cos x=\frac{\sqrt{3}}{2}=\cos 30°$

$\therefore x=\underline{\underline{30°}}$

(448)

$\sin A=-\frac{1}{\sqrt{2}}$ ヲ解ケ (0°ト8直角トノ間)

$-\frac{1}{\sqrt{2}}=\sin(-45°)$

$n=0$ トセバ $A=0\times180°+(-1)^0(-45°)=\cancel{-45°}$

$n=1$ トセバ $A=1\times180°+45°=\underline{\underline{225°}}$

$n=2$ トセバ $A=2\times180°-45°=\underline{\underline{315°}}$

$n=3$ トセバ $A=3\times180°+45°=\underline{\underline{585°}}$

$n=4$ トセバ $A=4\times180°-45°=\underline{\underline{675°}}$

(449)

$2\sin^2 x+3\cos x-3=0$ ヲ解ケ (正ノ360°以内)

$2(1-\cos^2 x)+3\cos x-3=0$

$2\cos^2 x-3\cos x+1=0$

$(2\cos x-1)(\cos x-1)=0$

$2\cos x-1=0,\ \cos x=\frac{1}{2} \quad \therefore x=60°$ 又ハ $300°$

$\cos x-1=0,\ \therefore x=0°$ 又ハ $360°$

答 0°, 60°, 300°, 360°

(450)

$\sin A=1$ ヲ解ケ $\quad 1=\sin 90°$

$\therefore A=n\times180°+(-1)^n\times90°$

今 n ヲ偶数 $2p$ トセバ

$A=2p\times180°+90°=(2p+\frac{1}{2})\times180°$

又 n ヲ奇数 $2q+1$ トセバ

$A=(2q+1)\times180°-90°=(2q+1-\frac{1}{2})\times180°$

$=(2q+\frac{1}{2})\times180°$

$\therefore A=\underline{\underline{(2n+\frac{1}{2})\times180°}}$

(451)

$\tan^2\theta-(\sqrt{3}+1)\tan\theta+\sqrt{3}=0$ ヲ解ケ (鋭角)

$(\tan\theta-1)(\tan\theta-\sqrt{3})=0$

$\tan\theta-1=0,\ \tan\theta=1=\tan 45°$

$\tan\theta-\sqrt{3}=0,\ \tan\theta=\sqrt{3}=\tan 60°$

$\therefore \theta=\underline{\underline{45°}}$ 又ハ $\underline{\underline{60°}}$

(452)

$\sin\theta=-\sin 25°$ ヲ解ケ (0°ヨリ720°マデノ間ノ角)

$-\sin 25°=\sin(-25°)$

$n=0$ トセバ $\theta=0\times180°+(-1)^0(-25°)=\cancel{-25°}$

$n=1$ トセバ $\theta=1\times180°-1(-25°)=\underline{\underline{205°}}$

$n=2$ トセバ $\theta=2\times180°+(-25°)=\underline{\underline{335°}}$

$n=3$ トセバ $\theta=3\times180°-(-25°)=\underline{\underline{565°}}$

$n=4$ トセバ $\theta=4\times180°+(-25°)=\underline{\underline{695°}}$

(453)

$\sec^3 x - \frac{2}{\cot^2 x} = 2$ ヲ解ケ(正ノ四直角以内)

$\frac{1}{\cos^3 x} - \frac{\sin 2x}{\cos^2 x} - 2 = 0$

$1 - 2\sin^2 x \cos x - 2\cos^3 x = 0$

$1 - 2(1-\cos^2 x)\cos x - 2\cos^3 x = 0$

$1 - 2\cos x + \cancel{2\cos^3 x} - \cancel{2\cos^3 x} = 0$

$\cos x = \frac{1}{2}$ $\therefore$ $x = \underline{60^\circ}$ 又 $\underline{300^\circ}$

(454)

$\cos A = -1$ ヲ解ケ $-1 = \cos 180^\circ$

$\therefore A = 2m \times 180^\circ \pm 180^\circ = (2m \pm 1) \times 180^\circ$

$= \underline{(2m+1) \times 180^\circ}$

(455)

$\sin \theta = \frac{1}{\sqrt{2}}$ ヲ解ケ(最小正角) 即 $\underline{45^\circ}$

(456)

$\sin x = -\frac{1}{2}$ ヲ解ケ(最小正角)

菊池氏三角法 88頁注意ニヨリテ $\frac{1}{2} = \sin 30^\circ$ ヲ得テ次ノ如シ

$30^\circ + 180^\circ = \underline{210^\circ}$

(457)

$4\sin^2 x + 2\sin x = 1$ ヲ解ケ (鋭角)

$4\sin^2 x + 2\sin x - 1 = 0$ ヲ解キテ

$\sin x = \frac{1}{4}(\sqrt{5}-1) = \sin 18^\circ$ (問題354)

$\therefore x = \underline{18^\circ}$

(458)

$\cos 3x + \cos 2x + \cos x = 0$ ヲ解ケ(二直角以内ノ正角)

$\cos 3x + \cos x + \cos 2x = 0$

$2\cos 2x \cos x + \cos 2x = 0$

$\cos 2x(2\cos x + 1) = 0$

即 $\cos 2x = 0$ ...(1) $2\cos x + 1 = 0$ ...(2)

(1)ヨリ $2x = 90^\circ$ 又 270°

$\therefore x = \underline{45^\circ}$ or $\underline{135^\circ}$

(2)ヨリ $\cos x = -\frac{1}{2}$ $\therefore x = \underline{120^\circ}$

(459)

$\cos A = \frac{1}{2}$ ヲ解キテ小ナル角四ツヲ答ヘヨ

$\frac{1}{2} = \cos 60^\circ$ $\therefore A = 2m \times 180^\circ \pm 60^\circ$

$m = 0$ トセバ $A = 2 \times 0 \times 180^\circ \pm 60^\circ = \underline{\pm 60^\circ}$

$m = 1$ トセバ $A = 2 \times 1 \times 180^\circ \pm 60^\circ$

$= 360^\circ \pm 60^\circ$ 即 $\cancel{420^\circ}$, $\underline{300^\circ}$

$m = -1$ トセバ $A = 2 \times (-1) \times 180^\circ \pm 60^\circ$

$= -360^\circ \pm 60^\circ$ 即 $\cancel{-420^\circ}$

$\underline{300^\circ}$

(460)

$\sin x + \cos x = \frac{1}{\sqrt{2}}$ ヲ解ケ(二直角以内ノ正角)

$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{2}$

$\sin 45^\circ \sin x + \cos 45^\circ \cos x = \frac{1}{2}$

$\cos(x - 45^\circ) = \cos 60^\circ$

$\therefore x - 45^\circ = 60^\circ$ $\therefore x = \underline{105^\circ}$

(461)

$\tan x = 1$ ヲ解ケ　$1 = \tan 45^\circ$

$x = n \times 180^\circ + 45^\circ$

$\tan x = -1$ ヲ解ケ　$-1 = \tan(-45^\circ)$

$x = n \times 180^\circ - 45^\circ$

(462)

$\cos 2x = (\sqrt{3}-\sqrt{2})\cos x + \frac{\sqrt{3}}{\sqrt{2}} - 1$ ヲ解ケ

$2\cos^2 x - 1 - (\sqrt{3}-\sqrt{2})\cos x - \frac{\sqrt{3}}{\sqrt{2}} + 1 = 0$

$2\cos^2 x - (\sqrt{3}-\sqrt{2})\cos x - \frac{\sqrt{3}}{\sqrt{2}} = 0$

$\cos x = \frac{\sqrt{3}-\sqrt{2} \pm \sqrt{5 - 2\sqrt{6} + \frac{4\sqrt{3}}{\sqrt{2}}}}{4}$

$= \frac{\sqrt{3}-\sqrt{2} \pm \sqrt{5 + 2\sqrt{6}}}{4} = \frac{\sqrt{3}-\sqrt{2} \pm \sqrt{2}+\sqrt{3}}{4}$

$\cos x = \frac{\sqrt{3}-\sqrt{2}+\sqrt{2}+\sqrt{3}}{4} = \frac{\sqrt{3}}{2} = \cos 30^\circ$

$\cos x = \frac{\sqrt{3}-\sqrt{2}-\sqrt{2}-\sqrt{3}}{4} = -\frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}$

$= \cos 135^\circ$

$\therefore x = \underline{\underline{30^\circ}}$ 又 $\underline{\underline{135^\circ}}$

(463)

$\cos^2 x + \sqrt{3}\cos x + \frac{3}{4} = 0$ (360°以内ノ正角)

$\cos x = \frac{-\sqrt{3} \pm \sqrt{3-3}}{2} = -\frac{\sqrt{3}}{2}$

$\therefore x = 180^\circ - 30^\circ = \underline{\underline{150^\circ}}$

及 $180^\circ + 30^\circ = \underline{\underline{210^\circ}}$

(464)

次ノ二ツノ方程式ヨリ x, y ノ最小角ヲ求メヨ

$\cos(2x+y) = \frac{1}{2} \cdots (1)$　$\sin(3x-y) = \frac{1}{2} \cdots (2)$

$\frac{1}{2} = \cos 60^\circ = \sin 30^\circ$

$\therefore 2x + y = 60^\circ,\quad 3x - y = 30^\circ$

此ニ方程式ヲ解キテ　$x = \underline{\underline{18^\circ}}$　$y = \underline{\underline{24^\circ}}$

(465)

$\frac{1}{\cot x} + \frac{1}{\tan x} = \frac{4}{\sqrt{3}}$ ヲ解ケ (四直角以内ノ正角)

$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{4}{\sqrt{3}},\quad \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{4}{\sqrt{3}}$

$\frac{\cancel{2}^{1}}{2\sin x \cos x} = \frac{\cancel{4}^{2}}{\sqrt{3}},\quad \frac{1}{\sin 2x} = \frac{2}{\sqrt{3}}$

$\sin 2x = \frac{\sqrt{3}}{2}$

$\therefore 2x = 60^\circ$ 又 120° 又 420° 又 480°

$\therefore x = \underline{\underline{30^\circ}}$ 又 $\underline{\underline{60^\circ}}$ 又 $\underline{\underline{210^\circ}}$ 又 $\underline{\underline{240^\circ}}$

(466)

$3\sin\theta - 2(1 - \sin^2\theta) = 0$ (鋭角ノ一ツ)

$2\sin^2\theta + 3\sin\theta - 2 = 0$

$(\sin\theta + 2)(2\sin\theta - 1) = 0$

$\sin\theta + 2 = 0,\quad \cancel{\sin\theta = -2}$

$2\sin\theta - 1 = 0,\quad \sin\theta = \frac{1}{2} = \sin 30^\circ$

$\therefore \theta = \underline{\underline{30^\circ}}$

(注意) 正弦ノ絶對値ニ於テ1ヨリ大ナルコトナシ故ニ -2 ハ不合理ナリ

(467)

$\sin^2 x = \cos^2 x$, $\sin^2 x - (1-\sin^2 x) = 0$

$2\sin^2 x = 1$, $\sin^2 x = \frac{1}{2}$, $\sin x = \pm\frac{1}{\sqrt{2}}$

即チ x ノ最小角ハ $45°$ 及 $-45°$

$\therefore\ x = \underline{\underline{n \times 180° \pm 45°}}$

(468)

$\frac{1}{\sin x} - 4\sin x = 2$ ヲ解キ(四直角以内ノ正角)

$4\sin^2 x + 2\sin x - 1 = 0$

$\sin x = \frac{-1 \pm \sqrt{1+5}}{4} = \frac{\sqrt{5}-1}{4}$ ($-\sqrt{5}-1$ ハ捨ツ)

$\therefore\ x = \underline{\underline{18°}}$ 又ハ $\underline{\underline{162°}}$

(469)

$\tan x = -\sqrt{3}$ ヲ解キ小ナル角四ツヲ出セ

$-\sqrt{3} = \tan(-60°)$ 此一般ナル解ハ

$x = n \times 180° - 60°$

$n=0$ トセバ $0 \times 180° - 60° = \underline{\underline{-60°}}$

$n=1$ トセバ $1 \times 180° - 60° = \underline{\underline{120°}}$

$n=2$ トセバ $2 \times 180° - 60° = \underline{\underline{300°}}$

$n=-1$ トセバ $-1 \times 180° - 60° = \underline{\underline{-240°}}$

(470)

$\frac{1}{\sec x} = \frac{1}{\sec 20°}$ ヲ解キテ絶対値ノ小ナルモノ三ツヲ出セ

$\cos x = \cos 20°$, $x = 2n \times 180° \pm 20°$

$n=0$ トセバ $2 \times 0 \times 180° \pm 20° = \underline{\underline{20°}}$ 又ハ $\underline{\underline{-20}}$

$n=1$ トセバ $2 \times 1 \times 180° \pm 20° = \cancel{340°}$ 又ハ $\underline{\underline{340°}}$

(471)

$\cos 3x + 8\cos^3 x = 0$ ヲ解ク(四直角以内ノ正角)

$4\cos^3 x - 3\cos x + 8\cos^3 x = 0$

$3\cos x(4\cos^2 x - 1) = 0$

$3\cos x(2\cos x - 1)(2\cos x + 1) = 0$

$\therefore\ \cos x = 0$ 又ハ $\frac{1}{2}$ 又ハ $-\frac{1}{2}$

$\therefore\ x = \underline{\underline{90°}}$ 又ハ $\underline{\underline{270°}}$ 又ハ $\underline{\underline{60°}}$ 又ハ $\underline{\underline{300°}}$ 又ハ $\underline{\underline{120°}}$

又ハ $\underline{\underline{240°}}$

(472)

$\frac{1}{\cot x} = \frac{1}{\cot 135°}$ ヲ解キ一般ナル解ヲ得テ之ヨリ小ナル角四ツヲ出セ

$\tan x = \tan 135°$, $x = n \times 180° + 135°$

$n=0$ トセバ $0 \times 180° + 135° = \underline{\underline{135°}}$

$n=1$ トセバ $1 \times 180° + 135° = \underline{\underline{315°}}$

$n=-1$ トセバ $-1 \times 180° + 135° = \underline{\underline{-45°}}$

$n=2$ トセバ $2 \times 180° + 135° = \cancel{495°}$

$n=-2$ トセバ $-2 \times 180° + 135° = \underline{\underline{-225}}$

(473)

$2(1-\cos^2 A) + \sqrt{2}\cos A = 2$ ノ二ツノ根ヲ求ム

$\cancel{2} - 2\cos^2 A + \sqrt{2}\cos A = \cancel{2}$

$\cos A(2\cos A - \sqrt{2}) = 0$

$\therefore\ \cos A = 0$ 又ハ $\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\therefore\ A = \underline{\underline{90°}}$ 又ハ $\underline{\underline{45°}}$

(474)

$$\frac{6}{\operatorname{cosec} x}+\operatorname{cosec} x=5$$

$$6\sin x+\frac{1}{\sin x}=5,\quad 6\sin x^2+1=5\sin x$$

$6\sin x^2-5\sin x+1=0$ 之ヲ解キテ

$\sin x=\frac{1}{2}$ .....(1)　$\sin x=\frac{1}{3}$ .....(2)

(1)ヨリ　$x=\sin^{-1}\frac{1}{2}=n\pi+(-1)^n\frac{\pi}{6}$

$$=\underline{\underline{\{6n+(-1)^n\}\frac{\pi}{6}}}$$

(2)ヨリ　$x=\sin^{-1}\frac{1}{3}=\underline{\underline{n\pi+(-1)^n\sin^{-1}\frac{1}{3}}}$

(475)

$4\sin^2A+\sqrt{8}\sin A-1=0$ ヲ解ケ (四直角以内ノ正角)

$4\sin^2A+2\sqrt{2}\sin A-1=0$

$\sin A=\frac{-\sqrt{2}\pm\sqrt{2+4}}{4}=\frac{\sqrt{6}-\sqrt{2}}{4}$ 又ハ $-\frac{\sqrt{2}+\sqrt{6}}{4}$

サテ $\frac{\sqrt{6}-\sqrt{2}}{4}=\sin 15^\circ$ ∴ $A=\underline{\underline{15^\circ}}$ 又ハ $\underline{\underline{165^\circ}}$

又ハ $\frac{\sqrt{6}+\sqrt{2}}{4}=-\sin 75^\circ=\sin(-75^\circ)$

∴ $A=-75^\circ$ 即 $\underline{\underline{285^\circ}}$, 又ハ $180^\circ-(-75^\circ)=\underline{\underline{255}}$

(476)

$\sin x=\frac{1}{4\sin x}$,　$4\sin^2x=1$

$2\sin x=\pm 1$,　$\sin x=\pm\frac{1}{2}=\sin(\pm 30^\circ)$

∴ $x=\underline{\underline{n\times 180^\circ\pm 30^\circ}}$

(477)

$1-\sin^2x+\frac{3}{2}\sin x=\frac{3}{2}$

$2\sin^2x-3\sin x+1=0$ 之ヲ解キテ

$\sin x=1$ .....(1)　$\sin x=\frac{1}{2}$ .....(2)

(1)ヨリ　$x=\sin^{-1}1=\underline{\underline{(4n+1)\frac{\pi}{2}}}$

遠藤氏三角法附録5頁例2ノ解ニヨル

(2)ヨリ　$x=\sin^{-1}\frac{1}{2}=n\pi+(-1)^n\frac{\pi}{6}=\underline{\underline{\{6n+(-1)^n\}\frac{\pi}{6}}}$

(478)

$2\cos^2A+4\sin^2A-5\sin A=0$

$2(1-\sin^2A)+4\sin^2A-5\sin A=0$

$2\sin^2A-5\sin A+2=0$

$(\cancel{\sin A-2})(2\sin A-1)=0$

$2\sin A-1=0$,　$\sin A=\frac{1}{2}$ 此最小角ハ 30°

∴ $A=\underline{\underline{n\times 180^\circ+(-1)^n\times 30^\circ}}$

(479)

$1-\cos^2x+\cos 2x=\cos x$

$\cancel{\sin^2x}+\cos^2x-\cancel{\sin^2x}=\cos x$

$\cos x(\cos x-1)=0$

$\cos x=0$ .....(1)　$\cos x=1$ .....(2)

(1)ヨリ　$x=\cos^{-1}0=2n\pi\pm\frac{\pi}{2}$

$=(4n\pm 1)\frac{\pi}{2}=\underline{\underline{(2n+1)\frac{\pi}{2}}}$

遠藤氏三角法附録7頁例1ニヨル

(2)ヨリ　$x=\cos^{-1}1=2n\pi\pm 0=\underline{\underline{2n\pi}}$

(480)

$\cos A+\cos 3A+\cos 5A=0$

$(\cos A+\cos 5A)+\cos 3A=0$

$2\cos 3A\cos 2A+\cos 3A=0$

$\cos 3A(2\cos 2A+1)=0$

$\cos 3A=0$ ....(1)　$\cos 2A=-\frac{1}{2}$ ....(2)

(1)ヨリ 3Aノ最小角ハ 90°ヲ得

$\therefore\ 3A=2n\times 180^\circ\pm 90^\circ=2n\times(2\times 90^\circ)\pm 90^\circ$

$=4n\times 90^\circ\pm 90^\circ=(4n\pm 1)\times 90^\circ$

$=(2n+1)90^\circ$

∴ {4n+1, 4n−1ニテ有ラユル奇数ヲ表ハスヲシテ 2n+1ト十スコトヲ得}

$\therefore\ A=\frac{(2n+1)\times 90^\circ}{3}=\underline{(2n+1)30^\circ}$

(2)ヨリ 2Aノ最小角 120°ヲ得

$\therefore\ 2A=2n\times 180^\circ\pm 120^\circ$

$=2n\times 180^\circ\pm\frac{2\times 180^\circ}{3}=(2n\pm\frac{2}{3})\times 180^\circ$

$=\frac{(6n\pm 2)\times 180^\circ}{3}=(6n\pm 2)60^\circ$

$\therefore\ A=\frac{(6n\pm 2)\times 60}{2}=\underline{(3n\pm 1)60^\circ}$

(481)

$2\sin(x+30^\circ)=\cos x$

$2(\sin x\cos 30^\circ+\sin 30^\circ\cos x)=\cos x$

$\cancel{2}\sin x\times\frac{\sqrt{3}}{\cancel{2}}+\cancel{2}\times\frac{1}{\cancel{2}}\cancel{\cos x}=\cancel{\cos x}$

$\sqrt{3}\sin x=0$ 即 $\sin x=0$

$\therefore\ x=\sin^{-1}0=n\pi+(-1)^n\times 0=\underline{n\pi}$

(482)

$3(1+\cot^2\theta)-8\cot\theta+2=0$

$3+3\cot^2\theta+8\cot\theta+2=0$

$3\cot^2\theta+8\cot\theta+5=0$ 之ヲ解キテ

$\cot\theta=\frac{5}{3}$ .....(1)　$\cot\theta=1$ .....(2)

(1)ヨリ $\theta=\cot^{-1}\frac{5}{3}=\underline{n\pi+\cot^{-1}\frac{5}{3}}$

(2)ヨリ $\theta=\cot^{-1}1=n\pi+\frac{\pi}{4}=\underline{(4n+1)\frac{\pi}{4}}$

(483)

$2\sin(90^\circ-x)=\sqrt{3}\cdot\frac{1}{\tan x}$ ノ小ナル二ツノ角ヲ求メヨ

$2\cos x-\sqrt{3}\cdot\frac{\cos x}{\sin x}=0$

$\cos x(2-\frac{\sqrt{3}}{\sin x})=0$　$\therefore\ \cos x=0$ ..(1)

$2-\frac{\sqrt{3}}{\sin x}=0$,　$\sin x=\frac{\sqrt{3}}{2}$ .....(2)

(1)ヨリ $x=\underline{90^\circ}$　(2)ヨリ $x=\underline{60^\circ}$

(484)

$5\sin(180^\circ-x)=\cos 2x+2$

$5\sin x=\cos^2 x-\sin^2 x+2$

$5\sin x=1-\sin^2 x-\sin^2 x+2$

$2\sin^2 x+5\sin x-3=0$

$(\sin x+3)(2\sin x-1)=0$

$\cancel{\sin x=-3}$,　$\sin x=\frac{1}{2}$

$\therefore\ x=\sin^{-1}\frac{1}{2}=n\pi+(-1)^n\frac{\pi}{6}$

$=\underline{\{6n+(-1)^n\}\frac{\pi}{6}}$

(485)

$\cos x = \frac{1}{2\cos x}$, $2\cos^2 x = 1$, $\cos^2 x = \frac{1}{2}$

$\cos x = \pm\sqrt{\frac{1}{2}}$ ∴ $x = 45^\circ$ 又ハ 135°

∴ $x = 2n\times 180^\circ \pm 45^\circ$ 又ハ $2n\times 180^\circ \pm 135^\circ$

此二式ハ何レモ次ノ式ニ含マル

$x = \underline{\underline{n\times 180^\circ \pm 45^\circ}}$

(486)

$x + y = 90^\circ$ ……(1) $\sin x + \cos y = \sin 60^\circ$ ……(2)

(1)ヨリ $y = 90^\circ - x = \frac{\pi}{2} - x$ ……(3)

(2)ヨリ $\sin x + \cos\left(\frac{\pi}{2} - x\right) = \sin 60^\circ$

即 $\sin x + \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x = \frac{\sqrt{3}}{2}$

$\sin x + 0 + \sin x = \frac{\sqrt{3}}{2}$

$2\sin x = \frac{\sqrt{3}}{2}$, $\sin x = \frac{\sqrt{3}}{4}$

∴ $x = \underline{\underline{\sin^{-1}\frac{\sqrt{3}}{4}}}$, $y = \underline{\underline{\frac{\pi}{2} - \sin^{-1}\frac{\sqrt{3}}{4}}}$

(487)

$\frac{1}{3\cot A} + \cot A = \frac{4}{3}$ ヲ解キ小ナル一角ヲ出セ

$\tan A + \frac{3}{\tan A} = 4$, $\tan^2 A + 3 - 4\tan A = 0$

$\tan^2 A - 4\tan A + 3 = 0$

$\tan A = \frac{2\pm\sqrt{4-3}}{1} = 2\pm 1$ 即 Ker 1

一ツノ角ヲ求ムルガ故ニ 3 ヲ割ル

$\tan A = 1$ ナレバ一ツノ角ハ次ノ如シ

$A = \underline{\underline{45^\circ}}$

(488)

$1 - \cos^2 x + \frac{1}{2}\sin^2 2x = 1$

$\cancel{2}\sin^2 x + \cancel{4}\sin^2 x\cos^2 x - \cancel{2} = 0$

$\sin^2 x + 2\sin^2 x(1 - \sin^2 x) - 1 = 0$

$\sin^2 x + 2\sin^2 x - 2\sin^4 x - 1 = 0$

$2\sin^4 x - 3\sin^2 x + 1 = 0$

$(\sin^2 x - 1)(2\sin^2 x - 1) = 0$

$\sin^2 x = 1$, $\sin x = \pm 1$ ……(1)

$\sin^2 x = \frac{1}{2}$, $\sin x = \pm\frac{1}{\sqrt{2}}$ ……(2)

(1)ノ＋ヲ採レバ $x = \sin^{-1} 1 = (4n+1)\frac{\pi}{2}$

(1)ノ－ヲ採レバ $x = \sin^{-1}(-1) = (4n-1)\frac{\pi}{2}$

以上二式ノ変化ハ遠藤氏三角法教科書 5,6 頁例 2,3 ヨリ

偖以上二式ハ次ノ式ニ含マル (問 480 ノ説明)

$x = \underline{\underline{(2n+1)\frac{\pi}{2}}}$

(2)ノ＋ヲ採レバ $x = \sin^{-1}\frac{1}{\sqrt{2}} = n\pi + (-1)^n\frac{\pi}{4}$

$= \{4n + (-1)^n\}\frac{\pi}{4}$

(2)ノ－ヲ採レバ $x = \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = n\pi + (-1)^n\frac{5}{4}\pi$

$= \{4n + (-1)^n\times 5\}\frac{\pi}{4} = \{4n - (-1)^n\}\frac{\pi}{4}$

偖 $4n + (-1)^n$, $4n - (-1)^n$ ハ何レモ奇數ヲ表ハシ又此二式ニテ総テノ奇數ヲ表ハス故ヘニ上ノ二式ハ次ノ式ヲ以テ表ハスコトヲ得

$\underline{\underline{(2n+1)\frac{\pi}{4}}}$

(489)

$\tan(4x+7y)=2+\sqrt{3},\quad \tan(2x-4y)=2-\sqrt{3}$

然 $\tan 75^\circ=2+\sqrt{3},\quad \tan 15^\circ=2-\sqrt{3}$

$\therefore\ 4x+7y=75^\circ,\quad 2x-4y=15^\circ$

此二式ヨリ $x=\underline{\underline{13.5^\circ}},\quad y=\underline{\underline{3^\circ}}$ ヲ得テ答トス

(490)

$2\cos^2x-1+\cos x=0,\quad 2\cos^2x+\cos x-1=0$

$(\cos x+1)(2\cos x-1)=0$

$\cos x=-1\ \cdots\cdots(1)\qquad \cos x=\frac{1}{2}\ \cdots\cdots(2)$

(1)ヨリ $x=\cos^{-1}(-1)=2n\pi\pm\pi=(2n\pm1)\pi$

$=\underline{\underline{(2n+1)\pi}}$

(2)ヨリ $x=\cos^{-1}\frac{1}{2}=2n\pi\pm\frac{\pi}{3}=\underline{\underline{(6n\pm1)\frac{\pi}{3}}}$

(491)

$\cos x+\frac{4}{3}\sin x=\frac{2}{3},\quad \frac{4}{3}=\tan 53^\circ8'$ ナルヲ以テ

$\cos x+\tan 53^\circ8'\sin x=\frac{2}{3}$

$\cos x+\frac{\sin 53^\circ8'}{\cos 53^\circ8'}\sin x=\frac{2}{3}$

$\cos x\cos 53^\circ8'+\sin x\sin 53^\circ8'=\frac{2}{3}\cos 53^\circ8'$

$\cos(x-53^\circ8')=\frac{2}{3}\cos 53^\circ8'$

而シテ $\cos 53^\circ8'=\cdot6018-\cdot0018=\cdot6=\frac{3}{5}$ ナルヲ以テ上式ハ次ノ如クナル

$\cos(x-53^\circ8')=\frac{2}{3}\times\frac{3}{5}=\frac{2}{5}$

而シテ $\cos 66^\circ25'=\frac{2}{5}$ ナルヲ以テ $x-53^\circ8'$ ノ最小角ハ $66^\circ25'$ ナリ故ニ

$x=\underline{\underline{2n\times180^\circ+53^\circ8'\pm66^\circ25'}}$

(492)

$\tan\theta+\frac{1}{3}\cot\theta=\frac{5}{3}\operatorname{cosec}\theta$

$\frac{3\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{5}{\sin\theta}$

$3\sin^2\theta+\cos^2\theta=5\cos\theta$

$3-3\cos^2\theta+\cos^2\theta=5\cos\theta$

$2\cos^2\theta+5\cos\theta-3=0$ ヲ解キテ

$\cos\theta=\frac{1}{2}$ 又ハ -3

$\therefore\ \theta=\cos^{-1}\frac{1}{2}=2n\pi\pm\cos^{-1}\frac{1}{2}$

$=2n\pi\pm\frac{\pi}{3}=\underline{\underline{(6n\pm1)\frac{\pi}{3}}}$

(493)

$\frac{1}{3}\tan x=\cot x$ 即 $\tan x=\frac{3}{\tan x}$

$\tan^2x=3,\quad \tan x=\pm\sqrt{3}$

$\therefore$ A ノ小角ハ 60° 又ハ 120°

$\therefore\ A=n\times180^\circ+60^\circ$ 又ハ $n\times180^\circ+120^\circ$

此両式ハ次ノ式ニ纏メラル $A=\underline{\underline{n\times180^\circ\pm60^\circ}}$

(494)

$\tan^2x-\frac{1}{3}\sec^2x=\frac{1}{3}$

$3\tan^2x-(\sec^2x-1)=1+1$

$3\tan^2x-\tan^2x=2$

$\cancel{2}\tan^2x=\cancel{2},\quad \tan x=\pm1$

$\therefore\ x=\tan^{-1}(\pm1)=n\pi\pm\frac{\pi}{4}=(4n\pm1)\frac{\pi}{4}$

$=(2n+1)\frac{\pi}{4}$

最後ノ式ニ変スルハ問題480ノ説明ニヨル

方程式

(495)

$\sin\theta - \frac{1}{\sqrt{3}}\cos\theta - \frac{\sqrt{2}}{\sqrt{3}} = 0$

$\sqrt{3}\sin\theta - \cos\theta - \sqrt{2} = 0$

$\tan 60^\circ \sin\theta - \cos\theta = \sqrt{2}$ $\left(\tan 60^\circ = \frac{\sin 60^\circ}{\cos 60^\circ}\right)$

$\sin 60^\circ \sin\theta - \cos 60^\circ \cos\theta = \sqrt{2} \times \frac{1}{2}$

$-\{\cos(\theta + 60^\circ)\} = \frac{1}{\sqrt{2}} = \cos 45^\circ$

$\therefore$ $\theta + 60^\circ$ ノ最小角ハ -45° 即 135°

$\therefore$ $\theta + 60^\circ = 2n \times 180^\circ \pm 135^\circ$

$\therefore$ $\theta = 2n \times 180^\circ - 60^\circ \pm 135^\circ$

$= n \times 180^\circ + 30^\circ + (-1)^n 45^\circ$

最後ノ式トナル理由ハ次ノ如シ

$2n \times 180^\circ - 60^\circ + 135^\circ$

$= 2n \times 180^\circ + 75^\circ = 2n \times 180^\circ + 30^\circ + 45^\circ$

$2n \times 180^\circ - 60^\circ - 135^\circ$

$= (2n-1) \times 180^\circ + 180^\circ - 195^\circ$

$= (2n-1) \times 180^\circ - 15^\circ = (2n-1) \times 180^\circ + 30^\circ - 45^\circ$

$\therefore$ $n \times 180^\circ + 30^\circ + (-1)^n 45^\circ$

(496)

$\sin A + 2\sin A \cos A = 0$ 即 $\sin A(1 + 2\cos A) = 0$

$\therefore$ $\sin A = 0 \ldots (1)$ $\cos A = -\frac{1}{2} \ldots (2)$

A ノ最小角ハ (1) ニテハ 0°, (2) ニテハ 120°

故ニ (1) ヨリ $A = \underline{\underline{n \times 180^\circ}}$

(2) ヨリ $A = \underline{\underline{2n \times 180^\circ \pm 120^\circ}}$

方程式

(497)

$\cos 2x - \cos 4x = \sin x$

$2\sin\frac{4x+2x}{2}\sin\frac{4x-2x}{2} = \sin x$

$2\sin 3x \sin x = \sin x$, $\sin x(2\sin 3x - 1) = 0$

$\therefore$ $\sin x = 0 \ldots (1)$ $\sin 3x = \frac{1}{2} \ldots (2)$

(1) ヨリ $x = \underline{\underline{n\pi}}$

(2) ヨリ $3x = \sin^{-1}\frac{1}{2} = n\pi + (-1)^n\frac{\pi}{6}$

$\therefore$ $x = \frac{n\pi}{3} + (-1)^n\frac{\pi}{18} = \underline{\underline{\{6n + (-1)^n\}\frac{\pi}{18}}}$

(498)

$\tan^2 x - \frac{4}{3}\sin^2 x - \frac{1}{3} = 0$

$\frac{3\sin^2 x}{1 - \sin^2 x} - 4\sin^2 x = 1$,

$\cancel{3\sin^2 x} - \cancel{4\sin^2 x} + 4\sin^4 x = 1 - \cancel{\sin^2 x}$

$4\sin^4 x = 1$, $2\sin^2 x = \pm 1$, $\sin^2 x = \pm\frac{1}{2}$

$\therefore$ $\sin x = \pm\sqrt{\frac{1}{2}}$ 又ハ $\pm\sqrt{\frac{1}{3}}$

即チ x ノ最小角ハ 45° 又ハ -45°

$\therefore$ $x = \underline{\underline{n \times 180^\circ \pm 45^\circ}}$

(499)

$\sin 2x - \sin(90^\circ - x) = 0$

$2\sin x \cos x - \cos x = 0$, $\cos x(2\sin x - 1) = 0$

$\cos x = 0 \ldots (1)$ $\sin x = \frac{1}{2} \ldots (2)$

(1) ヨリ $x = \cos^{-1} 0 = 2n\pi \pm \frac{\pi}{2} = \underline{\underline{(2n+1)\frac{\pi}{2}}}$

(2) ヨリ $x = \sin^{-1}\frac{1}{2} = n\pi + (-1)^n\frac{\pi}{6} = \underline{\underline{\{6n + (-1)^n\}\frac{\pi}{6}}}$

(500)

$\sin x+\tan 60^\circ\cos x=\cos 0^\circ$

$\sin x+\frac{\sin 60^\circ}{\cos 60^\circ}\cos x=1$

$\sin x\cos 60^\circ+\cos x\sin 60^\circ=\cos 60^\circ$

$\sin(x+60^\circ)=\cos 60^\circ=\frac{1}{2}$

故ニ $x+60^\circ$ ノ最小角ハ 30°

$\therefore\ x+60^\circ=n\times 180^\circ+(-1)^n\times 30^\circ$

$=2m\times 180^\circ+30^\circ$ ..... (1)

又ハ $(2m+1)\times 180^\circ-30^\circ$ ..... (2)

(1)ヨリ $x=2m\times 180^\circ+30^\circ-60^\circ$ ..... (3)

(2)ヨリ $x=(2m+1)\times 180^\circ-30^\circ-60^\circ$

$=2m\times 180^\circ+180^\circ-90^\circ$

$=2m\times 180^\circ+90^\circ$

$=2m\times 180^\circ+30^\circ+60^\circ$ ---- (4)

(3)(4)ヲ合併シテ $x=\underline{\underline{2m\times 180^\circ+30^\circ\pm 60^\circ}}$

(501)

$4\sin^2\theta-8\cos\theta+1=0$

$4(1-\cos^2\theta)-8\cos\theta+1=0$

$\cos^2\theta+2\cos\theta-\frac{5}{4}=0$

$\cos\theta=-1\pm\sqrt{1+\frac{5}{4}}=-1\pm\frac{3}{2}=\frac{1}{2}$ 又ハ $\cancel{-\frac{5}{2}}$

$\therefore\ \theta=\cos^{-1}\frac{1}{2}=2m\pi\pm\cos^{-1}\frac{1}{2}$

$=2m\pi\pm\frac{\pi}{3}=\underline{\underline{(6m\pm 1)\frac{\pi}{3}}}$

(502)

$2\sin x+5\sin(90^\circ-x)=2\tan 45^\circ$

$\sin x+2.5\cos x=1$

$\sin x+\tan 68^\circ 12'\cos x=1$

$\sin x\cos 68^\circ 12'+\sin 68^\circ 12'\cos x=\cos 68^\circ 12'$

$\sin(x+68^\circ 12')=\cos 68^\circ 12'=\sin 21^\circ 48'$

故ニ $x+68^\circ 12'$ ノ最小角ハ $21^\circ 48'$

$\therefore\ x+68^\circ 12'=n\times 180^\circ+(-1)^n\times 21^\circ 48'$

$\therefore\ x=\underline{\underline{n\times 180^\circ-68^\circ 12'+(-1)^n\times 21^\circ 48'}}$

(503)

$\sin 7x-\sin(180^\circ-x)=\sin 3x$

$\sin 7x-\sin x-\sin 3x=0$

$2\cos\frac{7+1}{2}x\sin\frac{7-1}{2}x-\sin 3x=0$

$\sin 3x(2\cos 4x-1)=0$

$\cos 4x=\frac{1}{2}$ --- (1) $\sin 3x=0$ --- (2)

(1)ヨリ $4x=\cos^{-1}\frac{1}{2}\ \therefore\ x=\frac{2}{4}n\pi\pm\frac{\pi}{12}$

$=\underline{\underline{(6n\pm 1)\frac{\pi}{12}}}$

(2)ヨリ $3x=\sin^{-1}0,\ \therefore\ x=\frac{1}{3}\{n\pi+(-1)^n\times 0\}$

$=\underline{\underline{\frac{1}{3}n\pi}}$

(504)

$\sin x=\frac{1}{2}$ ヲ解キ最小正角ヲ出セ

即チ $\underline{30^\circ}$ ヲ答トス

方程式

(505)

$\cot x - \tan x = \sec x - \operatorname{cosec} x$

$\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \frac{1}{\cos x} - \frac{1}{\sin x}$

$\cos^2 x - \sin^2 x = \sin x - \cos x$

$(\cos x + \sin x)(\cos x - \sin x) - (\sin x - \cos x) = 0$

$(\cos x + \sin x + 1)(\cos x - \sin x) = 0$

$\therefore \sin x + \cos x + 1 = 0 \ldots (1), \quad \cos x - \sin x = 0 \ldots (2)$

(1)ヨリ $\sin x + \cos x = -1$ 両辺ヲ平方シテ

$\cancel{\sin^2 x + \cos^2 x + 2\sin x \cos x = 1}$

$2\sin x \cos x = 0, \quad \sin 2x = 0, \quad x = n\pi$

$x = n\pi$ ヲ原方程式ニ検スルニ適合セズ故ニ捨ツ

(2)ヨリ $\sin x = \cos x, \quad \frac{\sin x}{\cos x} = 1, \quad \tan x = 1$

$\therefore x = \tan^{-1} 1 = n\pi + \frac{\pi}{4} = \underline{\underline{(4n+1)\frac{\pi}{4}}}$

(506)

$\sqrt{2}\sin x + \sqrt{2}\cos x = \sqrt{3}$

$\frac{\sqrt{2}}{2}\sin x + \frac{\sqrt{2}}{2}\cos x = \frac{\sqrt{3}}{2}$

$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{\sqrt{3}}{2}$

$\sin 45^\circ \sin x + \cos 45^\circ \cos x = \frac{\sqrt{3}}{2}$

$\cos(x - 45^\circ) = \frac{\sqrt{3}}{2}$

$\therefore x - 45^\circ$ ノ最小角ハ 30° ナリ

$\therefore x - 45^\circ = 2n \times 180^\circ \pm 30^\circ$

$\therefore x = 2n \times 180^\circ + 45^\circ \pm 30^\circ$

方程式

(507)

$2\sin(x+y) = \sqrt{3} \ldots (1) \qquad 2\cos(x-y) = \sqrt{3} \ldots (2)$

(1)(2)ヨリ $\sin(x+y) = \cos(x-y)$

即 $\cos\{\frac{\pi}{2} - (x+y)\} - \cos(x-y) = 0$

即 $-2\sin(\frac{\pi}{4} - y)\sin(\frac{\pi}{4} - x) = 0$

即 $\cos\{\frac{\pi}{2} - \frac{\pi}{4} + y\}\cos\{\frac{\pi}{2} - \frac{\pi}{4} + x\} = 0$

即 $\cos(\frac{\pi}{4} + y)\cos(\frac{\pi}{4} - x) = 0$

$\therefore \cos(\frac{\pi}{4} + x) = 0 \ldots (3), \quad \cos(\frac{\pi}{4} + y) = 0 \ldots (4)$

(3)ヨリ $\frac{\pi}{4} + x = 2n\pi \pm \cos^{-1} 0 = (2n+1)\frac{\pi}{2}$

$\therefore x = (2n+1)\frac{\pi}{2} - \frac{\pi}{4} = (4n+1)\frac{\pi}{4} \ldots\ldots (5)$

(4)ヨリ $\frac{\pi}{4} + y = 2n\pi \pm \cos^{-1} 0 = (2n+1)\frac{\pi}{2}$

$\therefore y = (2n+1)\frac{\pi}{2} - \frac{\pi}{4} = (4n+1)\frac{\pi}{4} \ldots\ldots (6)$

然ルニ(5),(6)ハ原方程式ニ適セマシ何ントナレバ

$\sin(x+y) = \cos(x-y)$ ハ恒等式ナレバナリ

故ニ(2)ヨリ $\cos(x-y) = \frac{\sqrt{3}}{2} = \cos(y-x)$

(5)ヲ入レテ $\cos\{y - (4n+1)\frac{\pi}{4}\} = \frac{\sqrt{3}}{2}$

即 $\cos(y - n\pi - \frac{\pi}{4}) = \frac{\sqrt{3}}{2}$

$\therefore y - n\pi - \frac{\pi}{4} = 2m\pi \pm \cos^{-1}\frac{\sqrt{3}}{2} = 2m\pi \pm \frac{\pi}{6}$

$\therefore y = 2m\pi + n\pi + \frac{\pi}{4} \pm \frac{\pi}{6}$

$= (24m + 12n + 3 \pm 2)\frac{\pi}{12}$

又(2),(6)ヨリ上ト同様ノ方法ニテ

$x = (24m + 12n + 3 \pm 2)\frac{\pi}{12}$ ヲ得

答 $\left\{\begin{array}{l} x = (4n+1)\frac{\pi}{4} \\ y = (24m + 12n + 3 \pm 2)\frac{\pi}{12} \end{array}\right\} \quad \left\{\begin{array}{l} y = (4n+1)\frac{\pi}{4} \\ x = (24m + 12n + 3 \pm 2)\frac{\pi}{12} \end{array}\right\}$

(508)

$\frac{1}{2}\sin x-\frac{1}{2}\cos x=\frac{1}{2}$ 両辺ニ$\sqrt{2}$ヲ掛テ約スレバ

$\frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x=\frac{1}{\sqrt{2}}$

$\sin 45^\circ\sin x-\cos 45^\circ\cos x=\frac{1}{\sqrt{2}}$

$-\{\cos(x+45^\circ)\}=\frac{1}{\sqrt{2}}$

$\therefore$ $x+45^\circ$ノ最小角ハ-45°即135°ナリ

$\therefore$ $x+45^\circ=2n\times180^\circ\pm135^\circ$

$\therefore$ $x=2n\times180^\circ-45^\circ\pm135^\circ$

(509)

$\cos(x+y)=\frac{1}{\sqrt{2}}$ ……(1) $\sin(x-y)=\frac{1}{2}$ ……(2)

(2)ヨリ $\cos(\frac{\pi}{2}-x+y)=\frac{1}{2}$

$\therefore$ $\frac{\pi}{2}-x+y=2n\pi\pm\cos^{-1}\frac{1}{2}=2n\pi\pm\frac{\pi}{3}$

$\therefore$ $y=2n\pi\pm\frac{\pi}{3}-\frac{\pi}{2}+x$ 之ヲ(1)ニ入レテ

$\cos(x+2n\pi\pm\frac{\pi}{3}-\frac{\pi}{2}+x)=\frac{1}{\sqrt{2}}$

即 $\cos(2x-\frac{\pi}{2}\pm\frac{\pi}{3})=\frac{1}{\sqrt{2}}$

$\therefore$ $2x-\frac{\pi}{2}\pm\frac{\pi}{3}=2n\pi\pm\cos^{-1}\frac{1}{\sqrt{2}}=2n\pi\pm\frac{\pi}{4}$

$\therefore$ $2x=2n\pi\pm\frac{\pi}{4}+\frac{3\pm2}{6}\pi=(8n\pm1)\frac{\pi}{4}\cdot(3\pm2)\frac{\pi}{6}$

$\therefore$ $x=(8n\pm1)\frac{\pi}{8}+(3\pm2)\frac{\pi}{12}$

又(1)ヨリ $x+y=2n\pi\pm\frac{\pi}{4}=(8n\pm1)\frac{\pi}{4}$

$\therefore$ $y=(8n\pm1)\frac{2\pi}{8}-x=2(8n\pm1)\frac{\pi}{8}$

$-\{(8n\pm1)\frac{\pi}{8}+(3\pm2)\frac{\pi}{12}\}$

$=(8n\pm1)\frac{\pi}{8}-(3\pm2)\frac{\pi}{12}$

答 $\left.\begin{array}{l}x=(8n\pm1)\frac{\pi}{8}+(3\pm2)\frac{\pi}{12}\\ y=(8n\pm1)\frac{\pi}{8}-(3\pm2)\frac{\pi}{12}\end{array}\right\}$

(510)

$\cos x-\frac{8}{3}\sin x=1$

$\cos x-\tan 69^\circ26'38''\sin x=1$

$\cos x\cos 69^\circ26'38''-\sin x\sin 69^\circ26'38''=\cos 69^\circ26'38''$

$\cos(x+69^\circ26'38'')=\cos 69^\circ26'38''$

$\therefore$ $x+69^\circ26'38''$ノ最小角ハ$69^\circ26'38''$ナリ

$\therefore$ $x+69^\circ26'38''=2n\times180^\circ\pm69^\circ26'38''$

$\therefore$ $x=2n\times180^\circ-69^\circ26'38''\pm69^\circ26'38''$

(511)

$\tan x+\tan(45^\circ+x)=2$

$\frac{\sin x}{\cos x}+\frac{\sin(45^\circ+x)}{\cos(45^\circ+x)}=2$

$\frac{\sin x\cos(45^\circ+x)+\cos x\sin(45^\circ+x)}{\cos x\cos(45^\circ+x)}=2$

$\frac{\sin(x+45^\circ+x)}{\cos x\cos(45^\circ+x)}=2$

$\sin(45^\circ+2x)=2\cos x\cos(45^\circ+x)$

$\sin(45^\circ+2x)=\cos(45^\circ+2x)+\cos 45^\circ$

$\sin(45^\circ+2x)-\cos(45^\circ+2x)=\frac{1}{\sqrt{2}}$

$\cancel{\sin 45^\circ\cos 2x}+\cos 45^\circ\sin 2x$

$\cancel{-\cos 45^\circ\cos 2x}+\sin 45^\circ\sin 2x=\frac{1}{\sqrt{2}}$

$\sin 2x(\cos 45^\circ+\sin 45^\circ)=\frac{1}{\sqrt{2}}$

$\sin 2x(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}})=\frac{1}{\sqrt{2}}$, $\frac{\cancel{1}}{\cancel{\sqrt{2}}}\times2\cdot\sin 2x=\frac{\cancel{1}}{\cancel{\sqrt{2}}}$

$2\sin 2x=1$, $\sin 2x=\frac{1}{2}$

$\therefore$ $2x=n\times180^\circ+(-1)^n\times30^\circ=\{6n+(-1)^n\}30^\circ$

$\therefore$ $x=\{6n+(-1)^n\}15^\circ$, 弧度デハ $\theta=\{6n+(-1)^n\}\frac{\pi}{12}$

方程式

(512)

$\cos x \cos 3x = \sin 3x \sin 5x$

$\cos x \cos 3x - \sin 3x \sin 5x = 0$

$(2\cos x \cos 3x - 2\sin 3x \sin 5x) = 0$

$\cos 4x + \cancel{\cos 2x} + \cos 8x - \cancel{\cos 2x} = 0$

$\cos 4x + \cos 8x = 0,\quad 2\cos 6x \cos 2x = 0$

$\therefore\ \cos 6x = 0 \ldots\ldots (1)\quad \cos 2x = 0 \ldots\ldots (2)$

(1)ヨリ $6x = 2m \times 180^\circ \pm 90^\circ$

$\therefore\ x = \dfrac{2m \times 180^\circ \pm 90^\circ}{6} = 2m \times 30^\circ \pm 15^\circ$

$= (2m \times 2 \pm 1)15^\circ = (2m+1)15^\circ \ldots\ldots (3)$

(2)ヨリ $2x = 2m \times 180^\circ \pm 90^\circ$

$\therefore\ x = \dfrac{2m \times 180^\circ \pm 90^\circ}{2} = 2m \times 90^\circ \pm 45^\circ$

$= (2m \times 2 \pm 1)45^\circ = (2m+1)45^\circ \ldots\ldots (4)$

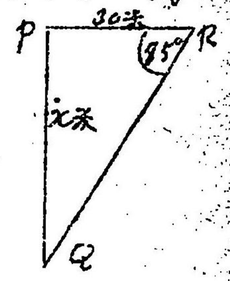
而シテ(4)ハ(3)ニ含マレルコト明カリ故ニ

$x = (2n+1)15^\circ$ 又ハ $(2m+1)\dfrac{\pi}{12}$ ヲ答トス

測量問題

(513)

二点P, Qアリテ相対ス, 或ル人Pヲ発シPQニ直角ニ進ムコト30米ニシテ一点Rニ達シ角PRQヲ測リテ75°ヲ得タリ PQノ距離ヲ問フ



PQヲx米トセバ

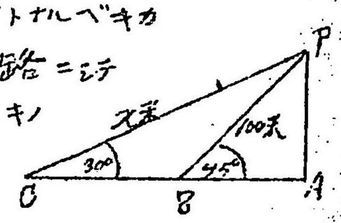
$\tan 75^\circ = \dfrac{x}{30}$ 即 $2+\sqrt{3} = \dfrac{x}{30}$

$\therefore\ x = 30(2+\sqrt{3})$

即 $PQ = \underline{\underline{30(2+\sqrt{3})\text{米}}}$

(514)

坂路アリ45°ノ傾斜ヲナシ其長サ100米ナリ, 若シ此傾斜ガ30°ナラバ坂路ノ長サ幾何トナルベキカ



APハ山ノ高サ, BPハ45°ノ坂路ニシテ其長サ100米, CPハ30°ナルトキノ坂路ニシテ其長サヲx米トス

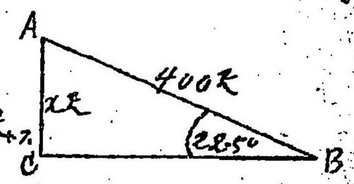
$AP = 100 \sin 45^\circ$

$= 100 \times \dfrac{1}{\sqrt{2}} = x \sin 30^\circ = \dfrac{1}{2}x$

$\therefore\ x = \dfrac{200}{\sqrt{2}} = 141.4$ 即 $CP = \underline{\underline{141.4\text{米}}}$

(515)

空中ニ揚リタル軽気球ヲ一地点ヨリ望見シタルニ其距離ガ400尺ニシテ其仰角ハ22.5°ナルコトヲ知レリ 軽気球ノ高サハ如何



Aハ軽気球ノ位置,

Bハ観望者ノ位置

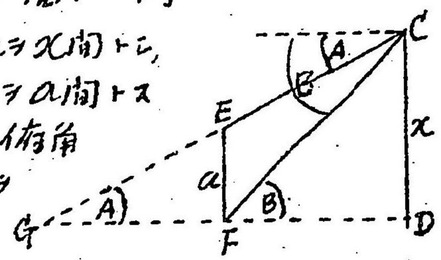
ACハ軽気球ノ高サニシテx尺トス

$\sin 22.5^\circ = \dfrac{x}{400}$

$\dfrac{1}{2}\sqrt{2-\sqrt{2}} = \dfrac{x}{400}$, $x = \underline{\underline{200\sqrt{2-\sqrt{2}}\ (\text{尺})}}$ ヲ答トス

(516)

山麓ニ立テル高サa間ノ塔ノ頂及ビ基礎ノ俯角ヲ山ノ頂ヨリ測リ夫々A, Bヲ得タリ, 山ノ高サ如何



CDハ山ノ高サニシテ之ヲx間トシ,
EFハ塔ノ高サニシテ之ヲa間トス
而シテCニテGヲ測リ俯角
A, Fヲ測リ俯角Bヲ
得タルヲ以テ錯角
ナル $C\hat{G}D=A$, $C\hat{F}D=B$ ナリ故ヘニ

$$x\cot A-x\cot B=a\cot A$$

$$(\cot A-\cot B)x=a\cot A$$

$$x=\frac{a\cot A}{\cot A-\cot B}\text{(間)}$$ ヲ答トス

(517)

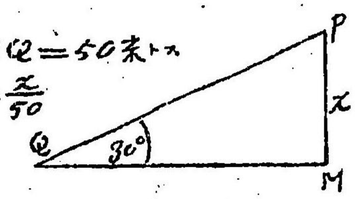
前問ニ於テa間$=100$尺, $A=15^\circ$, $B=30^\circ$トシテ計算セヨ

前問ト同様ノ式ニヨリテxヲ求メ得タルヲシテ直チニ前問ノ答ニ数値ヲ入ルレバ

$$x=\frac{100\cot 15^\circ}{\cot 15^\circ-\cot 30^\circ}=\frac{100\times(2+\sqrt{3})}{2+\sqrt{3}-\sqrt{3}}=\underline{\underline{50(2+\sqrt{3})\text{(尺)}}}$$

(518)

直立セル一樹ノ根ヲ距ルヿ50米ノ地ニ於テ樹ノ頂ノ仰角ヲ測リ30°ヲ得タリ樹ノ高ヲ問フ



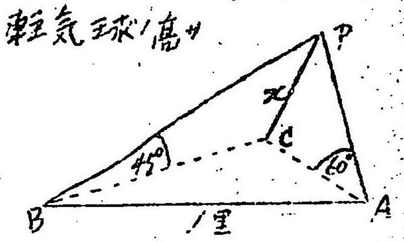
MPハ樹高ニシテx米トシ, $MQ=50$米トス

$$\tan 30^\circ=\frac{x}{50}\quad 即\ \frac{1}{\sqrt{3}}=\frac{x}{50}$$

$$\therefore\ x=\frac{1}{\sqrt{3}}\times 50=\underline{\underline{\frac{50}{\sqrt{3}}\text{(米)}}}$$

(519)

A, Bニ地ノ距離一里ニシテAハBノ東ニアリ空中ニアル軽気球ヲAニテ望メバ方位ハ北ニシテ仰角ハ60°, Bニテ望メバ仰角ハ45°ナリ, 依リテ軽気球ノ高サヲ問フ



Pハ軽気球ノ位置,
PCハ軽気球ノ高サニシテ
之ヲx里トシ
BC, ACハ地上ノ仮想線ニシテ $AC\perp AB$, $BC=PC$

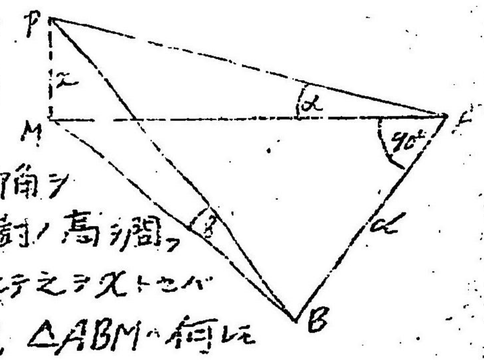
$$\cot 60^\circ=\frac{AC}{x},\quad \frac{1}{\sqrt{3}}=\frac{AC}{x},\quad \frac{1}{\sqrt{3}}x=AC$$

又 $$BC=\sqrt{\overline{AC}^2+\overline{AB}^2}=\sqrt{\frac{1}{3}x^2+1}=\sqrt{\frac{x^2+3}{3}}$$

$$\therefore\ x=\sqrt{\frac{x^2+3}{3}},\quad x^2=\frac{x^2+1}{3},\quad x=\underline{\underline{\sqrt{\frac{3}{2}}\text{(里)}}}$$

(520)

一樹アリ南ノ地ニ於テ其頂ノ仰角ヲ測リαヲ得夫レヨリ西方ニdナル距離ヲ歩ミ再ビ其ノ仰角ヲ測リシニβヲ得タリ樹ノ高サヲ問フ



PMハ樹ノ高サニシテ之ヲxトセバ
$\triangle PMA$, $\triangle PMB$, $\triangle ABM$ハ何レモ
直角三角形ナルヲシテ

$$MA=x\cot\alpha,\quad MB=\sqrt{\overline{MA}^2+\overline{AB}^2}=\sqrt{x^2\cot^2\alpha+d^2}$$

又 $$MB=x\cot\beta\quad\therefore\ x^2\cot^2\alpha+d^2=x^2\cot^2\beta$$

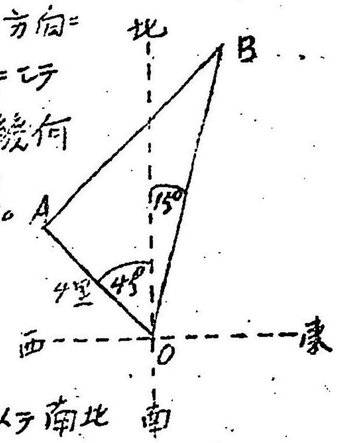
$$\therefore\ (\cot^2\alpha-\cot^2\beta)x^2=-d^2$$

$$\therefore\ x=\underline{\underline{\frac{d^2}{\sqrt{\cot^2\beta-\cot^2\alpha}}}}$$

測量問題

(521)

一燈台Oヨリ北西及ビ北15°東ノ方向ニ船A, Bアリ AB ノ方向ハ北東ニシテ OA ノ巨離ハ4里ナリ二船ノ巨離幾何



Oハ燈台ノ位置ニシテOヨリ北西ノ方向ハ南北線ト45°ノ角ヲナシ, Oヨリ北15°東ノ方向ハ南北線ト15°ノ角ヲナスヲ知ル

又ABノ方向ハ北東ナルヲ以テ南北線ト45°ノ角ヲナスベシ

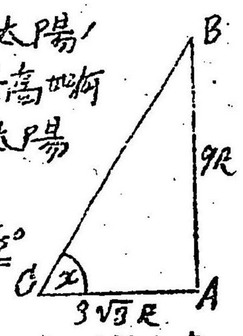
故ニ ABトAOトハAニ於テ直角ヲナス

又 $A\hat{O}B = 45° + 15° = 60°$ ナリ

偖 $AB = OA \tan 60° = 4\sqrt{3} = \underline{\underline{6.928\ (里)}}$

(522)

高サ9尺ノ直立セル竿ノ影ガ $3\sqrt{3}$ 尺ナレバ太陽ノ高度如何, 又塔ノ影ガ100尺ナレバ塔高如何



ABハ竿ニシテACハ其影ナリ然レバ太陽ノ高度ハ $\hat{C}$ ニシテ之ヲxトセバ

$\cot x = \frac{3\sqrt{3}}{9} = \frac{1}{\sqrt{3}} \therefore x = \underline{\underline{60°}}$

又直立セル物体ト其影トニテ作レル直角三角形ハ何レモ相似形ナルヲ以テ塔ノ高サヲx尺トシテ次式ヲ作ル

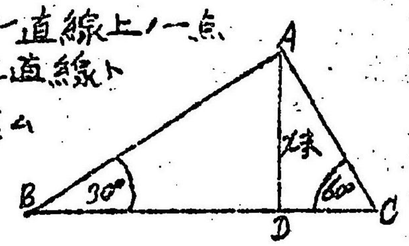
$\tan 60° = \frac{x}{100}, \quad \sqrt{3} = \frac{x}{100}$

$x = \underline{\underline{100\sqrt{3}\ (尺)}}$

測量問題

(523)

湖上ニ島アリ湖岸ニ沿ヒタル直線上ノ一点ヨリ之ヲ測レバ其方向ハ此直線ト30°ノ角ヲナス, 今此線ヲ進ムニト300米ニシテ再ビ其ノ方向ヲ測レバ60°ノ角ヲナセリ島ヨリ岸マデノ最小距離如何



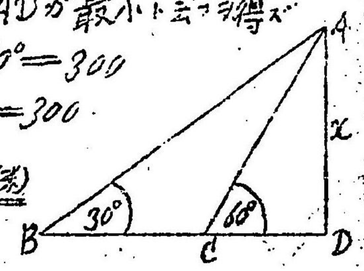
Aハ島ノ位置, BCハ湖岸ノ直線, ADハ所要ノ巨離ニシテ之ヲx米, BCノ巨離ハ300米

$\hat{A} = 180° - (60° + 30°) = 90°$

$\therefore AC = \frac{BC}{2} = 150$ 米

$AD = AC \sin 60° \therefore x = 150 \times \frac{\sqrt{3}}{2} = \underline{\underline{129.9\ (米)}}$

若シ次ノ圖ノ如クナルトキハADガ最小ト云フヲ得ズ



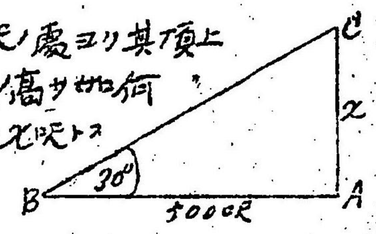
$x \cot 30° - x \cot 60° = 300$

$(\cot 30° - \cot 60°)x = 300$

$x = \frac{300}{\sqrt{3} - \frac{1}{\sqrt{3}}} = \underline{\underline{259.8\ (米)}}$

(524)

煙突ノ基礎ヲ距ルコト500呎ノ處ヨリ其頂上ヲ望ミ仰角30°ヲ得タリ煙突ノ高サ如何



ACハ煙突ノ高サニシテ之ヲx呎トス

$x \cot 30° = 500$

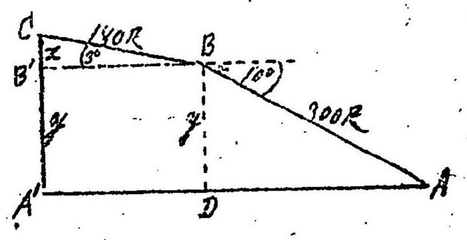
$\sqrt{3}x = 500$

$x = \frac{500}{1.732} = \underline{\underline{288.8\ (呎)}}$

測量問題

(525)

A, B, C ハ坂路上ノ三点ニシテ B ニテ測レバ A ノ俯角 10°, C ノ仰角 3° ナリ, 而シテ AB ハ 300尺, BC ハ 180尺ナリ 然レバ C ハ A ヨリ何程高キカ



AA' ハ A ニ於ケル水平線ニシテ CA' ハ C ヨリ AA' ヘノ高サ, CB' ハ C ヨリ BB' ヘノ高サ, BD ハ B ヨリ A'A ヘノ高サナリ

今 CB' ヲ x 尺, BD ヲ y 尺トセバ CA' ハ $(x+y)$ 尺ナリ

又 B ニ於ケル A ノ俯角ガ 10° ナル故ニ $\hat{A}=10°$ ナリ故ニ

$$\frac{x}{180}=\sin 30° \cdots\cdots (1) \qquad \frac{y}{300}=\sin 10° \cdots\cdots (2)$$

(1) ヨリ $x=180\times 0.0523=9.414$

(2) ヨリ $y=300\times 0.1736=52.08$

而シテ CA' ハ所要ノ高サニシテ $x+y$ ニ等シ

故ニ $x+y=9.414+52.08=61.5$

即チ C ハ A ヨリ高キコト <u>61.5尺</u> ナリ

(526)

山ノ北ニアリテ巨費 200間ナル二地ヨリ山ノ頂ヲ望ミ仰角 45°, 30° ヲ得タリ山ノ高サヲ求メヨ

問題 524 ノ図ニ於テ AP ヲ山ノ高サトシ之ヲ x 間トシ BC ヲ 200間トセバ

$$x\cot 30° - x\cot 45° = 200$$

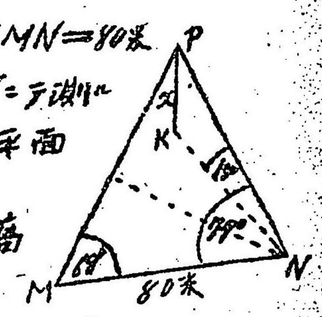
$$(\sqrt{3}-1)x=200, \quad x=\frac{200}{0.732}=273.2$$

即チ山ノ高サハ <u>273.2間</u>

測量問題

(527)

P ヲ山頂, M, N ヲ山麓ノ二点トシ, MN = 80米, $P\hat{M}N=68°$, $P\hat{N}M=79°$, N ニテ測ルP ノ仰角 13° ナルトキ N ヲ通ル水平面上ノ山ノ高サヲ問フ



PK ハ N ヲ通ル水平面上ノ山ノ高サニシテ之ヲ x 米トシ

$$M\hat{P}N=180°-(\hat{M}+\hat{N})=33°. \quad ND\perp MP,$$

$$\frac{ND}{MN}=\sin 68° \cdots\cdots (1) \qquad \frac{ND}{NP}=\sin 33° \cdots\cdots (2)$$

(1) ヲ (2) ニテ割レバ

$$\frac{NP}{MN}=\frac{\sin 68°}{\sin 33°} \text{ 即 } \frac{NP}{80}=\frac{0.9272}{0.5446}$$

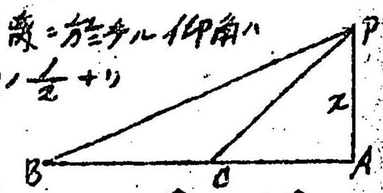
$$\therefore NP=\frac{0.9272\times 80}{0.5446}$$

$$x=NP\sin 13°=\frac{0.9272\times 80\times 0.225}{0.5446}$$

$$=\underline{30.6 \text{ (米)}}$$

(528)

塔アリ其基礎ヨリ 360米ノ距離ニ於ケル仰角ハ 135米ノ距離ニ於ケル仰角ノ $\frac{1}{2}$ ナリ塔ノ高サヲ問フ



PA ハ塔ノ高サニシテ之ヲ x 米トシ, AB = 360米, AC = 135米, $A\hat{C}P=2\hat{B}$ ナルヲ以テ △BCP ハ二等辺三角形ニシテ

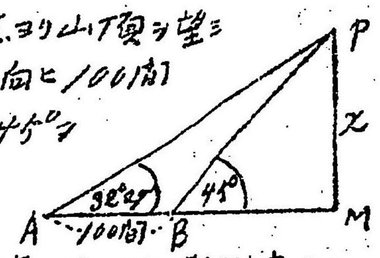
CP = CB = 360米 − 135米 = 225米 ナリ

故ニ $x=\sqrt{225^2-135^2}=\underline{180 \text{ (米)}}$

測量問題

(529)

一山ノ脚ヲ過ル水平面上ノ一点ヨリ山頂ヲ望ミ仰角32°27′ヲ得夫レヨリ山ニ向ヒ100間進ミテ再ビ之ヲ望ミテ仰角45°ヲ得タリ山ノ高ヲ問フ



PMハ山ノ高サニシテ之ヲx間トス，Aハ遠ノ観測点，Bハ後ノ観測点トス

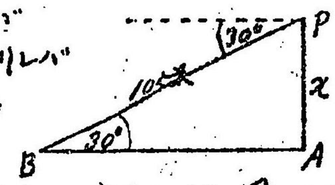
$$PM\cot 32°27' - PM\cot 45° = AB$$

$$x(1.573-1)=100$$

$$x=\frac{100}{0.573}=\underline{\underline{175\ (間)}}$$

(530)

坂路アリ其麓ヨリ頂上マデ登レバ105米アリテ頂上ニテ麓ノ俯角ヲ測リレバ30°ナリ頂上ハ平地ヨリ何米高キカ



APハ求メントスル高サニシテ之ヲx米トシ，Pニ於ケルBノ俯角ガ30°ナル故ニ$\hat{B}$ハ30°ナルベシ依リテ

$$x\operatorname{cosec}30°=105,\quad 即ち\ 2x=105$$

$$\therefore\ x=\underline{\underline{52.5(米)}}$$

(531)

問題523ニ於テ $BC=100$間，$\hat{B}=60°$ $\hat{C}=45°$ トシテCDヲ求メヨ

$$x\cot 60°+x\cot 45°=100.$$

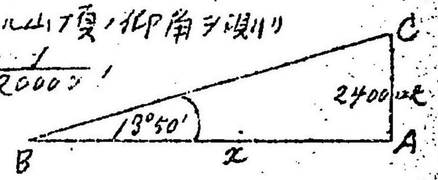
$$\therefore x+x=100,\quad (1+\sqrt{3})x=100\sqrt{3}$$

$$x=\frac{100\sqrt{3}}{1+\sqrt{3}}=\underline{\underline{50(3-\sqrt{3})\ (間)}}$$

測量問題

(532)

一地ヨリ2400呎ノ高サナル山頂ノ仰角ヲ測リ13°50′ヲ得タリ然ハトキハ$\frac{1}{20000}$ノ地図上ニテ此地ト山頂トノ距離幾何ナルヤ



CAハ山ノ高サ，CBハ山頂ノ観測地トノ距離，ABハ山ノ観測地トノ水平距離ニシテ地図上ニテハCBト一致スルモノナリ，今ABノ長サヲx呎トセバ

$$x\tan 13°50'=2400$$

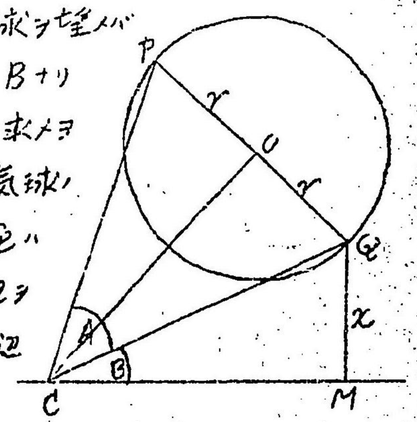
$$x=\frac{2400}{0.2462}=9748\ (呎)$$

然レバ地図上ニテBC即チBAノ長サハ

$$9748呎\times\frac{1}{20000}=\underline{\underline{0.487呎}}$$

(533)

地上ヨリ空中ニ揚レル軽気球ヲ望メバ其張ル角ハAニシテ其仰角ハBナリ今其半径ヲrトシテ其高サヲ求メヨ



地上ノ観測点ヲCトシ，気球ノ想定中心ヲOトセバPQハ直径トナル，CP，CO，CQヲ結ヘバ△CPQハ二等辺三角形ニシテCOハ$\hat{A}$ヲ二等分ス，又MQヲ気球ノ高サトシ其数ヲxトセバ

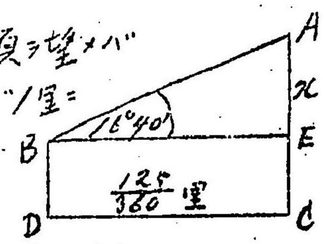
$$CQ=r\operatorname{cosec}\frac{A}{2},\quad MQ=CQ\sin B.$$

$$\therefore\ x=\underline{\underline{r\operatorname{cosec}\frac{A}{2}\sin B}}$$

測量問題

(534)

甲乙二山アリ乙山頂ヨリ甲山頂ヲ望メバ其仰角16°40′ナリ今3寸6分ガ1里ニ當ル地図上ニ於テ此二山ノ巨度ヲ測レバ1寸2分5厘ヲ得甲山ト乙山トノ高サノ差如何ナル



ACハ甲山ノ高サ, BDハ乙山ノ高サ, DCハ両山ノ巨度ニシテ $\frac{125}{360}$ 里ナリ, 而シテAEハ所要ノ高サノ差ニシテ之ヲx尺トセバ

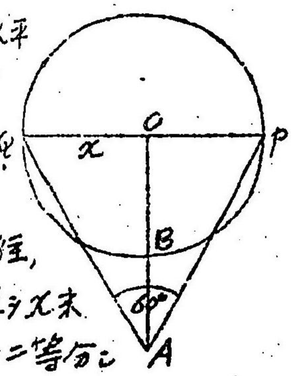
$$\tan 16°40' = \frac{x}{\frac{125}{360}}$$

$$\therefore\ x = \frac{125}{360} \times 0.2994 \times 12960 \quad (1里 = 12960尺)$$

$$= \underline{\underline{1347(尺)}}$$

(535)

円錐状ヲナセル小丘アリ脚ト同水平面上ノ一点ニ於テ脚ノ張ル角ハ60°ニシテ此点ト丘トノ最近巨度ハ15米ナリ丘脚ノ直径如何



Aハ観測点, PQハ円錐ノ底ノ直径, ABハ最近巨度(15米), 底ノ半径ヲx米トシ, AOハPQニ垂線ニシテAヲ二等分シ其長サガ(x+15)米ナリ

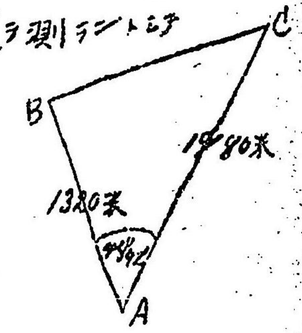
$$x = (x+15)\tan 30° = (x+15)\frac{1}{\sqrt{3}}$$

即 $\sqrt{3}x = x+15$ 即 $(\sqrt{3}-1)x = 15$

$$\therefore\ x = \frac{15}{\sqrt{3}-1} = 20.5 \quad \therefore 直径\ 20.5米 \times 2 = \underline{\underline{41米}}$$

(536)

港内ニ碇泊セル二船BCノ距離ヲ測ラントシテ陸上ノ一点AヨリB, Cニ對スル角ヲ測リ48°42′ヲ得タリ, 而シテ船中ノ号鐘ヲキクニBハ4秒間, Cハ6秒間ヲ要セリ, 今音響ハ毎秒時間330米ヲ走ルトセバ二船ノ巨度幾米ナルヤ



$$AB = 330米 \times 4 = 1320米$$

$$AC = 330米 \times 6 = 1980米$$

$$b-c = 1980-1320 = 660, \quad b+c = 3300$$

$$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\tan\frac{B+C}{2}$$

$$= \frac{660}{3300}\tan\frac{180°-48°42'}{2} = \frac{1}{5}2.296 = 0.4419$$

$$\therefore\ \frac{B-C}{2} = 23°51' \quad \therefore\ B = 89°30',\ C = 41°48'$$

然ルニ $a = \frac{b\sin A}{\sin B} = \frac{1980 \times 0.7513}{1.0000} = 1488$

即チ二船ノ巨度 $\underline{\underline{1488米}}$

(537)

問題522ニ於テ $AB = 6米$, $AC = 2\sqrt{3}$ ナルトキ太陽ノ高度ヲ求メヨ

$$\tan x = \frac{6}{2\sqrt{3}} = \sqrt{3}$$

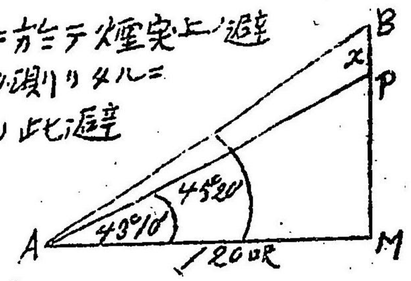
依テ $\sqrt{3} = \tan 60°$ ナルヲシテ, $x = 60°$

即チ 太陽ノ高度 $\underline{\underline{60°}}$

測量問題

(538)

煙突ノ基礎ヨリ120呎ノ地ニ於テ煙突上ノ避雷針ノ上部,下部ノ仰角ヲ測リタルニ45°20′, 43°10′ヲ得タリ此避雷針ノ長サヲ問フ



MPハ煙突, PBノ避雷針ニテ其長サヲx呎トス

$$MB = AM\tan 45°20' = 120\times 1.0117$$

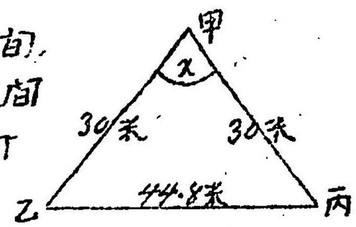
$$MP = AM\tan 43°10' = 120\times 0.938$$

$$PB = MB - MP \quad \therefore x = 120(1.0117-0.938)$$

$$= \underline{\underline{8.844\,(呎)}}$$

(539)

甲・乙・丙三点アリ其距離甲乙間, 甲丙間ハ何レモ30米ニテ乙丙間ハ44.8米ナリ甲ニ於ケル角如何



所要ノ角ヲxトセバ

$$S = \frac{30+30+44.8}{2} = 52.4$$

$$S-a = 7.6,\ S-b = S-c = 22.4$$

$$\tan\frac{x}{2} = \sqrt{\frac{22.4\times 22.4}{52.4\times 7.6}} \quad (範式48)$$

$$\log\tan\frac{x}{2} = \frac{1}{2}\{2\log 22.4 - \log 52.4 - \log 7.6\}$$

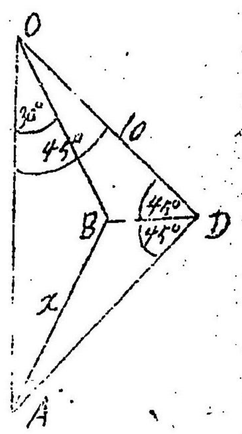
$$= \frac{1}{2}(2.7005 - 1.71983 - 0.88081) = 0.05018$$

$$\therefore \frac{x}{2} = 48°18' \quad \therefore x = \underline{\underline{96°36'}}$$

測量問題

(540)

人アリ南及南30°東ノ方向ニ二点A, Bヲ望ミ夫レヨリ南東ノ方向ニ10米ヲ進ミタレバABノ方向ハ南西及ビ西トナレリ, A, Bノ距離幾何カ



Oハ原位置, AハOヨリ南ニアリテ, BハOヨリ南30°東ニアル故ニ

$B\hat{O}A = 30°$ ナルベシ

又タOヨリ南東ニ向ヒテDニ到レバ

$A\hat{O}D = 45°$ ニテDニテAヲ望メバ

南西ニ方リ, Bヲ望メバ西ニ方ルヲ以テ $O\hat{D}A = 90°$

サル故ニ△DOAハ二等辺三角形ニテ DO=DA

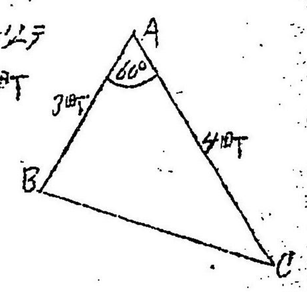
従ツテ△BOAモ二等辺三角形ナリ

偖テ $OB = \dfrac{OD\sin 45°}{\sin DBO} = \dfrac{10\times\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}} = 8.163$

而シテ AB=OB ∴ AB=$\underline{\underline{8.163米}}$

(541)

三村A, B, Cアリテ之ニ通ズル道路ヲ以テ三角形ヲ作ルニAB=3町, AC=4町ナルトキBCヲ問フ



菊池氏三角法ノ58頁95節ニヨリテ次式ヲ作ル

$$a^2 = b^2 + c^2 - 2bc\cos A$$

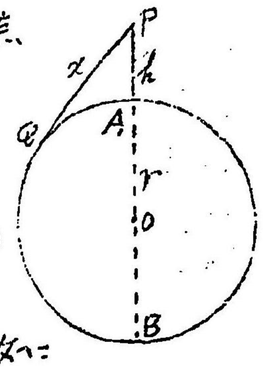
$$= 4^2 + 3^2 - 2\times 4\times 3\cos 60° = 25 - 24\times\frac{1}{2} = 13$$

$$\therefore a = \sqrt{13} \quad 即チ\ BC = \underline{\underline{\sqrt{13}\ 町}}$$

測量問題

(542)

地球ノ半径ヲrトセバ高サhナル一点ニ於テ視水平ノ距離ハ凡ソ幾何カ



PQヲ視水平ノ巨離トシ其数ヲxトシ, ABヲ地球ノ直径トセバ

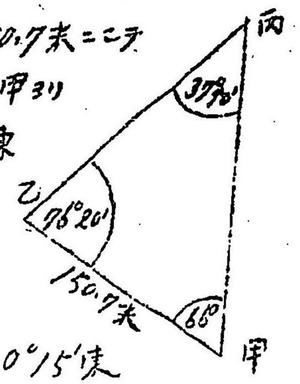
$$PQ^2 = PA \cdot PB \quad (\text{幾何定理})$$

PAハPBニ比シテ極メテ小ナルヲ以テ$PB = AB$トスルコトヲ得故ニ

$$x^2 = h \times 2r \quad \therefore x = \underline{\sqrt{2rh}}$$

(543)

甲乙丙ノ三村アリ甲ト乙トノ巨離ハ150.7米ニシテ乙ハ甲ヨリ北65°45′西ニ方リ, 丙ハ甲ヨリ北0°15′東ニ, 乙ヨリ北37°55′東ニ方レリ, 甲丙ノ巨離ハ幾何ナルカ



甲ヨリ乙, 丙ニ對スル角ハ66°ナリ何ントナレバ乙ハ甲ノ北65°45′西ニ方リ, 丙ハ甲ノ北0°15′東ニ方ル故ニ其ノ和ガ66°トナル

又乙ヨリ丙ノ方位ハ北37°55′東ナル故ニ乙ヨリ甲丙ニ對スル角ハ$180° - (65°45' + 37°55') = 76°20'$ニシテ, 丙ヨリ甲乙ニ對スル角ハ$180° - (66° + 76°20') = 37°40'$

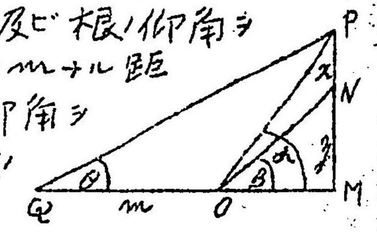
$$\text{甲丙巨離} = \frac{150.7 \times \sin 76°20'}{\sin 37°40'}$$

$$= \frac{150.7 \times 0.9717}{0.6111} = \underline{\underline{239.6\ (\text{米})}}$$

測量問題

(544)

山上ニ直立セル樹木ノ頂及ビ根ノ仰角ヲ測リα, βヲ得, 次ニ後方ニmナル距離ヲ退キテ再ビ山頂ノ仰角ヲ測リθヲ得タリ山及樹木ノ高サ各幾何ナルカ



山高ヲMNトシ, 樹高ヲNPトシ其数値ヲ夫々y, xトス, 也但メノ観測点ヲO, 後ノ観測点ヲQトセヨ

然レバ $m = (x+y)\cot\theta - (x+y)\cot\alpha$

$$= (x+y)(\cot\theta - \cot\alpha)$$

$$\therefore x + y = \frac{m}{\cot\theta - \cot\alpha} = \frac{m \sin\theta \sin\alpha}{\sin(\alpha - \theta)} \quad \cdots (1)$$

又 $OM = (x+y)\cot\alpha = \dfrac{m \sin\theta \cos\alpha}{\sin(\alpha - \theta)} \quad \cdots (2)$

而シテ $y = OM \tan\beta = \dfrac{m \sin\theta \cos\alpha}{\sin(\alpha - \theta)} \tan\beta$

即チ山高ハ $\underline{\underline{\dfrac{m \sin\theta \cos\alpha}{\sin(\alpha - \theta)} \tan\beta}}$

(1)ニyノ値ヲ入レテ $x + \dfrac{m \sin\theta \cos\alpha \tan\beta}{\sin(\alpha - \theta)} = \dfrac{m \sin\theta \sin\alpha}{\sin(\alpha - \theta)}$

$$\therefore x = \frac{m \sin\theta \sin\alpha}{\sin(\alpha - \theta)} - \frac{m \sin\theta \cos\alpha \tan\beta}{\sin(\alpha - \theta)}$$

$$= \frac{m \sin\theta \sin\alpha \cos\beta - m \sin\theta \cos\alpha \sin\beta}{\sin(\alpha - \theta) \cos\beta}$$

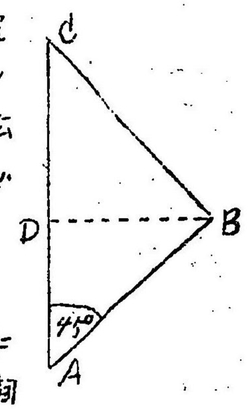
$$= \frac{m \sin\theta (\sin\alpha \cos\beta - \cos\alpha \sin\beta)}{\sin(\alpha - \theta) \cos\beta}$$

即樹高ハ $\underline{\underline{\dfrac{m \sin\theta \sin(\alpha - \beta)}{\cos\beta \sin(\alpha - \theta)}}}$

測量問題

(545)

甲乙二船一港ヲ発シ甲ハ毎時3浬ノ速サニテ北東ニ向ヒ，乙ハ毎時4浬ノ速サニテ北ニ向フ1.5時ノ後ニハ二船ノ巨離幾何トナルカ，又甲乙方位ハ如何



Aハ港ノ位置ニシテ甲ハABノ方向ニ，乙ハACノ方向ニ進メリ故ニ $B\hat{A}C=45°$ ナリ，而シテ1.5時間ニ甲ガABヲ進ミ，乙ガACヲ進ミタリトセバ

$AB=3浬\times1.5=4.5浬$

$AC=4浬\times1.5=6浬$ ナリ故ヘニ

偖テ $\frac{B+C}{2}=\frac{180°-45}{2}=67.5°$ ナルヲ以テ

$$\tan\frac{B-C}{2}=\frac{b-c}{b+c}\tan\frac{B+C}{2}$$

$$=\frac{1.5}{10.5}\tan 67°5$$

$$=\frac{1}{7}\times2.4142=0.3449$$

$\therefore\ \frac{B-C}{2}=19°2'\ \therefore\ B=86°32',\ C=48°28'$

又 $a=\frac{b\sin A}{\sin B}=\frac{6\times\frac{1}{\sqrt{2}}}{0.9981}=4.25$

即チ甲乙ノ巨離ハ <u>4.25浬</u>

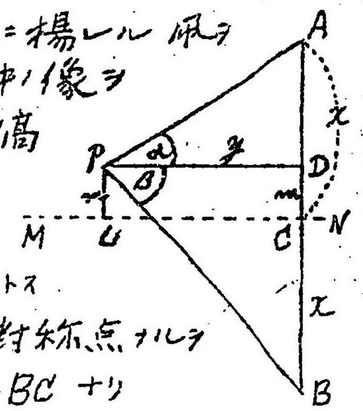
今BD⊥ACトセヨ然レバBDハ東西ノ線ニシテ $D\hat{B}C=90°-48°28'=41°32'$ ナルヲ以テ

BCノ方位ハ <u>北41°32′西ニ方</u>

測量問題

(546)

池水上高サm米ノ處ヨリ空中ニ揚レル凧ヲ望ミ仰角αヲ得同時ニ池中ノ像ヲ測リシニ俯角βヲ得タリ凧ノ高サ幾何ナルヤ



凧ノ位置ヲA，池水面ノ水平線ヲMN，Aノ像ヲBトス

偖テBハMNニ関シテAノ對称点ナルヲ以テ $AB\perp MN$，$AC=BC$ ナリ

又タ $PL\perp MN\ \therefore PL\parallel AB$

今 $PD\parallel MN$ トセバ $DC=PL$ ナリ

依テ $DB=CB+DC=x+m=y\tan\beta$

$AD=AC-DC=x-m=y\tan\alpha$

$\therefore\ \frac{x+m}{y+m}=\frac{y\tan\beta}{y\tan\alpha}$ 即チ $\frac{2x}{2m}=\frac{\tan\beta+\tan\alpha}{\tan\beta-\tan\alpha}$

即 $\frac{x}{m}=\frac{\frac{\sin\beta}{\cos\beta}+\frac{\sin\alpha}{\cos\alpha}}{\frac{\sin\beta}{\cos\beta}-\frac{\sin\alpha}{\cos\alpha}}=\frac{\sin(\beta+\alpha)}{\sin(\beta-\alpha)}$

$\therefore\ x=\frac{m\sin(\beta+\alpha)}{\sin(\beta-\alpha)}$ (米)

(547)

問題529ニ於テ $P\hat{B}M=72°10'$ $P\hat{A}M=43°30'$ $BA=85尺$ ナルトキMPヲ求メヨ

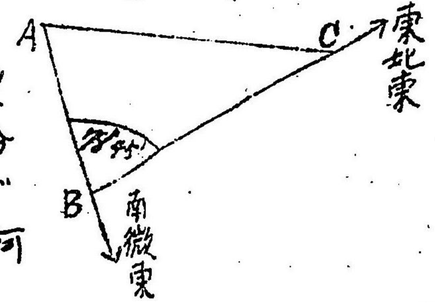
$x\cot 43°30'-x\cot 72°10'=85$

$(1.0538-0.3217)x=85,\ x=\frac{85}{0.7321}=116尺$

測量問題

(548)

南微東ノ方向ニ方リ9「ノット」ノ速サニテ東北東ノ針路ヲトリテ走リツヽアル汽船ヲ5哩ヲ距テ望見セリ，今之ヲ追ヒテ40分間ニ追及セントス，然レバ幾何ノ速サヲ以テ如何ナル方向ニ進ムベキヤ



A ハ望見者ノ位置，B ハ望觀シタル瞬間ノ汽船ノ位置，C ハ追及スル点，南微東線ト東北東線トハ $78°45'$ ノ角ヲナス，然レバ

$$BC = 9浬 \times \frac{40}{60} = 6浬$$

$$\frac{A+C}{2} = \frac{180° - 78°45'}{2} = 50°37.5'$$

$$\tan\frac{A-C}{2} = \frac{a-c}{a+c}\tan\frac{A+C}{2}, \quad a=6,\ c=5$$

$$= \frac{1}{11}\tan 50°37'.5 = \frac{1}{11}\times 1.2185 = 0.11077$$

$$\therefore \frac{A-C}{2} = 6°19' \quad \therefore A = 56°57'$$

即チ 追ヒ行ク方向ハ東南東ト東微南ノ間

又 $$AC(b) = \frac{6\times \sin 78°45'}{\sin 56°57}$$

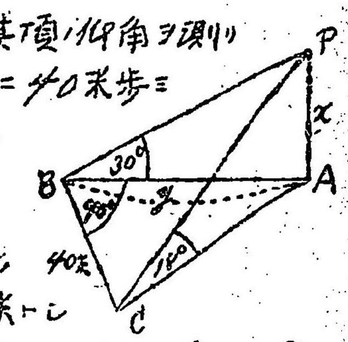
$$= \frac{6\times 0.9808}{0.8382} = 7.02$$

故ニ所要ノ速度ハ $7.02浬 \div \frac{40}{60} = \underline{\underline{10.5浬}}$

測量問題

(549)

直立セル煙突ノ北ノ一地ニ於テ其頂ノ仰角ヲ測リ30°ヲ得ツヽ此地ヨリ東ノ方向ニ40米歩ヲ進ミテ再ビ頂ノ仰角ヲ測リ18°ヲ得タリ，煙突ノ高サ幾何



AP ハ煙突ニテ之ヲ x 米トシ，B ハ北ノ一地ニテ AB ヲ y 米トシ C ハ B ヨリ西ノ方40ノ間ノ地トス，故ニ $A\hat{B}C = \hat{R}$ ナリ

依テ $y = x\cot 30° = \sqrt{3}x$

又 $AC = \sqrt{y^2+40^2}$

又 $AP = AC\tan 18° = \sqrt{y^2+1600}\tan 18°$

即 $$x = \sqrt{y^2+1600}\,\frac{\sin 18°}{\cos 18°}$$

$$\therefore x^2 = (3x^2+1600)\frac{\sin^2 18°}{1-\sin^2 18°}$$

$$= (3x^2+1600)\frac{\left(\frac{\sqrt{5}-1}{4}\right)^2}{1-\left(\frac{\sqrt{5}-1}{4}\right)^2}$$

$$= (3x^2+1600)\frac{3-\sqrt{5}}{5+\sqrt{5}}$$

即チ $(5+\sqrt{5})x^2 = (9-3\sqrt{5})x^2 + 1600(3-\sqrt{5})$

即チ $4(\sqrt{5}-1)x^2 = 1600(3-\sqrt{5})$

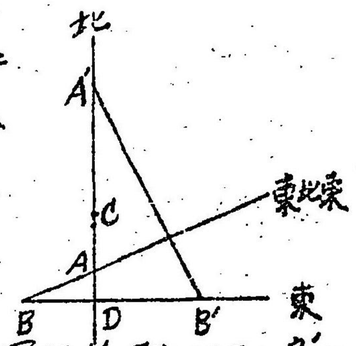
$$\therefore x^2 = \frac{1600(3-\sqrt{5})}{4(\sqrt{5}-1)} = \frac{400(3-\sqrt{5})(\sqrt{5}+1)}{5-1} = 100(2\sqrt{5}-2)$$

$$\therefore x = \sqrt{100(2\sqrt{5}-2)} = \underline{\underline{10\sqrt{2\sqrt{5}-2}}}$$

測量問題

(550)

甲乙二船ノ針路ハ夫々北ノ東ニテ其速サハ相等シ、今甲ハA ニ乙ハB ニアリテ乙ガ甲ノ方位ヲ測リ東北東ヲ得タリ夫ヨリ一浬進行シテ甲ハA′ ニ乙ハB′ ニ到リ乙ハ再ビ甲ノ方位ヲ測リシニ北々西ヲ得タリ、二船南北線上C, D ニアリシ時ノ距離如何



甲ガC ニ到レバ乙ハD ニ着スベク、甲ガA′ ニ到レバ乙ハB′ ニ着スベシ然レバ $AA'=BB'=1$浬、而シテ東北東線ハ東西線ト22°5ノ角ヲナシ、北々西線ハ南北線トモ22°5ノ角ヲナス故ニ $\widehat{B}$及ビ $A\widehat{A'}B'$ ハ何レモ22°5ノ角ナリ従テ $B\widehat{B}A'=67.5°$ ナリ

偖 $AD=BD\tan 22°5$

$A'D=AA'+BD\tan 22°5=(BB'-BD)\tan 67°5$

即 $1+BD\tan 22°5=(1-BD)\cot 22.5°$

即 $1+BD\tan 22°5=(1-BD)/\tan 22°5$

$\therefore\ \tan 22.5°+BD\tan^2 22.5°=1-BD$

即 $(1+\tan^2 22.5°)BD=1-\tan 22.5°$

$\therefore\ BD=\dfrac{1-\tan 22.5°}{\sec^2 22.5°}=(1-\tan 22.5°)\cos^2 22.5°$

而シテ $CD=AC+AD=BD+AD\overset{BD+}{=}BD\tan 22.5°$

$=\{(1-\tan 22.5°)\cos 22.5°\}(1+\tan 22.5°)$

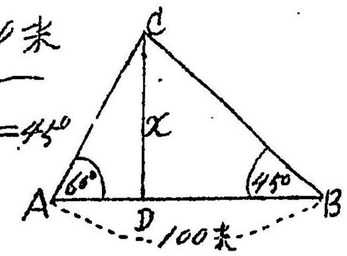
$=(1-\tan^2 22.5°)\cos 22.5°$

$=\cos^2 22.5°-\sin^2 22.5°=\cos 45°$

$=\underline{\dfrac{1}{\sqrt{2}}}$ (浬)

(551)

一直線状ヲナセル堤防アリ100米ノ距離ニアル二点A, Bヨリ他ノ一点Cヲ望ミ $C\widehat{A}B=60°$, $C\widehat{B}A=45°$ ナルヲ知リタリC点ヨリ堤防マデノ距離如何



所要ノ距離 CD ニテ之ヲ x 米トス

$AD=x\cot 60°,\quad BD=x\cot 45°$

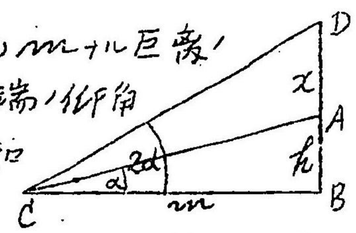
$\therefore\ AB=AD+BD=x(\cot 60°+\cot 45°)=100$

$\therefore\ \left(\dfrac{1}{\sqrt{3}}+1\right)x=100,\quad x=\dfrac{100\sqrt{3}}{1+\sqrt{3}}=\dfrac{173.2}{2.732}$

即チ CD ハ $\underline{\underline{63.4米}}$

(552)

高サ h ナル屋上ニ立テル竿アリ之ヨリ m ナル距離ノ一地ニ於テ之ヲ望ミシニ竿ノ上端ノ仰角ハ下端ノ仰角ノ二倍ナルコトヲ知レリ竿ノ長サ幾何ナルカ



AB ハ屋ノ高サ, AD ハ竿ノ長サニシテ其ノ数値ヲ夫々 h, x トシ竿ノ上端, 下端ノ仰角ヲ 2α, α トセバ

$x+h=m\tan 2\alpha=m\cdot\dfrac{2\tan\alpha}{1-\tan^2\alpha}$

$=\dfrac{m\cdot 2\cdot\frac{h}{m}}{1-\frac{h^2}{m^2}}=\dfrac{2m^2h}{m^2-h^2}$

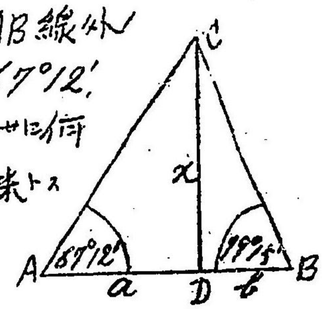
$\therefore\ x=\dfrac{2m^2h}{m^2-h^2}-h=h\left(\dfrac{2m^2}{m^2-h^2}-1\right)$

$=\underline{\underline{h\cdot\dfrac{m^2+h^2}{m^2-h^2}}}$

測量問題

(553)

A·B二点間ノ距離200米ニシテAB線外ノ一点CヲA,Bヨリ測レバ $B\hat{A}C=67°12'$, $A\hat{B}C=79°15'$ ナリ ABトCトノ距離ハ何程ノ距離ノ線ヲCDトシ之x米トス



又ADヲa米, BDヲb米トス

$x\cot 67°12'=a$,

$x\cot 79°15'=b$

$\therefore\ x(\cot 67°12'+\cot 79°15')=a+b=200$

即 $(0.4203+0.1898)x=200$

即 $0.6101x=200\quad\therefore\ x=\underline{328(米)}$

(554)

海上ヲ走ル船ノ64呎ノ高サノ檣頭ヨリ一ツノ山頂ヲ始メテ望見シ夫ヨリ之ニ向ヒテ半時間航走シテ16呎ノ高サノ甲板上ヨリ同山頂ヲ見得ルニ至ル, 今地球ノ半径ヲ4000哩トセバ船ノ一時間ノ速サハ幾何ナルカ

檣ノ高 $h=\frac{64}{5280}$哩, 甲板ノ高 $h'=\frac{16}{5280}$哩

問題542ノ答ヲ用ヒ

始メノ距離 $=\sqrt{2\times4000\times\frac{64}{5280}}=9.847$哩

後ノ距離 $=\sqrt{2\times4000\times\frac{16}{5280}}=4.923$哩

9.847哩 -4.923哩 $=4.924$哩

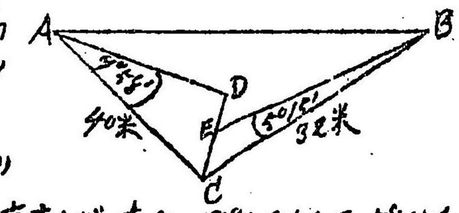
4.924哩 $\div.5=9.85$哩

即一時間ノ速サガ $\underline{9.85哩}$

測量問題

(555)

A,B二点アリテBハAヨリ高地ニアリ, 而シテ一樹Cヨリノ水平距離ハCA間ガ40米, CB間ガ32米ナリ



又A,BヨリCノ仰角ヲ測レバ夫々 9°58'.4及5°15' 得タリ, 然レバBハAヨリ何程高キカ

DハAト, EハBト同水平面上ノ点, CDハ樹ノ高サナリ

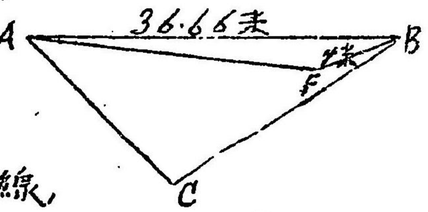
$DC=AC\sin 9°58'.4=40\times0.1732=6.928$

$EC=BC\sin 5°15'=32\times0.0915=2.928$

$ED=DC-EC=6.928-2.928=4$

即 BハAヨリ$\underline{4米高シ}$

又AヨリBノ仰角ヲ測レバ幾何ヲ得ルカ. 又 $B\hat{A}C$ ハ如何



BFハAノ水平面ヘ垂線,

$B\hat{A}F$ ハ所要ノ仰角, $\sin BAF=\frac{4}{36.66}=0.1091$

$\therefore\ B\hat{A}F=\underline{6°16'}$

又 $B\hat{A}C=180°-(B+C)=180°-(60°+70°54')$
$=\underline{49°6'}$

(556)

問題529ニ於テ $P\hat{B}M=46°10'$, $P\hat{A}M=24°40'$, $BA=100$尺ナルトキMPヲ求メヨ

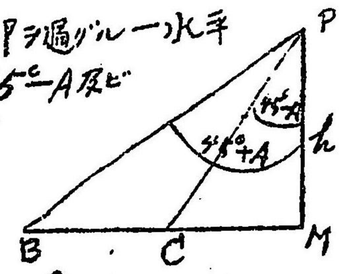
$x\cot 24°40'-x\cot 46°10'=100$

$(2.1775-0.9601)x=100,\ x=\frac{100}{1.2174}=\underline{82.14}$

測量問題

(557)

高サh呎ナル塔アリ其頂ヨリ塔脚ヲ過グル一水平線上ノ二点C, Bヲ測リ俯角$45°-A$及ビ$45°+A$ヲ得タリ二点ノ距離如何



$45°+A = P\hat{C}M$ (錯角ニテ)

$= B\hat{P}M$ (余角ニテ)

$45°-A = P\hat{B}M$ (錯角ニテ) $= C\hat{P}M$ (余角ニテ)

故 $CB = MB - MC$

$= h\tan(45°+A) - h\tan(45°-A)$

$= h\{\tan(45°+A) - \tan(45°-A)\}$

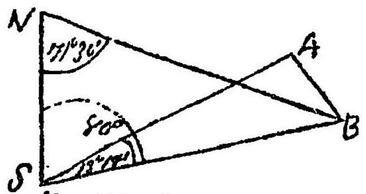
$= h\left\{\frac{1+\tan A}{1-\tan A} - \frac{1-\tan A}{1+\tan A}\right\}$ (公式20)

$= h\cdot\frac{4\tan A}{1-\tan^2 A} = h\left\{2\cdot\frac{2\tan A}{1-\tan^2 A}\right\}$

$= h(2\tan 2A)$ (公式32) $= \underline{\underline{2h\tan 2A}}$ (呎)

(558)

SNハ南北ノ水平線ニテ其長サ1000米ナリ, 今Sヨリ山頂Aノ仰角ヲ計レバ$13°14'$, 方向即チ$N\hat{S}B$ハ$80°$得, 次ニNヨリAノ方向即チ$N\hat{B}S$ヲ測レバ$71°30'$ヲ得タリ山ノ高サABハ何米ナルカ



$S\hat{B}N = 180° - (80° + 71°30') = 28°30'$

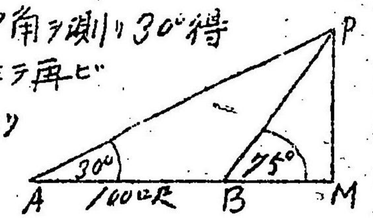
$BS = \frac{1000\sin 71°30'}{\sin 28°30'} = \frac{1000\times 0.9483}{0.4772} = 1987.2$

$AB = BS\tan 13°14' = 1987.2\times 0.2351 = \underline{\underline{467}}$ (米)

測量問題

(559)

人アリ其前方ニ立テル竿頂ノ仰角ヲ測リ30°得夫レヨリ竿ニ向ヒテ100呎ヲ進ミテ再ビ竿頂ノ仰角ヲ測リ75°ヲ得タリ竿ノ高サ及ビ始メノ観測点マデノ距離如何



MPヲ竿トシ, MAヲ所要ノ距離トス

$BP = \frac{AB\sin 30°}{\sin APB} = \frac{100\times\frac{1}{2}}{\sin(75°-30°)}$

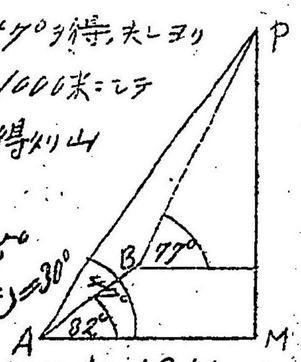
$MP = BP\sin 75° = \frac{50\sin 75°}{\sin 45°} = \frac{50\times 0.9659}{\frac{1}{\sqrt{2}}}$

$= \underline{\underline{68.3}}$ (米)

$MA = PM\cot 30° = 68.3\times\sqrt{3} = \underline{\underline{118.3}}$ (米)

(560)

山脚ノ一点Aヨリ山頂Pヲ測リ仰角$44°$ヲ得夫レヨリ山頂ニ向ヒ$32°$ノ角ヲナセル坂路ヲ上リ1000米ニテBニ達シ再ビPノ仰角ヲ測リ$77°$ヲ得タリ山高MPハ如何



$A\hat{B}P = 180° - 77° + 32° = 135°$

$B\hat{P}A = 180° - (44° - 32° + 135°) = 30°$

而シテ $PA = \frac{AB\sin PBA}{\sin BPA} = \frac{1000\sin 135°}{\sin 30°}$

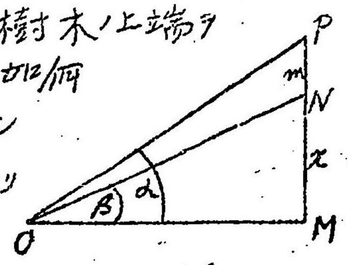
$= \frac{\frac{1000}{\sqrt{2}}}{\frac{1}{2}} = \frac{2000}{\sqrt{2}}$

故ニ $MP = PA\sin 44° = \frac{2000}{\sqrt{2}}\times 0.731 = \underline{\underline{1034}}$ (米)

測量問題

(561)

山頂及ビ其上ニ立テル高サm尺ノ樹木ノ上端ヲ望ミ仰角β及ビαヲ得タリ山ノ高サ如何



MNハ塔ニシテ其高サx尺トシ

NPハ樹ニシテ其高サm尺トリ

又觀測点ヲOトス

偖 $OM=(m+x)\cot\alpha=x\cot\beta$

即 $m\cot\alpha+x\cot\alpha=x\cot\beta$

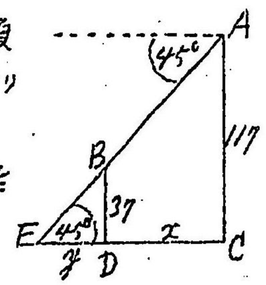
即 $(\cot\alpha-\cot\beta)x=-m\cot\alpha$

$$\therefore\ x=\frac{m\cot\alpha}{\cot\beta-\cot\alpha}=\frac{m\cdot\frac{\cos\alpha}{\sin\alpha}}{\frac{\cos\beta}{\sin\beta}-\frac{\cos\alpha}{\sin\alpha}}$$

$$=\frac{m\cos\alpha\sin\alpha\sin\beta}{\sin\alpha(\sin\alpha\cos\beta-\sin\beta\cos\alpha)}=\underline{\underline{\frac{m\sin\alpha\sin\beta}{\sin(\alpha-\beta)}}}\ (尺)$$

(562)

高サ117米, 37米ノ二丘アリ高丘ノ頂ヨリ低丘ノ頂ノ俯角ヲ測リ45°ヲ得タリ二丘ノ巨離ヲ問フ



ACハ高丘, BDハ低丘ニシテ, Aニ於ケルBノ俯角ガ45°ナル故ヘニ$\hat{E}$モ45°ナルベシ

又CDハ二丘ノ距離ニシテそヲx米, DEヲy米トスレバ

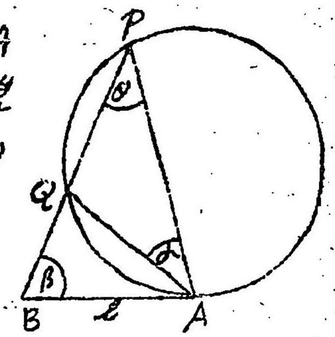
$\triangle AEC$, $\triangle BED$ハ何レモ直角二等辺三角形ナルヲ以テ

$EC=117=x+y$, $ED=37=y$

$\therefore\ x+37=117$, $\therefore\ x=\underline{\underline{80\ (米)}}$

(563)

一直線ノ鉄道AB上ノ一点Aニ於テ二電柱P, Qヲ最大角αニ望ミ其レヨリl丈ケ進ミテβニ到リPQ線上ニアリテ$P\hat{B}A=\beta$ナルヲ知レリ二電柱ノ距離如何



αハAB上ニアリテPQヲ張ル最大角ナルヲ以テA点ハ唯一ツニ限ル故ヘニABハA, Q, Pヲ過ギテ画ケル円ノ切線ナリ故ヘニ

$Q\hat{A}B=\hat{P}$ ナリ そヲθトセバ

$P\hat{Q}A=\beta+\theta=180^\circ-\alpha-\theta$

$\therefore\ 2\theta=180^\circ-(\alpha+\beta)$ ……(1)

偖 $\frac{AQ}{AB}=\frac{\sin\beta}{\sin(\beta+\theta)}$ $\therefore\ AQ=\frac{l\sin\beta}{\sin(\beta+\theta)}$

又 $\frac{QP}{AQ}=\frac{\sin\alpha}{\sin\theta}$

$$\therefore\ QP=\frac{l\sin\beta\sin\alpha}{\sin(\beta+\theta)\sin\theta}=\frac{2l\sin\alpha\sin\beta}{2\sin(\beta+\theta)\sin\theta}$$

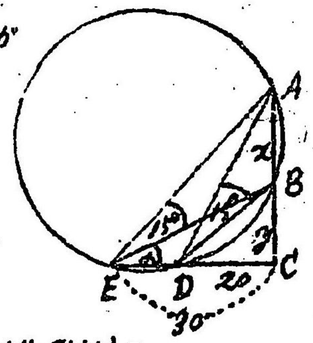
$$=\frac{2l\sin\alpha\sin\beta}{\cos\beta-\cos(2\theta+\beta)}\quad 分母ニ(1)ヲ入レテ$$

$$=\frac{2l\sin\alpha\sin\beta}{\cos\beta-\cos(180^\circ-\alpha)}$$

$$=\underline{\underline{\frac{2l\sin\alpha\sin\beta}{\cos\beta+\cos\alpha}}}$$

(564)

煙突BC上ニ立テル避雷針ABガ煙突基礎Cヲ過ル水平線上ニ於テCヨリ20呎,及ビ30呎ナル距離ニアル二点D,Eニ對シ等角15°ヲ張ルトキ避雷針ノ長サ幾何ナルヤ



題意ニヨリテ $A\hat{E}B = A\hat{D}B$ ナルヲシルシテ A, E, D, Bヲ過ル円ヲ画クコトヲ得

今 $B\hat{E}D$ ヲ α トセバ $D\hat{B}C = \alpha + 15°$

偖 $\frac{x}{20} = \tan BDC = \cot DBC = \cot(\alpha+15°)$ (1)

又 $\frac{y}{30} = \tan\alpha$ ..... (2)

而シテ $\frac{x+y}{20} = \tan ADC = \cot DAC = \cot\alpha$

即 $\frac{x}{20} + \frac{y}{20} = \cot\alpha$ 之ニ(1)ヲ入レテ

$\frac{x}{20} = \cot\alpha - \cot(\alpha+15°)$ ..... (3)

又 $\frac{x+y}{30} = \tan(\alpha+15°)$

即 $\frac{x}{30} + \frac{y}{30} = \tan(\alpha+15°)$ 之ニ(2)ヲ入レテ

$\frac{x}{30} = \tan(\alpha+15°) - \tan\alpha$ ..... (4)

(4)ノ右辺ヲ変化セバ

$$\frac{\sin(\alpha+15°)}{\cos(\alpha+15°)} - \frac{\sin\alpha}{\cos\alpha} = \frac{\sin(\alpha+15°)\cos\alpha - \sin\alpha\cos(\alpha+15°)}{\cos(\alpha+15°)\cos\alpha}$$

$$= \frac{\sin 15°}{\cos(\alpha+15°)\cos\alpha}$$

又(3)ノ右辺ヲ変化セバ

$$\frac{\cos\alpha}{\sin\alpha} - \frac{\cos(\alpha+15°)}{\sin(\alpha+15°)} = \frac{\sin 15°}{\sin(\alpha+15°)\sin\alpha}$$

故ニ(3),(4)ノ辺々相加フレバ(逆数ニシテ)

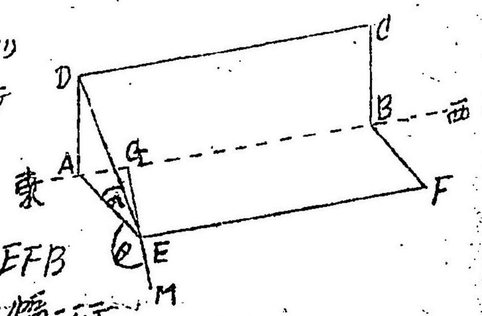
$$\frac{30}{x} + \frac{20}{x} = \frac{\cos(\alpha+15°)}{\sin 15°} + \frac{\sin(\alpha+15°)\sin\alpha}{\sin 15°}$$

$$\therefore \frac{50}{x} = \frac{\cos 15}{\sin 15°} = \cot 15°$$

$$\therefore x = \frac{50}{\cot 15°} = \frac{50}{3.732} = \underline{\underline{13.4\ (呎)}}$$

(565)

高サnノ家ノ東西ノ板塀アリ太陽ノ方位角ハθニシテ其高度ハαナレバ其影ノ幅ハ如何



ABCDハ板塀, AEFBハ其影, GEハ影ノ幅ニシテ南北線, ADハ塀ノ高サニシテ平地ニ直立シ来リ

$D\hat{A}E = \hat{R}$ ナルヲシルシテ $AE = n\cot\alpha$

$E\hat{G}A = \hat{R}$ ナルヲシルシテ $GE = -AE\cos\theta$

$\because G\hat{E}A$ ガθノ補角ナルヲシルシテ「−」ヲ附ス

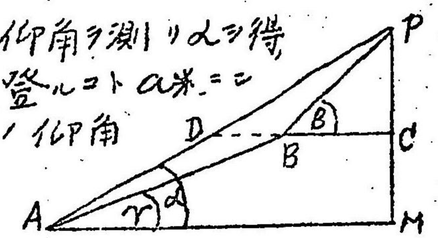
$$= \underline{\underline{-n\cot\alpha\cos\theta}}$$

$n = 20,\ \theta = 203°20',\ \alpha = 41°25'$ トセバ

$$GE = \underline{\underline{20.8}}$$

(566)

坂脚ノ一点Aニテ山頂ノ仰角ヲ測リαヲ得、次ニ傾斜角γナル坂ヲ登ルコトa米ニシテB点ニ達シ再ヒ山頂ノ仰角ヲ測リβヲ得タリ山ノ高サハ如何



MPハ山ノ高サニシテxヲ米トシ、CMハBヨリノ高サトス、次ニAPヲ結ビCBノ延長トDニ交ラシムレバ $P\hat{D}C=\alpha$, $B\hat{A}P=\alpha-\gamma$, $BPA=\beta-\alpha$, 且ツABハa米ナルヲ以テ

$$P\hat{B}A=180^\circ-(\alpha-\gamma+\beta-\alpha)=\beta-\gamma$$

$$\therefore\ \sin PBA=\sin(\beta-\gamma)$$

而シテ $\dfrac{AP}{\sin(\beta-\gamma)}=\dfrac{a}{\sin(\beta-\alpha)}$ (正弦比例)

$$\therefore\ AP=\frac{a\sin(\beta-\gamma)}{\sin(\beta-\alpha)}$$

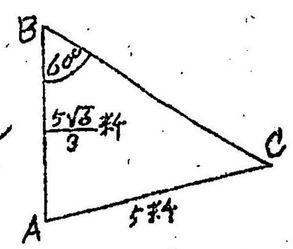
而シテ $MP=AP\sin\alpha=\underline{\underline{\dfrac{a\sin(\beta-\gamma)}{\sin(\beta-\alpha)}\sin\alpha}}$

今 $a=120$, $\alpha=24^\circ$, $\beta=46^\circ$, $\gamma=4^\circ$ トシテ計算セバ如何

$$\frac{120\times0.669}{0.3746}\times0.4067=\underline{\underline{87.2\ (米)}}$$

(567)

両軍廣原ノ二地A,Bニ於テ $\frac{5\sqrt{6}}{3}$ 粁ヲ距テ南北ニ對陣スルアリ或ル日北軍ノ逃走ヲ妨メタルヲ南軍望見セシニ其方向ハ南60°東ニ方ル依リテ南軍直チニ毎時15粁ヲ走ル騎兵



ヲシテ追擊セシメシニ20分ニシテCニ於テ追及セリ騎兵ノ方向及ヒ北軍ノ速度幾何ナルヤ

ACハ20分間ニ騎兵ガ行ク距離ニシテ5粁トナル

依 $\sin C=\dfrac{BA\sin B}{AC}=\dfrac{\frac{5\sqrt{6}}{3}\sin60^\circ}{5}=\dfrac{1}{\sqrt{2}}$

即 $\hat{C}=45^\circ$ 從テ $\hat{A}=180^\circ-(60^\circ+45^\circ)=75^\circ$

故ニ騎兵ノ方向ハ北75°東ナリ

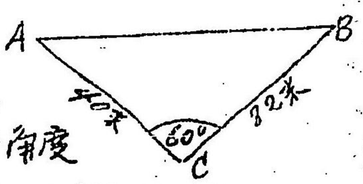
又 $BC=\dfrac{5\sin75^\circ}{\sin60^\circ}=5\times\dfrac{\sqrt{6}+\sqrt{2}}{4}\Big/\dfrac{\sqrt{3}}{2}=\dfrac{5(\sqrt{3}+1)}{\sqrt{6}}$

故ニ北軍ノ一時間ノ速サハ

$$\frac{5(\sqrt{3}+1)}{\sqrt{6}}\times\frac{1}{3}=\underline{\underline{18.6\ (粁)}}$$

(568)

城塞ノ左右ニアル二点A,Bノ距離ヲ知ランガタメニAヨリ40米、Bヨリ32米アル一地Cニ於テ角度ヲ測リ $A\hat{C}B=60^\circ$ ヲ得タリ然レバABハ如何



依 $\dfrac{B+A}{2}=\dfrac{180^\circ-60^\circ}{2}=60^\circ$

$b-a=40-32=8$, $b+a=40+32=72$

$$\therefore\ \tan\frac{B-A}{2}=\frac{8}{72}\tan60^\circ$$

$$=\frac{1}{9}\times1.7321=0.19245$$

$$\therefore\ \frac{B-A}{2}=10^\circ54'\qquad\therefore\ B=70^\circ54'$$

$$\therefore\ AB=\frac{40\sin60^\circ}{\sin75^\circ54}=\frac{40\times0.866}{0.945}=\underline{\underline{36.66\ (米)}}$$