MACHINE DESIGN

WALLACE

MACHINE DESIGN

A MANUAL OF PRACTICAL INSTRUCTION IN DESIGNING MACHINERY FOR SPECIFIC PURPOSES, INCLUDING SPECIFICATIONS FOR BELTS, SCREWS, PINS, GEARS, ETC., AND MANY WORKING HINTS AS TO OPERATION AND CARE OF MACHINES

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INTRODUCTION

MACHINE DESIGN is a subject which is often neglected by men who have the ambition to graduate from the drafting and machine work of our factories into the more important departments dealing with the design of tools and machines, and the numerous devices which are made by means of them. The draftsman is liable to forget that, in order to be a really good draftsman, he must add to his ability to draw objects the power of visualizing accurately the device or machine, the different views of which he is called upon to draw, and be a good enough mechanic to know "how things work" and how the machine is to be manufactured. Similarly, the machinist is liable to lose sight of the fact that he is all the better workman if he can couple with his skill in turning out the finished product machined to the proper size, the ability to read the draftsman's drawings down to the smallest detail and to know the why and wherefore of every element of the design.

¶ It is plain, therefore, that machine designing in its broadest sense demands a familiarity with the point of view of both draftsman and machinist; it demands a knowledge of materials—their strength, their characteristics, and their behavior under different machine operations. The designer must know which parts should be cast and which should be machined from steel or brass; he must also know the standard designs and specifications for bolts, screws, nuts, pins, keys, etc.; he must know the different types of transmission, the proportions of pulleys and gears, the strength of shafts, the design of bearings, and the methods of lubrication.

¶ It is with the idea of offering a simple treatment of these phases of machine design that this text was prepared. It is the hope of the publishers that the presentation will appeal to young men who are anxious to get ahead in their profession as well as to those of greater experience in the machine-design field.

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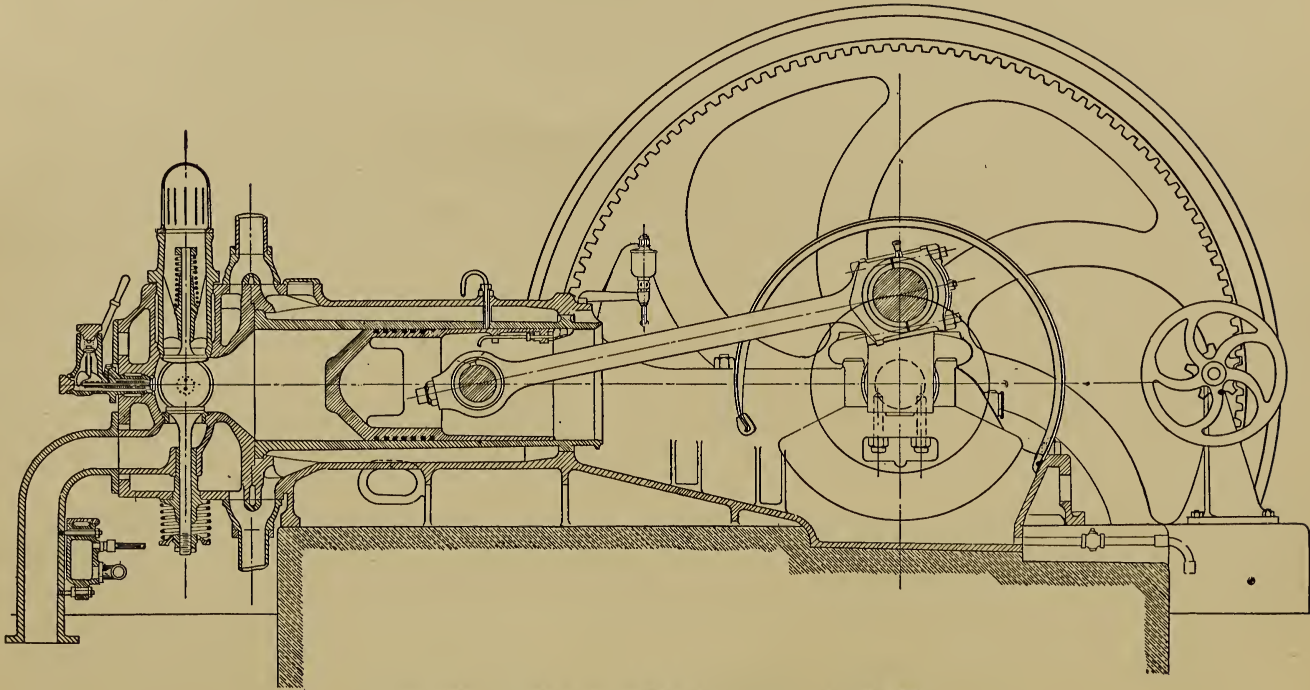


SECTIONAL ELEVATION OF NEW CROSSLEY OIL ENGINE
Courtesy of Crossley Brothers, Manchester, England

MACHINE DESIGN

PART I

INTRODUCTION

Definition. Machine Design is the art of mechanical thought, development, and specification.

It is an art, in that its routine processes may be analyzed and systematically applied. Proficiency in the art positively cannot be attained by any "short cut" method. There is nothing of a spectacular nature in the methods of Machine Design. Large results cannot be accomplished at a single bound, and success is possible only by a patient, step-by-step advance in accordance with well-established principles.

Mechanical thought means the thinking of things strictly from their mechanical side; a study of their mechanical theory, structure, production, and use; a consideration of their mechanical fitness as parts of a machine.

Mechanical development signifies the taking of an idea in the rough—in the crude form, for example, in which it comes from the inventor—working it out in detail, and refining and fixing it in shape by the designing process. Ideas in this way may become commercially practicable designs.

Mechanical specification implies the detailed description of designs in such exact form that the shop workmen are enabled to construct completely and to put in operation the machines represented in the designs.

Object. The object of Machine Design is the creation of machinery for specific purposes. Every department of a manufacturing plant is a controlling factor in the design and production of the machines built there. A successful design cannot be out of harmony with the organized methods of production. Hence, in the high development of the art of Machine Design is involved a knowledge of the operations in all the departments of a manufacturing plant. The student is therefore urged not only to familiarize himself with the direct production of machinery, but to study the relation thereto of the allied commercial departments. He should get into the spirit of business at the start, get into the shop atmosphere, execute his work just as though the resulting design were to be built and sold in competition. He should visit shops, work in them, if possible, and observe details of design and methods of finishing machine parts. In this way he will begin to store up bits of information, practical and commercial, which will have valuable bearing on his engineering study.

The labor involved in the design of a complicated automatic machine is evidenced by the designer's wonderful familiarity with its every detail as he stands before the completed machine in operation and explains its movements to an observer. The intricate mass of levers, shafts, pulleys, gears, cams, clutches, etc., packed into a small space, and confusing even to a mechanical mind, seems like a printed book to the designer of them.

This is so because it is a familiar journey for the designer's mind to run over a path which it has already traversed so many times that he can see every inch of it with his eyes shut. Every detail of that machine has been picked from a score or more of possible ideas. One by one, ideas have been worked out, laid aside, and others taken up. Little by little, the special fitness of certain devices has become established, but only by patient, careful consideration of others, which, at first, seemed equally good.

Every line, and corner, and surface of each piece, however small that piece may be, has been through the refining process of theoretical, practical, and commercial design. Every piece has been followed in the mind's eye of its designer from the crude material of which it is made, through the various processes of finishing, to its final location in the completed machine; thus its bodily existence at that point is but the realization of an old and familiar picture.

What wonder that the machine seems simple to the designer of it! As he looks back to the multitude of ideas invented, worked out, considered, and discarded, the machine in its final form is but a trifle. It merely represents a survival of the fittest.

No successful machine, however simple, was ever designed that did not go through this slow process of evolution. No machine ever just simply happened by accident to do the work for which it is valued. No other principle upon which the successful design of machinery depends is so important as this careful, patient consideration of detail. A machine is seldom unsuccessful because some main point of construction is wrong. The principal features of a machine are usually the easiest to determine. It is a failure because some little detail was overlooked, or hastily considered, or allowed to be neglected, because of the irksome labor necessary to work it out properly.

There is no task so tedious, for example, as the devising of the method of lubricating the parts of a complicated machine. Yet there is no point of design so vital to its life and operation as an absolute assurance of an adequate supply of oil for the moving parts at all times and under all circumstances. Suitable means often cannot be found, after the parts are together, hence the machine goes into service on a risky basis, with the result, perhaps, of early failure, due to "running dry." Good designers will not permit a design to leave their hands which does not provide practically automatic oiling, or at least such means of lubrication that the operator can offer no excuse for neglecting to oil his machine. This is but a single illustration of many which might be presented to impress the definite and detail character necessary in work in Machine Design.

Relation. The relation which Machine Design should correctly bear to the problems that it seeks to solve, is twofold; and there are, likewise, two points of view corresponding to this twofold relation, from which a study of the subject should be traced, viz, *theory* and *production.* Neither of these can be discarded and an efficient mastery of the art attained.

Theory. From this point of view, Machine Design is merely a skeleton or framework process, resulting in a representation of ideas of pure motion, fundamental shape, and ideal proportion. It implies a working knowledge of physical and mathematical laws. It is a strictly scientific solution of the problem at hand, and may be based purely on theory which has been reasoned out by calculation or deduced from experiment. This is the only sure foundation for intelligent design of any sort.

But it is not enough to view the subject from the standpoint of theory alone. If a stop were made here nothing would be obtained but mechanisms, mere laboratory machines, simply structures of ingenuity and examples of fine mechanical skill. A machine may be correct in the theory of its motions; it may be correct in the theoretical proportions of its parts; it may even be correct in its operation for the time being; and yet its complication, its misdirected and wasteful effort, its lack of adjustment, its expensive and irregular construction, its lack of compactness, its difficulty of ready repair, its inability to hold its own in competition—any of these may throw the balance to the side of failure. Such a machine, commercially considered, is of little value. No shop will build it, no machinery house will sell it, nobody will buy it if it is put on the market. Thus, aside from the theoretical correctness of principle, the design of a machine must satisfy certain other exacting requirements of a distinctly business nature.

Production. From this point of view, Machine Design is the practical, marketable development of mechanical ideas. Viewed thus, the theoretical, skeleton design must be clothed and shaped that its production may be cheap, involving simple and efficient processes of manufacture. It must be judged by the latest shop methods for exact and maximum output. It must possess all the good points of its competitor, and, withal, some novel and valuable ones of its own. In these days of keen competition it is only by carefully studied, well-directed effort toward rapid, efficient, and, therefore, cheap production that any machine can be brought to a commercial basis, no matter what its other merits may be. All this must be thought of and planned for in the design, and the final shapes arrived at are quite as much a result of this second point of view as of the first.

As a good illustration of this, may be cited the effect of the present somewhat remarkable development of the so-called *high-speed* steels. The speeds and feeds possible with tools made of these steels are such that the driving power, gearing, and feed mechanism of the ordinary lathe are wholly inadequate to the demands made upon them when working the tool to its limit. This means that the basis of design as used for the ordinary tool steel will not do if the machine is expected to stand up to the cuts possible with the new steels. Hence, while the old designs were right for the old standard, a new one has been set, and a thorough revision on a high-speed basis is imminent, else the market for them as machines of maximum output will be lost.

It is evident, therefore, that the designer must not only use all the theory at his command, but must continually inform himself on all processes and conditions of manufacture, and keep an eye on the tendency of the sales markets, both of raw material and the finished machinery product. Thought which is directed and controlled, not only by theoretical principle but by closely observed practice, is what, in the broadest sense, is meant by the term *mechanical thought.* From the feeblest pretenders of design to those engineers who consummate the boldest feats and control the largest enterprises, the process which produces results is always the same. Although experience is necessary for the best mechanical judgment, yet the student must at least begin to cultivate good mechanical sense very early in his study of design.

Invention. Invention is closely related to Machine Design, but is not design itself. Whatever is invented has yet to be designed and is of little value until it has been refined by the process of design.

Although original design is of an inventive nature, it is not strictly invention. Invention is the product of genius and though announced in a flash of brilliancy is the result of a long course of the most concentrated brain effort. It is not spontaneous; it is not thrown off like sparks from the blacksmith's anvil; but it is the result of hard and applied thinking. This is worth noting carefully, for the same effort which produces original design may develop a valuable invention. There is little possibility, however, of inventing anything except through exhaustive analysis and a clear interpretation of such analysis.

Handbooks and Empirical Data. The subject matter in these is often contradictory in its nature, but valuable nevertheless. Empirical data is data for certain fixed conditions and is not general. Hence, when handbook data is applied to some specific case of design, while the information should be used in the freest manner, yet it must not be forgotten that the case at hand is probably different, in some degree, from that upon which the data was based, and unlike any other case which ever existed or will ever again exist. Therefore the data should be applied with the greatest discretion, and when so applied will contribute to the success of the design at least as a check, if not as a positive factor.

The student should at the outset purchase one good handbook, and acquire the habit of consulting it on all occasions, checking and comparing his own calculations and designs therefrom. Care must be taken not to become tied to a handbook to such an extent that one's own results are wholly subordinated to it. Independence in design must be cultivated and one should not sacrifice his calculated results until they can be shown to be false or based on false assumption. Originality and confidence in design will be the result if this course be honestly pursued.

Calculations, Notes, and Records. Accurate calculations are the basis of correct proportions of machine parts. There is a right way to make calculations and a wrong way, and the student will usually take the wrong way unless he is cautioned at the start.

The wrong way of making calculations is the loose and shiftless fashion of scratching upon a scrap of detached paper marks and figures, arranged in haphazard form, and disconnected and incomplete. These calculations are in a few moments' time totally meaningless, even to the author of them himself, and are so easily lost or mislaid that when wanted they usually cannot be found.

Engineering calculations should always be made systematically, neatly, and in perfectly legible form, in some permanently bound blank book, so that reference may always be had to them at any future time for the purpose of checking or reviewing. Put all the data down. Do not leave in doubt the exact conditions under which the calculations were made. Note the date of calculation.

If a mistake in figures is made, or a change is found necessary, never rub out the figures or tear out the leaf, or in any way obliterate the figures. Simply draw a bold cross through the wrong part and begin again. Often a calculation which is supposed to be wrong is later shown to be right, or the fact which caused the error may be needed for investigation and comparison. Time which is spent in making figures is always valuable time, time too precious to be thrown away by destroying the record.

The recording of calculations in a permanent form, as just described, is the general practice in all modern engineering offices. This plan has been established purely as a business policy. In case of error it locates responsibility and settles dispute. Consistent designing is made possible through the records of past designs. Proposals, estimates, and bids may often be made instantly, on the basis of what these record books show of sizes and weights. This bookkeeping of calculations is as important a factor of systematic engineering as bookkeeping of business accounts is of financial success.

The student should procure for this purpose a good blank book with a firm binding, size of page not smaller than $6'' \times 8''$—perhaps $8'' \times 11''$ may be better—and every calculation, however small and apparently unimportant, should be made in it.

The development of a personal notebook is of great value to the designer of machinery. The facts of observation and experience recorded in proper form, bearing the imprint of intimate personal contact with the points recorded, cannot be equaled in value by those of any hand or reference book made by another. There is always a flavor about a personal notebook, a sort of guarantee, which makes the use of it by its author definite and sure.

The habit of taking and recording notes, or even knowing what notes to take, is an art in itself, and the student should begin early to make his notebook. Aside from the value of the notes themselves as a part of his personal equipment, the facility with which his eye will be trained to see and record mechanical things will be of great value in all of his study and work. How many men go through a shop and really see nothing of the operations going on therein, or, seeing them, remember nothing! An engineer, trained in this respect, will, to a surprising degree, be able to retain and sketch little details which fall under his eye for a brief moment only, while he is passing through a crowded shop.

Some draftsmen have the habit of copying all the standard tables of the various offices in which they work. While these are of some value in a few cases, yet this is not what is meant by a good notebook in the best sense. Ideas make a good notebook, not a mere tabulation of figures. If the basis upon which standards are founded can be transferred to permanent personal record, or novel methods of calculation, or simple features of construction, or data of mechanical tests, or efficient arrangement of machinery—if *these* can be preserved for reference, the notebook will be of greatest value.

Whatever is noted down, make clear and intelligible, illustrating by a sketch if possible. Make the note so clear that reference to it after a long space of years will bring the whole subject before the mind in an instant. If this is not done the author of the note himself will not have patience to dig out the meaning when it is needed; and the note will be of no value.

STRENGTH OF MATERIALS

In order to determine the sizes of members and the arrangement of parts of a machine it is necessary to know the character and the effect of the loads applied.

STRESS

If a force is applied to any body it tends to change the shape or volume of that body. Thus, if a bar of iron is fixed firmly at its upper end and a heavy weight is hung from the lower, the bar lengthens. If a weight is placed on the top of a block of wood or metal, the block is shortened. In the above examples, the weight or force called the *load* tends to change the shape or volume or both, and the forces within the body that tend to resist the change are called *stresses.* The word *stress* is also used as a name for the force. Engineers say that a body is strained or stressed when a load is placed upon it.

Stresses are measured in pounds, tons, grams, kilograms, etc. For example, if a weight of fifteen hundred pounds is suspended by a rope, the stress in the rope is fifteen hundred pounds. If a block of iron has a weight or force of forty tons on its top, the stress is forty tons. If the force tends to pull the particles of the body apart, it is called *tensile stress;* if it tends to crush the body, it is called *compressive stress;* and the stress that resists the slipping of one section of a body past another section is called a *shearing stress.*

Unit Stress. Unit strain or stress is the amount of stress on a unit area of section and is expressed as pounds per square inch, tons per square inch or per square foot, kilograms per square centimeter, etc. Let A equal the area in square inches and P the total stress in pounds; then the unit stress will be $\frac{P}{A}$. If S equals unit stress,

$$\frac{P}{A} = S, \quad P = AS, \quad A = \frac{P}{S} \tag{1}$$

Suppose a brick in a testing machine to crush at a stress of 10 tons, the section of the brick being 2×4 inches. The unit compressive strain is

$$\frac{P}{A} = \frac{20,000}{8} = 2,500 \text{ pounds}$$

If a rope supports a weight of 314.16 pounds, the unit tensile strain being 400 pounds per square inch, what is the size of the rope?

$$A = \frac{P}{S} = \frac{314.16}{400} = .7854 \text{ square inches}$$

Since the area is .7854 square inches, the diameter is 1 inch.

When a force or weight is applied to a body there is a change of form or volume. This change of shape is called a *strain* or *deformation* and may be measured in inches, centimeters, etc.

Unit Deformation. Unit deformation is the amount of lengthening or shortening per unit of length. Let L equal length in feet, b the elongation or shortening per unit of length, and B the total deformation. Then evidently $L \times b = B$, or $B = bL$. If a bar of iron 10 feet long lengthens .1 of an inch under a load of 20,000 pounds, the elongation per foot will be

$$b = \frac{B}{L} = \frac{.1}{10} = .01 \text{ inches} \qquad (2)$$

Experimental Laws. By means of experiments, together with experience, the following general laws have been established and may be taken as fundamental principles.

1. *When a small stress is applied to a body, a small deformation is produced and, on removal of the stress, the body returns to its original form.* Materials may be regarded as perfectly elastic for small stresses.

2. *Under small stresses the deformations are nearly proportional to those stresses, and also approximately proportional to the length of the body.*

3. *When the stress is sufficiently great, a deformation is produced which is partly permanent;* that is, the body does not spring back to its original shape when the stress or strain is removed. In such cases, the *elastic limit* is said to have been exceeded and *deformation is no longer proportional to stress.*

4. *If the stress is still further increased, deformation increases until the piece breaks.*

5. *A sudden stress or shock is more injurious than a steady stress gradually applied.*

The words *small* and *great* in the above laws have different values for different materials; *i. e.*, a large stress for wood would not be a large stress for steel.

Elasticity. According to the first law given above materials resume their original form after the stress is removed, provided that stress has not been too great. This resistance offered by a body to permanent change in form is called *elasticity*. When the load is large and the piece does not return to its original shape, it is said to have a *permanent set*. The unit stress at which permanent set is first visible is called the *elastic limit*.

The body being perfectly elastic within the elastic limit, laws describing its action can be formulated, but beyond the elastic limit, there being a permanent alteration of shape, these laws cannot be applied.

In testing a specimen of any material, it is easy to reduce its stress to unit stress by the formula $S = \frac{P}{A}$, and its deformation to unit deformation by the formula $b = \frac{B}{L}$. Suppose the unit stress and the unit deformation are known, then the ratio of these, called the *coefficient of elasticity*, may easily be found. This coefficient expressed algebraically is

$$\frac{S}{b} = E \qquad (3)$$

Examples. 1. Suppose the unit stress in a specimen under test is 45,000 pounds and the unit deformation is .0015 inch. What is E?

Solution.

$$E = \frac{S}{b} = \frac{45{,}000}{.0015} = 30{,}000{,}000$$

2. A flat cast-iron foundation ring, 4 inches high, whose area is 4 square feet, has a weight of 144 tons placed on the top. If the weight causes a shortening of .00016 of an inch, what is the coefficient of elasticity?

Solution.

$$E = \frac{S}{b}, \qquad S = \frac{P}{A}, \qquad b = \frac{B}{L}$$

Then substituting

$$E = \frac{PL}{AB} = \frac{144 \times 2000 \times 4}{4 \times 144 \times .00016} = 12,500,000$$

Ultimate Stress. When a bar is under stress exceeding its elastic limit, it is usually unsafe. As the load, or stress, is increased, the deformation is increased rapidly, and finally the bar ruptures. The ultimate strength of the bar is the unit strength which the bar offers just before rupture. Sometimes the strength shown by the bar just at the time when rupture occurs is less than that shown just before breaking, the latter being called the *breaking strength.*

Safe Working Strength. For machines and other structures, it would be unsafe to exert upon the bodies used an external force or load equal to their ultimate strength. It is not even advisable to allow the point to be reached at which permanent set occurs. The unit stress under which the body or material is to act or work, is called the *working strength.* This allowable working strength depends upon the character of the material and the load. It is usual to divide the ultimate strength by some number to determine the working strength. If the ultimate strength of steel is 60,000 pounds, the working strength may be $\frac{60,000}{10} = 6,000.$

Factor of Safety. The number by which the ultimate strength is divided to determine the working strength is called the *factor of safety.* Materials which are unreliable as to their ultimate strength, such as stone and cast iron, have a large factor of safety. Materials to which a steady load is applied do not require as high a factor of

TABLE I

Factor of Safety

Material	For Steady Load	For Varying Load	For Shock
Cast Iron	7	10	20
Wrought Iron	4	6	11
Steel	5	7	12
Soft Metals and Alloys	6	8	15
Ropes	7	10	12
Leather	9	12	15
Timber	9	12	15
Brick and Stone	12	20	30

safety as those which carry a moving load. The proper factor of safety can only be determined by good engineering judgment and experience. Average values for materials in extensive use are given in Table I.

Testing Machines. Materials are tested for strength and elasticity by placing specimens of standard shapes in machines designed for this purpose, and subjecting them to tension, compression, and bending. The stress and the resulting change in the material is shown by some indicating devices.

Tension. Tensile stress in a body is the resistance offered by its molecules to being pulled apart. The action of the forces may be investigated by subjecting a rod to a heavy downward pull until the molecular force is overcome.

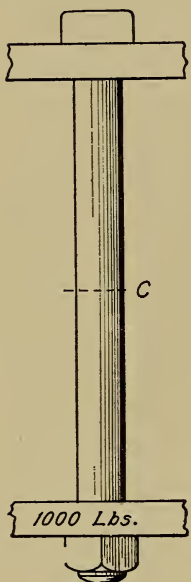


Fig. 1. Rod in Tension

Suppose the point where the rod may rupture is at the cross-section C, Fig. 1. At this point the downward pull of 1,000 pounds due to the load is resisted by the attraction for each other of the molecules about the section, each downward pull calling forth an equal upward pull from the molecules. At any other point along the rod a similar balanced condition may be found. Only when an increase in the load produces a downward pull which is greater than the maximum attractive force of the molecules, will a rupture occur.

If the load is P, the cross-sectional area A, and the safe tensile working stress S, then

$$A = \frac{P}{S}$$

Compression. The compressive or crushing strength of any material is the resistance offered by its molecules to being pushed nearer together. It is the opposite of tension. Fig. 2 represents a post carrying a load at its upper end. Consider any section as C. The load above this section tends to cause the molecules at the section to be pressed closer together, in this case assisting the molecular attraction along the vertical axis. The attraction of the molecules in all other directions, however, prevents the post from being crushed by the load. When the load becomes too great, the molecular resistance is overcome and the post breaks. Loaded posts, or struts, piers, etc., are under compressive stress.

In compression, the formula

$$A = \frac{P}{S}$$

also applies, A being the cross-sectional area, P the load applied, and S the safe compressive strength.

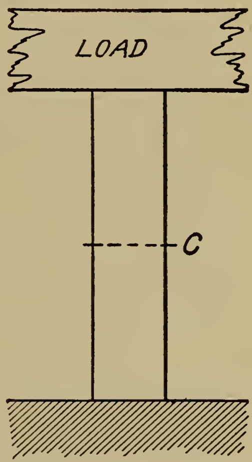


Fig. 2. Post in Compression

Shear. The shearing strength of a material is the resistance offered by its fibers to being *cut*, or, if not fibrous, the resistance offered by its molecules to being *slipped* by each other. Shear is somewhat different from tension and compression; it occurs when two forces tend to cut a body between them.

Let Fig. 3 represent a riveted joint and consider the section through the rivet at C as dividing it into two parts, A and B. The forces applied to the joint are such that A tends to slide to the left and B to the right; then the molecules on one side of section C exert an attraction on the molecules on the other side of the section which tends to prevent slipping. The force which produces the stress in the rivet is called a *shearing stress.*

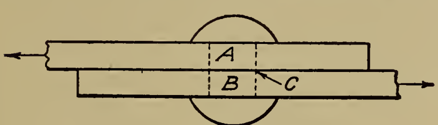


Fig. 3. Rivet in Shear

As in tension, the cross-sectional area A is found by the formula

$$A = \frac{P}{S}$$

where the symbol P represents the load and S, the safe compressive strength.

Beams. A bar supported in a horizontal position is called a *beam.* A *cantilever beam* is one resting on one support or fixed at one end, as in a wall, the other end being free. A *simple beam* is one resting on two supports.

Summation of Forces. The sum of all the forces acting in a given direction upon a body whose position relative to other bodies remains fixed, must be equal to the sum of all the forces acting in the opposite direction. If the forces acting in one direction are greater than those acting in the opposite direction, motion will result.

Moment of a Force. By moment of a force with respect to a point is meant its tendency to produce rotation about that point. Evidently the tendency depends on the magnitude of the force and on the perpendicular distance of the line of action of the force from the point; the greater the force and the perpendicular distance, the greater the tendency; hence the moment of a force with respect to a point equals the product of the force and the perpendicular distance from the force to the point.

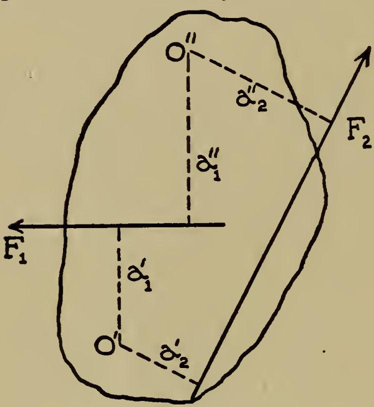


Fig. 4. Moments of Force

The point with respect to which the moment of one or more forces is taken is called an *origin* or *center of moments*, and the perpendicular distance from an origin of moments to the line of action of a force is called the *arm of the force* with respect to that origin. Thus, if F_1 and F_2, Fig. 4, are forces, their arms with respect to O' are a_1' and a_2' respectively, and their moments are F_1a_1' and F_2a_2'. With respect to O'' their arms are a_1'' and a_2'' respectively, and their moments are F_1a_1'' and F_2a_2''.

If the force is expressed in pounds and its arm in feet, the moment is in foot-pounds; if the force is expressed in pounds and its arm in inches, the moment is in inch-pounds.

A *sign* is given to the moment of a force for convenience. The rule used herein is as follows: *The moment of a force about a point is positive or negative according as it tends to turn the body about that point in the *clockwise or counter-clockwise direction.* Thus the moment, Fig. 4, of F_1 about O' is negative, about O'', positive; of F_2 about O' is negative, about O'', negative.

*By *clockwise direction* is meant that in which the hands of a clock rotate; and by *counter-clockwise*, the opposite direction.

Principle of Moments. If a line or point in a body is taken as an axis about which the body may be considered as tending to rotate, then the sum of the moments tending to produce rotation about that axis in one direction will be equal to the sum of the moments tending to produce rotation in the opposite direction if the body is at rest with respect to other bodies. If it is not at rest, the sum of the moments in opposite directions is not equal. In general, a single force of proper magnitude and direction can balance a number of forces.

All the forces acting upon a body which is at rest are said to be *balanced* or in *equilibrium.* No force is required to balance such forces and hence their equilibrant and resultant are zero. Since their resultant is zero, *the algebraic sum of the moments of any number of forces which are balanced or in equilibrium equals zero.*

This is known as the principle of moments for forces in equilibrium; or, briefly, *the principle of moments.*

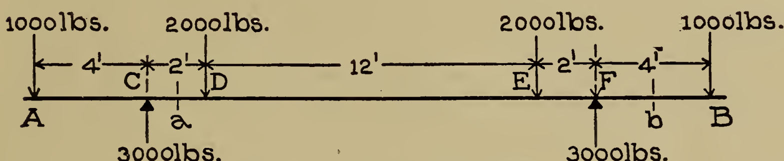


Fig. 5. Moments Acting in Beam

Example. Let *AB,* Fig. 5, be a beam resting on supports at *C* and *F.* Find the moments with respect to the supports.

Solution. It is evident from the symmetry of the loading that each reaction equals one-half of the whole load; *i. e.,* $\frac{1}{2}$ of 6,000 = 3,000 pounds. (The weight of the beam is neglected for simplicity.)

With respect to *C,* for example, the moments of the forces are, taking them in order from the left:

$$\begin{array}{rcrl}
-1{,}000 \times 4 & = & -\ 4{,}000 & \text{foot-pounds} \\
3{,}000 \times 0 & = & 0 & \quad \text{"} \\
2{,}000 \times 2 & = & 4{,}000 & \quad \text{"} \\
2{,}000 \times 14 & = & 28{,}000 & \quad \text{"} \\
-3{,}000 \times 16 & = & -48{,}000 & \quad \text{"} \\
1{,}000 \times 20 & = & 20{,}000 & \quad \text{"}
\end{array}$$

The algebraic sum of these moments is seen to equal zero.

Again, with respect to B, the moments are:

$$
\begin{aligned}
-1{,}000 \times 24 &= -24{,}000 \text{ foot-pounds} \\
3{,}000 \times 20 &= 60{,}000 \quad \text{"} \\
-2{,}000 \times 18 &= -36{,}000 \quad \text{"} \\
-2{,}000 \times 6 &= -12{,}000 \quad \text{"} \\
3{,}000 \times 4 &= 12{,}000 \quad \text{"} \\
1{,}000 \times 0 &= 0 \quad \text{"}
\end{aligned}
$$

The sum of these moments also equals zero. In fact, no matter where the center of moments is taken, it will be found in this and any other balanced system of forces that the algebraic sum of their moments equals zero. The chief use to be made of this principle will be in finding the supporting forces of loaded beams.

Determination of Reactions on Beams. The forces which the supports exert on a beam, *i. e.*, the "supporting forces," are called *reactions*, and will be considered chiefly in connection with simple beams. The reaction on a cantilever beam evidently equals the total load on the beam.

When the loads on a horizontal beam are all vertical—and this is the usual case—the supporting forces are also vertical and *the sum of the reactions equals the sum of the loads.* This principle is sometimes useful in determining reactions, but in the case of simple beams the principle of moments is sufficient. The general method of determining reactions is as follows:

1. Write out two equations of moments for all the forces—loads and reactions—acting on the beam with origins of moments at the supports.

2. Solve the equations for the reactions.

3. As a check, try if the sum of the reactions equals the sum of the loads.

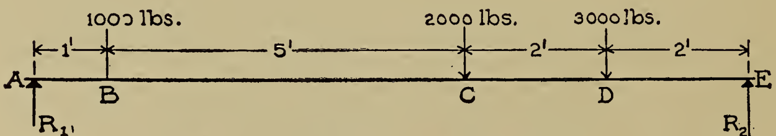


Fig. 6. Reactions at Supports of Simple Beam

Examples. 1. Fig. 6 represents a beam supported at its ends and sustaining three loads. Find the reactions due to these loads.

Solution. Let the reactions be denoted by R_1 and R_2, as shown; then the moment equations are:

For origin at A

$$(1{,}000 \times 1) + (2{,}000 \times 6) + (3{,}000 \times 8) - (R_2 \times 10) = 0$$

For origin at E

$$(R_1 \times 10) - (1{,}000 \times 9) - (2{,}000 \times 4) - (3{,}000 \times 2) = 0$$

The first equation reduces to

$$10\,R_2 = 1{,}000 + 12{,}000 + 24{,}000 = 37{,}000; \text{ or}$$
$$R_2 = 3{,}700 \text{ pounds}$$

The second equation reduces to

$$10\,R_1 = 9{,}000 + 8{,}000 + 6{,}000 = 23{,}000; \text{ or}$$
$$R_1 = 2{,}300 \text{ pounds}$$

The sum of the loads is 6,000 pounds and the sum of the reactions is the same; hence the computation is correct.

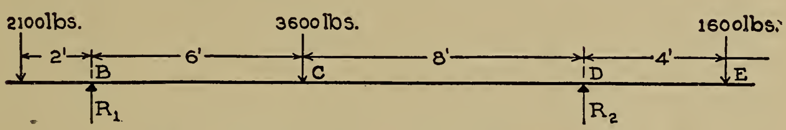


Fig. 7. Reactions in Beam with Overhanging Ends

2. Fig. 7 represents a beam supported at B and D—*i. e.*, it has overhanging ends—and sustaining three loads, as shown. Determine the reactions due to the loads.

Solution. Let R_1 and R_2 denote the reactions as shown; then the moment equations are:

For origin at B

$$(-2{,}100 \times 2) + 0 + (3{,}600 \times 6) - (R_2 \times 14) + (1{,}600 \times 18) = 0$$

For origin at D

$$(-2{,}100 \times 16) + (R_1 \times 14) - (3{,}600 \times 8) + 0 + (1{,}600 \times 4) = 0$$

The first equation reduces to

$$14\,R_2 = -4{,}200 + 21{,}600 + 28{,}800 = 46{,}200; \text{ or}$$
$$R_2 = 3{,}300 \text{ pounds}$$

The second equation reduces to

$$14\,R_1 = 33{,}600 + 28{,}800 - 6{,}400 = 56{,}000; \text{ or}$$
$$R_1 = 4{,}000 \text{ pounds}$$

The sum of the loads equals 7,300 pounds and the sum of the reactions is the same; hence the computation checks.

3. What are the total reactions in Example 1, if the beam weighs 400 pounds?

Solution. (1) Since the reactions due to the loads are already known, being 2,300 and 3,700 pounds at the left and right ends respectively, Example 1, it is only necessary to compute the reactions due to the weight of the beam and add. Evidently the reactions due to the weight equal 200 pounds each; hence the

left reaction $= 2{,}300 + 200 = 2{,}500$ pounds

and the

right reaction $= 3{,}700 + 200 = 3{,}900$ pounds.

(2) Or, the reactions due to the loads and weight of the beam might be computed together and directly. In figuring the moment due to the weight of the beam, or any uniformly distributed load, the *weight is considered as concentrated at the middle of the beam;* then its moments with respect to the left and right supports are (400×5) and $-(400 \times 5)$, respectively. The moment equations for origins at A and E are like those of Example 1 except that they contain one more term, the moment due to the weight; thus they are respectively

$$(1{,}000 \times 1) + (2{,}000 \times 6) + (3{,}000 \times 8) - (R_2 \times 10) + (400 \times 5) = 0$$
$$(R_1 \times 10) - (1{,}000 \times 9) - (2{,}000 \times 4) - (3{,}000 \times 2) - (400 \times 5) = 0$$

The first equation reduces to

$$10R_2 = 39{,}000, \text{ or } R_2 = 3{,}900 \text{ pounds}$$

The second equation reduces to

$$10R_1 = 25{,}000, \text{ or } R_1 = 2{,}500 \text{ pounds}$$

4. What are the total reactions in Example 2, if the beam weighs 42 pounds per foot?

Solution. As in Example 3, the reactions due to the weight might be computed and then added to the corresponding reactions due to the loads (already found in Example 2), but in the following solution the total reactions due to load and weight are determined directly.

The beam being 20 feet long, its weight is 42×20, or 840 pounds. Since the middle of the beam is 8 feet from the left and 6 feet from the right support, the moments of the weight with respect to the left and right supports are, respectively:

$$840 \times 8 = 6{,}720, \text{ and } -840 \times 6 = -5{,}040 \text{ foot-pounds}$$

The moment equations for all the forces applied to the beam for origins at B and D are like those in Example 2, with an additional term, the moment of the weight. They are, respectively:

$$(-2{,}100 \times 2) + 0 + (3{,}600 \times 6) - (R_2 \times 14) + (1{,}600 \times 18) + 6{,}720 = 0$$
$$(-2{,}100 \times 16) + (R_1 \times 14) - (3{,}600 \times 8) + 0 + (1{,}600 \times 4) - 5{,}040 = 0$$

The first equation reduces to

$$14\,R_2 = 52{,}920, \quad \text{or} \quad R_2 = 3{,}780 \text{ pounds}$$

The second equation reduces to

$$14\,R_1 = 61{,}040, \quad \text{or} \quad R_1 = 4{,}360 \text{ pounds}$$

The sum of the loads and weight of beam is 8,140 pounds; and since the sum of the reactions is the same, the computation checks.

EXTERNAL SHEAR AND BENDING MOMENT

On almost every cross-section of a loaded beam the three kinds of stress, viz, *tension*, *compression*, and *shear*, appear. Tension and compression are often called *fiber stresses* because they act along the real fibers of a wooden beam or the imaginary ones of which it may be supposed iron and steel beams are composed. Before taking up the subject of these stresses in beams it is desirable to study certain quantities relating to the loads, and on which the stresses in a beam depend. These quantities are called *external shear* and *bending moment.*

External Shear. By external shear at (or for) any section of a loaded beam is meant the algebraic sum of all the loads (including weight of beam) and reactions on either side of the section. This sum is called *external shear* because, as is shown later, it equals the shearing stress (internal) at the section. For brevity, the term "shear" will be used in this discussion when external shear is meant.

Rule of Signs. In computing external shears, it is customary to give the plus sign to the reactions and the minus sign to the loads. When the external shear is computed from the loads and reactions to the right *the sign of the sum is changed* in order to get the same sign for the external shear, as when computed from the left. Thus for section a of the beam in Fig. 5 the algebraic sum when computed from the left

$$= -1{,}000 + 3{,}000$$
$$= \quad 2{,}000 \text{ pounds}$$

and when computed from the right

$$= -1{,}000 + 3{,}000 - 2{,}000 - 2{,}000$$
$$= -2{,}000 \text{ pounds}$$

The external shear at section a is $+2{,}000$ pounds.

Again, for section b the algebraic sum when computed from the left

$$= -1{,}000 + 3{,}000 - 2{,}000 - 2{,}000 + 3{,}000$$
$$= +1{,}000 \text{ pounds}$$

and when computed from the right

$$= -1{,}000 \text{ pounds}$$

The external shear at the section is $+1{,}000$ pounds.

It is usually convenient to compute the shear at a section from the forces to the right or left according as there are fewer forces—loads and reactions—on the right or left sides of the section.

Units. It is customary to express external shears in pounds, but any other unit for expressing force and weights—as the ton—may be used.

Maximum Shear. It is sometimes desirable to know the greatest or maximum value of the shear in a given case. The maximum shear is given in Table II for the usual cases of loading.

In cantilevers fixed in a wall, the maximum shear occurs at the wall.

In simple beams, the maximum shear occurs at a section next to one cf the supports.

Bending Moment. By bending moment at (or for) a section of a loaded beam, is meant the algebraic sum of the moments of all the loads (including weight of beam) and reactions to the left or right of the section with respect to any point in the section.

Rule of Signs. The rule of signs previously stated, Page 14, is followed; but in order to get the same sign for the bending moment whether computed from the right or left, the sign of the sum of the moments is changed when computed from the loads and reactions on the right. Thus for section a, Fig. 5, the algebraic sum of the moments of the forces, when computed from the left

$$= (-1{,}000 \times 5) + (3{,}000 \times 1)$$
$$= -2{,}000 \text{ foot-pounds}$$

and when computed from the right

$$= (1{,}000 \times 19) - (3{,}000 \times 15) + (2{,}000 \times 13) + (2{,}000 \times 1)$$
$$= +2{,}000 \text{ foot-pounds}$$

The bending moment at section *a* is $-2{,}000$ foot-pounds.

Again, for section *b*, the algebraic sum of the moments of the forces, when computed from the left

$$= (-1{,}000 \times 22) + (3{,}000 \times 18) - (2{,}000 \times 16) - (2{,}000 \times 4) + (3{,}000 \times 2)$$
$$= -2{,}000 \text{ foot-pounds}$$

and when computed from the right

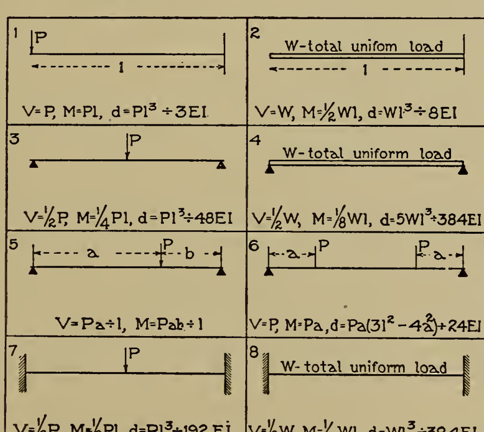
$$= \quad 1{,}000 \times 2$$
$$= -2{,}000 \text{ foot-pounds}$$

The bending moment at the section is $-2{,}000$ foot-pounds.

It is usually convenient to compute the bending moment for a section from the forces to the right or left according as there are fewer forces (loads and reactions) on the right or left side of the section.

TABLE II

Coefficient of Elasticity (E), Moment of Inertia (I), Values of Maximum Shear (V) Bending Moment (M), and Defection (d).



Units. It is customary to express bending moments in inch-pounds, but often the foot-pound unit is more convenient. *To reduce foot-pounds to inch-pounds, multiply by twelve.*

Maximum Bending Moment. It is sometimes desirable to know the greatest or maximum value of the bending moment in a given case. This may be obtained from Table II.

STRENGTH OF BEAMS

Resisting Moment of Beams. When a beam is bent the stresses acting over a cross-section are not the same at all points. The fibers on one side are stretched and therefore under tension, while those on the other side are shortened and under compression. In a simple beam the upper fibers are compressed and shortened and the lower fibers lengthened. In a cantilever beam the upper fibers are in tension, while the lower ones are compressed. *This change in length of fibers, and therefore in the intensity of stress, proceeds uniformly from the bottom to the top of the beam.* Since the character of the stress changes from tension to compression or compression to tension in passing from the top to bottom, there must be a point or layer in the beam which has neither tension nor compression exerted upon its fibers. This layer or surface is called the *neutral surface* and is represented in Fig. 8 by a plane $NNNN$. The central line OO of the neutral surface is called the *neutral axis* and passes through the center of gravity of the section. The resistance offered to bending by a beam is B, and is equal to the bending moment M, therefore

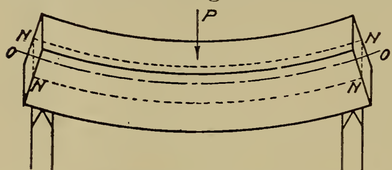


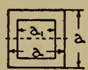
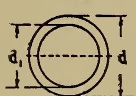
Fig. 8. Resistance of Beam to Bending

$$M = S\frac{I}{c} \tag{4}$$

where S is the unit stress in the fiber at the greatest distance from the neutral axis; c is the distance of this fiber from the axis; and I is the *moment of inertia,* a factor determined by higher mathematics. Taking S as a safe working stress of the beam, $S\frac{I}{c}$ should be equal to the greatest bending moment. The ratio $\frac{I}{c}$ is called the *section*

*TABLE III

Moments of Inertia, Section Moduli, and Radii of Gyration

Section	Moment of Inertia	Section Modulus
	$\frac{a^4}{12}$	$\frac{a^3}{6}$
	$\frac{a^4 - a_1^4}{12}$	$\frac{a^4 - a_1^4}{6a}$
	$\frac{ba^3}{12}$	$\frac{ba^2}{6}$
	$\frac{ba^3 - b_1a_1^3}{12}$	$\frac{ba^3 - b_1a_1^3}{6a}$
	$0.049d^4$	$0.098d^3$
	$0.049\ (d^4 - d_1^4)$	$0.098\frac{d^4 - d_1^4}{d}$

*In each case the axis is horizontal and passes through the center of gravity.

modulus. Values of the moment of inertia and the section modulus for different cross-sections are given in Table III.

Deflections of Beams. Sometimes it is desirable to know how much a given beam will deflect under a given load. In Table II formulas are given for the deflection in certain cases of beams and different kinds of loading.

Examples. 1. A wrought-iron cantilever beam having a square section, 2 inches on a side, is 10 feet long. If the beam weighs 12 pounds per foot and has a working fiber strength equal to 12,000 pounds, what load may be safely placed on its extremity?

Solution. Referring to Table II, the maximum bending moment for a uniform load is $\frac{1}{2}Wl$. The weight of the beam is a uniform load and $W = 12 \times 10 = 120$ pounds, $l = 12 \times 10 = 120$ inches, then

$$M_1 = \tfrac{1}{2}Wl = \tfrac{1}{2} \times 120 \times 120 = 7{,}200 \text{ inch-pounds.}$$

The resisting moment of the beam is $S\frac{I}{c}$. The value of $\frac{I}{c}$, the section modulus, may be obtained from Table III and for a square section is $\frac{a^3}{6}$, then

$$\frac{I}{c}=\frac{a^3}{6}=\frac{2^3}{6}=\frac{8}{6}$$

$$M = S\frac{I}{c} = 12{,}000 \times \frac{8}{6} = 16{,}000 \text{ inch-pounds.}$$

The resisting moment must oppose the moment due to the weight of the beam as well as the load and the resisting moment opposing the load is $M - M_1 = 16{,}000 - 7{,}200 = 8{,}800$ inch-pounds. From Table II the maximum bending moment due to the load is

$$M = Pl = P \times 120 \text{ inch-pounds.}$$

Equating

$$P \times 120 = 8{,}800$$

$$P = \frac{8{,}800}{120} = 73\tfrac{1}{3} \text{ pounds.}$$

2. What is the deflection of the above beam, due only to its own weight? The coefficient of elasticity is 25,000,000.

Solution. From Table II the deflection is expressed as

$$d = Wl^3 \div 8EI$$

From Table III

$$I=\frac{a^4}{12}=\frac{16}{12}=\frac{4}{3}$$

then

$$d = \frac{120 \times 120^3}{8 \times 25{,}000{,}000 \times \frac{4}{3}} = .77 \text{ inch}$$

STRENGTH OF SHAFTS

Shafts. A shaft is a part of a machine or system of machines, and is used to transmit power by virtue of its torsional strength, or resistance to twisting. Shafts which are nearly always made of metal are usually circular in cross-section and sometimes hollow.

Twisting Moment. Let AF, Fig. 9, represent a shaft with four pulleys on it. Suppose that D is the driving pulley and that B, C, and E are pulleys from which power is taken off to drive machines. When the shaft is transmitting power, the portions between the pulleys are twisted—by the twisting moment at any cross-section of the shaft is meant the algebraic sum of the moments of all the forces acting on the shaft on either side of the section, the moments being

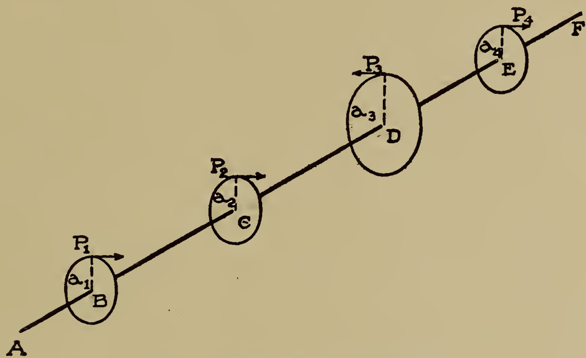


Fig. 9. Twisting Action on Shaft

taken with respect to the axis of the shaft. Thus, if the forces acting on the shaft (at the pulleys) are P_1, P_2, P_3, and P_4, as shown, and if the arms of the forces or radii of the pulleys are a_1, a_2, a_3, and a_4, respectively, then the twisting moment at any section

in BC $= P_1 a_1$
in CD $= P_1 a_1 + P_2 a_2$
in DE $= P_1 a_1 + P_2 a_2 - P_3 a_3$

Like bending moments, twisting moments are usually expressed in inch-pounds.

Example. Let $a_1 = a_2 = a_4 = 15$ inches, $a_3 = 30$ inches, $P_1 = 400$ pounds, $P_2 = 500$ pounds, $P_3 = 750$ pounds, and $P_4 = 600$ pounds.* What is the value of the greatest twisting moment in the shaft?

At any section between the first and second pulleys, the twisting moment is

$$400 \times 15 = 6{,}000 \text{ inch-pounds}$$

at any section between the second and third it is

$$(400 \times 15) + (500 \times 15) = 13{,}500 \text{ inch-pounds}$$

and at any section between the third and fourth it is

*These numbers were so chosen that the moment of P (driving moment) equals the sum of the moments of the other forces. This is always the case in a shaft rotating at constant speed; *i. e.*, the power given the shaft equals the power taken off.

$(400 \times 15) + (500 \times 15) - (750 \times 30) = -9{,}000$ inch-pounds
Hence, the greatest value is 13,500 inch-pounds.

Torsional Stress. The stresses in a twisted shaft are called *torsional* stresses. The torsional stress on a cross-section of a shaft is a shearing stress, as in Fig. 10, which represents a flange coupling in a shaft. Were it not for the bolts, one flange would slip over the other when either part of the shaft is turned; but the bolts prevent the slipping. Obviously there is a tendency to shear the bolts off unless they are screwed up very tight; *i. e.*, the material of the bolts is subjected to shearing stress.

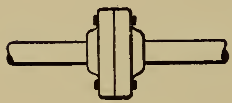


Fig. 10. Flange Coupling

Just so, at any section of the solid shaft there is a tendency for one part to slip past the other, and to prevent the slipping or shearing of the shaft, there arise shearing stresses at all parts of the cross-section. The shearing stress on the cross-section of a shaft is not a uniform stress, its value per unit-area being zero at the center of the section, and increasing toward the circumference. In circular sections, solid or hollow, the shearing stress per unit-area (unit-stress) varies directly as the distance from the center of the section, provided the elastic limit is not exceeded. Thus, if the shearing unit-stress at the circumference of a section is 1,000 pounds per square inch, and the diameter of the shaft is 2 inches, then, at $\frac{1}{2}$ inch from the center, the unit-stress is 500 pounds per square inch; and at $\frac{1}{4}$ inch from the center it is 250 pounds per square inch. In Fig. 11 the arrows indicate the values and the directions of the shearing stresses on very small portions of the cross-section of a shaft there represented.

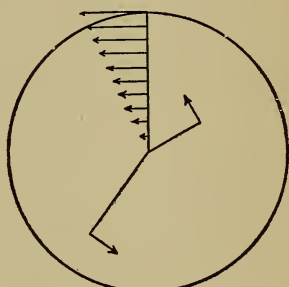


Fig. 11. Cross-Section of Shaft

Resisting Moment. By *resisting moment* at a section of a shaft is meant the sum of the moments of the shearing stresses on the cross-section about the axis of the shaft.

Let S_s denote the value of the shearing stress per unit-area (unit-stress) at the outer points of a section of a shaft; *d*, the diameter of the section—if the shaft is hollow, let *d* equal outside diameter and d_1 inside diameter. Then it may be shown that the resisting moment T is

$$T = \frac{S_s I}{c} \qquad (5)$$

where c is the distance of the most remote fiber from the neutral axis, and I is the polar moment of inertia. This is not the same as the direct moment of inertia, and must be determined by the aid of Calculus

Formula for the Strength of a Shaft. As in the case of beams, the resisting moment equals the twisting moment at any section. Determining the value of I by higher mathematics, the formula $T = \frac{S_s I}{c}$ becomes

for solid circular shafts, $$T = 0.1963\, S_s d^3 \qquad (6)$$

for hollow circular shafts, $$T = \frac{0.1963\, S_s (d^4 - d_1^4)}{d} \qquad (7)$$

In any portion of a shaft of constant diameter, the unit-shearing stress S_s is greatest where the twisting moment is greatest. Hence, to compute the greatest unit-shearing stress in a shaft, first determine the value of the greatest twisting moment and substitute its value in the first or second equation above, as the case may be, and solve for S_s. It is customary to express T in inch-pounds and the diameter in inches, S_s then being in pounds per square inch.

Examples. 1. Compute the value of the greatest shearing unit-stress in the portion of the shaft between the first and second pulleys represented in Fig. 9, assuming values of the forces and pulley radii as given in the example, Page 25. Suppose also that the shaft is solid, its diameter being 2 inches.

Solution. The twisting moment T at any section of the portion between the first and second pulleys is 6,000 inch-pounds. Hence, substituting in the first of the two formulas above

$$0.1963\, S_s \times 2^3 = 6{,}000$$

or

$$S_s = \frac{6{,}000}{0.1963 \times 8} = 3{,}820 \text{ pounds per square inch}$$

This is the value of the unit-stress at the outside portions of all sections between the first and second pulleys.

2. A hollow shaft is circular in cross-section, and its outer

and inner diameters are 16 and 8 inches respectively. If the working strength of the material in shear is 10,000 pounds per square inch, what twisting moment can the shaft safely sustain?

Solution. The problem requires merely the substitution of the values of S_s, d, and d_1 in the second of the above formulas, and solving for T. Thus

$$T=\frac{0.1963 \times 10{,}000\ (16^4-8^4)}{16}=7{,}537{,}920 \text{ inch-pounds}$$

PROBLEMS FOR PRACTICE

1. Compute the greatest value of the shearing unit-stress in the shaft represented in Fig. 9, using the values of the forces and pulley radii given in the example, Page 25, the diameter of the shaft being 2 inches.

Ans. 8,595 pounds per square inch.

2. A solid shaft is circular in cross-section and is 9.6 inches in diameter. If the working strength of the material in shear is 10,000 pounds per square inch, how large a twisting moment can the shaft safely sustain? (The area of the cross-section is practically the same as that of the hollow shaft of Example 2, Page 27.)

Ans. 1,736,736 inch-pounds.

COMBINED STRESSES

Formulas. Nearly all stresses may be reduced to simple tension, compression, shear, or torsion. In many cases complex combinations occur, which will not permit simple and direct application of the formulas for these, but it is essential to have perfect command of formulas 4 and 5. Assuming that the forces may be analyzed and the simple moment completed at the point where it is desired to find the strength of section, it remains only to insert the assumed working fiber stress of the material in the formula and to solve for the quantity desired.

In cases of combined stress, the relations become more complicated and difficult of analysis and solution. The most common case is that of a shaft transmitting power, and at the same time loaded transversely between bearings, which is a combination of bending and torsion—there are very few cases of shafts in machines, which, at some part of their length, do not have this combined stress. In this case the method of procedure is to find the simple bending moment and the simple torsional moment separately, in the ordinary way. Then the theory of elasticity furnishes a formula for an equivalent bending or an equivalent torsional moment which is supposed to produce the same effect upon the fibers of the material as the combined action of the two simple moments acting together. In other words, the separate moments combined in action, being impossible of solution in that form, are reduced to an equivalent simple moment and the solution then becomes the same as for the previous case.*

$$M_e = \frac{M}{2} + \tfrac{1}{2}\sqrt{M^2 + T^2} \qquad (8)$$

$$T_e = M + \sqrt{M^2 + T^2} \qquad (9)$$

M_e and T_e, found from these equations, are the external moments, and are to be equated to the internal moments of resistance of the section precisely as if they were simple bending or torsional moments. Although either formula may be used, the latter is more common for shafts.

STRENGTH OF PIPES AND CYLINDERS

Pipes and Cylinders. Water and steam exert equal pressures in all directions and tend to rupture a pipe or cylinder longitudinally or in the direction of its length. The material resists this rupturing stress.

Let p = pressure per square inch; D = diameter of the pipe; L = length of the pipe; t = thickness of the pipe; S = tensile stress. Then from a principle of hydraulics, the force which tends to cause rupture is equal to the pressure on a plane whose width is equal to the diameter of the pipe D and whose length is equal to L. Such a plane is indicated in Fig. 12 by *abcd*. The area of the plane is LD. The force acting to rupture the pipe or cylinder is

$$P = pLD \qquad (10)$$

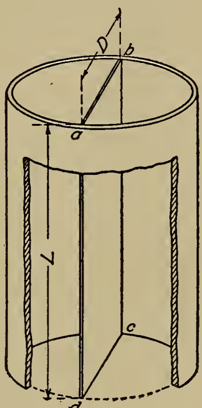


Fig. 12. Pipe Under Internal Pressure

Formulas. The resistance of each side is equal to the area of metal multiplied by the stress per square inch, or tLS, and for two sides, $2tLS$. Then, as the resistance must be at least equal to the force tending to rupture,

*The subscript "e" expresses separation from the simple moments.

$$pLD = 2tLS$$

or

$$pD = 2tS$$

Dividing by 2, the formula becomes

$$\frac{pD}{2} = tS$$

or, since $D = 2r$

$$pr = tS$$

and the equation may be written as

$$\frac{t}{r} = \frac{p}{S} \text{ or } \frac{\text{thickness}}{\text{radius}} = \frac{\text{pressure per square inch}}{\text{working stress}} \qquad (11)$$

In the solution of problems, the above ratio will be found to be a convenient one.

Example. What should be the thickness of a cast-iron pipe 10 inches in diameter, with an internal pressure of 150 pounds per square inch, and 1,250 pounds per square inch as the safe working stress?

Solution. $\frac{t}{r} = \frac{p}{S}$ or, $t = \frac{rp}{S} = \frac{5 \times 150}{1250} = .6$

The pipe should, therefore, be about $\frac{5}{8}$ of an inch thick.

METHOD OF DESIGN

The fundamental lines of thought and action which every designer follows in the solution of a problem in design are four in number. The expert may carry all these in mind at the same time, without definite separation into a step-by-step process; but the student must master them in their proper sequence, and thoroughly understand their application. In these four fundamentals is concentrated the entire art of Machine Design, and when they have become so familiar as to be instinctively applied on any and all occasions, good design is the result. Experience is the only other factor which will facilitate still further the design of good machinery; and it cannot be taught, it must be acquired by actual work.

ANALYSIS OF CONDITIONS AND FORCES

First, take a good square look at the problem to be solved. Study it from all sides, view it in all lights, note the worst conditions which can possibly exist, the average conditions of service, and any special or irregular service likely to be called for.

With these conditions in mind, make a careful analysis of all the forces, maximum as well as average, which may be brought into play. Although it is hard and sometimes impossible to determine exactly the forces acting on a given piece, their nature—whether sudden or slowly applied, rapid in action or only occurring at intervals—and their approximate magnitude, are always capable of analysis. Make a rough sketch of the piece under consideration, put in these forces, and go over the analysis carefully. A hasty and poor analysis will in the end be time lost, and, if the machine actually fails from this reason, heavy financial loss in material and labor will occur.

Machines are nothing but a collection of parts upon which forces are acting directly, or parts acting as loaded beams. Where the force has no leverage it acts directly on the sustaining part. Forces acting with leverage produce a moment; the sustaining member is a beam, and the effect produced therein depends on the theory of beams.

An example of the first is the load on a rope, the force acting without leverage, and the rope, therefore, having a direct pull put upon it.

An example of the second is a push of the hand on the crank of a grindstone. A moment is produced about the hub of the crank; the arm of the crank is a beam.

THEORETICAL DESIGN

After it is determined by careful analysis what stress the machine part has to sustain, the next step is so to design it that it will theoretically resist the applied forces with the least expenditure of material.

Machinery is often constructed with the metal of which it is made distributed in the worst possible manner. In places where the stress is heavy and a rigid member is needed, a weak, springy part is sometimes found; while in other parts, where there are no forces to be resisted, or vibration to be absorbed, there seems to be a waste of good material. Whether in such case the analysis of the forces was poor, or perhaps not made at all, or whether a knowledge of how to design so as to resist the given forces was wholly absent, cannot be told. At any rate, lack of either or both is clearly shown in the result.

Any member of a machine may vary in form from a solid block or chunk of material to an open ribbed structure. The solid chunk fills the requirement as far as strength is concerned, unless it is so heavy as to fail from its own weight. But such construction is poor design, except in cases where the concentration of heavy mass is necessary to absorb repeated blows like those of a hammer. The possibility of these blows should, however, have been determined in the analysis; and the solid, anvil construction then becomes theoretical design for that analysis.

For steadily applied loads an open, ribbed, or hollow box structure can be made which will distribute the metal where it is theoretically needed, and each fiber will then sustain its proper share of the load. In this way weight, cost, and appearance are heeded; and the service of the piece is as good as, and probably better than, it would be with the clumsy, solid form.

There is no such thing as putting too much theory into the design of machinery. The strongest trait which an engineer can have is absolute faith in his analysis and calculations, and their reproduction in his theoretical design. Theoretical design is an indication of scientific advance in the art, and some of the greatest steps of progress which have been made in recent years have been accomplished through a purely theoretical study of machine structure.

PRACTICAL MODIFICATION

It will never do to be satisfied with theoretical design when it is not in accord with modern commercial and manufacturing considerations. Hence the next step after the determination of the theoretical design is the study of it from the producing standpoint.

All theoretical design viewed from the business standpoint is worthless, unless it has been subjected to the test of cheap and efficient production. Each machine detail, though correct in theory, may yet be improperly shaped and unfit for the part it is to play in the general scheme of manufacture.

The conditions here involved are changeable. What is good design in this decade may be bad in the next. In this light the designer must be a close student of the signs of the times; he must follow the march of progress, closely applying existing resources, conditions, and facilities, otherwise he cannot produce up-to-date designs. The introduction of new raw materials, the cheapening of production of others, the changing of shop methods, the use of special machinery, the opening of new markets, the development of new motive agents—all these and many others are constantly demanding some modification in design to meet competition.

Illustrative of this, note the change which has been wrought by the development of electric power, the rise and decline of the bicycle business, the present manufacture of automobiles, the last named especially with reference to the development of the small motive unit, the gasoline engine, the steam engine, etc. The design of much machinery has been materially changed to meet the exacting demands of these new enterprises.

Practical modifications of design necessary to meet the limitations of construction in the pattern shop, foundry, and machine shop are of daily application in the designer's work. He must keep in his mind's eye at all times the workmen and the processes they use to create his designs in metal in the shop.

"How can this be made?" "Can it be made at all?" "Can it be made cheaply?" "Will it be simple in operation after it is made?" "Can it be readily removed for repair?" "Can it be lubricated?" "How can it be put in place?" "How can it be gotten out?" "Will it be made in small quantities or large?" "Will it sell as a special or standard machine?" etc., etc.

The consideration of such questions as these is a practical necessity from a business standpoint, as no other factor affects the design of machinery more. Designs which cannot be built as business propositions are no designs at all.

The student, it is true, may not have the extended shop knowledge which is essential to this; but he can do much for himself by visiting shops whenever possible, getting hold of shop ways of doing things, and invariably treating his work as a business matter. Though a man may not be a pattern maker, moulder, blacksmith, or machinist, yet he can soon gain ideas of the processes of each of these branches which will be of immense advantage to him in his designing work.

DELINEATION AND SPECIFICATION

Delineation and specification mean the clear and concise representation of the design by mechanical drawings, and are as much a part of the routine method of Machine Design as the three preceding fundamentals. The mere act of putting the results of mechanical thinking on paper is one of the greatest helps for bringing the thinking machinery into systematic and definite action. A designer never thinks very long without drawing something, and the student must bring himself to feel that a drawing in its first sense is a means of helping his own thought, and must freely use it as such.

In its second and final sense, the drawing is an order and specification sheet from the designer to the workman. Design which stops short of exact, finished delineation in the form of working shop drawings, is only half done. In fact, the possibility of a piece being thus exactly drawn is often the crucial test of its feasibility as a part of a machine. It is easy to make general outlines, but it is not so easy to get down to finished detail. It is safe to say that there is no one thing productive of more trouble, delay, and embarrassment, and waste of time and money in the shop, when there need be none from this cause, than a poor detail drawing. The efficiency of the process of design is not fully realized, and failures are often recorded where there should be success, merely because the indefiniteness permitted by the designer in the drawings naturally transmitted itself to the workman, and he in turn produced a part indefinite in form and operation.

The actual process of drawing in the development of a design may be outlined as follows:

Rough sketches merely representing ideas, not drawn to scale, are first made. These are of use only so far as the choice of mechanical ideas is concerned, and to carry preliminary dimensions.

Following these sketches, comes a layout to scale, of the favored sketch, a working out of the relative sizes and location of the parts. This drawing may be of a sketchy nature, carrying a principal dimension here and there to fix and control the detailed design. In this drawing the design is developed and general detail worked out. The minute detail of the individual parts is, however, left to the subsequent working drawing.

This layout drawing may now be turned over to an expert draftsman, or detail designer, who picks out each part, makes an exact drawing of it, studying every little detail of its shape, and finally adds complete dimensions and specifications so that the workman is positively informed as to every point of its construction.

Detail Drawings. Drawings of individual parts and sections showing the internal construction and sectional shape are sometimes necessary. These are called *detail drawings.*

Sometimes it is necessary to show sections in order that the internal construction or sectional shape may be easily understood. These sections are usually drawn through the axis, or center, but it is sometimes advisable to show sections of other portions. Where the drawing shows a section, the portions of metal or wood supposed to be cut are covered with parallel lines at equal distances and usually

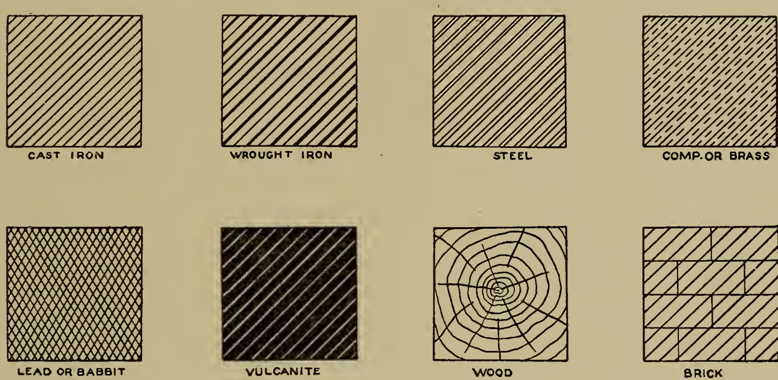


Fig. 13. Conventional Cross-Hatching

oblique. These sections are called *hatched, cross-hatched* or simply *sectioned.* The character of the lines—full, dotted, broken, light, or heavy—indicate the material supposed to be cut. One kind indicates cast iron, another steel, another brass, etc. There is no standard for cross-hatching, different draftsmen using lines of various character. There is likely to be a confusion unless the parts have the name of the material printed on or near it, or a key is provided.

Fig. 13 shows the lines as generally used, those representing cast iron, brass, wood, and lead being almost universal; the others are subject to more change. The lines may run from left to right, or right to left; in case two or more parts of the same metal are brought together it is necessary to avoid confusion by varying the direction and angles of the lines. If the cross-hatching were to be the same, the parting line would be confused and one might think it all one piece.

When drawing designs of the details, it is well to make them as large as convenient. The scales in general use are full size or half size; 3 inches or $1\frac{1}{2}$ inches = 1 foot. A drawing is seldom made by such scales as 2 inches or 4 inches = 1 foot.

A working drawing is one that shows all the dimensions of an object in such a manner that it may be made by reference to the drawing. Usually three views are sufficient—two elevations, taken at right angles to each other, and a horizontal projection or plan. Often the plan may be dispensed with if the object represented is of simple form. Sections to show the interior construction are sometimes added.

In this discussion, the dimensions of some of the drawings are given in letters instead of figures, so that the relation between the different dimensions may be explained. In general, however, this should not be done. The dimensions placed upon a drawing should be in figures, for letters without an explanatory table convey no idea of the size.

General Drawing. The last step in the process of design of a machine is the making of the assembled or general drawing. This is built up piece by piece from the detail drawings, and serves as a last check on the parts which are to be put together. This drawing may be a cross-section or an outside view. In any case it is not wise to try to show too much of the inside construction by dotted lines, for if this be attempted, the drawing soon loses its character of clearness, and becomes practically useless. A general drawing should clearly *hint at, but not specify,* detailed design. It is just as valuable a part of the design as the detail drawing, but it cannot be made to answer for both with any degree of success. A good general drawing has plenty of views and an abundance of cross-sections, but few dotted lines.

The functions which the general drawing may serve are many and varied. Its principal usefulness is, perhaps, in showing to the workman how the various parts go together, enabling him to sort out readily the finished detail parts and assemble them, finally producing the complete structure. Otherwise the making of a machine, even with the parts all at hand, would be like the putting together of the many parts of an intricate puzzle, and much time would be wasted in trying to make the several parts fit, with perhaps never complete success in giving each its absolutely correct location.

The general drawing also gives valuable information as to the total space occupied by the completed machine, enabling its location in a crowded manufacturing plant to be planned for, its connection to the main driving element arranged, and its convenience of operation studied.

In some classes of work it is a convenient practice to letter each part on the general drawing, and to note the same letters on the specification or order sheet, thus enabling the whole machine to be ordered from the general drawings. This, although a very excellent method in certain lines of work, makes the general drawing quite useless in others.

Merely as a basis for judgment of design, the general drawing fulfills an important function in any class of work, for it approaches the nearest possible to the actual appearance that the machine will have when finished. A good general drawing is, for critical purposes, of as much value to the expert eye of the mechanical engineer as the elaborate and colored sketch of the architect is to the house builder or landscape designer.

From the above it is readily understood that the general drawing, although a mere putting together of parts in illustration, is yet of great assistance in producing finished and exact Machine Design.

LUBRICATION

Friction. The parts of a machine which have no relative motion with regard to each other are not dependent upon lubrication of their surfaces for the proper performance of their functions. In cases where relative motion does occur—as between a planer bed and its ways, a shaft and its bearing, or a driving screw and its nut—friction, and consequent resistance to motion, will inevitably occur. Heat will be generated, and cutting or scoring of the surfaces will take place, if the surfaces are allowed to run together dry.

This difficulty, which exists with all materials, cannot be overcome, for it is a result of roughness of surface, characteristic of the material even when highly finished. The problem of the designer, then, is to take conditions as he finds them, and, as he cannot change the physical characteristics of materials, so choose those which are to rub together in the operation of the machine that friction will be reduced to the lowest possible limit. Now it fortunately happens that there are certain agents like oil and graphite, which seem to fill up the hollows in the surface of a solid material, and which themselves have very little friction on other substances. Hence, if a machine permits by its design an automatic supply of these lubricating agents to all surfaces having motion between them, friction may be reduced to the lowest limit.

If this full supply of lubricant be secured, and the parts still heat and cut, then the fault may be traced to other causes, such as springy surfaces, localization of pressure, or insufficient radiating surface to carry away the heat of friction as fast as it is generated.

Lubricant. Lubricating agents are of a nature running from the solid graphite form to a thick grease, then to a heavy dark oil, and finally to a thin, fluid oil flowing as freely as water. The solid and heavy lubricants are applicable to heavily loaded places where the pressure would squeeze out the lighter oils. Grease, forced between the surfaces by compression grease cups, is an admirable lubricator for heavy machinery under severe service. High-speed and accurate machinery, lightly loaded, requires a thin oil, as the fits would not allow room for the heavier lubricants to find their way to the desired spot. The ideal condition in any case is to have a film of lubricant always between the surfaces in contact, and it is this condition at which the designer is always aiming in his lubricating devices.

Means for Lubrication. Oil ways and channels should be direct, ample in size, readily accessible for cleaning, and distributing the oil by natural flow over the full extent of the surface. Hidden and remote bearings must be reached by pipes, the mouths of which should be clearly indicated and accessible to the operator of the machine. Such pipes must be straight, if possible, and readily cleaned.

There is one practical principle affecting the design of methods of lubrication of a machine which should be borne in mind. This is, "Neglect and carelessness by the operator *must* be provided for." It is of no use to say that the ruination of a surface or hidden bearing is due to neglect by the operator, if the means for such lubrication are not perfectly obvious. This is "locking the door after the horse is stolen." The designer has not done his duty until he has made the scheme of lubrication so plain that every part *must* receive its proper supply of oil, except by gross and willful negligence for which there can be no possible just excuse.

Materials Employed in Construction. When stress is induced in a piece, the strain is practically proportional to the stress for all values of the stress below the elastic limit of the material; and when the external load is removed the strain will entirely disappear, or the recovering power of the material will restore the piece to the original length.

Now it is found that if a piece is to last in service for a long time without danger of breakage, it must not be permitted to be stressed anywhere near the elastic limit value. If it is, although it will probably not break at once, it is in a dangerous condition, and not well suited to its requirements as a machine member. The technical name for this weakening effect is "*fatigue*." It is further found that the fatigue due to this repeated stress is reached at a lower limit when the stress is alternating in character than when it is not. In other words, if first a pull and then a push is exerted upon a piece it will be first in tension and then in compression; this alternation of stress repeated too near the elastic limit of the material will fatigue it, or wear out the fibers, and it will finally fail. If, however, a pull is first exerted on the piece with the same force as before, and then let go, the piece will be in tension and then entirely relieved; such repetition of stress will finally fatigue the material, but not so quickly as in the first case. Experiments indicate that it may take twice as many applications in the latter case as in the former.

The working stress of materials permissible in machines is based on the above facts. The breaking strength divided by a liberal factor of safety will not necessarily give a desirable working stress. The question to be answered is, "Will the assumed working fiber stress permit an indefinite number of applications of the load without fatiguing the material?"

Hence it is seen that the same material may be safely used under different assumptions of working stress. For example, a rotating shaft, heavily loaded between bearings, acts as a beam which in each revolution is having its particles subjected, first to a maximum tensile stress, and then to a maximum compressive stress. This is obviously a very different stress from that which the same piece would receive if it were a pin in a bridge truss. In the former is a case where the stress on each particle reverses at each revolution, while in the latter the same stress merely recurs at intervals, but never becomes of the opposite character. For ordinary steel, a value of 8,000 would be reasonable in the former case, while in the latter it may be much higher with safety, perhaps nearly double.

From the facts stated above, it is evident that exact values for working fiber stress cannot be assumed with certainty and applied broadly in all cases. If the elastic limit of the material is definitely known, a working value can be based on that. Data on the strength of materials is available in any of the handbooks, and should be consulted freely by the student; it will be found somewhat conflicting, but will assist the judgment in coming to a conclusion.

The principal materials used in the construction of machinery are cast and wrought iron, steel, copper, wood, brass, and other alloys.

Cast Iron. Cast iron is used to a considerable extent in the construction of machines. For the heavy, massive parts—the frames of lathes, steam-engines, planers, etc., for example—it is the best material. It is *not* suitable for parts requiring strength, elasticity, or those subjected to shocks. For this reason piston-rods, connecting-rods, shafts, etc., are usually made of steel or wrought iron.

Many complicated shapes that cannot be forged are readily cast. The ease with which parts may be given the desired shape makes cast iron valuable.

Cast iron contains 3 to $4\frac{1}{2}$ per cent of carbon with a little silicon. The hard and white varieties are used in the manufacture of wrought iron. The gray irons are used in the foundry.

Cast iron is made into the desired forms by melting it in a cupola and pouring into moulds. The moulds are made in sand or loam from patterns of pine wood. Patterns are made a little larger than the required casting because iron in solidifying contracts about $\frac{1}{8}$ inch per foot in each direction. This contraction is called *shrink-*

age. In making a pattern a shrinkage rule is used which is about $\frac{1}{8}$ inch longer per foot than the standard.

Castings are likely to be put into a state of internal stress because of contraction when cooling. If some parts of the casting contract more than others, the casting may become twisted. Thin parts of the castings solidify first. The contraction of the fluid parts strains the portions already set and their resistance to deformation causes stresses to be set up in the parts which are solidifying.

For example, the form shown in Fig. 14 has a rigid flange surrounding the inner part. If the contraction of the cross-piece takes place more slowly than the rim, it is likely to fracture. In a thick cylinder, as shown in section in Fig. 15, the outer portions solidify and begin the contraction. The contraction of the inner induces

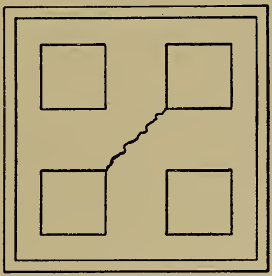


Fig. 14. Casting with Flange

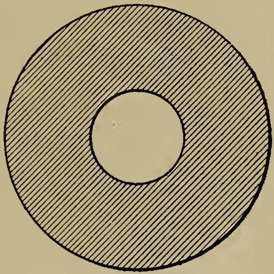


Fig. 15. Cylindrical Casting

pressure in the outer portion, which being rigid causes a resistance to contraction of the inner layers and puts them in tension. A cylinder so constructed is not strong to resist bursting pressure. If the cylinder is cast while water circulates through the core, the reverse distribution of initial strains is set up. This insures a stronger cylinder because the inner layers are in a state of compression and the outer portions are in tension.

The arms of pulleys may be broken by tension if the rim is thin and rigid. If the arms set first the rim may break near them. To have successful castings, the designer must carefully consider the dimensions of the various parts.

On account of these initial strains, that cannot be calculated, cast iron is unreliable. Cast-iron structures usually have excessive dimensions to insure safety.

Chilled castings are cooled rapidly during solidification, thus preventing the graphite from separating from the iron. This causes the iron to become harder. In order to chill the cast iron, the mould is made of, or lined with, this same material. The mould which is lined with loam for protection, is a good conductor of heat and the molten cast iron is cooled or chilled during solidification. The chilling usually extends to a depth of $\frac{1}{8}$ to $\frac{3}{8}$ of an inch from the surface, the interior remaining soft.

Malleable cast iron is made by surrounding castings with oxide of iron, powdered red hematite, or peroxide of manganese, keeping them at a high temperature for a considerable time according to the size of the casting. The elimination of carbon converts the cast iron into a crude form of wrought iron. Malleable castings will stand blows better than ordinary castings.

Cast iron is stronger than wrought iron when under pressure; but it is much weaker under tension and impact. It is such an uncertain metal on account of its variable structure that stresses are always kept low, say, from 3,000 to 4,000 for nonreversing stress, and 1,500 to 2,500 for reversing stress.

Wrought Iron. Wrought iron is made from cast iron by eliminating part of the carbon. It is strong and tough and can easily be welded. For these reasons it is used for parts of machines and structures requiring strength and of simple form. Wrought iron parts are shaped by forging and finished in the machine shop, steam hammers being used on the heavy portions.

Wrought iron is rolled into plates, round and square bars, angle, tee, channel, I-beam sections, etc. Large wrought iron structures are built up of bars or plates riveted or bolted together.

Wrought iron that has been rolled when cold has a greater tensile strength than before rolling; but its ductility and toughness is reduced. Annealing, or heating the iron to a red heat and allowing it to cool slowly, restores it to the original condition.

Compression of iron when cold increases its strength but reduces its ductility and toughness; annealing reduces strength and increases toughness and ductility. If the iron is rolled or hammered when hot, compression and annealing are carried on at the same time. Average wrought iron may be used for a load of from 8,000 to 10,000 lbs. per sq. in., nonreversing, and from 6,000 to 7,000 lbs. per sq. in., reversing.

Steel. Steel is by far the most useful material used in machines. It is not found in nature but is an alloy or mixture of iron, carbon, silicon, manganese, phosphorus, nickel, tungsten, etc. The strength, ductility, and other characteristics of steel depend upon the proportion of iron, carbon, and other materials in the mixture.

It is now successfully cast by the use of silicon, aluminum, and other elements and internal stresses are reduced by prolonged annealing. It can be welded, but greater care is necessary than in the welding of wrought iron.

Tempering greatly increases the usefulness of steel, since it becomes hard if heated and cooled suddenly. With good steel almost any desired hardness may be obtained. The steel is heated to the temperature indicated by the color of the oxide which forms at its surface and is then quenched in oil or water. Hardness makes it suitable for cutting tools. When tempered it is hard, strong, has high elastic limit and little ductility.

Alloy steels are now extensively used for special purposes. The chief alloy steels are manganese steel, tungsten steel, nickel steel, chrome steel, molybdenum steel, and vanadium steel.

Manganese steel is very ductile and possesses great hardness, but is too hard to be shaped by cutting tools. It is extensively used in rock crushing machinery.

Tungsten steel holds its temper very well under heat and retains magnetism.

Nickel steel has great hardness, strength, and ductility. It is much used for shafting.

Chrome steel when properly treated has a high elastic limit and great hardness. It is used in dies and in stamp mill machinery.

Molybdenum steel is very similar in its characteristics to tungsten steel.

Vanadium steel possesses endurance under shocks. Its strength under steady loads is about the same as that of carbon steel. At present vanadium steel is extensively used for automobiles.

With but a general knowledge of the elastic limit, ordinary steel is good for from 12,000 to 15,000 pounds per square inch non-reversing stress, and 8,000 to 10,000 reversing stress.

Copper. Copper is a reddish metal of great ductility and malleability. It is usually rolled or hammered into shape because it does not cast well. Copper can be welded, but as it requires considerable care to make a good joint, pieces are more often joined by brazing. It can be drawn into wire. The tensile strength of cast copper is about 20,000 pounds per square inch; of forged copper about 30,000 pounds per square inch.

Hammering, rolling, and wire-drawing increases the tensile strength, but makes it hard and brittle. It can be made soft and tough by annealing. It is expensive and is used for wire, fittings, and tubing. Its strength is less than that of wrought iron and decreases rapidly with rise of temperature.

Aluminum. Aluminum is a soft, ductile, malleable metal of bluish white color. It is very light; next to magnesium the lightest of the useful metals. Its strength is about one-third that of wrought iron. Aluminum casts well, the shrinkage being about the same as brass. The readiness with which aluminum unites with other metals makes it valuable for alloys. It can be electrically welded but does not solder well.

Bronze. Bronze, or gun-metal, is an alloy of copper and tin—about 90 parts copper and 10 parts tin. It makes good castings. Bronze is harder and less malleable than copper. It is used for bearings because it is softer and wears faster than wrought iron or steel shafts.

The hardness of bronze depends upon the proportion of tin; to increase hardness increase the amount of tin. An alloy of 92 parts copper and 3 parts tin is a soft bronze used for gear wheels.

Phosphor-bronze is made by mixing 2 or 3 per cent of phosphorus with ordinary bronze.

Manganese bronze, called *white bronze,* is an alloy of ordinary bronze and ferro-manganese. Like phosphor-bronze it is used in marine work, because it resists the corroding action of sea-water. Manganese bronze is equal in tensile strength and toughness to mild steel and can be easily forged.

Brass. The alloy of copper and zinc is called brass; sometimes tin and a little lead are added. For bearings it has about 60 per cent copper, 10 per cent zinc, and 30 per cent tin and lead. Naval brass has 62 per cent copper, 1 per cent tin and 37 per cent zinc. Red brass consists of about 37 per cent copper and for the rest about equal parts of tin, zinc, and lead. Brass is used for bearings, wire, fittings, and ornamental work. Its tensile strength is about 23,000 pounds per square inch.

Fusible Alloys. Fusible alloys are made of tin, lead, and bismuth. The melting point varies with the percentages of the various constituents. If made of 2 parts lead and 1 part tin, it melts at 475° F.; if 1 part lead, 1 part tin, and 4 parts bismuth, the melting point is about 200° F. An alloy of 1 part cadmium, 4 parts bismuth, 1 part tin, and 2 parts lead melts at 165° F.

Bearing Alloys. The principal constituents of bearing alloys are copper, tin, lead, zinc, antimony, and aluminum. The bronzes contain a large per cent of copper. A good bearing alloy is made of copper, 77 parts by weight, tin 3 parts, and lead 15 parts.

Babbitt metals have various proportions; hard babbitt having about 89 per cent tin, 4 per cent copper, and 7 per cent antimony.

There are many other alloys containing the metals in varying proportions according to the intended use.

Wood. Wood is but little used in machine construction. Soft woods like pine are used for patterns; hard varieties, oak and lignum-vitae for examples, are used for bearings. Sometimes levers are made of wood and the pulleys of some lathes are constructed of the same material. The cogs of mortise wheels are often made of beech or horn-beam.

With the foregoing information as a guide and the special conditions controlling each case carefully studied, reasonable limits may be assigned for working stresses of the various materials used in machines.

BOLTS, STUDS, NUTS, AND SCREWS

NOTATION—The following notation is used throughout the chapter on Bolts, Studs, Nuts, and Screws:

a = Area at root of thread (sq.in.)
d = Diameter of bolt (inches)
d_1 = Diameter at root of thread (inches)
e = Efficiency of screw and nut
H = Height of nut (inches)
I = Initial axial tension (lbs.)
k = Allowable bearing pressure on surface of thread(lbs.per sq.in.)
L = Lead, or distance nut advances along axis in one revolution (inches)
l = Length of wrench handle(inches)
n = Number of threads in nut $= \frac{H}{p}$
P = Axial load (lbs.)
p = Pitch of thread, or distance between similar points on adjacent threads, measured parallel to axis (inches)
S = Fiber stress (lbs. per sq. in.)
W = Load on bolt(lbs.)

Analysis. A *bolt* is simply a cylindrical bar of metal upset at one end to form a head, and having a thread at the other end,

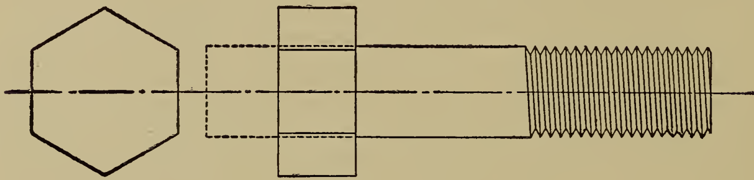


Fig. 16. Bolt

Fig. 16. A *stud* is a bolt in which the head is replaced by a thread; or it is a cylindrical bar threaded at both ends, usually having a small plain portion in the middle, Fig. 17. The object of bolts and studs is to clamp machine parts together, and yet permit these same parts to be readily disconnected. The bolt passes through the pieces to be connected, and, when tightened, causes surface compression between the parts, while the reactions on the head and nut produce

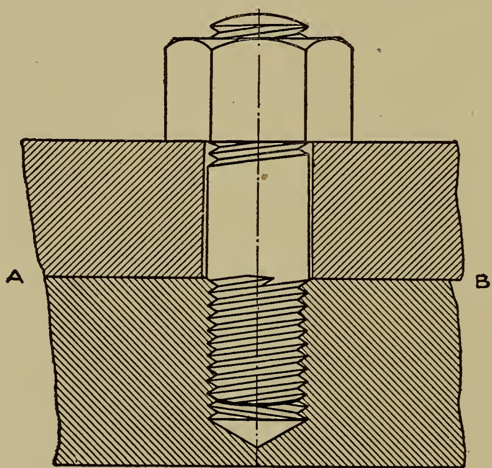


Fig. 17. Stud

tension in the bolt. Studs and tap bolts pass through one of the connected parts and are screwed into the other, the stud remaining in position when the parts are disconnected.

As all materials are elastic within certain limits, the action of a bolt in clamping two machine parts together, more especially if there is an elastic packing between them, may be represented diagrammatically by Fig. 18, in which a spring has been introduced to take the compression due to screwing up the nut. Evidently the tension in the bolt is equal to the force necessary to compress the spring. Now, suppose that two weights, each equal to $\frac{1}{2}W$, are placed symmetrically on either side of the bolt, then the tension in the bolt will be increased by the added weights if the bolt is perfectly rigid. The bolt, however, stretches; hence some of the compression on the spring is relieved and the total tension in the bolt is less than $W + I$, by an amount depending on the relative elasticity of the bolt and spring.

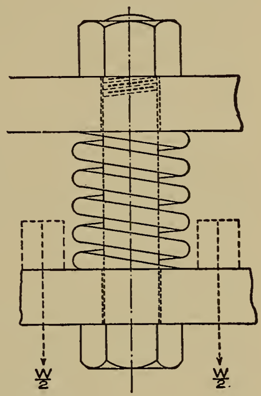


Fig. 18. Action of Elastic Packing

Suppose that the stud in Fig. 17 is one of the studs connecting the cover to the cylinder of a steam engine, and that the studs have a small initial tension; then the pressure of the steam loads each stud, and if the studs stretch enough to relieve the initial pressure between the two surfaces, then their stress is due to the steam pressure only; or, from Fig. 18, when $I = W$; the initial pressure due to the elasticity of the joint is entirely relieved by the assumed stretch of the studs. Except to prevent leakage, it is seldom necessary to consider the initial tension, for the stretch of the bolt may be counted on to relieve this force, and the working tension on the bolt is simply the load applied.

For shocks or blows, as in the case of the bolts found on the marine type of connecting-rod end, the stretch of the bolts acts like a spring to reduce the resulting tensions. So important is this feature that the body of the bolt is frequently turned down to the diameter of the bottom of the thread, thus uniformly distributing the stretch through the full length of the bolt, instead of localizing it at the threaded parts.

In tightening up a bolt, the friction at the surface of the thread produces a twisting moment, which increases the stress in the bolts, just as in the case of shafting under combined tension and torsion; but the increase is small in amount, and may readily be taken care of by permitting low values only for the fiber stress.

In a flange coupling, bolts are acted upon by forces perpendicular to the axis, and hence are under pure shearing stress. If the torque on the shaft becomes too great, failure will occur by the bolts shearing off at the joint of the coupling.

A bolt under tension communicates its load to the nut through the locking of the threads together. If the nut is thin, and the number of threads to take the load few, the threads may break or shear off at the root. With a V-thread there is produced a component force, perpendicular to the axis of the bolt, which tends to split the nut.

In screws for continuous transmission of motion and power, the thread may be compared to a rough inclined plane, on which a small block, the nut, is being pushed upward by a force parallel to the base of the plane. The angle at the bottom of the plane is the angle of the helix, or an angle whose tangent is the lead divided by the circumference of the screw. The horizontal force corresponds to the tangential force on the screw. The friction at the surface of the thread produces a twisting moment about the axis of the screw, which, combined with the axial load, subjects the screw to combined tension and torsion. Screws with square threads are generally used for this service, the sides of the thread exerting no bursting pressure on the nut. The proportions of screw thread for transmission of power depend more on the bearing pressure than on strength. If the bearing surface be too small and lubrication poor, the screw will cut and wear rapidly.

Theory. A direct tensile stress is induced in a bolt when it carries a load exerted along its axis. This load must be taken by the section of the bolt at the bottom of the thread. If the area at the root of the thread is $\frac{\pi d_1^2}{4}$, and if S is the allowable stress per square inch, then the internal resistance of the bolt is $\frac{S\pi d_1^2}{4}$. Equating the external load to the internal strength

$$W = Sa = \frac{S\pi d_1^2}{4} \qquad (12)$$

For bolts which are used to clamp two machine parts together so that they will not separate under the action of an applied load, the initial tension of the bolt must be at least equal to the applied load. If the applied load is W, then the parts are just about to separate when $I = W$. Therefore, the above relation for strength is applicable. As the initial tension to prevent separation should be a little greater than W, a value of S should be chosen so that there will be a margin of safety. For ordinary wrought iron and steel, S may be taken at 6,000 to 8,000.

If, however, the joints must be such that there is no leakage between the surfaces, as in the case of a steam cylinder head, and supposing that elastic packings are placed in the joints, then a much larger margin should be made, for the maximum load which may come on the bolt is $I + W$, where W is the proportional share of the internal pressure carried by the bolt. In such cases $S = 3{,}000$ to 5,000, using the lower value for bolts of less than $\frac{3}{4}$-inch diameter.

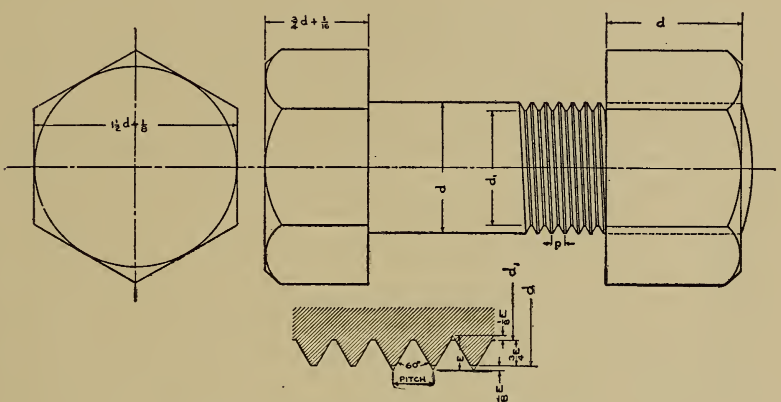


Fig. 19. Standard Bolt, Nut, and Thread

Table IV will be found very useful in proportioning bolts with U. S. standard thread for any desired fiber stress. Standard dimensions of bolt, nut, and thread are given in Fig. 19.

To find the initial tension due to screwing up the nut, the length of the handle of an ordinary wrench, measured from the center of the bolt, is assumed as about 16 times the diameter of the bolt. For one turn of the wrench a force F at the handle would pass over a distance $2\pi l$, and the work done is equal to the product of the force and space, or $F \times 2\pi l$. At the same time the axial load P would be moved a distance p along the axis. Assuming that there

TABLE IV

Strength of Bolts—*U. S. Standard Thread*

Bolt		Diameters		Areas		Approximate Tensile Strength at Bottom of Thread (In Hundreds of Pounds)					Approximate Shearing Strength Full Bolt Diameter (In Hundreds of Pounds)			
Diameter Inches	Threads per inch	Bottom of Thread	Tap Drill	Bolt Body	Bottom of Thread	At 4,000 lbs. per sq. in.	At 5,000 lbs. per sq. in.	At 6,000 lbs. per sq. in.	At 7,000 lbs. per sq. in.	At 10,000 lbs. per sq. in.	At 4,000 lbs. per sq. in.	At 5,000 lbs. per sq. in.	At 6,000 lbs. per sq. in.	At 7,000 lbs. per sq. in.
$\frac{1}{4}$	20	.18	$\frac{3}{16}$	.05	.03	1.08	1.35	1.60	1.88	2.69	1.96	2.45	2.95	3.43
$\frac{5}{16}$	18	.24	$\frac{1}{4}$	.08	.04	1.82	2.27	2.72	3.18	4.54	3.04	3.83	4.60	5.37
$\frac{3}{8}$	16	.29	$\frac{5}{16}$	.11	.07	2.71	3.39	4.07	4.75	6.78	4.40	5.52	6.62	7.73
$\frac{7}{16}$	14	.34	$\frac{23}{64}$	.15	.09	3.73	4.67	5.60	6.53	9.33	6.00	7.51	9.02	11.00
$\frac{1}{2}$	13	.40	$\frac{13}{32}$	.20	.13	5.00	6.25	7.50	8.75	12.00	7.84	9.81	12.00	14
$\frac{9}{16}$	12	.45	$\frac{15}{32}$	.25	.16	6.48	8.10	9.62	11.00	16	9.92	12.00	15	17
$\frac{5}{8}$	11	.51	$\frac{17}{32}$	.31	.20	8.04	10.00	12.00	14	20	12.00	15	18	21
$\frac{3}{4}$	10	.62	$\frac{5}{8}$	.44	.30	12.00	15	18	21	30	18	22	26	31
$\frac{7}{8}$	9	.73	$\frac{3}{4}$	.60	.42	17	24	25	29	42	24	30	36	42
1	8	.84	$\frac{27}{32}$	.78	.55	22	27	33	38	55	31	39	47	55
$1\frac{1}{8}$	7	.94	$\frac{31}{32}$	.99	.69	28	34	41	48	69	40	50	60	70
$1\frac{1}{4}$	7	1.06	$1\frac{3}{32}$	1.23	.89	31	39	47	55	78	49	61	74	86
$1\frac{3}{8}$	6	1.16	$1\frac{3}{16}$	1.48	1.05	42	53	64	72	106	59	74	89	104
$1\frac{1}{2}$	6	1.28	$1\frac{9}{32}$	1.77	1.29	51	64	77	90	128	71	88	106	124
$1\frac{5}{8}$	$5\frac{1}{2}$	1.39	$1\frac{13}{32}$	2.07	1.51	61	78	92	109	153	83	104	124	145
$1\frac{3}{4}$	5	1.49	$1\frac{1}{2}$	2.40	1.74	70	88	106	123	176	96	120	144	168
$1\frac{7}{8}$	5	1.61	$1\frac{5}{8}$	2.76	2.05	81	101	122	142	203	110	138	166	193
2	$4\frac{1}{2}$	1.71	$1\frac{3}{4}$	3.14	2.30	92	115	138	161	230	126	157	186	220
$2\frac{1}{4}$	$4\frac{1}{2}$	1.96	$1\frac{31}{32}$	3.98	3.02	125	156	187	218	312	159	199	238	278
$2\frac{1}{2}$	4	2.17	$2\frac{3}{16}$	4.91	3.72	148	185	222	259	370	196	245	294	344
$2\frac{3}{4}$	4	2.42	$2\frac{7}{16}$	5.94	4.62	184	230	276	322	460	237	297	356	416
3	$3\frac{1}{2}$	2.63	$2\frac{5}{8}$	7.07	5.43	218	272	326	381	544	283	353	424	495
$3\frac{1}{4}$	$3\frac{1}{2}$	2.88	$2\frac{29}{32}$	8.29	6.51	264	330	396	462	660	332	415	498	581
$3\frac{1}{2}$	$3\frac{1}{4}$	3.10	$3\frac{1}{32}$	9.62	7.55	302	376	452	528	754	385	481	577	673
$3\frac{3}{4}$	3	3.32	$3\frac{11}{32}$	11.04	8.64	344	430	516	602	860	442	552	663	773
4	3	3.57	$3\frac{19}{32}$	12.57	9.99	396	495	594	693	990	503	628	754	880
$4\frac{1}{4}$	$2\frac{7}{8}$	3.80	$3\frac{13}{16}$	14.19	11.33	452	565	678	791	1,130	567	709	851	993
$4\frac{1}{2}$	$2\frac{3}{4}$	4.03	$4\frac{1}{32}$	15.90	12.74	507	634	760	888	1,268	636	795	954	1,113
$4\frac{3}{4}$	$2\frac{5}{8}$	4.25	$4\frac{9}{32}$	17.72	14.22	567	709	851	1,993	1,420	709	886	1,063	1,240
5	$2\frac{1}{2}$	4.48	$4\frac{1}{2}$	19.63	15.76	630	788	946	1,103	1,676	785	982	1,178	1,374
$5\frac{1}{2}$	$2\frac{3}{8}$	4.95	$4\frac{31}{32}$	23.76	19.27	770	962	1,154	1,347	1,924	950	1,188	1,425	1,663
6	$2\frac{1}{4}$	5.42	$5\frac{7}{16}$	28.27	23.09	923	1,159	1,384	1,617	2,307	1,128	1,414	1,696	1,979

is no friction, the equation for the equality of the work at the handle and at the screw is

$$F2\pi l = Pp \tag{13}$$

Friction, however, is always present; hence the ratio of the useful work (Pp) to the work applied ($F2\pi l$) is not unity as above relations assume. From numerous experiments on the friction of screws and nuts, it has been found that the efficiency may be as low as 10 per cent. Introducing the efficiency in equation 13, it may be written

$$\frac{Pp}{F2\pi l} = e \tag{14}$$

Assuming that 50 pounds is exerted by a workman in tightening up the nut on a 1-inch bolt, the equation above shows that $P = 4{,}021$

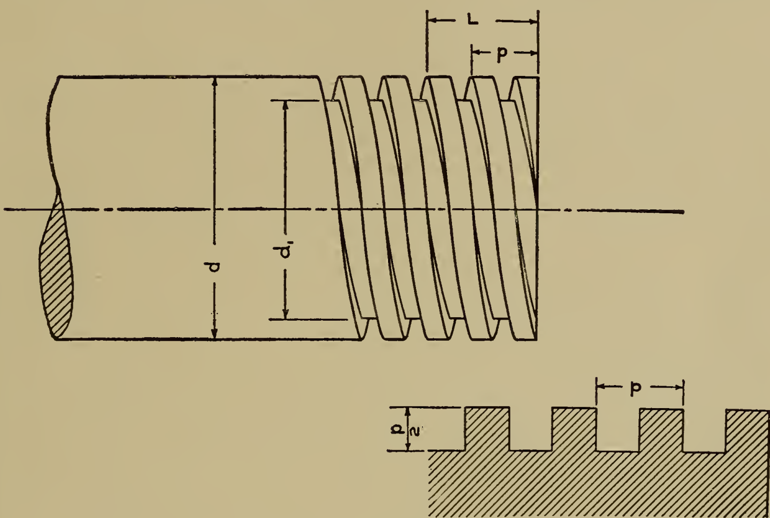


Fig. 20. Square Thread

pounds; or the initial tension is somewhat less than the tabular safe load shown for a 1-inch bolt, with S assumed at 10,000 pounds per square inch.

For shearing stresses the bolt should be fitted so that the body of the bolt, not the threads, resists the force tending to shear off the bolt perpendicular to its axis. The internal strength of the bolt to resist shear is the allowable stress S times the area of the bolt in shear, or $\frac{S\pi d^2}{4}$. If W represents the external force tending to shear the bolt the equality of the external force to the internal strength is

$$W = \frac{S\pi d^2}{4} \qquad (15)$$

Reference to Table IV on Page 50 for the shearing strength of bolts, may be made to save the labor of calculations.

Let Fig. 20 represent a square thread screw for the transmission of motion. The surface on which the axial pressure bears, if n is the number of threads in the cut, is $\frac{\pi}{4}(d^2 - d_1{}^2)n$. Suppose that a pressure of k pounds per square inch is allowed on the surface of the thread. Then the greatest permissible axial load P must not exceed the allowable pressure; or, equating,

$$P = k\frac{\pi}{4}(d^2 - d_1{}^2)\,n \qquad (16)$$

The value of k varies with the service required. If the motion be slow and the lubrication very good, k may be as high as 900. For rapid motion and doubtful lubrication, k may not be over 200. Between these two extremes the designer must use his judgment, remembering that the higher the speed the lower is the allowable bearing pressure.

Practical Modification. It will be noted in the formulas for bolt strengths that different values for S are assumed. This is necessary on account of the uncertain initial stresses which are produced in setting up the nuts. For cases of mere fastening, the safe tension is high, as just before the joint opens the tension is about equal to the load and yet the fastening is secure. On the other hand, bolts or studs fastening joints subjected to internal fluid pressure must be stressed initially to a greater amount than the working pressure which is to come on the bolt. As this initial stress is a matter of judgment on the part of the workman, the designer, in order to be on the safe side, should specify not less than $\frac{5}{8}$-inch or $\frac{3}{4}$-inch bolts for ordinary work, so that the bolts may not be broken off by a careless workman accidentally putting a greater force than necessary on the wrench handle. In making a steam-tight joint, the spacing of the bolts will generally determine their number; hence often an excess of bolt strength is found in joints of this character.

Through bolts are preferred to studs, and studs to tap bolts or cap screws. If possible, the design should be such that through bolts may be used. They are cheapest, are always in standard stock, and will resist rough usage in connecting and disconnecting. The threads in cast iron are weak and have a tendency to crumble; and if a through bolt cannot be used in such a case, a stud, which can be placed in position once for all, should be employed—not a tap bolt, which injures the thread in the casting every time it is removed.

The plain portion of a stud should be screwed up tight against the shoulder, and the tapped hole should be deep enough to prevent bottoming. To avoid breaking off the stud at the shoulder, a groove may be made at the lower end of the thread entering the nut.

To withstand shearing forces the bolts must be fitted so that no lost motion may occur, otherwise pure shearing will not be secured.

Nuts are generally made hexagonal, but for rough work are often made square. The hexagonal nut allows the wrench to turn through a smaller angle in tightening up, and is preferred to the square nut. Experiments and calculations show that the height of the nut with standard threads may be about $\frac{1}{2}$ the diameter of the bolt and still have the shearing strength of the thread equal to the tensile strength of the bolt at the root of the thread. Practically, however, it is difficult to apply such a thin wrench as this proportion would call for on ordinary bolts. More commonly the height of the nut is made equal to the diameter of the bolt so that the length of thread will guide the nut on the bolt, give a low bearing pressure on the threads, and enable a suitable wrench to be easily applied. The standard proportions for bolts and nuts may be found in any handbook. Not all manufacturers conform to the United States standard; nor do manufacturers in all cases conform to one another in practice.

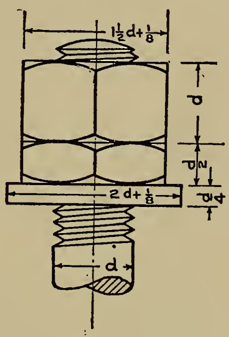


Fig. 21. Lock Nut

If the bolt is subject to vibration, the nuts have a tendency to loosen. A common method of preventing this is to use double nuts, or *lock nuts*, as they are called, Fig. 21. The under nut is screwed tightly against the surface, and held by a wrench while the second nut is screwed down tightly against the first. The effect is to cause the threads of the upper nut to bear against the under sides of the threads of the bolt. The load on the bolt is sustained,

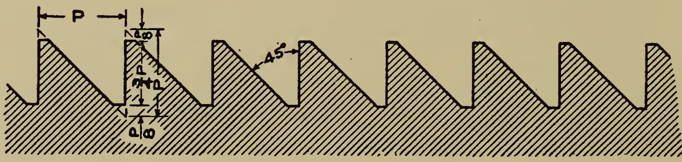


Fig. 22. Buttress Thread

therefore, by the upper nut, which should be the thicker of the two; but for convenience in applying wrenches the position of the nuts is often reversed.

The form of thread adapted to transmitting power is the square

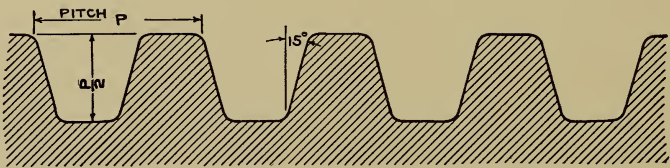


Fig. 23. Modification of Square Thread

thread, which, although giving less bursting pressure on the nut, is not as strong as the V-thread for a given length, since the total sec-

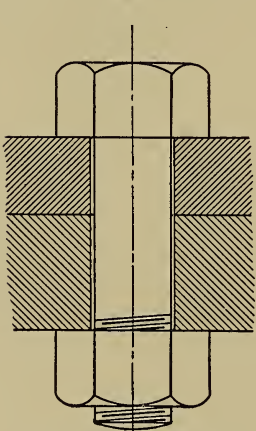


Fig. 24. Through Bolt

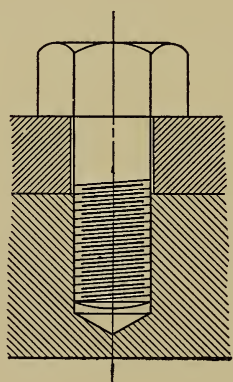


Fig. 25. Tap Bolt

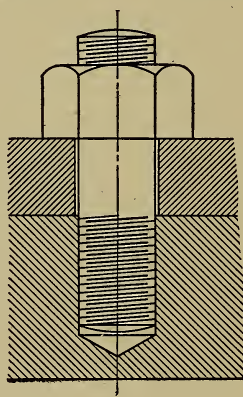


Fig. 26. Stud

tion of thread at the bottom is only $\frac{1}{2}$ as great. If the pressure is to be transmitted in but one direction, the two types may be combined advantageously to form the buttress thread of the proportions shown in Fig. 22. Often, as in the carriage of a lathe, to allow the split nut to be opened and closed over the lead screw, the sides of the

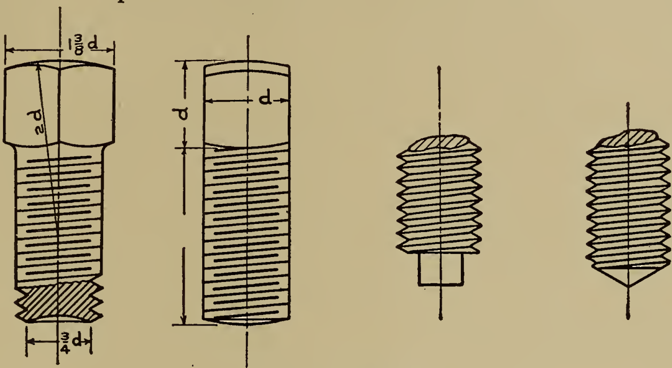


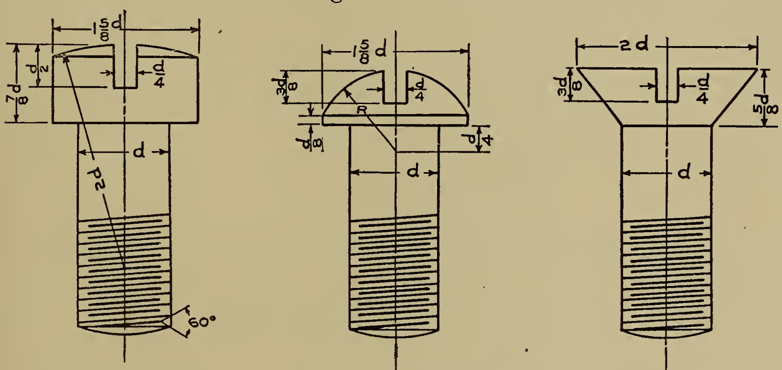
Fig. 27. Set Screws

thread are placed at a small angle, say 15°, to each other, as illustrated in Fig. 23.

The practical commercial forms of fastenings are usually included in five classes, as follows:

1. *Through bolts*, Fig. 24, usually rough stock, with square upset heads, and square or hexagonal nuts.

2. *Tap bolts*, Fig. 25, also called *cap screws*. These usually have hexagonal heads, and are found both in the rough form, and finished from the rolled hexagonal bar in the screw machine.



Fillister Head | Round Head | Flat Head

Fig. 28. Machine Screws

3. *Studs*, Fig. 26, rough or finished stock threaded in the screw machine.

4. *Set screws*, Fig. 27, usually with square heads and case-hardened points. Many varieties of set screws are made, the principal distinguishing feature of each being in the shape of the point. Thus, in addition to the plain beveled point, there are the "cupped," rounded, conical, and "teat" points.

5. *Machine screws*, Fig. 28, usually fillister, round, or flat head. Common proportions are indicated relative to diameter of body of screw.

Examples. 1. What is the working stress on a bolt $1\frac{1}{4}$ inches in diameter if the load on the bolt is 5,500 lbs.?

Solution. Referring to Table IV, it is found that a $1\frac{1}{4}$ in. bolt has an area at the bottom of the thread of .89 sq. in.

$$W = Sa \qquad S = \frac{W}{a}$$

$$S = \frac{5500}{.89} = 6{,}180 \text{ lbs., approximately}$$

2. A wrench 15 inches long is used to screw up a nut on a 1-inch bolt. If the efficiency is 12 per cent, what axial load is exerted when a force of 25 pounds is applied at one end of the wrench handle?

Solution. Using equation 14

$$\frac{Pp}{F2\pi l} = .12$$

or

$$P = \frac{.12F2\pi l}{p}$$

From Table IV, it is found that a 1-inch bolt has 8 threads per inch; then

$$p = \frac{1}{8} = .125$$

$$P = .12 \times \frac{25 \times 2 \times \pi \times 15}{.125} = 2262.0 \text{ lbs. nearly}$$

PROBLEMS FOR PRACTICE

1. Calculate the diameter of a bolt to sustain a load of 5,000 lbs.

2. The shearing force to be resisted by each of the bolts of a flange coupling is 1,200 lbs. What commercial size of bolt is required?

3. With a wrench 16 times the diameter of the bolt, and an

efficiency of 10 per cent, what axial load can a man exert on a standard $\frac{3}{4}$-inch bolt, if he pulls 40 lbs. at the end of the wrench handle?

4. A single, square-threaded screw of diameter 2 inches, lead, $\frac{1}{4}$ inch, depth of thread $\frac{1}{8}$ inch, length of nut 3 inches, is to be allowed a bearing pressure of 300 lbs. per square inch. What axial load can be carried?

5. Calculate the shearing stress at the root of the thread in Problem 4.

RIVETS AND RIVETED JOINTS

NOTATION—The following notation is used throughout the chapter on Rivets and Riveted Joints:

a = Net section of plate (square inches)
d = Diameter of rivet (inches)
n_1 = Number of rivets in a row
n_2 = Total number of rivets in a joint
n_3 = Total number of rivets in a lap joint and one-half the number of rivets in a butt joint
P_c = Strength of joint computed from bearing value of plate (lbs.)
P_s = Strength of joint computed from shearing volume (lbs.)
P_t = Strength of joint computed from tensile strength value (lbs.)
S_c = Compressive working strength of plate (lbs. per sq. in.)
S_s = Shearing working strength of the plate (lbs. per sq. in.)
S_t = Tensile working strength of the plate (lbs. per sq. in.)
t = Thickness of plate (inches)
w = Width of plate (inches)

Analysis. A rivet is a short bar of malleable metal with a head at each end, thus forming a bolt having its body head and nut in one piece. Rivets are used to fasten together parts where the straining force is parallel to the surfaces, as plates, beams, and girders. From this it is evident that rivets are placed in shear. The parts fastened together may fail due to the material crushing in front of the rivet holes, tearing between them, the rivets shearing, or a combination of the above.

When the joint is to be steam-, air-, or gas-tight, the joint must be tight as well as strong. This requires the rivets to be close together and near the edge in order to prevent opening of the edges. For purely structural purposes the joint is designed for strength.

Kinds of Joints. A *lap joint* is one in which the plates or bars joined overlap each other, Fig. 29. A *butt joint* is one in which the plates or bars that are joined butt against each other, Fig. 30. The thin side plates on butt joints are called *cover plates;* the plates or bars that are joined are called *main plates;* and the distance between the centers of consecutive holes in a row of rivets is called *pitch.*

Theory. When a lap joint is subjected to tension (*i. e.*, when P, Fig. 29, is a pull), and when it is subjected to compression (when P is a push), there is a tendency to cut or shear each rivet along the surface between the two plates. In butt joints with two cover plates, there is a tendency to cut or shear each rivet on two surfaces, Fig. 30. Therefore, the rivets in the lap joint are said to be in *single shear;* and those in the butt joint (two covers) are said to be in *double shear.*

The *shearing value* of a rivet means the resistance which it can safely offer to forces tending to shear it on its cross-section. This

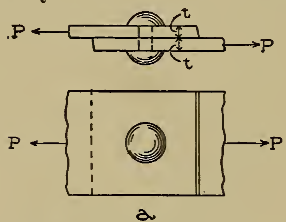


Fig. 29. Lap Joint

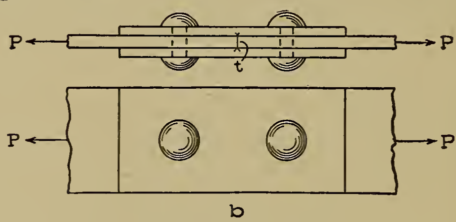


Fig. 30. Butt Joint

value depends on the area of the cross-section and on the working strength of the material. Then, since the area of the cross-section equals $0.7854\, d^2$, the shearing strength s of one rivet is

for single shear $\quad s = 0.7854\, d^2 S_s \quad$ (17)

for double shear $\quad s = 1.5708\, d^2 S_s \quad$ (18)

When a joint is subjected to tension or compression, each rivet presses against a part of the sides of the holes through which it passes. By *bearing value* of a plate, in this connection, is meant the pressure exerted by a rivet against the side of a hole in the plate, which the plate can safely stand. This value depends on the thickness of the plate, on the diameter of the rivet, and on the compressive working strength of the plate. Exactly how it depends on these three qualities is not known, but the bearing value is always computed from the expression tdS_c.

The holes punched or drilled in a plate or bar weaken its tensile strength, and to compute that strength it is necessary to allow for the holes. By *net section*, in this connection, is meant the smallest cross-section of the plate or bar; this is always a section along a line of rivet holes. Then the net section is

$$a = (w - n_1 d)t \tag{19}$$

The strength of the unriveted plate is wtS_t, and the equation for the reduced tensile strength is

$$P_t = (w - n_1 d)tS_t$$

The compressive strength of a plate is also lessened by the presence of holes; but when they are again filled up, as in a joint, the metal is replaced, as it were, and the compressive strength of the plate is restored. No allowance, therefore, is made for holes in figuring the compressive strength of a plate.

The strength of a joint is determined by either (1) the shearing value of the rivets; (2) the bearing value of the plate; or (3) the tensile strength of the riveted plate if the joint is in tension. Then, as before explained,

$$P_t = (w - n_1 d)tS_t \tag{20}$$

$$P_s = n_2\, 0.7854\, d^2 S_s \tag{21}$$

$$P_c = n_3\, tdS_c \tag{22}$$

Efficiency of a Joint. The ratio of the strength of a joint to that of the solid plate is called the *efficiency of the joint.* If ultimate strengths are used in computing the ratio, then the efficiency is called *ultimate efficiency;* and if working strengths are used, then it is called *working efficiency.* In the following, the values refer to working efficiency. An efficiency is sometimes expressed as a per cent. To express it thus, multiply the ratio, $\frac{\text{strength of joint}}{\text{strength of solid plate}}$, by 100.

Practical Modification. In a butt joint the cover plates are made not less than one-half the thickness of the main plates. Sometimes butt joints are made with only one cover plate; in such a case the thickness of the cover plate is never less than that of the main plate.

When wide bars or plates are riveted together, the rivets are placed in rows, always parallel to the "seam" and sometimes also perpendicular to the seam; but when a row of rivets is used in this discussion, a row parallel to the seam is meant. A lap joint with a single row of rivets is said to be *single-riveted;* and one with two rows of rivets, *double-riveted.* A butt joint with two rows of rivets (one on each side of the joint) is called *single-riveted,* and one with four rows (two on each side), *double-riveted.*

When a joint is subjected to tension or compression, there is a tendency to slippage between the faces of the plates of the joint. This tendency is overcome wholly or in part by frictional resistance between the plates. The frictional resistance in a well-made joint may be very large, for rivets are put into a joint hot, and are headed or capped before being cooled. In cooling they contract, drawing the plates of the joint tightly against each other, and producing a great pressure between them, which gives the joint a correspondingly large frictional strength. It is the opinion of some that all well-made joints perform their service by means of their frictional strength; that is to say, the rivets act only by pressing the plates together and are not under shearing stress, nor are the plates under compression at the sides of their holes. The *frictional strength* of a joint, how-

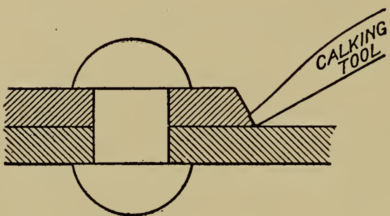


Fig. 31. Calking Iron

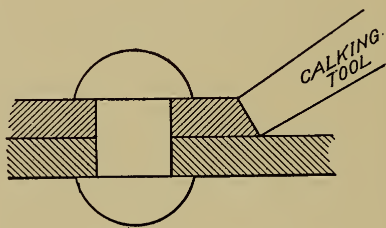


Fig. 32. Fullering Iron

ever, is usually regarded as uncertain, and generally no allowance is made for friction in computations on the strength of riveted joints.

To make sure of a good tight joint, it is closed up by burring down the edges of the plate. This is called *calking.* The calking tool, Fig. 31, resembles a chisel, except that the point is flat. The tool is forced into the plate by hand hammering and must be skillfully done in order to prevent injury to the plates. Sometimes a calking tool has a thickness equal to that of the plate, Fig. 32.

Examples. 1. Two half-inch plates, $7\frac{1}{2}$ inches wide, are connected by a single lap joint double-riveted, six rivets in two rows. If the diameter of the rivets is $\frac{3}{4}$ inch, and the working strengths are $S_t = 12{,}000$, $S_s = 7{,}500$, and $S_c = 15{,}000$ pounds per square inch, what is the safe tension which the joint can transmit?

Solution.

$$n_1 = 3,\ n_2 = 6, \text{ and } n_3 = 6$$

$$P_t = [7\tfrac{1}{2} - (3 \times \tfrac{3}{4})] \times \tfrac{1}{2} \times 12{,}000 = 31{,}500 \text{ pounds}$$
$$P_s = 6 \times 0.7854 \times (\tfrac{3}{4})^2 \times 7{,}500 = 19{,}880 \text{ pounds}$$
$$P_c = 6 \times \tfrac{1}{2} \times \tfrac{3}{4} \times 15{,}000 = 33{,}750 \text{ pounds}$$

Since P_s is the least of these three values, it determines the strength of the joint, viz, 19,880 pounds.

2. Suppose that the plates described in the preceding example are joined by means of a butt joint (two cover plates), and 12 rivets are used, being spaced as before. What is the safe tension which the joint can bear?

Solution. Here $n_1 = 3$, $n_2 = 12$, and $n_3 = 6$; hence, as in the preceding example,

$$P_t = 31{,}500; \text{ and } P_c = 33{,}750 \text{ pounds; but}$$
$$P_s = 12 \times 0.7854 \times (\tfrac{3}{4})^2 \times 7{,}500 = 39{,}760 \text{ pounds}$$

The strength equals 31,500 pounds, and the joint is stronger than the first.

3. Suppose that in the preceding example the rivets are arranged in rows of two. What is the tensile strength of the joint?

Solution. Here $n_1 = 2$, $n_2 = 12$, and $n_3 = 6$; hence, as in the preceding example,

$$P_s = 39{,}760; \text{ and } P_c = 33{,}750 \text{ pounds; but}$$
$$P_t = [7\tfrac{1}{2} - (2 \times \tfrac{3}{4})] \times \tfrac{1}{2} \times 12{,}000 = 36{,}000 \text{ pounds}$$

The strength equals 33,750 pounds, and this joint is stronger than either of the first two.

PROBLEMS FOR PRACTICE

$S_t = 12{,}000$, $S_s = 7{,}500$, and $S_c = 15{,}000$ pounds per square inch.

1. Two half-inch plates, 5 inches wide, are connected by a lap joint, with two $\tfrac{3}{4}$-inch rivets in a row. What is the safe strength of the joint?

2. Solve the preceding example supposing that four $\tfrac{3}{4}$-inch rivets are used, in two rows.

3. Solve Example 1 assuming three 1-inch rivets placed in a row lengthwise of the joint.

4. Two half-inch plates, 5 inches wide, are connected by a butt joint (two cover plates), and four $\tfrac{3}{4}$-inch rivets are used, in two rows. What is the strength of the joint?

5. It is required to compute the efficiencies of the joints described in the illustrative examples.

In each case the plate is $\frac{1}{2}$ inch thick and $7\frac{1}{2}$ inches wide; hence the tensile working strength of the solid plate is

$$7\tfrac{1}{2} \times \tfrac{1}{2} \times 12{,}000 = 45{,}000 \text{ pounds}$$

KEYS, PINS, AND COTTERS

NOTATION—The following notation is used throughout the chapter on Keys, Pins, and Cotters:

D = Average diameter of rod (inches)
D_1 = Outside diameter of socket (inches)
d = Diameter of shaft (inches)
L = Length of key (inches)
P = Driving force (lbs.)
P_1 = Axial load on rod (lbs.)
R = Radius at which P acts (inches)
S = Safe crushing fiber stress (lbs. per sq. in.)
S_s = Safe shearing fiber stress (lbs. per sq. in.)
S_t = Safe tensile fiber stress (lbs. per sq. in.)
T = Thickness of key or cotter (inches)
W = Width of key (inches)
w = Average width of cotter (inches)
w_1 = End of slot to end of rod (inches)
w_2 = End of slot to end of socket (inches)

KEYS AND PINS

Analysis. Keys and pins are used to prevent relative rotary motion between machine parts intended to act together as one piece. If a hole is drilled completely through a hub and across the shaft, and a tightly fitted pin is inserted, any rotary motion of the one will be transmitted to the other, provided the pin does not fail by shearing off at the joint between the shaft and the hub. The shearing area is the sum of the cross-sections of the pin at the joint.

A hole may be drilled in the joint, the axis of the hole being parallel to the axis of the shaft, and a pin may be driven in introducing a shearing area as before, but the area is now equal to the diameter of the pin multiplied by its length, and the pin is stressed sidewise, instead of across. It is evident in the sidewise case that the shearing area may be increased to anything desired without changing the diameter of the pin, merely by increasing the length of the pin.

As there are some manufacturing reasons why a round pin placed lengthwise in the joint is not always applicable, a rectangular pin may be used, in which case it is called a *key*.

When pins are driven across the shaft, as in the first instance, they are usually made taper. This is because it is easier to ream a taper hole to size than a straight hole, and a taper pin will drive more easily than a straight pin, it not being necessary to match the hole in hub and shaft so exactly in order that the pin may enter. The taper pin will draw the holes into line as it is driven, and can be backed out readily in removal.

Keys of the rectangular form are either straight or tapered, but for different reasons from those just stated for pins. Straight keys have working bearing only at the sides, driving purely by shear, crushing being exerted by the side of the key in both shaft and hub, over the area against the key. The key itself does not prevent end motion along the shaft; and if end motion is not desired, auxiliary means of some sort must be resorted to, as, for example, set screws through the hub jamming hard against the top of the key.

If end motion along the shaft is desired, the key is called a *spline*, and, while not jammed against the shaft, is yet prevented from

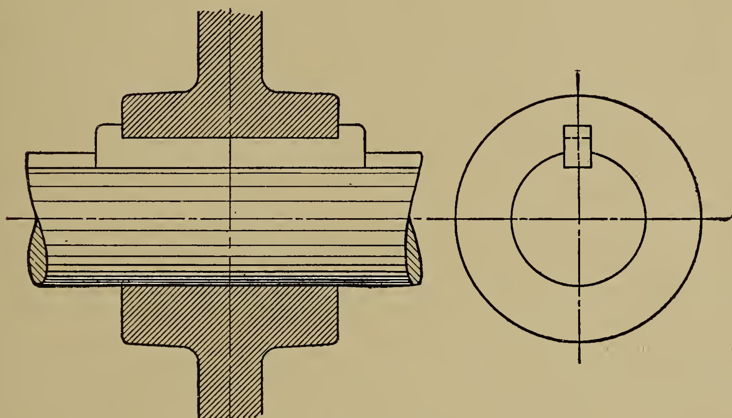


Fig. 33. Spline

changing its relation to the hub by some means such as illustrated in Fig. 33.

Taper keys not only drive through sidewise shearing strength, but prevent endwise motion by the wedging action exerted between the shaft and hub. These keys drive more like a strut from corner to corner; but this action is incidental rather than intentional, and the proportions of a taper key should be such that it will give its full resisting area in shearing and crushing, the same as a straight key.

Theory. Suppose that the pin illustrated in Fig. 34 passes through hub and shaft, and the driving force P acts at the radius

R; then the force which is exerted at the surface of the shaft to shear off the pin at the points A and B is $\frac{2PR}{d}$. If D_1 is the average diameter of the pin, its shearing strength is $\frac{2\pi D_1^2 S_s}{4}$. Equating the external force to the internal strength

$$\frac{2PR}{d} = \frac{2\pi D_1^2 S_s}{4}$$

or

$$D_1 = \sqrt{\frac{4PR}{\pi d S_s}} \tag{23}$$

In Fig. 35 a rectangular key is sunk half way in hub and shaft according to usual practice. Here the force at the surface of the

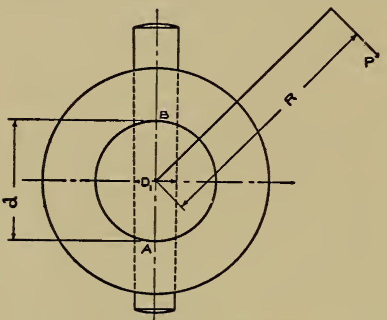


Fig. 34. Pin Driven into Fastening

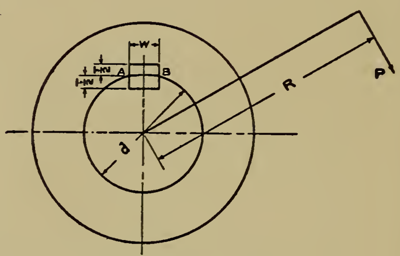


Fig. 35. Sunk Key

shaft, calculated the same as before, not only tends to shear off the key along the line AB, but tends to crush both the portion in the shaft and in the hub. The shearing strength along the line AB is LWS_s. Equating external force to internal strength

$$\frac{2PR}{d} = LWS_s$$

or

$$W = \frac{2PR}{dLS_s} \tag{24}$$

The crushing strength is, of course, that due to the weaker metal, whether in shaft or hub. Let S_c be this least safe crushing fiber stress. The crushing strength then is $\frac{LT}{2} S_c$, and, equating external force to internal strength,

$$\frac{2PR}{d} = \frac{LT}{2} S_c$$

or

$$T = \frac{4PR}{dLS_c} \qquad (25)$$

The proportions of the key must be such that the equations as above, both for shearing and for crushing, shall be satisfied.

Practical Modification. Pins across the shaft can be used to drive light work only, for the shearing area cannot be very large. A large pin cuts away too much area of the shaft, decreasing the latter's strength. Pins are useful in preventing end motion, but in this case are expected to take no shear, and may be of small diameter. The common split pin is especially adapted to this service, and is a standard commercial article.

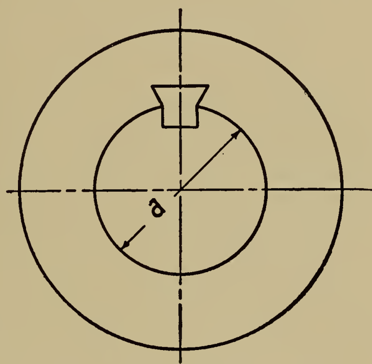


Fig. 36. Gibbed Key

Taper pins are usually listed according to the Morse standard taper, proportions of which may be found in any handbook. It is desirable to use *standard* taper pins in machine construction, as the reamers are a commercial article of accepted value, and readily obtainable in the machine-tool market.

With properly fitted keys, the shearing strength is usually the controlling element. For shafts of ordinary size, the standard proportions as given in Tables V and VI are safe enough without calculation, up to the limit of torsional strength of the shaft. For special cases of short hubs or heavy loads, a calculation is needed to check the size, and perhaps modify it.

Splines, also known as *feather keys,* require thickness greater than regular keys, on account of the sliding at the sides. Suggested proportions for splines are given in Table VI.

Though the spline may be either in the shaft or hub, it is generally *dovetailed,* Fig. 36, *gibbed,* or otherwise fastened in the hub, and a long spline way made in the shaft, in which it slides.

The straight key, accurately fitted, is the most desirable fastening device for accurate machines, such as machine tools, on account of the fact that there is absolutely no radial force exerted to throw the parts out of true. It, however, requires a tight fit of hub to shaft, as the key cannot be relied upon to take up any looseness.

The taper key, Fig. 37, by its wedging action, will take up some looseness, but in so doing throws the parts out slightly. Or,

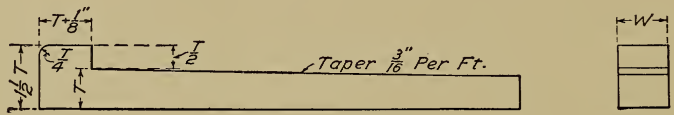


Fig. 37. Taper Key

even if the bored fit is good, if the taper key is not driven home with care, it will spring the hub, and make the parts run untrue. The great advantage, however, that the taper key has of holding the hub from endwise motion, renders it a very useful and practical article. It is usually provided with a head, or *gib,* which permits a draw hook to be used to wedge between the face of the hub and the key to facilitate starting the key from its seat.

Two keys at 90° from each other may be used in cases where one key will not suffice. The fine workmanship involved in spacing these keys so that they will drive equally makes this plan inadvisable except in case of positive and unavoidable necessity.

The "Woodruff" key, Fig. 38, is a useful patented article for

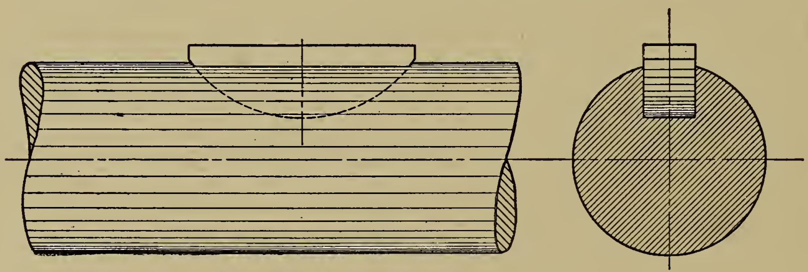


Fig. 38. Woodruff Key

certain locations. This key is a half-disk sunk in the shaft and the hub is slipped over it. A simple rotary cutter is dropped into the shaft to produce the key seat; because of the depth in the shaft, the tendency to rock sidewise is eliminated, and the drive is by shear.

TABLE V

Proportions for Gib Keys

Diameter of shaft (d) inches	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{5}{8}$	2	$2\frac{1}{2}$	$3\frac{1}{4}$	4	5	$6\frac{1}{2}$
Width (W) inches	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{11}{16}$	$\frac{7}{8}$	$1\frac{1}{16}$	$1\frac{5}{16}$	$1\frac{5}{8}$
Thickness (T) inches	$\frac{1}{4}$	$\frac{9}{32}$	$\frac{5}{16}$	$\frac{13}{32}$	$\frac{7}{16}$	$\frac{17}{32}$	$\frac{21}{32}$	$\frac{13}{16}$	1	$1\frac{1}{4}$

Keys may be milled out of solid stock, or drop-forged to within a small fraction of finished size. The drop-forged key is an excellent modern production and requires but a minimum amount of fitting. Any key, no matter how produced, requires some hand fitting and draw filing to bring it properly to its seat and give it full bearing.

It is good mechanical policy to avoid keyed fastenings whenever possible. This does not mean that keys may never be used, but that a key is not an ideal way to produce an absolutely positive drive, partly because it is an expensive device, and partly because the tendency of any key is to work itself loose, even if carefully fitted.

Tables V and VI are suggested as a guide to proportions of gib keys and feather keys, and will be found useful in the absence of any manufacturer's standard list.

TABLE VI

Proportions for Feather Keys

Diameter of shaft (d) inches	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$
Width (W) inches	$\frac{3}{16}$	$\frac{7}{32}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{5}{8}$
Thickness (T) inches	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{7}{8}$

COTTERS

Analysis. Cotters are used to fasten hubs to rods rather than to shafts, the distinction between a rod and a shaft being that a rod takes its load in the direction of its length and does not drive by rotation. A cotter, therefore, is nothing but a cross-pin of modified form, to take shearing and crushing stress in the direction of the axis of the rod, instead of perpendicular to it.

Referring to Fig. 39, one will see that the cotter is made long and thin—long, in order to get sufficient shearing area to resist shearing along lines A and B; thin, in order to cut as little cross-sectional area out of the body of the shaft as possible. The cotter itself tends to shear along the lines A and B, and crush along the surfaces K, G, and J. The socket tends to crush along the surfaces K and G. The rod end D tends to be sheared out along the lines $C\ H$ and $Q\ E$, and also to be crushed along the surface J. The socket tends to be sheared along the lines $V\ U$ and $X\ Y$.

The cotter is made taper on one side, thus enabling it to draw up the flange of the rod tightly against the head of the socket.

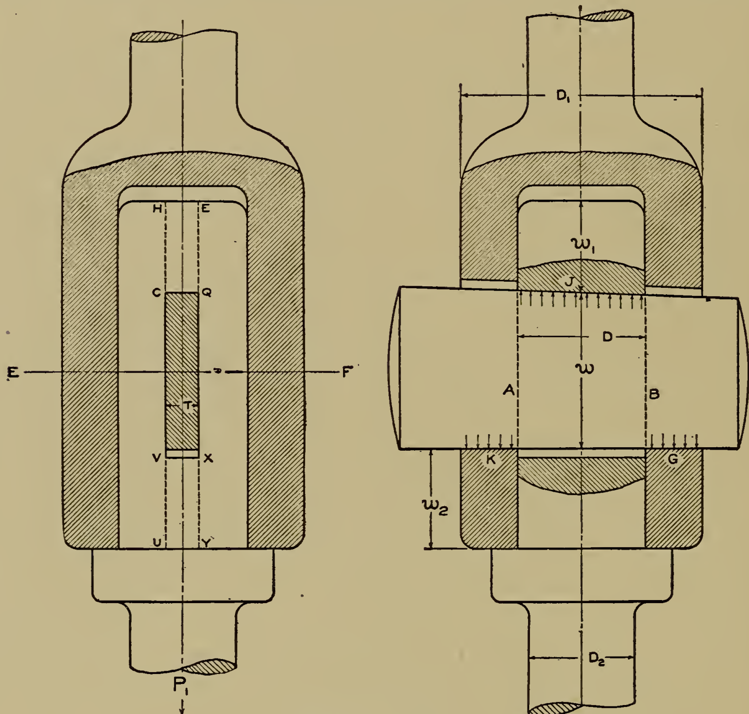


Fig. 39. Cotter

This taper must not be great enough to permit easy *backing out* and loosening of the cotter under load or vibration in the rod. In responsible situations this loosening cannot be safely guarded against except through some auxiliary locking device, such as lock nuts on the end of the cotter, Fig. 40.

Theory. Referring to Fig. 39, assume an axial load of P_1, as shown. The successive equations of external force to internal strength are enumerated below, for the different actions that take place.

For shearing along lines A and B

$$P_1 = 2TwS_s \tag{26}$$

in which w is the average width of cotter, and S_s its safe shearing stress.

For crushing along surfaces K and G

$$P_1 = T(D_1 - D)S_c \tag{27}$$

in which S_c is least safe crushing stress, whether of cotter or socket.

For crushing along surface J

$$P_1 = DTS_c \tag{28}$$

in which S_c is least safe crushing stress, whether of cotter or socket.

For shearing along surfaces CH and QE

$$P_1 = 2w_1 DS_s \tag{29}$$

in which S_s is the safe shearing stress of rod end, and w_1 end of slot to end of rod.

For tension in rod end at section across slot

$$P_1 = \left(\frac{\pi D^2}{4} - TD\right)S_t \tag{30}$$

in which S_t is safe tensile stress in rod end.

For tension in socket at section across slot

$$P_1 = \left[\frac{\pi D_1^2}{4} - \frac{\pi D^2}{4} - T(D_1 - D)\right]S_t \tag{31}$$

in which S_t is safe tensile stress in socket.

For shearing in socket along the lines VU and XY

$$P_1 = 2w_2(D_1 - D)S_s \tag{32}$$

in which S_s is safe shearing stress in the socket, and w_2 end of slot to end of socket.

The proportions of cotter and socket may be fixed to some extent by practical or assumed conditions. The dimensions may then be tested by the above equations, that the safe working stresses may not be exceeded, the dimensions being then modified accordingly.

The steel of which both cotter and rod would ordinarily be made has range of working fiber stress as follows:

Tension, 8,000 to 12,000 (lbs. per sq. in.).
Compression, 10,000 to 16,000 (lbs. per sq. in.).
Shear, 6,000 to 10,000 (lbs. per sq. in.).

The socket, if made of cast iron, will be weak as regards tension, tendency to shear out at the end, and tendency to split. The uncertainty of cast iron to resist these is so great that the hub or socket must be very clumsy in order to have enough surplus strength. This is always a noticeable feature of the cotter type of fastening, and cannot well be avoided.

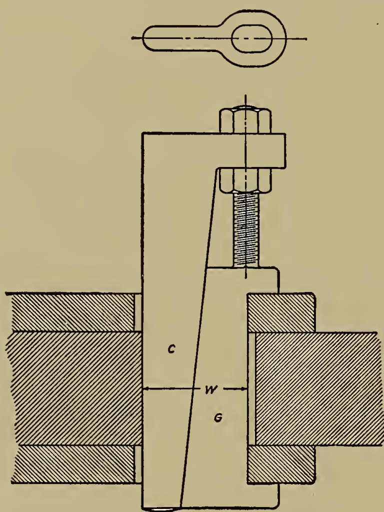


Fig. 40. Cotter with Locking Device

Practical Modification. The driving faces of the cotter are often made semicircular. This not only gives more shearing area at the sides of the slots, but makes the production of the slots easier in the shop. It also avoids the general objection to sharp corners —namely, a tendency to start cracks.

A practicable taper for cotters is $\frac{1}{2}$ inch per foot. This will, under ordinary circumstances, prevent the cotter from backing out under the action of the load. When set screws or lock nuts are used, as in Fig. 40, the taper may be greater than this, perhaps as much as $1\frac{1}{2}$ inches per foot.

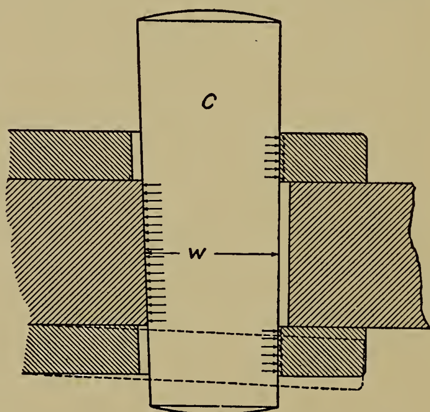


Fig. 41. Cotter at End of Connecting Rod

In the common use of the cotter for holding the strap at the ends of connecting rods, the strap acts like a modified form of socket. This is shown in Figs. 40 and 41. Here, in addition to holding the strap and rod together lengthwise, it may be necessary to prevent their spreading, and for this purpose an auxiliary piece G with gib ends is used. The tendency without this extra piece is shown by the dotted lines in Fig. 41.

The general mechanical fault with cottered joints is that the action of the load, especially when it constantly reverses, as in pump piston rods, always tends to work the cotter loose. Vibration also tends to produce the same effect. Once this looseness is started in the joint, the cotter loses its pure crushing and shearing action, and begins to partake of the nature of a hammer, and pounds itself and its bearing surfaces out of their true shape. Instead of a collar on the rod, a taper fit of the rod in the socket is often found; and any looseness in this case is still worse, for the rod then has end play in the socket, and by its *shucking* back and forth tends to split open the socket.

The only answer to these objections is to provide a positive locking device, and take up any looseness the instant it appears.

Examples. 1. A gear 60 inches in diameter has a load of 3,000 pounds at the pitch line. The shaft is 4 inches in diameter in a hub 5 inches long; and the key is a standard gib key as given in the table. Test its proportions for shearing.

Solution. $W = \dfrac{2PR}{dLS_s} \qquad W = 1\dfrac{1}{16} \qquad T = \dfrac{13}{16}$

$$1\frac{1}{16} = \frac{2 \times 3000 \times 30}{4 \times 5 \times S_s}$$

$$S_s = \frac{2 \times 3000 \times 30}{4 \times 5 \times 1.0625}$$

$$= 8450 \text{ lbs. per sq. in.}$$

9,000 is about the limit for safety. The key is, therefore, safe for shearing stresses.

2. A piston rod 2 inches in diameter carries a cotter $\frac{3}{8}$ inch thick, and has an axial load of 20,000 pounds. Calculate the average width of the cotter. $S_s = 9{,}000$.

Solution. $P_1 = 2TwS_s \qquad 20000 = 2 \times \frac{3}{8}w \times 9000$

$$w = \frac{20000}{2 \times .375 \times 9000} = 2.97, \text{ approximately 3 inches}$$

PROBLEMS FOR PRACTICE

1. Calculate the safe load in shear which can be carried on a key $\frac{1}{2}$ inch wide, $\frac{3}{8}$ inch thick, and 5 inches long. Assume $S_s = 6{,}000$.

2. Assuming the above key to be $\frac{3}{16}$ inch in hub and $\frac{3}{16}$ inch in shaft, test its proportions for crushing at $S_c = 16{,}000$.

3. A piston rod 2 inches in diameter carries a cotter $\frac{3}{8}$ inch, and has an axial load of 20,000 pounds. How far from the end of the rod must the end of the slot be?

4. Calculate the fiber stress in the rod in the preceding problem at a section through the slot.

COUPLINGS

NOTATION—The following notation is used throughout the chapter on Couplings:

D = Diameter of shaft (inches)
d = Diameter of bolt body (inches)
n = Number of bolts
R = Radius of bolt circle (inches)
S = Safe shearing fiber stress (lbs. per sq. in.)
S_c = Safe crushing fiber-stress (lbs. per sq. in.)
T = Twisting moment (inch-lbs.)
t = Thickness of flange (inches)
W = Load on bolts (lbs.)

Analysis. Rigid couplings are intended to make the shafts which they connect act as a solid, continuous shaft. In order that the shaft may be worked up to its full strength capacity, the coupling must be as strong in all respects as the shaft, or, in other words, it must transmit the same torsional moment. In the analysis of the forces which come upon these couplings, it is not considered that they are to take any side load, but that they are to act purely as torsional elements. It is doubtless true that in many cases they do have to provide some side strength and stiffness, but this is not their natural function, nor the one upon which their design is based.

Referring to Fig. 42, which is the type most convenient for analysis, the simplest form of *flange coupling* is shown. It consists merely of hubs keyed to the two portions, with flanges driving through shear on a series of bolts arranged concentrically about the shaft. The hubs, keys, and flanges are subject to the same conditions of design as the hubs, keys, and web of a gear or pulley, the key tending to shear and be crushed in the hub and shaft, and the hub tending to be torn or sheared from the flange. The driving bolts, which must be carefully fitted in reamed holes, are subject to a purely shearing stress over their full area at the joint, and at the same time tend to crush the metal in the flange, against which they bear, over their projected area. This latter stress is seldom of importance, the thickness of the flange, for practical reasons, being sufficient to make the crushing stress very low.

Theory. The theory of couplings, being the same as for keys, need not be repeated. The shearing stress on the bolts is the only new point to be studied.

In Fig. 42, for a twisting moment on the shaft of T, the

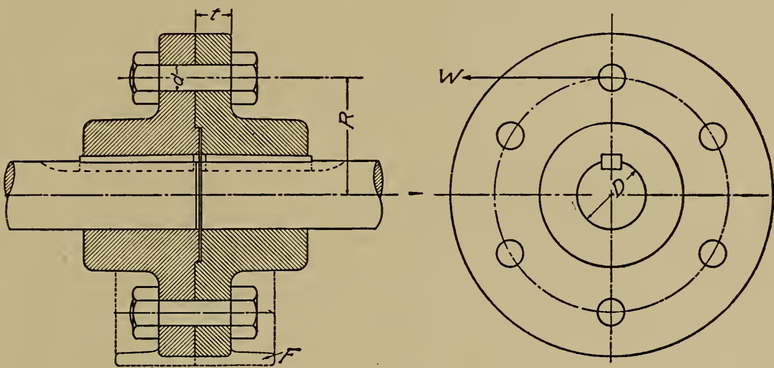


Fig. 42. Simple Flange Coupling

load at the bolt circle is $W = \frac{T}{R}$. If the number of bolts be n, equating the external force to the internal strength

$$W = \frac{T}{R} = \frac{S\pi d^2}{4}n \tag{33}$$

Although the crushing stress will seldom be used, yet for the sake of completeness its equation is given.

$$W = \frac{T}{R} = S_c dtn \tag{34}$$

The internal moment of resistance of the shaft is $\frac{SD^3}{5.1}$; hence the equation representing full equality of strength between the shaft and the coupling, depending upon the shearing strength of the bolts, is

$$\frac{SD^3}{5.1R} = \frac{S\pi d^2}{4}n \tag{35}$$

The theory of the other types of couplings is obscure, except as regards the proportions of the key, which are the same in all cases. The shell of the clamp coupling, Fig. 43, should be thick enough to give equal torsional strength with the shaft; but the exact function which the bolts perform is difficult to determine. In general the bolts clamp the coupling tightly on the shaft and provide rigidity, but the key does the principal amount of the driving. The bolt sizes, in these couplings, are based on judgment and relation to surrounding parts, rather than on theory.

Practical Modification. All couplings must be made with care and nicely fitted, for their tendency, otherwise, is to spring the shafts out of line. In the case of the flange coupling, the two halves may be keyed in place on the shafts, the latter then swung on centers in

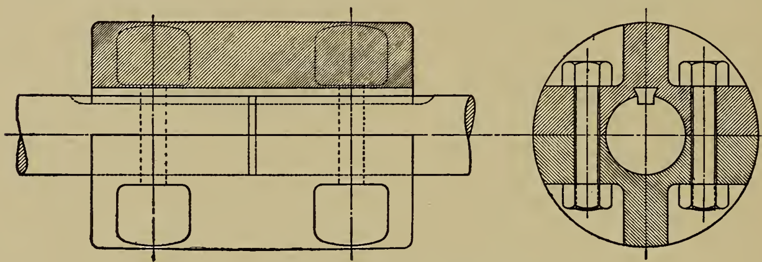


Fig. 43. Clamp Coupling

the lathe, and the joint faced off. Thus the joint will be true to the axis of the shaft; and, when it is clamped in position by the bolts, no springing out of line can take place.

A flange F, Fig. 42, is sometimes made on this form of coupling, in order to guard the bolts. It may be used, also, to take a light belt for driving machinery; but a side load is thereby thrown on the shaft at the joint, which is at the very point where it is desirable to avoid it.

The simplest form of rigid coupling, known as the *muff coupling*, consists of a plain sleeve slipped over from one shaft to the other, when the second is butted up against the first. This is a very satisfactory form of coupling, as it is perfectly smooth on the outside, and consists of only a sleeve and a key. It is, however, expensive to fit, difficult to remove, and requires an extra space of half its length on the shaft over which to be slipped back.

Where the flange coupling, Fig. 42, would be unnecessarily expensive, the *clamp coupling,* Fig. 43, which is simply a muff coupling split in halves and recessed for bolts, is a good coupling for moderate-sized shafts. It is cheap and it is easily applied and removed, even with a crowded shaft. If bored with a piece of paper in the joint, when it is clamped in position it will pinch the shaft

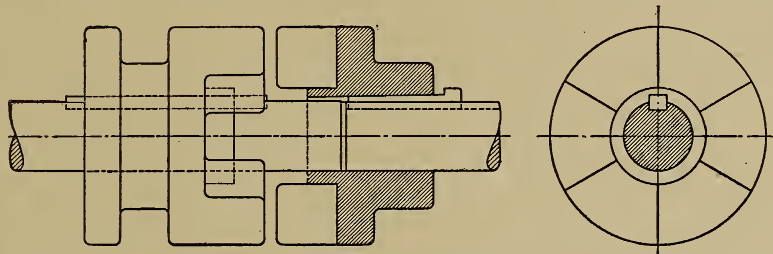


Fig. 44. Claw Coupling

tightly and make a rigid connection. It is desirable to have the bolt-heads protected as much as possible, and this may be accomplished by making the outside diameter large enough so that the bolts will not project. Often an additional shell is provided to encase the coupling completely after it is located.

There are many other special forms of couplings, some of them

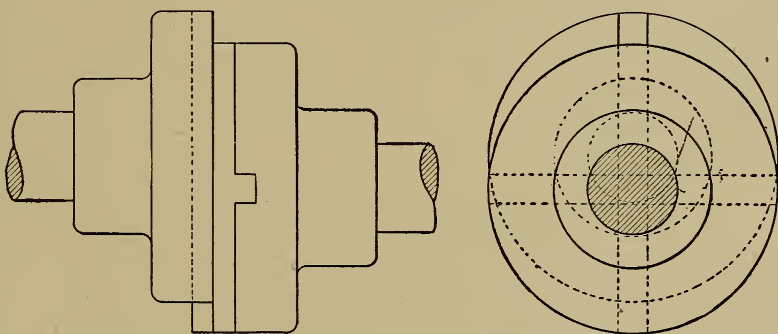


Fig. 45. Oldham Coupling

adjustable; most of them depending upon a wedging action exerted by taper cones, screws, or keys. Trade catalogues are to be sought for their description.

The *claw coupling,* Fig. 44, is nothing but a heavy flange coupling with interlocking claws or jaws on the faces of the flanges to take the place of the driving bolts. This coupling can be thrown in or out as desired, although it usually performs the service of a rigid coupling, as it is not suited to clutching-in during rapid motion, like a friction clutch.

Flexible couplings, which allow slight lack of alignment, are made by introducing between the flanges of a coupling a flexible disk, the one flange being fastened to the inner circle of the disk, the other to the outer circle. This is also accomplished by providing the faces of the flange coupling with pins that drive by pressure together or through leather straps wrapped round the pins. These devices are mostly of a special and often uncertain nature, lacking the positiveness which is one essential feature of a good coupling.

Oldham Coupling. Fig. 45 shows a form of coupling used when two shafts are parallel but not in line. A disk is keyed on the end of each shaft. Between these disks lies a third which has a feather on each side and at right angles to each other and fitting in a slot in the corresponding disk. The middle disk revolves around an axis parallel to the shafts and midway between them. The sliding of the feathers in the slots allows for the lack of alignment of the shafts. Since the feathers are at right angles to each other, the slots are held at right angles by the disk and both shafts must turn with the same angular velocity.

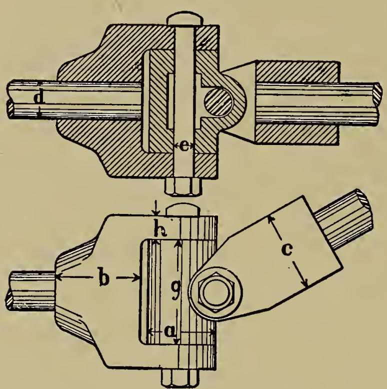


Fig. 46. Universal Coupling

Universal Couplings. In case two shafts are not in line and the angle between them is less than 45° they may be connected by a universal coupling, as shown in Fig. 46. There is an objection to this coupling, and that is that the velocity ratio of the two shafts is a constantly varying one, although they make quarter revolutions in equal times. The change in velocity ratio varies with the angle between the shafts, increasing as it increases.

Example. A flange coupling of the type shown in Fig. 42 is used on a shaft 2 inches in diamcter. The hub is 3 inches long and carries a standard key. The bolt circle is 7 inches in diameter, and it is desired to use $\frac{5}{8}$-inch bolts. How many bolts are needed to transmit 60,000 inch-pounds for a fiber stress in the bolt of 6,000?

Solution.

$$R = \frac{7}{2} = 3.5 \text{ in.} \quad T = 60000$$

$$W = \frac{T}{R} = \frac{60000}{3.5}$$

Shearing area of a $\frac{5}{8}$-in. bolt $= \frac{3.1416}{4} \times (\frac{5}{8})^2$

Total shearing area of n $\frac{5}{8}$-in. bolts $= \frac{3.1416 \times 25 \times n}{4 \times 8^2}$

$$W = \frac{3.1416 \times 25 \times n \times 6000}{4 \times 64}$$

$$\frac{60000}{3.5} = \frac{3.1416 \times 25 \times 6000 \times n}{4 \times 64}$$

$$n = \frac{60000 \times 4 \times 64}{3.5 \times 3.1416 \times 25 \times 6000} = 9.3$$

Ans. 10 bolts.

PROBLEMS FOR PRACTICE

1. If 6 bolts were used in the above example, what diameter of bolt would be required?

2. If four $\frac{3}{4}$-inch bolts were used on a circle of 8 inches diameter, what diameter of shaft would be used in the coupling to give equal strength with the bolts?

FRICTION CLUTCHES

NOTATION—The following notation is used throughout the chapter on Friction Clutches:

a = Angle between clutch face and axis of shaft (degrees)

H = Horse-power (33,000 ft.-lbs. per minute)

μ = Coefficient of friction (per cent)

N = Number of revolutions per minute

P = Force to hold clutch in gear to produce W (lbs.)

R = Mean radius of friction surface (inches)

T = Twisting moment about shaft axis (inch-lbs.)

V = Force normal to clutch face (lbs.)

W = Load at mean radius of friction surface (lbs.)

Analysis. The friction clutch is a device for connecting at will two separate pieces of shaft, thereby transmitting power from the driving shaft to the auxiliary shaft up to the full capacity of the clutch.

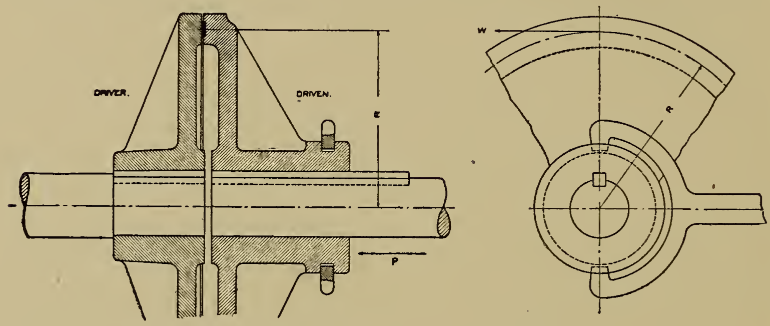


Fig. 47. Friction Clutch with Flat Face

The connection is usually accomplished while the driving shaft is under full speed, the slipping between the surfaces which occurs during the throwing-in of the clutch, permitting the driven shaft to pick up gradually the speed of the other. The disconnection is made in the same manner, the amount of slipping which occurs depending on the suddenness with which the clutch is thrown out.

The force of friction is the sole driving element, hence the problem is to secure as large a force of friction as possible by producing

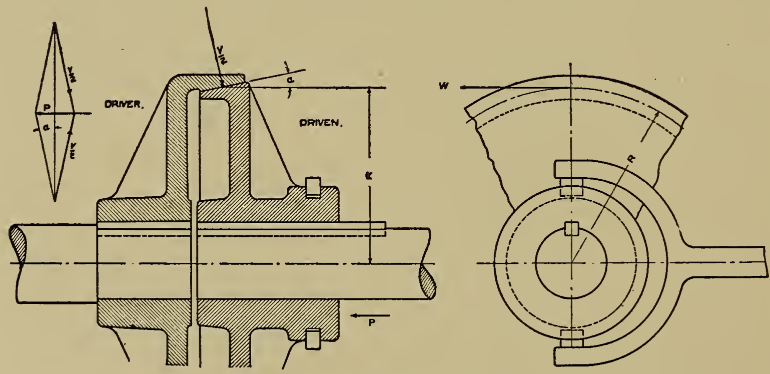


Fig. 48. Cone Friction Clutch

a heavy normal pressure between surfaces having a high coefficient of friction between them. The many varieties of friction clutches which are on the market or designed for some special purpose, are all devices for accomplishing one and the same effect, viz, the production of a heavy normal force or pressure between surfaces at such a radius from the driven axis, that the product of the force of friction thereby created and the radius shall equal the desired twisting moment about that axis. Three typical methods are shown in Figs. 47, 48, and 49. These drawings are not worked out in operative detail, but merely illustrate the principle, and are drawn in their simplest form.

In Fig. 47 the normal pressure is created in the simplest possible way, an absolutely direct push being exerted between the disks, due to the thrust P of the clutch fork.

By taking advantage of the wedge action of the inclined faces, Fig. 48, a less thrust P will produce the required normal pressure at the radius R.

In Fig. 49 the inclination of the faces is carried so far that the

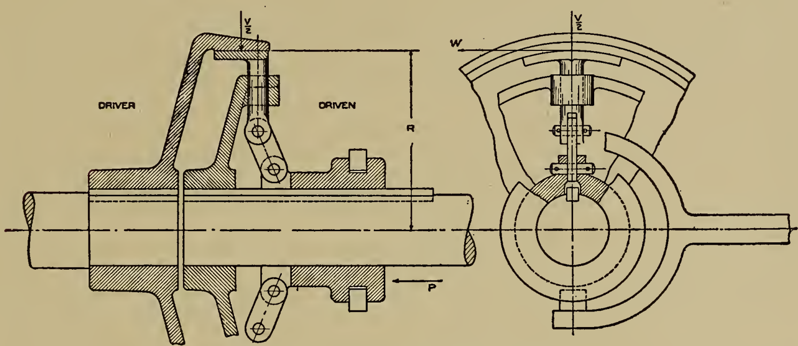


Fig. 49. Cylindrical Friction Clutch

angle a of Fig. 48 has become zero; and by the toggle-joint action of the link pivoted to the clutch collar, the normal force produced may be very great for a slight thrust P. By careful adjustment of the length of the link so that the jaw takes hold of the clutch surface, when the link stands nearly vertical, a very easy operating device is secured, and the thrust P is made a minimum.

Theory. In order to calculate the twisting moment, it must be remembered that the force of friction between two surfaces, Fig. 47, is equal to the normal pressure times the coefficient of friction. This, in the form of an equation, using the symbols of the figure, is

$$W = \mu P \tag{36}$$

Hence a force of magnitude μP may be considered as acting at the mean radius R of the clutch surface. The twisting moment will then be

$$T = WR = \mu PR \tag{37}$$

Referring to the equation which gives twisting moment in terms of horse-power, and putting the two expressions equal to each other

$$T = \frac{63025H}{N} = \mu PR$$

or

$$H = \frac{\mu NPR}{63025} \tag{38}$$

This expression gives at once the horse-power that the clutch will transmit with a given end thrust P.

In Fig. 48 the equilibrium of the forces is shown in the little sketch at the left of the figure. The clutch faces are supposed to be in gear, and the extra force necessary to slide the two together is not considered, as it is of small importance. The static equations then are

$$W = \mu V$$

$$T = WR = \mu VR$$

$$T = \frac{63025H}{N} = \mu VR$$

$$H = \frac{\mu VRN}{63025}$$

V may be determined by using a parallelogram of forces as shown in the upper left-hand corner of Fig. 48.

In Fig. 49, P would of course be variable, depending on the inclination of the little link. The amount of horse-power which this clutch would transmit would be the same as in the case of the device illustrated in Fig. 48, for an equal normal force V produced.

The further theoretical design of such clutches should be in accordance with the same principles as for arms and webs of pulleys, gears, etc. The length of the hubs must be liberal in order to prevent tipping on the shaft as a result of uneven wear. The end thrust is apt to be considerable; and extra side stiffness must be provided, as well as a rim that will not spring under the radial pressure.

Practical Modification. It is desirable to make the most complicated part of a friction clutch the driven part, for then the mechanism requiring the closest attention and adjustment may be brought to rest and kept in this condition when no transmission of power is desired.

Simplicity is an important practical requirement in clutches. The wearing surfaces are subjected to severe usage; and it is essential that they be made not only strong in the first place, but also capable of being readily replaced when worn out, as they are sure to be after some service.

Of the three forms of clutch shown, the one in Fig. 49 is the most efficient, although its commercial design is considerably different from that indicated. Usually the jaws grip both sides of the rim, pinching it between them. This relieves the clutch rim of the radial unbalanced thrust. Adjusting screws for taking up the wear in the jaws, and lock nuts for maintaining their position must be provided.

Theoretically, the rubbing surfaces should be of those materials whose coefficient of friction is the highest; but the practical question of wear comes in, and hence usually both surfaces are made of metal, cast iron being most common. For metal on metal the coefficient of friction μ cannot be safely assumed at more than 15 per cent, because the surfaces are sure to get oily.

A leather facing on one of the surfaces gives good results as to coefficient of friction, μ having a value, even for oily leather, of 20 per cent. Much slipping, however, is apt to burn the leather; and this is most likely to occur at high speeds.

Wood on cast iron gives a little higher coefficient of friction for an oily surface than metal on metal. Wood blocks can be so set in the face of the jaws as to be readily replaced when worn, and in such case make an excellent facing.

The angle a of a cone friction clutch, Fig. 48, may evidently be made so small that the two parts will wedge together tightly with a very slight pressure P; or it may be so large as to have little wedging action, and approach the condition illustrated in Fig. 47. Between these limits there is a practical value which neither gives a wedging action so great as to make the surfaces difficult to pull apart, nor, on the other hand, requires an objectionable end thrust along the shaft in order to make the clutch drive properly.

For a = about 15°, the surfaces will free themselves when P is relieved.
" a = " 12°, " " require slight pull to be freed.
" a = " 10°, " " cannot be freed by direct pull of the hand, but require some leverage to produce the necessary force P.

Example. What force must be exerted to hold in a friction clutch for transmitting 30 horse-power at 200 revolutions per minute, assuming working radius of clutch to be 12 inches; coefficient of friction 15 per cent; angle a = 15°?

Solution.

$$V = \frac{63025H}{\mu RN}$$

$$V = \frac{63025 \times 30}{.15 \times 12 \times 200} = 5252 \text{ pounds}$$

$$\frac{V}{2} = \frac{5252}{2} = 2626 \text{ pounds}$$

Using parallelogram of forces $P = 1313$ pounds

PROBLEMS FOR PRACTICE

1. With what force must a friction clutch of the form shown in Fig. 47 be held in, in order to transmit 20 horse-power? The speed to be 150 revolutions per minute, assuming the radius of the clutch to be 10 inches and coefficient of friction 15 per cent.

2. What horse-power could be transmitted if the working radius were decreased to 8 inches?

LEATHER BELTS

NOTATION—The following notation is used throughout the chapter on Belts:

A = Sectional area of belt (square inches) = bh
b = Width of belt (inches)
F = Force of friction at pulley rim (lbs.)
h = Thickness of belt (inches)
N = Number of revolutions of pulley per minute
P = Driving force at pulley rim (lbs.) $= F$
R = Radius of pulley (feet)
r = Radius of pulley (inches)
T = Initial tension (lbs.)
T_n = Total tension on tight side (lbs.)
T_o = Total tension on slack side (lbs.)
V = Velocity of belt (feet per minute)
w = Weight of belt per cubic inch (lbs.)

Analysis. When a belt stretched over a pair of pulleys is cut off at the proper length, and is laced together into an endless band, it is evident that as long as the belt is at rest there is a nearly uniform tension in it throughout its length, due to the tightness with which the lacing is drawn up. If the distance between the pulleys is considerable, the weight of the belt itself as it hangs between the pulleys will produce a slightly greater tension next to the pulleys than exists in the middle of the span. This increase of tension due to the weight of the belt would make but little difference in the unit-stress in the material of which the belt is made; hence it may safely be assumed that the tension in the belt when at rest is uniform throughout its entire length.

When by turning one of the pulleys, power is transmitted through the belt to the other pulley, the condition of stress in the belt is at once materially changed. The driving pulley can exert only a pull on the other pulley as the belt is a flexible member; the push, which is at the same time given to the other side of the belt, tends to make it sag or become slack. Hence, the immediate effect of the starting motion in a belt is to change the condition of equal tension to that of unequal tension in the two sides, the driving side being tight, and the other loose. The former has gained as much tension as the latter has lost, and the sum of the two is practically equal to the sum of the tensions in the two sides of the belt when at rest. This is not strictly true, as will be shown later, but is sufficiently accurate to form a good basis for the practical design, at least, of slow-speed belts.

The possibility of transmission of power is due, of course, to the friction existing between the belt and the pulleys, and the amount of pull that may be applied to the belt is, therefore, limited by the tension at which the belt slips around the pulley. Moreover, since the force of friction between the belt and the pulley is dependent upon the normal force with which the belt is pressed against the pulley, and the coefficient of friction between the two, it is evident that the tighter the belt is laced up, and the rougher the surfaces of the pulley and belt, the greater is the force that can be transmitted through the belt. This leads to the conclusion that it would be possible to transmit any amount of power through any belt, however small, if the belt were only laced up tight enough.

The above conclusion is literally true; but the important fact now comes in, that the strength of the material of which the belt is made is limited, and while theoretically it might be possible to accomplish this, practically it would be impossible, for at a certain point the belt would break under the strain. Other practical considerations also come in, which fix this limit of power transmission at a point far below the breaking strength of the material.

The complete analysis of the tension problem in belts is not quite as simple as the above, especially for high-speed belts. When the driving side of the belt becomes tight, it stretches and grows longer; and at the same time the other side of the belt becomes slack and grows shorter. It is not true, however, that the increase in the one side is the same as the decrease in the other, which explains why the sum of the tensions in motion is not quite the same as the sum of the tensions at rest.

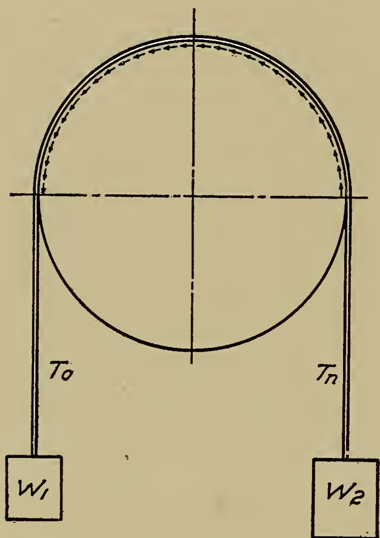


Fig. 50. Fixed Pulley with Belt

Again, when the belt, as it passes around the pulley, changes its motion from linear to circular, each particle of the belt—like a body whirling at the end of a cord about a center of rotation—tends, by centrifugal force, to fly away from the surface of the pulley, thereby decreasing the normal pressure, and hence the friction. The tensions in the belt between the pulleys are also changed somewhat by centrifugal force, and as this increases as the square of the linear velocity, it is evident that the effect is greater at high speeds than at moderate or low speeds.

A further circumstance that affects the driving power of a belt is the stiffness of the leather or other material of which it is made. The belt as it passes around the pulley assumes a circular form, and again straightens out as it leaves the pulley. Hence the theoretically perfect action is modified somewhat according to the sharpness of the bending and the thickness or flexibility of the belt; in other words, a small pulley carrying a thick belt would be the worst case for successful calculation on a theoretical basis.

Theory. The condition of the tight and loose sides of a belt transmitting power, is similar to that of the weighted strap and fixed pulley shown in Fig. 50. If motion is desired of the strap around the pulley, it is necessary to make the weight W_2 of such a magnitude that it will overcome not only the weight W_1, but also the friction between the strap and the pulley. The strap tension T_n is, of course, equal to W_2 and T_o to W_1. The equation showing the balance of forces for the condition when motion is about to occur, is

$$T_n - T_o = F = P \text{ (driving force)} \qquad (39)$$

If the pulley be free to turn on its axis, instead of being fixed as in Fig. 50, the strap by its friction on the pulley will turn the pulley, and the force of friction F becomes the driving force for the pulley as noted in equation 39.

In Fig. 51, it may be supposed that W is a weight representing

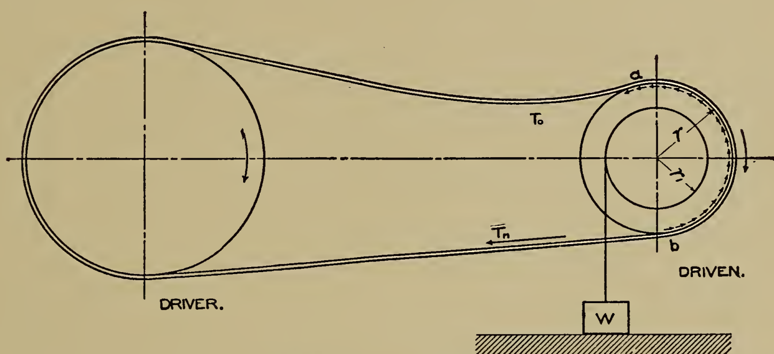


Fig. 51. Tension in Belt

the resistance to be overcome. The tensions T_n and T_o, which are equal at first owing to stretching the belt tightly over the pulleys at rest, change when an attempt is made to raise the weight by turning the larger pulley; and just as the weight leaves the floor, the equality of moments about the axis of the driven pulley gives the following equation:

$$(T_n - T_o)\, r = F \times r = P \times r = W \times r_1 \qquad (40)$$

This *equality of moments* remains as long as the motion of the weight is uniform, and represents closely the conditions under which belt pulleys work.

Although it may be determined from the above what the *difference* of the belt tensions is, and what this difference will do when applied to the surface of a given pulley, the values of neither T_n or T_o are yet actually known, and until they are known the belt cannot be correctly proportioned.

The tension on the tight side of the belt may be divided into three parts: tension doing useful work; tension from centrifugal force; and tension to keep belt from slipping. The tension upon the loose side may be divided into two parts: tension from centrifugal force and tension to keep belt from slipping.

By means of higher mathematics a relation between these quantities may be determined which, when combined with equations 39 and 40, will allow the determination of the values of T_n and T_o.

Practical Modification. In ordinary practice it has been found that a tension in the belt equal to the driving force will be sufficient to prevent slipping.

The tighter the belt is drawn up, the greater is the pressure against the pulley, and hence the greater is the force of friction. But if the belt is pulled up too tightly, when driving is begun, the tension on the tight side becomes too great, and the belt breaks or is under such stress that it wears out quickly. Moreover, the great side pressure on the bearings carrying the shaft produces excessive friction, and the drive is inefficient. This is why a narrow belt driven at high speed is more efficient than a wide belt driven at slow speed, for although the former cannot be pulled up as tightly as the latter without overstraining it, yet by running it at high speed the required power may be obtained from the narrow belt.

The centrifugal force is of small importance for low speeds, say of 3,000 feet per minute and less, and, therefore, it may usually be neglected.

The angle of contact of belt with pulley is important, as a large value gives a great difference between the tension on the tight and loose sides; and it is desirable to make this difference as great as possible, because thereby the driving force is increased. The loose side of a horizontal belt should always be above, as then the natural sag of the loose side due to its slackness tends to increase the angle of contact with the pulley, while the tightening up of the lower side acts against the sag to make the loss of wrap as little as possible. Vertical belts which have the driving pulley uppermost, utilize the weight of the belt to increase the pressure against the surface of the pulley, slightly increasing its capacity for driving. The angle of contact may be artifically increased by a *tightening pulley* which presses the belt further around the pulley than it would naturally lie. This device adds, however, the friction of its own bearing, and impairs the efficiency of the drive.

Since many of the factors upon which belt calculation depends must be assumed, the following empirical rules are generally used by American engineers, and if used with judgment give safe results.

$$\text{h. p.} = \frac{b \times V}{1{,}000} \tag{41}$$

For a double belt, assuming double strength, this becomes

$$\text{h. p.} = \frac{b \times V}{500} \tag{42}$$

With large pulleys and moderate velocities, this may hold good. With small pulleys and high velocities, however, the uncertain stresses induced by the bending of the fibers of the belt around the pulley, and the relatively great loss due to centrifugal force, modify this relation, and a safer value for a double belt of the ordinary kind is

$$\text{h. p.} = \frac{b \times V}{540} \tag{43}$$

or, still safer,

$$\text{h. p.} = \frac{b \times V}{700} \tag{44}$$

The theoretical value of horse-power for belt transmission is

$$\text{h. p.} = \frac{P \times V}{33000} \tag{45}$$

Putting this value equal to the empirical value

$$\text{h. p.} = \frac{P \times V}{33000} = \frac{b \times V}{1000}$$

and solving for P

$$P = 33b$$

This develops the fact that the empirical rule assumes a driving force of 33 pounds per inch of width of single belt.

Another way of expressing equation 42 is: A single belt will transmit one horse-power for every inch of width at a belt speed of 1,000 feet per minute.

Strength of Leather Belting. The breaking tensile strength of leather belting varies from 3,000 to 5,000 pounds per square inch. Joints are made by *lacing*, by *metal fasteners*, or by *cementing*. The strength of a laced joint may be about $\frac{7}{10}$, of a metal-fastened joint about $\frac{1}{2}$, and of a cemented joint about equal to the full strength of the cross-sectional area of the belt. The proper working strength of belting depends on the use to which the belt is put. A continuously running belt should have a low tension in order to have long life and a minimum loss of time for repairs. For double leather belting it has been shown that a working tension of 240 pounds per square inch of sectional area gives an annual cost—for repairs, maintenance, and renewals—of 14 per cent of first cost. At 400 pounds working tension, the annual expense becomes 37 per cent of first cost. These results apply to belts running continuously; larger values may be used where the full load comes on but a short time, as in the case of dynamos.

Good average values for working tensions of leather belts are:

Cemented joints, 400 pounds per square inch
Laced joints, 300 pounds per square inch
Metal joints, 250 pounds per square inch

It is evident from the equation for the theoretical value of horse-power that the horse-power of a belt depends upon two things, the driving force P and the velocity V, the driving force being the difference in tension on the tight and loose sides of the pulley. If either of these factors is increased, the horse-power is increased. Increasing P means a tight belt. Hence a tight belt and high speed together give maximum horse-power. But a tight belt means more side strain on shaft and journal. Therefore, from the standpoint of efficiency, *a narrow belt under low tension at as high a speed as possible is used.*

Speed of Belting. The most economical driving speed for belts is somewhere between 4,000 and 5,000 feet per minute. Above these values the life of the belt is shortened; also "flapping," "chasing," and centrifugal force cause considerable loss of power. The limit of speed with cast-iron pulleys is fixed at the same limit for bursting of the rim, which may be taken at one mile per minute.

Material of Belting. Oak-tanned leather, made from the part of the hide which covers the back of the ox, gives the best results for leather belting. The thickness of the leather varies from .18 to .25 inch. It weighs from .03 to .04 pound per cubic inch. The average thickness of double leather belts may be taken as .33 inch, although they may be ordered light, medium, or heavy, varying accordingly from $\frac{1}{4}$ inch to $\frac{7}{16}$ inch in thickness. Double leather belts are made by cementing the flesh sides of two thicknesses of belt together, leaving the grain or hair side exposed to surface wear. In a single-thickness belt, the grain side should be next to the pulley, as the flesh side is the stronger and is, therefore, better able to resist the tensile stress due to bending set up where the belt makes and leaves contact with the pulley face.

Raw hide and semi-raw hide belts have a slightly higher coefficient of friction than ordinary tanned belts. They are useful in damp places. The stress of these belts is about one and one-half times that of tanned leather.

Cotton, cotton-leather, rubber, and leather link belting are some of the forms on the market, each of which is especially adapted to certain uses. For their weights and their tensile and working strengths consult the manufacturers' catalogues.

The practice of a prominent manufacturer in regard to the sizes of leather belting will be found useful for comparison, and is indicated in Table VII.

Initial Tension in Belt. On the assumption that the sum of the tensions is unchanged, whether the belt be at rest or driving, there should be the following relation

$$T_n + T_o = 2T$$

whence

$$T = \frac{T_n + T_o}{2} \qquad (46)$$

This is not strictly true, however, as is stated in the "Analysis of Belts." It has been found that in a horizontal belt working at about 400 pounds tension per square inch on the tight side, and having 2 per cent slip on cast-iron pulleys, *i. e.*, the surface of the driven pulley moving 2 per cent slower than that of the driver, the increase

TABLE VII

Sizes of Leather Belting

Width	Thickness		Width	Thickness	
	Single	Double		Single	Double
1 inch	$\frac{5}{32}$ inch	$\frac{5}{16}$ inch	6 inch	$\frac{7}{32}$ inch	$\frac{3}{8}$ inch
2 "	$\frac{3}{16}$ "	$\frac{5}{16}$ "	10 "	$\frac{5}{16}$ "	$\frac{3}{8}$ "
3 "	$\frac{7}{32}$ "	$\frac{3}{8}$ "	12 "	....	$\frac{3}{8}$ "
4 "	$\frac{7}{32}$ "	$\frac{3}{8}$ "	14 "	....	$\frac{13}{32}$ "
5 "	$\frac{7}{32}$ "	$\frac{3}{8}$ "	20 "	....	$\frac{7}{16}$ "

of the sum of the tensions when in motion over the sum of the tensions at rest, may be taken at about $\frac{1}{3}$ the value of the tensions at rest. Expressing this in the form of an equation

$$T_n + T_o = \frac{4}{3}(2T) = \frac{8 \times T}{3}$$

$$T = \frac{3}{8}(T_n + T_o) \qquad (47)$$

The value of T thus found would be the pounds initial tension to which the belt should be pulled up when being laced, in order to produce T_n and T_o when driving. This value is not of very great practical importance, as the proper tightness of belt is usually secured by *trial*, by *tightening pulleys*, by *pulley adjustment* (as in motor drives), or by *shortening the belt* from time to time as needed. It is worth noting, however, that for the most economical life of the belt it would be very desirable in every case to weigh the tension by a spring balance when giving the belt its initial tension. This, however, is not always easy or even feasible; hence, it is a refinement with which good practice usually dispenses, except in the case of large and heavy belts.

Care of Belts. Leather belts should be well protected from water, steam, or other moisture, and only the firmest, finest grained leather should be used in damp localities. Oil should never be allowed to drop on belts as it destroys the life of the leather. In order to keep a belt in good condition, warm tallow should be applied whenever the leather appears very dry. When belts are to be used in damp places the addition of a little resin to the tallow will preserve their strength. Too much resin, however, will leave the belt sticky and will cause cracking.

Methods for Joining Ends of Belt. The ends of narrow, light belts such as are used on lathes, planers, and other machine tools, are generally fastened together by using strips of white leather tanned with alum. This form of joint is easily made, is flexible, and runs smoothly and noiselessly over the pulleys. Two common methods of lacing are shown in Figs. 52 and 53. To make a joint, first cut off the ends of the belt squarely. A punch should then be used for making the holes which must not come too near the ends or sides of the belt. Begin to lace in the middle of the belt and lace both sides

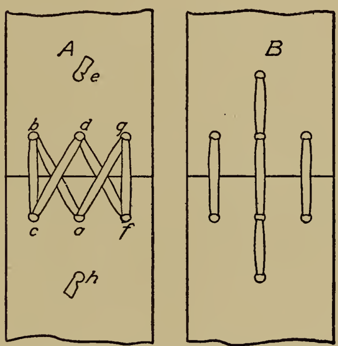


Fig. 52. Simple Laced Joint

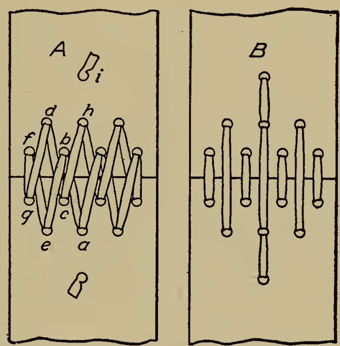


Fig. 53. Strong Laced Joint

with equal tightness. In Fig. 52, *A* is the outside and *B* the inside of the belt. Draw the lacing half way through the middle hole *a* from the under side, and then pass one end down through *b*, up through *c*, down through *b* and up through *c* again, then down through *d* to *e* where a cut is made upon the end of the lacing which acts as a barb to prevent unlacing. Lace the other side of the belt by carrying the other end up through *d*, down *f*, up *g*, down *f* and up *g* again, down *a* to *h*.

A somewhat stronger form of lacing is shown in Fig. 53. Bring the lacing up through *a*, and continue through *b*, *c*, *d*, *e*, *f*, *g*, *d*, *e*, *b*, *c*, *h*, and *i*. The edges of the holes should not come nearer than $\frac{7}{8}$ inch from the ends of the belts or $\frac{3}{4}$ inch from the sides; and when lacing with two rows of holes, the second row should be at least $1\frac{3}{4}$ inches from the end. For 3-inch belts the number of holes may be the same as in Fig. 52. A 6-inch belt should have seven holes—four in the row nearest the end—and a 10-inch belt, nine holes.

For any style of lacing, care should always be taken that the lacing on the side of the belt next the pulleys is parallel with the edge of the belt. When heavy belts transmit large amounts of power, their joints are generally made by gluing and riveting.

Example. The tension on the tight side of a belt is 1,059 pounds, and on the loose side 645 pounds; the pulley is 42 inches in diameter and the speed is 470 revolutions per minute. What is the horse-power transmitted and what should the width be for a double belt?

Solution.

$$P = 1059 - 645 = 414 \text{ pounds}$$

$$V = 470 \times 3.1416 \times \frac{42}{12} = 5168 \text{ ft. per minute}$$

$$\text{h. p.} = \frac{414 \times 5168}{33000} = .65 \text{ h. p., nearly}$$

Above 3,000 feet per minute, centrifugal force should be considered and, therefore, in order to obtain the width of the double belt, use the formula, $\text{h. p.} = \frac{b \times V}{700}$. Then

$$64 = \frac{b \times 5168}{700}$$

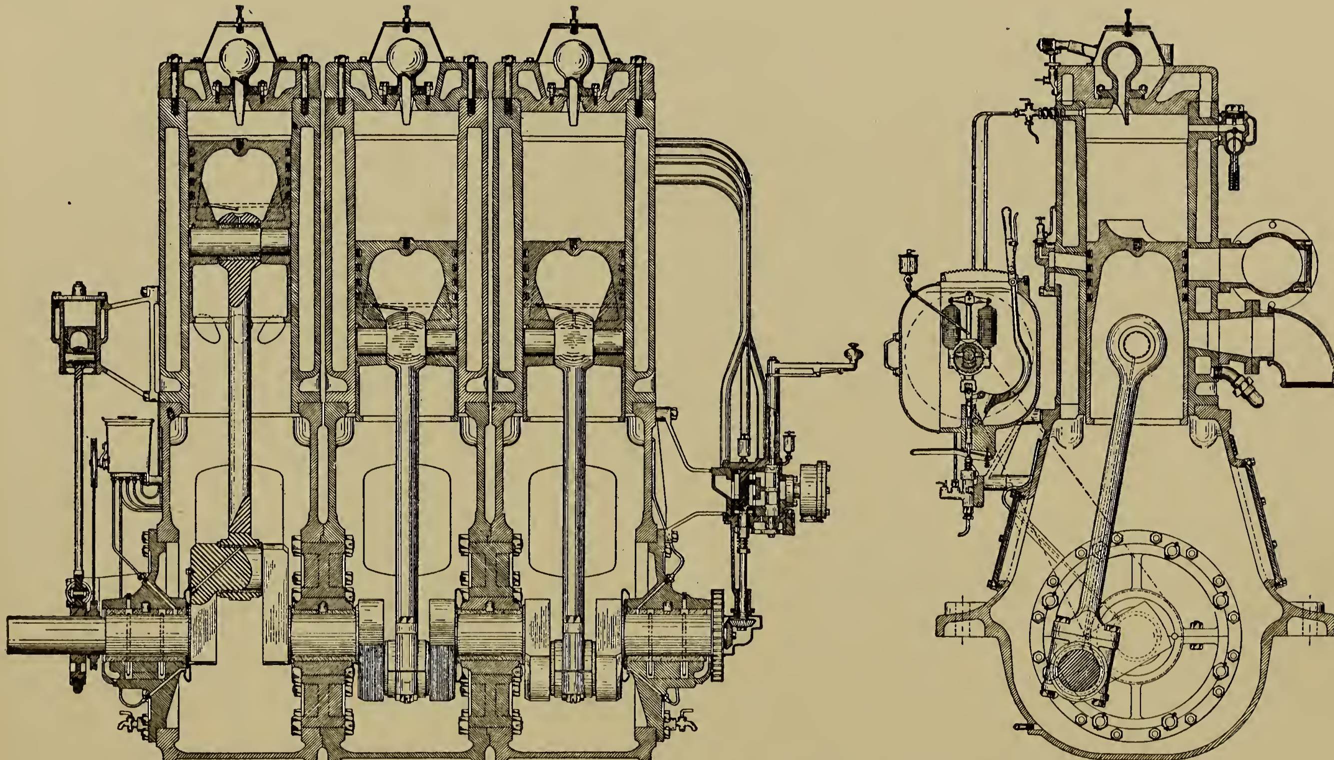
$$b = \frac{64 \times 700}{5168} = 8\tfrac{3}{4}'' \text{ nearly. Ans.}$$

PROBLEMS FOR PRACTICE

1. A double belt transmits 50 horse-power at 2,000 feet per minute. What should be the width?

2. A pulley 24 inches in diameter revolving at 250 revolutions per minute is to transmit 25 horse-power. Determine the width of belt.

3. A belt with a velocity of 3,500 feet per minute has a tension of 969 pounds in its tight side and 497 pounds in its loose side. What horse-power is it transmitting?



LONGITUDINAL AND TRANSVERSE SECTIONS OF MIETZ AND WEISS OIL ENGINE
Courtesy of August Mietz, New York City

MACHINE DESIGN

PART II

ROPE GEARING

NOTATION—The following notation is used throughout the chapter on Rope Gearing:

A = Net area of rope (sq. in.)
a = Diameter of wire in a rope (inches)
B = Total stress due to bending (lbs.)
C = Centrifugal force (lbs.)
D = Distance between sheaves (feet)
d = Diameter of the rope
E = Coefficient of elasticity
H = Horse-power
L = Distance between pulleys (feet)
μ = Coefficient of friction
P = Driving force at pulley rim (lbs.)
R = Radius of pulley (inches)
S = Sag of rope (feet)
T = Tension in rope (lbs.)
T_n = Total tension on tight side (lbs.)
T_o = Total tension on slack side (lbs.)
t = Tension, balancing slipping (lbs.)
V = Velocity of rope (ft. per sec.)
W = Weight of one foot of rope (lbs.)

HEMP AND COTTON ROPE

Analysis. The stresses acting upon a rope drive are practically the same as those acting upon a belt and the analysis for belts will apply to ropes.

Theory. The equation showing the balance between the actual working force and the tension in the rope is

$$P = T_n - T_o \qquad (48)$$

The formula for centrifugal force in belts is

$$C = \frac{WV^2}{32} \qquad (49)$$

The balance of the forces acting on the loose side is expressed by the equation

$$T_o = C + t \qquad (50)$$

Since one horse-power equals 550 foot-pounds per second

$$H = \frac{P \times V}{550} \qquad (51)$$

The rims of pulleys for hemp or cotton-rope gearing are usually grooved as shown in Fig. 54. In a semicircular groove, Fig. 55,

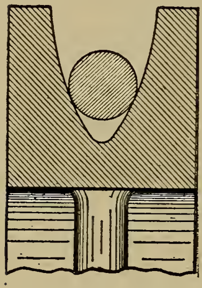


Fig. 54. Wedge Groove

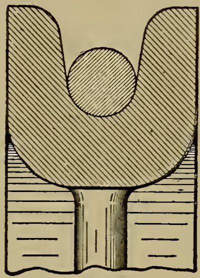


Fig. 55. Semicircular Groove

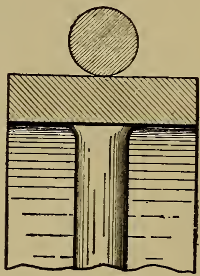


Fig. 56. Flat Run

the friction is very little more than that upon an ordinary flat cylindrical pulley of the shape shown in Fig. 56. The frictional resistance of the wedge-shaped groove may be determined from a parallelogram of forces. Referring to Fig. 56, it is evident that the pressure between the rope and pulley is equal to the radial force produced by the tension in the rope. In the wedge-shaped groove the radial force has been resolved into two components as shown in Fig. 57. Drawing a parallelogram of forces, Fig. 58 is obtained. If the rope slipped it would be compelled to slip on both sides, then the total resistance to slipping would be $\mu(K_1+K_2)$. It will be noted that $K_1 + K_2$ will always be greater than Q unless α is equal to 180° and the less angle α is, the greater will be $K_1 + K_2$ relative to Q.

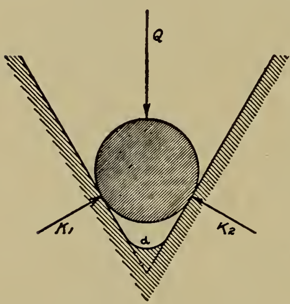


Fig. 57. Components of Forces

Practical Modification. It is usual in practice to make the sides of the groove incline at an angle of 45° to each other. Under these conditions experience has proved that the tension to prevent slipping will be sufficient when it is equal to one-half the force doing useful work, or $t = \frac{P}{2}$.

Transmission ropes wear internally, due to the fibers sliding on one another when the rope bends about the pulley or sheaves. In order to decrease this wear as much as possible it has been found advisable to use a sheave whose diameter is at least forty times that of the rope. The use of a large sheave not only adds to the life of the rope but offers a larger surface of contact and increases the driving force.

Wooden sheaves are not suitable for rope transmission because it is impossible to obtain such a sheave that will be of uniform density throughout, thus causing more wear in some portions of the groove than in others. This results in uneven running and increased wear on the rope.

Materials. Cotton rope is generally used on small machines as it is soft and flexible. Hemp rope possesses great strength, is durable, and withstands exposures to the weather. Ropes are usually made of three, four, or six strands. Their diameter is from 1 to 1¾ inches; however, the size is sometimes given by the girth or circumference. The net section is about .9 of the total area of the rope.

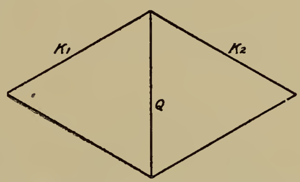


Fig. 58. Resolution of Force

It has been shown by experiment that the weight of dry rope may be determined by the formula

$$W = .3d^2$$

From the best information obtainable the breaking strength may be taken as 7,000 lbs. per sq. in., and the factor of safety 35 making the working strength 200 lbs. per sq. in.

It is not advisable to run ropes at a speed greater than 5,000 feet per minute as the centrifugal force begins to greatly reduce the friction between the rope and the pulley.

The sag on the slack side varies with the speed and the power transmitted. When at rest the sag is not as great as when running, being greater on the tight side or less on the slack side. The sag on the driving side when transmitting normal horse-power is the same, no matter what size of rope is used or what its velocity may be. Experience has shown that the sag may be determined by the formula

$$S = \frac{WD^2}{8T_n} \tag{52}$$

To determine the sag on the slack side use the formula

$$S_1 = \frac{WD^2}{8T_0} \tag{53}$$

Systems of Rope Driving. There are two systems of rope driving in general use, one known as the *multiple,* or *English,* and the other as the *continuous,* or *American.*

The *multiple system* consists of one or more independent ropes running side by side in the grooves of the sheaves. This system is suitable for the transmission of large power and gives best results where the shafts are parallel or nearly so and where the drive is sufficiently off the vertical to prevent the ropes when slack from leaving the pulley. If one rope breaks this does not cause stoppage of the entire drive.

The *continuous system* makes use of one rope which is wound around the driving and driven pulley several times. In this system, Fig. 59, the rope is conducted from the outside groove of one pulley

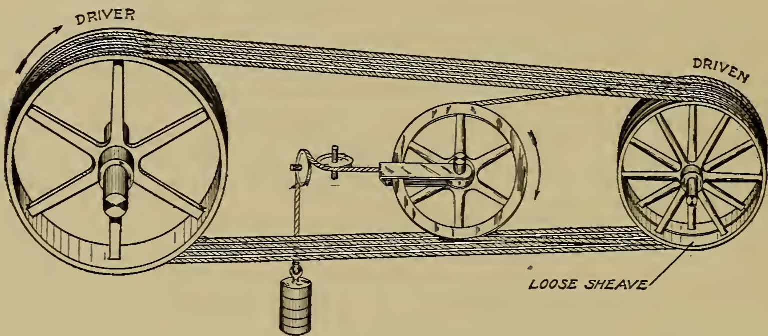


Fig. 59. Continuous Rope Drive

to the inside groove of the other. This is accomplished by using a traveling tension carriage or jockey. The carriage also serves to maintain a uniform tension throughout the rope and is arranged to travel back and forth, automatically regulating the slack; thus, the stretch in the rope and the inequalities in the load are taken care of.

A system patented by Joseph H. Hoadley uses a winding or idler pulley. The smaller pulley is provided with more grooves than the larger. After the rope has filled all the grooves of the larger and an equal number of grooves of the smaller, the rope is then carried around the grooves of the winding pulley. In this way the contact surface of the smaller pulley is increased and made more nearly equal to the larger.

WIRE ROPE

Theory. The forces acting in wire rope transmission are of the same character as those acting in belts. In addition, the stress due to the rope bending about the pulley must be considered and is represented by the equation

$$B = \frac{AEa}{2R} \qquad (54)$$

The total stress due to centrifugal force is as before

$$C = \frac{WV^2}{32}$$

The equation for the driving force is

$$P = T_n - T_o$$

The horse-power transmitted is

$$H = \frac{P \times V}{550}$$

The deflection of the rope may be calculated by means of the following formula—which will give results close enough for practice—

$$S = \frac{T}{2W} - \sqrt{\frac{T^2}{4W^2} - \frac{L^2}{8}} \qquad (55)$$

If it is wished to know the tension on the rope, the formula may be changed to read

$$T = \frac{WL^2}{8S} + WS \qquad (56)$$

Practical Modification. Wire rope is especially adapted to the transmission of large power over great distances. It is unsuitable for small power and short drives because of its rigidity and its short life when bent about pulleys of small diameter.

It has been found that with wire rope under best conditions the tension required to prevent slipping is equal to about $\frac{P}{2}$.

The force at which the rope is worked should not exceed the elastic limit which may be taken as 57,000 lbs. per square inch for steel and 28,500 lbs. per square inch for wrought iron. For the greatest safe working tension there is a certain ratio existing between the diameter of the sheave and the diameter of the wires composing the rope. The minimum diameters of sheaves for various ropes is usually given in rope manufacturer's catalogues. The coefficient of elasticity may be taken as 28,000,000 for both iron and steel.

Pulleys for Wire Ropes. Wire ropes are injured by the lateral crushing if run upon pulleys with V-shaped grooves. Hence, pulleys are constructed with wide grooves so that the rope may rest upon the bottom. The frictional resistance between the rope and the pulley is much increased by lining the bottom of the groove with gutta-percha, tarred hemp, rubber, leather, or wood. The material used is hammered into the groove, often dovetailed in section. Where the working tension is very great, wood filling is preferred. A cross-section of a wire rope sheave is shown in Fig. 60. The weight per foot of wire rope equals $1.43d^2$.

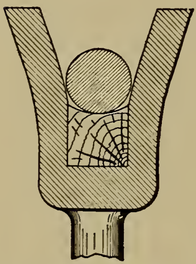


Fig. 60. Groove with Wood Filling

Examples. 1. A drive pulley 72 inches in diameter revolving at a speed of 100 revolutions per minute and having four grooves using $1\frac{1}{2}$-inch hemp rope will transmit what horse-power?

Solution. The problem may be solved by first finding the horse-power transmitted by one rope and then multiplying by 4 to find the total.

$$W = .3d^2 = .3 \times 1.5^2 = .675 \text{ lb. per foot}$$

Maximum tension allowed in the rope is

$$.7854 \times 1.5^2 \times .9 \times 200 = 318.1 \text{ lbs.}$$

$$V = 3.1416 \times \frac{72}{12} \times \frac{100}{60} = 31.4 \text{ feet per second}$$

$$C = \frac{.675 \times 31.4^2}{32} = 20.7 \text{ lbs.}$$

$$T_o = \frac{P}{2} + C$$

$$P = T_n - T_o = T_n - (\frac{P}{2} + C)$$

$$\frac{3}{2}P = T_n - C$$

$$P = \frac{2}{3}(T_n - C) = \frac{2}{3}(318.1 - 20.7) = 198.2 \text{ lbs.}$$

$$H = \frac{198.2 \times 31.4}{550} = 11.3$$

Total horse-power $= 11.3 \times 4 = 45$ nearly.

2. Power is transmitted by a $\frac{1}{2}$-inch 6-strand 19-wire steel rope. The driving pulley is 8 feet in diameter, the driven pulley 2 feet, and the speed of the rope 3,600 feet per minute. The net area of the wire is 0.097504 square inch, and the diameter of the wires composing the rope, 0.033 inch. The strength of wire is 57,000 pounds per square inch. Find the horsepower which may be transmitted.

Solution.

$$W = 1.43 \times .5^2 = .3575 \text{ lb. per foot}$$

$$V = \frac{3600}{60} = 60 \text{ feet per second}$$

The tension due to bending is greater at the small pulley.

$$B = .097504 \times \frac{28000000 \times .033}{2 \times 12} = 3754 \text{ lb.}$$

$$C = \frac{WV^2}{32} = \frac{.3575 \times 60^2}{32} = 40.2 \text{ lb.}$$

The tension on the slack side would be the sum of the tensions due to bending, centrifugal force, and that necessary to keep the rope from slipping, or

$$T_o = \frac{P}{2} + B + C = \frac{P}{2} + 3754 + 40.2 = \frac{P}{2} + 3794.2$$

$$P = T_n - T_o$$

$$P = T_n - (\frac{P}{2} + 3794.2) = T_n - \frac{P}{2} - 3794.2$$

$$\frac{3P}{2} = T_n - 3794.2 \qquad P = \frac{2}{3}(T_n - 3794.2)$$

$$T_n = 57000 \times .097504 = 5557.7 \text{ lb.}$$

$$P = \frac{2}{3}(5557.7 - 3794.2) = \frac{2}{3} \times 1763.5 = 1175.67$$

$$H = \frac{1175.67 \times 60}{550} = 128$$

What is the sag in the tight side of the wire if the distance between pulleys is 100 feet?

$$S = \frac{5557.7}{2 \times .3575} - \sqrt{\frac{5557.7^2}{4 \times .3575^2} - \frac{100^2}{8}}$$
$$= 7773 - 7772.92 = 0.08 \text{ feet} = 1 \text{ inch nearly}$$

PROBLEMS FOR PRACTICE

1. Find the horse-power which may be transmitted by a $\frac{3}{4}$-inch manila rope running at a speed of 3,000 feet per minute.

2. In the preceding problem if the distance between pulleys is 25 feet, what is the sag on both the tight and loose sides?

3. Will a $\frac{1}{4}$-inch 7-wire steel rope composed of wires .028 inch in diameter and having a net area of .025862 sq. in. transmit 45 horse-power? The velocity of the rope is 2,400 feet per minute and the small pulley is 24 inches in diameter.

4. A $\frac{1}{2}$-inch 19-wire iron rope transmits 150 horse-power. The small pulley is the driver, 5 feet in diameter, and operates at a speed of 250 revolutions per minute. The rope is composed of wires .033 inch in diameter and has a net area of .097504 sq. in. Is this iron rope sufficiently large to withstand the strain?

PULLEYS

NOTATION—The following notation is used throughout the chapter on Pulleys:

A = Area of rim (sq. in.)
a = Area of arm (sq. in.)
b = Center of pulley to center of belt (inches; practically equal to R)
C_1 = Total centrifugal force of rim (lbs.)
c = Distance from neutral axis to outer fiber (inches)
D = Diameter of pulley (inches)
D_1 = Diameter of hub (inches)
d_1 = Diameter of bolt at root of thread (inches)
d = Diameter of bolt holes (inches)
g = Acceleration due to gravity (ft. per sec.)
h = Width of arm at any section (inches)
I = Moment of inertia
L = Length of arm, center of belt to hub (inches)
L_1 = Length of rim flange of split pulley (inches)
l = Length of hub (inches)
N = Number of arms
n = Number of rim bolts, each side
P = Driving force of belt (lbs.)
P_1 = Force at circum. of shaft (lbs.)
P_2 = Force at circum. of hub (lbs.)
p = Stress in rim due to centrifugal force (lbs. per sq. in.)
R = Radius of pulley (inches)
S = Fiber stress (lbs. per sq. in.)
S_s = Shearing stress (lbs. per sq. in.)
T = Thickness of web (inches)
t = Thickness of rim (inches)
t_2 = Thickness of rim bolt flange (inches)
v = Velocity of rim (ft. per sec.)
w = Weight of material (lbs. per cu. in.)

Analysis. If a flexible band be wrapped *completely* about a pulley, and a heavy stress be put upon each end of the band, the rim of the pulley will tend to collapse just like a boiler tube with steam pressure on the outside of it. A compressive stress is induced which is very nearly evenly distributed over the cross-section of the rim, except at points where the arms are connected thereto. At these points the arms, acting like rigid posts, take this compressive stress. Now, a pulley never has a belt wrapped *completely* round it, the fraction of the circumference embraced by the belt being usually about $\frac{1}{2}$, and seldom reaching $\frac{3}{4}$, even with a tightener pulley. Assuming the wrap to be $\frac{1}{2}$ the circumference, and that all the side pull of the belt comes on the rim, none being transmitted through the arms to the hub, then one half of the rim is pressed hard against the other half by a force equal to the resultant of the belt tensions, which, in this case, would be the sum of them. Dividing the pulley by a plane through its center and perpendicular to the belt, the cross-section of the pulley rim cut by this plane has to take this compressive stress.

This analysis is satisfactory from an ideal standpoint only, for the intensity of stress due to the direct pull of the belt, with the usual practical proportions of rim, would be very small. Moreover, the element of speed has not been considered.

When the pulley is under speed, a set of conditions which complicates matters is introduced. The centrifugal force due to the weight of the rim and arms is no longer negligible, but has an important influence upon the design and material used. This centrifugal force acts against the effect of the belt-wrap, and tends to reduce the compressive stress, or, overcoming the latter entirely, sets up a tensional stress both in the rim and in the arms. It also tends to distort the rim from a true circle by bowing it out between the arms, thus producing a bending moment in the rim, which is at a maximum at the points where the rim joins each arm.

It can readily be imagined that the analysis in detail of these various stresses in the rim acting in conjunction with each other is quite complicated—far too much so in fact, to be introduced here. As in most cases of such design, however, one controlling influence can be separated out from the others, and the design based thereon with sufficient margin of strength to satisfy the more obscure conditions. This is rational treatment, and the "theory" will be studied accordingly.

The rim, being fastened to the ends of the arms, tends, when driving, to be sheared off, and the resistance to shear depends upon the areas of the cross-sections of the arms at their point of joining the rim. The force that produces this shearing tendency is the driving force of the belt, or the difference between the tensions of the tight and loose sides.

Again, at the point of connection of the arms to the hub, a shearing action takes place, so that, if this shearing tendency were carried to rupture, the hub would literally be torn out of the arms. Now, viewing the arms as beams loaded at the end with the driving force of the belt, and fixed at the hub, a heavy bending stress is set up, which is maximum at the point of connection to the hub. If the rim were stiff enough to distribute this driving force equally between the arms, each arm would take its proportional share of the load. The rim, however, is quite thin and flexible; and it is not safe to assume this perfect distribution. It is usual to consider that one half the whole number of arms take the full driving force.

Theory. *Pulley Rim.* Evidently it is practically impossible to make so thin a rim that it will collapse under the pull of a belt. As far as the *theory* of the rim is concerned, its proportion probably depends more upon the calculation for centrifugal force than upon anything else.

In order to separate this action from that of any other forces, let it be supposed that the rim is entirely free from the arms and hub, and is rotating about its center. Every particle, by centrifugal force, tends to fly radially outward from the center. The tendency with which one half of the rim tends to fly apart from the other half is indicated by the force C_1; and the relation between C_1 and the small radial force c for each unit-length of rim can readily be found from the principles of mechanics. The case is exactly like that of a boiler or a thin pipe subjected to uniform internal pressure, which, if carried to rupture, would split the rim along a longitudinal seam.

The tensile stress thus induced per square inch may be found, by simple mechanics, to be

$$p = \frac{12wv^2}{g} \tag{57}$$

or, since $w = 0.26$ pound for cast iron, and $g = 32.2$ feet per second,

$$p = 0.097\, v^2 \quad (\text{say}, \frac{v^2}{10}) \tag{58}$$

and, if p be taken equal to 1,000 pounds per square inch, which is as high as it is safe to work cast iron in this place,

$$v = 100 \text{ feet per second} \tag{59}$$

This shows the curious fact that the intensity of stress in the rim is directly proportional to the square of the linear velocity, and wholly independent of the area of cross-section. It is also to be noted that 100 feet per second is about the limit of speed for cast-iron pulleys to be safe against bursting.

By theoretically considering the rim together with the arms as actually connected to it, a much more complicated relation is obtained. This condition causes the rim to expand more than the arms and to bulge out between them. This makes the rim act something like a continuous beam uniformly loaded; but even then the resulting stress is not clearly defined on account of the variable stretch in the arms. Investigation on this basis is not needed further than to note that it is theoretically better, in the case of a split pulley, to make the joint close to the arms, rather than in the middle of a span.

Pulley Arms. The centrifugal force developed by the rim and arms tends to pull the arms from the hub. On the belt side, this is balanced to some extent by the belt wrap, which tends to compress the arm and to relieve the tension. On the side away from the belt, the centrifugal action has full play, but the arm is usually of such cross-section that the intensity of this stress is very low and may safely be neglected.

The rim being very thin in most cases, its distributing effect cannot be depended on, hence the driving force of the belt may be taken entirely by the arms immediately under the portion of the belt in contact with the pulley face. For a wrap of 180° this means that only one half of the pulley arms can be considered as effective in transmitting the turning effort to the hub. Each of these arms is a lever fixed at one end to the hub and loaded at the other. A lever of this description is called a *cantilever beam,* its maximum moment existing at its fixed end. The load that each of these beams may be subjected to is $\frac{P}{N}$, and, therefore, the maximum external moment at the hub is $\frac{2PL}{N}$. From mechanics, the internal moment of resistance of any beam section is $\frac{SI}{c}$, and equilibrium of the beam can be satisfied only when the external moment is equal to the internal moment of resistance of the beam section. Equating these two

$$\frac{2PL}{N} = \frac{SI}{c} \tag{60}$$

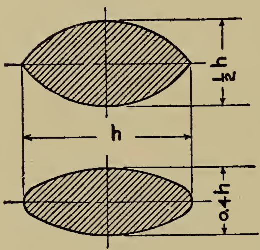


Fig. 61. Elliptical Arms

The arms of a pulley are usually of the elliptical or segmental cross-section, and may be of the proportions shown in Fig. 61. For either of these sections the fraction $\frac{I}{c}$ is approximately equal to $0.0393h^3$. For convenience—the error caused being on the safe side—L may be taken as equal to the full radius of the pulley R, whence

$$\frac{2PR}{N} = 0.0393Sh^3 \tag{61}$$

in which S may be from 2,000 to 2,250 for cast iron.

Taking moments about the center of the pulley, and solving for P_2, the force acting at the circumference of the hub.

$$\frac{2PR}{N} = \frac{P_2D_1}{2}$$

or

$$P_2 = \frac{4PR}{ND} \tag{62}$$

The area of an elliptical section is π times the product of the half axes. With the proportions of Fig. 61 this becomes

$$a = \pi \times 0.2h \times 0.5h = \frac{\pi h^2}{10} \tag{63}$$

Equating the external force to the internal shearing resistance

$$\frac{4PR}{ND_1} = \frac{\pi h^2 S_s}{10}$$

or

$$h = \sqrt{\frac{40PR}{D_1 N \pi S_s}} \tag{64}$$

in which the shearing stress S_s may run from 1,500 to 1,800 for cast iron.

Although both bending and shearing stresses as calculated above exist at the base of the arms, the bending is, in practically every case, the controlling factor in the design of the arms. An arm-section large enough to resist bending would have a very low intensity of shear.

If the number of arms be increased indefinitely, a continuous arm or web is finally reached, in which the bending action is eliminated. It may still shear off at the hub, where the area of metal is the least—*i. e.*, at minimum circumference. In this case the area under shearing stress is $\pi D_1 T$; and the force at the circumference of the hub is

$$\frac{PR}{\frac{D_1}{2}} = \frac{2PR}{D_1}$$

Equating external force to internal shearing resistance

$$\frac{2PR}{D_1} = \pi D_1 T S_s$$

or

$$S_s = \frac{2PR}{\pi D_1^2 T} \tag{65}$$

Pulley Hub. As in the case of the arms, centrifugal force does not play much part in the design of the hub of a pulley. The hub is designed principally to carry the key, and through it transmit the turning moment to the shaft. Considered thus, the hub may tear along the line of the key or crush in front of the key.

For example, in Fig. 62, if the connection with the lower arms be neglected, and the upper arms be held fast while a turning force P_1, at the surface of the shaft, is transmitted to the hub through the key, then the metal of the hub directly in front of the key is under crushing stress; and the metal along the line *eb*, from the corner to the outside, is under tensile stress. This condition is the worst that could possibly happen, because the bracing effect of the lower arms has been neglected, and the key is located between the arms.

Taking moments about the center of the shaft, the value of the force at the shaft circumference, or the "key pull," is

$$P_1 = \frac{PR}{r} \tag{66}$$

Now $\frac{P_1}{P_3} = \frac{k}{r}$, k being the distance from the center of the shaft to the center of eb, and the area of metal which is subjected to the tearing action P_3 is $l \times eb$. Equating the external force to the internal resistance, and assuming that the stress is equally distributed over the area $l \times eb$

$$P_3 = \frac{r}{k} P_1 = \frac{r}{k} \times \frac{PR}{r} = S \times l \times eb$$

or

$$S = \frac{PR}{k \times l \times eb} \tag{67}$$

The intensity of crushing on the metal in front of the key, due to force P_1, depends upon the thickness of the key, and is properly discussed later under "Keys."

Practical Modification. *Pulley Rim.* The theoretical calculation for the thickness of the rim may give a thickness that could not be cast in the foundry, and the section in that case will have to be increased. As light a section as can be readily cast will usually be found abundantly strong for the forces it has to resist. A minimum thickness at the edge of the rim is about $\frac{3}{16}$ inch; and as the pulleys increase in size, the rim also must be made thicker; otherwise the rim will cool so much more quickly than the arms, that the latter, on cooling, will develop shrinkage cracks at the point of junction.

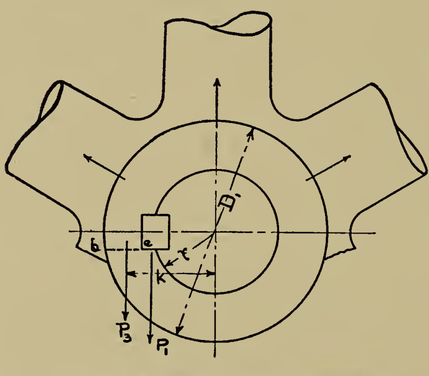


Fig. 62. Distribution of Stresses

For a velocity of 6,000 feet per minute, it is found that the tension in pounds per square inch, in the rim, due to centrifugal force, is

970. Though this in itself is a low value, yet the uncertain nature of cast iron, its condition of internal stress, due to casting, and the likely existence of hidden flaws and pockets, have established the usage of this figure as the highest safe limit for the peripheral speed of cast-iron pulleys. It is easily remembered that *cast-iron pulleys are safe for a linear velocity of about one mile per minute.*

To prevent the belt from running off the pulley, a "crown" or rounding surface is given the rim; however, a tapered face, which is more easily produced in the ordinary shop, may be used. This taper should be as little as possible, consistent with the belt staying on the pulley; $\frac{1}{2}$ inch per foot each way from the center is not too much for faces 4 inches wide and less; while above this width, $\frac{1}{4}$ inch

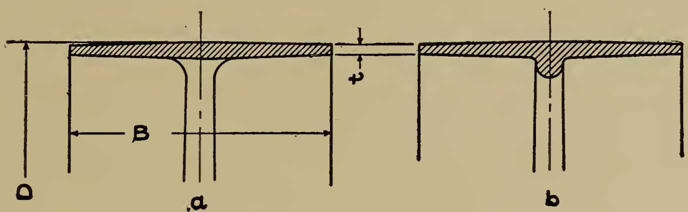


Fig. 63. Pulley Rims

per foot is enough. As little as $\frac{1}{8}$ inch total crown has been found to be sufficient on a 24-inch face, but this is probably too little for general service. Instead of being "crowned," the pulley may be flanged at the edges; but flanged pulley rims chafe and wear the edge of the belt.

The inside of the rim of a cast-iron pulley should have a taper of $\frac{1}{2}$ inch per foot to permit easy withdrawal from the foundry mould. This is known as *draft.* If the pattern be of metal, or if the pulley be machine-moulded, the greater truth of the casting does not require that the inside of the rim be turned, as the pulley, at low speeds, will be in sufficiently good balance to run smoothly. For roughly moulded pulleys, and for use at high speed, however, it is necessary that the rim be turned on the inside to give the pulley a running balance.

Fig. 63 shows a plain rim *a*, also one stiffened by a rib *b*. Where heavy arms are used this rib is essential so that there will not be too sudden change of section at the junction of rim and arm, and consequent cracks or spongy metal.

The following empirical formula is often used to determine the width of the rim

$$B = \frac{9}{8}(R + .4) \tag{68}$$

Split Pulleys. The split pulley is made in halves and provided with bolts through flanges and bosses on the hub for holding the two halves together. When the pulley is in place on the shaft, bolted up as one piece, it is subjected to the same forces as the simple pulley. Hence, its general design follows the same principles, and the fastening of the two halves, and the effect of this fastening on the detail of rim and hub, only need be studied. Practical considerations are chiefly responsible for the location of the joint in a split pulley between the arms instead of directly at the end of an arm, where theoretically it would seem to be required. It is usually more convenient in the foundry machine shop to have the joint between the arms; so it is generally placed there, and strength provided to permit this. It is possible, however, to provide a double arm, or a single split arm, in which case the joint of the pulley comes at the arm, and the "heeling" action of the rim flanges is prevented.

The rim bolts should be crowded as close as possible to the rim in order to reduce the stress on them, and also to reduce the stress in the flange itself. This practical point must not be forgotten, however, that the bolts must have sufficient clearance to be put into place beneath the rim.

While it is evident that the rim bolts are most effective in taking care of the centrifugal action on the halves of the pulley, yet in small split pulleys it is quite common to omit the rim bolts and to use the hub bolts for the double purpose of clamping the shaft and holding the two halves together. The pulley is cast with its rim continuous throughout the full circle, and it is machined in this form. It is then cracked in two by a well-directed blow of a cold chisel, the casting being especially arranged for this along the division line by cores so set that but a narrow fin of metal holds the two parts together. This provides sufficient strength for casting and turning, but permits the cold chisel to break the connection easily.

Pulley Arms. The arms should be well filleted at both rim and hub, to render the flow of metal free and uniform in the mould.

The general proportions of arms and connections to both hub and rim may perhaps be best developed by trial to scale on the drawing board. The base of the arm being determined, it may gradually taper to the rim, where it takes about the relation of $\frac{2}{3}$ to $\frac{3}{4}$ the dimensions chosen at the hub. The taper may be modified until it looks right, and then the sizes checked for strength.

Six arms are used in the great majority of pulleys. This number not only looks well, but is adapted to the standard three-jawed chucks and common clamping devices found in most shops. Elliptical arms look better than the segmental style; the flat, rectangular arm, which gives a very clumsy and heavy appearance, is seldom found except on the very cheapest work.

A double set of arms may be used on an excessively wide face, but it complicates the casting to some extent.

Although a web pulley may be calculated for shear at the hub, yet a thickness of web intermediate between the thickness of the rim and that of the hub—which will satisfy the casting requirements—will meet the requirements as to strength.

Pulley Hub. The hub should have a taper of $\frac{1}{2}$ inch per foot draft, similar to that of the inside of the rim. The length of the hub is arbitrary, but if made about $\frac{3}{4}$ the face width of the pulley will prevent rocking on the shaft.

The diameter of the hub, aside from the theoretical consideration given above, must be sufficient to take the wedging action of a taper key without splitting. This relation cannot well be calculated but probably the best rule that exists is the familiar one that *the hub should be twice the diameter of the shaft,* a rule which, however, cannot be literally adhered to, as it gives too small hubs for small shafts and too large ones for large shafts. It is always well to locate the key, if possible, underneath an arm instead of between the arms, thus gaining the additional strength due to the backing of the arm.

Special Forms of Pulleys. The plain, cast-iron pulley has been used in the foregoing discussion as a basis of design. A pulley is, however, such a common commercial article, and finds such universal use, that special forms, which can be bought in the open market, are not only cheaper but better than the plain, cast-iron pulley, at least, for regular line-shaft work.

Cast iron is a treacherous and uncertain material for rims of pulleys. It is not well suited to high fiber stresses; hence the range of speed permissible for pulley rims of cast iron is limited. Steel and wrought iron, having several times the tensional strength of cast iron, and being, moreover, much more nearly homogeneous in texture, are well suited for this work; one of the best pulleys on the market consists of a steel rim riveted to a cast-iron spider. Such an arrangement combines strength and lightness, without increasing complication or expense.

The all-steel pulley is a step further in this direction. Here the rim, arms, and hub are each pressed into shape by specially devised machinery, then riveted and bolted together. This pulley is strictly a manufactured article, which could not compete with the simpler form unless built in large quantities, enabling automatic machinery to be used. Large numbers of pulleys are built in this way, and are put on the market at reasonable prices.

Wood-rim pulleys have been made for many years, and, except for their clumsy appearance, are excellent in many respects. The rim is built up of segments in much the same way as an ordinary pattern is made, the segments being so arranged that they will not shrink or twist out of shape from moisture. The hubs may be of cast iron, bolted to wooden webs, and carrying hardwood split bushings, which may be varied in bore within certain limits so as to fit different sizes of shafting. The wooden pulley is readily and most often used in the split form, thus enabling it to be put in position easily at any point of a crowded shaft. It is often merely clamped in place, thus avoiding the use of keys or set screws, and not burring or roughening the shaft in any way.

Example. The driving force of a belt on a 36-inch pulley is 800 lbs. and the belt wrap, about 180°. Calculate proportions of the six elliptical arms to resist bending, the allowable fiber stress being 2,000 lbs. per sq. in.

Solution. Using equation (61)

$$\frac{2PR}{N} = .0393Sh^3$$

$$S = 2000 \qquad R = 18 \qquad N = 6 \qquad P = 800$$

and substituting the above values.

$$\frac{2 \times 800 \times 18}{6} = .0393 \times 2000h^3$$

$$h^3 = \frac{2 \times 800 \times 18}{6 \times .0393 \times 2000} = 61$$

$$h = \sqrt[3]{61} = 3.9365 = 4 \text{ inches, nearly.}$$

Then the thickness of the arm $= .4 \times 4 = 1.6$ or about $1\frac{5}{8}''$. Therefore, the dimensions of the arm should be $4'' \times 1\frac{5}{8}''$.

PROBLEMS FOR PRACTICE

1. Calculate the tensile stress due to centrifugal force in the rim of a 30-inch cast-iron pulley at 500 revolutions per minute.

2. A pulley 12 inches in diameter, $\frac{5}{8}$-inch web, 4-inch diameter hub, transmits 25 horse-power at a belt speed of 3,000 ft. per minute. Calculate the maximum shearing stress in the web.

SHAFTS

NOTATION—The following notation is used throughout the chapter on Shafts:

A° = Angular deflection (degrees)
B = Distance between bearings (feet)
c = Distance from neutral axis to outer fiber (inches)
d, d_0, d_2, d_3, d_4 = Diameters of shaft (inches)
d_1 = Internal diameter of shaft (inches)
E = Direct modulus of elasticity (a ratio)
e = Transverse deflection (inches)
G = Transverse modulus of elasticity (a ratio)
H = Horse-power (33,000 ft.-lbs. per minute)
I = Moment of inertia
K = Distance between bearings (inches)
L = Length along shaft (inches)
L_1, L_2 = Length of bearings (inches)
M = Simple bending moment (inch-lbs.)
M_e = Equivalent bending moment inch-lbs.)
N = Number of revolutions per minute
P = Driving force of belt (lbs.)
R = Radius at which load as stated acts (inches)
S = Fiber stress, tension, compression, or shearing (lbs. per sq. in.)
T = Simple twisting moment (inch-lbs.)
T_e = Equivalent twisting moment (inch-lbs.)
T_n = Tension in tight side of belt (lbs.)
T_o = Tension in loose side of belt (lbs.)
W = Load applied as stated (lbs.)

Analysis. The simplest case of shaft loading is shown in Fig. 64. The equal forces W, similarly applied to the disk at the distance R from its center, tend to twist the shaft off, the tendency being equal at all points of the length L between the disk and the post to which the shaft is rigidly fastened. The fastening to the post, of course, in this ideal case, takes the place of a resisting member of a machine. A state of pure torsion is induced in the shaft; and any element, such as *ca*, is distorted to the position *cb*, *aob* being the angular deflection for the distance *L*.

The case, Fig. 65, is illustrative of what occurs when a belt pulley is substituted for the simple disk. Here the twisting action is caused by the driving force of the belt, which is $T_n - T_o = P$, acting at the radius R. Torsion and angular deflection exist in the shaft, as in Fig. 64. In addition, however, another stress of a different kind has been introduced; for not only does the shaft tend to be twisted off, but the forces T_n and T_o, acting together, tend to bend the shaft, the bending moment varying with every section of the shaft, being nothing at the point *o*, and maximum at the point *c*. This combined action is the most common of any that is found in ordinary machinery.

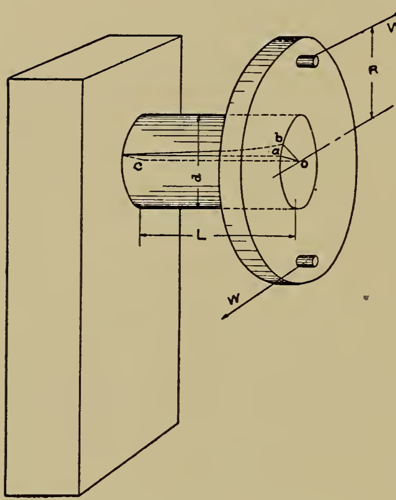


Fig. 64. Fixed Shaft with Disk

In Fig. 65, if the forces T_n and T_o be made equal, there will be no tendency at all to twist off the shaft, but the bending will remain, being maximum at the point *c*. This condition is illustrative of the case of all ordinary pins and studs in machines. In this sense, a pin or a stud is simply a shaft which is fixed to the frame of the machine, there being no tendency to turning of the pin or stud itself. The same condition would be realized if the disk in Fig. 65 were loose upon the shaft. In that case, the bending moment would be caused by $T_n + T_o$ acting with the leverage L. Of course there would have to be some resistance for $T_n - T_o$ to work against, in order that torsion should not be transmitted through the shaft. This condition might be introduced by having a similar disk lock with the first one by means of lugs on its face, thus receiving and transmitting the torsion.

If the distance L becomes very great, both the angular deflection due to twisting, and the sidewise deflection due to bending, become excessive, and not permissible in good design. This trouble is remedied by placing a bearing at some point closer to the disk, which, as it decreases L, decreases the bending moment and, therefore, the transverse deflection. The angular deflection can be decreased only by bringing the resistance and load nearer together.

Note. The above implies, of course, that the diameter of the shaft is not changed, it being obvious that increase of diameter means increase of strength and corresponding decrease of both angular and transverse deflection.

If the speed of the shaft be very high, and the distance between bearings, represented by L, be very great, the shaft will take

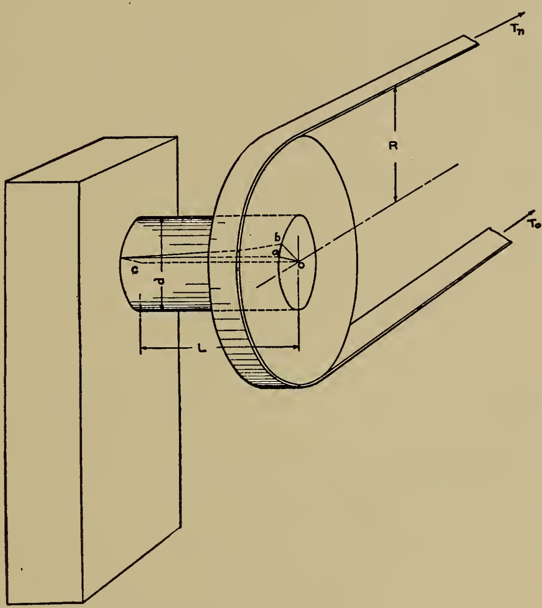


Fig. 65. Fixed Shaft with Belt

a shape like a bowstring when it is vibrated, and smooth action cannot be maintained.

It is necessary to carry the cases of Figs. 64 and 65 but a single step farther to illustrate the actual working conditions of shafting in machines. Suppose the rigid post to have the shaft passing clear through it, and to act as a bearing, so that the shaft can freely rotate in it, the resistance being exerted somewhere beyond. The twisting moment will be unchanged, also the bending moment; but the effect of the bending moment will be on each particle of the shaft in succession, first putting compression on a given particle, and then tension, then compression again, and so on, a complete cycle being performed for each revolution. This brings out a very important difference between the bending stress in pins and the bending stress in rotating shafts. In the one case the bending stress is non-reversing; in the other, reversing; and a much higher fiber stress is permissible in the former than in the latter.

Theory. *Simple Torsion.* In the case of simple torsion, the stress induced in the shaft is a shearing one. The external moment acts about the axis of the shaft, or is a polar moment; hence in the expression for the moment of the internal forces, the polar moment of inertia must be used. Now, from mechanics,

$$T = \frac{SI}{c}$$

and for circular section of diameter d

$$\frac{I}{c} = \frac{d^3}{5.1}$$

therefore

$$T = \frac{Sd^3}{5.1} \tag{69}$$

from which the diameter for any given twisting moment and fiber stress can readily be found.

For a hollow shaft this expression becomes

$$T = \frac{S(d_o^4 - d_1^4)}{5.1d_o} \tag{70}$$

Simple Bending. The stresses induced in a pin or shaft under simple bending are compression and tension. The external moment in this case is transverse, or about an axis across the shaft; hence the direct moment of inertia is applicable to the equation of forces.

$$M = \frac{SI}{c}$$

and for circular section of diameter d

$$\frac{I}{c} = \frac{d^3}{10.2}$$

therefore

$$M = \frac{Sd^3}{10.2} \tag{71}$$

For a hollow shaft or pin this expression becomes

$$M = \frac{S(d_o^4 - d_1^4)}{10.2d_o} \tag{72}$$

In the greater number of cases met with in practice, there are two or more simple stresses acting at the same time, and, although the shaft may be strong enough for any one of these alone, it may fail under their combined action. The most common cases are as follows:

Tension or Pressure Combined with Bending. In Fig. 66, the load W produces a tension acting over the whole area of d, due to its direct pull. It also produces a bending action due to the leverage R, which puts the fibers at B in tension and those at the opposite side in compression. It is evident, therefore, that by taking the algebraic sum of the stresses at either side the net stress may be obtained. It is evident that the greatest and also the controlling stress will occur on the side where the stresses add, *i. e.*, on the tension side. Hence, from mechanics,

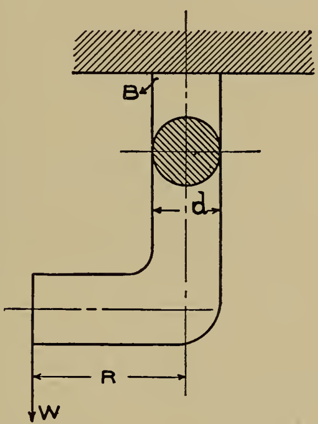


Fig. 66. Tension or Compression with Bending

$$W = \frac{\pi d^2 S}{4}$$

or due to direct tension

$$S = \frac{4W}{\pi d^2} \tag{73}$$

Also

$$WR = \frac{Sd^3}{10.2}$$

or due to bending

$$S = \frac{10.2\,WR}{d^3} \tag{74}$$

Hence, the combined tensional stress acting at the point B, or, in fact, at any point on the extreme outside of the vertical shaft toward the force W, is

$$S = \frac{4W}{\pi d^2} + \frac{10.2\,WR}{d^3} \tag{75}$$

If W acted in the opposite direction, the greatest stress would still be at the side B, but instead of a tension, would be a compression and of the same magnitude as before.

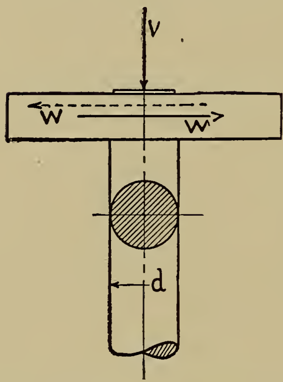


Fig. 67. Tension or Compression with Torsion

Tension or Compression with Torsion. In Fig. 67, V might be the end load on a vertical shaft; and the two forces W might act in conjunction with it, as in the case of Fig. 64, at the radius R. This case is not very often met with as it is usually possible to combine the moments, find an equivalent moment of a simple kind, and use the corresponding simple fiber stress. In the case in question a direct stress is combined with a shearing stress, and mechanics gives the following solution:

Let S_s = simple shearing stress (lbs. per sq. in.); S_c = simple compressive stress (lbs. per sq. in.); S_{rs} = resultant shearing stress (lbs. per sq. in.); S_{rc} = resultant compressive stress (lbs. per sq. in.), then

$$2WR = \frac{S_s d^3}{5.1}$$

or

$$S_s = \frac{5.1(2WR)}{d^3} \tag{76}$$

Also

$$V = \frac{\pi d^2 S_c}{4}$$

or

$$S_c = \frac{4}{\pi d^2} V \tag{77}$$

Now, from a solution given in simplest form in "Merriman's Mechanics"—which the student may consult, if desired—values for the resultant stresses may be found. Whichever of these is the critical one for the material used, should form the basis for its diameter.

$$S_{rs} = \sqrt{S_s^2 + \frac{S_c^2}{4}} \tag{78}$$

Also

$$S_{rc} = \frac{S_c}{2} + \sqrt{S_s^2 + \frac{S_c^2}{4}} \tag{79}$$

Bending Combined with Torsion. In Fig. 68, the load W acts not only to twist the shaft off, but also presses it sidewise against

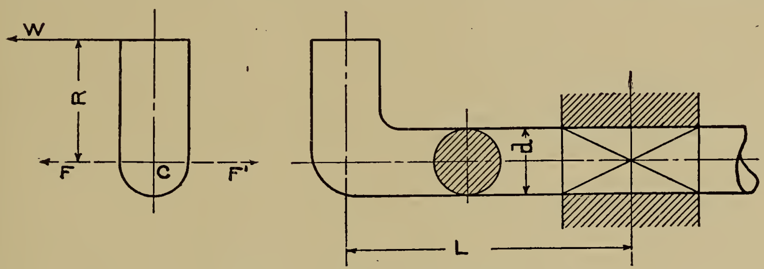


Fig. 68. Bending with Torsion

the bearings. As it is usually customary to figure the maximum moment as taking place at the center of the bearing, the length L, which determines the bending moment, is taken to that point. The theory of the stress induced in this case is complicated. In order to make the magnitude of the moments clearer, let the two equal and opposite forces F and F^1 be introduced, each equal to W, at the point C. Evidently this can be done without changing the equilibrium of the shaft in any way. W and F^1 act as a couple giving a twisting moment WR; and F acts with a leverage L, producing a bending moment $FL = WL$, at the middle of the bearing.

If, now, an equivalent twisting moment, or an equivalent bending moment is found which would produce the same effect on the fibers of the shaft as the two combined, the calculation of the diameter can be treated as a simple case, and the procedure would be the same as in the cases of simple torsion and simple bending considered above. This relation is given in mechanics as

$$M_e = \frac{M}{2} + \frac{1}{2}\sqrt{M^2 + T^2} \tag{80}$$

$$T_e = M + \sqrt{M^2 + T^2} \tag{81}$$

These expressions are true in relation to each other, on the assumption that the allowable fiber stress S is the same for tension, compression, and shearing. For the material of which shafts are usually made, this is near enough to the truth to give safe and practical results. Using the expressions for internal moments of resistance as previously noted for circular sections

$$M_e = \frac{Sd^3}{10.2} \tag{82}$$

Also

$$T_e = \frac{Sd^3}{5.1} \tag{83}$$

Either equation may be used and will give the same value for the diameter d. For the sake of simplicity, the torsion equations are generally used.

The expression $\sqrt{M^2 + T^2}$ is one that would sometimes be a tedious task to calculate. By inspection it is readily seen that this quantity may be graphically represented by means of a right-angled triangle having M and T as the sides. Lay off on a piece of paper to some convenient scale, the moments M and T, as the sides of a right-angled triangle; then, the measure of the hypothenuse gives the values of $\sqrt{M + T^2}$. Even if the drawing is made to a small scale, the accuracy of the reading will be sufficient to enable the value for d to be solved very closely.

Deflection. For a shaft subjected to pure torsion, as in Fig. 64, the angular deflection due to the load may be carried to a certain point before the limit of working fiber stress is exceeded. The equation worked out from mechanics for this condition, is

$$A^\circ = \frac{584\ TL}{Gd^4} \tag{84}$$

which gives the number of degrees of angular deflection for a shaft whose modulus of elasticity, torsional moment, and length are known.

Note. The shearing modulus of elasticity of ordinary shaft steel runs from 10,000,000 to 13,000,000, giving as an average about 12,000,000.

By the well-known relation of "Hooke's law"—stresses proportional to strains within the elastic limit of the material—

$$\frac{A°}{360°} = \frac{SL}{\pi Gd}$$

or

$$S = \frac{A° \pi Gd}{360L} \tag{85}$$

A twist of one degree in a length of twenty diameters is a usual allowance. Substituting $A = 1$, $L = 20d$, and $G = 12{,}000{,}000$,

$$S = 5{,}240, \text{ nearly} \tag{86}$$

This is a safe value for shearing fiber stress in steel. In fact, in calculations for strength, even for reversing stresses, the usual figure is 8,000 (lbs. per square inch), thus indicating that the relation of one degree to twenty diameters is well within the limit of strength.

For a hollow shaft the above formula becomes

$$A° = \frac{584\ TL}{G(d_o^4 - d_1^4)} \tag{87}$$

Transverse deflection occurs when the shaft is subjected to a bending moment. It may, therefore, exist alone or in conjunction with angular deflection. Transverse deflection of shafts, however, rarely exists up to the point of limiting fiber stress, because before that point is reached the alignment of the shaft is so disturbed that it is not practicable as a device for transmitting power. A transverse deflection of .01 inch per foot of length is a common allowance; but it is impossible to fix any general limit, as in many cases this figure, if exceeded, would do no harm, while in others—such as heavily loaded or high-speed bearings—even the figure given might be fatal to good operation.

The formula for transverse deflection, deduced from mechanics, varies with the system of loading. The three most common conditions only are given below, reference to the handbook being necessary if other conditions must be satisfied:

(1) *Fixed at one end, loaded at the other,*

$$e = \frac{WL^3}{3\,EI} \tag{88}$$

(2) *Supported at ends, loaded in middle,*

$$e = \frac{WL^3}{48\,EI} \tag{89}$$

(3) *Supported at ends, loaded uniformly,*

$$e = \frac{5WL^3}{384\,EI} \tag{90}$$

For transverse deflection the direct modulus of elasticity must be used, for the fibers are stretched or compressed, instead of being subjected to a shearing action. The most usual value of the direct modulus of elasticity for ordinary steel is 30,000,000, and is denoted in most books by the symbol E. Both the shearing and direct moduli of elasticity are really nothing but the ratio of the stress to the strain produced by that stress, it being assumed that the given material is perfectly elastic. A material is supposed to be perfectly elastic up to a certain limit of stress, and it is within this limit that the relation as above holds good. Expressed in the form of an equation this would be

$$E = \frac{S}{\frac{e}{L}} = \frac{SL}{e} \tag{91}$$

Centrifugal Whirling. If a line shaft deflects but slightly, due to its own weight, or the weight or pressure of other bodies upon it, and then be run at a high speed, the centrifugal force set up increases the deflection, and the shaft whirls about the geometrical line through the centers of the bearings, causing vibration and wear in the adjoining members. It is evident that the practical remedy for this tendency in a shaft of given diameter and speed is to locate the bearings sufficiently close to render the action of small effect.

Many formulas might be given for this relation, each being based on different assumptions. Perhaps as widely applied and as simple as any, is the Rankine formula, which sets the limit of length between bearings for shafts not greatly loaded by intermediate pulleys or side strains,

$$B = 175\sqrt{\frac{d}{N}} \tag{92}$$

Horse-Power of Shafting. Horse-power is a certain specific rate of doing work, viz, 33,000 foot-pounds per minute. Hence, to find the horse-power that a shaft will transmit, first, find the work done, and then relate it to the speed. Take, for example, the case of a pulley, using the symbols P = driving force at rim of pulley (lbs.); R = radius of pulley (inches); N = number of revolutions per minute; and H = horse-power. Then

$$\text{Work} = \text{force} \times \text{distance} = P \times (2\pi RN)$$

or

$$H = \frac{2\pi PRN}{33000 \times 12} \text{ and } PR = \frac{63025H}{N} \tag{93}$$

This is one of the most useful equations for calculations involving horse-power. By it the number of inch-pounds torsion for any horse-power can be at once ascertained.

It should be clearly noted, however, that in this equation the bending moment does not enter at all. Hence any shaft based in size on *horse-power alone,* is based on *torsional moment alone,* bending moment being entirely neglected. In many cases the bending moment is the controlling one as to limiting fiber stress. Hence empirical shafting formulas depending upon the horse-power relation are unsafe, unless it is definitely known just what torsional and bending moments have been assumed.

The only safe way to figure the size of a shaft is to find accurately what torsional moment and bending moment it has to sustain, and then combine them for the equivalent moment, introducing the element of speed as basis for assumption of a high or low working fiber stress.

Practical Modification. The practical methods of handling the theoretical shaft equations have reference to the fit of the shaft within the several pieces upon it. The running fit of a shaft in a bearing is usually considered to be so loose that the shaft could freely deflect to the center of the bearing. This is doubtless an extreme view of the case, but it is the only safe assumption. Hence a shaft running in bearings, Fig. 69, is supposed to be supported at the centers of those bearings, and its theoretical strength is based on this supposition.

For a tight or driving fit upon the shaft, a safe assumption to make is that there is looseness enough at the ends of the fit to permit the shaft to be stressed by the load a short distance within the faces of the hub, say from $\frac{1}{2}$ inch to 1 inch. Referring to Fig. 69, suppose P_1 to be the transverse load, exerted through a hub fast upon the part of the shaft d_3. Taking moments about the center of one bearing, and solving for the reaction at the center of the other,

$$P_1 u = R_1 K$$

or

$$R_1 = \frac{P_1 u}{K} \tag{94}$$

Also

$$P_1 t = R_2 K$$

or

$$R_2 = \frac{P_1 t}{K} \tag{95}$$

Now, as far as the part of shaft d_3 is concerned, it may depend for its size on the bending moment $R_2 b$, or on $R_1 a$. The reason the

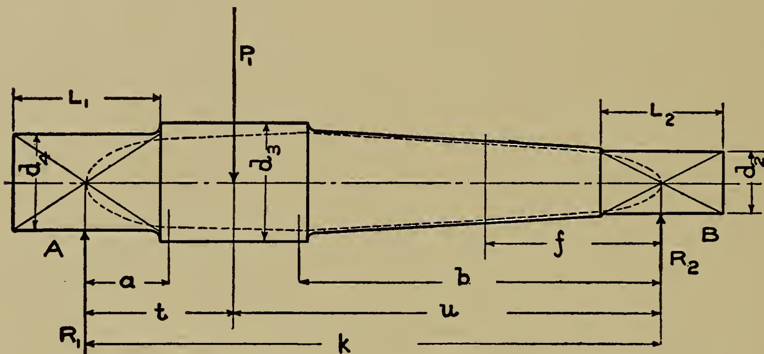


Fig. 69. Shaft with Load

lever arm is not taken to the point directly under the load P_1, is because it is not practically possible to break the shaft at that point on account of the reinforcement of the hub, which is tightly fitted upon it. Trying these moments to see which is the greater, it is found that the greater moment always occurs in connection with the longer lever arm. Hence $R_2 b$ will be greater than $R_1 a$. Writing the equation of *external moment* = *internal moment*

$$R_2 b = \frac{S\, d_3^{\,3}}{10.2}$$

or

$$d_3 = \sqrt[3]{\frac{10.2\, R_2 b}{S}} \tag{96}$$

For the size of bearing A, the maximum bending moment is

$$R_1\frac{L_1}{2}=\frac{Sd_4^3}{10.2}$$

or

$$d_4=\sqrt[3]{\frac{10.2\,R_1L_1}{2\,S}} \tag{97}$$

For the size of bearing B, the maximum moment is

$$R_2\frac{L_2}{2}=\frac{Sd_2^3}{10.2}$$

or

$$d_2=\sqrt[3]{\frac{10.2\,R_2L_2}{2\,S}} \tag{98}$$

NOTE. The above calculations are, of course, on the assumption that no torsion is transmitted either way through this axle. In that case combined torsion and bending occur. This has been made sufficiently clear in preceding paragraphs and in Part I, to require no further illustration.

The dotted line in Fig. 69 shows the theoretical shape the axle should take under the assumed conditions. The practical modification of this shape is obvious. At the shoulders of the shaft the corners should not be sharp, but carefully filleted, to avoid the possible starting of a crack at those points.

Often the diameter of certain parts of a shaft may be larger than strength actually calls for. For example, in Fig. 69, the part d_3 need only be as large as the dotted line; but it is obvious that unless the key is sunk in the body of the shaft, the hub could not be slipped into place over the part d_4. If, however, the diameter d_3 be made large enough so that the bottom of the key will clear d_4, the rotary cutter which forms the key way in d_3 will also clear d_4, and the key way can be more easily produced.

In cases where fits are not required to be snug, a straight shaft of cold-rolled steel is commonly used. Here any parts fastened on the middle of the shaft have to be driven over a considerable length of the shaft before they reach their final position. Moreover, there is no definite shoulder to stop against, and measurement has to be resorted to in locating them.

It does not pay to turn any portion of a cold-rolled shaft, unless it be the very ends, as the release of the "skin tension" in such material is sure to throw the shaft out of line and necessitate subsequent straightening.

Turned-steel shafts for machines may with advantage be slightly varied in diameter wherever the fit changes; and although the production of shoulders costs something, yet it assists greatly in bringing the parts to their exact location, and enables the workman to concentrate his best skill on the fine bearing fits, and to save time by rough-turning the parts that have no fits.

Hollow shafts are practicable only for large sizes. The advantages of removing the inner core of metal, aside from some specific requirement of the machine, are that it eliminates all possibility of cracks starting from the checks that may exist at the center, permits inspection of the material of a shaft, and, in case of hollow-forged shafts, gives an opening for the forging mandrel. In the last case, the material is improved by a rolling process.

The material most common for use in machine shafting is the ordinary *machinery steel,* made by the Bessemer process. This steel is apt to be "seamy," and often contains checks and flaws that are detected only upon sudden and unexpected breakage of a part apparently sound. This characteristic is a result of the process employed in the manufacture of the steel, and thus far has never been wholly eliminated. *Bessemer steel* is, nevertheless, a very useful material, and the above weakness is not so serious but that this kind of steel may be used with success in the great majority of cases.

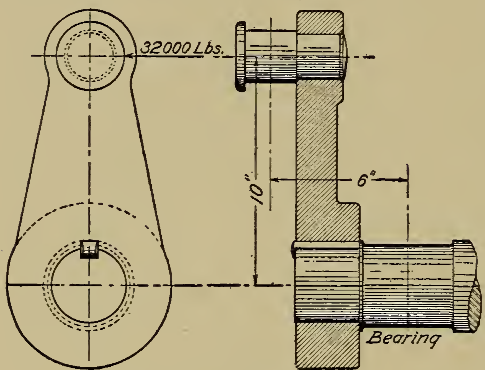


Fig. 70. Overhung Crank

When a more homogeneous shaft is desired, *open-hearth steel* is available. This is a more reliable material to use than the Bessemer, and costs somewhat more. It makes a stiff, true, fine-surfaced shaft, high-grade in every respect. It is usually specified for armature shafts of dynamos and motors.

Steels of special strength, toughness, and elasticity are made under numerous processes, of which *nickel steel* is perhaps the most conspicuous example. While this steel is very expensive, yet its great strength, in connection with other excellent qualities, makes it an extremely valuable material, wherever light weight is essential, or contracted space demands small size.

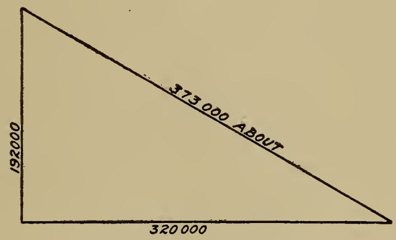


Fig. 71. Triangle of Forces

Example. The overhung crank of a steam engine shown in Fig. 70 has a force of 32,000 lbs. acting at the center of the crank pin. Assume S equal to 10,000 lbs. per sq. in. and calculate the diameter of the crank shaft.

Solution. This is an example of combined bending and torsion. There is torsion on the crank shaft due to a force 32,000 lbs. acting at the end of an arm 10 inches in length. The bending is also due to a force 32,000 lbs. acting on an arm 6 inches in length.

$$T_e = M + \sqrt{M^2 + T^2}$$

$$M_e = \frac{M}{2} + \frac{1}{2}\sqrt{M^2 + T^2}$$

$$M = 6 \times 32000 = 192000$$

$$T = 10 \times 32000 = 320000$$

The value $\sqrt{M^2 + T^2}$ may be found graphically by using a right triangle. Then drawing a right triangle to a scale such that one leg is proportional in length to 192,000 and the other to 320,000, as in Fig. 71, measure the hypothenuse and this will be found equal to 373,000 about.

$$T_e = 192000 + 373000 = 565000$$

$$M_e = \frac{192000}{2} + \frac{373000}{2} = 282500$$

By substituting the values for M_e and T_e in equations 82 and 83, respectively, the same result for the diameter is obtained.

$$M_e = \frac{Sd^3}{10.2}$$

$$282500 = \frac{10000d^3}{10.2}$$

$$\frac{282500 \times 10.2}{10000} = d^3$$

$$d^3 = 289.56$$

$$d = 6.6, \text{ say } 6\tfrac{5}{8}''$$

PROBLEMS FOR PRACTICE

1. Required the twisting moment on a shaft that transmits 30 horse-power at 120 revolutions per minute.

2. Find the diameter of a steel shaft designed to transmit 50 horse-power at 150 revolutions per minute.

3. Assuming same data as in Problem 1, find the diameters of a hollow shaft for a value of S = 8,000.

4. A belt on an idler pulley embraces an angle of 120 degrees. Assuming tension of belt 1,000 pounds on each side, and pulley located midway between bearings—which are 30 inches from center to center—what is the diameter of shaft required?

5. Calculate the diameter of a steel shaft designed to transmit a twisting moment of 400,000 inch-pounds and also to take a bending moment of 300,000 inch-pounds.

6. Find the angular deflection in a 4-inch shaft 20 feet long when subjected to a load of 5,500 pounds applied to an arm of 30-inch radius. Assume transverse modulus of elasticity equal to 12,000,000.

SPUR GEARS

NOTATION—The following notation is used throughout the chapter on Spur Gears:

b = Breadth of rectangular section of arm (inches)
C = Width of arm extended to pitch line (inches)
c = Distance from neutral axis to outer fiber (inches)
D = Pitch diameter of gear (inches)
F = Face of gear (inches)
f = Clearance of tooth at bottom (inches)
G = Thickness of arm extended to pitch line (inches)
H = Thickness of tooth at any section (inches)
μ = Coefficient of friction between teeth
N = Number of teeth
n = Number of arms
P = Diametral pitch (teeth per inch of diameter)
P^1 = Circular pitch (inches)
Q, Q_1 = Normal pressure between teeth (lbs.)
R, R_1 = Resultant pressure between teeth (lbs.)
r, r_1 = Radius of pitch circles (inches)
S = Fiber stress of material (lbs. per sq. in.)

h = Depth of rectangular section of arm (inches)
I = Moment of inertia
K = Thickness of rim (inches)
L = Distance from top of tooth to any section (inches)
M, M_1 = Revolutions per minute
s = Addendum of tooth (inches) = Dedendum of tooth
t = Thickness of tooth at pitch line (inches)
W = Load at pitch line (lbs.)
y = Coefficient for "Lewis" formula

Analysis. If a cylinder be placed on a plane surface, with its axis parallel to the plane, an attempt to rotate the cylinder about its axis would cause it to roll on the plane.

Again, if two cylinders be provided with axial bearings, and be slightly pressed together, motion of one about its axis will cause a similar motion of the other, the two surfaces rolling one on the other at their common tangent line. If moved with care, there will be no slipping in either of the above cases—which is explained by the fact that no matter how smooth the surfaces may appear to be, there is still sufficient roughness to make the little irregularities interlock and act like minute teeth.

The magnitude of the force possible to be transmitted depends not only on the roughness of the surfaces, but on the amount of pressure between them. Suppose that one cylinder is a part of a hoisting drum, on which is wound a rope with a weight attached. The weight can be made so great that, no matter how hard the two cylinders are pressed together, the driving cylinder will not turn the hoisting cylinder, but will slip past it. If now, instead of increasing the pressure, which is detrimental both to cylinders and bearings, the coarseness of the surfaces is increased, or, in other words, teeth of appreciable size are put on these surfaces, the desired result of positively driving without excessive side pressure is attained.

These artificial projections, or teeth, must fit into one another; hence, the surfaces of the original cylinders, which have been broken up into the alternate projections and hollows, have technically disappeared but nevertheless exist as ideal or imaginary surfaces. These roll together with the same surface velocities as if in bodily form, provided that the curves of the teeth are correctly formed. Several mathematical curves are available for use as tooth outlines, but in practice the *involute* and *cycloidal curves* are the only ones used for this purpose.

The ideal surfaces are known as *pitch cylinders* or *pitch circles.*

In Fig. 72 is shown an end view of such a pair of cylinders in contact at their pitch point P. In gear calculations it is assumed that there is no slip between the pitch circles acting as driving cylinders; hence the speeds of the two pitch circles at the pitch point are equal. If M and M_1 be the revolutions per minute of the cylinders, respectively, r and r_1 their radii, then

$$2\pi r M = 2\pi r_1 M_1$$

or

$$\frac{M}{M_1} = \frac{r_1}{r} \qquad (99)$$

That is, the number of revolutions varies inversely as the radii.

The simple calculation as above is the key to all calculations involving gear trains in reference to their speed ratio.

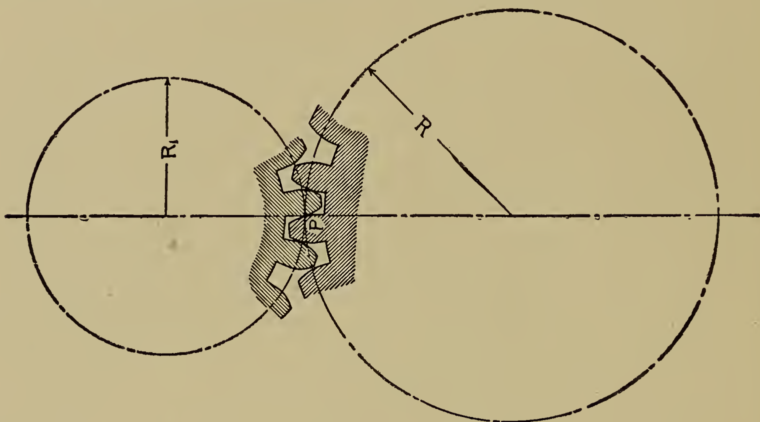


Fig. 72. Pitch Cylinders and Teeth

The distance from the center of one tooth to the center of the next measured along the pitch circle is called the *circular pitch*. The ratio of the number of teeth per inch of diameter is the *diametral pitch*. The relation of the diametral pitch, circular pitch, number of teeth, and pitch diameter is expressed in the following equations

$$P = \frac{3.1416}{P^1} \qquad (100)$$

$$D = \frac{N}{P} \qquad (101)$$

Fig. 73 represents cycloidal teeth in the two extreme positions of beginning and ending contact. The normal pressure Q or Q_1 between the teeth in each position acts through the pitch point O, as it must always do in order to insure the condition of ideal rolling of the pitch circles and the velocity ratio proportional to $\frac{r_1'}{r}$. As the surfaces of the teeth slide together, frictional resistance is produced at their point of contact. This force is widely variable, depending on the material and condition of the tooth surfaces, whether smooth and well lubricated, or rough and gritty. As this resistance acts in conjunction with the normal force between the teeth, a parallelogram of forces may be constructed on these two as a base, the resultant pressure between the teeth being slightly changed thereby, as shown in Fig. 73. It is true, however, that in nearly all cases in practice, the bending stress is the controlling one from a theoretical standpoint. Morover, the designer must consider the form and strength of the tooth when it is under the condition of maximum moment. This evidently occurs at the beginning of contact, for the follower teeth; and at the end of contact, for the driver teeth. In the particular case illustrated in Fig. 73, if the material in both gears were the same, tooth C, being the weaker at the root, would probably break before B; but if C were of steel, and B of cast iron, B might break first.

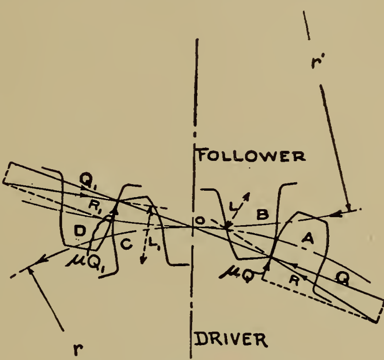


Fig. 73. Cycloidal Teeth

It will be noticed that R is nearly parallel to the top of the tooth; and it may easily happen that the friction may become of such a value that it will turn the direction of R until it lies along the top of the tooth exactly, which is the condition for maximum moment. For strength calculations it is usual to consider this condition as existing in all cases.

At the beginning of contact there is more or less shock when the teeth strike together, and this effect is much more evident at high speeds. There is also at the beginning of contact a sort of chattering action as the driving tooth rubs along the driven tooth.

Uniform distribution of pressure along the face of the tooth is often impaired by uneven wear of the bearings supporting the gear shafts, the pressure being localized on one corner of the tooth. The same effect is caused by the accidental presence of foreign material between the teeth. Again, in cast gearing, the spacing may be irregular, or, on account of draft on the pattern, the teeth may bear at the high points only. While it is usual to consider that the load is evenly distributed along the face of the tooth, yet the above considerations show that *an ample margin of strength must always be allowed* on account of these uncertainties.

When the number of teeth in the mating gears is high, the load will be distributed between several teeth; but, as it is almost certain that at some time the proper distribution of load will not exist, and that one tooth will receive the full load, it is considered that practically the only safe method is so to design the teeth that a single tooth may be relied upon to withstand the full load without failure.

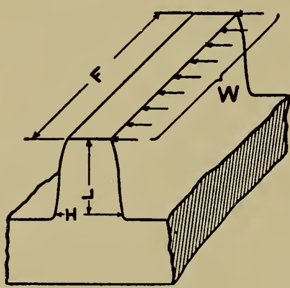


Fig. 74. Tooth as a Beam

Theory. Based on the analysis as given, the theory of gear teeth assumes that one tooth takes the whole load, and that this load is evenly distributed along the top of the tooth and acts parallel with its base, thus reducing the condition of the tooth to that of a cantilever beam. The magnitude of this load at the top of the tooth is assumed to be the same as the force transmitted at the pitch circle. This condition is shown in Fig. 74. Equating the external moment to the internal moment

$$WL = \frac{SI}{c} = \frac{SFH^2}{6} \qquad (102)$$

The thickness H is usually taken either at the pitch line or at the root of the tooth just before the fillet begins; and L, of course, is dependent on the tooth dimensions. The formula is most readily used when the outline of the tooth is either assumed or known, a trial calculation being made to see if it will stand the load, and a series of subsequent calculations followed out in the same way until a suitable tooth is found. This method is pursued because there are certain even pitches which it is desirable to use; and it is safe to say that any calculation figured the reverse way would result in fractional pitches. The latter course may be used, however, and the nearest even pitch chosen as the proper one.

As stated in the analysis, there are a great many circumstances attending the operation of gears which make impossible the purely theoretical application of the beam formulas. For this reason there is no one element of machinery which depends so much on experience and judgment for correct proportion as the tooth of a gear. Hence it is true that a rational formula based on the theoretical one is really of the greater practical value in tooth design.

If formula 102 is examined it is found that when solved for W

$$W = \frac{SFH^2}{6\,L} \tag{103}$$

Of these quantities, H and L are the only variables, the others being given any desired value. H and L depend upon the circular pitch

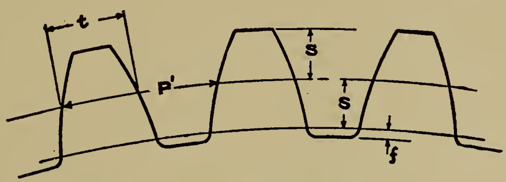


Fig. 75. American Standard Proportions

$$t = \frac{P'}{2}$$

$$f = \frac{t}{10}$$

$$s = \frac{P'}{\pi}$$

P^1 and the curvature and outline of the tooth. If now a standard system of teeth could be settled upon, a coefficient could be established to be used to take the place of the variable part of H and L, which depends on the outline of tooth, and thus an empirical formula which would be on a theoretical basis would be obtained. This, Wilfred Lewis has done; and this formula is more universally used and with more satisfactory practical results than any other formula, theoretical or practical, that has ever been devised. His coefficient is known as y, and was determined from many actual drawings of different forms of teeth showing the weakest section. This coefficient is worked out for the three most common systems as follows:

For 20° involute, $$y = 0.154 - \frac{0.912}{N} \tag{104}$$

For 15° involute, and cycloidal, $$y = 0.124 - \frac{0.684}{N} \tag{105}$$

For radial flanks, $$y = 0.075 - \frac{0.276}{N} \tag{106}$$

NOTE. The tooth upon which the above is based is the American standard, or Brown & Sharpe tooth, for which the proportions are shown in Fig. 75.

The "Lewis" formula* is

$$W = SP^1 Fy \tag{107}$$

In Table VIII may be found the value of S for different speeds.

TABLE VIII

Safe Working Stresses for Different Speeds

Speed of teeth, ft. per min.	100	200	300	600	900	1200	1800	2400
Cast iron	8000	6000	4800	4000	3000	2400	2000	1700
Steel	20000	15000	12000	10000	7500	6000	5000	4300

A usual relation of F to P^1 is:

for cast teeth $$F = 2P^1 \text{ to } 3P^1 \tag{108}$$

for cut teeth $$F = 3P^1 \text{ to } 4P^1 \tag{109}$$

The usual method of handling these formulas is as follows:

The pitch circles of the proposed gears are known or may be assumed; hence W is readily figured, as well as the speed of the teeth, whence S may be found in Table VIII. The desired relation of F to P^1 may be arbitrarily chosen, when P^1 and y become the only unknown quantities in the equation. A shrewd guess may be made for the number of teeth, and y calculated therefrom. Then solve the equation for P^1, which will undoubtedly be fractional. Choose the nearest even pitch, or, if it is desired to keep an even diametral pitch, the fractional pitch that will bring an even diametral pitch. Now, from this final and corrected pitch, and the diameter of the pitch circle, calculate the number of teeth N in the gear. Check the assumed value of y by this positive value of N.

Another good way of using this formula is to start with the pitch, and the face desired, and the diameter of the pitch circle. In this

*A full and convenient statement of the Lewis formula will be found in "Kent's Mechanical Engineers' Pocket-Book."

case W is the only unknown quantity, and when found should be compared with the load required to be carried. If it is too small, make another and successive calculations until the result approximates the required load.

RIM, ARMS, AND HUB

Analysis. The rim of a gear has to transmit the load on the teeth to the arms. It is thus in tension on one side of the teeth in action, and in compression on the other. The section of the rim,

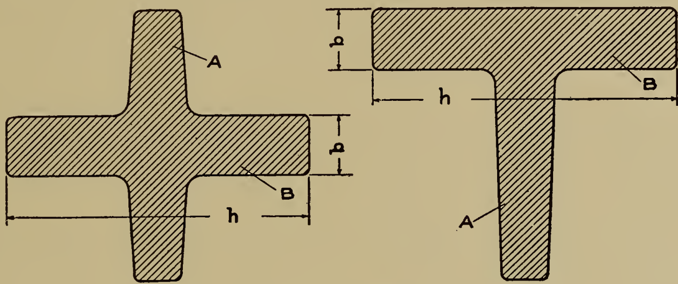


Fig. 76. Arms with Cross- and T-Sections

however, is so dependent on other practical considerations which call for an excess of strength in this respect, that it is not considered worth while to attempt a calculation on this basis.

Gears seldom run fast enough to make necessary a calculation for centrifugal force; and, in general, it may be said that the design of the rim is entirely dependent on practical considerations.

Theory. The arms of a gear are stressed the same as pulley arms, the same theory answering for both, except that a gear rim always being much heavier than a pulley rim, the distribution of load amongst the arms is better in the case of a gear than of a pulley, and it is usually safe to assume that each arm of a gear takes its full proportion of load; or, for an oval section, equating the external moment to the internal moment as in the case of pulleys,

$$\frac{WD}{2n} = 0.0393\, Sh^3 \qquad (110)$$

Heavy spur gears have the arms of a cross- or T-section, Fig. 76, the latter being especially applicable to the case of bevel gears where there is considerable side thrust. The simplest way of treating such sections is to consider that the whole bending moment is taken by the rectangular section whose greater dimension is in the direction of the load. The rest of the section, being close to the neutral axis of the section, is of little value in resisting the direct load, its function being to give sidewise stiffness. The equation for the cross- or T-style of arm, then is

$$\frac{W}{n} \times \frac{D}{2} = \frac{Sbh^2}{6} \tag{111}$$

Either b or h may be assumed, and the other determined. As a guide to the section, b may be taken at about the thickness of the tooth.

Gear hubs are in no wise different from the hubs of pulleys or other rotating pieces. The depth necessary for providing sufficient

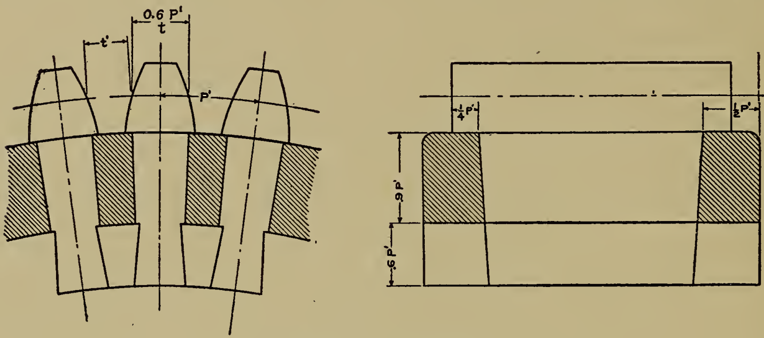


Fig. 77. Mortise Teeth

strength over the key to avoid splitting is the guiding element, and can usually be best determined by careful judgment.

Practical Modification. The practical requirements, which no theory will satisfy, are many and varied. Sudden and severe shock, excessive wear due to an atmosphere of grit and corrosive elements, abrupt reversal of the mechanism, the throwing-in of clutches and pawls, the action of brakes—these, and many other things have an important influence on gear design, but not one of them can be calculated. The only method of procedure in such cases is to base the design on analysis and theory as previously given, and then add to the face of gear, thickness of tooth, or pitch an amount which judgment and experience dictate as sufficient.

Excessive noise and vibration are difficult to prevent at high speeds. At 1,000 feet per minute, gears are apt to run with an unpleasant amount of noise. At speeds beyond this, it is often necessary to provide *mortise teeth,* or teeth of hard wood set into a cast-iron rim, Fig. 77. Rawhide pinions are useful in this regard. Fine

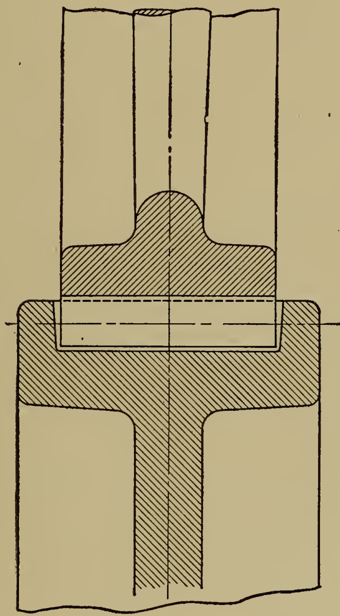


Fig. 78. Full-Shrouding

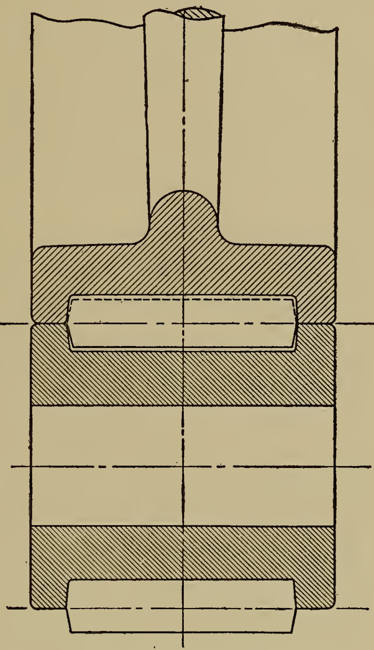


Fig. 79. Half-Shrouding

pitches with a long face of tooth run much more smoothly at high speeds than a coarse pitch and narrow-faced tooth of equal strength. Greater care in alignment of shafts, however, is necessary, also stiffer supports.

Should it be impracticable to use a standard tooth of sufficient strength there are several ways in which the carrying capacity can be increased without increasing the pitch, viz, use a stronger material, such as steel; shroud the teeth; use a hook tooth; use a stub tooth.

Shrouding a tooth consists in connecting the ends of the teeth with a rim of metal. When this rim is extended to the top of the tooth, the process is called *full-shrouding,* Fig. 78, and when carried only to the pitch line, it is termed *half-shrouding,* Fig. 79. The theoretical effect of shrouding is to make the tooth act like a short beam built in at the sides; and the tooth will practically have to be sheared out in order to fail. This modification of gear design requires the teeth to be cast, as the cutter cannot pass through the shrouding. The strength of the shrouded gear is estimated to be from 25 to 50 per cent above that of the plain-tooth type.

The *hook-tooth gear,* Fig. 80, is applicable only to cases where the load on the tooth does not reverse. The working side of the tooth is made of the usual standard curve, while the back is made of a curve of greater obliquity, resulting in a considerable increase of thickness at the root of the tooth. A comparison of strength

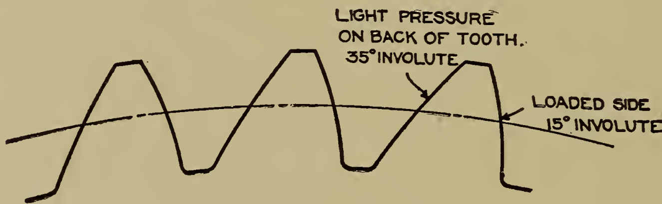


Fig. 80. Hook Teeth

between this form and the standard may be made by drawing the two teeth for a given pitch, measuring their thickness just at top of the fillet, and finding the relation of the squares of these dimensions. The truth of this relation is readily seen from an inspection of the formula of bending moment, equation (88).

The *stub tooth* merely involves the shortening of the height of the tooth in order to reduce the lever arm on which the load acts, thus reducing the moment, and thereby permitting a greater load to be carried for the same stress.

The rim of a gear is dependent for its proportions chiefly on questions of practical moulding and machining. It must bear a certain relation to the teeth and arms, so that, when it is cooling in the mould, serious shrinkage stresses will not be set up, forming pockets and cracks. Moreover, when under pressure of the cutter in the producing of the teeth, it must not chatter or spring. This condition is quite well attained in ordinary gears when the thickness of the rim below the base of the tooth is made about the same as the thickness of the tooth.

The stiffening ribs and arms must all be joined to the rim by

TABLE IX

*Gear Design Data

Diametral	P	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	5	6	8
Face	F	$6\frac{1}{4}$	$5\frac{1}{2}$	$4\frac{3}{4}$	$3\frac{3}{4}$	$3\frac{1}{4}$	$2\frac{3}{4}$	$2\frac{1}{2}$	$2\frac{1}{8}$	$1\frac{7}{8}$	$1\frac{1}{2}$
Thickness of arm when extended to pitch line	G	$1\frac{3}{8}$	$1\frac{1}{4}$	$1\frac{1}{8}$	1	$\frac{7}{8}$	$\frac{13}{16}$	$\frac{3}{4}$	$\frac{11}{16}$	$\frac{5}{8}$	$\frac{1}{2}$
Width of arm when extended to pitch line	C	4	$3\frac{1}{2}$	3	$2\frac{1}{2}$	$2\frac{1}{4}$	2	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{3}{8}$	$1\frac{1}{8}$
Thickness of rim	K	$2\frac{3}{4}$	$2\frac{3}{8}$	$2\frac{1}{8}$	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{3}{8}$	$1\frac{1}{4}$	1	$\frac{7}{8}$	$\frac{3}{4}$
Depth of rib	E	2	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{3}{8}$
Thickness of web	T	$1\frac{1}{8}$	1	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{9}{16}$	$\frac{1}{2}$	$\frac{7}{16}$	$\frac{3}{8}$	$\frac{5}{16}$

* Measurements given in inches. Letters refer to Fig. 81
Number of arms, 6.
Give inside of rims and hub a draft of $\frac{1}{2}$ inch per foot.

ample fillets, and the cross-section must be as uniform as possible, to prevent unequal cooling and consequent pulling-away of the

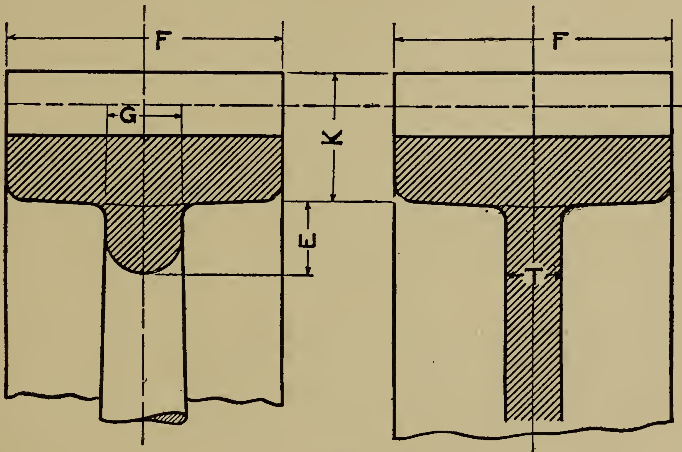


Fig. 81. Gear Proportions

arms from the rim or hub. Often the calculated size of the arms at both rim and hub has to be modified considerably to meet this requirement.

The arms are usually tapered to suit the designer's eye, a small gear requiring more taper per foot than a large one. Both rim and hub should be tapered $\frac{1}{2}$ inch per foot to permit easy drawing-out from the mould.

The proportions given in Table IX have been used with success as a basis of gear design in manufacturing practice. The table will serve as an excellent guide in laying out, and can be closely followed, in most cases with but slight modification. *Web gears* are introduced for small diameters where the arms begin to look awkward and clumsy.

BEVEL GEARS

Analysis. It is possible to consider bevel gears as the general case of which spur gears are a special form. The pitch surfaces of spur gears described as cylinders when mathematically considered, are cones whose vertices are infinitely distant, while bevel gears likewise are based on pitch cones, but with a vertex at some finite point, common to the mating pair. Hence, as might be expected, the laws of tooth action are similar in bevel gears to those in the case of spur gears. The profile of the tooth in the former case, however, is based, not on the real radius of the pitch cone, but on the radius of the normal cone; and in the development of the outline, the latter is treated just as though it were the radius of a spur gear. The tooth thus formed is wrapped back upon the normal cone face, and becomes the large end of the tapering bevel-gear tooth.

The teeth of bevel gears, being simply projections with bases on the pitch cones, have a varying cross-section decreasing toward the vertex; they also have a trapezoidal section of root, the latter section acting as a beam section to resist the cantilever moment due to the tooth load.

The arms must, as in the case of spur gears, transmit the load from the tooth to the shaft; in addition, the arms of a bevel gear are subjected to a side thrust due to the wedging action of the cones. Hence, sidewise stiffness of the arms is more essential in this type of gear than in the case of the spur gear.

Theory. It is evident that the calculations of tooth strength based on a trapezoidal section of root would be somewhat complicated; also that the trapezoid in most cases would be but little different from a true rectangle. Hence, the error will be but slight if the average cross-section of the tooth be taken to represent its strength, and the calculation made accordingly.

Fig. 82 shows a bevel-gear tooth with the average cross-section in dotted lines. For the purpose of calculation, the assumption is made that the section *A* is called the full length of the face of the gear, and that the load which this average tooth must carry is the calculated load at the pitch line of section *A*. This is equivalent to saying that the strength of a bevel-gear tooth is equal to that of a spur-gear tooth which has the same face, and a section identical with that cut out by a plane at the middle of the bevel tooth. The load, as in the case of the spur gear, should be taken at the top of the tooth; and its magnitude can be conveniently calculated at the mean pitch radius of the bevel face, without appreciable error.

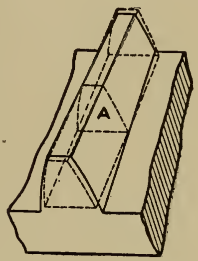


Fig. 82. Bevel Gear Tooth

This similarity to spur gears being borne in mind, the calculation for strength needs no further treatment. Once the average tooth is assumed or found by layout, a strict following-out of the methods pursued for spur-gear teeth will bring consistent results.

Practical Modification. The practical requirements to be met in transmission of power by bevel gears are the same as for spur gears; but in the case of bevel gears even greater care is necessary to provide stiffness, strength, true alignment, and rigid supports. As far as the gears themselves are concerned, a long face is desirable; but it is much more difficult to gain the advantage of its strength than in the case of spur gears, because full bearing along the length of the tooth is hard to guarantee.

The rim usually requires a series of ribs running to the hub to give required stiffness and strength against the side thrust which is always present in a pair of bevel gears. Instead of arms, the tendency of bevel-gear design, except for very large gears, is toward a web on account of the better and more uniform connection thereby secured between rim and hub. This web may be lightened by a number of holes, so that the resultant effect is that of a number of wide and flat arms.

The hubs naturally have to be fully as long as those of spur gears, because there is greater tendency to rock on the shaft, due to the side thrust from the teeth.

The teeth on small gears are cut with rotary cutters, at least two finishing cuts being necessary, one for each side of the tapering tooth. A more accurate method is to plane the teeth on a special gear planer, a method which is followed on all gears of any considerable size. The practical requirement here is that no portion of the hub shall project so as to interfere with the stroke of the planer tool. The requirements of gear planers vary somewhat in this regard.

Finally, after all that is possible has been done in the design of the gear itself to render it suitable to withstand the varied stresses, especial attention must be paid to the rigidity of the supporting shafts and bearings. Bearings should always be close up to the hubs of the gears, and, if possible, the bearing for both pinion and gear should be cast in the same piece. If this is not done, the tendency of the separate bearings to get out of line and destroy the full bearing of the teeth is difficult to control. Thrust washers are desirable against the hubs of both pinion and gear; also proper means of well lubricating the same.

With these considerations carefully met, bevel gears are not the bugbear of machine design that they are sometimes claimed to be. The common reason why bevel gears cut and fail to work smoothly, is that the gears and supports are not designed carefully enough in relation to each other. This is also true of spur gears, but the bevel gear will reveal imperfections in its design far the more quickly of the two.

WORM AND WORM GEAR

NOTATION—The following notation is used throughout the chapter on Worm and Worm Gear:

D = Pitch diameter of gear (inches)
E = Efficiency between worm shaft and gear shaft (per cent)
f = Clearance of tooth at bottom (in.)
i = Index of worm thread (1 for single, 2 for double, etc.)
L = Lead of worm thread (inches)
M = Revolutions of gear shaft per minute
P^1 = Circular pitch = Pitch of worm thread (inches)
R = Radius of pitch circle of worm gear (inches)
s = Addendum of tooth (inches)
T = Twisting moment on gear shaft (inch-lbs.)
T_w = Twisting moment on worm shaft (inch-lbs.)

M_w = Revolutions of worm shaft per minute
N = Number of teeth in gear
t = Thickness of tooth at pitch line (inches)
W = Load at pitch line (lbs.)

Analysis. The simplest way of analyzing the case of the worm and worm gear is to base it upon an ordinary screw and nut. Take for example, the lead screw of a common lathe. The carriage carries a nut through which the lead screw passes. By the rotation of the screw, the carriage, being constrained by the guides to travel lengthwise of the ways, is moved. This motion is, for a single-threaded screw, a distance per revolution equal to the lead of the screw.

Now, suppose that the carriage, instead of sliding along the ways, is compelled to turn about an axis at some point below the ways. Also, suppose the top of the nut to be cut off, and its length made endless by wrapping it around a circle struck from the center about which the carriage rotates. This reduces the nut to a peculiar kind of spur gear, the partial threads of the nut now having the appearance of twisted teeth.

This special form of spur gear, based on the idea of a threaded nut, is known as a *worm gear,* and the screw is termed a *worm.* The teeth are loaded similarly to those of a spur gear, but with the additional feature of a large amount of sliding along the tooth surfaces. This, of course, means considerable friction; and it is, in fact, possible to utilize the worm and worm gear as an efficient device only by running the teeth constantly in a bath of oil. Even then the pressures have to be kept well down to insure the required term of life of the tooth surfaces.

It is evident that for one revolution of a single-threaded worm, one tooth of the gear will be passed. The speed ratio between the worm gear and worm shaft will then be equal to the number of teeth in the gear, which is relatively great. Hence the worm and worm gear are principally useful in giving large speed reduction in a small amount of space.

Theory. The theory of worm-wheel teeth is complicated and obscure. The production of the teeth is simple, a dummy worm with cutting edges, called a *hob,* being allowed to carve its way into the worm-gear blank, thus producing the teeth and at the same time driving the worm gear about its axis.

It is clear that if the torsional moment on the worm-gear shaft and the pitch radius of the worm gear are known, the load on the teeth at the pitch line can be found by dividing the former by the latter. Expressed as an equation

$$WR = T$$

or

$$W = \frac{T}{R} \tag{112}$$

How this value of W is distributed on the teeth, is a question difficult to answer. The teeth not only are curved to embrace the worm, but are twisted across the face of the gear, so that it would be practically impossible to devise a purely theoretical method of exact calculation. The most reasonable thing to do is to assume the teeth as being equally as strong as spur-gear teeth of the same circular pitch, and to figure them accordingly. It is probably true, however, that the load is carried by more than one tooth, especially in a hobbed wheel; so it will be safe to assume that two—and, in case of large wheels, three—teeth divide the load between them. With these considerations borne in mind, the case reduces itself to that of a simple spur-gear tooth calculation, which has already been explained under "Spur Gears."

The worm teeth are probably always stronger than the worm-gear teeth; so no calculation for their strength need be made.

The twisting moment on the worm shaft is not determined so directly as in the case of spur gears. The relative number of revolutions of the two shafts depends upon the lead of the worm thread and the number of teeth in the gear. *Lead* L is the distance parallel to the axis of the worm which any point in the thread advances in one revolution of the worm. *Pitch* P^1 is the distance parallel to the axis of the worm between corresponding points on adjacent threads. The distinction between lead and pitch should be carefully observed, as the two are often confounded, one with the other.

The thread may be single, double, triple, etc., the index of the thread i, being 1, 2, 3, etc., in accordance therewith. The relation between lead and pitch may then be expressed by the equation

$$L = iP^1 \tag{113}$$

When the index of the thread is changed the speed ratio is changed, the relation being shown by the equation

$$\frac{M}{M_w} = \frac{i}{N} \tag{114}$$

If the efficiency were 100 per cent between the two shafts, the twisting moments would be inversely as the ratio of the speeds, thus

$$\frac{T_w}{T} = \frac{M}{M_w} = \frac{i}{N}$$

or

$$T_w = \frac{Ti}{N} \tag{115}$$

but for an efficiency E the equation would be

$$\frac{T_w}{T} = \frac{i}{EN}$$

or

$$T_w = \frac{Ti}{EN} \tag{116}$$

The diameter of the worm is arbitrary. A change of this diameter has no effect on the speed ratio but has a slight effect on the efficiency, the smaller worm giving a little higher efficiency. The diameter of the worm runs ordinarily from 3 to 10 times the circular pitch, an average value being $4P^1$ or $5P^1$.

A longitudinal cross-section through the axis of the worm cuts out a rack tooth, and this tooth section is usually made of the standard 15° involute form, shown in Fig. 83, for a rack.

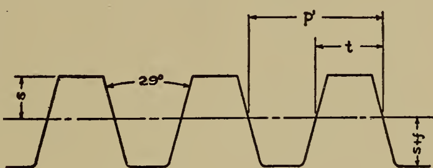


Fig. 83. Rack

The end thrust, of a magnitude practically equal to the pressure between the teeth, has to be taken by the hub of the worm against the face of the shaft bearing. A serious loss of efficiency from friction is likely to occur here. This is often reduced, however, by roller or ball bearings. With two worms on the same shaft, each driving into a separate worm gear, it is possible to make one of the worms right-hand thread, and the other left-hand, in which case the thrust is self-contained in the shaft itself, and there is absolutely no end thrust against the face of the bearing. This involves a double outfit throughout, and is not always practicable.

There are few mathematical equations necessary for the dimensioning of a worm and worm gear. The formulas for the tooth parts as given on pages 131 and 132 apply equally well in this case.

Practical Modification. The discussion of the efficiency E of the worm and worm gear is more of a practical than of a theoretical nature. It seems to be true from actual operation, as well as theory, that the steeper the threads the higher the efficiency. In actual practice there is seldom an opportunity to change the slope of the thread to get increased efficiency. The slope is usually settled from considerations of speed ratio, or available space, or some other condition. The usual practical problem is to take a given worm and worm gear, and to make out of it as efficient a device as possible. With hobbed gears running in oil baths, and with moderate pressures and speeds, the efficiency will range between 40 per cent and 70 per cent. The latter figure is higher than is usually attained.

To avoid cutting and to secure high efficiency, it seems essential to make the worm and the gear of different materials. The worm-thread surfaces being in contact a greater number of times than the gear teeth, should evidently be of the harder material. Hence, the worm is usually made of steel, and the gear of cast iron, brass, or bronze. To save the expense of a large and heavy bronze gear, it is common to make a cast-iron center and bolt a bronze rim to it.

The worm being the most liable to replacement from wear, it is desirable to so arrange its shaft fastening and general accessibility that it may be readily removed without disturbing the worm gear.

The circular pitch of the gear and the pitch of the worm thread must be the same, and the practical question comes in as to the threads per inch possible to be cut in the lathe in the production of the worm thread. The pitch must satisfy this requirement; hence the pitch will usually be fractional, and the diameter of the worm gear, to give the necessary number of teeth, must be brought to it. While it would perhaps be desirable to keep an even diametrical pitch for the worm gear, yet it would be poor design to specify a worm gear which could not be cut in a lathe.

The standard involute of 15°, and the standard proportions of teeth as given on page 132, are usually used for worm threads. This system requires the gear to have at least 30 teeth, for if fewer teeth are used the thread of the worm will interfere with the flanks of the gear teeth. This is a mathematical relation, and there are methods of preventing it by a change of tooth proportions or a change of angle of worm thread; but as there are few instances in which less than 30 teeth are required, it is not deemed worth while to go into a lengthy discussion of this point.

The angle of the worm embraced by the worm-gear teeth varies from 60° to 90°, and the general dimensions of rim are made about the same as for spur gears. The arms, or the web, have the same reasons for their size and shape. Probably web gears and cross-shaped arms are more common than oval or elliptical sections.

Worm gears sometimes have cast teeth, but they are for the roughest service only, and give but a point bearing at the middle of the tooth. An accurately hobbed worm gear will give a bearing clear across the face of the tooth, and, if properly set up and cared for, makes a good mechanical device although admittedly of somewhat low efficiency.

Example. A tooth load of 1,200 lbs. is transmitted between two spur gears of 12-inch and 30-inch diameter, respectively, the latter gear making 100 revolutions per minute. Calculate a suitable pitch and face of tooth by the "Lewis" formula.

Solution. Let a cast-iron gear be used. Speed of the gear

$= 100 \times 3.1416 \times \frac{30}{12} = 785+$ ft. per minute. (See Table VIII, Page 132.)

$$S = 3000 \text{ lbs. per sq. in.}$$

If the cycloidal tooth is used

$$y = .124 - \frac{.684}{N}$$

Assume $N = 90$, then

$$y = .124 - \frac{.684}{90} = .124 - .007 = .117$$

$$W = SP^1Fy$$

Use machine cut teeth; then

$$F = 3P^1$$

$$W = SP^1 3P^1 y = 3S(P^1)^2 y$$

$$P^1 = \sqrt{\frac{W}{3Sy}} = \sqrt{\frac{1200}{3 \times 3000 \times .117}} = \sqrt{1.14} = 1.07$$

Substituting the value of P^1 in equation 100

$$P = \frac{3.1416}{1.07} = 2.9$$

With 90 teeth the diametral pitch would be 3, and as 90 teeth were assumed in the above calculation it is not necessary to check the result any further

PROBLEMS FOR PRACTICE

1. Calculate proportions of a standard gear tooth of $1\frac{1}{2}$ diametral pitch, making a rough sketch and putting the dimensions on it.

2. Suppose the above tooth to be loaded at the top with 5,000 lbs. If the face be 6 inches, calculate the fiber stress at the pitch line, due to bending.

NOTE. In solving the next three problems use data given in the illustrative example.

3. Assuming a $\frac{1}{2}$-inch web on the 12-inch gear, calculate the shearing fiber stress at the outside of a hub 4 inches in diameter.

4. Design elliptical arms for the 30-inch gear, allowing S = 2,200.

5. Design cross-shaped arms for 30-inch gear.

BEARINGS, BRACKETS, AND STANDS

NOTATION—The following notation is used throughout the chapter on Bearings, Brackets, and Stands:

A = Area (square inches)
a = Distance between bolt centers (inches)
b = Width of bracket base (inches)
c = Distance of neutral axis from outer fiber (inches)
D = Diameter of shaft (inches)
d = Diameter of bolt body (inches)
d_1 = Diameter at root of thread (inches)
H = Horse-power
h = Thickness of cap at center (inches)
μ = Coefficient of friction (per cent)
N = Number of revolutions per minute
n = Number of bolts in cap
n_1 = Number of bolts in bracket base
P = Total pressure on bearing (lbs.)
p = Pressure per square inch of projected area (lbs.)
S = Safe tensile fiber stress (lbs.)
S_s = Safe shearing fiber stress (lbs.)
T = Total load on bolts at top of bracket (lbs.)
t = Thickness of bracket base (inches)
x = Distance from line of action of

I = Moment of inertia
L = Length of bearing (inches)

load to any section of bracket (inches)

Analysis. Machine surfaces taking weight and pressure of other parts in motion upon them are, in general, known as *bearings.* If the motion is rectilinear, the bearing is termed a *slide, guide,* or *way,* such as the cross-slide of a lathe, the cross-head guide of a steam engine, or the ways of a lathe bed.

If the motion is a rotary one, like that of the spindle of a lathe the simple word *bearing* is generally used.

In any bearing, sliding or rotary, there must be strength to carry the load, stiffness to distribute the pressure evenly over the full bearing surface, low intensity of such pressure to prevent the lubricant from being squeezed out and to minimize the wear, and sufficient radiating surface to carry away the heat generated by the friction of the surfaces as fast as it is generated. Sliding bearings are of such varied nature, and exist under conditions so peculiar to each case, that a general analysis is practically impossible.

Rotary bearings can be more definitely studied, as there are but two variable dimensions, diameter and length, and it is the proper relation between these two facts that determines a good bearing. The size of the shaft, as noted under "Shafts," is calculated by taking the bending moment at the center of the bearing, combining it with the twisting moment, and solving for the diameter consistent with the assumed fiber stress. But this size must then be tried for deflection due to the bending load, in order that the requirement for stiffness may be fulfilled. When this is accomplished, the friction at the bearing surface may still generate so much heat that the exposed surface of the bearing will not radiate it as fast as generated, in which case the bearing gets hotter and hotter, until it finally burns out the lubricant and melts the lining of the bearing, and ruin results.

The heat condition is usually the critical one, as it is very easy to make a short bearing which is strong enough and amply stiff for the load it carries, but which nevertheless is a failure as a bearing, because it has so small a radiating surface that it cannot run cool.

The side load, which causes the friction and the consequent development of heat, is due *to the pull of the belt* in the case of pulleys, *the load* on the teeth of gears, *the pull* on cranks and levers, *the weight of parts*, etc. If pure torsion could be exerted on shafts without any side pressure, and all the weight that comes on the shaft counteracted, trouble would not be encountered due to the development of heat in bearings; in fact, there would theoretically be no need of bearings, as the shafts would naturally spin about their axes, and would not need support.

It can be shown, theoretically, that the radiating surface of a bearing increases relatively to the heat generated by a given side load, *only when the length of the bearing is increased.* In other words, increasing the diameter and not the length, theoretically increases the heat generated per unit of time just as much as it increases the radiating surface; hence nothing is gained, and heat accumulates in the bearing as before. This important fact is verified by the design of high-speed bearings, which, it is always noted, are very long in proportion to their diameter, thus giving relatively high radiating power.

Bearings must be rigidly fastened to the body of the machine in some way, and the immediate support is termed a *bracket, frame,* or *housing*. Bracket is a very general term, and applies to the supports of other machine parts besides bearings. It is especially applicable to the more familiar types of bearing supports, and is here introduced to make the analysis complete.

The bracket must be strong enough as a beam to take the side load, the bending moment being figured at such points as are necessary to determine its outline. It may be of solid, box, or ribbed form, the latter being the most economical of material, and usually permitting the simplest pattern. The fastening of the bracket to the main body of the machine must be broad to give stability; the bolts act *partly in shear* to keep the bracket from sliding along its base, and *partly in tension* to resist its tendency to rotate about some one of its edges, due to the side pull of the belt, gear tooth, or lever load, as the case may be. The weight of the bracket itself and of the parts it sustains through the bearing, has likewise to be considered; and this acts, in conjunction with the working load on the bearing, to modify the direction and magnitude of the resultant load on the bracket and its fastening.

Stands are forms of brackets, and are subject to the same analysis.

The distinction is by no means well defined, although a stand is usually considered as having an upright or inverted position with reference to the ground. The ordinary "hanger" is a good example of an inverted stand; and the regular "floor stand," found on jack shafts in some power houses, is an example of the general class.

Theory. As the method of calculation of the diameter of the shaft, as well as its deflection, has been considered under "Shafts," the theoretical study of bearings may be assumed as starting on a given basis of shaft diameter D. The main problem then being one of heat control, let the amount of heat developed in a bearing by a given side load be first calculated. The force of friction acts at the circumference of the shaft, and is equal to the coefficient of friction times the normal force; or, for a given side load P, Fig. 84, the force of friction would be μP. The peripheral speed of the shaft for N revolutions per minute is $\frac{\pi DN}{12}$ feet per minute. As work is *force times distance*, the work wasted in friction is then $\frac{\mu P\pi DN}{12}$ foot-pounds per minute. One horse-power being equal to 33,000 foot-pounds per minute

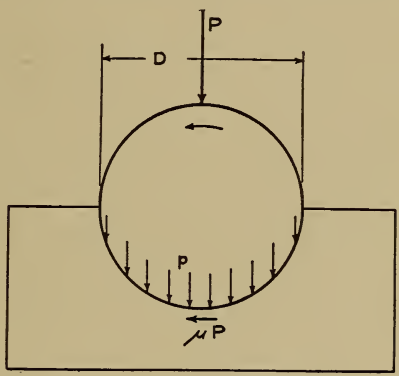


Fig. 84. Friction in Bearing

$$H = \frac{\mu P\pi DN}{12 \times 33000} \tag{117}$$

The value of μ for ordinary, well-lubricated bearings, may run as low as 5 per cent; but as the lubrication is often impaired, it quite commonly rises to 10 or 12 per cent. A value of 8 per cent is a fair average. This amount of horse-power is dissipated through the bearing in the form of heat. If the ability of each particle of the metal around the shaft to transmit the heat, or to pass it along to the outside of the casting could be determined and then if the ability of the particles of air surrounding the casting to receive and carry away this heat could be determined, just such proportions of the bearing and its casing as would never choke or retard this free transfer of heat from the running surface could be calculated.

Such refined theory is not practical, owing to the complicated shapes and conditions surrounding the bearing. The best that can be said is that for the usual proportions of bearings the side load may exist up to a certain intensity of "pressure per square inch of projected area" of bearing, or, in form of an equation,

$$P = pLD \tag{118}$$

The constant p is of a variable nature, depending on lubrication, speed, air contact, and other special conditions. For ordinary bearings having continuous pressure in one direction, and only fair lubrication, 400 to 500 is an average value. When the pressure

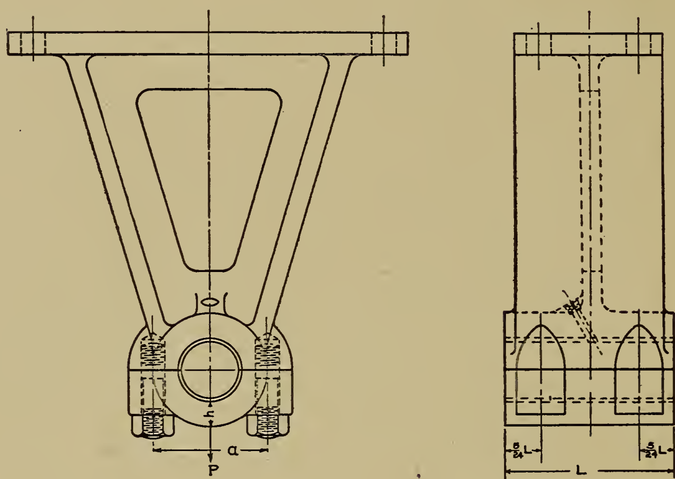


Fig. 85. Hanger

changes direction at every half-revolution, the lubricant has a better chance to work fully over the bearing surface, and a higher value is permissible, say, 500 to 800. In locations where mere oscillation takes place, not continuous rotation, and reversal of pressure occurs, as on the cross-head pin of a steam engine, p may run as high as 900 to 1,200. On the crank pins of locomotives, which have the reversal of pressure, and the benefit of high velocity through the air to facilitate cooling, the pressures may run equally high. On the eccentric crank pins of punching and shearing machines, where the pressure acts only for a brief instant and at intervals, the pressure ranges still higher without any dangerous heating action.

When a bearing, for practical reasons, is provided with a cap held in place by bolts or studs, the *theory* of the cap and bolts is of little importance, unless the load comes directly against the cap and bolts. Except in the latter case, the proportions of the cap and the size of the bolts are dependent upon general appearance and utility, it being manifestly desirable to provide a substantial design, even though some excess of strength is thereby introduced.

For the worst case of loading, however, which is when the cap is acted upon by the direct load, such as P in Fig. 85, there exists a condition of a centrally loaded beam supported at the bolts. It is probable that the beam is partially fixed at the ends by the clamping of the nut; also that the load P, instead of being concentrated at the center, is to some extent distributed. It is hardly fair to assume the external moment equal to $\frac{Pa}{8}$ or $\frac{Pa}{4}$, the one being too small, perhaps, and the other too large. It will be reasonable to take the external moment at $\frac{Pa}{6}$, in which case, equating the external moment to the internal moment of resistance,

$$\frac{Pa}{6}=\frac{SI}{c}=\frac{SLh^2}{6} \tag{119}$$

from which, the length of bearing being known, the thickness h may be calculated.

One bolt on each side is sufficient for bearings not more than 6 inches long, but for longer bearings usually two bolts on a side are used. The theoretical location for two bolts on a side, in order that the bearing may be equally strong at the bolts and at the center of the length, may be shown by the principles of mechanics to be $\frac{5}{24}L$ from each end, as indicated in Fig. 85.

The bolts are evidently in direct tension, and if equally loaded would each take their fractional share of the whole load P. This is difficult to guarantee, and it is safer to consider that $\frac{2}{3}P$ may be taken by the bolts on one side. On this basis, assuming a total number of bolts n, and equaling the external force to the internal resistance of the bolts

$$\frac{2}{3}P = \frac{S\pi d_1^2}{4} \times \frac{n}{2} \tag{120}$$

from which the proper commercial diameter may be readily found.

The bracket may have the shape shown in Fig. 86. The portion at B is under direct shearing stress; and if A be the area at the

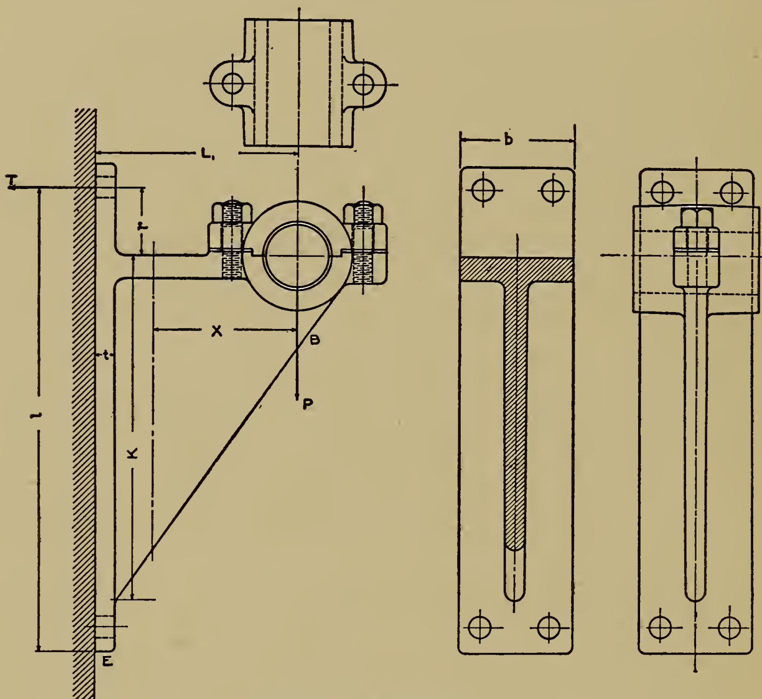


Fig. 86. Wall Bearing

point, and S_s the safe shearing stress, then, equating the external force to the internal shearing resistance,

$$P = AS_s \tag{121}$$

The same shear comes on all parts of the bracket to the left of the load, but there is an excess of shearing strength at these points.

At the point of fastening, the bolts are in shear, due to the same load, for which the equation is

$$P = \frac{\pi d^2}{4} n_1 S_s \tag{122}$$

For the upper bolts, the case is that of direct tension, assuming that the whole bracket tends to rotate about the lower edge E. To find the load T on these bolts, moments about the point E are taken as follows:

$$PL_1 = Tl$$

or

$$T = \frac{PL_1}{l} \tag{123}$$

Then, equating the external force to the internal resistance,

$$T = \frac{PL_1}{l} = \frac{\pi d_1^2}{4} \times \frac{n_1}{2} S \tag{124}$$

The upper flange is loaded with the bolt load T, and tends to break off at the point of connection to the main body of the bracket, the external moment, therefore, being Tr. The section of the flange is rectangular; hence the equation of external and internal moments is

$$Tr = \frac{PL_1}{l} r = \frac{Sbt^2}{6} \tag{125}$$

It may be noted that the lower bolts act on such a small leverage about E, that they would stretch and thus permit all the load to be thrown on the upper bolts; this is the reason why they are not subject to calculation for tension.

The section of the bracket to the left of the load P is dependent upon the bending moment, for, if this section is large enough to take the bending moment properly, the shear may be disregarded. It should be calculated at several points, to make sure that the fiber stress is within allowable limits. The general expression for the equation of moments is, for any section at leverage x,

$$Px = \frac{SI}{c} \tag{126}$$

from which, by the proper substitution of the moment of inertia of the section, the fiber stress may be calculated. The moment of inertia for simple ribbed sections may be found in most handbooks. The process of solution of the above equation, though simple, is apt to be tedious, and is not considered necessary.

Practical Modification. *Adjustment.* Adjustment is an important practical feature of bearings. Unless the proportions are so ample that wear is inappreciable, simple and ready adjustment must be provided. The taper bushing, Fig. 87, is neat and satisfac-

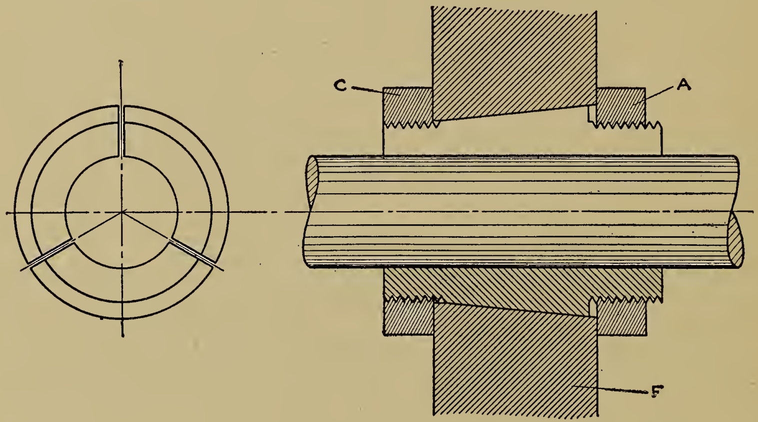


Fig. 87. Taper Bushing

tory for machinery in which expense and refinement are permissible. Though this is true of some machine tools, it is not true of the general run of bearings. The most common form of adjustment is secured by the plain cap—which may or may not be tongued into the bracket—with liners placed in the joint when new, which may subsequently be removed or reduced so as to allow the cap to close down upon the shaft. Several forms of cap bearings are illustrated in Figs. 88, 89, and 90.

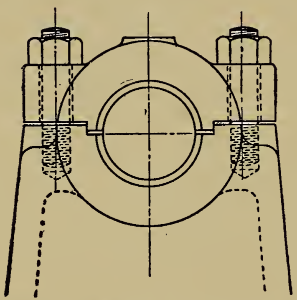


Fig. 88. Tongued Cap Bearing

Large engine shaft bearings have special forms of adjustment such as wedges and screws, which take up the wear in all directions at the same time accurately preserving the alignment of the shafts; but this refinement is seldom required for shafts of ordinary machinery.

In cases where the cap bearing is not applicable, a simple bushing may be used. This may be removed when worn, and a new one inserted, the exact alignment being maintained, as the outside will be concentric with the original axis of shaft, regardless of the wear which has taken place in the bore.

Lubrication. The lubrication of bearings is a part of the design, in that the lubricant should be introduced at the proper point, and pains taken to guarantee its distribution to all points of the running surface. The method of lubrication should be so certain that no excuse for its failure would be possible. Grease is a successful lubricator for heavy loads and slow speeds, oil for light loads and high speeds.

In order to insure the lubricant reaching the sliding surfaces and entering between them, it must be introduced at a point where the pressure is moderate, and where the motion of the parts will

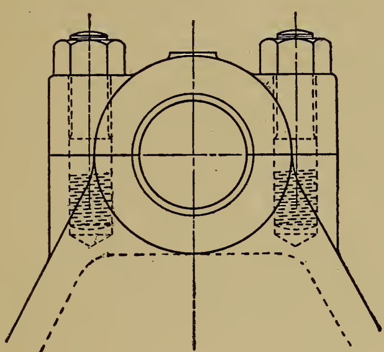


Fig. 89. Plain Cap Bearing

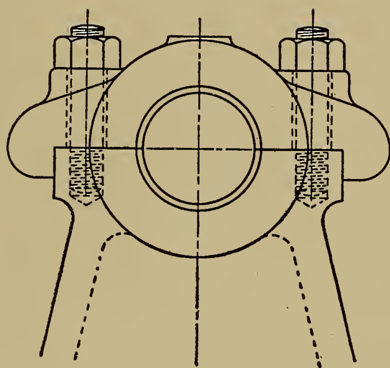


Fig. 90. Bracket Forming Tongue

naturally lead it to all points of the bearing. Grooves and channels of ample size assist in this regard. A special form of bearing uses a ring riding on the shaft to carry the oil constantly from a small reservoir beneath the shaft up to the top, where it is distributed along the bearing and finally flows back to the reservoir and is used again.

Materials. The materials of which bearings are made vary with the service required and with the refinement of the bearing. *Cast iron* makes an excellent bearing for light loads and slow speeds, but it is very apt to "seize" the shaft in case the lubrication is in the least degree impaired. *Bronze,* in its many forms of density and hardness, is extensively used for high-grade bearings, but it also has little natural lubricating power, and requires careful attention to keep it in good condition. *Babbitt,* a composition metal of varying degrees of hardness, is the most universal and satisfactory material for ordinary bearings. It affords a cheap method of production, being poured in molten form around a mandrel, and firmly retained in its casing or shell through dovetailed pockets into which the metal flows and hardens. It requires no boring or extensive fitting; some scraping to uniform bearing is necessary in most cases, but this is easily and cheaply done. Babbitt is a durable material, and has some natural lubricating power, so that it has less tendency to heat with scanty lubrication than any of the materials previously mentioned. Almost any grade of bearing may be produced with babbitt. In its finest form the babbitt is hammered, or pened, into the shell of the bearing, and then bored out nearly to size, a slightly tapered mandrel being subsequently drawn through, compressing the babbitt and giving a polished surface.

A combination bearing of babbitt and bronze is sometimes used. In this the bronze lies in strips from end to end of the bearing, and the babbitt fills in between the strips. The shell, being of bronze, gives the required stiffness, and the babbitt the favorable running quality.

Examples. 1. The journals on the tender of a locomotive are $3\frac{1}{2} \times 7$ inches. The total weight of the tender and load is 60,000 pounds. If there are 8 journals, what is the pressure per square inch of the projected area?

Solution. Projected area of one journal $= 3\frac{1}{2} \times 7 = 24\frac{1}{2}$ sq. in. Total projected area $= 8 \times 24\frac{1}{2} = 196$ sq. in

$$\frac{60000}{196} = 306 \text{ lbs. per sq. in.}$$

2. What horse-power is lost in friction at the circumference of a 3-inch bearing carrying a load of 6,000 pounds, if the number of revolutions per minute is 150 and the coefficient of friction is assumed to be 5 per cent?

Solution.

$$H = \frac{\mu P \pi D N}{12 \times 33000}$$

$$H = \frac{.05 \times 6000 \times 3.1416 \times 3 \times 150}{12 \times 33000} = 1.07$$

PROBLEMS FOR PRACTICE

1. The allowable pressure on a bearing is 300 pounds per square inch of projected area. What is the required length of the bearing if the total load is 4,500 pounds and the diameter is 3 inches?

2. The cross-head pin of a steam engine must be 2.5 inches in diameter to withstand the shearing strain. If the maximum pressure is 10,000 pounds, what length should be given to the pin?

3. The cast-iron bracket in Fig. 86 has a load P of 1,000 pounds. Determine the fiber stress in the web section at the base of the bracket if the thickness is taken at $\frac{1}{2}$ inch, and $L_1 = 12$ inches; $l = 20$ inches; $k = 11$ inches.

4. Calculate the diameter of the bolts at the top of the bracket.

5. Assuming r equal to 6 inches, what is the fiber stress at the root of flange?

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