THE PRODUCTION OF ELLIPTIC INTERFERENCES IN RELATION TO INTERFEROMETRY

PART III

By CARL BARUS
Hazard Professor of Physics and Dean of the Graduate Department in Brown University

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PREFACE.

The accompanying report contains a variety of investigations made at widely different times, but in all of which the displacement interferometer was used as a basis of measurement. It is thus a continuation of the work in the Carnegie Publications No. 149, 1911 and 1912.

In Chapter XII, the distance apart of the doublet interference patterns, obtained with plates of doubly-refracting crystals, is used for computing the corresponding ordinary and extraordinary indices of refraction of these crystals, in the case of light traveling through them in different directions. As quartz gives particularly beautiful and well-placed images, measurements were made in greatest number by means of it, though other bodies were also studied. It is an essential feature of the method that the center of ellipses is never lost, but may be brought back into coincidence with the fiducial sodium line of the spectrum by the proportionate displacement of the mirror on the micrometer. Hence plates of glass or crystal of any thickness, or indeed columns of any length, are available for measurement. These wide limits of application are peculiarly favorable to certain accurate measurements. For instance, if a glass column 1 meter long is inserted, and the micrometer reads to but 0.5×10^{-4} centimeter, $\mu - 1$ (where μ is the index of refraction) would be measurable to one part in a million. The method should therefore be useful to answer refined questions in refraction.

Chapter XIII contains a number of miscellaneous measurements made with the displacement interferometer, or bearing upon it. Thus the ease with which it lends itself to the comparison of screws of any length is illustrated by many trials; a simple type of long screw micrometer is designed and investigated; a method for the measurement of very small increments of angle, consisting essentially of mounting two small mirrors in parallel and symmetrically to the axis, is tested; advantages in obtaining elliptic interferences when the reciprocating opaque mirrors are concave are discussed, etc. Furthermore, investigation is made of the availability of the interferometer in measuring certain acoustic displacements, such as occur in the Dvorak and Mayer experiments and telephone plates. It is then shown that by replacing the opaque mirrors of the interferometer by mirrors attached to the synchronized and reciprocating plates of modified telephones an induction balance of a peculiar kind may be constructed. If, for instance, one of these mirrors vibrates synchronously with the other towards and away from it, the ellipses remain in the field without displacement; if, however, the mirrors move in the same direction, the ellipses necessarily vanish. Hence, if there is a gradually increasing retardation in the electric circuit of one telephone only, the ellipses will alternately appear and vanish for identical intervals of retardation. The experiments attest the feasibility of so mounting the telephone mirrors on the interferometer that the balance may be realized. Finally, by the above method, an attempt is made to study double refraction resulting from dielectric polarization, in solids and in liquids, while in allied sections the prismatic spectrum is studied on a Rowland spectrometer and certain peculiar results are shown in the resolution of interference fringes resembling the elliptic type.

Chapter XIV treats briefly of the interferometry of highly exhausted air carrying electric current. The adjustment used is that originally proposed by Mach, in which the rays do not retrace their path, but move along the sides of a parallelogram. The long exhaustion tube carrying current is placed in one of these sides, the light passing from end to end, and the effect of presence and absence of current on the interference pattern is determined. Thus far the exhaustions available were insufficient, so that the experiment in its final developments will be resumed at some time in the future. It is worth an inquiry to ascertain whether the motion of the cathode rays in a given direction may not produce some preponderating modification of the properties of the ether in that direction.

Chapter XV deals with the refraction of air at high temperatures investigated by the displacement interferometer (provided with water circulation), by comparing the refraction of a vacuum and of a plenum of air at each given temperature. As the system is not very sensitive to pressure, very perfect exhaustion need not be reached, but the need of sealed apparatus makes the experiment difficult at high temperatures. This chapter is but a beginning of experiments of this kind, which are naturally very difficult. It is curious that the refraction of a Bunsen flame through which a beam of light passes symmetrically may be approximately found by this means.

Finally, in Chapter XVI the displacement interferometer is applied to the electrometer. A large number of forms of the instrument are designed and the results of each are tested in detail. All of these are comprehended by the closed cylindric electrometer, in which a closed cylindric charged needle is capable of displacement along the axis of a closed cylinder, consisting of two symmetrically insulated halves, oppositely charged. The suspension of the needle is of the pendulum type. If the ends of each cylinder are removed, the instrument reduces to the cylindrical pattern, in which the movable cylinder may be within or without the fixed cylinder. If the cylindrical mantle is removed, the instrument is of the disk form, the simplest case of which is the absolute electrometer. All of these instruments may, with less sensitiveness, be used idiostatically. None of them, however, are exceptionally sensitive, the disk form, which is most so, and at the same time most treacherous, being characterized by a displacement of about 10^{-3} centimeters per volt, in favorable cases.

Far greater sensitiveness is therefore secured by adapting the device of two small light parallel mirrors for the measurement of angle (as referred to above) to the needle of the quadrant electrometer. When this is properly done, an instrument capable of measuring 10 to 20 millionths of a volt per vanishing interference ring may be constructed, provided a satisfactory environment is at hand. The chapter contains a great variety of experiments with each of these forms of apparatus.

My thanks are due to Miss A. I. Burton for efficient assistance both in the observations and in the editorial work and drawing.

CARL BARUS.

BROWN UNIVERSITY, *February 1, 1914*.

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CHAPTER XII.

THE INDICES OF DOUBLY-REFRACTING CRYSTALS, DETERMINED BY DISPLACEMENT INTERFEROMETRY.

98. Introduction. Method.—In the following paper I shall make a further application* of the displacement interferometer, in endeavoring to investigate a method for the determination of the two refractions of a crystalline plate, with a degree of accuracy approaching the spectrometer and eventually to exceed it. The work is to be extended to all directions of the ordinary and extraordinary ray within the crystal. Inasmuch as the center of ellipses is never lost, but may always be brought back again to the fiducial and coincident sodium lines by moving the micrometer, however remote the center may be, a crystal (or indeed a column of any thickness) may be placed in one of the component beams and the micrometer equivalent of the difference resulting from presence and absence of the crystal may be ascertained. This makes the method quite sensitive; for the two micrometer readings for the presence of air and crystal, respectively, may differ by many centimeters, if desirable, and each reading may be taken to 0.00005 centimeter. If it were easily possible to determine the thickness of the crystal with the same degree of precision, the conditions would be enhanced; but this is not at once attainable, for ordinary plates, to more than 0.0001 centimeter, even apart from the errors of the two micrometer screws involved. The case of columns is more favorable.

Furthermore, the method in its direct applications requires an accurately plane parallel plate, however thick. If the faces are at even a slight angle, the beam passing through them will be deflected by the introduction of the crystal and will not be returned in its own path. This requires a readjustment and therefore a correction for the displacement of the mirror difficult to estimate without special appliances. It is to be accentuated, however, that the method for normal incidence admits of a plate of any thickness, *i.e.*, of a long column of glass with plane parallel end faces, so that this method of measuring the index of refraction must ultimately exceed in sensitiveness the method of the spectrometer, while even the importance of the readjustment in question becomes of less consequence. It will thus be available for answering refined questions relative to this index in glass, etc., under conditions where the spectrometer fails of application. It is in this direction that the results are to be prosecuted as soon as the preliminaries discussed in the present paper have been clarified.

It is also probable that if the deviation of the ray *mm* due to non-parallel faces in the crystal is corrected, not at the micrometer mirror *M*, but at

* The earlier papers on elliptic interferometry will be found in the Carnegie Publications, No. 149, 1911, Part I, and 1912, Part II.

the other opaque mirror N, beyond the grating, the discrepancy in question will vanish. For in this case, on restoring the interferences by rotating N, the air path of *both* component beams mm and nn is equally incremented.

The plate crystal X is mounted on a horizontal axle ab, normal to the interfering beam mm in question. The angle of incidence of the beam is changed by rotating the plate in definite steps and the amount of rotation read off on the attached fixed graduated circle C, with its plane parallel to the beam and its axis normal to it. Angles of 0°, 10°, 20°, 30°, 40°, and even 50° were usually available on either side of the normal position. The crystal, mounted in a cork plate, was at first conveniently held by a spring clip; but later* more elaborate means for mounting the plate parallel to the axis of rotation were adopted. The normal position is found with certainty by slightly rotating the crystal around the horizontal axis in either direction, until the interference rings in the field of view change their direction of motion. This is the minimum thickness e. The same operation is then performed around the vertical axis, both adjustments being usually easy and the result satisfactory.

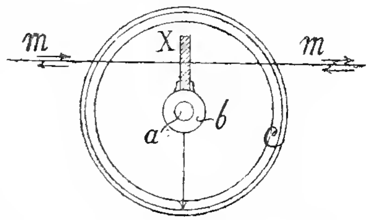


FIG. 65.—Apparatus for rotating crystals.

With the angle i given, the corresponding angle of refraction r, *i.e.*, the angle of the ray with the optic axis of the crystal cut normally to this axis, may be computed when the index of refraction for the given case is known. Hence the refraction in different directions r, both for the ordinary and extraordinary rays, follows, as far as about $r=30°$ from the optic axis, in the above apparatus. Plates are not usually large enough for a larger internal angle on the one hand, while the errors of measurement multiply on the other; but by selecting a plate cut parallel to the axis, the final 30° of the quadrant may be found. To obtain the intermediate 30° a plate cut obliquely to the axis or else a plate of large area would be necessary.

When the plate is cut parallel to the axis and the difference of the refractive indices (as in quartz) is not too large, the two interferometer ellipses, corresponding to the ordinary and extraordinary rays, are in the field at once, making an exceedingly beautiful design. Their distance apart may then be found micrometrically in terms of the refraction of the ordinary ray, giving an acceptable datum even in case of an imperfect, *i.e.*, slightly wedge-shaped, plate. In other words, the ordinary and extraordinary rays encounter the same conditions and their difference of path remains practically unimpaired.

To distinguish between the two families of intersecting ellipses it is necessary to polarize the incident beam in the vertical and horizontal

* It will be shown below that the eccentric mounting of the crystal, as in fig. 65, is not, as a rule, advisable.

planes respectively, seeing that the crystal is revolvable about the horizontal or vertical axis. When the plate is true and the mounting trustworthy, there need never be difficulties due to elliptical polarization, for the vertical and horizontal vibrations pass unchanged. If, however, rotary polarization occurs, the case of the oblique path is more complicated, but it does not interfere with the measurement.

The present method inherently presupposes a knowledge of the dispersion of the crystal; but as this term is of the nature of a correction, the coefficient in the simplified Cauchy equation of two terms may at the outset be inserted. For accurate work Cauchy's equation as far as the fourth power of wave-length must be used. It will be shown, moreover, that the method is self-contained, by affording means for determining the coefficients of the equation in question. Initially, no more than a measurement of the refraction of the extraordinary ray in terms of the ordinary ray will be attempted. The dispersion constant found for these rays from data for two Fraunhofer lines may then be used for all wave-lengths lying between.

99. Equations.—If e', I, R, μ', denote the thickness, the angles of incidence and refraction, and the index of refraction of the grating, and e, i, r, μ, the corresponding quantities for the plane parallel plate inserted in one of the component beams of the interferometer; if, furthermore, b' and b denote the corresponding coefficients in the simplified dispersion equation of Cauchy, $\mu = a + b/\lambda^2$, so that

$$-\lambda d\mu/d\lambda = 2b/\lambda^2$$

the coördinates of the micrometer in the absence of the plate will be, if μ_a is the absolute index of refraction of air,

$$N' = e'\mu'\left(\cos R + \frac{2b'}{\lambda^2\mu' \cos R}\right) + e\mu_a \qquad (1)$$

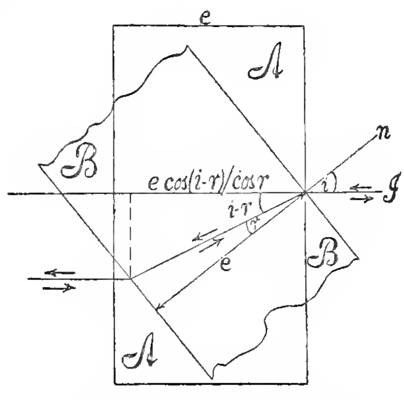


FIG. 66.—Diagram showing path increments.

where $e\mu_a$ is the air path coextensive with the thickness of the plate in its normal position AA, fig. 66, I being the incident beam of light. If the plate is inserted at an angle of incidence i, in the position given by BB, and the center of ellipses is then returned to the same fiducial line D of the spectrum, by moving the opaque mirror of the micrometer over a distance ΔN, nothing has been changed at the grating and the first term of the right-hand member of equation (1) remains unchanged. It therefore vanishes so far as ΔN is concerned. Hence we may write

$$\mu_a(e + \Delta N) = \frac{e}{\cos r}\left(\mu + \frac{2b}{\lambda^2}\right) + e\mu_a\left(1 - \frac{\cos(i-r)}{\cos r}\right) \qquad (2)$$

as fig. 66 shows. For not only has the path e been increased by the introduction of the oblique plate, but the air-path is also longer. Hence, on removing $e\mu_a$,

$$\mu=\mu_a\left(\frac{\Delta N \cos r}{e}+\cos(i-r)\right)-\frac{2b}{\lambda^2} \tag{3}$$

In this equation the quantities μ, r, refer to the same fiducial spectrum line of wave-length λ, to which the center of ellipses is always to be restored by moving the micrometer. Whenever the incidence at the plate is normal, $i=r=0$,

$$\mu=\mu_a\left(\frac{\Delta N}{e}+1\right)-\frac{2b}{\lambda^2} \tag{4}$$

an equation more generally useful and particularly so in case of long columns.

If the dispersion of air can not be disregarded, μ_a must be replaced by $\mu_a(1+2b''/\lambda^2\mu_a)$, μ_a and b'' referring to a plenum of air. Thus if all quantities are to be considered, the equation is

$$\mu=\mu_a\left(\frac{\Delta N \cos r}{e}+\cos(i-r)\right)-\frac{2}{\lambda^2}(b-b'') \tag{5}$$

The value of b for the crystals is usually of the order of 4×10^{-11}, whereas the value for air is of the order of 1.6×10^{-14}, so that the dispersive effect of air is usually negligible, being not more than 0.00001. Again, the value of ΔN for a plate about 0.6 cm. in thickness is usually of the order of 0.3 to 0.6 cm. and a single reading should be correct to 0.00005 cm. The error should not be larger, therefore, than a few parts in 10,000, so that the minimum error of μ should not exceed 5 units in the fourth place.

Finally, the last term $2b/\lambda^2$ is of the order of 0.02 and if it is determinable by the method of §100 to about 1 per cent, this error also can only affect the fourth place. There remains the case of cos r, which in the most unfavorable case of large incidence can not be wrong by more than 0.4 per cent per degree misread, and the effect would even then be but 2 units in the third place of μ. Whenever $i=r=0$, *i.e.*, for normal incidence, the values of μ are acceptable at once. Table 69, for instance, gives data under varying conditions.

TABLE 69.—True plate of quartz cut normal to the axis. $e=0.5996$ cm. $i=0°$ (normal incidence). $\mu_0=\mu_a+\Delta N/e-2b/\lambda^2$. D line fiducial. Sunlight. $10^{10}\times b=0.421$

Method.	$\Delta N\times 10^4$	μ_0
Short collimator. Ellipses centered	0.34025	1.54350
No collimator. Sunlight and slit: Ellipses not centered	.34000	1.54309
Ellipses centered but distorted	.34020	1.54342
Ellipses centered	.34040	1.54376
Weak objective as collimator: Ellipses less distorted	.34015	1.54334
With large telescope	.34050	1.54392
Short collimator. Sharp ellipses	.34040	1.54376
Concave lenses added to collimator	.34060	1.54408

In this table the absolute value of μ depends upon the value of e (which was calipered), on the relation of the two screws of the micrometer and of the caliper, on the alignment of the micrometer screw which is difficult to adjust, on the value accepted for the dispersion constant b, etc. The fact that it is here appreciably different from the normal value, 1.54423, has therefore no significance. The values of ΔN are the chief exhibit. They possibly show some difference with the obliquity (convergence and divergence in the vertical plane) of the incident rays of light. It is probable, however, that the slight difference of values of ΔN found is due simply to the distortion of ellipses in the different cases. In some of these it was hard to define the center. One may argue that in case of a clear center the obliquity of rays is without influence on the result.

The beam, in case of sunlight, is sheet-like, with its long dimension vertical. Much of it is apt to be lost on successive reflection. Results obtained with the arc lamp and a collimator objective which places the focus relatively to the vertical plane on the opaque mirrors M and N furnishes a rapier-like beam, which may easily be made to penetrate tubes and columns. It is thus far preferable and the results better. The sodium line is invariably present as a line of reference for measurement.

Since $\cos r = \sqrt{\mu^2 - \sin^2 i}/\mu$, equation (5) is essentially of the fourth degree and can be solved only by approximation. If $\Delta N/e = x$ and $\sin i = y$ the equation reads, when b'' is neglected,

$$\mu - \mu_a \cos (i-r) = x\mu_a\sqrt{1 - y^2/\mu^2 - 2b/\lambda^2} \qquad (6)$$

The approximation is therefore rapid, since $\mu - 1$ (nearly) is computed successively in terms of μ, which is always relatively much more accurate than $\mu - 1$.

100. Dispersion constants.—In the equation in the normal form (4) for two wave-lengths λ and λ', b may be eliminated or determined; for if $y=0$ (normal incidence, detected with certainty by the inversion in the motion of the rings, as described above) and $\mu = A + b/\lambda^2$ is assumed for a sufficiently narrow range of the spectrum near the D line,

$$A = \mu_a(1 + x\lambda^2 - x_1\lambda_1^2)/(\lambda^2 - \lambda_1^2) \qquad 3b = \mu_a\lambda^2(x - x_1)/(1 - (\lambda/\lambda_1)^2) \qquad (7)$$

If the value of x is restored, since for the two wave-length s$\Delta N = N' - N$ and $\Delta N_1 = N_1' - N_1$, where N' refers to glass and N to air,

$$e(x_1 - x) = N_1' - N_1 - (N' - N) = N_1' - N' - (N_1 - N)$$

or briefly

$$e(x_1 - x) = \delta N' - \delta N$$

whence

$$b = \frac{\mu_a\lambda^2}{3e} \quad \frac{\delta N' - \delta N}{1 - (\lambda/\lambda_1)^2} \qquad (8)$$

Here $\delta N'$ and δN are the differences of micrometer readings for the two spectrum lines λ and λ', for glass in place and for glass removed, respectively. ΔN_1 and ΔN need not therefore be known, *i.e.*, the glass plate need not be scrupulously plane parallel, as it is merely necessary to get the glass difference and the air difference for two successive lines, however far apart the air and glass positions may be for either line. Equation (8) also follows at once from (3) by inserting the equivalent of μ for the two spectrum lines. Table 70 gives a series of data obtained in this way.

TABLE 70.—Data for the dispersion constant b in $\mu = a + b/\lambda^2$ for a series of media.

Medium.	e	Line.	$10^3 \times \Delta N'$	$10^3 \times \Delta N$	$\lambda \times 10^6$	$\lambda' \times 10^6$	$b \times 10^{11}$
Common plate	0.7641	$D-b_1$	16.55	7.80	58.93	51.84	4.53
Do......	.7641	$D-b_1$	....	....	58.93	51.84	4.60
		$E-F$	....	....	52.70	48.61	5.46
Flint glass....	.6720	$D-b_1$	24.05	7.90	58.93	51.84	9.52
		$E-F$	18.97	6.70	52.70	48.61	9.64
Crown glass..	.6946	$D-b_1$	15.95	7.90	58.93	51.84	4.70
		$E-F$	12.35	6.70	52.70	48.61	4.26

Granting the approximate form of Cauchy's equation, the error of b thus obtained should be well within 1 per cent. Its influence on μ should not exceed one unit in the fourth place. Unfortunately, however, the equation in question is not warranted to this extent, when the lines of the spectrum are as far apart as D and E, for instance. The question relative to b will be resumed in §107. Meanwhile I may observe that the equation with three constants, $\mu = A + b/\lambda^2 + c/\lambda^4$, leads to

$$3b\left(\frac{1}{\lambda_1^2} - \frac{1}{\lambda_2^2}\right) + 5c\left(\frac{1}{\lambda_1^4} - \frac{1}{\lambda_2^4}\right) = \mu_a \frac{\Delta N_1 - \Delta N_2}{e} \qquad (8')$$

and would need three spectrum lines for the determination of b and c. This is a cumbersome procedure, but from the nature of the problem is nevertheless probably the only resort, particularly in the case where a long column of glass, crystal, or liquid is to be investigated, for exceptionally accurate indices of refraction.

In conclusion, it is worth while to determine the value of μ_e in terms of μ_0 found from the normal position, when the difference of these indices (as in quartz) is not large. In fact, since $d(\Delta N) = e dx$,

$$\frac{d\mu}{d(\Delta N)} = \frac{\mu \cos^2 r}{e\mu \cos r/\mu_a - \Delta N \sin^2 r + e \sin r \sin (i-r)} \qquad (9)$$

Equation (9) may be reduced, with questionable advantage, by replacing μ in the denominator by its value from (3) to

$$\frac{d\mu}{d(\Delta N)} = \frac{\mu \cos^2 r}{(e+\Delta N) \cos 2r - 2eb \cos r/\mu_a \lambda^2}$$

If $i = r = 0$, this reduces to

$$\frac{d\mu}{d(\Delta N)} = \frac{\mu_a}{e} \qquad (10)$$

If in equation (9) the value of μ_0 is introduced, the corresponding value of μ_e may be computed from the ΔN and $\Delta N'$ which put the two ellipses, respectively, on the fiducial sodium line. The result gives a fair value for μ_e, even when μ_0 varies with the incidence i due to errors of adjustment, seeing that μ_0 must be constant and is found from the normal position. See equation (11).

101. Oblique incidence.—In the first experiments no distinction was made between the ordinary and extraordinary rays, and these data may therefore be waived here.

In table 71 the object used was a plate cut normal to the axis, $e = 0.6100$ cm. thick, but not sufficiently plane parallel. In other words, it required angular adjustment of the opaque mirror when the plate was inserted and again when it was removed. This introduces an indeterminable error, as plane parallelism is a prime condition for the accurate application of the present method. The incident light was polarized in the vertical plane, before reaching the slit, the D line being as usual fiducial. When viewed through an analyzer, the light reaching the telescope in case of the more oblique positions of the plate on its horizontal axis was elliptically polarized in one of the superimposed spectra, the latter showing the usual channeled structure with more dark bands as the effective thickness increased. With obliquity decreasing through zero, moreover, the black bands between crossed Nicols moved in opposite directions when the normal position was passed. For the polarizer at 45° both families of ellipses were in the field at once. Either one or the other set could be excluded by rotating the analyzer. In the absence of this, the field showed the ordinary or the extraordinary set of ellipses separately, on rotating the polarizer 90°. In the further absence of the polarizer, one frequently observes dislocated rings, the dislocation being in multiple along equidistant vertical planes. In such loci the ordinary and extraordinary rings, therefore, successively passed into each other. In general, a figure which is blurred without polarizer is resolved into two clean figures by the aid of it, one being definitely displaced with reference to the other.

TABLE 71.—Double refraction of normal quartz. Rays not parallel in vertical plane. Opaque mirrors silvered on back. Plate thick, oblong, and slightly wedge-shaped. $e = 0.6100$ cm. Horizontal axis of rotation for i. $\mu_0 = 1.54423$; $\mu_e = 1.55338$; $b \times 10^{10} = 0.421$.

i	r	ΔN_0	Observed μ_0	ΔN_e	Observed μ_e	$10^5 \times \delta\mu$	Computed $10^5 \times \delta\mu$	Corrected $\mu_e = \mu_o + \delta\mu$
0°	0°	0.34855	1.54759	0.34855	1.54759	0	0	1.54423
10	6.47	.35140	1.54671	.35158	1.54684	13	29	1.54436
20	12.80	.36103	1.54545	.36130	1.54588	43	43	1.54466
30	18.88	.37868	1.54485	.37935	1.54588	103	104	1.54526
40	24.60	.40300	1.54100	.40405	1.54258	158	157	1.54581
50	29.73	.43833	1.53820	.43978	1.54027	207	207	1.54630

Seeing that the plate was not quite plane parallel and that in addition to displacement ΔN, a slight rotation of the micrometer mirror was needed to restore the ellipses on inserting the plate (equivalent to an appreciable increment of ΔN), the large value of μ for the normal position is to be anticipated. I have not thus far tested the way to correct this discrepancy.* One would have to evaluate the rotation of mirror horizontal or vertical in terms of the displacement ΔN or else make the compensation instrumentally.

The second feature is the very rapid increase of ΔN and therefore of μ with the angle of incidence i. The reason for this has yet to be succinctly stated, but it is due to the slightly wedge-shaped plate. When this is rotated on an eccentric axis, as in fig. 65, even if the axis is parallel to the plate, different parts of the crystal, of successively increasing thickness, are penetrated by the beam of light. Nevertheless both μ_0 and μ_e may be found, since the former is constant and would be given for a plate, while the latter is determinable from equation (9). Hence

$$\mu_e = \mu_0 + \mu \cos^2 r (\Delta N_e - \Delta N_0)/(e\mu \cos r - \Delta N \sin^2 r + e \sin r \sin (i - r)) \quad (11)$$

Again μ_0 and μ_e may be computed as in the table and μ_e found by adding the difference for a given i to the constant value of μ_0. Values of $\delta\mu$ found by both of these methods are frequently instanced in the tables, the one found by equation (11) being called computed $\delta\mu$.

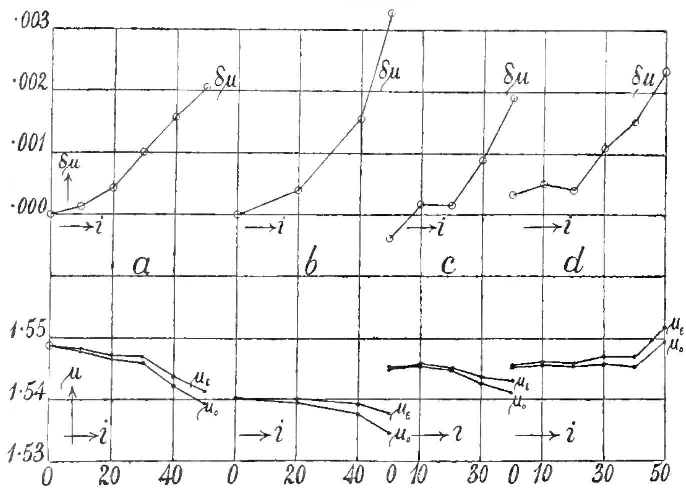


FIG. 67.—Observed indices of refraction and variation at different incidences.

The difference is slight and due to the fact that the direct computation of μ_e, as compared with the method by way of equation (9), has not been carried to a sufficient degree of successive approximation.

If in table 71, μ_0, μ_e, $\delta\mu = \mu_e - \mu_0$, be constructed as depending upon r, the angle between the axis and the ordinary ray (or more directly upon i,

* It is probable that if the adjustment for non-parallel faces is made, not at the micrometer mirror M, but at the distant mirror N, no appreciable discrepancy will be introduced. But the distant mirror is not easily accessible at the position of the observer.

the angle of incidence), the data appear as in fig. 67, *a*. The irregularity in μ at 30° is due to some error in the adjustment; nevertheless the values of $\delta\mu$ are not appreciably influenced by it. The results are naturally less trustworthy as i is larger.

It will be seen that the observations for $\delta\mu$ as such make a satisfactorily smooth series, even beyond the fourth place of decimals. If v refers to the velocity of light in the crystal, and o and e refer to ordinary and extraordinary rays,

$$v_0/v_e = 1 + \delta\mu/\mu_0$$

The observed order of change is apparently sufficient. As a whole and for large r the interpretation is complicated because of the occurrence of rotary polarization along the axis.

102. Oblique incidence. Round quartz cut normal to the axis.—This plate was 0.3100 cm. thick and cut approximately normal to the axis. Like the preceding plate it was slightly wedge-shaped. It therefore required rotation of the micrometer mirror, as well as the displacement ΔN on being inserted into the beam of light, in order to bring the ellipses back to the fiducial line. Table 72 shows the results on the same plan as the preceding. The second series is reproduced in fig. 67, *b*.

TABLE 72.—Thin quartz plate, round, cut normal to the axis. $e = 0.3100$ cm. Horizontal axis of rotation. $\mu_0 = 1.54423$; $b \times 10^{10} = 0.421$.

I.				II (later).		
i	r	ΔN_0	Obs. μ_0	i	r	ΔN_0
0°	0°	0.17280	1.53360	0°	0°	0.17450
10	6.47	.17470	1.53427	20	21.80	.18128
20	12.80	.17995	1.53436	40	24.60	.20331
30	18.88	.18855	1.53289	50	29.73	.22101
40	24.60	.20210	1.53307			

II (later).					
Obs. μ_0	ΔN_e	Obs. μ_e	$10^5 \times \delta\mu$	Comp. $10^5 \times \delta\mu$	Corr. $\mu_e = \mu_0 + \delta\mu$
1.53910	0.17450	1.53910	0	0	1.54423
1.53855	.18141	1.53895	40	41	1.54463
1.53662	.20385	1.53819	157	159	1.54580
1.53330	.22218	1.53657	327	329	1.54750

This plate was at first supposed to be too thin to admit of the determination of the difference of μ_0 and μ_e, satisfactorily; in fact, the ordinary ray in the first series shows the usual increase of index with i, regarding which the remarks of the preceding paragraph apply. However, the second series of data, found at a later date for both ordinary and extraordinary rays, is in good agreement with the preceding table, particularly in relation to $\delta\mu$. The data are smooth and consistent, even as far as $i=50°$, which is not the case in table 71. This is probably owing to the large surface of the present plate.

103. Oblique incidence. Quartz rouge, cut normal to the axis, rhomboidal.—This is the only quartz plate found to be adequately plane parallel to admit of both of the measurements for ΔN (presence and absence of plate), without readjusting the mirrors by rotation. It would thus have been possible to obtain the true ΔN without appreciable error, but unfortunately the surface was too small for much inclination. The data are given in table 73.

TABLE 73.—Thick (rouge) quartz, lozenge-shaped, cut normal to the axis. $e=0.5996$ cm. Plane parallel plate. Rays not parallel in vertical plane. Opaque mirrors silvered on back. $\mu_0=1.54423$; $b\times 10^{10}=0.421$.

	i	r	ΔN_o	Obs. μ_0	ΔN_e	Obs. μ_e	$10^5\times\delta\mu$	Comp. $10^5\times\delta\mu$	Corr. $\mu_e=\mu_0+\delta\mu$
I. Horizontal axis of rotation.	0°	0°	0.34059	1.54422	0.34037	1.54385	−37	−37	1.54386
	10	6.47	·34395	1.54431	·34407	1.54448	+18	+20	1.54441
	20	12.80	·35379	1.54369	·35389	1.54385	16	16	1.54439
	30	18.88	·37011	1.54150	·37067	1.54239	89	88	1.54512
	40	24.60	·39543	1.53993	·39679	1.54183	190	207	1.54613
III. Vertical axis of rotation.	0°	0°	0.34049	1.54406	0.34069	1.54439	33	33	1.54456
	10	6.47	·34404	1.54445	·34434	1.54495	50	50	1.54473
	20	12.80	·35414	1.54426	·35439	1.54466	40	41	1.54463
	30	18.88	·37211	1.54465	·37280	1.54575	110	109	1.54533
	40	24.60	·39825	1.54422	·39925	1.54575	153	152	1.54576
	50	29.73	·43781	1.54827	·43944	1.55063	236	237	1.54659
IV. Vertical axis of rotation.	0°	0°	0.34065	1.54433	0.34065	1.54433	0	....	1.54423
	10	6.47	·34388	1.54419	·34388	1.54419	0	....	1.54423
	20	12.80	·35371	1.54356	·35407	1.54414	58	....	1.54481
	30	18.88	·37153	1.54375	·37217	1.54475	100	....	1.54523
	40	24.60	·39750	1.54309	·39818	1.54412	103	....	1.54526
V. Vertical axis of rotation.	0°	0°	0.34048	1.54403	0.34048	1.54403	0	....	1.54423
	30	18.88	·37198	1.54445	·37248	1.54525	80	....	1.54503
	50	29.73	·43423	1.54308	·43583	1.54540	232	....	1.54655
VI. Vertical axis of rotation. Sunlight.	0°	0°	0.34040	1.54390	0.34040	1.54390	0	....	1.54423
	20	12.80	·35400	1.54402	·35400	1.54402	0	....	1.54423
	30	18.88	·37153	1.54375	·37208	1.54462	87	....	1.54510
	40	24.60	·39770	1.54339	·39870	1.54490	151	....	1.54574
VII. Vertical axis of rotation. Electric arc.	0°	0°	0.34040	1.54390	0.34040	1.54390	0	....	1.54423
	20	12.80	·35360	1.54338	·35394	1.54393	55	....	1.54478
	30	18.88	·37109	1.54305	·37183	1.54422	117	....	1.54540
	40	24.60	·39723	1.54268	·39823	1.54418	150	....	1.54573
VIII. Biquartz. Line of separation horizontal. Vertical axis of rotation. $e=0.748$, Small face.	0°	0°	0.42503	1.54442	0.42503	1.54442	0	....	1.54423
	20	12.80	·44233	1.54497	·44248	1.54515	18	....	1.54441
	30	18.88	·46425	1.54472	·46468	1.54525	53	....	1.54476

A variety of normal values was adduced in table 69. The earlier results of table 73 are constructed in fig. 67 under *c*. In case of the third series (fig. 67, *d*, and fig. 69), fourth series (fig. 68, *a*, and fig. 69), and fifth series (fig. 68, *b*, and fig. 69), the apparatus used had been totally changed. The quartz plate was now rotated around a vertical axis and a more accurate circle was installed for the measurement of the angle of incidence, *i*. Furthermore, the endeavor was made to mount the crystal parallel to the axis of rotation, by providing the axle with a fine axial perforation. A straight, snugly fitting steel wire could be sunk into this, against which the quartz plate was to be pressed, before fastening it. The wire was then removed. The position normal to the ray of light was found as before, by horizontal and vertical rotation, until the motion of ellipses changed sign. It would

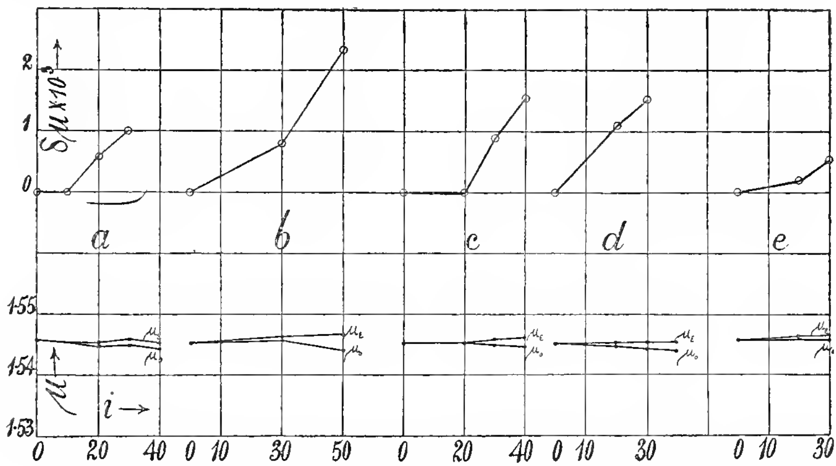


FIG. 68.—Observed indices of refraction and variation at different incidences.

have been a further improvement to mount the crystal so that the axis, instead of passing through one face, would pass symmetrically between the faces. The spot of light would not then move along the face appreciably. The data as a whole are of the same order of value and show the same character as the preceding results, obtained with a horizontal axis of rotation. No effect is therefore to be ascribed to the adjustment, though from a theoretical point of view one might be inclined to look for a difference. Practically the vertical axis, being parallel to the slit, is in some respects advantageous, as less surface of glass is required. The circle for measuring *i* was accurately graduated.

In Series III the change of μ as far as $i=40°$ is but one unit in the fourth place; beyond this, for $i=50°$, light passed partially outside the crystal and the data are approximate. Series IV and Series V show similar behavior, even as far as $i=50°$. In fact Series III to V, both as to μ_0 and μ_e, are a marked improvement over Series I and II, in so far as the indices themselves are concerned. It is curious that the corresponding values of $\delta\mu$, obtained by difference, did not come out as smoothly as in the earlier tables, though the order of value is throughout the same. Probably slight dislocations of the parts of the interferometer itself in an agitated laboratory are the cause of these discrepancies.

Final measurements, Series VI and VII of table 73, were made with the apparatus further modified, as stated in §106. In particular the mirrors used were silvered in front, so as to obviate discrepancies (though such an effect is here probably negligible, since the spot of light does not move on the plane of the mirror) from the two wedges of glass which lie in front of

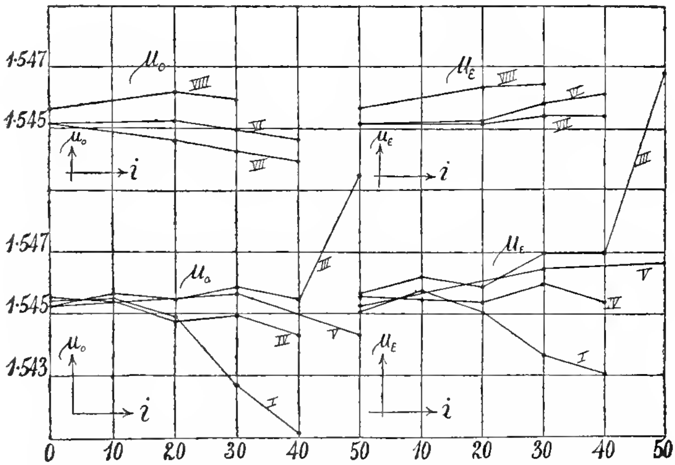


FIG. 69.—Observed indices of refraction and variation at different incidences.

the silver surface in the former case. The axis for varying i was vertical. A contrast is made between sunlight (using no collimator objective) and the electric arc.

The new results for μ are smoother, but in general not much of an improvement on preceding work. They represent the average limits of the method and show that the mounting of the crystal parallel to the axis, preferably with the latter passing symmetrically between the faces, finally becomes the chief obstacle to further development. The direction of this axis is immaterial. The results are given in fig. 68, c and d, and fig. 69.

Finally, certain measurements, table 73, VIII, fig. 68, e, made with a biquartz "rouge," may be inserted here. The right and left turning crystals were identically thick and respectively above and below the horizontal plane corresponding to the center of ellipses. The crystal was too small for satisfactory work, but no difference could be detected in the ellipses corresponding to right-handed and left-handed rotation, the biquartz behaving like a single homogeneous crystal.

104. Oblique incidence. Quartz cut parallel to the axis.—Table 74 and fig. 70, *a*, *b*, *c*, give the results for the parallel quartz. The index of refraction should, therefore, in the normal position, $i=0°$, show maximum difference, $\delta\mu$. The only available plate was not quite plane parallel and rotational adjustment was required between the two readings for ΔN. This again throws the absolute values out. The mean thickness of the plate was $e=0.4918$ cm. The surface was too small for much rotation.

TABLE 74.—Rectangular quartz plate parallel to the axis. $e=0.4918$ cm. Rays not parallel in vertical plane. Opaque mirrors silvered on back. Horizontal axis of rotation. $\mu_0=1.54423$; $\mu_e=1.55338$; $b \times 10^{10}=0.421$.

i	r	ΔN_o	Obs. μ_o	ΔN_e
0°	0°	0.27919	1.54388	0.28431
10	6.47	.28165	1.54336	.28653
20	12.80	.28930	1.54193	.29395
30	18.88	.30315	1.54069	.30785
35	21.80	.31260	1.53991	.31730

The differences $\delta\mu$ between the corrected ordinary and extraordinary index are somewhat in excess of the true value, the observed ordinary index being about 4 units in the fourth place too large and the observed extraordinary 6 units too small. Apart from these absolute differences, however, the values of $\delta\mu$ make a smooth curve to the fifth place of decimals,

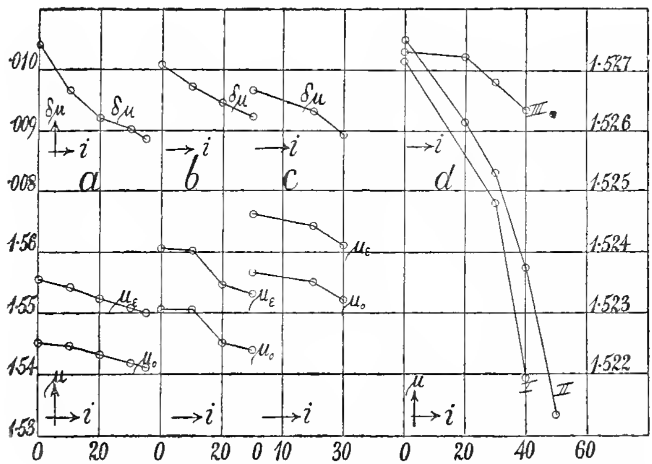


FIG. 70.—Observed indices of refraction and variation at different incidences.

as shown in fig. 70, *a*, and $\delta\mu$ decreases with the obliquity, *i.e.*, from the maximum difference of μ_0 and μ_e, as it should. Unfortunately the need of readjustment for air and the quartz medium leaves an uncertain element in these data.

The work was repeated shortly after this with the results given in table 75 and fig. 70, *b*.

TABLE 75.—Quartz plate parallel to the axis. Conditions as in table 74. $\mu_0=1.54423$; $\mu_e=1.55338$; $b\times10^{10}=0.421$.

i	r	ΔN_0	Obs. μ_0	ΔN_e
0°	0°	0.28193	1.54946	0.28690
*10	6.47	.28468	1.54949	.28960
20	12.80	.29038	1.54408	.29516
30	18.88	.30425	1.54280	.30904

*Readjusted.

This series is poor in relation to μ_0 and μ_e, as it was necessary to readjust the mirror twice. The values of $\delta\mu$, however, are nevertheless in accord with the data of the preceding table.

In the following experiments, table 76 and fig. 70, *c*, with the same plate of quartz, the lens of the collimator was removed and sunlight thrown directly on the slit. Light rays parallel in the vertical plane thus pass through the plate; but the ellipses in this case were neither sharp nor easily identified.

TABLE 76.—Quartz plate parallel to the axis. Sunlight and slit without collimator lens. $\mu_0=1.54423$; $\mu_e=1.55338$; $b\times10^{10}=0.421$.

i	r	ΔN_o	Obs. μ_e	ΔN_e	Obs. μ_e
0°	0°	0.28490	1.55549	0.28965	1.56515
20	12.80	.29535	1.55393	.30005	1.56325
30	18.88	.30855	1.55109	.31320	1.56002

In view of the difficulty of observation, these results are not easily interpreted in so far as the corrected absolute values are concerned. The data for $\mu_e-\mu_0$ are too low as compared with the preceding series, but they come nearest to the true value, while the individual indices of refraction μ_0 and μ_e are much too high. As a whole, the results for the plate cut parallel to the axis have failed to turn out well, owing to the fact that this plate was appreciably wedge-shaped.

105. Data for crown and for flint glass.—To exclude the peculiarities due to double refraction a few experiments were made with rectangular plates of crown and of flint glass. Neither of them was adequately plane parallel and separate adjustment (rotation of the micrometer mirror) for the air and for the plate medium was necessary.

To determine the dispersion constant, it is necessary to use the method of equation (8), §100, since the total reflectometer, in case of flint glass at least, is quite unavailable. The results, which were also discussed in §100, were crown glass, $10^{10}b=0.470$; flint glass, $10^{10}b=0.952$.

The results of table 77, like those for the ordinary ray in the preceding table, show the same variation of μ with r, the cause of which has been ascribed to the appreciably wedge-shaped glass plates. Furthermore, a dispersion equation with three constants would have been preferable, if not necessary.

TABLE 77.—Plates of crown glass, $e=0.6946$ cm., and of flint glass, $e=0.6720$ cm. $b \times 10^{10}$ (crown glass) $=0.4698$; $b \times 10^{10}$ (flint glass) $=0.952$.

Crown glass			Flint glass	
i	ΔN	μ	ΔN	μ
0°	0.38876	1.53307	0.42965	1.58501
10	.39255	1.53306	.43355	1.58482
20	.40366	1.53221	.44672	1.58600
30	.42328	1.53125	.46880	1.58698
40	.45384	1.53157	.50103	1.58764

106. Apparatus remodeled.—The change in the direction of the axes of rotation for i, from the horizontal to the vertical, seems to be without effect, as has already been tested in table 73. Hence the vertical axis is to be preferred throughout the measurements. Being parallel to the slit, it requires a smaller face of crystal for a corresponding variation of i and none of the rays need be cut off on rotation. In any case, the axis should pass as nearly as possible through the line of symmetry of the crystal, *i.e.*, midway between the faces, to avoid the wedge-effect. The plate is made parallel to the axis by the device of the perforated axle and normal to the beam by aid of the motion of the interference rings for two planes of rotation. Errors introduced by inequalities of the screws for the measurement of ΔN and e will here be disregarded, as they are not significant and their effect may be corrected at the end of the work. (See Chapter XIII, p. 192.)

The first improvement consisted in providing all the mirrors and plates with a firmer spring, so that a tremor of the table might effect no permanent disadjustment, however small. Flexure of the table due to a weight must be scrupulously avoided during the measurements. The feet of the micrometer should be provided with vertical and horizontal (parallel to the mirror) adjustment screws, so that the final placing of the mirror may be made elastically, straining the arm which holds the micrometer to a very small extent. This secures much finer and easier adjustment than is possible by actuating the three leveling screws of the mirror. The ellipses may now be placed with certainty in the center of the field and they appear with maximum sharpness. Moreover, if the support of the micrometer is under strain, it is clutched more securely, so that the micrometer screw may be manipulated with the hand with less tremor to the image.

The opaque mirrors, which in the above experiments were silvered on the rear, were replaced by a similar set silvered on the front face. In this way the possibility of a sharp edge of glass at either mirror is avoided, care being taken to obtain a specially even coat of silver. Finally, tests were made with parallel rays of light, the objective of the collimator being removed and the slit used alone either with sunlight or a distant arc light. The rays are then parallel in their projection on a vertical plane, but not in relation to a horizontal plane, because of diffraction of the slit.

Results so obtained are given in table 78 and fig. 70, *d*. The measurements were made with a selected piece of crown glass cut from a large sheet of common plate, which had been tested throughout its length and a suitable part taken. For this purpose it is merely necessary to examine the white undeviated slit images. If they are not displaced appreciably in any direction by the introduction of the plate into one of the component beams, the part of the plate used will probably be serviceable. The final test may be made with the ellipses themselves, which must be clear for both positions of the micrometer N and N_0, in the presence and absence of the plate.

TABLE 78.—Selected plate glass. $e=0.7641$ cm. $b\times10^{10}=0.453$; $2b/\lambda^2=0.2609$.

	I. Sunlight. Horizontal axis.			II. Electric arc. Horizontal axis.			III. Electric arc. Vertical axis.		
i	r	ΔN	μ	r	ΔN	μ	r	ΔN	μ
0°	0°	0.42238	1.52713	0°	0.42265	1.52749	0°	0.42250	1.52729
20	....	....	....	12.80	.43855	1.52614	12.80	.43938	1.52720
30	18.88	.45965	1.52481	18.18	.46006	1.52530	18.88	.46125	1.52679
40	24.60	.49033	1.52194	24.60	.49184	1.52374	24.60	.49403	1.52634
50	....	....	....	29.73	.53586	1.52134	....	....	....

In table 78 the first series was obtained, as already intimated, with parallel rays of sunlight, the objective of the collimator being removed. The ellipses in this case are liable to be small and cramped, so that this method (without lenses) is otherwise much inferior to that of the series in which the arc lamp was used in the usual way. The axis of rotation for varying i for the plate was eccentrically horizontal, even here an undesirable adjustment.

The values of μ obtained in the first series with sunlight are definitely below the second with electric light. They agree more nearly as i approaches zero, so that the effect is probably not due to obliquity of the rays, but to the eccentric axis. At $i=0°$ and as far as 30° the indices of refraction are alike to four units in the fourth place, which would be satisfactory.

In the belief that the marked variation of μ with i in Series I and II is referable to the eccentric axis of rotation, even in nearly plane parallel plates, the experiments were repeated with a carefully constructed vertical axis adjustable parallel to the faces. The graduated circle, moreover, was much finer, showing the values of i to tenths of a degree.

The new facilities in adjustments for rotation to secure different angles of incidence, taken as usual on both sides of $i=0°$, the mean value being used, have so much improved the values of μ (see fig. 70, *d*), that the suspicion aroused against the eccentric horizontal axis of rotation is verified. In fact, in the latter case different parts of the plate are successively put into the beam obliquely when i changes. Any variation in thickness of the plate thus enters the data as a systematic discrepancy. In Series III, the values of μ at $i=0°$ and $i=20°$ differ by less than one unit of the fourth place, which is an altogether satisfactory result. Beyond 20° the error increases rapidly, as is to be expected.

It thus appears, in conclusion, that for very long columns of glass the values of the index of refraction should be obtainable with any refined degree of accuracy desirable, as suggested in §98.

107. Results for different spectrum lines.—In §100 the method for computing the approximate dispersion constant b was indicated. There is, however, a difficulty inherent in this method, inasmuch as for the case where μ is to be found for four or more places of decimals the abbreviated Cauchy equation is insufficient. It is necessary, in other words, to use the more extended form $\mu=A+\frac{b}{\lambda^2}+\frac{c}{\lambda^4}$.

Leaving this for the present, it is interesting to compare the results obtainable by the interferometer with the standard data of Mascart, de Lépinay, and others.* For this purpose the quartz of table 73 was mounted at normal incidence and the micrometer readings for the C, D, E, b, F lines of the spectrum taken in succession, both for the presence and absence (air) of the quartz plate. The differences of corresponding values (quartz and air), δN for the same line, were then reduced to the value of ΔN for the D line in table 73, so that the ΔN for each spectrum line was obtained in succession for the computation of μ. In other words, the values of δN are shifted so that for the sodium line $\delta N_D=0$.

The table shows that the value of μ thus computed differs from the value of Mascart μ_M by 14 units in the fourth place in case of the extreme C and F lines. The reason of this is the approximate value of b taken. To find what the value of b should have been in each case put $\mu_M=M$ and $\mu_a(\Delta N/e+1)=S$; then

$$(M-S)\lambda^2=-2b \qquad (12)$$

These data are also given in the table and show a very appreciable variation in the march of b from the C to the F line.

Table 79 contains three series of values made at different times. The agreement of the first two is very good, but the difficulty in recognizing the Fraunhofer lines among the stationary interferences has not been quite overcome.

If the mean value of b is computed from Mascart's data between the C and D and the D and E lines and compared with the computed mean of the table, the results are

	C to D	D to E
Mascart	$b\times10^{10}=0.421$	0.407
Interferometer	$b\times10^{10}=0.433$	0.412

* See Landolt and Boernstein's Tables.

a difference of about 3 and 1 per cent, respectively. But the C line in the interferometer is difficult to observe, as the stationary interferences are very prominent in the red. Between D and E, however, the agreement is as close as the method warrants.

TABLE 79.—Values of b as compared with the data of Mascart. Quartz as in table 73. $\Delta N_D = 0.34059$ cm. $i=r=0°$. $\mu=\mu_a(\Delta N/e+1)-2b/\lambda^2=S-2b/\lambda^2$. Assumed $b\times 10^{10}=0.415$.

		δN	$\delta N-\delta N_D$	ΔN	S	μ	M	Difference	$S-M$	$b\times 10^{10}$
I	C	0.01079	−0.00353	0.33706	1.56259	1.54332	1.54188	−0.00141	0.02071	0.446
	D	.01432	.0	.34059	1.56847	1.54457	1.54423	− .00034	.02424	.421
	E	.01897	+ .00465	.34524	1.57623	1.54635	1.54718	+ .00083	.02905	.403
	b	.01995	.00563	.34622	1.57785	1.54697	1.54770	+ .00073	.03015	.405
	F	.02325	.00893	.34952	1.58337	1.54825	1.54966	+ .00141	.03371	.398
II	C	.01008	− .00362	.33697	1.56243	1.54316	1.54188	− .00128	.02055	.443
	D	.01370	.0	.34059	1.56847	1.54457	1.54423	− .00034	.02424	.421
	E	.01838	+ .00468	.34527	1.57628	1.54640	1.54718	+ .00078	.02910	.404
	b	.01940	.00570	.34629	1.57798	1.54710	1.54770	+ .00060	.03028	.407
	F	.02255	.00885	.34944	1.58323	1.54811	1.54966	+ .00155	.03357	.397
III	C	.00317	− .00353	.33706	1.56259	1.54332	1.54188	− .00144	.02071	.446
	D	.00670	.0	.34059	1.56847	1.54457	1.54423	− .00034	.02424	.421
	E	.02515	+ .00437	.34496	1.57577	1.54589	1.54718	+ .00129	.02859	.397
	b	.02725	− .00535	.34594	1.57739	1.54651	1.54770	+ .00119	.02969	.399
	F	.03550	.00855	.34914	1.58273	1.54761	1.54966	+ .00205	.03307	.391

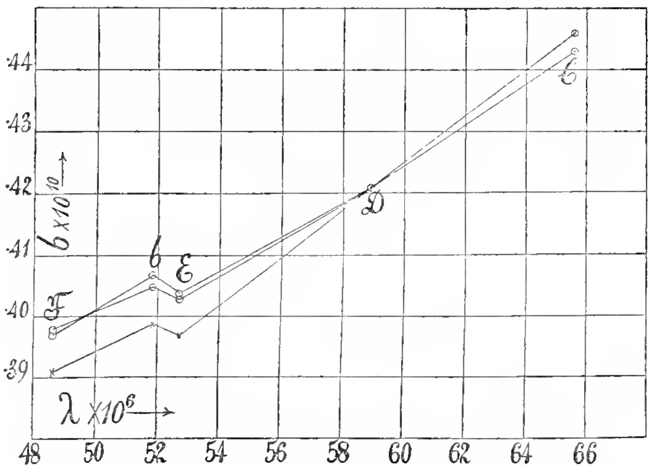


FIG. 71.—Variation of the dispersion coefficient with wave-length.

The results of b so found show the difficulty encountered in using the method of §100. The equivalent of the distance apart of C and D and D and E on the micrometer is too great to warrant the simplified Cauchy equation there used. It also shows, however, that the outstanding error of the above results is referable to the approximate value of b used. For the

D line in table 79 the mean result for *b* is $10^{-10}\times 0.421$. This value was therefore used in reducing the quartz data above.

108. Conclusion.—The above results, taken as a whole, bear out the surmise that it must be possible to determine the refraction of solid and liquid media by means of displacement interferometry with any degree of accuracy desirable, depending ultimately on the length of the refracting column employed. The center of ellipses is never lost but may be brought back to the fiducial Fraunhofer line if the micrometer screw is sufficiently long and the end faces of the medium are plane parallel. If the latter is not the case, and the angle of wedge is not too large, the discrepancy may be corrected by compensation. For this purpose the distant mirror *N* and not the one at the micrometer *M* and controlling the beam passing through the medium is to be rotated to restore the ellipses.

The coefficient $d\mu/d\lambda$ may be obtained by the same method with an adequate degree of accuracy, by using successive Fraunhofer lines. In the case of work of precision, the Cauchy equation as far as the fourth power of wave-length is necessary and three Fraunhofer lines are needed to determine the two essential constants. For this purpose it is immaterial whether the plate is slightly wedge-shaped or not.

If the optical constants of doubly refracting media are to be observed, the plates are to be mounted for rotation around an axis parallel to the slit and passing symmetrically between the faces. Otherwise the wedge effect is liable to produce excessive distortion in the results. If columns are to be used, plane parallel faces are to be cut at a definite angle to the optic axes, as the columns can not be rotated.

The position of minimum thickness of the plate or crystal may be recognized from two rotations at right angles to each other, and the reversal of the motion of the interference rings. It is thus easy to place the axis of rotation at right angles to the beam.

The interferometer used in the above experiments was an improvised apparatus, made of ¼-inch gas-pipe, through which water continually circulated. The angle between the component or interfering beams being but 30° or less, the micrometer was sufficiently close to the observing telescope to admit of easy manipulation. The arms have since been increased to over 1 meter in length, each, and the available free depth below the beam is 15 cm. There seems to be no difficulty of increasing these dimensions in any degree. In spite of the lightness of the apparatus, tremor of ellipses does not seriously hamper the observations. Suppose now a glass column 1 meter long with plane parallel end faces is placed in one of the beams. The micrometer displacement which restores the center of ellipses to the fiducial sodium line will be of the order of 50 cm., measurable, so far as the interferences are concerned, to 5×10^{-5} centimeter. Hence $\mu-1$ must be measurable with an accuracy of 1 part in a million.

* Described in Am. Journ. of Sci., XXXIII, 1912, pp. 109–219.

CHAPTER XIII.

SUBSIDIARY EXPERIMENTS.

THE PRISM SPECTRUM ON THE ROWLAND SPECTROMETER.

109. Introductory. Apparatus.—Having certain experiments to make with the spectra of prisms of small angle, I attempted to find the indices in terms of wave-length on a modified Rowland adjustment, as described in a preceding paper.* In such a case the wave-length of the grating spectrum would at once be given in terms of the position of the carriage which supports the ocular. The refraction of the prism is found in terms of the same data, provided the Fraunhofer lines are available for adjustment.

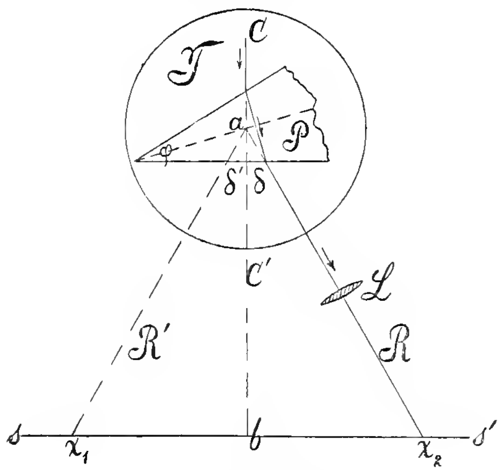


FIG. 72.—Adjustment of prism.

In fig. 72, ss' and CC' are the two fixed rails, symmetrically normal to each other, and R the sliding oblique rail of fixed length, swiveled at the axles a and x. If the grating were placed at a, normal to CC' or to R, the wave-lengths would be computed from

$$\lambda = D(x_2 - x_1)/2R \qquad (1)$$

which is either $D \sin \theta$ or $D \sin i$, respectively. I may therefore insert a slight digression here. In the figure the light is supposed to come from a white source beyond C, and to be collimated in the direction CC'. L is a weak achromatic lens carried by the rail R and placed so that Lx is the principal focal distance. For a transmission film grating of about 15,000

* American Journal of Sci., C. and M. Barus, xxxi, pp. 85–95, 1911.

lines to the inch, such as those made by Mr. Ives, the rail R is easily adjusted to a suitable length, while at the same time equation (1) reduces to

$$10^6\lambda = (x_2 - x_1)/2$$

This is a great convenience in practice. The rail in such a case is slightly less than 167 centimeters long. For a grating of 10,000 lines to the inch, it would be about 250 centimeters long, while about half this length would make $10^6\lambda = x_2 - x_1$.

By providing the ends of the rail aR with a right and left screw respectively or a stiff turnbuckle, the adjustment is secured with considerable precision. Table 80 shows a few measurements with the spectrometer so adjusted, sunlight being used.

TABLE 80.—Data for $x_2 - x_1$ and λ

Line.	I.			II.			III.		
	$x_2 - x_1$	$10^6 \times \lambda$ approx.	$10^6 \times \lambda$ comp.	$x_2 - x_1$	$10^6 \times \lambda$ approx.	$10^6 \times \lambda$ comp.	$x_2 - x_1$	$10^6 \times \lambda$ approx.	$10^6 \times \lambda$ comp.
B	137.35	68.68	68.77	137.00	68.50	68.65	137.35	68.68	68.68
C	131.10	65.55	65.64	131.00	65.50	65.65	131.20	65.60	65.60
D	117.70	58.85	58.93	117.60	58.80	58.93	117.85	58.93	58.93
E	105.20	52.60	52.67	105.15	52.58	52.69	105.40	52.70	52.70
b	103.45	51.73	51.80	103.40	51.70	51.81	103.60	51.80	51.80
F	97.00	48.50	48.57	96.95	48.48	48.58	97.20	48.60	48.60
G	86.10	43.05	43.11	86.00	43.00	43.10	86.05	43.03	43.03

The successive series show a different degree of approach to the true value and in the last the requirements have been met quite adequately. The parts of the apparatus were improved from available laboratory furniture and the scale was read to 0.1 mm. by estimation. For the identification of unknown lines, an approximation of this kind has proved invaluable.

Suppose now the grating is removed and the prism P placed on the horizontal table T, so that the plane bisecting the prism angle contains the axis a, at one end of the rod R. Let the prism be placed so that the angle δ may be the angle of minimum deviation for the given color, or the ray within the prism be normal to the place $a\varphi$. Finally, let R' and δ' be the corresponding position on the other side of the undeviated ray Cb, the prism having been suitably reversed and being again at minimum deviation. Then

$$\sin \delta = (x_2 - x_1)/2R \qquad (2)$$

For comparison, since $\lambda = D(x_2 - x_1)/2R$ would be the corresponding wavelength of the grating spectrum,

$$D = \lambda/\sin \delta \qquad (3)$$

is the grating space of the normal spectrum, coincident at the given Fraunhofer line λ, with the prismatic spectrum. Naturally the two spectra are turned in opposite directions.

To determine φ, the prism is rotated until the plane φa of the prism coincides with the direction $C'C$ and the positions x_1' and x_2' of the rays reflected from the two faces of the prism, respectively, are taken. Then in the same way

$$\sin \varphi = (x_2' - x_1')/2R \tag{4}$$

From equations (2) and (4), which give δ and φ, respectively, the index of refraction μ is as usual

$$\mu = \frac{\sin (\varphi + \delta)/2}{\sin \varphi/2}. \tag{5}$$

110. Data.—The following observations were made to illustrate this equation.

TABLE 81.—Refraction of a prism. $\varphi = 30°$; $R = 168.3$ cm. $10^6 D$ (computed) = 176.55; $1/D = 5664$ lines /cm².

Line.	I				II			
	$10^6 \times \lambda$	$x_2 - x_1$	δ	μ	$10^6 \times \lambda$	$x_2 - x_1$	δ	μ
B	.....	......	.....	.....	68.7	111.25	19.30°	1.611
C	65.63	111.35	19.32°	1.612	65.63	111.65	19.37	1.614
D	58.93	112.35	19.50	1.618	58.93	112.45	19.52	1.618
E	52.70	113.55	19.72	1.624	52.70	113.75	19.75	1.626
b	51.84	113.80	19.76	1.626	51.84	114.00	19.80	1.627
F	48.61	114.60	19.91	1.630	48.61	114.85	19.95	1.632
G	43.08	116.70	20.28	1.641	43.08	116.90	20.32	1.643
	III				IV			
B	.....	......	.....	.....	.....	......	.....	.....
C	65.63	111.35	19.32°	1.612	65.63	111.05	19.26°	1.610
D	58.93	112.50	19.53	1.619	58.93	112.25	19.48	1.617
E	52.70	113.65	19.73	1.625	52.70	113.60	19.72	1.624
b	51.84	114.00	19.80	1.627	51.84	113.90	19.78	1.626
F	48.61	114.80	19.94	1.631	48.61	114.80	19.94	1.631
G	43.08	117.00	20.34	1.643	43.08	116.70	20.28	1.641

The chief difficulty here was the arrangement of ocular until the lines in the small spectrum of the 30° prism could be distinctly seen for measurement. This was finally accomplished by suitable magnification, as is shown by the successive series in fig. 73, in which Series II and IV are raised to keep them apart. The curves show the uncertainty in identifying the fine lines of the spectrum.

No attempt has here been made at accuracy, the scale being a millimeter scale on brass and the fractions of a millimeter estimated. Obviously the precision could now be increased at pleasure by sharper scale-reading. The difficulty in the work as a whole will ultimately be the measurement of R, the length between axles; but here, as this is about 169 centimeters, a reading of a little less than 0.2 millimeter would be needed to vouch for the fourth place in μ, a result which is easily warranted. If a good grating is at hand whose space D is known, R may be found indirectly from the Fraunhofer lines of the solar spectrum.

Usually $x_2 - x_1$ is inconveniently large for so large a value of R, and this is particularly true when prisms of large angle are to be treated. Hence a smaller value of R is in general preferable, so long as its accurate length is still determinable. All this applies in an even greater degree to the measurement of the prism angle φ, in which case $x_2 - x_1$ is still larger than

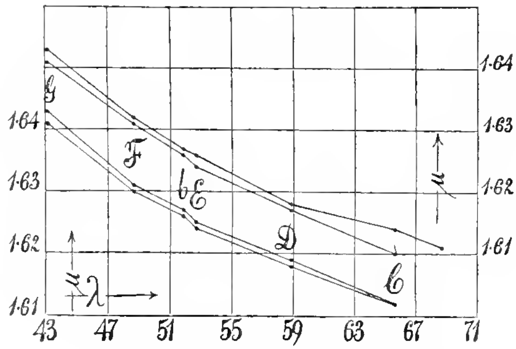


FIG. 73.—Dispersion and wave-length.

for δ. A plausible suggestion that an adjustment like the above might suffice without sunlight for the rectification of the spectrum does not seem feasible.

THE COMPARISON OF TWO SCREWS.

111. Introductory.—In work like the above, where two lengths are to be compared, a displacement ΔN measured with one micrometer, for instance, and a thickness e with another (a caliper screw), it becomes of importance to coördinate these data directly. This may be done with facility by mounting the opaque mirrors M and N on the two screws in question, shifting the ellipses away from the spectrum line with the first screw at N and returning them to the identical line with the other screw at M, through successive consecutive steps of their length. The screws must now be identically reversed in relation to the fixed beam of light and a similar series of complete data investigated. The true relation is the geometric mean of the two results at each step.

112. Method.—In fig. 74, let M and N be the opaque mirrors, actuated by the micrometer screws at angles α and α', respectively, to the beams of light m and n, *i.e.*, the normals of the mirrors. C is the collimator, G the grating, and T the telescope. Let the latter be clamped, so that the direction TG is fixed throughout the experiment, as is also CG. It must be possible to remove the mirrors M and N, together with their micrometer screws, without changing the angles α and α', in order that they may be identically reversed.

Let S be the pitch of the screw at M, for instance, s the corresponding equivalent (approximate pitch) at N. Then

$$S \cos \alpha = s \cos \alpha' \tag{1}$$

Now let the mirrors be reversed, the angles α and α' and the beams m and n being reproduced, in virtue of the fixed direction GT. In this case

$$S \cos \alpha' = s \cos \alpha \tag{2}$$

Hence

$$S^2 = ss' = s^2(1+k). \tag{3}$$

Since k is very small,

$$S = s(1+k/2) = \frac{1}{2}(s+s'), \text{ nearly.} \tag{4}$$

In order that the reversal may be properly made, the tablets carrying the micrometers M and N with the attached mirrors should be truly horizontal, so that merely an adjustment of each in azimuth is necessary. The sharp white line which is the direct reflection from the mirror N is

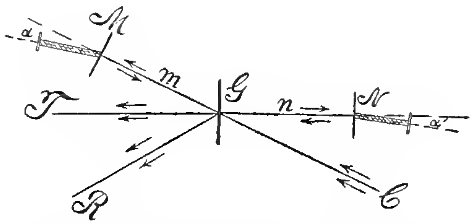
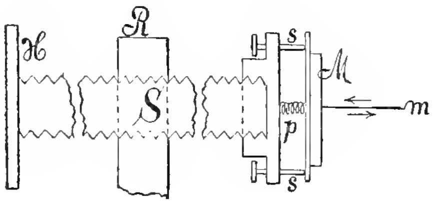


FIG. 74.—Interferometer for comparing screws.

first restored to the cross-hairs of the fixed telescope, after which the reflection from M is restored to the same image, the two lines merging into a single vertical sharp line. The slit images should now also coincide horizontally in the field, *i.e.*, the two images of any specks of dust or cross lines in the slit should cover each other. If this is not the case the tablets carrying M and N must be provided with three adjustment screws, so that condition of mirror normal to the beams of light may be reëstablished without changing α and α'. In other words, the adjustment screws of the mirrors on the micrometer must be left untouched.

When this is done, the ellipses will be seen in the direction of the diffracted rays R as soon as the proper distances m and n have also been reëstablished, by moving either mirror alone. The arrangement of mirrors on the tablet should therefore be as nearly as possible symmetric, even when different forms of screws are compared.

A simple type of micrometer screw, or attachment suitable to any screw to be tested, will also be used below and may be described here. S, fig. 75, is the micrometer screw in question, with its graduated head at H, revolvable in the socket R, this being adjustable with three screws as usual, on the tablet at either N or M, fig. 74. The mirror M, fig. 75, is separately adjustable with three screws (horizontal and vertical axes), s, s, and a strong spring is suggested at p. The whole arrangement Mps is firmly fixed to the end of the screw S. In this case the mirror rotates with the screw as it advances and the latter is therefore necessarily normal to the component beam m, an adjustment secured by the set screws at s in the usual way, the effect of the operations being observed in the telescope T in fig. 74. The approximate adjustment is conveniently made with sunlight returned by reflection from the mirror to a screen 20 or 30 feet distant. Suppose, fig. 76, the mirror is nearly normal to the screw; let the intersection of the incident ray, the screw axis, the normals, and the reflected rays with the



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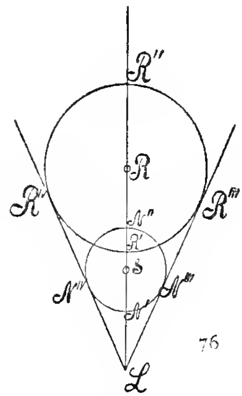

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FIG. 75.—Simple screw micrometer with slide.
FIG. 76.—Diagram for adjustment.

screen, all lines prolonged if necessary, be at $L\,S\,N'$ and R, respectively. If the screw is rotated 360° the normal will trace a circle N' to N^{iv}, the reflected ray a similar circle R' to R^{iv}. If the normal N' coincides with the axis of the screw S, the reflected ray will be at the center R of the circle $R'\,R''$. Hence the center is to be sought by rotating the screw. The mirror is now adjusted so that the reflected ray is at R. Then N is at S, as required. With successive trials this succeeds very well.

113. Data.—Table 82 contains a comparison of the screws of two slide micrometers, one of them old but with an excellent slide, the other with a good new screw but an imperfect slide. As a result of this it was impossible to keep the ellipses adequately sharp throughout the whole motion of the screw. In fact, it was necessary to make a readjustment of mirrors M and N from time to time, *i.e.*, to take a fresh start, so that the comparison proceeded rather in sections. Moreover, the motion of the slide not strictly parallel to itself appears in the data as an error of the screw, and virtually it is so.

The discrepancy may, however, be corrected without reëstablishing a fresh zero, by removing the effect of any slight rotation at one mirror (due to an imperfect slide or other causes) by the adjustment screws at the tablet of the other mirror, until the sharp lines of the ellipses are quite restored. This is, in fact, a method of compensation by which the excess of length due to incidental rotation of one mirror (at M) is manually imparted to the other (at N). Care must be taken to keep the angles α and α' unchanged.

TABLE 82.—Comparison of two micrometer screws.

I. Clockwise.			II. Counterclockwise.		
New.	Old	Diff.	New.	Old.	Diff.
cm.	*cm.*	*cm.*	*cm.*	*cm.*	*cm.*
0.00	0.0	0.04910	1.35	1.34645	0.04940
.05	.04910	.04940	1.30	1.29705	.04970
.10	.09850	.05020	1.25	1.24735	.04965
.15	.14870	.05020	1.20	1.19770	.05010
.20	.19890	.04970	1.15	1.14760	.05000
.25	.24860	.04995	1.10	1.09760	.04980
.30	.29855	.05050	1.05	1.04780	.05015
.35	.34905	.04995	1.00	.99765	.04935
.40	.39900	.04975	.95	.94830	.04960
.45	.44875	.04985	.90	.89870	.05035
.50	.49860	.05010	.85	.84835	.05020
.55	.54870	.04950	.80	.79815	.05015
.60	.59820	.04970	.75	.74800	.04990
.65	.64790	.04975	.70	.69810	.05020
.70	.69765	.05025	.65	.64790	
.75	.74790	.04965	III. Mirrors reversed		
.80	.79755	.04985	0.75	0.74790	0.05180
.85	.84740	.04935	.80	.79970	.05130
.90	.89675	.04970	.85	.85000	.04970
.95	.94645	.04980	.90	.89970	.04975
1.00	.99625	.04985	.95	.94945	.04995
1.05	1.04610	.04905	1.00	.99940	.04965
1.10	1.09515	.04985	1.05	1.04905	.04985
1.15	1.14500	.04940	1.10	1.09890	.04995
1.20	1.19440	.04965	1.15	1.14885	.05000
1.25	1.24405	.04965	1.20	1.19885	.05015
1.30	1.29370	.04900	1.25	1.24900	.05010
1.35	1.34270	.04965	1.30	1.29910	.05015
1.40	1.39235		1.35	1.34925	

In other respects the comparison, so far as the experiment is concerned, was satisfactory (see fig. 77), very little difficulty in adjusting and reversing being encountered. With two good slides the work would have proceeded smoothly from end to end of the screws, which were several inches long.

The first screw had a pitch of 0.05 cm., the micrometer reading to 0.00010 cm.; the second a pitch of 0.025 cm., the micrometer reading to 0.00005 cm. The former was turned in steps of a single pitch, and the value which brought the center of ellipses, originally coincident with the given Fraunhofer line, back to coincide with it, was read off on the finer screw.

In table 82 the successive sections have to be coördinated, as a fresh fiducial mark was determined in terms of the preceding whenever the ellipses lost adequate clearness. To find the relation of the two screws, the mean of the initial and final halves of each series was computed by deduct-

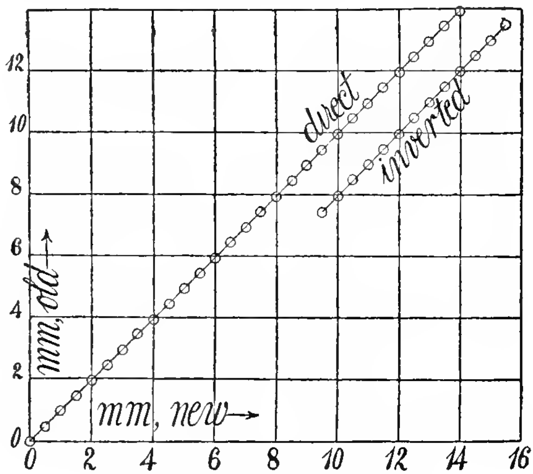

FIG. 77.—Comparison of screws.

ing the corresponding observations from each other and averaging the result. In this way the ratio r of the old appears in

Series	I clockwise	$r' = 0.9962$
"	III mirrors reversed	$r'' = 0.9997$

Thus the true relation is finally

$$r = (r' + r'')/2 = 0.998$$

Usually a large part of the difference of values of the two screws is to be ascribed to the angles of alignment α and α', if no special means are taken to orientate them accurately. With the ratio $r = S/s$ given, the ratio of the alignment angles would follow, but this is of little value. In fact, if $\alpha' = \alpha + d\alpha$, then

$$d\alpha = (1 - r)\frac{2 - \alpha^2}{2\alpha}$$

where α^2 may be neglected in comparison with 2. Hence $d\alpha$ increases as α decreases, numerically. The real problem of finding α for the measuring micrometer is naturally not touched by such a method, in case of two slide micrometers. It is given at once, however, when one of the micrometers is of the form of fig. 75, where α' (say) is necessarily zero. Hence if the screws are identical $\cos \alpha = 1/r$. Virtually, however, $S \cos \alpha = s$ is the effective absolute value of the micrometer screw calibrated in this way in terms of s. This is a particular reason for the development of such an apparatus, fig. 75, to serve the purposes of comparison and standardization.

114. Conclusion.—The advantage of the present method is that steps of any size, quite arbitrarily, are admissible and there is no danger of ever losing count. A rigorously linear slide is presupposed. If the slide is slightly circular, even with very large radius, the ellipses are soon blurred and lost. To restore the ellipses by rotation of mirror is possible, but in any case precarious.

OBSERVATIONS WITH THE SIMPLE SCREW MICROMETER.

115. Apparatus.—The screw micrometer without slide of the type of fig. 75 was made out of an old millimeter micrometer screw, which I used many years ago in determining the expansion of molten magmas. The screw was somewhat worn, though, held in place by an adjustable lock nut (not shown in fig. 75), it was still serviceable. The tripod of the micrometer was first adjusted, so as to bring the nearly normal reflected ray in the direction through the grating, roughly. Thereafter the final adjustment was made at the rotating mirror (special screws) at the end of the screw. Calling this mirror N, the corresponding mirror M (Fraunhofer slide) of the interferometer was placed so that the spot (cross-wire) in the direct image of the slit from M fell at the center of motion of the corresponding spot of the image from N, when the micrometer screw of the latter was rotated. The mirror N was now adjusted for coincidence of the two spots, using its adjustment screws for horizontal and vertical axes. The operation was then repeated. It is difficult to keep the spots from N quite stationary. One has better success on merely endeavoring to let the image from N rotate in a very small circle in the field of the telescope and return to its initial position after a complete rotation. In this way the following data were found, an example of many similar results sufficing the present purposes. The screw at N was turned in steps of about 1 mm. and their value found at the micrometer at M, by shifting the displaced center of ellipses back to the sodium line. In fig. 78 the observations have been raised to the same line at the points between which fresh adjustment was made. This line, curiously enough, is not horizontal, as it should be, but inclined by the amount of about 0.0037 cm. per centimeter of length of either screw. The explanation of this seems to be that the trial screw is not straight but slightly bent, so that at each turn slightly more of the screw is needed to bring the reflected image again into coincidence in consequence of flexure. The amount of this superposed error is 0.0037 cm. per centimeter of length. Otherwise, coincidence of the reflected images would ensue after each complete turn of the trial screw.

The object of the present test was in the main an endeavor to find some means of replacing the slide of a micrometer, as these slides are difficult to grind sufficiently true for interference work. In fact, the results obtained are encouraging enough to suggest that a screw mounted not on one but on *two supports*, *a* and *b*, fig. 79 (the sockets of the latter to be adjustable by being horizontally split), would probably meet the severe requirements of the interference problem, in the complete absence of a slide. The parts of this screw *SabS*, if of metal, must of course be ground together. The endeavor will therefore presently be made to avoid metal sockets. The effect of errors in the obliquity of faces of the grooves of the screws, or

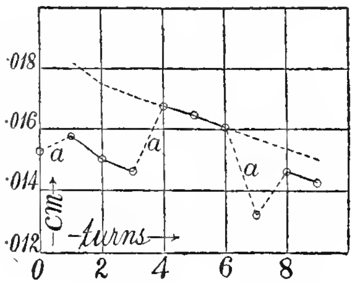

FIG. 78.—Comparison of screws.

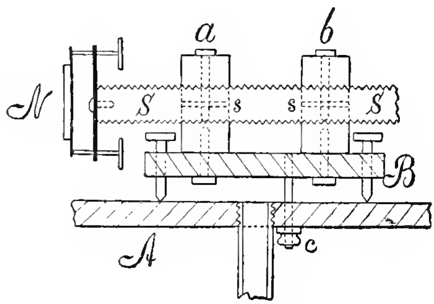

FIG. 79.—Simple screw micrometer with two lugs.

a residual wobble such as appears in fig. 75, would thus be minimized. Naturally the mirror *N* should be a good plate and reflection take place from the central part opposite the axis of the screw.

116. Observation.—A large number of experiments was now made with a screw of the type fig. 79, principally with brass screws, carefully cut but not ground in the laboratory and at first running in brass sockets.

The results with these screws were only moderately successful, so long as an unbroken socket was used. With the tools at hand it was not possible to cut the screw sufficiently loose to move easily for micrometer purposes, and at the same time sufficiently tight to be free from wobble. A single result (fig. 80), among many, may be shown as an example, obtained with the same screw as above, but readjusted on two sockets, the abscissas denoting the individual turns determined by one rotation as seen in the telescope, the ordinates the difference of value read off on the two drums of the screws. Here again

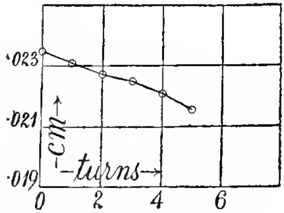

FIG. 80.—Comparison of screws.

the difference of reading of the drums of the screws is not constant, but diminishes by about 0.0038 cm. per linear centimeter, due to some superimposed error, attributable (as above) to flexure of the screw axis. The reflected images do not register a complete turn for successive coincidences. Though the above datum is somewhat smaller, the difference is probably referable to the total change of mounting needed in the second case.

Success was eventually reached by slotting the sockets (no longer of metal) parallel to the axis of the screw on one side and drawing the two halves of the nut together or pushing them apart by a set screw. This slot at *s* should run nearly across the lugs *ab* in fig. 79 so that the two parts are held together elastically. The first experiments were made with screw No 5, in which the lugs are of indurated fiber. Screws so cut (each nut one-quarter to one-half inch thick) are usually too tight at first, but they cling admirably. The necessary ease of motion was secured by the set screws at *a* and *b*, fig. 79, which here slightly push the socket apart. The ease with which a brass screw 10 inches long and a pair of sockets of the kind in question may be cut and mounted and their admirable precision of motion when tested by the interferometer are astonishing.

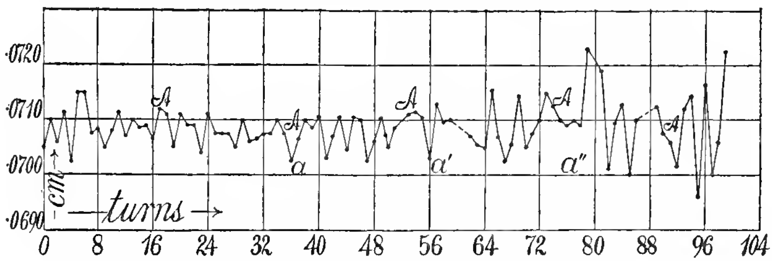

FIG. 81.—Comparison of screws.

Fig. 81 shows a comparison of screw No. 5 with the Fraunhofer micrometer, using about 3 inches of the former. The adjustment of the mirror of the latter (No. 5) was left incomplete, so that the directly reflected ray described a small circle in the telescope. The coincidence of images of the two mirrors *M* and *N* was secured at the beginning of each of the turns, by the initial adjustment, so that the ellipses reappeared with each complete turn, after the Fraunhofer micrometer had also been displaced. Owing to the length (3 inches) of the new screw used, it was necessary to use the Fraunhofer screw four times in succession, by introducing thick glass compensators into the ray coming from the new screw. These were made with thick plates of glass and added at the places indicated by a, a', a'', in the figure. The total thicknesses of the piles of plates were: at a, 2.10 cm., 3 plates; at a', 5.60 cm., 8 plates; at a'', 8.60 cm., 12 plates.

Owing to the number of reflections (24 faces in the last case), the image gradually became more colored and less intense, so that the adjustment was gradually less certain. At the same time the shift of ellipses was less sensitive. It was, however, wholly due to dimness of the interferences that adjustment was more difficult, a result which could have been avoided by using a single plate of thick glass (about 9 cm.); but this was not at hand.

Finally a few adjustments of the Fraunhofer mirror were made for incidental reasons not connected with the precision of motion of the screw (as shown at *A* in figure). In other words, through the 3 inches of screw used no effect of flexure or other serious discrepancy could be detected, the ellipses returning in full strength after each complete turn.

The fluctuation of the curve which represents the equivalent of 1/36 inch in centimeters (*i.e.*, 0.0705 cm.) is largely within 0.0005 cm. at the beginning of the work, when seeing was good. This is about one division of the drum of the micrometer screw, or about 1/141 of the pitch of the screw No. 5 and is largely referable to the difficulty of setting the head of the new screw at zero, the disk and graduation having been improvised. It furthermore contains all errors of the Fraunhofer screw, and finally all errors in judgment in bringing the centers of ellipses back to the sodium line. Nevertheless, throughout the whole length examined the new screw retains a satisfactory mean pitch. The whole operation of comparison can easily be completed in one afternoon.

A similar screw with the two lugs but ¼ inch thick and about 3½ inches apart was now tested. The mirror mechanism was made lighter and the adjustment screws finer (56 threads to the inch), the object being to quite eliminate, if possible, the circular motion of the reflected ray. This was not quite accomplished, owing perhaps to an insufficiently plane mirror; but on rotating the screw in successive arcs of 90° the ellipses were available in three of the positions, though they were too blurred in the fourth for use. For final adjustment, besides a more perfect mirror (which need be only ½ inch in diameter), finer adjustment screws than the above would be desirable. The new comparison as a whole is given in fig. 82

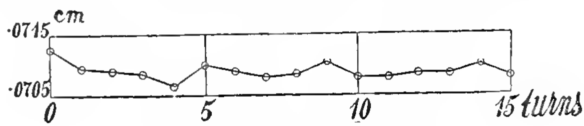

FIG. 82.—Comparison of screws.

in the same way as above. The fluctuation is on the average within about 0.0003 cm., or half a scale part of the drum of the Fraunhofer micrometer and 1/236 of the circumference of the drum of the new screw. This uncertainty is almost wholly due to the difficulty of setting the latter with this precision, as the head was improvised in the laboratory for the purposes here in view.

To summarize: There seems to be no doubt, therefore, that a screw, satisfying the requirements of the interferometer and trustworthy to about 0.0001 cm. and a length of even a foot or more, could be constructed by the above method. It is necessary to begin with a straight rod for this purpose (or preferably with a tube) of a larger diameter, say 1 or 2 cm. The adjustable mirror at the end should be light, which was not the case in the above apparatus, ordinary thick plate glass 1½ inch square being used. The adjustment screw with orthogonal axes must be of very fine pitch if the image is to be stationary. Such a screw of low pitch, carefully cut in brass and running in sockets of indurated fiber, can be made in almost any laboratory.

The final advantage of this type of micrometer is the fact that the normal to the mirror is necessarily a prolongation of the axis of the screw and also coincides very nearly with the incident and reflected ray. There are thus no unknown angles between screw axis and normal to the mirror and exchange of screws at M and N is no longer necessary.

THE RESOLUTION OF INTERFERENCE FRINGES.

117. Experiment.—The following remarks apply to interference fringes produced preferably by the bi-prism, though Young's slits and the other methods of course show the same results. The experiment is straightforward, as is given in fig. 83, but the effect obtained is at first quite unexpected.

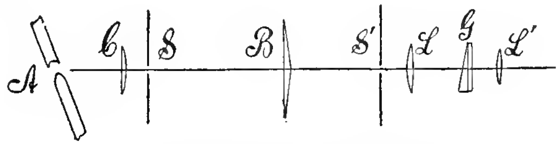

FIG. 83.—Adjustment of bi-prism.

In fig. 83 the arc light A passing the condenser C and the slit S, falls on the bi-prism B and then upon the direct-vision spectroscope $S'G$ (preferably of the grating form like that of Mr. Ives), where S' is the slit, G the grating with attached prism, L and L' the collimator and eyepiece, respectively. Both slits S and S' must be as fine as possible, the spectrum seen being just short of darkness to obtain the sharpest effects, though the results are perfectly distinct, if more washed, for wider slits.

What one would expect to see as the spectroscope $S'G$ is moved across the field from left to right is the usual form of channeled spectrum, with

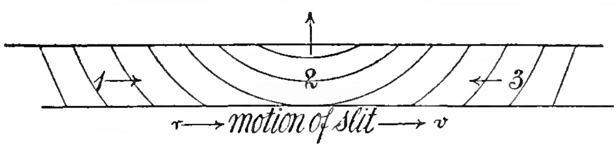

FIG. 84.—Spectrum with circular interferences.

the fringes moving horizontally from end to end of it. What actually appears, however, is a succession of intercepts between an upper and a lower horizontal, of broad concentrically circular or oval absorption bands, all of a very large radius, the displacement being on the plan shown in fig. 84, or the reverse. In other words, the spectroscope reveals the similarly intercepted arcs of large absorption bands. The arcs if essentially horizontal move up and down, if essentially vertical right and left. They are never quite vertical, and they become thinner and more crowded toward the ends of the spectrum. The groups 1, 2, 3 appear in succession.

The phenomenon is exceptionally sensitive to changes in the approximate verticality of the slit, so that if the spectroscope is slightly rotated on its axis, all the groups may reappear in turn.

118. Explanation.—To interpret this phenomenon it is convenient to plot the order n of a given fringe in terms of its distance x from the center of fringes, where $n=(c/r(\lambda/2))x$, r being the virtual distance of the fringes on the screen, here the slit of the spectroscope, from the virtual position of the two slit images for the wave-length λ. There will be dark bands for a given color λ, as fig. 85 shows, whenever $n=1, 3, 5$, etc. The two lines drawn show the limits of the spectrum for any distance of fringe x and the heavy horizontal lines the number of dark bands to be expected. Thus

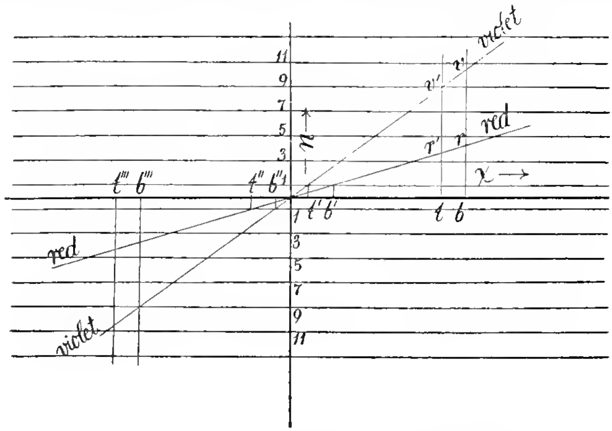

FIG. 85.—Diagram showing color distribution.

for the value of x corresponding to vr, there will be three black bands in the spectrum, their color distribution depending upon their position between v and r. As the slit moves from right to left from the positive to the negative values of x, bands will enter the red end and leave the violet end, for a positive value of x, and do just the reverse for a negative value of x.

The question now arises as to what will happen if the fine slit is not quite parallel to the fine interference fringes, crowded together as they are in a vertical band about half an inch in breadth. The oblique but fine slit in such a case corresponds to a succession of values of x depending upon the obliquity and length of the slit, and the diagram, fig. 85, therefore, shows the case for only one point in the length of the slit. It is thus necessary to introduce the third dimension corresponding to the breadth of spectrum, at right angles to the plane of fig. 85, which plane may be supposed to correspond to the bottom point of the slit. If the horizontal projection of the oblique line of the effective slit is *tb*, the spectrum corresponding to the top point of the slit will be $v'r'$, so that all the bands have been displaced toward the violet. Since the internal points of the slit correspond to the intermediate spectra between *vr* and $v'r'$, the spectrum will thus contain curved black bands with the tops toward the violet and their bottoms toward the red. Precisely the opposite will be the case for a negative value of x, *i.e.*, on the other side of the center of interference fringes. If the slit moves, the general motion of bands will remain as already explained, only there must now be successive changes of form.

Thus if the slit takes the position $t'b'$ (same distance apart from top to bottom of spectrum) there can be but one black band in the spectrum, running from the red at the bottom to the violet at the top and being therefore nearly horizontal. At t'' b'' a single band must run from the violet at the bottom to the red at the top, *i.e.*, with reversed slope, so that clearly with the projected slit symmetrical to $x=0$ the bands if appearing (as they do in multiple in a proper position of the bi-prism B, fig. 83, close to the spectroscope $S'G$) must be quite horizontal in the middle and curved upwards, or the reverse, at both ends.

Following fig. 85 for a given obliquity and length of slit, all conditions may be easily computed. It is clear, moreover, that the bands can never be closed curves, but are limited to arcs cut off by the band of spectrum from a series of concentric closed curves. In this respect they differ from the elliptic interferences, which they in many respects recall; but the elliptics are essentially closed curves moving as a whole in the same direction.

THE MEASUREMENT OF SMALL INCREMENTS OF ANGLE.

119. Apparatus.—In connection with electrometry (see below, §160 and elsewhere), the measurement of small angles or small variations of an angle is often necessary and the following method was tested with such an end in view. The adjustments are simple in design, but in application they often give some difficulty. Two light mirrors, m and n, are attached to the rotating body, *bb*, pivoted in such a way that their faces are as nearly as possible parallel, so that the incoming component beam of light L, from the interferometer, is reflected at the same angle i at each mirror. The beam then strikes the fixed mirror N of the interferometer normally and retraces its path, meeting the component beam from the mirror M at the grating in the usual way.

To obtain sufficient parallelism in the two mirrors, a beam of sunlight usually suffices and it is merely necessary to make the undeviated beam L and the reflected beam L' coincide at a distant screen, the mirror N being absent. In other words, the light reflected from the mirror m is to cover the shadow of the disk. If the revolving body is substantial, m and n may be adjusted around the vertical and horizontal axes, as usual, by aid of three screws. In case of a light body it was found convenient to clutch the two stems of the mirrors m and n in slots cut in the ends of bb with a fine scroll saw, and after adjustment to hold them in place with wax, applied by melting.

The angle i, which is to change by a very small quantity only, must be determined by rotating bb on a divided arc, for instance until n is parallel to L, which is sufficient, $90°-i$ being thus given. Usually it is possible and far preferable to rotate the mirror n on bb, until the beam retraces its path toward the grating. In such a case i may be read off directly. From this, a, the normal distance apart of the mirrors, may be found from their fixed oblique distance, mn.

Since $\delta=2a\cos i$, the variation of the angle i in terms of the variation of the path difference δ for different orientations m and n is found to be

$$-di=d\delta/2a\sin i.$$

Hence for the given value of the increment di, $d\delta$ will be larger as $2a\sin i$ is numerically larger. Hence a and i should have the largest values available. It is more convenient to make i about 45° and to increase the sensitiveness by increasing a. If i is measured in degrees,

$$-d\delta/di=0.035a\sin i$$

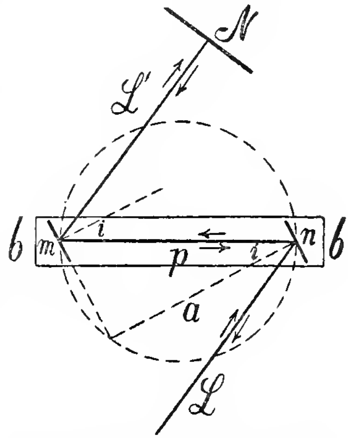

FIG. 86.—Parallel mirror adjustment for micrometry of angles.

120. Experiments.—To put this apparatus to a practical test, a pair of light mirrors m, n was mounted and made parallel by sunlight as stated, on the index arm bb of a divided circle. The latter was clamped on one arm of the interferometer, in front of the stationary mirror M, this being about 1 meter from the grating. Table 83 gives an example of the results obtained, the total range of the angle i being about 10°, after which the reflected images passed beyond the limits of the small mirrors. The index with its mirrors was moved in steps of about 0.5, 1, or 2 degrees, in different cases, steps which were read off directly on the circle, and therefore necessarily unequal as compared with the refined measurement on the interferometer, the circle being only about 3 inches in diameter. The normal distance apart of the mirrors was 4.5 cm. and the angle i on the average about 50°. The initial value of i was found by normal reflection.

Table 83 shows the successive angles of incidence i, the successive displacements ΔN of the micrometer between each, and finally the observed and computed value of displacement per degree of arc.

The irregularities in table 83 are merely due to the setting of the index arm of the divided circle, where an uncertainty of 0.1° was inevitable. It was not thought worth while to repeat the work without a definite purpose. It appears from the table that if 10^{-4} centimeter is guaranteed on the interferometer, the twelve-hundredth part of a degree, *i.e.*, about 3 seconds of arc, is guaranteed; or if the installation is sufficient for the use of the interference rings, 1 second of arc per ring would be the sensitiveness of the above apparatus, where $a=4.5$ cm. and i has the value of

TABLE 83.—Comparison of the displacement at the micrometer and the increment of angle.

i	$\Delta N \times 10^3$	Observed $d\delta/d\,i$	Computed $d\,\delta/d\,i$
54°, 52	0.2408	0.1204	0.1258
52, 50	.2438	.1219	.1224
50, 48	.2212	.1106	.1189
48, 47	.1151	.1151	.1161
53, 55	.2529	.1264	.1274

about 50°. As a may without inconvenience easily be increased ten times, there would thus be no difficulty in measuring well within 0.1 second of arc, the displacement being 0.1 second of arc per ring.

ELLIPTIC INTERFERENCES WITH CONCAVE MIRRORS.

121. Apparatus.—In the course of the work, the production of elliptic interferences with the opaque mirrors identically concave presented itself and it was thought worth while to test the adjustment. Figure 87 is a

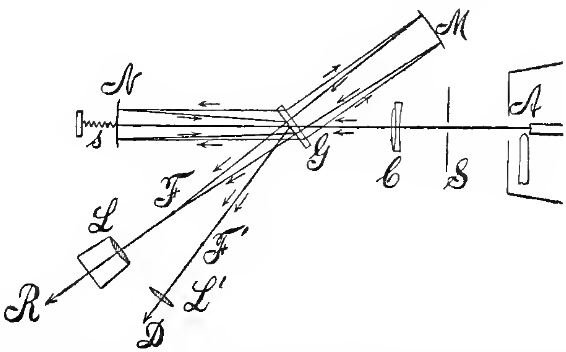

FIG. 87.—Adjustment for elliptic interferences with concave mirrors.

diagram of the parts of the interferometer, S being the slit illuminated by the arc lamp (screened), A and C the collimating lens, G the grating with its ruled face toward the impinging light. The beam reflected from G strikes the concave mirror M and is then reflected to the focus at F. The transmitted beam reaches the concave mirror N on the micrometer screw, and is in its turn doubly reflected to the same focus F. If now the undeviated beams R are in superposition, horizontally and vertically (fine wire across slit to facilitate the former), and the mirrors M and N are adjusted for equivalent distances, the diffracted beam D coming to the focus F' will be made up of two superimposed spectra which show the interferences in question. Lenses L and L' are available for magnifying the phenomena respectively; or an observing telescope with reinforced objectives for short distance vision may be used with advantage.

The mirrors used had an area of only about ⅛ inch square, each, and were cut from the common concave mirrors used in connection with Kelvin's galvanometer, each mirror being a quadrant. Nevertheless the spectra and ellipses obtained were adequately sharp and luminous for practical use in the portion of the interference pattern near the center of ellipses. The remote parts of the pattern, *i.e.*, the fine interference lines, did not come out well, for reasons doubtless connected with the quality of the mirrors. It is probable that the parts of a silvered concave spectacle lens would have been preferable throughout. However, it is astonishing that mirrors so small and necessarily not of high grade should give the phenomenon as satisfactorily as was the case.

Though the adjustment is easily made, two difficulties were encountered which are not incident to the case of plane mirrors and parallel rays. If the axis of the micrometer screw at N is not parallel to the axis of the beam, the mirror M will shift laterally and with it the focus at F, destroying the coincidence of spectra and the interferences. Again, if the mirrors are as usual silvered on the back, there are many slit images from both surfaces and they are not very unequally bright when the mirrors are small. One soon succeeds, however, in recognizing the bluish image from M and the yellowish image from N in correspondence with the position of the face of the grating, which are more serviceable for effective coincidence than the others. In view of the near foci F F', the images in the telescope are highly magnified and the ellipses are apt to be stretched in their vertical dimension.

If the grating G is replaced by a half-silvered plate of glass and the mirrors M and N replaced by as identically concave gratings as possible, set at the same fixed angle of incidence, a variation of the present method well worth the trouble of special test suggests itself. F would then be the focus of spectra and the interferences observed at L. These interferences would naturally be on a large scale and this fact suggests many applications.

ACOUSTIC DISPLACEMENTS.

122. Mayer and Dvorak's experiment.—A variety of acoustic experiments was made for the detection of small forces, in cases where the interferometer seemed to be particularly adapted. The forces acting on a sounding resonator, discovered by A. M. Mayer in this country and by Dvorak abroad, seemed to fall within the scope of the method. Accordingly a paper cylinder (resonator), open at one end and closed at the other, was suspended horizontally from the bifilar pendulum shown in fig. 98, so that any longitudinal displacement due to acoustic pressure could be detected and the corresponding forces estimated. The cylinder used in the final experiments was 6 cm. in diameter; hence with an area of 28.3 sq. cm. It was about 14 cm. long, so as to be in resonance with a König tuning fork of 522 vibrations. The mass of the cylinder and appurtenances was 3.132 grams and the pendulum suspension 18.5 cm. long. The mirror was adjustably attached at the closed end.

When placed on the interferometer the ellipses were easily obtained, though the tremors of the laboratory kept them in continual slight motion, so that measurement by the interference rings was out of the question. The resonator was excited by placing the mouth of the box of the tuning fork near the mouth of the suspended resonator. During this interval the ellipses vanished at once, as a whole, showing that the vibrations of the *body* of the resonator were so intense as to destroy the visibility of the interference pattern. They reappeared, however, again at once, as soon as the tuning fork ceased to sound, without showing the occurrence of any discernible vibration. Hence there could not have been any appreciable deflection of the resonator or the forces, if any, are below the limit of measurement.

The experiment was now modified by making the head of the resonator thicker and more rigid. For this purpose a disk of cardboard was fixed within the resonator by cement. The cylinder was also provided with a mica vane damper submerged in oil, and the whole apparatus surrounded by a case of tin plate to guard against air-currents. To avoid interferences due to the nearness of the operator, the resonator was excited with the tuning fork *at a distance*, the sound being conveyed through a hole in the case, after having passed through a 2-inch metal pipe about 10 feet long.

The results, however, were not different from the above. When the tuning fork sounded, the ellipses vanished, owing to the tremor of the whole system of mirror and resonator. When the fork was suddenly stopped, the ellipses reappeared at once, and in place, showing no gradually decreasing oscillation to the point of equilibrium, such as would have occurred had there been any appreciable deflection.

It appears, therefore, that the deflection of the resonator, if any, could not have been larger than $\Delta N = 10^{-4}$ centimeter, and since in the first experiment $M = 3.132$ grams, the deflecting force could not have exceeded $F = Mg\Delta N/l = 0.017$ dyne, equivalent to a pressure $p = 0.0006$ dyne per square centimeter. In the second experiment, since $M = 8.316$ grams,

$$F = 0.045 \text{ dyne}, \quad p = 0.0016 \text{ dyne/cm}^2$$

would have been recognized. Acoustic pressure difference would therefore have to be less than this value.

In the later experiments of Dvorak and in the work of Rayleigh, Lebedew, M. Wien, and others the neck of the resonator is the seat of the forces actually observed. The above resonator was therefore closed with a disk of cardboard having a hole about 2 cm. in diameter at its center. The experiments made were like the preceding and showed the same negative results. The work was therefore abandoned as not sufficiently promising for further research by this method. The method of Davis of detecting acoustic pressure within the organ pipe seemed equally difficult of adaptation.

123. Telephonic displacements.—The endeavor was then made to determine the displacement of the telephone plate, by attaching a mirror at its center and fixing it on the interferometer confronting the micrometer mirror. It was supposed that ellipses might be visible even when the plate vibrated, at the elongations of the displacements, but this was not the case, even after several attempts in relocating the mirror. The vibrations

TABLE 84.—Deflection of telephone blade. 2 strong cells. $L=20$; $b=1.1$; $d=0.062$; $E=2.1\times 10^{12}$; $F=7.2\times 10^{3}\Delta N$.

R ohms.	i amperes.	$\Delta N\times 10^{3}$ cm.	r ohms.	F dynes.
500	0.0066	0.6	100	4.3
60	.024	2.25		16.2
200	.103	1.35	87	9.7
40	.027	3.05		22.0
100	.019	2.05	117	14.8
20	.032	3.25		23.4
80	.021	2.30	124	16.6
10	.034	3.50		25.2

were always accompanied by rotation of mirror, as a result of which the interferences were lost. With direct current of reasonable intensity the ellipses were not appreciably displaced. The tremors of the laboratory were too great to admit of the use of interference rings and strong currents were apt to rotate the mirror.

The plate was then replaced by a steel hack-saw blade, 20 cm. long, 1.1 cm. wide, and 0.062 cm. thick, held by two horizontal arms clutching the ends of the blade, immediately in front of the magnet of the telephone (see fig. 88). The mirror was cemented to the center of the blade near the magnet so that the mirror moved parallel to itself. But even here the ellipses vanished when alternating currents were used to put the mirror in vibration, showing that rotation accompanies the vibrating plate; otherwise the ellipses should have been glimpsed at the elongations. It is probable that the whole supporting adjustment securing the telephone, as well as part of the interferometer, is put into vibration. Direct currents, however, now gave deflections ΔN of a reasonable order of value, as may be seen from table 84, chosen from many similar results. R ohms is the resistance in circuit, and r that of the telephone coils, ΔN centimeters is the displacement of mirror for the current i in amperes. If ΔN varies as i, $(R+r)\Delta N$ is constant. This appears to be nearly the case, the discrepancies being rather due to incidental disturbances in the adjustment and to difficulties in reading the ellipses. It is interesting to compute the forces F involved, supposing the saw blade vertically loose at its end (which was nearly the case),

$$F = 4Eb(d^3/L^3)\Delta N$$

The values of F are given in the table, so that the table may be summarized by putting the displacement about $\Delta N = 0.11$ cm. and $F = 740$ dynes per ampere of current, when ΔN is less than 0.002 cm., after which a rapid decrease of sensitiveness begins. A number of experiments of the same kind gave practically identical results. It is curious that for such small displacements i and ΔN are not proportional throughout.

In place of the saw blade a narrow strip of thin tin plate was next used, the strip being 20 cm. long, 8 cm. broad, and 0.029 cm. thick. The experiment showed $\Delta N = 0.001$ cm. for $i = 0.04$ ampere, which is equivalent to a sensitiveness of 0.072 cm. per ampere, not a marked advance on the preceding case when the distance from the magnet is taken into account. This means that less iron is available for attraction in the thin spring, or that, if it were the purpose to increase the sensitiveness, an iron armature should be mounted on the spring just in front of the magnet.

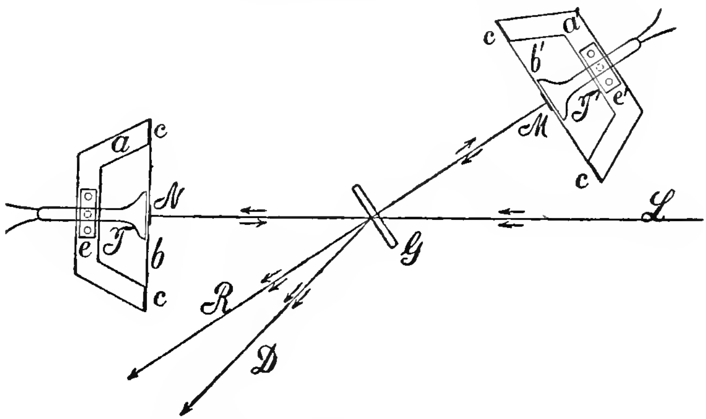

FIG. 88.—Telephonic induction balance on interferometer.

124. Reciprocating and synchronized telephones. Apparatus.—The next step in development was the insertion of the second telephone T, carrying the micrometer N (fig. 88) and furnishing with the mirror M, also on a telephone plate, the two component beams of the interferometer. The two identical saw blades of the preceding case were used in preference. In the diagram L is the entering beam of white light, G the grating, R the undeviated, and D the diffracted rays. The telephone T is held in a wooden yoke a, eventually brass-faced at cc. To these plates the saw blades bb * are screwed down. The telephones are adjustably secured at e. The

* Barus, American Journal (3), III, 1897, pp. 107–116.

yokes a, a' are fixed in short vertical ends of gas pipe clamped to the arms of the interferometer, admitting of sufficient vertical and horizontal rotation for rough adjustment. The fine adjustment for horizontal and vertical coincidence of the slits is made at a by three leveling screws (horizontal and vertical axes), and the yoke is additionally mounted on the plate of the micrometer screw. It is this contrivance which is to be specially designed and which at present constitutes the difficulty of the method; for the telephone T must be rigidly placed and yet be delicately adjustable and movable on a micrometer.

The electrical interferometer or induction balance * so obtained should be capable of exhibiting the phenomena illustrated by fig. 89, where the telephone T is within the resistances R and R', whereas T' is in immediate connection with the battery by the key K. Hence, if $R=R'=0$, both telephones are simultaneously affected and the plates vibrate symmetrically towards and from each other as at a. The ellipses should therefore remain in the field. When $R=R'$ is increased sufficiently the time must arrive when both plates move in the same direction in parallel as at b, the lag being half a period. The ellipses should therefore vanish. For a further increase of $R=R'$ the lag will become equal to a whole period and case a is reproduced, etc.; or two different circuits coupled together may act separately on T and T' and the time element of the coupling be determined, since the ellipses vanish for lags of an even number of half periods and appear for lags of an odd number of half periods.

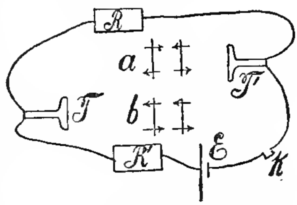

FIG. 89.—Diagram of induction balance.

125. Reciprocating and synchronized telephones. Measurements.—In the first experiments with the apparatus, the blades were screwed down on wood surfaces, not brass-faced, and purposely fixed just short of loosely. Table 85 contains an example of the results which are of the same nature as the preceding.

TABLE 85.—Two telephones. Deflection $\Delta N+\Delta N'$ summational. 2 storage cells. Blades as in table 84.

R	i	$\Delta N \times 10^3$	r
400	0.0067	0.80	122
100	.0133	2.00	
300	.0080	1.15	231
60	.0155	2.10	
200	.0100	1.80	171
0	.0200	2.75	

Some of the observations are not good, as is shown in the values of r, which should be constant. Probably the blade was too loose. The sen-

* Barus, American Journal (3), III, 1897, pp. 219–222. Note on the excursion of the diaphragm of a telephone.

sitiveness is nearly constant and equivalent to about $\Delta N = 0.14$ cm. per ampere, not twice the preceding value, because the blades were probably further from the magnet.

On connecting the two telephones differentially so that the deflections proceeded in opposite directions, practically no displacements of ellipses was obtained. Thus at $R = 100$ ohms, $\Delta N = 0.0001$ cm.; at $R = 50$, $\Delta N = 0.00015$ cm., etc.—a difference which could have been removed by more carefully spacing the saw blades in front of the magnets of the telephones.

The expectation that, on applying alternating currents, the ellipses would remain in place in case of differential adjustment, was, however, frustrated. They practically vanished, either because the support as a whole was put in vibration whenever the telephone sounded, or because the upper harmonics play along the length of the saw blade. In either case the practical solution is not yet at hand, even for the case of identical counter-motion of the two mirrors.

In the following experiments the yoke holding the saw blade was faced with brass and the blades rigidly screwed in place. An example of the results is given in table 86. Sensitiveness was increased by approaching the blades closer to the magnet of the telephone. This table still shows the same uncertainty as to $r = \dfrac{R\Delta N - R'\Delta N'}{\Delta N' - \Delta N}$, which can only be ascribed to the small value of the denominator of the equation, as the current i is roughly proportional to ΔN. Thus half a drum division, which may easily be due to slight changes of adjustment, would have corrected the first value of r. The sensitiveness is greater than before, being $\Delta N = 0.185$ cm. per ampere, owing to smaller distances between magnets and armatures.

TABLE 86.—Yokes brass-faced. Rigid blades. $E = 2$ volts. Mirrors traveling in same direction.

R	i	$\Delta N \times 10^3$	r
300	0.0040	0.55	112
50	.0080	1.40	
220	.0050	.85	171
20	.0091	1.65	
100	.0067	1.20	200
0	.0100	1.80	

On commutation, *i.e.*, when the mirrors moved in opposite directions, the displacement ΔN showed small values similar to the preceding case. Alternating currents, however, did not now quite wipe out the ellipses. In the differential adjustment they were fairly clear, showing the advantages of the new form of apparatus.

In the final experiments the steel springs were iron-faced (armatures in the rear and just in front of the telephone magnet and immediately behind the small mirrors). This armature of soft iron, 3 cm. long, 1 cm. wide, 0.3 cm. thick, increased the strength of the forces involved and probably had a tendency to wipe out the possibility of nodes in the middle of the spring, *i.e.* at the mirrors, and thus of eliminating rotation at this place. The results given in table 87 seem to bear out this surmise. The irregularity of the values of r has not been overcome, although the observations were more definite than heretofore. It fails of explanation even if the currents are supposed to be not proportional to the displacements. It is rather due to some actual but irregular difference in the position of the mirrors before and after the displacement produced by the current. When the mirrors travel in opposite directions, ΔN is not quite zero, which simply means that one blade is a little nearer the magnet than the other.

TABLE 87.—Iron-faced springs. $E=2$ volts.

I. Mirrors traveling in same direction. Telephones in series.				II. Mirrors traveling in opposite directions. Telephones in series.		
R	$i\times10^3$	$\Delta N\times10^3$	r	$i\times10^3$	$\Delta N\times10^3$	r
400	3.3	0.60	177	3.3	0.00	...
100	6.7	1.25		6.7	.25	...
300	4.0	.88	252	4.0	.05	...
50	8.0	1.60		8.0	.40	...
200	5.0	1.07	191	5.0	.20	...
0	10.0	2.20		10.0	.50	...
III. N cut out; M alone vibrating.				**IV. M cut out; N alone vibrating.**		
R	$i\times10^3$	$\Delta N\times10^3$	r	$i\times10^3$	$\Delta N\times10^3$	r
400	4.0	0.45	59	4.0	0.15	...
100	10.0	1.30		10.0	.73	
300	5.0	.75	129	5.0	.25	33
50	13.3	1.80		13.3	1.00	
200	6.7	.90	81	6.7	.57	102
0	20.0	3.10		20.0	1.70	

In Series I the sensitiveness is about $\Delta N=0.2$ cm. per ampere and the data are fairly regular. In Series II, however, the case of opposed vibrations, the displacements begin after the current has reached a relatively large value, and the ensuing sensitiveness would be about $\Delta N=0.07$ cm. per ampere. Correspondingly the telephone M showed a reasonably regular march with the sensitiveness of 0.14 cm. per ampere, while for the fixed telephone N the displacement does not begin with the current, the mean sensitiveness being thereafter about 0.1 cm. per ampere. No doubt the way in which the blades are incidentally clamped, whether with or without a small initial strain, has much to do with this. The sensitiveness for the two telephones in series is naturally reduced by the double resistance. It appears from this that for small currents (less than 0.003 ampere) the telephone if differentially connected should show no displacement. This was in fact the case on introducing the alternating current from a small induction coil. The ellipses were nearly as clear for two telephones in series as in the absence of current, if the mirrors vibrated in opposite directions; while the ellipses vanished almost completely when the mirrors vibrated in the same direction. Unfortunately the case of telephones in parallel was not tried, though it would probably have made no difference.

It was now attempted to equalize the displacements of the two telephones with the results for mirrors traveling in opposite directions and the telephones in series (2 volts applied)

$$R=0 \qquad i=0.01 \qquad \Delta N=0.0025 \text{ cm.}$$

The sensitiveness is increased to 0.25 cm. per ampere, because the weaker action of one of the telephones was strengthened. On using alternating currents, however, there was not much advantage observed for the new adjustment, probably because only weak currents could be used in both this and the preceding case.

To conclude: It has been possible to devise an induction balance consisting of two modified telephone plates vibrating synchronously on the interferometer in such a way that if the vibrations are in opposite directions the ellipses remain in the field, while they gradually vanish for vibrations more and more in the same sense. Some experiments to apply this apparatus in various practical directions were begun, but the work was discontinued for the present, as it is not in line with the main purpose of the present paper.

CHANGE OF REFRACTION RESULTING FROM DIELECTRIC POLARIZATION.

126. Introductory.—The experiments made on the value of Kerr's constant subsequently to the fundamental researches of the discoverer of the phenomenon are very numerous. It will suffice here to refer to the summary by Prof. L. Graetz in Winkelmann's Handbuch, vol. 4, p. 168 *et seq.* If d is the optical retardation due to an electric field f in a column of length e

$$d=Bef^2 \tag{1}$$

where B is Kerr's constant. If μ_0 and μ_e be the indices of refraction for the two rays due to the presence of the electrical field

$$d=e(\mu_0-\mu_e) \tag{2}$$

or

$$\mu_0-\mu_e=Bf^2 \tag{3}$$

If $\mu_0-\mu_e$ can be measured as above, the value of B would depend only on the measurement of f in volts/cm.

The present paper, however, gives merely a superior limit to the amount of double refraction produced by electrostriction in glass. The accuracy with which the index of refraction may be measured in case of long columns of solids, or of liquids, with plane parallel ends, made it seem probable that the increase of index at right angles to the lines of electric force in a glass condenser might be detected. If so, it could at once be measured absolutely. But the amount in the solid is so small that in the first experiments no displacement of the ellipses or motion of the rings was visible when a strong electric field was alternately applied and removed.

127. Apparatus.—Liquids were inclosed in glass tubes about an inch or more in diameter with a tubulure normal to the axis in the middle. The ends were ground flat and plate glass fitted to the ends. One of these plates was cemented on either with glue or resinous cement, as the occasion required; the other, when rubber was permissible, was put in place by pressing it against a thick rubber cushion, intervening, with three adjustment screws. In this way it was possible to put the two faces adequately in parallel by observations on the interferometer, *i.e.*, the direct images of the slit should coincide horizontally or vertically, no matter whether the column is present or absent in one of the interfering beams. In the later experiments, selected troughs of plate glass, to be described below, were used.

In case of glass, plates about 0.7641 cm. in thickness, 6 cm. long and 2.5 cm. wide, were placed with their solid faces in contact, as shown in fig. 90. A column about 11.4 cm. long was thus built up, with 15 plates

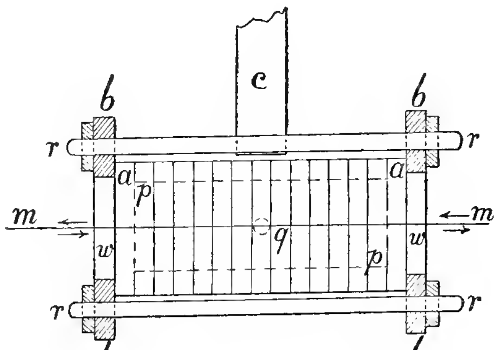

FIG. 90.—Pile of plates for Kerr effect, side view.

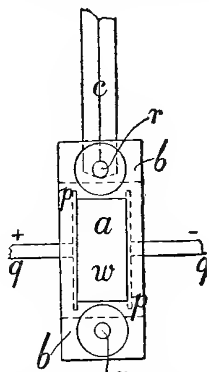

FIG. 91.—Pile of plates for Kerr effect, front view.

in contact and compressed by a rectangular screw clamp, *bbbb* (fig. 90), of hard rubber. The clutch of the longitudinal rods *rr* could be shortened by hard-rubber nuts at their ends. The rubber end-plates *bb* were provided with a window each, so that the interferometer beam could be passed through the column, before and after reflection at the opaque mirror on the micrometer. The whole arrangement was firmly held by a hard-rubber clutch *c*, suspended from a wall bracket. Suitable clamps admitted of raising and lowering and of two rotations. To supply the electrical field, the side faces of the glass plates were partially covered by copper plates *pp*, *pp*, confronting each other, and provided with terminals *q*, *q*. These plates were held in place with rubber cement (picein), so that *ppapp* together form a plate condenser, the glass dielectric being 2.5 cm. thick. The terminals q and q (see fig. 91) communicated with the prime conductors of a Holtz machine, and the insulation was sufficient to admit of sparks as long as 1.5 cm., even in the summer. The conductors of the Holtz machine were then pulled apart, so that a potential difference of 40,000 volts was assured. The maximum electrostatic field applied was thus about 16,000 volts/cm.

On placing the pile of plates between crossed Nicols with their principal sections at 45° to the lines of electrical force, no effect due to the presence or absence of the field could be discerned. There was usually a little permanent brightness due to stress in the insufficiently annealed glass.

When the plates were placed on the interferometer a sharp series of multiannular small ellipses was easily obtained. This part of the experiment was in every way satisfactory. The screw of the micrometer, however, was but 3.5 cm. long between the end positions of the slide (maximum displacement). The glass column would require a displacement of nearly twice this for the measurement of the index of refraction (shift of micrometer corresponding to presence and absence of the plate). But this index could be obtained with sufficient accuracy for the present purposes from a single plate, so that a thick compensator was introduced into the other component beam, n, of the interferometer, the compensator consisting of a pile of plates about 7 cm. thick. This reduced the effective thickness of the column to about 4.4 cm., requiring a play of screw of a little over 2 cm. to bring the center of ellipses to the fiducial sodium line, in case of the simultaneous presence and absence of the condenser and the compensator. On the other hand, the sensitiveness of displacement is not diminished, but corresponds to a column of glass 11.4 cm. long.

128. Equations.—Let the index of the uncharged transparent insulating column in the direction normal to the lines of electric force be μ_0; let the second index, due to electric stress of the charged plate, be μ_e. Let b be the constant of the simplified Cauchy equation for the wave-length λ. In the absence of the column let the center of ellipses coincide with the sodium line of the spectrum (air position); and let ΔN_0 and ΔN_e be the micrometer displacements which bring the center of ellipses back to the fiducial line when the transparent column is inserted, charged and uncharged, respectively. Then

$$\mu_0 - 1 + 2b/\lambda^2 = \Delta N_0/e \tag{4}$$

$$\mu_e - 1 + 2b/\lambda^2 = \Delta N_e/e \tag{5}$$

where e is the length of the column. Hence

$$\mu_0 - \mu_e = (\Delta N_0 - \Delta N_e)/e \tag{6}$$

and

$$(\mu_0 - \mu_e)/\mu = (\Delta N_0 - \Delta N_e)/e\mu \tag{7}$$

if μ is the mean index of the column (practically equal to either index).

As the evanescence of a ring corresponds to a shift $\Delta N = \lambda/2$, or roughly 30×10^{-6} cm., and as a change of one-fifth of this may certainly be detected, the limit of discernible $\Delta N_0 - \Delta N_e$ may be reckoned as 6×10^{-6} cm.

129. Data. Solid.—As has been stated, it is not necessary for the present purpose to determine the air position for long columns. For a plate $e=0.7641$ cm. in thickness showed a displacement of 0.4225 cm.; and hence by equation (1) the displacement for 15 similar plates, or a thickness of 11.46 cm., would be $\Delta N = 6.34$ cm. Since no radial motion of ellipses was discernible, due to presence and absence of electric field,

$$\Delta N_0 - \Delta N_e < 6\times 10^{-6}/6.34 = 9.5\times 10^{-7}$$

Hence

$$\mu_0 - \mu_e < 9.5\times 10^{-7}/11.46$$

or less than 8.3×10^{-8}. The index of refraction μ computed by equation (1) was roughly $\mu = 1.53$; whence

$$(\mu_0 - \mu_e)/\mu = 5.4\times 10^{-8}$$

Thus the change of index of refraction normal to the lines of force, in case of a field over 1 kilovolt per centimeter, is probably less than 10^{-8} of its value and quite beyond the scope of even the present sensitive adjustment of the interferometer.

These experiments were made in the summer, and considerable annoyance was encountered owing to the dampness of the laboratory. It is purposed, however, to repeat them in the dry laboratory in the winter months, and a report will then be made.

130. Data. Liquid.—In case of carbon disulphide, the absolute value of B is known from Lemoine's measurements and equal to $B=3.70\times 10^{-7}$, if the field is given in electrostatic units. In case of a volt per centimeter B would then be roughly 4.1×10^{-12}. Hence for the above field of $f=16\times 10^{-3}$

$$\mu_0 - \mu_e = 4\times 10^{-12}\times 256\times 10^{9} = 10^{-3}, \text{ nearly,}$$

which should be easily observable. In other words, since

$$\Delta N_0 - \Delta N_e = e\Delta\mu = 10.46\times 10^{-3} = 10^{-2}, \text{ nearly,}$$

displacement would be

$$10^{-2}/5\times 10^{-5}$$

or 200 times the least perceptible value.

Unfortunately the expectations were doomed to disappointment, in view of the large variation of carbon disulphide with temperature. It was found impossible to obtain an adequately sharp image of the slit for the case of a beam of light passing twice through the column of carbon disulphide, in consequence of the convection currents set in motion by the heat of the beam. The slit image was invariably washed and unsteady, and to obtain the interferences under these conditions was out of the question. As these troughs were 10 cm. long and high, but only 1 cm. broad, it is possible that horizontal partitions of mica may in a measure remove these difficulties, but the complications introduced in this way are so great that further trials were for the present abandoned.

CHAPTER XIV.

THE INTERFEROMETRY OF AIR CARRYING ELECTRICAL CURRENT.

131. Introduction.—The following experiments, though leading (as was to be anticipated) to negative results, are nevertheless sufficiently interesting in their details to deserve to be reported. The object in view was direct test as to whether a rarefied column of air, through which a current of electricity is flowing, shows any perceptible change of its index of refraction. Such an effect might result from the occurrence of ionization, or from rise of temperature, or, finally (in the extreme case), from a possible influence of rapidly moving corpuscles on the velocity of light traveling in the same direction as the corpuscles. The means of exhaustion at present at my disposal were not sufficient to carry the vacuum much below 1 mm. of mercury; neither were exceptionally large potential differences employed, so that the experiment has not been pushed to a limit at which it might possibly show results. I shall hope to return to the work at some other time. The present paper attempts therefore to do no more than to describe the adaptability of the displacement interferometer for present purposes.

132. First experiments. Apparatus.—At the outset the interferometer was used without other modification than a marked elongation of the arms *GM* and *GN*, where *G* is the grating, *M* (the micrometer) and *N* the opaque mirrors. The component beams of light *m* and *n* were now 1 meter in length, each, so that a glass tube *t*, hermetically sealed at its ends with plate-glass windows, provided with a tubulure for exhaustion at *E* (the tube being nearly 1 meter in length), could be inserted in either beam. The arc light or sunlight enters at the slit *S* of the collimator *C*. The direct beam for adjusting the interferences is observed with the telescope at *R*, and the interfering diffraction spectra by the same telescope on being rotated into the position *D*. The sodium line from the two coincident spectra is an admirable fiducial mark to which the centers of ellipses, as they move across the field, are always referred. A fine horizontal wire across the slit enables the observer to place the spectra in coincidence both horizontally and vertically.

As in the earlier instrument, the arms *m* and *n* were made of gas-pipe, through which a current of water continually circulates, when necessary. Tremors, of course, could not be wholly eliminated; but the instrument, in spite of its lightness, was made firm by the aid of horizontal and vertical leveling screws, which imparted slight strains to the gas-pipe arms *m* and *n*; *i.e.*, the ends at *M* and *N* were pushed away from the wall by horizontal screws, until sufficient strain of parts resulted, while at the same time they were lifted above the table carrying the tripod (below *G*), by similar screws acting vertically. The latter, moreover, were advantageous in bringing (raising or lowering) the center of ellipses into the center of the field, after an approximate adjustment had been made. Similarly the foot of the revolving arm, which supports the telescope at *D*, could be used for the same purpose with advantage.

An open collimator at *C*, *i.e.*, objective and slit each in opaque screens, is specially convenient, inasmuch as it allows the image of the slit *S* reflected at *M* to be visibly reproduced on the jaws of the slit. The occurrence of parallel light and of a beam of light normal to *M* are thus both put in evidence. If *m* is cut off, the same applies to the mirror *N* and beam *n*, though this is liable to be too dark, and the rough and fine adjustments are best made at the telescope, using a wide slit first. The collimator *C*

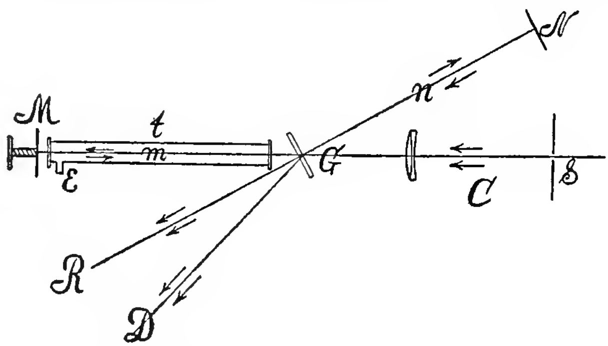

FIG. 92.—Diagram showing vacuum tube on interferometer.

should be long; in other words, the lens of long focal distance, so that the beam *m*, passing through the tube, may remain very short or spot-like in its vertical dimensions. Otherwise too much is cut off by the tube and reflection from *G* before the beam enters the telescope.

With the arc lamp the spectrum may be darkened, so that the sodium lines stand out clearly, by raising or lowering the arc behind the black screen, with a hole about 1 cm. in diameter for the passage of light. A small swiveled screen in front of the objective of the telescope, and approached from one side, greatly sharpens the interferences and the Fraunhofer lines by cutting off undesirable light and particularly the stationary interferences. Possibly the latter would disappear if the plate of the grating were made very thick. When water circulation is used, some time must elapse before the temperature conditions are adequately stationary. With distances *m* and *n* as long as the above, the ellipses are rarely perfect and frequently appear coarse and distorted. It is nevertheless easy to adjust the center of ellipses with an error not larger than 0.00005 cm., at the micrometer screw *M*, *i.e.*, to about the mean wave-length of light.

133. First experiments. Results.—The sealed glass tube, t, of length $e=89.49$ cm. within, having been inserted, the data for the refraction of air, found by alternately exhausting and filling it, were about as given in table 88.

TABLE 88.—Glass tube with plate-glass ends; diameter 4.5 cm.; length, 89.49 cm. Barometer, 75.59 cm. at 17.5°; temperature about 20°.

p_0	p	$10^5\Delta N$	$\mu-1$
75.59	0.32	2505	280
	.10	2495	279

Here ΔN is the displacement of the micrometer corresponding to the pressure difference p_0-p, at the constant temperature given, and μ the index of refraction of the air contained.

The induction coil was now attached and a current from 3 to 4 storage cells sent through the apparatus. Not the slightest shifting of ellipses or interference lines could be detected when the secondary current was alternately made and broken. The lines were very clear. The experiment was then repeated and the air-pump (Geryk) connected directly with the tube without accessory apparatus between. A coarsely stratified column and some cathode dark space was obtained. But the effect at the interferometer was again definitely negative.

The absence of an effect due to the current was to be expected, since the positive and negative currents and the direct and returning beam of light both traveled in opposite directions. The same is true for the ionization, which is too slight; yet it was supposed that some evidence of a temperature effect might be obtained; but there was none.

Since

$$p=C(\mu-1)\vartheta=C\Delta N\vartheta/e$$

where C is constant and ϑ denotes absolute temperature, therefore at constant pressure

$$\frac{d(\Delta N)}{\Delta N}=\frac{d\vartheta}{\vartheta}$$

If $d(\Delta N)=5\times10^{-5}$ cm., $\Delta N=25\times10^{-3}$ cm., $\vartheta=293°$, $d\vartheta=0.59°$; so that whatever rise of temperature may have occurred must have been much less than 0.6° C.

134. Second experiments. Apparatus.—In this experiment the direct and return beams m and n were separated, as shown in fig. 93, into m and m' and n and n', the beams traveling along contiguous sides of a rhombus. M and N are again the two opaque mirrors, the former with micrometer; C is the collimator, open as in fig. 92. The grating above the tripod of the interferometer has been removed and replaced by a plate of glass P, of about the same thickness, while the grating is at G diametrically opposite, with its ruled face toward N. The normals of the mirrors are shown at p and p', and the telescope is either at R (direct ray) or at D (diffracted ray). It is here convenient to have two telescopes, one set at R and the other at D.

To secure this adjustment, the interferometer of fig. 92 is provided with a cross-arm of gas-pipe under PG, which can be clamped firmly in place on the standard of the tripod, below P in fig. 93, or G in fig. 92. The end of the arm below G, fig. 93, has a foot with a vertical set screw for imparting strain, and is braced with slight strain against rotation around the cross-arm. In the same way M must be laterally braced with slight strain against rotation around the arm under N or NP.

The arm of the telescope R, in fig. 92, is in this case turned about P, the telescope being simultaneously rotated until it occupies the position D in fig. 93. It is for this reason that two telescopes are used, as it would be much less convenient to provide a special axle moving about the foot of G.

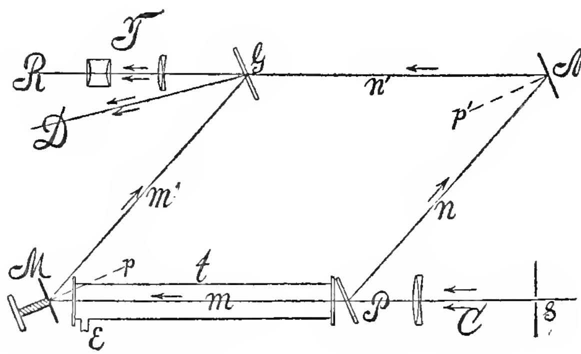

FIG. 93.—Displacement interferometer with separated pencils and vacuum tubes.

To adjust the apparatus the arms, m' and n', are first made roughly equal with a scale. P and G are then placed in the same line by a straight edge; the slit is widened so that the beam m passes directly through central parts of P and the reflection (spot) from the central parts of the mirror M is seen to strike the central part of the grating G. Proceeding thence, the beam is reflected into the telescope T approximately in position (both M and G being rotated horizontally and vertically for this purpose, finally by the adjusting screws). Next the beam reflected from P is made to strike the central parts of the mirror N (which it should do at once on very slight rotation of P), from which it is reflected to the central parts of the grating G, so that the two beams m' and n' may be across the same vertical line. Finally G is adjusted, until the two direct images coincide accurately (both horizontally and vertically) in the telescope, a fine hair having been drawn across the slit, as above.

Under these circumstances there will be four direct images in the telescope at *R*, due to front and rear reflection at *P* and *G*, neither of which is optical plate; but there will be but three spectra or three sodium lines visible at *D*, since the ray reflected at the rear face of *G* is not diffracted. To obtain the interferences, a single spectrum line from *M* must be placed fully in coincidence with either of the two lines from *N*; but to find them is nevertheless a matter of considerable difficulty. Since reflection takes place not from one and the same face of glass, but from the two independent faces *P* and *G*, the ellipses in the field of view are liable to be very eccentric. They thus appear as the merest hair lines, easily overlooked, even when *M* has (by trial) been moved into the correct position. In fact if the sodium lines be called Na_m, Na_n, $Na_{n'}$, I have only been able to get interferences from two of these, say Na_m, Na_n, but not from Na_m $Na_{n'}$, after wasting much patience in the attempt. There may be some other reason for this which has escaped me. Na_m passes through three thicknesses of glass, whereas Na_n and $Na_{n'}$ pass through 1 and 3 thicknesses, respectively, and I have not been able to ascertain whether the self-compensating case or the other is the one which succeeds. In fact, the four lines m, m', n, n' are not necessarily coplanar, but will lie on a ruled surface, which is to be made as nearly plane as possible. Thus the plates *P* and *G* may be clamped to a long strip of plate glass, or other devices employed. These succeed adequately after some readjustment, but I have not been able to place the center quite in the center of the field of view. Experimentally this is not necessary, for the eye is quite as sensitive in placing the horizontal element of the circle in coincidence with the fiducial sodium lines (which it intersects at right angles), the inclination on the two sides being in opposite directions. Naturally the interference bands should be strong.

The micrometer screw of *M*, in this case, bisects the acute angle of the rhombus of $\varphi=30°$, and the motion of the mirror over ΔN here cuts off $2\Delta N/\cos 15°$, from the beam of light. Moreover, at the grating the beam of light is shifted laterally when *M* advances, by an amount $\Delta N \tan 15°$, so that $2\Delta N \sec 15° \sin^2 15°$ is restored. Thus the path difference produced is generally $2\Delta y = 2\Delta N \cos \varphi/2$.

135. Second experiments. Results.—The data obtained in case of the rhombus are given in table 89.

TABLE 89.—Glass tube as in table 88. Barometer, 76.0 centimeters.

p_0	p	$10^5\Delta N$	$10^5\Delta y$	$\mu-1$
76.0	0.4	1285	1241	287
....	....	1280	....	286

The data are of the same order as the preceding table, remembering that the tube is not twice traversed by the beam of light, as is the case with the adjustment of table 88.

The current from the induction coil was now passed through the exhausted tube and the circuit made and broken. Not the slightest effect could be observed, the interferences remaining stationary in all parts of the field, just as in the preceding case.

The endeavor must now be made to exhaust the tube to the highest degree possible, and to pass the current between electrodes of very high potential difference. To this I hope to return at some other opportunity.

CHAPTER XV.

THE REFRACTION OF HOT GASES.

THE REFRACTION OF AIR AT HIGH TEMPERATURES.

136. Introductory.—Inasmuch as the displacement interferometer admits of the determination of the index of refraction, no matter how large the excursions of the micrometer screw may be, it was thought interesting to examine the refraction of air and other gases at temperatures as high as may be applied. The gas for this purpose should be inclosed in a long hermetically sealed tube of metal (brass or iron) or porcelain, provided with glass plates at the ends. At very high temperatures this desideratum will have to be modified.

A number of difficulties at once present themselves because the effect of temperature on the glass plates, as well as the irregular distribution of temperature in the surrounding air, can not be eliminated unless the tube admits of exhaustion. In such a case difference readings (N_0, the position of the micrometer for a plenum of air, and N, corresponding to the position for a vacuum), at the same temperature, at once eliminate all discrepancies. The exhausted condition may alternate with the full condition of the tube and the result is enhanced, since the index is not very sensitive to pressure, *i.e.*, very perfect exhaustions need not be reached if the degree of exhaustion is known. Nevertheless the need of hermetically sealed windows is imperative, and this makes the experiment exceedingly difficult for temperatures approaching or exceeding red heat. In the present paper it is not the endeavor to go as far as this.

If the exhaustion at high temperature is not admissible, it is then necessary to determine the effect of temperature on the plates separately—an equally difficult problem, since difference readings can not be made. Changes of temperature in the air surrounding the tube become increasingly important. The cold-air column must also be kept in a water bath. Moreover, it is now absolutely essential that the arms of the interferometer be of absolutely invariable length for long periods of time. As in the preceding paper, they were made of gas-pipe provided with a continually circulating current of water from the hydrant.

The tube is inserted (fig. 94) so that one of the component beams *mm'* of the interferometer may pass through it axially. It is desirable for this purpose to have the collimator of long focus. The beam in such a case remains slender and narrow throughout its extent within the tube, and the clear space at the windows is not much over 2 cm. in diameter, when the outer diameter of the tube is over 3 cm. It is perhaps best to have the focus of condensation in the vertical plane at the micrometer mirror. In its lateral dimensions the beam is almost without breadth, being sheet-like.

The tube and apparatus for heating were suspended from a wall bracket, quite free from the interferometer. To obtain the low temperature a current of cold water was circulated through the annular space *BB*—the same current, in fact, which traversed the arms of the interferometer being used. This water circulation might also be employed to keep the counter column of air at a constant low temperature. However, when the method of exhaustion is applicable such a safeguard is not needed; for in this case the time consumed is small enough to admit of the assumption of an average mean temperature in the region as a whole, during the interval of measurement.

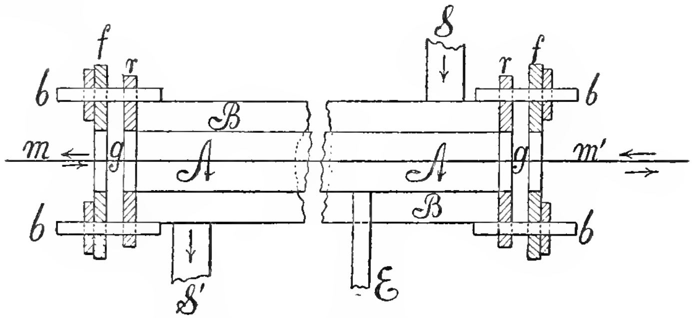

FIG. 94.—Heating apparatus for interferometry of air.

The high temperature was obtained from a current of steam taken from the steam-heating plant of the building. In later experiments it was found advantageous to generate the steam more quietly in a special copper boiler. The air was dried over phosphorus pentoxide.

To obtain a temperature of 200°, the limiting value in the present paper, a brazed vapor bath of naphthalene was available. In all these cases measurements may still be made with a mercury thermometer. At higher temperatures the direct determination by means of a thermo-couple will have to be resorted to, which introduces an additional difficulty, since the junction of the couple will also have to enter the hermetically sealed tube.

137. Small apparatus.—The interferometer is the same as that used in the preceding paper, the available length of arm being about 1 meter, and the free radius or vertical depth below the beam nearly 15 cm. It would be easy to increase these dimensions, but the apparatus in such a case becomes more sensitive to tremors and a greater distance is not needed. In the first experiments a short tube of brass 25.34 cm. long was used, adapted for measurements up to 100° C. only. The parts were soldered together, as shown in fig. 94, in longitudinal section. *A* is the cylindrical air-chamber provided with a tubulure *E* for exhaustion. *B* is the annular cylindrical steam or water chamber, the vapor or liquid entering at *S* and escaping at S'. The tube should be jacketed without. The ring-shaped ends of the tube are turned smooth and provided with glass plates g pressed against rubber gaskets r by rings of fiber f and bolts bb, the inner ends of which are soldered longitudinally to the outside of the tube. It was at first difficult to keep the system quite tight, but it becomes so in the course of time.

138. Equations.—If we use the extended equation of Mascart, μ and μ_0 being the index of refraction at $t°$ and 0° C., respectively, at the pressure p in centimeters of mercury, and α the thermal coefficient

$$\frac{\mu-1}{\mu_0-1}=\frac{p}{76}\,\frac{1+\beta p}{1+\alpha t} \qquad (1)$$

Since the coefficient β is very small ($\beta=0.0000017$, relative to centimeters of mercury), if the two observations at t and t' be made at about the same pressure p, the binomial $1+\beta p$ may be neglected so that, $p-p'=0$

$$\frac{\mu-1}{\mu'-1}=\frac{p}{p'}(1+\beta(p-p'))\frac{1+\alpha t'}{1+\alpha t}=\frac{N-N_0}{N'-N_0'} \qquad (2)$$

where N_0 and N, N_0' and N' are the two positions of the micrometer for a plenum and a vacuum at the same temperature, at t and t', respectively. If for brevity the observed ratio

$$\frac{N-N_0}{N'-N_0'}=\frac{\Delta N}{\Delta N'}=R' \text{ and } \frac{N-N_0}{N'-N_0'}\,\frac{p'}{p}=R \qquad (3)$$

$$\alpha=\frac{R-1}{t'-Rt} \qquad (4)$$

If it were admissible to put $N_0=N_0'$ for the plenum of air and $\delta t=t'-t$, $\Delta N=N-N_0$, $\delta N=N-N'$, the equation

$$\alpha=\frac{\delta N}{\Delta N\delta t-t'\delta N}=\frac{1}{\Delta N\delta t/\delta N-t'} \qquad (5)$$

might be used; but this is not the case. It shows, however, the difficulty of measurement, as δN is of the order of 0.0012 cm., ΔN of the order of 0.007 cm., if δt is about 80° C. Hence, since the smallest error of N is about 0.00005, the value of δN can not be trustworthy to more than 5 per cent, quite apart from the manifold other difficulties of measurement. Hence the need of using longer tubes than the present.

Equation (5) may, however, be remodeled by replacing

$N-N_0$ by $(N-N_0)/p=\Delta N/p$ $\qquad$ $N'-N_0'$ by $(N'-N_0')/p'=\Delta N'/p'$ (6)

and putting $\delta t=t'-t$ and $\delta N=\Delta N-\Delta N'$, in which case equation (5) is generally applicable in the form given.

139. Data.—The results for α in table 90 show considerable differences for the reasons stated.

TABLE 90.—Refraction of air. Short tube, length $e=25.342$ cm. $\Delta N = N - N_0$ at low temperature, pressure interval p; $\Delta N' = N' - N_0'$ at high temperature, pressure interval p'. μ_0 refers to 0° centigrade and 76 cm.

t	p	$\Delta N \times 10^3$	$10^6 \times$ $\mu - 1$	$10^6 \times$ $\mu_0 - 1$	t'	p'	$\Delta N' \times 10^3$	R	$10^3 \times \alpha$
15.8°	67.94	5.56	219.4	260.1	100°	67.94	4.24	1.312	3.93
....	....	....	....	....	100	74.55	5.36	1.305	3.82
14.0	74.55	7.00	276.2	296.5	100	74.55	5.43	1.288	3.52
....	....	....	....	....	*100	76.91	5.62	1.286	3.48
18.9	75.98	6.90	272.1	291.6	100	75.98	5.48	1.259	3.40
16.2	75.93	7.01	276.4	293.6	100	75.98	5.48	1.280	3.54
18.9	74.67	6.95	274.1	298.9	100	74.67	5.41	1.284	3.75
....	....	....	....	....	100	74.67	5.50	1.263	3.45
....	....	....	....	....	100	74.67	5.41	1.284	3.75
....	....	....	....	....	100	74.67	5.39	1.290	3.83

*Corrected for p'/p; cold measurements not made on same day.

These observations are obtained from triplets, two at high pressure (barometer) alternating with one at low pressure (partial vacuum). The observations for ΔN, of the order of 0.007 cm., and those for $\Delta N'$, of the order of 0.0055 cm., should each be correct to about 1 per cent; but their difference δN, of the order of 0.0014 cm., will not be reliable to more than 4 per cent. Since

$$\frac{d\alpha}{d(\delta N)} = \left(\frac{\alpha}{\delta N}\right)^2 \Delta N \delta t$$

the effect on α is of the order of 5 per cent, when $d(\delta N)$ denotes the limiting precision of 5×10^{-5} cm.

The values of α obtained from the successive series on different days, if we neglect the first crude results after which the details of the apparatus were gradually more and more perfected, range from $\alpha = 0.0034$ to $\alpha = 0.0038$, *i.e.*, over about 10 per cent, an interval twice as large as the estimated minimum error. It is difficult to find a reason for this. Undoubtedly the arms of the apparatus are continually varying in length, in view of the micrometric scale on which the observations of length measurement are made and in spite of the flow of cold water at nearly constant temperature. The atmospheric temperature also, both in front of and behind the steam tube as well as in the other arm of the apparatus (*GN* in fig. 92, Chap. XIV), is continually fluctuating around a mean value. The temperature and refraction of the plate-glass caps of the apparatus are changing slightly; but it does not seem probable that, in the short interval of time between the occurrence of a plenum of air and a vacuum in the refraction tube, there should be any discrepancy sufficient to increase the minimum error two times. The possibility of discrepancies from adiabatic expansion, moreover, can be disregarded in so thin a tube, and in the case of a vacuum they vanish naturally.

It is true that in the course of the work the ellipses gradually undergo change of form or of clearness. Some part of the apparatus or the supporting table is continually undergoing slight change of form. Similarly on successive days (different series in table 90) the values of α, though nearly identical and consistent on the same day, show marked differences due to undetected differences of adjustment, etc. The ellipses, if the steam tube is turbulent, are liable to be in incessant vibration, though in the last series this difficulty was practically conquered. Finally, the actual error may possibly reach twice the minimum error in the case of two single readings.

The only way of overcoming the elusive causes of error referred to must therefore consist in the installation of a long tube and the use of longer ranges of temperature. This is attempted in the next section, where a tube about 28 inches long shows about $\Delta N = 38.4$ scale parts or about 0.019 cm. at ordinary temperature, being thus nearly three times as large as the above. On the same scale of errors, it should therefore be possible to obtain α to a few per cent. Moreover, the rubber gasket, which at 100° and low pressures may essentially modify the air contained in the tube by vitiating it with sulphur fumes, should be abandoned. It is not impossible that such discrepancies may have entered the above data; but they were not considered serious, because at high exhaustions the difficulty cures itself, as there is an absence of gas; whereas at the large pressures sulphur is insufficiently volatile at 100° to be a menace. Anticipating the results of the next section, it is possible that the difference of values on successive days is due to the steam supply. To my surprise it was difficult to heat the air in the tube with glass ends, to 100° permanently, in spite of the jacketed steam bath, fed with a steady flow from the radiator pipe. Measurement of temperature on the inside of the tube at this stage of the work seemed superfluous and no such tests were made.

140. Long tubes.—To obtain a larger value of $\delta N = N - N_0 - (N' - N_0')$, longer tubes must be resorted to and the present experiments were therefore made with staunch brass gas-pipe, 28 inches long and 1 inch inside

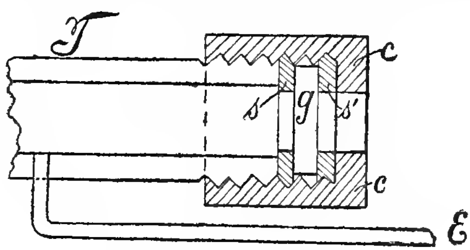

FIG. 95.—End of air tube.

diameter. To obtain a gas-tight joint for the plates of glass at the ends, the brass caps C (fig. 95) of the tube were perforated with axial holes about three-fourths inch in diameter and the inside of the ends turned smooth.

In this way they secured a round disk of plate glass *g* between gaskets *s* and *s'* of fiber. Before adjusting in place the inner gasket was slightly soaked in water and after placing the parts the cap was screwed down till the joint appeared air-tight and then left in this position to dry. The drying operation was assisted by repeated exhaustion over calcic chloride by the aid of the air-pump. In proportion as the paper gasket dries, it shrinks, and the screws must be tightened. Eventually a good joint is obtained, practically tight at 100° and even at 200°. There is no further danger from contamination by gases which arise in the gasket. They seem, however, to shrink after exposure to high temperature. Although the joint is rarely quite tight, it is sufficiently so for work of the present character, where the phenomenon is not very sensitive to variations of pressure.

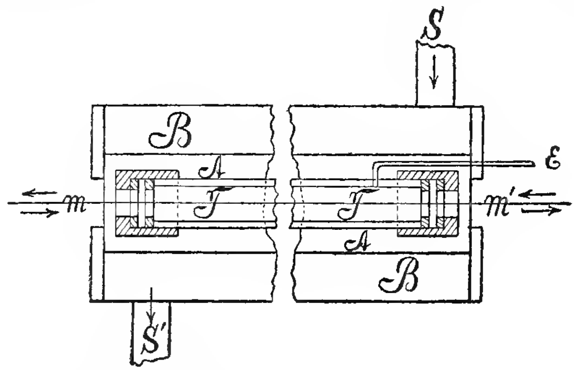

FIG. 96.—Heating apparatus for higher temperatures.

Fig. 96 shows the air-tube *TT* in place in the cylindrical axial channel of the annular vapor bath *BB*. In case of steam, the vapor entered at *S* and left at *S'*. The bath was jacketed with felt. A small hole was left in the end for the beam of light. The inner tube *TT*, wedged in place, was provided with a bent exhaust-pipe *E*, of thin brass or steel tubing, screwed into *T*. To make a tight joint, the end of *E* is covered with an adhesive coating of lead or a solder fusing at high temperature and then screwed into *T* before the latter is put into the chamber. The copper pipe may thus be fitted practically tight even without lead.

The vapor bath is suspended (with the tube inclosed) from an independent wall bracket as usual. There seemed to be no difficulty in causing the beam to traverse the tube twice, remembering that the collimator objective must be of long focus. For higher temperatures a brazed annular bath of sheet iron, similar in form to fig. 96 and containing the liquid to be boiled, replaced *BB*. This molten liquid was heated by burners placed below *BB* and a central pipe (not shown) on top provided for the escape and recondensation of the vapor. The bath was jacketed with asbestos on the sides and ends with the exception only of the small holes for the beam of light, *mm*, of the interferometer.

The glass ends of the tube extend nearly to the ends of the vapor bath. So long as the exhaustion method is applicable, there need be no correction for hot air at the ends of the chamber and outside of the tube, the distance being small. For the irregular temperature, distribution between the ends of the tube and the mirror and grating, respectively, remains constant on the average during the short time between the exhaustion and the reoccurrence of full supply of air in the tube. So also the arms, in view of water circulation, are of constant length for short intervals of time. In case of long periods, however, a time correction obtained from length measurements before and after on the interferometer would be needed; *i.e.*, in the absence of exhaustion. The inevitable flickering of the ellipses, due to air currents, proves no serious annoyance. They are steady enough for measurement and the change of aspect is rather a convenience, as it assists in bisecting the ellipses with the fiducial sodium line.

141. Data.—The results obtained at low temperatures, 22.5°, present no anomaly. They are given in table 91, as obtained on different occasions, a thermometer placed in the inner chamber of the cold steam bath showing the temperature. Each observation refers to a triplet, the observation for exhausted air lying between two observations for a plenum of air, in regular succession.

TABLE 91.—Refraction of air. Long tube, length $e=71.75$ cm. (air column), diameter 2.5 cm., windows 2 cm.

Barometer	t	p	$10^3\times \Delta N$	$10^6\times \mu-1$	$10^6\times \mu_0-1$	t'	p'	$10^3\times \Delta N'$	R	$10^3\times \alpha$
75.37 at 22°	22.5°	74.87	19.34	269.5	297.0	99.58°	74.87	17.27	1.13	1.75
....	....	....	19.41	270.6	298.1	....	....	17.10	1.14	1.91
....	....	....	19.80	276.0	304.1	....	....	16.94	1.15	2.07
....	....	....	....	....	....	....	....	16.55	1.18	2.46
....	....	....	....	....	....	....	....	16.30	1.20	2.71
....	....	....	....	....	....	....	....	16.22	1.20	2.80
76.66 at 22°	22.5	74.87	19.52	272.0	299.7	95.5	75.96	16.09	1.23	3.40

The table also contains the high-temperature results, t' being the boiling-point of water for the day and p' the effective pressure.

The data of the table are exceedingly disappointing, as the errors made in measuring δN can not be greater than a few tenths of 1 per cent. Moreover, a long time was allowed to secure constancy of temperature and to reach the maximum value. The observations lasted thereafter for about an hour, and the bath was well jacketed on all sides, including the ends. Nevertheless the temperature of the steam was not even approximately reached by the air within the tube and the successively increasing values of α from 1.8 to 2.8 show that, even at the end of the experiments, temperature on the inside of the air-tube is still markedly rising. In fact, if the constant α be taken at 3.6, the initial and final temperatures would be 61°

and 80° instead of 99.6°. Unfortunately, so confident were the observers of the temperature constancy throughout the steam bath that no thermometer was put inside of the tube.

The experiments contained in the lower part of table 91 were made with great care as to the temperature limits to be reached. Thermometers at the end of the brass tube registered the heating there, and it is obvious that, for parts within, the temperature must be higher and probably normal. Hence, when the temperature of the ends and the boiling-point for the day differed by but a few degrees, observations were commenced and the mean values taken as the actual temperature. Thus t' can not be too large; and since

$$d\alpha/dt' = -\alpha^2\left(\frac{\Delta N}{\delta t} - 1\right)$$

α can not be too small. If all temperatures agree within a single degree the error can not much exceed 1 per cent.

Unfortunately the apparatus began to leak as the experiments proceeded, so that only the first result is reasonably trustworthy. The effect of this leak was to decrease the refraction of the plenum of air at 100° and hence to enormously exaggerate α, very probably owing to the ingress of water-vapor present in the air. These results were therefore discarded.

In the next experiments, table 92, a special copper boiler was installed and all steam tubes, etc., were jacketed. Efflux steam was more carefully condensed in order to leave the atmosphere without dry, and decrease the burden on the drying tube. The air was taken from some distance from the apparatus. An additional advantage was secured in the use of the boiler, as the whole apparatus showed much greater freedom from tremors. Indeed, the experiment proceeded very smoothly throughout and, though it took some time before an adequately constant temperature was reached, exceptionally trustworthy results were anticipated. After two hours the temperature was still about a degree below normal at the end of the tube, the effect of which upon α would be about -1.6 per cent. This is not, therefore, the difficulty encountered. In other words, the observations for α in table 92 remain astonishingly small and present a degree of irregularity which can not easily be made out.

TABLE 92.—Tube as in table 91. Barometer, 75.50 at 25°.

t	p	$10^3\times\Delta N$	$10^6\times$ $\mu-1$	$10^6\times$ μ_0-1	t'	p'	$10^3\times$ $\Delta N'$	R	$10^3\times\alpha$
22.7°	75.71	19.43	270.8	295.3	95.0°	75.15	16.15	1.19	2.87
....	....	....	....	....	96.0	75.11	16.03	1.20	2.96
....	....	....	....	....	96.5	75.15	15.90	1.21	3.09
....	....	....	....	....	96.5	75.17	15.96	1.21	3.03
....	....	....	....	....	97.0	75.15	16.02	1.20	2.93
....	....	....	....	....	97.0	75.10	15.80	1.22	3.18
....	....	....	....	....	97.5	75.14	15.54	1.24	3.48
....	....	....	....	....	98.0	75.17	15.67	1.23	3.31

Even at the end of the experiments, however, the value of α has not yet reached a limiting value, as fig. 97, in which α and ΔN are graphically shown for successive observations, clearly evidences. To obtain such a result the experiment will have to be prolonged about half a day, and even greater safeguards taken at the ends of the tube.

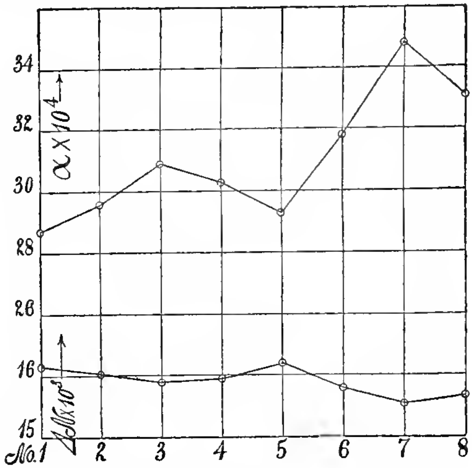

FIG. 97.—Displacement and temperature coefficients in successive experiments.

THE REFRACTION OF THE BUNSEN FLAME.

142. Method.—The following experiment is interesting as a rough application of the method. It aims at measuring the refraction of flaming gases and, so far as I am aware, is the first example of results of this kind. The attempt to determine the temperature of the flame would not be warranted so long as the temperature coefficient of the gases undergoing chemical ignition can not be known. The first result shows, however, that the refraction of flame gases is apparently peculiar, inasmuch as the flame behaves as if its index of refraction were less than 1. Such an outcome must be seriously considered, even if it can eventually be explained away.

The method consisted in first finding the micrometer displacement on the interferometer per centimeter of length of the column of air; thereafter, the displacement per linear centimeter for the given mixture of air and gas is ascertained by direct comparison with air; finally, the displacement of a column of the hot flaming gases per centimeter of length is found by comparison with cold air.

The cold-air tube at 20° C., for the length $e=25.3$ cm., showed a total displacement of $\Delta N=12.1$ scale parts or 0.00605 cm., as the equivalent of the difference in refraction between a plenum of air and a vacuum. Thus the air displacement per centimeter is $\frac{0.00605}{25.3}=0.000239$, which is nearly equal to $\mu-1$ at the given temperature (cold).

The gases of the burner, without being ignited, were now passed into a similar tube, 33.4 cm. long, at 21°. This tube was provided with glass plates at its ends, but there was room between them and the end of the tube for the escape of gas. The displacement at the micrometer due to the difference between the gas-content and the air-content of the tube was now found to be successively (observations being made in triplets) 0.87 and 0.92 scale parts, or 0.00045 cm., for the tube-length of 33.4 cm.; *i.e.*, $\frac{0.00045}{33.4} = 0.0000135$ per linear centimeter of gas column. The total displacement for the gas column per centimeter of length would be the sum, or $\Delta N = 0.000239 + 0.0000135 = 0.0002526$ cm. It was interesting to note the motion of the ellipses, while the contents of the tube changed from gas to air and back again. They are naturally both deformed and displaced during the transfer. The method must thus be applicable in case of a suitably designed apparatus for the measurement of the diffusion of gases into gases.

Finally the same beam of light of the interferometer was passed through the flame directly and at the top of the blue cone, the external mantle of flame being about 2.3 cm. in diameter. It was astonishing to find that the ellipses, though quivering violently, were still visible and available for rough measurement; but this is only the case when the beam of light passes normally through the flame. If it passes through or grazes either side of the mantle, it is naturally refracted away from one side or the other, respectively. The white slit image of the undeviated beam shows the nature of the result clearly. One of the two coincident images is drawn to one side of the stationary image, the latter corresponding to the beam not passing through the flame, and appears as a narrow band of white light, fluted with fine interference lines.

The displacement due to the flame was successively as follows:

Air.	Flame.	Diff.
45.7 } 46.8 } 46.8 }	44.6 } 45.2 }	−1.6

equivalent to −0.00080 cm. for the flame diameter 2.3 cm.; or $\delta N = -\frac{0.00080}{2.3} = -0.00035$ per linear cm. The gradual advance of the air value is due to the expansion of the arms of the apparatus, resulting from the radiation of the flame. The value of δN is the mean of two triplets.

Comparing this with the preceding result,

$$\frac{\Delta N}{\delta N} = -\frac{0.0002526}{0.00035} = -0.72$$

Now since the flame temperature t' is given in terms of the room temperature t by the equation

$$t' = \frac{1/\alpha + t\Delta N/\delta N}{\Delta N/\delta N - 1}$$

where α is the thermal coefficient of the gas, the result $\delta N > \Delta N$ is impossible, since $\delta N = \Delta N$ would make $t' = \infty$

Hence the depth of flame (2.3 cm.) should be increased by the region of hot air around it, or $\delta N = \frac{1.6}{2.3 + x}$, where x is the increment of diameter, which is again apparently impossible, since the increase would have to be at least 2.3+1.7 cm., or about 75 per cent. To avoid this assumption we must suppose, alternatively, that associated flame gases have an abnormally small refraction.

143. Conclusion.—Since the equation for the high temperature t' of the flame is of the form just given, δN can at most be equal to ΔN or $\Delta N/\delta N > 1$. Since experimentally the reverse of this is apparently the case and since

$$\frac{\mu - 1}{\mu' - 1} = \frac{\Delta N}{\Delta N - \delta N} = \frac{1}{1 - \delta N/\Delta N}$$

$\mu' - 1$ must be effectively negative or apparently $\mu' < 1$. Before endeavoring to account for this anomalous result, two other results may be mentioned. For a flame about 2 cm. in diameter the micrometer positions for air and flame were, respectively, 44.5 and 43.5 scale parts. This makes, per linear centimeter, $\delta N = 0.000375$ cm.

In the next experiment with a somewhat larger flame, water at constant temperature rapidly circulated through the arms of the interferometer. The micrometer positions in scale parts were successively

Air.	Flame.
9.7 } 9.7	7.5
9.7 } 9.7	7.4

The difference is equivalent to 0.00113 cm., or $\delta N = \frac{0.00113}{2.3} = 0.00049$ cm. per linear centimeter of flame. As $\Delta N = 0.000253$ is the same value as above, the anomalously excessive value of δN again appears, in an even more pronounced degree. One should note that here there can be no thermal expansion of the arms of the apparatus as the result of flame radiation. Such an effect, moreover, is excluded when the measurements are made in triplets, as was done throughout.

To account for this result in the simplest manner and in the absence of a value for α, one may assume, as has been done, that the flame acts effectively beyond the visible mantle (diameter e). Hence the limit of activity must be greater than $x/2$ beyond the flame on either side when

$$\Delta N = \frac{\delta N}{e+x}$$

In the last experiment, therefore,

$$x = \frac{1130}{253} - 2.3 = 2.2 \text{ cm.}$$

The flame at the excessively high temperature ($t = \infty$) would thus have to extend effective much more than 1 cm. beyond the mantle, which seems to be unreasonable.

The other alternative is the occurrence of $\mu < 1$ for the ignited gases, an even more anomalous result. It would mean that light travels more rapidly through a region of violent chemical reaction than through a vacuum, for which there is no other correlative instance. The question will have to be attacked in a different manner at some other opportunity.

CHAPTER XVI.

ELECTROMETRY WITH THE DISPLACEMENT INTERFEROMETER.

CYLINDRICAL AND DISK ELECTROMETERS.

144. Introductory.—The possibility of compensating any displacement of the mirror N by a corresponding displacement of the micrometer mirror M, *i.e.*, the fact that the ellipses are never lost in a properly adjusted apparatus, for any amount of motion of the mirror N, however sudden it may be, suggests the use of this method for electrometry. Some time ago I made experiments of this kind, using the device of the absolute electrometer for the purpose. A mirror was attached to the disk and the force of restitution was obtained from a pendulum suspension for parallel motion. Michelson's interferometer was employed. It appeared that for difference of potential of 1 volt the excursion, ΔN, would correspond to a distance apart of the condenser plates, d, in the given apparatus, as follows:

$d=$	1.0	0.1	0.01,
$10\Delta N=$	0.3	35	3,520, etc.

so that the apparatus was insensitive for small potentials, even with the use of the interferometer.

At first sight it seemed promising to change to the method of the quadrant electrometer. Here, however, the mirror can not be mounted on the needle unless a particular device of two mirrors is adopted; for even the slightest rotation is fatal to the interference measurements contemplated. A cylindric or lamellar form, such as is usually referred to in constructing the theory of the instrument, seemed to be alone available, but the sensitiveness here also, if torsion systems are excluded, is unexpectedly small, unless (as in Thomson's original apparatus) excessive charges are given to the needle, a procedure which introduces its own difficulties. Nevertheless I constructed the apparatus in a variety of forms, each of which will be described below, in turn, in order to ascertain in how far the sensitiveness which may be estimated from theory may be actually realized in practice; *i.e.*, to actually confront the instrumental difficulties of the problem, with a view to the use of the method in absolute measurements of potential.

145. Cylindrical electrometer. Movable cylinder without.—Figures 98 and 99, both sectional elevations, show the first form of electrometer in which the movable cylinder (usually highly charged) is external to the fixed cylinders h and i, carrying opposite charges. The latter are supported on hard-rubber pillars, u and v, rising from the adjustable feet q, of which r and s are the leveling screws and the set screw. The whole arises from a narrow base of brass plate AA. On this base the braced scaffolding of very thin brass pipe efg is also mounted, grasped above by the light cross-rod *d*. This makes a light but very firm support for the two parallel horizontal rods of hard rubber *cc*, secured at *b* and carrying the revolvable brass rollers or reels *aa* snugly. From *aa* the threads of very thin copper wires *yy*, 0.007 cm. in diameter, depend, their lowest point being attached to the cylinder *kk*, making of it a pendulum. To obviate torsional vibration the wires *yy* converge on their front elevation from *aa* to the top *tt* of the cylinder, while in their side elevation they are rigorously parallel and

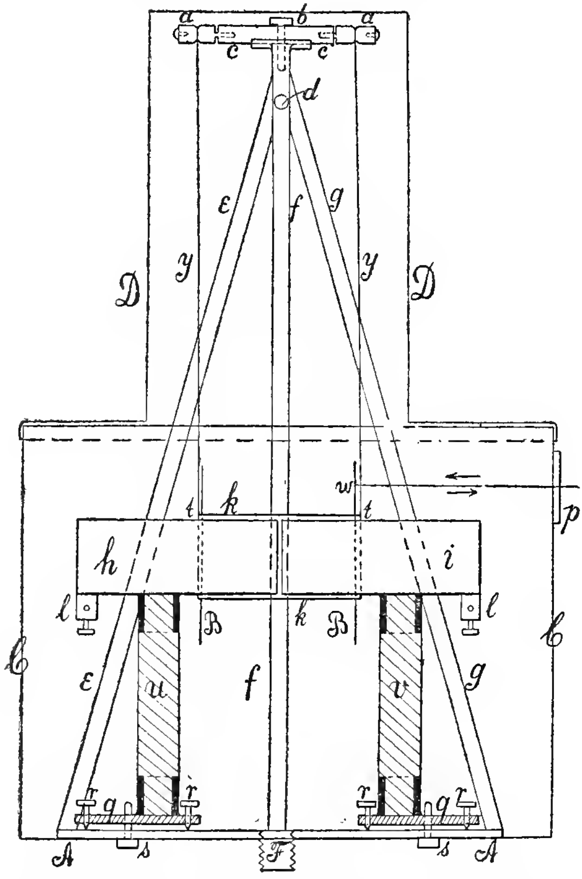

FIG. 98.—Cylindrical electrometer. Side view. Cylinder outside.

equally long. They may be lengthened or shortened by turning the close-fitting rollers *aa*, so that the cylinder *kk* may be placed accurately concentric to the cylinders *ih*. The latter may be mounted with a cylinder of wood passed quite through them, which is withdrawn after they are fastened. The clamp screws *ll* and those at the end of *a* are convenient for charging, the wires passing out of the light tin-plate cover *CCDD*, through hard-rubber cylinders at *Z*. The top *DD* may be lifted off. When there is no danger of induction, *DD* may be removed. The ellipses of the interferometer, in a room free from draft, are quite adequately stationary even without *DD*, a surprising result.

The cylinder *kk* is either of very thin aluminum tubing, or preferably of gilt paper. The wires *yy* pass through small holes at the top ends. To secure adequate damping, thin circular rings of mica or of paper, *BB*, surround the ends, and the front one carries the light mirror *w*, which

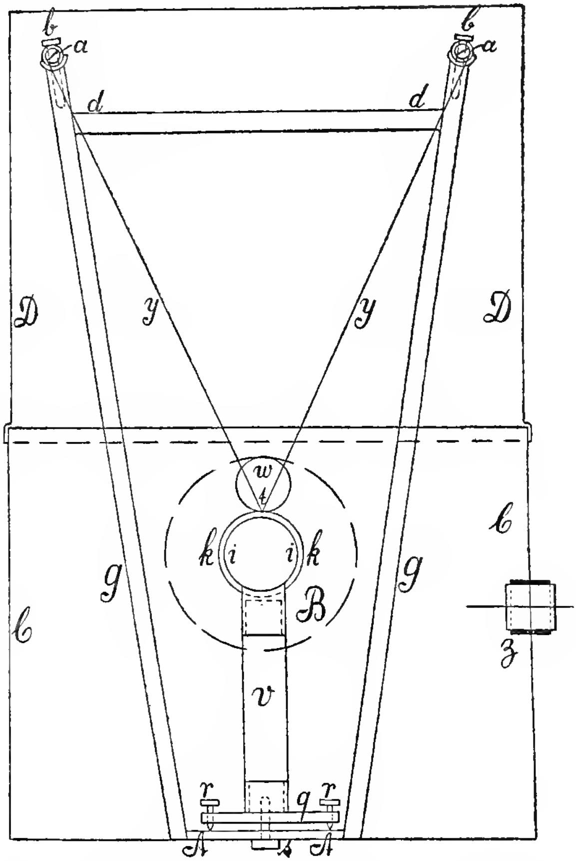

FIG. 99.—Cylindrical electrometer. End view. Cylinder outside.

reflects one of the component beams of the interferometer, as indicated at the glass window *p*. If liquid damping is desired, in the case of a heavy cylinder, wires ending in a vane may be dropped from one of these disks with the vane in a box of oil below. This was tried without apparent disadvantage, but a light paper cylinder and air damping is of course preferable, being far more sensitive. The mica *B* should be somewhat adjustable, so that the beam *p* may be given the proper direction, roughly. The methods for doing this will be described below.

The whole apparatus covered, at least as far as *C*, was then mounted on one arm of the interferometer by the aid of a large clamp of the ordinary pattern, the clamp having been screwed into the base *A* at *F*. As the arm is of gas-pipe, *F* is an ordinary nipple of suitable length. It is advisable to further support the tin case with wood uprights from the table to avoid the effect of tremors due to the vicinity of an active laboratory; but no great difficulty was here encountered. In fact, the ellipses of the interferometer were obtained in all cases after a preliminary rough adjustment of the mirrors with surprising ease, and they were quite as stable and available for measurement as if the mirrors had been rigidly fixed. Naturally this was quite contrary to our anticipations, but it is shown by the large number of experiments made, each of which required a new adjustment.

The measurement consists merely in a compensation of the displacement of the movable mirror on the electrometer, by the micrometric displacement ΔN of the mirror belonging to the other component beam. The center of the displaced ellipses in the field of the telescope is thus brought back to the fiducial sodium line. The compensation admits of an accuracy of about $\Delta N = 0.0001$ cm. without resorting to the interference rings. Other remarks on the optical method are given in the next paragraph.

146. Cylindrical electrometer. Movable cylinder within.—The framework and the mounting here are in general the same as in the preceding case and the movable cylinder *kk* is the same, but differently suspended. The fixed cylinders of the former apparatus, however, have been removed and replaced by the cylinders *h* and *i* of the present figures 100 and 101. These are mounted on higher hard-rubber supports *uv*, so that the interferometer beam *p* may have the same level as before.

The movable cylinder *kk* is fixed in the middle of the thin steel rod *tt* and suspended bifilarly from the threads *yy*, which are parallel in their side elevation, but triangular in their front elevation, as before. It must therefore have longer hard-rubber rods *cc*, though the suspension is in other respects the same. The interferometer mirror *w* was at first mounted in the end of the cylinder *kk*, where it meets the beam *p*. In a later construction it was thought advisable to mount the thin rod *tt* in the axis of the cylinder *kk*, the ends of the latter being closed. The mirror is then fixed to the end of *tt*. This has several advantages. The closed cylinder is very light and, together with the cylinders *h* and *i*, partially closed at their ends, admits of sufficient air damping; the mirror is accessible for adjustment and the axial rod *tt* is less subject to harmful induction. Other forms of instrument (disk pattern, closed field pattern) will be described below.

The general treatment of the displacement interferometer has been given in my earlier papers. The field of the telescope, after the appropriate two (of the four) undeviated images of the slit, from front and rear of the grating plate, have been put into coincidence horizontally and vertically (the images being usually bluish and yellowish), contains but three superposed spectra, although there are four white images, two from the front and two from the rear of the plate. For the component beam reflected from the rear face of the grating is not diffracted. It is thus advantageous to eliminate the third spectrum also, and to keep only the two which are superimposed for interference in the field. This may be easily done as follows: the two component beams reflected from the mirror facing the ruled side of the grating plate usually appear as separated images on the mirror,

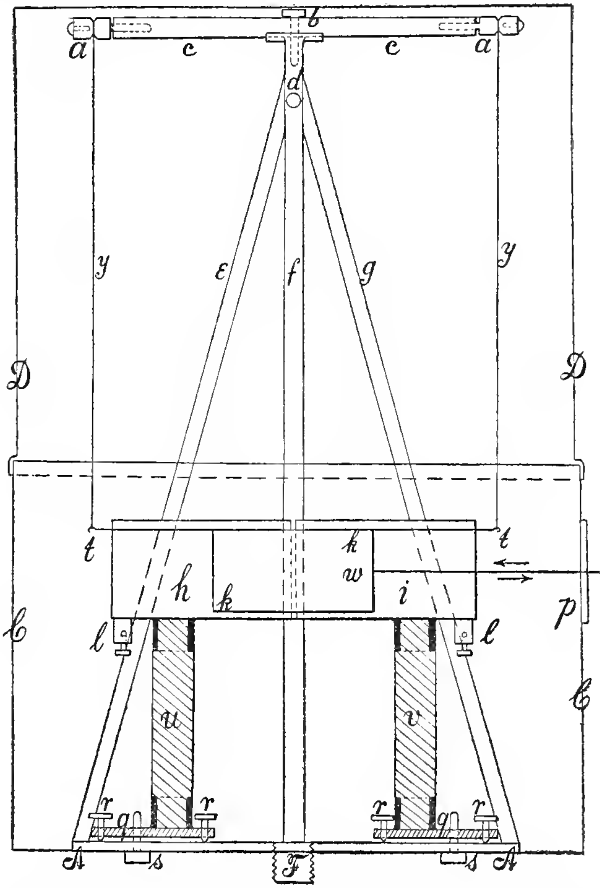

FIG. 100.—Cylindrical electrometer. Side view. Cylinder inside.

about one-eighth inch apart, and the one yellowish in color may be blotted out by a small screen near the mirror. As a result of this the stationary interferences which are due to the two beams from the same mirror also vanish. The spectrum is now clear and contains only the interfering superpositions, giving the ellipses their maximum definition.

In other respects the mounting on the interferometer is the same as before, and very little difficulty was experienced in obtaining the ellipses

The top of the case *DD*, here necessarily a square box, may usually be removed without disadvantage. If liquid damping is necessary in case of a heavy cylinder *kk*, the wire with vane may be dropped into a vessel of oil from the rear end of *tt*, the front carrying the mirror.

The usual method of connecting was used for potential measurements, one pole of the experimental cells being alternately earthed as well as one pole of the charging battery. A Mascart key was very useful. When the

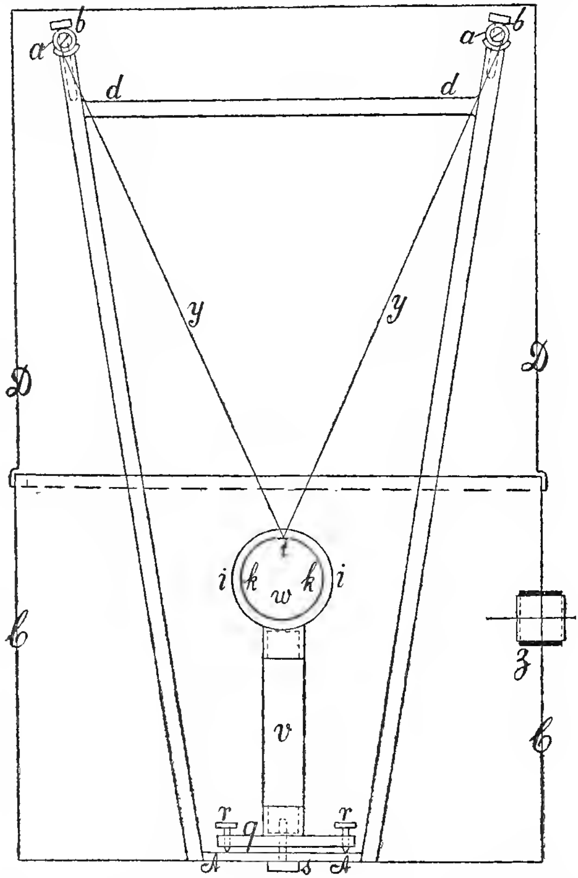

FIG. 101.—Cylindrical electrometer. End view. Cylinder inside.

needle swings to and fro the ellipses are visible at the extreme elongations of the deflection (if not too large) of the vibrating cylinder. They flash into the field of the telescope at the two ends of it, but are invisible between. The distance apart of the positions of visibility gradually diminishes until they coincide and the cylinder is stationary. If the ellipses are clear, slight motion does not interfere with the measurements, as the mean position is readily determinable in the displacement method. This would not, however, be the case had the evanescence of rings been made the basis of measurement, as in Michelson's interferometer.

When the needle or quadrants carry isolated charges to be removed by leakage, the electrometer is caged, and a caged key, shown in figs. 102 and 103, is necessary. This is a mercury key, *c* being the cup attached by a hard-rubber cylinder *h* and screw *i* to the base *B*, *d* being the clamp screw. The metallic connector is *a* supported on the hard-rubber stem *g* and screw *k*. The connector *a* turns in the axle *b* held by the brass cap *e*, the axle being connected by a thin flexible wire to the clamp screw *f*. The axle *b* is controlled by the observer at a distance by a long stick passing out of a hole in the cage. The cage is earthed.

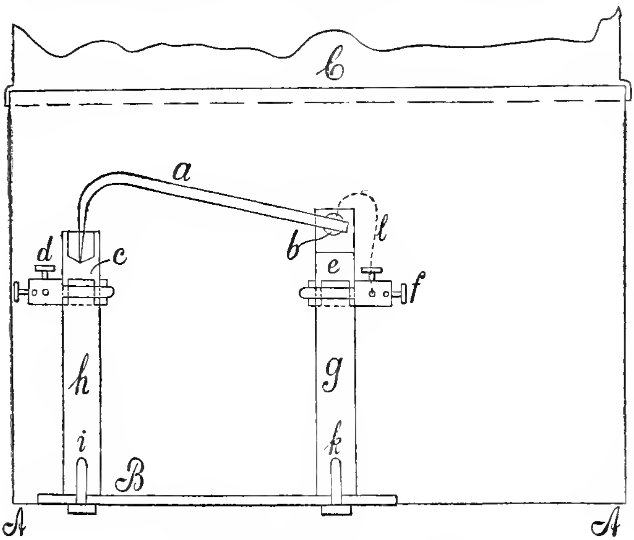

FIG. 102.—Electrometer key. Side view.

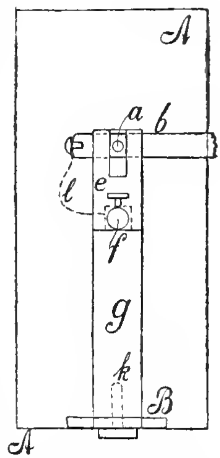

FIG. 103.—Electrometer key. End view.

147. Equations. Absolute electrometer.—It is convenient to include, for comparison, the equations for the absolute electrometer as used in the older paper (*l.c.*). If the condenser plates are at a distance *d* apart and *A* is the area of the disk; and further, *m* is the mass of the disk with appurtenances (mirror, etc.), *l* is the pendulum suspension, ΔN the micrometer displacement of the mirror corresponding to $V_2 - V_1$ or the potential difference in volts

$$\Delta N = 4.51 \times 10^{-10} \frac{Al(V_2 - V_1)^2}{d^2 m}$$

If the thickness and density of the disk are *t* and ρ

$$\Delta N = 4.51 \times 10^{-10} \, l(V_2 - V_1)/d^2 t\rho$$

which is independent of *A* if the disk is large. The apparatus is thus extremely insensitive for small potentials in spite of the interferometer, but its readings are absolute. If ΔN is to remain small in comparison with *d*, the latter can not be made much less than 0.05 cm. For the case of a silvered sheet of mica one may estimate

$$t=0.2 \text{ cm.} \qquad \rho=3 \qquad l=100 \text{ cm.} \quad \text{or} \quad \Delta N=3\times 10^{-4} \text{ cm.}$$

per volt is about the limit of sensitiveness. Hence about one-third volt could be measured by displacement interferometry, or one-tenth volt per ring.

The small value of d could be eliminated as usual from two measurements where $d-d'$ is known.

148. Equations. Cylindrical electrometer.—The equations may be found in the usual manner, and as so little of the field is displaced during deflection they apply with acceptable accuracy. Virtually a small cylindrical shell of uniform field is supposed to disappear on one side to reappear on the other. Hence, let V_3, V_2, and V_1 be the potentials of the two fixed and a movable cylinder (needle) respectively. Let f_{12} denote the electric force between cylinders (2) and (1), f_{13} the force between cylinders (3) and (1), R the common radius of the fixed cylinders, and r that of the movable cylinder. Hence if $R-r$ is small, then

$$f_{12}=(V_1-V_2)/(R-r), \qquad f_{13}=(V_1-V_3)/(R-r)$$

If the displacement of the cylinder is dx, the changes of potential energy on the two sides are

$$dW_{12}=\frac{R+r}{R-r}\,\frac{V_1-V_2}{8}\,dx \qquad\qquad dW_{13}=-\frac{R+r}{R-r}\,\frac{V_1-V_3}{8}\,dx,$$

and hence the force acting in the direction x (if $W=W_{13}+W_{12}$) is after reduction

$$X=\frac{1}{8}\,\frac{R+r}{R-r}(V_3-V_2)\,\big(2V_1-(V_2+V_3)\big)$$

On the other hand, the restoring force is for small displacements

$$X=Mgdx/l$$

where M is the mass of the cylinder, l the length of the pendulum suspension; whence if $dx=\Delta N$, in electrostatic units,

$$(V_3-V_2)\left(V_1-\frac{V_3+V_2}{2}\right)=\frac{4Mg(R-r)}{l(R+r)}\,\Delta N$$

which is the required equation. Since, if t is the thickness, L the length, and ρ the density of the movable cylinder,

$$(V_3-V_2)\left(V_1-\frac{V_3+V_2}{2}\right)=\frac{8\pi Lt\rho(R-r)}{l(1+R/r)}\,\Delta N$$

the appurtenances, like mirror, etc., being disregarded, and where $1+R/r$ is slightly larger than 2. A given deflection ΔN will therefore correspond to smaller V_3-V_2 as $R-r$, L, t, are smaller and l and V are larger.

If the instrument is used idiostatically (electrostatic units)

$$(V_3-V_2)^2=\frac{8Mg(R-r)}{l(R+r)}\Delta N$$

If one pole of the cell V_3-V_2 is earthed, $V_2=0$, and the deflections on commutation are ΔN and $-\Delta N'$,

$$V_3=300^2\frac{4Mg(R-r)}{V_1l(R+r)}\,\frac{\Delta N+\Delta N'}{2}=300^2\frac{8\pi Lt\rho(R-r)}{V_1l(1+R/r)}\,\frac{\Delta N+\Delta N'}{2}\text{ volts.}$$

If the cylinder V_1 is made of paper, the following dimensions seem to be reasonably convenient limits, so that, for $\Delta N=0.0001$ cm.,

$L=5$ cm. $t=0.2$ cm. $\rho=1$ $l=100$ cm. $V_1=300$ volts

whence

$V_3=3.8\times10^{-4}$ volts, or about 10^{-4} volts per ring.

One should thus be able to measure voltages even within 4×10^{-4}; but on trial the actual limits fall much within this. The use of wide apparatus has the advantage, only, of diminishing the effect of appurtenances like the mirror, etc. In the practical work below, however, these limits could not be reached, for reasons which will appear.

The idiostatic method under the same circumstances, $\Delta N=0.0001$ cm., should just measure $V_3=0.5$ volt, so that alternating currents much within 1 volt will apparently be measurable. This method is naturally much less sensitive than the other, but its absolute character and its adaptation to alternating currents are of interest. The sensitiveness of the idiostatic electrometer as here designed and in the above case of the absolute electrometer would thus appear to have the same practical limit. In the way of absolute measurement nothing has been gained.

149. Equations. Disk electrometer.—In this case a charged disk with a guard ring (the apparatus to be described hereafter, §155, figs. 112 and 113) at potential V_1 is suspended bifilarly, as above, between the two concentric parallel disks of the condenser at potentials V_3 and V_2. The adjustment may be regarded as the extreme case of the cylinder electrometer with closed ends, and the equations will be, necessarily, nearly the same. Let d be the distance apart of the plates of potential V_3 and V_1, D the distance between the plates of potential V_3 and V_2, r the radius of the movable disk. The total energy W of the variable system is then, after reduction,

$$W=\frac{r^2}{8\pi}\left\{\frac{(V_3-V_1)^2}{d}+\frac{(V_1-V_2)^2}{D-d}\right\}$$

and the corresponding mechanical force will be

$$X=\frac{r^2}{8}\left(\frac{V_1-V^2}{D-d}+\frac{V_3-V_1}{d}\right)\left(\frac{V_1-V_2}{D-d}-\frac{V_3-V_1}{d}\right)$$

which depends essentially on d as well as on D.

Let the disk be in the middle, or $2d=D$, and put $X=Mg\Delta N/l$ as above, where M is now the mass of the disk and appurtenances, ΔN the interferometer displacement, l the length of the bifilar pendulum, and g the acceleration of gravity. Then

$$(V_3-V_2)\left(V_1-\frac{V_2+V_3}{2}\right)=\frac{MgD^2}{lr^2}\,\Delta N$$

Let $V_2=0$ and $M=\pi r^2 t\rho$, t being the thickness and ρ the density of the disk, the latter being sufficiently large so that the appurtenances may be ignored. Then on commutation and in electrostatic units, ΔN being the mean displacement,

$$V_3=\frac{MgD^2}{V_1 l r^2}\Delta N=\frac{\pi t\rho g D^2}{V_1 l}\Delta N$$

which (for a sufficiently large disk) is independent of its area.

To estimate the limiting sensitiveness the following apparently reasonable values may be inserted:

$V_1=\frac{1}{3}$ els. $t=0.05$ cm. $\rho=1$ $D=0.1$ cm. $\Delta N=10^{-4}$ cm. $l=10^2$ cm.

whence

$$V_3=1.4\times 10^{-3} \text{ volts}$$

The limiting sensitiveness is thus of the same order as in the cylindrical case for the same V_1, as might have been anticipated. It is probable, however, that the present conditions may be more nearly realized in practice, as the apparatus is essentially simpler.

If $V_2=V_1=0$, the idiostatic case is identical with the absolute electrometer of §147.

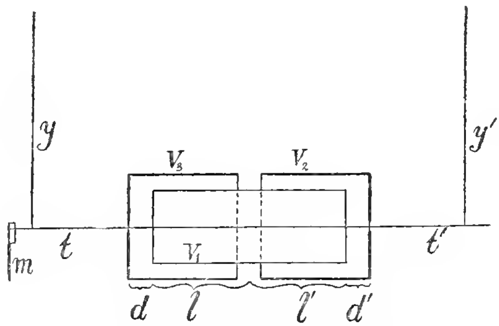

FIG. 104.—Closed field electrometer.

150. The closed field electrometer (disk and cylinder combined).—If the two cases of the preceding paragraphs be combined as in fig. 104, where the symbols of potential and distance are the same as before, the energy of the system is, in electrostatic units,

$$W=\frac{1}{8}\left\{(V_3-V_2)^2\left(\frac{r^2}{d}+L\frac{R+r}{R-r}\right)+(V_1-V_2)^2\left(\frac{r^2}{d'}+L'\frac{R+r}{R-r}\right)\right\}$$

and hence the forces become

$$X=\frac{1}{4}\left(\frac{r^2}{d^2}+\frac{R+r}{R-r}\right)(V_3-V_2)\left(V_1-\frac{V_2+V_3}{2}\right)=\frac{Mg}{l}\Delta N$$

If $V_2=0$, then

$$V_3=\frac{4Mg\Delta N}{V_1(r^2/d^2+(R+r)/(R-r))l}$$

after commutation, where L is the length of the cylinder. The sensitiveness of this apparatus depends very largely on d—that is, upon the disk—in any practical case; but it should be more nearly absolute. Experiments were not made. In fig. 104 the distances d, l, l', d', are used in the equations. Also

V_1 = the closed cylinder.
V_3, V_2 = the hollow half-cylinders with their open ends confronting each other, coaxially with V_1.
tt' = the horizontal steel wire fixed to V_1 and passing through perforations in the closed ends of V_2 and V_3.
m = the mirror (slot suspension).
y, y' = the bifilar pendulum supports.

151. Corrections.—The most important consideration to be made here is the effect attributable to the want of symmetry in the orientation of the disk or movable charge, supposing the latter and the plates are quite parallel. Let k be the axial distance of the uncharged disk from the plane midway between the condenser plates (guard ring). In such a case, if the electric displacement is ΔN, the quantity $\Delta = k + \Delta N$ must be added on one side and subtracted on the other, so that

$$d = \frac{D}{2} + \Delta \qquad d' = \frac{D}{2} - \Delta$$

Thus the displacing force is, after reduction:

$$X = \frac{r^2}{2}\left(\frac{V_1 - V_2}{D - 2\Delta} + \frac{V_3 - V_1}{D + 2\Delta}\right)\left(\frac{V_1 - V_2}{D - 2\Delta} - \frac{V_3 - V_1}{D + 2\Delta}\right)$$

which, if $a = V_3 - V_2$ and $b = 2V_1 - (V_2 + V_3)$, may be further reduced to

$$X = \frac{ab}{2}\frac{r^2}{D^2}\left(1 + 2\frac{\Delta}{D}\left(\frac{a}{b} + \frac{b}{a}\right)\right)$$

if $4\Delta^2$ is neglected in comparison with D, and where $X = Mg\Delta N/l$. If $\Delta = 0$, this reduces to the equation above.

If nothing is neglected and under the assumption of a uniform field, the equation is, for a positive charge, V_3, corresponding to a displacement $\Delta N'$ with $V_2 = 0$,

$$\frac{2Mg\Delta N'}{lr^2} = V_3(2V_1 - V_3)\frac{D^2 + 4\Delta^2}{(D^2 - 4\Delta^2)^2}\left(1 + \frac{2D\Delta}{D^2 + 4\Delta^2}\left(\frac{2V_1 - V_3}{V_3} + \frac{V_3}{2V_1 - V_3}\right)\right)$$

and for a negative charge, $-V_3$, corresponding to a displacement, $-\Delta N''$,

$$\frac{2Mg\Delta N''}{lr^2} = V_3(2V_1 + V_3)\frac{D^2 + 4\Delta^2}{(D^2 - 4\Delta^2)^2}\left(1 - \frac{2D\Delta}{D^2 + 4\Delta^2}\left(\frac{2V_1 - V_3}{V_3} + \frac{V_3}{2V_1 - V_3}\right)\right)$$

The mean of the equations, $\overline{\Delta N} = \frac{1}{2}(\Delta N' + \Delta N'')$, after reduction becomes

$$\frac{Mg\overline{\Delta N}}{l}\frac{D^2}{r^2}\frac{(1 + 2\Delta/D)^2}{V_1} = V_3$$

the corrected equation required, reducing to the above case, when $\Delta=0$. The correction factor is thus $1/F=(1+2\Delta/D)^2$ and has been given in table 93 and fig. 105 for both positive and negative values of Δ.

TABLE 93.—Values of the factors F and $1/F$. $M=1.0865$ g.; $D=1.04$ cm.; $l=23.5$ cm.; $r=3.065$ cm.; $V_1=250$ volts.

$+\Delta/D$	F	$1/F$	$-\Delta/D$	F	$1/F$
0.10	1.44	0.69	0.10	0.64	1.56
.15	1.69	.59	.15	.49	2.04
.20	1.96	.51	.20	.36	2.78
.25	2.25	.44	.25	.25	4.00
.30	2.56	.39	.30	.16	6.25
.35	2.89	.35	.35	.09	11.11
.40	3.24	.31	.40	.04	25.00
.45	3.61	.28	.45	.01	100.00

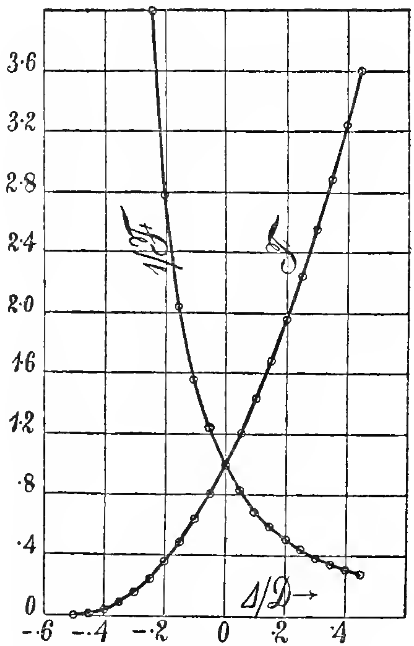

FIG. 105.—Graph showing reduction factors for different displacements.

It is now easy to discuss the conditions of equilibrium, for the forces X are given by the equations (second member referring to the pendulum and the third member to the electrical forces)

$$X=\frac{Mg\Delta N}{l}=V_1V_3\frac{r^2}{D^2}\frac{1}{(1+2\Delta/D)^2}$$

Hence, if $\Delta N = \Delta$, $k = 0$, the condition under which the disk just moves, without interruption, from the guard ring to the condenser plates, *i.e.*, the limiting value of the potential products $(V_1 V_3)$, for stable positions for the disk, since

$$\frac{dX}{d(\Delta/D)} = \frac{MgD}{l} = (V_1 V_3)\,\frac{r^2}{D^2}\,\frac{-4}{(1+2\Delta/D)^3}$$

is (in electrostatic units)

$$(V_1 V_3) = -\frac{Mg}{4l}\,\frac{D^3}{r^2}\,\frac{(1+2\Delta/D)^3}{\Delta/D}$$

Assuming $V_1 = 250$ volts, the forces X for the pendulum and the electric field have been given in table 95 and fig. 106.

TABLE 94.—Electric forces. V_3 charged, $V_2 = 0$, $V_1 = 250$ volts. $M = 1.0865$ g.; $D = 104$ cm.; $l = 2.3.5$ cm.; $r = 3.065$.

V_3	$\frac{\Delta}{D}$	X	V_3	$\frac{-\Delta}{D}$	X
20	0.10	0.335	20	0.10	0.756
	.15	.286		.15	.985
	.20	.246		.20	1.340
	.25	.215		.25	1.930
	.30	.189		.30	3.016
	.35	.167		.35	5.361
	.40	.149		.40	12.063
	.45	.132		.45	48.252
100	.10	1.676	100	.10	3.779
	.15	1.428		.15	4.924
	.20	1.231		.20	6.700
	.25	1.073		.25	9.650
	.30	.943		.30	15.080
	.35	.835		.35	26.605
	.40	.745		.40	60.315
	.45	.659		.45	241.260
140	.10	2.346	140	.10	5.290
	.15	1.999		.15	6.893
	.20	1.723		.20	9.380
	.25	1.502		.25	13.510
	.30	1.320		.30	21.110
	.35	1.169		.35	37.327
	.40	1.042		.40	84.441
	.45	.923		.45	337.760

If, now, we insert the value of V_3 from the above equation and reduce, $-6\Delta/D = 1$ or $-\Delta = D/6$

Hence the value of V_3 which corresponds to tangency is

$$V_3 = \frac{Mg\,D^3}{lV_1\,r^2}\left(-\frac{1}{6}\right)\left(\frac{2}{3}\right)^2 = \frac{2}{27}\,\frac{MgD^3}{lV_1 r^2}$$

or $V_3 = 145$ volts, above which charge the disk passes continuously from guard ring to plate.

In fig. 106 the case of tangency is well indicated, the line referring to the gravitational forces and the curves to the electrical forces, supposing $V_1 = 250$ volts.

If the suspension is provided with a horizontal micrometer by which it can be shifted as a whole from k to k', taking the needle with it, k may be eliminated. But the expression is not simple.

Let $\sqrt{\Delta N}=x$, $\sqrt{\Delta N'}=x'$, and $k'-k=\kappa$. Then

$$k=\kappa x'/(x-x')+\left(\frac{D}{2}+x^2+x'x+x'^2\right)$$

It would be more convenient to find k from known potential V_1 and V_3.

The idiostatic method needs a corresponding correction, and if Δ^2 is neglected in comparison with D, then

$$\left(1-2\frac{\Delta}{D}\right)^2\frac{2MgD^2}{lr^2}\Delta N=V_3^2$$

As the new factor is practically k or constant, V_3^2 is linear with ΔN. If, as above indicated, the suspension is provided with a horizontal micrometer by which it can be shifted as a whole from k to k', taking the needle with it, k may be eliminated. But the expression is not simple. Let

$$\sqrt{\Delta N}=x,\ \sqrt{\Delta N'}=x' \quad\text{and}\quad k'-k=\kappa$$

Then

$$k=\kappa x'/(x-x')+\left(\frac{D}{2}-\left(x^2+x'x+x'^2\right)\right)$$

It would be more convenient to find k from known potential V_1 and V_3.

If $V_2=V_3=0$, and the disk V_1 is charged, the equation reduces to

$$\frac{2Mg\,(D^2-4\Delta^2)^2}{lr^2(D^2+4\Delta^2)}\Delta N=2V_1\frac{2D\Delta}{D^2+4\Delta^2}2V_1$$

or

$$V_3^2=MgD^2(1-4\Delta^2/D^2)^2\Delta N/4lr^2(\Delta/D)$$

The equation may be written in terms of the two forces X (gravitational and electric)

$$X=\frac{Mg\Delta N}{l}=\frac{4r^2}{D^2}V_1^2\frac{\Delta/D}{(1-4\Delta^2/D^2)}=\frac{4V_1^2r^2}{D^2}F'$$

where

$$F'=\frac{\Delta/D}{(1-4\Delta^2/D^2)^2}\quad\text{and}\quad V_1^2=\frac{MgD^2}{4lr^2F'}\Delta N$$

The value of the factor F' is shown in table 95 and fig. 107.

TABLE 95.—Values of the factor F' and $1/F'$. $M=1.055$ g.; $D=0.612$ cm.; $l=23.5$ cm.; $r=3.065$ cm.; $V_3=V_2=0$.

Δ/D	F'	$1/F'$
0.10	0.109	9.174
.15	.181	5.525
.20	.283	3.534
.25	.445	2.247
.30	.732	1.366
.35	1.345	.741
.40	3.086	.324
.45	12.465	.080

With the aid of table 95 the two forces X may be at once computed, and they are given for different values of V_1 in table 96 and fig. 108 under the supposition that k is equal to zero or $\Delta = \Delta N$; *i.e.*, that the disk is initially symmetric.

TABLE 96.—Electric forces. $V_3 = V_2 = 0$. V_1 charged. $M = 1.055$ g.; $D = 0.612$ cm.; $l = 23.5$ cm.; $r = 3.065$ cm.

V_1	Δ/D	X/r^2	V_1	Δ/D	X/r^2	V_1	Δ/D	X/r^2	V_1	Δ/D	X/r^2
20	0.10	0.005	40	0.10	0.021	70	0.10	0.064	100	0.10	0.130
	.15	.009		.15	.034		.15	.105		.15	.215
	.20	.013		.20	.054		.20	.164		.20	.335
	.25	.021		.25	.084		.25	.259		.25	.528
	.30	.035		.30	.139		.30	.426		.30	.870
	.35	.064		.35	.254		.35	.779		.35	1.590
	.40	.146		.40	.583		.40	1.786		.40	3.645
	.45	.592		.45	2.367		.45	7.248		.45	14.793

The curves, fig. 108, indicate a tangency of the curves X/r^2, above V_1, at least 100 volts. Below this, therefore, there are necessarily two available points of equilibrium of the disk, one unstable at $\Delta N = 0$ and the other in

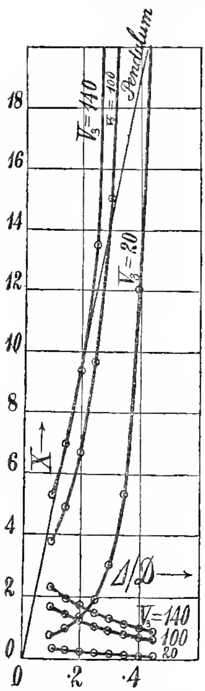

FIG. 106.—Graphs showing pendulum and electric forces at different potentials and displacements.

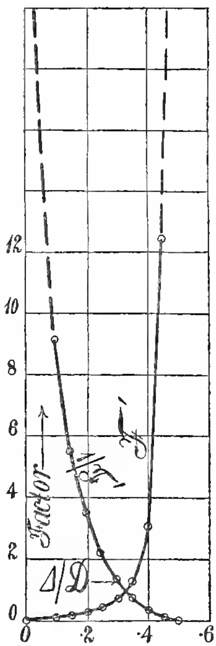

FIG. 107.—Graphs showing reduction factors at different displacements.

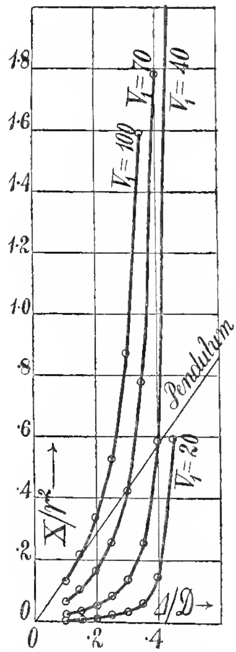

FIG. 108.—Graphs showing pendulum and electric forces at different potentials and displacements.

practice not very far from it. For high potentials the disk sweeps uninterruptedly from $\Delta N=0$ into contact with the nearer plate, as the electric forces are now throughout in excess of the gravitational forces of restitution. In fact, the condition may be precisely stated, inasmuch as the values $dX/d\Delta$ may be easily derived. They are for the pendulum Mg/l and for the electric field

$$\frac{dX}{d\Delta}=\frac{4V_1^2r^2}{D^3}\,\frac{1+12\Delta^2D^2}{1-4\Delta^2/D^2}$$

Equating them, we derive the voltage of transition, to be characterized by (V_1), as

$$(V_1)^2=\frac{MgD^3}{4lr^2}$$

since $\Delta=0$ at the origin. Hence above $(V_1)=156$ volts the disk would move on any slight disturbance without interruption from the guard ring, $\Delta=0$, to either plate of the condenser; but within (V_1) the apparatus is available for accurate measurement of V_1, if k is equal to zero by trial.

152. Experiments. Cylindrical electrometer; movable cylinder within.—Tests of the apparatus described were begun by using a rather heavy cylinder of aluminum, damped with a vane submerged in a cup of oil. The constants of the apparatus were as follows:

$M=10.4$ g.	$l=21$ cm.	$2R=4.15$ cm.
$2r=3.46$ cm.	$L=6.7$ cm.	$V_1=250$ volts

The cylinder itself weighed but 6.546 grams; but air damping was quite inadequate, though the cylinder might easily have been etched to a more appropriate degree of thinness in acid.

TABLE 97.—Cylindrical electrometer; movable cylinder within (aluminum). $M=10.4$ g.; $l=21$ cm.; $r=1.73$ cm.; $R=2.075$ cm.; $V_1=250$ volts; $V_2=0$.

I			II			
Drum.	$10^3\Delta N$	V_3	Drum.	$10^3\Delta N$	V_3	Voltmeter.
5.2	1.29	95	4.7	1.18	74	70
4.1	1.02	75	3.3	.82	52	48
3.0	.74	54	2.5	.63	39	36
			1.4	.23	22	19
			.1	.03	2	0

The observations are given in table 97, showing the drum reading on the micrometer, the displacement ΔN on the micrometer and the voltages corresponding. The displacement is doubled by commutation. Compared with a voltmeter the data were too high, and the idiostatic method was inapplicable. In the second series of this table, the comparison with the voltmeter shows that the observations are distributed as if an initial potential of 3 or 4 volts were superimposed upon them. These data are merely given to point out the difficulties in the use of a metallic cylinder, however thin. The apparatus containing such a cylinder is very insensitive, if liquid damping is resorted to, otherwise the swing of the needle is almost indefinitely long. It is interesting, nevertheless, to follow the ellipses which flash out at the extreme elongations of the vibrating cylinder.

It is therefore necessary to make use of paper cylinders, even if the possibility of securing more nearly accurate values of the diameter of cylinder is the advantage of the metallic appurtenance.

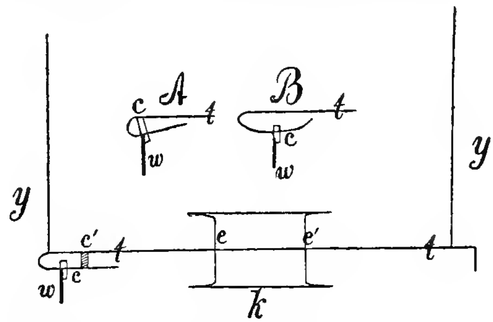

FIG. 109.—Suspension of inside cylinder of electrometer.

The gilt paper cylinder *k* was mounted as shown in fig. 109, *yy* being the bifilar suspension. The ends of the cylinder are closed by the reëntrant partitions *e* and *e′*, the object being to avoid the disk effect, by screening off the disk by the ends of the cylinder. The steel wire *tt* passes through the centers of the disks *ee′*, a drop of mucilage being sufficient for electrical contact and to keep the cylinder in place. The mirror *w* is attached by a bit of cork *c* to the bent end of the elastic rod. As the mirror must be adjustable around the horizontal axis (the apparatus, as a whole, may be rotated around the vertical), a second little piece of cork *c′* serves, when moved from left to right, to open the spring hook and rotate the mirror. Fig. 109, *A* and *B*, are other devices for this purpose. In the former a single piece of cork opens the spring and holds the mirror; in the latter the lower part of the hook is circular and the mirror is rotated on sliding the cork. Many other devices were tried, but probably a small piece of cork, slit vertically, and mounted on a straight rod, is as good as any. The difficulty in devising these parts is that both cork and mirror must be exceedingly light. No trouble was encountered in using a light plane mirror about one-fourth inch in diameter. To sharpen the ellipses the yellow image at the movable mirror should be screened off, leaving but the blue image and but two superposed spectra.

The first experiments were made with a cylinder of relatively small diameter having the constants, and showing the results as given in Series I of table 98.

TABLE 98.—Cylindrical electrometer; movable cylinder (gilt paper) within. $M=1.613$ g.; $l=21$ cm.; $R=2.075$ cm.; $r=1.66$ cm.; $V_1=250$ volts; $V_2=0$.

I.			II.			
Drum.	$10^3\Delta N$	V_3	Drum.	$10^3\Delta N$	V_3	Volt-meter.
16.4	4.1	55	6.6	1.65	22.0	19
10.3	2.6	34	9.2	2.29	30.6	29
9.7	2.4	32	9.4	2.36	31.6	30
5.3	1.3	18	10.0	2.50	33.4	32
.35	.1	1	16.1	4.02	53.7	52

Further experiments with the same instrument (somewhat improved), on being compared with the voltmeter, gave the values contained in Series II of the table.

A lighter gilt-paper cylinder, weighing with mirror, etc., $M=1.049$ grams, $2R=4.15$, $2r=3.32$, was thereafter inserted, the other constants remaining the same as before. The behavior now was satisfactory throughout and the increased sensitiveness is shown in table 99, Series I.

TABLE 99.—Cylindrical electrometer; movable cylinder (gilt paper) within. $M=1.049$ g.; $l=21$ cm.; $R=2.075$ cm.; $r=1.66$ cm.

I. $V_1=250$ volts. $V_2=0$				II. Idiostatic method. $V_3=V_2=0$			
Drum.	$10^3\Delta N$	V_3	Volt-meter.	Drum.	$10^3\Delta N$	V_1	Volt-meter.
9.1	2.27	19.7	15	4.77	2.38	102	101
18.8	4.71	40.9	41	2.83	1.41	78	81
28.7	7.17	62.3	61	1.81	.90	63	61
37.2	9.41	81.7	81	.73	.36	40	41
47.5	11.88	103.2	101				

The second part of the table contains the idiostatic results, in which the instrument, though less sensitive, is absolute.

The endeavor was then made to secure greater sensitiveness by increasing the diameter of the movable cylinder, and its constants were

$$M=1.138\text{ g.} \qquad 2R=4.15 \qquad 2r=3.83 \qquad L=5\text{ cm.}$$

so that the shell-like space between the cylinders was only $R-r=0.16$ cm. thick. But with a charge of 250 volts the cylinder was pulled aside in contact with the stationary cylinders and observations had to be abandoned. With the idiostatic method, the results of table 100 were secured.

TABLE 100.—Cylindrical electrometer; movable cylinder (gilt paper) within. $M=1.138$ g.; $l=21$ cm.; $R=2.075$ cm.; $r=1.915$ cm. Idiostatic method; $V_2=V_3=0$.

Drum.	$10^3\Delta N$	V_1
20.1	10.0	124
14.0	7.0	104
8.4	4.2	80
4.9	2.4	62
.8	.4	25

The march of results is fairly linear with the actual voltages, though the computed values of V_3 are necessarily inaccurate, actually in excess, owing to the fact that the small value, $R-r=0.16$ cm., is not vouched for in the case of the paper cylinder. The use of a smaller charging potential was tried with similar results.

TABLE 101.—Cylindrical electrometer; movable cylinder (gilt paper) within. $M=1.1050$ g.; $l=21$ cm.; $R=2.075$ cm.; $r=1.75$ cm.

I. $V_1=250$ volts. $V_2=0$.				II. Idiostatic method. $V_3=V_2=0$.			
Drum.	$10^3\Delta N$	V_3	Volt-meter.	Drum.	$10^3\Delta N$	V_1	Volt-meter.
0.8	0.2	1	2	0.15	0.08	15	15
2.9	.7	5	6	.50	.25	28	29
5.3	1.3	8	9	1.2	.60	43	41
9.3	2.3	15	15	2.2	1.12	60	61
23.3	5.8	37	41	3.7	1.87	77	81
33.9	8.5	53	61				
44.0	11.0	69	81				

The next cylinder to be inserted was of smaller diameter, and its constants and the records of results are given in table 101. In case of the charged needle the sensitiveness increases at the higher voltages, due no doubt to the growing asymmetry of the needle. Similarly the idiostatic method, fig. 110, *b*, in view of its small deflections, shows fairly regular results, though they diminish at high voltages.

In another adjustment, carefully made with reference to freedom and symmetry of cylinder, it was found that a charge of 250 volts could actually be carried, without deflecting the cylinder laterally into contact with the walls. The observations in Series II of table 102 were carried out in this way, and show the maximum sensitiveness obtained with short suspension.

TABLE 102.—Cylindrical electrometer; movable cylinder (gilt paper) within. $M=1.189$ g.; $l=21$ cm.; $R=2.075$ cm.; $r=1.90$ cm.

I. Idiostatic method. $V_3=V_2=0$				II. $V_1=250$ volts.			
Drum.	$10^3\Delta N$	V_1	Volt-meter.	Drum.	$10^3\Delta N$	V_3	Volt-meter.
8.75	4.37	87.8	88.0	7.0	1.74	6.1	5
5.35	2.67	68.6	68	8.7	2.17	7.6	7
5.80	2.90	71.5	70.5	13.2	3.31	11.6	11.5
2.90	1.45	50.5	50	18.1	4.53	16.0	15
1.70	.85	38	38	23.4	5.85	20.6	19

With a final closely fitting cylinder of gilt paper of a diameter probably as large as practicable, the data of table 102 and fig. 110, *a*, were found by the idiostatic method. These results for high voltages are quite satisfactory and the differences are more liable to be sought for in the voltmeter than in the electrometer. It indicates the sufficiency of the equation assumed, apart from corrections for the ends of the cylinder, so that the latter can not be menacing.

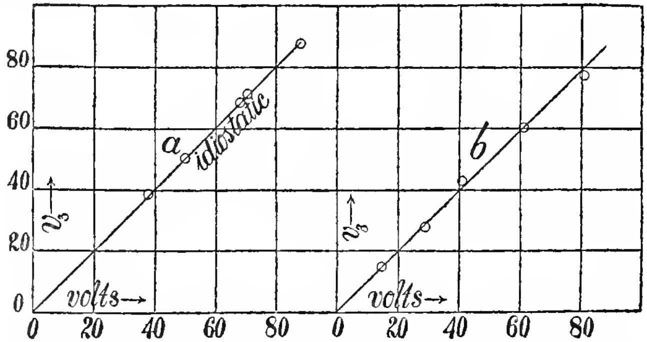

FIG. 110.—Observations with cylindrical electrometer. Cylinder inside.

Besides this, the method of the charged needle was tried. In this case, 250 volts was an excessive charge and the needle could no longer be freed from the stationary cylinder. With a charge of 87 volts (storage battery) and a single battery for V_3, the results were

$$10^3 \times \Delta N = 0.108 \qquad V_3 = 1.1$$

instead of 1.3 volts, the sensitiveness being thus 2×0.4 drum parts of double deflection per volt.

In addition to these experiments, a long suspension ($l = 150$ cm.), in which the sensitiveness would have been increased seven times or equivalent to 5×10^{-3} cm. per volt or 0.006 volt per vanishing ring, was installed. The ellipses were easily found, but were continually in motion, owing to the friction of air-currents moving across the wires. To make the adjustment available it would have been necessary to build a closed case around the bifilar suspension over 1.5 meters in height. It was not thought worth while to do this, and further experiments were abandoned.

153. Cylindrical electrometer; movable cylinder within. Summary.— The above experiments were made in the midst of the turmoil of a large city and near the engineering laboratories of a university. It is rather remarkable, therefore, that the ellipses were so easily found throughout and so easily made use of; but it was quite out of question to use the evanescence of rings by which the sensitiveness could have been increased over twenty-fold. Though the work was carefully done, it is intended merely to exhibit the general character of the method, inasmuch as the storage battery which was drawn upon was unavoidably in use elsewhere in the laboratory and the potential may have fluctuated.

The design of the apparatus with a movable inside cylinder is probably the least interesting of those used. Thus it is difficult to keep a paper cylinder quite smooth or to give it a rigorous cylindrical shape, and the correction for the ends can scarcely be estimated. For high charges any asymmetry of the movable cylinder is liable to place it in contact with the fixed cylinders. Thus there is a limit of sensitiveness, from a purely instrumental point of view, not contemplated in theory. In the above experiments with short suspension (21 cm.), 7×10^{-4} cm. per volt was the largest double displacement practically obtained, which would mean about 0.04 volt per vanishing ring. With the long suspension 0.006 volt per ring may be estimated. It is far short of the theoretical datum of §148.

154. Experiments. Cylindrical electrometer; movable cylinder without. After a few preliminary experiments with the metal cylinder of aluminum, the gilt-paper cylinder was tried and was at once successful. The mirror was attached to the paper wing needed for damping by the aid of a bent piece of thin steel wire, cemented on so as to give a horizontal axis on passing through the perforation of the bit of cork holding the mirror. For a vertical axis the whole apparatus is rotated (*cf.* figs. 98 and 99). The results are given in table 103 and fig. 111, *a*.

TABLE 103.—Cylindrical electrometer; movable cylinder (gilt paper) without. $M=2.075$ g.; $R=1.75$ cm.; $r=1.59$ cm.; $l=21$ cm.; $V_1=250$ volts.

I.				II. $M=1.0930$ g.			
Drum.	$10^3\Delta N$	V_3	Volt-meter.	Drum.	$10^3\Delta N$	V_3	Volt-meter.
11.2	2.79	18.7	18	24.9	6.22	21.9	21
22.9	5.72	38.2	38	47.3	11.82	41.6	41
35.3	8.83	59.1	58				
27.6	6.90	46.1	48				

The differences in this table are more liable to be of the voltmeter than of the electrometer. The needle easily carried a potential of $V_1=250$ volts. With $V_3=100$ volts, the needle became asymmetric and observations could no longer be taken. The sensitiveness is about 0.0003 cm. of double displacement per volt for the short suspension, so that 0.1 volt per ring is the equivalent datum. The present form is thus easily made as sensitive as the preceding form, besides being much simpler in general design and installation.

The experiment was now pushed a step further in the direction of sensitiveness by using an even lighter paper cylinder with paper wings, the needle having the constants given in the second series of table 103 and fig. 111, *b*. The sensitiveness is now about 0.0006 cm. of double displacement per volt, twice that of the preceding case, as it should be. This apparatus did not, however, behave as satisfactorily as the other, there being greater tendency of the movable cylinder to cling to the stable parts, whenever V_1 was high. In case of smaller potentials or of the idiostatic method there would be no difficulty in this respect. No doubt if greater time and patience were spent on the work (a cylinder of celluloid suggests itself) the final result could be somewhat improved; but the limit found (6×10^{-4} cm. or 20 rings per volt) will not easily be exceeded.

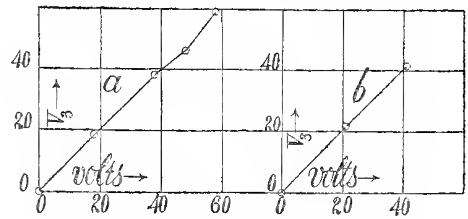

FIG. 111.—Observations with cylindrical electrometer. Cylinder outside.

155. Disk electrometer. Apparatus.—The disk electrometer deserves special attention because it is simpler in design and practically more sensitive than the cylinder types; but at the same time it is more treacherous, and without special precautions there is danger of short-circuiting the highly charged disk, while the sensitiveness is enormously variable in response to any unsymmetrical position of the disk. The instrument used is shown in figs. 112 and 113, in sectional side and front elevation. V_1 is

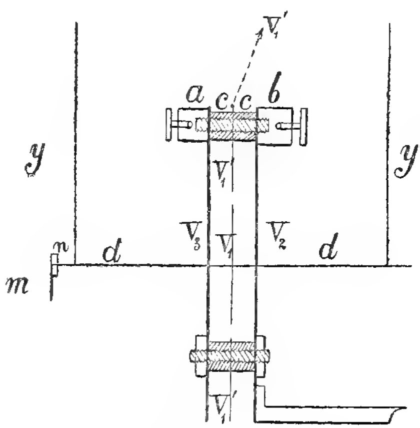

FIG. 112.—Disk electrometer.
End view.

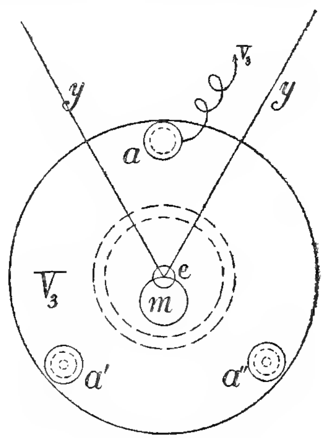

FIG. 113.—Disk electrometer.
Front view.

the charged disk on the bifilar suspension of very thin copper wire yy', 0.007 cm. in diameter. The disk is made of thin mica, silver-plated, carried by the horizontal axial steel rod dd, and surrounded by the guard ring of very thin copper V_1'. Parallel to the disk and equidistant (d) from it are the plates V_3 and V_2 of the condenser at a distance D apart, so that D is twice d. The plate V_2 is earthed and firmly held on the arm of the interferometer. The plates V_3 and V_2 are spaced at three points by hard-rubber gaskets, *cc*, through the holes of which hard-rubber screws are passed, the plates being ultimately secured by nuts *a*, *a'*, and *a''* on the outside of the plates. The nuts *a* terminate in clamp screws. The guard ring V_1' and the suspension wires *yy'* are in metallic connection at the tops of the suspension, the latter being the same already shown in figs. 98 and 99. The small mirror, *m*, is attached to a small plate of cork *n*, which is slotted parallel to the rod *dd*, the latter being clutched by the jaws of cork which make up the sides of the slot. In this way *m* may be rotated around a horizontal axis, while the apparatus as a whole may be revolved about a vertical axis.

If the grating is capable of being raised or lowered, it is not difficult to adjust the apparatus and find the ellipses. The damping of the disk is naturally good, though it may be improved by surrounding it with the case shown in fig. 98.

156. Experiments with the disk electrometer.—The experiments were begun with but a small distance, *D*, between the plates of the electrometer. In such a case the disk can carry but a slight potential before it is drawn across to the condenser plates and short-circuited. In fact, though the mica disks may be made very light, the annoyance of short-circuiting is correspondingly increased and the great advantage of sensitiveness can not for this reason be realized unless the idiostatic method is used. This is the case in table 104 and fig. 114, *e*.

TABLE 104.—Disk electrometer. $M=1.055$ g.; $D=0.612$ cm.; $l=23.5$ cm.; $r=3.065$ cm. Idiostatic method: $V_3=V_2=0$.

	Drum.	$10^3\Delta N$	V_1	Volt-meter.	10^3k
I	11.7	5.84	22.3	18	82
	79.7	39.82	58.3	38	207
II	2.38	1.19	19.4	18	25
	9.75	4.87	39.3	38	10
	22.47	11.23	59.6	57.5	11
	51.5	25.75	90.2	84	23
	82	41.01	113.9	104	27

Larger voltages threw out the coincidence of spectra, so that the ellipses could not be obtained because of asymmetry. The table shows that whereas the magnitude V_3 is of fair order of value for small potentials, it rapidly becomes excessive, owing to the fact that $k+\Delta N$ is no longer negligible in comparison with *D*. The ΔN in such a case is algebraically added to the original asymmetry of the disk, if *d* is greater or less than $D/2$. The importance of this superposition increases as the square of the potential and is thus serious at high voltages. A second and more careful adjustment of the same disk showed (by the use of the idiostatic method) a much more favorable succession of values (fig. 114, *f*), while at the same time there is necessarily marked reduction in sensitiveness. The uncorrected V_3 is thus much nearer the actual value and more gradually fails as the high voltages are approached. The constants k computed for the first series are not only large, but differ unexpectedly. In a measure it is probably indicated that the larger deflection proceeds from a position of increased asymmetry. In agreement with this, data for k in the second series are

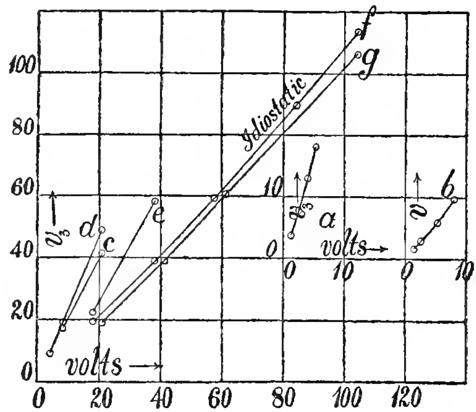

FIG. 114.—Observations with disk electrometer.

satisfactory and but slowly increase. The use of the apparatus for absolute measurement of potentials above 20 volts in case of a carefully adjusted disk is therefore not inadmissible.

TABLE 105.—Disk electrometer. $M=1.055$ g.; $D=0.612$ cm.; $l=23.5$ cm.; $r=3.065$ cm.; $V_1=101$ volts.

Drum.	$10^3\Delta N$	V_3	Volt-meter.
9.6	2.4	3.8	1.3
20.4	5.1	7.9	2.6
33.4	8.36	13.1	4.0
46.8	11.7	18.3	5.3

The disk was now charged with the storage battery to 101 volts and a number of small potentials measured at the plates. The results are given in table 105 and fig. 114, a, showing that the computed and uncorrected values are about three times as large as the true voltages. The sensitiveness is here surprisingly large, almost 8 drum parts of 2×10^{-3} cm. per volt, *i.e.*, 70 rings per volt. The distribution of results is nevertheless obviously linear and proportional to the true voltages. To account for this result by aid of the equation

$$V_3=\frac{Mg\Delta N}{lV_1}\,\frac{D^2}{r^2}(1-2\Delta/D)^2$$

is not impossible, if the asymmetry is such as to make the terms in Δ/D essentially negative. Yet in the case of a large value of Δ, the results are so lacking in probability that a lack of uniformity or of variation in the field is more liable to be in question.

On spacing the plates further, $D = 1.04$ cm., and carefully adjusting the disk it carried a charge equivalent to $V_1 = 250$ volts easily, so long as V_3 did not exceed about 20 volts. On increasing the potential above this, short-circuiting accidentally occurred which destroyed the apparatus. The single datum obtained was $\Delta N = 0.0273$, for which $V_3 = 49.8$ instead of 18 volts, the excessive result being due to causes of the kind just discussed.

A new and lighter disk was thereupon introduced, weighing but 0.607 gram. As it failed to carry a charge quite as large as $V_1 = 250$ volts, the only available smaller one, $V_1 = 104.5$ volts, was given to it. The data found are contained in table 106 and reproduced in fig. 114, *b*.

TABLE 106.—Disk electrometer. $D = 1.04$ cm.; $l = 23.5$ cm.; $r = 3.065$ cm.

	Drum.	$10^3 \Delta N$	V_3	Volt-meter.
I. $V_1 = 104.5$ volts. $M = 0.607$ g.	2.9	0.73	1.8	1.3
	5.0	1.25	3.1	2.5
	9.9	2.48	6.2	5.2
	15.7	3.92	9.9	8.0
II. $V_1 = 250$ volts. $M = 1.086$ g.	88.9	22.2	41.8	21
	36.3	9.1	17.1	8.2
III. $V_1 = 250$ volts. $M = 1.086$ g.	104.1	26.02	49.0	21
	40.6	10.14	19.1	8.3
	19.2	4.79	9.0	4.1
IV. Idiostatic method; $V_3 = V_2 = 0$. $M = 1.086$ g.	0.77	0.38	19.1	21
	3.30	1.65	39.4	41
	7.95	3.97	61.1	61
	24.20	12.10	106.7	104

The uncorrected results for V_3 are here much nearer the true values, *i.e.*, about 1.2 times too large, than was the case in the preceding table, so that the correction for asymmetry is probably applicable. The mean trend of the locus is through zero. Sensitiveness is naturally much lower than above, showing $\Delta N = 5 \times 10^{-4}$ cm. per volt The experiment well indicates how markedly dependent the constants of the instrument are on the position of the disk. Similar examples will be given below.

The same disk was now weighted with a rider, making the total mass 1.086 grams, with the object of charging it to 250 volts. The data for V_3 are given in the second series of table 106 and in fig. 114, *c*. The uncorrected results, while maintaining proportionality, are about twice too large.

A later adjustment gave the values in Series III and fig. 114, *b*, which are again two times too large. In spite of the effective weight of the disk, sensitiveness has risen in both cases to about 10^{-3} cm. per volt, which is quite out of proportion with M/V_1 and in contrast with the nearly normal values of V_3 in the first series.

The data obtained idiostatically, where the deflections are essentially small, are contained in the fourth series of table 103 and show an excellent agreement with the true voltages, as is indicated in fig. 114, *g*. Discrepancies are as liable to be in the voltmeter as in the electrometer. The values of k given for this table show how nearly a true condition of symmetry was attained, as its mean value from the equation

$$V_3^2 = \frac{2MgD^2}{lr^2}\Delta N\left(1-2\frac{\Delta}{D}\right)^2$$

is but −0.0085 cm.

The data of table 106, as a whole, illustrate very well the peculiarities of the disk method, giving evidence both of its relatively great sensitiveness (the double displacements being even 2×10^{-3} centimeters per volt) on the one hand, and the variation of sensitiveness as the result of a more or less unsymmetric position of the disk on the other. The ratio of observed and actual voltages is fairly constant. The question then arises as to the degree to which the latter difference may be corrected. There is, in many cases, a peculiar shift of the zero of displacement for the uncharged apparatus which is not easily accounted for; though, from another point of view, it is truly astonishing that a suspended disk should adjust itself to a given position with an accuracy comparable with the wave-length of light. In the experiments detailed, such difficulties were eliminated by taking mean results, but usually the position in question was actually stable. If the shortcomings in question can be overcome, the practical limits of the apparatus as here constructed should, for short suspensions (23 cm.), be about 0.015 volt per vanishing ring.

157. Further experiments with the disk electrometer.—The interesting feature of the preceding result is the large range of variability in the sensitiveness of the apparatus. It was therefore thought worth while to throw further light upon the investigation by purposely tipping the apparatus, in order that the disk might lie on one side or the other of the guard ring. The same voltage (4 Leclanche cells, 5.7 volts) was evaluated for each position of the disk, which, being light, did not admit of a potential much in excess of $V_1 = 101$ volts. Table 107 is a record of the results in which the deflections were also taken for the case $V_3 = V_2 = 0$, with the total significant potential on the disk.

TABLE 107.—Experiments with eccentric disk. $M = 0.6075$ g.; $D = 1.04$ cm.

$V_2 = V_3 = 0$; $V_1 = 101$ volts.				$V_1 = 101$ volts; $V_2 = 0$			
Position of disk.	Drum.	$10^3\Delta N$	$10^3 k$	Drum.	$10^3\Delta N$	V^3	Volt-meter.
Tipped forward...	+28.5	+14.2	+81	13.82	3.46	9.0	5.7
Intermediate.....	−25.0	−12.5	−71	7.33	1.83	4.8	5.7
Tipped rearward..	−71.2	−35.6	−203	7.3	1.82	4.8	5.7
Slightly forward..	+16.2	+ 8.1	+46	7.5	1.87	4.9	5.7
More forward....	+11.2	+ 5.6	+32	11.8	2.95	7.7	5.7

Applying to the first half of the table the approximate equation

$$k/\Delta N = \frac{MgD^3}{4lr^2V_1^2} - 1$$

the results were as shown in the column under k. It is not, however, quite as consistent as was anticipated, inasmuch as the same value of k did not reappear when the apparatus was tipped back apparently to its original position. Moreover, the central position does not clearly correspond to $k=0$. Hence it seems that $k=0$ is determined by the geometry of a non-uniform field. Neither do the values of V_3 correspond very closely to the values of k. The apparatus is more sensitive when the disk V_1 is on the side of the plate V_3 and about equally so when central or when on the side of $V_2=0$. In the latter cases V_3 as computed comes out for the first time distinctly less than the datum given by the voltmeter, the usual tendency being in the direction of excessive values.

TABLE 108.—Experiments with eccentric disk. $M=1.086$ g.; $D=1.04$ cm.

$V_2=V_3=0$; $V_1=250$ volts.				$V_2=0$; $V_1=250$ volts.			
	Drum.	$10^3\Delta N$	10^3k	Drum.	$10^3\Delta N$	V_3	Volt-meter.
I	+29.05	+14.52	+13.9	126.9	31.72	59.9	20.5
				49.1	12.27	23.0	8.2
				23.2	5.81	10.9	4.1
				12.6	3.15	5.9	2.0
II	+35.15	+17.57	+16.7	103.0	25.75	48.5	20.5
				39.4	9.84	18.5	8.2
				18.3	4.58	8.0	4.1
III	−45.1	−22.55	−21.5	107.75	26.94	50.7	20.5
				42.9	10.72	20.2	8.2

The light disk is difficult to manipulate, as its weight is scarcely sufficient to stretch the wires. In the experiments of table 108 and fig. 115, *I*, the disk was therefore weighted with a rider and a similar series of data was taken, although $V_1=250$ volts was carried by the disk only in cases where its position was nearly central. Consequently the adjustments made contain but a slight departure from that position.

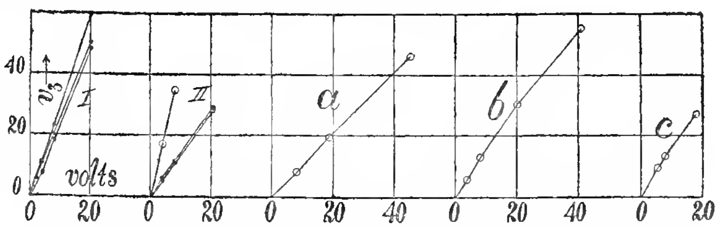

FIG. 115.—Observations with disk electrometer.

The disk in the three adjustments was therefore successively placed on the two sides of the central position within fractions of a millimeter. The addition of a rider did not make it essentially more stable, as slightly greater displacements drove it quite across, into contact with one or the other of the plates of the condenser. This was even the case on the addition of heavier riders. The computed values of V_3 are in all cases two or three times the true values, though the proportionality is maintained except in the second series. The effect of k on V_3 does not appear.

Finally, the disk was made still heavier by the addition of a second rider of 0.709 gram. Nevertheless (as indicated) it was still unstable, except very near the symmetrical position. It was possible, however, to secure a change of sign in deflection. The results are given in table 109 and fig. 115, *II*.

TABLE 109.—Experiments with eccentric disk. $M=1.791$ g.; $D=1.04$ cm.

$V_2=V_3=0$; $V_1=250$ volts.				$V_2=0$; $V_1=250$ volts.			
	Drum.	$10^3\Delta N$	10^3k	Drum.	$10^3\Delta N$	V_3	Volt-meter.
I	+110.25	+55.12	+122.6	21.85	5.46	16.9	4.1
				45.0	11.25	34.8	8.2
II	− 24.5	−12.25	− 27.2	7.6	1.90	5.9	4.1
				14.4	3.60	11.2	8.2
				37.8	9.46	29.3	21.0
III	− 47.27	−23.63	− 52.6	6.8	1.71	5.3	4.1
				14.0	3.51	10.9	8.2
				36.4	9.11	28.2	21.0

In the first of these adjustments k comes out over a millimeter distant from the symmetrical position in one case and about 0.5 mm. on the other side in another. In the first case, V_3 is over four times the true value, the highest ratio heretofore obtained, although the sensitiveness (2.8×10^{-3} cm. of double deflection per volt) is not much in excess of the earlier values. The second and third series are again near the true value in spite of not inconsiderable values of k. Apart from the abnormally large sensitiveness the curves are regular in proportion to the voltage.

158. Case of the inclosed disk.—With the object of obtaining a more uniform field, the disk electrometer was now modified as shown in fig. 116, where V_3 and V_2 are the plates of the condenser and V_1 is the disk inclosed in the short cylindric or drum-shaped tube within the guard ring V_1'. The disk, as before, is supported by the axial fine steel wire tt', passing through perforations in V_3 and V_2 and suspended by the bifilar system yy'. The disk is again well damped naturally. V_2 is usually earthed. The difficulty with this design is the impossibility of observing the disk, with a view to its symmetric adjustment. By pushing tt' longitudinally, the disk may be seen in the space between plate and tube and some adjustment made; but this is not quite adequate. The plates and guard ring are spaced as before by three hard-

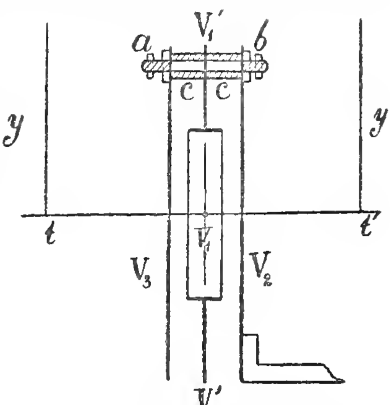

FIG. 116.—Modified disk electrometer.

rubber rings *cc*, with a hard-rubber screw passed through the perforations. On the outside are the brass connectors *a* and *b*. V_1' and yy' are in metallic connection at all times.

Experiments were begun with the light disk, $M=0.607$ gram, which was charged to 110 volts, as it would not carry an appreciably higher voltage. Consequently a rider was thereafter added, making $M=1.086$ grams, and the disk charged as far as 250 volts. The results are given in table 110 and fig. 115, *a*.

TABLE 110.—Experiments with inclosed disk. $M=1.086$ g.; $D=1.04$ cm.; $l=23.5$ cm.; $r=3.065$ cm.

V_1	Drum.	$10^3\Delta N$	V_3	Volt-meter.
34	4.2	1.05	8.1	8.5
250	42.0	10.5	19.7	19.0
	97.9	24.5	46.0	45.5

These results, though obtained under very different conditions, are practically coincident and the discrepancies are as liable to be in one instrument (voltmeter) as the other.

Some time after, an accidental earth developed in the storage battery and destroyed this electrometer. The old disk was then again inserted and the experiments detailed in table 111 were made.

TABLE 111.—Experiments with inclosed disk. $M=1.060$ g.; $D=1.04$ cm.; $l=23.5$ cm.; $r=3.065$ cm. $V_1=250$ volts.

	Drum.	$10^3\Delta N$	V_3	Volt-meter.
I...	6.4	1.59	5.8	4.0
	14.2	3.55	13.0	8.2
	33.3	8.32	30.6	20.5
	60.4	15.10	55.4	41.0
II...	10.2	5.12	9.4	6.0
	14.6	7.31	13.4	8.6
	29.5	14.77	27.1	18.5

The error of asymmetry in the first series is marked, showing inadequate adjustment. The curve, fig. 115, *b*, moreover, is not straight, as the disk did not move quite parallel to itself. An improvement of those imperfections was then attempted, as shown in the second series of the table and in fig. 115, *c*, but not successfully. The curve is again not quite straight. The experiments as a whole are unsatisfactory, inasmuch as the disk only just sustained the potential of 250 volts and became unstable on slight readjustment. An appropriately lower potential was not available. Further work was therefore abandoned. It is probable that table 110, where the conditions were more favorable, gives the better account of the behavior of the present apparatus.

To summarize: the enormous variation of the sensitiveness of the disk electrometers as depending upon the position of the disk, k, has therefore failed of interpretation; *i.e.*, the correction for V_3 does not seem to follow the above equations. The field is therefore probably far from uniform, possibly consisting of a conical tube of force between plate and disk on the side toward which the disk leans and of a more nearly cylindric tube on the opposite side, where the tube must pass through the circular perforation in the guard ring. The result is an empiric instrument in which the deflections are proportional to the voltages to be measured, of enormously increased sensitiveness, even 2×10^{-3} cm. per volt, but not yet sensitive enough to be immediately valuable for practical purposes. The idiostatic instrument, for voltages above 20 volts (after the disk has been adjusted for $k=0$ by comparing the positions of the charged and uncharged disk), may in some cases be useful.

159. Case of the unsymmetric disk.—The cause of the departure from linearity in the preceding experiments is to be referred to a slight asymmetry in the disk, whereby an effect varying as the square of voltage of the disk is superimposed on an effect varying as its first power. To eliminate the non-linear term, it should be sufficient to obtain no deflection in the case of earthed plates, when the needle is successively charged and uncharged. In fact, the charged non-symmetrical needle between earthed plates introduces an interesting method of electrometry as follows.

Let k be the amount of non-symmetry for a disk whose mean distance from the plates D cm. apart would be $d=D/2$. Hence for any displacements ΔN, the distance of the disk from the condenser plates will be

$$d=D/2+(k+\Delta N) \qquad\qquad D-d=D/2-(k+\Delta N)$$

Hence the displacement force is (after reduction)

$$X=\frac{V_1^2r^2}{2}\,\frac{k+\Delta N}{D/2}\,\frac{1}{D^2/4-(k+\Delta N)^2}=\frac{Mg\Delta N}{l}$$

As $(k+\Delta N)^2$ may usually be neglected in comparison with $D^2/4$ the equation becomes

$$V_1^2\equiv=\frac{MgD^3}{4lr^2}\,\frac{\Delta N}{k+\Delta N}$$

This equation fails when $k+\Delta N=0$, which practically means that $k=0$. When ΔN is also zero, V_1^2 is indeterminate and the sign of the deflection depends upon chance conditions. If k is large as compared with ΔN, which will usually be the case, V_1^2 will vary linearly with ΔN.

To use this method, k must therefore be known, and it may be determined with the aid of a given voltage preliminarily. The following experiments have been made in this way.

TABLE 112.—Disk electrometer. $M = 1.055$ g.; $D = 0.612$ cm.; $l = 23.5$ cm.; $r = 3.065$ cm.; $V_2 = V_3 = 0$; V_1 increasing. Mean $k = 0.04771$.

Drum.	$10^3\Delta N$	10^3k	Volt-meter.	Corrected V_3
1.38	0.69	50.9	18	18.6
5.85	2.92	46.0	38	37.4
14.60	7.30	46.1	57	56.6
40.75	20.37	47.8	85	85.1
94.9	47.45	65.0	104	110.0

TABLE 113.—Disk electrometer. $M = 1.055$ g.; $D = 0.674$ cm.; $l = 23.5$ cm.; $r = 3.065$ cm.; $V_2 = V_3 = 0$; V_1 increasing. Mean $k = 0.04778$.

Drum.	$10^3\Delta N$	10^3k	Volt-meter.	Corrected V_3
1.02	0.51	50.6	18	18.5
4.17	2.09	44.6	38	36.8
10.47	5.24	45.9	57	56.5
26.82	13.41	46.5	85	84.1
47.27	23.64	51.2	104	103.4

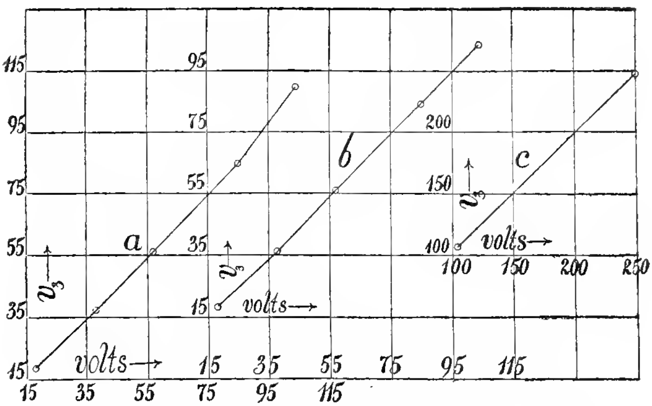

FIG. 117.—Observations with unsymmetric disk electrometer.

In both these tables and fig. 117, a and b, the asymmetry of the disk was about 0.5 mm., for the plates were over 6 mm. apart. The equation employed answers the requirements as closely as the observations could be made. At high voltages (100 volts) there is liable to be divergence, while at low voltages the displacement ΔN is too small for accurate work, seeing that the underlying equation is quadratic.

In tables 114 (fig. 117, *c*) and 115, tentative experiments are recorded as obtained with a light disk, $M=0.6075$ gram. In table 114 it was weighted by a rider, so as to be available in measuring 250 volts and over. In table 115 the disk was used without a rider. The distance apart of the plates is over a centimeter.

TABLE 114.—Disk electrometer. $M=1.0865$ g.; $D=1.04$ cm.; $l=23.5$ cm.; $r=3.065$ cm.; $V_2=V_3=0$.

	Drum.	$10^3\Delta N$	10^3k	Volt-meter.	Corrected V_3
V_1 decreasing: Mean $k=0.005468$ cm.	10.85	5.42	5.2	250	247
	1.13	.56	5.8	104	107

TABLE 115.—Disk electrometer. $M=1.0865$ g.; $D=1.04$ cm.; $l=23.5$ cm.; $r=3.065$ cm.; $V_2=V_3=0$.

	Drum.	$10^3\Delta N$	10^3k	Volt-meter.	Corrected V_3
V_1 increasing:	3.0	1.50	25.6	57	57
I. Mean $k=0.030$ cm.	13.1	6.58	34.6	104	110
II. Mean $k=0.049$ cm.	4.9	2.47	44.0	57	57
	17.0	8.49	54.6	104	100

The asymmetry occurring in table 114 is slight and the apparatus is therefore insensitive. In the second series of table 115 the disk was purposely made unsymmetric, without, however, increasing the sensitiveness or value of k as much as would have been supposed. For if ΔN is small as compared with k, $dV/d(\Delta N)$ should increase as $1/k$. In general, the *angular* orientation of the disk and irregularities at the edge of the guard ring and disk may introduce discrepancies of this kind. An apparatus like the present, intended for actual measurement, should be provided with a micrometer suspension, for shifting the disk as a whole in the direction of its axis, and with more elaborate means for sighting with a view to horizontal and vertical parallelism of the disk with the plates of the condenser than was the case with the improvised apparatus here treated. It has been stated that it is an inversion of the present displacement which is superimposed on the displacement in case of a permanently charged needle, and which thus demands an apparatus tested for absence of non-symmetry, if the voltages are to be proportional to the displacements.

THE QUADRANT ELECTROMETER.

160. Apparatus.—The method of measuring small angles given in §119 may be used in measuring very small voltages or small increments of potential, by attaching a pair of light mirrors, in parallel, to the needle of a quadrant electrometer. In the first experiments this was an improvised instrument constructed by myself, the quadrants being of sheet copper fitted and soldered together and supported on cylinders of hard rubber. The bottom of the stem of the needle was submerged in sulphuric acid, as in Kelvin's instrument, and the suspension was bifilar. The insulation was throughout excellent. The needle was kept charged to about 200 volts with a Zamboni pile, any variation of charge being indicated by Elster and Geitel's electroscope.

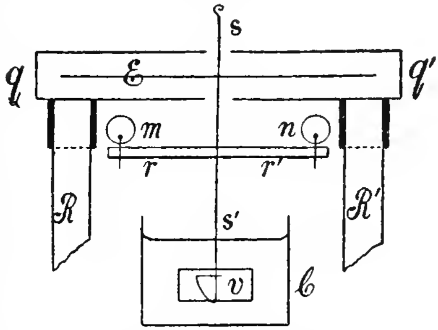

FIG. 118.—Quadrant electrometer adapted for parallel-mirror interferometry.

In fig. 118, qq' shows a pair of quadrants in vertical section, E the needle on the stem ss', the lower end of which is platinum, bent as shown, thus making a clip to hold the light mica vane v (if necessary) submerged in sulphuric acid of the vessel C charged by the Zamboni pile; RR are the hard-rubber supports of the quadrants. At a suitable distance below them the light parallel mirrors m and n (less than 1 cm. in diameter each) are supported by the light cross-piece of hard rubber rr attached to the stem ss' of the needle. The axial line of the needle E is parallel to the line rr between the mirrors, and the latter are placed at a horizontal angle of about 45° to rr.

To adjust m and n to adequate parallelism, each is supported by an attached fine needle, fitting snugly in a vertical groove in the ends of rr. The needle, as a whole, is to be clamped at rr in a suitable support and sunlight is to be used. When the horizontal beam reflected from m to n, and thence to a distant white screen, falls within the direct shadow of n, the needles are fixed in place by resinous cement. The mirrors should be equally high.

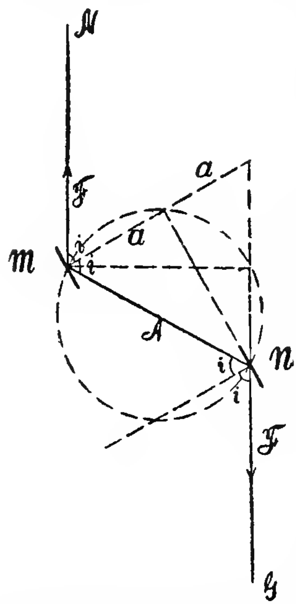

FIG. 119.—Diagram showing displacement and deflection in parallel-mirror interferometry.

The quadrant electrometer with the needle in position was now placed with the aid of three long foot-screws on a circular platform just below the iron arm holding the fixed interferometer mirror M, and the reflection took place as shown in fig. 119 or in fig. 86 above. In effect the arc light from a collimator reflected from the front face of the grating (blue image due to scattering) is next reflected at n, thence to m, whence it goes to N and returns by the same route, passing, however, from n through the grating to the observer's telescope. The case surrounding the electrometer must therefore be provided with a front and rear window. With so many reflections (altogether ten, including the one at the grating and the effect of the two windows) this beam is considerably weakened, and it is advisable to use black glass and not a silver mirror at the micrometer M. It is preferable, moreover, to use the glass side of the mirrors at m and n, as the silvering is liable to be brighter in the rear and the effect of the thin glass plate is of no consequence. This, in fact, was the greatest difficulty encountered, as the mirrors m and n were not at first adequately silvered and polished. Even when the direct image of the slit is clear the spectrum is apt to be dull and the ellipses are hard to find.

The adjustments, when so many conditions have to be met, were not easy of attainment. The electrical installation should first be completed and the needle in place between the quadrants, the electrometer being placed so that the beam from the interferometer strikes n (see figs. 118 and 119). A white screen behind n, catching the light passing beyond the edges and showing the shadow of n, facilitates this adjustment, the slit being opened wide. If possible, sunlight should here be used. The electrometer is now rotated as a whole around the vertical, until the light reflected from n strikes m, and a similar screen behind the latter in the line mn is necessary here. Next the mirror N is adjusted to normality, a white screen behind m in the direction Nm being essential, whereupon the light is reflected from n again toward the grating. As the pencil must pass through the grating (G) again, this is mounted to be capable of being raised or lowered and rotated around the vertical and horizontal (adjustment screws); but these are the usual adjustments on the interferometer. It follows that the electrometer must also be capable of being raised and lowered on its long foot screws, as already indicated.

Naturally sunlight, being steadier, is preferable to arc light, seeing that the range in which reflection takes place is limited by the area of the small one-fourth inch mirrors. Direct sunlight, without concentration, is adequate. It has been stated that both mirrors must be well silvered. Moreover, the spectrum should be so placed by raising and lowering the source of light and inclining the grating together with N that the higher orders of direct spectra are not in the same plane with the spectra superposed for interference. When the adjustments described are well made, there is no further difficulty in finding the solitary ellipses. As there is compensation owing to the two glass plate windows in case of the electrometer, the ellipses are liable to be enormous, practically vertical straight lines which are displaced with correspondingly great rapidity by the micrometer screw and are therefore hard to find. Hence some counter compensation at the micrometer mirror is desirable, in order that centers may appear and the displacement may be slower. A compensating plate about 1 cm. or less in thickness, with the *vertical* focus of the light from the slit on the stationary mirror, produces very clear and sharp ellipses, admirably adapted for measurement.

161. Observations.—The stems of the mirrors m and n on rr were about 4.5 cm. apart. Hence, since $i=45°$, $a=4.5/\sqrt{2}$ and if δ is the path difference

$$\frac{d\delta}{di}=0.0175\times 2a \sin i=0.079 \text{ cm. per degree.}$$

Supposing that 1 volt gave a deflection of 45° and that 10^{-4} cm. are measurable at the micrometer, the sensitiveness should be $10^{-4}/45\times 0.079$ or about 3×10^{-5} volts; *i.e.*, about 10^{-5} volts per vanishing interference ring. It would thus seem probable that with a lighter needle and a more delicate suspension the possibility of measuring 10^{-6} volts would not be out of the question. The following observations show that these surmises are correct so far as the method is concerned.

The great hardship encountered in the present work was the unavoidable agitation of the laboratory, and this unfortunately is insuperable. After the ellipses were found they were always in motion, so that the displacement work was bound to be rough and the use of interference rings out of the question. As the rings, however, may easily be obtained and used under suitable conditions, and as the purpose of the present paper is merely to test the method, the annoyances in question are of less consequence.

To obtain small potentials, a thin bare German-silver wire about 1 meter long was stretched and insulated on a board and the ends supplied with a constant potential difference of one volt. Two points of the wire, 1 or more centimeters apart, were then used as a source of potential, the wires from these points leading to a Mascart key, which suitably earths and commutates the charged wires leading to the electrometer. A thermocouple might have been used; but the long wire is preferable because of its simplicity.

In the first experiments made, the suspension was a bifilar about 10 cm. long (each strand of several silk fibers) and about 0.5 to 1 mm. apart. No doubt the torsional stress of the fiber was here of an order commensurate with the bifilar force. The needle, being damped in concentrated sulphuric acid, moved very slowly and about 2 minutes were allowed for each deflection. It was thus possible to follow the ellipses on commutation, from one to the other extreme elongation, by moving the micrometer screw proportionally to the displacement of ellipses. As a rule the ellipses were quite clear all the way, showing that the adjustment for parallelism of mirrors on the needle by the aid of sunlight is adequate. The potential of the charge on the needle was of the order of 200 volts.

Table 116 gives the results for the case when the points tapped on the wire were 10 and 18 cm., respectively, apart, the equivalent potential differences being 0.10 and 0.18 volt. The deflections on commutation at the Mascart key were nearly on opposite sides of the middle position of the needle.

The zero of the needle shows the usual tendency to wander, due apparently to air-currents, etc., but also to variations of the Zamboni pile, as the electrometer was an improvised instrument, in which the electrometric adjustments were not quite perfect. In the same way the displacement per volt, $\Delta N/\Delta V$, varies somewhat, though here it is also a question as to whether in all cases adequate time was allowed before reading the micrometer; for the needle damped by concentrated sulphuric acid was quite aperiodic. There may be some residual effect

TABLE 116.—Potentials of quadrant electrometer read off by displacement interferometer. Mirrors at about 45° and 4.5 cm. apart along the needle.

P. D. applied.	Sign.	cm.	ΔN	$\Delta N/\Delta V$	P. D. applied.	Sign.	cm.	ΔN	$\Delta N/\Delta V$
0.18	±	0.0755	0.1025	0.5700	0.10	−	0.0183	0.0614	0.6140
	−	.0256				+	.0797		
	+	.1281				−	.0224	0.0569	0.5690
	±	.0810				+	.0793		
	±	.0790	0.1018	0.5650		−	.0232	0.0558	0.5580
	−	.0194				+	.0790		
	+	.1212							
	+	.1232	0.1046	0.5810		−	.0226	0.0564	0.5640
	−	.0186				+	.0790		

due to the viscosity of the relatively thick silk-fiber suspension. The sensitiveness obtained in the two series of experiments was as nearly the same as was to be expected, though there must have been some instrumental discrepancy in the first adjustment of the second series. We may, therefore, put $\Delta N/\Delta V=0.57$ cm. on the micrometer, so that $\Delta N=10^{-4}$ cm. corresponds to 0.000170 volt, or the sensitiveness is about 0.000060 per vanishing interference ring.

The instrument was now improved by inserting a longer bifilar suspension, consisting of a single fiber of silk. The damper and other details were retained. The results are given in table 117, potentials being tapped from points of the long wire, which were respectively 5 and 10 cm. apart. Owing to the very large deflection, the ellipses were not equally clear throughout the whole displacement from elongation to elongation.

TABLE 117.—The same. Long (20 cm.) suspension. Single fiber of silk.

P. D. applied.	Sign.	cm.	ΔN	$\Delta N/\Delta V$	P. D. applied.	Sign.	cm.	ΔN	$\Delta N/\Delta V$
0.10	−	0.0158	0.1538	1.538	0.05	−	0.0015	0.1495	2.990
	+	.1696				+	.1510		
	−	.0136	0.1595	1.595					
	+	.1731	0.1609	1.609		−	.0093	0.1437	2.874
	−	.0122				+	.1530		

The instrument behaved much better in the present case and the sensitiveness has been much increased, particularly in the second series of observations. Since $\Delta N/\Delta V$ is of the order of 2.9 cm. per volt, 10^{-4}

cm. corresponds to 0.000035 volt, *i.e.*, to about 0.000010 volt per ring, so that under favorable circumstances a few millionths of a volt should already be discernible.

162. Water damper.—The endeavor was now made to dispense with the damper of concentrated sulphuric acid, the motion of the needle being too slow in such a case. Water was next used, which as a damping medium admitted of several small oscillations of the needle. The curious fact, however, was here encountered that the illuminated mirrors were rarely at rest and that it was nearly impossible to determine the amount of displacement. In some cases the limit of motion of the uncharged needle was of the same order as the displacement to be measured.

To give examples of this behavior table 118 has been inserted, and it shows the consecutive values found for a needle the vane of which was damped in water. It was not quite aperiodic, but nearly so. The sensitiveness was low because of the low charge on the needle. The irregularity of displacement is the result of the superposition of the motion of the needle on the deflection to be measured.

Thus the mean sensitiveness is 0.4300 cm. per volt or 1/12900, about 0.000076 volt per vanishing interference ring. In spite of many trials the drift of the zero could not be removed. The results were about the same on the ensuing day.

Table 118.—Needle with a water damper. Potential = 170 volts. Potential difference 0.1 volt.

Shortly after charging $10^3 \Delta N$ cm.	33.4	38.4	36.6	53.8
	49.1	46.7	39.3	37.5
	53.0	40.5	45.3	
Next day............	40.5	48.0	51.5	

The instrument was now made more sensitive by charging the needle to a higher voltage. The results are given in table 119.

Table 119.—Needle with water damper. Potential = 220 volts. Potential difference 0.1 volt.

$10^3 \Delta N$...........	82.0	85.0	73.0	83.0

The sensitiveness is thus 0.81 cm. per volt or 0.000040 volt per vanishing ring. The sensitiveness has not reached the above case, but the needle moves more rapidly.

A variety of other experiments led to similar results and need not be given here. The next step in advance will therefore be an investigation of the drift in the charged and uncharged instrument.

163. Drift forces.—The presence, in other words, of a beam of light impinging upon the mirrors seemed to be accompanied by torque tending to move the mirrors on the needle *into the beam*, *i.e.*, from the dark to the bright side of the mirrors. Very little control of this force was possible in case of the water damper, so that a return to the sulphuric acid seemed advisable and a variety of experiments were made in this way. The quadrants were both put to earth, but the needle was charged.

It frequently appeared that when the illumination commenced the ellipses gradually traveled in such a way as to show a decrease of path at the needle, a decrement which had to be compensated at the micrometer. The motion was very slow, so that the ellipses could easily be kept in the field and at times even the rings observed. The speed of the displacements, although slow at first, increased gradually to a maximum, precisely as one would expect it to do from the nature of the damper. The motion then continued at a steady pace for over 15 minutes, during which the micrometer displacement often amounted to 0.04 cm. On turning off the illumination of the mirrors and allowing them to remain without interference, the ellipses frequently returned to their original position. Leaving the interpretation of this result, whether real or illusory, for further experimentation, it is interesting to here compute the forces involved in such motion, to whatever cause they may be due.

Since the path difference, $\delta = 2a \cos i$, or $-di = d\delta/2a \sin i$ (see fig. 119), a decrease of δ, as observed, means an increase of i, or a motion of each mirror from the dark to the bright side. If we call this force in the direction of the beam F, the couple on the needle is $T' = F\,2a \sin i$, where a is the normal distance apart of mirrors and i the angle of incidence. Again, we may write for the torque on the bifilar

$$T = mg\frac{ll'}{L}\theta$$

m being the mass of the needle, L the length, $2l$ and $2l'$ the distance apart of the fibers above and below, and θ the deflection. Thus we eventually obtain, when the needle has reached equilibrium so that $T = T'$,

$$2Fa \sin i = mgll'\theta/L$$

But for small angles

$$\theta = di = \Delta N/2a \sin i \qquad \delta = \Delta N,$$

so that

$$F = mg\frac{ll'}{L}\,\frac{\Delta N}{(2a \sin i)^2}$$

If A is the distance between the centers of the mirrors,

$$a = A \cos i \qquad F = \frac{mgll'}{L(A \sin 2i)^2}\Delta N$$

Though this is not generally a large force in comparison with the electric forces to be measured, it is nevertheless necessary to carefully investigate it before further progress can be made.

In case of the given instrument,

$$m = 1.65 \text{ grams} \qquad g = 981 \qquad 2l = 0.11 \text{ cm.} \qquad 2l' = 0.055 \text{ cm.}$$
$$L = 13.5 \text{ cm.} \qquad A = 4.5 \text{ cm.} \qquad \sin 2i = 1$$

Hence

$$F = 0.0082\ \Delta N \text{ dyne.}$$

The above unusually large displacement of $\Delta N=0.04$ cm. would therefore be equivalent to a force $F=0.00033$ dyne, acting from the dark to the front side of the small mirrors in the direction of the impinging light.

In a variety of other similar experiments made with the needle uncharged, the results were not constantly in favor of its association with the presence of a beam of light, though the drift remained of about the same order. Unfortunately, in the midst of the work, the needle was accidentally destroyed and it seemed advisable to construct an entirely new instrument, specially adapted for the present experiments. The work was therefore temporarily discontinued.

164. Conclusion.—It has been shown that by attaching a pair of light parallel mirrors to the two ends (directly or indirectly) of the needle of the quadrant electrometer and observing the deflection of the needle by displacement interferometry, an instrument may be made capable of measuring a few millionths of a volt and possibly much smaller potentials.