ELEMENTS

OF

MACHINE DESIGN

BY

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PREFACE

THIS book is the outgrowth of the experience of the authors in teaching Machine Design to engineering students in Sibley College, Cornell University. It presupposes a knowledge of Mechanism and Mechanics of Engineering. While the former subject is a logical part of Machine Design, it may be, and usually is, for convenience, treated separately and in advance of that portion of the subject which treats of the proportioning of machine parts so that they will withstand the loads applied. The same logical order is usually followed in actual designing, as it is, ordinarily, necessary and convenient to outline the mechanism before proportioning the various members.

With the mechanism determined, the remainder of the work of designing a machine consists of two distinct parts:

(a) Consideration of the energy changes in the machine, and the maximum forces resulting therefrom.

(b) Proportioning the various parts to withstand these forces.

This logical procedure, and the fundamental principles underlying the first part (a), are seldom made clear to the student, in works of this character; and such information as is given on energy transformation in machines is, in general, that relating to special cases or types. A thorough understanding of these general principles is, however, in most cases, essential to successful design, since a consideration of the machine as a *whole* necessarily precedes consideration of details. A very brief discussion of typical energy and force problems is given, therefore, in Chapter II, in the hope of making this important matter somewhat clearer to the beginner.

While the treatment presented presupposes a knowledge of Mechanics of Materials, a brief discussion of the more important straining actions is given in Chapter III, partly to make the application of the various formulæ to engineering problems somewhat more definite, and partly to present such rational theory as is of assistance in selecting working stresses and factors of safety. This discussion serves also to show why certain equations have been selected in preference to others, and also to collect in concise form the more important equations relating to stress and strain with which the designer needs to be familiar.

The general principles of lubrication and efficiency are discussed in Chapter IV. Both of these are of prime importance to the engineer; and while the discussion is necessarily brief it is believed that the fundamental principles are fully covered.

The remainder of the book is devoted to the discussion of some of the more important machine details, with a view of showing how the theoretical considerations and equations discussed in the first part of the work are applied and modified in practice. The treatise is, in no sense, a hand-book, neither is it a manual for the drafting room, but is a discussion of the fundamental principles of design, and only such practical data have been collected as are needed to verify or modify logical theory. It is hoped that the illustrative numerical examples which are introduced throughout the work may, in conjunction with the analytical methods given, suggest proper treatment of practical problems in design. The treatment of all topics is necessarily brief, as it was desired to obtain a text-book which could be conveniently covered in one college year and yet present the salient features of the subject needed by the student as a preparation and basis for more advanced work. While intended primarily for engineering students it is hoped that it may also prove of some interest to the practising designer. It has been the endeavor in the preparation of the book not only to develop rational analytical treatment, with due regard to constructive considerations and other practical limitations, but to reduce the analysis to such forms and terms that definite numerical results can be obtained in concrete problems.

Considerable of the matter contained in the book has already been published, specially for the use of students in Sibley College, under the title of "Special Topics on the Design of Machine Elements," by John H. Barr, and also in "Elements of Machine

Design," Part I, by the Authors. The writers have availed themselves freely of the work of many others in the field, for which due credit is given in the text.

The authors are especially indebted to Professor G. F. Blessing of Swarthmore College, Professors W. N. Barnard, L. A. Darling, and C. D. Albert of Sibley College, Cornell University, all of whom have given instruction in the course at various times, and also to Mr. A. J. Briggs, for many helpful suggestions and criticisms. They will be very grateful for further suggestions or criticisms which will improve the book.

D. S. K.
J. H. B.

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MACHINE DESIGN

CHAPTER I

INTRODUCTORY

1. The purpose of machinery is to transform energy obtained directly or indirectly from natural sources into useful work for human needs. Useful work involves both *motion* and *force*, hence the basis of Machine Design is the laws that govern motion and force.

The term useful work carries with it the idea of definite *motion* and definite *force*, for work itself is always of a definite or measurable character. An examination of any machine will show that its parts are so put together as to give definite *constrained* motion suitable for the work to be done. The constrainment of motion is determined by the moving parts, the stationary frame and the nature of the connections between them.

Mechanics is the science which treats of the relative motions of bodies, solid, liquid, or gaseous, and of the forces acting upon them.

Mechanics of Machinery is that portion of pure mechanics which is involved in the design, construction, and operation of machinery. It has been noted that the consideration of a machine involves *constrained* motion, hence that portion of pure mechanics is mostly needed in Machine Design which deals with stationary structures and constrained motion. While the laws of Mechanics of Machinery give us the underlying principles on which machine action rests, their practical application brings in many modifying conditions.

Machine Design therefore may be defined as the practical application of Mechanics of Machinery to the design and construction of machines.

A Mechanism is a combination of material bodies so connected that motion of any member involves definite, relative, constrained motion of the other members. A mechanism or combination of mechanisms which is constructed not only for modifying motion but also for the transmission of definite forces and for the performance of useful work is called a **machine.** A machine consists of one or more mechanisms; a *mechanism,* however, is not necessarily a machine. Many mechanisms transmit no energy except that required to overcome their own frictional resistance, and are used only to modify motion as in the case of most engineering instruments, watches, models, etc.

A brief reflection will show that the same mechanism will serve for different machines (see any treatise on Kinematics) and within limits the design of the mechanism for a given machine may usually be carried out, so far as motion is concerned, with little regard to the amount of energy to be transmitted. This, of course, does not apply to such mechanisms as centrifugal governors, or in general where inertia or other kinetic actions affect constrainment of motion. Except for the limitations of such cases as those just noted, the design of any machine may be divided into two main parts:

(1) Design of the mechanism to give the required motion.

(2) Proportioning of the parts so that they will carry the necessary loads due to transmitting the energy, without undue distortion or practical departure from the required constrained motion.

(1) The design or selection of the mechanism for a machine is governed by the manner in which the energy is supplied and the character of the work to be done; for energy may be supplied in one form of motion and the work may have to be done with quite a different one. If mechanisms already exist which will accomplish the desired result the problem is one of selection and arrangement of parts. But if a new type of machine is to be built, or a new mechanism is desired, the solution of the motion problem borders on or may indeed be of the nature of invention. While it is true that in most cases the mechanism and the relative proportions of its parts can be designed to suit the work

to be done without reference to the energy transmitted, in general it is necessary to know something about the energy transmitted before any *definite dimensions* of the parts of the mechanism can be fixed, and frequently before the nature of the mechanism is determined. Furthermore, the methods and available facilities of construction control the design to a large extent. Thus in designing a steam engine the size of the cylinder must be first fixed before the length of crank and connecting-rod can be fixed, and in general while the mechanism can be treated apart from the energy problem it is necessary to keep the latter constantly in mind.

(2) The problem of proportioning the various parts of a machine so that they will carry their loads without excessive or undue deformation may conveniently be divided into two parts:

(a) Solution as a whole, of the energy and force problem in the mechanism.

(b) Assigning of dimensions to the various parts based on the forces acting upon them.

(a) When the type and proportions of the mechanism have been fixed the relative velocity of any point in the mechanism may be found. If then the energy which the mechanism must transmit is known, it is possible, in general, to find the forces acting at any point since the law of Conservation of Energy underlies all machines; or the product of velocity multiplied by force is constant throughout the train. If the forces acting on a machine member and the manner in which it is connected are known, these may serve as a basis for the assigning of definite dimensions to the part. A fuller discussion of this important principle is given in Chapter III.

(b) If the forces acting on a machine member can be determined it would seem easy to choose the material and assign proportions to it based on the laws of Mechanics, and such is the case when the stresses are simple and the conditions fully known. Thus a machine member subjected to simple tension within known limits, can be intelligently proportioned in this manner. But in many cases the forces acting are very complex, the theoretical design is not always clear, and our knowledge of materials and their laws is limited in many respects. Recourse must therefore often be made to judgment or to empirical data, the result of experience. Even when the conditions are clear, theoretical design must always be tempered with practical modification and by constructive considerations, etc. The logical method of proportioning machine elements where theory is applicable is, therefore, as follows:

(a) Make as close an analysis as possible of all forces acting and proportion parts according to theoretical principles.

(b) Modify such design by judgment and a consideration of the practical production of the part.

In the case of details and unimportant parts, judgment and empirical data are commonly the best guides.

Summing up then, the logical steps in the design of a machine are as follows:

(I) Selection of the mechanism.
(II) Solution of the energy and force problem.
(III) Design of the various machine members so they will not unduly distort or break under the loads carried.
(IV) Specification and Drawing.

The last step, Specification and Drawing, is a necessary and important adjunct to the process of design; it is a powerful aid to the designer's mental process and is the best way of showing the workman what is to be done to construct the machine in question, and also of making a record of what has actually been done. It is not machine design of itself, however, as machines may be designed and built without any drawings. It is, nevertheless, an indispensable part of the designer's equipment. Very often written specifications accompanying the drawings are not only useful but necessary. In fact the highest skill on the part of the designer is often needed to clearly and fully specify in writing just what is to be done, as the writing of specifications presupposes the most intimate knowledge of theory of design, and selection of materials.

From the foregoing it is seen that the part of Machine Design included in Mechanism can be and generally is for convenience taught as a separate subject, and the student is expected to have a knowledge of Mechanism, Mechanical Drawing, Mechanics of Engineering, and Materials of Engineering as a preparation for the work contained in this book. The chapters that follow deal therefore with the solution of the Energy and Force Problem, and the Design of Machine Elements.

CHAPTER II

THE ENERGY AND FORCE PROBLEM

2. From the law of Conservation of Energy it is known that energy can be transformed or dissipated but not destroyed. Therefore all the energy supplied to any machine must be expended as either useful or lost work. Since frictional resistances, and frequently other losses, occur in all machines, the useful work done must always be less than the energy received. The useful work delivered divided by the energy received is called the **efficiency** of the machine. This expression is different for different machines and is evidently a fraction or less than unity. In the discussion which follows in this chapter, frictional losses are neglected, unless otherwise stated.

A Kinematic Cycle is made by a machine when its moving parts start from any given set of simultaneous positions, pass through all positions possible for them to occupy, and ultimately return to their original positions.

The energy received by a machine during a kinematic cycle may or may not be equal to the work done plus frictional losses. Thus the energy supplied during a number of cycles may be stored in some heavy moving part and then be given out during some succeeding kinematic cycle, as in the case of a punching machine with a heavy flywheel.

An Energy Cycle is made by a machine when its moving parts start from any given set of simultaneous energy conditions, pass through a series of energy changes, and ultimately return to their original energy conditions.

Thus the complete mechanism of a four-stroke gas engine makes one kinematic cycle every two revolutions of the crank shaft. The slider-crank mechanism of the engine considered separately makes a complete kinematic cycle every revolution of the crank. The engine makes one energy cycle every two revolutions of the crank. If a punching machine driven by a belt and running continuously, punches a hole every fourth stroke of the punch, it will be making a complete kinematic cycle every stroke and a complete energy cycle every four strokes.

Therefore, during a kinematic cycle,

Energy received = useful work + lost work ± stored energy.

And during an energy cycle,

Energy received = useful work + lost work.

Generally speaking, the useful work to be done and also the character of the source of energy are known and the problem of design is, therefore, to select the mechanism which will transform the motion of the source of energy into the required motion, to determine the capacity of the driving device, and to proportion the machine members.

The proportions of any machine part depend, as regards strength and rigidity, on the maximum force it must carry; and this maximum force may be due to the direct action of the driving device, or it may result from the inertia effect of some member which has a capacity for storing energy, and in such a case may be greatly in excess of any direct force that the driving device may deliver. Before this maximum force can be determined for any member it is therefore necessary to make a complete solution of the energy problem including the determination of the driving device.

A knowledge of the *quantity* of energy required to do the desired work during a complete energy cycle is not always sufficient information upon which to base the design of the machine or the capacity of its driving device.

A machine may receive energy at either a **uniform** or **variable rate** and may be called upon to do work at either a uniform or variable rate. **Power** or *rate of doing work* being the product obtained by multiplying together simultaneous values of velocity and force, it follows that in making any energy transformations both the force and the velocity factors must be kept in mind. While the mechanism chosen may transform the motion of the source of energy into the *desired motion*, it may not necessarily so modify the *energy* as to give a distribution of *force* at the point where work is being done which exactly or even approximately fulfils the required conditions. Again, some of the moving machine parts may have to be very heavy in order to carry the required loads, and during one part of the cycle they may absorb energy, thus reducing the operating force, while at another part of the cycle they may give up energy, thus increasing the operating force. Such a condition may make an entirely different distribution of the forces acting on the members of the mechanism, from that which would occur were the parts light or the motion of the machine very slow, and may materially modify the design.

If it is predetermined that some device is to be used for storing energy when the effort is in excess, and for giving it out when the effort is deficient, the *capacity* of the driving device need only be such as will supply during the energy cycle an amount of energy equal to the useful work and lost work during that cycle. But in many machines such devices are not desirable and in many others they cannot be applied.

Two such cases may be noted. (a) In many machines under **continuous operation,** where flywheels are not desirable, it is found that if the driving device is proportioned so as to supply energy at a uniform rate equal to the *average* rate required throughout the energy cycle, the force at the operating point is sometimes greater and sometimes less than that required. If simultaneous values of the force and velocity at the working point are multiplied together, their product is the rate at which work will be done at the point considered. The maximum product thus obtained will be the **maximum rate** at which work will be done and also at which energy must be supplied by the source. It is evident that the capacity of the driving device will be greater in such a case than if based on the average rate of energy required per energy cycle. If the driving device under the above conditions should be too large or expensive, as is liable to be the case in large work, recourse must be had to a different mechanism or to the use of flywheels or other means of storing and redistributing energy. (b) Again, consider any

hoisting mechanism. Not only must the driving device supply during the cycle of operations (the raising of the load) energy equal to the work done, but it must also be able to *start* and *sustain* the load at any point. It is evident that in such cases the *torque* of the driving device on the hoisting drum, must be at least equal to that of the load, and if the torque of the driving device should be variable, its minimum torque must be equal to that of the load when referred to the same shaft.* If this minimum torque should be small compared to the maximum, the driving device chosen might have to be excessively large and this condition might preclude the use of the driving device first selected.

In any of these cases, after the form and capacity of the driving device have been determined, the **maximum force** that may come on any member may also be determined.

It is to be noted that the choice of mechanism and the capacity of the driving device are governed largely by the relative manner in which energy is to be received and work done, and it may be well to enumerate the combinations that can occur, before applying the above principles to the discussion of illustrative problems.

In any machine *under continuous operation* energy may be received and work may be done in one of the following ways:

(a) **Energy may be received at a constant rate and work be done at a constant rate.**

(b) **Energy may be received at a constant rate and work be done at a variable rate.**

(c) **Energy may be received at a variable rate and work be done at a constant rate.**

(d) **Energy may be received at a variable rate and work be done at a variable rate.**

3. **Case** (a). As an example of this case, where energy is received at a constant rate and work done at a constant rate, consider a steam turbine running a centrifugal pump raising water to a fixed level. Evidently the rate at which energy is

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* In certain hoisting devices friction is utilized to sustain the load or prevent overhauling; this statement does not apply broadly to such cases.

supplied must just equal the rate at which work is done plus frictional and other losses, for any given period, and the capacity of the turbine is very easily determined.

4. **Case (b).** As an example of this case (energy received at a constant rate and work done at a variable rate) consider the case of a machine for punching holes in boiler plate. Here the driving belt can supply energy at a constant rate while the useful work, which is of considerable magnitude, is delivered intermittently. If the driving belt were designed with sufficient capacity to force the punch through the plate by direct pull it would have to be very large. The machine runs idly a large portion of the time, while the plate is being shifted, and in a machine of this kind a device for storing energy, such as a flywheel, can be used to advantage. The total capacity of the driving belt need only be sufficient to supply, during the energy cycle, an amount of energy equal to the useful work plus the lost work. When a hole is punched the velocity of the wheel is reduced, the wheel giving up stored energy. During the time that the machine is running idly the belt can store up energy in the flywheel by bringing its velocity up to normal. The maximum *force* that may be transmitted by the machine members will be based on the maximum force at the tool and will be transmitted only by the members that lie between the tool and the flywheel.

As a second example of these conditions, take the design of a small shaping machine. Here the useful work is done during the forward stroke of the ram. During the return stroke frictional resistances only are to be overcome. The resistance of the cut during the forward stroke is uniform and the speed of cutting is limited by the character of the metal to be cut. During the return stroke, however, the velocity may be greatly increased, the limiting velocity depending on the mass of the moving parts, as these should be brought to rest at the end of the stroke without shock. The machine is driven by a belt which *can* supply energy at a uniform rate and, as noted above, the work is done at a variable rate.

Numerous mechanisms have been devised to meet these conditions. Suppose a mechanism such as shown in Fig. 1 has been selected. The maximum length of the stroke is fixed by the work to be done and the minimum length of stroke should be 3 or 4 inches. Continuous rotary motion is imparted to the crank *a* through the gear *b* of which it forms a part. The gear *b* is in turn driven by the pinion *c* which is rigidly attached to the shaft *d*. On the other end of *d* is a stepped pulley having diameters

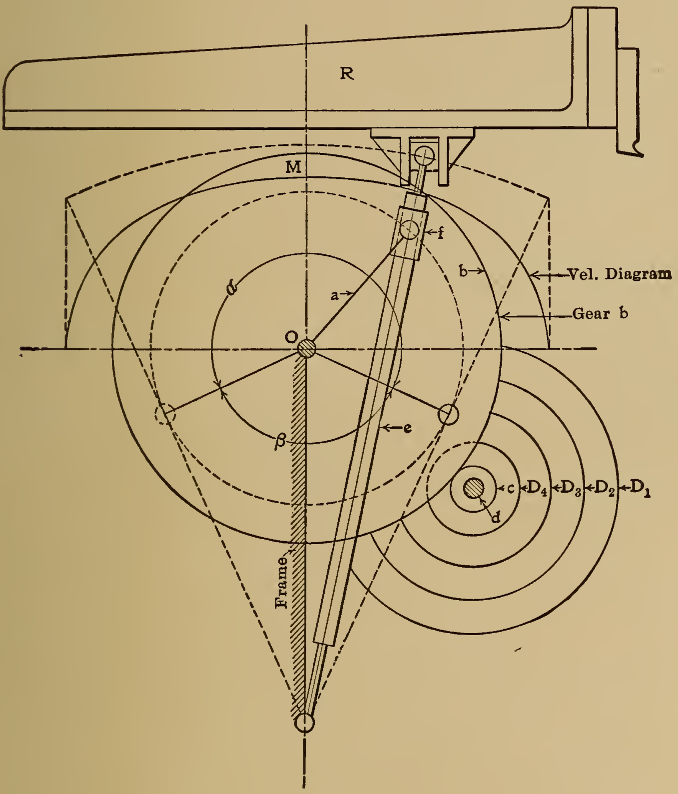

FIG. 1.

D_1 D_2 D_3 D_4. On the countershaft overhead is a mating stepped pulley so placed that when the belt is on the largest step of the machine D_1 it is also on the smallest step of the countershaft pulley. The crank pin on *a* is adjustable and can be moved from the outer position as shown toward the centre of the crank, so that the vibrator *e* can be made to give the ram *R* any length of stroke from the maximum (20 inches in this example) to a minimum of 3 or 4 inches. The range of velocity of the tool for any length of stroke must be such that it can be lowered to the cutting velocity of hard cast iron or tool steel and raised to the economical cutting velocity of brass. With the pin in its extreme outer position and the belt on the large step D_1 the speed of the ram will be a maximum for that position of the belt. As the crank is drawn toward the centre (the belt remaining in its original position) the velocity of the ram is obviously decreased. If now the belt is shifted to a smaller step as D_2 the velocity of the ram will be increased, so that at any stroke variable speed may be obtained to suit the metal to be cut. It is not desirable to use a flywheel, the inertia of the moving parts is small, and the problem is therefore to design the driving belt and proportion the machine members on the basis of the maximum pull which the belt may be able to exert.

The mechanism transforms the uniform rotary motion of the line shaft into the required reciprocating motion. Consider the crank pin at its extreme outward position and the belt on D_1. The velocity diagram for full forward stroke under these conditions is shown, the ordinates of the diagram* representing the velocity of the ram to the scale that the crank length represents the uniform velocity of the crank pin. The diagram for the backward stroke is not drawn since it is not needed in the solution of the energy problem; but it should in general be drawn to make sure that the change in velocity at the extreme ends of the stroke is not excessive. If the belt supplies energy at a constant rate the force which it can deliver at the tool will vary inversely as its cutting velocity. The cutting resistance, however, is uniform so that while the mechanism produces the desired transformation in *motion* it may not give the distribution of *force* desired.

To design the driving device (or belt) for such a mechanism

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* For a full discussion of these so called *quick-return* mechanisms and the methods of drawing velocity diagrams see "Kinematics of Machinery" by John H. Barr, "Machine Design" by Smith and Marx and "Machine Design," Part I., by F. R. Jones.

the operating conditions of the machine when the belt has both its maximum and minimum velocity must be investigated. The maximum pull which a belt can give is $T_1 - T_2$ where T_1 is the allowable tension on the tight side of the belt. (See Church's "Mechanics," page 182.) The *power** that a belt can give out is therefore $V(T_1 - T_2)$ where V is the velocity of the belt. Since $T_1 - T_2$ has, at all moderate belt speeds, a constant maximum value for a given belt, the power that a belt can deliver will vary directly with its velocity. The belt receives its energy from a shaft running at constant speed and when the belt is on the smallest step of the countershaft cone it will also be on the largest step D_1 of the machine cone and will in consequence be running at its lowest velocity, under which condition its capacity for delivering energy is a minimum.

The maximum power required for small machine tools is approximately constant at all speeds; for since the heating effect which governs the cutting capacity of the tool is proportional to the work done, it follows that as the cutting speed is increased the resistance of the cut must be decreased and *vice versa*, thus keeping their product approximately constant. If then the belt is designed to have sufficient capacity when the ram is making full stroke and the belt is on D_1, and hence at the lowest belt velocity, it will have excess capacity when in any other position. If a softer metal is to be cut the velocity of the ram may be increased, but this can only be done by shifting the belt to a position where its velocity and hence its capacity will be greater.

As before noted, the effect of moving the crank pin inward, the belt remaining in the same position, is to decrease the average velocity of the ram. Therefore as the stroke is made shorter the velocity of the crank, to maintain a given cutting speed, must be increased by shifting the belt to a smaller step of the machine cone. The other limiting condition is when the ram is making its shortest stroke and giving a cutting velocity high enough for the softest metal to be worked. The belt should then be on the smallest diameter D_4, and hence at its highest speed.

* A full discussion of the power transmitted by belting is given in chap. 12.

An inspection of the velocity diagram when the ram is making full stroke shows that its velocity is a maximum when the ram is in mid position. Neglecting friction and inertia, which here are small, the force exerted on the ram will be a minimum where the velocity of the ram is a maximum at any given belt velocity, because, for a given belt pull since no flywheel is used, force at belt × velocity of belt = force at tool × velocity of tool. If, therefore, with the ram making full stroke, the capacity of the belt when running on D_1 is made great enough to give a force at mid position of the ram equal to the required cutting force, it will have excess capacity at any other position; and if this condition does not give too large a belt the driving device will be satisfactory. The maximum *force* that any member may have to sustain will be based on the maximum torque of the belt, which will occur when it is running on D_1; for since the inertia forces are small this torque will be transmitted directly to the members, and the resulting stresses may be easily computed.

Example:

Let the greatest resistance of cut	=	800	lbs.
" " maximum stroke of ram	=	20	inches.
" " minimum stroke of ram	=	4	inches.
" " maximum length of crank	=	6½	"
" " minimum " " "	=	1½	"

" " max. cutting speed on shortest stroke and highest belt speed = 60 ft. per min.

" " max. cutting speed on full stroke and lowest belt speed = 25 ft. per min.

Then in general,

$$\frac{\text{linear velocity of crank}}{\text{length of crank}} = \frac{\text{max. linear velocity of ram}}{\text{max. ordinate of diagram*}}$$

Hence in this example when the ram is making full stroke at lowest speed,

* In the mechanism here chosen the position of the ram for maximum velocity can be located by inspection and the value of the velocity determined without drawing the complete diagram. In general, however, the diagram must be drawn in order to locate the maximum ordinate.

$$\text{Linear vel. of crank} = \frac{25' \times 6\frac{1}{2}}{7} = 23.5 \text{ ft. per min.}$$

$$\therefore \text{R.P.M. of crank} = \frac{23.5 \times 12}{2 \times \pi \times 6\frac{1}{2}} = 6.9.$$

In a similar way when the ram is making the shortest stroke at highest speed,

Linear velocity of crank = 42.5 ft. per min.

$$\text{Therefore, R.P.M. of crank} = \frac{42.5 \times 12}{2 \times \pi \times 1\frac{1}{2}} = 54.1.$$

Let the gear ratio be 8 to 1. Then the minimum and maximum R.P.M. of shaft $d = 55.2$ and 432.8 respectively. A 14″ pulley is a convenient diameter for D_1.

$$\therefore \text{velocity of belt on low speed} = \frac{14 \times \pi \times 55.2}{12}$$

$$= 204 \text{ ft. per min.}$$

If the efficiency of the machine be 85 per cent, the maximum rate of doing work at this position of belt is the cutting resistance multiplied by the maximum velocity of the ram, divided by the efficiency, or $\frac{800}{.85} \times 25 = 23{,}500$ ft. lbs. per minute.

$$\therefore \text{effective pull at belt} = \frac{23{,}500}{204} = 115 \text{ lbs. approximately.}$$

The effective pull of single-ply belt per inch of width may be taken at 40 to 45 lbs.

$$\therefore \text{width of belt} = \frac{115}{45} = 2\tfrac{1}{2}'' \text{ nearly.}$$

If the cone pulleys on machine and countershaft are alike, as is the usual case in metal-working tools, then

$$\frac{D_1}{D_4} = \sqrt{\frac{\text{Max. R.P.M. of Machine Cone}}{\text{Min. R.P.M. of Machine Cone}}};$$

$$\therefore D_4 = D_1 \sqrt{\frac{\text{Min. R.P.M. of Machine Cone}}{\text{Max. R.P.M. of Machine Cone}}};$$

and hence, in the example if $D_1 = 14$, $D_4 = 14\sqrt{\frac{55.2}{432.8}} = 5''$ nearly.

The **maximum force** that may be applied to any member will be based on the maximum torque of the driving belt, which occurs when the belt is on D_1 the largest step of the machine cone. The difference in this respect between this case and the punching machine discussed above should be noted, for, while the driving mechanisms of both can deliver energy at a uniform rate and while both do work at a variable rate, the maximum load is applied in entirely different ways.

During the complete energy* cycle of the machine the total work done, neglecting friction, is equal to the length of stroke multiplied by the uniform resistance of the cut, or

$800 \times \frac{20}{12} = 1333$ ft. lbs. For every cycle of the machine the shaft *d* makes 8 revolutions; hence the amount of energy that the belt could deliver if work were done uniformly during one cycle is $8 \times \frac{14 \times \pi}{12} \times 115 = 3370$ ft. lbs.

The **capacity** of the belt is therefore two and one-half times as great as it would need to be if a device for equalizing the energy, such as a flywheel, had been used. Where a small machine is belt-driven, as in the case discussed, this added first cost is not serious. But when the power needed is great, or in such cases as direct driving by electric motor, the additional cost of a driving device so greatly in excess of average requirements needs to be carefully considered. This, in fact, is one of the most important elements to be considered in fixing the size of motors needed for direct-driven machine tools, sometimes making it desirable to introduce a flywheel to reduce the size of motor.

5. Case (c). One of the best examples of Case (c) where energy is received at a variable rate and work is performed at a uniform rate is found in the reciprocating **steam engine,** and since this machine is of such great importance to the engineer it will be discussed somewhat in detail. Here the energy is supplied in the form of steam pressure, and after cutoff occurs and the steam expands in the cylinder the pressure falls from

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* The kinematic and energy cycle are, in this case, simultaneous.

the "initial" or boiler pressure to somewhat above exhaust or atmospheric pressure. The energy is therefore supplied at a **varying rate.** But the engine is required to deliver energy at the driving belt at a **uniform rate.** The mechanism used will

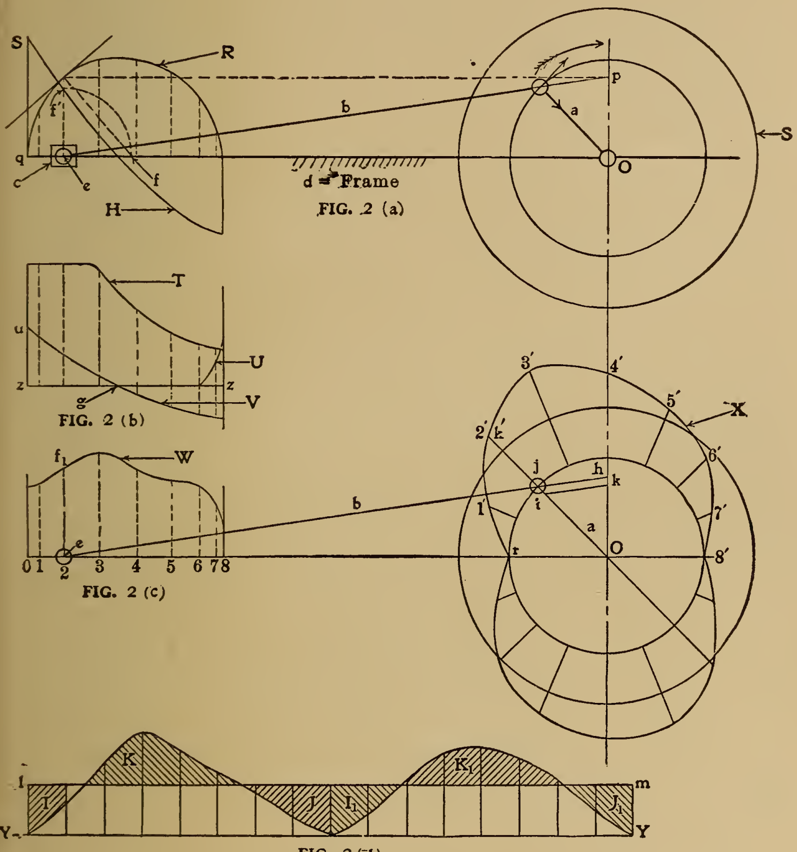

FIG. 2 (a)
FIG. 2 (b)
FIG. 2 (c)
FIG. 2 (d)

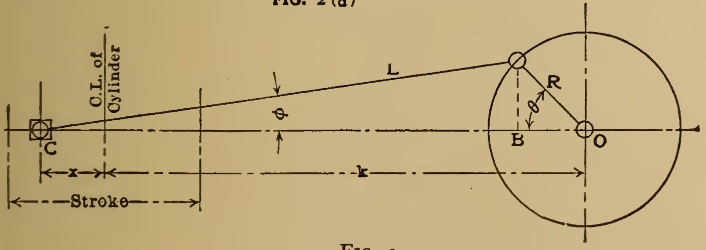

FIG. 3.

produce the required transformation of the reciprocating motion of the piston into the rotary motion of the crank shaft. But the distribution of the driving force in the form of torque or tangential effort will not be uniform but it will be a maximum somewhere near the position at which the crank is at right angles to the connecting-rod, and it becomes zero when the crank is on the dead centre. The turning effort will therefore sometimes be greater and sometimes less than the resisting effort of the driving belt and the machine will stop unless a redistributing device, such as a flywheel, is used. The reciprocating parts, such as the piston and crosshead, and also the connecting-rod, are heavy and their maximum velocity is considerable; hence the forces due to their inertia cannot be neglected.

Referring to Fig. 2 (*a*), the crank *a* is required to rotate around the center *O* with uniform velocity and to give a uniform force at the driving belt. The moment at the driving belt is equal to the average moment at the crank pin, hence the equivalent uniform force at the crank pin may be derived from that at the belt. This required driving force at the crank pin may be plotted radially from the crank circle as a base, forming a polar diagram of the required force at the pin, as shown by circle *S*. The crosshead *C* moves at a varying rate of speed. If the velocity of the crank pin be represented by the length of the crank, the intercept *Op* made by the connecting-rod on the vertical through *O* will represent the simultaneous velocity of the crosshead to the same scale. These intercepts may be plotted at the corresponding positions of the crosshead, thus outlining the curve whose ordinates represent the velocity of the crosshead at any point.

The forces acting upon the piston and which must be transmitted to the crank are,

(1) The **steam pressure** which is represented at any point by the ordinates of the curve *T*, Fig. 2 (*b*).

(2) The **back pressure*** on the other side of the piston, act-

* This generally amounts to 2 or 3 pounds per sq. in. above atmospheric pressure in non-condensing engines.

ing against the steam pressure, and represented by the exhaust pressure line *z z* and the compression curve *U*.

(3) The **inertia forces** due to accelerating and retarding the heavy reciprocating parts.

During the first part of the stroke these inertia forces tend to reduce the effective pressure transmitted to the crank pin, and during the latter part they increase the effective force on the rod. They can be represented graphically by such a curve as *V*. The first two curves can be found by the well-known methods of drawing indicator cards, and the third can be found either by mathematical deduction or by graphic methods* based on the velocity diagram. It is believed that the analytical method is the most satisfactory, and such a method is presented in a succeeding article.

If the acceleration is known the force necessary to produce the acceleration is also known since accelerating force = mass × acceleration, and the force at any point (reduced to pounds per sq. in. of piston) may be plotted as shown by curve *V*, Fig. 2 (*b*). When the reciprocating parts reach their maximum velocity their acceleration is zero, hence the curve of acceleration forces crosses the axis at a point *g* corresponding to the point of maximum velocity. This point is very nearly at the position where the crank and the connecting-rod are at right angles and the error introduced by assuming this to be so is small with ordinary ratios of crank to connecting-rod length. Beyond *g* the reciprocating parts are retarded, hence the inertia forces increase the effective crank-pin pressure from that point on. The compression curve (*U*) tends to decrease the effective pressure on the piston and hence its ordinates must be subtracted from the forward pressure. The algebraic sum of the curves *T*, *U*, and *V* will give a resultant pressure curve *W*, Fig. 2 (*c*), whose ordinates at any point represent the effective pressure acting on the piston rod at that point. This effective pressure is transmitted to the crank by the connecting-rod *b*. The pressure of the rod against the crank pin may be resolved into two components, one **tangential**

* For a full discussion of this matter see "Kinematics of Machinery" by J. H. Barr, page 71, paragraph 42.

to the crank circle and tending to produce rotative motion, and one **radial** along the crank tending to produce compression or tension in the crank and friction in the main bearing. Only the tangential force can do useful work. If friction be neglected the rate at which work is done by this force at the crank must equal the rate at which work is being done at the piston. Now the curves R and W, Fig. 2 (a) and 2 (c) respectively, give the simultaneous values of force and velocity at every point of the stroke. If such simultaneous values be multiplied together and divided by the uniform velocity of the crank (all in the proper units) the quotient is the tangential force at the pin, and this may be plotted radially on the crank circle as a base, thus giving what is called a radial crank-effort diagram, Fig. 2 (c), Curve X.

These values of the tangential force can be found more easily graphically. It will be remembered that the ordinates of the velocity diagram (R), as drawn in Fig. 2 (a), represent the velocity of the crosshead to the same scale as the length of the crank represents the velocity of the crank pin. In Fig. 2 (c), the connecting-rod extended, if necessary, cuts the perpendicular through O in the point h. Therefore $O\,h$ = velocity of crosshead when $O\,j$ = velocity of crank pin. Neglecting friction, the rate of work at the crank pin is equal to the rate of work at the crosshead, hence the velocity of the crank pin multiplied by the force at the crank pin is equal to the velocity of the crosshead multiplied by the force at the crosshead, or the tangential force $\times O\,j = e_1 f_1 \times O\,h$.

$$\therefore \text{ tangential force } = \frac{e_1 f_1 \times O\,h}{O\,j}$$

Lay off $O\,i = e_1 f_1$ and draw $i\,k$ parallel to b. Then, $\dfrac{O\,k}{O\,h} = \dfrac{O\,i}{O\,j}$.

$$\text{Therefore, } O\,k = \frac{O\,i \times O\,h}{O\,j} = \frac{e_1 f_1 \times O\,h}{O\,j} = \text{tangential force.}$$

Therefore $O\,k$ may be laid off radially from j as an ordinate of the required curve as $j\,k'$. The construction for the return stroke is performed in a similar manner.

It will be noted that the distribution of force as represented by this diagram is less uniform than the original curve of pressure at the crosshead. By the conditions of the problem, however, the mechanism must produce a uniform turning effort at the driving belt or such as would be given by a crank-effort diagram like S, Fig. 2 (*a*). A flywheel must therefore be used to store energy when the crank effort is in excess and to give out energy when the crank effort is deficient. Fig. 2 (*d*) shows the crank-effort diagram rectified with rectangular ordinates equal to the polar ordinates of curve X. The base YY is equal to the circumference of the crank circle and the ordinates of the line $l\ m$ are equal to the ordinates of the required uniform crank-effort curve S. Since the abscissas represent *space* and the ordinates represent *force*, the areas I, K, J, I_1, K_1, etc., represent *work*. The work represented by $K+K_1$ is that which the flywheel must absorb and the area represented by $I+J+I_1+J_1$ that which it must give up in one revolution. Manifestly $I+J+I_1+J_1$ must equal $K+K_1$. A full discussion of the design of the flywheel will be given in a later chapter.

The **maximum force** that may come upon the crosshead can be seen from an inspection of the force diagram W. It is to be noted in this regard that if the engine is designed for variable cutoff, an indicator diagram at late cutoff should be drawn for the purpose of locating this maximum force, as an earlier cutoff will not give the maximum value. The method of analysis developed above will enable the designer to determine the maximum straining action on any member of the mechanism.

The graphical method of finding the inertia curves, while convenient, are open to criticism on account of their inaccuracy because the tangents or sub-normals to the curve, on which these graphic methods depend, are difficult to construct with accuracy and are at some points indeterminate. In general, therefore, it is thought that the following method or some similar one is more satisfactory.

Referring to Fig. 3 (page 17),

Let a = acceleration at any point,
" R = length of crank in feet,
" L = " " connecting-rod in feet,

Let N = Rev. per min.,

" θ and φ = angles made with centre line by the crank and connecting-rod respectively at any position measured from the crank position $O\,r$,

Let k = distance from centre of crank shaft to mid position of crosshead,

Let x = displacement of crosshead from mid position,

" $n = \frac{L}{R}$,

" v = velocity of crosshead at any point x,

" t = time elapsed corresponding to v.

" ω = angular velocity in radians per second,

Then $x + k = O\,B + B\,C = R.\cos\theta + L\cos\varphi$,

But $L\cos\varphi = \sqrt{L^2 - R^2\sin^2\theta} = R\sqrt{\frac{L^2}{R^2} - \sin^2\theta}$

$$= R\sqrt{n^2 - \sin^2\theta}$$

$$\therefore\ x + k = R\,(\cos\theta + \sqrt{n^2 - \sin^2\theta}). \quad \text{(1)}$$

Expanding the radical by the binomial theorem and omitting all terms beyond the second (which can be done without appreciable error with the limiting proportions ordinarily used) equation (1) becomes,

$$x + k = R\left[\cos\theta + \left(n - \frac{\sin^2\theta}{2\,n}\right)\right] \quad \text{(2)}$$

Now x = the distance moved through by the crosshead, from mid stroke and velocity at $x = \frac{d\,x}{d\,t}$; and therefore differentiating (2) with reference to t

$$v = \frac{d\,x}{d\,t} = -R\left(\sin\theta + \frac{\sin 2\,\theta}{2\,n}\right)\frac{d\,\theta}{d\,t} \quad \text{(3)}$$

The acceleration =

$$a = \frac{d\,v}{d\,t} = \frac{d^2\,x}{d\,t^2} = -R\left(\cos\theta + \frac{\cos 2\,\theta}{n}\right)\left(\frac{d\,\theta}{d\,t}\right)^2 \quad \text{(4)}$$

but $\frac{d\,\theta}{d\,t}$ = angular velocity in radians per sec. $= \frac{2\,\pi\,N}{60}$,

hence $a = -\left(\frac{2\,\pi\,N^2}{60}\right)R\left(\cos\theta + \frac{\cos 2\,\theta}{n}\right)$, ... (5)

which is the general expression for acceleration of the reciprocating parts.

If the weights of parts be called W, from Mechanics it is known that the force necessary to produce an acceleration (a) is

$$P = \frac{W}{g} a \text{ where } g = 32.2 \text{ in English units; therefore}$$

$$P = -\frac{W}{g} R \left(\frac{2 \pi N^2}{60}\right)\left(\cos \theta + \frac{\cos 2 \theta}{n}\right) \text{ where } R \text{ is in ft.} \quad . \quad (6)$$

or reducing,

$$P = -\frac{W r N^2}{35{,}200}\left(\cos \theta + \frac{\cos 2 \theta}{n}\right) \text{ where } r \text{ is in inches.} \quad . \quad (7)$$

When the solution of the above expression gives a negative result the force of inertia is acting away from the crank and when positive, toward the crank. It is also to be noted that the expression $\frac{W}{g} R \left(\frac{2 \pi N^2}{60}\right)$ is the centrifugal force of a weight equal to that of the reciprocating parts concentrated at the crank pin since centrifugal force in general is equal to $\frac{W R \omega^2}{g}$.

By means of equation (7) all points on the acceleration curve could be found and plotted. In general, however, the exact characteristics of the curve are not essential and it is sufficient to make the three most simple solutions as follows, and a curve drawn through the three points thus located is sufficiently accurate for all ordinary purposes. In cases of extremely high speed with small ratios of connecting-rod to crank a more accurate determination of the curve may be desired.

$$\text{When } \theta = 0,\ P = -\left(\frac{W r N^2}{35200}\right)\left(1 + \frac{1}{n}\right) \quad . \quad . \quad (8)$$

$$\text{When } \theta = 180^\circ,\ P = +\left(\frac{W r N^2}{35200}\right)\left(1 - \frac{1}{n}\right). \quad . \quad (9)$$

$$\text{When } \theta = 90^\circ \text{ or } 270^\circ{*}\ P = \left(\frac{W r N^2}{35200}\right)\left(\frac{1}{n}\right). \quad . \quad (10)$$

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* The piston is *not* at half stroke.

If the inertia forces are to be combined with the steam pressures, as shown graphically in Fig. 2 (*b*), they must be reduced to **pounds per square inch of piston** to give correct diagrams.

An **example** may serve to make these points clearer. Let it be required to design a steam engine to deliver 150 H.P. with the following data:

Steam pressure = 90 lbs. gauge. Cutoff at $\frac{3}{8}$ stroke.

Ratio of crank to connecting-rod = 1 to 5.

Piston speed = strokes per minute multiplied by length of stroke = 640 ft.

Here something must be known about the size of cylinder necessary, before definite dimensions are assigned to the various members. Let a theoretical indicator card be drawn as in Fig. 2 (b), neglecting for the present the inertia curve V since this only tends to redistribute the energy and does not affect its quantity. The distance $z\ z$ represents the piston travel and the ordinates of the curve T represent piston pressures; therefore the area between $z\ z$ and the curve T represents the work done by the steam pressure during the stroke. In a similar way the area under curve U represents the work of compression due to back pressure. The difference of these areas is the net work done per stroke of piston and the mean ordinate corresponding to this area represents to the proper scale the average pressure per sq. inch on the piston during stroke. In the case given $z\ z = 2''$. Area under T minus area under $U = 1.75$ sq. in. Therefore mean ordinate $= \frac{1 \cdot 75}{2} = .875''$. The scale of pressures taken is $1'' = 70$ lbs. Therefore mean pressure during stroke $= 70 \times .875 = 62$ lbs.

Let A = area of piston.

P = mean effective pressure per sq. in.

L = length of stroke in feet.

N = number of revolutions per minute.

$H.P.$ = horse power required.

Then H.P. $= \frac{2\,P\,L\,A\,N}{33000}$. Here P, $N \times L$ and H.P. are known.

Whence $A = \frac{H.P. \times 33000}{P \times 2NL} = \frac{150 \times 33000}{62 \times 640} = 132$ square inches, or a diameter of cylinder of 13 inches.

If the stroke be taken at about twice the diameter of the cylinder, or say 24 inches, the proportions will be good.

Hence since $2L \times N = 640$, $N = 160$ R.P.M. The mechanism can now be laid out to scale. This has been done in Fig. 2 (a and c),* the space scale being 1″ = 1 ft.

As before stated, the location of the three points, namely, where θ is respectively 0°, 180°, and 90° or 270° (Fig. 3), is sufficient to locate the inertia curve. In the above example $W = 3.5$, $n = 5$, and $N = 160$.

The general expression for the inertia force is, for $\theta = 0$.

$P = \frac{WrN^2}{35,200}\left(1 + \frac{1}{n}\right) = C\left(1 + \frac{1}{n}\right)$ where C is a constant and here equal to $\frac{3.5 \times 12 \times 160^2}{35,200} = 30.5$.

Therefore, When $\theta = 0°$, $P = 30.5\left(1 + \frac{1}{5}\right) = 36.6$ lbs.

$$\text{When } \theta = 90°, \quad P = 30.5\left(\frac{1}{5}\right) = 6.1 \text{ lbs.}$$

$$\text{When } \theta = 180°, \quad P = 30.5\left(1 - \frac{1}{5}\right) = 24.4 \text{ lbs.}$$

These values serve to locate the curve as in Fig. (2).

The resultant of T U and V, curve W, Fig. 2 (c), can now be drawn and the crank-effort diagram X plotted. The crank-effort curve can be rectified as in Fig. 2 (d) and the mean ordinate Yl drawn. The area $I + J = K$ will be proportional to the energy to be absorbed and delivered by the flywheel. One inch of ordinate here = 70 lbs. per sq. in. of piston and one inch of abscissa = 1 ft.; therefore one sq. in. of area = 70 ft. lbs.

The area of $K = .5$ sq. in. and area of piston = 132 sq. in. Hence, if E = energy to be absorbed,

$E = .5 \times 70 \times 132 = 4,620$ ft. lbs. on which the design of the flywheel can be based.

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* Reduced in reproduction about one-half.

The maximum pressure that can occur on the piston is the initial or boiler pressure as the ordinates of W are at all points less than those of T. Hence, when running, the parts will be subjected to less load than in starting up, when full boiler pressure may be applied before inertia forces become noticeable.

6. Case D. A good example of energy supplied at a varying rate and work done at a varying rate is found in **a direct-driven air compressor.** Here the varying steam pressure in the steam cylinder is opposed by a varying air pressure in the air cylinder as shown in Fig. 5 (a). The area of the cylinders are, for simplicity, assumed to be equal. The steam cylinder takes steam at 80 lbs. pressure and the air compressor cylinder delivers air at 100 lbs. pressure. The **efficiency** of the system shown is taken at 80 per cent. and hence the area of the compressor card is 80 per cent. of the steam card.* If both the pistons were rigidly attached to the same rod it is evident that the maximum steam pressure will occur where the air pressure is a minimum. If, however, each cylinder is independently connected to a common shaft by means of a crank and connecting-rod mechanism, the maximum and minimum pressures of the cards may be made to coincide more closely by placing the crank pins at the proper angular distance apart. In other words the mechanism may be so designed that energy will be delivered at the working point more nearly at the rate required by the work to be done. The loss by friction, etc., is about 20 per cent. Part of this is lost on the steam side and part on the air-compressor side. It can be assumed, without great error, that the losses can be evenly divided between the two slider-crank chains and also that the loss is at a uniform rate throughout the stroke. Thus the loss on the steam side can be represented by the line a b, Fig. 5 (a), which *reduces* the effective pressure at every point by a fixed

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* In the general case, where the cylinders are of different diameter and area, the diagrams which represent pounds per square inch of piston area would not have a ratio equal to the efficiency. The mean effective pressure of the air cylinder multiplied by the area of the air cylinder, divided by the mean effective pressure of the steam cylinder multiplied by the area of the steam cylinder, would, in this case, equal the efficiency.

amount. In a similar way ordinates to the line *c d increase* the effective resistance of the air diagram. The area of the diagrams modified in this way will be equal and all energy supplied will be accounted for.

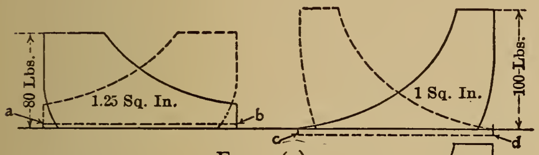

FIG. 5 (a).

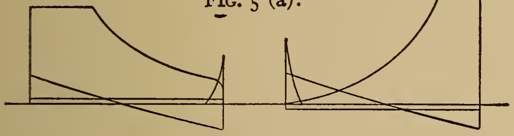

FIG. 5 (b).

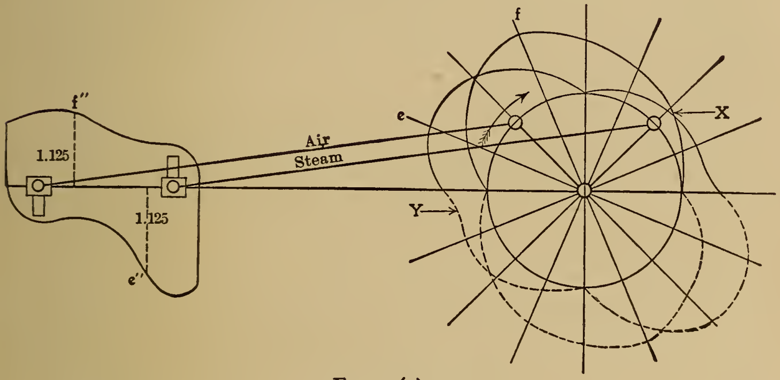

FIG. 5 (c).

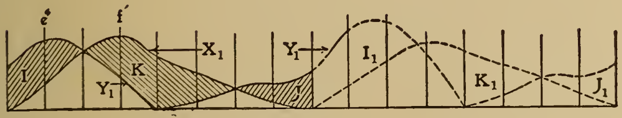

FIG. 5 (d).

Since the moving parts of both slider-crank chains will be heavy, the effect of inertia cannot be neglected. In Fig. 5 (*b*) the air and steam cards are shown with the inertia curve, the friction line, and the compression curves in their correct relationship. Fig. 5 (*c*) shows the resultant pressure curves, the curve of air pressures being plotted below the base line for convenience. The crank-effort curve of the steam cylinder is represented by X, and the resisting crank-effort curve of the air cylinder is represented by Y. The cranks are here placed 90° apart, the steam crank being in advance, a common arrangement in practice. It is evident, however, that this is not the most advantageous angle, for if the point e on the air curve is made to correspond with f on the steam curve, Fig. 5 (*c*), the excess and deficiency of effort will be still further reduced. This would place the cranks at 45° apart. This is even more clearly shown in Fig. 5 (*d*), on the rectified curve of crank effort. Here the area $K+K_1$ is the amount of energy to be absorbed and $I+J+I_1+J_1$ the amount to be given up by the flywheel during one revolution. In the steam slider-crank mechanism the greatest pressure is, as before, that due to the initial steam pressure, while on the air side it will be that due to the terminal air pressure.

6.1. In the four cases discussed above the action of the machine has in all instances been supposed to be continuous, and all machines which operate continuously will belong to one of these classes. Where the action of the machine is **intermittent** or **irregular,** these general solutions will not always hold and the design of the machine cannot be based on the energy given or received, but will depend on the **maximum force** or **maximum torque** or, in other words, on the **mechanical advantage** which the motor must possess. Thus the motor on an auto car has a certain maximum capacity for delivering power. On a level road it can propel the car at a high rate of speed, the engine making only a few turns to every revolution of the wheels. But on a steep hill the gears must be shifted so that the engine has a greater mechanical advantage, and gives a greater torque on the axle, the engine making many revolutions to every one of the wheels. Another example of this is the case of hoisting mechanisms already discussed somewhat (see article 2). An engine or a motor might be capable of giving out energy at a rate equal to that required to lift the load in a given time, and it might be able, running continuously, to raise the load to the required height. But its ability to *start* and *sustain* the load at any point will depend on whether it has a *mechanical advantage* at that point and *not* on its *capacity*. Where the torque of the load is constantly changing, as in deep mine hoisting, the design of the hoisting devices becomes quite complicated and is beyond the scope of the present treatise. It will be noted, however, that in such cases the minimum torque of the motor or engine must always exceed the maximum torque of the load when referred to the same shaft. This general principle must be kept in mind in designing hoisting devices and similar machines which act intermittently and slowly, or where redistributing devices are undesirable or impossible.

6.2. Redistribution of Energy and Inertia Effects. Devices for storing and redistributing energy are very common in transmis-

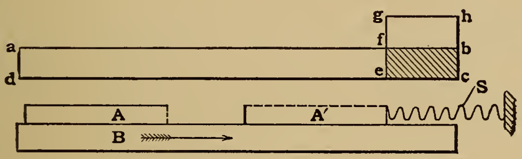

FIG. 5 (e).

sion systems. Thus, in hydraulic distribution, the excess supply of power is stored in an **accumulator,** and given out again when the supply is deficient. In electrical distribution a **storage battery** is sometimes used for the same purpose. In transmission of power by compressed air a large **reservoir** is sometimes employed as a store-house of energy. In the case of a single machine, the redistribution is effected by compressing a gas, by using a spring, or by accelerating and retarding some heavy moving part. Thus in the steam engine the piston compresses steam in the clearance space at the end of its stroke, and the energy so absorbed is returned to it during the next stroke. Again, when the energy supplied by the steam is in excess of the effort required, the fly-wheel absorbs the excess and thereby has its velocity (and hence its kinetic energy) increased. When the effort is in excess, the wheel gives up the stored energy at the expense of its velocity.

It does not necessarily follow, however, that all heavy moving parts simply *redistribute* the absorbed energy as *useful* work, as the action may be a positive source of loss. In Fig. 5 (*e*) let A be the platen of a large planing machine, and suppose it to be making its return stroke, moving from left to right. The force just necessary to slowly move the platen may be represented by the vertical ordinates of the diagram $a\ b\ c\ d$. Suppose now, that a greater force is applied, in order to hasten the operation, so that at the position A', the platen has been accelerated till its kinetic energy is equal to the rectangle $e\ g\ h\ c$. Evidently the platen will not stop at the end of the stroke if the actuating force be removed at A', as the work of friction during the remainder of the stroke is less than the stored energy. If, therefore, the "return" belt is removed at A' and the "driving" belt applied, the latter will slip upon the driving pulley till the excess of energy is absorbed and dissipated as heat. If the point A' has been properly chosen the platen will just stop at the end of the stroke and the energy absorbed by the belt will equal the area $f\ g\ h\ b$. If a spring, S, were fitted to the machine, so that the work of compression from the position A' to the end of the stroke just equalled the excess kinetic energy of the platen, at that position, the return belt could be thrown off at A', and the platen would stop at the end of the stroke. The energy stored in the spring would then be returned to the platen on the forward stroke. This latter action is identical with that of compression in the steam-engine cylinder, Fig. 2, the energy under the curve U being returned to the reciprocating parts on the next stroke. It is to be noted in this last case, that even if the work of compression is not quite equal to the energy to be absorbed during the latter part of the stroke, there is no loss of energy (friction neglected), as what is not absorbed by compression is absorbed at the crank pin in useful effort.

CHAPTER III

STRAINING ACTIONS IN MACHINE ELEMENTS

7. Nature of Forces acting in Machines. From the foregoing chapter it is clear that machine members which transmit energy are subjected to forces of a varying character and intensity. Since the various parts of a machine must be constrained to move in fixed paths it is important that they should neither break or be distorted appreciably under the loads carried; that is, the members must be not only *strong* but also *stiff*. The proportioning of machine elements as dictated by various methods of loading is therefore most important, and will be considered in this chapter.

The forces acting on a machine element may be one or several of the following:

(a) The **useful load** due to the energy transmitted.
(b) Forces due to **frictional resistances.**
(c) The **weight** of the part itself or of other parts.
(d) **Inertia forces** due to change of velocity.
(e) **Centrifugal or inertia forces.**
(f) Forces due to change of **temperature.**
(g) **Magnetic attractions,** as in electrical machinery.

These forces or loads may be applied to a machine in several ways. They may act steadily in one direction; they may act intermittently in one direction, or they may be applied first in one direction and then in the reverse; they may be applied gradually, or suddenly in the nature of a shock.

A **steady** or **dead load** is one which is always applied steadily in the same direction. A **live load** is one which is alternately applied and removed. A **suddenly applied load** is one imposed instantaneously but without initial velocity. If the load is applied with initial velocity as in the case of a blow from a falling body, the member is subjected to **impact**.

8. Nature of Straining Actions, Stress, and Strain. Since all materials of construction are more or less elastic a machine element must change its form to some extent whenever subjected to a load. This change of form may be very small and temporary; it may be a permanent distortion; or if the load applied be heavy enough the element may even be ruptured. Such change of form, whether temporary or permanent, is called a **strain**. When a machine member is thus distorted under a load certain molecular reactions, equal and opposite to the load applied, are set up within the material and resist the deformation. **Stress** is the term applied to this internal reaction and is to be clearly distinguished from **strain**, stress being in the nature of a force and strain being a dimension.

The character of the straining action and of the stress which results from a given load depend upon the direction and point of application of the load (or forces), and upon the form, the position, and the arrangement of the supports of the member. A given load may produce tension, compression, shearing, flexure, or torsion or a combination of these. Of course tension and compression cannot both exist at the same time between any pair of molecules. Flexure is a combination of tensile and compressive stresses between different sets of molecules; or, as it is often expressed, in different fibres* of the same body. Torsion is a special form of shearing stress. Owing to the frequent occurrence of flexure and torsion it is convenient to treat these as elementary forms of stress.

The stresses due to tension, compression, and flexure are essentially molecular actions normal to the planes separating adjacent sets of interacting molecules; that is, the stresses increase or decrease the distances between these molecules along lines connecting them.

The primary straining effect of shearing and torsional actions is displacement of adjacent molecules, between which the stress

* It should be noted that the term fibre is used in a conventional sense when discussing homogeneous metals, such as iron and steel.

acts, tangentially to the planes separating such molecules. In uniform shear the interacting molecules move or are strained relatively with a rectilinear translation. In torsional action the adjacent molecules each side of a plane of stress have a relative motion or strain about an axis. A brief reflection will show that in reality only two kinds of strain exist, namely, elongation (contraction if negative) and shearing. In a similar way only two corresponding kinds of stress are met with, namely, normal or direct, and tangential or shearing. But for convenience it is much more desirable to treat the special cases previously mentioned, separately as elementary stresses. (See Church's Mechanics, page 201.)

Machine members are often subjected to combinations of these simple stresses, as flexure and torsion. Such stresses are called **Compound Stresses** and will be more fully treated later.

When a load is applied to a piece of material the strain which results is a function of the load and of the character of the material involved. In general for a given loading the deformation is different for different materials but constant in its relation to stress for any one material. These relations have been determined experimentally for all the ordinary materials used in engineering, and works on mechanics of materials treat of the subject fully. Enough will be inserted here to make the discussion complete.

If a bar of metal is tested under an increasing tensile load and the strain caused by each successive load is accurately observed the relation between stress and strain can be shown graphically as at $O\ a\ d\ e$ Fig. 6; such a diagram is called a **stress-strain** diagram.

If axes $O\ X$ and $O\ Y$ are chosen and the stresses plotted as ordinates and strains as abscissas, it will be found that up to a certain point as a, either in tension or compression, the curve so formed is sensibly a straight line; that is, **stress is proportional to strain.** Further, if at any point below a the stress is released, the piece returns to its original shape. But above a this relation ceases, strain usually increases* faster than stress, till finally

*Ordinary rubber is an exception to this general rule, strain decreasing as stress increases.

3

rupture occurs. If at any point beyond a the stress is released, it is found that the piece no longer returns to its original dimensions but has been permanently distorted.

If at any point on the curve below a the stress be divided by the strain a ratio is obtained which is constant for all points below a. This ratio is called the **modulus or coefficient of elasticity.** If, therefore, this modulus of elasticity is known for a given material, the strain corresponding to any given load may be calculated, providing it does not exceed the value corresponding to the point a.

The point a is called the **elastic limit** and is well-defined in most materials. Cast iron has, however, no well-defined elastic

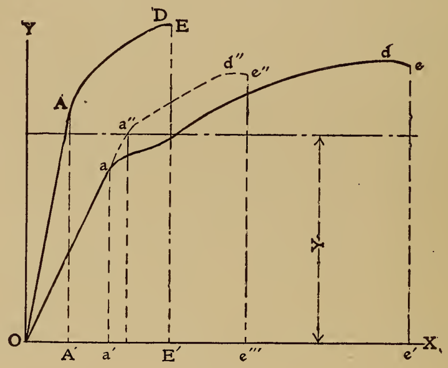

FIG. 6.

limit and little permanent elongation. Materials of this kind are said to be *brittle.*

If sufficient tensile stress is applied to a test piece its elongation increases until finally it "necks down" at its weakest point and rupture occurs. The load per unit area under which a bar breaks is called its **ultimate strength** and the corresponding stress or load per unit area is called the **ultimate stress.** Similar phenomena are observed when a piece is tested in compression or torsion, etc.

It is evident that the working stress of a machine member must be less than the elastic limit if the piece is to retain permanency of form. The stress at which a member is designed to be operated is called the **working stress** and the ratio of the ultimate stress to the working stress is called the **factor of safety.** It is to be especially noted that not only must the working stress in the member be kept below the value where permanent deformation takes place, but also so low that the resulting *strain*, whatever it may be, shall be so small as not to destroy the proper alignment of the piece, or cause unnecessary friction through distortion. A machine member may be amply strong enough to carry the load with perfect safety, and yet distort so badly under the load as to render it unfit for the service desired. Both *strength* and *stiffness* should therefore be kept in mind in designing a machine part, as sometimes one and sometimes the other will dictate the form and dimensions to be used. A short discussion will now be given of the relations which exist between load, stress, and strain for the cases most often met and of their bearing on the selection of the form and size of a machine member. In this discussion it will be assumed that the load is a dead load applied without shock, and the modifying effect of suddenly applied and repeated loads will be considered after the fundamental relations between load and stress are established.

9. **Tension.** Let p be the stress in the section, P the load, and A the area of cross section. The relation which exists between them in simple tension is

$$p = \frac{P}{A} \quad \ldots \ldots \quad (A)$$

And if E be the coefficient of elasticity and l the length of the member, the total elongation $\triangle$ is given by the equation

$$\triangle = \frac{P\,l}{A\,E} \quad \ldots \ldots \quad (B)$$

The elongation per unit of length or the strain $= \frac{\triangle}{l}$.

If, then, a tension member is to be designed to join two machine parts, the formula for strength dictates a piece of uniform cross section without regard to any particular form. Hence the most convenient or cheapest form would be used, avoiding thin, wide sections where concentrated stress at the edge might cause undue strain.

Suppose it is required to hold the two surfaces within certain limits, as is often the case in machine tools where accuracy is desired. If the tension member is long it may yield more than is desirable, though the working stress may be well below the elastic limit and a greater area may be necessary to reduce $\triangle$ to the desired value.

Example. Let $P=20{,}000$ lbs., let the allowable stress $p=10{,}000$ lbs., let $E=30{,}000{,}000$, let $l=40''$, and let it be required to keep $\triangle$ within $.001''$. If the design is based on allowable stress alone,

$$A=\frac{P}{p}=\frac{20{,}000}{10{,}000}=2 \text{ square inches.}$$

But for $\triangle = .001$, $A \doteq \dfrac{Pl}{\triangle E}=\dfrac{20{,}000\times 40}{.001\times 30{,}000{,}000}=26$ sq. in.

In general, therefore, where tension members are of any considerable length and distortion under load is of importance, they should be checked as above.

10. Compression. If the member under consideration be subjected to compression, the remarks of the last paragraph apply equally well if the member can be considered a short column, *i.e.*, one whose length is not greater than six times its least diameter. If longer than this it must be considered as a long column and the conditions governing its design will be more fully treated hereafter. (See Art. 20.)

11. Shear. If the member is subjected to simple shear the expressions for the relations existing between the stress, area, and load are similar to those for tension or

$$p_s=\frac{P}{A} \quad \ldots\ldots \quad (C)$$

12. Torsion. If the member is subjected to a torsional stress, the following relations exist:

Let P = load applied in pounds.

a = arm of load in inches.

Let I_p = polar moment of inertia of the section in biquadratic inches.

p_s = shearing stress in lbs. per unit area at outer fibre.

e = distance from neutral axis to outer fibre in inches.

l = length of member in inches.

θ = angle of deformation in radians.

T = twisting moment applied to member in inch pounds,

E_s = transverse coefficient of elasticity.

Then for torsional strength in general,

$$P\,a = T = \frac{p_s I_p}{e} \quad . \quad . \quad . \quad . \quad . \quad (D)$$

For a circular shaft of solid section,

$$T = \frac{p_s \pi d^3}{16} \quad . \quad . \quad . \quad . \quad . \quad . \quad (E)$$

For a hollow circular section whose outside and inside diameters are d_1 and d_2 respectively,

$$T = \frac{p_s \pi (d_1^4 - d_2^4)}{16\, d_1} \quad . \quad . \quad . \quad . \quad . \quad (F)$$

For deformation under stress for a solid circular section, which is the most common case,

$$\theta = \frac{32\, T l}{\pi E_s d^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad (G)$$

and for a hollow circular section,

$$\theta = \frac{32\, T l}{\pi E_s (d_1^4 - d_2^4)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (H)$$

An inspection of equation (D) shows that the torsional resistance for a given stress is proportional to the polar moment of inertia divided by the distance from the neutral axis to the outer fibre. Examination of equations (E) and (F) shows that for circular sections torsional strength is proportional to the third power of the outer diameter. Equations (G) and (H) show that torsional deformation is inversely proportional to the fourth power of the outer diameter, hence torsional stiffness is directly proportional to the fourth power of the outer diameter.

For a given amount of material that section in which this material is distributed farthest from the gravity axis will be strongest and stiffest as long as the walls of the section do not become so thin and weak as to yield locally from other causes. The hollow circular and hollow rectangular sections, commonly called the "box section," Fig. 7, are best adapted, therefore, to resist tor-

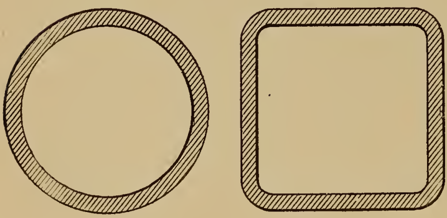

Fig. 7.

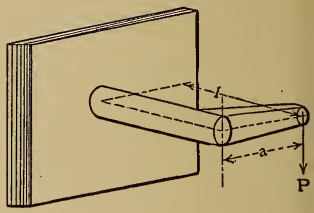

Fig. 8.

sional strains. The box section is peculiarly useful in machine construction, as many machine members must carry a combination of stresses. Machine frames may be subjected to tension, compression, or shearing, combined with torsion, and the box section, while equally good for simple stresses, is, as has been noted, vastly superior in torsion. Furthermore, the box section is well adapted to resist combined flexure and torsion. The flat sides of a box section also afford facilities for attaching auxiliary parts and its appearance is one of strength and stability. The thickness of the walls being thinner in hollow than in solid forms insures a better quality of metal in castings and also more skin surface, where the greatest strength of cast iron lies. An advantage not to be overlooked in some lines of work is the ease with which hollow sections can be strengthened by increasing the thickness of the walls by changing the core without changing the external dimensions. The cost of pattern work is about the same, in general, for hollow sections as for I or other sections, while the work in the foundry is, in general, a little greater.

Example. A circular cast iron boring bar 60 inches long carries a solid circular boring head 60 inches in diameter. The bar is subjected to a torsional moment of 60,000 inch pounds which is applied at one end. It is desired to keep the torsional deflection of the tool below $\frac{1}{32}''$ when the bar is transmitting power through its entire length, in order to prevent chattering of the tool. What should be the diameter of the bar if the working stress be taken as 3,000 pounds per square inch and E_s be taken as 6,000,000.

For torsional strength from formula *E*,

$$d^3 = \frac{60{,}000 \times 16}{3{,}000 \times \pi} = 100$$

$$\therefore d = 4.6''.$$

For torsional stiffness $\theta^* = \dfrac{\frac{1}{32}}{30} = \frac{1}{960}$ since θ is in radians and the length of an arc $= r\theta$, where r = radius.

$$\therefore \text{ from } G,\ d^4 = \frac{32 \times 60{,}000 \times 60}{\pi \times 6{,}000{,}000 \times \frac{1}{960}} = 5{,}870$$

hence $d = 8.8''$. It is evident that the shaft will be amply *strong* if designed for stiffness, therefore the last value would be used.

If the section is made hollow less metal can be used. In this case either the inside or outside diameter or the ratio between them can be assumed. Let

$$\frac{d_2}{d_1} = \frac{3}{4} \text{ or } d_2 = \frac{3d_1}{4}$$

$$\text{whence, } d_2^4 = \frac{81\, d_1^4}{256} \text{ and } d_1^4 - d_2^4 = \frac{175}{256} d_1^4$$

Substituting in *H*,

$$d_1^4 - d_2^4 = \frac{175}{256} d_1^4 = \frac{32 \times 60{,}000 \times 60}{\pi \times 6{,}000{,}000 \times \frac{1}{960}}$$

$$\therefore d_1^4 = 8{,}550 \text{ and } d_1 = 9.6'', \text{ hence } d_2 = 7.2''.$$

The area of the hollow shaft $= 31.67$ sq. in., while the area of

* The angular deflection or twist of a shaft in degrees $= 57.296 \times$ (Angular deflection in radians).

the solid shaft = 60.84 sq. in., so that with a small increase in diameter one half the metal secures, by using the hollow section, the same stiffness.

13. **Compound Stresses.** In the cases of simple loading just discussed only one form of stress is brought on the member and the design of the cross-section can be safely based on this stress. When, however, the loads applied induce stresses of several kinds, it is no longer possible in general to base the design on any one stress, but regard must be had to the *combination of stresses* that may occur. In many cases one or more of the stresses are so small, or their action is such, that they may be neglected in designing the member, though they should always be borne in mind. The stress on which the design of the member is based may be called the **predominating or primary stress** and it may be a simple stress or a combination of simple stresses. The latter will be called a **Compound Stress.**

14. **Flexure.** When a beam is subjected to simple bending the principal stresses that are induced are (a) a tension on one side of the neutral axis, (b) a compression on the other side of the neutral axis, and (c) a shearing stress which acts on every section of the beam at right angles to the tension and compression. Generally speaking, the shearing stress is small compared with the tension or compression and can often be neglected. It must never be forgotten, however, and where the beam is designed to withstand the bending moment only, care should be exercised that the sections which are subjected to a small bending moment are not made so small as to yield under shear. The predominating stress will be the tension or compression depending on the material and the form of section.

When a beam is subjected to simple flexure,—

Let M = bending moment at any section in inch pounds.
I = moment of inertia of section in biquadratic inches.
e = distance from neutral axis to outermost fibre in inches.
$\triangle$ = deflection at any point in inches.
P = load applied in pounds.
p = maximum stress at outer fibre in lbs. per sq. inch.
E = coefficient of elasticity.

Then for strength, in general, within the elastic limit,

$$M = \frac{p\,I}{e}\,* \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (J)$$

Every beam when loaded deflects somewhat, depending on the shape of its cross-section, the material, the way in which it is supported, and the load applied. The curve assumed by a beam loaded within the elastic limit is called the elastic curve and is of course different for different combinations of the above conditions. The general equation of the elastic curve, whatever the shape of the beam may be, the load, or manner of support is, $\frac{d^2 y}{d x^2} = \frac{M}{E I}$. To find the particular equation for any case, M must be expressed in terms of x and the expression integrated twice. The ordinate y, which is the deflection, can then be found for any value of x and its greatest value is the maximum deflection. This integration has been performed for all the cases usually met with in practice, and the results are tabulated in Table I. It is to be noted that this tabulation is for beams of *uniform section* and for stresses within the elastic limit. Here, as in other classes of machine members, the design of the part may be based on strength or stiffness, depending on the conditions, and in general both should be considered.

Example. A steel I beam 20 ft. long and supported at the ends is used as a track for a crane trolley carrying 4,000 lbs. Select a standard rolled I beam that will carry the load with a deflection of not more than $\frac{3}{16}''$ at the centre and a maximum stress of not more than 8,000 lbs.

From Table I,

$$\triangle = \frac{3''}{16} = \frac{P\,l^3}{48\,E\,I} = \frac{4{,}000 \times 240^3}{48 \times 30{,}000{,}000 \times I}$$

$$\text{whence } I = \frac{4{,}000 \times 240^3 \times 16}{48 \times 30{,}000{,}000 \times 3} = 205.$$

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* The expression $\frac{I}{e}$ is sometimes called the modulus of the section and is generally indicated by the letter Z. It should be noted, however, that this expression is applicable only to symmetrical sections as e may have two values for other sections. $\frac{p\,I}{e}$ is termed the resisting moment.

From handbooks on structural shapes it is found that the moment of inertia of a 12″ I beam weighing 31.5 lbs. per foot is 215.8. Let such a beam be chosen. Then from formula J, the stress $p = \frac{Me}{I} = \frac{2{,}000 \times 10 \times 12 \times 6}{205} = 7{,}000$ lbs. nearly. The section therefore is satisfactory.

15. Beams of Uniform Strength. The values in Table I refer to beams of uniform cross-section. In nearly all cases the bending moment, which is usually the basis of design, varies and if, therefore, the beam is made strong enough at its most strained section and uniform in cross-section throughout its length it will have an excess of material at every other section.* Sometimes it is desirable to have the cross-section uniform, while in other cases the metal can be so distributed that every section shall have the necessary strength to resist the bending moment and no more. In the latter cases the shearing stress must be looked after carefully. Table II gives a few of the forms most usually met and an example may make their application clear.

Example. A cantilever of rectangular section 30 inches long carries at its outer end a load of 1,000 lbs. It is to have a uniform thickness. What is its vertical outline so as to have uniform strength?

Let the thickness $= b$ and the variable height $= y$. Then the moment at any section at a distance x (Fig. I, Table 2) is Px, and this must be equal to the resisting moment of the section at each point, hence

$$Px = \frac{pI}{e} = \frac{pby^2}{6} \quad \text{or} \quad y^2 = \frac{6Px}{pb}$$

which is the equation of a parabola whose vertex is at the outer end of the beam. In the problem assumed let $b = 1.5$ inches and let $p = 4{,}000$ lbs. Then when $x = 30$, $y = h = 5.5''$. In a similar way other points may be found or the curve may be laid out by graphical method. The shearing load at any point is P, and hence the shearing stress increases as the cross-section of the

* This of course does not cover the possible case where the effect of shearing or other stresses may exceed that due to flexure.

beam decreases. When $x=0$, $y=0$, and in general when x is small, y is very small; therefore the outer end of the member must be modified so as to safely carry the shearing stress. Reference will be made to this again under the section dealing with machine attachments (see chap. 16). It is to be especially noted that these theoretical shapes are based on certain assumptions and unless these are observed in the design, the theoretical outlines do not apply. Thus in the cantilever example above, if the thickness of the beam is not kept uniform the outline for uniform strength is *not* a parabola. *The mistake of using a parabola when the thickness is not uniform is often made* when *I* or *T* sections are used instead of uniform thickness or depth. It is evident, that, whatever may be the form of section adopted, by means of the bending moment and shearing load the correct depth of section can be found for a number of points and a curve plotted that will answer the requirements of uniform strength.

16. **Combined Flexure and Torsion.** Let the force P, Fig. 8, act upon a rod with an arm a at a distance from the support equal to l. Then the stresses induced in the section close to the support are

(a) flexure due to the bending moment Pl
(b) torsion " " " twisting " Pa
(c) shearing " " " direct load and equal to $\frac{P}{A}$.

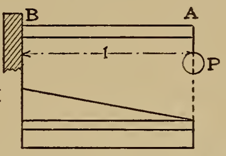
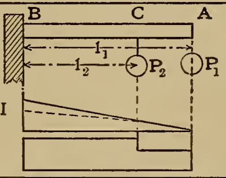
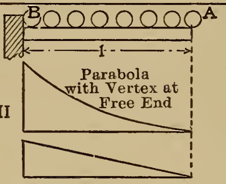
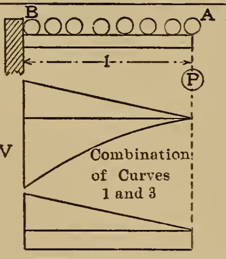
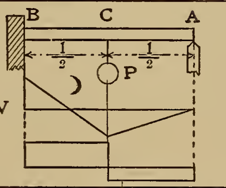
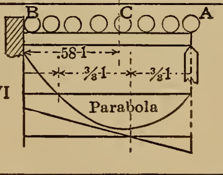
The shearing stress is usually very small compared to that due to bending and twisting, and can be neglected; the predominating stress therefore is that due to the combined action of the bending and twisting moments.

It can be shown that if a bar or rod is subjected to a longitudinal tensile or compressive stress and at the same time to a shearing stress at right angles to its length, the combination of these stresses may produce similar stresses greater than either and acting along planes other than those along which the original stresses act.*

* Church's "Mechanics," page 317.

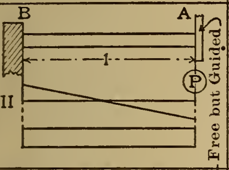
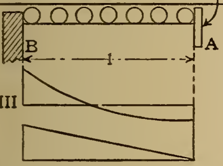
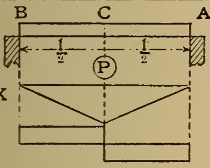
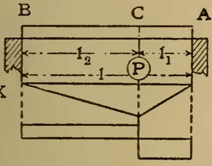
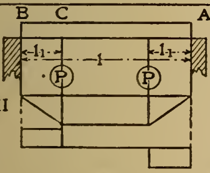
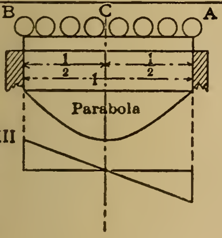
TABLE I

BEAMS OF UNIFORM SECTION

Diagram of Loads, Bending Moments and Shear		Greatest Bending Moment M	Location of M	Greatest Deflection Δ	Location of Δ	Maximum Shearing Force	Section where Shear is Max.
I 	Cantilever	Pl	B	$\frac{Pl^3}{3EI}$	A	P	Any
II 	Cantilever	$P_1l_1+P_2l_2$	B		A	P_1+P_2	From C to B
III 	Cantilever	$\frac{wl^2}{2}$	B	$\frac{Wl^3}{8EI}$	A	$wl = W$	B
IV 	Cantilever	$\frac{wl^2}{2}+Pl$	B	$\frac{l^3}{EI}\left[\frac{P}{3}+\frac{wl}{8}\right]$	A	$wl + P$	B
V 	Fixed at B Supported at A	$-\frac{5}{32}Pl \rightarrow$ C $+\frac{3}{16}Pl \rightarrow$ B	C B	$\frac{Pl^3}{107EI}$ At	0.45l from A	$\frac{11}{16}P$ $\frac{5}{16}P$	B to C C to A
VI 	Fixed at B Supported at A	$-\frac{9wl^2}{128} \rightarrow$ C $+\frac{wl}{8} \rightarrow$ B	C B	$\frac{wl^3}{185EI}$	C	$\frac{3wl}{8}$ $\frac{5wl}{8}$	A B

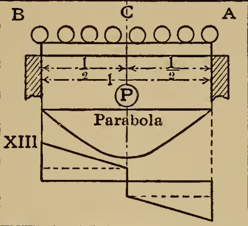
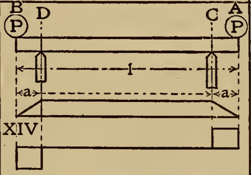
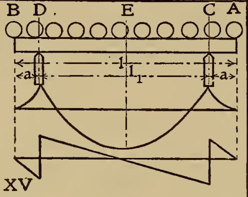
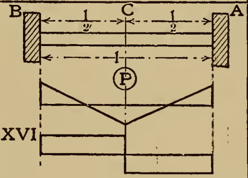
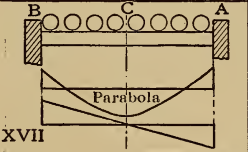
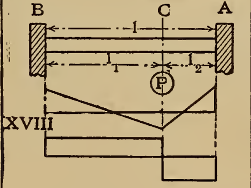
w = load per unit length. W = total distributed load. P = concentrated load.

TABLE I—*Continued*

Diagram of Loads, Bending Moments and Shear		Greatest Bending Moment M	Location of M.	Greatest Deflection Δ	Location of Δ	Maximum Shearing Force	Section where Shear is Max.
VII 	One End Fixed Other Free but Guided.	$\frac{Pl}{2}$	A and B	$\frac{Pl^3}{12\,EI}$	A	P	Equal in All
VIII 	One End Fixed Other Free but Guided.	$\frac{wl}{3}$ $-\frac{wl}{6}$ 0	At B At A At 0.4231 from B	$\frac{Wl^3}{24\,EI}$	A	W	B
IX 	Supported at both Ends Load Central.	$\frac{Pl}{4}$	C	$\frac{Pl^3}{48\,EI}$	C	$\frac{P}{2}$ $-\frac{P}{2}$	B to C C to A
X 	Supported at both Ends, Load not Central.	$\frac{Pl_1l_2}{l}$	C	$\frac{Pl_1^2l_2^2}{3l\,EI}$ Not Max.	At C	$\frac{Pl_2}{l}$ $-\frac{Pl_1}{l}$	B to C C to A
XI 	Supported at both Ends, Two Symmetrical Loads.	Pl_1	C to D	$\frac{PA}{24\,EI} \times \left[3l^2-4l_1^2\right]$	Centre	P	B to C D to A
XII 	Supported at both Ends, Uniform Load.	$\frac{wl^2}{8}$	Centre	$\frac{5\,Wl^3}{384\,EI}$	Centre	$\frac{wl}{2}$	A or B

w = load per unit length. W = total distributed load. P = concentrated load.

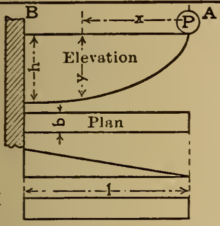
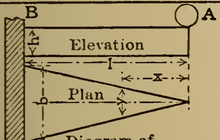
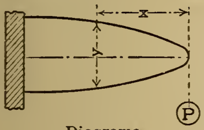
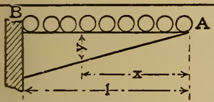
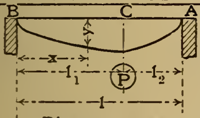
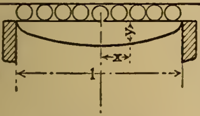
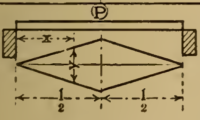
TABLE I—*Continued*

Diagram of Loads Bending Moments and Shear		Greatest Bending Moment M	Location of M	Greatest Deflection Δ	Location of Δ	Maximum Shearing Stress	Section where Shear is Max.
 XIII	Supported at both Ends	$[P+\frac{wl}{2}]\frac{l}{4}$	C	$(P+\frac{5}{8}W)\times\frac{l^3}{48EI}$	C	$\frac{P}{2}+\frac{W}{2}$	A or B
 XIV	Two Symmetrical Loads Overhanging Supports	Pa	D to C			P	in a
 XV	Uniform Load Symmetrical Supports	$\frac{W}{l}\left[\frac{l_1^2}{8}-\frac{a^2}{2}\right]\rightarrow E$ $+\frac{Wa^2}{2l}$	C and D			$\frac{W}{2}-wa$	C and D
 XVI	Fixed at both ends One Load at Centre	$\frac{Pl}{8}$	at A.B. and C.	$\frac{Pl^3}{192EI}$	C	$\frac{P}{2}$ $-\frac{P}{2}$	B to C C to A
 XVII	Fixed at both ends Uniform Load	$\frac{wl^2}{12}$	A or B	$\frac{wl^4}{384EI}$	C	$\frac{wl}{2}$	A or B
 XVIII	Fixed at both ends Uniform Load	$\frac{Pl_1l_2^2}{l^2}$ $\frac{2Pl_1^2l_2^2}{l^3}$ $\frac{Pl_2l_1^2}{l^2}$	at B at C at A	$\frac{2Pl_1^2l_2^2}{3EI(l+2l_1)^2}$			

w = load per unit length. W = total distributed load. P = concentrated load.

TABLE II

BEAMS OF UNIFORM STRENGTH

	Outline of Beam	Greatest Bending Moment M	Location of M	Greatest Deflection Δ	Location of Δ	Maximum Shearing Stress	Section where Shear is Max.
I 	Parabola with Vertex at P. Rectangular Section. Equation, $y^2 = \frac{6Px}{bp}$	Pl	B	$\frac{8Pl^3}{Ebh^3}$	A	P	any
II  Diagram of Moments and Shear same as I	Rectangular Section, Equation of Triangular Plan, $y = \frac{6P}{h^2 p} x$	Pl	B	$\frac{6Pl^3}{Ebh^3}$	A	P	any
III  Diagrams as in I	Circular Section of variable Diameter $y^3 = \frac{32P}{\pi p} x$					P	any
IV  Diagrams as in No. III Table I	Breadth b Uniform $y^2 = \frac{3w}{pb} x^2$	$\frac{wl^2}{2}$	B			W	B
V  Diagrams same as No. X Table I	Breadth, Uniform Two Parabolas $y^2 = \frac{6Pl_1}{bpl} x$ $y^2 = \frac{6Pl_2}{bpl} x$	$\frac{Pl_1 l_2}{l}$	C	when $l_1 = l_2$ $\Delta = \frac{Pl^3}{2Ebd^3}$	C		
VI  Diagrams same as No. XII Table I	Breadth, Uniform Elliptical Outline $y^2 = \frac{3w}{4pb}\left[l^2 - 4x^2\right]$	$\frac{wl^2}{8}$	Centre				
VII  Diagrams same as No. IX Table I	Depth, Uniform $y = \frac{6P}{2h^2 p} x$	$\frac{Pl}{4}$	Centre	$\frac{3Pl^3}{8Ebd^3}$			

w = load per unit length. W = total distributed load. P = concentrated load.

If t be the greatest *direct* tensile or compressive stress and s the greatest *direct* shearing stress applied to the bar, then the maximum tensile or compressive stress p due to t and s is given by the following equation:

$$p = \tfrac{1}{2}[t + \sqrt{t^2 + 4s^2}] \quad . \quad . \quad . \quad . \quad (1)$$

and the maximum shearing stress p_s due to t and s is

$$p_s = \tfrac{1}{2}\sqrt{t^2 + 4s^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It is evident that the numerical value of p will always exceed that of p_s and therefore if the material used has approximately the same tensile and shearing strength the design can be safely based on (1). But should the allowable shearing strength of the material be less than the tensile strength, as is usually the case, it may happen that the shearing stress p_s as found by (2) would dictate a larger section than that required by p as found by (1).

If the tensile stress is due to a bending moment and the shearing stress is due to a twisting moment the values of s and t can be found from equations J and D respectively and p and p_s obtained as above in equations (1) and (2) respectively.

Example. A certain section of a circular cast iron shaft is subjected to a bending moment M of 10,000 inch lbs. and a twisting moment T of 60,000 inch lbs. The allowable tensile stress p, is 2,000 lbs. per square inch and the allowable shearing stress p_s, is 1,600 lbs. per square inch. It is required to design the cross-section of the shaft.

$$\text{From } J,\ t = \frac{M e}{I} = \frac{32\,M}{\pi\,d^3} = \frac{32 \times 10{,}000}{\pi \times d^3} = \frac{100{,}000}{d^3} \text{ nearly}$$

$$\text{and from } D,\ s = \frac{T e}{I_p} = \frac{16\,T}{\pi\,d^3} = \frac{16 \times 60{,}000}{\pi\,d^3} = \frac{300{,}000}{d^3} \text{ nearly}$$

$$\text{hence from (1)},\ p, = \frac{350{,}000}{d^3} \text{ and since } p = 2{,}000$$

$$\therefore\ d^3 = \frac{350{,}000}{2{,}000}$$

$$\text{or } d = 5.55''$$

$$\text{From (2)},\ p_s = \frac{300{,}000}{d^3} \text{ and since } p_s = 1{,}600,\ d^3 = \frac{300{,}000}{1{,}600}$$

$\therefore d = 5.8''$ or $\frac{1}{4}''$ greater than that given by (1). It is evident that the last value should be taken.

Equations (1) and (2) are general, and applicable to any and all sections, but for circular shafts operating under conditions that produce both bending and twisting it has been found convenient to make use of what may be called an **equivalent or ideal bending moment** which may be derived from equation (1) as follows.

Let M_e = the equivalent bending which will produce the maximum direct stress p.

Let M = the bending moment producing the direct stress t.
" T = the twisting moment producing the shearing stress s.
" r = radius of shaft.

$$\text{From } J,\ M = \frac{t\,I}{r} \text{ and } M_e = \frac{p\,I}{r},$$

$$\text{and from } D,\ T = \frac{s\,I_p}{r} = \frac{2\,s\,I}{r}$$

(Since $I_p = 2I$ for circular or other sections for which the moments of inertia about two perpendicular axes are equal.)

Multiply equation (1) through by $\frac{I}{r}$, whence

$$\frac{p\,I}{r} = \frac{1}{2}\left[\frac{t\,I}{r} + \sqrt{\frac{t^2\,I^2}{r^2} + \frac{4\,s^2\,I^2}{r^2}}\right] = M_e$$

$$\therefore \frac{p\,I}{r} = M_e = \tfrac{1}{2}M + \tfrac{1}{2}\sqrt{M^2 + T^2} \quad . \quad . \quad . \ (K)$$

In a similar manner an *equivalent twisting* moment can be deduced from (2) thus,

$$\frac{2\,p_s\,I}{r} = T_e = \sqrt{M^2 + T^2} \quad . \quad . \quad . \quad . \ (K_1)$$

The quantities M and T are usually large and the numerical work involved in solving K and K_1 can be simplified by writing $M = x\,T$, where x for any particular problem will be a known quantity.

Whence K reduces to,

$$M_e = \tfrac{1}{2}\,T\,[x + \sqrt{x^2 + 1}] \quad . \quad . \quad . \quad . \ (K_2)$$

and K_1 reduces to,

$$T_e = T\sqrt{x^2 + 1} \quad . \quad . \quad . \quad . \quad . \quad . \ (K_3)$$

4

It is to be especially noted that M_e and T_e are equivalent moments in a numerical sense only; that is, if a bending moment M and a twisting moment T are applied to a shaft, producing a tensile stress t and a shearing stress s respectively, then M_e is a bending moment which will give a stress equal to the maximum resultant tensile or compressive stress p, and T_e is a twisting moment which will give a stress equal to the maximum resultant shearing stress p_s, reference being made to the same section.

The application of these equations to the investigation of any existing shaft subjected to a bending moment M and a twisting moment T is obvious, and it remains to consider their application to the design of new shafts. It has been pointed out that the greater numerical value given by equation (1) does not necessarily indicate that a larger section will result from its adoption than would result from the use of equation (2). For the same reasons the greater numerical value of M_e, obtained from K may not give a larger section than would be obtained from T_e by applying K_1. It is necessary therefore to determine under what conditions each should be used for designing in order that the *maximum* diameter of shaft shall be found in all cases.

From J,
$$M_e = \frac{pI}{r} = \frac{p\pi d^3}{32}$$

Whence
$$d^3 = \frac{32\,M_e}{\pi p} = \frac{16}{\pi} \times \frac{2\,M_e}{p} \quad \text{. . . . . (3)}$$

In a similar way from E

$$d^3 = \frac{16}{\pi} \times \frac{T_e}{p_s} \quad \text{. . . . . . (4)}$$

Since in any given problem M and T are always known, M_e and T_e can always be found from K and K_1 (or K_2 and K_3) and since the allowable values of p and p_s can always be assigned, the diameter of the shaft d can always be determined from both equations (3) and (4) and the larger value selected as in the problem previously solved. It is, however, desirable to know, for any set of conditions, whether equation (3) or equation (4) will give the greater value of d without the necessity of solving both equations.

It is evident that in order that equations (3) and (4) may give the *same* diameter of shaft $\frac{2M_e}{p}$ must equal $\frac{T_e}{p_s}$ or $\frac{T_e}{2M_e} = \frac{p_s}{p}$ and that for conditions other than these, either equation (3) or equation (4) may give the greater diameter. It is therefore necessary to investigate the relations existing between $\frac{T_e}{2M_e}$ and $\frac{p_s}{p}$ for three sets of conditions.

(1) When equations (3) and (4) will give equal values of d.
(2) " equation (3) will give the greatest value of d.
(3) " " (4) " " " " " " "

(1) It has already been shown that equations (3) and (4) will give equal values of d when $\frac{p_s}{p} = \frac{T_e}{2M_e}$ or if $\frac{p_s}{p}$ be called y, then

$$y = \frac{T_e}{2M_e} = \frac{\sqrt{M^2 + T^2}}{M + \sqrt{M^2 + T^2}} \quad . \quad . \quad . \quad . \quad (5)$$

is the equation of a curve which expresses all the simultaneous values of $\frac{p_s}{p}$ and $\frac{T_e}{2M_e}$ for which equations (3) and (4) will give equal values of d. The value of either M or T in equation 5 may vary from zero to infinity and the most convenient way of plotting *simultaneous* values of M and T is to plot their *ratio*. If then, in equation (5), the relations as given in K_2 and K_3 be substituted for those in K and K_1 the equation becomes

$$y = \frac{T_e}{2M_e} = \frac{\sqrt{x^2 + 1}}{x + \sqrt{x^2 + 1}} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

which is the equation of a curve expressing all the simultaneous values of y $\left(\text{or } \frac{p_s}{p}\right)$ and x $\left(\text{or } \frac{M}{T}\right)$ for which equations K_2 and K_3 will give equal diameters of shaft.

It is desirable before plotting the curve to examine the limits between which x and y may vary. It is clear that for $M = 0$ $x = 0$, and for $T = 0$ $x = \infty$, hence the limits of x are 0, and ∞.

Using these same limits for M and T in equation (5) it is found that

$$\text{when } M = 0,\ y = 1 \text{ and } M_e = \frac{T_e}{2}$$

$$\text{and when } T = 0,\ y = \tfrac{1}{2} \text{ and } M_e = T_e$$

That is for all materials where the ratio of allowable shearing to tensile stress lies between 1 and ½ there are always simultaneous values of M and T for which equations (3) and (4) will give equal values of d. The curve giving these simultaneous values is shown in Fig. 9 and has been plotted from equation (6).

(2) If for any given value of $\frac{p_s}{p}$ within the limits 1 and ½ a

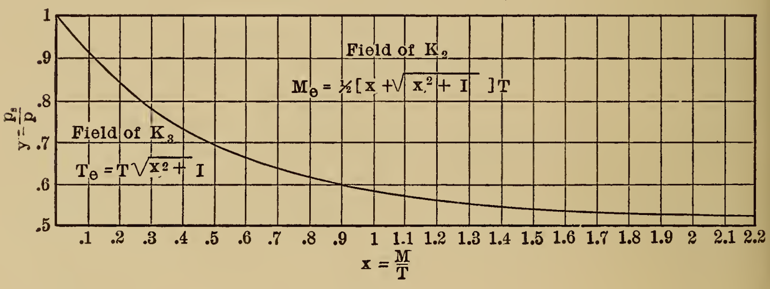


FIG. 9.

ratio of $\frac{M}{T}$ be taken greater than the simultaneous value given by the curve (or in other words if the co-ordinates chosen intersect above the curve) equation (3) will give the largest value of d. For the value of $\frac{M}{T}$ can be increased only by making M greater relatively to T and an examination of K and K_1 shows that increasing M increases K more rapidly than it does K_1. Hence in such cases K (or K_2) applies and equation (3) which is based on them will give the largest value.

Further, for values of $\frac{p_s}{p}$ equal to or greater than unity, equation (3) will also give the largest value of d. For it has just been shown that M_e can never be less than $\frac{T_e}{2}$ and only equals this when $M=0$. *For all finite values of M, therefore, M_e must be greater than* $\frac{T_e}{2}$; and it is evident from equations (3) and (4) that for values of $p_s = p$ and $M_e > \frac{T_e}{2}$ equation (3), which is based on K (or K_2), will give the greatest value of d.

(3) In a similar way it can be shown that for all simultaneous values of $\frac{p_s}{p}$ and $\frac{M}{T}$ which intersects below the curve and within the limits $y=1$ and $y=\frac{1}{2}$; or for all materials where $\frac{p_s}{p}$ is less than $\frac{1}{2}$, equation (4), which is based on K_1 (or K_3), will give the greatest value of d.

Summary. Equations K_2 and K_3 are the most convenient forms of equivalent moments and will be used in this work. It is to be particularly noted that they are applicable only to circular or other sections where the polar moment of inertia is equal to the sum of the rectangular moments of inertia around perpendicular axes (see page 50). Where the section to be considered is more complex the solution must be based on the original equations (1) and (2) in a similar manner to that employed in the example on page 49. Equation K_2 should be used where the simultaneous values of $\frac{p_s}{p}$ and $\frac{M}{T}$ intersect above the curve which is always the case whenever $\frac{p_s}{p} > 1$. Equation K_3 should be used where the simultaneous values of $\frac{p_s}{p}$ and $\frac{M}{T}$ intersect below the curve, which is always the case whenever $\frac{p_s}{p} < \frac{1}{2}$.

Example 1. An engine cylinder is 16″×24″ (piston 16″ in diameter and stroke of 24″), steam pressure = 100 lbs. per square inch. The centre of the crank pin overhangs the centre of the main journal by 15″ (measured parallel to the axis of shaft). Assume that the pressure on the crank pin may be equal to 100 lbs. unbalanced pressure per square inch of the piston when the connecting-rod is perpendicular to the crank radius. Allowing 8,000 pounds as the maximum allowable direct stress and 6,400 as the maximum allowable shearing stress, compute the diameter of the shaft.

Area of piston = 200 sq. inches; radius of crank (arm of maximum twisting moment) $r = 12''$; arm of bending moment $a = 15''$

$$\therefore \quad T = 200 \times 100 \times 12 = 240{,}000 \text{ inch lbs. Also}$$

$$M = 200 \times 100 \times 15 = 300{,}000$$

$$x = \frac{M}{T} = 15 \div 12 = 1.25; \quad \frac{p_s}{p} = \frac{6{,}400}{8{,}000} = .8 = y.$$

By referring to Fig. 9 it is seen that for $y = \frac{p_s}{p} = .8$ and $x = 1.25$ the ordinates intersect above the curve, hence K_2 should be used.

From K_2, $M_e = \frac{1}{2}\left[1.25 + \sqrt{1.25^2 + 1}\right] 240{,}000$
$= 342{,}000$ inch lbs.

From (3), $$d^3 = \frac{342{,}000 \times 32}{8{,}000 \times \pi} = 435$$

$$\therefore \quad d = 7.58''$$

Example 2.—A circular cast iron shaft is subjected to a twisting moment of 250,000 inch lbs. and a bending moment of 62,500 inch lbs. The allowable tensile stress is 2,000 lbs. per sq. inch and the allowable shearing stress 1,400 lbs. Determine the diameter of the shaft.

Here $y = \frac{p_s}{p} = \frac{1{,}400}{2{,}000} = .7$ and $x = \frac{62{,}500}{250{,}000} = .25.$

From the curve, Fig. 9, it is seen that for $y = \frac{p_s}{p} = .7$ and $x = .25$ the intersection of the ordinates falls below the curve, hence K_3 should be used.

Then $T_e = \left[\sqrt{(.25)^2 + 1}\right] 250{,}000 = 257{,}500$

$$d^3 = \frac{16 \times 257{,}500}{\pi \times 1{,}400} = 935$$

$$d = 9.75 \text{ inches.}$$

Suppose, however, that K_2 should be used.

Then, $M_e = \frac{1}{2}[.25 + \sqrt{(.25)^2 + 1}]\ 250{,}000 = 160{,}000$

$$d^3 = \frac{32 \times 160{,}000}{\pi \times 2{,}000} = 830$$

$\therefore$ $d = 9.4$ inches or $.35''$ less than the value given by K_2.

A convenient graphical solution of K_2 and K_3 is shown in Fig. 9 (a) which may be used as follows:

For K_3 make Oa = unity; lay off $Ob = x$ to the same scale on the vertical axis. Draw ab extending it beyond b for a length somewhat greater than x;

then $ab = \sqrt{Ob^2 + Oa^2} = \sqrt{x^2 + 1}$;

hence $T_e = ab \times T$.

For equation K_2, lay off $bc = Ob = x$ along the extension of ab. Then $bc + ab = ac = x + \sqrt{x^2 + 1}$;

Hence $M_e = \frac{1}{2}[ac \times T]$.

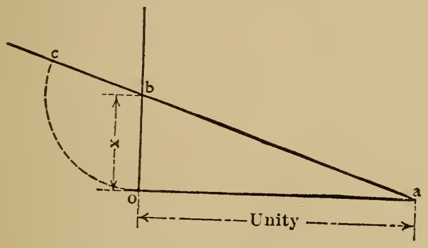


FIG. 9 (a).

17. Other Formulas.—Equation K is sometimes transformed into an *equivalent twisting moment.* Since in general $M = \frac{pI}{r}$ and $T = \frac{2p_sI}{r}$, for an equal intensity of stress (that is, $p_s = p$) $T = 2M$ for the same section. If therefore it is considered more convenient to use an equivalent twisting moment instead of an equivalent bending moment it is allowable to substitute for M_e (the bending moment, equivalent to the combined bending and twisting moment), $\frac{1}{2}T_e$ (a *twisting moment* equivalent to the combined bending and twisting moments) provided the same allowable *direct* stress is used with T_e in solving for the diameter of shaft.

$$\therefore T_e = 2M_e = M + \sqrt{M^2 + T^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (K_4)$$

Equations K_2, K_3, and K_4 are all different forms of Rankine's formula for combined bending and twisting. Other authorities give slightly different coefficients. Thus Grashof gives

$$M_e = \tfrac{3}{8} M + \tfrac{5}{8} \sqrt{M^2 + T^2} \quad . \quad . \quad . \quad . \quad (7)$$

While others give

$$M_e = 0.35\, M + 0.65 \sqrt{M^2 + T^2} \quad . \quad . \quad (8)$$

The diameter of shaft given by equations (7) and (8) will not differ much from that given by K_2, for any set of conditions, except where the bending moment is very small. At the limit where the bending moment M is equal to zero, Grashof's formula gives a value of M_e, 25 per cent greater than that given by K_2. But it may be noted that in general for all materials whose shearing strength is less than their tensile strength (and this is the case for most materials used in engineering) that when M is small or, in other words, when the shearing stress predominates, it is safer to use K_3 in preference to K_2. It will be found that for the range where equations (7) and (8) give values greater than K_2, that these values will still be less than those obtained from K_3 or at least not enough greater to warrant the use of a different formula in place of K_3. Take for example steel where $\frac{p_s}{p} = .8$ and $x = .1$, which is down close to the limit where Grashof's formula gives the greatest value compared to K_2. Expressing d^3 in terms of T as in equations (3) and (4),

$$\text{from } K_2 \quad d = 1.77 \sqrt[3]{\frac{T}{p}}$$

$$\text{from } K_3 \quad d = 1.84 \sqrt[3]{\frac{T}{p}}$$

$$\text{from Grashof's formula} \quad d = 1.88 \sqrt[3]{\frac{T}{p}}.$$

from which it is seen that the difference between d as determined by K_3 and Grashof's formula is negligible. The same evidently applies to equation (8) which differs but little from Grashof's. As the value of x increases, the difference between these equivalent *bending* moments decreases, and any variation is more than covered by the factor of safety which must be used.

18. Combined Torsion and Compression. Propeller shafts of steamers and vertical shafts carrying considerable weight are subjected to combined **twist** and **thrust.** The span, or distance between bearings, is frequently so small that the shaft may be considered as subjected to simple compression, so far as the action of the thrust is concerned.

The intensity of this compressive stress in such cases is

$$c = \frac{4P}{\pi d^2},$$

in which P = the thrust, and d = the diameter of the (solid circular) shaft.

If T = the twisting moment on the shaft, $r = \frac{1}{2}\,d$ = the radius of the shaft, I_p = the polar moment of inertia = $2\,I$ (= 2 times the rectangular moment of inertia), and s = the intensity of shearing stress due to T, then

$$T = \frac{s\,I_p}{r} = \frac{4\,s\,I}{d} \quad \therefore\ s = \frac{T\,d}{4\,I} = \frac{16\,T}{\pi\,d^3}$$

for solid circular shafts.

The resultant maximum stresses are those due to the combined actions of a normal stress (compression) and a tangential stress (shear) as in the case of combined bending and twisting (Art. 17); hence equations (1) and (2) of the preceding article apply and may be used to find the maximum compressive or maximum shearing stress; or if c be the compressive stress due to P, s be the shearing stress due to T, p_c the **maximum resultant compressive stress,** and p_s the **maximum resultant shearing stress,** then

$$p_c = \frac{1}{2}c + \frac{1}{2}\sqrt{c^2 + 4\,s^2} \text{ and } p_s = \frac{1}{2}\sqrt{c^2 + 4\,s^2}$$

$$\text{or } p_c = \frac{2\,P}{\pi\,d^2} + \frac{1}{2}\sqrt{\frac{16\,P^2}{\pi^2\,d^4} + \frac{4(16)^2\,T^2}{\pi^2\,d^6}} \text{ and } p_s = \frac{1}{2}\sqrt{\frac{16\,P^2}{\pi^2\,d^4} + \frac{4\,(16)^2\,T^2}{\pi^2\,d^6}}$$

$$\text{or } p_c = \frac{2}{\pi\,d^2}\left[P + \sqrt{P^2 + \frac{64\,T^2}{d^2}}\right] \quad \ldots\ldots\ldots \quad (L)$$

$$\text{and } p_s = \frac{2}{\pi\,d^2}\sqrt{P^2 + \frac{64\,T^2}{d^2}} \quad \ldots\ldots\ldots \quad (L_1)$$

It is difficult to find the value of d for a given value of p_c or p_s from the above equations, and it is much more convenient to assume a trial diameter d and then check for the values of p_c and p_s to see that they do not exceed the allowable compressive and shearing stresses of the material under consideration.

If the span of the shaft between bearings is so great that the shaft must be considered as a column likely to buckle, the trial diameter of the shaft may be taken so as to bring the mean compressive stress c well below the allowable value, and after solving for p_c and p_s the shaft may also be checked as a long column (Art. 20). In steel shafting it is necessary usually to apply equation (L) only, but it is well to check the shearing stress p_s against the allowable stress by applying (L_1).

19. Flexure Combined with Direct Stress. If the section XY, Fig. 10, be acted on by a force P at a distance from its gravity axis O equal to a, the stresses induced in the section will be:—

(a) A **uniformly distributed stress** due to the load P and equal to $\frac{P}{A}$ per unit area. This will be tensile or compressive, depending on the direction of P.

(b) A **flexural stress** due to the bending moment $P\,a$. This flexural stress will be a tensile stress on one side of the gravity axis which is at right angles to a, and compressive on the other.

If the direct stress induced in the section by the load P is tensile, then the flexural stress on the side toward the load is tensile. If the direct stress induced is compressive, the flexural stress on the side toward the load is compressive. The maximum stress will be the greatest algebraic sum of these combined stresses at the outer fibres at X or Y. The distribution of these stresses for both cases is shown graphically in Fig. 10, where tensile stresses are plotted above the line $U\,V$ and the compressive stress below; the ordinates under $r\,s$ representing the flexural stresses, and those under $m\,n$ the direct stresses. An inspection will show where the algebraic sum is greatest. In the case shown the *combined* compressive or *combined* tensile stresses at X are the greatest which may come on the section, depending on the direction of P. This is not necessarily so, as a brief reflection will show that if O be located near enough to X the reverse of the above conditions may exist. The form of section and location of the gravity axis should be fixed with reference to the relative tensile and compressive strength of the material used.

Let p' = the direct stress due to P,

Let p'' = the tensile or compressive stress due to $P\,a$,

Let p = the maximum stress in the section at X or Y.

Then from formula A, $p' = \dfrac{P}{A}$, and from formula J, $p'' = \dfrac{P\,a\,e}{I}$.

Therefore,
$$p = p' + p'' = \frac{P}{A} + \frac{P\,a\,e}{I}\text{*} \quad . \quad . \quad . \quad (M)$$

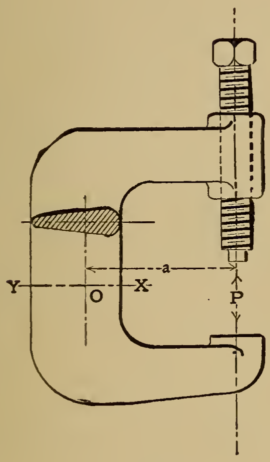


FIG. 10 (a).

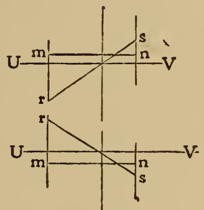


FIG. 10 (b).

where e is the distance from O to the outer fibre at either X or Y, depending on which is under consideration. If the material used is equally strong in tension and compression the gravity axis should not be far from central, but where cast iron is used it is advantageous to distribute the metal more toward the tension side, thus drawing the gravity axis toward that side. This increases e on the compression side, and hence increases the

* See Church's "Mechanics," p. 362.

compressive stress. It decreases e on the tension side, and hence decreases the tensile stress. Cast iron is much stronger in compression than in tension, and therefore a greater moment can be withstood by a given cross-sectional area when distributed in this manner.

It is not practicable, in general, to solve equation (M) for the direct determination of the dimensions of a cross section to sustain a given eccentric load P with an assigned intensity of stress p, because both A, I, and e are functions of the required dimensions; and with any but the simplest sections complicated functions result. With solid, square, or circular sections, or in general where only one dimension is unknown, it is possible to reduce M to a form which can be solved; but the algebraic expression is a troublesome cubic equation. The practical way is to assume a trial section and check this for P or p.

Example 1. A small crane (Fig. 11) has a clear swing of 28 inches. The section at $m\,n$ is shown by Fig. 11 (b). Find the load corresponding to a maximum fibre stress (compression) of 9,000 lbs. per square inch at n.

$$p = \frac{P}{A} + \frac{P\,a\,e}{I} \qquad \therefore\ P = \frac{p\,A\,I}{I + A\,a\,e}$$

$$a = 28 + 2 = 30 \qquad A = 2 \times 4 - 1.5 \times 3. = 3.5$$

$$I = \tfrac{1}{12}\,(2 \times 64 - 1.5 + 27) = 7.3$$

$$\therefore\ P = \frac{9{,}000 \times 3.5 \times 7.3}{7.3 + 3.5 \times 30 \times 2} = 1{,}060 \text{ lbs.}$$

Example 2. A punching machine (Fig. 12) has a reach of 22 inches. Maximum force P acting at the punch is taken at 70,000 lbs. Design the section $m\,n$ so that the maximum fibre stress at n (tension) shall be about 2,400 pounds per square inch, and check the compressive stress at m.

The general form of section best adapted to this case is that shown in Fig. 12 (b). Taking the trial dimensions as in Fig. 12 (b), the neutral axis is found to be 8″ from n.

$\therefore\ a = 22 + 8 = 30.$ It is also found that $A = 216$ and $I = 7{,}680.$

$\therefore$ at n,

$$p = \frac{70,000}{216} + \frac{70,000 \times 30 \times 8}{7,680} = 325 + 2,200 = 2,525 \text{ lbs.}$$

and at m,

$$p_c = -\frac{70,000}{216} + \frac{70,000 \times 30 \times 11}{7,680} = -325 + 3,000 = 2,675 \text{ lbs.}$$

The tensile stress is slightly greater than the limit assigned. If this excess is not considered permissible a stronger section must be taken. It is evident that this can be accomplished by massing the metal still further toward the tension side as the compressive stress is very low.

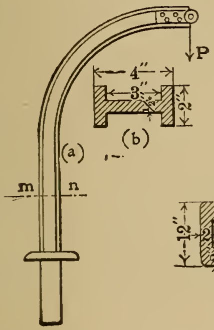


FIG. 11.

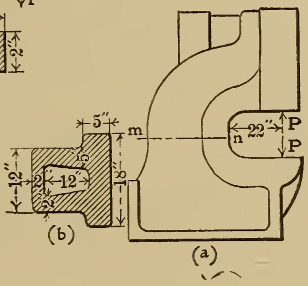


FIG. 12.

The theory regarding the position of the neutral axis given above is that in general use for such cases. Recent investigations have pointed out the fact that this theory is not absolutely rigid for curved beams. For the usual case of design it is believed that the above is sufficiently accurate.

20. Stresses in Columns or Long Struts. When a short bar is subjected to an **axial compressive load** the stress induced in each section is simple compression (see Art. 20), and the value of the stress p is given by formula (A) or

$$p = \frac{W}{A}.$$

If, however, the bar is **more than 4 to 6 times** as long as its least diameter, the above equation does not apply, as the bar will, if proportioned as above, deflect laterally under the load and will ultimately break under a compound stress due to compression and lateral bending. Such a member is called a **column.**

Theoretical equations for the design of columns were first developed by Euler. Other formulæ were later developed experimentally by Hodgkinson and Tredgold. Gordon and Rankine have also proposed equations for the design of this class of members. The student is referred to any good treatise on the Mechanics of Materials for a fuller discussion of these expressions than can be given in this work.

Let l = the length of the column in inches,
ρ = the least radius of gyration of cross-section,
I = the least moment of inertia of cross-section,
A = the area of the cross-section in square inches,
P_c = the *breaking* load on the column in pounds,
p_c' = the *mean* intensity of stress under the *breaking* load, or the *unit* breaking load, $= P_c \div A$.
p_c = the crushing strength of the material, or unit stress at the yield point. This is the maximum intensity of stress in the column when the mean intensity of stress is p_c',
n = the factor of safety,
P = the *working* load on the column in pounds, $P_c \div n$,
p' = the *mean* intensity of *working* stress, or *unit* working load, $= p_c' \div n = P \div A$,
p = the intensity of working stress in the column $(= p_c \div n)$. This is the maximum intensity of stress in the column when the mean intensity of stress is p_c'.
m = a coefficient for the end conditions as shown in Table 3.

Then **Euler's formula** for long columns is

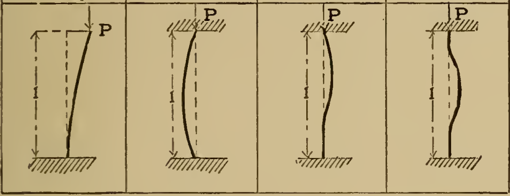
$$P_c = m \frac{\pi^2 E I}{l^2}.$$

It is to be especially noted that Euler's equation is rational and deduced from the theory of elasticity. The coefficient m is also rational and applicable to other forms of column formulæ. As will be shown later, the equation is strictly applicable only to very long columns.

Very short compression members, of ductile material, fail under stresses corresponding to, or only slightly in excess of, the

TABLE III

VALUES OF m FOR DIFFERENT END CONDITIONS			
CASE I	CASE II	CASE III	CASE IV
Fixed at one end, free at other.	"Pin Ended" Both ends free, but guided.	"Pin & Square" One end fixed, the other guided.	"Square Ended" Both ends fixed.
$m = \frac{1}{4}$	$m = 1$	$m = \frac{9}{4}$	$m = 4$



apparent elastic limit, or yield point; for when this stress is reached the metal flows, although it does not actually break. Very long columns may approximate the resistance as given by Euler's formula. Columns of lengths intermediate between compression members which yield by simple crushing and those which fail by pure flexure are weaker than the former and stronger than the latter. If a column is initially exactly straight, perfectly homogeneous, and subjected to an absolutely concentric load (that is, if it is an ideal column) there seems to be no reason why its strength should diminish rapidly with an increase of length, other conditions remaining the same.

However, even an ideal very long column would reach the condition of unstable equilibrium when subjected to a certain critical load (the greatest load consistent with stability). If the load is increased beyond this limit and a deflection is caused in any way, the deflection will increase until the stress due to flexure produces failure of the column. If a deflection is caused while the column is under a load less than this greatest load consistent with stability, the elasticity of the material tends to make the column regain its normal form. Initial defects in the form or structure of a column or eccentric application of load tend to produce such a deflection; hence long struts fail under smaller loads than short struts of similar material and cross-section, for the ideal conditions are not realized in practice. Or, in other words, for equal safety under a given load long columns must have a greater cross-section, and lower mean, or nominal, working stress.* Even in columns of moderate length, if of ductile material, the flow at the yield point causes buckling.

Merriman says that if the length of a compression member be only from four to six times its least "diameter," it may be treated as one which will yield by simple compression. Johnson gives limits within which the Euler formula should not be applied as $l \div \rho = 150$ for pin-ended, and $= 200$ for square-ended columns. Other authorities give somewhat different limits; but nearly all agree that most of the columns in ordinary structures and machines are intermediate between simple compression members and those to which Euler's formulæ apply. There have been a great many column formulæ proposed. A graphical representation of several of these formulæ is shown in Fig. 13. In this diagram, abscissas represent ratios of the length of column to the least radius of gyration of the cross-section, and the ordinates represent the nominal (mean) intensity of compressive stress. Or,

$$x = l \div \rho = l \div \sqrt{I \div A}, \text{ and } y = p'_c = P_c \div A.$$

The diagram is drawn for the ultimate resistance of pin-ended columns with a material having a crushing resistance, p_c (yield point) of 36,000 pounds per square inch, and a modulus of elasticity, E, of 29,400,000. The value of p'_c is 36,000 for a very short compression member, and it is evident that a long column could not be expected to have a greater strength; hence no formula should be used which would give a value of p_c' in excess of

* Owing to the flexure of the long column, the stress is not uniform across the section. The maximum intensity of stress must be kept within the compressive strength of the material; hence the mean stress is less than for shorter compression members, in which the mean stress is more nearly equal to the maximum.

the crushing resistance p_c. Referring to the diagram, it will appear that the Euler formula (represented by the curve $E\,E_1\,E_2$) cannot apply to pin-ended columns (of this particular material) in which $l \div \rho < 90$. If columns with a ratio of l to ρ less than this limit yielded by simple crushing, and those with a greater ratio of l to ρ followed Euler's formula, the straight line $F\,F_1$ and the curve $F_1\,E_1\,E_2$ would give the laws for all lengths of columns. It is not reasonable to expect such an abrupt change of law in passing this limit ($l \div \rho = 90$); and, as already stated, columns of moderate length fail under a mean stress considerably less than the simple crushing resistance of the material; or the strength

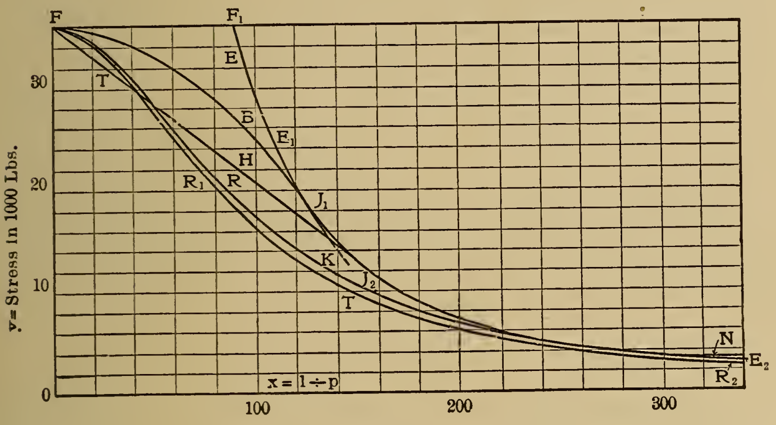


FIG. 13.

of columns is inversely as some function of the length divided by the "least diameter."

Mr. Thomas H. Johnson has developed a formula which is based on the assumption that the strength of the column may be taken inversely as $l \div \rho$. This expression is

$$p_c' = \frac{P_c}{A} = p_c - k\frac{l}{\rho} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which the coefficient k has the value,

$$k = \frac{p_c}{3}\sqrt{\frac{4\,p_c}{3\,m\,\pi^2 E}}.$$

This formula is represented by the straight line $T\ H\ J_2$ in Fig. 13. It will be noted that this line is tangent to the Euler curve at J_2, and the equation of the latter is to be used, should the columns exceed the length corresponding to this point of tangency ($l \div \rho > 150$). This expression is very simple, after k has been determined. It is very convenient in making a large number of computations for columns of any one material, and it is employed in structural work to a considerable extent. It does not appear to have any advantage, on the ground of simplicity, when some particular value of k does not apply to several computations.

For determination of nominal working stress, p_c' (as computed above) may be divided by a suitable factor of safety, n. Or if $p_c' \div n = p'$, the expression may be put in the following form for direct computation of mean working stress.

$$p' = \frac{p_c}{n} - \frac{k' l}{n \rho} = p - \frac{p}{3}\sqrt{\frac{4 p_c}{3 m \pi^2 E}}\frac{l}{\rho} \quad . \quad . \quad . \quad (2)$$

Professor J. B. Johnson has derived a formula from the results of the very careful experiments of Considère and Tetmajer. His formula is:

$$p_c' = p_c - \frac{p_c^2}{4\pi^2 E}\left(\frac{l}{\rho}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

for pin-ended columns. The curve $F\ B\ J_1$ (Fig. 13) represents this expression. This curve is a parabola tangent to the Euler curve, and with its vertex in the axis of ordinates at F, the direct crushing stress of the material. For columns having $l \div \rho$ greater than the value corresponding to the point of tangency J_1 (should such be used), the Euler formula is to be employed. The formula of Professor Johnson's is empirical, but it agrees remarkably well with very refined experiments on *breaking* loads. It gives considerably higher values for allowable stress than other generally accepted formulas, probably because it is based upon more refined tests, or upon conditions further removed from those in practice.

Professor Johnson says ("Materials of Construction," pages 301–302) that both Bauschinger and Tetmajer "mounted their columns with cone or knife-edge bearings at the *computed* gravity axis, while M. Considère mounted his with lateral screw adjustments, and arranged a very delicate electric contact at the side so as to indicate a lateral deflection as small as 0.001 mm. He then applied moderate loads to the columns and adjusted the end bearings until they stood under such loads rigidly vertical, with no lateral movement whatever."*

It would appear that this precaution tends to make the test one of the *material and not of a long strut;* for the eccentricity of the load (relative to the nominal geometric axis) compensates, in a measure, for the lack of homogeneity of the material. Had the correction been made under greater load, the results of the tests, if plotted in Fig. 13, would probably be still nearer the line $F F_1$, and the difference between these test columns and columns as used in practice would be greater, requiring a higher contingency factor in the latter for safety.

For determining the working stress, the value of p_c' (as computed from the above form of Johnson's expression) should be divided by a suitable factor of safety n. Or, the formula may be put in the following form for computing nominal working stress:

$$p' = p - \frac{p_c p}{4\pi^2 E}\left(\frac{l}{\rho}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The **Rankine** or **Gordon** formula (see Church's "Mechanics," pages 372–376) has been extensively used for columns. It may be expressed as follows:

$$p_c' = \frac{P_c}{A} = \frac{p_c}{1 + \frac{1}{m}\beta\left(\frac{l}{\rho}\right)^2} \quad . \quad . \quad . \quad (5)$$

The above formula is based upon experiments on the *breaking* strength of columns. The coefficient β is purely empirical, and this fact limits its usefulness, for it leaves much uncertainty as to how this coefficient should be modified for materials different from those which have been actually tested as columns. The

* "This precaution is essential to a perfect test of the *material*. . . . Only in this way can other sources of weakness be eliminated."—[J. B. J.]

mean intensity of working stress, p', might be inferred by dividing p_c' by n, or the expression can be written:

$$p' = \frac{p}{1 + \frac{1}{m}\beta\left(\frac{l}{\rho}\right)^2} \quad . \quad . \quad . \quad . \quad (6)$$

but it is not entirely satisfactory to assume the action for stresses within the elastic limit, from the results of tests for breaking strength. The *form* of the Rankine expression is rational, but the coefficient β is not.

Professor Merriman says, in his "Mechanics of Materials," page (129): "Several attempts have been made to establish a formula for columns which shall be theoretically correct. . . . The most successful attempt is that of **Ritter,** who, in 1873, proposed the formula

$$p' = \frac{P}{A} = \frac{p}{1 + \frac{p_c}{m\pi^2 E}\left(\frac{l}{\rho}\right)^2} \quad . \quad . \quad . \quad (N)$$

"The form of this formula is the same as that of Rankine's formula, . . . but it deserves a special name because it completes the deduction of the latter formula by finding for β a value which is closely correct when the stress p does not exceed the elastic limit p_c." The above notation is changed to agree with that previously used in this article. The ratio $p_c \div p$ is the factor of safety. For ultimate strength, this formula might be written:

$$p'_c = \frac{P_c}{A} = \frac{p_c}{1 + \frac{p_c}{m\pi^2 E}\left(\frac{l}{\rho}\right)^2} \quad . \quad . \quad . \quad (N_1)$$

but the first form (eq. N) is the more important. The curve $R_1\ T\ R_2$ (Fig. 13) is the graphical representation of the last expression, eq. N^1.*

Merriman gives the Euler formula for a factor of safety of $n = p_c \div p$, which is

$$p' = \frac{P_c}{A\,n} = \frac{p}{p_c} m\pi^2 E\left(\frac{\rho}{l}\right)^2 \quad . \quad . \quad . \quad . \quad (9)$$

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* Professor Merriman developed equation N_1 independently, but later than Ritter. He gives Ritter sole credit for the formula in the 1897 edition of his "Mechanics of Materials."

Failure occurs if $p \geqq p_c$. The Ritter formula (eq. N) reduces to this last expression for columns so long that the term unity in the denominator is negligible; strictly speaking, this is only the case when $l \div \rho$ = infinity. Professor Merriman also shows, mathematically, that the two curves, $E\,E_1\,E_2$ and $R_1\,T\,R_2$, are tangent to each other when $l \div \rho$ = infinity.

If $l \div \rho = 0$, the Ritter formula reduces to $p' = P \div A$, which is the ordinary formula for short compression members.

The facts that this formula is rational in form, that it gives the correct values at the limits $l \div \rho = \infty$ and $l \div \rho = 0$, and that it lies wholly within the boundary $F\,F_1\,E_1\,E_2$ (Fig. 13), all justify its use, and it will be adopted in this work. It will be noted from Fig. 13 that the Ritter and Rankine formulas agree very closely for the material taken for illustration; but the fact that the curve of the latter crosses the Euler curve near the right-hand limit of the diagram indicates that its constant β is not theoretically correct.

Exception may be taken to the use of the Ritter formula for cast iron, since it involves the use of the stress at the elastic limit, and the coefficient of elasticity, both of which have no definite fixed values for cast iron. But the same criticism applies to the use of any rational formula founded on the elastic theory, as far as cast iron is concerned. Thus the expressions for deflection in simple beams contain E which, for cast iron, may vary from 15,000,000 to 20,000,000. Since cast-iron columns designed simply for strength are very rare in machine design it therefore seems best to use the formula since otherwise it fulfils all needs better than any other.

If it is desired to design a **cast-iron column** with great accuracy values of $\dfrac{p_c}{m\,\pi^2\,E}$ may be taken which will give results in accordance with experiment and which practically transforms the equation into Rankine's formula. If $\dfrac{p_c}{m\,\pi^2\,E} = q$, then for cast-iron columns with fixed ends $q = \dfrac{1}{5{,}000}$, for one end fixed and the other free but guided $q = \dfrac{1.78}{5{,}000}$, and for both ends free but guided $q = \dfrac{4}{5{,}000}$. In addition the student should consult treatises on the strength of materials treating fully of this subject.

All of the above formulas give the value of the mean ultimate stress ($p_c' = P_c \div A$), or the mean working stress ($p' = P \div A$), corresponding to a maximum ultimate stress p_c or a maximum working stress p, respectively. However, the ordinary problem

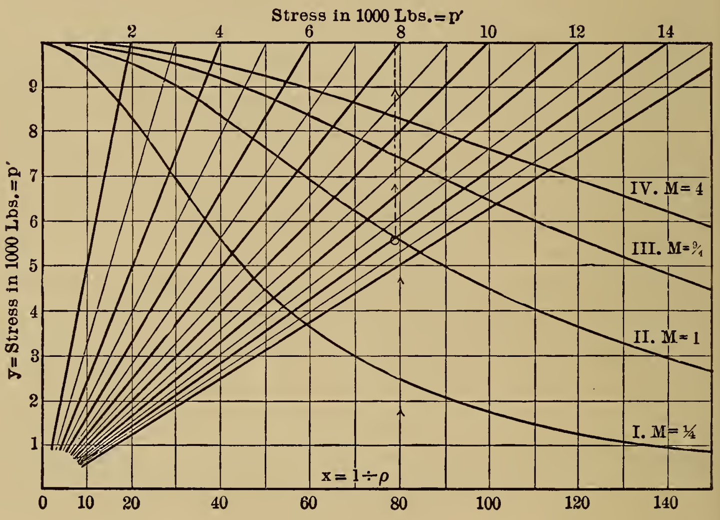

FIG. 13 (a).

of design is to assign proper dimensions for the member under the given load. It is not practicable to solve directly, for the area in such expressions as those given in this article as p' (or p) and ρ are both functions of the area of the cross-section. It is usual to assume a section somewhat larger than that demanded for simple crushing, and then to check for the ultimate load P, or the working load P'. Professor W. N. Barnard has devised a diagram which is very convenient for these computations for steel or wrought-iron columns. It is shown, to a reduced scale, in Fig. 13 (a). The four curves are for the four end conditions given in Table III, page 63. They are plotted for a maximum working stress of 10,000 pounds per square inch, and a value of $\frac{p_c}{E} = \frac{36,000}{29,500,000}$ which is an average value for steel. The curves should not, however, be used for cast iron, wood, or other materials where the ratio $\frac{p_c}{E}$ will give values far different from the above, but such cases may be solved directly by equation N. They may be used for any other stress by proceeding as follows: Assume a trial cross-section, which fixes ρ. Divide l by this value of ρ; take this quotient on the lower scale and pass directly upward to the proper curve for the given end conditions; then pass horizontally to that one of the radiating diagonals which is numbered to correspond with the selected stress; from this last point pass upward to the horizontal scale at the top of the diagram, where the value of the unit load or mean working stress (p') is read off.* If this value of p' agrees sufficiently well with the quotient of the load divided by the trial area, the section may be considered as satisfactory.

In the case of a square-ended column, or when the supporting action of the ends is equal in all possible planes of flexure, it is sufficient to take the least radius of gyration of the section; or to take ρ for the axis about which the section is weakest. In case of a pin-ended column, as a connecting-rod, the cylindrical supporting pins make it equivalent to a square-ended column against flexure in the plane of the axes of the pins, provided these bear symmetrically with reference to the axis of the column; while the column is pin-ended with reference to a plane perpendicular to the axes of the pins. If the cross-section of such a column has equal dimensions in these two planes (circular, square sections, etc.), the column need only be computed for the latter plane. If the pin-ended column has an oblong section (elliptical, rectangular but not square, I section, etc.), it may be weaker in

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* The method of using the diagram is indicated by the arrows, for an example in which $l \div \rho = 80$ and the maximum working stress $= 14,000$ (pin-ended). In this case, p' is found to be about 7,900.

either of these two planes, notwithstanding the difference in end conditions relative to them; and it may be necessary to compute for both planes, unless the section is obviously stronger in one of them. If a rectangular, or elliptical, column has a section in which the dimension in the plane of the pins is more than one-half the dimension in the plane perpendicular to the pins, it will suffice to compute as a pin-ended column against flexure in the latter plane, and *vice versa.*

In the preceding discussion, the various formulæ have been given both for breaking and for working loads. The Euler and Ritter formulæ are derived from the theory of elasticity; hence these are proper for computations pertaining to working loads, in which the stress should never exceed the elastic limit.* It does not follow that these two rational formulas will agree with experiments on the ultimate resistance of columns or for materials which do not follow Hooke's law of proportionality of stress to strain. These expressions are, in this respect, like the common beam formulæ. Such formulæ as Rankine's and J. B. Johnson's, derived from tests of ultimate resistance of columns, are, for similar reasons, less rigidly applicable to working loads and stresses.

Example. The connecting-rod of a steam engine is 5 feet long and is subjected to a load of 20,000 lbs. If the maximum allowable stress is 9,000 lbs. per sq. in., determine the diameter of a circular section at the centre of the rod. Take $E=30{,}000{,}000$, and the elastic limit $p_c=36{,}000$ lbs. per sq. in. The rod may be considered a pin-ended column. Hence $m=1$.

If the rod were designed as a short column, the required area would be $A = \dfrac{20{,}000}{9{,}000} = 2.2$ sq. ins. or a diameter of $1\frac{11}{16}$ inches; and it is evident that for a long column the diameter must be greater than this. Assume $2\frac{1}{2}$ inches as a trial diameter.

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* The Euler formula is not applicable for practical applications, except for quite long columns.

Then $A = 4.9$, $\rho = \frac{d}{4} = \frac{5}{8}$ $l = 60''$, whence in N

$$p' = \frac{9,000}{1 + \frac{36,000}{1 \times \pi^2 \times 30,000,000}\left[\frac{60}{\frac{5}{8}}\right]^2} = 4,300$$

$\therefore P = 4,300 \times 4.9 = 21,070$ lbs. which is a little more than the required load and the section will fulfil the requirements. The student should also follow the solution through on the diagram.

21. Eccentric Loading of Long Columns. In the preceding discussion of columns it has been assumed that the load has been applied axially. This is obviously the best way of applying the load, but cases often occur where it must be applied at a distance a from the axis of the column. In such a case the column is said to carry an **eccentric** load, and the arm a is called the **eccentricity.** If the length of the column be less than 4 or 6 times its least diameter, that is, if the ratio $\frac{l}{\rho}$ be less than about 25, the member may be treated by the method outlined in paragraph (10) and formula M will apply or $p = \frac{P}{A} + \frac{Pae}{I}$.

If, however, the column be longer than 4 to 6 times its least diameter, it can no longer be assumed that the direct stress $\frac{P}{A}$ due to the load is uniformly distributed over the section, as it has been shown by the discussion on long columns that such is not the case.

In addition, if the load is applied eccentrically, it is obvious that the column will deflect somewhat more than it would if the load were applied axially. This will have the effect of adding to the original lever arm a an additional amount α, due to this deflection.

The stresses therefore acting on an eccentrically loaded column are—

(a) A **compressive stress** p_1, such as would be induced if the load were axial.

(b) A **flexural stress** p_2, due to the eccentricity and proportional to the bending moment $P\ (a+\alpha)$.

For the first from Ritter's formula (N)

$$p_1 = \frac{P}{A}\left[1 + \frac{p_c}{m\pi^2 E}\left(\frac{l}{\rho}\right)^2\right]$$

and for the second from (J)

$$p_2 = \frac{P\ (a+\alpha)\ e}{I} = \frac{P\ (a+\alpha)\ e}{A\ \rho^2}$$

Therefore the maximum compressive stress* in the section is

$$p = p_1 + p_2 = \frac{P}{A}\left[1 + \frac{p_c}{m\pi^2 E}\left(\frac{l}{\rho}\right)^2 + \frac{(a+\alpha)\ e}{a^2}\right]. \quad . \quad (O)$$

For columns whose ratio of $\frac{l}{\rho}$ is less than 100, and working stresses such as must be used in machine design, the deflection α may be neglected. For columns longer than this, or where the stress is necessarily high, α can be determined by the theory of elasticity. For a full discussion of the manner of computation see Merriman's "Mechanics of Materials," 1905 edition, page 217. For the ordinary cases of machine design this refinement may be omitted.

Example. A circular wooden pole 30 feet high is required to carry a transformer weighing 800 pounds, with an eccentricity of 10 inches. What must be the diameter at the middle in order that the stress due to this load shall not exceed 500 pounds per square inch? Let $p_c = 3{,}000$ pounds per square inch and $E =$ 1,500,000. Also $m = \frac{1}{4}$. (See Table III.)

---

* The stress induced on the convex side of an eccentrically loaded column may be either tensile or compressive, but will always be less than the stress on the concave side. For materials whose elastic strength is about the same in either tension or compression the stress on the convex side is of no importance. If, however, the column is made of a material, such as cast iron, whose tensile strength is much less than its compressive strength, the character and magnitude of the stress on the convex side should be investigated. If e' be the distance from the neutral axis to the outer fibre on the convex side, then the stress (p) on the convex side is, $p = \frac{P}{A}\left[\frac{p_c}{m\ \pi^2\ E}\left(\frac{l}{\rho}\right)^2 - 1 + \frac{(a+\alpha)\ e'}{\rho^2}\right]$ If p is positive the stress is tensile; if p is negative the stress is compressive.

Assumed a diameter of 8″. Then $\rho = 2$ and $A = 50$

$$\text{Whence } p = \frac{800}{50}\left[1 + \frac{3{,}000}{\frac{1}{4} \times \pi^2 \times 1{,}500{,}000}\left(\frac{360}{2}\right)^2 + \frac{10 \times 4}{4}\right]$$

$$= \frac{800}{50}\left[1 + 26 + 10\right]$$

$= 592$ pounds per square inch.

If this excess is considered too great, a second approximation can be made.

22. **Stress Due to Change of Temperature.** Practically all metals expand when heated, and contract again when cooled. The amount which a bar expands per unit of length, for a rise of one degree in temperature, is called its **coefficient of linear expansion,** and will be denoted by C. The following table gives values of C for various substances for one degree Fahrenheit:

Hard Steel	$C =$ .0000074
Soft Steel	$C =$ .0000065
Cast Iron	$C =$ .0000062
Wrought Iron........	$C =$ .0000068

If a bar of metal is held at the ends, so as to prevent it from expanding or contracting, stresses are produced in it which are called **temperature stresses**; the effect being the same as though the bar had been compressed, or elongated, an amount corresponding to its expansion or contraction due to the change in temperature.

Let t = change in temperature in degrees.

Let p = stress induced per unit area.

$$\text{Since } E = \frac{\text{stress}}{\text{strain}} = \frac{p}{Ct} \therefore p = CtE$$

Example. A bar of wrought iron 2″ square is raised to a temperature of 100 degrees above its normal. If held so that it cannot expand, what stress will be induced in it, and what force must oppose it to prevent expansion?

$$\text{Let } E = 30{,}000{,}000$$

$$p = C\,t\,E = .0000068 \times 100 \times 30{,}000{,}000 = 20{,}400 \text{ lbs.}$$

and the total opposing force P will be

$$P = 20{,}400 \times 4 = 81{,}600 \text{ lbs.}$$

23. **Resilience.** In all the previous discussions on the various straining actions to which a member may be subjected, it has been assumed that the load was a simple dead load and applied without initial velocity or impulse. But, as already pointed out, the load may be applied impulsively; or it may be applied in any way, and removed and applied again and again repeatedly. The application of a load in an impulsive manner, or the repeated application of a load, does not affect the *character* of the straining action, but does affect the *amount* of stress or strain. In order to more clearly discuss the effect of impulsive loading it will be necessary to consider the straining effect of a load somewhat more fully; the discussion of repeated loads will be given in a succeeding section.

If a material is distorted by a straining action, it is capable of doing a certain amount of work as it recovers its original form. If the deformation does not exceed the elastic strain, this amount of work is equal to the work done upon the material in producing such deformation. If the material is strained beyond the elastic limit, it returns work only equal to that expended in producing elastic deformation; and the energy required to cause the plastic deformation, or set, is not recovered, as it is not stored but has been expended in producing such permanent change of form. Ordinary springs illustrate the first case; the shaping of ductile metals by forging, rolling, wire-drawing, etc., are processes in which nearly all of the energy is expended in producing permanent deformation.

The work required to produce a strain in a member is called the **work of deformation.** If the strain produced is equal to the deformation at the true elastic limit, the energy expended is called *elastic resilience.** If the piece is ruptured, the energy

* When the term resilience is used without qualifying context, elastic resilience is to be understood.

expended in breaking it is called **total work of deformation.** If $O\ a\ d\ e$ (Fig. 6) is the stress-strain diagram for a given material, the area $O\ a\ a'$ represents the elastic resilience, and $O\ a\ d\ e\ e'$ represents the total work of deformation *per cubic inch of the material.*

In such materials as have well-marked elastic limits (proportionality between stress and strain through a definite range) the line Oa is a sensibly straight line, and the elastic resilience $Oaa' = \frac{1}{2}aa' \times Oa'$; *or, the elastic resilience equals the elastic strain* (Oa') *multiplied by one-half the elastic stress* ($\frac{1}{2}\ aa'$). *The area* $Oadee'$ equals the base (Oe') multiplied by the mean ordinate (y) of the curve $Oade$; or, if the quotient of this mean ordinate of the curve divided by the maximum ordinate be called k, the work of deformation equals the ultimate strain multiplied by k times the maximum stress. It is evident that for a straining action beyond the elastic limit, $k > \frac{1}{2}$ and $k < 1$.

The curve $OADEE'$ represents the stress-strain diagram of a material having higher elastic and ultimate strength than the former. The greater inclination of the elastic line (OA) with the axis of strain (OX) shows, in the second case, a higher modulus of elasticity, as this modulus equals the elastic stress divided by the elastic strain. In the first case $E_1 = \dfrac{a\ a'}{O\ a'}$, in the second case, $E_2 = \dfrac{A\ A'}{O\ A'}$.

The stress-strain diagram $OADEE'$ shows that of two materials one may have both the higher elastic and ultimate strength, and still have less elastic resilience and less total work of deformation. If the curve $O\ a''\ d''\ e''$ is the stress-strain diagram of a third material (having a modulus of elasticity similar to the first), it appears that this third material possesses greater elastic resilience, but less total work of deformation than the first.

A comparison of these illustrative stress-strain diagrams (for quite different materials) also shows that, *for a given stress*, the more ductile, less rigid material may have the greater resilience. Hence, when a member must absorb considerable energy, as in case of severe shock, a comparatively weak yielding material may be safer than a stronger, stiffer material. This is frequently recognized in drawing specifications. The principle is similar to that involved in the use of springs to avoid undue stress from shock. In fact springs differ from the so-called rigid members only in the *degree* of distortions under loads, or in having much greater resilience for a given maximum load.

If a material is strained beyond its elastic limit, as to a' (Fig. 13 b), upon removal of the load it will be found to have such a permanent set as $O\,O'$. Upon again applying load, its elastic curve will be $O'\,a'$; but beyond the point a' its stress-strain diagram will fall in with the curve which would have been produced by continuing the first test (*i.e.*, $a'\,d\,e$). Similarly, if loaded to a'', the permanent set is $O\,O''$, and upon again applying load, the stress-strain diagram becomes $O''\,a''\,d\,e$. The elastic limit a'' of the overstrained material is evidently higher than the original elastic limit, a; while the original total work of deformation, $O\,a\,d\,e$, is considerably greater than the total work of deformation of the overstrained material, $O''\,a''\,d\,e$. The effects of strain beyond the elastic limit are thus seen to be:

I. Elevation of the elastic strength and increase of the elastic resilience.

II. Reduction of the total work of deformation.

These facts have an important influence on resistance to repeated shock. The above noted elevation of the elastic limit by overstraining can usually be largely or wholly removed by annealing.

24. Suddenly applied Load, Impact, Shock. It will perhaps be well to first consider the general case of a load impinging on the member, with an initial velocity; this velocity (v) corresponding to a free fall through the height h. For simplicity, the discussion will be confined to a load producing a tensile stress; but the formulæ will apply equally well to uniform compressive and shearing stresses, and all except (5) apply directly to cases of torsion and flexure.

W = static value of load applied to member.

h = height corresponding to velocity with which load is applied.

δ = total distortion of member due to impulsive load.

p = maximum *intensity* of resulting stress.

A = area of cross-section of the member.

$P = p\,A$ = total max. stress due to load as applied suddenly.

λ = total distortion of member due to static load, W.

$x = h \div \lambda$ (for convenience).

k = a constant; its value is ½ if *E. L.* is not passed; but if *E. L.* is exceeded $k > \frac{1}{2}$ and $k < 1$.

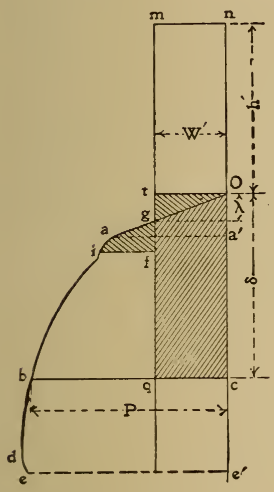


FIG. 13 (c).

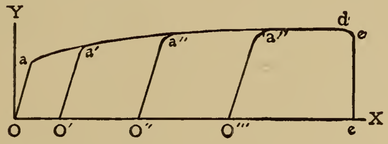


FIG. 13 (b).

The energy to be absorbed by the member due to the impulsive application of the load is $W\,(h+\delta)$; the work of deformation is $k\,P\delta$. (See preceding article, Resilience.)

Case I.—Maximum Stress within Elastic Limit.

$$W\,(h+\delta) = k\,P\,\delta = \tfrac{1}{2}\,P\,\delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\delta : \lambda :: P : W \quad \therefore \quad \delta = \frac{P\,\lambda}{W} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$P = \frac{2\,W\,h}{\delta} + 2\,W = \frac{2\,W^2\,h}{P\,\lambda} + 2\,W \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\therefore P^2 = \frac{2\,W^2 h}{\lambda} + 2\,W P \quad \therefore P = W\,(1 + \sqrt{1 + 2\,x}) \quad . \quad (4)$$

$$p = \frac{P}{A} = \frac{W}{A}\,(1 + \sqrt{1 + 2\,x}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\delta = \frac{P\,\lambda}{W} = \lambda\,(1 + \sqrt{1 + 2\,x}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The elongation at the elastic limit equals $F \div E$, in which E= modulus of elasticity and F=intensity of stress at the elastic limit.

If L=length of the member,

$$(\lambda \div L) : (F \div E) :: (W \div A) : F; \quad \therefore \lambda = W L \div A E. \quad . \quad (7)$$

As λ is small for metals (except in the forms of springs) a moderate impinging velocity may produce very severe stress. It will be evident that λ and δ are directly proportional to the length of the member; hence the stress produced by a given velocity of impact (height h) is reduced by using as long a member as possible.

If the load is applied instantaneously, but without initial velocity, h=0 and x=0; whence

$$P = W\,(1 + \sqrt{1 + 0}) = 2\,W \quad . \quad . \quad . \quad . \quad . \quad . \quad (3')$$

$$p = \frac{P}{A} = \frac{2\,W}{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5')$$

$$\delta = \lambda\,(1 + \sqrt{1 + 0}) = 2\,\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6')$$

Case II.—Maximum Stress Beyond the Elastic Limit. If the maximum stress exceeds the elastic limit, the constant k of equation (1) is between ½ and 1 (see Art. 23, Resilience), and its exact value cannot be determined in the absence of the stress-strain diagram for the particular material. Thus (Fig. 13 c), $W\,(h+\delta)$, is represented by the rectangle $m\,n\,c\,q$; and this area must equal the area $O\,a\,b\,c$; the latter being greater than the elastic resilience, $O\,a\,a'$, and less than the total work of deformation $O\,a\,d\,e\,e'$, in this illustration.

When the stress-strain diagram is known, the following problems can be readily solved:—

(a) Determination of the velocity of impinging of a given load (or corresponding value of h) to produce a given stress, or strain.

(b) Determination of the load which will produce any particular stress, or strain, when impinging with a given velocity.

(c) Determination of the stress, or strain, produced by a given load impinging with a given velocity.

Let the work of deformation corresponding to the known stress, or strain, in (a) and (b), be called $R = k\,P\,\delta$. If the stress-strain diagram is for stress per unit of sectional area and strain per unit of length of the member, let W' be the load per unit of sectional area; h' the height due the velocity of impinging divided by the total acting length of the member; δ' the distortion per unit of length of the member due to impulsive load; and R' the resilience for unit of volume, or the modulus of resilience.

$$\text{(a)} : W'\,(h' + \delta') = k\,p\,\delta' = R' . \;\therefore\; h' = \frac{R'}{W'} - \delta' \quad . \quad . \quad . \quad (7)$$

$$\text{(b)} : W' = \frac{R'}{h' + \delta'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

(c): The solution of this problem is not quite so definite, in the general case, as the preceding; but it can be easily accomplished, graphically, with sufficient accuracy. Draw the line $g\,q$ (Fig. 13 c) (indefinitely), parallel to $O\,e'$, and at a distance from it equal to W'; take out the area $f\,i\,g = g\,O\,t$. Whatever the value of δ', the shaded area $O\,c\,q\,f\,i\,g\,O = W'\,\delta'$; hence the unshaded area under the stress-strain curve must equal $W'\,h'$. A few trials will suffice to locate the limiting line $b\,q\,c$ which will give $f\,i\,b\,q\,f = m\,n\,O\,t = W'\,h'$.

The case in which the maximum stress is within the elastic limit is by far the most important, as it is almost always desired to keep the maximum intensity of stress, $P \div A$, within the elastic limit, especially as every overstrain (beyond this limit) raises the elastic limit and decreases the total resilience (see Fig. 13). The effect of a shock which strains a member beyond the elastic limit is to reduce its margin of safety for subsequent similar loads, because of reduction in its ultimate resilience. Numerous successive reductions of the total resilience by such actions may finally cause the member to break under a load which it has often previously sustained.

6

No doubt many cases of failure can be accounted for by the effects just discussed; but there is another and quite different kind of deterioration of material, which is treated in the following article.

Dr. Thurston has shown that the prolonged application of a dead load may produce rupture, in time, with an intensity of stress considerably below the ordinary static ultimate strength but above the elastic stress. It is well known that an appreciable time is necessary for a ductile metal to flow, as it does flow when its section is changed under stress; hence, a test piece will show greater apparent strength by quickly applying the load than by applying it more slowly, provided the application of load is not so rapid as to become impulsive.

The kind of failure which is the subject of the next topic is due to a real permanent deterioration of the metal, and it is due to distinctly different causes from those mentioned above.

25. On the Peculiar Action of Live Load. Fatigue of Metals. It has been found by experience and experiment, that materials which are subjected to continuous variation of load cannot be depended upon to resist as great stress as they will carry if applied but once, or only a few times. When the load is suddenly applied, and frequently repeated, the decline of strength or of the power of endurance may perhaps be ascribed, in part at least, to the elevation of the elastic limit and reduction of the ultimate resilience, as discussed in Art. 24. But apart from this cause, with repeated loads, even in the absence of appreciable shock, a decided deterioration of the material very frequently occurs. This effect has been called the **Fatigue of Materials,** although some authorities restrict this term to the kind of deterioration already referred to as the simple result of a decrease of resilience. The term fatigue implies a weakening of the material due to a general change of structure. It was formerly supposed that the repeated variation of stress caused such change of the general structure, possibly owing to slight departure from perfect elasticity under stress much below that ordinarily designated as the elastic limit. The crystalline appearance of the fracture sustained this view; but numerous tests of pieces from a member ruptured in this way, (taken as near as possible to the break), fail to show such crystalline fracture, and it is difficult to reconcile the normal appearance and behavior of such test pieces with the theory of general change of structure.

A theory which has been largely accepted is that every piece of metal contains innumerable minute flaws or imperfections, often originally too small to be detected by ordinary means. These "**micro-flaws**" tend to extend across the section under variation of stress, and may, in time, reduce the net sound section so greatly that the intensity of stress in the fibres which remain intact becomes equal to the normal breaking strength of the material. Professor Johnson suggests: "**the gradual fracture of metals**" as a more appropriate term than "fatigue." Many men of large practical experience still prefer wrought iron to mild steel for various members which are subject to constantly reversing stress.

It is probable that the prejudice against steel is largely the result of unskilful manipulation of this more sensitive material; and the product of the best steel makers of to-day is much stronger and more reliable than wrought iron.

However, it is just possible that the very lack of homogeneity in wrought iron renders it safer under varying stress (*other things being equal*), as the fibres are more or less separated by the streaks of slag, and a flaw is less apt to extend across the entire section than it is in the continuous structure of steel. Wrought iron may be likened to a wire rope, in which a fracture in one wire does not directly extend to adjacent wires.

The "gradual fracture" through extension of "micro-flaws" seems to accord with the observed facts more closely than the older theory of general change of structure.

In the *American Machinist* (Sept. 27, 1906) will be found an account of recent researches tending to show that metals are made up of *grains*, each grain consisting of many *crystals*, and that when deformation takes place in a metal these crystals move relatively to each other along "*gliding planes*." If the stress producing such sliding is repeated often enough the contact at the gliding planes weakens and finally passes into a crack or series of cracks which extend across the section.

The theory of the subject is, as yet, too incomplete to permit of derivation of rational formulæ to account for the effects of repeated live loads; and if the "micro-flaw" theory is correct, it is not probable that such rational analysis can ever be satisfactorily applied.

All of the formulæ that have been derived for computation of breaking strength under known variations of load, or stress, are empirical ones which have been adjusted to fit the experimentally determined facts.

Consult: Johnson's "Materials of Construction."
Merriman's "Mechanics of Materials."
Unwin's "Testing of Materials."
Weyrauch (Du Bois): "Structure of Iron and Steel."

Experiment has shown that the breaking strength under repeated loading, or the "carrying strength," is a function of the magnitude of the variation of stress and of the number of repetitions of such varying stress. Furthermore, this function is different for different materials; and there are authentic observations on record which go to show that, as between different materials, the one with the higher static breaking strength does not always possess the greater endurance under repeated loading. In general, however, the carrying strength under repeated loads is a function of the static strength.

The allowable working stress usually depends upon: (a) The number of applications of the load. This should be considered as indefinite, or practically infinite, in many machine members. (b) The range of load. This is frequently either from zero to a maximum; or between equal plus and minus values. (c) The static breaking strength or the elastic strength.

The first systematic experiments upon the effect of repeated loading were conducted by **Wöhler** [1859 to 1870]. He found, for example, that a bar of wrought iron, subjected to tensile stress varying from zero to the maximum, was ruptured by:

800 repetitions	from	0 to 52,800	lbs. per sq. in.		
107,000	"	"	0 to 48,000	"	"
450,000	"	"	0 to 39,000	"	"
10,140,000	"	"	0 to 35,000	"	"

—*Merriman, page* 191.

It was found that the stress could be varied from zero up to something less than the elastic limit an indefinite number of times (several millions) before rupture occurred; but with complete reversal of stress, or alternate equal and opposite stresses, (tension and compression), it could be broken, by a sufficient

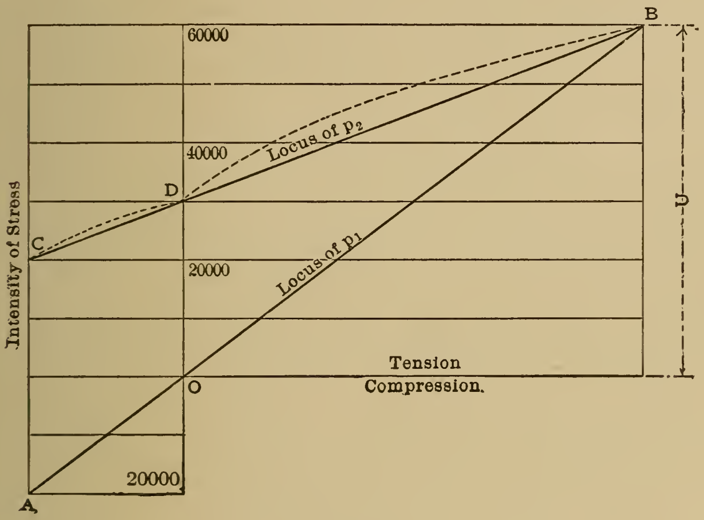


FIG. 13 (d).

number of applications, when the maximum stress was only about one-half to two-thirds the stress at the elastic limit.

A number of efforts have been made to deduce from the experiments of Wöhler, formulæ which could be applied to the design of machine members (see Unwin, page 36). One of the best of these formulæ is that of Professor Johnson as it is easily applied to all cases that will arise; it is simpler than most of those previously proposed; and it is probably as reliable as any yet offered.

Two formulæ which have been very generally accepted for computing the probable carrying strength are: Launhardt's for varying stress of one kind only, and **Weyrauch's** for stress which changes sign.

Suppose a material to have a static ultimate strength u of 60,000 lbs. per sq. in. If the minimum unit strength be plotted as a straight line, $A\ O\ B$ (Fig. 13 d), the locus of the maximum unit stress, from the Launhardt formula, is the broken curve from B to D. That is, for example, when the minimum tensile stress is 12,500, the maximum tensile carrying stress would be about 40,000; or the material could be expected to stand an indefinite number of loadings if the range of stress did not exceed 15,000 to 40,000 pounds per square inch in tension. In a similar way, the broken curve from D to C is the locus of maximum tension, from the Weyrauch formula, when the locus of minimum stress (negative tension, or compression) is the straight line $A\ O$. It will appear that the straight line $C\ D\ B$ agrees fairly well with these two curves. Inasmuch as it seems unreasonable to expect an abrupt change of law when the minimum stress passes through zero, and as there is no rational basis for the Launhardt and Weyrauch formulæ, it appears reasonable to adopt the upper straight line as the locus of the maximum stress. Owing to the discrepancies in the observations (which must be expected from the probable cause of the deterioration of the metal), this straight line may be accepted as representing the law as accurately as could be expected of any empirical line. These are, in substance, the reasons given by Professor Johnson for basing his formula on the straight line $C\ D\ B$. For full discussion and derivation of the following formula, see Johnson's "Materials of Construction," pages 545–547.

Let p_2 = maximum intensity of stress.
p_1 = minimum intensity of stress.
u = ultimate (static) intensity of stress.

Then in general:

$$p_2 = \frac{\frac{1}{2} u}{1 - \frac{1}{2}\frac{p_1}{p_2}}. \qquad (1)$$

As the expressions contain the *ratio* of the minimum to maximum intensities of stress, instead of their *difference*, they are applicable when the area of cross-section of the member is unknown; for whatever this area, the ratio of the stresses is the same as the ratio of the loads producing these stresses. In substituting values of p_1 and p_2, care must be taken to use proper signs; thus, if tension is taken as positive, compression is negative; or, if the stress varies between tension and compression p_2 is positive and p_1 is negative.

For dead load, $p_1 = p_2$;

$$\therefore\ p_2 = \frac{\frac{1}{2}u}{1 - \frac{1}{2}\frac{p_2}{p_2}} = \frac{\frac{1}{2}u}{\frac{1}{2}} = u. \qquad (2)$$

For repeated load when $p_1 = 0$, $\frac{p_1}{p_2} = 0$

$$\therefore\ p_2 = \frac{\frac{1}{2}u}{1 - 0} = \frac{1}{2}u. \qquad (3)$$

For complete reversal of load, $p_1 = -p_2$

$$\therefore\ p_2 = \frac{\frac{1}{2}u}{1 - \frac{1}{2}\frac{-p_2}{+p_2}} = \frac{\frac{1}{2}u}{1 + \frac{1}{2}} = \frac{1}{3}u. \qquad (4)$$

The three special cases (2), (3), and (4), are those most commonly met with in designing, but the general expression (1) should not be lost sight of.

Example. A bar of steel, whose ultimate *static* tensile strength is 70,000 lbs. per sq. inch, is subjected to a repeated load whose minimum value is one half the maximum value. What is the maximum *stress* that can be carried by the bar for an indefinite number of repetitions?

Since the stress will be proportional to the load $p_1 = \frac{p_2}{2}$

Hence substituting in equation (1), $p_2 = \dfrac{\frac{1}{2} \times 70{,}000}{1 - \frac{1}{2}\frac{p_2}{2\,p_2}} = 47{,}000.$

It is to be noted that the allowable maximum stress is above the original elastic limit of most steel, and if the piece were designed to be stressed to 47,000 lbs. the result would be that the first application of the load would raise the elastic limit to that value.

But the piece would take permanent set and be in most cases of no further use. A **factor of safety** must therefore be used in order that the maximum stress may be well below the elastic limit.

The experiments of Wöhler, and his successor in the field, Baushinger, were conducted on a very limited variety of materials; so that while the above discussion points out what may be expected in a general way from most materials, they are not sufficiently conclusive to make it possible to pick out the exact factor of safety to be used in all cases. They do, however, throw much light on the apparently high factors of safety which must sometimes be used, and for which no other satisfactory explanation has been found.

26. The Factor of Safety. The preceding paragraphs (articles 9 to 26) have considered the effect that different methods of applying the load will have on a member, and the relations which exist between a given *dead* load and the resulting stress and strain. It has been shown in Art. 24 that if the load is applied suddenly the resulting stress and strain will be twice as great as for a dead load. And finally in Art. 25 it has been shown that the maximum stress that can with safety be induced repeatedly in a member, will depend on the range of stress. It would seem as though a member designed in accordance with these logical theories would be satisfactory. But it must be remembered that these theories are not absolute, that the information regarding the characteristics of materials is still very incomplete; that flaws and hidden defects always exist; and finally that there is always danger of accidental overloading.

In addition, it is generally essential that a machine member be not only strong enough to avoid breaking under the regular maximum working load, but also that it shall not receive a permanent set; for a machine member ordinarily becomes useless if it takes such set after it has been given the required form. In many cases a temporary strain, even considerably below that corresponding to the elastic limit, would seriously impair the accuracy of operation; and in such cases the member often requires great excess of strength to secure sufficient rigidity. It follows, therefore, from these considerations that if the design of a machine member were based on the maximum allowable stress, as indicated by Wöhler's experiments (such stress being modified by the theory of suddenly applied loading, should it be present), there would be no margin to allow for the uncertainties and unknown defects enumerated above; and in many cases leave no assurance that the elastic limit would not be exceeded. So that while stresses fixed in accordance with these theories form a good basis, they must in general be reduced by means of a **factor of safety** so that the *working stress* is enough lower to provide for these uncertainties.

The factor of safety is generally defined as the quotient of the ultimate static strength divided by the working stress. A consideration of Wöhler's experiments shows that such a definition is misleading. For a factor of safety of 2, for instance, might be perfectly safe for a dead load; but for a repeated load with stress in one direction it would leave no margin at all for contingencies. The **apparent factor of safety** would seem to be a better term, and the **real factor of safety** may be defined as the quotient of the **carrying strength,** or maximum allowable stress as given by Wöhler's experiments, divided by the working stress.

The factor of safety has been called the "factor of ignorance," and, as it is too often applied, it is perhaps little else. Thus very often it is specified that all the members of a machine shall be designed with a certain fixed factor of safety without regard to the conditions under which the various members may have to act. A factor of safety applied in this manner is, generally speaking, a factor of ignorance. It is probable that the factor of safety will always retain an element of ignorance, for it can hardly be hoped that the powers of analysis will ever permit the prediction of the exact effect of every possible straining action, due to regular service and accident. Neither can it be expected that the methods of manufacture, and inspection, will become so perfect as to eliminate or measure precisely every possible defect in materials or workmanship. But a careful study of the conditions of each particular case and a proper attention to the effects which *may* be weighed (at least approximately) should, with the knowledge now to be had, enable the designer to make a fairly accurate application of the factor of safety, an intelligent choice of which is the most important part of design.

Most of the formulæ of Mechanics which are applicable to the design of machine members, are based on theoretical treatment of the stresses induced by the action of given forces within the elastic limit upon the member under consideration; and the theoretical conclusions so reached are amply verified by practical experiment. When, therefore, the conditions under which the member is to work can be analyzed, and the laws of Mechanics applied to its design, such methods as outlined in this chapter are perfectly rational, if intelligent allowance is made for contingencies. Many machine members, however, are subjected to such a complicated system of stress that analysis cannot be strictly applied, and less satisfactory approximations or assumptions are unavoidable in the present state of knowledge. When such is the case, the designer must either base the design on the *predominating stress*, if there is such, allowing such a margin or factor of safety as experience or experiment may show, to provide for the minor uncertain stresses; or, if the case considered be beyond such treatment, recourse must be had to empirical methods or judgment. (See Art. 1.)

While therefore mathematical treatment of any case will serve as a good guide to correct proportions, such treatment must always be tempered with judgment, a high development of which is necessary to successful design, as in all other branches of engineering.

While, also, no fixed rules for selecting the factor of safety can be laid down, a knowledge of Wöhler's experiments, and the effect of suddenly applied loads, will greatly aid the designer in the matter. Thus when it is known that the load is to be a dead load, an apparent factor of safety of 3 will, for wrought iron, or steel, bring the working stress well below the elastic limit and allow something for contingencies. If, however, the load be a repeated load, the stress varying from zero to a maximum tensile stress, the apparent factor of safety for steel must at least be 5, to allow a good margin below the elastic limit;

and in either case, if in addition the load is to be suddenly applied, these factors must be multiplied by 2 to insure safety.

Example. A steel beam is subjected to a suddenly applied load which alternately induces an equal tensile and compressive stress; if the ultimate strength be 60,000 lbs. per sq. in., what apparent factor of safety should be used, and what will be the real factor of safety?

Since the stress is a reversed one, the maximum allowable stress or *carrying strength* is by Wöhler's experiments one-third of ultimate strength or 20,000 lbs. If the working stress is one-half of this value or 10,000 lbs., it will leave a good margin for contingencies, disregarding the impulsive effect. But the load is applied suddenly and, by Art. 24, this value (10,000) must be again divided by 2, making the working stress 5,000 lbs. per sq. in. Therefore the apparent factor of safety is $\frac{60,000}{5,000} = 12$, while the real factor based on Wöhler's law is $\frac{20,000}{5,000} = 4$.

If the member should have to work under extremely trying conditions, or if shock or other stresses which could not be analyzed were present, this value might have to be still further reduced.

TABLE IV

FACTORS OF SAFETY

Character of Material.	Dead Load.	Repeated Stress in One Direction.		Repeated Reversed Stress.	
		Gradually Applied Load.	Suddenly Applied Load.	Gradually Applied Load.	Suddenly Applied Load.
Wrought Iron, Steel, or other Ductile Metals..........	3	5	10	6	12
Cast Iron, or other Brittle Metals....	4	6	12	10	20

Table 4 contains factors of safety such as are used in practice and which agree fairly well with the foregoing theory. They are, of course, average values and must be used with judgment;

but in the absence of trained judgment, or as an aid to its development, they may be found useful.

Table 5 contains values of the ultimate strengths and elastic limits of the materials most used in engineering. They, also, are average values such as the designer must use in the absence of exact information regarding the material to be employed, and in general such exact information is lacking.

It may be observed that an increased factor of safety may not always in the case of cast metals give a stronger member. If the increased dimensions give sections so thick that sponginess results, the gain in strength may be negative; and when internal pressure, such as is found in hydraulic work, is to be withstood, it is often necessary to do with a smaller factor of safety to insure soundness.

TABLE V

ULTIMATE AND ELASTIC STRENGTH

Material.	Ultimate Strength.			Elastic Strength.			Direct Coefficient of Elasticity. E	Transverse Coefficient of Elasticity. E_s
	Tension.	Comp.	Shear.	Tension.	Comp.	Shear.		
Cast Iron	20,000	95,000	20,000	10,000*	25,000	8,000	15,000,000	6,000,000
Malleable Iron	35,000	42,000	20,000	....	....	....	....	....
Wrought Iron	55,000	....	40,000	30,000	28,000	22,000	28,000,000	10,000,000
Steel, 0.15 Carbon	63,000	....	48,000	42,000	40,000	....	30,000,000	10,000,000
Steel, 0.50 Carbon	80,000	....	57,000	48,000	46,000	....	30,000,000	10,000,000
Steel, 0.70 Carbon	89,000	....	60,000	53,000	53,000	....	30,000,000	10,000,000
Steel, 0.80 Carbon	103,000	....	80,000	57,000	63,000	....	30,000,000	10,000,000
Steel, 0.96 Carbon	118,000	....	83,000	69,000	71,000	....	30,000,000	10,000,000
Steel, Boiler Plate	60,000	....	48,000	30,000	....	....	30,000,000	....
Crucible Steel	116,000	....	....	80,000	80,000	....	31,000,000	12,400,000
Steel Castings	50,000	....	40,000	30,000	30,000	....	25,000,000	....
Nickel Steel	100,000	....	....	60,000	....	....	31,000,000	....
Copper Castings	22,000	60,000	....	6,000	....	....	12,000,000	....
Rolled Copper	31,000	....	....	6,000	....	....	15,000,000	....
Brass Castings	20,000	12,000	....	....	....	....	10,000,000	....
Bronze, Gun Metal	35,000	....	....	....	....	....	12,000,000	....
Bronze, Phosphor	50,000	....	....	20,000	....	....	14,000,000	....
Tobin Metal	80,000	....	....	55,000	....	....	....	....
Aluminum Castings	15,000	12,000	12,000	6,500	3,500	....	11,000,000	....

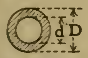
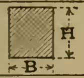
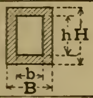
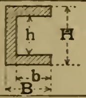
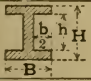
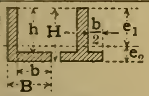
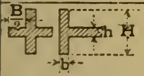
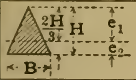
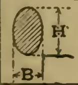
* Cast Iron has no true elastic limit.

TABLE VI

	Character of Stress or Strain.	Formula.
A	Stress in Ten. or Comp.....	$p = \frac{P}{A}$
B	Strain in Ten. or Comp.....	$\Delta = \frac{P\,l}{A\,E}$
C	Stress in Shear............	$p_s = \frac{P}{A}$
D	Torsional Stress...........	$P\,a = T = \frac{p_s\,I_p}{e}$
E	Torsional Stress, Solid Circular Shaft.............	$P\,a = T = \frac{p_s\,\pi\,d^3}{16}$
F	Torsional Stress, Hollow Circular Shaft..............	$P\,a = T = \frac{p_s\,\pi\,(d_1^4 - d_2^4)}{16\,d_1}$
G	Torsional Strain, Solid Circular Shaft..............	$\theta = \frac{32\,T\,l}{\pi\,E_s\,d^4}$
H	Torsional Strain, Hollow Circular Shaft..............	$\theta = \frac{32\,T\,l}{\pi\,E_s\,(d_1^4 - d_2^4)}$
I	Deflection in Bending.......	See Table I.
J	Stress due to Flexure.......	$M = \frac{p\,I}{e}$ See Table I.
K	Combined Bend'g and Twist'g	$M_e = \frac{1}{2}\,M + \frac{1}{2}\sqrt{M^2 + T^2}$
K_1	" " " "	$T_e = \sqrt{M^2 + T^2}$
K_2	" " " "	$M_e = \frac{1}{2}\,[x + \sqrt{x^2 + 1}\,]\,T$
K_3	" " " "	$T_e = T\sqrt{x^2 + 1}$
L	Combined Torsion and Compression................	$p_c = \frac{2}{\pi\,d^2}\left[P + \sqrt{P^2 + \frac{64\,T^2}{d^2}}\right]$
L_1	Combined Torsion and Compression................	$p_s = \frac{2}{\pi\,d^2}\sqrt{P^2 + \frac{64\,T^2}{d^2}}$
M	Combined Flexure and Direct Stress..................	$p = \frac{P}{A} + \frac{P\,a\,e}{I}$
N	Long Column..............	$p' = \frac{P}{A} = \frac{p}{1 + \frac{p_c}{m\,\pi^2\,E}\left(\frac{l}{\rho}\right)^2}$
O	Eccentric Loading of Long Columns................	$p = \frac{P}{A}\left[1 + \frac{p_c}{m\,\pi^2\,E}\left(\frac{l}{\rho}\right)^2 + \frac{(a + a)\,e}{\rho^2}\right]$

TABLE VII

PROPERTIES OF SECTIONS

Shape of Section.	Moment of Inertia. I	Modulus of Section. $\frac{I}{e}$	Square of Radius of Gyration. $\rho^2=\frac{I}{A}$	Polar Moment of Inertia. J
	$\frac{\pi d^4}{64}=.049d^4$	$\frac{\pi D^3}{32}=.098D^3$	$\frac{D^2}{16}$	$\frac{\pi D^4}{32}$
	$\frac{\pi}{64}\left[D^4-d^4\right]$	$\frac{\pi}{32}\left[\frac{D^4-d^4}{D}\right]$	$\frac{D^2+d^2}{16}$	$\frac{\pi(D^4-d^4)}{32}$
	$\frac{BH^3}{12}$	$\frac{BH^2}{6}$	$\frac{H^2}{12}$	$\frac{BH(B^2+H^2)}{12}$
	$\frac{1}{12}\left[BH^3-bh^3\right]$	$\frac{1}{6H}\left[BH^3-bh^3\right]$	$\frac{1}{12}\left[\frac{BH^3-bh^3}{BH-bh}\right]$	
	$\frac{1}{12}\left[BH^3-bh^3\right]$	$\frac{1}{6H}\left[BH^3-bh^3\right]$	$\frac{1}{12}\left[\frac{BH^3-bh^3}{BH\quad bh}\right]$	
	$\frac{1}{12}\left[BH^3-bh^3\right]$	$\frac{1}{6H}\left[BH^3-bh^3\right]$	$\frac{1}{12}\left[\frac{BH^3-bh^3}{BH-bh}\right]$	
	$I=\frac{(BH^2-bh^2)-4BHbh(H-h)^2}{12(BH-bh)}$		$\frac{I}{e_1}=\frac{(BH^2-bh^2)-4BHbh(H-h)^2}{6(BH^2-bh^2)}$ $\frac{I}{e_2}=\frac{(BH^2-bh^2)-4BHbh(H-h)^2}{6(BH^2-2bhH+bh^2)}$	
	$\frac{1}{12}\left[bH^3+Bh^3\right]$	$\frac{1}{6H}\left[bH^3+Bh^3\right]$	$\frac{bH^3+Bh^3}{12(bH+Bh)}$	
	$\frac{BH^3}{36}$	$\frac{I}{e_1}=\frac{BH^2}{24}$ $\frac{I}{e_2}=\frac{BH^2}{12}$	$\frac{H^2}{18}$	
	$\frac{\pi BH^3}{64}$	$\frac{\pi BH^2}{32}$	$\frac{H^2}{16}$	$\frac{\pi(BH^3+HB^3}{64}$

CHAPTER IV

GENERAL THEORY OF FRICTION, LUBRICATION, AND EFFICIENCY

27. Friction in General. When two solid surfaces are held in contact by any appreciable force, any effort tending to move them relatively to each other is met by a resisting force acting tangentially to the surface of separation of the two bodies. This resistance to relative motion is due to the interlocking of the minute depressions and elevations which exist even in the smoothest surfaces and will, of course, vary with different properties of materials and different qualities of finish. Thus, unsurfaced cast iron will show a very great resistance to relative motion, while two hardened and ground surfaces of steel will move over each other with much more ease. If the two surfaces are very carefully fitted together without any foreign matter in between, they will, in the case of many substances, *adhere* firmly together, which still further increases the resistance to relative motion. If oils or lubricants of any kind are interposed between the surfaces, the resistance to relative motion is, to a considerable extent, overcome.

This tendency to resist relative motion is sometimes a desirable feature and sometimes not. In the case of bearing and rubbing surfaces generally, such frictional resistances result in loss of power and should be reduced to a minimum; while in the case of friction clutches, brake straps, keys, screw fastenings, etc., frictional resistance is of great utility and every effort is made to insure its presence. The laws of friction, and the manner of their application therefore, are of prime importance to the engineer.

These laws are at present rather imperfectly understood, though considerable experimental work has been done. It has been found that many of the older theories based on experimental work are true only for the range of conditions covered by the experiments, and that conditions different from these show entirely different results.

The ratio of frictional resistance F, to the normal load P, is called the **coefficient of friction**; or if this ratio be denoted by f, then *for flat surfaces*, $f = \frac{F}{P}$ or $F = fP$.

In the case of circular surfaces, such as journals and bearings, the distribution of the normal pressure is variable and dependent on the manner in which the surfaces are fitted together. In such cases it is customary for convenience to define the coefficient of friction in a similar way as in flat surfaces, and consider that it has special values for circular surfaces. The **coefficient of friction for circular surfaces** will be denoted by μ. Hence as before, $F = \mu P$.

The *intensity* of normal pressure on circular surfaces, as before stated, is difficult of accurate determination and it is therefore customary to take as the normal pressure the **intensity of pressure per unit of projected area.** Or, if d = diameter of shaft, and l = length of bearing, then the intensity of pressure per unit of projected area, $w = \frac{P}{d\,l}$.

The energy absorbed by frictional resistance is transformed into heat which is conducted away by conduction and radiation to the air, or, in the case of certain kinds of bearings, by water circulation or other means. The **work of friction** is often therefore an important factor in the design of rubbing surfaces. For flat plates the foot pounds of energy absorbed per minute is $E = fPV$, where V the velocity is in feet per minute and P is in pounds. For circular surfaces, if N be the number of revolutions per minute, and d the diameter of shaft in inches,

$$*\,E = \frac{\pi \mu d N P}{12} = .2618\,\mu d N P.$$

* For other forms of surfaces see Kent's "Engineer's Pocketbook," page 938, and Thurston's "Friction and Lost Work," page 40.

If then f or μ be known, for any pair of rubbing surfaces, the frictional resistance and the energy absorbed for any load P may be calculated. Values of f and μ have been obtained experimentally for many of the materials and conditions met with in engineering, but the data so far available are still incomplete.

The consideration of the laws of friction, as applied to machinery, naturally divides itself into two parts.

(a) **Friction of Dry or Unlubricated Surfaces.**

(b) **Friction of Lubricated Surfaces.**

28. Friction of Unlubricated Surfaces. The experiments of Morin, Rennie, Coulomb,* and many others, furnish the following laws for dry or very slightly lubricated surfaces.

(1) **The frictional resistance is approximately proportional to the normal load.**

(2) **The frictional resistance is approximately independent of the extent of the surfaces.**

(3) **The frictional resistance, except at very low speeds, decreases as the velocity increases.**

It was formerly supposed that an abrupt change took place in the value of f when the body passed from a state of motion to one of rest. It seems now, however, that while the coefficient of rest is in general greater than that of motion, the change in value is gradual and the value at rest is not far different from that at very slow motion. As the velocity increases, the value of f materially decreases and this must be taken account of in designing machinery where friction is involved. Unfortunately the information regarding high or even moderate speeds is also very incomplete.

The following values of f must, in view of the incomplete information, and also because of variations which come with slight changes of conditions, be looked on as approximate values only. Unless it is positively known that the surfaces will be kept free from even slight contamination by oily substances, these values must be used with judgment.

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* See "Lubrication and Lubricants": Archbutt & Deely, for a full discussion of these points.

Coefficients of Friction (f) for Dry or Slightly Lubricated Surfaces.

Wood on Wood—Static or very low velocity	....	.3 to .5
Wood on Metals " " " " "	....	.2 to .6
Leather on Metals " " " " "	....	.3 to .6
Leather on Wood " " " " "	....	.3 to .5
Metal on Metal " " " " "	(average)	.3
Cast Iron on Steel—velocity = 440 feet per minute	...	.32
" " " " " = 2640 " " "	...	.2
" " " " " = 5280 " " "	...	.06

There are no experimental data giving the decrease in the value of f at high speeds, for combinations such as wood or leather on metals. The data for cast iron on steel will, however, serve as a rough guide to what may be expected to occur. It is to be particularly noted that, in designing brake shoes or other friction machinery where great velocities are involved, allowance must be made for the decrease in the value of the coefficient.

29. Dry Rolling Friction. When a curved body rolls upon a plane or curved surface, it has been found that the so-called frictional resistance due to the rolling action is much less than that due to sliding, for the same load. If P = the load; F = the horizontal force required at the axis of a circular body to produce and sustain uniform motion; and r = radius of rolling body, it has been found that $F = \frac{kP}{r}$ where k is a coefficient to be determined experimentally. If r be expressed in inches k is found to have a value of about .02 for iron or steel rolling on iron or steel.

Neither the coefficient k nor the exact theory of rolling friction is at present very accurately known. The most important use of rolling friction is, as far as the present discussion is concerned, in connection with roller bearings for shafting, and a fuller discussion of these will be given later.

30. Friction of Lubricated Surfaces. When a lubricant is interposed between a pair of rubbing surfaces, the frictional resistance is materially reduced because the surfaces are wholly or partially separated from each other by the lubricant. The lubricant may be fed to the surfaces in a number of ways. If the motion is intermittent, and other conditions will allow, a simple oil hole leading to the rubbing surfaces is often used. If the motion is continuous, some form of oil cup which will give a continuous supply is better. Fig. 14 (a) shows a cup of the simpler type where a wick of cotton or wool draws up the oil by capillary attraction and feeds it slowly into the oil hole. This is sometimes called siphon feed. Fig. 14 (b) shows a so-called sight feed cup where the oil falling by gravity from the cup can be seen as it passes the hole *e* and the flow can be regulated by the screw *d*. Centrifugal action is also used to some extent to feed oil to rotating parts. Sometimes an opening is made in the bearing so that a pad saturated with lubricant can be kept pressed up against the moving surface, thus lubricating the whole length of the journal continuously. For heavy lubricants, such as greases, where very heavy pressures are carried on the rubbing surfaces, so-called compression cups are often used and are constructed so as to force the lubricant in between the surfaces. Fig. 14 (c) shows a "ring oiled" bearing. The ring *r* running loose on the shaft *s* dips into the pocket below the shaft. The friction of the ring on the shaft causes it to rotate and draw up oil from the pocket. Sometimes chains are used instead of solid rings. For the most efficient lubrication the journal itself runs in a bath of oil (Fig. 15) or is flooded with oil supplied under pressure. The relative merits of these various methods of supplying the lubricant will be more apparent after a discussion of the general laws of lubrication.

The effect of friction, and the efficiency of lubrication of so-called lubricated surfaces, may conveniently be treated under three heads:—

(a) Static Friction of Lubricated Surfaces.
(b) Friction of Imperfectly Lubricated Surfaces.
(c) Friction of Perfectly Lubricated Surfaces.

31. Static Friction and Lubrication. When a pair of lubricated surfaces are pressed together by a load, the pressure tends to slowly expel the lubricant from between the surfaces.

Experiments and experience show that it is very difficult even with limited areas and heavy pressures completely to expel the lubricant. If ordinary machinery, however, is allowed to stand at rest for a short period of time, this action is sufficient to expel so much of the lubricant that may have been between the surfaces while running as to allow the metallic surfaces to come more or less in contact. The static coefficient of friction of lubricated surfaces is hence very much higher than that of surfaces which move even very slowly; for it will be seen presently that even at low velocities the surfaces tend to draw in the lubricant by their motion. It is a well-known fact that heavy machinery always offers a great resistance to starting after lying idle a short time and often the rubbing surfaces, if not oiled before starting, will

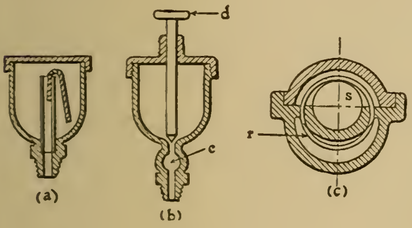

FIG. 14.

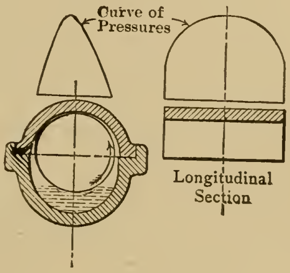

FIG. 15.

abrade each other before the lubricating action due to running begins to take effect. The materials therefore for the rubbing surfaces of heavy machinery should be carefully chosen for their antifriction qualities, and oil grooves should be carefully provided so that lubricant can be applied as near the point of greatest pressure as possible before motion begins.

The coefficient of static friction for lubricated surfaces is not very accurately known and it varies somewhat with the pressure and character of the lubricant. A fair average value for metal surfaces and pressures ranging from 75 to 500 lbs. per sq. in. is .15.*

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*See Thurston's "Friction and Lost Work," pages 316–317.

32. Imperfect Lubrication. When one lubricated surface slides over another, the moving surface, even at low velocities, tends to carry the lubricant, if properly applied, in between the surfaces. Thus the layer of oil which touches the surface of a journal adheres to it and is carried along under the bearing. This layer in turn tends to carry along the layer which next adjoins it, because the viscosity of the lubricant opposes the shearing action which results between layers on account of the action of the moving surface of the journal. In plane sliding surfaces the lubricant is generally applied to the stationary surface and tends to cling to it in spite of the tendency of the slider to rub it off. The action of the sliding surfaces in drawing in the lubricant is similar to that of the rotating journal, but in a much less marked degree as would naturally be expected from the nature of the case. If the velocity of rubbing be very low, or the pressure very high, or the supply of lubricant limited, the quantity of lubricant that is carried in is very small and the surfaces in contact are very slightly lubricated and may even be in actual metallic contact. The materials, therefore, for the rubbing surfaces of slow-moving machinery should also be carefully chosen for their antifriction qualities, as even after the machinery has been successfully set in motion metallic contact may occur between them.

If the velocity of rubbing and the supply of lubricant be increased, the load remaining the same, more and more lubricant is thrust between the surfaces by the action noted above till, at a point depending on the pressure, velocity of rubbing, and viscosity of the lubricant, the metallic surfaces are completely separated and the friction becomes only that due to the fluid friction of the lubricant itself. This last state is known as **perfect lubrication.** The formation of this separating film with increasing speed is probably gradual and the character of the contact most probably passes through a gradual change, from contact which is nearly metallic through successive stages of partially fluid contact to complete fluid separation. The exact point at which perfect lubrication occurs for any given load, velocity, and lubricant is not accurately known, but what data are available will be given in connection with the discussion of perfect lubrication which follows. It is known, however, that perfect lubrication cannot be obtained without a plentiful supply of the lubricant, as in the case where a journal runs in an oil bath, or is supplied by so-called forced lubrication where the lubricant is delivered under pressure. It is impossible or inconvenient, however, to lubricate the greater part of the rubbing surfaces of machines in this manner and, therefore, all surfaces lubricated by such means as simple oil holes, oil cups, oily pads, etc., where the supply of lubricant is in any way restricted must be considered as imperfectly lubricated.

As already noted, the exact condition which will exist between such surfaces depends on the pressure, the velocity of rubbing, the supply and character of the lubricant, and the temperature of the bearing as affecting the viscosity of the oil. Naturally where so many variables exist, experimental results are very discordant, and while an immense amount of work has been done, the results only serve to emphasize the great variation in conditions with change of these variables. It is evident, for instance, that if velocity and pressure remain constant, almost any condition may be produced from metallic contact to perfect lubrication simply by varying the supply of lubricant. The law of variation of the coefficient of friction, with either varying pressure or velocity, is also found to be modified by the rate at which oil is supplied. The generally accepted theories for imperfectly lubricated bearings running under average conditions, *i.e.*, at normal temperature, and with good oil supply from cups or pads, are as follows: *

(a) Starting from rest with constant load, the coefficient of friction first increases slightly with increasing velocity and then decreases, until at a velocity somewhere below 200 feet per minute (and depending upon the oil supply) a minimum value is reached (see Fig. 16).† With further increase of velocity the

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* See Archbutt and Deeley, page 58, and Thurston's "Friction and Lost Work," pages 296–312.

† It is to be noted that this discussion and the coefficients given refer to circular bearings and friction of rotation.

coefficient increases till the temperature affects the viscosity of the lubricant to such an extent that abrasion and failure occur.

(b) With constant velocity and very light loads (see Fig. 17) the coefficient of friction is very high. As the load is increased, the coefficient decreases very rapidly at first, and then more slowly till pressures of about 100 to 200 lbs. per square inch are obtained when the coefficient again slowly increases.

(c) The law of variation of friction with temperature is very complex and not well defined. Its general characteristics, however, may be expressed as follows: every combination of pressure and velocity requires a lubricant of a certain viscosity for best results. At high speeds and light loads, a light, thin oil

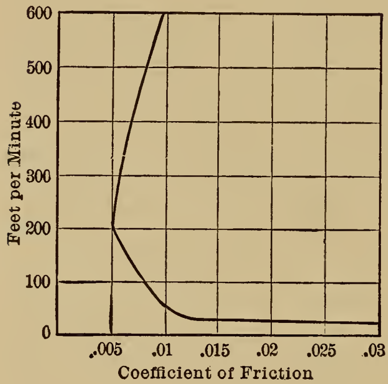

FIG. 16.

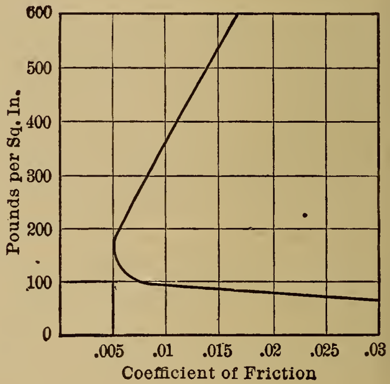

FIG. 17.

will be readily drawn in between the bearings, and its fluid friction, which constitutes the greater part of the resistance in such cases, will be less than that of a heavier oil. Increasing the temperature of a lubricant decreases its viscosity and, in the above case therefore, would cause a decrease in friction. In the case of the heavier loads and lower velocities, usually met with in machines, an increase of temperature decreases the viscosity and may, owing to the expulsion of the lubricant, give an increase in friction.

Care should therefore be used to obtain an oil suited to the case in hand, for sometimes a change of lubricant is sufficient to cause great trouble or, on the other hand, to reduce the temperature of a bearing that is heating. The failure of imperfectly lubricated bearings generally results from the lowering of the viscosity by increased temperature, so that the oil film is no longer maintained and metallic contact and abrasion ensue.

From the foregoing it is evident that the coefficient of friction for imperfect lubrication will necessarily be a variable quantity. Figs. 16 and 17 show the variation of μ for varying velocities and pressures. With good lubrication and moderate velocity it may be as low as .005, and again with low velocity and poor lubrication it may rise to .05 or more. When the velocity is exceedingly low, the coefficient approaches that of static friction of lubricated surfaces, the average value of which is .15. A fair average range for pressures from 50 to 500 lbs., and velocities from 50 to 500 ft. per minute, is from .02 to .008 and, for purposes of design of ordinary machinery, may be taken at .015. It is to be noted that with imperfectly lubricated surfaces and low velocities the coefficient of friction is less dependent on the character of the lubricant, and more dependent on the character of the rubbing surfaces. The curves Figs. 16 and 17 are composite curves taken from a number of actual experimental results. They are not to be taken as giving exact values of the coefficient μ, but serve to show graphically the general laws by which it varies. In interpreting such curves as Fig. 17 it must be kept in mind that, while the coefficient is decreasing or increasing, the actual *frictional resistance* may not be changing in like manner. The frictional resistance is the product of the load and the coefficient of friction. If, for instance, the coefficient *decreases* as fast as the load *increases*, the frictional resistance will remain constant. The curves show, however, where best results may be expected when designing new machinery, and throw some light on proposed changes in running speed of machinery already installed. They also indicate the complexity of the relation which exists between velocity, pressure, and the coefficient of friction. When it is considered that the temperature also greatly affects these relations, it is evident that a statement of these relations for imperfect lubrication, in the form of a general law or mathematical expression, is impracticable, and all such expressions are misleading.

33. **Perfect Lubrication.** It has been shown in the last article that any rotating journal will, by means of the molecular attraction between it and the lubricant, combined with the viscosity of the lubricant, draw more or less of the lubricant in between the journal and bearing, the amount so drawn in depending on the velocity and pressure. If the journal be allowed to run in an oil bath, or is otherwise plentifully supplied with oil, and the velocity be high enough for the pressure carried, it is found that this action is so marked that the rubbing surfaces are completely separated by a thin film of lubricant and the friction becomes only that due to the fluid friction of the lubricant itself.

Mr. Beaucamp Tower experimenting with journal friction (see Proceedings of Institution of Mechanical Engineers, 1883) found that with a journal and bearing arranged as in Fig. 15, the above action was so marked as to form a **film of oil under pressure** such that the load was completely **fluid borne.** The distribution of the pressure in this film was found to be as indicated by the diagrams above the cross-sections, rising to a maximum at the middle and falling to zero at the edges of the bearing. Mr. Tower succeeded in this way in carrying a load of 625 pounds per square inch of projected area at a velocity of 471 ft. per minute. With a load of about 330 lbs. per sq. inch, and a velocity of about 150 ft. per minute, a maximum oil pressure of 625 lbs. was found near the middle point of the bearing. It has been proved mathematically, and verified experimentally, that the conditions which exist in a bearing running under these conditions are as follows: the journal, being slightly smaller than the bore of the bearing, tends to be crowded back from the side where the lubricant is carried in, as shown in an exaggerated manner in the figure, giving a wedging effect. The pressure is consequently greatest at a point a little more than half way beyond the centre of loading where the distance between surfaces is least.

The exact relation which must exist between velocity and pressure, to allow this pressure film to form, is not known nor is it likely that exact limits can ever be set. Enough is known, however, to serve as a general guide for average conditions.

Professor H. F. Moore found that for circular journals the minimum limiting values of pressure and velocity where the film will just form may be approximately expressed by the expression $w = 7.47 \sqrt{v}$,* where w is in pounds per square inch, and v in feet per minute. The values given by this expression are plotted in Fig. 18, Curve No. 1, and seem to check fairly well with considerable other data. Curve (2), Fig. 18, shows the simultaneous values obtained by Tower with olive oil, where frictional resistance was a minimum, indicating that the film was at least well

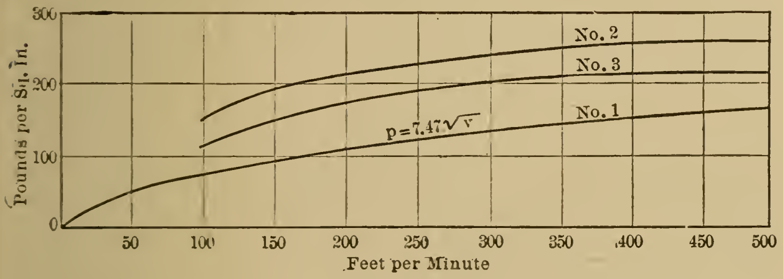

FIG. 18.

formed. Curve (3) shows similar values for mineral oil. The values obtained by Moore are on the safe side judged by Tower's work, which is accepted as accurate, and probably do not indicate the very lowest point at which a film will form. Tower found that a film would form considerably below the values given in Curve (2). In Moore's experiments, as in Tower's, the temperature was constant at 90°. Moore's experiments were on mineral oils. The results of Tower's experiments are very concordant and conclusive, and show that the laws of friction for perfectly lubricated surfaces, for ordinary speeds and pressures, are quite definite, the coefficient of friction varying as the square root of the velocity and inversely as the pressure, very nearly.

* See *American Machinist*, Sept. 16, 1903.

Thus for olive oil the relation is expressed very closely by $\mu = .2 \frac{\sqrt{v}}{w}$. It follows from this, that for any fixed velocity and temperature the product of μ and w will be a constant. That is, the frictional resistance is practically constant with change of load, for any velocity. This was actually found to be the case in the experiments, a variation of pressure per square inch from 100 to 500 not appreciably affecting the resistance. Table VIII will serve to show the remarkable regularity of the results, and the low values of the coefficient as compared with imperfectly lubricated surfaces. Much lower values have since been attained in oil-testing machines, under more ideal conditions, but such low values must not be considered as attainable under ordinary practical working conditions, while there is no good reason why such coefficients as given below cannot be obtained in well-constructed machinery.

TABLE VIII

BATH OF RAPESEED OIL

Load in lbs. per sq. Inch.	Coefficients of Friction for Speeds as Below.							
	105 ft. per Min.	157 ft. per Min.	209 ft. per Min.	262 ft. per Min.	314 ft. per Min.	366 ft. per Min.	419 ft. per Min.	471 ft. per Min.
573	......	.00102	.00108	.00118	.00126	.00132	.00139	
520	......	.00095	.00105	.00115	.00125	.00133	.00142	.00148
415	......	.00093	.00107	.00119	.00130	.00140	.00149	.00158
363	......	.00084	.00960	.00110	.00122	.00134	.00147	.00155
258	.00107	.00139	.00162	.00178	.00195	.00213	.00227	.00243
153	.00162	.00200	.00239	.00267	.00300	.00334	.00367	.00396
100	.00277	.00357	.00423	.00503	.00576	.00619	.00663	.00714

Tower's experiments have been amply verified and may be accepted as reliable for the range which they cover. The experiments of Stribeck and Lasché (see Chap. X) have extended the range of knowledge on this point to velocities over 2,000 ft. per minute. Their experiments show that for velocities between 500 and 2,000 ft. per minute the coefficient of friction, for a given load and temperature, varies as the 5th root of the velocity; and beyond 2,000 ft. is independent of the velocity. This point is discussed still further in Chap. X in connection with the design of bearings, where its principal application is found.

34. **Summary.** From the foregoing discussion the following statements may be made:

(a) The friction of imperfectly lubricated surfaces depends partly on the character of the surfaces themselves, and in a greater degree on the character and amount of the lubricant supplied.

(b) The load that can be successfully carried on an imperfectly lubricated surface will vary greatly with the amount of lubricant supplied, and must be kept very low where this supply is restricted.

(c) The friction of perfectly lubricated surfaces depends very little on the character of the rubbing surfaces, but depends mainly on the character of the lubricant.

(d) The frictional resistance of perfectly lubricated surfaces is, within the ordinary limits, independent of the intensity of pressure and dependent only on the velocity.

(e) The coefficient of friction of perfectly lubricated surfaces, for any given pressure and temperature, varies very nearly as the **square root** of the velocity for velocities up to 500 ft. per minute; approximately as the **fifth root** of the velocity for velocities between 500 and 2,000 ft. per minute; and is practically **independent** of the velocity for values above 2,000 ft. per minute.

35. **Efficiency.** It has been pointed out that all the energy supplied to a machine is not transformed into useful work, but that some of it is always lost in overcoming frictional resistances and doing useless work. There are many ways in which energy losses may occur in machines, and a careful distinction must be made between certain of these ways in order to get a clear definition of the term efficiency. Thus the steam engine receives its supply of heat in the form of steam under pressure. A considerable portion of the heat so received is lost by condensation of steam on the cooler cylinder walls, and some escapes by radiation without doing any work whatever on the piston. Of the energy actually applied to the piston, part is transformed into useful work at the driving belt, and part is lost in over-

coming the frictional resistances just discussed at the various constraining surfaces.

The gas engine is subject to similar losses; a large part of the heat of combustion escaping to the jacket water or to the atmosphere by radiation, and doing no work on the piston; while only a part of the energy actually applied to the piston reappears as useful work. Hydraulic and electric machinery have similar elements of loss. The first class of these energy losses might be called **leakage losses,** as they are of the same character as losses by actual leakage of the medium which is used to transmit the energy. The losses in the machine itself are known as **frictional losses** and are common to all machines; and no machine can transform all the energy supplied into useful work, but must lose some of it in friction or other wasteful resistances.

Efficiency has been defined (Art. 2) as the ratio of useful work to energy supplied; and from the above it appears that a machine may have two efficiencies depending on whether reference is had to total energy supplied, or to that portion only of the total energy which the machine transforms into useful and useless work. These efficiencies are respectively known as the **Absolute Efficiency** and the **Mechanical Efficiency.** Thus, if a gas engine is supplied with 1,000 thermal units, and transforms 200 units into useful work, and 50 units into the useless work of friction, its absolute efficiency is $\frac{200}{1,000} = .20$, and the mechanical efficiency is $\frac{200}{250} = .80$. The consideration of absolute efficiency is beyond the scope of this work; for the design of many machines it does not need to be considered; but the mechanical efficiency can seldom be neglected, since, in general, the amount of work to be done is fixed, and the source of energy must supply enough more energy than this to compensate for the frictional losses of the machine.

The mechanical efficiency of any train of mechanism is the continued product of the efficiencies* of all the several pairs of

* It may be noted in passing that the term efficiency is used in a number of ways other than as the ratio of work done to energy expended. Thus the strength of a

constraining surfaces in the train at which frictional losses occur. Let any machine have n pairs of such surfaces, and let their respective efficiencies be $e, e_1, e_2, e_3, e_4, ——— e_n$. Let E be the mechanical efficiency of the whole machine, and let K be the total amount of energy available for transformation into either useful or useless work. Then, the amount of energy which the first pair of constraining surfaces delivers to the second is $K \times e$, and the amount which the second delivers to the third is $Ke \times e_1$, and so on, till the amount of energy delivered by the last element (or the work done) is $K (e \times e_1 \times e_2 ——— e_n)$. But the mechanical efficiency of the train is

$$E = \frac{\text{work done}}{\text{energy supplied}} = \frac{K (e \times e_1 \times e_2 ———— e_n)}{K}$$

$$= (e \times e_1 \times e_2 ——— e_n).$$

A machine may consist of several trains of mechanism. If these several trains are arranged in series so that the energy passes from one to another consecutively, the efficiency of the whole machine, by reasoning similar to that in the last paragraph, is the continued product of the efficiencies of the several trains of mechanism. If, however, the trains are arranged in parallel so that the total energy is transmitted simultaneously through several trains of mechanism, each train transmitting only a portion of the energy, the above reasoning for the efficiency of the whole machine does not hold. If the amount of energy supplied to each train is known, the amount of work which it will deliver can be computed as above. The sum of all the work, delivered by all the trains, divided by the total energy supplied, will be the efficiency of the whole machine.

If, therefore, the efficiencies of the several constraining surfaces of a machine are known, the mechanical efficiency of the

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riveted joint, compared to the strength of the original unpunched plate, is called the *efficiency* of the joint, when what really is meant is its *relative strength*. Again, in an air compressor, the ratio of the air actually discharged per stroke, to the whole amount raised to the required pressure per stroke, is called the volumetric efficiency. It is evident that such efficiencies are of a different character from those discussed above and do not enter into the calculations of the efficiency of the machine, as a whole, in the manner indicated above.

whole machine can be calculated. The mechanical efficiency of any machine element is, however, a variable quantity; for the coefficient of friction of any pair of constraining surfaces will vary with the lubricant and its method of application, the temperature, the alignment of the surfaces, the velocity of rubbing, and the bearing pressure. Furthermore, when all other conditions are constant, the same pair of constraining surfaces will have an entirely different efficiency for the same amount of power transmitted, depending on the manner in which the load is applied. Thus, consider a simple wheel and axle driven by a belt on the periphery of the wheel. With a given diameter of wheel, the transmission of a given amount of power will bring a certain definite frictional load on the bearings. If, however, the diameter of the wheel is doubled, the belt speed is increased in a like ratio, and the belt tension will, for the same power transmitted, be one-half of the former value; and, as a consequence, the frictional resistance at the bearings will be reduced to one-half the original value, the revolutions remaining constant.

In general, therefore, it is impossible to calculate precisely from the analysis of a design what the mechanical efficiency will be, particularly if the mechanism is at all complicated, though a reasonable approximation is possible. If machines of a similar type have been built, it is far more accurate to base the design of new ones on efficiency tests made on those already in existence. For all standard machines such tests have been made, and the recorded results form a valuable basis for the design of new machines of like characteristics. But when a machine of a new type is to be designed, and no recorded tests are to be had that will give any information as to the probable efficiency, an estimate must often be made and the efficiency calculated as outlined above. In general, a close approximation can be made, and the making of such estimates is a great aid to the development of that judgment in such matters, which comes only with experience. In such cases a knowledge of the efficiencies of various machine elements becomes necessary. If the coefficient of friction for any constraining surface could be accurately determined, it would be possible to calculate its efficiency with some degree of certainty. But, as before noted, the quantity varies with the velocity of rubbing, with changes in bearing pressures, etc., and such methods of computation are necessarily cumbersome and to be attempted only where a very close estimate is required.

The following are rough average values of the efficiencies of the most common elements. For more accurate values the student is referred to the respective discussions of these various elements which follow:

Common Bearing, singly	96–98
Common Bearing, long lines of shafting	95
Roller Bearing	98
Ball Bearings	99
Spur Gear Cast Teeth, including bearings	93
Spur Gear Cut Teeth, including bearings	96
Bevel Gear Cast Teeth, including bearings	92
Bevel Gear Cut Teeth, including bearings	95
Worm Gear, varies with thread angle, see Art. 54	
Belting	96–98
Pin-connected Chains, as used on bicycles	95–97
High Grade Transmission Chains	97–99

CHAPTER V

SPRINGS

36. Distinguishing Characteristic of Springs. Springs are characterized by a considerable distortion under a moderate load. Every machine member is, in a sense, a spring, for no material is absolutely rigid and the application of a load always produces stress and accompanying strain. By proper selection and distribution of material it is possible to control (within wide limits) the degree of distortion under a given load.

An absolutely rigid material would be practically unfit for the construction of any member subject to other than a perfectly quiescent load; for (as shown in Art. 24) the stress due to a suddenly applied load would be infinite if the corresponding distortion of the member were zero.

While it is usually desirable to restrict the distortions of most machine parts to very small magnitudes, there are many cases in which considerable distortion under moderate load is desirable or essential. To meet this last requirement the member is often given some one of the forms commonly called springs.

37. The Principal Applications of Springs. Springs are in common use:

I. For weighing forces; as in spring balances, dynamometers, etc.

II. For controlling the motions of members of a mechanism which would otherwise be incompletely constrained; for example, in maintaining contact between a cam and its follower. This constitutes what Reuleaux has called "force closure."

III. For absorbing energy due to the sudden application of a force (shock); as in the springs of railway cars, etc.

IV. As a means of storing energy, or as a secondary source of energy; as in clocks, etc.

An important class of mechanisms in which springs are used to weigh forces is a common type of governor for regulating the speed of engines or other motors. In those governors which use springs to oppose the centrifugal, or other inertia actions, the springs automatically weigh forces which are functions of speed, or of change of speed. The links, or other connections, which move relative to the shaft with any variation of the above forces, correspond to the indicating mechanism of ordinary weighing devices.

The first of the above-mentioned applications—the weighing of forces—is usually the most exacting as to the relation between the load and the distortion of the spring throughout the range of action. In the second and third classes of application, it is frequently only required that the maximum load and distortion shall lie within certain limits, which often need not be very precisely defined. The use of springs for storing energy (as the term spring is ordinarily understood) is almost wholly confined to light mechanisms or pieces of apparatus requiring but little power to operate them.

38. Materials of Springs. Springs are usually of metal; although other solid substances, as wood, are sometimes used. A high grade of steel, designated as spring steel, is the most common material for heavy springs, but brass (or some other alloy) is often used for lighter ones.

A confined quantity of air, or other compressible fluid, is used in many important applications to perform the office of a spring. The air-chamber of a pump with its inclosed air is a familiar example of what may be called a fluid spring used to reduce shock ("water hammer"). The characteristic distortion of the solid springs is a change in *form* rather than of volume; while the fluid springs are characterized by a change of *volume* with incidental change of form.

Soft-rubber cushions, or buffers, are not infrequently employed as springs, and these are in some respects intermediate in their action to the two classes mentioned above. It is usually not necessary, in these simple buffers, or cushions, to secure a very exact relation between the loads and the distortions under such loads.

The discussion of the confined gases (fluid springs) is not within the scope of the present work; hence the following treatment will be limited to solid springs.

39. Forms of Solid Springs. Springs may be subjected to actions which extend, shorten, twist, or bend them, producing

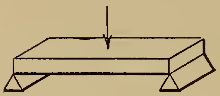

FIG. 19.

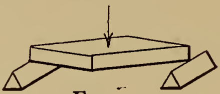

FIG. 20.

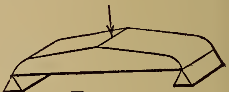

FIG. 21.

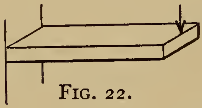

FIG. 22.

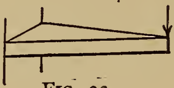

FIG. 23.

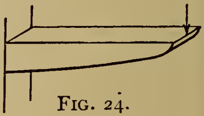

FIG. 24.

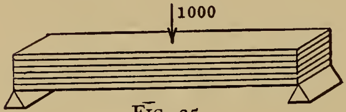

FIG. 25.

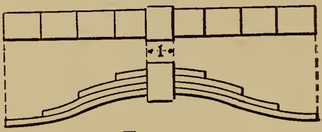

FIG. 27.

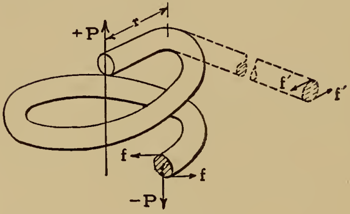

FIG. 29.

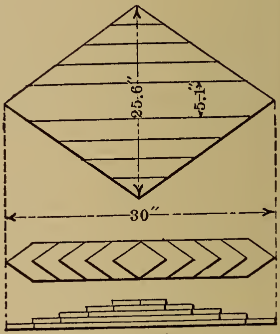

FIG. 26.

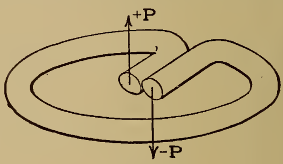

FIG. 28.

stresses in the material, the character of which depends both upon the form of the spring and upon the manner of applying the load.

I. *Flat Springs* are essentially beams, either cantilevers, or with more than one support. These springs are subjected to flexure when the load is applied, and the resultant stresses are tension in certain portions of the material, and compression in others, with a transverse shear, as in all beams; the shear may usually be neglected in computations. The ordinary beam formulæ for strength and rigidity may be applied to flat springs, with constants appropriate to the particular material and form of beam used.

Flat springs may be simple prismatic strips, of uniform cross-section (Fig. 19 or 22), although it is preferable that the form of such springs approximate those of the "uniform strength" beams (Figs. 20 or 21; 23 or 24).

It is often desirable or practically necessary to build up these springs of several layers, leaves, or plates, producing a laminated spring. It will appear from the discussion of these laminated

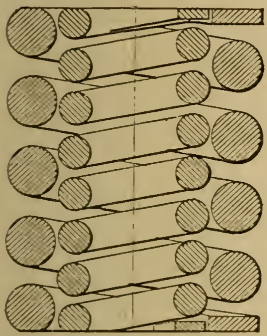

FIG. 30.

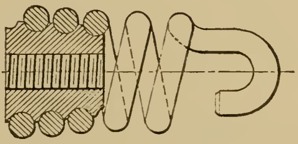

FIG. 30 (a).

springs that they may be properly treated as a modification of one form of "uniform strength" beam. The neutral surface of the beam used as a spring may be initially curved, either to clear other bodies, or to give the spring an advantageous form when it is under normal load. See Fig. 27.

Two or more springs may be compounded, as in the "elliptical" springs or in the platform springs frequently used under carriages. In such cases, each spring may be computed separately, and the total deflection is the sum of the deflections of the separate springs of the set.

II. *Helical, or Coil Springs* are most commonly used to resist actions which extend, shorten, or twist the spring relatively to its longitudinal axis. These are sometimes improperly called spiral springs.

The stress in the wire (or rod) of which a helical spring is made is somewhat complex, consisting of torsion combined with tension or compression, or both. In a "pull spring," one which is extended longitudinally under the load, the predominating stress (with ordinary proportions) is a torsion, and there is a secondary tensile stress in the wire. In a "push spring," one which is shortened by the load, the predominating stress is torsion, with a secondary compressive stress. When the helical spring is subjected to an action which twists the spring (as a whole) the principal stress in the wire is that due to flexure (tension and compression in opposite fibres) and the secondary stress is torsion.

Helical springs are sometimes arranged in "nests," springs of smaller diameter being placed within those of larger diameter, (Fig. 30). In these cases, the different springs of a set are computed separately. This last arrangement is common practice in car trucks.

III. *Spiral Springs*, properly so called are those of the form of the familiar clock spring. These are best adapted for a twist relative to the axis of the spiral, and are usually employed when a very large angle of torsion between the two connections is necessary. In this form of spring, the stress in the material is that due to flexure: or tensile and compressive stress on opposite sides of the neutral axis.

IV. *Helico-Spiral Springs*. The form of spring represented by the common upholstery spring may be looked upon as a spiral spring which has been elongated, and given a permanent set, in the direction of its axis; or it may be considered as a modified helical spring in which the radii of the successive coils are not equal. It is thus intermediate between the two preceding classes. This last form is not usual in machine construction; though it has the advantage over the common helical spring of considerable lateral resistance, and it may be employed to advantage where it is difficult or undesirable otherwise to constrain the spring against buckling. This spring is used only as a push spring, to resist a compressive action. The springs used on the ordinary disc valves of pumps are often of this form, as they will close up flat between the valve and guard. Car springs are sometimes made of a flat strip or ribbon of steel wound in this general form, with the edges of the strip parallel to the axis of the spring.

V. Occasionally straight rods, usually of circular or rectangular cross-sections, are employed to resist torsion relative to their longitudinal axis. These are comparatively stiff springs, and the stress is, of course, torsional. Every line of shafting is necessarily a spring, in this sense.

The following summary gives the ordinary forms of solid springs; the kinds of loading to which they are subjected; and the predominating stresses resulting from the different loads.

GENERAL SUMMARY OF SPRINGS

Form of Spring.	Load Action.	Predominating Stress.
Flat Spring.	Flexure or Bending.	Tension and Compression.
Helical Spring.	Extension, Pull.	Torsion (plus).
" "	Compression, Push.	Torsion (minus).
" "	Torsion, Twist.	Tension and Compression.
Spiral "	Torsion, Twist.	Tension and Compression.

41. Computations of Simple Flat Springs. The following notation will be used in treating of flat springs with rectangular cross-sections.

P = load applied to the spring.

l = free length of the spring.

p = intensity of stress in outer fibres.

I = moment of inertia of most strained section.

h = dimension of this section in plane of flexure.

b = dimension of this section perpendicular to plane of flexure.

E = modulus of elasticity of material.

δ = deflection of the spring.

The six forms of rectangular section beams, shown by Figs. 19 to 24, are the most important of those used as simple flat springs. These will be designated Type I, II, etc., as in the following table, which gives the constants to be substituted in the general formulæ for computations relating to each type.

TABLE IX

	Type.	Coefficients.				
		A	β	B	K	C
I	As per Fig. 19	$\frac{2}{3}$	$\frac{1}{48}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{3}{2}$
II	" " " 20	$\frac{2}{3}$	$\frac{1}{32}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{3}{2}$
III	" " " 21	$\frac{2}{3}$	$\frac{1}{24}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{3}{2}$
IV	" " " 22	$\frac{1}{6}$	$\frac{1}{3}$	4	$\frac{2}{3}$	6
V	" " " 23	$\frac{1}{6}$	$\frac{1}{2}$	6	1	6
VI	" " " 24	$\frac{1}{6}$	$\frac{2}{3}$	8	$\frac{4}{3}$	6

The theory of strength against flexure (equation J and tables 1 and 2) gives: For rectangular section beams supported at the ends and loaded at the middle (Types I, II, III).

$$\frac{1}{4} P l = \frac{1}{6} p b h^2 \therefore P l = \frac{2}{3} p b h^2 \quad . \quad . \quad . \quad (1)$$

For the rectangular section cantilevers, with load at free end,

$$P l = \frac{1}{6} p b h^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Or the general formula for the strength of rectangular section beams may be written

$$P l = A p b h^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In which the coefficient A has the values given in the Table.

The theory of elasticity of beams gives

$$\delta = \beta . \frac{P l^3}{E I}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

or for rectangular cross-sections

$$\delta = B \frac{P l^3}{E b h^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In which β and B are as given in the Table, for the types under consideration.

The last equation (5) may be used for all computations as to rigidity of flat springs (beams), provided the elastic limit is not exceeded. The only constant for the *material* which enters this expression is the modulus of elasticity (E); this is simply the *ratio* of stress to strain which holds up to, but not beyond, the elastic limit; hence any computation made by this formula should be checked for safety. Equation (3) may be used for this purpose. **To illustrate,** assume that a rectangular section prismatic spring (Type I) has a length between supports of $l=30''$; the load at the middle is $P=1{,}000$ lbs.; the deflection under this load is to be $\delta=1.5$ inches; and the spring is made of a single strip of steel $\frac{3}{8}$ inch thick (h). Required the breadth (b) of the spring, assuming the modulus of elasticity, $E=30{,}000{,}000$.

From eq. (5):—

$$b = B\frac{P\,l^3}{E\,\delta\,h^3} = \frac{1}{4} \times \frac{1{,}000 \times 27{,}000 \times 512}{30{,}000{,}000 \times 1.5 \times 27} = 2.84 + \text{ inches.}$$

This gives the width of spring for the required relation of the deflection to load; that is, it gives a spring of the required stiffness, provided the stress produced does not exceed the elastic limit. It is necessary to check the spring as found above, for if the elastic stress is passed, the spring not only takes a permanent set, but the required ratio of the load to the deflection will not be secured. On the other hand, it is often important for economy of material to use as light a spring as is consistent with safety; or, in other words, it is important not to have too low a working stress under the maximum load.

From eq. (3):—

$$p = \frac{P\,l}{A\,b\,h^2} = \frac{3 \times 1000 \times 30 \times 64}{2 \times 2.84 \times 9} = 112{,}500 \text{ lbs. per sq. inch.}$$

This stress is beyond the elastic limit of any ordinary grade of steel, hence it is probable that some different form of spring should be used. A change could be assumed, as in the thickness of the plate, and new computations made with the new data. A thinner plate would reduce the stress, but it would demand a wider spring for the required stiffness. A more general method will now be given, by which it is possible to determine the proper spring for given requirements without the necessity of successive trial computations.

From eq. (3):—

$$b\,h^2 = \frac{P\,l}{A\,p} \therefore b\,h^3 = \frac{P\,l\,h}{A\,p} \quad . \quad . \quad . \quad . \quad (6)$$

From eq. (5):—

$$b\,h^3 = \frac{B\,P\,l^3}{E\,\delta} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

From eqs. (6) and (7):—

$$\frac{P\,l\,h}{A\,p} = \frac{B\,P\,l^3}{E\,\delta};$$

$$\therefore h = A\,B\frac{p\,l^2}{E\,\delta} = K\frac{p\,l^2}{E\,\delta} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

From eq. (3):—

$$b = \frac{1}{A}\,\frac{P\,l}{p\,h^2} = C\,\frac{P\,l}{p\,h^2} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The two equations (8) and (9) are in convenient form for designing a flat spring when the span (l), deflection (δ), load (P), and the material are given. **Example:** The span of a rectangular section prismatic flat spring (Type I) is 30 inches; and a load of 1,000 lbs. applied at the middle is to cause a deflection of 1.5 inches.

If the modulus of elasticity be 30,000,000 and the safe maximum working stress be taken at 50,000 lbs. per sq. in.,* required the dimensions of the cross-section, h and b.

From eq. (8):—

$$h = K\,\frac{p\,l^2}{E\,\delta} = \frac{1}{6} \times \frac{50{,}000 \times 900}{30{,}000{,}000 \times 1.5} = \frac{1}{6}\text{ inch.}$$

Taking $h = \frac{5}{32}$ inch, to use a regular size of stock, p will be somewhat less than 50,000, or

$$p : 50{,}000 :: \tfrac{5}{32} : \frac{1}{6}; \therefore p = 47{,}000.$$

From eq. (9):—

$$b = C\,\frac{P\,l}{p\,h^2} = \frac{3}{2} \times \frac{1000 \times 30 \times 1024}{47{,}000 \times 25} = 39.2\text{ inches.}$$

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* If the spring is provided with stops to prevent deflection beyond a certain amount, the stress due to such deflection may be nearly equal to the elastic limit of the material. A very small factor of safety is all that is necessary.

If this width is inadmissible, a laminated or plate spring may be used. See next article.

It will be noted that equation (8) does not directly involve either the load P or the breadth of spring b. It is evident that if a beam (flat spring) of given span (l), and thickness (h), is caused to deflect a given amount (δ), the outer fibres will undergo a definite strain which is not dependent upon the width of the beam (b), nor upon the force required to produce this change in relative positions of the molecules. As the unit strain multiplied by the modulus of elasticity equals the unit stress, it follows that this stress may be computed from l, h, and δ (which determine the strain), in connection with E. If the breadth of the beam (b) is increased, the force (P) required to produce the given deflection (δ) will be proportionately increased, but the intensity of stress is not affected by these changes alone.

This same conclusion may be reached from the following relation,* in which ρ = the radius of curvature due to load.

$$\rho = \frac{EI}{M} = EI \div \frac{pI}{\frac{1}{2}h} = \frac{Eh}{2p} \quad . \quad . \quad . \quad (10)$$

$$\therefore p = \frac{Eh}{2\rho} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

It appears from eq. (11) that the stress is simply proportional to the thickness (h) and the radius of curvature (ρ), for any given value of E. The span l, and the deflection δ, determine ρ, so that eq. (10) or (11) may take the place of eq. (8). Equations (10) and (11) are important in connection with the theory of laminated springs.

42. Laminated, or Plate, Springs. It was shown in the preceding article that the maximum thickness of a simple flat spring is fixed when the span, deflection, and modulus of elasticity are known, and the intensity or working stress has been assigned. [See eq. (8).] With the value of the thickness (h) thus limited it will frequently happen that a simple spring will require excessive breadth (b) to sustain the given load, and it is often necessary to use a spring built up of several plates or leaves.

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* See Church's "Mechanics," page 250.

Example: $P=1{,}000$ lbs.; $l=30''$; $p=60{,}000$ lbs. per sq. in.; $\delta=2''$, and $E=30{,}000{,}000$. A simple prismatic spring of rectangular section, with load at the middle of the span (Type I), to meet the above requirements would have:

$$h = K\frac{p\,l^2}{E\,\delta} = \frac{1}{6} \times \frac{60{,}000 \times 900}{30{,}000{,}000 \times 2} = .15 \text{ inch.}$$

$$b = C\frac{P\,l}{p\,h^2} = \frac{3}{2} \times \frac{1{,}000 \times 30}{60{,}000 \times .0225} = 33\tfrac{1}{3} \text{ inches.}$$

This spring, consisting of a plate .15 inch thick and 33⅓ inches wide, with a span of 30 inches, is evidently an impracticable one for any ordinary case. Suppose this plate be split into six strips of equal width, each $33.3 \div 6 = 5.5''$ wide, and that these strips are piled upon each other as in Fig. 25; then, except for friction between the various strips, the spring would be exactly equivalent, as to stiffness and intensity of stress, to the simple spring computed above. While the form of laminated spring which has just been developed might answer in some cases, another form, based upon the "uniform strength" beam (Type II), is much better for the ordinary conditions. It may be developed as follows, taking the same data as the preceding example except that the spring is to be of Type II, Fig. 20.

In the simple spring, Type II, Table IX

$$h = K\frac{p\,l^2}{E\,\delta} = \frac{1}{4} \times \frac{60{,}000 \times 900}{30{,}000{,}000 \times 2} = .225 \text{ inches.}$$

$$b = C.\frac{P\,l}{p\,h^2} = \frac{3}{2} \times \frac{1{,}000 \times 30}{60{,}000 \times .0506} = 14.8 \text{ inches.}$$

A laminated spring for the case under consideration may be derived from this simple spring by imagining the lozenge-shaped plate to be cut into strips which are piled one upon another as indicated in Fig. 26. The thickness of .225 inches does not correspond to a regular commercial size of stock, however, and it will usually be better to modify the spring to permit using standard stock. If a thickness of ¼″ be assumed for the leaves or plates, the stress, as found from eq. (8) of the preceding article becomes:

$$p = \frac{h E \delta}{K l^2} = \frac{4 \times .25 \times 30{,}000{,}000 \times 2}{900} = 66{,}700.$$

If this stress is considered too great, steel $\frac{3}{16}''$ thick might be used, when $p = \dfrac{4 \times 3 \times 30{,}000{,}000 \times 2}{16 \times 900} = 50{,}000.$

With $h = \frac{3}{16}''$, and $p = 50{,}000$,

$$b = C\frac{P l}{p h^2} = \frac{3}{2} \times \frac{1{,}000 \times 30 \times 256}{50{,}000 \times 9} = 25.6''.$$

If this spring, 30″ span, $\frac{3}{16}''$ thick, and 25.6″ wide at the middle, be replaced by 5 equivalent strips, each $25.6 \div 5 = 5.11''$ wide (nearly $5\frac{1}{8}''$), see Fig. 26, a laminated spring of good form and practical dimensions will result. In cases where the maximum allowable width of spring is fixed, a larger number of plates may be necessary. Thus, in the preceding problem, if the spring width must be kept within $4\frac{1}{2}''$, it is necessary to use 6 plates, each $25.6 \div 6 = 4.27''$ wide. In actual springs, the usual construction is that shown by Fig. 27, in which the several plates have the ends cut square across instead of terminating in triangles. These springs approximate uniform strength beams, and may be computed by equations (8) and (9) of Art. 41, remembering that b is the breadth of the equivalent simple spring. Or, if n is the number of plates and b_1 the breadth of each plate in the laminated spring, $n\, b_1 = b$.

The last of these formulæ, eq. (9), is not strictly applicable when the ends of the plates are cut square across; but it may generally be used with sufficient accuracy, provided the successive plates are regularly shortened by uniform amounts. It is quite common practice to have two or more of the plates extend the full length of the spring. This construction makes the spring a combination of the triangular and prismatic types (Type II and Type I, or Type V and Type IV, depending upon whether the spring is supported at the ends, or is a cantilever). Mr. G. R. Henderson in discussing the cantilever form (Trans. A. S.

M. E., Vol. XVI), says:—"For a spring with all the plates full length we would have (see eq. 5)

$$\delta = \frac{4 P l^3}{E n b_1 h^3}$$

so for one-fourth of the leaves full length, the deflection would be decreased approximately one-fourth of the difference between

$$\frac{6 P l^3}{E n b_1 h^3} \text{ and } \frac{4 P l^3}{E n b h^3} \text{ or } \frac{5.5 P l^3}{E n b h^3}."$$

By similar reasoning, for a spring loaded at the middle and supported at the ends, with one-fourth the plates extending the whole length of the spring,

$$\delta = \frac{11}{32} \frac{P l^3}{E n b_1 h^3}.$$

This may be otherwise stated as follows:

When the number of full-length leaves is one-fourth the total number of leaves in the spring, use $\frac{11}{12} B$ instead of B and $\frac{11}{12} K$ instead of K in equations (5) and (8) of the preceding article; the values of B and K being those given for the triangular forms, Type II or Type V, as the case may be.

The spring shown in Fig. 27 is initially curved (when free), which is common practice. The best results are obtained by having the plates straight when the spring is under its normal full load (if this is practicable) because the sliding of the plates upon each other, with the vibrations, is then reduced to a minimum. The several plates of a laminated spring are usually secured by a band shrunk around them at the middle of the span. This band stiffens the spring at the middle, and one-half the length of the band (½ l, Fig. 27) may be deducted from the full span to give the effective span to be used as l in the above formulæ. It is not uncommon to make the longest plate thicker than the others, if but one plate is given the full length of the spring. This cannot be looked upon as desirable practice, however, as all of the plates are subjected to the same change in radius of curvature; hence the thicker plate is subjected to the greater stress. See equation (11).

The following formulæ (derived from the preceding) may be used in computing flat springs; but it must be remembered that there is always liability of considerable variation in the modulus of elasticity, hence such computations can only be expected to give approximations to the deflections which will be observed by tests of actual springs. These computations will be sufficiently exact for many purposes; but when it is important accurately to determine the scale of the spring (ratio of deflection to load), actual tests must be made. In using these formulæ the following rules should be observed.

I. When the several plates are secured by a band shrunk, or forced, over them, one-half the length of the band is to be subtracted from the length of the spring to get the effective length of the spring.

II. When the plates have different thicknesses, the stress should be computed for the plate having the maximum thickness.

III. If more than one plate has the full length of the spring, an appropriate modification of the values of the coefficients B and K should be made. Thus, when one-fourth of the total number of plates are full length, $\frac{1}{1}\frac{1}{2}$ B and $\frac{1}{1}\frac{1}{2}$ K should be used instead of B and K (Type II or V) in equations I, II, III, and IV, below.

EQUATIONS.

$$\delta = B \frac{P\, l^3}{E\, n\, b_1\, h^3} \qquad \text{(I)}$$

$$P = \frac{E\, n\, b_1\, h^3\, \delta}{B\, l^3} \qquad \text{(II)}$$

$$h = A\, B \frac{p\, l^2}{E\, \delta} = K \frac{p\, l^2}{E\, \delta} \qquad \text{(III)}$$

$$p = \frac{E\, \delta\, h}{K\, l^2} \qquad \text{(IV)}$$

$$p = \frac{E\, h}{2\, \rho} \qquad \text{(V)}$$

$$P = \frac{A\, p\, n\, b_1\, h^2}{l} \qquad \text{(VI)}$$

$$p = \frac{C\, P\, l}{n\, b_1\, h^2} \qquad \text{(VII)}$$

$$b_1 = \frac{P\, l}{A\, n\, p\, h^2} = \frac{C\, P\, l}{n\, p\, h^2} \qquad \text{(VIII)}$$

Experience shows that thin plates have a higher elastic limit than thick plates of similar grade of material. In the practice of a prominent eastern railway company, the values allowed for the maximum intensity of stress in flat steel springs are, for:

Plates	$\frac{1}{4}$	inch	thick,	$p = 90{,}000$	lbs.	sq. in.
"	$\frac{5}{16}$	"	"	$p = 84{,}000$	"	"
"	$\frac{3}{8}$	"	"	$p = 80{,}000$	"	"
"	$\frac{7}{16}$	"	"	$p = 77{,}000$	"	"
"	$\frac{1}{2}$	"	"	$p = 75{,}000$	"	"

The above values are satisfied by the equation $p = 60{,}000 + \frac{7{,}500}{h}$, in which h is the thickness of plate in inches.

These values are for the greatest stress to which the material can be subjected, as when the spring is deflected down against the stops.

The modulus of elasticity, E, may vary considerably; but its value may be assumed at about 30,000,000 in the absence of more definite data.

In designing a new spring, the value of h is to be found from equation (III); then b_1 is found by equation (VIII). The other formulæ are useful in checking springs already constructed, for deflection due to a given load, or the reverse; for safety, etc.

43. **Helical Springs.** If a rod or wire be wound into a flat ring with the ends bent in to the centre, Fig. 28, and two equal and opposite forces, $+P$ and $-P$, be applied to these ends (perpendicular to the plane of the ring) as indicated, the rod will be subjected to torsion.

If a longer rod be wound into a helix, with the two ends turned in radially to the axis, the typical helical spring is produced. If two equal and opposite forces, $+P$ and $-P$, act on these ends, along the axis of the helix, they induce a similar stress (torsion) in the rod, but as the coils do not lie in planes perpendicular to the line of the forces, there is a component of direct stress along the rod. This direct stress increases as the pitch of the coils increases relative to their diameter; but with ordinary proportions of springs, the torsion alone need be considered, when the external forces lie along the axis of the helix.

The following notation will be used in treating of helical springs of circular wire, subjected to an axial load:

P = the force acting along the axis.
r = the radius of the coils, to center of wire.
d = the diameter of wire.
p = the maximum intensity of stress in wire (torsion).
I_p = the polar moment of inertia of wire.
E_s = the transverse modulus of elasticity.
δ = the "deflection" (elongation or shortening) of spring.
n = the number of coils in the spring.
l = the length of wire in the helix $= 2\,\pi\, r\, n$ (approximately).

Suppose a helical spring under an axial load to be cut across the wire at any section, and the portion on one side of this section to be considered as a free body, Fig. 29. Neglecting the direct stress, equilibrium demands that the moment (Pr) of the external force shall equal the stress couple, or moment of resistance $\left(\frac{\pi}{16}\, p\, d^3 \text{ for circular section}\right)$.

If this free portion of the helix is straightened out, as indicated by the broken lines in Fig. 29, till its direction is perpendicular to the radial end, it will appear that the moment Pr still equals the moment of resistance, $\frac{\pi}{16}\, p\, d^3$. Since the stress and strain are the same in this helix and the straight rod, it appears that the energy expended against the resilience is the same in both cases (the length of wire affected remaining constant). Or, as the force (P) and the arm (r) are the same in both conditions, the distances through which this force acts to produce a given torsional stress (p) are equal. If a straight rod of length l is subjected to a torsional moment Pr, the angle of twist being α (in π measure), then

$$P\, r = \frac{\alpha\, I_p\, E_s}{l}$$

[See Church's "Mechanics," page 236].

The energy expended on the rod is the mean force applied multiplied by the distance through which this force acts. If the load is gradually applied, this energy is ½ $P r a$. In the case of the corresponding helical spring, the mean force (½P) acts through a distance equal to the "deflection" of the spring (δ), or the energy expended is ½ $P\delta$. As pointed out above, the energy expended in the two cases is the same, or

$$\tfrac{1}{2} P r a = \tfrac{1}{2} P \delta \therefore a = \frac{\delta}{r}$$

$$\therefore P r = \frac{a I_p E_s}{l} = \frac{\delta}{r} \times \frac{\pi d^4}{32} \times \frac{E_s}{2 \pi r n} = \frac{\delta d^4 E_s}{64 r^2 n}$$

$$\therefore P = \frac{\delta d^4 E_s}{64 r^3 n} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Equation (1) may be used for finding the load corresponding to an assigned deflection in a given spring. The equation can be put in the following form for finding the deflection due to a given load:

$$\delta = \frac{64 P r^3 n}{d^4 E_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Or the equation may be employed for designing a spring in which the load and deflection are given, by assuming any two of the three quantities, r, d and n. The most convenient form for this latter purpose is usually,

$$n = \frac{\delta d^4 E_s}{64 P r^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

These equations for *rigidity* hold good only within the elastic limit of the material, as E_s is simply a *ratio* between stress and strain within this limit. It therefore becomes necessary to check any of the above indicated computations for strength, and it will often be found, after thus checking, that the stress is either too high for safety, or too low for economy.

The formula for the *strength* of a solid circular-section rod under torsion is

$$P\,r = \frac{\pi}{16} p\, d^3 \;\therefore\; P = \frac{\pi\, p\, d^3}{16\, r};$$
$$r = \frac{\pi\, p\, d^3}{16\, P}; \qquad p = \frac{16\, P\, r}{\pi\, d^3} \tag{4}$$

It is to be remembered that as equation (4) is for safe strength, the load (P) should be the maximum load to which the spring can be subjected; but equation (3) may be used with any load and the corresponding deflection.

Example: The load on a helical spring is 1600 lbs., and the corresponding deflection is to be 4″. Transverse modulus of elasticity of material = 11,000,000, and the maximum intensity of safe torsional stress = 60,000 lbs., wire of circular section. To design the spring, assume $d = 5/8''$, and $r = 1\frac{1}{2}''$; from eq. (3),

$$n = \frac{4 \times 625 \times 11{,}000{,}000 \times 8}{4{,}096 \times 64 \times 1{,}600 \times 27} = 19.4.$$

Checking for the stress by the last equation in group (4),

$$p = \frac{16 \times 1{,}600 \times 1.5 \times 512}{\pi \times 125} = 50{,}200 \text{ lbs.}$$

This stress is found to be safe, but is considerably below the limit assigned, and it may be desirable to work up to a somewhat higher stress. Another computation can be made (with a smaller d or larger r), and by a series of trials, the desired spring can be found. The following order of procedure avoids this element of uncertainty. The load being given, assume a diameter of wire and value of safe stress, then solve in eq. (4) for the radius of coil. Make this radius some convenient dimensions (not exceeding that computed if the assumed stress is considered the maximum safe value). Next substitute these values of d and r (with those given for P, δ and E_s) in eq. (3) to find the number of coils. Thus, with the data of the preceding example, assuming $d = 5/8''$;

$$r = \frac{\pi}{16} \frac{p\, d^3}{P} = \frac{\pi \times 60{,}000 \times 125}{16 \times 1{,}600 \times 512} = 1.79''.$$

If the $5/8''$ rod is wound on an arbor 3″ diameter, the radius to the centre of coils will be about 1.81″; and the corresponding stress would be 60,500 lbs. per square inch. This is so slightly in excess of the assigned value that it may be permitted, especially as this value is a moderate one for spring steel. Substituting in eq. (3),

$$n = \frac{\delta\, d^4\, E_s}{64\, P\, r^3} = \frac{4 \times 11{,}000{,}000 \times 625}{64 \times 1{,}600 \times 5.93 \times 4{,}096} = 11.1.$$

It may be desirable to fix upon the radius of coil, rather than the diameter of wire, in the first computation, in designing a spring. From eq. (4):

$$d^3 = \frac{16\, P\, r}{\pi\, p}, \therefore d = 1.72\sqrt[3]{\frac{P\, r}{p}} \quad . \quad . \quad . \quad (5)$$

In other cases, it may be desirable to assume the ratio of the radius of coil to the diameter of wire, then from eq. (4):

$$d^2 = \frac{16\, P\, r}{\pi\, p\, d}, \therefore d = 2.26\sqrt{\frac{P}{p}\left(\frac{r}{d}\right)} \quad . \quad . \quad (6)$$

In either of the preceding conditions, a standard size of wire should be chosen.

In checking a given spring, it may be required to determine either the safe load, or the safe deflection. If the former is the case, eq. (4) may be used directly. If it is required to find the safe deflection, substitute the value of P from eq. (4) in eq. (2) and the result is

$$\delta = \frac{12.57\, n\, r^2\, p}{E_s\, d} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The weight of a spring is a matter of some importance, as the material is expensive. The following discussion shows that the weight varies directly as the product of the load and the deflection, inversely as the square of the intensity of stress in the wire, and directly as the transverse modulus of elasticity. Hence for a given load and deflection, economy calls for a high working stress and a low modulus of elasticity. From eq. (4):

$P = \dfrac{\pi}{16}\, p\, \dfrac{d^3}{r}$; also for a member under torsion,

$p = \dfrac{d}{2} \times \dfrac{\alpha\, E_s}{l}$ [Church's "Mechanics," p. 235].

$$\therefore\ p = \frac{d}{2} \times \frac{\delta}{r} \times \frac{E_s}{2\,\pi\,r\,n} = \frac{d\,\delta\,E_s}{4\,\pi\,r^2\,n}$$

$$\therefore\ \delta = \frac{4\,\pi\,r^2\,n\,p}{d\,E_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\therefore\ P\,\delta = \frac{\pi^2\,d^2\,r\,n\,p^2}{4\,E_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

But the volume of the spring is

$$v = \tfrac{1}{4}\,\pi\,d^2\,l = \tfrac{1}{2}\,\pi^2\,d^2\,r\,n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$\therefore\ P\,\delta = \frac{p^2\,v}{2\,E_s},\ \therefore\ v = \frac{2\,E_s}{p^2}\,P\,\delta \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The weight is directly proportional to the volume; hence for given values of E_s and p, the weight varies simply as the product of the load and the deflection. All possible helical springs (of similar section of wire) have the same weight for a given load and deflection, if of the same material and worked to the same stress. It can be shown that a helical spring of square wire must have 50 per cent greater volume than one of round wire, the stress and modulus of elasticity being the same in both. The round section is generally admitted to be best for helical springs under ordinary conditions.

A small wire of any given steel usually has a higher elastic limit than a larger one, while there is not a corresponding change in the modulus of elasticity with change in diameter. This suggests the use of as light a wire as is consistent with other requirements.

An extensive set of tests of springs, conducted by Mr. E. T. Adams, in the Sibley College Laboratories, indicates that the steel such as is used in governor springs may be subjected to stress varying from about 60,000 lbs. per square inch with ¾″ wire to 80,000 lbs. per square inch (or more) in wire ⅜″ diameter. The following expression may be used to find the safe stress in such springs:

$$p = 40{,}000 + \frac{15{,}000}{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Mr. J. W. Cloud presented a most valuable paper on Helical Springs before the Am. Society of Mechanical Engineers (Trans., Vol. V, page 173), in which he shows that for rods used in railway springs (¾″ to $1\frac{5}{16}$″ diam.), the stress may be as high as 80,000 lbs. per square inch, and that the transverse modulus of elasticity is about 12,600,000.

Two or more helical springs are often used in a concentric nest (the smaller inside the larger); all being subjected to the same deflection. This is common practice in railway trucks, where the springs are under compression when loaded. If these springs have the same "free" height (when not loaded), and if they are of equal height when closed down "solid," Mr. Cloud shows that the length of wire should be the same in each spring of the set for equal intensity of stress. The "solid" height of a spring is $H = d\,n$, and the length of wire is $l = 2\,\pi\,r\,n$; hence the numbers of coils of the separate springs of the set are inversely as the diameters of the wire and inversely as the radii of the coils; or the ratio of r to d is the same in each spring of the nest. This conclusion may be somewhat modified when it is remembered that the wire of smaller diameter may usually be subjected to somewhat higher working stress than the larger wire of the outer helices; and also that the wire of these compression springs is commonly flattened at the end to secure a better bearing against the seats. See Fig. 30.

SUMMARY OF HELICAL SPRING FORMULÆ.

$$P\,r = \frac{\pi}{16}\,p\,d^3 \quad \text{(I)}$$

$$r = \frac{\pi\,p\,d^3}{16\,P} \quad \text{(II)}$$

$$d = 1.72\sqrt[3]{\frac{P\,r}{p}} \quad \text{(III)}$$

$$d = 2.26\sqrt{\frac{P}{p}\left(\frac{r}{d}\right)} \quad \text{(IV)}$$

$$p = \frac{16\,P\,r}{\pi\,d^3} \quad \text{(V)}$$

$$P = \frac{\pi\,p\,d^3}{16\,r} \quad \text{(VI)}$$

$$\delta = \frac{12.57\,n\,r^2\,p}{E_s\,d} \quad \text{(VII)}$$

$$P = \frac{\delta\,d^4\,E_s}{64\,r^3\,n} \quad \text{(VIII)}$$

$$\delta = \frac{64\,P r^3 n}{d^4\,E_s} \quad \text{(IX)}$$

$$n = \frac{\delta\,d^4\,E_s}{64\,P r^3} \quad \text{(X)}$$

$$v = \frac{p^2}{2\,E_s}\,P\,\delta \quad \text{(XI)}$$

Two common methods of attaching "pull" springs are shown in Fig. 30 (a). One end of the spring shows a plug with a screw thread to fit the wire of the spring. This plug is usually tapered slightly, and the coils of the spring are somewhat enlarged by screwing it in. The other end of the spring shows the wire bent inward to a hook which lies along the axis of the helix. The former method is usually preferable for heavy springs.

Formulæ (I) to (VII), inclusive, relate to strength; (VIII) to (X), inclusive, relate to rigidity, or elasticity.

In the absence of more exact information as to the properties of the material of which a steel helical spring is made, the following values may be taken:

$$E_s = 12{,}000{,}000,$$

$$p = 40{,}000 + \frac{15{,}000}{d}.$$

44. **Spiral or Helical Springs in Torsion.** The following formulæ for either true spiral or helical springs subjected to torsion are derived from "The Constructor," by Professor Reuleaux.

$$\varphi = \frac{PRl}{EI}; \; p = \frac{PR}{Z},$$

In which

P = load applied to rotate axle,
R = lever arm of this load,
φ = angle through which axle turns,
l = length of effective coils,
E = modulus of elasticity (direct),
I = moment of inertia of the section.

CHAPTER VI

RIVETED FASTENINGS

45. **General Considerations.** The simplest form of fastening is the rivet. It consists of a head *a* (Fig. 31), a straight shank *b*, and a second head *c*, which is formed while hot and known as a *point*. When it is desired to rivet two pieces *M N* together, mating holes are punched or drilled as shown, the rivet is heated white hot and pushed into the hole which is purposely made a little larger in diameter. The head is held up firmly against the plate by a heavy bar or sledge and the point may be formed with a hand hammer, or with the aid of a forming tool or set. In riveting on a large scale this operation is performed by hydraulic or pneumatic machines. The relative merits of the two methods will be more apparent after further discussion. The rivet is a permanent fastening and cannot be removed without the destruction of either head or point. It is largely used in structures such as bridges, the framing of buildings, ship work, boilers, tanks, etc.

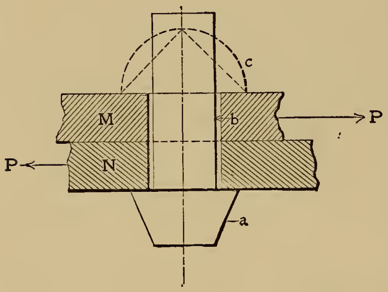


Fig. 31.

Fig. 32 shows various forms of rivet heads and points. The form shown at *B* is most commonly used for small rivets up to $\frac{5}{16}$ in. diameter, which are driven without heating, for such work as light tank and smokestack work. The form at *C* is much used in ship work, or wherever smooth exterior surfaces are desired. In machine work, where great accuracy is required, the holes are reamed, and the rivet carefully fitted so as completely to fill the hole; both heads in such cases are usually countersunk and formed cold.

When a rivet is "driven" hot it shrinks in cooling, drawing the riveted parts firmly together. When cold it is under a tensile stress due to this shrinking, and for the same reason it is always a little smaller than the hole which it originally completely filled when hot. The tensile stress due to this cooling effect cannot be accurately determined as it depends on the temperature of the rivet, and the manner in which it is driven. Rivets are, for this reason, unreliable as tension members and are seldom so used. In most cases the parts M and N (Fig. 31) have the load P applied as shown, and the tendency is to shear off the rivet and produce relative sliding between M and N. The normal load, P', due to the tensile stress in the rivet, holding the surfaces of M and N firmly in contact, sets up a frictional resistance equal to $\mu P'$ which opposes the action of P. From experiments made

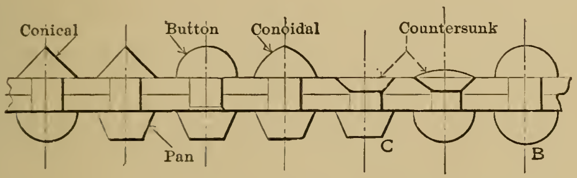


FIG. 32.

by Stoney* it appears that this frictional resistance may be taken at about 10,000 lbs. per square inch of rivet area. Experiments by Bach, and others, show a much higher resistance, but it is evident that if the normal pressure of the rivet is such that a stress equal to or greater than the elastic limit is induced, the permancy of the resistance cannot be relied upon.

In some French and German practice the design of the joint is based entirely upon the frictional resistance, but in England and America it is neglected, and the design based upon the tensile and shearing strength of the plates and rivets.

46. Forms of Joints. Riveted joints are of many forms depending on the character of the work to which they are applied. In structural work, such as bridges, they are used simply to

* "Strength of Riveted Joints," page 75.

resist direct loads; but in boiler construction, and similar work, they must not only resist direct loading but must also be tight against fluid pressure. This last requirement materially affects the proportions of the joint, and makes the design of joints for withstanding fluid pressure most important. Riveted joints are divided into two general forms.

(a) **Lap Joints**, where the sheets to be joined are lapped on each other and riveted as shown in Fig. 33 (a).

(b) **Butt Joints**, where the edges of the sheets to be joined abut against each other, and have auxiliary butt straps or cover plates riveted to the edge of each, as shown in Fig. 33 (e) and (f).

A lap joint may have one or more rows or **seams** of rivets, and these rows way be arranged in the form of "chain" riveting, Fig. 33 (b), or in the form of **zigzag** or **staggered** riveting, Fig. 33 (c).

A butt joint may have one or more seams of rivets on each side of the joint, and these may also be arranged in either chain or staggered form, as shown in Fig. 33 (f) and (g).

The combinations that may be thus made up are very numerous, and the student is referred to any treatise on boiler work for fuller information on this point.

The distance between rivets along the seam is called the **pitch** or **spacing**, and will be denoted by s, Fig. 33 (b). An examination of any riveted joint will show that the arrangement of rivets, or pattern as it may be called, continually repeats itself as the "seam" extends along the joint, the repetition occurring with the greatest pitch, where the pitch of the various seams is unequal as in Fig. 33 (h). A **unit strip** is equal in width to the pitch, the maximum pitch being taken when the pitch of all seams is not the same. The **transverse pitch** is the distance between the centre lines of adjacent seams Fig. 33 (b) and will be denoted by s_t. The **diagonal pitch** is the distance between the centre of a rivet and that of the one nearest to it diagonally, in the next row, and will be denoted by s_d Fig. 33 (d). The **margin** is the distance from the edge of the plate to the center line of the nearest row of rivets, as e Fig. 33 (d). It is sometimes defined as the distance from the edge of the plate to the edge of the rivet hole.

47. Stresses in Riveted Joints. The stresses that exist in the various members of riveted joints are complex, and do not admit of refined calculation. Not only are the plates subjected to the apparent direct stresses of tension and compression, and

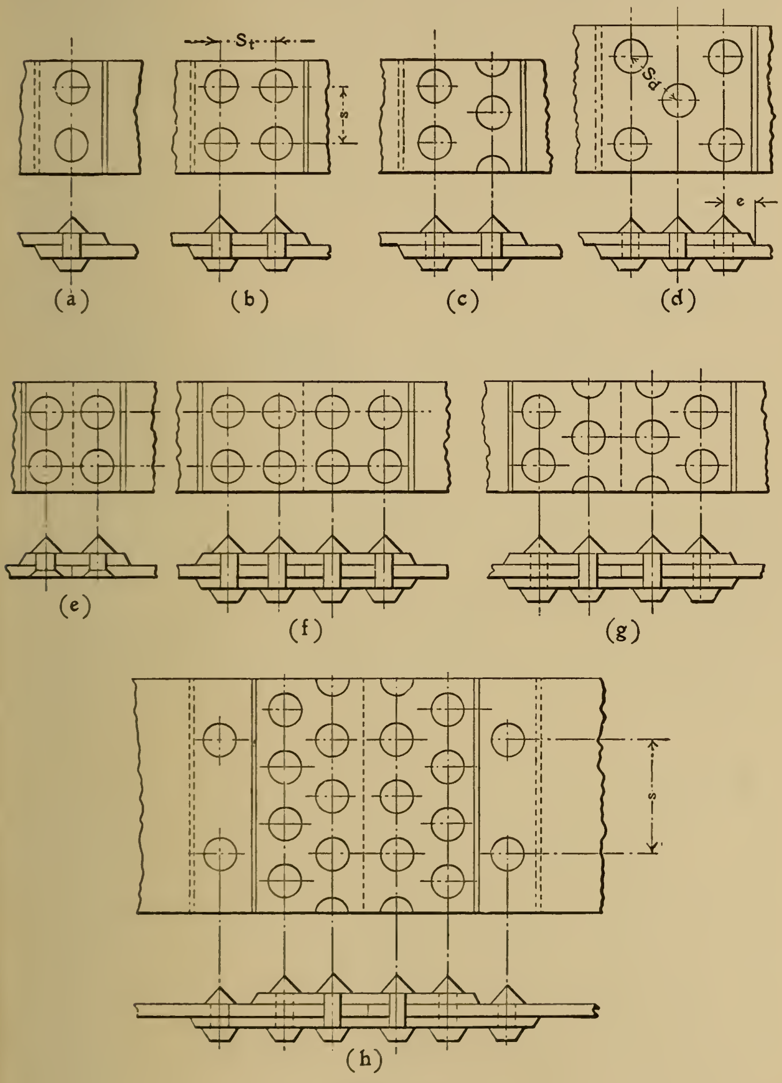


FIG. 33.
(a, b, c, d, e, f, g, h.)

the rivets to shear and compression, but often there are also bending actions which are difficult to analyze and provide for mathematically. Thus a simple lap joint, as that shown in Fig. 33 (a), when subjected to a load, tends to take the form shown in Fig. 34. The force applied tends to draw the plates into the same plane, putting a bending action on the plate and rivet, a greater tensile stress on the rivet head, and a concentrated crushing load on the corners of the sheets. The frictional resistance is entirely destroyed when the conditions illustrated in Fig. 34 exist.

The above defects are more marked in the lap joint than in the double strapped butt joint, as in the latter the plates are ini-

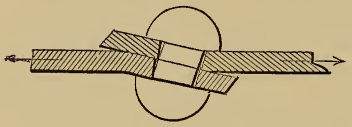


Fig. 34.

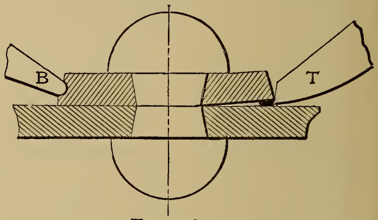


Fig. 36.

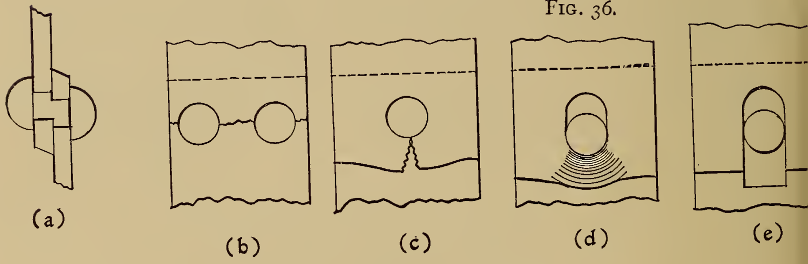


Fig. 35.

tially in line and the condition shown in Fig. 34 cannot occur. But even here the rivets do not completely fill the holes when cold, and hence some bending of the rivet and concentrated crushing on the plate must result. Again, while the quality of the material forming the joint may be well known or determined, the workmanship is not so easily controlled and may be very defective and yet not show on the exterior; and while there have been many tests * made to find the ultimate strength of riveted joints,

* Proceedings Institute of Mechanical Engineers, 1881, 1882, 1885, 1888. Watertown Arsenal Reports, 1885, 1886, 1887, 1891, 1895, 1896.

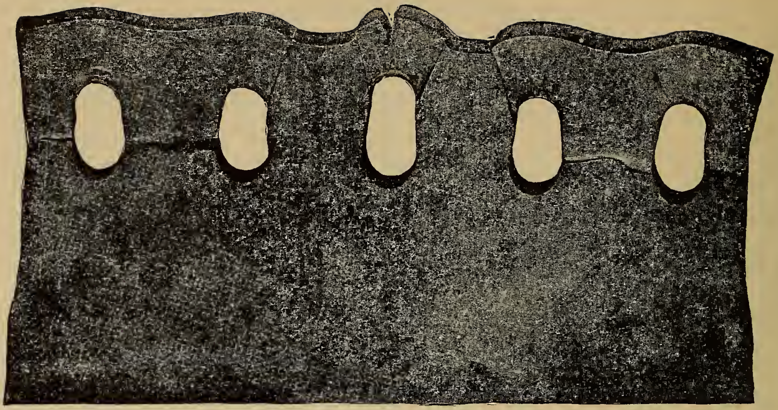
such tests show only the stress at which a certain element of the joint failed, and do not throw any light on the distribution and progress of the stresses in the various individual members during the test. Such tests have usually been performed on joints made of straight plates while in practice these are often curved. These experiments, therefore, while giving the only data available relative to the ultimate strength, should be used with judgment in designing. For these reasons the theoretical formulæ deduced for the design of riveted joints, as a rule, take cognizance only of the *apparent* simple stresses and provide for the unknown by means of a factor of safety.

It has been found that riveted joints may fail in one of the following ways:

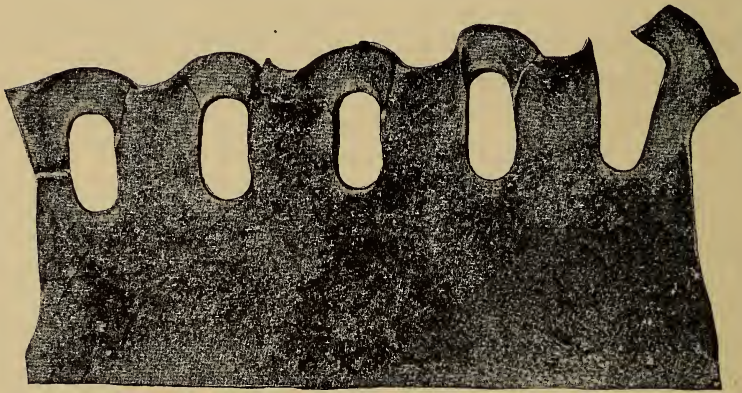
(a) Shearing of the rivet as in Fig. 35 (a).
(b) Rupturing of the plate by tension as in Fig. 35 (b).
(c) Tearing of the margin as in Fig. 35 (c).
(d) Shearing of the margin as in Fig. 35 (e).
(e) Crushing the plate, or rivet as in Fig. 35 (d).
(f) Rupturing of the plate diagonally between rivet holes by tension, in staggered riveting.

Where the joint is complex in form, ultimate failure may be due to one or more of the above causes. The Watertown Arsenal reports include cuts of ruptured joints which are very instructive on this point. Figures (*a*) and (*b*), on Plate I, are reproduced from these reports, and show very clearly all the ways in which failure may occur in the plate. Fig. (*c*) shows a rivet that has been tested to destruction in single shear, while Fig. (*d*) shows one that has been similarly tested in double shear.

It is obvious that no riveted joint can be as strong as the unperforated plate, since the very fact of making holes in it reduces the cross-sectional area in the line of the rivet holes. The ratio of the strength of the weakest element of the joint, to the strength of the unperforated plate, is called the **relative strength** or **efficiency** of the joint. The first expression is more suggestive and will be used in this work. It is desirable to reduce the strength of the plates as little as possible by perforation; and if,



(a)



(b)

(c)

(d)

Plate I.

therefore, the correct relation between the size of rivet and cross section of perforated plate, for equal strength, is established, an excess of strength in other directions, as marginal distance, is not a defect but good design, as it insures that the full strength of the perforated plate will be in service before rupture can occur. A well-designed joint should hence fail by tearing of the sheet along the line of the rivet holes, at about the same load as will destroy the rivets; and the relative strength of a well-designed joint should be the ratio of the cross section of the perforated to that of the unperforated plate, the shearing and crushing resistance of the rivets being equal to the former. If this equality does not exist, the relative strength of the joint can be made greater by strengthening the weaker of these elements at the expense of the stronger.

48. Marginal Strength. The width of margin is independent of the proportions of the other elements, and hence can be made sufficient to prevent tearing or shearing, as in Fig. 35 (c) and (e). It has been found that, with the usual proportions, if the margin be made equal to one and one half the diameter of the rivet, it will be safe against both shearing and tearing from rivet pressure. A Committee of the Master Steam Boiler Makers Association recently recommended, as a result of experiments, that the distance from the center of the rivet to the edge of the plate be made twice the diameter of the rivet, in order to insure excess strength enough to either shear the rivets or rupture the plate by tension. It is important, however, that the margin be not excessive in boiler work as this makes it more difficult to make a steam-tight joint by calking the edge of the plate. The consideration of the marginal strength can hence be omitted as far as its influence on the relative strength of the joint is concerned, as it can be made to depend on the diameter of the rivet.

49. Transverse and Diagonal Pitch. It has been determined, in a similar way as above, that in chain riveting the transverse pitch should not be less than twice the diameter of the rivet or $2d$, where d is the diameter of the rivet, and $2.5\ d$ is better. It has also been demonstrated mathematically (see Cathcart's "Machine Design," page 148) that in staggered riveting the transverse pitch should not be less than 0.4 times the pitch along the seam, in order to avoid rupture along the diagonal pitch, and a greater distance is recommended for safety. Unwin (page 123) gives 2 d as the minimum diagonal pitch in staggered riveting, which would make the transverse pitch 1.7 d, and recommends that somewhat greater distances be used for added strength. An examination of the practice of several boiler-making and insurance concerns shows that these values check fairly well with practice. It appears from the above that the transverse pitch can also be made to depend on the diameter of the rivet, though it is not a direct function of the rivet diameter.

50. Theoretical Strength of Riveted Joints. Since the margin and transverse pitch can be assigned from the diameter of the rivet, three of the ways in which a joint may fail, namely c, d and f, page 141, can be omitted from the theoretical discussion of the strength of riveted joints, leaving a, b and e to be considered; the problem being so to proportion the rivet and the pitch along the seam as to give equal strength against failure in any of these three ways. Let,—

d = diameter of rivet in inches.
s = pitch of rivets in inches.
p_t = tensile strength of plates in pounds per sq. inch.
p_c = crushing strength of plates or rivets in pounds per sq. inch, if rivets are in single shear.
p'_c = crushing strength of plates or rivets in pounds per inch, where rivets are in double shear.
p_s = shearing strength of rivets in pounds per sq. inch, when in single shear.
p'_s = shearing strength of rivets in pounds per sq. inch, when in double shear.

It is known that the unit shearing resistance of a rivet is greater in single shear than in double shear, while the unit crushing resistance is less in single shear than in double shear.

Consider first a simple lap joint (see Fig. 33 a). The tensile strength of the unperforated strip is

$$P = s\,t\,p_t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The tensile strength of the perforated strip along the seam of rivets is

$$T = (s - d)\, t\, p_t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In the simple lap joint there is but one rivet per unit strip and its shearing strength is,

$$S = \frac{\pi d^2}{4} p_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The resistance to crushing of the rivet or the plate against which it bears is,

$$C = d\, t\, p_c{}^{*} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

For uniform strength against rupture and hence for greatest relative strength,

$$T = S = C$$

Equating (3) and (4)

$$\frac{\pi d^2}{4} p_s = d\, t\, p_c$$

$$\therefore d = 1.27\, t \frac{p_c}{p_s}$$

Equating (2) and (3)

$$(s - d)\, t\, p_t = \frac{\pi d^2}{4} p_s$$

$$\therefore s = \frac{.7854\, d^2 p_s}{t\, p_t} + d$$

The relative strength $= \dfrac{T}{P} = \dfrac{(s - d)\, t\, p_t}{s\, t\, p_t} = \dfrac{s - d}{s}$

Double-Riveted Lap Joints. In a similar way equations may be developed for any other form of joint. Thus for double-riveted lap joints

$$d = 1.27\, t \frac{p_c}{p_s}$$

$$\text{and } s = \frac{1.57\, d^2 p_s}{t\, p_t} + d$$

---

* It is known from experiments on indentation that the resistance to indentation depends very little on the form of the indenting body but mainly on its projected area. Hence it is customary to take the resistance of rivets to crushing as proportional to their projected area.

The relative strength $= \frac{s - d}{s}$ and will be greater than in the case of the single-riveted lap joint since s is greater in proportion to d.

Single-Riveted Butt Joints with double cover plates.

Here,

$$d = .64\, t \frac{p'_c}{p'_s}$$

$$s = \frac{1.57\, d^2 p'_s}{t\, p_t} + d$$

and the relative strength $= \frac{s - d}{s}$

Double-Riveted Butt Joints with either chain or staggered riveting and double cover plate.

Here,

$$d = .64\, t \frac{p'_c}{p'_s}$$

$$s = \frac{3.14\, d^2 p'_s}{t\, p_t} + d$$

Note that the relative strength $= \frac{s - d}{s}$, and compare the relative values of s and d in this case with those in single riveting.

Thickness of Cover Plates. It is evident that where only one cover plate is used its thickness should not be less than that of the main plate; and in practice, single cover plates are made a little thicker than the main plate to insure an excess of strength. A butt joint with a single cover plate is shown at Fig. 33 (e). A joint of this kind is really equivalent to two lap joints. They are used where a smooth surface is desired or in such places as the longitudinal seams of steam boilers where a lap joint has sufficient strength. Double butt straps should not be made less than half the thickness of the main plate, and, for the same reason as above, it is not unusual to increase their thickness to about $\frac{8}{10}\, t$ where the cover plates are the same width. Where the outer cover plate is narrower than the inner plate, as in

Fig. 33 (h), the outer cover plate is often of the same thickness as the main plate and the inner one from $\frac{7}{10}$ to $\frac{8}{10}\,t$.

51. General Equations for Riveted Joints. The fundamental equations for riveted joints may be put in a more general form. The unit strip, as before, is of width equal to the pitch; the maximum pitch being taken for such width of unit strip if all rows do not have the same pitch.

Let $k_1 = p_c \div p_s$; $k_2 = p'_c \div p'_s$; n = number of rivets per unit strip in single shear and m = number of rivets per unit strip in double shear per unit strip of joint.

The general expression for the net tensile strength of the unit strip is

$$T = (s - d)\, t\, p_t\,^* \qquad . \quad . \quad . \quad . \quad . \quad (1)$$

The general expression for resistance to shearing of the rivets in the unit strip is

$$S = \frac{n\,\pi\, d^2}{4}\, p_s + \frac{2\, m\, \pi\, d^2}{4}\, p'_s \qquad . \quad . \quad . \quad . \quad (2)$$

The general expression for resistance to crushing of the rivets in the unit strip is

$$C = n\, d\, t\, k_1\, p_s + m\, d\, t\, k_2\, p'_s \qquad . \quad . \quad . \quad . \quad (3)$$

The tensile resistance of the solid strip is

$$P = s\, t\, p \; . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Equating S and C, eqs. (2) and (3) and solving for d

$$d = \frac{4}{\pi} \times \frac{n\, k_1\, p_s + m\, k_2\, p_s'}{n\, p_s + 2\, m\, p_s'}\, t \qquad . \quad . \quad . \quad . \quad (5)$$

which gives the proper diameter of rivets for a given thickness of plate, when the number of rivets in single shear and the number in double shear and the corresponding shearing and crushing resistances are known.

---

* Where the rows of rivets do not all have the same pitch, as in some forms of butt joints, the outer row or that farthest from the edge of the sheet has the greatest pitch (see Fig. 33 h). It is evident that if the sheet yield at all by tearing, it will yield along this outer row of rivets; for it cannot tear along an inner row without shearing the outer row of rivets, and it cannot shear one row of rivets without shearing all, in which case the joint would yield by shearing of the rivets and not by tearing.

Equating T and S, eqs. (1) and (2), and solving for s

$$s = \frac{\pi}{4} d^2 \left(\frac{n\, p_s + 2\, m\, p_s'}{t\, p_t} \right) + d \quad . \quad . \quad . \quad . \qquad (6)$$

Equating S and C, eqs. (2) and (3)

$$\frac{\pi\, d^2}{4} (n\, p_s + 2\, m\, p_s') = d\, t\, (n\, k_1\, p_s + m\, k_2\, p'_s)$$

$$\therefore d\, t = \frac{\pi\, d^2\, (n\, p_s + 2\, m\, p_s')}{4\, (n\, k_1\, p_s + m\, k_2\, p_s')} \quad . \quad . \quad . \quad . \qquad (7)$$

Equating T and S, eqs. (1) and (2) and solving

$$s\, t\, p_t = \frac{\pi\, d^2}{4} (n\, p_s + 2\, m\, p_s') + d\, t\, p_t = P. \quad . \quad . \qquad (8)$$

If the joint is designed for maximum relative strength, $T = S = C$, hence any one of these three quantities divided by P gives the relative strength (E) of the ideal joint, for any given form, or dividing (2) by (8)

$$E = \frac{S}{P} = \frac{\frac{\pi\, d^2}{4}(n\, p_s + 2\, m\, p_s')}{\frac{\pi\, d^2}{4}\, (n\, p_s + 2\, m\, p_s') + d\, t\, p_t}$$

Substituting the value of $d\, t$ as given by eq. (7) and dividing numerator and denominator by $\frac{\pi\, d^2}{4}(n\, p_s + 2\, m\, p_s')$,

$$E = \frac{1}{1 + \frac{p_t}{n\, k_1\, p_s + m\, k_2\, p_s'}} \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

If all rivets are in single shear,

$$E = \frac{1}{1 + \frac{p_t}{n\, k_1\, p_s}} \quad . \quad . \quad . \quad . \quad . \qquad (9')$$

If all the rivets are in double shear,

$$E = \frac{1}{1 + \frac{p_t}{m\, k_2\, p'_s}} \quad . \quad . \quad . \quad . \quad . \qquad (9'')$$

Equation (9) applies to any form of riveted joint. It is useful in finding the limiting relative strength of joint for any form and materials; the actual proportions adopted may give a lower relative strength, but can never give higher relative strength.

These general equations were originally due to Professor William N. Barnard, who also suggested the above expressions for the maximum relative strength in the general case.

The forms in which eqs. (9), (9′) and (9)″ are now given are due to Professor H. F. Moore.

The following are rough average values of the relative strength of joints as made in practice for boiler work:

Single riveted lap joints	.55
Double riveted lap joints	.70
Single riveted butt joints	.65
Double riveted butt joints	.75
Triple riveted butt joints	.80
Quadruple riveted butt joints	.85

52. Practical Considerations Affecting Proportions of Riveted Joints. It is to be especially noted that the proportions of riveted joints as given by the foregoing equations are based on equal strength of rivet and plate, and that any variation therefrom will destroy this theoretical equality. It is apparent also that any variation in the strength of the material used would affect the proportions as given by these equations, and that a table of rivet diameters and pitches would have to be very extensive to cover the entire range of practice. It is an advantage in practice, however, to adopt regular diameters and pitches for a given thickness of plate and form of joint. It has also been found that as the thickness of the plate increases, the corresponding theoretical diameter of the rivet sometimes becomes too large to be easily driven, especially in the case of simple lap joints. In the case of boilers, or wherever fluid pressure must be withstood, the theoretical spacing must sometimes be modified in order that it may not be so great as to prevent the making of a steam-tight joint. Wherever such variations are made the general expressions for T, S and C can always be used to check the strength and show in what direction the joint may be strengthened, with the fundamental object of making it strong enough in all other directions to insure full service out of the plate itself. It may be noted that the bearing resistance of a rivet varies with the diameter, while the shearing resistance varies with the square of the diameter. If, therefore, the rivet chosen be smaller in diameter than would be given by the theoretical equations, the shearing resistance alone need be regarded; while if the diameter of the rivet be greater than the theoretical diameter, the bearing pressure only need be considered.

If the joint to be made does not have to withstand fluid or gaseous pressure, the design can, for ordinary thickness of plate, be made to conform closely to the theoretical proportions for equal strength; but when fluid or gaseous pressure must be withstood, as in boiler work, the spacing of the rivets for thick plates must be less than the theoretical spacing to insure tightness; and in all cases as the plates increase in thickness, the diameter of the rivet, as already noted, is for practical reasons reduced in diameter from that required for equal strength. The relation between diameter of rivet and thickness of plate as fixed by average practice may be expressed by the equation,

$$d = 1.2\sqrt{t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

For plates above $\frac{3}{8}''$ thick this equation will give rivets smaller in diameter than required for equal strength in all directions. As before pointed out, the rivets in such cases need only be checked for shearing strength. If the diameter of the rivet be determined by equation (10), and the pitch so chosen as to make the tensile strength of the perforated plate equal to the shearing strength of the rivet, the maximum relative strength of joint possible with the rivet chosen will be obtained, and it will be found that the joints will be steam-tight.

Example. It is required to design a riveted joint, as shown in Fig. 33 (h), for a boiler shell in which the force tending to pull the joint apart is 6,000 pounds per inch of length of shell. The plate is to be of steel of 60,000 lbs. tensile strength, the rivets are to be of steel and are to have a shearing strength of 49,000 lbs. per square inch in single shear, and 42,000 lbs. per square inch in double shear. The factor of safety is to be 5.

The allowable stress per inch of length of the shell outside the joint is $\frac{60,000}{5} = 12,000$ lbs. If the joint were as strong as the unperforated plate the thickness of the plate would be $\frac{6,000}{12,000} = \frac{1}{2}$ inch. The relative strength of the joint will not be less than .80, and hence the thickness of the plate must be

$$\frac{\frac{1}{2}}{.80} = \frac{5''}{8}.$$

The diameter of the rivets will $= 1.2\sqrt{\frac{5}{8}} = \frac{7''}{8}$ nearly.

The size of the punched hole and the diameter of the driven rivet will be $\frac{15}{16}''$. Equation (6) of this chapter gives the relation between pitch and diameter of rivet for equal strength against shearing of the rivets and tearing of the plate.

Thus, $s = \frac{\pi d^2}{4}\left[\frac{n p_s + 2 m p_s'}{t p_t}\right] + d$

Here, $n = 1$ and $m = 4$.

Hence, $s = \frac{\pi (\frac{15}{16})^2}{4}\left[\frac{49,000 + (2 \times 4 \times 42,000)}{\frac{5}{8} \times 60,000}\right] + \frac{15}{16}$

$\therefore s = 8$ inches nearly.

The relative strength of the joint is

$$\frac{8 - \frac{15}{16}}{8} = 88\%, \text{ hence the design is safe.}$$

If the pitch found as above should be considered too great, either on account of very high steam pressure or because it is desired to make the structure stiffer, a smaller pitch could be used, but the relative strength would be less.

Where no fluid pressure is to be withstood the above methods will always give satisfactory results for joints in *tension*. For joints in *compression* the student is referred to treatises on structural work.

52.1. The Making of Riveted Joints. It is evident that the following precautions must be observed in making first-class riveted joints.

(a) The plates must be in close contact before the rivet is driven, to prevent a fin from forming between them and thus making a tight joint impossible.

(b) The mating holes must be "fair"; that is, they must be in perfect alignment to insure full cross section of the rivet at the junction of the plates.

(c) The rivet must completely fill the hole.

(d) The rivet should be carefully driven so that its strength, or that of the plate, will not be weakened by poor workmanship.

(a) In hand riveting the plates are drawn up together, before the rivet is driven, by a bolt placed in a hole near that in which the rivet is to be driven. With comparatively thin plates this method will accomplish the result if the holes have been accurately spaced, and if the plates have been properly rolled and fit well. For heavy work, where for other reasons machine riveting is necessary, the riveter is sometimes provided with a power-driven closing device which holds the plates up till the rivet nips from cooling.

(b) The rivet holes in the plate may be either punched or drilled. They are generally made about $\frac{1}{16}$ inch larger than the rivet. Generally speaking it is cheaper to punch the holes than to drill them, and hence in the cheaper kinds of work, and with thin plates, punching is almost always resorted to. In structural work the holes are generally punched. There are, however, some serious objections to punching. When the punch is forced through a plate the amount of metal which it removes in the form of a "plug" is not equal to the amount that originally filled the hole. This is accounted for by the fact that punching is not a pure shearing action, but that during the process there is a flow of metal from under the punch to the walls of the hole, setting up a stress in the material, and, if the metal is at all hard, seriously impairing the strength of the plate. It is found that this action is confined to a thin ring next to the hole, and that by either reaming out the hole about $\frac{1}{16}$ inch all around, or annealing the plate, this weakening effect disappears. The process of punching is apt to make inaccurate work and, therefore, when the plates are brought together the mating holes are not fair. The old practice of driving a taper drift pin into such holes and drawing them into line by force is now largely prohibited, the injury thus done to the plate being often very serious. If, however, the holes are punched a little small, and put together and reamed to the proper size, the difficulties due to punching are largely overcome. Thin plates, in which the effect of punching is small, are punched and used without reaming or annealing. Plates more than ½ inch thick should always be either annealed or have the holes reamed after punching. Heavy plates are always better if drilled, and all first-class boiler work requires the holes to be drilled in place and all burrs carefully removed. This last is important, as the burr, if allowed to remain, may seriously impair the strength of the head. A small countersink, on the other hand, materially contributes to the strength of the rivet. When plates are annealed the work should be properly done; for if the plate be overheated structural changes take place that materially weaken the metal. The heating should not be too rapid nor the temperature above a medium cherry red. The holes made by punching are necessarily somewhat tapering in form, and where they are used as they come from the punching machine the rivet holes should be punched so that they will come together as in Fig. 36; for the rivet drives better and the tapering rivet has a tendency to relieve the head of part of the tensile stress.

(c) Since it is necessary to have the hole a little larger than the rivet, it is clear that the rivet when driven must be upset throughout its entire length in order that it may completely fill the hole. Large rivets should therefore be machine driven, as it is difficult to upset heavy rivets, especially if of great length, by hand. If a rivet does not completely fill the hole an undue concentrated bearing, or shearing stress, may be brought on its neighbor.

(d) On the other hand, care must be exercised in machine riveting that the pressure applied does not create such a flow in the rivet as unduly to strain the plate, and also that the pressure applied is not great enough to crush the plate directly. Practice allows about 80 tons per square inch of rivet area. Machine riveting, when well done, is superior to hand work, the plates being held up firmer, and also because the impact from hand riveting, especially if the rivet is worked too cold, is liable to result in the breaking off of the head. In either case care should be exercised that the point is formed on the rivet concentrically; if the dies are not properly set eccentric riveting will occur. In machine riveting the pressure should be retained on the rivet till cold enough to hold the plate firmly. This is sometimes recognized in writing specifications.

If the spacing of the rivet is correct and the riveting well done, the joint will be tight against ordinary pressures. Where tight joints do not result the edge of the plate is "calked." This is often done, as shown in Fig. 36, by means of a sharp-nosed tool *T* which tucks the sharp bevelled edge of the plate underneath, as shown in an exaggerated manner. There is liability of injuring the lower plate in using a sharp-nosed tool as *T*, and the method shown at *B*, Fig. 36, is preferable. The plates should be bevelled before riveting, as the method of hand bevelling after riveting, as often done in practice, is almost sure to result in some injury to the lower plate.

52.2. Strength of Materials for Riveted Joints. It is well known that the strength of rivets is different in single and double shear. The following may be used as average values.

	p_s*	p_c†
Iron rivets single shear	40,000	60,000
Iron " double "	39,000	72,000
Steel " single "	49,000	80,000
Steel " double "	42,000	100,000

Steel Plates for Boiler Work are generally specified to have a tensile strength of not less than 55,000 lbs. per square inch, and not more than 65,000 lbs. per square inch, for if the tensile strength is too high and the metal is hard they are liable to crack while being worked. For structural steel construction the student is referred to handbooks on structural work. For **iron plates** an average value may be taken as 45,000 pounds per

* Master Boiler Makers' Association Rules, page 150.

† Proceedings Inst. Mech. Engineers, 1885, and Unwin's "Machine Design," page 132.

square inch. It is shown by experiment that the metal between the rivet holes has a higher apparent tensile strength than that of the unperforated plate; this increase being sometimes as high as 20%. It is questionable, however, if this should be taken account of in designing, especially where the holes are punched, as the operation of punching may more than offset this peculiar increase.

52.3. **Factor of Safety.** In boiler work the factor of safety is taken at about 5 which, of course, brings the working stress well below the elastic limit. If the joint is to be subjected to hydraulic pressures, where heavy shocks may have to be withstood, this factor should be increased.

52.4. **Practical Rules.** It has already been noted that practical considerations make it necessary to modify the theoretical equations for uniform strength. There are many sets of practical rules for designing riveted joints a number of which will be found in the references given below.

Rules of the Hartford Steam Boiler and Inspection Co.
Rules of the American Bureau of Shipping.
Rules of the Master Steam Boiler Makers' Association.
Rules of the U. S. Board of Supervising Inspectors.
See also Cathcart's "Machine Design."
Proceedings of Inst. of Mech. Engineers, 1885.
Unwin's "Machine Design."
Wm. M. Barr's "Boilers and Furnaces."
"Steam Boiler Construction," W. S. Hutton.

CHAPTER VII

SCREWS AND SCREW FASTENINGS

53. Form of Screws. Screws, as used in machines, may be divided into two classes.

(a) **Screw fastenings.**

(b) **Screws for transmitting power.**

The form of the thread depends upon the service required. Thus, for screw fastenings, the full *V* as shown in Fig. 37 (a), or modified forms of *V* threads, as shown in Fig. 37 (b) and (c), are most used because they are strong and easily cut by machine dies. They are inefficient for transmitting power, but this is a desirable quality in fastenings, as it reduces the liability of unscrewing. For transmitting power the square thread, Fig. 37 (d), is most used, since its efficiency is higher than that of any other form. It cannot be cut with a die, however, and it is difficult to compensate for wear with this form of thread. For these reasons the half *V* thread, Fig. 37 (e), is often used for transmitting power when wear is an important factor. In Fig. 37 (e) the Acme standard thread of this form is shown. The efficiency of this form of thread is a little less than the square thread but it can be cut with a die and wear can be compensated for by means of a longitudinally split nut; this compensation making it very desirable for such service as lead screws of lathes, etc. Fig. 37 (f) illustrates the buttress thread, which is often used to exert pressure in one direction only. The pressure face is perpendicular to the axis of the screw, and the back face usually makes an angle of 45° with this axis. This screw has, therefore, the efficiency of the square thread and the strength of the *V* thread. The underlying principles of all screws are the same, and before discussing the various forms and classes in detail the fundamental equations relating to their action will be developed.

54. Friction and Efficiency of Square Threaded Screws. In Fig. 38 (b), let N represent a nut moving on a square thread, under the action of a tangential force P, acting at the mean radius of the thread. Let this force P be applied by means of a couple, so that there is no lateral pressure against the screw. Let W represent the load under which the nut is moved, and consider that it can move only in an axial direction, hence there is friction between N and W. This frictional force F_1 (Fig. 38 b) may or may not act at the same radius as P, and the work due to this frictional force will vary with the radius at which it acts. It can be considered as forming a resisting moment opposing the

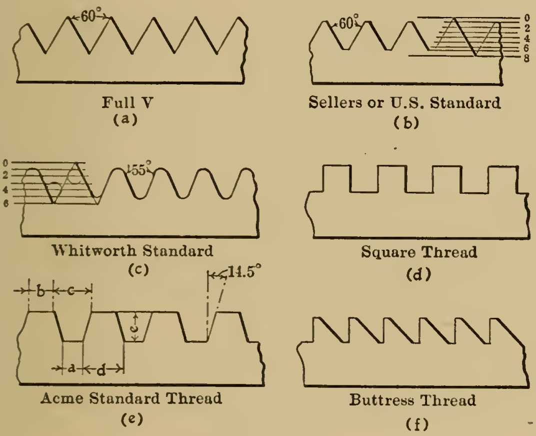


FIG. 37.

turning moment due to P, hence, when computing the required turning moment of P, is to be added to that value. It can therefore, for simplicity, be omitted temporarily from the discussion.

Let μ = coefficient of friction between thread and nut.
μ_1 = coefficient of friction between load and collar.
d = nominal or external diameter of screw.
r = nominal or external radius of screw.
d_1 = diameter of screw at bottom of thread.
r_1 = radius of screw at bottom of thread.

r_c = frictional radius of collar.

d_m = mean diam. of thread $= \dfrac{d + d_1}{2}$.

r_m = mean radius of thread $= \dfrac{r + r_1}{2}$.

s = pitch, or angular advance of thread per turn.

a = angle made by thread with a plane perpendicular to the axis of screw.

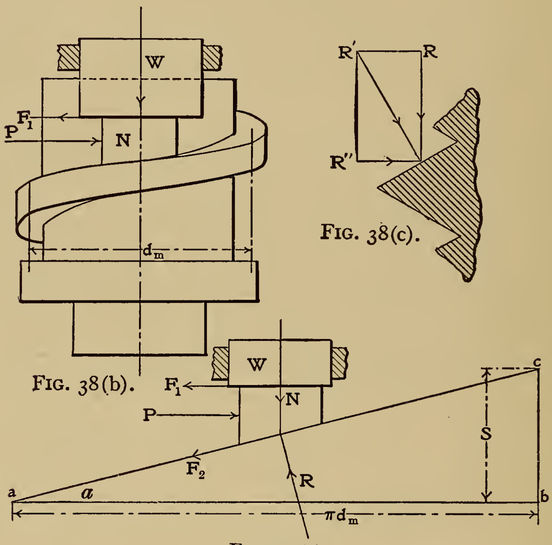


FIG. 38(b).

FIG. 38(c).

FIG. 38(a).

If now the thread be developed as in Fig. 38 (a), it is seen, since the thread is a true helix, that the action of the thread and nut is identical with that of a body N sliding up an inclined plane of length πd_m, and vertical height s equal to the pitch, and carrying a load W which is free to move vertically only. Omitting F_1 temporarily, the forces acting are the load W, the friction F_2 between the thread and nut, the driving force P, and the normal reaction R. It is required to determine for any angle a, the value of P required to slide the body N (turn the nut) up the incline. The frictional resistance $F_2 = \mu R$. Hence, resolving all forces parallel to ac

$$P \cos \alpha - \mu R = W \sin \alpha \quad . \quad . \quad . \quad . \quad (1)$$

$$\therefore R = \frac{P \cos \alpha - W \sin \alpha}{\mu} \quad . \quad . \quad . \quad . \quad (2)$$

Resolving all forces perpendicular to *ac*

$$R - P \sin \alpha = W \cos \alpha \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Substituting in (3) the value of R obtained in (2)

$$P = W \left[\frac{\sin \alpha + \mu \cos \alpha}{\cos \alpha - \mu \sin \alpha}\right] \quad . \quad . \quad . \quad . \quad (4)$$

Since (Fig. 38 a) $\cos \alpha = \frac{\pi d_m}{a c}$ and $\sin \alpha = \frac{s}{a c}$, equation (4) may be written

$$P = W \left[\frac{s + \mu \pi d_m}{\pi d - \mu s}\right] \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The friction, $F_1, = \mu_1 W$, and if r_c be the radius at which F_1 acts, the moment of F_1 around the axis of the screw $= \mu_1 W r_c$; and when this resistance is considered the total moment of P around the axis is

$$P r_m = W r_m \left[\frac{s + \mu \pi d_m}{\pi d_m - \mu s}\right] + \mu_1 W r_c \quad . \quad . \quad . \quad (6)$$

If the load is being lowered, the directions of F_1 and F_2 are reversed, and in this case the turning moment that must be applied is

$$P_1 r_m = W r_m \left[\frac{\mu \pi d_m - s}{\pi d_m + \mu s}\right] + \mu_1 W r_c \quad . \quad . \quad . \quad (6')$$

In equation 6′ the first term of the right-hand side of the equation is the moment of the resistance at the thread, while the second term is the moment of the collar friction. If $\mu \pi d_m = s$, that is if $\frac{s}{\pi d_m} = \tan \alpha = \mu$, the moment of the resistance at the thread will be zero, and if there is no collar friction, or if it is very small, as in the case of ball-bearing thrusts, this will give a condition of equilibrium, the friction of the thread alone just sustaining the load, and P_1 will be equal to zero. If the pitch s is made greater than $\mu \pi d_m$, the moment of the resistance at the thread becomes negative; and if increased till its numerical value is equal to the moment of collar friction, the entire right-hand side of the equation will be equal to zero and the load will just be sustained by the friction of the thread and collar combined. If the pitch is still further increased, the entire right-hand side of the equation becomes negative, and the moment $P_1 r_m$ must be applied in the direction of raising the load, or the screw will "overhaul," the nut exerting a turning moment in the downward direction.

To find the limiting value of α where the nut will not overhaul, equate the right-hand side of the equation (6′) to zero and solve for $\frac{s}{\pi d_m}$, whence

$$\frac{s}{\pi d_m} = \tan \alpha = \frac{\mu r_m + \mu_1 r_c}{r_m - \mu_1 \mu r_c} \quad . \quad . \quad . \quad . \quad (7)$$

If $r_c = 0$, in which case there is no moment due to collar friction, $\tan \alpha = \mu$ as before.

If $r_m = r_c$ and $\mu = \mu_1$

$$\tan \alpha = \frac{2\mu}{1 - \mu^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and if μ be taken as .1 (see Art. 65) $\tan \alpha = .2$ whence $\alpha = 11°$.

To find the **efficiency** of the screw, consider that the load has been raised a distance equal to the pitch, that $\mu_1 = \mu$, and $r_c = r_m$, then

$$e = \text{efficiency} = \frac{\text{work done}}{\text{energy expended}} = \frac{W s}{2 \pi P r_m} = \frac{W s}{\pi d_m P} \text{ or}$$

since $s = \pi d_m \tan \alpha$, and inserting the value of P from equation (5)

$$e = \frac{W \pi d_m \tan \alpha}{W \pi d_m \left[\frac{s + \mu \pi d_m}{\pi d_m - \mu s}\right] + \mu W \pi d_m}.$$

$$= \frac{\tan \alpha (1 - \mu \tan \alpha)}{\tan \alpha + 2\mu} \text{ nearly} \quad . \quad . \quad . \quad . \quad (9)$$

If the collar friction is zero, or very small, as in the case of ball-bearing thrust collars

$$e = \frac{\tan \alpha \, (1 - \mu \tan \alpha)}{\tan \alpha + \mu} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

If, in equation (9), μ be taken as .1, and $\tan \alpha$ as .2 as before, e will equal 50 per cent (nearly), and a brief reflection will show that in no case can the efficiency of a self-sustaining hoisting screw exceed 50 per cent. Suppose the load (Fig. 38) to be just sustained by the frictional resistance of lowering, that is $\tan \alpha = \mu$ or α = angle of repose. If now a force P is applied, just sufficient to relieve this frictional resistance, the load will be sustained by the force and the reaction R. If the frictional resistance of *raising* were zero, the slightest addition to P would move the body up to plane. But the frictional resistance of raising is equal to that of lowering, and consequently, before the body can be started up the plane, a force $2P$ must be applied; which is twice the force required to balance the frictional resistance, and the efficiency would then be 50 per cent. A similar reasoning will apply to other hoisting devices which are barely self-sustaining on account of friction, namely, that the force which must be applied to *start* the load is equal to the friction due to lowering plus the friction due to raising. Hence the maximum efficiency for such self-sustaining mechanisms is 50 per cent.

In designing screws for power transmission it is desirable to know the pitch angle that will give maximum efficiency for the conditions taken. If the first differential of equation (9) be taken and equated to zero, it is found that the maximum efficiency when collar friction is considered will occur when

$$\tan \alpha = \sqrt{2 + 4\mu^2} - 2\mu$$

If $\mu = .1$ as in transmission screws where lubrication is imperfect, $\tan \alpha = 1.23$ and $\alpha = 51°$.

In the case of oil bath lubrication, as in worm gearing, μ may be as low as .05 when $\tan \alpha$ for maximum efficiency $= 1.318$ or $\alpha = 52° - 49'$.

In a similar way from equation (10) for maximum efficiency

$$\tan \alpha = \sqrt{1 + \mu^2} - \mu$$

11

Whence for $\mu = .1$, $\alpha = 42°$
and for $\mu = .05$, $\alpha = 43°-34'$

The effect of the pitch angle on the efficiency of the screw is of great importance in designing screws for power transmission, and is more fully discussed in Arts. 64 and 65.

55. Friction and Efficiency of Triangular Threaded Screws. With triangular threaded screws the normal pressure at the threads is greater than with square threads; hence the friction at the threads is greater, other things being equal. In Fig. 38 (c) the normal pressure for a square thread is indicated by R, while the normal pressure for a triangular thread is $R' = R \sec \varphi$, in which φ = half the angle between the adjacent faces of a thread. R'' represents the radial crushing action on the thread of the screw, and its equal and opposite reaction tends to burst the nut. With 60° angular thread, as in the Sellers system, or the common V thread, $R' = R \sec 30° = 1.15R$. The friction increases directly as the normal pressure; or it is about 15 per cent greater in the 60° angular thread than in the square thread.

If in equations (1) and (3), $R \sec \varphi$ be substituted for R, then by a similar method of reasoning as in square threads, when collar friction is neglected,

$$P = W\left[\frac{s + \mu \pi d_m \sec \varphi}{\pi d_m - s \sec \varphi}\right] \quad . \quad . \quad . \quad . \quad (11)$$

If the collar friction is considered, the moments around the axis when raising the load may be written

$$P r_m = W r_m \left[\frac{s + \mu \pi d_m \sec \varphi}{\pi d_m - \mu s \sec \varphi}\right] + \mu_1 W r_c \quad . \quad . \quad (12)$$

If $\varphi = 30°$

$$P r_m = W r_m \left[\frac{s + 1.15 \mu \pi d_m}{\pi d_m - 1.15 \mu s}\right] + \mu_1 W r_c \quad . \quad . \quad (13)$$

The efficiency of a triangular threaded screw, following the same reasoning as for square threaded screws, taking $r_m = r_c$, and $\mu_1 = \mu$, is

$$e = \frac{W s}{2 \pi r_m P} = \frac{W \pi d_m \tan \alpha}{\pi d_m P} =$$

$$\frac{W \pi d_m \tan \alpha}{W \pi d_m \left[\frac{s + \mu \pi d_m \sec \varphi}{\pi d_m - \mu s \sec \varphi}\right] + \mu W \pi d_m}$$

$$\text{or } e = \frac{\tan \alpha (1 - \mu \tan \alpha \sec \varphi)}{\tan \alpha + \mu \sec \varphi + \mu} \text{ nearly. . . . . } \quad (13a)$$

For the thread on a one-inch bolt in the Sellers system $\tan \alpha = .04$, and taking $\mu = .1$, $e = 11\%$. The **efficiency of the threads on standard bolts** is hence seen to be very low and this, as

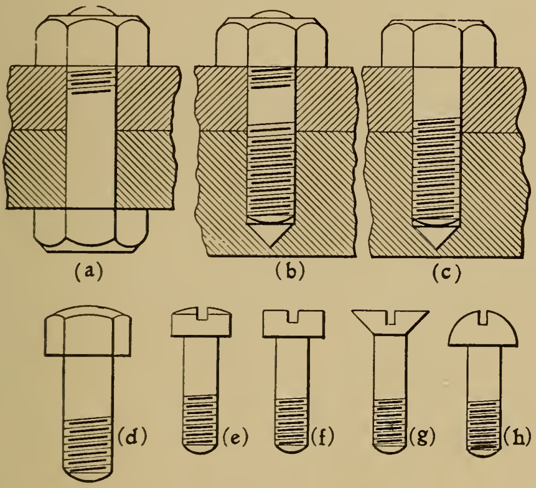


FIG. 39.

has been pointed out, is an advantage in fastenings as it tends to prevent them from unscrewing.

56. **Screw Fastenings.** Screw fastenings are used for holding two machine parts together in permanent position, or for adjusting one part relatively to another. There is a great variety of screw fastenings but all may be roughly classified as follows: 1. Through-Bolts; 2. Studs; 3. Tap-Bolts and Cap-Screws; 4. Machine Screws; 5. Set Screws.

Through-Bolts. A through-bolt, or "bolt" as it is commonly called, Fig. 39 (a), has a solid head on one end and a nut on the other. It is the best form of screw fastening and should always be used when the hole can be drilled completely through the two pieces to be held together.

Studs. Sometimes it is not possible or desirable to drill a hole entirely through both pieces which are to be held together, and in such cases a stud-bolt, or "stud," is often used. A stud, Fig. 39 (b), is a circular bar having a thread cut on each end. A hole is tapped in the part that cannot be drilled clear through, and one end of the stud is screwed firmly into the hole. The part that can be drilled through is slipped over the stud, and a nut on the outer end clamps the two parts firmly together. Where a through-bolt cannot be used the stud is the next best fastening. It should be a tight fit in the tapped hole, and when once screwed in should not be taken out, especially if the hole is tapped into cast iron, as repeated removal wears out the threads. The length of the tapped hole should be at least one and one half times the diameter of the stud, in order to secure ample frictional resistance against turning when the nut is unscrewed.

Tap-bolts and Cap-Screws. Tap-bolts, Fig. 39 (c), and cap-screws, Fig. 39 (d), have a solid head on one end and a thread on the other. They are used under exactly the same circumstances as the stud but are not as good a fastening, as they necessarily must be unscrewed from the tapped hole whenever they are removed. Where they have to be frequently unscrewed, and especially if the hole is tapped into cast iron, they should be avoided. The only difference between tap-bolts and cap-screws is in the size and form of the head, the tap-bolt having a standard head (see next article), and the cap-screw for the same size of bolt having a smaller head slightly rounded on top. Tap-bolts are much used in such work as securing patches on boilers, where a large head is desirable. Cap-screws are a little more ornamental and are much used in cheaper grades of machinery. They are a standard article in the market and hence can be bought very cheaply.

Machine Screws. Under the term "machine screws" are included many forms of small screws usually provided with a slotted head so that they may be set up with a screw-driver. The most usual forms of machine screws are shown in Fig. (39 *e*, f, g and h). At *e* is shown an **oval fillister** head; at *f* a **flat fillister** head; at *g* a **flat countersunk** head; and at *h* a **round head.**

Machine screws are designated, for convenience, by **numbers,** the larger numbers indicating the larger diameters. Thus the smallest size, as given in Brown and Sharp's catalogue, is number 000 the diameter of which is .03152. The difference in diameter between consecutive numbers is .01316. The diameter of a number 0 screw is .0578, so that the diameter of any number larger than this is given by the formula $d = .0131\ N + .0578$; where d is the diameter in inches, and N the serial number of the screw. The number N is not to be confused with the number of threads per inch n. Machine screws larger than number 16, which is about ¼″ in diameter, are not much used in machine work, another standard, to be discussed later, being used for sizes above that diameter.

Manufacturers have, so far, been unable to agree upon standard numbers of threads per inch, for a given diameter of machine screw. Thus a number 12 machine screw may have 20 or 24 threads per inch, so that these screws are usually specified by naming the size number first, followed by the number of threads per inch. Thus, an 18–20 machine screw means *size* 18 and 20 threads per inch. Because of the great confusion now existing regarding this point, the American Society of Mechanical Engineers appointed a committee to establish, if possible, a system of standards for machine screws. This committee has reported and their recommendations can be found in Vol. 28 of the Transactions.

Set Screws. Set screws are a form of screw-fastening frequently used to prevent relative rotation of two machine parts. Thus in Fig. 40 the hub *a* is prevented from revolving on the shaft *b* by the set screw *c*. The head of the set screw is square while the point may be cup-shaped as in Fig. 40 (a), round as in Fig. 40 (b), or conical as at *c* in Fig. 40. When the set screw is made in the form shown at Fig. 40 (a), the point is hardened to enable it to cut into the shaft, thus increasing its holding power. If the screw is made of tool steel the hardening may be done by the ordinary process of tempering; if made of wrought iron the same result may be obtained by case hardening. The objection to the cup-shaped end is that it makes a burr on the shaft which sometimes greatly interferes with the removal of the hub. To obviate this a small conical depression is sometimes made in the shaft with the end of a drill and the form shown at Fig. 40 used. Set screws are not reliable for heavy work and should be used only when the load is light.

Standard Screw Threads. Sellers or U. S. System. Screw fastenings larger than ¼ inch diameter are made according to some standard system in order to secure interchangeability. The first system of this kind was that introduced into England

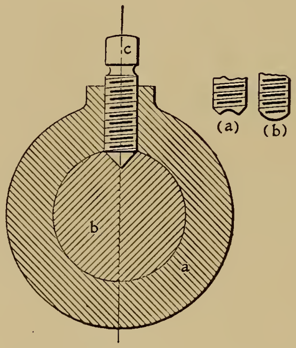


Fig. 40.

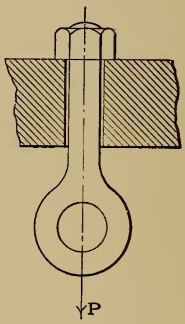


Fig. 41.

by Sir Joseph Whitworth. The form of the **Whitworth thread** is shown in Fig. 37 (c). The thread angle is 55° and the top and bottom of thread are rounded off as shown.

The recognized standard screw thread in the United States is the Sellers, U. S., or Franklin Institute thread. The form of this thread is shown in Fig. 39 (b). The thread angle is 60°; the top of the thread is cut off and the bottom of the thread filled in as shown. This standard is not used exclusively in this country, however, but a full *V* thread, as shown in Fig. 37 (a), without the flattened tops and bottoms is also in common use. The angle of such V thread is generally 60° in machine bolts and the number of threads per inch usually corresponds to those of the Sellers system, but there are many variations in this particular. Where the Sellers standard is not strictly adhered to it is advisable, therefore, to buy machine bolts of one manufacturer only or so specify as to insure interchangeability.

The Sellers screws have much greater tensile strength than screws with full V threads of equal angles and pitch, because the thread of the former is only three-fourths as deep owing to the flattening at the tops and bottoms.

TABLE X

SELLERS, U. S., OR FRANKLIN INSTITUTE STANDARD BOLTS

Bolts and Threads.				Hex. Nuts and Heads.				Sq. Nuts and Heads.	
Diameter of Bolt.	Threads per Inch.	Diameter at Root of Thread, d_1.	Area at Root of Thread.	Width of Nut Between Parallel Sides.	Width Across Corners.	Thickness of Nut.	Thickness of Head.	Width Between Parallel Sides.	Width Across Corners.
Inches.	No.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
1/4	20	.185	.027	1/2	37/64	1/4	3/16	1/2	7/10
5/16	18	.240	.045	19/32	11/16	5/16	1/4	19/32	10/12
3/8	16	.294	.068	11/16	51/64	3/8	5/16	11/16	63/64
7/16	14	.344	.093	25/32	9/10	7/16	3/8	25/32	1 7/64
1/2	13	.400	.126	7/8	1	1/2	7/16	7/8	1 15/64
9/16	12	.454	.162	31/32	1 1/8	9/16	1/2	31/32	1 23/64
5/8	11	.507	.202	1 1/16	1 7/32	5/8	9/16	1 1/16	1 1/2
3/4	10	.620	.302	1 1/4	1 7/16	3/4	11/16	1 1/4	1 49/64
7/8	9	.731	.420	1 7/16	1 21/32	7/8	13/16	1 7/16	2 1/32
1	8	.837	.550	1 5/8	1 7/8	1	15/16	1 5/8	2 19/64
1 1/8	7	.940	.692	1 13/16	2 3/32	1 1/8	1 1/16	1 13/16	2 9/16
1 1/4	7	1.065	.890	2	2 5/16	1 1/4	1 3/16	2	2 53/64
1 3/8	6	1.160	1.057	2 3/16	2 17/32	1 3/8	1 5/16	2 3/16	3 3/32
1 1/2	6	1.284	1.293	2 3/8	2 3/4	1 1/2	1 7/16	2 3/8	3 23/64
1 5/8	5 1/2	1.389	1.510	2 9/16	2 31/32	1 5/8	1 9/16	2 9/16	3 5/8
1 3/4	5	1.491	1.741	2 3/4	3 3/16	1 3/4	1 11/16	2 3/4	3 57/64
1 7/8	5	1.610	2.050	2 15/16	3 13/32	1 7/8	1 13/16	2 15/16	4 5/32
2	4 1/2	1.712	2.300	3 1/8	3 5/8	2	1 15/16	3 1/8	4 27/64
2 1/4	4 1/2	1.962	3.030	3 1/2	4 1/16	2 1/4	2 3/16	3 1/2	4 61/64
2 1/2	4	2.176	3.719	3 7/8	4 1/2	2 1/2	2 7/16	3 7/8	5 31/64
2 3/4	4	2.426	4.620	4 1/4	4 29/32	2 3/4	2 11/16	4 1/4	6
3	3 1/2	2.629	5.428	4 5/8	5 3/8	3	2 15/16	4 5/8	6 27/32

The area of a 1″ full 60° thread is .482 square inches, while the area at the bottom of a Sellers thread is .55 square inches, or 14 per cent greater.

The Whitworth system of threads differs from the Sellers in the shape of the thread, as noted above, and the number of threads per inch is also different for some diameters. Thus the

Sellers system gives 13 threads per inch for ½″ bolts while the Whitworth gives 12. The Whitworth system gives somewhat stronger screws as the diameter at the root of the thread is greater for the same size of bolt and the rounded shape at the root is stronger than the flat root of the Sellers thread. The Sellers thread is, however, much easier to produce than the Whitworth.

Table X gives the proportion of screws as fixed by the Sellers standard for bolts up to 2¾″. Above this size the standard is not adhered to rigidly, as the size and pitch of the screw becomes rather large for convenience. Thus a 6″ bolt in the Sellers system will have 2¼ threads per inch. It is common, therefore, to make these larger sizes, which are comparatively rare, with 4 threads to the inch.

In Germany, France, and other European countries other systems are in use, some of which are based on metric units.

57. Pipe Threads. The **Briggs** system of pipe threads is the established standard in the United States. The numbers of threads per inch for the various sizes of pipe are given below:

⅛″ pipe, 27 threads per inch.
¼″ and ⅜″ pipe, 18 threads per inch.
½″ and ¾″ pipe, 14 threads per inch.
1″ to 2″ pipe, 11½ threads per inch.
2½″ and over, 8 threads per inch.

For form of threads and other details as to Briggs system, see Trans. A. S. M. E., Vol. VIII, page 29.

58. Straining Action in Bolts due to External Load. The load applied to the bolts is generally one which tends to separate the connected members, in the direction of the axis of the bolt, and this action is resisted by a tensile stress in the bolts; but bolts are sometimes used to prevent the relative translation of two or more pieces, when a shearing stress is produced in the bolts. When the action of the load is oblique to the axis, the stress in the bolt may be combined tension and shearing.

If any screw is tightened up under load there is an initial direct stress (tension or compression) and usually a torsional stress due to friction between the threads of the screw and the nut. With bolts or studs screwed up hard, as in making a steam-tight joint, the initial tension due to screwing up may be much in excess of that due to the working load. This will be treated more fully later.

If the load applied to the bolt produces a shearing action, the bolt shank should accurately fit the holes in the connected pieces, at least for the portions near the joint; and if P is the load per bolt, d the diameter of the bolt (shank), and p_s the shearing stress,

$$P = \frac{\pi}{4} d^2 p_s \therefore p_s = \frac{4P}{\pi d^2}$$

In a bolt subjected to a load which produces tension, the minimum cross section sustains the greatest stress. This smallest cross section, in common bolts, is through the bottoms of the threads. Thus if a load P be applied to an eye-bolt, as in Fig. 41, the only stress that will be induced in the bolt will be that due to the external load P. If p be the tensile stress due to the load P, and d_1 the diameter of the bolt at the bottom or root of the threads,

$$P = \frac{\pi}{4} d_1^2 p \therefore p = \frac{P4}{\pi d_1^2} \quad . \quad . \quad . \quad . \quad (14)$$

Values of d_1 are given in Table X, page 167, for the various sizes of Sellers screws. For a given diameter and pitch of screw the area at the bottom of threads would be considerably less with full V threads.

59. Initial Tension in Bolts due to Screwing up. If the bolt is used simply to hold two machine parts together, as in Fig. 39 (a), and there is no external load tending to separate the parts, the stress in the bolt will be the resultant of the tensile stress due to screwing up the nut, and the torsional stress due to the frictional resistance at the thread.

In the Sellers system the pitch angle of the thread (α) varies from about 3° in a ½″ screw, to 1°–50′ in a 3″ screw; or tan α varies from .054 to .032 in this same range. If, therefore, in equation (12), r_c be taken equal to $\frac{4}{3}$ r_m, μ be taken at .1, and μ_1 .15, it appears that P varies from .35 W with a ½″ screw to .32

W with a 3″ screw. The coefficients of friction will vary much more than this, so it may be assumed that for the ordinary range of screw fastenings

$$P = .33\ W \text{ approximately}$$

or the tension in the screw $W = \dfrac{P}{.33}$.

The turning moment, Pr_m, due to the wrench pull, is resisted by the frictional moment of the nut or collar and the frictional moment at the thread. This frictional moment at the thread is transmitted to the body of the bolt, so that the bolt itself is subjected to a twisting moment equal to Pr_m minus the frictional moment at the nut or collar. The resultant stress, therefore, under these circumstances, is that due to combined twisting and direct stress, and it can be shown (see Art. 67) that the resultant stress as determined by equation (1), page 49, is from 15 to 20 per cent greater than the direct stress alone.

Refined calculations regarding the resultant stress in bolts due to screwing up are, in general, useless and misleading, especially in the case of fastenings less than ⅞″ in diameter. Since a mechanic using a wrench of ordinary proportions can easily rupture any of these small fastenings, it follows that the actual stress in such bolts depends entirely on the judgment of the mechanic.

A series of experiments was made in the Sibley College Laboratory, a few years ago, to determine directly the probable load produced in standard bolts when making a tight joint. The sizes of bolts used were ½″, ¾″, 1″ and 1¼″. One set of experiments was made with rough nuts and washers, and another set with the nuts and their seats on the washers faced off. A bolt was placed in a testing machine, so that the axial force upon it could be weighed after it was screwed up. Each of twelve experienced mechanics was asked to select his own wrench and then to screw up the nut as if making a steam-tight joint, and the resulting load on the bolt was weighed. Each man repeated the test three times for every size of bolt, and each had a helper on the 1″ and 1¼″ sizes, The sizes of wrenches used were 10″ or 12″ on the ½″ bolts up to 18″ and 22″ on the 1¼″ bolts. The results were rather discordant, as should be expected; the loads in the different tests were rather more uniform, as well as higher, with the faced nuts and washers. The general results indicate: (a) that the initial load due to screwing up for a tight joint varies about as the diameter of the bolt; that is, a mehanic will graduate the pull on the wrench in about that ratio. (b) That the load produced may be estimated at 16,000 lbs. per inch of diameter of bolt, or

$$W = 16{,}000\,d \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

in which W is the initial load in pounds due to screwing up, and d is the nominal (outside) diameter of the screw thread. This value of W is rather above the average for the tests; but it is considerably below the maximum, and it is probably not in excess of the load which may reasonably be expected in making a tight joint.

If the initial load due to screwing up be divided by the cross-sectional area of the bolt at the bottom of the threads, the initial intensity of the tensile stress is obtained. The above experiments indicate that this intensity of stress varies, approximately, inversely as the nominal diameter (d) of the bolt; and that it may frequently equal or exceed

$$p = \frac{30{,}000}{d} \text{ lbs. per sq. in.} \quad . \quad . \quad . \quad . \quad (16)$$

In addition to this tensile stress there is, as before stated, a considerable twisting action on the bolt. Equation (16) would give a stress of 60,000 lbs. per square inch on a ½-inch bolt; and this result is substantiated by the fact that steel bolts of this size were broken in the course of the experiments. It also agrees with common experience which forbids the use of screws as small as ½-inch for cases requiring the nuts to be screwed up hard.

In these experiments, the average effective lever arm of the wrench was not far from 15 times the diameter, or 30 times the radius, of the screw; hence, if it be assumed as in the previous paragraph that the turning force acting at the radius of the screw is $P = .33W$ the force applied at the wrench is, in pounds, about

$$P_1 = \frac{P}{30} = \frac{.33W}{30} = \frac{.33 \times 16{,}000\,d}{30} = 176\,d.$$

The above discussion indicates that the factor of safety should be increased as the size of the screw decreases, and of course this factor should be varied with the conditions of the case, as in some applications the nuts are much more apt to be screwed up hard than in others.

A set of experiments was made by Mr. James McBride (Trans. A. S. M. E., Vol. XII, page 781), which show that the factor of safety, as bolts are frequently used, is very low, even with a very moderate external load. One case cited by Mr. McBride indicates that the stress due to screwing up a 3⅛-inch bolt was nearly one-half the ultimate strength, or probably very near the elastic limit. His direct determinations of the efficiency of a standard 2-inch screw bolt shows an average of only 10.19 per cent. It is probably this low efficiency which saves many screws from being broken, as the frictional loss reduces the tension produced in the bolt by screwing up. The excessive friction makes the screw bolt a useful fastening, as it reduces the tendency to "overhaul" or unscrew.

60. Resultant Stress on Bolts due to Combined Initial Tension and External Load. It was shown, in Art. 59, that bolts may be subjected to a high tensile stress by screwing up the nuts. The question often arises as to the effect of the combined action of this initial tension and the external, or useful, load. It is stated by some that the resultant load on the bolt is simply the sum of the initial and the external loads. Others contend that the application of the external load does not change the stress in the bolt, unless this external load exceeds the initial load due to screwing up; that is, that the resultant load is equal to the initial load alone, or to the external load alone, whichever is the greater.

Neither of these views is entirely correct for conditions attained in practice. They represent the extreme limiting cases and every actual case lies between them.

If the bolt itself could be absolutely rigid while the members forced together in screwing it up yielded under pressure, the total load on the bolt would be equal to the sum of the initial load and the external load. If, however, the members pressed together were absolutely rigid, only the bolt yielding, the total (resultant) load on the bolt would be the initial load alone, or the external load alone, whichever is the greater.

The first of the above conditions is approached by the arrangement shown in Fig. 42. Screwing up the nut compresses the spring interposed between *A* and *B*. Assume that an axial force of 2,000 pounds will compress this spring 1 inch; then if the nut is screwed up till the spring is 2 inches shorter than its free length, the load on the bolt, due to screwing up, must equal the reaction

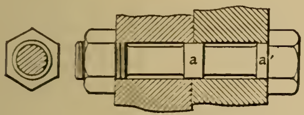


FIG. 46.

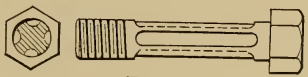


FIG. 47.

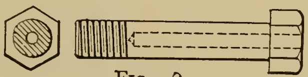


FIG. 48.

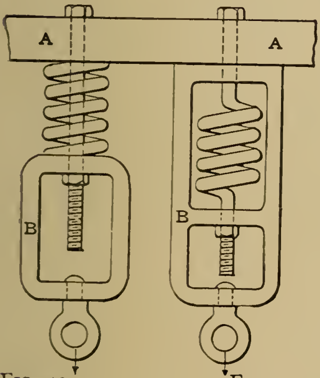


FIG. 42. FIG. 43.

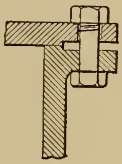


FIG. 44.

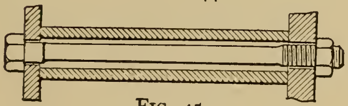


FIG. 45.

of the spring, or 4,000 lbs. Assume, also, that the extension of the bolt under this screwing-up action, or under the initial load of 4,000 lbs., is .02 inch. Now, if an external axial load of say 2,000 lbs. be applied to the eye at the bottom of *B*, this added load would tend further to increase the length of the bolt by about .01 inch; but this further extension of the bolt would reduce the compression on the spring by a corresponding amount and thus slightly diminish the spring reaction. With such great difference between the rigidity of the bolt and of the connected members, the load on the bolt becomes practically the sum of the initial and the external loads, but the resultant load is necessarily somewhat less than this sum in any possible case.

The arrangement shown in Fig. 43 is one which approaches the other limiting case mentioned above. Suppose the bolt to be a spring which is subjected to an axial load of 4,000 lbs. in screwing the nut up two inches, and that the corresponding yielding of the member *B* is .02 inch. The initial load on the bolt (which is the spring in this case) is 4,000 lbs., and the pressure between the contact surfaces of *A* and *B* is equal to it. If an external axial load be now applied to the eye in *B*, the pressure between the contact surfaces is reduced by an amount nearly equal to this external load. But, unless the external load exceeds the initial load, the bolt will not elongate enough to separate these contact surfaces and entirely remove the pressure between them, because the load on the bolt (spring) cannot change without changing the length of the bolt, and with the above data the bolt would have to stretch an additional .02 inch (equal to the initial yielding of the connected members) before the contact surfaces would be entirely relieved of pressure. It therefore appears that the addition of an external load in this case does not materially affect the resultant tension on the bolt as long as this external load does not exceed the initial load. If the external load is greater than the initial load (say 6,000 lbs.), the elongation of the bolt increases (to 3 inches); the resultant load on the bolt will be simply the external load alone, because the latter is sufficient entirely to relieve the pressure produced between the contact surfaces in screwing up.

In all ordinary practical cases the difference in rigidity between the bolt and the connected members is much less than in the extreme conditions considered above. The resultant load on a bolt may be anything between the *sum of the initial and the external loads* as a maximum, and the *greater of these two loads alone* as a minimum. This resultant load approaches the maximum limit when the bolts are rigid relative to the connected members as in Fig. 44; and this resultant approaches the minimum limit when the bolts are relatively yielding, as in Fig. 45. In any particular case the designer can tell which limit is the more nearly approached, and he should be governed accordingly.

The *Locomotive* (Nov., 1897) contains an excellent article on the resultant load on bolts, and a relation is derived from which the following method of treatment has been developed: The application of this method depends simply upon the *ratio* of the yield of the connected members to the yield of the bolts. It will usually not be difficult to assign a sufficiently close value to this ratio even when the actual magnitudes of yielding are unknown; in fact, only a rough approximation to the value of this ratio is necessary. Let this ratio be called y and let $\frac{y}{y+1} = x$; call the initial load on the bolt due to screwing up W_1; the external (useful) load W_2; and the total (resultant) load W. Then it can be shown that

$$W = W_1 + x\,W_2.$$

If the yield ratio (y) is known, the value of x is at once found by the above relation of x and y. If the yield of the connected members is between 1 and 5 times that of the bolt, the resultant load is equal to the initial load added to from 0.5 to 0.8, the external load. If a tight joint is made with short rigid bolts or studs, connecting flanges which are separated by an elastic packing, or with a metal contact at some distance from the center line of the bolts, as indicated in Fig. 44, the applied load is an important consideration since the value of y is relatively great. In some other cases the external load may be a minor consideration as affecting the strength of the bolt.

When the conditions are such that the nut is not apt to be screwed up hard, that is when the initial load may be safely neglected, design for the external load alone.

The following suggestions may serve as a guide in practical problems involving the resultant load on bolts when the initial load due to screwing up is apt to be considerable.

(a) If a bolt is manifestly very much more yielding than the connected members, design the bolt simply for the initial load or for the external load, whichever is the greater.

(b) If the probable yield of the bolt is from one-half to once that of the connected members, consider the resultant load as the initial load plus from one-fourth to one-half the external load.

(c) If the yield of the connected members is probably four or five times that of the bolts, take the resultant load as the initial load plus about three-fourths the external load.

(d) In case of extreme relative yielding of the connected members, the resultant load may be assumed at nearly the sum of the initial and external loads.

61. Allowable Stress in Screw Fastenings. From the foregoing it is seen that small screw fastenings are very liable to be heavily overstrained by the initial load due to screwing up the nut. While the body of the bolt is well designed to resist heavy loads a source of weakness is found in the threaded portion. The reduced area, due to cutting the thread, localizes the greatest stress, and cracks are very liable to start from the roots of the threads, especially where the thread is of the full *V* form.

For these reasons the ordinary apparent fibre stresses allowed in most machine members cannot be permitted in screw fastenings. For ordinary purposes, where overstraining is not likely to occur, or for large bolts, 8,000 to 10,000 lbs. per square inch may be allowed, for steel. For such work as steam and hydraulic joints, where the initial stress may be large, from 6,000 to 8,000 lbs. per square inch should be allowed, depending on the conditions and quality of material employed, and if shocks are liable to occur, stresses as low as 3,000 to 4,000 are often preferable.

Example 1: The cylinder of the steam engine is 12 inches in diameter, and the cylinder head is held in place by 10 steel *through* bolts. The maximum steam pressure is 100 pounds per square inch. If the contact surfaces of the head and cylinder are ground together so that no packing is necessary, what must be the diameter of the bolts so that the maximum stress in the bolt necessary to insure a steam-tight joint will not exceed 7,000 lbs. per square inch?

In this case it is evident that the bolts are much more yielding than the parts which they hold together and the conditions are those of case *a* in the previous paragraph. It is also clear that the initial load on the bolt must be greater than the external load due to the steam pressure in order to insure a steam-tight joint. If this initial load be taken at twice the external load a fair margin of safety is secured. If W_1 be the initial load and W_2 the external load per bolt then

$$W_2 = \frac{\pi\, 12^2}{4 \times 10} \times 100 = 1{,}130 \text{ lbs.}$$

whence $W_1 = 2\, W_2 = 2{,}260$ lbs.

Whence if d_1 be the diameter at the root of the thread

$$\frac{\pi\, d_1^2}{4} \times 7{,}000 = 2{,}260.$$

Therefore $d_1 = .64$ inch (at root of thread) which corresponds closely to a ¾″ screw. It is to be noted that while a total stress of 7,000 lbs. per square inch of section is *sufficient* to insure a tight joint, a much greater stress may be induced by the workman in screwing up the nut, if he is careless or inexperienced.

Example 2: If in the above example steel *studs* are used and rubber packing ⅛″ thick be placed between the contact surfaces, what must be the diameter of the studs?

Here the parts held together are more elastic than the studs and the conditions may be taken as corresponding to those of case *c*. As before, the initial load W_1 may be taken at twice the external load.

$$\text{Then } W_2 = \frac{\pi \times 12^2}{4 \times 10} \times 100 = 1{,}130$$

$$\text{and } W_1 = 2\, W_2 = 2{,}260.$$

From (*c*) paragraph 60 the total load

$$W = W_1 + \tfrac{3}{4}\, W_2$$

$$= 2{,}260 + (\tfrac{3}{4} \times 1{,}130) = 3{,}107 \text{ lbs.}$$

$$\text{Whence } \frac{\pi\, d_1^2}{4} \times 7{,}000 = 3{,}107$$

$$\text{and } d_1 = .76 \text{ inch (at root of thread)}$$

which corresponds closely to a ⅞ inch screw.

The maximum stress which the workman may, perhaps, induce in the stud by screwing up the nut is

$$p = \frac{30,000}{d} = \frac{30,000}{\frac{7}{8}} = 34,000 \text{ lbs. approximately,}$$

which will be increased a little by the external load. This is close to the elastic limit; but it may be noted that even should the elastic limit be slightly exceeded the efficiency of the fastening is not impaired, since here permanency of form is not so essential as in machine parts which transmit motion.

62. Resilience of Bolts with Impulsive Load.

In bridge work and other cases requiring long bolts, it is very common to make the cross-section through the body of the bolt about equal to the section at the bottom of the threads. This may be done by upsetting the ends where the thread is to be cut, or by welding on ends made from stock somewhat larger than that used for the main length of the bolt.

The most apparent result of this practice is to economize material without sacrifice of strength (as the shank still has an area of cross-section equal to the threaded portion), and if the weld (when the ends are welded) is perfect, the strength of the bolt is not reduced. It seems probable that this reason is responsible for the original adoption of this practice, since it has been most generally used in long tie rods. However, in case of bolts liable to shock, there is an even more important reason for such construction; since it can be shown that the reduced section not only maintains the full strength under static load, but it very greatly increases the capacity of the bolt to resist shock. This last fact has not been very generally recognized, as appears from the common application of such reduced shank bolts only to structures, rather than to machines.

It has been seen that the resistance of a tension member under a static load is determined solely by its weakest section; while, in a member subjected to shock, impact, or impulsive load, the resistance depends upon the total extent of distortion of the member due to a given intensity of stress.

As shown in Art. 24, the maximum stress with impulsive load is

$$p = \frac{W\,(h + \delta)}{k\,\delta\,A}$$

For a stress within the elastic limit

$$p = \frac{2\,W}{A}\left(\frac{h + \delta}{\delta}\right) = \frac{2\,W}{A}\left(\frac{h}{\delta} + 1\right)$$

This shows clearly that for a given load, W, applied suddenly or with impact, the stress produced in a member of sectional area, A, is greater as δ becomes less relative to h. Hence, if δ is increased, the stress produced becomes less for a given impulsive action; or the resistance to such action is greater for a given value of the maximum stress.

If an ordinary bolt is subjected to shock in a direction to produce tension, the stress will be a maximum at the sections through the bottom of the threads; the bolt will elongate, but the elongation will be confined largely to the very short reduced (threaded) sections, hence the stress will be much less in the larger portion of the bolt. In a Sellers bolt of one inch diameter the area A of the shank is .78 sq. inches, while the area A' at the bottom of threads is only .55 sq. inches. Therefore a stress on A' of 30,000 lbs. per sq. in. $= \dfrac{30{,}000 \times .55}{.78} = 21{,}000$ on the full sections. Suppose the elongation per inch of length at a stress of 30,000 (taken as the elastic limit) is $\frac{1}{1000}''$. Each inch of section A' will elongate $\frac{1}{1000}''$, while each inch of full section A $(=.78$ sq. in.) will have a stress of only 21,000 lbs., with a corresponding elongation of $\frac{21}{30} \times \frac{1}{1000} = .0007''$. Assume the thread to be $1''$ long, and the remainder of the bolt to be $5''$ long. It will appear that the mean stress on the threaded portion $(1'')$ is about the mean of 30,000 and 21,000, or say 25,500 lbs. per square inch; as the mean section is an average of .55 and .78 square inches. Hence the elongation for this threaded 1 inch, when the stress on $A' = 30{,}000$, is $.00085''$, while the other $5''$ (of area A) will elongate under this load $5 \times .0007 = .0035''$. The total elongation will then be $\delta = .00085 + .0035 = .00435$ inches.

$$\text{If } h = \tfrac{1}{10}'',\ W = \frac{A'p}{2}\,\frac{\delta}{h+\delta}$$

$$\frac{.55 \times 30{,}000}{2} \times \frac{.00435}{.10435} = 8250 \times .0416 = 344 \text{ lbs.}$$

Now, suppose the 5″ shank of this bolt were reduced in section to an area $A' = .55$. Then the elongation of this portion under the above load would be $5 \times .001 = .005''$, instead of $.0035''$ and the total elongation would be $\delta = .00085 + .005 = .00585$.

$$\therefore W = \frac{.55 \times 30{,}000}{2} \times \frac{.00585}{.10585} = 8250 \times 0.553 = 457 \text{ lbs.}$$

This latter load is 33 per cent greater than the preceding.

The preceding example shows that the *elastic resilience* of the bolt was increased 33 per cent by reducing the body of the bolt to A'. Of course the gain would be still greater with a longer bolt. It may be well to remember that the "long specimen" is more apt to contain a weak section than is a short specimen; but, on the other hand, the sharp notching of the threads is quite liable to start a fracture at their roots.

If the bolt is strained beyond the elastic limit, the portion thus strained yields at a much greater rate, relative to the stress, than that given above. With a load which would produce a stress of 30,000 lbs. per sq. in. in the larger portion (area A), the stress in the reduced portion (area A') will be $\dfrac{30{,}000 \times .785}{.55} = 43{,}000$ lbs. per sq. inch. Hence, the effect of a long section in resisting shock *without rupture* is much greater even than that shown for elastic deformation only.

The section of the shank of the bolt may be reduced as in Fig. 46, by turning down the body of the bolt to about the diameter at the bottoms of the threads. The collars a and a' may be left to form a fit in the hole. This form is easy to make, but does not fit the hole throughout its length, and it is weak in torsion.

Fig. 47 is somewhat more expensive, but fits the hole better, and is somewhat stronger in torsion. Fig. 48 is the form which gives the best fit, and is also the strongest in torsion. If very long it is difficult to make; otherwise it is perhaps the best.

These high resilience bolts only increase the resistance to impulsive load, not to dead load. They are good forms to use in such cases as the so-called "marine type" of connecting rod, where the bolts are subjected to considerable shock.

For cylinder head bolts, and other cases where a tight joint is the main consideration, this form of bolt may be entirely unsuited.

Professor Sweet prepared, for tests, some bolts such as are used in the connecting rod of the Straight Line Engine; of these, half were solid (ordinary form) bolts, and the other half were of the form shown in Fig. 48.

Tests of a pair of these bolts, one of each kind, showed an elongation at rupture of .25″ for the solid bolt, which broke in the thread; while the drilled bolt elongated 2.25″, or 9 times as much, and it broke through the shank, the net section of which was a trifle less than that at the bottom of the threads. Drop tests showed similar results. These tests indicate the superior ultimate resilience of the reduced shank bolts.

It was shown in Art. 24, page 77, that where a machine member must absorb considerable shock, a rather weak yielding material might be safer than one which is stronger and stiffer, because of the greater elastic resilience of the weaker and more ductile material. This principle is of importance in designing fastenings which are subjected to shock where they must necessarily work under high stress.

63. Location of Fastenings. As previously stated, screw fastenings are generally intended to be tension members only, and from the foregoing discussion it appears that even when used in this manner alone they are subjected to very high stresses. The conditions under which a fastening is to be used should therefore be carefully considered in order that all forces acting upon it may be provided for. Further, the **location** of the fastening may or may not be advantageous, thus greatly affecting its required size. Thus in Fig. 49, if the bolts alone are depended upon to resist the downward force P, they must be carefully fitted, to insure that each bolt receives its full share of this shearing load. Through bolts only can be used in such a case as studs or tap bolts cannot be accurately fitted. If the downward force is resisted by a projecting ledge, as at A, which is the preferable way, the bolts need not fit the holes closely and either studs or tap bolts can be used. The bracket now tends to rotate around A and the moment of the load $P\,l$ must equal the sum of the moments of the bolts round the same point. It is evident that the lower bolt must be considerably larger than the upper bolt, to be equally effective. In small work it is convenient to make all bolts the same size, the sum of their resisting moments being made equal to $P\,l$. In large work the bolt at C is often made large enough to exert a moment equal to $P\,l$, and the bolts near A serve only to insure correct location. The upper bolt should, in any case, be located as far away from A as possible.

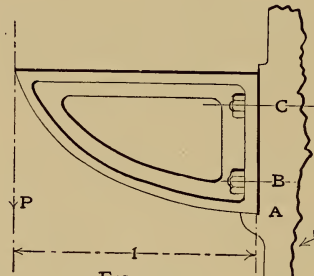


Fig. 49.

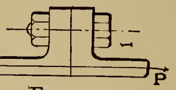


Fig. 50.

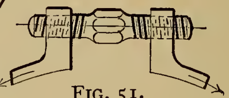


Fig. 51.

In many machine parts, such as flywheel rims and brake bands, it often occurs that the bolts cannot be placed directly in line with the applied force but must be at a distance l (Fig. 50) from its line of action. The bolts in such cases may be subjected to both flexure and direct stress. Thus in Fig. 50, if the bolts fit the holes in the lugs tightly such a combination of stresses will be induced. In such parts as brake bands the connecting bolts are often used as a means of adjustment against wear, as shown in Fig. 51. If in such a case the lugs be weak and yielding, the threaded portion of the bolt will be subjected to both flexure and direct stress. The threaded portion of the bolt is particularly weak against flexural stress because cracks are easily started at the root of the threads, and where screws are used in this manner they should be designed with a large factor of safety.

64. Screws for the Transmission of Power. It has been pointed out, in Art. 53, that the square thread is most used for transmitting power because of its higher efficiency, and that when wear must be compensated for the half V thread is most serviceable (see Figs. 37 d and e). Where the thread angle of the half V thread is small, as in the Acme thread, the general equations which have been deduced for the square thread may be used without great error.

Equation (6), Art. 54, expresses the relation which exists between the turning moment which must be applied to the screw, and the moments due to the load, friction at the thread and at the thrust collar. An examination of this equation shows that for a given applied force P, the load W which can be overcome decreases with an increase in the value of the pitch s, and increases with a decrease in the value of s, since s is added to the numerator of the fraction and subtracted from the denominator. This can be seen in another way by considering the energy supplied and the work performed. If the force P be applied through a complete revolution, or a distance of πd_m, the load will be raised a distance equal to the pitch s. Evidently, if s is decreased, a greater load can be raised by a given force P; since (neglecting friction), the force applied, multiplied by the space through which it moves, must be equal to the load multiplied by the space through which it is raised. In other words, the **mechanical advantage** of the screw can be varied by reducing the pitch angle; and it is evident that by reducing the pitch angle a small force *applied at a long radius* may be made to raise a great load.

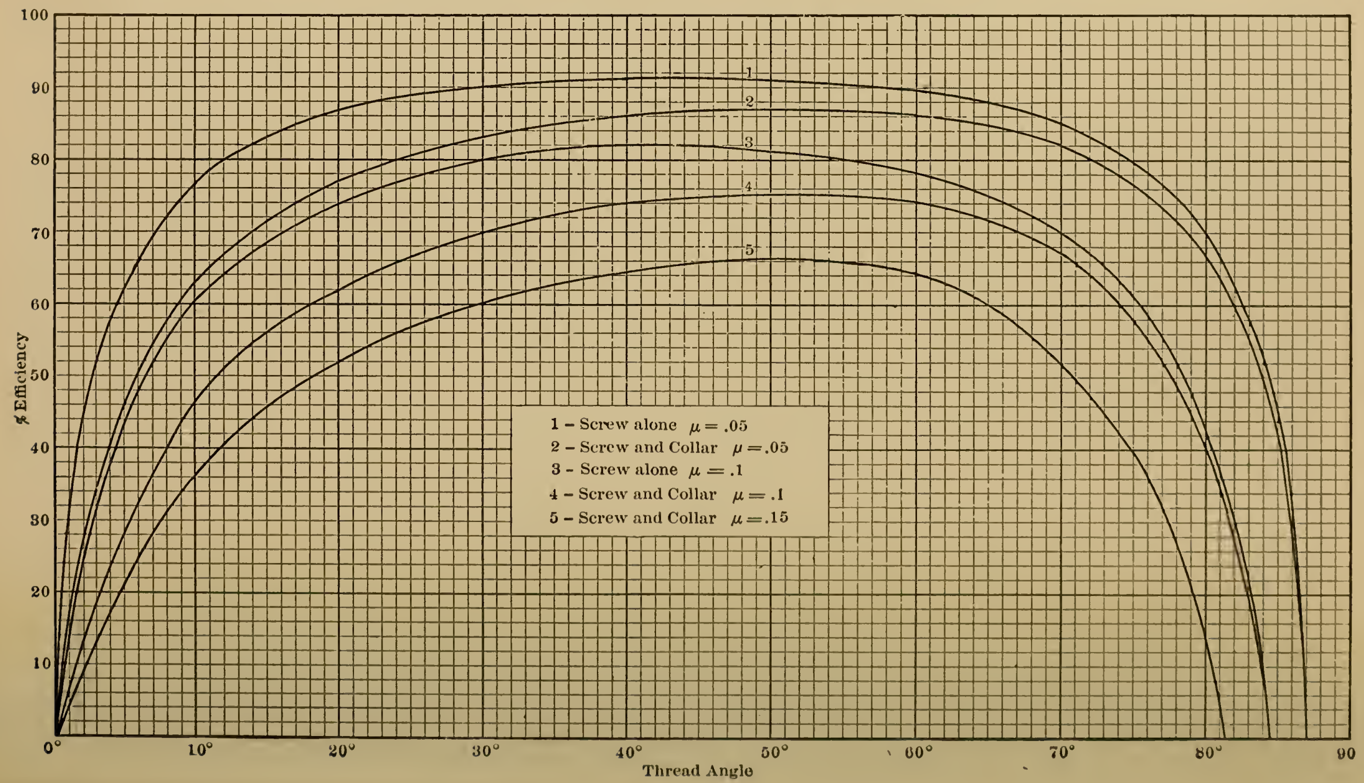
In order that the thread on the screw and nut may be equally strong, with similar materials, the thread and space on the screw are made equal to each other and therefore equal to half the pitch. As the pitch is increased the axial width of both thread and space are necessarily increased, and if it is desired to keep the section of the thread square in form, this soon results in a very heavy thread when compared to the cylinder on which it is formed. If the depth of the space is reduced, to avoid reducing the diameter of the cylinder, the bearing surface of the screw and nut is reduced, which is not desirable. It is customary in such cases to divide the axial width of the thread and space into several equal parts, arranged alternately round the axis of the screw, thus forming several parallel threads and spaces. The depth of the space can, by this means, be greatly reduced and ample wearing surface be provided. Such screws are called **multiple threaded screws** and may have two, three, or more parallel threads. The theory of such screws is evidently identical with that of the single threaded screw.

65. Friction and Efficiency of Screws for Power Transmission. Equations (9) and (10), while expressing the general relations which exist between efficiency and the pitch angle, do not show clearly the effect upon the efficiency due to varying this angle. In Fig. 52 these equations have been plotted for various constant values of μ, and these curves show graphically the effect of varying the pitch angle. An examination of this figure shows that for the value of μ chosen the efficiency increases rapidly, as the angle increases, up to 15° or 20°, attaining a maximum between 40° and 50°, and then decreasing with an increase in the angle, becoming zero again near 90°.

It is to be noted that between 20° and 60° the efficiency does not vary materially with change of angle, and that when the efficiency of the screw alone is considered, steep pitched threads, as from 30° to 50° pitch angle, give maximum efficiency and hence a more durable thread. It is seldom feasible to use such pitches in practice, for reasons that will be presently discussed. The curves in Fig. 52 will be found useful in making trial assumptions for the efficiency of screws.

Screws for transmitting power are usually difficult to lubricate freely, hence, in general, their rubbing surfaces are imperfectly lubricated (see Art. 28). *The coefficient of friction for screws* working under pressures ranging from 3,000 to 10,000 lbs. per square inch, and at low velocities, has been experimentally determined * by Professor Albert Kingsbury. From his experiments it appears that, *for these conditions*, the value of μ may be

* Transactions of American Society of Mechanical Engineers, Vol. 17.


% Efficiency
Thread Angle
1 – Screw alone $\mu = .05$
2 – Screw and Collar $\mu = .05$
3 – Screw alone $\mu = .1$
4 – Screw and Collar $\mu = .1$
5 – Screw and Collar $\mu = .15$

taken at .15. For pressures lower than 3,000 lbs. per square inch, and velocities above 50 ft. per minute, the value of μ may be assumed at .1, if fair lubrication is maintained.

The **bearing pressure** per unit area of thread surface that may be carried on a screw thread, will vary greatly with the conditions of service. If the velocity is low, and wear not an important factor, as in the case of jack screws, very heavy pressures may be carried; but where accuracy of form is important, and where the velocity exceeds 50 ft. per minute, the pressure per unit area should not exceed 200 lbs., and for such service as lead screws, where maintenance of form is essential, it should be as low as possible.

If W = load carried.

p = intensity of pressure per unit of projected area of thread.
n = number of threads per inch.
l = length of nut in inches.
d = outside diameter of thread.
d_1 = inside diameter of thread.

$$\text{Then } W = p\,n\,l\,\frac{\pi}{4}\left[d^2 - d_1^2\right] \quad . \quad . \quad . \quad . \quad (17)$$

The load per unit of projected area is the same as the load per unit of true area, since the projected area is equal to the true area multiplied by cos α, and the axial or projected pressure is equal to the normal or true pressure multiplied by the same function.

66. Stresses in Transmission Screws. It has been shown in Art. 59 that the resisting moment at the thread is, from equation (6), equal to $W\,r_m\left[\frac{s + \mu\pi d_m}{\pi d_m - s\mu}\right]$ or to the total turning moment applied, minus the frictional moment at the collar. This moment induces a torsional stress* in the screw. The direct action of the load is to induce a tensile or compressive

* This statement applies strictly to the most usual case only, where the thrust collar is located at that end of the screw to which the power is applied. If the collar is not located at the end to which the power is applied, the total torque, Pr_m, of equation (6) is transmitted through the body of the screw.

stress in the screw equal to $\frac{W}{A}$ (where A is the area at the root of the thread) if the screw is short. If the screw is over six times as long as its least diameter d_1, the compressive stress, if the screw is in compression, will be that due to W, considering the screw as a long column. Equations (1) and (2) of Chapter III, page 48, and their discussion in Art. 18, are therefore, applicable to the design of such screws.

67. Design of Screws for Power Transmission. An inspection of Fig. 52 shows that screws of small pitch have very low efficiency, and it would seem desirable for that reason to keep the pitch as great as possible. On the other hand, it was pointed out in Art. 64 that the mechanical advantage of a screw increases as the pitch decreases. It was also shown in Art. 54 that a self-sustaining screw could not have an efficiency of over 50 per cent, and for perfect safety against overhauling it should be much less than this value. The best pitch, for a given set of conditions, may therefore be a compromise between these conflicting requirements. Thus if the turning moment which can be applied to a screw is limited (as is often the case with hand power) a low pitch must be selected in order to attain mechanical advantage. In such a case it is obvious that care should be used in selecting the materials of the screw and nut, so as to obtain as low a coefficient of friction as possible. Thus a bronze nut will run well on a steel screw with imperfect lubrication. (See Art. 28.) Again in such cases as the screws in certain machine tools, as plate planers, a more efficient pitch may be taken. If there is no tendency for the screw to overhaul, and the necessary moment can be applied, the pitch of maximum efficiency can be selected.*

Example. The force required to open or close a certain submerged sliding water gate is estimated at 6,000 lbs. It is required to design a single-threaded steel screw such that one man

* In certain forms of saw-mill carriages these conditions exist, and the screws for setting the log over to the saw may be and are made with a very efficient pitch.

exerting a pull of 60 lbs. at the periphery of a hand wheel 40 inches in diameter, attached directly to the screw, can operate the gate. The greatest unsupported length of the screw is found to be 4 ft. Let the coefficient of friction = .15, the crushing strength of the material = 30,000 lbs. per square inch, the maximum working tensile or compressive stress = 10,000 lbs. per square inch, and the coefficient of elasticity = 30,000,000.

Since the screw is in compression in closing the gate it must be designed as a long column square at both ends (case 4), and if so designed it will have surplus strength when in tension. The effect of the thread in stiffening the screw is small and will be neglected. The *total maximum* unit stress in the screw is to be 10,000 lbs. and it is evident that the maximum compressive stress p will form the larger part of the total stress; p may therefore be taken at 9,000 lbs. per square inch, and the *mean* compressive stress p' assumed at 6,000 lbs. per square inch: whence the trial area of the screw at the root of the thread $= \frac{P}{p'} = \frac{6{,}000}{6{,}000} = 1$ sq. inch, or a diameter of $1\frac{1}{8}$ inches. Checking this assumption by formula N page 94

$$p' = \frac{P}{A} = \frac{p}{1 + \frac{p_c}{m\pi^2 E}\left(\frac{l}{\rho}\right)^2} = \frac{9{,}000}{1 + \frac{30{,}000}{4 \times \pi^2 \times 30{,}000{,}000}\left(\frac{48}{.28}\right)^2} = 5{,}300 \text{ lbs.}$$

which checks closely enough with the mean stress assumed.

Assume the efficiency of the screw and collar at 15 per cent. The energy which the operator can supply in one complete revolution of the wheel $= \pi \times 40 \times 60 = 7{,}600$ in. lbs. Hence the energy delivered at the nut will be $7{,}600 \times .15 = 1{,}140$ inch lbs. But during one revolution of the screw the gate must move a distance equal to the pitch s, against a force of 6,000 lbs.

Hence $s \times 6{,}000 = 1{,}140$

$\therefore s = \frac{1{,}140}{6{,}000} = .19''$ or say $\frac{1}{5}'' = .2''$ so that the thread may be easily cut in a lathe.

Since the thread is square the outer diameter of the screw will be $d_1 + s = 1\frac{1}{8}'' + .2 = 1.325$, or in order to use a standard tap the outer diameter may be taken as $1\frac{1}{2}''$. The corrected diameter of the screw at the root of the thread will be $1.5 - .2 = 1.3''$ and the corrected mean diameter will be $1.5 - .1 = 1.4''$. For these values $\tan \alpha = \dfrac{.2}{\pi \times 1.4} = .046$ which corresponds to $\alpha = 2° - 40'$. From curve 5 (Fig. 52) the efficiency of the screw is about 13 per cent, and the original assumption of efficiency is sufficiently close.

The twisting moment applied to the screw $= 60 \times 20 = 1{,}200$ in. lbs. The frictional moment at the collar is approximately $\mu\, W r_m = .15 \times 6{,}000 \times .7 = 630$ in. lbs. Hence the torsional moment at the nut*

$$T = 60 \times 20 - 630 = 570 \text{ in. lbs.}$$

The torsional stress due to this moment is by equation E, page 91,

$$p_s = \frac{16\, T}{\pi\, d_1^{\,3}} = \frac{16 \times 570}{\pi \times (1.3)^3} = 1{,}320 \text{ lbs.}$$

$\therefore$ by equation (I), page (48), the maximum direct stress, $p_m = \frac{1}{2}[p + \sqrt{p^2 + 4\, p_s^{\,2}}] = \frac{1}{2}[9{,}000 + \sqrt{9{,}000^2 + (4 \times 1{,}320)^2}]$ $= 9{,}600$ lbs., which is less than the assigned limit and the design is therefore correct.

The increase in the maximum stress due to the torsional moment is here only about 6 per cent. Where the screw is short, so that a much greater mean direct stress can be carried, or where the screw is only in tension and hence admits of a high tensile stress, this increase may be from 15 to 20 per cent. If the screw is made of cast iron it should also be checked for shearing by equation (2) of Art. 16.

* It is assumed that the collar is at the upper end where the power is applied.

CHAPTER VIII

KEYS, COTTERS, AND FORCE FITS

68. Forms of Keys. Keys are wedge-shaped pieces, generally made of steel, which are used primarily to prevent relative rotation between shafts and the pulleys, gears, etc., which they carry. On account of the frictional resistance which they induce between the surface of the shaft and the member which is keyed to it, they also often prevent relative sliding of the parts. Keys are most usually rectangular in cross-section; but occasionally they are made of circular form. A **saddle key** is shown in Fig. 53. This form of key does not require the shaft to be cut; but its holding power is so small that it is used only for light work. For small loads, or as a safeguard when the hub is shrunk on, a round pin as shown in Fig. 54 (a) is often used. Figs. 54 (b) and 54 (c) show two other methods of applying round taper pins as a substitute for keys. A **flat key** is shown in Fig. 55. This form requires a small portion of the shaft to be cut away, and its holding power is much greater than that of the saddle key. The **sunk key**, Fig. 56, is the most secure form of key fastening, and is more used than any other. It is so called because it is sunk into a keyway or groove cut in the shaft. It thus requires more metal to be cut away from the shaft than the flat key, and this must be taken into account in designing shafting since the metal is removed from the outer fibre where it is most serviceable for resisting applied loads. The keyway cut in a shaft for a sunk key is made parallel to the axis of the shaft; but the keyway in the hub of the pulley or gear which is to be made fast is cut tapering as shown in Fig. 56 (b). The sides of the key are parallel, as shown, and should fit well in both shaft and hub. When the key is driven in, the shaft and hub are drawn tightly together on the side of the shaft opposite to the key, and the frictional resistance thus set up helps to prevent relative sliding of the parts lengthwise of the shaft. If the bore of the hub is tapering, or if the key fits

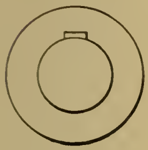


FIG. 53.

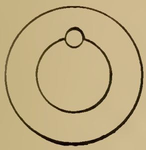


FIG. 54 (a).

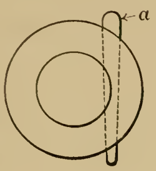


FIG. 54 (b).

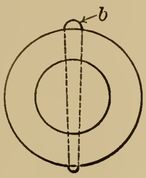


FIG. 54 (c).

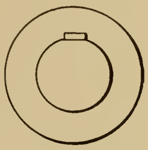


FIG. 55.

more tightly at one end than at the other, the part keyed on may be thrown out of alignment so that its plane is not perpendicular to the axis of the shaft. Where great accuracy is required, as in flanged couplings on shafting, owing to this tendency, the faces of the flange or part secured to the shaft should be faced in place after the key is driven. If the part keyed on does not have to be removed often, the hub may be made a tight or press fit on the shaft, thereby preventing largely the tilting action of the key should such occur. In the **Woodruff system** (Fig. 57), the key

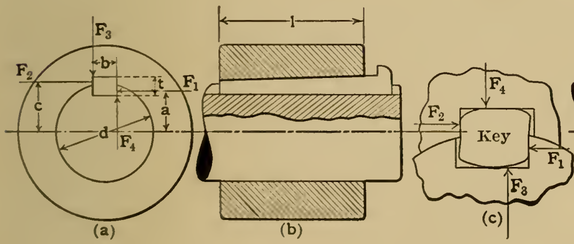


FIG. 56.

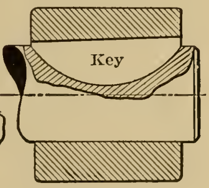


FIG. 57.

is a circular segment and the keyway may be cut with a milling cutter. This allows the key to adjust itself to the taper of the keyway in the hub, hence it will not throw the keyed part out of perpendicular alignment. With this system, the hub must be forced on over the key. These keys are used largely in machine tools.

In general, the part to be secured on the shaft is placed in position and the key driven in. This makes it necessary to extend the keyway along the shaft at least the length of the key, (except when the hub is at the end of the shaft) unless the diameter of the shaft is enlarged under the hub, sufficiently to allow the keyway to be cut without cutting into the shaft proper. Where it is desirable to withdraw the key occasionally, it is often provided with a head, as shown in Fig. 56 (b), in which case it is called a **draw key.*** Sometimes, however, it is not desirable to extend the keyway beyond the hub, in which case the keyway in the shaft is made the same length as the key, and the hub is driven over the key into its correct position. Much more force is necessary to drive the hub into place in this manner than to drive the key, on account of the friction between the shaft and the hub. When the hub is a sliding or an easy fit on the shaft, and only one key is used, there is a tendency to throw the hub eccentric to the shaft. Under these circumstances there is a tendency for the hub to rock and work loose on the shaft, especially if the direction of motion be reversed. In such cases two keys set 90° apart make a much more secure fastening as this gives three lines of contact and prevents rocking. If one of these keys is a saddle key, as shown in Fig. 53, the fitting is greatly facilitated and the fastening is almost as secure as with two sunk keys.

69. Stresses in Sunk Keys. Since keys are designed to prevent relative rotation, it is evident that every key must transmit a certain torsional moment or torque. This torsional moment may be equal to the total torque transmitted by the shaft, or the key may be required to transmit only a part of it. This would indicate that keys of different sizes should be used with any given diameter of shaft, depending on the load which the key must transmit. For practical reasons, however, such as standardiza-

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* Where a draw key cannot be used the point of the key is sometimes case-hardened so that it will not upset so readily in being driven out.

tion and interchangeability, it is desirable that the dimensions of the shaft and key should bear a fixed relation to each other. All practical systems of keys, therefore, give a fixed size of key for each diameter of shaft, the dimensions of the key, presumably, being such that its strength is equal to the torsional strength of the shaft. Shafts are usually designed for torsional stiffness rather than torsional strength, which results in a shaft considerably larger than necessary as far as strength is concerned. If, under these circumstances, the key is designed as indicated above it will also have excess strength. Where the shaft is short and is designed for strength alone, the key should be more carefully considered.

Keys resisting a torsional moment are subjected to simple crushing, or to crushing and shearing, depending on the manner of their application and manner of fitting. The ordinary sunk key (Fig. 56 a), is subjected to a force, F_1, due to the pressure from the shaft, and to a resisting force, F_2, due to the reaction from the hub which it secures. The effect of these two forces is to set up a shearing stress along the middle section of the key at the outer surface of the shaft. They also form a couple which tends to rotate the key in the keyway. This tendency to rotate should be, for best results, resisted by the pressure of the hub and shaft against the top and bottom of the key. If the key is not a tight fit on the top and bottom these resisting pressures, F_3 and F_4, will be concentrated near the corners. This concentrated pressure may be sufficient to crush the key at these points, and allow it to roll in the keyway, deforming both the keyway and key and subjecting the key to a severe crushing action rather than simple shear. If the conditions of service require a continual reversal of motion a state similar to that shown in Fig. 56 (c) is induced, where the resisting forces F_3 and F_4 have been moved inward and their moment arm made so short that their magnitude must be very great to hold the key in position. This may bring a severe bursting stress on the hub. It is evident, therefore, that keys which fit sidewise only, cannot be depended on to carry as great a load as those which fit well on the top and bottom. Where great accuracy is required, as

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in machine tool construction, the hub is often made a force fit on the shaft and the key fitted only on the sides, so that it cannot throw the parts out of relative alignment by radial pressure.

Referring to Fig. 56 (a)

Let l = the length of the key or hub

" t = " thickness of the key

" b = " breadth of the key

" T = " torsional moment applied to the shaft

" P = " force acting at the radius of the shaft so that $P\frac{d}{2} = T$

Then for shearing stress p_s

$$P = p_s l b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and since the torsional moment applied to the shaft must equal the moment of the crushing load applied to the side of the key

$$T = P\frac{d}{2} = F_1 a$$

$$\text{or } P\frac{d}{2} = p_c l \frac{t}{2}\left(\frac{d}{2} - \frac{t}{4}\right) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If F_1 be considered to act at the radius of the shaft (which can be done without serious error for keys as ordinarily proportioned) equation (2) reduces to

$$P = p_c l \frac{t}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Equations (1) and (3) may be used to compute the stresses in any sunk key.

If the shearing resistance of the key is to equal the crushing resistance, then from (1) and (3)

$$p_s l b = p_c l \frac{t}{2}$$

$$\therefore \frac{b}{t} = \frac{p_c}{2\,p_s} \text{ or } t = b\frac{2\,p_s}{p_c} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If $p_c = 2\,p_s$, $t = b$, and the key is square for equal resistance to shearing and crushing. For machinery steel, such as is generally used in keys, $\frac{p_s}{p_c} = .8$, and hence from (4) for equal strength in shearing and compression $t = 1.6b$. If, in addition, the moment of the shearing resistance of the key is to be equal to the torsional resisting moment of the shaft, then

$$T = p_s\, l\, b\, \frac{d}{2} = p'_s\, \frac{\pi d^3}{16} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where p'_s is the shearing stress in the outer fibre of the shaft. For steel shafts and keys, which are most common, $p_s = p'_s$ whence from (5)

$$l\, b\, \frac{d}{2} = \frac{\pi d^3}{16} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The minimum length of hub (l), as determined by practice, which is necessary to give a good grip on the shaft, should not be less than $\frac{3d}{2}$. Substituting this value of l in equation (6)

$$\frac{3\, b\, d^2}{4} = \frac{\pi d^3}{16}$$

$$\therefore b = \frac{\pi}{12} d = \frac{d}{4} \text{ nearly} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The above would, therefore, give keys of breadth $b = \frac{d}{4}$, depth or thickness $t = 1.6\, b = .4\, d$, and minimum length $\frac{3}{2}\, d$. Keys as used in practice conform closely to these rules as far as length and breadth are concerned; but, to avoid cutting away so much of the shaft, the thickness is usually much less than that given above. An average value of the thickness may be taken at $\frac{5}{8}\, b$. This gives a key considerably thinner than it is wide and makes it weakest in crushing. The crushing resistance can, however, be increased by lengthening the key or by using a hard grade of steel.

Keys designed as above usually have an excess of strength, since the friction between the shaft and the hub materially decreases the load actually brought upon the key. In addition, as has been pointed out, shafts are most usually designed for stiffness or angular distortion, and therefore are greater in diameter than would be required for strength alone. If the key is made proportional to the shaft diameter as above, it must, therefore, have excess strength against rupture; and such keys seldom fail unless subjected to severe shock or extraordinary loads.

There are no fixed standards for the dimensions of keys, various machine builders having their own standards.* The following table may be taken as representing average practice when the length of the key is not less than $\frac{3}{2}d$. If the length must be less than this value, the crushing stress should be computed, as it may be necessary to use two keys.

TABLE XI

DIMENSIONS OF FLAT KEYS IN INCHES

Diam. of Shaft d....	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	5	6	7	8	9	10
Breadth of Key b....	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$
Thickness of Key t..	$\frac{5}{32}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{9}{32}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{13}{16}$	$\frac{7}{8}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$

The **taper of sunk keys** is usually about ⅛″ per foot of length.

Another form of sunk key is shown in Fig. 58. This key drives by compression or as a strut. The keyways are more difficult to cut, the keys more difficult to fit, and the shaft is cut deeper than for the common form. It has been used with great success on very heavy work.

70. Feathers or Splines. Sometimes it is desirable to have the hub free to slide axially along the shaft, but constrained to rotate with it. In such cases a **feather** or **spline** is used. The sides of the spline are parallel and it may be either fastened rigidly to the shaft or it may move with the hub. Small splines are frequently dovetailed into the shaft (or hub), as shown in Fig. 59 (a), while larger ones are often held in place by means of countersunk screws (Fig. 59 b), or rivets.

A common way of securing the feather so that it will move

* See Kent's "Mechanical Engineer's Handbook," page 977.

with the hub is shown in Fig. 60. Splines are subjected to a shearing stress across the mid-section at the radius of the shaft, and to a crushing stress on the sides in the same way as sunk keys. Being fitted loosely on the top and bottom, they do not produce any friction between the hub and the shaft and, therefore, offer much less resistance than sunk keys to the rolling action imposed upon them (see Art. 69). This rolling action tends to bring a concentrated crushing force at *a* and *b* (Fig. 59 a), if the

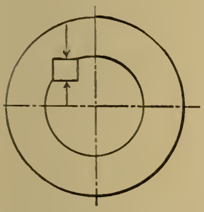

Fig. 58.

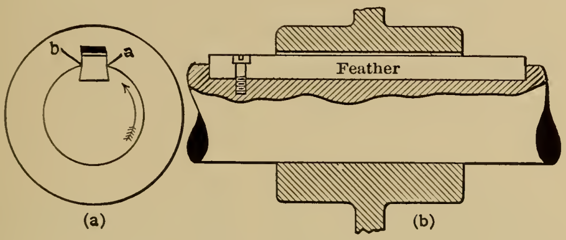

Fig. 59.

feather is not rigidly secured to either the hub or the shaft. For this reason, and in order also to provide ample wearing surfaces, feathers are usually given a greater radial depth than sunk keys, and from their general proportions are often distinguished as square keys. It is evident that the holding power of splines is not equal to that of sunk keys.

The following table gives dimensions of feathers which agree with common practice:

TABLE XII

DIMENSIONS OF FEATHER KEYS IN INCHES

Diam. of Shaft d....	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	5	6	7	8	9	10
Breadth of Feather b.	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$
Thickness of Feather t	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{5}{8}$	$1\frac{3}{4}$	2	$2\frac{1}{2}$	$2\frac{3}{4}$

The length of feather keys is, in general, greater than that of sunk keys, for the same size of shaft, in order to reduce the bearing pressure and increase the wearing surface on the sides.

71. Cotters. A cotter is a form of key used to prevent relative sliding between two members. Fig. 61 shows a method of securing a piston rod to a piston by means of a cotter. In this case the connection is permanent in character, the cotter being removed only when the piston or piston rod is repaired or renewed. In other forms of cottered joints of this character the rod is not tapered, but is prevented from sliding into the boss by means of a shoulder or by the cotter alone. The cotter is usually rectangular in section, but sometimes the edges are rounded so as to avoid sharp corners in the opening cut through the rod or to facilitate machining. In light work a taper pin of circular section is often used as a cotter. Fig. 62 shows an arrange-

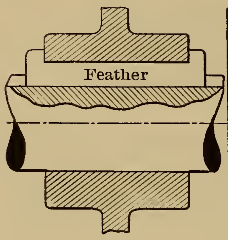

Fig. 60.

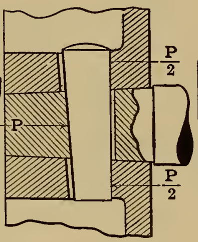

Fig. 61.

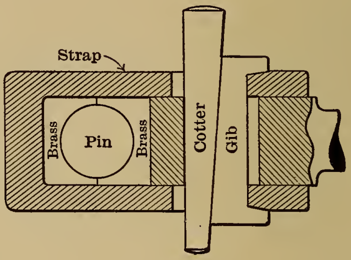

Fig. 62.

ment of a **gib and cotter** (commonly known as a **gib and key**), such as is used on the ends of the connecting-rod of steam engines. The function of the gib is to prevent spreading of the strap. This arrangement permits a small amount of adjustment between the strap and the connecting-rod for taking up wear on the pin and brasses.

72. Stresses in Cotters. A cotter of the form shown in Fig. 61 is a beam supported at the ends. The exact distribution of the loading is indeterminate, as the bending of the cotter tends to concentrate the load near the points of support. It is sufficiently accurate, however, to consider the load as uniformly distributed. The area of the surface of the cotter where it bears on the rod, and also on the hub, should be sufficiently great to prevent crushing of the material. This indicates that the diameter of the hub should, for similar materials, be twice that of the rod, which is the usual proportion. The section of the cotter at the point of support should be great enough to prevent shearing, and in many cases it is sufficient to compute the section for shear alone, neglecting the bending action.

When a cottered joint of this character is made, the cotter must be driven in tight enough to prevent its backing out. This is especially true when the load is a reversed one as in the case of the steam-engine piston. This induces an initial stress in the cotter and rod, over and above that due to the load P. The conditions, in fact, are somewhat similar to those which exist in screwed fastenings (see Art. 60). The initial stress due to the driving of the cotter cannot be accurately computed, though it may be very great. For this reason all calculations of dimensions based on the maximum applied load should be modified to suit the conditions of service and the materials of which the joint is made. Thus if the rod be of brass and the hub or boss of steel, as is common in pump work, the proportions would be different from those employed if all the materials were of steel or of steel and cast iron.

Let d = the diameter of the rod where the cotter passes through.
" t = thickness of cotter.
" b = breadth of cotter.

Then, in order that the net cross-section of the rod may be as strong in tension as the cotter and rod, where they bear upon each other, are in crushing,

$$p_t\left(\frac{\pi d^2}{4} - t\,d\right) = t\,d\,p_c$$

$$\therefore\ t = \frac{\pi d}{4\left(1 + \frac{p_c}{p_t}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For a steel rod and steel cotter where $p_c = .8\,p_t$, $t = .44d$. (2)

Good practice gives $b = 4\,t = 1.76\,d$ . . . . (3)

The taper of cotters, as shown in Fig. 61, should be so small that there is no danger of backing out and should not exceed ½ inch per foot of length. An auxiliary locking device is often used in arrangements such as shown in Fig. 62, in which case the taper may be as great as 1 in 8.

In the form of cotter shown in Fig. 62, the stress due to driving the key may be disregarded, and the design based on the maximum applied load. The student is referred to treatises on steam-engine design for relative proportions of this form.

It is often necessary to allow a rather high bearing pressure on the cotter to avoid large and clumsy proportions. An examination of successful practice shows an allowable pressure of 15,000 pounds per square inch as computed from the applied load.

FORCE AND SHRINKAGE FITS

73. General Considerations. Crank discs, the hubs of heavy fly-wheels, impulse water wheels, and work in general which is to be subjected to shock or vibration, must be fastened to the shaft more securely than can be accomplished with a key, when the hub is a sliding fit on the shaft. In such cases the bore of the hub is made slightly smaller than the diameter of the shaft, and the shaft is forced cold into the hub; or the hub is expanded by heating till the bore is slightly larger than the shaft, then slipped over the shaft and allowed to cool in place. The first method is known as a **force or pressure fit,** and the second as a **shrinkage fit.** The degree of tightness or "grip" required between shaft and hub depends largely on the service. Thus, with shafts up to three or four inches in diameter, a difference between the diameter of the shaft and the bore such that the parts may be driven together with a hand sledge is often satisfactory. Such a fit is called a **driving fit,** and the difference between the shaft diameter and the bore is very small. With such work as armature spiders and fly-wheel hubs, the allowance for the press fit depends largely on the facilities for erection. If the parts can be forced together in the shop, where adequate means, in the form of a powerful hydraulic press is to be had, an allowance requiring a pressure of one hundred tons or more may be made. But if the parts must be erected in the field, this allowance may have to be reduced on account of the difficulties of erection. It is usually possible in the case of armature spiders, fly-wheel hubs, etc., to obtain a sufficiently tight grip on the shaft by means of a press fit without inducing undue stress in the parts. Dependence for preventing relative rotation may be, in a large measure, placed upon the key in all such cases.

In such work as crank shafts when built up from separate parts, it is often necessary to insure as strong a grip upon the shaft as is possible without inducing undue stress. A greater difference between the shaft diameter and the bore of the hub is then allowed than in forced fits and the parts are usually put together by shrinking. In the latter cases the stresses induced are of importance and should be carefully considered.

74. Stresses Due to Force Fits. If x be the elongation or contraction of any radius r, then $2\pi x$ is the corresponding elongation or contraction of the circumference $2\pi r$. The elongation or contraction of the circumference per unit of length is $\frac{2\pi x}{2\pi r}$. If p be the stress which would induce this change of length of circumference, and E be the coefficient of elasticity of the material, then

$$E = \frac{p}{\frac{2\pi x}{2\pi r}} \quad \text{or} \quad x = \frac{pr}{E} \quad \ldots \ldots \quad (1)$$

In Fig. 63, let A represent a hollow shaft on which has been forced or shrunk a hub or boss B, the radius of the contact surface being r_2. Before the operation of pressing, the outer radius of the shaft was r_2+e_2, and the inner radius of the hub was $r_2-e'_2$. The hub B is, therefore, in the condition of a thick cylinder subjected to an internal pressure, and the shaft A is in the condition of a thick cylinder subjected to an external pressure. The greatest tensile stress will be found at the inside surface of the hub, and the greatest compressive stress at the inside surface of the shaft. If, therefore, e be the difference between the outer radius of the shaft and the inner radius of the hub, before pressing, then $e=e_2+e'_2$.

Let p_t be the unit tensile stress in the hub at a radius r_2.
Let p_c be the unit compressive stress in the shaft at a radius r_2.
Let w_2 be the unit radial pressure between A and B.
Let r_1 be the internal radius of the shaft.
Let r_3 be the external radius of the hub.

Then from (1)

$$e = \frac{p_t}{E} r_2 + \frac{p_c}{E} r_2 \quad \text{or} \quad p_t + p_c = \frac{Ee}{r_2} \quad . \quad . \quad . \quad (2)$$

The general equation for the stress in thick cylinders of this kind is,—

$$* \; p = \frac{2 r_1^2 w_1 - 2 r_2^2 w_2 + \dfrac{4 r_1^2 r_2^2}{r^2}(w_1 - w_2)}{3 (r_2^2 - r_1^2)} \quad . \quad . \quad . \quad (3)$$

Where r_1 is the inner radius of the cylinder, r_2 the outer radius, w_1 the internal unit pressure, w_2 the external unit pressure and p the tensile or compressive stress at any radius r. Applying this equation to the shaft, $w_1 = 0$, $r = r_2$, whence the compressive stress at the surface of the shaft is

$$p_c = \frac{w_2 (2 r_2^2 + 4 r_1^2)}{3 (r_2^2 - r_1^2)} = a w_2 \quad . \quad . \quad . \quad . \quad (4)$$

In a similar way substituting in the general equation r_1 for r_2, r_2 for r_3, w_1 for w_2 and w_2 for w_3, the unit tensile stress on the inner surface of the hub is

$$p_t = \frac{w_2 (2 r_2^2 + 3 r_3^2)}{3 (r_3^2 - r_2^2)} = \beta w_2 \quad . \quad . \quad . \quad . \quad (5)$$

Dividing (4) by (5)

$$\frac{p_c}{p_t} = \frac{a}{\beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

From (2) and (6)

$$p_t = \frac{E e \beta}{r_2 (a + \beta)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\text{and } p_c = \frac{E e a}{r_2 (a + \beta)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

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* The following treatment is from Professor Merriman's "Mechanics of Materials," 1906 edition, page 396. The notation has been changed to agree with that adopted in this work.

When the shaft is solid, r_1 in the above equation becomes zero and the equations are much simplified.

Example. A hollow steel shaft 10 inches outside diameter and 2 inches inside diameter is to have a steel crank shrunk upon its end. The hub of the crank is 18 inches in diameter. What must be the difference between the diameter of the shaft and the bore of the crank so that the tensile stress at the inner surface of the hub shall not exceed 20,000 lbs. per square inch? What will be the corresponding compressive stresses at the outer and inner surfaces of the shaft? Take $E = 30{,}000{,}000$.

Here $r_1 = 1, \quad r_2 = 5, \quad r_3 = 9 \quad$ and $\quad p_t = 20{,}000$

$$\text{Whence } \alpha = \frac{2r_2^2 + 4r_1^2}{3(r_2^2 - r_1^2)} = \frac{(2 \times 5^2) + (4 \times 1^2)}{3(5^2 - 1^2)} = \frac{3}{4}$$

$$\text{and } \beta = \frac{2r_2^2 + 4r_3^2}{3(r_3^2 - r_2^2)} = \frac{(2 \times 5^2) + (4 \times 9^2)}{3(9^2 - 5^2)} = 2.23$$

Then from (7).

$$e = \frac{p_t r_2 (\alpha + \beta)}{E\beta} = \frac{20{,}000 \times 5 \left(\frac{3}{4} + 2.23\right)}{30{,}000{,}000 \times 2.23} = .0044$$

From (8)

$$p = \frac{Ee\alpha}{r_2(\alpha + \beta)} = \frac{30{,}000{,}000 \times .0044 \times .75}{5 \times 2.98} = 6{,}700$$

From (4)

$$w_2 = \frac{p_c}{a} = \frac{6{,}700}{\frac{3}{4}} = 8{,}900 \text{ lbs.}$$

and substituting this value in (3), making $r = r_1$ and $w_1 = 0$, it is found that the compressive stress at the inner surface of the shaft is 18,500 lbs. per square inch.

It is evident that if e be assumed, which is usually the case, the resulting pressure and stresses can be computed. It should be noted that p_t must be well within the elastic limit to prevent the hub yielding and relieving the pressure. It appears, as pointed out by Professor Merriman, that the allowances made in practice for force fits, induce stresses which should be considered if other stresses are to act on the members. Thus, in the example given, the total allowance or difference between the *diameter* of the shaft and the bore of the hub would be $2 \times .0044 = .0088$; and the allowance per inch of diameter would be $\frac{.0088}{10} = .00088''$, which is close to average practice for *force* fits, where $.001''$ per inch of diameter is often allowed. A somewhat greater allowance

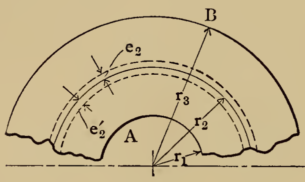

FIG. 63.

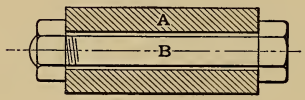

FIG. 64.

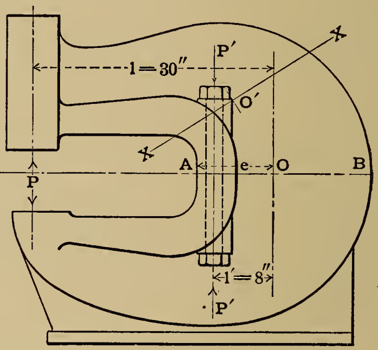

FIG. 65.

is generally made for shrinkage fits, as here the difficulty of forcing on the hub does not occur.

75. Practical Considerations in Force and Shrink Fits. The foregoing equations, while giving the probable stresses and radial pressure resulting from a force or shrink fit made with an allowance e, are limited in their application to the practical making of *force* fits. There is, generally speaking, no difficulty in making shrink fits, with any practical allowance, as far as getting the parts together is concerned; although greater skill is required in handling shrink fits than force fits. In making force fits, however, the amount of pressure that can be applied to the parts is often a controlling factor. The probable radial pressure between the shaft and hub (w_2) may be found as above, but little is known of the coefficient of friction in such work, and it is evident that this quantity will vary greatly with the character of the material, the finish of the surface, and the lubricant applied. Experimental data are lacking on this point, hence it is almost impossible to estimate the resistance to slipping offered by force or shrink fits. In general, shrink fits are superior to force fits since their surfaces are very dry and unlubricated, while those of a force fit are lubricated. Total dependence is, therefore, seldom placed on the fit itself, but a key is also used for safety.

Experience shows that the pressure required to make a force fit will vary for any given diameter.

(a) Directly as the length of the hub
(b) Directly as the allowance e
(c) As some function of the radial thickness of hub
(d) With the character of the materials and the finish of the surfaces.

It is evident that a mathematical expression accurately expressing these relations would not be practicable, and recourse must be had to successful practice.

An allowance of .001″ per inch of diameter will represent average practice in this country for such work as crank shafts, crank pins, and in general where a tight fit is required. For armature spiders, or fly-wheels, one-half this allowance is often sufficient. For shrink fits a greater allowance is often made, although the foregoing discussion indicates that this should not be much exceeded considering the stresses induced.

For further information and practical data the student is referred to the following:

Transactions of the American Society of Mechanical Engineers, Vol. XXIV.

"Machine Design" by Forrest R. Jones.

"Machine Design" by W. L. Cathcart.

Machinery, Vol. III, No. 9, May, 1897.

76. Thin Bands or Hoops. If the ring or band which is forced or shrunk on to a member be thin, radially, compared to its diameter, the assumption can be made, without appreciable error, that the stress is uniform throughout the cross-section of the ring. The change of form in the member on which the band is placed due to compression is so small in such cases that it may be neglected, and the stress in the band may be taken as that due to stretching it over an incompressible body. This is practically applicable to any ordinary shape of band, but rigidly true for circular shapes only. Thin bands of this character are usually shrunk into position.

Example. A thin steel band is to be shrunk on to a casting whose external linear dimension where the band is to be placed is 48 inches. What must be the length of the inside face of the band so that the stress per unit area due to shrinking will be 30,000 lbs.? What will be the area of the cross-section of the band in order that the total stress in the band may be 60,000 lbs.?

Let l = the length of band before shrinking.

Then $48 - l$ = total amount of elongation of band.

and $\frac{48 - l}{l}$ = unit elongation of band.

Whence, if E, the coefficient of elasticity, be taken as 30,000,000,

then, $E = 30{,}000{,}000 = \frac{\text{unit stress}}{\text{unit strain}} = \frac{30{,}000}{\frac{48 - l}{l}}$, or $l = 47.95$ ins.

The total area of the cross-section of the band will be $A = \frac{60{,}000}{30{,}000}$ = 2 square inches which may be distributed in any convenient proportions.

If the part on which the band is to be shrunk is circular in form, the band is in the condition of a thin cylinder subjected to an internal pressure w per unit area, where w is the radial pressure between the band and the part on which it is shrunk.

Therefore by Art. 78, $wd = 2P$, where P is the *total* stress per unit width of the band or $w = \frac{2P}{d}$. Thus in the above problem let the band be shrunk upon a circular hub of diameter $\frac{48}{\pi}$, and let the cross-section of the band be ½″ x 4″. Then $P = \frac{60{,}000}{4} = 15{,}000$, and $w = \frac{2P}{d} = \frac{2 \times 15{,}000}{\frac{48}{\pi}} = 1{,}962$ lbs. per square inch.

The steel tires of locomotive driving wheels are usually shrunk on with an allowance for shrinkage of .001″ per inch of diameter which gives .001 inches elongation ( ) per inch of circumference.

Taking $E = 30,000,000$, and considering the tire a thin band, the unit stress in the tire is

$$p = E \triangle = 30,000,000 \times .001 = 30,000 \text{ lbs.}$$

77. Other Forms of Shrink Fits. Many machine parts such as fly-wheel rims are held together by steel links or bands shrunk into place. The theory outlined in the preceding article is clearly applicable to these members, and their dimensions should be carefully calculated so that they will not be overstrained by the shrinking alone. If such members are so designed that they will be stressed up to the elastic limit from shrinkage alone, they are liable to be strained beyond the elastic limit, when an external load greater than the total shrinkage stress is applied to the parts which they hold together, and the link, taking a permanent set, becomes ineffective. In computing the dimensions of such links allowance must sometimes be made for the compression of the parts held together, but ordinarily this is small and may be neglected.

Occasionally a bolt or link is used to **reinforce** a cast-iron member against tensile stress. Thus in open frames, Fig. 65, a large bolt is sometimes placed on each side of the throat as shown. These bolts are usually put in hot and allowed to cool in place. As ordinarily applied, the benefit derived from them is questionable. If they are designed and fitted so as to put the frame in compression at A, an amount equal to the tension induced by the working load P at this same point, without being themselves strained beyond the elastic limit when the load is applied, then no stress can come upon the frame itself from the force P. If, however, the bolts and frame are each to carry part of the load, care should be exercised that the stress induced in the bolts by the *initial* load due to shrinking is so low that the additional stress due to the external load does not raise this initial stress beyond the elastic limit, thus giving the bolts a permanent set and destroying their usefulness.

Let A, Fig. 64, represent a cast-iron member of uniform cross-section which is to be reinforced against tensile stress by the bolt B. Suppose, first, that the nut is screwed up till it just bears firmly on the casting. If now an external tensile load is applied to the casting, the bolt and casting will be elongated the same amount Δ. But the coefficient of elasticity of cast iron is only about one-half that of steel. Hence, since $p=E\lambda$, the stress per *unit area* in the casting will only be one-half that in the steel. If 2,000 lbs. is the allowable unit stress in the casting, 4,000 lbs. per unit area is all that can be thus obtained in the bolt. This would lead to unnecessarily large bolts.

Suppose, however, that the nut is set up till a total compressive load W is applied to the cast iron. The bolt will be elongated* and the casting compressed, the amount of elongation or compression depending on the cross-section of the respective members. The unit stress induced in the bolt and casting will also be proportional to the area of their respective cross-sections. If now an external tensile load W' is applied to the *bolt*, the tendency is to relieve the compressive stress in the casting and to increase the tensile stress in the bolt. When the load applied is sufficient to elongate the bolt as much as the casting was originally compressed, the casting will be relieved of all stress. If the external load W' is applied to the bolt through the casting itself, it is evident that practically the same result is obtained; and after the compressive stress in the casting is fully relieved any further addition to W' induces a tensile stress in the casting and still further increases the tension in the bolt. Usually the cross-sectional area of the casting is very much greater than that of the bolt. Furthermore the compressive stress induced in the casting by the initial load on the bolt is usually very small compared to the tensile stress induced by the working load. For these reasons the compressive deformation in the casting can usually be neglected without appreciable error; and the bolt may be designed on the basis of the external load alone. (See Art. 60, Case *a*.)

Example. In Fig. 65 let the section AB be stressed by the load P whose arm is l. Let O be the location of the gravity axis of the section AB. It is desired to keep the stress at A not

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* See Art. 59.

greater than 3,000 lbs. per square inch. The material is to be cast iron. Let $P = 60{,}000$.

" I = moment of inertia of section = 4,500.

" $e = 10$ inches.

Also let the area of the section be 200 square inches. Then from (M), page 91, the *tensile* stress at A due to P is $p = \left(\frac{P}{A} + \frac{P l e}{I}\right)$ $= \left(\frac{60{,}000}{200} + \frac{60{,}000 \times 30 \times 10}{4{,}500}\right) = 4{,}300$ lbs., and it is desired to reduce this to 3,000 lbs. by reinforcing bolts. These reinforcing bolts serve the double purpose of increasing the factor of safety by reducing the fibre stress, and also of decreasing the deflection of the frame at the point where the work is done. Let these bolts be located 8″ from O. Then the *compressive* stress induced at A by P' is

$$p' = \left(\frac{P'}{A} + \frac{P' l' e}{I}\right) = \left(\frac{P'}{200} + \frac{P' \times 8 \times 10}{4{,}500}\right)$$

But $p - p'$ must equal 3,000; therefore

$$4{,}300 - \left(\frac{P'}{200} + \frac{P' \times 8}{450}\right) = 3{,}000.$$

Whence $P' = 57{,}000$. This is the total tensile load on both bolts, when the full working load P is applied. If the maximum stress at the root of the thread be taken at 15,000 lbs., then the area of each bolt at the root of the thread is $\frac{57{,}000}{2 \times 15{,}000} = 1.9$ sq. in., which corresponds closely to a 1¾″ bolt. The area of the body of a 1¾″ bolt, where most of the stretching takes place, is 2.4 square inches. Hence the working stress in the body of the bolt is $\frac{57{,}000}{2 \times 2.4} = 11{,}880$ lbs. per sq. in. That portion of the boss which immediately adjoins the throat is subjected to an average tensile stress nearly equal to the fibre stress at the surface of the throat or 3,000 lbs. per square inch. The upper and lower portions of the boss have little or no tensile stress induced in them, as a consideration of a section such as XX whose gravity axis is at O', will show. It will be reasonable to estimate that the stress in the boss is equivalent to the full stress of 3,000 lbs. per square inch through 14 inches of its length, the total length being 19″. Neglecting the compressive deformation of the boss due to the initial load from the bolts, the stress induced in the bolt when the stress in the boss is 3,000 lbs., will be $3,000 \times 2 \times 14/19 = 4,500$ lbs. (remembering that the coefficient of elasticity of steel is twice that of cast iron). Whence the initial stress in the bolt will be $11,880 - 4,500 = 7,380$. The allowance for shrinkage necessary to give this initial stress will be

$$\triangle = \frac{lp}{E} = \frac{19 \times 7,380}{30,000,000} = .0046''$$

The number of threads per inch on a $1\frac{3}{4}''$ bolt is 5. Hence after the nut has been set up snugly it should be given $\dfrac{.0046}{\frac{1}{5}} = .023$ of a turn, or should be turned through $360 \times .023 = 8.2$ degrees. This is most easily done in the case of large bolts by first marking the nut with reference to the bolt when set up snug in a cold state, and then heating the body of the bolt, if necessary, and rotating the nut the desired amount, allowing it to cool in position.

It is to be especially noted that a very small shrinkage allowance is needed to induce a great stress in the bolt. If too great an allowance is made, the bolts may be stressed beyond the elastic limit, and take permanent set the first time the external load is applied. When the external load is again applied, a force much smaller than the total load P will strain the casting to the point where the bolt becomes effective. The total load P will strain the casting further than it did originally, and even if the stresses induced are not sufficient to rupture the casting, the stiffness of the frame is materially decreased.

CHAPTER IX

TUBES, PIPES, CYLINDERS, FLUES, AND THIN PLATES

78. Resistance of Thin Cylinders to Internal Pressure. If a hollow circular cylinder, whose walls are very thin compared to its diameter, is subjected to an internal bursting pressure, a tensile stress is induced in the walls. This tensile stress is reduced near the ends by the action of the ends themselves which tend to hold the walls together. Let Fig. 66 represent one-half of a portion of a thin cylinder so far removed from the ends that their effect may be neglected.

Let w = the unit internal pressure
" d = the diameter of the cylinder
" r = the radius of the cylinder
" t = the thickness of the cylinder walls
" p = the unit tensile stress in the longitudinal section
" p_t = the unit tensile stress in the transverse section
" l = the length of the part considered

Consider the half of the cylinder as a free body, and resolve all forces perpendicular to the cutting plane. The normal pressure on a longitudinal strip of length l and width $r d\theta$ is $w\, l\, r\, d\theta$. The component of this force perpendicular to the cutting plane is $w\, l\, r\, d\theta \sin \theta$. The total pressure normal to this plane is

$$\int_0^\pi w\, l\, r\, d\theta \sin \theta = w\, l\, r \int_0^\pi \sin \theta\, d\theta = 2\, w\, l\, r = w\, l\, d.$$

For equilibrium this normal force must equal the resisting stress in the two sides of the cylinder. Hence

$$2\, p\, t\, l = w\, l\, d$$

$$\text{or } p = \frac{w\, d}{2\, t} \quad (1); \quad w = \frac{2\, p\, t}{d} \quad (2); \quad \text{or } t = \frac{w\, d}{2\, p} \quad (3)$$

In other words, the unit longitudinal stress in the walls of a thin

cylinder is equal to the product of the diameter into the unit internal pressure, divided by twice the thickness of the cylinder walls, and is independent of the length of the cylinder.

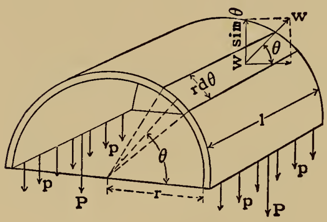

FIG. 66.

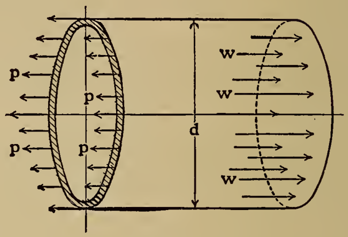

FIG. 67.

If a transverse section of the cylinder (Fig. 67) be considered, it will be seen that the total pressure on the head, which tends to cause rupture along a transverse section, is $\frac{\pi d^2 w}{4}$, and this must be equal to the intensity of the transverse stress produced multiplied by the area of the metal in such a section, or,

$$\frac{\pi d^2 w}{4} = \pi d t p_t$$

$$\therefore {}^* p_t = \frac{w d}{4t} \quad . \; . \; (4) \qquad w = \frac{4 p_t t}{d} \quad . \; . \; (5) \qquad \text{or } t = \frac{w d}{4 p_t} \quad . \; . \; (6)$$

A comparison of (1) and (4) shows the stress in transverse sections to be only one-half of that in longitudinal sections. For this reason it is very common practice to make the circumferential seams of a boiler shell with a single riveted joint, when the longitudinal seams are double or triple riveted.

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* In this discussion the mutual interaction of the longitudinal and transverse stresses is neglected. If a tensile stress p_t is induced in a body, the body contracts laterally as if acted upon by a stress λp_t acting at right angles to the line of action of p_t where λ is Poisson's ratio. (See Merriman's "Mechanics of Materials," 1906 edition, page 359.) Therefore the true longitudinal stress p_l in the above case (since λ equals $\frac{1}{3}$ for steel) is

$$p_l = p - \frac{1}{3} p_t = \frac{w d}{2t} - \frac{1}{3} \frac{w d}{4t} = .85 \frac{w d}{2t}.$$

This gives a lower value than equation (1) and hence the latter is on the side of safety.

79. Thin Spheres. Since all the meridian sections of a sphere are the same as the transverse section of a cylinder of equal diameter, it is evident that the stress in the walls of a sphere is given by (4). If spherical heads, of the same thickness as the shell, are placed on a cylinder which is to withstand internal pressure, they will be subjected to a maximum stress equal to the transverse stress in the shell.

80. Resistance of Non-Circular Thin Cylinders to Internal Pressure. Suppose a cylinder to have a cross-section made up of circular arcs as in Fig. 68. Take the upper half as a free body (section along the major axis). Let the resultants of the components of pressure which are normal to the plane of the section be W_1, W_2, W_3, for the portion marked I, II, III, respectively. Then these resultant forces per unit of length of the cylinder are as follows:—

$$W_1 = wr\int_0^{\varphi'} \sin\varphi\, d\varphi = wr\,(-\cos 0 + \cos\varphi') = w\,m_1$$

$$W_2 = wR\int_{\theta''}^{\theta'} \sin\theta\, d\theta = wR\,(-\cos\theta' + \cos\theta'') = w\,m_2$$

$$W_3 = wr\int_{\varphi''}^{\pi} \sin\varphi\, d\varphi = wr\,(-\cos\pi + \cos\varphi'') = w\,m_3$$

Therefore $W_1 + W_2 + W_3 = w\,(m_1 + m_2 + m_3) = wA$

In a similar way, if the section is taken along the minor axis, the resultant force normal to this axis is found to be wB. In like manner the resultant force normal to any section is (per unit of length of cylinder) equal to the intensity of pressure multiplied by the axis of that section. As B is less than A, the resultant force wB is less than wA; or the force tending to elongate the minor axis is greater than the force tending to elongate the major axis. If the tube were perfectly flexible, its form of cross-section would become, under pressure, one in which all axes are equal, or circular. A rigid material offers resistance to such change of form, and a flexural stress is produced in addition to the direct tension, but it approaches nearer to the circular form as the pressure increases. The existence of this flexural stress in a non-circular cylinder becomes apparent from a comparison of Figs. 69

and 70. In Fig. 69 (circular section) the lines of normal pressure all pass through a single point (the centre of the circle); the resultant (P_r) of the tensions (P_1 and P_2) also passes through this same point, hence these forces form a concurrent system, and they are in equilibrium. In Fig. 70, however, the pressures do not in themselves form a concurrent, nor parallel, system of forces, hence they cannot be balanced by a single force (as the resultant P_r), but there must be a moment, or moments, of stress for equilibrium. A similar course of reasoning could be applied to a cylinder of any non-circular cross-section; for such a section (Fig. 71) could be considered as made up of circular arcs, each of which could be treated (like the special case of Fig. 68) by inte-

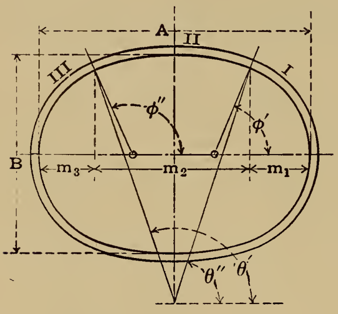

FIG. 68.

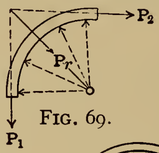

FIG. 69.

FIG. 70.

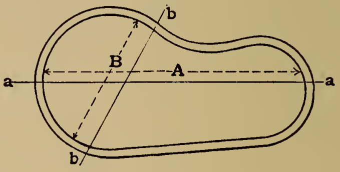

FIG. 71.

grating between proper limits. A direct inspection will also show that in any non-circular section cylinder, subjected to internal pressure, the pressure tends to reduce the cylinder to a circular cross-section. Suppose the original cylinder (Fig. 71) to be cut along the greatest axis of its cross-section, and that a flat bottom coinciding with this section-plane be secured to it, the lower portion of the cylinder being entirely removed. The total pressure on this bottom evidently balances the components of the pressure on the curved surface which lie normally to this flat bottom; hence, the resultant of these normal components of pressure equals w (a . . a) $= w\,A$, per unit of length of cylinder. In a similar way, the resultant of components of pressure acting normally to any other section (as b . . b, Fig. 71) equals

$w\ (b \quad . \quad . \quad b) = w\,B < w\,A$. This direct method might have been used in the preceding cases (Figs. 66 and 68) without recourse to the calculus.

It is apparent, then, that any cylinder under internal pressure tends to assume a circular cross-section. A cylinder of nominal circular section, but departing from the true form to some extent, tends to correct this departure under internal pressure; or if a circular cylinder under internal pressure is deformed by any external force, it tends to resume its circular shape. Thus a circular cylinder under internal pressure is in "stable equilibrium." If the section is other than a true circle there is a flexural stress, as well as tension, when under pressure.

RESISTANCE OF THIN CYLINDERS TO EXTERNAL PRESSURE

81. Theoretical Considerations. If a thin hollow cylinder of circular section is subjected to an external pressure, it is obvious that a course of reasoning similar to that in Art. 78 will show that a compressive stress is induced in the walls of the cylinder, the value of which will be given by formula (1) Art. 78 or

$$p = \frac{w\,d}{2\,t} \text{ where } p \text{ is a compressive stress.}$$

If the cylinder were perfectly cylindrical, of uniform thickness, and of homogeneous material, there seems to be no reason why failure should occur until the compressive stress reaches the yield point of the material. But tubes are never absolutely circular in form, uniform in thickness, or homogeneous in character; and hence failure occurs long before the compressive yield point is reached. A tube which fails under external pressure is said to **collapse,** and the forms of collapsed tubes are very characteristic. Fig. 72 shows the form of cross-section of collapsed tubes, and Unwin* has shown that the number of lobes depends on the ratio of length to diameter, the smaller this ratio the greater being the number of lobes. This peculiarity is undoubtedly due to the influence of the heads placed in the ends. For values of $\frac{l}{d}$

* See "Elements of Machine Design," page 101, 1901 edition.

greater than about 4 to 6, only the forms of collapse shown at *c* and *d*, Fig. 72, appear.

If the non-circular cylinders of either Fig. 68 or 71 be considered as subjected to external pressure, the force tending to increase the major axis will be seen to be greater than that tending to increase the minor axis; hence the external pressure will cause collapse, unless the flexural rigidity of the material is sufficient to prevent this action. In a cylinder of nominal circular section any departure from the ideal section will be increased by the external pressure. Or, if a cylinder of true circular section is deformed in any way while under external pressure, this pressure will tend still further to increase the deformation.

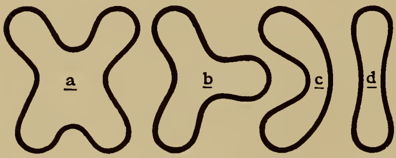

FIG. 72.

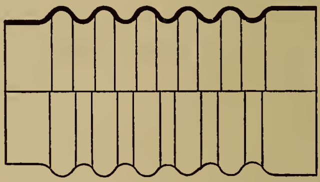

FIG. 73.

In other words, a cylinder under external pressure is in "unstable equilibrium." As perfectly true circular sections and homogeneous materials are not attainable in practice the danger of collapse must be taken into consideration in designing pipes, tubes, or flues to withstand external fluid pressure.

Since the wall of an ideal thin tube is subjected to a uniform compressive stress, it may be considered as being in the same condition as a long column; and theoretical equations expressing the relation between the external pressure, the stress, and the dimensions of the tube have been developed on this basis. In view of the fact that the theory of long columns is itself most unsatisfactory, it is not surprising that such equations do not accord with actual results, and they may be safely disregarded, but the analogy between long compression members and tubes subjected to external pressure is instructive. Other deductions based upon the theory of elasticity, while throwing some light on the form of rational equations expressing these relations, are not as yet applicable to practical problems.

82. Long Tubes, Pipes, etc. Until very recently the only experimental results on the collapse of thin tubes were those due to Sir William Fairbairn, who, in 1858, made a series of careful experiments on **short tubes** and deduced therefrom the following formula:

$$w = 9{,}675{,}600 \frac{t^{2.19}}{l\,d} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where w is the unit external collapsing pressure in pounds per square inch and t, l, and d are the thickness, length, and outside diameter respectively in inches. Fairbairn himself modified this equation, for simplicity, to the form

$$w = 9{,}675{,}600 \frac{t^2}{l\,d} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Many other equations have been deduced from the experiments of Fairbairn, usually of the same form but with different exponents. Thus Professor Unwin gives the following as the result of a careful résumé of Fairbairn's work:

For tubes with a **longitudinal lap-joint**

$$w = 7{,}363{,}000 \frac{t^{2.21}}{l^{.9}\,d^{1.16}} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For tubes with a **longitudinal butt-joint**

$$w = 9{,}614{,}000 \frac{t^{2.21}}{l^{.9}\,d^{1.16}} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

For tubes with **longitudinal and cross joints** like an ordinary boiler flue

$$w = 15{,}547{,}000 \frac{t^{2.35}}{l^{.9}\,d^{1.16}} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Other writers have deduced similar equations from the same data.

Fairbairn's experiments were conducted with tubes whose lengths were small compared to their diameters. In such tubes the effect of the supporting action of the head is noticeable; hence his equations make the allowable pressure vary inversely as some function of the length. Now it is reasonable to suppose that if the tube were long enough the head would have no effect, except near the ends, and the collapsing pressure would be independent of the length. In a similar way if the tube were very short, the walls should theoretically yield by crushing, and the intensity of the compressive stress would be given by formula (1) or, $p = \frac{w\,d}{2\,t}$. In 1906 Professor A. P. Carman published* the results of a set of experiments made at the Engineering Experiment Station of the University of Illinois, which prove conclusively that Fairbairn's equations hold only for tubes whose lengths are from four to six times their diameters; and that beyond that ratio the collapsing pressure is independent of the length. He found that the results of his experiments could not be well expressed by a single equation, but devised two equations to cover the range; these equations expressing the relation which exists between w and $\frac{t}{d}$. Thus for values of $\frac{t}{d} > .025$, and length greater than 4 to 6 times the diameter, he gives $w = k\frac{t}{d} - c$ where k and c are constants to be determined experimentally and depending upon the material.

For brass tubes

$$w = 93{,}365\,\frac{t}{d} - 2{,}474 \quad . \quad . \quad . \quad . \quad . \quad (6)$$

For seamless drawn cold steel

$$w = 95{,}520\,\frac{t}{d} - 2{,}090 \quad . \quad . \quad . \quad . \quad . \quad (7)$$

For lap-welded steel

$$w = 83{,}270\,\frac{t}{d} - 1{,}025 \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Professor R. T. Stewart,† in an elaborate set of experiments

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* See Bulletin of the University of Illinois Engineering Experiment Station Vol. III, No. 17, June, 1906.

† See Transactions of American Society of Mechanical Engineers, Vol. XXVII, 1906.

on lap-welded steel boiler tubes made for the National Tube Company, found that for values of $\frac{t}{d} > .023$ the results of his work could be expressed by the following:

$$w = 86{,}670\frac{t}{d} - 1{,}386 \quad . \quad . \quad . \quad . \quad . \quad (9)$$

which corresponds closely with (8) of Professor Carman's work, showing the accuracy of the experimental work.

For values of $\frac{t}{d} < .025$ Professor Carman found that the results of his work could be expressed by an equation of the form

$$w = k'\left(\frac{t}{d}\right)^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where k' as before is an experimentally determined constant, whose value for thin brass tubes is 25,150,000, and for thin cold-drawn seamless steel tubes 50,200,000.

Professor Stewart found that for values of $\frac{t}{d}$ below .023, or practically the same limit as above, his results were expressed by

$$w = 1{,}000\left(1 - \sqrt{1 - 1{,}600\frac{t^2}{d^2}}\right) \quad . \quad . \quad . \quad (11)$$

The value of w for $\frac{t}{d} = .023$ is about 600 lbs., which corresponds closely with the upper limiting value of w obtained from (10).

For values of $\frac{t}{d}$ less than .023, the corresponding values of w, as found by either (10) or (11), do not differ materially. Furthermore, tubes in which $\frac{t}{d} < .02$ are not much used in engineering work under external pressure, and for convenience therefore equation (10) will be adopted.

83. Summary of Equations for Long Tubes. The works of Stewart and Carman deal entirely with tubes which are so long that the supporting effect of the heads is negligible, or in which the length is at least four times the diameter. Their experiments, while conducted separately, supplement and corroborate each other. As given above, the equations are not in the most convenient form for use by the designer, since usually l, d and w are known and t is required. Transposing these equations, therefore, they may be written as follows:

For values of $\frac{t}{d} < .023$ and pressures less than 600 equation 10 becomes

$$t = d\sqrt[3]{\frac{w}{k}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where $k = 25{,}150{,}000$ for thin brass tubes, and 50,200,000 for thin cold-drawn seamless tubes or lap-welded steel tubes.

For values of $\frac{t}{d} > .023$ and pressures greater than 600 lbs., equation 6 becomes

$$t = \frac{d(w + c)}{k} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where for brass tubes	$k = 93{,}365$ and $c = 2{,}474$
" " seamless cold-drawn steel . . .	$k = 95{,}520$ and $c = 2{,}090$
" " lap-welded steel . .	$k = 83{,}270$ and $c = 1{,}025$

The following approximate formula, which covers practically the whole range of values of $\frac{t}{d}$, is suggested by Professor Carman as useful in making rough calculations.

$$w = k''\left(\frac{t^2}{d}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (13a)$$

where $k'' = 1{,}000{,}000$ for cold-drawn seamless tubes and 1,250,000 for lap-welded steel tubes. From this, the following usually more convenient formula can be derived:

$$t = \sqrt{\frac{wd}{k''}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13b)$$

Example. A lap-welded steel boiler tube 4 inches outside diameter and 10 feet long, is subjected to an external pressure of 300 pounds per square inch. What must the thickness be in order to have a factor of safety of at least 6?

Here the assumed collapsing pressure is

$$300 \times 6 = 1{,}800 \text{ lbs. per square inch.}$$

Applying equation (13)

$$t = \frac{d\,(w + c)}{k} = \frac{4\,(1{,}800 + 1{,}025)}{83{,}270} = .14 \text{ inch.}$$

Here the ratio $\frac{t}{d} = \frac{.14}{4} = .035$, and hence equation (13) applies. In case this ratio should be less than .023, which will seldom occur, a second solution, using equation 12, should be made.

84. Short Cylinders, Flues, etc. When the cylinder or flue is short, *i.e.*, $\frac{l}{d} < 4$ to 6, the effect of the heads should not be neglected as in Carman's and Stewart's work, and Fairbairn's approximate equation is applicable, or

$$w = 9{,}675{,}600 \frac{t^2}{l\,d} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

or transposing

$$t = \sqrt{\frac{w\,l\,d}{9{,}675{,}600}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

If a cylinder under external pressure could be depended upon to fail only by actual crushing, instead of through collapse (buckling), then the formula $w = \frac{2\,p\,t}{d}$ would apply, as in internal pressure; remembering that under external pressure the stress p is compression. If this equation gives a lower working pressure than (14) the flue designed by it will be safe against collapse. The rules of the Lloyd's Marine Register allow the following pressure in boiler flues: $w = \frac{1{,}075{,}200\, t^2}{l\,d}$. This is Fairbairn's equation with a factor of safety of 9. The British Board of Trade rules allow a working stress in furnaces and flues of about 4,000 when computed by the equation $w = \frac{2\,p\,t}{d}$. This is a little less than that allowed by the U. S. Board of Supervising Inspectors.

Hence for the same allowable pressure under these two rules

$$w = \frac{2\,p\,t}{d} = \frac{8{,}000\,t}{d} = \frac{1{,}075{,}200\,t^2}{l\,d}$$

$$\text{or } l = 134.4\,t$$

If, therefore, $l < 134.4\,t$ equations (1), (2), and (3) may be safely used.

It will be observed that this relation limits the use of these equations to comparatively short flues. Thus a flue ¼″ thick could only be 34″ long to have these equations applicable. In practice long flues of large diameter are reinforced at short intervals by heavy rings of rolled or other section, known as **collapse rings,** thus making the flue consist virtually of a series of short flues, to which equations (1), (2), and (3) may be applied.

Various Insurance and Government inspection departments give rules for proportioning flues and furnaces. These rules change from time to time, and if the boiler is to be insured in any company the specific rules prescribed by it should be consulted.

Thus Lloyd's Register for 1906–7 gives

$$w = \frac{1{,}075{,}200\,t^2}{l \times d} \quad \text{when } l > 120\,t$$

$$\text{and } w = \frac{50\,(300\,t - l)}{d} \quad \text{when } l < 120\,t$$

where l, t and d are all in inches.

Various other authorities give similar equations with practically the same coefficients.

85. Corrugated Furnace Flues. Flues corrugated as in Fig. 73 are very much stiffer against collapse than plain cylindrical flues, and with proper dimensions of corrugations may be safely made of any desired length. Their peculiar shape also permits of expansion and contraction under the influence of heat. When the corrugations are not less than 1½ inches deep, and not more than 8 inches from centre to centre of corrugations, and plain portions at the ends do not exceed 9 inches, the U. S. B. S. I. allows a working pressure of

$$w = \frac{14,000\,t}{d}$$

This is also the formula of the British Board of Trade.

Lloyd's Register for 1907–8 gives a number of rules for designing various types of flues.

The following references contain valuable practical information on this subject:

Lloyd's Register of British and Foreign Shipping.

"Steam Boilers," by Peabody and Miller.

Rules and Regulations of U. S. Board of Supervising Inspectors.

Rules and Regulations of the American Bureau of Shipping.

Rules of the British Board of Trade.

Rules of the Bureau Veritas.

Seaton and Rounthwaite's Pocket Book.

THICK CYLINDERS

86. When the wall of a cylinder, which is subjected to internal or external fluid pressure, is thick relatively to the internal diameter, it can no longer be assumed that the stress in the wall is uniformly distributed over the cross-section, but it is greater at the inner surface and decreases to a minimum at the outer surface whether the pressure is internal or external. When the pressure is internal the stress is tensile, and when the pressure is external the stress is compressive.

Many formulæ have been deduced to express the relations between pressure, stress, and cylinder thickness. Of these, that of Lame, deduced in 1833, is perhaps best known. Clavarino's* modification of Lame's formula, which was published in 1880, is now much used and will be adopted in this work.

Ordinarily the cylinder is subjected to either external or internal pressure alone; but in a gun tube, for example, which

* The student is advised to read the discussion of thick cylinders given in Merriman's "Mechanics of Materials," edition of 1906.

has a hoop shrunk upon it, the more general case occurs in which the cylinder is subjected to both internal and external pressure.

Let w_1 = the internal unit pressure.
" w_2 = the external " "
" r_1 = the internal radius of the cylinder.
" r_2 = the external " " "
" p_1 = the unit stress at the inner surface.
" p_2 = the " " " outer "

Then by Clavarino's equation the unit stress at any radius r is

$$p = \frac{\left[r_1^2 w_1 - r_2^2 w_2 + \frac{4 r_1^2 r_2^2}{r^2}(w_1 - w_2)\right]}{3(r_2^2 - r_1^2)} \quad . \quad (16)$$

If the external pressure w_2 be zero, which is the most usual case, the greatest tensile stress is at the inner surface, and is

$$p_1 = \frac{w_1}{3}\left[\frac{r_1^2 + 4 r_2^2}{r_2^2 - r_1^2}\right] \quad . \quad . \quad . \quad . \quad . \quad (17)$$

$$\text{or } r_2 = r_1\left[\frac{3 p_1 + w_1}{3 p_1 - 4 w_1}\right]^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (18)$$

Example. A cast-iron cylinder 20 inches in internal diameter, is to withstand an internal pressure of 1,000 lbs. per square inch. How thick must the wall be in order that the stress at the inner surface may not exceed 4,000 lbs. per square inch?

Here $r = 10$, $w_1 = 1{,}000$ and $p_1 = 4{,}000$. Hence substituting in (18)

$$r_2 = r_1\left[\frac{3 p_1 + w_1}{3 p_1 - 4 w_1}\right]^{\frac{1}{2}} = 10\left[\frac{3 \times 4{,}000 + 1{,}000}{3 \times 4{,}000 - 4 \times 1{,}000}\right] = 12.8''$$

or the cylinder walls must be 2.8″ thick.

From (16) it is found that p_2, the stress in the cylinder walls at the outer fibre, is 2,620 lbs.

PRACTICAL CONSIDERATIONS

87. Cast-iron pipes are widely used for underground water pipes and to some extent also for gas pipes, largely on account of their durability against corrosion. For steam, or for high pressures generally, cast-iron pipes are now seldom used

because of their unreliability. For all ordinary purposes pipes made of wrought iron or steel are most used, although in special cases, such as marine work, copper and brass are preferred.

Wrought-iron or steel pipes may be either lap-welded or butt-welded, the latter being commonly used for the smaller diameters, while steel piping may be "drawn" so that there is no seam, in which case it is known as "seamless drawn tubing."

Standard Piping is designated by its nominal internal diameter. Thus standard 1-inch gas pipe has a nominal internal diameter of 1 inch, and an external diameter of 1.315 inches. So-called standard wrought-iron piping may be used for pressures up to 100 lbs. with safety. For still higher pressures, such as are found in high-class steam plants, thicker pipes, known as **extra strong,** are used. For hydraulic work, where pressures up to several thousand lbs. per square inch must be withstood, still thicker piping, known as **double extra strong,** is used. These heavy pipes are made by decreasing the internal diameter of the standard pipe, thus keeping the outside diameter and hence the screw threads for the flanges to one standard.* Thus an extra strong 1-inch pipe (nominal size) would have an internal diameter of .95 inches, and a double extra strong of the same nominal size would have an internal diameter of .587 inches, the external diameter remaining 1.315 inches in all cases.

For large cylinders both for steam and hydraulic service, cast iron is still much used and probably will be for some time yet, on account of the ease with which complicated iron castings can be made and machined. In the case of steam-engine cylinders the thickness of the walls is fixed by considerations other than those of strength, such as stiffness and securing good castings. The proportions of steam cylinders as fixed by practice are the best guide. An examination of current practice shows the average thickness of low-speed engines to be given by the

* The student is referred to Kent's "Engineer's Pocket Book," or similar works, for full tables of standard sizes of pipes, flanges, etc. See also current trade catalogues.

following, $t = .05d + .3$ inch,* where t = thickness and d = diameter in inches, when the steam pressure does not exceed 125 lbs. per sq. inch.

Kent's "Mechanical Engineer's Pocket Book" gives the following as representing current practice, $t = .0004dp + .3$, where d = diameter in inches and p = pressure in pounds per square inch. If p be taken as 125 pounds this equation reduces to that given by Barr.

Cast iron is also much used for the cylinders of hydraulic machines, although steel castings are better in general. In such cases equations (16) to (18) developed above, in common with all equations based on the theory of elasticity, should be used with caution when cast iron is selected for the cylinder. Furthermore it must be borne in mind that the thicker the cylinder walls, the more liable are they to be porous in the interior, where made of castings. It is safer, therefore, as a rule, to carry a high working stress, within safe limits, and insure sound castings, than to design thick walls which are open to suspicion, in order to get a theoretically lower stress. A 3-inch wall, for instance, with a working stress of 5,000 pounds per square inch is preferable to a 4-inch wall with a working stress of 3,000 pounds per square inch. Care should also be exercised in cylinders made of castings to avoid excessive thickness of metal at any point, thus insuring sound castings. Thick castings of any metal are very liable to give trouble by leaking on account of porosity, if subjected to high pressures, and cast-iron cylinders are often lined with brass or bronze liners to obviate this difficulty.

87.1. Pipe Couplings, Flanges, etc. Methods for securing the ends of pipes together have become of greater importance as higher steam pressures have been employed. The most usual method for accomplishing this purpose has been to thread the ends of the pipes (see Art. 57) and secure them together with either a cylindrical pipe **coupling,** a pipe **union,** or a pair of pipe **flanges.** All of these are in very common use. For pressures up to 100 pounds per square inch and pipes not over 12 inches in

* See "Current Practice in Engine Proportions," by J. H. Barr; Transactions A. S. M. E., Vol. XVIII.

diameter these may be used with success, but for higher pressures and larger diameters they are not so satisfactory. The union is used on small pipes only.

In the ordinary screwed fitting of large size it is difficult to cut the thread accurately, and to screw the fitting on tight enough to prevent leakage at *A*, Fig. 73 (a). This can be remedied to some extent by making the threaded portion of the pipe long enough to project through the flange slightly, and then facing off pipe and flange so as to make a smooth surface, and permitting the packing or gasket (*P*) to cover up the screwed joint, as shown at *B*, Fig. 73 (a). Even this joint, however, is liable to leak if the workmanship is poor or if the flanges do not align properly.

To obviate the difficulties of the screwed joint on pipe of

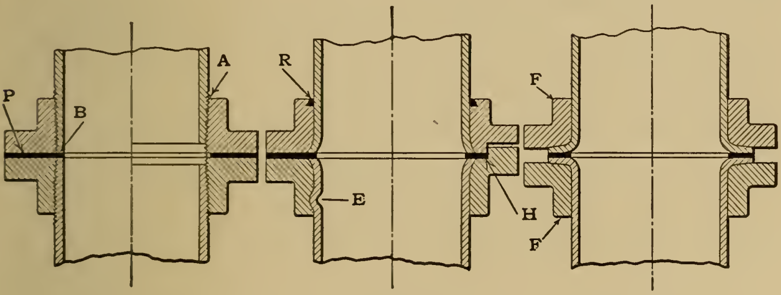

FIG. 73 (a). FIG. 73 (b). FIG. 73 (c).

larger diameter, the flanges are sometimes shrunk on as shown in Fig. 73 (b) (see also Art. 73). In order to insure tightness, and secure a firmer grip on the flange, the end of the pipe is usually expanded into the flange, as shown in Fig. 73 (b). The gasket usually covers up the joint between the pipe and the flange. This form of coupling is not well suited for high-pressure work, however, especially if the pipe is not machined on the outside before the flange is shrunk on. When subjected to the heavy straining action incident to expansion and contraction, as in heavy steam mains, the pipe is sure to work in the flange, and leaking will ensue. These flanges are sometimes fitted with a recess, *R* (Fig. 73 b), into which a strip of soft metal, such as copper, can be caulked to check small leaks; but this can hardly be considered satisfactory in high-grade work. A somewhat better grip of the flange is sometimes obtained by rolling the pipe into a groove in the flange, as shown at *E*, Fig. 73 (b).

In the so-called Van Stone joint (Fig. 73 c), the ends of the pipes themselves are flanged over and the joint made between the flanges so formed. The flanges *F*, *F* become clamps for holding the flanges proper together, and may be loose on the pipe. This last feature is a very useful one, as it greatly facilitates erection. This form of joint has been used with success in high-pressure work.

For the highest grade of work, wrought-steel flanges are welded to the pipe, making the pipe and its flanges one piece. This construction, while expensive, is almost essential for large pipe and the highest pressures.

To prevent the packing from blowing out, the flanges are sometimes fitted with a recess and tongue as shown at *H*, Fig. 73 (b). This construction is essential for very high pressures, as in hydraulic work, but should be avoided if possible in steam lines, as it makes it difficult to renew the packing.*

Although several efforts have been made to establish standard dimensions for pipe flanges, several systems are in common use in this country. The most important of these systems are the standards adopted by the Flange Standardization Committee, of the A. S. M. E., and that known as the Manufacturers' Standard. The first is used for pressures up to 125 pounds per square inch and the second for pressures up to 250 pounds per square inch. The student is referred to standard handbooks, and the catalogues of various manufacturers for details of these several systems. See also Transactions A. S. M. E., Vol. XXI.

THIN PLATES

88. General Theory. The theory of the stresses induced in thin plates, when subjected to load pressures, is one of the most uncertain portions of the mechanics of materials, due in part to

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* For a fuller description of Van Stone joints see articles by W. F. Fischer, *Power*, Feb. 23, 1909, and March 2, 1909.

the complexity of the problem, and in part to the scarcity of corroborative experimental data. The subject has been investigated mathematically by Grashof, Bach, Unwin, Merriman, and others, and the experimental work of Bach, Benjamin, and others verifies in a measure some of their conclusions. The mathematical results obtained by various authorities differ mainly in the coefficients, the general form of the equations being in most cases similar. Those due to Merriman* will be used in this treatise as they are simple, easy to apply, and give values as safe, generally, as any others.

Let t = the thickness of the plate in inches; r = the radius of circular plate in inches; l = length and b = breadth of rectangular plate in inches; p = maximum tensile stress; w = load per unit area in lbs.; and P = concentrated load in lbs. Then

For flat circular plates supported but not fixed at the edges, carrying a distributed load

$$t = r\sqrt{\frac{w}{p}} \text{ for wrought iron or steel} \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{and } t = r\sqrt{\frac{9}{8}\,\frac{w}{p}} \text{ for cast iron} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For flat circular plates fixed at the edges (encastre), carrying a distributed load

$$t = r\sqrt{\frac{2}{3}\,\frac{w}{p}} \text{ for wrought iron or steel} \quad . \quad . \quad . \quad . \quad (3)$$

$$\text{and } t = r\sqrt{\frac{3}{4}\,\frac{w}{p}} \text{ for cast iron} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

For flat circular plates supported but not fixed at the edges, carrying a concentrated load P, which is applied to the centre of the plate over a small circle of radius r_0 so that $P = \pi\, r_0^2\, w_0$

$$t = r_0\sqrt{\left[1 + 2\, log_e \frac{r}{r_0}\right]\frac{w_0}{p}} \text{ for steel or wrought iron} \quad (5)$$

$$\text{and } t = r_0\sqrt{\left[\frac{9}{8} + \frac{9}{4}\, log_e \frac{r}{r_0}\right]\frac{w_0}{p}} \text{ for cast iron} \quad . \quad . \quad . \quad (6)$$

* See Merriman's "Mechanics of Materials," 1907 edition, page 409.

For flat circular plates encastre carrying a concentrated load P, which is applied to the centre of the plate over a small circle of radius r_0.

$$t = r_0 \sqrt{\left[\frac{2}{3} + \frac{4}{3} log_e \frac{r}{r_0}\right] \frac{w_0}{p}} \text{ for steel or wrought iron} \quad (7)$$

$$\text{and } t = r \sqrt{\left[\frac{3}{4} + \frac{3}{2} log_e \frac{r}{r_0}\right] \frac{w_0}{p}} \text{ for cast iron} \quad . \quad . \quad . \quad (8)$$

In equations (5), (6), (7) and (8) w_0 is the pressure per unit area on the small circle whose radius is r_0, *i.e.*, $w_0 = \frac{P}{\pi r_0^2}$. The value of w_0 should not exceed the elastic strength of the material.

Example. A circular cast-iron plate 20″ in diameter supports a load of 4,000 lbs. at its centre, the load being applied by a bolt whose head is 2″ in diameter. How thick must the plate be, if simply supported, in order that the tensile strength in it shall not exceed 6,000 lbs. per square inch?

Here $P = 4{,}000$; $r = 10$; $r_0 = 1$ and $p = 6{,}000$.

$$\therefore w_0 = \frac{P}{\pi r_0^2} = \frac{4{,}000}{\pi \times 1^2} = 1{,}265 \text{ lbs.}$$

$$\text{and } log_e \frac{r}{r_0} = log_e \frac{10}{1} = 2.3 \text{ whence from equation (6)}$$

$$t = 1 \sqrt{\left[\frac{9}{8} + \frac{9}{4} \times 2.3\right] \frac{1{,}240}{6{,}000}} = 1.14''$$

Rectangular Plates. If $2l$ and $2b$ are the length and breadth of a rectangular plate, then for plates supported but not fixed, and for a uniformly distributed load w per square inch

$$t = \frac{3}{2} l b \sqrt{\frac{w}{(l^2 + b^2) p}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and for fixed edges

$$t = l b \sqrt{\frac{3 w}{2 (l^2 + b^2) p}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

If $l = b$ equation (9) reduces to

$$t = l\sqrt{\frac{9}{8}\frac{w}{p}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and equation (10) reduces to

$$t = l\sqrt{\frac{3}{4}\frac{w}{p}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The above equations, as before stated, are not to be relied on implicitly, but will serve as approximate guides only. This is particularly true in cast materials where heavy ribbing is used and where trained judgment is perhaps the best guide.

89. Flat Stayed Surfaces. One of the most important cases of flat plates occurs in boiler work, where large flat areas are held against pressure by stays at regular intervals over the surface. These stays are usually screwed into the plate and the projecting end is slightly riveted over to insure steam tightness. The various Inspection Bureaus and Insurance Companies give practical formulæ for the design of such plates, and these can be safely used. Thus the U. S. Board of Supervising Inspectors* and the American Boiler Makers' Association rules give for steel plates

$$w = \frac{C \times t^2}{s^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Where w = pressure in lbs. per sq. in., t = thickness of plate in sixteenths of an inch, s = greatest pitch of stays in inches, and C = a constant as below given

$C = 112$ for plates $\frac{7}{16}''$ thick and under.
$C = 120$ " " over $\frac{7}{16}''$ thick.
$C = 140$ " " with stays having a nut inside and outside.
$C = 160$ " " " " " washers .5 as thick as the plate and of a diameter at least .5 the greatest pitch.

* See General Rules and Regulations of U. S. Supervising Inspectors; also Rules of American Bureau of Shipping.

CHAPTER X

CONSTRAINING SURFACES

90. General Considerations. As the various members of a machine must move with definite relative motion, they must be retained in correct position by **constraining surfaces.** Thus a shaft is held in position by bearings which locate its axis of rotation, and by collars which prevent motion endwise. The relative motion of a pair of constrained members may be that of **sliding,** as in the case of an engine crosshead and its guide; **rotation,** as in the case of a shaft journal and its bearing; **rolling,** as in roller and ball bearings; or a combination of some of these as illustrated in certain forms of cams, where both sliding and rolling exist. Dry metallic surfaces, under any appreciable load, even when smoothly machined, will not slide over each other without abrasion. It is therefore necessary to keep rubbing surfaces separated by a thin film of some kind of lubricant, and the whole subject of the design of constraining surfaces is closely connected with the theory of lubrication.*

It has been pointed out in Chapter IV, that when bath or forced lubrication is maintained, the friction between two rubbing surfaces is independent of the *character* of the material of which the surfaces are composed; but when the surfaces are "imperfectly" lubricated the frictional resistance depends somewhat on the metals used. Experience has shown that *like* metals usually do not rub together well. Thus steel on steel (except when hardened), steel on wrought iron, or cast iron on cast iron, are poor combinations except where the velocity is low and the pressure light. If two rubbing surfaces of cast iron can be run together for some time without cutting, they take on hard glazed surfaces which will run well together. This is well illustrated in slide valves and pistons of

* See Chapter IV.

steam engines. Care must be exercised that the surfaces are well lubricated when first put in service. Soft steel and wrought iron will both run well on hardened steel, and hardened steel may be run on hardened steel at very high pressures and velocities, if the surfaces are ground true, and polished. Steel and wrought iron will run very well on brass or bronze. The alloys of copper, tin, zinc, antimony, lead, etc., commonly known as **anti-friction** or **babbitt metals,** run extremely well with steel or wrought-iron journals.* Innumerable alloys of this kind are upon the market under different names. They can be made of any degree of hardness, depending largely upon the proportion of antimony used. Very hard alloys of this kind are sometimes known as **white brass.** In using babbitt metal for heavy pressures, care should be exercised that the particular alloy selected is hard enough so as not to flow under the applied pressure. Other materials, such as wood, are sometimes used for rubbing surfaces. The conditions which influence the selection of materials for rubbing surfaces, and the practical considerations governing their application, will be more fully discussed in connection with the several forms of constraining surfaces.

The most common forms of motion in machines are rectilinear translation and rotation; therefore the most important forms of constraining surfaces are

(**a**) Sliding surfaces, for the constrainment of rectilinear motion.

(**b**) Journals and bearings, for the constrainment of motion of rotation.

SLIDING SURFACES

91. Forms of Sliding Pairs. The stationary member of a pair of surfaces, which have relative sliding motion, is usually called the **guide,** while the moving part has various names depending on the service, as the **ram** of a shaping machine, the **table** of a planing machine, or the crosshead of an engine. The general term **sliding member** will be used here to denote the moving

* See Kent's "Mechanical Engineer's Pocket Book" for detailed analysis and properties of some of the best known alloys.

member. Sliding pairs may be classified by the degree of **lateral constrainment** afforded the slider by the guides, and this may be

(a) Partial lateral constrainment.

(b) Complete lateral constrainment.

In either case the rubbing surfaces of the guide and sliding member may be either **square, angular, or circular.** Thus Fig. 74 (a) shows a form of angular guide much used on planing machines while Fig. 74 (b) shows a set of square guides for a similar purpose. In each case the lateral constrainment is only partial, the tendency of the platen to raise being resisted by gravity. Fig. 75 (a) shows the crosshead of a steam engine with an angular guide. Here, lateral constrainment is complete. Fig. 75 (b) is also a steam-engine crosshead with circular guiding surfaces. This form of surface may be considered as a special form of the

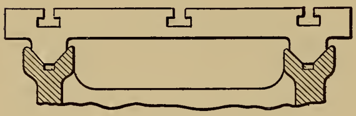


Fig. 74 (a).

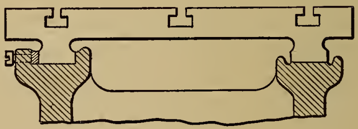
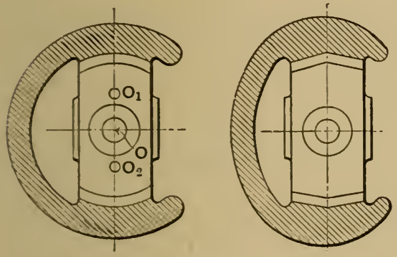


Fig. 74 (b).

angular type. If the circular guiding surfaces have a common centre at O, the crosshead is prevented from rotating around O only by the connecting-rod; and as long as it is so held from rotating the lateral constrainment is complete. If the surfaces have different centres as O_1 O_2, it is obvious that rotation cannot take place. Figs. 76 (a) and 76 (b) show square and angular guides where constrainment is complete.

The characteristic which distinguishes the square guide from the angular one is that in the square guide two sets of adjustments must be made to compensate for wear, while in the case of the angular guide one set only is needed. Thus in Fig. 76 (a), vertical wear must be compensated for by lowering the piece A, while lateral wear is taken up by the set screws C which press against the wearing strip or gib B. In Fig. 76 (b) lateral and vertical wear are both compensated for by the set screws C which press upon the gib D. Sometimes D is made tapering and provided with a screw adjustment so that it can be moved endwise, thus compensating for wear. In such cases the set screws *C* are omitted.

As to the relative merits of square and angular guiding surfaces, it may be said, in general, that square surfaces are easier to machine and fit than the angular ones. There are many places, however, such as the cross slides of lathe carriages, where the angular guide is much more convenient. In places such as lathe beds the *V* guides commonly used have the advantage of automatically taking up lost motion, no matter how badly they are worn. But, as a rule, the bearing surfaces of such *V* guides are very small and wear soon begins to be apparent, especially as the wear from the carriage is usually concentrated on a short por-



(b) FIG. 75. (a)

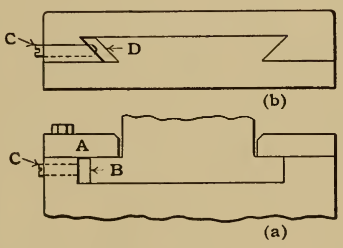


FIG. 76.

tion of the bed. There is a tendency among manufacturers to discard the *V* guide in favor of flat surfaces. English practice, especially in large tools, is in advance of American practice in this particular. A combination of *V* and flat guides is also often used.

92. General Principles. If a short block, Fig. 77, slides backward and forward upon another member *B*, carrying a fixed load *P*, it is evident that, if the material in *A* and *B* were homogeneous and the velocity were uniform throughout the stroke, the frictional resistance and consequent wear would be practically uniform over the whole surface of *B*. These conditions are difficult to attain and seldom occur in practice. Since *A* must be stopped and started at each end of the stroke, it follows that the velocity cannot be uniform; although in some machines such as plate planers this condition is approximated. Usually, however, the velocity varies from zero at the beginning to a maximum somewhere near the middle of the stroke, as in the case of engine crossheads, shaping machines, etc. Again, the load P may vary greatly. Thus in the steam engine, the normal pressure P between the crosshead and its guide is zero at each end of the stroke and a maximum near mid-stroke. The velocity of the crosshead also varies from zero at each end of the stroke to a maximum near mid-stroke. In the ordinary case the greatest frictional resistance* and wear will occur near mid-stroke, because both velocity and normal pressure between the bearing surfaces are greatest at this position. If the crosshead could be made the same length as the guide, the unit bearing pressure, at the middle of the stroke, would be practically uniform over the whole surface, and would be small compared to the unit normal pressure attained when the crosshead is short. For positions of the crosshead near mid-stroke the wear would be approximately equal over the whole surface, and much less than when the crosshead is very short, but still theoretically greater than at the end positions, when both velocity and normal pressure are zero. It has been found by experience that when the sliding block and guide are made the same length, the wear, even under varying load and velocity, is very small, and more uniform over the entire contact surfaces.

It is seldom possible, however, to make the sliding member the same length as the guide. Thus in lathe carriages, the rams of shaping machines, and the tables of planing machines, the sliding member is, in some machines, shorter than the guide, and in other machines longer. In most cases of this kind the wear is liable to be greater on one part of the guide, or sliding member, than on another. Thus in the case of a shaping machine the ram seldom operates at full stroke, and the wear on the back end of the ram is very small, the result being that when appreciable wear takes place on the forward end of the ram, and the guides are readjusted to compensate for the same, the back end of the ram will not pass through the guides at all, hence the

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* See Art. 32.

adjustment must be somewhat slack, and accurate work cannot be done. In other machines the excessive local wear comes on the guide, and a similar result occurs. Professor Sweet* has corrected this difficulty, in certain machines which he has built, by reducing the wearing surface on that portion of the sliding member or guide, as the case may be, where the tendency to wear is least. He has suggested the following convenient method of laying out the wearing strips on the surface of a sliding member. Fig. 78 shows a sliding surface such as is found on the ram of a shaping machine, where little wear occurs on the back or right-hand end as here shown. The shaded portions represent the parts of the surface which have been relieved, leaving the wearing

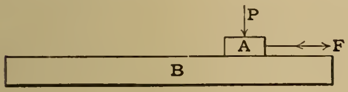

FIG. 77.

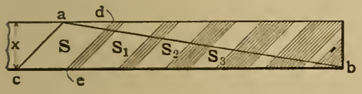

FIG. 78.

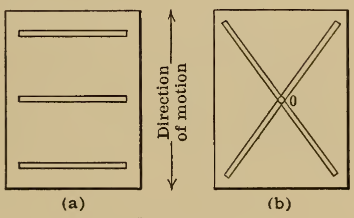

FIG. 79.

strips S, S_1 and S_2, etc. To lay off the surface, draw the diagonal *a*, *b* across the surface to be relieved. From *a* draw the line *ac*, making any convenient angle with the horizontal. Lay off *ce* equal to the width of the face *x*. Draw *de* parallel to *ac* and take the vertical distance above the point of intersection of *ab* and *de* for the first gap, and the corresponding vertical distance below the point of intersection for the first wearing strip, repeating this operation to the end of the surface. Similar wearing strips should be cut in the opposite direction on the other member, if it is comparatively long; but where a short block slides in a long guide, the guide only need be relieved.

93. Bearing Pressures on Sliding Surfaces. It is pointed out in Article 32 that the tendency of a loaded flat surface to expel the lubricant is resisted to a certain degree by the viscosity of

* Professor Sweet has embodied some of his experience in this line in a little book called "Things that are Usually Wrong," which will well repay reading.

the lubricant, and its power to adhere to the stationary member. This resisting power is much less marked in sliding surfaces than in rotating surfaces, as here the motion is intermittent. It is difficult therefore to lubricate sliding surfaces as efficiently as rotating surfaces, and, in general, they must be considered as "imperfectly" lubricated surfaces. The unit bearing pressure that can be sustained by sliding surfaces is, therefore, much less than can be borne by rotating journals. Further, it is difficult to obtain initially true sliding surfaces and, as pointed out above, very difficult to maintain their accuracy under service. The sliding part, and also the guides themselves, should, therefore, be designed for rigidity; in fact considerations of strength seldom need to be considered, but the guides should be so stiff that localized pressure will not occur. It is not surprising, in view of these considerations, that the allowable bearing pressures as fixed by practice vary greatly, even with similar classes of work. Owing to the difficulties of lubrication and compensation for wear, it may be stated, as a general principle, that the bearing pressure must be kept so low that wear is inappreciable, if accurate surfaces are to be maintained.

The following are average values of bearing pressures for different forms of sliding surfaces, as fixed by practice:

Crossheads,* stationary slow-speed engines	30 lbs.	to	50 lbs.
" " high " "	10 "	"	30 "
" marine engines	50 "	"	75 "

94. Lubrication of Sliding Surfaces. Sliding surfaces are very difficult to lubricate efficiently on account of the "wiping" action of the sliding member. In high-speed engines, bath lubrication is commonly obtained by enclosing the running parts, and allowing them to run in what practically amounts to an oil bath.

Where this cannot be done, care must be exercised in the manner in which the lubricant is supplied. If possible, when the guide is horizontal, the lubricant should be supplied near the middle of the guide. The oil grooves in the moving part should also be given careful consideration. From the theory of lubrica-

* See Trans. A. S. M. E., Vol. XVIII, page 753.

tion it is evident that the oil channels on all constraining surfaces should be *at right angles* to the direction of motion, wherever the velocity is great enough to draw lubricant between the surfaces. If made otherwise their effect is to relieve any tendency to form a pressure film. The grooves in crossheads, and other sliding members, should, therefore, be made as in Fig. 79 (a) and not as in 79 (b). In either case the grooves should be stopped some distance from the edge of the surface so as not to facilitate the escape of the oil. When the load is so heavy that forced lubrication must be used, the system of grooves shown in Fig. 79 (b) is correct; the oil being forced in at *O*. Care should also be taken that the outer edges of the slider, and the edges of the oil grooves, are chamfered so as to assist the entrance of the lubricant. If the edges are square and sharp their scraping effect may seriously impair the lubrication. Where the guiding surfaces are very long, as in planing machines, oiling devices such as rollers dipping in an oil pocket, placed at intervals along the guides, are very effective.

JOURNALS AND BEARINGS

BEARINGS

95. Forms of Bearings. The part of a machine frame, or other member, which constrains a rotating member, such as a shaft, is known as a **bearing.** That portion of the rotating member which engages with the bearing is known as a **journal.** Journals are necessarily circular in all cross-sections, but their profile may be cylindrical, conical, spherical, or even more complex in form, as in the case of thrust bearings. (See Art. 105.)

One or more of the following considerations affect the design of the bearing proper:—

(a) Rigidity, in order that the alignment may not be seriously affected by deflection.

(b) Strength, to resist rupture under the greatest loads.

(c) Adjustment, to compensate for wear.

(d) Formation and maintenance of an oil film.

(e) Automatic adjustment, to insure alignment.

(a and b). The inside diameter, or bore of the bearing, and also its length are fixed by the dimensions of the journal which engages with it; and the required strength and rigidity may be secured by a proper distribution of metal in accordance with the general principles discussed in Chapter III, and which apply to all forms of bearings, as far as strength and stiffness are concerned.

Usually the question of strength does not enter into the design of the main part of the bearing. If, however, the cap A, Fig. 80, should be called upon to carry the load, as is often the case, its dimensions should, in general, be checked for strength, and its design should be such that stiffness is secured. The exact distribution of the pressure over a bearing is not known; but the assumption that the cap is a beam loaded at the centre and of a length equal to the distance between the cap bolts will give dimensions on the safe side for strength and deflection. The greatest bending moment and deflection for such beams are given in Case IX, Table 1. It is impossible to adjust the cap bolts so as to be sure that the load is uniformly distributed among them, and the uncertainty of the initial stress due to screwing up the nuts makes the problem more difficult. For this reason the cap bolts should be designed to carry more than the apparent load. If only two bolts are used each should be designed for two-thirds of the total load; if four are used each should be able to carry one-third of the load with an apparent stress of not more than 6,000 lbs. per square inch.

The last three items, *c*, *d* and *e*, affect the *form* of the bearing. Consider first *c* and *d*. It is evident that the metal of the bearing will wear away most rapidly in the *line of greatest pressure*, hence adjustment for wear should also be along this line. It follows also that the bearing should be parted at *right angles* to the line of greatest pressure. Thus, if the load on the shaft be a simple vertical load P, as in Fig. 80, wear will take place only on the bottom half of the bearing. If this wear is so small as not to interfere with the alignment of the shaft, or if all the bearings on the shaft wear uniformly, adjustment may be made by lowering the cap A. If the shaft must occupy a fixed position relative to the frame of the machine, alignment must be maintained by raising the lower bearing surface. Where this is desirable the lower wearing surface is usually made separate from the pillow-block, as in Fig. 82, thus allowing the bearing to remain fixed in position, while the wearing part may be raised to compensate for the wear. If the load P, Fig. 80, be in an upward direction, all necessary adjustment may be made by means of the cap.

It was shown in Articles 32 and 33 that a journal will automatically tend to form a film of lubricant between itself and the bearing. If the conditions under which the lubricant is supplied are correct, fluid pressure may thus be created between the journal and bearing *provided the surface of the bearing is continuous for some distance on each side of the line of action of the*

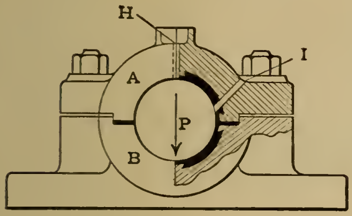

FIG. 80.

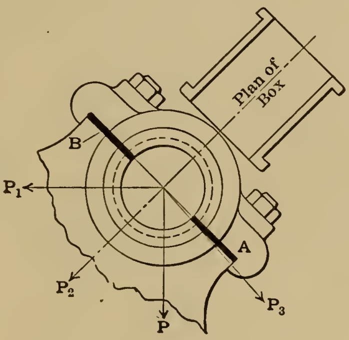

FIG. 81.

load. The greatest pressure will be found near this line of action. It is evident that the bearing shown in Fig. 80 fulfills both these requirements for vertical load either upward or downward; but is unsuited for lateral pressure from the standpoints both of adjustment for wear and lubrication.

Suppose, however, that the journal carries a heavy vertical load P (Fig. 81), and is subjected at the same time to a heavy horizontal belt pull P_1. The resultant of these forces is P_2, and the arrangement of parts shown in Fig. 81 is correct for motion of rotation in either direction. If P_1 be reversed in direction the resultant of P_1 and P_2 will be P_3, and the arrangement is not correct for adjustment against wear, and very defective as far as lubrication is concerned, as the surface is broken near the point where the greatest film pressure should exist. Bearings of this form are often used in steam-engine work, and in such cases the force P_1 due to the steam pressure on the piston, is continually reversed in direction. Another adjustment for a similar case is shown in Fig. 82. Here the shoe or bottom "brass" can be raised up by introducing thin "shims," or liners, underneath it; while lateral wear can be taken up by setting out the "cheek pieces" *B*, by means of the wedges *D*. Provision is thus made, by this arrangement, for taking up wear in all directions and keeping the shaft accurately aligned and located. For horizontal pressures in either direction the resultant P_3 passes close to the point at which the bearing is parted; and hence the best conditions for lubrication do not exist. Pressure films more

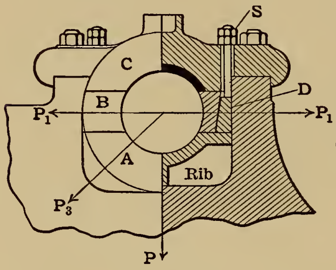

FIG. 82.

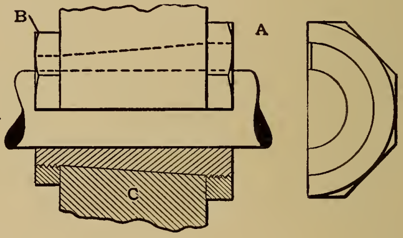

FIG. 83.

or less perfect, depending on the oil supply, will form on the lower shoe, but the continual reversing of the lateral pressure *P*, hardly allows time for the formation of pressure films on the cheeks. These reversals in pressure, however, allow the lubricant to be *carried* by the shaft, first under one cheek, and then under the other, thus lubricating them effectively.

Sometimes a bearing consists of a conical bushing split at some convenient place, as shown in Fig. 83. By releasing the nut *A*, and screwing up on *B*, the bushing may be forced into the frame *C*, thus closing the bore of the bushing slightly and compensating for wear. It is obvious that once the bore of the bushing is worn eccentric, no amount of taking up can rectify its shape; in fact taking up wear in this manner tends to destroy the fit of the journal in the bearing. Occasionally the journal itself is made conical, and adjustment for wear is made by moving the shaft endwise. The application of such bearings is limited to short shafts, such as machine-tool spindles.

Machine bearings are made in many forms, depending on the location and service. The bearings are sometimes split into three pieces, and various other means of compensating for wear are used, but the fundamental principles outlined above regarding the point where the bearing should be parted apply to all forms.

Consider the last item (*e*, automatic adjustment). In long lines of shafting, which tend rapidly to get out of adjustment, it is desirable that the bearing be so constructed as to adjust itself automatically to the changing position of the shaft, in order to avoid localized pressure, which would result in heating. In fast-running machinery, also, such as countershafts, dynamos, and motors, where perfect alignment is necessary, self-adjusting bearings have been found almost essential. Fig. 84 shows a bearing of this kind as used in dynamo and motor bearings. The sleeve *A* has a spherical surface turned upon the outside, the centre of the surface being at *O*. This surface engages with a similar surface bored in the outer casing *B*. The sleeve may swivel in any direction, but the centre line of the shaft must always pass through *O*. When a shaft has only two bearings of this kind it is evident that perfect alignment can be secured, within the range of motion of the sleeves. Similar devices are used in the case of long shafting, where many bearings must be used. It is obvious that the fundamental principles regarding adjustment for wear and maintenance of the oil film, apply to all bearings of this form also.

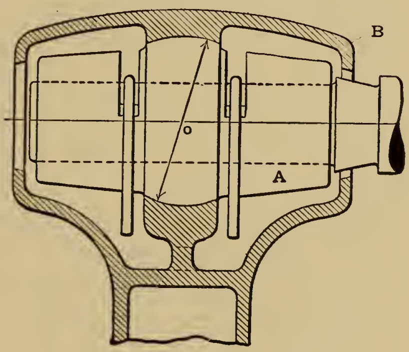

FIG. 84.

96. Practical Construction of Bearings. It was shown in

Article 88 that metal such as brass, bronze, and the white alloys make excellent bearing surfaces for wrought-iron or steel journals, on account of their anti-friction qualities. It is to be noted that even in the case of perfect lubrication, where the character of the rubbing surfaces is less important *once the oil film is established*, care must be exercised in the selection of the material for the bearing surface, in order that abrasion may not occur *before* the film is formed, or in case of failure of the film. There is a further advantage in having the bearing surface softer than the journal, in that it is very desirable to have the journal maintain its form against wear, which it is more likely to do when rubbing against a soft surface than it would against one harder than itself. The bearing itself should be rigid, so as to insure proper alignment of the shaft. Rigidity, against even moderate pressure, could not ordinarily be attained if the entire bearing member were made of the white alloys, and economy prohibits the use of brass and bronze for the entire bearing. It is customary, therefore, to make the main body of the bearing of cast iron (or sometimes a steel casting), and to fit into it wearing surfaces of the softer metals. These wearing surfaces may be either rigidly attached to the main castings or may be removable. In Fig. 80 is shown a bearing of the type commonly used for heavy shafts when the babbitt-metal lining is rigidly attached by means of dovetail shaped recesses, into which the babbitt is poured in a molten state. The necessary shrinkage due to cooling, which would leave the lining loose in the recesses, is usually overcome by hammering the babbitt, when cold, till it again fills the recesses, and then boring the babbitt to size. For cheap work the lining is often cast to size on a metal mandrel and no further work put upon it, but for all good work the bore of the lining is cast small enough to allow of hammering or peening, and then boring to a smooth surface. Fig. 81 shows removable linings of brass or bronze which are circular in section, and are prevented from turning when in place by the parting piece *B*. This parting piece, or "liner," also permits taking up wear by reducing its thickness as occasion requires. Fig. 82 shows an arrangement of wearing surfaces common on horizontal steam-engine bearings. The cap *C* is babbitted with some form of cheap metal since there is no wear upon it, all the pressure being either downward or sidewise. The "quarter boxes" *B*, and the lower box or shoe *A*, may be of brass or bronze, or of cast iron lined with babbitt. Where there is danger of the boxes breaking, through pounding by the shaft, and where it is desired to use a babbitt metal, they may be made of brass or bronze and babbitt-lined. When cast-iron wearing surfaces are used, and compensation for wear is important, as in the case of machine tools, it is customary to make the wearing surfaces removable as indicated in Fig. 81. For less accurate work the bearing surface is part of the main casting itself, machined to the required size. Hardened steel bearing surfaces are obtained by making circular shells or "bushings," of the required internal diameter, and of sufficient thickness to insure strength. These bushings are forced into openings in the main casting and no provision for taking up wear is made. If the forcing operation closes the bore of the bushing, it is "lapped" out with emery and oil to the required size. Where the bearing must work under water, as in the case of a propeller shaft or the lower bearing of a vertical turbine water wheel, a lining of lignum vitæ or other hard wood is often used. The surrounding water furnishes the only lubricant necessary in such cases. A detailed description of the many arrangements of bearing surfaces is beyond the scope of this treatise.

When the bearing must work under trying conditions, as on shipboard or in a heated room, and there is some question as to whether the heat of friction will be dissipated by radiation, the bearing is cast hollow so that water may be circulated around it, thus carrying off the heat and maintaining the lubrication. In an emergency, water may be allowed to run over the outside of the bearing, accomplishing the same purpose. High-grade marine work, and large stationary-engine installations, are often equipped with a complete system of water circulation on the most important bearings.

JOURNALS

97. Theoretical Design of Journals. The considerations affecting the design of any journal are one or more of the following:

(a) Strength to resist rupture.
(b) Rigidity, or stiffness, to prevent undue yielding.
(c) Maintenance of form against wear.
(d) Maintenance of lubrication.
(e) Radiation of the heat due to frictional resistance.

The first two considerations, strength and rigidity, are covered by the general principles laid down in Chapter III, and are more fully considered in Chapter XI, where the special problems in connection with shafts are discussed. Economy of material dictates that the *minimum* diameter of shaft be consistent with the applied bending and twisting moments.

The third consideration (c) particularly affects such journals as those on the spindles of grinding machines and machine tools generally, where the accuracy of the product depends on the accuracy of the journals. Usually, in such cases, the wearing surface must be so great, in order to reduce the wear to an inappreciable amount, that the consideration of strength does not enter into the computations.

The considerations, (d) and (e), are closely correlated. It was shown in Articles 32 and 33 that if the unit bearing pressure on the journal is not too great, the lubricant, because of its viscosity, may be drawn in between the journal and the bearing, thereby reducing the frictional resistance. This frictional resistance can never be reduced to zero even with perfect lubrication. The energy thus absorbed appears as heat, and is radiated to the surrounding air by the metallic surfaces of the bearing, the temperature of which rises till the rate of radiation equals that at which heat is being generated. In well-designed machinery the temperature of the bearing should not exceed 150° F. The raising of the temperature of the bearing has a tendency to lower the viscosity of the lubricant, and if the bearing becomes too hot, the lubricant becomes so thin that the pressure squeezes it out completely, and failure of the bearing by abrasion occurs. It is evident, therefore, that a journal of given dimensions may carry a given load very satisfactorily under certain conditions, and fail absolutely under others, the same lubricant being used in each case. The consideration of the proper radiation of the heat generated is, therefore, most important. It may be assumed, without serious error, that the rate of radiation of heat is proportional to the projected area of the bearing. The number of heat units which will be radiated from a unit of surface, at any given difference in temperature between the bearing and the surrounding air, is a fixed quantity for any set of conditions; and if the heat of friction per unit area is greater than can be radiated at the desired bearing temperature, the temperature of the bearing must rise till equilibrium is obtained. It follows therefore that for any desired bearing temperature the work of friction per unit of projected area of bearing must not exceed the rate of radiation per unit of projected area, or

$$\mu w V = K \quad \text{or} \quad w V = \frac{K}{\mu} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where μ is the coefficient of friction, w the load in pounds per unit of projected area, V the velocity of rubbing in feet per minute, and K the rate of radiation per unit of projected area in foot pounds per minute, to be determined experimentally.

It is to be especially noted that if μ be considered as constant, increasing the diameter of a journal (the number of revolutions and the total load remaining constant) does not materially affect the development or dissipation of heat, since the velocity of rubbing is increased in the same ratio as radiating surface is increased. If, however, the bearing be lengthened, the radiating surface is increased and the work of friction remains unchanged, with the same total load as before. This last statement, while true for imperfectly lubricated surfaces, is only approximately true for bearings with perfect lubrication as will be seen presently.

The amount of heat which will be radiated from a bearing has been experimentally determined by Lasche.* The curves shown in Fig. 84 (a) are those shown in his Fig. 57, transformed into English units, and with the scale of radiation further modified so as to read in foot pounds per square inch of projected area per second, instead of per square inch of actual bearing surface.

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* See *Traction and Transmission*, January, 1903, page 52.

Curve 1 represents actual experimental results, with bearings of the usual proportions, in still air. Curve 2 is for bearings which are connected to large iron masses, or which are ventilated by air currents. Curve 3 was calculated from theoretical considera-

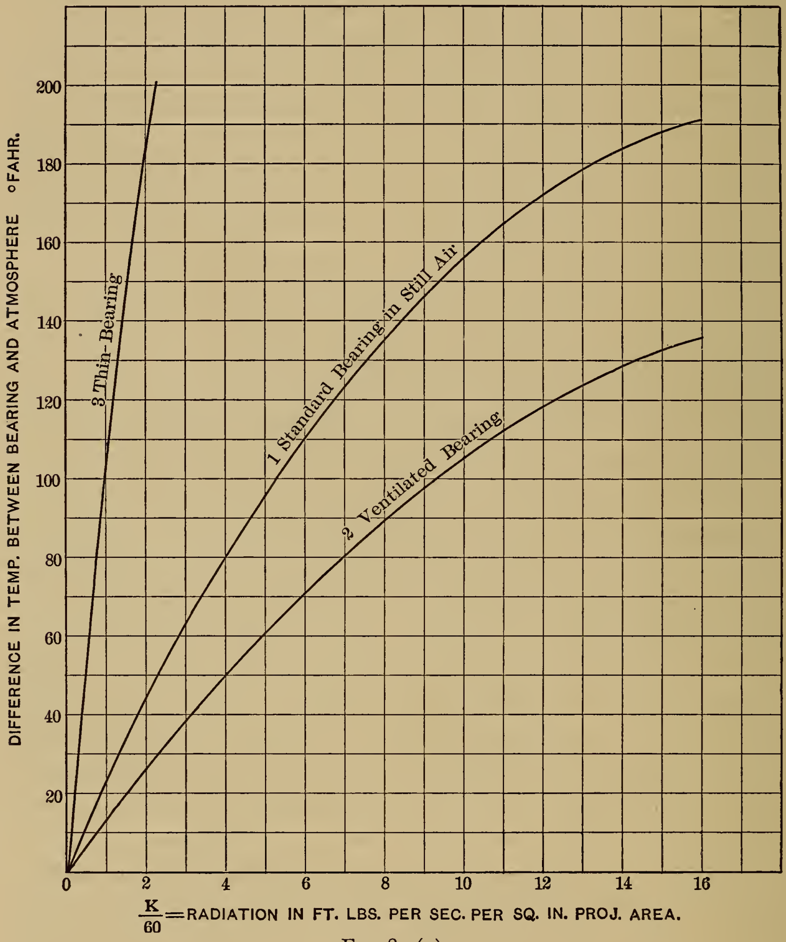

FIG. 84 (a).

tions. It gives the radiation from a very thin bearing or sleeve and indicates that radiation is more effective as the bearing becomes thicker, as might be expected; for metal is a better conductor of heat than air, and hence the thick bearing more easily carries the heat away to a greater radiating surface. The values obtained from these curves may therefore be used for K in equation (1). Lasche points out that though these experiments represent only a limited variety of conditions, they are probably on the safe side and will serve at least as a very useful check in designing.

If, in designing a journal, the value of μ can be determined, equation (1) and Fig. 84 (a) give the relations which must exist between the velocity and pressure in order that the safe bearing temperature may not be exceeded; or if the pressure and velocity are fixed by other circumstances, Fig. 84 (a) indicates whether radiation must be assisted by artificial means, such as water circulation or currents of air.

98. Imperfectly Lubricated Journals. It has been shown in Articles 32 and 33 that the value of μ, for imperfectly lubricated surfaces, is a very variable quantity, even for the same simultaneous values of velocity and pressure. Not only does it vary with velocity, pressure, and temperature, but the regularity of the oil supply (over which the designer has little control) affects it much more seriously. Further, bearings running under the same nominal load and velocity give widely different values of frictional resistance and temperature rise, depending on whether the load is constant or intermittent, or whether the motion is steady or vibratory, etc. Notwithstanding this, equation (1) may be made to serve as a useful check in doubtful cases by assuming a safe value of μ.

The assumption is sometimes made that μ is a constant; and formulæ of the form $wV = \frac{K}{\mu} = C$, where C is a constant that has been determined from practice, are much used. Thus if w be expressed in pounds per square inch of projected area and V in feet per minute, Mr. Fred W. Taylor* gives for mill work $C = 24{,}000$; and says that $C = 12{,}000$ is not safe for cast-iron bearings with ordinary lubrication. If the rise of temperature in the bearing be taken as 75° and μ be taken as .015, which is ordinarily a safe

* Transactions A. S. M. E., Vol. XXVII.

value, then from curve 1, Fig. 84 (a), $K = 222$, whence $C = \frac{K}{\mu} = \frac{222}{.015} = 15{,}000$. From curve 2, $K = 384$ whence for ventilated bearings $C = \frac{384}{.015} = 25{,}600$. These values agree with Mr. Taylor's limits better than would be expected.

All formulæ of this empirical form must be considered, as far as imperfectly lubricated journals are concerned, as applying only to the conditions and range for which they have been found true, and for which μ is apparently constant. This is more evident when the wide variation of the value of such constants as determined by practice is considered. Thus Mr. H. G. Reist gives, as the practice of the General Electric Company on generator bearings, a limiting value of $C = 50{,}000$ for bearing pressures from 30 to 80 pounds per square inch. Mr. H. P. Been gives the practice of one of the largest Corliss engine builders as $C =$ 60,000 to 78,000 for bearing pressures not higher than 140 pounds per square inch.

Unwin, page 249, gives values of a similar constant, β, which corresponds to the following values of C:

Locomotive Crank Pins	250,000 to 375,000
Locomotive Axles	200,000
Marine Engine Crank Pins	50,000 to 75,000
Stationary Engine Crank Pins	15,000 to 50,000
Railway Carriage Axles	75,000 to 100,000
Crank Shaft Bearings	7,500 to 20,000

The great variation in these values of C is no more than might be expected in view of the foregoing, and also in view of the difference in lubrication and in radiating capacities of bearings, due to material, form, and location. While, therefore, these coefficients may form a guide, and while doubtful cases may be checked for heating by equation (1), care should be exercised that the bearing pressure is kept within the limits which will admit of good lubrication. The allowable bearing pressures as fixed by practice for various classes of machines are given in the following table, and it may be noted that these are *more accurately* *known than the values of μ, or the values of the coefficient of radiation K.*

Economy in the use of material and the importance of minimizing the work of friction suggest that the diameter of the journal shall be as small as is consistent with strength and stiffness. With the diameter of the journal determined by these

TABLE XIII

BEARING PRESSURES FOR VARIOUS CLASSES OF BEARINGS

Class of Bearing and Condition of Operation.	Allowable Bearing Pressure in lbs. per Square Inch.
Bearings for very slow speed as in turntables in bridge work	7000 to 9000
Bearings for slow speed and intermittent load as in punch presses	3000 to 4000
Locomotive Wrist Pins	3000 to 4000
Locomotive Crank Pins	1500 to 1700
Locomotive Driving Journals	190 to 220
Railway Car Axles	300 to 325
Marine Engine Main Bearings, Naval Practice	275 to 400
Marine Engine Main Bearings, Merchant Practice	400 to 500
Marine Engine Crank Pins	400 to 500
Stationary Engine Main Bearings (high speed) for dead load.*	60 to 120
Stationary Engine Main Bearings (high speed) for steam load.	150 to 250
Stationary Engine Crank Pins (high speed) overhung crank	900 to 1500
Stationary Engine Crank Pins (high speed) centre crank	400 to 600
Stationary Engine Wrist Pins (high speed)	1000 to 1800
Stationary Engine Main Bearings (slow speed) for dead load.*	80 to 140
Stationary Engine Main Bearings (slow speed) for steam load.	200 to 400
Stationary Engine Crank Pins (slow speed)	800 to 1300
Stationary Engine Wrist Pins (slow speed)	1000 to 1500
Gas Engines, Main Bearings	500 to 700
Gas Engines, Crank Pins	1500 to 1800
Gas Engines, Wrist Pins	1500 to 2000
Heavy Line Shaft Brass or Babbitt Lining	100 to 150
Light Line Shaft Cast Iron Bearing Surfaces	15 to 25
Generator and Dynamo Bearings	30 to 80

considerations, it is evident that the length of the journal must be such that the bearing pressure is within the allowable limit. It may be, however, that the length of the journal thus determined will be so great that localized pressure may result; or it may be that the type of machine will not allow space enough for

* Weight of shaft, flywheels, etc.

such a length of bearing. In such cases the diameter must be made larger and the length may be correspondingly decreased.

While practice shows wide variations, it is found that the ratio of the length of the journal to its diameter $\left(\frac{l}{d}\right)$ is fairly well defined for any given class of machinery. It often occurs, therefore, that, when journals are designed with the ratio as fixed by practice, they have an excess of strength while barely satisfying the conditions as to bearing pressure.

The following are average values of $\frac{l}{d}$ as found in good practice:

TABLE XIV

TYPE OF BEARING	Values of $\frac{l}{d}$
Marine Engine Main Bearings..................	1 to 1.5
Marine Engine Crank Pins....................	1 to 1.5
Stationary Engine Main Journals...............	$1\frac{1}{2}$ to 2.5
Stationary Engine Crank Pins..................	1
Stationary Engine Crosshead Pins.............	1 to 1.5
Ordinary Heavy Shafting with Fixed Bearings....	2 to 3
Ordinary Shafting with Self-adjusting Bearings...	3 to 4
Generator Bearings..........................	3

99. Summary. From the foregoing the following statements may be made regarding **imperfectly lubricated journals:**

(a) The minimum diameter of a journal is fixed by the considerations of **strength** and **stiffness** under the loads applied.

(b) The smaller the diameter of the journal for a given coefficient of friction, the less is the work of friction and consequent liability to heating.

(c) The tendency of the bearing to heat, other things equal, is not materially affected by changing the diameter of the journal, but is reduced by increasing the length.

(d) The projected area of the journal must be such that the bearing pressure will be kept within the allowable limits for the particular conditions; and the ratio of length to diameter must not be so great that severe localization of bearing pressure is liable to result. These considerations may require a larger bearing than the previous requirements alone would demand.

(e) The work of friction, per unit area, must not exceed the rate of radiation, per unit area, for the allowable bearing temperature.

100. Perfectly Lubricated Journals. It was shown in Article 33 that if a journal is supplied with sufficient lubricant, of proper viscosity, the journal itself may draw in the lubricant till a film is formed under such a pressure that the load will be entirely fluid-borne. With any given set of conditions, therefore, and perfect lubrication, a definite journal velocity will permit the carrying of a definite load per unit area upon the journal, and once the relation is established between the load, velocity, and coefficient of friction, it is constant, and not unstable, as in the case of imperfectly lubricated surfaces.

It was further shown that the following statements are true regarding perfectly lubricated surfaces.

(a) The friction of perfectly lubricated surfaces for a given velocity depends very little on the materials which form the rubbing surfaces, but does depend largely on the character of the lubricant.

(b) The **frictional resistance** of perfectly lubricated journals for any given velocity is, within the limits of pressure under which the oil film may be maintained, independent of the pressure; (that is, $\mu\, w =$ a constant).

(c) The **coefficient of friction** of perfectly lubricated surfaces, for any given pressure, varies very nearly as the **square root** of velocity, for velocities up to 500 ft. per minute; approximately as the **fifth root** of the velocity for velocities between 500 and 2,000 ft. per minute; and is practically **independent** of the velocity for values above 2,000 ft. per minute.

The first of these statements has been abundantly verified by experiment and is discussed more fully in Article 90.

The following table, which is one of the several given in Tower's report, shows clearly the truth of the second statement, for the frictional resistance is seen to remain practically constant with all loads at any fixed velocity. The frictional resistance is also seen to vary very nearly as the square root of the velocity. Table VIII, Article 33, which was deduced from Table XV, shows the coefficients of friction for the range of pressures and velocities given, the latter not exceeding 500 ft. per minute.

TABLE XV

BATH LUBRICATION

Load in lbs. per sq. in. of projected area.	Frictional resistance in pounds per square inch of projected area of bearing surface $= \mu w$, for velocities in feet per minute as below. Temperature $= 90°$ F.							
	105 ft.	157 ft.	209 ft.	262 ft.	314 ft.	366 ft.	419 ft.	471 ft.
573	....	.583	.62	.678	.721	.758	.794	Seized
520	....	.496	.546	.597	.648	.691	.735	.771
415	....	.386	.445	.495	.539	.582	.619	.655
363	....	.306	.35	.401	.444	.488	.532	.561
258	.277	.357	.416	.459	.503	.547	.583	.626
153	.248	.306	.364	.408	.459	.510	.561	.605
100	.277	.357	.423	.503	.576	.619	.663	.714

Tower's experiments at different temperatures show that the coefficient of friction, for the above range of pressure and velocities, decreases as the temperature increases. His principal experiments, from which Table XV is taken, were conducted at 90° F. and without artificial means of cooling the bearing. The difference between the coefficients of friction obtained at 90° F. and those obtained at temperatures as high as are usually allowed in practice, can be neglected, as far as designing is concerned, especially since those at 90° are on the safe side. Tables VIII and XV may, therefore, be taken as representing fairly well the relation existing between pressure, velocity, and frictional resistance for this range, which fortunately covers the most usual conditions in practice. It is to be noted that at the greatest pressure and highest velocity, the bearing seized, indicating that with such velocity a lower pressure must be assigned, if a perfect oil film is to be maintained, or that with this greatest load a lower velocity must be assigned, if the bearing is to radiate the heat of friction. The work of friction at 471 ft. per minute with a load of 520 lbs. is seen to be $.771 \times 471 = 365$ ft. lbs. per minute, or about 6 ft. lbs. per second. From curve I, Fig. 84 (a), it appears that to radiate this amount of energy, the bearing must attain a temperature of over 110° F. above that of the surrounding atmosphere, or a total temperature of at least 180° F.

It is to be especially noted that within the limits of pressure where a perfect oil film will form, the frictional resistance, for a given velocity, is practically constant and independent of the pressure. (See Table XV.)

The frictional resistance, and coefficient of friction, for bearings running at velocities of over 2,000 ft. per minute with perfect lubrication, have been quite fully determined by Wm. O. Lasche.* The experimental work was very extensive, the results very conclusive, and should be carefully read by designers of high-speed machinery. A discussion of these experiments is beyond the scope of this treatise, but a few of the most important results will be considered. Lasche found that at these high velocities the coefficient of friction was practically independent of the velocity, but varied inversely with the pressure as in the Tower experiments, and also varied inversely with the temperature. He found that if w be the bearing pressure in pounds per square inch, and t the temperature of the bearing in Fahrenheit degrees, then

$$\mu w (t - 32) = 51.2 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{or } \mu w = \frac{51.2}{(t - 32)} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For velocities between 500 and 2,000 ft. per minute the coefficient of friction varies about as the fifth root of the velocity, as shown by the experiments of Stribeck. As far as designing is concerned, the difference between the coefficients for this range and those found by Lasche for the higher velocities, may be neglected, and Lasche's equation may be applied, without serious error, to all velocities above 500 ft. per minute.

Equation (3) may be written

$$\mu w V = \frac{51.2 V}{(t - 32)} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where V is the rubbing velocity in ft. per minute. Since $\mu w V$ is the frictional loss per unit of projected area in foot pounds per minute, equation (4) may be used to compute the heat which a

* See *Traction and Transmission*, January, 1903.

perfectly-lubricated high-speed journal can radiate per square inch of projected area, and not rise above a temperature t.

The limiting values of the pressure under which a perfect oil film can be maintained, at these high velocities, have not been fully determined. In Lasche's experiments a load of 213 pounds per square inch of projected area was carried at a velocity of 1,968 ft. per minute. In Kingsbury's* experiments loads from 80 to 86 pounds per square inch were repeatedly carried at velocities up to 1,990 ft. per minute. In both Kingsbury's and Lasche's work either the oil circulation, or the bearing itself, was artificially cooled, thus materially assisting the radiation.

The values given by these experiments were obtained on experimental machines and may be looked upon as limiting values. Successful practice in the design of steam turbine bearings gives velocities ranging from 1,800 to 3,000 ft. per minute, with pressures inversely as the velocity ranging from 80 to 50 pounds per square inch. Where the pressure is as high as 90 pounds per square inch, it is found that the velocity must be kept below 1,800 ft.† per minute. The empirical equation $w\,V = 150{,}000$ is much used for this class of work, and gives values agreeing with those just quoted. It is evident that with these high velocites the radiation must be assisted. Thus let $V = 2{,}000$ and $w = 75$ in accordance with the empirical rule just given, and let it be required to keep the temperature t at 150° F. or a temperature of say 75° F. above the atmosphere.

Then by equation (4) the frictional work is,

$$\mu\,w\,V = \frac{51.2 \times 2{,}000}{(150 - 32)} = 867 \text{ ft. lbs. per minute or } 14.5 \text{ ft. lbs.}$$

per second, whereas the bearing alone, if connected to a heavy iron frame, will, from Curve 2, Fig. 84 (a), radiate only 6.4 ft. lbs. per second. Since the specific heat of both water and oil are known the supply of either necessary to carry off the excess heat of friction can be calculated.

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* Transactions A. S. M. E., Vol. XXVII, page 425.

† See "Steam Turbines," by Frank Foster, page 181.

It is to be noted that with perfect lubrication the product μw for any velocity is a constant quantity. It follows therefore that for any given total load W the unit bearing pressure should be kept as high as possible provided it does not exceed the maximum allowable value for the given conditions. For if the unit bearing pressure is decreased, either by increasing the diameter or length of the bearing, the coefficient of friction is correspondingly increased; hence the total frictional resistance μW is also increased. Care should, of course, be exercised in any case that the heat of friction is properly carried away.

101. Examples of Journal Design. Journals generally form an integral part of a shaft or spindle, and the determination of the stresses acting upon them is a part of the solution of the

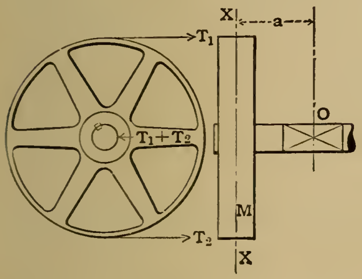


FIG. 85.

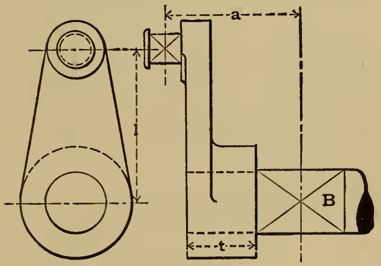


FIG. 86.

stresses in the shaft itself. It is desirable, however, to point out some of the special features of journal design.

The actual distribution of pressure over a journal, in the direction of the axis, is not known; but there is every reason to believe that the distribution is fairly uniform. Thus bearings, as a rule, wear quite uniformly over their entire length, where fair alignment is maintained. It is customary, in the absence of exact data, to assume for computations as to strength and rigidity that the load on the journal is concentrated at the middle of its length. This assumption is on the safe side, and will sometimes give shaft diameters excessively large as far as strength is concerned.

The following examples (a, b and c) show the most important cases of journal design. It is assumed in each case that the bear-

17

ings are imperfectly lubricated, which is the most common condition, but the application of the theory to perfectly lubricated journals is obvious.

Example (a). This case is illustrated in Fig. 85. Here the centre of the bearing is fixed at O, by the construction of the machine. The centre line of the pulley M is also fixed at XX, by the location which the belt must occupy, so that the pulley overhangs the bearing by the distance a. The diameter of the pulley d is 40 inches, $a = 10$ inches, the pull on the tight side of the belt is 500 lbs., and the pull on the slack side is 300 lbs. It is required to determine the dimensions of the journal.

The stresses induced in the journal are, torsional stress due to the twisting moment $(T_1 - T_2)\dfrac{d}{2}$, flexural stress due to the bending moment $(T_1 + T_2)\,a$, and shear due to the direct pull $T_1 + T_2$. The last is small and is usually neglected (see Art. 26), and the journal may be considered as subjected to a combined bending and twisting moment. Formula K_2 or K_3 (page 49), therefore, applies.

The bending moment $M = (T_1 + T_2)\,a = (500 + 300)\,10 = 8{,}000$

The twisting moment $T = (T_1 - T_2)\dfrac{d}{2} = (500 - 300)\,20 = 4{,}000$

Hence $\dfrac{M}{T} = \dfrac{8{,}000}{4{,}000} = 2 = x$ and taking $\dfrac{p_s}{p} = .8$ it is found from Figure 9 that equation K_2 applies

$$\therefore M_e = \frac{1}{2}\left[x + \sqrt{x^2 + 1}\right]T = \frac{1}{2}\left[2 + \sqrt{2^2 + 1}\right]4{,}000 = 8{,}480$$

From equation J, page 94, $M_e = \dfrac{pI}{e} = \dfrac{p\pi d^3}{32}$.

$$\text{or } d^3 = \frac{32\,M_e}{\pi p} = \frac{32 \times 8{,}480}{\pi \times 10{,}000} = 8.63$$

$$\therefore d = 2\tfrac{1}{8} \text{ inches (nearly), or say } 2\tfrac{1}{4}''$$

If the length of the bearing be taken at 7 inches (see Table XIV), the bearing pressure will be $\dfrac{T_1 + T_2}{2\frac{1}{4} \times 7} = \dfrac{500 + 300}{15.75} = 50$ lbs., which is a safe value.

If the number of revolutions be 300 per minute, and μ be taken as .015, the work of friction per unit of projected area will be $50 \times .015 \times 300 \times \frac{\pi \times 2.25}{12} = 133$ ft. lbs. per min., or 2.2 ft. lbs. per sec. From Curve 1, Fig. 84 (a), it is seen that to radiate this amount of energy the temperature of the bearing will rise about 50° above the surrounding air. This is a safe value and the design is satisfactory.

Example (b). Let the line of action of the load pass through the centre line of the journal, as in the case of the steam-engine crank pin in Fig. 86. Let the length of the crank be 18 inches, and the total maximum pressure on the crank pin be 25,000 pounds. What should be the dimensions of the crank pin in order to be safe against rupture and overheating?

Referring to Table XIV, it is seen that journals of this character are short compared to their diameter, and hence are usually strong enough and stiff enough if designed for a bearing pressure low enough to prevent overheating. Let $\frac{l}{d}$ be taken as 1.25. From Table XIII it is seen that 900 lbs. per square inch may be safely carried on this type of pin. If d be the diameter of the pin and l the length, then the projected area of the pin is $l \times d = 1.25\, d \times d = 1.25 d^2$.

$$\text{Whence } 1.25 d^2 \times 900 = 25{,}000$$
$$\text{or } d^2 = 22.2$$
$$\therefore d = 4.7 \text{ or say 5 inches,}$$
$$\text{and } l = 5 \times 1.25 = 6.25 \text{ inches.}$$

The pin may now be checked for strength. In a short pin of this kind it is more accurate to assume the load uniformly distributed along the pin, than to assume it as concentrated at the middle. The pin may, therefore, be considered as a cantilever uniformly loaded with a load $W = 25{,}000$.

Whence from Table I, case 3, the maximum bending moment

$$M = \frac{W l}{2} = \frac{25{,}000 \times 6.25}{2} = 78{,}125 \text{ inch pounds}$$

$\therefore$ from equation J, page 94, $M = \dfrac{pI}{e}$ or $p = \dfrac{Me}{I} = \dfrac{32\,M}{\pi\, d^3}$

or $p = \dfrac{32 \times 78{,}125}{\pi \times 5^3} = 6{,}400$ pounds nearly, which is a safe value.

In a similar way the pin may be checked for deflection, if desired, by means of case 3, Table I.

Example (c). Sometimes the location of the bearing is dependent on the diameter of the shaft, which is unknown, and in such case a tentative method must be adopted. Thus in Fig. 86 neither the length of the bearing *B*, nor the thickness of the crank hub *t*, can be definitely decided upon till something is known about the diameter of the journal. The diameter must therefore be assumed, and then checked by the equations which apply to the case. Usually a close estimate can be made from existing machines of similar type. In the case of the steam-engine shaft, for example, it is known that the main journal is frequently about one-half the diameter of the cylinder. The data taken in example (*b*) correspond to a cylinder diameter of about 18 inches, and the journal diameter may therefore be assumed as 9 inches. From Table XIV, the length of the journal may be taken as 20 inches. The length of the hub should be at least 8 inches, for this diameter. The boss under the pin may be taken as $\frac{3}{8}''$ in height and since the pin, from case (*b*), is 6.25 inches long, the total distance from the centre of the crank pin to the centre of the shaft may be assumed as $21\frac{1}{2}$ inches. The projected area of the journal is $9 \times 20 = 180$ square inches, which gives a bearing pressure of $\dfrac{25{,}000}{180} = 140$ pounds per square inch; and from Table XIII it is seen that this is a safe value as far as the load due to steam pressure is concerned. If the shaft also carries a heavy fly-wheel this must be taken into account (see next chapter).

The stresses induced in the journal are of the same character as in case (*a*). Taking the length of crank $l = 18$ inches, and the pressure on the pin $= 25{,}000$ as before, then the bending moment $M = 25{,}000 \times 21\frac{1}{2} = 537{,}500$ inch pounds, the twisting moment

$T = 25{,}000 \times 18 = 450{,}000$ inch pounds, whence $\frac{M}{T} = 1.19 = x$, and taking $\frac{p_s}{p} = .8$, equation K_2 is found by Fig. 9 to apply to the case.

$$\therefore M_e = \frac{1}{2}[x + \sqrt{x^2 + 1}]\,T = \frac{1}{2}[1.19 + \sqrt{1.19^2 + 1}]\,450{,}000 = 616{,}500.$$

From J (as in example a) $p = \frac{32\,M_e}{\pi\,d^3} = \frac{32 \times 616{,}500}{\pi \times 9^3} = 8{,}600$, which is a safe value and the design is satisfactory.

102. Lubrication of Journals. The **point of application of the lubricant** is of utmost importance, and the method of supplying the lubricant to the journal sometimes materially affects the design of the bearing. The most common methods of feeding lubricants to rubbing surfaces as given in Article 30 apply fully to journals and may be classified as follows:

Imperfect Lubrication	Common oil hole. Common wick or siphon feed cup. Common drop sight feed cup. Oily pad against journal. Ring or chain feed. Centrifugal oiler. Compression grease cup.
Perfect Lubrication	Bath lubrication. Flooded lubrication. Forced lubrication.

In flooded lubrication (sometimes erroneously called forced lubrication), the oil is supplied to the bearing under a low pressure which insures that the journal is always flooded at the point of application, as in bath lubrication, but it does not force the lubricant between the surfaces. In forced lubrication the oil is supplied at a pressure in *excess of the film pressure* at the point of application, and is thus forced in between the surfaces, no reliance being placed on the tendency of the journal to draw in the lubricant. The compression grease cup, while supplying the lubricant under slight pressure, gives only imperfect lubrication as the supply of lubricant is not copious as in the case of forced lubrication.

In applying any of these methods of lubrication, therefore, except the compression grease cup and forced lubrication, care should be exercised that the point of application is at, or near, a point of lowest pressure and at the place where the journal will naturally draw in the lubricant. Thus, in Fig. 80, if the pressure is always downward, lubricant can be supplied at *H* for motion in either direction. If the pressure were upward, an oil hole at *H* would not only be useless for supplying lubricant but would be fatal to good lubrication, as any tendency for a pressure film to form would be destroyed by relief of the pressure at the hole. In such a case the lubricant should be supplied from underneath, or if the direction of rotation were anti-clockwise an oil hole as shown at *I* would be good design. In forced lubrication the point of application should be the point of greatest bearing pressure, and the hydraulic pressure under which the oil is supplied should be *greater* than the maximum bearing pressure.

While the decreased friction due to perfect lubrication is evident, it does not follow that an effort should be made to design every bearing so as to secure this advantage. In some places a simple oil hole is sufficient, in others a constant supply from a wick feed will suffice, while again, with greater speeds, a ring oiling device is necessary. In many modern power installations, with either steam turbines or reciprocating engines, very complete apparatus for supplying flooded lubrication will be found. The bearings are constructed so as to catch all the oil, as it leaves the journal, and pipes convey it to a central receiver. A pump continually circulates the oil to the various bearings, and in the best installations the oil is filtered and cooled during the circuit. The same results are obtained by flooded lubrication as with bath lubrication. Forced lubrication is resorted to only where the bearing pressures are excessive and beyond those which can be supported by the natural action of the film formed by rotation of the journal. (See Art. 33.)

The **location and character of the oil grooves** deserve special attention. If the velocity of the journal is so low as to draw in little lubricant the oil grooves should be so cut as to allow the lubricant to flow in near the points of greatest pressure. Grooves, or scores on the journal itself, have been found helpful in drawing in the lubricant under such circumstances; especially where the lubricant is heavy. But where the velocity is above 25 feet per minute (see Fig. 16), and for ordinary pressures, care should be used that no oil grooves are cut that will tend to prevent the formation of the pressure film. If the lubricant is delivered at H, Fig. 80, and the pressure is downward, oil grooves of any kind running from H which will distribute the oil over the surface of the journal, are allowable so long as they terminate at a little distance from the edge of the bearing. If the oil is delivered at I, and the pressure is either downward or upward, the grooves should be cut *at right angles to the direction of motion*, so as to distribute the oil along the entire length of the bearing. If cut diagonally they will extend under the journal toward the point of greatest oil pressure, thus relieving any tendency to the formation of a pressure film and the lubrication will not be as good as it would be if no grooves were present.

The sharp edges of all oil grooves should be carefully removed to facilitate the passage of the oil under the journal. The sharp edges of the bearings themselves should also be filed or scraped away for the same reason. Where one bearing surface encircles nearly one-half of the shaft, as in Fig. 80, the surfaces should be relieved for some little distance from the parting line to help the wedging action of the oil and to insure the journal against side pressure due to springing of the bearing under the load. A bearing which binds sidewise will not lubricate properly.

THRUST BEARINGS

103. General Considerations. When a shaft is subjected to a heavy end thrust, either from the weight of the parts carried or on account of the power transmitted, the simple collars which are used to prevent end thrust in ordinary shafting will not suffice, and bearings of special form, known as **thrust bearings,** must be provided. If the bearing is designed so that the thrust is taken on the end of the shaft it is called a **step-bearing** or

footstep-bearing. If the thrust bearing must be placed at some distance from the end of the shaft it is called a **collar bearing.**

104. **Step-Bearings.** If the motion of rotation is very slow, as is the case in swinging cranes and similar work, a simple cast-iron step, as shown in Fig. 87, will meet the requirements, even if the pressure is heavy. If, however, the velocity is high, this simple arrangement will not give good results, even when the pressure per unit area is low. It may be assumed, without great error, that the unit pressure between the faces of a newly fitted step-bearing is uniform at all points. The velocity of rubbing, however, is a maximum at the outer edge, and, theoretically, it is zero at the geometric centre of the pivot. Since the wear is proportional to the product of pressure and velocity, it follows that the surface will wear unevenly, the greater wear taking place at the outer edge. This will bring a concentrated pressure at other points, and heating and cutting may result. It is always advisable in heavy work, for this reason, to remove the wearing surface near the center, where the motion is slowest, and where eventually the greatest concentration of pressure is likely to be produced (see Fig. 87). Decreasing the bearing pressure by increasing the surface, is effective within limits, since the area increases as the square of the diameter while the velocity of rubbing increases directly as the diameter. Increasing the radius, however, increases the average moment arm of the frictional resistance, and hence increases the lost energy. It is often better, therefore, to carry a higher bearing pressure, and thus keep the diameter of the pivot small.

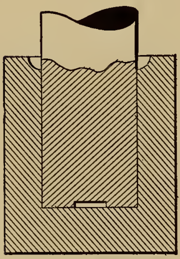


FIG. 87.

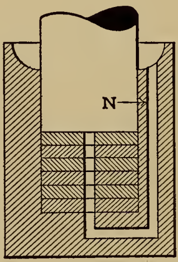


FIG. 88.

If a number of discs are placed between the step, or pivot, and the bearing (Fig. 88), they have the effect of reducing the relative velocity between adjacent surfaces; and if the rotative velocity of the pivot is high, they are very useful as a safeguard against cutting; for if abrasion should begin between any pair of discs, motion will cease at that point till the lubrication became effective again. These washers are usually made alternately of steel and brass, or some other metal, and the upper and lower washers are fastened to the shaft and bearing respectively. An oil hole passes through the centre of the washers, and radial grooves cut across the faces permit a flow of oil between the surfaces, centrifugal action assisting the lubrication. If the top of the bearing is connected to the bottom by an oil passage, as shown at *N* (Fig. 88), the centrifugal action will set up a continuous circulation of the oil, making the lubrication effective. The unit pressure between washers is the same as between the shaft and the first washer, but the relative motion between the surfaces is decreased and the wear thus reduced. A combination

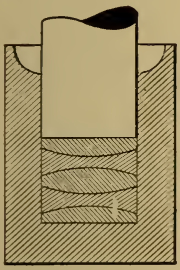


FIG. 89.

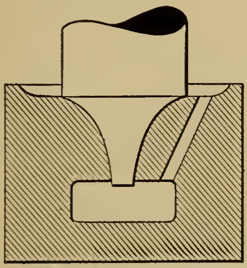


FIG. 90.

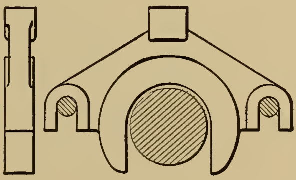


FIG. 91.

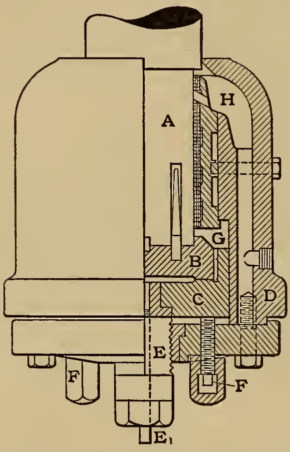
of hardened and ground steel washers, alternating with brass or bronze washers, makes an effective bearing. Sometimes the washers are made lenticular in shape, as shown in Fig. 89, in order to allow the shaft automatically to adjust its alignment. For very light work the shaft sometimes rests on a pair of hardened steel buttons, or a hardened steel ball which runs between hardened steel surfaces is introduced. In the submerged step-bearings of water turbines the shaft, which is often capped with bronze, rests on a lignum vitæ step and lubrication is effected by the surrounding water.

If the outline of a step-bearing be made that of a **tractrix*** (Fig. 90), it is found that the tendency to wear in an axial direction

* See Church's "Mechanics," page 181.

is uniform at all points; in fact if two homogeneous flat surfaces are rotated together they tend to wear into the form of a tractrix as has been proven by experiment. This is, therefore, the correct shape, theoretically, for all step-bearings; but on account of the difficulty and expense of machining the surfaces, it is seldom used. The tractrix has been called **Schiele's Anti-friction Curve** after the discoverer of the above property. This is a misnomer, however, for the friction of a tractrix-shaped step is much higher than that of a plain pivot.

It is evident that the rubbing surfaces of all the step-bearings which have been discussed can be submerged in an oil bath. The lubrication thus obtained is not to be confused with that obtained on horizontal rotating bearings discussed formerly. While centrifugal force does drive the oil from the centre to the outside, there is little action on the part of the surfaces themselves tending, on account of its viscosity, to draw the lubricant between them, as in horizontal bearings. Such lubrication cannot therefore be looked on as perfect lubrication although giving excellent results. The experiments of Beaucamp Tower* on a steel foot step, three inches in diameter, gives considerable information on this subject. It was found that a single diametral oil groove was better than more, and pressures up to 160 pounds per square inch were successfully carried at 128 revolutions per minute. The foot step was freely lubricated, and rested directly on the bearing, no washers being interposed. At 240 pounds per square inch the bearing seized.



THRUST BEARING OF CURTISS VERTICAL STEAM TURBINE

FIG. 92.

If under heavy loads the maintenance of lubrication is im-

* Transactions of the Institute of Mechanical Engineers, 1891, page 111.

portant, the lubricant should be supplied at the centre of the step-bearing under a pressure such that the metallic surfaces are forced apart and the load is fluid-borne. Fig. 92 shows a recent form of the step-bearing used on the Curtiss steam turbine. The vertical shaft A, which supports the heavy rotating parts of both turbine and generator, is carried on the disc B which rotates with it. The lower disc C can be adjusted vertically, by means of the screw E, and is prevented from rocking on E by the screws F. Oil is forced between the discs through the central pipe E_1, forcing the discs apart and escaping into the cavity G. The load is thus completely fluid-borne and perfect lubrication is maintained.

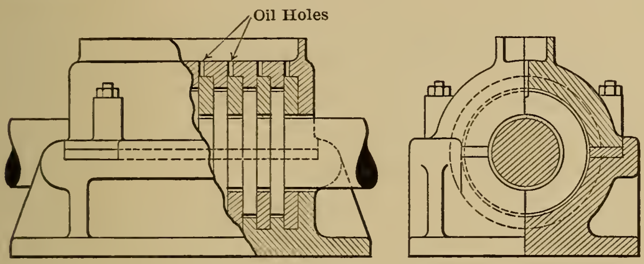


FIG. 93.

The oil passes from G upward through the guide bearing escaping at H.

105. Collar Thrust Bearings. When the thrust bearing must be placed at some distance from the end of the shaft, the shaft is provided with collars integral with itself, which bear against the resisting surfaces as shown in Fig. 93, which illustrates a thrust bearing as used for marine work. In cheap work, or where the load is small, a single collar is sometimes used. Occasionally a series of washers, as in Fig. 88, are interposed between the collar and the bearing ring. The objection to the single-collar bearing for heavy loads is that the large diameter necessary to obtain a practical bearing pressure increases the work of friction, due to the increased velocity, and the difference between the rubbing velocities of the ring at the shaft and at its outer diameter results in unequal wear. The outer diameter of the ring, or collar, is usually, therefore, not more than one and one-half times the diameter of the shaft, which limits the width of face of the collar even in large shafts to a few inches; and the necessary area is obtained by using a number of rings.

In small or cheap work, the bearing surfaces of the thrust block are sometimes made integral with the bearing proper; but usually they are made detachable. Thus the main casting of the block may be of cast iron and the bearing rings of brass are inserted and held in place by radial grooves cut in the block. These rings must be scraped until each collar on the shaft bears properly against its mating ring, so that the thrust is uniformly distributed. The most modern practice in marine work is to make the bearing rings horseshoe-shaped, as in Fig. 91, so that each ring can be withdrawn without disturbing any other portion of the bearing or shaft. Occasionally the horseshoe collars are adjustable along the shaft so as to be more easily brought to a proper bearing. In first-class work each horseshoe has its own independent water circulation, so that local heating may be prevented, and the lower part of the bearing constitutes an oil bath into which the collars dip. This oil bath also has a water circulation for cooling the oil.

106. Friction and Efficiency of Thrust Bearings. If P be the total load on a flat circular pivot of radius r_1 and μ be the coefficient of friction, then the **frictional moment** resisting rotation is

$$M = \frac{2}{3} \mu P r_1 * \qquad (1)$$

If r_1 be in inches and P be in pounds then the energy lost per minute in foot pounds is

$$E = \frac{2}{3} \mu P r_1 \times \frac{2 \pi N}{12} = .349 \mu P r_1 N \qquad (2)$$

where N is the number of revolutions per minute.

In a similar manner if the thrust be taken on a collar of outside radius r_1, and inside radius r_2, then

$$M = \frac{2}{3} \mu P \left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right) \qquad (3)$$

$$\text{and } E = .349 \mu P N \left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right) \qquad (4)$$

* Church's "Mechanics," page 180.

The efficiency of a thrust bearing cannot always be expressed as a function of the power transmitted. Thus in the case of a vertical shaft carrying a heavy load of gears, the frictional resistance of the step has little to do with the power transmitted. In the case of the thrust bearing of a steamship the frictional moment and energy loss are directly proportional to the driving force *P*. In either case, however, the frictional moment or the energy loss must be added to the turning moment or the energy supplied, as the case may be.

The following coefficients of friction are taken from Tower's experiments:

TABLE XVI

Pressures in lbs. per Unit Area.	Coefficients of Friction of Flat Pivots for the Revolutions per Minute as given below.				
	50 R.P.M.	128 R.P.M.	194 R.P.M.	290 R.P.M.	353 R.P.M.
20	.0196	.0080	.0102	.0178	.0167
40	.0147	.0054	.0061	.0107	.0096
80	.0181	.0063	.0045	.0064	.0063
120	.0221	.0083	.0052	.0048	.0053
140		.0093	.0062	.0046	.0054

At 50 and 128 R.P.M., the oil supply was restricted, but at the other velocites the bearing was flooded. In all cases the coefficient increased at revolutions below 40 R.P.M., which was probably due to the decrease of the centrifugal force (the bearing being oiled from the centre). This would seem to indicate that devices for reducing relative rubbing velocity, such as multiple washers (Art. 102), may be carried to an extreme, causing more friction than a plain flat pivot where centrifugal action is effective. In the case of thrust collars, such as shown in Fig. 91, running in an oil bath, the surfaces themselves tend to draw in lubricant in a way similar to that of the ordinary journal. The coefficients of friction for this class of thrust should therefore be as low at least as those given above.

107. Bearing Pressures on Thrust Bearings. Where the velocity of rubbing is very low and wear is not important, as in the case of swinging cranes, very heavy unit loads may be put upon pivot bearings, especially if they rotate in an oil bath. Where the velocity is high, or even moderate, and wear is important, much lower pressures must be carried with imperfect lubrication, than on ordinary bearings running at the same velocity. With forced lubrication, as in the step-bearing shown in Fig. 92, it is evident that very heavy pressure may be maintained. If, on the other hand, too many collars are used on a collar thrust bearing, in an effort to keep the bearing pressure down to a low value, there is danger that all of the collars will not bear simultaneously. The following are average values of bearing pressures, for thrust bearings, as found in practice:

TABLE XVII

Mean Velocity in ft. per Min.	Character of Lubrication.	Bearing Pressure in lbs. per Square Inch.
Very slow as in hand cranes	Bath as in Fig. 87	2,000 to 3,000
Up to 50 ft.	Bath as in Fig. 88	200
50 to 125	Bath as in Fig. 88	150
125 to 200	Bath as in Fig. 88	100
200 to 500	Bath as in Fig. 88	50.
500 to 800	Thrust Bearing and Bath Lubrication as in Fig. 93.	75 to 50

Example. Design the thrust journal for a steamship having the following data, and estimate the frictional loss in the thrust bearing.

Speed in knots (1 knot = 6,080 ft. per hour)	15
Indicated horse-power of one engine	5,000
Inside diameter of thrust collars	14″
Outside diameter of thrust collars	21″
Allowable pressure per sq. in. of surface	40 lbs.
Revolutions of the shaft per minute	120

Owing to frictional losses in the engine, propeller, and shaft only about two-thirds of the indicated power is delivered to the thrust block. The pressure against the thrust block multiplied by the distance through which the ship moves per minute must equal the energy delivered to the block per minute; or if P be the thrust, S the speed of the ship in knots per hour, and the indicated horse-power be denoted by $I.\ H.\ P.$, then $\frac{2}{3} \times I.H.P. \times 33{,}000$

$$= \frac{P \times S \times 6{,}080}{60} \text{ or } P = \frac{2 \times I.H.P. \times 33{,}000 \times 60}{3 \times S \times 6{,}080} = \frac{I.H.P. \times 217}{S}.$$

Hence in the above example $P = \dfrac{5{,}000 \times 217}{15} = 72{,}300$. The area of each thrust collar $= \dfrac{\pi}{4}(21^2 - 14^2) = 192$ sq. in. Therefore the total allowable pressure on each collar $= 192 \times 40 = 7{,}680$ and the number of collars $= \dfrac{72{,}300}{7{,}680} = 9.5$ or say 10.

If the bearing runs in an oil bath, the coefficient of friction will not be more than .01 under the worst ordinary conditions.

Therefore from (4) $E = .349\,\mu P N \left[\dfrac{r_1^3 - r_2}{r_1^2 - r_2^2}\right]$

$$= .349 \times .01 \times 72{,}300 \times 120 \left[\frac{10.5^3 - 7^3}{10.5^2 - 7^2}\right]$$

$$= 405{,}000 \text{ ft. lbs. per min. or } 12.3 \text{ H.P.}$$

ROLLER AND BALL BEARINGS

108. General Consideration of Rolling. It was pointed out in Article 29 that the resistance due to rolling friction was much less than that due to sliding friction, for a given load. The application of this principle to very heavy loads and low speeds, as in the case of moving heavy bodies on rollers, is of great antiquity; but only in recent years have mechanics been able to produce surfaces of such a character as could carry even very light loads at high speeds on either roller or ball bearings. At present, however, bearings of this character can be obtained which will run well under very severe conditions.

When a curved surface rolls upon any other surface with which it theoretically makes line or point contact, the two surfaces tend mutually to deform each other, the amount of deformation depending on the character and hardness of the materials forming the surfaces, and the intensity of the load sustained. If the surfaces of both members are very hard, and the load is very light, the deformation is negligible and true rolling can be practically attained. When, however, any appreciable load is to be carried the mutual deformation of the surfaces destroys the theoretical line or point contact and the load is borne on a small surface. This occurs even when the surfaces are very hard, and the action instead of being that of pure rolling, is a combination of rolling and sliding.* The true theory of this action, which is very complex, has not been fully demonstrated and is beyond the scope of this treatise. It can readily be seen that it is closely connected with the elastic properties of materials, on which much research work has been done. Undoubtedly the work of this character, which is of most value in the design of roller or ball bearings, is that of Professor Stribeck whose masterly report has been translated into English by Mr. Henry Hess,† and to this translation reference will be made hereafter.

If the intensity of pressure be such that the elastic limit of the materials is exceeded, permanent deformation will occur. In the case of roller or ball bearings this may result in the destruction of the surfaces either by flaking off locally, or by simply crushing out of shape. In either case continued action of this character is destructive to the bearing. Experiments on either balls or rollers to determine the ultimate crushing load are, therefore, misleading and useless as far as the design of such bearings is concerned. It appears from experiment and experience that bearings of this character can be constructed to carry fairly heavy loads at high speeds for a long period of time provided the intensity of pressure is not too great. It is obvious from the foregoing that the materials used in such bearings must be homogeneous, and of uniform hardness. The success of the modern ball and roller bearing has been made possible by improved materials and workmanship rather than by new theories.

Referring to Fig. 94, it is evident that when two adjacent rollers or balls, *A* and *B*, touch each other, the directions of motion of the common points of contact are in opposite directions. It is often stated that this results in considerable frictional loss;

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* The student may demonstrate this action by rolling a round lead pencil on a piece of soft rubber under pressure.

† See Transactions A. S. M. E., Vol. XXVIII.

and sometimes small intermediate balls, or rollers, are used as shown at *C*, Fig. 94, to obviate the supposed loss. Such intermediate balls or rollers must be kept in place by a cage such as *E*, Fig. 94, and this cage will give rise to a greater frictional loss than that which it is expected to remedy. A brief reflection will show that very little pressure can possibly exist between *A* and *B*. The only pressures that can be exerted by the guiding surfaces upon the balls or rollers are in a radial direction or normal to the surfaces, and these have no component tending to force the adjacent rollers or balls together. Sometimes the rollers or balls are separated by a guiding cage (see Fig. 95), and if any appreciable pressure could exist between adjacent rollers or balls the same would necêssarily exist between them and this guiding cage. This theory is not borne out by experience, as these cages, in well-built roller bearings, do not wear appreciably. The frictional loss from this source is undoubtedly very small.

The friction of roller and ball bearings while at rest is very small, and this is a very important point in the design of heavy, slow-moving machinery where, with ordinary sliding bearings, it often takes a much greater effort to start the machinery from rest than to maintain motion at full speed.

ROLLER BEARINGS

109. Forms of Bearings. Roller bearings, in common with the ordinary bearing, are classified as **radial** or **thrust** bearings, according to the manner in which the load is sustained. A typical form of construction of roller bearings for radial loading is shown in Fig. 95. A shell of hardened steel, *B*, surrounds the shaft *A*, and is secured firmly to it. The rollers *C* bear against this shell *B*, and against an outer shell *D*, which is secured to the bearing proper, *E*. Both rollers and shells are usually made of high carbon steel hardened and ground, or of mild steel case-hardened. The rollers are held parallel with the axis of the shaft by means of a cage *F* which is made of brass or other soft material. Some form of cage is necessary in all roller bearings on account of the tendency of the rollers to twist out of line with the shaft, thus replacing the theoretical line contact with point contact, and also causing an end pressure and cramping on the rollers. This tendency to end thrust is sometimes provided for by putting a small ball at each end of the roller to act as a thrust bearing. If the axis of the roller is not parallel to that of the shaft, it cannot make line contact with the shaft unless it assumes a spiral form.

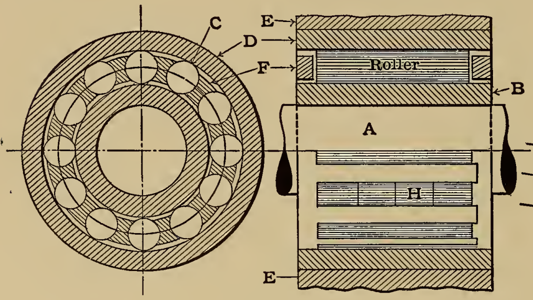

FIG. 95.

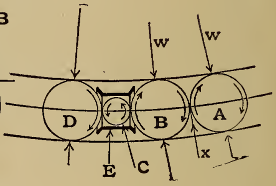

FIG. 94.

If the surfaces which confine the roller are accurately made, and the clearance is very small, as it should be, the roller cannot get out of parallelism with the shaft without being *bent* into a spiral form. If the rollers are hardened this may result in fracturing them, especially if they are relatively long. To obviate this

FIG. 96.

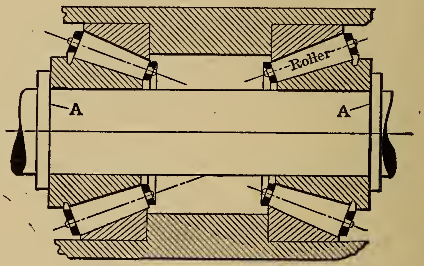

FIG. 97.

trouble the rollers are sometimes made in short lengths, as shown at *H*, in Fig. 95, or the roller is made flexible as illustrated by the Hyatt roller shown in Fig. 96. This roller is made by winding steel strip spirally upon a mandrel, thus making a hollow flexible roller. It is to be especially noted that neither of these methods will compensate for inaccurate workmanship. For continuous line contact the outer and inner shells must be machined with great accuracy, placed in very accurate alignment, and the rollers must be guided so as to remain perfectly parallel to the shaft. These conditions are difficult to obtain initially, and almost impossible to maintain with great accuracy under continuous service. The rollers in bearings for radial loading may be cylindrical or they may be conical as in the Grant bearing shown in Fig. 97. The construction here shown permits of adjustment for wear, which is difficult to obtain where the roller is cylindrical.

If the direction of the load to be carried is axial, roller thrust bearings of the form shown in Fig. 98 are often used. The shaft

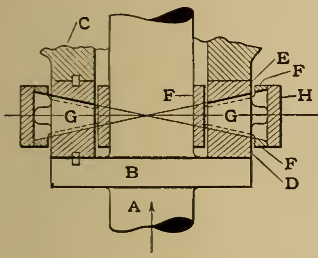

FIG. 98.

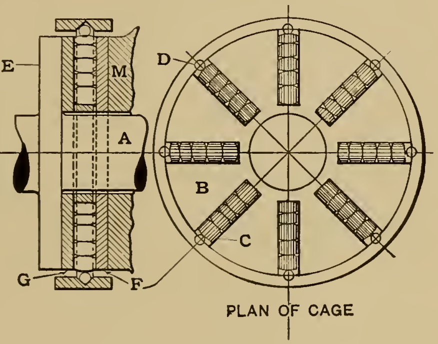

FIG. 99.

A carries a thrust collar *B* and the thrust is taken on the frame of the machine by a corresponding collar *C*. A hardened steel ring *D* is attached to *B* and rotates with it, while a similar ring *E* is fastened to the stationary part *C*. The conical rollers *G* roll between these rings, carrying with them the cage *F*. A thrust ring *H* prevents the rollers from moving radially outward. The apex angle of the roller should not exceed 15°, and in most cases is kept down to 6° or 7° to prevent serious end pressure against this retaining ring. It is evident that where the roller is conical in form, *the apex of the cone lying in the centre line of the shaft*, the velocity of any point in its periphery is proportional to its distance from the axis of the shaft and, theoretically, true rolling will be obtained.

Bearings of this character with conical rollers are expensive to make in an accurate manner, and a simpler form, as shown in Fig. 99, is sometimes used. Here the rollers are cylindrical in form and are made in short lengths so as to reduce relative slipping. The outer rollers rotate faster than the inner rollers, and the lengths and arrangement of the rollers are such that ridges are not worn in the seat.

Space does not permit of discussion of the many forms of roller bearings on the market; but their fundamental principles are the same, and the student is referred to current trade catalogues for variations in methods of construction.

110. Allowable Bearing Pressures. It is evident that the bearing pressure in roller bearings must not be great enough to stress the material of either roller or bearing surface beyond the elastic limit, but theoretical considerations are of little service in the actual designing of such bearings. The most reliable experimental data bearing on the subject are the results of Stribeck's work. In roller bearings under radial pressure the load is considered as carried on one-fifth of the total number of rollers; and the quantity equivalent to the projected area of the ordinary bearing, as far as carrying capacity is concerned, is considered as the product of length and diameter of a single roller, multiplied by one-fifth the total number of rollers in the bearing. Thus, according to Stribeck, for cylindrical bearings, if

N = total number of rollers.
W = total load on bearing in lbs.
w = load on one roller in lbs.
d = diameter of roller in inches (mean diameter for conical rollers).
l = length of roller in inches.
k = a constant to be determined experimentally.

Then $$w = k\,l\,d \qquad (1)$$

and $$W = k\,l\,d\,\frac{N}{5} \qquad (2)$$

From Stribeck's* experiments k has a value of 550 for unhardened rollers and bearing surfaces and 1,000 for hardened surfaces.

* See Transactions A. S. M. E., Vol. XXVII, page 444.

In the case of thrust bearings the load may be considered as distributed over the total number of rollers. Bearings of the type shown in Fig. 99 have been constructed to carry a load of 156,000 pounds at 250 revolutions per minute.

BALL BEARINGS

111. **Theoretical Considerations.** Let the ball *A*, Fig. 100 (b), roll along the circular path* *B*, with pure rolling motion, making point contact with the path. Let the path *B* be parallel to the plane *CD*, and suppose also that the ball as it rolls remains a fixed distance from this same plane. Then it is evident that if *A* rolls with pure rolling motion along *B*, it will rotate around

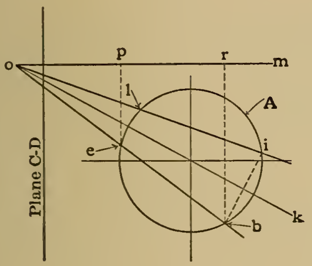

Fig. 100 (a).

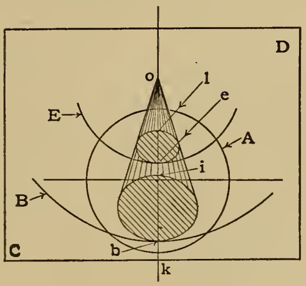

Fig. 100 (b).

some one of its diameters, at right angles to *B*, as an axis, and will make contact with *B* along the edges of such a disc as would be cut from it by a plane passing through the point of contact *b* perpendicular to the diameter around which the ball rotates. Thus the ball may rotate around *Ok* as an axis, and roll along the edges of the disc *b i*. It is clear, however, that the ball can rotate around only one diameter at a time, and preserve true rolling contact with *B*. If the ball has two concentric paths of contact as *B* and *E*, Fig. 100 (b) whose points of contact with the ball are *b* and *e* (Fig. 100 a) respectively, then it must roll along two discs *b-i* and *e-l*, and these discs must have a common axis of rotation *O k* perpendicular to their planes and passing through the centre of the ball. Further, the discs must be so placed that

* The guiding surfaces of ball bearings are almost invariably circular in form.

the lines *i l* and *b e* intersect on the line *o m*, passing through the common centre of *B* and *E;* for then

$$\frac{p\,e}{e\,l} = \frac{r\,b}{b\,i},$$

or the circumferences of the rolling discs are proportional to the circumferences of the paths of contact, and true rolling may be attained. It is not possible to have more than two points of contact between the ball and one of its guiding surfaces, with pure rolling, as the proportionality given above is not true for any other points on the line *o b* except those given. The above principles are fundamental and apply to all ball bearings with circular guiding surfaces.

112. Spinning. Usually one of the guiding members is fixed and the other rotates, the friction between the moving member and the ball causing the latter to roll. If the load carried is so small that no distortion of the surfaces takes place, and true point contact exists, this frictional force will act tangent to the outer circumference of the disc of contact and be parallel to its plane. Such theoretical conditions never exist in practice, as the surfaces of contact are deformed, even under light loads, and the load is carried on a small area instead of a point. The frictional force rotating the ball is, hence, indeterminate and in general has components which tend to rotate the ball about other axes than the one which will give pure rolling motion. It is clear that inaccurate workmanship will give the same result. This action is known as **spinning** and is necessarily accompanied by friction.

113. Forms of Bearings. Ball bearings are divided into three types, according to the character of the load and the way it is sustained by the bearing:

(a) Radial bearings, for loads acting at right angles to the shaft.
(b) Thrust bearings, for loads acting parallel to the axis of the shaft.
(c) Angular bearings, for taking loads both perpendicular and parallel to the axis of the shaft.

Each of these types may be either a **two-point, three-point** or **four-point** bearing, depending on the number of points of contact made by the ball on the guiding surfaces.

114. Radial Bearings. Figure 101 (a) shows a two-point radial bearing. The race *B* is secured to the shaft *A*, while the race *F* is secured to the other member *C*. Either *A* or *C* may be the rotating part. In order to place the balls in the raceway an opening is often cut in the side of one of the races, as shown at *E*, and the opening then closed with a filling piece as shown. If the race *F* is stationary this filling piece can be located on the unloaded side and no wear brought upon it. If *B* is stationary the opening must be cut in it, and the same care used in locating the

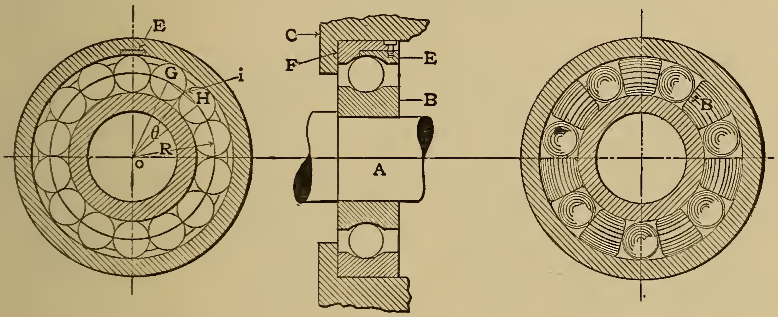

FIG. 101 (a). FIG. 101 (b).

filling piece with reference to the load. If both the shaft *A* and hub *C* rotate this cannot be accomplished, and the full load is brought upon this filling piece, thus decreasing the capacity of the bearing to sustain a load, on account of the break in the surface of the race. If the velocity of the rotating member is high this break in continuity of the race is destructive to the bearing.

If about half the total number of balls necessary completely to fill the race is used, each race may be made of one solid piece. In such cases the bearing is assembled by moving the inner race over eccentrically to the outer race, filling in the balls and then distributing them. Separators of elastic material are then pushed in between the balls to maintain correct spacing. These separators, also, often act as reservoirs for lubricant. They may be of felt or such soft material or may be made in the form of a helical spring. This construction is shown in Fig. 101 (b). The lessened number of balls is compensated for by using balls of larger diameter and hence greater carrying capacity.

The carrying capacity of radial ball bearings, according to Stribeck's experiments, is not affected materially by velocity, within reasonable limits, so long as the velocity of rotation is uniform; but sharp variations of velocity at high speed reduce the capacity.

115. Thrust Bearings. Fig. 102 illustrates a four-point thrust bearing. Here there is no difficulty in filling in the balls when the races are solid. In Fig. 102 the angles ϕ and ϕ' are equal, but this is not necessary as it is evident that any line drawn through

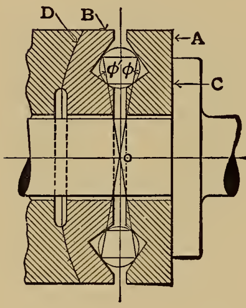

Fig. 102.

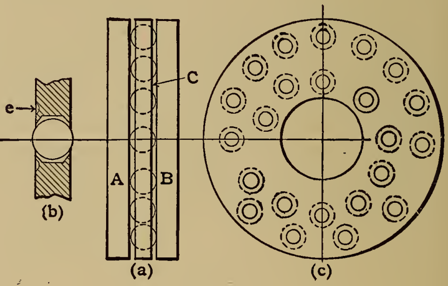

Fig. 103.

O and intersecting the ball circle will locate a pair of rolling discs which will roll on *B*, without interfering with the pair shown which may roll on *A*.

The surfaces *C* and *D* are sometimes made both flat and parallel. It is difficult, however, to obtain absolute parallelism, initially, between *C*, *D* and the ball races, and much more difficult to maintain this parallelism under running conditions. An error in alignment, either from poor workmanship or deflection under load, of less than one thousandth of an inch will cause concentrated loading of the balls on one side. If possible, therefore, such bearings should be seated on spherical surfaces, as shown at *D*, thus allowing the races to adjust themselves correctly.

Mr. Henry Hess states that speed is an important factor in such bearings and he gives 1,500 revolutions per minute as a maximum.

A simple form of ball thrust bearing is shown in Fig. 103. Here the balls run against flat hardened surfaces, *A* and *B*, and are kept in position by a cage *C* made of some soft alloy. The cage may be made to retain the ball loosely by drilling the openings for the balls almost through as shown in Fig. 103 (b), inserting the ball and then closing down the upper edge a little with a set as shown at *e*, Fig. 103 (b).

116. Angular Bearings. If possible, radial loads should be supported by radial bearings, axial loads by thrust bearings, and angular bearings should be avoided. Radial bearings should

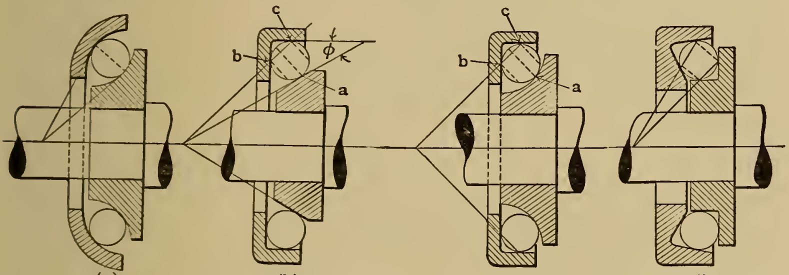

FIG. 104.

not sustain heavy axial loads and thrust bearings should not be loaded axially. For light loads the angular bearing will sustain pressure in either of these directions. There are innumerable forms of angular bearings. Fig. 104 (a), (b), (c), and (d) may be taken as typical of two-, three-, and four-point angular bearings. The races can be made continuous in all cases, and are often adjustable. This last feature, while sometimes necessary and often claimed to be an advantage, is really a detriment as it puts the bearing at the mercy of an unskilled person. Properly designed ball bearings do not wear appreciably, and if wear does take place it will occur on the loaded side only; and adjustment cannot compensate for this, but only hastens the failure of the bearing.

It is evident that all the arrangements shown in Fig. 104

fulfill the requirements for pure rolling contact as outlined in Art. 109. The path of the ball is not so definitely determined at *a*, Fig. 104, as in the other forms. For this reason the radius of the ball races should, in order to prevent wedging of the ball, not be greater than three-quarters the diameter of the ball. For the same reason the angle ϕ in Fig. 104 (b) should not be less than about 25°. In Fig. 104 (b) and 104 (c) the point *a* may, theoretically, be anywhere, as long as it lies between the discs which roll on the outer raceway. It should be so placed, however, as nearly to equalize the loads at *b* and *c*.

117. Allowable Load. The allowable load which may be put upon a ball bearing will depend on the following:

(a) **The character of the materials forming the balls and races.**

(b) **The shape of the raceways.**

(c) **The diameter of the balls.**

(a) Ball bearings fail by overstressing the material of the raceways or balls. If the stress induced is far beyond the elastic limit, and often repeated, the surfaces will flake off and failure will occur. Experiments on the *crushing* strength of balls or races are useless and misleading as the life of the bearing depends on the *elastic* and not the *crushing* strength. Evidently none but hard materials can be used for appreciable loads and these must be homogeneous in texture. Case-hardened materials are of doubtful value for severe service. For most trying circumstances special steels and alloys will no doubt be much used.

(b) Theoretically, a ball supports the load on a point, but practically the unavoidable distortion of the material increases the point to a small surface. It can be demonstrated mathematically, and is evident on reflection, that a greater bearing surface will be formed for a given distortion of ball and ball race the more closely the cross-section of the ball race corresponds to the cross-section of the ball. On the other hand, and as a direct consequence of this increase of surface, it is found that the friction increases as the cross-section of the races approaches the cross-section of the ball, a result to be expected.

It is almost impossible to machine and adjust ball bearings of three- or four-point contact so that the load is uniformly distributed at the various points of contact. It is borne out by experiment and it is well known that two-point bearings can carry heavier loads, than any other form for a given diameter of ball.

(c) The allowable load which a ball can carry varies with the square of the diameter.

These statements have been proven experimentally by Stribeck, who found that the carrying capacity of a ball could be expressed by

$$w = k\,d^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where w = greatest load on one ball in pounds.

k = a constant depending on the material and shape of ball races.

d = diameter of ball in inches.

Stribeck assumes that the total load is carried on one-fifth of the total number of balls. If, therefore, W be the total load in pounds on one row of balls, and N the total number of balls,

$$W = w\frac{N}{5} = k\,d^2\frac{N}{5} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For hardened steel races made of good quality of steel

k = 450 to 750 for flat or conical races, three- or four-point contact.

k = 1,500 for two-point contact and raceways whose radius of curvature equals $\frac{2}{3}\,d$.

With more perfect materials Stribeck states that these values may be increased fifty per cent.

118. **Practical Considerations.** It is clear that in order to insure an even distribution of load, initially, the workmanship on both balls and races must be very accurate; and in order to maintain this distribution the material must be uniform in quality and hardness throughout. It is also found that, for best results, the surfaces must be highly polished and free from scratches. The bearing must be kept free from acid and rust and provision made for excluding dust and grit and for retaining a supply of lubricant, the function of the lubricant being largely to prevent rusting.

As before stated, it has been found better to carry the load on one row of balls, if possible. Where this cannot be done special provision should be made to insure that each of the several rows of balls carry its proportionate load. This usually leads to some form of equalizing-device which complicates the design.

The minimum diameter of the shaft is fixed, approximately, by the load carried, and balls are made, commercially, in standard sizes. In designing a bearing for a full number of balls a tentative computation must generally be made to fix the proper number and diameter of the balls. Knowing these, the exact diameter of the circle passing through the centre of the balls can be determined as follows:

Referring to Fig. 101 (a), draw a line connecting the centres of any two balls in contact as G and H, and draw the radii $O\,G$, $O\,H$ and $O\,i$, as shown. Also let r be the radius of the ball and R the radius of the ball circle.

$$\text{Then } r = R \sin \theta$$

$$\text{and } \theta = \frac{180^\circ}{N}$$

$$\therefore R = \frac{r}{\sin \dfrac{180^\circ}{N}}$$

This value must be increased sufficiently to allow for the necessary clearance.

The methods of applying ball bearings are so numerous and varied, that no attempt can be made here to illustrate them, and the student is referred to the following sources of information on this point:

Transactions of A. S. M. E., Vol. XXVII and Vol. XXVIII.

Trade publications generally.

CHAPTER XI

AXLES, SHAFTS, AND SHAFT COUPLINGS

119. General. The terms **axle, shaft,** and **spindle** are applied somewhat indiscriminately to machine members which are so constrained by journals and bearings as to admit of motion of rotation. These rotating members may be subjected to simple torsion or bending, or to combinations of torsion and bending. Shear, also, usually exists as in the case of loaded beams. Rotating members may be classified roughly as follows, according to the predominating stress (see Art. 26), or to the particular purpose for which they are intended.

(a) **Axles,** loaded transversely and subjected principally to bending.

(b) **Shafts,** subjected to torsion or combined torsion and bending.

(c) **Spindles,** or short shafts which directly carry a tool for actually doing work, and which as a consequence must have accurate motion.

The axles of railway freight cars are good examples of case (a); transmission shafting in factories, or the shafts of steam engines are good examples of (b); while lathe and milling-machine spindles illustrate (c).

Considerations of strength seldom enter into the design of spindles. In these members torsional stiffness and accuracy of form in the bearings are, usually, the most important considerations. When the spindle is designed with these latter requirements in view, there is usually an excess of strength against rupture. The discussions given in Art. 12 apply in this case, and it will not be considered further here.

120. Axles. Let A (Fig. 105) be an axle which carries the loads P_1, P_2 and P_3, but is not subjected to any torsional stress except that due to negligible bearing friction. Suppose the axle to be supported by the bearings N and N. The distribution of the bearing reactions is indeterminate, as explained in Art. 95, and the assumption is usually made that they are concentrated at the middle of the bearings, as indicated. This assumption is on the safe side, so far as the strength of the shaft is concerned, as the slightest deflection of the shaft tends to concentrate the reaction at the inner edge of the bearing. The axle can, therefore, be treated as a simple beam (Art. 14). If the load P_2 were zero, and the loads P_1 and P_3 were equal and symmetrically placed (which is the most usual condition, as in car axles), the case

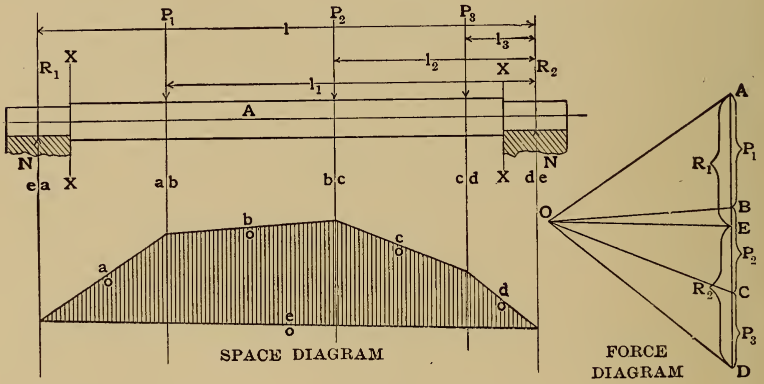

FIG. 105.

would be identical with Case XIV of Table I. It will be instructive, however, to make a solution of the general case given above.

The principal stress to which the axle is subjected is simple bending. Shear also exists in every section; but from the general theory of beams (Art. 14) it is known that, usually, this latter may be neglected in the body of the shaft. If, however, the shaft is short, and consequently need not be large to withstand the applied bending moment, the section of the bearing at XX should be checked for shearing stress. The **dangerous section** of the shaft will be where the bending moment is a maximum, and hence it is necessary to determine this maximum moment, which also involves the determination of the unknown reactions. The reactions may be determined mathematically by taking moments around R_2.

Then, $R_1 l = P_1 l_1 + P_2 l_2 + P_3 l_3$

$$\therefore R_1 = \frac{P_1 l_1 + P_2 l_2 + P_3 l_3}{l}$$

$$\text{and } R_2 = P_1 + P_2 + P_3 - R_1$$

The bending moment at any section is the algebraic sum of all the moments at either side of the plane considered. Thus the bending moment at $P_2 = M_2 = R_1 (l - l_2) - P_1 (l_1 - l_2)$ and this value may be used in equation J of Table VI $\left(M_2 = \frac{pI}{e}\right)$ to determine the stress for a given cross section, or to determine the cross section for an assumed stress.

A graphical solution is much more convenient, as it shows at once where the *maximum* bending moment is located. In Fig. 105, denote the forces P_1, P_2, etc., thus, *ab*, *bc*, *cd*, etc., and draw the corresponding force diagram as shown, making $AB = P_1$, $BC = P_2$, and $CD = P_3$, to any convenient scale. It is to be noted that these forces are drawn consecutively downward, since they act in that direction, and their sum, AD, must equal the sum of the reactions, or vertical forces. Take any convenient pole, as O, and draw OA, OB, OC and OD. From any point on *ab*, in the space diagram, draw *oa* and *ob*, parallel respectively to OA and OB in the force diagram. From the intersection of *ob* and *bc* draw *oc*, parallel to OC, and in similar manner draw *od*. Join the intersection of *oa* and *ea* with the intersection of *od* and *de*, thus locating the closing string *oe*. Draw OE parallel to *oe*, locating E. Then in the force diagram $DE = R_2$, and $EA = R_1$ to the assumed scale of the force diagram.

The vertical ordinates of the space polygon are proportional to the bending moments at the points considered. The numerical value of any bending moment is the continued product of the length of the ordinate, the perpendicular distance of O from AD, the *reciprocal* of the scale of the space diagram, and the *reciprocal* of the scale of the force diagram. Thus if the ordinate at some point be 2″ long, the pole distance be 2½″, the space scale be 1½″ to 1 ft., or ⅛ size, and 1″ = 5,000 lbs. on the force diagram; then the bending moment at the point considered is $M = 2 \times 2\frac{1}{2} \times 8 \times 5{,}000 = 200{,}000$ inch lbs.; and from this moment the diameter of the shaft may be computed.

121. **Shafts Subjected Principally to Torsion.** The fundamental relations existing in a shaft which is subjected to torsion only have been fully discussed in Article 12, and for such cases or where other stresses, such as those due to bending, are negligible, Article 12 is applicable. Shafts subjected to pure torsion rarely occur in practice, as bending is almost always present due to the weight of the shaft itself, and to the weight of pulleys which it supports, as well as to belt pull, etc. There are many cases, however, where the torsional stress is predominant, and where the secondary bending effect is difficult to compute. Thus in long factory shafting, where the power is supplied to the shaft at one point, and is given off in small increments at short intervals all along the shaft, the bending due to the pull of the belts is small. This is especially true if care is exercised to place the pulleys as close to the bearings as possible.

If the shaft is of considerable length, the angular distortion is of importance, and it may often occur that a shaft having sufficient torsional *strength* will not have proper torsional *stiffness*. If the power is applied at one end of the shaft, and taken off at the other end, computations for both strength and stiffness are easily made and may be of service. In nearly all cases, however, power is delivered in varying quantities all along the shaft, and such computations are not only difficult to make but would indicate that the diameter of the shaft should vary at different parts of its length. This would be undesirable, as it is important that shafting, hangers, etc., should, as far as possible, be uniform and interchangeable for convenience and economy; and the practice of reducing the diameter of the shaft as it extends from the driving point is confined to larger shafting (say over 3″ in diameter). The design of shafts subjected principally to torsion, therefore, is usually based on the formula for torsional strength, modified by practical coefficients which experience has shown will provide for stiffness against torsion and bending.

Referring to equation E, Article 12,

$$d^3 = \frac{16\ T}{\pi\ p_s} \text{ or } d = \sqrt[3]{\frac{16\ T}{\pi\ p_s}} \quad . \quad . \quad . \quad . \quad (1)$$

If P be the equivalent force applied at the periphery of the shaft, so that $T = Pr$, where r is the radius of the shaft in inches; and if N be the number of revolutions of the shaft per minute; then the horse power transmitted will be

$$H.P. = \frac{2\,P\,r\,\pi\,N}{33{,}000 \times 12} = \frac{2\,T\,\pi\,N}{33{,}000 \times 12}$$

$$\text{or } T = \frac{33{,}000 \times 12 \times H.P.}{2\,\pi\,N} = \frac{63{,}024\ H.P.}{N}$$

Substituting this value of T in (1) above,

$$d = \sqrt[3]{\frac{321{,}400}{p_s} \times \frac{H.P.}{N}} = k\sqrt[3]{\frac{H.P.}{N}} \quad . \quad . \quad . \quad (2)$$

where k is a constant depending on the stress assigned. If shearing stress alone were to be considered, p_s might be taken as high as 9,000 lbs. per square inch, for steel shafting. In order to secure stiffness, and to provide for the indeterminate bending in line shafts, it is customary to assume a lower stress (or higher factor of safety), depending on the material used, and the service for which the shaft is intended. The larger and more important the shaft, the lower should be the working stress, as the failure of a head shaft or shaft of a prime mover is accompanied by great inconvenience and expense. The following factors of safety are indicated by successful practice:—

For head shafts	15
" line shafts carrying pulleys . . .	10
" small short shafts, countershafts, etc.	7

For steel shafting, the allowable stress for the above factors would be about 4,000, 6,000, and 8,500 respectively, whence

For head shafts,

$$d = 4.3 \sqrt[3]{\frac{H.P.}{N}} \quad \ldots\ldots \quad (3)$$

For line shafting carrying pulleys,

$$d = 3.75 \sqrt[3]{\frac{H.P.}{N}} \quad \ldots\ldots \quad (4)$$

For small short shafts, countershafts, etc.,

$$d = 3.36 \sqrt[3]{\frac{H.P.}{N}} \quad \ldots\ldots \quad (5)$$

It must, however, be borne in mind that a universal rule cannot be laid down for any class of shafting; and cases will always arise which need further consideration than given by the above equations. For example, in the span of shafting where the power is applied by a large belt the bending action may be excessive, and this particular span may have to be of larger diameter than the remainder of the shaft. The student is referred to handbooks* for tabulated data on the size of transmission shafts for various purposes. It is to be especially noted that a shaft carrying a transverse load, which applies a bending moment to the shaft, is subjected to a reversed stress as the shaft rotates. If, in addition, the twisting moment varies in magnitude, the factor of safety, owing to complete or partial reversal of stress (see Arts. 25 and 26), must be high, and this accounts for the low stresses allowable with such shafts.

122. Shafts Subjected to Torsion and Bending. In engine shafts, head shafts driven by heavy belts, and many others, the torsional stress is not predominant and may, in fact, be less than that due to bending. A full discussion of the relations which exist in this case has been given in Article 16 and it remains to show the application of this discussion to actual cases of design.

From Article 16 (equations K, K_1 and Fig. 9), it appears that if the bending and twisting moments can be determined for any section, the theoretical diameter of the shaft at that section

* See Kent's "Mechanical Engineer's Pocket Book," page 869.

can be found. Usually the twisting moment can be determined without difficulty, but the bending moment is often difficult to determine, and sometimes the designer must be content with an approximation. One of the greatest sources of uncertainty is the location of the reactions at the bearings. Usually, as already pointed out, the safe assumption is made that these reactions are concentrated at the centre line of the bearing. When the shaft is of appreciable length (15 or 20 diameters), the error is small; but in such cases as the crank shafts of multiple-cylinder engines, where the distance between the centres of bearings is only four or five diameters, or less, it is evident that the assumption is in the direction of excessive safety.

In line shafting, particularly with the usual swivel bearings, the error from this source is small, and at first sight the conditions of such shafting would appear to approximate those of a continuous beam. While such an assumption might be safely made when the shafting has been put in perfect alignment, it would not be safe as a general principle, as perfect alignment, even under best conditions, is of short duration, and bending stresses soon appear as a result of lack thereof. It appears, therefore, that, in this case, the safest procedure would be to treat each span as if disconnected at the bearing, when computing bending moments.

A typical case of combined twisting and bending is the engine shaft shown in Fig. 106 (a), the data taken being those of the example in Case (c), Art. 5. Here the shaft is supported by the bearings at the points X and X', as indicated, and carries a heavy generator spider at Y. The weight of this spider, and that of the shaft itself, with the probable magnetic pull which may occur when the shaft wears downward a little, is estimated at 22,000 lbs. The maximum pressure (P) on the crank pin, due to the steam pressure, is 25,000 lbs. This force is a maximum when the crank is about vertical, and, at that position, it exerts a twisting moment on the shaft from the crank to the point Y* where power is delivered, and also a bending moment on the shaft in a

* The reinforcing effect of the hub of the spider is neglected.

horizontal direction. The weight of the generator, etc., exerts a simple bending moment in a downward direction, and at right

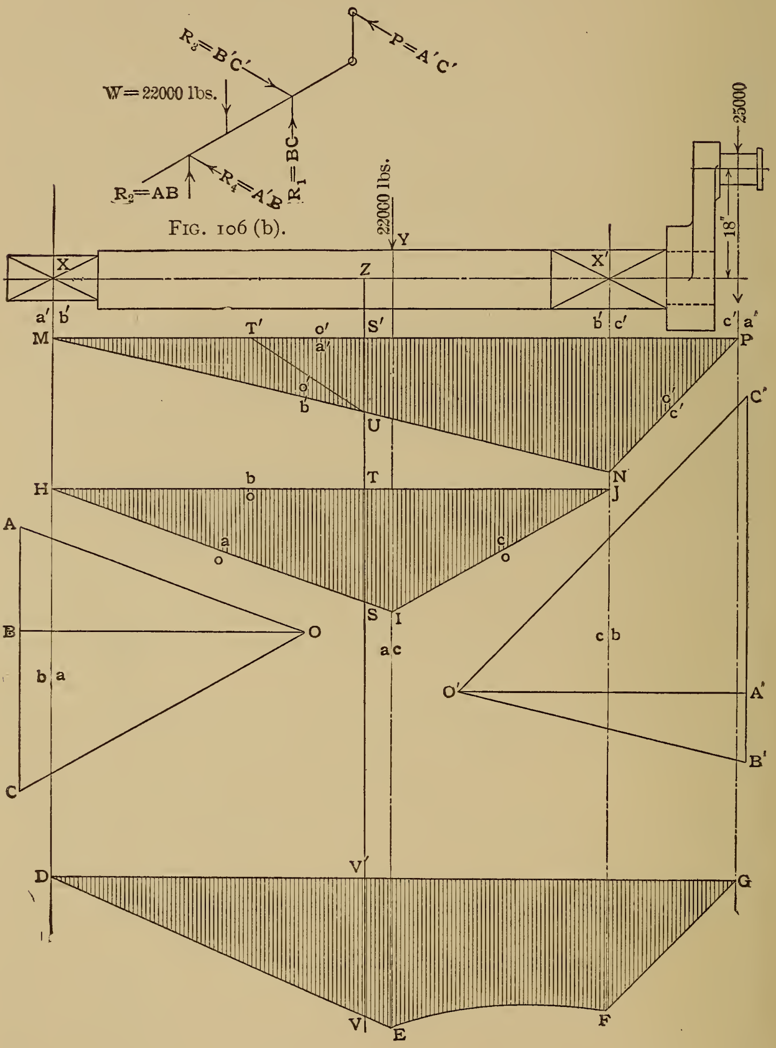

FIG. 106 (b).

FIG. 106 (a).

angles to that induced by P. Fig. 106 (b) shows, isometrically, the direction and point of application of the various forces and reactions, and it is required to find the maximum *equivalent* bending moment on the shaft.

It was shown in Example (c), Article 99, how a tentative solution could be made for the diameter and length of the main journal, thus fixing the distance of its centre line from the centre line of the crank pin at 21½″. Other data fix the distance between bearings as 7′ − 9″.

Graphical Analysis is here very convenient, and the order of procedure will be as follows:—

(a) Find the bending moment due to the steam pressure *P*.

(b) Find the bending moment due to the dead load *W*.

(c) Combine these bending moments to find the maximum resultant bending moment.

(a) Consider, for convenience, that the force *P*, and all the reactions due to it, have been rotated into the plane of the paper so that *P* is represented as acting vertically. Draw the force* diagram, *O′ B′ A′ C′* for force *P*, and the reactions due to it, to a convenient scale, here taken as 8,000 lbs. per inch, taking *O′* on a horizontal line through *A′*, thus making the closing string of the space diagram also horizontal, which is convenient for later work. Draw the space diagram, *M N P* for force *P*, and the reactions due to it, as shown. The scale of the space diagram is ¾″ to 1 ft. or $\frac{1}{16}$ size.

(b) In a similar manner construct the force diagram *A B C O*, for force *W*, and the corresponding space diagram *H T J I*, for the force *W*, *making the pole distance* = *A′ O′*; taken here as 3″.†

(c) To combine the bending moments at any section, as *Z*, take the intercept *S T*, on *H I J*, and lay it off as *S′ T′* on the diagram *M N P*. The distance *T′ U* is proportional to the combined bending moments and may be used as an ordinate *V V* in the diagram of combined bending moments *D G F E*.

It often occurs that the shaft carries a heavy flywheel at *Y*,

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* See also Article 120.

† Reduced to one-half size in cut.

instead of a generator, and a heavy belt may also run on the wheel. It is evident that the resultant force due to the weight of the wheel and the pull of the belt, can be determined, both in magnitude and direction. In general, the direction of this force will not be vertical, but will make an angle, ϕ, less than 90° with the direction of the force P. In such a case the moments may be combined by the triangle of forces taking into consideration the angle ϕ.

The numerical value of any moment is the continued product of the ordinate which represents it, the pole distance, the reciprocal of the scale of the space diagram, and the reciprocal of the scale of the force diagram. Thus the maximum bending moment, which occurs at

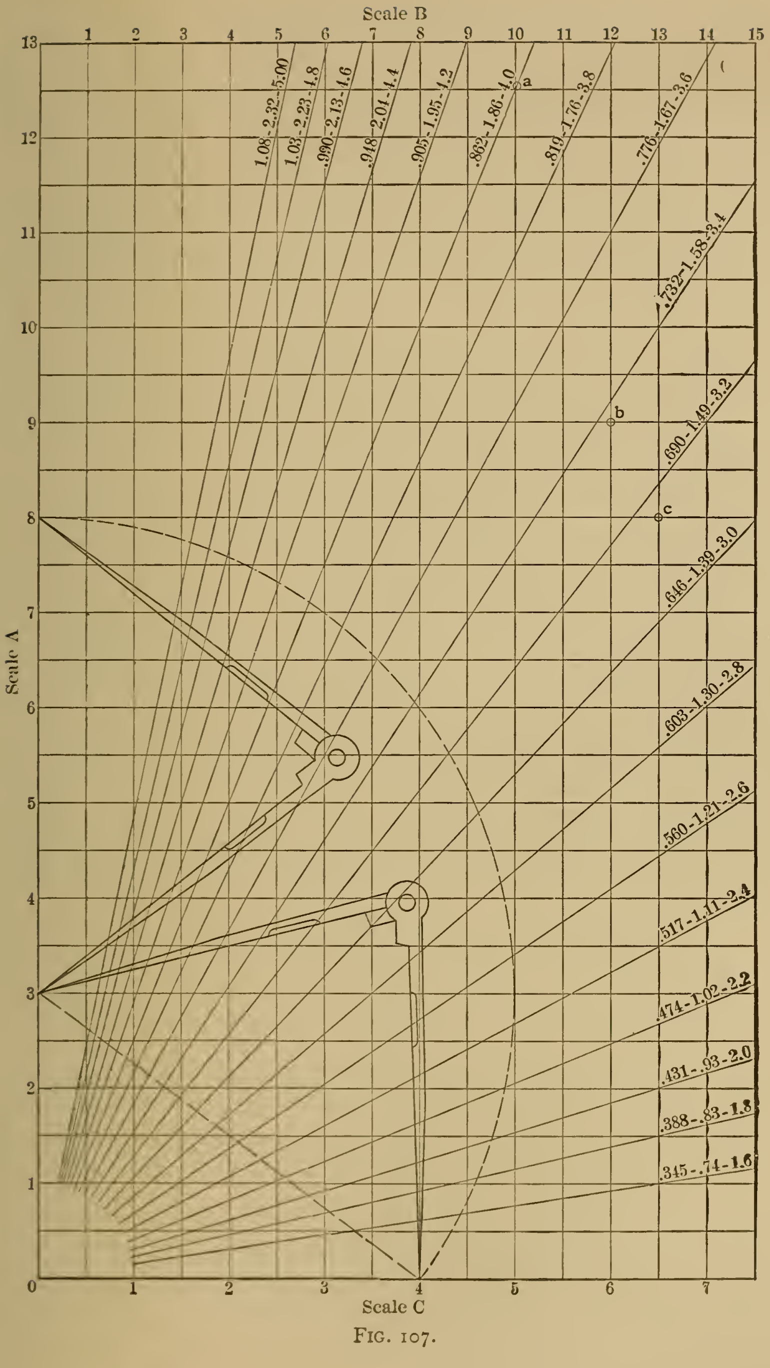
$$Y = 1\tfrac{5}{16}'' \times 3 \times \tfrac{16}{1} \times \frac{8,000}{1} = 485,400 \text{ inch pounds.}$$

The twisting moment is seen by inspection to be uniform over the whole length of the shaft which it affects. Its numerical value is, as before, $25,000 \times 18 = 450,000$ inch lbs.; and these two moments may be combined, to determine the safe diameter of the main part of the shaft according to the methods of Article 16. A graphical method will be given later, which somewhat facilitates the numerical work of this computation.

The methods outlined above are clearly applicable to any shaft which has not more than two points of support since in such cases the reactions can be readily found.

A convenient diagram is shown in Fig. 107 for determining the diameter of a shaft, of solid circular cross-section, subjected to any moment, and with any intensity of fibre stress from zero to 15,000 lbs. per sq. inch. This diagram can be used for either simple bending or twisting moments, or for combined bending and twisting actions. Its use in connection with problems involving simple twisting moments will be discussed first.

If T is the twisting moment, d the diameter of the solid circular shaft, and p the intensity of stress in the most strained fibres, $T = \frac{\pi}{16} p\, d^3$. Therefore, for a given diameter of shaft, T



Scale B
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Scale A
13 12 11 10 9 8 7 6 5 4 3 2 1 0
Scale C
1 2 3 4 5 6 7
1.08-2.32-5.00
1.03-2.23-4.8
.990-2.13-4.6
.948-2.01-4.4
.905-1.95-4.2
.862-1.86-4.0
.819-1.76-3.8
.776-1.67-3.6
.732-1.58-3.4
.690-1.49-3.2
.646-1.39-3.0
.603-1.30-2.8
.560-1.21-2.6
.517-1.11-2.4
.474-1.02-2.2
.431-.93-2.0
.388-.83-1.8
.345-.74-1.6
a
b
c

FIG. 107.

is directly proportional to p. Thus, if $d = 4''$, $d^3 = 64$, and $T = .196 \times 64p = 12.57p$. If p be taken as 10,000, $T = 125,700$ inch lbs. In Fig. 107 if ordinates represent moments (to the scale "A," of 500 inch lbs. to each division); and if abscissas represent intensity of stress (to the upper scale, "B," of 1,000 lbs. per sq. inch to each division), the point a corresponds to $T = 125,700$, $p = 10,000$, $d = 4''$. As the moment varies directly as the intensity of stress, *for any given diameter of shaft*, the relations between corresponding values of T and p (for a $4''$ shaft) will be represented by the straight line through the point a, and the origin O. In a similar manner straight lines through the origin are drawn for other shaft diameters.

To determine the diameter of shaft for a moment of 90,000 inch lbs., with a fibre stress of 12,000 lbs. per sq. inch, pass along the horizontal through the point marked "9" (or $T = 90,000$) on scale "A," to the vertical line through the point marked "12" (or $p = 12,000$) on scale "B." The intersection of this horizontal and vertical (b) lies a little below the diagonal marked 3.4 at its outer end; or the shaft should be about $3.37''$ (or $3\frac{3}{8}''$) diameter to give a stress of 12,000 lbs. per sq. inch.

The oblique line nearest to the point located in the last example bears three figures, viz.: ".732 – 1.58 – 3.4," and the other diagonals each bear three separate figures. The significance of these designations will be explained by further illustrations.

If $T = \frac{1}{10}$ of 90,000, or 9,000 inch lbs., and $p = 12,000$,

$$d = \sqrt[3]{\frac{16\,T}{\pi\,p}} = \sqrt[3]{\frac{16}{12,000\,\pi}}\sqrt[3]{\frac{90,000}{10}} = 3.37 \div \sqrt[3]{10} = 1.56'';$$

since d varies as the cube root of T, and when $T = 90,000$, $d = 3.37''$.

In a similar way, if $T = 900$, or $\frac{1}{100}$ of 90,000, $d = 3.37 \div \sqrt[3]{100} = .726''$.

To use the diagram when $T = 900$, and $p = 12,000$, consider scale "A" as representing the moment in 100 inch lbs.; pass along the horizontal through 9 on this scale to the vertical, through 12 of scale "B," as before, to the point "b," and take the *first* figure borne by the nearest diagonal (.732) as the approximate diameter of the shaft; or, by interpolation, find the diameter = .726″.

If $T = 9{,}000$, $p = 12{,}000$; consider scale "*A*" as representing the moment in 1,000 inch lbs., and read the *middle* figure on the nearest diagonal (1.58) as the required approximate diameter of the shaft; or, by interpolation, the diameter is found to be 1.56″.

If the moment is greater than 130,000, the diagram is quite as applicable as for smaller moments. Thus if $T = 900{,}000$ and $p = 12{,}000$, consider scale "*A*," as representing the moment in 100,000 inch lbs. The horizontal through 9 of scale "*A*" and the vertical through 12 of scale "*B*" intersect at "*b*" as before. The required diameter is about 7.26″; because the diameter was found to be about .726 for a moment of 900, and it must be 10 times as great for a moment of $10^3 \times 900 = 900{,}000$. For $p = 12{,}000$ with a moment of 9,000,000 inch lbs. ($= 10^3 \times 9{,}000$), the diameter is $10 \times 1.57 = 15.7''$, etc. It thus appears that the diagram covers all moments, without being of such impracticable size as it would be if it were not for the peculiar designation of the oblique lines and the method of using scale "*A*." The diagram can also be used for simple bending moments. The expression for the bending moment in a shaft of solid circular section is

$$M = \frac{\pi}{32} p d^3;$$

while the expression for a twisting moment is, as given above,

$$T = \frac{\pi}{16} p d^3.$$

Therefore, with a given diameter and numerically equal fibre stress, T is numerically equal to $2M$. To determine d for given values of p and M, multiply M by 2 to get the equivalent T, and with this value of T, proceed as in the former examples.

For finding the diameter appropriate to a combined bending and twisting moment, the equivalent twisting moment,

$$T_e = M + \sqrt{M^2 + T^2}$$

is to be determined; see Art. 16, equation K_4. This equivalent twisting moment is readily determined from the diagram by the use of scale "C" at the bottom of Fig. 107 and a pair of dividers, when the simple bending moment (M) and the simple twisting moment (T) are given. Example: Suppose $M = 30{,}000$; $T = 40{,}000$; and $p = 13{,}000$. Consider scales "A" and "C" to measure moments in 10,000 inch lbs. Take M at 3 *on scale* "A" with one point of the dividers, and T at 4 *on scale* "C" with the other point of the dividers; then the distance between 3 on scale "A" and 4 on scale "C" represents $\sqrt{M^2 + T^2}$. Swing the dividers about the point at 3 on scale "A" as a centre until the other point reaches scale "A" (at point 8); then o. .8 on scale "A" = $0.\ldots.3 + 3.\ldots.8 = M + \sqrt{M^2 + T^2} = T_e$. With the value of T_e, found in this way, proceed as in case of a simple twisting moment. The intersection of the horizontal through 8 (T_e) and the vertical through 13 (p) is at point "c." Since the moments correspond to units of 10,000 inch lbs. on scale "A," the largest figures of the diagonals are to be read in determining the diameter. The point "c" therefore indicates a diameter of between 3.0″ and 3.2″; by interpolation the diameter is taken as 3.15″. By computation the diameter is found to be 3.14″. A shaft $3\frac{3}{16}$″ diameter would be proper for this case. The use of the diagram in connection with equations K and K_1 of Table VI is obvious from the above.

The diagram of Fig. 107 is equally convenient for finding the intensity of stress in a given shaft under a known moment; or the moment on a given shaft corresponding to any intensity of stress. Thus, if a $7\frac{3}{4}$″ shaft is subjected to moment of 1,000,000 inch lbs., consider the moment units as 100,000 inch lbs., pass horizontally from 10 on scale "A" to a point slightly below the diagonal marked .776 (7.76″ diameter), and then vertically upward to scale "B," where the stress is read as about 11,000 lbs. per sq. inch.

If it is required to find the twisting moment corresponding to an intensity of stress of 9,000 lbs. per sq. inch on a shaft $1\frac{1}{2}$″ diameter; pass vertically downward from "9" on scale "B" to a point slightly above the diagonal marked "1.49"; then horizontally to 5.9 on scale "A." As 1.49 is the middle number on the diagonal, the moment units are 1,000 inch lbs.; therefore $T = 5.9 \times 1{,}000 = 5{,}900$ inch lbs.

123. **Torsional Stiffness and Deflection of Shafting.** When a shaft has considerable length, the matter of torsional stiffness is important. A rule, common in practice, is to limit the twist in the shaft to one degree for every 20 diameters in length. Another rule limits the twisting to 0.075 degree for every foot in length. The **lateral deflection** of the shaft should not exceed $\frac{1}{100}''$ per foot of length, to insure proper contact at the bearings. Theoretical considerations, however, do not enter so largely into the spacing of bearings of line shafting, as does the construction of the framework to which the bearings are fastened. Care should be exercised in laying out such structures, that provision is made for fastening the hangers close enough together to avoid excessive deflection. For the average range of velocities found in practice the following formulæ* can be used for ordinary small shafting.

$$L = 7\sqrt[3]{d^2} \text{ for shaft without pulleys} \quad . \quad . \quad . \quad (1)$$

$$L = 5\sqrt[3]{d^2} \text{ for shaft carrying pulleys} \quad . \quad . \quad . \quad (2)$$

where L = distance between hangers in feet and d = diameter of shaft in inches.

If T be the twisting moment in foot lbs. applied to a shaft, then the power transmitted at N revolutions per minute is $2T\pi N$; from which it appears that the greater the velocity of the shaft, the smaller is the required turning moment, for a given amount of power transmitted.

If a slightly deflected shaft is rotated, centrifugal force, acting on the eccentric mass of the shaft, tends to equalize the forces which hold the shaft deflected in one plane and to **whirl** the shaft as a whole around the axis of rotation. At low speeds the action of centrifugal force is small, and the deflecting force will hold the shaft deflected in its plane. As the effect of centrifugal force increases with the velocity, while the effect of the deflecting force is constant, it is clear that as the speed is increased the centri-

* See also Kent's "Mechanical Engineer's Pocket Book," page 869.

fugal force will, at some speed, balance the effect of the deflecting force, and the shaft will become unstable. Beyond this speed the shaft will whirl about the central axis. For a given diameter of shaft there is one definite speed within which it will maintain a stable condition with a given deflection.

If L = distance between bearings in feet, d = diameter of shaft in inches, and N = the revolutions per minute, then for the critical speed *

$$L = 175\sqrt{\frac{d}{N}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This equation refers to the bare shaft only and it determines the maximum safe span. Where pulleys are carried at some distance from the bearings, the span, L, must be less than the value given by equation (3) on account of the added mass of the pulleys, and the great liability of the latter to be unbalanced. The speed of shafting in practice is, almost always, considerably below the critical speed.

124. Practical Considerations, Hollow Shafting, etc. Shafting up to 3″ in diameter is, in this country, made of cold-rolled steel. Such shafting is true and straight and needs no turning whatever. If keyways are cut the shaft must, in general, be carefully straightened afterward, as the cutting relieves, locally, the skin tension due to the cold-rolling † thus causing the shaft to warp. Larger sizes of shafting are forged and machined.

The use of hollow shafts not only reduces the weight for a given strength, but the removal of the metal from the core of a steel shaft (or of the ingot from which it is made) very greatly increases its reliability under repeated application of stress.

Shortly after a steel ingot is cast, the exterior solidifies and becomes comparatively cool while the internal portion is still fluid. The subsequent contraction, during complete cooling, is much less in the exterior walls than it is in the hotter interior mass. Unless the interior is "fed" during this period, it will be less dense than the outer portions and shrinkage cavities are apt to

* See Rankine's "Millwork," page 549.

† See Article 12.

be present near the centre of the ingot. Numerous expedients have been adopted to reduce this evil, among which is "fluid compression," or subjecting the ingot to heavy pressure immediately after it is poured. The difficulty is not entirely overcome by such means, however, as the walls of large ingots become too rigid to yield to the pressure before the interior is entirely solidified. The external walls "freeze," after which the internal shrinkage is made up by metal flowing from the upper portion toward the bottom as long as any of it remains fluid. This leaves a shrinkage cavity at the upper end of the ingot. Gas liberated during cooling collects in this cavity also. The result of these two actions is to form what is called the "pipe," which frequently extends to a considerable depth. The top end of the ingot is cut off and remelted, but this does not insure removal of all of the pipe, and it also involves much expense. If the portion cut off is not sufficient to remove all of the pipe, a piece rolled or forged from the ingot contains a flaw near the centre which is drawn out into a long crack if the ingot is worked into a long piece. The rolling and forging may squeeze the sides of the cavity together so that it is not easily detected at any section, but as this work is done at a temperature much below that corresponding to welding, the defect is not removed. This flaw is more or less irregular or ragged; hence its form is favorable to starting a fracture, under variations of stress, which may finally extend far enough to cause rupture.

If the ingot is bored out, the pipe is effectually removed, and the metal remaining is superior to that of a solid shaft. It will be evident that casting a hollow ingot is not the equivalent of boring out one which was cast solid; for if the ingot is cast hollow the outer and inner walls cool before the intermediate mass does, and the shrinkage effect takes place in the latter. In fact, a shaft made from a hollow ingot is worse than the solid shaft, in the respect that the former has the defective material nearer the outer fibres where the stress is greater.

COUPLINGS AND CLUTCHES

125. General Description. Couplings are machine members which fasten together the ends of two shafts, so that rotary motion of one causes rotary motion of the other. Where the connection is to be broken only at rare intervals, as in making of repairs, the couplings are generally constructed so that they must be partially or wholly dismantled to separate the shafts. Such couplings are known as **permanent couplings.** When it is desired to disengage the shafts at will, the coupling is of a different construction and is generally known as a **clutch.*** The use of clutches is not, however, confined to securing together the ends of shafting, but they are much used for engaging and disengaging pulleys at will, in connection with the shafts on which they are placed. For this service clutches making use of friction are much used, and this particular type is discussed in Chapter XIII.

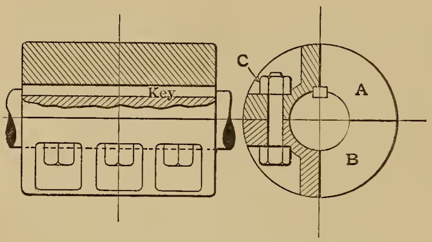


FIG. 108.

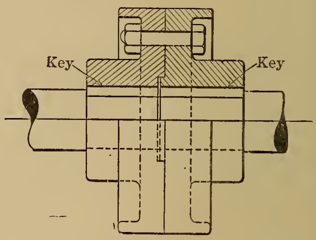


FIG. 109.

Couplings should be placed near a bearing, so as to bring the joint in the shaft near a supported point, and should be placed on the side of the bearing farthest away from the point where power is applied, so that when the shaft is disconnected the running part is supported near the end.

126. Permanent Couplings. Where the axes of the two shafts to be connected are parallel and coincident, couplings such as are shown in Figures 108, 109, 110, and 111 are used. Fig. 108 illustrates a type of coupling known as a **split-muff** coupling.

* See Transactions A. S. M. E., 1908, for a full description and discussion of various forms of clutches.

The parts A and B are separated by a small space and can, therefore, be clamped to the shaft by the bolts C. For heavy work a key as shown is provided, but in lighter shafting friction alone may suffice to prevent relative rotation.

Fig. 110 shows the **Sellers Muff Coupling.** Here the circular tapered wedges B, B, are drawn inward by the bolts C. The wedges are split as shown at D, hence the tighter they are drawn inward the more firmly they clasp the shaft. For light work no key is necessary, but for the full capacity of the shaft keys are advisable.

Couplings such as shown in Figures 108 and 110 are regularly

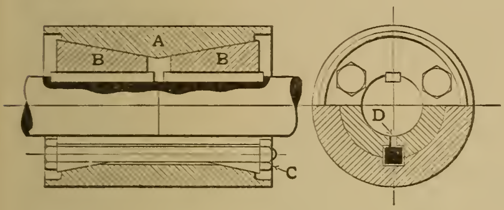


FIG. 110.

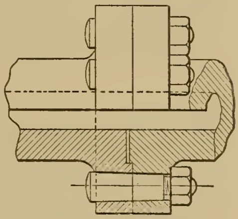


FIG. 111.

manufactured in standard sizes, and the student is referred to the trade catalogues of manufacturers for dimensions and capacities of such couplings.

The **Flange Coupling,** Fig. 109, is one of the most common and also one of the most effective forms of permanent couplings. The general proportions are usually designed empirically, but the bolts should be designed so that their combined resistance to a torsional moment, around the axis of the shaft, will be at least as great as the torsional strength of the shaft itself; and the bolts should be accurately fitted so as to distribute the load evenly among them.

Let D = diameter of the shaft in inches
d = diameter of the bolt in inches
n = the number of bolts
r = radius of bolt circle in inches
p_s = allowable shearing stress per square inch, for steel.

$$\text{Then } \frac{\pi D^3}{16} p_s = \frac{\pi d^2}{4} n r p_s$$

$$\text{whence } d = .5 \sqrt{\frac{D^3}{n r}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Good practice gives $n = 3 + \frac{D}{2}$, but this number may be modified for convenience in spacing, etc. The bolts should be carefully fitted to insure that each one carries its full share of the load. The projecting outer flange is an important feature as it covers the revolving bolt heads, thus protecting workmen from becoming entangled. For best results the flanges should be pressed on to the shaft and the faces trued up in place, thus

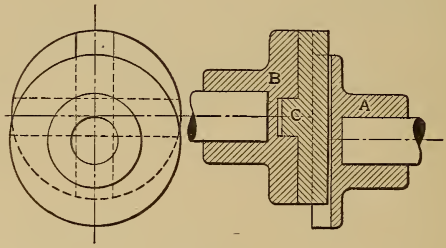


Fig. 112.

insuring greater accuracy of alignment. This should be done in all good work.

When great strength and reliability are desired, as in marine work, the flange is sometimes forged solid with the shaft, as in Fig. 111. Here the bolt holes are sometimes bored tapering, and reamed after the flanges are placed together, thus insuring a perfect fit for the bolts, and also facilitating their withdrawal.

When the axes of the two shafts are parallel, but not coincident, or when there is danger of parallel and coincident axes wearing out of coincidence, **Oldham's Coupling**, Fig. 112, is often used. It consists of two heavy flanges (*A* and *B*), each keyed fast to its own respective shaft, and an intermediate disc *C*. The disc has a tongue running diametrically across each face, these tongues being placed at right angles to each other and fitting into grooves cut in the flanges. With this coupling the rate of rotation of the driven shaft is identical with that of the driver, or, in other words, the angular velocity is the same. The coupling is often used on the propeller shafts of small power boats.

If the axes of the two shafts, *A* and *B*, Fig. 113, intersect and make an angle θ with each other they may be coupled together by means of a **Hook's Coupling** or **Universal Joint,** as it is often called. In this coupling each shaft is fitted with a jaw *D* which is pin-connected to an intermediate member *F*. The holes in this intermediate member for receiving the pins *G* are at right angles to each other. With this arrangement the angular velocity of the driven shaft is not the same at all points of the revolution as that of the driver.* The construction shown in Fig. 113

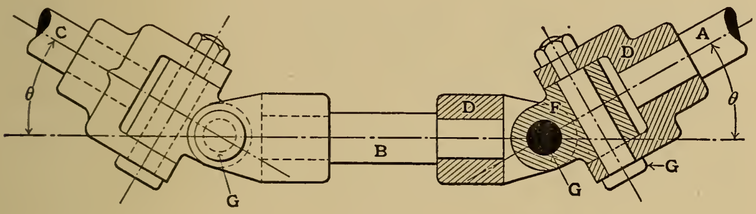


FIG. 113.

is very common, but the difference between the angular velocity of the driver and that of the driven shaft is less when the construction is such that the axes of the pins *G* intersect. The construction required to make the axes of the pins intersect is usually more complicated than that shown in Fig. 113, and hence in rough work the simpler design is adopted.

If another shaft *C* be coupled to *B* so that *A* and *C* make the same angle θ with *B;* if also the pins *G*, *G* in *B* are parallel to each other and all three shafts lie in the same plane; then the angular velocity of *C* will be identical with that of *A* and *vice versa.* Empirical practice makes the diameter of the pin *G* equal to one-half the diameter of the shaft.

128. Positive Clutches. Positive clutches are much used for

* See "Kinematics of Machinery," J. H. Barr, page 198.

starting and stopping such machines as punch presses which must work intermittently. They are made in so many forms that a description of them would be beyond the scope of this work. A very full description of many forms is given in the Transactions of the A. S. M. E., Vol. XXX, to which reference has already been made. Fig. 114 illustrates the most common form of disengaging coupling for heavy work. The part *B* is made fast to the shaft to be driven, while part *A*, which is compelled to rotate by the feather *F*, can be moved axially along the driving shaft. A ring *R*, fitting the groove *G* loosely in a radial direction, is connected by the pins *P* to an operating lever

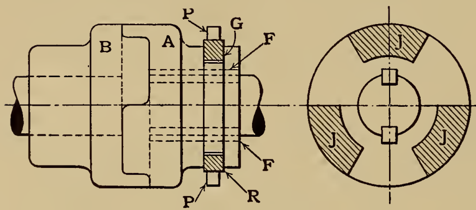


FIG. 114.

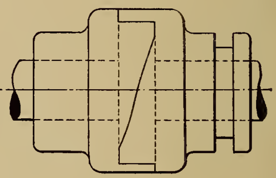


FIG. 115.

which is not shown. When the part *A* is moved forward till the jaws *J* engage, *A* will drive *B* positively in either direction. In order to facilitate the engaging of the jaws they are often made as in Fig. 115, but in this case the driving can be in one direction only. The total cross-sectional area of the jaws must be such that they will not shear off under the load, and the area of the jaw faces must be sufficient to prevent crushing.

Frequently, for light work, only one feather is used, but two feathers are, in general, better, both on account of the driving effort and for ease of operation.

129. Flexible Couplings. Where it is desirable to have a small amount of flexibility in a shaft, a **flexible coupling,** such as is shown in Fig. 116, is employed. These members are much used for connecting rapidly revolving machines to prime movers, as in the case of a dynamo directly coupled to a steam engine, the object being to prevent undue stress, or bearing pressure, from lack of accurate alignment of the two shafts. In the construction shown, the shafts *A* and *B* are fitted with heavy flanges, *F*, which carry pins, *P*. Links of leather or other elastic material connect pins on one flange with pins on the other, there being as many links as there are pins in each flange. This arrangement

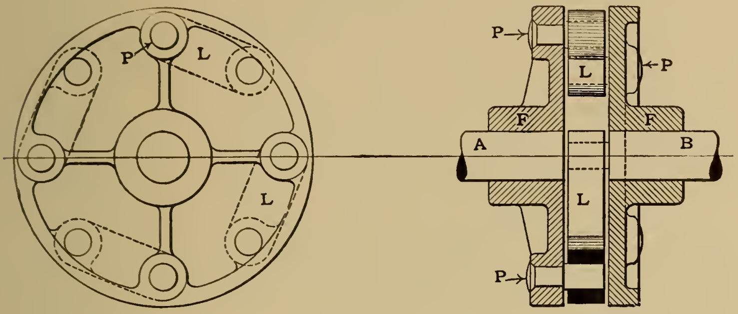


FIG. 116.

allows for a slight angle between the axes of the shafts, or for a small lack of coincidence in the axes. The pins in one disc are sometimes placed on a smaller diameter than those on the other, so that in case of failure of the links the pins will not strike and cause breakage.

CHAPTER XII

BELT, ROPE, AND CHAIN TRANSMISSION

130. General Considerations. When power is to be transmitted from one shaft to another, especially when such shafts are not far apart, in such a manner that the velocity ratio of the two must be constant, some form of toothed gearing is usually employed. When, however, it is not necessary that the velocity ratio remain constant, flexible elastic connectors are much used. When the distance through which power is to be transmitted is comparatively short (50 feet or less), flat belts, or ropes of cotton or manila, are most common; while for longer distances steel ropes have certain advantages. For small amounts of power, round belts of leather are much used. Chain drives, which are virtually flexible connectors running on toothed wheels, have lately come into extended use for transmitting power over comparatively short distances. They are very efficient, maintain positive velocity ratio between the two shafts, and can be used when the distance between shafts is too great for convenient use of gears.

Leather belts are made by cementing, sewing, or riveting together strips of leather cut from oak-tanned ox-hides. Where only one thickness is used they are known as **single leather** belts; where two, three, or four thicknesses are needed to obtain a heavy belt, they are known respectively as **double, triple,** and **quadruple** belts. Cotton belts are made either by weaving in a loom, or are built up of several layers of canvas, sewed together, with a special composition between each fold. They are very little used in this country. Rubber belts are made of several layers of canvas, held together with, and completely covered by a rubber composition. They are very effective in wet places. Belts of rawhide are also used to some extent.

The ends of all belts are joined, to make them continuous, either by lacing or sewing, or by some kind of special fastening of which there are many on the market, or by making a permanent joint by cementing and riveting. The latter method is much preferable where it can be applied, as it makes the joint practically as strong as the rest of the belt, and gives a smooth surface which runs better than any joint. Other kinds of joints reduce the strength of the belt from 60 to 75 per cent, but inasmuch as the lacing can be replaced and the belt itself has its life prolonged by reduced load, this initial loss of efficient strength is not as wasteful as it at first appears.

131. Theoretical Consideration of Belts and Ropes. In Fig. 117, let A represent a pulley whose centre is at O, and which is

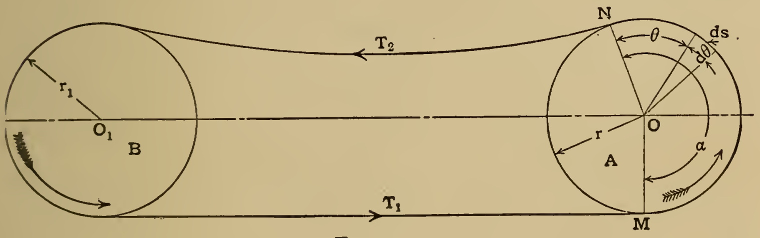


FIG. 117.

connected by a belt as shown to the pulley B, whose centre is at O_1. When no turning moment is applied to the driving pulley A, the tensions in the two parts of the belt are the same, except possibly for friction of the bearings, and is that due to the initial tension with which the belt is placed upon the pulleys. Let this *total* initial tension on each side of the belt be called T_3.

It is evident that this initial tension will cause the belt to exert a pressure upon the pulley, and this pressure will induce a frictional resistance opposing relative sliding between the belt and the pulley. If now a turning moment is applied to A, and a resisting moment to B, the pull upon the belt due to this frictional resistance will *increase* the tension in the lower part of the belt, and *decrease* the tension in the upper part. Let these new *total* tensions be called T_1 and T_2 respectively. It is evident that the tendency of the belt to slip around the pulley, owing to the difference in tension on the two parts of the belt, is resisted by the frictional resistance between the belt and pulley. The difference in tensions tends to rotate the pulley B, and when the turning moment $(T_1 - T_2)r_1$ becomes equal to the resisting moment applied to B, rotation will take place.

If the difference between T_1 and T_2 which is necessary to overcome the resisting moment, is small compared to the frictional resistance between the pulley and belt, no slipping of the belt on the pulley will occur. To obtain this result in practice, would necessitate the use of very large belts, relatively, for the power transmitted. It has been found to be better practice to use smaller belts and allow the belt to **slip** somewhat.

In addition to the slipping action noted above, all belts are subjected to what is known as **creep.** Referring again to Fig. 117 consider a piece of the belt of unit length moving on to the pulley under a tension T_1. As this piece of belt, of unit length, moves around with the pulley from M to N, the tension to which it is subjected decreases from T_1 to T_2 and the piece, owing to its elasticity, shrinks in length accordingly. The pulley A, therefore, continually receives a *greater* length of belt than it delivers, and the velocity of the surface of the pulley is *faster* than that of the belt which moves over it. In a similar way the pulley B receives a *lesser* length of belt than it delivers, and its surface velocity is *slower* than that of the belt which moves over its surface. This creeping of the belt, as it moves over the pulley, results in some loss of power, and is unavoidable. The total loss of speed due to both slip and creep should not exceed 3%; that is, the surface speed of the driving pulley should not exceed that of the driven pulley by more than 3%. Good practice limits this value to about 2%. When the total slip approaches 20%, there is danger of the belt sliding off of the pulley entirely.

Since the pulling power of a belt is proportional to the difference between T_1 and T_2, it is necessary to know the relation which exists between these quantities.

Let $t =$ the tension per square inch of belt *section* at *any* point on the pulley.

t_1 = the tension per square inch of belt *section* on the *tight* side in pounds.

t_2 = the tension per square inch of belt *section* on the *slack* side in pounds.

p = the maximum allowable tension per square inch of belt in pounds.

f = effective pull of belt per square inch of cross-section $= (t_1 - t_2)$, in pounds.

v = the velocity of the belt in feet per second.

w = the weight of one cubic inch of belt in pounds.

q = the reaction of pulley against one linear inch of belt of the width considered, in pounds.

c = the centrifugal force of one cubic inch of belt in pounds at the given speed.

μ = the coefficient of friction between belt and pulley.

r = the radius of the pulley in inches.

a = the angle of belt contact in degrees.

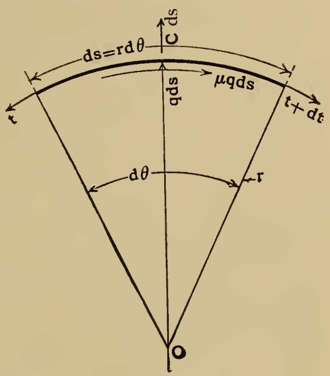


FIG. 118.

θ = the angle of belt contact in radians $= .0175\ a$

The centrifugal force of one cubic inch of belt will be $c = \dfrac{12\,w v^2}{g\,r}$; hence the centrifugal force of one linear inch of belt having 1 square inch of cross section will be $\dfrac{12\,w\,v^2}{g\,r}$.

Let the cross-sectional area of the belt be one square inch and consider an elemental portion of its length as shown in Fig. 118. It is held in equilibrium, when slipping is impending, by the following forces:—

(*a*) The centrifugal force $= c\ ds$

(*b*) The radial reaction of the pulley against the belt $= q\ ds$.

(*c*) The frictional force $= \mu\ q\ ds$.

(*d*) The tensions t and $t + dt$.

Resolving all forces vertically

$$q\,ds + c\,ds = t \sin \frac{d\,\theta}{2} + (t + d\,t) \sin \frac{d\,\theta}{2} \quad . \quad . \quad (1)$$

Here $d\,\theta$ is so small that $\sin \frac{d\,\theta}{2}$ may be taken as equal to $\frac{d\,\theta}{2}$ in radians, without appreciable error, and the product of $d\,t$ and $\sin \frac{d\,\theta}{2}$ may be neglected.

Hence (1) may be written

$$q\,ds + c\,ds = t\,d\theta \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{but } c = \frac{12\,w\,v^2}{g\,r} \text{ and } d\,s = r\,d\,\theta$$

$$\therefore c\,ds = \frac{12\,w\,v^2}{g\,r}\,ds = \frac{12\,w\,v^2}{g}\,d\,\theta = z\,d\,\theta \text{ for convenience.}$$

Hence from (2) $q\,ds = t\,d\theta\ - z\,d\,\theta = (t - z)\,d\,\theta \quad . \quad . \quad . \quad (3)$

From equality of moments around O

$$t + dt = t + \mu\,q\,ds$$

$$\therefore d\,t = \mu\,q\,ds \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Substituting in (4) the value of $q\,ds$ obtained from (3)

$$d\,t = \mu\,(t - z)\,d\,\theta$$

$$\therefore \int_{t_2}^{t_1} \frac{d\,t}{t - z} = \mu \int_0^{\theta} d\,\theta$$

$$\text{or } \log_e \frac{t_1 - z}{t_2 - z} = \mu\,\theta \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\text{and common } \log \frac{t_1 - z}{t_2 - z} = 0.439\,\mu\,\theta$$

$$\therefore \frac{t_1 - z}{t_2 - z} = 10^{0.434\,\mu\,\theta} = 10^{0.0076\,\mu\,a} = 10^{k} \text{ for convenience} \quad . \quad (6)$$

Now $f = t_1 - t_2 \quad \therefore t_2 = t_1 - f$ and substituting this value of t_2 in (6) and reducing

$$t_1 = \frac{[f + z]\,10^k - z}{10^k - 1} = \frac{f}{C} + z \quad . \quad . \quad . \quad . \quad (7)$$

$$\text{where } C = \frac{10^k - 1}{10^k}$$

$$\text{and } f = [t_1 - z]\left[\frac{10^k - 1}{10^k}\right] = [t_1 - z]\,C \quad . \quad . \quad (8)$$

If (8) be multiplied through by $\frac{v}{550}$ it will express the horse-power (h.p.) which a belt of one square inch cross-sectional area will transmit or,

$$\frac{fv}{550} = h.p. = [t_1 - z]\,\frac{Cv}{550} \quad . \quad . \quad . \quad . \quad (9)$$

132. Practical Coefficients. In the above equations the following quantities a, μ and z, must be known or assumed before a solution for t_1 or f can be made. The **angle of contact,** a, can be taken from the drawing of the drive in question, and some allowance should be made for the conditions of operation. Thus if the belt is to run in a horizontal position, with the slack side on top, the full theoretical value of a may be taken. If, however, the slack side must be on the bottom (an arrangement which should be avoided if possible) or if the belt is to be run in a vertical position, some reduction must often be made in the theoretical value of a to allow for sagging of the belt. This also applies to belts running at high speed, where centrifugal force tends to lessen the arc of contact.

The **coefficient of friction** μ is an exceedingly variable quantity, changing with the character and the condition of the surfaces of contact, the initial tension of the belt, and the rate of slip. It has been found by experiment that, within reasonable limits, the coefficient increases with the slip and that, as before stated, a maximum rate of slip, including creep, not in excess of about 3 per cent is good practice. Experiments made by Professor Diederichs in the laboratories of Sibley College gave the values of μ

shown in the first column of the following table. Allowing for the difference between conditions in the laboratory and those found in practice, the value shown in the second column may be used in designing leather belts.

For pulleys made of pulp, μ = 0.29............0.20
For pulleys made of wood, μ = 0.31............0.22
For pulleys made of cast iron, μ = 0.46............0.30

Values considerably above these were found for paper pulleys of special construction.

The quantity z is proportional to the **weight of the belt per cubic inch.** For ordinary leather (which is most commonly used), w may be taken from 0.03 to 0.04, an average value being 0.035 pounds.

Table XVIII has been calculated with a value of $w = 0.035$, while Table XIX is abbreviated from "Transmission of Power by Belting"* by Wilfred Lewis.

TABLE XVIII

Values of $z = \frac{12\, w\, v^2}{g}$, for v = ft. per sec., or V = ft. per minute, w = .035.

v	30	40	50	60	70	80	90	100	110	120	130	140
V	1,800	2,400	3,000	3,600	4,200	4,800	5,400	6,000	6,600	7,200	7,800	8,400
z	11.75	20.9	32.5	47.0	64.2	83.4	105.5	130.5	157.6	187.6	220.2	255.5

Example. Design a belt to operate a dynamo of 15 H.P. capacity, when the belt velocity is 2,400 ft. per minute. Assume $\mu = 0.30$, $a = 180°$ and $t_1 = 200$ lbs.

From equation (9) the horse-power transmitted by a belt having a cross-sectional area of one square inch is for these conditions:

$$h.p. = [t_1 - z]\ \frac{C\,v}{550} = [200 - 20.9]\ \frac{.61 \times 40}{550} = 7.9$$

$$\therefore \text{ the cross-section required } = \frac{15}{7.9} = 1.9 \text{ sq. in.}$$

which is equivalent to a belt $\frac{7}{32}''$ thick and 8″ wide.

* Transactions A. S. M. E., Vol. VII, page 579.

The total tension (T_1) in the tight side of the belt will be $1.9 \times 200 = 380$ lbs. The total tension (T_2) in the slack side will be this value minus the required effective pull, P, which is found by dividing the foot pounds of work to be done by the velocity of the belt or, $P = \frac{15 \times 33{,}000}{2{,}400} = 206$. Hence $T_2 = T_1 - P = 380 - 206 = 174$ pounds.

TABLE XIX

Values of $C = \frac{10^k - 1}{10^k}$ (Nagle)

μ	Degrees of Contact = α									
	90	100	110	120	130	140	150	160	170	180
.15	.210	.230	.250	.270	.288	.307	.325	.342	.359	.376
.20	.270	.295	.319	.342	.364	.386	.408	.428	.448	.467
.25	.325	.354	.381	.407	.432	.457	.480	.503	.524	.544
.30	.376	.408	.438	.467	.494	.520	.544	.567	.590	.610
.35	.423	.457	.489	.520	.548	.575	.600	.624	.646	.667
.40	.467	.502	.536	.567	.597	.624	.649	.673	.695	.715
.45	.507	.544	.579	.610	.640	.667	.692	.715	.737	.757
.55	.578	.617	.652	.684	.713	.739	.763	.785	.805	.822

Equations (7) and (8) involve the relations which exist between T_1 and T_2 for a given set of conditions, but they do not indicate the relation between them and the initial tension T_3. It was formerly supposed that the sum of T_1 and T_2 was constant and equal to $2T_3$; and this relation may still be used for very rough calculations. Mr. Wilfred Lewis* has shown, experimentally, that this is not true. The ratio of stress to strain in leather and rubber *increases* with the strain instead of being *proportional* to it as in ductile metals. When a belt transmits power the tension is *increased* on the tight side and *decreased* on the slack side till the difference in tension is equal to the required driving force.

* See Transactions A. S. M. E., Vol. VII, page 566.

This is accomplished by what virtually amounts to shortening the belt on the tight side, a given amount, by transferring this amount to the slack side. Because, however, of the relation between stress and strain noted above, the increase of tension on the tight side, due to this amount of shortening, is greater than the decrease of tension on the slack side due to an equal amount of lengthening, and, as a consequence, the sum of the two tensions is increased* as the effective pull is increased. Suggestion: Place a rubber band over the fingers of the two hands and stretch it moderately; then twist one of the hands in either direction and the increase of force tending to bring the hands together will be apparent.

In the case of a long horizontal belt the increase in the sum of the tensions is still further augmented in driving, because the tension on the slack side (with a proper initial tension in the belt) is largely due to the sag of the belt from its own weight; and thus the tension on the slack side tends to remain nearly constant, while the tension on the tight side increases with the power transmitted, at a given speed. It is found that the sum of the tensions on the two sides, when driving, may exceed the sum of the initial tensions by about 33 per cent in vertical belts, and in horizontal belts the increase may be limited only by the strength of the belt. In addition to the causes discussed, the tension on both parts of the belt are increased by the centrifugal action due to the mass of that portion of the belt which is rotating round the pulley axis. This latter cause increases the stresses on both the tight and slack sides of the belt, and decreases adhesion between the belt and the pulley, but does not increase the loads on the shafts which produce pressure at the bearings and flexure of the shafts.

Large belts should therefore be put on with care, as to initial tension. Ordinarily, the initial tension is left to trained judgment, but it would seem that the more advanced practice of splicing the belt under a known initial tension will add to the life of large and important belts.

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* See Transactions A. S. M. E., Vol. VII, page 569.

133. Strength of Belting. The ultimate strength of good leather belting will vary from 3,500 to 6,000 pounds per square inch. Professor Benjamin* gives the strength of cotton belting as about the same as good leather. He also found that four-ply rubber belting had a tensile strength of from 840 to 930 pounds per inch of width. The ultimate strength of belting seldom enters as a factor in belt design, as the real strength of the belt is in the joint. Where the ends of the belt are laced together, a maximum working stress of 200 to 300 pounds per sq. inch is found to be good practice; and where the belt is cemented together, thus making it "endless," a working stress of 400 pounds per square inch may be used. The thickness of leather belting varies from $\frac{3}{16}$ to $\frac{7}{32}$ inch for single leather, and from ⅜ to ½ inch for double leather. Hence for single leather,

p = 50 to 75 pounds per inch of width for laced belts.

p = 100 pounds per inch of width for cemented belts.

For double leather belts p may be taken at twice these values. Lower stresses than these are often advocated, and undoubtedly lower stresses increase the life of the belt.

134. Velocity of Belting. In equation (8) when $z = t_1$, $f = 0$ and the belt will exert no turning force, the centrifugal force relieving all frictional resistance between the belt and pulley.

If t_1 be taken as high as 400 pounds, and $w = .035$ this will occur when $z = 400$ or when $\frac{12\,w\,v^2}{g} = 400$ whence $v = 175$ ft. per second or 10,500 feet per minute.

If equation (8) be multiplied through by v, the velocity of the belt, it will express the rate at which energy is being delivered, or

$$f\,v = v\,[t_1 - z]\;\;C = v\left[t_1 - \frac{12\,w\,v^2}{g}\right]C$$

If now $\mu = .3$, $w = .035$, $a = 180$, which are average conditions, the equation becomes

$$f\,v = v\,[t_1 - .013\,v^2] \times 0.6 = 0.6\,t_1\,v - .0078\,v^3$$

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* See "Machine Design," by Benjamin, page 186.

Differentiating the right-hand side with respect to v and equating to zero

$$0.6\,t_1 - .0234\,v^2 = 0 \quad \text{or} \quad v = 5.1\sqrt{t_1} \quad . \quad . \quad (10)$$

which gives the relation between v and t_1 for maximum power. When $t_1 = 400$, $v = 102$ feet per second or 6,120 feet per minute and when $t_1 = 275$ pounds, $v = 85$ feet per second, or 5,100 feet per minute. It is often necessary to run belts at much lower speeds than these; but it is not economical to exceed these limits. A speed of a mile per minute may be taken as about the economical maximum limit; and it so happens that this is also about the limit of safety for ordinary cast-iron pulley rims. For durability combined with efficiency, a speed of 3,000 to 4,000 feet per minute may be taken as a fair value, though practical limitations such as speed of shafting and diameter of pulleys often fix belt velocities at much lower values.

135. Efficiency of Belting. The losses of power in belt transmission consist of the loss due to slip and creep, that due to bending the belt over the pulley, and the frictional losses at the shaft bearings, due to belt pull. The first two, slip and creep, should not exceed 3 per cent, and 2 per cent is better. The loss due to bending the belt is, usually, negligible although the effect on the life of thick belting running on small pulleys is important. The losses at the bearings may be considerable if the belt must be laced on under great initial tension in order to carry the load, and this condition should be avoided except where it is absolutely necessary to use a short belt. A well-designed belt transmission should have an efficiency at least as high as 95 per cent, and it may be as high as 97 per cent including bearing losses.

136. Other Equations, Common Rules. If in equation (9), w be taken as 0.032 and t_1 as 305 pounds the equation reduces to

$$h.\,p. = [\,.55 - 0.0000216\,v^2]\,v\,C \quad . \quad . \quad . \quad (11)$$

where $C = \dfrac{10^k - 1}{10^k}$ as before and $h.\,p.$ = horse-power per square inch of belt area. If the equation be multiplied by A, the area of the belt cross-section, it will express the total horse-power transmitted, or $\text{H.P.} = [\,.55 - 0.0000216\,v^2]\,v\,C\,A \quad . \quad . \quad . \quad . \quad (12)$

Professor Diederichs has pointed out that equation (12) is identical with that reported by Mr. Nagle to the A. S. M. E.* and commonly known by his name. Values of C have already been given in Table XIX.

In the transactions of the American Society of Mechanical Engineers, January, 1909, Mr. Carl Barth presents a more extended mathematical treatment of the driving capacity of belts. He also presents scientific methods for measuring the tension in belting. Many other formulæ of a strictly empirical character are given by different authorities and some of them are very convenient. In general these last formulæ neglect centrifugal action and are hence applicable only to belt speeds below 2,500 feet per minute. Thus a common rule is that a single leather belt one inch wide traveling 1,000 feet per minute will transmit 1 H.P. Kent's "Mechanical Engineer's Pocket Book," page 877, gives a number of these so-called practical rules.

137. Practical Considerations. One of the most valuable contributions to the literature of the subject is **"Notes on Belting,"** by Mr. F. M. Taylor, in Vol. XV of the Transactions of the American Society of Mechanical Engineers. Mr. Taylor kept an accurate record of measurements and observations on belts in use at the Midvale Steel Co.'s works, for nine years, and gives many valuable facts and practical suggestions in his paper. A satisfactory abstract of it is not possible here. Mr. Taylor advocates thick narrow belts rather than thin wide belts.† He sums up his investigation in 36 "Conclusions," among which are:

"A double leather belt having an arc of 180° will give an effective pull on the face of the pulley per inch of width of belt of 35 pounds for oak-tanned and fulled leather, or 30 pounds for other types of leather belts and 6- to 7-ply rubber belts."

"The number of lineal feet of double belting, 1 inch wide,

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* Vol. II, page 91.

† While in general this conclusion is justifiable, care should be taken that it is not carried to the extreme where the life of the belt may be shortened by excessive bending.

passing around a pulley per minute, required to transmit one horse-power is 950 feet for oak-tanned and fulled leather belt, and 1,100 feet for other types of leather belts, and 6- to 7-ply rubber belts."

"The most economical average total load for double belting, is 65 to 73 pounds per inch of width, *i.e.*, 200 to 225 pounds per square inch of section. This corresponds to an effective pulling power of 30 pounds per inch of width."

"The speed at which belting runs has comparatively little effect on its life, till it passes 2,500 or 3,000 feet per minute."

"The belt speed for maximum economy should be from 4,000 to 4,500 feet per minute."

It should be especially noted that Mr. Taylor advocates a maximum belt tension of about one-half that ordinarily used. This would, of course, increase the first cost of the installation materially. His values, however, are not based on the minimum size of belt required to simply transmit a given horse-power, but on the size of belt which will transmit that horse-power for a given time with minimum wear and loss of time due to breakage or taking up to restore tension. Whether his practice is followed or not, it indicates the true aspect of the problem, and is a step in advance.

In laying out belt drives, care should be taken to keep the diameters of pulleys reasonably large. The constant bending action to which the belt is subjected as it runs around the pulley is a great source of wear, and where the pulley is very small, compared to the thickness of the belt, this may be excessive. For this reason also it is probably better to run the hair side of the belt next to the face of the pulley as this side is more easily cracked by bending, than the flesh side, which is more soft and pliable. Mr. Taylor says it is safe to run double leather belts on pulleys 12 inches in diameter.

The total length of the belt or distance between shaft centres also deserves attention. A belt being elastic, acts like a spring when tension is applied to it. The longer the belt the greater will be the total stretch for a given load. Suddenly applied loads, therefore, produce less stress in long belts than in short ones (see Art. 24). If, however, the distance between centres is too great, compared to the size of the belt, the belt is liable to flap and run unevenly on the pulleys. For small, narrow belts a maximum distance of 15 feet is good practice, while for heavier belts 25 feet is found satisfactory.

A number of important investigations of belt transmission have been reported to the American Society of Mechanical Engineers. See the following papers in the transactions of the Society by: Mr. A. F. Nagle, Vol. II, page 91; Professor G. Lanza, Vol. VII, page 347; Mr. Wilfred Lewis, Vol. VII, page 549; Mr. F. W. Taylor, Vol. XV, page 204; Professor W. S. Aldrich, Vol. XX, page 136. Abstracts of these as well as other valuable data are given in Kent's "Mechanical Engineer's Pocket Book," pages 876 to 887.

FIBROUS ROPE DRIVES

138. General Considerations. When the amount of power to be transmitted is large, the width of belt required may be excessive, even when the belt is made very thick. To run wide belts successfully, the shafting must be kept in perfect parallel alignment, and the distance between shaft centres must not be too great. For these reasons rope drives have been found very satisfactory where the amount of power to be transmitted is large, and the distance of transmission relatively great. They are also particularly serviceable for connecting shafts which are not parallel, as in the case of "quarter-turn" drives, especially where a belt would have to be of considerable width and would, as a consequence, run badly.

In all fibrous rope drives the surfaces of the pulleys or "sheaves" are provided with wedge-shaped grooves to receive the rope and thereby give the rope a better grip on the sheave. For drives of moderate length, 40 to 150 feet, fibrous ropes of cotton, hemp or manila fibre are chiefly employed. For transmitting power comparatively great distances, wire rope is more common, although fibrous ropes are also used for comparatively long transmissions. In all long-distance transmission the rope must be supported at intervals by idler pulleys.

Fig. 119* shows a typical rope drive where the line shafting of each floor of a mill is driven by its own rope drive from the main shaft of the engine.

139. Materials for Fibrous Ropes. Round ropes of **leather,** or **rawhide,** are used to a limited extent, when the amount of power to be transmitted is small. Rawhide is especially useful in damp places, but since it costs about six times as much as vegetable fibre rope, its application is very limited. Leather belts or ropes of square† or wedge-shaped section have also been

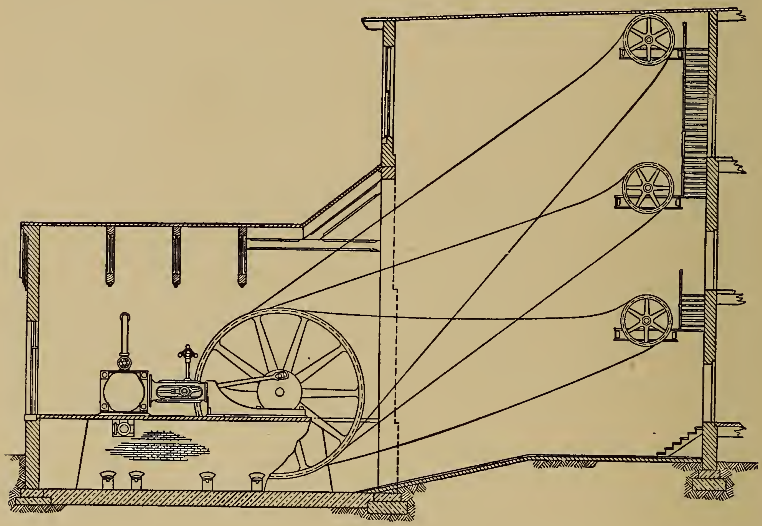

FIG. 119.

used to a limited extent. In certain localities in Great Britain, **hemp,** which is a local product, is quite extensively used; but **cotton** and **manila** fibre are by far the most common for transmissions of any considerable size. In this country manila fibre is used almost exclusively, while in England and on the Continent cotton rope is also much employed.

It is obvious that as a twisted rope of any fibrous material bends while passing over the sheave, there must be a certain

* Reproduced by permission from "The Blue Book of Rope Transmission."

† For a fuller discussion of such ropes see "Machine Design," by H. J. Spooner.

amount of internal friction. The result of this action is very noticeable in any old manila rope which has been used without lubrication. When such a rope is broken open it is found to be filled with powdered fibre, due to the internal chafing. For this reason manila fibre, which is naturally rough, is usually lubricated, while being twisted into rope, with tallow, paraffine, soapstone, graphite, or some such lubricant.

Cotton fibres, on the other hand, are smoother and hence give rise to less internal friction. They are, therefore, usually laid up dry into rope, a dressing or lubricant being applied to the exterior to prevent small fibres from rising on the outside, thus starting the rope to fraying. This dressing also excludes moisture and retains the natural oils in the interior fibres. Cotton fibre is not as strong as manila.

Professor Flather* makes the following comparison between cotton and manila rope: "As compared with manila, then, the advantages of cotton ropes of the same diameter are: Greater flexibility, greater elasticity, less internal wear and loss of power due to bending of the fibres, and the use of smaller pulleys for a given diameter of rope. Its disadvantages are: Greater first cost, lesser strength, and possibly a greater loss of power due to pulling the ungreased rope out of the groove—in any case this is usually small with speeds over 2,000 feet per minute."

140. Theoretical Considerations. The general equations (7), (8), and (9), of Art. 131, which were deduced for flat belts hold also for round ropes if the proper notation be substituted. In these equations the unit mass of belt was taken as one cubic inch. With ropes it is more convenient to take a piece of rope one inch in length and one inch in diameter. With the following exceptions, therefore, the notation used here will be the same as that used in Art. 131.

Let w' = the weight of a piece of rope 1 inch in diameter and 1 inch long.

Let $z' = \dfrac{12\, w' v^2}{g}$ where w' has the value above.

* "Rope Driving," by J. J. Flather, page 81.

Let t'_1 = the tension in a rope of 1 inch diameter on the tight side.

Let C' = a new coefficient = C modified on account of wedging effect of groove.

Then equations (8) and (9) become

$$f = [t'_1 - z']C' \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$\text{and } h.p. = [t'_1 - z']\frac{C'v}{550} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

In equations (8) and (9) the frictional force between the pulley and the belt for a flat belt is taken as μq where q is the radial pressure between the pulley and the belt. In a grooved pulley the pressure between the pulley and the rope is greater than the radial pressure in the ratio of cosec $\frac{\theta}{2}$ to unity, where θ is the angle between the sides of the groove. The frictional resistance between the rope and sheave is therefore μq cosec $\frac{\theta}{2}$. If μ *cosec* $\frac{\theta}{2}$ be substituted for μ in the quantity C (equations 8 and 9) the result C' may be used as indicated in equations (13) and (14) for rope drives. The value of μ for rope sheaves has not been determined with any degree of accuracy. Professor Flather* after reviewing what experimental data there is on the subject, concludes that 0.12 is a fair value and computes the following values of $\phi = \mu\ cosec\ \frac{\theta}{2} = 0.12\ cosec\ \frac{\theta}{2}$

TABLE XX

ϕ = coefficient of friction = 0.12 *cosec* $\frac{\theta}{2}$

Angle of groove.	30°	35°	40°	45°	50°	55°	60°
ϕ	.46	.40	.35	.31	.28	.26	.24

It is obvious that if ϕ be used instead of μ in Table XIX, the corresponding values of C in Table XIX will be the new constant

* "Rope Driving," page 112.

C'. Thus if $\theta = 45°$, $\phi = .31$. If also $a = 180°$, C' from Table XIX = .61 about. The angle 45° has been found to be the most satisfactory and is most commonly used. If the angle θ be less than 45°, the wedging action, hence the pulling capacity is increased, but the power loss and wear of rope due to drawing it out of the grooves is greater. For such sheaves, with $\theta = 45°$ and $a = 180°$

$$h.p = .61\,[t'_1 - z']\,\frac{v}{550} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

As before stated, reliable data on the coefficient of friction for ropes are scarce, and designing engineers have approached the problem of rope drives without regard to this coefficient. One of the most important contributions to the subject is that of Mr. C. W. Hunt (see Transactions A. S. M. E., Vol. XII). The notation of Mr. Hunt's article has been changed somewhat to correspond with that used in this text.

Let d = diameter of the rope in inches.
δ = sag of rope in inches.
L = distance between pulleys in feet.
w' = weight of one inch of rope *of one-inch diameter*.
W = weight of one foot of rope of diameter d.
T_1 = total tension in rope on *tight* side.
T_2 = total tension in rope on *slack* side.
T_0 = tension necessary to give the rope adhesion.
K = the total tension applied to each side of the rope due to centrifugal force.
P = effective turning force = $T_1 - T_2$

Then $T_1 = T_0 + K + P$

and $T_2 = T_0 + K$

Mr. Hunt says that "when a rope runs in a groove whose sides are inclined toward each other at an angle of 45° there is sufficient adhesion when $T_1 \div T_2 = 2$. However, he assumes a somewhat different ratio in the development of his equation, for which he assumes "that the tension on the slack side necessary for giving adhesion is equal to one-half the force doing useful work on the driving side of the rope."

$$\text{Or } T_0 = \frac{P}{2} \text{ and } T_1 = T_0 + K + P = \frac{P}{2} + K + P = \frac{3}{2}P + K$$

$$\text{and } T_2 = T_0 + K = \frac{P}{2} + K \text{ by assumption.}$$

$$\therefore P = \frac{2}{3}[T_1 - K] \quad . \quad . \quad . \quad . \quad . \quad (16)$$

If equation (16) be multiplied through by $\frac{v}{550}$ it will express the total horse-power transmitted or

$$H.P. = \frac{2}{3}[T_1 - K]\frac{v}{550} \quad . \quad . \quad . \quad . \quad (17)$$

The tension K on each side of the rope for an arc of contact of 180° and a rope of *one inch diameter* is $\frac{12\,w'\,v^2}{g}$, which is identical with the constant z' in equation (14). Mr. Hunt's formula therefore may be written

$$h.p. = \frac{2}{3}[t'_1 - z']\frac{v}{550} = \frac{v}{825}[t'_1 - z'] \quad . \quad . \quad (18)$$

where h.p. is the horse-power transmitted by a rope one inch in diameter. This is identical in form with the theoretical equation (15) and differs from it only by a negligible amount in the value of the coefficient.

It would seem therefore that Mr. Hunt's assumptions give results very close to those obtained by using the value 0.12 for μ as recommended by Professor Flather.

It is to be noted that the values of z given in Table XVIII may be used in computing values of z'. The quantities are the same except for the weight w'. In Table XVIII, w = the weight of *one cubic inch of leather* = .035. In equation (18), w' = the *weight of one inch of rope of one inch diameter* = .028 for manila rope and .022 for cotton rope. If, therefore, the values given in Table XVIII are multiplied by $\frac{4}{5}$ they are applicable to manila ropes, and if multiplied by $\frac{3}{5}$ they may be used for cotton ropes.

Example. What diameter of manila rope is necessary to transmit 25 H.P. when running 4,000 feet per minute, in grooves having an angle of 45°. Take $t'_1 = 200$ pounds, and $w' = .028$. From Table XVIII, z, for the given velocity = 64 nearly. $\therefore z' = 64 \times \frac{4}{5} = 51$. From equation (18) the horse-power which a rope one inch in diameter will deliver under these conditions is

$$h.p. = [t'_1 - z'] \frac{v}{825} = [200 - 51] \frac{66\frac{2}{3}}{825} = 12.1 \ h.p.$$

$\therefore$ the cross-section required $= \frac{25}{12.1}$ = twice the area of a one-inch rope which corresponds to a rope $1\frac{3}{8}''$ in diameter.

Fig. 120 * shows curves based on equation (17), giving the *total* horse-power transmitted by ropes of various sizes for $T_1 = 200d^2$, and will be found convenient for making calculations.

141. Strength of Fibrous Ropes. The ultimate strength of manila transmission ropes may be taken as about $7{,}000d^2$ and for cotton rope as about $4{,}600d^2$ where d = diameter of rope in inches. The working stress must be taken very much less than these values or otherwise the life of the rope is much shortened. For manila rope Mr. Hunt recommends that the working tension (T_1) be not over $200\,d^2$. The same factor of safety would give $130\,d^2$ as the allowable working tension for cotton ropes; but since cotton ropes are somewhat less affected by internal chafing the working tension may, perhaps, be safely taken at a rather higher value.

142. Velocity of Fibrous Ropes. The centrifugal force produces a tension in a rope of one inch diameter of $z' = \frac{12\,w'\,v^2}{g}$ or in a rope of diameter d the centrifugal force $= \frac{12\,w'\,d^2\,v^2}{g}$. The allowable stress in the rope is $200\,d^2$. The centrifugal force will equal the allowable tensile stress when $\frac{12\,w'\,d^2\,v^2}{g} = 200\,d^2$ or

* From "The Blue Book of Rope Transmission," by the American Mfg. Co.

when $v = 140$ feet per second, at which speed the effective pull becomes zero for this allowable working stress.

If equation (18) be differentiated and the differential be equated to zero as in Art. 134, the resultant equation will give the value

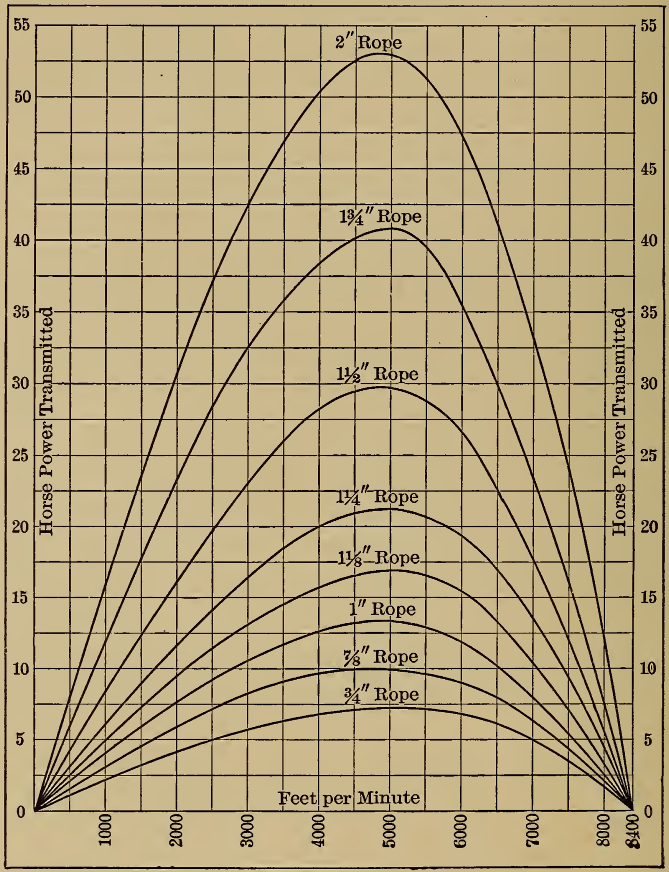

FIG. 120.

of the velocity where the work done is a maximum, for a rope of one inch in diameter. This is found to be about 4,900 feet per minute. Since the centrifugal force, and the total working stress, both vary as the area of the rope this limiting velocity applies to all sizes of ropes, a conclusion which is borne out by the curves of Fig. 120.

It has been found, in practice, that the most economical speed for ropes is from 4,000 to 5,000 feet per minute. If speeds greater than this are used, the wear on the rope is excessive. For a fixed value of $T_1 = 200\,d^2$ the first cost of a rope is a minimum at about 4,900 feet as above, and this first cost is greater by 10 per cent if the velocity is increased to 6,000, or decreased to 3,700 feet per minute. The first cost is increased 50 per cent when the velocity is reduced to 2,400 feet per minute with

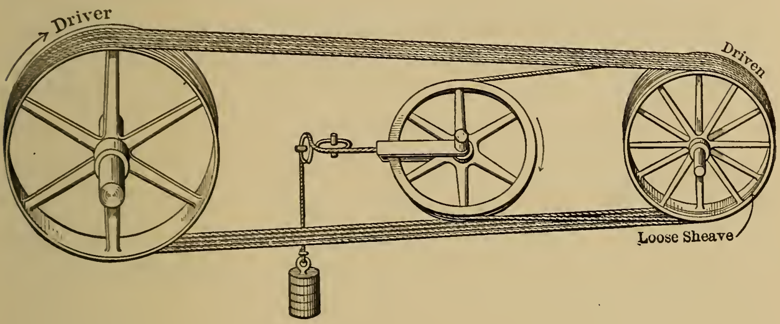

FIG. 121.

$T_1 = 200\,d^2$ but the reduction in speed increases the life of the rope.

143. Systems of Rope-Driving. There are two methods of placing fibrous ropes on the sheaves. In the **Multiple, or English** system, several separate ropes run side by side, each rope forming a closed circuit in exactly the same manner as a flat belt, and running constantly in its own particular groove on each pulley. In the **Continuous** or **American** system one rope only is used, the rope being carried continuously from one pulley to the other till all the grooves are filled, and it is then spliced; so that the rope as it leaves the last groove of the driven sheave is returned to the first groove of the driver, or driving pulley, by means of an idler, or guiding sheave. This idler is usually arranged so that through it a suitable tension may be put upon the rope (see Fig. 121).

Regarding the merits of the two systems it may be said that the multiple system is the simpler, and that it also provides considerable security against the loss of time due to breakdowns, as it is not likely that more than one rope will break at a time. When failure of a rope does occur, the broken rope may be removed and repaired at a more convenient opportunity, allowing the other ropes to carry the load temporarily. Occasionally, however, the breaking of a rope in the multiple system may cause great delay, on account of the broken rope becoming entangled in one of the rope sheaves and winding up upon it before the machinery can be stopped. In this system the individual ropes must be respliced occasionally to take up the sag in the rope due to stretching. The velocity ratio transmitted by a new rope will be different from that transmitted by an old one which has worn smaller, and hence fits down farther into the grooves, thereby changing its effective radius. The velocity ratio of the two sheaves can, however, have but one value, and, therefore, the tendency will be for either the old or the new ropes to carry the whole load. When the driving sheave is the larger, this will result in a tendency to throw more load on the old ropes; when the driving sheave is the smaller the tendency is to throw more load on the larger and new ropes. The unequal speed of the ropes, of course, leads to unequal stress; and slipping and consequent wear are sure to occur.

The continuous system is more flexible in its application than the multiple system; for, owing to the limited sag in the ropes due to the action of the weighted idler, the rope may be run safely at any angle. This form of drive is, therefore, much used for vertical and quarter-turn drives, and, generally, where the transmission is of a complicated nature. The principal objections to the system are the danger of loss of time due to a breakdown, and the unequal straining of the various spans of the rope particularly with a varying load or inequality of grooves. When a load is suddenly applied to the continuous system all the spans on the slack side become slacker except that which runs over the idler and which is kept at a fixed tension. A much greater load is hence brought on the driving span of rope next to the idler and some time must elapse before this load can be equalized over all the spans. Mr. T. Spencer Miller* has pointed out that the general tendency to unequal straining may be somewhat obviated, where the sheaves are of different diameters, by making the angle of the groove in the small sheave somewhat sharper than that in the larger, so that the product of the arc of contact and the cosecant of half the groove angle are equal; thus making the tendency to slip equal.

The above are the principal points of difference between the two systems. The particular conditions of the installation must be considered in making a choice between them.

144. Sheaves for Fibrous Ropes. The sheaves over which ropes are to run deserve special attention. Care should be taken

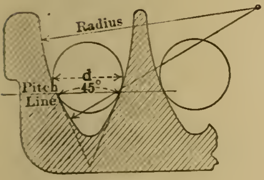

FIG. 122 (a).

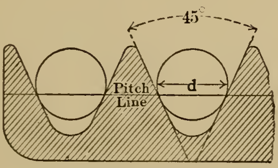

FIG. 122 (b).

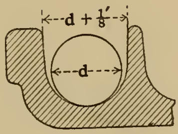

FIG. 122 (c).

that the form of the grooves, and, the effective diameters are the same for all grooves of the same sheave and the surfaces should be accurately finished and well polished, as any roughness or unevenness seriously affects the life of the rope. As the result of much experimentation two forms of grooves as shown in Fig. 122 (a) and 122 (b) have become most common. In Fig. 122 (b) the sides of the groove are straight while in 122 (a) the sides are curved. This curving of the sides makes the angle of the groove somewhat flatter at the bottom and hence when the rope has been reduced in diameter from wear it lies lower in the groove and will slip a little more readily than when it is new and occupies a higher position. This is of importance in relieving the old rope of a tendency to pull harder as indicated in the preceding article. The curved outline is also said to assist the rope to roll in the

* Transactions A. S. M. E. Vol. XII, page 243.

groove, a very desirable feature since it distributes the wear on the rope. The curved groove is therefore much used in the multiple system. In the continuous system the rope necessarily rotates as it passes round the idler to the first groove.

The angle of the groove, as before stated, is usually 45°. The grooves of idler pulleys for simply supporting the rope when the stretch is great are not made v-shaped but as shown in Fig. 122 (c).

The wear of fibrous ropes is both internal and external, the internal wear being due largely to chafing of the fibres on each other in bending the rope over the sheaves. For this reason sheaves should be as large as possible, and, in general, should not have a diameter less than forty diameters of the rope.

145. Deflection or Sag. Where the span between the pulleys is considerable the amount of deflection is sometimes of importance. Since the deflection varies with the distance between pulleys, the size and speed of the rope and the difference in elevation of the pulleys, it is impossible to express the relation existing between them in a single formula. For the simple case of the horizontal drive the approximate deflection on the *driving side* may be determined both for the continuous and multiple systems and also the deflection of the *slack* side of the continuous system, where uniform tension is maintained by a tension weight. In the multiple system, however, ample allowance must be made on the slack side, as new ropes stretch very rapidly, and the deflection may become excessive before resplicing can be performed. Mr. Hunt gives the following equation (transformed), for computing the deflection in horizontal drives:

$$\triangle = \frac{T}{2W} - \sqrt{\frac{T^2}{4W^2} - \frac{L^2}{8}} \quad . \quad . \quad . \quad . \quad (19)$$

Where T is the total tension on either the slack or tight side depending on the side for which it is desired to compute the deflection, W the weight of rope per foot, L the span in feet and $\triangle$ the deflection in feet. Where the tension on the driving side is assumed to be equal to 200 d^2, regardless of speed, the deflection on the driving side will be constant for a given span. As the tension in the rope due to centrifugal action increases as the square of the velocity, there is an increasing total tension T_2 on the slack side for a fixed value of T_1; and hence the deflection on the slack side decreases with the velocity, the span remaining constant. The value of T_2 may be computed and substituted in equation (19) to find the deflection.

Mr. Frederick Green * gives the following approximate formula for computing the deflection:

$$\triangle = \frac{W \times L^2}{8T} \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Where the symbols are the same as in equation (19), and from which he has calculated the following table on the assumption that $T_1 = 200\, d^2$.

TABLE XXI

Distance between Pulleys, Feet.	Sag on Driving Side, All Speeds, Feet.	Sag on Slack Side. Velocity, Feet per Minute. 3,000	4,000	4,500	5,000	5,500
30	.19	.45	.39	.36	.33	.30
40	.34	.80	.69	.64	.59	.53
50	.53	1.2	1.1	1.0	.92	.84
60	.76	1.8	1.7	1.4	1.3	1.2
70	1.0	2.4	2.1	1.9	1.7	1.6
80	1.4	3.2	2.9	2.5	2.3	2.1
90	1.7	4.0	3.5	3.2	3.0	2.7
100	2.1	5.0	4.3	4.0	3.7	3.3
120	3.0	7.2	6.2	5.7	5.3	4.8
140	4.1	9.9	8.5	7.8	7.2	6.6
160	5.4	12.9	11.1	10.2	9.5	8.6

WIRE-ROPE TRANSMISSION

146. General. Ropes made of iron or steel wire have been used to a considerable extent for transmitting power over comparatively great distances. The introduction of electrical transmission has, however, greatly curtailed the field as far as power transmission is concerned; although wire ropes are still much

* See "The Blue Book of Rope Transmission," by American Mfg. Co.

used for conveying materials such as coal, rock, etc., by means of buckets attached at intervals along the rope. The rope in such installations moves at very low velocities and constitutes a different problem from that of power transmission. Wire ropes are also much used for hoisting work such as elevator and mine work and for carrying static loads as in supporting smokestacks, masts and suspension bridges.

147. Materials for Wire Ropes. Wire ropes are usually made of wrought iron, open hearth steel, or crucible steel. For very severe work especially strong crucible steel known as **plough steel** is used. For a few special cases, copper and bronze are employed.

The John A. Roebling's Sons Co. publications give the following values for the tensile strength of various kinds of wire.

Swedish Iron	45,000 to 100,000 lbs. per sq. in.
Open Hearth Steel	50,000 to 130,000 lbs. per sq. in.
Crucible Steel	130,000 to 190,000 lbs. per sq. in.
Plough Steel	190,000 to 350,000 lbs. per sq. in.

They also state that it is difficult to obtain from a sample of rope in a testing machine, more than 90 per cent of the aggregate strength of all the wires. This is due to the difficulty of getting a perfect grip on the rope so that all the wires will carry their full share of the load; and also because the inner wires of a strand are shorter than the outer wires and are therefore more quickly overloaded. The wires, on account of the twisted construction, also tend to mutually cut into each other, thus rendering them more liable to fracture under heavy loads. On account of this latter action ropes made with a short twist break at a lower percentage of their full strength than those of a longer twist.

148. Power Transmission by Wire Rope. Wire ropes for power transmission are usually made of iron or soft steel and are laid up with a soft core of hemp in order to give greater flexibility. They cannot be run on metallic surfaces and the sheaves must be lined at the bottom with soft rubber or similar yielding material. Great care must be taken that the rope does not chafe and, unlike the sheaves for fibrous ropes, the grooves in sheaves used for wire rope are so formed that the sides of the groove do not compress the ropes. In wire-rope sheaves, the radius at the bottom of the groove is always greater than that of the rope itself so that wire-ropes drive, like flat belts, simply through the friction on the bottom of the groove, due to the tension of the rope. The lining of the bottom of the groove (leather, wood, or some other comparatively soft material) gives increased friction as well as less wear of the rope. The sheaves should be as large as possible to minimize the bending effect on the rope; one hundred rope diameters being often taken as the minimum diameter of the sheave.

The general theory and equations developed for fibrous rope hold also for wire rope, proper constants being substituted. It is evident from this discussion that wire ropes can safely transmit a greater amount of power than fibrous ropes of the same diameter, because of the much higher allowable tensile stress.

The table on the following page, which is taken from a circular of the John A. Roebling's Sons Co., shows the power that may be transmitted by iron ropes of various sizes with sheaves of different diameters and rotative speeds. These values are for a rope made with six strands around a hemp core, each strand consisting of seven wires. This table does not make allowances for the change of stress due to the change of centrifugal force at various speeds; but it does consider the influence of the sheave diameter on the bending stress. **For example:** a ⅝″ rope on an eight-foot sheave running 100 r.p.m., transmits only 32 H.P.; while the same rope transmits 64 H.P., when running on a ten-foot sheave at 80 r.p.m. or at the same linear velocity. By referring to Fig. 120 it is seen that a manila rope of 1½″ diameter transmits only 30 H.P., at the most economical velocity, or at about twice the velocity in the above instance.

For hoisting and for transmission, if the sheave diameters must be much smaller than those given in the preceding table, a more flexible rope is used. This consists of six strands around a hemp core, but each strand is made up of 19 wires, which are, of course, of smaller diameter than those used for corresponding sizes of seven-wire strands. The lining of the bottoms of the grooves in the sheaves should be maintained in good repair. If it becomes irregular, through wear, the rope may be bent at a sharp

TABLE XXII.

TABLE OF TRANSMISSION OF POWER BY WIRE ROPES *

Diameter of Wheel in Feet.	Number of Revolutions.	Trade Number of Rope.	Diameter of Rope.	Horse-Power.	Diameter of Wheel in Feet.	Number of Revolutions.	Trade Number of Rope.	Diameter of Rope.	Horse-Power.
3	80	23	$\frac{3}{8}$	3	7	140	20	$\frac{9}{16}$	35
3	100	23	$\frac{3}{8}$	$3\frac{1}{2}$	8	80	19	$\frac{5}{8}$	26
3	120	23	$\frac{3}{8}$	4	8	100	19	$\frac{5}{8}$	32
3	140	23	$\frac{3}{8}$	$4\frac{1}{2}$	8	120	19	$\frac{5}{8}$	39
4	80	23	$\frac{3}{8}$	4	8	140	19	$\frac{5}{8}$	45
4	100	23	$\frac{3}{8}$	5	9	80	20 19	$\frac{9}{16}$ $\frac{5}{8}$	47 48
4	120	23	$\frac{3}{8}$	6	9	100	20 19	$\frac{9}{16}$ $\frac{5}{8}$	58 60
4	140	23	$\frac{3}{8}$	7	9	120	20 19	$\frac{9}{16}$ $\frac{5}{8}$	69 73
5	80	22	$\frac{7}{16}$	9	9	140	20 19	$\frac{9}{16}$ $\frac{5}{8}$	82 84
5	100	22	$\frac{7}{16}$	11	10	80	19 18	$\frac{5}{8}$ $\frac{11}{16}$	64 68
5	120	22	$\frac{7}{16}$	13	10	100	19 18	$\frac{5}{8}$ $\frac{11}{16}$	80 85
5	140	22	$\frac{7}{16}$	15	10	120	19 18	$\frac{5}{8}$ $\frac{11}{16}$	96 102
6	80	21	$\frac{1}{2}$	14	10	140	19 18	$\frac{5}{8}$ $\frac{11}{16}$	112 119
6	100	21	$\frac{1}{2}$	17	12	80	18 17	$\frac{11}{16}$ $\frac{3}{4}$	93 99
6	120	21	$\frac{1}{2}$	20	12	100	18 17	$\frac{11}{16}$ $\frac{3}{4}$	116 124
6	140	21	$\frac{1}{2}$	23	12	120	18 17	$\frac{11}{16}$ $\frac{3}{4}$	140 149
7	80	20	$\frac{9}{16}$	20	12	120	16	$\frac{7}{8}$	173
7	100	20	$\frac{9}{16}$	25	14	80	8 7	1 $1\frac{1}{8}$	141 148
7	120	20	$\frac{9}{16}$	30	14	100	8 7	1 $1\frac{1}{8}$	176 185

* Taken from a publication of the John A. Roebling's Sons Company, of Trenton, N. J.

The above table gives the power transmitted by Patent Rubber-lined Wheels and Wire Belt Ropes, at various speeds.

Horse-powers given in this table are calculated with a liberal margin for any temporary increase of work.

angle in passing over the high spots of the lining, with a resultant increase in the stress of the wires. This last action, however, is not equivalent, so far as the life of the rope is concerned, to running over a correspondingly smaller sheave, for every portion of each wire is bent around each sheave once during every circuit of the rope; while it is not likely that the same portion of the rope will frequently come in contact with any single irregularity in the lining.

ROPES AND CABLES FOR HOISTING

149. Fibrous Ropes for Hoisting. In power transmission it is usually possible to install sheaves large enough to prevent the bending action from seriously affecting the life of the rope; but in hoisting work this is not always possible, on account of the size and clumsiness of the resulting tackle. Thus, a manila rope of 1 inch diameter, if used for power transmission, should run over a sheave at least 40 inches in diameter but if used for hoisting it might be required to run over a block sheave 12 inches or even 8 inches in diameter. The internal friction and external chafing are, in such cases, very great and the life of the rope, even when working at a lower stress, is greatly shortened; but in hoisting tackle, the frequency with which any portion of the rope passes over the sheaves is much less than is ordinarily the case in power transmission, on account of lower speed.

Theoretical considerations are of little or no help in hoisting installations, and recourse must be had to successful practice on which, fortunately, there are considerable data. The following table, from a paper presented by Mr. C. W. Hunt, before the A. S. M. E., gives the results of a long series of observations, and indicates the most economical size of rope for a given load. It has been found, by experience, that ropes larger or smaller than those recommended in the table are shorter-lived under the load indicated. The speeds indicated in the table are defined as follows:

"Slow"—Derrick, crane, and quarry work; 50 to 100 feet per minute.

"Medium"—Wharf and cargo work; 150 to 300 feet per minute.

"Rapid"—400 to 800 feet per minute.

TABLE XXIII

WORKING LOAD FOR MANILA ROPE

A.	B.	C.	D.	E.	F.	G.	H.
Diameter of Rope, Inches.	Ultimate Strength, Pounds.	Working Load in Pounds.			Minimum Diameter of Sheaves in Inches.		
		Rapid.	Medium.	Slow.	Rapid.	Medium.	Slow.
1	7,100	200	400	1,000	40	12	8
$1\frac{1}{8}$	9,000	250	500	1,250	45	13	9
$1\frac{1}{4}$	11,000	300	600	1,500	50	14	10
$1\frac{3}{8}$	13,400	380	750	1,900	55	15	11
$1\frac{1}{2}$	15,800	450	900	2,200	60	16	12
$1\frac{5}{8}$	18,800	530	1,100	2,600	65	17	13
$1\frac{3}{4}$	21,800	620	1,250	3,000	70	18	14

150. Wire Hoisting Ropes. On overhead travelling cranes, elevators and mine work, iron or steel cables are used almost exclusively, as here it is usually possible to install sheaves or drums of large diameter. For rough service, deep mine work or wherever great strength is necessary, these ropes are sometimes made of crucible or plough steel. Great care should be exercised in installing such ropes and it is well, in general, to obtain the advice of the manufacturers before selecting any rope made of crucible steel, especially if great safety is desired. A factor of safety of at least 5 should be used in ordinary work, and for elevator, or similar work, a factor as high as 10 or 15 is sometimes desirable. Table XXIV, taken from a publication of the John A. Roebling's Sons Co., gives data on standard hoisting ropes. For open-hearth steel the strength as given for iron rope may be increased 25 per cent. It will be noticed that these tables are based on a factor of safety of 5.

CHAINS AND CHAIN TRANSMISSION

151. Chains may be conveniently divided into three classes:

(a) Chains for raising and supporting loads.

(b) Chains for conveying purposes.

(c) Chains for power-transmission purposes.

Chains for Hoists. In the first class are such chains as are used on cranes and hoisting appliances. Chains of this character are made with elliptical-shaped links and should be manufactured of the best **wrought iron** to insure perfect welding where the link is joined. The links themselves should be as small as possible to minimize the collapsing action or bending due to the pull of the adjacent links, and also that due to winding the chain upon a circular drum. Such chains are sometimes called **short-link, close, or crane chains.**

TABLE XXIV

STRENGTH OF IRON AND STEEL HOISTING ROPES

Swedish Iron.					Crucible Steel.				
Diameter in Inches.	Weight per Foot in lbs.	Approximate Breaking Strength in Tons.	Working Loads in Tons.	Minimum Diameter of Sheaves in Feet.	Diameter in Inches.	Weight per Foot in lbs.	Approximate Breaking Strength in Tons.	Working Load in Tons.	Minimum Diameter of Sheaves in Feet.
$2\frac{3}{4}$	11.95	114	22.8	16	$2\frac{3}{4}$	11.95	228	45.6	10
$2\frac{1}{2}$	9.85	95	18.9	15	$2\frac{1}{2}$	9.85	190	37.9	$9\frac{1}{2}$
$2\frac{1}{4}$	8.00	78	15.60	13	$2\frac{1}{4}$	8.00	156	31.2	$8\frac{1}{2}$
2	6.30	62	12.40	12	2	6.30	124	24.8	8
$1\frac{3}{4}$	4.85	48	9.60	10	$1\frac{3}{4}$	4.85	96	19.2	$7\frac{1}{4}$
$1\frac{5}{8}$	4.15	42	8.40	$8\frac{1}{2}$	$1\frac{5}{8}$	4.15	84	16.8	$6\frac{1}{4}$
$1\frac{1}{2}$	3.55	36	7.20	$7\frac{1}{2}$	$1\frac{1}{2}$	3.55	72	14.4	$5\frac{3}{4}$
$1\frac{3}{8}$	3.00	31	6.20	7	$1\frac{3}{8}$	3.00	62	12.4	$5\frac{1}{2}$
$1\frac{1}{4}$	2.45	25	5.00	$6\frac{1}{2}$	$1\frac{1}{4}$	2.45	50	10.0	5
$1\frac{1}{8}$	2.00	21	4.20	6	$1\frac{1}{8}$	2.00	42	8.4	$4\frac{1}{2}$
1	1.58	17	3.40	$5\frac{1}{4}$	1	1.58	34	6.8	4
$\frac{7}{8}$	1.20	13	2.60	$4\frac{1}{2}$	$\frac{7}{8}$	1.20	26	5.2	$3\frac{1}{2}$
$\frac{3}{4}$	0.89	9.7	1.94	4	$\frac{3}{4}$	0.89	19.4	3.88	3
$\frac{5}{8}$	0.62	6.8	1.36	$3\frac{1}{2}$	$\frac{5}{8}$	0.62	13.6	2.72	$2\frac{1}{4}$
$\frac{9}{16}$	0.50	5.5	1.10	$2\frac{3}{4}$	$\frac{9}{16}$	0.50	11.0	2.20	$1\frac{3}{4}$
$\frac{1}{2}$	0.39	4.4	0.88	$2\frac{1}{4}$	$\frac{1}{2}$	0.39	8.8	1.76	$1\frac{1}{2}$
$\frac{7}{16}$	0.30	3.4	0.68	2	$\frac{7}{16}$	0.30	6.8	1.36	$1\frac{1}{4}$
$\frac{3}{8}$	0.22	2.5	0.50	$1\frac{1}{2}$	$\frac{3}{8}$	0.22	5.0	1.00	1
$\frac{5}{16}$	0.15	1.7	0.34	1	$\frac{5}{16}$	0.15	3.4	0.68	$\frac{2}{3}$
$\frac{1}{4}$	0.10	1.2	0.24	$\frac{3}{4}$	$\frac{1}{4}$	0.10	2.4	0.48	$\frac{1}{2}$

The strength of a chain link in tension is less than twice the strength of a bar of the iron from which the chain is made on account of the bending action due to the manner in which the load is applied, and also on account of the weld. If W = the breaking load in pounds, and d = the diameter in inches of the bar from which the link is made, then the following empirical equation may be used for iron crane chains.

$$W = 54{,}000\, d^2 \quad \ldots \quad (1)$$

The working load (W') should not be more than one-third this value or $W' = 18{,}000\, d^2 \quad \ldots \quad (2)$

In many cases a lower stress than indicated by (2) should be adopted. Whenever the load is not a direct pull, but severe bending stresses are also induced, as in chain "slings" for handling heavy iron castings, the chain should have great excess of strength. Chains should be carefully inspected and tested or "proved" before using. The "**proof**" usually applied is one-half the ultimate load. Where chains are used for hoisting work, they are likely to become badly strained. Annealing by heating allows a readjustment of the structure of the iron, and this should be done periodically with all such chains, particularly chains used for slings. This also affords an opportunity to thoroughly inspect chains which are greased in operation. The uncertainty regarding the exact condition of a chain in service, and the fact that it gives no warning of weakness, but may break at a load below the normal working load, have caused them to be largely replaced, on such appliances as overhead cranes, by steel rope. The state of the strength of the latter is more easily determined by inspection.

Weldless* steel chain rolled from a bar of special shape has lately come into use to some extent. The chain is made in lengths of from 60 to 90 feet, and the lengths are joined together by a link made of special welding steel. They are said to be much stronger than iron chains.

152. Chain Drums and Sheaves. Drums on which crane chains are to wind should be carefully grooved so that alternate links lie flat on the surface of the drum; and should have sufficient capacity to receive the chain in one layer, as overwinding brings severe stresses on the parts wound upon the drum. The diameter of the drum should in no case be less than twenty times the diameter of the chain used, and thirty times this diameter is better.

* See "Machine Design," by H. J. Spooner, page 452.

Where it is not possible to have the chain wind upon a drum, **pocket chain wheels** are often used. These wheels are made with pockets around the periphery into which the links fit. The links are prevented from coming out by a guide over a portion of the wheel; and hence cannot slip on the sheave. Anchor chains, and the chains of certain forms of chain blocks for raising weights, run over such sheaves.

153. **Hoisting-Hooks.** The hooks used for raising heavy weights deserve special attention. They are usually made of

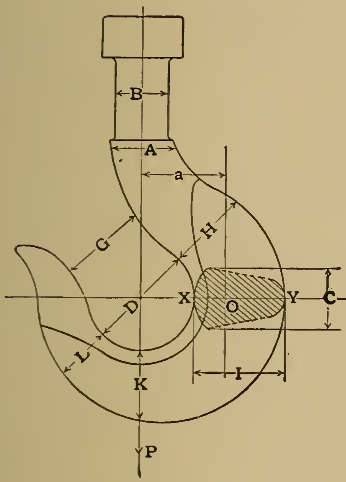

FIG. 123 (a).

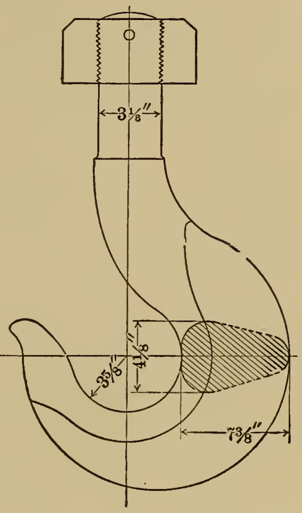

FIG. 123 (b).

steel or iron forgings although steel castings are employed to some extent. If the stress in the hook can be kept low the use of steel castings may be justified; but where the load is great and the fibre stress in the hook necessarily high, to avoid clumsy proportions, the hook should be forged from ductile material.

Let the hook in Fig. 123 (a) be subjected to a vertical load P; then XY, the most dangerous section, is apparently acted upon by a direct stress $p' = \frac{P}{A}$ (where A is the area of the section) and by a flexural stress p'' due to the moment Pa; the stress p'' being tensile at X and compressive at Y. The theory of article 19, therefore applies, apparently, and equation M (Table VI) may be used to design the section, or

$$p = p' + p'' = \frac{P}{A} + \frac{Pae}{I}$$

Experience shows that members of this kind, even when made of materials whose tensile and compressive elastic limits are about the same, almost invariably yield to rupture on the tension side; and the section is usually made as shown, the gravity axis being located nearer the load, thus decreasing the tensile stress and increasing the compressive stress, as computed by the above equation. Recent investigations* have shown that in a curved beam loaded in this manner, the neutral axis does not coincide with the gravity axis, as in straight beams, but is located nearer the tension side, and the above theory is therefore defective, as the true tensile stress is greater than that given by equation M. That this is true is borne out by the fact mentioned above, that hooks fail in tension when designed with an apparent compressive stress considerably above the tensile stress, although the elastic limit for either stress is about the same. The application† of the more accurate theory is, however, somewhat complicated and it is believed that equation M may be safely used if due care is taken in assigning the limits of stress.

Hooks for small cranes and hoists are much more likely to be loaded frequently to their full capacity than hooks for raising large loads; thus a hook on a five-ton crane may be loaded to its full capacity several times every day, while the hook of a twenty-ton crane would be thus loaded at rare intervals. The stresses in small hooks must therefore be kept low, and fortunately this can be done without making the hook clumsy. As the size of the hook increases, however, the stresses must necessarily be increased to avoid clumsiness, but the larger the hook the less frequently will it be fully loaded and a working stress as high as

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* See "Strength of Materials," by Slocum and Hancock.

† See "On a Theory of the Stresses in Crane and Coupling Hooks. With Experimental Comparisons with Existing Theory," by Professor Karl Pearson and Mr. E. S. Andrews. Messrs. Dulan & Co.

15,000 pounds per square inch, or more, is as safe in a fifty-ton hook as 10,000 pounds per square inch would be in a ten-ton hook. (See Art. 25.)

The most valuable data on crane hooks is that given by Mr. Henry R. Towne in his "Treatise on Cranes," as a result of both mathematical and experimental work. Fig. 123 (a) and the following formulæ give the most important dimensions of a hook according to this work, and these proportions have been much used with uniform success. The basis for each size is a commercial size of round iron or dimension A. In the following formula $\triangle$ is the nominal capacity of the hook in tons of 2,000 pounds. The dimension D is assumed arbitrarily but so as to provide ample room for the slings. The following measurements are then expressed in inches:

$$D = .5 \triangle + 1.25 \qquad H = 1.08\,A \qquad K = 1.13\,A$$

$$G = .75\,D \qquad I = 1.33\,A \qquad L = 1.05\,A$$

The following gives the capacity of the hooks made from various sizes of bar stock:

TABLE XXV

Capacity of hook in tons	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	3	4	5	6	8	10
Size of bar A in inches	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$1\frac{1}{16}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{7}{8}$	$3\frac{1}{4}$

It is to be noticed that the stresses allowed by Mr. Towne's proportions are very low. Thus in a ten-ton hook the dimension A is $3\frac{1}{4}''$ or, allowing for finishing, the dimension B may be taken as $3''$, which would give a tensile stress in the shank of only 3,000 pounds per square inch. It should be borne in mind, however, that hooks are subjected to much abuse and the designer has no assurance that they will always be loaded with a true axial load, for improper arrangement of the sling often throws the load more toward the point of the hook and the member is called upon to carry a bending moment greatly in excess of that for which it is intended.

When, however, hooks larger than those covered by Mr. Towne's work are to be designed his proportions lead to clumsy dimensions. Thus a twenty-ton hook would require a shank

$4\frac{1}{2}''$ in diameter and a fifty-ton shank would be $6\frac{1}{2}''$ in diameter. Fig. 123 (b) shows a twenty-ton hook of Norway iron which has been successfully used in practice. The threaded shank being $3\frac{1}{8}''$ in diameter is therefore stressed to about 6,000 pounds per square inch but yet is only as large as the shank of a ten-ton hook as given by Mr. Towne's dimensions. Examination of current practice and measurements taken from a number of large hooks in successful service indicate an allowable tensile stress at X, as computed by equation M of Table VI, ranging from 10,000 lbs. per square inch in ten-ton hooks, to 15,000 lbs. per square inch in fifty-ton hooks.

154. Conveyor Chains. Chains for conveying and elevating materials, such as grain, coal, ashes, etc., are usually made of

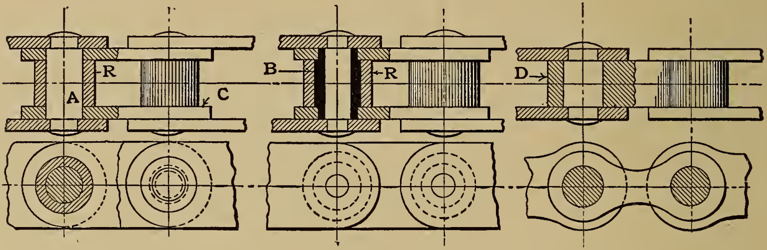

FIG. 124 (a). FIG. 124 (b). FIG. 124 (c).

malleable iron, the links *hooking* together in some manner. This style of chain is known as link belt. On account of the diverse purposes to which they are applied, they are made in many forms, and the selection of the particular form for a given problem is usually made in conference with the manufacturer or taken from trade catalogues giving the desired information. This form of chain is also extensively used in rough machinery, such as agricultural implements, for the transmission of power. In such cases the chains must be run at low speeds, as they become noisy and unreliable even at moderate velocities.

155. Chains for Power Transmission. The chains heretofore discussed move, necessarily, at low velocities, but of late a demand has arisen for chains which may be run at high speeds for the purpose of transmitting power. Such chains are used when a positive velocity ratio must be maintained between the connected shafts, and where the distance between shafts is so great as to make tooth gearing inconvenient. Of this class there are at present three principal types, namely, **roller chains, block chains,** and so-called **silent chains.** Fig. 124 (a) illustrates the simplest form of roller chain in which the pin *A* is riveted fast in the outer links, and rotates in the inner links. The roller *R* lessens the friction against the tooth. In this form of chain the wear between the pin and the inner link is excessive, and for this reason it is now little used for power transmission. It is sometimes made without the roller and with several inside links and is then known as **stud chain.** In this form it is used for very low velocities only.

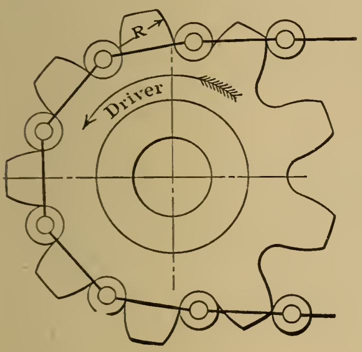

FIG. 125.

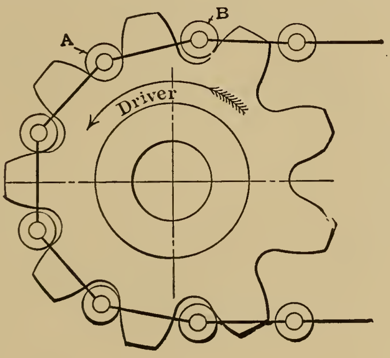

FIG. 126.

The form shown in Fig. 124 (b) is most common. Here the bushing *B* is pressed into the inner links, and the pin, which is riveted fast to the outer links, bears over the whole length of the bushing. The roller *R* rotates on the bushing. In the block chain, Fig. 124 (c), the pin also bears over the whole thickness of the block *D*, but since the roller is necessarily omitted, there is more friction against the tooth. Roller chains may be used for velocities up to about 800 feet per minute, and block chains up to about 500 feet per minute.

The defect in the operation of the roller or block chain may be seen by referring to Figs. 125 and 126. When the chain is new, and has the same pitch as the wheel, it fits down on the wheel as shown in Fig. 125, but in a very short time the chain stretches slightly, due to wear of the joints, thus increasing the pitch of the links. The wheel, on the other hand, may *wear* but this does not change the *pitch*. The operation of the chain is then as shown in Fig. 126, the increased pitch causing the rollers to ride higher and higher on the back of the tooth as they move round the sprocket. The roller *A* is shown fully seated while *B* is just coming down to its seat. Before *B* can become fully seated *A* must rise, and this action takes place when *A* and *B* are carrying full load. As a consequence the chain does not run quietly and smoothly and the wear is excessive, thus limiting the speed at which the chain may be run. This difficulty is sometimes met by the arrangement shown in Fig. 127. Here the pitch of the

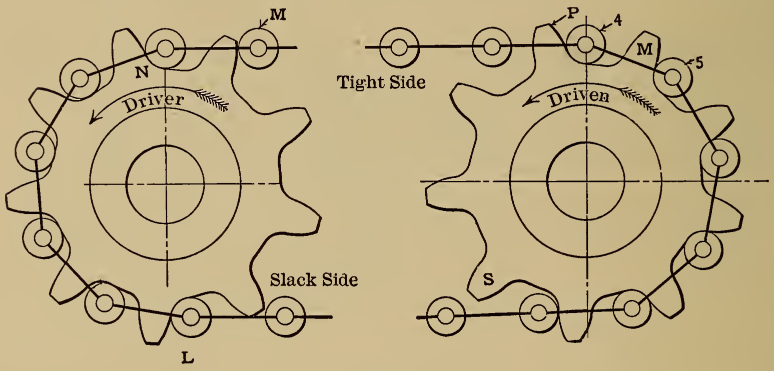

FIG. 127.

chain when new is made a little *less* than the pitch of the driving sprocket, and clearance is allowed between the roller and the tooth, so that the driving is done by the last tooth *L*; the pitch of the chain being such that the incoming roller *M* just clears the *back* of the first tooth and seats itself close to it at the root as at *N*. As the chain stretches, the rollers move backward toward the *faces* of the teeth, till a condition like that in Fig. 125 is reached, and riding commences. The pitch of the driven sprocket wheel is made equal to that of the chain, and the condition when new is that shown in Fig. 127. As the chain stretches, the rollers move gradually backward *away* from the driving faces of the tooth, the driving being done on the last tooth *P*. It is evident that this construction extends the time preceding the condition shown in Fig. 126. When this construction is used, the form of the tooth must be slightly modified. Referring to Fig. 128 it is obvious that if the outline of the tooth *M* be an arc of a circle struck from the centre of the roller (2), this roller will swing from its position 1′ rolling on the face of the tooth, and this is the usual outline. But before roller (3) can take the load, which (2) is about to give up, it must be fully rooted against the next tooth; whereas (from Fig. 128), a small distance now separates the two. Therefore as (2) rolls up the curve of the tooth it should allow (3) to slowly settle back in place. The tooth outline is therefore struck (as shown on *M*), from a point a little inside the pitch polygon so as to give a curve tangent to the first and last positions

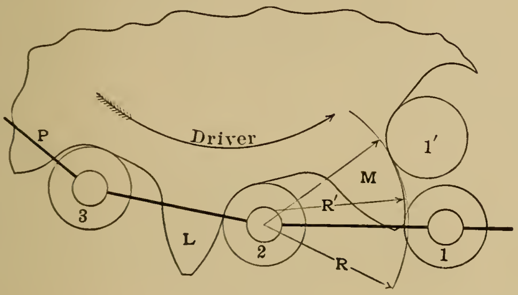

FIG. 128.

of the roller. This outline is also necessary for the back of the tooth in order to allow the incoming roller to swing in without striking. The velocity of the chain is, therefore, a little *less* than the theoretical velocity on account of this continual slipping backward. Brief reflection will show that the tooth outlines of the driven sprocket may be struck from the centre of the roller when rooted in place; and that when the chain is stretched a little it will *creep* as it is wound upon the driven sprocket.

When the roller (4), Fig. 127, is about to roll up the face of *P*, roller (5) is not in contact with *M* (wear having begun); hence the chain will move ahead till (5) is in full contact with *M*.

The greatest defect in this construction is the fact that the load is carried entirely on one tooth and hence the wear is excessive. This may be so great that the chain creeps forward on the driven wheel so as to cause the incoming roller to strike the tooth S, Fig. 127.

The above difficulties are overcome in the so-called *silent chains*. In these chains the inevitable stretching of the links is compensated for in a peculiar manner. The true theory of the action of these chains is very complex; but the general action is as follows:—as the chain stretches, the links continually tend to take up a position farther and farther away from the centre of the sprocket; thus increasing the length of the sides of the pitch polygon to suit the elongation of the link. Each link therefore remains in constant contact with its own tooth, from the time of

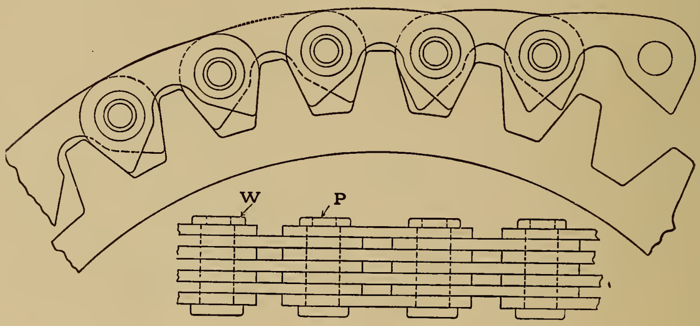

FIG. 129.

engagement till release takes place. The links seat themselves without sliding action and the operation is nearly noiseless.

In the Renold chain of this type, Fig. 129, the links move relative to each other on a round pin P, the shouldered ends of which are riveted into a washer W, thus holding the chain together. In a later form half bushings of bronze are so fitted to the links that the pin has a bearing over its full length; but the relative motion of the pin to the bush is still a *sliding* motion. In the Morse chain this sliding is eliminated by an ingenious form of rocker joint shown in Fig. 130. The hardened steel parts, A and B, are fitted respectively to the sets of links, D and C. While keeping contact along a fixed line they *rock* on each other as the links C and D move relatively to each other, and sliding is thus eliminated. When transmitting simple tension between the sprockets, the parts *A* and *B* are in contact on flat surfaces as shown at *E*. This construction has the advantage of requiring little or no lubrication, hence the chain may be run at higher speeds than others requiring lubrication, the speeds of which are limited by the velocity at which centrifugal action throws off the lubricant. The Morse chains also work well in dusty places.

The efficiency of both of these chains is very high, the makers of the Morse chain claiming an efficiency of nearly 99 per cent.

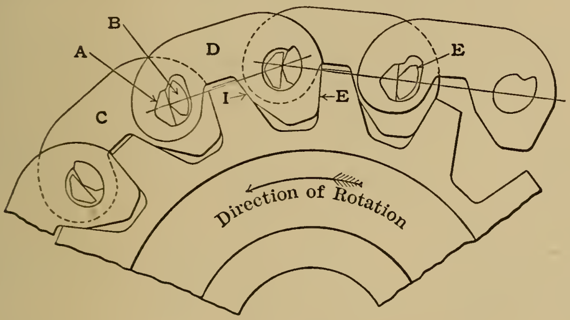

FIG. 130.

Such chains are particularly useful for connecting shafts which are too far apart for gearing, and not far enough for a belt, and in places where positive connection is desirable, as in motors driving heavy machine tools. It is to be especially noted that this form of transmission requires no definite tension on the slack side of the chain to produce a certain driving force on the tight side; and hence the pressure on the bearings is much reduced, for a given effective pull on the wheel rim.

CHAPTER XIII

APPLICATIONS OF FRICTION

Friction Wheels for Power Transmission

156. General Considerations. When it is required to drive a rotating member intermittently, and the rate of driving is not necessarily positive, friction wheels have been found very useful. They are particularly applicable where the amount of power is comparatively small, as in feed mechanisms, but they may also be used for heavy work when properly constructed. For continuous driving the transverse sections of friction wheels

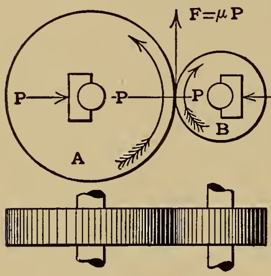

FIG. 131.

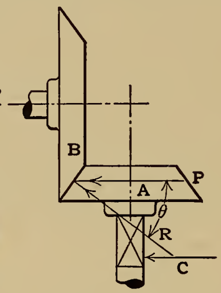

FIG. 132.

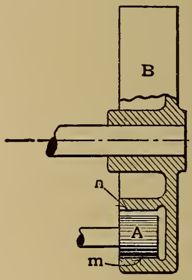

FIG. 133.

must be circular in cross-section, and this form, only, is used in practice.

Figures 131 and 132 show common forms of friction wheels. In Fig. 131 let A be the driving wheel which rotates continuously, and let B be the driven wheel which is required to be driven intermittently. The shaft of A is so mounted that, by means of a lever attached to the bearing, A may be pressed up against B with a force P, or it can be moved slightly away from B until no contact exists. If now the force P is applied to the bearing (which should be close to A), an equal and opposite force is set up in the bearing of B, and the wheels are pressed together at the line of contact. The resistance to slipping at the line of contact will be μP, where μ is the coefficient of friction of the materials of which the wheels are made; and if μP is equal to, or greater than, the resisting force at the surface of B, A will cause B to rotate. Theoretically, A and B will roll together with pure rolling motion, but practically this cannot be attained, as even with very hard materials the wheels flatten slightly at the line of contact. (See Art. 108.)

Fig. 132 illustrates the application of friction wheels to shafts which are not parallel to each other, the wheels here having the form of rolling cones. Obviously the principle is of wide application and many combinations of friction wheels are used. Fig. 133 illustrates a friction wheel arranged so that the driver A

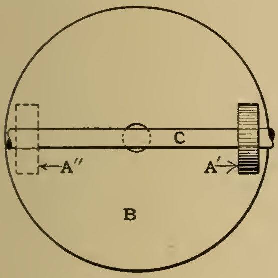

FIG. 134.

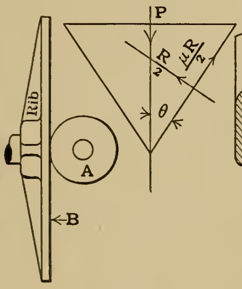

FIG. 135 (b).

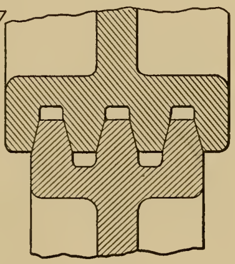

FIG. 135 (a).

can rotate the driven wheel B in either direction, depending on whether it is pressed against the surface m or the surface n.

Fig. 134 shows a form of friction mechanism much used for imparting variable speed to the driven shaft. The driver A may be moved along the shaft C at will. When at A' the angular velocity of B is a minimum. As A is moved inward, the rotative velocity of B increases. When A is moved across the centre of B to the other side, the direction of rotation of B is reversed. If A were infinitely thin, it would, theoretically, roll upon B with pure rolling motion. Since, however, it must have an appreciable width of face, and since the velocity of B varies with the radius, it is evident that there must be some sliding at the line of contact. For this reason the thickness of A must, for best results, be kept small compared to the radius of B.

157. Materials for Friction Wheels. The driven wheels of friction devices should always be made of a harder material than the driver, for the reason that the driven wheel is likely at any time to be held stationary by the load, while the driving wheel revolves against it under pressure. This action, while severe on the driver, does not tend to wear it out locally, while it does rapidly wear flat spots on the driven wheel. Driven wheels are, therefore, almost universally made of iron, and driving wheels of wood, leather, paper, rubber or of some composition of these; the most common being leather and various forms of paper.

158. Practical Coefficients. The tangential force F, exerted by A upon B, Fig. 131, is dependent on the pressure P and the coefficient of friction μ. It is, therefore, necessary to know the allowable pressure per unit of length along the contact elements and also the value of μ for the particular materials used. The most comprehensive investigation of these relations is that made by Professor Goss,* whose experiments cover a variety of materials. He recommends the following pressures, which are about one-fifth of the ultimate crushing strength of the respective materials.

Safe Working Pressures per Inch of Contact.

Material.	*Pressure.*
Straw fibre	150
Leather fibre	240
Tarred fibre	240
Leather	150
Wood†	100 to 150

Professor Goss found that the coefficient of friction for all the wheels tested approached a maximum value when the slip between the two wheels was about 2 per cent, and, within narrow limits, was practically independent of the pressure of contact. He found these values to range for different combinations from low values up to .515. In these experiments the friction due to the bearings was neglected. The bearings, however, were of the roller type and, probably, absorbed less power than the ordinary

* See Transactions A. S. M. E. Vol. XXIX.

† The value for wood is not from Professor Goss's paper.

bearing. Making due allowance for the difference between laboratory conditions and those found in practice, Professor Goss recommends the following approximate values of μ* for the various combinations. In this connection it is to be noted that allowance must be made for a decrease in the value of this coefficient when the linear velocity of the driver is great, in the case where the driver is starting the driven wheel under load (see Art. 28).

WORKING VALUES OF COEFFICIENT OF FRICTION.

Materials.	*Coefficient of Friction.*
Straw fibre and cast iron	0.26
Straw fibre and aluminum	0.27
Leather fibre and cast iron	0.31
Leather fibre and aluminum	0.30
Tarred fibre and cast iron	0.15
Tarred fibre and aluminum	0.18
Leather and cast iron	0.14
Leather and aluminum	0.22
Leather and typemetal	0.25
Wood and metal	0.25

159. Power Transmitted by Friction Wheels. If V be the velocity of the surface of the friction wheels in feet per minute, P the total normal pressure in pounds, F the resulting tangential force, and μ the coefficient of friction; then since $F = \mu P$, the rate at which power is transmitted in foot pounds per minute is $\mu P\,V$, and the horse-power is

$$H\,P = \frac{\mu\,P\,V}{33{,}000} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

of if d be the diameter of the driver in inches, l the length of face in inches, w the allowable load per inch of face, and N the number of revolutions per minute, the horse-power is

$$H.\,P. = \frac{\mu\,w\,l \times \pi\,d\,N}{12 \times 33{,}000} = 0.000008\,\mu\,w\,l\,d\,N \quad . \quad . \quad (2)$$

Example. How many horse-power can be transmitted by a straw-fibre friction pulley of 8″ diameter and 6″ face, when running at 500 r.p.m., the driven wheel to be of cast iron?

* The coefficient for wood is not from Professor Goss' paper.

Here $d = 8''$, $l = 6''$, $N = 500$, $\mu = 0.26$, $w = 150$

$\therefore$ H. P. $= .000008 \times 0.26 \times 150 \times 6 \times 8 \times 500 = 7.5$

It may be noted that the horse-power per inch of width of face is a little more than unity, for a surface speed of 1,000 feet, as in the above example. This corresponds closely to the empirical rule given for belts in Art. 136; and corroborates the empirical rule often used that the same width of face is necessary for a friction wheel as for a belt, to transmit a given horse-power at the given speed.

In the case of bevel wheels (see Fig. 132) the component R of the applied force P presses the wheels together and $R = \dfrac{P}{\cos\ \theta}$

The velocity of the mean circumference of the driver may be taken as the velocity of transmission.

In face friction driving as in Fig. 134, the width of the driving wheels should be kept as narrow as possible for best results. If the velocity of the outer edge of the driving wheel is not more than 4 per cent greater than that of the inner edge, the above coefficients may be used. Where the driver must, at times, drive at a short distance from the centre, lower values of the coefficient of friction must be taken.

The faces of a pair of metal friction wheels are sometimes formed as shown in Fig. 135 (a), and are then known as **wedge-faced friction wheels.** The object of this construction is to secure a greater resistance to slipping, with a given radial pressure. It is to be noted that the number of wedges does not affect this ratio, but decreases the wear by distributing it over several surfaces. This last item is important, as it is easily seen that the contact surfaces of the driver and the driven wheel can have the same velocity at one point only, and that at all other points slipping or a grinding action occurs and wear must result.* The teeth therefore should not be very long.

In Fig. 135 (b), let P be the radial force applied to the wedged surface, F the tangential force transmitted, $\frac{R}{2}$ the reaction on

* See "Kinematics of Machinery," by John H. Barr, page 10 .

each face and 2θ the angle of the wedge; then the wedge is held in equilibrium by the force P, the reactions $\frac{R}{2}$ and the frictional resistances $\mu\frac{R}{2}$ due to the wedging action. Equating vertical forces

$$P = 2\left(\frac{R}{2} \sin\theta + \frac{\mu R}{2} \cos\theta\right) \text{ or since } F = 2\left(\frac{\mu R}{2}\right) \text{ or } R = \frac{F}{\mu}$$

$$P = \frac{F \sin\theta}{\mu} + F \cos\theta \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{or } F = \frac{\mu P}{\sin\theta + \mu\cos\theta} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

To avoid sticking the angle 2θ should not be less than 30°.

FRICTION BRAKES

160. Friction brakes are used for controlling and stopping machinery by absorbing energy through frictional resistance from some moving part, and dissipating it as heat. Brakes used in heavy work, and as dynamometers for measuring energy, must often be fitted with water circulation to carry away the heat. The student is referred to treatises on power measurement for a discussion of dynamometers.

161. **Block Brakes.** The simplest form of brake is the block brake as shown in Fig. 136. Here the force P, acting on the lever A, presses the block C against the wheel B. Let the reaction between the wheel and the block be R. Then if B be rotating, a tangential frictional resistance $\mu R = F$ will oppose its motion. With the arrangement shown in Fig. 136, the line of action of F passes through O the centre of the fulcrum of A. Considering A as a free body and taking moments around O, then for rotation in either direction

$$P(a + b) = R\,b \quad \text{or since } R = \frac{F}{\mu}$$

$$P = \frac{F\,b}{\mu(a + b)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In Fig. 137 the line of action of F does not pass through O and therefore in writing the equation for the equilibrium of A its effect must be considered, whence

$$P = \frac{F\,b}{a+b}\left[\frac{1}{\mu} \pm \frac{c}{b}\right] \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The minus sign is to be used for rotation in a clockwise direction, *for the arrangement shown*, and the plus sign for rota-

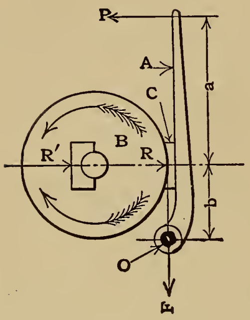

FIG. 136.

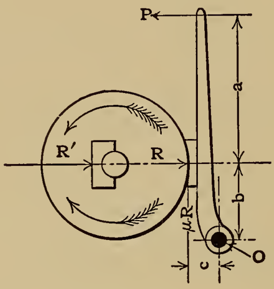

FIG. 137.

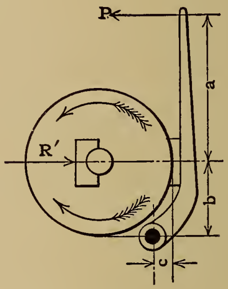

FIG. 138.

tion in the opposite direction. It is to be especially noted that for clockwise rotation when $\frac{1}{\mu} = \frac{c}{b}$, or when $b = \mu\, c$, $P = 0$; that is, the brake is *self-acting* and if put in contact the moment of the frictional force will apply it with ever-increasing pressure. Obviously such proportions should be avoided.

In a similar manner for Fig. 138

$$P = \frac{F\,b}{a+b}\left[\frac{1}{\mu} \pm \frac{c}{b}\right] \quad . \quad . \quad . \quad . \quad . \quad (3)$$

the plus sign referring to clockwise rotation, *for the arrangement shown*, and the minus sign to rotation in the opposite direction.

In this class of brakes the pressure of the brake R against the wheel is opposed by an equal force R' at the bearing near the wheel. In the calculations above, the braking effect due to friction of the journal is neglected, as its lever arm is, usually, small. It cannot be neglected in designing the bearing, and for this reason this form of brake is not well adapted to heavy work.

Fig. 139 shows an arrangement of brake beams for heavy work such as is used in mining machinery. The force W, which may be applied by a steam cylinder, acting on the system of levers, causes the brake beams B and B to press equally on opposite sides of the wheel, and causes no pressure on the bearings of the drum. If $\frac{T}{2}$ be the tension in each of the rods A and A, the frictional force exerted on the wheel is $F = 2\mu T$. If the pin O is so located that when the load W is applied it moves to O', and the centre

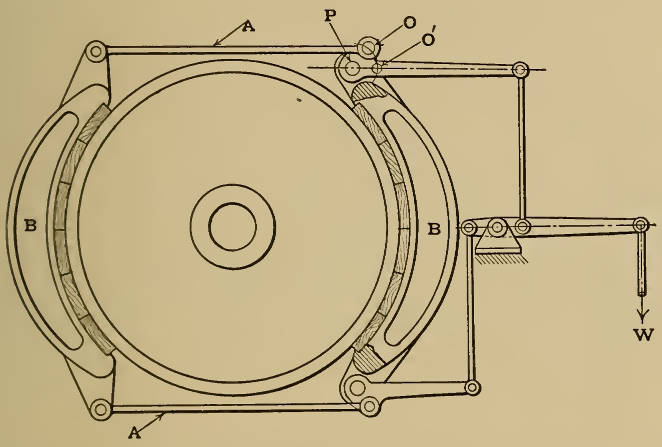

FIG. 139.

line of the rod A passes through the centre of the pin P, a toggle effect is obtained and the tension in the rods A and A may be made any desirable value; in fact with such an arrangement care must be exercised in adjusting the brake that such pressures are not brought on the pins as will cause failure by shearing. When O moves down to O' the brake is "locked" in position and the operating force may be removed. This last feature is often a valuable quality in a brake. Brakes of this character are generally lined with wooden blocks as shown.

162. Strap Brakes. If the effect of centrifugal force is neglected (see Art. 131), and the total tensions in the band (T_1

and T_2) be taken instead of the tensions per inch of width, equation (6) of that article reduces to

$$\frac{t_1}{t_2} = \frac{T_1}{T_2} = 10^k \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Where $k = 0.0076\ \mu\ a$, a being the arc of contact in *degrees*. If, also, F is the total frictional force exerted by the band upon the wheel,

$$F = T_1 - T_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It is obvious that these equations are applicable to the discussion of band brakes. Figs. 140, 141, and 142 show the most usual arrangement of band brakes. In Fig. 140 the end of the strap

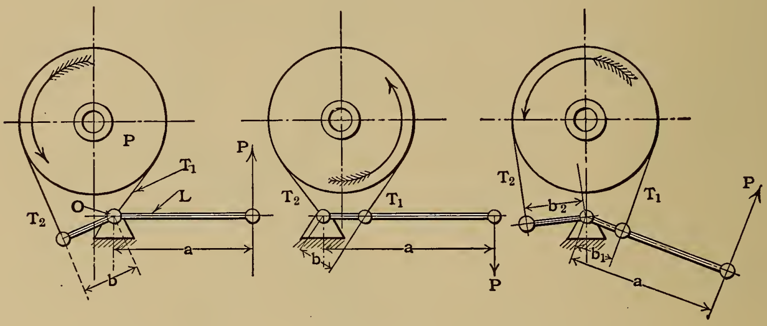

FIG. 140. FIG. 141. FIG. 142.

which is subjected to the greatest tension T_1 is anchored, for convenience, at the pin which serves as a fulcrum for the operating lever L; it could be anchored to any other convenient part of the frame.

$$\text{From (1) and (2),} \quad T_2 = \frac{F}{10^k - 1}$$

Taking moments around O

$$P\,a = T_2\,b = \frac{F\,b}{10^k - 1}$$

$$\text{or } P = \frac{F\,b}{a\,(10^k - 1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

which expresses the relation between the applied force P and the frictional resistance applied to the wheel.

In Fig. 141 the end under greatest tension is attached to the lever and the end of least tension is anchored, hence for this case

$$P = \frac{F b}{a}\left[\frac{10^k}{10^k - 1}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In Fig. 142 the end under greatest tension is anchored to the lever at a *shorter* radius than the end of least tension; so that the force which it exerts *assists* the operating force P. This is known as a **differential brake.** For this case in a similar manner as above

$$P = \frac{F}{a}\left[\frac{b_2 - 10^k b_1}{10^k - 1}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

It is to be especially noted that if $10^k b_1 = b_2$, $P = 0$, and the band will brake automatically; that is if any force is applied to the lever, the brake will continue to set itself up with ever increasing force till motion ceases or rupture occurs. This form of brake is exceedingly dangerous on account of its tendency to "grab," especially if μ is materially increased through a change in the character of the friction surfaces.

Strap brakes are usually made of wrought iron or steel. In light work they may engage with a cast-iron surface or may be lined with leather; but in very heavy work they should be lined with wood.

FRICTION CLUTCHES AND FRICTION PULLEYS

163. Friction clutches though made in a great variety of forms can, in a large measure, be classified under four principal types, namely, **Conical, Radially Expanding, Disc, and Band.** A well-designed clutch should start its load quickly but without shock, and should disengage quickly. It should be "self-sustained," that is, when the clutch is driving, no *external* force should be necessary to hold the contact surfaces together. In addition, it is often necessary that the clutch should "lock" in place, after the manner of the brake in Fig. 139.

164. **Conical Clutches.** Fig. 143 shows the elements of a conical clutch which is self-sustained. The cone F is fast to the shaft S and rotates with it. The pulley H rotates upon F and

carries with it the levers E. When the thimble B is forced under the rollers C, the levers E force the cone surfaces in contact. Heavy springs at G (not shown) throw the surfaces apart when the thimble is withdrawn. The relation between the transmitted frictional force F and the force P applied to the cone, in a direction parallel to the axis, is the same as that of the wedge gearing in Art. 159, or

$$F = \frac{\mu P}{\sin \theta + \mu \cos \theta} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The angle θ should not be less than 10°, unless some mechanism is provided for separating the cone surfaces, positively, when

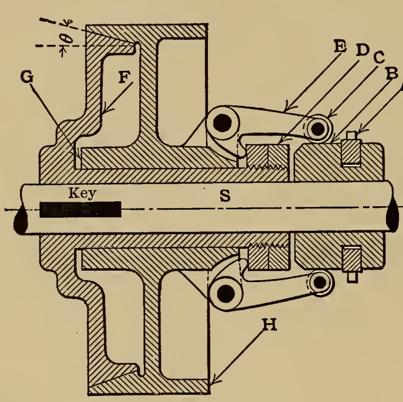

FIG. 143.

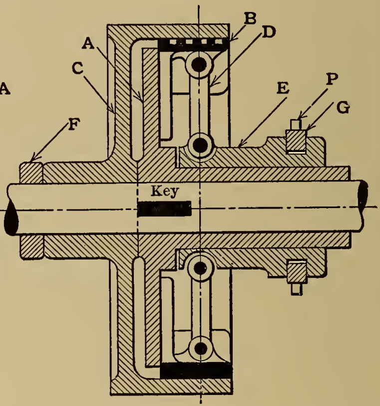

FIG. 144.

desired. For clutches that do not operate frequently, metal surfaces are often used; but where the operation of clutching is frequent, one surface is usually lined with wood, cork, or leather.

165. Radially Expanding Clutches. Fig. 144 shows the elements of a radially expanding, self-sustained clutch. The clutch body A is keyed to the shaft, while the pulley C rotates loosely upon the shaft. The circular segment B, which fits the inner surface of C, can be moved radially upon A. The loose ring G is operated axially by a forked lever fitting on the pins P. When the sleeve E is forced inward by the ring G, the links D force the segments B outward against C. In the arrangement shown the links have a toggle effect and can exert enormous pressure against

B, hence adjustment must be carefully performed. This is usually accomplished by making the length of the link D adjustable, by means of turn-buckles or similar devices, which also provide a means of compensating for wear. Usually the sleeve has motion enough to carry the inner end of the link slightly past the centre position shown, thus locking the clutch in place.

166. Disc Clutches. Fig. 145 shows the elements of a **multiple-disc clutch** as sometimes used in automobile work for connecting the engine to the transmission shaft, A being fast to the engine shaft and B to the transmission shaft. The part A carries a number of discs, C, which fit loosely in a radial direction but are prevented from rotating relatively to A by bolts E which also hold L, the cover of the case, in place. A second set of discs D, placed alternately between the discs C are carried on the part B and compelled to rotate with it by the keys G. A heavy helical spring F (sometimes made of rectangular section as shown) presses the two sets of discs together with a known load P, when the clutch is in and the shafts connected. The sleeve B while compelled by the feather S to rotate with the transmission shaft N, can be moved axially by means of the grooved collar I and the ring J; I being made fast to B but built separately from it for constructive purposes only. When B is moved to the right the spring is compressed and the pressure on the discs relieved. The discs often run in an oil bath to prevent "grabbing." It is readily seen that while the force, P, which presses each pair of contact surfaces together is the same, the total frictional force transmitted is proportional to the number of pairs of contact surfaces n or

$$F = \mu n P \qquad \qquad (7)$$

If the mean friction radius of the discs be r, the frictional moment transmitted is $Fr = \mu nPr$. In Fig. 145, $n = 7$. The above form of clutch is known as the Weston clutch. Obviously any number of pairs of discs may be used. For large work the discs are sometimes made of iron and wood (or wood-faced). For small work, alternate discs of steel and brass are employed. Many pairs of contact surfaces are then used and the discs run in oil to prevent "grabbing." The width of the wearing faces of the discs should be made small to prevent undue wear toward the outer edges of the discs *D*, as in a thrust block (Art. 105). It is better to use more discs of a smaller diameter than a few of great face.

167. Band Clutches. Fig. 146 illustrates the elements of a **band clutch.** The clutch wheel *A* (which may be fast to one shaft) carries the wood-lined band *C*. When the thimble *F* (which slides on the shaft) is forced under the lever *E*, the iron band *C* is tightened and clutches the rim of the driven wheel *B*.

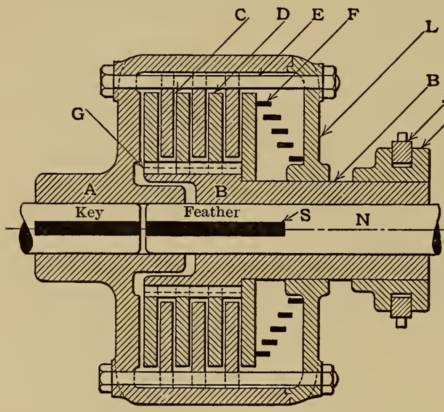

FIG. 145.

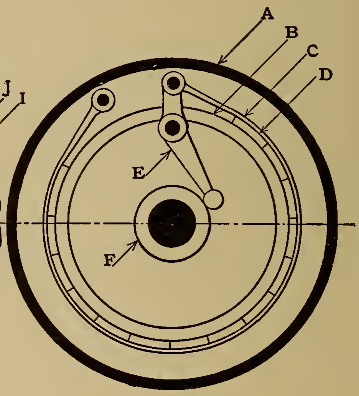

FIG. 146.

Obviously the principles involved are identical with those of the strap brake, Fig. 140 of Art. 162. For light work the band may be lined with leather, but in heavy work, such as mine hoisting, blocks of bass wood, or other soft wood, are used. The wood lining is usually made fast to the strap, though occasionally on very large diameters they are attached to the wheel so that they may be turned true in place. These clutches are made self-locking by arranging for a toggle effect in some one of the operating levers.

Occasionally the band is made to *expand inside* of the rim of the wheel to be driven. It is to be noted that this case is not the same as the one just discussed, but is a special case of a radially expanding clutch. The outward force exerted by the band may be computed by the theory of Art. 78, considering the band as a thin cylinder under compression, the compressive stress at any section being that due to the pressure applied by the operating lever.

168. **Magnetic Clutches.** A number of clutches have recently appeared which are operated magnetically. These are most generally of the disc type. Evidently the general principles above, regarding transmissive power, apply also to these clutches. In magnetic brakes, the load is usually applied by a spring, or weight, and released by magnetic action, thus insuring safety against accident should the electric service fail.

Practical Coefficients for Brakes and Clutches. The most usual combinations of friction surfaces for brakes and clutches are wood, leather, or cork with iron; and iron with iron. In the multiple-disc type, brass or bronze on iron or steel are sometimes used. Mr. C. W. Hunt gives the following values of μ as the result of considerable experience in designing clutches, namely: cork on iron, 0.35; leather on iron, 0.3; and for wood on iron 0.2. For iron on iron μ may be taken as 0.25 to 0.3. It should be remembered that if the friction surfaces are to be engaged at high velocity, lower values must be assumed than for lower speeds (see Art. 28).

The pressure per unit area of surface is also an important feature in the design of friction machinery, for if this is taken too high, excessive wear will result. Thus in disc clutches the pressure is usually taken at not more than 25 to 30 pounds per square inch and lower values are desirable. Wooden surfaces should not be loaded beyond 20 to 25 pounds per square inch. If the clutch or brake is to operate frequently, ample surface must be provided to properly radiate the heat generated.

References:—

Transactions A. S. M. E., Vol. XXX, 1908.
Transactions Inst. Mechanical Engineers, July, 1903.

CHAPTER XIV

TOOTHED GEARING

169. General Principles. When it is necessary that rotation of one shaft shall produce definite and positive rotation of another, it is evident that friction wheels, as discussed in the preceding chapter, will not suffice where any considerable amount of power is to be transmitted. In such cases the peripheral surfaces of the transmission wheels are provided with teeth, so that the motion shall be positive. It is evident that any pair of surfaces which will roll together with pure rolling motion, so as to give the required velocity ratio, may serve as a basis for the design of a pair of toothed gears; and works on mechanism treat fully of the methods of drawing the sections of such surfaces for various conditions and velocity ratios. Whether the elements of the surface thus outlined shall be parallel or otherwise will depend on the angle which the shafts make with each other, as in the case of friction wheels, and tooth gearing may be classified* according to the character of the pitch surfaces, and the relation of the axes, thus:

Kind.	Relation of Axes.	Pitch Surfaces.
Spur	Parallel	Cylinders
Bevel	Intersecting	Cones
Screw	Not in one plane	Cylinders
Skew	Not in one plane	Hyperboloids
Twisted	Any	Any of the above
Face	Any	None, strictly

The most important of these are spur, bevel, and a few special forms of twisted and screw gears. The motion transmitted by a pair of properly designed toothed gears is identical with that of the base curves or surfaces rolling together. If r_1 and r_2 be

* See "Kinematics of Machinery," by John H. Barr, page 110.

the instantaneous radii of such a pair of surfaces at the point of contact, and ω_1 and ω_2 be their instantaneous angular velocities, then $\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1}$. In the most common case the angular velocity of both shafts is constant and hence r_1 and r_2 are constant, and the rolling surfaces are circular in cross-section. Thus Fig. 147 shows a portion of two gears whose rolling surfaces are a pair of circular cylinders, represented in cross-section by the circles *C* and *D*. If the teeth are properly proportioned the motion transmitted will be identical with that produced by the rolling of *C* on *D*. It can be shown that the condition which such tooth outlines

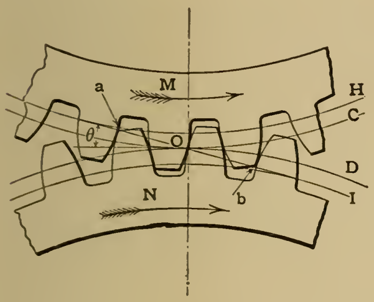

FIG. 147.

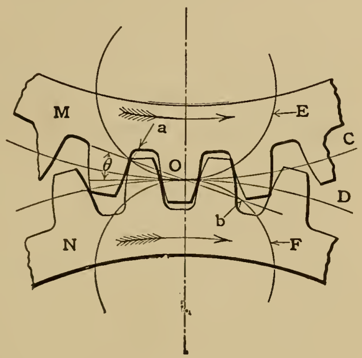

FIG. 148.

must fulfil in order that the velocity ratio may be constant, is that *the common normal to the tooth outlines at the point of contact must always pass through the point of tangency of the rolling circles*. There are many curves which can be used for tooth outlines, and which would fulfil the condition, but in practice only two are commonly employed, namely, the **involute** and the **cycloid.**

Fig. 147 illustrates a portion of two gears with involute teeth. The upper wheel, *M*, is the driver. Contact between two teeth has just begun at *a*, and the common normal to the point of contact *a O b* passes through the pitch point *O*. As the wheels rotate the point of contact will move along the line *a O b* till contact ceases at *b*. Hence in the involute system the normal to the point of contact makes a fixed angle with the common tangent to the pitch circles.

Fig. 148 shows a portion of two gears with cycloidal teeth. Contact is just beginning at *a*, and as the gears rotate the point of contact will move along the curved path *a O b*, contact ceasing at *b*. The normal to the first point of contact is drawn, and it is clear that the inclination of the normal to the common tangent of the pitch circles, is a maximum at this point, and continually varies in direction though always passing through the point *O*. It can be shown that in the involute system the angular velocity ratio will remain constant, within the limits of action, whether the pitch circles are tangent or not; but for the transmission of constant velocity ratio with cycloidal gearing the pitch circles must remain tangent. The involute gear, therefore, has a decided advantage for general use and it has practically superseded the cycloidal for most work. A fuller treatment of the theory of gear-tooth outlines, which is beyond the scope of this work, will be found in treatises on mechanism.*

170. Interchangeable Systems of Gearing: Standard Forms. It is desirable in practical work that any gear of a given pitch shall run properly with any other gear of the same pitch. In order that this may be so, certain limitations must be placed upon the form and dimensions of the tooth. In the cycloidal system interchangeability may be accomplished, as far as the tooth outlines are concerned, by keeping the diameter of the describing circle constant for all gears of the series.

Any involute tooth outline will run properly with any other similar outline; and any gear with involute teeth will run with any other gear having similar teeth, as far as the length of the involute outlines will permit, providing the thickness of teeth will allow them to mesh. In order to obtain involute outlines of sufficient length, and a series of gears with fixed nominal pitch circles, the angle θ, Fig. 147, made by the line of action with the common tangent to the pitch circles must have a proper value, and be constant for all gears of the series. In the systems in

* See "Kinematics of Machinery," by J. H. Barr, page 111; also, "Machine Design," part 1, by F. R. Jones.

most common use this angle is $14\frac{1}{2}°$, though there is a tendency in modern work toward a greater angle.

It is found undesirable in practice to make gears with less than twelve teeth; and in some cycloidal systems the radius of a twelve-tooth gear of the required pitch is taken as the diameter of the describing circle. For a twelve-tooth gear this will result in radial lines for the tooth outlines below the pitch circle, *i.e.*, the tooth will have radial **flanks.** In the practice of the Brown & Sharpe Mfg. Co., the diameter of the describing circle is the radius of the fifteen-tooth gear of the series. This gives spaces between the flanks of the teeth on the twelve-tooth, or smallest gear, so nearly parallel that they may be cut with a rotary cutter.

It is evident from Figs. 147 and 148 that the tooth outlines of any system may be extended both above and below the pitch line till they meet. It is also clear that the longer the teeth the earlier will they engage with each other, the greater will be the arc of contact, and the greater will be the number of teeth continually in contact. The distribution of the load over a number of pairs of teeth is in itself conducive to smooth running; but on the other hand, extending the arc of contact away from the pitch point, increases the sliding between teeth, and also the velocity with which the teeth approach each other. The tooth also becomes weaker as it is lengthened, the thickness remaining the same, and for these reasons a practical limit is placed on the length of teeth. The length of tooth adopted in practice is, therefore, a compromise between conflicting conditions, which experience has shown will give good results.

The distance along the pitch line from any point on a tooth to a corresponding point on the next tooth, is called the **circular pitch**; and will be noted by s. The **thickness of the tooth** along the pitch line will be denoted by t, Fig. 151. In the case of cut gears, where no clearance is allowed between teeth, $t = \frac{s}{2}$. In some forms of gears, such as shown in Fig. 150, where a metal pinion engages with a gear having wooden teeth, the pitch may not be equally divided, but the metal tooth may be thinner than the wooden tooth. If N be the number of teeth and D the diameter, then evidently $Ns = \pi D$. If the number of teeth N be divided by the diameter, the quotient, or *the teeth per inch of diameter*, is called the **diametral pitch** and will be denoted by S. Since $S = \frac{N}{D}$ and $s = \frac{\pi D}{N}$, $S \times s = \pi$ $\therefore S = \frac{\pi}{s}$ and $s = \frac{\pi}{S}$. The diametral pitch is, ordinarily, the most convenient for use, and in this country practically all interchangeable systems are based upon the diametral pitch. Thus a gear 24″ in diameter and 3 diametral pitch would have $24 \times 3 = 72$ teeth, and the circular pitch would be $\frac{\pi}{3} = 1.05$ inches. In the system of teeth adopted by the Brown & Sharpe Mfg. Co., and which is used very extensively in America, the following proportions are given for *cut* teeth. See Fig. 151.

Let D_1 = the outside diameter of the gear.
" D = the pitch diameter of the gear.
" D_2 = the diameter of a circle through bottom of space.
" S = the diametral pitch.
" s = the circular pitch.
" a = the addendum = height of tooth above pitch line.
" c = the clearance between top of tooth and bottom of space when gears are in mesh.
" d = the dedendum, or total depth of space below pitch line.
" t = the thickness of tooth on pitch line = width of space on pitch line in cut teeth.
" N = the number of teeth in gear.
" h = the total height of tooth.

Then $N = D\,S = \frac{\pi D}{s}$,

$$t = \frac{s}{2} = \frac{\pi}{2S}$$

$$c = \frac{t}{10} = \frac{\pi}{20S}$$

$$a = \frac{1}{S}$$
$$d = a + c$$
$$h = 2a + c$$
$$D_1 = \frac{N + 2}{S} \text{ and } D_2 = D - 2(a + c).$$

In the case of *rough* gear teeth, cast from a wooden pattern, the thickness of the tooth must be less than the width of the space,* and the clearance at the bottom of the space must be greater than in cut teeth. If the gears are **machine-moulded,** the difference need not be quite so great as in pattern-moulded gears. For pattern-moulded gears good practice gives $t = 0.45\,s$ for large gears, to $0.47\,s$ for small gears, and the corresponding width of the space would be $0.55\,s$ to $0.53\,s$. For machine-moulded gears $t = 0.46s$ to $0.48\,s$ and the corresponding space would be $0.54\,s$ to $0.52\,s$.

Table XXVI gives dimensions of gear teeth for cut spur gears, in accordance with the standards of the Brown & Sharpe Mfg. Co.

171. **Methods of Making Gear Teeth.** Metallic gear wheels are either cast from a pattern, or the rim is cast or forged solid, and the teeth are cut from the solid metal by rotary or reciprocating cutters. Where the gear teeth are cast, it is very important that the pattern itself be very accurately made; for even with the greatest care in moulding, it is impossible to obtain true spacing, on account of shrinkage and displacement due to "rapping" the pattern in the sand. For this reason, and on account of the difficulty of obtaining smooth surfaces, greater clearance must be allowed in cast gears than in cut gears, as already noted. Wooden patterns are very unreliable for such work, on account of their tendency to warp and shrink, and permanent patterns should be made of metal. If the pattern for a spur gear is withdrawn from the sand with a movement *parallel* to the length of the tooth, the tooth pattern must have draft, or be slightly tapering to facilitate drawing, and consequently the cast tooth must also be tapering. Care should be taken in assembling such gears, that the tapers in

* The difference between the thickness of the tooth and the width of the space is commonly called "backlash."

the two gears are reversed to avoid having the thick ends of both sets of teeth come together, thus concentrating the pressure at one end. Rough cast gears, of the kind described above, are used only for rough or large work, and not for high speed. The particular defect of spur gears due to draft does not exist in bevel gearing.

In gear-moulding machines the pattern consists of a segment of the gear pattern, carrying several teeth. The pattern is

TABLE XXVI

PROPORTIONS OF GEAR TEETH

Diametral Pitch. S	Circular Pitch. s	Thickness of Tooth. t	Addendum $\frac{1}{S}=a$. a	Depth of Space Below Pitch Line. $a+c$	Total Depth of Tooth. $2a+c$
1	3.1416	1.5708	1.0000	1.1571	2.1571
$1\frac{1}{4}$	2.5133	1.2566	.8000	.9257	1.7257
$1\frac{1}{2}$	2.0944	1.0472	.6666	.7714	1.4381
$1\frac{3}{4}$	1.7952	.8976	.5714	.6612	1.2326
2	1.5708	.7854	.5000	.5785	1.0785
$2\frac{1}{4}$	1.3963	.6981	.4444	.5143	.9587
$2\frac{1}{2}$	1.2566	.6283	.4000	.4628	.8628
$2\frac{3}{4}$	1.1424	.5712	.3636	.4208	.7844
3	1.0472	.5236	.3333	.3857	.7190
$3\frac{1}{2}$	.8976	.4488	.2857	.3306	.6163
4	.7854	.3927	.2500	.2893	.5393
5	.6283	.3142	.2000	.2314	.4314
6	.5236	.2618	.1666	.1928	.3595
7	.4488	.2244	.1429	.1653	.3082
8	.3927	.1963	.1250	.1446	.2696
9	.3491	.1745	.1111	.1286	.2397
10	.3142	.1571	.1000	.1157	.2157
11	.2856	.1428	.0909	.1052	.1961
12	.2618	.1309	.0833	.0964	.1798
13	.2417	.1208	.0769	.0890	.1659
14	.2244	.1122	.0714	.0826	.1541
15	.2094	.1047	.0666	.0771	.1437
16	.1963	.0982	.0625	.0723	.1348
17	.1848	.0924	.0588	.0681	.1269
18	.1745	.0873	.0555	.0643	.1198
19	.1653	.0827	.0526	.0609	.1135
20	.1571	.0785	.0500	.0579	.1079

mounted on an axis in such a manner that it can be rotated accurately through any portion of a complete revolution, or "indexed." In forming the mould the segmental pattern is placed in position and sand is rammed around it. The pattern is then withdrawn *radially* and rotated to the next succeeding position (the indexing device insuring accurate spacing), the operation being repeated till the whole circumference is moulded. The mould for the hub and arms is then completed, in large work this last being often accomplished by means of cores. If machine moulding is well done the results are far superior to those obtained by pattern moulding, and gears may be made that can be run at moderately high speeds. Obviously, however, all cast gears are much more inaccurate than cut gears, and the latter are preferable where high speeds and smoothness of action are required.

Metallic gearing, even when accurately cut and aligned, is inclined to be very noisy when run at a peripheral speed of more than 1,200 feet per minute, especially if any appreciable "back-lash" exists. Relieving the points of the teeth, slightly, reduces the tendency to produce noise. Where high speeds are unavoidable the teeth of one of the mating gears is sometimes made of wood or rawhide. Wheels with wooden teeth are known as **mortise wheels.** They are not as much used as formerly, because modern methods of gear-cutting produce metallic gears of such accurate form that they may be run in places where mortise gears were formerly considered indispensable. In making mortise wheels the wooden teeth are roughed out and the shank is fitted into openings cast in the rim of the wheel, as shown in Figs. 149 and 150. The teeth are held in place by the keys, *K*, or pins, *P*,

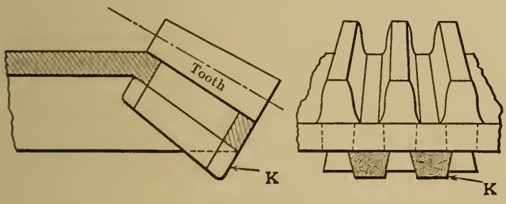

FIG. 149.

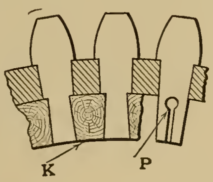

FIG. 150.

as shown. The teeth proper are dressed to correct form with hand tools or by special machines using a fine circular saw for a cutter.

Usually the large gear, only, is made with wooden or "mortise" teeth, the pinion being made of metal. This is rational since the pinion, on account of the shape of its teeth, is the weaker of the two, and also because the teeth of the pinion come into contact more frequently, and hence suffer greater wear. In such combinations, the metal gear frequently has teeth of thickness less than $\frac{s}{2}$ and the wooden gear teeth of thickness greater than $\frac{s}{2}$, to equalize strength. See Fig. 150. In recent years gears made of rawhide have been much used for high speeds. The blanks for rawhide gears are made by cementing specially prepared rawhide discs together under great pressure. Metallic discs, on each side of the blank, held together by rivets passing through the blank, assist the rawhide teeth in retaining their form. The teeth are cut in the blank in the same manner that metallic teeth are cut. In using rawhide gearing the pinion is almost always made of rawhide and the larger gear of cast iron or brass. Such a combination may be run at a very high rate of speed, 3,000 feet per minute being a not unusual velocity. Rawhide gears are almost noiseless in operation but care must be used that they are not subjected to extreme moisture nor run in too dry an atmosphere.

Formerly it was cheaper to cast gear teeth, but the development of gear-cutting machinery has changed the situation where a large number of gears with small teeth are to be made. Modern methods of gear-cutting produce teeth of great accuracy, and have also so greatly reduced the cost of production that for high speeds, and where smoothness of action is necessary, cut gears have largely superseded cast gears even in large work.

There are many methods of cutting gear teeth in practical operation, the most common method of cutting spur gears being by the use of a rotating cutter.* The outlines of gear teeth vary with the number of teeth in the gear, the pitch or thickness of tooth remaining constant, and, theoretically, a different cutter is required for every different diameter of gear in a series of the same pitch. To meet this requirement would lead to an excessive number of cutters for each pitch. It is found in practice, however, that the same cutter can be used, without serious error, for

* See "Gear-Cutting Machinery," by Ralph E. Flanders.

several sizes of gears of a given pitch. In the system adopted by the Brown & Sharpe Mfg. Co., only 24 cutters are used for each pitch in the cycloidal system, and only 8 cutters for each pitch in the involute system, as given below. The letters and numbers in the first column are the manufacturer's designations, for purposes of ordering cutters.

TABLE XXVII

CUTTERS FOR CYCLOIDAL TEETH

Cutter	A	cuts	12 teeth.	Cutter	M	cuts	27	to 29	teeth.
"	B	"	13 "	"	N	"	30	" 33	"
"	C	"	14 "	"	O	"	34	" 37	"
"	D	"	15 "	"	P	"	38	" 42	"
"	E	"	16 "	"	Q	"	43	" 49	"
"	F	"	17 "	"	R	"	50	" 59	"
"	G	"	18 "	"	S	"	60	" 74	"
"	H	"	19 "	"	T	"	75	" 99	"
"	I	"	20 "	"	U	"	100	" 149	"
"	J	"	21 to 22 teeth.	"	V	"	150	" 249	"
"	K	"	23 " 24 "	"	W	"	250	or more	"
"	L	"	25 " 26 "	"	X	"	Rack.		

TABLE XXVIII

CUTTERS FOR INVOLUTE TEETH

Cutter	No. 1	cuts	from	134 teeth to rack.	
"	" 2	"	"	55 to 134	teeth.
"	" 3	"	"	35 " 54	"
"	" 4	"	"	26 " 34	"
"	" 5	"	"	21 " 25	"
"	" 6	"	"	17 " 20	"
"	" 7	"	"	14 " 16	"
"	" 8	"	"	12 " 13	"

When gear-cutting is carefully done, very accurate work may be accomplished. It is to be noted, however, that the form of the teeth when cut with a set of cutters, as above, are not all theoretically correct;* and even in best practice the error in the gear-cutting machine itself, coupled with that due to dullness of cutters and deviation due to different degrees of hardness in the metal, may be considerable.

172. Forces Acting on Spur Gears. In Fig. 151 let the gear A drive the gear B. Let V_a be the velocity of the pitch circle of A; and V_b be the velocity of the pitch circle of B. Also let W_a be the *equivalent driving force* acting at the pitch circle of A, and let W_b be the *equivalent resisting force* acting at the pitch circle

* There are gear-cutting machines which, theoretically, generate correct forms of teeth for all gears of a series.

of B. If now the tooth outlines are properly constructed, the line of action of the *actual* driving force W_1 will always pass through the pitch point and the angular velocity ratio of A to B will be constant. The action of the pitch circles will be as though they rolled upon each other and their linear velocity will be the same or $V_a = V_b$. From the principle of work

$$W_a \; V_a = W_b \; V_b \quad \text{Therefore } W_a = W_b$$

The tangential driving force exerted by one gear upon another is, therefore, independent of the angle of pressure, in any correct

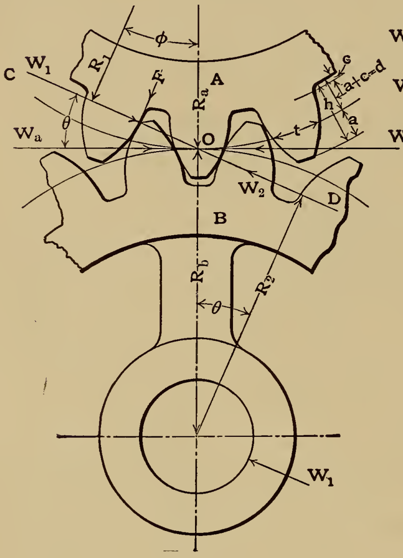

FIG. 151.

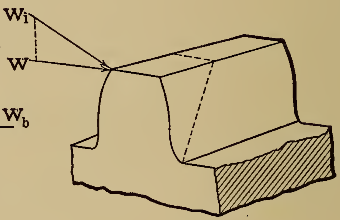

FIG. 152.

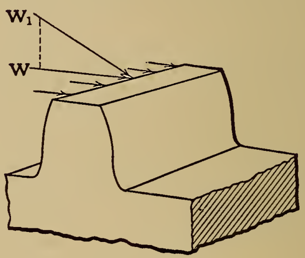

FIG. 153.

system of gearing, and the action is, in this respect, the same as if a pair of teeth were continually in action at the pitch point.

The distribution of the reaction at the bearings due to the pressure between teeth (W_1, Fig. 151), and its bending effect on the shaft which supports B, will depend upon the relative positions of the gear and bearings; but the latter will, in any case, be directly proportional to W_1. As the obliquity of the line of action $C \, D$ is increased, the angle θ (Fig. 151) is increased and hence sec θ is also increased. Therefore, since $W_1 = W_a \sec \theta$, the pressure on the bearings is increased with an increase on the obliquity of the line of action; but the torque on the driven shaft remains unchanged.

In cycloidal gearing the obliquity varies from a maximum at the beginning of the contact to zero when the contact point lies in the line of centres; and, during the arc of recess, it increases to a maximum at the end of contact. The maximum value of the angle θ, Fig. 148, is about 22°, with usual forms of cycloidal teeth. When θ equals 22°, sec θ equals 1.08, or the maximum normal pressure is about 8 per cent greater than the tangential rotative force.

The obliquity is constant throughout the arc of action in involute gears, and the angle θ, Fig. 147, is usually 14½° or 15°. When $\theta = 15°$, sec $\theta = 1.035$, or the normal pressure is 3½ per cent greater than the tangential force. In the above discussion the influence of friction has been neglected. During the arc of approach the frictional force F (Fig. 151) deflects the line of action of W_1 in such a way as to increase the effective obliquity. During the arc of recess it acts in the opposite direction and decreases the obliquity. The influence of this frictional force is small and may, usually, be neglected, but its action accounts, to a certain degree, for the well-known fact that gears run more smoothly during recess than during approach.

It is usually intended that more than one pair of teeth shall be in action at all times, but, owing to the unavoidable inaccuracy of form and spacing previously noted, it is not safe to depend upon a distribution of the load between two or more teeth of a gear. It is safest to provide sufficient strength for carrying the entire load on a single tooth. In the rougher classes of work, this load may be concentrated at one end of the tooth, as indicated in Fig. 152, and all such gears should be carefully inspected and corrected, if intended to carry heavy and important loads. With well supported bearings, and machine-moulded or cut gears, it is not unreasonable to consider the load as fairly well distributed across the face of the gear, if the face does not exceed in width about three times the circular pitch (see Fig. 153).

The obliquity of the line of pressure gives rise to a crushing action on the teeth (due to the radial component of the normal force), in addition to the flexural stress which results from the tangential component. This crushing component, with the ordinary proportions of teeth, does not exceed 10 per cent of the normal pressure. Its effect is to reduce the tensile stress due to flexure, and to increase the compressive stress. Since cast iron is far stronger in compression than in tension, this may be neglected in gears made of that metal, while in the case of steel, or composition gears, the margin of safety assumed usually makes it unnecessary to consider this component.

173. Strength of Spur Gear Teeth. The assumption often made that the teeth of spur gears can be considered as rectangular cantilevers, in determining their strength, is not satisfactory, especially when treating pinions having a small number of teeth. Fig. 154 shows four gear teeth which have the same thickness at the

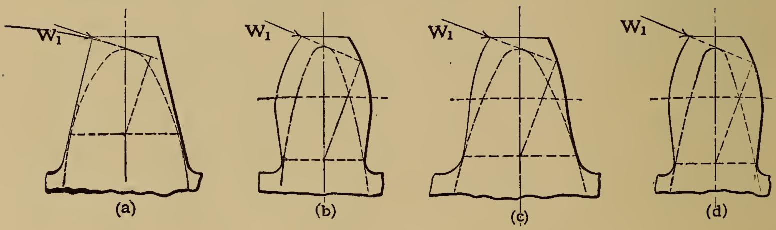

FIG. 154

pitch line and the same height. The tooth marked (a) is one of an involute rack; (b) is one of an involute pinion having 12 teeth;* (c) is one of a cycloidal gear having 30 teeth; (d) is one of a cycloidal pinion of 12 teeth.

Mr. Wilfred Lewis, of Wm. Sellers & Co., seems to have been the first to investigate the strength of gear teeth with due regard to the actual forms used in modern gearing. His work was published originally in the Proceedings of the Engineers' Club of Philadelphia, in January, 1893, and his method of investigation was as follows: Accurate drawings of gear teeth were made on a large scale, and the line of action of the normal force, when acting on the point of a tooth was drawn in; see Fig. 155. From the

* The 12-tooth involute pinion may have its teeth weakened by a correction for interference; but it is usually better to correct the points of the mating gear.

intersection of this line of action with the centre line of the tooth, a parabola was drawn tangent to the sides of the tooth, thus locating a beam of uniform strength equal to the effective strength of the tooth (see Article 15). The points of tangency a, a, locate the weakest section of the tooth, and the bending moment applied to this section is $W\,l$. Then from equation J, page 94.

$$W\,l = p\frac{I}{e} = \frac{p\,b\,(2\,h)^2}{6} = \tfrac{2}{3}\,b\,p\,h^2 = b\,p\,s\left(\tfrac{2}{3}\,\frac{h^2}{s}\right)$$

$$\text{or } W = b\,p\,s\left(\tfrac{2}{3}\,\frac{h^2}{s\,l}\right) = b\,p\,s\,(y) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Where b = the breadth of the tooth in inches, p = the tensile stress, and s = the circular pitch. The factor y is a variable, depending on the shape of the tooth. Mr. Lewis found that its value is practically independent of the pitch (since s, h and l are proportional to the pitch), but dependent mainly on the number of teeth in the gear. Tabulated values of this coefficient may be found in Kent's "Mechanical Engineers' Pocketbook," page 901. From these tabulated values, Mr. Lewis deduced the following equations in which N = the number of teeth in the gear.

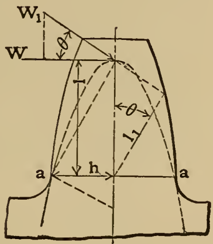

FIG. 155.

For the 15° involute system and the cycloidal system using a generating circle whose diameter equals the radius of the 12-tooth pinion,

$$W = b\,p\,s\left(0.124 - \frac{0.684}{N}\right) \quad . \quad . \quad . \quad . \quad (2)$$

For the 20° involute system,

$$W = b\,p\,s\left(.154 - \frac{.912}{N}\right) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Mr. Lewis' investigations on cycloidal gears were made on a system using the radius of the 12-tooth pinion as the diameter of the describing circle. Modern practice sometimes makes the radius of the 15-tooth pinion the diameter of the describing circle, which gives somewhat weaker teeth than the first system. The difference is small, however, compared to the variation in the assumed stress, p, and since cycloidal teeth are now little used for small and moderate-sized gears, equation (2) will be adopted in this work for standard gears.

The Lewis' formula is convenient for determining W, b, s, or p, where the number of teeth (N) is known; but a very common problem in design is to determine the *pitch* (s), when the pitch diameter of the gear is given and the *number of the teeth is unknown.* The formula may be adapted to this last stated problem as follows.* To accord with modern practice, circular pitch will also be transformed to diametral pitch.

Let D = the pitch diameter.

" w = the load per inch of face.

" S = the diametral pitch $= \frac{\pi}{s}$ or $s = \frac{\pi}{S}$

Then $N = D \times S$

$$\text{Therefore } W = b\,s\,p\left(.124 - \frac{.684}{N}\right) = b \times \frac{\pi}{S} \times p\left(.124 - \frac{.684}{D\,S}\right)$$

$$\text{or } W = b\,p\left(\frac{.389}{S} - \frac{2.15}{D\,S^2}\right) \qquad (4)$$

or since $w = W \div b$

$$w = p\left(\frac{.389}{S} - \frac{2.15}{D\,S^2}\right) \qquad (5)$$

$$\text{and therefore } S = \frac{p}{w}\left(.194 + \sqrt{.038 - \frac{2.15\,w}{p\,D}}\right) \qquad (6)$$

The pitch can be found from equation (6) for any values of w, D, and p, when the face of the gear is known or assumed. A common problem is as follows: The distance between two shafts and their velocity ratio is known; required the pitch of spur gears to connect these shafts for a given load and working stress on the teeth. The centre distance of the shafts, and the velocity ratio fix the diameter of the gears. The face of the gears may be governed by the space available, or it may be assumed by the designer upon other considerations. To illustrate; suppose

* See a discussion by John H. Barr, Trans. A. S. M. E., Vol. XVIII, page 766.

W = 15,000 lbs., p = 8,000 lbs. per square inch, and that the smaller gear is to be 40 inches diameter. Assume also that the face of the gear may be taken as 6 inches. The load per inch of face is $w = 15{,}000 \div 6 = 2{,}500$ lbs., hence,

$$S = \frac{8{,}000}{2{,}500}\left(.194 + \sqrt{.038 - \frac{2.15 \times 2{,}500}{8{,}000 \times 40}}\right) = 1.1 \text{ or say } 1$$

diametral pitch.

The diagrams shown in Figs. 156 and 157 are plotted from equation (5). That in Fig. 156 covers the range from 12 to 6 diametral pitch and Fig. 157 covers the range from 5 to 1 diametral pitch. The abscissas (Scale A) represent pitch diameters of gears in inches, and the ordinates (Scale B) the load in pounds *per inch of width of face, for a stress of* 6,000 *pounds* per square inch. Any other stress could have been taken for plotting the diagrams, and any other may be used in solving problems by them. A curve is drawn for each pitch; **to illustrate,** let $S = 1.5$, let $p = 6{,}000$. Substituting these values in (5)

$$w = 6{,}000\left(\frac{.389}{1.5} - \frac{2.15}{D \times 2.25}\right)$$

$$\text{or} \quad w = \left(1{,}556 - \frac{5{,}730}{D}\right)$$

hence when $D = 3.7$, $w = 0$; $D = 10$, $w = 983$; $D = 20$, $w = 1{,}270$, etc.

Plotting these corresponding values of D and w as abscissas and ordinates, respectively, the curve for a diametral pitch = 1½ is drawn through the points. The other curves are constructed in a similar manner. If, then, the diameter of the gear is known, the allowable load per inch of face *for a stress of* 6,000 *pounds per square inch* may be found by passing vertically upward from the given diameter on scale A, to the curve corresponding to the pitch, and then moving horizontally to the left-hand scale B, which gives the required load per inch of face. Scale B is reproduced at the top of the diagram, as scale C, and a 45° diagonal marked 6,000 is drawn from the lower right-hand corner of the diagram to scale C. If, then, instead of moving horizontally from the pitch curve to scale B on the left, the movement be

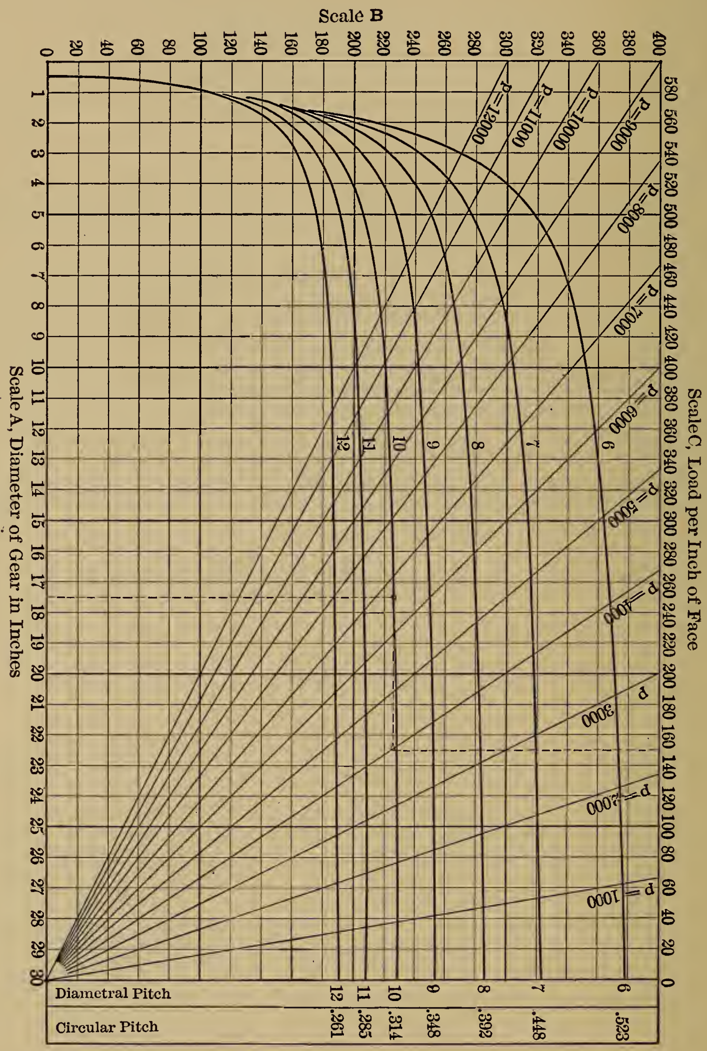

Fig. 156.

horizontally to the right (or left) to the diagonal marked 6,000, and then vertically upward to the scale C, the same reading will be obtained on C as originally found on B.

Furthermore, if other diagonals be drawn, as shown, from various points on scale C, they may be used to read loads per inch

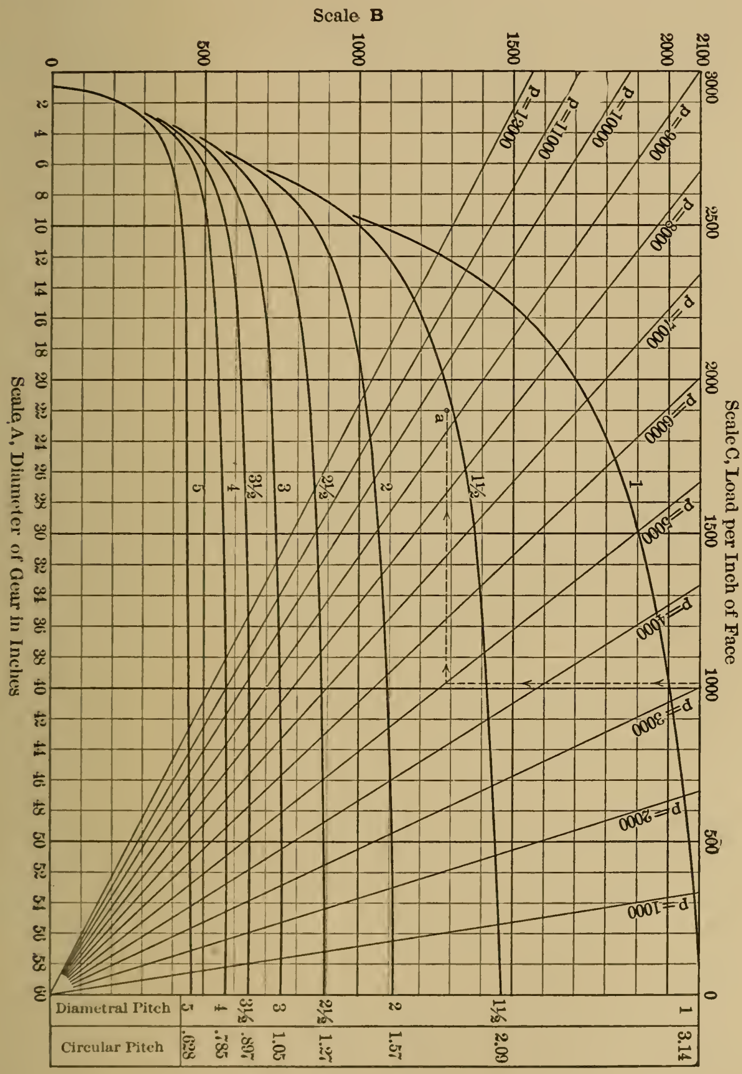

FIG. 157.

of face for stresses corresponding to these respective points on this scale, since from equation (4) it appears that the stress varies directly with the load. These stresses are indicated along the several diagonals. Thus to find the pitch when $w = 2,500$, $p = 8,000$, $D = 40$, from 2,500 on scale (C), Fig. 157, pass vertically downward to the diagonal marked $p = 8,000$; then horizontally to a point on the vertical rising from $D = 40''$, on scale A. The pitch curve nearest this point is for 1 diametral pitch which would be the required pitch.

If it is required to find the load per inch of face for a gear of given diameter and pitch, with an assigned stress, start at the point on scale (A), corresponding to the diameter; pass upward to the given pitch curve; thence horizontally (right or left as the case may be), to the proper stress diagonal; thence upward to scale (C), where the unit load may be read off. (See Fig. 156 or 157.)

If the diameter, pitch, and unit load are known quantities, pass upward from the diameter reading on scale A, to the proper pitch curve; thence horizontally to a point under the unit load on scale C, and the stress may be found by interpolating between the adjoining diagonals.

It may be noticed that the different pitch curves in either Fig. 156 or 157 have a common tangent through the origin O. The points of tangency correspond to the diameters of gear at which cycloidal teeth have radial flanks for their respective pitches; *i.e.*, a 12-tooth gear. The various curves have not been extended far beyond this point as in that case they intersect, and mar the clearness of the diagram. Since this intersection occurs after the diameter of the 12-tooth gear is reached, it is evident that the remainder of the curve is of no practical importance. In fact, increase of pitch, or a less number of teeth *for a given diameter*, beyond this point will not give additional strength, because the length of tooth will be increased, and the flanks will be undercut to an extent which more than compensates for the added thickness of tooth.

The data may be such that a point corresponding to a, Fig. 157, will lie to the left and above all the pitch curves, *i.e.*, above the common tangent through O. If the same data were substituted in equation (4) an imaginary quantity would result. This means that the unit load taken cannot be carried with the stress and diameter assumed, by *any possible pitch*. The only recourse for a gear, of the given diameter and total load W, is to increase the face, and thus reduce w, or to use a material which permits a higher intensity of stress.

It should be noted that the teeth of the smaller gear of a mating pair are weaker in form than those of the larger. The wear, also, is greater on the teeth of the smaller gear since they come in contact more frequently. Hence, in general, if the small gear is properly designed the larger gear will have sufficient strength.

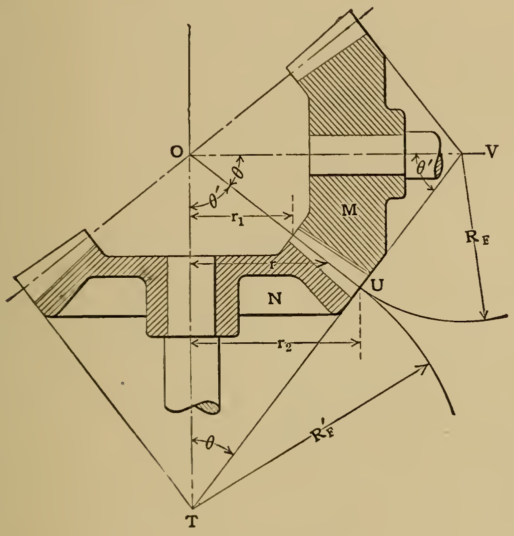

FIG. 158.

This does not apply to certain forms of reinforced or "shrouded" teeth discussed later, nor, necessarily, where the thickness of teeth and spaces are unequal, nor where the mating gears are of different material.

174. Strength of Bevel Gear Teeth. If a pair of bevel gear teeth, Fig. 158, have just come into contact as shown at *a*, Fig. 147, then the driving force is applied to the *point* of the driven tooth by the *root* of the driver. The tooth of the driven wheel will be deflected a certain amount, while the deflection of the driving tooth will be negligible. Since the deflection of the driven tooth is caused by the rotative effort of the driving gear, the magnitude of this deflection at any point on the line of contact of the two teeth will be proportional to the movement of the corresponding contact point of the driver or to its distance from the axis of rotation of the driver; hence, from similar triangles, will also be proportional to the distance from its own axis of rotation. Now the cross-sectional outlines of the tooth are similar at all points, and it can be shown that in the case of simple cantilevers of similar form the load applied is proportional to deflection. It has just been shown, however, that the deflection of the tooth at any point is proportional to the distance of that point from the axis of rotation. Hence the load on the tooth at any point must also be proportional to the distance from the axis, being least at the small end, and greatest at the large end, the *mean* value being at the middle of the tooth. Therefore a spur gear which has the same width of face, and teeth of the same form and pitch as the *mean* section, will have, theoretically, the same strength as the bevel gear. It can also be shown that in simple cantilevers of *equal breadth* and *similar outline*, the stresses induced at corresponding points on the cantilevers are equal, if the load applied is proportional to the linear dimensions. Hence the maximum stresses are the same at all sections of the tooth.

It is evident that the relation established above between the mean section of a bevel gear and a spur gear with similar teeth, may be (and often is) used as a means of designing bevel gears. It is much more convenient, however, to deal with the teeth at the outer or large end. If, also, the pitch radii are used instead of the addendum radii, the error will not be great.

Let r_1 = the pitch radius at the small end of the tooth.
" r_2 = the pitch radius at the large end of the tooth.
" r = the mean pitch radius of the tooth.
" b = the width of the face of the tooth along the elements.
" w = the load per inch of face at the radius r.
" w_2 = the load per inch of face at the radius r_2.
" W = the resultant load on the tooth = $w\,b$.
" w_e = the equivalent load per inch of face which, if acting at a radius r_2, would produce the same rotative effect as the actual load.

Since the load on the tooth varies as the radii, the *total resultant* load will act at a radius $R = \frac{2}{3}\frac{(r_2^3 - r_1^3)}{(r_2^2 - r_1^2)}$, and the torsional moment due to the resultant force, W, will be

$$W R = \frac{2}{3} w\, b \frac{(r_2^3 - r_1^3)}{(r_2^2 - r_1^2)}.$$

Now by definition $w_e\, b\, r_2 = W R$.

$$\text{Therefore, } w_e = \frac{2}{3} w \frac{(r_2^3 - r_1^3)}{(r_2^2 - r_1^2)\, r_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Also, since the load varies with the radius,

$$\frac{w_2}{r_2} = \frac{w}{r} = \frac{w}{\frac{r_2 + r_1}{2}} = \frac{2\, w}{r_2 + r_1} \qquad \therefore w_2 = \frac{2\, w\, r_2}{r_2 + r_1} \quad . \quad . \quad . \quad (8)$$

And from (7) and (8)

$$\frac{w_2}{w_e} = \frac{2\, w\ r_2}{r_2 + r_1} \div \frac{2}{3} w \frac{(r_2^3 - r_1^3)}{(r_2^2 - r_1^2)\, r_2} = \frac{3\,(r_2 - r_1)\ r_2^2\ *}{(r_2^3 - r_1^3)} = k \quad (9)$$

The actual load, w_2, will always be greater than w^e in the ratio shown above. A bevel gear, therefore, will be more heavily loaded at the large end than a spur gear of the same diameter, and carrying the same torque, in the ratio shown above. If, however, w_e is known, w_2 can be computed, and used in equations (5) and (6) of Art. 173 instead of w. Usually r_1 is made not less than $\frac{2}{3} r_2$. When $r_1 = \frac{2}{3} r_2$, $\frac{w_2}{w_e} = k = 1.4$, and this value can be used in computing w_2 unless the face of the gear is excessively long. It should be especially noted that in solving problems in bevel gearing, either by the diagrams of Figs. 156 and 157 or by equations (5) and (6) of Art. 173, the diameter D, which must be substituted therein, is that corresponding to the *formative circle*, whose radius is $R_f = r_2 \sec \theta$, Fig. 158, as the *form* of the tooth is fixed by this radius and *not* by the radius r_2. The

* See also Mr. Lewis's article, Proceedings Engineers' Club of Philadelphia, Jan., 1893.

computations should be made for the smaller of the two gears, as in the case of spur gears.

Example. Design a pair of bevel gears to transmit 50 H.-P. with a velocity ratio of 3 to 2; the gears to be of cast iron, and the maximum fibre stress to be 4,000 pounds per square inch. The revolutions per minute of the shafts are to be 300 and 200 respectively.

Lay off the axes OV and OT, Fig. 158, and draw OU so that corresponding radii of N and M are in the proportion of 3 to 2. Then it is found that $\theta = 34°$ and $\theta' = 56°$. Assume tentatively that r_2 for the large gear $= 15''$ and r_2 for the small gear $= 10''$, and take a trial width of face of $4''$.

The velocity at the radius $r_2 = \dfrac{2 \times \pi \times 10 \times 300}{12} = 1{,}575$ feet per minute. Hence the equivalent *total* load at a radius r_2

$$= W_e = \frac{50 \times 33{,}000}{1{,}575} = 1{,}050, \quad \text{or } w_e = \frac{1{,}050}{4} = 262 \text{ pounds.}$$

Therefore $w_2 = w_e \times k = 262 \times 1.4 = 367$ pounds per inch of face. Since $\sec \theta = \sec 34° = 1.2$, the diameter of the formative circle is $20 \times 1.2 = 24''$ and from the diagram, Fig. 157, or from equation (6), it is found that for $D = 24''$, $p = 4{,}000$ pounds, and $w = 367$ pounds, the diametral pitch is very nearly $3\frac{1}{2}$, which may therefore be selected. This would give 70 teeth for the small gear and 105 for the large gear. The width of face is a little more than three times the circular pitch and is therefore in accordance with good practice.

175. Stresses in Gear Teeth. Gear teeth of all kinds are likely to be subjected to shock, unless running at a very low velocity, and the danger from shock increases as the velocity increases; hence the allowable stress must be reduced as the velocity is increased. Reliable experimental data on the allowable stress in gear teeth are lacking, although many empirical rules are to be found in treatises on the subject. The values given by Mr. Lewis, in the paper already quoted, are probably as reliable as any, for teeth that bear along their entire length. The following equations have been deduced from Lewis's work:

$$\text{For cast iron, } p = 8{,}000\left(\frac{600}{600 + V}\right) \quad . \quad . \quad (10)$$

$$\text{for steel } p = 20{,}000\left(\frac{600}{600 + V}\right) \quad . \quad . \quad (11)$$

where V = the velocity at pitch line in feet per minute. It would be probably safe to take the stress

$$\text{for bronze as } p = 12{,}000\left(\frac{600}{600 + V}\right) \quad . \quad . \quad (12)$$

since the resilience of bronze is greater than that of cast iron. An old empirical rule for rough cast teeth is

$$W = 200 \times s \times b \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where W, as before, is the *total* load, s the circular pitch, and b the width of face of tooth. The strength of wooden mortise teeth, made of beech or maple, may be taken as about one-half that of cast iron, under the same circumstances; and the strength of good rawhide gears may be taken as equal to that of similar gears made of cast iron. It is to be noted that a rawhide gear will endure considerable more shock than one made of cast iron.

While rough cast teeth are more likely to bear on one corner only, they are stronger than cut teeth of the same pitch, which compensates in a measure for this defect; furthermore, there is, usually, an excess of strength, to allow for wear, in all new gears, and the subsequent wear tends to correct the initial unequal bearing, along the elements.

On account of the increased liability to shock, with increase of speed, and also because of the noise of operation at high speeds, there is a **limit to the speed** at which any form of gear may be safely and conveniently operated. Mr. A. Fowler, in *Engineering*, April, 1889, gives the following as maximum values at which gearing may be successfully operated:

	Ft. per Min.
Ordinary cast-iron gears	1,800
Helical cast-iron gears	2,400
Mortise wheel and cast-iron pinion	2,400
Ordinary cast-steel gears	2,400
Helical cast-steel gears	3,000
Special cast-iron machine-cut gears	3,000

Although higher velocities are occasionally found in practice, these are undoubtedly maximum average values and, in general, the velocity should not be more than two-thirds the values given above, on account of noise and wear. Rawhide gearing, which operates almost noiselessly, may be run satisfactorily up to 3,000 feet per minute.

176. Width of Face of Gears. Equation (5) of Art. 173 gives the load *per inch of face* that may be applied to a tooth of the given form and pitch, the *total* load depending on the width of face as shown by equation (4) of the same article. The durability of a tooth for a given load is, therefore, theoretically increased with increase of face. The difficulty of securing uniform distribution of the load along the contact element increases, however, as the width of face is increased, and this imposes a practical limit to the width of the face. On the other hand, if the intensity of the load on the tooth is too great, excessive wear may result. The equations given above do not take wear into account, the allowable load being fixed with reference to the *stress* alone. On this basis a large tooth may carry a much higher load per inch of face, but the wear will be proportionally greater, the velocity being the same. The empirical rule given in equation (13) of Art. 175 assigns a load of 200 pounds per inch of face, per inch of circular pitch. For a tooth of 1″ circular pitch this load will give, by the Lewis equation, a stress of only 2,000 pounds per square inch, for moderate-sized gears. This is a very low stress, for ordinary speeds, so that this rule would give more *durable* teeth than the Lewis equation, as ordinarily applied.

Experimental data on the durability of teeth are lacking. It is evident, however, that the allowable load will depend largely on the character of the service, velocity of rubbing, lubrication, and the material used. Thus, for ordinary cut cast-iron teeth under constant service, the value given above (200 lbs.) is probably conservative; while with teeth of high-grade steel much greater loads may be carried. Cases are on record where loads of over 2,000 pounds per inch of face were successfully carried, with a peripheral velocity of over 2,000 feet per minute, the pinion being of forged steel and the gear a steel casting, 4.92″ circular pitch. Well-made **gears of rawhide** may be loaded up to 150 pounds per inch of face, per inch of circular pitch; but in no case should the load exceed 250 pounds per inch of face.*

In the case of machines such as punching-machines which work intermittently, and whose operation extends over a short space of time, the element of wear is not so important in the design of the teeth; but in such gears as those connecting street-railway or automobile motors with the driving axles, where the work is both continuous and severe, wearing qualities may be fully as important as strength; and gears made of steel or other hard materials may have to be used solely on this account.

Good practice makes the face of the tooth **about three times the circular pitch**; but in fixing the pitch and width of face, in extreme cases, the points discussed above should be considered.

177. Other Forms of Gear Teeth. Gear teeth made according to the Brown & Sharpe standard, on which the foregoing discussion is based, have been found very satisfactory for average conditions, and are in most common use in this country. For extreme conditions, however, it has been found necessary to reinforce such teeth, or to use teeth of a different form.

A very common way of reinforcing teeth of cast gears is by **shrouding,** which consists in casting an annular ring of metal on one or both ends of the teeth, as shown in Fig. 159. This ring is cast as an integral part of the gear casting, and hence strengthens the gear tooth by practically twice the shearing strength of the cross-section of the tooth, when both ends are shrouded to the top. The teeth of the pinion are, from their outline, always weaker than those of the gear, and the wear on them is also greatest. The shrouding should, therefore, be put on the pinion; and if carried to the top of the tooth on both ends it will give them an excess of strength over those of the gear, with usual widths of face. If the gears to be reinforced do not differ greatly in diameter, the teeth of both may be shrouded half way up.

* Private communication from the New Process Raw Hide Co.

Shrouding is used mostly on rough cast gears, the shroud practically prohibiting the cutting of the teeth by the usual methods.

If the gears are to run in one direction only, and where very heavy pressures are to be withstood, a form of tooth as shown in Fig. 160, and known as a **buttress** tooth, may be, but seldom is, employed. The driving face, *A*, is made of correct theoretical outline, while the back face *B* may be of any outline* that will give the required strength, and clear the teeth of the mating gear. The front face should be of standard cycloidal or involute form and the backs are preferably involute forms, with a much greater obliquity of generator than would be permissible in driving.

For some time past there has been a marked tendency† on the part of the designers of gearing for extremely trying service to depart from the Brown & Sharpe standard, and to use teeth somewhat shorter than those given by that standard. In some instances the same angle of pressure has been retained, while in others this angle has been increased. Mr. C. W. Hunt reported to the A. S. M. E. in 1897 (Vol. XVIII) the results of the adoption of such a system and gives full information for their design. A few other manufacturers have adopted similar systems. The need of a small gear of great strength, in automobile work, has increased the demand for a stronger form of tooth, and it would seem that the old standard must be modified or a second standard adopted for extreme service. The most prominent form of these so-called "**stub teeth**," at present, is that advocated by the Fellows Gear Shaper Co. In this system an involute tooth with a pressure angle of 20° is used, the addendum being about 0.8 as high as that of the Brown & Sharpe standard. This gives a tooth nearly twice as strong as the old standard. Sometimes these stub teeth are given the height of a standard tooth of smaller pitch; thus a 6-pitch stub tooth may have the length of a standard 8-pitch tooth, in which case the gear is sometimes described as a 6–8 gear. Notwithstanding the fact that the arc of contact in stub-tooth gears is, generally speaking, less than in

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* See "Kinematics of Machinery," by John H. Barr, page 131.

† See a paper by R. E. Flanders, Trans. A. S. M. E., Vol. XXX, résumé of other systems.

the old standard, they run well and will undoubtedly be more used in the future.

178. Strength of Gear Rims and Arms. The rim of the gear wheel must not only be strong enough to resist the forces brought upon it, but stiff enough also to prevent improper action of the teeth due to springing of the rim. A section of rim between two arms may be considered as a beam fixed at the ends and carrying a load at the middle, the value of which is $W_1 \sin \theta$, Fig. 151. Good practice makes the thickness of the rim at least 1.25 t, where t is the thickness of the tooth on the pitch line. For small gears this proportion gives ample stiffness, but for very large gears stiffening ribs are also sometimes necessary. In many cases the thickness should be sufficient to allow of dovetail-

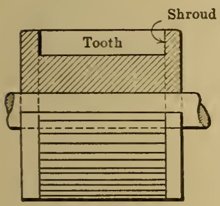

FIG. 159.

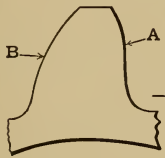

FIG. 160.

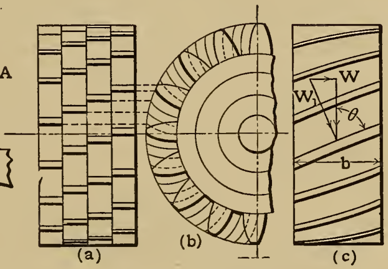

FIG. 161.

ing a tooth into the rim, in case of accidental breakage of one or more teeth. Gear wheels are seldom run at peripheral velocities which induce dangerous centrifugal stresses. The principles governing the design of such wheels are discussed, however, in Chap. XV.

The arms of gear wheels may be treated as cantilevers, assuming that each arm carries a load $\frac{W}{n}$, where n is the number of arms, and W the tangential load. Computations for strength of either arms or rims must, however, be considered as giving minimum dimensions, stiffness being the prime requirement, and due regard must be paid to proportions of rim, arms, and hub, to minimize shrinkage stresses due to cooling.

179. Efficiency of Spur Gearing. The experimental data on the efficiency of spur gearing are very meagre. Probably the best available data are those obtained by Mr. Wilfred Lewis, for details of which see Trans. A. S. M. E., Vol. VII. His investigation was made with a cut spur pinion of 12 teeth meshing with a gear of 39 teeth. The circular pitch was 1½ inches and the face 3⅜ inches. The load varied from 430 pounds to 2,500 pounds per tooth, and the peripheral speed ranged from 3 feet to 200 feet per minute. The measurements included the friction at the teeth, and the friction at the bearings. The efficiency, as observed, varied from 90 per cent at a velocity of 3 feet per minute to over 98 per cent at 200 feet per minute. It appears that the friction at the teeth is a small part of the loss with good cut gears, the greater portion of the loss being at the journals. The efficiency of bevel gears is somewhat less than that of spur gears, on account of the axial thrust, which induces friction between the hub of the gear and the collar at the supporting bearing.

HELICAL OR TWISTED GEARING

180. General Principles. Suppose a spur gear to be cut into n small sections by a series of planes perpendicular to the axis of rotation. If each section be then placed a proper distance ahead or behind the adjacent section, Fig. 161 (a), it is evident that they may be so arranged that some one section is just coming into contact with its mating section when the n^{th} section in advance of it is in contact at the pitch point. With such an arrangement some section will always be in contact near the pitch point, and there will always be approximately n points of contact with the mating gear between the pitch point and the point which marks the beginning of tooth action. Since the action of gear teeth is smoothest when contact is near the pitch point, this arrangement of gearing runs more quietly and smoothly than ordinary spur gearing, and it was at one time used in machine tool and similar work where smooth action is very desirable.

As the number of sections is increased, the total width of the gear remaining the same, the spacing of these sections being kept uniform as before, the form of the stepped tooth approaches that shown in Fig. 161 (c). When the number becomes infinite the teeth become helical in form, and contact is continuous along that portion of the face which is within the arc of contact. It is evident, however, that since the relative position of adjacent laminæ is arbitrary, and may follow any desired law, the outline of the tooth in an axial direction is not necessarily helical, but may have any desired shape; although these teeth are most usually made helical, this form being more practical to cut. This form of gearing is also known as *twisted gearing*, for an obvious reason. The action of such gears is identical with that of common spur gearing, and should not be confused with that of screw gearing, though certain limiting forms of the latter are also twisted gears. A screw gear must have regular or uniform helical teeth, while a twisted gear does not necessarily have this limitation.

Since the pressure, W, between mating teeth must be normal to the surface, there is a component, Fig. 161 (c), which tends to move the gear in an axial direction, causing end thrust on the shaft collars. This can be obviated by making two sets of helical teeth on each gear, one right-hand and one left-hand, as shown in Fig. 162. When it is desired to use cut teeth the wheel is sometimes made in two parts and fastened together, or the wheel may be made in one piece and the two sets of teeth staggered so as to allow them to be cut; but in both of these constructions there is some loss of strength due to the absence of the reinforcing action of teeth cast solid as in Fig. 162. Gears of this type are also called **herring-bone gears.** With the arrangement shown in Fig. 162, care must be used that the alignment in an axial direction is accurate, or end play must be provided so that the middle plane of both gears coincide; otherwise the full load will be thrown on one-half the gear and the object of the double gear defeated.

181. Strength of Twisted Gears. If the effective load which one tooth of a twisted gear transmits to its mate be W, Fig. 161 (c), then the total load normal to the face is $W_1 = W \operatorname{cosec} \theta$. If the length of the tooth be denoted by l, and the breadth of the gear by b, then $l = b \operatorname{cosec} \theta$. Hence the load per inch of face on a twisted tooth $= \frac{W_1}{l} = \frac{W \operatorname{cosec} \theta}{b \operatorname{cosec} \theta} = \frac{W}{b}$ or the same as in a spur gear of face b. This would be strictly true if all points in the line of contact were at the same distance from the axis of rotation as in a spur gear. This is never so, in twisted gears, the line of contact always extending diagonally across the tooth face. The error due to this, however, is small, and on the side of safety, and it may be assumed that the load per inch of face in twisted gears is the same as that of a spur gear of equal width and equally loaded. This diagonal distribution of the load across the tooth face, decreases the lever arm of the force which tends to break the tooth; the amount of decrease depending

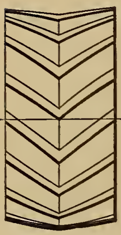

FIG. 162.

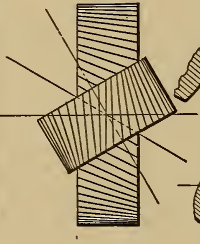

FIG. 163.

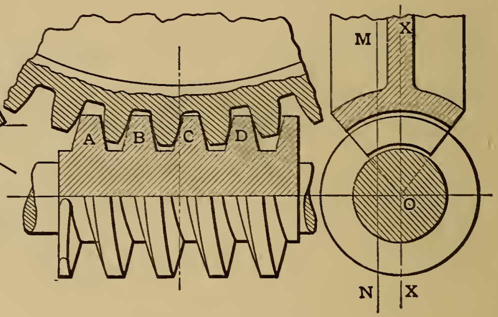

FIG. 164.

on the amount of twist in the tooth. If the twist is so great that when the end in advance is going out of contact the other end is just coming into contact, the line of contact will run diagonally across the tooth from point to flank, and the average arm of the driving force will be about one-half the height of the tooth. If the twist be made equal to the pitch, tooth action is continuous at every point of the arc of action and this proportion is the one most used. It is clear, however, that the assumption often made that twisted teeth are twice as strong as spur teeth of the same pitch is not true for teeth of usual proportions, a difference of 25 per cent being, perhaps, as much as can safely be assumed. On account of continuous tooth action and consequent smoother operation in twisted gears, the effect of shock is lessened somewhat. Twisted gears have been used with success on heavy winding and hoisting engines, the teeth being often rough cast and both gear and pinion half shrouded, making a very strong tooth.

SCREW GEARING

182. Forms of Screw Gears. When the axis of two shafts are not parallel and do not intersect, it is possible to lay out contact surfaces on which gear teeth may be constructed which will give line contact. Gears of this kind are known as **skew-bevel** gears. They are difficult to construct, and are very rarely used. If the load can be carried on point contact, pitch cylinders may be described on the axes, Fig. 163, and on these surfaces helical teeth may be constructed which will transmit the desired motion. Such gears are known as **screw** or **spiral*** gears, the latter name being really a misnomer. While the teeth of such gears resemble those of helical twisted gears, their theory and action are quite different; for, in addition to the conjugate rolling and sliding action, as in spur gears, there is also a sliding component along the elements between contact surfaces. The action of screw gearing is very smooth. The special case where the axes are at right angles, and where a large wheel having many helical teeth meshes with a small one having a very few helical teeth, is an important one on account of the great reduction in velocity ratio that may thus be obtained. This last arrangement is commonly known as a **worm and worm-wheel.** Fig. 165 illustrates such a worm and worm-wheel, the teeth on the worm wheel being truly helical in form and cut at an angle to suit the worm thread or helix. The same result is sometimes obtained by using a plain spur gear, and setting the axis of the worm at the proper angle with the plane of the gear.† The contact in these cases is point contact, and on the worm wheel tooth is confined to points in a line cut from the working surface of the tooth by a plane passing

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* For a full discussion of the methods of laying out and producing so-called spiral gears, see a "Practical Treatise on Gearing," by Brown & Sharpe Mfg. Co., and also "Worm and Spiral Gearing," by F. A. Halsey.

† A highly successful form of this arrangement is the worm-and-rack drives on planing machines, first used by Wm. Sellers & Co.

through the axis of the worm at right angles to the axis of the worm wheel. In practice the point of contact becomes a limited area. The advantage of this form of worm wheel, like all spur gears, is that the teeth can be cut with a rotary cutter, and patterns for rough cast teeth are comparatively easy to construct.

It is possible, however, to construct a worm wheel in such a manner as to secure line contact, as in spur gearing. Referring to Fig. 164, it can be seen that when the single-threaded worm shown is rotated through 360°, any median section as A is moved forward an amount equal to the pitch of the worm wheel to a position B; and that rotation of the worm, in general, is equivalent to a translation of these sections backward or forward. The action is equivalent to translating a rack of similar proportions, and, in fact, if the worm itself is moved axially it will engage with the teeth of the worm wheel in the same manner as a rack does with a gear. In the involute system of gear teeth the rack has straight sides,* and this property is usually taken advantage of in making worm gearing, since a worm thread of such a cross-section is easily machined. The sides of the involute rack face are at right angles to the line of contact, $a\ O\ b$, Fig. 147, and hence the inclination of the sides to each other is $2\ \theta$, Fig. 147, and in the standard system $2\ \theta = 29°$. If other planes such as $M\ N$ be passed through the worm and worm wheel parallel to the median plane $X\ X$, Fig. 164, it will cut a trapezoid from the worm somewhat different from that cut by the median plane. The rack-like action of these trapezoids would, however, be similar to those on the median plane, and it is clear that the shape of the worm-wheel tooth in the plane $M\ N$ may be so made as to mesh correctly with this new trapezoidal section. It is evident that if enough such sections be taken, a complete tooth outline may be formed that will give line contact with a worm across its full face. It is evident also that any other form cf worm thread may be similarly treated.

The preceding discussion demonstrates the possibility of line contact in screw gearing, and suggests a method by which

* See "Kinematics of Machinery," by John H. Barr, page 125.

the teeth of such gearing could be drawn, and hence constructed. There is no practical value in actually making such drawings; but teeth having this property of line contact are automatically produced by what is known as the **hobbing process.** A worm wheel of tool steel is made of the exact form of the desired worm. This worm is made into a cutter by cutting flutes across the face as in Fig. 168. This is known as a **hob**; and when hardened and tempered it is used as a milling cutter. The wheel blank, which has been turned to correspond to the outside of the teeth, is mounted in a gear cutter, or a special hobbing machine, and the

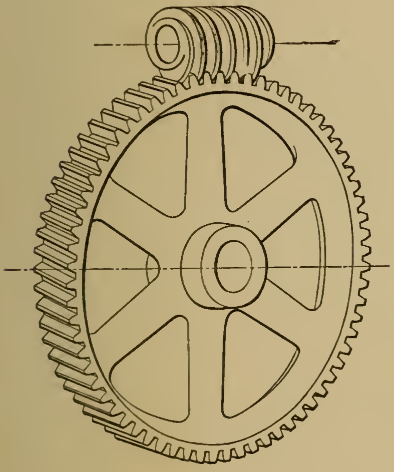

FIG. 165.

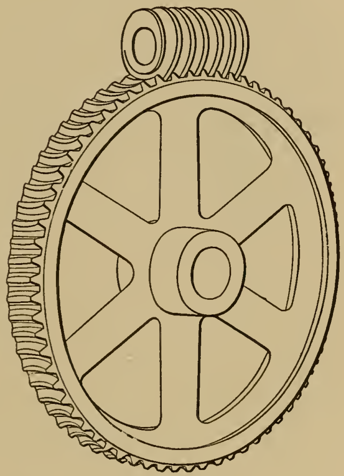

FIG. 166.

hob is also mounted in correct relation to the wheel, but with the axes of the wheels a little greater distance apart than the required final distance. The hob is then rotated and at the same time fed toward the worm wheel till the proper distance between the axes is reached, thus cutting the teeth in the worm wheel in a very accurate manner. Sometimes the wheel is caused to rotate simply by the action of the hob, but much better results are obtained if it is driven positively, with the proper velocity ratio, from the cutter spindle by means of positive gearing. In heavy work the teeth of the wheel are roughed out or "gashed" before hobbing.

Fig. 166 shows a worm wheel which has been hobbed, and its mating worm. Fig. 167* shows a form of wheel occasionally used where the wheel is sometimes rotated by hand or when the projecting teeth are undesirable. Such wheels may be hobbed, but are usually cut by the approximate method shown in Fig. 169, where a cutter is fed radially inward toward the axis of the worm wheel, producing what is known as a **drop-cut** wheel. In the **Hindley** worm the pitch line of the worm is curved to coincide with the pitch line of the wheel, thus obtaining contact on several teeth at the same time.†

183. **Velocity Ratio of Worm Gearing.** The axial advance per turn of the worm thread is called the **lead.** Thus in Fig. 164 the lead of the single-threaded worm shown is the distance, parallel to the axis, from any point on the tooth section A, to a corresponding point on the section B, and is equal to the circumferential pitch of the worm wheel. If the worm were double-threaded the lead would be twice this amount, or equal to the distance between corresponding points on A and C, and would then be twice the pitch of the worm wheel. The lead of the triple-threaded worm would be three times the pitch, and so on. If a single-threaded worm makes one revolution, a tooth of the worm wheel is moved a distance equal to the pitch. In the case of a double-threaded worm the tooth would be moved twice the pitch; and in general if N be the number of teeth in the worm wheel, and n the number of threads on the worm, then, $\dfrac{\text{angular velocity of worm}}{\text{angular vel. of worm-wheel}} = \dfrac{N}{n}$.

Evidently a very great velocity ratio is possible with a comparatively small worm-wheel. It is to be especially noted that the angular velocity ratio is independent of the diameter of the worm. The pitch of the worm wheel, which must be decided upon by consideration of the strength of the teeth, fixes the radius of the worm wheel for a given number of teeth; but the radius of the worm may then be varied to suit other conditions.

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* Figs. 165, 166, and 167 are reproduced from Browne & Sharpe's "Treatise on Gearing."

† See "Worm and Spiral Gearing," by F. A. Halsey.

184. Efficiency of Worm Gearing. The general expressions for the efficiency of screws, deduced in Art. 54 of Chap. VII, apply also to worm-gearing. Since the worm thread is, usually, a so-called angular thread, equation 13 (*a*) of that Article would strictly apply. However, the inclination of the face of worm threads is so small that the error introduced in using the simpler equations (9) and (10) of that article, which were deduced from the square thread, is small. These equations show that the efficiency of all screw gears is a function of the angle which the thread makes with a plane perpendicular to the axis, and of

FIG. 167.

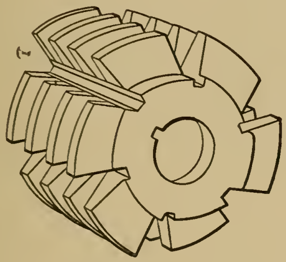

FIG. 168.

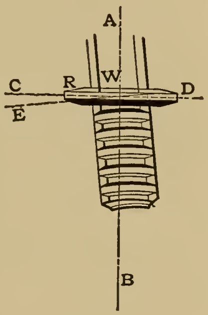

FIG. 169.

the coefficient of friction, assuming that the coefficient of friction at the thrust collar is the same as at the tooth.

One of the most valuable contributions to this subject is the experimental work of Mr. Wilfred Lewis.* The full lines in Fig. 170 have been plotted from the diagram on which he has summarized his results. They show clearly the increase of efficiency with increase of thread angle at all velocities. They also show a remarkable agreement with the theoretical equations of Art. 54. The dotted curve is reproduced from curve (2) of Fig. 52, and its close agreement with Mr. Lewis' curves is to be noted. This dotted curve was plotted for a value of $\mu = 0.05$.

* Trans. A. S. M. E., Vol. VII, page 297.

Mr. Lewis' calculated average value of this coefficient for a velocity of 20 feet per minute is 0.059 and for 10 feet per minute 0.074. Curves (4) and (5) in Fig. 52 may, therefore, be taken as supplementary to those in Fig. 170, and may be used, as they were intended, for designing slow-moving and poorly lubricated screws. A theoretical curve plotted from equation (9), Art. 54, with a value of μ = 0.014 (which would be obtained only at high speeds), will coincide very closely with curve 1, Fig. 170.

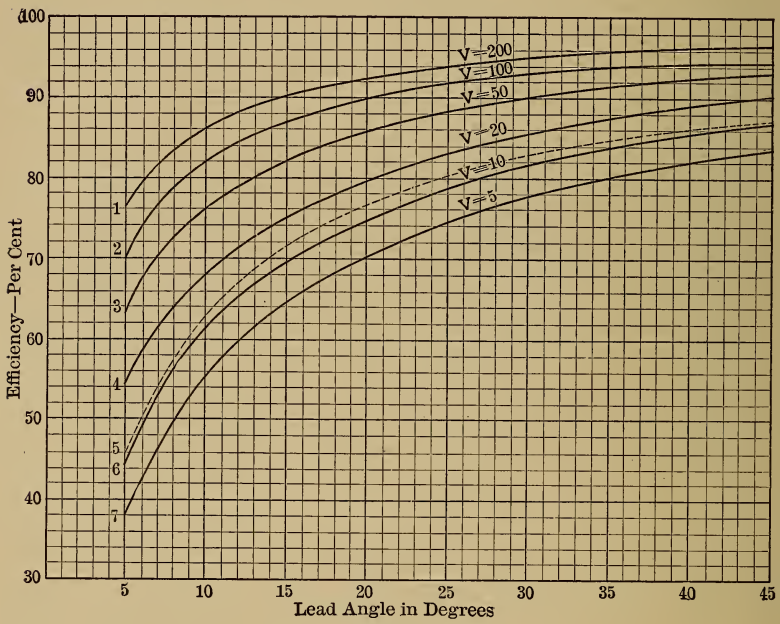

FIG. 170.

This coincidence is closer than might be expected from the nature of the problem and the assumptions on which equation (9) is based. Mr. Lewis' value of μ for these velocities * (200 feet per minute) ranged from 0.026 to 0.015, his average value being 0.02.

Mr. Halsey † has examined the design of a number of success-

* Velocity here means velocity of rubbing at the point of contact between worm and worm wheel.

† See "Worm and Spiral Gearing," page 38.

ful and unsuccessful worms used for transmitting power and found that every worm among those examined whose lead angle was greater than 12° – 30′ was successful, and every worm whose lead angle was less than 9° was unsuccessful, and quotes Mr. James Christie, who has had considerable experience with this form of gearing, as giving 17° – 15′ as the lower limit for successful design, which still further corroborates the general theory given. It is to be noted, on the other hand, that there is little to be gained in using a pitch angle above 30°, the increase in efficiency being very small, while the side thrust on the wheel is increased. It is not to be understood that it is never proper to design a worm with a lead angle less than 9°; for there are many cases, not primarily for power transmission, and where the velocity is low, in which worms of less pitch are not only effective but necessary. In Mr. Lewis' experiments the worms ran in a bath of oil, and the efficiencies given include journal friction, the thrust being taken at the end of the worm shaft by a loose brass washer running between two hardened and ground steel washers (see Art. 104).

The effect of the velocity of rubbing on the coefficient of friction of imperfectly lubricated surfaces, was noted in Art. 32, and Fig. 17 of that article indicates, in a general way, what may be expected with sliding *surfaces:* all experimental results going to show that the lowest coefficient was obtained at about 200 feet per minute. Mr. Lewis, as the result of his work, fixes 200 feet per minute as the point of maximum efficiency of worm gearing, which is in perfect accord with the general theory of lubrication. The surfaces of worm gearing, although running in an oil bath, must, from the nature of the contact, be classified as imperfectly lubricated surfaces. An increase of velocity may, up to a certain limit, decrease the coefficient of friction, but it is not possible at any speed, with the small amount of surface contact obtainable in screw gearing, to create a true oil film so that the load would be fluid-borne (Art. 33).

185. Limiting Pressures and Velocities in Worm Gearing. It was stated in the last two articles that the best results are obtained from worm gearing when the rubbing velocity is about 200 feet per minute and the lead angle not less than $12° - 30'$. It is not always possible, however, to keep the design within these limits. Thus in order to obtain *mechanical advantage* (see Art. 64), it may be necessary to use a worm with a very small lead angle, and kinematic requirements may necessitate a much higher velocity than 200 feet at the pitch line.

The allowable axial load that may be applied to a worm under varying velocities has not been very accurately determined, the law undoubtedly being complex (see Art. 32). Enough experimental work has been done, however, to show that the pressure varies, approximately, inversely with the velocity; or the law may be roughly expressed as $W V = K$, where W = the axial load on the worm, V = the velocity of rubbing in feet per minute, and K = a constant to be determined by experiment (see also Art. 98). In Lewis' experiments, made on cast-iron worms and worm wheels, running in an oil bath, it was found that the *limiting* value of K, *i.e.*, where cutting began, was about 1,500,000. Smith and Marx* quote corresponding pressures and velocities, attributed to Stribeck, obtained with hardened steel worm and bronze worm wheel running in an oil bath, which give an average allowable value of 690,000 for K. Bach and Roser, experimenting with soft-steel worms and bronze worm wheels, succeeded in carrying a pressure of 800 pounds at a velocity of 1,700 feet per minute, which gives K = 1,360,000. It would appear, therefore, that for average conditions and bath lubrication of the worm it will be safe, for velocities up to 1,500 feet per minute, to take

$$W V = 750{,}000 \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The above discussion has reference to worms as ordinarily constructed with straight-sided threads. Mr. Robert Bruce† has shown that if the sides of the worm are made *concave* a much greater load may be carried. With improved threads of this form he has succeeded in carrying 25 tons at a velocity of 120 feet per minute, corresponding to K = 6,720,000. This great gain is due,

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* "Machine Design," page 301.

† Proceedings of Institution of Mechanical Engineers (British), page 57 of the year 1906.

without doubt, to the improved lubrication obtained by what practically amounts to surface contact, between the mating convex and concave surfaces of the teeth.

186. Design of Worm Gearing. In general, the strength of the worm exceeds the strength of the teeth in the worm-wheel; and where the worm is made of a harder material, which is the usual case, the wear is greatest on the worm-wheel teeth. It is usually sufficient, therefore, to design the wheel teeth alone, considering them as simple spur gear teeth as in Art. 173. In the case of rough-cast, or drop-cut teeth, it must be assumed that the entire load is carried by a single tooth; but in hobbed gearing it is safe to assume that the load is distributed between two, or even three, teeth, depending on the number of teeth in the wheel.

Example. Design a worm gear to connect two shafts which are 11 inches apart, and to transmit 7½ H.-P. The velocity ratio is to be 20 to 1, the worm shaft is to make 320 R.P.M., the lead angle is not to be less than 15°, and the worm wheel is to be cut with a hob.

The solution of problems in worm gearing must, generally, be tentative. If the velocity ratio is to be 20 to 1, the worm-wheel will have 20, 40, or 60 teeth, depending on whether the worm is single-, double-, or triple-threaded. It is difficult to obtain a high lead angle with a single-threaded worm without making a very large thread, therefore a trial assumption will be made with a triple-threaded worm, and 60 teeth in the wheel. Twenty inches may be taken as a trial diameter for the wheel, and the trial pitch circumference will therefore be 63 inches approximately. If the circumferential pitch be taken as one inch, the lead of the worm thread will be three inches, and can therefore be easily cut in a lathe. The corrected circumference of the wheel will then be 60″, corresponding to a pitch diameter of 19.11″. The pitch diameter of the worm, with the given distance between centres, will be 2.9″; hence the tangent of the lead angle $= \dfrac{3}{\pi \times 2.9} =$ 0.33, or the lead angle is 18° – 15′, which is an efficient angle.

The number of revolutions per minute of the worm wheel will be $\frac{320}{20} = 16$. Hence the velocity of the *worm wheel* at the pitch line $= \frac{60 \times 16}{12} = 80$ feet per minute. The total axial thrust on the worm will be $\frac{7\frac{1}{2} \times 33{,}000}{80} = 3{,}100$ pounds. The velocity of rubbing equals the length of one turn of the worm thread multiplied by the number of revolutions per minute, or

$$V = \frac{\pi \times 2.9 \times 320}{(\cos 18' - 15'') \times 12} = \frac{\pi \times 2.9 \times 320}{0.95 \times 12} = 255 \text{ ft. per minute.}$$

The product of velocity and axial pressure on the worm $= 255 \times 3{,}100 = 790{,}000$ which by equation (14) is a safe value, although somewhat high.

The load may be considered as distributed between two teeth, and each tooth will have a face or length at the root at least equal to the pitch of the worm (see Fig. 164), or say 2.75″. Hence the load per inch of face of tooth $= \frac{3{,}100}{2 \times 2.75} = 560$ pounds. From the diagram, Fig. 158, it is seen that this load corresponds to a fibre stress of about 5,000 pounds per square inch with 1 inch circular pitch. From equation (10) of Art. 175, however, it is seen that for the velocity, 80 feet, the allowable stress is 7,000 pounds, hence the tooth has an excess of strength to provide against the wear, which falls heaviest on the worm wheel.

From curve (1), Fig. 170, it is found that the efficiency is about 90 per cent; hence the horse-power which must be supplied to furnish 7.5 H.-P. at the worm-wheel shaft will be $\frac{7.5}{0.90} = 8.4$ H.-P. $= 277{,}200$ foot-pounds per minute, or 866 foot-pounds per revolution of the worm. The torque T, which must be applied to the worm-wheel shaft, will be $T = \frac{866 \times 12}{2\pi} = 1{,}650$ inch-pounds. The depth below the pitch line of a standard tooth of one inch circular pitch is, from Table XXV, 0.3857 inches; therefore the diameter of the worm at the root of the thread $= 2.9 - (2 \times 0.3857) = 2.13''$, and from equation E, page 94, the torsional stress $p_s = \frac{16T}{\pi d^3} = \frac{16 \times 1,650}{\pi \times (2.13)^3} = 850$ pounds per square inch, which is very low. The design may, therefore, be considered satisfactory if the worm is to be cut integral with the shaft. If, however, it is to be bored out and fitted over the shaft, further calculation as to the strength of the shaft which may be fitted is necessary.

187. Thrust Bearings for Worms. An important frictional loss in worm gearing occurs in the thrust bearing, which therefore deserves special attention. The general discussion in Art. 104 applies in this case. The type of bearing shown in Fig. 88 is much used, and of late ball bearings have met with considerable success in such places.

CHAPTER XV

FLYWHEELS AND PULLEYS

188. Capacity of Flywheels.—There are two distinct types of flywheels; namely, those whose sole function is to absorb and redistribute energy, as noted in Articles 2, 4, and 6.2, and those which also act as a pulley or band wheel and transmit power continuously. When a flywheel is attached to a train of mechanism in which the supply of energy varies it tends to absorb any excess energy, thus having its velocity increased. When the work to be done is in excess of the energy supply, the wheel tends to furnish the deficiency at the expense of its kinetic energy, with a resulting reduction of velocity. Flywheels, therefore, to be effective must vary in velocity; the allowable amount of variation depending on the conditions of the case. Thus in engines driving electric generators, the variation from normal speed may be limited to one-half of one per cent, or less, while in such machines as punching machines, the variation may be as great as twenty per cent.

If W be the weight in pounds of a body moving with a velocity of v feet per second, then the kinetic energy in *foot-pounds* which the body possesses is $K = \frac{W v^2}{2 g}$ where $g = 32.2$. If the velocity of the body be changed from v_1 to v_2, the change in kinetic energy is the work which the body will do, or the energy it will absorb, depending on whether its velocity is decreased or increased. If, then, the work to be done or the energy to be absorbed with a given change in velocity is known, the necessary weight of the body may be found; for if K_1 be the kinetic energy of the body when moving with a velocity v_1, and K_2 be the kinetic energy at a velocity v_2, then the energy delivered or absorbed during a change of velocity is

$$E = K_1 - K_2 = \frac{W v_1^2}{2g} - \frac{W v_2^2}{2g} = \frac{W}{2g}(v_1^2 - v_2^2) \quad . \quad (1)$$

If the body is rotating around a fixed axis, the velocities of different points in the body vary as the distance of these points from the axis. For this case the kinetic energy of the body is $\frac{W}{2g}\rho^2 \omega^2$ where ρ is the radius of gyration, and ω the angular velocity.

Hence for rotating bodies equation (1) may be written

$$E = K_1 - K_2 = \frac{W \rho^2}{2g}(\omega_1^2 - \omega_2^2) \quad . \quad . \quad . \quad (2)$$

or since $\frac{W \rho^2}{g} = I$, the moment of inertia * of the body,

$$E = K_1 - K_2 = \frac{I}{2}(\omega_1^2 - \omega_2^2) \quad . \quad . \quad . \quad . \quad (3)$$

In all cases of flywheel design the effect of the hub may be neglected, and in nearly all cases the effect of the arms is so small as to be negligible, and the rim only need be considered. When such is the case it is sufficiently accurate to take the mean radius of the rim R as the radius of gyration, and equation (2) becomes identical with equation (1) since, in general, $R\omega = v$. In the case of wheels with many heavy arms, or heavy disc wheels, and where it is desirable to compute the inertia effect of the wheel closely, as in direct driving of electric generators, equation (2) or equation (3) is applicable. In the case of a wheel with arms whose sides are parallel, or nearly so, it is to be noted that the square of the radius of gyration of the arms or ρ^2 is very nearly equal to $\frac{1}{3} R^2$. Hence for this case, if W be the weight of the rim and W_a the total weight of the arms

$$E = K_1 - K_2 = \frac{R^2}{2g}(W + \tfrac{1}{3} W_a)(\omega_1^2 - \omega_2^2) \quad . \quad (4)$$

Example (1). A punching machine is to make 30 strokes per minute and is to punch holes ¾″ in diameter in steel plate ½″

* The student should distinguish clearly between the moment of inertia of a solid body and the moment of inertia of an *area*. See Church's "Mechanics," Art. 86, page 91. In the case of a circular disc $I = \frac{W R^2}{2g}$.

thick. Since the machine may be used for shearing also, it should be capable of punching a hole, or of doing the equivalent amount of work in shearing, at every stroke of the punch, continuously. The belt speed is to be about 600 feet per minute, and, from existing machines of the same type, it is known that the efficiency will not exceed 85 per cent. It is required to find the cross-section of the flywheel rim.

Let Fig. 171 represent the machine under discussion. The mechanism in the head, *A*, is a slotted crosshead;* so that the punch *M* moves with harmonic motion. Let the diagram, Fig. 171 (a), represent the path of the pin *F*, in the crosshead. When

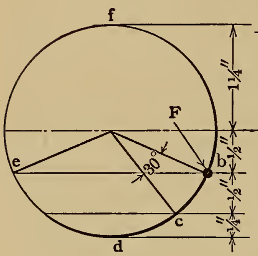

FIG. 171 (a).

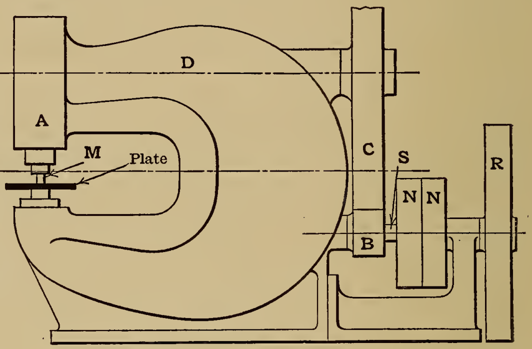

FIG. 171.

the pin is at *b* the punch enters the plate, and emerges from the lower face of the plate when the pin is at *c*. When the pin is at *d* the punch is at its lowest position, and has entered the die ¼″; at *e* the punch is withdrawn from the plate and at *f* is at its highest position. The pin, therefore, moves through an angle of 30° while the work of punching is being performed.

The preliminary layout also shows that the diameter of the driving pulley *N* should not exceed 18″, and the mean diameter of the flywheel rim should not exceed 42″. A preliminary estimate also fixes the ratio of the diameter of the pinion *B* to the diameter of the gear *C* as 1 to 6; hence the driving shaft will make 30 × 6 = 180 R.P.M. The *circumference* of the driving

* See "Kinematics of Machinery," by John H. Barr, page 184.

pulley $= \frac{\text{belt speed}}{180} = \frac{600}{180} = 3.34$ feet and, therefore, the diameter of the driving pulley will be 13″ which is well within the limit set. The machine makes an energy cycle (see Art. 4) every stroke of the punch, or every six revolutions of the driving shaft. While the hole is being punched the flywheel is giving up energy to assist the belt, and during the remainder of the cycle the belt withdraws the punch from the sheet, and restores the wheel to normal speed.

The greatest pressure which the punch must exert on the plate will be at the beginning of the punching operation, and will be equal to the area of the metal in shear multiplied by the shearing resistance; or $P = \pi \times \frac{3}{4} \times \frac{1}{2} \times 60{,}000^{*} = 70{,}800$ lbs. If the belt had to exert this effort unaided, it would have to be of double leather 15 inches wide; hence the need of a flywheel. As the punch passes through the plate the shearing resistance decreases, until it becomes zero as the punch passes out. The *average* pressure may therefore be taken as half the maximum, and the total work performed in punching is $\frac{70{,}800}{2} \times \frac{1}{2} \times \frac{1}{12} = 1480$ ft.-lbs. The work of withdrawing the punch from the sheet is small and may be considered as part of the frictional loss. Since the efficiency of the machine is 85 per cent the belt must supply $\frac{1{,}480}{0.85} = 1740$ ft.-lbs. every energy cycle. The energy delivered by the belt per cycle is the product of the difference of the belt tensions $(T_1 - T_2)$ multiplied by the distance through which the belt moves (see Art. 131). Since the speed of the belt is 600 feet per minute, and the *time* of the cycle is $\frac{1}{30}$ of a minute, the belt moves a distance $= \frac{600}{30} = 20$ feet per cycle. Hence $(T_1 - T_2)\,20 = 1740$ or $T_1 - T_2 = 87$ lbs. The effect of centrifugal force may be neglected at this belt speed, hence equation (8), Art. 131, gives $f = 0.6t$ for an arc of contact of 180° and $\mu =$

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* The shearing strength of a plate in *punching* is about equal to its tensile strength.

0.3, where f = the effective pull per inch of width of belt and t = tension per inch of width of belt on the tight side. In this class of machinery where excessive slipping of the belt is sure to occur, t should be taken at not more than 40 lbs. per inch of width; whence $f = 0.6 \times 40 = 24$ lbs., and the width of the belt = $\frac{87}{24} = 3\frac{1}{2}$ inches.

Since the pin, Fig. 171 (a), moves through 30°, or $\frac{1}{12}$ of a revolution, during the operation of punching, and since the punch makes 30 strokes per minute, the *time* consumed during the operation of punching will be $\frac{1}{12} \times \frac{1}{30} = \frac{1}{360}$ of a minute. The belt, therefore, moves $600 \times \frac{1}{360} = 1.7$ ft. during the operation and supplies $87 \times 1.7 = 150$ ft.-lbs., leaving $1750 - 150 = 1600$ ft.-lbs. to be supplied by the flywheel. The driving shaft makes 180 R.P.M., or 3 revolutions per second and, therefore, since the mean radius of the wheel = 21 inches,

$$v_1 = \frac{2 \times \pi \times 21 \times 3}{12} = 33 \text{ feet per second.}$$

The allowable variation in velocity may be taken as 10 per cent, hence $v_2 = 33 \times 0.90 = 30$ feet per second. Therefore, neglecting the effect of the hub and arms, the weight of the rim is from equation (1)

$$W = \frac{2gE}{v_1^2 - v_2^2} = \frac{2 \times 32.2 \times 1{,}600}{33^2 - 30^2} = 545 \text{ lbs.}$$

One cubic inch of cast iron weighs 0.26 lbs., hence the number of cubic inches in the rim will be $\frac{545}{0.26} = 2{,}100$. The mean circumference of the rim $= \pi \times 42 = 132$ inches; therefore the cross-section $= \frac{2{,}100}{132} = 16$ square inches, or a section $3\frac{3}{8}$ inches wide by 5 inches deep may be selected.

In the above example it is quite easy to compute the amount of energy to be redistributed, from the conditions of the problem. In the more general case this cannot be done so readily, and methods such as those outlined in Articles 5 and 6 of Chapter

II must be employed, where the diagram representing work to be done is superimposed upon that representing the energy supplied, in such a manner that the excess and deficiency may be measured. Care should also be exercised that the solution covers a complete energy cycle in order that the solution may be based on the *greatest excess or deficiency*. Thus in Fig. 2 (d) the flywheel redistributes energy on both strokes, but the maximum excess of effort is represented by K and not by K_1. Again in such machines as internal-combustion engines with "hit-and-miss" governors, giving a very variable energy supply, the design of the flywheel may have to be based on a hypothetical performance of the engine covering a number of successive strokes, or in other cases may be based on empirical constants which are the result of experience.

In some machinery, such as steam engines, it is desirable to limit the variation in velocity to a definite amount both *above* and *below* the *mean* velocity. If v be the mean velocity, v_1 and v_2 the maximum and minimum velocities respectively, then it is sufficiently accurate for most work to take $v = \frac{1}{2}(v_1 + v_2)$, but the true relation between these quantities depends on the manner in which the velocity changes. For very exact work, as in parallel operation of alternating generators, it may be necessary to take this into account.*

Example (2). Find the weight of a cast-iron flywheel necessary to limit the speed of the engine discussed in Art. 5 to a *total* variation of not more than 0.01, *i.e.*, 0.005 above or below the mean speed. The wheel is also to act as a belt wheel, and the belt speed is to be about 4,000 feet per minute. The sides of the arms are to be parallel and their effect is to be considered.

Since the engine makes 160 R.P.M., the circumference of the wheel $= \frac{4,000}{160} = 25$ feet. Therefore the diameter of the wheel $= 8$ feet, which may be taken without great error as a *mean* diameter of the wheel rim, since the thickness of the rim will not be great. The face of the wheel should be at least 16

* See a paper by I. J. Astrom, Trans. A. S. M. E., Vol. XXII, page 972.

inches wide to give the necessary width of belt. A preliminary layout gives 6 arms, each weighing 170 pounds, or $W_a = 170 \times 6 = 1{,}020$. Let n = the revolutions per second $= \frac{160}{60}$ and R = the radius = 4 feet; from Art. 5, $E = 4{,}620$ foot-pounds. Then $\omega = 2\pi n = 2\pi \times \frac{160}{60} = 16.8$ radians.

Now, $\omega = \frac{\omega_1 + \omega_2}{2}$ and hence from the conditions of the problem, $\omega_1 = 1.005\omega = 1.005 \times 16.8 = 16.88$; and $\omega_2 = 0.995\,\omega = 16.7$. Therefore

$$(\omega_1^2 - \omega_2^2) = (16.88^2 - 16.7^2) = 6.1$$

From Equation (4), $(W + \tfrac{1}{3} W_a) = \frac{2gE}{R^2(\omega_1^2 - \omega_2^2)}$

or $W = \frac{2gE}{R^2(\omega_1^2 - \omega_2^2)} - \frac{W_a}{3} = \frac{2 \times 32.2 \times 4{,}620}{16 \times 6.1} - \frac{1{,}020}{3} = 2{,}720.$

Therefore $W + Wa = 2{,}720 + 1{,}020 = 3{,}740$ lbs., to which must be added the weight of the hub to obtain the total weight of the wheel. Since the rim is to be 16 inches wide and the mean diameter is to be 96 inches the thickness will be $\frac{2{,}720}{0.26 \times \pi \times 96 \times 16} = 2.2$ inches or say $2\tfrac{1}{4}$ inches.

189. Practical Coefficients. It is evident that the ratio of the energy to be absorbed, to the total energy supplied per energy cycle, will vary in different machines, and also in the same machine under different conditions. Thus in the punching machine the flywheel absorbs and redistributes nearly the entire energy supply per cycle, while in the steam-engine example the amount absorbed is about one-third the total. It is readily seen that in the steam engine this ratio will vary with the point of cut-off, steam pressure, weight of reciprocating parts, etc., and therefore, in general, tabulated values of this ratio are deceptive unless they refer to specific conditions. It is to be noted that the weight of the flywheel is directly proportional to the energy to be absorbed and inversely proportional to $(v_1^2 - v_2^2)$. The latter is usually a small quantity and, therefore, if E is large the weight of the flywheel may be excessive, which is undesirable because of the cost, and also because heavy wheels bring great loads on the bearings, causing frictional losses. For this reason it is always desirable so to arrange the sequence of events in the energy supply and work to be done as to minimize the excess energy to be absorbed. This is illustrated in Art. 6, Fig. 5, where the area K may be greatly decreased (or increased) by changing the relative positions of the crank pins. This procedure is of great importance to avoid wheels of great weight in large steam engines when variation in velocity must be closely restricted.

The allowable variation in velocity is fixed with reference to the character of the work to be done. It is evident that some classes of work require much more constant velocity than others, and experience has shown what the limits in variation of velocity may be for successful operation. The following limiting values of the proportionate variation $\frac{v_1 - v_2}{v}$ represent average practice. The particular case of direct driving of alternating generators in parallel must, in general, be treated with reference to the allowable variation per pole, and when, therefore, the number of poles is great the total allowable variation is correspondingly small.

TABLE XXIX

Values of $\frac{v_1 - v_2}{v}$

For punching machines and similar machines	0.10 to 0.15
For engines driving stamps, crushers, etc.	0.20
For engines driving pumps, saw mills	0.03 to 0.05
For engines driving machine tools, weaving and paper mills	0.025 to 0.03
For engines driving spinning mills for coarse thread	0.016 to 0.025
For engines driving spinning mills for fine thread	0.01 to 0.02
For engines driving single dynamos	0.007
For engines driving alternators in parallel	0.003 to 0.0003

190. Stresses in Flywheels. The velocity of the rims of all flywheels is, from the nature of their requirements, very high. If the wheel is to act as a band wheel, the desirability of obtaining high belt speed (Art. 134) brings the peripheral velocity up to 4,000 or 5,000 feet per minute. It has been shown that the *capacity* of a given wheel is proportional to the square of its velocity and, therefore, when the wheel is to act as a flywheel alone, economy in the use of material, or the limiting of the external dimensions, makes high speed very desirable. Great care should be used in the design of such wheels, for a flywheel which breaks at normal speed is exceedingly dangerous to life and limb, and when such wheels "explode" or break from overspeeding, the results are usually very disastrous.

Unfortunately, mathematical analysis of the stresses in flywheels and pulleys is not satisfactory or conclusive. In the case of small wheels cast in one piece, unknown shrinkage stresses of great magnitude may exist, which render useless any refined calculations. In large wheels built up of sections, the presence of joints vitiates any calculations based on the elastic theory of the strength of materials; and when the parts are of cast materials and of large sectional area, there is no assurance that the character of the material is uniform throughout. It is important, however, to understand fully the general *character* of the stresses even when no accurate computations can be made as to their magnitude.

Consider that the rim of the wheel in Fig. 172 is free to expand radially, the arms exerting no restraining force in a radial direction. If the wheel be rotated on its axis the action of centrifugal force is such as to cause an outward pressure on every part of the rim, in exactly the same manner as in a boiler shell acted on by an internal pressure (see Art. 78); the rim expanding until the tensile stress induced in any section $A\ A$, Fig. 172, balances the tendency of the wheel to separate along that section. If, on the other hand, the arms are rigidly attached to both hub and rim, and are so inelastic that their stretch, under the action of the centrifugal pull due to their own mass and that of the rim, is negligible, it is clear that they may be placed so close together that the rim cannot expand, and practically no stress will exist in the rim, the centrifugal action being balanced by the stress in the arms.

Flywheels approximating both of these conditions are sometimes built, but in the most usual case the arms stretch a certain amount and are not placed close together, so that a condition results similar to that shown, in an exaggerated manner, in Fig. 173. Here, the arms, though stretching somewhat, do not stretch enough to allow the rim to expand freely, and, therefore, the hoop tension is somewhat less than that in the free ring. The section of rim between each pair of arms is so long that it becomes a beam fixed at the ends and loaded uniformly by the unbalanced centrifugal action; the greatest bending moment being at the arm, and a bending moment of half the maximum occurring at the centre

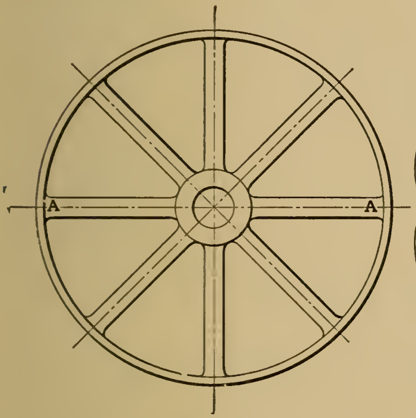

FIG. 172.

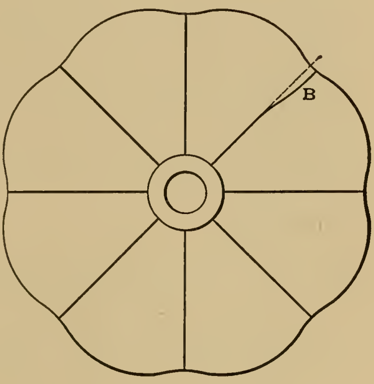

FIG. 173.

of the span. The maximum tensile stress will be the sum of the hoop tension and the tensile stress due to the bending action. The relative values of the hoop tension and bending stress will, evidently, depend upon the amount which the arms stretch. If they should stretch enough owing to their own centrifugal force, so that the rim expands freely, no bending action will occur; while if they are so inelastic as completely to restrain the rim, no hoop tension will be induced, but the full centrifugal force will be applied to bend the rim. With any intermediate amount of stretch of the arms the rim will be held in equilibrium, partly by the hoop tension and partly by the restraining action of the arms, the latter being a measure of the unbalanced centrifugal force of the rim, and of the bending stress caused thereby. Since the expansion of the rim is directly proportional to the stretch of the arms it is clear that the hoop tension is also directly proportional to the stretch. If, for instance, the arms stretch one quarter the amount necessary for free expansion, the hoop tension will be one-quarter that due to free expansion, and the bending stress will be proportional to three-quarters of the centrifugal force of the rim. The mathematical relation which exists between these stresses is complex, and will of course vary with the relative size and shape of the rim and the arms. If the rim is of a wide thin section, and the arms are few, the bending stress may be very serious. Professor Lanza* has shown that, with the proportions ordinarily used, the arm, theoretically, stretches about **three-quarters** the amount necessary for free expansion. It is also to be noted that if the wheel is to act as a band wheel, and has a wide thin rim, the bending action on the arms as at *B*, Fig. 173, still further distorts the rim and increases the bending on the forward side.

Let D = the mean diameter of the rim in feet.
Let R = the mean radius of the rim in feet.
Let t = the thickness of the rim in inches.
Let v = the velocity of the rim in feet per second.
Let w = the weight of the material per cubic inch.
Let l = the length of the rim between arms in inches.

Consider a section of the rim one inch wide on the face. The centrifugal force per unit of length (1″), circumferentially, of this section is $c = \frac{w\,t\,v^2}{R\,g}$ and, therefore, by Art. 78, the total load which tends to separate such a ring along a diameter is $\frac{w\,t\,v^2}{R\,g} \times 12D$, and the unit stress in the section, if no bending exists, is therefore,

$$p_1 = \frac{12\,w\,t\,v^2\,D}{2\,t\,R\,g} = \frac{12\,w\,v^2}{g} = \frac{v^2}{10}, \text{ nearly, for iron wheels} \quad . \quad (5)$$

* Trans. A. S. M. E., Vol. XVI, page 208.

The maximum bending moment in the rim occurs at the arms, and its value is $M = \frac{c l^2}{12}$ * considering the rim as a straight beam. The stress due to this bending moment when no hoop tension exists is therefore,

$$p_2 = \frac{M e}{I} = \frac{c l^2 e}{12 I} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where e is the distance to the outer fibre, and I † the moment of inertia of the *cross-section of the elementary ring.*

If now the stretch of the arms be taken as three-quarters that necessary for free expansion of the rim, the total unit tension in the rim will be

$$p = \tfrac{3}{4} p_1 + \tfrac{1}{4} p_2 = \left(\frac{3 v^2}{40} + \frac{c l^2 e}{48 I}\right) \quad . \quad . \quad . \quad (7)$$

if n be the number of arms in the wheel, $l = \frac{12 \pi D}{n}$, and if the cross-section of the rim be rectangular $\frac{e}{I} = \frac{6}{t^2}$ whence equation (7) reduces to

$$p = \left(\frac{3 v^2}{40} + \frac{3 D v^2}{t n^2}\right) = 3 v^2 \left(\frac{D}{t n^2} + 0.025\right) \quad . \quad . \quad (8)$$

For $t = \frac{9}{16}$, $v = 88$, $D = 4$ ft., and $n = 6$. Professor Lanza finds the stress due to hoop tension $= 575$ and the stress due to bending $= 5{,}060$ or p the total stress $= 5{,}635$. For the same data equation (8) gives $\tfrac{3}{4} p_1 = 581$ and $\tfrac{1}{4} p_2 = 4{,}600$, or a total stress $p = 5{,}811$, which agrees quite closely.

The above equation may be used for checking, roughly, the allowable stress in flywheel rims, but implicit faith must not be placed upon it for the reasons given in the first paragraph of this article, and all results obtained from this or similar formulæ should be checked by successful practice wherever a doubt arises. The equation does, however, show clearly that in wheels having

* See Table I, case 17.

† It should be noted that this I is for a unit (1″) of width of rim and not for the entire cross-section.

thin rims, or few arms, the bending stress is much greater than that due to hoop tension, and care should be exercised accordingly when such wheels must run at high speed. Equation (5) is often taken as a basis for the design of flywheels, using, therewith, a large factor of safety to cover uncertainties. If p_1 in equation (5) be taken as 1,000 (a factor of safety = 20), then v = 6,000 feet per minute, and this is found to be a safe peripheral speed for ordinary cast iron wheels. It is to be noted, however, that this speed is safe only because experience has shown it to be so, and not, as will be seen, because the stress is necessarily as low as 1,000 pounds.

Example. Compute the stress in the rim of the cast-iron flywheel discussed in example (2) of Art. 188, assuming that the arms stretch three-quarters the amount necessary for free expansion of the rim. Here $n = 6$, $t = 2.2$, $D = 8$ and $v = \frac{4{,}000}{60} = 66.6$.

$$\therefore \text{ from (8)}, \; p = 3\,v^2\left(\frac{D}{t\,n^2} + 0.025\right) =$$

$$3 \times 66.6^2\left(\frac{8}{2.2 \times 6^2} + 0.025\right) = 1{,}668 \text{ lbs. per sq. in.}$$

The stress, if based on equation (5), would be 444 pounds per square inch.

When a flywheel is being accelerated from rest, or when the energy supply is suddenly cut off, as it may be in a steam engine, the arms may be called upon to carry the full torque load. Each arm of a wheel with a very stiff rim approximates a cantilever beam fixed at one end, free but guided at the other, and carrying a concentrated load at the free end (see Table I, case 7). If the rim is thin and flexible, the arms approximate a simple cantilever loaded at the free end. In addition, the arm is subjected to a tensile stress due to the centrifugal action of its own weight, and that part of the rim which it supports, so that apparently equation M (Table VI) applies. The direct stress is difficult to compute, however, and since the bending stress in the simple cantilever is twice that of a cantilever with the free end guided, it is considered sufficiently accurate to compute the arm as a simple cantilever and neglect the direct stress.

Let P = the greatest force due to the belt pull at the rim.

Let a = the length of the arm.

Let n = the number of arms.

Then from J (Table VI):

$$p = \frac{Pae}{nI} \quad \ldots \ldots \quad (9)$$

from which the stress p, or the moment of inertia I, may be determined. The stress allowed should not exceed 2,000 pounds per square inch, for cast iron, on account of the uncertainties of the case, and a lower value is sometimes desirable. The statement sometimes made that the arms should be as strong against bending stress as the shaft is against torsional stress, is misleading as, in general, shafts are designed for *stiffness*, and not for torsional strength. The shaft of a steam engine may have to be very large to avoid excessive deflection and, as a consequence, may have great excess of torsional strength.

191. Construction of Wheels. Flywheels and band wheels, for velocities below 5,000 feet per minute, are usually made of cast iron on account of low cost. For higher velocities steel castings are used, and in extreme cases wheels made of steel plates, or wire-wound wheels have been constructed. Equation (5) may be written $v = 1.64\sqrt{\frac{p_1}{w}}$. The allowable unit tensile strength divided by the weight per cubic unit is, therefore, a measure of the value of the material for this purpose. For this reason some woods are superior to cast iron for wheel rims, and cast-iron wheels which have burst have been successfully replaced with wheels having rims made of wood.*

Difficulties in transportation limit the diameter of wheels cast in one piece to about ten feet, and the diameter of wheels cast in two parts to about twenty feet. Wheels from about sixteen feet in diameter upward are usually made in several sections. Small flywheels and band wheels are usually cast in one piece, or

* Trans. A. S. M. E., Vol. XIII, page 618.

made in two parts for convenience in erecting. In either of the latter cases unknown shrinkage stresses will most probably exist. These shrinkage stresses are sometimes relieved by casting the hub in several pieces, each piece being cast integral with one or more arms. The openings between the parts are afterward filled with lead, and rings are shrunk upon the hub to hold the parts in place. Experience shows that solid cast-iron wheels, when properly proportioned, are safe up to 6,000 feet per minute which, fortunately, is also about the limit of efficient belt speed. If, however, the wheel has a very wide thin rim it cannot be considered safe at this speed, particularly if balance weights are attached to the rim between the arms, thus increasing the centrifugal bending

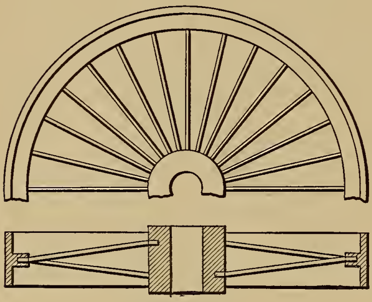

Fig. 174.

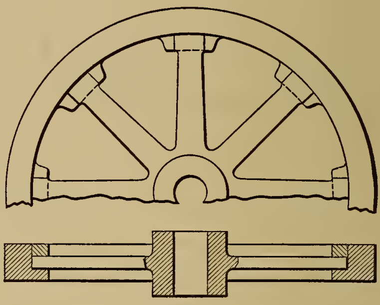

Fig. 175.

force. If joints exist in the rim, their relative strength must be considered. Band wheels of wrought-steel construction can now be obtained up to about 4 feet in diameter; they are light and strong, and are rapidly coming into favor.

Where speeds above 6,000 feet per minute are necessary, wheels such as shown in Fig. 174 are sometimes built. Here the rim and hub are of cast iron, each cast in one piece, and the spokes are of steel. The spokes are placed in the mould, and the metal poured around them, so that on cooling they are gripped very firmly. The spokes are placed close together so that there is practically no bending of the rim, and the rim is also prevented from expanding freely. Wheels of this construction are used for large band saws at velocities above 10,000 feet per minute, under heavy service, with perfect success.

In Fig. 175 the rim is cast separately in one or more pieces. The arms do not constrain the rim radially, but leave it free to expand. The stresses in the rim when cast in several pieces so that shrinkage is not a factor are those due to centrifugal force only, and the arms are simple cantilevers. Wheels of this character have been used with success in rolling-mill work.* Figs. 174 and 175 illustrate wheels which correspond closely to the limiting types discussed in Art. 190. The construction of most wheels lies between these types. Fig. 176 illustrates a band wheel with the arms and hub cast in one piece and the rim in sections. The joints in the rim are simple flange joints, placed midway between the arms. This is the most dangerous location possible, on account of the added bending effect due to the centrifugal force

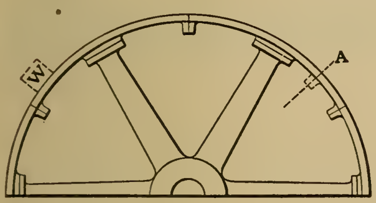

FIG. 176.

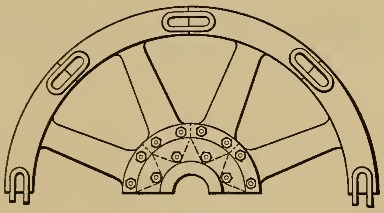

FIG. 177.

of the flanges which add to the mass without contributing to the strength. The best location is at the arm, and many wheels are built thus, the arm being bolted to each segment, and the segments themselves bolted together as well. Where the joint is placed between the arms, it should be about one-quarter the length of the span away from the arm, as at *A*, Fig. 176, where, by the theory of elasticity, the bending moment is zero. Fig. 177 shows a heavy flywheel with an arm and a segment of the rim cast together. The arms are secured in the hub by means of fitted bolts. The hub may be solid or the flange on one side may be movable axially so as more firmly to clamp the arms. The segments are held together at the rim by means of links of rectangular cross-section shrunk in place. This construction is very common. Occasionally links are also

* Trans. A. S. M. E., Vol. XX, page 944.

shrunk into recesses on the outer face of the wheel. In Fig. 178 the segments are held together by T-headed links, sometimes called "prisoners," shrunk in place. The segments are joined at the arms, which are fastened to them by through bolts. This construction is simple and the machining is easier than with flanged connections. The construction of the hub is similar to that in Fig. 177.

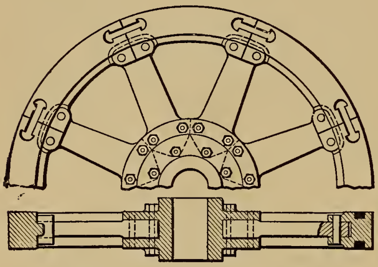

Fig. 178.

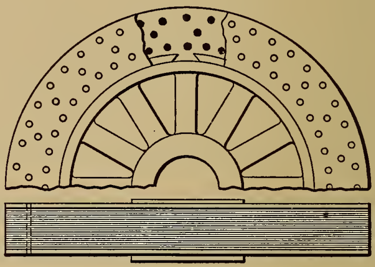

Fig. 179.

It is evident that the manner of joining segments in built-up wheels is most important. Wheels seldom fail at the hub. Wheels with thin, wide sections are almost always joined by flanges as shown in Fig. 180. When such joints are used they should be well ribbed for stiffness, as indicated, and the bolts should be placed as near the rim as possible, so that the lever arm *a* shall be as great as possible compared to the arm *b* (see

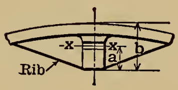

Fig. 180.

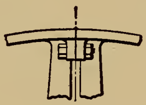

Fig. 181.

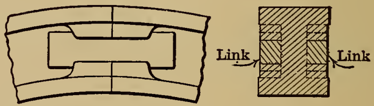

Fig. 182.

Art. 63). A much better arrangement is shown in Fig. 181, where an arm is placed on each side of the joint. This is particularly applicable to wheels cast in two parts. It may be noted that thin rims are often stiffened by light *circumferential* ribs at the outer edges. Mr. A. K. Mansfield has pointed out (Trans. A. S. M. E., Vol. XX) that these ribs *may* be a source of weakness.

The greatest bending moment is near the arm where these ribs are on the *tension* side of the beam. A rim having such ribs is not necessarily as strong against bending in this direction, as one of rectangular cross-section having the same area ; and when ribs are used the section modulus should be calculated.

The prisoner link shown in Fig. 178 has certain advantages over the link shown in Fig. 177. It is evident that the depth of the recesses in Fig. 177 is limited, while in Fig. 178 the slot can extend entirely across the section and the link can be made as wide as the rim itself. Furthermore, it is possible to machine both wheel and link in Fig. 178 accurately, which is difficult to do with the construction in Fig. 177. This permits of greater accuracy in computing the initial stress induced in the link by shrinking it in place, the importance of which has been noted in Art. 77. If the rim be made I-shaped,* as in Fig. 182, the links can be so proportioned that the joint will be as strong as the rim proper or even stronger.

While, evidently, the relative strength of the joint compared to the solid rim will vary with the exact proportions selected, average practice gives the following apparent values:

Flanged joint, bolted, midway between arms	.25
Flanged joint, bolted, at end of arms	.50
Linked joint as in Fig. 177	.60
Linked joint as in Fig. 178	.65
Linked joint as in Fig. 182	1.00
Solid rim	1.00

It must not be inferred from the above that a solid rim is necessarily the best; as, obviously, a wide thin rim with unknown shrinkage strain may not be as safe as a narrow deep rim of the same sectional area if held together by a good joint.

For extreme velocities, wheels built up of steel plates, or wheels with rims made of plates fastened to a central spider made of steel castings, are now used. Fig. 179 shows a flywheel of the latter type used in rolling-mill work (see *Power*, Feb. 4, 1908). The rim is made of laminations held to the spider by dovetails,

* See Trans. A. S. M. E., Vol. XX, page 944.

as shown, the laminations being assembled with overlapping joints. Heavy outside plates clamp the whole structure together by means of through bolts. In the particular case noted above, the velocity of the wheel rim is 250 feet per second. Descriptions of a number of examples of such wheels are to be found in the Transactions of the A. S. M. E., the magazine *Power*, and other periodicals. Wheels for great speed have also been constructed by winding the rim with many turns of steel wire.

The rotors of some forms of electric generators, steam turbine rotors, and similar revolving members are often loaded as shown at *W*, Fig. 176. Such loads add to the centrifugal force acting on the rim, but do not add to the strength of the rim. Due allowance should be made in such cases; particularly if the load or loads are placed near a joint as shown in Fig. 176. The teeth of gear wheels constitute such a load, and if the wheel is large, and the peripheral speed high, this should be considered. Balance weights, placed between the arms, should be carefully considered, especially when the rim is thin and the velocity high.

192. Experiments on Flywheels. The best experimental data upon the strength of flywheels are from tests conducted by Professor Benjamin and reported to the A. S. M. E.* While these experiments were made to determine the *bursting speed* of small cast-iron wheels only, and throw no light on the increase of stress with an increase of speed, they are very valuable as indicating the manner in which various types of wheels fail. Being conducted on small wheels, due allowance must be made for the difference in quality between the metal of small and large castings in estimating probable bursting stresses. These experiments go to show that solid cast wheels will burst at a peripheral velocity somewhere near 400 feet per second, and such wheels are safe only at a velocity of not more than 100 feet per second. Rim joints midway between the arms, particularly the common flange joints, were found to reduce the strength materially. The strength of various joints was found to be about as tabulated in Art. 191.

* See Trans., Vols. XX and XXIII. See also "Machine Design," by C. H. Benjamin.

Extra loads, such as balance weights located between the arms, were found to be very dangerous, on account of the added bending effect.

193. Rotating Discs. If the radial depth of a wheel rim be great compared to its axial width, the equations deduced in the preceding articles do not apply, the difference being analogous to that existing between thick and thin cylinders. Mathematical analysis of the stresses in a rotating disc, in common with those existing in thick cylinders under internal pressure, are complicated and not altogether satisfactory. Experimental data, corroborating the theories, are also lacking. A full mathematical treatment of these stresses is beyond the scope of this treatise, and only enough will be inserted to show the general character of the problem.

When a disc of uniform thickness is rapidly rotated on its axis, the principal stresses induced are a tangential tension, and a radial stress, at every point in the disc.

Let r_2 = the outer radius of the disc in inches.
" r_1 = the inner radius of the disc in inches.
" r = the radius at any point.
" λ = Poisson's ratio = $\frac{1}{3}$ for steel and $\frac{1}{4}$ for cast iron.
" N = revolutions per minute.
" w = weight of one cubic inch of the material.
" p = the tangential stress at any radius r.
" p' = the radial stress at any radius r.

Then it can be shown* that for a flat disc of uniform thickness, having a hole at the centre of radius r_1,

$$p = 0.00000355\, w N^2 \left[(3+\lambda)\left(r_2^2 + r_1^2 + \frac{r_2^2 r_1^2}{r^2}\right) - (1+3\lambda) r^2 \right] \quad (11)$$

and $$p' = 0.00000355\, w N^2 \left[(3+\lambda)\left(r_2^2 + r_1^2 - \frac{r_2^2 r_1^2}{r^2} - r^2\right) \right] \quad (12)$$

For a solid disc

$$p = 0.00000355\, w N^2 \left[(3+\lambda) r_2^2 - (1+3\lambda) r^2 \right] \quad . \quad (13)$$

and $$p' = 0.00000355\, w N^2 \left[(3+\lambda)(r_2^2 - r^2) \right] \quad . \quad (14)$$

---

* See "Theory of the Steam Turbine," by A. Jude, pages 192 and 204. The notation and units have been changed to correspond with those used in this text.

It is to be noted that the radial stress is less at any point than the corresponding tangential stress; and an examination of equation (11) shows that this tangential stress is a maximum at the surface of the bore and a minimum at the outer periphery. At the surface of the bore or where $r = r_1$ the stress

$$p = 0.00000355\, \omega N^2 [\, (3 + \lambda)\ (2\, r_2^2 + r_1^2) - (1 + 3\, \lambda)\, r_1^2]$$

If now r_1 be taken so small that r_1^2 is negligible, it appears that the tangential stress is

$$p = 2 \times 0.00000355\, \omega N^2 [\, (3 + \lambda)\, r_2^2]$$

which is just twice that obtained by making $r = o$ in equation (13). The effect of even a very small hole at the centre of a rotating disc is, therefore, to increase the stresses greatly.

Example. A circular steel saw ¼ inch in thickness and 80 inches in diameter has a hole 4 inches in diameter in the centre and runs at the rate of 500 R.P.M. Determine the tangential stress at rim and also at the hole.

Here $N = 500$, $w = 0.28$, $\lambda = ⅓$, $r_2 = 40$, and $r_1 = 2$. Whence in (11) making $r = r_2 = 40$ the tangential stress at the rim is

$$p = 0.00000355 \times 0.28 \times 500^2 [\, (3 + ⅓)\ (40^2 + 2^2 + 2^2) - (1 + 1)\ 40^2] = 535 \text{ lbs. per sq. in.}$$

and at the hole making $r = r_1 = 2$.

$$p = 0.00000355 \times 0.28 \times 500^2 [\, (3 + ⅓)\ (40^2 + 2^2 + 40^2) - (1 + 1)\ 2^2] = 2{,}643 \text{ lbs. per sq. in.}$$

The foregoing equations, (11) to (14), are deduced on the hypothesis that the material is perfectly elastic and homogeneous. It is clear that they cannot be intelligently applied to built-up wheels of the disc type, and must also be applied with caution to brittle materials. They are of great value, however, in showing the general character of these stresses and the location of the greatest stress, thus indicating the shape which discs should have for uniform strength; for a brief reflection will show that such discs must be thickened at the centre to reduce the stress at that point. For complete mathematical analysis of discs of different shapes, reference may be made to the various works on the steam turbine. It is evident that great care should be used in selecting and working the material for high-speed discs. Rolled sheets are not good for very high speeds on account of their seamy structure, which is conducive to incipient cracks, and cast materials of brittle structure must be of first-class quality. Discs forged down from much thicker ingots give the safest construction.

References:

"The Theory of the Steam Turbine," by A. Jude.
"Steam Turbines," by L. French.
"The Steam Turbine," by Dr. A. Stodola.

CHAPTER XVI

MACHINE FRAMES AND ATTACHMENTS

194. Stresses in Machine Frames. Since machine frames must, in general, receive the reactions from the forces applied to the various moving members by the energy transmitted, it is obvious that the stresses induced in frame members are, in most cases, very complex and beyond mathematical analysis. If it is essential that the moving members be held in accurate alignment, as in the case of machine tools, the predominating requirement for the frame is *stiffness* and not strength. For these reasons the design of machine frames, in general, must be governed largely by judgment and experience, the cases where complete mathematical analysis is possible being rare. However, even in cases where judgment must be the guide, it is not only helpful, but sometimes necessary to check, as closely as possible, the stresses in certain important sections, by applying those fundamental formulæ of Table VI, page 94, which apparently fit the circumstances. In all cases, what may be termed a "qualitative analysis" of the frame is very desirable as a guide in properly distributing the material, and in determining the *forms* of the various sections.

If the character, value, and line of action of every force acting upon a given section are known, the stresses in the section can be determined by applying the fundamental requirements for static equilibrium of the section, namely:—

(*a*) The algebraic sum of all horizontal component forces must = 0.

(*b*) The algebraic sum of all vertical component forces must = 0.

(*c*) The algebraic sum of all the moments must = 0

The stress, in any direction, at any point, will be the algebraic sum of all the stresses acting in that direction, at that point, as found by applying (*a*), (*b*), and (*c*). It is impossible to make a classification of machine frames that would be of any service, but the principles will be illustrated by applying them to typical cases. It is to be noted that it is seldom possible to find the required dimensions of a section, directly, by solving the particular equations from Table VI which apply; but, in general, the section must be assumed from the conditions given, and then checked for strength or stiffness.

Fig. 183 illustrates a type of frame which is quite common and known as an **open frame.** It is one of the few types where

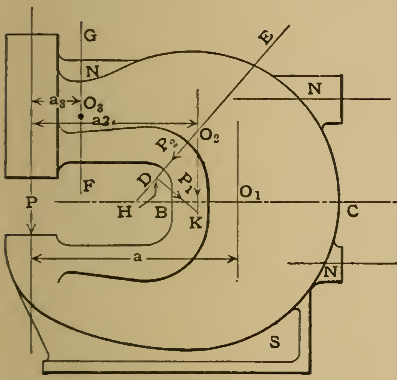

FIG. 183.

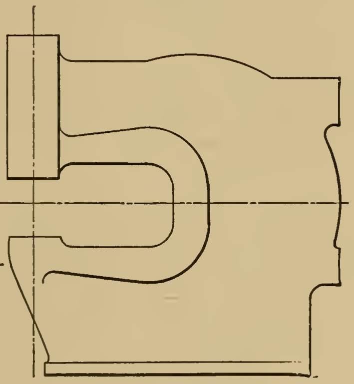

FIG. 184.

a mathematical analysis can be made with some degree of completeness. In the case of a punching-machine frame as illustrated in Fig. 183, great stiffness is not essential and the design may be based on the strength required. Suppose the frame to be outlined as shown so that the dimensions of the cross-section at any place may be assigned. Evidently, if the stresses are checked at the sections *BC*, *DE*, and *FG*, the strength of the frame will be fully determined.

In the section *BC*, whose gravity axis is at O_1, consider the portion of the frame above *BC* as a **free body.** It is in equilibrium under the action of the *exterior* force *P*, due to punching, and the *internal* forces exerted upon it by the lower half of the frame. There are no horizontal forces. The vertical force P must be balanced by an equal and opposite force at the section BC, which induces a tensile stress uniformly distributed over the section, the intensity of which is $p_1 = \frac{P}{A}$ pounds per square inch, where A is the area of the section. The only moment acting on the part is Pa, due to the action of P, which tends to rotate the upper part of the frame around O_1, the gravity axis of the section, causing a resisting tension at B, and a resisting compression at C. The maximum intensity of these flexural stresses is given by the fundamental equation for flexure in beams (see J, Table VI), or $p_2 = \frac{Pae}{I_1}$ where e is the distance from O_1 to the outer fibre and I_1 is the moment of inertia of the cross-section around the axis O_1. The greatest tension will therefore be at B and its value will be $p = p_1 + p_2 = \frac{P}{A} + \frac{PaI_1}{e}$, which is equation M of Table VI. This is, therefore, a case of combined flexure and direct stress, which is fully discussed in Art. 19, Chapter III.

Consider next the section DE, whose gravity axis is at O_2, and suppose the part of the frame at the left of DE to be a free body. There are no horizontal forces and the vertical force P must be balanced by a vertical tensile pull upon the upper part of the frame by the lower part. The resultant of this tensile pull, which is distributed uniformly over the whole section, may be represented by $O_2 K$ acting at the centre of gravity. This force may be resolved into the components $HK = P_1$ perpendicular to DE, and producing a tensile stress at right angles to the section and $P_2 = O_2 H$ parallel to DE and producing a shearing stress along the section. The only moment acting upon the section is that due, as before, to P, whose moment arm is a_2. The tensile and compression stress due to this moment, as determined by equation J, Table VI, may be combined with the direct stress P_1, as in the section BC, to find the maximum tensile or compression stress. The shearing stress is

$$p_s = \frac{P_2}{A_2} \text{ where } A_2 = \text{the area of the section } DE.$$

This is usually small and may be neglected except near the ends of the beam as in the section FG (see Art. 14, Chapter III).

Consider last the section FG. As before, there are no horizontal forces, but the vertical force P must be balanced by a vertical resisting force which induces a shearing stress at the section. The intensity of this shearing stress is $\frac{P}{A_3}$, where A_3 is the area of the section. Since the area of the section is much smaller than at DE or BC, it is advisable to compute its value. The moment Pa_3 is balanced as before by the resisting moment of the section and the resulting stress may be computed by equation J, of Table VI. Evidently these general principles may be applied to any section.

Fig. 185 illustrates an open frame as applied to a power riveter. The rivet which is to be "driven" is placed between the dies D and D_1, and pressure is applied to the movable die D, by means of the power cylinder R. The pressure which is applied may be very great (150 tons or more), and unless the jaws are properly designed they may spring so much that the dies will fail to align properly, and faulty work will result (see Art. 53). Stiffness and not strength is, therefore, the essential factor in the design; for if the parts are stiff enough they will be, in general, strong enough. The yielding which most affects the alignment is that due to the bending of the frame B, and the stake C, and that which may result from the elongation of the bolts which hold these members together. When the riveting pressure P is applied, the beams B and C tend to rotate around the point O, this tendency being resisted by the tension in the bolts. The load which may be applied to the bolts by the force P will be $P_1 = \frac{P(a+b)}{b}$. If the nuts on the bolts are set up so that a combined total initial tension somewhat greater than P_1 is induced in the bolts, the stretching of the bolts, and the consequent opening up between the frame and the stake, will be negligible.

(See Art. 60 and Fig. 43 and also Art. 77.) The *intensity* of stress in the bolts should not exceed 6,000 pounds per square inch. The upper part of the frame, B, approximates a cantilever of *uniform strength* of length a. (See Art. 15 and Case 1 of Table II.) The maximum deflection which occurs at D may, therefore, be computed and the maximum stress which occurs at EF may be checked by Equation J of Table VI. The stake, C, approximates a cantilever of *uniform cross-section,* and may therefore be

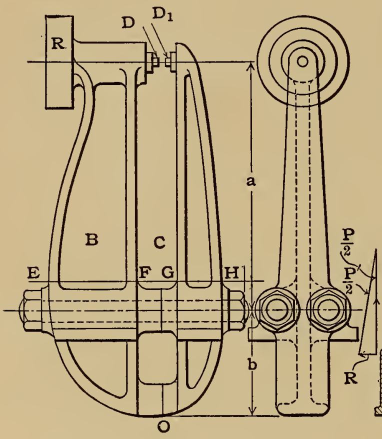

Fig. 185

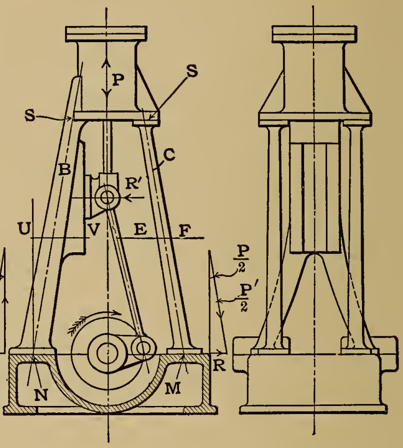

Fig. 186.

treated in a similar manner. (See Case 1, Table I, and Equation J, Table VI.)

Fig. 186 illustrates a **closed frame** as applied to a vertical steam engine. The back column, B, which carries the crosshead guide is of cast iron, while the front columns, C, are of steel. It is required to check the stresses in these columns when the piston is ascending and also when it is descending, the rotation of the engine to be taken in a clockwise direction as indicated.

When the piston is ascending, the steam pressure tends to draw the cylinder and bed closer together. This tendency is resisted by P', the combined thrust on all the columns, the vertical component of which must equal P, the total steam pressure on the piston. It may be reasonably assumed that the back column carries one-half of the total thrust, and that each of the front columns carries one-quarter. The thrust of the back column, $\frac{P'}{2}$, may be resolved into components perpendicular and parallel to the face of the foot. The vertical component will equal $\frac{P}{2}$. The horizontal component R tends to spread the foot of the column outward and induce a bending stress in it. The column should, therefore, be secured to the bed by fitted bolts, or, if the bolts are loose in the holes, the foot should be well dowelled to the bed; or, better still, the foot should fit against a ledge cast on the bed plate. R will then be balanced by an equal and opposite reaction at the feet of the front columns, thus setting up a negligible tension in the bed and leaving a compressive force only on the column. By similar reasoning each front column is subjected to a compressive load $\frac{P'}{4}$ and the total horizontal component R is balanced by that of the back column through the bed.

The tension or compression in the piston-rod and connecting-rod, either ascending or descending, have a resultant R' normal to the guide, which may have a large value where the connecting-rod is short compared to the crank. This resultant tends to bend B, and hence C also, in a left-hand direction, the bending being resisted by the fastenings at the feet. The columns and cylinder, however, constitute a very stiff structure, and except where the frame is made up of light construction this effect may be neglected. This reaction, R', however, also bends the column B locally, that is as a beam encastré at S and N, the effect of R being greatest when the crosshead is near half stroke. (See Case 18, Table I.) If then it be desired to check the central section UV of the column, the long column stress due to $\frac{P'}{2}$ must be added to the flexural stress due to R'. The sum of these stresses should not, of course be greater than the allowable stress for the material used. The columns, C, need only be checked as long columns (see equation N, Table VI).

When the piston is descending the steam pressure tends to separate the bed and the cylinder. The reactions at M and N are reversed in direction and the columns are put in tension, the horizontal components inducing negligible compression in the bed. The most dangerous section in this case will be under R' and the stress will be that due to $\frac{P'}{2}$ plus the tensile stress due to the bending effect of R'. The fastenings of the columns to the cylinder and to the bed plate must, of course, be sufficiently strong in tension to resist the force tending to separate the cylinder and bed.

In the foregoing examples the lines of action of all forces acting on the section considered, lay in a *plane* of *symmetry* of the section, and the section tended to rotate around a gravity axis at right angles to this plane. While this is the most usual case, occasionally the force or forces acting are not in a plane of symmetry. Thus Fig. 187 may represent the cross-section of the column of a radial drilling machine, in which it is required to check the stresses when the force P, due to drilling, is in the position shown. If C be the centre of gravity of the section, the tendency to rotate will be around the axis $X' X'$ at right angles to PC, the arm of the force P, and the resistance of the section against such rotation will be measured by the moment of inertia of the section with reference to this axis. The maximum tensile and compressive stresses will occur at the fibre farthest removed from $X' X'$ or at M and N, the stress at M being tensile when the direction of P is upward to the plane of the paper, and compressive when its direction is downward. The centre of gravity, C, may be located readily, by finding the intersection of any pair of gravity axes. If the section has an axis of symmetry, as UV, Fig. 187, it is necessary only to find the axis at right angles to UV. This is most readily done graphically as follows: Divide the section into small areas, as shown by dotted lines at $x x$ in Fig. 187. From the centre of gravity of each area draw parallel lines ab, bc, cd, preferably at right angles to the known axis UV. In Fig. 187 (a), lay off AB, BC, etc., proportional to the respective areas whose gravity axes are *ab*, *bc*, etc. Take any pole *O* and draw *AO*, *BO*, etc. From any point on *ab*, draw *ao* indefinitely, parallel to *AO*. From the same point draw *ob* parallel to *OB*. From the intersection of *ob* and *bc* draw *co* parallel to *CO*, and from its intersection with *cd*, draw *od* parallel to *OD*. The intersection of *ao* and *od* locates the gravity axis *XX* (see also Art. 120). It is evident that this method may be applied when both axes are unknown.

The moment of inertia of the section around $X' X'$ may be most readily found by transforming the area of the figure into an *equivalent figure* with *RP* as a base, as follows: Draw lines,

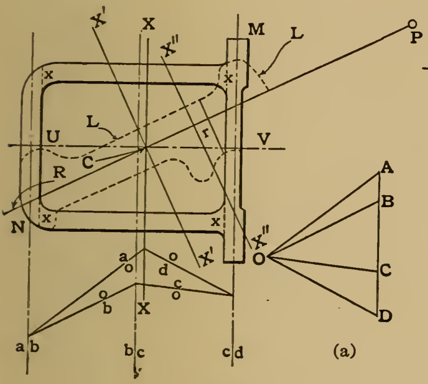

FIG. 187.

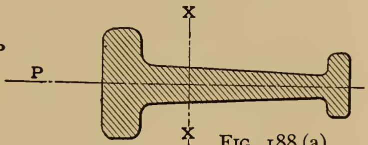

FIG. 188 (a).

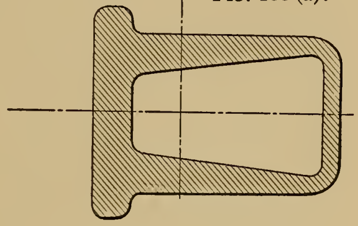

FIG. 188 (b).

as $X'' X''$, parallel to $X' X'$, and plot the intercepts made by it on the given section, on each side of *CB* as ordinates of an equivalent section, shown in Fig. 187 by the dotted line *L*. The accuracy of the work may be checked with a planimeter, as it is evident that the area of the transformed section will be equal to that of the original. Divide this equivalent figure into approximate rectangles, by lines drawn parallel to $X' X'$, as shown at *r*. Then the moment of inertia of *r* around the axis $X' X'$ will be its moment of inertia round its own gravity axis parallel to $X' X'$, plus its area into the square of the distance between these axes. The sum of the moments of inertia of all such small areas will be the required moment of inertia of the section.

195. Distribution of Metal in Frames. Machine frames are usually made of castings, on account of their complicated shapes, cast iron being the material most used, while steel castings are rapidly coming into use for severe work. In addition to the stresses induced in the frame by the energy transmitted by the machine, it may also be subjected to severe *accidental* stresses due to such causes as shrinkage, or the settling of a part of the foundation. Both these classes of stresses are, in general, very complex and generally beyond mathematical analysis, and the problem must frequently be left to the judgment of the designer, especially if stiffness is a large factor. Economy in the use of metal, however, demands that its distribution throughout the frame shall be in accord with the best analysis possible, and, therefore, the general principles governing the forms of sections must be kept in mind.

The most trying stresses to which a frame may be subjected are torsion, flexure, or a combination of these. It has been noted in Art. 12 that the hollow section (Fig. 7) is most effective for resisting torsion, and, if this be the predominating stress sections such as are shown in Fig. 7, or modified sections as shown in Fig. 187, are correct. It was also noted in Art. 19 (Fig. 10) that in the case of cast iron, or other metal whose tensile strength is much less than its compressive strength, a great saving of material is effected by massing the metal on the tension side as shown in Fig. 188 (a); thus making the tensile and compressive stresses more in proportion to the strength of the material. If then the predominating stress in a frame is simple flexure (in a given plane), a section like that shown in Fig. 188 (a) is allowable, but if, in addition, torsional strength must be withstood, or if the plane of flexure may change, a section similar to that shown in Fig. 188 (b) is better design, since it combines the merits of both Figs. 187 and 188 (a). Sometimes it is better to make the section so large that the flexural stress can be safely withstood by a wall of uniform thickness, as in Fig. 187, as the construction of the pattern is simpler and the shrinkage stresses less serious than in such sections as shown in Fig. 188. The metal in the walls will be much sounder, also, as the thick sections of Fig. 188 are very likely to have a porous interior, due to shrinkage. Cast-iron parts more than four or five inches thick are almost sure to be defective in this manner. The walls of such sections as shown in Fig. 188 should taper uniformly from the thick part to the thin parts, and all corners should be well rounded, and filleted, to minimize as far as possible the concentration of shrinkage stresses. Thin wide flanges or webs should not be cast integral with thick heavy parts, as unequal shrinkage and porosity are sure to result. This is especially true of thin ribs cast on the tension side of large sections, as the edge of the rib is liable to crack through shrinkage, thus starting rupture across the entire section. Small brackets or other attachments of thin sections should never be cast on a large frame, as they seldom cast well. A section of moderate thickness is often stronger than a thicker one, since the greatest strength of cast iron is in the outer skin. It should also be remembered that even when a frame is both strong and stiff enough to do the required work at low speeds, it may not have mass enough to absorb the vibrations set up when running more rapidly. This may call for more metal in the frame than is dictated by other requirements. Openings for supporting or removing cores should be placed near the gravity axis so as to reduce the strength as little as possible (see Fig. 191).

196. Attachments and Supports. The general appearance of a machine is affected more by the outline of the main frame than by that of any other member. This outline should, therefore, be clearly shown, and not obliterated at places by the various attachments which restrain the moving parts or support the frame. In Fig. 183 is shown the outline of a frame in which the various sections have been proportioned in accordance with the loads brought upon them, and the various bosses N and the support S appear as attachments to the main member. Fig. 184 illustrates the same machine with the attachments merged into the main member, thereby destroying the character of the design, and also making it more difficult to judge of the relative strength of various sections of the frame.

The form of an attachment will, of course, be governed by the service it is required to render and the manner in which it is loaded and supported. If the outline of the attachment is based on theoretical considerations, care should be exercised that all the modifying influences are duly considered. Thus if parabolic outlines are given to an attachment, such as the housings *H* for supporting the tool in Fig. 192, the upper end of the housing must be modified from the theoretical parabolic outline indicated by the bending effect of the force *P*, so as to provide for the shearing effect at the upper end, which is frequently neglected. (See also Article 15.)

If the frame rests directly on the floor its outlines should be carried down to the floor in such a manner as will give an appear-

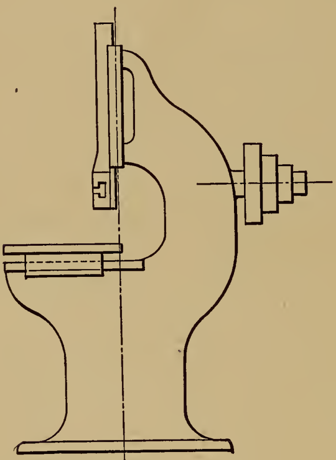

FIG. 189.

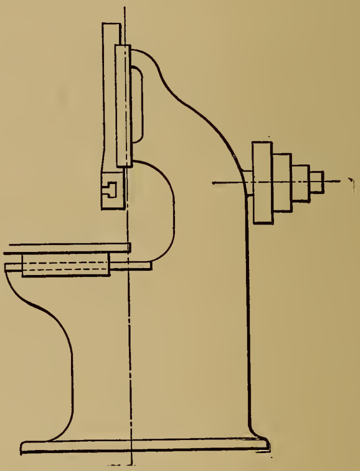

FIG. 190.

ance of stability. Thus Fig. 189 shows such a machine frame on which the vertical outline of the back of the frame is undercut. Fig. 190 shows the same machine with the outline carried straight to the floor and the improvement in appearance, so far as stability is concerned, is obvious. Fig. 191 shows the outline of a planing machine in which the upright, *U*, is carried to the floor at *V*, in the form of a leg. This construction is not correct, as *U* is an attachment to the bed, designed to resist the force of the cut and transfer it to the bed, which should itself be stiff enough to withstand all such stress thus brought upon it. Any settling of the foundation might affect the alignment of

U and hence the arrangement shown in Fig. 192 is more nearly correct.

In large machines the frame usually rests directly on the foundation, and should have sufficient stiffness to resist distortion due to the settling of the foundation, since the latter is very difficult to avoid. In smaller machines the frame is carried on supports, which may be of two general types, (*a*) cabinet or box pillar supports (Fig. 192), and (*b*) legs as shown in Fig. 193. The choice of support will, of course, depend on the type and size of the machines. In any case the number of points of support should be as few as possible. If the machine can be supported on three points it is evident that the frame cannot be affected by settling of the foundation. It is difficult, in general,

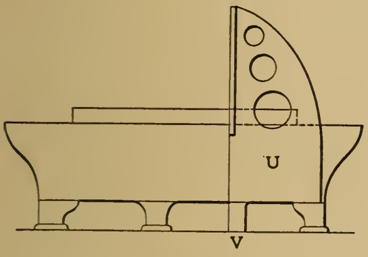

FIG. 191.

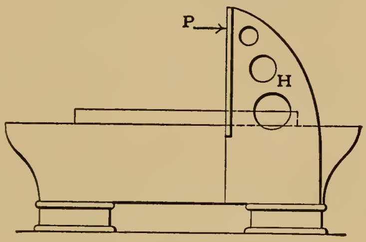

FIG. 192.

to obtain three-point support, but it is seldom necessary to place supports as close together as in Fig. 191 (which is taken from an actual design), where the frame is carried on eight points. Fig. 192 shows the same frame properly carried on box supports, the supports themselves being so stiff as materially to assist the frame and practically reducing the support to so-called two-point support. Small machines can often be supported on a single box-pillar, the overhanging parts of the frame having a parabolic outline as suggested in Fig. 189. If the box pillar is of considerable height the sides should taper slightly toward the top; for if made parallel the pillar will appear wider at the top than at the bottom. It is preferable to use one form of support throughout, *i.e.*, all box pillars or all legs, and not one or more of each.

When the frame must be supported on legs, as in Fig. 195, these should not curve outward as in Fig. 193, unless it is absolutely essential in order to obtain stability. Spreading the legs as in Fig. 193 lengthens the distance between the reactions, *R*, *R*, and, therefore, increases the bending effect on the bed and legs as a whole. The leg shown in profile in Fig. 194 is better and much easier to make. The legs should be so placed that the outline *L* forms a continuation of the principal vertical outline *L′* of the frame, as shown in Fig. 194. The same remarks apply to the end view of the legs as shown in Figs. 195 and 196. The complex curves and ornate features of Fig. 195 are not only use-

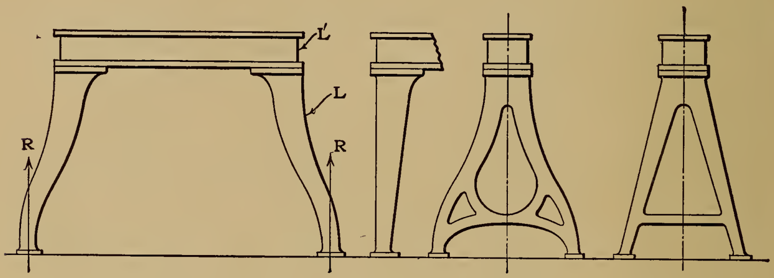

FIG. 193. FIG. 194. FIG. 195. FIG. 196.

less but expensive. It is not always possible or desirable to make machine frames and supports with simple straight-line outlines; but where curves are necessary they should be as simple as possible; and in general the best results can be obtained by using arcs of circles or parabolas. Ornamentation of a fanciful nature is not permissible anywhere, as it really detracts from the appearance of the machine, and adds to the cost of production. *Harmony* of design can be attained by making the various members of correct proportions to withstand the loads brought upon them, and by using the simplest and most direct design with smooth transition curves between straight lines which intersect. It is a proverb in design that "what is right looks right."

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