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PARTICLE DYNAMICS AROUND BLACK HOLE IN EINSTEIN-MAXWELL-SCALAR GRAVITY

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I. INTRODUCTION

General relativity (GR) is the standard theory of gravity which is proposed by Einstein in 1915 and has been successfully tested using different experiments and observations. Recently, observation of gravitational waves [1] and shadow of M87 [2] can be considered as a test of GR in strong field regime. However, GR has few limitations in sense of describing the nature of the singularity during the gravitational collapse, cosmic acceleration, rotation curve of galaxies, incompatibility of GR with quantum field theory, etc. In order to resolve these problems one needs to either break the symmetries of GR or introduce modifications or alternative theories of gravity. Indeed, there are few experimental and theoretical studies of Lorentz symmetry violations [3]. Among the different modifications and extensions of GR built on symmetry breaks the Einstein-Maxwell-Scalar theory became attractive for theoretical studies.

II. GEODESICS OF A PARTICLE AROUND BLACK HOLES IN EINSTEIN-

MAXWELL-SCALAR THEORY

The exact solution of field equations in the Einstein-Maxwell-Scalar theory describing the spacetime of spherically symmetric black hole has the following form in the spherical polar coordinates [4]:

$$ds^2 = -Udt^2 + \frac{dr^2}{U} + f^2(r)(d\theta^2 + sin^2\theta d\varphi^2) \quad (1)$$

with

$$U = \left(1 - \frac{b}{r}\right)\left(1 - \frac{a}{r}\right)^{1-\gamma} + \frac{\beta Q^2}{f^2} \quad (2)$$

$$f(r) = r\left(1 - \frac{a}{r}\right)^{\frac{\gamma}{2}} \quad (3)$$

Here *a* and *b* are related to the mass *M* and electric charge *Q* of the black hole by

$$\mathrm{M} = \frac{1}{2}[\mathrm{b} + (1 - \gamma)\mathrm{a}], \quad Q^2 = \left(1 - \frac{\gamma}{2}\right)ab \quad (4)$$

Observing the expressions of *U* and f in Eq. (2) and Eq. (3), we see the metric combines the dilaton part $\left(1 - \frac{b}{r}\right)\left(1 - \frac{a}{r}\right)^{1-\gamma}$ and RN part $\frac{\beta Q^2}{f^2}$ together.

First we study massive particle motion around the regular black hole in Einstein-Maxwell-Scalar gravity. In order to find the trajectory of the test particle one needs to consider Lagrangian for the test particle of mass *m* of the following form

$$L' = \frac{1}{2} g_{\mu\nu} u^\mu u^\nu \quad \text{and} \quad u^\mu = \frac{dx^\mu}{d\tau} \quad (5)$$

where τ is an affine parameter, x^μ and u^μ are the coordinates and four-velocity of the test particle, respectively. The conservative quantities of the motion such as the energy ε and the angular momentum *L* of the test particle can be written in the following form

$$\varepsilon = \frac{\partial L\prime}{\partial u^t} = -f(r)\frac{dt}{d\tau}, \qquad L = \frac{\partial L\prime}{\partial u^\varphi} = r^2 sin^2\theta \frac{d\varphi}{d\tau} \quad (6)$$

By inserting expressions in (6) into the normalization condition $g_{\mu\nu}u^\mu u^\nu = -\epsilon$ one can easily find the equations of motion of test particle in the following form:

$$\frac{dr}{d\tau} = \sqrt{\varepsilon^2 - f(r)(\epsilon + \frac{K}{r^2})} \quad (7)$$

$$\frac{d\theta}{d\tau} = \frac{1}{r^2}\sqrt{K - \frac{L^2}{sin^2\theta}} \qquad (8)$$

$$\frac{d\varphi}{d\tau} = \frac{L^2}{r^2 sin^2\theta} \qquad (9)$$

$$\frac{dt}{d\tau} = \frac{\varepsilon}{f(r)} \qquad (10)$$

where K is the Carter constant and the parameter is defined as:

$$\epsilon = \begin{cases} 1, & for\ timelike\ geodesics \\ 0, & for\ null\ geodesics \\ -1, & for\ spacelike\ geodesics \end{cases} \qquad (11)$$

For simplicity, one can consider the motion of the particles in the equatorial plane in which $\theta = \frac{\pi}{2}$ and $\frac{d\theta}{d\tau} = 0$. In this special case Carter constant takes form $K = L^2$ and the equation for radial motion takes the following form

$$\left(\frac{dr}{d\tau}\right)^2 = \varepsilon^2 - V_{eff}(r) = \varepsilon^2 - f(r)\left(1 + \frac{L^2}{r^2}\right) \qquad (12)$$

with

$$V_{eff}(r) = f(r)\left(1 + \frac{L^2}{r^2}\right)$$

being the effective potential of the radial motion of test particles. The radial dependence of the effective potential for the massive test particle around BH in Einstein-Maxwell-Scalar theory for different values of Q, γ and β parameters is represented in Fig. 1.

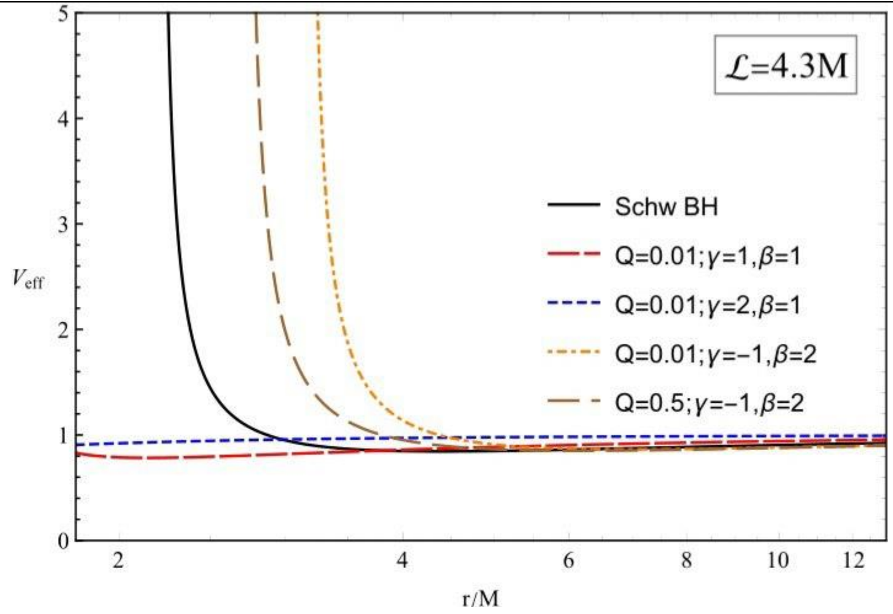

Fig. 1. Dependence of V_{eff} on radial coordinate r of massive particle for the different values of Einstein-Maxwell-Scalar BH parameters Q, γ and β

Now we may consider the inner stable circular orbit r_{ISCO}. In order to find ISCO radius one needs to use the following conditions:

$$\begin{cases} V'_{eff} = 0 \\ V''_{eff} = 0 \end{cases}$$

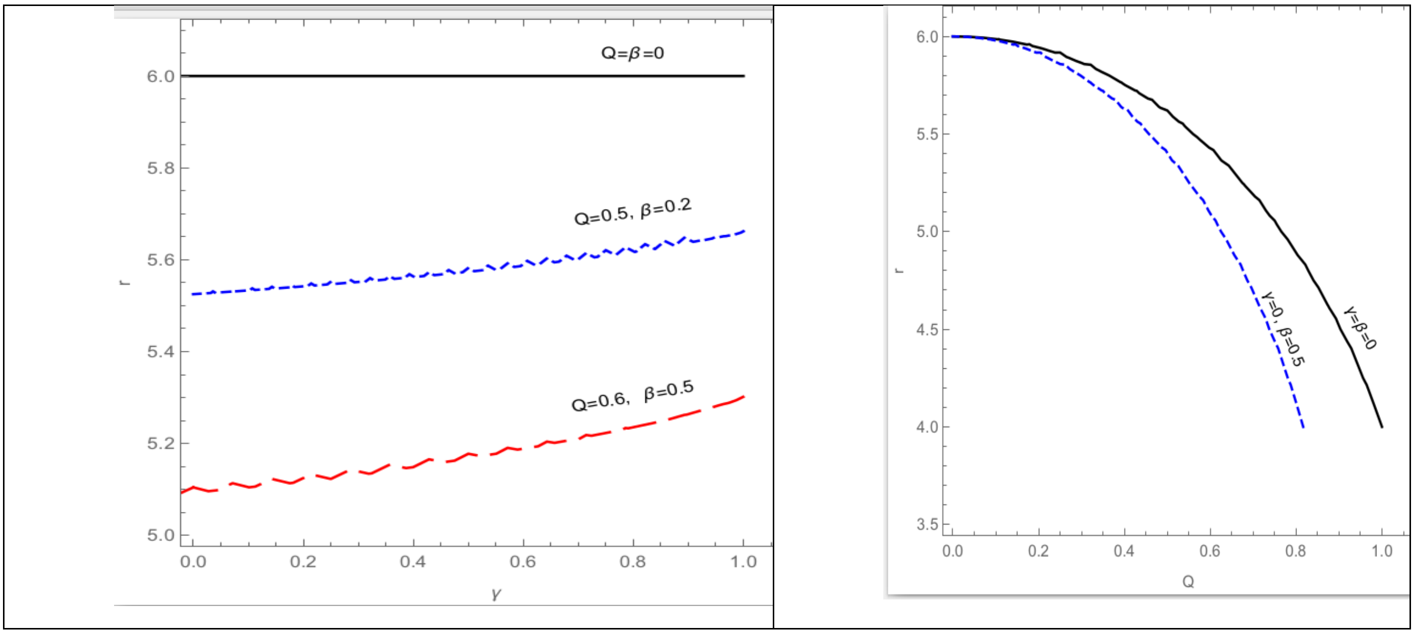

Fig. 2. Relation of r_{ISCO} on parameters Q and γ in Einstein-Maxwell-Scalar gravity.

Due to the complex form of the effective potential we cannot get exact analytical expression of r_{ISCO} . However, one may represent the radius of ISCO using the graphs and it is represented for various parameters Q and γ in Fig. 2. From this one can get information about inner stable circular orbits dependence on parameter Q and γ in Einstein-Maxwell-Scalar. Particularly, ISCO radius increases with the increase of parameter γ. ISCO radius decreases with the increase of parameter$Q$ Fig.2.

III. CONCLUSION

In this work, we studied the motion of the test particle around black hole in Einstein-Maxwell-Scalar gravity. We could not find the exact form of ISCO. But we drew a graph of its dependence on various parameters. Studying similar theories will help us understand our vast universe.

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