Esté

# Design a SSV

# SMALL SOLAR VEHICLE

## Case SSV part 2

2012-2013

# 1. New and improved Sankey-Diagram

For making a more correct Sankey-diagram, the solar vehicle needs to be tested in real life. This means measuring the distance that the vehicle can travel when released from the top of the slope. By this the real friction coefficient $C_{rr}$ can be determined.

For the next calculations the formula for conservation of energy will be used along with the formulas for potential and kinetic energy:

$$E_{begin} - E_{end} = E_{non\ conservative}$$

$$U = m * g * h$$

$$T = \frac{m * v^2}{2}$$

When the vehicle is at the top of the slope it has a potential energy of:

$$U = 0{,}850 * 9{,}81 * 0{,}5 = 4{,}169\ J$$

When the vehicle is standing still it has neither potential nor kinetic energy, thereby:

$$E_{end} = 0$$

This means that all the potential energy that the vehicle had at his starting point went to losses during the test. The losses working on the vehicle are frictions due to air and rolling resistance, given by:

Air resistance:

$$P = 0{,}5 * A * C_w * \rho * v^3$$

With:

P = power
v = linear velocity
$C_w$ = friction coefficient
A = frontal surface
ρ = density of air

Rolling resistance:

$$P = m * g * C_{rr} * v$$

With:

P = power
m = mass
g = gravity constant (9,81 $m/s^2$)
v = linear velocity
$C_{rr}$ = rolling resistance coefficient

The formula of conservation of energy is known:

$$m * g * h = \frac{m * g * C_{rr} * v}{t} + \frac{\rho * A * C_w * v^3}{2 * t} + E_{transmission}$$

In this formula the forces are multiplied by the maximum velocity to get the power losses. Those losses are divided by the resulting time of the test to receive the energy losses. The maximum velocity can be found by the conservation of energy between the top of the slope where the potential energy is at his maximum but the kinetic energy is zero, and the bottom where the kinetic energy reaches its maximum and the potential energy is zero:

$$m * g * h = \frac{m * {v_{max}}^2}{2}$$

$$v_{max} = 3{,}13\ {}^{m}/_{s}$$

When releasing the vehicle from the slope it travels a distance of:

$$x = \ 8{,}5\ m$$

Thereby:

$$x = \frac{v}{t} = 8{,}5\ m$$

When solving the above general formula for the conservation of energy taking the loses of the transmission as zero:

$$C_{rr} = 0{,}0564$$

With this new value of the rolling resistance a new Sankey-diagram can be constructed with the values in table 1. The remainder values stay the same; also the mass of the vehicle because this is accidentally the same is in the simulation.

| | |
|---|---|
| Maximum linear velocity (v) | 4,57 (m/s) |
| Radius SSV wheels | 0,04 (m) |
| Gear ratio | 8 |
| Rolling resistance coefficient (Crr) | 0,0564 |
| Friction coefficient (Cw) | 0,5 |
| Density of air (ρ) | 1,293 (kg/m$^3$) |
| Torque constant (Ke) | 8,55 (mNm/A) |
| Frontal surface (A) | 0,02 (m) |
| Mass (m) | 0,850 (kg) |

**Table 1: Values used for calculations Sankey diagram**

The biggest changes in our sankey are the losses due to rolling resistance because of the changed value of $C_{rr}$. The loses due to the rolling friction are given by:

$$P = m * g * C_{rr} * v$$

With:

P = power
m = mass
g = gravity constant (9,81 m/s²)
v = linear velocity
$C_{rr}$ = rolling resistance coefficient

$$P_{rolling} = m * g * C_{rr} * v = 0{,}850\ kg * 9{,}81\ \frac{m}{s^2} * 0{,}0564 * 4{,}57 \frac{m}{s} = 2{,}149\ W$$

The total power when riding at maximum speed is, while taking the motor efficiency of 84 % into account (conform SSV Part one):

$$P = U_a * I_a = \big((7{,}815 + 3{,}32 * 0{,}31799\,)V * 0{,}31799\ A\big) * 0{,}84 = 2{,}36947\ W$$

The losses when the SSV is riding at maximum speed are:

$$P_{Air\ friction} = 0{,}5 * A * C_w * \rho * v^3 = 0{,}5 * 0{,}02\ m^2 * 0{,}5 * 1{,}293 \frac{kg}{m^3} * 4{,}57^3 \frac{m}{s} = 0{,}666\ W$$

$$P_{rolling} = m * g * C_{rr} * v = 0{,}850\ kg * 9{,}81\ \frac{m}{s^2} * 0{,}0564 * 4{,}57 \frac{m}{s} = 2{,}1492\ W$$

When taking these losses and the gear efficiency of 95% into account, the resulting power is:

$$P_{resulting} = P_{initial} - P_{losses} = (2{,}36947 - 0{,}666 - 2.14923) * 0{,}95 = -0{,}41\ W$$

The resulting power is negative. The reason for this can be the neglecting of the transmission energy during the calculations of $C_{rr}$. Thereby the value of the rolling resistance would have been lower and the resulting power would lay more around zero to give a more correct power calculation.

With the calculated powers a new Sankey-diagram can be constructed:

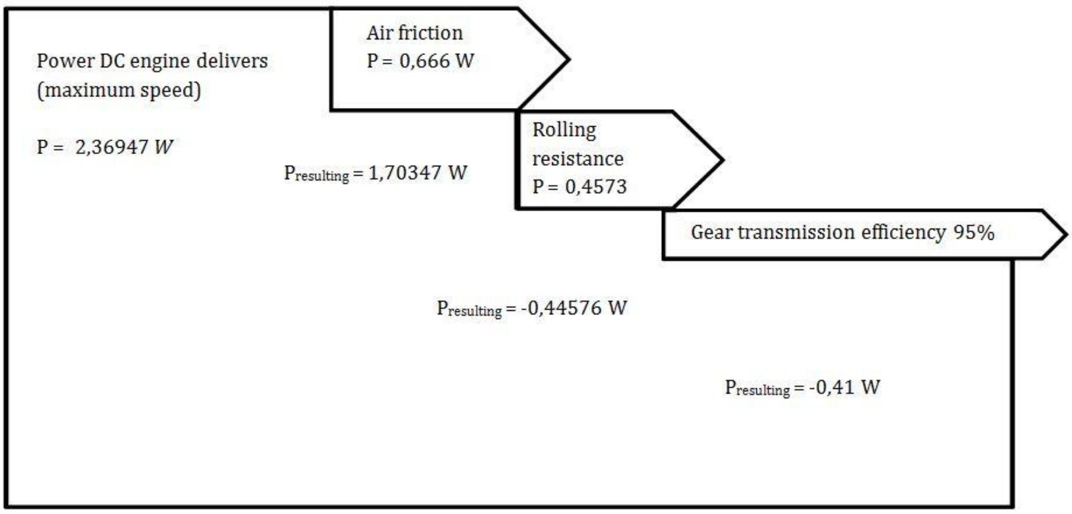



Figure 1: new Sankey-Diagram

## 2. Analysis of the drive shaft

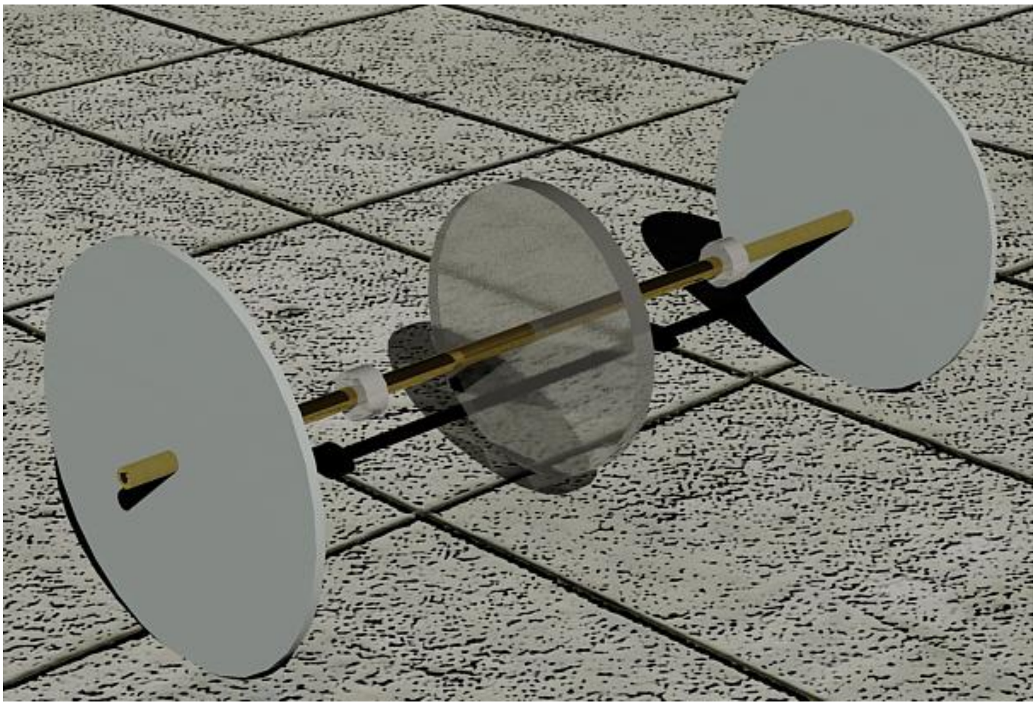


Figure 2: 3D image of rear shaft

**Situation A – The SSV accelerates from standstill and the motor delivers maximum torque. The wheels do not slip.**

In the first part of the analysis all the forces and moments are calculated.

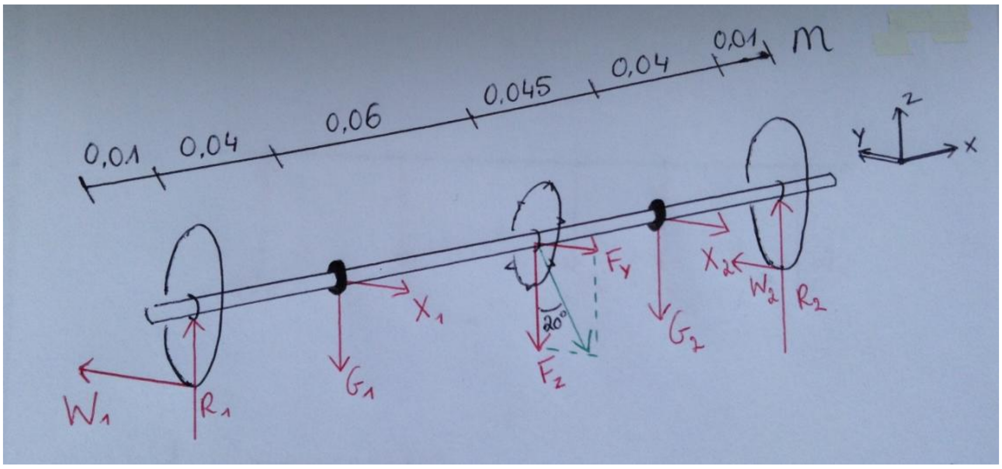



Figure 3: forces on rear shaft

The maximum torque of the motor is: 6,22 $mNm$. Because the power losses between the gear transmissions can be neglected, next formula can be used:

$$P = T_1 * \omega_1 = T_2 * \omega_2$$

With this the torque of the final gear can be calculated as followed:

$$T_2 = T_1 * \frac{\omega_1}{\omega_2} = 6{,}22\,mNm * 8 = 49{,}76\,mNm$$

Through the torque, the force on the gear can be calculated:

$$F = \frac{T}{r} = \frac{49{,}76\,mNm}{0{,}02\,m} = 2{,}488\,N$$

This force lies under an angle of 20° with the z-axle. Therefore it can be divided into two forces, one in the direction of y and one in the direction of z. The values of these forces are:

$$F_y = \sin(20°) * F = \sin(20°) * 2{,}488\,N = 0{,}85\,N$$

$$F_z = \cos(20°) * F = \cos(20°) * 2{,}488\,N = 2{,}34\,N$$

The weight of the car lies on bearings but isn't equally distributed over these two. By measuring the weight on each side, the gravitational force is calculated as followed:

$$G_1 = \mathrm{m_{right}} * g = 297{,}31\,\mathrm{g} * 9{,}81\frac{N}{kg} = 2{,}92\,N$$

$$G_2 = \mathrm{m_{left}} * g = 161{,}44\,\mathrm{g} * 9{,}81\frac{N}{kg} = 1{,}58\,N$$

The static friction coefficient of plastic on rubber is approximately equal to 0,84. This give friction forces on the wheels of:

$$W_1 = \mu_s * \mathrm{m_{right}} * g = 0{,}84 * 297{,}31\,\mathrm{g} * 9{,}81\frac{N}{kg} = 2{,}45\,N$$

$$W_2 = \mu_s * \mathrm{m_{left}} * g = 0{,}84 * 161{,}44\,\mathrm{g} * 9{,}81\frac{N}{kg} = 1{,}33\,N$$

First the forces in the xz-plane are calculated.

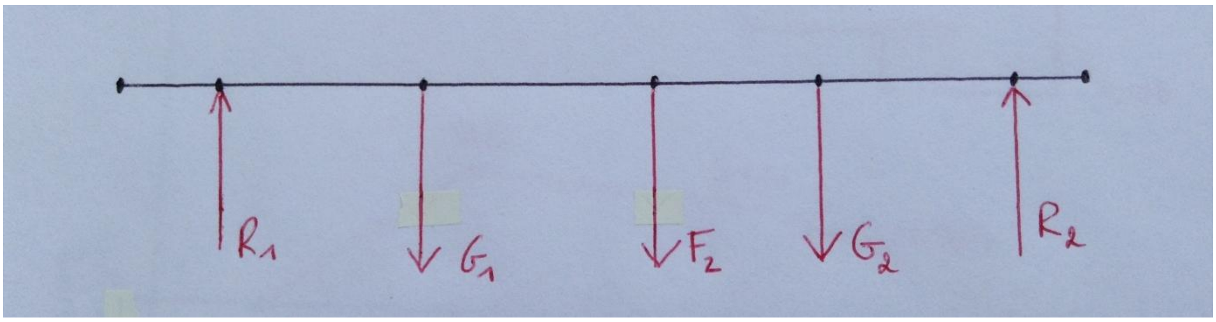



Figure 4: forces in xz-plane

- $\sum F_z = 0$:

$$R_1 + R_2 = G_1 + F_z + G_2$$

$$R_1 + R_2 = 6{,}84\,N$$

■ $\sum M_{R_1} = 0$:

$$-0{,}04 * G_1 - 0{,}10 * F_z - 0{,}145 * G_2 + 0{,}185 * R_2 = 0$$

When solving these equations, the values for $R_1$ and $R_2$ are:

$$R_1 = 3{,}71\ N;\ R_2 = 3{,}13\ N$$

This results in following diagrams:

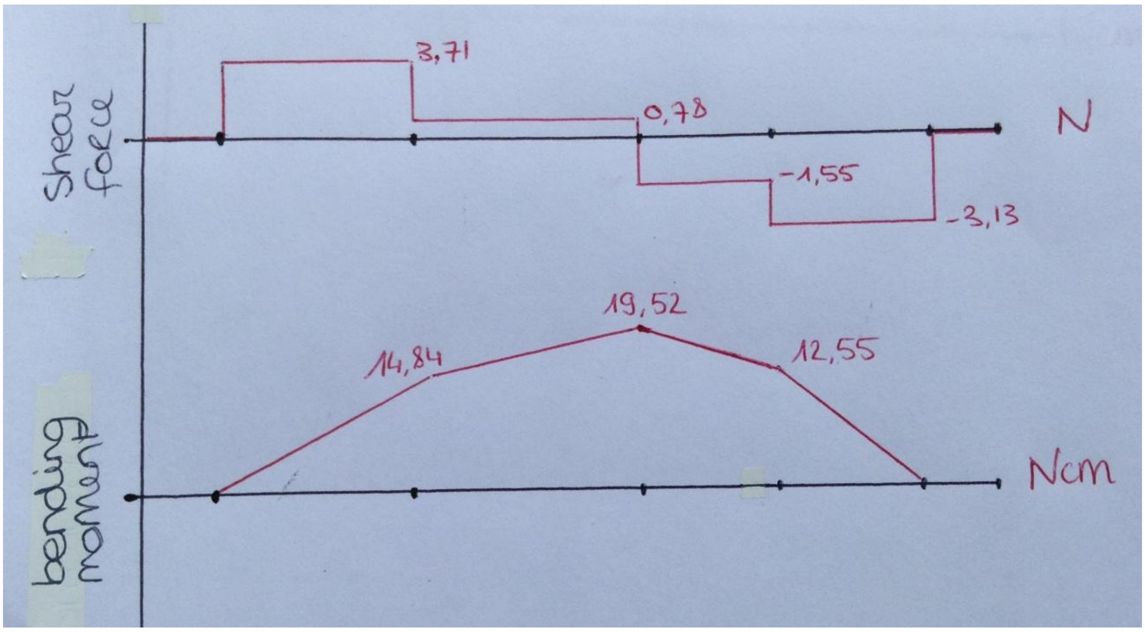



Figure 5: force diagram and moment diagram

Now the forces in the xy-plane are calculated.

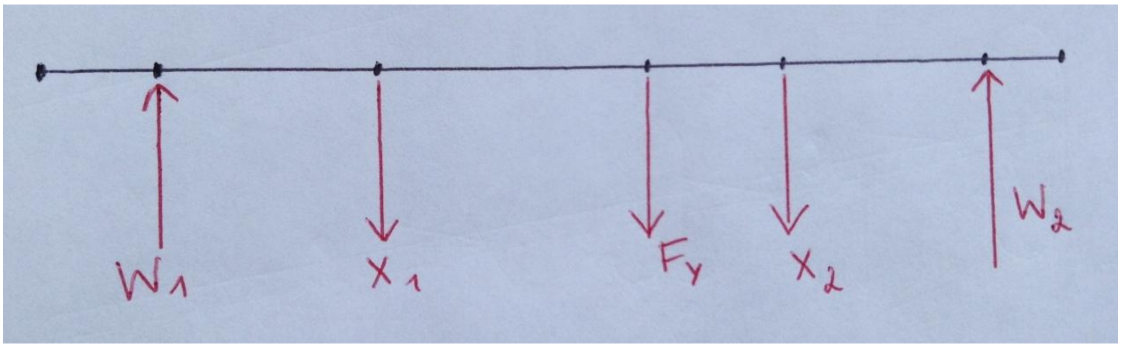



Figure 6: forces in xy-plane

■ $\sum F_y = 0$:

$$X_1 + X_2 = W_1 + W_2 - F_y$$

$$X_1 + X_2 = 2{,}93\ N$$

■ $\sum M_{X_1} = 0$:

$$-0{,}04 * W_1 - 0{,}06 * F_y - 0{,}105 * X_2 + 0{,}145 * W_2 = 0$$

When solving these equations, the values for $X_1$ and $X_2$ are:

$$X_1 = 2{,}51\ N;\ X_2 = 0{,}418\ N$$

This results in following diagrams:

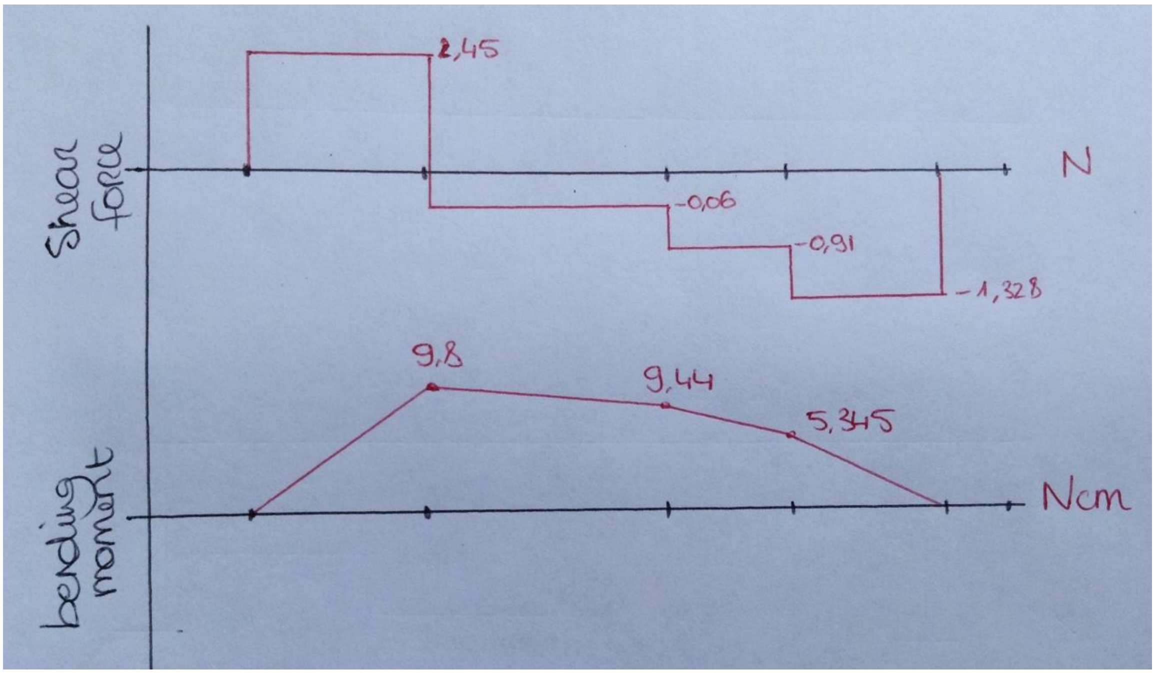



Figure 7: force diagram and moment diagram

As seen on the diagrams there are 3 crucial places on the shaft; at bearing 1, at the gear and at bearing 2. So the equivalent stress is calculated with Von Mises at these three places.

The magnitudes of the moments are: $M_1 = 0{,}1778\ Nm;\ M_2 = 0{,}2168\ Nm;\ M_3 = 0{,}1364\ Nm$

First calculate the principal stress:

$$\sigma_b = \frac{M * R}{I} = \frac{M * 0{,}002\ m}{\frac{\pi(0{,}004^4)}{64}}$$

This gives following results for the three crucial places:

$$\sigma_{b_1} = 28{,}30\ GPa;\ \sigma_{b_2} = 34{,}50\ GPa;\ \sigma_{b_3} = 21{,}71\ GPa$$

Now the shear stress is calculated:

$$\tau = \frac{M * R}{I} = \frac{M * 0{,}002\ m}{\frac{\pi(0{,}004^4 - 0{,}002^4)}{32}}$$

This gives following results for the three crucial places:

$$\tau_1 = 15{,}10\ GPa;\ \tau_2 = 18{,}40\ GPa;\ \tau_3 = 11{,}58\ GPa$$

Through difficult calculations the formula of Von Mises can be simplified from:

$$\sigma_e = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}$$

to:

$$\sigma_e = \sqrt{{\sigma_b}^2 + 3 * \tau^2}$$

This gives following results for the equivalent stresses:

$$\sigma_{e_1} = 38{,}53\ GPa;\ \sigma_{e_2} = 46{,}97\ GPa;\ \sigma_{e_3} = 29{,}56\ GPa$$

When comparing these results with the maximum allowed stress, it is obvious that no problem will occur:

$$Maximum\ allowed\ stress = 100 - 125\ Gpa \gg\ \sigma_{e_1}, \sigma_{e_2}, \sigma_{e_3}$$

**Situation B – The speed is maximal; the torque is smaller.**

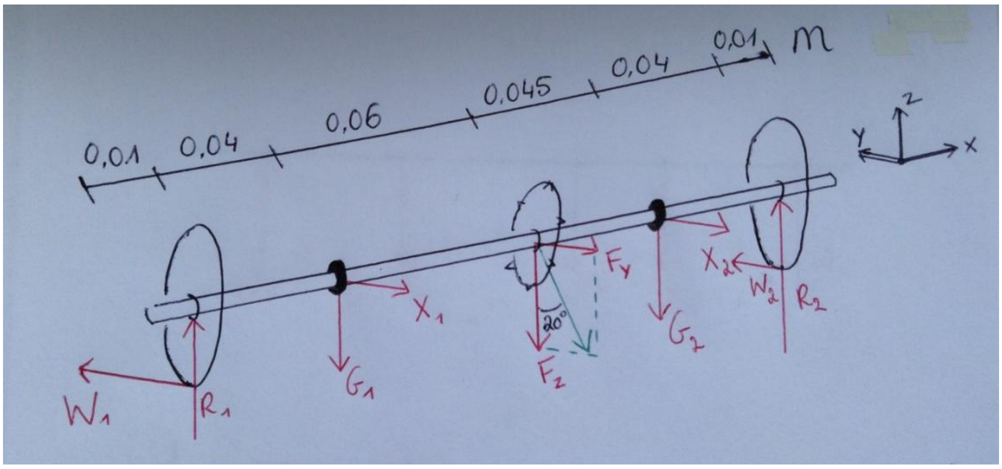



Figure 8: forces on rear shaft

The torque of the motor is smaller when driving at maximum speed. This can be calculated with help of the current at maximum speed.

$$v_{max} = 4{,}58\ m/s$$

$$I_{\max speed} = 0{,}32743\ A$$

$$T_{motor} = I_{\max speed} * torque\ constant$$

$$T_{motor} = 0{,}32743\ A * 8{,}55 \frac{mNm}{A} = 2{,}80\ mNm$$

All the formulas stay the same, so the force on the gear is:

$$F = 1{,}12 \text{ N}$$

$$F_y = \sin(20°) * F = \sin(20°) * 1{,}12\ N = 0{,}383\ N$$

$$F_Z = \cos(20°) * F = \cos(20°) * 1{,}12\ N = 1{,}052\ N$$

The gravitational forces stay the same because does weight doesn't change:

$$G_1 = 2{,}92\ N;\ G_2 = 1{,}58\ N$$

The friction forces are different since the car is moving. As seen before a new rolling resistance is calculated through measurements. So the friction forces can be calculated as followed:

$$W_1 = \mathrm{C_{rr}} * \mathrm{m_{right}} * g = 0{,}0564 * 297{,}31\ \mathrm{g} * 9{,}81\frac{N}{kg} = 0{,}164\ N$$

$$W_2 = \mathrm{C_{rr}} * \mathrm{m_{left}} * g = 0{,}0564 * \ 161{,}44\ \mathrm{g} * 9{,}81\frac{N}{kg} = 0{,}089\ N$$

First the forces in the xz-plane are calculated.

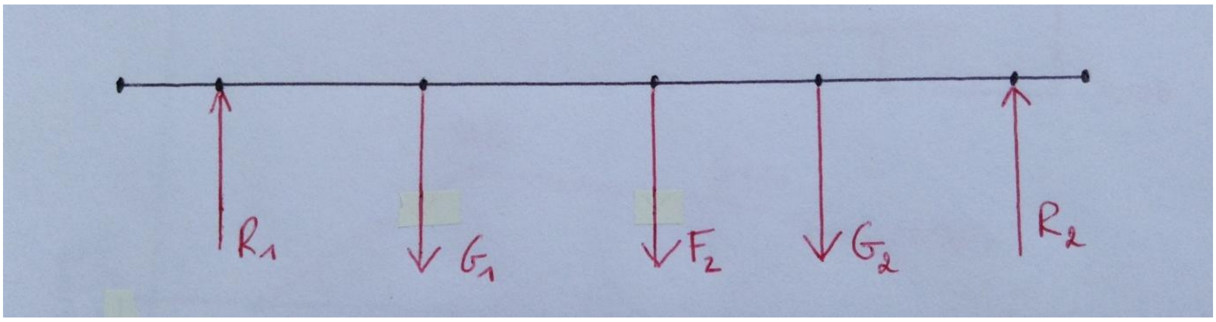



Figure 9: forces in xz-plane

- $\sum F_z = 0$:

$$R_1 + R_2 = G_1 + F_z + G_2$$

$$R_1 + R_2 = 5{,}552\ N$$

- $\sum M_{R_1} = 0$:

$$-0{,}04 * G_1 - 0{,}10 * F_z - 0{,}145 * G_2 + 0{,}185 * R_2 = 0$$

When solving these equations, the values for $R_1$ and $R_2$ are:

$$R_1 = 3{,}11\ N;\ R_2 = 2{,}44\ N$$

This results in following diagrams:

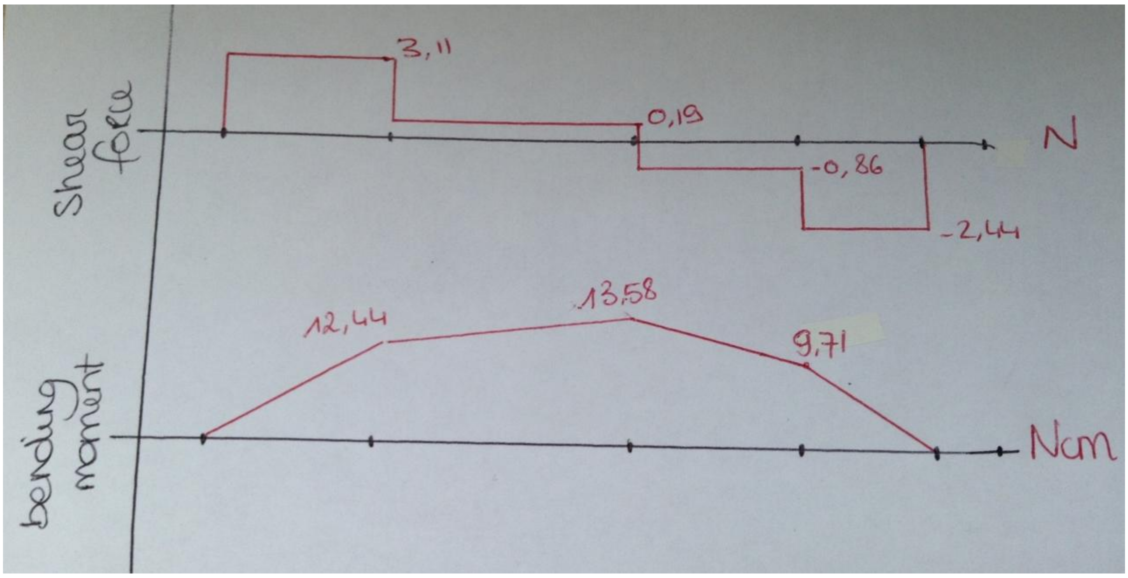



Figure 10: force diagram and moment diagram

Now the forces in the xy-plane are calculated.

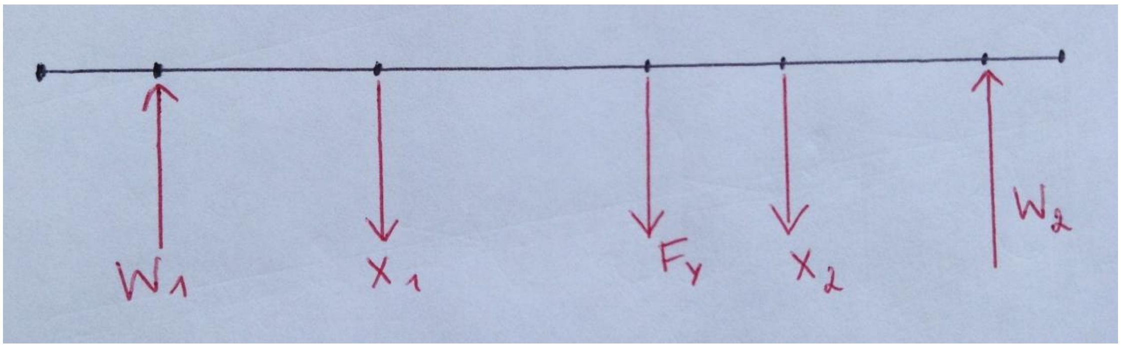



Figure 11: forces in xy-plane

- $\sum F_y = 0$:

$$X_1 + X_2 = W_1 + W_2 - F_y$$

$$X_1 + X_2 = -0{,}13\ N$$

- $\sum M_{X_1} = 0$:

$$-0{,}04 * W_1 - 0{,}06 * F_y - 0{,}105 * X_2 + 0{,}145 * W_2 = 0$$

When solving these equations, the values for $X_1$ and $X_2$ are:

$$X_1 = 0{,}028\ N;\ X_2 = -0{,}158\ N$$

This results in following diagrams:

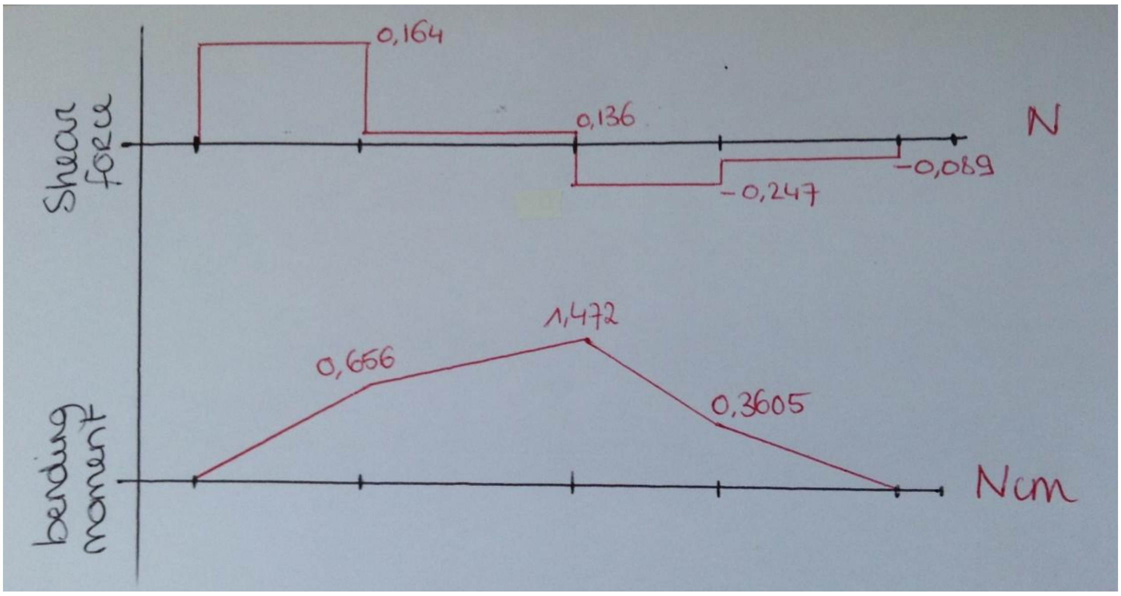



Figure 12: force diagram and moment diagram

The calculation of the equivalent stress in the three crucial places is the same as seen before.

The magnitudes of the moments are: $M_1 = 0{,}1246\,Nm;\ M_2 = 0{,}1366\,Nm;\ M_3 = 0{,}0972\,Nm$

First calculate the principal stress:

$$\sigma_b = \frac{M * R}{I} = \frac{M * 0{,}002\,m}{\frac{\pi(0{,}004^4)}{64}}$$

This gives following results for the three crucial places:

$$\sigma_{b_1} = 19{,}83\,GPa;\ \sigma_{b_2} = 21{,}74\,GPa;\ \sigma_{b_3} = 15{,}47\,GPa$$

Now the shear stress is calculated:

$$\tau = \frac{M * R}{I} = \frac{M * 0{,}002\,m}{\frac{\pi(0{,}004^4 - 0{,}002^4)}{32}}$$

This gives following results for the three crucial places:

$$\tau_1 = 10{,}58\,GPa;\ \tau_2 = 11{,}59\,GPa;\ \tau_3 = 8{,}25\,GPa$$

Through difficult calculations the formula of Von Mises can be simplified from:

$$\sigma_e = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}$$

to:

$$\sigma_e = \sqrt{{\sigma_b}^2 + 3 * \tau^2}$$

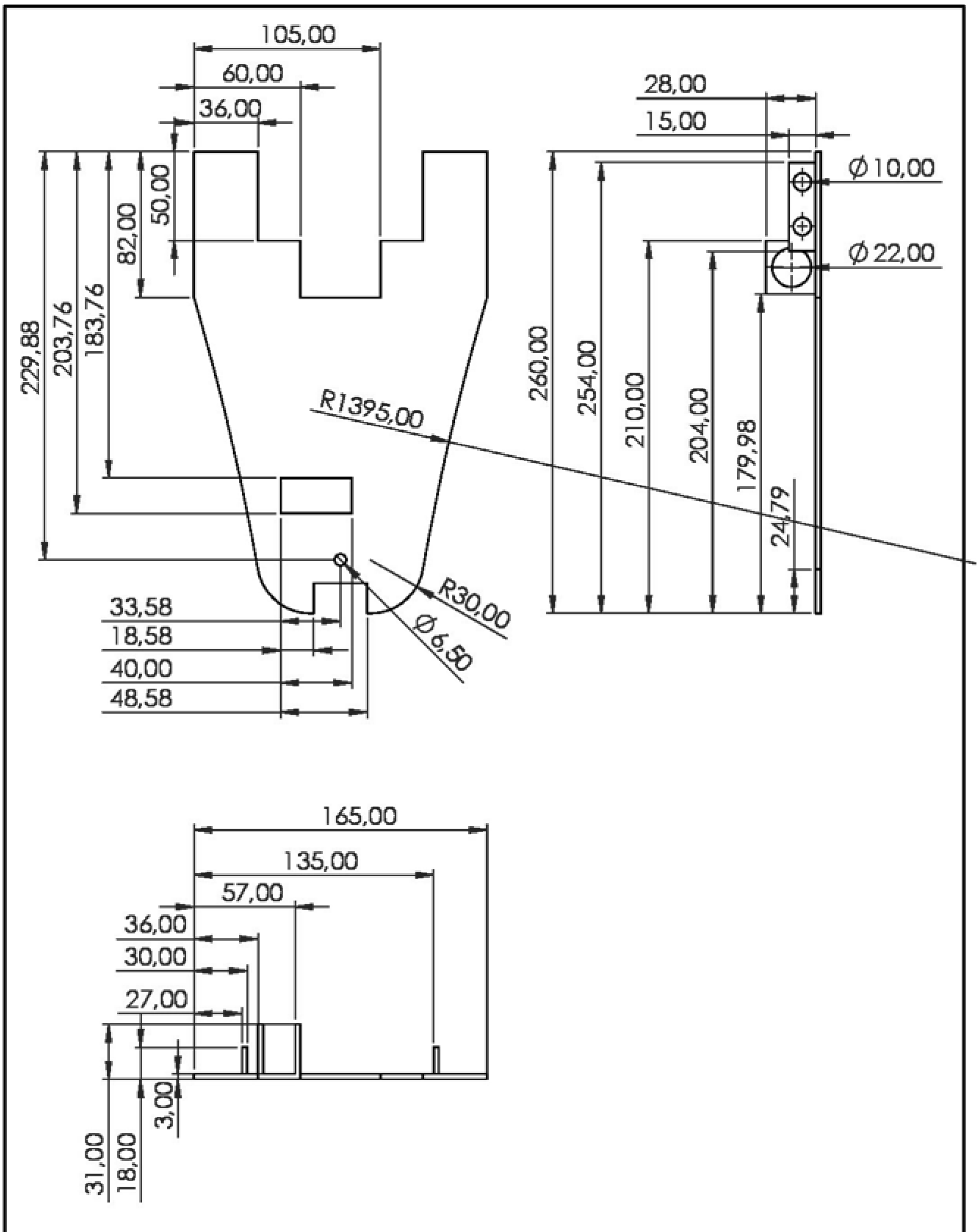
This gives following results for the equivalent stresses:

$$\sigma_{e_1} = 27{,}00\ GPa;\ \sigma_{e_2} = 29{,}59\ GPa;\ \sigma_{e_3} = 21{,}06\ GPa$$

When comparing these results with the maximum allowed stress, it is obvious that no problem will occur:

$$Maximum\ allowed\ stress = 100 - 125\ Gpa \gg\ \sigma_{e_1}, \sigma_{e_2}, \sigma_{e_3}$$

# 3. Technical drawing




| | | | |
|---|---|---|---|
| RUWHEID VOLGENS NEN 3634 | MAATTOLERANTIES VOLGENS NEN 2365 | MATERIAAL Material <not specified> | VORM- EN PLAATSTOL. VOLGENS NEN 3311 |
| PROJECTIE | SCHAAL 1:3 | GETEKEND -- | OPMERKINGEN |
| | MAATEENHEID mm | GROEPSL. -- | -- |
| | DATUM 7/05/2013 | GECONTR. -- | |
| GROUP T | frame! | NUMMER -- | A4 |

## 4. Exercise 1

**Your SSV collides with the side of the track on the flat part at maximum speed under an angle of 10°. What is the impulse, if you assume an elastic collision? How long does the collision need to last for the force to remain below 10 N?**

Figure 13 displays the situation both before the elastic collision and after the collision. From the results of previous simulations we conclude $v_1 = v_{max} = 4{,}58$ m/s. Elastic collision means the magnitude of the speed will stay the same after the collision but the direction of the speed will change. The mass of the vehicle is 0,800 kg.

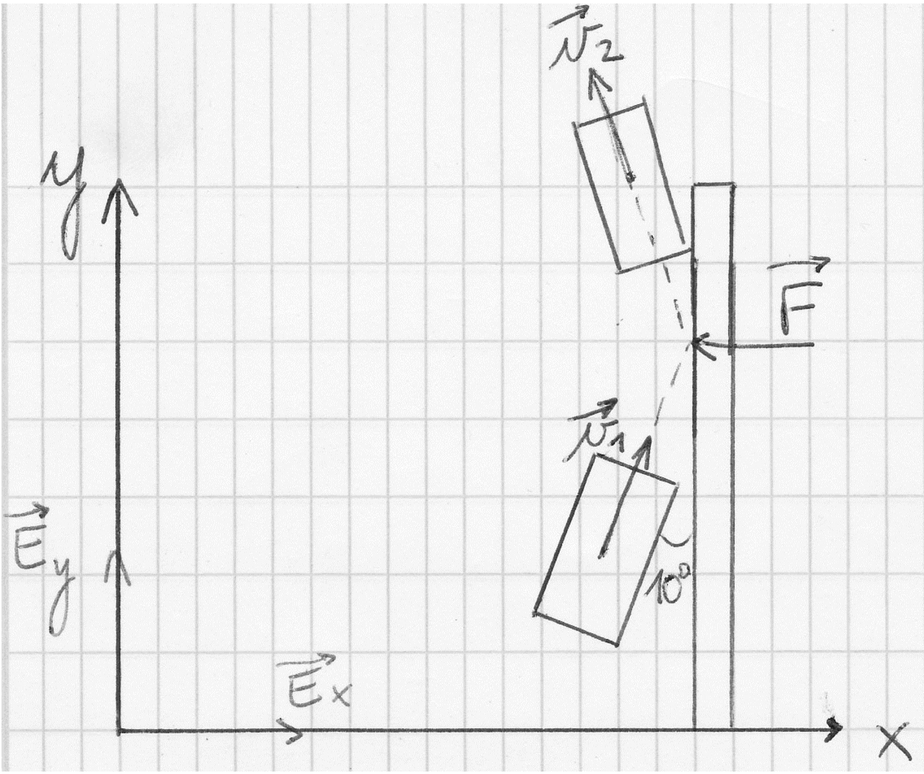



Figure 13

Considering the direction the vector becomes:

$$\vec{v_1} = v_1 * \sin(10°)\,\vec{E_x} + v_1 * \cos(10°)\,\vec{E}_y\ m/s$$

$$= 0{,}795\,\vec{E}_x + 4{,}51\,\vec{E}_y\ m/s$$

And so the speed after the collision:

$$\vec{v_2} = -0{,}795\,\vec{E_x} + 4{,}51\,\vec{E_y}\ m/s$$

If we only consider the x-direction (there is no difference in speed between before and after the collision in the y-direction):

$$L = m * v_{1,x} - m * v_{2,x}$$

$$= 1{,}27\ Ns$$

This impulse must equal the average force during the collision multiplied by the time fragment of the collision:

$$L = F_{avg} * \Delta t$$

So when the average force needs to be limited the during of the collision must be larger than:

$$\Delta t = \frac{L}{F_{avg}} = 0{,}127\ s$$

## 5. Exercise 2

**A cyclist is riding at a speed of 50 km/h. He arrives at a crossroad and needs to turn left. The radius of the turn is 10 m. What is the necessary inclination angle? Does he have to reduce his speed to make a safe turn? What is the maximum possible speed? Mass of the cyclist: 60 kg; mass of the bicycle: 12 kg; distance between ground and centre of gravity: 1,5 m (when he is riding vertically); static coefficient of friction between wheels and ground: 0,3.**

**Given values:**

- $v_{cyclist} = 50\ km/h$
- $m_{total} = 60\ kg\ (cyclist) + 12\ kg\ (bicycle)$
- $d = 1{,}5\ m$
- $\mu_s = 0{,}3$
- $\rho = 10\ m$

**Sketch:**

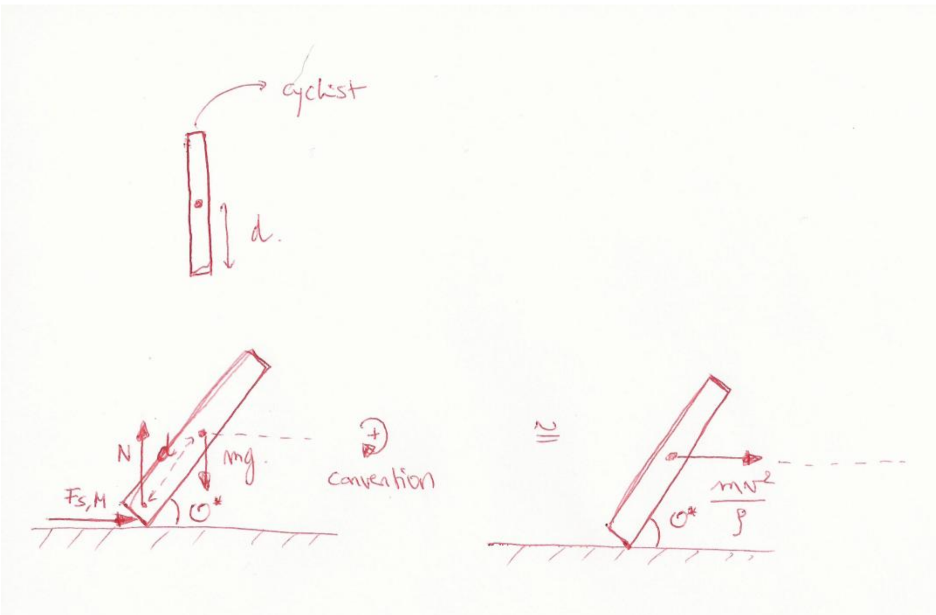



Figure 14

**Values to search**:

- $v_{max} = ?$
- $\theta^* = ?$

**Solution:**

1) $\sum F_y = \cdots$

   Applying on the sketch: $-mg + N = 0$

   with: $N = mg$

   $F_{S,M} = \mu_s * N$

   gives: $F_{S,M} = \mu_s * m * g$

2) Summation of n components

   Applying on the sketch: $F_{S,M} = \frac{m*v^2}{\rho}$

   with: $F_{S,M} = \mu_s * m * g$

   gives: $\mu_s * m * g = \frac{m*v^2}{\rho}$

   $v = \sqrt{\mu_s * \rho * g}$

   with the given values:

   $v_{max} = 5{,}42\ m/s$

   $\boldsymbol{v_{max} = 19,53\ km/h}$

The cyclist is riding way to fast to take the turn in a safe way.

3) $M_A = \cdots$ A is the point on the ground when you would draw a straight vertical line from the cyclist's center of gravity to the ground.

   Applying on the sketch: $m * g * d * \cos(\theta^*) = \frac{m*v^2}{\rho} * d * \sin(\theta^*)$

   with: $\frac{m*v^2}{\rho} = \mu_s * m * g$

   gives: $m * g * \cos(\theta^*) = \mu_s * m * g * \sin(\theta^*)$

   $\cos(\theta^*) = \mu_s * \sin(\theta^*)$

   $\tan(\theta^*) = \frac{1}{\mu_s}$

   with the given values for $\mu_s$:

   $\boldsymbol{\theta^* = 73,3°}$