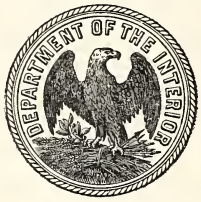
POPULAR SERIES

ELEMENTARY ARITHMETIC

AMERICAN · BOOK · COMPANY

NEW YORK

CINCINNATI CHICAGO



6—1132

POPULAR SERIES

ELEMENTARY ARITHMETIC

MENTAL AND WRITTEN EXERCISES

NEW YORK ·:· CINCINNATI ·:· CHICAGO
AMERICAN BOOK COMPANY

PREFACE.

In the preparation of this work, the author has kept constantly in view **certain needs** which actual work in the school-room and a wide intercourse with the teachers of our public schools have made apparent.

At the outset the **ideas** are so **simple** and the **words** so **easy** that a child who has but a slight knowledge of reading will not find it difficult to study the book.

Since it is not possible for a teacher of a large school to give orally that instruction which fits a child for the study of more advanced arithmetic, **the child needs a text-book** in his hand, which in a simple manner gives him a thorough drill in the fundamental rules, the common tables, fractions, and decimals.

The **importance of mental arithmetic** can not be stated in too emphatic a manner. It is questionable whether the pupils of our schools are as accurate and rapid in arithmetical work as those who were in the schools twenty years ago. The author thinks this is partly due to a lack of drill in mental arithmetic, and has, therefore, put into this book a large number of mental problems.

Again, **poor oral instruction** will never accomplish as much as text-book study. When it is deemed best not to have a text-book in the hands of the pupils for the first

work in arithmetic, **teachers would be aided by having an elementary arithmetic**; and yet the author knows many teachers who own but one arithmetic—the more advanced one, which they use in preparation for teachers' examinations.

Addition and Subtraction are first presented together, and later Multiplication and Division. This is in accordance with the advice of experienced teachers, who say they have been **able to bring about better results** with this mode of presentation than by presenting the four fundamental processes with each number.

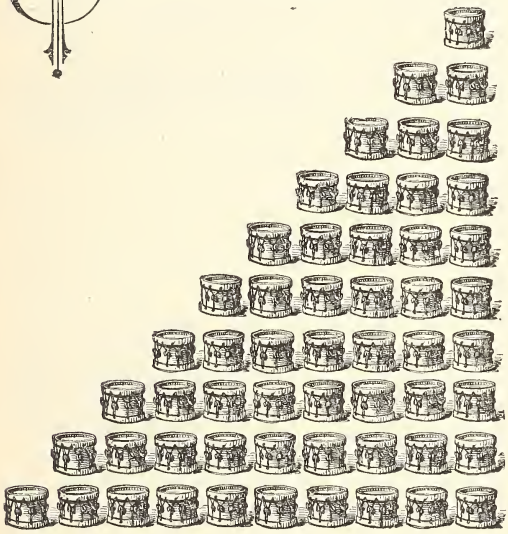
In Part I. no number exceeds 100, and the greater part of the work is mental. Some written work, however, is introduced that the **children may be led gradually** to its use; and, also, that they may make out their own addition, subtraction, multiplication, and division tables. The only case in Part I. in which these tables are filled out is in the 8's and 9's of the multiplication and division tables, where the products and quotients have been supplied, so that the children may have something by which to test this more difficult work.

While a **great variety of objects** should be used in the first presentation of number, their use can be carried to an extent which produces intellectual slowness.

Thoughtful teachers have stated that the difficulty in understanding a subject in arithmetic is often increased by the size of the numbers with which the pupil is required to work. As this dealing with large numbers is rarely required for business transactions, this **unnecessary difficulty is removed** from the present work.

Fundamental Operations.

Lesson I.



one	1
two	2
three	3
four	4
five	5
six	6
seven	7
eight	8
nine	9
ten	10

ADDITION AND SUBTRACTION.

Lesson II.

1. Nat had 1 dog, and his father bought him 1 more: how many dogs had he then?

SOLUTION.—Nat had then 1 dog and 1 dog, which are 2 dogs.

2. Mary had 3 roses, and gave her mother 1: how many roses had she left?

SOLUTION.—Mary had left 3 roses less 1 rose, which are 2 roses.

3. How many are 2 pigs, and 1 pig, and 1 pig?

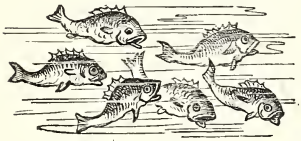
4. Rita had 4 apples, and gave 2 apples away: how many apples had she left?

5. Harry had 4 lambs, and sold 3: how many lambs had he then?

6. I have 3 marbles in my hand, and 3 in my pocket: how many marbles have I?

7. Kitty had 4 dolls; she gave two of them to Mary: how many had she left?

8. William had 4 fishes, and



John gave him 2 more: how many fishes had William then?

9. Five birds were on a tree; 1 flew away: how many birds remained?

10. There were 3 rats in one trap, and 2 in another: how many rats in both traps?

11. Lucy had 5 ducks, but 2 of them died: how many had she then?

12. Rose had 5 apples, and lost

3: how many apples had she left?

13. Here are 5 children; if 3 leave the room, how many will be left?

Lesson III.

1. How many are 2 and 1? One taken away from 3 leaves how many?

2. How many are 1 and 3? How many are 2 and 2?

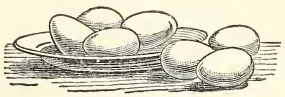
3. One taken from 4 leaves how many? Three taken from 4 leaves how many? Two taken from 4 leaves how many?

4. How many are 4 and 1? One taken from 5 leaves how many?

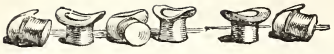
5. How many are 3 and 2? Three taken from 5 leaves how many? Two taken from 5 leaves how many?

Lesson IV.

1. Rob found 6 eggs one day, and 1 egg the next day: how many eggs did he find in the two days?



2. Nell had 6 cherries, and ate 2: how many cherries had she left?



3. I see 4 hats, and 2 hats: how many hats do I see?

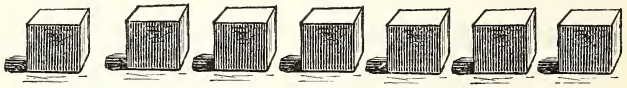
4. Ben had 6 marbles, and lost 2 of them: how many had he left?

5. Fanny is 6 years old, and Bess is 4: how many years older than Bess is Fanny?

6. Francis has 3 pears in his hand, and 3 in his pocket: how many pears has he?

7. Tom had 6 cherries, and ate 3 of them: how many cherries were left?

8. Helen had 5 blocks, and her brother gave her 2 more: how many had she then?



9. Dick bought 7 oranges, and ate 2 of them: how many had he then?

10. Kate drew 4 lines on the board, and Lucy drew 3: how many lines were then on the board?

11. There were 7 chickens in a field, now there are only 4: how many ran away?

Lesson V.

1. How many are 5 and 1? 4 and 2? 3 and 3? 5 and 2? 4 and 3?

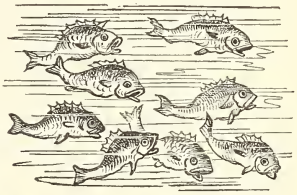
2. One from 6 leaves how many? Two from 6? Four from 6? Three from 6?

3. Five from 7 leaves how many? Two from 7? Three from 7? Four from 7?

4. How many are 7 and 1? Six from 7 leaves how many? Seven from 8 leaves how many? One from 8?

Lesson VI.

1. Tom put 6 fishes in a tub, and afterward John put in 2 more: how many fishes were then in the tub?

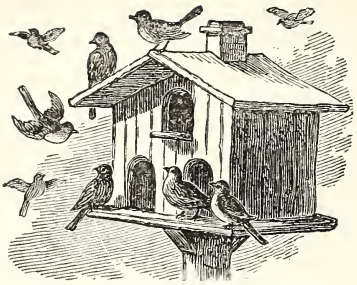


2. A man had 8 drums, and sold 6: how many had he left?

3. Here are 8 acorns in a row: if you take 2 of them away, how many will remain?

4. Mary spent 5 cents for thread, and 3 cents for a thimble: how much money did she spend?

5. Kate picked 7 flowers; 4 were pinks, and the others were tulips: how many tulips did she pick?



6. Here is a dove-cot and some doves. Some of the doves are flying in the air, some are sitting on the roof, and some are on the shelf. How many are flying? How many are on the roof? How many are on the shelf? How many are there in all?

7. Will has 4 marbles in his hand, and 4 in his pocket: how many has he in all?

8. There are 8 cherries on the table: if I take 5 away,

how many will still be there?

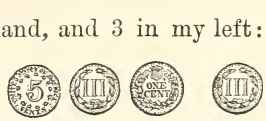
9. Henry bought 7 pretty roses, and his sister bought 2: how many did they both buy?

10. Ellen made 9 marks on her slate, and then rubbed out 2: how many were left on the slate?

11. A farmer has 6 lambs in one field, and 3 in another: how many lambs in both fields?

12. Tom had 9 berries, and he gave 6 to Jack: how many had he left?

13. I have 9 cents in my right hand, and 3 in my left: how many more have I in my right hand than in my left? How much have I altogether?



14. Robert gave 5 cents for a whistle, and 4 cents for a top: how much did he give for both?

Lesson VII.

1. How many are 6 and 2? 5 and 3? 4 and 4?

2. How many is 8 less 6? 8 less 2? 8 less 5? 8 less 3? 8 less 4?

3. How many are 8 and 1? 7 and 2? 6 and 3? 5 and 4?

4. How many is 9 less 2? 9 less 7? 9 less 3? 9 less 6?

5. How many are 4 and 5? Five from 9 leaves how many? Four from 9 leaves how many?

Lesson VIII.

We can now make the figures 1, 2, 3, 4, 5, 6, 7, 8, 9.

Let us now learn to make a naught, also called zero, and cipher, thus, 0. With these figures all numbers can be written.

Let us take little sticks—tooth-picks will do—for counters, and tie them up in bundles, with ten in each bundle. If we should write 1 alone, that would mean 1 counter; but if we want to write 1 bundle and no counters besides, we shall write it 10.

How shall we write 2 bundles and no counters? 3 bundles and no counters? 4 bundles and no counters? 5 bundles and no counters? 6 bundles and no counters? 7 bundles and no counters? 8 bundles and no counters? 9 bundles and no counters?

We have written twenty, thirty, forty, fifty, sixty, seventy, eighty, and ninety.

Lesson IX.

Read the following numbers: 30, 50, 20, 60, 40, 10, 90, 70, 80.

Write: Twenty; seventy; thirty; eighty; forty; sixty; fifty; ninety.

Lesson X.

How shall we write 1 bundle and 1 counter?

1 bundle and 2 counters? 1 bundle and 3 counters?

1 bundle and 4 counters? 1 bundle and 5 counters?

1 bundle and 6 counters? 1 bundle and 7 counters?

1 bundle and 8 counters? 1 bundle and 9 counters?

We have written eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen.

Lesson XI.

Read the following numbers: 13, 19, 16, 12, 11, 14, 17, 15, 18.

Write: Eleven; thirteen; nineteen; twelve; sixteen; eighteen; fourteen; fifteen; seventeen.

NOTE.—There will now be no difficulty in teaching the writing and reading of all numbers up to 100. Give daily drill in writing numbers until they are written with ease.

Lesson XII.

1. Frank had 9 marbles, and found 2 more: how many did he then have?

2. Ella had 10 plums, and she gave 2 to her sister: how many had she left?

3. Jane has 10 cents: if she gives 8 cents for a book, how many cents will she then have?

4. James has 7 pets, and his sister has 3: how many pets have both?

5. Ned had 8 oranges, and gave away 6 of them: how many had he left?

6. I have 4 grains of corn in one hand, and 6 in the other: how many grains have I in both hands?

7. Lucy received 5 reward cards one week, and 5 the next week: how many did she receive in the two weeks?

8. A boy counted 8 birds on a tree; after some had

flown away, he counted 2 remaining: how many flew away?

9. Tom had 10 cents, and spent 5 cents for a whip: how many cents had he left?

10. Six from 10 leaves how many? Four from 10 leaves how many?

Lesson XIII.

1. Mary has 3 berries in one hand, and 8 in the other: how many berries has she in both hands?

2. Kate gave 3 cents for paper, and 9 cents for a book: how much money did she spend?

3. Susan bought 11 peaches, and gave away 8: how many had she left?

4. I had 11 cents, and spent 9 cents for nuts: how many cents had I left?

5. There were 12 pupils in a class; three of them were boys: how many girls in the class?

6. Henry put 7 dots on the board, and · · · · · · · James put 4: how many dots were there · · · · on the board?

7. A cat caught 9 mice one day, and 2 the next: how many did she catch in both days?

8. If you have 6 plums, and your brother gives you 6 more, how many will you then have?

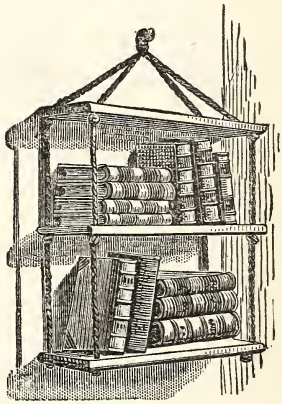
9. A man had 11 dollars, and spent 5 dollars for a vest: how much money had he left?

10. If 7 is taken from 11, how many will remain?

11. There are 7 chairs in one room, and 5 in another: how many chairs in both rooms?

12. There are 8 geese in one pond, and 4 in another: how many geese in both ponds?

13. There are 7 books on one shelf, and 5 on another: how many books on both shelves?



Lesson XIV.

1. How many are 10 and 1? 9 and 2? 8 and 3? 7 and 4? 6 and 5? 4 and 7? 3 and 8? 2 and 9? 1 and 10?

2. One from 11 leaves how many? 2 from 11? 3 from 11? 8 from 11? 7 from 11? 4 from 11? 6 from 11? 5 from 11? 9 from 11?

3. How many are 10 and 2? 11 and 1? 9 and 3? 7 and 5? 8 and 4? 6 and 6?

4. Three from 12 leaves how many? 5 from 12? 2 from 12? 6 from 12? 4 from 12? 7 from 12? 8 from 12? 9 from 12?

Lesson XV.

Copy on your slate and read the following numbers:

15	42	81	95	25
27	51	24	59	87
34	72	36	78	53
18	43	63	35	17
69	75	99	41	13

Lesson XVI.

The Sign of Addition is $+$. It is called *plus*, which means *more*. Placed between two numbers, it shows that they are to be added.

The Sign of Equality is $=$. It denotes that the quantities between which it stands are *equal*.

Thus, $2 + 6 = 8$ is read, 2 plus 6 equals 8, and it means that 2 and 6 are 8.

Let us now make what we may call an Addition Table, by writing what the given numbers together equal.

$2 + 1 =$	$2 + 6 =$	$3 + 1 =$	$3 + 6 =$
$2 + 2 =$	$2 + 7 =$	$3 + 2 =$	$3 + 7 =$
$2 + 3 =$	$2 + 8 =$	$3 + 3 =$	$3 + 8 =$
$2 + 4 =$	$2 + 9 =$	$3 + 4 =$	$3 + 9 =$
$2 + 5 =$	$2 + 10 =$	$3 + 5 =$	$3 + 10 =$

Lesson XVII.

The Sign of Subtraction is $-$. It is called *minus*, which means *less*. Placed between two numbers, it shows

that the one on the right is to be taken from the one on the left. Thus, $6 - 3 = 3$ is read 6 minus 3 equals 3.

Copy the following on slate, and fill out so as to make what we may call a Subtraction Table :

$2 - 2 =$	$7 - 2 =$	$3 - 3 =$	$8 - 3 =$
$3 - 2 =$	$8 - 2 =$	$4 - 3 =$	$9 - 3 =$
$4 - 2 =$	$9 - 2 =$	$5 - 3 =$	$10 - 3 =$
$5 - 2 =$	$10 - 2 =$	$6 - 3 =$	$11 - 3 =$
$6 - 2 =$	$11 - 2 =$	$7 - 3 =$	$12 - 3 =$

Lesson XVIII.

1. How many are 6 flowers and 8 flowers ?

2. William spent 9 cents for pears, and 5 cents for plums : how many cents did he spend ?

3. My tablet cost 10 cents, and my pencil 3 cents : how much did both cost ?

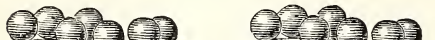
4. A man had 13 chickens, and sold 4 of them : how many had he left ?

5. I owed 13 dollars, and paid all but 8 dollars : how much did I pay ?

6. George had 14 cents, and spent 9 : how many cents had he left ?

7. Samuel wrote 10 words on his slate correctly, and 4 incorrectly : how many words did he write ?

8. There are 6 letters in my name, and 8 in yours : how many letters in both names ?



9. If you put 7 balls on one end of the table, and 7 balls on the other end, how many balls on the table ?

10. How many are 15 cars and 4 cars?

11. A man earned 9 dollars while his son earned 4 dollars: how much did both earn?

12. One word contains 5 letters, and another 8: how many letters in both words?

13. Lucy is 13 years old; her sister Bess is 5 years old: how much older is Lucy than Bess?

14. John had 14 oranges, and gave away 5: how many had he left?

15. Write in figures: Twenty-nine; thirty-six; forty-eight; seventy-three; fifty-four; ninety-two; sixty-five; eighty-seven.

Lesson XIX.

Copy and complete:

$4 + 0 =$	$4 + 5 =$	$5 + 0 =$	$5 + 5 =$
$4 + 1 =$	$4 + 6 =$	$5 + 1 =$	$5 + 6 =$
$4 + 2 =$	$4 + 7 =$	$5 + 2 =$	$5 + 7 =$
$4 + 3 =$	$4 + 8 =$	$5 + 3 =$	$5 + 8 =$
$4 + 4 =$	$4 + 9 =$	$5 + 4 =$	$5 + 9 =$

1. How many are 4 and 10? 5 and 10? 6 and 10?

2. Four from 14 leaves how many? 10 from 14 leaves how many? 5 from 15 leaves how many? 10 from 15 leaves how many? 6 from 16 leaves how many? 10 from 16 leaves how many?

Lesson XX.

Copy and complete:

$4 - 4 =$	$9 - 4 =$	$5 - 5 =$	$10 - 5 =$
$5 - 4 =$	$10 - 4 =$	$6 - 5 =$	$11 - 5 =$
$6 - 4 =$	$11 - 4 =$	$7 - 5 =$	$12 - 5 =$
$7 - 4 =$	$12 - 4 =$	$8 - 5 =$	$13 - 5 =$
$8 - 4 =$	$13 - 4 =$	$9 - 5 =$	$14 - 5 =$

1. Begin with 2 and add 2 successively until the sum equals 30.

2. Begin with 1 and add 2 successively until the sum equals 29.

Lesson XXI.

1. A lady traveled 9 miles by water, and 6 miles by land: how far did she travel?

2. Jack had 15 flowers, and gave his brother 6: how many had Jack then?

3. Mary is 14 years old, and Anna is 6: how much older is Mary than Anna?

4. I bought a melon for 7 cents, and then paid 7 cents for some grapes: how much money did I spend?

5. If 7 words be rubbed out from a list of 14 words, how many words will remain?

6. Lucy gave 7 cents to one poor man, and 8 cents to another: how many cents did she give to both?

7. Nat had 15 marbles, and lost 6: how many marbles had he then?

8. Fanny paid 7 cents for ribbon, and 9 cents for buttons: how much money did she spend?

9. My mother gave me 16 cents, and I spent 7 and put the rest in my bank: how much did I put in my bank?

10. Mary had 19 cents, and she spent 8 for pencils: how many cents had she left?

11. I learned 9 new words to-day, and 8 yesterday: how many did I learn in both days?

WRITTEN EXERCISES.

1.	Add	2	3	2	4	5	6
		5	5	6	4	1	3
2.	From	7	8	8	8	6	9
	Take	2	3	2	4	1	3

Lesson XXII.

Copy and complete:

$6 + 0 =$	$6 + 5 =$	$7 + 0 =$	$7 + 5 =$
$6 + 1 =$	$6 + 6 =$	$7 + 1 =$	$7 + 6 =$
$6 + 2 =$	$6 + 7 =$	$7 + 2 =$	$7 + 7 =$
$6 + 3 =$	$6 + 8 =$	$7 + 3 =$	$7 + 8 =$
$6 + 4 =$	$6 + 9 =$	$7 + 4 =$	$7 + 9 =$

1. Begin with 2 and add 3 successively until the sum equals 41.

2. Begin with 3 and add 3 successively until the sum equals 51.

Lesson XXIII.

Copy and complete:

$6 - 6 =$	$11 - 6 =$	$7 - 7 =$	$12 - 7 =$
$7 - 6 =$	$12 - 6 =$	$8 - 7 =$	$13 - 7 =$
$8 - 6 =$	$13 - 6 =$	$9 - 7 =$	$14 - 7 =$
$9 - 6 =$	$14 - 6 =$	$10 - 7 =$	$15 - 7 =$
$10 - 6 =$	$15 - 6 =$	$11 - 7 =$	$16 - 7 =$

1. Begin with 3 and add 4 successively until the sum equals 55.

2. Begin with 30 and add 2 successively until the sum equals 60.

Lesson XXIV.

1. I bought 8 yards of blue cloth, and 8 yards of black: how many yards of cloth did I buy?

2. If 16 persons are in a room, and 8 of them leave, how many remain?

3. Anna is 9 years old, and Alice is 8 years older than Anna: how old is Alice?

4. There are 17 pupils in a class; 8 of them are boys: how many girls are in the class?

5. George wants to earn 16 cents; when he has earned 9, how many more must he earn to get the number he wishes?

6. Nine and how many make 17? 8 and how many make 15?

7. Jack bought a kite for 9 cents, and sold it for 18 cents: how much did he make?

WRITTEN EXERCISES.

1.	Add	8	6	7	9	8	6
		2	2	2	2	3	4
2.	From	10	8	9	11	11	9
	Take	2	6	7	2	8	6

3. Express in figures: eighteen; twenty-eight; thirty-eight; forty-eight.

Lesson XXV.

Copy and complete:

$8 + 0 =$	$8 + 5 =$	$9 + 0 =$	$9 + 5 =$
$8 + 1 =$	$8 + 6 =$	$9 + 1 =$	$9 + 6 =$
$8 + 2 =$	$8 + 7 =$	$9 + 2 =$	$9 + 7 =$
$8 + 3 =$	$8 + 8 =$	$9 + 3 =$	$9 + 8 =$
$8 + 4 =$	$8 + 9 =$	$9 + 4 =$	$9 + 9 =$

1. Begin with 4 and add 4 successively until the sum equals 56.

2. Begin with 5 and add 5 successively until the sum equals 75.

Lesson XXVI.

Copy and complete :

8 — 8 =	13 — 8 =	9 — 9 =	14 — 9 =
9 — 8 =	14 — 8 =	10 — 9 =	15 — 9 =
10 — 8 =	15 — 8 =	11 — 9 =	16 — 9 =
11 — 8 =	16 — 8 =	12 — 9 =	17 — 9 =
12 — 8 =	17 — 8 =	13 — 9 =	18 — 9 =

1. Robert gave 7 cents for a top, 4 cents for some marbles, and then had 4 cents left: how many cents had he at first?

2. Francis had 12 cents; he bought an orange for 5 cents, and a lemon for 3 cents: how many cents had he left?

Lesson XXVII.

1. Add	3	3	4	4	2
	12	14	15	11	17
2. From	15	17	19	15	19
Take	3	3	4	4	2

3. Two numbers added together make 11; one of them is 7: what is the other?

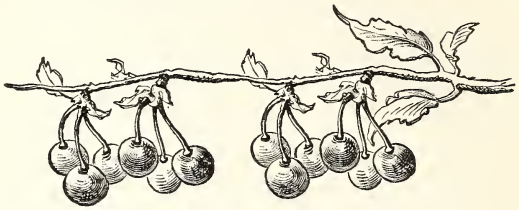
4. Three numbers together make 10; the first is 5, the second is 3: what is the third?

5. Frank had 8 cents; after he had spent 5, his mother gave him 4: how many had he then?

6. John is 17 years old and James 8: how much older is John than James?

Lesson XXVIII.

1. Mary had 12 cherries; she gave 4 to Lucy, and 5 to Nancy: how many had she left?



2. If John were 5 years older, he would be 12: how old is he?

3. Emma paid 5 cents for thread, 3 cents for tape, and 5 cents for needles; her mother had given her 15 cents: how much change did she bring back?

4. How many are 7 and 5, less 8?

5. How many are 5 and 8, less 6?

6. Five birds were on one tree and 6 on another; 3 of them flew away: how many were left?

WRITTEN EXERCISES.

	(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)
Add	1	2	4	3	2
	2	2	1	3	1
	3	2	2	3	3
	1	2	5	3	5
	3	2	3	3	4
	2	2	6	3	6
	1	2	1	3	1

Use the same columns for mental drill by adding from left to right, and from right to left, and naming results.

Lesson XXIX.

1. Begin with 2 and add 5 successively until the sum equals 47.

2. John had 25 cents; he bought a loaf of bread for 8 cents, and some cakes for 10 cents: how much change did he receive?

3. How many pecks are 3 pecks and 8 pecks? 13 pecks and 8 pecks? 23 pecks and 8 pecks?

4. How many are 7 and 9, less 8?

5. Mary had 30 cents; she spent 15 cents for a doll, and 5 cents for candy: how much had she left?

6. How many are 9 and 9, less 9? 6 and 6, less 4? 7 and 8, less 3?

WRITTEN EXERCISES.

(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)
4	1	5	2	6
4	4	5	3	6
4	6	5	1	6
4	3	5	4	6
4	2	5	5	6
4	5	5	7	6
4	7	5	6	6

Use the same columns for mental drill by adding from left to right, and from right to left, and naming results.

Lesson XXX.

1. How many are 2 and 3? 12 and 3? 22 and 3? 13 and 3? 23 and 3?

2. How many are 4 and 3? 14 and 3? 24 and 3? 64 and 3? 54 and 3?

3. There are 24 pupils in a school: if 4 of them leave, how many pupils will remain?

4. How many lines are 14 lines less 4 lines? 34 less 4?

WRITTEN EXERCISES.

1.	Add	5	5	6	7	5
		13	9	12	11	10
2.	From	18	14	18	18	15
	Take	5	5	6	7	5

Lesson XXXI.

1. How many are 7 dollars, 4 dollars, and 2 dollars?

2. Herbert had 4 apples; his brother gave him 3, and his sister 2: how many did he then have?

3. A merchant had 19 barrels of flour; he sold 10 barrels, and kept the rest: how many barrels did he keep?

4. How many is 10 less 3? 30 less 3? 40 less 3? 50 less 3?

WRITTEN EXERCISES.

1.	Add	21	32	43	64
		22	42	23	22
2.	From	43	74	66	86
	Take	21	32	43	64

Lesson XXXII.

1. There were 14 books on one shelf of a book-case, and 8 on another: how many books on the two shelves?

2. How many are 24 and 8? 34 and 8? 44 and 8?

3. Begin with 28 and subtract by 2's to 0.

4. Begin with 30 and subtract by 3's to 0.

WRITTEN EXERCISES.

	(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)	(*6.*)
Add	7	1	7	8	6	9
	7	2	6	8	4	9
	7	3	5	8	3	9
	7	4	4	8	8	9
	7	5	3	8	7	9
	7	6	2	8	5	9
	7	7	1	8	1	9

Lesson XXXIII.

1. How many are 20 and 1? 2 and 20? 20 and 3? 4 and 20? 20 and 5? 6 and 20? 20 and 7? 8 and 20? 20 and 9? 10 and 20?

2. One taken from 21 leaves how many? 2 from 22? 3 from 23? 4 from 24? 5 from 25? 6 from 26? 10 from 30?

3. How many are 7 and 4? 17 and 4? 27 and 4? 37 and 4? 47 and 4? 57 and 4?

4. Four from 11 leaves how many? 4 from 21? 4 from 31? 4 from 41? 4 from 61?

WRITTEN EXERCISES.

	(*1.*)	(*2.*)	(*3.*)	(*4.*)
Add	25	31	20	45
	13	16	45	11
	31	42	12	43

	(*5.*)	(*6.*)	(*7.*)	(*8.*)
From	46	56	67	88
Take	14	13	25	14

Lesson XXXIV.

1. How many are 5, 3, 4, and 2? How many are 7, 5, 6, and 3? How many are 8, 6, 3, and 1?

2. Henry's father gave him 8 cents, and his mother enough to make 24 cents: how many cents did his mother give him?

3. John had 50 cents; he spent 25 cents for a book and 10 cents for candy: how much money had he left?

WRITTEN EXERCISES.

	(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)
Add	10	37	23	22	13
	25	11	32	11	34
	43	30	21	20	10

	(*6.*)	(*7.*)	(*8.*)	(*9.*)	(*10.*)
From	25	37	99	76	89
Take	13	27	54	56	25

MENTAL EXERCISES.

At sight name the sum, then the difference:

(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)	(*6.*)
15	22	34	46	55	68
7	9	8	7	4	5

(*7.*)	(*8.*)	(*9.*)	(*10.*)	(*11.*)	(*12.*)
28	34	68	72	14	59
9	7	8	5	8	4

MULTIPLICATION AND DIVISION.

Lesson XXXV.

1. At a cent apiece, what will 2 apples cost?

SOLUTION.—If one apple costs 1 cent, 2 apples will cost 2 times 1 cent, which are 2 cents.

2. Henry paid 1 cent for a slate pencil; at that rate, how much will 3 pencils cost?

3. John bought 4 figs at 1 cent each: how much did they cost?

4. If one yard of tape costs 1 cent, what will be the cost of 5 yards?

5. How many are 6 times 1? 7 times 1? 9 times 1? 8 times 1? 10 times 1?

6. How many times can you take 1 from 2? Then how many times 1 in 2? How many times can you take 1 from 3? How many times 1 in 3? How many times 1 in 4?

7. How many times can you take 1 from 6? How many times 1 in 6? 1 in 7? 1 in 8? 1 in 9? 1 in 10?

8. What will be the cost of 6 yards of silk, if one yard costs 1 dollar?

9. If a car travels 1 mile in a minute, how far will it travel in 7 minutes?

10. How many pears at 1 cent each, can be bought for 6 cents?

SOLUTION.—As many pears can be bought as 1 cent is contained times in 6 cents, which are 6.

11. How many plums at 1 cent each, can be bought for 8 cents?

12. If a sheet of paper costs 1 cent, how many sheets can be bought for 10 cents?

Lesson XXXVI.

If we want to add 1, 1, 1, and 1, we write it $1+1+1+1=4$. By using the sign $\times$ between any two numbers we can show that either number is taken as many times as there are ones in the other. Thus, $1\times4=4$, shows us that if 1 be taken 4 times, it will equal 4. $1\times7=7$, shows us that if 1 be taken 7 times, it will equal 7.

Form the multiplication table of ones.

$1\times1=$	$4\times1=$	$7\times1=$	$10\times1=$
$2\times1=$	$5\times1=$	$8\times1=$	$11\times1=$
$3\times1=$	$6\times1=$	$9\times1=$	$12\times1=$

A short way of showing that 1 can be taken from 6, six times, is to write it, $6\div1=6$; that 2 can be taken from 6, three times, $6\div2=3$. The sign $\div$ placed between two numbers denotes that the one on the left is to be divided by the one on the right.

Form the division table of ones.

$1\div1=$	$4\div1=$	$7\div1=$	$10\div1=$
$2\div1=$	$5\div1=$	$8\div1=$	$11\div1=$
$3\div1=$	$6\div1=$	$9\div1=$	$12\div1=$

Lesson XXXVII.

1. When peaches are selling at 2 cents each, how much will 2 peaches cost?

2. I have 3 marbles in each hand: how many marbles have I?

3. How many apples at 2 cents each, can you buy for 4 cents?

4. How many times 2 eggs in 6 eggs ?

5. How many times 2 acorns in 10 acorns ?

6. How much will 2 pounds of sugar cost, at 5 cents a pound ?

7. How many lemons at 2 cents each, can you buy for 8 cents ?

8. How many peaches at 2 cents each, can you buy for 10 cents ?

Form the multiplication and the division table of twos.

$1 \times 2 =$	$6 \times 2 =$	$2 \div 2 =$	$12 \div 2 =$
$2 \times 2 =$	$7 \times 2 =$	$4 \div 2 =$	$14 \div 2 =$
$3 \times 2 =$	$8 \times 2 =$	$6 \div 2 =$	$16 \div 2 =$
$4 \times 2 =$	$9 \times 2 =$	$8 \div 2 =$	$18 \div 2 =$
$5 \times 2 =$	$10 \times 2 =$	$10 \div 2 =$	$20 \div 2 =$

Lesson XXXVIII.

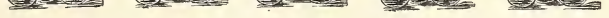
1. How many are 2 times 3 sheep ?

2. James bought 3 tops at 3 cents each : what did he pay for them ?

3. How many times 3 in 6 ? 3 in 9 ?

4. There are 3 feet in a yard : how many yards are there in 12 feet ?

5. How many groups of marbles, with 3 in each group,



can you make out of 15 marbles ?

6. If one peach is worth 3 plums, how many plums are 6 peaches worth?

7. How much will 7 yards of tape cost at 3 cents a yard?

8. How many times 3 in 18? 3 in 21?

9. How many are 8 times 3? 9 times 3? 10 times 3?

10. How much silk at 3 dollars a yard, can be bought for 24 dollars?

11. How many buttons at 3 cents apiece, can you buy for 30 cents?

Form the multiplication table and the division table of threes.

$1 \times 3 =$	$6 \times 3 =$	$3 \div 3 =$	$18 \div 3 =$
$2 \times 3 =$	$7 \times 3 =$	$6 \div 3 =$	$21 \div 3 =$
$3 \times 3 =$	$8 \times 3 =$	$9 \div 3 =$	$24 \div 3 =$
$4 \times 3 =$	$9 \times 3 =$	$12 \div 3 =$	$27 \div 3 =$
$5 \times 3 =$	$10 \times 3 =$	$15 \div 3 =$	$30 \div 3 =$

Lesson XXXIX.

1. Rita has 2 kittens, and each one has 4 feet: how many feet have both?

2. Thomas has 3 pigeons, and James has 4 times as many: how many has James?

3. How many times 4 in 8? 4 in 12? 4 in 16?

4. How many fingers on 4 hands?

5. There are 4 pecks in a bushel: how many bushels in 20 pecks?

6. How many are 4 times 4? 4 times 5? 4 times 6?

7. If 4 sheets of paper make a copy-book, how many copy-books will 24 sheets make?

8. Francis bought 7 oranges at 4 cents each: what did they cost?

9. How many times 4 in 32? 4 in 36? 4 in 40?

10. A slate cost 10 cents, and a book 4 times as much: what did the book cost?

Form the multiplication table and the division table of fours.

$1 \times 4 =$	$6 \times 4 =$	$4 \div 4 =$	$24 \div 4 =$
$2 \times 4 =$	$7 \times 4 =$	$8 \div 4 =$	$28 \div 4 =$
$3 \times 4 =$	$8 \times 4 =$	$12 \div 4 =$	$32 \div 4 =$
$4 \times 4 =$	$9 \times 4 =$	$16 \div 4 =$	$36 \div 4 =$
$5 \times 4 =$	$10 \times 4 =$	$20 \div 4 =$	$40 \div 4 =$

	(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)	(*6.*)
Multiply	3	4	6	5	8	10
	2	2	2	2	2	2
	(*7.*)	(*8.*)	(*9.*)	(*10.*)	(*11.*)	(*12.*)
Divide	2)12	2)8	2)6	2)10	2)16	2)20

Lesson XL.

1. How many acorns are there in 2 bunches, if there are 5 in each bunch?

2. If a woman sells 3 quarts of milk in one day, how many quarts does she sell in 5 days?

3. How many oranges at 5 cents each, can you buy for 10 cents?

4. How many oranges at 5 cents each, can you buy for 15 cents?

5. How many are 5 times 4? 6 times 4? How many times 5 are there in 20? 5 in 25?

6. How many melons at 5 cents each, can you buy for 30 cents?

7. What will be the cost of 7 kites, at 5 cents apiece?

8. Lulu has 5 hens, and each hen has 8 chickens: how many chickens have all the hens?

9. How many tops at 5 cents each, can you buy for 45 cents?

10. How many slates at 5 cents each, can be bought for 50 cents?

Form the multiplication and the division table of fives.

$1 \times 5 =$	$6 \times 5 =$	$5 \div 5 =$	$30 \div 5 =$
$2 \times 5 =$	$7 \times 5 =$	$10 \div 5 =$	$35 \div 5 =$
$3 \times 5 =$	$8 \times 5 =$	$15 \div 5 =$	$40 \div 5 =$
$4 \times 5 =$	$9 \times 5 =$	$20 \div 5 =$	$45 \div 5 =$
$5 \times 5 =$	$10 \times 5 =$	$25 \div 5 =$	$50 \div 5 =$

	(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)	(*6.*)
Multiply	12	22	32	21	13	51
	3	3	3	3	3	3

	(*7.*)	(*8.*)	(*9.*)	(*10.*)	(*11.*)	(*12.*)
Divide	3)36	3)66	3)96	3)63	3)39	3)18

Lesson XLI.

1. How many hands have 6 children?

2. How many quarts of milk at 6 cents a quart, can you buy for 12 cents?

3. If one quart of berries costs 4 cents, what will 6 quarts cost?

4. If it takes 6 yards of cloth to make a cloak, how many cloaks will 24 yards make?

5. Here are 6 flowers, and each flower has 5 petals: how many petals have they all?

6. If one pencil costs 6 cents, how many pencils can be bought for 30 cents?

7. How many are 6 times 6? 7 times 6? 8 times 6?

8. How many times 6 in 36? 6 in 42? 6 in 48?

9. When melons are selling at 9 cents each, how much will 6 melons cost?

10. If one orange is worth 10 plums, how many plums are 6 oranges worth?

11. How many rings at 6 dimes each, can you buy for 54 dimes?

12. A farmer sold 6 loads of hay for 60 dollars: what did he get for each load?

13. If one slate-pencil costs 2 cents, what will 6 slate-pencils cost?

14. How many pears at 6 cents each, can you buy for 42 cents?

Form the multiplication table and the division table of sixes.

$1 \times 6 =$	$6 \times 6 =$	$6 \div 6 =$	$36 \div 6 =$
$2 \times 6 =$	$7 \times 6 =$	$12 \div 6 =$	$42 \div 6 =$
$3 \times 6 =$	$8 \times 6 =$	$18 \div 6 =$	$48 \div 6 =$
$4 \times 6 =$	$9 \times 6 =$	$24 \div 6 =$	$54 \div 6 =$
$5 \times 6 =$	$10 \times 6 =$	$30 \div 6 =$	$60 \div 6 =$

	(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)	(*6.*)
Multiply	10	21	12	20	11	22
	4	4	4	4	4	4
	(*7.*)	(*8.*)	(*9.*)	(*10.*)	(*11.*)	(*12.*)
Divide	$4)\overline{36}$	$4)\overline{40}$	$4)\overline{48}$	$4)\overline{80}$	$4)\overline{44}$	$4)\overline{88}$

Lesson XLII.

1. How many are 7 and 7? How many are 7 times 2? 7 from 14 leaves how many? How many times 7 in 14?

2. How many times 7 dots in 14 dots?

3. How many are 7 and 14? How many are 7 times 3? 7 from 21 leaves how many? How many times 7 in 21?

4. There are 7 days in a week: how many days in 4 weeks?

5. If a man walks 4 miles an hour, how long will it take him to walk 28 miles?

6. Begin with 28 and add 7 successively until the sum equals 70.

7. Begin with 70 and subtract 7 successively until 28 remains.

Form the multiplication and the division table of sevens.

$1 \times 7 =$	$6 \times 7 =$	$7 \div 7 =$	$42 \div 7 =$
$2 \times 7 =$	$7 \times 7 =$	$14 \div 7 =$	$49 \div 7 =$
$3 \times 7 =$	$8 \times 7 =$	$21 \div 7 =$	$56 \div 7 =$
$4 \times 7 =$	$9 \times 7 =$	$28 \div 7 =$	$63 \div 7 =$
$5 \times 7 =$	$10 \times 7 =$	$35 \div 7 =$	$70 \div 7 =$

	(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)	(*6.*)
Multiply	10	11	12	10	10	13
	5	5	4	6	7	3

	(*7.*)	(*8.*)	(*9.*)	(*10.*)	(*11.*)	(*12.*)
Divide	$5)\underline{50}$	$5)\underline{55}$	$5)\underline{35}$	$5)\underline{30}$	$7)\underline{70}$	$3)\underline{39}$

Lesson XLIII.

$1 \times 8 = 8$	$6 \times 8 = 48$	$8 \div 8 = 1$	$48 \div 8 = 6$
$2 \times 8 = 16$	$7 \times 8 = 56$	$16 \div 8 = 2$	$56 \div 8 = 7$
$3 \times 8 = 24$	$8 \times 8 = 64$	$24 \div 8 = 3$	$64 \div 8 = 8$
$4 \times 8 = 32$	$9 \times 8 = 72$	$32 \div 8 = 4$	$72 \div 8 = 9$
$5 \times 8 = 40$	$10 \times 8 = 80$	$40 \div 8 = 5$	$80 \div 8 = 10$

1. If Tillie goes to school 5 days each week, how many days does she go in 8 weeks?

2. When butter is selling at 8 cents a pound, how much will 3 pounds cost?

3. There are 8 quarts in a peck: how many pecks are there in 16 quarts?

4. If 8 yards of calico cost 40 cents, how much will one yard cost?

5. James gave 2 marbles to each one of 8 friends: how many marbles did he give away?

6. I have 24 cents to give to 8 little girls: how many cents can I give to each girl?

7. There are 4 quarts in a gallon: how many quarts are there in 8 gallons?

8. How many pencils at 8 cents each, can you buy for 32 cents?

9. If one lemon costs 6 cents, what will 8 lemons cost?

10. If one cane costs 8 dimes, how many canes can be bought for 48 dimes?

11. How many are 7 times 8? 8 times 8? 9 times 8? 10 times 8?

12. How many times 8 in 56? 8 in 64? 8 in 72? 8 in 80?

Lesson XLIV.

$1 \times 9 = 9$	$6 \times 9 = 54$	$9 \div 9 = 1$	$54 \div 9 = 6$
$2 \times 9 = 18$	$7 \times 9 = 63$	$18 \div 9 = 2$	$63 \div 9 = 7$
$3 \times 9 = 27$	$8 \times 9 = 72$	$27 \div 9 = 3$	$72 \div 9 = 8$
$4 \times 9 = 36$	$9 \times 9 = 81$	$36 \div 9 = 4$	$81 \div 9 = 9$
$5 \times 9 = 45$	$10 \times 9 = 90$	$45 \div 9 = 5$	$90 \div 9 = 10$

1. How many are 9 times 9? 9 times 2? 9 times 5? 9 times 3?

2. How many times 9 in 81? 9 in 45? 9 in 18? 9 in 27?

3. If there are 6 trees in a row, how many trees are there in 9 rows?

4. Mary gave 54 cents for 9 spools of thread: how much did she give for each spool?

5. A boy rode 63 miles in 9 days: how many miles did he ride in one day?

6. How many are 9 times 4? 9 times 7? 9 times 10? 9 times 8?

7. If 81 blocks are placed in 9 rows, how many blocks will there be in each row?

8. Melons were sold at the rate of 4 for 36 cents: how much was that apiece?

9. At 9 cents a yard, how many yards of ribbon can be bought for 72 cents?

Lesson XLV.

1. I bought 10 pens at 2 cents each: how much did they cost?

2. How many pen-knives at 10 cents each, can you buy for 20 cents?

3. If George earns 3 dollars in one week, how much will he earn in 10 weeks?

4. If one quince is worth 10 apples, how many quinces can you get for 30 apples?

5. How many are 10 times 4? 10 times 5? 10 times 6? 10 times 7? 10 times 8? 10 times 9?

6. How many times 10 in 40? 10 in 50? 10 in 60? 10 in 70? 10 in 80? 10 in 90?

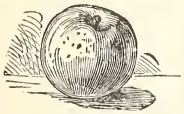
Copy and complete:

$1 \times 10 =$	$6 \times 10 =$	$10 \div 10 =$	$60 \div 10 =$
$2 \times 10 =$	$7 \times 10 =$	$20 \div 10 =$	$70 \div 10 =$
$3 \times 10 =$	$8 \times 10 =$	$30 \div 10 =$	$80 \div 10 =$
$4 \times 10 =$	$9 \times 10 =$	$40 \div 10 =$	$90 \div 10 =$
$5 \times 10 =$	$10 \times 10 =$	$50 \div 10 =$	$100 \div 10 =$

Lesson XLVI

EQUAL PARTS OF NUMBERS.

1. If an apple is cut into two equal pieces, what part of the apple will each piece be?



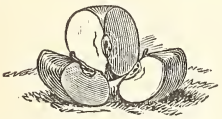
2. If a pile of books is divided into two equal parts, what part of the pile will each of these parts be?

3. One half of 4 apples are how many apples? One half of 6 apples? One half of 12 apples? One half of 20 apples?

4. If an orange is divided into 2 equal parts, what is each part called? If I give you one half, how much will I have left?

5. If an apple is divided into 4 equal parts, what is each part called?

6. One fourth of 8 apples are how many apples? One fourth of 16 apples? One fourth of 12 apples? One fourth of 20 apples?



7. If an apple is divided into three equal parts, what is each part called?

8. What is one third of 9 cents? One third of 15 cents? One third of 21 cents? One third of 12 cents?

9. What is one of the five equal parts of an object called? One of the six equal parts? One of the eight equal parts? One of the ten equal parts?

10. What is one fifth of 15 cents?

11. What is one fifth of 20? Of 30? 35? 40? 50? 60? 70? 75?

12. What is one sixth of 12? Of 24? 36? 48? 30? 18?

13. How much is one eighth of 16? Of 24? 48? 40? 32? 56?

14. How much is one tenth of 20? Of 60? 40? 80? 30? 90?

Lesson XLVII.

REVIEW EXERCISES.

1. How many are 20+2? 3+30? 40+4? 50+6? 60+8? 7+70? 5+80? 90+9? 10+15?

2. How many are 21+2? 31+4? 6+41? 41+8? 10+41? 3+51? 61+5? 71+7? 9+81?

3. How many are 22+2? 5+32? 4+42? 52+7? 3+62? 72+6? 8+82? 12+10? 52+9?

4. How many are 13+2? 33+3? 4+23? 43+5? 6+53? 63+7? 8+83? 73+9? 93+6?

5. How many are 24+1? 3+34? 44+5? 7+54? 64+9? 2+74? 84+4? 6+14? 8+24?

6. How many are 25+2? 4+35? 45+6? 8+55? 65+10? 3+75? 85+5? 95+3?

7. How many are 26+1? 3+36? 46+5? 7+56? 66+9? 2+76? 86+4? 6+76?

8. How many are 27+2? 17+4? 37+6? 47+8? 57+10? 1+67? 3+77? 5+87? 3+97? 7+37? 47+9?

9. How many are 1+28? 38+3? 5+48? 7+58? 68+9? 2+78? 88+4? 18+6? 1+98? 7+68? 6+88? 9+38? 6+48? 7+68? 38+9? 5+28? 8+18?

10. How many are 29+2? 4+39? 49+6? 8+59? 89+1? 79+3? 69+5? 59+7? 39+7? 59+9? 89+3? 39+5? 89+6? 49+8? 9+29? 69+9? 7+19?

Lesson XLVIII.

REVIEW EXERCISES.

1. How many are 5 + 9 + 3 — 4?
2. How many are 6 + 10 + 2 — 3?
3. How many are 4 + 9 + 5 — 6?
4. How many are 5 + 7 + 8 — 4?
5. How many are 8 + 3 + 10 — 3?
6. How many are 10 + 4 + 8 — 4?
7. How many are 9 + 8 + 5 — 6?
8. How many are 7 + 9 + 7 — 6?
9. How many are 9 + 6 + 8 — 7?
10. How many are 8 + 8 + 10 — 9?
11. How many are 9 + 9 + 9 — 10?
12. How many are 11 + 2 + 3 — 4?
13. How many are 11 + 5 + 2 — 7?
14. How many are 10 + 11 + 1 — 10?
15. How many are 10 + 9 + 8 — 7?
16. How many are 10 + 8 + 6 — 5?
17. How many are 9 + 8 + 7 — 10?
18. How many are 8 + 9 + 10 — 8?
19. How many are 9 + 11 + 10 — 8?
20. How many are 8 + 11 + 8 — 9?
21. How many are 20 + 2 + 4 — 6?
22. How many are 20 + 6 + 4 — 10?
23. How many are 29 + 4 + 8 — 6?
24. How many are 26 + 7 + 9 — 7?
25. How many are 24 + 9 + 10 — 8?
26. How many are 40 + 3 + 7 — 4?
27. How many are 18 + 8 + 10 — 6?
28. How many are 40 + 4 + 5 — 6?
29. How many are 44 + 4 + 8 — 7?
30. How many are 35 + 9 + 10 — 6?
31. How many are 16 + 4 + 5 — 5?

Lesson XLIX.

REVIEW EXERCISES.

1. How many are 3 × 3 — 4?
2. How many are 4 × 4 — 5?
3. How many are 5 × 5 — 6?
4. How many are 6 × 6 — 7?
5. How many are 7 × 7 — 8?
6. How many are 8 × 8 — 9?
7. How many are 9 × 9 — 10?
8. How many are 3 × 5 — 3?
9. How many are 3 × 7 — 4?
10. How many are 4 × 5 — 5?
11. How many are 5 × 4 — 8?
12. How many are 5 × 6 — 10?
13. How many are 5 × 8 — 10?
14. How many are 6 × 3 — 2?
15. How many are 5 × 7 — 5?
16. How many are 6 × 8 — 6?
17. How many are 6 × 10 — 3?
18. How many are 4 × 10 — 4?
19. How many are 5 × 10 — 6?
20. How many are 4 × 9 — 7?
21. How many are 7 × 4 — 3?
22. How many are 6 × 7 — 3?
23. How many are 8 × 4 — 5?
24. How many are 9 × 2 — 7?
25. How many are 10 × 2 — 5?
26. How many are 8 × 5 — 10?
27. How many are 9 × 5 — 6?
28. How many are 10 × 3 — 8?
29. How many are 8 × 9 — 4?
30. How many are 10 × 5 — 9?
31. How many are 9 × 3 — 6?

Lesson L.

1. How many are $3 \times 10 \div 5$? $7 \times 4 \div 7$?
2. How many are $6 \times 3 \div 2$? $2 \times 6 \div 4$?
3. How many are $6 \times 5 \div 3$? $8 \times 5 \div 4$?
4. How many are $2 \times 9 \div 6$? $4 \times 10 \div 8$?
5. How many are $5 \times 9 \div 5$? $6 \times 10 \div 6$?
6. How many are $7 \times 9 \div 7$? $5 \times 6 \div 5$?
7. How many are $8 \times 7 \div 8$? $5 \times 8 \div 4$?
8. How many are $6 \times 10 \div 6$? $9 \times 8 \div 9$?
9. How many are $2 \times 7 \div 2$? $3 \times 7 \div 3$?
10. How many are $36 - 6, \div 5$? $28 - 4, \div 6$?
11. How many are $30 + 5, \div 7$? $41 + 7, \div 8$?
12. How many are $76 + 5, \div 9$? $43 - 7, \div 4$?
13. How many are $31 - 4, \div 3$? $25 - 7, \div 2$?
14. How many are $37 + 3, \div 10$? $67 - 3, \div 8$?
15. How many are $77 - 5, \div 9$? $49 - 9, \div 10$?

Read rapidly the results only:

One half of	One third of	One fourth of	One fifth of
4	6	12	25
8	12	20	35
16	24	28	20
24	9	16	40
14	18	32	55

Make problems by filling the blanks:

1. Mary had —— cents; she bought —— yards of lawn at —— cents a yard. How much money had she left?

2. Tom had —— cents; he paid —— cents for a top; —— cents for a pen-knife. How much money had he left?

3. A boy went to the grocery store and bought —— pounds of butter at —— a pound; —— dozen of eggs at —— a dozen. He gave the grocer ——. How much change ought he to receive?

4. Rita spent —— cents for a tablet; —— cents for a lead-pencil; —— cents for a book. How much money did she spend?

5. I bought —— pounds of oatmeal at —— cents a pound; —— pounds of sugar at —— cents a pound. I gave the grocer a half dollar: how much change shall I get?

6. If —— pencils can be bought for —— cents, how many pencils can be bought for —— cents?

7. At —— dollars a pair, how many pairs of boots can be bought for —— dollars?

8. —— × —— = 33.
9. —— ÷ —— = 6.
10. —— × —— = 50.
11. —— ÷ —— = 12.

12. —— + —— = 60.
13. —— + —— = 44.
14. —— − —— = 25.
15. —— − —— = 35.

PART II.

DEFINITIONS

1. A **Unit** is a single thing; as, one book, one slate, one desk, one.

2. A **Number** is a unit or a collection of units.

3. **Arithmetic** is the science of numbers and the art of using them.

4. **Notation** is the art of expressing numbers.

5. In the Arabic method of Notation, which is commonly used, numbers are expressed by figures.

6. **Figures** are characters used to represent numbers.

The following figures are used: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

The first nine are called *digits*, or *significant* figures.

The cipher is used to give different values to the significant figures by changing their location.

All numbers can be expressed by these figures, or by combinations of them.

7. **Numeration** is the art of reading numbers.

Any number less than ten is read by naming the figure representing it; all other numbers, by naming the figures, the places they occupy, and the period in which they stand.

Nine is the largest number that can be represented by one figure; ninety-nine, by two figures; nine hundred and ninety-nine (999), by three.

8. A **Period** may consist of three figures.

In every period, the *right-hand* figure represents *units*, and occupies the *first* place in the period.

The *middle* figure represents *tens*, and occupies the *second* place in the period.

The *left-hand* figure represents *hundreds,* and occupies the *third* place in the period.

In the number 564, 4 in the first place is 4 units ; 6 in the second place, 6 tens, or sixty units ; 5 in the third place, 5 hundreds, or 50 tens, or 5 hundred units.

9. The number expressed by any figure depends upon the order or place which it occupies.

Thus, 2 in the first order is 2 units ; in the second order, 2 tens, or twenty ; in the third order, 2 hundreds, etc.

10. The value of a figure increases from right to left *tenfold ;* thus, ten units of the first place make one unit of the second ; ten units of the second place make one unit of the third ; and

INVARIABLY, *Ten units of any place or order make one unit of the next higher.*

NUMERATION TABLE.

5	5	5,	5	5	5,	5	5	5,	5	5	5,	5	5	5,	5	5	5,	5	5	5,	5	5	5,
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units
of			of			of			of			of			of			of			of		
Trillions, or 5th Period.			Billions, or 4th Period.			Millions, or 3d Period.			Thousands, or 2d Period.			Units, or 1st Period.											

To THE TEACHER.—Frequent and thorough drills upon this Table are indispensable. The pupils should be exercised upon it until able to name promptly and accurately all the places and periods.

NOTE.—It is deemed unnecessary to present in an elementary work exercises extending beyond the fifth period, or trillions. For exercises upon the higher periods, see the Advanced Arithmetic.

11. Each period contains three places or orders.

If any *order* is wanting, fill its place with a cipher; if a whole *period* is wanting, fill its place with three ciphers.

For convenience, a comma is used to separate the different periods.

RULE FOR NOTATION.—*Begin at the left, write the highest period first, then the lower periods in their order, filling all vacant periods and places with ciphers.*

RULE FOR NUMERATION.—*Begin at the right, and separate the given numbers into periods of three figures each.*

Begin at the left, and read each period as if it stood alone, then give the name of the period.

NUMBERS TO BE WRITTEN.

1. Write one; ten; eleven; one hundred; one hundred and one; one hundred and eleven.

2. Write two hundred; two hundred and two; two hundred and twenty-two; three hundred and three; three hundred and thirty-three.

3. Write nine; ninety; ninety-nine; nine hundred; nine hundred and nine; nine hundred and ninety-nine.

4. Write 1 hundred and 1 ten; 1 hundred and 1 unit; 1 hundred, 1 ten, and 1 unit.

5. Write 2 hundreds; 2 hundreds and 2 tens; 2 hundreds and 2 units; 2 hundreds, 2 tens, and 2 units.

6. Write 3 hundreds; 3 hundreds and 3 tens; 3 hundreds and 3 units; 3 hundreds, 3 tens, and 3 units.

7. Write 4 hundreds; 4 hundreds and 4 tens; 4 hundreds and 4 units; 4 hundreds, 4 tens, and 4 units; 4 tens and 4 units.

8. Write 5 units; 5 tens; 5 tens and 5 units; 5 hundreds; 5 hundreds and 5 tens; 5 hundreds and 5 units; 5 hundreds, 5 tens, and 5 units.

REMARK.—Remember that numbers are read by periods, that each period contains three places, and that in writing numbers all vacant places and periods are filled with ciphers.

9. Write 4 *units* in the first period; 4 units in the second period; 4 units in the third period; 4 units in the fourth period.

10. Write 4 *tens* in the first period; 4 tens in the second period; 4 tens in the third period; 4 tens in the fourth period.

11. Write 4 *hundreds* in the first period; 4 hundreds in the second period; in the third period; in the fourth.

12. Write 4 hundreds and 4 units in the first period; 4 hundreds and 4 units in the second period; in the third period; in the fourth.

13. Write 4 hundreds, 4 tens, and 4 units in the first period; in the second period; in the third period.

14. Write 4 hundreds in the third period; 4 hundreds and 4 units in the second period; and 4 hundreds, 4 tens, and 4 units in the first period.

15. Write 4 tens and 4 units in the third period; 4 units in the second period; and 4 tens in the first period.

Write the following in figures, and read them:

16. Seven hundred and eighty-four.
17. Six hundred and five.
18. Two thousand, seven hundred and four.
19. Eight thousand, and one.
20. Five thousand, and ten.
21. Fourteen thousand, nine hundred and nine.
22. Twenty thousand, and two.
23. Twenty-two thousand, and two hundred.

24. Eleven thousand, and eleven.
25. Thirty-seven thousand, eight hundred and thirteen.
26. Seventy-three thousand, and thirty-one.
27. Ninety thousand, and seventy.
28. Nine hundred and sixty-two thousand.
29. Seven hundred and one thousand.
30. One hundred thousand, and seven hundred.
31. Eight hundred thousand, and seventy-three.
32. Three hundred and one thousand, and twenty-one.
33. Five hundred thousand, and four.
34. Six hundred and twenty thousand, and forty.
35. Four millions, seven hundred and fifty-two thousand, four hundred and fifty-eight.
36. Write 107 millions, 17 thousand, and 17.
37. Write 10 billions, 100 millions, and 1 thousand.
38. Write 509 thousand, and 90.

Read the following numbers :

1. 189	*10.* 2306	*19.* 812345	*28.* 5210374
2. 203	*11.* 3054	*20.* 905432	*29.* 6387049
3. 356	*12.* 4120	*21.* 810678	*30.* 7569103
4. 550	*13.* 5387	*22.* 753029	*31.* 8789320
5. 679	*14.* 12345	*23.* 687401	*32.* 3030605
6. 800	*15.* 60789	*24.* 789350	*33.* 2008713
7. 903	*16.* 73094	*25.* 2123567	*34.* 2001001
8. 1000	*17.* 85107	*26.* 3098765	*35.* 3000003
9. 1234	*18.* 96130	*27.* 4305678	*36.* 3050050

ROMAN NOTATION.

12. In the Roman method of Notation, numbers are expressed by seven capital letters ; these are : I, V, X, L, C, D, and M.

I denotes one ; V, five ; X, ten ; L, fifty ; C, one hundred ; D, five hundred ; and M, one thousand.

Repeating a letter repeats its value; thus, II denotes two; XXX, thirty; CCCC, four hundred.

If a letter is placed *before* one of greater value, the less is taken from the greater; but if placed *after*, its value is added to the greater; thus, IV denotes four, while VI denotes six.

TABLE OF ROMAN NOTATION.

I	One.	XXI	Twenty-one.
II	Two.	XXX	Thirty.
III	Three.	XL	Forty.
IV	Four.	L	Fifty.
V	Five.	LX	Sixty.
VI	Six.	XC	Ninety.
IX	Nine.	C	One hundred.
X	Ten.	CCCC	Four hundred.
XI	Eleven.	D	Five hundred.
XIV	Fourteen.	DC	Six hundred.
XV	Fifteen.	DCC	Seven hundred.
XVI	Sixteen.	DCCC	Eight hundred.
XVIII	Eighteen.	DCCCC	Nine hundred.
XIX	Nineteen.	M	One thousand.
XX	Twenty.	MM	Two thousand.

Read the following numbers:

1. XXII.
2. XLIX.
3. LXIV.
4. XCVIII.
5. XIX.
6. XXXVII.
7. CXVI.
8. LV.
9. LXXI.
10. LXXXIX.
11. CC.
12. XLVI.

Write the following in Roman Notation:

1. Twelve.
2. Eighteen.
3. Nineteen.
4. Twenty-seven.
5. Thirty-eight.
6. Eighty.
7. Sixty.
8. Ninety-five.
9. Four hundred.
10. Five hundred.
11. Forty.
12. One thousand.

ADDITION

13. **Addition** is the process of finding the sum of two or more numbers.

14. The **Sum**, or **Amount**, is the number which contains as many units as all the numbers combined.

15. The **Sign of Addition** is +. It is called *plus*, which means *more*. Placed between two numbers, it shows that they are to be added.

16. The **Sign of Equality** is =. It denotes that the quantities between which it stands are equal.

17. Only like numbers, that is, those of the same kind or denomination, can be added.

The Sum, or Amount, is of the same denomination as the numbers added.

MENTAL EXERCISES.

1. How many are 7 dollars, 4 dollars, and 2 dollars?

2. Jack had 5 apples; his brother gave him 3, and his sister 2: how many did he then have?

3. A man gave 9 dollars for a plow, and 6 for a harrow: how much did he give for both?

4. In one window there are 9 panes of glass; in another, 6: how many panes are there in both?

5. Susan had 8 pears, her father gave her 5, and her mother 3: how many had she then?

6. How many are 7, 5, and 6?

7. Jane paid 5 cents for a spool of thread, 8 cents for pins, and 4 cents for tape: how much did she pay for all?

8. Mabel read 7 pages of her new book on Monday, 9 on Tuesday, and 10 on Wednesday: how many pages did she read in the three days?

9. Tom carried 7 sticks of wood at one load, 8 at another, and 6 at another: how many sticks did he carry in all?

10. How many are 11 and 7? 11 and 9? 11 and 12? 11 and 14? 11 and 16?

11. How many are 12 and 8? 10 and 12? 12 and 12? 14 and 12? 12 and 13?

12. How many are 13 and 10? 11 and 13? 13 and 13? 13 and 15? 13 and 16?

13. How many are 8, 7, and 5? 6, 5, and 3? 1, 9, and 7? 9, 8, and 7? 8, 6, and 9?

14. How many are 3, 2, and 9? 2, 4, and 8? 3, 5, and 7? 9, 4, and 7? 8, 7, and 5?

15. Begin with 1 and add 2 successively until the sum = 101.

16. Begin with 2 and add 2 successively until the sum = 100.

17. Begin with 2 and add 3 successively until the sum = 101.

18. Begin with 3 and add 3 successively until the sum = 102.

19. $4+2+3+7+1+5+9+6+3 =$ what?

WRITTEN EXERCISES.

MODEL SOLUTION OF A PROBLEM.

18. EXAMPLE.—What is the amount of 204, 849, 698, and 532?

SOLUTION.—Since only numbers of the same kind can be added, write units under units, tens under tens, and hundreds under hundreds; and draw a line beneath.

Begin at the units' column and add, naming results only; thus, 2, 10, 19, 23: 23 units = 2 tens and 3 units. Write the 3 units under the column of units, and add the 2 tens to the column of tens; thus, 2, 5, 14, 18: 18 tens = 1 hundred and 8 tens. Write the 8 tens under the column of tens, and add the 1 hundred to the column

OPERATION.

Hund's.	Tens.	Units.
2	0	4
8	4	9
6	9	8
5	3	2
2	2	8	3

of hundreds; thus, 1, 6, 12, 20, 22 : 22 hundreds = 2 thousand and 2 hundreds. Write the 2 hundreds under the column of hundreds, and the 2 thousand in the thousands' place.

Conclusion.—Therefore, the amount of 204, 849, 698, and 532 is 2283. Hence, the

Rule for Addition.—*Write the numbers to be added, so that figures of the same order may stand in the same column, units under units, tens under tens, etc., and draw a line underneath.*

Begin at the right, and add each column separately. Place the units obtained by adding each column under it, and add the tens, if any, to the next higher order.

Write the entire sum of the last column.

Proof.—Add the columns downward. If the result is the same, the work is probably correct.

Note.—In adding, give the results without naming the figures. Thus, in the example given, say 2, 10, 19, etc.; *not* 2 and 8 are 10, and 9 are 19, etc.

Copy and add the following numbers:

(*20.*)	(*21.*)	(*22.*)	(*23.*)	(*24.*)	(*25.*)
5432	7654	5346	6135	4523	7840
2364	1235	2453	3844	2236	2105

(*26.*)	(*27.*)	(*28.*)	(*29.*)	(*30.*)	(*31.*)
2253	1122	1065	5645	5831	5555
4314	2203	5416	8354	5673	7777
2432	3322	2518	8649	8624	9999

(*32.*)	(*33.*)	(*34.*)	(*35.*)	(*36.*)	(*37.*)
2045	3725	7336	5555	9742	3249
5183	9534	5943	6666	4324	7185
6317	6102	8204	7777	1762	6227
7102	4261	6427	8888	5483	2539

(*38.*)	(*39.*)	(*40.*)	(*41.*)
6574	7604	25	3779
3865	4596	437	4005
4936	1009	3401	218
5457	8888	9778	28
20832	22097	13641	8030

(*42.*)	(*43.*)	(*44.*)	(*45.*)
	6079	744	47653
9	983	7243	33646
83	72	63	72835
107	8154	7005	34467
4973	6713	3111	287
5172	22001	18166	188888

46. Find the sum of 9777, 675, 9799, 5583, 8999, 5569, and 8995. *Ans.* 49397.

47. 6472 + 509 + 6006 + 66 + 8 = ? *Ans.* 13061.

48. Add 668, 4497, 58, 6759, 85, and 7203. *Ans.* 19270.

49. Add 5687, 7794, 8649, 3867, 7868, 7769, and 9786. *Ans.* 51420.

50. 9847 + 4562 + 8443 + 7987 + 6657 + 9667 + 7999 + 7854 = ? *Ans.* 63016.

51. A farmer raised 9742 bu. of wheat, and bought 1568 bu.: how much wheat did he then have? *Ans.* 11310 bu.

52. A carriage cost $1250; a pair of horses, $980; and a set of harness, $136: how much did they all cost? *Ans.* $2366.

REMARK.—The sign $ denotes dollars, and, when used, is placed *before* the figures.

53. A man paid $5409 for a house, $1209 for furniture, and $807 for improvements: what did he pay for all? *Ans.* $7425.

54. I owe one man $542, another $327, another $487, and another $6983 : how much do I owe? *Ans.* $8339.

55. From A to B is 212 miles, from B to C 4501 miles, from C to D 908 miles, and from D to E 8976 miles : how many miles from A to E? *Ans.* 14597 miles.

56. A farmer had 963 sheep in one field, 208 in another, 427 in another, and 530 in another : how many had he in all? *Ans.* 2128 sheep.

57. Washington was born in 1732, and lived 67 years : in what year did he die? *Ans.* In 1799.

58. On my land are 1640 oak-trees, 748 ash-trees, 639 beeches, 184 birches, 597 firs, 48 poplars, 186 apple, and 247 pear-trees : how many trees in all? *Ans.* 4289 trees.

59. In one year a farmer made by his horses $364, by his cows $785, by his sheep $1064, by his pigs $184, by his corn $304, by his hay $97, and by his oats $46 : what did he make in all? *Ans.* $2844.

60. One shepherd has 327 sheep, another has 25 more than the first : how many have both? *Ans.* 679 sheep.

61. By selling a farm for $6479, the owner lost $2734 : what did it cost at first? *Ans.* $9213.

62. James was born in 1826 : in what year was he 48 years old? *Ans.* In 1874.

63. September has 30 days ; October, 31 ; November, 30 : how many days in these months of the fall?

Ans. 91.

64. A man bought four loads of hay ; the first weighing 2030 pounds ; the second, 2212 pounds ; the third, 2200 pounds ; and the fourth, 2322 pounds : how many pounds of hay in the four loads? *Ans.* 8764.

65. In a city having six wards, there are 348 voters in the first ward ; 443, in the second ; 305, in the third ; 516, in the fourth ; 410, in the fifth ; and 418 in the sixth : how many voters in the city? *Ans.* 2440.

66. A man's sheep are worth $8076, and his cows are worth $1096 more than his sheep: what are the cows and sheep together worth? *Ans.* $17248.

67. Add twenty-six thousand three hundred and sixteen; forty-two thousand six hundred and twenty-seven; five thousand and five; three hundred and seven thousand and four; three hundred. *Ans.* 381252.

PROBLEMS TO MAKE CHANGE AS USUALLY MADE.

Having 1 ct., 5 ct., 10 ct., 25 ct., and 50 ct. pieces, and $1, $2, and $5 bills, count out the proper change in each of the transactions given below:

1. I have $1; buy gloves for 35 cts.; ribbon for 22 cts.

METHOD.—The clerk will most probably use three 1 ct., one 5 ct., one 10 ct., and one 25 ct. pieces; and, since the cost of the goods is 57 cts., he will say, while handing the change, 60, 65, 75, $1.

2. Have 25 cts.; buy an illustrated paper for 10 cts.

3. Have 50 cts.; buy a bottle of ink, 8 cts.; a bottle of mucilage, 25 cts.

4. Have $1; buy a slate, 25 cts.; a tablet, 10 cts.; an eraser, 3 cts.

NOTE.—$1 and 25 cts. is written thus, $1.25; $1 and 13 cts., thus, $1.13; $1 and 56 cts., thus, $1.56. The two numbers placed after the dot represent cents.

5. Have $3; buy Lowell's Poems, $1.25; paper and envelopes, 75 cts.; a magazine, 35 cts.

6. Have $5; buy kid gloves, $1.50; handkerchiefs, $2; ribbon, 38 cts.

7. Have $5; buy a pair of shoes, $3.50; a pair of slippers, 75 cts.

8. Have $5; buy sugar, $1.50; strawberries, $1.32; sealing wax, 15 cts.

9. Have $10; buy dress goods, $7.80; buttons, 75 cts.

SUBTRACTION.

19. **Subtraction** is the process of finding the difference between two numbers.

20. The **Minuend** is the number to be diminished.

21. The **Subtrahend** is the number to be subtracted.

22. The **Difference,** or **Remainder,** is the number left after subtraction.

23. The **Sign of Subtraction** is —. It is called *minus,* which means *less.* Placed between two numbers, it shows that the one on the right is to be taken from the one on the left.

NOTE.—The Minuend and Subtrahend must be *like* numbers, that is, of the same kind or denomination. The Difference, or Remainder, is of the same denomination as the Minuend and Subtrahend.

MENTAL EXERCISES.

1. How many is 10 less 2? 20 less 2? 30 less 3? 40 less 3? 50 less 4? 60 less 4?

2. How many is 70 less 5? 90 less 5? 80 less 5? 20 less 6? 30 less 6?

3. How many is 20 less 7? 30 less 7? 50 less 8? 40 less 8? 50 less 9? 60 less 9?

4. How many is 11 less 2? 31 less 2? 21 less 3? 41 less 3? 51 less 4? 61 less 4?

5. How many is 71 less 5? 91 less 5? 81 less 6? 11 less 7? 21 less 7? 41 less 8? 51 less 8?

6. How many is 71 less 9? 61 less 9? 12 less 3? 22 less 3? 42 less 4? 32 less 4?

7. How many is 52 less 5? 72 less 5? 62 less 6? 82 less 6? 92 less 7? 22 less 7?

8. How many is 13 less 4? 23 less 5? 33 less 4?

9. How many is 53 less 7? 83 less 7? 93 less 8? 33 less 8? 14 less 5? 24 less 5?

10. How many is 24 less 6? 34 less 6? 54 less 7? 44 less 7? 64 less 8? 84 less 8?

11. William bought 16 marbles; he gave his brother 7, and kept the rest: how many did he keep?

12. Mary had 15 problems to solve; she finished 9: how many were not finished?

13. How many are 13 books less 7 books?

14. George had 17 cents; he lost 9 cents, and spent the rest for paper: what did the paper cost?

15. A man sold a cart for 18 dollars; he received for it a barrel of flour worth 9 dollars, and the rest in money: how much money did he receive?

16. James' father gave him 10 cents, his mother gave him 9; he spent 8 cents: how much had he left?

17. From 101 subtract 2 successively until only 1 remains.

18. From 100 subtract 3 successively until only 1 remains.

19. From 100 subtract 4 successively until nothing remains.

20. From 100 subtract 5 successively until nothing remains.

WRITTEN EXERCISES.

When no figure of the subtrahend has a greater value than the corresponding figure of the minuend.

1. From 768 subtract 524.

ANALYSIS.—For convenience, the less number is written under the greater, units under units, tens under tens, hundreds under hundreds.

Beginning at the right, each order in the subtrahend is subtracted separately from the same order in the minuend.

Minuend	768
Subtrahend	524
	244

Thus, 4 units taken from 8 units leaves 4 units, which are written under the units.

2 tens from 6 tens leaves 4 tens, which are written under the tens.

5 hundreds from 7 hundreds leaves 2 hundreds, which are written under the hundreds. Hence, the remainder is 244.

PROOF.—244, the remainder, plus 524, the subtrahend, equals 768, the minuend. Hence, the result is correct.

Copy, subtract, and prove:

(*2.*)	(*3.*)	(*4.*)	(*5.*)	(*6.*)	(*7.*)
298	572	987	925	898	495
176	460	354	613	777	311

(*8.*)	(*9.*)	(*10.*)	(*11.*)	(*12.*)	(*13.*)
7693	7546	8997	8456	7936	9879
3252	4243	7276	3242	6724	3627

MENTAL EXERCISES.

1. If I have 3 marbles in one hand, and 5 in the other, how many more have I in one hand than in the other?

2. If I take 1 more marble into each hand, what will be the difference between the number of marbles in each hand?

3. If I have 14 marbles in one hand, and 9 in the other, how many more have I in the one hand than in the other?

4. If I take 10 more marbles into each hand, how many shall I have in each hand?

5. How many more will there be in one hand than in the other?

6. What change is made in the remainder when one is added both to minuend and subtrahend?

7. What change is made in the remainder when ten, or *one ten*, is added both to minuend and subtrahend?

8. I have one dollar in my right hand, and one dollar and fifty cents in my left: how much more money have I in my left hand than in my right?

9. If I take one dollar more into each hand, how much more will then be in my left than in my right hand?

10. How many cents in one dollar?

11. What change is made in the remainder when one hundred is added both to minuend and subtrahend?

We have then learned that to add the same number to both minuend and subtrahend does not alter the remainder.

Since ten units of any place or order make one unit of the next higher, adding ten units is the same as adding one ten; or adding ten tens is the same as adding one hundred.

WRITTEN EXERCISES.

MODEL SOLUTION.

12. EXAMPLE.—What is the difference between 934 and 576?

Write units under units, tens under tens, etc.

Since 6 units can not be taken from 4 units, add 10 units to the 4 units, making 14 units, and take 6 units from the 14 units, and write 8 units (the difference) below. Since we have added 10 units to the minuend, we must add 1 ten to the 7 tens, making 8 tens. We can not take 8 tens from 3 tens, so we add 10 tens to the 3 tens, making 13 tens, and take 8 tens from 13 tens, and write 5 tens below. Having added 10 tens to the minuend, we must add 1 hundred (equal to 10 tens) to the 5 hundreds, making 6 hundreds, and take 6 hundreds from 9 hundreds, and write 3 hundreds (the difference) below. The difference is 358. Hence the

9	3	4
5	7	6
3	5	8

RULE FOR SUBTRACTION.—*Write the less number under the greater, placing units under units, tens under tens, etc., and draw a line underneath.*

Begin at the right hand, subtract each figure from the one above it, and write the difference underneath.

When any term of the subtrahend is greater than the term above it, add 10 to the upper term, and then subtract, and write the difference underneath.

When 10 has been added to the upper term, add 1 to the next higher term of the subtrahend before subtracting.

PROOF.—Add the remainder and subtrahend; if their sum equals the minuend, the work is right.

NOTE.—When the process is understood, pupils should not longer be required to explain problems in Subtraction; but should be taught to work rapidly, saying, as in the model problem, 6 from 14, 8; 8 from 13, 5; 6 from 9, 3.

(*13.*)	(*14.*)	(*15.*)	(*16.*)
9524	7627	2378	8472
6382	1994	1769	5887
3142	5633	609	2585

(*17.*)	(*18.*)	(*19.*)	(*20.*)
5223	8745	14406	20670
2996	4996	8375	6537
2227	3749	6031	14133

(*21.*)	(*22.*)	(*23.*)	(*24.*)
35678	10001	300000	987654
16745	5674	10001	321987
18933	4327	289999	665667

25. Subtract 34675 from 600000. *Ans.* 565325.

26. Take 93299 from 100603. *Ans.* 7304.

27. How much less is 36759 than 95763?

Ans. 59004.

28. Find the difference between 373737 and 736736.

Ans. 362999.

29. How much greater is 77777 than 69898?

Ans. 7879.

30. A grocer bought 675 pounds of sugar, and sold 342 pounds: how many pounds had he left?

Ans. 333 pounds.

31. A house cost $3705 ; the owner sold it, losing $596 : what did he receive for it ? *Ans.* $3109.

32. A lot was worth $685 in 1850, and $7261 in 1860 : how much had it increased in value in ten years ?

Ans. $6576.

33. What year was 95 years before 1860 ? *Ans.* 1765.

34. A man had 73300 bushels of coal, and sold 18008 : how many bushels had he left ? *Ans.* 55292 bu.

35. A man whose income is $1675, spends annually $1480 : how much does he lay up ? *Ans.* $195.

36. A merchant having $13325 in a bank, drew out $789 : how much remained in the bank ? *Ans.* $12536.

EXAMPLES COMBINING ADDITION AND SUBTRACTION.

1. I had 153 yards of cloth ; I sold 27 yards to A, and 48 yards to B : how much have I now ? *Ans.* 78 yd.

2. From 43021, subtract the sum of 3798 and 30247 : what remains ? *Ans.* 8976.

3. A man worth $19000, lost at one time $3427, and at another $1946 : how much had he remaining ?

Ans. $13627.

4. A man had 640 acres of land ; he bought 266 acres, and then sold 512 acres : how many acres had he then ?

Ans. 394 acres.

5. From the difference between 10001 and 667 take the sum of 2347 and 5169. *Ans.* 1818.

6. A bought 463 barrels of flour ; B bought 95 barrels less than A : how many barrels did both buy ?

Ans. 831 bbl.

7. To the sum of 7749 and 5928 add their difference.

Ans. 15498.

8. From the sum of 7654 and 1234 subtract their difference. *Ans.* 2468.

MULTIPLICATION

DEFINITIONS.

24. **Multiplication** is the process of taking one number as many times as there are units in another.

25. The **Multiplicand** is the number taken.

26. The **Multiplier** is the number denoting how many times the multiplicand is to be taken.

27. The **Product** is the result obtained.

The multiplicand and multiplier are together called *Factors*, because they make or produce the product.

28. The **Sign of Multiplication** is $\times$. Placed between two numbers, it denotes that they are to be multiplied together.

29. The product of two numbers is not altered by changing the order of the factors. Thus, $4 \times 3 = 12$, and $3 \times 4 = 12$; hence, in performing a multiplication, we generally use for a multiplier the one containing the fewest figures, since it is more convenient to do so.

In indicating the multiplication, however, it is best to write that word before the sign ($\times$) which properly comes before the word times; thus, three times thirty-six dollars, $3 \times \$36$, etc.

30. The multiplier shows the *number of times* the multiplicand is to be taken; hence, it must always be considered simply a number. The product is always of the same denomination as the multiplicand.

Thus, $\$12 \times 3 = \$36$, three times twelve *dollars* are thirty-six *dollars*.

NOTE.—See that the following table is thoroughly committed to memory:

MULTIPLICATION TABLE.

1×1 = 1	1× 2 = 2	1× 3 = 3	1× 4 = 4
2×1 = 2	2× 2 = 4	2× 3 = 6	2× 4 = 8
3×1 = 3	3× 2 = 6	3× 3 = 9	3× 4 = 12
4×1 = 4	4× 2 = 8	4× 3 = 12	4× 4 = 16
5×1 = 5	5× 2 = 10	5× 3 = 15	5× 4 = 20
6×1 = 6	6× 2 = 12	6× 3 = 18	6× 4 = 24
7×1 = 7	7× 2 = 14	7× 3 = 21	7× 4 = 28
8×1 = 8	8× 2 = 16	8× 3 = 24	8× 4 = 32
9×1 = 9	9× 2 = 18	9× 3 = 27	9× 4 = 36
10×1 = 10	10× 2 = 20	10× 3 = 30	10× 4 = 40
11×1 = 11	11× 2 = 22	11× 3 = 33	11× 4 = 44
12×1 = 12	12× 2 = 24	12× 3 = 36	12× 4 = 48
1×5 = 5	1× 6 = 6	1× 7 = 7	1× 8 = 8
2×5 = 10	2× 6 = 12	2× 7 = 14	2× 8 = 16
3×5 = 15	3× 6 = 18	3× 7 = 21	3× 8 = 24
4×5 = 20	4× 6 = 24	4× 7 = 28	4× 8 = 32
5×5 = 25	5× 6 = 30	5× 7 = 35	5× 8 = 40
6×5 = 30	6× 6 = 36	6× 7 = 42	6× 8 = 48
7×5 = 35	7× 6 = 42	7× 7 = 49	7× 8 = 56
8×5 = 40	8× 6 = 48	8× 7 = 56	8× 8 = 64
9×5 = 45	9× 6 = 54	9× 7 = 63	9× 8 = 72
10×5 = 50	10× 6 = 60	10× 7 = 70	10× 8 = 80
11×5 = 55	11× 6 = 66	11× 7 = 77	11× 8 = 88
12×5 = 60	12× 6 = 72	12× 7 = 84	12× 8 = 96
1×9 = 9	1×10 = 10	1×11 = 11	1×12 = 12
2×9 = 18	2×10 = 20	2×11 = 22	2×12 = 24
3×9 = 27	3×10 = 30	3×11 = 33	3×12 = 36
4×9 = 36	4×10 = 40	4×11 = 44	4×12 = 48
5×9 = 45	5×10 = 50	5×11 = 55	5×12 = 60
6×9 = 54	6×10 = 60	6×11 = 66	6×12 = 72
7×9 = 63	7×10 = 70	7×11 = 77	7×12 = 84
8×9 = 72	8×10 = 80	8×11 = 88	8×12 = 96
9×9 = 81	9×10 = 90	9×11 = 99	9×12 = 108
10×9 = 90	10×10 = 100	10×11 = 110	10×12 = 120
11×9 = 99	11×10 = 110	11×11 = 121	11×12 = 132
12×9 = 108	12×10 = 120	12×11 = 132	12×12 = 144

MENTAL EXERCISES.

1. How many are 6 times 2? 8 times 2? 10 times 2? 11 times 2? 5 times 2? 12 times 2? 7 times 2?

2. How many are 5 times 3? 7 times 3? 9 times 3? 11 times 3? 12 times 3? 10 times 3? 8 times 3?

3. How many are 6 times 4? 8 times 4? 4 times 4? 10 times 4? 12 times 4? 9 times 4? 11 times 4?

4. How many are 7 times 5? 5 times 5? 3 times 5? 11 times 5? 12 times 5? 10 times 5? 8 times 5?

5. How many are 3 times 6? 5 times 6? 7 times 6? 9 times 6? 11 times 6? 10 times 6? 12 times 6?

6. How many are 5 times 7? 7 times 7? 9 times 7? 11 times 7? 8 times 7? 12 times 7? 10 times 7?

7. How many are 5 times 8? 7 times 8? 11 times 8? 10 times 8? 12 times 8? 9 times 8? 8 times 8?

8. How many are 4 times 9? 6 times 9? 8 times 9? 10 times 9? 12 times 9? 5 times 9? 11 times 9?

9. How many are 5 times 10? 7 times 10? 9 times 10? 11 times 10? 12 times 10? 8 times 10? 6 times 10?

10. How many are 3 times 11? 5 times 11? 7 times 11? 9 times 11? 11 times 11? 10 times 11? 12 times 11?

11. How many are 3 times 12? 5 times 12? 7 times 12? 9 times 12? 11 times 12? 2 times 12? 4 times 12? 6 times 12? 8 times 12? 10 times 12? 12 times 12?

12. How many are 2 times 8? 3 times 9? 4 times 10? 2 times 11? 4 times 11? 3 times 10? 6 times 11?

13. John bought 2 pears at 3 cents apiece: how much did they cost?

14. At 6 cents a pound, what will 2 pounds of beef cost?

15. What is the cost of 3 yards of cloth at 3 dollars a yard?

16. How many are four times 3 books?

17. If one barrel of apples costs 4 dollars, what will 3 barrels cost ?

18. If one pound of iron costs 5 cents, what is the cost of 3 pounds ?

19. If a farmer gets 5 pounds of wool from one sheep, how much does he get from 4 sheep ?

20. If Henry goes to school 5 days each week, how many days will he go in 5 weeks?

21. Frank can throw a stone 6 yards, his father can throw one 5 times as far : how far can his father throw ?

22. Susan had 5 peaches ; Mary had seven times as many as Susan : how many had Mary ?

23. In one mile there are 8 furlongs : how many furlongs in 6 miles ?

24. In one peck there are 8 quarts : how many quarts in 7 pecks ?

25. How many are 7 times 7 bushels ?

26. There are 7 days in one week : how many days in 9 weeks ?

27. If a man earns $3 a day, how many dollars will he earn in 5 days ?

28. If a man walks 6 miles a day, how many miles will he walk in 7 days ?

29. There are 10 dimes in one dollar : how many dimes are there in 7 dollars ?

30. There are 4 quarts in a gallon : how many quarts in 6 gallons ? In 8 gallons ? In 10 gallons ?

31. If a dressmaker earns $12 a week, how much will she earn in 7 weeks ?

32. There are 7 rows of trees in an orchard, and 7 trees in each row : how many trees in the orchard ?

33. What will 9 pieces of braid cost at 8 cents apiece ?

34. What will 8 yards of muslin cost at 8 cents a yard ? 12 yards at the same price ?

35. What will 8 quarts of berries cost at 11 cents a quart?

36. What will 9 yards of cloth cost at $6 a yard?

37. If a boy is in school 5 hours each day, how many hours will he spend in school in 4 days?

38. One dozen of eggs is worth 11 cents: how much will 12 dozen be worth?

39. If one pound of butter costs 12 cents, what will 8 pounds cost?

WRITTEN EXERCISES.

31. When the multiplier is expressed by one figure.

EXAMPLE.—What is the product of 659 multiplied by 7?

SOLUTION.—Write the multiplier under the multiplicand, placing units under units, and draw a line beneath.

Begin at the right hand to multiply. 7 times 9 units are 63 units = 6 tens and 3 units; write the 3 units in the units' place, and add the 6 tens to the product of tens. 7 times 5 tens are 35 tens + 6 tens are 41 tens = 4 hundreds and 1 ten; write the 1 ten in the tens' place, and add the 4 hundreds to the product of hundreds. 7 times 6 hundreds are 42 hundreds + 4 hundreds are 46 hundreds = 4 thousands and 6 hundreds; write the 6 hundreds in the hundreds' place, and the 4 thousands in the place of thousands.

OPERATION.

Hund's.	Tens.	Units.	
6	5	9	Multiplicand.
		7	Multiplier.
4	6	1	3 Product.

CONCLUSION.—Therefore, the product of 659 multiplied by 7 is 4613.

1. Multiply 59 by 3. *Ans.* 177.

2. Multiply 68 by 4. *Ans.* 272.

3. Multiply 123 by 5. *Ans.* 615.

4. Multiply 456 by 6. *Ans.* 2736.

5. Multiply 789 by 7. *Ans.* 5523.

6. Multiply 654 by 8. *Ans.* 5232.

7. Multiply 321 by 9. *Ans.* 2889.

8. Multiply 1321 by 2. *Ans.* 2642.

9. Multiply 2172 by 3. *Ans.* 6516.
10. Multiply 3785 by 4. *Ans.* 15140.
11. Multiply 5006 by 5. *Ans.* 25030.
12. Multiply 7208 by 6. *Ans.* 43248.
13. Multiply 53124 by 7. *Ans.* 371868.
14. Multiply 64005 by 8. *Ans.* 512040.
15. Multiply 54321 by 9. *Ans.* 488889.
16. What will 2 houses cost at $2345 each? *Ans.* $4690.
17. What will 3 houses cost at $4465 each? *Ans.* $13395.
18. What will 5 horses cost at $275 each? *Ans.* $1375.

EXAMPLE.—What is the product of 659 multiplied by 87?

SOLUTION.—Write the multiplier under the multiplicand, placing units under units and tens under tens and draw a line beneath.

Multiply each figure in the multiplicand by the unit figure of the multiplier, as in the preceding example. Then multiply by the second, or *tens*, figure of the multiplier. 8 tens times 9 units are 72 *tens* = 7 hundreds and 2 tens; write the 2 tens in the tens' place, and add the 7 hundreds to the product of hundreds. 8 tens times 5 tens are 40 *hundreds*, + 7 hundreds = 47 hundreds = 4 thousands and 7 hundreds; write the 7 hundreds in the hundreds' place, and add the 4 thousands to the product of thousands. 8 tens times 6 hundreds are 48 thousands, + 4 thousands are 52 thousands = 5 ten-thousands and 2 thousands; write the 2 thousands in the thousands' place, and the 5 ten-thousands in the place of ten-thousands.

OPERATION.

	Hund's.	Tens.	Units.	
	6	5	9	Multiplicand.
		8	7	Multiplier.
	4	6	1	3
5	2	7	2	
5	7	3	3	3 Product.

Draw a line beneath, and add these partial products, which gives the entire product, 57333.

CONCLUSION.—Therefore, the product of 659 multiplied by 87 is 57333.

EXAMPLE.—What is the product of 3842 multiplied by 408?

SOLUTION.—Write the multiplier under the multiplicand, as before. Multiply first by the first term of the multiplier, and write the product as before. Then, since units multiplied by hundreds produce *hundreds*, write the first figure of the second partial product in hundreds' order. In 408 there are no tens to be used as a multiplier.

OPERATION.

```
  3 8 4 2
      4 0 8
  -------
  3 0 7 3 6
1 5 3 6 8
-----------
1 5 6 7 5 3 6
```

GENERAL RULE.—1. *Write the multiplier under the multiplicand, placing figures of the same order in a column.*

2. *Multiply the multiplicand by each figure of the multiplier in succession, beginning with units, always placing the right-hand figure of each product under that figure of the multiplier which produces it.*

3. *Add the partial products together; their sum will be the product sought.*

PROOF.—Multiply the multiplier by the multiplicand; if the product thus obtained is the same as the first product, the work is probably correct.

(*19.*)	(*20.*)	(*21.*)	(*22.*)	(*23.*)
235	346	425	1234	5678
13	19	29	506	708
705	3114	3825	7404	45424
235	346	850	6170	39746
3055	6574	12325	624404	4020024

24. Multiply 4606 by 75. *Ans.* 345450.

25. Multiply 2327 by 84. *Ans.* 195468.

26. Multiply 1234 by 88. *Ans.* 108592.

27. Multiply 987 by 568. *Ans.* 559616.

28. Multiply 776 by 478. *Ans.* 370928.

29. Multiply 208 by 409. *Ans.* 85072.
30. Multiply 6704 by 568. *Ans.* 3807872.

NOTE.—The removal of a figure *one* place to the left increases its value ten-fold; *two* places, a hundred-fold; *three* places, a thousand-fold, etc.; hence, to multiply by 10, 100, 1000, etc., annex as many ciphers to the multiplicand as there are ciphers in the multiplier.

31. Multiply 123 by 10. *Ans.* 1230.
32. Multiply 456 by 100. *Ans.* 45600.
33. Multiply 789 by 1000. *Ans.* 789000.
34. Multiply 654 by 10000. *Ans.* 6540000.
35. Multiply 6535 by 100. *Ans.* 653500.

NOTE.—When there are ciphers on the right of one or both factors, multiply without regarding the ciphers, and annex to the product as many ciphers as there are on the right of both factors. (Teacher explain.)

36. Multiply 123 by 20. *Ans.* 2460.
37. Multiply 456 by 300. *Ans.* 136800.
38. Multiply 789 by 4000. *Ans.* 3156000.
39. Multiply 600 by 24. *Ans.* 14400.
40. Multiply 530 by 70. *Ans.* 37100.
41. Multiply 5798 by 20. *Ans.* 115960.
42. Multiply 7658 by 900. *Ans.* 6892200.
43. Multiply 8760 by 73. *Ans.* 639480.
44. Multiply 4869 by 509. *Ans.* 2478321.
45. Multiply 9873 by 654. *Ans.* 6456942.
46. What cost 9 farms at $457 each? *Ans.* $4113.
47. What cost 78 bonds worth $4096 each?
Ans. $319488.

48. How many gallons in 308 casks, each containing 59 gallons? *Ans.* 18172 gallons.

49. How many bushels of coal in 7009 loads, each containing 28 bushels? *Ans.* 196252 bushels.

50. A merchant sold 4090 bales of cotton at $78 a bale: how much did he receive for the lot? *Ans.* $319020.

51. A drover bought 356 horses at $207 apiece: what did he pay for all? *Ans.* $73692.

52. Find the weight of a train of 1057 cars, each weighing 9260 pounds. *Ans.* 9787820 pounds.

53. If there are 1040 men in each regiment, how many are there in 308 regiments? *Ans.* 320320 men.

54. A house contains 4 stories of six rooms each: what will the papering cost, at $17 a room? *Ans.* $408.

55. A farm cost me $5064: what would 5 such farms cost? *Ans.* $25320.

56. What will 9 lots cost at $4500 apiece?

Ans. $40500.

57. If one horse costs $85, what will 125 such horses cost? *Ans.* $10625.

58. Find the cost of 95 sewing machines at $75 each.

Ans. $7125.

59. What will be the cost of 18 pianos at $375 each?

Ans. $6750.

60. What will 212 cows cost at $48 each?

Ans. $10176.

61. If an engine runs 425 miles a day, how far will it run in 35 days? *Ans.* 14875 miles.

62. If a ship sails 216 miles a day, how far will it sail in 55 days? *Ans.* 11880 miles.

63. A father divided his estate among four sons, giving to each $5265: what was the value of the estate?

Ans. $21060.

64. There are 366 days in leap-year: how many days in 18 leap-years? *Ans.* $6588 days.

65. If a garrison of soldiers consumes 4878 pounds of bread a day, how many pounds will supply the garrison 515 days? *Ans.* 2512170 pounds.

66. There are 5280 feet in a mile: how many feet in 765 miles? *Ans.* 4039200 feet.

67. The average daily expenses of a manufactory were $8359. To what would the expenses amount in 173 days?

Ans. $1446107.

68. There are 480 sheets of paper in a ream: how many sheets in 125 reams? *Ans.* 60000 sheets.

69. A merchant purchased 26 pieces of broadcloth, each containing 48 yards, at $6 per yard. How much did he pay for the whole? *Ans.* $7488.

70. A man sold a farm of 476 acres at $95 per acre. How much did he receive for it? *Ans.* $45220.

EXAMPLES COMBINING ADDITION, SUBTRACTION, AND MULTIPLICATION.

1. Multiply the sum of 402 and 940 by the difference of 102 and 47. *Ans.* 73810.

2. Multiply the sum of 5080 and 6890 by the difference of 739 and 1806. *Ans.* 12771990.

3. Multiply the difference between 1004 and 806 by the sum of 87 and 263. *Ans.* 69300.

4. A man having 207 hogs, bought 65 more, and then sold all at $14 apiece: what was the amount realized?

Ans. $3808.

5. A man bought 45 acres of land at $375 an acre, and sold the entire piece for $15000: how much did he lose?

Ans. $1875.

6. At $507 apiece, how much more will 297 carriages cost than 133? *Ans.* $83148.

7. Bought 491 yards of cloth at 81 cents a yard; used 29 yards, and sold the rest at 95 cents a yard: how much did I gain? *Ans.* $64.68.

8. A clerk receives $100 a month, and spends $66 a month: how much does he save each year? *Ans.* $408.

9. What are 389 horses worth at $139 apiece; and 876 cows at $67 apiece? *Ans.* $112763.

DIVISION

32. **Division** is the process of finding how many times one number is contained in another, or of finding one factor of a number when the other factor is known.

33. The **Dividend** is the number to be divided.

34. The **Divisor** is the number by which we divide.

35. The **Quotient** is the number of times the divisor is contained in the dividend.

36. The **Remainder** is the part of the dividend remaining undivided.

NOTE.—Since the remainder is a part of the dividend, it must be of the same kind or denomination; that is, if the dividend is dollars, the remainder will be dollars; if pounds, the remainder will be pounds.

SIGNS OF DIVISION.

37. There are *three methods* of indicating division.

1. By a *horizontal line* with a *point* or *period* above and below it; thus, $\div$. This sign, when standing between two numbers, shows that the first is to be divided by the second; thus, $6 \div 2$ is read 6 divided by 2.

2. By a *horizontal line* with a *dividend written above,* and the *divisor below;* thus, $\frac{6}{2}$ is read 6 divided by 2.

3. By a *curved line* with the *divisor written at the left,* and the *dividend at the right;* thus, $2\overline{)6}$ is read 6 divided by 2.

NOTE.—If the following table is thoroughly committed to memory, it will prove of great benefit in rapid calculations:

DIVISION TABLE.

1 ÷ 1 = 1	2 ÷ 2 = 1	3 ÷ 3 = 1	4 ÷ 4 = 1
2 ÷ 1 = 2	4 ÷ 2 = 2	6 ÷ 3 = 2	8 ÷ 4 = 2
3 ÷ 1 = 3	6 ÷ 2 = 3	9 ÷ 3 = 3	12 ÷ 4 = 3
4 ÷ 1 = 4	8 ÷ 2 = 4	12 ÷ 3 = 4	16 ÷ 4 = 4
5 ÷ 1 = 5	10 ÷ 2 = 5	15 ÷ 3 = 5	20 ÷ 4 = 5
6 ÷ 1 = 6	12 ÷ 2 = 6	18 ÷ 3 = 6	24 ÷ 4 = 6
7 ÷ 1 = 7	14 ÷ 2 = 7	21 ÷ 3 = 7	28 ÷ 4 = 7
8 ÷ 1 = 8	16 ÷ 2 = 8	24 ÷ 3 = 8	32 ÷ 4 = 8
9 ÷ 1 = 9	18 ÷ 2 = 9	27 ÷ 3 = 9	36 ÷ 4 = 9
10 ÷ 1 = 10	20 ÷ 2 = 10	30 ÷ 3 = 10	40 ÷ 4 = 10
11 ÷ 1 = 11	22 ÷ 2 = 11	33 ÷ 3 = 11	44 ÷ 4 = 11
12 ÷ 1 = 12	24 ÷ 2 = 12	36 ÷ 3 = 12	48 ÷ 4 = 12
5 ÷ 5 = 1	6 ÷ 6 = 1	7 ÷ 7 = 1	8 ÷ 8 = 1
10 ÷ 5 = 2	12 ÷ 6 = 2	14 ÷ 7 = 2	16 ÷ 8 = 2
15 ÷ 5 = 3	18 ÷ 6 = 3	21 ÷ 7 = 3	24 ÷ 8 = 3
20 ÷ 5 = 4	24 ÷ 6 = 4	28 ÷ 7 = 4	32 ÷ 8 = 4
25 ÷ 5 = 5	30 ÷ 6 = 5	35 ÷ 7 = 5	40 ÷ 8 = 5
30 ÷ 5 = 6	36 ÷ 6 = 6	42 ÷ 7 = 6	48 ÷ 8 = 6
35 ÷ 5 = 7	42 ÷ 6 = 7	49 ÷ 7 = 7	56 ÷ 8 = 7
40 ÷ 5 = 8	48 ÷ 6 = 8	56 ÷ 7 = 8	64 ÷ 8 = 8
45 ÷ 5 = 9	54 ÷ 6 = 9	63 ÷ 7 = 9	72 ÷ 8 = 9
50 ÷ 5 = 10	60 ÷ 6 = 10	70 ÷ 7 = 10	80 ÷ 8 = 10
55 ÷ 5 = 11	66 ÷ 6 = 11	77 ÷ 7 = 11	88 ÷ 8 = 11
60 ÷ 5 = 12	72 ÷ 6 = 12	84 ÷ 7 = 12	96 ÷ 8 = 12
9 ÷ 9 = 1	10 ÷ 10 = 1	11 ÷ 11 = 1	12 ÷ 12 = 1
18 ÷ 9 = 2	20 ÷ 10 = 2	22 ÷ 11 = 2	24 ÷ 12 = 2
27 ÷ 9 = 3	30 ÷ 10 = 3	33 ÷ 11 = 3	36 ÷ 12 = 3
36 ÷ 9 = 4	40 ÷ 10 = 4	44 ÷ 11 = 4	48 ÷ 12 = 4
45 ÷ 9 = 5	50 ÷ 10 = 5	55 ÷ 11 = 5	60 ÷ 12 = 5
54 ÷ 9 = 6	60 ÷ 10 = 6	66 ÷ 11 = 6	72 ÷ 12 = 6
63 ÷ 9 = 7	70 ÷ 10 = 7	77 ÷ 11 = 7	84 ÷ 12 = 7
72 ÷ 9 = 8	80 ÷ 10 = 8	88 ÷ 11 = 8	96 ÷ 12 = 8
81 ÷ 9 = 9	90 ÷ 10 = 9	99 ÷ 11 = 9	108 ÷ 12 = 9
90 ÷ 9 = 10	100 ÷ 10 = 10	110 ÷ 11 = 10	120 ÷ 12 = 10
99 ÷ 9 = 11	110 ÷ 10 = 11	121 ÷ 11 = 11	132 ÷ 12 = 11
108 ÷ 9 = 12	120 ÷ 10 = 12	132 ÷ 11 = 12	144 ÷ 12 = 12

MENTAL EXERCISES.

1. How many times is 2 contained in 8? 2 in 10? 2 in 14? 2 in 18? 2 in 20? 2 in 24? 2 in 22?

2. How many times 3 in 6? 3 in 9? 3 in 21? 3 in 27? 3 in 33? 3 in 30? 3 in 36?

3. How many times 4 in 16? 4 in 24? 4 in 36? 4 in 44? 4 in 48? 4 in 40? 4 in 20?

4. How many times 5 in 15? 5 in 25? 5 in 35? 5 in 50? 5 in 60? 5 in 55? 5 in 45?

5. How many times 6 in 24? 6 in 36? 6 in 30? 6 in 60? 6 in 72? 6 in 66? 6 in 54?

6. How many times 7 in 28? 7 in 35? 7 in 49? 7 in 70? 7 in 84? 7 in 77? 7 in 63?

7. How many times 8 in 32? 8 in 40? 8 in 64? 8 in 88? 8 in 80? 8 in 96? 8 in 72?

8. How many times 9 in 36? 9 in 54? 9 in 63? 9 in 90? 9 in 108? 9 in 45? 9 in 99?

9. How many times 10 in 50? 10 in 70? 10 in 40? 10 in 100? 10 in 120? 10 in 90? 10 in 110?

10. How many times 11 in 22? 11 in 44? 11 in 88? 11 in 110? 11 in 132? 11 in 121? 11 in 99?

11. How many times 12 in 48? 12 in 60? 12 in 72? 12 in 144? 12 in 120? 12 in 132? 12 in 108?

12. How many times 9 in 36? 6 in 36? 4 in 36? 3 in 36? 12 in 36?

EXAMPLE.—If pears are worth 3 cents each, how many can be bought for 6 cents?

SOLUTION.—If one pear costs 3 cents, as many pears can be bought for 6 cents as 3 cents is contained times in 6 cents, which is 2.

CONCLUSION.—Therefore, if pears are worth 3 cents each, 2 can be bought for 6 cents.

13. How many apples at 3 dollars a barrel, can be bought for 9 dollars?

14. If 3 sheep cost 12 dollars, what is the cost of one?

15. If there are 3 feet in one yard, how many yards in 18 feet?

16. How many times is 4 contained in 16?

17. How much cloth at 5 dollars a yard, can be bought for 20 dollars?

18. John spent 24 cents for oranges, at 4 cents apiece: how many did he buy?

19. Mary bought six papers of pins for 30 cents: what was the cost of one paper?

20. In one week there are 7 days: how many weeks in 21 days?

21. What is the price of one barrel of flour if 6 barrels cost 36 dollars?

22. How many books at 10 cents each, can be bought for 50 cents?

23. In one bushel there are 4 pecks: how many bushels in 32 pecks?

24. A school of 40 pupils is divided into five classes: how many pupils in each class?

25. Mary has a basket that holds 7 apples: how many times can she fill her basket with 42 apples?

26. Charles bought 7 cocoa-nuts for 56 cents: what did one cost?

27. How many eggs at 9 cents a dozen, must be given for 6 yards of muslin at 12 cents a yard?

28. Bought 6 yards of velvet at 5 dollars a yard, and 4 yards of broadcloth at 6 dollars a yard; gave in exchange 6 barrels of flour: what was the price per barrel?

29. A merchant bought 5 dozen eggs at 10 cents a dozen, and 7 dozen at 11 cents; he sold the whole so as to gain 17 cents: what did he receive per dozen?

30. Six pine-apples were sold for 72 cents, and 12 cents were gained: what did each cost?

SHORT DIVISION.

There are two methods of division, Long Division and Short Division.

38. In **Short Division**, the work of dividing is performed mentally, and the result only is written.

MODEL SOLUTION.

EXAMPLE.—What is the quotient of 2635 divided by 5 ?

SOLUTION.—Write the divisor at the left of the dividend with a curved line between, and draw a horizontal line under the dividend. Begin at the left; 5 is not contained in 2 (thousands), but 2 (thousands) = 20 (hundreds), which added to the 6 (hundreds), make 26 (hundreds); 5 is contained in 26 (hundreds), 5 (hundred) times, with 1 (hundred) remaining. Write the 5 (hundreds) in the hundreds' place under the dividend.

OPERATION.

Dividend.

5) 2635

527 Quotient.

The 1 hundred remaining = 10 tens, which added to the 3 tens make 13 tens ; 5 is contained in 13 tens, 2 tens times, with 3 tens remaining. Write the 2 tens in tens' place. The 3 tens remaining = 30 units, which with the 5 units make 35 units ; 5 is contained in 35 units, 7 units times, which write in units' place.

CONCLUSION.—Therefore, the quotient of 2635 divided by 5 is 527. Hence, the

RULE FOR SHORT DIVISION.—*Write the divisor at the left of the dividend with a curved line between, and draw a horizontal line under the dividend.*

Begin at the left hand; divide each figure of the dividend by the divisor, and write the result in the same order of the quotient.

If there is a remainder after dividing any figure, prefix it to the figure in the next lower order, and divide as before.

If the number in any order does not contain the divisor,

place a cipher in the same order of the quotient, prefix the number to the figure in the next order, and proceed as before.

PROOF.—Multiply the divisor by the quotient, to the product add the remainder, if any, and if the result equals the dividend the work is right.

NOTE.—If there is a remainder after dividing the last figure of the dividend, write it over the divisor with a line between, and place it at the right of the quotient.

EXAMPLES FOR PRACTICE.

1. Divide 226 by 2. *Ans.* 113.
2. Divide 268 by 2. *Ans.* 134.
3. Divide 480 by 2. *Ans.* 240.
4. Divide 548 by 2. *Ans.* 274.
5. Divide 732 by 2. *Ans.* 366.
6. Divide 872 by 2. *Ans.* 436.
7. Divide 1136 by 2. *Ans.* 568.
8. Divide 1348 by 2. *Ans.* 674.
9. Divide 1652 by 2. *Ans.* 826.
10. Divide 2062 by 2. *Ans.* 1031.
11. Divide 3104 by 2. *Ans.* 1552.
12. Divide 4792 by 2. *Ans.* 2396.

(*13.*)	(*14.*)	(*15.*)
2) 42682	2) 64208	2) 450026
21341	32104	225013

16. Divide 930690 by 3. *Ans.* 310230.
17. Divide 753693 by 3. *Ans.* 251231.
18. Divide 88416 by 4. *Ans.* 22104.
19. Divide 648428 by 4. *Ans.* 162107.
20. Divide 123575 by 5. *Ans.* 24715.
21. Divide 386480 by 5. *Ans.* 77296.

22. Divide 364854 by 6. *Ans.* 60809.
23. Divide 120607 by 6. *Ans.* $20101\frac{1}{6}$.
24. Divide 87507 by 7. *Ans.* 12501.
25. Divide 123456 by 7. *Ans.* $17636\frac{4}{7}$.
26. Divide 789324 by 8. *Ans.* $98665\frac{4}{8}$.
27. Divide 80608 by 8. *Ans.* 10076.
28. Divide 90729 by 9. *Ans.* 10081.
29. Divide 198364 by 9. *Ans.* $22040\frac{4}{9}$.
30. Divide 543020 by 10. *Ans.* 54302.
31. Divide 678360 by 10. *Ans.* 67836.
32. Divide 736121 by 11. *Ans.* $66920\frac{1}{11}$.
33. Divide 924132 by 11. *Ans.* 84012.
34. Divide 264384 by 12. *Ans.* 22032.
35. Divide 648378 by 12. *Ans.* $54031\frac{6}{12}$.

36. If 2 wagons carry 2448 pounds, how many pounds will one wagon carry? *Ans.* 1224 pounds.

37. If 3 houses are worth $47055, how much is one house worth? *Ans.* $15685.

38. If 4 times a certain number is 180488, what is the number? *Ans.* 45122.

39. A township of 60145 acres is to be divided among 5 persons: what is each one's share? *Ans.* 12029 acres.

40. Six men have 268116 bushels of corn: how much has each man? *Ans.* 44686 bushels.

41. In a field containing 7 acres, there are 42336 hills of corn: how many hills in one acre? *Ans.* 6048 hills.

42. A township of 64144 acres is to be divided among 8 persons: what is each one's share? *Ans.* 8018 acres.

43. If 9 square feet make one square yard, how many square yards in 209790 square feet? *Ans.* 23310 sq. yds.

LONG DIVISION.

39. In **Long Division**, the result of each step of the operation is written.

Short Division is generally used when the divisor does not exeeed 12 ; *long division,* when it does exceed 12.

MODEL SOLUTION.

EXAMPLE.—What is the quotient of 15664 divided by 24 ?

SOLUTION.—Write the divisor at the left of the dividend with a curved line between, and draw a curved line to the right of the dividend.

As 24 is not contained in one, nor in 15, take the first three figures: 24 is contained in 156 hundreds, 6 hundreds times. Write the 6 in hundreds' place in the quotient ; multiply the divisor by this figure, and subtract, and 12 hundreds remain; 12 hundreds = 120 tens, which with 6 tens brought down from the dividend make 126 tens.

OPERATION.

Dividend.	Quotient.
24) 15664 (	$652\frac{16}{24}$
144	
126	
120	
64	
48	
16	Remainder.

24 is contained in 126 tens, 5 tens times, which write in tens' place of the quotient ; multiply the divisor by this figure, and subtract, and 6 tens remain ; 6 tens = 60 units, which with 4 units brought down make 64 units.

24 is contained in 64 units, 2 units times, which place in units' place of the quotient ; multiply and subtract, and 16 units remain, which place over the divisor, thus, $\frac{16}{24}$, and annex it to the quotient.

CONCLUSION.—Therefore, the quotient of 15664 divided by 24 is $652\frac{16}{24}$. Hence, the

RULE FOR LONG DIVISION.—*Write the divisor at the left of the dividend with a curved line between, and draw a curved line to the right of the dividend.*

Find how many times the divisor is contained in the fewest left-hand figures of the dividend that will contain it, and place this number in the quotient.

Multiply the divisor by this quotient figure, and place the product under the part of the dividend from which it was obtained.

Subtract this product from the figures above it; to the remainder, bring down the next figure of the dividend, and divide as before, until all the figures of the dividend are brought down.

If at any time, after bringing down a figure, the number thus formed is too small to contain the divisor, place a cipher in the quotient, and bring down another figure, after which divide as before.

PROOF.—The same as in Short Division.

44. Divide 34188 by 84.

OPERATION.

Dividend. Quotient.

84) 34188 (407

336

588

588

NOTE.—Teachers must impress upon the minds of the pupils that every time a figure is brought down some number or a cipher *must* be placed in the quotient. The neglecting to place the cipher in the quotient is one of the most common errors made by children.

45. Divide 815678 by 21. *Ans.* $38841\frac{17}{21}$.

46. Divide 896758 by 86. *Ans.* $10427\frac{36}{86}$.

47. Divide 72984 by 24. *Ans.* 3041.

48. Divide 9859 by 27. *Ans.* $365\frac{4}{27}$.

49. Divide 64087 by 31. *Ans.* $2067\frac{10}{31}$.

50. Divide 123456 by 64. *Ans.* 1929.

51. Divide 29990 by 68. *Ans.* $441\frac{2}{68}$.

52. Divide 89773 by 123. *Ans.* $729\frac{106}{123}$.

53. Divide 86400 by 216. *Ans.* 400.

54. Divide 76800 by 256. *Ans.* 300.

55. Divide 625000 by 125. *Ans.* 5000.

56. Divide 987654 by 356. *Ans.* $2774\frac{110}{356}$.

57. Divide 321456 by 403. *Ans.* $797\frac{265}{403}$.

58. Divide 625125 by 125. *Ans.* 5001.
59. Divide 54002 by 88. *Ans.* $613\frac{58}{88}$.
60. Divide 78567 by 219. *Ans.* $358\frac{165}{219}$.
61. Divide 250 by 10. *Ans.* 25.

NOTE.—Since removing a figure one place to the right decreases its value tenfold, to divide by 10, 100, 1000, etc., cut off as many places from the dividend as there are ciphers in the divisor. The figures left form the quotient ; those cut off, the remainder.

62. Divide 3500 by 100. *Ans.* 35.
63. Divide 75000 by 1000. *Ans.* 75.
64. Divide 1234560 by 10. *Ans.* 123456.
65. Divide 789240 by 100. *Ans.* $7892\frac{40}{100}$.
66. Divide 936560 by 1000. *Ans.* $936\frac{560}{1000}$.
67. Divide 144000 by 2400. *Ans.* 60.

NOTE.—First divide both divisor and dividend by 100, which is done by cutting off the two right-hand figures. Then divide 1440 by 24. The quotient is 60.

68. Divide 256000 by 6400. *Ans.* 40.
69. Divide 512000 by 8000. *Ans.* 64.
70. Divide 7290000 by 24300. *Ans.* 300.
71. If 54054 books are packed in 91 boxes, how many books in each box? *Ans.* 594 books.
72. A man owned 19 acres of land worth $71919 : what was the value of one acre ? *Ans.* $\$3785\frac{4}{19}$.
73. The amount of toll received at a bridge in one year was $21170 : how much was received each day, allowing 365 days to a year ? *Ans.* $58.
74. In 14 rooms of a warehouse 710008 bushels of wheat are stored : how many bushels in each room ?
Ans. $50714\frac{2}{14}$ bushels.
75. There are 100 cents in a dollar : how many dollars in 385400 cents? *Ans.* $3854.
76. If 11818 peach-trees are planted in 311 rows, how many trees in a row? *Ans.* 38 trees.

77. The circumference of the earth is about 25000 miles: how long would it take a car to go round it, going 312 miles a day? *Ans.* $80\frac{40}{312}$ days.

78. A garrison of 7000 men consumed 4099130 pounds of flour in 217 days: how much did they consume in 1 day? *Ans.* 18890 pounds.

79. A certain factory expends $212998 in 52 weeks: how much is that a week? *Ans.* $4096\frac{6}{52}$.

80. How much land can be purchased for $36425, if an acre costs $35? *Ans.* $1040\frac{25}{35}$ acres.

81. A farmer raised 23652 bushels of corn on 324 acres of land: how much was raised on one acre?

Ans. 73 bushels.

82. How many barrels will 171814 apples fill, if there are 271 in each barrel? *Ans.* 634 barrels.

83. The dividend of a certain example is 120119; the divisor is 9061: what is the remainder? *Ans.* 2326.

84. The cost of 463 mules was $73154: what was the cost of one? *Ans.* $158.

85. The total equipment of a regiment of 1200 cavalry cost $236400: what did it cost to equip each man?

Ans. $197.

EXAMPLES COMBINING ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION.

1. A company of 45 men bought an estate of 963 acres at $35 an acre: how much did each man pay?

Ans. $749.

2. How many bushels of oats at 35 cents a bushel, must be given for 28 bushels of wheat at 85 cents a bushel?

Ans. 68 bushels.

3. The subtrahend of an example is 15091, the remainder is 3218: what is the minuend? *Ans.* 18309.

4. How many barrels of cider at $8 a barrel, should be given for 31 tons of hay at $32 a ton ?

Ans. 124 barrels.

5. From a lot of cloth containing 572 yards, there were made 39 suits of 11 yards each : how many yards were left ?

Ans. 143 yards.

6. Multiply 480 by 1008, and divide 7 times the product by 1329 : what is the remainder ? *Ans.* 588.

7. A man having $4700, bought 33 acres of land at $40 an acre, and expended the remainder for molasses at $20 a barrel : how much molasses did he buy ?

Ans. 169 barrels.

8. A man earns $1800 a year ; his expenses are $4 a day: how much will he save in 9 years of 365 days each ?

Ans. $3060.

9. In 44 hogsheads of sugar, each containing 1446 pounds, how many pounds ? How many barrels of 132 pounds each, can be filled from them ? *Ans.* 482 barrels.

10. In a certain township there are 32 square miles ; in one square mile there are 640 acres : how many farms of 128 acres each, in the township ? *Ans.* 160 farms.

11. A man left $5500 to his wife and three children ; his wife received $2500, and the children each an equal amount : how much did each child receive ?

Ans. $1000.

12. Add 247 and 3195 ; subtract the result from 10000 ; multiply the remainder by 25, and divide the product by 5 : what is the quotient ? *Ans.* 32790.

13. In 4 bags there are $500 ; in the first, $96 ; in the second, $120 ; in the third, $55 : what sum in the fourth bag ? *Ans.* $229.

14. How many yards of woolen cloth at $6 a yard, will it take to pay for 8 horses at $60 each, and 14 cows at $45 each. *Ans.* 185 yd.

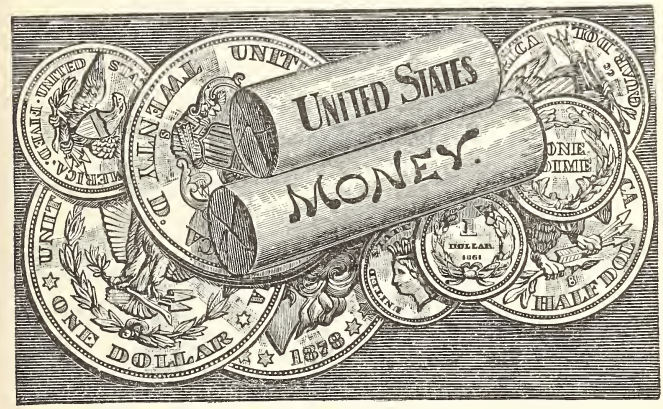
COMPOUND NUMBERS

DEFINITIONS.

40. With reference to their *unit values,* numbers are divided into two classes, **Simple** and **Compound.**

41. A **Simple Number** is one that denotes things of the same unit value ; as, 3 yards, 5 dollars, 3 pints.

42. A **Compound Number** is a number made up of different denominations ; as, 5 dollars 3 cents, 3 feet 5 inches.



43. **United States Money** is the legal money of our country.

The **Denominations** are mills, cents, dimes, and dollars.

Ten units of each denomination make one of the next higher ; hence, the

TABLE.

10 mills, marked m,	make	1 cent,	marked ct.
10 cents	"	1 dime,	" d.
10 dimes	"	1 dollar,	" $.

44. A **Coin** is a piece of metal, stamped by authority of government, to be used as money.

The coins of the United States are made of gold, silver, nickel, and bronze; and are as follows:

GOLD.—The double-eagle, $20; eagle, $10; half-eagle, $5; quarter-eagle, $2.50; and dollar, $1.

SILVER.—The dollar, half-dollar, quarter-dollar, dime, and half-dime.

NICKEL.—The five-cent piece and three-cent piece.

BRONZE.—One cent.

The mill is used only in making calculations.

Paper money consists of notes issued by the United States, called *Treasury Notes*, and bank notes issued by National Banks.

NOTATION AND NUMERATION OF U. S. MONEY.

45. Since ten units of one denomination make one unit of the next higher, United States money is written like simple numbers.

Dollars are separated from cents by a period. The cents and dimes are read together as cents and tens of cents, and occupy two places at the right of the period.

RULE FOR NUMERATION.—*Read the number at the left of the period as dollars; the first two figures at the right as cents; and the third, if any, as mills.*

EXAMPLES TO BE COPIED AND READ.

1. $1.23.	*7.* $16.10.	*13.* $20.00.
2. $5.12.	*8.* $90.00.	*14.* $856.04.
3. $21.09.	*9.* $52.00.	*15.* $100.00.
4. $35.05.	*10.* $371.42.	*16.* $3607.02.
5. $2.01.	*11.* $99.90.	*17.* $13067.75.
6. $94.02.	*12.* $167.87.	*18.* $10100.10.

46. Rule for Notation.—*Write the dollars as in simple numbers, with a period at the right; in the next two places write the cents.*

Note.—If the cents are less than ten, put a cipher next to the dollars; if there are *no* cents, put two ciphers in the place of cents.

EXAMPLES TO BE WRITTEN.

1. Five dollars and twenty-three cents. $5.23.
2. Ten dollars and nineteen cents. $10.19.
3. Fifteen dollars and twenty cents. $15.20.
4. One hundred dollars and fifty cents. $100.50.
5. Nineteen dollars and nine cents. $19.09.
6. Eighty dollars and eight cents. $80.08.
7. Eleven dollars and one cent. $11.01.
8. Six dollars and six cents. $6.06.
9. Six hundred dollars and ten cents. $600.10.
10. Nine hundred and seven dollars and one cent. $907.01.
11. One thousand and nine dollars and thirty cents. $1009.30.
12. Twelve hundred and twelve dollars and sixty cents. $1212.60.
13. Fifteen hundred and one dollars. $1501.00.
14. Seventeen hundred dollars and one cent. $1700.01.

REDUCTION.

47. **Reduction** is changing a number from one denomination to another without altering its value.

Reduction may be either, (1.) Changing a number from a *higher* denomination to a *lower;* or, (2.) Changing a number from a *lower* denomination to a *higher.*

MENTAL EXERCISES.

Example.—How many cents are there in 5 dimes?

SOLUTION.—Since there are 10 cents in 1 dime, in 5 dimes there are 5 times 10 cents, which are 50 cents.

1. How many dimes in 2 dollars? In 4 dollars? In 8?
2. How many dollars in 1 eagle? In 5 eagles? In 9?
3. How many cents in 1 dollar? In 2 dollars? In 6?

EXAMPLE.—How many dimes are there in 30 cents?

SOLUTION.—Since 10 cents make one dime, 30 cents will make as many dimes as 10 is contained times in 30, which is 3 times.

CONCLUSION.—Therefore, in 30 cents there are 3 dimes.

4. How many dimes in 40 cents? In 70 cents? In 90?
5. How many dollars in 60 dimes? In 50 dimes? In 80? In 20?
6. How many eagles in 10 dollars? In 40 dollars? In 70? In 100?
7. How many dollars in 200 cents? In 500 cents? In 800? In 1000?
8. How many dimes in 3 dollars? In 10 dollars? In 2 eagles?

48. MODEL SOLUTION.

EXAMPLE.—How many cents in 87 dollars?

SOLUTION.—Since there are 100 cents in \$1, in \$87 there are 87 times 100 cents = 8700 cents. Hence, the

OPERATION.

\$87
100
8700 cents.

RULE.—*To reduce dollars to cents, multiply by 100, or annex two ciphers.*

NOTE.—Dollars and cents are reduced to cents by removing the period and the dollar sign.

EXAMPLES FOR PRACTICE.

1. How many cents in \$83? In \$94? \$107? \$239?
2. How many cents in \$529? In \$600? \$712? \$720?

3. How many cents in $960 ? In $1000? $1050? $1200 ?

49. MODEL SOLUTION.

EXAMPLE.—How many dollars in 2145 cents ?

SOLUTION.—Since 100 cents make $1, 2145 cents will make as many dollars as 100 is contained *times* in 2145, which is 21, with a remainder of 45 cents; that is, $21.45.

OPERATION.

Cts.

1 | 00) 21 | 45 ($21.45

CONCLUSION.—Therefore, in 2145 cents there are $21.45. Hence, the

RULE.—*To reduce cents to dollars, divide by 100, or cut off two figures.*

NOTES.—1. As all the operations in Reduction of United States Money consist in multiplying or dividing by 10 and 100, the work can be shortened by simply moving the point.

2. In multiplying, move the point as many places to the *right* as there are ciphers in the multiplier. Thus : $12.50 = 1250 cents.

3. In dividing, move the point as many places to the *left* as there are ciphers in the divisor. Thus : 275 cents = $2.75.

EXAMPLES FOR PRACTICE.

1. How many dollars in 145 cents? In 235? 416?
2. How many dollars in 2500 cents ? In 2803 ? 3565 ?
3. How many dollars in 7512 cents ? In 9009 ? 10025 ?
4. How many cents in $46.15 ? In $55.60 ? $40.00 ?

ADDITION OF U. S. MONEY.

EXAMPLE.—What is the sum of $26.15, $3.72, $25.12, $136.84, and $5.68 ?

OPERATION.

$26.15
3.72
25.12
136.84
5.68
$197.51

50. RULE.—*Write the numbers to be added, placing units of the same denomination in the same column. Add as in simple numbers, and place the period under the points above.*

PROOF.—Same as in Addition of Simple Numbers.

EXAMPLES FOR PRACTICE.

1. What is the sum of 8 dollars and 23 cents, 10 dollars and 18 cents, 45 dollars and 72 cents, 43 dollars and 65 cents, and 29 dollars and 29 cents? *Ans.* $137.07.

2. What is the sum of 23 dollars and 12 cents, 59 dollars and 47 cents, 100 dollars and 75 cents, and 87 dollars and 16 cents? *Ans.* $270.50.

3. What is the sum of 850 cents, 10 dollars, 1200 cents, 45 dollars, and 325 cents? *Ans.* $78.75.

4. A farmer sold a horse for $257, a cow for $76, a wagon for $37.87, and produce for $320.17: how much did he receive? *Ans.* $691.04.

5. What is the sum of $116.23, $270.03, $4.00, $100.05, and $.75? *Ans.* $491.06.

6. What is the sum of $1010.10, $490.37, $13.07, $2.50, $10, $400.03, and $1? *Ans.* $1927.07.

7. A man bought a carriage for $450, a horse for $175, a harness for $50.35, and a whip for $.87: what did he pay for all? *Ans.* $676.22.

8. A man collected of A, $390.35; of B, $29.05; of C, $50.10; of D, $37.12; of E, $112.87: how much did he collect in all? *Ans.* $619.49.

9. The property of Wm. Bushnell was valued as follows: land, $15600; houses, $25350; horses, $625; carriage, $275; money in bank, $1025.75. How much was he worth? *Ans.* $42875.75.

10. Mrs. Smith bought a set of furniture for the parlor, costing $415.75; one for the sitting-room, costing $285; one for a bedroom, costing $175.25; one for the dining-room, costing $215.75. What was the cost of the four sets? *Ans.* $1091.75.

11. Add $347.43, $5249, and $29.37.

Ans. $5625.80.

SUBTRACTION OF U. S. MONEY.

EXAMPLE.—What is the difference between $200.56 and $132.15?

51. RULE.—*Write the less number under the greater, dollars under dollars, and cents under cents. Subtract as in simple numbers, placing the period directly under the points above.*

OPERATION.

$200.56
132.15
$68.41

PROOF.—The same as in Subtraction of Simple Numbers.

EXAMPLES FOR PRACTICE.

1. From $189.25 subtract $132.10. *Ans.* $57.15.

2. From $200.01 subtract $75.19. *Ans.* $124.82.

3. From $700 subtract $516.25. *Ans.* $183.75.

4. From $658.37 subtract $400. *Ans.* $258.37.

5. From $905.00 subtract $431.75. *Ans.* $473.25.

6. From $340.75 subtract $29.99. *Ans.* $310.76.

7. From $9000 subtract $9.99. *Ans.* $8990.01.

8. From $77.17 subtract $.17. *Ans.* $77.

9. From one hundred dollars take ninety-nine dollars and seventy-two cents. *Ans.* $.28.

10. From one hundred and ninety dollars take nineteen cents. *Ans.* $189.81.

11. From $707 take ninety-seven cents. *Ans.* $706.03.

12. From $672.15 take one hundred and seventeen dollars. *Ans.* $555.15.

13. From $70 take seventy cents. *Ans.* $69.30.

14. A man had $365.25; he spent $136.27: how much had he left? *Ans.* $228.98.

15. Bought a horse for $185.50; sold it for $163.75: how much did I lose? *Ans.* $21.75.

16. Bought a house for $4560.37; sold it for $4800: how much did I gain? *Ans.* $239.63.

17. A man had $200; he bought a horse for $155, a saddle for $27.50, a bridle for $7.37, and a pair of spurs for $1.25: how much money had he left? *Ans.* $8.88.

MULTIPLICATION OF U. S. MONEY.

EXAMPLE.—What will 19 acres of land cost at $24.75 an acre?

SOLUTION.—$24.75 equal 2475 cents; and since 1 acre costs 2475 cents, 19 acres will cost 19 times 2475 cents, which are 47025 cents. Reduce the cents to dollars by pointing off two figures. Hence, the

OPERATION.

$24.75
19
222 75
247 5
$470.25

52. RULE.—*Multiply as in simple numbers; the product will be the answer in the lowest denomination of the multiplicand, which may be reduced to dollars by pointing.*

PROOF.—Same as in Multiplication of Simple Numbers.

EXAMPLES.

1. Multiply $17.25 by 9. *Ans.* $155.25.

2. Multiply $112.17 by 17. *Ans.* $1906.89.

3. Multiply $93.90 by 72. *Ans.* $6760.80.

4. Multiply fifty-nine dollars and five cents by one hundred and twenty. *Ans.* $7086.

5. Multiply eighteen cents by four hundred and eighty. *Ans.* $86.40.

6. What is the cost of 12 quarts of strawberries at 15 cents a quart? *Ans.* $1.80.

7. What is the cost of 36 bushels of apples at $1.50 a bushel? *Ans.* $54.

8. What will 144 arithmetics cost at $.38 each? *Ans.* $54.72.

9. If a man earns $4.75 a week, how much will he earn in 32 weeks? *Ans.* $152.

10. A drover sold 56 cows at $34.15 a head: how much did he receive for them? *Ans.* $1912.40.

11. What will 90 bushels of wheat cost at $1.62 a bushel? *Ans.* $145.80.

12. What will 126 barrels of flour cost at $6.25 a barrel? *Ans.* $787.50.

13. Mr. James sold 814 shade trees at $.55 each: how much did he receive for the lot? *Ans.* $447.70.

14. A farmer sold 87 bushels of corn at $.52 a bushel; 720 bushels of wheat at $1.42 a bushel; and 42 chickens at $.45 each: what did he receive for all?

Ans. $1086.54.

DIVISION OF U. S. MONEY.

EXAMPLE.—At 12 cents a pound, how much sugar can be bought for $3.00?

SOLUTION.—$3.00 = 300 cents; and since 1 pound can be bought for 12 cents, as many pounds can be bought for 300 cents as 12 is contained *times* in 300, which is 25 times.

CONCLUSION.—Therefore, if 1 pound of sugar costs 12 cents, 25 pounds can be bought for $3.00.

OPERATION.

Cts. Pounds. Times.

12) 300 (25

24

60

60

EXAMPLE.—If 8 acres of land cost $92, what is the cost of 1 acre?

SOLUTION.—If 8 acres of land cost $92, 1 acre will cost one eighth of $92, or 9200 cents; one eighth of 9200 cents is 1150 cents = $11.50. Hence, the

53. RULES.—*To divide one sum of money by another, reduce both to the same denomination, and divide as in simple numbers.*

To divide a sum of money into equal parts, reduce it to cents, then divide; the quotient will be cents, which reduce to dollars.

PROOF.—The same as in Division of Simple Numbers.

NOTE.—If there is a remainder after dividing cents, place the sign + after the quotient.

EXAMPLES.

1. Divide \$345 equally among 3 men. *Ans.* \$115.
2. Divide \$63 equally among 25 men. *Ans.* \$2.52.
3. Divide \$7807 by 37. *Ans.* \$211.
4. Divide \$143.22 by 62. *Ans.* \$2.31.
5. Divide \$1040.06 by 133. *Ans.* \$7.82.
6. Divide \$16.80 into 12 parts. *Ans.* \$1.40.
7. Divide \$80.24 into 8 parts. *Ans.* \$10.03.
8. Divide \$128.04 into 22 parts. *Ans.* \$5.82.
9. \$527.40 ÷ 45 = what? *Ans.* \$11.72.
10. \$907.46 ÷ 225 = what? *Ans.* \$4.03+.
11. \$20000 ÷ 640 = what? *Ans.* \$31.25.
12. How much land at \$5.75 an acre, can be purchased for \$529? *Ans.* 92 acres.
13. A man paid \$57.85 for cheese at 8 cents a pound: how many pounds did he buy? *Ans.* 723⅛ pounds.
14. Paid \$800 for cloth at \$1.25 a yard: how many yards did I buy? *Ans.* 640 yards.
15. By what must \$18.55 be multiplied to make \$3784.20? *Ans.* 204.

MISCELLANEOUS EXAMPLES.

1. A laborer worked 80 days at \$1.75 a day: how much did he earn? *Ans.* \$140.
2. A man had \$2500; he spent \$2.51 a day for 87 days: how much had he remaining? *Ans.* \$2281.63.

3. A owed B $920, and worked for him 102 days, at $2.75 a day: how much did he then owe? *Ans.* $639.50.

4. A farmer sold 300 bushels of corn at 54 cents a bushel, and bought 42 yards of cloth at $3.85 a yard: how much money had he left? *Ans.* 30 cents.

5. A load of coal contains 25 bushels: what will 90 loads cost at 14 cents a bushel? *Ans.* $315.

6. A man whose expenses were $1.70 a day, saved $46.50 in 30 days: what did he earn a day? *Ans.* $3.25.

7. A man sold 19 horses for $152.58 each; he spent $43.02 for clothing; and with the remainder bought sugar at $21 a barrel: how many barrels did he buy?

Ans. 136 barrels.

8. A farmer sold 6 cows at $37.50 each, and with the proceeds bought sheep at $2.50 each: how many sheep did he buy? *Ans.* 90 sheep.

9. A man bought 6 bales of cloth, each bale containing 12 pieces, for $5265.72: how much did each piece cost?

Ans. $73.13+.

10. At $9.24 a barrel, how many barrels of flour can be bought for $970.20? *Ans.* 105 barrels.

ACCOUNTS AND BILLS.

54. A **Bill** is an account of goods bought or sold, the number or quantity of each article, with its price and the entire cost.

The person who buys is called *Debtor,* and the one who sells is called *Creditor.*

The character @ is used as an abbreviation for *at,* and is followed by the price of a single article or specified quantity.

55. The **Footing** of the bill is the amount of the several items.

56. The **Receipt** is the signature of the seller after the words *Received Payment* at the foot of the bill.

NOTE.—In performing the following examples, let the pupil be required to copy the items, carrying out the cost of each, and finding the footing of the bill.

(*1.*)

CINCINNATI, O., Oct. 30, 1889.

Mr. GEORGE GRAHAM,

Bought of JOHN BROWN & Co.

12 lb. Coffee,	@ 25c.	$3.00
6 lb. Butter,	@ 28c.	1.68
5 lb. Cheese,	@ 18c.	.90
10 lb. Sugar,	@ 13c.	1.30
		$

Received Payment,

JOHN BROWN & Co.

(*2.*)

COLUMBUS, O., Nov. 14, 1889.

Mr. CHARLES LONG,

Bought of DAVID WHITE.

12 yd. Silk, ...	@ $2.25,	$
18 yd. Calico, ...	@ .08,	
20 yd. Muslin, ...	@ .10,	
6 yd. Flannel, ...	@ .75,	
10 yd. Ribbon, ...	@ .15,	
	Amount,	$39.44

Received Payment,

DAVID WHITE.

(*3.*)

CHICAGO, Ill., Dec. 10, 1889.

THOMAS JOHNSON,

Bought of MOSES BLACK,

28 yd. Brussels Carpeting, . .	@ $2.50,	$
24 yd. Ingrain Carpeting,. . .	@ 1.50,	
10 yd. Oil Cloth, ...	@ 1.20,	
20 yd. Matting, ...	@ 1.10,	
	Amount,	$140.00

Received Payment,

MOSES BLACK.

MEASURES

DRY MEASURE.

57. Dry Measure is used in measuring grain, vegetables, fruit, coal, etc.

TABLE.

2 pints (pt.)	. . make	1 quart,	. . . marked	qt.
8 quarts	"	1 peck,	. . . "	pk.
4 pecks	"	1 bushel,	. . "	bu.

NOTE.—The standard of dry measure is the bushel, which contains $2150\frac{2}{5}$ cubic inches.

MENTAL EXERCISES.

EXAMPLE.—How many pints are there in 4 quarts?

SOLUTION.—Since in 1 quart there are 2 pints, in 4 quarts there are 4 times 2 pints, which are 8 pints.

1. How many pints in 3 quarts? In 5 qt.? In 6 qt.? In 7 qt.?

2. How many quarts in 2 pecks? In 3 pk.? In 4 pk.? In 5 pk.?

3. How many pecks in 2 bushels? In 4 bu.? In 3 bu.? In 5 bu.?

4. How many pints in 8 quarts or 1 peck? In 2 pk.? In 4 pk.?

5. How many pecks in 6 bushels? In 9 bu.? In 8 bu.? In 10 bu.?

6. How many quarts in a bushel? How many pints in a bushel?

7. Place the proper numbers after the sign of equality in the following:

1 bu. = pk. = qt. = pt.

EXAMPLE.—How many quarts are there in 6 pints?

SOLUTION.—In 6 pints there are as many quarts as 2 pints are contained times in 6 pints, which are 3 times. Therefore, in 6 pints there are 3 quarts.

8. How many quarts in 4 pints? In 8 pt.? In 14 pt.? In 20 pt.?

9. How many pecks in 8 quarts? In 32 qt.? In 48 qt.? In 64 qt.?

10. How many bushels in 8 pecks? In 32 pk.? In 16 pk.? In 24 pk.?

11. How many bushels in 12 pk.? In 20 pk.? In 32 qt.? In 64 qt.?

12. What part of a quart is a pint?

13. What part of a peck is a quart?

14. What part of a bushel is a peck?

15. A lady bought half a bushel of strawberries: how many quarts did she buy?

WRITTEN EXERCISES.

EXAMPLE.—Reduce 4 bu. 1 pk. 7 qt. 1 pt. to pints.

SOLUTION.—Since in 1 bu. there are 4 pk., in 4 bu. there are 4 times 4 pk., which are 16 pk.; 16 pk. + 1 pk. = 17 pk.

Since in 1 pk. there are 8 qt., in 17 pk. there are 17 times 8 qt., which are 136 qt.; 136 qt. + 7 qt. = 143 qt.

Since in 1 qt. there are 2 pt., in 143 qt. there are 143 times 2 pt., which are 286 pt.; 286 pt + 1 pt. = 287 pt.

CONCLUSION.—Therefore, in 4 bu. 1 pk. 7 qt. 1 pt. there are 287 pt.

Hence we deduce the following:

OPERATION.

bu.	pk.	qt.	pt.
4	1	7	1
4			
16 pk.			
1			
17 pk.			
8			
136 qt.			
7			
143 qt.			
2			
286 pt.			
1			
287 pt.			

RULE FOR REDUCTION FROM HIGHER TO LOWER DENOMINATIONS.—*Multiply the highest denomination given, by that number which it takes of the next lower denomination to make one of this higher; add to the product the number, if any, of the next lower denomination.*

Proceed in like manner with the result obtained, until the whole is reduced to the required denomination.

NOTE TO TEACHER.—When the pupil understands the process given above he should be taught to make the additions of the lower denominations mentally.

EXAMPLE.—Reduce 177 pints to higher denominations.

SOLUTION.—Since 2 pt. make 1 qt., 177 pt. make as many qt. as 2 is contained times in 177, = 88 qt. and 1 pt. remaining.

Since 8 qt. make 1 pk., 88 qt. make as many pk. as 8 is contained times in 88, = 11 pk.

Since 4 peck make 1 bu., 11 pk. make as many bu. as 4 is contained times in 11, = 2 bu. and 3 pk. remaining.

OPERATION.

$$\begin{array}{r} 2\,)\,177 \text{ pt.} \\ 8\,)\,88 \text{ qt. } 1 \text{ pt.} \\ 4\,)\,11 \text{ pk.} \\ \hline 2 \text{ bu. } 3 \text{ pk. } 1 \text{ pt. } \textit{Ans.} \end{array}$$

CONCLUSION.—Therefore, in 177 pt. there are 2 bu. 3 pk. 1 pt. Hence, we deduce the following

RULE FOR REDUCTION FROM LOWER TO HIGHER DENOMINATIONS.—*Divide the given quantity by the number of units of its own denomination which make one of the next higher.*

Proceed in like manner with the quotient thus obtained, till the whole is reduced to the required denomination.

The last quotient, with the several remainders, if any, annexed, will be the answer.

PROOF.—Reverse the operation, that is, reduce the answer back to the denomination from which it was derived.

WRITTEN EXERCISES.

1. Reduce 2 bu. to pints. *Ans.* 128 pt.

2. Reduce 172 pt. to bushels. *Ans.* 2 bu. 2 pk. 6 qt.

3. Reduce 5 bu. 1 pk. to pints. *Ans.* 336 pt.

4. Reduce 408 pt. to bushels. *Ans.* 6 bu. 1 pk. 4 qt.

5. Reduce 1 bu. 1 pk. 1 qt. to quarts. *Ans.* 41 qt.

6. Reduce 18 bu. 3 pk. to pints. *Ans.* 1200 pt.

7. Reduce 1803 pt. to bushels. *Ans.* 28 bu. 5 qt. 1 pt.

8. Reduce 12 bu. 1 pk. 3 qt. to pints. *Ans.* 790 pt.

9. Reduce 21132 qt. to bushels.

Ans. 660 bu. 1 pk. 4 qt.

10. Reduce 24188 pt. to pecks. *Ans.* 1511 pk. 6 qt.

11. Bought 10 bushels of plums at 5 cents a quart: what did they cost? *Ans.* $16.

12. Bought 8 bu. 3 pk. of blackberries at 8 cents a quart, and sold them for 10 cents a quart: how much did I gain? *Ans.* $5.60.

13. A man sold 108 bu. 3 qt. of corn at the rate of one cent a pint: how much did he receive?

Ans. $69.18.

14. How many bushels of wheat can be purchased for $17.30, at 2 cents a pint? *Ans.* 13 bu. 2 pk. 1 pt.

15. Mrs. Black bought half a bushel of strawberries at 8 cents a quart, and 10 pounds of sugar at 9 cents a pound: how much did she pay for both sugar and berries?

Ans. $2.18.

16. A boy received 2 cents a quart for picking berries: when he had earned $1.28, how many bushels had he picked? *Ans.* 2 bu.

LIQUID MEASURE.

58. **Liquid Measure** is used in measuring liquids; as, oil, milk, etc.

TABLE.

4 gills (gi.)	. . make	1 pint,	. . marked pt.
2 pints	. . . "	1 quart,	. . " qt.
4 quarts	. . . "	1 gallon,	. " gal.

NOTE.—The standard liquid gallon contains 231 cubic inches.

MENTAL EXERCISES.

1. How many gills in 3 pints? In 8 pt.? In 9 pt.?
2. How many pints in 12 gills? In 32 gi.? In 40 gi.?
3. How many pints in 6 quarts? In 8 qt.? In 11 qt.?
4. How many quarts in 12 pints? In 18 pt.?
5. How many quarts in 6 gallons? In 8 gal.?
6. How many pints in 1 gallon?
7. How many gills in 1 gallon?
8. How many quarts in half a gallon?
9. How many pints in half a gallon?

WRITTEN EXERCISES.

1. Reduce 14 gal. 1 qt. 1 pt. 3 gi. to gills.
Ans. 463 gi.
2. Reduce 22 gal. 3 qt. 1 gi. to gills. *Ans.* 729 gi.
3. Reduce 307 pt. to gallons. *Ans.* 38 gal. 1 qt. 1 pt.
4. Reduce 840 gi. to gallons. *Ans.* 26 gal. 1 qt.
5. How many gallons in 5313 cubic inches?
Ans. 23 gal.
6. How many cubic inches in 231 gallons?
Ans. 53361 cu. in.
7. What is the cost of 53 gal. 1 gi. of vinegar at 1 cent a gill? $16.97.
8. What will 5 gal. 3 qt. of plums cost at 10 cents a quart? *Ans.* $2.30.
9. What will 512 pints of maple syrup cost at $1.20 a gallon? *Ans.* $76.80.
10. How many vials holding 2 gills each, can be filled from 6 gallons of alcohol? *Ans.* 96 vials.

AVOIRDUPOIS WEIGHT.

59. **Avoirdupois Weight** is used in weighing all ordinary merchandise, as groceries, metals, etc.

TABLE.

16 ounces (oz.) . .	make	1 pound, . . .	marked lb.
100 pounds	"	1 hundred-weight,	" cwt.
20 cwt., or 2000 lb.,	"	1 ton, ...	" T.

MENTAL EXERCISES.

1. How many ounces in 3 pounds? In 2 lb.? In 4 lb.?

2. How many pounds in 48 ounces? In 32 oz.?

3. How many ounces in one half a pound?

4. How many ounces in one quarter of a pound? How many in two quarters? How many in three quarters?

5. How many pounds in 3 hundred-weight? In 7 cwt.? In 4 cwt.?

6. How many hundred-weight in 300 lb.? In 700 lb.?

7. How many hundred-weight in 2 tons? In 3 tons?

WRITTEN EXERCISES.

1. Reduce 2 T. to ounces. *Ans.* 64000 oz.

2. Reduce 32000 oz. to tons. *Ans.* 1 T.

3. Reduce 425 lb. to ounces. *Ans.* 6800 oz.

4. Reduce 1000224 oz. to tons. *Ans.* 31 T. 514 lb.

5. Reduce 360 cwt. 5 lb. to ounces. *Ans.* 576080 oz.

6. Reduce 8374160 oz. to tons. *Ans.* 261 T. 1385 lb.

7. Reduce 17460 lb. to tons. *Ans.* 8 T. 1460 lb.

8. What is the cost of 3014 lb. of sugar at 15 cents a pound? *Ans.* $452.10.

9. A man bought 15 cwt. of lead at 10 cents a pound, and paid for it with sugar at 15 cents a pound: how much sugar did it take? *Ans.* 1000 lb.

10. At 25 cents a pound, what quantity of grapes can be bought for $450? *Ans.* 1800 lb.

LONG MEASURE.

60. Long Measure is used in measuring distances, or length, in any direction.

TABLE.

12	inches (in.) . .	make 1 foot, ...	marked ft.
3	feet	" 1 yard, ...	" yd.
$5\frac{1}{2}$	yards	" 1 rod, ...	" rd.
320	rods	" 1 mile, ...	" mi.

NOTES.—1. In a mile there are 5280 feet. The mile is commonly divided into halves, fourths, eighths, etc.

2. In measuring cloth, ribbon, etc., the width is not considered, and the yard is divided into halves, fourths, eighths, etc.

MENTAL EXERCISES.

1. How many inches in 2 feet? In 4 ft.? In 3 ft.? In 5 ft.?

2. How many feet in 48 inches? In 72 in.? In 120 in.? In 96 in.?

3. How many feet in 5 yards? In 7 yd.? In 6 yd.? In 12 yd.?

4. How many yards in 12 feet? In 21 ft.? In 18 ft.? In 27 ft.?

5. How many yards in 5 rods?

SOLUTION.—Since in 1 rod there are $5\frac{1}{2}$ yards, in 5 rods there are 5 times $5\frac{1}{2}$ yards. Five times 5 yards are 25 yards, and five times $\frac{1}{2}$ yard $= \frac{5}{2}$ yards, or $2\frac{1}{2}$ yards, which added to 25 yards make $27\frac{1}{2}$ yards.

CONCLUSION.—Therefore, in 5 rods there are $27\frac{1}{2}$ yards.

6. How many yards in 4 rods? In 6 rd.? In 9 rd.?

7. How many rods in 15 yards?

SOLUTION.—Since $5\frac{1}{2}$ yards make 1 rod, 15 yards will make as many rods as $5\frac{1}{2}$ is contained times in 15.

Since 1 yard = 2 halves, $5\frac{1}{2}$ yards = $5\frac{1}{2}$ times 2 halves, which is 11 halves; and 15 yards = 15 times 2 halves, which is 30 halves. 11 halves are contained in 30 halves 2 times, with a remainder of 8 half yards = 4 yards.

Conclusion.—Therefore, in 15 yards there are 2 rods and 4 yards.

8. How many rods in 11 yd.? In 22 yd.? In $16\frac{1}{2}$ yd.?

9. How many rods in 12 yards? In 20 yd.?

WRITTEN EXERCISES.

1. Reduce 2 mi. to rods. *Ans.* 640 rd.

2. Reduce 5 yd. 1 ft. 6 in. to inches. *Ans.* 198 in.

3. Reduce 2340 rd. to miles. *Ans.* 7 mi. 100 rd.

4. Reduce 2 mi. to ft. *Ans.* 10560 ft.

5. Reduce 15840 ft. to miles. *Ans.* 3 mi.

6. How many feet in half a mile? *Ans.* 2640 ft.

7. How many feet in one fourth of a mile? *Ans.* 1320 ft.

8. A steam-boat moves at the rate of 70 feet in a second: how far will it go in 3630 seconds? *Ans.* 48 mi. 40 rd.

9. If a horse can travel 1 mile in 6 minutes, how many yards will he travel in an hour? *Ans.* 17600 yd.

10. How many rods from Cincinnati to Columbus, the distance being 120 miles? *Ans.* 38400 rd.

TIME MEASURE.

61. Time Measure is used in measuring time or duration.

TABLE.

60	seconds (sec.). .	make	1	minute, . .	marked min.
60	minutes . . .	"	1	hour, . . .	" hr.
24	hours	"	1	day, . . .	" da.
$365\frac{1}{4}$	days	"	1	year, . . .	" yr.
100	years	"	1	century, . .	" cen.
Also, 7	days	"	1	week, . . .	" wk.
4	weeks	"	1	month (nearly),	" mo.
12	calendar months	"	1	year, . . .	" yr.

NOTES.—1. The exact length of the SOLAR year is 365 days, 5 hours, 48 minutes, 48 seconds ; but it is usually considered to be 365 days, 6 hours, or $365\frac{1}{4}$ days. This $\frac{1}{4}$ day in 4 years amounts to one whole day; therefore, *every fourth year*, or *every year divisible by 4, consists of 366 days*, and is called a *leap*-year.

The additional day is added to the month of February, which has 28 days in ordinary years, and 29 days in leap-years.

2. Of the hundreds of years, or centuries, only those which are divisible by 400 are considered leap-years.

3. The year, as divided by most civilized nations, consists of 12 calendar months, as follows :

January,	1st	month,	31	days.	July,	7th	month,	31	days.
February,	2d	"	28	"	August,	8th	"	31	"
March,	3d	"	31	"	September,	9th	"	30	"
April,	4th	"	30	"	October,	10th	"	31	"
May,	5th	"	31	"	November,	11th	"	30	"
June,	6th	"	30	"	December,	12th	"	31	"

4. The following lines will aid in fixing in the mind the number of days in each month :

Thirty days hath September,
April, June, and November ;
All the rest have thirty-one,
Save February, which alone
Hath twenty-eight ; and, to be fine,
One year in four hath twenty-nine.

5. In most business transactions, 30 days are considered a month.

6. The year is divided into four seasons, of three months each, as follows :

Spring, { March, April, May.
Autumn, or Fall, { September, October, November.
Summer, { June, July, August.
Winter, { December, January, February.

MENTAL EXERCISES.

1. How many seconds in 2 minutes ? In 4 min. ? In 3 min. ?

2. How many minutes in 3 hours? In 5 hr.? In 10 hr.?

3. How many minutes in 120 seconds? In 180 sec.? In 600 sec.?

4. How many hours in 180 minutes? In 300 min.? In 600 min.?

5. How many hours in 2 days? In 3 da.? In 5 da.?

6. How many days in 48 hours? In 72 hr.? In 120 hr.?

7. How many days in 5 weeks? In 7 weeks? In 10 weeks?

8. How many weeks in 21 days? In 56 da.? In 84 da.?

9. How many calendar months in 3 years? In 5 yr.? In 2 yr.?

10. Is the present year a leap-year?

WRITTEN EXERCISES.

1. How many seconds in 10 hours? *Ans.* 36000 sec.

2. How many hours in 57600 seconds? *Ans.* 16 hr.

3. Reduce 4 da. 13 hr. 20 min. to seconds.

Ans. 393600 sec.

4. Reduce 30600 minutes to higher denominations.

Ans. 21 da. 6 hr.

5. How many minutes in a leap-year?

Ans. 527040 min.

6. How many minutes in a common year?

Ans. 525600 min.

7. Reduce 1 wk. 3 da. 2 hr. to seconds.

Ans. 871200 sec.

8. Reduce 806472 hr. to years. *Ans.* 92 yr.

9. How many seconds in the three months, January, February, and March, of the year 1888?

Ans. 7862400 sec.

10. How many minutes in the months of April, May, and June? *Ans.* 131040 min.

62. MISCELLANEOUS TABLE.

24 sheets of paper	make 1 quire.
20 quires	" 1 ream.
2 reams	" 1 bundle.
12 things	make 1 dozen.
12 dozen	" 1 gross.
20 things	" 1 score.
100 pounds of nails	make 1 keg.
196 pounds of flour	" 1 barrel.
200 pounds of beef or pork	" 1 barrel.

MISCELLANEOUS EXAMPLES.

1. What will 9 bu. of wheat cost at 40 ct. a peck?
Ans. $14.40.

2. What will 12 pounds of castings cost at 8 ct. an ounce? *Ans.* $15.36.

3. What will 41 yd. of fence cost at 96 ct. a foot?
Ans. $118.08.

4. What will 125 gal. of vinegar cost at 5 ct. a quart?
Ans. $25.

5. What is the cost of 40 cwt. of iron at 6 ct. a pound?
Ans. $240.

6. What will 312 sheets of paper cost at 60 ct. a quire?
Ans. $7.80.

7. What will 3 tons 25 pounds of rice cost at 12 ct. a pound? *Ans.* $723.

8. What will it cost to build 140 miles of railroad at $48 a rod? *Ans.* $2150400.

9. How many files, each weighing 3 oz., can be made from 19 lb. 5 oz. of steel? *Ans.* 103 files.

10. How many dozen toys at 75 ct. apiece, can be purchased for $45? *Ans.* 5 doz.

11. What will be the cost of 5 boxes of tea, each containing 95 lb., at $1.55 a pound? *Ans.* $736.25.

12. What is the cost of 12 barrels of flour at 5 ct. a pound? $117.60.

13. A merchant packed 122 bbl. of beef in 305 boxes: how many pounds in each box? *Ans.* 80 lb.

14. Find the cost of 5 reams of paper at 1 ct. for every two sheets. *Ans.* $12.

15. What will be the cost of 176 gross of pens at 15 ct. a dozen? *Ans.* $316.80.

16. How much time would a man who rises at 7 A.M. gain in 40 years of 365 days each, by rising 25 minutes before 7 A.M. each day? *Ans.* 253 da. 11 hr. 20 min.

17. If a horse eats 10 lb. of hay in 1 day, how many cwt. will he eat in the year 1892? *Ans.* 36 cwt. 60 lb.

18. A farmer has a flock of 320 sheep: how many bushels of oats must he give them, so that each sheep shall have 1 pint? *Ans.* 5 bu.

19. What is the difference in weight of 20 bbl. of pork and 21 bbl. of flour? *Ans.* 116 lb.

20. A boy had 4 score and 8 apples; he sold 4 dozen: how many had he left? *Ans.* 40 apples.

ADDITION OF COMPOUND NUMBERS.

63. Addition of Compound Numbers is the process of finding the sum of two or more compound numbers of the same kind.

NOTE.—In adding Simple Numbers, we carry one for every ten, because ten units of a lower denomination always make one of the next higher; but

In Compound Numbers we carry one for the number of units of each denomination which make one of the next higher, and these numbers are different in the different tables.

MODEL SOLUTION.

EXAMPLE.—What is the sum of 4 bu. 2 pk. 6 qt.; 9 bu. 3 pk. 7 qt. 1 pt.; 13 bu. 1 pk. 1 pt.; and 6 bu. 3 pk. 4 qt. 1 pt.?

SOLUTION.—Since only numbers of the same kind can be added, write units of the same denomination under each other.

Begin with the lowest denomination. The sum of the first column is 3 pt. = 1 qt. 1 pt.

Write the 1 pt. under the column of pt., and add the 1 qt. to the sum of the column of qt., which makes 18 qt. = 2 pk. 2 qt.

OPERATION.

bu.	pk.	qt.	pt
4	2	6	0
9	3	7	1
13	1	0	1
6	3	4	1
34	3	2	1

Write the 2 qt. under the column of qt., and add the 2 pk. to the sum of the column of pk., which makes 11 pk. = 2 bu. 3 pk.

Write 3 pk. under the column of pk., and add the 2 bu. to the sum of the column of bu., which makes 34 bu.

CONCLUSION.—Therefore, the sum of 4 bu. 2 pk. 6 qt.; 9 bu. 3 pk. 7 qt. 1 pt.; 13 bu. 1 pk. 1 pt.; and 6 bu. 3 pk. 4 qt. 1 pt., is 34 bu. 3 pk. 2 qt. 1 pt. Hence, the

RULE FOR ADDITION OF COMPOUND NUMBERS.—*Write the numbers to be added, placing units of the same denomination in the same column.*

Add the lowest denomination, and carry one for as many units of the denomination added as it takes to make a unit of the next higher denomination.

Add the other columns in the same manner; at the last, set down the whole amount.

PROOF.—The same as in Simple Numbers.

(*1.*)

bu.	pk.	qt.	pt.
18	1	3	0
8	3	7	1
12	0	6	0
3	2	4	0
23	1	3	1
13	0	5	1
11	1	0	1
90	3	6	0

(*2.*)

gal.	qt.	pt.	gi.
60	2	1	1
13	0	1	1
23	3	0	2
14	1	1	3
3	0	0	1
6	2	1	0
10	0	0	1
131	3	0	1

3. Add 7 T. 13 cwt. 48 lb. 5 oz.; 3 T. 5 cwt. 92 lb. 12 oz.; 8 T. 17 cwt. 64 lb. 10 oz.; 16 T. 10 cwt. 20 lb. 11 oz.; and 21 T. 37 lb. 10 oz. *Ans.* 57 T. 7 cwt. 64 lb.

4. What is the sum of 8 mi. 290 rd. + 7 mi. 38 rd. + 8 mi. 240 rd. + 9 mi. 200 rd. + 3 mi. 18 rd.?

Ans. 37 mi. 146 rd.

5. What is the sum in yards, feet, and inches of 7 yd. 2 ft. 8 in. + 9 yd. 1 ft. 10 in. + 3 yd. 2 ft. 7 in. + 13 yd. 5 in. + 20 yd. 11 in. + 17 yd. 2 ft.?

Ans. 72 yd. 1 ft. 5 in.

6. A gentleman has 6 fields of wheat; from the first, he obtained 310 bu. 2 pk. 6 qt.; from the second, 220 bu. 1 pk. 4 qt.; from the third, 400 bu. 7 qt.; from the fourth, 85 bu. 3 pk.; from the fifth, 95 bu. 1 pk. 3 qt.; and from the sixth, 103 bu. 2 pk. 6 qt.: how much wheat did he obtain? *Ans.* 1216 bu. 2 qt.

7. What is the sum of 15 wk. 2 da. 20 hr. 15 min.; 6 wk. 3 da. 15 hr. 24 sec.; 7 wk. 10 hr. 45 min. 23 sec.; and 1 wk. 3 da. 5 hr. 30 min. 30 sec.?

Ans. 30 wk. 3 da. 3 hr. 31 min. 17 sec.

SUBTRACTION OF COMPOUND NUMBERS.

64. Subtraction of Compound Numbers is the process of finding the difference between two compound numbers of the same kind.

MODEL SOLUTION.

EXAMPLE.—What is the difference between 10 bu. 1 pk. 7 qt., and 6 bu. 3 pk. 3 qt.?

SOLUTION.—Write the less number under the greater, placing units of the same denomination in the same column.

Take 3 qt. from 7 qt. and 4 qt. remain, which write under the column of qt.

OPERATION.

bu.	pk.	qt.
10	1	7
6	3	3
3	2	4

Since 3 pk. can not be subtracted from 1 pk., add 4 pk. to 1 pk., making 5 pk. for the term of the minuend, and take 3 pk. from 5 pk., and write 2 pk., the difference, under the column of pk.

Since 4 pk. were added to the minuend, add 1 bu. (which equals 4 pk.) to the 6 bu. of the subtrahend, making 7 bu., and take 7 bu. from 10 bu., and write 3 bu., the difference, under the column of bu. The difference is 3 bu. 2 pk. 4 qt. Hence, the

RULE.—*Write the less number under the greater, placing units of the same denomination in the same column.*

Begin with the lowest denomination, and, if possible, take each lower number from the one above it.

But if any lower number is greater than the one above it, increase the minuend by as many units of that denomination as it takes to make one of the next higher; subtract as before; then add one to the next higher denomination of the subtrahend, and subtract as before. Proceed in like manner with each denomination.

PROOF.—Same as in Subtraction of Simple Numbers.

EXAMPLES.

(*1.*)

	bu.	pk.	qt.	pt.
From	18	1	3	1
Take	12	1	7	1
	5	3	4	0

(*2.*)

	mi.	rd.	yd.	ft.
From	81	300	5	2
Take	65	315	3	1
	15	305	2	1

(*3.*)

	gal.	qt.	pt.	gi.
From	34	2	1	3
Take	27	3	1	2
	6	3	0	1

(*4.*)

	cwt.	lb.	oz.
From	56	73	11
Take	26	35	15
	30	37	12

5. A wagon loaded with hay weighs 38 cwt. 70 lb.; the wagon alone weighs 20 cwt. 98 lb.: what is the weight of the hay? *Ans.* 17 cwt. 72 lb.

65. To find the time between any two dates.

EXAMPLE.—What is the time from March 17th, 1884 to January 12th, 1886?

SOLUTION.—Write the first date under the last, numbering the months in their order; subtract as in subtraction of other compound numbers, remembering that 30 days are considered a month.

OPERATION.

yr.	mo.	da.
1886	1	12
1884	3	17
1	9	25

6. How long from June 5th, 1883 to September 21st, 1889? *Ans.* 6 yr. 3 mo. 16 da.

7. A note given June 15th, 1881 was paid Aug. 9th, 1886: how long was it on interest?

Ans. 5 yr. 1 mo. 24 da.

8. How long a time from July 4th, 1776 to April 12th, 1861? *Ans.* 84 yr. 9 mo. 8 da.

9. A certain man was born June 15th, 1837: what was his age Sept. 1st, 1861? *Ans.* 24 yr. 2 mo. 16 da.

10. The American Revolution began April 19th, 1775 and ended Jan. 20th, 1783: how long did it continue?

Ans. 7 yr. 9 mo. 1 da.

11. America was discovered Oct. 12th, 1492: how long a time from then until now?

12. The Declaration of Independence was signed July 4th, 1776: how much time has elapsed since then?

MULTIPLICATION OF COMPOUND NUMBERS.

66. Multiplication of Compound Numbers is the process of taking a compound number as many times as there are units in the multiplier.

MODEL SOLUTION.

EXAMPLE.—A merchant sold 7 bags of flour, each containing 1 bu. 3 pk. 5 qt.: how much did he sell in all?

SOLUTION.—Since one bag contains 1 bu. 3 pk. 5 qt., 7 bags will contain 7 times 1 bu. 3 pk. 5 qt.

Seven times 5 qt. are 35 qt. = 4 pk. 3 qt. Write the 3 qt. under the qt., and add the 4 pk. to the product of pk.; 7 times 3 pk. are 21 pk.; 21 pk. + 4 pk. = 25 pk.; 25 pk. = 6 bu. 1 pk. Write the 1 pk. under the pk., and add the 6 bu. to the product of bu. 7 times 1 bu. are 7 bu.; 7 bu. + 6 bu. = 13 bu., which write under bu.

OPERATION.

bu.	pk.	qt.
1	3	5
		7
13	1	3

CONCLUSION.—Therefore, if a merchant sold 7 bags of flour, each containing 1 bu. 3 pk. 5 qt., he sold 13 bu. 1 pk. 3 qt. Hence, the

RULE.—*Write the multiplier under the lowest denomination of the multiplicand.*

Multiply as in simple numbers, and reduce and carry as in addition of compound numbers.

PROOF.—The same as in Multiplication of Simple Numbers.

EXAMPLES.

1. Multiply 3 tons, 10 hundred-weight, 37 pounds, 11 ounces by 9. *Ans.* 31 T. 13 cwt. 39 lb. 3 oz.

2. How much grain in 15 bins, each containing 122 bu. 3 pk. 1 qt.? *Ans.* 1841 bu. 2 pk. 7 qt.

3. What distance can a man walk in 14 hours, at the rate of 3 mi. 125 rd. an hour? *Ans.* 47 mi. 150 rd.

4. What are the contents of 32 boxes, each holding 3 bu. 2 qt.? *Ans.* 98 bu.

5. If one load of hay weighs 1 T. 23 lb., what is the weight of 30 such loads? *Ans.* 30 T. 6 cwt. 90 lb.

6. If a barrel of flour will support a family 3 wk. 5 da., how long will 12 barrels support it? *Ans.* 44 wk. 4 da.

7. If one keg holds 5 gal. 1 qt. 1 gi., how much will 24 such kegs hold? *Ans.* 126 gal. 3 qt.

8. If a man sleeps 7 hr. 45 min. 20 sec. each day, how long will he sleep in 24 days? *Ans.* 7 da. 18 hr. 8 min.

9. A man walks 5 ft. 10 in. in a second : how far will he walk in 10 minutes? *Ans.* 212 rd. 2 ft.

10. A hotel uses daily 3 doz. and 6 eggs : how many will it use in a month? *Ans.* 105 doz.

DIVISION OF COMPOUND NUMBERS.

67. Division of Compound Numbers is the process of finding one of the equal parts of a compound number.

MODEL SOLUTION.

EXAMPLE.—What is one fifth of 17 bu. 3 pk. 2 qt. ?

SOLUTION.—Divide the highest denomination first, so that if there is a remainder it may be reduced to the next lower denomination, and added to it.

OPERATION.

	bu.	pk.	qt.
5)	17	3	2
	3	2	2

One fifth of 17 bu. is 3 bu., with 2 bu. remaining ; place the 3 bu. under bu., and reduce the 2 bu. to pk.; 2 bu. = 8 pk.; 8 pk. + 3 pk. = 11 pk.

One fifth of 11 pk. is 2 pk., with 1 pk. remaining ; place the 2 pk. under pk., and reduce the 1 pk. to qt.; 1 pk. = 8 qt.; 8 qt. + 2 qt. = 10 qt.

One fifth of 10 qt. is 2 qt., which write under qt.

CONCLUSION.—Therefore, the quotient of 17 bu. 3 pk. 2 qt. divided by 5, is 3 bu. 2 pk. 2 qt. Hence, the

RULE FOR DIVISION OF COMPOUND NUMBERS.—*Beginning with the highest denomination, divide each number separately, and write the quotient above.*

If a remainder occurs after dividing any denomination, reduce and add it to the next lower denomination, and divide as before.

PROOF.—The same as in Simple Numbers.

NOTE.—In dividing one compound number by another, reduce both quantities to the same denomination. The process then becomes one of Simple Numbers.

EXAMPLES.

1. What is one fourth of 33 wk. 5 da. 23 hr. 21 min. 20 sec.? *Ans.* 8 wk. 3 da. 5 hr. 50 min. 20 sec.

2. What is one seventh of 24 bu. 1 pk. 4 qt. 1 pt.?

Ans. 3 bu. 1 pk. 7 qt. 1 pt.

3. What is one ninth of 973 gal. 1 pt.?

Ans. 108 gal. 1 pt.

4. What is one twelfth of 48 cwt. 64 lb. 8 oz.?

Ans. 4 cwt. 5 lb. 6 oz.

5. Divide 144 da. 44 min. 16 sec. by 22.

Ans. 6 da. 13 hr. 7 min. 28 sec.

6. A man walks 35 mi. 40 rd. in 8 hours: how far does he walk in 1 hour? *Ans.* 4 mi. 125 rd.

7. The weight of 10 boxes of books is 2 T. 15 cwt. 65 lb.; what is the weight of each? *Ans.* 5 cwt. 56 lb. 8 oz.

8. How many bottles holding 3 qt. 1 pt. each, can be filled from a cask containing 45 gal. 2 qt.?

Ans. 52 bottles.

9. How many castings weighing 12 lb. 8 oz. each, can be made from 11 cwt. of iron? *Ans.* 88 castings.

10. If a man can dig a ditch 32 rd. 2 yd. 2 ft. long in 8 days, how much of it can he dig in one day?

Ans. 4 rd. 1 ft.

68. MISCELLANEOUS EXAMPLES.

MENTAL EXERCISES.

1. What will 6 quarts of strawberries cost at 11 cents a quart? At 12½ ct. a quart?

2. A man carried 3 pecks of cherries to market, and sold them at 10 cents a quart: how much did he receive?

3. When apples sell at 25 cents a peck, what are they worth a bushel?

4. A boy sold 2 pecks of plums at 10 cents a quart: how much did he receive?

5. What will 8 quarts of milk cost at 3 cents a pint ?

6. If maple syrup costs $1.20 a gallon, what is the price per quart ?

7. How many half-pint tumblers will 3 gallons of jelly fill ?

8. How many quart baskets will 2 pk. 3 qt. of blackberries fill ?

WRITTEN EXERCISES.

1. A farmer sold 158 bu. 2 pk. of rye, 1040 pk. of corn, 197 bu. 1 pk. 5 qt. of wheat : how much grain did he sell?

Ans. 615 bu. 3 pk. 5 qt.

2. A has 57 bu. 1 pk. 3 qt. of oats, B has 6 times as much : how much have both ?

Ans. 401 bu. 1 pk. 5 qt.

3. If a locomotive runs 210 mi. 156 rd. in 9 hours, how far will it run in one hour ? *Ans.* 23 mi. 124 rd.

4. What is the weight of 154 hogsheads of sugar, each weighing 11 cwt. 91 lb. ? *Ans.* 91 T. 14 cwt. 14 lb.

5. How many quarts of cranberries in 4 barrels, each containing 2 bu. 3 pk. ? *Ans.* 352 qt.

6. A man bought 165 firkins of butter, each containing 56 lb., at 38 cents a pound : what was the cost?

Ans. $3511.20 ?

7. Reduce 3 wk. 5 da. 41 sec. to seconds.

Ans. 2246441 sec.

8. A grocer buys 587 gallons of molasses at 42 ct. a gallon, and sells it for 51 ct. a gal. : how much does he gain ? *Ans.* $52.83.

9. What would be the cost of 172 bags of coffee, each weighing 78 lb., at 32 ct. a lb. ? *Ans.* $4293.12.

10. Reduce 14500 pints to gallons.

Ans. 1812 gal. 2 qt.

11. How many times will a wheel, 18 ft. 4 in. in circumference, turn in going 15 miles ? *Ans.* 4320 times.

PROPERTIES of NUMBERS

69. An **Integer** is a whole number; as, 1, 5, 7, 21, etc.

Integers are divided into two classes: *prime* and *composite.*

A **Prime Number** is one that can be exactly divided *only* by itself and 1; as, 1, 3, 5, 7, 11, etc.

A **Composite Number** is one that can be exactly divided by some other number; as, 4, 6, 8, 9, etc.

NOTE.—Two numbers are *prime to each other* when 1 is the only number that will exactly divide *both.* Thus, 4 and 5 are prime to each other; 8 and 15 are prime to each other, though each is a composite number.

70. A **Factor** of a number is a number that will exactly divide it. Thus, 3 is a factor of 6, 9, 18, etc.

A **Prime Factor** of a number is a *prime* number that will exactly divide it. Thus, 3 is a prime factor of 24; and 5 of 30.

A **Composite Factor** of a number is a *composite* number that will exactly divide it. Thus, 4 is a composite factor of 24; and 6 of 30.

A **Divisor** of a number is a number that will exactly divide it. Thus, 3 is a divisor of 12; and 5 of 10.

NOTE.—The terms *factor, divisor,* and *measure* all mean the same thing. Every composite number is divisible by its factors or by any product of them.

A **Multiple** of a number is a number that can be divided by it *without a remainder.* Thus, 12 is a multiple of 3; and 15 of 5.

MENTAL EXERCISES.

1. What are the factors of 10? 15? 30? 45? 55? 21? 27? 36?

2. What are the factors of 16? 20? 28? 32? 56? 50? 63? 77?

3. Give all the pairs of factors of 16.

SOLUTION.—16 is 4 times 4 or 8 times 2.

4. Give all the pairs of factors of 12; 18; 30; 28; 36; 48.

5. Name all the prime numbers from 0 to 15; 15 to 30; 30 to 45.

6. Name all the composite numbers from 0 to 15; 15 to 30; 30 to 45.

7. Give all the prime factors of 24, not including 1.

SOLUTION.—2, 2, 2, and 3; $2 \times 2 \times 2 \times 3 = 24$.

8. Give all the prime factors of 12; 16; 18; 21; 28; 36; 40.

9. Of what number are 2, 2, 2, and 3 the prime factors? 2, 3, and 3? 2, 3, and 5? 2, 3, and 7? 2, 3, and 11?

10. Give all the prime factors of 15; 30; 25; 50; 60.

WRITTEN EXERCISES.

To resolve composite numbers into prime factors.

71. The **Prime Factors** of any number are all the prime numbers that will divide it.

EXAMPLE.—What are the prime factors of 60?

SOLUTION.—On trial, it is found that 60 can be divided by the prime factor 2, leaving the composite factor 30.

Again, it is found that 30 can be divided by 2, leaving the composite factor 15.

It is also found that 15 can be divided by the prime factor 3, giving the prime factor 5.

OPERATION.

$$\begin{array}{r} 2\,)\,60 \\ 2\,)\,30 \\ 3\,)\,15 \\ \hline 5 \end{array}$$

CONCLUSION.—Therefore, the prime factors of 60 are 2, 2, 3, and 5. Hence, the

RULE.—*Divide the composite number by any prime number that will exactly divide it; divide the quotient in the same manner, and so continue to divide until a quotient is obtained which is a prime number.*

The last quotient and the several divisors will be the prime factors of the given number.

The following propositions will assist the pupil in determining what to divide by:

PROPOSITION 1.—Every number ending with 0, 2, 4, 6, or 8 is divisible by 2.

PROPOSITION 2.—Every number is divisible by 4, when the number indicated by its two right-hand figures is divisible by 4.

PROPOSITION 3.—Every number is divisible by 5, when its right-hand figure is 0 or 5.

PROPOSITION 4.—Every number the sum of whose figures is divisible by 3 or 9, is divisible by 3 or 9.

Resolve the following into their prime factors:

1.	48. 2, 2, 2, 2, 3.	*15.*	1820. 2, 2, 5, 7, 13.
2.	91. 7, 13.	*16.*	1850. 2, 5, 5, 37.
3.	105. 3, 5, 7.	*17.*	2992. 2, 2, 2, 2, 11, 17.
4.	104. 2, 2, 2, 13.	*18.*	2625. 3, 5, 5, 5, 7.
5.	154. 2, 7, 11.	*19.*	1768. 2, 2, 2, 13, 17.
6.	242. 2, 11, 11.	*20.*	1470. 2, 3, 5, 7, 7.
7.	252. 2, 2, 3, 3, 7.	*21.*	3430. 2, 5, 7, 7, 7.
8.	625. 5, 5, 5, 5.	*22.*	6250. 2, 5, 5, 5, 5, 5.
9.	900. 2, 2, 3, 3, 5, 5.	*23.*	7375. 5, 5, 5, 59.
10.	1080. 2, 2, 2, 3, 3, 3, 5.	*24.*	5250. 2, 3, 5, 5, 5, 7.
11.	1492. 2, 2, 373.	*25.*	7000. 2, 2, 2, 5, 5, 5, 7.
12.	2436. 2, 2, 3, 7, 29.	*26.*	2958. 2, 3, 17, 29.
13.	1050. 2, 3, 5, 5, 7.	*27.*	4325. 5, 5, 173.
14.	1155. 3, 5, 7, 11.	*28.*	5000. 2, 2, 2, 5, 5, 5, 5.

CANCELLATION

DEFINITIONS.

72. **Cancellation** is a method of shortening the operations of arithmetic by omitting equal factors of the divisor and dividend.

NOTE.—To cancel is to suppress or erase. When the same factor is omitted in both dividend and divisor, it is canceled.

Cancellation depends on the following principle:

Omitting an equal factor in both dividend and divisor does not alter the quotient.

Thus, $12 \times 6 \div 9 = 8$, and $12 \times 2 \div 3 = 8$; the quotient being the same in both cases, though the factor 3 is omitted in the second operation.

EXAMPLE.—Divide the product of 16, 18, and 12 by the product of 24, 8, 9, and 3.

SOLUTION.—For convenience, arrange the dividend above a line, and the divisor below. Since 9 is a factor of both dividend and divisor, cancel the 9 in the divisor and the 18 in the dividend, placing 2, the remaining factor of 18, above it. Cancel 12 and 24 in the same manner, placing the 2 beneath the 24. Cancel 8 and 16, placing 2, the remaining factor, above the 16. Then cancel the 2 in the dividend and divisor. The factor left in the dividend is 2, and in the divisor 3. Hence, the quotient is $\frac{2}{3}$.

OPERATION.

$$\frac{\overset{2}{\cancel{16}} \times \cancel{18}^{2} \times \cancel{12}}{\underset{2}{\cancel{24}} \times \cancel{8} \times \cancel{9} \times 3} = \frac{2}{3}$$

CONCLUSION.—Therefore, the product of 16, 18, and 12 divided by the product of 24, 8, 9, and 3 is $\frac{2}{3}$. Hence, the

RULE.—*Arrange the dividend above a horizontal line, and the divisor below.*

Cancel all the factors common to both, and divide the product of the remaining factors of the dividend by those of the divisor.

NOTES.—Cancellation is merely *dividing* both dividend and divisor by the same number. The numbers divided are usually indicated by drawing a line through them.

In any case in which all the figures in the dividend have been canceled, and all in the divisor, the quotient will be 1. If all have been canceled in the dividend, and any remain in the divisor, be careful to place 1 in the dividend.

EXAMPLES FOR PRACTICE.

1. Divide the product of 56 and 27 by 36. *Ans.* 42.

2. Divide the product of 48 and 49 by 42. *Ans.* 56.

3. Divide the product of 96 and 40 by 120. *Ans.* 32.

4. Divide $36 \times 27 \times 15$ by $9 \times 6 \times 10$. *Ans.* 27.

5. Divide $16 \times 64 \times 81$ by $72 \times 3 \times 8 \times 6$. *Ans.* 8.

6. Divide $42 \times 18 \times 132$ by $44 \times 136 \times 6$. *Ans.* $2\frac{5}{6}\frac{3}{8}$.

7. Divide $63 \times 60 \times 112$ by 66×80. *Ans.* $80\frac{2}{11}$.

8. A farmer hired 8 men for 6 months at \$18 a month; he paid them with hay at \$12 a ton: how many tons did they receive? *Ans.* 72 T.

9. A merchant bought 12 casks of rice, each containing 95 lb., at 10 cents a pound, and paid for it with 3 barrels of butter at 19 cents a pound: how many pounds in a barrel? *Ans.* 200 lb.

10. Divide $3 \times 15 \times 33 \times 8 \times 2 \times 17$ by $20 \times 34 \times 22 \times 27$. *Ans.* 1.

11. Divide $21 \times 6 \times 13$ by $26 \times 3 \times 7 \times 6$. *Ans.* $\frac{1}{2}$.

GREATEST COMMON DIVISOR.

73. A **Common Divisor** of two or more numbers is any number that will divide each without a remainder.

The **Greatest Common Divisor** of two or more num-

bers is the *greatest* number that will divide each without a remainder.

NOTE.—Numbers may have several *common* divisors, but only one *greatest* common divisor.

MENTAL EXERCISES.

1. What numbers are divisors of 18? 21? 28? 35? 63? 48?

2. What are the common divisors of 12 and 16? 30 and 45? 40 and 72?

3. What is the greatest common divisor of 15 and 30? 18 and 54? 24 and 36?

EXAMPLE.—What is the greatest common divisor of 42 and 54?

SOLUTION.—Resolve each number into prime factors. The prime factors of 42 are 2, 3, and 7; the prime factors of 54 are 2, 3, 3, and 3. The factors 2 and 3 are the *only* ones common to both numbers; hence, their product, 6, is the greatest number that will divide each of them.

OPERATION.

$42 = 2 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3$

$2 \times 3 = 6$

CONCLUSION.—Therefore, the greatest common divisor of 42 and 54 is 6. Hence, the

RULE.—*Resolve the given numbers into their prime factors. Multiply the common factors, and their product will be the greatest common divisor.*

WRITTEN EXERCISES.

Find the greatest common divisor of the following numbers:

1. 72 and 99. *Ans.* 9.

2. 96 and 132. *Ans.* 12.

3. 221 and 323. *Ans.* 17.

4. 576 and 276. *Ans.* 12.
5. 312 and 429. *Ans.* 39.
6. 630 and 585. *Ans.* 45.
7. 24, 36, and 60. *Ans.* 12.
8. 40, 80, and 100. *Ans.* 20.
9. 36, 144, and 162. *Ans.* 18.
10. 72, 108, 144, 180. *Ans.* 36.
11. 75, 125, 150. *Ans.* 25.
12. 63, 84, 105, 168. *Ans.* 21.

LEAST COMMON MULTIPLE.

74. **A Common Multiple** of two or more numbers is a number that can be divided by each without a remainder. Thus, 90 is a common multiple of 9 and 15.

The **Least Common Multiple** of two or more numbers is the *least* number that can be divided by each without a remainder. Thus, 15 is the least common multiple of 3 and 5.

MENTAL EXERCISES.

1. Of what numbers is 21 a multiple? 18? 25? 24? 32? 60?

2. What number is a multiple of 3? 5? 7? 11? 8?

3. What is a common multiple of 3 and 4? 4 and 5? 5 and 6? 7 and 8?

4. What is a common multiple of 2, 3, and 5? 3, 4, and 8? 2, 5, and 10? 3, 4, and 5?

5. What is the least common multiple of 2 and 6? 3 and 5? 4 and 10? 6 and 8?

A *common* multiple must contain the prime factors of each number. Thus, 90 contains 3×3, the factors of 9; and 3×5, the factors of 15.

The *least* common multiple must contain each factor only so many times as it occurs in any one number. Thus,

90 is *not* the *least* common multiple of 9 and 15, because it contains the factor 2, which is not found in 9 or 15.

EXAMPLE.—What is the least common multiple of 8, 12, and 15?

SOLUTION.—Resolve the numbers into their prime factors. The least common multiple of 8, 12, and 15 must contain 2, 2, 2, 3, and 5; or, it is the product of $2 \times 2 \times 2 \times 3 \times 5 = 120$.

CONCLUSION.—Therefore, the least common multiple of 8, 12 and 15 is 120. Hence, the

OPERATION.

$$8 = 2 \times 2 \times 2$$
$$12 = 2 \times 2 \times 3$$
$$15 = 3 \times 5$$
$$2 \times 2 \times 2 \times 3 \times 5 = 120.$$

RULE.—*Resolve the numbers into their prime factors. The product of each factor, taken as many times as it occurs in any one number, will be the least common multiple.*

EXAMPLES FOR PRACTICE.

Find the least common multiple

1. Of 12 and 20. *Ans.* 60.
2. Of 15 and 35. *Ans.* 105.
3. Of 52 and 63. *Ans.* 3276.
4. Of 70 and 84. *Ans.* 420.
5. Of 14, 18, and 28. *Ans.* 252.
6. Of 32, 48, and 80. *Ans.* 480.
7. Of 18, 36, 54, and 72. *Ans.* 216.
8. Of 25, 125, and 250. *Ans.* 250.
9. Of 15, 75, 150, and 300. *Ans.* 300.
10. Of 12, 24, 48, and 96. *Ans.* 96.
11. Of 7, 14, 21, and 35. *Ans.* 210.
12. Of 75 and 45. *Ans.* 225.
13. Of 16, 32, and 64. *Ans.* 64.
14. Of 9, 18, 27, and 108. *Ans.* 108.
15. Of 2, 3, 6, 9, and 54. *Ans.* 54.
16. Of 88, 126, and 330. *Ans.* 27720.

FRACTIONS.

MENTAL EXERCISES.

1. If an apple is cut into two *equal* pieces, what part of the apple will one piece be?

2. If a piece of chalk is broken into two *equal* pieces, what part of the whole piece will one of these pieces be?

3. How many halves in an apple? How many halves in any thing?

4. If an orange is cut into four equal pieces, what part of the orange will one piece be? Two pieces? Three pieces?

5. How many fourths in an orange? How many fourths in any thing?

6. Which would you rather have, one half or one fourth of an orange? Why?

7. If you put two fourths of an apple together, what part of an apple will you have?

8. How many fourths will equal one half? How many fourths will be more than one half an apple, and less than the whole apple?

9. If a sheet of paper is cut into three equal pieces, what part of the sheet will one piece be? Two pieces?

10. How many thirds in any thing?

11. If an orange is cut into six equal pieces, what part of the orange will one piece be? Three pieces? Five pieces?

12. How many sixths in any thing?

13. Which is the greater, one third or one sixth of an orange?

14. If you take one third of an orange and cut it into two equal parts, what part of the whole orange will each of these parts be?

15. If a unit is divided into three equal parts, what is each part called? What, then, is meant by one third?

16. What is meant by two thirds? One fourth? Two fourths? Three fourths? Five sixths?

17. What is meant by one fifth? Four fifths? Three fifths? Two fifths?

18. Which is greater, three fourths or a unit? Five thirds or a unit? Six fourths or a unit? Four fourths or a unit?

Such parts of a unit as we have been thinking about are called FRACTIONS.

They may be written as follows: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{3}{6}$, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{1}{5}$, $\frac{4}{5}$, $\frac{3}{5}$, $\frac{2}{5}$, $\frac{5}{3}$, $\frac{6}{4}$, $\frac{4}{4}$.

19. How many numbers have we taken to express a fraction, and how have they been written?

The number below the line is the number of equal parts into which the unit is divided.

The number above the line is the number of equal parts taken.

Read the following fractions, and in each case tell into how many equal parts the unit is divided, and how many parts are taken:

(*20.*)	(*21.*)	(*22.*)	(*23.*)	(*24.*)	(*25.*)
$\frac{2}{4}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{7}{12}$	$\frac{13}{25}$	$\frac{13}{33}$
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{8}{9}$	$\frac{8}{13}$	$\frac{14}{19}$	$\frac{15}{37}$
$\frac{2}{3}$	$\frac{1}{6}$	$\frac{9}{10}$	$\frac{5}{14}$	$\frac{13}{20}$	$\frac{16}{43}$

Write the following fractions in figures:

(*1.*)	(*2.*)
Three fifths.	Twenty-three fiftieths.
Eleven fifths.	Thirty-three twelfths.
Seven sixths.	Nineteen fortieths.
Three tenths.	Forty-nine thirtieths.
Five twelfths.	Thirty seventeenths.

DEFINITIONS.

75. A **Fraction** is one or more of the equal parts of a unit.

The **Denominator** of a fraction is that number which shows into how many parts a unit is divided.

The **Numerator** is the number that shows how many of these parts are taken.

The numerator and denominator are called the *terms* of a fraction.

76. A **Proper Fraction** is one whose numerator is less than its denominator; as, $\frac{1}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

An **Improper Fraction** is one whose numerator is equal to or greater than its denominator; as, $\frac{4}{4}$, $\frac{10}{7}$.

The value of a proper fraction is less than one; and the value of an improper fraction is equal to one or greater than one.

77. A **Mixed Number** is a fraction joined to a whole number; as, $5\frac{1}{2}$, $3\frac{7}{8}$.

78. **To reduce integers or mixed numbers to fractions.**

MENTAL EXERCISES.

1. How many halves in an apple? In 3 apples? In 6 apples?

2. How many thirds in an apple? In 2 apples? In 3 apples?

3. How many fourths in an orange? In 4 oranges?

SOLUTION.—In 1 orange there are 4 fourths, and in 4 oranges there are 4 times 4 fourths, which are 16 fourths. There are 16 fourths in 4 oranges.

4. How many fifths in 3 peaches? In 5 peaches? In 7 peaches?

5. How many sixths in 2 pears? In 4 pears? In 6 pears? In 7 pears?

6. How many fourths in 3? In 5? 7? 9?

7. How many sevenths in 4? In 6? 9? 11?

8. How many thirds in 4 and 1 third cakes?

SOLUTION.—In 4 cakes there are 4 times 3 thirds, which are 12 thirds, and 12 thirds and 1 third are 13 thirds. There are 13 thirds in 4 and 1 third cakes.

9. How many fourths in 3? In 3 and 3 fourths?

10. How many fifths in 3? In 3 and 2 fifths?

11. How many sixths in 2? In 2 and 5 sixths?

12. How many sevenths in 3? In 3 and 1 seventh?

Reduce the following to improper fractions:

13. $6\frac{2}{3}$.	*17.* $8\frac{3}{7}$.	*21.* $12\frac{1}{8}$.
14. $9\frac{1}{4}$.	*18.* $7\frac{4}{6}$.	*22.* $11\frac{1}{9}$.
15. $6\frac{3}{4}$.	*19.* $5\frac{3}{11}$.	*23.* $9\frac{8}{9}$.
16. $10\frac{1}{5}$.	*20.* $10\frac{6}{7}$.	*24.* $7\frac{6}{12}$.

WRITTEN EXERCISES.

EXAMPLE.—How many fourths in $14\frac{3}{4}$?

SOLUTION.—Since in one unit there are 4 fourths, in 14 units there are 14 times 4 fourths = 56 fourths; 56 fourths + 3 fourths = 59 fourths.

CONCLUSION.—Therefore, $14\frac{3}{4}$ equal $\frac{59}{4}$. Hence, the

OPERATION.

$$14\tfrac{3}{4} = \tfrac{59}{4}, \textit{ Ans.}$$

$$\begin{array}{r} 4 \\ \hline 56 \\ 3 \\ \hline 59 \end{array}$$

RULE.—*Multiply the whole number by the given denominator; to the product add the numerator, if any, and place the sum over the denominator.*

1. How many eighths in 13? *Ans.* $\frac{104}{8}$.

2. How many twelfths in 26? *Ans.* $\frac{312}{12}$.

3. How many fourths in 17? In $17\frac{3}{4}$? *Ans.* $\frac{68}{4}$; $\frac{71}{4}$.

4. How many halves in 42? In $19\frac{1}{2}$? *Ans.* $\frac{84}{2}$; $\frac{39}{2}$.

5. How many fifths in 898? *Ans.* $\frac{4490}{5}$.

6. How many seventeenths in 29? *Ans.* $\frac{493}{17}$.

7. How many twenty-seconds in 28? *Ans.* $\frac{616}{22}$.

8. Reduce $29\frac{3}{4}$ to an improper fraction. *Ans.* $\frac{119}{4}$.

9. Reduce 17 to a fraction whose denominator is 73.

Ans. $\frac{1241}{73}$.

10. Reduce 586 to thirteenths. *Ans.* $\frac{7618}{13}$.

11. Reduce $18\frac{56}{502}$ to an improper fraction.

Ans. $\frac{9092}{502}$.

12. Reduce $27\frac{5}{107}$ to an improper fraction.

Ans. $\frac{2894}{107}$.

13. How many thirty-thirds in 52? *Ans.* $\frac{1716}{33}$.

14. Reduce $\$51\frac{3}{19}$ to an improper fraction.

Ans. $\$\frac{972}{19}$.

15. Reduce $28\frac{32}{279}$ to an improper fraction.

Ans. $\frac{7844}{279}$.

16. How many seventy-seconds in 58? *Ans.* $\frac{4176}{72}$.

17. Reduce 54 to an improper fraction whose denominator is 65. *Ans.* $\frac{3510}{65}$.

18. Reduce $49\frac{17}{100}$ to an improper fraction.

Ans. $\frac{4917}{100}$.

19. Reduce $83\frac{7}{200}$ to an improper fraction.

Ans. $\frac{16607}{200}$.

20. Reduce $155\frac{3}{100}$ to an improper fraction.

Ans. $\frac{15503}{100}$.

21. Reduce $111\frac{1}{9}$ to an improper fraction.

Ans. $\frac{1000}{9}$.

22. Reduce $5\frac{1}{108}$ to an improper fraction. *Ans.* $\frac{541}{108}$.

23. Reduce 73 to a fraction whose denominator is 50.

Ans. $\frac{3650}{50}$.

24. Reduce 85 to a fraction whose denominator is 100.

Ans. $\frac{8500}{100}$.

25. Reduce $116\frac{9}{29}$ to an improper fraction.

Ans. $\frac{3373}{29}$.

79. **To reduce an improper fraction to an integer or mixed number.**

MENTAL EXERCISES.

1. How many oranges in 8 half oranges? In 6 half oranges?

2. How many apples in 10 half apples? In 14 half apples?

3. How many weeks in 7 half weeks?

SOLUTION.—In 7 half weeks there are as many weeks as 2 half weeks are contained times in 7 half weeks, which are $3\frac{1}{2}$ times. There are $3\frac{1}{2}$ weeks in 7 half weeks.

4. How many yards in 11 eighths of a yard? In 13 eighths of a yard? In 21 eighths of a yard?

5. How many yards in 21 fourths of a yard? In 17 fourths of a yard?

6. How many ones in 25 fifths? In 32 fifths?

Reduce the following improper fractions to integers or mixed numbers:

7. $\frac{24}{5}$.	*12.* $\frac{29}{9}$.	*17.* $\frac{39}{4}$.
8. $\frac{31}{6}$.	*13.* $\frac{35}{8}$.	*18.* $\frac{49}{5}$.
9. $\frac{29}{7}$.	*14.* $\frac{37}{6}$.	*19.* $\frac{73}{6}$.
10. $\frac{30}{5}$.	*15.* $\frac{36}{9}$.	*20.* $\frac{84}{12}$.
11. $\frac{32}{8}$.	*16.* $\frac{33}{10}$.	*21.* $\frac{108}{6}$.

WRITTEN EXERCISES.

EXAMPLE.—How many units in $\frac{291}{8}$?

SOLUTION.—Since $\frac{1}{8}$ equal one unit, $\frac{291}{8}$ equal as many units as 8 is contained times in 291. $291 \div 8 = 36\frac{3}{8}$.

CONCLUSION.—Therefore, $\frac{291}{8}$ equal $36\frac{3}{8}$. Hence, the

OPERATION.

$8\overline{)291}$

$36\frac{3}{8}$, *Ans.*

RULE.—*Divide the numerator by the denominator; the quotient will be the integer or mixed number.*

Reduce to an integer or mixed number:

1. $\frac{100}{13}$.	*Ans.* $7\frac{9}{13}$.	*10.* $\frac{905}{44}$.	*Ans.* $20\frac{25}{44}$.
2. $\frac{113}{17}$.	*Ans.* $6\frac{11}{17}$.	*11.* $\frac{800}{13}$.	*Ans.* $61\frac{7}{13}$.
3. $\frac{120}{24}$.	*Ans.* 5.	*12.* $\frac{320}{41}$.	*Ans.* $7\frac{33}{41}$.
4. $\frac{146}{146}$.	*Ans.* 1.	*13.* $\frac{100000}{49}$.	*Ans.* $204\frac{4}{49}$.
5. $\frac{204}{23}$.	*Ans.* $8\frac{20}{23}$.	*14.* $\frac{83325}{9}$.	*Ans.* 925.
6. $\frac{263}{27}$.	*Ans.* $9\frac{20}{27}$.	*15.* $\frac{99990}{370}$.	*Ans.* $27\frac{9}{370}$.
7. $\frac{214}{19}$.	*Ans.* $11\frac{5}{19}$.	*16.* $\frac{83325}{19}$.	*Ans.* $438\frac{3}{19}$.
8. $\frac{295}{18}$.	*Ans.* $16\frac{7}{18}$.	*17.* $\frac{85994}{21}$.	*Ans.* $409\frac{5}{21}$.
9. $\frac{435}{28}$.	*Ans.* $15\frac{15}{28}$.	*18.* $\frac{62007}{209}$.	*Ans.* $291\frac{46}{209}$.

80. **To reduce a fraction to higher terms.**

1. If you have an apple and cut it into 2 pieces, what part of an apple will each piece be?

2. If each of these halves is cut into 2 pieces, what part of an apple will each of those smaller pieces be?

3. How many fourths of an apple in 1 half? In 2 halves?

4. If each of these fourths is cut into 2 pieces, what part of the apple will 1 piece be?

5. How many eighths in 1 fourth? In 2 fourths? In 3 fourths?

6. How many sixths in 1 third? In 2 thirds?

7. How many ninths in 1 third? In 2 thirds?

SOLUTION.—1 third is 3 ninths, and 2 thirds are 2 times 3 ninths, which is 6 ninths.

8. How many tenths in $\frac{2}{5}$? $\frac{4}{5}$? $\frac{6}{5}$? $\frac{5}{5}$?

9. How many fourteenths in $\frac{2}{7}$? $\frac{5}{7}$? $\frac{3}{7}$? $\frac{6}{7}$? $\frac{4}{7}$?

10. Change $\frac{1}{2}$ and $\frac{1}{3}$ each to sixths.

11. Change $\frac{1}{4}$ and $\frac{1}{3}$ each to twelfths.

12. Change $\frac{3}{5}$ and $\frac{3}{4}$ each to twentieths.

13. Reduce $\frac{1}{8}$, $\frac{5}{6}$, $\frac{7}{12}$ to twenty-fourths.

14. Reduce $\frac{2}{5}$ and $\frac{3}{10}$ to thirtieths.

NOTE TO TEACHERS.—Having shown, by cutting each half of an apple into 2 parts, that $\frac{2}{4}$ has the same value as $\frac{1}{2}$, and $\frac{4}{4}$ the same as 2 halves, that $\frac{1}{3} = \frac{2}{6}$, etc., the pupils can be led to see

PRINCIPLE.—*The value of a fraction is not changed by multiplying or dividing both its terms by the same number.*

WRITTEN EXERCISES.

Reduce $\frac{3}{10}$ to fortieths.

SOLUTION.—40 divided by 10 is 4. Multiplying both terms of $\frac{3}{10}$ by 4, the result is $\frac{12}{40}$. Hence, the

OPERATION.

$$40 \div 10 = 4$$

$$\frac{3 \times 4}{10 \times 4} = \frac{12}{40}, \textit{ Ans.}$$

RULE.—*Divide the required denominator by the denominator of the given fraction.*

Multiply both terms of the fraction by the quotient; the result will be the required fraction.

1. Change $\frac{17}{24}$ to forty-eighths. *Ans.* $\frac{34}{48}$.
2. Change $\frac{17}{39}$ to seventy-eighths. *Ans.* $\frac{34}{78}$.
3. Change $\frac{23}{24}$ to ninety-sixths. *Ans.* $\frac{92}{96}$.
4. Change $\frac{5}{21}$ and $\frac{13}{42}$ each to eighty-fourths.
   Ans. $\frac{20}{84}$, $\frac{26}{84}$.
5. Change $\frac{3}{8}$, $\frac{11}{12}$, $\frac{23}{24}$ each to forty-eighths.
   Ans. $\frac{18}{48}$, $\frac{44}{48}$, $\frac{46}{48}$.
6. Change $\frac{9}{10}$, $\frac{9}{20}$, $\frac{9}{50}$ each to hundredths.
   Ans. $\frac{90}{100}$, $\frac{45}{100}$, $\frac{18}{100}$.
7. Reduce $\frac{22}{43}$ to a fraction whose denominator is 215.
   Ans. $\frac{110}{215}$.
8. Reduce $\frac{35}{41}$ to a fraction whose denominator is 246.
   Ans. $\frac{210}{246}$.
9. Reduce $\frac{77}{81}$ to a fraction whose denominator is 567.
   Ans. $\frac{539}{567}$.
10. Reduce $\frac{15}{16}$ to a fraction whose denominator is 256.
    Ans. $\frac{240}{256}$.

81. To reduce a fraction to lower terms.

1. If we take 2 fourths of an apple and put them together, what part of an apple shall we have?

2. How many sixths shall we put together to have 1 half?

3. How many eighths shall we put together to have 1 half?

4. How many sixths will equal 1 third? How many thirds will 4 sixths equal?

5. How many halves in 2 fourths? How many halves in 3 sixths?

6. How many halves in 4 eighths? How many fourths in 4 eighths?

7. How many fifths in $\frac{2}{10}$? $\frac{6}{10}$? $\frac{8}{10}$? $\frac{12}{10}$? $\frac{18}{10}$?

8. How many sevenths in $\frac{6}{21}$? $\frac{12}{21}$? $\frac{24}{21}$? $\frac{9}{21}$? $\frac{18}{21}$?

9. How many sixths in $\frac{12}{36}$? $\frac{18}{36}$? $\frac{30}{36}$? $\frac{24}{36}$? $\frac{60}{36}$?

10. Reduce $\frac{30}{100}$, $\frac{10}{100}$, $\frac{70}{100}$ each to tenths.

A fraction is in its *lowest terms* when no integer except 1 will exactly divide both numerator and denominator.

WRITTEN EXERCISES.

Reduce $\frac{24}{40}$ to its lowest terms.

SOLUTION.—Since the value of a fraction is not changed by dividing both terms by the same number, divide the numerator and denominator by their common factors. Dividing the terms of $\frac{24}{40}$ by 2 three times in succession, or dividing by 8, we have $\frac{3}{5}$, which gives us a fraction in its lowest terms, since no integer except 1 will exactly divide 3 and 5.

OPERATION.

$$\frac{24 \div 2}{40 \div 2} = \frac{12}{20}$$

$$\frac{12 \div 2}{20 \div 2} = \frac{6}{10}$$

$$\frac{6 \div 2}{10 \div 2} = \frac{3}{5}$$

Or,

$$\frac{24 \div 8}{40 \div 8} = \frac{3}{5}$$

RULE.—*Divide the terms of the fraction by any common factor; divide the resulting fractions in like manner until no number greater than 1 will exactly divide both terms.*

1. Reduce $\frac{375}{150}$ to halves. *Ans.* $\frac{5}{2}$.
2. Reduce $\frac{120}{160}$ to fourths. *Ans.* $\frac{3}{4}$.
3. Reduce $\frac{45}{75}$ to fifths. *Ans.* $\frac{3}{5}$.
4. Reduce $\frac{49}{70}$ to its lowest terms. *Ans.* $\frac{7}{10}$.
5. Reduce $\frac{39}{42}$ to its lowest terms. *Ans.* $\frac{13}{14}$.
6. Reduce $\frac{522}{126}$ to its lowest terms. *Ans.* $\frac{26}{63}$.
7. Reduce $\frac{48}{68}$ to its lowest terms. *Ans.* $\frac{12}{17}$.
8. Reduce $\frac{49}{161}$ to its lowest terms. *Ans.* $\frac{7}{23}$.
9. Reduce $\frac{105}{135}$ to its lowest terms. *Ans.* $\frac{7}{9}$.
10. Reduce $\frac{1442}{3752}$ to its lowest terms. *Ans.* $\frac{12}{31}$.
11. Reduce $\frac{1822}{2880}$ to its lowest terms. *Ans.* $\frac{13}{20}$.
12. Reduce $\frac{1006}{1906}$ to its lowest terms. *Ans.* $\frac{25}{49}$.
13. Reduce $\frac{2221}{3223}$ to its lowest terms. *Ans.* $\frac{13}{19}$.
14. Reduce $\frac{2225}{3225}$ to its lowest terms. *Ans.* $\frac{9}{13}$.
15. Reduce $\frac{2455}{3755}$ to its lowest terms. *Ans.* $\frac{49}{75}$.
16. Reduce $\frac{2311}{3411}$ to its lowest terms. *Ans.* $\frac{21}{31}$.
17. Reduce $\frac{180}{360}$ to its lowest terms. *Ans.* $\frac{1}{2}$.
18. Reduce $\frac{360}{480}$ to its lowest terms. *Ans.* $\frac{3}{4}$.
19. Reduce $\frac{104}{136}$ to its lowest terms. *Ans.* $\frac{13}{17}$.
20. Reduce $\frac{9801}{24001}$ to its lowest terms. *Ans.* $\frac{20}{49}$.

82. To reduce fractions to the least common denominator.

Several fractions having the same denominator are said to have a *common denominator*.

When this common denominator is the smallest denominator these fractions can have, it is their *least common denominator*.

MENTAL EXERCISES.

1. Change $\frac{1}{2}$ and $\frac{2}{3}$ each to sixths.
2. Change $\frac{1}{4}$ and $\frac{2}{3}$ each to twelfths.
3. Change $\frac{5}{6}$ and $\frac{3}{4}$ each to twelfths.
4. Change $\frac{1}{3}$ and $\frac{2}{5}$ each to fifteenths.
5. Change $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{9}$ each to eighteenths.
6. Change $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{4}{9}$ each to eighteenths.

7. Reduce $\frac{1}{2}$, $\frac{2}{7}$, and $\frac{3}{14}$ to the least common denominator.

8. Reduce $\frac{1}{3}$ and $\frac{3}{10}$ to the least common denominator.

WRITTEN EXERCISES.

Reduce $\frac{3}{4}$, $\frac{5}{6}$, $\frac{2}{9}$, $\frac{5}{12}$ to their least common denominator.

OPERATION.

$3 \mid 4,\ 6,\ 9,\ 12$

$3 \quad 4$

$3 \times 3 \times 4 = 36$

$36 \div 4 = 9 \qquad 36 \div 6 = 6$

$\frac{3 \times 9}{4 \times 9} = \frac{27}{36} \qquad \frac{5 \times 6}{6 \times 6} = \frac{30}{36}$

$36 \div 9 = 4 \qquad 36 \div 12 = 3$

$\frac{2 \times 4}{9 \times 4} = \frac{8}{36} \qquad \frac{5 \times 3}{12 \times 3} = \frac{15}{36}$

$\frac{27}{36}$, $\frac{30}{36}$, $\frac{8}{36}$, $\frac{15}{36}$, *Ans.*

SOLUTION.—The least common denominator of several fractions is the least common multiple of all the denominators. The L. C. M. of 4, 6, 9, and 12 is 36. Each fraction, then, must be reduced to thirty-sixths according to the method learned for reducing a fraction to higher terms. Hence, the

RULE.—*Find the least common multiple of the fractions for their least common denominator;*

Reduce each fraction to an equivalent fraction having this denominator.

NOTE.—Each fraction should be reduced to its lowest terms before beginning the operation, and all mixed numbers to improper fractions.

Reduce to their least common denominator:

1. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{4}$. *Ans.* $\frac{6}{12}$, $\frac{4}{12}$, $\frac{9}{12}$.

2. $\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{9}$. *Ans.* $\frac{12}{18}$, $\frac{3}{18}$, $\frac{10}{18}$.

3. $\frac{3}{2}$, $\frac{1}{5}$, $\frac{3}{4}$. *Ans.* $\frac{30}{20}$, $\frac{4}{20}$, $\frac{15}{20}$.

4. $\frac{3}{8}$, $\frac{5}{12}$, $\frac{5}{6}$. *Ans.* $\frac{9}{24}$, $\frac{10}{24}$, $\frac{20}{24}$.

5. $\frac{3}{7}$, $\frac{1}{4}$, $\frac{3}{14}$. *Ans.* $\frac{12}{28}$, $\frac{7}{28}$, $\frac{6}{28}$.

6. $\frac{7}{12}$, $\frac{1}{18}$, $\frac{5}{6}$. *Ans.* $\frac{21}{36}$, $\frac{2}{36}$, $\frac{30}{36}$.

7. $\frac{2}{9}$, $\frac{3}{5}$, $\frac{1}{8}$, $\frac{7}{10}$. *Ans.* $\frac{80}{360}$, $\frac{216}{360}$, $\frac{45}{360}$, $\frac{252}{360}$.

8. $\frac{1}{11}$, $\frac{1}{2}$, $\frac{3}{22}$, $\frac{1}{4}$. *Ans.* $\frac{4}{44}$, $\frac{22}{44}$, $\frac{6}{44}$, $\frac{11}{44}$.

9. $1\frac{3}{26}$, $\frac{3}{6}$, $\frac{14}{28}$, $\frac{5}{10}$. *Ans.* $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$.

10. $1\frac{1}{6}$, $\frac{5}{12}$, $\frac{3}{4}$, $\frac{5}{9}$. *Ans.* $\frac{42}{36}$, $\frac{15}{36}$, $\frac{27}{36}$, $\frac{20}{36}$.

11. $3\frac{1}{2}$, $5\frac{1}{3}$, $6\frac{1}{4}$, $7\frac{1}{5}$. *Ans.* $\frac{210}{60}$, $\frac{320}{60}$, $\frac{375}{60}$, $\frac{432}{60}$.

12. $\frac{11}{20}$, $\frac{13}{40}$, $\frac{17}{60}$, $\frac{19}{80}$. *Ans.* $\frac{132}{240}$, $\frac{78}{240}$, $\frac{68}{240}$, $\frac{57}{240}$.

83. ADDITION OF FRACTIONS.

MENTAL EXERCISES.

1. I bought $\frac{1}{2}$ lb. of candy and had $\frac{1}{4}$ lb. given to me: how much candy had I then?

2. Yesterday I worked $\frac{1}{4}$ of a day and the day before $\frac{1}{6}$ of a day. What part of a day did I work in the two days?

3. Bessie bought $\frac{3}{8}$ of a yard of blue ribbon, and $\frac{1}{4}$ of a yard of white: how much ribbon did she buy?

4. How much is $\frac{1}{8} + \frac{2}{8} + \frac{5}{8}$?

5. How many sevenths in $\frac{1}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, and $\frac{5}{7}$?

6. How many tenths in $\frac{1}{10}$, $\frac{3}{10}$, and $\frac{1}{5}$?

7. How many twelfths in $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$?

WRITTEN EXERCISES.

EXAMPLE.—What is the sum of $\frac{3}{4}$ and $\frac{2}{3}$?

SOLUTION.—Since *only* quantities of the same denomination can be added, first reduce the fractions to a common denominator.

$\frac{3}{4}$ equal $\frac{9}{12}$, and $\frac{2}{3}$ equal $\frac{8}{12}$. As they are now of the same denomination, they can be added; $\frac{9}{12} + \frac{8}{12} = \frac{17}{12}$; and, reducing to a mixed number, $\frac{17}{12} = 1\frac{5}{12}$.

OPERATION.

$$\frac{3}{4} = \frac{9}{12},$$
$$\frac{2}{3} = \frac{8}{12},$$
$$\frac{9}{12} + \frac{8}{12} = \frac{17}{12};$$
$$\frac{17}{12} = 1\frac{5}{12}, \textit{Ans.}$$

CONCLUSION.—Therefore, the sum of $\frac{3}{4}$ and $\frac{2}{3}$ is $1\frac{5}{12}$. Hence, the

RULE.—*Reduce the fractions to a common denominator, add their numerators, and place the sum over the common denominator.*

NOTE.—After adding, reduce the result to its lowest terms.

1. What is the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{1}{2}$? *Ans.* $3\frac{5}{8}$.

2. What is the sum of $\frac{5}{12}$, $\frac{1}{6}$, $\frac{7}{24}$, $\frac{5}{9}$, and $\frac{1}{3}$? *Ans.* $1\frac{55}{72}$.

3. What is the sum of $\frac{5}{16}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{24}$, and $\frac{7}{8}$? *Ans.* $1\frac{47}{48}$.

4. Add $\frac{5}{18}$, $\frac{9}{18}$, $\frac{7}{18}$, $\frac{13}{18}$, $\frac{2}{18}$, $\frac{15}{18}$, $\frac{7}{18}$ *Ans.* $3\frac{2}{9}$.

5. Add $\frac{1}{2}\frac{7}{7}$, $\frac{1}{2}\frac{9}{7}$, $\frac{1}{2}\frac{5}{7}$, $\frac{1}{2}\frac{3}{7}$, $\frac{9}{2}\frac{}{7}$, $\frac{1}{2}\frac{1}{7}$, $\frac{7}{2}\frac{}{7}$, $\frac{6}{2}\frac{}{7}$, $\frac{2}{2}\frac{5}{7}$. *Ans.* $4\frac{1}{2}\frac{4}{7}$.

6. Add $\frac{2}{3}$, $\frac{2}{5}$, $\frac{1}{4}$, $\frac{7}{9}$, $\frac{7}{1}\frac{}{8}$. *Ans.* $2\frac{2}{6}\frac{3}{0}$.

7. Add $\frac{2}{3}$, $\frac{9}{1}\frac{}{4}$, $\frac{1}{2}\frac{7}{1}$, $\frac{1}{4}\frac{3}{2}$. *Ans.* $2\frac{3}{7}$.

8. Add $\frac{5}{6}$, $\frac{3}{8}$, $\frac{5}{1}\frac{}{2}$, $\frac{1}{2}\frac{9}{4}$, $\frac{3}{4}\frac{7}{8}$. *Ans.* $3\frac{3}{1}\frac{}{6}$.

9. Add $\frac{3}{1}\frac{}{0}$, $\frac{1}{3}\frac{1}{0}$, $\frac{3}{2}\frac{}{0}$, $\frac{2}{1}\frac{}{5}$, $\frac{1}{6}\frac{7}{0}$. *Ans.* $1\frac{7}{3}\frac{}{0}$.

10. Add $\frac{2}{3}$, $\frac{5}{9}$, $\frac{6}{2}\frac{}{7}$, $\frac{1}{5}\frac{3}{4}$, $\frac{3}{1}\frac{3}{0}\frac{}{8}$. *Ans.* $1\frac{1}{1}\frac{0}{0}\frac{7}{8}$.

11. Add $2\frac{1}{2}$, $3\frac{1}{4}$, $2\frac{5}{8}$.

OPERATION.

$$\begin{array}{r} 2\frac{1}{2} \quad \frac{4}{8} \\ 3\frac{1}{4} \quad \frac{2}{8} \\ 2\frac{5}{8} \quad \frac{5}{8} \\ \hline 8\frac{3}{8} \end{array}$$

SOLUTION.—The sum of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{5}{8}$ is $\frac{11}{8}$; $\frac{11}{8} = 1\frac{3}{8}$; write the $\frac{3}{8}$ under the column of fractions, and add the 1 to the column of integers. The sum of 1, 2, 3, and 2 is 8.

12. Add $\frac{9}{1}\frac{}{1}$, $3\frac{1}{2}$, $5\frac{1}{3}$. *Ans.* $9\frac{4}{6}\frac{3}{6}$.

13. Add $7\frac{1}{5}$, $9\frac{2}{7}$, $5\frac{3}{8}$. *Ans.* $21\frac{2}{2}\frac{4}{8}\frac{1}{0}$.

14. Find the sum of $7\frac{2}{3}$, $5\frac{3}{4}$, $7\frac{8}{1}\frac{}{1}$, and $5\frac{1}{2}$. *Ans.* $26\frac{8}{1}\frac{5}{3}\frac{}{2}$.

15. Find the sum of $\frac{7}{1}\frac{}{0}$, 40, $\frac{5}{9}$, $\frac{2}{3}$, and $\frac{3}{2}\frac{}{0}$. *Ans.* $42\frac{1}{1}\frac{3}{8}\frac{}{0}$.

16. From what must $13\frac{1}{7}$ be subtracted to give $102\frac{7}{9}$?

Ans. $115\frac{5}{6}\frac{8}{3}$.

17. A man worked for Mr. B. $3\frac{5}{8}$ days; for Mr. D., $4\frac{5}{6}$ days; for Mr. J., $5\frac{1}{1}\frac{}{2}$ days; for Mr. W., $5\frac{1}{4}$ days. How many days did he work for all? *Ans.* $19\frac{5}{8}$ days.

18. What is the sum of $7\frac{1}{9}$, $3\frac{1}{2}$, $3\frac{2}{3}$, and $2\frac{1}{1}\frac{1}{8}$?

Ans. $16\frac{8}{9}$.

19. Add 1, $12\frac{3}{4}$, $6\frac{2}{3}$, $9\frac{3}{7}$. *Ans.* $29\frac{7}{8}\frac{1}{4}$.

20. Add $16\frac{2}{3}$, $12\frac{3}{4}$, $8\frac{3}{5}$, $2\frac{4}{5}$. *Ans.* $40\frac{3}{6}\frac{1}{0}$.

84. SUBTRACTION OF FRACTIONS.

MENTAL EXERCISES.

1. John had $\frac{3}{4}$ of an apple, and he gave $\frac{1}{4}$ to James: how many fourths had he left?

2. How much is $\frac{3}{4}$ less $\frac{1}{4}$? $\frac{3}{4}$ less $\frac{2}{4}$? $\frac{4}{4}$ less $\frac{1}{4}$? $\frac{4}{4}$ less $\frac{3}{4}$?

3. Mary had $\frac{3}{5}$ of a dollar, and she spent $\frac{1}{5}$: what part of a dollar had she left?

4. How much is $\frac{3}{5}$ less $\frac{1}{5}$? $\frac{3}{5}$ less $\frac{2}{5}$? $\frac{5}{5}$ less $\frac{3}{5}$? $\frac{5}{5}$ less $\frac{1}{5}$? $\frac{5}{5}$ less $\frac{4}{5}$?

5. Subtract $\frac{3}{7}$ from $\frac{6}{7}$.
6. Subtract $\frac{5}{8}$ from $\frac{7}{8}$.
7. Subtract $\frac{7}{13}$ from $\frac{11}{13}$.
8. Subtract $\frac{1}{2}$ from $\frac{5}{8}$.
9. Subtract $\frac{1}{3}$ from $\frac{5}{6}$.
10. Subtract $\frac{1}{5}$ from $\frac{7}{10}$.

WRITTEN EXERCISES.

EXAMPLE.—What is the difference between $\frac{7}{9}$ and $\frac{3}{5}$?

SOLUTION.—Since *only* quantities of the same denomination can be subtracted, first reduce the fractions to a common denominator.

$\frac{7}{9} = \frac{35}{45}$, and $\frac{3}{5} = \frac{27}{45}$. As they are now of the same denomination, they can be subtracted: $\frac{35}{45} - \frac{27}{45} = \frac{8}{45}$.

OPERATION.

$\frac{7}{9} = \frac{35}{45}$,
$\frac{3}{5} = \frac{27}{45}$,
$\frac{35}{45} - \frac{27}{45} = \frac{8}{45}$, *Ans.*

CONCLUSION.—Therefore, the difference between $\frac{7}{9}$ and $\frac{3}{5}$ is $\frac{8}{45}$. Hence, the

RULE.—*Reduce the fractions to a common denominator, find the difference of their numerators, and place it over the common denominator.*

EXAMPLES.

1. $\frac{1}{2} - \frac{1}{3} =$ what? *Ans.* $\frac{1}{6}$.
2. $\frac{1}{3} - \frac{1}{4} =$ what? *Ans.* $\frac{1}{12}$.
3. $\frac{2}{3} - \frac{1}{2} =$ what? *Ans.* $\frac{1}{6}$.
4. $\frac{4}{5} - \frac{2}{3} =$ what? *Ans.* $\frac{2}{15}$.
5. $\frac{5}{6} - \frac{3}{4} =$ what? *Ans.* $\frac{1}{12}$.
6. $\frac{7}{8} - \frac{4}{5} =$ what? *Ans.* $\frac{3}{40}$.
7. $\frac{5}{16} - \frac{1}{8} =$ what? *Ans.* $\frac{3}{16}$.
8. $\frac{9}{10} - \frac{4}{5} =$ what? *Ans.* $\frac{1}{10}$.
9. $\frac{1}{7} - \frac{1}{11} =$ what? *Ans.* $\frac{4}{77}$.
10. $\frac{8}{15} - \frac{1}{12} =$ what? *Ans.* $\frac{9}{20}$.
11. $\frac{8}{21} - \frac{3}{14} =$ what? *Ans.* $\frac{1}{6}$.
12. $\frac{17}{18} - \frac{1}{9} =$ what? *Ans.* $\frac{5}{6}$.
13. $\frac{3}{5} - \frac{6}{11} =$ what? *Ans.* $\frac{3}{55}$.

14. $3\frac{2}{3} - \frac{3}{4}$ = what? *Ans.* $5\frac{9}{12}$.

15. From $5\frac{2}{9}$ subtract $1\frac{5}{6}$.

SOLUTION.—$\frac{2}{9} = \frac{4}{18}$; and $\frac{5}{6} = \frac{15}{18}$. Since $\frac{15}{18}$ can not be taken from $\frac{4}{18}$, add 1 or $\frac{18}{18}$ to the $\frac{4}{18}$, making $\frac{22}{18}$. $\frac{22}{18} - \frac{15}{18} = \frac{7}{18}$. Since 1 has been added to the minuend, 1 must be added to the subtrahend; 2 from 5 leaves 3. Therefore, $5\frac{2}{9} - 1\frac{5}{6} = 3\frac{7}{18}$.

OPERATION.

$$\begin{array}{ll} 5\frac{2}{9} & \frac{4}{18} \\ 1\frac{5}{6} & \frac{15}{18} \\ \hline 3\frac{7}{18}, & \textit{Ans.} \end{array}$$

16. From $4\frac{1}{2}$ subtract $1\frac{4}{5}$. *Ans.* $2\frac{7}{10}$.

17. From $3\frac{1}{4}$ subtract $2\frac{1}{8}$. *Ans.* $1\frac{1}{8}$.

18. From $12\frac{1}{6}$ subtract $6\frac{1}{4}$. *Ans.* $5\frac{11}{12}$.

19. From $26\frac{1}{2}$ subtract $8\frac{3}{4}$. *Ans.* $17\frac{3}{4}$.

20. From 19 subtract $9\frac{5}{9}$. *Ans.* $9\frac{4}{9}$.

21. A tailor bought a piece of cloth measuring $36\frac{3}{8}$ yd. After he had shrunk the cloth it measured $34\frac{1}{3}$ yd. How much had it shrunk? *Ans.* $2\frac{1}{24}$ yd.

22. I bought a barrel of flour weighing $218\frac{7}{16}$ lb. The weight of the barrel alone was $22\frac{3}{4}$ lb. How much did the flour weigh? *Ans.* $195\frac{11}{16}$ lb.

23. A man had 2 ten-dollar bills, 3 five-dollar bills, and 5 one-dollar bills; he spent $\$10\frac{3}{4}$: how much money had he left? *Ans.* $\$29\frac{1}{4}$.

24. A farmer had $123\frac{1}{3}$ acres of land; he bought $37\frac{1}{4}$ acres, and gave his son $49\frac{3}{8}$ acres: how many acres had he left? *Ans.* $111\frac{5}{24}$ acres.

25. The sum of two mixed numbers is $55\frac{3}{5}$, and one of the numbers is $26\frac{5}{7}$: what is the other? *Ans.* $28\frac{31}{35}$.

MULTIPLICATION OF FRACTIONS.

85. To multiply a fraction by an integer.

MENTAL EXERCISES.

1. Two times one half yard equals what?

2. Three times one half yard equals what? Five times one fourth yard?

3. At 3 fifths of a dollar per yard, what will 3 yards of alpaca cost?

4. At $\frac{5}{8}$ of a dollar per pound, what will 2 pounds of tea cost?

5. If a man earns $\frac{1}{5}$ of a dollar per hour, how much will he earn in 8 hours?

6. If a man spends $\$\frac{3}{4}$ per day, how much will he spend in 12 days?

7. If a man pays $\$\frac{9}{10}$ for a book, how much will 10 such books cost?

8. How much is 3 times $\frac{3}{4}$? 6 times $\frac{3}{4}$? 8 times $\frac{3}{4}$?

9. If a man earns $\$2\frac{1}{2}$ per day, how much will he earn in 8 days?

10. If sugar is worth $8\frac{1}{2}$ cents per pound, how much are 7 pounds worth?

WRITTEN EXERCISES.

1. Multiply $13\frac{3}{24}$ by 8.

SOLUTION.—8 times 13 twenty-fourths are 104 twenty-fourths, or $4\frac{1}{3}$.

Or, using cancellation, since 8 is contained in 24 three times, the result is $1\frac{3}{3}$, or $4\frac{1}{3}$. Hence, the

OPERATION.

$$13\tfrac{3}{24} \times 8 = \tfrac{104}{24} = 4\tfrac{1}{3}$$

Or,

$$13\tfrac{3}{24} \times 8 = \tfrac{13 \times \overset{1}{\cancel{8}}}{\underset{3}{\cancel{24}}} = \tfrac{13}{3} = 4\tfrac{1}{3}$$

RULE.—*Multiply the numerator or divide the denominator of the fraction by the integer.*

NOTE.—In multiplying mixed numbers, multiply the integers and fractions separately, and add the results.

Multiply:

2. $12\frac{7}{5}$ by 5. *Ans.* $3\frac{2}{5}$.

3. $\frac{5}{28}$ by 4. *Ans.* $\frac{5}{7}$.

4. $12\frac{3}{7}$ by 6. *Ans.* $3\frac{5}{7}$.

5. $13\frac{7}{6}$ by 9. *Ans.* $4\frac{1}{4}$.

6. $11\frac{1}{24}$ by 12. *Ans.* $5\frac{1}{2}$.

Multiply:

7. $12\frac{7}{3}$ by 2. *Ans.* $11\frac{1}{3}$.

8. $2\frac{3}{4}$ by 4. *Ans.* 11.

9. $6\frac{5}{9}$ by 5. *Ans.* $32\frac{7}{9}$.

10. $19\frac{2}{7}$ by 6. *Ans.* $115\frac{5}{7}$.

11. $20\frac{3}{10}$ by 7. *Ans.* $142\frac{1}{10}$.

12. What is the cost of 8 yards of muslin at $9\frac{1}{2}$ cents a yard? *Ans.* 76 cents.

13. What is the cost of 7 pounds of cheese at $12\frac{1}{2}$ cents a pound? *Ans.* $87\frac{1}{2}$ cents.

14. If a man walks $2\frac{7}{8}$ miles per hour, how far will he walk in 8 hours? *Ans.* 23 miles.

15. Find the value of 64 acres of land at $\$10\frac{3}{8}$ per acre. *Ans.* \$664.

16. What will 17 tons of anthracite coal cost at $\$6\frac{4}{5}$ per ton? *Ans.* $\$115\frac{3}{5}$.

86. To multiply an integer by a fraction.

MENTAL EXERCISES.

1. Mary had 9 oranges, but gave Kate $\frac{1}{3}$ of them. How many did she give Kate?

2. Tom had 12 marbles, but lost $\frac{1}{4}$ of them. How many did he lose?

3. Rita had 15 cents, but spent $\frac{1}{3}$ of her money. How many cents did she spend?

4. How much will $\frac{1}{4}$ of a yard of ribbon cost at 16 cents a yard? How much will $\frac{3}{4}$ of a yard cost?

5. If a yard of muslin costs 10 cents, what will $\frac{3}{5}$ of a yard cost?

6. A man paid \$8 for coal, and $2\frac{1}{4}$ times as much for food and children's shoes. How much did he pay for food and shoes?

7. How much is $\frac{1}{3}$ of 21? $\frac{2}{3}$ times 21? $\frac{2}{3}$ times 24? $\frac{2}{3}$ times 18?

8. How much is $\frac{1}{4}$ of 12? $\frac{3}{4}$ of 12? $\frac{3}{4}$ times 16? $\frac{3}{4}$ times 20?

9. How much is $\frac{1}{5}$ of 15? $\frac{2}{5}$ of 15? $\frac{3}{5}$ times 15? $\frac{3}{5}$ times 20?

10. What is 5 times $3\frac{2}{5}$? 6 times $4\frac{1}{6}$?

WRITTEN EXERCISES.

1. Multiply 125 by $\frac{3}{5}$.

SOLUTION.—$\frac{3}{5}$ times 125 is 3 times $\frac{1}{5}$ of 125. Or, $\frac{3}{5}$ times 125 is $\frac{1}{5}$ of 3 times 125. The same result is obtained by cancellation.

$$125 \times \tfrac{3}{5} = \frac{\overset{25}{\cancel{125}} \times 3}{\cancel{5}} = 75.$$

OPERATION.

$5)125$

$25 \times 3 = 75$

Or,

$125 \times 3 = 375 \div 5 = 75$

RULE.—*Divide the integer by the denominator of the fraction, and multiply the quotient by the numerator. Or, multiply the integer by the numerator of the fraction, and divide the product by the denominator.*

Multiply :

2. 16 by $\frac{3}{4}$. *Ans.* 12.

3. 20 by $\frac{4}{5}$. *Ans.* 16.

4. 48 by $\frac{7}{8}$. *Ans.* 56.

5. 85 by $\frac{4}{9}$. *Ans.* $37\frac{7}{9}$.

6. 87 by $\frac{3}{2\,9}$. *Ans.* 9.

Multiply :

7. 142 by $\frac{8}{1\,1}$. *Ans.* $103\frac{3}{11}$.

8. 300 by $\frac{11}{1\,5}$. *Ans.* 220.

9. 216 by $\frac{7}{9}$. *Ans.* 168.

10. 625 by $\frac{3}{2\,5}$. *Ans.* 75.

11. 348 by $\frac{12}{2\,9}$. *Ans.* $167\frac{1}{2\,5}$

12. Multiply 225 by $5\frac{2}{9}$.

SOLUTION.—First multiply by 5, and then by $\frac{2}{9}$, and add the products.

OPERATION.

$$\begin{array}{r} 225 \\ \times\ 5\tfrac{2}{9} \\ \hline 1125 \\ 50 \\ \hline 1175 \end{array}$$

13. What will $7\frac{2}{3}$ lb. of tea cost, at 73 cents a lb.?

Ans. $\$5.59\frac{2}{3}$.

14. A railroad train travels at the rate of 38 miles per hour. How far will it go in $5\frac{2}{3}$ hours? *Ans.* $198\frac{4}{9}$ miles.

15. If a mowing machine costs \$70, and a reaper $3\frac{1}{4}$ times as much, how much does a reaper cost?

Ans. $\$227\frac{1}{2}$.

16. At \$1.25 a yard, what will $12\frac{3}{4}$ yards of cloth cost?

Ans. $\$15.93\frac{3}{4}$.

87. **To multiply a fraction by a fraction.**

MENTAL EXERCISES.

1. If $\frac{1}{2}$ of an apple is divided into 2 equal parts, what part of an apple will each of these parts be? How much is $\frac{1}{2}$ of $\frac{1}{2}$?

2. If $\frac{1}{2}$ of an inch is divided into 3 equal parts, what part of an inch will each of these parts be? How much is $\frac{1}{3}$ of $\frac{1}{2}$?

3. If $\frac{1}{4}$ of an orange is divided into 2 equal parts, what part of an orange will each part be? What is $\frac{1}{2}$ of $\frac{1}{4}$?

4. If $\frac{1}{4}$ of an orange is divided into 3 equal parts, what part of an orange will each part be? What is $\frac{1}{3}$ of $\frac{1}{4}$?

5. How much is $\frac{1}{2}$ of $\frac{1}{5}$? $\frac{1}{5}$ of $\frac{1}{2}$? $\frac{2}{5}$ of $\frac{1}{2}$?

6. How much is $\frac{1}{2}$ of $\frac{1}{6}$? $\frac{1}{6}$ of $\frac{1}{2}$? $\frac{5}{6}$ of $\frac{1}{2}$?

WRITTEN EXERCISES.

1. What is the product of $\frac{5}{7}$ by $\frac{2}{3}$?

SOLUTION.—$1 \times \frac{2}{3} = \frac{2}{3}$. $\frac{1}{7} \times \frac{2}{3}$ would then equal $\frac{1}{7}$ of $\frac{2}{3}$, or $\frac{2}{21}$. $\frac{5}{7} \times \frac{2}{3}$ would equal 5 times $\frac{2}{21}$, or $\frac{10}{21}$. The same result is obtained by multiplying the numerators together, and the denominators together. Hence, the

OPERATION.

$$\frac{5}{7} \times \frac{2}{3} = \frac{10}{21}$$

RULE—*Multiply the numerators together for the numerator of the product, and the denominators together for its denominator.*

NOTES.—1. After indicating the operation, cancel when possible.

2. When mixed numbers are to be multiplied by mixed numbers, they should first be reduced to improper fractions.

Multiply:

2. $\frac{6}{5}$ by $\frac{8}{10}$. *Ans.* $1\frac{2}{25}$.

3. $\frac{9}{7}$ by $\frac{14}{27}$. *Ans.* $\frac{2}{3}$.

4. $\frac{5}{7}$ by $\frac{3}{16}$. *Ans.* $\frac{15}{112}$.

5. $\frac{13}{11}$ by $\frac{12}{52}$. *Ans.* $2\frac{3}{4}$.

6. $\frac{16}{17}$ by $\frac{85}{96}$. *Ans.* $\frac{5}{6}$.

Multiply:

7. $\frac{50}{87}$ by $\frac{63}{125}$. *Ans.* $\frac{42}{145}$.

8. $\frac{7}{12}$ by $\frac{16}{21}$. *Ans.* $\frac{4}{9}$.

9. $\frac{4}{17}$ by $\frac{85}{19}$. *Ans.* $1\frac{1}{19}$.

10. $2\frac{1}{3}$ by $4\frac{1}{5}$. *Ans.* $9\frac{4}{5}$.

11. $6\frac{1}{2}$ by $7\frac{1}{3}$. *Ans.* $47\frac{2}{3}$.

12. What will $8\frac{3}{4}$ pounds of rice cost, at $3\frac{2}{3}$ ct. a pound? *Ans.* $32\frac{1}{12}$ ct.

13. Find the value of $\frac{7}{11} \times \frac{2}{3} \times \frac{8}{21} \times \frac{33}{35}$. *Ans.* $\frac{16}{105}$.

14. What will $2\frac{1}{4}$ tons of hay cost, at $\$16\frac{1}{2}$ a ton?

Ans. $\$37\frac{1}{8}$.

15. What will $7\frac{3}{4}$ cords of wood cost, at $\$8\frac{3}{4}$ a cord?

Ans. $\$67\frac{13}{16}$.

16. What number divided by $9\frac{3}{5}$ will give $17\frac{1}{4}$ for a quotient? *Ans.* $165\frac{3}{5}$.

DIVISION OF FRACTIONS.

88. To divide a fraction by an integer.

1. If I divide $\frac{3}{4}$ of an apple equally among 3 pupils, what part of the apple will each pupil receive? How much is $\frac{3}{4} \div 3$?

2. A father divided $\frac{4}{5}$ of a dollar equally among his 4 children: what part of a dollar did each child receive? How much is $\frac{4}{5} \div 4$?

3. How much is $\frac{2}{5} \div 2$? $\frac{3}{5} \div 3$? $\frac{5}{5} \div 5$?

4. If I divide $\frac{1}{5}$ of a dollar equally between 2 children, what part of a dollar will each receive? How much is $\frac{1}{5} \div 2$?

5. How much is $\frac{1}{5} \div 3$? $\frac{1}{5} \div 4$? $\frac{1}{5} \div 5$?

6. How much is $\frac{2}{5} \div 3$? $\frac{2}{5} \div 4$? $\frac{2}{5} \div 5$?

7. If $\frac{1}{3}$ of an orange is divided equally among 3 boys, what part of an orange will each boy receive? How much is $\frac{1}{3}$ divided by 3?

8. How much is $\frac{2}{3} \div 3$? $\frac{1}{3} \div 2$? $\frac{2}{3} \div 2$?

9. How much is $\frac{3}{7} \div 3$? $\frac{3}{7} \div 2$? $\frac{5}{7} \div 5$? $\frac{1}{7} \div 2$?

10. How much is $\frac{3}{8} \div 3$? $\frac{1}{8} \div 3$? $\frac{5}{8} \div 5$? $\frac{1}{8} \div 5$? $\frac{3}{8} \div 5$?

PRINCIPLE.—*A fraction is divided by dividing the numerator or by multiplying the denominator.*

WRITTEN EXERCISES.

1. Divide $\frac{7}{11}$ by 7.

SOLUTION.—Since dividing the numerator of a fraction divides the fraction, $\frac{7}{11}$ may be divided by 7, by dividing the numerator by 7. Hence the result is $\frac{1}{11}$.

OPERATION.

$\frac{7}{11} \div 7 = \frac{1}{11}$

2. Divide $\frac{6}{7}$ by 5.

SOLUTION. — Since multiplying the denominator divides the fraction, the fraction $\frac{6}{7}$ may be divided by 5, by multiplying the denominator by 5. Hence, the

OPERATION.

$\frac{6}{7} \div 5$ is $\frac{1}{5}$ of $\frac{6}{7}$, which is $\frac{6}{5 \times 7}$, or $\frac{6}{35}$.

RULE.—*Divide the numerator or multiply the denominator of the fraction by the given integer.*

Divide :		Divide :	
3. $\frac{15}{17}$ by 5.	*Ans.* $\frac{3}{17}$.	*7.* $\frac{9}{5}$ by 6.	*Ans.* $\frac{3}{10}$.
4. $\frac{16}{19}$ by 4.	*Ans.* $\frac{4}{19}$.	*8.* $\frac{3}{16}$ by 4	*Ans.* $\frac{3}{64}$.
5. $\frac{12}{13}$ by 6.	*Ans* $\frac{2}{13}$.	*9.* $\frac{85}{97}$ by 17.	*Ans.* $\frac{5}{97}$.
6. $\frac{5}{8}$ by 8.	*Ans.* $\frac{5}{64}$.	*10.* $\frac{39}{51}$ by 13.	*Ans.* $\frac{1}{17}$.

11. Divide $13\frac{1}{3}$ by 4.

SOLUTION.—4 is contained in $13\frac{1}{3}$ 3 times, with a remainder of $1\frac{1}{3}$, or $\frac{4}{3}$; but $\frac{4}{3}$ divided by 4 equals $\frac{1}{3}$. Therefore, $13\frac{1}{3}$ divided by 4 equals $3\frac{1}{3}$. Hence, the

OPERATION.

$4 \overline{)\, 13\frac{1}{3}}$

$3\frac{1}{3}$

RULE.—*Divide the whole number, and then the remaining fraction by the given integer.*

Divide :		Divide :	
12. $18\frac{4}{7}$ by 6.	*Ans.* $3\frac{2}{21}$.	*15.* $27\frac{3}{8}$ by 9.	*Ans.* $3\frac{1}{24}$.
13. $12\frac{2}{5}$ by 4.	*Ans.* $3\frac{1}{10}$.	*16.* $16\frac{2}{3}$ by 5.	*Ans.* $3\frac{1}{3}$.
14. $15\frac{5}{7}$ by 5.	*Ans.* $3\frac{1}{7}$.	*17.* $25\frac{3}{5}$ by 6.	*Ans.* $4\frac{4}{15}$.

18. If a man walks $32\frac{2}{5}$ miles in 8 hours, how many miles can he walk in 1 hour? *Ans.* $4\frac{1}{20}$ miles.

19. A man earned $\$20\frac{1}{4}$ in 9 days. How much did he earn each day? *Ans.* $\$2\frac{1}{4}$.

20. A woman paid $82\frac{1}{2}$ cents for 5 pounds of butter: how much did she pay per pound? *Ans.* $16\frac{1}{2}$ cents.

89. **To divide an integer by a fraction.**

MENTAL EXERCISES.

1. How many halves are there in 1 dollar? If a concert ticket costs $\$\frac{1}{2}$, how many tickets can be bought for $1? For $2?

2. If a peck of apples costs $\$\frac{1}{4}$, how many pecks can be bought for $1? For $2?

3. If a pound of butter costs $\$\frac{1}{3}$, how many pounds can be bought for $1? For $3?

4. If a man earns $\$\frac{1}{5}$ in one hour, in how many hours can he earn $1? How long will it take him to earn $2?

5. How many times $\frac{2}{3}$ in 2? $\frac{2}{3}$ in 4?

6. How many times $\frac{1}{8}$ in 1? $\frac{1}{8}$ in 2? $\frac{3}{8}$ in 1?

7. Divide 10 by $\frac{1}{5}$; 10 by $\frac{1}{6}$; 10 by $\frac{1}{7}$.

8. Divide 6 by $\frac{1}{4}$; 6 by $\frac{3}{4}$; 6 by $\frac{5}{4}$.

WRITTEN EXERCISES.

1. Divide 12 by $\frac{5}{6}$.

SOLUTION. — $\frac{1}{6}$ is contained in 1, six times; in 12 it is contained 12 times 6, or 72 times; and $\frac{5}{6}$ is contained in 12 $\frac{1}{5}$ of 72 times, or $14\frac{2}{5}$ times. Hence, the

OPERATION.

$$12 \div \tfrac{5}{6} = \frac{12 \times 6}{5} = 14\tfrac{2}{5}$$

RULE.—*Multiply the integer by the denominator of the fraction, and divide the product by the numerator.*

NOTES.—1. When possible, use cancellation.

2. When the divisor is a mixed number, reduce it to an improper fraction.

Divide:		Divide:	
2. 4 by $\frac{3}{4}$.	*Ans.* $5\frac{1}{3}$.	*6.* 36 by $\frac{4}{9}$.	*Ans.* 81.
3. 29 by $\frac{2}{5}$.	*Ans.* $72\frac{1}{2}$.	*7.* 42 by $\frac{6}{7}$.	*Ans.* 49.
4. 36 by $\frac{7}{8}$.	*Ans.* $41\frac{1}{7}$.	*8.* 27 by $\frac{9}{10}$.	*Ans.* 30.
5. 46 by $\frac{9}{10}$.	*Ans.* $51\frac{1}{9}$.	*9.* 72 by $\frac{9}{13}$.	*Ans.* 104.

10. How many times is $6\frac{1}{2}$ contained in 28?
Ans. $4\frac{4}{13}$.

11. How many times is $8\frac{2}{3}$ contained in 60?
Ans. $6\frac{12}{13}$.

12. How many times is $3\frac{1}{5}$ contained in 48? *Ans.* 15.

13. How many times is $3\frac{5}{6}$ contained in 17?
Ans. $4\frac{6}{23}$.

14. Divide 21 by $20\frac{1}{2}$. *Ans.* $1\frac{1}{41}$.

15. Divide 15 by $4\frac{2}{7}$. *Ans.* $3\frac{1}{2}$.

16. Divide 83 by $9\frac{3}{5}$. *Ans.* $8\frac{31}{48}$.

17. When hay is worth $\$9\frac{3}{4}$ per ton, how many tons can be bought for \$78? *Ans.* 8 tons.

18. How many cords of wood at $\$3\frac{7}{8}$ per cord, can be bought for \$31? *Ans.* 8 cords.

19. At $37\frac{1}{2}$ cents per pair, how many pairs of stockings can be bought for \$3.00? *Ans.* 8 pairs.

90. To divide a fraction by a fraction.

MENTAL EXERCISES.

1. I have $\$\frac{3}{4}$. To how many children can I give $\$\frac{1}{4}$ each?

2. A little girl has $\frac{7}{8}$ of a yard of lawn for dolls' dresses. If each dress requires $\frac{1}{8}$ of a yard to make it, how many dresses can she make?

3. Into how many pieces, each $\frac{3}{8}$ of a yard long, can a ribbon $\frac{21}{8}$ of a yard long be cut?

4. How many times $\frac{1}{3}$ in $\frac{2}{3}$? $\frac{1}{3}$ in $\frac{4}{3}$? $\frac{1}{3}$ in $\frac{5}{3}$?

5. How many times $\frac{1}{5}$ in $\frac{2}{5}$? $\frac{1}{5}$ in $\frac{3}{5}$? $\frac{1}{5}$ in $\frac{4}{5}$?

6. How many times $\frac{2}{5}$ in $\frac{4}{5}$? $\frac{2}{5}$ in $\frac{6}{5}$? $\frac{2}{5}$ in $\frac{14}{5}$?

7. How many times $\frac{5}{6}$ in $\frac{10}{6}$? $\frac{5}{6}$ in $\frac{15}{6}$? $\frac{5}{6}$ in $\frac{25}{6}$?

8. How many times $\frac{1}{6}$ in $\frac{1}{3}$? $\frac{1}{6}$ in $\frac{2}{3}$? $\frac{1}{6}$ in $\frac{4}{3}$?

SOLUTION.—Since in $\frac{1}{3}$ there are two sixths, $\frac{1}{6}$ is contained in $\frac{1}{3}$ twice.

9. How many times $\frac{1}{4}$ in $\frac{1}{2}$? $\frac{1}{4}$ in $\frac{3}{2}$? $\frac{3}{4}$ in $\frac{5}{2}$?

10. How many times $\frac{1}{10}$ in $\frac{1}{5}$? $\frac{3}{10}$ in $\frac{3}{5}$? $\frac{7}{10}$ in $\frac{7}{5}$?

WRITTEN EXERCISES.

1. Divide $\frac{3}{4}$ by $\frac{2}{3}$.

SOLUTION.—Since $\frac{3}{4} = \frac{9}{12}$, and $\frac{2}{3} = \frac{8}{12}$, we have $\frac{9}{12} \div \frac{8}{12} = \frac{9}{8}$, or $1\frac{1}{8}$. It is, therefore, plain that inverting the divisor and proceeding as in multiplication is simply a *short method* of reducing the fractions to a common denominator, and then dividing the numerator of the dividend by the numerator of the divisor. Hence, the

OPERATION.

$$\frac{3}{4} \div \frac{2}{3} = \frac{3 \times 3}{4 \times 2} = \frac{9}{8} = 1\frac{1}{8}$$

RULE.—*Multiply the dividend by the divisor with its terms inverted.*

NOTES.—1. When possible, use cancellation.

2. Reduce mixed numbers to improper fractions.

Divide:		Divide:	
2. $\frac{4}{7}$ by $\frac{2}{3}$.	*Ans.* $\frac{6}{7}$.	*7.* $\frac{11}{13}$ by $\frac{13}{11}$.	*Ans.* $1\frac{21}{169}$.
3. $\frac{5}{9}$ by $\frac{1}{5}$.	*Ans.* $2\frac{7}{9}$.	*8.* $\frac{21}{25}$ by $\frac{6}{10}$.	*Ans.* $1\frac{2}{5}$.
4. $\frac{5}{12}$ by $\frac{3}{4}$.	*Ans.* $\frac{5}{9}$.	*9.* $\frac{31}{40}$ by $\frac{5}{8}$.	*Ans.* $1\frac{6}{25}$.
5. $\frac{7}{16}$ by $\frac{8}{9}$.	*Ans.* $\frac{63}{128}$.	*10.* $4\frac{1}{5}$ by $\frac{2}{3}$.	*Ans.* $6\frac{3}{10}$.
6. $1\frac{2}{15}$ by $\frac{6}{9}$.	*Ans.* $1\frac{1}{5}$.	*11.* $8\frac{1}{6}$ by $1\frac{1}{4}$.	*Ans.* $6\frac{8}{15}$.

12. How many bottles, each containing $2\frac{1}{4}$ pints, may be filled from a vessel containing $166\frac{1}{2}$ pints?

Ans. 74 bottles.

13. How many yards of cloth, worth $\$2\frac{5}{6}$ per yard, may be bought for $\$61\frac{5}{8}$? *Ans.* $21\frac{3}{4}$ yards.

14. At $\$6\frac{1}{4}$ a barrel, how many barrels of flour can be bought for $\$112\frac{1}{2}$? *Ans.* 18 barrels.

91. To find a number when a fractional part of it is given.

MENTAL EXERCISES.

1. \$3 is $\frac{1}{3}$ of how many dollars? 4 cents is $\frac{1}{2}$ of how many cents?

2. A woman spent \$3 for shoes, which was $\frac{1}{5}$ of all the money she had. How much money had she?

3. A boy earned \$5 per week, which was $\frac{1}{4}$ as much as his father earned. How much did his father earn?

4. There are 15 boys in a school, which is $\frac{1}{2}$ of the number of pupils in the school? How many pupils in the school?

5. In one row there are 4 desks, which is $\frac{1}{10}$ of the number of desks in the room. How many desks in the school-room?

6. If 8 is 2 thirds of a number, what is $\frac{1}{3}$ of the number?

7. If 10 is 2 fifths of a number, what is $\frac{1}{5}$ of the number? If 5 is $\frac{1}{5}$ of the number, what is the number?

8. If 15 is 3 sevenths of a number, what is $\frac{1}{7}$ of the number? If 5 is $\frac{1}{7}$ of the number, what is the number?

9. Of what number is 4 two fifths?

10. Of what number is 9 three eighths?

WRITTEN EXERCISES.

1. Of what number is 120 eight ninths?

SOLUTION.—Since 120 is $\frac{8}{9}$ of a certain number, one ninth of that number is $\frac{1}{8}$ of 120, or 15; and since 15 is $\frac{1}{9}$ of the number, the number must be 9 times 15, or 135. Hence, 120 is $\frac{8}{9}$ of 135.

OPERATION.

$8 \overline{)\,120}$

$15 \times 9 = 135$

Find the number of which:

2. 60 is $\frac{3}{4}$. *Ans.* 80.

3. 45 is $\frac{5}{8}$. *Ans.* 72.

4. 72 is $\frac{9}{15}$. *Ans.* 120.

5. 36 is $\frac{9}{10}$. *Ans.* 40.

6. 63 is $\frac{9}{14}$. *Ans.* 98.

7. 65 is $\frac{5}{13}$. *Ans.* 169.

8. 160 is $\frac{4}{5}$. *Ans.* 200.

9. 108 is $\frac{9}{20}$. *Ans.* 240.

10. 110 is $\frac{11}{30}$. *Ans.* 300.

11. 144 is $\frac{12}{25}$. *Ans.* 300.

12. 132 is $\frac{12}{19}$. *Ans.* 209.

13. 169 is $\frac{13}{20}$. *Ans.* 260.

14. If \$50 is $\frac{5}{12}$ of my money, how much money have I? *Ans.* \$120.

15. If a man has \$8000 invested in real estate, and that is $\frac{4}{5}$ of his property, what is the value of his property? *Ans.* \$10000.

16. If $\frac{4}{7}$ of a ship is worth \$16000, what is the whole ship worth? *Ans.* \$28000.

MISCELLANEOUS EXAMPLES.

1. A merchant bought 5 pieces of cloth; the first contained $13\frac{1}{6}$ yd.; the second, $18\frac{1}{5}$ yd.; the third, $16\frac{1}{4}$ yd.; the fourth, $15\frac{3}{4}$ yd.; and the fifth, $21\frac{3}{5}$ yd.: what amount of cloth did he purchase? *Ans.* $84\frac{29}{30}$ yd.

2. A boy bought a slate for $\frac{7}{20}$ of a dollar, an arithmetic for $\frac{7}{8}$ of a dollar, a dictionary for $3\frac{1}{2}$ dollars, and a reader for $\frac{9}{50}$ of a dollar: what did he expend? *Ans.* $\$4\frac{81}{200}$.

3. A farmer divided his estate among his four sons; to the eldest son he gave $\frac{3}{8}$, to the second $\frac{5}{16}$, to the third $\frac{7}{32}$, and the remainder to the youngest son: how much did the youngest son receive? *Ans.* $\frac{3}{32}$.

4. A traveler went $\frac{1}{9}$ of a journey on foot, $\frac{2}{15}$ on horseback, $\frac{1}{4}$ by rail, and the rest by coach: what part did he go by coach? *Ans.* $\frac{91}{180}$.

5. If $\frac{1}{5}$ of a pole is blue, $\frac{3}{7}$ red, and the rest white, what part is white? *Ans.* $\frac{13}{35}$.

6. I bought $\frac{3}{4}$ of a piece of cloth, and sold $\frac{1}{4}$ of what I purchased: how much had I left? *Ans.* $\frac{9}{16}$.

7. What will $7\frac{3}{4}$ lb. of coffee cost at $26\frac{2}{3}$ ct. per lb.? *Ans.* $\$2.06\frac{2}{3}$.

8. If I walk $3\frac{5}{16}$ miles in one hour, how far can I walk in 12 hours? *Ans.* $39\frac{3}{4}$ mi.

9. At $\$\frac{3}{5}$ a bushel, how many bushels of potatoes can be bought for \$210? *Ans.* 350 bu.

10. How many words can a man write in $15\frac{1}{2}$ minutes, if he writes 159 words in 3 minutes? *Ans.* $821\frac{1}{2}$ words.

11. If the product of two fractions is $11\frac{1}{20}$, and the multiplicand is $1\frac{5}{12}$, what is the multiplier? *Ans.* $7\frac{4}{5}$.

12. If $\frac{5}{8}$ of a farm is worth \$5780, what is the value of $\frac{3}{4}$ of the farm? *Ans.* \$6936.

DECIMALS

92. The value of integers decreases from left to right tenfold. Thus, in 222 the 2 hundreds are ten times 2 tens, and the 2 tens are ten times 2 units. (See Art. 11.)

ORDERS OF DECIMALS.

The orders may be continued to the right beyond units' order by the same law of decrease.

Let the point (.) be used to separate the order of units from the order next following.

Then, in the number 2.22, since the 2 at the left of the point is 2 units, the 2 at the right is 2 tenths, and the second order to the right is 2 hundredths.

DEFINITIONS.

93. A **Decimal Fraction** is one whose denominator is 10, 100, 1000, etc., or a *unit* with one or more ciphers annexed.

The *denominator* of a decimal fraction is not usually expressed.

The *numerator* of a decimal fraction is written in such a manner as to indicate the denominator.

In decimal fractions, the unit is divided into tenths, hundredths, thousandths, etc.; and, since ten units of any place or order make one unit of the next higher order,

Decimal Fractions are written in the same manner as simple numbers.

The decimal is separated from the whole number by a period, called the *decimal point;* thus, (.).

NUMERATION TABLE.

Millions			Thousands			UNITS					Thousandths			Millionths		
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Tenths	Hundredths	Units	Tenths	Hundredths	Units	Tenths	Hundredths
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

NOTE.—The periods may be extended in both directions far beyond the numbers indicated.

94. RULE FOR NUMERATION OF DECIMALS.—*Read the decimal as a whole number, then add the name of the right-hand order.*

Thus, .325 is read three hundred and twenty-five thousandths.

Read the following

DECIMAL FRACTIONS.

.6	6.015	.145	8.462
.05	2.004	.0145	.0021
.75	.014	6.7543	.00372
4.28	.009	.1706	.4124
.14	34.072	1.002	16.240006
.014	.016	.1509	.0047534
16.307	.14	203.0107	.1743219

NOTES.—1. Mixed *decimal* numbers are read the same as other *mixed* numbers.

2. As in whole numbers, the *value* of each figure depends upon its place from units.

3. Removing a figure one place further from units, decreases its value ten times.

4. Removing a figure one place nearer units, increases its value ten times.

95. RULE FOR NOTATION OF DECIMALS.—*Write the numerator as a whole number, and prefix ciphers, if necessary, until the right-hand figure stands in the required order; then, place the decimal point at the left.*

Write the following in the form of decimals:

1. Six *tenths.* Nine *thousandths.* Eighteen *thousandths.*

2. Nine *tenths.* Fourteen *hundredths.* Twenty-nine *hundredths.*

3. Thirty-four *hundredths.* Six *thousandths.* Fourteen *thousandths.*

4. 4965 *units* and 44 *thousandths.* 1000 *units* and 43 *millionths.*

5. 145 *hundred-thousandths.* 1540 ten *millionths.*

Change the following common fractions to decimals:

$\frac{105}{10000}$	$\frac{7}{10}$	$\frac{7}{10000000}$
$\frac{75}{1000}$	$\frac{7}{100}$	$\frac{7}{100000000}$
$\frac{63}{100000}$	$\frac{7}{1000}$	$\frac{2520}{10000}$
$\frac{9}{1000000}$	$\frac{7}{10000}$	$\frac{3250790}{10000000}$
$\frac{145609}{1000000}$	$\frac{7}{100000}$	$\frac{19}{10000}$

If a decimal fraction occupies only one place, the denominator is 10; if two places, 100; three places, 1000; and so on.

The denominator of a decimal fraction is always 1 with as many ciphers annexed as there are places in the decimal.

A mixed decimal may be read as an improper fraction. Thus, 3.4 may be read as 34 tenths.

Cents and mills may be considered the decimal fractions of a dollar.

ADDITION OF DECIMALS.

EXAMPLE.—What is the sum of 15.07, 349.002, 8.15, and 16.452?

SOLUTION.—Since *only* numbers of the same denomination can be added, write figures of the same order in the same column.

Since ten units of one denomination make one unit of the next higher, add as in simple numbers, and place the decimal point between units and tenths.

OPERATION.

15.07
349.002
8.15
16.452
388.674, *Ans.*

CONCLUSION. — Therefore, the sum of 15.07, 349.002, 8.15, and 16.452 is 388.674. Hence, the

96. RULE FOR ADDITION OF DECIMAL FRACTIONS.— *Write numbers of the same order in the same column, tenths under tenths, etc., and draw a line beneath.*

Add as in simple numbers, and place the decimal point directly under the points above.

PROOF.—Same as in Addition of Simple Numbers.

EXAMPLES.

1. Add 6.5, 67.75, 999.475, 35.232, and .1234.
Ans. 1109.0804.
2. Add 75.432, 8.45, 9943.1, and 47264.
Ans. 57290.982.
3. Add 346.2, 79.476, 3.1245, and 753.
Ans. 1181.8005.
4. Add $94.37, $149.43, $17.50, $25756, and $.97.
Ans. $26018.27.

5. Add 13.005, 4234, 1.75274, .007219, 651.004212, 5.314, and 9521. *Ans.* 14426.093171.

6. Add 500369, 30.80624, 4578.70014, .345, 187.9542, and 145.12345. *Ans.* 505311.92903.

7. What is the sum of 4.74 yd., 367.54 yd., .857 yd., and 945000. yd.? *Ans.* 945373.137 yd.

8. Add 15.901, 47652, .79642, 52654.001, and .000005. *Ans.* 100322.698425.

9. Add 154.06 bu., 2479 bu., .009 bu., and 252790.195. bu. *Ans.* 255423.264 bu.

10. What is the sum of 1000 and .1000? *Ans.* 1000.1.

11. What is the sum of 10000 and .00001? *Ans.* 10000.00001.

SUBTRACTION OF DECIMALS.

EXAMPLE.—What is the difference between 4.7386 and 2.4542?

SOLUTION.—Since *only* numbers of the same denomination can be subtracted, write figures of the same order in the same column. Subtract as in simple numbers, and place the decimal point directly under the points above.

OPERATION.

4.7386
2.4542
2.2844, *Ans.*

CONCLUSION.—Therefore, the difference between 4.7386 and 2.4542 is 2.2844. Hence, the

97. RULE FOR SUBTRACTION OF DECIMAL FRACTIONS. —*Write the less number under the greater, tenths under tenths, hundredths under hundredths, etc., and draw a line beneath. Subtract as in simple numbers, and place the decimal point directly under the points above.*

PROOF.—Same as in Subtraction of Simple Numbers.

NOTE.—If there are fewer decimal places in the minuend than in the subtrahend, annex ciphers to the minuend. This will not alter

the value of the decimal, since it does not remove any figure of the minuend further from the units' place.

EXAMPLES.

1. What is the difference between 8.4726 and 5.3019? *Ans.* 3.1707.

2. Subtract 13.0695 from 38.0723. *Ans.* 25.0028.

3. Subtract .00891 from 1.1. *Ans.* 1.09109.

4. Subtract 637.52 from 1000. *Ans.* 362.48.

5. Subtract 19 units and 567 ten-thousandths from 20 units and 419 thousandths. *Ans.* 1.3623.

6. Subtract \$18.456 from \$103.05. *Ans.* \$84.594.

7. Subtract 1 ten-thousandth from 1 hundredth. *Ans.* .0099.

8. Subtract 1 unit and 11 millionths from 19 tenths. *Ans.* .899989.

9. I had 59 A. of land, from which I sold 6.75 A.: how much had I left? *Ans.* 52.25 A.

10. A man bought two pieces of cloth, one containing 67.1036 yd., the other, 34.35 yd.; he sold 71.4167 yd.: how much had he left? *Ans.* 30.0369 yd.

MULTIPLICATION OF DECIMALS.

EXAMPLE.—What is the product of 3.15 multiplied by 1.7?

SOLUTION.—Three units and 15 hundredths = 315 hundredths, or $\frac{315}{100}$; one unit and 7 tenths = 17 tenths, or $\frac{17}{10}$; $\frac{315}{100} \times \frac{17}{10} = \frac{5355}{1000} = 5.355$.

CONCLUSION.—Therefore, the product of 3.15 multiplied by 1.7 is 5.355.

OPERATION.

$$\begin{array}{r} 3.15 \\ 1.7 \\ \hline 2\ 205 \\ 3\ 15 \\ \hline 5.355, \textit{Ans.} \end{array}$$

It will be seen that the product is the numerator of a fraction having for its denominator 1, with as many

ciphers annexed as there are decimal places in both factors. Hence, the following rule:

98. Rule for Multiplication of Decimals.—*Multiply as in simple numbers: from the right of the product, point off as many figures for decimals as there are decimal places in both multiplicand and multiplier. If there are not so many places in the product, supply the deficiency by prefixing ciphers.*

Proof.—Same as in Multiplication of Simple Numbers.

EXAMPLES.

1. Multiply 42.145 by 6. *Ans.* 252.87.

2. Multiply 73.241 by 25. *Ans.* 1831.025.

3. Multiply 149.149 by 149. *Ans.* 22223.201.

4. What is the product of 34950 by .69 ? *Ans.* 24115.50.

5. What is the product of 3.45 by 1.93 ? *Ans.* 6.6585.

6. What is the product of .457 by .03 ? *Ans.* .01371.

Find the products of :

7. $17.02 \times .075$. *Ans.* 1.2765.

8. $.001 \times .1$. *Ans.* .0001.

9. $.07 \times .1256$. *Ans.* .008792.

10. $.356 \times .065$. *Ans.* .023140.

11. $42 \times .0075$. *Ans.* .3150.

12. $.195 \times .00034$. *Ans.* .0000663.

13. $28.65 \times .357$. *Ans.* 10.22805.

14. 1825×17.435. *Ans.* 31818.875.

15. $.0009 \times .0534$. *Ans.* .00004806.

16. $18.25 \times .70054$. *Ans.* 12.784855.

17. What will be the cost of 59 yards of cloth at $1.87 a yard ? *Ans.* $110.33.

18. I am 4.075 times as old as my sister; she is 4 years old: how old am I? *Ans.* 16.3 years.

19. At $.75 a bu., what is the cost of 5240 bu. of potatoes? *Ans.* $3930.

20. If one lb. of sugar costs $.125, what will 23 lb. cost? *Ans.* $2.875.

21. What is the cost of 18.25 bu. of corn at $.67 a bu.? *Ans.* $12.2275.

DIVISION OF DECIMALS.

EXAMPLE.—What is the quotient of 7.236 divided by 1.34?

SOLUTION.—Divide as in whole numbers, and, since the product of the divisor and quotient equals the dividend, the number of decimal places in the dividend must equal the number of decimal places in *both* divisor and quotient; therefore, the quotient must contain as many decimal places as the decimal places in the dividend *exceed* those in the divisor.

OPERATION.

1.34) 7.236 (5.4, *Ans.*
6 70
536
536

As there are *three* decimal places in the dividend and *two* in the divisor, the number of decimal places in the quotient must be equal to their difference, or *one*. Pointing off one place from the right of the quotient, gives 5.4.

CONCLUSION.—Therefore, the quotient of 7.236 divided by 1.34 is 5.4. Hence, the

99. RULE FOR DIVISION OF DECIMALS.—*Divide as in simple numbers, and from the right of the quotient point off as many places for decimals as the decimal places in the dividend exceed those in the divisor; if there are not so many places, supply the deficiency by prefixing ciphers.*

PROOF.—Same as in Division of Simple Numbers.

NOTES.—1. If the divisor has *more* decimals than the dividend, annex ciphers to the dividend, until its decimal places *equal* those of the divisor; the quotient will be a whole number.

2. If there is a remainder after division, annex ciphers to it, and continue to divide until there is no remainder, or the quotient is sufficiently exact. In pointing, regard the ciphers annexed as decimal places.

3. When the division can not be *exactly* performed, carry it to a *sufficient* number of decimal places, and annex the sign +.

EXAMPLES.

Find the quotients of :

1. .4÷8.	*Ans.* .05.	*13.* 32.6÷4.	*Ans.* 8.15.
2. .6÷5.	*Ans.* .12.	*14.* 188.49÷60.	*Ans.* 3.1415.
3. .4÷4.	*Ans.* .1.	*15.* 102048÷31.89.	*Ans.* 3200.
4. .04÷2.	*Ans.* .02.	*16.* 351÷.117.	*Ans.* 3000.
5. 5÷.4.	*Ans.* 12.5.	*17.* 4.87625÷25.	*Ans.* .19505.
6. 4÷.02.	*Ans.* 200.	*18.* 43÷.04.	*Ans.* 1075.
7. .48÷12.	*Ans.* .04.	*19.* .0456÷.04.	*Ans.* 1.14.
8. .84÷4.	*Ans.* .21.	*20.* 12.6÷.0012.	*Ans.* 10500.
9. .49÷7.	*Ans.* .07.	*21.* 15.63386÷4.367.	*Ans.* 3.58.
10. .16÷8.	*Ans.* .02.	*22.* 8.024÷2.006.	*Ans.* 4.
11. 99÷.11.	*Ans.* 900.	*23.* 1.6875÷6.75.	*Ans.* .25.
12. .99÷11.	*Ans.* .09.	*24.* 36÷.00036.	*Ans.* 100000.

25. If a boat sails 34.78 miles in 3.7 hours, how far does she sail in 1 hour? *Ans.* 9.4 mi.

26. If 45 barrels of apples cost $157.50, what will one barrel cost? *Ans.* $3.50.

27. How many barrels of apples at $3.50 a barrel, must I sell to receive $15.75? *Ans.* 4.5 bbl.

28. The circumference of a circle is 3.141 times the diameter. Find the diameter of a circle whose circumference is 141.345 in. *Ans.* 45 in.

29. If 46.45 yd. of lace cost $274.055, what will one yard cost? *Ans.* $5.90.

30. How many lb. of tea at $1.125 a lb. can be bought for $7.20? *Ans.* 6.4 lb.

31. A steam-engine is to run 28755 miles. At the rate of 35.5 miles an hour, how many hours will it take to go the distance? *Ans.* 810 h.

32. How many yards of silk at $1.75 a yard, can be bought for $271.25? *Ans.* 155.

33. A laborer earned $1340 in 1 year and 35 days: how much would he earn, at the same rate, in 5 years and 20 days? *Ans.* $6180.75.

REDUCTION OF DECIMALS.

To reduce a common fraction to a decimal.

EXAMPLE.—Reduce $\frac{5}{8}$ to a decimal fraction.

SOLUTION.—Reduce the numerator to tenths; 5 units = 50 tenths; $\frac{1}{8}$ of 50 tenths = 6 tenths, with 2 tenths remaining.

OPERATION.

$$8\overline{)5.000}$$
$$.625, \textit{Ans.}$$

Reduce the 2 tenths to hundredths; 2 tenths = 20 hundredths; $\frac{1}{8}$ of 20 hundredths = 2 hundredths, with 4 hundredths remaining.

Reduce the 4 hundredths to thousandths; 4 hundredths = 40 thousandths; $\frac{1}{8}$ of 40 thousandths = 5 thousandths.

6 tenths, 2 hundredths, and 5 thousandths = .625.

CONCLUSION.—Therefore, $\frac{5}{8}$ = .625. Hence, the

100. RULE FOR REDUCING A COMMON TO A DECIMAL FRACTION.—*Annex ciphers to the numerator, divide by the denominator, and from the right of the quotient point off as many places for decimals as there are ciphers annexed to the numerator.*

NOTE.—There are many common fractions that can not be reduced exactly to equivalent decimals. In such cases, the division must be carried out to three or four places, and the mark + must be used.

Reduce the following to decimal fractions:

1. $\frac{4}{5}$.	*5.* $\frac{19}{32}$.	*9.* $\frac{9}{45}$.	*13.* $\frac{8}{64}$.
2. $\frac{7}{8}$.	*6.* $\frac{4}{25}$.	*10.* $\frac{3}{15}$.	*14.* $\frac{17}{19}$.
3. $\frac{9}{20}$.	*7.* $\frac{18}{625}$.	*11.* $\frac{5}{6}$.	*15.* $\frac{18}{64}$.
4. $\frac{16}{25}$.	*8.* $\frac{10}{17}$.	*12.* $\frac{3}{22}$.	*16.* $\frac{1}{25}$.

To reduce a decimal to a common fraction.

EXAMPLE.—Reduce .35 to a common fraction.

SOLUTION.—Write the denominator under the numerator; .35 becomes $\frac{35}{100}$, and reducing to its lowest terms, it becomes $\frac{7}{20}$.

CONCLUSION.—Therefore, .35$=\frac{7}{20}$. Hence, the

OPERATION.

$.35=\frac{35}{100}$

$\frac{35}{100}=\frac{7}{20}$, *Ans.*

101. RULE FOR REDUCING A DECIMAL TO A COMMON FRACTION.—*Write the denominator under the numerator of the decimal, and reduce this common fraction to its lowest terms.*

Express the following in lowest terms of common fractions:

1. .4. *Ans.* $\frac{2}{5}$.	*5.* .375. *Ans.* $\frac{3}{8}$.	*9.* .5625. *Ans.* $\frac{9}{16}$.
2. .6. *Ans.* $\frac{3}{5}$.	*6.* .875. *Ans.* $\frac{7}{8}$.	*10.* .0016. *Ans.* $\frac{1}{625}$.
3. .25. *Ans.* $\frac{1}{4}$.	*7.* .4375. *Ans.* $\frac{7}{16}$.	*11.* .9375. *Ans.* $\frac{15}{16}$.
4. .16. *Ans.* $\frac{4}{25}$.	*8.* .04. *Ans.* $\frac{1}{25}$.	*12.* .7435. *Ans.* $\frac{1487}{2000}$.

MISCELLANEOUS EXAMPLES.

1. If a man walks 3.789 miles an hour, how far will he walk in 5 hours? *Ans.* 18.945 mi.

2. If 18.2 lb. of butter cost $6.825, what does 1 pound cost? *Ans.* $.375.

3. Add .34 yd., 1.07 ft. and 8.92 in.

Ans. 2 ft. 10 in.

4. A had 42 gallons of a mixture of alcohol and water; .15 of the whole was water: what was the cost of the alcohol at $3.42 a gallon? *Ans.* $122.094.

5. Express as a decimal $\frac{14}{15}$ of $5\frac{1}{2}$ yd. *Ans.* 5.133 + yd.

6. A grocer bought 15.4 lb. of butter at $.375 a pound, and sold the lot for $7.225: how much did he gain by the bargain? *Ans.* $1.45.

7. What is the dividend when the divisor is 38.125 and the quotient, 5.25? *Ans.* 200.15625.

8. If a man travels 34.2 miles the first day, 19.9 miles the second day, 18.87 miles the third day, and 19.7 miles the fourth day, how far does he travel altogether?

Ans. 92.67 mi.

9. What will be the cost of 12.375 gallons of wine at $3.44 a gallon? *Ans.* $42.57.

10. If a railroad train runs 27.125 miles an hour, in what time will it run 303.6 miles? *Ans.* 11 h. 11 min. +.

11. If I give 2.12 bu. of corn for 1 day's work, how much should I give for 14.5 days' work? *Ans.* 30.74 bu. How much would it be worth at $.84 a bushel?

Ans. $25.8216.

12. The product of two numbers is 6.23; one of the numbers is 124.6: what is the other number?

Ans. .05.

13. If 6.5 lb. of sugar cost 74.75 cents, what do 3.7 lb. cost? *Ans.* $42.55.

14. From .41 da. subtract .16 h.

Ans. 9 h. 40 min. 48 sec.

15. What is the cost of 9 yd. of flannel at $.40 per yard, and 12 yd. at $.75 per yard? *Ans.* $12.60.

16. Find the cost of 25 gross of slates at $.75 a dozen?

Ans. $225.

17. What is the sum of 4.1 acres, 3.72 acres, 4.82 acres, and 8.15 acres? *Ans.* 20.79 acres.

RETAIL PRICE CENTS