XXIV. *On the Indian Arc of Meridian.* *By the Venerable* J. H. PRATT, *M.A.*, *Archdeacon of Calcutta.* *Communicated by Professor* STOKES, *Sec. R.S.*

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IT is with pleasure that I request the attention of the Royal Society to the present communication, in continuation and completion of my former papers, because I think that the anomalies which the Indian Arc has appeared to present are here traced to the true causes.

1. I will explain what those anomalies were. On completing a laborious and well-executed survey of the two northern portions of the Indian Arc of Meridian, between Kaliana (29° 30′ 48″) and Kalianpur (24° 7′ 11″), and Kalianpur and Damargida (18° 3′ 15″), Colonel EVEREST found that their astronomical and geodetical amplitudes differed considerably; in the higher arc the geodetic amplitude he found to be in excess by 5″·236, in the lower of the two arcs in defect by 3″·791*. The three stations had been selected with great care, and were finally chosen as being apparently free from all disturbing causes. Indeed, a fourth station which had been at one time adopted, Takal Khera in Central India, was rejected by Colonel EVEREST because a neighbouring hill-range was discovered on calculation to produce a deflection of about 5″. Kaliana had been chosen nearly sixty miles from the lower hills at the foot of the Himmalaya Mountains, in the full conviction that it would be free from mountain influence. The surprise was therefore great when, on the completion of the survey of the two arcs in question, these two errors were brought to light. The first was attributed to the influence of the Himmalayas, but without any calculation; but the second, with its negative sign, received no interpretation. At this stage I devised a method of calculating the effect of the Himmalayas by a direct process; and found that the deflections produced are far greater than the errors which had to be explained, and the negative sign was left altogether unaccounted for. Thus the perplexity was increased. It next occurred to me that the vast Ocean to the south of India might have some influence on the plumb-line. On making the necessary calculations the effect of this cause was found, as the mountain attraction had been, to be far greater than had been anticipated; the negative sign was still unexplained, and the difficulties were not cleared up. No other cause of disturbance was apparent at the surface. But I showed by calculation that in the crust below one *might* exist sufficient to reduce the large deflections occasioned by the Mountains and the Ocean, and make them accord with the results deduced by Colonel EVEREST

* Colonel EVEREST used a mean figure of the earth somewhat different to that used in this paper, which is taken from the volume of the British Survey lately published, and makes the errors somewhat less than those above given. See EVEREST 'On the Indian Arc,' 1847, p. clxxvi.

from the arcs themselves. But, being hidden from our sight, neither the magnitude nor indeed the existence of this cause could be *à priori* ascertained, much less reduced to calculation. Whether, moreover, the errors brought to light by Colonel EVEREST arose solely from local attraction, or from local attraction combined with some local peculiarity in the curvature of the Indian Arc, was not apparent; so that the subject of local attraction, and its influence on geodetic operations in this country, was still involved in obscurity, and the anomalies of the Indian Arc remained unexplained in the papers which I have hitherto forwarded to the Society. In the present communication I think ambiguity is removed. It is demonstrated that no peculiarity in the curvature of the arc can produce any part of the errors brought to light by Colonel EVEREST; that those errors arise solely from local attraction; that they are in fact the exact measure of the difference of the resultant local attraction at the two extremities of each arc, from whatever causes the attraction may arise—mountains, ocean, or crust; lastly, it is proved that there *are* hidden causes in the crust below the Indian Arc, and the differences of their resultant effect upon the stations of the arc are computed. An inference from these results is, that the relative position of places in a Map, laid down from geodetic operations, is accurate, being altogether unaffected by local attraction; though the position of the Map itself on the terrestrial spheroid will be dependent upon the observed latitude of some one station in it, and that observed latitude will be affected by the local attraction at that place. To determine the absolute latitude in some one station connected with the geodetic operations is still a desideratum.

§ 1. *Summary of the Results of former Papers.*

2. The results of my former papers I may briefly sum up as follows:—

(1) In the first of them* I calculated the effect of the Mountain-Region north of India upon the plumb-line at the three principal stations of the northern portion of the Indian Arc; viz. Kaliana (29° 30′ 48″), Kalianpur (24° 7′ 11″), and Damargida (18° 3′ 15″). The deflections towards the north were found to be 27″·98, 12″·05, 6″·79; and in consequence of these, the observed astronomical amplitudes would be 15″·93 and 5″·26 less than the true amplitudes determined by normals to the meridian line in the meridian plane.

These quantities, as I showed †, are not materially affected by new information regarding the mountain mass communicated to me by Lieut.-Colonel STRACHEY.

(2) In my second paper ‡ I calculated the effect on the plumb-line of a slight but wide-spread deviation of density in the crust of the earth, in excess or defect, from

* Philosophical Transactions, 1855; also 1859, p. 770.

† Ibid. 1859, p. 774. The reader is requested to make the following corrections in that paper:—

Page 761, *line* 4, *for* multiply *read* modify.

— 767, — 2 *ab imo, for* $\frac{r}{r-d}$ *read* $\frac{2r}{r-d}$.

— 781, — 26, *for* require *read* requires.

Page 782, *line* 16 *ab imo, after* by *insert* 0·185 and by

— 794, — 17, *for* 15·88 *read* 62·60

— 795, — 4 *ab imo, for* 15·88 *read* 62·60

— — — — *for* 514·57 *read* 561·29

‡ Philosophical Transactions, 1859, p. 745.

that required by the fluid-law, and showed that the effect might be very sensible and important. The results of the calculation were embodied in the following Table:—

TABLE OF DEFLECTIONS, caused by an excess or defect of matter throughout a semi-cubic space of 4 millions of cubic miles,—the mean density of the excess or defect being $\frac{1}{100}$th part of the density of the earth at the depth of the centre of the cubic space.

Depth of the centre of the semi-cubic Space.	Distance, measured along the chord, from the station to the point where the radius of the earth drawn through the middle point of the semi-cubic space meets the earth's surface.				
	379 miles.	581 miles.	781 miles.	980 miles.	1173 miles.
50 miles	$1''\cdot940$	$0''\cdot835$	$0''\cdot457$	$0''\cdot248$	$0''\cdot118$
150 miles	1·621	0·803	0·456	0·252	0·120
250 miles	1·383	0·782	0·483	0·272	0·131
350 miles	1·067	0·749	0·490	0·286	0·142
450 miles	0·663	0·713	0·425	0·277	0·145

(3) In my third paper * I calculated the effect on the plumb-line of the deficiency of attracting matter in the Ocean. I assumed the following law, as giving an average representation of the mass of water; viz. that the depths at the middle of the Bay of Bengal and of the Arabian Sea in the latitude of Cape Comorin, and at the mid-point between Madagascar and Australia, are severally $\frac{3}{4}$, 1 and 3 miles, and that the bottom slopes from the shores to these points, or to lines joining the first two with the third, or to other lines drawn northwards from those two points.

The meridian deflections toward the north at the three stations were made to be $6''\cdot18$, $9''\cdot00$, and $10''\cdot44$, causing an increase in the amplitudes equal to $2''\cdot81$ and $1''\cdot44$.

On combining these with the effect of the mountains, the deflections are $34''\cdot16$, $21''\cdot05$, and $17''\cdot23$, and therefore the true amplitudes $13''\cdot11$ and $3''\cdot82$ greater than the observed or astronomical amplitudes.

These are the main results of my calculations.

3. In the Great Trigonometrical Survey of India, which has been conducted with so much care and ability, the amplitudes of the two arcs in question, calculated geodetically on the supposition of the Indian Arc being curved like the mean arc, came out, the first $5''\cdot236$ in excess, and the second $3''\cdot791$ in defect, of the amplitudes observed astronomically. Neither the attraction of the mountains, nor that of the ocean combined with it, as appears from the last paragraph, would account for these, and especially for the negative sign. The other cause treated of (a variation in the density of the crust) being purely hypothetical and, if existent, yet altogether unknown in position and extent, it seemed hopeless to look for any precise explanation of the deviations of the plumb-line from that quarter, although the sufficiency of the cause to produce a sensible deflection was demonstrated.

I therefore attempted in each paper to explain the difference by attributing it to the

* Philosophical Transactions, 1859, p. 779.

Indian Arc being curved differently to the mean meridian of the earth. As each new disturbing cause—first the mountains, secondly the possible variation in density below, thirdly the ocean—was thought of and the effect calculated, the resulting curvature, of course, came out differently.

In the present communication, however, I shall demonstrate that no change in the curvature of the arc, within reasonable and indeed wide limits, can possibly have any appreciable effect on the calculated amplitude. It is this fact which leads to the result I have announced in the first paragraph. I will explain how this result did not flow from my former calculations. The length of the arc s between two stations is

$$a(1-\tfrac{1}{2}\varepsilon)\lambda-\tfrac{3}{2}a\varepsilon\sin\lambda\cos 2m,$$

λ and m being the amplitude and middle latitude, and a, ε the semi-major-axis and ellipticity. In order to find the effect produced on the dimensions of the ellipse passing through the two stations by increasing or decreasing the amplitude, this was differentiated, s and m being considered constant. This gives an equation connecting da and $d\varepsilon$ with $d\lambda$, the change of the amplitude. A relation was then assumed (in the absence of a better method) between da and db, viz. that the mean value of a and b is the same in the two ellipses*. The calculation which I now give rests upon the fact, that the length of the chord of the arc must be the same in both the ellipses, the local and the mean, drawn through the stations at the extremities of the arc. There was a difficulty in following this course before, which I have now overcome. I find the length of the arc in terms of the unknown chord and semi-axes, and then differentiate with respect to the semi-axes, remembering that the chord is constant. All the terms now being small, an approximate value may be used in them for the chord in terms of either semi-axis and the observed latitudes of the extremities of the arc.

§ 2. *Statement of the several calculations which have been made of the form of the northern portion of the Indian Arc.*

4. I will bring together the results of the various measures and calculations which have been made of the arc between Kaliana and Damargida, divided near the middle by Kalianpur.

I. By a comparison of the two portions of the arc, Colonel Everest, taking the observed amplitudes, got the following results (a and b being the semi-axes of the ellipse):—

$$a=20985260 \text{ feet}, \ b=20875737, \ \varepsilon=\frac{1}{192};$$

and he found that the amplitudes, calculated on the supposition of the arc being part of the mean ellipse, are $5''\cdot236$ in excess and $3''\cdot791$ in defect of the observed amplitudes †.

II. Captain Clarke, in his Chapter on the Figure of the Earth, in the volume of the British Ordnance Survey lately published, gives formulæ which enable us to calculate

* Philosophical Transactions, 1859, p. 762, Note.

† See Colonel Everest's Volume, 1847, p. 428; also p. clxxvii of the Preface.

the form of the Indian Arc, making use of the observed data. He shows* that the following quantities must be added to the observed latitudes of the three stations to make the arcs measured by the Survey fit an ellipse of which the axes are given by

$$a-b=\frac{1}{300}\left(1-\frac{u}{10000}+\frac{v}{50}\right)20890000,\ a+b=2\left(1-\frac{u}{10000}\right)20890000 \text{ feet},$$

viz. for Kaliana . . . $0{\cdot}403+4{\cdot}1251u+2{\cdot}7756v+x$

Kalianpur . . $-4{\cdot}085+2{\cdot}1831u+1{\cdot}6203v+x$

Damargida x

He finds for the mean ellipse of the whole earth $u=-0''{\cdot}3856$, $v=1''{\cdot}0620$, $x=0''{\cdot}050$, and therefore the corrections of the observed latitudes are, by the above formulæ, $1''{\cdot}810$, $-3''{\cdot}156$, $0''{\cdot}050$. Hence the amplitudes thus determined are $4''{\cdot}966$ in excess, and $3''{\cdot}206$ in defect of the observed amplitudes.

The form of the local ellipse can be determined from the data by putting the corrections for the latitudes equal to zero. This gives

$$4{\cdot}1251u+2{\cdot}7756v=-0{\cdot}403,$$
$$2{\cdot}1831u+1{\cdot}6203v=\ \ 4{\cdot}085,$$

which give

$$u=-19{\cdot}2015,\ v=28{\cdot}3920,$$

and thence

$$a=20984066 \text{ feet},\ b=20876151,\ \varepsilon=\frac{1}{193}.$$

These results agree well with those of Colonel Everest noticed above.

III. The third measure is that determined by a comparison of the two arcs, the amplitudes being corrected for mountain and ocean attraction. Let s, s', λ, λ', m, m' be the lengths, amplitudes, and middle-latitudes of two arcs, of which the amplitudes are not large—as in this instance. Then

$$\frac{s}{\lambda}=\frac{a+b}{2}-3\frac{a-b}{2}\cos 2m,\ \frac{s'}{\lambda'}=\frac{a+b}{2}-3\frac{a-b}{2}\cos 2m',$$

$$\frac{a-b}{2}=\frac{1}{3}\,\frac{\frac{s}{\lambda}-\frac{s'}{\lambda'}}{\cos 2m'-\cos 2m},\quad \frac{a+b}{2}=\frac{\frac{s}{\lambda}\cos 2m'-\frac{s'}{\lambda'}\cos 2m}{\cos 2m'-\cos 2m}.$$

By what I have stated in paragraph 2, the increase to be made to the amplitudes to correct for mountain and ocean attraction is $13''{\cdot}11$ and $3''{\cdot}82$. The values of λ and λ' are therefore

$$\lambda=5^\circ\ 23'\ 37''+13''=5^\circ\ 23'\ 50'',\ \lambda'=6^\circ\ 3'\ 56''+4''=6^\circ\ 4'\ 0'',$$

also

$$s=1961157 \text{ feet},\quad s'=2202926\dagger.$$

These lead to

$$a=20906792,\quad b=20843795,\quad \varepsilon=\frac{1}{332}.$$

* See Ordnance Survey, pp. 737, 741, 767.

† See Colonel Everest's Volume, 1847, p. 427.

IV. The fourth measure of the arc is one proposed by Captain CLARKE in the volume of the Ordnance Survey. He suggests that by the principle of least squares the ellipse should be found which departs least from the mean ellipse in form, and at the same time gives deflections of the normal from the normal of the mean ellipse most in accordance with the calculated deflections. He finds this ellipse, taking account of mountain attraction only; the amount of ocean-attraction not having then been ascertained. The following recalculation, according to Captain CLARKE'S method, takes account of both.

Let l_1, l_2, l_3 be the latitudes of the three stations referred to the mean ellipse. Then $l_1-1''\cdot 81$, $l_2+3''\cdot 16$, $l_3-0''\cdot 05$ are the observed latitudes (see the calculation under II.). Let l_1+e_1, l_2+e_2, l_3+e_3 be the latitudes of the three places referred to any other ellipse, the axes being given by the formulæ in u and v under II. Then $e_1+1''\cdot 81$, $e_2-3''\cdot 16$, $e_3+0''\cdot 05$ are the corrections which must be added to the observed latitudes to make them accord with this new ellipse. The dimensions, then, of this ellipse are determined by solving these equations:—

$$\begin{aligned} e_1+1\cdot 81 &= 0\cdot 403+4\cdot 1251u+2\cdot 7756v+x, \\ e_2-3\cdot 16 &= -4\cdot 085+2\cdot 1831u+1\cdot 6203v+x, \\ e_3+0\cdot 05 &= x. \end{aligned}$$

These equations give

$$\begin{aligned} u &= -0\cdot 3856+2\cdot 5946e_1-4\cdot 4446e_2+1\cdot 8500e_3, \\ v &= 1\cdot 0620-3\cdot 4958e_1+6\cdot 6056e_2-3\cdot 1098e_3. \end{aligned}$$

Suppose d_1, d_2, d_3 are the angles of deflection caused by the mountains and ocean. Then the most probable ellipse to measure the curvature of the Indian Arc (supposing there are no other causes of deflection of the vertical) is that which makes

$$(e_1-d_1)^2+(e_2-d_2)^2+(e_3-d_3)^2$$
$$+(2\cdot 5946e_1-4\cdot 4446e_2+1\cdot 8500e_3)^2+(3\cdot 4958e_1-6\cdot 6056e_2+3\cdot 1098e_3)^2$$

a minimum. By differentiation with respect to e_1, e_2, and e_3 we obtain three equations, which after transformation become

$$\begin{aligned} e_1 &= 0\cdot 76873d_1+0\cdot 35715d_2-0\cdot 12584d_3, \\ e_2 &= 0\cdot 35716d_1+0\cdot 34199d_2+0\cdot 30086d_3, \\ e_3 &= -0\cdot 12583d_1+0\cdot 30087d_2+0\cdot 82493d_3. \end{aligned}$$

These give

$$\begin{aligned} u &= -0\cdot 3856+0\cdot 17432d_1-0\cdot 03671d_2-0\cdot 13760d_3, \\ v &= 1\cdot 0620+0\cdot 06325d_1+0\cdot 07484d_2-0\cdot 13808d_3. \end{aligned}$$

The values of the calculated deflections d_1, d_2, d_3 are $34''\cdot 16$, $21''\cdot 05$, $17''\cdot 23$. When these are substituted in the above formulæ, we have

$$e_1=31''\cdot 61,\ e_2=24''\cdot 58,\ e_3=16''\cdot 25,\ u=2\cdot 4255,\ v=2\cdot 4189.$$

Hence the errors in the observed latitudes as affected by deflection (or $e_1+1\cdot 81$, $e_2-3\cdot 16$,

$e_3+0{\cdot}05$) are $33''{\cdot}42$, $21''{\cdot}42$, and $16''{\cdot}30$, which are very nearly equal to the calculated deflection. Also the values of u and v give the following results for the semi-axes and ellipticity:—

$$a=20919988, \quad b=20846981, \quad \varepsilon=\frac{1}{287}.$$

5. The mean ellipse, as determined in the British Ordnance Survey Volume, gives

$$a=20926500, \quad b=20855400, \quad \varepsilon=\frac{1}{294}.$$

If δa, δb be the excess (or, in case of a negative sign, the defect) of the semi-axes of any of the four ellipses described above, compared with the mean ellipse, then the following are true:—

	Arc I.	Arc II.	Arc III.	Arc IV.
$\delta a =$	11·13 miles,	10·90 miles,	−3·73 miles,	−1·23 miles.
$\delta b =$	3·85 miles,	3·93 miles,	−2·20 miles,	−1·60 miles.

§ 3. *The deviation of the local elliptic arc from the form of the mean ellipse.*

6. The four several ellipses enumerated in the last section, representing the form of the arc between Kaliana and Damargida under different data and methods of calculation, are not necessarily concentric with the mean ellipse; but they must have their axes parallel to those of the mean ellipse, because the latitudes are measured from the same or parallel lines.

Suppose one of these four ellipses drawn through the extremities of the arc, Kaliana—Damargida, and an ellipse equal to the mean ellipse also drawn through those two fixed points, with the axes of the ellipses parallel to each other. Let a, b, ε be the semi-axes and ellipticity of the first of these, α and β the coordinates to its centre measured from some fixed point near that centre, and therefore near the centre of the earth. The squares and products of $a-b$, ε, α and β may be neglected. Let s be the length of the arc, R the distance of the point of the arc in mid-latitude from the origin of coordinates, l and l' the observed latitudes of the extremities (viz. 29° 30′ 48″ and 18° 3′ 15″), λ and m the amplitude and middle-latitude of the arc. I proceed to find the difference in length, and the distance at the mid-latitude of the local and mean arcs lying between the two stations, and also the distance of the centre of the local ellipse from that of the ellipse equal in dimensions to the mean ellipse, but drawn through the two stations at the extremities of the arc, as described above.

7. *First.* The difference in length of the arcs.

$$s=\frac{1}{2}(a+b)\lambda-\frac{3}{2}(a-b)\sin\lambda\cos 2m.$$

Let c be the chord, r and θ, r' and θ' the polar coordinates from the centre of the ellipse to the extremities of the arc;

$$\therefore\ c^2=r^2+r'^2-2rr'\cos(\theta-\theta')=2rr'\{1-\cos(\theta-\theta')\}+(r-r')^2,$$

$$r=a(1-\varepsilon\sin^2 l), \quad r'=a(1-\varepsilon\sin^2 l').$$

Also
$$\tan\theta=(1-2\varepsilon)\tan l,$$

$$\therefore\ \theta=l-\varepsilon\sin 2l,\quad \theta-\theta'=\lambda-2\varepsilon\sin\lambda\cos 2m;$$

$$\therefore\ 1-\cos(\theta-\theta')=1-\cos\lambda-2\varepsilon\sin^2\lambda\cos 2m=2\sin^2\tfrac{1}{2}\lambda\Big\{1-2\varepsilon(1+\cos\lambda)\cos 2m\Big\};$$

$$\therefore\ c^2=4a^2\sin^2\tfrac{1}{2}\lambda\Big\{1-2\varepsilon(1+\cos\lambda)\cos 2m-\varepsilon(\sin^2 l+\sin^2 l')\Big\}$$

$$=4a^2\sin^2\tfrac{1}{2}\lambda\Big\{1-\varepsilon\{1+(2+\cos\lambda)\cos 2m\}\Big\};$$

$$\therefore\ \sin\tfrac{1}{2}\lambda=\frac{c}{2a}\Big\{1+\tfrac{1}{2}\varepsilon\{1+(2+\cos\lambda)\cos 2m\}\Big\};$$

$$\therefore\ \frac{\lambda}{2}=\sin^{-1}\frac{c}{2a}+\frac{\varepsilon}{2}\Big\{1+(2+\cos\lambda)\cos 2m\Big\}\frac{c}{\sqrt{4a^2-c^2}}$$

$$=\sin^{-1}\frac{c}{2a}+\frac{\varepsilon}{2}\Big\{1+(2+\cos\lambda)\cos 2m\Big\}\tan\frac{\lambda}{2}.$$

Hence $s=a\left(-\frac{1}{2}\varepsilon\right)\lambda-\frac{3}{2}a\varepsilon\sin\lambda\cos 2m$

$$=a(2-\varepsilon)\sin^{-1}\frac{c}{2a}+a\varepsilon\Big\{1+(2+\cos\lambda)\cos 2m\Big\}\tan\frac{\lambda}{2}-\frac{3}{2}a\varepsilon\sin\lambda\cos 2m$$

$$=(a+b)\sin^{-1}\frac{c}{2a}+(a-b)\Big\{1+\tfrac{1}{2}(1-\cos\lambda)\cos 2m\Big\}\tan\tfrac{1}{2}\lambda.$$

Taking the variation with respect to the axes,

$$\delta s=(\delta a+\delta b)\sin^{-1}\frac{c}{2a}-\frac{a+b}{a}\,\frac{c\delta a}{\sqrt{4a^2-c^2}}+(\delta a-\delta b)\Big\{1+\tfrac{1}{2}(1-\cos\lambda)\cos 2m\Big\}\tan\frac{\lambda}{2}.$$

Since the terms are small, we may use approximate values;

$$\therefore\ \delta s=(\delta a+\delta b)\tfrac{1}{2}\lambda-2\tan\tfrac{1}{2}\lambda.\delta a+(\delta a-\delta b)\Big\{1+\tfrac{1}{2}(1-\cos\lambda)\cos 2m\Big\}\tan\tfrac{1}{2}\lambda$$

$$=(\delta a+\delta b)\left(\tfrac{1}{2}\lambda-\tan\tfrac{1}{2}\lambda\right)+(\delta a-\delta b)\tfrac{1}{2}\tan\tfrac{1}{2}\lambda(1-\cos 2\lambda)\cos 2m.$$

Applying this to the case in hand, we have $\lambda=11°\ 27'\ 33''$ and $2m=47°\ 34'\ 3''$. These lead to

$$\delta s=-0{\cdot}0003350(\delta a+\delta b)+0{\cdot}0006747(\delta a-\delta b)$$
$$=\ 0{\cdot}0003397\delta a-0{\cdot}0010097\delta b.$$

8. *Secondly.* The distance between the arcs at the mid-latitude.

The equation to the local ellipse is

$$\frac{(x-\alpha)^2}{a^2}+\frac{(y-\beta)^2}{b^2}=1.$$

Neglecting small quantities of the second order,

$$x^2+y^2=a^2+2\alpha x+2\beta y-2\varepsilon(a^2-x^2),$$

$$\therefore\ r^2=a^2+2a\alpha\cos\theta+2a\beta\sin\theta-2a^2\varepsilon\sin^2\theta,$$

$$\therefore\ r=a+\alpha\cos\theta+\beta\sin\theta-a\varepsilon\sin^2\theta.$$

Let R, C, C′ be the values of r at the mid-latitude and at the extremities of the arc;

$$\therefore\ R=a+\alpha\cos m+\beta\sin m-(a-b)\sin^2 m,$$
$$C=a+\alpha\cos l+\beta\sin l-(a-b)\sin^2 l,$$
$$C'=a+\alpha\cos l'+\beta\sin l'-(a-b)\sin^2 l'.$$

Multiply by 1, M, N, add and make the coefficients of α and β vanish;

$$\therefore\ \cos m+M\cos l+N\cos l'=0,\quad \sin m+M\sin l+N\sin l'=0;$$
$$\therefore\ M=-\frac{\sin(m-l)}{\sin(l'-l)}=-\frac{1}{2}\sec\frac{1}{2}\lambda=N;$$
$$R+MC+NC'=a(1+M+N)-(a-b)(\sin^2 m+M\sin^2 l+N\sin^2 l')$$
$$=a(1+2M)-\frac{1}{2}(a-b)\{1-\cos 2m+2M(1-\cos\lambda\cos 2m)\}$$
$$=\frac{1}{2}(a+b)(1+2M)+\frac{1}{2}(a-b)(1+2M\cos\lambda)\cos 2m$$
$$=\frac{1}{2}(a+b)\left(1-\sec\frac{1}{2}\lambda\right)+\frac{1}{2}(a-b)\left(1-\cos\lambda\sec\frac{1}{2}\lambda\right)\cos 2m.$$

Taking the variation with respect to the axes,

$$\delta R=\frac{1}{2}(\delta a+\delta b)\left(1-\sec\frac{1}{2}\lambda\right)+\frac{1}{2}(\delta a-\delta b)\left(1-\cos\lambda\sec\frac{1}{2}\lambda\right)\cos 2m.$$

Put $\lambda=11°\ 27'\ 11''$, $2m=47°\ 34'\ 25''$,

$$\delta R=-0{\cdot}0025078(\delta a+\delta b)+0{\cdot}0050586(\delta a-\delta b)$$
$$=\ 0{\cdot}0025508\delta a-0{\cdot}0075664\delta b.$$

9. *Third.* The coordinates to the centre of the local ellipse from the centre of the ellipse equal to the mean ellipse drawn through the extremities of the arc.

By eliminating β from the two equations which give C and C′, we have

$$\alpha=\frac{(C'-a)\sin l-(C-a)\sin l'-(a-b)\sin l\sin l'(\sin l'-\sin l)}{\sin(l-l')}$$
$$=\frac{C'\sin l-C\sin l'}{\sin(l-l')}-\frac{\sin l-\sin l'}{\sin(l-l')}\{a+(a-b)\sin l\sin l'\}$$
$$=\frac{C'\sin l-C\sin l'}{\sin(l-l')}-\frac{\cos m}{\cos\frac{1}{2}\lambda}\left\{a+\frac{1}{2}(a-b)(\cos\lambda-\cos 2m)\right\}.$$

Also

$$\beta=\frac{(C-a)\cos l'-(C'-a)\cos l+(a-b)(\sin^2 l\cos l'-\sin^2 l'\cos}{\sin(l-l')}$$
$$=\frac{C\cos l'-C'\cos l}{\sin(l-l')}-\frac{\cos l'-\cos l}{\sin(l-l')}\left\{a-(a-b)(1+\cos l\cos l')\right\}$$
$$=\frac{C\cos l'-C'\cos l}{\sin(l-l')}-\frac{\sin m}{\sin\frac{1}{2}\lambda}\left\{b-\frac{1}{2}(a-b)(\cos\lambda+\cos 2m)\right\}.$$

Taking the variations with respect to a and b,

$$\delta\alpha = -\frac{\cos m}{\cos\frac{1}{2}\lambda}\left\{\delta a + \frac{1}{2}(\delta a - \delta b)(\cos\lambda - \cos 2m)\right\}$$

$$\delta\beta = -\frac{\sin m}{\cos\frac{1}{2}\lambda}\left\{\delta b - \frac{1}{2}(\delta a - \delta b)(\cos\lambda + \cos 2m)\right\}.$$

Put $\lambda = 11° \ 27' \ 11''$, and $2m = 47° \ 34' \ 25''$,

$$\delta\alpha = -0{\cdot}9196678\delta a - 0{\cdot}1404082(\delta a - \delta b)$$
$$= -1{\cdot}0600760\delta a + 0{\cdot}1404082\delta b,$$
$$\delta\beta = -0{\cdot}4053130\delta b + 0{\cdot}3353533\,(\delta a - \delta b)$$
$$= \ 0{\cdot}3353533\delta a - 0{\cdot}7406663\delta b.$$

10. The formulæ I have thus obtained are as follows:—

$$\delta s = \ 0{\cdot}0003397\delta a - 0{\cdot}0010097\delta b,$$
$$\delta R = \ 0{\cdot}0025536\delta a - 0{\cdot}0075756\delta b,$$
$$\delta\alpha = -1{\cdot}0600760\delta a + 0{\cdot}1404082\delta b,$$
$$\delta\beta = \ 0{\cdot}3353533\delta a - 0{\cdot}7406663\delta b.$$

The values of δa, δb have been found in paragraph 5 for the Four Ellipses. By substituting them in these formulæ we are able to compare the ellipses with the mean ellipse. The results of this substitution are gathered together in the following Table, which contains also the semi-axes and ellipticities:—

	Mean Arc.	Arc I.	Arc II.	Arc III.	Arc IV.
	feet.	feet.	feet.	feet.	feet.
a =	20926500	20985260	20984066	20906792	20919988
b =	20855400	20875737	20876151	20843795	20846981
ε =	$\frac{1}{294}$	$\frac{1}{192}$	$\frac{1}{193}$	$\frac{1}{332}$	$\frac{1}{287}$
		miles.	miles.	miles.	miles.
δa =	....	11·13	10·90	−3·73	−1·23
δb =	....	3·85	3·93	−2·20	−1·60
δs =	....	− 0·0000761	− 0·0002654	0·0009542	0·0011977
δR =	....	− 0·0007445	− 0·0019379	0·0071414	0·0089801
$\delta\alpha$ =	....	−11·26	−11·00	3·65	1·08
$\delta\beta$ =	....	0·88	0·74	0·38	0·77

§ 4. *The difference between the geodetic and astronomical amplitudes, in the Indian Arc between Kaliana and Damargida, arises solely from local attraction affecting the plumb-line, and in no degree whatever from any deviation of the curvature of the arc from that of the mean arc.*

11. The differences of the length of the four arcs, and of that of the mean arc between Kaliana and Damargida, are, by the Table in the last paragraph,

$$-0{\cdot}0000761 \text{ mile}, \ -0{\cdot}0002654, \ 0{\cdot}0009542, \ 0{\cdot}0011977.$$

These, converted into seconds, at the estimate of 69·5 miles to one degree, or 1 mile to $51''\cdot 8$, are

$$-0''\cdot 00394,\quad -0''\cdot 01375,\quad 0''\cdot 04943,\quad 0''\cdot 06204,$$

which are absolutely insensible.

From this it follows, that the length of the arc lying between its two extremities will be sensibly the same, whether it coincide with the mean ellipse in curvature, or with any of the other ellipses enumerated in § 2. Of course, on determining the geodetic amplitude from the formula

$$s=\frac{1}{2}(a+b)\lambda-\frac{3}{2}(a-b)\sin\lambda\cos 2m,$$

the amplitude will come out differently for these different ellipses, although s is the same. But the fact, that the length of the arc is the same whatever the curvature of the arc (within the recognized limits), leads to this result—that the geodetic amplitude calculated from the length measured by the Survey, by means of the above formula applied to the mean ellipse, will be the amplitude corresponding to the mean ellipse, however much the actual arc differs from the mean ellipse owing to geological or other causes. Hence the deflection of the plumb-line in India from the normal to the mean ellipse can in no degree be attributed to the possible or probable circumstance of the curvature of the arc differing from the mean ellipse, as it may differ materially from it without producing this effect.

I may illustrate this still further by finding what amount of deviation in the curvature there may be without producing even $1''$ in the calculated length. The formulæ of paragraph 10 give

$$\delta s=0\cdot 0003397\delta a-0\cdot 0010097\delta b,$$

$$\delta R=0\cdot 0025536\delta a-0\cdot 0075756\delta b.$$

Eliminate δa, and these give

$$\delta R=7\cdot 517\delta s+0\cdot 000015\delta b.$$

Putting $\delta s=1''=\frac{1}{51\cdot 8}$ mile, and neglecting the second term,

$$\delta R=0\cdot 1451 \text{ mile}=\tfrac{1}{7}\text{th of a mile}.$$

The surface of the earth may, therefore, be elevated or depressed through one-seventh part of a mile at the middle parts of the arc (about 800 miles long) without producing more than $1''$ difference in the length of the arc.

The Table in paragraph 10 shows that the length of the Indian Arc, according to no one of the four different measures which have been made of it, differs by even $\frac{1}{800}$th of a mile, and in three cases even by much less than this, from the ellipse equal to the mean ellipse.

12. The deviations brought out by the Indian Survey must arise, therefore, altogether from local attraction. The effect of the two visible causes—the Mountain-Mass and the

Ocean—have been calculated approximately, and are found to produce the deviations $13''\cdot11$ and $3''\cdot82$, making the astronomical amplitudes so much less than those calculated geodetically. But the Survey makes these deviations $5''\cdot24$ and $-3''\cdot79$. There must, therefore, be some other source of attraction which increases the amplitudes by the differences of these, viz. by $7''\cdot87$ and $7''\cdot61$. We must attribute this to those hidden and unknown causes which lie below in the crust of the earth, where, as I have shown, causes, sufficient to produce a sensible deflection in the plumb-line, calculation proves may easily be supposed to reside. The following appears to me to be the simplest hypothesis regarding the variation below to account for these quantities, $7''\cdot87$ and $7''\cdot61$, which, it will be observed, are nearly equal to each other, that appertaining to the northern portion of the arc being somewhat larger than the other.

If the density of the crust deviates by $\frac{1}{100}$th part from the density given by the fluid-law through a cubic space, measuring 200 miles parallel and at right angles to the meridian and 200 miles deep, and situated with the centre of its upper surface at Kalianpur, then the Table in par. 2 shows that the deflections caused by the attraction of the upper and lower halves of this mass at a distance 379 miles from the centre of the upper surface on a point on the surface of the earth, along the chord of the arc, are $1''\cdot94+1''\cdot62=3''\cdot56$.

If the deviation in density be twice this, viz. $\frac{1}{50}$th part of the fluid-density, this deflection becomes $7''\cdot12$, very nearly the quantities to be accounted for. Now Kaliana is 371 miles, and Damargida is 430 miles from Kalianpur: these do not differ much from 379.

It is very conceivable, therefore, that the deflections $7''\cdot87$ and $7''\cdot61$, which have to be accounted for, arise from a slight excess of density of about $\frac{1}{50}$th part prevailing through a circuit of about 100 or 120 miles around Kalianpur, and to a depth of about 200 miles. Of course an indefinite number of other similar hypotheses might be framed to account for the deflections, but hardly one so simple as this. If we adopt the hypothesis of deficiency of matter beneath the Mountain Mass, we must suppose a similar deficiency to exist south of Damargida towards Cape Comorin and the Ocean; but as two independent hypotheses are here necessary, this solution is not so simple as the one I have adopted above.

13. Had I foreseen the result of the demonstration given in this communication, that a deviation in the curvature is altogether inadequate to account for any part even of the errors in the amplitudes, it would have been at once perceived, as it is now, that the errors $5''\cdot24$, $-3''\cdot79$ brought out by the Survey, must arise solely from local attraction affecting the plumb-line; and these errors would have been taken, as they now must be, to be the accurate measures of the differences of total local attraction at the three stations.

The calculations of Himmalayan and Ocean Attraction are nevertheless of considerable importance. Without them we should most probably have remained ignorant of the large amount of deflection due to that cause.

14. The numerical values of δR at the end of § 3 show that the Indian Arc, as represented by the first and second of the four measures I have enumerated, is slightly flatter than the mean ellipse in the corresponding parts; but as represented by the third and fourth, is somewhat more curved*.

15. The calculations of the Survey bring to light, with considerable exactness, the errors in the amplitudes, or *differences* of latitude of the stations, but do not at all help us to discover what the total deflections at the stations are. These can be found only by a direct calculation of the effect of the causes, such as I have given in my former papers. If these are not determined and allowed for, the latitude of a place determined by an observation of the sun or other heavenly body must always be erroneous to the extent of the deflection of the plumb-line at the place of observation. Thus, if the estimate of the hidden cause of disturbance in the instance of the Indian Arc, as above given, be accepted, the deflections at the three stations are—

Arising from the Mountains . .	$27''{\cdot}98$	$12''{\cdot}05$	$6''{\cdot}79$
Arising from the Ocean	6·18	9·00	10·44
Arising from the Hidden-cause .	−7·87	0·00	7·61
Totals	26·29	21·05	24·84

* In this Paper only the two northern portions of the Arc are considered. In this note I will give the results of a comparison of the three great divisions of the whole arc from Kaliana to Punnœ near Cape Comorin. The Curve of the Sea-level—for this is, of course, what we mean by the Arc, as it is this which the Survey calculates—is defined by the angles between the normals (that is, the plumb-lines) at the extremities of its three divisions, the lengths of the intervening arcs in feet, and the middle latitudes. The data are as follows: they are taken from p. 757 of the Volume of the British Survey lately published. They differ slightly from those given at p. 427 of Colonel EVEREST's Volume of 1847: the results of the first comparison below therefore differ slightly from those in the text of this Paper, in the first of the four measures of the Indian Arc enumerated in § 2.

	Astronomical amplitude.	Length in feet.	Twice the middle latitude.
Arc I. Kaliana to Kalianpur	$(\lambda)\ 5^\circ\ 23'\ 37''{\cdot}060$,	(s) 1961138,	$(2m)\ 53^\circ\ 38'\ 0''$
Arc II. Kalianpur to Damargida ...	(λ') 6 3 55·970,	(s') 2202905,	$(2m')$ 42 10 27
Arc III. Damargida to Punnœ	(λ'') 9 53 44·160,	(s'') 3591784,	$(2m'')$ 26 12 46

Putting these in the formulæ

$$\frac{a-b}{2}=\frac{1}{3}\,\frac{\frac{s}{\lambda}-\frac{s'}{\lambda'}}{\cos 2m'-\cos 2m},\quad \frac{a+b}{2}=\frac{\frac{s}{\lambda}\cos 2m'-\frac{s'}{\lambda'}\cos 2m}{\cos 2m'-\cos 2m}$$

and

$$\delta R=\frac{1}{2}(\delta a+\delta b)\left(1-\sec\frac{1}{2}\lambda\right)+\frac{1}{2}(\delta a-\delta b)\left(1-\cos\lambda\sec\frac{1}{2}\lambda\right)\cos 2m,$$

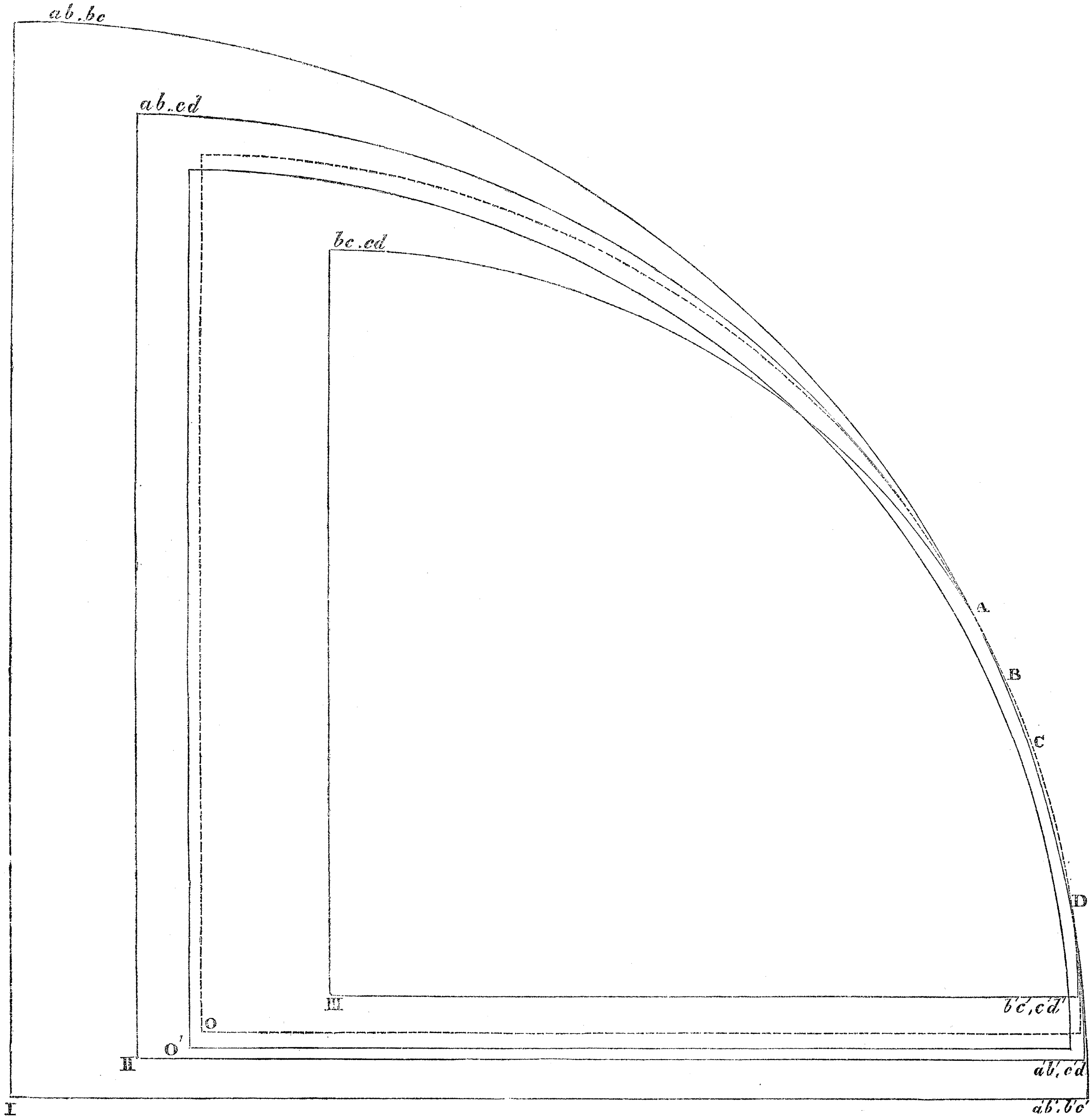
we obtain

	Arc I.	Arc II.	Arc III.	Mean ellipse.
a =	20983139 feet,	20943697,	20893936,	20926500
b =	20875011 feet,	20863963,	20841164,	20855400
ε =	$\frac{1}{194}$	$\frac{1}{262}$	$\frac{1}{396}$	$\frac{1}{294}$
δa =	56639 feet,	17197,	−32564	
δb =	19611 feet,	8563,	−14236	
δR =	−5·8 feet,	−4·6 feet,	−4·4 feet.	

These results are exhibited in the following diagram. ABCD is the actual arc or sea-level curve connecting

By these quantities will the latitudes of the three places, as determined by observations of the heavenly bodies, be wrong. The existence of such large discrepancies would not

Kaliana, Kalianpur, Damargida, and Punnœ; *ab*, *bc*ABC*a'b'*, *b'c'* is the ellipse I drawn through A, B, C, and having its semi-axes parallel to the semi-mean-axes, and 56639 and 19611 feet respectively longer than



those semi-axes. In the same way ellipses II. and III., as determined above, are drawn. (The parts of these three ellipses between A and D are not drawn in the diagram. The differences in the lengths of the semi-axes are made about eighty times out of proportion, that the differences between the ellipses may be visible.) The dotted ellipse about the centre O is an ellipse equal to the mean ellipse, and so drawn as to have been known, or perhaps suspected, if the actual calculation of the attractions had not been made.

16. It will be observed that the four several elliptic arcs which I have examined, as representing the Indian Arc, have been compared, not with the mean ellipse itself, but with an ellipse equal to the mean ellipse, and supposed to be drawn through the extremities of the arc. By the calculations of one part only of a meridian line it is impossible to determine the position of the centre of the mean ellipse, and therefore to ascertain how much the arc may have been upheaved or depressed with reference to the original centre of the earth when in a fluid state. It would require the survey of the whole meridian from pole to pole to determine this.

§ 5. *Conclusions from the whole investigation regarding the Indian Arc.*

17. The results finally arrived at may be stated as follows:—

(1) Colonel EVEREST discovered that the astronomical amplitudes of the two portions of the Indian Arc between Kaliana and Kalianpur, and between Kalianpur and Damargida, are, the first less by $5''\cdot24$, and the second greater by $3''\cdot79$, than the geodetic amplitude calculated with the mean semi-axes and ellipticity of the earth.

(2) The geodetic amplitudes of these two portions of the arc, calculated from the measured lengths and with the mean axes, will come out sensibly the same, even should the curvature of the arc differ from that of the mean meridian within reasonable but wide limits—a thing which geology teaches us to be very likely the case.

(3) Hence the geodetic measurements of the Survey being without sensible error, as is known by the tests applied, the discrepancy mentioned in (1) can arise only from local attraction affecting the vertical line, and so changing the astronomical amplitudes.

(4) Two great visible causes of disturbance of the vertical by attraction are, the Mountain Mass on the north of India, and the Ocean on the south. The influence of both of these is felt all over India; the first producing a northerly deflection varying from $27''\cdot98$ at Kaliana to a sensible angle (probably about $3''$, but this I have not calculated) at Cape Comorin; the second producing also a northerly deflection, varying from about $19''\cdot71$ at Cape Comorin to $6''\cdot18$ at Kaliana.

(5) The combined effect of these two visible causes is to make the astronomical amplitudes of the upper arc $13''\cdot11$ too small, and of the lower $3''\cdot82$ also too small. They are therefore insufficient to account for the discrepancies pointed out by Colonel

pass through A and D, the extremities of the whole arc. The ellipse about the centre O′ is the mean ellipse itself, the position of which with respect to our starting line (viz. the arc ABCD) is not known, except that its axes are parallel to those of the other ellipses. The values of δR above deduced show how very little the Indian Arc differs in curvature from the curvature of the mean ellipse in the same latitudes; and that it is very slightly flatter. How much other parts of the meridian may differ in curvature from the mean ellipse it is impossible to say, and therefore how much the Indian continent may be bodily upheaved or depressed below the mean ellipse. To determine this would require, as stated in the text, the survey of a whole meridian line.

EVEREST. Some other cause must exist tending to increase the upper astronomical amplitude by $13''\cdot 11-5''\cdot 24=7''\cdot 87$, and also to increase the lower amplitude by $3''\cdot 82+3''\cdot 79=7''\cdot 61$.

(6) It has been demonstrated that a slight but wide-spread variation in the density of the crust from that deduced from the fluid-theory, either in excess or defect, such as there is no difficulty in conceiving to exist, is sufficient to account for deflections such as these. For example, an excess of density amounting only to $\frac{1}{50}$th part, extending through a circuit of about 120 miles around the mid-point of the whole arc between Kaliana and Damargida (and therefore not far from Kalianpur), and to a depth of about 200 miles, will produce this effect, and make the calculated deflections from the three causes—the Mountains, the Ocean, and this Hidden Cause below—exactly accord with the observed errors in the astronomical amplitudes.

(7) The resulting total deflections at Kaliana, Kalianpur, and Damargida, arising from the three causes, are $26''\cdot 29$, $21''\cdot 05$, and $24''\cdot 84$: these make the two astronomical amplitudes, the one $5''\cdot 24$ smaller, and the other $3''\cdot 79$ larger than the geodetic amplitudes, the errors brought to light by Colonel EVEREST.

(8) No sensible error can arise in the relative situation of places determined by geodetic measurements, and arranged in a map. But the position assigned to the map itself on the surface of the mean spheroid will be affected by local attraction; viz. by the error at the station the latitude of which is observed in order to fix the map. This error may amount to as much as half a mile. Any station afterwards inserted in the map, from an observation of the sun, will be out of its place on the map, by the difference of the errors arising from local attraction at that station and at the principal station which fixes the position of the map on the spheroid. The calculation shows that this error may amount in some places to as much as one-tenth of a mile.